Net Allwave Radiation as a Hydrologic Input for the Computation of Snow-melt Hydrographs from a Small Watershed

by

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ABSTRACT

The purpose of this research was an initial study of the use of the net, allwave radiation as the hydrologic input for snow-melt runoff prediction.

Hydrographs of snow-melt runoff have been obtained for the spring melting periods of 1970 on a rural basin near Ottawa, Canada. Using a mathematical model written in matrix form, the IBM 360/65 digital computer was used to obtain the distribution graphs from each of the hydrographs for days of high net,allwave radiation. This method has been adapted from the technique for obtaining unit hydrographs from complex storms. Through a least-squares solution a distribution graph was obtained which was used to produce a computed hydrograph . This in turn was compared with the observed snow-melt hydrograph and adjustments made in the distribution graph until the computed hydrograph was obtained within certain prescribed limits. In the process of the solution the original estimate of the effective radiation inputs were adjusted by computer to improve the fit of the solutions.

Although the distribution graph altered as the melt season progressed, the distribution graphs derived by this model for individual events show reasonable agreement. It is concluded that use of effective net, allwave radiation as the input for deriving distribution graphs shows considerable promise and can be a valuable tool in further research on the runoff processes from snow-melt.

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Notation

a	:	constant
b	:	constant
d	:	rainfall depth in inches
D	:	depth of water melted from snow
D_f	:	diffuse sky radiation
e	:	vapor pressure of air
E	:	emissivity
$\{E\}$	:	error vector
H	:	stage in feet
$[I]$	:	input matrix
k	:	constant
k_1	:	constant
k_a	:	absorption coefficient
k_o	:	von karman's constant
k_e	:	eddy transfer coefficient
k_a	:	thermal conductivity of soil
M	:	basin snow-melt runoff
p	:	atmospheric pressure
Q	:	snow-melt discharges
ΣQ	:	the algebraic sum of all heat components
Q_{sw}	:	net shortwave radiation
Q_{lw}	:	net longwave radiation
Q_c	:	sensible heat transfer
Q_e	:	latent heat transfer

Notation

Q_g	:	net heat transfer by conduction
Q_r	:	transfer of heat from rain to snowpack
Q_d	:	shortwave radiation penetrates at depth d
Q_{eff}	:	effective net radiation
T	:	temperature of air
U	:	wind velocity
Z_1	:	elevation of anemometer
Z_2	:	elevation of hygrothermograph
Z_o	:	roughness parameter
σ	:	stefan constant
ρ	:	density of air

CHAPTER ONE

INTRODUCTION

In Canada, spring runoff caused by the melting of snow provides the primary source for surface storage necessary to supply projected water requirements. Groundwater supplies are also replenished at the time of spring melt. At the same time, for many streams and rivers, this period is a critical one in regard to serious flooding. Due to the general condition of storage being filled in this period and possible low infiltration rates in frozen soil, high melting rates alone, or these combined with rainfall, may result in the maximum flood peak for a given stream. This potentiality must be considered in choosing a design flood for rivers subject to spring flooding from snow-melt.

One of the principal aims of recent water resources efforts has been to establish the economic utilization of the water supplies in rivers, lakes, and groundwater reservoirs. The operation of hydroelectric power plants, municipal supply systems, or recreational reservoir system requires forecasts of supply. The manner in which snow accumulates throughout the winter to be available for replenishment of supplies in the spring enhances the possibility of successful management of this source of water.

To forecast runoff from rainfall accurately, reliable meterological forecasts of precipitation are required. These, at the present time, are available only a few days in advance. In contrast, prediction of runoff volumes from snow-melt is possible much further in advance based upon the knowledge of the amount of accumulated snow. Therefore, it can be seen that management of water resources, which is dependent upon accurate forecasting, has a better chance of sucess when the source is from the melting of an accumulated snowpack.

Some examples of forecasting and frequency prediction follow. Reservoirs may be drawn down to produce hydroelectric power during the high-demand winter months. Then, the snow-melt runoff will replenish the reservoir supply in the spring periods. Inaccurate forecasts may result in wastage of water or shortage in the succeeding dry periods. Reliable methods of predicting both the amount of runoff and its time distribution are needed to formulate good methods of forecasting. For spillway design or flood design, a design flood must be selected so that the capacity may be expected to be rarely exceeded in the design period. A method of predicting the magnitude of floods from snow-melt is required that is similar to the unit hydrograph method for rainstorms.

Procedures have been formulated for estimating snow-melt from which sequential streamflow can be synthesized by basin routing, but the classical system analysis of input-output relations has not been applied. In this method, a lumped, linear, time-invariant unit hydrograph has been widely used as the input-output response model. Consideration of the general problem of the prediction of runoff hydrographs will show how the methods presently used for storm rainfall have been extended in this study for snow-melt and what limitations are imposed.

The problem of the prediction of runoff hydrographs has usually been divided into two parts. The first problem has to do with the determination of the volume of runoff to be expected and the second one with the computation of the time distribution of the runoff. It is the second problem which is of principal interest here, although some attention will be given to the first one. To determine the amount of runoff from rainfall, losses due to infiltration, interception, depression storage, and evaporation have to be considered. To determine the amount of melt runoff from heat input, heat losses must be considered as well.However, the use of the effective rainfall, which is only that part of the rainfall contributing to the runoff makes it possible to treat the second problem of time distribution of runoff separately. In this study an effective heat input is considered which is that heat necessary to produce the melt runoff that is actually observed. This separates the two problems of amount of runoff and its time distribution in a similar way.

The time distribution of the input, effective rainfall or effective heat, is required. In the original unit hydrograph theory this problem was solved by using short intense storms which were isolated from other rainfall so that resulting runoff could be clearly identified. Recently, generalizations of the unit hydrograph theory have produced methods which make it possible to derive unit hydrographs from complex storms. In this study such a method is used since sufficiently isolated events of snow-melt runoff do not occur.

To estimate the time distribution of the effective heat input, the time distribution of the net, allwave radiation is used. This is the major source of heat input for snow-melt in the study reported here.

In this study of snow-melt then, the successive values of the effective heat will be treated as the input function during the days on which the net, allwave radiation is the primary source of heat input. The small watershed is considered as a hydrogic system and the snow-melt runoff records are the output values. Base flow is separated by a consistent and logical procedure. The resulting surface-runoff volumes were used to determine the effective input. The first estimate of the distribution of this input was taken from the distribution of the total net, allwave radiation. Then, using the high-capacity digital computer, the dimensionless unit hydrographs or distribution graphs for a specified duration were calculated from selected hydrographs. From this distribution graph a computed hydrograph then compared to the actual snow-melt hydrograph. If they compare favorably, the computer will give the desired dimemsionless unit hydrograph or distribution graph. This process is repeated until a favorable comparison is obtained. Corrections in the estimated distribution of input are also made by computer to improve the fit of the hydrograph.

Thus, the results of the analysis are a distribution graph for each day of melt and an estimate of the distribution of the effective heat.Just as the classical unit hydrograph represents the response of the basin, the derived distribution graphs of this study represent the response of the snow-covered valley under study. Although the response is derived from inputs based upon the net, allwave radiation, it may be possible to apply them to inputs of sensible or latent heat or to inputs of rainfall upon snow. This possibility awaits testing from data on runoff under these conditions. Also, a study of these results shows changes in the hydrologic response with time and may be helpful in making an improved model of the snow-melt runoff process.

CHAPTER TWO

PREVIOUS INVESTIGATION

Numerous investigation of snow-melt and snow-melt runoff have been carried out based on simple empirical methods or thermodynamic principles for estimates of seasonal volume, stream-flow forecasting, design of flood control systems or for other purposes.

In 1941, W. T. Wilson (1) discussed certain principles of thermodynamics that applied to the ripening and melting of snow. He presented the following points. The temperature of the air was the most significant single index of snow-melt conditions. The rate of snow-melt could be expressed as a function of air temperature measured in term of degree-days above freezing. Mechanisms of heat transfer for melting snow are radiation, sensible heat transfer, latent heat transfer and conduction. If melting conditions exist during a short period of a day, melt will occur, regardless of how cold the rest of the day is, or what the average temperature of the day may be.

For runoff forecasting, two important factors tending to lag the runoff from melting snow must be considered. They are the storage in the snow and the percolation rate through the snowpack.

The net effect of the radiation is reflected in the air temperature, and under certain conditions the radiation effect can be expressed as a simple function of air temperature. Because the direct measurement of net, allwave radiation was not generally available at that time, most of the radiation values had to be calculated from meteorological parameters.

The turbulent exchange, or convection, of heat at a snow surface can be expressed as function of wind velocity and air temperature as follows:

$$D = K U (T-32) \quad \text{..................}(2.1)$$

where D is depth of water melted from snow in six hours, U is wind-velocity in m.p.h., T is dry-bulb temperature F, and K is a constant involving the latent heat of ice, exposures of instruments, air density, conversion of units, and certain considerations involved in the theory of turbulence exchange. With K=0.001, the depth of water per six hours could be defined.

The snow-melt due to melt from condensation, also a turbulent exchange process, can be expressed as follows:

$$D = K_1 U (e-6.11) \quad \text{...............}(2.2)$$

where D is depth of water melted from snow in six hours plus condensate added to snow, U is wind velocity in m.p.h., e is vapor pressure in mbs, and K_1 is constant. If e is measured at the same height above the ground as T, K_1 is approximately 3.2 times K .

The rate of heat loss or gain in snow due to conduction from the soil is very small and can be neglected.

In the same year, P. Light (2) presented a theoretical method of calculating heat and moisture exchange between the snowpack and the atmosphere. The methods were based on the theory of turbulent exchange in the lower (close to the snow surface) layer of atmosphere. He found that for high melting rates of snow, the heat contributed by convection and condensation of moisture through turbulent diffusion of warm moist air are the important heat-sources. The theoretical melting formula he derived is

$$D = \frac{\rho K_o^2}{80 \ln(Z_1/Z_o) \ln(Z_2/Z_o)} U \left[C_p T - (e - 6.11) \frac{423}{P} \right] \quad \text{......}(2.3)$$

where ρ = density of air (1.25×10^{-3} g/cm^3)

K_0 = Von Karman's constant = 0.38

Z_1 = elevation of anemometer

Z_2 = elevation of hygrothermograph

Z_0 = roughness-parameter

C_p = sepecific heat of air at constant pressure

(0.24 cal / gm degree)

T = air temperature at hygrothermograph level

U = wind-velocity

e = vapor-pressure of air in mbs

P = atmospheric-pressure in mbs

All the above values are expressed in cgs units. The roughness parameter, Z_0, depends on wind velocity, the depth of the snow cover and the Richardson number. The value of Z_0= 0.25 was used in his study (Eq.2.3).

In order to apply the theoretical method to an actual drainage basin, it is necessary to develop a procedure for determining areal melting rates. Two modifying factors must be considered. One is the surface roughness, and another is the type and percentage of forest cover within the study basin. The method Light adopted was to group these two effects of surface roughness and forest cover into a single constant, such as :

$$M = K\,D \qquad (2.4)$$

where M is the basin snow-melt. To establish the value of K for an individual basin it is necessary to correlate observed snow-melt with the above theoretical Equation (2.3).

In 1943, Linsley (4) developed a method of forecasting snow-melt runoff for San Joaquin river basin in the Central Valley of Califorinia by using a weighted degree-day. The basin was divided into subareas according to elevation and degree-day relationship was developed for each subarea. Then the degree-days were weighted in proportion to area and a weighted values relationship for degree-days in the entire basin was obtained.

In 1945, the Corps of Engineers and the Weather Bureau formulated a joint research program, organized as the cooperative snow investigation. Since the time of that organization, the investigation has dealt with the analysis of individual snow hydrology problems. A few years later, two remarkably complete books, " Snow Hydrology " and " Runoff From Snow-melt " (3, 5), were published. They presented a detailed, comprehensive theory of snow-melt and snow-melt runoff, together with procedures for applying the theories of snow-melt to actual basin snow hydrology problems. These books have become standard references for further work in snow-melt hydrology research.

In the past few years various investigators (6, 7, 8, 9, 10) have studied the energy budget equation and tried to assess the various components of this equation for a snowpack. This equation can be written in a general form as :

$$\Sigma Q = Q_{sw} \pm Q_{Lw} \pm Q_c \pm Q_e + Q_g + Q_r \qquad (2.5)$$

where

ΣQ = the algebraic sum of all heat components, in cal/cm^2 per unit time

Q_{sw} = absorbed net shortwave radiation. (including solar and diffuse sky radiation)

Q_{Lw} = net longwave radiation exchange between the snowpack and its environment

Q_c = sensible heat transfer

Q_e = the gain or loss of latent heat caused by evaporation, condensation,or sublimation

Q_g = net heat transfer by conduction at the snow-ground interface

Q_r = the transfer of heat from rain to snowpack

The positive signs represent energy transferred to the snowpack ; the negative signs represent energy taken away from it.

It might be expected that the net, allwave radiation (i.e. $Q_{SW} \pm Q_{Lw}$) would be a good index of snow-melt among all the components of Equation (2.5). H. Landsberg (11) stated that the melting of snow or ice depends upon the amount of energy absorbed from solar and sky radiation in his experiment. D. H. Miller (12) concluded from the results of his experimental research in Sierra Nevada that most of the heat available to melt the snow during melting periods was directly from solar radiation. The Corps of Engineers (3) using the net shortwave radiation measured in the open area and the calculated net longwave radiation exchange, found good relationships between the snow-melt and net radiation. M. L. Johnson (13) carried out research at Danville, in Northern New England and stated that during the snow-melt periods a large amount of surface runoff was due to the amount of solar radiation. M. F. Megahan (14) installed nine lysimeters in an open research area to measure the snow-melt volume during the melting periods in the Rocky Mountains. On each day that he compared the net, allwave radiation with snow-melt volume, he found that about 95 per cents of snow-melt was due to the net, allwave radiation.

It has only been recently that instruments to give a reliable measure of net radiation have been available. No evidence was found in the literature of attempts to forecast the time distribution of snow-melt runoff by using the net, allwave radiation as an index of the effective input. Therefore, the purpose of this research is to initiate such a study of the net, allwave radiation as the hydrologic input to a snow-covered basin.

CHAPTER THREE

THEORY OF SNOW-MELT

In theory, snow-melt determination is a problem concerning several different processes of heat transfer, specifically radiation, convection, conduction and condensation. Wilson (1) was one of the first to approach the computation of snow-melt through an energy budget. The quantity of snow-melt produced as runoff is, moreover, dependent upon the condition of the snowpack and its local environment. As a consequence, the rigorous determination of snow-melt amount would be quite complex and probably unjustified. Therefore, approximate methods and simplifying assumptions are in order and certain such assumptions will be made in this study.

The principal fluxes of heat energy transferred by the different processes from or to the snowpack must be equal to the heat required to melt or cool the snow. The upper surface of the snowpack is subjected to the net, allwave radiation, sensible heat transfer, latent heat transfer and rain falling on the snowpack. Certain features of these energy fluxes are of note.

The solar radiation can penetrate several inches below the snow surface. The bottom surface of the snowpack is subjected to conduction of heat from the ground. Shortwave radiation and rain always added heat to snowpack whereas net longwave radiation, latent heat transfer, and sensible heat transfer may result in either a gain or loss of heat to the snowpack.

The combination of shortwave radiation and net, longwave radiation are called the net, allwave radiation.

<u>Shortwave radiation</u> : Shortwave radiation is associated with sunlight

and is made up of wave-lengths in the range from about 0.15 to 5 μ. When shortwave radiation strikes the snow surface, part of it is absorbed and part of it is reflected. The percentage of incoming shortwave radiation reflected is called the 'Albedo' of the snow surface. In general, the brighter the snow surface, the higher the albedo. The approximate values of albedo on snow were given by Williams (8) :

	Albedo
new snow	0.8-0.9
old snow	0.6-0.8
melting snow	0.4-0.6

The above values show that during the melting periods less shortwave radiation is reflected than for other snow because of physical change in the nature of the snow surface during melting. Hoeck (15) showed that positive air temperature (>32°F) and intense solar radiation would hasten the snow-melt and decrease of the albedo.

Because of the translucent nature of snow, the solar radiation is absorbed by the snowpack not at the snow surface but it also penetrates to some depth. Gerdel (20) has shown that about 80 % of the solar radiation absorbed by snow penetrates as far as 2 to 6 inches for snow densities of 26 to 40 %.

If the snowpack is assumed to be a homogeneous structure, the penetration of solar radiation in the snowpack is given by Beer's Law (16). For the radiation , Q_d , at the depth d, we have

$$Q_d = Q_{sw} \cdot e^{-k_a d} \quad \text{.....................} \quad (3.1)$$

where Q_{sw} is the intensity of shortwave radiation trnasmitted through the snow surface. It can be expressed as :

$$Q_{sw} = S \sin(h) + D_f - R \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.2)$$

where

$S \sin(h)$ = the vertical component of solar radiation(S = total radiation ; h = Sun's height)

D_f = diffuse sky radiation

R = radiation reflected by the snow

K_a is the absorption coefficient or extinction coefficient; the higher the density, the lower the value of K_a .

Longwave radiation : For snow-melt runoff studies, longwave radiation components can also be separated into two broad categories; radiation emitted by the snowpack and back radiation to the snowpack. Snow is very nearly a perfect black body with respect to longwave radiation. It absorbs all the incoming radiation incident upon it and emitts the maximum possible radiation. Since it radiates as a black body, in accordance with Stefan's law, this can be expressed by the equation :

$$Q_L = E \sigma T^4 \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \quad (3.3)$$

where Q_L is the intensity of the radiation, σ is the stefan constant, which is equal to 8.26×10^{-11} ly/min/ K , E is the emissivity (for a black body it is equal to 1.0). T is the black body absolute temperature. Since the maximum temperature of the snow is always considered to be 32°F, the maximum intensity of radiation that may be emitted by snow is 0.459 ly/min. It is obvious from Equation (3.3), if the snow cover temperature decreases,its outgoing radiation decreases.

The back radiation to the snowpack from earth's atmosphere,clouds, and forest cover do not radiate as a black body. so, it is necessary to develop an empirical relationship for them. A cloud, containing 0.05mm of water will absorb all the terrestial radiation coming from the earth. This is illustrated by the fact that cloudy nights are not as cold as clear night. With clear skies in open area, the U. S. Corps of Engineers (3) expresses the back radiation as follows :

$$Q_b = 0.76\,\sigma\,T^4 \quad (\text{lys/min}) \quad \ldots\ldots\ldots\ldots\ldots (3.4)$$

where T is the air temperature. The net exchange by longwave radiation is then :

$$Q_{Lw} = 0.76\,\sigma\,T^4 - 0.459 \quad (\text{lys/min}) \quad \ldots\ldots\ldots (3.5)$$

Another type of equation proposed by Brunt (17) estimates net longwave radiation from the mean air temperature, mean vapor pressure and cloudiness values.

All the above empirical formulae give the best estimate of the net longwave radiation but cannot be expected to be highly reliable. In recent years radiation instruments have been developed to measure the combined net, shortwave and longwave radiation. As might be expected, these instruments would give the most accurate results for the net, allwave radiation received at the snow surface.

Sensible heat transfer : During spring snow-melt periods, energy exchange by sensible heat transfer is usually not so important as radiation. When the overlying air is warmer than the snow surface, there is a direct transfer of heat from the air to the snow. Heat is transferred from the snow when the temperature of the snow is greater than the air. This transfer process is directly dependent upon the temperature gradient. According to turbulent transfer theory, the vertical heat transfer above the snow can be expressed as :

$$Q_c = \rho\,C_p\,K_e\,dt/dh \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (3.6)$$

where ρ is the air density equal to 1.45×10^{-3} g/cm^3 , C_p is the specific heat of dry air at constant pressure which is equal to 0.25 cal/gram degree K, K_e is the eddy transfer coefficient, and dt/dh is the gradient of temperature with elevation. Equation (3.6) can be reduced to the following equation :

$$Q_c = b (T - T_0) U \quad \text{.......................} \quad (3.7)$$

in which b is the constant, T is the air temperature, T_0 is the snow surface temperature and U is the wind velocity. Since during actual melting periods, the snow surface must be 32° F, Equation (3.7) becomes

$$Q_c = b (T - 32) U \quad \text{........................} \quad (3.8)$$

For an open site, such as the area in this study, the U. S. Army Corps of Engineers (3) has stated that the melt contribution due to sensible heat transfer is so small as to be insignificant compared with radiation.

Latent heat transfer : When the snow is melting, evaporation or condensation, depending on the direction of the vapor pressure gradient, takes place. When the snow is not melting, there is a phase change from vapor to solid or vice versa. This involves the latent heat of sublimation.When the vapor pressure in the air is greater than the vapor pressure at the surface of the snowpack, there is a transfer of moisture from the air to the snowpack, which results in a release of latent heat. When the vapor pressure gradient is reversed, the transfer process will also be reversed resulting in a heat loss to the snow. This transfer of moisture can be expressed as :

$$Q_e = 680 \rho K_e \, dq/dh \quad \text{.......................} \quad (3.9)$$

where ρ and K_e are the same as in Equation (3.6) and dq/dh is the gradient of the specific humidity with height at the snow surface. This equation can also be reduced to :

$$Q_e = a (e_a - e_s) U \quad .... \quad (3.10)$$

in which a is a constant, U is the wind velocity, e_a is the vapor pressure of the air and e_s is the vapor pressure at the snow surface. It is usually assumed that e_s is equal to saturation vapor pressure at the temperature of the snow surface(32° F) during melting periods. Therefore, Equation(3.10) can be written as :

$$Q_e = a (e_a - 6.11) U \quad .... \quad (3.11)$$

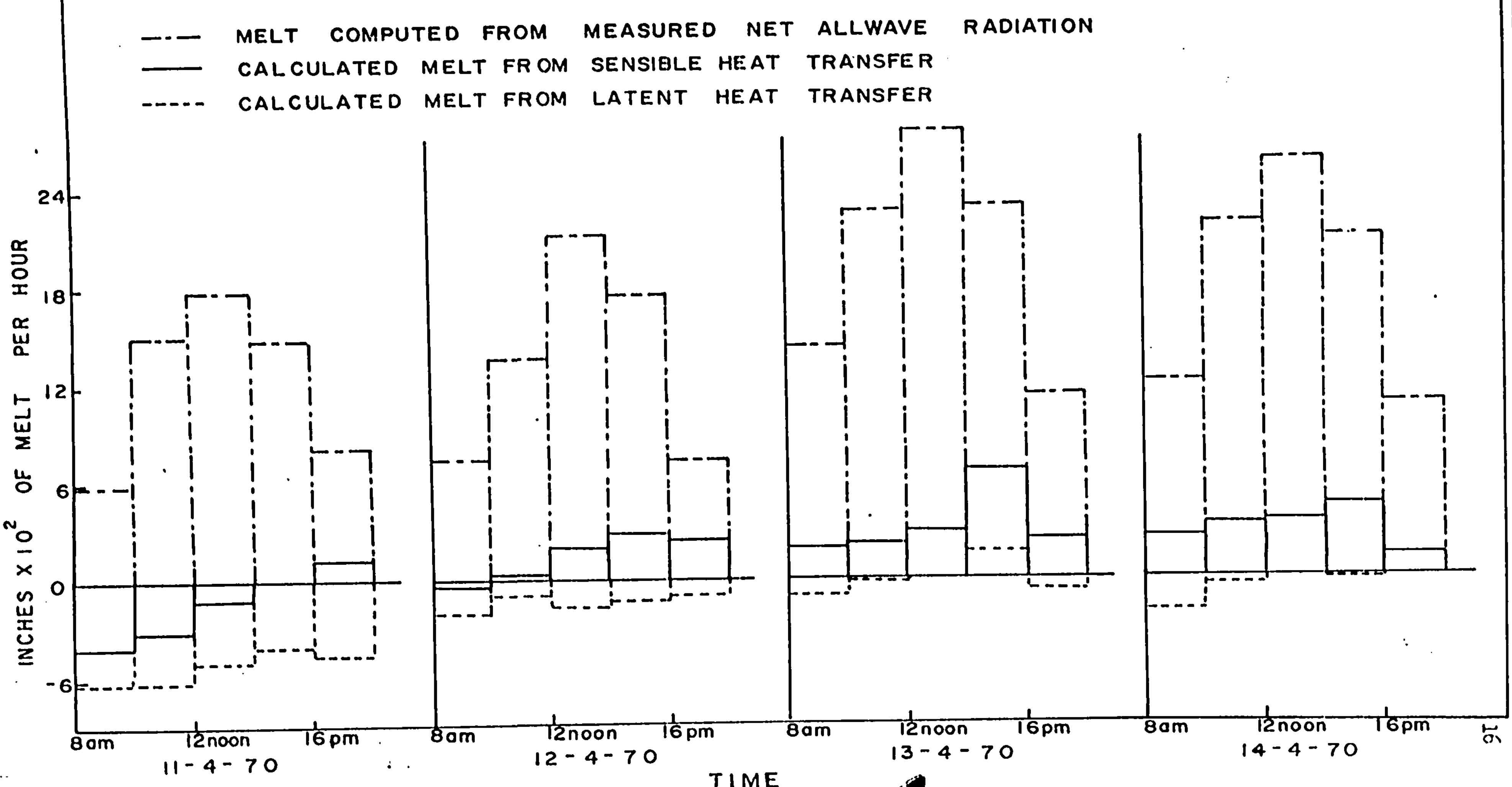
Using constant a and b in Equation (3.8) and (3.11) from the U.S. Army Corps of Engineers(5) corrected for height of temperature and wind velocity measurement, the snow-melt due to latent heat transfer, sensible heat transfer, and the measurement of net, allwave radiation in the study area have been plotted in Figure 3.1 . It is found that during the high melting periods, such as 11th to 14th April, the net, allwave radiation has been the most important heat transfer process.

Heat transfer by conduction : During the melting periods, heat transfer by conduction is usually negligible. With the melting of the snowpack cover on the frozen soil, the temperature of the upper soil layer rapidly rises to almost 32°F. This is due to the penetration into its pores of snow-melt water. In this condition, no appreciable heat exchange occurs between the snow and soil. When the snowpack melts on unfrozen soil, if we assume the heat flow from the soil to the snowpack is in a steady state, then the energy transfer will be :

$$Q_g = K_a \, dt/dh \quad .... \quad (3.12)$$

where K_a is the thermal conductivity of the soil. dt/dh is the vertical temperature gradient of soil under the snow cover.

FIG. 3-1



MELT COMPUTED FROM MEASURED NET ALLWAVE RADIATION
CALCULATED MELT FROM SENSIBLE HEAT TRANSFER
CALCULATED MELT FROM LATENT HEAT TRANSFER
INCHES X 10² OF MELT PER HOUR
24
18
12
6
0
-6
8am
12noon
16pm
11-4-70
12-4-70
13-4-70
14-4-70
TIME

Rain falling on the snowpack : The temperature of rain is usually higher than the snowpack, therefore, there will be a transfer of heat from the raindrops to the snowpack. The amount of heat transferred to the snow is directly proportional to the mass of the rainwater and to its temperature excess above that of the snowpack. This can be written as :

$$Q_r = 1.41\, d\, (T_r - 32) \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.13)$$

where d is the rainfall depth and T_r is the temperature of the rain. Because of the near saturation of the atmosphere during rainstorms, the wet bulb temperature is usually used as the temperature of the rain. One of the most important fact about rain falling on the snowpack is that it speeds the physical change of the snowpack and affects the structure of the snow to some depth and it may create drainage channels in the snowpack.

Since the net, allwave radiation is the primary source of heat input to melt the snow (Fig. 3.1), the 'effective' net, allwave is assumed to be the best single significant index for snow-melt runoff prediction for this particular study area. Other sources of heat would be treated as the gain or loss to the net, allwave radiation. The energy budget equation (Equation 2.5) could then be written as :

$$Q_{eff} = Q_{net} \pm \Sigma Q_{other} \qquad \ldots\ldots\ldots\ldots\ldots\ldots(3.14)$$

in which Q_{eff} is the effective heat input to melt the volume of snow observed as outflow. Q_{net} is the net, allwave radiation which can be measured by the net radiometer. ΣQ_{other} is all other heat contributions or losses.

CHAPTER FOUR

THE STUDY BASIN

4-1. Physical characteristics

The snow-melt runoff data were collected in an open grassy valley at Kanata, Ontario, west of the centre of Ottawa (Figure 4-1), about seventeen miles from The University of Ottawa.

The valley outline with contours of one-foot interval and the stream channel system are shown in Figure 4-2. It has approximately a south to north orientation. The stream flows generally in a northerly direction, and the channel length within the watershed is approximately five hundred feet. The difference in elevation between the weir at the downstream end and the upstream end is about eight feet. The valley has a vee shape with fairly steep slopes on both sides. The side slopes gradually flatten in the upland portion of the valley.

The entire basin is farm land and most of it is pasture. Soil survey of the area by the Department of Chemistry, Ontario Agricultural College, Guelph, and the Experimental Farms Service, Dominion Department of Argiculture, Ottawa, indicates that the soil consists predominantly of grey-brown clay called Rideau clay.

4-2. Instrumentation

An inexpensive weir was constructed of plywood. It is located at the downstream end of the study watershed where the valley joins the valley of a larger stream, Watts Creek. The weir has a total crest length of thirteen and a half feet. In order to provide a sharp-crested control, in the middle of the weir, a 60-degree triangular notch brass plate with a total vertical altitude of one foot from apex to base has been used.

FIG 4-1 LOCATION MAP OF RESEARCH BASIN

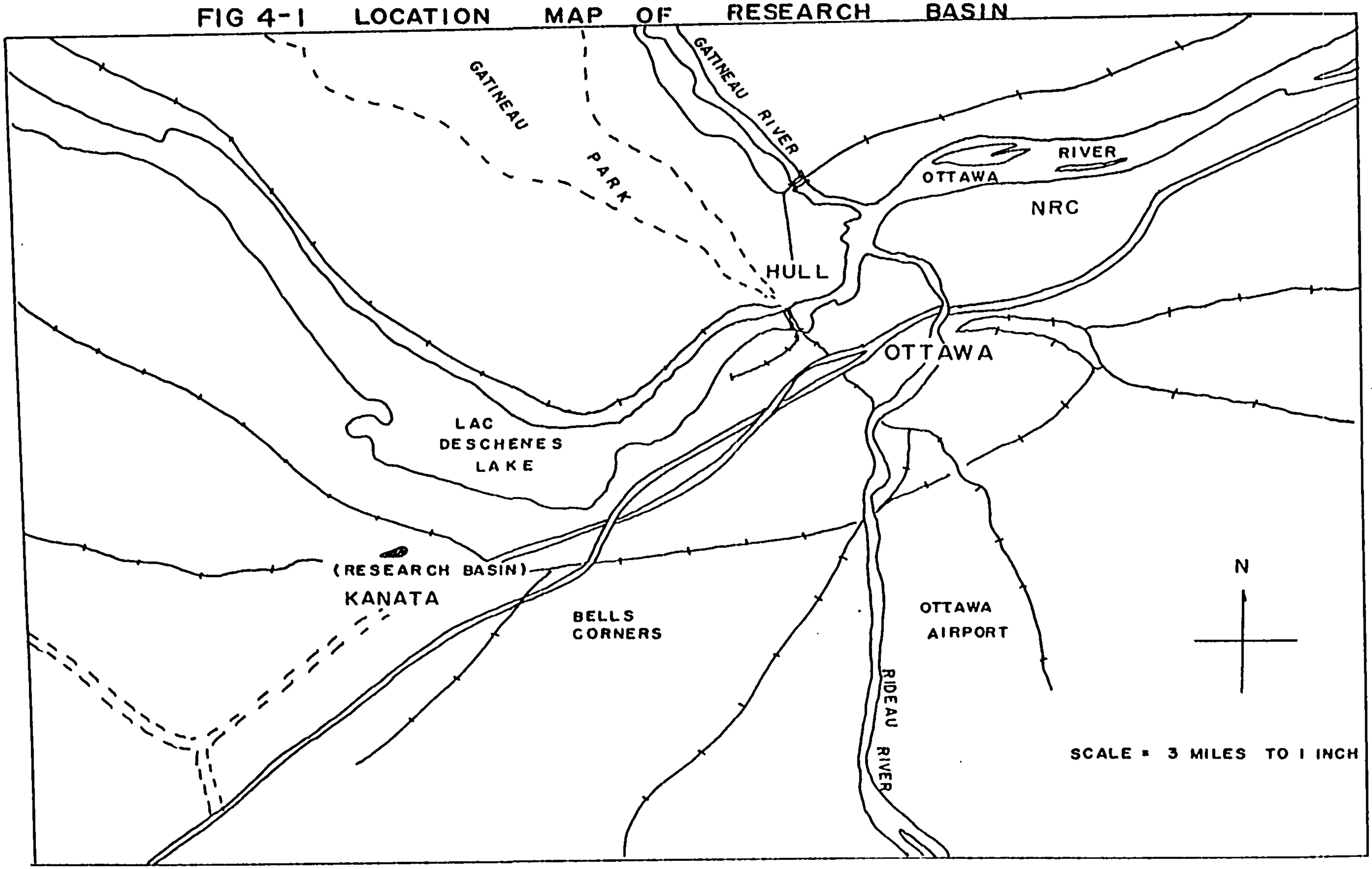
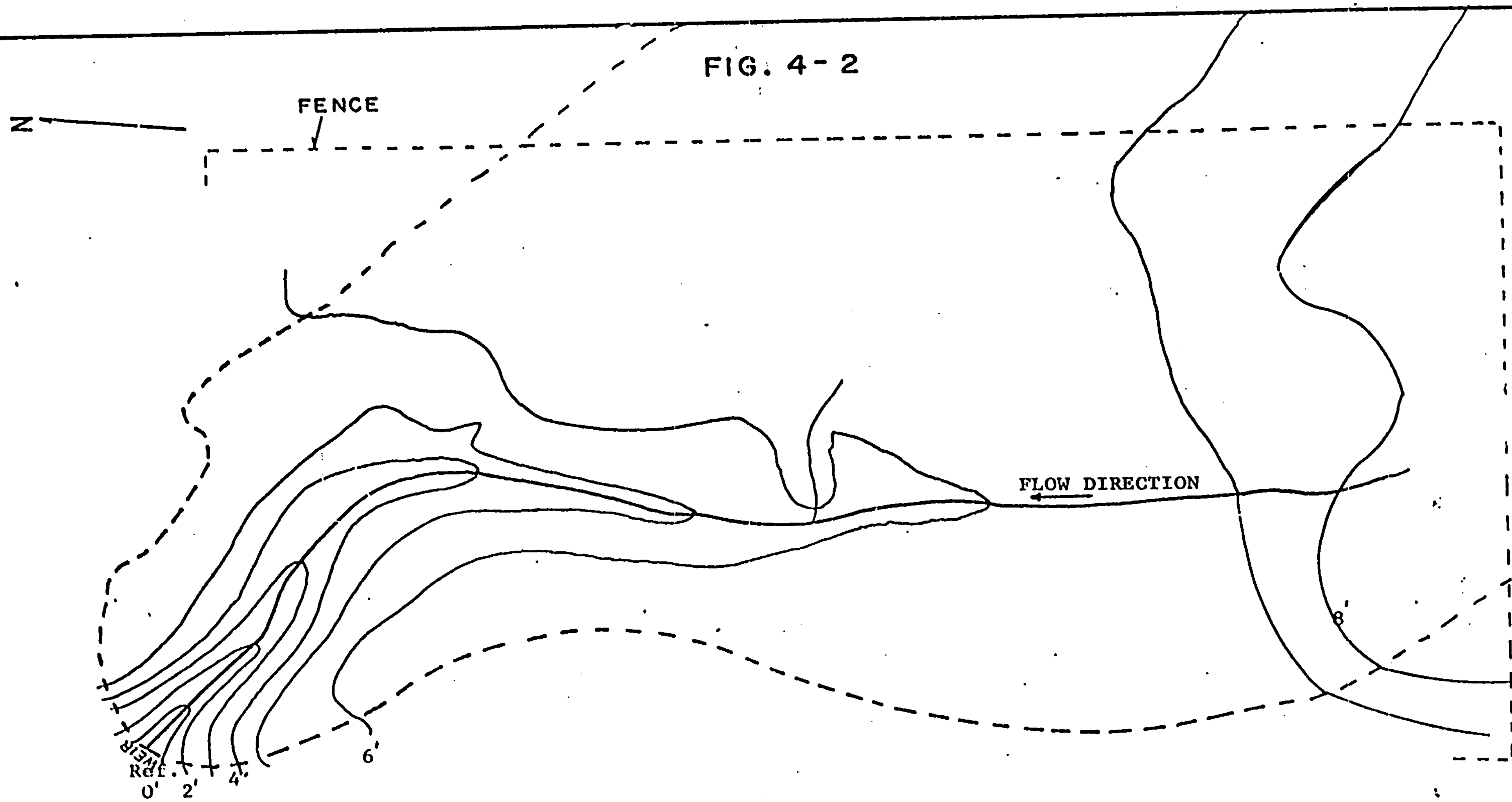


FIG. 4-2



EXPERIMENTAL VALLEY

CONTOUR INTERVAL = 1'
SCALE 0.02" = 1'

The brass plate is one-quarter inch thick and in a Vee-shape which is three inches wide. Holes were drilled in the plate for bolting it to the plywood. To prevent leakage, the bottom of the plywood was completely sealed with a concrete mixture and each connecting joint was coated with asphalt.

The stilling well was constructed approximately five feet upstream from the weir. A corrugated steel pipe with diameter of one and a half feet was used. An eight-inch intake pipe was connected to the stilling well pipe with a small slope towards the stilling well. A plywood shelter was built on the top of the corrugated steel pipe in order to protect the water-level recorder. Fiberglass insulation was used inside the shelter to reduce heat loss from it. A propane burner was used inside the shelter to reduce freezing problems.

A strip-chart automatic water-level recorder was used to measure the stage of the stream. A total of about thirty different measurements of stage and discharge were made to obtain a calibration for the 60-degree Vee-notch weir. This relation is shown in Figure 4-3 and may be expressed by the equation :

$$Q = 1.48 H^{2.97} \qquad \text{..................... (4.1)}$$

where Q is in cfs and H is feet. Although the calibration was made for low discharges and extrapolated to discharge beyond this range,it is felt that this procedure is justified because of the excellent control provided by a sharp-crested weir and because the primary information desired is the time distribution of the runoff rather than its total volume. Upon installation of a permanent weir, measurements of discharge at higher stages will be made.

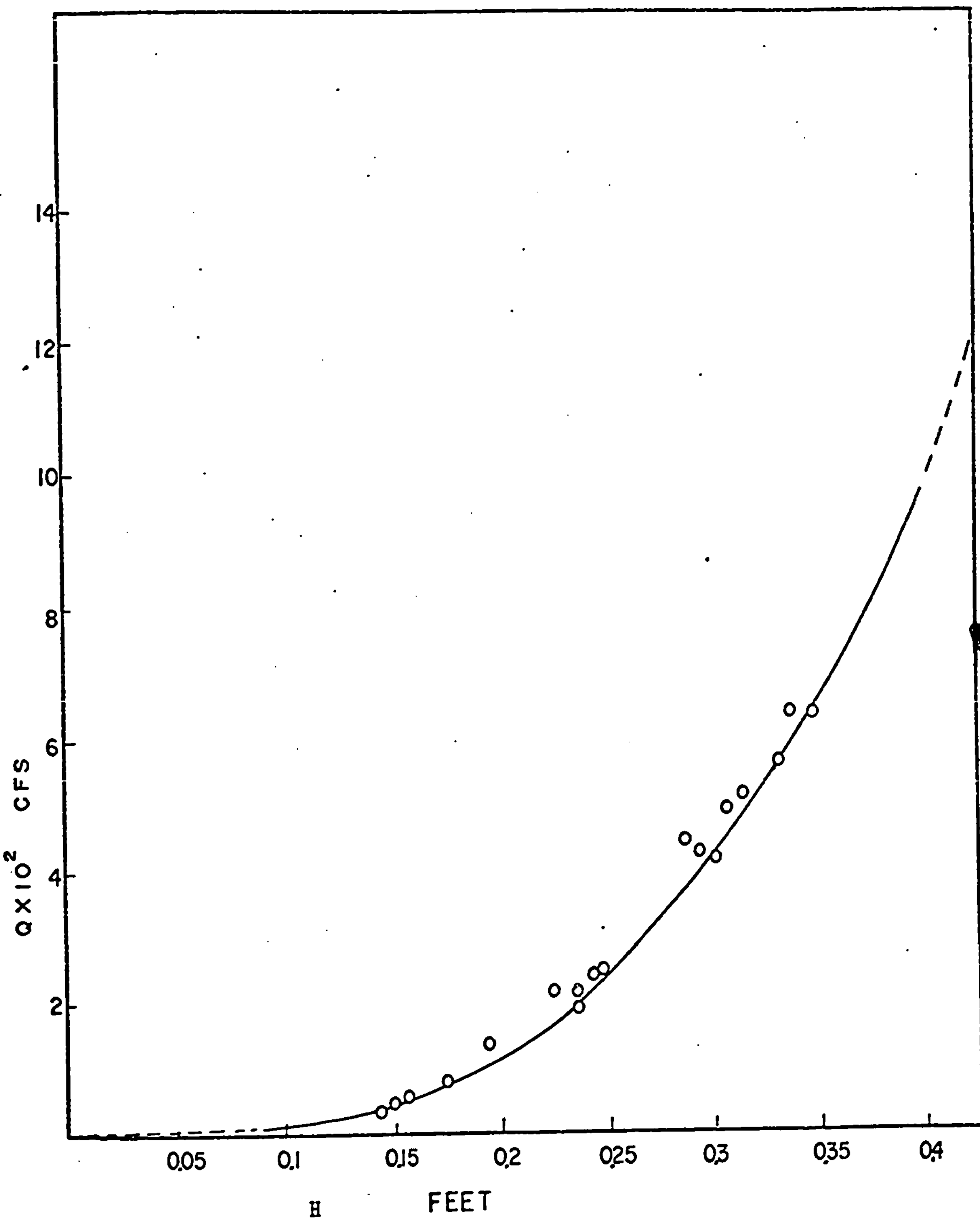


FIG. 4-3 RATING CURVE

A Bendix-Friez hygrothermograph was used to give a continous record of the relative humidity and temperature of the air throughout the whole study period. The relative humidity was measured with a psychrometer once a week so that accurate humidity values could be obtained from the hygrothermograph. The instrument recorded relative humidity and temperature continuously on a rectangular chart. The hygrothermograph recorder was put inside the standard IS-1 instrument shelter which was located at the downstream end of the valley near the weir.

Measurements of net,allwave radiation were attempted with a C.S.I.R.O. Nett Radiometer. The components for these measure were : (a) direct shortwave radiation from the sun, (b) diffuse shortwave radiation from the sky, (c) reflected shortwave radiation from the snowpack, (d) longwave radiation from sky,cloud,etc., and (e) longwave radiation emitted from snowpack. To obtain the total sum of the values of the above components, the Nett Radiometer was connected to a Cole-Parmer potentionmeter recorder. Unfortunately, due to the failure of obtaining measurements with this instrument, the net,allwave radiation data from The Division of Building Research, N.R.C.,Ottawa (Figure 4.1) had to be used. The surroundings of the site at the N.R.C. are similar to those at the study area in respect to radiation measurements (27). In Appendix B, the data of wind velocity are given which were obtained from the Interational Airport, Ottawa (Figure 4.1), measured at 33 feet above the ground.

All the data of temperature, relative humidity, net, allwave radiation and snow-melt discharges are given in Appendix B. In Plate 4.1A, the measuring site is shown at the beginning of snow-melt runoff . In Plate 4.1B is shown the site shortly after snow-melt was ended. Runoff occuring is predominantly from rainfall.

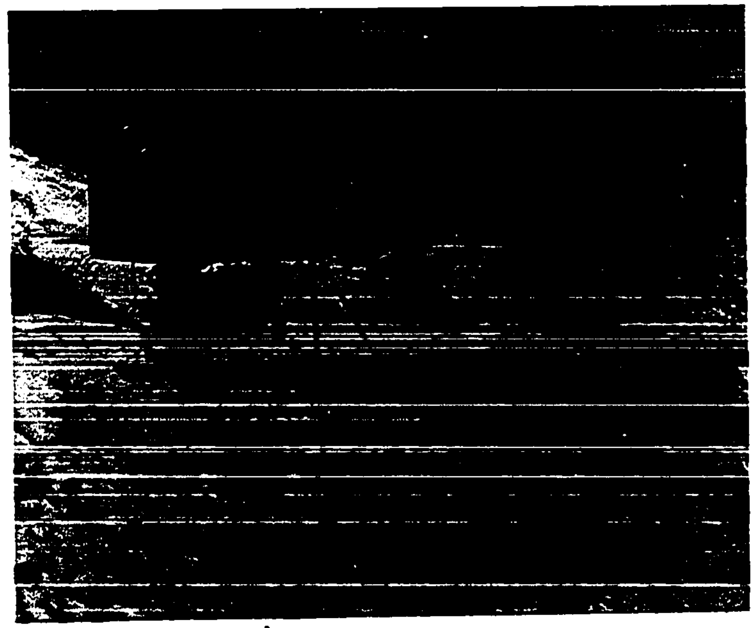


Plate 4-1A Date : 27th March 1970

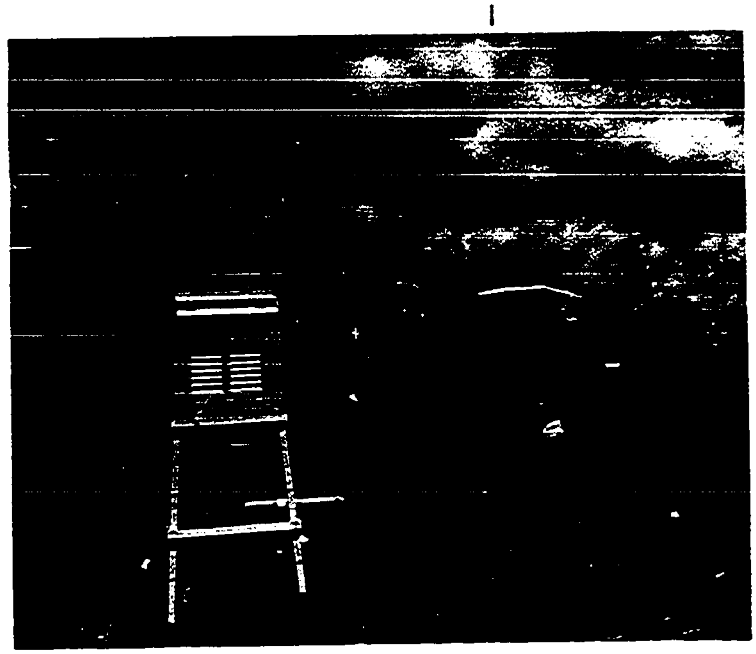


Plate 4-1B Date : 17th April 1970

CHAPTER FIVE

ANALYSIS OF THE SNOW-MELT HYDROGRAPHS

5-1. Hydrograph analysis

It is an attractive hypothesis to separate runoff into two components : (a) direct runoff and (b) base or groundwater flow. There obviously is no way of distinguishing between the direct and groundwater flow in a stream at any instant and the definition of these two components is rather arbitrary. In this study it was observed the flow receded rather rapidly after a day of active melt but that a low flow was maintained even when several successive days passed without melt. Therefore, in the computation of the daily snow-melt runoff throughout this study, a constant base flow of 0.001 cfs was assumed, below which the recession curve was not applied.

The recession curves were computed from Barnes (23) recession equation, $Q = Q_0 K^t$, until the base flow of 0.001 cfs is reached. This method was proposed by W.U. Garstka (19) and has been found to give consistent results in this study. The recession factors were derived from the snow-melt periods as follows. Each discharge on the recession for a two-hour interval was plotted against the immediately preceding two-hour interval discharge for the recession during the melting periods. A recession factor was determined by finding the slope of the line, as shown in Figure 5-1a to 5-1c. The recession factors, as computed from these plots, were found to be 0.852 for flows above 0.1 cfs, 0.892 for flows between 0.02 to 0.1 cfs, and for flows below 0.02 cfs the factor is 0.926. In a similar way, Garstka (19) in his research on the St. Louis Creek Basin in Colorado used different recession coefficient for different flows.

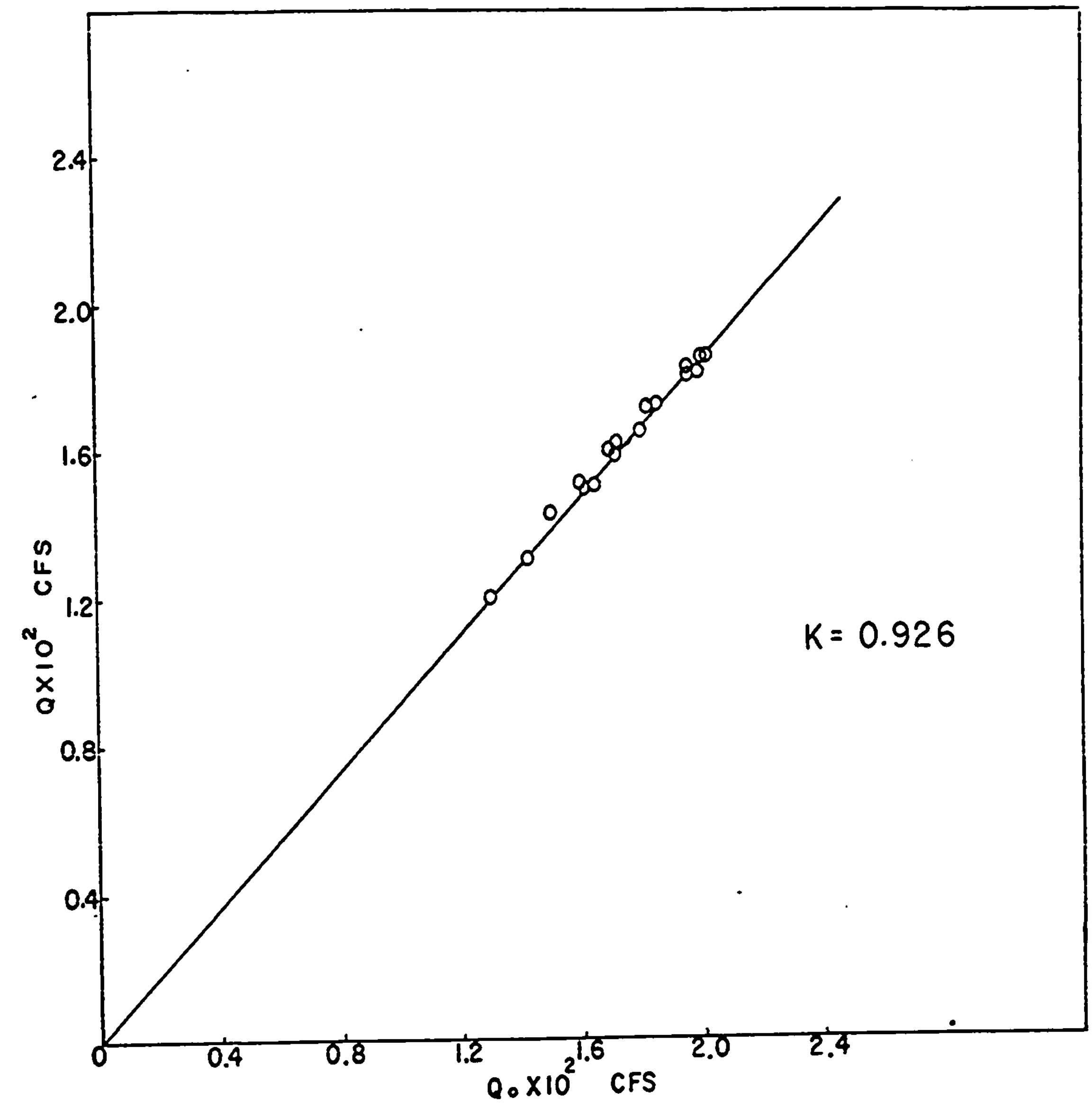


FIG. 5-1A

TWO-HOUR RECESSION ANALYSIS

FLOWS BELOW 0.02 CFS

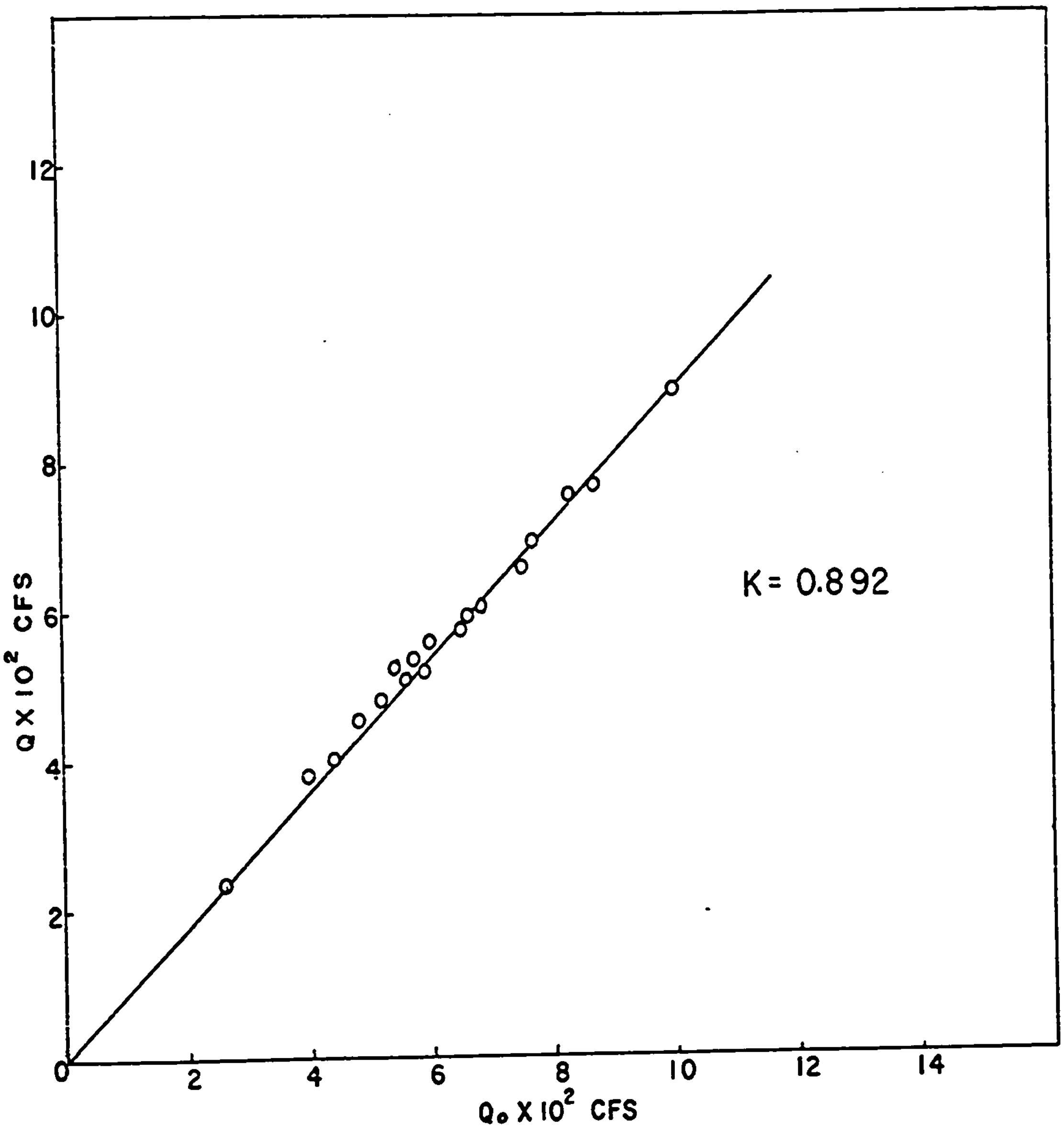


FIG. 5-1B

TWO-HOUR RECESSION ANALYSIS

FLOWS BETWEEN 0.02 TO 0.1 CFS

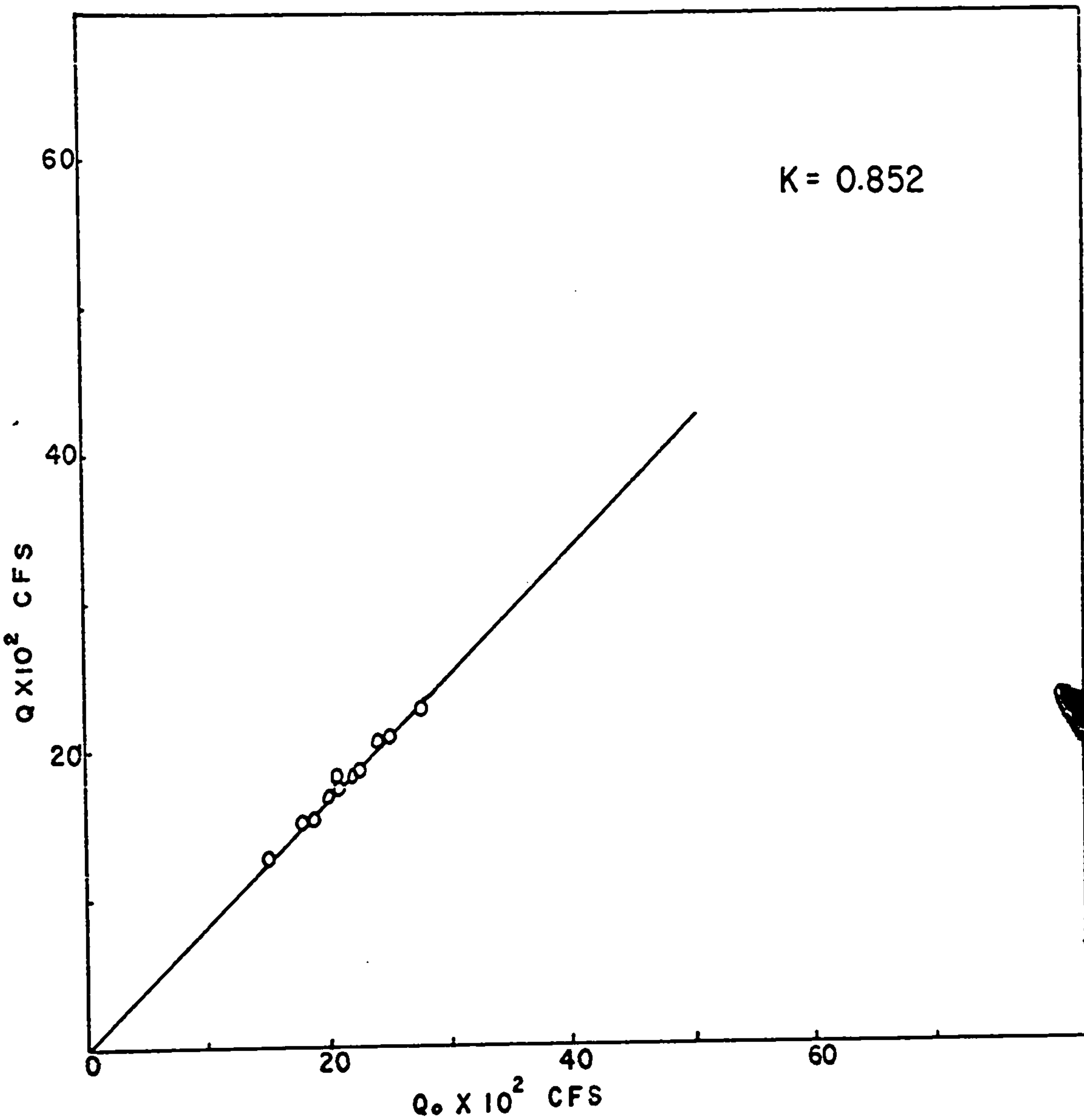


FIG. 5-1C

TWO-HOUR RECESSION ANALYSIS

FLOWS ABOVE 0.1 CFS

The above three recession factors have been found to give consistent results in this study.

The snow-melt volume from the heat input for a given day was as shown diagramatically in Figure 5-2. The volume of snow-melt runoff is to be computed for Day 2.

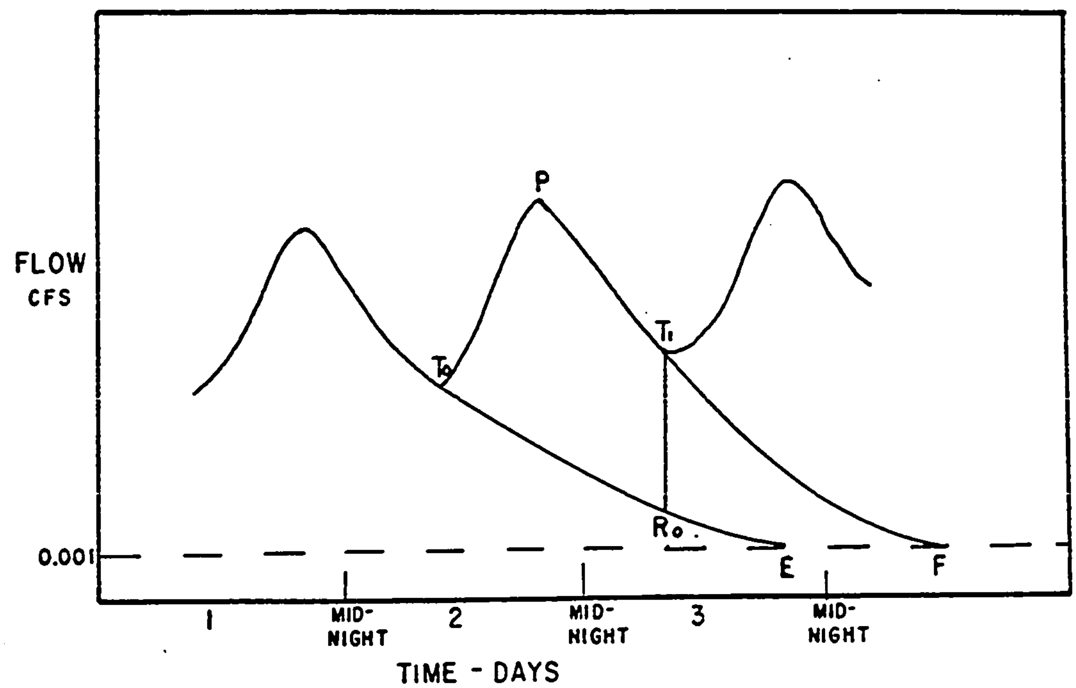


Figure 5-2 METHOD OF COMPUTATION FOR DAILY VOLUME OF SNOW-MELT RUNOFF

From Fig. (5-2), T_0 , is the preceding trough in the hydrograph - that is the point of inflection at which runoff begins in the given day under the influence of the effective heat input. Point P is the peak of the snow-melt runoff and point T_1 is its trough. The lines from T_0 to E and T_1 to F are the recession lines of the preceding day and the given day respectively. The area $T_0PT_1R_0$ is the 'direct' contribution to runoff from the

the heat input of the given day. The volume of the ' recession ' contribution, T_1FER_0 , expresses the volume between the two recession curves which resulted from the day's contribution of snow-melt not appearing as runoff on that day but which is added in the following days.The total volume for the contribution of a given day to the snow-melt runoff is the sum of the direct contribution and the recession contribution volumes . The contribution of each individual day to the snow-melt runoff is shown in Table 5-1.

In this study area, it was found that about 80 per cent of the runoff was released on a given day, and the remaining part came from the basin as recession flow. The daily peak of snow-melt runoff occured between 4 to 6 pm on each day, and the trough of the hydrograph occured between 8 to 10 in the next morning.

5-2. Distribution Graph

In 1932, Sherman proposed the well-known theory of unit hydrograph. This theory has been further developed and is widely used in applied hydrology. He reasoned that, since the physical characteristics of the watershed remain essentially constant, the shape of the hydrographs resulting from storms with similar characteristics should also be similar. In 1935, Bernard (21) presented the distribution graph which provides a most useful tool for the determination of the hydrograph of direct runoff that results from any given amount of precipitation excess. It is obvious that the distribution graph simply represents the characteristic shape of the unit hydrograph plotted with dismensionless ordinates.

The unit hydrograph or distribution graph has not been widely used in snow hydrology. It is proposed here that in a small watershed, the

TABLE 5-1 (SEE FIGURE 5-2)

<u>Daily Volume of Snow-melt Runoff</u>

DATE	VOLUME OF DIRECT CONTRIBUTION ABOVE RECESSION CFS x 10^2 HR	VOLUME OF RECESSION CONTRIBUTION CFS x 10^2 HR	TOTAL OF THE DAY'S SNOW-MELT RUNOFF CFS x 10^2 HR
28-3-70	104.39	47.34	151.74
29-3-70	188.83	78.83	267.66
30-3-70	180.82	60.75	241.57
31-3-70	79.52	47.63	127.15
1-4-70	110.87	24.90	135.78
2-4-70	19.39	14.80	34.2
3-4-70	22.54	20.91	43.46
4-4-70	37.71	18.82	56.54
5-4-70	143.90	20.09	163.99
6-4-70	78.24	12.54	90.78
7-4-70	93.10	24.42	117.52
8-4-70	132.2	21.19	153.39
9-4-70	–	–	–
10-4-70	–	–	–

TABLE 5-1

(CONTINUED)

DATE	VOLUME OF DIRECT CONTRIBUTION ABOVE RECESSION CFS x 10^2 HR	VOLUME OF RECESSION CONTRIBUTION CFS x10^2 HR	TOTAL OF THE DAY'S SNOW-MELT RUNOFF CFS x 10^2 HR
11-4-70	993.56	140.73	1134.30
12-4-70	1147.46	167.66	1315.12
13-4-70	1423.26	113.00	1536.27
14-4-70	1417.55	148.74	1566.29

unit hydrograph could also be used to derive a snow-melt hydrograph. To obtain the unit hydrograph, however, the techniques developed for the analysis of complex storms must be used. Also the time distribution of the inputs must be properly defined.

The unit hydrograph or distribution graph is based on the following principal assumptions :

(a) Lumped : The watershed is treated as a system. A gross representation of the system is determined from the input and output data belonging to the system, but no concern is given to the detailed processes within the system. In this lumped system-model, position and space are not important and all components of the system being simulated are located at a single point in space.

(b) Time-invariance : The runoff hydrograph from the drainage basin resulting from a given pattern of effective input at whatever time it occures is invariable and reflects all the combined physical characteristic of the basin. This states that the parameters do not change with time. It is believed that under snow-melt conditions, this assumption cannot be satisfied over an entire snow-melt season. However, it may approximately hold true during high melting periods.

(c) Linearity or Superposition : The runoff hydrograph resulting from a given pattern of effective input can be built up from the separate amounts of effective input occurring in each unit period. This includes the principle of proportionality in which the ordinates of the runoff are proportional to the total amount of direct runoff.

Under natural precipitation and watershed conditions the above assumptions cannot be satisfied perfectly. The first assumption simplifies the problem considerably by removing the spatial variation of the system. This may be inapplicable when the system varies spatially. The asumption of the time invariance is not fulfilled if the physical factors of a watershed change with time. Similarly, the volume of the snow-melt in a given storm may not necessarily be proportional to the volume of effective input and the parameters may vary with the intensity of the input. Treatment of the above system models is, of course, more complicated and difficult. At the present time, our knowlege in watershed hydrology is limited. No definition of the system components is necessary in a lumped system model; the time-invariant may approximately hold true for the watershed over a geologically short period of time; the assumption of linearity may be valid for some physical systems. In practice, if the system models can effectively serve its purpose, it is thought to be worthwhile to consider a simpler lumped, linear, time-invariant system.

5-3 Mathematical model :

The mathematical model being proposed, then, consists of a lumped, linear, time-invariant system. A snow-melt distribution graph was determined by isolating the snow-melt runoff hydrograph from one day's melt in a similar way as used in determing a rainfall unit hydrograph. The ordinate of the distribution graph having a unit duration T at any time, t, is represented by u(T,t) where t is any time after the beginning of effective input. The effective net, allwave radiation is considered as the effective input. Each block of the effective input was represented as a percentage of the total snow-melt. They can be treated as i blocks of different intensity I_i (percent of total snow-melt runoff per unit time) and with the same duration equal to T. The subscript i denotes the number of the block.

Let I(1), I(2), I(3), I(4) .... I(i) be the intensity of the effective net, allwave radiation as in Figure 5-3 (after Nash(18)) .

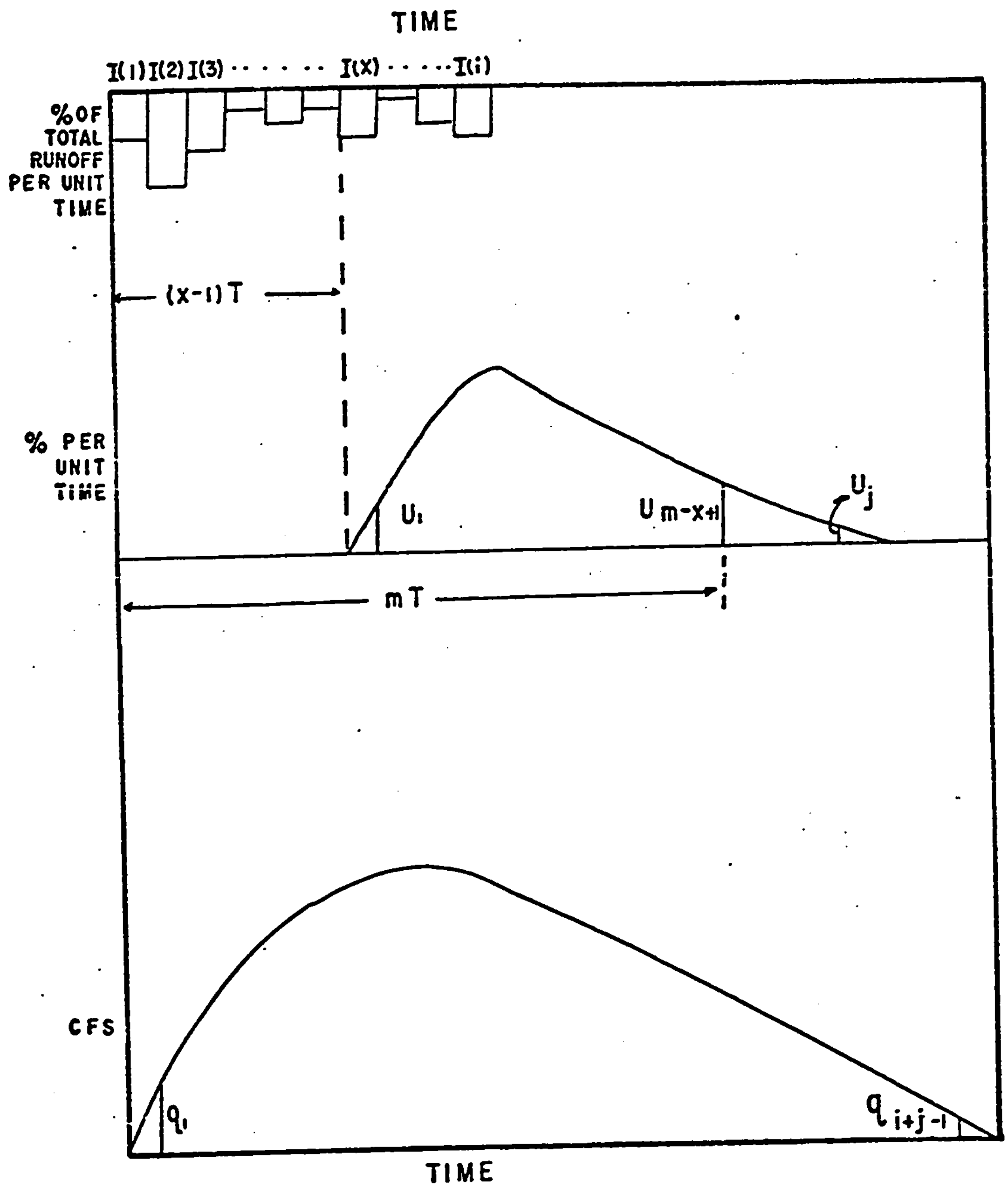


FIGURE 5-3 MODEL OF THE SYSTEM

The non-zero ordinates of the pulse response, or the distribution graph at time t = T, 2T, 3T, 4T .... iT, after the beginning of the pulse (t = 0) are u_1, u_2, u_3, u_4 .... u_j, in which the output of the snow-melt runoff is defined by the ordinates q_1, q_2, q_3, q_4 .... q_{i+j-1}, with the same duration T.

Let us consider a certain time t = mT (Figure 5-3). The ordinate Δq (mT), a contribution due to the input I(x), may be expressed as :

$$\Delta q\ (mT) = I(x) * T * U_{m-x+1} \quad \cdots\cdots\cdots\cdots \ (5.1)$$

where $U_{m-x+1} = U\ (T,\ t = (m-x+1)\ T)$.

Because of the superposition principle, the output of snow-melt runoff due to the several inputs of effective net, allwave radiation is the sum of those due to each ; i.e.

$$q\ (mT) = \sum_{x=1}^{m} I(x) * T * U_{m-x+1} \cdots\cdots\cdots \ (5.2)$$

It is understood in the summation that if x exceeds i, then I(x) = 0 and if m-x+1 exceeds j, $U_{m-x+1} = 0$. From Equation (5.2), it is obvious that once the pulse response is known, the snow-melt runoff due to any series of blocks of effective net, allwave radiation could be obtained. For convenience, we can write Equation (5.2) in tabular form as shown in Table 5-2 :

I_1, I_2, I_3 .... I_i	:	Intensity of effective input in interval of time T.
U_1, U_2, U_3 ....U_j	:	The ordinates of the distribution graph with duration T after the beginning of the effective input
q_1, q_2, q_3 ....q_{i+j-1}	:	The ordinates of snow-melt runoff with duration T beginning at t = 0.

TABLE 5-2 SNOW-MELT RUNOFF COMPUTATION

TIME		U_1	U_2	...	....	U_j	
0	I_1	...	...				q_0
T	I_2	$I_1 U_1$	.....	....			q_1
2T	I_3	$I_2 U_1$	$I_1 U_2$	....			q_2
3T	I_4	$I_3 U_1$	$I_2 U_2$	$I_1 U_1$	.....		q_3
:	:	:	:	:		.	:
(i-1)T	:	$I_{i-1}U_1$	$I_{i-2}U_2$	....	I_1U_{j-1}		q_i
iT		$I_i U_1$	$I_{i-1}U_2$	....	....	$I_1 U_j$	:
(i+1)T			I_iU_2	....	$I_{i-2}U_{j-1}$	.....	:
.....					$I_{i-1}U_{j-1}$	$I_{i-2}U_j$	q_{i+j-3}
(i+j-2)					$I_i U_{j-1}$	$I_{i-1}U_j$	q_{i+j-2}
(i+j-1)T						I_iU_j	q_{i+j-1}

From Table 5-2, the q's are equal to the sums of the terms in corresponding row ; for example,

$$\left.\begin{aligned} q_1 &= I_1U_1 \\ q_2 &= I_2U_1 + I_1U_2 \\ &\text{------------------------} \\ q_{i+j-1} &= \qquad\qquad I_iU_j \end{aligned}\right\} \quad \ldots\ (5.3)$$

From Equation 5-3, the answer is simply the product of a matrix times a vector. It can be written as :

$$\begin{bmatrix} q_1 \\ q_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ q_{i+j-1} \end{bmatrix} = \begin{bmatrix} I_1 & 0 & 0 & \cdots & 0 \\ I_2 & I_1 & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ \cdot & \cdot & \cdot & & \cdot \\ I_i & I_{i-1} & \cdots & I_1 & 0 \\ 0 & I_i & I_{i-1} & \cdots & I_1 \\ \cdot & \cdot & \cdot & & \cdot \\ 0 & 0 & 0 & \cdots & I_i \end{bmatrix} \begin{bmatrix} U_1 \\ U_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ U_j \end{bmatrix} \quad \ldots\ldots (5.4)$$

i.e.

$$\{Q\} = [I]\{U\} \quad \ldots\ldots\ldots\ldots (5\text{-}4a)$$

where $\{Q\}$ and $\{U\}$ are column matrices and $[I]$ has i+j-1 rows and j columns, in which i+j-1 is the number of snow-melt runoff hydrograph ordinates and j is the number of distribution graph ordinates with the same duration.

As with the continuous convolution integral the discrete case, such as Equation (5-4), can also be written as :

$$\begin{bmatrix} U_1 & 0 & 0 & \cdots & 0 \\ U_2 & U_1 & 0 & \cdots & 0 \\ \cdot & \cdot & \cdot & & \cdot \\ U_i & \cdot & \cdots & U_2 & U_1 \\ U_{i+1} & \cdots & & \cdots & U_2 \\ \cdot & \cdot & \cdot & & \\ U_j & U_{j-1} & \cdots & & U_{j-i+1} \\ \cdot\cdot & \cdot & \cdot & & \\ 0 & 0 & \cdots & & U_j \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ I_i \end{bmatrix} = \begin{bmatrix} q_1 \\ q_2 \\ q_3 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ q_{i+j-1} \end{bmatrix} \ldots (5\text{-}5)$$

i.e.

$$\left[U \right] \left\{ I \right\} = \left\{ Q \right\} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5.5a)$$

In this case, the effective input values and runoff values are column vectors and matrix $\left[U \right]$ has i+j-1 rows and i columns.

Equation (5.4) can be solved for any snow-melt runoff unit hydrograph or distribution graph ordinates by multiplying the equation by the inverse matrix $\left[I \right]^{-1}$. The inverse matrix $\left[I \right]^{-1}$ exists if, and only if, $[I]$ is a square matrix and is non-singular(i.e., has a nonzero determinant). Generally, the matrix $\left[I \right]$ is not a square matrix. However, by premultiplying $\left[I \right]$ by its transpose $\left[I \right]^T$, a square symmetrical matrix can easily be generated as :

$$\left[I \right]^T \left[I \right] \left\{ U \right\} = \left[I \right]^T \left\{ Q \right\}$$

$$\text{or} \qquad \left\{ U \right\} = \left[\left[I \right]^T \left[I \right] \right]^{-1} \left[I \right]^T \left\{ Q \right\} \quad \ldots\ldots\ldots\ldots (5.6)$$

The terms $\left[I \right]^T \left[I \right]$, generate the sums of squares and products of the independent variable in the simultaneous normal equations for a conventional least-squares solution and $\left[I \right]^T \left\{ Q \right\}$ forms the mixed products of independent and dependent variables . Therefore , Equation (5.6) gives a direct solution for a unit hydrograph or distribution graph based on a matrix operation. It is convenient to solve Equa-ion (5.6) by electronic digital computers because subroutines for matrix multiplication and matrix inversion are available in the computer subroutine libraries .

Theoretically, the solution of Equation (5.6) assumes that the time distribution of the effective net, allwave radiation is known exactly. In fact, it is not known how this effective net, allwave radiation is distributed with time. It can be computed in the matrix operation shown before. Adding an error term to each of the effective net, allwave radiation input, $\{I\}$, in Equation (5.5) and multiplying the effective net,allwave radiation plus an error term by the pulse response, the snow-melt runoff hydrograph Q in Equation (5.5) would be generated as :

$$[U]\ \{I + E\} = [U]\ \{I\} + [U]\ \{E\} = \{Q\} \quad \ldots\ldots\ (5.7)$$

Equation (5.7) shows that if $[U]$ and $\{I + E\}$ are correct, the observed snow-melt hydrograph Q can be computed exactly. Therefore, substituting $\hat{Q}$ for $[U]\{I\}$, Equation (5.7) gives :

$$\hat{Q} + [U]\ \{E\} = Q$$

or

$$[U]\ \{E\} = Q - \hat{Q} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ (5.8)$$

As in Equation (5.6), the error vector can be generated as :

$$[U]^T [U]\{E\} = [U]^T\ \{Q - Q\}$$

or

$$\{E\} = \left[\ [U]^T [U]\ \right]^{-1}\ [U]^T \{Q - \hat{Q}\} \ldots\ldots\ (5.9)$$

This equation expresses the computation of an error vector. Element by element, the error vector gives a correction to the effective input values. This error vector allows the effective input values to be adjusted to optimize the fit of the computed hydrograph to the observed one.

The initial estimate of effective input in each block was assumed from the rate of the net,allwave radiation in that block. Figures (5-4) and (5-5) show four individual, daily snow-melt runoff hydrograph as observed and computed . The computed hydrograph based on the correction to the initial effective input derived in Equation (5.7) is found to be much closer to the observed hydrograph than the one with the initial estimated input. This shows that the error vector given in Equation (5.9) provides an important correction to the time distribution of effective input .

Sometimes, instability of the system may give rise to a poor

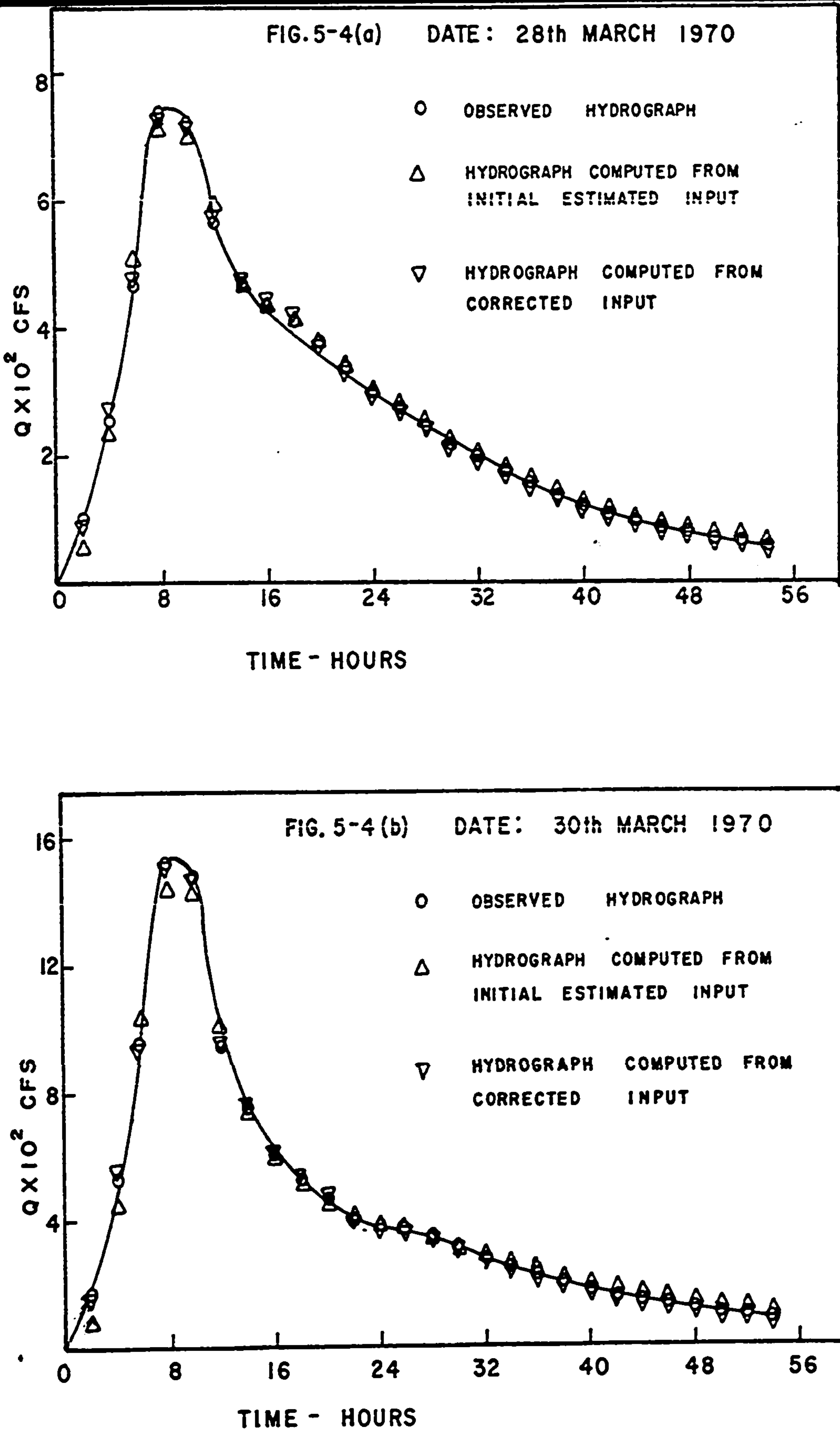
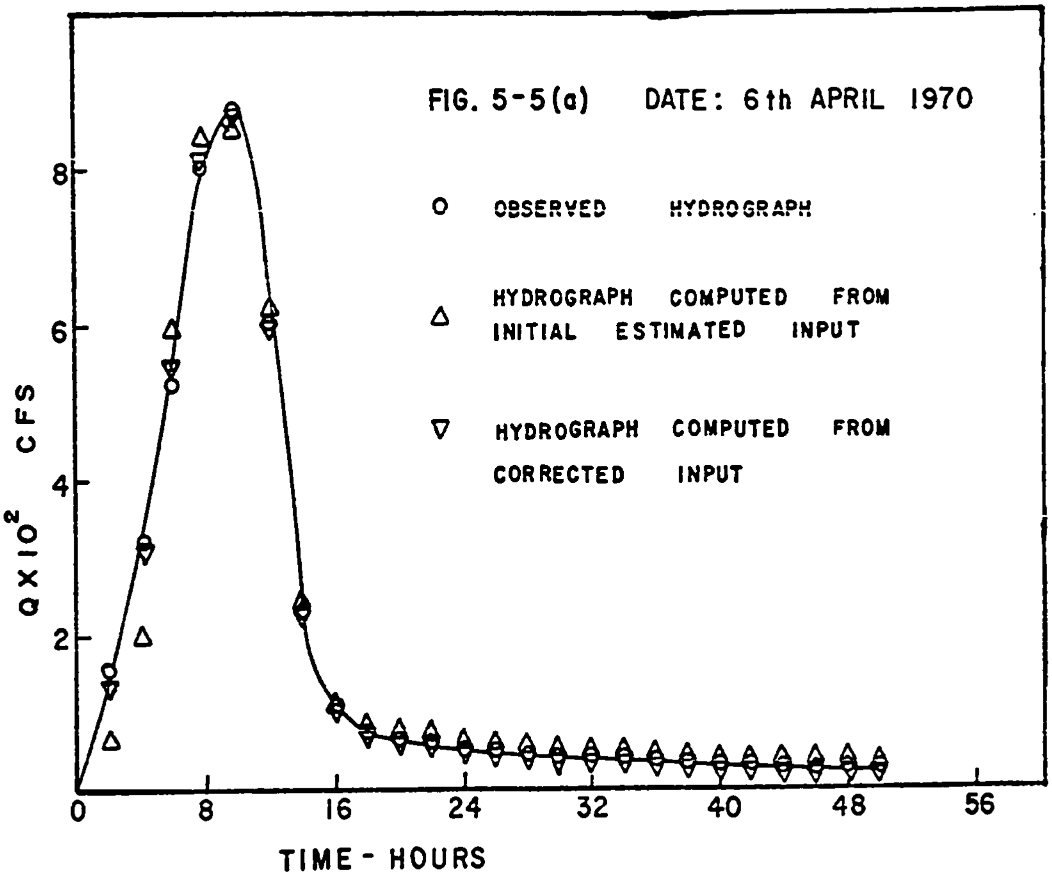


FIG. 5-4 COMPARISON OF HYDROGRAPHS



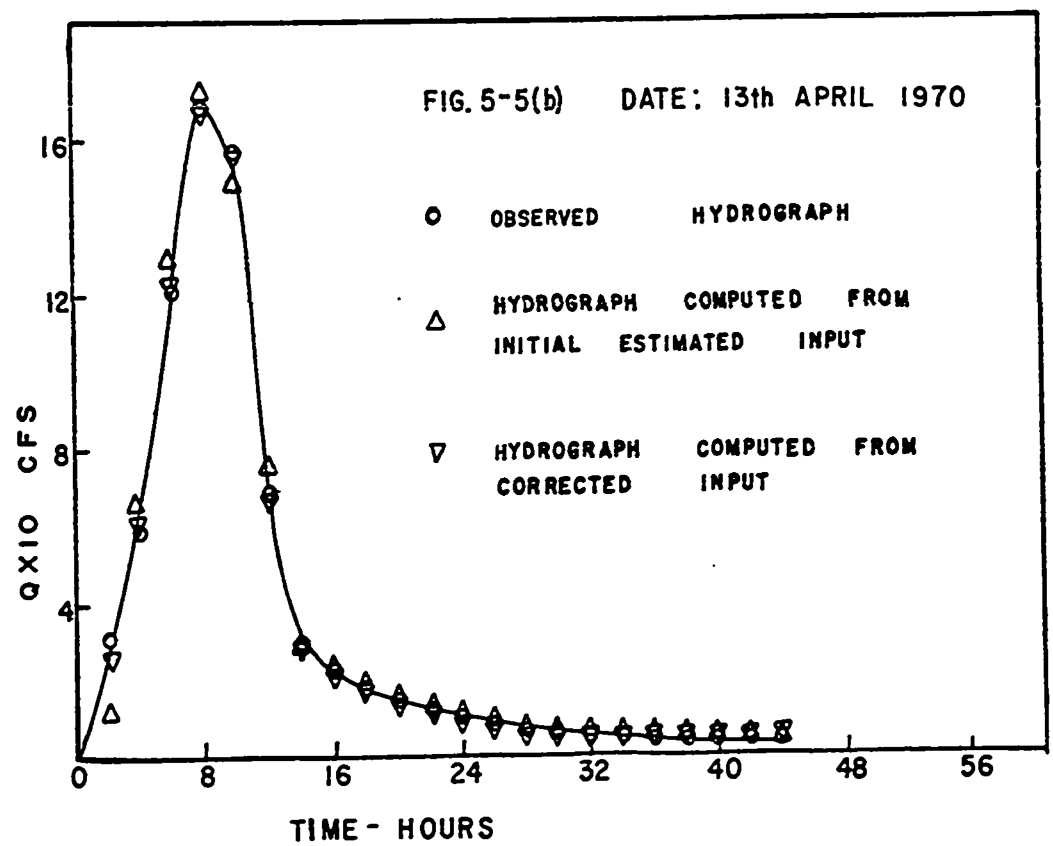


FIG. 5-5 COMPARISON OF HYDROGRAPHS

solution in this mathematical model, this is partly because of the limited number of observations for estimating the pulse response. There are only i-1 more output values than the pulse response ordinates or i-1 more observations than the unknowns. To get a mathematical solution that is more realistic hydrologically, the sub-programs which were suggested by D, W. Newton (22) were modified for use in this research.

5-4. Computer procedures

The high speed digital computer system 360/65 was used to perform the operations necessary for the above mathematical model. In order to achieve satisfactory results in using this program, care must be taken in both the initial estimate of effective net allwave radiation and the 'List' values of the subroutine program. In many cases, there are large segments of the distribution graph recession that can be approximated by a linear relationship. The subroutine program therefore transforms the I matrix by reducing the number of columns. These transform values are based on the 'List' data read in the main program.

The data required in the program are only the values of snow-melt runoff and net, allwave radiation. The total estimate of effective input must equal the volume of the total snow-melt runoff. The desired snow-melt distribution graph duration is chosen first, then, all the data of effective input or snow-melt hydrograph must be expressed in terms of the duration of the distribution graph. The choice of duration is somewhat arbitrary. It depends on the judgement of the hydrologist; decreasing the duration increases the work involved but may not give better results. A two-hour duration was used throughout this study.

In order to insure as consistent a result as possible, the following criteria were followed in choosing the snow-melt runoff hydrographs and in making the initial estimate of the effective input used to determine the distribution graph : (1) The net, allwave radiation in a given day was high. (2) The effective heat input for two hours was not less than 4 ly. (3) The net, allwave radiation occuring after the peak of the snow-melt runoff was neglected. (4) The list values used in the subroutine program were used only after the peak of the snow-melt runoff.

Base flow separation was made as mentioned before . The time base of the snow-melt runoff for each effective input becomes the equation $L = j + i - 1$, in which L is the number of ordinates in the snow-melt hydrograph, j is the number of ordinates in the distribution graph and i is the number of blocks of estimated effective input.

The program first solves for the distribution graph, and then uses this distribution graph to estimate the effective input. The proper volumes are checked and adjusted by computer. Then, using the computer-corrected effective input, the new distribution graph is computed. This process is continued for a pre-determined number of iterations or until the average error between the computed snow-melt hydrograph and observed hydrograph reaches a designated value.

The subroutine called 'Bak' was used to develop a reduced matrix of snow-melt runoff ordinates. This subroutine was based on the list data which was read in the main program. The intermediate values between the list data were determined by interpolation. The purpose of this subroutine program is to smooth the recession side of the computed distribution graph. After the mathematical model was solved by the Gaussian Elimination method, the subroutine called 'snow' was then used to expand the distribution graph ordinates generated by the least square solution to the original number of values. This extrapolation process was again based on the list data.

Suitable 'List' selection in the subroutine program can lead to smoothing of fluctuations in the recession part of the computed distribution graph. Experience with the computer program has shown that if the list ordinates begin before the peak of the snow-melt runoff hydrograph, the distribution graphs will vary erratically from one ordinate to the next. This is because the interpolation used in the subroutine program cuts across the shape of the hydrograph. Figures 5-6(a) and (b) indicate different distribution graphs derived from different list values. All the 1st list ordinates were selected after the peak hydrograph. In all cases, the results seemed to bring the distribution graph to a more reasonable shape.

Experience has shown that if within five iteration the answers still do not converge to an apparent best solution, then, obviously, something is wrong in the initial estimate of the effective heat input or the transform values. Smoothness of the distribution graph appears to indicate a reasonable initial input estimation and the best transform values. In this study, five iterations and the average error of ±5% have been used as the criteria for successful values. The steps to use the programs are described in the Appendix A.

5-5 Analysis and discussion of the results

As mentioned previously, because of the restrictive assumptions in the model, the distribution graph cannot be expected to represent completely the response of a watershed. For this reason it is instructive to examine the distribution graphs derived for successive days. Similarities

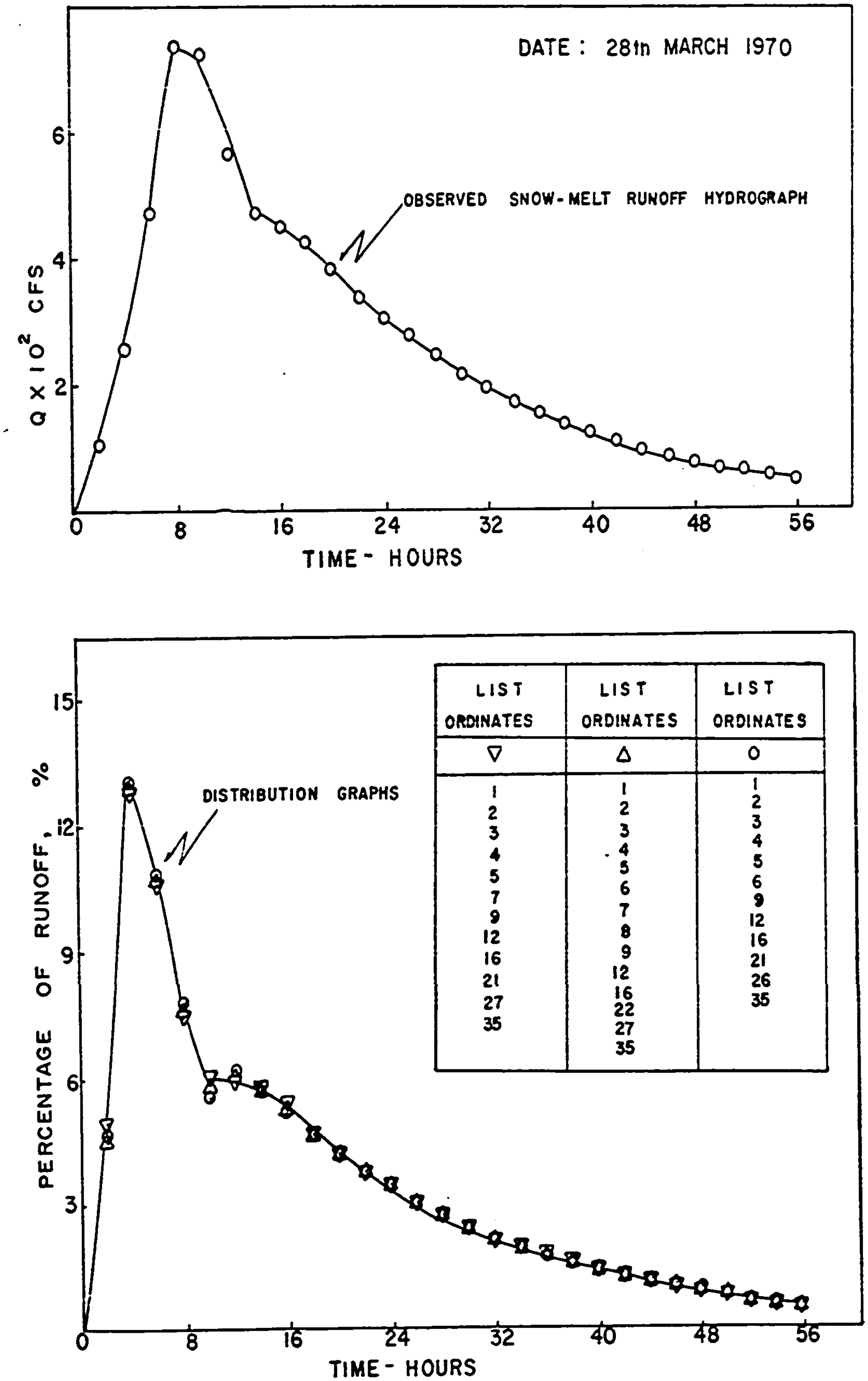


FIG. 5-6(a) EFFECT OF LIST ON DISTRIBUTION GRAPHS DERIVED FROM AN OBSERVED HYDROGRAPH

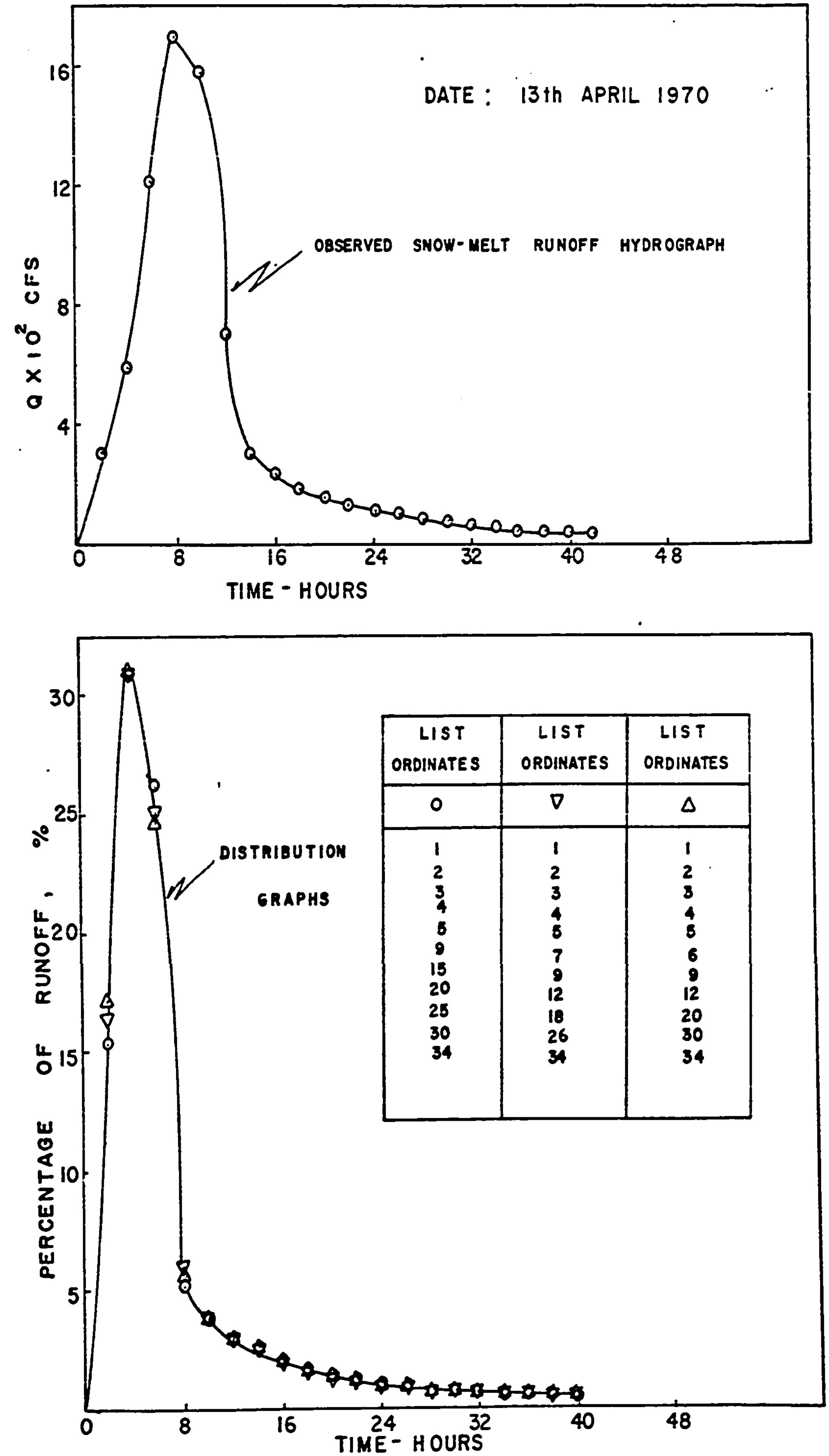


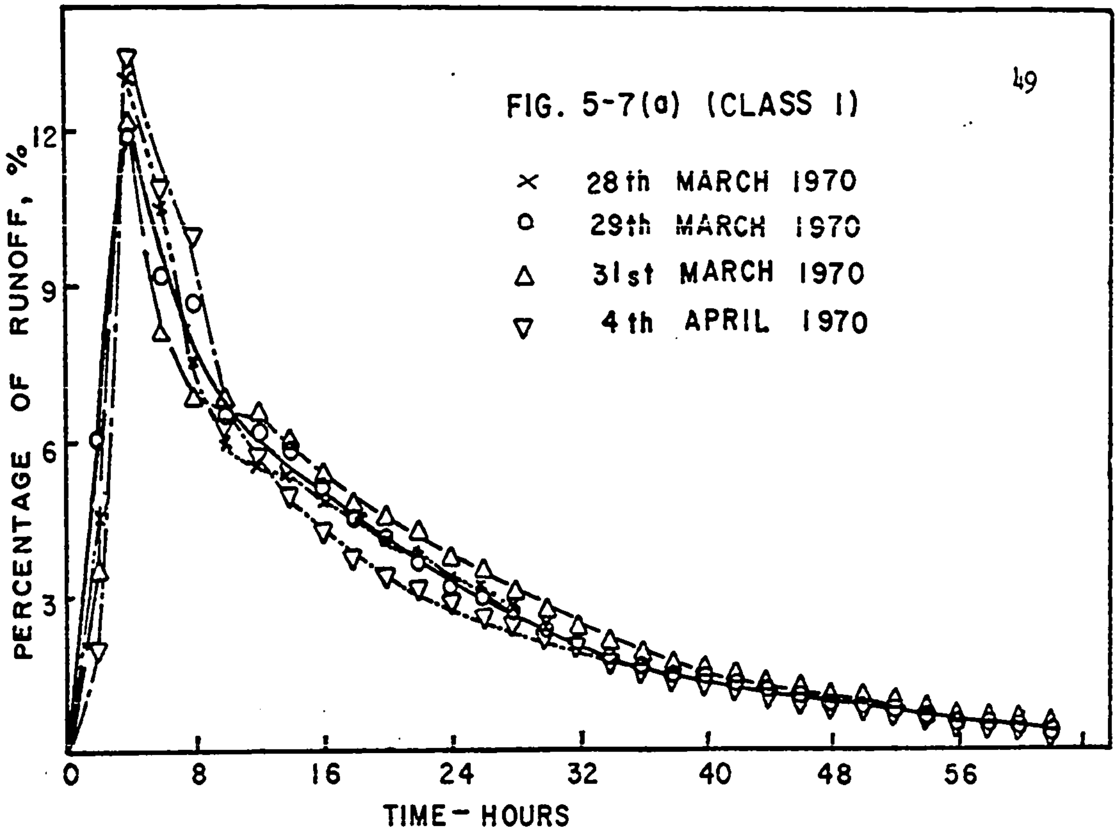
FIG. 5-6 (b) EFFECT OF LIST ON DISTRIBUTION GRAPHS DERIVED FROM AN OBSERVED HYDROGRAPH and differences in them can aid in explaning what factors are important in the snow-melt runoff process.

During the period studied, on the 2nd and 3rd of April, the net, allwave radiation was low and the wind velocity was very high (Appendix B). These hydrographs were not used for the determination of the distribution graphs. On the 9th and 10th of April, the distribution graphs could not be derived due to the loss of data.

The resulting distribution graphs derived from the snow-melt hydrographs for rain-free periods are given in Figures (5-7) and (5-8). According to the peak of the distribution graphs, they may be divided into two classes. The first class, for the period 28th of March to 4th of April, is plotted in Figure (5-7). This shows a flow peak which is much smaller than the peak for the class 2 graphs shown in Figure (5-8). For class 1, the range of the flow peak is about 12 to 13 % of the total runoff except on the days of 30th March and 1st April. On both of these days, the flow peak rose up to 20 % of total runoff. The second class is shown in Figure (5-8). The flow peak is approximately in the range of 24 to 31 % of total runoff.

From Figures (5-7) and (5-8), it appears that the peak percentage generally increases as the melting season progresses. Factors which might cause this effect are as follow :

(1) Changes in snowpack characteristics : As snow-melt progresses and velocities of flow increase, it may be expected that well-defined channels develop in the snow and as a result transport of the snow-melt is more rapid. The result would tend to be higher peak percentages. It is noted, however, that the 31st of March and 4th of April had low peak percentages, while the 30th of March and 1st of April had rather high peaks.



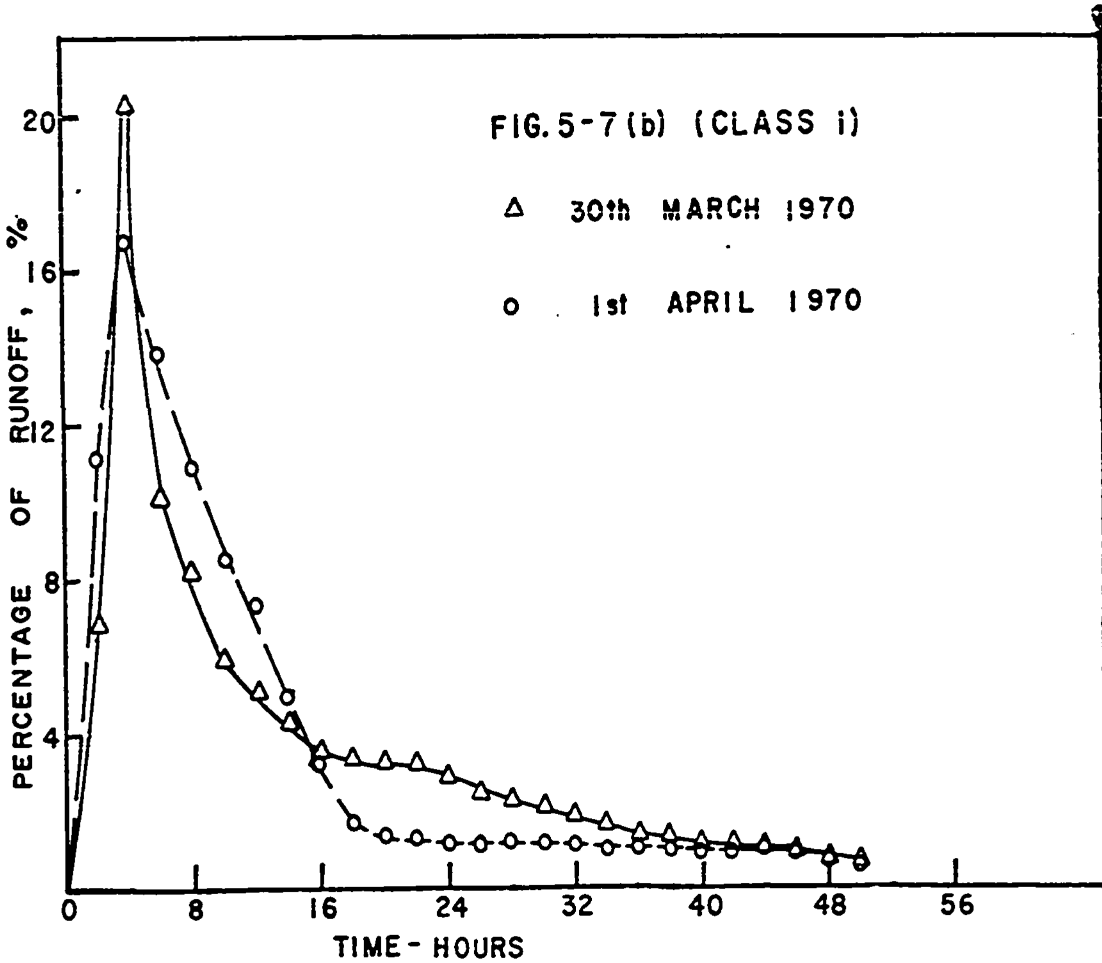


FIG. 5-7 SNOW-MELT DISTRIBUTION GRAPHS

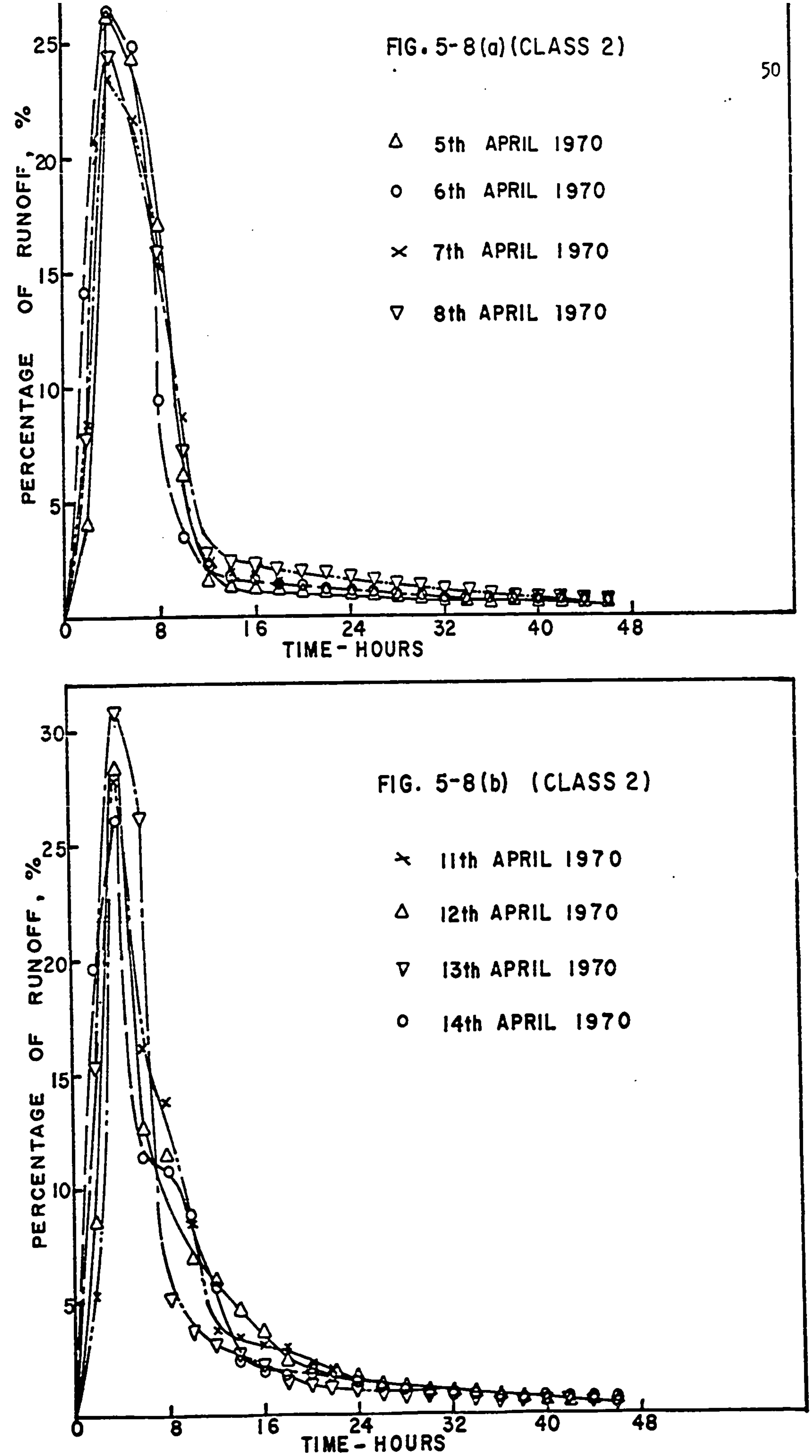


FIG. 5-8 SNOW-MELT DISTRIBUTION GRAPHS

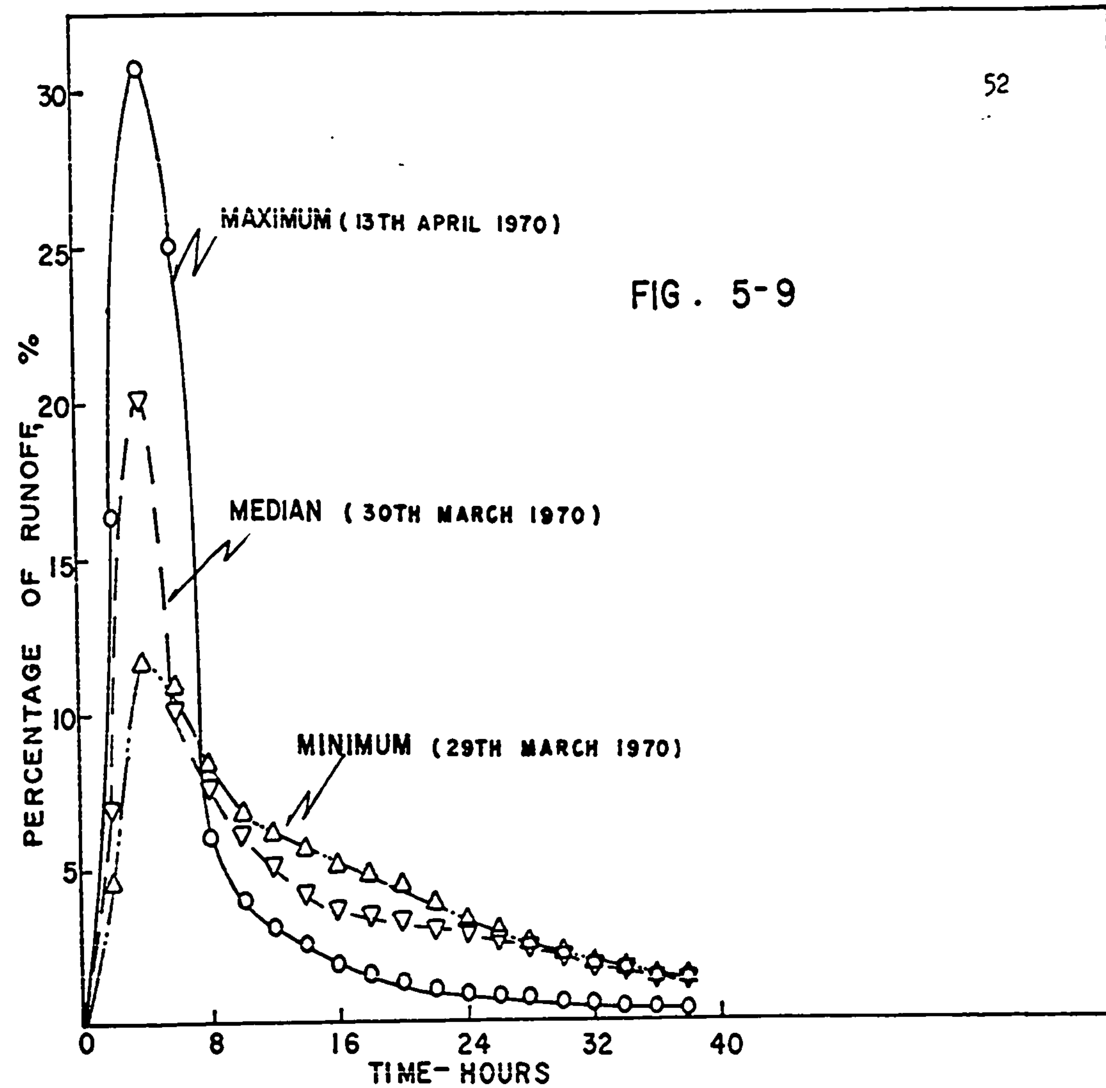
This would suggest that this effect may not be very appreciable.

(2) Effect of runoff volume : It has been noted in other studies(24) that floods with higher volumes of flow produce distribution graphs with higher peak percentages. In fact, reasoning from basic hydraulic equations shows that the peak flow is dependent upon the volume of runoff (25). This effect is apparent here as well since the floods with greater amounts of snow-melt volume in the later part of the season show higher peaks.

Design should properly be based upon the best estimate of the maximum possible discharge. Figure (5-9) indicates three different flow peaks of distribution graphs ; these are the graphs with maximum, median and minimum peak percentage which were obtained from all the distribution graphs shown in Figures (5-7) and (5-8). The maximum peak of the season occurred on the 13th of April. On that day, the net, allwave radiation was the highest of all days for which snow-melt runoff data were collected. From Figure (5-9), it is seen, as expected, that the higher the peak flow of the distribution graph, the steeper the rising limb will be and the quicker will be the recession . Figure (5-10) shows the distribution graph derived from one rainfall which occurred after the end of the snow-melt. While the peak percentage is not particularly high, the rising limb of the graph is much steeper. This difference may be due to the retording effect of the snow for the snow-melt hydrographs.

Table (5-3) shows the difference between the time distribution of the initial estimated input and computer adjusted values. The initial estimate was based on the total value of net, allwave radiation. Several reasons which might cause this variation are disscussed as follow :

(1) The drainage basin transforms the effective input into out-flow



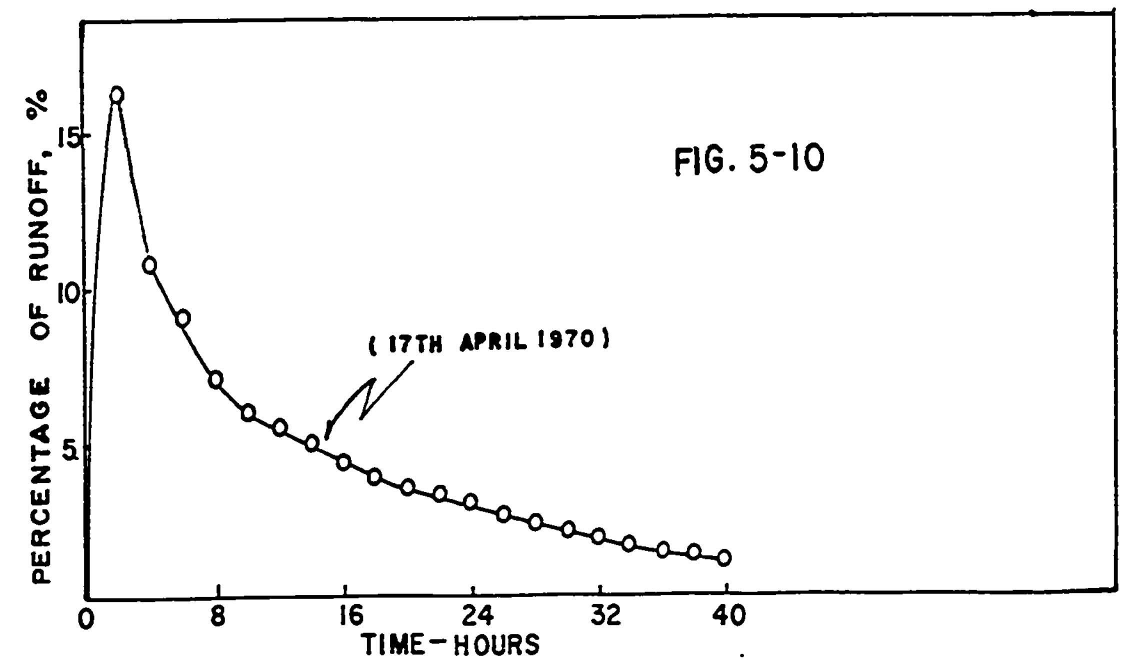


FIG. 5-9 EXTREME AND MEDIAN DISTRIBUTION GRAPHS

FIG. 5-10 RAINFALL DISTRIBUTION GRAPH

TABLE 5-3 TIME-DISTRIBUTION OF EFFECTIVE INPUT

DATE	CLOCK TIME	INITIAL ESTIMATED INPUT IN C.F.S.x10	COMPUTER ADJUSTED INPUT IN C.F.S.x10
28-3-70	10 a.m.	1.2939	1.7182
	12noon	2.0333	1.1229
	14 p.m.	2.9576	3.1458
	16 p.m.	1.2939	1.5918
29-3-70	10 a.m.	3.4588	3.4588
	12noon	5.0729	5.0729
	14 p.m.	3.4588	3.4588
	16 p.m.	1.3835	1.3835
30-3-70	10 a.m.	1.3228	2.1431
	12noon	3.3070	1.7746
	14 p.m.	4.1337	4.4907
	16 p.m.	3.3070	3.6619
31-3-70	12noon	1.3803	0.7983
	14 p.m.	2.2085	2.4616
	16 p.m.	2.7606	3.0895
1-4- 70	10 a.m.	0.6988	1.3980
	12noon	1.9966	0.1612
	14 p.m.	2.4957	4.6544
	16 p.m.	1.5972	0.5747
4-4-70	10 a.m.	0.5421	0.3219
	12noon	1.1925	1.8890
	14 p.m.	1.0814	0.6069

Note : 10 a.m. represent the average rate from 8 a.m. to 10 a.m.

TABLE 5-3 (CONTINUED)

5-4-70	10 a.m.	1.3651	0.8705
	12noon	4.2904	5.0717
	14 p.m.	2.5352	2.2485
6-4-70	10 a.m.	0.3627	0.9571
	12noon	0.7254	0.4694
	14 p.m.	2.0856	1.3405
	16 p.m.	1.3602	1.7670
7-4-70	10 a.m.	0.3306	0.5562
	12noon	1.9008	1.1135
	14 p.m.	2.3140	1.7367
	16 p.m.	1.3223	2.4613
8-4-70	10 a.m.	0.8839	1.7354
	12noon	1.3750	0.3496
	14 p.m.	1.9642	0.3001
	16 p.m.	2.0624	5.0370
	18 p.m.	1.3750	0.2384
11-4-70	10 a.m.	5.4436	3.9734
	12noon	13.8359	18.4367
	16 p.m.	13.6091	14.2457
	18 p.m.	7.4850	8.2366
12-4-70	10 a.m.	8.1814	10.7948
	12noon	15.0042	4.2232
	14 p.m.	23.4610	31.1815
	16 p.m.	19.0962	19.5460

TABLE 5-3 (CONTINUED)

13-4-70	10 a.m.	11.6303	15.8051
	12noon	20.1884	8.3611
	14 p.m.	24.5772	36.3595
	16 p.m.	20.4079	16.2782
14-4-70	10 a.m.	11.7693	15.6218
	12noon	21.1367	9.9799
	14 p.m.	24.9798	32.2307
	16 p.m.	20.4161	20.4696

runoff by means of storage constituents. The storage -flow relation for the watershed might have a distinct effect for each effective input. Due to the assumptions of linearity and time-invariance in the model used, an unrealistic distribution of the effective input may be obtained.

(2) The initial estimate of the effective input in each block was based on the rate of the net, allwave radiation in that block.The resulting hydrograph, however, is the resultant of all the heat inputs to the snow. Accordingly, it would expected that adjusted inputs might not agree with the recorded net, allwave radiation.

(3) As long as the entire region is covered with snow, the orientation and aspect of the valley and its slope play a decisive role in its irradiation . The overhead sun at noon on the horizontal centre of the study area might not be as effective as the solar radiation earlier and later in the day on the larger sloping side areas.This would explain the result, shown in Table (5-3), of the second block of the computer adjusted input being much smaller than the initial estimated values.

(4) The time distribution of the net, allwave radiation from N.R.C. might be somewhat different from the actual values because of the different location.

The relation between the positive daily net, allwave radiation and snow-melt runoff volume are shown in Figure 5-11. A good relationship between them is obviously shown after a simple linear regression analysis. The resulting regression equation is

$$Y = 16.92 + 22.24X$$

where X is the snow-melt runoff in c.f.s.-hours and Y is the net, allwave radiation in langleys. The correlation coefficient for this equation

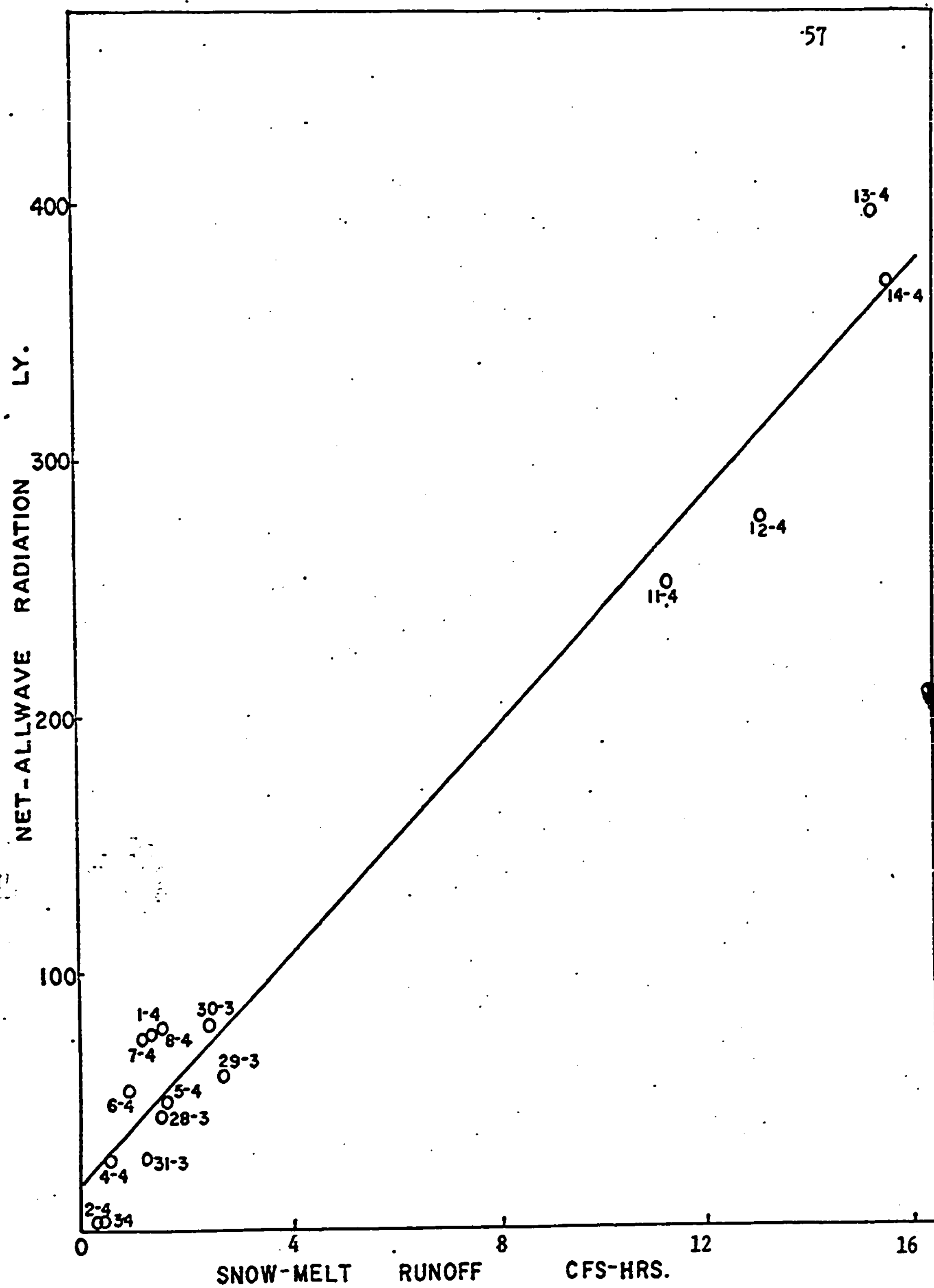


FIG. 5-11 DAILY POSITIVE NET-ALLWAVE RADIATION

vs DAILY SNOW-MELT RUNOFF

is 0.915. This indicates that the net, allwave radiation used to predict the snow-melt runoff would provide a reasonable result for this valley under the condition that the primary source of heat is radiation. Since this result is an empirical one for this particular study valley, it was not felt advisable to generalize it to depth of melt. However, it supports the contention that the net, allwave radiation is a good index of the snow-melt in this case.

CHAPTER SIX

SUMMARY AND CONCLUSIONS

The assumption of a simple lumped, linear, time-invariant system has been used to analyze snow-melt hydrographs from a watershed of about two acres. The input has been taken to be the effective net, allwave radiation occurring between the start of runoff and the peak. Distribution graphs derived from the hydrographs represent the response of the hydrologic system. The method used to derive the response is one adapted from a method for determing unit hydrographs from complex rainstorms. This consists of a least-squares technique proposed by Snyder (26) . Net, allwave radiation has been used to estimate the effective net radiation with a correction made by the use of an error term.

Fourteen distribution graphs were obtained in the study period for days of high net, allwave radiation. A study of these results leads to the following conclusions :

(1) The results of the method for days of high net, allwave radiation are quite reasonable, showing agreement as close as that often obtained for unit hydrographs from rainstorms.

(2) Distribution graphs varied as melt progressed with the later hydrographs which had higher volumes showing higher peak percentages.

(3) The time from the beginning of runoff to the peak flow for the distribution graphs found was four hours and remained constant throughout the melt period.

(4) Use of the radiation data from a measuring station some distance from the watershed appeared to be satisfactory for deriving the response function.

(5) Total volume of runoff correlated well with daily net, allwave radiation with a correlation coefficient of 0.915.

(6) Correction to the effective net, allwave radiation input which improves the fit of the computed hydrograph to the observed one may result in somewhat unrealistic patterns of input.

(7) The model used has the advantage of simplicity for application. Computer subroutines required are standard packages. About 15 seconds of computer time were required to derive a distribution graph.

Further areas for study have presented themselves in the course of this research. Some of these are as follows :

(1) The data may be used to derive more sophisticated models of snow-melt runoff. These should take into account the variation of the response throughout the day and the variation as the snow-melt season progresses.

(2) Through application of these results from a source area of snow-melt to other source areas and the use of routing techniques, it may be possible to predict flows in larger streams.

(3) Analysis of more complex heat inputs such as combined radiation and sensible heat may be possible using the derived response of the watershed to radiation inputs alone. Also, combined heat inputs and rainfall may be analysed by using the derived response function. Study of these situations awaits collection of data on the study from runoff events due to these factors.

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APPENDICES

APPENDIX A

COMPUTER FLOW CHART

INPUT DATA

NGRAPH : The number of daily snow-melt hydrograph to be put into the program.

ERROR : Average allowable error between computed and observed hydrograph.

FACTOR : A factor for reducing the effective heat input adjustment.

NITER : The maximum number of iterations desired if the error factor is not reached.

NLIST : The number of ' List ' ordinates .

LIST(I): The reduced matrix of snow-melt runoff hydrograph ordinates.

TIME : The distribution graph duration.

N : The number of effective input ordinates.

NI : The number of snow-melt runoff hydrograph ordinates.

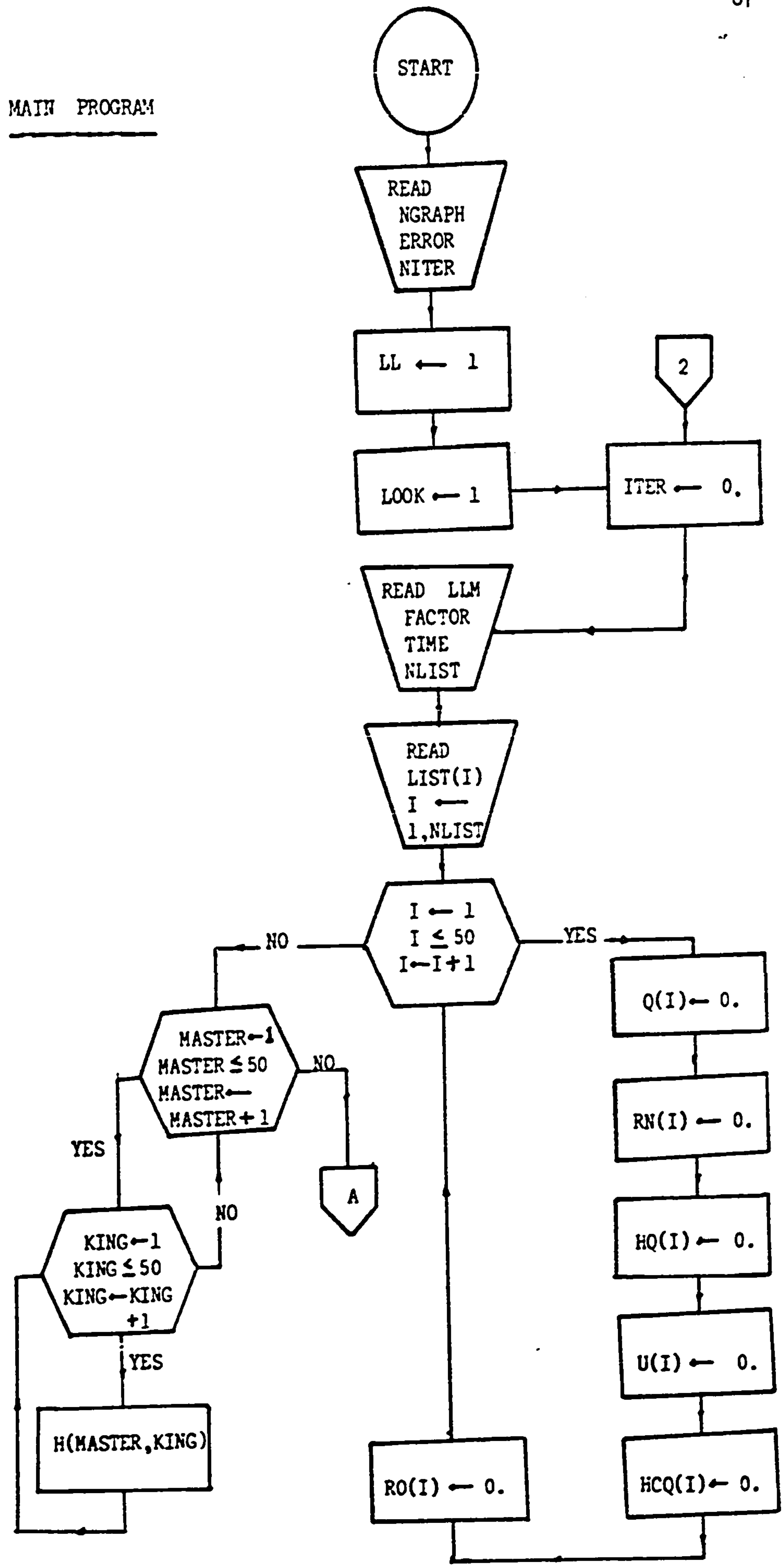
Q(I) : The values of snow-melt runoff hydrograph in c.f.s.

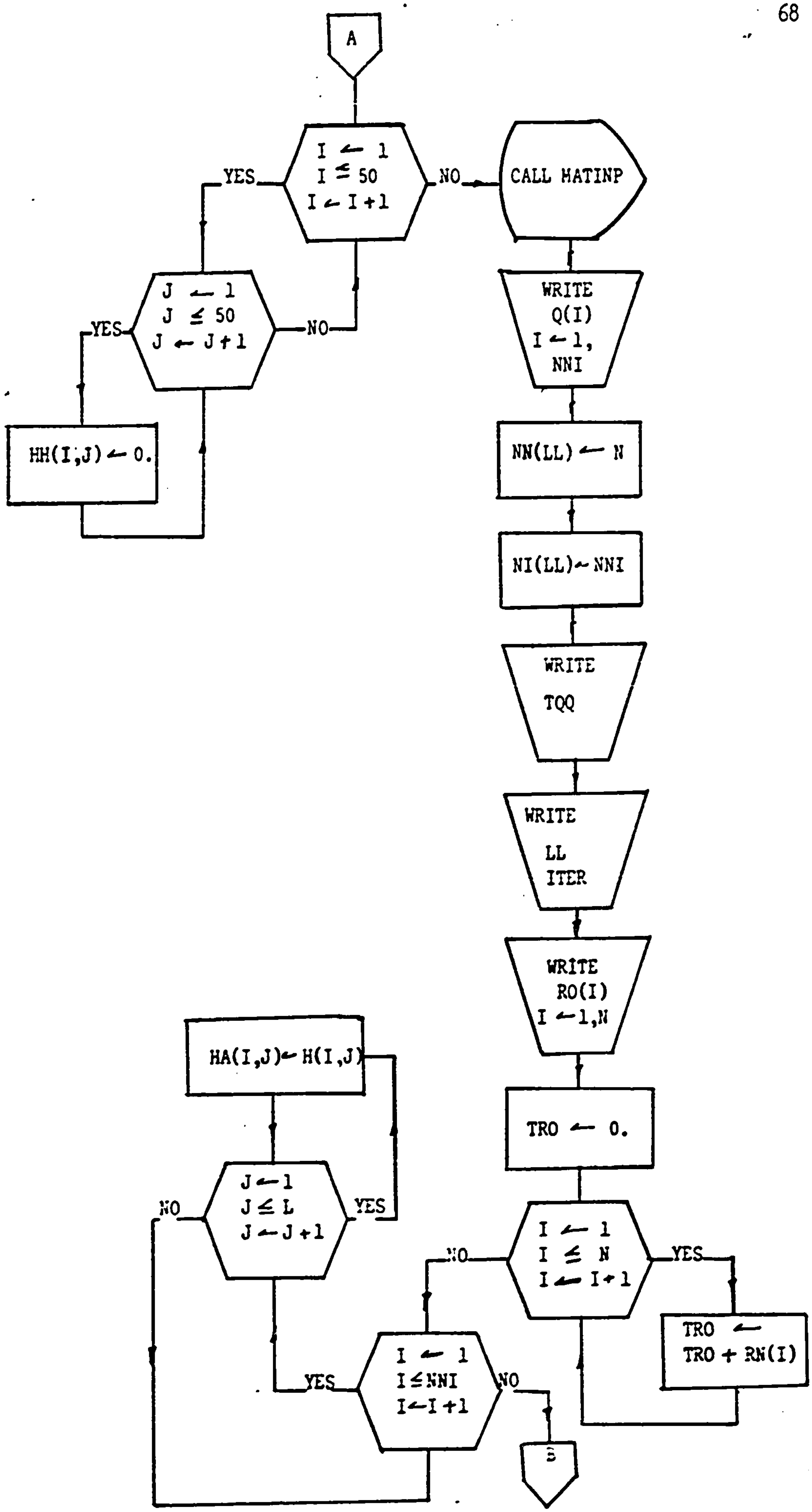
RAD(I) : The initial estimate of effective heat input.

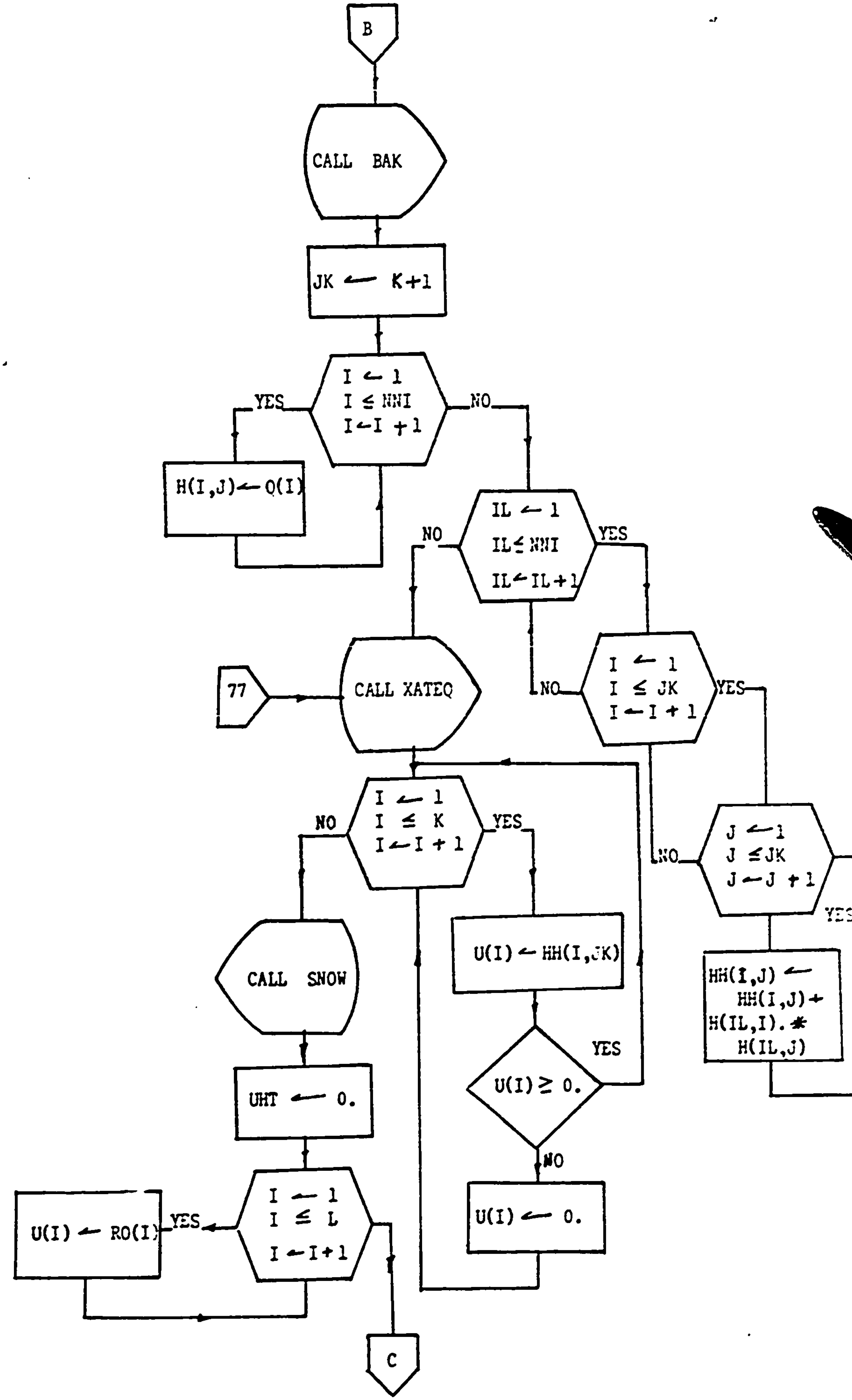
DATE : The date which the distribution graph is obtained.

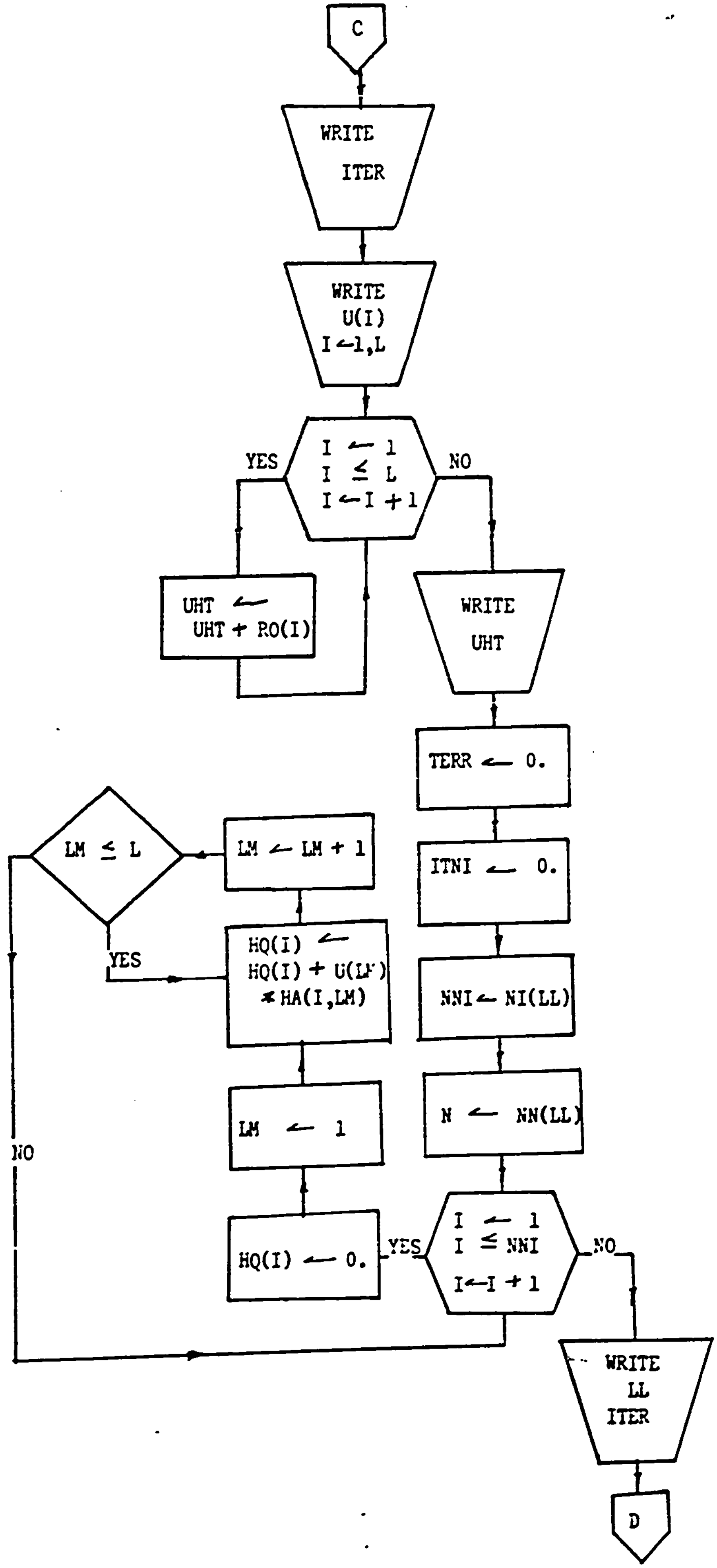
LIM : The number of snow-melt runoff hydrograph ordinates , same as the symbol NI.

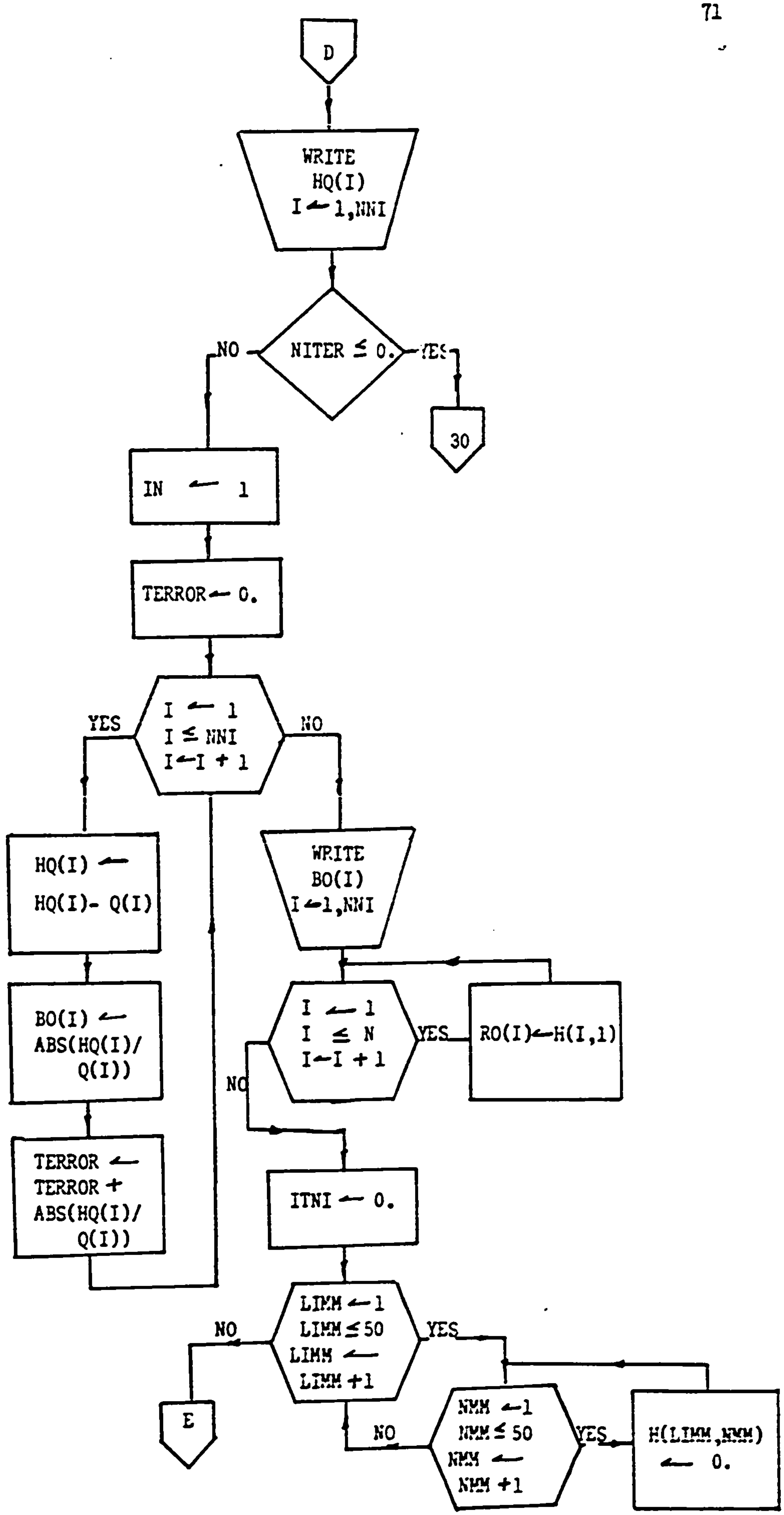
MAIN PROGRAM

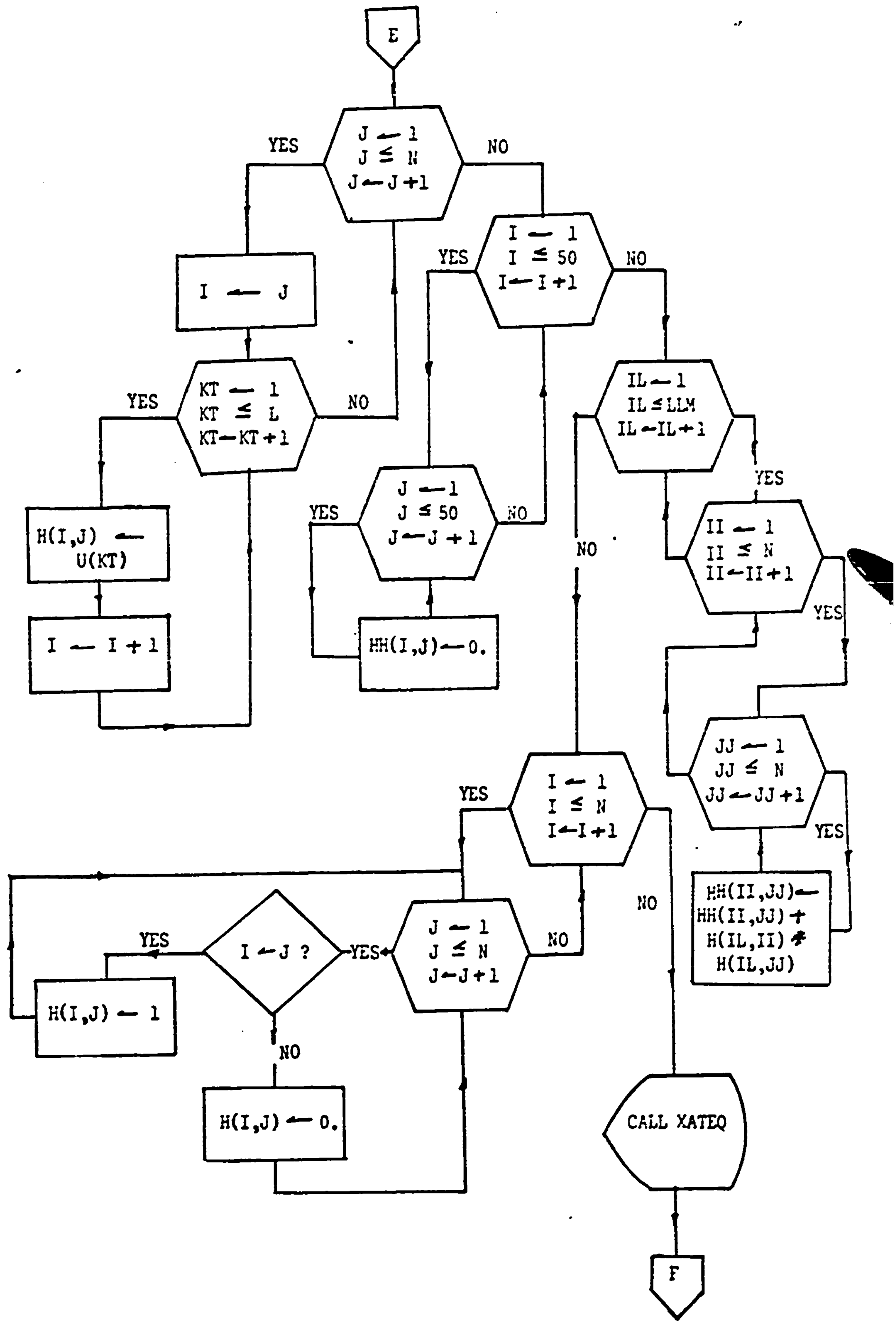


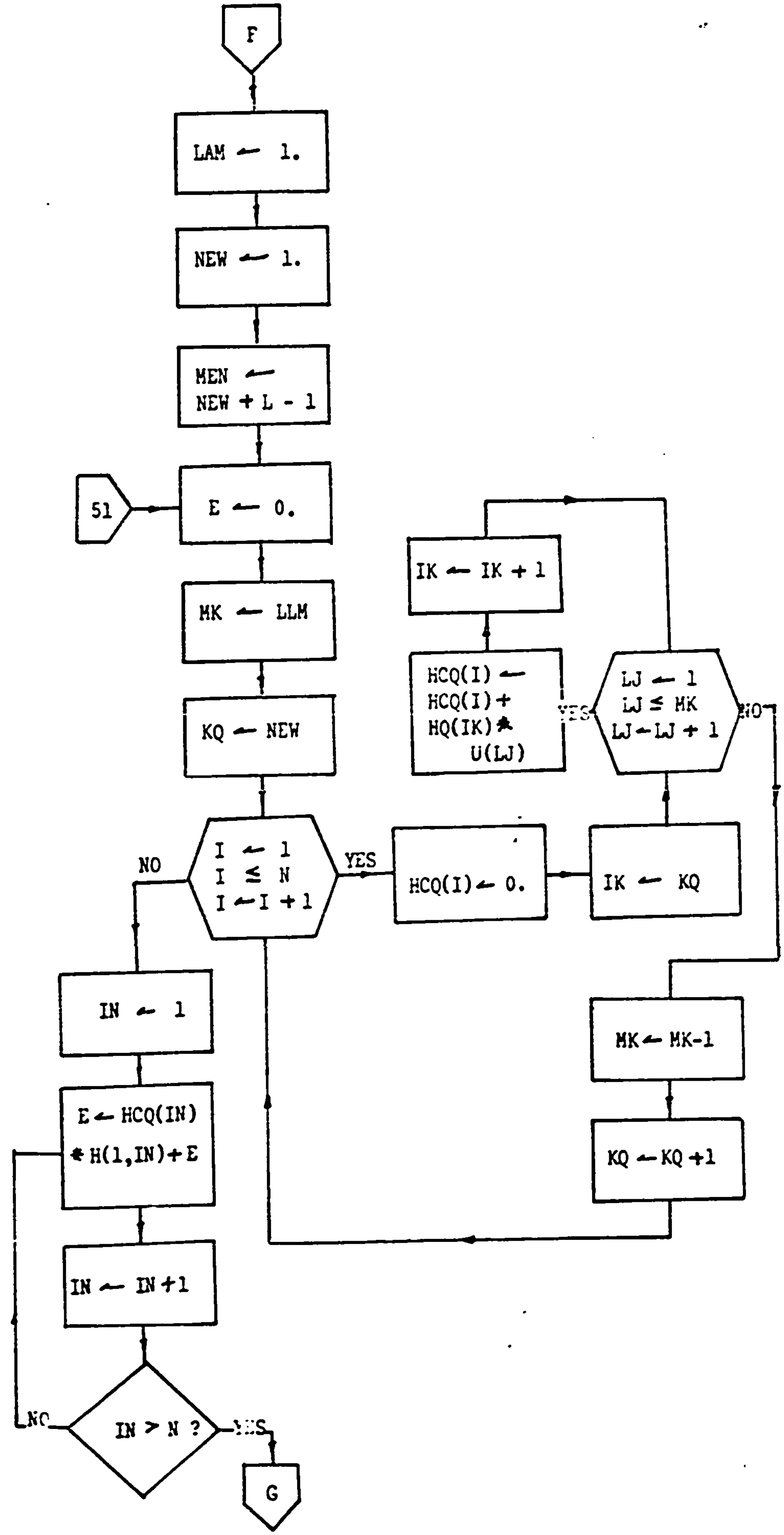

A
I ← 1
I ≤ 50
I ← I + 1
YES
NO
CALL MATINP
J ← 1
J ≤ 50
J ← J + 1
YES
NO
HH(I,J) ← 0.
WRITE
Q(I)
I ← 1,
NNI
NN(LL) ← N
NI(LL) ← NNI
WRITE
TQQ
WRITE
LL
ITER
WRITE
RO(I)
I ← 1,N
TRO ← 0.
HA(I,J) ← H(I,J)
J ← 1
J ≤ L
J ← J + 1
NO
YES
I ← 1
I ≤ N
I ← I + 1
NO
YES
TRO ←
TRO + RN(I)
I ← 1
I ≤ NNI
I ← I + 1
YES
NO
B

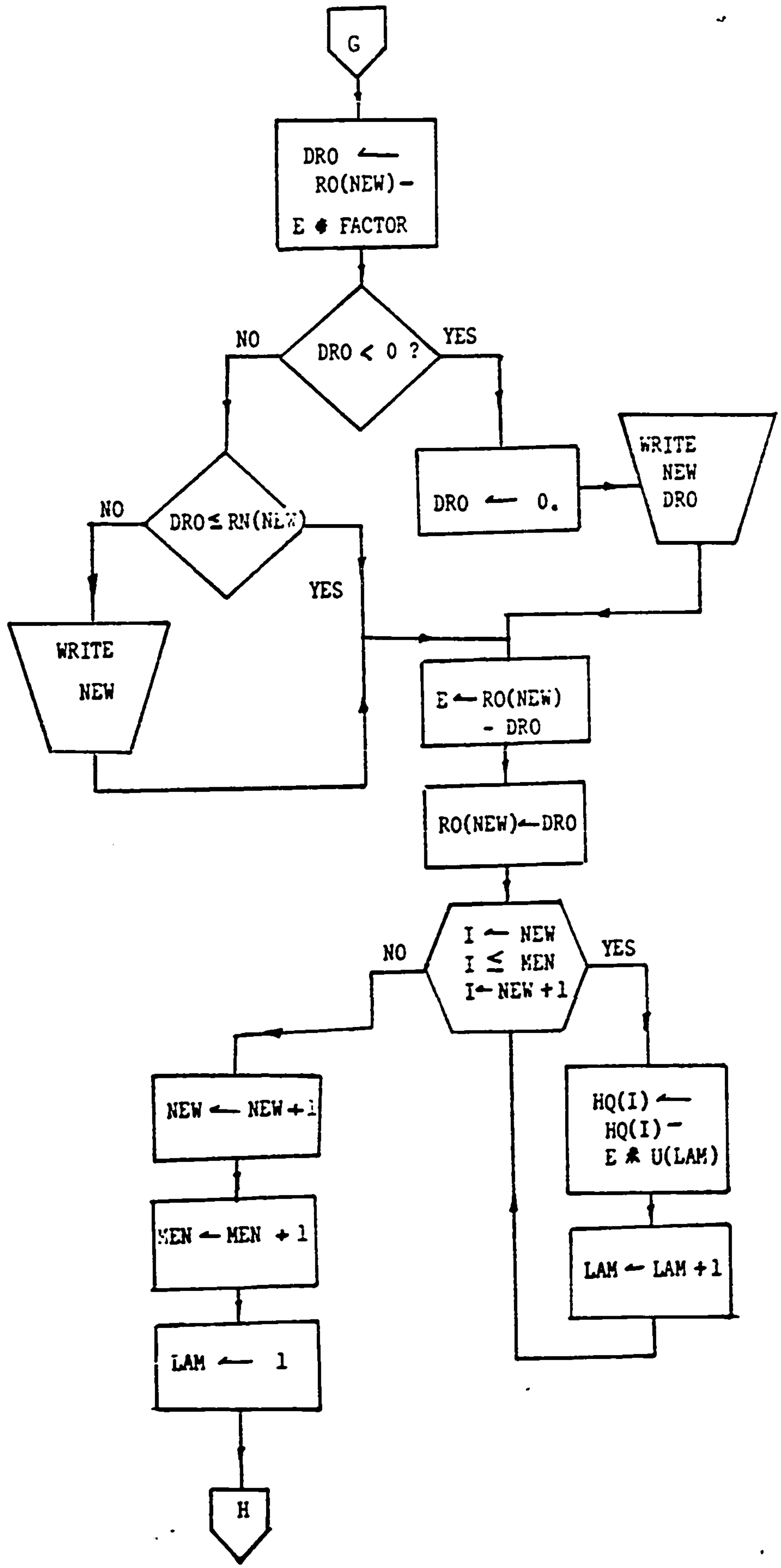

B
CALL BAK
JK ← K+1
I ← 1
I ≤ NNI
I ← I + 1
YES
NO
H(I,J) ← Q(I)
IL ← 1
IL ≤ NNI
IL ← IL + 1
NO
YES
77
CALL XATEQ
NO
I ← 1
I ≤ JK
I ← I + 1
YES
I ← 1
I ≤ K
I ← I + 1
NO
YES
J ← 1
J ≤ JK
J ← J + 1
NO
YES
CALL SNOW
U(I) ← HH(I,JK)
HH(I,J) ←
HH(I,J) +
H(IL,I). *
H(IL,J)
YES
U(I) ≥ 0.
UHT ← 0.
NO
I ← 1
I ≤ L
I ← I + 1
U(I) ← RO(I)
YES
U(I) ← 0.
C

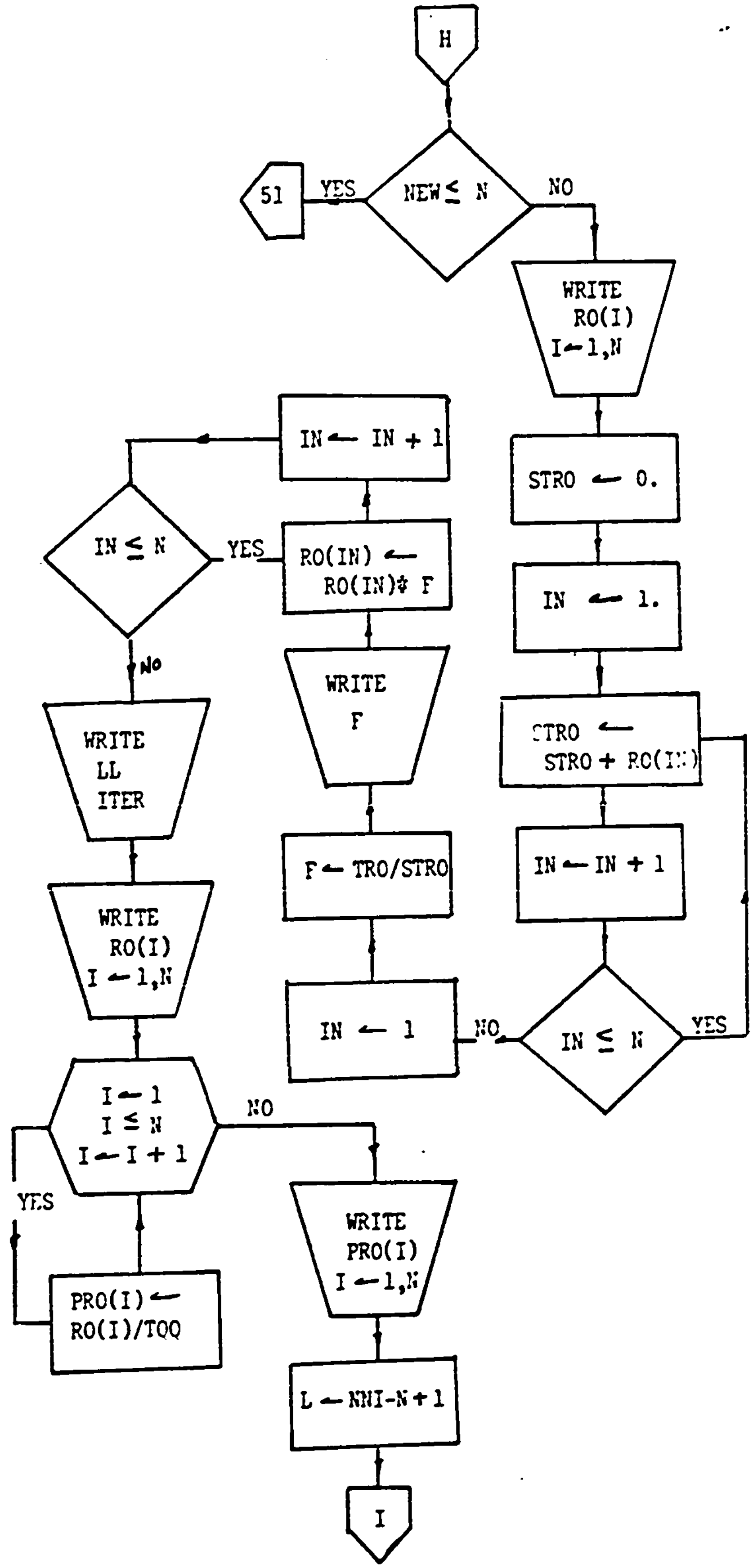

C
WRITE
ITER
WRITE
U(I)
I ← 1,L
YES
I ← 1
I ≤ L
I ← I + 1
NO
UHT ←
UHT + RO(I)
WRITE
UHT
TERR ← 0.
LM ≤ L
LM ← LM + 1
ITNI ← 0.
YES
HQ(I) ←
HQ(I) + U(LM)
* HA(I,LM)
NNI ← NI(LL)
LM ← 1
N ← NN(LL)
NO
HQ(I) ← 0.
YES
I ← 1
I ≤ NNI
I ← I + 1
NO
WRITE
LL
ITER
D

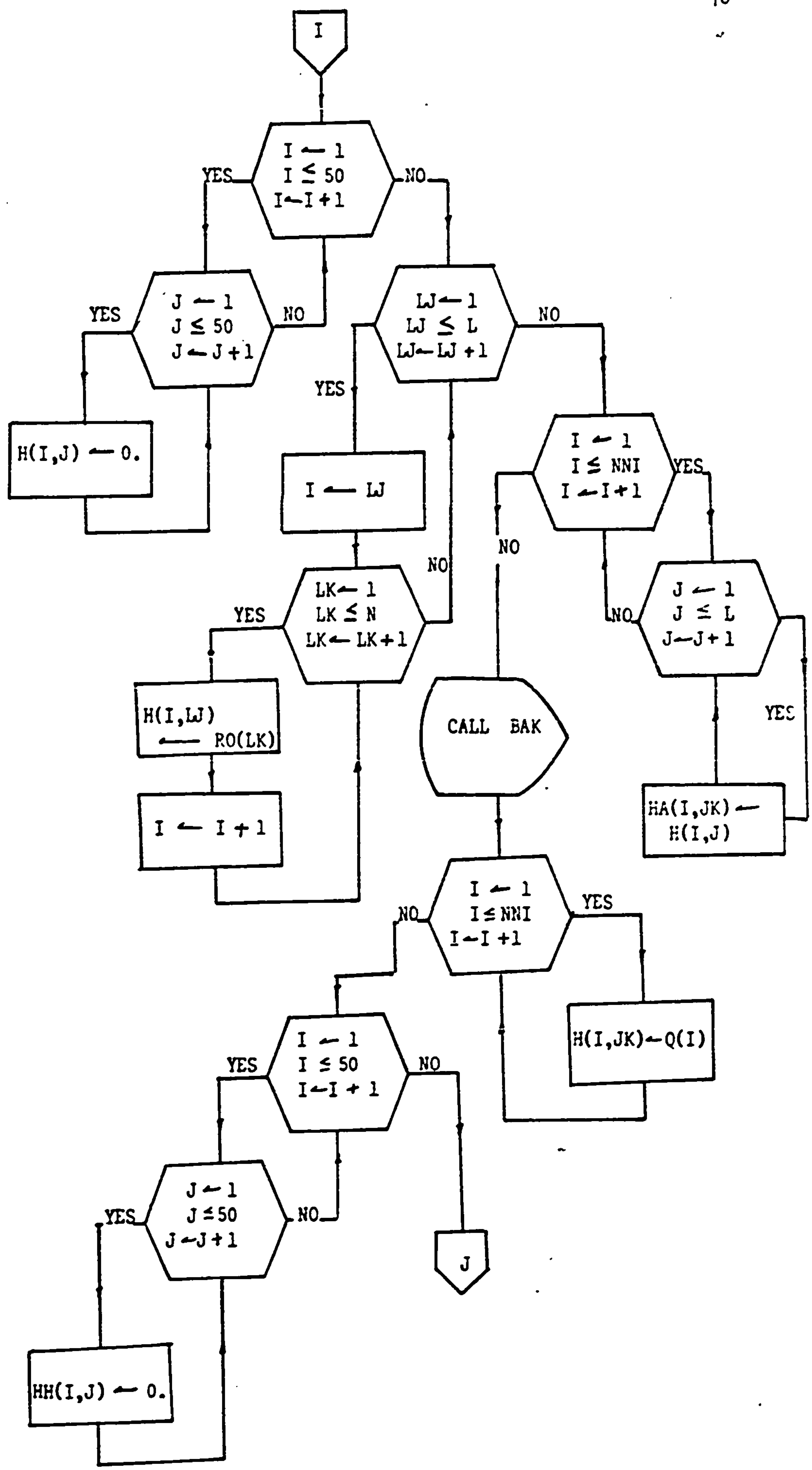

D
WRITE
HQ(I)
I ← 1,NNI
NITER ≤ 0.
NO
YES
30
IN ← 1
TERROR ← 0.
YES
I ← 1
I ≤ NNI
I←I + 1
NO
HQ(I) ←
HQ(I)- Q(I)
WRITE
BO(I)
I←1,NNI
BO(I) ←
ABS(HQ(I)/
Q(I))
I ← 1
I ≤ N
I←I + 1
YES
RO(I)←H(I,1)
NO
TERROR ←
TERROR +
ABS(HQ(I)/
Q(I))
ITNI ← 0.
NO
LIMM ←1
LIMM≤50
LIMM ←
LIMM +1
YES
E
NO
NMM ←1
NMM≤ 50
NMM ←
NMM +1
YES
H(LIMM,NMM)
← 0.

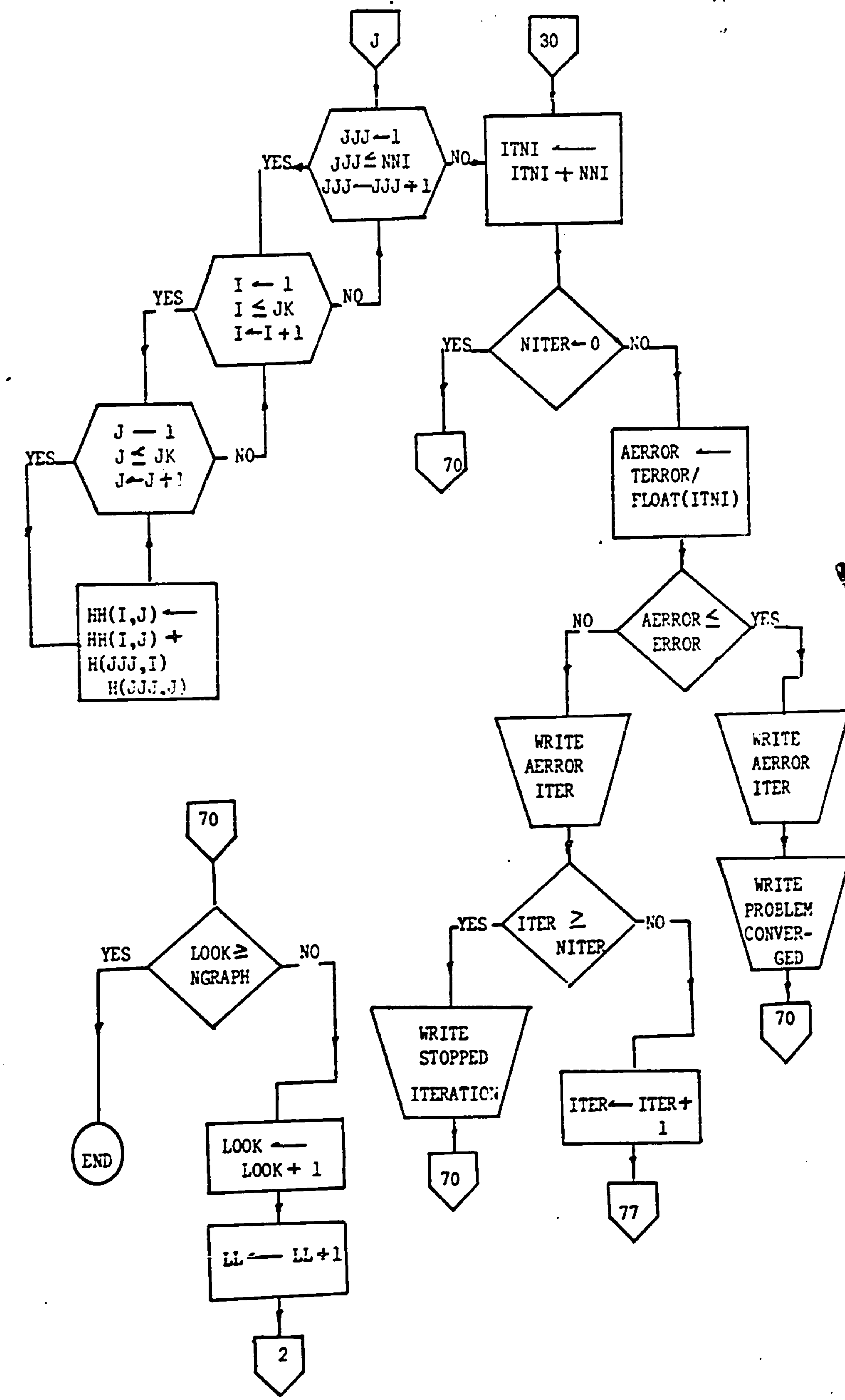

E
J ← 1
J ≤ N
J ← J + 1
YES
NO
I ← J
I ← 1
I ≤ 50
I ← I + 1
YES
NO
KT ← 1
KT ≤ L
KT ← KT + 1
YES
NO
IL ← 1
IL ≤ LLM
IL ← IL + 1
J ← 1
J ≤ 50
J ← J + 1
YES
NO
H(I,J) ← U(KT)
I ← I + 1
HH(I,J) ← 0.
NO
YES
II ← 1
II ≤ N
II ← II + 1
YES
JJ ← 1
JJ ≤ N
JJ ← JJ + 1
YES
I ← 1
I ≤ N
I ← I + 1
YES
NO
HH(II,JJ) ← HH(II,JJ) + H(IL,II) * H(IL,JJ)
I ← J ?
YES
YES
J ← 1
J ≤ N
J ← J + 1
NO
H(I,J) ← 1
NO
H(I,J) ← 0.
CALL XATEQ
F


F
LAM ← 1.
NEW ← 1.
MEN ← NEW + L - 1
51
E ← 0.
MK ← LLM
KQ ← NEW
I ← 1
I ≤ N
I ← I + 1
NO
YES
HCQ(I) ← 0.
IK ← KQ
LJ ← 1
LJ ≤ MK
LJ ← LJ + 1
YES
NO
HCQ(I) ← HCQ(I) + HQ(IK) * U(LJ)
IK ← IK + 1
MK ← MK-1
KQ ← KQ +1
IN ← 1
E ← HCQ(IN) * H(1,IN) + E
IN ← IN +1
IN > N ?
NO
YES
G

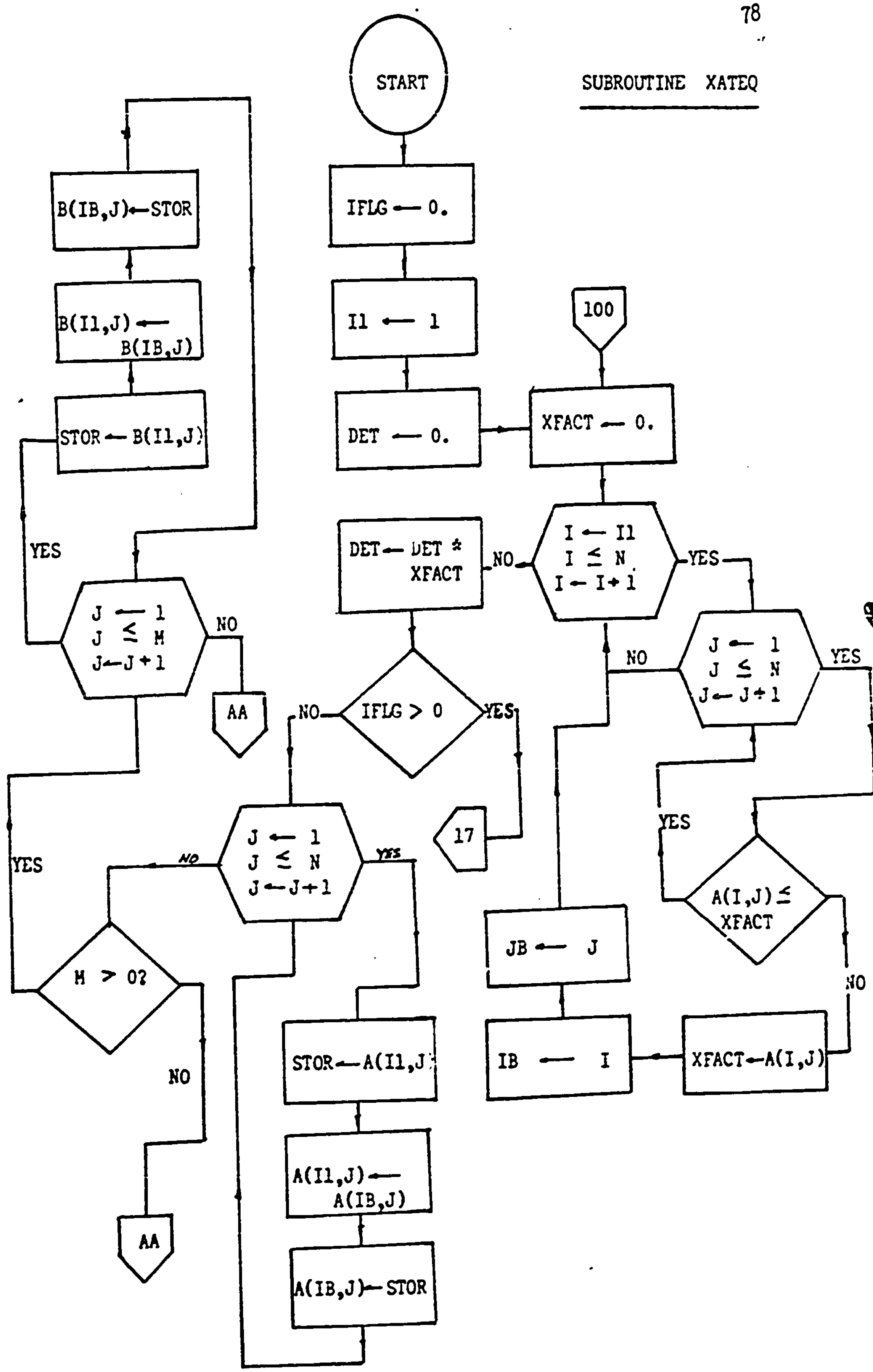

G
DRO ←
RO(NEW) –
E * FACTOR
DRO < 0 ?
NO
YES
DRO ← 0.
WRITE
NEW
DRO
DRO ≤ RN(NEW)
NO
YES
WRITE
NEW
E ← RO(NEW)
– DRO
RO(NEW) ← DRO
I ← NEW
I ≤ MEN
I ← NEW + 1
NO
YES
NEW ← NEW + 1
MEN ← MEN + 1
LAM ← 1
HQ(I) ←
HQ(I) –
E * U(LAM)
LAM ← LAM + 1
H

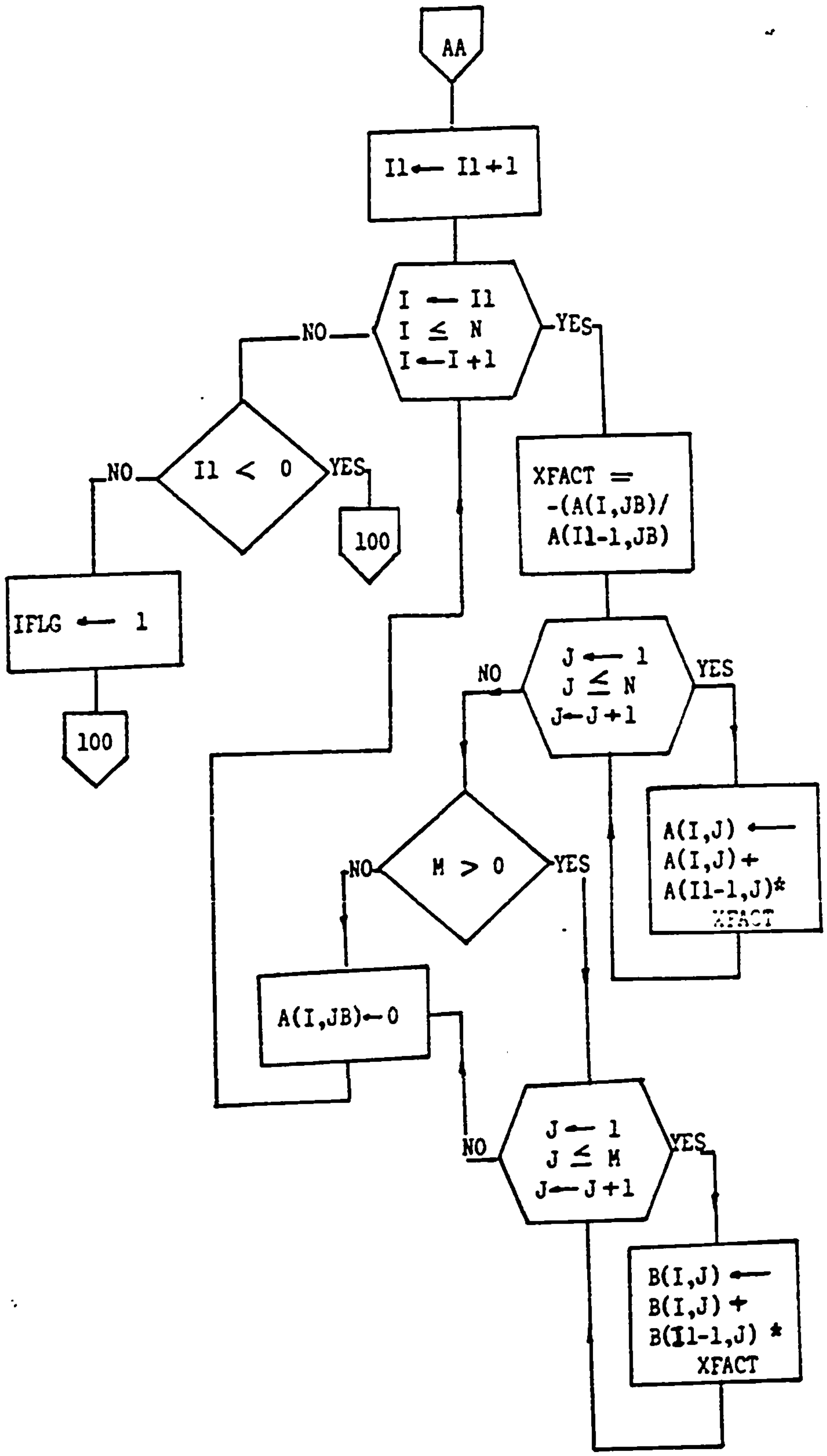

H
NEW ≤ N
YES
51
NO
WRITE
RO(I)
I ← 1,N
STRO ← 0.
IN ← 1.
STRO ←
STRO + RO(IN)
IN ← IN + 1
IN ≤ N
YES
NO
IN ← 1
F ← TRO/STRO
WRITE
F
RO(IN) ←
RO(IN)* F
IN ← IN + 1
IN ≤ N
YES
NO
WRITE
LL
ITER
WRITE
RO(I)
I ← 1,N
I ← 1
I ≤ N
I ← I + 1
YES
NO
PRO(I) ←
RO(I)/TOQ
WRITE
PRO(I)
I ← 1,N
L ← NNI-N + 1
I


I
I ← 1
I ≤ 50
I ← I + 1
YES
NO
J ← 1
J ≤ 50
J ← J + 1
YES
NO
H(I,J) ← 0.
LJ ← 1
LJ ≤ L
LJ ← LJ + 1
YES
NO
I ← LJ
LK ← 1
LK ≤ N
LK ← LK + 1
YES
NO
H(I,LJ) ← RO(LK)
I ← I + 1
I ← 1
I ≤ NNI
I ← I + 1
YES
NO
J ← 1
J ≤ L
J ← J + 1
YES
NO
HA(I,JK) ← H(I,J)
CALL BAK
I ← 1
I ≤ NNI
I ← I + 1
YES
NO
H(I,JK) ← Q(I)
I ← 1
I ≤ 50
I ← I + 1
YES
NO
J ← 1
J ≤ 50
J ← J + 1
YES
NO
HH(I,J) ← 0.
J

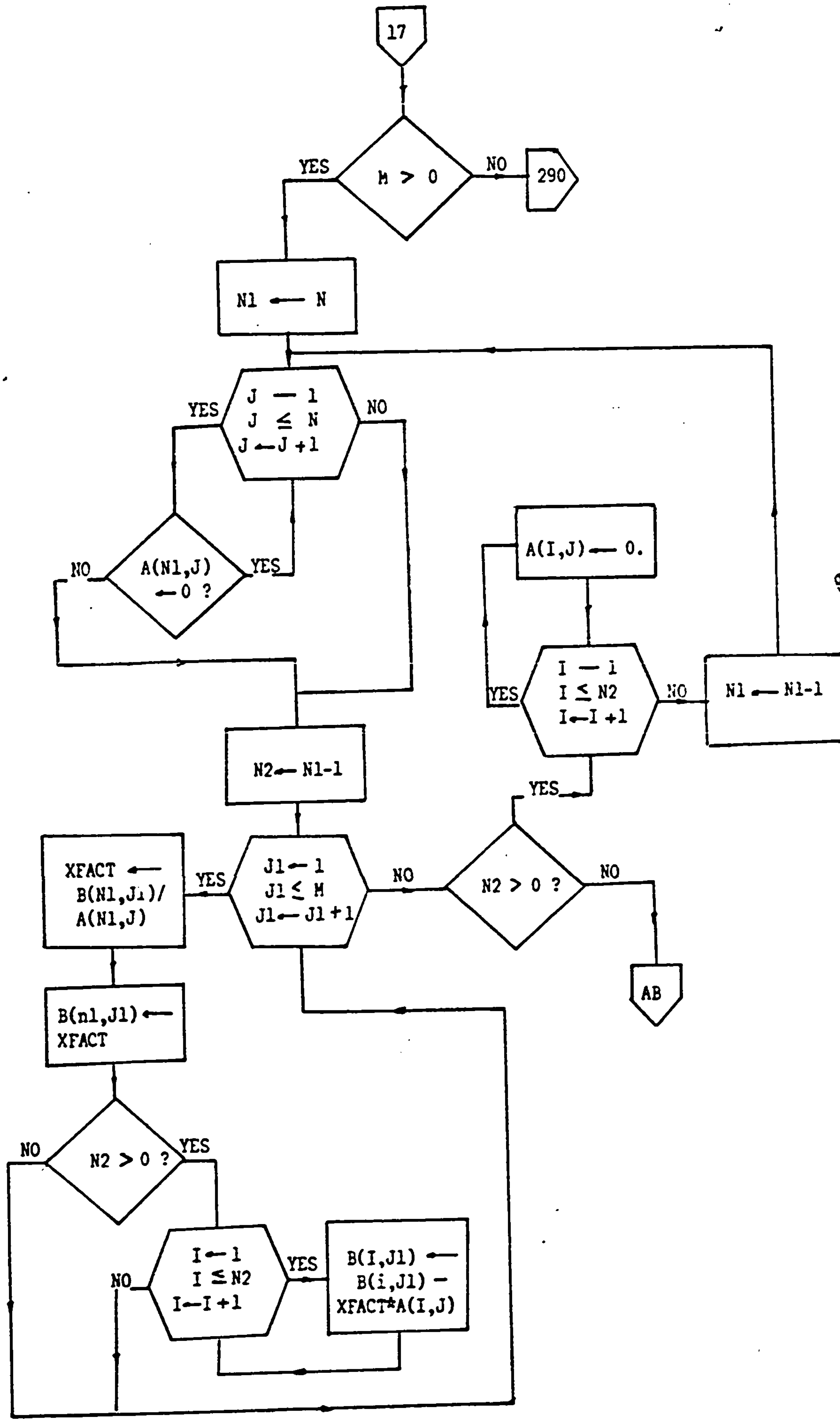

J
30
JJJ←1
JJJ≤NNI
JJJ←JJJ+1
YES
NO
ITNI ←
ITNI + NNI
I ← 1
I ≤ JK
I←I+1
YES
NO
NITER←0
YES
NO
70
J ← 1
J ≤ JK
J←J+1
YES
NO
AERROR ←
TERROR/
FLOAT(ITNI)
HH(I,J) ←
HH(I,J) +
H(JJJ,I)
H(JJJ,J)
AERROR ≤
ERROR
NO
YES
WRITE
AERROR
ITER
WRITE
AERROR
ITER
70
LOOK≥
NGRAPH
YES
NO
ITER ≥
NITER
YES
NO
WRITE
PROBLEM
CONVER-
GED
70
WRITE
STOPPED
ITERATION
END
LOOK ←
LOOK + 1
ITER← ITER+
1
70
77
LL← LL+1
2

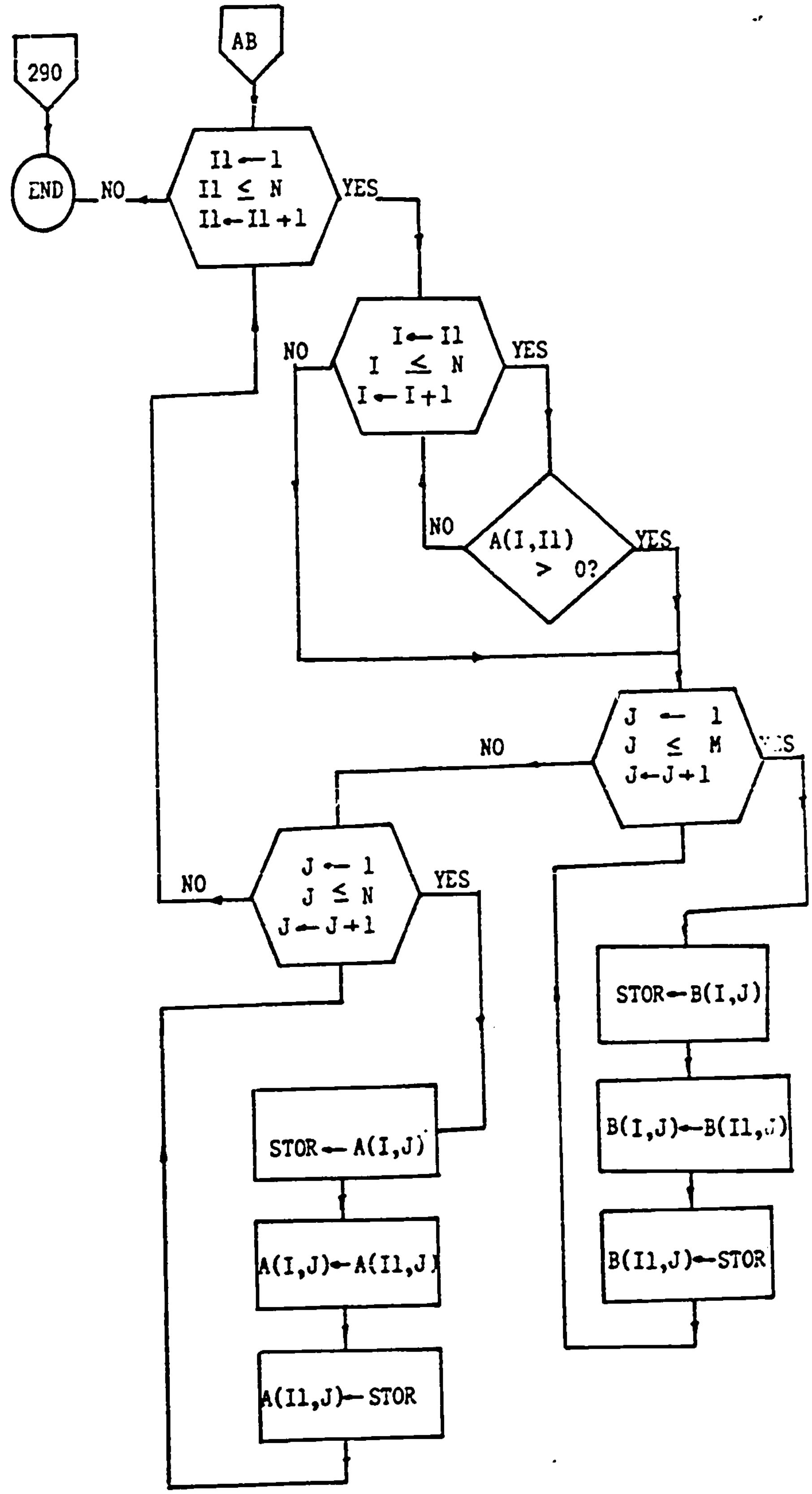
SUBROUTINE XATEQ



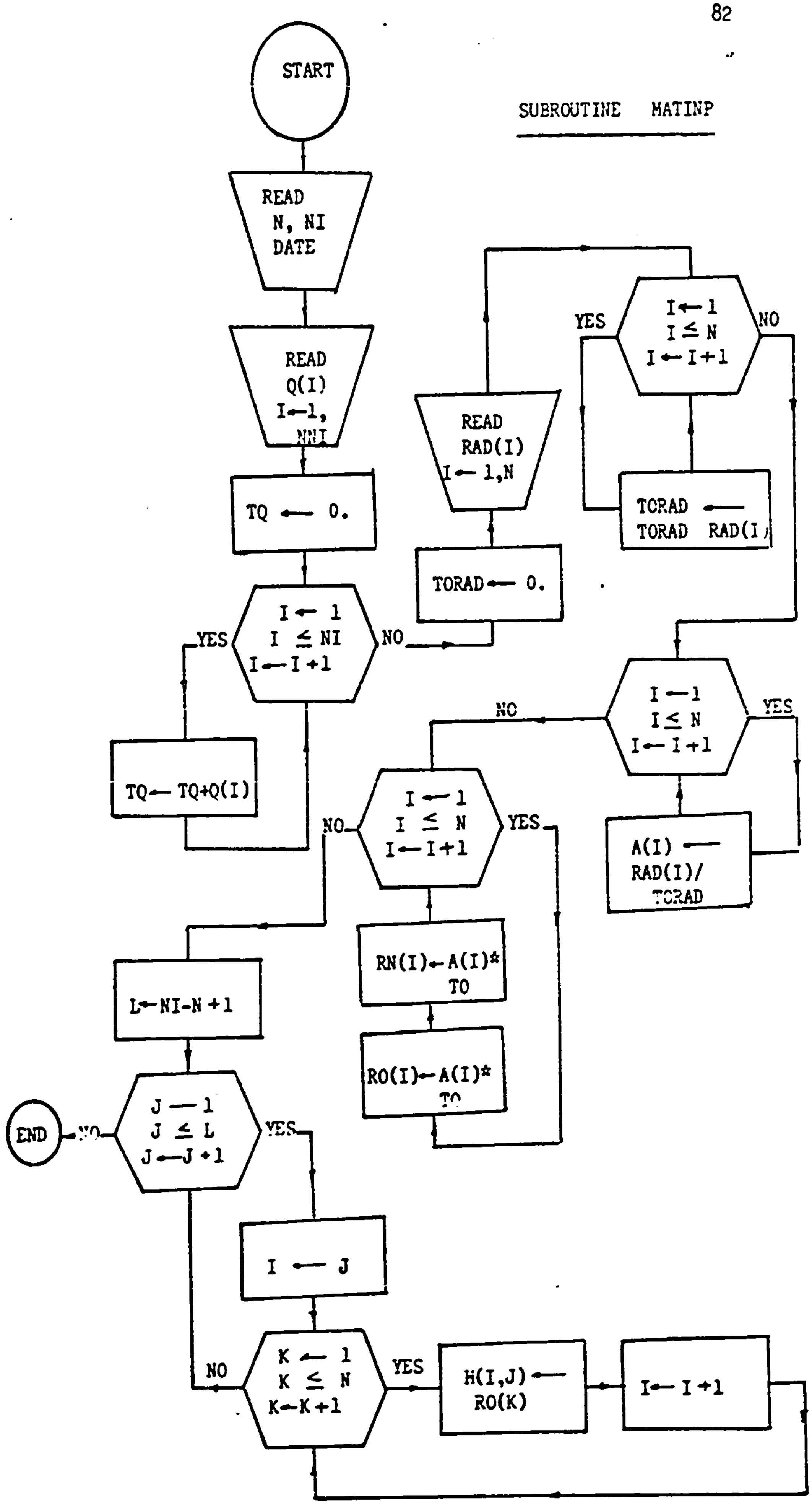


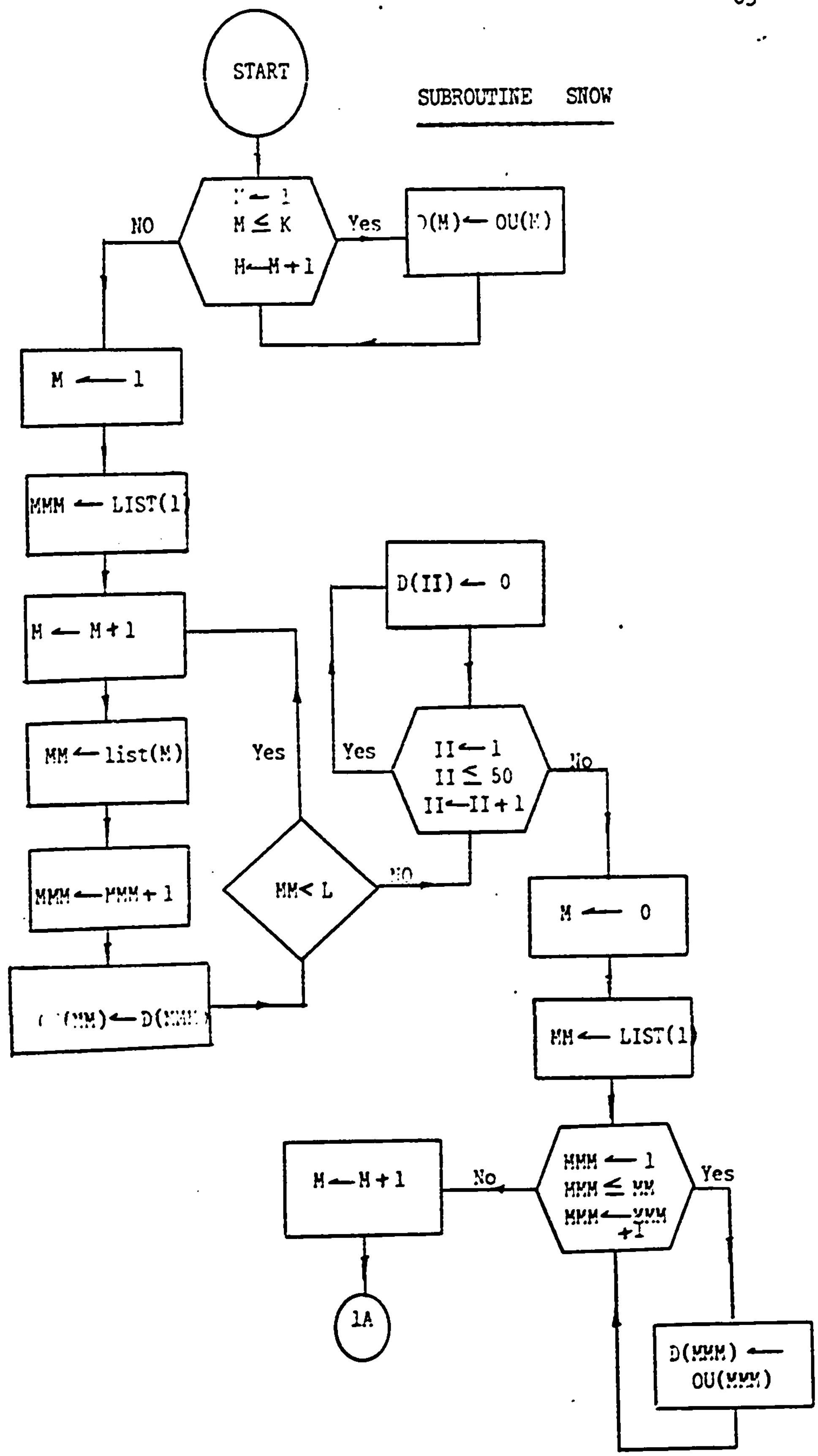
AA
I1 ← I1 + 1
I ← I1
I ≤ N
I ← I + 1
NO
YES
I1 < 0
NO
YES
100
IFLG ← 1
100
XFACT =
-(A(I,JB)/
A(I1-1,JB)
J ← 1
J ≤ N
J ← J + 1
NO
YES
A(I,J) ←
A(I,J) +
A(I1-1,J)*
XFACT
M > 0
NO
YES
A(I,JB) ← 0
J ← 1
J ≤ M
J ← J + 1
NO
YES
B(I,J) ←
B(I,J) +
B(I1-1,J) *
XFACT

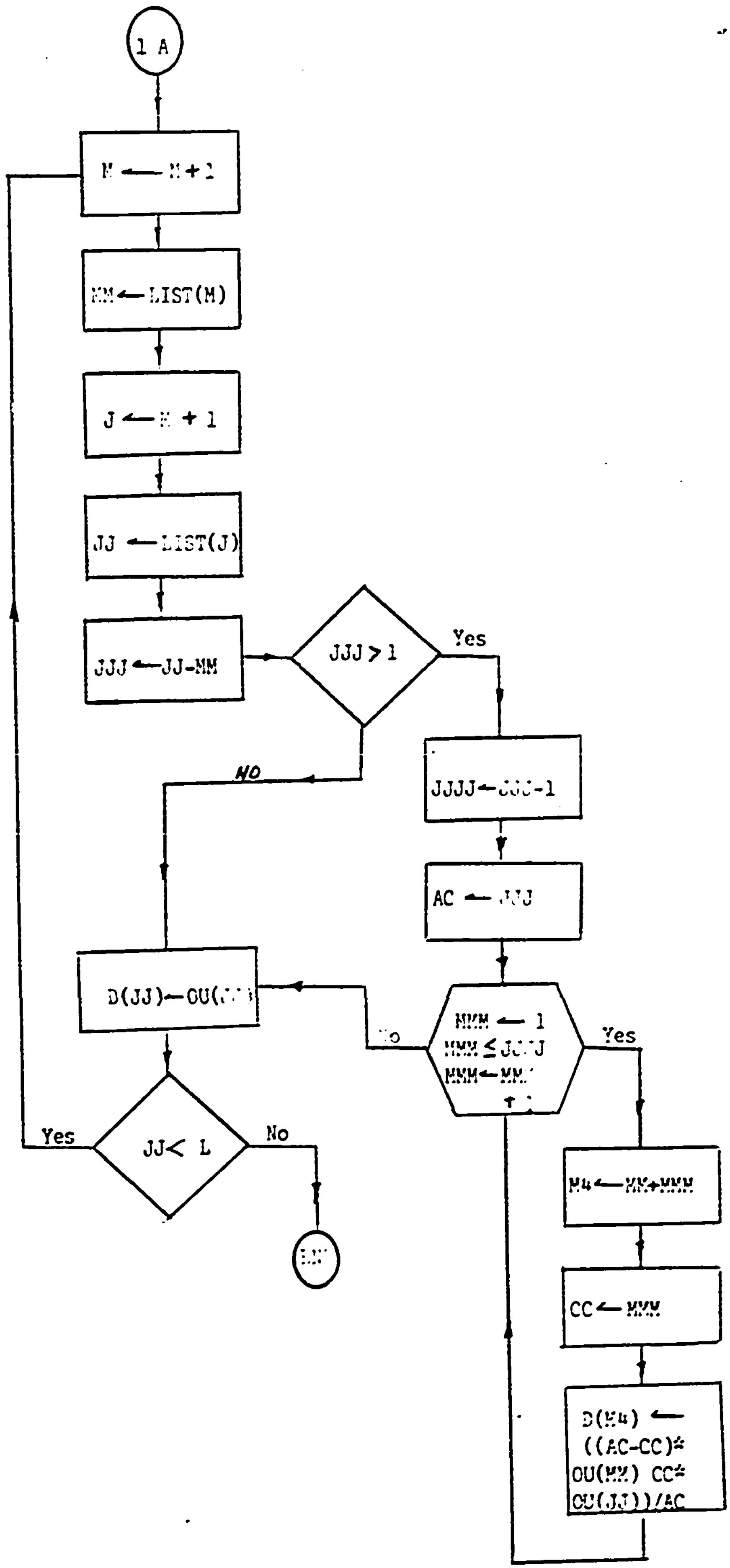

17
M > 0
YES
NO
290
N1 ← N
J ← 1
J ≤ N
J ← J + 1
YES
NO
A(N1,J)
← 0 ?
NO
YES
A(I,J) ← 0.
I ← 1
I ≤ N2
I ← I + 1
YES
NO
N1 ← N1-1
N2 ← N1-1
YES
XFACT ←
B(N1,J1)/
A(N1,J)
YES
J1 ← 1
J1 ≤ M
J1 ← J1 + 1
NO
N2 > 0 ?
NO
AB
B(n1,J1) ←
XFACT
NO
N2 > 0 ?
YES
I ← 1
I ≤ N2
I ← I + 1
NO
YES
B(I,J1) ←
B(i,J1) −
XFACT*A(I,J)


AB
290
END
NO
I1 ← 1
I1 ≤ N
I1 ← I1 + 1
YES
NO
I ← I1
I ≤ N
I ← I + 1
YES
NO
A(I,I1) > 0?
YES
NO
J ← 1
J ≤ M
J ← J + 1
NO
J ← 1
J ≤ N
J ← J + 1
YES
STOR ← B(I,J)
STOR ← A(I,J)
A(I,J) ← A(I1,J)
A(I1,J) ← STOR
B(I1,J) ← STOR

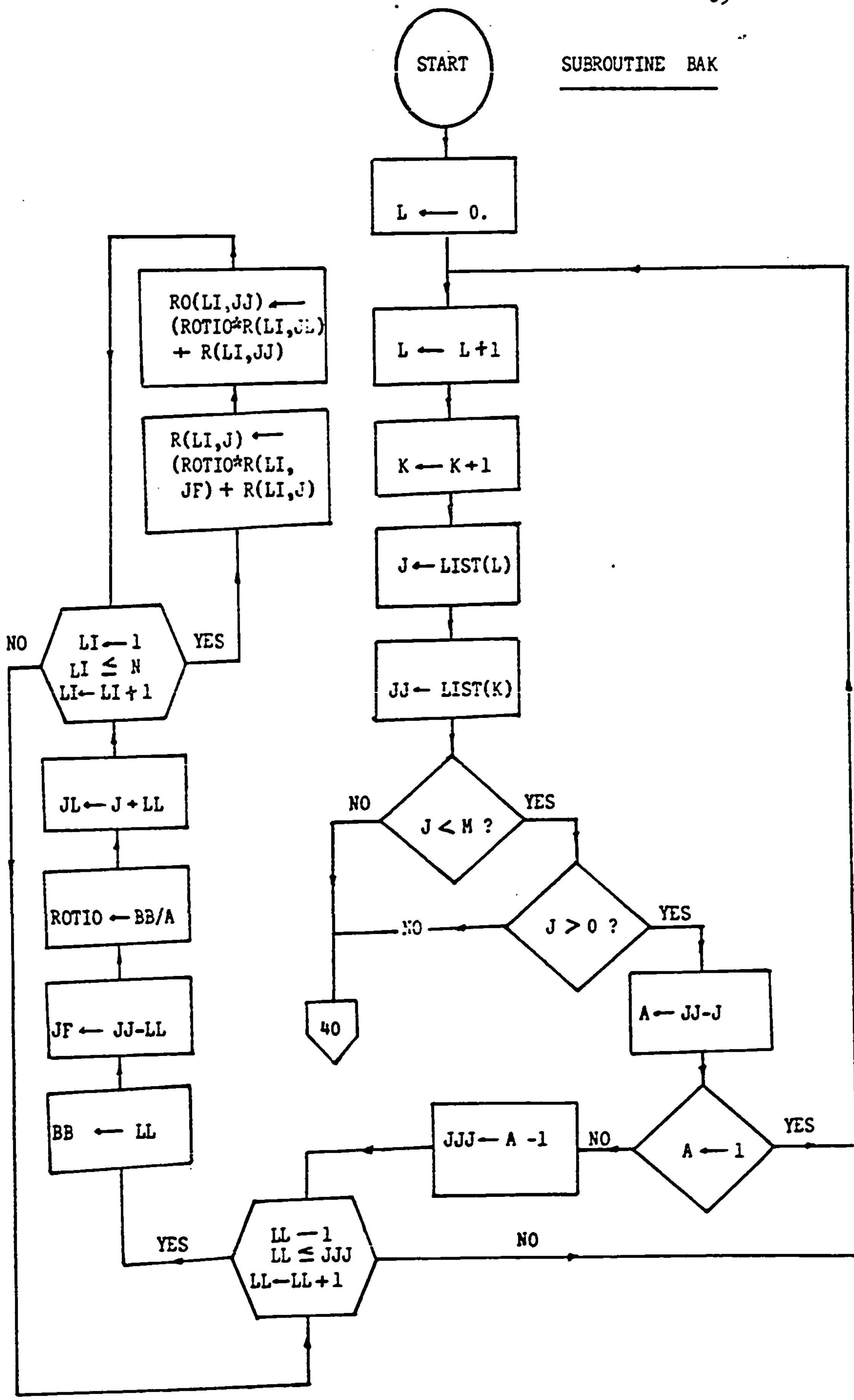
SUBROUTINE MATINP

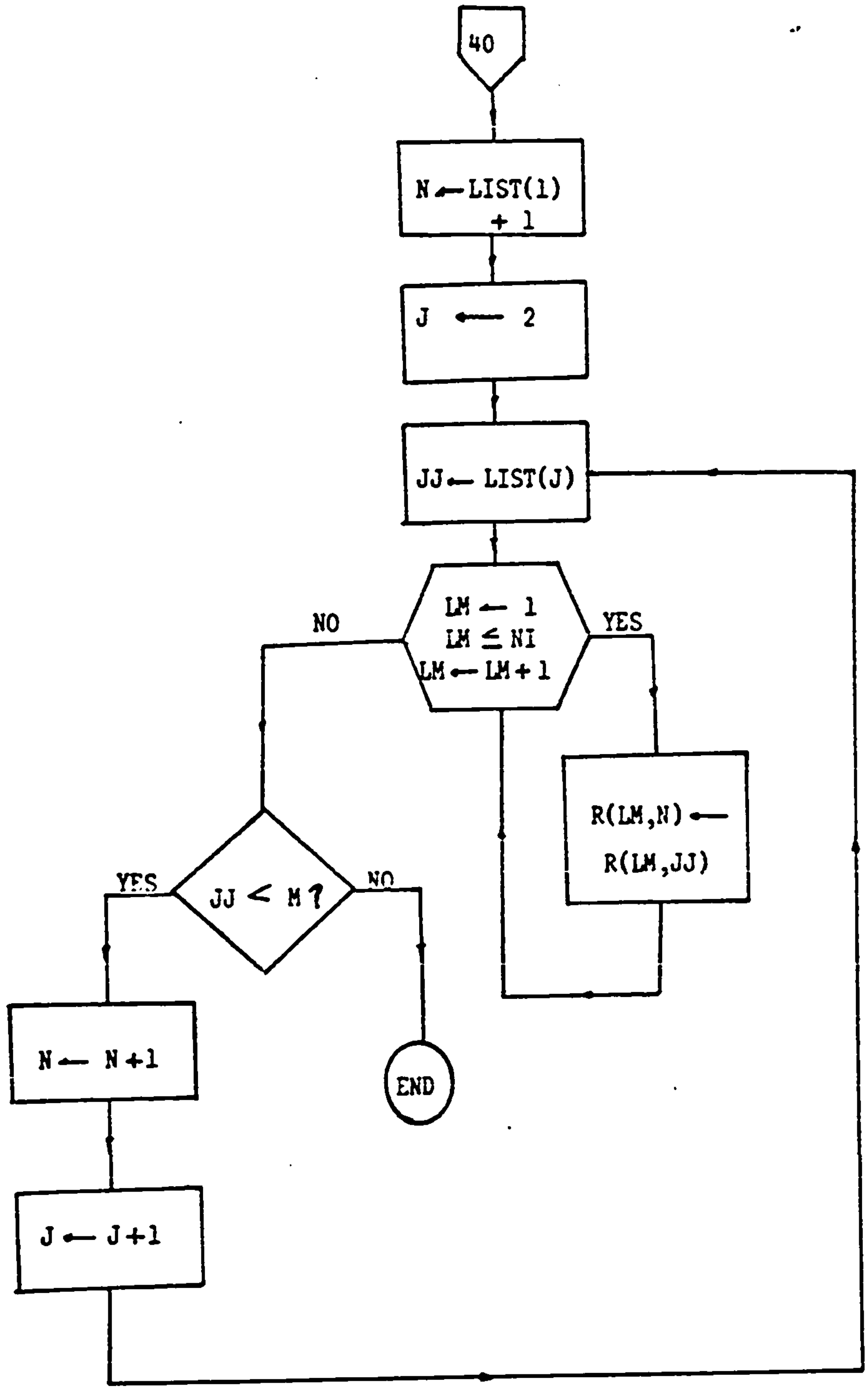



START
SUBROUTINE SNOW
M ≤ K
H←M+1
NO
Yes
OU(M)
M ← 1
MMM ← LIST(1)
M ← M+1
MM ← list(M)
MMM ← MMM+1
D(MMM)
Yes
MM< L
D(II) ← 0
Yes
II←1
II ≤ 50
II←II+1
No
M ← 0
MM ← LIST(1)
MMM ← 1
MMM ≤ MM
+1
Yes
No
M←M+1
1A
D(MMM) ←
OU(MMM)


1 A
JJ ←LIST(J)
JJJ ← JJ-MM
JJJ > 1
Yes
NO
AC ←
D(JJ)← OU(
Yes
JJ< L
No
Yes
CC ← MMM
((AC-CC)*

SUBROUTINE BAK





40
N ← LIST(1) + 1
J ← 2
JJ ← LIST(J)
LM ← 1
LM ≤ NI
LM ← LM + 1
NO
YES
R(LM,N) ←
R(LM,JJ)
JJ < M ?
YES
NO
N ← N+1
END
J ← J+1

APPENDIX B

HYDROMETEOROLOGICAL DATA

Time	Temperature F	Relative Humidity %	Net Radiation ly.	Q x 10 [2] c.f.s.	Wind-Velocity m.p.h.
28-3-70					
2 a.m.	26	96	-7	5.4	8
4 a.m.	27	96	-2	4.85	12
6 a.m.	27	96	-2	4.20	7
8 a.m.	30	96	0	3.80	11
10 a.m.	31	96	7	4.40	11
12noon	35	68	11	5.60	15
14 p.m.	37	70	16	7.40	20
16 p.m.	34	96	7	9.80	17
18 p.m.	29	89	3	9.40	17
20 p.m.	27	88	-4	7.60	14
22 p.m.	23	68	-8	6.40	9
24 p.m.	22	64	-8	6.00	8
29-3-70					
2 a.m.	16	68	-8	5.60	11
4 a.m.	10	87	-8	5.00	6
6 a.m.	3	94	-8	4.42	8
8 a.m.	-1	95	-4	4.00	4
10 a.m.	10	70	15	5.6	7
12 noon	17	49	22	10.30	6

(CONTINUED)

29-3-70					
14 a.m.	23	45	15	14.00	4
16 a.m.	22	42	6	15.00	9
18 a.m.	23	40	-10	13.70	9
20 a.m.	14	50	-10	11.70	11
22 p.m.	12	60	-12	10.20	11
24 p.m.	8	78	-10	9.40	8
30-3-70					
2 a.m.	3	90	-10	8.70	10
4 a.m.	1	96	-10	7.65	10
6 a.m.	-1	96	-9	6.82	9
8 a.m.	1	95	-6	6.00	7
10 a.m.	10	76	8	7.20	10
12noon	21	47	20	10.00	13
14 p.m.	23	45	25	13.00	9
16 p.m.	26	47	20	19.05	5
18 p.m.	25	56	7	18.20	12
20 p.m.	19	60	-5	12,40	9
22 p.m.	16	62	-8	10.30	7
24 p.m.	14	64	-8	8.30	12

(CONTINUED)

31-3-70					
2 a.m.	13	69	-8	7.50	8
4. a.m.	12	68	-8	6.50	9
6 a.m.	10	76	-8	5.70	8
8 a.m.	6	90	-4	5.40	8
10 a.m.	17	64	-4	5.20	4
12noon	25	46	5	5.44	-
14 p.m.	33	38	8	5.70	4
16 p.m.	35	38	10	8.30	4
18 p.m.	33	41	3	9.50	3
20 p.m.	30	65	-4	7.60	5
22 p.m.	24	78	-6	7.00	3
24 p.m.	23	87	-4	6.60	4
1-4-70					
2 a.m.	22	88	-8	5.90	8
4 a.m.	26	71	-10	5.15	13
6 a.m.	19	87	-10	4.80	8
8 a.m.	18	90	-8	4.50	8
10 a.m.	27	67	7	5.57	14
12noon	32	57	20	6.00	12

(CONTINUED)

1-4-70					
14 p.m.	36	40	25	10.60	11
16 p.m.	38	36	16	12.86	12
18 p.m.	38	45	8	11.27	6
20 p.m.	31	66	-5	9.20	5
22 p.m.	27	83	-6	7.70	4
24 p.m.	28	85	-3	6.00	4
2-4-70					
2 a.m.	30	70	-2	4.00	11
4 a.m.	30	74	-2	3.30	12
6 a.m.	30	70	-1	2.60	16
8 a.m.	29	85	0	2.25	26
10 a.m.	29	96	0	1.95	28
12noon	30	96	2	1.80	26
14 p.m.	30	96	0	2.00	32
16 p.m.	30	96	0	2.20	21
18 a.m.	32	96	0	2.40	22
20 p.m.	31	96	0	2.37	17
22 p.m.	30	96	0	2.20	7
24 p.m.	29	96	0	2.10	11

(CONTINUED)

3-4-70					
2 a.m.	28	91	0	1.95	14
4 a.m.	27	94	0	1.82	15
6 a.m.	27	92	0	1.71	18
8 a.m.	26	86	-2	1.62	19
10 a.m.	26	71	-4	1.50	19
12noon	27	63	-1	1.40	18
14 p.m.	31	56	2	2.00	24
16 p.m.	34	44	-1	2.30	26
18 p.m.	36	34	-2	2.50	20
20 p.m.	35	33	-6	2.40	18
22 p.m.	33	41	-10	2.30	9
24 p.m.	28	44	-9	2.10	9
4-4-70					
2 a.m.	28	51	-6	1.95	9
4 a.m.	24	57	-7	1.82	11
6 a.m.	21	76	-7	1.70	9
8 a.m.	12	92	-6	1.60	6
10 a.m.	23	67	5	1.90	9
12noon	31	44	11	2.10	12

(CONTINUED)

4-4-70					
14 p.m.	33	35	10	4.25	17
16 p.m.	34	28	-2	4.27	20
18 p.m.	35	29	-3	3.80	16
20 p.m.	33	33	-7	3.00	9
22 p.m.	24	54	-10	2.50	6
24 p.m.	14	74	-8	2.20	5
5-4-70					
2 a.m.	14	81	-9	1.95	6
4 a.m.	17	89	-6	1.80	12
6 a.m.	22	78	-10	1.65	16
8 a.m.	20	73	-9	1.50	14
10 a.m.	20	75	7	4.20	20
12noon	22	57	22	5.17	21
14 p.m.	25	46	13	17.60	20
16 p.m.	29	40	3	20.80	16
18 p.m.	32	37	3	15.70	12
20 p.m.	31	44	-5	8.00	4
22 p.m.	18	73	-8	3.10	6
24 p.m.	17	77	-5	2.00	7

(CONTINUED)

6-4-70					
2 a.m.	21	91	-2	1.85	4
4 a.m.	22	93	-3	1.72	6
6 a.m.	21	94	-5	1.60	9
8 a.m.	20	93	-3	1.52	12
10 a.m.	23	85	4	2.91	13
12noon	29	57	8	4.56	12
14 p.m.	38	49	23	6.60	14
16 p.m.	38	56	15	9.20	15
18 p.m.	37	62	3	9.79	12
20 p.m.	36	83	-1	7.00	7
22 p.m.	34	98	0	3.20	5
24 p.m.	31	98	-2	1.90	4
7-4-70					
2 a.m.	28	98	-5	1.50	3
4 a.m.	21	98	-7	1.42	-
6 a.m.	19	98	-3	1.30	4
8 a.m.	22	98	0	1.20	9
10 a.m.	31	77	4	1.92	11
12noon	36	53	23	2.92	13

(CONTINUED)

7-4-70					
14 p.m.	39	46	28	5.53	10
16 p.m.	40	41	16	9.50	13
18 p.m.	39	40	3	11.18	10
20 p.m.	34	79	-5	7.80	6
22 p.m.	30	80	-9	6.30	5
24 p.m.	32	77	-6	3.30	8
8-4-70					
2 a.m.	35	72	0	2.00	8
4 a.m.	34	76	0	1.85	9
6 a.m.	35	85	0	1.70	11
8 a.m.	35	95	0	4.09	17
10 a.m.	38	87	9	5.49	12
12noon	45	68	14	6.48	12
14 p.m.	48	62	20	9.27	11
16 p.m.	51	60	21	16.57	10
18 p.m.	50	54	14	13.56	13
20 p.m.	44	77	1	9.86	9
22 p.m.	44	74	-2	5.95	9
24 p.m.	41	76	0	2.30	6

(CONTINUED)

9-4-70					
2.a.m.	37	91	-4	1.95	7
4 a.m.	40	90	2	1.79	6
6 a.m.	36	93	-1	1.60	7
8 a.m.	42	85	-2	8.00	7
10 a.m.	46	56	21	34.04	12
12noon	50	44	46	-	13
14 p.m.	51	40	46	-	13
16 p.m.	52	38	37	-	15
18 p.m.	51	43	16	-	14
20 p.m.	45	55	-1	-	16
22 p.m.	39	64	-6	-	18
24 p.m.	36	74	-2	-	19
10-4-70					
2 a.m.	33	79	-2	-	18
4 a.m.	32	84	-2	-	9
6 a.m.	31	90	-2	-	11
8 a.m.	31	92	-1	-	12
10 a.m.	32	94	11	-	16
12noon	33	92	26	-	14

(CONTINUED)

10-4-70					
14 p.m.	33	94	24	-	16
16 p.m.	33	85	29	-	15
18 p.m.	32	68	12	-	18
20 p.m.	29	76	0	-	20
22 p.m.	25	78	-7	-	20
24 p.m.	24	78	-10	-	15
11-4-70					
2 a.m.	22	77	-13	42.00	14
4 a.m.	20	79	-14	26.00	13
6 a.m.	18	88	-14	20.50	16
8 a.m.	18	91	-7	17.50	14
10 a.m.	20	89	24	22.74	13
12noon	24	83	61	33.00	15
14 p.m.	29	77	72	76.02	15
16 p.m.	32	69	60	84.42	14
18 p.m.	35	62	33	100.20	15
20 p.m.	36	65	-2	87.00	11
22 p.m.	32	76	-6	57.40	8
24 p.m.	32	80	-13	38.50	8

(CONTINUED)

12-4-70					
2 a.m.	30	88	-8	27.50	8
4 a.m.	30	84	-4	22.50	12
6 a.m.	28	89	-4	18.50	12
8 a.m.	28	90	4	15.00	12
10 a.m.	30	86	30	27.00	10
12noon	34	80	55	44.00	7
14 p.m.	39	65	86	59.97	11
16 p.m.	44	54	70	132.13	9
18 p.m.	45	52	30	115.05	7
20 p.m.	41	58	-2	75.86	6
22 p.m.	37	66	-2	56.39	7
24 p.m.	35	61	-2	42.97	6
13-4-70					
2 a.m.	34	73	-3	32.88	8
4 a.m.	31	92	-13	24.00	8
6 a.m.	28	94	-13	20.20	8
8 a.m.	30	80	-1	17.00	9
10 a.m.	41	62	53	45.18	8
12noon	48	52	92	71.30	5

(CONTINUED)

13-4-70					
14 p.m.	50	50	112	130.99	6
16 p.m.	64	36	93	177.70	8
18 p.m.	55	34	46	165.00	4
20 p.m.	46	68	-3	75.00	3
22 p.m.	34	96	-12	35.00	-
24 p.m.	29	96	-12	27.50	7
14-4-70					
2 a.m.	27	96	-12	22.00	7
4 a.m.	27	95	-13	18.00	8
6 a.m.	29	93	-14	15.00	9
8 a.m.	30	92	-1	12.50	10
10 a.m.	40	61	49	38.50	13
12noon	50	53	88	68.00	7
14 p.m.	54	43	104	116.00	6
16 p.m.	56	39	85	159.50	7
18 p.m.	56	40	43	120.00	2
20 p.m.	50	55	-3	80.00	6
22 p.m.	37	91	-14	64.50	7
24 p.m.	31	96	-14	45.00	8

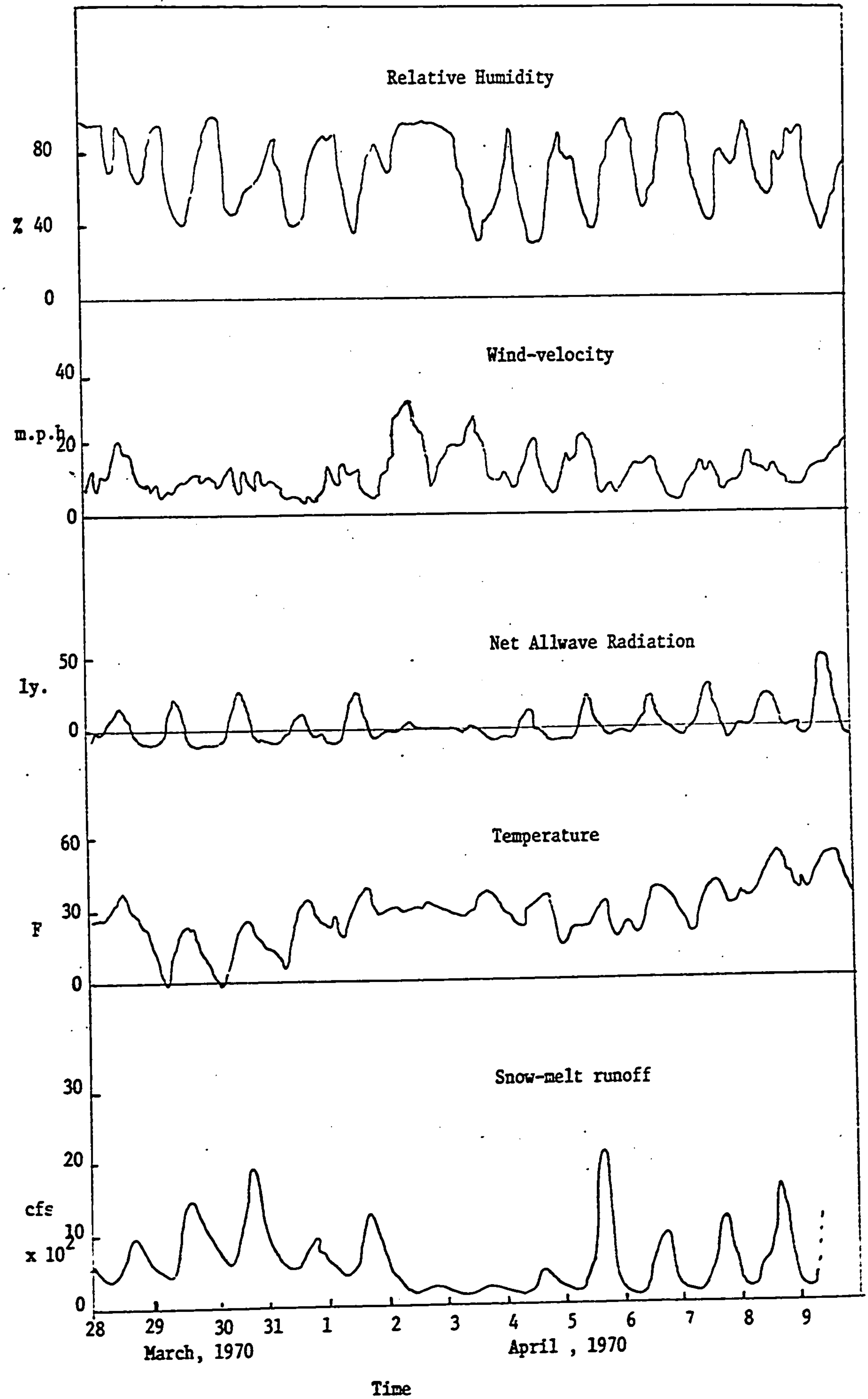
(CONTINUED)

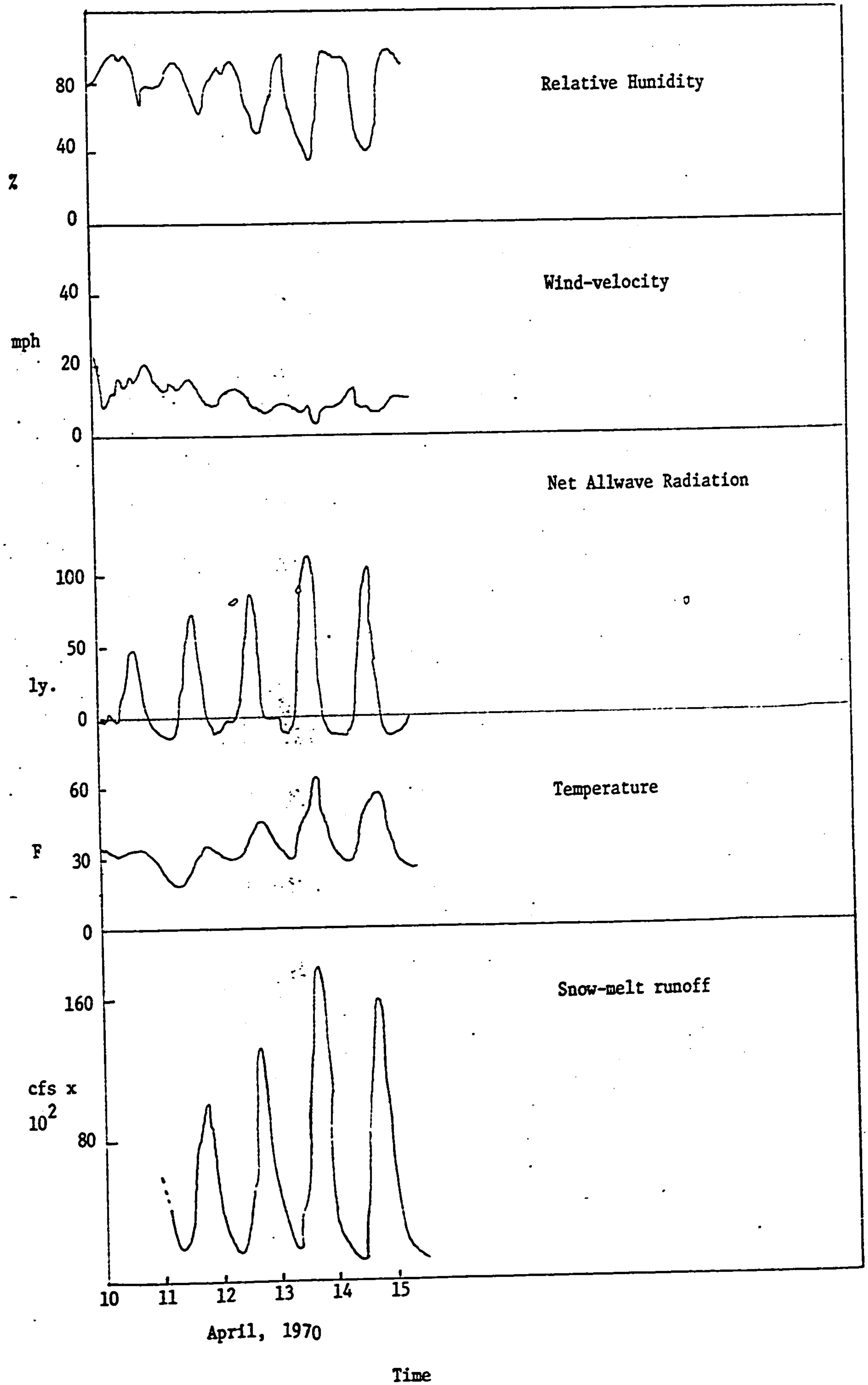
15-4-70					
2 a.m.	28	96	-14	25.00	10
4 a.m.	26	96	-14	20.70	10
6 a.m.	25	96	-14	18.00	10
8 a.m.	26	90	-2	15.00	10

17-4-70 rainfall 0.07 inches

APPENDIX C

FIGURE C.1

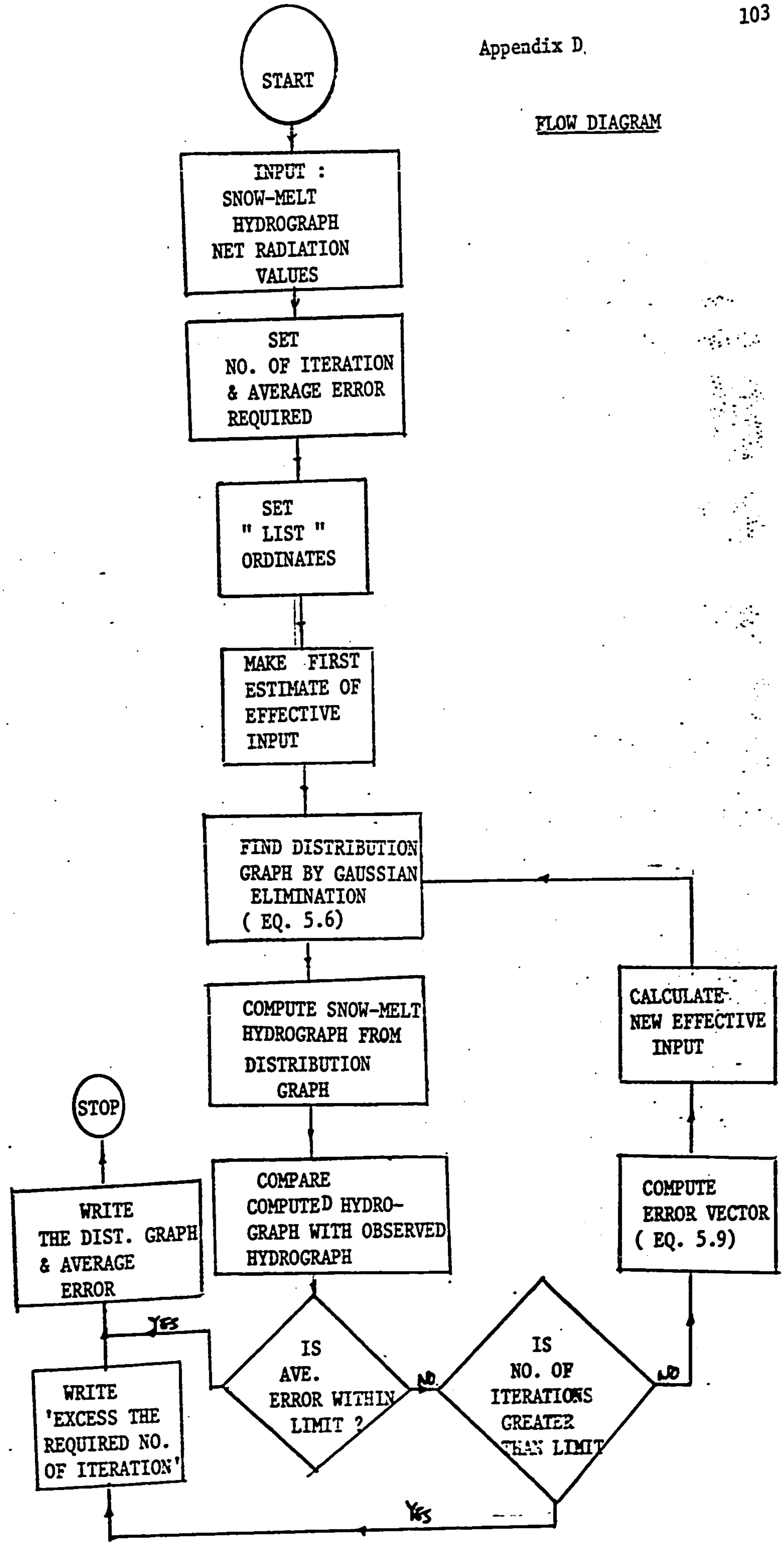

Relative Humidity
80
% 40
0
Wind-velocity
40
m.p.h. 20
0
Net Allwave Radiation
50
ly.
0
Temperature
60
30
F
0
Snow-melt runoff
30
20
cfs
x 10^2 10
0
28
29
30
31
1
2
3
4
5
6
7
8
9
March, 1970
April , 1970
Time


Relative Hunidity
80
40
%
0
Wind-velocity
40
mph
20
0
Net Allwave Radiation
100
50
ly.
0
Temperature
60
F
30
0
Snow-melt runoff
160
cfs x 10^2
80
10
11
12
13
14
15
April, 1970
Time

APPENDIX D

Appendix D.

FLOW DIAGRAM



```
C*****************************************************************************
C      SNCW-MELT RUNOFF RESEARCH                                              *
C      TO SOLVE THE PULSE RESPONSE OF THE WATERSHELD DUE TO SNOW-MELTS        *
C      A MATHEMATICAL MODEL  IS WRITTEN IN  MATRIX FORM                       *
C      NGRAPH: NO. CF DAILY SNOWMELT RUNOFF HYDROGRAPH TO BE COMPUTED         *
C      ERROR : IS PERCENT ERROR BETWEEN COMPUTED AND OBSERVED SNOWMELT        *
C      RUNOFF HYDRCGRAPH REQUIRED                                             *
C      FACTOR: REDUCE ERROR FACTOR                                            *
C      NITER: NO. OF ITERATIONS TO BE COMPUTED                                *
C      NLIST: NO. OF LISTS TO BE COMPUTED                                     *
C*****************************************************************************
       DIMENSION H(50,50),Q(50),U(50),HH(50,50)
       DIMENSICN PPO(10),BO(50)
       DIMENSION HA(50,50)
       DIMENSION HC(50),RO(50)
       DIMENSICN RN(50),NI(50),NN(50),HCQ(50),LIST(50),GG(50)
       READ(1,1)NGRAPH,ERROR
1      FCRMAT(I6,F10.2)
       LL=1
       LOCK=1
2      ITER=0
       DO 200 I=1,50
       C(I)=0.
       RN(I)=0.
       HQ(I)=0.
       U(I)=0.
       HCQ(I)=0.
200    RO(I)=0.
3      READ(1,4)FACTOR,TIME,NITER,LLM,NLIST
4      FORMAT(2F10.3,3I6)
       READ(1,400)(LIST(I),I=1,NLIST)
400    FCRMAT(36I2)
       WRITE(3,401)(LIST(I),I=1,NLIST)
401    FORMAT(1H1,2X,'CCMPUTE ORDINATE ',36I3,/)
C      ARRANGE RO IN MATRIX FORM
       DO 5 MASTER=1,50
       DO 5 KING=1,50
5      H(MASTER,KING)=0.
       DO 500 I=1,50
       DO 500 J=1,50
500    HH(I,J)=0.
       CALL MATINP(H(1,1),RO(1),Q(1),N,NNI,L,RN(1),TOO)
       WRITE(3,503)
```

```
503   FORMAT(//10X,'SNOWMELT RUNOFF OBSERVED HYDROGRAPH IN C.F.S.' )
      WRITE(3,7)(Q(I),I=1,NNI)
      NN(LL)=N
      NI(LL)=NNI
7     FORMAT(/2X,10F10.4)
      WRITE(3,181)TQQ
181   FORMAT(//2X,'TQQ=TOTAL OF SNOWMELTS RUNOFF IN C.F.S.*2HR.= ',F10.4
     &)
      WRITE(3,6)LL,ITER
6     FORMAT(///10X,'SNOWMELTS RUNOFF HYDROGRAPH NO.= ',I5,10X,'ITERATIO
     &N NO.= ',I5,//)
      WRITE(3,9)
9     FORMAT(10X,'EFFECTIVE INPUT VOLUME IN C.F.S * 2HR. ')
      WRITE(3,10)(RO(I),I=1,N)
10    FORMAT(12X,10F10.4,//)
C     TOTAL OF INPUT
      TRO=0.
      DO 11 I=1,N
11    TRO=TRO+RN(I)
      DO 112 I=1,NNI
      DO 112 J=1,L
112   HA(I,J)=H(I,J)
C     THE SUBROUTINE PROGRAMS WERE USED  FROM N.W. NEWTON
      CALL BAK(H(1,1),NNI,L,K,LIST(1))
      JK=K+1
      DO 12 I=1,NNI
12    H(I,JK)=Q(I)
C     CALCULATE ROT*RO AND ROT*Q
      DO 13 IL=1,NNI
      DO 14 I=1,JK
      DO 14 J=1,JK
14    HH(I,J)=HH(I,J)+H(IL,I)*H(IL,J)
13    CONTINUE
C     SUBROUTINE XATEQ: GASSIAN ELIMINATION,EXPLAIN IN SUB-PROGRAM
77    CALL XATEQ(HH(1,1),HH(1,JK),K,1,DET)
      DO 19 I=1,K
      U(I)=HH(I,JK)
      IF(U(I))16,19,19
16    U(I)=0.
19    CONTINUE
      CALL  SNOW(U(1),RO(1),K,L,LIST(1))
      UHT=0.
      DO 21 I=1,L
```

```
21    U(I)=RC(I)
      WRITE(3,22)ITER
22    FORMAT(///10X,' UNIT GRAPH ;ITERATION NO.= ',I5)
      WRITE(3,23)(U(I),I=1,L)
23    FORMAT(8X,10F10.4,//)
      DO 20 I=1,L
20    UHT=UHT+RC(I)
C     CALCULATE UH
      WRITE(3,201)UHT
201   FORMAT(//10X,'TOTAL OF UNIT GRAPH= ',F10.4,///)
      TERR=0.
      ITNI=0
      NNI=NI(LL)
      N=NN(LL)
C     COMPUTE RO*U
240   DO 26 I=1,NNI
      HQ(I)=0.
      LM=1
25    HQ(I)=HQ(I)+U(LM)*HA(I,LM)
      LM=LM+1
      IF(LM-L)25,25,26
26    CONTINUE
      WRITE(3,27)LL,ITER
27    FORMAT(10X,'COMPUTER SNOWMELT RUNOFF HYDROGRAPH NO.= ',I5,5X,'ITER
     &ATION NO.= ',I5,/)
      WRITE(3,28)
28    FORMAT(10X,'COMPUTER SNOWMELT HYDROGRAPH IN C. F. S.  ',/)
      WRITE(3,29)(HQ(I),I=1,NNI)
29    FORMAT(10X,10F10.4,/)
C     COMPUTER THE DIFFERENT BETWEEN OBSERVED & COMPUTER HYDROGRAPH
      IF (NITER)30,30,31
31    IN=1
      TERROR=0.
      DO 32 I=1,NNI
      HQ(I)=HQ(I)-Q(I)
      BO(I)=ABS(HQ(I)/Q(I))
32    TERROR=TERROR+ABS(HQ(I)/Q(I))
      WRITE(3,323)
323   FORMAT(/2X,'EACH ORDINATE COMPARE ERROR= ')
      WRITE(3,322)(BO(I),I=1,NNI)
322   FORMAT(5X,10F10.4)
321   DO 33 I=1,N
33    RO(I)=H(I,1)
```

```
      ITNI=0
      DO 34 LIMM=1,50
      DO 34 NMM=1,50
34    H(LIMM,NMM)=0.
      DO 35 J=1,N
      I=J
      DO 36 KT=1,L
      H(I,J)=U(KT)
36    I=I+1
35    CONTINUE
C     ERASE HH
      DO 37 I=1,50
      DO 37 J=1,50
37    HH(I,J)=0.
      DO 38 IL=1,LLM
      DO 38 II=1,N
      DO 38 JJ=1,N
38    HH(II,JJ)=HH(II,JJ)+H(IL,II)*H(IL,JJ)
C     IDENTITY MATRIX
      DO 39 I=1,N
      DO 39 J=1,N
      IF(I-J)40,41,40
40    H(I,J)=0.
      GO TO 39
41    H(I,J)=1.
39    CONTINUE
      CALL XATEQ(HH(1,1),H(1,1),N,N,DET)
      LAM=1
      NEW=1
      MEN=NEW+L-1
51    E=0.
      MK=LLM
      KQ=NEW
      DO 511 I=1,N
      HCQ(I)=0.
      IK=KQ
      DO 42 LJ=1,MK
      HCQ(I)=HCQ(I)+HQ(IK)*U(LJ)
42    IK=IK+1
      MK=MK-1
      KQ=KQ+1
511   CONTINUE
      IN=1
```

```
43    E=HCQ(IN)*H(1,IN)+E
      IN=IN+1
      IF(IN-N)43,43,44
44    DRO=RO(NEW)-E*FACTOR
      IF(DRO)45,46,46
45    DRO=0.
      WRITE(3,47)NEW,DRO
47    FORMAT(//5X,'SOLUTION IS NEGATIVE FOR EFFECTIVE INPUT NO.= ',I4,15
     &X,'INPUT= ',F10.4)
      GO TO 48
46    IF(DRO-RN(NEW))48,48,49
49    WRITE(3,50)NEW
50    FORMAT(//5X,'RUNOFF EXCESSED EFFECTIVE INPUT AT NO.= ',I4)
48    E=RO(NEW)-DRO
      RO(NEW)=DRO
      DO 52 I=NEW,MEN
      HQ(I)=HQ(I)-E*U(LAM)
52    LAM=LAM+1
      NEW=NEW+1
      MEN=MEN+1
      LAM=1
      IF(NEW-N)51,51,54
54    WRITE(3,55)
55    FORMAT(//5X,'CORRECTED RUNOFF VOLUME BEFORE ADJUSTMENT',//)
      WRITE(3,10)(RO(I),I=1,N)
      STRO=0.
      IN=1
56    STRO=STRO+RO(IN)
      IN=IN+1
      IF(IN-N)56,56,57
57    IN=1
      F=TRO/STRO
      WRITE(3,58)F
58    FORMAT(//5X,'RUNOFF VOLUME ADJUSTMENT FACTOR= ',F10.3)
59    RO(IN)=RO(IN)*F
      IN=IN+1
      IF(IN-N)59,59,60
60    WRITE(3,61)LL,ITER
61    FORMAT(//10X,'ADJUSTED SNOWMELT RUNOFF HYDROGRAPH NO.= ',I5,5X,'IT
     &ERATION NO.= ',I5,/)
      WRITE(3,62)(RO(I),I=1,N)
62    FORMAT(//2X,'ADJUST NEW INPUT= ',10F10.4)
      DO 621 I=1,N
```

```
621    PRO(I)=RC(I)/TCQ
       WRITE(3,623)
623    FCRMAT(/2X,'PERCENT NEW INPUT')
       WRITE(3,622)(PRO(I),I=1,N)
622    FCRMAT(/2X,10(F10.3,2X))
       L=NNI-N+1
C      ERASE H
       DC 63 I=1,50
       DO 63 J=1,50
63     H(I,J)=C.
C      ARRANGE RO IN MATRIX FORM
       DO 64 LJ=1,L
       I=LJ
       DO 64 LK=1,N
       H(I,LJ)=RC(LK)
64     I=I+1
       DO 640 I=1,NNI
       DC 640 J=1,L
640    HA(I,J)=H(I,J)
652    CALL BAK(H(1,1),NNI,L,K,LIST(1))
       DO 66 I=1,NNI
66     H(I,JK)=Q(I)
C      ERASE HH
       DO 67 I=1,50
       DO 67 J=1,50
67     HH(I,J)=0.
C      CALCULATE RCT*RO AND ROT*Q WITH NEW RO
       DC 68 JJJ=1,NNI
       DO 69 I=1,JK
       DO 69 J=1,JK
69     HH(I,J)=HH(I,J)+H(JJJ,I)*H(JJJ,J)
68     CONTINUE
30     ITNI=ITNI+NNI
       IF(NITER.EQ.0)GO TO 70
       WRITE(3,301)TERROR
301    FOFMAT(/2X,'TOTAL ERROR= ',F10.4)
       AERRCR=TERROR/FLCAT(ITNI)
       IF(AERROR-ERROR)71,71,72
72     WRITE(3,73)AERROR,ITER
73     FCRMAT(//10X,'AVERAGE ERROR= ',F10.4,5X,'FOR ITERATICN NO.= ',I5)
730    IF(ITER-NITER)74,75,75
75     WRITE(3,76)
76     FORMAT(/2X,'STOPPED BY ITERATION TEST ')
```

```
      GO TO 70
74    ITER=ITER+1
      WRITE(3,741)
741   FORMAT(1H1)
      GO TO 77
71    WRITE(3,73)AERROR,ITER
      WRITE(3,78)
78    FORMAT(//5X,'PROBLEM CONVERGED')
781   WRITE(3,780)
780   FORMAT(1H1,5X,'UNIT GRAPH TEST BY COMPUTER ')
      WRITE(3,23)(U(I),I=1,L)
70    IF(LOOK-NGRAPH)79,80,80
79    LOOK=LOOK+1
      LL=LL+1
      GO TO 2
80    RETURN
      DEBUG SUBCHK
      END
```

```
      SUBROUTINE BAK(R,NI,M,N,LIST)
      DIMENSION R(50,50),LIST(50)
      L=0
1     L=L+1
      K=L+1
      J=LIST(L)
      JJ=LIST(K)
      IF(J-M)20,40,40
20    IF(J)40,40,16
16    A=JJ-J
      IF(A-1.0)10,1,10
10    JJJ=A-1.
      DO 5 LL=1,JJJ
      BB=LL
      JF=JJ-LL
      ROTIO=BB/A
      JL=J+LL
      DO 7 LI=1,NI
      R(LI,J)=(ROTIO*R(LI,JF))+R(LI,J)
7     R(LI,JJ)=(ROTIO*R(LI,JL))+R(LI,JJ)
5     CONTINUE
      GO TO 1
40    N=LIST(1)+1
      J=2
15    JJ=LIST(J)
      DO 100 LM=1,NI
100   R(LM,N)=R(LM,JJ)
      IF(JJ-M)75,60,60
75    N=N+1
      J=J+1
      GO TO 15
60    RETURN
      DEBUG SUBCHK
      END
```

```
      SLBROUTINE MATINP(H,RO,Q,N,NI,L,RN,TQ)
C**********************************************************************
C      N: THE NUMBER OF EFFECTIVE NET RADIATICN ORDINATES.
C      THE TIME PERIOD FOR THESE ORDIATES MUST BE EQUAL TO THE          *
C      CURATICN OF THE COMPUTER UNIT HYDROGRAPH.                        *
C      NI: THE NO. OF SNOWMELTS RUNOFF HYDROGRAPH ORDIATES. THE TIME    *
C      PERICD FCR THESE ORDINATES MUST BE EQUAL TO THE DORATION OF THE  *
C       COMPUTED UNIT HYDROGRAPH.                                       *
C      RN,RO : EFFECTIVE INPUT IN C.F.S
C      L: NO. OF CRDINATES IN UNIT HYDROGRAPH                           *
C**********************************************************************
      DIMENSION H(50,50),RO(50),Q(50),RN(50)
      DIMENSICN RAD(20)
      DIMENSICN DATE(20)
      DIMENSICN A(20)
 3    FORMAT(8F10.4)
 4    FORMAT(4I3)
      READ(1,4)N,NI
      READ(1,3)(Q(I),I=1,NI)
      READ(1,2)DATE
 2    FORMAT(1X,20A1)
      WRITE(3,2)DATE
      DO 6 I=1,NI
 6    Q(I)=Q(I)/10.
      TQ=0.
      DO 9 I=1,NI
 9    TQ=TQ+Q(I)
C     INITIAL INPUT ASSUMED
      TORAD=0.
      READ(1,15)(RAD(I),I=1,N)
 15   FORMAT(8F10.0)
      WRITE(3,150)
 150  FORMAT(2X,'INITIAL NET RADIATION IN LY.')
      WRITE(3,151)(RAD(I),I=1,N)
 151  FORMAT(2X,10F10.4)
      DO 16 I=1,N
 16   TORAD=TORAD+RAD(I)
      DO 17 I=1,N
 17   A(I)=RAD(I)/TORAD
      WRITE(3,12)(A(I),I=1,N)
 12   FORMAT(/2X,'PERCENT INPUT= ',10F10.4)
      DO 11 I=1,N
      RO(I)=A(I)*TQ

 11   RN(I)=A(I)*TQ
      L=NI-N+1
C     ARRANGE INPUT IN MATRIX FORM
      DO 5 J=1,L
      I=J
      DO 5 K=1,N
      H(I,J)=RO(K)
 5    I=I+1
      RETURN
      DEBUG SUBCHK
      END
```

```
      SUBROUTINE SNOW(OU,D,K,L,LIST)
      DIMENSION OU(50),D(50),LIST(50)
      DO 1 M=1,K
1     D(M)=OU(M)
      M=1
      MMM=LIST(1)
2     M=M+1
      MM=LIST(M)
      MMM=MMM+1
      OU(MM)=D(MMM)
      IF(MM-L)2,3,3
3     DO 50 II=1,50
50    D(II)=0.
      M=0
      MM=LIST(1)
      DO 5 MMM=1,MM
5     D(MMM)=OU(MMM)
10    M=M+1
      MM=LIST(M)
      J=M+1
      JJ=LIST(J)
      JJJ=JJ-MM
      IF(JJJ-1)4,4,6
6     JJJJ=JJJ-1
      AC=JJJ
      DO 7 MMM=1,JJJJ
      M4=MM+MMM
      CC=MMM
7     D(M4)=((AC-CC)*OU(MM)+CC*OU(JJ))/AC
4     D(JJ)=OU(JJ)
      IF(JJ-L)10,8,8
8     RETURN
      END
```

```
      SUBROUTINE XATEQ(A,B,N,M,DET)
      DIMENSION A(50,50),B(50,50)
      IFLG=0
      I1=1
      DET=1.
100   XFACT=0.
      DO 10 I=I1,N
      DO 10 J=1,N
      IF(A(I,J)-XFACT)10,10,1
1     XFACT=A(I,J)
      IB=I
      JB=J
10    CONTINUE
      DET=DET*XFACT
      IF(IFLG)1701,1701,17
1701  DO 11 J=1,N
      STOR=A(I1,J)
      A(I1,J)=A(IB,J)
      A(IB,J)=STOR
11    CONTINUE
      IF(M)201,201,200
200   DO 12 J=1,M
       STOR=B(I1,J)
      B(I1,J)=B(IB,J)
      B(IB,J)=STOR
12    CONTINUE
201   I1=I1+1
      DO 16 I=I1,N
      IF(A(I1-1,JB).EQ.0)GO TO 2000
      XFACT=-(A(I,JB)/A(I1-1,JB))
      DO 14 J=1,N
      A(I,J)=A(I,J)+A(I1-1,J)*XFACT
14    CONTINUE
      IF(M)203,203,202
202   DO 15 J=1,M
      B(I,J)=B(I,J)+B(I1-1,J)*XFACT
15    CONTINUE
203   A(I,JB)=0.
16    CONTINUE
      IF(I1-N)100,1700,1700
1700  IFLG=1
      GO TO 100
17    IF(M)290,290,170
```

```
170    N1=N
23     DO 18 J=1,N
       IF(A(N1,J))19,18,19
18     CONTINUE
19     N2=N1-1
       DO 21 J1=1,M
       IF(A(N1,J).EQ.0)GO TO 2000
       XFACT=B(N1,J1)/A(N1,J)
       B(N1,J1)=XFACT
       IF(N2)21,21,210
210    DO 20 I=1,N2
       B(I,J1)=B(I,J1)-XFACT*A(I,J)
20     CONTINUE
21     CONTINUE
       IF(N2)25,25,220
220    DO 22 I=1,N2
       A(I,J)=0.
22     CONTINUE
       N1=N1-1
       GO TO 23
25     DO 29 I1=1,N
28     DO 30 I=I1,N
       IF(ABS(A(I,I1)))30,30,26
30     CONTINUE
26     DO 27 J=1,M
       STOR=B(I,J)
       B(I,J)=B(I1,J)
       B(I1,J)=STOR
27     CONTINUE
       DO 300 J=1,N
       STOR=A(I,J)
       A(I,J)=A(I1,J)
       A(I1,J)=STOR
300    CONTINUE
29     CONTINUE
       GO TO 290
2000   WRITE(3,2001)
2001   FORMAT(//5X,'ILL CONDITIONAL MATRIX')
290    RETURN
       END
```