RELIABILITY AND PRODUCTION COSTS OF INTERCONNECTED SYSTEM IMPACT OF LOAD MANAGEMENT AND JOINT OWNERSHIP OF GENERATION

by

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ABSTRACT

In generation expansion planning, reliability evaluation as well as economic analysis of all feasible plans, is essential in order to identify a plan which impacts on the utility as a whole in the most favourable manner. Among utilities the rationale of interconnection is well understood.

This thesis presents an efficient technique to evaluate the reliability of two interconnected systems. The technique is based on the segmentation method. The method is compared with the existing methods in terms of accuracy and computational efficiency.

This thesis also presents a methodology to evaluate the production costs of two interconnected systems. The method utilizes exact as well as approximate computational techniques, and the results are compared. The proposed methodology is applied to evaluate the expected energy generation, unserved energy as well as the economic benefits of two interconnected systems.

The impact of different load management strategies, joint ownership of generation, time zone difference and increased forced outage rates of the generating units on reliability as well as on production costs of two interconnected systems is also investigated.

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INTRODUCTION

1.1 BACKGROUND AND MOTIVATION

Electricity demand follows customer through breakfast,work,and dinner, over weekends,through air conditioned summers, and electrically heated winters. But while it takes customers only a flick of a switch to create demand, it takes far more than that for utilities to meet demand. Utilities make plans long before to meet customers' demands.

Generation planning begins with estimates of peak demands and associated electrical energy consumption [1]. After identifying the need for generating capacity additions, the planner develops a number of feasible expansion alternatives on the basis of

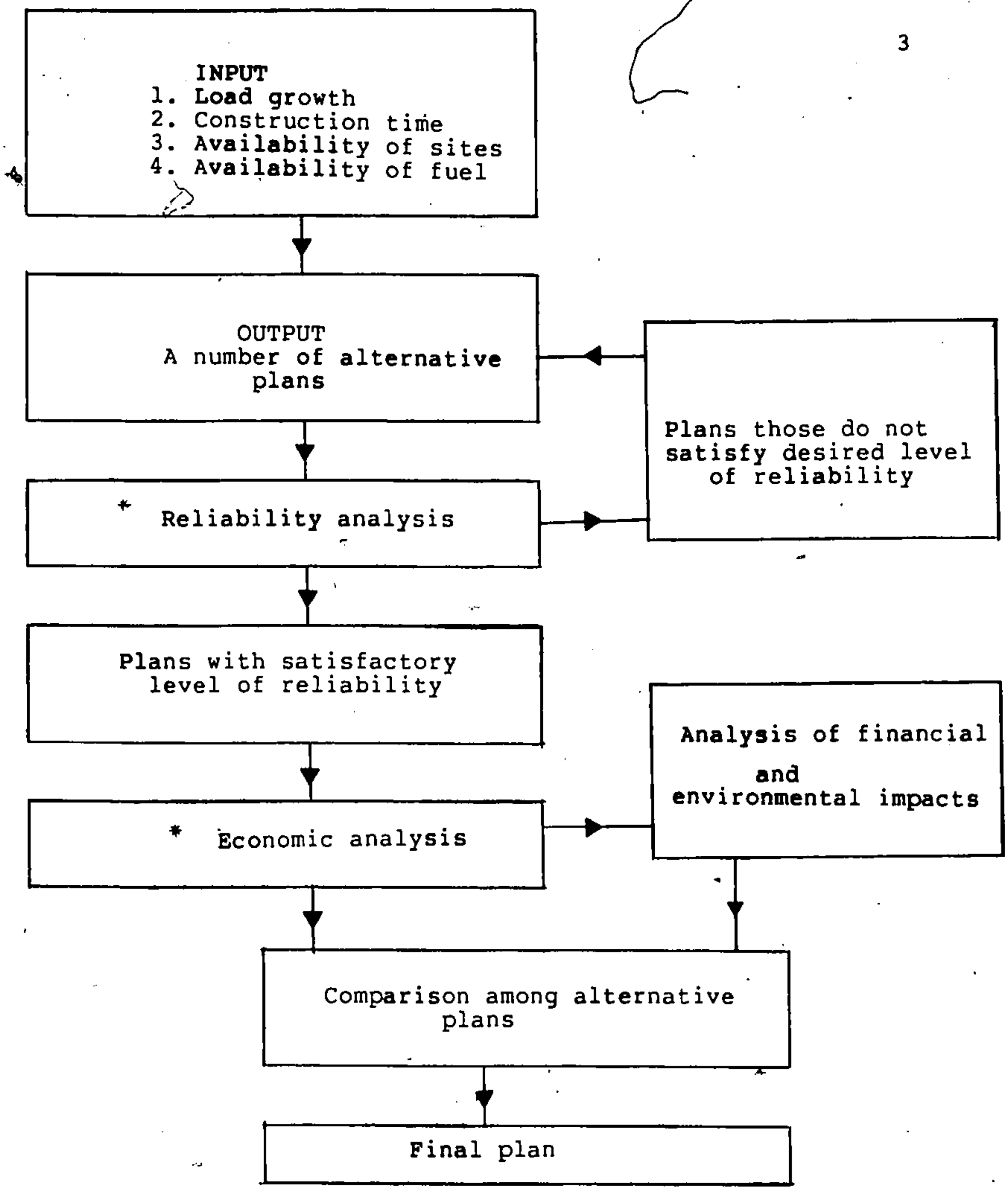
1. Load growth,
2. Construction time,
3. Availability of sites,
4. Availability of fuel.

Given these alternative plans, it is common to subject each plan to a detailed reliability analysis to ensure that all plans satisfy the desired level of reliability [1]. Plans that do not meet the reliability criteria are eliminated or appropriately modified, and plans which satisfy the required reliability level are evaluated on the basis of economics. For each potential plan, financial and environmental impacts are analyzed. Finally, the alternative plans are compared in order to identify the one that impacts on the utility as a whole in the most favourable manner. In Figure 1 the planning process is depicted in the form of block diagrams.

Another approach [2], used to some extent in the power industry, is a semiautomated procedure in which capacity requirements are determined over the horizon period using analytical methods. With this information, various expansion plans can be determined by varying the type and the timing of unit additions.

Obviously, it is an enormous task to discuss in detail the entire planning process. Therefore, it is chosen to concentrate on the reliability and the economic analysis of a generation expansion plan devised by the planner.

The reliability of an electric power supply system is defined [3] as the probability of providing the users with continuous service of satisfactory quality. The quality constraint refers to the requirement that the frequency and the voltage of the power supply should remain within prescribed tolerances. Several measures have been devised



Generation expansion planning process

to evaluate the reliability performance of a given expansion plan. The simplest and most common of all is the loss of load probability (LOLP) [4,5].

The main factors which enter into the cost analysis of a given plan are:

1. Capacity cost,
2. Production cost,
3. Operating and maintenance cost.

Besides these three major factors, timing of unit additions is also important in cost analysis. The evaluation of the energy production cost is by far the most complex part of cost analysis associated with a particular expansion plan.

The reliability and the economic evaluation of the interconnected systems is different from that of a single system. Although the simplest way of evaluating the interconnected systems is to consider them as a single system, however, there are very good reasons for keeping the identities of the constituent systems separate. The reasons are the following.

1. A utility is primarily interested in the benefits that its own system can obtain from interconnection.
2. The ties forming the interconnections are usually limited in capacity, and in addition, are subject to failures.

3. The load characteristics in the various interconnected networks may be different.
4. There may be some firm agreement between the interconnected systems to provide capacity assistance during certain times.

Interest in power system reliability became evident about 1933. The first significant contributions appeared in 1947 with the introduction of early computers. Some of the methods in use at the present time are based on the basic concepts proposed in 1947 by Calabrese [6], Lyman [7], Seelye [8], and by Loane and Watchorn [9]. The development of power system reliability methods and techniques has been documented in an ever-increasing number of publications. The orientation of the vast literature that has accumulated over the years is provided in references [10], [11] and [12].

In 1967, Baleriaux et al. [13] introduced a probabilistic simulation technique for estimating the expected fuel cost or the production cost. Booth [14] incoporated a dynamic programing algorithm with the method [13] to evaluate a generation expansion plan. Jenkins and Joy [15] used the method [14] to estimate the operating cost of the generating units assuming constant heat rate curves. Zahavi et al. [16] introduced the incremental loading of units rather than merit order loading. Later Schenk [17] and Rau et al. [18] introduced the cumulant method as a fast method for probabilistic simulation by approximating the discrete distribution as a continuous function through the Gram-Charlier series expansion. Recently Schenk et al. [19] proposed the segmentation method to evaluate the production cost as well as loss of load probability (LOLP). The segmentation method avoids the use of series expansion but still retains a similar computational efficiency as the cumulant method.

The above mentioned works deal with single utility systems only. In 1963, Cook et al. [5] proposed the basic method for evaluating the LOLPs of two interconnected systems. The authors calculate the exact probability outages of each system and multiplying them together a two dimensional array is obtained which carries the information of simultaneous outages in each system. The LOLPs of two interconnected systems are obtained by summing the appropriate zones of the array. In [5] load diversity is considered by evaluating the LOLP for three different conditions,

1. Time of system 1 peak,
2. Time of system 2 peak,
3. Time of combined systems peak,

and taking the maximum of these probabilities as the daily LOLP of the interconnected system. This approach essentially takes into account the correlation of load of two systems externally to the model.

In 1972, Vassell and Tibberts [20] introduced the concept of capacity margin to determine any deficiencies or surpluses by utilizing a 'S' shaped cumulative distribution. In [20], the two dimensional capacity-margin probability array to evaluate the reliability of two interconnected systems is used. The method does not consider the correlation among the loads of the interconnected systems. Billinton and Singh [21] proposed the frequency and duration approach to evaluate the reliability of two interconnected systems considering load correlation. Additional work in multiarea reliability evaluation has been reported by Pang et al. [22] as well as by Pang and Dhar [23]. In all cases [5,20-23], a peak load model is used.

In 1982, Rau et al. [24] proposed a computationally fast method to evaluate the LOLPs of two interconnected systems considering load correlation between the two systems. The method avoids a peak load model by using an hourly load model. Although the method is computationally fast and utilizes a realistic load model, it is based on the bivariate Gram-Charlier series expansion which approximates the discrete distribution as a continuous one.

As noted earlier, the techniques for estimating the production cost of single utility system are reasonably well developed; however, methods for estimating the production costs for interconnected systems are not that developed. Recently, only the techniques to evaluate the economic benefits of interconnection from production cost savings are reported in [25] and [26].

In [25], first the production cost for each system in isolation is estimated. Then the production cost for the entire pool is estimated without transmission limitations. This information is then used in a computer program, in which transfer capacity and system's incremental cost curve are determined separately for each system, to estimate the benefits for a given tie line capacity. This method clearly disregards load correlation among the interconnected systems.

In [26], a method is also proposed to evaluate the expected savings of the energy production cost of two interconnected systems. This method is based on the representation of the incremental cost as affected by generating unit outages. From the knowledge of the distribution of the correlated load of two interconnected systems a bivariate distribution for a logarithmic function of the incremental cost, including the effect of outages of all units, is obtained. Integrating the appropriate zones of the bivariate distribution, subject to tie line constraints, the economic benefit is evaluated. The bivariate Gram-Charlier expansion is utilized in this evaluation.

Very recently, almost simultaneously with the reported work in this thesis, Noyes [27] proposed a method to calculate the production costs of two interconnected systems. The author introduces the concept of residual demand and the export or import are decided by dividing the residual demand plane into zones, according to the magnitude of the residual demand. In this method, the correlation between the loads of the two interconnected systems and also the joint ownership of generation are considered. In modeling the jointly owned unit the maximum capacity of the unit is assumed to be limited by the capacity of the tie line, which implies that the unit must be located in either of the two systems. The method is based on the notion of the bivariate Gram-Charlier expansion. Thus the method is prone to the inherent inaccuracies of such series expansions. Clearly, the method does not consider the evaluation of economic benefits.

The tumultuous events of the past decade have brought great financial pressure to bear on electrical utilities. The installation of the peaking type of generator to meet the peak load is becoming difficult. Many utilities are opting for load management as an alternative to a new unit to meet the peak load. Recently a large group of papers [28-35] appeared in the literature, in which mainly different load management approaches are described. But there has appeared almost none investigating the impacts of load management on the reliability and the production costs, especially for two interconnected systems.

Joint ownership of generation facilities as becoming more common as utilities are finding themselves pressed for capital. A comprehensive model for a jointly owned unit and the investigations regarding the impacts of the jointly owned unit on reliability as well as on production costs will be useful to a power system planner.

1.2 THESIS ORGANIZATION

The work presented in this thesis may be divided into two parts.

1.
   a) The development of a computationally efficient as well as accurate technique to evaluate a given plan of two interconnected systems in terms of reliability.
   b) The development of a methodology, to evaluate a given plan of two interconnected systems in terms of production costs and economic benefits.
2. The application of the above methodologies to a realistic system.

In what follows, an outline of the research work of different chapters of this thesis is briefly given.

In Chapter I, the background of this research and an overview of this thesis are given.

A technique has been developed in Chapter II [36] to evaluate the reliability of two interconnected systems. The method is capable of considering the loads between the two systems as correlated as well as independent. The method is compared in terms of accuracy and computational efficiency with the existing methods. The sensitivity of the proposed method is also investigated. The LOLPs, obtained for two interconnected systems using an hourly load model, are compared with those obtained by using a peak load model in this chapter. The studies regarding the impacts of time zone difference of two interconnected systems and the impacts of the varying forced outage rates of the generating units, on the reliability of two interconnected systems are also included in Chapter II.

In Chapter III, a method [37,38] has been proposed to evaluate the production costs and the expected energy generation of two interconnected systems. The method also evaluates the economic benefits in terms of production cost savings and the reliability in terms of expected unserved energies. The proposed method is applied to calculate the production costs and the expected energy generation utilizing the segmentation and the cumulant techniques separately. The results as obtained by these two techniques are compared. The investigations, regarding the impacts of tie line capacity variation, time zone difference between the two systems, and the varying FORs of the units on the production costs of two interconnected systems, are included in this chapter.

Chapter IV includes the applications [39,40] of the methods, proposed in Chapter II and Chapter III, regarding the impacts of different load management strategies applied in the constituent systems on the reliability, production costs and on the economic benefits of two interconnected systems. The possibilities of the use of load management strategies as an alternative to an interconnection with the neighbouring utility are discussed in this chapter.

Further, a model of a jointly owned generating unit, suitable for long term probabilistic simulation, is developed in Chapter V[39,40]. The impacts of joint ownership of generation on the reliability as well as on the production costs of two interconnected systems are investigated in this chapter.

The last chapter of the thesis includes the conclusion of this research work. In addition, suggestions for further research that need elaboration are included in this chapter.

In Apppendix A, the generating unit model used for this research is described in detail. Data for the hourly load and for the generating system utilized in the numerical simulation is given in Appendix B. Appendix C includes the development of the bivariate Gram-Charlier series. The convolution process of a bivariate distribution with the univariate distribution, based on the notion of cumulants, is presented in Appendix C. In this convolution process, an elegant advantage of cumulants is derived.

The methods proposed in this thesis will be useful to the power system planner in regional planning. As mentioned earlier, the reliability evaluation and economic analysis of a given plan are the vital steps in screening a number of alternative plans. The proposed method for the reliability evaluation has been found to be computationally fast as well as accurate. The methods, proposed in this thesis for reliability evaluation and for the evaluation of production costs,are conceptually straight forward and rather easy for a planner to apply to a practical system.

Chapter II

RELIABILITY EVALUATION OF TWO INTERCONNECTED SYSTEMS

2.1 INTRODUCTION

Improvement of reliability, or equivalently, maintaining a specified level of reliability by reducing the reserve reqirements is one of the major reasons of interconnection between utilities. Subject to tie-line limitations the participating system will be able to receive additional generation from others, should its own generation be unable to meet its own load due to unit random failures, unexpected load increases, or both as well as maintenance requirements.

Several measures have been devised to evaluate the reliability performance of a given electric power system .The most common reliability indices are:

1. Loss of load probability (LOLP),
2. Loss of energy probability (LOEP),
3. Frequency and duration (FAD).

The simplest and most common of all is the loss of load probability [4,5]. For a system interconnected with others LOLP is the probability that the available generation of that system and the possible capacity assistance from the connected utilities will be insufficient to meet its own demand. The LOLP is usually expressed in terms of days per year, hours per day, or as a percentage of time of insufficient generation. The disadvantage of this index of reliability is that it gives no indication of the duration of a possible outage or the amount of load that may be lost-but only how often the load will exceed generation. It also offers no clue, to which element has the greatest impact on system reliability, or what areas offer the least costly improvement in reliability.

The benefits of interconnection accrue mainly because of varying incremental costs, load diversity due to time zone differences, reserve capacity margins, and planned maintenance outages. However, as is often the case, with the use of yearly, seasonal, or monthly peak variations in interconnection studies, the planner may arrive at an unrealistic decision. For example, it is possible to incorrectly allow for the reserves of system X in the first week of July to supply the deficits of system Y in the last week of that month. By performing studies on a hourly basis, the load diversity problem can be handled more realistically. The planner will, therefore, be more confident that a generation expansion plan is not chosen on the basis of using the July reserves of a neighbor to supply August's deficiencies.

Before proceeding any further it is important to have the central problem in mind. What is wanted is to find out an efficient simulation technique which can be used in screening and selecting a number of alternative plans of two area interconnected systems. As was mentioned in Section 1.1, the first step of this screening process is the evaluation of the reliability index of a particular plan. If the plan satisfies the desired level of reliability then it is considered for economic analysis.

In this chapter, a computationally efficient and accurate technique referred to as the segmentation method [36] has been proposed to calculate the LOLPs of two interconnected systems. The method derives its name from the fact that it is based on the segmentation of unit capacities by a common factor. It is capable of considering loads between the two systems as independent, or as correlated. For correlated loads two dimensional segments are utilised corresponding to a joint load distribution. After convolving all the units the joint probability density function (PDF) of the two systems' equivalent load is obtained. Summing the joint probabilities under appropriate limits gives the LOLPs for the two area interconnected systems. For the case of independent loads one dimensional segments are considered for each system, and the PDF of equivalent load of each system is obtained by convolving the units with its own distribution of load. The joint probabilities of the two systems' equivalent loads is obtained by properly multiplying the probabilities of the corresponding segments of the two systems. As in the case of correlated loads, the LOLPs are obtained by summing the joint probability values of the segments under appropriate limits.

The proposed method has been applied to two realistic interconnected systems. The results are compared in terms of accuracy and computational efficiency to the commmonly used recursive method [3,41]. In case of correlated loads the results are also compared to the cumulant method [24]. The sensitivity study of the segmentation method is also carried out to investigate the effect of the segment size on the accuracy of the results as well as on the computational efficiency.

Different studies have been made to investigate the effects on LOLPs of two interconnected systems such as

1. Time zone difference in the interconnected systems,
2. The use of daily peak loads for the load model,
3. The application of different load management strategies,
4. The incorporation of joint ownership of generation

The last two studies will be discussed in Chapter IV and Chapter V, respectively.

For completeness, a brief description of the recursive as well as of the cumulant methods is given in this chapter. However, before going into a detailed description of these two methods a model for a generating unit as well as load is presented. These models are suitable for reliability analysis.

2.2 GENERATING CAPACITY MODEL

The life history of a generating unit is not as simple as described by the so called two-state model shown in Figure 2, where it is assumed that the unit may be in the up state with full capacity, or in the down state with zero MW output. In reality the generating units are complex pieces of machinery, and in addition to complete failures, they may experience partial failures, when they continue to operate but at reduced capacity levels. For different derated capacity levels a multi state model [18] of a generating unit may be considered. But for the long term planning the binary representation of a generating unit is usually considered for most applications.

In Figure 2, λ and μ represent the failure and repair rates of the generating unit, respectively. The values of λ and μ may be obtained from the history of the unit. Under the following assumptions,

1. Changes of state (up or down) are possible at any time,

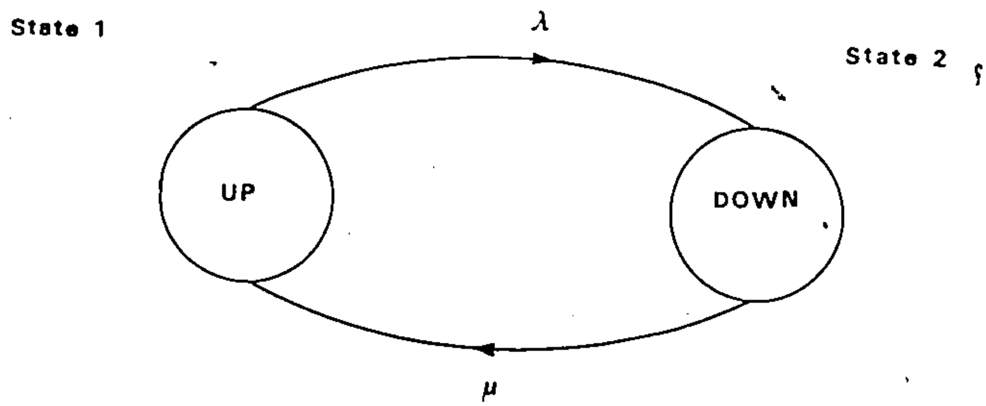


Figure 2: Generating unit state space diagram

2. The probability of departure from a state depends only on the current state, and is independent of time,
3. The probability of more than one change of state during a small interval Δt is negligible,

the long term probabilities are given by (a detailed analysis is given in Appendix A)

$$P(\text{up state}) = P = \frac{\mu}{\lambda + \mu} \qquad (2.1)$$

$$P(\text{down state}) = q = \frac{\lambda}{\lambda + \mu} \qquad (2.2)$$

so that $p+q=1$. The expressions (2.1) and (2.2) are used to define the avilability p and the forced outage rate (FOR) q of a generating unit, respectively. (Data on FOR are collected by several utilities and much of this information is presented in several publications of the Edison Electric Institute; for example, in reference [42])

For a generating unit of capacity C MW, the two state failure model may be represented by a probability density function (PDF) of forced outage capacity as shown in Figure 3.

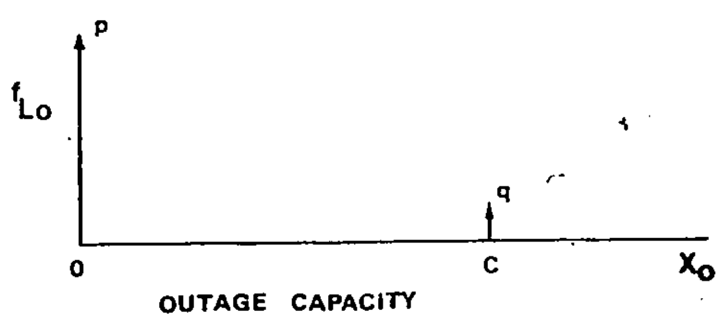


Figure 3: PDF of forced outage capacity

This PDF of forced outage capacity may be conveniently expressed by the relation

$$f_{Lo}(X_o) = p\ \delta(X_o) + q\ \delta(X_o - C) \tag{2.3}$$

where $\delta(.)$ is the dirac-delta function. Figure 3 may be interpreted as follows: the unit is operating p% of the time and has failed q% of the time.

2.3 LOAD MODEL

Proper modeling of load is an important factor in the evaluation of a reliability index of two interconnected systems, since load diversity is one of the major contributors in improving the reliability of the two interconnected utilities. Thus, when the load is small in one system it may provide assistance to the other system which may be experiencing at the same time peak load condition. In what follows an hourly load will be used in the analysis.

To a utility the load L is a random variable by nature. Hence, the load may take up any value between base load and peak load. The chronological load curve may be considered as one relization of this random variable. Usually, however, one is dealing with historical data which is clearly deterministic. There does not seem to be any randomness questions which can be answered because history has already happened, there is no uncertainty involved. The conceptual trick when dealing with past history is to imagine one is at unknown time in history,and use this information to project into the future.

An easier way to conceive of L, the random variable for load, is to look at the problem from the forecasting point of view. Imagine a forecaster making hourly forecasts over a given time interval. At each hour the forecaster gives an estimate of the mean load for the particular hour. These mean forecasts give a trajectory for the load over this interval. However, the forecaster understands that as history passes and the time interval goes by, that there are an infinite number of trajectories which could be realized. What the forecaster presumes for is that the actual trajectory will lie in a band around the forecasts of the means. Figure 4 gives a graphical representation of the process.

Considering past history once again, one can conceptualize the seemingly deterministic event in an analogous fashion. If one had made hourly forecast of the load,then one could imagine the forecast as being the mean hourly values for that year and the actual history as a single realization.

Clearly, the use of historical hourly data is well established [19,24,27,43]. In any case, a random component L_R can always be added to the hourly data which will represent the forecast error as well as the random consumption pattern of consumers of electricity.

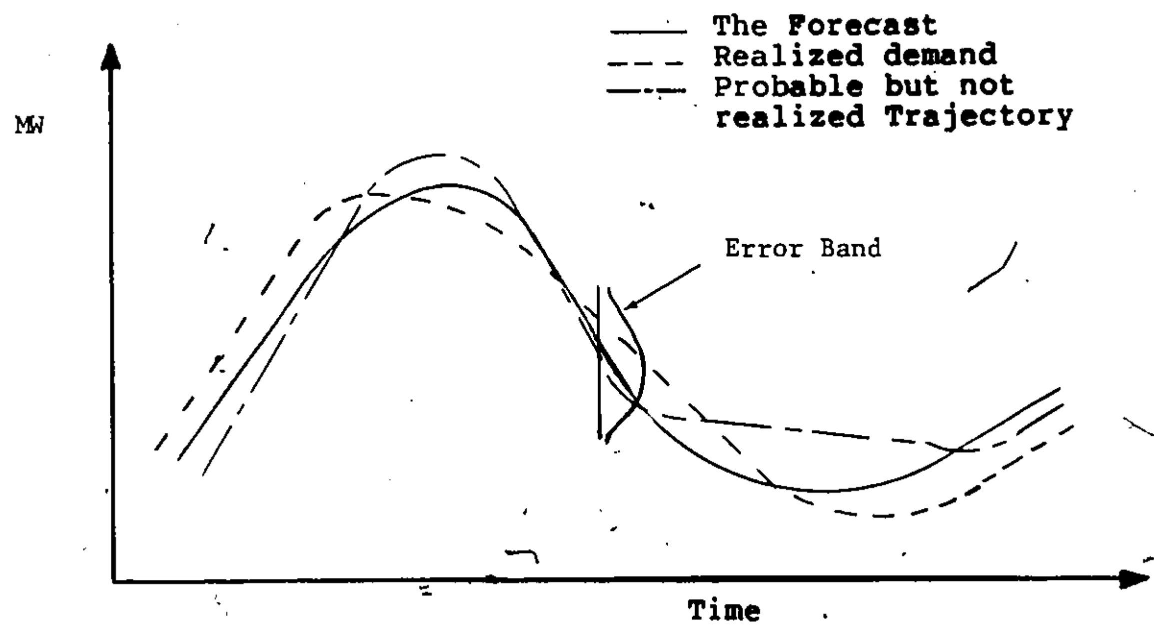


Figure 4: Mean forecast vs. possible trajectories

The often used load model in generation expansion planning is the load duration curve(LDC). The LDC is a function which relates the consumer's load level to the duration of time that each load level is equalled or exceeded. The data required to develop such a model are readily available, since continuous readings of system demand and energy are usually obtained on a routine basis by electric utilities. If a recording of instantaneous demands were plotted for a particular historical day, a curve such as depicted in Figure 5a might result. From this curve, the load duration curve as shown in Figure 5b is easily constructed. The load duration curve is created by determining what percentage of time the demand exceeded a particular level, where percent time is shown on the x-axis and demand level on the y-axis. Although detail is lost in the conversion, the load duration curve is easier to work with for time periods longer than a day and for time periods for which there is not enough information to create hourly loads.

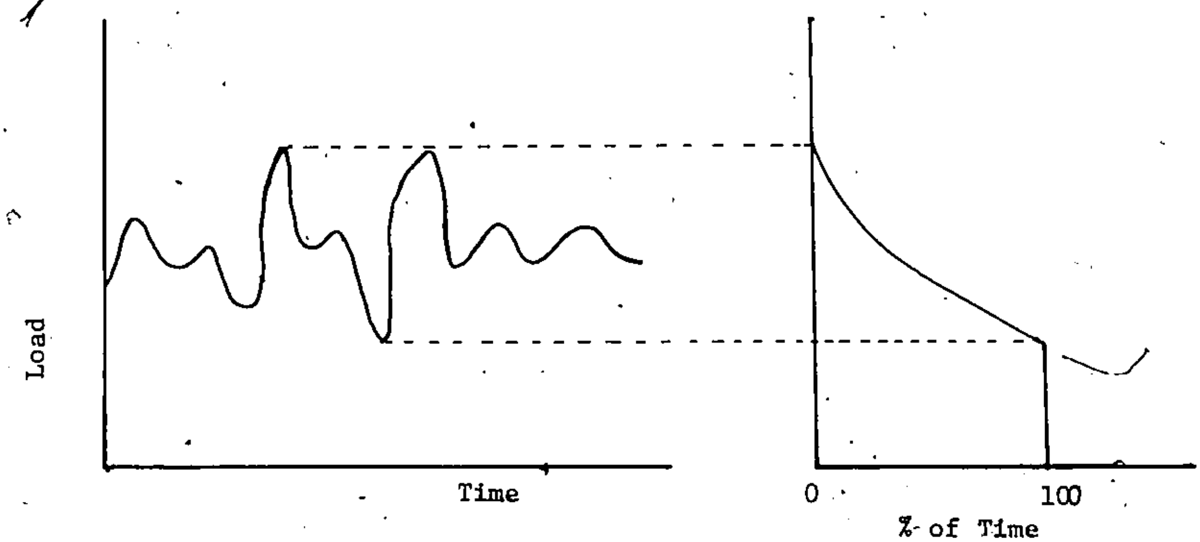


Figure 5: a: chronological load curve. b: Load duration curve

The load model that best suits the segmentation method is the model which assumes that the mean value of the load for any suitable interval of time stays constant for that particular interval. In the hourly load model, an hour interval is assumed. A typical hourly load model is depicted in Figure 6. The load is then sampled every hour or any other suitable interval, and assigned equal probability of occurrence. Clearly, coincident load points will give rise to the summation of the probabilities. The probability distribution of load obtained from the hourly load of Figure 6 by sampling at every hour is depicted in Figure 7 .

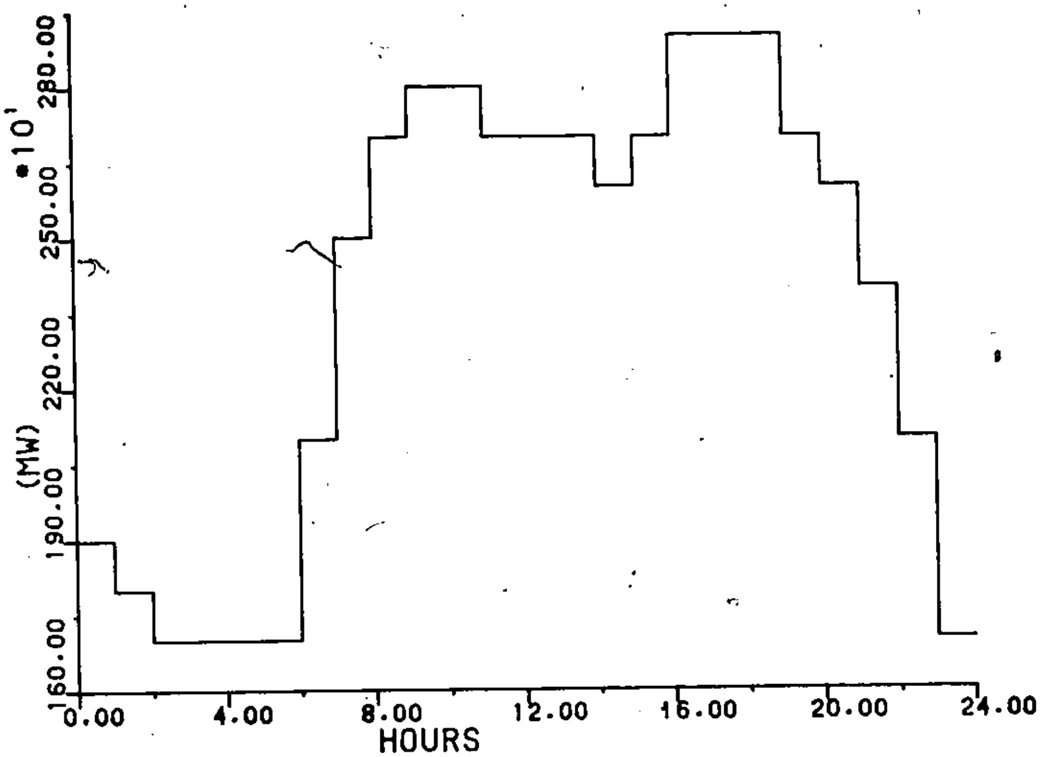


Figure 6: Hourly load variation

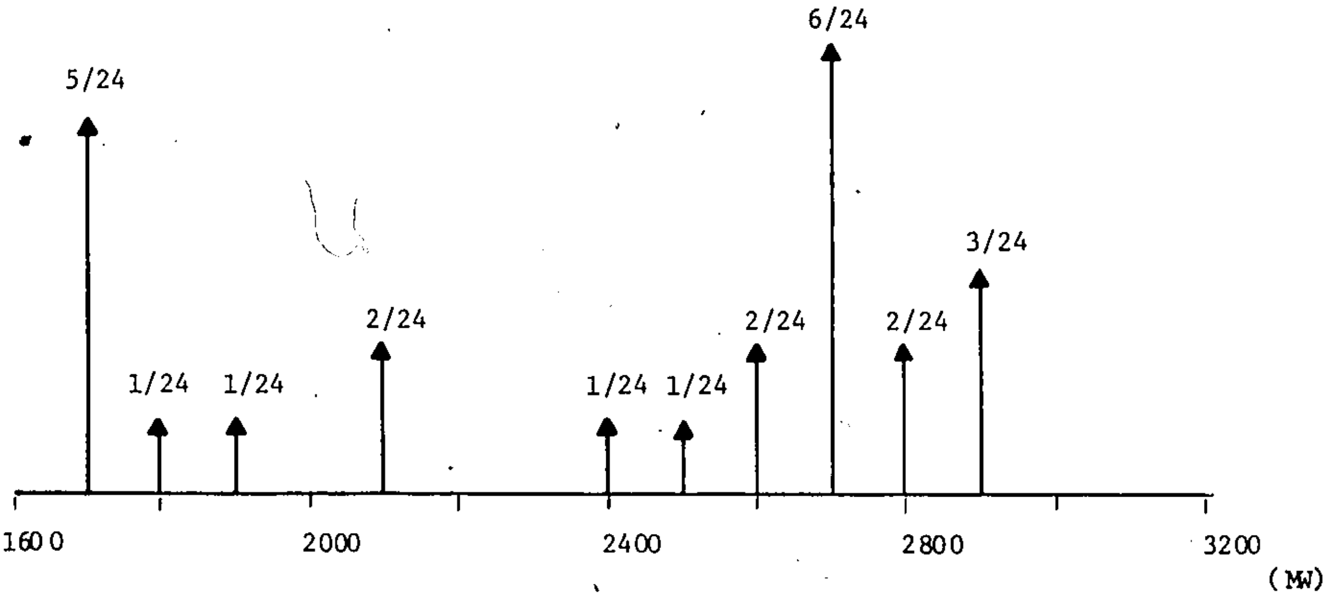


Figure 7: PDF Of Load

2.4 LOSS OF LOAD PROBABILITY (LOLP)

The loss of load probability is the probability that the available generation capacity of an utility will fail to meet its demand. That is,

$$LOLP = P\left\{Ga < L\right\} \tag{2.4}$$

where Ga and L are the available generating capacity and the load respectively. The availability of the utility Rs may be defined as

$$Rs = 1 - LOLP \tag{2.5}$$

2.5 EQUIVALENT LOAD

The randomness in the availability of generation capacity is taken into consideration by defining a ficticious load, known as equivalent load, Le [13]. If Lo_i represents the random outage load corresponding to the i-th unit, Le may be expressed as

$$Le = L + \sum_{i=1}^{n} Lo_i \tag{2.6}$$

where n is the total number of generating units. In this case, the actual generating unit will be replaced by a ficticious unit which is 100% reliable and a ficticious random load will be added to the system load ,whose probability density functions are the outage capacity density functions of the unit. That is, when $Lo_i = C_i$, the net demand injected into the system is zero for the i-th unit, just as it would be if the actual unit of capacity C_i were forced off-line. This is depicted in Figure 8 with the system load.

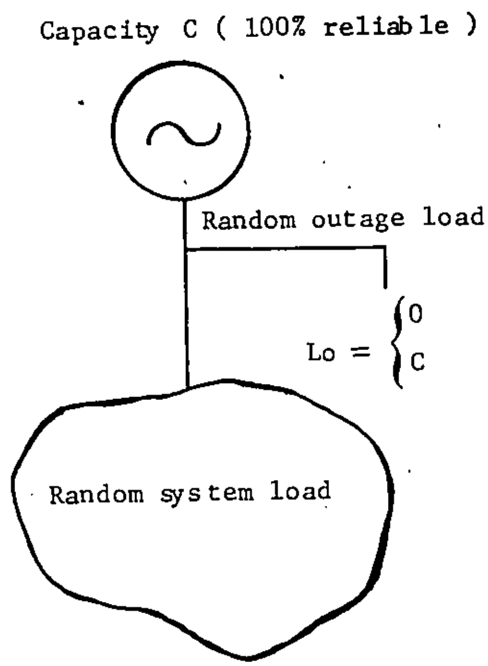


Figure 8: Fictitious generating unit and system load

Now the LOLP may be redefined as

$$\text{LOLP} = P\left\{ Le > IC \right\} \tag{2.7}$$

where IC is the installed capacity and may be expressed as

$$IC = \sum_{i=1}^{n} C_i \tag{2.8}$$

The outages of the generating units may be assumed independent of the system load. Then the distribution of the equivalent load will be the outcome of the convolution of two distributions: f_{Lo} and f_L representing the PDF of outages of machine and the PDF of system load, respectively. For the discrete case the PDFs, f_L and f_{Lo}, respectively, may be written as

$$f_L\ (l) = \sum_i P_{L_i}\ \delta(\ l - l_i\ ) \tag{2.9}$$

$$f_{Lo}\ (lo) = \sum_j P_{Lo_j}\ \delta(\ lo - lo_j) \tag{2.10}$$

Then the PDF of equivalent load f_{Le} may be given as

$$\begin{aligned} f_{Le}\ (le\ ) &= f_L\ (l\ ) * f_{Lo}(lo\ ) \\ &= \sum_{i,j} P_{L_i}\ P_{Lo_j}\ \delta(\ le - (l_i + lo_j)) \end{aligned} \tag{2.11}$$

where the star indicates the convolution.

For a binary representation of the generating unit as mentioned in Section 2.2 and for m number of load impulses, it is obtained from equation (2.11) that the number of impulses of equivalent load, after convolving a generating unit, will be 2 x m . Similarly, for n number of generators the total no of impulses of the equivalent load will be 2^n x m. Of course, the coincident impulses will decrease the number of impulses. The PDF of the equivalent load is depicted in Figure 9. It is obtained after convolving one generating unit of capacity C=200 MW and FOR q=0.5 with the PDF of load shown in Figure 7.

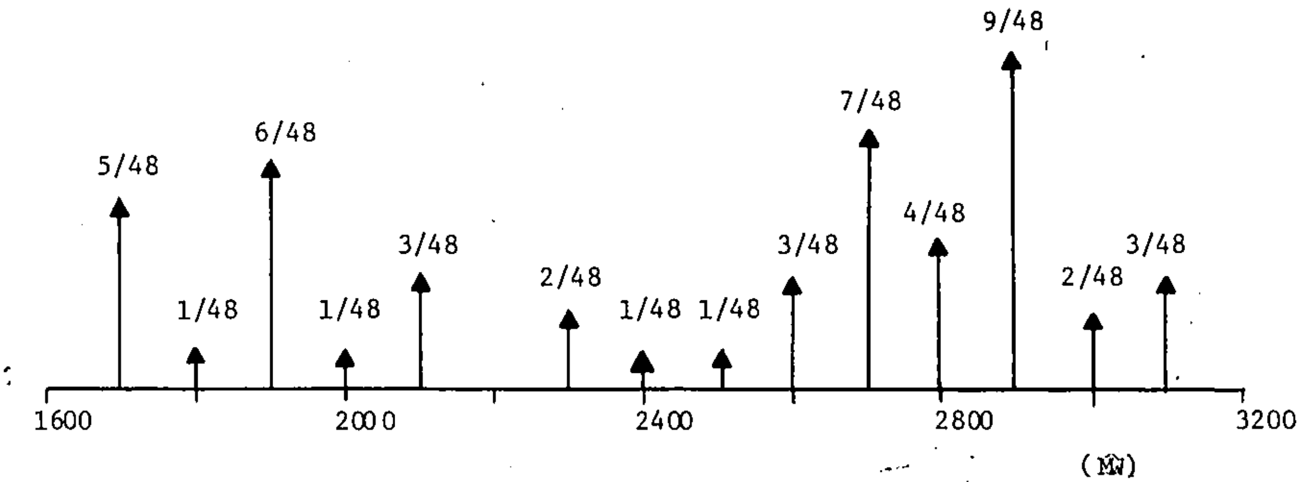


Figure 9: PDF Of Equivalent Load

In order to evaluate the reliability index (LOLP) the distribution of equivalent load incorporating the outages of all the generating units in the system is required. It clearly shows that the required distribution may be obtained at the cost of huge computer memory and excessive CPU time.

To solve the above problem of large computer memory and long CPU time requirements Schenk et al. [19] proposed the segmentation method .

2.6 SEGMENTATION METHOD

The starting point of the segmentation method is the formation of segments of equal size by dividing the demand axis. The size of the segments depends on the largest common factor of the generating unit capacities. To each segment a probability value is attached which is equal to the sum of the probabilities of the impulses lying in the range of the particular segment. The distribution of segments, obtained from the PDF of load of Figure 7, is depicted in Figure 10.

One segment beyond the installed capacity (IC) is considered. Note that the probability attached to this last segment in the final distribution is the LOLP. Recall that the LOLP is obtained when the equivalent load is larger than the installed capacity. Since the probability of occurrence of any load lower than the base load is zero the formation of segments starts from the base load. Clearly it shows that the numerous number of impulses have been reduced to a few number of segments.

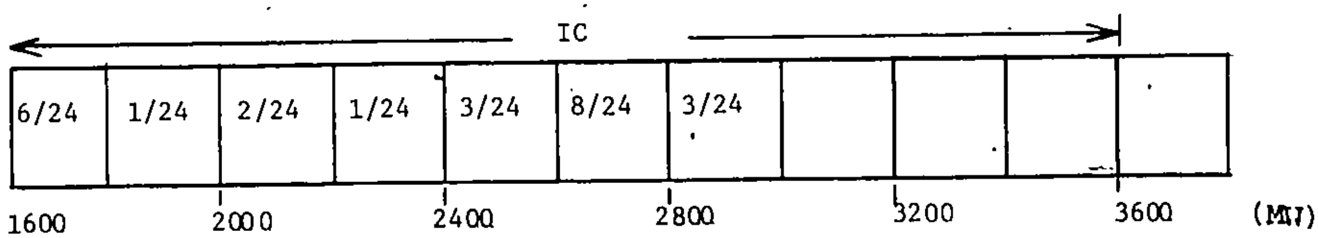


Figure 10: Distribution of segments

In order to account for the random outages of units it is necessary to get a new distribution of segments incorporating the outages of all units. Considering the k-th segment and assuming a generating unit of capacity C MW and FOR=q, to be convolved, the probability of the k-th segment, after the convolution may be expressed as

$$\tilde{P}_k = P_k \times (1-q) + \hat{P}_k \times q \qquad (2.12)$$

Where $\tilde{P}_k$ = Probability of the k-th segment after the convolution

$\hat{P}_k$ = Probability of the k-th segment after the shift

P_k = Probability of the k-th segment before convolving the unit

The convolution process is schematically represented for one generating unit in Figure 11. The procedure to be followed in convolving a generating unit may be described as follows:

1. Multiply the original distribution of segments (Figure 10) by the availability of the units (1-q) (shown in Figure 11a)
2. Shift the original distribution by the unit capacity and multiply by the FOR of the unit q (shown in Figure 11b)
3. Add the values of the corresponding segments of Figure 11 a and Figure 11b (shown in Figure 11c).

Note that the probability value of the last segment is the sum of the probabilities of all the segments exceeding the installed capacity. Also note that the segments below the already committed capacity can be deleted, since the probability values of these segments will not further contribute to the value of the last segment. Therefore, as the convolution process proceeds, the number of segments decrease.

In what follows the segmentation method will be extended to evaluate the LOLPS of two interconnected systems for independent as well as correlated loads.

For two interconnected systems the method also starts with sampling daily chronological load curves for the period under study. The demand is subdivided into segments of equal capacity. The size of the segments depends on the largest

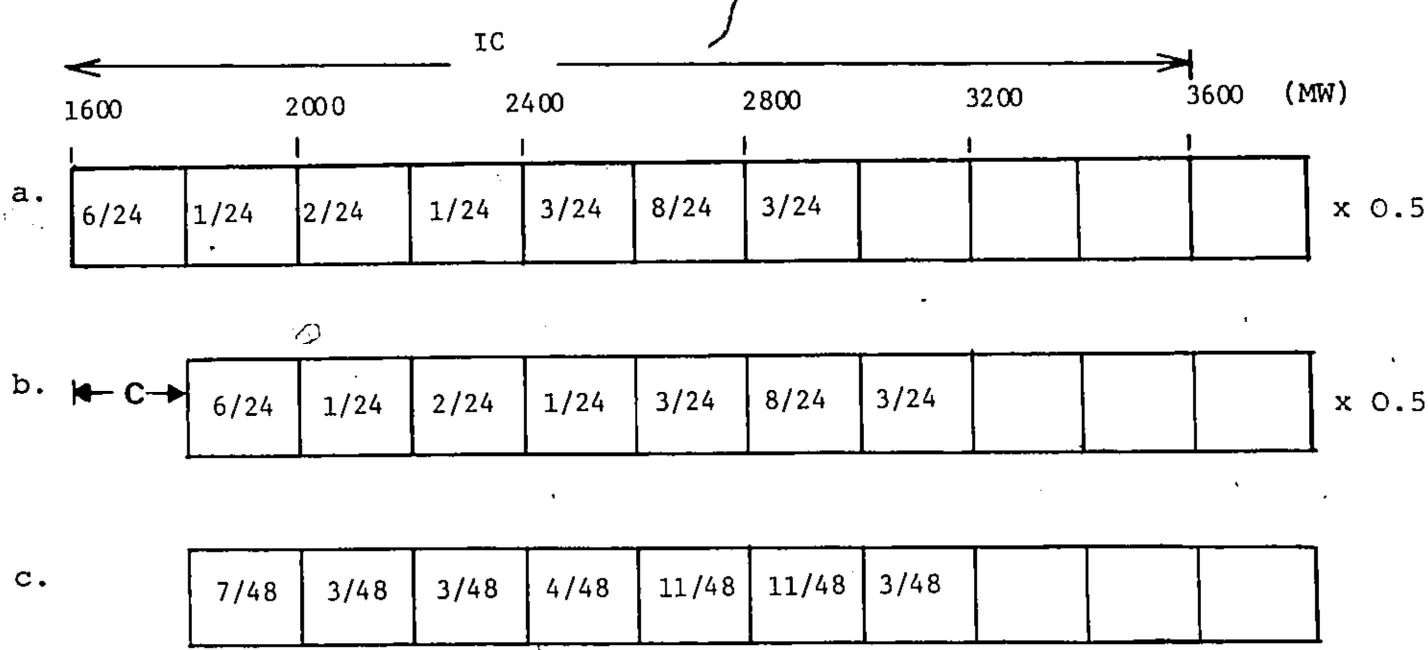


Figure 11: Schematic representation of the convolution process

common factor of the generating unit capacities and the tie line capacity. In most cases this common factor corresponds to the capacity of the smallest unit. For disparate unit sizes 1 MW segment must be considered. Fractional segments may be considered but not really required in long term planning studies. To decrease the computational requirements, rounding off unit capacity may be considered. In any case, even with a 1 MW segment, the computational requirements are very reasonable.

To each segment the probability value corresponding to the respective impulse lying in the range of the particular segment is attached. Units are then convolved and LOLPs are obtained from the final distribution using the Venn-diagram [5] approach.

In what follows the computational steps for cases of independent and correlated loads will be fully described with a simple but revealing example.

Consider two systems, system X and system Y connected through a 10 MW tie line, shown in Figure 12. Consider also the hourly load model, depicted in Figure 13 for the two interconnected systems.

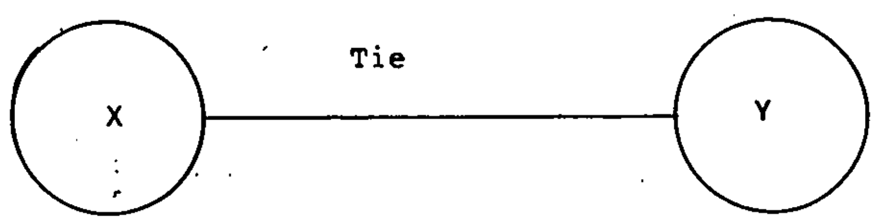


Figure 12: Two Interconnected Systems

For brevity assume that the time period under consideration is only 4 hours. The generating unit model is described in Table 1. The segment size is chosen to be 5 MW using the largest common factor of the generating unit capacities and the tie line capacity.

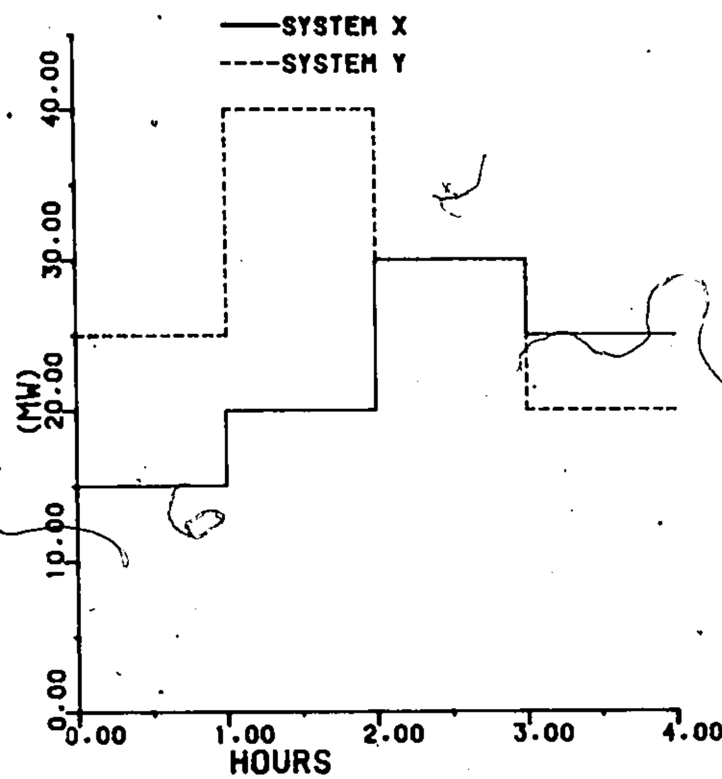


Figure 13: Load representation

TABLE 1

Generation system description

	system X			system Y		
	No of units	capacity (MW)	FOR	No of units	capacity (MW)	FOR
	2	10	0.2	2	15	0.1
	1	30	0.1	1	25	0.3

2.6.1 Independent loads

In this case, the loads of the two interconnected utilities are assumed independent. This means that for a given load in one utility any load in the other utility may occur. Although this does not sound realistic especially for two neighboring utilities where normally interconnections are made, the independence assumption in the load model has been considered just to compare the results of the proposed method with those of the existing technique based on the recursive algorithm.

The process of convolution in the segmentation method demands that as each unit is committed that each segment be shifted by the unit capacity and the probability values be multiplied by the unit unavailability (FOR), as mentioned earlier.

During the process of convolution for the single area probabilistic simulation the deletion of segments is possible up to the installed capacity [19]. In addition, every time a new unit is loaded, the segments below the capacity equal to the total capacity of the committed units are deleted. But in the evaluation of the reliability of two area interconnected systems the deletion of segments is possible up to a certain limit. This limiting capacity is given by

$$LPDP = IC - TC - C_m \tag{2.13}$$

Where

LPDP = The limiting capacity up to which the deletion of blocks is possible

IC = Installed capacity

TC = Tie line capacity

C_m = Capacity of the largest unit

In the proposed method after convolving all the units of a system the probabilities of the segments up to the capacity equal to (IC - TC) in the form of a sum is obtained by subtracting the sum of the probabilities of the segments above (IC - TC) capacity from unity. This probability will be considered as the probability of the initial segment. That is,

$$P_1 = 1 - \sum_{k=K_1}^{K_2} P_k \qquad (2.14)$$

where P_1 is the probability of the initial segment and the limits K_1 and K_2 can be obtained from the following relations

$$K_1 = (IC - TC)/\Delta C + 1 \qquad (2.15)$$

$$K_2 = (IC + TC)/\Delta C + 1 \qquad (2.16)$$

in which ΔC is the chosen common factor of generating unit capacities of both systems and of the tie line capacity. It is worth mentioning that the loading order is immaterial in the evaluation of LOLP.

To exemplify the above mentioned process obtain the PDF of loads shown in Figure 14(a) and Figure 14(b) by assigning to each sampled hour load of Figure 13 an equal probability i.e. (1/4) in this case.

The installed capacity of system X plus the tie line capacity (60 MW) is divided into 12 equal segments each spanning 5MW .Out of these 12 segments two initial segments are omited since there are no impulses below 15 MW. One additional segment must be considered after the sum of installed capacity and tie line capacity as shown in Figure 15(a). The last segment will carry the values of the LOLPs

(a) -PDF of load for system X (b)-PDF of load for system Y

Figure 14: PDFs of demand for the two interconnected systems

as will become clearer later. The probability value of each segment corresponding to the respective impulse of Figure 14(a), lying in the range of the particular segment is also shown in Figure 15(a). Note that the probability values in Figure 15 are four times of the actual values. The value of LPDP (see equation (2.13)) here is 10 MW. So in this particular example there will be no deletion of segments in the process of convolution. The steps of convolution of generating units for system X are depicted in Figure 15. Note that probability values are shifted toward the last segment and a number of them may be accumulated in this special segment. Thus, the last segment of Figure 15(f) is the sum of the last seven segments of Figure 15(e).

The LOLP of the system X (LOLP) alone is simply the sum of probability values above the installed capacity. Thus,

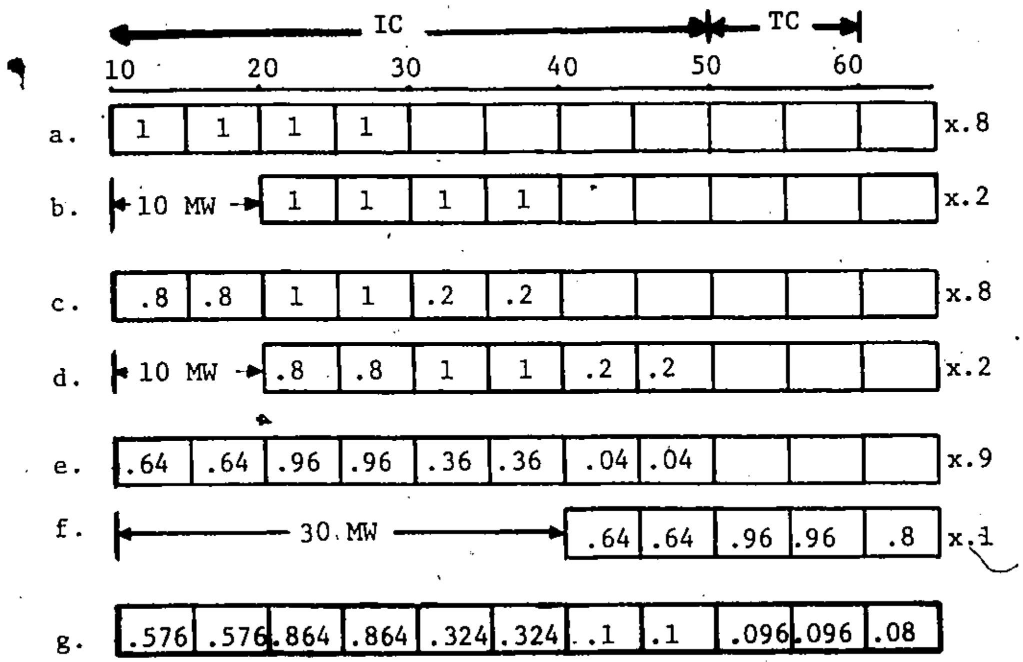


Figure 15: Schematic of convolution procedure of system X (All numbers in the boxes to be divided by 4)

$$LOLP_X = \frac{0.096+0.096+0.08}{4} = 0.068$$

In a similar fashion the PDF of equivalent load for system Y is obtained. To obtain the joint PDF of the interconnected system multiplication of the probabilities of system X with the corresponding probabilities of system Y is required as depicted in Figure 16. Note that the resulting two area joint probability matrix shown in Figure 16 is only for illustrative purposes. In the computer algorithm the joint probability matrix is not required thus substantially reducing the computational requirements.

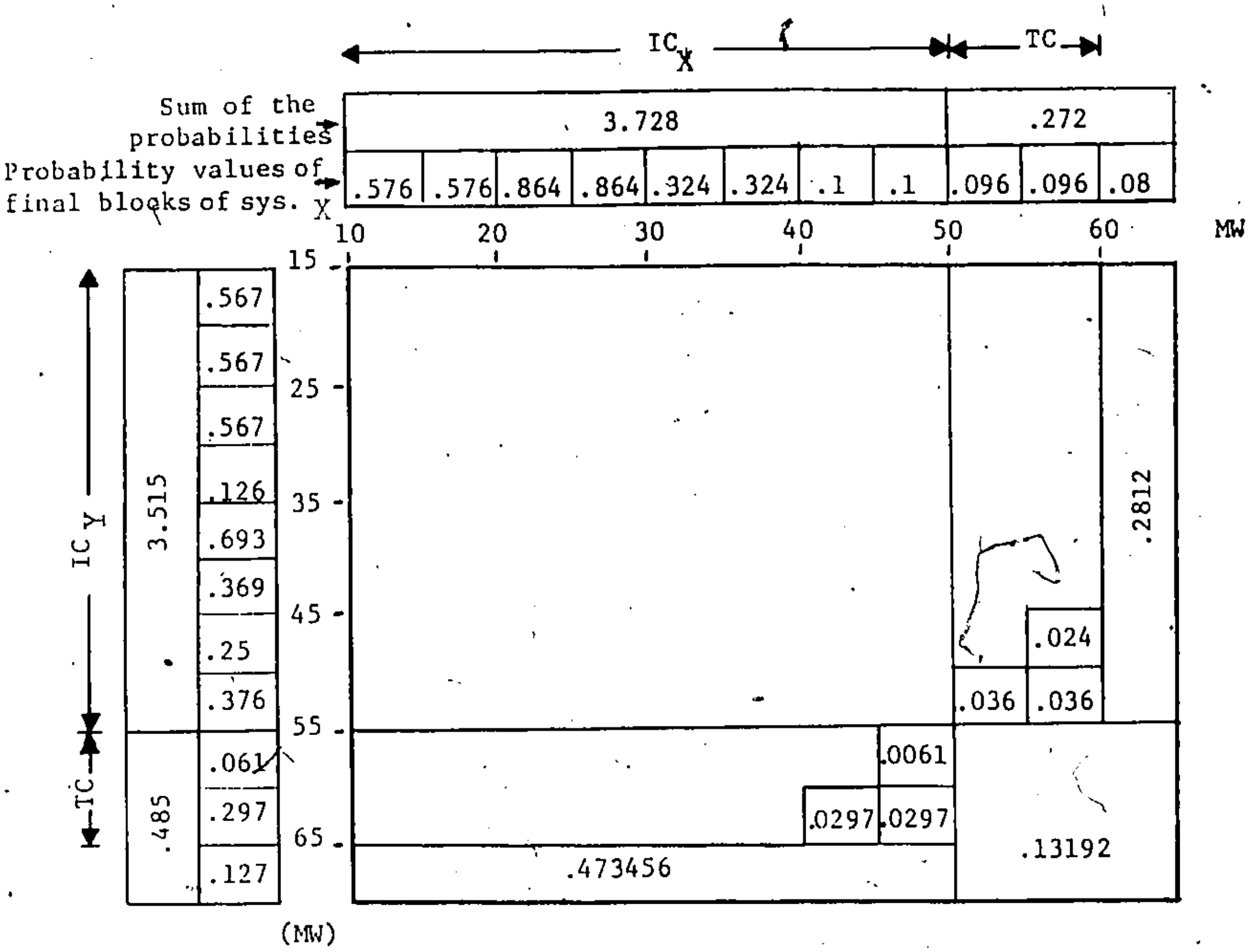


Figure 16: Schematic representation of two area joint probabilities (All numbers inside the joint probability matrix to be divided by 4x4 and all probability values outside the matrix to be divided by 4)

To calculate the LOLPs of the two interconnected systems one simply has to add the joint probabilities spanning the area above the installed capacity of each system by taking into consideration the possible assistance through the tie line. Therefore,

$LOLP_X$: Loss of load probability of system X

$LOLP_Y$: Loss of load probability of system Y

$LOLP_{X|Y}$: Loss of load probability of system X assisted by system Y

$LOLP_{Y|X}$: Loss of load probability of system Y assisted by system X -are equal to:

$$LOLP_X = 0.272/4 = 0.068$$

$$LOLP_Y = 0.485/4 = 0.12125$$

$$LOLP_{X|Y} = \frac{0.13192+0.036096+0.036096+0.024+0.2812}{4\times4} = 0.031832$$

$$LOLP_{Y|X} = \frac{.13192+0.0061+0.0297+0.0297+0.473456}{4\times4} = 0.04192975$$

2.6.2 Correlated loads

It is more realistic to consider the loads in the interconnected utilities as correlated. A load level for a particular hour in system X corresponds to only one load level in system Y. The degree of correlation varies depending on the geographical location of the interconnected systems and the weather sensitivity of the loads and the measure of this proportion is the so called correlation coefficient expressed as

$$\rho(l_X , l_Y) = \frac{\mu_{11}}{\sigma_X \quad \sigma_Y} \tag{2.17}$$

where L_X and L_Y represent the loads of the two interconnected systems, X and Y, and $\sigma(X)$ and $\sigma(Y)$ are the standard deviations of the two RVs L_X and L_Y, respectively. In equation (2.17) μ_{11} represents covariance.

For correlated loads the expected value of the product of the two RVs L_X and L_Y is not equal to the product of expected values. That is

$$E(l_X, l_Y) \neq E(l_X)\, E(l_Y) \tag{2.18}$$

Consider the interconnected systems again shown in Figure 12 and assume that the loads of the two systems are correlated. Consider the load model and the generation model, shown in Figure 13 and Table 1 respectively, for these two systems.

In the correlated load case, two dimensional segments are required, unlike the independent load case. Each segment has four sides of equal size and the side of each segment corresponds to the largest common factor of the generating unit capacities of both systems as well as the tie line capacity.

Assigning to each sampled hourly load equal probability (1/4) the joint probability matrix shown in Figure 17 is obtained. Note that all numbers in the boxes of Figure 17 must be divided by 4 to get the actual probability values. Thus the joint probability of 15 MW load in system X and the corresponding 25 MW load in system Y is 1/4. In the same manner the rest of the joint probabilities are obtained. As in the independent load case there is no need to depict the segments below base load. In this example the joint probability segment construction starts with the 15 MW load in system X and the 20 MW load in system Y, as shown in Figure 17.

The process of convolution requires the shifting of each segment as it is described for the case of independent loads. However, in this case, the direction of shift depends on which system is being considered. The convolution of the generating units from system X will shift the segments of Figure 17 in the direction of the x-axis while the convolution of the generating units from system Y will shift the segments in the direction of the y-axis. That is, to convolve the first unit of system X the probability value of Figure 17 is shifted 10 MW along the x-axis as shown in Figure 18.

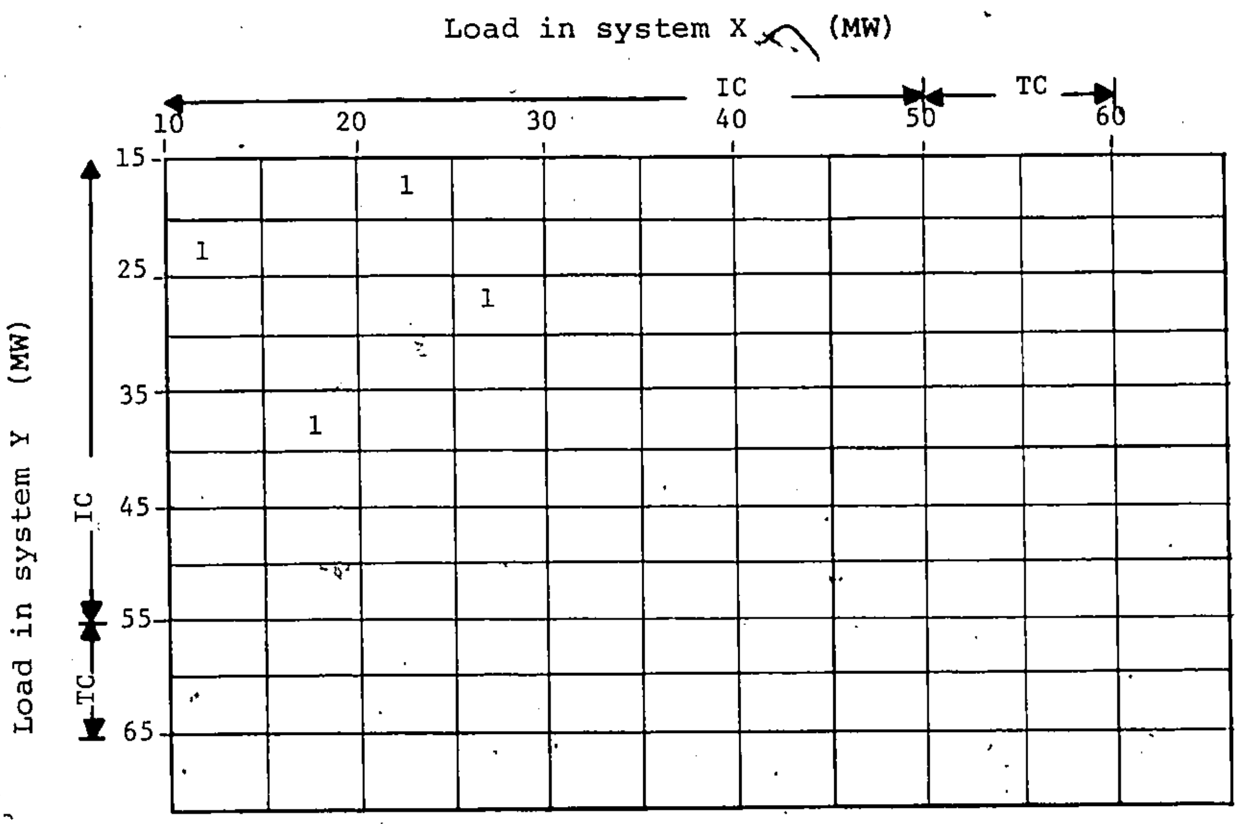


Figure 17: Joint probability matrix for systems with correlated load (All numbers in the boxes to be divided by 4)

The new distribution of demand shown in Figure 19 is obtained by adding the corresponding probability values of Figure 17 and Figure 18 after multiplying by the availability (p=0.8) and unavailability (q= 0. 2) of the unit, respectively. In this process of convolution it is found to be convenient to convolve all the units of one system first. Deletion of segments starts when the units of

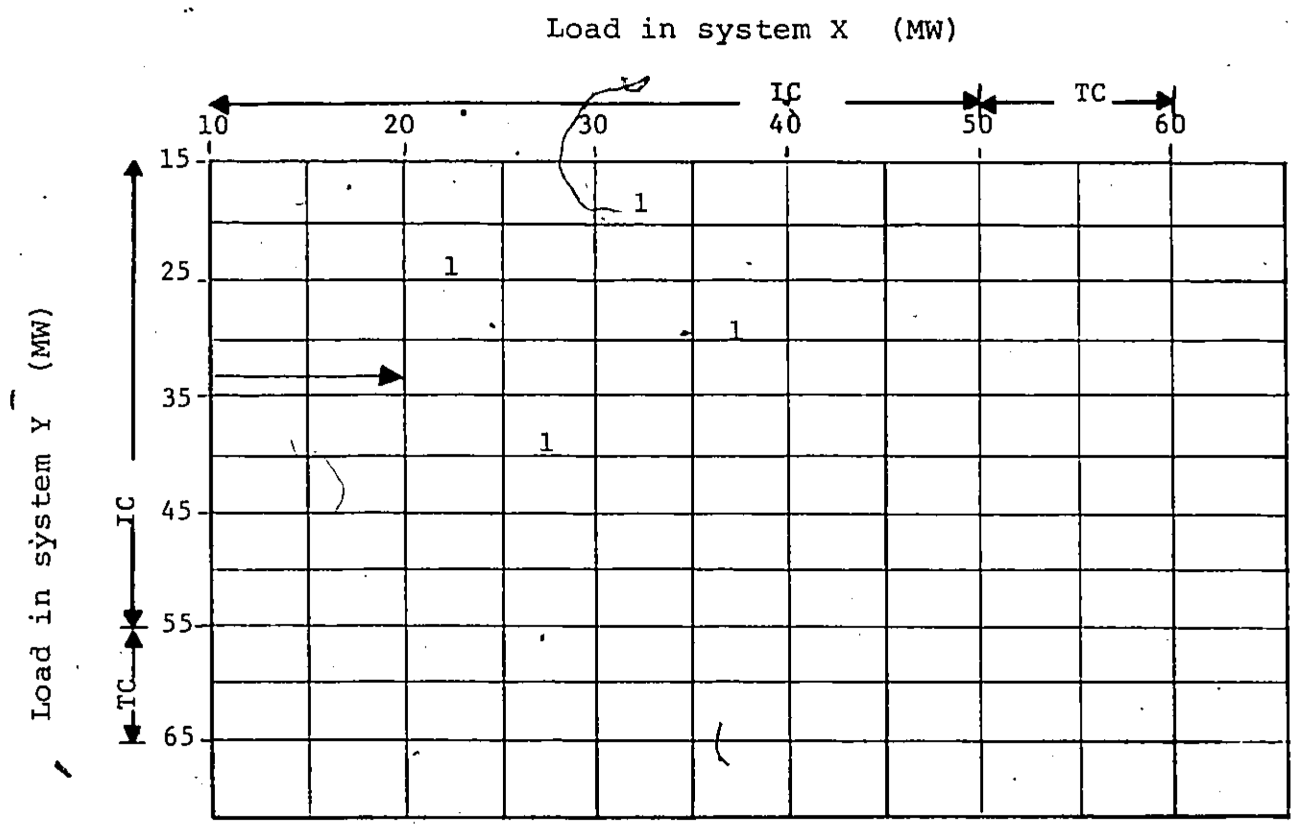


Figure 18: Shifted joint PDF by loading a unit in system X

the second system are convolved, i.e. the segments below the total capacity of the committed units of the second system are deleted and the limits of the deletion zones are the installed capacities of either system.

Figure 20 shows the joint probability matrix after convolving all the the units in the two systems. Note that all the segments below the installed capacity of either system are deleted in the process of convolution. Also note

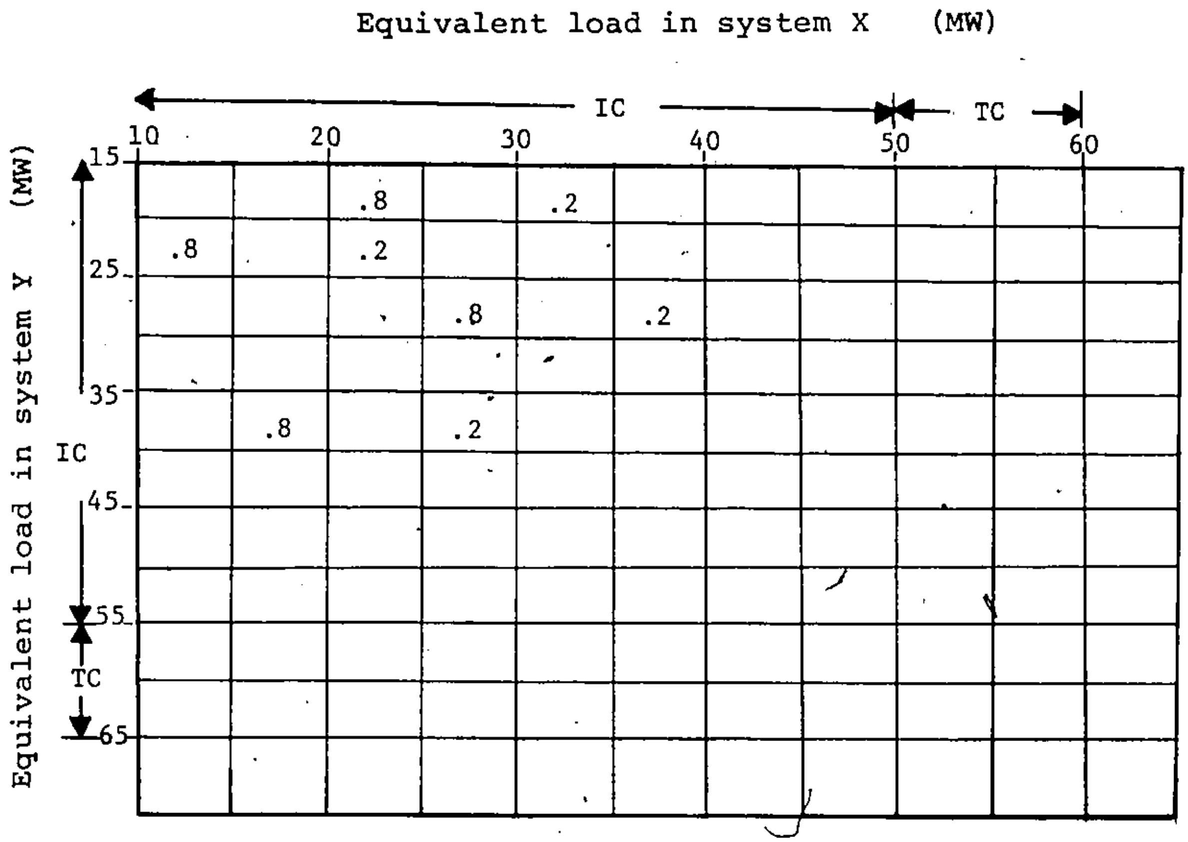


Figure 19: Joint probability matrix after convolving one 10 MW unit of system X. (All numbers in the boxes to be divided by 4)

that the probability values in the last segments are the sum of the probabilities above installed plus tie line capacity. As the convolution process continues the number of segments always decrease to a minimum number which is required to calculate the LOLPs. Summation of the joint probabilities above the installed capacity of each system will result in sub-total joint probabilities spanning only the area of

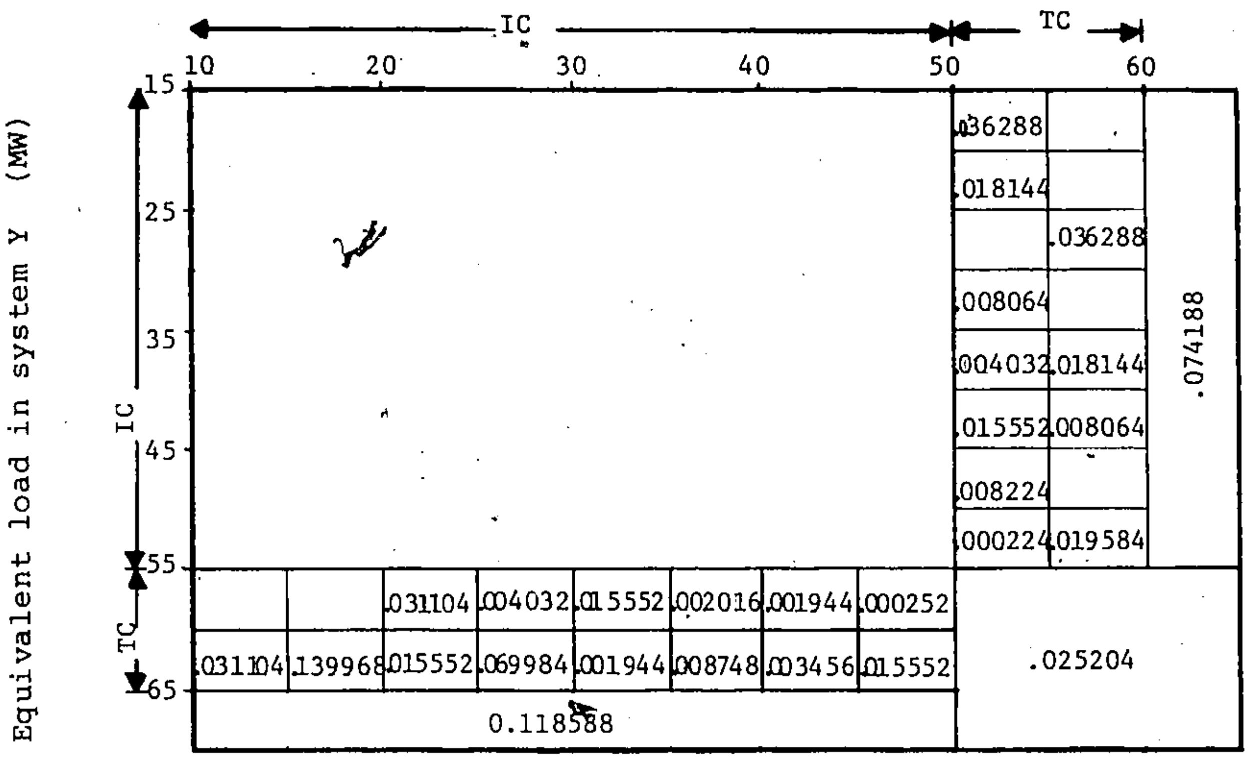


Figure 20: Joint probability matrix after convolving all the units of the two systems (All numbers in the boxes to be divided by 4)

interest which is above the installed capacity for each system. Recall that the LOLP is obtained when the equivalent demand is larger than the installed capacity. The LOLPs for the system are:

$$LOLP_X = (.025204+.074188+.036288+....+.019584)/4 = 0.068$$

$$LOLP_Y = (.025204+.118588+.031104+....+.000252)/4 = 0.12125$$

$$LOLP_{X|Y} = (.025204+.074188+.000224+.019584)/4 = 0.0298$$

$$LOLP_{Y|X}=(.025204+.118588+.003456+.015552+.000252)/4$$
$$=0.040763$$

2.7 RECURSIVE METHOD

In what follows the method based on the recursive algorithm will be briefly described separately for cases of independent and correlated loads.

2.7.1 Independent loads

The first step in this approach is to develop the capacity model of each utility of the interconnected system separately. This is normally expressed in the form commonly known as the capacity outage table. The capacity outage table expresses the probability that various amounts of generating capacity will be unavailable. The table is usually given in terms of exact and cumulative probabilities. A recursive algorithm is commonly used in obtaining the cumulative probabilities.

$$P(x) = P(x)\cdot(1 - q) + P(x-C)\, q \tag{2.19}$$

where

q = FOR of the generating unit to be added

$\tilde{P}(X)$ = Probability of a capacity outage of X MW or greater after a unit of capacity C MW is added

$P(X)$ = Probability of a capacity outage of X MW or greater before a unit of capacity C MW is added

To combine the generation and the load models a capacity margin table is developed on the basis of independence of events.

A margin state M_j is the combination of the load state L_i and the capacity state C_k, where

$$M_j = C_k - L_i \tag{2.20}$$

The probability of the margin state P_{M_j} is the product of the capacity state and load state availabilities. That is

$$P_{M_j} = P_{C_k} \times P_{L_i} \tag{2.21}$$

where

P_{C_K} = The probability of the k-th state in the capacity outage table

P_{L_i} = The probability of the i-th level of load

In the capacity margin table identical margin states are combined simply by adding the probability values. In this table the state with a capacity margin greater than or equal to zero MW represents success while a negative margin state represents failure of the system.

To incorporate the possible assistance from the neighboring utility through the tie line a combined state diagram is developed. This diagram is constructed from the arrays of the capacity margin table for systems X and Y; the one for system X shown to the left and that for Y in the top as shown in the Figure 21. The probability $P_{M_{ij}}$ of any state (ij) of the combined diagram is given as

$$P_{M_{ij}} = P_{X_i} \times P_{Y_j} \tag{2.22}$$

where

P_{X_i} = The probability of i-th margin state of system X

P_{Y_j} = The probability of j-th margin state of system Y

This simple formula (2.22) reflects the assumption that both the generation and the load models for the two systems are independent.

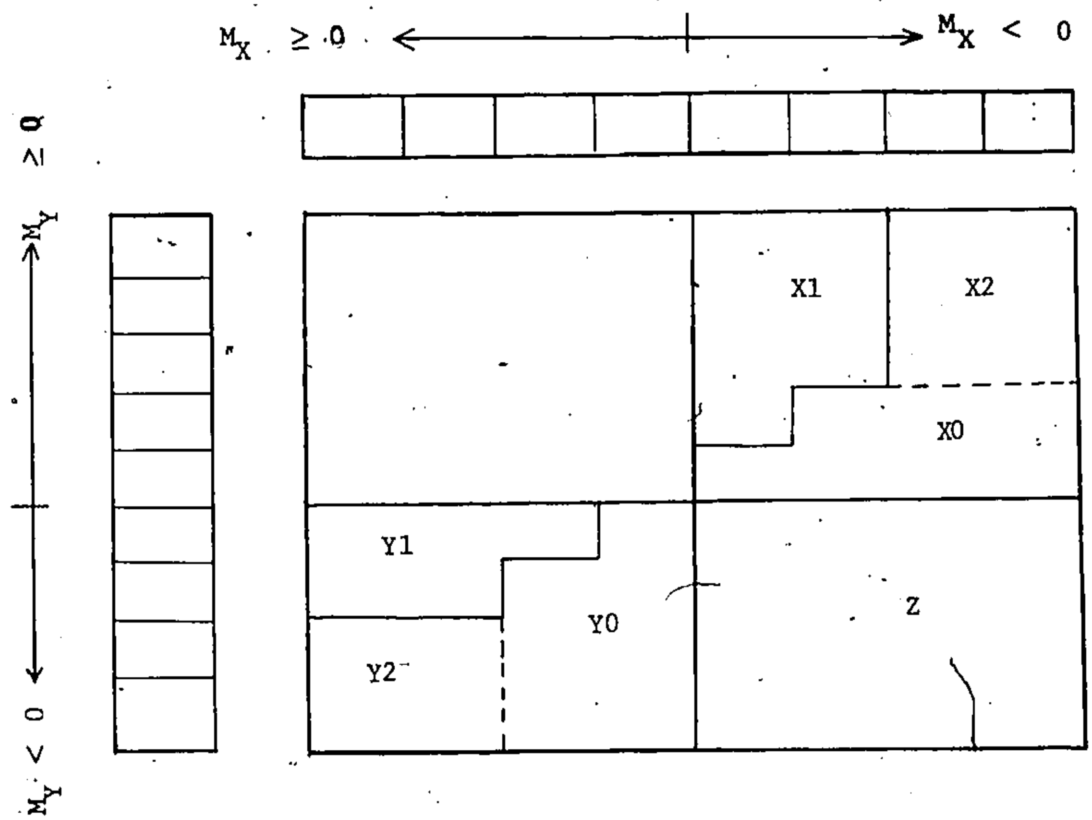


Figure 21: Schematic representation of joint probability matrix for the two interconnected systems

From Figure 21 one can easily compute the LOLPs of the two systems with and without interconnection simply by summing the probability values of the appropriate zones. That is,

$$LOLP_X = X0 + X1 + X2 + Z$$

$$LOLP_Y = Y0 + Y1 + \cancel{Y2 + Z}$$

$$LOLP_{X|Y} = X0 + X2 + Z$$

$$LOLP_{Y|X} = Y0 + Y2 + Z$$

2.7.2 Correlated load

The load model used in this case for the interconnected systems is based on the assumption that the loads in system X and system Y are correlated. In this case, the reliability of the two interconnected systems can be evaluated with the help of a series of combined state diagrams, one for each load level i.e. one load in system X and the corresponding load in system Y. The combined state diagram is constructed from the two capacity outage tables each of which is obtained from the individual system. The joint probability P_{ij} of any state (i,j) in the combined state diagram is given as

$$P_{ij} = P_{Xo_i} \times P_{Yo_j} \tag{2.23}$$

where

P_{Xo_i} = Probability of i-th state of the capacity outage table of system X.

P_{Yo_j} = Probability of j-th state of the capacity outage table of system Y.

For each pair of load levels the reserve in each system, which is the difference between the installed capacity and the load, will be different. Therefore, the dividing lines

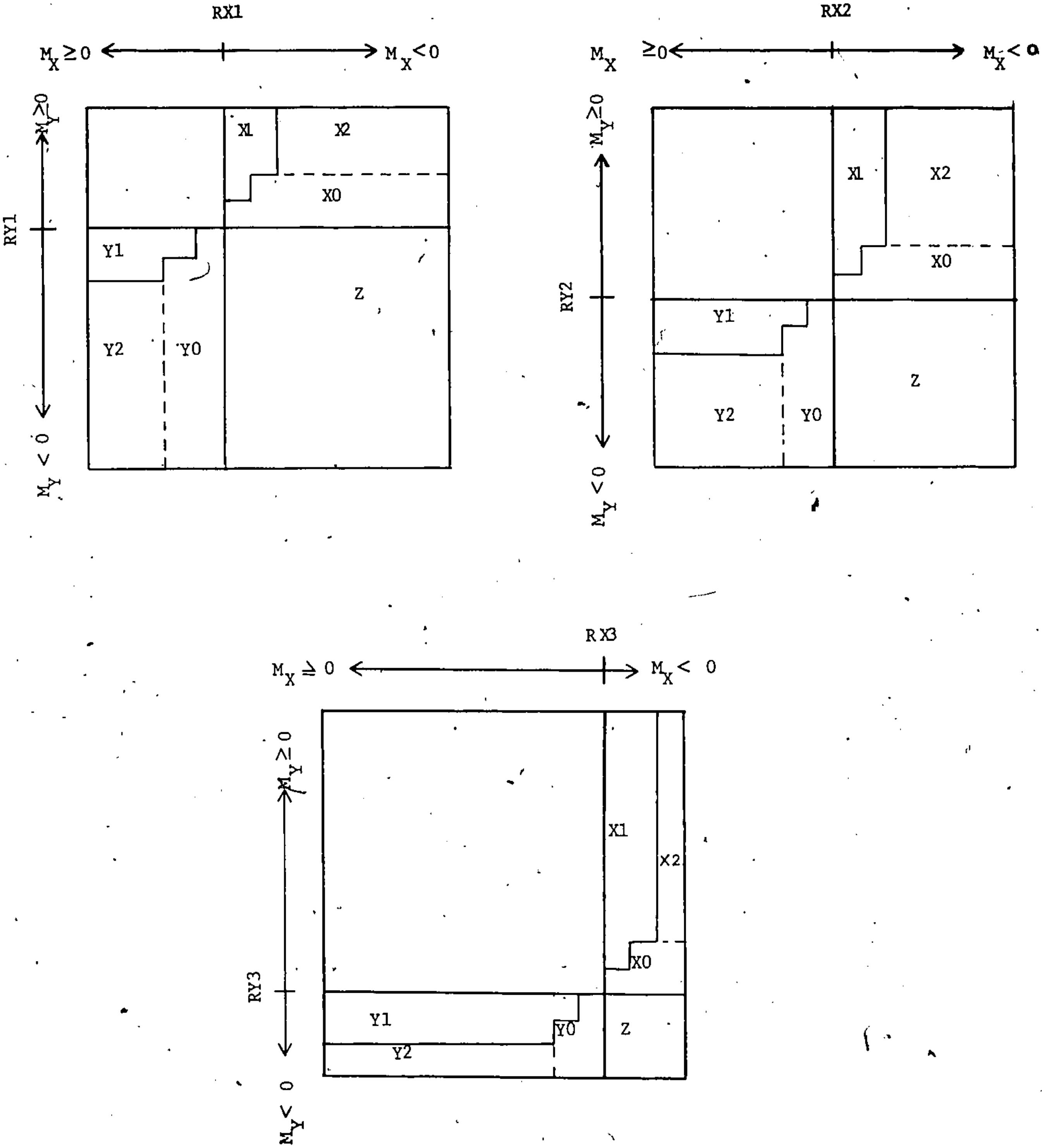


Figure 22: A space of combined state digrams, one for each load level

in the joint probability matrix to evaluate the LOLPs will be different for different load levels as shown in Figure 22. R_{X_i} and R_{Y_i} in Figure 22 represent the reserve in system X and in system Y, respectively, corresponding to the i-th load pair. The final value of LOLP is obtained by summing all the LOLPs calculated separately for each pair of load and multiplying by the avilability of the corresponding load. That is,

$$LOLP = \sum_{i} LOLP_i \times P_{L_i} \tag{2.24}$$

where

$LOLP_i$ = The loss of load probability evaluated from the combined state diagram corresponding to i-th load pair

P_{L_i} = The probability of occurrence of i-th load pair

The value of P_{L_i} is obtained from the joint distribution of load.

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2.8 CUMULANT METHOD

Like the segmentation method the joint distribution of demand is obtained in this method by sampling the historical demand curve at every hour. Let two random variables L_X and L_Y represent the loads of two systems, X and Y, respectively and $P_{L_X,L_Y}(l_X, l_Y)$ represents the joint probability density function of loads. The joint probability function $P_{L_X,L_Y}(l_X, l_Y)$ may be approxmited by a continuous function $f_{L_X,L_Y}(l_X, l_Y)$ in terms of bivariate Gram-Charlier expansions [44] as follows.

$$f_{L_X,L_Y}(l_X, l_Y) = (1/\sigma_X \sigma_Y) \sum_{i=0}^{\infty}\sum_{j=0}^{\infty} (-1)^{i+j} D_{ij} H_{ij} \quad \varphi(X) \qquad (2.25)$$

in which σ_X and σ_Y are the standard deviations of the marginal distributions of L_X and L_Y, respectively; D_{ij} are coefficients involving the central moments and H_{ij} are Hermite polynomials which are functions of the co-ordinates (l_X, l_Y) at any point. The bivariate Gram-Charlier series is described in details in Appendix C.

The outages of machines in system X and system Y are considered independent and random as well as independent of the load.

The basic procedure of this technique is to modify the moments of the marginal frequency distribution of demands of each system by the moments of the RVs corresponding to forced outages capacities for the set of generating units in the system. In this process of modification it was found convenient to utilize the additive property of cumulants to get the distribution of all the units of one system [24]. These cumulants are converted to moments which are used to get the joint moments of equivalent load. Using these joint moments in the bivariate Gram-Charlier expansion [44] the joint frequency distribution function of equivalent load $f_{EL_X,EL_Y}(el_X, el_Y)$ is obtained. Integration of this joint frequency distribution over appropriate limits gives the LOLPs of the two interconnected systems.

2.9 NUMERICAL EVALUATION

The proposed segmentation method is applied to two realistic interconnected systems and the numerical results obtained by using the above mentioned techniques will be compared in terms of computational efficiency and accuracy. The sensitivity study of the proposed method is included in this section. Also included in this section are two studies on the impact on the reliability of two interconnected systems. One study assumes a time difference between the two systems and the other considers only the daily peak of the two systems instead of considering the hourly loads.

2.9.1 System model

One of the two systems chosen in the numerical evaluation is the IEEE reliability test system (IEEE-RTS) [45]. However, some of the unit capacities are rounded off to give an installed capacity of 3400 MW. The second system is a hypothetical system with an installed capacity of 2950 MW. Generation data of the two systems are given in Appendix B. The modified IEEE-RTS and the hypothetical system in what follows are referred to as system X and system Y, respectively.

For the load model the IEEE-RTS hourly data are used. For system X thirteen Winter weeks and for system Y thirteen Summer weeks are utilized assuming that these loads occur in the same period. The hourly data of IEEE-RTS is given in Appendix B.

Note that the common factor for the generating unit capacities, considered in this numerical example, is 10 MW. Therefore, in the segmentation method the size of the segments must be 10 MW in case of independent loads and 10 MW x 10 MW in case of correlated loads. In the application of the recursive algorithm a 5 MW step increment is utilized in case of independent loads while for correlated loads, capacity outage tables are developed using 10 MW step increment. Capacity states with cumulative probabilities less than 10 E-10 are neglected. The grid size for the numerical integration was 50 MW X50 MW in the case of the cumulant method.

2.9.2 Comparison of results obtained by using different techniques

Table 2 shows the results obtained by using the segmentation method and the recursive method assuming the loads of the two systems as independent. In this case loads are slightly modified to avoid the enormous capacity margin states in the recursive method. The modified loads are such that they are divisible by 5 i.e. the IEEE-RTS hourly loads are rounded off to the nearest value divisible by 5.

Table 3 shows the comparison of results using the recursive method, the cumulant method and the segmentation method assuming that the loads are correlated. Note that in this case the exact values of loads are used. Note also that the cumulant method gives negative values of LOLP (this is a drawback of methods utilizing the Gram-charlier or similar series expansions).

Yet in another example, the FORs of some of the units of IEEE-RTS are increased: the FORs of two 400 MW nuclear units, one 350 MW coal unit and three 200 MW oil units are increased to 20%, 15% and 12%, respectively. The LOLPs are again calculated using the three different methods:

TABLE 2

Comparison of LOLPs for the case of independent loads(loads rounded off to nearest value divisible by 5)

tie line capacity	recursive method		segmentation method	
(MW)	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$
0.0	0.00274	0.06543	0.00274	0.06543
100	0.00143	0.04533	0.00143	0.04533
200	0.00079	0.03026	0.00079	0.03026
300	0.00050	0.01942	0.00050	0.01942
400	0.00037	0.01219	0.00037	0.01219
500	0.00032	0.00775	0.00032	0.00775
600	0.00031	0.00505	0.00031	0.00505
700	0.00030	0.00337	0.00030	0.00337
800	0.00030	0.00243	0.00030	0.00243

TABLE 3

Comparison of LOLPs for the case of correlated loads (actual loads)

	recursive method		cumulant method		segmentation method	
tie line (MW)	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$
0.0	.00280	.06609	.00153	.06876	.00280	.06610
100	.00156	.04583	.00033	.04556	.00156	.04585
200	.00097	.03076	-.00022	.02880	.00097	.03077
300	.00072	.02000	-.00045	.01737	.00072	.02000
400	.00062	.01283	-.00053	.01002	.00062	.01284
500	.00058	.00850	-.00055	.00558	.00058	.00850
600	.00057	.00594	-.00056	.00305	.00057	.00594
700	.00057	.00441	-.00056	.00170	.00057	.00442

segmentation, recursive and cumulant methods. The results are compared in Table 4. Note that in this case the negative LOLPs obtained by the cumulant method are avoided. The results obtained by the segmentation method are almost equal to those obtained by the commonly used recursive technique. However, note the computer times as given in Table 5. The CPU times, as shown in Table 5, are required for the different methods in the computer AMDAHL 470/V78.

TABLE 4

Comparison of LOLPs for the case of correlated loads with increased FORs

	recursive		cumulant		segmentation	
tie line (MW)	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$
0.0	.01124	.06608	.00983	.06876	.01124	.06610
100	.00704	.04626	.00561	.04624	.00704	.04628
200	.00473	.03171	.00338	.03029	.00473	.03171
300	.00354	.02148	.00230	.01969	.00354	.02149
400	.00299	.01484	.00184	.01308	.00299	.01485
500	.00275	.01096	.00166	.00922	.00275	.01097
600	.00266	.00875	.00160	.00711	.00266	.00875
700	.00263	.00750	.00159	.00604	.00263	.00750

TABLE 5

Comparison of CPU times

	Independent load		Correlated load		
	Recusive method	Segmentation method	Recursive method	Cumulant method	Segmentation method
avg. CPU time (Sec)	150	1.8	193	11	17

2.9.3 Sensitivity study of the segmentation method

The effects of the segment size on the accuracy of the results and on the computational efficiency are investigated in this section. The systems described in Section 2.9.1 are also used for this study and the correlated loads are considered. Table 6 offers ~~a~~ comparison of CPU times for different segment sizes for a tie line capacity of 200 MW.

TABLE 6

Comparison of CPU times for different segment sizes at 200 MW tie line capacity

$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	segment size	CPU times (sec)
.00097	.03077	5 MW x 5 MW	43.13
.00097	.03077	10 MW x 10 MW	11.87
.00087	.03011	20 MW x 20 MW	3.68
.00918	.03080	50 MW x 50 MW	1.26
.00055	.02897	100 MW x 100 MW	0.90

2.9.4 Effect of using daily peak load in the load model

The system model described in Section 2.9.1 is also considered here. But instead of using the hourly load model the daily peak loads are utilized to evaluate the LOLPs. From each working day's load profile two pairs of loads are used; one for the daily peak of system X and the corresponding load of system Y and other for the daily peak of system Y and the corresponding load of system X. The correlation between the two systems peak loads are considered.

In Table 7, the results obtained by using the daily peak loads are compared with those obtained by using the hourly loads.

TABLE 7

Comparison of results obtained for daily peak and for the hourly loads

	Daily peak load		Hourly load	
Tie line capacity	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$
0.0	.00963	.17316	.00280	.06610
100	.00561	.12348	.00156	.04585
200	.00363	.08611	.00097	.03077
300	.00274	.05759	.00072	.02000
400	.00241	.03778	.00062	.01284
500	.00228	.02589	.00058	.00850
600	.00222	.01891	.00057	.00594

2.9.5 Effect of time difference between two systems

Another study is made to incorporate time differences between the two systems. The time difference simply means that the geographical distance is such that at the same instant the clock time in two systems is not the same. In this case the peaks in two systems may not occur at the same instant for the same type of loads. To simulate this time difference all the loads of system Y considered in Section 2.9.1 are shifted by two hours retaining the loads of system X in the original positions. LOLPs are also evaluated by shifting the loads of system Y by four hours. The results for these two load models are shown in Table 8. The LOLPs obtained by using the original loads are also shown in the same Table for comparison purposes.

TABLE 8

Change in LOLPs due to time difference

	original load		Load diversity			
			Two hours shift		Four hours shift	
tie line capacity	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$
0.0	.00280	.06610	.00280	.06610	.00280	.06610
100	.00156	.04585	.00155	.04584	.00153	.04583
200	.00097	.03077	.00096	.03075	.00093	.03072
300	.00072	.02000	.00070	.01997	.00066	.01991
400	.00062	.01284	.00060	.01278	.00056	.01270
500	.00058	.00850	.00056	.00843	.00052	.00833

2.10 OBSERVATIONS AND DISCUSSION

Tables 2,3 and 4 clearly indicate that the results obtained by using the segmentation method are the same to the ~~results obtained~~ ained by using the recursive method.

It is also clearly observed from Table 5 that the segmentation method is computationally much faster than the conventional method. The average CPU time of the proposed method for the independent load case is about 80 times faster than the recursive method and for the correlated load case the segmentation method is about 10 times faster. The cumulant method, takes less time than the proposed method. The segmentation method takes about 1.2 times more CPU time than the cumulant method. In many cases, however, the results obtained by the cumulant method can not be trusted since it may yield negative numbers. This is especially true for systems with low FORs (< 5%) [46].

The results obtained by the segmentation method for a two area interconnected system show that when independency between the loads of the two systems is assumed the method is much faster than the existing technique. However, by considering the correlation between the loads of the two systems some computational efficiency of the method is lost . In any case the segmentation method is much faster compared to the commonly used method based on the recursive algorithm. Furthermore, the method is almost as computationally efficint as the cumulant method based on the bivariate Gram-Charlier expansion but without the inacuracies of such expansion.

It is observed from Table 6 that the coarser segment size in the proposed method produces approximate results but increases computational efficiency. Note that for a coarse segment size of 100 MW x 100 MW the errors are 43.21% and 5.84% for $LOLP_{X|Y}$ and $LOLP_{Y|X}$, respectively. However, the computational time is 13 times less than that for a segment size of 10 MW x 10 MW (recall, that 10 MW x 10 MW is the proper segment size for this numerical example). As expected the results for a 5 MW x 5 MW and 10 MW x 10 MW segment sizes are identical since both segment sizes are common factors of generating unit capacities.

From Table 7 it is observed that there is a large difference in the results when daily peak loads are utilized instead of using the hourly load variation.

Time zone differences between the two interconnected systems improve the reliability as can be expected, as shown in Table 8. It indicates that the diversity in the peaks allows for more assistance between the two systems over the tie line. It is also observed from Table 8 that as the diversity in the loads of the two interconnected systems increases the improvement in the reliability also increases. For example, at 500 MW tie line capacity, $LOLP_{X|Y}$ and $LOLP_{Y|X}$ are decreased (comparing with original load) by 3.44% and 0.82% ,respectively for the shift of the load by two hours and for four hours shift, $LOLP_{X|Y}$ and $LOLP_{Y|X}$ are decreased by 10.34% and 2.0%, respectively.

Chapter III

EXPECTED ENERGY PRODUCTION COSTS OF TWO INTERCONNECTED SYSTEMS

3.1 INTRODUCTION

Interconnection of electric utilities is one way in which economic benefits can be achieved. Such benefits include lower production cost due to energy interchange and displacement of costly and/or scarce resources as well as deferral of plant construction due to reduced reserve requirements.

In generation expansion planning the evaluation of production costs is one of the most important aspects whether the system is interconnected with others or meeting its own demand. It is mentioned in Section 1.1 that the plans which satisfy the desired reliability level must be evaluated on the basis of economics in order to identify a plan that impacts on the utility as a whole in the most favourable manner, and the evaluation of production costs is one of the major factors in cost analysis. However, an accurate method does not exist to evaluate the production costs of interconnected systems although the methods to evaluate the production cost of a single system are reasonably well developed.

The simplest way to evaluate the interconnected systems would be to consider them as a single system where the number of generating units is the sum of the units in the constituent systems, and the load is the total of the loads in these systems. However, by treating the interconnected systems as a single system it is not possible to simulate the tie line limitation, contractual limitation, and load correlation between the interconnected systems, and also it is not possible to compute the benefits of interconnection for individual systems. Therefore, a different approach of evaluation seems necessary.

In this chapter, an efficient method is proposed to evaluate the production costs of two interconnected systems considering loads between the two systems correlated. The proposed methodology is based on the notions of the modified demand and the residual tie line capacity. The segmentation technique [19] is utilized in this method to evaluate the expected energy generation, the expected unserved energy, the economic benefits and the production costs of two interconnected systems. The method is also applied to evaluate the production cost as well as the expected energy generation utilizing the bivariate Gram-Charlier expansion. The results obtained by utilizing the above two techniques are compared. Studies have also been made to investigate the effects on production costs of two interconnected systems such as:

1. Tie line capacity variation;
2. Time zone difference;
3. Increased FORs of generating units;
4. Different load management schemes;
5. Joint ownership of generation.

The last two studies will be discussed in Chapter IV and V, respectively.

3.2 COST ANALYSIS

The main components which enter into the determination of revenue requirements for a given alternative plan are:

1. Capacity cost;
2. Production cost;
3. Operating and maintenance cost.

Besides these three major factors, timing of unit additions is also important in cost analysis.

3.2.1 Capacity cost

Utilities require a very high investment in plant and equipment in comparison to annual revenues and annual operating cost. The relative high plant investment is the most distinguishing characteristic of an electric utility. Factors which affect capacity costs include type of generating unit, unit size, site, depreciation, taxes, labor costs, environmental requirements, capital and financing costs.

The capacity cost of unit i, denoted by CC_i, is usually defined as follows [2]:

$$CC_i = FCR_i \; UC_i \; C_i \quad (\$) \tag{3.1}$$

where

FCR_i = fixed charge rate,

UC_i = unit capacity cost (\$/MW),

C_i = capacity of i-th unit (MW).

The fixed charge rate reflects the annual amount of revenue requirement to pay for the facility over its lifetime, and is designed to meet the annual costs associated with capital investment. In general, the FCR consists of depreciation, income taxes, and annual return (interest on debt, dividents on preferred stock and earnings on common equity).

Unit capacity cost is self-explanatory; however, it is important to be aware of the relative capacity cost for various types of units. The capital cost of hydro units varies considerably depending upon such variables as unit size, site and others. Their capital cost, however, tends to be higher than that of corresponding thermal units. In order to convert the capacity cost into equivalent annual revenue requirements, a carrying charge rate is used. The carrying charge reflects the levelized annual amount of revenue required to pay for the facility over its life span. The carrying charge rate begins to flow once construction begins, which is not uniform over time.

3.2.2 Production cost

The estimation of the energy production cost of an electric utility is by far the most complex part of cost analysis associated with a particular expansion plan. It depends on the loading order procedure, availability of units, and the demand for electric energy in the day to day operation of the system, which are highly variable and unpredictable, especially when the calculation extends far into the future.

The production cost associated with a given generation plan includes both fuel cost (FC), which reflects the cost of fuel, and start-up and shut-down costs. Start-up and shut-down costs are fixed operating costs and can be estimated quite accurately from past data and they are added to the system fuel cost at the end. In what follows the expected fuel cost is referred to as the production cost.

Production cost can be accurately determined only if:

1. A realistic load model for the future period under consideration can be developed.
2. The commitment of the units to supply load can be realized in such a way that reflects the actual operating procedures and conditions.

Various methods have been developed for predicting future production costs, among which probabilistic load duration method by far is the most popular method in North America. The method represents the actual operation of the system more accurately than other methods. It considers the forced outage rates and maintenance scheduling of the generating units as well as the merit order loading.

In the probabilistic method, it is recognized that the energy generated by a particular unit is represented by the area it occupies beneath the load duration curve. The position of the unit depends, however, on the availability of the more efficient units which take priority in loading. If a large base load unit is unavailable, a less efficient unit is likely to generate more than otherwise.

The objective of the probabilistic method is to determine the expected amount of energy generated by each unit, by taking into account the availability reflected through vertical shifts [14,15] in the load duration curve weighted by the associated probabilities.

In Section 3.3 a probabilistic method will be developed to evaluate the production costs of two interconnected systems.

3.2.3 Operating and Maintenance cost

These costs include all fixed non-production costs such as labour, supplies and materials required to maintain and operate a plant, and in the case of nuclear plants also the heavy water upkeep plus any additional security and insurance costs, etc. The operation and maintenance cost also include all variable costs associated with preparing the unit for daily operation. Essentially these costs do not depend on the amount of energy produced. Data regarding operating and maintenance cost are estimated on an annual basis in order to be integrated into the economic analysis.

3.3 METHODOLOGY OF EVALUATION OF PRODUCTION COSTS OF TWO INTERCONNECTED SYSTEMS

This section presents a method to evaluate the production costs of two interconnected systems. The first step in the method is to decide on the loading order of the units.

3.3.1 Merit order loading

In Chapter II, the full capacity of each unit was (committed) convolved without following any order because the interest was to find LOLP, which is independent of the way units are convolved. But here, the order of the units is important to achieve the lowest production cost.

The basic tenet in the loading order procedure is that the generating units are loaded in the order of their average incremental costs. The most efficient generating unit is the one with the lowest incremental cost; this generating unit is loaded first. Next in line are generating units with higher average incremental costs.

The economics of the generating units are governed by performance parameters such as the heat rate and price of fuel used by the units. In actual operation, it is seldom economical to commit one generating unit fully before another unit is loaded. To simulate this fact the generating unit capacities are segmented into several capacity blocks. Typically, lower segments of large units are committed before committing any particular unit completely.

The reason for segmented commitments is the basic shape of the heat rate (HR) curve; a typical heat rate curve is shown in Figure 23.

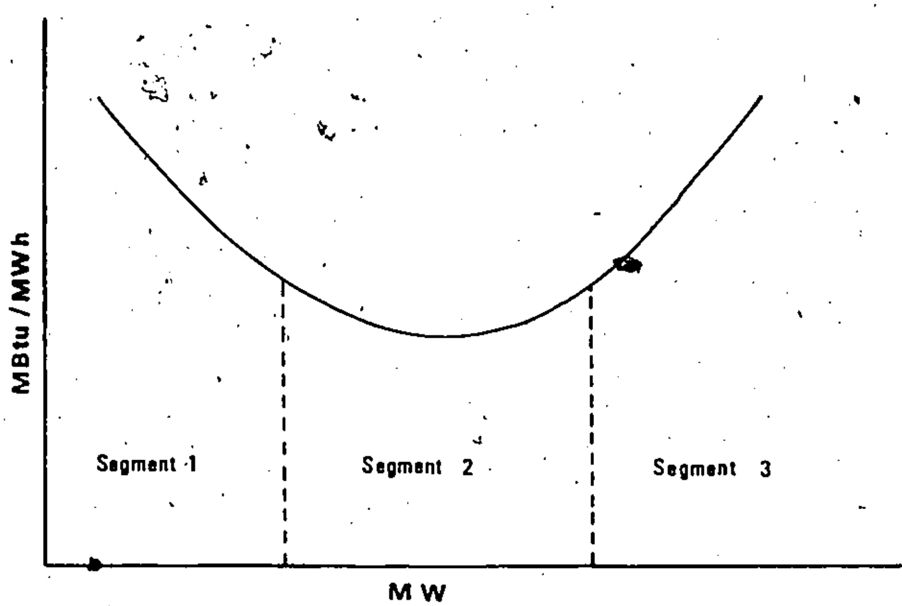

Figure 23: Typical heat rate curve

It is clear from the shape of the heat rate curve that the second segment corresponds to higher efficiency, since fewer Btus are required for each MWh of energy produced. From this basic HR curve the input/output ,I/O, curve can be obtained by multiplying every y-axis value by its corresponding x-axis value. A typical I/O curve is depicted in Figure 24.

By differenciating the I/O curve with respect to load (L), the incremental heat rate (IHR) curve is obtained. A typical IHR curve is depicted in Figure 25.

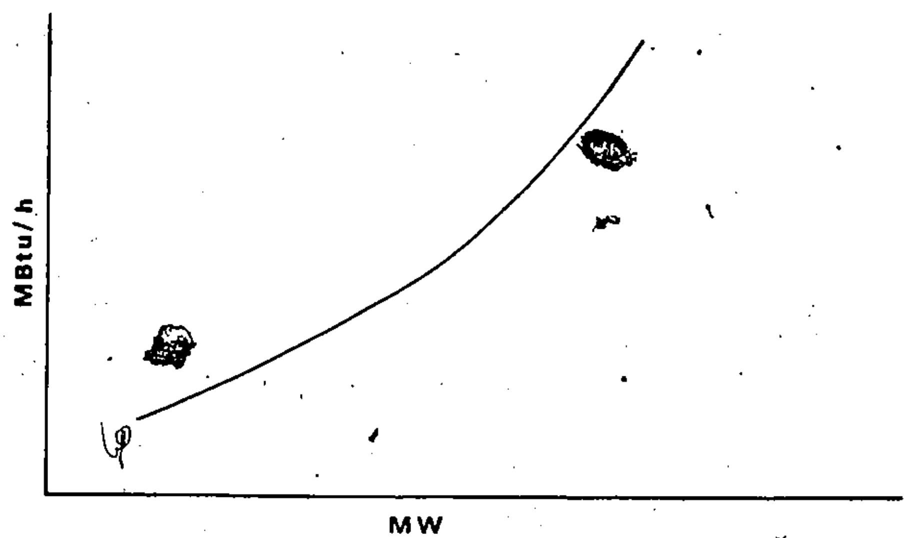

Figure 24: Typical Input/Output curve

To quantify these relationships, one has for unit i,

$$I/O_i = l_i \, HR(l_i) \tag{3.2}$$

and

$$IHR_i(l_i) = \frac{d\, I/O_i}{d\, l_i} \tag{3.3}$$

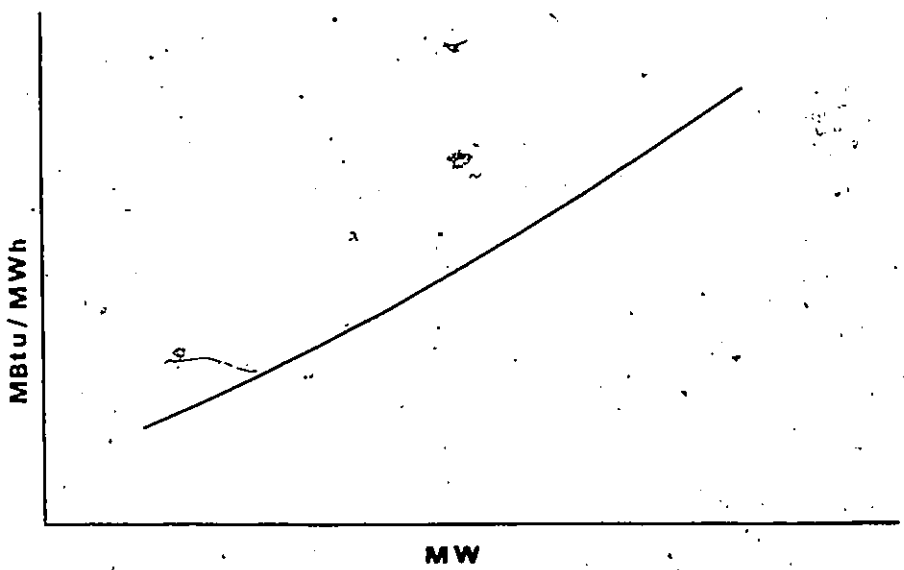

Figure 25: Typical incremental heat rate (IHR) curve

In a special case when the HR curve is assumed to be constant then

$$IHR_i = HR_i \tag{3.4}$$

From the incremental heat rate the incremental fuel cost (IFC) can be calculated for unit i as

$$IFC_i = \frac{IHR_i \; UFC}{HV} \qquad (3.5)$$

where

IFC_i = Incremental fuel cost of unit i in \$/ MWh,

UFC = Unit fuel cost in \$/bbl or \$/ton,

HV = Heat value in MBtu/bbl or MBtu/ton.

In the commitment of capacity blocks of a generating unit it is important to note that lower capacity block should be committed before any higher capacity block is loaded, since physically it is not possible to commit any higher capacity blocks before committing all the lower capacity blocks of that unit.

In the case where the segment to be committed, the segments already convolved in must be deconvolved out, then combined with the segments about to be committed, and the combination convolved back into the load distribution. The reason for this cumbersome procedure is to avoid making any unit more reliable than it really is. If unit i of capacity C_i is divided into three segments, each having an availability of p_i and FOR of q_i, then the probability of loosing the capacity C_i of the total unit is q_i^3 . But $q_i^3 < q_i$, thus the unit looks more reliable than it really is. To avoid this problem, all lower segments of the same unit committed before commiting a higher segment must be deconvolved out from the equivalent load distribution, and combined with the segment about to be committed. For example, if the capacity of the unit j is split into two segments of capacities C_{j1} and C_{j2}, each with a FOR of q_j and availability P_j then according to equation (2.11), the PDFs of the equivalent loads are as follows:

$$f_{j1}(le) = \sum_i P_{L_i} \left\{ P_j \, \delta(le - l_i) + q_j \, \delta(le - (l_i + C_{j1})) \right\} \tag{3.6}$$

$$f_{j2}(le) = \sum_i P_{L_i} \left\{ P_j^2 \, \delta(le - l_i) + P_j \, q_j \, [\, \delta(le - (l_i + C_{j1})) + \delta(le - (l_i + C_{j2}))\,] + q_j^2 \, \delta(le - (l_i + C_j)) \right\} \tag{3.7}$$

where

$f_{j1}(le)$ = PDF of the equivalent load after committing the first block of j-th unit,

$f_{j2}(le)$ = PDF of the equivalent load after commiting the second block of the j-th unit ,

P_{L_i} = The probability of occurrence of i-th load .

However, if the entire unit is convolved without segmenting its capacity, then the PDF of the equivalent load f_j (Le) becomes

$$f_j\ (le) = \sum_i P_{L_i} \left\{ P_j\ \delta(le - l_i) + q_j\ \delta(le-(l_i + C_j)) \right\} \tag{3.8}$$

Clearly, the two expressions (3.7) and (3.8) are not the same, and in particular the coefficients of the last terms are not the same. In the first case, where the FOR = q_j^2, the two segments look more reliable than the unit itself, which is absurd. As it is stated earlier, the solution is to deconvolve out the first segment, combine its capacity with the second, and convolve the combination back.

The merit order loading procedure to be followed in the case of two interconnected systems is to select the generating unit, from among the set of generating units in both systems, with the lowest average incremental cost, to be loaded first. This is followed by the next most efficient unit among this set and so on. That is, if S^X generators from a total of n generators of system X and S^Y generators from a total of m generators of system Y are already committed, then the average incremental cost of the most efficient unit, i, in the set of the units which are to be committed, λ_i is expressed as

$$\lambda_i \overset{\Delta}{=} \lambda_{X \wedge Y} = \text{Min} \left\{ \lambda_{X_k} \; , \; \lambda_{Y_k} \right\}_{k=1..(n-S^X) \quad k=1..(m-S^Y)} \qquad (3.9)$$

where

$$i=(S^X+1) \quad \text{OR} \quad (S^Y+1) \qquad (3.10)$$

For i as given in equation (3.10), the corresponding value of j will be

$$j= S^Y \quad \text{OR} \quad S^X \qquad (3.11)$$

where, j represents the number of units already commited in the other system (later this system will be referred to as the importing system).

3.3.2 Expected Export or Import

The fundamental strategy subsumed in the evaluation of export is that each system must keep its own interest paramount. That is, a utility will only export power to another as long as it has excess capacity after having met its own load. An electric utility experiencing loss of load will therefore not undertake export of capacity.

The principal factors which affect the capacity transactions between the two interconnected systems are:

1. The unserved load or demand of the exporting system,
2. The unserved load or demand of the importing system,
3. The capacity of the committing (being committed) unit, and
4. The residual tie line capacity of the exporting system.

In what follows the system in which a generating unit is being committed is referred to as the exporting system, and the other system will be referred to as the importing system.

The residual tie line capacity for a system is that capacity of the tie line that remains at any stage of the loading process after having been utilized by the previously committed generating unit or units from the system. Let the random variables RTC and e represent the residual tie line capacity and export or import, respectively . Then the residual tie line capacity may be expressed by the following equation

$$RTC = TC - e \tag{3.12}$$

where TC is the tie line capacity.

The possible export may be calculated from the relation

$$e = \text{Min} \left\{ Z_1 , Z_2 , Z_3 \right\} \tag{3.13}$$

where $e \geq 0$,

and the random variables (RVs) Z_1, Z_2, Z_3 represent excess generation of the exporting system, residual tie line capacity of the exporting system, and unserved demand of the importing system, respectively. Clearly, Z_1 can be obtained by subtructing the equivalent load from the capacity of already committed units plus the capacity of the unit being committed of the exporting system, while Z_3 can be obtained by subtructing the capacity of already committed units from the equivalent load of the importig system. These three RVs (Z_1, Z_2, Z_3) are given by

$$Z_1 = \sum_{k=1}^{S^E+1} C_k^E - EL^E \qquad (3.14)$$

$$Z_2 = RTC^E \qquad (3.15)$$

$$Z_3 = EL^I - \sum_{k=1}^{S^I} C_k^I \qquad (3.16)$$

The superscript E and I refer to the exporting and importing systems, and EL refers to the random variable (RV) of the equivalent load, and S is the total number of units already committed.

3.3.3 Expected energy generation

The expected energy generation by a given generating unit is obtained by evaluating the difference in unserved energies before and after commitment of the generating unit. This energy generation clearly must include the energy required to meet its own demand as well as demand for export.

Therefore, to incorporate the export or import the equivalent load (before commitment of any unit it is simply load) of exporting as well as of importing system must be modified. The modified equivalent load of exporting system, $\tilde{EL}^{E}$, can be expressed as

$$\tilde{EL}^{E} = EL^{E} + e \qquad (3.17)$$

Similarly, the modified demand of the importing system, $\tilde{EL}^{I}$, can be obtained by subtructing the import (export) from the present load or equivalent load, i.e

$$\tilde{EL}^{I} = EL^{I} - e \qquad (3.18)$$

Modification of the demand due to the export or import may be exemplified as follows: Consider the chronological demand of system X and system Y as shown in Figure 13. Sample the load at every hour which will give rise to the distribution of demand as depicted in Figure 26.

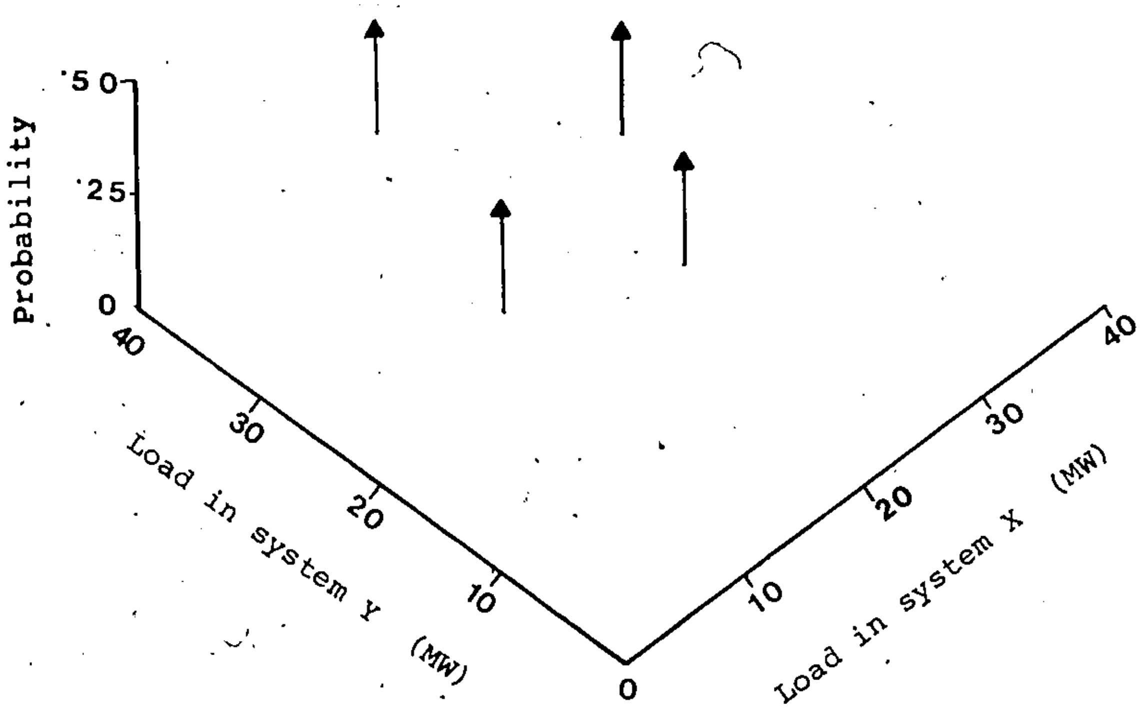

Figure 26: Joint distribution of demand

Consider also the generating system as given in Table 9. In this system of generating units, the 20 MW unit of system X

TABLE 9

Generation System Description

System X				System Y			
No of units	Capacity (MW)	FOR	Avg. Incremental cost ($/MWh)	No of units	Capacity (MW)	FOR	Avg. Incremental cost ($/MWh)
1	20	0.2	5	1	30	0.1	6
1	30	0.1	7	1	25	0.3	9

has the lowest average incremental cost. Therefore, this 20 MW unit will be the first in the merit order i.e. ~~this unit~~ should be loaded first. This is followed by the 30 MW unit of system Y, and so on.

The possible export from system X to system Y must be evaluated before convolving the selected unit. The only load point for which the export is possible is the one which corresponds to 15 MW load ~~in~~ system X and 25 MW load in system Y. For this load point 5 MW capacity can be exported from system X to system Y. To incorporate this export the demand of exporting as well as the importing system must be modified according to the equations (3.17) and (3.18), respectively. The modified demand ~~is~~ depicted in Figure 27. The impulse shown by the dotted line represents the impulse which has been shifted to a new position. The impulse at (15,25) is shifted to (20,20). But the probability value attached to this impulse remains unchanged. After the modification of the demand the unit is convolved.

In a similar vein the rest of the generating units in the merit order must be loaded.

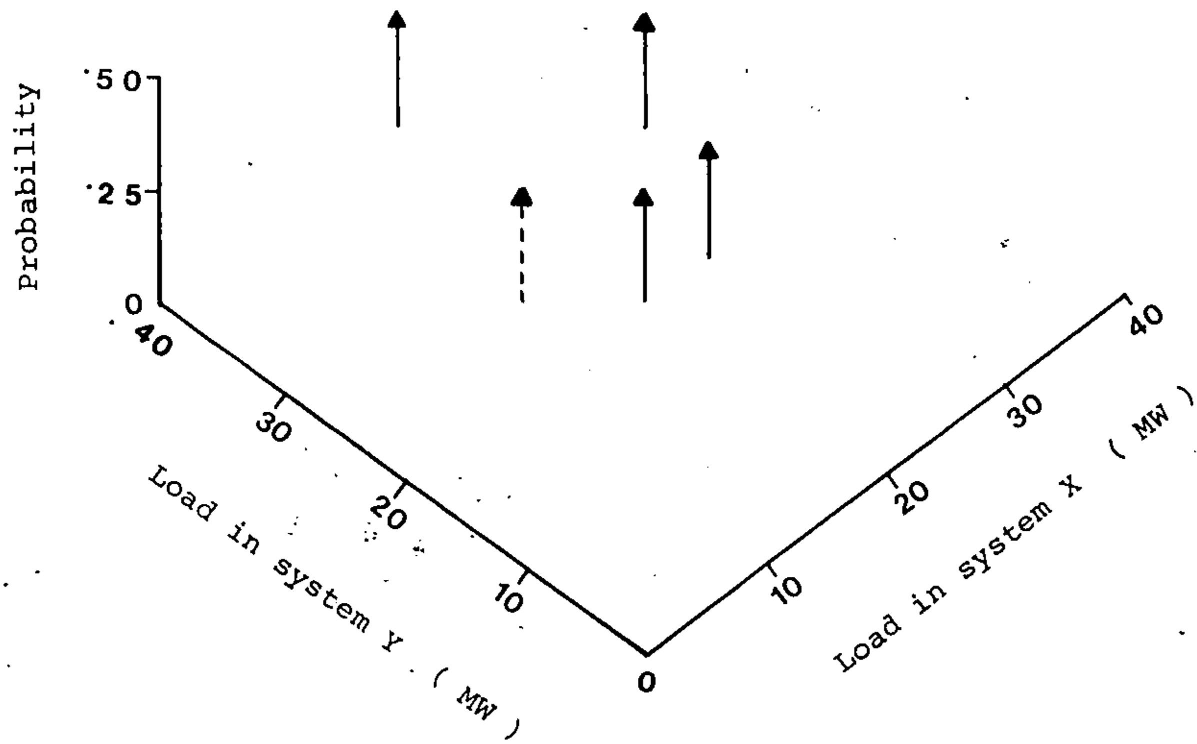

Figure 27: Modified demand

The next step in this method is to calculate the expected energy generated by the committing unit. In order to compute the expected energy generation the expected unserved demands before and after convolving the unit must be computed. The expected unserved demand of the exporting system before commitment of its i-th unit is given by

$$USD_{i-} = \int_{C_{t_1}^E}^{UP_1^E} \int_0^{UP_1^I} (el^E - C_{t_1}^E) \; f_{i-1,j}(el^E, el^I) \, del^E \, del^I \, , \tag{3.19}$$

where, $f_{i-1,j}(el^E, el^I)$ is the probability density function of the equivalent load before commitment of the i-th unit and $C_{t_1}^E$ is the total commited capacity in the exporting system. $C_{t_1}^E$ can be expressed as

$$C_{t_1}^E = \sum_{k=1}^{S^E} C_k^E \tag{3.20}$$

The upper limit of equation (3.19) is given by

$$UP_1^E = PK^E + C_{t_1}^E \tag{3.21}$$

$$UP_1^I = PK^I + \sum_{k=1}^{S^I} C_k^I \tag{3.22}$$

where PK refers to the peak load of the system. After commitment of the i-th unit in the exporting system, its unserved demand USD_i must be recalculated. USD_i can be expressed as

$$USD_i = \int_{C_{t_2}^E}^{UP_2^E} \int_0^{UP_1^I} (el^E - C_{t_2}^E) \; f_{i,j}(el^E, el^I) \, del^E \, del^I \tag{3.23}$$

Clearly, after commitment of the i-th unit in the exporting system total committed capacity should be increased by the capacity of the committing unit, i.e

$$C^E_{t_2} = \sum_{k=1}^{S^E+1} C^E_k \qquad (3.24)$$

The upper limit UP^E_2 in equation (3.23) is given by

$$UP^E_2 = PK^E + C^E_{t_2} \qquad (3.25)$$

and $f_{i,j}(el^E, el^I)$ is the PDF of the equivalent load after commitment of i-th unit. The expected energy, ES_i, supplied by the i-th unit would be the product of time period of study, T, and the difference of unserved demands calculated in equation (3.19) and in equation (3.23).

Therefore, ES_i can be expressed as

$$ES_i = T\ [\ USD_i - USD_i\ ] \qquad (3.26)$$

The global expected energy generation, GES, is equal to the sum of the energies supplied by all the units of two interconnected systems. That is,

$$GES = \sum_{i=1}^{n+m} ES_i \tag{3.27}$$

Finally, the production cost (expected fuel cost) for i-th generating unit is

$$EC_i = \lambda_i \quad ES_i \tag{3.28}$$

The global production costs are then as follows:

$$GEC = \sum_{i=1}^{n+m} EC_i \tag{3.29}$$

Global production cost savings at any tie line capacity are the difference between the global production costs at that tie line and at zero MW tie line. That is,

$$GS_{TC} = GEC_{TC_0} - GEC_{TC} \tag{3.30}$$

where

GS_{TC} = Global production cost savings at tie line capacity TC MW,

GEC_{TC_0} = Global production costs at zero MW tie line capacity

GEC_{TC} = Global production costs at tie line capacity TC MW.

As mentioned in Section 2.5, for S^E and S^I generating units in the exporting and in the importing systems, respectively, the total number of impulses, to be dealt with, is equal to the number of load impulses times $2^{S^E} \times 2^{S^I}$. From such a joint distribution of equivalent load the evaluation of production cost, by keeping track of residual tie line capacity and export or import, is a formidable task. Therefore, in what follows the segmentation technique which is proven to be faster and accurate [19] will be extended and utilized to evaluate the production cost by applying the above mentioned method. The proposed method will also be applied to evaluate production costs and expected energy generation using cumulant technique.

The segmentation and the cumulant techniques (methods), to evaluate the production costs of two interconnected systems, will be developed seperately in Sections 3.4 and 3.5, respectively.

3.4 SEGMENTATION TECHNIQUE

The starting point of the segmentation technique (method) for evaluating the production costs of two interconnected electric power systems is the chronological load for each system for the time period under consideration. As mentioned in Section 2.6, these loads are sampled every hour (or any other appropriate time interval) and assigned equal probability of occurrence. For example, assuming a choronological load for a day, the joint occurrence of each sampled load point is assigned a probability value of 1/24.

The next step in the application of the segmentation technique is the selection of the segment size. In this case load plane is subdivided into a grid structure. Each grid or segment has sides of equal size. Thus, the load (equivalent load) plane is covered with squares. The side of each square corresponds to the largest common factor of the generating unit capacities of both system as well as the tie line capacity. Note that a segment has been defined as having two dimensions.

As the loads of the two interconnected systems are sampled, the segments are filled up. Each segment contains the following parameters:

1. The zeroth moment, or probability, of load which corresponds to the joint occurrence of the load sample; this corresponds to the joint segment probability.
2. The first moments, or expected values, of load for each system corresponding to load samples in the range of particular segment.

3. The first moments, or expected values, of residual tie line capacity for each system.

Initially the first moment of residual tie line capacity for each system is set equal to the product of the corresponding tie line capacity and the joint probability of the particular segment. For convenience, the joint probability of the load points spanning a segment is attached to the probability of occurrence of the particular segment.

Although the loads of the two interconnected systems are assumed correlated, the random failures of the respective generating units are assumed independent of each other as well as independent of load. The random failures of the generating units are taken into account through the equivalent load approach mentioned in Section 2.5. In this approach the capacity on outage of a generating unit, expressed as a random variable, is added to the load to give rise to an equivalent load.

3.4.1 Process of Convolution

In the segmentation method, the process of convolution is simply effected by shifting each segment appropriately as a generating unit, in the loading order, is committed to meet the equivalent load. For two interconnected systems a two dimensional approach is necessary; that is, the direction of shift depends on the system that the generating unit belongs to. Assuming that the X-direction is attributed to system X and the Y-directon to system Y, then the generating unit in system X will shift the corresponding moments along the x-axis. Conversely, a generating unit belonging to system Y will shift the corresponding moment along the Y-axis.

Considering the (i,j)th segment and assuming a generating unit of capacity C MW belonging to system X to be committed, the shifted first moment of load (or equivalent load) of system X of the $(i,j+w_X)$th segment may be expressed as,

$$\hat{m}_{i,j+w_X} = m_{i,j} + C \times P_{i,j} \qquad (3.31)$$

where

$$w_X = C/\Delta C \qquad (3.32)$$

and

$P_{i,j}$ = The joint probability of the (i,j)th segment before the shift.

ΔC = The largest common factor of capacity, selected from all the generating units of both systems, and the tie line capacity.

$m_{i,j}$ = First moment, or expected value, of load

corresponding to the (i,j)th segment
before the shift.

Clearly, the first moments of load of system Y remain unchanged by the commitment of a generating unit belonging to system X. Note that in what follows the zeroth moment or the first moment of a segment are referred to as the unnormalized ones. Note also that the segment probability stay unchanged by the shift since the zeroth order moments are not affected by the shift.

To obtain the final parameters for each segment, after a generating unit i is loaded, the procedure is as follows:

1. Multiply the segment's parameters, before the generating unit is committed, by the generating unit's availability (1-q).
2. Multiply the shifted segment's parameters by the generating unit's FOR =q .
3. Sum the two previous results for each segment to obtain the final parameters after commitment of the i-th unit.

The procedure described above is same as in Section 2.6 .

3.4.2 Export or Import

The possible export $e_{i,j}$ at any segment (i,j), may be calculated using the equation (3.13) . But the variables $\tilde{Z}_1$, $\tilde{Z}_2$ and $\tilde{Z}_3$ representing the possible export of capacity are redefined as follows,

$$\tilde{Z}_1 = \sum_{k=1}^{S^E+1} C_k^E \; P_{i,j} - m_{i,j}^E (EL) \tag{3.33}$$

$$\tilde{Z}_2 = m_{i,j}^E (RTC) \tag{3.34}$$

$$\tilde{Z}_3 = m_{i,j}^I (EL) - \sum_{k=1}^{S^I} C_k^I \; P_{i,j} \tag{3.35}$$

In equation (3.34), $m_{i,j}^E (RTC)$ is the first moment of residual tie line capacity of the exporting system. It is obtained initially (before commiting any generating unit) by multiplying the segment's probability by the tie line capacity.

3.4.3 Expected energy generation

As is mentioned earlier, the expected energy generation by a given generating unit is obtained by evaluating the difference in unserved energies before and after the commitment of the generating unit. This energy generation must include export. To accomplish this the first moments of load must properly be modified. This will be done in what follows.

Assume that the (k-1) generating units has been committed (from either system). At this stage, the modified first moments of load (equivalent load) for the exporting and importing system, $\tilde{m}^{E}_{i,j}(EL)$ and $\tilde{m}^{I}_{i,j}(EL)$, respectively, are given by

$$\tilde{m}^{E}_{i,j}(EL) = m^{E}_{i,j}(EL) + e_{i,j} \tag{3.36}$$

$$\tilde{m}^{I}_{i,j}(EL) = m^{I}_{i,j}(EL) - e_{i,j} \tag{3.37}$$

The first moments of residual tie line capacity must also be modified in those segments where transactions take place. The modification of the first moments of residual tie line capacity is necessary to carry the information to the corresponding segment of equivalent load.

The modified first moments of residual tie line capacity are

$$\tilde{m}^{E}_{i,j}(RTC) = m^{E}_{i,j}(RTC) - e_{i,j} \tag{3.38}$$

$$\tilde{m}^{I}_{i,j}(RTC) = m^{I}_{i,j}(RTC) + e_{i,j} \tag{3.39}$$

where it is assumed that simultaneous transactions in both directions cannot occur.

The expected unserved energy of the exporting system before committing the k-th generating unit is given by

$$USE_{k^-} = T \sum_{i,j=k_1}^{i=t_Y,\, j=t_X} \left(\tilde{m}^{E}_{i,j}(EL) - C^{E}_{t} \times P_{i,j}\right) \tag{3.40}$$

where C^E_t is the total capacity of the already committed generating units of the exporting system, and T is the time period under consideration.

In equation (3.40), C^E_t is given by the expression

$$C^E_t = \sum_{k=1}^{S^E} C^E_k \tag{3.41}$$

The limits K_1 , t_X and t_Y in equation (3.40) are given by

$$K_1 = \min \left\{ w^E , w^I \right\} \text{ but } K_1 \geq 1 \tag{3.42}$$

where

$$w^E = \sum_{k=1}^{S^E} C^E_k / \Delta C \tag{3,43}$$

$$w^I = \sum_{k=1}^{S^I} C^I_k / \Delta C \tag{3,44}$$

$$t_X = \frac{\sum_{k=1}^{n} C_k^{(X)} + TC}{\Delta C} + 1 \tag{3.45}$$

$$t_Y = \frac{\sum_{k=1}^{m} C_k^{(Y)} + TC}{\Delta C} + 1 \tag{3.46}$$

where $C_k^{(X)}$ and $C_k^{(Y)}$ are in reference to generating units in system X and Y, respectively.

Note that for any segment (i,j) the unserved demand must be positive. That is,

$$(\tilde{m}_{i,j}^{E}(EL) - C_t^{E} \times P_{i,j}) \geq 0 \tag{3.47}$$

The unserved energy, after commitment of the k-th generating unit USE_k, may be evaluated by using equation (3.40). However, the parameters must include this new generatig unit,

The expected energy generation by the k-th generating unit is given by

$$ES_k = USE_{k-} - USE_k \tag{3.48}$$

3.4.4 Application of Segmentation technique for clarification

The segmentation technique may be exemplified in reference to the load profile shown in Figure 13. Consider the generation system shown in Table 9.

Assigning to each sampled hourly load equal probability (1/4 in this case), the joint probabilities of the sampled loads are obtained. For instance, the joint occurrence of the 20 MW load in system X and the 40 MW of load in system Y has probability of 1/4 .

The segment size corresponds to a square with sides of 5 MW. Thus ΔC = 5 MW. These segments are filled up as the load is sampled and the final result is shown in Figure 28. Segments below base load are not required .

The first, second and third rows of each segment, as exemplified in Figure 28, contain the following:

1. First row -the segment's probability,
2. Second row - the first moments of load for both systems(system X shown first) ,
3. Third row - the first moments of residual tie line capacity for both systems (system X shown first).

Recall that the first moment is the product of the probability and the numerical value of the RV.

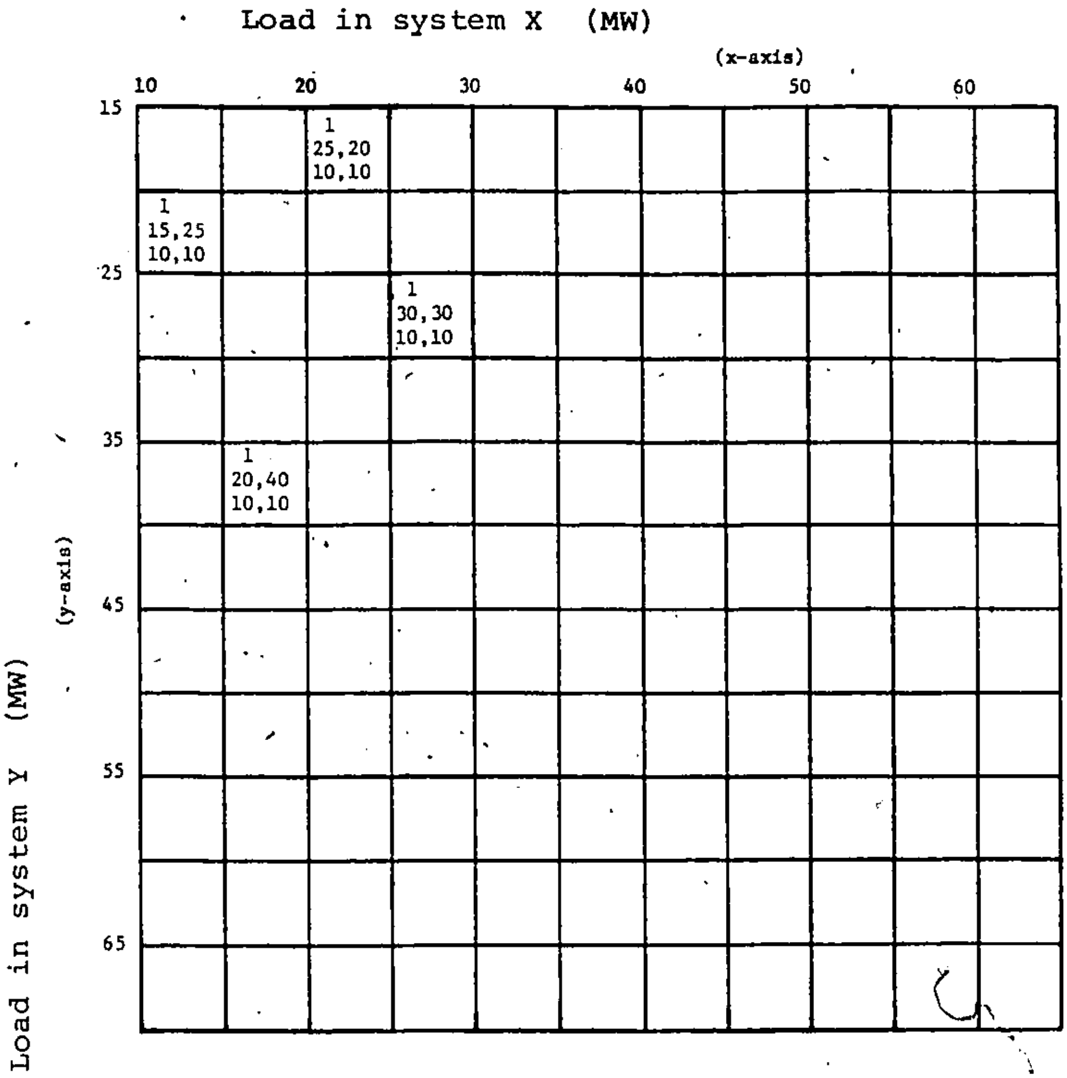

Figure 28: Schematic representation of joint probabilities and corresponding first moments (all numbers in the boxes to be divided by 4)

From Table 9 the loading order can be easily deduced. The 20 MW generating unit of system X is loaded first. This is followed by the 30 MW generating unit of system Y; then the 30 MW generating unit of system X and finally the 25 MW generating unit of system Y.

Before convolving the 20 MW generating unit the possible export must be evaluated using equation (3.13) and equation (3.33) to equation (3.35). The only segment for which export is possible is the one which corresponds to the 15 MW load in system X: the first segment in second row. For this segment, 5 MW can be exported from system X to system Y. The modified first moments of load are, therefore,

$$(15+5)/4 = 20/4 \text{ for system X}$$

and

$$(25-5)/4 = 20/4 \text{ for system Y .}$$

Since system X is exporting 5 MW over the tie line, its modified first moment of residual tie line capacity is

$$(10-5)/4 = 5/4 \quad \text{(in accordance with (3.38))}$$

Similarly, for system Y the modified first moment of residual tie line capacity is

$$(10+5)/4 = 15/4 \quad \text{(in accordance with (3.39))}$$

Considering the modified first moments of load, the expected unserved energy of system X before committing the 20 MW generating unit is

$$USE_{X_{\bar{1}}} = 4(25+20+30+20)/4$$
$$= 95 \text{ MWh}$$

The convolution process is shown in Figure 29. Only the first moments of load of system X are modified according to equation (3.31)

In Figure 30 the distribution of load (equivalent load) and the corresponding first moments after the convolution of the 20 MW generating unit are shown. From Figure 30 the unserved energy is recalculated. Thus, the expected unserved energy, by equation (3.40) is given by

$$USE_{X_1} = 4 \times [\ (20+9+16+8+24+10+16+8)-$$
$$20 \times (0.8+0.2+0.8+0.2+0.8+0.2+0.8+0.2)]/4$$
$$= 31 \text{ MWh}$$

The expected energy generation of the 20 MW generating unit of system X is from equation (3.48) equal to

$$ES_{X_1} = 95-31 = 64.0 \text{ MWh}$$

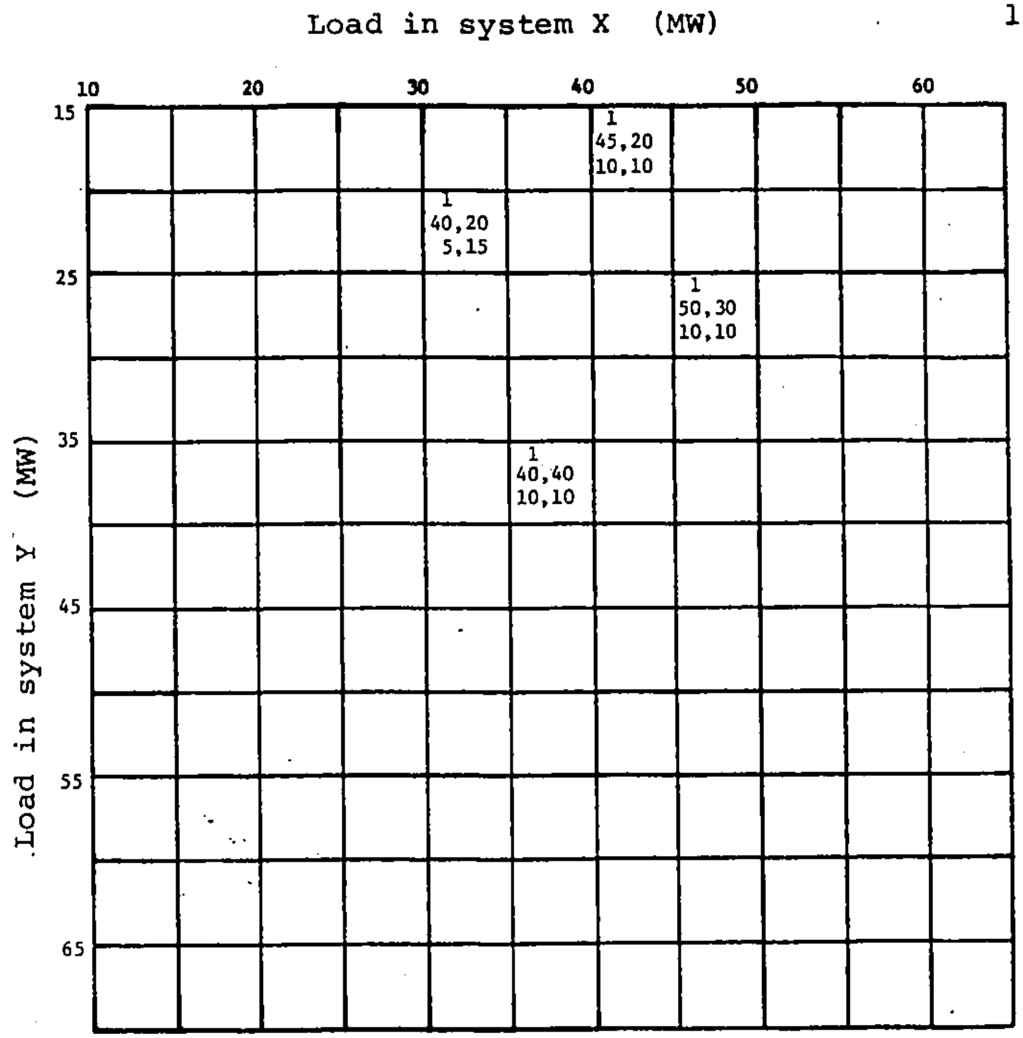

Figure 29: Shift of joint probability and first moments during the convolution of the 20 MW unit of system X (all numbers in the boxes to be divided by 4)

In a similar vein the rest of the generating units are loaded. The expected energies of the remaining units are given by

ES_{Y_1} = 97.20 MWh (30 MW unit of system Y)

ES_{X_2} = 32.40 MWh (30 MW unit of system X)

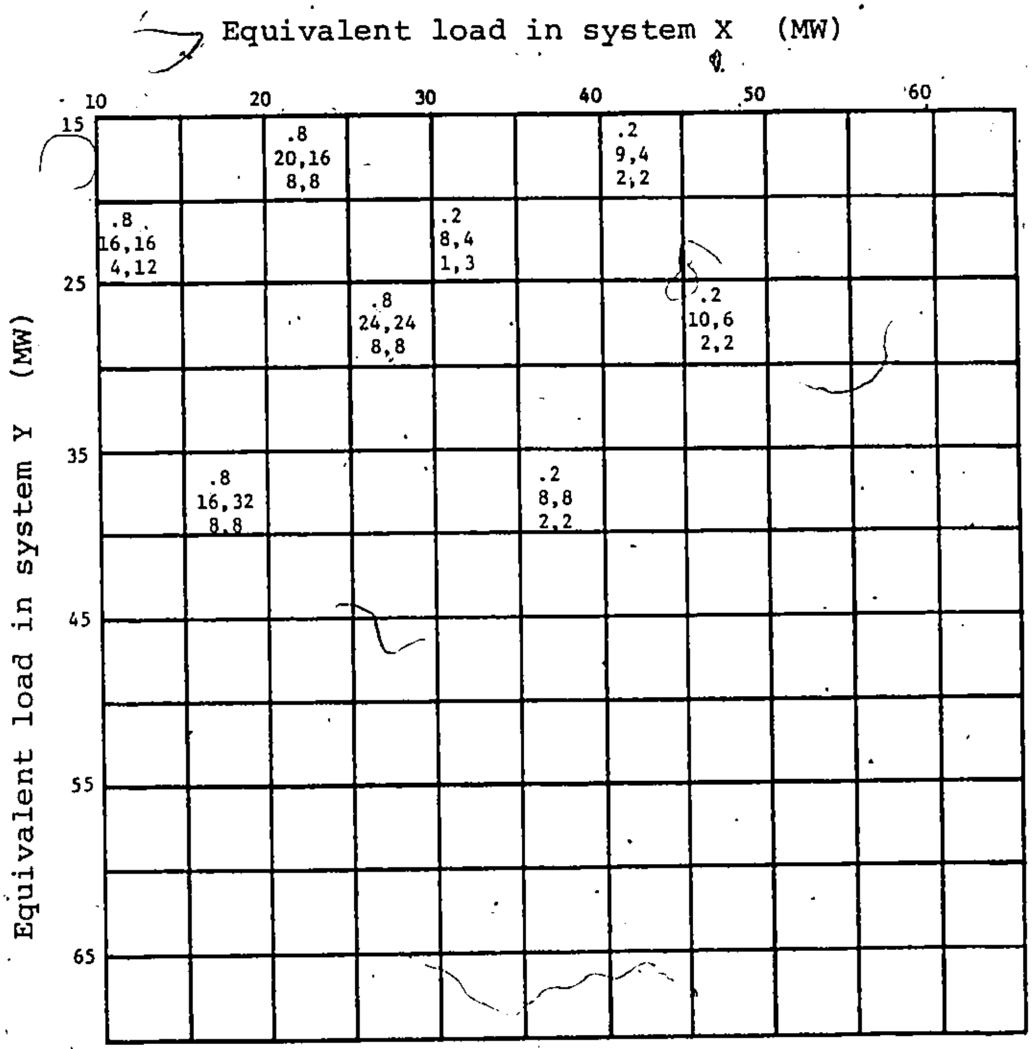

Figure 30: Distribution of load and resulting first moments after convolving the 20 MW unit (all numbers in the boxes to be divided by 4)

ES_{Y_2} = 6.66 MWh (25 MW unit of system Y)

3.5 CUMULANT TECHNIQUE

In this technique the discrete probability distribution of correlated load (equivalent load) $P_{L_X,L_Y}(l_X, l_Y)$ of system X and system Y is approximated by a continuous function $f_{L_X,L_Y}(l_X, l_Y)$ in terms of the bivariate Gram-Charlier expansion.

$$f_{L_X,L_Y}(l_X, l_Y) = 1/(\sigma_X, \sigma_Y) \sum_{i=0}^{\infty} \sum_{j=0}^{\infty} (-1)^{i+j} D_{i,j} H_{i,j} \varphi(Z) \tag{3.49}$$

where

$H_{i,j}$ = The Hermite Polynomials, functions of the coordinates (l_X, l_Y) at any load point.

$D_{i,j}$ = The coefficients involving the joint cumulants of L_X and L_Y.

$\varphi(Z)$ = The binormal function.

The detailed description of the bivariate series is given in Appendix C.

The demand plane is then subdivided into a number of grids whose size depends on the size of the system. The probability value of each grid is calculated from the continuous function, and the initial PDF of demand is replaced by another with impulses at the centre of these grids. The value of the impulse of each grid corresponds to its probability value.

In what follows the computational steps to be followed in the cumulant technique are described.

1. Step 1 From the chronological load curve evaluate the joint probability mass function, and then calculate the load cumulants.
2. Step 2 Select the most efficient unit, from the set of the units of both systems, which are to be commmitted, using equation (3.9).
3. Step 3 Compute the possible export from the equation (3.13) and modify the corresponding loads of the exporting and the importing system using equations (3.17) and (3.18), respectively. Also modify the residual tie line capacities of the exporting and the importing systems simultaneously. Modify the residual tie line of the importing system by adding the import with the residual tie line capacity (initially tie line).
4. step 4 Calculate the unserved demand of the exporting system using equation (3.19). Convolve the generating unit by adding the machine cumulants to the load (equivalent load) cumulants. The convolution procedure is described in the Appendix C.
5. Step 5 Recalculate the unserved demand using equation (3.23) and then calculate the expected energy supplied by the committing unit from (3.26).

3.6 NUMERICAL EXAMPLE

The proposed method is applied to two interconnected systems. The generation parameters of these two systems are the same as those of Section 2.9.1 with the exception that the installed capacity of system Y has been increased from 2950 MW to 3150 MW by increasing the capacity of the 150 MW oil fired unit to 250 MW and adding an additional 100 MW hydro unit. Load models considered for these interconnected systems are also the same as those mentioned in Section 2.9.1. Generating system data and hourly load data are given in Appendix B.

3.6.1 Expected Energy generation and Production Cost

The results shown in Table 10 and in Table 11 are obtained by using segmentation technique. The variations of expected energy generation and production costs for each system and for the global system with the tie line, are shown in Table 10 and Table 11, respectively. The results for unlimited tie line capacity corresponds to the case where the two systems are merged into one pool. The number of generating units of this pool is the sum of the generating units of system X and system Y, and the load of this pool is the sum of the loads of two constituent systems. Clearly, two constituent systems loose their identity. The last two columns of Table 10 are based on 'split-the-savings' principle (usual practice in North America). The results of sixth column in this table are obtained using the following relation;

$$\widehat{EC} = EC_O - 1/2\ GS$$

where,

$\widehat{EC}$ = Production cost shared by system X,

EC_O = Production cost of system X at zero MW tie line capacity,

GS = Global savings

Similarly, the last column of Table 10 is obtained from ;

$$\widetilde{EC} = EC_Y + EC_X - EC_O + 1/2\ GS$$

where,

$\widetilde{EC}$ = Production cost shared by system Y,

EC_Y = Production cost of system Y,

EC_X = Production cost of system X.

TABLE 10

Expected energy generation and production costs of individual systems

Tie line	Exp. Energy generation (GWh)		Production Cost (M$)		Production cost shared (M$)	
(MW)	system X	System Y	System X	System Y	System X	System Y
0.0	4162.6505	3949.5223	33.234	44.593	33.234	44.593
100	4332.9653	3785.3003	35.558	40.819	32.508	43.869
200	4487.9633	3634.0474	37.744	37.526	31.955	43.315
300	4616.8758	3507.3935	39.597	34.864	31.550	42.911
400	4715.6351	3409.9606	41.044	32.843	31.264	42.624
500	4780.6178	3345.7208	42.035	31.478	31.076	42.437
600	4815.2336	3311.4950	42.598	30.701	30.969	42.339
700	4830.0331	3296.8930	42.867	30.324	30.916	42.276

In Table 11, the decrease of global production costs with the increase of tie line capacity is clearly observed. It indicates that the use of cheap resources in the global system increases as the transfer facility increases. The saturation effect as the tie line is increased, can also be clearly observed from the same table. The decrease of global production costs is M$ 1.107 when the tie line capacity is increased from 100 MW to 200 MW, while the decrease of global production costs is only M$ 0.107 when the tie line is incresed from 600 to 700 MW. Since system X has, on average, lower FORs and incremental costs, it is expected

TABLE 11

Expected energy generation and production costs of the global system (* pooling operation)

Tie line (MW)	Expected energy generation (GWh)	Expected unserved Energy (GWh)	production cost (M$)
0.0	8112.1727	15.4503	77.828
100	8118.2657	9.3569	76.377
200	8122.0107	5.6123	75.270
300	8124.2693	3.3537	74.461
400	8125.5957	2.0273	73.888
500	8126.3386	1.2844	73.513
600	8126.7286	0.8940	73.299
700	8126.9235	0.6991	73.192
.	.	.	.
.	.	.	.
.	.	.	.
*	8127.0849	0.5376	73.093

that system X would export major part of the time. Table 10 confirms this; the expected energy generation and the production costs of system X increase with the increase of tie while those of system Y decrease.

In Figure 31, the economic benefit, in terms of global production costs savings and the global expected unserved energies are depicted. The global benefit in terms global production cost savings at any tie line capacity is obtained by comparing the production cost at that tie line and at zero MW tie line capacity. Note that there are other economic benefits that are not taken into consideration explicitly in this approach of computing economic benefit.

Among these are deferral of unit addition and decrease in service interruption because of increased reliability.

The maximum global production cost savings and the minimum global unserved energy are obtained when the tie line capacity is infinite. The upper line in Figure 31 represents the maximum expected saving or the upper limit of saving, and the lower line represents the minimum expected unserved energy or the lower limit of unserved energy. These two limits are obtained by considering two interconnected systems as a pool. From Figure 31 the saturation is also clearly observed as the tie line capacity is increased. The reliability of the global system reaches close to the lower limit. The global benefit also reaches close to the upper limit.

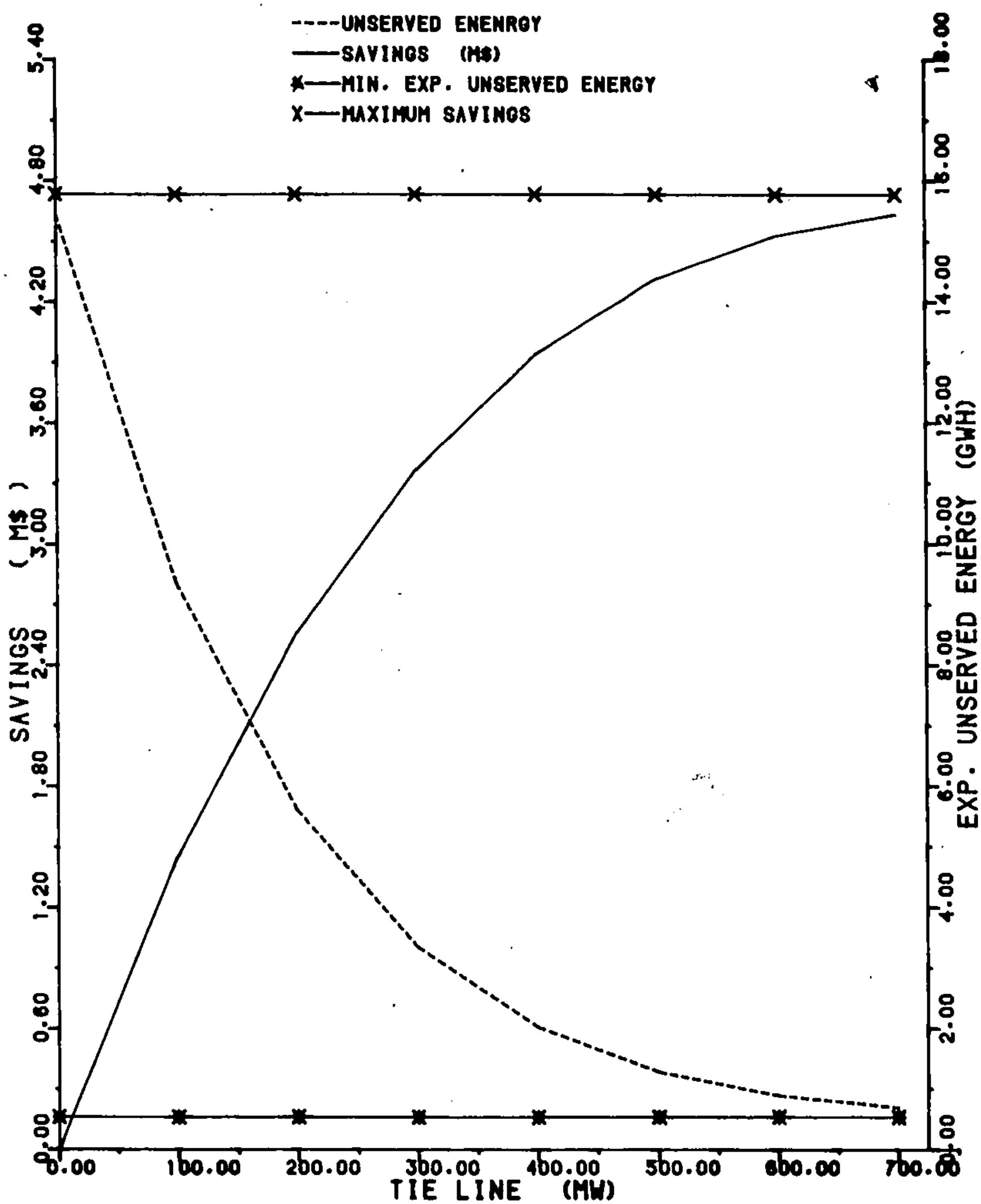

Figure 31: Global expected savings and expected unserved energy Vs. tie line capacity

3.6.2 Comparison of results obtained by cumulant and the segmentation technique

In this section, the expected energy generation and the production cost of individual system as well as global system, obtained by using the two different techniques, the segmentation and the cumulant, are compared.

In the cumulant method, 50 MW x 50 MW grids are used for the numerical evaluation. Since the common factor of the generating unit capacities and tie line capacity, considered in this numerical example, is 10 MW the segment size must be 10 MW x 10 MW for the segmentation method.

TABLE 12

Comparison of expected energies obtained by using the segmentation and the cumulant techniques

	Segmentation technique			Cumulant technique		
Tie	System X	System Y	Global	System X	system Y	Global
(MW)	(GWh)	(GWh)	(GWh)	(GWh)	(GWh)	(GWh)
0.0	4162.65	3949.52	8112.17	4091.14	3795.14	7886.28
100	4332.96	3785.30	8118.26	4184.34	3710.00	7894.34
200	4487.96	3634.04	8122.01	4261.64	3642.54	7904.18
300	4616.87	3507.39	8124.26	4277.64	3632.79	7910.43
400	4715.63	3409.96	8125.59	4285.16	3627.83	7912.99
500	4780.61	3345.72	8126.33	4285.16	3627.83	7912.99

TABLE 13

Comparison of production costs obtained by using the segmentation and the cumulant techniques

	Segmentation technique			Cumulant technique		
Tie (mW)	System X (M$)	System Y (M$)	Global (M$)	System X (M$)	system Y (M$)	global (M$)
0.0	33.234	44.593	77.828	32.1932	42.5443	74.7375
100	35.558	40.819	76.377	33.5008	40.6572	74.1580
200	37.744	37.526	75.270	34.5838	39.3019	73.8857
300	39.597	34.864	74.461	34.7651	39.1586	73.9237
400	41.044	32.843	73.888	34.8536	39.0838	73.9370
500	42.035	31.478	73.513	34.8536	39.0833	73.9370

From Table 12 and from Table 13 it is clearly observed that the cumulant technique gives only approximate results. In case of global production costs the results vary up to 3.97% and in case of global expected energy generation the results vary up to 2.78%, from the results obtained by the segmentation technique.

The two interconnected systems with zero MW tie line capacity are equivalent to two systems in isolation. Note that at zero MW tie line capacity the results obtained by the segmentation method are the same as those by the standard Booth-Baleriaux method [14]. It is also observed in case of the cumulant technique that the increase of global expected energy generation and the decrease of global production costs saturates at 400 MW tie line capacity while the small changes are observed even at 700 MW tie line capacity when the segmentation technique is used (shown in Table 10 and Table 11). The average CPU times are almost the same for the two methods. The segmentation method requires about 1 minute 29 seconds and the cumulant method requires about 1 minute 44 seconds.

since the results obtained by the segmentation method are accurate and the two methods require almost the same CPU times the segmentation method will be used in the later part of the thesis.

3.6.3 Effect of time zone difference and effect of varying forced outage rate

In this section, the effect of time zone difference between the two interconnected systems as well as the effect of varying forced outage rate (FOR) of generating units has been investigated.

As mentioned in Section 2.9.5, to simulate the time difference between the two systems all the original loads of system Y are shifted by two and four hours respectively, retaining the loads of system X in the original position.

On the average, the FORs of the generating units of system X are lower than those of system Y. The FORs of some of the generating units of system X are increased . This is shown in Table 14.

TABLE 14

Changed FORs of some of the generatng units of system X

type	Number of units	capacity (MW)	original FOR	increased FOR
Nuclear	2	400	0.12	0.20
Coal	1	350	0.08	0.15
Oil	3	200	0.05	0.12

The global production costs are calculated for the case of time zone difference and also for the case of increased FORs. The results for these two cases are shown in Table 15. The results for the original loads and original generating system are also shown in the same Table for comparison purposes.

The variation of global production cost savings with the increase of tie line capacity, for the time zone difference ,for the FORs variation and for the original systems is depicted in Figure 32.

Note that the effect of increased FORs of the generating units is to increase the global production costs or to decrease the production cost savings. For example, at 500 MW tie line capacity the production cost savings are reduced from 5.54% to 4.76%. Conversely, the effect of load diversity is to decrease the production costs i.e. to increase the the production cost savings. Also note that the

TABLE 15

Global production costs for shifted loads and for increased FORs

Original system		Load diversity: Load shifted by 2 hours	Load diversity: Load shifted by 4 hours	Increased FOR
Tie	Production cost	Production cost	production cost	Production cost
(MW)	(M$)	(M$)	(M$)	(M$)
0.0	77.828	77.828	77.828	79.461
100	76.377	76.316	76.237	78.149
200	75.270	75.152	74.999	77.167
300	74.461	74.296	74.078	76.465
400	73.888	73.683	73.412	75.981
500	73.513	73.273	72.956	75.675

increase in the load diversity increases the production cost savings. For example, at 500 MW tie line capacity the production cost savings are increased from 5.54% to 5.85% for the shift of loads by two hours and to 6.26% for the shift by four hours.

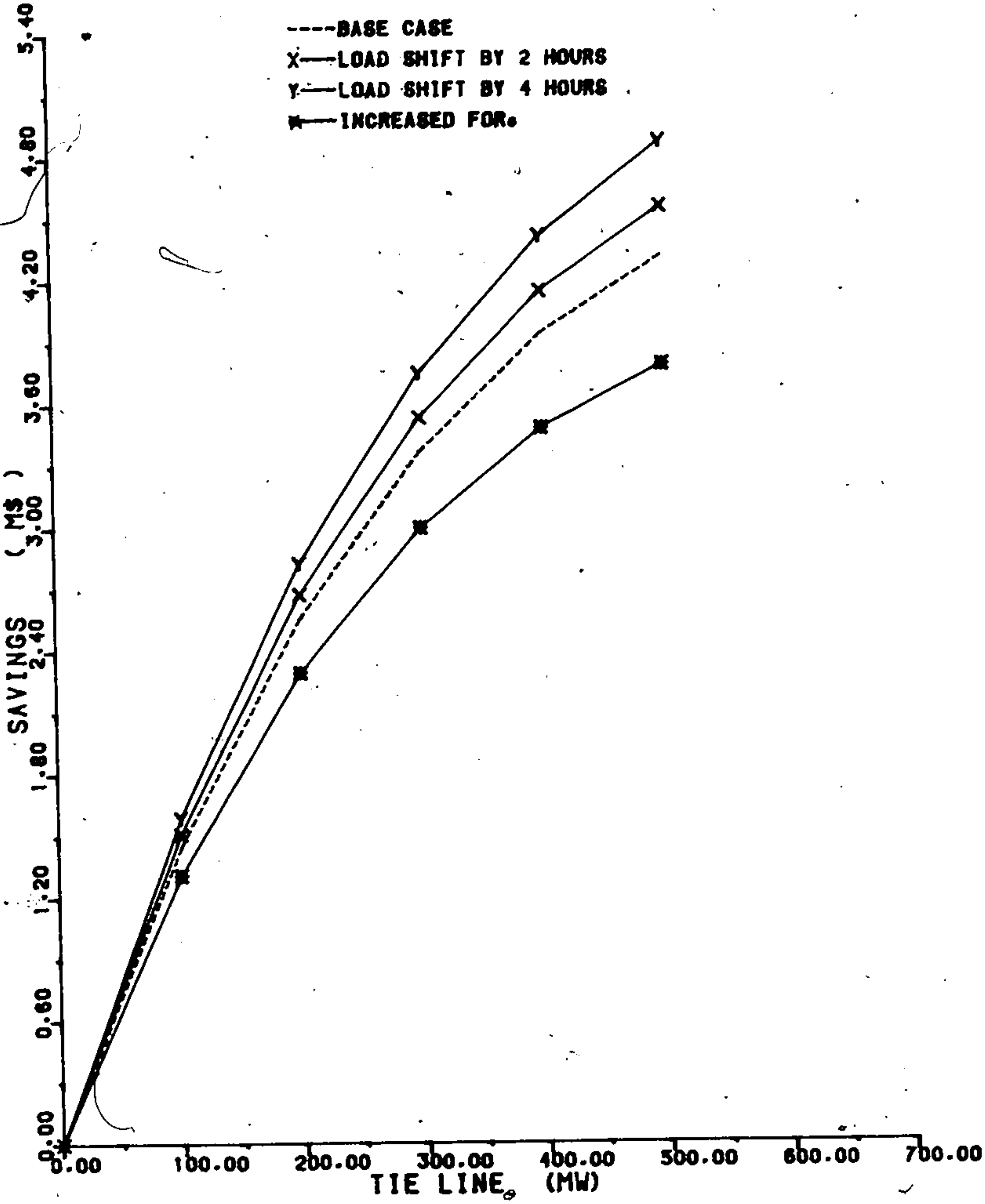

Figure 32: Global expected savings Vs. tie line capacity

3.7 DISCUSSION

In this chapter, a probabilistic method, to simulate the production cost of two interconnected systems considering load correlation between the two systems is proposed. The proposed method utilizes the segmentation technique to caculate the production cost and the expected energy generation for the global system as well as for the constituent systems. The production cost and expected energy generation are also calculated utilizing the bivariate Gram-Charlier expansion in the proposed method. The results obtained by utilizing the techniques, the segmentation and the cumulant, are compared. It is found that the segmentation method is better in terms of accuracy.

The proposed method is applied to two realistic interconnected systems. The decrease in global production costs and in global unserved energy with the increase of tie line capacity, as expected, is clearly observed from the results.

The method can be used, as seen from the results, not only to calculate the production cost and the expected energy generation of the global system and of the constituent systems, but also to calculate the reliability in terms of unserved energy and the economic benefits in terms of production cost savings.

The effect of time zone differences and the effect of variation of FORs of the generating units on production cost are investigated utilizing the proposed method. The increase in global production cost savings is observed in case of load diversity, as expected. The decrease in global production cost savings is observed in case of incresed FORs, as also expected.

Chapter IV

IMPACT OF LOAD MANAGEMENT ON RELIABILITY AND PRODUCTION COSTS

4.1 INTRODUCTION

Load management is the deliberate control or influencing of customer load in order to shift the time of use of electric power and energy. Reducing the system load at the time of peak by load management techniques is an alternative to installing peaking type of generating capacity. Any load that can be supplied as an off peak load instead of an on peak load will reduce the need for an increment of generating, transmission and distribution capacity.

Load management is not a new idea; it has been used by number of utilities for many years. But with a new economic situations and new technologies, more and more utilities are venturing into load management. An annual EPRI-DOE survey [28] shows that over 100 utilities are involved in variety of load management programs. Load management may not be attractive as a new power plant ; but recent studies [28-35] show that it is a resource that deserves utility consideration.

In this chapter, the methods outlined in Chapter II and Chapter III are applied to different load management schemes and the impacts, on the production costs and on the LOLPs of two interconnected systems, are studied.

4.2 REASONS OF LOAD MANAGEMENT

The classic arrangement of base-load, intermediate, and peaking units has long permitted utilities to meet the customer electricity at the least cost. But customer use patterns have been changing in recent years and electricity demand changing with them. Air conditioning, once a luxury, has become commonplace and air conditioners are major contributors to summer peak demand in most utilities. Its daily load shape is mostly dependent on outdoor temperature as shown in Figure 33. The air conditioner demand is also dependent on its size in relation to the space to be cooled.

Space- and water-heating demand has continued to grow, and many utilities' winter peak has become sharper as a result. A typical water heater load characteristic is shown in Figure 34. The load characteristic of an electric water-heater depends on utilization, capacity, inlet water temperature and thermostat setting .

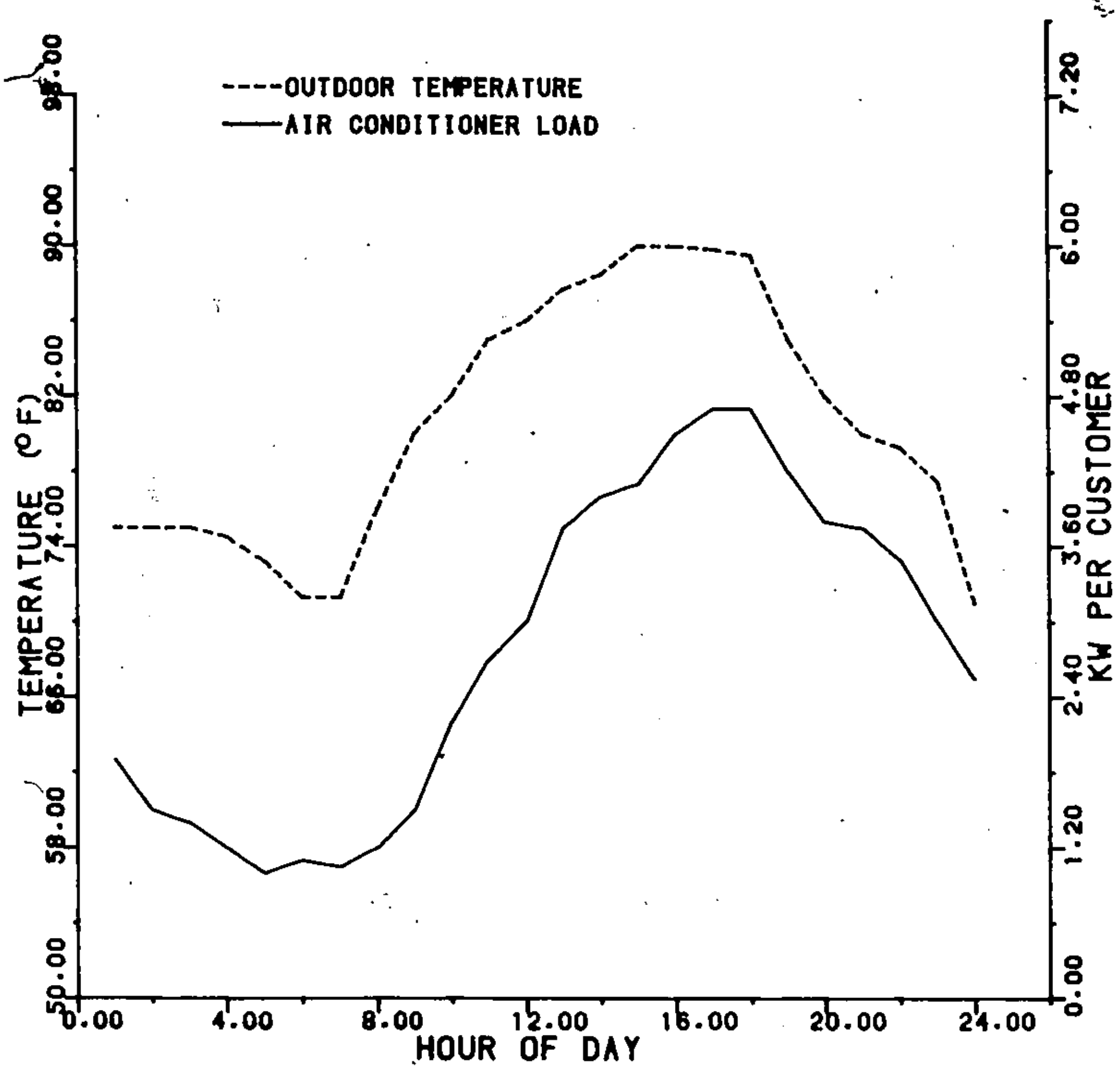

Figure 33: A typical air conditioner load

Traditional solutions to deal with growing peak periods are:

1. Construction of new power plants,
2. Interconnection with other systems,
3. Reducing system voltage (Brownouts).

But the economics of power plant construction have been changing . New plant construction costs more than ever because of materials, labor and, specially, financing costs.

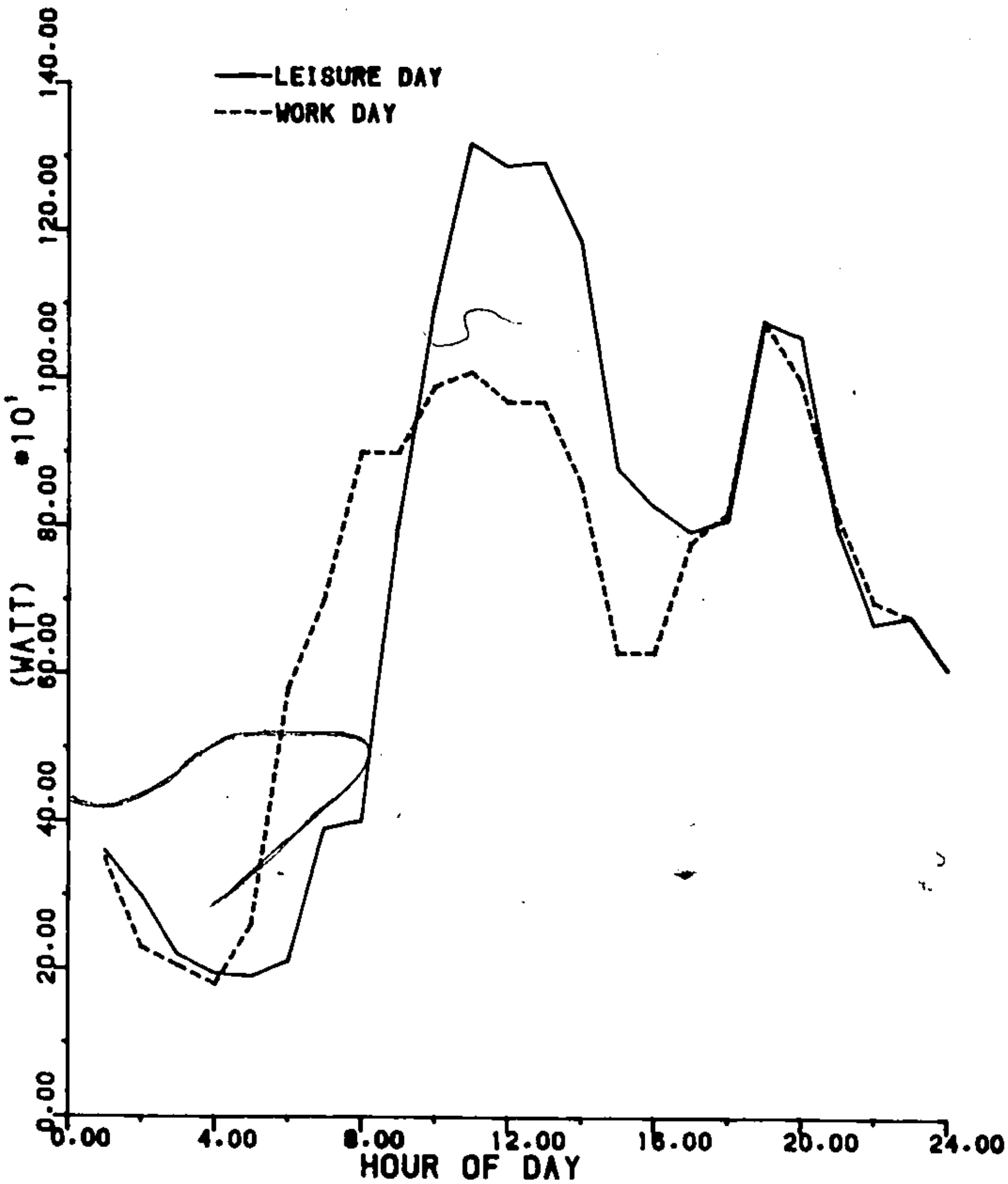

Figure 34: Load characteristic of a typical water heater

Plant licensing is increasingly difficult because of regulatory road blocks. In the past, the installation of oil fired equipment was a low cost means of supplying peak loads. Now the future prospects of availability and price of fuel oil may inhibit the advisability of major investments in combustion turbines. There are considerable environmental and legal obstacles to power plant siting and construction.

In case of interconnected systems, the peak in the interconnected systems may occur at the same time. Surplus energy of exporting system may not be sufficient to meet the peak demand of the importing system. It may also happen that it is not economically feasible to interconnect with other utility/utilities, located far from the system.

While the major reason for voltage reduction is transmission inadequacy, it may also be applied in cases when the reserve on line and spinning, is insufficient to accommodate the demand for electric power. But a voltage reduction [4] beyond certain percentage (maximum 8%) exceeds the tolerance built in many appliances. The major impact of brownouts is overheating and lower performance of electric motors, less light from light bulbs, and similar inconveniences. The sophisticated industry, the computer industry for instance, can be seriously affected by voltage fluctuations.

4.3 OBJECTIVE OF LOAD MANAGEMENT

Load management is attractive in terms of its potential for conserving energy and capital in production costing and distribution of electric power; for shifting a significant amount of fuel base from oil and gas to coal, nuclear and renewable resources; and holding down the cost of electricity. The objective of Load management is to alter the real or apparent pattern of electricity use in order to :

1. Improve the efficiency as well as the utilization of generation, transmission and distribution system,
2. Shift fuel dependency from limited to abundant energy resources,
3. Lower the reserve requirements of generation and transmission capacity,
4. Improve the reliability of service to essential loads,
5. Reduce the cost/benefit ratio as well as the average cost of electricity.

4.4 CONDITION OF LOAD MANAGEMENT

Load management as a strategy may not work for every utility. Certain conditions are needed before load management can even be considered.

One condition is the 'peakiness' of the utility's load profile, i.e. a low load factor - the ratio of average demand to peak demand. Obviously, a utility with a flatter load profile will not need load management because its power plants are fully used.

The second condition is the utility's generation mix. If a utility has a coal or nuclear base load, but burns oil or gas to meet sharp peaks, load management could spare some of that costly oil or gas . But if a utility meets its peaks with less costly unit, such as pumped hydro, load management may not be necessary. If a utility burns oil or gas to meet both peak and off-peak load, load management will not provide any economic benefits because there is no inexpensive base load energy to shift to. If a utility has new plants well under consideration, load management may not be attractive .

The third condition is the contractual limitation . If a utility has firm purchase contracts during the peak periods with the connected utilities, any load management scheme may not be beneficial.

The fourth condition for load management is the customers' ability to reduce peak load or to shift peak load to off peak periods. This depends partly on the saturation level of devices such as water heaters and air conditioners, and partly on whether customers can use those devices at off-peak periods.

Therefore, load management schemes are not applicable to all utilities. Careful consideration of such factors as load profile, generation mix, ability to shift load, and customers' attitude is necessary before a utility can adopt a load management program.

4.5 APPROACHES TO LOAD MANAGEMENT

There are three basic approaches to load management

1. Direct control ,
2. Indirect control or customer incentives ,
3. Energy storage .

Direct load control is defined as the utility control, of customer loads. These loads are of two types: interruptive customer load such as air conditioner and deferrable customer loads such as water heaters, space heating systems, and swimming pool pumps. In direct load control approach the utility uses switching devices such as ripple control and others. Direct load control is attractive to utilities because they can plan for specific demand levels.

Indirect control or voluntary control is defined as the customer control of load in response to price signals or incentives. Customer incentives, such as time of day rates, encourage customers to shift electricity demand to off-peak periods, typically by charging less for off-peak electricity . The customer decides which appliances should be used and when. Nonessential appliances may be used during off peak periods when rates are low, or the customer may choose to pay higher peak rates. However, customer incentives may not guarantee the utility a definite demand level, and the utility must plan accordingly.

Energy storage is the use of electricity during off-peak hours to store energy for use during on-peak period. Pumped-hydro storage is at present almost the only form of storage system available to large utilities. But sites for pumped hydro are hard to find and license. Newer storage systems such as compressed air, storage batteries, flywheels, and superconducting magnets are still under development.

4.5.1 Specific Studies

To investigate the effect of different load management schemes on the reliability and the production costs of the interconnected systems the following studies are performed.

1. Direct load management is simulated in two different ways.

   a) Loads of any one of the two interconnected systems or both systems are reduced by a certain amount during the peak hours of the day. The managed load of the system, in which load management scheme is applied, may be expressed as

$$\tilde{L}(t) = L(t) - (a\ L(t))\chi_{(t_1,t_2)}(t) \qquad (4.1)$$
$$0 \leq a \leq 1$$

where

$$\chi_{(t_1,t_2)}(t) = \begin{cases} 1 & \text{if} \quad t_1 \le t \le t_2 \\ 0 & \text{elsewhere} \end{cases} \tag{4.2}$$

and

$\tilde{L}(t)$ = The managed load of the system

$L(t)$ = The demand of the system

t_1 = The starting time of the load management scheme which lies during the 24 hours interval of the day

t_2 = The final time of the load management scheme which lies during the 24 hours interval of the day.

b) The second direct load control approach is to reduce the demand whenever it exceeds a prefixed value. This load control technique may be referred to as the constant peak technique and may be applied to any one of the two interconnected systems or both systems. In this case, the managed load of the system, in which this load management technique is applied, may be expressed as

$$\tilde{L}(t) = L(t) - (\ L(t) - CPK\ ) \quad \chi(L(t)) \tag{4.3}$$

where

$$\chi(L(t)) = \begin{cases} 1 & \text{if } L(t) > CPK \\ 0 & \text{otherwise} \end{cases} \tag{4.4}$$

and CPK is a prefixed constant peak demand.

2. Energy storage approach is simulated using the pumped storage unit. The energy is stored by pumping water into the reservoir during the low demand period of the day and this water is utilized during the peak demand period of the day to generate electric power. This technique may be applied to any one of the two interconnected systems or to both systems simultaneously. The managed load of the system, in which the energy storage approach is applied, can be expressed as

$$\tilde{L}(t) = L(t) - (a\,L(t))\chi_{(t_1,t_2)}(t) + (b\,L(t))\mathbb{1}_{(t_3,t_4)}(t)$$

$$0 \le a \le 1 \tag{4.5}$$

where

$$\chi_{(t_1,t_2)}(t) = \begin{cases} 1 & \text{if } t_1 \le t \le t_2 \\ 0 & \text{otherwise} \end{cases} \tag{4.6}$$

and

$$\mathbb{1}_{(t_3,t_4)}(t) = \begin{cases} 1 & \text{if } t_3 \le t \le t_4 \\ 0 & \text{otherwise} \end{cases} \tag{4.7}$$

and

t_1 = Starting time of pumping the water

t_2 = Final time for pumping the water

t_3 = Starting time of the utilization of the pumped water

t_4 = Final time of the utilization of the pumped water

t_1, t_2, t_3, t_4 lie in the 24 hours interval of the day and the value of b depends on the capacity of the pump and the reservoir.

4.6 RESULTS

In this study, the generating system and the load model, used to investigate the impacts of load management schemes on reliability, are the same as those used in Section 2.9.1. For the investigation of the impacts of load management schemes on the production costs of two interconnected systems the load model and the generating system, mentioned in Section 3.6, are used. In the energy storage scheme, the efficiency of the pump is assumed to be 100% for this study.

Demand in the system may increase in addition to the usual demand when the load reduction period is over. Note that this type of load increase is not considered for this study.

4.6.1 Reliability

In the first scheme of direct load control, the load is reduced in system X from 3 P.M to 8 P.M by 10%. That is in equation (4.1) 'a' is assumed to be 0.1 and t_1 and t_2 are assumed to be 3 P.M. and 8 P.M. respectively. In system Y the load is also reduced by 10%, but from 10 A.M to 3 P.M. In reference to the load model given in the Appendix B it can be mentioned that the peak in system X occurs during 3 P.M to 8 P.M (in general) and in system Y the peak occurs from 10 A.M to 3 P.M (in general). The LOLPs obtained at different tie line capacities for the individual system and for the global system are given in Table 16 and Table 17 respectively. In what follows the base case is reffered to the condition where no load management schemes is applied to the system.

In the second scheme of direct load control the load in system X is set to 2565.0 MW whenever it exceeds this value and in system Y the load is set to 2308.5 MW whenever it exceeds this value. That is, in equation (4.3) CPK for system X is 2565.0 MW and for system Y CPK is 2308.5 MW.

TABLE 16

LOLPs of the individual system for the reduced load (Reduced by 10% during the peak hours)

Tie	Load reduced in System X only		Load reduced in System Y only		Load reduced in both systems	
	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$
0.0	.00150	.06610	.00280	.04927	.00150	.04927
100	.00081	.04579	.00151	.03274	.00077	.03267
200	.00051	.03062	.00089	.02090	.00043	.02075
300	.00038	.01975	.00062	.01312	.00029	.01287
400	.00034	.01247	.00052	.00843	.00023	.00806
500	.00032	.00803	.00047	.00550	.00021	.00503

TABLE 17

LOLPs of the global system for the reduced load (reduced by 10% during the peak hours)

Tie	Base case	Load reduced in System X only	Load reduced in System Y only	Load reduced in both systems
0.0	.06856	0.06741	0.05181	0.05066
100	.04707	0.04641	0.03399	0.03332
200	.03141	0.03094	0.02153	0.02106
300	.02039	0.01994	0.01348	0.01304
400	.01312	0.01261	0.00868	0.00818
500	.00875	0.00816	0.00572	0.00513

LOLPs for the individual system and for the global system are given in Table 18 and Table 19 respectively.

TABLE 18

LOLPs of the individual system for the prefixed peak demand

Tie	Load management applied to System X only		Load management applied to System Y only		Load management applied to both systems	
	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$
0.0	.00199	.06610	.00280	.05832	.00199	.05832
100	.00112	.04582	.00154	.04170	.00110	.04167
200	.00066	.03071	.00094	.02629	.00063	.02623
300	.00051	.01990	.00068	.01593	.00047	.01582
400	.00044	.01268	.00058	.01042	.00040	.01028
500	.00042	.00832	.00054	.00697	.00038	.00680

TABLE 19

LOLPs of the global system for the prefixed peak demand

Tie	Base case	Load management applied to System X only	Load management applied to System Y only	Load management applied to both systems
0.0	.06856	0.06784	0.06081	0.06009
100	.04707	0.04669	0.04293	0.04254
200	.03141	0.03112	0.02692	0.02664
300	.02039	0.02016	0.01630	0.01608
400	.01312	0.01287	0.01069	0.01045
500	.00875	0.00849	0.00720	0.00696

In the energy storage scheme the load in system X is reduced by 10% from 3 P.M. to 8 P.M. and the same amount of load is increased from 1 A.M. to 6 A.M. That is, in equation (4.5) t_1, t_2, t_3, t_4 are 3 P.M., 8 P.M., 1 A.M. and 6

A.M. respectively and a is 0.1 and b is selected such that the total amount of load reduced in 24 hours periods is equal to the total amount of load increased. In system Y the load is also reduced by 10% from 10 A.M to 3 P.M. and the same amount of load is increased from 1 A.M. to 6 A.M. LOLPs obtained for the individual system and for the global system are shown in Table 20 and Table 21, respectively.

TABLE 20

LOLPs of the individual system for the energy storage scheme

Tie	Load management applied to System X only		Load management applied to System Y only		Load management applied to both systems	
	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$
0.0	.00150	.06610	.00280	.05001	.00150	.05001
100	.00081	.04579	.00151	.03313	.00077	.03307
200	.00051	.03062	.00089	.02112	.00043	.02097
300	.00038	.01975	.00062	.01323	.00029	.01298
400	.00034	.01247	.00052	.00848	.00023	.00811
500	.00032	.00803	.00047	.00553	.00021	.00506

TABLE 21

LOLPs of the global system for the energy storage scheme

Tie	Base case	Load management applied to System X only	Load management applied to System Y only	Load management applied to both systems
0.0	.06856	0.06741	0.05255	0.05140
100	.04707	0.04641	0.03438	0.03372
200	.03141	0.03094	0.02175	0.02128
300	.02039	0.01994	0.01360	0.01315
400	.01312	0.01261	0.00873	0.00823
500	.00875	0.00816	0.00574	0.00515

4.6.2 Production Cost

The results shown in Table 22 and Table 23 are obtained by reducing the load by 10%:

1. in system X from 3 P.M. to 8 P.M while retaining the load in system Y
2. in system Y from 10 A.M. to 3 P.M. while retaining the load in system X
3. in system X from 3 P.M. to 8 P.M. and in system Y from 10 A.M. to 3 P.M. simultaneously.

The corresponding global benefits in terms of production cost savings are depicted in Figure 35. The results for the base case are also depicted in Figure 35.

In the second scheme of direct load management the load is reduced whenever it exceeds certain prefixed peak demand.

TABLE 22

Production costs of the individual system for the reduced load (load reduced by 10% during the peak hours)

Tie	Load management applied to System X only		Load management applied to System Y only		Load management applied to both systems	
	Production cost (M$)		Production cost (M$)		Production cost (M$)	
	'X'	'Y'	'X'	'Y'	'X'	'Y'
0.0	31.575	44.593	33.234	42.117	31.575	42.117
100	33.962	40.676	35.288	38.729	33.692	38.588
200	36.233	37.227	37.251	35.775	35.740	35.477
300	38.176	34.417	38.948	33.350	37.529	32.904
400	39.700	32.280	40.255	31.537	38.912	30.974
500	40.742	30.837	41.097	30.387	39.804	29.747

TABLE 23

Production costs of the global system for the reduced load (load reduced by 10% during the peak hours)

Tie	Base case	Global production cost (Load management applied to System X only)	Global production cost (Load management applied to System Y only)	Global production cost Load management applied to both systems)
	(M$)	(M$)	(M$)	(M$)
0.0	77.828	76.168	75.352	73.693
100	76.377	74.639	74.018	72.280
200	75.270	73.461	73.026	71.218
300	74.461	72.594	72.299	70.433
400	73.888	71.981	71.793	69.886
500	73.513	71.580	71.485	69.552

The prefixed peak demand in system X is set to 2565.0 MW and

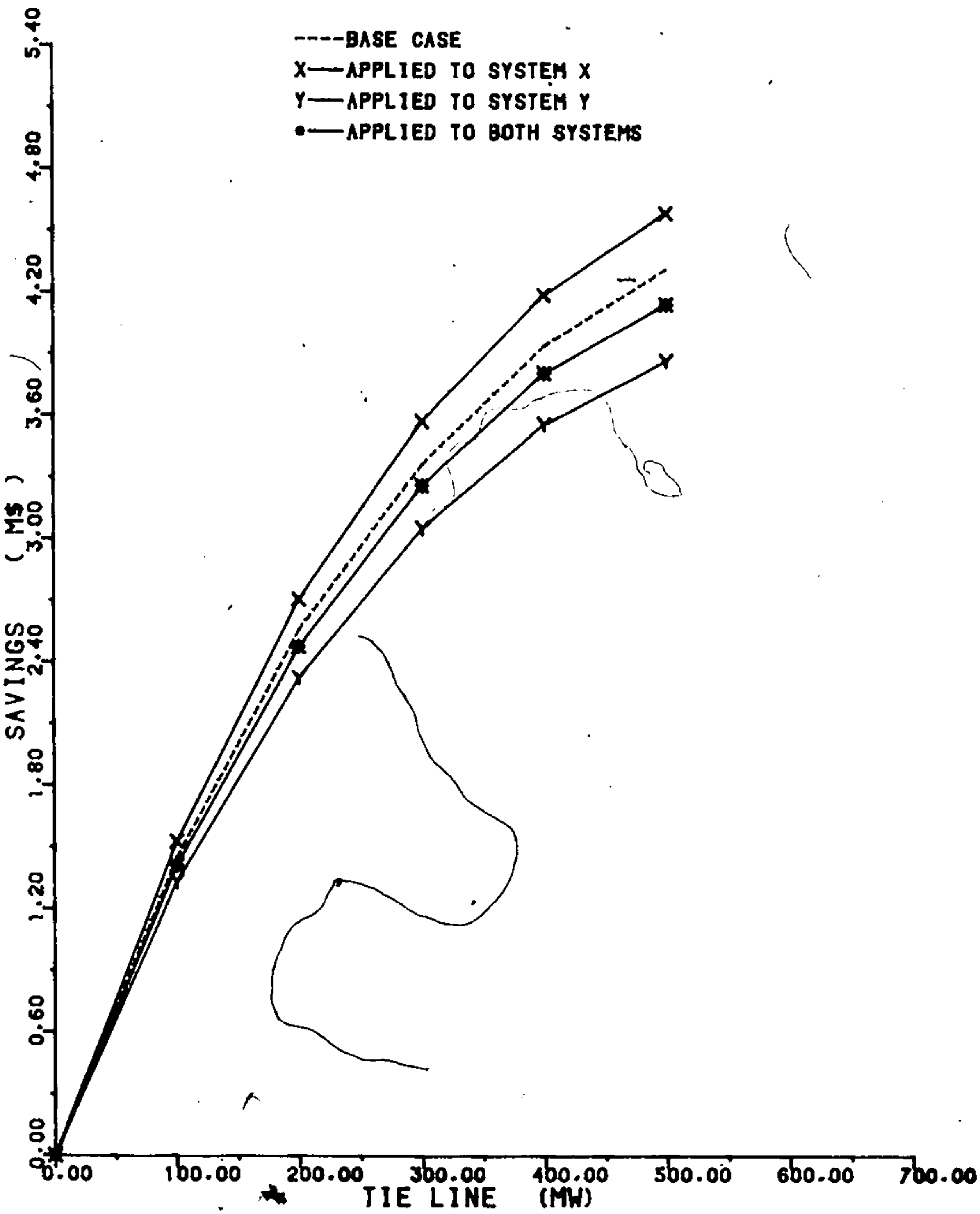

Figure 35: Global benefits for direct load management (load reduced during peak hours)

in system Y the prefixed peak demand is set to 2308.5 MW . The production costs of the individual system and the production costs of the global system at different tie line capacities are shown in Table 24 and Table 25 respectively. For the prefixed peak demand of load management scheme the global benefits in terms of production cost savings are depicted in Figure 36. The base case results are also included in the same figure.

TABLE 24

Production costs of the individual systems for the prefixed peak demand

Tie	Load management applied to System X only		Load management applied to System Y only		Load management applied to both systems	
	Production cost (M$)		Production cost (M$)		Production cost (M$)	
	'X'	'Y'	'X'	'Y'	'X'	'Y'
0.0	33.063	44.593	33.234	43.841	33.063	43.841
100	35.390	40.810	35.557	40.034	35.390	40.026
200	37.580	37.508	37.594	36.987	37.431	36.969
300	39.440	34.834	39.385	34.427	39.228	34.398
400	40.893	32.803	40.816	32.429	40.665	32.390
500	41.886	31.433	41.779	31.101	41.630	31.057

In the energy storage scheme of load management, the load is reduced by 10% in system X from 3 P.M. to 8 P.M. and the same amount of load is increased during 1 A.M. to 6 A.M.

TABLE 25

Production costs of the global system for the prefixed peak demand

Tie	Base case	Global production cost (Load management applied to System X only)	Global production cost (Load management applied to System Y only)	Global production cost Load management applied to both systems)
	(M$)	(M$)	(M$)	(M$)
0.0	77.828	77.656	77.075	76.904
100	76.377	76.201	75.592	75.416
200	75.270	75.088	74.582	74.401
300	74.461	74.274	73.812	73.626
400	73.888	73.697	73.246	73.055
500	73.513	73.320	72.880	72.688

Similarly in system Y the load is reduced by 10% from 10 A.M. to 3 P.M. and the same amount of load is increased during 1 A.M. to 6 A.M. Production costs for the individual system and for the global system are shown in Table 26 and Table 27 respectively. The corresponding benefits in terms of production cost savings are compared with the base case in Figure 37.

The average global production costs, at 200 MW tie line capacity obtained from different load management schemes are compared with those of the base case in Table 28. The average global production costs are obtained at any tie line capacity by dividing the global production costs with the global energy generation at that tie line. In this case, the load management schemes are applied to both systems simultaneously.

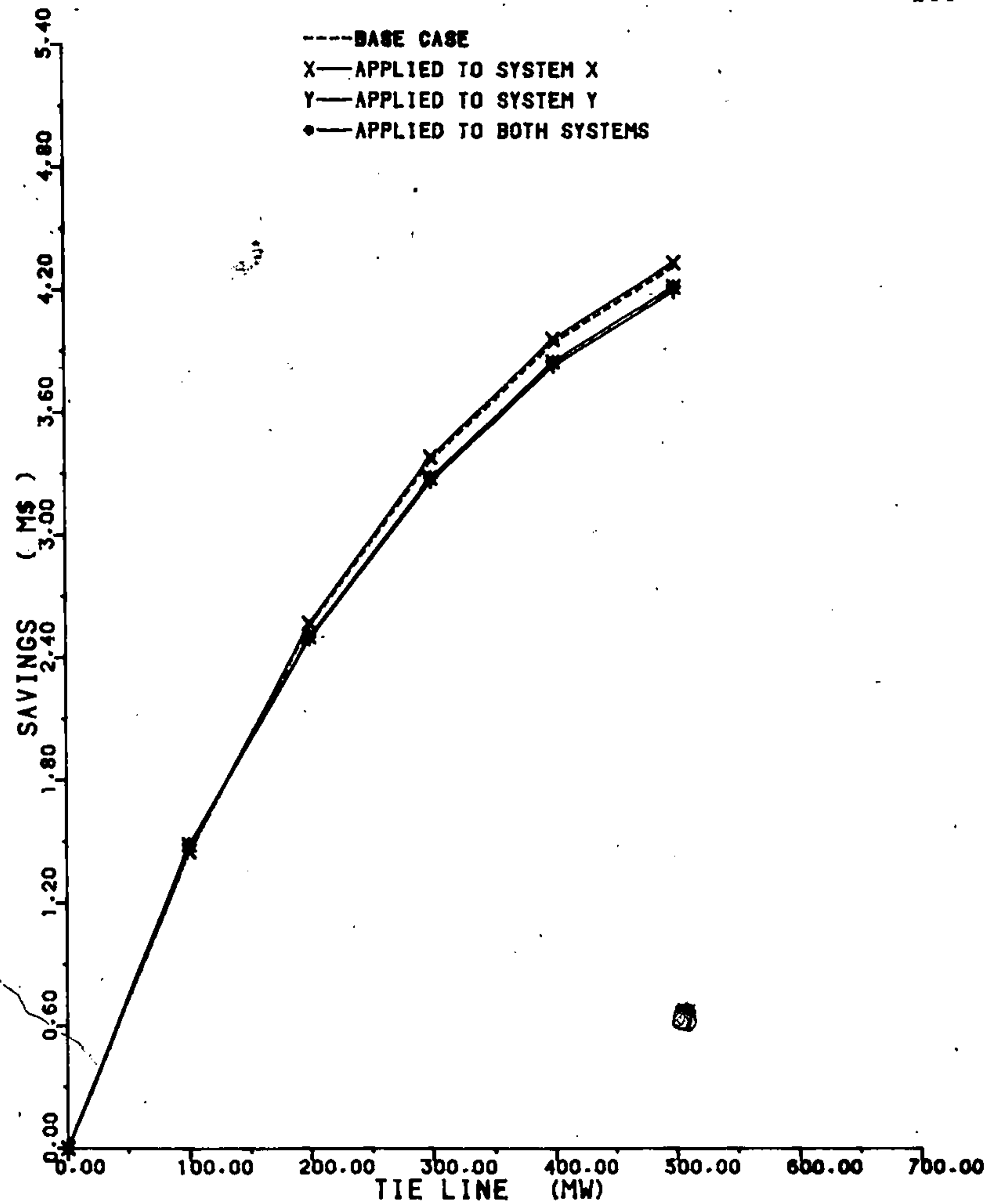

Figure 36: Global benefit for prefixed peak demand

TABLE 26

Production costs of the individual system for the energy storage scheme

Tie	Load management applied to System X only Production cost (M$)		Load management applied to System Y only Production cost (M$)		Load management applied to both systems Production cost (M$)	
	'X'	'Y'	'X'	'Y'	'X'	'Y'
0.0	32.765	44.593	33.234	43.680	32.765	43.680
100	35.049	40.798	35.303	40.222	34.830	40.170
200	37.213	37.485	37.282	37.213	36.819	37.112
300	39.031	34.836	39.007	34.734	38.537	34.620
400	40.410	32.884	40.368	32.841	39.841	32.790
500	41.307	31.625	41.309	31.560	40.665	31.651

TABLE 27

Production costs of the global system for the energy storage scheme

Tie	Base case	Global production cost (Load management applied to System X only)	Global production cost (Load management applied to System Y only)	Global production cost Load management applied to both systems)
	(M$)	(M$)	(M$)	(M$)
0.0	77.828	77.359	76.914	76.446
100	76.377	75.848	75.525	75.000
200	75.270	74.699	74.495	73.931
300	74.461	73.868	73.741	73.157
400	73.888	73.294	73.210	72.631
500	73.513	72.932	72.870	72.317

The global production costs obtained for different load

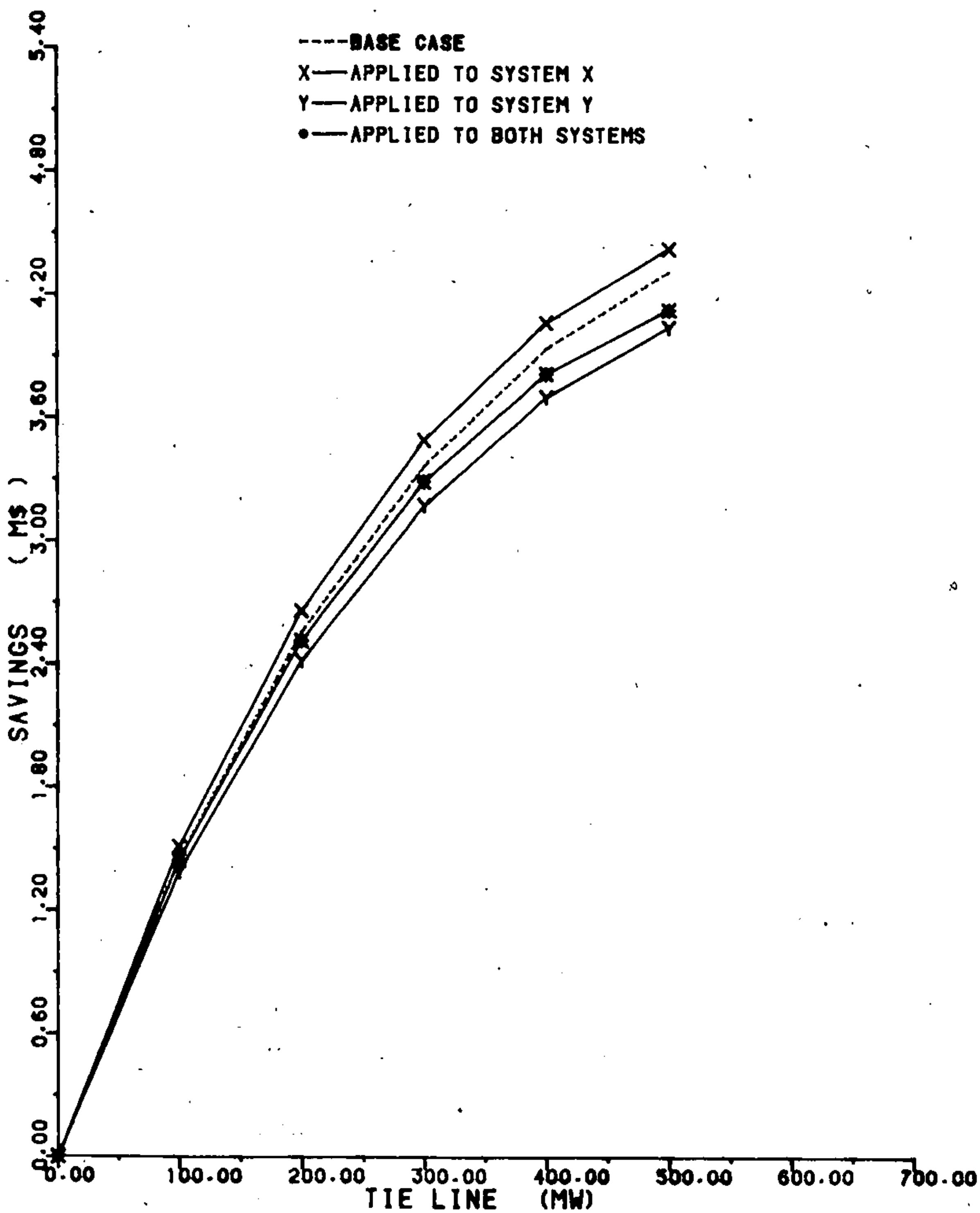

Figure 37: Global benefit for energy storage scheme

TABLE 28

Average cost of generation ($/MWh) for the global system (at 200 MW tie line)

Base case	Direc load management		Energy storage
	Load reduced by 10%	load reduced whenever it exceeds the fixed value	
9.267	8.986	9.197	9.100

management schemes at different tie line capacities are compared graphically in Figure 38 with the base case. Note that in this case load mangement schemes are applied to both systems simultaneously.

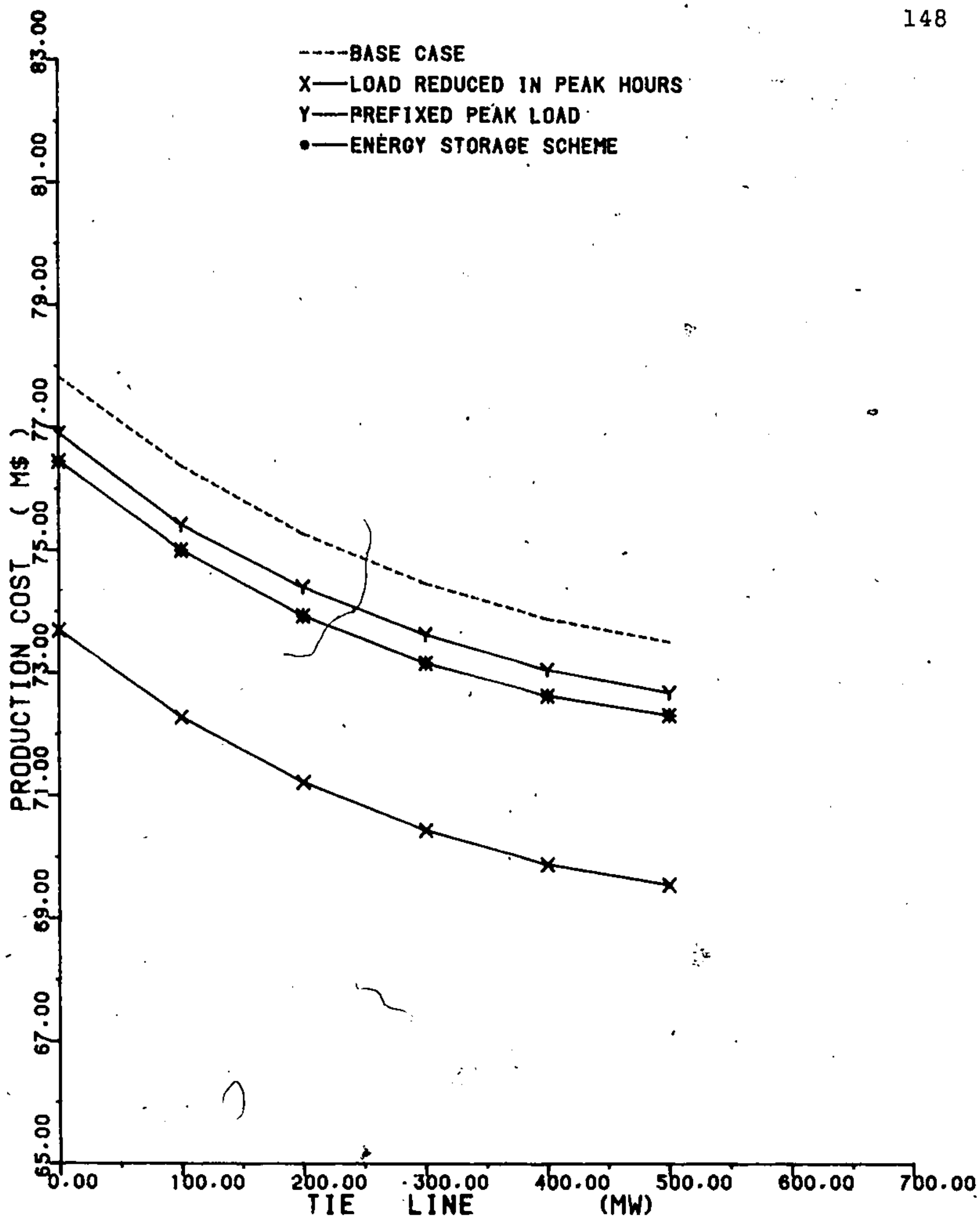

Figure 38: Global production costs VS tie line capacity for different load management schemes and also for the base case

4.7 OBSERVATIONS AND DISCUSSION

In this chapter, the impacts of different load management schemes on reliability as well as on production costs are studied.

Regarding the impacts of load management on the reliability of interconnected systems the following observations are made.

1. Comparing the results in Table 16 and Table 18 with the results in Table 3 (obtained by using the segmentation or the recursive technique) and from the results of Table 17 and Table 19, it is found that the reduction of demand during the peak hours decreases the LOLPs i.e. increases the system reliability, as expected.
2. It is observed from the Table 17, Table 19 and Table 21 that the improvement in the reliability of the global system is more when the load management is applied to the system of higher FOR units. Note that system Y has on average higher FOR units.
3. It is also observed, by comparing the results of Table 17, that the 10% reduction of demand in one system during the peak hours of the day is equivalent to the increment of approximately 100 MW tie line capacity between the two systems. For instance, in

the base case, the LOLP of the global system at 400 MW tie line capacity is 0.01312 while that in the case of reduced peak demand, the LOLP of the global system is 0.01994 or 0.01348 at 300 MW tie line capacity.

4. By comparing the results of Table 17 and Table 19, it is found that the improvement in the global system reliability is less when the prefixed peak scheme is applied instead of reducing the load during the peak hours.

5. In the energy storage scheme, the total energy supplied is the same as that of the base case. But it is observed from Table 21 that the global system reliability improves by applying the energy storage scheme. This indicates that the unutilized capacity during the off-peak hours may be useful in improving the system reliability by utilizing the energy storage scheme.

6. In case of direct load management scheme the load is reduced while in case of the energy storage scheme the demand is not reduced. But from Table 17 and Table 21 it is observed that the LOLPs in both cases are approximately same.

Regarding the impact of load management schemes on production costs the following observations are made.

1. Comparing the results in Table 22 and Table 24 with the results of Table 10 and also from Tables 23,25 and from Figure 38 it is found that direct load management schemes decrease the production costs of the individual as well as global system, as expected, because of the reduced energy supply.
2. In case of energy storage scheme, comparing the results of table 26 with the results of Table 10, it is observed that the increase in the production costs of system X is less compared to the base case. Recall that system X has, on average, lower FORs and incremental costs, it is expected that system X would export major part of the time. It is also observed from Table 27 and from Figure 38 that the global production costs are lower compared to the base case. It indicates that the energy storage scheme reduces the export/import over the tie line. But the cumulative effect of the interconnection and the energy storage scheme reduces global production costs.
3. The decrease of global production costs is more when the load management scheme is applied to the system of higher incremental cost and higher FOR units, which is clearly observed from Table 23, Table 25 and Table 27.

4. From Table 28 , it is found that the average production cost of generation due to the load management schemes decreases. That is, the load management increases the efficiency of the system in terms of production costs.

5. From Figure 38 , it is observed that it would be the same global production cost if the energy storage scheme is applied to both systems instead of interconnecting the system through 100 MW tie line capacity (approximately). It is also observed that reduction of load during the peak hours of the day results the minimum production cost and the energy storage scheme provides lower global production costs than that for the prefixed peak demand scheme, although the total energy supply is more in case of the energy storage scheme. Note that the penalty due to not meeting the demand is not included in the cost calculation. Obviously this would increse the cost.

6. Although from Figure 38 it is observed that load management strategies reduce the global production costs compared to the base case; but Figures. 35, 36 and 37 show that the global benefits (savings) are not always higher than that of the base case, especially when the load management is applied to system Y (system with higher incremental cost and higher FOR units) or both systems simultaneously. It

is observed that the maximum global benefits are obtained when the load management scheme is applied to system X (system with lower incremental cost and lower FOR units).

From these observations, one can conclude that the load management is an alternative in generation expansion planning in deciding whether to purchase power from the neighbouring utility.

Chapter V

IMPACT OF JOINT OWNERSHIP OF GENERATION ON RELIABILITY AND ON PRODUCTION COSTS

5.1 INTRODUCTION

Construction of a new power plant needs a large investment. Billions of dollars are required before the power plant can start to generate electricity. The Diablo Canyon nuclear plant in California, recently completed, had a cost of $4.4 billon; $2 billion is spent only to construct the dam of the Revelstok hydroelctric project at British Columbia [47]. Construction costs are increasing with time very rapidly. In the early 1970s, the capital cost of nuclear power plants were as high as $200/KW, and of coal plants around $175/KW; where as by the late 1970s these costs had increased to $700/KW and $500/KW, respectively. Presently nuclear power plant capital cost may be as high as $3000/KW and the coal plants cost $1200/KW. Including the cost of financing, the final cost may be still higher.

Some utilities are solving the problem by sharing the plant among themselves. For example, the interconnected utilities may jointly install a generating unit in which interconnected utilities may have different or shame shares; or one of the interconnected utilities may buy a certain percentage of the output of a generating unit which already exits in the other utility. Therefore, the failure of a jointly owned generating unit will cause a decrease in the available capacities of all the sharing utilities simultaneously.

It is more realistic that the jointly owned unit is located in one of the interconnected systems. However, for the unconventional energy sources, like wind power, tidal power, the generating plant may be located far from the utilities' load centre. The location of the jointly owned unit will be discussed in Section 5.4.

In this chapter, a model is developed to represent the generating unit jointly owned by two systems. The impacts of the jointly owned unit on LOLPs and production costs have been investigated considering the location of the unit outside the two interconnected systems.

5.2 JOINTLY OWNED GENERATING UNIT MODEL

Consider a generating unit of C MW, jointly owned by two systems; system X and system Y. Consider the share of system X as SH_X MW, and the share of system Y as SH_Y MW such that

$$SH_X + SH_Y = C \tag{5.1}$$

This jointly owned generating unit may be modeled as two separate generating units [48,49] of capacities SH_X and SH_Y MW, assuming that both the units are completely correlated. That is, the failure of SH_X MW in system X and the failure of SH_Y MW in system Y or the availability of SH_X MW in system X and the availability of SH_Y MW in system Y occur simultaneously.

Consider two correlated random variables, X_1 and X_2, and let X_1 correspond to the share of system X, and X_2 corresponds to the share of system Y. Consider two possible states for the jointly owned generating unit: either operating or has failed. That is, X_1 takes either the value SH_X MW or zero MW and X_2 takes either the value SH_Y MW or zero MW. If the probability that the jointly owned unit is in the failed state is q, the probability that the jointly owned unit in the working state is (1 - q), and then the probability density function (PDF) of the outages of the jointly owned unit can be graphically represented as in Figure 39. Figure 39 may be interpreted as follows: the jointly owned generating unit is operating (1 - q)% of the time providing SH_X MW generation to system X and SH_Y MW to system Y, and has failed q% of the time.

The PDF of forced outage capacity of the jointly owned unit can also be expressed as,

$$f_{X_1X_2}(x_1, x_2) = (1 - q)\ \delta(x_1)\ \delta(x_2) + q\ \delta(x_1 - SH_X)\ \delta(x_2 - SH_Y) \tag{5.2}$$

where $\delta(.)$ is the dirac delta function.

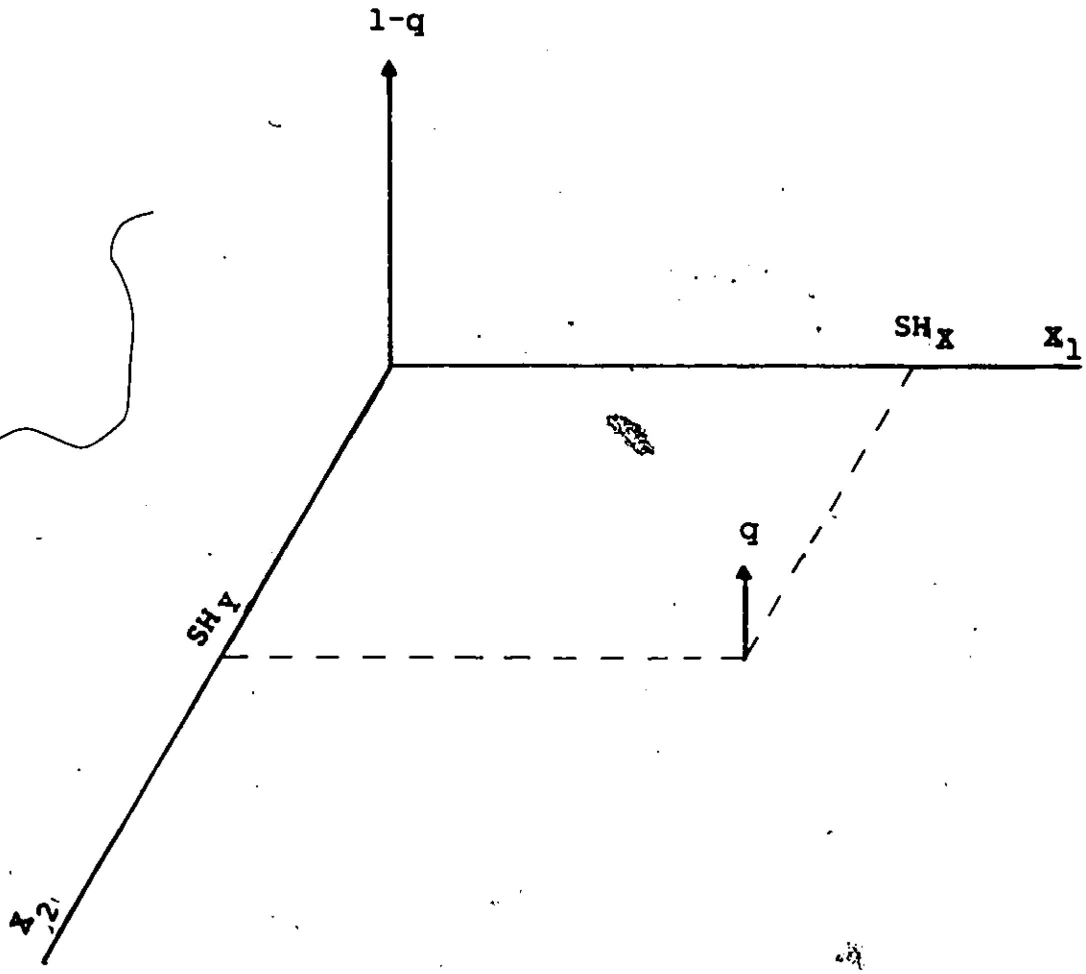

Figure 39: PDF of outage capacity of a jointly owned generating unit

5.3 CONVOLUTION

The convolution procedure described in Sections 2.6.2 and 3.4.1 is the convolution of the joint distribution of load (equivalent load) with the univariate distribution of the forced outages of the unit. The process of convolution is effected by shifting each segment appropriately, and the direction of shift depends on the system that the generating unit, to be convolved, belongs to.

The PDF of the jointly owned unit is represented by the bivariate function. Therefore, the convolution of a jointly owned unit with the correlated demand is equivalent to the convolution of two bivariate distributions. To accomplish this, each segment must be shifted in both directions simultaneously. The amount of shift in each direction depends on the share of each system from the jointly owned unit. For example, assume a jointly owned unit of C MW, and also assume that the share of system X is SH_X and the share of system Y is SH_Y;, then the shift in the X direction would be SH_X MW, and in the Y direction the shift would be SH_Y MW, considering that the X direction is attributed to system X and the Y direction is attributed to system Y.

Considering the (i,j)th segment and assuming a SH_X MW share belonging to system X and a SH_Y MW share belonging to system Y of the jointly owned unit of capacity C MW, the probability and the first moment of load corresponding to $(i+w_Y, j+w_X)$th segment of system X are expressed by equations (5.3) and (5.4), respectively,

$$\tilde{P}_{i+w_Y,j+w_X} = P_{i+w_Y,j+w_X}(1-q) + \hat{P}_{i+w_Y,j+w_X}\, q \tag{5.3}$$

$$\hat{m}_{i+w_Y,j+w_X} = m_{i,j} + SH_X \times P_{i,j} \tag{5.4}$$

where

$$w_X = SH_X / \Delta C \tag{5.5}$$

$$w_Y = SH_Y / \Delta C \tag{5.6}$$

$\tilde{P}$ and $\hat{P}$ are the probabilities of $(i+w_Y , j+w_X)$th segment after convolution and after the shift, respectively. P represents the probability value of the corresponding segment before convolution. m and $\hat{m}$ are the first moments, or the expected values, of load of the corresponding segments before and after the shift, respectively. ΔC is the largest common factor of the generating unit capacities of two interconnected systems and the tie line capacity.

The corresponding first moment of load of system Y due to the shift of the segments may be obtained from equation (5.4) by changing SH_X to SH_Y. Note that the equation (5.3) and (5.4) are the modified forms of the equations (2.12) and (3.31), respectively. The rest of the convolution procedure is the same as Section 3.4.1.

5.4 LOCATION

In the modeling of a jointly owned generating unit, it may be located inside of any one of the two systems which are interconnected, or it may be located outside the two systems as depicted in Figure 40. The connecting line, in Figure 40, is used to transmit power from the jointly owned unit to the main grid of the respective system.

In the former case, if the jointly owned generating unit is located in system X, the tie line capacity must be greater than or equal to SH_Y MW, since if the unit is available in system X with SH_X MW, it must be available in system Y with capacity SH_Y MW. Similarly, if the jointly owned unit is available in system Y, the tie line capacity must be greater than or equal to SH_X MW.

In the latter case, if the jointly owned unit is located outside the two systems, there are two possible ways of exporting excess capacity from the jointly owned unit:

1. through the tie line;
2. through the connecting line.

The export of excess capacity of the jointly owned unit through the tie line only, implies that the capacity of the respective connecting line is limited to its share. That is,

$$CC_i = SH_i \tag{5.7}$$

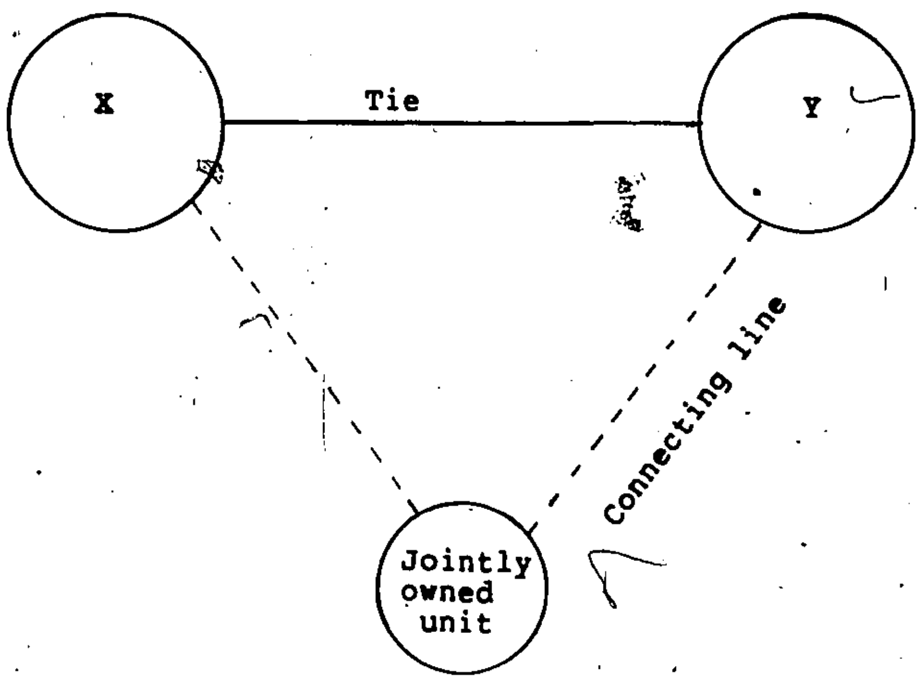

Figure 40: Jointly owned generating unit located outside the two systems

where,

CC_i = Capacity of the connecting line from the jointly owned unit to the i-th system

SH_i = Share of the i-th system of the jointly owned unit

Note that at any (i,j)-th segment, import is accomplished only when the expected committed capacity with its share of the jointly owned unit is less than the expected demand of the importing system at that segment. That is,

$$\left(\sum_{i=1}^{S^I} C_i^I + SH^I \right) P_{i,j} < m_{i,j}^I(EL) \qquad (5.8)$$

where

S^I = Number of units already committed in the importing system ;

$m_{i,j}^I(EL)$ = Expected demand of the importing system at (i,j)th segment .

Therefore,when equation (5.7) is valid, the export of excess capacity through the connecting line is not possible during the availability of the jointly owned unit, since the connecting line towards the importing system will be fully occupied by the share of the importing system.

If the connecting line acts also as a tie line to import additional capacity from the jointly owned unit , the connecting line towards the importing system must be greater than the share of the importing system. If C is the capacity of the jointly owned unit, then

$$SH^I < CC^I \leq C \qquad (5.9)$$

assuming that only the excess capacity of the jointly owned unit is transfered through the connecting line.

To investigate the impacts of the joinly owned unit on the production cost as well as on LOLPs of the interconnected system the following assumptions are made in this thesis.

1. The jointly owned unit is located outside the two interconnected systems.
2. The capacity of the connecting line towards any system is equal to the share of that system.
3. The export of the excess expected capacity from the units other than the jointly owned unit through the connecting line is not economical.

The above assumptions are made in order to compare the results directly with the base case. The base case is referred to the system without jointly owned unit.

5.5 EXPORT OR IMPORT BY THE JOINTLY OWNED UNIT

As mentioned in Section 3.3.1, the system in which the generating unit is being committed is referred to as the exporting system and the other system is referred to as the importing system. However, by the turn of the jointly owned unit in the loading order, both systems may enjoy the status of being the exporting or the importing system, since both systems have the shares from this unit.

It has been mentioned in Section 3.3.2 that the principal factors which affect the excess capacity transactions between the two interconnected systems are :

1. The unserved demand of the exporting system .
2. The unserved demand of the importing system .
3. The capacity of the committing unit, and
4. The residual tie line capacity of the exporting system.

In the loading stage of the jointly owned unit, the expected export at any segment (i,j) may be calculated using relation (3.13). Equation (3.13) is rewritten as

$$e_{i,j} = \mathrm{Min}\ (\tilde{Z}_1, \tilde{Z}_2, \tilde{Z}_3) \qquad (5.10)$$

The expected excess generation , Z_1, may be calculated as a normal generating unit replacing the committing unit capacity by the share of the exporting system. That is, equation (3.33) may be modified as

$$\tilde{Z}_1 = (\sum_{k=1}^{S^E} C_k^E + SH^E) P_{i,j} - m_{i,j}^E (EL) \qquad (5.11)$$

Note that in equation (5.11) the upper limit has been reduced to S^E, and the share of the exporting system is added.

The expected residual tie line capacity of the exporting system, $\tilde{Z}_2$, may be expressed using equation (3.34) without any modification. That is,

$$\tilde{Z}_2 = m^{E}_{i,j}\,(RTC) \tag{5.12}$$

In calculating the expected unserved demand of the importing system, Z_3 , the share of the importing system must be considered, since both shares are available at the same time, and the average incremental cost is the same. Therefore, Z_3 can be expressed as

$$\tilde{Z}_3 = m^{I}_{i,j}\,(EL) - \Big(\sum_{k=1}^{S^I} C^{I}_{k} + SH^{I}\Big)\, P_{i,j} \tag{5.13}$$

The rest of the procedure in evaluating the expected energy generation by the jointly owned unit includes :

1. The modification of the demand
2. The modification of the residual tie line capacity
3. The calculation of the unserved energies before and after convolution.

These three steps are the same as the normal units and have already been presented in Section 3.4.3.

It can also be mentioned here that the method to evaluate the LOLPs of the interconnected system, including the jointly owned unit , is the same as the method developed in Chapter II, except the convolution process for the jointly owned unit (mentioned in Section 5.3). In calculating the installed capacity of any one of the two systems, the capacity share from the jointly owned unit must be included.

5.6 NUMERICAL EVALUATION

The proposed model has been applied to two interconnected systems and the impact of joint ownership of generation on system reliability as well as on production costs has been investigated.

In the studies of the impact on reliability, the generation and the load model used are the same as those used in Section 2.9.1 except that one coal unit of 350 MW capacity from system X is combined with one coal unit of 150 MW capacity from system Y to provide a 500 MW jointly owned generating unit. The forced outage rate of this jointly owned generator is 0.08. A share of 70% from the jointly owned generator is considered for system X and the remaining share is considered for system Y.

The LOLPs of the individual system and of the global system, as obtained by considering the joint ownership of generation, are shown in Table 29. The results for the base case are also shown in the same table. Here, the base case is referred to as the system without any jointly owned unit.

The generation and the load model, used to study the impacts on production costs, are also the same as those used

TABLE 29

Comparison of LOLPs of the system with the jointly owned unit and those of the base case

Tie	Base case			Joint ownership		
(MW)	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	LOLPS (Global)	$LOLP_{X\|Y}$	$LOLP_{Y\|X}$	LOLPS (Global)
0.0	.00280	.06610	.06856	.00280	.06610	.06856
100	.00156	.04585	.04707	.00161	.04589	.04709
200	.00097	.03077	.03141	.00106	.03085	.03148
300	.00072	.02000	.02039	.00083	.02013	.02053
400	.00062	.01284	.01312	.00075	.01302	.01333
500	.00058	.00850	.00875	.00072	.00877	.00905

in Section 3.6, except that the 350 MW coal unit of system X is combined with the 250 MW coal unit of system Y. This 600 MW jointly owned generating unit is assumed to have an average incremental cost of $11.4/MWh and a FOR of 0.08. A 58.33% share from this jointly owned unit is considered for system X,and the remaining share is considered for system Y.

The production costs, obtained by considering the jointly owned unit, are compared with those for the base case in Table 30. Note that the average incremental costs of the 250 MW coal unit of system Y, in fact, has been reduced to $11.4/MWh from $18.6/MWh while the jointly owned unit is considered.

The global benefits in terms of production costs at different tie line capacities are depicted in Figure 41. The global benefits for the base case are also depicted in the same figure for comparison.

TABLE 30

Comparison of the production costs of the system with the joinly owned unit with those of the base case

Tie	Base case			Joint ownership		
	system X (M$)	system Y (M$)	global (M$)	system X (M$)	system Y (M$)	global (M$)
0.0	33.234	44.593	77.828	33.234	42.014	75.248
100	35.558	40.819	76.377	35.431	38.451	73.882
200	37.744	37.526	75.270	37.480	35.391	72.871
300	39.597	34.864	74.461	39.148	33.013	72.161
400	41.044	32.843	73.888	40.303	31.381	71.684
500	42.035	31.478	73.513	40.993	30.392	71.385

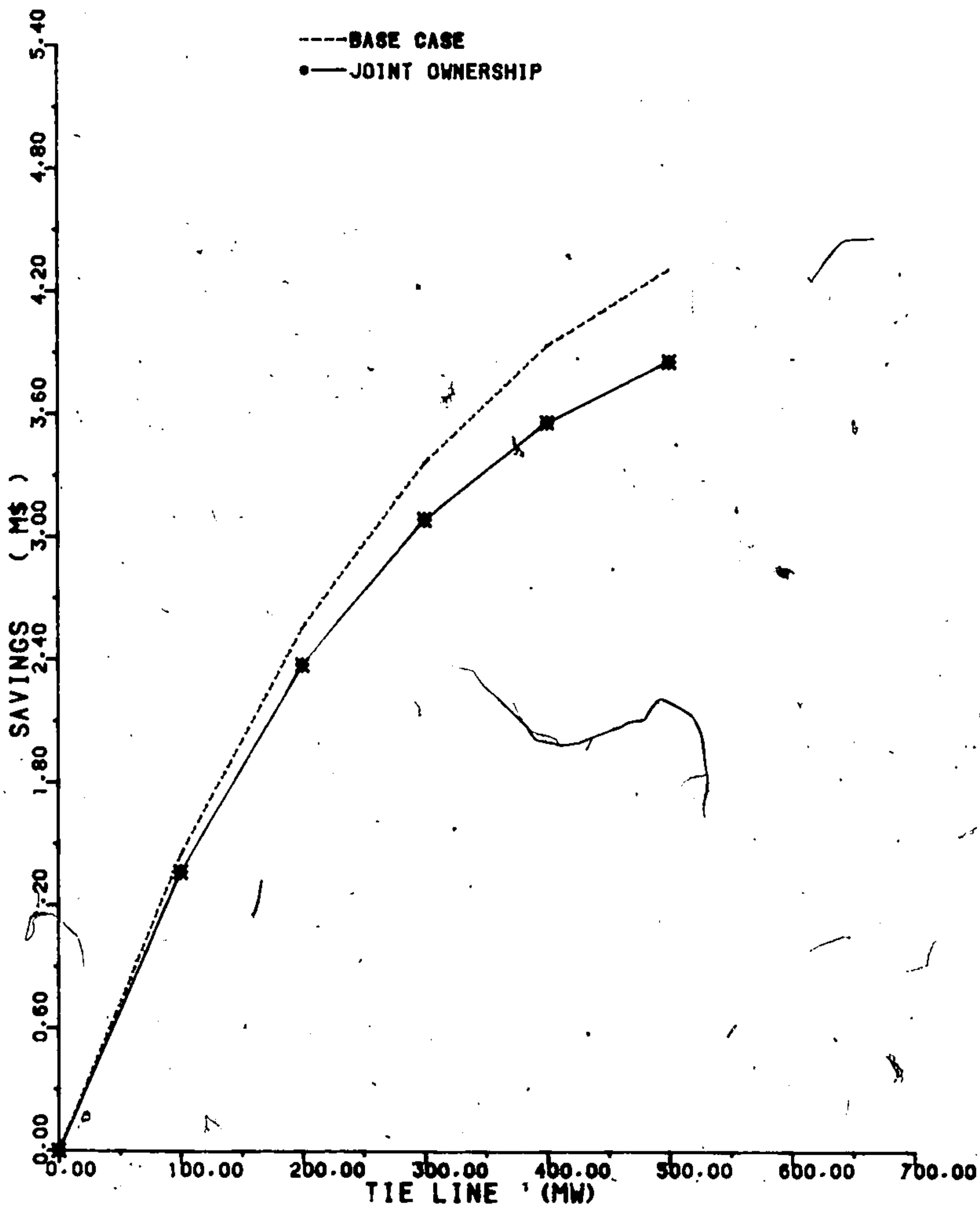

Figure 41: Global savings VS. tie line capacity

5.7 OBSERVATIONS AND DISCUSSION

From Table 29, it is clearly observed that, due to the consideration of the jointly owned unit, the improvement in the reliability of the individual systems as well as of the global system is less with the increase of tie line capacity. For example, in the case of joint ownership of generation at 500 MW tie line capacity, the decrease of LOLP of the global system is 86.79% while in the base case the decrease is 87.23%. Similarly for system X and for system Y the decrease in LOLPs at 500 MW tie line capacity is 74.28% and 86.73%, respectively, while for the base case the decrease is 78.95% and 87.14% respectively. The lower improvement in the reliability with the increase of tie line capacity indicates that the assistance between the connected utilities decreases when the diversity in FORs decreases. Recall that in modeling the jointly owned unit the unavailability of the shares of two interconnected systems is considered to occur simultaneously.

It is observed, from Table 30, that the increase in the production costs of system X, and the decrease in the production cost of system Y with the increase of tie line capacity are less when the jointly owned generating unit is considered. Note that the production costs of system Y as well as the global production costs are not equal in the base case and in the case of joint ownership of generation at zero MW tie line capacity. This occurs due to the changed average incremental cost of the constituent units of the jointly owned generator. It is clearly observed from the Table 30 and from the Figure 41 that the effect of joint ownership of generation is to decrease the economic benefits. For example, at 500 MW tie line capacity the savings are reduced from 5.54% to 5.13%.

The increase in LOLPs and the decrease in production cost savings in case of jointly owned unit are the results of combining two units together to form a jointly owned unit and also the reduced diversity of the forced outage rates of the constituent units.

Chapter VI

CONCLUDING REMARKS AND SUGGESIONS FOR FURTHER RESEARCH

6.1. CONCLUSIONS

In the planning process, reliability evaluation and an economic analysis of all feasible generation expansion plans are very important. The methods for computing the reliability index and the production costs of a single system are reasonably well developed. However, the method for the evaluation of the production costs of interconnected systems does not exist ,and the existing methods for computing the reliability index of the interconnected systems either follow the approximate technique, or suffer from the large computer memory and long computer time.

The objectives of this thesis are:

1. To develop a method, computationally efficient and accurate, to evaluate a plan devised by the planner for the two interconnected systems in terms of reliability.
2. To develop a method to evaluate a given plan of two interconnected systems in terms of production costs and economic benefits.

The first objective has been achieved by extending the segmentation technique to evaluate LOLPs of two interconnected systems. Comparing all other existing methods it is found that the developed technique is the most efficient one. From the sensitivity study of the proposed method it is observed that a coarser segment size ,not the common factor, provides approximate results but increases computational efficiency.

The second objective has also been achieved by developing a method for evaluating the expected energy generation, expected fuel costs, and expected economic benefits. The method is quite flexible. It can be applied to any existing systems using an approximate as well as accurate computational techniques, namely: cumulant method and segmentation method, respectively. The proposed method can also be used to evaluate the reliability of the given plan in terms of the expected unserved energies.

Studies regarding the impacts of different load management strategies indicate that load management is a resource that deserves the utilities' considerations in deciding whether to purchase power from the neighbouring utilities or implement a load management scheme to meet the peak load.

The studies of the impact of a jointly owned unit usually show that joint ownership of generation reduces the benefits of interconnection by reducing the improvement in the reliability and by decreasing the production cost savings.

The studies of time zone difference and varying FORs indicate that the diversity in the peak load and the diversity of FORs of the units of two systems increases the exchange of cheaper resources between the connected utilities, thus improves the reliability of the system and decreases the production costs.

6.2 SUGGESTIONS FOR FURTHER RESEARCH

In most of the cases the interconnections are made between more than two utilities. The proposed methods in this thesis has been found to be efficient and accurate. Therefore, it is highly recommended to extend these methods for multiarea interconnected systems.

In this thesis, the impacts of joint ownership of generation have been investigated by considering the unit outside the two systems. Further investigations can be made considering different locations of the units. These studies will be helpful in the optimum site selection for the jointly owned unit.

In this thesis a 100 percent reliable tie line has been considered. The extension of the proposed methods for random tie line availability can be made.

The proposed segmentation method presents an elegant procedure of convolving two or more discrete distributions. The application of this method is recommended in cases where convolution of discrete distributions is required.

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Appendix A

GENERATING UNIT MODEL

Different types of generating units are in use today and all types of units are randomly forced off-line because of technical problems during normal period of operation. The random outages of a unit can be taken into account by determining the probability density function that describes the probability that the unit will be forced off-line or will be available during its normal period of operation. For example, assume on the basis of historical data, that the availability of the generating capacity of a given unit may be graphically represented as shown in Figure 42.

The system alternates between an operating state, or up state, followed by a failed state, in which repair is effected. For i-th cycle, let

$$m_i = \text{UP time}$$

$$r_i = \text{DOWN time} \tag{A-1}$$

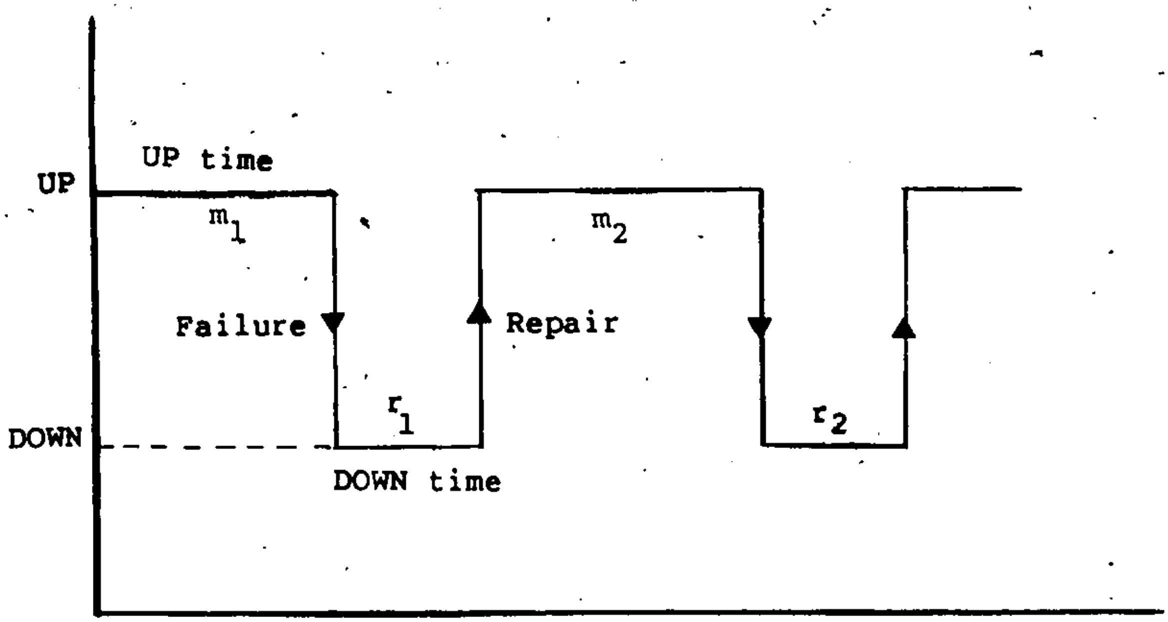

Figure 42: Run-fail-repair-run cycle for a generating unit

The random unit history may be represented in terms of an average UP time and average DOWN time as follows:

$$m = \text{mean UP time} = (1/N)\sum_i m_i$$

$$r = \text{mean down time} = (1/N)\sum_i r_i \qquad \text{(A-2)}$$

where N is the total Run-Fail-Repair-Run cycles. Thus, the unit failure rate λ and the repair rate μ may be expressed, respectively as,

$$\lambda = (1/m)$$

$$\mu = (1/r) \tag{A-3}$$

This simplest model for a generating unit expresses the idea that the random failure and the random repair of a unit can be defined as a two state stochastic process. A stochastic process is defined as a process that develops in time in a manner controlled by probabilistic laws [2]. That is, if an experiment specified by its outcome w forming the space S, by certain subsets of S called events, and by the probabilities of these events, then to every outcome w, according to a certain rule, a time function

$$\xi(t,w)$$

can be assigned. This family of time functions is called a stochastic process, and for a fixed w the function $\xi(t,w)$ is called a trajectory, or a sample path of the stochastic process.

The so-called generating unit state-space diagram for the stochastic process, shown in Figure 2, expresses that a unit may be in state 1(the state corresponding to maximum available capacity) and then randomly transfer to state 2 (the state corresponding to zero available capacity) and vice versa.

Consider a special type of stochastic process, called zero order, discrete state, continuous transition Markov process. A stochastic process $\xi(t,w)$ is called a Markov process if

$$P\left\{ \xi(t,w) \middle| \mathcal{F} \right\} = P\left\{ \xi(t,w) \middle| \xi(s,w) \right\} \tag{A-4}$$

where $\mathcal{F}$ is the σ-algebra of events generated by all events of the form

$$\left\{ \xi(u,w) \in S \quad , \quad (u \leq s) \right\} \tag{A-5}$$

Such a stochastic process has the following properties:

1. The system describes the available capacity of a generating unit, can be characterized as being in one of a set of mutually exclusive, collectively

exhaustive, discrete states $\xi_1, \xi_2, \ldots \xi_n$ at any time. The states of the system are mutually exclusive and discrete, because the generating unit can be in either the UP or DOWN state but not in both simultaneously, and the states are collectively exhaustive since only possible assumed states for a generating unit are the UP or DOWN states.

2. Changes of states are possible at any time.
3. The probabilities of departure from a state depends only on the current state and is independent of time.
4. The probability of more than one change of state during a small interval Δt is negligible.

Let

$p_1(t + \Delta t)$ = Probability that the unit will be in state 1 at time $t + \Delta t$.
= Probability of being in state 1 at time t and not leaving that state during interval Δt. + Probability of being in state 2 at time t and moving to state 1 during interval Δt.

(A-6)

If the probability of a unit is described by an exponential distribution, then

$$e^{-\lambda t} = \text{Probability of unit being available up to time } t$$
$$= 1 - \lambda\triangle t + (\lambda^2 \triangle t/2!) + \ldots$$
$$\simeq 1 - \lambda \triangle t$$
$$= \text{Probability of unit being available during time } \triangle t \qquad \text{(A-7)}$$

where

$\lambda \triangle t$ = Probability of transfering from state 1 to state 2 in time $\triangle t$.

Similarly the probability of unit being unavailable up to time t can be expressed in terms of the repair rate μ.

$$e^{-\mu t} = \text{Probability of unit being unavailable up to time } t$$
$$= 1 - \mu\triangle t + (\mu^2 \triangle t/2!) + \ldots$$
$$\simeq 1 - \mu\triangle t$$
$$= \text{Probability of unit being unavailable in time } \triangle t. \qquad \text{(A-8)}$$

where

$\mu\triangle t$ = Probability of transfering from state 2 to state 1 in time $\triangle t$.

These probabilities are shown in Figure 43.

If $P_i(t)$ is defined as the probability that a unit is in the i-th state at time t, then the equation (A-6) may be written as follows:

$$P_1(t + \triangle t) = P_1(t)\,(1 - \lambda\triangle t) + P_2(t)\,\mu\triangle t \qquad \text{(A-9)}$$

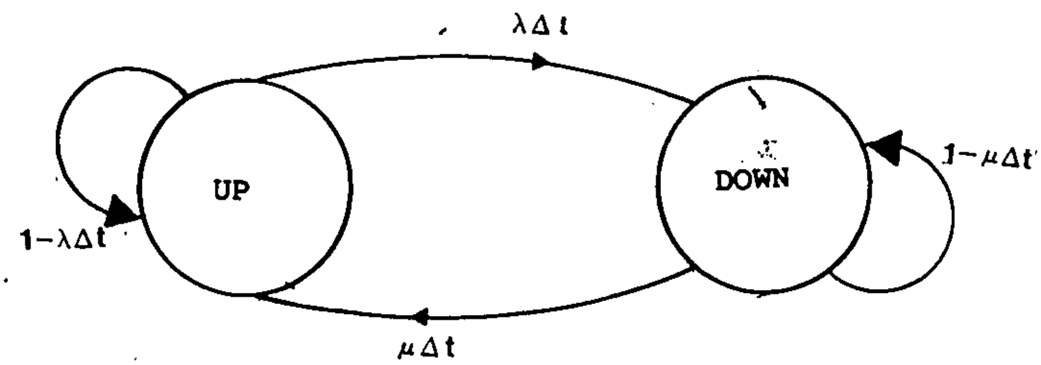

Figure 43: A two state Markov model

Similarly

$$P_2(t + \Delta t) = P_2(t)(1 - \mu \Delta t) + P_1(t)\lambda \Delta t \tag{A-10}$$

Equations (A-9) and (A-10) may be rearranged as

$$\frac{P_1(t + \Delta t) - P_1(t)}{\Delta t} = -\lambda P_1(t) + \mu P_2(t) \tag{A-11}$$

$$\frac{P_2(t + \Delta t) - P_2(t)}{\Delta t} = \lambda P_1(t) - \mu P_2(t) \tag{A-12}$$

Let $\Delta t \to 0$, then one obtains the system of differential equations.

$$\frac{dP_1(t)}{dt} = -\lambda P_1(t) + \mu P_2(t) \qquad \text{(A-13)}$$

$$\frac{dP_2(t)}{dt} = \lambda P_1(t) - \mu P_2(t) \qquad \text{(A-14)}$$

In matrix form

$$\begin{bmatrix} \dot{P}_1(t) \\ \dot{P}_2(t) \end{bmatrix} = \begin{bmatrix} P_1(t) & P_2(t) \end{bmatrix} \begin{bmatrix} -\lambda & \lambda \\ \mu & -\mu \end{bmatrix}$$

(A-15)

Solving for $P_1(t)$ and $P_2(t)$, we have

$$P_1(t) = \frac{\mu}{\lambda+\mu}\left\{P_1(0) + P_2(0)\right\} + \frac{e^{-(\lambda+\mu)t}}{\lambda+\mu}\left\{\lambda P_1(0) - \mu P_2(0)\right\}$$

(A-16)

$$P_2(t) = \frac{\lambda}{\lambda + \mu}\left\{P_1(0) + P_2(0)\right\} + \frac{e^{-(\lambda+\mu)t}}{\lambda + \mu}\left\{\mu P_2(0) - \lambda P_1(0)\right\}$$

(A-17)

where $P_1(0)$ and $P_2(0)$ are the initial conditions and

$$P_1(0) + P_2(0) = 1 \quad \ldots \tag{A-18}$$

If the process starts in state 1, i.e. at time zero the system is in the operating condition, then

$$P_1(0) = 1 \text{ and } P_2(0) = 0 \tag{A-19}$$

Now the equations (A-16) and (A-17) become

$$P_1(t) = \frac{\mu}{\lambda+\mu} + \frac{\lambda\, e^{-(\lambda+\mu)t}}{\lambda + \mu} \tag{A-20}$$

$$P_2(t) = \frac{\lambda}{\lambda + \mu} - \frac{\lambda \quad e^{-(\lambda + \mu)t}}{\lambda + \mu} \qquad \text{(A-21)}$$

In generation expansion planning what is required is the long term (steady state) probabilities. That is,

As $t \to \infty$

$$P_1 (\infty) = \mu / (\lambda + \mu) = p \qquad \text{(A-22)}$$

$$P_2 (\infty) = \lambda / (\lambda + \mu) = q \qquad \text{(A-23)}$$

which are the expressions used to define the availability p and forced outage rate q of a generating unit.

Appendix B

HOURLY LOAD MODEL AND THE GENERATION SYSTEM

In this Appendix, the load model and the generation system, utilized to carry out this research are described. The hourly load model resembles the hourly load of the IEEE reliability test system (IEEE-RTS) [45]. The generation data for system X are the same as the IEEE-RTS with rounded-off capacities of some of the units. The generation data of system Y are hypothetical.

Load Model

Table 31 gives data on weekly peak loads in percent of the annual peak load. The data in this table show a typical pattern, with two seasonal peaks. The annual peak occurs in week 51. The second peak is in week 23.

Table 32 gives a daily peak load cycle, in per cent of the weekly peak. The same weekly peak load cycle is assumed to apply for all seasons.

TABLE 31

Weekly peak load in percent of annual peak

week	peak load	week	peak load	week	peak load	week	peak load
1	86.2	14	75.0	27	75.5	40	72.4
2	90.0	15	72.1	28	81.6	41	74.3
3	87.8	16	80.0	29	80.1	42	74.4
4	83.4	17	75.4	30	88.0	43	80.0
5	88.0	18	83.7	31	72.2	44	88.1
6	84.1	19	87.0	32	77.6	45	88.5
7	83.2	20	88.0	33	80.0	46	90.9
8	80.6	21	85.6	34	72.9	47	94.0
9	74.0	22	81.1	35	72.6	48	89.0
10	73.7	23	90.0	36	70.5	49	94.2
11	71.5	24	88.7	37	78.0	50	97.0
12	72.7	25	89.6	38	69.5	51	100.0
13	70.4	26	86.1	39	72.4	52	95.2

TABLE 32

Daily peak load in percent of weekly peak

day	peak load
Monday	93
Tuesday	100
Wednesday	98
Thursday	96
Friday	94
Saturday	77
Sunday	75

Table 33 gives weekday and weekend hourly ~~load~~ model for each of three seasons. The interval of weeks is given for each season. The first two columns reflect a winter season (evening peak), while the next two columns reflect a summer season (afternoon peak).

TABLE 33

Hourly peak load in percent of daily peak

	Winter weeks (1-8 & 44-52)		Summer weeks (18-30)		Spring/Fall weeks (9-17 & 31-43)	
Hour	week day	week-end	week day	week-end	week day	weekend
12-1 am	67	78	64	74	63	75
1-2	63	72	60	70	62	73
2-3	60	68	58	66	60	69
3-4	59	66	56	65	58	66
4-5	59	64	56	64	59	65
5-6	60	65	58	62	65	65
6-7	74	66	64	62	72	68
7-8	86	70	76	66	85	74
8-9	95	80	87	81	95	83
9-10	96	88	95	86	99	89
10-11	96	90	99	91	100	92
11-noon	95	91	100	93	99	94
noon-1pm	95	90	99	93	93	91
1-2	95	88	100	92	92	90
2-3	93	87	100	91	90	90
3-4	94	87	97	91	88	86
4-5	99	91	96	92	90	85
5-6	100	100	96	94	92	88
6-7	100	99	93	95	96	92
7-8	96	97	92	95	98	100
8-9	91	94	92	100	96	97
9-10	83	92	93	93	90	95
10-11	73	87	87	88	80	90
11-12	63	81	72	80	70	85

Combination of Tables 31,32, and 33 with the annual peak load defines an hourly load model of 364x24 = 8736 hours.

The load of the i-th hour of the j-th day of the k-th week of the year may be expressed in terms of data of Tables 31,32,and 33 as

$$L_i = PK\ (WPK_k/100)\ (DPK_j/100)\ (HPK_{i,j,k}/100)\ MW \qquad (B-1)$$

where

PK = Yearly peak in MW

WPK_k = The peak load of the k-th week in per cent of the annual peak

DPK_j = The peak load of the j-th day in per cent of the weekly peak.

$HPK_{i,j,k}$ = The peak load of ith hour of j-th day of k-th week in percent of daily peak

The annual peak load for this system is 2850 MW. Using equation (B-1) the load at any hour of the year can be calculated in reference to the data of Tables 31,32,33. For example, the load of the first week of the year on Monday at 6 O' clock in the evening is equal to:

$$2850 \times 0.862 \times 0.93 \times 1.0$$
$$= 2284.731\ MW$$

To carry out the research for this thesis, 1-8 and 48-52 weeks hourly load is considered for system X and 18-30 weeks hourly load is considered for system Y, assuming that these loads occur in the same period.

Generation system

In Table 34 and Table 35, the generating unit size, type, number, and the average incremental cost of the units of system X and system Y are given, respectively. System X has 32 generating units with an installed capacity of 3400 MW, and system Y has 14 generating units with an installed capacity of 2950 MW.

TABLE 34

Generation data for system X

Type of units	No. of units	capacity (MW)	FOR	Avg. incremental cost ($/MWh)
Nuclear	2	400	0.12	5.592
Coal	4	150	0.04	11.160
Coal	1	350	0.08	11.400
Coal	4	80	0.02	14.882
Oil	3	200	0.05	19.870
Oil	3	100	0.04	20.080
Oil	5	10	0.02	28.558
Oil	4	20	0.10	37.500
Hydro	6	50	0.01	0.0
Total	32	3400		

TABLE 35

Generation data for system Y

Type of units	No. of units	capacity (MW)	FOR	Avg. incremental cost ($/MWh)
Nuclear	1	500	0.13	4.500
Coal	2	400	0.13	14.300
Coal	1	350	0.13	15.100
Coal	1	150	0.08	18.600
Oil	1	350	0.14	30.400
Oil	2	200	0.10	35.000
Oil	4	50	0.11	43.200
Hydro	2	100	0.01	0.0
Total	14	2950		

In both the systems hydro units are used as base loaded units.

Appendix C

BIVARIATE GRAM-CHARLIER SERIES AND USE OF CUMULANTS

In this Appendix, the bivariate Gram-Charlier series is presented and the convolution of the bivariate distribution with the univariate distribution is analysed. The convolution process is based on the notion of cumulants. In order to present the bivariate Gram-Charlier series and the convolution process it is necessary to discuss statistical cumulants first.

Cumulants

Besides the more common place statistical moments associated with a PDF are a set of descriptive parameters known as cumulants. The cumulants K_1 , K_2 , ... K_r, ... are implicitly defined by the identity in t as

$$\text{EXP}\left\{K_1 t + \frac{K_2 t^2}{2!} + \ldots + \frac{K_r t^r}{r!} + \ldots\right\} = 1 + \mu'_1 t + \frac{\mu'_2 t^2}{2!} + \ldots + \frac{\mu'_r t^r}{r!} + \ldots \qquad \text{(C-1)}$$

where μ'_r is the moment of order r about the point 'a' and 'a' is any point. The r th moment of the random variable X with PDF $f_X(x)$ may be expressed as

$$\mu'_r = \int_{-\infty}^{\infty} (X - a)^r f_x(x)\,dx \qquad r = 0,1,2,...$$

(C-2)

Equivalently the equation (C - 1) may be written in it as

$$\mathrm{EXP}\left\{K_1(it) + K_2\frac{(it)^2}{2!} + ... + K_r\frac{(it)^r}{r!} + ...\right\} = 1 + \mu_1\frac{(it)}{1}$$

$$+ \mu_2\frac{(it)^2}{2!} + ... + \mu_r\frac{(it)^r}{r!} + ...$$

$$= E\left\{e^{itx}\right\}$$

$$= \varphi(t) \qquad \text{(C-3)}$$

wrere $\varphi(t)$ is the characteristic function.

Expanding the left hand side of the equation (C-1) or the equation (C-3) and equating the coefficients of t's or (it)'s the expressions for cumulants in terms of μ may be obtained. The first eight cumulants are

$K_1 = \overline{X}$

$K_2 = \mu_2$

$$\begin{aligned}
K_3 &= \mu_3 \\
K_4 &= \mu_4 - 3\mu_2^2 \\
K_5 &= \mu_5 - 10\mu_3\mu_2 \\
K_6 &= \mu_6 - 15\mu_4\mu_2 - 10\mu_3^2 + 30\mu_2^3 \\
K_7 &= \mu_7 - 21\mu_5\mu_2 - 35\mu_4\mu_3 + 210\mu_3\mu_2^2 \\
K_8 &= \mu_8 - 28\mu_6\mu_2 - 56\mu_5\mu_3 - 35\mu_4^2 + 420\mu_4\mu_2^2 \\
&\quad + 560\mu_3^2\mu_2 - 630\mu_2^4
\end{aligned} \tag{C-4}$$

The central moments μ_r may be expressed in terms of μ_r' as

$$\mu_r = \sum_{j=0}^{r} \binom{r}{j} \mu_{r-j}' (-\mu_1')^j, \qquad r = 1, 2, 3 \ldots \tag{C-5}$$

The bivariate or joint cumulants are required in the bivariate Gram-Charlier expansion. The bivariate analogue of equation (C-1) may be written as

$$\text{EXP}\left\{\frac{K_{10}}{1!0!} t_1 t_2^0 + \frac{K_{01}}{0!1!} t_1^0 t_2 + \frac{K_{11}}{1!1!} t_1 t_2 + \ldots + \frac{K_{rs}}{r!s!} t_1^r t_2^s + \ldots\right\}$$

$$= 1 + \frac{\mu_{10}'}{1!0!} t_1 t_2^0 + \frac{\mu_{01}'}{0!1!} t_1^0 t_2 + \frac{\mu_{11}'}{1!1!} t_1 t_2 + \ldots + \frac{\mu_{rs}'}{r!s!} t_1^r t_2^s + \ldots \tag{C-6}$$

K_{rs} is the (r+s)th order joint cumulant. Again expanding the left hand side of (C-6) and equating the coefficients of $t_1 t_2$ the expressions for joint cumulants can be derived.

Joint cumulants can also be derived from the univariate cumulants by replacing order of cumulants and central moments, m as function of r^m and operating by $s \frac{\partial}{\partial r}$. Then replace the power of r and s by suffixes relating to the first and second variate. For example, in (C-4) the fifth order cumulant has the expression,

$$K_5 = \mu_5 - 10\mu_3\mu_2 \qquad \text{(C-7)}$$

Replace the cumulant and the moment as function r^m, m being the order of cumulant or moment;

$$K(r^5) = \mu(r^5) - 10\mu(r^3)\mu(r^2) \qquad \text{(C-8)}$$

Now operate by $s \frac{\partial}{\partial r}$

$$5K(r^4 s) = 5\mu(r^4 s) - 30\mu(r^2 s)\mu(r^2) - 20\mu(r^3)\mu(rs) \qquad \text{(C-9)}$$

Replacing the power of r and s as suffixes one gets

$$K_{41} = \mu_{41} - 6\mu_{21}\mu_{20} - 4\mu_{30}\mu_{11} \tag{C-10}$$

The expressions of joint cumulants, up to $(r+s) \leq 6$ in terms of joint central moments are given below.

$K_{00} = 1$

$K_{01} = 0$

$K_{02} = \mu_{02}$

$K_{03} = \mu_{03}$

$K_{04} = \mu_{04} - 3\mu_{02}^2$

$K_{05} = \mu_{05} - 10\mu_{03}\mu_{02}$

$K_{06} = \mu_{06} - 15\mu_{04}\mu_{02} - 10\mu_{03}^2 + 30\mu_{02}^3$

$K_{10} = 0$

$K_{11} = \mu_{11}$

$K_{12} = \mu_{12}$

$K_{13} = \mu_{13} - 3\mu_{02}\mu_{11}$

$K_{14} = \mu_{14} - 4\mu_{03}\mu_{11} - 6\mu_{12}\mu_{02}$

$K_{15} = \mu_{15} - 5\mu_{04}\mu_{11} - 10\mu_{13}\mu_{02} - 10\mu_{03}\mu_{12} + 30\mu_{02}^2\mu_{11}$

$K_{20} = \mu_{20}$

$K_{21} = \mu_{21}$

$K_{22} = \mu_{22} - \mu_{20}\mu_{02} - 2\mu_{11}^2$

$$\kappa_{23} = \mu_{23} - \mu_{03}\mu_{20} - 6\mu_{12}\mu_{11} - 3\mu_{02}\mu_{21}$$
$$\kappa_{24} = \mu_{24} - \mu_{04}\mu_{20} - 8\mu_{13}\mu_{11} - 4\mu_{03}\mu_{21} - 6\mu_{22}\mu_{02} - 6\mu_{12}^2 + 6\mu_{02}^2\mu_{20} + 24\mu_{02}\mu_{11}^2$$
$$\kappa_{30} = \mu_{30}$$
$$\kappa_{31} = \mu_{31} - 3\mu_{20}\mu_{11}$$
$$\kappa_{32} = \mu_{32} - \mu_{30}\mu_{02} - 6\mu_{21}\mu_{11} - 3\mu_{20}\mu_{12}$$
$$\kappa_{33} = \mu_{33} - 3\mu_{31}\mu_{02} - \mu_{30}\mu_{03} - 9\mu_{22}\mu_{11} - 9\mu_{21}\mu_{12} - 3\mu_{20}\mu_{13} + 18\mu_{20}\mu_{11}\mu_{02} + 12\mu_{11}^3$$
$$\kappa_{40} = \mu_{40} - 3\mu_{20}^2$$
$$\kappa_{41} = \mu_{41} - 4\mu_{30}\mu_{11} - 6\mu_{21}\mu_{20}$$
$$\kappa_{42} = \mu_{42} - \mu_{40}\mu_{02} - 8\mu_{31}\mu_{11} - 4\mu_{30}\mu_{12} - 6\mu_{22}\mu_{20} - 6\mu_{21}^2 + 6\mu_{20}^2\mu_{02} + 24\mu_{20}\mu_{11}^2$$
$$\kappa_{50} = \mu_{50} - 10\mu_{30}\mu_{20}$$
$$\kappa_{51} = \mu_{51} - 5\mu_{40}\mu_{11} - 10\mu_{31}\mu_{20} - 10\mu_{30}\mu_{21} + 30\mu_{20}^2\mu_{11}$$
$$\kappa_{60} = \mu_{60} - 15\mu_{40}\mu_{20} - 10\mu_{30}^2 + 30\mu_{20}^3$$

(C-11)

where the joint central moments μ_{rs} can be expressed in terms μ'_{rs} as

$$\mu'_{rs} = (\mu + \overline{X}_1)^r \; (\mu + \overline{X}_2)^s \qquad \text{(C-12)}$$

In equation (C-12), the product $\mu^r\mu^s$ on the right hand side is to be replaced by μ_{rs} and $\bar{X}_1$ and $\bar{X}_2$ are the means of the RVs X_1 and X_2 respectively.

Bivariate Gram-Charlier Series

Consider two correlated random variables X_1 and X_2 with marginal dispersion σ_1 and σ_2 and marginal mean μ_1 and μ_2 respectively. Also consider the correlated determinant expressed as

$$R = \begin{vmatrix} 1 & \rho \\ \rho & 1 \end{vmatrix} \qquad \text{(C-13)}$$

If μ_{11} is the covariance of the RVs X_1 and X_2, the correlation coefficient is given as

$$\rho = \frac{\mu_{11}}{\sigma_1\sigma_2} \qquad \text{(C-14)}$$

Then the binormal frequency function has the expression

$$\varphi(Z) = \frac{\mathrm{EXP}(-1/2\ g(Z))}{2\pi\sqrt{R}} \qquad \text{(C-15)}$$

where

$$g(Z) = a_{11}Z_1^2 + 2a_{12}Z_1 Z_2 + a_{22}Z_2^2 \tag{C-16}$$

In equation (C-16), Z_1 and Z_2 are the standardized RVs corresponding to X_1 and X_2, expressed as

$$Z_1 = \frac{X_1 - \mu_1}{\sigma_1} \tag{C-17}$$

$$Z_2 = \frac{X_2 - \mu_2}{\sigma_2} \tag{C-18}$$

and

$$a_{ij} = \frac{R_{ij}}{R} \tag{C-19}$$

R_{ij} is the cofactor of the correlation coefficient .

A set of polynomials may be defined by (C-16) as

$$H_{ij}(Z) = (-1)^{i+j} \mathrm{EXP}(1/2\ g(Z)) \frac{\partial^{i+j}}{\partial Z_1^i \partial Z_2^j} \mathrm{EXP}(-1/2\ g(z)) \tag{C-20}$$

The polynomials defined by (C-20) are known as Hermite polynomials and they are adjoint in respect to each other.

The Hermite polynomials $H_{ij}(Z)$ for $(i+j) \leq 6$, calculated according to (C-20), have the expressions,

$$H_{00} = 1$$
$$H_{10} = \nu_1$$
$$H_{20} = \nu_1^2 - a_{11}$$
$$H_{30} = \nu_1^3 - 3a_{11}\nu_1$$
$$H_{40} = \nu_1^4 - 6a_{11}\nu_1^2 + 3a_{11}^2$$
$$H_{50} = \nu_1^5 - 10a_{11}\nu_1^3 + 15a_{11}^2\nu_1$$
$$H_{60} = \nu_1^6 - 15a_{11}\nu_1^4 + 45a_{11}^2\nu_1^2 - 15a_{11}^3$$

$$H_{01} = \nu_2$$
$$H_{02} = \nu_2^2 - a_{22}$$
$$H_{03} = \nu_2^3 - 3a_{22}\nu_2$$
$$H_{04} = \nu_2^4 - 6a_{22}\nu_2^2 + 3a_{22}^2$$
$$H_{05} = \nu_2^5 - 10a_{22}\nu_2^3 + 15a_{22}^2\nu_2$$
$$H_{06} = \nu_2^6 - 15a_{22}\nu_2^4 + 45a_{22}^2\nu_2^2 - 15a_{22}^3$$

$$H_{11} = \nu_1\nu_2 - a_{12}$$
$$H_{12} = \nu_1\nu_2^2 - a_{22}\nu_1 - 2a_{12}\nu_2$$

$$H_{13} = \nu_1\nu_2^3 - 3a_{12}\nu_2^2 - 3a_{22}\nu_1\nu_2 + 3a_{12}a_{22}$$
$$H_{14} = \nu_1\nu_2^4 - 4a_{12}\nu_2^3 - 6a_{22}\nu_1\nu_2^2 + 12a_{12}a_{22}\nu_2 + 3a_{22}^2\nu_1$$
$$H_{15} = \nu_1\nu_2^5 - 5a_{12}\nu_2^4 - 10a_{22}\nu_1\nu_2^3 + 30a_{12}a_{22}\nu_2^2 + 15a_{22}^2\nu_1\nu_2 - 15a_{12}a_{22}^2$$
$$H_{21} = \nu_1^2\nu_2 - 2a_{12}\nu_1 - a_{11}\nu_2$$
$$H_{22} = \nu_1^2\nu_2^2 - a_{22}\nu_1^2 - a_{11}\nu_2^2 - 4a_{12}\nu_1\nu_2 + a_{11}a_{12} + 2a_{12}^2$$
$$H_{23} = \nu_1^2\nu_2^3 - 3a_{22}\nu_1^2\nu_2 - a_{11}\nu_2^3 - 6a_{12}\nu_1\nu_2^2 + 6a_{12}a_{22}\nu_1 + 3a_{11}a_{22}\nu_2 + 6a_{12}^2\nu_2$$
$$H_{24} = \nu_1^2\nu_2^4 - 6a_{22}\nu_1^2\nu_2^2 - 8a_{12}\nu_1\nu_2^3 + 24a_{12}a_{22}\nu_1\nu_2 - a_{11}\nu_2^4 + 12a_{12}^2\nu_2^2 + 6a_{11}a_{22}\nu_2^2 + 3a_{22}^2\nu_1^2 - 12a_{22}^2a_{22} + 3a_{11}a_{22}^2$$

$$H_{31} = \nu_1^3\nu_2 - 3a_{12}\nu_1^2 - 3a_{11}\nu_1\nu_2 + 3a_{11}a_{12}$$
$$H_{32} = \nu_1^3\nu_2^2 - a_{22}\nu_1^3 - 6a_{12}\nu_1^2\nu_2 - 3a_{11}\nu_1\nu_2^2 + 6a_{11}a_{12}\nu_2 + 6a_{12}^2\nu_1 + 3a_{11}a_{22}\nu_1$$
$$H_{33} = \nu_1^3\nu_2^3 - 3a_{22}\nu_1^3\nu_2 - 9a_{12}\nu_1^2\nu_2^2 - 3\nu_1\nu_2^3 + 9a_{11}a_{12}\nu_2^2 + 18a_{12}^2\nu_1\nu_2 + 9a_{11}a_{22}\nu_1\nu_2 + 9a_{12}a_{22}\nu_1^2 - 9a_{11}a_{12}a_{22} - 6a_{12}^3$$

$$H_{41} = \nu_1^4\nu_2 - 4a_{12}\nu_1^3 - 6a_{11}\nu_1^2\nu_2 + 12a_{11}a_{12}\nu_1 + 3a_{11}^2\nu_2$$
$$H_{42} = \nu_1^4\nu_2^2 - a_{22}\nu_1^4 - 8a_{12}\nu_1^3\nu_2 - 6a_{11}\nu_1^2\nu_2^2 + 12a_{12}^2\nu_1^2 + 6a_{11}a_{22}\nu_1^2 + 24a_{11}a_{12}\nu_1\nu_2 + 3a_{11}^2\nu_2^2 - 12a_{11}a_{12}^2 - 3a_{11}^2a_{22}$$

$$H_{51} = \nu_1^5\nu_2 - 5a_{12}\nu_1^4 - 10a_{11}\nu_1^3\nu_2 + 30a_{11}a_{12}\nu_1^2 + 15a_{11}^2\nu_1\nu_2 - 15a_{11}^2a_{12} \tag{C-21}$$

where ν_i are the functions of standarized RVs given by

$$\nu_i = \frac{1}{2} \frac{\partial\, g(Z)}{\partial\, Z_i} \tag{C-22}$$

Another set of polynomials $G_{ij}(\nu)$ may be defined as

$$G_{ij}(\nu) = (-1)^{i+j} \mathrm{EXP}(1/2\gamma(\nu)) \frac{\partial^{i+j}}{\partial\nu_1^i \partial\nu_2^j} \mathrm{EXP}(-1/2\,\gamma(\nu)) \tag{C-23}$$

The expressions for $G_{ij}(\nu)$ may be obtained from (C-21) by replacing a_{ij} and ν_i by (A_{ij}/Δ) and Z_i respectively without repeating the calculus. $\Delta = \det(a_{ij})$.

Now the frequency function $f_Z(Z)$ can be expanded formally in a series which has the form

$$f_Z(z) = (1/\sigma_1\sigma_2) \sum_{i,j=0}^{\infty} (-1)^{i+j} D_{ij}\; H_{ij}\,\varphi(Z) \tag{C-24}$$

This series is known as the bivariate Gram-Charlier series of type A.

The D-coefficients in (C-24) are given by

$$D_{ij} = \frac{(-1)^{i+j}}{i!\ j!} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} G_{ij}(Z)\ f_Z(z)\ dz_1 dz_2 \qquad \text{(C-25)}$$

By substituting the expressions of $G_{ij}(\nu)$ in (C-25) the D-coefficients can be expressed in terms of joint cumulants. The coefficients up to sixth order have the expressions as

$$D_{00} = C_{00}$$

$$D_{10} = C_{10}$$

$$D_{11} = C_{11}$$

$$D_{12} = C_{12}$$

$$D_{13} = C_{13} + (1/2)(K_{02}\ K_{11}/\sigma_1\sigma_2^3)$$

$$D_{14} = C_{14} + (1/24) + (4\rho K_{03}/\sigma_2^3) - ((4K_{03}K_{11} + 6K_{12}\ K_{02})/\sigma_1\sigma_2^4)]$$

$$D_{15} = C_{15} + (1/120)\ [(1/\sigma_1\sigma_2^5)\ \left\{5K_{04}K_{11} + 10K_{13}K_{02} + 10K_{03}K_{12} + 15K_{02}^2\ K_{11}\right\} - 5(\rho/\sigma_2^4)\ (K_{04} + 3K_{02}^2) - (30K_{02}K_{11}/\sigma_1\sigma_2^3)$$

$$D_{20} = C_{20}$$

$$D_{21} = C_{21}$$

$$D_{22} = C_{22} + (1/4)\ (K_{20}K_{02}/\sigma_1^2\sigma_2^2)$$

$$D_{23} = C_{23} - (1/12)\ (1/\sigma_1^2\sigma_2^3)(K_{03}K_{20} + 3K_{02}K_{21}) - (K_{03}/\sigma_2^3)$$

$$D_{24} = C_{24} + (1/48)(1/\sigma_1^2\sigma_2^4)(K_{04}K_{20} + 4K_{03}K_{21} + 6K_{22}K_{02} + 3K_{02}^2\ K_{20} + 12K_{02}K_{11}^2\ ) - (6K_{20}K_{02}\ /\sigma_1^2\sigma_2^2\ ) - (24\rho K_{02}K_{11}/\sigma_1\sigma_2^3) - (1/\sigma_2^4)(K_{04} + 3K_{02}^2\ )]$$

$$D_{30} = C_{30} - (K_{30}/6\sigma_1^3)$$

$$D_{31} = C_{31} + (1/2)[K_{20}K_{11}/\sigma_1^3\sigma_2]$$

$$D_{32} = C_{32} - (1/12)[(1/\sigma_1^3\sigma_2^2)(K_{30}K_{02} + 3K_{20}K_{12}) - K_{30}/\sigma_1^3]$$

$$D_{33} = C_{33} + (1/36)[(1/\sigma_1^3\sigma_2^3)(3K_{31}K_{02} + K_{30}K_{03} + 3K_{20}K_{13} + 9K_{20}K_{11}K_{02}) - (9K_{20}K_{11}/\sigma_1^3\sigma_2) - (9\rho K_{20}K_{02}/\sigma_1^2\sigma_2^2) - (9K_{02}K_{11}/\sigma_1\sigma_2^3)$$

$$D_{40} = C_{40} + (1/24\sigma_1^4)[K_{40} + 3K_{20}^2]$$

$$D_{41} = C_{41} - (1/24)[(1/\sigma_1^4\sigma_2)(4K_{30}K_{11} + 6K_{21}K_{20}) - 4\rho K_{30}/\sigma_1^3]$$

$$D_{42} = C_{42} + (1/48)[(1/\sigma_1^4\sigma_2^2)(K_{40}K_{02} + 4K_{30}K_{12} + 6K_{22}K_{20} + 3K_{20}^2K_{02} + 12K_{20}K_{11}^2) - (1/\sigma_1^4)(K_{40} + 3K_{20}^2) - (24\rho K_{20}K_{11}/\sigma_1^3\sigma_2) - (6K_{20}K_{02}/\sigma_1^2\sigma_2^2)]$$

$$D_{50} = C_{50} - (1/120)[(1/\sigma_1^5)(K_{50} + 10K_{30}K_{20}) - (10K_{30}/\sigma_1^3)]$$

$$D_{51} = C_{51} + (1/120)[(1/\sigma_1^5\sigma_2)(5K_{40}K_{11} + 10K_{31}K_{20} + 10K_{30}K_{21} + 15K_{20}^2K_{11}) - (5\rho/\sigma_1^4)(K_{40} + 3K_{20}^2) - (30K_{20}K_{11}/\sigma_1^3\sigma_2)]$$

$$D_{60} = C_{60} + (1/720)[(1/\sigma_1^6)(K_{60} + 15K_{40}K_{20} + 10K_{30}^2 + 15K_{20}^3) - (15/\sigma_1^4)(K_{40} + 3K_{20}^2)]$$

$$D_{01} = C_{01}$$

$$D_{02} = C_{02}$$

$$D_{03} = C_{03} - (K_{03}/6\sigma_2^3)$$

$$D_{04} = C_{04} + (K_{04} + 3K_{02}^2)/24\sigma_2^4$$

$$D_{05} = C_{05} + (1/120)[10K_{03}/\sigma_2^3) - (1/\sigma_2^5)(K_{05} + 10K_{03}K_{02})]$$

$$D_{06}= C_{06}+(1/720)[(1/\sigma_2^6)(K_{06}+15K_{04}K_{02}+10K_{03}^2+15K_{02}^3) - (15/\sigma_2^4)(K_{04}+3K_{02}^2)]$$

(C-26)

where the C_{ij} coefficients are invarient. These coefficients do not change with the convolution of machines. The expressions for the C_{ij} coefficients up to 6th order are given below.

$C_{00}= 1$
$C_{10}= 0$
$C_{11}= 0$
$C_{12}= - (K_{12}/2\sigma_1\sigma_2^2)$
$C_{13}= K_{13}/6\sigma_1\sigma_2^3 - (\rho/2)$
$C_{14}= (1/24)[(6K_{12}/\sigma_1\sigma_2^2) - K_{14}/\sigma_1\sigma_2^4]$
$C_{15}= (1/120)[(K_{15}/\sigma_1\sigma_2^5) - (10K_{13}/\sigma_1\sigma_2^3)] + \rho/4$

$C_{20}= 0$
$C_{21}= - (K_{21}/2\sigma_1^2\sigma_2)$
$C_{22}=(1/4)[(1/\sigma_1^2\sigma_2^2)(K_{22}+2K_{11}^2) - 2\rho^2 - 1]$
$C_{23}= -(1/12)[(1/\sigma_1^2\sigma_2^3)(K_{23}+ 6K_{12}K_{11}) - (3K_{21}/\sigma_1^2\sigma_2) - 6\rho(K_{12}/\sigma_1\sigma_2^2)$
$C_{24}= (1/48)[(1/\sigma_1^2\sigma_2^4)(K_{24}+ 8K_{13}K_{11}+ 6K_{12}^2) - (6/\sigma_1^2\sigma_2^2)(K_{22}+2K_{11}^2) - (8\rho K_{13}/\sigma_1\sigma_2^3)+ 24\rho^2+6]$

$C_{30} = 0$

$C_{31} = (K_{31}/6\sigma_1^3\sigma_2) - \rho/2$

$C_{32} = -(1/12)[(1/\sigma_1^3\sigma_2^2)(K_{32} + 6K_{21}K_{11}) - 6\rho(K_{21}/\sigma_1^2\sigma_2)$
$-(3K_{12}/\sigma_1\sigma_2^2)]$

$C_{33} = (1/36)[(1/\sigma_1^3\sigma_2^3)(K_{33} + 9K_{22}K_{11} + 9K_{21}K_{12} + 6K_{11}^3)$
$-(3K_{31}/\sigma_1^3\sigma_2) - (9\rho/\sigma_1^2\sigma_2^2)(K_{22} + 2K_{11}^2) - (3K_{13}/\sigma_1\sigma_2^3)$
$+12\rho^3 + 18\rho\ ]$

$C_{40} = -(1/8)$

$C_{41} = -(1/24)[K_{41}/\sigma_1^4\sigma_2 - 6K_{21}/\sigma_1^2\sigma_2]$

$C_{42} = (1/48)[(1/\sigma_1^4\sigma_2^2)(K_{42} + 8K_{31}K_{11} + 6K_{21}^2) - (8\rho K_{31}/\sigma_1^3\sigma_2)$
$-(6/\sigma_1^2\sigma_2^2)(K_{22} + 2K_{11}^2) + 24\rho^2 + 6]$

$C_{50} = 0$

$C_{51} = (1/120)[(K_{51}/\sigma_1^5\sigma_2) - 10K_{31}/\sigma_1^3\sigma_2 + 30\rho\ ]$

$C_{60} = (1/24)$

$C_{01} = 0$

$C_{02} = 0$

$C_{03} = 0$

$C_{04} = (1/8)$

$C_{05} = 0$

$C_{06} = (1/24)$

(C-27)

Convolution of Bivariate distribution with Univariate Distribution

Consider two correlated random variables (RV), L_X and L_Y, representing the load of system X and the load of system Y respectively. Also consider another two RVs, C_X and C_Y representing the outages of a unit of system X and system Y respectively.

Recall that in this research, the loads of the two interconnected systems are assumed correlated but the random outages of the respective generating units are assumed independent of each other as well as independent of load. As mentioned in Section 2.5, the random outages of the generating units are taken into account through the equivalent load approach. In this approach, the capacity on outage of a generating unit is added to the system load to give rise an equivalent load. Therefore, the equivalent load of system X may be expressed as

$$Le_X = L_X + C_X \tag{C-28}$$

If the probability density function (PDF) of the correlated loads of the two systems is $f_{L_X,L_Y}(\ell_X,\ell_Y)$ and the PDF of the RV, C_X is $f_{C_X}(c_X)$ then the PDF of the equivalent load, $f_{Le_X,L_Y}(\ell e_X,\ell_Y)$ may be obtained by convolving the two distributions. That is,

$$f_{Le_X,L_Y}(\ell e_X,\ell_Y) = f_{L_X,L_Y}(\ell_X,\ell_Y) * f_{C_X}(c_X)$$

$$= \int_{\infty}^{\infty} f_{L_X,L_Y}(Le_X - C_X\ ,\ \ell_Y)\ f_{C_X}(c_X)\,d\ell_X \tag{C-29}$$

Now, the central moments of order (r+s) of the equivalent load can be expressed in terms of the central moment of the load and the central moments of the outages of the unit of system X as

$$\mu_{rs}(Le_X,L_Y) = E\ [\ (L_X - \overline{L}_X) + (\ C_X - \overline{C}_X)]^r (L_Y - \overline{L}_Y)^s \tag{C-30}$$

where $\overline{L}_X$, $\overline{L}_Y$ and $\overline{C}_X$ are the expected values or the means of the RVs L_X, L_Y and C_X respectively. Expanding the right hand side of the equation (C-30) as binomial expansion it becomes

$$\mu_{rs}(Le_X, L_Y) = E\ [(L_Y - \overline{L}_Y)^s [\binom{r}{0}(L_X - \overline{L}_X)^r + \binom{r}{1}(L_X - \overline{L}_X)^{r-1}$$

$$(C_X - \overline{C}_X) + \binom{r}{2}(L_X - \overline{L}_X)^{r-2}(C_X - \overline{C}_X)^2 + \ldots + \binom{r}{r}(C_X - \overline{C}_X)^r]$$

$$= \sum_{i=0}^{r} \binom{r}{i} \mu_{(r-i)s}(L_X, L_Y)\mu_i(C_X)$$

(C-31)

The cumulants of the equivalent load can be expressed in terms of the cumulants of the load (L_X, L_Y) and the outages of the unit of system X, C_X using the relation (C-31). For example, the fourth order cumulant of the equivalent load $K_{40}(Le_X, L_Y)$ may be expressed using (C-11) in terms of the central moments of the equivalent as

$$K_{40}(Le_X, L_Y) = \mu_{40}(Le_X, L_Y) - 3\mu_{20}^2(Le_X, L_Y) \qquad \text{(C-32)}$$

Substitute $\mu_{40}(Le_X, L_Y)$ and $\mu_{20}(Le_X, L_Y)$ from (C-31) in (C-32).

$$\begin{aligned}
K_{40}(Le_X, L_Y) &= \mu_{40}(L_X, L_Y) + 6\mu_{20}(L_X, L_Y)\, \mu_2(C_X) + \mu_4(C_X) \\
&\quad -3\, [\, \mu_{20}(L_X, L_Y) + 2\mu_{20}(L_X, L_Y)\, \mu_2(C_X) \\
&\quad + \mu_2^2(C_X) \\
&= [\mu_{40}(L_X, L_Y) - 3\mu_{20}^2(L_X, L_Y)] + [\mu_4(C_X) \\
&\quad -3\mu_2^2(C_X) \\
&= K_{40}(L_X, L_Y) + K_4(C_X)
\end{aligned} \tag{C-33}$$

Equation (C-33) reveals the fact that the fourth order cumulant of the equivalent load ($K_{40}(Le_X, L_Y)$) is the sum of the fourth order cumulant of load $K_{40}(\ell_X, \ell_Y)$ and the fourth order cumulant of the outages of the unit. It is also true for any order of cumulants. That is

$$K_{rs}(Le_X, \tilde{L}_Y) = K_{rs}(L_X, L_Y) + K_{(r+s)}(C_X) \tag{C-34}$$

Another important advantage of the utilization of the cumulants in the process of convolution of the bivariate distribution with the univariate distribution is that most of the cumulants of the equivalent load are equal to the cumulant of load. For example, from (C-11) the fifth order cumulant of the equivalent load may be written as

$$K_{23}(Le_X,L_Y) = \mu_{23}(Le_X, L_Y) - \mu_{03}(Le_X,L_Y)\mu_{20}(Le_X, L_Y) - 6\mu_{12}(Le_X,L_Y)\,\mu_{11}(Le_X,L_Y) - 3\mu_{02}(Le_X,L_Y)\,\mu_{21}(Le_X, L_Y) \tag{C-35}$$

using the relation (C-31) the equation (C-35) may be written as

$$\begin{aligned} K_{23}(Le_X,L_Y) &= \mu_{23}(L_X, L_Y) + \mu_{03}(L_X, L_Y)\,\mu_2(C_X) \\ &\quad - \mu_{03}(L_X,L_Y)[\mu_{20}(L_X,L_Y) + \mu_2(C_X)] \\ &\quad - 6\mu_{12}(L_X,L_Y)\mu_{11}(L_X,L_Y) - 3\mu_{02}(L_X,L_Y)\mu_{21}(L_X,L_Y) \\ &= K_{23}(L_X, L_Y) \end{aligned} \tag{C-36}$$

In (C-33) it has appeared that the cumulant of the equivalent load is the sum of the cumulant of the load and the cumulant of of the outages of the generating unit. But the relation (C-36) shows that the cumulant of the equivalent load is equal to the cumulant of the load.

In a similar way if the other cumulants are treated it can be shown that [37]

$$K_{rs}(Le_X, L_Y) = K_{rs}(L_X, L_Y) \qquad \text{if } s \neq 0 \qquad \text{(C-37)}$$

Following the similar process the relations for the convolution of the unit of system Y can be obtained.