Preservice Teachers' Perceptions of Integrating Immersive Virtual Reality (VR) to Teach Vector Operations

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Legend

VR: Immersive Virtual Reality

MathVR: The immersive virtual reality application used in this study for learning vector operations

MAET: Multimodal Analysis for Embodied Technologies

TPACK: Technological Pedagogical Content Knowledge

TK: Technological Knowledge

PK: Pedagogical Knowledge

CK: Content Knowledge

PCK: Pedagogical Content Knowledge

TCK: Technological Content Knowledge

TPK: Technological Pedagogical Knowledge

TPCK: Technological Pedagogical Content Knowledge

RQ1: Research Question One

RQ2: Research Question Two

RQ3: Research Question Three

Meta Quest 2: The virtual reality headset used during the immersive sessions

AR: Augmented Reality

CAMIL: Cognitive Affective Model of Immersive Learning

CAS: Computer Algebra System

DGS: Dynamic Geometry Software

HMD: Head-Mounted Display

SoE: Sense of Embodiment

UTAUT2: Extended Unified Theory of Acceptance and Use of Technology

XK: Contextual Knowledge

Abstract

This doctoral thesis investigates how preservice mathematics teachers experienced immersive virtual reality in relation to vector learning and pedagogy. Although virtual reality is often discussed as a promising tool for mathematics education, less is known about how preservice teachers make sense of such experiences as learners of mathematics and as future teachers. This study therefore examined the insights preservice teachers provided while performing vector operations in MathVR (Cabanag, 2023), how they then described the affordances and constraints of MathVR for teaching Grade 12 vectors in Ontario, and what those insights and descriptions revealed about their emergent Technological Pedagogical Content Knowledge (TPACK).

The study used a qualitative exploratory case study design. Ten preservice mathematics teacher candidates participated in a structured session that included an orientation, an immersive MathVR activity, a focused freewriting task, and a focus group discussion. Data sources included VR session recordings with audio, observational field notes, written reflections, and focus group transcripts. Analysis was organized by research question. VR session data were examined using Multimodal Analysis for Embodied Technologies. Freewriting and focus group data were examined through thematic analysis. Findings across data sources were then interpreted through the TPACK framework as a synthesis lens.

The findings show that participants used gesture, spatial repositioning, and real time visual feedback to make sense of vector operations in three-dimensional space. They identified conceptual affordances related to visualization, projection, direction, and magnitude, while also noting interface complexity, cognitive demands, sensory discomfort, and equity concerns. The

study further shows that participants' TPACK appeared as emerging and uneven rather than complete, becoming most visible through reflection and discussion after immersive experience.

Overall, the thesis contributes an embodied and multimodal account of how immersive virtual reality may support early pedagogical reasoning in mathematics teacher education.

Résumé

Cette thèse de doctorat examine la manière dont de futurs enseignants et enseignantes de mathématiques ont vécu une expérience de réalité virtuelle immersive en lien avec l'apprentissage et l'enseignement des vecteurs. Bien que la réalité virtuelle soit souvent présentée comme un outil prometteur pour l'enseignement des mathématiques, on connaît moins bien la façon dont les futurs enseignants et enseignantes interprètent de telles expériences, à la fois comme apprenants et apprenantes de mathématiques et comme futurs praticiens et praticiennes. La présente étude a donc examiné les observations que ces personnes ont formulées en effectuant des opérations sur les vecteurs dans MathVR (Cabanag, 2023), la manière dont elles ont ensuite décrit les possibilités et les contraintes de cet environnement pour l'enseignement des vecteurs en 12e année en Ontario, et ce que ces observations et descriptions révèlent de leur TPACK émergent, c'est-à-dire de leurs connaissances technologiques, pédagogiques et disciplinaires en formation.

L'étude repose sur un devis d'étude de cas qualitative exploratoire. Dix personnes candidates à l'enseignement des mathématiques ont participé à une séance structurée comprenant une initiation à l'équipement, une activité immersive dans MathVR, une tâche d'écriture libre ciblée et un entretien de groupe. Les données proviennent d'enregistrements d'écran des séances de réalité virtuelle accompagnés de la bande sonore, de notes d'observation, de réflexions écrites et de transcriptions des entretiens de groupe. L'analyse a été organisée en fonction de chaque question de recherche. Les données des séances de réalité virtuelle ont été analysées à l'aide de l'analyse multimodale pour les technologies incarnées (Multimodal Analysis for Embodied Technologies, MAET). Les données issues de l'écriture libre et des entretiens de groupe ont fait l'objet d'une analyse thématique. Les résultats provenant de ces différentes sources ont ensuite

été interprétés à travers le modèle TPACK (Technological Pedagogical Content Knowledge, soit les connaissances technologiques, pédagogiques et disciplinaires), utilisé comme cadre de synthèse.

Les résultats montrent que les personnes participantes ont mobilisé le geste, le repositionnement spatial et la rétroaction visuelle en temps réel pour donner du sens aux opérations sur les vecteurs dans l'espace tridimensionnel. Elles ont relevé des possibilités conceptuelles liées à la visualisation, à la projection, à la direction et à la norme, tout en signalant la complexité de l'interface, les exigences cognitives, l'inconfort sensoriel ainsi que des préoccupations relatives à l'équité. L'étude montre en outre que le TPACK des personnes participantes s'est manifesté sous une forme émergente et inégale plutôt qu'achevée, et qu'il est devenu le plus visible dans la réflexion et la discussion qui ont suivi l'expérience immersive.

Dans l'ensemble, la thèse propose une description incarnée et multimodale de la manière dont la réalité virtuelle immersive peut soutenir l'émergence du raisonnement pédagogique dans la formation initiale à l'enseignement des mathématiques.

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This research received ethics approval from the University of Ottawa Research Ethics Board (Ethics File Number S-11-24-11007), and all procedures followed the approved protocol. I am grateful to the Office of Research Ethics and Integrity for its review, and to the Faculty of Education for the institutional support that made the study possible.

My greatest debt is to the ten preservice teachers who agreed to put on a headset, work through a set of unfamiliar tasks while being recorded, and then speak candidly about what had and had not made sense. They gave their time during a demanding period of their teacher education program and asked for nothing in return. Their willingness to think aloud, to admit confusion, and to imagine how they might one day teach with such a tool is the substance of this dissertation. I also thank the teacher candidates who took part in the pilot session, whose feedback led to changes in the task sequence and the orientation procedure that improved the study for everyone who followed. To all: the pseudonyms in these pages conceal your identities but not, I hope, the quality of your thinking.

The immersive environment used in this study, MathVR, was developed by Cabanag (2023), and I am grateful for permission to use the application and to reproduce images of its interface, and for the developer's work in making an application of this kind available for educational use.

Generative AI tools were used in a limited supporting role. AI assisted search tools helped identify potentially relevant peer reviewed literature, and every source located in this way was retrieved from its original publication platform through the uOttawa and Queen's University library databases and read in full before being cited. These tools were also used occasionally to clarify terminology and to test my own understanding while reading. They were not used to generate or analyze research data, to conduct coding, to produce results, or to make methodological decisions. The research design, data collection, analysis, and interpretation are my own, and so is the writing. Responsibility for the accuracy of the citations, the interpretation of the evidence, and the integrity of the written work rests with me.

Finally, I thank my family. A doctorate is not completed alone, and the years this work has taken were shared by the people closest to me. My wife, Jessica, carried more than her share of our life so that I could carry this one, with a patience that held through every late night and every weekend lost to writing. My parents gave me, long before I knew what a thesis was, the habits of work and the belief in education that made this one possible, and their encouragement never faltered. Hazel, our Irish Setter, sat under the desk through most of these pages and made sure that no writing day ended without a walk. And I thank my little one, whose presence over the last two years motivated me to finish this journey stronger than ever; every page written in that time was written with a better reason.

Chapter One

Introduction

Digital Technology in Education

Students attending elementary and secondary schooling today are immersed in a digitally connected world, and so are their teachers. Globally, an estimated 6 billion people, about three quarters of the world's population, now use the Internet (International Telecommunication Union, 2025). In Canada, 77 percent of youth have their own smartphone and 97 percent report using the Internet to stay connected with friends and family (MediaSmarts, 2022). At school, students use laptops to access content posted to learning management systems through board provided Internet connections (MediaSmarts, 2022), and teachers are expected to use digital technologies in ways that support student learning (International Society for Technology in Education [ISTE], 2024).

Despite widespread claims that digital technologies are essential to twenty first century schooling (OECD, 2023), critical research has shown that the presence of digital systems in schools is not, on its own, a strong predictor of better learning outcomes for students (Watters, 2021). Digital technologies have become part of everyday school life, but they are rarely found to cause learning by themselves.

Instead, the strongest evidence for digital technologies as supports for student learning tends to come from studies where teachers integrate a digital tool within a well understood, high impact teaching practice to teach a specific disciplinary concept or skill (e.g., Chueh & Cheng, 2026; Mishra & Koehler, 2006).

In mathematics education specifically, early evidence suggests that immersive virtual reality, a computer-generated three-dimensional environment experienced through a head-

mounted display in which a user's own head and hand movements are tracked and reflected within the environment in real time, may support mathematical reasoning when it is used by skilled teachers, particularly the coordination of spatial, visual, and symbolic representations of the same underlying concept (National Council of Teachers of Mathematics, 2014; Radianti et al., 2020). Very little research to date, however, has explored how teachers themselves come to understand immersive virtual reality as a pedagogical tool. Vectors, mathematical objects defined by both magnitude and direction, place exactly this kind of spatial and representational demand on learners, making them a natural focus for this inquiry.

The Persistent Challenge of Learning Vectors

Vectors are foundational in disciplines such as physics, engineering, computer graphics (Oliveira et al., 2025), and navigation, yet they remain persistently difficult for students to learn and for instructors to teach (Chang & Beilock, 2016; Ellis et al., 2016; Khan et al., 2018). Research in mathematics and physics education has shown recurring challenges in vector learning among university students (e.g., Barniol & Zavala, 2014; Bollen et al., 2017; Cuevas-Vallejo et al., 2023; Sabah, 2023; Sirait & Oktavianty, 2017) and, in some contexts, for preservice teachers (e.g., Tamba, 2023), that include difficulty coordinating direction and magnitude, moving between graphical and algebraic representations, and reasoning about vectors in two-dimensional and three-dimensional contexts. Even after instruction, learners often retain misconceptions, such as treating vectors as scalars (Appova & Berezovski, 2013) or struggling to decompose vectors into components (Bani-Salameh et al., 2020; Barniol & Zavala, 2014). A scalar is a quantity fully described by magnitude alone, such as speed or temperature. A vector is a quantity that requires both magnitude and direction to be fully described, such as velocity or

displacement. This distinction, whether a quantity needs a direction attached to be meaningful, underlies much of the difficulty documented in this section.

This persistence suggests that lecture and textbook based approaches, which often rely on static two-dimensional diagrams and procedural calculation with limited opportunities for interactive, spatial, and representational coordination, do not reliably support the conceptual and representational demands of vector reasoning (Appova & Berezovski, 2013; Barniol & Zavala, 2014; Bollen et al., 2017; Sabah, 2023). This educational problem is well recognized; studies have found high rates of errors and misunderstandings in basic vector operations among both high school and undergraduate students (Appova & Berezovski, 2013; Zhao, 2024). Collectively, this literature positions vector learning as a well-established and enduring instructional problem, particularly where spatial reasoning and representational coordination are central. Representational form refers to one of these specific external ways a vector is represented and operated on, such as an arrow in a diagram, an ordered pair or triple of components, a column vector, or a vector equation; representational coordination is the interpretive work required to connect those particular forms to direction, magnitude, and spatial meaning, and to one another.

Instruction that relies on static, two-dimensional media such as textbook diagrams, lecture slides, and board drawings may be ill equipped to address these difficulties. While such representations support procedural practice, they offer limited opportunities for dynamic manipulation, perspective shifting, or embodied exploration, particularly when students are expected to reason about three-dimensional relationships (Schutera et al., 2021). For example, vector problems in geometry are often taught in an abstract, purely theoretical manner, since it is hard to physically demonstrate or dynamically manipulate vectors using only chalkboard sketches (Schutera et al., 2021). In these conditions, learners may rely on memorized procedures

rather than developing coherent spatial understanding, and they may struggle to connect symbolic operations to geometric meaning (Bollen et al., 2017; Sirait & Oktavianty, 2017). The persistence of misconceptions documented in studies of vector teaching and learning (e.g., Appova & Berezovski, 2013; Mut, 2015; Nguyen & Meltzer, 2003; Van Deventer & Wittmann, 2007) suggests the need for learning experiences that better support visualization, action, and meaningful coordination across representations (Barniol & Zavala, 2014; Bollen et al., 2017; Cuevas-Vallejo et al., 2023).

Immersive Virtual Reality as a Possible Response

Immersive virtual reality (VR) offers one possible response to the representational and spatial challenges of vector learning. For topics such as vectors, virtual reality may make mathematical relationships more perceptible by allowing learners to view, manipulate, and move around objects in space, affordances that align with embodied perspectives that treat perception and action as integral to conceptual understanding (Farina, 2021; Makransky & Petersen, 2021). Early evidence also suggests that such environments may be especially useful for spatially demanding content. For example, Campos et al. (2022) found that students using VR to learn three-dimensional vectors showed stronger performance on visualization related tasks, while broader review level work suggests that technology enhanced embodied learning can support conceptual understanding when learners engage with objects through movement and interaction (Georgiou & Ioannou, 2019).

This study is positioned within the embodied mathematics education scholarship that precedes it. Gerofsky (2010, 2011) studied the gestures secondary students produced when asked to enact graphs of functions with their bodies and found that these gestures could be placed along a continuum between two viewpoints. From an observer viewpoint, a student depicted the graph

as a small object held at a distance in front of the body, seeing the graph from outside. From a character viewpoint, a student enacted the graph at the scale of the whole body, moving as though travelling along it, being the graph rather than seeing it. Gerofsky reports that the position students took on this continuum corresponded closely to their teachers' judgements of how engaged they were and how well they attended to the mathematically salient features of the graph (Gerofsky, 2010, 2011), and she has since situated this work within a broader account of embodied approaches to mathematics learning (Gerofsky, 2016). Immersive virtual reality turns this continuum from a matter of individual disposition into a property of the learning environment. In MathVR, a learner can step back and inspect a vector configuration from outside, as the observer viewpoint describes, but can also walk into the vector space, reach for a vector at arm's length, and act on it directly, which places the encounter at the character viewpoint end of Gerofsky's continuum. The present study is positioned at that end. My interest is in the insights preservice teachers provide when they act within the mathematical space itself rather than gesture toward a representation of it, and I treat the two viewpoints as different positions from which mathematics is encountered rather than as a lesser and a greater form of embodiment. Chapter Three develops this positioning, naming the first position representational embodiment and the second immersive first-person embodiment.

Despite this promise, the use of VR in mathematics classrooms, and specifically for teaching vectors, remains underexplored (Campos et al., 2022). Campos et al. (2022) argue more generally that VR can provide "a suitable platform for understanding mathematical concepts and their relationship with the physical world" (p. 2), and they then situate vectors as a core mathematical concept for describing physical quantities and motivate their study by evaluating a VR tool for learning three-dimensional vectors in a university physics course (Campos et al.,

2022). Consistent with their framing, studies that examine VR based vector instruction remain relatively scarce. In contrast, VR integration is far more common in science education; Abutayeh et al. (2022) found that roughly 69.7% of VR and augmented reality (AR) studies address science, with only about 24.2% focusing on mathematics. Cevikbas et al. (2023) similarly observe that evidence for AR/VR in mathematics learning is very limited, noting a "discernible lack of research" in this area. As a result, VR is still rarely adopted in everyday mathematics teaching (Lai & Cheong, 2022; Su et al., 2022), and its potential benefits for topics like vectors are not yet well understood. This gap may be due in part to practical barriers (cost, limited access to equipment) as well as pedagogical uncertainty, with teachers unsure how to use VR effectively for mathematics or unclear about its potential benefits. Technology adoption also depends on teachers' beliefs, confidence, and perceptions of value. If educators are unfamiliar with virtual reality, uncertain about its instructional relevance, or concerned about feasibility, they may hesitate to implement it even when they recognize potential benefits (Ndlovu et al., 2020; Xie et al., 2022). For these reasons, it is important not only to examine what immersive tools can enable for learning, but also to investigate how future teachers interpret those tools as pedagogical resources and how they weigh potential affordances against constraints.

Preservice Teachers as a Population for Study

Because the classroom value of immersive virtual reality depends not only on what the technology can do but also on how teachers interpret its usefulness, feasibility, and relevance, preservice teachers, meaning teacher candidates in initial teacher education before full-time professional practice, are an especially important population for study (Howard et al., 2019; Ndlovu et al., 2020; Xie et al., 2022). In this thesis, the terms preservice teacher and teacher candidate are used interchangeably; teacher candidate is the designation used in the Ontario teacher

education program from which participants were recruited, and it appears where that institutional context is in view. As future educators who are still developing their pedagogical reasoning and technological confidence, preservice teachers are in a formative stage in their professional development and early experiences with immersive tools may therefore influence how they later evaluate and use such technologies in their instructional practice (Howard et al., 2019). Existing research suggests that preservice teachers often view VR positively for visualization and engagement, while also raising concerns related to access, preparedness, and classroom integration (Alkaabi et al., 2025; Figueroa-Flores & Huffman, 2020). This makes their perspectives particularly important for understanding not only the perceived affordances of VR for teaching abstract topics such as vectors, but also the practical and pedagogical conditions under which they consider it meaningful for mathematics instruction.

Rationale for This Study

This study was motivated by the intersection of these issues. First, vectors represent a persistent and well documented learning challenge, particularly where spatial visualization and representational coordination are required (Barniol & Zavala, 2014; Bollen et al., 2017; Nguyen & Meltzer, 2003; Sabah, 2023; Zhao, 2024). Second, immersive virtual reality offers interactive and embodied affordances that may address these challenges, yet its use in mathematics education, and specifically for vectors, remains underexplored (Lai & Cheong, 2022; Su et al., 2022). Third, there is a need to understand how preservice mathematics teachers perceive such technology as a potential instructional resource, including the conditions under which they view it as feasible, equitable, and pedagogically meaningful (Ndlovu et al., 2020; Xie et al., 2022). By examining preservice teachers' embodied interaction with a vector focused immersive environment, their subsequent reflective descriptions of its affordances and constraints for

teaching, and what both reveal about their emergent TPACK, this study is positioned at the intersection of these concerns.

A final dimension of this study's motivation is personal and scholarly. My earlier work examined how fictive motion, the use of spatially dynamic language and gesture to describe static mathematical concepts such as limits and continuity, allows learners to engage with ideas that are formally defined in disembodied, symbolic terms, asking how bodily action participates in making such concepts graspable (Khatin-Zadeh et al., 2021). Working within that line of inquiry, I became increasingly curious about whether an immersive virtual reality environment, one in which learners do not merely gesture toward mathematical objects but move within and act upon them directly, might extend those questions into a new domain. These experiences gave rise to questions that formal study could not yet answer: how do teachers actually make sense of a technology like VR when they encounter it as both learners of mathematics and as developing professionals? And what does that encounter reveal about the relationship between embodied experience and the kind of integrated knowledge teaching demands? This study is, in part, an attempt to pursue those questions rigorously, and to contribute to a conversation in mathematics teacher education that is consequential precisely because it is happening now, as these tools become available and practices around them are still being formed.

Statement of the Research Problem

Immersive virtual reality has attracted sustained attention as a potential tool for making abstract mathematical concepts tangible through direct, embodied interaction with three-dimensional representations (Campos et al., 2022; Cevikbas et al., 2023; Johnson-Glenberg, 2023; Wu & Tarng, 2025). Research in this area suggests that immersive environments allow learners to manipulate mathematical objects, observe relationships from multiple spatial

perspectives and coordinate symbolic and geometric understanding through direct manipulation and multiple viewpoints in ways that lecture-based, textbook-based, or static two-dimensional software instruction supports less readily (Campos et al., 2022; Su et al., 2022; Walkington et al., 2025). These affordances are particularly relevant for topics that depend on spatial reasoning, including vectors, where persistent learning difficulties have been documented across secondary and postsecondary settings (Barniol & Zavala, 2014; Sabah, 2023). The educational promise of immersive VR, however, does not translate into classroom practice on its own. It depends on how teachers experience these environments, what they come to see as pedagogically meaningful about them, and how they learn to connect the technological possibilities of VR to the specific demands of their content and their students. Research has largely focused on student outcomes or on measuring teacher attitudes toward VR in general terms, leaving comparatively little understanding of how teachers themselves make sense of immersive tools as learners and as developing practitioners (Botes, 2025; Hamilton et al., 2021; Selvakumar & Sivakumar, 2025).

This situation presents a specific challenge for mathematics teacher education. Preservice teachers are encountering immersive technologies at a formative stage in their professional development, when their beliefs, pedagogical reasoning, and sense of content knowledge are still taking shape (Howard et al., 2019). The professional orientations they develop now, including their understanding of what immersive VR can and cannot offer for mathematics instruction, may influence how they approach these tools throughout their careers. Teacher education programs are beginning to incorporate immersive technologies without yet having a strong empirical foundation for understanding what such experiences contribute to professional development (Han et al., 2025). Whether preservice teachers who engage with a content-specific VR environment develop richer, more integrated professional knowledge, or whether the

experience remains disconnected from their pedagogical reasoning, is a question that research has not yet examined in enough detail to be useful for program design. The Technological Pedagogical Content Knowledge framework (Mishra, 2019; Mishra & Koehler, 2006) offers a productive lens for this examination, because it attends precisely to how technological, pedagogical, and content-related knowledge come into relation in teaching practice. Yet few studies have used this framework to interpret immersive VR experiences in mathematics teacher education, with Rodríguez and García López (2026) a notable exception, and none has examined embodied interaction, reflective articulation, and professional knowledge development together in a single study. The present study responds to this situation by examining the insights preservice teachers provided through their embodied interaction with a mathematics-specific immersive VR environment, how they then described its affordances and constraints for teaching vectors, and what those insights and descriptions reveal about their emergent TPACK. Understanding these interactions, and the insights preservice teachers draw from them, is necessary for the design of teacher education experiences that support meaningful and grounded integration of immersive tools into mathematics instruction.

Purpose of the Study

The purpose of this study is to develop an empirically grounded account of how preservice mathematics teachers encounter and make sense of a content-specific immersive virtual reality environment, and what those encounters reveal about their developing professional knowledge. As noted above, teacher education programs are beginning to incorporate immersive technologies at a moment when the empirical understanding of how such experiences shape professional development remains limited (Alkaabi et al., 2025; Figueroa-Flores & Huffman, 2020). This study is an effort to contribute empirically informed understandings through close,

multimodal examinations of a small group of preservice teachers, attending to how embodied engagement in VR and subsequent structured reflection together contribute to the integration of technological, pedagogical, and content-related reasoning. The study is exploratory and interpretive in design. Its aim is to describe and interpret the forms of professional knowing that a content-specific immersive VR experience can generate and, in doing so, to open a line of inquiry that future research can extend, compare across contexts, and refine.

Specifically, the study follows preservice teachers' encounter with MathVR at three moments, and these three moments structure the research questions stated in Chapter Three. The first is embodied interaction: the insights preservice teachers provided, through what they did and said, while performing vector operations in MathVR. The second is reflective description: how, after the session, they described the affordances and constraints of MathVR for teaching Grade 12 vectors in Ontario. The third is emergent TPACK: what those insights and descriptions reveal about the integrated technological, pedagogical, and content knowledge (Mishra, 2019; Mishra & Koehler, 2006) that participants were beginning to form. I use these three terms, embodied interaction, reflective description, and emergent TPACK, whenever I refer to the purpose of the study, so that the same language carries from the research questions through the analyses to the discussion. Participants' virtual embodied activities (Farina, 2021; Johnson-Glenberg, 2018) and their reflective accounts are interpreted through TPACK as a synthesis lens, alongside virtual embodied cognition, which extends embodied views of learning into immersive digital environments and emphasizes how bodily action, perception, and spatial interaction shape meaning making in virtual space. Both perspectives serve as orienting concepts in this chapter and are developed in greater depth in Chapter Three.

Organization of the Thesis

This thesis is organized into six chapters that move from conceptual framing to theoretical grounding, to methodological design, to layered analysis, and finally to interpretation and contribution. Together, the chapters present an argument about how immersive, embodied experiences in MathVR seem to shape preservice mathematics teachers' emerging professional knowledge. Chapter One has introduced the study.

In Chapter Two, I review the literature and situate the study at the intersection of three bodies of scholarship:

1. Research on learning and teaching vectors, including persistent misconceptions and representational challenges;
2. Research on digital technology in mathematics education, with attention to immersive and embodied environments;
3. Research on teacher knowledge and technology integration, including TPACK and preservice teachers' developing professional reasoning.

Drawing on literature reviews of immersive virtual reality as a pedagogical tool in education, descriptive reviews of digital technologies in mathematics education, and conceptual scholarship on embodiment, presence, and teacher knowledge, the chapter establishes the conceptual and empirical foundation for the theoretical framework, the research questions, and the methodological design. In it, I also clarify how embodiment and presence, as theorized in immersive learning models such as that of Makransky and Petersen (2021), intersect with mathematics specific representational challenges, and I close the chapter with a summary of the review.

In Chapter Three, I set out the theoretical framework that guides the study, which rests on two complementary lenses. Technological Pedagogical Content Knowledge (TPACK) characterizes the integrated professional knowledge teachers draw on when they teach with technology, and virtual embodied cognition characterizes how self-location, agency, and bodily action within an immersive environment participate in mathematical meaning making. I then apply virtual embodiment to the specific domain of vectors, distinguishing the representational embodiment available when learners view and reason about vector relationships from an external vantage point from the immersive first-person embodiment available when learners act within the vector space itself. This distinction, which extends Gerofsky's (2010) contrast between observer viewpoint and character viewpoint, anchors the study's account of what immersive VR may afford that other representational technologies do not. The chapter closes with the three research questions that follow from this framing.

In Chapter Four, I detail the methodological design. I describe the research design, a qualitative exploratory case study informed by both Stake (1995) and Yin (2018), and the relationship among the three research questions. I then describe the context of the research and the ten participants, and the research methods, covering the scaled pilot implementation, the main session structure and chronology, and my role during data collection. Four data sources were generated: VR session screen recordings with audio, observational field notes, focused freewriting responses, and focus group discussion. After clarifying the frameworks operating at different analytic levels, I describe the data analyses aligned with each research question. Embodied VR interaction data were analyzed using Multimodal Analysis for Embodied Technologies, or MAET (Walkington et al., 2024a). Participants' reflective accounts in the focused freewriting and focus group data were analyzed using thematic analysis in the reflexive

tradition of Braun and Clarke (2006, 2021). Evidence from both was then integrated through TPACK as a synthesis lens. The chapter closes with the strategies I used to establish trustworthiness and analytic rigor by maintaining traceability between raw data and interpretive claims, and with the ethical considerations governing the study.

In Chapter Five, I present the results organized by research question and analytic approach: the embodied interaction analysis for Research Question One, presented as participant accounts and session matrices; the thematic analysis of the freewriting and focus group data for Research Question Two, organized around affordances, constraints, pedagogical implications, and equity considerations; and the TPACK synthesis for Research Question Three, which proceeds domain by domain through the six TPACK components before drawing the strands together in a cross modal synthesis and in four cross participant profiles of emergent TPACK.

In Chapter Six, I discuss the findings in relation to the literature and conceptual framing. I articulate implications for mathematics education research, embodied learning research, and teacher education, identify the study's limitations and boundary conditions, and propose directions for future research (e.g., longitudinal work and classroom-based interventions) grounded in the analytic structure established in Chapters Two through Five.

Chapter Two

Literature Review

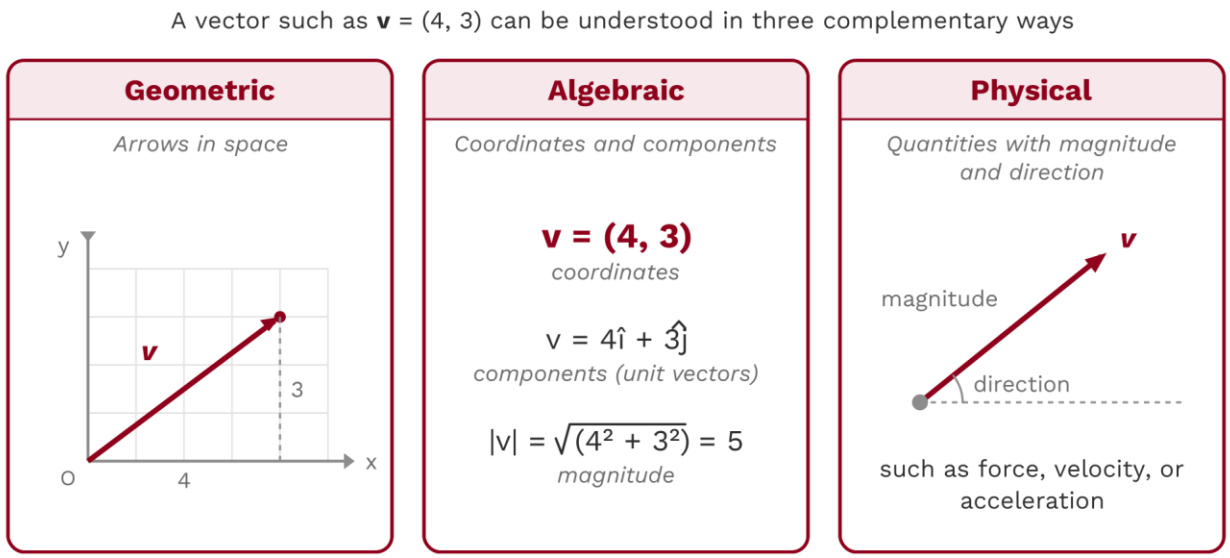
Learning and Teaching Vectors in School and Postsecondary Contexts

Chapter One established vectors as a persistent instructional challenge. This section develops that challenge in greater detail by examining how vector learning has been understood across school and postsecondary contexts, with particular attention to representational form, disciplinary framing, and recurring difficulties in conceptual interpretation. As introduced in Chapter One, representational form refers to the external form through which vector meaning is expressed and interpreted, such as arrow diagrams, component notation, column vectors, or vector equations, together with the interpretive work needed to connect those forms to direction, magnitude, and spatial meaning.

Vectors are inherently multifaceted. They may be understood geometrically as arrows in space, algebraically through coordinates or components, and physically as quantities such as force or velocity (see Figure 1). Research suggests that this conceptual range contributes to student confusion, particularly when learners encounter vectors across mathematics, physics, and engineering without developing a coherent understanding that transfers across contexts (Bollen et al., 2015; Cuevas-Vallejo et al., 2023; Demetriadou & Tzanakis, 2011; Latifa et al., 2021). According to the Ontario curriculum for Grades 11 and 12, this challenge is also curricular. Students may encounter vectors first as contextualized physical quantities in Grade 11 physics (Ontario Ministry of Education, 2007) before encountering their more formal geometric and algebraic treatment in Grade 12 mathematics (Ontario Ministry of Education, 2007). This sequencing can create a disciplinary asymmetry that complicates the coordination between physical interpretation and abstract mathematical structure.

Figure 1

Three Representations of a Vector



Each panel shows the same object: a vector carries both magnitude and direction, whether drawn as an arrow, written as coordinates, or realized as a physical quantity.

Empirical research consistently documents difficulties in students' reasoning about magnitude, direction, components, and vector operations across secondary and undergraduate settings (Appova & Berezovski, 2013; Barniol & Zavala, 2012, 2014; Nguyen & Meltzer, 2003; Sabah, 2023). These difficulties are not well explained as isolated misconceptions. Instead, they often arise from coordination problems. Learners may successfully manipulate one representation, such as component form, while misunderstanding what that representation implies for direction, magnitude, or geometric meaning, and they can perform an operation procedurally while failing to interpret the resulting vector as a spatial object in relation to others (García-Salmerón et al., 2025; Sabah, 2023; Tairab et al., 2020).

For this reason, recent scholarship increasingly frames vector learning as a challenge of coordinating meaning across representations, contexts, and forms of reasoning rather than simply mastering discrete rules or calculations.

This next section synthesizes recent empirical work around six interrelated themes. Rather than treating the research as a catalog of what students can and cannot do, the synthesis is organized to move between challenge and intructional response, tracing the difficulties that have been documented and the approaches that research suggests can address them.

Visualization challenges and the problem of "seeing" vectors

A first recurring theme in recent research is that vectors are challenging not simply because they are abstract, but because their defining features are distributed across visual and spatial conventions that are easy to misinterpret. In school settings, students may recognize that vectors "have direction," yet struggle to determine direction consistently, interpret magnitude from diagrams, or add vectors graphically using head-to-tail reasoning. In a study of Grade 11 science students, learners were generally able to distinguish vectors from scalars, but encountered substantial difficulty with tasks that required graphical addition, determining magnitude and direction, or performing scalar multiplication (Tairab et al., 2020). These tasks rely less on recalling a formula and more on interpreting arrow conventions accurately. As explained by Tairab et al. (2020), the broader implication is that diagrammatic vector work is not automatically intuitive. It involves learning a representational system with its own rules, grammar, and constraints.

According to the Ontario curriculum for Grade 12 (Ontario Ministry of Education, 2007), explicit and formal instruction on vectors in mathematics is concentrated in the Grade 12 university preparation course MCV4U Calculus and Vectors. In this course, the Geometry and Algebra of Vectors strand uses prior geometry and trigonometry to develop vector concepts for problem solving, with students representing vectors in two space and three space, performing operations such as addition, subtraction, and scalar multiplication, and connecting vector

representations to the equations of lines and planes in analytic contexts. In contrast, exposure to vectors in physics occurs earlier. In the Grade 11 university preparation course SPH3U Physics, vectors arise within the kinematics strand as students distinguish scalar and vector quantities and solve motion problems using tools such as scale vector diagrams and vector component. By Grade 12 in SPH4U Physics, vector work becomes more explicit and analytically demanding in dynamics, where students add and subtract two-dimensional vector quantities using vector diagrams, vector components, and algebraic methods, and apply these tools to motion and forces. This sequencing creates a disciplinary asymmetry in which students may meet vectors first as contextualized physical quantities in physics before encountering the more formal geometric and algebraic treatment in mathematics, making the coordination of physical interpretation and abstract structure an implicit curricular challenge rather than an assured developmental progression.

At the end of secondary schooling, vector visualization becomes even more complex when vectors are used to depict fields rather than single displacements or forces (Ontario Ministry of Education, 2007, p. 133). Hoyer and Girwidz (2024) show that many learners can generate vector representations (i.e., place arrows following instructions) while still struggling to interpret what the representation implies about field structure, that is, about how the strength and direction of the field vary from point to point across the region the arrows depict. Their study identifies specific interpretive errors, including treating the number of arrows as a proxy for field strength and misreading how density and direction communicate geometry. Importantly, the authors argue that drawing and interpreting are distinct skills that must be practiced in integrated ways, and they demonstrate that changing representational conventions (e.g., centering unit vectors) can significantly affect readability and geometric inference (Hoyer & Girwidz, 2024).

This is not a minor design issue; it underscores that visualizing vectors is not simply about adding pictures to instruction, but about carefully selecting and sequencing representational systems that reduce ambiguity while maintaining mathematically meaningful structure.

Visualization difficulties persist at the university level as well. Research using vector concept assessments shows that undergraduates often demonstrate uneven understanding across vector properties. Students may successfully identify components yet struggle with direction in unit vector notation, with constructing graphical representations from symbolic expressions, or with interpreting scalar multiplication geometrically (García-Salmerón et al., 2025; Sabah, 2023). This pattern suggests that visualization difficulties persist even as instruction becomes more algebraically advanced, likely because formalism, the ability to manipulate symbolic and algebraic expressions, correctly according to their rules, can conceal representational weakness; students may be able to carry out calculations while still lacking a clear spatial understanding of what the numbers actually represent (García-Salmerón et al., 2025).

Representational coordination and translation difficulties

A second major theme centers on the challenges learners face when translating across representational registers. Contemporary physics education studies explicitly frame these issues through the lens of semiotic transformation, the act of converting a mathematical idea from one representational system into another, for instance from an algebraic expression into a graph. Campos et al. (2020) examined how students recognize and convert among three common representations of electric fields: algebraic notation, vector field plots, and field line diagrams. They found that success depended strongly on the direction of translation. Students showed relatively strong coordination between algebraic notation and vector plots, but conversions involving field lines were far more fragile, particularly when magnitude had to be inferred from

line density. These findings align with a broader claim from semiotic representation theory that conceptual understanding depends on coordinating meaning across registers rather than remaining within a single representational system (Duval, 2006). Recent work extends these findings by focusing on the vector-field plot specifically. Campos et al. (2026) report that students' interpretation and construction of vector-field plots are often dominated by surface features and contextual assumptions, and that conversion tasks introduce additional interaction effects between source and target representations, producing difficulty even when students can handle each representation in isolation. In other words, difficulty does not always lie in the representation itself, but in the act of moving between them.

A parallel pattern appears in research on vectors themselves, not only vector fields. Using the Test of Understanding of Vectors, Sabah (2023) documents that university science and engineering students show systematic difficulties across vector properties, reinforcing the view that representational competence is not uniformly acquired through prior schooling. García-Salmerón et al. (2025) similarly operationalize vector understanding as multidimensional, emphasizing that students' performance profiles vary across defining, distinguishing, and applying the vector concept; suggesting that successful work in one dimension (e.g., applying procedures) does not guarantee flexible recognition or translation across contexts and notations.

Importantly, these findings challenge the assumption that representational competence is a background skill that students either possess or lack. Küchemann et al. (2021) developed and validated an inventory to assess representational competence for vector fields specifically, highlighting that students confuse conventions between field lines and vector plots and struggle with translation between them.

Further, Edelsbrunner et al. (2023) show that representational competence and conceptual knowledge are related but distinguishable constructs in undergraduate electromagnetism learning. Students can display high representational competence, reading and producing field diagrams and vector plots correctly, without corresponding conceptual mastery, that is, without a secure grasp of the physical relationships those diagrams stand for. This supports the idea that representational fluency, the ability to read, produce, and move between representations of the same object, is necessary but insufficient for deep understanding. Together, these studies shift the explanatory focus from a claim that students do not understand vectors to a claim that students struggle to coordinate what counts as information across representations, a framing that has direct relevance for instructional design (Küchemann et al., 2021).

Magnitude versus direction misconceptions and representational overgeneralization

A third theme concerns persistent confusions about magnitude and direction. Although vectors are routinely defined as quantities with magnitude and direction, empirical research shows how easily students overgeneralize visible features of representations and misassign meaning (Bollen et al., 2017; Heckler & Scaife, 2015; Mikula & Heckler, 2014; Tairab et al., 2020). In secondary settings, students may approach direction as a simple angle reading task detached from a reference frame, or interpret negative components as indicating a negative magnitude. These misunderstandings often surface when students shift between graphical and algebraic forms (Tairab et al., 2020). At the level of vector fields, Hoyer and Girwidz (2024) found that students sometimes infer strength from the number of arrows rather than their length or scale. This error reflects a reasonable perceptual strategy applied to a representational system whose conventions are not yet fully understood. Such findings resonate with earlier work arguing that standard arrow diagrams can invite systematic misunderstandings in vector addition

and subtraction when students interpret them as literal pictures rather than symbolic depictions of directed quantities (Heckler & Scaife, 2015).

Magnitude-direction issues also appear in postsecondary assessments that foreground unit-vector notation and dot/cross products. García-Salmerón et al. (2025) note broad student difficulty with dot and cross product development, even when basic operations show more stability, suggesting that the move from geometric intuition ("projection," "perpendicular") to formal computation remains under-supported for many learners. Sabah (2023) similarly found that tasks involving scalar multiplication or unit vectors often pose difficulty. Although students may carry out these operations symbolically, they frequently struggle to connect the procedures to underlying geometric transformations. These findings reinforce a central point: magnitude and direction are not two independent attributes to be "remembered," but relations that must be understood in coordination with reference frames, scaling, and operations that transform vectors.

Tensions between algebraic and geometric reasoning

A fourth theme concerns the tension between algebraic and geometric reasoning, especially in postsecondary linear algebra, where vectors are treated as elements of vector spaces and as objects related to span, independence, and transformation. Research shows that many students rely heavily on algebraic procedures, such as solving systems of equations, performing row reduction, or computing determinants, without developing stable geometric interpretations of these operations (Bernier & Zandieh, 2024; Cuevas-Vallejo et al., 2023; Khemane et al., 2024). Bouhjar et al. (2021) provide empirical evidence that instructional approach makes a difference. In their study, students in inquiry oriented linear algebra classrooms developed concept images of span and linear independence that were more consistent with formal definitions and showed more coordinated reasoning than students in comparison classrooms,

even though performance differences varied by concept. These findings suggest that geometric understanding is not an optional supplement to algebraic skill. It can be intentionally developed through tasks that emphasize meaning making rather than procedural completion (Bouhjar et al., 2021).

This tension between algebraic and geometric reasoning also appears in studies that compare student strategies across different task environments. Bernier and Zandieh (2024) examined undergraduate students' approaches to a game-based vector task and a traditional pen and paper task. They identified multiple distinct strategy patterns rather than a single dominant form of reasoning. Their analysis suggests that different environments can cue different types of thinking, including exploratory and representational approaches as well as procedural ones. Students do not automatically integrate these modes into a coherent vector concept without instructional support. Although game-based environments differ from immersive VR, they point to a similar conclusion: design decisions influence which features of vectors become visible and which forms of reasoning are encouraged (Bernier & Zandieh, 2024).

Research in teacher education further illuminates this issue. Altınbaş et al. (2025) studied preservice teachers' learning in linear algebra using multiple representations based and problem-based instruction. They examined outcomes in terms of dimensions of understanding and self-efficacy. While their work does not focus exclusively on vectors, it shows that instruction that deliberately connects representations and problem structures can shape both conceptual development and teachers' confidence. This insight is particularly relevant for the present dissertation, which examines preservice teachers learning vectors in an environment where shifting representations and spatial reasoning are central.

Cross-disciplinary differences between mathematics and physics education research

A fifth theme concerns differences in how vector learning is conceptualized in mathematics and physics education research. Physics education research frequently emphasizes vectors as quantities in context (forces, velocities, fields) where learning difficulties often emerge from the interaction between mathematical form and physical interpretation. For example, Campos et al. (2020) show that students' difficulties with representational conversion are closely tied to their struggle to adopt a field-based perspective. In this framing, representation is not a neutral tool but is intertwined with disciplinary ontology, particularly the shift from viewing force as localized to understanding fields as distributed quantities. In contrast, mathematics education research, especially in linear algebra, tends to treat vectors as abstract objects within formal systems. Here, learning challenges are often framed in terms of definitions, classification, such as subspace and span, and the coordination of symbolic and geometric meanings (Bouhjar et al., 2021).

Recent studies highlight these disciplinary contrasts more directly. Khemane et al. (2024), in a study of engineering students' understanding of Stokes' theorem, document misconceptions in prerequisite ideas such as vectors, curl, and parametrization. Using representational transformation diagrams, they compare textbook treatments in mathematics and physics and identify distinct anchor approaches that shape how line and surface integrals are introduced. Their analysis identifies distinct anchor approaches that shape how students are oriented to the same formal expressions. These approaches position vectors differently within explanatory systems, influencing how students interpret symbolic expressions and geometric configurations. They suggest that vector learning problems are not solely cognitive. They are also curricular and discursive. Students encounter different explanatory norms, representational emphases, and

application contexts depending on whether vectors are taught within mathematics or physics sequences. For school mathematics education, this distinction is significant. Vectors are often introduced with an implicit orientation toward later applications in physics and calculus. Yet the transfer between these disciplinary framings cannot be assumed. Differences in how vectors are defined, visualized, and contextualized may influence how students coordinate meaning across subjects.

Instructional approaches that support vector learning

A sixth theme addresses what instructional approaches show promise for supporting the forms of spatial and representational reasoning that vector learning demands. The emerging picture from recent research is that explicit coordination of representations, structured scaffolding of interpretive skills, and tasks that make spatial structure open to discussion can support meaningful improvements in student understanding, while also revealing the trade-offs that instruction must navigate between conceptual depth and cognitive demand.

In vector field learning, Hahn and Klein (2023) developed multi-representational learning tasks that integrate sketching activities and interactive simulations to help students interpret vector field diagrams and connect them to algebraic representations. Importantly, they treated representational competence as something to be developed through instruction rather than assumed in advance. Their findings show improvements in both conceptual understanding and representational performance. However, these gains were accompanied by higher reported cognitive load, as students had to coordinate multiple forms of representation simultaneously. A subsequent field implementation of the same approach produced similar results. Students achieved more, but they also experienced increased cognitive load, highlighting the ongoing

design challenge of balancing conceptual depth with manageable complexity (Hahn & Klein, 2025).

Other interventions locate the primary challenge in coordinating equations and diagrams. Klein et al. (2021) designed a serious game to support students in connecting vector field equations with graphical representations in introductory physics. This work reflects a broader shift toward interactive environments that position representational translation as a central learning activity rather than a test of prior understanding. Across studies, a consistent finding emerges. Translation does not develop automatically through exposure to multiple representations. It must be deliberately designed, practiced, and scaffolded (Klein et al., 2021; Küchemann et al., 2021).

In mathematics education contexts, technology-supported instruction has also been investigated as a means to strengthen conceptual and procedural understanding of vectors. Aliu et al. (2025) report that ninth-grade students who learned vectors using GeoGebra demonstrated stronger conceptual and procedural performance than a comparison class taught through teacher-centred explanation using charts, paper, and pencil, where students continued to describe vectors as simple arrows and reported difficulty visualizing them in space even after instruction. At the same time, they note practical constraints such as teacher preparation and access to technology. Although these findings are context specific, they support a central claim of this dissertation. Vector understanding depends heavily on how learners engage with representations. Tools that allow dynamic manipulation may open new possibilities for coordinating meaning, but only when they are embedded within carefully structured instructional sequences.

Synthesis: Coordinating Representations as a Precondition for Immersive Approaches

Together these six themes in the research on vector learning point toward a pattern: students struggle most when they must coordinate representations, especially when connecting algebraic expressions, geometric interpretations, and spatial reference frames. Visualization is a necessary condition for understanding what vectors are and how vector operations function. At the same time, visualization alone does not guarantee understanding. Students may draw arrows without grasping their meaning, compute components without attending to direction, or move between representations only when tasks are tightly scaffolded (Campos et al., 2020; Hoyer & Girwidz, 2024; Tairab et al., 2020). Intervention studies reinforce this interpretation. When instruction deliberately structures representational translation through sketching, simulation, inquiry-oriented tasks, and explicit attention to conventions, students' conceptual and representational performance can improve (Edelsbrunner et al., 2023; Hahn & Klein, 2023; Küchemann et al., 2021).

These findings suggest a need for teaching and learning approaches that make spatial relationships more visible and allow learners to act directly on vectors. In such environments, moving, orienting, and manipulating representations become central to learning rather than secondary features. Even without yet focusing on immersive VR, the literature reviewed seems to point toward the potential value of environments that reshape how vectors are seen, handled, and connected across forms by students. The next sections therefore turn to research on digital tools that enhance visualization and interaction, as well as to immersive and embodied learning theories that help explain when and why such tools may meaningfully support mathematical understanding (Drijvers & Sinclair, 2024).

Technology in Mathematics Education

A second body of literature relevant to the Geometry and Algebra of Vectors strand of the Ontario Grade 12 curriculum focuses on digital technologies in mathematics education, not as neutral tools for delivering content, but as environments that reshape what becomes visible, manipulable, and open to discussion in mathematical activity. When learners drag points, rotate objects, or dynamically transform representations, there is a shift in what can be seen and discussed. Recent syntheses emphasize that digital technologies can open new mathematical actions (dragging, tracing, dynamically transforming objects) and new learning spaces, but that these possibilities depend on how tools are designed and how teachers guide their use (Drijvers & Sinclair, 2024). Broader reviews of technology enhanced mathematics learning likewise emphasize that learning gains cannot be attributed to technology alone but emerge from the alignment between technological affordances, representational tasks, and pedagogical structures (St Omer et al., 2025).

This overview serves a specific analytic purpose in the present study. The preservice teachers in this research were asked to judge whether an immersive environment could support the teaching of vectors, and the terms in which such a judgement can responsibly be made are not self-evident. Mathematics education research has already established a set of findings about digital tools that constrain what any claim about a new technology can reasonably assert: that learning gains cannot be attributed to a tool considered in itself (Drijvers & Sinclair, 2024; St Omer et al., 2025), that representational richness generates interpretive demands as well as opportunities (Cameron et al., 2024; Edelsbrunner et al., 2023; Pedersen et al., 2025), that instrumented technique reshapes what counts as mathematical work rather than simply accelerating it (Clark-Wilson et al., 2020; Engelbrecht & Borba, 2023; Pedersen et al., 2025),

and that teachers' task design and framing determine much of what a tool makes available (Clark-Wilson et al., 2020; Komatsu & Jones, 2020). Reviewing this body of work therefore does two things here. It supplies the criteria against which the claims made for immersive virtual reality later in this chapter should be assessed, and it establishes that immersive virtual reality is best understood not as a departure from this landscape but as an intensification of features the field has already identified as consequential (Dimmel et al., 2021; Drijvers & Sinclair, 2024; Walkington et al., 2024b). The sections that follow move through that evidence base before returning, at the close, to the question of where immersive virtual reality sits within it.

In parallel, large scale mapping work has examined how the research field itself has evolved. Hwang et al. (2023) use a scoping review and topic modeling approach to analyze 2,433 peer reviewed studies published between 1981 and March 2022. They report sustained growth in research attention and demonstrate that technology use in mathematics education has developed into a distinctive area with multiple trajectories rather than a single unified trend. Together, these reviews position digital technology not as a peripheral add on, but as a central driver of both scholarly and pedagogical development.

Recent synthesis work also emphasizes that the educational value of digital tools depends on the purposes for which they are designed and used, and on whose goals and perspectives are at stake. In a narrative overview, Drijvers and Sinclair (2024) propose a five-purpose framework for understanding how digital technologies are used in mathematics education research and practice. The five purposes are: to improve mathematics learning; to understand how digital technologies affect mathematics learning and teaching; to design new approaches to mathematics learning; to provide equitable access to mathematics learning; and to change mathematics curricula and teaching and assessment practices (Drijvers & Sinclair, 2024, p. 240). This

framework argues that the question of whether digital technology is effective in mathematics education cannot be answered in the abstract, because the answer depends entirely on which purpose is being pursued, under what theoretical assumptions, and from whose perspective. The most extensively studied purpose has been improvement, supported by a substantial meta-analytic base showing positive effects, while purposes related to equity and curriculum reform remain comparatively underexplored. The framework therefore invites a more precise research orientation: rather than asking whether technology works in mathematics education, it asks which purpose is served, for whom, and under what conditions. This purpose-oriented view helps explain the pattern Hwang et al. (2023) document: growth in research volume without a single unifying trajectory, since different studies are often answering different questions about purpose rather than converging on one.

The purposes framework helps make sense of a related observation from Engelbrecht and Borba (2023). They note the increasing use of computer algebra systems and dynamic geometry software, alongside new multimodal learning spaces, and argue that digital technologies are reshaping what counts as mathematical practice and where it occurs. Read through Drijvers and Sinclair's framework, this single observation touches at least two purposes at once, designing new approaches to mathematics learning and changing curricula, teaching, and assessment practices, which is itself consistent with their argument that a tool's effects rarely reduce to one purpose in isolation.

A longstanding tension in the literature concerns whether digital tools primarily enhance procedural efficiency or support conceptual development. Automation, rapid calculation, and symbolic manipulation can streamline mathematical work, but conceptual engagement requires interpretation and justification of outputs. Engelbrecht and Borba (2023) suggest that

contemporary systems allow educators to redirect focus from manual execution toward structural understanding. Yet such a shift is not automatic. It depends on how tasks are designed and how teachers position tool outputs within classroom discourse.

The potential of digital environments to link symbolic, graphical, numeric, and geometric registers does not automatically translate into deeper representational understanding (Edelsbrunner et al., 2023). Learners must actively make sense of these connections, treating the outputs of digital tools not as finished answers but as representations that require interpretation. This demand is significant because representational fluency functions simultaneously as a goal of instruction and as a prerequisite for benefiting from the learning environment: without strategies for interpreting dynamic displays, the richness of a multi-register environment can overwhelm rather than support. Empirical work on computer algebra systems (CAS), one of the tool categories through which this representational demand plays out concretely, illustrates this clearly. In a repeated-measures study, Cameron et al. (2024) show how Year 11 students' beliefs about whether CAS constitutes "proper mathematics" shaped whether they treated its outputs as representations worthy of interpretation or as shortcuts bypassing the mathematical work. The implication is that digital representational environments require learners to adopt interpretive stances toward what the tools produce, and those stances are shaped not only by cognitive readiness but by the affective and cultural dimensions of how students understand what mathematics is and what it means to do it.

Research on CAS further illustrates how technology can reveal conceptual gaps rather than simply repair them. In an empirical study with Danish upper secondary students, Pedersen et al. (2025) examined how students interpreted the universal quantifier for all in differentiability tasks while using CAS. They found that CAS use could clash with students' interpretations and

reveal gaps between what students thought a statement meant and how they acted on it. In this sense, instrumented techniques can reshape how mathematical meaning is constructed. This evidence challenges the common assumption that CAS simply supports learning. Rather than serving only as a computational aid, CAS can play a diagnostic role by surfacing conceptual tensions, especially in tasks that require students to coordinate local and global reasoning or move between symbolic expressions and graphical representations.

These findings point to the need for theoretical lenses that treat tool use as learned, structured activity rather than a neutral channel for thinking. In their survey for the journal ZDM Mathematics Education, Clark-Wilson et al. (2020) explain that classroom practice with technology is often theorized through instrumental genesis and orchestration, because teaching with tools involves configuring resources, exploiting affordances, and responding to unfolding interactions; they further note that dynamism is now increasingly interpreted within multimodal, embodied environments rather than as an on-screen property alone. They also caution that the field still lacks sufficient mid- to large-scale naturalistic studies of classroom technology use, which limits generalization across tools, contexts, and populations. From this perspective, conceptual use of a digital tool is not a fixed feature of the software but an achievement of classroom practice, shaped by tasks, norms, and teachers' moment-to-moment decisions.

Dynamic geometry systems and broader dynamic mathematics software occupy a related but distinct position. While CAS is typically associated with algebraic and analytic work, DGS emphasizes geometric relationships through dynamic manipulation, dragging, and invariant preserving transformations. Meta-analytic evidence suggests that DGS environments such as GeoGebra are frequently linked with positive learning outcomes, though effects vary by context and design (Juandi et al., 2021; Suparman et al., 2024). Together, these reviews suggest that DGS

can strengthen visualization and relational reasoning, but the nature and size of the benefits depend on the instructional ecology in which the tools are embedded.

At the same time, limitations in technology integration remain well documented. Drijvers and Sinclair (2024) argue that issues of equity in access and use are important yet under researched and that digital technologies continue to challenge curriculum, teaching, and assessment practices. Their review suggests that comprehensive accounts of how classrooms sustain meaningful tool use under practical constraints are still emerging. At the level of classroom practice, empirical studies similarly reveal how teachers' instructional decisions shape, and sometimes restrict, what digital tools make possible. Mailizar and Fan (2021), in a qualitative analysis of ten upper-secondary teachers, report that participants often "failed to take advantage of pedagogical opportunities" that could have supported students' knowledge construction. Their findings emphasize that technology alone does not generate conceptual growth; rather, tasks must be deliberately designed to activate the affordances of digital tools. These critiques do not reject digital technology in mathematics education. Instead, they call for more precise theorization of the relationships among tools, tasks, teacher decision-making, and students' meaning-making practices.

Dynamic Geometry Systems and Relational Understanding

There is a long history of research on digital technologies and their usefulness for mathematics teaching and learning. One category of digital technology that has shown particular promise for the kind of spatial and representational reasoning at stake in this dissertation is Dynamic Geometry Systems (DGS). This section reviews that history because DGS represents the most immediate precursor to immersive VR: both share the goal of making mathematical relationships manipulable rather than static, and DGS research therefore identifies both what a

manipulable environment can offer vector learning and where two-dimensional, screen-bound manipulation reaches its limits, limits that immersive VR is positioned to address.

Dynamic Geometry Systems have long occupied a central place in digital technology research in mathematics education. With DGS, learners can construct geometric objects and then transform them while preserving structural relationships. This feature positions DGS as an important precursor to newer digital visualization environments, since these applications frame mathematical ideas as manipulable relations rather than static diagrams. In a systematic review of secondary geometry teaching, Sunzuma (2023) identifies GeoGebra as one of the most frequently used tools across studies from 2010 to 2022. Most of these studies focus on effectiveness and outcomes, suggesting a strong empirical base but also pointing to the need for deeper theorization of how and why DGS supports conceptual change (Sunzuma, 2023).

A defining feature of DGS is dragging, which is the ability to move points or objects while preserving construction constraints. In an empirical study, Yao (2020) describes drag mode, macros, and loci as core features that make a geometry tool a "dynamic geometry environment," emphasizing that responsiveness and continuous feedback can support exploration of relationships among geometric objects. This responsiveness is pedagogically significant because it allows learners to generate multiple instances quickly, observe variation, and search for invariants (Komatsu & Jones, 2020; Yao, 2020). Unlike static diagrams, DGS representations can embody a family of configurations within a single construction, encouraging learners to treat geometry as relational structure rather than as isolated figures (Dimmel et al., 2021; Komatsu & Jones, 2020). However, Yao (2020) emphasizes that these affordances do not operate automatically. Learners must develop purposeful ways of using the tool, and conceptual growth depends on emerging schemes of interaction.

Research on conjecturing and proving further illustrates how relational reasoning is both supported and constrained within DGS environments. In an empirical classroom analysis, Komatsu and Jones (2020) analyzed how Grade 9 students used a dynamic geometry environment alongside paper and pencil work during generalization and proof. They found that the dynamic environment supported conjecture formation, but students initially struggled to construct formal proofs within the software. Proof development improved when students shifted to paper and pencil representations. The authors interpret this pattern as evidence of a division of representational labour; DGS can powerfully support conjecture generation and testing, while formal proof may require additional symbolic articulation and theorem-based justification. The implication is not that DGS is deficient, but that different representational environments contribute distinct resources for relational reasoning (Komatsu & Jones, 2020).

This complementarity bears directly on relational understanding, which involves explaining why a relationship holds and how it persists under transformation rather than observing that it holds in one particular case. Dragging can make invariance an explicit object of mathematical attention, but perceptual recognition of invariance does not by itself constitute relational understanding (Komatsu & Jones, 2020). The division of representational labour is what matters for the present study. Komatsu and Jones's students consolidated by turning away from the software toward paper, and a learner wearing a head-mounted display cannot turn away. Immersive environments therefore inherit this problem in a sharper form, since the consolidation work that DGS research locates outside the environment must be designed into immersive activity or scheduled to follow it. This study's sequencing of headset use, focused freewriting, and focus group discussion follows directly from that requirement.

The role of the teacher becomes even more visible when DGS is used for exploratory learning. Although DGS environments enable students to generate and test conjectures efficiently, inquiry does not arise automatically from access to these affordances. Komatsu and Jones (2020) demonstrate that productive interplay between dynamic geometry and paper-based representations did not occur spontaneously; rather, it was deliberately stimulated through teacher configurations and exploitation modes aligned with instrumental orchestration. This finding has two implications for DGS research. First, exploration should be conceptualized as socially organized activity, structured by classroom norms, task framing, and teacher interventions, rather than as an individual cognitive response to software features. Second, DGS based learning is best understood as a coordinated system of tools and representations. Transitions across media and modalities, including movement from screen to paper, from dynamic manipulation to static inscription, and from visual perception to verbal or symbolic articulation, constitute central sites of mathematical meaning making rather than peripheral shifts in format.

Meta analytic research clarifies the scope of learning outcomes associated with DGS, particularly in relation to visualization and spatial structuring. Suparman et al. (2024) report a strong positive effect of GeoGebra assisted geometry instruction on spatial visualization, while also noting that effect sizes vary according to participant characteristics and educational level. More broadly, Juandi et al. (2021) identify positive average effects of GeoGebra based learning on mathematical abilities across included studies, suggesting that dynamic visualization environments can contribute to learning gains when implemented under supportive conditions. These reviews should be interpreted cautiously in this synthesis, because they combine studies that differ in design, measures, and instructional contexts. At the same time, they point to a

consistent pattern. Across varied settings, DGS environments are associated with gains in geometric thinking that depend on visualization, transformation, and attention to structural relationships. These forms of reasoning are closely aligned with relational understanding, which requires learners to recognize how properties are maintained and how relationships are organized across changing configurations.

At the same time, DGS research has consistently highlighted limitations and unresolved questions, particularly as mathematics education has increasingly engaged with three-dimensional concepts and with embodied participation in mathematical activity (Abrahamson et al., 2020; Alberto et al., 2022; Dimmel et al., 2021; Drijvers & Sinclair, 2024). Traditional DGS environments are typically screen-bound and two-dimensional, which can constrain how learners perceive depth, scale, and viewpoint. Recent research has therefore explored how DGS might evolve in ways that integrate richer spatial interaction while preserving the representational strengths of dynamic constructions. For instance, in an empirical paper, Walkington et al. (2024b), analyzed video data from high school students using an augmented reality version of GeoGebra and document new forms of interaction related to perspective, scale, and three-dimensional reasoning, including gesture and whole-body movement. Although this research moves beyond conventional two-dimensional DGS, it remains conceptually continuous with the DGS tradition. It demonstrates that the dynamic dimension of geometry can be reconceptualized as embodied engagement with mathematical objects rather than limited to cursor-based manipulation. This line of research helps clarify a boundary condition of classic DGS; although it supports invariant reasoning through dragging and transformation, DGS may offer limited support for viewpoint-dependent spatial reasoning unless the interaction space is expanded.

The augmented reality extensions of dynamic geometry described above, in which learners shift perspective, work at varying scales, and use whole-body movement to manipulate figures, make visible a theoretical link that was present in dynamic geometry from the outset but easy to overlook. Even in traditional DGS environments, meaning making is not purely visual or symbolic. Learners drag points, trace paths, pause, compare configurations, and reorient constructions as they look for stable relationships. In this sense, DGS provides an early example of how mathematical concepts can be developed through structured action within a representational environment. Broader technology reviews increasingly recognize this shift toward multimodality and embodiment (Cevikbas et al., 2023; Ng et al., 2025). For instance, Drijvers and Sinclair (2024) argue that digital technology research has produced new design paradigms that seek to align tool features with learners' coordinated perception, action, and reasoning. Together, these perspectives position DGS as part of a wider movement toward understanding mathematical learning as embodied and multimodal activity.

These insights are particularly relevant for topics such as vectors, which require spatial structuring, coordination of magnitude and direction, and interpretation of operations as transformations in space. The DGS research tradition therefore lays important conceptual groundwork for examining immersive and embodied environments. It clarifies what structured dynamic interaction can support, where it encounters boundaries, and why mediation and guidance remain essential for developing relational understanding (Yao, 2020).

Augmented Reality and Immersive Virtual Reality in Mathematics Education

Augmented reality and immersive virtual reality have shifted from peripheral innovation topics to sustained areas of research in mathematics education, particularly in domains that require learners to coordinate spatial structure, multiple representations, and dynamic change.

Both technologies expand what can count as mathematical representation and mathematical action. Learners can perceive, manipulate, and even inhabit three-dimensional objects and coordinate systems in ways that differ from paper, static screens, or conventional dynamic geometry environments. In broad terms, AR overlays digital objects or inscriptions onto the physical world while leaving learners able to see the room and the people in it, whereas immersive VR replaces that view with a fully virtual environment delivered through a head-mounted display (HMD) (Makransky & Petersen, 2021; Radianti et al., 2020; Walkington et al., 2024b).

As the preceding section established, dynamic geometry supports relational invariance through constrained dragging within a two-dimensional screen space. Augmented reality and immersive virtual reality represent the next stage in that progression, extending interaction into three-dimensional fields and allowing learners to engage with mathematical objects in expanded spatial contexts. This shift reflects a broader trajectory in mathematics education technology, moving from screen-based manipulation toward environments in which learners can move within, around, and through mathematical objects. The integration of immersive technologies into mathematics education has therefore been framed as an attempt to overcome limitations of two-dimensional representation, particularly in the visualization of complex three-dimensional geometrical concepts (Cevikbas et al., 2023; Lai & Cheong, 2022; Su et al., 2022; Tan & Tay, 2022).

These developments matter because mathematical meaning often depends on coordinating visual structure with symbolic relations and operations, particularly in geometry, vectors, transformations, and calculus. At the same time, AR and VR research cautions against treating "more immersive" as automatically "more effective" (Buchner & Mulders, 2026; Coban

et al., 2022; Makransky & Petersen, 2021; Radianti et al., 2020). Findings vary across contexts and designs, and familiar implementation challenges (technical reliability, classroom orchestration, accessibility, teacher readiness) function as boundary conditions that shape what AR and VR can realistically contribute to mathematics teaching and learning (Walkington et al., 2024b).

A recent review of AR in mathematics education illustrates both the expansion of the field and its current contours. Bulut and Borromeo Ferri (2023) analyzed 42 studies and found that AR research concentrates heavily on geometry and spatial reasoning. Reported benefits frequently involve visualization and engagement, yet persistent challenges include hardware and software limitations and the need for more robust evidence on task design. Importantly, the review's scope highlights a methodological pattern: many AR studies are short in duration and evaluate outcomes at the level of immediate performance or attitudes, leaving underexplored questions about durability of learning, representational transfer, and how teachers mediate AR-supported activity

A related development is the growing tendency to frame AR and VR not merely as instructional media but as environments that reorganize mathematical activity. In introducing a special issue on augmented reality for calculus, Swidan et al. (2025) frame the problem as one of mathematical meaning making, emphasizing how learners' gestures, tracing, and attention to feedback become part of conceptual development rather than peripheral supports for motivation. This signals a shift from asking whether AR works to asking what kinds of mathematical work AR makes possible and at what cost.

Three-Dimensional Visualization and Spatial Cognition

The most consistent justification for AR and VR in mathematics education is representational. Three-dimensional visualization is seen as a way to narrow the gap between abstract symbolic descriptions and the spatial structures they describe. This rationale is especially prominent for geometry and vector-related topics, where static diagrams and two-dimensional displays often require learners to infer depth, orientation, and relative position from limited visual cues. Schutera et al. (2021), for example, describe the development of an AR application called cleARmaths designed to support upper secondary vector geometry. Their argument is grounded in the idea that instruction relying on static representations such as chalkboard sketches can restrict students' ability to visualize and interact with three-dimensional problems, and that AR environments can provide more direct and intuitive spatial access to vector configurations. Because cleARmaths is reported as a design case rather than an efficacy study, Schutera et al. (2021) establish a rationale for spatial AR rather than evidence of its effect. The rationale itself recurs throughout this literature: AR may be most justifiable where the mathematical content is intrinsically spatial and where static, two-dimensional representations systematically hide structure, as in three-dimensional vector configurations, projections, and spatial relationships. Whether that rationale is borne out empirically is a separate question, taken up in the studies reviewed below.

Similarly, tools such as Construct3D, Geo+ (Rossano et al., 2020), SketchUp software (Auliya & Munasiah, 2019), and markerless AR systems such as ScholAR (Sarkar et al., 2020) demonstrate practical applications of AR for interacting with three-dimensional models. These technologies allow learners to rotate, reposition, and examine objects spatially, potentially making vector addition, scalar multiplication, and spatial decomposition perceptually available

rather than purely inferential processes (Chi-Poot & Martin-Gonzalez, 2014; Kounlaxay et al., 2021).

Empirical studies in geometry typically operationalize spatial benefit through achievement measures on three-dimensional geometry tasks or standardized spatial ability scales. For example, Koparan et al. (2023) developed an AR learning material focused on properties of three-dimensional shapes and reported improved learning outcomes along with generally positive student acceptance in a case study design. Although the study was not a large randomized trial, it reflects a common pattern in AR research: AR is used to externalize three-dimensional structure so that students can inspect, rotate, and interpret geometric configurations more directly, and achievement gains are interpreted in relation to this representational support.

Other studies shift attention from performance outcomes to changes in thinking processes. Wu et al. (2024) examined AR based mathematical picture books and reported differences in geometric thinking, cognitive load, and flow experience. This line of work treats affective and cognitive experience as potential mechanisms rather than as secondary outcomes. AR is assumed to reorganize attention and reduce the effort required to coordinate multiple geometric relationships by embedding them in perceptually accessible forms. At the same time, because this line of research often integrates novelty (AR books) with pedagogy (story-based tasks), attributing gains to AR alone is methodologically difficult. In many studies, immersive technology is introduced alongside novel pedagogical elements, such as story-based tasks or redesigned instructional sequences, which makes it difficult to disentangle the effects of the technology itself from the effects of task design and instructional innovation (Na & Sung, 2025; Yang et al., 2026). This methodological entanglement is compounded by the well-documented

novelty effect, in which short-term performance gains may reflect the excitement of a new environment rather than durable learning (Georgiou & Ioannou, 2021).

Immersive VR research similarly leverages full three-dimensional immersion for geometric reasoning. Su et al. (2022) compared a VR immersive geometry system against traditional paper-based instruction with 40 secondary students, targeting geometric ideas in which embodied viewpoint change and object manipulation are central, such as reasoning about volume relationships and deriving centers of gravity. They found that students using the VR system showed significantly higher learning motivation than the control group ($p = .001$, a medium effect), alongside higher post-test performance on the geometry content (Su et al., 2022, p. 6). This pattern, stronger motivation and improved conceptual performance on spatially demanding material, is consistent with findings from other VR studies in mathematics: Campos et al. (2022) and Lai and Cheong (2022) similarly report improved comprehension of abstract mathematical concepts in VR-supported environments, and Cevikbas et al. (2023) emphasize enhanced conceptualization and visualization in immersive contexts. Building on the viewpoint-change and manipulation affordances these studies identify, immersive tools in vector-specific contexts allow learners to manipulate and observe vector properties from multiple perspectives, which may support deeper structural understanding of direction, magnitude, and projection.

Together, these studies establish an empirical foundation within the VR literature: when mathematical content involves spatial invariants and transformations, immersive environments can create a perceptual field in which learners' movements, such as turning, approaching, or rotating, become part of the mathematical work itself. In these settings, evidence suggest that bodily action does more than navigate the interface. Bodily movement contributes to how

evidence is generated and interpreted, potentially reshaping how learners coordinate spatial relationships and conceptual structure (Farina, 2021).

Some of the most analytically valuable studies are those that avoid treating three dimensionality as automatically beneficial and instead investigate when it supports learning and when it creates friction. Walkington et al. (2025), for example, compared geometry learning in a tablet based dynamic geometry environment and a head-mounted display-based AR condition. They examined how the dimensionality of shapes moderated both learning outcomes and interaction patterns. In their design, dimensionality is treated as a conditional affordance rather than a general advantage meaning increased dimensionality may support initial sense making when learners are working with genuinely three-dimensional objects, yet it may offer little added value, or even introduce additional cognitive overhead, when tasks are effectively two dimensional. This conditional framing is especially important for mathematics education because it shifts the conversation away from broad claims that AR or VR is good for mathematics and toward more precise, content sensitive arguments: Immersive technologies may be most productive when they address specific representational challenges under carefully designed tasks and their value depends on the nature of the mathematical structure at stake and on how dimensionality interacts with learners' reasoning demands.

Interaction and Embodied Manipulation

A second major thematic strand centers on interaction and embodied manipulation. AR and VR do more than display enhanced images. They expand the range of possible actions on mathematical objects. Learners can grasp, resize, trace, reorient, walk around, and coordinate gestures with symbolic inscriptions in ways that are not possible with static diagrams or conventional screens.

One line of research treats embodied interaction as a resource for conceptual grounding. Immersive VR has been described as enabling students to explore and construct knowledge through direct experiential engagement (Figueroa-Flores & Huffman, 2020; Meyer et al., 2019; Piovesan et al., 2012). Bos et al. (2022), for example, studied an augmented reality sandbox designed to support calculus concepts such as plane equations and gradients. Their tasks deliberately guided learners through perception based and action-based engagements, encouraging students to shape and observe surfaces before connecting these experiences to formal mathematical constructs. Their case-based analysis suggests that AR can function as a bridge between sensorimotor activity and symbolic meaning when that bridge is intentionally designed and orchestrated. Embodied interaction does not automatically produce abstraction. In other words, it should be structured so that bodily experience becomes mathematically consequential.

A second line of work approaches embodied interaction as a feature of collective mathematical activity. Although much mathematics AR/VR research remains focused on individual learners, emerging work develops analytic tools to capture how gesture, viewpoint, and object manipulation become shared resources in collaborative reasoning. Walkington et al. (2024a) propose Multimodal Analysis for Embodied Technologies (MAET) as a method adapted to environments where learners' gestures, body positioning, and talk unfold together with dynamic virtual objects and shifting perspectives. In the context of AR/VR mathematics studies, this methodological move is important because it positions embodied action as legitimate evidence of mathematical thinking rather than as mere engagement behaviour. It also reinforces a design point: AR/VR systems that support meaningful mathematics learning typically do so by enabling learners to coordinate multiple modalities (gesture, speech, viewpoint changes, object

transformations) in ways that paper-based or screen-based classroom tools without motion tracking often cannot capture or stabilize.

Motivation, Engagement, and Learning Gains

Motivation and engagement constitute a third prominent theme in mathematics AR and VR research. Many studies report that AR/VR environments increase situational interest, enjoyment, or student willingness to participate, especially compared with paper-based or lecture-based instruction (e.g. Bulut & Borromeo Ferri, 2023; Su et al., 2022; Wu et al., 2024). In mathematics contexts, these outcomes are often interpreted as addressing a persistent participation challenge. Students may be more inclined to explore geometric or functional relationships when they can manipulate and visually see them in spatially rich settings. Su et al. (2022) document motivational gains alongside performance improvements. Wu et al. (2024) link AR experiences to flow states. These findings align with broader immersive learning research suggesting that immersive media increase presence and enjoyment (Makransky & Petersen, 2021; Radianti et al., 2020). At the same time, the literature consistently cautions that motivational enhancement does not automatically translate into conceptual advancement (Coban et al., 2022; Makransky & Mayer, 2022; Parong & Mayer, 2018). Increased enjoyment and presence can create conditions of attention, curiosity, and persistence, but these affective states must be converted into mathematical learning through carefully designed tasks, structured prompts, and explicit connections to formal representations. Without such mediation, engagement may remain experiential rather than analytic (Su et al., 2022).

This tension becomes clearer when mathematics specific findings are situated within broader immersive learning research. Studies across domains indicate that immersive media often increase presence and enjoyment, whereas learning gains depend more strongly on

instructional design than on medium alone (Buchner & Mulders, 2026; Coban et al., 2022; Radianti et al., 2020). Although this evidence base extends beyond mathematics, it offers an important disciplinary caution: in mathematics, where progress depends on representational coordination, abstraction, and justification, the value of AR and VR lies not in immersion itself but in how immersive interaction is directed toward mathematical noticing, explanation, and reasoning and justifications (Makransky & Petersen, 2021).

Design Challenges and Pedagogical Distinctions

A fourth theme concerns reported learning gains and the inconsistencies that accompany them. In mathematics education, positive effects are frequently reported in studies focused on geometry and spatial reasoning. However, outcomes vary depending on the measures used, the duration of the intervention, and the comparison conditions. For example, Walkington et al. (2025) analyzed a VR-based distance tutoring intervention and identified distinctive affordances, including gesture-rich reasoning, mathematical play, and interactions with dimensionality, while also documenting constraints such as connection problems, spatial orientation issues, and off-task behaviour. Although preliminary achievement findings appeared promising, the authors emphasized exploratory interpretation because sample sizes were modest and control conditions limited. This pattern underscores a broader issue in the field that effectiveness claims in AR and VR mathematics education are highly sensitive to design features and classroom realities, suggesting caution in generalizing results.

Design and implementation challenges therefore remain central concerns. Across AR mathematics studies, recurring practical and pedagogical constraints include technical instability, the instructional time required to learn interfaces, limited device access, and uneven teacher readiness. Schutera et al. (2021) highlight technological entry barriers and the scarcity of robust,

school ready AR applications for vector geometry. Koparan et al. (2023) similarly note the time and resource demands involved in developing AR materials. In classroom settings, these challenges are not peripheral details. They shape whether AR and VR are feasible at all and influence who can realistically benefit from their affordances (Schutera et al., 2021).

Immersive Virtual Reality and Educational Potential

Finally, mathematics education research increasingly distinguishes augmented reality from fully immersive virtual reality not only in technical terms but also in pedagogical implications (Bulut & Borromeo Ferri, 2023; Cevikbas et al., 2023; Walkington et al., 2024b). Whereas AR maintains learners' contact with the physical classroom, fully immersive VR can create a strong sense of being inside a mathematical space (Dimmel et al., 2021; Makransky & Petersen, 2021). Because AR learners can still see peers, the teacher, and physical artefacts, AR remains relatively compatible with shared discussion, teacher guidance, and conventional inscriptions such as whiteboards and notebooks, whereas immersive VR removes those resources from view and requires that they be reintroduced before or after the immersive activity (Beauchamp & Walkington, 2024; Walkington et al., 2025). AR accordingly supports interaction at the scale of the desk and the room, while immersive VR permits embodied viewpoint shifts and interaction with objects at scales that would otherwise be impossible (Dimmel et al., 2021; Walkington et al., 2024b). Each configuration carries its own technical demand: AR must manage alignment between digital overlays and the physical environment (Cevikbas et al., 2023), while immersive VR must manage the attention, navigation, and spatial orientation costs that full immersion introduces (Makransky & Petersen, 2021; Radianti et al., 2020). Each also carries a distinct consequence for the teacher, since AR leaves student activity directly

observable, whereas full immersion limits teachers' visibility into what students are doing (Beauchamp & Walkington, 2024; Walkington et al., 2025).

These contrasts are not only practical, since they imply different theories of action for mathematics instruction. AR may be understood as an enriched representational layer embedded within classroom discourse, whereas immersive VR may function as a temporary mathematical world that requires structured entry, monitoring, and debriefing to reconnect experience with formal mathematical ideas (Walkington et al., 2024b). Because the present study uses a fully immersive environment, the remainder of this section concentrates on immersive virtual reality. Within mathematics education, immersive VR represents a significant expansion of possible representational environments, and by leveraging dynamic and visually rich simulations, educators may create opportunities for students to explore mathematical principles through direct manipulation and multiple viewpoints (Campos et al., 2022; Lai & Cheong, 2022; Su et al., 2022).

Despite the growing AR and VR research base, the role of preservice teachers and teacher learning remains comparatively underexplored. Much of the literature centers on student outcomes and tool effectiveness. Although some recent studies foreground mathematical activity and multimodality, fewer investigate how teacher candidates interpret AR and VR affordances, anticipate constraints, and develop the pedagogical reasoning required to integrate such environments into instruction. This gap is significant. Without attention to teacher learning, AR and VR research risks generating tools and demonstrations that are difficult to translate into sustained classroom practice (Bulut & Borromeo Ferri, 2023).

Synthesis: Locating Immersive Virtual Reality Within the Technology Landscape

Read together, the evidence reviewed in this section converges on a consistent claim. The educational value of a digital tool in mathematics is determined less by what the tool can display than by the mathematical actions it makes available, the interpretive work it requires of learners, and the pedagogical structures within which it is placed (Drijvers & Sinclair, 2024; St Omer et al., 2025). Dynamic geometry systems support relational reasoning when dragging is treated as an epistemic act rather than a visual effect (Dimmel et al., 2021; Komatsu & Jones, 2020; Yao, 2020). Computer algebra systems can surface conceptual gaps as readily as they can bypass the mathematical work, depending on the stance learners adopt toward their outputs (Cameron et al., 2024; Pedersen et al., 2025). Across tool categories and across three decades of research, the same conditional pattern recurs: the question is never whether a technology works, but for which purpose, under which task design, and with what teacher mediation (Clark-Wilson et al., 2020; Drijvers & Sinclair, 2024; St Omer et al., 2025).

Immersive virtual reality enters this landscape as a particularly consequential case rather than as an exception to it (Drijvers & Sinclair, 2024; Makransky & Petersen, 2021). It intensifies precisely the dimensions this literature identifies as decisive, namely interaction, movement, perspective shifting, and spatial presence (Dimmel et al., 2021; Johnson-Glenberg, 2018; Walkington et al., 2024b), all of which bear directly on vector reasoning, where direction, magnitude, and three-dimensional relationships must be coordinated across registers (Bollen et al., 2017; Hoyer & Girwidz, 2024). But intensification cuts in both directions. The same immersion that places a learner inside a mathematical space also removes that learner from the classroom inscriptions, peer talk, and teacher monitoring through which mathematical meaning is ordinarily stabilized (Beauchamp & Walkington, 2024; Walkington et al., 2025), and it raises

in sharper form the questions of cognitive load, usability, and instructional scaffolding that this literature has already posed of far less immersive tools (Albus et al., 2021; Makransky & Petersen, 2021; Parong & Mayer, 2018). This is why the present study asks how preservice teachers reason about the conditions under which immersive virtual reality might work for teaching vectors, rather than whether it works, which is the question this body of research suggests is the answerable one (Drijvers & Sinclair, 2024; Makransky & Petersen, 2021).

Embodied Cognition and Technology Enhanced Learning

As described above, the place of the body and movement in mathematical understanding in immersive virtual environments raises important questions. In this section of my review of literature, I examine the theoretical perspective of embodied cognition and review evidence for how bodily action, perception, and interaction support mathematical meaning making in technology rich environments. Theories of embodied cognition take the view that thinking is inseparable from perception, action, and the material and social environment (Farina, 2021). This view stands in contrast to information processing perspectives of cognition that separate the brain and memory from the body (e.g., Atkinson & Shiffrin, 1968). In mathematics education, embodied perspectives are especially relevant because mathematical understanding often depends on perceiving structure and coordinating it across multiple representational forms, including symbols, diagrams, graphs, gestures, and physical or digital models (Way & Ginns, 2024). From this view, bodily activity does more than accompany learning. It can function as a semiotic and cognitive resource that helps learners notice relations, stabilize attention, and connect abstract ideas to perceptual and motor experience (Nathan, 2022; Way & Ginns, 2024).

Rather than treating the body as secondary to cognition, embodied accounts argue that cognition is shaped through the same sensorimotor systems by which people move, gesture,

orient, and interact with the world, a claim increasingly supported in mathematics and science education research (Georgiou et al., 2019; Khatin Zadeh et al., 2022; Macedonia, 2019; Mansour et al., 2025). In educational research, this perspective has been articulated through ideas such as perception and action coupling, gesture as representational activity, spatial grounding of abstract concepts, and distributed or extended cognition across tools and settings, including digital and immersive environments (Chang & Aberash, 2026; Lehrman, 2025; Way & Ginns, 2024). These principles are explanatory rather than merely philosophical; recent studies show that coordinated bodily action and representation can reorganize attention, stabilise working memory, and support conceptual generalization across domains, from early mathematics and science to language learning and immersive virtual reality (Chang & Aberash, 2026; Mansour et al., 2024; Rudling et al., 2026).

The account of embodiment that guides this study belongs to the cognitive science tradition, and it is worth setting out what that tradition claims before considering how immersive technology might extend it. Lakoff and Núñez (2000) argue that mathematical ideas are structured by conceptual metaphors that recruit recurring patterns of bodily experience, so that notions such as magnitude, direction, containment, and path carry traces of sensorimotor experience into formal reasoning. Wilson (2002) cautions that embodied cognition is not a single thesis but a family of six distinguishable claims, among them the claims that cognition is situated, that people off-load cognitive work onto the environment, and that off-line cognition is body based, and she argues that these claims are supported to differing degrees and should be evaluated separately rather than accepted together. Shapiro (2019) develops the same point systematically, separating conceptualization, replacement, and constitution research programs and showing that the great majority of empirical work in education belongs to the first of these.

Farina (2021) situates this body of work within the wider landscape of embodiment research and its applications. Overall, these accounts supply the working assumption used throughout this dissertation: bodily action and perception are resources that ground and scaffold conceptual understanding, so that a change in what learners are able to do with their bodies can produce a change in what they are able to notice and, subsequently, to articulate.

Gesture research provides one of the strongest empirical bridges between embodied theory and classroom learning. Seccia and Goldin-Meadow (2024) synthesizes evidence that producing and observing gestures can support mathematics learning through multiple mechanisms including directing attention, reducing cognitive load, and revealing implicit strategies that are not yet articulated in speech. Importantly for mathematics education, gesture also serves as a diagnostic window into students' emerging understanding, enabling more responsive instruction. At the same time, Seccia and Goldin-Meadow (2024) note that much of this research has been conducted in controlled or one-on-one settings, which leaves open the question of how gesture informed practices can be translated into the realities of classrooms (Seccia & Goldin-Meadow, 2024, p. 5). As the authors themselves put it, "gesture research offers a low-cost, low-stakes way of bringing researchers and teachers together" (Seccia & Goldin-Meadow, 2024, p. 8).

Related empirical work further complicates simplified assumptions about bodily action and learning. Kersey et al. (2024), for example, compared gesture-based and action-based instruction for mathematical equivalence and found that both can support learning, but that gesture-based instruction may promote stronger longer-term retention. Their findings suggest that embodiment and abstraction should not be treated as opposites. Instead, effective instruction

may involve sequencing experiences that move learners from physical manipulation toward representational gesture and symbolic reasoning (Kersey et al., 2024).

The relationship between embodiment and technology becomes especially salient when technologies are designed not only to show representations but to invite bodily enactment of mathematical relations (Shvarts et al., 2021; Shvarts & Van Helden, 2023). Over the last decade, research on embodied design has explored how learning environments can be structured so that students' movements correspond to specific conceptual relationships and later become resources for reflection and formalization (Abrahamson & Bakker, 2016; Alberto et al., 2022).

Abrahamson and colleagues (2020) describe their work as a program of design-based research in which the mathematics learning process is conceptualized as the development of new sensorimotor perceptual structures, that is, as learning to move and to perceive in new ways within environments engineered around a mathematical relationship (pp. 18–19). They also treat embodiment as social and instructional rather than purely individual. Their framework draws on sociocultural theory alongside cognitive developmental theory, and the role assigned to social interaction is one of the dimensions along which they distinguish the six research programs they survey (p. 3). They plot those programs along an axis running from individual to collective coordination of solution enactment (p. 19), and they locate the pedagogical work of embodied design in the negotiation between tutor and student over what a disciplinary display signifies (p. 4). This body of research suggests the value of embodied interaction depends on how tasks connect movement to invariant structure and on how teachers guide the transition from action to explicit reasoning (Abrahamson et al., 2020; Abrahamson & Bakker, 2016; Shvarts et al., 2021).

The term invariant structure is used throughout this dissertation to name the mathematical relationship within a task that holds constant while the surface features of a learner's enactment

vary. The Mathematics Imagery Trainer offers the clearest illustration. Students are asked to move two hands so that the screen turns green, and the screen responds only when the heights of the two hands stand in a fixed ratio. A great many hand positions satisfy that condition, so no single position is the answer. What students eventually notice, and what the instructor never names for them, is the interval between the hands and the fact that it must grow as the hands rise (Abrahamson et al., 2020, p. 5). Abrahamson and colleagues describe such a noticed relationship as an attentional anchor, a perceptual structure that emerges in the service of motor control and that can subsequently be described, measured, reconstructed, and symbolized in other media (pp. 5–6). Invariant structure, in this sense, is what remains the same across the variation a learner produces, and the pedagogical claim is that it becomes available to reflection precisely because the learner has had to sustain it in action.

In the context of the present study, the invariant structure of vector operations includes the magnitude and direction that a vector retains when it is translated to a new location, the head to tail relation that determines a resultant regardless of where the addends are positioned, the proportional relation between a vector and its scalar multiples, and the perpendicularity that holds between a cross product and the plane of its two factors regardless of the angle from which a learner views them. Each of these is a relationship that survives changes in position, orientation, and viewpoint. Each is therefore something that an immersive environment can render available to a learner who moves, walks around a representation, or reaches into the space in which it is displayed, and it is in this specific sense that the tasks examined in this study can be said to connect movement to invariant structure.

A review focused specifically on action-based embodied technologies for mathematics learning further clarifies these design considerations. Alberto et al. (2022) examine a decade of

embodied mathematics learning variations and emphasize that embodied environments are not automatically meaningful simply because they are active or hands on. Their effectiveness depends on how perceptual cues, actions, and feedback guide learners' attention to mathematical relationships. A central implication is that embodied technologies must be carefully aligned with mathematical goals. The actions learners perform in the environment should directly support the ideas they are meant to notice. When this alignment is weak, movement risks becoming mere activity without conceptual value; when it is strong, movement can serve as a cognitive scaffold that supports deeper understanding and later symbolic articulation (Alberto et al., 2022).

Technology-enhanced embodiment also raises questions about how learning is distributed across bodies, artefacts, and representations. Perspectives on distributed and extended cognition argue that thinking is not confined to internal mental processes but unfolds across external resources such as diagrams, tools, and coordinated social action (Clark & Chalmers, 1998; Hutchins, 1995). This is the position Wilson (2002) treats as the most contested of the six she distinguishes, and the one Shapiro (2019) assigns to constitution research rather than to the more widely supported claim that bodily experience shapes conceptual content. In mathematics education, Walkington et al. (2019) give the position empirical form, showing that when learners prove geometric conjectures together, the gestural work that carries the reasoning is distributed across several bodies and is not reducible to the sum of the individual contributions.

In technology rich environments, this distribution becomes more visible because digital systems can capture, transform, and represent learners' actions as feedback. Walkington et al. (2024b) situate this point in their work on embodied technologies including augmented and virtual reality and argue that that gesture, viewpoint, and object manipulation should be treated as central components of reasoning rather than as secondary behaviours. Their MAET

framework reflects a broader theoretical shift: if cognition is understood as embodied and distributed, then analyses of learning must examine how learners coordinate multiple modalities over time and across space.

Work in the materialist tradition in mathematics education speaks directly to these concerns and merits engagement. Drawing on Châtelet's account of mathematical invention, Sinclair et al. (2013) treat mathematical diagrams as arising from the movement of the hand rather than as static representations that the hand later illustrates (p. 239), and they study inventiveness at the local level of the classroom, which is to say the introduction of ideas that are new to the learners involved rather than new to the discipline (p. 239). They characterize a creative act as one that brings into being something that was not present before, that departs from established norms of behaviour, that is unscripted, and whose meaning is not exhausted by existing meanings (pp. 241–242). In a grade one classroom working with dynamic geometry software, children's arm and hand movements brought into being an intersection point lying beyond the edge of the visible screen, extending the plane past what the display could show (pp. 243–246). In a grade four classroom working with a motion capture environment, children gestured into existence a vertical position and time graph that no real motion could produce, and in doing so reasoned their way to why it was impossible (pp. 246–250).

Three features of this account bear on the present study. First, the authors decline to locate creativity in any individual child, attributing it instead to the collective activity of children, teacher, projected image, and software (p. 246), which is a stronger version of the distributed account of cognition described above. Second, they are explicit that digital technology is neither necessary nor sufficient for such moments, and that what matters is the specific way a technology is used, with particular tasks, around particular mathematical situations, and through

particular patterns of interaction between teachers and students (p. 250). That claim is close in substance to the argument developed in this dissertation, since it locates the educational significance of a tool in the intersection of technology, content, and pedagogy rather than in the tool itself. Third, they observe that mobility carries particular weight in mathematics, both because formal mathematics is systematically detemporalized and because mathematical objects are ordinarily inaccessible to being moved at all (p. 251). That observation applies with unusual force to vectors rendered at body scale in a head-mounted display, where objects conventionally encountered as static inscriptions can be approached, walked around, and recomposed.

Sinclair and colleagues position their contribution as an alternative and complementary perspective on existing gesture and diagram research rather than as a replacement for it (p. 251), and it is taken up here in that spirit. Although this scholarship was developed in relation to two-dimensional dynamic geometry and motion capture environments rather than immersive head-mounted displays, its central claim, that bodies, digital tools, and mathematical concepts become entangled in ways that exceed a model of the body as a vehicle for grounding cognition, bears directly on what is at stake when a learner's hand movements are tracked and rendered as a vector inside MathVR. The new materialist foundations of that claim, developed for mathematics education by de Freitas and Sinclair (2014), are treated here as an important body of theory rather than as a marginal one.

The primary theoretical lens of this dissertation nonetheless remains the cognitive science account of embodiment set out earlier in this chapter, in which bodily action and perception are treated as resources that ground and scaffold conceptual understanding rather than as constitutive elements of an assemblage that includes the mathematical concept itself. The cognitive science tradition offers a closer fit with the study's interest in how specific embodied actions, such as

gesturing at a vector or repositioning to view a projection from another angle, supported specific mathematical insights that participants could later articulate in their own words.

It should be acknowledged, however, that the account of immersive embodiment drawn from Johnson-Glenberg et al. (2014) and Johnson-Glenberg (2018) carries a distributed orientation of its own. Their taxonomy grades a learning experience by three components: the amount of motoric engagement it requires, the gestural congruency between a learner's movements and the content to be learned, and the learner's perception of immersion (Johnson-Glenberg et al., 2014, p. 89). None of these is a property of a learner considered alone. Gestural congruency is a relation between a movement and a designed representation, and the perception of immersion is described explicitly as subjective and strongly contingent on the type and configuration of the display (p. 90). The unit of analysis is therefore already the learner in interaction with a designed environment. The authors make this explicit about their own findings, reporting that they cannot identify which kinesthetic or multimodal act was most predictive of the learning gains, nor disentangle the contribution of embodiment from that of the co-located collaboration, because the lessons were complex classroom situations rather than isolated variables (p. 101).

Some readers might regard this as a limitation, since it means that the framework used here does not adjudicate between a view in which the body grounds a concept and a view in which the body is part of what the concept is. It is nevertheless consistent with the study's use of TPACK, which likewise treats teacher knowledge as arising from the interaction of content, pedagogy, and technology within particular contexts rather than as three separable bodies of knowledge held independently by an individual (Mishra & Koehler, 2006). Both frameworks locate what matters in the relation among a learner, a task, and a technology rather than in the

learner considered in isolation, and the analysis that follows is conducted at that level throughout.

Two traditions are acknowledged here without being adopted. The first is the phenomenological tradition, in which the lived body is treated as the primary and irreducible condition of meaning making rather than as a resource supporting a separable cognitive process (Dreyfus, 1992; Merleau-Ponty, 1945/2012). The second is the materialist tradition discussed above. A study designed within either tradition would ask different questions of the same data. It would attend less to what participants could subsequently articulate about vector operations and more to what was brought into being in the moment of enactment, and it would treat the headset, the controllers, and the rendered vector as participants in that activity rather than as conditions for it. These are productive directions rather than gaps in the present design, and they are returned to in Chapter Six.

Empirical research on technology enhanced embodied learning shows both its potential and its fragility when moved from controlled studies into real classrooms. Georgiou and Ioannou (2021), in a large-scale classroom implementation of a technology enhanced embodied learning design for mathematics, address the gap between laboratory success and classroom complexity. They emphasize that orchestration and learning design are crucial. Embodied technologies do not automatically produce learning gains when introduced into classrooms. They require carefully structured activity sequences, established routines, and active teacher mediation so that physical engagement is translated into mathematical understanding. This point is methodologically significant because it shifts the focus away from the novelty of the tool itself and toward the instructional ecology in which it is used.

The broader learning sciences literature continues to explore the boundary conditions of embodied learning. A recent narrative review by Castro-Alonso et al. (2024) synthesizes evidence across multiple research avenues and argues that embodied effects are not uniform; they depend on instructional alignment, learner characteristics, and the nature of the task. For mathematics education, this suggests that embodiment should be conceptualized as a design constraint rather than a guaranteed mechanism. In other words, the body can support meaning-making, but only when tasks make the relevant invariants perceptible through action and when instruction supports learners in articulating those invariants in more formal mathematical term (Castro-Alonso et al., 2024).

Overall, the embodied cognition literature establishes a conceptual bridge toward immersive and interactive digital environments without implying that immersion is inherently beneficial. Its strongest claims are conditional. Embodied activity may enhance mathematical understanding when perception, action, feedback, and reflection are coherently organized. Environments must enable learners to notice structure through movement and to formalize that structure through discourse and inscription. This conditional framing is particularly relevant for AR and VR contexts, where opportunities for gesture, viewpoint shifts, and manipulation are amplified and where the demands for intentional instructional design and scaffolding are correspondingly intensified. The next section considers how immersive virtual reality may amplify these embodied possibilities in mathematics specific contexts.

Immersive Virtual Reality and the Promise of Embodied Learning

Building on embodied perspectives on learning, this section examines why immersive virtual reality may be especially relevant for mathematics education. Immersive virtual reality places learners within computer-generated three-dimensional environments where they can

move, gesture, manipulate virtual objects, and change viewpoint in ways that are not possible through static diagrams or conventional screen-based representations (Engelbrecht & Borba, 2023). For mathematics topics that depend on spatial reasoning, including geometry and vectors, these affordances may make abstract relations more perceptible by allowing learners to act on mathematical objects and observe the consequences of those actions in real time (Campos et al., 2022; Washington et al., 2024). In this sense, immersive VR is relevant not simply because it is novel, but because it can intensify forms of embodied interaction that are already understood to matter for mathematical meaning making.

Immersive virtual reality (VR) creates a simulated 3D environment that "engulfs a user, providing the illusion of being physically present in a different world" (Engelbrecht & Borba, 2023, p. 7). Using head-mounted displays and motion-tracking, VR surrounds the learner with visual, auditory, and sometimes haptic stimuli, allowing direct interaction with virtual objects. Its core features; immersion, interaction, and involvement, distinguish it from traditional media. As Freina and Ott (2015) explain, VR rests on three basic principles, immersion, interaction, and user involvement with the environment and narrative, which together make learning experiences more motivating and engaging. In practice, immersion in VR means a learner feels spatially "present" in the scene (Saran, 2025), while interaction (through handheld controllers or gestures) allows manipulating virtual elements (Johnson et al., 2022). Involvement, often discussed as agency, supports learners in actively navigating and exploring content rather than passively receiving information, and this agency is closely tied to how immersive and interactive features shape the learning experience (Philipsen et al., 2022).

From a learning perspective, VR's multisensory and interactive affordances (combining sight, sound, and body movement) are especially significant for domains involving spatial and

abstract reasoning (Coban et al., 2022; Gabay & Schonberg, 2026; Lehrman, 2025). Immersion supports the perception of depth, orientation, and scale, while interaction allows learners to act on objects and immediately observe the consequences of those actions (Makransky & Petersen, 2021; Sun et al., 2025). Agency further enables learners to control pacing, viewpoint, and exploration paths, fostering active engagement with content (Makransky & Petersen, 2021). Together, these affordances support a learning environment in which abstract entities can be experienced as manipulable objects. For example, in a VR geometry or vector lesson, learners may physically rotate shapes, translate vectors through space, or observe relationships from multiple vantage points, engaging visual perception and bodily movement simultaneously in ways that are not possible with two-dimensional representations (Washington et al., 2024).

These affordances align closely with theories of embodied cognition, which posit that conceptual understanding is grounded in sensorimotor experience rather than abstract symbol manipulation alone (Halbig & Latoschik, 2025; Wilson, 2002). In VR, learners use gestures and movement to work with ideas. Lehrman (2025) notes that VR opens new opportunities for expressing ideas through gesture, spatial navigation, and environmental manipulation, echoing the classic embodied cognition claim that interactions between body and world facilitate cognition (Wilson, 2002). Similarly, an education report points out that AR and VR environments allow intuitive hand gestures to act directly on geometric objects, giving real-time feedback as students manipulate shapes in three dimensions (Washington et al., 2024). These actions engage the brain's motor system, and recent research shows that expressing mathematical ideas through gesture or movement activates neural circuits that help ground even highly abstract concepts (Khatin-Zadeh et al., 2022). In other words, when students in VR physically push, rotate, or draw

vectors and shapes, they embody those operations; a process shown to enhance understanding of mathematics concepts (Khatin-Zadeh et al., 2022).

Empirical research increasingly demonstrates that embodied interaction in immersive technologies can enhance learning outcomes. A systematic review by Georgiou and Ioannou (2019) found that technology-enhanced embodied learning environments often lead to improved conceptual understanding, especially when learning tasks require spatial reasoning or transformation. Campos et al. (2022) found that engineering students using a VR app to learn 3D vectors outperformed peers on visualization tasks and overwhelmingly rated VR as enhancing their experience. In this qualitative feedback, students reported that VR helped them better identify and understand vectors spatially, underscoring VR's ability to make the connection between mathematical representations and physical meaning more concrete. Likewise, surveys of AR and VR mathematics environments note that the ability to manipulate shapes and get real-time feedback is a "profound affordance" of immersive tech (Washington et al., 2024). These findings suggest that engaging the body through gesture, movement, and spatial interaction carry cognitive consequences, not only motivational ones.

Beyond performance outcomes, research on VR implementation clarifies the conditions under which such gains are realized. Students report high engagement and autonomy in VR lessons, which the three-dimensional spatial qualities of these environments appear to support (Rodríguez & García López, 2026). Researchers are also translating these findings into design guidance. Rodríguez et al. (2026), for example, describe structuring VR training to overcome technical hurdles and build teacher TPACK, emphasizing that "VR learning depends largely on teacher knowledge and training" (p. 2). In other words, VR's value may lie not in novelty, but in

its use of embodiment to make abstract mathematics, such as vectors, more concrete and intuitively understandable.

In summary, immersive virtual reality offers a multisensory, interactive, and embodied learning environment in which learners can move their bodies and gesture to explore mathematical entities in space. Through embodied interaction, abstract objects such as vectors can be experienced not merely as arrows on paper but as manipulable spatial entities that can be grasped, oriented, and transformed through bodily action (Campos et al., 2022). From an embodied cognition perspective, such sensorimotor activity can support meaning making by linking conceptual understanding to perception and action, a connection that is central to embodied accounts of learning and is taken up in Johnson-Glenberg's (2018) embodied design principles for VR. Consistent with this view, review level evidence suggests that technology enhanced embodied learning can support learning in STEM contexts (Georgiou & Ioannou, 2019). VR therefore stands on solid theoretical ground as a learning medium: it extends the core idea that understanding arises from doing (Johnson et al., 2022; Johnson-Glenberg, 2018; Yang et al., 2023) into the digital realm, transforming abstract content into objects that can be touched and moved. Framed in this way, VR is not a technological novelty but a learning environment that extends long-standing insights about how understanding emerges through perception and action. These qualities make immersive VR a promising medium for addressing persistent challenges in teaching and learning abstract mathematical concepts such as vectors. These mathematics specific affordances help explain why immersive virtual reality has attracted attention as a medium for spatially demanding concepts such as vectors. The next section situates this promise within the broader educational literature on immersive VR, including its mechanisms, limitations, and design conditions.

Immersive Virtual Reality in Education

Having considered the mathematics specific promise of immersive virtual reality, this section situates VR within the broader educational literature. Across subject areas, a consistent conclusion is emerging: the educational value of immersive virtual reality is better explained by the learning mechanisms it activates, such as presence, agency, attention, and self-regulation, rather than by simple comparisons between media (Makransky & Petersen, 2021; Philipsen et al., 2022). Meta-analyses and systematic reviews therefore frame immersive virtual reality not as a uniformly effective intervention, but as a learning context whose value depends on task design, instructional support, and the management of boundary conditions such as overload, discomfort, and usability (Breves & Stein, 2023; Coban et al., 2022; Wu et al., 2020). This broader literature is important for the present study because it clarifies why immersive virtual reality may support learning in some designs while constraining it in others.

A prominent synthesis framework that reflects this mechanism focused perspective is the Cognitive Affective Model of Immersive Learning (CAMIL) proposed by Makransky and Petersen (2021). CAMIL identifies presence and agency as central psychological affordances of immersive virtual reality. According to the model, features such as immersion, representational fidelity, and user control do not influence learning directly. Instead, they shape learning outcomes through affective and cognitive pathways, including interest, motivation, embodiment, cognitive load, and self regulation (Makransky & Petersen, 2021). The value of the model for educational research lies in its ability to explain why immersive virtual reality can both increase engagement and create risks of overload. Greater presence and agency may strengthen involvement in learning, but they can also direct cognitive resources toward navigating the environment instead of processing the intended content (Makransky & Mayer, 2022; Makransky

& Petersen, 2021). The model is applied in Chapter Six, where it provides the interpretive frame for participants' accounts of what MathVR made visible and what it demanded of them.

Empirical studies have begun to examine and refine the claims advanced in CAMIL. Philipsen et al. (2022) investigated how immersion and interactivity shape learning processes in virtual reality and reported effects on cognitive load, situational interest, presence, and agency, suggesting that technological features interact in complex ways rather than contributing independently to learning outcomes. Classroom based research has extended this line of inquiry. Makransky and Mayer (2022), in testing the immersion principle within multimedia learning, examined how learners' subjective experience of a virtual environment influences learning processes rather than focusing solely on posttest performance. Their study involved 102 secondary school students aged 13 to 16 years old, including 82 eighth grade and 20 ninth grade students from four public schools in a European country. Participants were assigned to a between subjects experimental design in which 49 students experienced a virtual field trip through a three-dimensional head-mounted display, while 53 students viewed the same content as a two-dimensional classroom projected video. This body of work is significant because it reframes immersive virtual reality as an attentional and self-regulatory environment. Researchers increasingly analyze how VR shapes where learners direct attention, how they manage cognitive resources, and how they regulate engagement over time (Breves & Stein, 2023; Hsu & Lee, 2026).

At a population level, meta-analytic findings offer a cautious endorsement of immersive virtual reality while underscoring modest effect sizes and high variability. Wu et al. (2020) meta-analysis synthesized 35 experimental and quasi experimental studies conducted across Asia, Europe, and the Americas, encompassing 1,847 learners from K 12 and post-secondary contexts,

with most studies situated in higher education and science related domains. Across these studies, head-mounted display based immersive virtual reality yielded a small but statistically significant overall effect on learning performance with substantial heterogeneity, indicating meaningful variability across contexts and implementations. Moderator patterns suggested that effects varied by educational level, subject area, hardware, comparison condition, and instructional duration. Coban et al. (2022) meta-analysis similarly reported a small overall effect and emphasized differences across educational levels and implementations. These results converge on an important point for educational interpretation: immersive virtual reality is not a uniformly powerful intervention but a context whose effectiveness depends on design, task type, and how learners are supported in managing attention and coordinating representations (Coban et al., 2022; Wu et al., 2020).

Motivation and engagement appear to be more consistent outcomes than achievement gains. Makransky and Mayer (2022) tested what they term the immersion principle in multimedia learning, which proposes that learners achieve better outcomes when instruction is delivered through immersive media, such as a three dimensional video experienced through a head-mounted display, rather than less immersive formats, such as an equivalent two dimensional screen presentation, provided that the "immersive lessons is designed in accordance with established multimedia design principles and the specific affordances of the media" (p. 1772). In other words, immersion is treated as an instructional variable whose effectiveness depends not only on whether it works, but on how and why it works in relation to cognitive and affective processes. Consistent with this principle, Makransky and Mayer (2022) found that immersive virtual reality increased learners' sense of presence and enjoyment and was associated with improved retention relative to a less immersive condition. More broadly, meta-analytic

syntheses indicate that learners tend to evaluate immersive environments more positively, even when knowledge gains are comparable to other media (Coban et al., 2022; Wu et al., 2020). This pattern reinforces a mechanism-oriented interpretation. Immersive virtual reality reliably alters affective experience, including presence, enjoyment, and situational interest. However, learning gains depend on whether this heightened engagement is channeled into generative cognitive processing rather than absorbed by navigation demands and environmental management (Breves & Stein, 2023; Makransky and Petersen, 2021).

The cognitive load studies help to specify why this conversion from engagement to learning is not automatic. Immersive environments can impose additional extraneous load due to heightened sensory input, interface complexity, and orientation demands, particularly when learners must divide attention between content and environmental control (Albus et al., 2021; Elme et al., 2022; Parong & Mayer, 2018; Philipsen et al., 2022). Breves and Stein (2023) provide a mechanism level contribution in an experiment comparing a 360-degree video experienced via head-mounted display and laptop and found that immersive virtual reality increased presence, cognitive load, and cybersickness but only cybersickness mediated strained cognitive capacity. This finding complicates the assumption that presence itself is the primary cognitive cost of immersion. Instead, bodily discomfort and sensory conflict emerge as critical boundary conditions that can undermine learning even under high perceived presence (Biswas et al., 2024; Cossio et al., 2025).

Cybersickness has therefore become a significant constraint in immersive virtual reality education research. Reviews in human computer interaction document that susceptibility is shaped by both individual differences and design variables such as locomotion method, visual flow, and frame rate and it can limit session duration and reduce attention to instructional content

(Biswas et al., 2024; Cossio et al., 2025). For educational research, this introduces a physiological dimension to validity. If some learners experience discomfort, their performance may reflect coping strategies rather than conceptual processing (Breves & Stein, 2023; Wu et al., 2020).

Another mechanism that has gained increasing attention is self-regulation. CAMIL explicitly identifies self-regulation as a pathway through which presence and agency influence learning, proposing that immersive virtual reality environments require learners to manage attention, persistence, and strategy use under conditions of heightened stimulation (Makransky & Petersen, 2021). Empirical studies have increasingly integrated self-regulated learning supports into immersive environments. Hsu and Lee (2026), for example, integrated self-regulation scaffolds into virtual reality science activities and reported relationships among these supports, cognitive load, and learning outcomes. These findings imply that immersive virtual reality designs may need to incorporate explicit prompts for planning, monitoring, and evaluation, particularly when tasks are exploratory or open ended (Hsu & Lee, 2026; Philipsen et al., 2022).

Collaboration and co-presence add a further layer, because immersive virtual reality does not inherently isolate learners. Whether it supports or constrains social learning depends on design, and two complementary sources bear on this point. De Back et al. (2020) compared collaborative learning of neuroanatomy in a CAVE, a room-sized system in which the virtual environment is projected onto the walls so that learners remain visible to one another, against learning from a conventional textbook. They reported substantially higher learning gains in the CAVE condition, and, notably for spatially demanding content, learners with lower spatial ability benefited most from the strong spatial cues the environment provided, which raised their performance to the level of learners with higher spatial ability. Van der Meer et al. (2023), in a

systematic review of 139 studies of virtual reality for collaborative learning, found that head-mounted displays accounted for only a small proportion of this work, since roughly three quarters of the studies used monitor-based systems rather than immersive hardware. They also identified a set of features that collaborative virtual environments require if they are to support learning, among them shared communicative tools, the ability to manipulate objects in the environment, a degree of learner autonomy, and deliberate support for socialization. Read together, these studies indicate that co-presence is an achievement of design rather than a property of the medium, and that the same social demands can increase extraneous load when an environment is not structured to manage it through clear instructions, equal access to information, visible interaction cues, and coordinated turn taking.

This line of work is relevant because it reframes a common concern, that immersive virtual reality isolates learners, as a design choice. Johnson-Glenberg et al. (2014) observe that educational technology configured around a single user model can produce an isolating experience that runs counter to the highly collaborative ways students ordinarily work with physical tools (p. 102), and the same worry attaches with greater force to head-mounted displays, which remove the peers, teacher, and classroom inscriptions that remain available in augmented reality (Walkington et al., 2024b). The research reviewed here suggests that some implementations do isolate learners, while others deliberately design for multiuser interaction, shared objects, and communicative gestures. In educational contexts, collaboration can amplify the benefits of agency through shared exploration and articulation of strategies, but it can also amplify demands as learners coordinate turns, negotiate viewpoints, and manage group attention (de Back et al., 2020; van der Meer et al., 2023).

Across the studies reviewed, several design tradeoffs become apparent. Increasing immersion and interactivity can heighten presence and agency, but may also raise cognitive load and distraction, especially when environments include salient but instructionally irrelevant features (Breves & Stein, 2023; Elme et al., 2022). Reducing interactivity can lower overload but may also diminish the sense of agency that often justifies immersive virtual reality use (Makransky & Petersen, 2021). The implication is not that immersion should be minimized, but that design must calibrate interactivity, pacing, feedback, and opportunities for reflection (Makransky & Mayer, 2022; Philipsen et al., 2022). The strongest learning outcomes tend to emerge in designs that situate immersive virtual reality within a broader instructional sequence, including preparatory orientation, guided tasks, and post experience consolidation rather than relying on immersion alone (Coban et al., 2022; Wu et al., 2020).

When narrowed to mathematics education, this mechanism focused synthesis has two important implications. First, mathematics learning often depends on coordinating visual structure with symbolic reasoning. Immersive virtual reality can intensify spatial and visual experience, but it can also introduce representational complexity that should be carefully managed so learners attend to structural invariants rather than surface features (Makransky & Petersen, 2021; Philipsen et al., 2022). Second, mathematics topics vary substantially in their spatial and dynamic demands. Immersive virtual reality may be most theoretically aligned with content where embodied viewpoint change and spatial interaction provide epistemic access to relationships that are otherwise difficult to perceive (Coban et al., 2022; Wu et al., 2020). It may be less aligned with topics where symbolic manipulation and procedural precision dominate and where immersion adds little beyond additional cognitive demands (Coban et al., 2022; Wu et al., 2020). If immersive virtual reality supports learning only under particular design and

instructional conditions, then its classroom value depends not only on the medium itself but also on how teachers interpret its affordances, constraints, and pedagogical fit (Makransky & Mayer, 2022; Makransky & Petersen, 2021). The next section therefore turns from immersive virtual reality as a learning environment to teachers as the decision makers who mediate whether, how, and for what purposes it is used.

Teachers, Technology, and Pedagogical Decision Making

If immersive virtual reality is to move from experimental demonstrations toward sustained classroom use, teachers' beliefs, perceptions, and situated pedagogical judgements become decisive factors. Research on technology integration consistently shows that adoption depends not only on perceived usefulness and ease of use, but also on confidence, access, and alignment with curricular and instructional goals (Al Zboon et al., 2021; Granić & Marangunić, 2019; Ndlovu et al., 2020). In this sense, technology does not enter classrooms simply because it exists or appears pedagogically promising. It enters through teachers' judgements about whether it is feasible, valuable, and worth integrating into practice. Studies of immersive technologies reinforce this point by showing that enthusiasm for VR is often shaped by how teachers and teacher candidates perceive its usability, instructional relevance, and practical constraints (Cardullo & Wang, 2021; Xie et al., 2022).

This issue is especially important in teacher education, where preservice teachers are still developing the professional knowledge, technological confidence, and pedagogical reasoning that may later shape how they use emerging tools in their own classrooms (Howard et al., 2019). As participants in teacher education programs, preservice teachers occupy a formative stage in which early experiences with immersive technologies may influence later instructional decisions. Their judgements therefore matter not only because they indicate whether immersive tools are

viewed positively, but also because they reveal the practical and pedagogical conditions under which such tools are considered meaningful for instruction.

This teacher education context also connects closely to frameworks of professional knowledge such as Technological Pedagogical Content Knowledge, or TPACK, which conceptualizes effective technology integration at the intersection of funds of knowledge that are technological, pedagogical, and content based (Mishra & Koehler, 2006). More recent scholarship has also emphasized the importance of context within this framework, with Mishra's updated model identifying contextual knowledge as a dimension that shapes how technology integration is interpreted and enacted in practice (Mishra, 2019; Petko et al., 2025). For immersive VR, this means that pedagogical decisions are likely shaped not only by what the technology can do, but also by classroom realities, curricular aims, student needs, and teachers' confidence in aligning technological affordances with disciplinary goals.

These issues are particularly relevant in the Ontario context of the present study, where vectors are taught in the Grade 12 Calculus and Vectors course (Ontario Ministry of Education, 2007). Preservice secondary mathematics teachers are therefore a particularly relevant population through which to examine how immersive technologies may be interpreted in relation to abstract and spatially demanding content. When preservice teachers have opportunities to explore virtual reality alongside methods coursework, they may develop more nuanced judgements about when and how immersive technologies can support topics such as vectors, a pattern echoed in studies reporting increased acceptance, self efficacy, and preparedness after structured virtual reality experiences (Hur, 2025; Ogegbo et al., 2024).

Across these findings, this literature suggests that the educational value of immersive virtual reality depends not only on the design of the medium itself, but also on the judgements

teachers make about its pedagogical fit and feasibility. For that reason, preservice teachers are a particularly important population for understanding how immersive technologies may be interpreted before they reach sustained classroom use. The next section therefore turns specifically to research on preservice teachers' perceptions of VR and educational technology, with attention to how they evaluate usefulness, preparedness, pedagogical fit, and implementation constraints.

Preservice Teachers' Perceptions of Virtual Reality and Educational Technology

Research on preservice teachers' perceptions of virtual reality has expanded considerably in recent years, reflecting growing recognition that teacher beliefs and readiness play a central role in whether emerging technologies are adopted and how they are used pedagogically, and whether they become instructionally meaningful rather than occasional or novelty driven tools (Bower et al., 2020; Li et al., 2023; Thohir et al., 2023). Across international studies, preservice teachers' perceptions tend to revolve around three related questions: whether VR is useful for learning, particularly within a specific subject area; whether it is manageable for teachers and students from logistical and pedagogical perspectives, whether it is compatible with everyday classroom realities such as time constraints, resource availability, and student needs (Hickethier et al., 2026; Ogegbo et al., 2024; Taggart et al., 2023).

A central finding across this literature is that preservice teachers' attitudes toward VR are rarely stable or predetermined. Rather, perceptions are shaped by experience, framing, and the kinds of pedagogical tasks preservice teachers are asked to perform when interacting with the technology (Hickethier et al., 2026; Li et al., 2022; Taggart et al., 2023). This observation could be particularly relevant for studies focused on vectors, because vectors represent a mathematical domain in which many learners struggle with spatial visualization and the coordination of

graphical and symbolic representations (Bollen et al., 2017; Mikula & Heckler, 2013; Nguyen & Meltzer, 2003; Sabah, 2023). Preservice teachers' judgements about VR may therefore depend less on general claims about engagement and more on whether they perceive a meaningful alignment between the technology and the conceptual demands of the content being taught (Hickethier et al., 2026; Rodríguez & García López, 2026).

One prominent body of research examines preservice teachers' perceptions through technology acceptance frameworks such as the Technology Acceptance Model (TAM) and the extended Unified Theory of Acceptance and Use of Technology (UTAUT2). These models emphasize perceived usefulness and perceived ease of use as central predictors of adoption, helping explain why teachers may express interest in VR conceptually while hesitating when implementation appears complex or resource intensive (Thohir et al., 2023). Studies applying these frameworks to immersive VR in teacher education consistently find that behavioural intention to use VR is associated with perceived usefulness, enjoyment motivation, and facilitating conditions such as the availability of resources and institutional support (Bower et al., 2020; Li et al., 2022; Xie et al., 2024).

Empirical research illustrates how these factors interact in practice. Li et al. (2023) used a pre and post mixed methods design with questionnaires from 58 preservice teachers and semi structured interviews with 44 participants in rural China, framed through UTAUT2. They reported increases in intention-related variables after exposure to VR learning activities. At the same time, participants continued to identify cost and access issues as barriers, demonstrating how structural constraints can remain salient even when teachers become more positive about the pedagogical potential of VR. Adelana et al. (2023) examined preservice teachers in Nigeria using a mixed method approach grounded in UTAUT2, combining an online survey with qualitative

data and analysing results using variance based structural equation modelling and qualitative coding. Their findings show that perceived usefulness strongly predicts preservice teachers' readiness and behavioural intention to use VR, suggesting that adoption depends both on the technology's perceived instructional value and on the conditions that support its practical use. Ogegbo et al. (2024) surveyed 83 preservice science teachers at a large metropolitan university in Gauteng Province, South Africa, about using a VR classroom for microteaching, using UTAUT2 and regression analysis. They found high acceptance and intention to use VR classrooms, with behavioural intention especially associated with social influence and technology self-assurance.

Large scale work further illustrates how perceptions of infrastructure and pedagogy intersect. Thohir et al. (2023) surveyed 406 preservice teachers across 12 universities and examined relationships among TPACK, facility conditions, and technology acceptance model constructs. Their analysis showed that perceived usefulness and perceived ease of use were both significant predictors of behavioural intention to use VR. Facility conditions and TPACK were significantly associated with perceived ease of use, and facility conditions were also strongly related to TPACK. These findings suggest that preservice teachers' judgements about adopting VR emerge from interconnected considerations. In other words, teachers evaluate whether the technology appears instructionally valuable while simultaneously assessing whether available resources, institutional support, and infrastructure make classroom implementation feasible (Thohir et al., 2023).

Importantly, acceptance research also indicates that experience with VR does not necessarily produce uniformly positive attitudes. Exposure can increase confidence while simultaneously making constraints more visible (Li et al., 2023; Taggart et al., 2023). From a teacher education perspective, this implies that VR experiences should not be framed solely as

demonstrations of technological potential and they may also serve as structured opportunities for preservice teachers to identify operational challenges such as time requirements, classroom management issues, and technical setup demands while developing more grounded pedagogical judgements (Ogegbo et al., 2024; Taggart et al., 2023).

A second major theme concerns preservice teachers' self-efficacy, digital competence, and sense of preparedness. Preservice teachers frequently report interest in VR and curiosity about its classroom potential but also describe limited hands-on experience and uncertainty about how to integrate it into real lessons (Bower et al., 2020; Taggart et al., 2023). Taggart et al. (2023), for example, used a convergent mixed methods design across four initial teacher education providers in Ireland and Northern Ireland. In Stage 1, 114 postgraduate preservice secondary teachers completed a baseline survey about VR and AR, and a subgroup of 38 then participated in a four-hour hands-on workshop followed by a post workshop survey and focus groups. They found that participants were generally aware of VR and AR but had limited direct experience using them, and that the workshop was associated with increased willingness to incorporate innovative ICT and greater confidence in using digital technology purposefully in teaching. At the same time, participants continued to identify practical barriers, most notably cost and funding, lack of resources such as reliable Wi Fi and headsets, and limited time to embed and evaluate technology in real school settings. This pattern aligns with broader findings in mathematics teacher education showing that professional digital competence develops through situated engagement and reflective practice rather than through abstract discussion of tools (Dilling et al., 2024).

Recent research also highlights the role of structured training and guided design work. Rodríguez and García López (2026) followed 33 preservice secondary mathematics teachers in a

teacher education course where they learned to use NeoTrie VR through an initial hands-on introduction, scaffolded practice tasks, and a sequence of curriculum aligned design activities. Participants first explored the tool, then completed guided training sessions designed to build fluency with NeoTrie's features before drafting individual learning situations linked to curriculum standards. They then worked in groups to design full teaching units, reviewed another group's unit using a TPACK based rubric, and presented a revised version in a final whole class session. Across cycles, the authors report improvement in TPACK related performance, especially in the technological dimensions, suggesting that structured support helped participants move beyond early usability issues and design more curriculum aligned VR lessons. At the same time, the study shows that pedagogical and content related reasoning did not improve evenly across groups, with some teaching units showing limited mathematical depth or insufficient guidance for learners even when the VR tool was used successfully. This matters for vectors because VR can make three-dimensional representations available for exploration, but designing effective vector lessons still requires pedagogical and mathematical resources that connect embodied interaction to formal reasoning about vector operations and representations, rather than relying on visualization alone (Dilling et al., 2024; Rodríguez and García López, 2026).

Evidence from mathematics specific VR studies reinforces this point. In a mixed methods study conducted through an international teacher education collaboration, Hickethier et al. (2026) examined preservice teachers' perceptions after they completed a VR simulation designed to make growth tangible through a storyline about the spread of infectious disease. The study involved 57 student teachers from four countries, Finland, Germany, Israel, and Turkey, who completed a short pre and post questionnaire around perceived usefulness and intention to use VR in later practice. Participants then worked through the VR task individually, typically for

about 60 to 90 minutes, using a head-mounted display and hand controllers, and afterwards took part in a guided interview, with interview data from 41 participants included in the qualitative analysis. Quantitative results showed a significant positive shift in perceived usefulness from pre to post, and the change in perceived usefulness significantly predicted intention to use VR in future teaching. Qualitative interviews added nuance by showing that participants valued VR for motivation and for making abstract content feel more concrete and classroom relevant, while also raising practical concerns about time demands, costs and maintenance, classroom logistics, and age appropriateness, including worries that younger students might not yet have the abstraction needed for effective use. These findings suggest that preservice teachers often infer usefulness from whether a VR task makes a mathematical idea more tangible and teachable within everyday constraints, rather than from general claims about technological innovation (Bower et al., 2020; Hickethier et al., 2026).

A third theme concerns preservice teachers' pedagogical reasoning about how VR might function within classroom instruction. Across studies, preservice teachers frequently identify visualization and student engagement as key strengths of virtual reality but also report uncertainty about how to integrate virtual reality activities into lesson sequences, align them with curricular goals, and assess learning (Alkaabi et al., 2025; Bower et al., 2020; Ogegbo et al., 2024; Rodríguez, 2024; Van der Want & Visscher, 2024). In mathematics education, this issue is particularly important because technology integration is evaluated not only in terms of engagement but also in terms of how it supports mathematical practices such as representing, reasoning, generalizing, and justifying (Drijvers & Sinclair, 2024). Mathematics focused research shows that preservice teachers tend to frame immersive tools as supplementary enrichments unless they are explicitly supported to design tasks, connect virtual actions to formal

notation, and use the environment to surface student thinking (Alkaabi et al., 2025; Davis et al., 2022; Olawale, 2024; Rodríguez, 2024; Rodríguez & García López, 2026). When teacher education programmes provide structured opportunities for preservice teachers to design, test, and reflect on virtual reality based teaching sequences, along with feedback related to TPACK and professional noticing, preservice teachers begin to develop more nuanced pedagogical reasoning and rather than framing VR primarily in terms of engagement, they start to articulate clearer justifications for when and how it can support specific instructional goals (Han et al., 2025; Huang et al., 2025; Kosko et al., 2020; Van der Want & Visscher, 2024).

A fourth theme concerns institutional and contextual constraints perceived as barriers to adoption. Preservice teachers may view VR as valuable while anticipating practical barriers such as limited headsets, insufficient training, and time constraints (Hur, 2025; Li et al., 2023; Ogegbo et al., 2024; Taggart et al., 2023; Thohir et al., 2023). Similar concerns appear in K–12 implementation research, where teachers describe challenges related to equipment, preparation time, classroom management, and the feasibility of scaling VR activities (Aziz & Younes, 2025; Schott et al., 2024). From a technology acceptance perspective, these issues correspond to facilitating conditions and first order barriers, which influence whether teachers view VR as realistically implementable even when they hold positive beliefs about its pedagogical value (Al Breiki et al., 2023; Hur, 2025; Ogegbo et al., 2024). In acceptance models, these concerns are typically described as facilitating conditions. However, qualitative research shows that they often function as moral and pedagogical considerations as well (Li et al., 2022; Schott et al., 2024). Teachers question whether it is fair to design activities that only some students can access, whether time spent in VR displaces other important instruction, and how learning can remain

visible and assessable when students are immersed in headset-based environments (Li et al., 2022; Schott et al., 2024).

These perceived constraints correspond to well documented limitations in the wider immersive virtual reality literature. Reviews consistently identify hardware availability, connectivity, maintenance, and the logistical difficulty of equipping a whole class as barriers to sustained use, particularly outside specialized laboratories (Di Natale et al., 2020; Radianti et al., 2020; Schott et al., 2024). Physical tolerance imposes a further limit. Alongside the cybersickness discussed earlier, Ratan et al. (2025) found in a multi-week study that students benefited from longer sessions in virtual reality up to a point, but that certain outcomes declined beyond roughly forty-five minutes, with substantial variation between individuals. For school mathematics, where periods are short and tolerance varies, this suggests that briefer immersive episodes combined with non-immersive consolidation activities are likely to be more feasible than extended sessions.

Methodological limitations further complicate what can be claimed. Reviews note continued reliance on small samples, short interventions, self-report measures, and simple media comparisons, which makes it difficult to determine which features of virtual reality contribute to learning outcomes (Buchner & Mulders, 2026; Di Natale et al., 2020; Radianti et al., 2020). Taken together with the perceptions research reviewed above, these constraints do not undermine the potential of immersive virtual reality. They clarify the conditions under which it may support meaningful learning, and they form the practical context within which preservice mathematics teachers form judgements about a specific tool used for a specific mathematical topic.

These patterns reveal an important gap that directly motivates research on preservice teachers integrating immersive VR in the teaching of vectors. Much of the existing reviewed

studies focus on three main areas: general beliefs about VR, the use of VR for teacher rehearsal or classroom management training, and studies examining student learning outcomes (Daltoè et al., 2024; King et al., 2022; Schott et al., 2024). In contrast, comparatively little research follows preservice teachers as they interact with a specific immersive VR tool situated within a particular mathematics content domain that is conceptually aligned with spatial embodiment. Few studies examine how such experiences influence preservice teachers' developing interpretations of technological affordances, perceived constraints, and pedagogical possibilities over time and through structured reflection (Bower et al., 2020; Hickethier et al., 2026). In other words, while research suggests that situated experience and reflection matter for how teachers develop mathematics specific digital competencies and make decisions about technology integration, including in VR supported vector instruction (Dilling et al., 2024), it remains insufficiently understood how content specific VR experiences, such as manipulating vector operations in three dimensional space, may influence preservice teachers' pedagogical and technological judgements about teaching and learning mathematics. Studies examining VR based training for preservice teachers show promising developments in their TPACK related competencies (Rodríguez & García López, 2026), but they rarely examine how such experiences shape teachers' decisions about instructional feasibility within specific mathematical topics. The current study endeavours to respond to this identified gap in research on preservice mathematics teachers' understandings of immersive VR as a teaching tool by first documenting their experiences in MathVR, an immersive environment that enables the exploration of vector operations.

Chapter Summary

In this chapter, I reviewed three bodies of scholarship that together frame the study. The first concerns the learning and teaching of vectors. Six recurring themes in that research

converge on a single pattern: students struggle most when they must coordinate representations, connecting arrows, components, and equations across spatial reference frames, and visualization, although necessary, does not by itself secure understanding. Instruction that deliberately structures representational translation improves conceptual and representational performance, which points toward learning environments in which seeing, handling, and connecting vectors across forms become central to the activity rather than incidental to it.

The second body of literature concerns digital technology in mathematics education. From dynamic geometry systems and computer algebra systems to augmented and immersive virtual reality, the evidence converges on a conditional claim: the value of a tool lies in the mathematical actions it makes available, the interpretive work it demands of learners, and the pedagogical structures within which it is placed. Immersive virtual reality intensifies the dimensions this literature identifies as decisive, namely interaction, movement, perspective shifting, and spatial presence, but the same immersion removes the learner from the inscriptions, peer talk, and teacher monitoring through which mathematical meaning is ordinarily stabilized, and it raises questions of cognitive load, usability, and scaffolding in sharper form. Embodied perspectives on learning, extended into immersive environments through research on virtual embodiment and through the Cognitive Affective Model of Immersive Learning, explain why these affordances and risks arrive together.

The third body of literature concerns teachers. Technology enters classrooms through teachers' judgements about feasibility, value, and fit, and research on preservice teachers' perceptions of virtual reality shows that those judgements are shaped by experience, by the framing of the task, and by whether a tool is seen to align with the conceptual demands of particular content, while remaining bounded by cost, access, time, and the visibility of learning

when students are immersed. Read together, these literatures identify the gap the study addresses. Research on vector learning explains why an environment that lets learners act within three-dimensional space is worth examining; research on technology and embodiment explains the conditions under which such an environment may support or constrain mathematical meaning making; and research on teachers explains why preservice teachers' judgements about such an environment matter. What is missing is work that follows preservice teachers through a content-specific immersive tool aligned with spatial embodiment and examines how the experience, together with structured reflection, shapes their interpretations of its affordances and constraints and their reasoning about its place in instruction. Studies of VR based training report promising developments in TPACK related competencies, but they rarely examine judgements of instructional feasibility at the level of a specific mathematical topic.

The review also sets the terms for the theoretical framework that follows. Because the evidence shows that technology integration depends on mathematics specific knowledge for teaching and on context, the study needs a framework of professional knowledge that treats content, pedagogy, technology, and context as integrated, which Chapter Three develops through TPACK. Because the evidence on embodiment shows that the position from which mathematics is encountered matters, the study also needs an account of how bodily action within an immersive space participates in meaning making, which Chapter Three develops through virtual embodied cognition and the distinction between representational embodiment and immersive first-person embodiment. The conditional conclusion of this review, that the productive question is never whether a technology works but for which purposes, under what task design, and with what teacher mediation, is carried into the three research questions stated at the end of Chapter Three: the insights preservice teachers provide during embodied interaction with MathVR, their

reflective descriptions of its affordances and constraints, and what both reveal about their emergent TPACK.

Chapter Three

Theoretical Framework

The study is grounded in a theoretical framework that integrates the TPACK model of teacher knowledge and the emerging concept of virtual embodied cognition. Together, these lenses provide a comprehensive perspective to understand how preservice teachers perceive and use immersive virtual reality (VR) in their teaching practices, specifically for teaching mathematics. TPACK emphasizes the interdependent relationship among technology, pedagogy, and content knowledge in teaching practice, increasingly acknowledging the importance of contextual factors (Petko et al., 2025). Virtual embodied cognition focuses on how bodily experience, including movement, gesture, and spatial perception within virtual environments, shapes cognitive processes (Farina, 2021; Johnson-Glenberg, 2018; Kilteni et al., 2012).

By bringing these frameworks together, the study is positioned to examine not only what preservice teachers learn about teaching mathematics with VR, but also how that knowledge is constructed through technological engagement and embodied experience. Each framework contributes a distinct but complementary lens for interpreting participants' experiences and reflections.

Technological Pedagogical Content Knowledge (TPACK)

Effective teaching with technology requires more than discrete knowledge of subject matter, instructional strategies, or digital tools. It requires an integrated form of professional knowledge that enables teachers to make principled decisions about how particular technologies can support particular content through appropriate pedagogical approaches. Technological Pedagogical Content Knowledge (TPACK) provides a theoretical framework for understanding this integrated knowledge base. Building on Shulman's (1986) concept of Pedagogical Content

Knowledge (PCK), Mishra and Koehler (2006) extended the model to explicitly account for technological knowledge as a central component of contemporary teaching expertise (Petko et al., 2025). Shulman's (1986) original framework proposed that expert teachers hold a distinctive form of knowledge that blends subject matter expertise with pedagogical reasoning about how that content can be represented, sequenced, and made accessible to particular learners.

This insight was developed further within mathematics education by Ball et al. (2008), whose framework for Mathematical Knowledge for Teaching divides the knowledge that teaching requires into six domains (p. 403). On the subject matter side they distinguish common content knowledge, the mathematics that any educated adult might hold, from specialized content knowledge, the unpacking of mathematics that teaching demands and that other uses of mathematics do not, and from horizon content knowledge, a sense of how a topic sits within the wider discipline. On the pedagogical side they distinguish knowledge of content and students, which concerns how learners are likely to reason about a particular idea, from knowledge of content and teaching, which concerns the representations, sequences, and analogies that make an idea accessible, and from knowledge of content and curriculum. Their central contribution is the demonstration that content knowledge for teaching means knowing mathematics in the ways that teaching requires. A teacher who knows vectors as a mathematician may still lack the specialized understanding of how students tend to confuse direction with magnitude, where graphical and algebraic registers create interpretive difficulties, and which representational choices make three-dimensional spatial relationships more visible.

TPACK extends this same logic by one further step. Just as Ball and colleagues argue that knowledge for teaching mathematics is not mathematical knowledge with pedagogy added to it but a distinct integration of the two, Mishra and Koehler (2006) argue that knowledge for

teaching with technology is not that integration with technical skill added to it. Effective technology integration emerges from the dynamic intersection of Content Knowledge (CK), Pedagogical Knowledge (PK), and Technological Knowledge (TK) rather than from the accumulation of these domains in isolation (Petko et al., 2025). For a study of preservice teachers and vector instruction, this parallel establishes what the C in TPACK must mean. It is not mathematical competence in the general sense but the mathematics specific knowledge that Ball and colleagues describe, and that knowledge is not set aside when a technology enters the classroom. It becomes the resource a teacher draws on in judging whether an immersive representation of a vector makes a relationship visible or merely makes it vivid.

A core assumption of TPACK is that teacher knowledge is situated and relational. Teachers can, and often do, learn to operate a technology in general terms, through play, tutorials, or everyday use, and the participants in this study did so during the orientation to MathVR. What TPACK describes is a different achievement: the understanding of how a specific technology can be used to teach specific content in pedagogically meaningful ways within particular instructional contexts, which develops only when the technology is taken up with pedagogical intention. From this perspective, knowledge of technology is inseparable from knowledge of teaching and learning. This assumption is particularly relevant for mathematics education, where abstract concepts often challenge learners' visualization and conceptualization (Campos et al., 2022), and it is here that the domains described above become analytically useful. The demands that Ball et al. (2008) identify do not disappear when a technology is introduced. A teacher deciding whether MathVR is suitable for vector addition is exercising knowledge of content and teaching, since the question is which representation serves the mathematical purpose, and knowledge of content and students, since the question is equally how

learners are likely to read that representation. What changes is the material on which those judgements operate, which is now a manipulable three-dimensional object rather than a diagram on a page. Immersive virtual reality therefore offers new representational possibilities, but they are realizable only by a teacher who can already reason about which representations serve which mathematical purposes and how students are likely to interpret them.

In practical terms, a mathematics teacher developing strong TPACK for immersive virtual reality would draw on several of these domains at once. Knowing how to compute with vectors, which Ball et al. (2008) would classify as common content knowledge, would not be sufficient. The teacher would also need the specialized understanding that teaching requires, including recognizing why a student who computes a resultant correctly may still treat magnitude and direction as independent properties, and anticipating where a three-dimensional display will clarify that relationship and where it will only add visual complexity. They would need to judge which MathVR interactions carry mathematical purpose, such as placing the tail of one vector at the head of another and moving around the resulting configuration to inspect it, and which merely occupy the learner. They would need to locate the activity within the sequence in which vectors are encountered in the Grade 12 curriculum. The emphasis within TPACK is therefore not on technical proficiency but on the pedagogical deployment of technology in the service of conceptual understanding, and the knowledge that makes such deployment possible is mathematics specific rather than generic. This distinction is central to the present study, which examines how preservice teachers reason about virtual reality as a pedagogical tool for teaching vectors rather than how they learn to operate the technology.

Recent scholarship has further refined the TPACK framework by foregrounding the role of context in technology integration. In earlier representations, context appeared as a surrounding

condition rather than a core component of teacher knowledge. Petko et al. (2025) argue that this framing underrepresents the extent to which teachers' instructional decisions are shaped by contextual knowledge, including institutional constraints, curriculum requirements, access to resources, and student characteristics. They propose an expanded model that explicitly includes Contextual Knowledge (XK) alongside CK, PK, and TK. This reconceptualization is particularly relevant for immersive technologies such as VR, whose classroom use is shaped by factors including physical space, equipment availability, school policies, and student readiness.

In the present study, contextual knowledge plays a significant role in shaping participants' engagement with VR. Preservice teachers encountered MathVR within the structured environment of a teacher education program and in relation to the Ontario Grade 12 mathematics curriculum. Their reflections on immersive VR were frequently intertwined with considerations of classroom feasibility, student accessibility, and curricular alignment. Understanding TPACK as contextualized knowledge allows these considerations to be interpreted not as peripheral concerns, but as integral to how teachers develop capacity for technology integration. From this perspective, growth in TPACK involves learning to navigate the intersections of content, pedagogy, technology, and context in practice.

Another key assumption underpinning TPACK is that this integrated knowledge develops through social interaction and reflective practice rather than through individual skill acquisition alone. In a systematic review of professional development research, Jiménez Sierra et al. (2023) found that collaborative learning structures, such as lesson study and shared design activities, provide contextualized opportunities for teachers to build and refine their TPACK. These approaches emphasize dialogue, experimentation, and reflection, allowing teachers to negotiate meanings about how technology supports teaching and learning.

Empirical research further underscores the importance of attending to teachers' confidence and perceptions in relation to TPACK development. Saltan and Arslan (2017) found that preservice teachers generally report lower overall TPACK self-confidence than in-service teachers, despite often expressing relatively high confidence in their technological skills. Notably, their study identified that preservice mathematics teachers demonstrated significantly lower TPACK self-confidence compared to their peers in other subject areas. The authors suggest that this may reflect the traditionally abstract nature of mathematics instruction and a tendency toward established pedagogical routines that do not readily incorporate technology.

These findings are particularly relevant to the present study. If preservice mathematics teachers begin with lower confidence in their ability to integrate technology meaningfully, immersive VR experiences may function as both a challenge and an opportunity. Saltan and Arslan (2017) also caution that frequent use of technology does not automatically result in higher quality TPACK. Instead, growth depends on structured opportunities to integrate technology with pedagogical intent and content specific goals. The MathVR intervention examined in this study was designed with this principle in mind. Participants were not merely exposed to VR technology, but were guided to reflect on how the tool supported or complicated the teaching of vector concepts.

In summary, TPACK provides a coherent framework for examining how preservice teachers reason about the integration of immersive VR into mathematics instruction. It foregrounds the interdependence of content, pedagogy, technology, and context. In this study, TPACK informs both the design of the data collection instruments and the analytic lens used to interpret participants' reflections.

Virtual Embodied Cognition

Building on the embodied perspectives reviewed earlier in this chapter, this study adopts virtual embodied cognition as a more specific theoretical lens for examining how preservice teachers learn mathematics through immersive virtual reality. Whereas the earlier literature review established that bodily action, perception, and interaction may support mathematical meaning making across technology rich environments, virtual embodied cognition focuses more specifically on how these processes operate within immersive virtual spaces. In this study, the framework is used to examine how movement, gesture, spatial navigation, and interaction with virtual mathematical objects shape participants' conceptual and pedagogical sense making.

A central concept within this framework is the Sense of Embodiment (SoE) in virtual reality. SoE refers to the subjective experience of having and controlling a body, even when that body is virtual (Guy et al., 2023; Kilteni et al., 2012). In immersive VR, users typically perceive virtual hands or an avatar that responds to their movements, allowing them to act within the environment as if it were physical. Guy et al. (2023) conceptualize sense of embodiment as comprising three interrelated components: sense of agency, the feeling that one is causing actions in the virtual world; sense of body ownership, the feeling that the virtual body or hands are one's own; and sense of self-location, the feeling of being physically situated within the virtual environment. When these dimensions are strongly aligned, users may experience heightened sense of presence and bodily involvement, which supports more natural, goal directed interaction in VR (Arlati et al., 2025).

From an educational standpoint, this implies that learning in immersive VR may be shaped not only by visual immersion but also by the learner's bodily participation in meaning making (Arlati et al., 2025; Khorasani et al., 2023). When learners move, gesture, manipulate

objects, and reposition themselves in virtual space, those actions may become part of the cognitive process rather than merely a way of navigating the environment. Virtual embodied cognition therefore treats learning in immersive VR as experiential and action based, while still recognizing that embodiment alone does not guarantee understanding.

This perspective is especially relevant for mathematics education, where instruction has long relied heavily on symbolic representations and static inscriptions, even as research on technology use in mathematics education has expanded substantially over the past four decades (Hwang et al., 2023). In vector instruction specifically, learners often encounter vectors through diagrams and formulas that can obscure their dynamic and relational properties, particularly when students must coordinate meaning across representations (Barniol & Zavala, 2014; Bollen et al., 2017; Sabah, 2023). Immersive VR offers an alternative mode of engagement by allowing learners to experience mathematical structures through bodily action (Daughrity et al., 2025).

Within this framework, bodily action is treated as a resource for sense making. For example, aligning vectors tip to tail in three-dimensional space or walking along a vector direction may support intuitive understanding of vector addition or magnitude in ways that differ from viewing static representations. In this sense, virtual embodied cognition does not claim that embodiment alone guarantees learning. Rather, it suggests that embodied interaction provides a resource for sense making that can be leveraged through pedagogical design and reflection.

Research on immersive environments further suggests that embodied interaction may influence not only individual understanding but also socially mediated meaning making. Studies of virtual embodiment have examined how immersive environments affect cognition, perspective, and interpretation (Bailey et al., 2016; Maister et al., 2015; Romano et al., 2016), while more recent work in mathematics education has shown that embodied interaction in AR

and VR can support collaborative geometric reasoning and mathematical noticing (Walkington et al., 2021, 2024b).

In the present study, virtual embodied cognition provides a lens for examining how preservice teachers' bodily actions in MathVR intersect with their cognitive and pedagogical reasoning. Rather than treating VR as a neutral delivery mechanism, this framework foregrounds the role of movement, gesture, spatial navigation, and presence in shaping participants' experiences of learning vectors. By engaging with vectors through embodied interaction, participants encountered mathematical ideas as dynamic and manipulable rather than purely symbolic. Their reflections may therefore offer insight into how embodied experience informs not only conceptual understanding but also pedagogical thinking about how such ideas might be taught.

In sum, virtual embodied cognition extends broader theories of embodied learning into immersive digital environments. It provides a study specific lens for examining how participants act within virtual mathematical space, how those actions may support conceptual grounding, and how embodied engagement may inform pedagogical sense making in teacher education.

Application of Virtual Embodiment in Mathematics Education

To operationalize the concept of virtual embodied cognition, this section considers its application to a specific mathematical domain, namely vectors, which form the focus of the present study. Vectors are abstract mathematical objects characterized by magnitude and direction and are commonly represented as arrows in two- or three-dimensional space. They are often difficult for students because they require spatial reasoning and coordination across graphical, symbolic, and algebraic forms.

Immersive virtual reality may provide a more tangible way of engaging with these relationships. In a virtual environment, a learner can create a vector, move it through space, align it with another vector, or change its direction and length while immediately observing the result. These interactions are relevant to embodied learning because the learner's physical movements correspond to mathematical operations and explorations. Rather than only viewing vector relationships from an external perspective, learners can act within the mathematical space itself.

It is worth naming this shift precisely, because it is the distinction on which this study's account of immersive virtual reality rests. Embodied approaches to mathematics education have long established that gesture, movement, and spatial action participate in mathematical meaning making rather than merely accompanying it, including in settings where the mathematics itself is presented on paper or on a screen (Alibali & Nathan, 2012; de Freitas & Sinclair, 2014; Lakoff & Núñez, 2000; Nathan & Walkington, 2017). In those settings the learner's body works upon a representation held at a distance. A hand traces the path of a projection, an arm sweeps to indicate a resultant, and a gesture stands in for a relationship the learner is reasoning about but is not inside. Gerofsky (2010, 2011) describes this position directly in her analysis of students' gestured graphs of functions, distinguishing observer viewpoint gestures, in which the hand depicts a mathematical object viewed from outside, from character viewpoint gestures, in which the learner's own body enacts the object, and organizing the contrast around the difference between seeing the graph and being the graph. She reports that this distinction corresponded closely to teachers' assessments of students' mathematical engagement and understanding. As noted in Chapter One, the present study refers to the first of these positions as representational embodiment; Gerofsky (2016) situates the same contrast within a broader account of embodied approaches to mathematics learning.

Immersive virtual reality makes a second position available. When learners enter a three-dimensional environment at the scale of their own bodies, vectors are no longer objects gestured about from outside but objects occupying the same space as the learner, approachable from any side and manipulable at arm's length. Dimmel et al. (2021) characterize this as the capacity to look within and to look along a mathematical figure rather than only upon it. Studying pairs of learners in a shared augmented reality geometry environment, Walkington et al. (2024b) likewise document students walking around figures rather than through them, positioning themselves inside enlarged shapes in order to perceive their surfaces from the interior, and handling projected mathematical objects as though these were tangible. In the terms introduced below, what immersion supplies is self-location and agency with respect to the mathematical object itself rather than with respect to a depiction of it (Guy et al., 2023; Kilteni et al., 2012). The present study refers to this second position as immersive first-person embodiment.

The distinction is one of position rather than degree. Both involve the body, and representational embodiment remains a full form of embodiment in its own right, since the evidence that gesture supports mathematical reasoning in conventional settings is substantial and is not displaced by the availability of immersive environments (Alibali & Nathan, 2012). What changes is the position from which the mathematics is encountered, and specifically whether the learner reasons about a represented space or acts within one. This matters for vectors in particular, because several of the difficulties documented in Chapter Two are difficulties of position as much as of content. Determining direction and interpreting magnitude from arrow conventions requires reading a spatial relationship from a fixed external vantage point (Heckler & Scaife, 2015; Sabah, 2023; Tairab et al., 2020). Coordinating graphical with algebraic and symbolic registers requires conversions between representational systems that do not share a

common perceptual basis (Bouhjar et al., 2021; Duval, 2006). Reasoning about three-dimensional vector relationships is frequently attempted through two-dimensional depictions that offer no opportunity for perspective change (Schutera et al., 2021). A projection onto a plane viewed from a single fixed vantage point must be inferred; a projection walked around can be inspected. Whether that difference produces mathematical or pedagogical consequences for preservice teachers is an empirical question rather than a theoretical guarantee, and it is the question this study takes up.

A defining feature of immersive VR is the learner's sense of self-location and agency, key components of the sense of embodiment. Instead of observing vectors from a fixed outside viewpoint, learners can reposition themselves in relation to the objects they are studying. This may support spatial reasoning in ways that static diagrams cannot. For example, understanding a vector projection onto a plane may be supported when a learner changes viewpoint and examines the relationship from multiple angles. Similarly, vector addition may become more perceptible when learners place the tail of one vector at the head of another and move around the resulting configuration to inspect the relationship spatially.

Overall, these two perspectives position the analysis to attend to more than technology acceptance. TPACK supplies the account of professional knowledge, and, following Ball et al. (2008), the content dimension of that knowledge is understood here as mathematics specific rather than generic: not competence with vectors as such, but the specialized understanding of how vector concepts can be represented, sequenced, and made accessible to learners who are likely to misread them. Virtual embodied cognition supplies the account of how the technology is encountered, distinguishing the representational embodiment of gesture toward a depiction from the immersive first-person embodiment of acting within the represented space. The two lenses

meet in a single question: whether preservice teachers who have experienced vectors at body scale come to see immersive first-person embodiment as offering something their students would need, and whether the pedagogical reasoning through which they reach that judgement draws on the mathematics specific knowledge that teaching vectors demands. Framing the research in this way allows the investigation to move beyond surface level evaluations of technology acceptance and toward a deeper examination of how immersive virtual reality may shape mathematical understanding and teaching practice.

Research Questions

Given the limited empirical work exploring preservice mathematics teachers' experiences of immersive virtual reality, and given two open questions that motivate this study, namely whether immersive first-person embodiment offers instructional affordances that representational embodiment does not, and how an immersive mathematical experience is taken up as professional knowledge integrating technology, pedagogy, and content, this study is guided by the following three research questions:

Research Question 1

While performing vector operations in MathVR, what insights do preservice teachers provide about the technology and its pedagogical possibilities for teaching vector concepts?

Research Question 2

After performing vector operations using MathVR, how do preservice teachers describe the affordances and constraints of MathVR for teaching vector operations to Grade 12 students in Ontario?

Research Question 3

How do these insights and descriptions inform understanding of preservice teachers' emergent TPACK, particularly in relation to a technology that enables visualization and virtual embodied experience within mathematics education?

In Chapter Four, I outline the methodological design of the study, including the research context, participants, data sources, and analytic procedures used to examine preservice teachers' embodied interaction with MathVR, their reflective descriptions of its affordances and constraints, and their emergent TPACK.

Chapter Four

Methodology

This chapter outlines the methodological design used to examine the insights preservice mathematics teachers provided while performing vector operations in MathVR, how they subsequently described the affordances and constraints of MathVR for teaching Grade 12 vectors, and what those insights and descriptions reveal about their emergent TPACK. The study was designed to capture embodied interaction during a structured MathVR session and participants' reflective description of the experience as a potential teaching resource, and to bring the two together in a synthesis of their emergent TPACK.

Research Design

This study uses a qualitative exploratory case study design (Baxter & Jack, 2008; Harrison et al., 2017; Stake, 1995; Yazan, 2015; Yin, 2018). It is descriptive and interpretive in its approach. It was designed to document what preservice mathematics teachers did during a single structured immersive virtual reality session focused on vector operations and how they later interpreted the experience in relation to teaching and learning vectors. Its object is therefore participants' in the moment interaction with MathVR and their subsequent interpretation of the experience as a potential teaching resource, examined within one setting and one session.

The case, in this research, is the collective experience of a bounded group of preservice mathematics teachers in one teacher education program as they engage in an immersive VR session centred on vector operations and then reflect on the instructional implications of that experience. This framing aligns with case study scholarship that defines a case as a bounded, contextually situated system and uses multiple sources of evidence to develop an in-depth and context sensitive account of a contemporary phenomenon (Baxter & Jack, 2008; Harrison et al.,

2017; Stake, 1995; Yazan, 2015; Yin, 2018). Hyett et al. (2014) further emphasize that strong case study reporting depends on coherence among the case definition, contextual boundaries, data sources, and interpretive claims.

Following Hyett et al.'s (2014) elements of case definition, contextual boundaries, data sources, and interpretive claims, this case is defined as the collective experience of ten preservice mathematics teachers engaging with a single structured MathVR session on vector operations, bounded by that session and its immediate reflective follow-up, with data sources spanning VR session recordings, written reflections, and focus group discussion, and interpretive claims directed at the embodied interaction and reflection those sources make visible. This is a single-occasion elicitation design rather than an iterative one: unlike design-based research, which cycles through design, enactment, analysis, and redesign across multiple rounds to refine an intervention (Anderson & Shattuck, 2012), the MathVR session here was designed once, delivered once, and treated as the occasion for generating the data the study analyses, not as an intervention under progressive refinement.

Two traditions of case study scholarship inform this design, and they inform it in different ways. Yin (2018) treats case study as a form of empirical inquiry that investigates a contemporary phenomenon in depth within its real world context, and he places the burden of rigor on design: a clearly bounded unit of analysis, a stated purpose or propositions that connect the research questions to the evidence, multiple sources of evidence that converge on the same questions, and a documented chain of evidence linking questions to data to conclusions. Stake (1995) approaches the case differently, as a bounded system whose meaning is constructed by the people within it and interpreted by a researcher who stays close to its particulars. His emphasis falls on issues rather than hypotheses, on direct interpretation of individual instances alongside

the aggregation of instances into categories, and on the naturalistic generalizations that readers form when a case is described richly enough for them to recognize its resonance with their own settings. Yazan (2015) shows that these positions rest on different epistemological commitments, Yin's closer to post-positivism and Stake's to constructivism, and that researchers who draw on both need to be explicit about what they take from each.

This study takes its interpretive stance from Stake and its design discipline from Yin. The questions it asks are about meaning: what preservice teachers did and said as they encountered MathVR, how they later described it as a teaching resource, and what both reveal about their emergent professional knowledge. These are questions about participants' constructions, which Stake's conception of the case was built to address, and the study is instrumental in his sense because the case is studied for what it reveals about a larger issue, the formation of professional knowledge through immersive experience, rather than for its own sake.

At the same time, a case assembled from four data sources and three analytic procedures needs the structural safeguards that Yin provides. His requirement that the case be bounded explicitly is met by the case definition above; his insistence on multiple sources of evidence is met by the combination of session recordings, field notes, freewriting, and focus group discussion; and his chain of evidence is realized in the analytic matrices, memos, and audit records described under Trustworthiness and Analytic Rigor later in this chapter, where Stake's complementary requirement that interpretations remain traceable to the particulars of the case is also taken up. In Yin's typology the study is exploratory, because it describes and interprets a phenomenon about which little is yet known rather than testing explanations of it; in Stake's typology it is instrumental and, because the ten participants are treated as one bounded case rather than as ten cases, it is a single case rather than a collective one. Drawing on both authors is

therefore a considered division of labour rather than an eclectic citation practice: Yin governs how the case is bounded, evidenced, and audited, and Stake governs what the case is taken to be and how its meaning is interpreted.

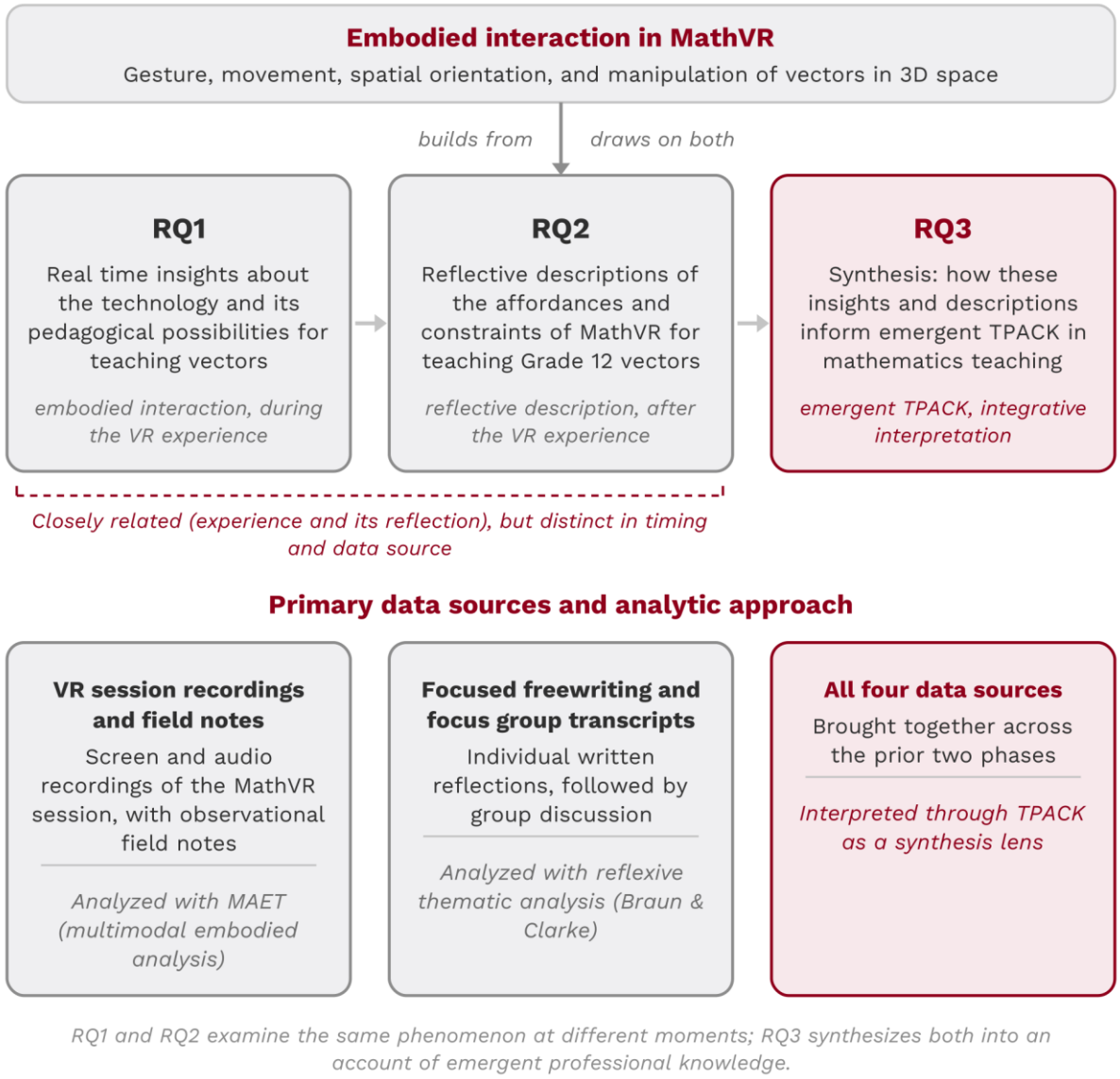
The design is informed by Jacobsen and McKenney's (2023) distinction among research for interventions, research on interventions, and research through interventions. Although the present study is not an educational design research project, it aligns conceptually with research through interventions in the limited sense that a structured intervention is used as the context for examining the responses it elicits and the meaning making that unfolds around it (Jacobsen & McKenney, 2023). In this study, the intervention is a single, structured VR session in which participants engage in vector focused tasks in three-dimensional space, followed by written reflection and group discussion. Data were collected from multiple sources, including VR session screen recordings with audio, observational field notes, post experience freewriting, and focus group discussion. Together, these sources support a rich account of how participants navigated the activity, what they attended to, and how they evaluated instructional value, feasibility, and pedagogical integration after the experience.

Relationship Among the Research Questions

Because the study addresses three research questions that examine the same phenomenon at different moments and through different data sources, the relationship among the questions warrants explicit description. Figure 2 represents this relationship.

Figure 2

Relationship Among the Three Research Questions



RQ1 and RQ2 examine the same phenomenon at different moments; RQ3 synthesizes both into an account of emergent professional knowledge.

The three questions are connected by a shared object of inquiry, namely the preservice teachers' encounters with vector operations in MathVR. They differ, however, in their moments of focus. Research Question One concerns embodied interaction: the insights participants

provided, through what they did and said, during the immersive VR experience itself. As described below in detail, the primary data used to answer this question are VR session recordings with audio and observational field notes, analyzed using Multimodal Analysis for Embodied Technologies (Walkington et al., 2024a). Research Question Two concerns reflective description: how participants described the affordances and constraints of MathVR for teaching Grade 12 vectors after the immersive experience had concluded. I used focused freewriting responses and focus group transcripts, analyzed thematically, to explore this question. Research Question Three concerns emergent TPACK: it prompts synthesis across the prior two questions by interpreting how the embodied and reflective accounts together inform an understanding of participants' emergent TPACK.

Context of the Research

The study was conducted in the Faculty of Education at the University of Ottawa within a teacher education program that prepares preservice teachers for intermediate and senior mathematics instruction. The study focused on their engagement with Grade 12 vector concepts, because vectors are a central topic in the Ontario Grade 12 mathematics curriculum (Ontario Ministry of Education, 2007) and are commonly positioned as conceptually and representationally demanding within both mathematics and physics contexts.

Data collection took place in edstudiO, a university-based learning space equipped to support immersive technology use. This setting was selected because it provided the physical space, technical infrastructure, and safety conditions required for head-mounted VR sessions, including boundary calibration for movement, reliable casting of the VR view to an external display for observation and screen recording, and a controlled environment for audio recording during group discussion.

The research activities were not embedded within participants' routine coursework or practicum classrooms. Instead, participants attended a structured research session outside of regular class time. This approach ensured that all participants engaged with the same VR tasks and prompts under similar conditions, which supported comparison across accounts and reduced variability introduced by differences in classroom placements, school resources, or instructor practices. At the same time, because the experience occurred in a university setting rather than a school classroom, participants' pedagogical reflections were oriented to anticipated classroom implementation rather than immediate enactment with secondary students.

The intervention used MathVR (Cabanag, 2023) on Meta Quest 2 headsets. The MathVR session was designed to provide a shared experience of manipulating and interpreting vectors in three-dimensional space and to support subsequent reflection on how such interaction might translate into instructional practice for Grade 12 students. The context of edstudiO therefore functioned as a controlled teacher education research environment where immersive technology use could be safely facilitated and documented through multimodal recording.

Participants

All participants were preservice teacher candidates enrolled in the English-language teacher education program at the Faculty of Education, University of Ottawa. They were all seeking certification for Intermediate and Senior mathematics teaching. All participants had an academic and professional orientation toward mathematics education, which supported content specific discussion of Grade 12 vectors and reduced disciplinary variability in how the tasks were interpreted.

Participants were recruited through multiple channels within the Faculty of Education. A research poster was posted on Faculty notice boards, and a recruitment announcement was

shared through Brightspace in preservice teacher course spaces. The Brightspace announcement and accompanying registration materials outlined the purpose of the study, the voluntary nature of participation, and the requirements for participation, including the immersive virtual reality session, written reflections, and focus group discussion. Recruitment materials advertised the honorarium provided for participation ($50.00).

Interested individuals contacted me through registration forms to express interest. Participants were selected on a first come, first served basis, with a waitlist established in case of withdrawals. This recruitment approach supported transparency and was feasible within the logistical constraints of scheduling immersive virtual reality sessions.

Timing of participation

A pilot study was conducted on January 21, 2025 with two participants to test the feasibility of the protocol, including orientation procedures, task pacing, recording methods, and the clarity of freewriting and discussion prompts. Pilot data were used to refine the session procedures but were not included in the main analysis.

The main study included ten participants who took part across three data collection dates.

Three participants (all names are pseudonyms), Nora, Amaya, and Adrian, participated on February 13, 2025. Nili and Violet participated on February 21, 2025, and Maren, Cassia, Karim, Kai, and Philip participated on February 27, 2025. Participants were divided into three small groups to support safe headset management and to facilitate meaningful group discussion.

Table 1 reports the background information collected for each participant. As the table makes visible, the study did not systematically collect information about participants' age, gender, cultural or linguistic background, or prior experience with immersive technology beyond

what individual participants stated in registration forms and volunteered in conversation. This boundary is addressed further in the Limitations section of Chapter Six.

Table 1

Participant Background Information

Pseudonym	Stream	Teachable Subject(s)	Expected Graduation	Relevant Background Noted
Nora	Intermediate-Senior	Physics, Mathematics	2026	Physics background informed interpretation of projection
Amaya	Intermediate-Senior	Mathematics, French, English, Sciences	2025	Approached the session with an expressive, artistic orientation
Adrian	Intermediate-Senior	Mathematics, English	2026	First-time VR user
Violet	Intermediate-Senior	Chemistry (primary), Mathematics (secondary)	2026	First-time VR user
Nili	Junior-Intermediate	Mathematics	2025	Prior experience working with children; attentive to IEP-related needs
Maren	Intermediate-Senior	Mathematics	2026	Not specified
Cassia	Intermediate-Senior	Mathematics	2026	Not specified
Karim	Intermediate-Senior (second year)	Mathematics, Science	2026	Prior experience with educational

				technology; no prior VR experience
Kai	Intermediate-Senior	Not specified	2026	Not specified
Philip	Intermediate-Senior	Mathematics, Physics	2025	One prior VR experience

Rationale for Size of Participant Group

The use of a small, purposeful sample is consistent with this study's qualitative, exploratory case study design, which prioritizes depth of understanding over statistical generalizability. Qualitative studies often rely on intensive data collection with a limited number of participants in order to capture rich, detailed accounts of experience, since what matters for qualitative rigor is not the number of participants itself but whether the data collected can richly represent the phenomenon under study (Hennink & Kaiser, 2022). In a systematic review of 23 empirical studies assessing saturation in qualitative research, Hennink and Kaiser (2022) found that "saturation can be achieved in a narrow range of interviews (9–17) or focus group discussions (4–8)" (p. 9), particularly in studies with relatively homogenous study populations and narrowly defined objectives, both of which characterize the present study. Accordingly, the sample size of ten participants was considered sufficient to allow for in depth analysis of individual experiences while still capturing variation in perspectives, reflections, and responses to the VR intervention.

Description of Research Methods

This section describes the procedures used to generate the study data and presents the chronology of each research session. Data collection was organized as a single structured session per participant group, lasting approximately two and a half hours. Each session was designed to

provide a shared immersive VR experience with vector tasks and to elicit both immediate interactional evidence and post experience reflection on pedagogical use.

Scaled Implementation (Pilot Study)

Prior to the main study, a pilot study was conducted as a scaled implementation of the research design. The pilot involved two preservice mathematics teacher candidates and followed the same basic structure as the main study. Its purpose was to test the clarity of instructions, the functionality of the VR equipment and MathVR application, the sequencing of activities, and the feasibility of the data collection procedures.

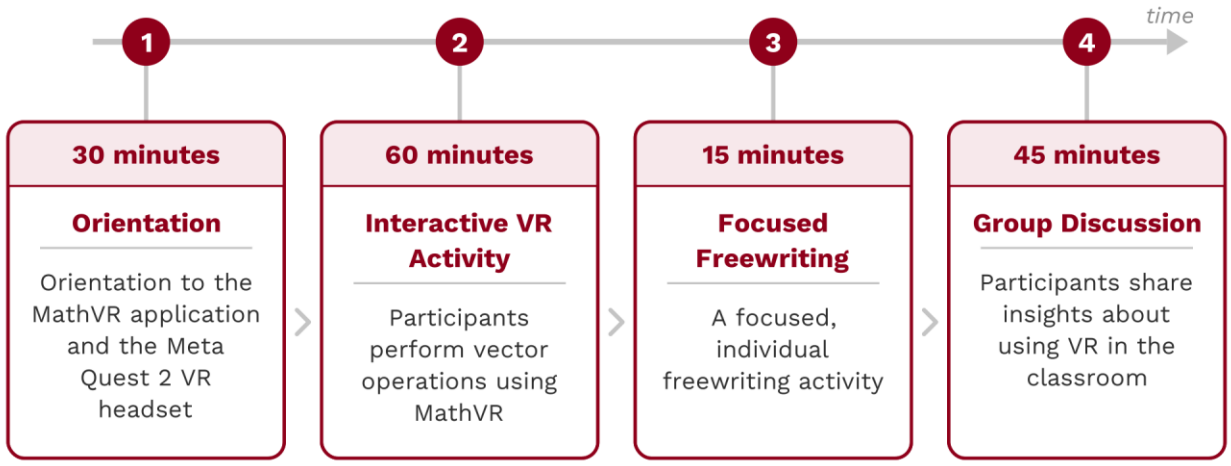
The pilot also provided an opportunity to refine the reflective prompts and focus group questions. Minor adjustments were made following the pilot in order to improve clarity and flow. Because these adjustments were made after the pilot, pilot data were not included in the main analysis.

Main Session Structure and Chronology

Each session followed the same sequence. Figure 3 summarizes the overall research sequence.

Figure 3

Research Design Chronology



Total duration approximately 150 minutes (2.5 hours).

Orientation to VR equipment and MathVR

The first part of each session was an orientation to the Meta Quest 2 headset, hand controllers, and the MathVR application. The orientation included headset fitting, comfort and safety guidance, calibration of the play boundary, and basic navigation within MathVR. Participants were introduced to how to use the controllers to select tools and interact with objects and were given a brief opportunity to practice movement and basic actions. The goal was to establish baseline operational familiarity so that the subsequent activity would reflect participants' engagement with the mathematical tasks rather than preventable confusion about device use.

Immersive VR activity focused on vector operations

The second part comprised the 60 minute interactive VR activity, in which participants engaged in a structured sequence of vector focused tasks in MathVR. The tasks were designed to prompt interaction with vectors as spatial objects and to surface how participants interpreted

vector operations through embodied manipulation in three-dimensional space. Across the activity, participants constructed vectors using given components, translated vectors, scaled vectors, and explored vector addition and subtraction through spatial arrangement, such as aligning vectors tail to head and interpreting resultant vectors. Participants also engaged in tasks that required perspective shifting, such as repositioning their bodies or changing viewpoint to interpret a configuration from different angles.

Participants interacted using the controllers and moved in the physical space as needed to change viewpoint in the virtual environment. Depending on equipment availability and session logistics, participants worked individually or in pairs, but all participants had hands-on interaction with the environment. I facilitated the session by providing procedural guidance about the task sequence and offering technical assistance when necessary to prevent interruptions, while avoiding conceptual instruction on vectors that would replace participants' own reasoning during the activity.

During the VR activity, the virtual view was screen recorded with audio, capturing what participants saw and did inside MathVR as well as their in the moment comments, questions, and observations. Field notes were also recorded to document notable episodes, including moments of ease or difficulty, spontaneous pedagogical remarks, or points where participants explicitly connected the experience to teaching.

Focused freewriting

The third part was a focused freewriting task, completed immediately after the VR activity. Prompts invited participants to describe what they perceived as the main affordances and constraints of MathVR for teaching Grade 12 vectors, what aspects of the experience supported or hindered mathematical sense making, and what conditions they believed would be

necessary for classroom feasibility. The full set of freewriting prompts is provided in Appendix A.

Focused freewriting was chosen as the first reflective instrument for three reasons. Freewriting, as Elbow (1973, 1998) developed it, asks writers to write continuously for a set time without stopping to edit, so that thinking is captured as it unfolds rather than after it has been tidied for an audience; focused freewriting applies the same rule to a given topic or prompt (Elbow, 1998; Li, 2007). First, this made the task suited to recording participants' initial sense making while the embodied experience was still fresh, before composing a polished account, or hearing other accounts, had reorganized it. Second, writing continuously to a prompt immediately after an experience has been found to aid comprehension and the consolidation of ideas (Hinkle & Hinkle, 1990), and focused freewriting has been used as a learning tool beyond writing instruction because it draws out thinking that writers have not yet formulated (Elbow, 1989; Li, 2007), which is the kind of emerging insight the study set out to document. Third, because every participant wrote before anyone spoke, the freewriting preserved ten individual accounts that could later be read against the collective account produced in the focus group, where group dynamics shape what is said and how it is said (Morgan, 2011). Written reflection also has a long-standing place in teacher education as a means by which preservice teachers make their reasoning about practice available for examination (Hatton & Smith, 1995), so the task was a familiar genre for participants rather than an unfamiliar research procedure.

Focus group discussion

The fourth and final part was a semi-structured focus group that I facilitated. The discussion built on the VR experience and the freewriting and focused on the affordances, constraints, and pedagogical implications of VR for teaching vectors, including how VR might

fit within a lesson sequence, what kinds of student thinking might be supported or obscured, how learning could be assessed when students are immersed, and what equity and access concerns would shape implementation (see Appendix B). The focus group format was selected because it is well suited to eliciting comparison of perspectives, the articulation of shared concerns, and divergent interpretations through group dialogue (Morgan, 2011). It also serves a specific function within a case study design. Case study research is defined by the use of multiple sources of evidence converging on a bounded system rather than by any single instrument (Baxter & Jack, 2008; Yin, 2018), and Stake (1995) characterizes the qualitative case study as an intensive account of a particular instance in which the researcher seeks the meanings participants themselves attach to their experience. The focus group contributes the collective dimension of that account. Where the freewriting captures individual sense making before it is shaped by discussion, the focus group makes visible where participants converged, where they disagreed, and how they revised their judgements in response to one another, which is evidence about the case that no individual instrument can supply. This methodological approach parallels other studies of preservice teachers' encounters with immersive technology, such as Taggart et al. (2023), who similarly followed a hands-on technology workshop with post-workshop focus groups to capture how participants compared and negotiated their interpretations of a shared experience. The discussion was audio recorded and later transcribed for analysis, and the focus group guide is provided in Appendix B.

Researcher role during data collection

Throughout the session, I acted as facilitator and observer. During the orientation and VR activity, I ensured safety, supported equipment operation, and guided participants through the task sequence. Technical support was provided when needed to avoid session breakdowns. I also

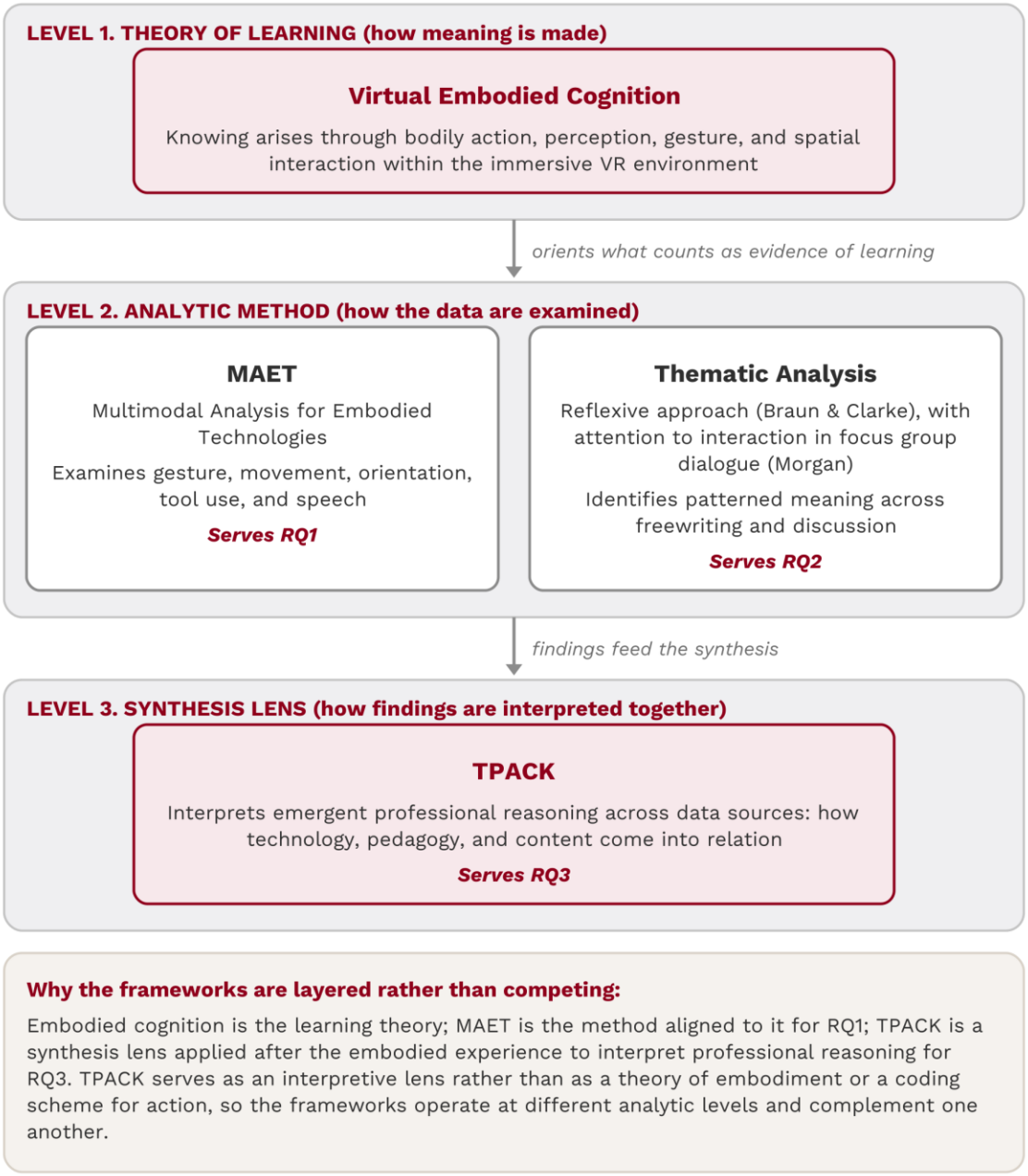
monitored participant comfort and reminded participants that they could pause or stop the VR activity at any time without penalty. The VR sessions were screen recorded from within the headsets by the participants, following my instructions, and I cast the VR display to an external screen, which allowed me to synchronize observed actions with participants' later reflections. During freewriting, I provided the prompts to participants and maintained a quiet writing environment. During the focus group, I used the discussion guide to ensure that key topics were addressed while allowing participants to pursue issues they identified as salient.

Frameworks Operating at Different Analytic Levels

The study draws on several frameworks. Their relationships to one another and to the research questions are worth making explicit before the analytic procedures are described. Virtual embodied cognition, Multimodal Analysis for Embodied Technologies, thematic analysis, and TPACK are all part of the study, but they do not occupy the same analytic level and they are not in competition as theories of knowledge. Virtual embodied cognition functions as the theory of learning. Multimodal Analysis for Embodied Technologies and thematic analysis function as analytic methods aligned to specific research questions. TPACK functions as a synthesis lens for the integrative interpretation. In Figure 4, I provide a visualization of these relationships.

Figure 4

Conceptual Framework



Virtual embodied cognition operates at the first level, as the theory of learning. It orients the study to what counts as evidence of meaning making, foregrounding learning as grounded in bodily action, perception, and spatial interaction within immersive environments (Farina, 2021;

Johnson-Glenberg, 2018). Multimodal Analysis for Embodied Technologies and thematic analysis operate at the second level, as analytic methods aligned to particular research questions and particular forms of data, the first to the interaction data generated during immersion and the second to the written and spoken reflection that followed. TPACK operates at the third level, as a synthesis lens for the integrative interpretation addressing Research Question Three (Mishra, 2019; Mishra & Koehler, 2006). Because TPACK enters only at the synthesis stage, and addresses a different question and a different form of evidence than Multimodal Analysis for Embodied Technologies, the two frameworks do not compete. Each is described in detail in the section that follows, at the point where its procedures are set out.

Description of Data Analyses

Building on the layered structure described above, this section outlines how each analytic procedure was carried out in practice. Because the study generated different forms of qualitative data, including immersive virtual reality interaction data, written reflections, and focus group discussions, different analytic approaches were used for different parts of the dataset. The analyses moved from embodied action to reflective interpretation, to integrative synthesis.

Analysis for Research Question One

Research Question One asks: While performing vector operations in MathVR, what insights do preservice teachers provide about the technology and its pedagogical possibilities for teaching vector concepts?

In this study, an insight refers to a moment, whether verbal, gestural, or both, in which a participant demonstrates emerging awareness of a technological affordance or constraint of MathVR, a mathematical relationship involving vectors, or a pedagogical possibility for teaching vectors with the tool. An insight is therefore identified not by a participant's certainty or

correctness, but by a discernible shift in attention or understanding that becomes visible in the data, for example when a participant notices an unexpected system behaviour, articulates a spatial relationship they had not previously stated, or connects an action in the virtual environment to a possible classroom use. This definition follows directly from the MAET framework's premise that gesture, movement, and speech are coordinated evidence of reasoning rather than speech alone (Walkington et al., 2024a), and it is this coordinated evidence, not verbal report in isolation, that the coding in this chapter treats as an insight.

Screen recordings of participants' views in MathVR, audio recorded during the sessions, and observational field notes were analyzed using Multimodal Analysis for Embodied Technologies, or MAET (Walkington et al., 2024a). MAET was selected because it is designed for analyzing interactional data generated in embodied educational technologies. Rather than analyzing speech alone, MAET provides a systematic method for the coordination of bodily movement, perception, speech, spatial orientation, and tool mediated action as learners engage with interactive digital environments (Walkington et al., 2024a). This made it particularly appropriate for the present study, since MathVR required participants to manipulate vectors in three-dimensional space through movement, gesture, and viewpoint shifts.

Following this methodological orientation, MAET was used as an orienting framework, rather than as a fixed or exhaustive coding taxonomy, that directed analytic attention to the embodied and multimodal features of interaction most relevant to MathVR. Four broad domains guided the analysis: gesture, body movement and positioning, speech in coordination with action, and actions on virtual objects and tools. Within these domains, analytic attention was refined through repeated viewing of the recordings. These dimensions included participants' embodied orientation within the virtual space, perceptual exploration through head and body

repositioning, actions on virtual vectors and interface tools, coordination between speech and action, responsiveness to system feedback, attention management, metacognitive monitoring, and affective displays such as hesitation, surprise, satisfaction, or frustration.

Consistent with Walkington et al. (2024a), spoken language was analyzed in relation to embodied action rather than in isolation. Verbal utterances were therefore interpreted alongside gesture, movement, and actions on virtual objects. The analysis also followed the broader two stage logic described in the MAET framework. First, I identified task-based episodes across the virtual reality sessions. Second, I selected salient moments for more fine-grained interpretation, especially where gesture, movement, tool use, and verbal reasoning converged in ways that illuminated mathematical sense making or pedagogical noticing.

Following this coding process, participant level profiles were constructed. Each account took the form of a narrative account of an individual participant's engagement with MathVR, organized chronologically and grounded in time stamped excerpts from the recordings. These accounts brought together participants' utterances and concurrent embodied actions in order to preserve the flow and context of interaction. To support cross participant comparison, a technological and mathematical insights matrix was then developed (see Tables 4 through 13).

Together, MAET coding, participant accounts construction, and the comparative matrix allowed Research Question One to be addressed through participants' real-time embodied interactions and in the moment comments during the MathVR experience.

To support the traceability of the technological and mathematical classification used throughout the Research Question One tables, Tables 2 and 3 summarize the decision rule applied to each category, together with an illustrative example from the dataset.

Table 2

Decision Rules for Technological and Mathematical Classification of Utterances and Actions

Classification	Decision Rule	Illustrative Example
Technological	Assigned when the utterance or action is directed primarily at the VR system itself: locating, activating, or troubleshooting a tool; following interface instructions; recovering from an error or reset; or monitoring task completion status.	Cassia's "What the hell is happening?" while reorienting the task board. This is coded as *technological* because her attention is directed at making sense of the interface, not at a mathematical relationship.
Mathematical	Assigned when the utterance or action occurs in the context of performing a vector operation, manipulating a mathematical object, or attending to a visible change in a vector, plane, or projection configuration.	Cassia's "Oh, that's really cool" while observing the projection line appear is coded mathematical because her attention has shifted to the mathematical result the system has produced.

The MAET coding used a binary distinction between technological and mathematical attention. A separate content-specific code was not applied, because every task in MathVR was content-specific by design; content was therefore treated as continuously present, and the codes marked where a participant's attention settled in each moment, on the tool or on the mathematics. This differs from the TPACK framework, in which content knowledge is a distinct component, and the distinction is intentional: the MAET analysis characterizes moment-to-moment attention during embodied interaction, whereas TPACK characterizes the integrated professional

knowledge that emerged across the full sequence of immersion and reflection. Pedagogical noticing was not treated as a third primary category. Attending to teaching is not a locus of attention in the same sense as the tool or the mathematics but a stance taken toward what has already been noticed, and it was therefore recorded in the secondary descriptive layer summarized in Table 3, where the code pedagogical imagination is applied when a participant connects a system behaviour to a hypothetical classroom use. A single moment can consequently carry a primary code of technological or mathematical together with a secondary code of pedagogical imagination, as when Nora watched the Generate menu produce vectors and observed that "Kids would love this." Reading the two layers together, rather than the primary code alone, is therefore necessary to recover the pedagogical dimension of what participants noticed during immersion.

Table 3

Secondary Descriptive Codes Applied Within the Technological and Mathematical Categories

Code	Decision Rule	Example
System orientation / exploratory action	Applied when a participant tries an unfamiliar feature or navigates the interface for the first time, typically early in a session and often accompanied by uncertainty.	Philip asking "How do I generate a vector?" before experimenting with spawning one.
Affective uncertainty / affective play / affective payoff / mastery acknowledgment	Applied to moments where tone or wording signals an emotional response: hesitation or confusion (uncertainty), open enjoyment or experimentation (play), a positive response to a visible result (payoff),	Cassia reading the system's explanation of the yellow line and concluding it represents the height of the vector.

Code	Decision Rule	Example
	or explicit recognition of having succeeded (acknowledgment).	
Metacognitive monitoring / full-body engagement	Applied when a participant comments on their own actions or bodily state while acting, including noticing their own movement or pausing to check their own understanding.	Cassia noting that she had been swinging her arms a lot.
Cause and effect noticing / pedagogical imagination	Applied when a participant connects a system behaviour to a hypothetical classroom use.	Nora's "Kids would love this" while testing the Generate menu.
Embodied reasoning / direction comparison / metaphorical reasoning / visual analogy	Applied when a participant uses gesture, spatial comparison, or a perceptual metaphor to work through a mathematical relationship	Philip describing projection as "seeing the shadow that is cast."

These secondary codes were applied as descriptive qualifiers within the primary technological and mathematical classification rather than as a separate coding tier with independently defined boundaries, and they are documented here for transparency rather than presented as an exhaustive taxonomy.

Analysis for Research Question Two

Research Question Two asks: After performing vector operations using MathVR, how do preservice teachers describe the affordances and constraints of MathVR for teaching vector operations to Grade 12 students in Ontario?

To address this question, I analyzed participants' focused freewriting responses and focus group transcripts using reflexive thematic analysis as developed by Braun and Clarke (2006, 2019, 2021), supplemented by Morgan's (2011) guidance on attending to the interactional features of focus group dialogue. Thematic analysis was selected because it offers a systematic yet flexible way of identifying and interpreting patterned meaning across textual data, and it was especially appropriate here because Research Question Two concerns participants' articulated evaluations, judgements, and pedagogical reflections after the virtual reality experience.

Braun and Clarke's reflexive approach rests on commitments that shaped how the analysis was conducted and how its quality should be judged. Themes are understood as patterns of shared meaning organized around a central concept, generated by the researcher through sustained engagement with the data, rather than as topics that pre-exist the analysis or as summaries of everything said about a subject (Braun & Clarke, 2019, 2021). Coding is treated as an organic and evolving process rather than the application of a fixed instrument, and the researcher's subjectivity is regarded as a resource for interpretation, to be documented and reflected on, rather than a source of error to be removed through multiple coders or agreement statistics (Braun & Clarke, 2021). For this reason, I coded the data alone and did not calculate inter-coder agreement; quality was pursued instead through the transparency of the analytic record and through the reflexive practices described below and under Trustworthiness and Analytic Rigor.

The analysis followed the six phases that Braun and Clarke (2006) set out and later refined (Braun & Clarke, 2021), moving back and forth between phases rather than through them in a single pass. In the familiarization phase, I read the freewriting responses and focus group transcripts several times, re-listened to the focus group recordings, and recorded initial

observations in the analytic journal. In the coding phase, I worked systematically through both data sources, labelling segments that captured recurring ideas, evaluative judgements, emotional responses, and pedagogical reflections. Coding combined inductive and deductive orientations: many codes were data derived and stayed close to participants' own language, such as what they found helpful, confusing, engaging, difficult, or pedagogically significant, while others were theory informed by the study's conceptual concerns with embodiment, technological affordances and constraints, conceptual understanding, pedagogical application, and classroom feasibility. Codes were applied at both semantic and latent levels, so that Amaya's explicit statement that the headset made her dizzy was coded for what it stated, while Cassia's remark that she had finished tasks without understanding how was also coded for its latent concern that the system's feedback had come apart from her comprehension. In the phase of generating initial themes, I compared codes across participants and across the two data sources, clustered related codes, and drafted candidate themes that expressed a shared meaning rather than a shared topic. In the reviewing phase, I tested each candidate theme against the coded extracts and against the full dataset, splitting, merging, or setting aside themes that were not internally coherent, not distinct from one another, or not adequately supported. In the defining and naming phase, I wrote a short definition for each theme and chose names that convey its central organizing concept, such as Embodied Spatial Reasoning or Interface Complexity and Learning Curve. Writing the results chapter was the final phase and continued the analysis, since arranging extracts and interpretive commentary tested once more whether each theme held together.

Two features of how the themes are reported in Chapter Five follow from this process. First, the four headings under which the results are organized, affordances, constraints, pedagogical implications, and equity considerations, are the domains set by Research Question

Two and by the freewriting and focus group prompts. They are a deductive organizing frame rather than themes in Braun and Clarke's sense; the themes themselves sit within these domains and were generated inductively. Second, because the data came from two sources, I coded the freewriting and the focus group transcripts within one analytic process rather than separately, which allowed themes to draw on both the individual sense making recorded before discussion and the collective sense making produced in it. Following Morgan (2011), I treated moments of agreement, disagreement, and revision in the focus groups as analytically meaningful in their own right, and I noted in the analytic journal where a theme rested mainly on one source or on one or two participants.

Reflexivity was maintained throughout in the analytic journal, in which I recorded coding definitions and their revisions, interpretive decisions, and my own reactions to the data, including the pull toward reading participants' enthusiasm as evidence of learning, which the journal helped me to hold at a distance. Periodic discussions with my supervisor served as a check on these interpretations in the manner of a critical dialogue rather than as a test of coding agreement. The evolving code list that these records contain functioned as a trace of analytic development rather than as a fixed codebook (Braun & Clarke, 2021).

The analysis also remained attentive to embodied references within participants' written and spoken accounts. When participants described actions such as reaching, grabbing, walking around, or seeing vectors from different angles, these descriptions were treated as analytically meaningful because they revealed how participants retrospectively interpreted embodied engagement as part of their learning experience.

To support cross participant comparison, a thematic matrix was developed (see Table 14 in Chapter Five). In this matrix, each row represented an individual participant, while columns

documented perceived affordances of MathVR, perceived constraints of MathVR, representative quotations or summaries, pedagogical implications for teaching vectors, and equity or accessibility considerations. This matrix did not replace the thematic analysis itself, but it supported analytic movement from individual reflections to broader claims about how MathVR was being interpreted as a pedagogical tool.

This approach allowed Research Question Two to be addressed through participants' reflective and dialogic evaluations of MathVR, grounded in their post experience reasoning about affordances, constraints, and classroom feasibility.

Analysis for Research Question Three

Research Question Three asks: How do these insights and descriptions inform understanding of preservice teachers' emergent Technological Pedagogical Content Knowledge (TPACK), particularly in relation to a technology that enables visualization and virtual embodied experience within mathematics education?

To address this question, findings from the first two analytic phases were brought together through the Technological Pedagogical Content Knowledge (TPACK) framework as an interpretive synthesis lens. Unlike Research Questions One and Two, which examined specific data sources through distinct analytic approaches, Research Question Three was integrative.

While the Multimodal Analysis for Embodied Technologies (MAET) captured how participants engaged with mathematical ideas through embodied action in VR, and thematic analysis illuminated how participants articulated and reflected on those experiences, TPACK provided a conceptual structure for interpreting how these dimensions converged as forms of professional knowledge. Specifically, TPACK enabled examination of how participants'

technological experiences in VR were connected to their pedagogical considerations and mathematical understanding, rather than treating these domains as isolated outcomes.

TPACK was used as a framework for interpretation rather than as an action level coding scheme. Individual gestures, utterances, and interactional moves were not coded as discrete instances of technological, pedagogical, or content knowledge, since TPACK is intended to describe integrated knowledge structures rather than moment-to-moment behaviour, and fragmenting it into isolated indicators can misrepresent its holistic and transformative character (Angeli & Valanides, 2009; Cox & Graham, 2009; Willermark, 2018). A synthesis matrix structured around the major TPACK domains served as the principal analytic tool for this phase, treating technological, pedagogical, and content knowledge and their intersections as interpretive categories rather than coding labels (Mishra & Koehler, 2006). Evidence from the virtual reality sessions, focused freewriting responses, and focus group discussions was aligned within this structure to support comparison across modalities.

The analysis examined how participants' embodied experiences in MathVR, their conceptual reflections on vectors, and their pedagogical judgements about classroom use converged. A participant who described how physically manipulating vectors clarified addition, direction, or projection was interpreted as showing interaction between technological affordances and content understanding; where the same participant also reflected on how such experiences might support student learning, that reasoning was examined as pedagogical. Approaching the synthesis this way kept findings from being separated by data type, with physical actions on one side and verbal reflections on the other, and supported a unified interpretation of how immersive experiences contributed to the integration of technology, pedagogy, and content knowledge (Willermark, 2018). Anchoring the synthesis in an established framework for studying teacher

knowledge and technology integration also positioned the findings within a recognized body of scholarship on preservice teacher development and technology rich pedagogy (Schmidt et al., 2009; Voogt et al., 2013; Wang et al., 2018).

Trustworthiness and Analytic Rigor

To support analytic rigor and trustworthiness, several procedures were embedded throughout the research process, addressing the criteria of credibility, dependability, confirmability, and transferability that Lincoln and Guba (1985) proposed as the qualitative counterparts to reliability and validity, and that Nowell et al. (2017) operationalize specifically for thematic analysis. These procedures respond to the demands of both analytic traditions the study draws on. Braun and Clarke (2006, 2021) treat thematic analysis as a situated interpretive practice in which quality depends on the transparency of the analytic process rather than on the elimination of researcher subjectivity, so the procedures below document how codes and themes were developed rather than claiming a neutral reading of the data. Case study methodology adds a further requirement, since Yin (2018) argues that a case study's credibility rests on a documented chain of evidence linking questions to data to conclusions, and Stake (1995) emphasizes that interpretations must remain traceable to the particulars of the case. Particular attention was therefore given to documenting analytic decisions, maintaining consistency across data sources, and ensuring that interpretations remained closely grounded in the data.

Analytic decisions were systematically documented through analytic memos, evolving codebooks, and structured audit records. Throughout the analysis, an analytic journal was maintained in which coding definitions, revisions, and interpretive decisions were recorded. This documentation created a clear audit trail linking raw data to analytic interpretations. By preserving a detailed record of how codes and themes were developed, revised, and applied, the

analysis process remained fully traceable. This level of transparency allows readers to follow the progression from data generation to analytic claims and supports the credibility of the findings (Nowell et al., 2017).

Coding procedures were applied consistently across all relevant data sources. I conducted the initial coding and refined the codes iteratively as the analysis progressed. When codes were revised or clarified, I revisited previously coded data to ensure consistency across the dataset. Although I conducted the analysis alone, which reflexive thematic analysis treats as a legitimate design rather than a limitation (Braun & Clarke, 2021), I discussed analytic interpretations and coding decisions periodically with my supervisor. These discussions functioned as a form of external analytic checking, creating space to question assumptions, refine interpretations, and strengthen the dependability of the analysis.

Coded data were also organized using analytic matrices that explicitly linked data excerpts to codes, themes, and interpretive claims. The complete MAET session matrices for all ten participants are provided in Tables 4 through 13, and the TPACK Evidence Matrix synthesizing across them is provided in Appendix C. For example, matrices documented how specific utterances, embodied actions, or reflective statements supported particular themes or knowledge domains. These matrices functioned both as analytic tools and as records of decision making, allowing the relationship between evidence and interpretation to be examined clearly. In practice, presenting data in this matrix form makes it easy to see how "the association between the raw data and the categorization matrix is clearly shown and easily followed," which strengthens trustworthiness (Assarroudi et al., 2018, p. 51). By making visible how conclusions were derived from the data, the matrices strengthened confirmability and provided a structured means of cross participant comparison (see Appendix C).

Throughout the analytic process, close alignment was maintained between data excerpts and analytic claims. Analytic memos documented how specific excerpts supported themes or interpretive conclusions. Coding tables and matrices were reviewed regularly to ensure coherence between evidence and interpretation. This ongoing attention to alignment helped ensure that findings were grounded in participants' actions and accounts rather than imposed by me.

Ethical Considerations

Ethical approval for this study was obtained from the University of Ottawa Research Ethics Board (Ethics File Number S-11-24-11007) prior to recruitment and data collection, and all procedures followed the approved protocol. Informed consent was obtained from every participant. Participants were told the purpose of the research, the data to be collected, and their right to withdraw at any time without penalty. Participation was voluntary, and it was made clear that declining or withdrawing would not affect participants' academic standing or relationship with the university.

Confidentiality was maintained at every stage. Pseudonyms were used in all transcripts, reports, and publications, and audio and video recordings were stored on password-protected, encrypted devices accessible only to me and used solely for analysis.

Because the study used immersive virtual reality, particular attention was paid to comfort, well-being, and accessibility. Participants were told in advance that VR can cause discomfort such as dizziness or motion sickness, and that they could pause or stop at any time without any effect on their compensation or continued participation in the written and discussion activities. Headsets were adjusted to accommodate participants who wear glasses. These measures were

intended to minimize physical discomfort and to create an inclusive and respectful research environment.

Chapter Summary

This chapter has outlined the methodological design and analytic procedures used in the study, with particular emphasis on how the research questions were addressed through carefully aligned analytic frameworks. The chapter detailed the research design, data collection processes, and analytic strategies used to examine preservice teachers' embodied interaction with MathVR, their reflective descriptions of its affordances and constraints for teaching Grade 12 vectors, and their emergent TPACK.

Each analytic procedure was explicitly linked to a specific research question. Multimodal analysis was used to examine embodied interaction during VR use, thematic analysis was applied to reflective and discursive data, and a TPACK oriented synthesis was used to integrate insights across data sources. Procedures for ensuring trustworthiness and analytic rigor were also described, demonstrating how transparency, consistency, and alignment between data and interpretation were maintained throughout the study.

By making analytic decisions explicit and grounding interpretations in multiple forms of evidence, this chapter establishes a strong methodological foundation for the findings that follow. Chapter Five presents the results of these analyses, organized by research question and analytic approach. The transparent and systematic procedures described in this chapter underpin the credibility of those findings and support the study's contributions to understanding embodied learning and teacher knowledge development in immersive VR contexts.

Chapter Five

Results

Research Question One: Participant Accounts and Individual Trajectories in MathVR

RQ1: While performing vector operations in MathVR, what insights do preservice teachers provide about the technology and its pedagogical possibilities for teaching vector concepts?

The participant accounts presented here show how individual participants engaged with MathVR and made sense of vector operations through embodied interaction while using VR. By focusing on each participant, I show how the interplay of affect, cognition, and pedagogy unfolded in real time. Each account is a description of the participant's experiences, supported by excerpts from VR transcripts. In presenting these accounts, the aim is to foreground the richness and variability of participants' perspectives, while preparing the ground for the subsequent MAET coding and cross participant synthesis that build on these individual accounts.

As noted above (refer to Table 2 in Chapter Four) utterances and actions are described as *technological* when they primarily involve orienting to the VR system, navigating interface elements, managing tools, or responding to system feedback. Utterances and actions are described as *mathematical* when they occur in the context of performing vector operations, manipulating mathematical objects, or attending to visible changes in vector configurations. The length of the descriptive accounts varies across participants due to differences in the duration of available VR recordings.

Nora

Nora was a preservice teacher in the Intermediate-Senior stream with teachables in Physics and Mathematics. She anticipated graduating in 2026. Her background in physics gave

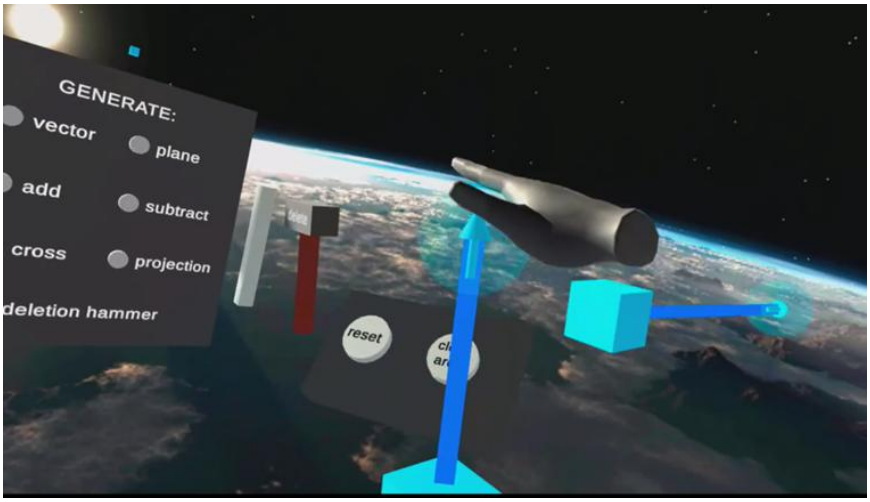
her a strong conceptual foundation, and she frequently drew on this during her MathVR session to interpret operations involving planes and vector projection.

Nora entered the MathVR space with immediate curiosity. The first moments showed her testing the relationship between button presses and the appearance of vectors. Her early utterance, “Multiplication gives multiple vectors,” indicated that she was trying to build a conceptual map between mathematical terminology and VR behaviours. She tapped the generator repeatedly, noticing how new arrows appeared in space, and commented with excitement that “kids would love this,” showing her spontaneous pedagogical imagination emerging directly from the tactile experience.

Nora moved her hand toward the floating tools on her left (Figure 5). The tools included a white delete hammer and a red delete hammer. Reset and clear all buttons floated beside them. She looked at the tools and then moved her hand away without selecting them.

Figure 5

Nora Reaching Toward the Generate Panel and Tool Buttons

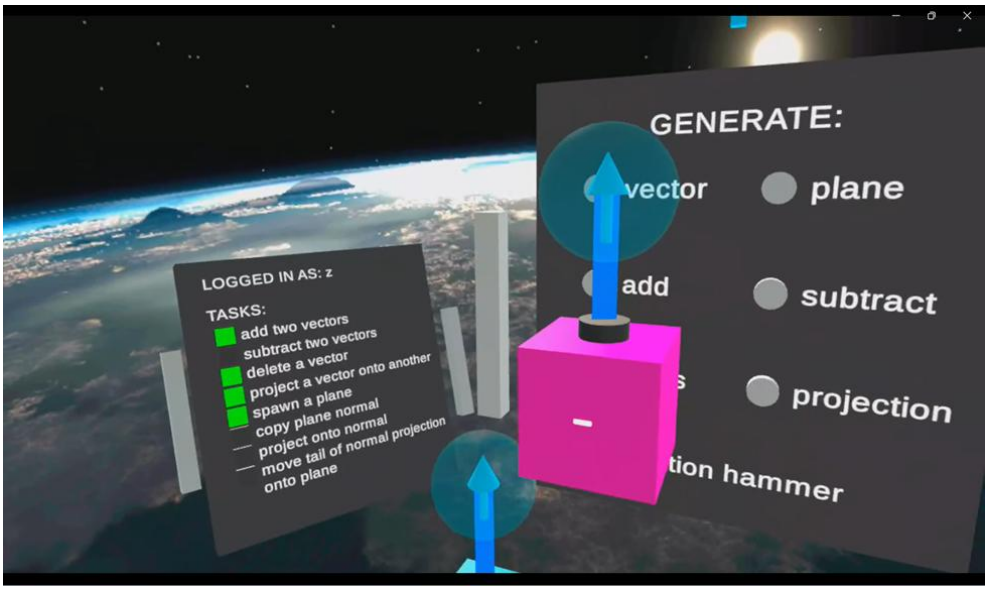


Note. Screen capture from Nora’s headset recording.

She stood still and said, “I can be in this environment forever.” Her hand remained near the vector. Her head tilted slightly while she continued looking at the arrow (Figure 6).

Figure 6

Nora’s Task Board With Completed Tasks and a Subtraction Block



Note. Screen capture from Nora’s headset recording.

Several minutes later she was positioned in front of the task board. The board showed multiple completed tasks with green check marks. The completed tasks included add two vectors, project a vector onto another, spawn a plane, and copy plane normal. The remaining tasks were not checked. She adjusted the arrow by pulling it upward. The arrow lifted. She rotated the arrow. To clarify how Nora’s interaction unfolded over time, Table 4 presents selected moments from her VR session, including the timestamp, her utterance-based insight, her action-based insight, and the analytic classification assigned to each moment. The table helps show how technological exploration and mathematical noticing were intertwined during the session

Table 4

Nora's VR Session Matrix (RQ1)

Timestamps	Utterance-Based Insight	Action-Based Insight	Technological or Mathematical
00:00:02	"Multiplication gives multiple vectors."	Presses generator repeatedly and observes new vectors appearing in sequence	Technological (testing system behaviour)
00:00:06	"Whenever I press this one, it generates? Kids would love this."	Leans toward Generate menu and uses hand to test multiple buttons	Mathematical (pedagogical imagination prompted by system behaviour)
00:00:18	"That is very interesting."	Moves around vector objects, visually inspects how each new vector behaves in space	Mathematical (attending to vector behaviour)
00:00:40	"I can be in this environment forever."	Slow exploratory movement, extended hand hovering over vector tip with careful wrist rotations	Technological (exploratory interface engagement)
00:00:46	(Silent)	Tilts head and shifts body while inspecting multiple	Technological (tool inspection)

		tools such as delete hammer and reset buttons	
00:00:55	(Silent)	Grasps and drags the endpoint of a turquoise vector, testing responsiveness and directionality	Mathematical (manipulating a vector)
00:03:05	(Silent)	Approaches task board, studies remaining tasks, then realigns tall blue vector on pink cube with controlled hand positioning	Mathematical (aligning a vector relative to a plane)

Across Nora's session, a clear balance emerged between technological and mathematical considerations. Technologically, she navigated moments of hesitation, spatial disorientation, and interface learning, gradually building fluency with alignment, rotation, and projection tools. Mathematically, her VR actions repeatedly supported conceptual noticing, including seeing head to tail relationships, observing how vectors behave on a plane, and connecting embodied alignment with her own established problem-solving habits. For Nora, the technological environment acted as a gateway that enabled deeper visualization and clarification of vector

relations, and her comments signaled that the immersive space supported both engagement and conceptual grounding.

Amaya

Amaya was an Intermediate-Senior preservice teacher with teachable subjects in Mathematics, French, English, and Sciences. She was set to graduate in 2025. Amaya approached the VR experience with an artistic and expressive mindset, often referring to the emotional and sensory impact of the virtual space.

During the VR session, Amaya showed strong signs of immersion in the virtual environment. Early in the exploration, she commented aloud that the space felt intensely real and she paused to steady herself when she experienced a brief moment of dizziness. As she began manipulating vectors, she used wide, expressive arm movements and stepped forward and backward to reposition her body in relation to the objects. Her gestures became increasingly fluid as she experimented with spatial arrangements, often rotating her torso and tilting her head to align her view with the vectors. At several points she vocalized her sense of being fully inside the environment, remarking that it felt like “real reality” while continuing to adjust her movement to maintain balance and perspective. Her actions suggest sustained engagement with the virtual space and a strong embodied response to the depth and scale of the environment. To clarify how Amaya’s interaction unfolded over time, Table 5 highlights selected moments from her VR session, showing how her utterances, actions, and insights reflected both embodied engagement and technological sense making.

Table 5

Amaya's VR Session Matrix (RQ1)

Timestamps	Utterance-Based Insight	Action-Based Insight	Technological or Mathematical
00:00:01.01	"The one that says subtract has two coming out. It already has two."	Notices two vectors already attached to "subtract" too	Technological (Expressing confusion with tool's initial state)
00:00:36.18	"I do not know how to subtract if it has two."	Begins interacting with vector menu	Technological (Expresses difficulty understanding system logic. Cognitive load on digital tool use)
00:01:02.06	"So subtract. I will add subtract here, right?"	Successfully selects the "Subtract" option	Technological (Verifies her process. Demonstrates trial-and-error learning with interface)
00:01:08.01	"Now I have a pink one with the blue vector."	Places pink vector next to blue vector	Mathematical (Begins constructing subtraction operation visually. Shows initial conceptual mapping)
00:01:21.29	"I forgot how to add vector."	Forgets how to add a new vector	Technological (Reflects momentary breakdown in

			task flow and tool familiarity)
00:01:55.21	“Not sure if I did it correctly.”	Unsure about correctness of subtraction	Mathematical (Engaged in self-evaluation. Suggests metacognitive monitoring of conceptual understanding)
00:02:29.26	“So I move the pink one?”	Adjusts pink box location to make room	Technological (Responds to spatial clutter. Learns spatial management within VR interface)
00:02:37.24	“So it has blue, blue and then pink there.”	Mentions layout of vectors on screen	Mathematical (Summarizes the arrangement. Demonstrates spatial reasoning and mapping)

Amaya’s trajectory showed a strong interplay between the sensory impact of VR and her emerging pedagogical insights. Technologically, she experienced confusion with tool states, trial and error exploration of the interface, and breakdowns in recalling steps for addition or subtraction. Mathematically, these challenges coexisted with moments of expressive meaning making as she visualized vector relationships and arranged operations spatially. Her descriptions suggest that the immersive qualities of VR shaped how she attended to the mathematics, with

emotional engagement amplifying her curiosity and concentration even when technical difficulties interrupted her workflow.

Adrian

Adrian was an Intermediate-Senior preservice teacher with teachables in Mathematics and English. He planned to graduate in 2026. During his MathVR session, Adrian approached the environment cautiously and methodically, with much of his early interaction focused on orienting himself to the interface and understanding task structure.

Adrian began by confirming that the system was active, stating, “Yeah, now it’s on. Okay.” He searched the floating application menu and asked, “Would it be in there?” while scanning the interface. As he located MathVR, he verbalized the spatial layout of the menu, noting that it should be “on the right side at the bottom,” before selecting the application and acknowledging that it was opening. He read the login prompt aloud and completed the setup by following on screen instructions.

Once inside the environment, Adrian directed his attention to the task panel. He read the listed tasks aloud, stating, “Okay, task, add two vectors, subtract. These are the tasks,” and proceeded one step at a time. He generated a vector and explicitly selected subsequent tasks, asking, “Can we start this. Subtract two vectors,” and responding to system feedback with brief affirmations such as “Perfect.”

As he moved through the tasks, Adrian frequently narrated the interface options visible to him, including subtraction and projection tools. He read advanced task labels aloud, such as “Spawn a plane. Copy a plane normal. Project onto normal,” while selecting corresponding actions. During projection, his verbal output decreased as he focused on executing the task. After successful completion, visual feedback indicated progress.

Near the end of the session, Adrian paused to review the task checklist and sought confirmation from the facilitator. He reflected on his progress by stating, “I did three out of the five… now there’s only the delete and then the X button left.” His session concluded with attention to task completion and system state rather than continued exploration. Table 6 follows this movement across the session, from his early difficulty locating controls to his closing review of the checklist.

Table 6

Adrian’s VR Session Matrix (RQ1)

Timestamps	Utterance-Based Insight	Action-Based Insight	Technological or Mathematical
00:00:00.23	Yeah, now it’s on. Okay.	Adjusts headset and orients body to VR space, signaling readiness to begin	Technological (system activation)
00:00:22.16	Do you see MathVR? Yes. Login Z. Yeah.	Reads login prompt aloud and aligns input with task progression	Technological (interface navigation)
00:00:38.18	Generate the vector.	Activates vector generation tool, initiating mathematical interaction	Mathematical (initiating a vector operation)
00:00:46.17	Okay, task, add two vectors, subtract. These are the tasks	Reads task list aloud while visually tracking task panel	Technological (orienting to task structure)

00:02:10.08	Spawn a plane. Copy a plane normal. Project onto normal.	Reads advanced task options, preparing for three-dimensional operations	Mathematical (anticipating upcoming operations)
00:03:29.24	Project onto normal, please.	Repeats projection attempt, treating system as interactive agent	Mathematical (executing a vector operation)
00:03:54.11	No verbal utterance	Observes visual confirmation of successful projection on plane	Mathematical (confirming a mathematical result)
00:04:08.27	I did three out of the five… now there's only the delete and the X button left.	Reviews task checklist and evaluates system state	Technological (monitoring progress)

Across Adrian's session, a clear balance emerged between technological and mathematical considerations. Technologically, much of his interaction focused on locating tools, reading interface labels, interpreting task prompts, and monitoring system feedback through the task checklist. He frequently verbalized on screen information and used it to guide his actions, indicating sustained attention to navigating and understanding the structure of the MathVR environment.

Mathematically, these technological interactions supported his engagement with vector operations, particularly as he progressed from generating vectors to completing subtraction and projection tasks. His step-by-step approach to selecting operations and observing visual confirmation allowed him to complete tasks in sequence and attend to how mathematical actions were represented in the environment. Together, Adrian's interaction reflects how technological navigation and mathematical engagement were intertwined, with careful system orientation enabling his participation in the mathematical tasks presented in MathVR.

Violet

Violet was an Intermediate-Senior teacher candidate with a primary teachable in Chemistry and a secondary in Mathematics. She expected to graduate in 2026. During her MathVR session, Violet's engagement was marked by careful manipulation of vectors and close attention to visual feedback from the environment (Figures 7 and 8).

Figure 7

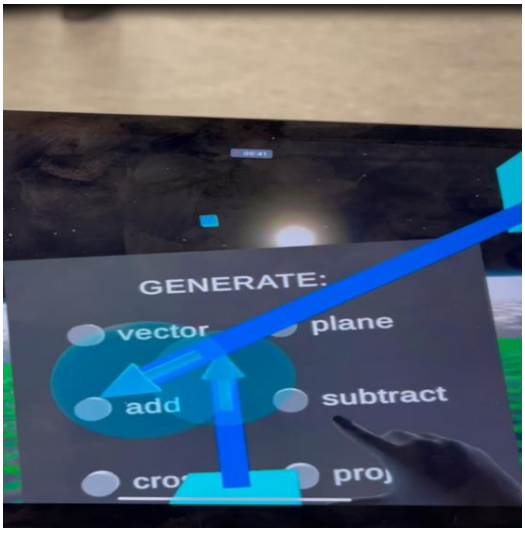
The Task Board Cast to the Tablet During Violet's Session



Note. Screen capture of the headset view cast to the tablet during Violet's session.

Figure 8

Violet Pointing Toward the Generate Panel in the Cast View



Note. Screen capture of the headset view cast to the tablet during Violet's session.

While working with vector addition and manipulation on the grid plane, Violet used both controllers to adjust a cyan vector, holding the base cube with one hand and guiding the shaft with the other. As she repositioned the vector relative to the plane, she commented, "That's cool," responding to the way the vector's movement and alignment changed in real time. Her hands remained actively engaged as she pulled and adjusted the vector while visually tracking its behaviour on the plane.

As she moved into projection tasks, Violet aligned her hand with the projection tool and activated it. She observed the resulting changes in the vector configuration as they appeared in the environment. I noted that she had already completed similar tasks earlier, and Violet proceeded to repeat them, reestablishing the task sequence after the reset.

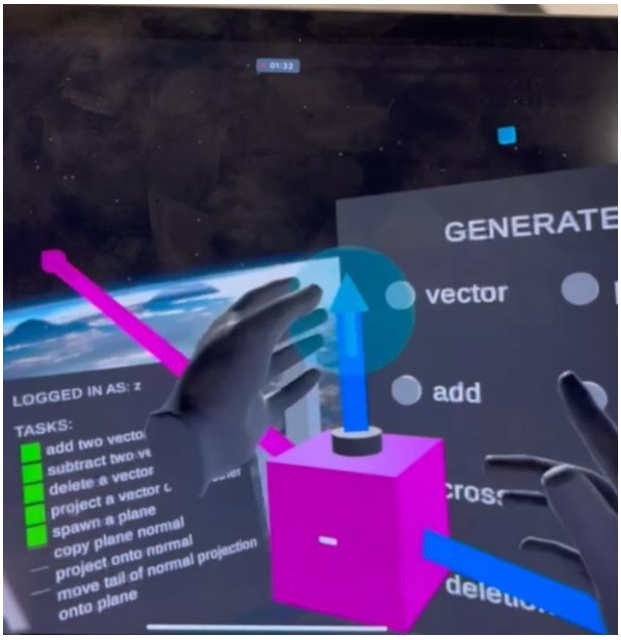
Violet continued working through vector subtraction and plane related tasks, physically manipulating the vectors and monitoring the task list as progress indicators reappeared. Her

interaction showed sustained coordination between hand movements, visual confirmation, and task completion cues within the interface.

Overall, Violet's session was characterized by precise, controlled manipulation of virtual objects and steady engagement with the task structure. Her brief verbal responses, including "That's cool," occurred at moments where changes in the visual representation aligned with her physical actions, highlighting how meaning making emerged through coordinated movement and observation in the MathVR environment (Figure 9). Table 7 presents selected moments from Violet's VR session, including the timestamp, her utterance-based insight, action-based insight, and the analytic classification assigned to each moment.

Figure 9

Violet Holding the Subtraction Block With Attached Vectors



Note. Screen capture from Violet's headset recording.

Table 7

Violet's VR Session Matrix (RQ1)

Timestamps	Utterance-Based Insight	Action-Based Insight	Technological or Mathematical
00:00:27.08	"That's cool."	Violet physically manipulates a cyan vector anchored to the grid plane. Her hands (visible in VR as controllers) are gripping and adjusting the vector: one hand on the base "X" cube, the other stabilizing the shaft.	Mathematical (responding to vector manipulation)
00:00:56.08	[No direct quote]	Violet accidentally resets the VR environment, which erases all prior task progress. The completed tasks (with green checkmarks) disappear, and the	Technological (unintended system reset)

		scene returns to its default layout.	
00:01:04.00	[No direct quote]	Violet completes the vector projection task. She aligns her virtual hand with the projection command and activates it. Her hand motion and gaze track the result as it populates.	Technological (activating the projection command)
00:01:10	No verbal utterance	Aligns virtual hand with the projection tool and activates the projection command	Mathematical (executing the projection operation)
00:01:12	No verbal utterance	Observes the projected vector populate in the environment	Mathematical (observing the mathematical result)
00:02:05.00	[No direct quote]	She successfully completes vector subtraction and the	Technological (system state update

	task of spawning a plane. Her hands manipulate the virtual objects into alignment while the task list updates accordingly.	following task completion)

Across Violet's session, her interaction involved both technological navigation and engagement with vector related tasks. Technologically, she attended to interface elements such as the task list, reset function, and projection tools, responding to system changes as they occurred. She experienced a reset that cleared the workspace and task indicators and then continued interacting with the environment.

Mathematically, her actions included manipulating vectors, aligning them with a grid plane, activating projection and subtraction tools, and monitoring task completion cues as they appeared. Her verbal responses, including "That's cool," occurred alongside moments of visible change in the vector configuration. Throughout the session, her interaction consisted of coordinated hand movements, visual monitoring of the environment, and sequential engagement with the tasks presented in MathVR.

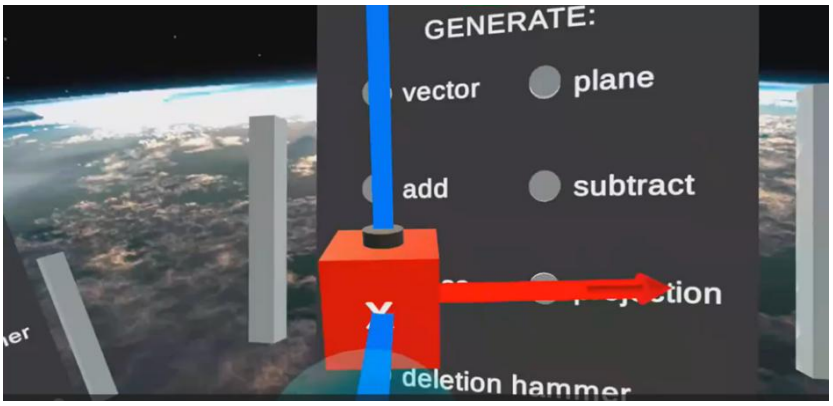
Nili

Nili was completing her Junior-Intermediate teacher education in Mathematics and was on track to graduate in 2025. As someone who had prior experience working with children, she showed a strong awareness of how learners with Individualized Education Programs (IEP) or attention difficulties might benefit from embodied learning in VR.

Nili began the session by spawning a red vector using the "generate" menu and naming it “x,” which was clearly aligned along the positive x-axis (Figure 10). She proceeded to work through the list of assigned activities displayed in-world.

Figure 10

Nili’s Red Vector Labelled x Beside the Generate Panel



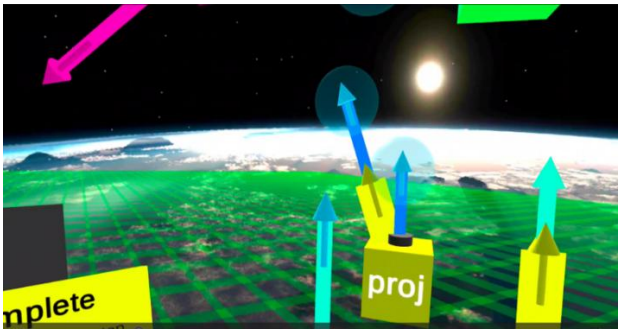
Note. Screen capture from Nili’s headset recording.

The first completed action was vector deletion, immediately confirmed by a green checkmark on the task list. She then proceeded to engage with the projection task by pressing the “projection” button in the generate menu, spawning a new vector in the positive y-direction and shortly after used the “+” add button with precision.

Over the next few moments, Nili systematically generated a green vector and attempted several transformations as the checkmarks for “project a vector onto another” and “add two vectors” appeared successively. The interface view confirmed her location in MathVR application, and she continued to create new vector sets while refining spatial orientation, notably aligning their tails accurately to execute a successful projection onto another vector block labeled “proj” (Figure 11).

Figure 11

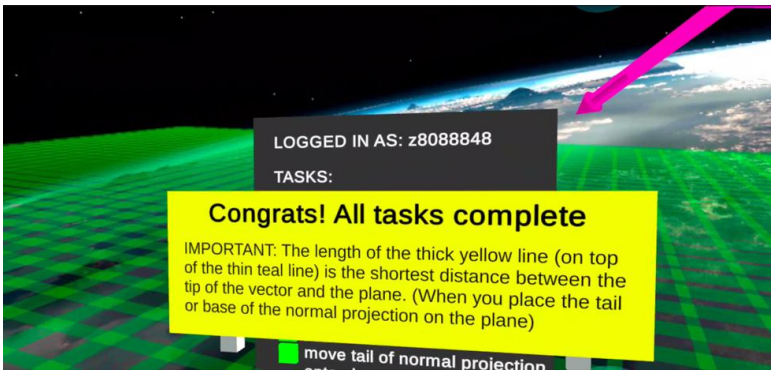
Nili's Projection Block Beneath Vectors With a Completion Banner



Note. Screen capture from Nili's headset recording.

Figure 12

The All Tasks Complete Banner at the End of Nili's Session



Note. Screen capture from Nili's headset recording.

After this, she demonstrated familiarity with the environment by adding and subtracting vectors in quick succession. Her hand gestures indicated confident navigation between tools and an intuitive understanding of the virtual physics engine embedded in MathVR.

As she manipulated multiple vectors in 3D space, she concluded the task successfully. A pop-up confirmed: "All tasks complete" (Figure 12), reinforcing her spatial success with projection and subtraction. Table 8 presents selected moments from Nili's VR session, including

the timestamp, her utterance-based insight, action-based insight, and the analytic classification assigned to each moment.

Table 8

Nili's VR Session Matrix (RQ1)

Timestamps	Utterance-Based Insight	Action-Based Insight	Technological or Mathematical
00:00:20.02	"I need a vector. Give me a vector." "Okay. Bring the vector here and shorten the length of this one…"	Repeated attempts to select a vector from the VR menu (see Screenshots 1 and 2) Manually dragged and resized a vector on the grid (see Screenshot 3)	Technological (locating and resizing a tool)
00:00:40.08	"We'll just put everything over here."	Arranged multiple vectors spatially within the VR workspace (Screenshot 4 shows scattered objects on grid)	Mathematical (organizing vectors spatially)
00:01:11.00	"Okay. Should I quit now?"	Moment of hesitation or completion signal; visible in Screenshot 8 where work appears paused or finalized	Mathematical (signaling completion of the task sequence)
00:01:30.00	[No direct utterance – focused on action]	Moved vector origin using grab-and-drag technique to	Technological (repositioning using interface controls)

		align with a specific coordinate	
00:01:45.00	[No direct utterance – focused on action]	Rotated the 3D grid for better spatial visualization of vectors	Technological (adjusting viewpoint via interface controls)
00:02:16.00	[No direct utterance – focused on action]	Constructed two vectors with different directions and lengths to explore magnitude and orientation	Mathematical (exploring magnitude and direction)
00:02:50.00	"Resume"	Resumed recording while navigating VR; a meta-cognitive interaction with tech interface	Technological (resuming the recording session)

For Nili, technological and mathematical considerations were tightly interconnected. Technologically, she demonstrated increasing fluency as she generated, repositioned, resized, and aligned vectors in quick succession. Mathematically, this fluency translated into purposeful manipulation of vector properties and the creation of spatially meaningful configurations. Her consistent movement between tools and her sensitivity to alignment and magnitude suggest that the VR interface may have supported her conceptual reasoning, enabling her to complete tasks confidently and anticipate how learners with varied needs might experience the technology.

Maren

Maren was in the Intermediate-Senior program with Mathematics as her teachable and a projected graduation year of 2026. She brought energy and playfulness to the VR session, often vocalizing her interactions. Her comfort with exploration made her reflections rich in affective and sensory detail.

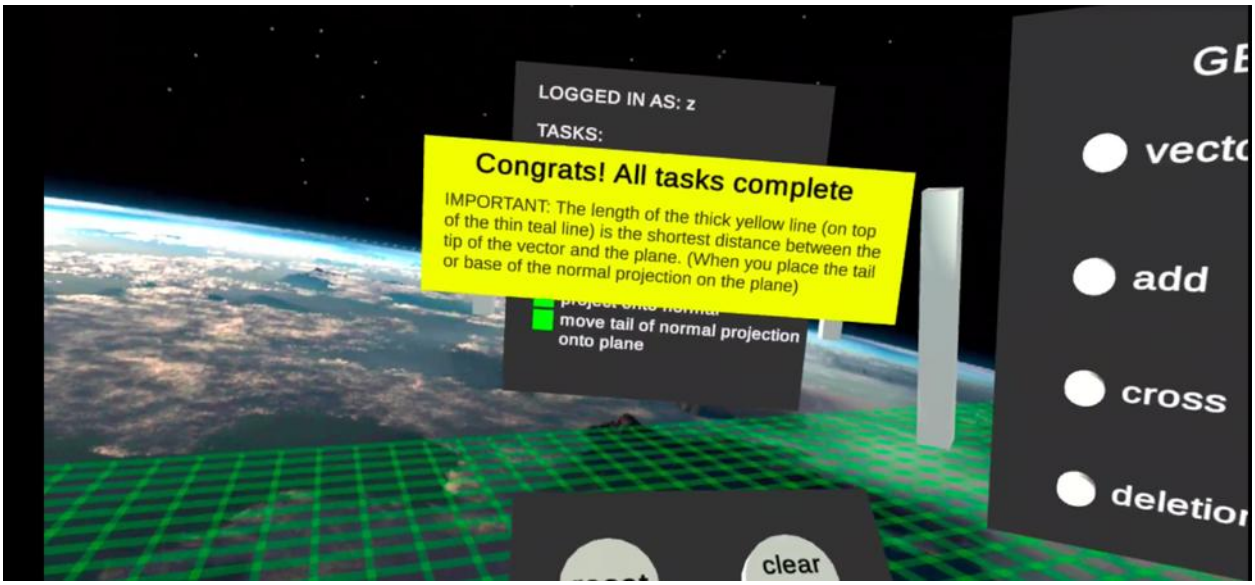
Maren's session was marked by playful immersion, emotional highs, and moments of conceptual clarity. Early on, she voiced discomfort with the spatial boundaries; "I don't like being so close to the wall," before quickly shifting into exploratory joy, deleting and stretching vectors with sound effects like "poh poh poh." This embodied play helped lower entry anxiety and led to a realization during addition: "Longer it is, shorter the addition goes. Whoa, this is amazing."

She added and stretched vectors while speaking aloud, asking how to determine the length and confirming vector addition with excitement. Maren observed changes in vector length and reacted with amazement when the task was marked complete. She proceeded to vector subtraction, selecting the appropriate tool, stretching vectors, and verbalizing her understanding of the process. After confirming that both addition and subtraction tasks were marked complete, she deleted the vectors and shifted to the projection task. During this stage, she expressed confusion over unexpected movements in the environment and struggled to remember how to project a vector. She spawned a plane and expressed enthusiasm while manipulating it, although she accidentally deleted it. She attempted to copy the plane normal, became frustrated when items disappeared, and continued experimenting. Maren returned to projection, stated that all tasks were marked complete, and expressed surprise and confusion about how that had occurred (Figure 13). She questioned the presence of measurement tools, interacted with a peer, and made

additional vectors. She continued to explore the projection feature and plane tool, repeating some steps and showing enjoyment. Her session concluded with discussion about resetting the program and recording resources.

Figure 13

The All Tasks Complete Banner at the End of Maren's Session



Note. Screen capture from Maren's headset recording.

Maren also encountered breakdowns, especially during projection, when she admitted, "It's glitchy… I don't remember projecting." In response, she spawned and rotated a plane, exclaiming, "I love the plane," using it as a repair tool that restored her sense of control. Table 9 shows how these breakdowns and repairs alternated across the session.

Table 9

Maren's VR Session Matrix (RQ1)

Timestamps	Utterance-Based Insight	Action-Based Insight	Technological or Mathematical
00:00:10	"Okay. I feel like these are so close. I don't like being so close to the wall."	Repositions herself in the VR space.	Technological (adjusting physical position in the play space)
00:00:16	"Look at that there. Oh, my goodness. What are these things?"	Begins deleting objects using the delete hammer.	Technological (exploring an interface tool)
00:01:20	"Then if I go like this... Longer it is, shorter the addition goes. Whoa, this is amazing."	Manipulates vector length and observes effect on addition.	Mathematical (noticing a relationship between magnitude and the resultant)
00:01:52	"Okay, I want a vector. Okay. Okay. Subtract. See if I remember how to subtract vectors."	Selects subtraction tool and stretches vectors.	Mathematical (recalling and performing a vector operation)
00:04:22	"It's weird. It's glitchy. Why did that move?"	Experiences glitch during projection task.	Technological (responding to unexpected system behaviour)

00:04:30	“I don't remember what projecting a vector is.”	Attempts to recall projection task.	Mathematical (content recall gap)
00:13:01	“How come I don't see any measurements? Do you see measurements on yours?”	Asks peer about measurement visibility.	Technological (identifying a missing interface feature)
00:15:39	“Yeah, I did all mine, too. Yeah, okay. I just reset it.”	Confirms reset and task completion.	Technological (system reset and progress confirmation)

Maren’s session revealed moments where technological uncertainty and playful exploration worked together to shape her conceptual trajectories. She experienced glitches, confusion, and accidental deletions, yet these breakdowns coexisted with curiosity, expressive engagement, and spontaneous insight during vector addition and plane manipulation. Mathematically, her excitement during recognition moments shows how embodied experimentation contributed to her understanding. The technological environment therefore served both as a site of challenge and as a catalyst for conceptual clarity.

Cassia

Cassia was an Intermediate-Senior teacher candidate specializing in Mathematics. She planned to graduate in 2026. Cassia began her MathVR session with visible curiosity and a moment of confusion, vocalizing, “What the hell is happening?” as she attempted to reposition the task board and surrounding elements. She quickly transitioned into active engagement, completing the “add two vectors” task and saying, “This is really cool.” In the early moments,

Cassia's vocal playfulness ("Doo Doo Doo") and physical manipulation of vectors indicated her comfort with the interface. She completed vector subtraction, deletion, and projection tasks while repeatedly reaching out toward virtual elements and using hand gestures to stretch, adjust, and align vectors.

She noted, "I've been swinging my arms a lot," while physically managing vector placements, and commented with phrases like "Oh, cool" and "Oh, that's really cool" as projection operations became visible in the environment. She reset the environment at one point and reorganized it by bringing the task board and vector interface back together.

Cassia encountered challenges with tool visibility and control, such as when multiple deletion hammers spawned or when she struggled to complete the "move tail of normal projection onto plane" task. She frequently verbalized her actions, posed questions aloud, and sought clarification from her peer Maren. Through multiple resets and experimentation, Cassia eventually completed the final task, reacting with phrases like "That's pretty fucking cool" and "Oh, it turns green" as the task status updated. Her concluding comments and physical reorganization of the scene reflected her ongoing interaction with the spatial and mathematical affordances of the MathVR environment. Table 10 presents selected moments from Cassia's VR session, including the timestamp, her utterance-based insight, action-based insight, and the analytic classification assigned to each moment.

Table 10

Cassia's VR Session Matrix (RQ1)

Timestamps	**Utterance-Based Insight**	**Action-Based Insight**	**Technological or Mathematical**
00:00:15	"What the hell is happening?"	Turns head rapidly and moves hands while	Technological (struggling with interface orientation)

		attempting to reorient the VR space; repositions the task board	
00:01:05	"This is really cool."	Completes the add two vectors task by moving and stretching two vectors until the system registers completion	Mathematical (completing a vector operation)
00:01:47	"Doo Doo Doo"	Vocalizes playfully while swinging delete hammer to remove extra vectors.	Technological (using a deletion tool)
00:02:10	"I've been swinging my arms a lot."	Gestures widely, creating and repositioning vectors in open space.	Technological (self-monitoring bodily movement during interface use)
00:03:20	"Oh, that's really cool."	Completes vector projection task and observes the yellow projection line appear.	Mathematical (observing a projection result)
00:06:41	"That's pretty f..... cool."	Completes all tasks; reacts to visual feedback and confirmation message in VR.	Technological (responding to system confirmation feedback)

00:07:55	"How do I get rid of this yellow box?"	Attempts to reset or hide measurement overlay; gestures and searches interface.	Technological (troubleshooting an interface element)

Across Cassia's session, her interaction involved ongoing shifts between technological navigation and engagement with vector operations in the VR environment. Technologically, she managed interface elements such as task lists, deletion tools, projection boxes, and reset functions, encountering moments of tool duplication, visibility challenges, and workspace clutter. She responded to these moments by reorganizing the virtual space, resetting the environment, and verbally tracking system states such as task completion indicators.

In terms of task engagement, Cassia repeatedly manipulated vectors through addition, subtraction, and projection operations, using wide arm movements, reaching gestures, and repositioning actions to complete required tasks. Her verbal responses, including "What the hell is happening?", "Oh, that's really cool," and "Why are we not green?", occurred alongside visible changes in vector configuration and system feedback. Throughout the session, her interaction was characterized by trial and error, repeated attempts, and continued engagement with the mathematical operations presented, even during extended periods of confusion or interface breakdowns.

Karim

Karim was a second-year preservice teacher in the Intermediate-Senior stream, preparing to teach mathematics and science at the secondary level. He entered the study with prior experience using educational technology but no direct exposure to immersive VR environments.

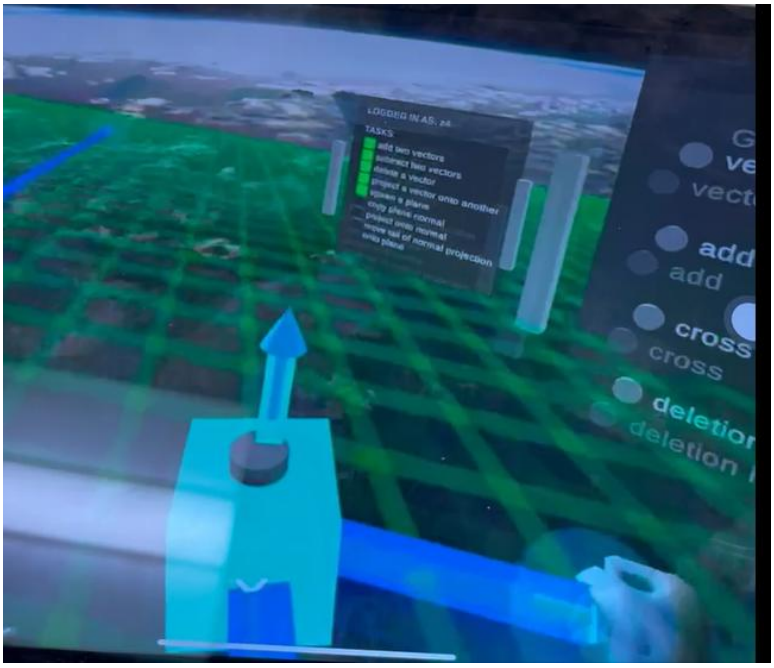
He demonstrated fluency and confidence with VR, approaching the tasks with humour and creativity.

Karim entered the MathVR environment already familiar with its interface, with six out of eight tasks on his board showing as completed. The session began with him confidently creating and manipulating vectors, completing the “add two vectors” task almost immediately. His tone was relaxed and conversational as he adjusted his workspace, saying, “I’m going to make that right,” while repositioning and aligning the vectors in front of him. As the tasks progressed, he added playful sounds and gestures while interacting with the objects, signaling comfort with the system’s controls. He rapidly completed the foundational operations such as subtraction, deletion, and projection, and spawned a plane without difficulty.

When prompted by his peer Kai and by me, Karim paused briefly at the “copy plane normal” task, repeating its title aloud as he searched for the correct command (Figure 14).

Figure 14

Karim’s Cast View While Locating the Copy Plane Normal Task



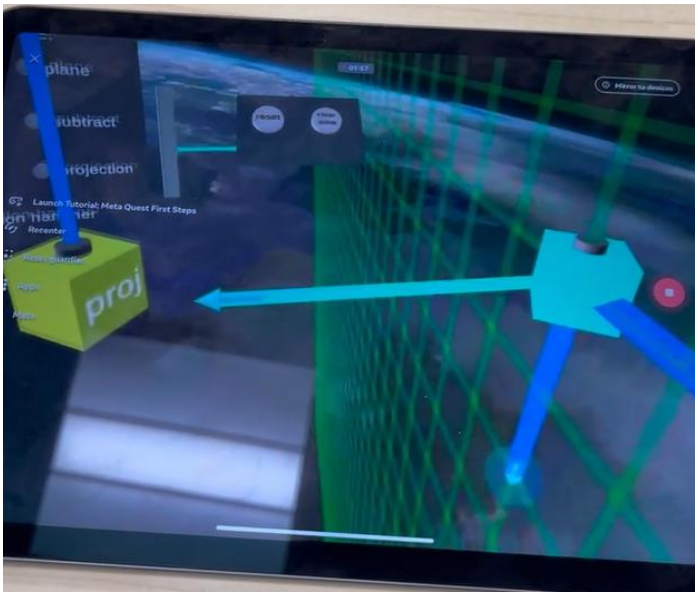
Note. Screen capture of the headset view cast to the tablet during Karim's session.

After identifying the visual cue for "plane normal," he proceeded to generate new projection vectors, experimenting with attaching and aligning them to existing vector additions. While attempting the "project onto normal" task, Karim first placed the yellow projection box onto a previously created vector before deleting and redoing the action, generating a new projection that triggered a completion checkmark on the board. He acknowledged this completion with a short verbal cue and continued adjusting the configuration, stretching and rotating the vectors in space to observe their alignment.

As the session continued, he engaged in conversation with Kai about the next steps, asking, "How do you project onto a normal just mathematically again?" before testing various manipulations of the plane and vector blocks. His dialogue reflected continuous switching between the VR actions and the on-screen task list. He repeatedly spawned new planes and projections, experimenting with their orientation and proximity (Figure 15). Together with his peer, he explored how moving or placing objects triggered completion cues, and when instructed to "move the tail of the normal projection onto the plane," he read the instruction aloud and leaned forward to check his progress.

Figure 15

Karim's Projection Block and Vectors in the Cast View



Note. Screen capture of the headset view cast to the tablet during Karim's session.

The final moments of his session showed him standing close to the virtual board, reviewing which tasks had turned green while continuing to move projection and plane objects around the space. His movements remained deliberate and controlled as he adjusted, deleted, and recreated components before concluding the session without completing the final task. Table 11 presents selected moments from Karim's VR session, including the timestamp, his utterance-based insight, action-based insight, and the analytic classification assigned to each moment.

Table 11

Karim's VR Session Matrix (RQ1)

Timestamps	Utterance-Based Insight	Action-Based Insight	Technological or Mathematical
00:01	"I think the issue is I haven't had a refresher on any calculus."	Inspects task board showing completed vector tasks, indicates familiarity and readiness.	Technological (orienting to the task board)
00:00:19.00	Okay so. I got two vectors.	Completes the first task "add two vectors"; task list updates with a green checkmark.	Mathematical (completing a vector operation)
00:01:09.22	Just make it fun. Just make it fun. Do that. Or do that where there's a street of problems. Do that. Do that. I want to draw this out. I want to spawn. I want to spawn.	Moves actively around workspace; gestures toward vector and plane elements while initiating a new "spawn" action.	Technological (initiating a system action)
00:01:47.05	That's just so cool. Okay. Copy. Copy plain normal. How do I copy it? I don't even remember how to copy it.	Attempts "copy plane normal" task; pauses to recall prior method; interacts with vector-plane control box.	Technological (seeking the correct interface procedure)

00:01:56.29	Oh, yeah. That's a Plane Normal.	Recognizes the plane normal object on screen and identifies it correctly.	Mathematical (identifying a mathematical object)
00:02:04.02	Oh, it gets a name. Now I remember what Plane Normal was. Is that what it was? Is that what it was? Oh, this is so cool. Okay.	Engages with visual label of “Plane Normal”; connects terminology to visual representation in VR.	Mathematical (connecting terminology to a visual representation)
00:03:52.05	Oh, I'm still on that one. I don't remember it from my universe of case.	Struggles to recall the mathematical logic behind projection.	Mathematical (content recall gap)
00:04:08.05	I'm going to name it myself. How do you project onto a normal just mathematically again? Not VR-wise, just math-wise.	Steps back from physical interaction to recall theoretical definition of projection.	Mathematical (seeking the mathematical definition)
00:04:25.11	Yeah, maybe. Yeah, that makes sense. Cross and a vector.	Confirms and connects peer’s explanation with vector mathematics operations.	Mathematical (confirming peer-scaffolded mathematical reasoning)
00:05:26.10	Wait, I just did a plane onto a projection. I think that's it. It's like, hide to this. Oh, no. I don't even think this is an issue with the VR. I think that's just	Reflects on whether the problem is due to interface or conceptual misunderstanding; continues adjusting plane-projection link.	Technological (diagnosing whether a difficulty is interface- or content-related)

	an issue with me, this part.		
00:06:31.00	Project onto normal? Project onto normal, yeah. Okay.	Reads and confirms the current task aloud; resumes interaction with projection.	Technological (confirming the current task)
00:08:27.20	Oh, God. I'm going to need a moment to digest. Move tail of the normal projection onto the plane.	Reads final task aloud; pauses to process its abstract spatial instruction.	Technological (processing task instructions)

Karim's session blended technological confidence with ongoing mathematical reasoning. He demonstrated ease navigating menus, spawning objects, and correcting errors, yet still encountered moments of uncertainty regarding projection and plane tasks. Mathematically, he moved seamlessly between VR actions and mathematical explanations, using gestures, peer dialogue, and conceptual recall to interpret vector relationships. His comments reveal how the technological space allowed him to test and revise his understanding, integrating procedural actions with conceptual inquiry.

Kai

Kai was an Intermediate-Senior teacher candidate, with a projected graduation year of 2026. He drew on both mathematical and social perspectives, often commenting on the physical and ethical dynamics of VR learning. His critical take on equity and access added depth to the focus group discussions.

In this session, Kai was positioned in the same physical room as Karim, both working in the shared MathVR environment using separate headsets. On-screen, the task board displayed the

list of operations: “add two vectors,” “subtract,” “delete,” “project onto another,” and “spawn a plane,” while the control panel labeled Generate offered options such as vector, plane, add, subtract, cross, projection, and deletion hammer. At the beginning of the clip, Karim moved quickly through his tasks, while Kai watched and occasionally interacted verbally, reflecting moments of shared discovery and confusion. As seen in Figure 16, a green grid environment formed the base, with a yellow projection cube labeled “proj” suspended above it, and vectors extending vertically and horizontally.

Figure 16

The Shared MathVR Environment in the Cast View During Kai and Karim’s Session



Note. Screen capture of the headset view cast to the tablet during the paired session.

Kai’s first notable intervention occurred around the one-minute mark, when he responded to Karim’s uncertainty about copying the plane normal by saying, “I don’t know,” then later recognized a projection event with the brief encouragement, “Awesome, you project.” Shortly

after, he expressed his own difficulty, “I don’t get that, I’m confused by this right now,” while manipulating projection objects, possibly similar to the yellow cube visible in Figure 16. His actions seemed exploratory, creating and discarding elements while testing how the system registered changes. When the deletion hammer was discussed, Kai noted, “You can throw them in the space, but you can’t actually get rid of them,” suggesting that he was experimenting with the physical affordance of throwing or discarding objects.

As the sequence continued, Kai’s contributions showed his effort to make sense of the logic behind “project onto normal.” He recalled prior mathematical rules, stating, “That would be the dot product of the cross product, no? So you find your normal through the cross, and then you do a dot product of a vector on that,” bridging symbolic mathematics knowledge with what he observed on screen. Later, when tasks were reversed, “plane onto projection” instead of “projection onto plane,” he remarked, “That feels backwards,” voicing how the virtual manipulation did not always align with textbook procedures. Eventually, he guided Karim toward the correct sequence: “Did you put a vector in the projection? … You need to have a vector in there,” which triggered task completion on both headsets. This progression revealed Kai’s growing procedural fluency as he alternated between reasoning aloud, responding to Karim’s cues, and testing the VR tools visible on the tablet interface. Table 12 captures this progression, including the exchanges with Karim that shaped it.

Table 12

Kai's VR Session Matrix (RQ1)

Timestamps	Utterance-Based Insight	Action-Based Insight	Technological or Mathematical
00:01:54.28	I don't know.	Responds to Karim's uncertainty about how to copy the plane normal; observes task without direct manipulation.	Mathematical (peer scaffolding context, content uncertainty)
00:02:34.02	I don't get that. I don't get that. I'm confused by this right now. I made a new one. I made a throw your away.	Expresses confusion while testing object creation and deletion; likely manipulating vectors or projections in the shared VR environment.	Mathematical (content confusion during manipulation)
00:03:17.26	Oh, the vector is good, but the hammer is down.	Distinguishes between two objects; vector and hammer, confirming their independent behaviours after a deletion attempt.	Technological (diagnosing tool state)
00:03:21.17	You can use a deletion hammer on a deletion. That one makes you sad. How do you make a projection on a plane?	Jokes about the hammer's functionality, then redirects conversation toward understanding the projection operation.	Mathematical (redirecting toward a mathematical operation)

00:03:51.17	The second last one.	Refers to the “project onto normal” task on the task board, helping Karim orient to the correct instruction.	Mathematical (orienting peer to the relevant task)
00:04:15.08	I can't remember. That would be the dot product of the cross product, no? So you find your normal through the cross, and then you do a dot product of a vector on that.	Recalls formal mathematical operations for projection; verbalizes conceptual reasoning while observing VR manipulation.	Mathematical (recalling formal mathematical definitions)
00:05:18.14	Yeah, I feel like it should be project on plane rather than plane on projection.	Comments on naming logic of task; compares conceptual expectation with VR implementation	Mathematical (critiquing task framing against mathematical convention)
00:06:41.19	In terms of actual, mechanics-wise, it's put the plane onto the plane. Not projection on plane.	Explains task procedure verbally while testing spatial arrangement of plane elements.	Mathematical (explaining a procedural mathematical step)
00:07:11.04	Oh, does that work? No, it didn’t. Okay. Yeah. Grab the plane itself, the box of the plane, and put it on the projection box.	Directs Karim step-by-step; identifies error and proposes alternate manipulation.	Mathematical (peer-directed procedural troubleshooting)
00:07:25.20	That’s what worked for me. I’m going to double-check projection. How did	Confirms what led to task success earlier; attempts	Mathematical (confirming a successful procedure)

	that. I don't know where you are. Over here.	to replicate procedure in shared workspace.	
00:08:16.17	Yes, because the tail of the normal projection onto the plane happens the moment you make your projection because the projection or the vector starts with the tail on the plane already.	Explains underlying system logic; connects procedural behaviour to conceptual structure of projection.	Mathematical (connecting procedure to mathematical structure)

Kai's trajectory highlights a prominent interplay between technological experimentation and mathematical reasoning. Technologically, he tested object behaviours, differentiated between tool functions, and helped diagnose interface interactions during shared VR moments. Mathematically, he drew heavily on mathematical definitions, justified operations through formal rules, and suggested renaming or reordering tasks to better align with conceptual logic. His reflections show how the VR environment prompted both critique of its structure and deeper engagement with vector principles.

Philip

Philip was an Intermediate-Senior preservice teacher with teachables in Mathematics and Physics, expected to graduate in 2025. His background in technical problem-solving was evident in his detailed observations of spatial manipulation and interface usability. He often brought up concerns about accessibility and practical implementation.

In this session, Philip engaged individually with the MathVR environment, systematically moving through each task on the virtual task board. As the session began, he read the task aloud:

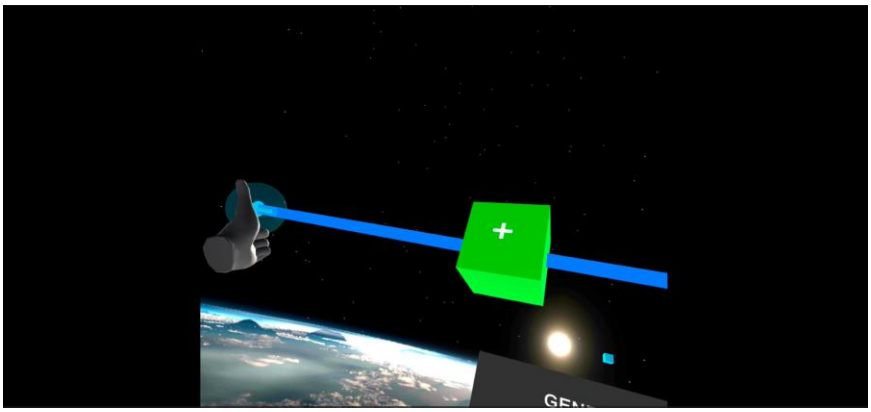
"Task. I," before asking, "How do I generate a vector?" He quickly experimented with spawning one, saying, "Vector. I put one there. There's another one. Okay, let's do that," suggesting that he was learning through repetition and manipulation. The environment visible on screen (green grid floor, control panel on the right, and floating yellow cube labeled proj) provided spatial context as Philip tested how objects appeared and responded.

After generating his first two vectors, Philip announced, "Then I add. This plus this gives this," marking his first completed operation. He reflected on their orientation: "Let's make them all linear… almost anti parallel," while fine tuning the direction and scale of the vectors. His verbal pacing slowed as he adjusted, noting, "It's a little awkward making them exactly the same," which indicated how the immersive interface required continuous physical calibration.

When moving to subtraction, Philip narrated each step: "I make a vector here, and I make another vector here, and I subtract." He paused, saying, "If they make exactly the same, it should be zero… If they're anti parallel, it'll be that," as he reasoned aloud through vector properties. His speech reflected metacognitive engagement, checking logic against real time visual output (Figure 17).

Figure 17

Philip's Addition Block Joining Two Vectors



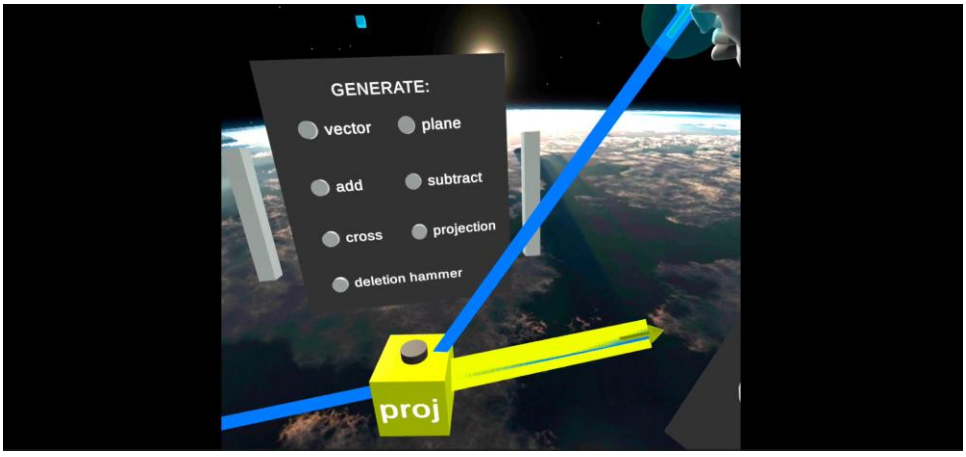
Note. Screen capture from Philip's headset recording.

As Philip continued, he engaged with the plane generation and projection tasks. While performing the projection operation, he said, “Okay, project a vector onto another,” and pointed toward the yellow projection block and the attached vector. He then added, “I can imagine a case where I’m looking at two factors and then just seeing the shadow,” verbalizing an analogy as he observed the projected vector in the scene. His hand remained steady as he gestured toward the projection, indicating that the visual elements were stable relative to his movement.

During this interaction, as can be seen in Figure 18, Philip noticed a change in the visual environment and commented, “The flickering of this light is interesting.” At this moment, his attention shifted from the vector configuration to the behaviour of the visual display itself. He continued pointing toward the projected vector and surrounding elements while monitoring the flicker, suggesting concurrent attention to both the task outcome and the system’s visual feedback.

Figure 18

Philip’s Projection Block and the Projected Vector

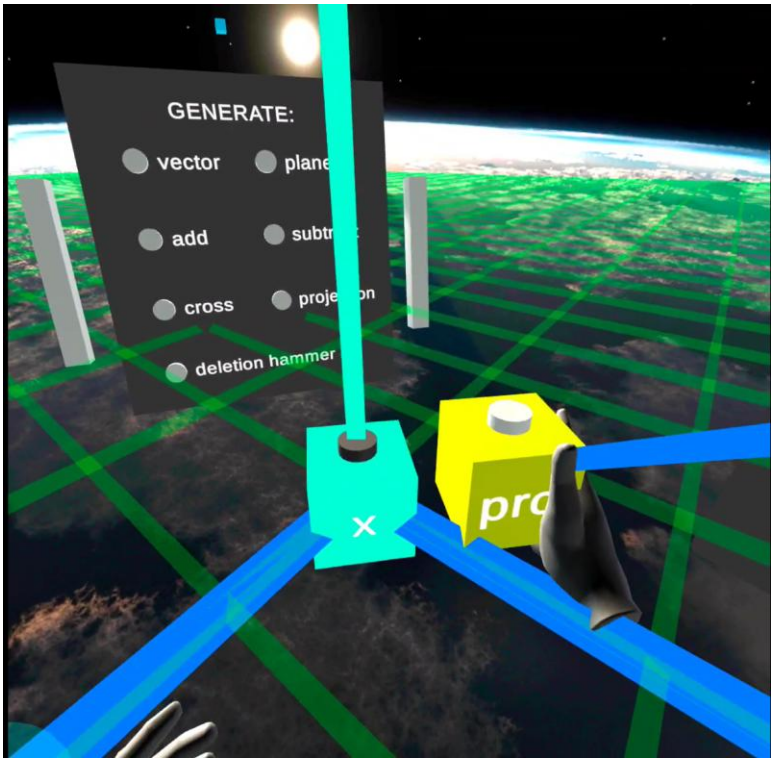


Note. Screen capture from Philip’s headset recording.

Later, as he encountered the “project onto normal” and “move tail of normal projection onto plane” tasks, Philip grew audibly puzzled: “What does that mean?… This is not working. Why is this not working?” and cycled through tool commands, trying cross products and projection blocks (Figure 19). His frustration built toward humour and persistence: “Am I stupid?… Oh my God, how do you start this?” before settling into discovery once more: “Move tail of normal projection onto the plane… This shows that the length of the thick yellow line on top of the thin tail line is the shortest distance between the tip of the vector and the plane.”

Figure 19

Philip’s Cross Product and Projection Blocks During the Normal Projection Task



Note. Screen capture from Philip’s headset recording.

This final reflection captured Philip’s moment of conceptual integration; he articulated the mathematical principle directly from the visual, embodied interaction. Though the task took several minutes, his concluding statement, “I don’t know why this took so long for me to do

that," revealed a shift from confusion to understanding. Throughout, Philip alternated between exploratory trial and error, self-talk, and real time analysis of spatial relations, illustrating how embodied manipulation supported his comprehension of vector operations within immersive 3D space. Table 13 presents selected moments from Philip's VR session, including the timestamp, his utterance-based insight, action-based insight, and the analytic classification assigned to each moment.

Table 13

Philip's *VR Session Matrix (RQ1)*

Timestamps	Utterance-Based Insight	Action-Based Insight	Technological or Mathematical
00:00:27	How do I generate a vector?	Searches for the vector generation tool; likely selects "vector" on the control panel and spawns a new one in the 3D space.	Technological (locating an interface tool)
00:00:57	Okay, let's make them all linear.	Attempts to align the two vectors along the same line (collinear), likely by adjusting their direction.	Technological (manual spatial alignment via controls)
00:01:00	It's a little awkward making them exactly the same, but that makes sense.	Describes difficulty in precisely matching vector orientation; comments on manual spatial control.	Technological (describing control precision difficulty)

00:01:15	Oh, wow. This is really cool.	Expresses excitement at the interactive display of vectors in VR.	Technological (affective response to the interface)
00:01:19	What the heck? There's all kinds of manipulations I can do here.	Recognizes multiple interactive affordances in the environment; experimenting freely.	Technological (discovering interface affordances)
00:02:15	If they make exactly the same, it should be zero.	Verbalizes conceptual rule for vector subtraction; relates visual feedback to theory.	Mathematical (stating a mathematical rule)
00:02:20	If they're anti-parallel, it'll be that.	Tests outcome for opposite direction vectors; confirms consistency with logic.	Mathematical (testing a mathematical hypothesis)
00:04:04	A case where I am looking at two factors and then just seeing the shadow.	Describes conceptual metaphor for projection; visualizes it as a shadow relation.	Mathematical (constructing a perceptual metaphor for projection)
00:05:19	But it'd be nice to have a little way to interchange the two.	Suggests potential design improvement for interchanging vector order.	Technological (proposing an interface improvement)
00:08:40	This is the normal vector. Or maybe I do a cross product.	Identifies vector incorrectly, then tries alternative operation.	Mathematical (testing mathematical identification)

00:15:08	Projecting onto the normal. It's like this.	Restates process while visualizing projection alignment.	Mathematical (restating a mathematical process)
00:21:22	The length of the thick yellow line on top of the thin tail line is the shortest distance between the tip of the vector and the plane.	Articulates visual-mathematical insight; identifies geometric principle of projection distance.	Mathematical (articulating a geometric conclusion)
00:21:43	I don't know why this took so long for me to do that.	Reflects on difficulty; recognizes completion and understanding.	Mathematical (metacognitive reflection on understanding)

Philip's session revealed a continuous movement between technological troubleshooting and conceptual reasoning. Technologically, he experimented with orientation, struggled with precision alignment, encountered interface constraints, and persisted through multi step trial and error sequences. Mathematically, he articulated vector properties aloud, used metaphors such as shadows to reason about projection, and verbalized conceptual breakthroughs once the spatial interactions made sense. The VR environment acted as both a challenge and a scaffold, enabling him to translate embodied manipulation into explicit mathematical insight.

Overall, the descriptive accounts in Research Question One document how preservice teachers interacted with MathVR while performing vector operations, highlighting patterns in how participants navigated the interface, engaged with mathematical tasks, and responded to system feedback during real time VR use. This analysis foregrounds embodied interaction as it unfolded moment by moment within the VR environment. The next section shifts analytic focus

away from in situ interaction and toward participants' retrospective accounts. Research Question Two examines how preservice teachers described the affordances and constraints of MathVR after completing the VR tasks, drawing on freewriting and focus group data to capture their reflections on the technology and its potential use in Grade 12 mathematics instruction.

Research Question Two: Affordances and Constraints of MathVR

RQ2: After performing vector operations using MathVR, how do preservice teachers describe the affordances and constraints of MathVR for teaching vector operations to Grade 12 students in Ontario?

Overview of Data Sources and Analytic Approach

Following the embodied interactional analysis presented above for Research Question One, this section turns to participants' reflective accounts to examine how they described the affordances and constraints of MathVR after completing the vector operation tasks. Whereas the previous analysis captured real time gestures, orientations, and object manipulations through the Multimodal Analysis for Embodied Technologies (MAET) framework, the present inquiry focuses on post experience reflections drawn from participants' freewriting and focus group discussions. These discursive forms of data represent participants' retrospective meaning making processes as they articulated what the technology enabled and what it limited in the context of teaching vector operations to Grade 12 students. Because these accounts do not include embodied visual or audio traces, a targeted thematic analysis was employed instead of MAET to identify recurring patterns in how participants perceived the learning potential and practical constraints of MathVR. The analysis remains attentive to embodied language; references to movement, space, touch, and perspective, while emphasizing Mathematical and equity implications. The findings are organized around key themes of affordance and constraint,

summarized through a visual matrix that integrates direct quotations and interpretive commentary. This structure highlights how preservice teachers envisioned MathVR's promise for supporting conceptual understanding and the challenges they anticipated in translating this technology into equitable classroom practice.

To support transparency and provide direct access to participant evidence, Table 14 below presents a thematic matrix summarizing key affordances and constraints identified across participants' freewriting and focus group data. The table includes representative quotations and brief summaries that anchor each theme in participants' own words. The thematic discussions that follow draw directly from this matrix and elaborate on patterns observed across the data.

Table 14

RQ2 Thematic Matrix

Participant	Affordance Noted	Constraint Noted	Direct Quote or Summary	Pedagogical Implication	Equity Consideration
Nora	Strong engagement and 3D visualization enhanced comprehension of spatial and vector relations.	Limited scaffolding for specific operations (e.g., subtraction) and space requirements in classrooms.	Freewriting: "I find this MathVR tool really engaging… I feel that I need a bit of explanations regarding the subtraction one."	MathVR facilitates embodied understanding of 3D operations, allowing teachers to demonstrate abstract vector relationships through visualization.	Classroom setup, physical movement, and safety boundaries may restrict accessibility; could require adapted spaces or rotation systems.

Amaya	High motivational value and accessibility for diverse learners; supports visualization and differentiation.	Physical discomfort and cognitive load due to headset use and lack of sufficient tutorials.	“Students who have visual intelligence can appreciate the app as it allows them to visualize the abstract conception… Yet, there are a few limitations… Wearing the VR glasses for more than 10 minutes made me feel dizzy.”	Enhances affective engagement and conceptual clarity through embodied exploration; may promote inclusion of visual learners.	Highlights health and ergonomic concerns; calls for structured breaks and potential adaptation for students with sensory sensitivities.
Adrian	Interaction-based learning increased motivation and understanding; encouraged curiosity and focus.	Tool manipulation difficulties for new users limited depth of engagemen.	“The benefit was the interaction part which gives motivation to the user to learn… The constraint was… not everyone can adapt quickly to fully manipulate the tool.”	VR interactivity can foster agency and active learning when paired with teacher scaffolding; suggests VR as an enhancer of inquiry-based pedagogy.	New learners may require training or guided onboarding to ensure equitable participation; scaffolding can mitigate tech familiarity gaps.

Nili	Fun, engaging visualization of abstract vector problems; supported visual and experiential learning.	Limited hardware availability and lack of built-in tutorials for classroom use.	"It's a really fun and enjoyable way to solve vector problems… but there might not be a lot of devices available at school level. Also, the app should have a demo first or tutorial."	Enhances spatial understanding and engagement; integrating VR can transform dull topics into exploratory experiences.	Highlights need for equitable access to VR hardware and guided introductions to prevent exclusion of students with less tech familiarity.
Violet	Promoted conceptual visualization and deeper questioning about projection and cross product.	Limited numeric data visibility and initial difficulty with controls.	"I appreciated seeing the concepts come together in a visual way that I could interact with… I wasn't paying attention to any numerical data… even by the end, I still found the controls challenging."	Strengthens conceptual reasoning by focusing on process over memorization; supports inquiry-based vector teaching.	Suggests need for multimodal feedback (numeric and visual) for diverse learners, ensuring accessibility for students who rely on symbolic representations.

Cassia	Allowed creative manipulation of vectors; inspired ideas for customizing VR lessons across grades.	Confusion about task feedback and inability to connect actions to outcomes.	"Tasks were completed without me understanding how I did them… There should be an option showing how the task was completed successfully."	Customizable VR options could allow differentiated instruction; visual task feedback supports metacognition in learning.	Teachers should be able to tailor environments (checklists, visuals, difficulty) to accommodate varying student abilities and learning styles.
Kai	Physical interaction deepened spatial understanding and made abstract ideas tangible.	Limited left-hand control, inconsistent button functions, and need to restart for certain operations.	"Physicality to vectors… Helps with making planes and the 3D/higher dimensionality make sense… Can't grab/manipulate with left hand… had to restart app to make/destroy more blocks."	Reinforces spatial reasoning through embodied exploration; supports inquiry-based mathematics pedagogy emphasizing manipulation.	Device design and control limitations may disadvantage left-handed users or those with motor impairments.
Karim	Hands-on and visual exploration	Limited tutorials, precision	"The benefit of MathVR is that it provides a hands-	Encourages experimentation and discovery	Users with lower digital literacy may struggle

	enhanced conceptual understanding of vectors and planes.	tools, and deletion options within the interface.	on and visual approach… limitations include limited tutorials and no ability for precise measuring or removing vectors."	learning; underscores need for embedded guidance tools.	without scaffolding; accessibility requires simplified user interfaces.
Maren	Appreciated ease of manipulating pre-existing vectors and planes; preferred brighter, open space design.	Difficulty adjusting orb size and inputting specific coordinates.	"I loved that the vectors were already created for me… I would have liked the ability to input a specific coordinate and have the vector take that position."	Streamlined design and pre-set vectors can support early-stage learners; customizable coordinates promote accuracy and engagement.	Accessibility improves with visual clarity and interface simplicity; coordinate input could aid students with numeric learning preferences
Philip	Highly immersive and self-directed; clear object labeling supported	High cost, accessibility barriers for visually impaired students,	"Kinesthetic activities make sure a student is learning… Only accessible to those who are not	Immersive VR fosters autonomy and visualization; labeling aids conceptual	Equity concerns include cost, vision accessibility, and potential isolation of learners needing

independent exploration.	unclear button functions.	visually impaired… Buttons for projection and cross product were unclear even with tutorials."	mapping between representations.	collaboration or verbal guidance.

The themes discussed below were constructed using participants' written reflections and contributions to group discussions. Each subsection highlights a specific pattern in how participants described their experiences with MathVR and remains grounded in the quotations presented in Table 14. Together, these themes show how participants characterized MathVR as both a pedagogical resource and a technological system with practical constraints.

Affordances: Embodied Visualization, Engagement, and Conceptual Access

Across participants, MathVR was described by participants as a tool that brings abstract mathematical ideas into tangible, manipulable form. The virtual environment seemed to allow preservice teachers to see, move, and construct vector relationships in ways that supported embodied sense-making.

Embodied Spatial Reasoning

In their freewriting, Nora, Amaya, and Adrian wrote about how the three-dimensional environment supported their spatial understanding of vector operations. Nora noted that she could "see where it's pointing," drawing attention to how direction became visually apparent in the VR space. Adrian described the experience as being like "working with Legos," emphasizing the hands-on and constructive nature of manipulating vectors. Across these accounts, participants

described how interacting with vectors in three dimensions made aspects of direction, magnitude, and spatial relationships more noticeable and accessible during the activity.

Playful Engagement and Motivation

Playful engagement and heightened motivation were explicitly articulated by three participants: Amaya, Kai, and Karim. In her focused freewriting, Amaya described the MathVR experience as engaging and psychologically reassuring, noting that it made mathematics feel less intimidating and more accessible. During the focus group discussion, Karim stated that working with vectors in VR was "a lot more fun" than writing them on paper and emphasized that the ability to stretch, toss, and manipulate vectors increased his enjoyment and willingness to persist with the task. Kai similarly highlighted the enjoyment of interacting with tools such as the hammer and experimenting freely within the virtual space, describing the experience as fun and exploratory.

Across these three accounts, playfulness was not framed as distraction but as a condition that supported sustained engagement and lowered affective barriers to working with vectors. The data indicate that, for these participants, enjoyment emerged from active bodily interaction and exploratory freedom rather than from game-like rewards or competition.

Support Through Visual and Embodied Interaction

In both freewriting and focus group discussions, participants described how seeing and physically manipulating vectors in MathVR supported their understanding of vector operations. Rather than referring to learner types, participants emphasized how visual feedback and bodily interaction made abstract relationships observable.

In her freewriting, Nili wrote that MathVR allowed her to "actually see how it works," explaining that actions typically performed on paper could instead be carried out "in reality,"

which helped her make sense of vector relationships as she moved them in space. Similarly, Amaya reflected in the focus group that "you are not separate from the reality," noting that being inside the environment made the experience feel fundamentally different from interacting with two-dimensional representations.

Several participants highlighted the role of movement in shaping understanding. During the third focus group session, Karim explained that in MathVR he could "stretch it out and then toss it," describing how changing a vector's length and direction through gesture made the mathematics feel more accessible than writing symbols alone. Cassia similarly emphasized the value of seeing vectors respond immediately to movement, stating that being able to watch vectors change "in real time" helped her understand how operations were constructed.

Participants also commented on how visual cues supported reasoning. Philip noted in the focus group that he appreciated how "the resultant vectors change in real time," allowing him to observe relationships as he adjusted inputs. In another moment, Cassia described how seeing vectors arranged tip to tail helped her understand addition, explaining that the visual alignment clarified how values combined across axes.

Across these accounts, drawn from both freewriting and focus group data, participants consistently pointed to visual responsiveness and physical interaction as central features of MathVR. Their descriptions focus on what the environment made visible and manipulable, highlighting how interaction and perception worked together to support sense making during vector tasks.

Pedagogical Creativity

Participants described MathVR as offering room for teacher customization and adaptable lesson design, especially through flexible modes, representational options, and task structures

that could be tuned to different learners and grade levels. In her focused freewriting, Cassia emphasized this design flexibility directly, writing, “I think there should be more options teachers can customize for their students,” and linking this to concrete instructional choices such as enabling tip to tail vector placement, offering either an open creative mode or a checklist mode, and allowing different background environments to support engagement for younger learners.

This theme also surfaced in focus group talk as participants imagined extending MathVR beyond practice tasks into broader classroom uses such as assessment and collaborative work. During the third focus group session, Cassia suggested assessment possibilities by proposing that students could “work through it on the whiteboard or whatever in VR, and then they could build it with the vectors to illustrate their math,” positioning MathVR as a space where learners could both show process and construct representations.

Constraints: Technological Frictions, Cognitive Load, and Access Barriers

While the participants’ reflections highlighted clear pedagogical potential, they also underscored several recurring constraints that limit MathVR’s feasibility for classroom use.

Interface Complexity and Learning Curve

Participants such as Karim, Kai, and Cassia noted that unclear tool functions and inconsistent feedback made it difficult to know whether tasks were completed correctly. Cassia explained in her focused freewriting activity that she “finished tasks without understanding how,” reflecting a disconnect between user action and task comprehension. Kai echoed this frustration in his freewriting, stating, “I had to restart the app to make/destroy more blocks,” and added that “buttons sometimes didn’t work or were confusing to make work.” These experiences underscore how lack of intuitive interface design and inconsistent system responses can interrupt

the learning flow. Karim described a related issue in the focus group: “I grabbed the addition thing and I created maybe 15 duplicates that all scattered… which made us no longer add blocks to it.” Together, these accounts highlight the importance of user-centred, streamlined design in educational VR environments, particularly when the goal is to support novice learners exploring complex mathematical concepts.

Physical and Sensory Fatigue

References to physical sensation and comfort emerged in both focused freewriting and focus group discussions. In her focused freewriting), Amaya wrote, “I felt dizzy with the goggles on.” During the focus group discussion the same day, she added, “At the beginning, I felt dizzy… I was moving my hands and moving my mind and my head.” Philip, speaking in the focus group discussion reflected on prior experience with VR, stating, “My first experience using VR was a year ago, and the first time I felt a little nauseated because of different sensory stuff. I was expecting myself to be a bit nauseated again, but this… I wasn’t nauseous at all.”

Cognitive Load and Lack of Scaffolding

A few participants referenced the absence of clear instructions or guidance while interacting with MathVR. In the focus group discussion, Karim noted difficulty understanding how to perform actions within the environment, stating, “You couldn’t take a block out of the adding or subtraction, or if there was, there was no way for me to figure it out,” and later added, “We would have trouble sometimes duplicating.” In the same discussion, he suggested additional supports, explaining that “having some form, two things. One would be maybe a tutorial, not just on the controls, but also on vectors that they can access.”

Similar concerns appeared in Nili’s focused freewriting where she wrote “The app in itself should have a demo first or a tutorial of some basic concepts of vectors before letting them

do everything on their own." During the focus group discussion, Nili also described early moments of uncertainty, noting that "there's vectors over here, there's vectors over here, and then there's some addition, subtraction, projection things in the middle," and expressed a desire for "more structured instructions in the beginning."

Hardware and Equity Limitations

Participants frequently raised concerns about the material and structural conditions required to implement MathVR in school settings. Cost and device availability were cited as major constraints. In her freewriting, Nili questioned feasibility at the school level, noting that "there might not be a lot of devices available at school level" and emphasizing that "the cost associated with VR devices is even huge" (Focus group discussion). During the focus group, Cassia similarly expressed uncertainty about institutional buy in, stating, "I don't know how this would be accessible to schools and how I would convince admin to spend money on it" and later adding, "maybe we only get one headset per class or one headset for the whole school".

Participants also highlighted spatial limitations within classrooms. Kai pointed to physical space as a barrier, explaining that "space availability in schools can sometimes be lacking" and that "having to move all of the desks… is already a lot" (Focus group discussion), Nili echoed this concern, noting that students would need "space around them so they're not bumping into each other," suggesting that classroom size could constrain implementation (Focus group discussion).

Accessibility concerns were also raised. Philip explicitly noted that the technology was not universally accessible, writing that MathVR was "only accessible to those who are not visually impaired" (Focused Freewriting). He also identified management challenges related to

classroom use, describing VR as “expensive to implement in the classroom, and to manage it in the classroom” (Focused Freewriting).

Together, these accounts document participant concerns related to cost, space, device availability, and accessibility as possible constraints on the use of MathVR in school contexts.

Pedagogical Implications: Scaffolding Embodied Learning

Participants frequently described MathVR as a tool that would be used alongside other instructional approaches rather than as a standalone replacement. Cassia emphasized that VR would need to be integrated with more traditional methods, explaining that “maybe you’re teaching in a more traditional way or using 2D manipulatives… and then maybe kids take turns with the headset” (Focus Group). Similarly, Nili noted that students would need prior conceptual grounding before entering the VR environment, stating that “the students need to be familiarized with the concept beforehand to actually go in and solve problems” (Freewriting activity).

Several participants described MathVR as a way to engage with vectors through physical interaction before articulating mathematical ideas symbolically. Karim highlighted the contrast with paper-based methods, explaining that in VR he could “stretch it out and then toss it… and just throw another thing in there,” describing the experience as “a lot more enjoyable” than writing vectors out (Focus Group). Cassia connected embodied interaction to later explanation, noting that being able to see numbers and visual elements together would allow a teacher to ask students, “How do you think we got to this number?” and then “explain it a little bit more so that they’re able to actually understand it and see it in real time at the same time” (Focus Group).

These accounts indicate that participants discussed MathVR in relation to existing instructional practices and described its use as part of a broader sequence that included physical interaction, discussion, and explanation.

Pedagogical Structuring Described by Participants

Participants described several ways MathVR would need to be situated within instruction in order for students to use it meaningfully. These descriptions appeared across both focused freewriting activities and focus group discussions, though they were not expressed by all participants.

Need for initial orientation or guidance

Several participants explicitly described difficulty navigating tasks without clear instructions and suggested the need for introductory guidance. In focused freewriting, Cassia wrote that "tasks were completed without me understanding how I did them" indicating uncertainty about how actions led to outcomes. Similarly, Violet noted that she was "not sure how I completed like the last two or three items on the checklist" and questioned whether instructions were available or simply "not intuitive how to find" in her focused freewriting. In the focus group discussions, Karim suggested the inclusion of tutorials and guided supports, stating that "there was no way for me to figure it out" when interacting with certain blocks. These accounts point to multiple participants identifying a need for some form of initial orientation or instructional support before independent exploration.

Sequencing of embodied interaction and explanation

A smaller group of participants described MathVR as something they would use in combination with other instructional moments rather than as a standalone activity. In the third focus group session, Cassia described a scenario in which students might first explore vectors in VR and then engage in teacher led explanation, noting that "you could then explain to the students how do you think we got to this number" while referencing real time visualization. Nora similarly described using VR early in instruction to help students see vector relationships before

continuing with further explanation and discussion (focus group). These descriptions reflect participants articulating a sequence that moves between embodied interaction and subsequent explanation, rather than remaining solely within the VR environment.

Post activity reflection and discussion

Several participants explicitly referenced the value of discussion or reflection following VR use. In the first focus group session, Nora described taking students back to a board after the VR activity to "start discussing what we have done" and to "fill the gaps" in understanding through conversation. Violet similarly described noticing unexpected outcomes during projection tasks and emphasized that these moments would prompt class discussion about why the vectors behaved as they did (focus group). These comments indicate that participants envisioned reflection and discussion as occurring after the VR activity to support sense making.

Read together, these participant accounts show that pedagogical structuring was described by several participants across both freewriting and focus group data. The insights consistently referenced preparation, embodied interaction, and post activity discussion as interconnected components of how MathVR might be used within instruction.

Equity Considerations: Accessibility, Inclusion, and Design Adaptation

Equity related concerns emerged most clearly in the third focus group discussions, which included five participants (Cassia, Kai, Karim, Philip, and Maren), and were reinforced by focused freewriting reflections from multiple participants across the data collection period.

Device access, cost, and institutional constraints

Several participants raised concerns about the feasibility of implementing MathVR in typical school settings due to cost and limited device availability. During the focus group discussion, Cassia stated, "I don't know how this would be accessible to schools," noting that

most classrooms would not realistically have enough headsets for all students. She later added that even having "one headset for a class of anywhere from 25 to 30 students" would require rotational use and careful scheduling. Kai similarly pointed to cost comparisons with existing school technologies, noting that while VR headsets were expensive, they were comparable to other resources such as LEGO kits already used in schools.

These concerns were echoed in focused freewriting. Nili wrote that "there might not be a lot of devices available at school level," describing hardware availability as a primary limitation when imagining classroom use. Philip also identified expense as a constraint, writing that VR "can be expensive to implement in the classroom, and to manage it in the classroom."

Physical space and classroom logistics

Participants also identified classroom space as a potential barrier to equitable participation. In the focus group discussion, Kai noted that "space availability in schools can sometimes be lacking," explaining that moving desks or finding open areas such as gyms might be required to use VR safely. Nora similarly described the need for "free space" to avoid students bumping into one another and suggested the use of gyms or clearly marked physical boundaries.

These comments reflect participants' attention to the physical demands of embodied interaction and the ways existing classroom layouts may constrain access for all students simultaneously.

Accessibility and sensory considerations

Some participants explicitly identified accessibility limitations related to physical and sensory differences. In his focused freewriting, Philip stated that MathVR was "only accessible to those who are not visually impaired," identifying a clear limitation for students who rely on nonvisual modalities. Other participants noted sensory responses that could affect participation.

In focused freewriting, Amaya wrote that "wearing the VR glasses for more than 10 minutes made me feel dizzy and short of breath," describing a physical response that could limit duration of use.

During the focus group discussion, Philip contrasted this experience by explaining that while he felt nauseated during his first VR experience a year earlier, he "wasn't nauseous at all" during the MathVR session, attributing this difference to prior familiarity with VR. Cassia also described feeling dizzy when using controller-based movement in other VR games, but noted that physically walking within the MathVR space reduced this sensation.

Interface design and differentiated access

Participants further discussed how interface design could either support or limit access, particularly for users with less prior experience using VR or video games. In the third focus group discussion session, Cassia explained that closely spaced buttons caused her to "reset myself so many times," adding that this was "really easy" to do for someone who had "only used VR once." In the same session, Kai and Karim both described difficulties duplicating or deleting objects, which sometimes disrupted task completion.

Several participants suggested design adaptations to address these issues. In focused freewriting, Karim proposed clearer menus, visual cues, and optional animations to show how vector operations are performed, writing that "many students have trouble abstracting numbers" and that visual assistance could help bridge this gap. In her freewriting, Cassia similarly suggested showing how a task was completed successfully after completion to "ensure student comprehension."

In combination, these data show that equity related considerations were raised by multiple participants across both focus group discussions and focused freewriting activities, with

attention to cost, space, accessibility, sensory experience, and interface design. These concerns were articulated in concrete, situational terms rather than abstract principles, reflecting participants' attempts to imagine how MathVR might function within real classroom contexts.

Across freewriting and focus group discussions, participants identified patterns related to visualization, engagement, instructional flexibility, interface challenges, physical comfort, and equity considerations. These descriptions remain grounded in participants' own accounts and do not evaluate effectiveness or outcomes. The next section builds on these findings by examining how participants' embodied interactions and reflective accounts intersect in relation to technological, pedagogical, and content considerations.

Research Question Three (RQ3): Synthesis Through the TPACK Framework

RQ3: How do these insights and descriptions inform understanding of preservice teachers' emergent TPACK, particularly in relation to a technology that enables visualization and a virtual embodied experience within mathematics education?

Introduction and Rationale for the Synthesis Layer

This section presents the synthesis layer of my Results chapter. Here, I present integrated insights from participants' embodied actions during the VR sessions, their written reflections, and our focus group discussions. While the previous two research questions centred on describing what participants did in the virtual environment and how they interpreted those experiences, the focus of this section is on how their overall engagement with MathVR informed their emerging professional knowledge. To examine this learning across modalities, I use the TPACK framework as an interpretive structure.

TPACK offers a conceptual lens for tracing how preservice teachers brought together technology, pedagogy and mathematical content during and after their immersive encounters

with vectors. Although the MAET framework was essential for analyzing the embodied and multimodal dimensions of participants' VR interactions in RQ1, TPACK is more appropriate for RQ3 because it highlights the knowledge domains that shape instructional reasoning. MAET helped uncover how participants oriented their bodies, manipulated objects, and expressed affective responses. These insights laid the groundwork for understanding how VR mediation shaped their actions. However, the aim of RQ3 is not to code action but to interpret how participants transformed those experiences into meaningful insights about teaching and learning mathematics.

The shift to TPACK aligns with the nature of the available data for this research question. Participants' freewriting activities and focus group conversations reveal how they made sense of the pedagogical and conceptual potential of MathVR. Their reflections extend well beyond procedural mastery of the interface which is technological knowledge. They articulate how VR might influence student engagement, how it might support clarity in teaching vector concepts, and how it might require different forms of classroom organization.

By organizing the cross-participant synthesis in this way, I show how participants' knowledge developed across multiple communicative modes. The immersive sessions provided a space to experience mathematical structures through movement and manipulation. The freewriting tasks allowed participants to slow down their thinking and articulate conceptual or logistical concerns. The focus group discussions encouraged collaborative meaning making about how VR might fit into the realities of classroom practice. Using TPACK as a frame makes it possible to braid these insights into a coherent account of how preservice teachers were beginning to integrate technological, pedagogical and content considerations in relation to an emerging form of mathematical visualization.

TPACK Matrix

To support the cross-participant synthesis for RQ3, I created the matrix below in Table 15 that brings together the main insights from participants' VR sessions, freewriting activities and focus group reflections. This matrix summarizes emerging patterns across the six domains of the TPACK framework and provides a visual representation of how participants connected technology, pedagogy and mathematical content. Each cell includes a theme, an illustrative participant example and a brief comment that explains how this insight reflects an aspect of their developing teacher knowledge. See Appendix C for TPACK evidence matrix based on participants' VR Experience for more details.

Table 15

TPACK Synthesis Matrix for RQ3

TPACK Domain	Theme or Insight	Participant Example	Supporting Data Source	Integration Comment
Technological Knowledge	Developing fluency with immersive controls	Kai realized how to re orient the interface only after several attempts. Amaya struggled to add or subtract vectors until she relearned the menu sequence. Kai and Karim asked repeatedly how to copy or project normals saying "How do I copy it… I got it… That is Plane Normal" (Kai's VR experience) Amaya struggled to recall the add sequence saying "I forgot how to add vector" (Amaya's VR experience) Karim attempted deletion and reset saying "I think for me the issue is I cannot delete them… I do not know what I am doing" (Karim's VR experience)	VR Experience Freewriting reflections	Participants learned through repeated attempts and corrections demonstrating developing technological fluency and adaptive action during immersive tasks.
Pedagogical Knowledge	Designing lesson routines that support movement, safety and consolidation	Nora suggested following the VR activity with a board discussion to fill conceptual gaps. (Freewriting Activity)	VR Experience Freewriting Activity	Participants reasoned about classroom flow, transitions and consolidation

		Adrian proposed warm up and cool down routines to ease transitions. (Freewriting). Nora expressed sustained engagement saying "I can be in this environment forever" (Nora 's VR experience) which contributed to her later argument for a post VR board discussion. Nora followed complex instructions across movement zones showing how VR demands structured transitions Nora 's VR experience] Adrian and others proposed warm up and cool down routines during focus group discussions to manage pacing.	Focus groups	without relying on the technology itself.
Content Knowledge	Revealing both strengths and gaps in vector understanding	Karim noted he no longer remembered the mathematical definition of projection (VR experience) Maren observed that stretching vectors changed the resulting addition. Karim openly identified his content gap saying "I do not remember what projection even means" [Karim, VR experience] Maren discovered the effect of magnitude on addition saying "Longer it is shorter the addition goes" [Maren, VR experience] Philip explored projection behaviour saying "I can imagine a case where I am looking at two vectors and then just seeing the shadow cast" [Philip, VR experience]	VR Experience Freewriting reflections	VR actions surfaced both conceptual gaps and intuitive insights revealing uneven prior knowledge and opportunities for conceptual strengthening.
Technological Pedagogical Knowledge	Understanding how VR influences attention, engagement and classroom behaviour	Nora described how VR removed external distractions and supported focus (Freewriting) Cassia noted that students would enjoy the gesture rich movements (Focus group discussion) Cassia noted gesture rich interaction saying "I have a lot of hammers now… Come back… This is really cool" showing awareness of how embodied engagement shapes student experience (Cassia's VR experience) Participants in the third focus group session later discussed in focus groups that such	Focus groups Freewriting reflections VR Experience	Participants envisioned how the immersive form of the technology shapes student motivation and the structure of a mathematics lesson.

		immersive movements could influence lesson flow and behavioural expectations.		
Technological Content Knowledge	Recognizing VR's affordances for visualizing vector operations	Nora wrote that VR helped her see projection and distance more clearly than chalkboard drawings. (Freewriting activity) Violet explored how the add tool changed vectors in real time (VR experience). Violet manipulated the add tool saying "I am playing with the add tool right now… It changes it in real time" (VR experience). Philip explored projection and plane relationships saying "If they are anti parallel it will be that… This is really cool" (VR experience) Nora noted that pressing generate creates new projected vectors saying "Whenever I press this one it generates… Kids would love this" (VR experience)	Freewriting reflections VR Experience	Participants connected specific mathematical structures to the spatial and dynamic affordances of the VR environment.
Technological Pedagogical Content Knowledge	Integrating embodied action, visualization and instruction	Nora imagined asking students to use their arms to represent direction. Participants proposed debriefing sessions after VR to translate movement-based insights into formal mathematical language (Focus Group) Nora used whole body movement to explore vector placement saying "Multiplication gives multiple vectors" while rotating and repositioning objects (VR Experience) Maren worked through operations sequentially and verbally narrating her thinking "Project a vector onto another… Oh interesting… This is amazing" (VR Experience) Participants in focus groups proposed translating these embodied actions into post VR debriefs where students articulate mathematical meaning based on movements completed inside VR.	Focus groups Freewriting reflections VR Experience	Participants showed early TPCK development by coordinating VR mediated manipulation with instructional aims such as formalizing vector operations through movement-based experiences.

In the following section, I present the synthesis across the six components of TPACK: Technological Knowledge, Pedagogical Knowledge, Content Knowledge, Technological Pedagogical Knowledge, Technological Content Knowledge and Technological Pedagogical

Content Knowledge. For each domain, I integrate evidence from all three data sources and provide illustrative quotations that show how participants' thinking developed across modalities.

TPACK Domain by Domain Synthesis

Technological Knowledge

Across the data sources, participants demonstrated a developing familiarity with immersive tools and interfaces. The VR sessions required trial and error, and many learned through repeated attempts rather than through explicit instruction. Kai's experience captured this gradual sense of mastery as he experimented with menu selections, object manipulation and recalibration. His comment in VR session that he realized how to re-orient the interface only after several attempts reflected the emerging nature of this form of technological fluency. Others described similar learning moments. For example, Karim discovered how to spawn and duplicate blocks unintentionally by gripping the controller in a particular way, which highlighted the sensitivity of the interface but also his persistence in learning to correct the system's response. Participants often experienced friction during onboarding, especially with tools for projection, plane manipulation or resets.

These moments reveal that participants did not simply acquire procedural knowledge of the device. Instead, they began developing adaptive strategies for working with immersive interfaces. In their freewriting activities, they also recognized limitations that shaped their experience such as limited tutorials, unclear controls, or uncertainty about how to perform basic actions. Amaya's difficulty recalling how to add or subtract vectors within the interface illustrated how cognitive demands of the interface could overshadow the mathematical task. Together, these insights indicate that participants were beginning to build foundational

technological knowledge while also identifying barriers that might affect classroom implementation.

Pedagogical Knowledge

Participants also articulated pedagogical considerations related to lesson flow and student support when discussing the use of VR in mathematics instruction. In the third focus group session participants emphasized that movement and bodily engagement could serve as meaningful learning supports. In her freewriting, Violet described how this type of mathematics could appeal to students who learn by doing rather than through static representations.

During focus group discussions, participants also imagined scaffolding practices. Several noted the need for pre and post VR discussions to help consolidate learning and to fill gaps for students who may not have understood everything inside the virtual space. For example, Nora explained that she would take students to a non-digital board after the VR sequence to talk through their observations and build shared understanding. Others highlighted safety and structure, suggesting warm up and cool down routines to ease transitions into and out of the headset.

These reflections show that participants were reasoning pedagogically about how to organize learning activities, support students, and maintain instructional coherence when incorporating VR into mathematics lessons. While these considerations were informed by awareness of mathematical content and the features of the VR environment, participants' comments centred on instructional decisions related to classroom management, sequencing, and student support within an immersive learning context.

Content Knowledge

The VR sessions surfaced both strengths and gaps in participants' mathematical understanding. Some participants expressed clear conceptual awareness of vector operations, including addition, subtraction and basic manipulations of magnitude and direction. Maren showed this type of emerging understanding as she noticed how stretching a vector changed the resulting addition and commented on the relationship between magnitude and the combined structure. This suggests that VR helped her notice relationships she might not have observed in a static environment.

Other participants expressed content uncertainty. In his VR session, Karim openly stated that he no longer remembered what a projection meant. During the VR tasks he, at times, attempted to recall formal definitions while manipulating objects. Kai expressed confusion about placing a plane in relation to a projection, commenting that it felt backwards. Amaya described needing more support with subtraction or projection inside the virtual environment. These difficulties highlight authentic CK gaps. They also suggest that VR tasks sometimes revealed conceptual ambiguities rather than resolving them.

Taken together, the data show that VR provided a space in which both participants' consolidated understandings and their gaps or limitations in understanding vector concepts became visible. The virtual environment supported moments of intuitive insight while also revealing areas where formal mathematical ideas, such as projection or subtraction, were uncertain or inconsistently applied. In this sense, the technology could support intuitive insights, but it could also expose underlying misconceptions that required pedagogical intervention.

Technological Pedagogical Knowledge

Participants frequently linked the immersive qualities of VR with student engagement, motivation and lesson design. VR was described as an environment that can remove external distractions and support sustained focus. During focus group discussion, Nora explained that when she was inside the headset she no longer noticed anything around her, which she interpreted as a potential benefit for students who struggle to stay on task.

Participants also saw how the interactive nature of VR could motivate learners. In his freewriting, Adrian described the tool as something that increases motivation to learn new skills. Cassia noted that students would enjoy the gesture rich movements required for vector tasks. Others raised concerns related to classroom use. Nora reflected that if the VR lesson is not structured well the experience could lead to chaos since students might attempt to explore unrelated applications or become distracted by the interface itself (Freewriting). Others reflected on classroom dynamics. In the first focus group discussion, Nora described how students often become “restless” during existing mathematics lessons and suggested that VR could create opportunities for purposeful movement, reflecting pedagogical reasoning about how technology might support classroom engagement.

These reflections represent early forms of technological pedagogical knowledge. Participants were analyzing not only what the technology does, but how it might shape engagement patterns, classroom management strategies and student behaviour.

Technological Content Knowledge

Participants showed strong awareness of how VR helped them see vector concepts differently. Many expressed that the three-dimensional environment provided access to visual structures that would be difficult to communicate through drawings or static textbook diagrams.

Nora wrote that VR helped her visualize operations such as projection or distance calculations in ways that felt clearer than relying on chalkboard sketches. Others described the ways real time manipulation of vectors enhanced their understanding of magnitude, rotation and plane relationships.

Participants frequently commented on the spatial affordances of VR. Maren described how manipulating a plane in VR felt powerful and intuitive. Violet noted that using the add tool allowed her to explore vector combination dynamically. These affordances helped participants see mathematical structures unfold through movement, which supported conceptual connections that might not appear in traditional environments.

These comments show that participants were forming preliminary technological content knowledge. They understood that the immersive environment does not merely display mathematical objects but transforms how those objects can be explored, compared and reasoned about.

Technological Pedagogical Content Knowledge

The deepest forms of integration appeared when participants talked about how VR could support new instructional approaches in teaching vectors. These moments revealed how technology, pedagogy and content were mutually informing rather than treated as separate domains.

Another example of integrated technological pedagogical content knowledge emerged when participants described how VR based vector activities would need to be followed by teacher guided consolidation outside the headset. For example, during the focus group discussion, Nora described transitioning students from the VR environment to a physical board to talk through what they had observed and to make sense of vector relationships collectively.

She explained that after students interacted with vectors in VR, she would use the board to revisit direction, magnitude, and relative positioning, supporting shared understanding through discussion and representation. This reflects a TPCK insight in which the teacher uses technology to surface concepts through embodied experience and then uses existing pedagogical tools to refine and formalize understanding.

Participants also recognized that the technology could not replace clear pedagogy. Several noted that although VR enhances visualization, it does not automatically clarify underlying mathematical relationships. For example, Cassia reflected that "tasks were completed without me understanding how I did them," highlighting moments where successful interaction did not necessarily correspond with conceptual understanding. Similarly, Philip noted that key operations such as projection and copying normals were "unclear even with tutorials," requiring experimentation to infer how the system worked. Violet similarly described moments where unexpected system behaviour prompted the need for classroom discussion, noting that such moments could be used to ask, "What did you notice when you were doing this," and to "address those as a class and come up with explanations together."

These examples show that participants were beginning to view VR as part of a broader instructional design that required coordination between technology, mathematical content, and pedagogical support. Rather than treating VR as a standalone solution, participants positioned it as a tool whose educational value depended on teacher guidance, structured discussion, and deliberate integration into mathematics instruction. This recognition that technology requires teacher mediation is a hallmark of TPCK.

Synthesis and TPACK Profiles

The findings across TPACK domains reveal how preservice teachers began to weave together technological, pedagogical and mathematical considerations through their immersive work with vectors in MathVR. Although participants entered the study with differing levels of confidence in both vector content and digital tools, the three data sources show a gradual entanglement of knowledge that extended beyond the boundaries of the headset. Their experiences illustrate that the development of professional knowledge in this immersive environment was neither linear nor uniform. Instead, their knowledges emerged through oscillations between action, reflection and conversation.

Across accounts, technological knowledge provided the initial anchor point for learning. Participants needed to navigate the interface before they could engage deeply with mathematical ideas. The VR experience data show that early interactions were often dominated by troubleshooting, experimentation and re-attempts at basic manipulation. Yet these moments formed the foundation for subsequent insights. Once participants learned how to grip, spawn, adjust or reset objects, the technology moved from being an obstacle to a tool that enabled conceptual exploration. This shift is visible in the freewriting activities where participants describe how the interface supported new ways of seeing vector addition, subtraction or projection.

Pedagogical knowledge emerged most clearly in the focus group conversations where participants shifted from describing their own learning to imagining the learning of others. They considered how VR would interact with classroom routines, student needs and lesson structures. Their comments about warm up exercises, space requirements, safety routines and post VR consolidation sessions reveal that participants were beginning to reason about VR not simply as a

learning tool but as something that must be coordinated with intentional pedagogy. These considerations reflect a movement from personal experience toward pedagogical planning grounded in participants' own developing understandings of teaching.

Content knowledge was illuminated through participants' efforts to recall definitions, interpret vector tasks or resolve moments of confusion. VR sometimes amplified these gaps, especially when manipulations required precise understanding of mathematical relationships such as projection onto a plane or the interpretation of normal vectors. At other times, VR provided footholds for sense making by allowing participants to observe how objects changed in relation to one another when stretched, combined or rotated. These oscillations suggest that VR can both reveal and support the reconstruction of conceptual knowledge for preservice teachers, depending on their prior familiarity with vector concepts and the level of scaffolding available during the activity.

The strongest moments of synthesis appeared in participants' developing technological content knowledge and technological pedagogical content knowledge. When participants described how movement could reveal mathematical relationships, they were not only commenting on the benefits of VR but on how embodied interaction can shape conceptual understanding. When they imagined asking students to use their arms to represent direction or when they proposed debriefing sessions that help translate virtual actions into formal mathematical language, they demonstrated early forms of TPCK. These moments show participants beginning to articulate how VR based interaction might be used to support connections between intuitive, embodied experiences and more formal mathematical representations.

Across accounts, a clear pattern emerges. Participants did not treat technology, pedagogy and content as isolated domains. Instead, their insights reveal an evolving sense of interdependence among them. VR made vector concepts visible in new ways. These new forms of visibility prompted pedagogical ideas about engagement, structure and assessment. These pedagogical ideas, in turn, led participants to identify what kinds of mathematical knowledge needed refinement or consolidation. The entanglement of these domains is what gives shape to their emerging TPACK.

Cross Modal Synthesis

Across participants, technological and content-related reasoning was most strongly evidenced during embodied VR interaction, where manipulation of vectors, planes, and projections made mathematical relationships visible. Pedagogical and technological pedagogical reasoning was articulated most fully in reflective contexts, particularly in the focused freewriting and focus group discussions, where participants raised concerns about scaffolding, accessibility, and classroom implementation. Two features of the analysis qualify these statements. First, as set out in Chapter Four, the MAET layer coded where a participant's attention settled at a given moment, on the tool or on the mathematics, rather than assigning utterances to TPACK domains; content was treated as continuously present because every task in MathVR was content-specific by design. Claims about content knowledge are therefore inferred from the substance of the mathematical-attention moments, from what participants said and did while attending to a vector's direction, magnitude, or projection, rather than read off a separate content code. Second, pedagogical reasoning was not absent from immersion. It was recorded in the secondary descriptive layer through the code pedagogical imagination, and Nora's remark at the Generate menu is the clearest instance in the dataset. The pattern described here is therefore one of relative

emphasis across modalities rather than a claim that any domain was confined to one phase of the study.

Full technological pedagogical content knowledge was uneven and rarely evidenced in a fully integrated form. When present, it appeared through reflective articulation rather than in real time VR interaction. This pattern suggests that integrated teacher knowledge did not emerge instantaneously within the immersive environment, but rather through the interaction of embodied experience and subsequent reflection. The mapping highlights TPACK as an emergent configuration shaped across modalities, rather than a stable attribute demonstrated within a single activity. Overall, these cross-modal patterns raise a further question: whether the emergence of coordinated reasoning followed a single trajectory across participants or took recognizably different forms.

Participant Profiles: Pathways through Embodied Experience to Emergent TPACK

Given these overarching insights, I conducted an additional analysis to determine whether visible trends or patterns in TPACK emergence were evident. In Figure 20, I present a set of four profiles, constructed through the detailed TPACK analysis presented above. The term profile is used in a descriptive and non-evaluative sense. It does not rank participants or assign fixed categories. It names a recurrent relationship among three analytical dimensions: the affective and bodily stance a participant took up during immersion, the attentional orientation observable in the MAET-coded VR session data, and the dominant configuration of technological, pedagogical, and content-related reasoning evident in the reflective data.

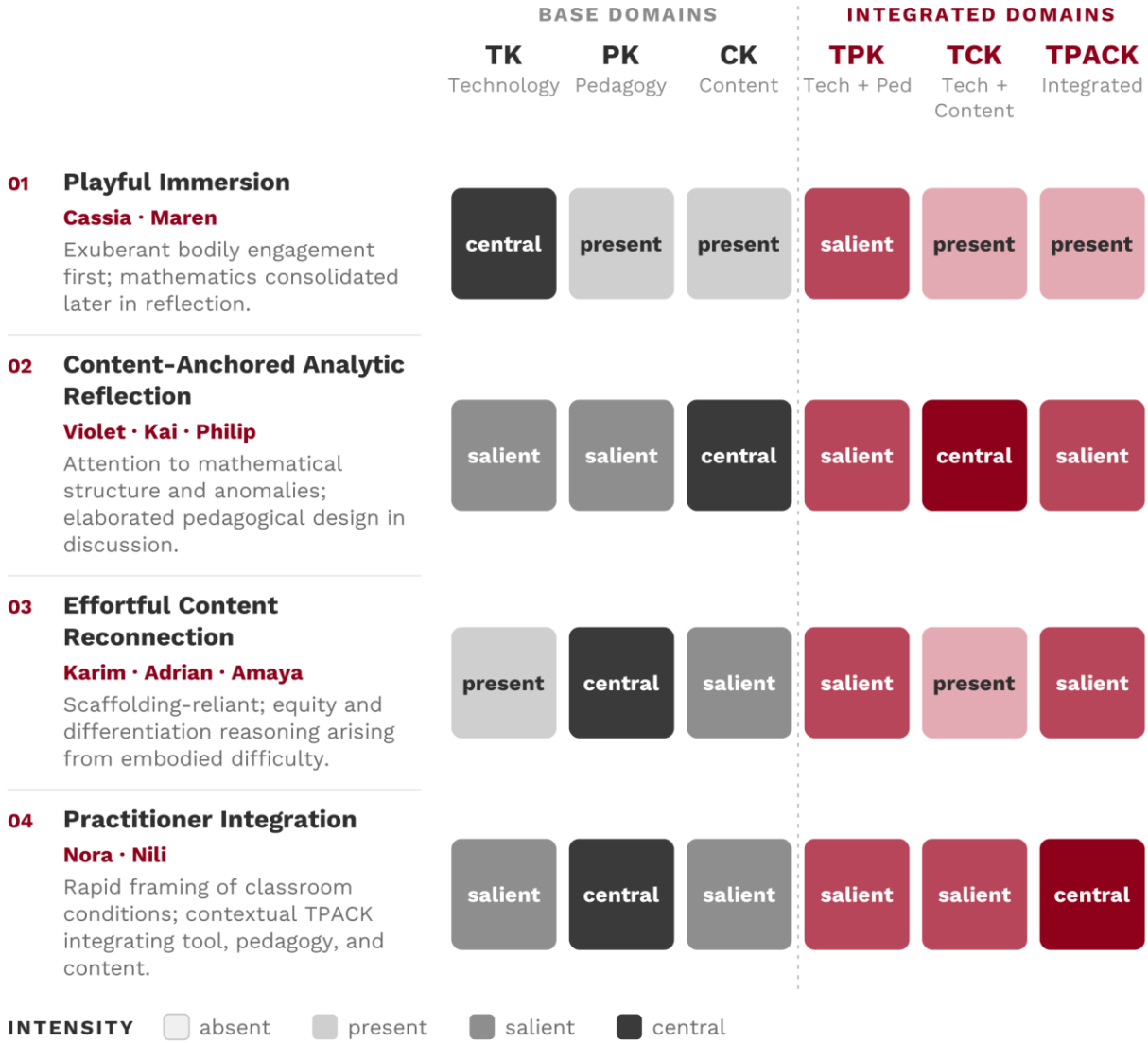
Four profiles are identified. Several participants exhibit features of more than one profile, and these bridging accounts are noted explicitly. Consistent with the study's qualitative exploratory case study design, the profiles are offered as a provisional and heuristic typology

rather than as a classification of participants: they specify the forms TPACK emergence took within this bounded context, and they identify dimensions along which future research might extend or revise these findings.

Figure 20 summarizes the four profiles visually, showing the relative emphasis each knowledge dimension received within each configuration; the descriptions that follow provide the evidence underlying that summary.

Figure 20

Four Configurations of Emergent TPACK: Participant Profiles



Note. Cell shading indicates how strongly each knowledge dimension was expressed within a profile, from absent to central. Grey marks the three base domains (technological, pedagogical, and content knowledge); garnet marks the integrated domains (technological-pedagogical,

technological-content, and full TPACK). The grid reports emphasis rather than proportion: the dimensions are independent, so a single profile may register several as central.

Read across the grid, the figure makes visible a pattern described below: the base technological and content dimensions were most present during immersion in immersive VR, while the integrated domains, where technology, pedagogy, and content combine, reached their fullest expression in the profiles whose participants brought either secure content knowledge or prior teaching experience to the session.

Profile 1: Playful Immersion

Cassia and Maren experienced the immersion as physically exuberant and high in positive affect, treating the environment as a space for embodied play; for both, mathematical understanding arrived after sustained play rather than before it, and became articulable as pedagogical reasoning only in the reflective activities that followed.

- **Affective and bodily stance:** exuberant, physically expansive, high positive affect; both new to VR
- **Attentional orientation:** attention on the physicality of interaction; mathematical meaning consolidated in later discussion, not during immersion
- **Emergent TPACK in VR:** technological knowledge and technological-content knowledge central during VR; pedagogical knowledge absent until reflection
- **Emergent TPACK in reflection:** technological-pedagogical, organized around engagement, motivation, and the affective conditions of learning
- **Defining evidence:** Cassia reformulating the system's "shortest distance" explanation as "the height of the vector" only after completing all tasks; Maren stating in the focus group that she had not assembled the projection idea until that moment of discussion
- **Distinctive note:** content knowledge is emerging rather than secure; playfulness is the affective signature that distinguishes this profile

Profile 2: Content-Anchored Analytic Reflection

Violet, Kai, and Philip brought secure prior knowledge of vectors into their sessions and settled their attention, from the outset and consistently throughout, on the mathematical structure of what they were manipulating rather than on interface mastery or affective engagement; their reflective reasoning produced the most specific and elaborated pedagogical designs in the dataset, anchored in particular features of the tool.

- **Affective and bodily stance:** focused curiosity, affectively stable throughout; secure incoming content knowledge
- **Attentional orientation:** attention on mathematical structure, conceptual anomalies, and relationships from the outset; the richest mathematical verbalization in the dataset
- **Emergent TPACK in VR:** content knowledge and technological-content knowledge recurrent during immersion; mathematical reasoning voiced continuously
- **Emergent TPACK in reflection:** content-technological, joined to specific, elaborated pedagogical design tied to particular tool features rather than general claims about immersive technology
- **Defining evidence:** Philip describing projection as "seeing the shadow that is cast" and systematically testing vector configurations; Violet reasoning independently that the projection reversal beyond ninety degrees was mathematically correct, then turning it into a whole-class discussion design; Kai proposing an in-VR calculation surface and identifying the projection toggle as an affordance for students who struggle to visualize projection
- **Distinctive note:** Violet's projection-reversal moment is a fully integrated TPACK instance in reflection, where a content realization enabled by the tool becomes the material for a specific instructional move

Profile 3: Effortful Content Reconnection

Karim, Adrian, and Amaya encountered the vector content as distant or unfamiliar and required peer or researcher scaffolding to proceed; what distinguishes the profile is that content difficulty did not suppress pedagogical reasoning in reflection but generated it, as each participant drew on their own experience of difficulty as a resource for anticipating what learners would need.

- **Affective and bodily stance:** effortful, scaffolding-reliant, with affective recovery over the session; content distant from current fluency (Karim five years from calculus, Adrian first-time VR user, Amaya no prior vectors and longstanding mathematics anxiety)
- **Attentional orientation:** interface management and scaffolded procedural progress; content reconnection rather than fluent manipulation
- **Emergent TPACK in VR:** partial or emerging content knowledge; pedagogical knowledge absent from verbalizations during immersion
- **Emergent TPACK in reflection:** predominantly pedagogical, with a marked equity and differentiation orientation generated from within the participant's own experienced difficulty
- **Defining evidence:** Karim designing a networked support class in which the teacher monitors students' avatars in shared space, directly addressing the help-giving problem he had faced; Adrian observing that shared unfamiliarity with VR lowers the social cost of asking questions; Amaya organizing her reasoning around visual learners and students with disabilities, and citing her own shift from anxiety to engagement
- **Distinctive note:** Amaya is a bridging account between the Playful Immersion and Effortful Content Reconnection profiles; her affective arc resembles the former, but her content insecurity persisted across the session, and she is placed here on the analytical

weight of her pedagogical and equity reasoning rather than a categorical judgement about affect

Profile 4: Practitioner Integration

Nora and Nili entered as practicing secondary mathematics teachers and moved rapidly and concretely from the affective experience of immersion to the contextual and institutional conditions of classroom use; their reasoning integrated technological, pedagogical, and content considerations with the practical realities of deploying VR in a real school.

- **Affective and bodily stance:** strong absorption arc; rapid shift into a practitioner framing; both first-time VR users with prior teaching experience
- **Attentional orientation:** contextual conditions of classroom use; attention moving quickly from the tool to its deployment
- **Emergent TPACK in VR:** Nora is the sole participant to voice pedagogical imagination during immersion itself ("kids would love this" at the Generate menu, coded as cause-and-effect noticing, affective engagement, and pedagogical imagination), rather than only in reflection
- **Emergent TPACK in reflection:** contextual TPACK integrating technological, pedagogical, and content reasoning with resource constraints, professional readiness, spatial requirements, and curriculum alignment
- **Defining evidence:** Nora characterizing the environment as a "thinking station" and designing a post-immersion whiteboard consolidation sequence, a direct articulation of the cross-modal logic; Nili reasoning about the tablet cast function for monitoring headsets, a staged demonstration-to-exploration sequence, and device cost, fragility, and professional development, using Chromebook adoption as a reference point

- **Distinctive note:** incoming professional experience is the signature of this profile; the pedagogical reasoning is institutionally specific in a way the other profiles are not.

Taken together, the three sets of findings describe a movement from embodied action, through individual reflection, to collective articulation, with the four profiles showing that this movement did not take a single form across participants. What the results establish is that preservice teachers encountering vector operations at body scale noticed different things, brought different affective and bodily stances to the encounter, and drew on those experiences differently when they turned to questions of teaching. What those differences mean for embodied learning theory, for the TPACK framework, and for mathematics teacher education is the work of Chapter Six, which interprets these findings in relation to the literature reviewed in Chapter Two and the frameworks set out in Chapter Three.

Chapter Six

Discussion

This chapter interprets the findings presented in Chapter Five and situates them within the broader literature on embodied learning, immersive technologies, and mathematics teacher education. Where the previous chapter described participants' embodied interaction with MathVR, their reflective descriptions of its affordances and constraints, and the emergent TPACK visible in their accounts, this chapter examines what those results mean for theory, research, and teaching practice. Taken as a whole, the participants' accounts show that the immersive MathVR environment did more than present mathematical content: it created opportunities for them to reorganize how they perceived, enacted, and reasoned about mathematical ideas. The chapter proceeds by returning to each research question in turn, interpreting the findings in relation to relevant theory and prior research. It then identifies cross-cutting themes that connect the three questions, and concludes with the study's contributions, implications, limitations, and directions for future research.

Interpretation of Findings for RQ1

Research Question One asked what insights preservice teachers offered about the technological and pedagogical possibilities of MathVR while they were using it to work with vectors. Drawing on the participant accounts (see pages 140–181), this section interprets how embodied action, gesture, and spatial reasoning shaped participants' conceptual and pedagogical insights, read through embodied cognition, immersive learning theory, and multimodal interaction.

Embodied Interaction as a Foundation for Mathematical Reasoning

Across accounts, participants' gestures, body alignments, and spatial manipulations functioned as part of the reasoning process itself rather than as background activity accompanying it. Participants frequently used whole body orientation, arm extensions, and hand adjustments to align with vector directions, often before they verbalized their thinking. In other words, movement was one of the means through which they explored and interpreted vector relationships, not simply a physical by product of being in VR. These behaviours are consistent with the view that cognition is deeply rooted in the body's sensorimotor engagement with the environment (Alibali & Nathan, 2012; Wilson, 2002). This form of enactment aligns with embodied cognition theory, which holds that mathematical ideas are not solely abstract but are often first encountered through perceptual and physical experience (Lakoff & Núñez, 2000; Shapiro, 2019).

Participants such as Nora and Philip rotated their heads, adjusted wrist angles, and shifted stance to inspect vector projections. These actions reflect what Johnson-Glenberg (2018) terms sensorimotor grounding, meaning the use of bodily action to anchor conceptual processing in immersive environments. Similarly, Cassia and Maren's broad arm gestures, playfulness, and affective commentary suggest a high degree of presence and engagement, allowing them to test hypotheses through movement and adjust based on visual feedback (Johnson-Glenberg, 2018).

Gestures also served as tools for externalizing thinking. As participants manipulated vector directions or compared vector magnitudes, they used their hands to enact mathematical operations before articulating them. Research has shown that such depictive gestures are both reflective and generative, revealing current understanding and prompting new insights (Alibali & Nathan, 2012; Goldin-Meadow, 2003). In particular, gestures allowed participants to create

intuitive representations of vector addition, subtraction, and projection, sometimes before they could recall formal definitions. These patterns echo findings from Walkington et al. (2019), who demonstrated that coordinated bodily gestures support geometric reasoning by freeing cognitive resources and enabling learners to focus on structural relationships.

Perception Action Cycles and the Affordances of VR

Participants moved between manipulating an object and watching what it did, in continuous cycles of perception and action. As Karim and Violet adjusted vectors, they paused to observe resulting movements or check whether actions had triggered visual confirmations. These iterative cycles are consistent with the enactive learning model, which describes cognition as emerging from ongoing loops of sensory input and motor adjustment (Abrahamson & Sánchez García, 2016).

The affordances of VR, including real time responsiveness, immersive visuals, and dynamic control, intensified these cycles. Participants like Philip and Kai repeatedly attempted, observed, and recalibrated their vector manipulations, with moments of insight often emerging after several rounds of exploratory trial and error. This embodied experimentation may have supported the development of what Shvarts and Abrahamson (2023) describe as "attentional anchors," that is, perceptual structures learners use to stabilize their actions around emerging concepts. For instance, when Maren aligned vectors and noticed consistent triangle formations during vector addition, she seemed to form a perceptual cue that preceded her conceptual articulation of the rule. Future research is needed to examine how patterns of movement in MathVR relate to the emergence of attentional anchors and how these perceptual reference points support preservice teachers' developing understandings of mathematical concepts across repeated interactions.

MathVR also seemed to amplify learners' engagement by placing them "inside" mathematics. Participants frequently commented on their sense of immersion, with Amaya and Nora noting that the environment felt real or absorbing. This sense of presence has been identified as a driver of motivation and sustained attention in educational VR (Makransky et al., 2019). When learners feel situated within a task, evidence suggests they are more likely to persist, explore, and attend to structural relationships, which are essential conditions for deep mathematical reasoning (Essoe et al., 2022; Johnson-Glenberg, 2018).

However, the immersive qualities of MathVR also presented challenges. Several participants experienced initial disorientation, interface confusion, or sensory overload. Amaya and Cassia struggled with tool visibility or control states, while others, like Maren, were frustrated by perceived glitches. These challenges reflect findings from Parong and Mayer (2018), who note that immersive media can increase cognitive load, particularly when learners are managing novel physical interactions alongside conceptual demands.

Scaffolding the Transition from Embodied Experience to Conceptual Insight

While physical manipulation opened space for discovery in this study, participants' clearest mathematical insights usually did not appear at the exact moment of movement alone. They tended to emerge when movement was followed by reflection, dialogue, or some form of guided interpretation. In other words, the embodied experience seemed to provide the first contact with the idea, but not always its full mathematical articulation. This finding helps clarify why scaffolding matters in embodied learning environments. Abrahamson and Bakker argue that indirect movement instruction is productive because it gives learners time, space, and license to discover new sensorimotor schemes and attentional anchors through action, rather than simply reproducing prescribed movements. At the same time, their account also implies that such

movement becomes educationally powerful when learners can subsequently describe and model what they have enacted as mathematical relations (Abrahamson & Bakker, 2016).

This pattern was visible in the present study. During the VR sessions, participants often acted first and explained later. Karim's exchange with Kai shows this clearly. Their manipulation of projection and plane tools did not immediately produce a conceptual account, but peer dialogue helped transform uncertainty into a more explicit interpretation of the mathematical relationship. Philip's progression offers an even clearer example. He began with an exploratory metaphor, describing projection as "just seeing the shadow," but later reformulated this observation as "the shortest distance between the tip of the vector and the plane." What changed was not only that he had more time with the tool, but that the embodied interaction was followed by reflective verbalization. These episodes suggest that movement alone was not sufficient. What mattered was the transition from enacted noticing to articulated interpretation.

This is why the embodied actions in the study should not be understood as self-sufficient evidence of conceptual understanding. Rather, they functioned as a first layer of meaning making that needed to be extended through prompts, comparison, explanation, or debrief. This interpretation is consistent with Johnson-Glenberg's discussion of high embodiment learning, where bodily activity is most educationally powerful when the movements are meaningfully mapped onto the target concept rather than added as a decorative or motivational feature. In this study, participants' actions became more mathematically productive when those actions were tied to explicit opportunities to name relationships, interpret spatial patterns, and connect movement to disciplinary language.

The implication for future research is that studies of embodied VR learning should examine not only whether learners move, but how instructional supports help them convert

movement into mathematical explanation. Future work could compare different forms of scaffolding, such as peer dialogue, teacher questioning, in app prompts, or post VR consolidation tasks, to determine which kinds of support are most effective for helping learners move from sensorimotor exploration to formal mathematical reasoning.The findings here suggest that the value of immersive embodied environments may lie not simply in giving learners something to do with their bodies, but in designing structures that help them interpret what those bodily actions reveal. This aligns with embodied design principles, which recommend combining physical tasks with prompts that support noticing, naming, and formalizing patterns (Abrahamson & Sánchez García, 2016).

Summary

The findings from the analyses framed by the first research question show that preservice teachers used movement, gesture, repositioning, and real time visual feedback to make sense of vector operations in MathVR. Across the accounts, participants frequently turned their bodies to inspect objects from new angles, adjusted vectors through repeated hand actions, and used the environment's responsiveness to test relationships among direction, magnitude, addition, subtraction, and projection. In participants such as Nora, Maren, Cassia, and Philip, exploratory movement was closely tied to moments of noticing, including seeing how vectors aligned, how projection behaved, or how changing magnitude altered a result.

At the same time, the findings also show that embodied interaction on its own did not always produce clear mathematical interpretation. Participants such as Karim, Amaya, and Philip often reached more explicit understanding only after they paused, verbalized what they were seeing, or discussed the task with a peer or the facilitator. In this study, the most productive moments were not simply those in which participants moved, but those in which movement was

followed by explanation, comparison, or reflection. Read in relation to the literature, these findings support the view that embodied interaction can play a meaningful role in mathematical reasoning, while also showing the importance of scaffolding that helps learners connect action to disciplinary interpretation (Abrahamson & Bakker, 2016; Elme et al., 2022; Huang et al., 2024; Nathan & Walkington, 2017; Shvarts et al., 2021).

For preservice teacher education, this suggests that immersive work with vectors may be most useful when exploration is paired with structured prompts, peer discussion, and post activity debriefs. Recent work in immersive learning and teacher education likewise suggests that virtual reality is more likely to support meaningful pedagogical development when it is accompanied by self explanation, self regulated learning supports, and structured guidance that helps preservice teachers connect immersive activity to instructional goals and content specific reasoning (Alkaabi et al., 2025; Elme et al., 2022; Hsu & Lee, 2026; Rodríguez & García López, 2026; Van der Want & Visscher, 2024). Future research could build on this by examining which kinds of scaffolding best support the shift from exploratory interaction to explicit mathematical explanation in immersive environments.

Interpretation of Findings for RQ2: Affordances and Constraints of MathVR

Recall that RQ2 asks how preservice teachers described the affordances and constraints of MathVR for teaching vector operations to Grade 12 students in Ontario after completing the VR experience. This section interprets those findings by drawing on participants' focused freewriting and focus group discussions, which captured how they reflected on MathVR once they had completed the vector tasks. Whereas the discussion of RQ1 examined what participants did and noticed during immersive interaction, this section focuses on how they later interpreted the instructional value, practical limitations, and classroom relevance of that experience.

This section therefore examines how participants described MathVR as a possible teaching tool, rather than revisiting their moment to moment actions in VR. Their reflective accounts revealed both enthusiasm and caution. Participants frequently described MathVR as supporting three-dimensional visualization, embodied exploration, and student engagement, while also identifying concerns related to usability, discomfort, cost, access, and classroom implementation. These findings are interpreted here in relation to scholarship on immersive learning, educational VR, and mathematics pedagogy, with particular attention to how participants weighed conceptual promise against practical and equity related constraints.

Embodied Visualization and Conceptual Understanding

A pattern in the RQ2 data is that participants did not find MathVR only engaging, but that they identified particular representational features that seemed to support their understanding of vectors. In focused freewriting, Nora wrote that the tool provided students with "other ways of visualizing problems" and made it easier to grasp relationships such as the distance between a point and a plane, rather than relying only on teacher explanation or board drawings. Amaya similarly emphasized that MathVR allowed learners to "visualize the abstract conception" of operations such as addition and subtraction and to better understand "space" and "projection." Adrian also described "the interaction part" as what motivated learning, even though he noted that difficulty manipulating the tool could sometimes limit full understanding for beginners. In the focus group data, Kai explained that seeing vectors in three dimensions helped him understand why a third dimension matters in ways that paper does not, Philip highlighted that "the resultant vectors change in real time," and Cassia said that seeing a gray line "as if the vectors were tip to tail" helped her understand addition while the simultaneous display of coordinates could support teacher explanation. Maren likewise described plane manipulation as

something that made her “appreciate vectors in a new way.” Across these findings, these accounts point not to immersion in a general sense, but to specific affordances participants associated with conceptual access: manipulability, real time transformation, multiple perspectives, visible coordinate relations, and dynamic representation of operations.

Interpreted in relation to prior scholarship, these findings align with research showing that embodied and immersive environments can support mathematical learning by making structures available through movement, perception, and interaction rather than through static symbolic display alone (Beauchamp & Walkington, 2024; Dimmel et al., 2021; Johnson-Glenberg, 2018; Walkington et al., 2024a). They also echo work suggesting that VR can support presence, agency, and sensorimotor engagement in ways that make abstract content more accessible (Johnson-Glenberg, 2018; Lehrman, 2025; Philipsen et al., 2022). What is analytically interesting here is that preservice teachers themselves named which features of MathVR seemed to matter for understanding vectors. Their comments did not stop at saying that VR was engaging or immersive. They referred to watching resultants change as vectors were adjusted, seeing tip to tail relationships appear visually, manipulating a plane directly, and observing the coordinate system expand around them. In that sense, this study contributes a more fine-grained account of how preservice teachers interpreted the conceptual value of immersive mathematical representations, especially in relation to direction, magnitude, projection, and spatial positioning.

Engagement and Affective Benefits

A smaller set of participants explicitly described MathVR in affective terms, particularly as enjoyable, engaging, or less intimidating than more conventional approaches to vectors. In the focus group, Karim described working with vectors in VR as “a lot more fun than writing them on paper,” linking enjoyment to the ability to stretch, toss, and manipulate vectors directly in

space. In focused freewriting, Nili similarly described MathVR as “a really fun and enjoyable way to solve vector problems.” Amaya also framed the experience in affective terms, writing that the VR activity made mathematics feel more accessible and less intimidating for her. These comments do not demonstrate a general reduction in affective barriers across the participant group, nor do they establish a measured change in motivation or anxiety. They do show, however, that some participants experienced MathVR as affectively supportive and described its interactive qualities as making vector work feel more approachable than paper-based methods.

This more limited interpretation aligns with prior research suggesting that immersive virtual reality can promote enjoyment, interest, and situational engagement during learning activities, even when broader claims about learning outcomes or affective change require more direct evidence (Makransky et al., 2019; Makransky & Petersen, 2021; Parong & Mayer, 2018). Several participants described the experience as enjoyable, engaging and, in Amaya’s case, less intimidating, although the data do not allow a conclusion that MathVR reduced affective barriers. This suggests that immersive interaction may help some preservice teachers approach vector learning with greater openness, while also indicating the need for future research that examines affective responses more directly and systematically.

Participants’ reflections, the study’s evidence on affect, suggest that affective response was tied to particular features of the environment rather than to novelty alone. Enjoyment was linked to being able to manipulate vectors directly, observe changes in real time, and explore spatial relationships from multiple viewpoints. In that sense, the affective value participants described appears alongside the perceptual and conceptual supports they also identified elsewhere in the study, especially clearer access to direction, magnitude, projection, and spatial

positioning. Future research would need more direct measures to determine whether and how these affective responses contribute to sustained engagement or to longer term learning.

Interface Learning Curve and Cognitive Load

Counterbalancing the above affordances, participants identified notable usability issues and cognitive load challenges with MathVR. Several preservice teachers admitted struggling with the VR controls and feedback, resulting in confusion about how to perform certain operations or whether a task was completed correctly. Cassia's remark that she finished tasks "without me understanding how I did them" exemplifies this disconnect. Similarly, Karim noted that "there was no way for me to figure it out" when trying to duplicate or remove objects. In these moments, attention appeared to shift away from vector reasoning and toward managing the interface itself.

This pattern suggests that, for some participants, the demands of learning how to operate the VR system competed with the demands of learning the mathematics. In cognitive load terms, the effort required to interpret controls, tool functions, and system feedback may have introduced additional processing demands that were not directly related to understanding vectors (Albus et al., 2021; Parong & Mayer, 2018). Participants' requests for clearer tutorials, more explicit instructions, and better feedback therefore matter not only as usability preferences, but as indicators that the interface was sometimes drawing cognitive resources away from the mathematical task. This interpretation is consistent with research showing that immersive environments can become instructionally counterproductive when learners must devote too much attention to navigating the system rather than to the target concept (Makransky et al., 2019; Philipsen et al., 2022; Yu et al., 2025). This concern also appears in classroom-based VR research. Walkington et al. (2025), in their study of VR mathematics tutoring, reported that

students sometimes lost orientation, encountered technical disruptions, and moved off task while trying to work within the immersive environment. The present findings resonate with those observations. Cassia questioned how tasks had been completed, Karim described not being able to figure out how to duplicate or remove objects, and several participants called for tutorials or clearer onboarding. Although the current study was not conducted in a secondary classroom, these preservice teachers were already identifying the same kinds of breakdowns that classroom research has documented.

Participants' accounts across the dataset reinforced this pattern. Cassia questioned how tasks had been completed and other participants referred to the need for tutorials, demonstrations, or more structured onboarding before independent exploration. These findings reinforce the importance of user centred design and instructional scaffolding in educational VR environments (Albus et al., 2021). When interface elements are unintuitive (e.g. inconsistent button functions or lack of feedback), students may become "lost" in the virtual environment, a phenomenon Makransky et al. (2019) warn can diminish the sense of presence and hinder learning (Philipsen et al., 2022). In MathVR, such issues manifested as frustration (having to restart the app, accidentally spawning too many objects. These moments show how affordances can become constraints when the technology is not transparently aligned with users' actions. In interpreting these accounts, it becomes clear that the learning curve of the interface shaped how quickly participants could shift from managing the system to reasoning about vectors.

The implication is that the pedagogical value of MathVR depends partly on how transparent the system is to novice users. In this study, design features such as unclear button functions, limited feedback, or uncertainty about task completion sometimes constrained participants' ability to focus on vector relationships. This suggests that design revisions, such as

clearer controller mapping, more explicit visual feedback, and a built-in tutorial mode, may reduce these interface related demands. It also suggests that pedagogical intervention matters. Brief orientation, guided modelling, and staged task entry may help learners direct more of their attention to the mathematics itself. Further research is needed to examine which kinds of design support and instructional scaffolding are most effective in helping preservice teachers move more smoothly from interface management to conceptual engagement.

Necessity of Scaffolding and Pedagogical Integration

Preservice teachers in this study did not see VR as a standalone solution. Rather, they consistently positioned it as one part of a broader teaching sequence that would require preparation, guidance, and follow up discussion if it were to support meaningful mathematical learning. This pattern was visible across participants' reflections. Nora suggested that students would need some familiarity with vector ideas before entering the VR environment, and several participants described the need to return to whole class discussion or board work after the headset activity in order to consolidate what had been noticed in motion and space. Nili expressed this especially clearly in focused freewriting, stating that students would need to be familiarized with the concept beforehand and that the app should include a demo or tutorial of basic vector ideas before students were left to explore on their own. Violet made a similar point in the focus group when she explained that a tutorial mode or shadow guide could help students who were struggling with the required movements, particularly when a teacher was not immediately available beside them. Together, these accounts suggest that participants were not rejecting MathVR. They were identifying the pedagogical work required to make embodied experience instructionally productive. A similar pattern appears in Rodríguez's (2024) study of preservice teachers learning to use a virtual reality geometry tool. There, participants were also

enthusiastic about VR's visual and motivational possibilities, yet they struggled to imagine how to structure lessons around it. The present findings extend that pattern into mathematics teacher education with vectors. Participants such as Nora, Nili, Cassia, and Adrian repeatedly returned to questions of preparation, onboarding, discussion, and follow up, suggesting that preservice teachers' positive perceptions of VR are closely tied to their recognition that the technology requires pedagogical framing to become instructionally meaningful.

This interpretation can be connected to Johnson-Glenberg's account of immersive learning, where the educational value of embodiment lies not in movement alone but in how bodily action is meaningfully mapped onto the target concept and supported through instructional design. In the present study, participants repeatedly pointed toward that need for design. Cassia for example, did not simply praise the visual and interactive qualities of the environment. She also proposed that students should be able to revisit what happened, explain how a task was completed, and connect the immersive action to mathematical understanding. Similarly, Nora imagined using discussion after the VR experience to revisit vector relationships and address gaps in understanding that might remain after embodied exploration. Karim extended this pedagogical line of thought by suggesting optional animations that would visually show how vectors add together, so that students who struggle with abstraction could see the operation unfold rather than being expected to infer it independently. These reflections suggest that participants were already reasoning beyond the novelty of immersion and toward the instructional conditions under which immersive experience could be transformed into disciplinary understanding.

The relevance of this interpretation is strengthened by Beauchamp and Walkington's analysis of Prisms Math VR, which similarly identifies both the affordances and the limits of

immersive mathematics learning. Their findings show that some students were able to move productively among dynamic representations in VR, whereas others were overwhelmed by the sensory and representational demands of the environment. On this basis, they argue that scaffolds in VR mathematics settings should focus learners' attention, reduce visual and sensory complexity, and help teachers understand what students are experiencing from the first-person perspective. This argument resonates closely with the present findings. Participants in this study likewise identified the need for clearer preparation, more explicit tutorials, and instructional sequencing before independent exploration. Their reflections suggest that the pedagogical question is not whether VR can offer mathematically meaningful experience, but how such experience must be organized if it is to support understanding rather than confusion.

The present study extends this discussion by showing how such concerns are relevant within preservice teacher education. Whereas Beauchamp and Walkington examine students' reasoning within immersive simulations, the participants here responded as future teachers, reflecting not only on their own learning, but also on the conditions under which similar environments might be used with Grade 12 students. Their comments moved beyond immediate experience toward questions of instructional design, including when immersive activity should occur within a lesson, what prior knowledge students would need, how long they should remain in the environment, and what forms of discussion or consolidation should follow. Cassia imagined a rotational model in which some students use VR while others work with more conventional or two-dimensional materials, and then the class reconvenes to discuss what was observed. Adrian likewise suggested that the interactive quality of the tool could motivate learning but noted that not everyone would adapt quickly enough to manipulate the system independently without support. In this sense, the data show preservice teachers beginning to

conceptualize VR not as a self-contained instructional event, but as an experience whose educational value depends on its integration within a larger pedagogical structure.

The study also points toward an issue that Beauchamp and Walkington (2024) identify as particularly significant in immersive learning environments, namely the difficulty teachers may face in interpreting students' activity when the student's first-person perspective is not fully visible. They report that facilitators sometimes struggled to provide meaningful support because they could not adequately access what learners were seeing or attending to within the headset. A related concern surfaced in the present study, where participants valued the possibility of casting the headset view to an external display as a means of supporting teacher monitoring, feedback, and intervention. This suggests that pedagogical integration in immersive mathematics environments like MathVR depends not only on task design, but also on the extent to which teachers can remain instructionally present while students are immersed.

Interpreted collectively, these findings suggest that participants viewed MathVR as a powerful visualization and exploration tool that requires thoughtful instructional design to realize its educational potential. They anticipated the need for clear initial guidance to reduce disorientation and for embedding MathVR activities within a coherent pedagogical sequence. This underscores a central principle of immersive learning: embodied experiences in VR gain educational value when coupled with scaffolding and reflection, enabling learners to connect immersive actions with underlying concepts (Lehrman, 2025).

The findings also suggest several directions for future research. One important line of inquiry would be to examine how different forms of support before, during, and after immersive activity shape learners' ability to connect embodied interaction with formal mathematical reasoning. Another would be to investigate how teachers monitor, interpret, and respond to

students' activity within VR, particularly whether live casting, shared displays, or embedded analytic supports can help address the first-person visibility problem identified in prior research and echoed in participants' reflections here (Beauchamp & Walkington, 2024). Through these lines of inquiry, future scholarship could move beyond broad claims about scaffolding and specify more precisely the instructional conditions under which immersive mathematics environments become educationally productive.

Equity, Access, and Contextual Constraints

A significant aspect of participants' reflections on constraints involved the practical realities of classroom implementation, particularly cost, access, and inclusivity. Preservice teachers were quick to note that MathVR's benefits could only be realized if schools can overcome several barriers. Hardware availability and cost emerged as primary concerns, with several participants doubting that a typical secondary school would have sufficient VR headsets for an entire class or the budget to acquire them. This concern closely mirrors findings in the broader literature, where teachers frequently identify financial limitations and resource scarcity as major obstacles to adopting immersive VR in formal education settings (Campos et al., 2022). VR equipment, especially fully immersive headsets, remains prohibitively expensive for many schools, and participants realistically imagined scenarios such as "maybe we only get one headset for the whole school." Such limited access would necessitate rotational use and careful scheduling, potentially introducing equity concerns if opportunities to engage with VR are unevenly distributed across students.

Physical space was another constraint repeatedly raised by participants. Effective use of room scale VR requires open and safe movement areas, yet many classrooms are densely furnished. As Kai observed, rearranging desks alone could be "already a lot," pointing to

logistical challenges that could make VR activities disruptive or unsafe if insufficiently planned. It is noteworthy that teachers in Walkington et al.'s (2021) study likewise expressed worries about managing student movement and behaviour in VR, especially when multiple learners are involved (Walkington et al., 2025). Participants' reflections demonstrate an early awareness that the affordance of embodied movement in VR is inseparable from the responsibility of maintaining classroom order and physical safety. This anticipatory concern aligns with classroom-based VR research showing that immersive mathematics activity can introduce management challenges alongside conceptual opportunities. Walkington et al. (2025), for example, note issues related to student orientation, technical interruptions, and staying on task during VR tutoring. In the present study, similar concerns appeared in participants' reflections about monitoring, space, and the possibility that VR use could become difficult to manage if it were not carefully structured.

Equity considerations also extend to student characteristics and needs. Kai pointed out that MathVR, by its very nature, is a highly visual medium "only accessible to those who are not visually impaired," as written in Kai's freewriting activity. This underscores a limitation of current VR technology: it inherently excludes learners with severe vision impairments and may also pose challenges for those with certain motor disabilities or vestibular disorders. While emerging research is exploring non-visual or audio-based VR for inclusion, mainstream educational VR still lacks robust accommodations for the visually impaired (Wolf et al., 2025). Even among sighted students, sensory and health factors can constrain VR usage. Amaya's experience of dizziness after 10 minutes in the headset is a reminder that VR can induce motion sickness or fatigue in some users (Cardullo & Wang, 2022; Souchet et al., 2023). Such side

effects vary individually and can improve with acclimation, but they impose practical limits on session length and raise the need for frequent breaks and alternative activities.

In interpreting the findings, it's important to note that participants weren't merely listing personal complaints; but were actively considering how such issues would scale in diverse classrooms. For example, one participant suggested that students with sensory sensitivities might need special accommodations (shorter exposure or modified devices), and another noted left-handed users struggled with the right-hand-biased controls, indicating potential universal design improvements needed in MathVR. These reflections align closely with principles of Universal Design for Learning, which emphasize the importance of providing multiple means of engagement and access to accommodate learner variability. They also echo concerns raised in the literature that emerging technologies, including VR, risk exacerbating existing digital inequities if access is uneven or accommodations are insufficient (Stanney et al., 2020).

Preservice teachers' reflections can be interpreted as equity-oriented calls to action. They suggest that for MathVR and similar tools to be implemented responsibly and at scale, issues of funding, physical infrastructure, safety, and accessibility must be addressed at the institutional and system level. Without such supports, the pedagogical affordances of immersive VR may remain available only to a limited subset of students, undermining broader goals of inclusive and equitable mathematics education.

Balancing Affordances with Constraints

Taken together, participants did not describe MathVR as simply beneficial or simply problematic. Rather, they described an instructional environment in which the same immersive features that opened up mathematical possibilities also generated new forms of difficulty. Participants such as Nora, Violet, and Philip emphasized the value of real time visual feedback,

direct manipulation of vectors, and the ability to inspect projection, direction, and magnitude from multiple viewpoints. At the same time, participants such as Cassia, Karim, and Amaya described moments when interface demands, uncertainty about task completion, or sensory strain competed with attention to the mathematics. The central issue, then, was not whether immersion was good or bad in itself, but how its affordances and constraints were experienced together within the same activity.

This interpretation is consistent with Makransky and Petersen's (2021) Cognitive Affective Model of Immersive Learning, which argues that immersive virtual reality is not inherently more effective than less immersive media. Rather, its educational value depends on whether instructional methods are aligned with the affordances of presence and agency while also managing cognitive load. In their review, Makransky and Petersen synthesize several studies showing that immersive environments can support learning more effectively when paired with specific instructional supports. They cite Meyer et al.'s (2019) experiment, in which pretraining improved knowledge retention, transfer, and self efficacy in immersive VR but not in video, as well as studies showing that enactment and teaching activities improved learning outcomes in immersive VR when compared with less immersive conditions. Their point is not that VR is automatically transformative, but that it can become instructionally powerful when learners are given supports that help them interpret and regulate the experience.

That argument helps clarify what participants were describing here. In this study, MathVR's visualization strengths were specific rather than abstract. Participants valued being able to manipulate vectors directly, observe changes in real time, inspect spatial relationships from different angles, and make features such as projection and vector direction more visible than they would be on paper. Yet these same immersive conditions also produced interface

related demands. Cassia described completing tasks without understanding how she had done them. Karim described not being able to figure out how to duplicate or remove objects. Amaya and others described moments in which bodily engagement and system management made the activity feel demanding rather than transparent. These are not incidental usability complaints. They indicate that part of the learner's effort was sometimes redirected away from vector reasoning and toward making sense of the system itself.

Research on immersive learning supports this interpretation. Meyer et al. (2019) found that pretraining had a positive effect on knowledge, transfer, and self efficacy in the immersive VR condition, but not in the video condition, suggesting that instructional preparation may be especially important when learners must manage the added demands of immersive media. Albus et al. (2021) similarly found that signaling through textual annotations in VR improved recall and germane cognitive load, even though it did not improve comprehension, transfer, or reduce extraneous cognitive load. Their findings suggest that learners in VR may benefit from cues that help direct attention toward relevant features, even if such supports do not solve all processing challenges (Albus et al., 2021). In other words, the literature does not support a simple claim that VR transforms learning by itself. It shows, more specifically, that immersive environments become more educationally productive when design and pedagogy help learners use their affordances without being overwhelmed by their demands.

In summary, the findings for Research Question Two show that participants weighed what MathVR made possible against what it made more difficult in a secondary mathematics context. Their accounts emphasized embodied visualization, engagement, and creative pedagogical possibilities, all of which are supported in contemporary immersive learning research (Beauchamp & Walkington, 2024; Lehrman, 2025). At the same time, they identified

constraints including interface difficulty, cognitive load, equity concerns, and logistical barriers, concerns that also appear in prior research (Cardullo & Wang, 2022; Makransky et al., 2019; Souchet et al., 2023). These findings suggest that the participants were neither uncritically enthusiastic nor dismissive about VR. Rather, they understood MathVR as a tool whose instructional promise depended on how its affordances and constraints were managed within classroom practice. Their accounts therefore point to the importance of usability, accessibility, and pedagogical alignment when considering immersive technologies for secondary mathematics teaching.

Interpretation of Findings for RQ3 through TPACK

Emergent Integration of Knowledge Domains

Recall that RQ3 asks: How do these insights and descriptions inform understanding of preservice teachers' emergent Technological Pedagogical Content Knowledge, particularly in relation to a technology that enables visualization and virtual embodied experience within mathematics education? To address this question, this section interprets those findings by drawing on all three data sources used in the study: participants' embodied interaction during the MathVR sessions, their focused freewriting reflections, and the focus group discussions that followed.

The findings suggest that participants were beginning to move beyond experiencing MathVR as a novel digital tool and instead were starting to interpret it in relation to teaching and learning vectors. In several accounts, technological interaction, mathematical noticing, and pedagogical reflection became visibly linked. For example, Philip's VR session showed him working through projection by trial, error, and repeated manipulation until he articulated that the yellow line represented "the shortest distance between the tip of the vector and the plane." That

moment was not only a content related realization about projection. It later fed into his reflective comments about the value of immersive visualization for supporting mathematical understanding. Similarly, Nora 's interaction with generation, projection, and vector positioning in VR was accompanied by immediate pedagogical noticing, such as her comment that "kids would love this," and later by reflective suggestions that students would need prior conceptual preparation and post VR discussion to consolidate understanding. In both accounts, participants were not dealing with technology, pedagogy, and content as separate categories. Rather, the experience of interacting with the mathematical objects in VR appeared to prompt increasingly integrated judgements about what the tool was showing, how the mathematics worked, and how such an experience might need to be taught.

This pattern aligns with the foundational view of TPACK as an integrated form of teacher knowledge in which content, pedagogy, and technology become meaningful in relation to one another rather than as isolated competencies (Koehler et al., 2014; Mishra & Koehler, 2006). In the present study, MathVR functioned as a setting in which these interdependencies became visible. They did not merely acquire isolated skills related to operating VR tools, nor did they simply rehearse definitions of vector concepts. Instead, they began to integrate these knowledge domains in ways that reflected developing professional judgement. Participants first had to engage with the technological environment, but as they worked through vector operations and reflected on their experience, they began to articulate how the technological affordances of manipulation, immersion, and dynamic visualization related to mathematical understanding and classroom use. Cassia offers a clear example of this movement. In her reflections, she did not simply note that the tool was visually interesting. She also questioned how students would know why a task had been completed and suggested that the environment should allow users to revisit

or see how success was achieved. Her comment points to a developing understanding that effective use of technology in mathematics teaching depends not only on what the technology can display, but also on how it supports explanation, feedback, and conceptual transparency for learners.

The data further suggest that participants' emerging knowledge was often most visible at the intersections among TPACK domains. In several accounts, technological content knowledge and technological pedagogical knowledge appeared to develop before more fully integrated TPACK. Kai, for instance, sed the immersive environment to reason about projection, plane relationships, and normal vectors, while also critiquing the way the tasks were structured and named in the interface. His reflections showed that he was not only trying to understand the mathematics but also evaluating how the technology shaped that understanding. Likewise, Karim moved between content related uncertainty, such as trying to recall projection mathematically, and pedagogical proposals, such as using animation or clearer visual supports to help future students see how vector operations unfold. These examples suggest that preservice teachers' experiences with MathVR supported early, partial, and uneven forms of integration across knowledge domains. This is consistent with prior research indicating that teachers often begin by developing understanding at the intersections of technology, pedagogy, and content before demonstrating more stable and fully integrated TPACK (Adipat, 2021; Koehler et al., 2014).

VR Affordances and Content Understanding (TCK)

One of the most salient outcomes of the VR experience was the way MathVR's affordances supported participants' technological content knowledge. By immersing themselves in a three-dimensional vector environment, preservice teachers were able to manipulate mathematical objects in ways that are not possible in existing classroom settings. Direct actions

such as stretching vectors, projecting onto planes, or combining arrows in mid-air enabled participants to observe vector operations as dynamic processes rather than static symbols. As describe earlier, these observed interactions seemed to help participants by making abstract mathematical relationships perceptually accessible.

Such experiences align with research demonstrating that immersive VR can support content understanding by allowing learners to embody and visualize abstract concepts through gesture, spatial movement, and first-person interaction (Dimmel, 2021; Walkington et al., 2025). In the present study, Maren's realization that changing vector magnitude altered the result of vector addition, or Philip's experimentation with vector projections, exemplify how embodied interaction supported mathematical insight. These hands-on discoveries suggest that VR can function as a bridge between intuitive, sensorimotor insight and formal mathematical understanding.

Prior research on VR in math education similarly highlights that being "immersed with dynamic mathematical objects" enables interactions and visualizations "in ways impossible in the real world" (Walkington et al., 2025). From a TPACK perspective, this indicates that the technology is not merely an instructional supplement, but an active mediator of content knowledge. For preservice teachers, recognizing these affordances is a step toward technological content knowledge: understanding what the technology does for the content. Participants began to articulate, for instance, that vector projection in VR allowed them to see a kind of "shadow" cast from one vector onto another, a spatial analogy that is considerably less intuitive in two-dimensional representations. Such reflections signal developing awareness of how technological representations can transform content understanding, suggesting that participants' TCK may have been emerging in tandem with their VR engagement.

Pedagogical Reflections and Classroom Feasibility (TPK)

Alongside their exploration of vector concepts, participants were also reasoning about how MathVR would need to be taught. Their freewriting and focus group responses show emerging technological pedagogical knowledge because they were no longer speaking only as learners using a tool, but as future teachers considering lesson flow, student readiness, transitions, and classroom management. Nora, for example, described the immersive environment as highly absorbing, yet later argued that students would need prior familiarity with vector concepts and a follow up board discussion to consolidate what had been noticed in VR. Adrian similarly proposed warm up and cool down routines around headset use, suggesting that VR lessons would require pacing and framing rather than simply inserting the technology into an existing lesson to explain pedagogical judgements about the conditions under which the technology might support learning.

These findings can be interpreted in relation to work that treats VR implementation as a pedagogical and organizational challenge rather than a purely technical one. Saito and Lee (2023) argue that VR instruction requires teachers to plan for sufficient space, device setup, prior experience with the system, and the alignment of educational goals and assessment with the specific character of VR activity. The present findings support this argument. The preservice teachers in this study anticipated challenges associated with this heightened engagement, including the need for structured onboarding, explicit safety protocols, clear instructional objectives surrounding VR activities, explicit transitions into and out of the immersive environment, and some form of post activity consolidation. Such anticipatory thinking reflects developing pedagogical judgement rather than uncritical enthusiasm.

This interpretation also resonates with Walkington et al.'s (2025) analysis of VR mathematics tutoring, which found that embodied affordances such as gesture, whole body movement, and interaction with dynamic objects created rich opportunities for reasoning, but also introduced constraints related to connection problems, spatial orientation, body positioning, and distraction in the VR environment. A similar tension appeared in this study. Participants valued the immersive and interactive qualities of MathVR, but they also anticipated the need for teacher guidance, clear objectives, and classroom structures that would help students stay oriented to the mathematics rather than only to the interface. In that sense, their reflections suggest emerging TPK because they were beginning to understand that teaching with VR involves designing for both the affordances and the demands of the medium.

As for the implications for future research, as preservice teachers are already identifying pacing, orientation, consolidation, and logistical planning as central to classroom feasibility, future studies should examine how these pedagogical supports shape what students are actually able to learn in immersive mathematics environments. It would be important to compare different forms of lesson framing, such as pre-briefing, guided in app prompts, paired discussion, and post VR consolidation, in order to determine which kinds of pedagogical support most effectively help teachers translate immersive activity into mathematically focused classroom instruction.

Challenges as Catalysts for Professional Growth

Notably, the difficulties participants encountered while using MathVR did not function solely as barriers. Instead, they became catalysts for professional learning that supported deeper integration across TPACK domains. For instance, struggles with the interface (e.g. Amaya forgetting how to add vectors, or Karim inadvertently duplicating objects) highlighted the need

for robust technological knowledge (TK). Early in the VR sessions, such technical stumbles often dominated the experience. However, through trial and error, participants built strategies to overcome these issues; effectively expanding their TK with each attempt. This process underscores that technological fluency is iterative and essential; teachers must first be comfortable with a tool before they can leverage it instructionally.

This iterative process aligns with research demonstrating that teachers' confidence and competence with technology form a necessary foundation for higher order TPACK development (Koehler et al., 2014; Saito & Lee, 2023). In the present study, once participants achieved a basic level of comfort with MathVR, such as Kai learning how to reset the interface or Violet mastering the add tool, they were better able to attend to pedagogical and content related considerations. Several participants later reflected that insufficient orientation or lack of built-in tutorials was a significant barrier, suggesting that teacher education contexts should intentionally scaffold technological learning so that it does not overshadow instructional goals.
Participants also confronted gaps in their content knowledge, as when Karim realized he'd forgotten what "projection" meant mathematically. These moments of uncertainty prompted reflection and re-engagement with content, sometimes during the VR session itself. The visibility of their content gaps in an active context likely made the need for pedagogical support evident. For example, Nora 's idea of a post-VR debrief on the board was directly aimed at solidifying content understanding that VR play alone might not guarantee.

These moments illustrate the dynamic nature of TPACK development. Challenges in one domain, such as technological complexity or content misconceptions, prompted compensatory reasoning in others, including pedagogical strategies for scaffolding or re-framing content using the technology differently. Rather than viewing difficulties as failures, participants treated them

as signals informing how VR would need to be supported instructionally. In this way, challenges served as productive learning moments that catalyzed deeper integration across technological, pedagogical, and content knowledge. Participants were not only learning how to use MathVR, but also what conditions are necessary for VR to function as an effective instructional tool in mathematics classrooms.

What the Profiles Show: Embodied Stance as the Distinguishing Dimension

Considered together, the four profiles provide a more precise answer to Research Question Three than a general claim of situatedness. The configurations of emergent TPACK documented in this study were not arbitrarily distributed across participants. They were consistently connected to two analytically identifiable features of each participant's VR experience: the affective and bodily stance they adopted during immersion and the attentional orientation that the MAET-coded session data makes visible. Playful, physically expansive engagement in the presence of emerging content knowledge was associated with technological-pedagogical reasoning organized around engagement and affective conditions of learning. Secure content knowledge and attention to mathematical structure were associated with content-technological reasoning and specific, elaborated pedagogical design. Effortful navigation of unfamiliar content, supported by scaffolding, was associated with pedagogical and equity reasoning derived from the participant's own embodied difficulty. Professional teaching experience was associated with rapid and concrete movement toward the contextual and institutional conditions of classroom implementation.

Content knowledge does not define a separate profile because it functioned differently within each of the four. In Profile 2 it anchored attention from the outset and created the cognitive space for design-level pedagogical reasoning. In Profile 3, the distance of the content from participants' current fluency generated the empathic resource from which pedagogical insight

developed. In Profiles 1 and 4 it was present at different levels of security but was not the primary determinant of the emerging TPACK configuration. This cross-profile variability follows from how the coding was constructed. The technological and mathematical codes defined in Chapter Four marked attentional salience within a content-saturated environment, so the synthesis is more precisely described in terms of technological-content and pedagogical-content configurations than as a separation of content knowledge from the other dimensions.

The dimension that consistently distinguished the four profiles was not a property of the technology considered in the abstract but the bodily and affective character of each participant's engagement with the immersive environment. This is the point at which the account of immersive first-person embodiment developed in Chapter Three contributes directly to the synthesis. All participants brought their bodies to the mathematical encounter: gesture, movement, and spatial reasoning are part of mathematical sense making across a wide range of instructional contexts, including those involving static or two-dimensional representations. The profiles differ because the immersive environment placed participants inside the represented mathematical space at the scale of their own bodies, and the stance each participant took up within that space, whether playful or analytic or effortful or professionally oriented, shaped what they went on to notice, to experience affectively, and to articulate as teachers. Embodiment in this sense is not a background condition of the VR experience but a foreground analytical category that the TPACK synthesis requires.

These profiles are provisional. They are generated from a single bounded case involving ten preservice teachers in one Ontario teacher education program at one point in time. Whether these configurations recur in other populations is a question for further research. Within the constraints of the design, however, they offer the most analytically grounded and empirically specific response available to Research Question Three. Preservice teachers encountering MathVR

did not arrive at a single form of emergent TPACK. They arrived at different configurations of it, shaped by the embodied and affective dimensions of their immersive experience and made articulable through the reflective activities that followed.

The findings suggest that participants were beginning to coordinate technological, pedagogical, and content related considerations, but this coordination appeared in uneven and context dependent ways rather than as a single, fully formed body of knowledge. In other words, what the study shows is preservice teachers articulating partial, emerging, and sometimes tentative integrations shaped by the specific conditions of the MathVR experience, rather than a simple movement from separate knowledge domains toward a final, seamless state of TPCK. This interpretation aligns more closely with scholarship that treats TPACK as dynamic, situated, and continually developing across contexts rather than as a fixed endpoint that teachers eventually "achieve" once and for all (Angeli & Valanides, 2009).

As the analyses presented in Figure 20 show, emergent TPACK in this study appeared as a set of recognizable configurations, each representing a characteristic way of integrating technological, pedagogical, and content-related knowledge, and those configurations were not randomly distributed across participants. They were patterned, and the pattern seemed connected to how participants inhabited the immersive environment bodily and affectively.

During the VR sessions, pedagogical knowledge and fully integrated TPACK were largely absent from participants' verbalizations. With the one exception of Nora's spontaneous pedagogical imagining while manipulating the Generate menu (see pages 140-145), nine participants demonstrated technological knowledge, emerging or secure content knowledge, and technological-content knowledge through their embodied actions during immersion. The pedagogical dimensions of their thinking surfaced in the focused freewriting activities and the

focus group discussions, once the bodily and affective experience of the immersion could be put into words. The reflective architecture surrounding the immersive experience was not a supplement to learning. The freewriting and discussions created the conditions through which TPACK integration became expressible. Fully integrated TPACK, where technological, pedagogical, and content considerations converge, seemed to emerge when participants had the opportunity to narrate, in writing and in dialogue, what their bodies had already enacted. This matters because it locates TPACK integration not in the immersive experience alone but in the movement between doing and articulating: the reflective activities were where embodied experience became professional knowledge a teacher could name and act on.

This interpretation is consistent with research suggesting that preservice teachers often develop technology related professional knowledge unevenly and in relation to particular tasks, tools, and settings rather than as an immediately holistic capacity practice (Rodríguez, 2024; Saito & Lee, 2023). Rodríguez (2024), for example, found that preservice teachers recognized the motivational and visual promise of VR while also struggling with the practical work of lesson design. A similar tension is visible here. Participants could identify what MathVR made possible for teaching vectors, but they were still working out how such experiences might be sequenced, scaffolded, and adapted for actual classrooms.

This pattern also aligns with Saito and Lee's (2023) work that frames VR integration as requiring simultaneous attention to technological conditions such as setup, interface transparency, and physical space, and pedagogical conditions such as task sequencing, diversity, and assessment. In the present study, participants were beginning to make those connections, but they were doing so from within an exploratory teacher education setting rather than through repeated classroom enactment. Their comments therefore reveal provisional and contextually

imagined integrations of technology, pedagogy, and content shaped by the opportunities and limits of the study design.

These findings point to an important implication for teacher education. Immersive experiences such as MathVR may create conditions for preservice teachers to notice new relations among content, pedagogy, and technology, but those relations do not become explicit automatically. In this study, structured reflection activities were important because they created space for participants to interpret what had happened in the headset and to translate that experience into pedagogical reasoning. The significance of the findings, then, is not that participants had or had not reached some final stage of TPACK. It is that the study captured how embodied technological experience, when paired with opportunities for reflection and discussion, can support the early formation of more integrated professional judgement about teaching mathematics with immersive tools.

This is the distinctive contribution of findings in response to Research Question Three. Emergent TPACK is generally understood to be situated, partial, and context-dependent (Koehler et al., 2013; Mishra & Koehler, 2006; Rosenberg & Koehler, 2015). Shulman's (1986) foundational account of pedagogical content knowledge and the integrative TPACK framework developed by Mishra and Koehler (2006) both predict that professional knowledge of this kind will be emergent and relational rather than fixed. What the present study adds is an empirical demonstration, within a domain where research on immersive VR in preservice mathematics teacher education remains scarce, that the emergence took recognizable and distinguishable forms. The participant profiles reported in Chapter Five give those forms names and analytical grounding in the data. They constitute the most specific answer available within this study's design to the question of how an immersive VR encounter with MathVR informs preservice

teachers' emergent TPACK, and they provide an initial foundation for future comparative and longitudinal investigation.

Implications for Teacher Education

The profiles suggest that preservice teachers encountering VR in a mathematics education context are likely to develop TPACK along more than one recognizable pathway, and that the pathway a given teacher candidate follows is connected to their affective and bodily engagement with the technology and to their incoming content security with the mathematical domain. A program design that treats preservice teachers as a homogeneous group, assumes that all participants will arrive at the same form of integrated professional reasoning, or omits the structured reflective activities that the cross-modal analysis identifies as necessary for TPACK integration, would be poorly matched to the heterogeneous reality that this study documents. The reflective architecture surrounding the immersion was not incidental. Across all four profiles, it was the condition under which the VR experience became TPACK.

Interpreting Research Question Three through the TPACK lens suggests that immersive mathematical activity, when followed by structured opportunities for writing and discussion, can make early connections among technology, pedagogy, and content more visible in preservice teacher learning. In this study, participants did not remain at the level of describing MathVR as a novel tool. Through focused freewriting and focus group discussion, several began to reinterpret their VR experiences in pedagogical terms. Nora, for example, moved from immediate enthusiasm during the VR session, including the comment that "kids would love this," to a more developed instructional position in which students would need some prior familiarity with vector ideas and a post VR board discussion to consolidate what they had noticed. Cassia likewise moved from describing real time visual change and physical interaction in the headset to arguing

that students should be able to revisit how a task was completed, so that success in the system would be connected to explanation and understanding. Adrian, in turn, emphasized that not all learners would adapt quickly enough to manipulate the tool independently, which led him to frame VR use in terms of support, pacing, and readiness. These examples suggest that firsthand immersive experience, when paired with guided reflection, can help preservice teachers begin to articulate how an emerging technology might function instructionally rather than encountering it only as a technical skill.

This matters for teacher education because the study placed participants in a situation where they had to interpret what the technology was showing mathematically and then consider what that experience might mean for students, rather than asking them merely to learn how to operate MathVR. In that sense, the pedagogical value of the session lay not only in the VR activity itself, but also in the reflective structures surrounding it. The freewriting and focus group phases created space for participants to translate embodied interaction into professional reasoning. Philip provides a useful example. In VR, he worked through projection by repeated manipulation and eventually described the projected relation as “the shortest distance between the tip of the vector and the plane.” In later reflection, he extended that experience into commentary about the value of labeling, self exploration, and visual support for student learning. What this suggests for teacher education is not that immersive experience automatically produces integrated teacher knowledge, but that it can serve as a productive starting point when paired with opportunities to interpret, question, and pedagogically reframe what happened in the headset.

At the same time, the unevenness of participants’ responses underscores the importance of developmental expectations. The study revealed different configurations of emerging

knowledge rather than one uniform level of TPACK across the group. Some participants, such as Philip and Kai, demonstrated relatively strong technological content related reasoning as they reflected on how MathVR made projection, spatial orientation, and vector relations more perceptible. Others, such as Nora, Adrian, and Cassia, showed more explicit technological pedagogical reasoning by focusing on student readiness, lesson sequencing, explanation, and follow up discussion. This interpretation aligns with work by Mishra and Koehler (2006), who define TPACK as knowledge that emerges through the interaction of technology, pedagogy, and content rather than through isolated competencies, and with Angeli and Valanides (2009), who argue that such knowledge is context bound and develops through experience rather than appearing all at once as a finished capacity. It also resonates with Saito and Lee's (2023) finding that teachers working with VR must reason simultaneously about technical setup, physical space, learner readiness, and instructional goals. In the present study, participants were beginning to make those connections, but they were doing so within an exploratory teacher education setting rather than through repeated classroom enactment.

Read together, the RQ3 findings suggest that teacher education can benefit from giving preservice teachers opportunities to experience MathVR as an emerging technology in ways that are immediately connected to content specific reasoning and pedagogical reflection. The significance of the findings is not that participants arrived at a final or complete form of TPACK, but that the study captured early moments in which embodied technological experience was translated into more integrated professional judgement about teaching mathematics. This is consistent with research on preservice teacher learning that describes technology integration as situated, developmental, and shaped by the specific instructional contexts in which teachers encounter tools and reflect on their use (Angeli & Valanides, 2009; Rodríguez, 2024; Saito &

Lee, 2023). In this study, MathVR created a setting in which those early integrations became visible. Participants began to connect what the tool allowed them to see and do, what students might need in order to learn with it, and what kinds of mathematical understanding it might support or obscure. For teacher education, that is an important finding because it suggests that MathVR can function not only as spaces for student learning, but also as spaces in which preservice teachers begin to form more contextually grounded judgements about technology rich mathematics teaching.

Cross Cutting Synthesis

When the findings from analyses framed by all three research questions are considered together, a consistent pattern becomes visible. Preservice teachers' engagement with MathVR unfolded across three linked processes: embodied interaction in the headset, reflective description of the tool's affordances and constraints after the activity, and the emergent TPACK visible in their pedagogical imagining about future classroom use. What this study shows is that participants' mathematical noticing often began through movement, spatial repositioning, and direct manipulation of vectors, but became more pedagogically and conceptually explicit through focused freewriting and discussion. Across the dataset, the most fully articulated insights were usually not produced during headset use alone. They emerged when embodied experience was followed by opportunities to explain, compare, question, and reinterpret what had happened.

A second cross cutting pattern concerns variability. Participants displayed different forms of learning and different levels of developing professional knowledge, and the four profiles reported in Chapter Five give that variability a structure. What differed across participants was not how much TPACK each had accumulated but which configuration emerged, and the configuration tracked both what participants brought to the session and how they inhabited it.

Read alongside the findings for the first two research questions, this suggests that responses to immersive mathematics environments are uneven, situated, and shaped by prior knowledge and by the reflective structures surrounding the experience rather than by the properties of the technology itself.

Seen in relation to the study's theoretical framing, these patterns suggest that embodied mathematical activity in VR became most educationally significant when it was connected to reflection and pedagogical interpretation. The contribution of this study, therefore, is not only that MathVR made vector relationships more visible, but that it provided a setting in which preservice teachers began to connect embodied experience, conceptual reasoning, and instructional judgement in observable ways. This point provides the basis for the contributions discussed in the next section.

Contributions of the Study

This study contributes to scholarship in four interrelated ways.

First, the study contributes theoretically by naming and specifying a distinction that embodied mathematics education research has largely left implicit. Work on gesture, motion, and spatial action has established that mathematical reasoning is bodily, but it has tended to treat immersive environments as a further instance of that general claim rather than as a change in kind. This study distinguishes representational embodiment, in which the learner acts upon a mathematical representation held at a distance, from immersive first-person embodiment, in which the learner acts within the represented space at body scale. The participants' accounts give that distinction empirical content. They did not report seeing vectors more clearly. They reported walking around a projection, positioning themselves relative to a plane, and inspecting a resultant from vantage points a diagram cannot offer, and the four profiles differed according to

the stance each participant took up within that space rather than according to whether they engaged their bodies at all. Naming the difference matters because it specifies what immersive virtual reality offers that dynamic but screen-bound technologies do not, and it does so in terms that are open to empirical test rather than to promotional claim.

Second, the study contributes to research on immersive virtual reality in mathematics education by focusing on preservice teachers as learners of mathematics and as future teachers reasoning about how mathematics might be taught through immersive technology, rather than on K-12 student outcomes. Across the VR sessions, focused freewriting, and focus group discussions, participants went beyond reporting that MathVR was engaging or innovative: they identified specific representational affordances that they associated with clearer access to vector concepts, especially direction, magnitude, projection, and spatial relationships. They referred to direct manipulation, real time visual change, multiple viewpoints, and the ability to inspect vector relations in three dimensions. In this respect, the study extends research showing that immersive environments can support mathematical reasoning by making structures available through movement, perception, and dynamic representation rather than through static symbolic display alone (Beauchamp & Walkington, 2024; Johnson-Glenberg, 2018; Walkington et al., 2021). Its contribution is therefore a more fine-grained account of how preservice teachers themselves interpreted the conceptual value of immersive mathematical representation while engaging with vector operations.

Third, the study contributes conceptually to TPACK scholarship by showing that, in this context, preservice teachers' technological, pedagogical, and content related reasoning did not appear as a finished or uniform body of knowledge. Instead, it emerged in partial, uneven, and context dependent ways across participants and across data sources. Some participants showed

stronger technological content related insight, especially when reflecting on how MathVR made vector relationships more visible. Others showed stronger technological pedagogical reasoning, especially when discussing preparation, scaffolding, pacing, classroom feasibility, and post VR consolidation. More integrated reasoning was usually articulated not during headset use alone, but when embodied experience was followed by opportunities for focused writing and discussion. In that sense, the study offers a situated interpretation of TPACK as developing through embodied encounter, reflection, and pedagogical imagination rather than as a stable capacity that teachers either possess or lack (Angeli & Valanides, 2009; Mishra & Koehler, 2006). This contributes a more nuanced understanding of how preservice teachers begin to think pedagogically with immersive technology in relation to specific mathematical content.

Fourth, the study contributes methodologically by showing the value of examining teacher learning in immersive environments through a multimodal design that connects embodied interaction with later reflective articulation. Rather than relying only on what participants said after the experience, the study also analyzed what they noticed, how they moved, how they manipulated vectors, how they responded to the environment in real time, and how those experiences shaped their later reflections. By bringing together VR session data, focused freewriting, and focus group discussions, the study makes visible dimensions of teacher learning that would be much harder to capture through interviews or surveys alone. In this respect, it demonstrates the value of combining a multimodal analytic approach for embodied interaction with a conceptual lens for interpreting emerging professional knowledge in immersive mathematics learning contexts (Angeli & Valanides, 2009; Walkington et al., 2024a).

Overall, these contributions show that the significance of the study lies in how participants' pedagogical thinking about teaching vectors with immersive technology began to

develop across embodied interaction, reflection, and discussion, rather than in whether they liked or disliked MathVR. For teacher education, this is important because it suggests that immersive environments such as MathVR may be most productive not when they are treated as stand alone technological experiences, but when they are embedded within opportunities for interpretation, pedagogical questioning, and content specific reflection. That way, the study contributes a more detailed and contextually grounded account of how immersive virtual reality may support the early formation of professional judgement in mathematics teacher education.

Implications for Mathematics Teacher Education

The findings suggest several implications for mathematics teacher education, particularly for methods work related to spatial and representationally demanding topics such as vectors. First, immersive experiences such as MathVR appear to be most productive when preservice teachers engage with them first as learners and then reflect on them as future teachers. In this study, many of the most explicit pedagogical insights did not arise during headset use alone, but during focused freewriting and discussion after the immersive activity. This suggests that the educational value of VR in teacher education may lie not only in the experience itself, but in the structured reflection that follows it.

Second, the findings suggest that immersive mathematical activity should not be treated as self sufficient. Participants repeatedly identified the need for onboarding, guided entry, and post activity consolidation. Nora, Nili, Violet, and others described the need for prior familiarity with vector ideas, tutorial support, or follow up discussion to help connect embodied interaction to explicit mathematical understanding. For mathematics teacher education, this implies that immersive tools such as MathVR may be most useful when embedded within a broader sequence that includes preparation, exploration, and debrief rather than used as isolated experiences.

Third, the findings suggest that preservice teachers' developing knowledge about immersive technology is uneven and context dependent. Participants did not all move toward integrated technological, pedagogical, and content related reasoning in the same way. Some focused more strongly on representational affordances; others foregrounded learner support, pacing, safety, and accessibility. This suggests that teacher education should treat immersive technology integration as developmental and interpretive rather than assuming that a single exposure will generate stable or fully integrated professional knowledge. In that sense, MathVR may be valuable less as a standalone innovation than as a setting in which preservice teachers can begin to make more contextually grounded judgements about technology rich mathematics teaching.

Limitations of the Study

This study should be interpreted within the boundaries of its design and context. It was conducted in a single teacher education setting with a small group of preservice mathematics teachers at one institution. The findings therefore speak most directly to how this particular group experienced, interpreted, and reflected on MathVR within a structured teacher education session.

A related boundary concerns the cultural and curricular specificity of the study. Participants were drawn from a single teacher education program in Ontario, and their engagement with vector content was shaped by the particular sequencing of vectors across the Ontario secondary mathematics and physics curricula described in Chapter Two. The findings are therefore situated within this institutional, provincial, and curricular context, and the extent to which similar configurations of emergent TPACK would appear among preservice teachers working within different curricula, different cultural contexts, or different national teacher

education systems remains an open empirical question. This is not a flaw in the design so much as a feature of a bounded case study, but it does mean that the participant profiles developed in Chapter Five should be read as a starting typology rather than as claims about preservice teachers in general. Comparative research across institutional and cultural contexts would help clarify which aspects of the findings are specific to this setting and which extend more broadly.

A further boundary concerns the depth of demographic information collected about participants. The study gathered information about participants' program stream, teachable subjects, and expected graduation year, but did not systematically collect data on participants' age, gender, cultural or linguistic background, or prior experience with immersive technology beyond what individual participants volunteered in conversation. Richer demographic information might have allowed the participant profiles to be examined in relation to a wider set of background factors, and its absence here limits the interpretive reach of the profiles to the dimensions, affective stance, attentional orientation, and content security, that the available data could support.

A third boundary concerns the content focus of the study. The investigation centred specifically on three-dimensional vectors, a topic that is especially compatible with spatial visualization and embodied interaction. Because MathVR was used here in relation to vectors, the findings cannot simply be assumed to apply in the same way to other areas of mathematics that may rely less directly on spatial reasoning or immersive manipulation.

A fourth boundary concerns the nature of the activity itself. Participants engaged with MathVR as learners and then reflected on its pedagogical possibilities, but they did not enact VR based lessons with secondary students in classroom settings. For that reason, the study illuminates preservice teachers' embodied experiences, reflections, and anticipations about

teaching with MathVR, rather than observed classroom practice. Questions of how such ideas would unfold in real lessons, across diverse student groups, under ordinary school conditions, remain beyond the scope of the present study.

Finally, the study was shaped by the practical conditions of immersive VR use. Participants encountered interface learning demands, occasional uncertainty about controls and task completion, and, in some accounts, physical discomfort. These conditions were part of the phenomenon under study, since they influenced how participants experienced MathVR and how they later judged its pedagogical value. At the same time, they also mark an important boundary of interpretation, because participants' reflections were formed in relation to a specific platform, a specific session design, and a specific level of familiarity with immersive VR.

Directions for Future Research

The findings point to five important directions for future research. One direction is to examine how different forms of scaffolding before, during, and after immersive activity shape learners' movement from embodied interaction to explicit mathematical explanation. The present study suggests that participants often noticed mathematical relationships through action first and articulated them more clearly only after reflection, discussion, or guided interpretation. Future studies could therefore compare forms of support such as prebriefing, in app prompts, peer dialogue, teacher questioning, and post VR consolidation in order to clarify which combinations most effectively help learners connect embodied exploration with formal mathematical reasoning.

A second direction is to investigate how preservice teachers or practicing teachers actually enact VR based mathematics lessons in classroom settings. The present study showed that participants were already reasoning about issues such as pacing, onboarding, monitoring,

safety, limited headset access, and the need to connect immersive activity to board work, notation, and discussion. Classroom based research is needed to examine how these pedagogical concerns play out in practice and how teachers manage the relationship between immersive activity and more conventional mathematical representation, explanation, and assessment.

A third direction concerns the attentional data that could be captured through more recent hardware. The Meta Quest 2 headsets used in this study do not support eye tracking, so the analysis of where participants directed their attention relied entirely on behavioral evidence such as head and body orientation, gesture, and verbal commentary. Headsets equipped with eye tracking, together with computational methods for predicting visual attention from gaze and scene features, could complement this behavioral evidence with a more direct record of where participants looked while reasoning about a vector configuration, and could help test whether the attentional orientations proposed in the four participant profiles correspond to measurable differences in gaze behaviour.

A fourth direction concerns the integration of generative artificial intelligence within immersive mathematics environments. Several participants in this study identified the absence of immediate feedback or guidance as a constraint, proposing tutorials, demonstrations, or in-environment supports that a human instructor would otherwise need to provide. An AI-supported scaffold embedded within MathVR, capable of offering prompts, answering procedural questions, or inviting reflection at the moment a participant appears uncertain, could address this constraint directly and would be a natural extension of the present design.

A fifth direction concerns haptic feedback. The controllers used in this study provided no tactile information about contact, resistance, or surface, so participants' sense of manipulating a vector or plane was visual and auditory rather than tactile. Haptic devices capable of guiding

hand movement, rather than only signaling contact through vibration, represent an emerging direction for technological knowledge development within immersive mathematics education and could be examined for their effect on the kind of sensorimotor grounding that the theoretical framework in Chapter Three associates with embodied learning.

Conclusion to Chapter Six

This chapter has interpreted the findings of the study in relation to the three research questions and the broader scholarship on embodied learning, immersive technology, and teacher knowledge. Across the discussion, the findings suggest that preservice teachers' experiences with MathVR were shaped not only by what they could see and do in the virtual environment, but also by how they later reflected on those experiences as future teachers. Embodied interaction supported mathematical noticing, reflective discussion helped participants articulate pedagogical implications, and the synthesis through TPACK showed that their professional knowledge was emerging in partial, uneven, and context dependent ways rather than as a finished or stable capacity.

Overall, the discussion has shown that MathVR was experienced by participants as a tool with genuine conceptual and pedagogical promise, but also as one whose value depended on scaffolding, interface transparency, accessibility, and integration within a larger instructional sequence. The chapter has therefore moved from description to interpretation by clarifying what these findings contribute to understanding preservice teacher learning with immersive mathematics technology. With this foundation in place, the thesis closes where it began: with preservice teachers' embodied interaction with MathVR, their reflective descriptions of its affordances and constraints, and the emergent TPACK that those experiences made visible, as the

three moments through which immersive virtual reality may come to matter in mathematics teacher education.

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Appendix A. Focused Freewriting Activity

Objective

To encourage participants to reflect individually and express their insights regarding the use of Math VR for teaching and learning vector concepts. The freewriting prompts guide participants to consider the affordances, constraints, and pedagogical possibilities of using VR technology in teaching vector operations.

Instructions to Participants

1. **Introduction to the Activity (5 minutes)**

- Thank you for participating in this focused freewriting activity. The purpose of this activity is to give you an opportunity to reflect on your experiences with MathVR and to capture your thoughts in writing. You will have 15 minutes to respond to the prompts provided. There are no right or wrong answers, and you are encouraged to write freely and honestly. Your responses will be used to help us understand your individual perspective before we engage in group discussions.

- Please write continuously for the full 15 minutes. Don't worry too much about spelling or grammar; what's important is capturing your thoughts as they come to you.

2. **Freewriting Prompts (10 minutes)**

   Prompt: Reflect on the technological aspects of Math VR. What were the affordances (benefits) and constraints (limitations) you encountered? How did the technology enhance or challenge your understanding of vectors?

Data Collection

Participants' freewriting responses were collected at the end of the activity, with each response linked to its respective participant. This allows for a more nuanced analysis, as it enablesd me to trace insights back to individual participants, which is essential for understanding each pre-service teacher’s unique perspective. During the analysis phase, the responses were anonymized to ensure confidentiality. The responses then underwent thematic analysis to identify common themes and unique insights, capturing participants' perceptions of the technological, pedagogical, and content-related dimensions of using VR for teaching vector concepts.

Appendix B. Focus Group Protocol

Introduction

Thank you so much for your participation and valuable contributions to this study. Your involvement, especially in completing the focused freewriting activity, has been instrumental in providing us with insightful data regarding your teaching practices and your perspectives on immersive virtual reality (VR).

Before we dive into our focus group discussion, I'd like to take a moment to express my appreciation for your time and effort. Your engagement and feedback are essential in helping us understand the impact and potential of VR in educational settings, especially in the context of mathematics instruction.

This focus group session is an opportunity for us to engage in a collaborative and reflective dialogue.

The purpose of today's discussion is twofold: Firstly, it's a platform for all of us to share our experiences, insights, and thoughts about using MathVR app in a VR setting. This is a chance for you to voice your perceptions, challenges, successes, and any transformative moments you experienced during the use of this technology.

Secondly, this session is designed to create a supportive environment for collective growth and development. As we discuss and reflect, we aim to deepen our understanding not only as users of VR but also as critical thinkers about its role and potential in mathematics instruction. Your insights will be invaluable in shaping a better understanding of how VR can be integrated into educational practices effectively.

As we proceed, please feel free to express your thoughts openly and honestly. There are no right or wrong answers here; your unique experiences and perspectives are what make this discussion truly enriching. Let's try to build on each other's ideas and experiences.

With that said, let's begin our discussion. The first question we will explore today is about your direct experience with using MathVR app in the VR environment.

Exploring Experiences with Immersive Virtual Reality

1. Let's build a definition of immersive virtual reality together. Based on our experience today, what key elements do we think characterize a VR experience?
2. If someone has used VR before, could you explain how MathVR differs from other digital technologies or educational tools you've used? What makes it unique or engaging?

Virtual Embodied Experience in VR

3. Now that you've experienced VR using MathVR, can you describe your initial reactions? What stood out to you the most during this experience?
4. Before the session, you might have had certain expectations about VR. How did your actual experience compare with those expectations?
5. How did the physical aspect of using VR, like moving and interacting in a virtual space, influence your understanding of vector concepts? Did it enhance your learning experience?
6. In what ways did the immersive nature of VR impact your engagement with the learning material? Did you feel more connected to the concepts of vectors?

7. Let's discuss the 'feel' of teaching Mathematics in a VR space. How does the embodied/physical experience of being in a VR classroom differ from a physical classroom? What are the implications for teaching and learning Mathematics?
8. How did using MathVR feel in terms of embodying the concept of vector? Did you experience any moments where the physical interaction with VR changed your understanding of vectors?
9. Can anyone share how they felt differently as a learner while being in this immersive environment?

TPACK Questions

10. What are some new insights you've gained about teaching Mathematics that are unique to the VR experience?
11. Was there any specific moment during your experience in the immersive environment of VR with MathVR where you had a sudden realization or 'aha' moment that enhanced your understanding of vector concepts?
12. How does your experience of learning vectors through VR compare to more traditional methods (using body language, white/black board, PowerPoint, video presentation etc.)? Were there any differences in how you conceptualized vectors? Did VR offer any new perspectives or insights?
13. Given your experience, how do you envision integrating VR technologies like MathVR into your future teaching practices? What potential do you see?

14. In your opinion, what are the potential benefits of using VR, specifically MathVR, to teach the concept of vector?
15. How could you use this technology to teach students the concept of vector?
16. How do you perceive the impact of immersive virtual reality on your teaching methods and course design? How do you think these factors influence the pedagogical design of your course or subject area?
17. How do you anticipate students will interact with vectors through VR? What pedagogical benefits do you expect from this interaction?
18. Do you feel that VR, particularly MathVR, enhanced your understanding of vector concepts? How might this influence your teaching strategies?
19. Do you think VR can enhance students' understanding of the concept of vector compared to traditional teaching methods?
20. Do you think using MathVR helped to improve your understanding of the concept of vectors through visualization? If so, how?
21. What feedback do you have about the MathVR application in terms of user-friendliness, functionality, and educational value?
22. What was your experience using MathVR? How did you find the technology to be effective in teaching the concept of vector?
23. Do you think the technology was easy to use and understand? Would you feel comfortable using it in your future teaching practice?
24. Based on your experience, what do you perceive as the main benefits and challenges of using VR, specifically MathVR, for teaching complex subjects like vectors? How can we overcome the challenges?

25. Looking back at your experience with using MathVR to teach the concept of vector, do you have any final thoughts or insights you'd like to share about how this technology could impact teaching and learning in mathematics?

End of Session

Appendix C. TPACK Evidence Matrix

Note: This section does not assume that all preservice teachers will demonstrate every TPACK dimension within a single data source. VR interaction data primarily captures enacted technological and content related knowledge, while pedagogical dimensions may remain implicit or absent for some participants. Where evidence for specific TPACK domains is limited or unavailable in the VR transcripts, this absence is treated as analytically meaningful and addressed through cross modal synthesis with freewriting and focus group data. This approach foregrounds entanglement across modalities rather than completeness within any one dataset.

Table 16

TPACK Evidence Matrix Based on VR Experience

Participant	Content Knowledge (CK)	Pedagogical Knowledge (PK)	Technological Knowledge (TK)	Technological Content Knowledge (TCK)	Technological Pedagogical Knowledge (TPK)	Technological Pedagogical Content Knowledge (TPACK)
Nora	Confident manipulation of vectors with attention to direction and magnitude during addition and subtraction;	Explicit learner oriented comment, "Kids would love this," indicating attention to student engagement and classroom appeal.	Noticed system cause and effect, "Whenever I press this one, it generates?" used menus and task board to guide actions and	Used MathVR functions to carry out vector multiplication, projection, and manipulation of vectors anchored to cubes while	Linked a system feature to learner engagement while interacting with the Generate menu and tracking task progression,	Integrated content actions with technology use and learner oriented framing during task work, combining vector operations and

	labeled operation outcome, "Multiplication gives multiple vectors."		monitor progress.	observing outcomes in real time.	"Kids would love this."	interface driven feedback with the classroom reference, "Kids would love this."
Adrian	Named and sequenced vector operations including projection and projection onto normal, "Project on to normal," "Spawn a plane. Copy a plane normal. Project onto normal."	No explicit evidence in VR data. Indirect evidence limited to goal setting and progress monitoring, "These are the tasks," "I did three out of the five."	Activated headset, navigated menu, mapped interface location, read interface text and tool labels aloud, monitored system state via task panel.	Used system tools to generate vectors and complete operations through interface affordances, including plane and normal related projection within the task sequence.	No explicit evidence in VR data. Indirect evidence limited to self regulation through interface structure and task panel monitoring.	Partial evidence. Integration is enacted through technology guided sequencing of mathematically meaningful operations and progress monitoring, but no learner or teaching oriented framing appears in VR data.
Amaya	Partial and uncertain understanding of subtraction, "I do not know how to subtract if it has	No evidence observed in VR data.	Limited and guidance dependent use of reset, login, vector tool, and subtract tool;	Emerging evidence through placing vectors in the subtraction structure and manipulating VR	No evidence observed in VR data.	No evidence observed in VR data. Engagement remains procedural and facilitator guided

	two." Evidence limited to subtraction attempts.		relied on repeated facilitator prompts.	objects representing subtraction, with attention to object positions and changes after reset.		without integrated pedagogical framing.
Nili	Worked with vector direction, length, and placement during operations, "Bring the vector here and shorten the length of this one."	No evidence observed in VR data.	Used Generate menu, system level panels, pause and resume controls, and monitored task status, "Resume," "Okay. Should I quit now?"	Used MathVR tools to generate vectors, adjust length, and enact subtraction through the subtraction cube with visible resulting arrow.	No evidence observed in VR data.	No evidence observed in VR data. Successful task completion occurred without learner oriented or teaching oriented talk.
Violet	Completed addition, subtraction, projection, and plane related tasks; enacted spatial reasoning during manipulation and affirmed	No explicit evidence in VR data. Responsiveness to instructional guidance appears through following sequencing support, "Subtract, projection…"	Procedural understanding of system consequences through reset event and recovery; continued task work after workspace reset.	Strong evidence through embodied manipulation of vectors on a plane and successful projection and subtraction with	Indirect evidence through the pedagogical implications of reset disrupting task flow and requiring reorientation support; not	Partial evidence. Integration is enacted through coordinated action during projection and subtraction, but pedagogical reasoning and learner framing

	alignment, "That's cool."			visual confirmation.	participant articulated.	are not verbalized in VR data.
Maren	Engaged with addition, subtraction, and projection while making sense of outcomes, "That makes sense," and noted uncertainty, "I don't remember what projecting a vector is."	No evidence observed in VR data	Active tool use across delete, reset, vector, subtract, projection, plane, and copy plane normal; responded to completion feedback, "Congrats all tasks complete."	Used VR tools to explore relationships by stretching, rotating, and projecting vectors while observing changes, "Oh, interesting."	No evidence observed in VR data.	No evidence observed in VR data. No teaching or learner oriented framing appears in VR data.
Cassia	Named and completed complex operations including projection onto normal and shortest distance, "The length of the thick yellow line is the shortest distance…" "Oh,	No evidence observed in VR data.	Managed workspace by moving task board, reorganizing tools, clearing area, and recovering from tool confusion, "Where'd my hammer go?"	Strong evidence through using add, subtract, project, plane tools, and normal related tasks to produce and interpret mathematical visuals and distances.	No explicit evidence in VR data. Interface structure and feedback supported task flow but was not framed as teaching strategy.	Partial evidence. Integrated engagement with content through technology and task structure is clear, but learner or teaching framing is absent in VR data.

	it's the height of the vector."		"Oh, it turns green."			
Karim	Partial content knowledge with recall attempts, "I don't remember what projection even means," "How do you project onto a normal just mathematically again?"	No evidence observed in VR data.	Used vectors, planes, projection boxes, deletion hammer, and task board monitoring; inferred system behaviour through action outcomes, "I think it deletes once it gets too far."	Used VR tools to add vectors, project onto vectors and plane normals, and adjust placement to satisfy task conditions, "I put a vector in a projection and finished."	No evidence observed in VR data.	Partial evidence. Coordinated tool use with task structure and mathematical actions is present, but teaching or learner framing is absent in VR data.
Kai	Partial content knowledge with stated gap, "I don't remember what projection even means," attempted projection alignment but mispositioned elements and left	No evidence observed in VR data.	Used task board, green checkmarks, planes, projection boxes, deletion hammer, and viewpoint shifting to inspect alignment.	Used VR tools to add vectors, create projections, spawn planes, and adjust length and position through repeated trial and error.	No evidence observed in VR data.	No evidence observed in VR data. Integration remains technological and procedural without articulated pedagogy, and task completion

	final task incomplete.					was incomplete in VR data.
Philip	Strong verbalized reasoning about vector relations, “This plus this gives this,” “If they make exactly the same, it should be zero,” projection as “the shadow that is cast,” and shortest distance explanation read aloud.	No evidence observed in VR data.	Oriented to task board, used Generate menu and controls, searched for tools, used “Clear” and “Clean,” and read system explanation aloud.	Used add, subtract, projection, plane tools to produce visuals aligned with conceptual reasoning and interpreted system generated explanation.	Minimal evidence via usability focused suggestions, “It’ll be nice to have different colors,” “is there a way to label this?” not framed as teaching.	No evidence observed in VR data. Deep content engagement occurs through technology, but teaching and learner framing is not present in VR data.

Appendix D. Ethics Approval Document

14/11/2024

Université d'Ottawa
Bureau d'éthique et d'intégrité de la recherche

University of Ottawa
Office of Research Ethics and Integrity

CERTIFICAT D'APPROBATION ÉTHIQUE | CERTIFICATE OF ETHICS APPROVAL

Numéro du dossier / Ethics File Number	S-11-24-11007
Titre du projet / Project Title	Pre-service Teachers' Perception of Integrating Immersive Virtual Reality (VR) in Mathematics Pedagogy
Type de projet / Project Type	Thèse de doctorat / Doctoral thesis
Statut du projet / Project Status	Approuvé / Approved
Date d'approbation (jj/mm/aaaa) / Approval Date (dd/mm/yyyy)	14/11/2024
Date d'expiration (jj/mm/aaaa) / Expiry Date (dd/mm/yyyy)	13/11/2025

Équipe de recherche / Research Team

Chercheur / Researcher	Affiliation	Role
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Conditions spéciales ou commentaires / Special conditions or comments

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14/11/2024

Université d'Ottawa
Bureau d'éthique et d'intégrité de la recherche

University of Ottawa
Office of Research Ethics and Integrity

Le Comité d'éthique de la recherche (CÉR) de l'Université d'Ottawa, opérant conformément à l'*Énoncé de politique des Trois conseils* (2014) et toutes autres lois et tous règlements applicables, a examiné et approuvé la demande d'éthique du projet de recherche ci-nommé.

L'approbation est valide pour la durée indiquée plus haut et est sujette aux conditions énumérées dans la section intitulée "Conditions Spéciales ou Commentaires". Le formulaire « Renouvellement ou Fermeture de Projet » doit être complété quatre semaines avant la date d'échéance indiquée ci-haut afin de demander un renouvellement de cette approbation éthique ou afin de fermer le dossier.

Toutes modifications apportées au projet doivent être approuvées par le CÉR avant leur mise en place, sauf si le participant doit être retiré en raison d'un danger immédiat ou s'il s'agit d'un changement ayant trait à des éléments administratifs ou logistiques du projet. Les chercheurs doivent aviser le CÉR dans les plus brefs délais de tout changement pouvant augmenter le niveau de risque aux participants ou pouvant affecter considérablement le déroulement du projet, rapporter tout évènement imprévu ou indésirable et soumettre toute nouvelle information pouvant nuire à la conduite du projet ou à la sécurité des participants.

The University of Ottawa Research Ethics Board, which operates in accordance with the *Tri-Council Policy Statement* (2014) and other applicable laws and regulations, has examined and approved the ethics application for the above-named research project.

Ethics approval is valid for the period indicated above and is subject to the conditions listed in the section entitled "Special Conditions or Comments". The "Renewal/Project Closure" form must be completedfour weeks before the above-referenced expiry date to request a renewal of this ethics approval or closure of the file.

Any changes made to the project must be approved by the REB before being implemented, except when necessary to remove participants from immediate endangerment or when the modification(s) only pertain to administrative or logistical components of the project. Investigators must also promptly alert the REB of any changes that increase the risk to participant(s), any changes that considerably affect the conduct of the project, all unanticipated and harmful events that occur, and new information that may negatively affect the conduct of the project or the safety of the participant(s).

Riana MARCOTTE
Responsable d'éthique en recherche / Protocol Officer
Pour/For **Barbara GRAVES** Président(e) du/ Chair of the **Comité d'éthique de la recherche en sciences sociales et humanités / Social Sciences and Humanities Research Ethics Board**

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