MANIFESTATIONS OF
GEOMETRICAL CONCEPTS IN MESOAMERICA

A thesis submitted

by

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to

The School of Graduate Studies

of the University of Ottawa

in partial fulfillment of the requirements

for the degree of

Master of Science

in the subject of

Mathematics

December 1982

ABSTRACT

This study presents an examination of manifestations of geometrical concepts in Mesoamerica. Since there is no written evidence of an exact knowledge of geometry, as is the case for the ancient Greek and Egyptian civilizations, different approaches are necessary in order to determine Mesoamerican geometrical knowledge.

Symmetry analyses, among which Washburn's is described in detail, are presented here as new methods to classify regularized patterns (found on pottery, paintings and woven materials). They may also be used to further anthropological studies.

Artifacts such as paintings, sculptures and monuments are analyzed in order to find geometrical notions that can be extracted from them. Some compositional structures are examined and the problems involved with respect to imposing knowledge rather than extracting it, are noted.

Previous studies in archaeoastronomy, geomagnetism, and geomancy, are considered and serve to justify, to some extent, the non-coincidental nature of the geometrical concepts displayed in the disposition of monuments at Mesoamerican sites.

Finally, possible units of measure are examined from ethnohistorical sources and linguistic analysis. Values of units of measure resulting from statistical studies and actual field measurements are also reported.

ACKNOWLEDGEMENT

I would like to express my sincere thanks to Dr. M.P. Closs for his guidance and valuable advice during this research and my thanks also to the Department of Mathematics. I wish to express my appreciation to Dr. B. Drewitt of the University of Toronto for his helpful comments.

I am also very grateful to my parents for their help and support at all times. Finally, I wish to thank my friends and relatives, especially Antonino and my sister Danielle, for their help and encouragement.

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INTRODUCTION

The non-existence of documents dealing specifically with geometrical knowledge in Mesoamerica, raises a problem in methodology. The written records available to us consist of a small number of native codices and numerous monumental inscriptions as well as Spanish texts written shortly after the Conquest. None of these contain explicit data addressing the question of an indigenous geometry. Any attempt to determine such knowledge in Mesoamerica, requires scrupulous care on the part of the researcher, since this knowledge has to be extracted from physical evidence of the use of geometrical concepts, rather than reported from written primary sources.

In my thesis, I propose to examine manifestations of a variety of geometrical concepts. These include symmetry transformations such as translations, reflexions and rotations which generate regularized patterns, certain elements of Euclidean geometry such as right angles and special triangles, and the notion of measuring. This research does not attempt to deduce the specific knowledge of Mesoamerican geometry, rather it reports on and assesses the literature in this area which deals with such manifestations. I also comment on inherent problems in the literature, indicating, when necessary, the need for improvement. In doing so, my research is intended to be a preliminary study for further investigations leading towards a more precise evaluation of geometrical knowledge in Mesoamerica.

In the first chapter, I examine several analyses of the geometrical patterns found on pottery, paintings, and woven material. Among these, I focus on Washburn's symmetry analysis on the regularized patterns of Upper Gila ceramics. Based on the theory of groups of symmetry, her method distinguishes between horizontal and vertical displacement of figures, between basic units and design units, as well as monochrome and counterchanged figures. These distinctions prove to be very useful in giving a better classification of geometrical designs.

I next consider the validity of such a "mathematical" analysis of geometrical designs in permitting , by itself, anthropological conclusions relating to the interractions of tribes. I also comment on the need to supplement an analysis such as Washburn's with studies in other disciplines.

In chapter 2, I examine the manifestations of geometrical concepts in artifacts. Studies such as Miller's analysis of Teotihuacan paintings and Sanders's comparison of the twin Stelae of Seibal, will be presented. These produce evidence supporting the hypothesis that the Mesoamericans used patterns and planned their artistic use of space.

Special attention will be given to research into compositional structures which claims to exhibit the use of specific geometrical elements in the formation of selected artifacts. Although this study is restricted to Mesoamerica, the analysis of the Stelae of Chavin done by Hébert-Stevens provides a good example of the problems involved when trying to extract geometrical knowledge from artifacts.

Chapter 3 discusses recent developments in the study of orientations inherent in the arrangement of Mesoamerican structures at various sites. Although most site plans appear to be rather disorganized when first viewed, a closer examination reveals numerous instances of the apparent use of geometrical concepts. This chapter deals with studies in the field of archaeoastromony, geomagnetism, ritual and symbolism, geomancy and hierophanies, in search of arguments to justify the intentional nature of the geometrical concepts displayed by certain dispositions of monuments.

Finally, in ~~Chapter~~ 4, an analysis of the search for a Mesoamerican unit of measure is attempted. The notion of measuring has been a basic concept leading to developments in geometry in other regions and should be examined in the Mesoamerican context. I begin by presenting ethnohistorical information on this matter and follow it with linguistic data. I also consider two previous studies which have proposed precise units of measure, Drewitt's field survey of Teotihuacan and Cramer's statistical analysis based on measures collected at Uxmal. Kendall's analysis of the problems involved in the search for a quantum using statistical methods is discussed. Although his work was not based on Mesoamerican data, it raises important methodological points that should be taken into account in any statistical approach to this problem.

CHAPTER 1

ANALYSE DE DESSINS GEOMETRIQUES

Une figure symétrique est le résultat de répétitions d'une section assymétrique appelée <u>figure de base</u> ou <u>figure origine</u>. La classification de figures symétriques demande premièrement l'identification de la figure origine et ensuite l'étude des différents déplacements de cette dernière afin de former le motif étudié. Ces déplacements sont appelés indifférement <u>opérations de symétrie</u>, <u>isométries</u>, <u>transformations</u> ou encore <u>fonctions génératrices</u>. On distingue quatre différentes isométries:

1. <u>Translation</u>: C'est la plus simple transformation. La figure de base est déplacée le long d'un axe en gardant son orientation.

2. <u>Réflexion pure</u> ou <u>réflexion</u>: La figure de base est réfléchie par rapport à un axe en changeant son orientation. Une réflexion est appelée <u>longitudinale</u> si l'axe de réflexion est parallèle à l'axe de mediane de la bande ornementale: autrement elle est dite <u>transversale</u>. Selon que l'axe de réflexion est parallèle à l'axe des x ou à l'axe des y, on aura une réflexion <u>horizontale</u> ou une réflexion <u>verticale</u>. Par conséquent, parler de réflexion horizontale ou verticale lors de l'étude d'une bande décorative, impliquera l'orientation de la bande. Si l'axe de réflexion n'est pas

perpendiculaire à la mediane de la bande, la réflexion est dite oblique.

3. Rotation: La figure de base tourne autour d'un axe à un angle fixe. L'ordre de rotation indique le nombre de rotations de même angle nécessaire pour qu'une figure de base revienne à l'identité.

Figure 1. Exemples de rotations d'ordre 2: la figure origine fait deux demi-tours complets avant d'être superposable.

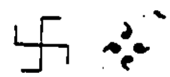


Figure 2. Exemples de rotations d'ordre 4.

4. Réflexion translatoire ou réflexion longitudinale avec glissement: Cette opération est le produit d'une réflexion suivie d'une translation le long d'un même axe.

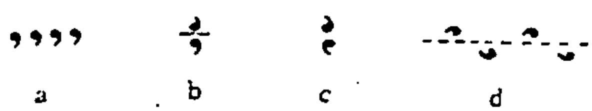


Figure 3. Illustration des différentes opérations.

(a) Translation.
(b) Réflexion horizontale. Le trait noir entre les virgules indique l'axe de réflexion.
(c) Rotation de 180° ou d'ordre 2. Le point entre les virgules indique le point ou l'axe de rotation.
(d) Réflexion translatoire. Les pointillés entre les virgules indiquent l'axe de réflexion et de translation.

Quelque soit l'opération de symétrie employée lors du déplacement de la figure origine, la distance entre les répétitions successives de cette unité est constante: les axes de réflexion et/ou de rotation sont donc équidistants. Les translations et les rotations font partie de la classe des opérations de <u>première espèce</u> (direct isometry): les figures sont équidistantes et superposables, c'est-à-dire congruentes. Les opérations de <u>deuxième espèce</u> (opposite isometry), telles les réflexions pures et les réflexions translatoires, sont des isométries qui maintiennent les distances entre les figures, égales mais changent leur orientation. Les figures sont alors énantiomorphes.

Aucune mention n'a été faite jusqu'à maintenant quant à la couleur des figures. Les motifs où la forme et la couleur de la figure de base sont inchangées, sont appelés <u>purs</u> ou <u>monochromes</u>. On nomme <u>motif contrechangé</u>, tout motif où la figure de base se déplace en changeant de couleur. Chaque isométrie peut être contrechangée; par contre, certains groupes de symétrie ne possèdent pas de forme contrechangée (ex. les rotations d'ordre 3).

a b

Figure 4. Exemples de contrechange.

On distingue, en outre, trois catégories de motifs monochromes ou contrechangés: les motifs finis, uni-dimensionnels et bi-dimensionnels. Les motifs finis sont des motifs constitués d'une seule figure: les seules opérations de symétrie sont les réflexions et les rotations. Les motifs uni-dimensionnels sont formés par la répétition de figure de base se déplaçant le long d'un seul axe ou médiane. Les bandes ornementales appartiennent à cette catégories. Les figures origines des motifs bi-dimensionnels se déplacent le long des axes des x et des y. Toutes les transformations décrites ci-dessus peuvent être employées dans la formation de motifs appartenant aux deux dernières catégories. De plus, ces motifs sont qualifiés d'infinis, par opposition aux motifs finis, car leur figure de base peut, en théorie, se répéter indéfiniment le long des axes.

D'autres restrictions quant aux possibles isométries sont imposées lorsqu'on étudie les bandes ornementales. Le déplacement de la figure origine des motifs uni-dimensionnels doit se faire le long d'une ligne droite. La figure ci-dessous explique pourquoi les réflexions obliques et les rotations autres que l'ordre 2 sont interdites.

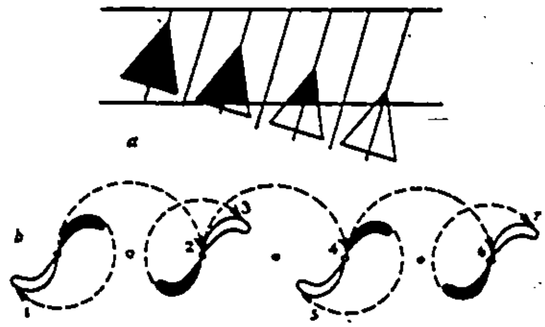


Figure 5. (a) La réflexion oblique fait sortir la figure de base de la bande ornementale. En fait, elle implique la formation d'une oblique où le déplacement de la figure de base est une réflexion transversale.

(b) Description des sept axes de rotations d'ordre 2 de la figure origine. Le mouvement se fait en ligne droite, formant ainsi une bande horizontale. Si on utilisait un autre angle de rotation que l'angle plat, la figure de base serait rejetée hors de la bande.

Notation

Le système de notation de D.K. Washburn (1977) est un système où les opérations de symétrie sont représentées par trois chiffres. La position de chaque chiffre indique par quelle isométrie la figure de base se déplace pour former le motif à classifier. L'avantage de ce système consiste à nous informer d'une part s'il y a présence de contrechange ou non, et d'autre part, si le déplacement s'effectue le long de tel axe ou tel autre.

Les principes de notation s'appliquent aux différentes catégories de figures. Cependant, une distinction est faite par l'addition de 1- ou de 2-, selon que la figure est uni-dimensionnelle ou bi-dimensionnelle. L'absence de chiffres devant les symboles indique que la figure à noter appartient à la première catégorie, c'est-à-dire est un motif fini.

La colonne des centaines désigne le nombre de fois qu'une figure de base tourne autour d'un axe avant de revenir à l'identité. L'absence de rotation (translation) est indiquée par un 1. Comme on l'a expliqué précédement, le seul cas possible de rotation de figures uni-dimensionnelles est une rotation d'ordre 2: elle est notée par un 2 dans la colonne des centaines. Les rotations de 180°, 120°, 90° et de 60° de figures bi-dimensionnelles sont notées dans cette colonne par un 2, 3, 4 et 6 respectivement. Ainsi la classe 1-200 représente les figures uni-dimensionnelles avec rotation de 180°, alors que les motifs bi-dimensionnels avec rotation d'ordre 3 appartiennent à la classe 2-300.

Un 1 ou un 0 signale la présence ou l'absence de réflexion. La colonne des dizaines est réservée pour l'indication de réflexion horizontale, la colonne des unités pour l'indication de réflexion verticale. Ainsi, une figure uni-dimensionnelle comportant uniquement une réflexion horizontale appartient à la classe 1-110, tandis qu'une figure comportant une réflexion verticale fait partie de la classe 1-101. A noter cependant le cas représentant une rotation d'ordre k ou d'ordre 2k (k impair) où le 1 dans la colonne des unités indique uniquement la présence de réflexion: la position est purement arbitraire.

La présence de réflexion translatoire est notée par un x et/ou par un y placés à la suite des trois chiffres décrits auparavant: un x si le déplacement de la réflexion et de la translation se fait le long de l'axe des x et un y s'il se fait le long de l'axe des y. Le déplacement peut se faire le long d'un axe de glissement ou encore entre deux axes de glissement, comme l'explique la Figure 6.

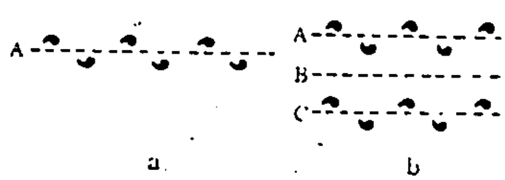


Figure 6. (a) Motif uni-dimensionnel: déplacement de la figure de base le long de l'axe A.

(b) Motif bi-dimensionnel; déplacement le long des axes A et C, mais aussi le long de l'axe B.

Un sur-indice 1 accompagnant le x ou le y implique que l'axe de la réflexion translatoire est le même que l'axe de déplacement de la figure origine. Un sous-indice 1 suivant le x ou le y implique que l'axe de la réflexion translatoire se trouve entre les unités. Ainsi, dans la Figure 6(a), le déplacement de la figure origine du motif uni-dimensionnel, est noté $1\text{-}100x^{1}$. Dans la Figure 6(b), le déplacement des figures se fait le long et entre les axes des x; il est représenté par la classe $2\text{-}100x^{1}_{1}$.

Notation pour les motifs contrechangés

On appelle module (design unit) une figure symétrique composée de deux ou plusieurs figures origines. Prenons comme exemple un triangle rectangle que l'on déplace suivant une rotation d'ordre 2. Les deux triangles rectangles ainsi constituent un module. Le système de notation de D.K. Washburn permet de distinguer l'existence de contrechange à l'intérieur même du module, ainsi que l'existence de contrechange entre les modules déplacés le long d'un seul axe ou de deux axes.

Un sur-indice 2 placé à la suite du symbole désignant une rotation, une réflexion ou une réflexion translatoire d'un module, indique la présence de contrechange à l'intérieur du module. Un sous-indice 2 placé à la suite du symbole désignant une réflexion ou une réflexion translatoire, indique la présence de contrechange entre les modules ainsi déplacés.

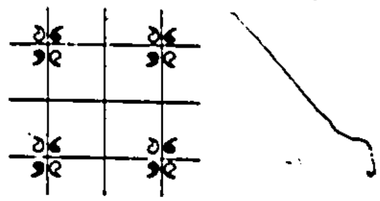
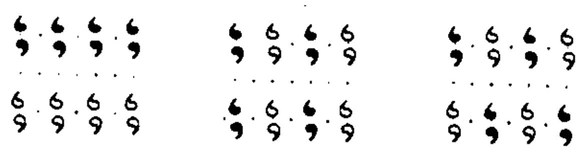


Figure 7. Motif de classe $2\text{-}21^2_2 1^2_2$; contrechange à l'intérieur et entre les modules, par rapport aux deux axes de réflexions.

Trois indices différents placés à la suite du symbole désignant une rotation (ou une translation équivalent à l'absence de rotation), indiquent le contrechange entre modules. Un 1 indique le contrechange autour de l'axe des y, un 2 pour le contrechange autour de l'axe des x et un 3 pour le contrechange autour des axes des x et des y.



a $2\text{-}2_1 00$ b $2\text{-}2_2 00$ c $2\text{-}2_3 00$

Figure 8. (a) Exemple de contrechange autour de l'axe des y.
(b) Exemple de contrechange autour de l'axe des x.
(c) Exemple de contrechange autour des axes des x et des y.

Différentes classes dans les trois catégories

La section suivante est consacrée à l'étude plus détaillée de chaque catégorie. Elle comprend en outre, des tableaux comparant les notations de Coxeter (1961), Loeb (1971) et Shubnikov (1966 et 1974), qui ont considéré mathématiquement la théorie des groupes de symétrie et son application dans les domaines scientifiques et artistiques, avec la notation de D.K. Washburn qui, analysant uniquement les motifs décoratifs, se permet une rigueur mathématique moindre dans le but d'une classification plus précise. Cette comparaison montre les problèmes soulevés par l'analyse scientifique des arts décoratifs. Pour ce faire, je présenterai brièvement la notation de Loeb: l'équivalence de cette notation et celle de Shubnikov, déjà démontrée dans "Color and Symmetry", sera rapportée pour référence.

L'approche de la théorie des groupes de symétrie de Loeb diffère de celle de Coxeter et de Shubnikov, du fait qu'elle examine le nombre d'axes de rotation d'un motif: une série de théorèmes détermine ensuite la possibilité d'existence d'une autre isométrie (réflexion horizontale, verticale et/ou translatoire), à l'intérieur de ce même motif. La présence d'un seul axe de rotation indique une rotation d'ordre k. Si k=1, le motif considéré est une figure assymétrique. Si k=∞, le motif est déplacé le long d'un seul axe par une translation. Deux axes de rotation impliquent l'existence d'un troisième: ces axes k, L, m obéissent à l'équation diaphantienne :

$$\frac{1}{k} + \frac{1}{L} + \frac{1}{m} = 1 \qquad k \leq L \leq m$$

Les solutions de cette équation détermineront les groupes de symétrie suivants:

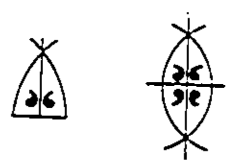
k=1	L=m=∞		Translation le long de deux axes non-parallèles;
k=L=2	m=∞		rotation d'ordre 2 d'un motif uni-dimensionnel.
k=2	L=6	m=6	Motifs bi-dimensionnels.
k=2	l=m=4		
k=L=m=3			

L'addition d'un quatrième axe de rotation n'est possible que dans le cas ou k=L=2 et m=∞, créant ainsi le groupe 2222, formé de quatre axes de rotation d'ordre 2.

Les axes de rotations peuvent être placés de différentes manières, impliquant la présence de réflexion (horizontale, verticale, translatoire). Des théorèmes limitent le choix de ces positions, déterminant ainsi le reste des groupes de symétrie. Dans la notation de Loeb, un prime (') ou un double prime ("), indiquent deux ou trois axes distincts, un accent circonflexe (^) la présence d'énantiomorphisme. De plus, un axe de rotation situé sur un axe de réflexion est souligné. Si deux axes de réflexions sont présents, la présence ou l'absence de diagonale (/) indiquera que ces derniers sont perpendiculaires ou parallèles: un m désignant un axe de réflexion horizontale ou verticale et un g un axe de réflexion translatoire. La présence de contrechange est notée par un r (exemples $2\hat{2}'\hat{2}'g/g'$ et $1\infty\infty'mm'$).

1. Motifs finis

Les translations et les réflexions translatoires ne pouvant être employées lors de la formation des motifs finis, seules restent les réflexions, les rotations et la combinaison de ces deux opérations.



a b

Figure 9. (a) Classe 101 indiquant une réflexion verticale.
(b) Classe 211 indiquant une réflexion horizontale et verticale ainsi qu'une rotation d'ordre 2.

Etant donné que le nombre de rotations et de réflexions possibles à l'intérieur d'un motif fini est illimité, la liste des classes pures ou contrechangées est infinie.

Il est à noter, cependant, que non toutes les classes pures possèdent une forme contrechangée: ainsi la création de la classe contrechangée correspondant à la classe 300 (rotation d'ordre 3 et absence de réflexion) est impossible, alors que la classe 211 possède trois classes contrechangées $22^2 1^2$, $2^2 11^2$ et $2^2 1^2 1$. La Table I donne quelques exemples de classes de motifs finis.

.2. Motifs uni-dimensionnels

Il existe seulement sept classes de motifs uni-dimensionnels. Cinq sont formées par l'utilisation d'une seule opération parmi les isométries suivantes; translation, rotation d'ordre 2, réflexion horizontale, verticale et translatoire. Les deux autres sont le produit de ces dernières transformations. La Table II dresse la liste de ces sept groupes de symétrie. De plus, elle comporte des remarques utiles pour l'étude des bandes ornementales. L'existence de plusieurs isométries intrinsèques à un motif souligne l'importance du choix de la figure origine: il est donc nécessaire de bien distinguer figure de base et module afin de classer de manière unique le motif étudié.

Coxeter ne fait pas de classification spéciale pour les motifs contrechangés, alors que D.K. Washburn et Loeb (ainsi que Shubnikov) ont étudié les différentes possibilités de contrechange de chaque groupe de symétrie. Ils distinguent respectivement 21 et 17 classes de motifs contrechangés. Cette différence dans le nombre de classes est due à l'importance accordée par D.K. Washburn à la distinction entre figure de base et module. Ainsi, les groupes $\infty^r m^r m'$ et $\infty^r mm'^r$ représentent, selon Loeb, la même forme contrechangée du groupe $\infty mm'$, alors que D.K. Washburn distingue le contrechange à l'intérieur du module ($1\text{-}101^2$) du contrechange entre deux modules consécutifs ($1\text{-}101_2$). La liste des classes des motifs uni-dimensionnels est résumée dans la Table III.

3. Motifs bi-dimensionnels

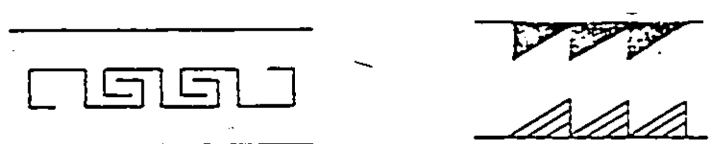
Les figures origines des motifs appartenant à cette catégorie se déplacent le long des axes des x et des y. L'importance de la différence d'orientation des motifs oblige la création par D.K. Washburn de six paires de classes monochromes, augmentant ainsi de 17 à 23, le nombre de classes pures établi par Loeb, qui considère équivalents les déplacements horizontaux et les déplacements verticaux. Les six paires de nouvelles classes sont les suivantes: les classes 2-110 et 2-101, $2\text{-}100x_1^1$ et $2\text{-}100y_1^1$ $2\text{-}110x_1$ et $2\text{-}101y_1$, $2\text{-}201x_1^1$ et $2\text{-}210y_1^1$, $2\text{-}301y_1$ et $2\text{-}310x_1$, $2\text{-}310y_1$ et $2\text{-}301x_1$. Dans la Table IV la classe correspondant à la rotation de 90° d'une classe de la colonne a est listée dans la colonne b. La classe 2-101 et sa correspondante

2-110 après rotation de 90°, représente le même groupe $1\infty\infty$'mm' selon la classification de Loeb. L'augmentation du nombre de classes pures, ainsi que la distinction entre contrechange à l'intérieur du module et contrechange entre deux modules, impliquent une différence dans le nombre des classes contrechangées: de 80 il passe à 94. Ajoutons cependant aux 94 classes de D.K. Washburn, la classe $2\text{-}2_1 1_2 1$ et la correspondante $2\text{-}2_2 1 1_2$, non mentionnées par D.K. Washburn (voir Appendice)

Différents niveaux de classification

1. La symétrie de la structure du motif

Les lignes de construction de base divisant la surface à décorer (design area) définissant la structure du motif. En vue d'une classification, il est nécessaire d'étudier séparement la symétrie de la structure et la symétrie du motif. Ces symétries seront identiques si, par exemple, les lignes de division coïncident avec les lignes de construction de base de la figure origine d'un motif monochrome.



a   b

Figure 10. (a) Exemple où la symétrie de structure est identique à celle du motif monochrome.
(b) Exemple où la symétrie de structure serait identique à celle du motif s'il n'y avait pas de contrechange.

Prenons maintenant le cas où la surface à décorer est divisée en compartiments pouvant contenir un motif. La symétrie de ces compartiments peut alors différer de celle du motif contenu dans chaque compartiment, comme la figure ci-dessous explique.

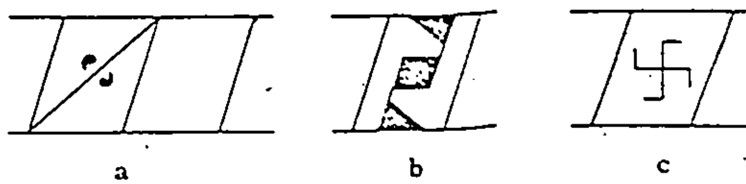


Figure 11. (a) Illustration de la symétrie de classe 200 des parallélogrammes divisant la surface à décorer.

(b) Motif de classe 200 à l'intérieur du parallélogramme; les symétries de la structure et du motif sont identiques.

(c) Motif de classe 400 à l'intérieur d'une structure de classe 1-200. La symétrie de l'ensemble est de classe 1-200, la transformation déplaçant le motif au complet étant une rotation de 180°.

2. La symétrie des éléments et des motifs insérés dans les espaces structuraux

Un élément est une forme géométrique élémentaire telle un triangle, un cercle, un carré. Un motif est une figure composée de deux ou de plusieurs éléments: la symétrie n'est pas une propriété inhérente du motif. Les mêmes éléments peuvent être combinés de plusieurs manières, formant ainsi des motifs de classes différentes. Dans la

Figure 12, les motifs a et b, de classe 101, sont formés du triangles isocèles et de hachures. La symétrie du motif a reste de classe 101 après l'addition de hachures verticales. Par contre, la réflexion verticale n'est plus l'opération de symétrie du motif b où les hachures sont en diagonale.

a b

Figure 12.

On distingue les <u>motifs principaux</u> des <u>motifs mineurs</u> ou <u>secondaires</u> (fillers). Les premiers occupent une place importante dans une division de la surface à décorer et sont les premiers à y être placés. Les motifs secondaires ne sont généralement que de simples éléments dont la symétrie duplique très souvent celle du motif.

3. <u>La symétrie du motif au complet</u>

L'étude des bandes ornementales comprend deux phases: d'une part l'identification de la classe de symétrie du module et d'autre part, le type de déplacement effectué par ce module à l'intérieur de la bande. Certaines isométries ne générant pas de motifs uni-dimensionnels, la classification du motif au complet impliquera une réduction de symétrie. Ceci est illustré dans la Figure 13.

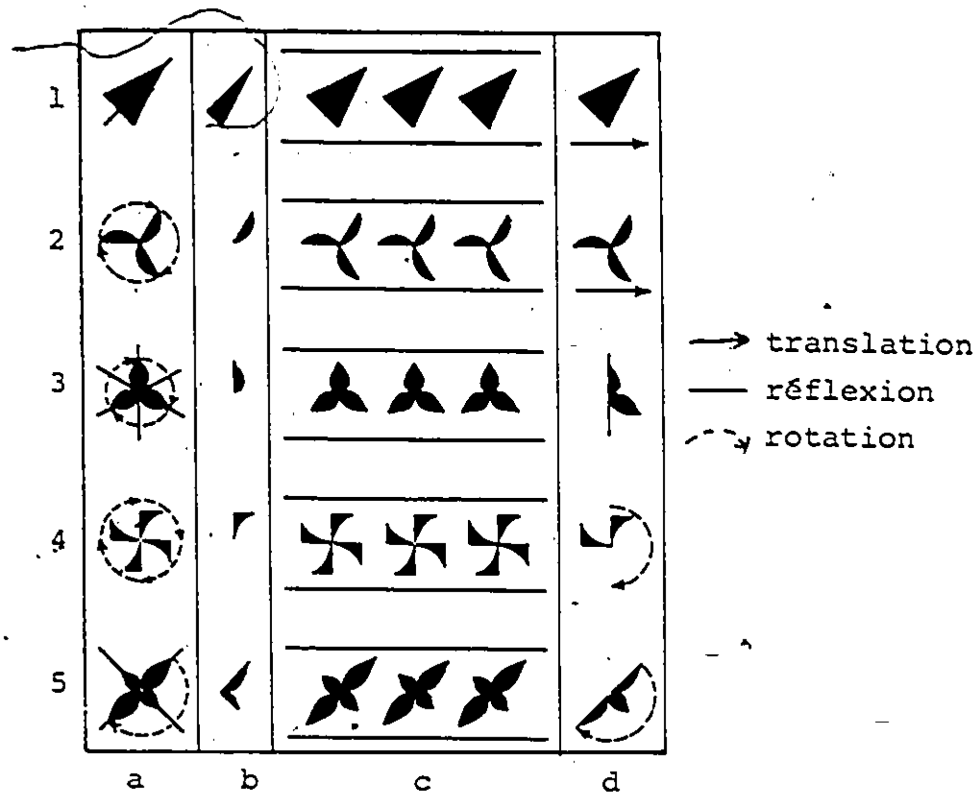


Figure 13. Description des colonnes.

(a) Isométrie du module inutilisable dans la formation de motifs uni-dimensionnels.
(b) Figure origine du module de la colonne a.
(c) Motif obtenu en déplaçant le module de la colonne a.
(d) Figure origine du motif de la colonne a et le type de déplacement de cette figure origine.

Description des rangées: réduction de symétrie

(1) Réflexion oblique à translation.
(2) Rotation d'ordre 3 à translation.
(3) Produit de réflexion verticale et de rotation d'ordre 3 à réflexion verticale.
(4) Rotation d'ordre 4 à rotation d'ordre 2.
(5) Produit de réflexion oblique et de rotation d'ordre 4 à rotation d'ordre 2.

La Table V compare les motifs finis et les motifs uni-dimensionnels, indiquant du même coup, les différentes réductions de symétrie.

L'addition de détails décoratifs entraîne parfois une réduction de symétrie lors de la classification du motif complet. Par exemple, si un élément rectangulaire est hachuré horizontalement, la symétrie de l'élément a et celle du motif b de la Figure 14, appartiennent toutes les deux à la classe 200. Il n'y a donc pas de réduction de symétrie. Ajoutons maintenant des points et des hachures horizontales. Après une rotation de 180°, les deux motifs c et d n'étant pas superposables, la répétition du motif c réduira la symétrie du motif au complet à une translation.

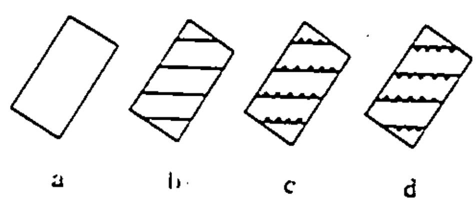


Figure 14.

On considère séparement les déplacements de chaque figure d'une bande ornementale caractérisée par l'alternation des motifs, avant de classer la symétrie du motif au complet. Shepard (1974) distingue les trois cas suivant:

Cas 1 - Symétries des deux motifs différentes et absence d'isométrie commune. La symétrie de la bande est une translation, les deux motifs formant la figure de base (voir Figure 15(a)).

Cas 2 - Symétries des deux motifs identiques. La symétrie de la bande appartient à la même classe que leur isométrie commune. La figure de base est composée des moitiés adjacentes des motifs (voir Figure 15(c)).

Cas 3 - Symétries des deux motifs différentes et présence d'isométrie commune. La symétrie de la bande appartient à la même classe que cette isométrie commune. La figure de base est composée des deux moitiés adjacentes des motifs (voir Figure 15(b) et (d)).

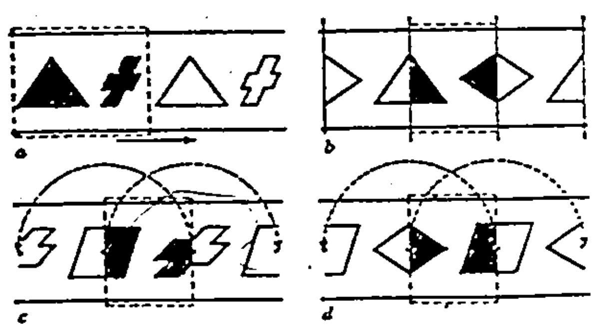


Figure 15.

Il est certain que l'artisan n'exécutait pas son oeuvre en termes des différentes isométries pouvant être employées pour déplacer les figures origines de ses motifs. On peut alors questionner la validité d'une analyse "mathématique" des dessins géométriques, comme le fait Sheppard (1956): "Is it merely an artificial system that we impose, or is it something that man is aware of in his environment and copies intuitively as he uses natural forms as models, or does it follow from the structure of design itself and from regular arrangement?" (Sheppard, 1956, p. 271). L'intention de symétrie n'est pas la seule considération lors de l'exécution des motifs: d'autres critères, tels la présence de dynamisme et de rythme de la distinction entre figure et fond, rentrent en ligne de compte dans la composition de l'oeuvre. En effet, on remarque le choix des formes et leur orientation, la continuité des lignes, l'opération de symétrie, affectent l'équilibre et le rythme du motif. Une réflexion suivie d'une rotation ajoutera un effet dynamique, alors qu'un motif dont l'isométrie est une réflexion semblera statique. De même, un motif à ligne continue formé par la réflexion verticale d'une figure de base paraîtra plus dynamique que le motif obtenu par la rotation de figure de base semblable, mais cette fois, avec un espace entre les modules. L'artisan peut aussi jouer avec des contrastes effectués par la couleur, les hachures, etc., et ainsi empêcher la distinction entre figure et fond d'un motif, ou encore donner diverses illusions optiques.

D'autres facteurs compliquent la classification d'un motif mettant en doute parfois, l'intention de symétrie de l'artisan. L'imperfection de la symétrie peut être due à des erreurs de la part de l'artisan, qui, accordant plus d'importance à la fonction (rituelle ou non) de son oeuvre, oublie momentanément la logique de l'exécution. On peut aussi supposer le manque d'expérience ou d'habilité de l'artisan pour expliquer certaines erreurs. Mais ces irrégularités telles la disposition disproportionnelle sur la surface à décorer (vase, peinture et autre) des motifs répétés, ou encore les oublis ou additions d'éléments de décoration, permettent difficilement une classification unique de ces dessins géométriques. Aussi, l'utilité d'un système tel que le traité de D.K. Washburn pour tirer des conclusions quant aux influences ou interactions d'un peuple à l'autre, est-elle mise en doute. Une étude anthropologique en ce qui a trait aux relations, ne peut être suffisante si elle s'en tient uniquement à l'expression graphique; elle doit être étayée d'autres preuves d'ordre politique, économique, social, religieux ainsi qu'une analyse des matériaux, ce qui dépasse sans conteste le cadre mathématique.

Citons brièvement les études de Alcina Franch et de Field qui bien que différentes des analyses semblables au traité de D.K. Washburn, présentent néanmoins un élément très important non mentionné jusqu'à maintenant, c'est-à-dire le symbolisme rattaché aux motifs. L'analyse de Alcina Franch des "Pintaderas" du Mexique regroupe les motifs selon leur ressemblances: formes circulaires, formes à spirales, grecques échelonnées, formes géométriques diverses, etc. Certains motifs, que l'on pourrait classer selon une analyse semblable à celle de D.K. Washburn sont interpretés comme étant des représentations schématiques de Quetzalcoatl, le serpent emplumé, ou attribués à d'autres deités. L'étude de Field sur les estampes mexicaines divise les motifs selon qu'ils représentent certaines figures (personnages, animaux, plantes, etc.) ou encore selon la ressemblance des formes (net patterns, Ollin, etc.). Les estampes à motifs répétés ainsi que les motifs "net patterns" et "stepped fret" présentent des dessins géométriques dont les opérations de symétrie pourraient être analysées suivant une notation telle celle de D.K. Washburn. Les répétitions de motifs non-géométriques (personnages, animaux) démontrent parfois une préférence pour une isométrie particulière associée au motif. En effet, on remarque, spécialement en Amérique du Nord, qu'une certaine figure se déplacera selon une seule opération de symétrie, alors qu'un motif différent se déplacera selon une isométrie différente, laissant supposer ainsi une correspondance entre figures (ou symboles) et diverses opérations de symétrie. Par conséquent, une classification suivant la méthode de D.K. Washburn ajouterait certainement d'autres arguments dans le cadre d'une étude pour des fins comparatives.

Table I

EXEMPLES DE MOTIFS FINIS

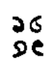
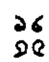
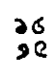
Monochromes			Contrechangés			
D.K. Washburn		Loeb	D.K. Washburn		Loeb	Remarques
	100	1				Figure assymétrique
	101	1m		101^2	$1m^r$	Réflexion verticale
	110			11^20		Réflexion horizontale
	200			2^200		
	300	k (I.T.C_k)		4^200	$(2K)^r$ k=2K	Rotation d'ordre k
	400			6^200		
	600					
	211			21^21^2	$(2K)m^rm'$	
				41^21^2		
	411			2^21^21		
		km (I.T.D_k)		2^211^2	$(2K)^rm^rm'$	produit de rotation et de réflexions
				4^211		
				$4^21^21^2$		
	301			301^2	km^r k impair	

Table II

Colonne a: Nombre de déplacements avant la translation du module
Colonne b: Nombre de figures de base dans un module

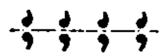
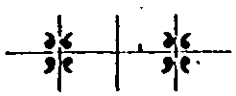
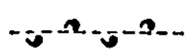
Illustration du déplacement	Classe	Description du déplacement selon D.K. Washburn	Description du déplacement selon Coxeter	a	b	Autres isométries inhérentes au motif
	1-100	Translation	1 translation	0	1	0
	1-110	Réflexion horizontale	1 translation 1 réflexion	1	2	Réflexion translatoire
	1-101	Réflexion verticale	2 réflexions	1	2	0
	1-200	Rotation de 180°	2 rotations de 180°	1	2	0
	1-211	Réflexion horizontale et verticale	3 réflexions	2	4	Rotation de 180° et réflexion translatoire
	1-100x[1]	Réflexion translatoire	1 réflexion translatoire	1	2	0
	1-201	Rotation de 180° et réflexion verticale	1 réflexion 1 rotation de 180°	2	4	Réflexion translatoire

Table III

MOTIFS UNI-DIMENSIONNELS

Monochromes		Contrechangés		Shubnikov/
Washburn	Loeb	Washburn	Loeb	Belov
1-100	∞	$1\text{-}1_2 00$	∞^r	p'1
1-110	∞m	$1\text{-}11^2 0$	∞m^r	p1m'1
		$1\text{-}1_2 10$	$\infty^r m$	p'1m1
		$1\text{-}1_2 1^2 0$	$\infty^r m^r$	p'1a1
1-101	$\infty mm'$	$1\text{-}101^2$ $1\text{-}101_2$	$\infty^r m^r m'$	p'm11
		$1\text{-}101^2_2$	$\infty m^r m'^r$	pm'11
$1\text{-}100x^1$	∞g	$1\text{-}100x^2$	∞g^r	p1a'1
1-200	$22'\infty$	$1\text{-}2^2 00$ $1\text{-}2_2 00$	$2^r 2'\infty^r$	p'112
		$1\text{-}2^2_2 00$	$2^r 2'^r \infty$	p112'
1-201	$2\hat{2}\infty$	$1\text{-}201_2$	$2^r \hat{2}^r \infty m^r/g$	pm'a2'
		$1\text{-}2^2 01$	$2^r \hat{2}^r \infty m/g^r$	pma'2'
		$1\text{-}2^2 01_2$	$2\hat{2}\infty m^r/g^r$	pm'a'2
1-211	$22'\infty$	$1\text{-}21^2 1^2_2$	$22'\infty m^r m'^r/m''^r$	pm'm'2
		$1\text{-}2^2 11^2$ $1\text{-}2_2 11_2$	$22'^r \infty^r mm'^r/m''$	p'mm2
		$1\text{-}2^2 1^2 1_2$ $1\text{-}2_2 1^2 1^2$	$2^r 2'\infty^r mm'^r/m''^r$	p'ma2
		$1\text{-}2^2_2 1^2 1$	$2^r 2'^r \infty mm'/m''^r$	pmm'2'
		$1\text{-}2^2_2 11^2_2$	$2^r 2'^r \infty m^r m'^r/m''$	pm'm2'

Table IV

MOTIFS BI-DIMENSIONNELS

Monochromes				Contrechangés			Shubnikov/
Washburn		Loeb	I.T.	Washburn		Loeb	Belov
a	b			a	b		
2-100		$1_{\infty\infty}'$	p1	$2\text{-}1_100$	$2\text{-}1\infty00$	$1_\infty{}^r{}_\infty{}'{}^r$	$p_b'1$
				$2\text{-}1_300$			
2-101	2-110	$1_{\infty\infty}'mm'$	pm	$2\text{-}101^2$	$2\text{-}11^20$	$1_\infty{}^r{}_\infty{}'mm'{}^r$	$p_b'1m$
				$2\text{-}101_2$	$2\text{-}11_20$		
				$2\text{-}101_2^2$	$2\text{-}11_2^20$	$1_{\infty\infty}'m^rm'{}^r$	pm'
				$2\text{-}1_101$	$2\text{-}1_210$	$1_{\infty\infty}'{}^rmm'$	$p_b'm$ (≡g')
				$2\text{-}1_101^2$	$2\text{-}1_21^20$	$1_\infty{}^r{}_\infty{}'{}^rmm'{}^r$	c'm
				$2\text{-}1_101_2$	$2\text{-}1_21_20$		
				$2\text{-}1_101_2^2$	$2\text{-}1_21_2^20$	$1_{\infty\infty}'{}^rm^rm'{}^r$	$p_b'g$ (≡m')
$2\text{-}101y_1$	$2\text{-}110x_1$	$1_{\infty\infty}'mg$	cm	$2\text{-}101y_2$	$2\text{-}110x_2$	$1_\infty{}^r{}_\infty{}'mg^r$	$p_c'm$
				$2\text{-}101^2y_1$	$2\text{-}11^20x_1$	$1_\infty{}^r{}_\infty{}'m^rg$	$p_c'g$
				$2\text{-}101^2y_2$	$2\text{-}11^20x_2$	$1_{\infty\infty}'m^rg^r$	cm'

$2\text{-}100y_1^1$	$2\text{-}100x_1^1$	$1^{\infty\infty\infty\prime}gg'$	pg	$2\text{-}100y_1^2$ $2\text{-}100y_2^1$	$2\text{-}100x_1^2$ $2\text{-}100x_2^1$	$1\infty^{r}\infty' g^{r}g'$	$P_b'1g$
				$2\text{-}100y_2^2$	$2\text{-}100x_2^2$	$1^{\infty\infty\infty\prime}g^{r}g'^{r}$	pg'
2-200		$22'2''2'''$	p2	$2\text{-}2_100$ $2\text{-}2_300$	$2\text{-}2_200$	$22'^{r}2''^{r}2'''$	$P_b'2$
				$2\text{-}2_1^200$ $2\text{-}2_3^200$	$2\text{-}2_2^200$	$2^{r}2'^{r}2''^{r}2'''^{r}$	p2'
2-211		$\underline{\underline{2}}\underline{\underline{2}}'\underline{\underline{2}}''\underline{\underline{2}}'''$	pmm'	$2\text{-}21_2^21_2^2$		$\underline{\underline{2}}\underline{\underline{2}}'\underline{\underline{2}}''\underline{\underline{2}}'''m^{r}m^{r}/m^{r}m^{r}$	pm'm'
				$2\text{-}2_1 1_2 1$ $2\text{-}2_1^2 1 1^2$	$2\text{-}2_2 1 1_2$ $2\text{-}2_2^2 1^2 1$	$\underline{\underline{2}}\underline{\underline{2}}'\underline{\underline{2}}''^{r}\underline{\underline{2}}'''^{r}mm/mm^{r}$	$P_b'mm$
				$2\text{-}2_3^2 1 1_2^2$	$2\text{-}2_3^2 1_2^2 1$	$\underline{\underline{2}}^{r}\underline{\underline{2}}'^{r}\underline{\underline{2}}''^{r}\underline{\underline{2}}'''^{r}mm/m^{r}m^{r}$	pmm'
				$2\text{-}2^2 1_2 1^2$ $2\text{-}2_3 1_2 1_2$ $2\text{-}2_3 1^2 1^2$	$2\text{-}2^2 1^2 1_2$	$\underline{\underline{2}}\underline{\underline{2}}'^{r}\underline{\underline{2}}''\underline{\underline{2}}'''^{r}mm^{r}/mm^{r}$	c'mm
				$2\text{-}2_1^2 1_2^2 1_2$ $2\text{-}2_1 1^2 1_2^2$	$2\text{-}2_2^2 1_2 1_2^2$ $2\text{-}2_2 1_2^2 1^2$	$\underline{\underline{2}}^{r}\underline{\underline{2}}'\underline{\underline{2}}''\underline{\underline{2}}'''^{r}mm^{r}/m^{r}m^{r}$	$P_b'gm$
$2\text{-}200x_1y_1$		$\hat{2}22'\hat{2}'g/g'$	pgg	$2\text{-}200x_2y_2$		$\hat{2}22'\hat{2}'g^{r}/g'^{r}$	pg'g'
				$2\text{-}2_3^200x_1y_2$	$2\text{-}2_3^200x_2y_1$	$2^{r}\hat{2}'2'^{r}\hat{2}'^{r}g^{r}/g'$	pgg'

$2\text{-}210y_1^1$	$2\text{-}201x_1^1$	$2\hat{2}2'\hat{2}'m/g$	pmg	$2\text{-}21_2\dot{0}y_2^2$	$2\text{-}201_2x_2^2$	$2\hat{2}2'\hat{2}'m^r/g^rg'^r$	pm'g'
				$2\text{-}2^210y_1^2$	$2\text{-}2^201x_1^2$	$2^r\hat{2}^r2'\hat{2}'m/g^rg'$	p_b'mg
				$2\text{-}2_210y_2^1$	$2\text{-}2_101x_2^1$		
				$2\text{-}2^21_20y_2^1$	$2\text{-}2^201_2x_2^1$	$2^r\hat{2}^r2'\hat{2}'m^r/g^rg'$	p_b'gg
				$2\text{-}2_21_20y_1^2$	$2\text{-}2_101_2x_1^2$		
				$2\text{-}2_2^210y_2^2$	$2\text{-}2_1^201x_2^2$	$2^r\hat{2}^r2'^r\hat{2}'^rm/g^rg'^r$	pmg'
				$2\text{-}2_2^21_20y_1^1$	$2\text{-}2_1^201_2x_1^1$	$2^r\hat{2}^r2'^r\hat{2}'^rm^r/gg'$	pm'g
$2\text{-}211x_1y_1$		$2\hat{2}\underline{2}'\underline{2}''$	cmm	$2\text{-}211x_2y_2$		$2^r\hat{2}^r\underline{2}'\underline{2}''m/m'$	p_c'mm
				$2\text{-}21_2^21_2^2x_1y_1$		$2^r\hat{2}^r\underline{2}'\underline{2}''m^r/m'^r$	p_c'gg
				$2\text{-}21_2^21_2^2x_2y_2$		$2\hat{2}\underline{2}'\underline{2}''m^r/m'^r$	cm'm'
				$2\text{-}2_3^211_2^2x_1y_2$	$2\text{-}2_3^21_2^21x_2y_1$	$2^r\hat{2}^r\underline{2}'^r\underline{2}''^rm/m'^r$	cmm'
				$2\text{-}2_3^211_2^2x_2y_1$	$2\text{-}2_3^21_2^21x_1y_2$	$2\hat{2}\underline{2}'^r\underline{2}''^rm/m'^r$	p_c'mg
$2\text{-}300$		$33'3''$	p3				
$2\text{-}301y_1$	$2\text{-}310x_1$	$\underline{3}\underline{3}'\underline{3}''$	p3m1	$2\text{-}301^2y_2$	$2\text{-}31^20x_2$	$\underline{3}\underline{3}'\underline{3}''m^r$	p3m'
$2\text{-}310y_1$	$2\text{-}301x_1$	$3\hat{3}\underline{3}'$	p31m	$2\text{-}31^20y_2$	$2\text{-}301^2x_2$	$3\hat{3}\underline{3}'m^r$	p31m'

2-400	244'	p4	$2\text{-}4^2 00$	$24^r4'^r$	p4'
			$2\text{-}4_3 00$ $2\text{-}4_3^2 00$	$2^r4^r4'$	$p_c'4$
2-411	$\underline{\underline{244}}'$	p4m	$2\text{-}41_2^2 1_2^2$	$\underline{\underline{244}}'m^r_{44'}m^r_{24}m^r_{24'}$	p4m'm'
			$2\text{-}4^2 11$	$\underline{24}^r\underline{4}'^r m^r_{44'} m_{24} m_{24'}$	p4'mm'
			$2\text{-}4^2 1_2^2 1_2^2$	$\underline{24}^r\underline{4}'^r m_{44'} m^r_{24} m^r_{24'}$	p4'm'm
			$2\text{-}4_3 1^2 1^2$ $2\text{-}4_3^2 1_2 1_2$	$\underline{2}^r\underline{4}^r\underline{4}' m^r_{44'} m_{24} m^r_{24'}$	p'4gm
			$2\text{-}4_3 1_2 1_2$ $2\text{-}4_3^2 1^2 1^2$	$\underline{2}^r\underline{44}'^r m_{44'} m_{24} m^r_{24'}$	$p_c'4mm$
$2\text{-}400x_1y_1$	$\underline{2}\hat{44}$	p4g	$2\text{-}400x_2y_2$	$\underline{2}\hat{44}m^r$	p4g'm'
			$2\text{-}4^2 00x_1y_1$	$\underline{2}4^r\hat{4}^r m^r$	p4'gm'
			$2\text{-}4^2 00x_2y_2$	$\underline{2}4^r\hat{4}^r m$	p4'g'm
2-600	236	p6	$2\text{-}6^2 00$	2^r36^r	p6'
$2\text{-}611x_1y_1$	$\underline{\underline{236}}$	p6m	$2\text{-}61_2^2 1_2^2 x_2y_2$	$\underline{\underline{236}} m^r_{26} m^r_{236}$	p6m'm'
			$2\text{-}6^2 11_2^2 x_1y_2$	$\underline{2}^r\underline{36}^r m^r_{26} m_{236}$	p6'm'm
			$2\text{-}6^2 1_2^2 1x_2y_1$	$\underline{2}^r\underline{36}^r m_{26} m^r_{236}$	p6'mm'

Table V

COMPARAISON DES MOTIFS FINIS ET
DES MOTIFS UNI-DIMENSIONNELS

Motif fini	Isométrie du motif uni-dimensionnel formé par la répétition du motif fini
1. Réflexion	
Verticale	Réflexion transversale
Horizontale	Réflexion longitudinale
Oblique	Translation
2. Rotation	
D'ordre 2	Rotation de 180°
D'ordre impair ≥ 3	Translation
D'ordre pair ≥ 4	Rotation de 180°
3. Produit de réflexion et de rotation	
a) Nombre impair d'axes	
Un axe vertical	Réflexion transversale
Un axe horizontal	Réflexion longitudinale
Tous les axes obliques	Translation
b) Nombre pair d'axes	
Rotation d'ordre 2, axes verticaux et horizontaux	Réflexion transversale et longitudinale
Rotation d'ordre ≥ 4, axes verticaux et horizontaux	Réflexion transversale et longitudinale
Rotation d'ordre ≥ 2, tous les axes obliques	Rotation de 180°

CHAPTER 2

ELEMENTS DE GEOMETRIE EXTRAITS DES PEINTURES, DES SCULPTURES ET DES MONUMENTS

1. Utilisation de patrons et planification de l'espace

Plusieurs auteurs, tels que Miller (1973) et Sanders (1977) ont soulevé la possibilité d'utilisation de patrons pour effectuer les sculptures et les peintures. Après avoir constaté les nombreuses répétitions de motifs dans les murales de Teotihuacan, Miller propose l'emploi de modèles comme technique de peinture. Il suppose que l'artiste plaçait sur le plâtre humide, ces patrons ou motifs dessinés sur du matériel périssable et traçait ensuite les contours du motif à l'aide de stylet. Cependant, les motifs répétés ne sont pas toujours identiques, on note des variations (inférieures à 12 cm) dans les mesures des répétitions d'un motif. Aussi, Miller en conclut-il que si un modèle était utilisé lors de l'exécution d'une murale, ce modèle n'était pas rigoureusement suivi. De même, Sanders, ayant vérifié à l'aide de mesures la ressemblance de deux paires de stèles, stèles 5 et 7 et stèles 10 et 11 de Seibal (ressemblance se situant au niveau de détails vestimentaires et corporels), suggère l'emploi de patrons pour la création de certaines parties d'une sculpture. Il indique non seulement les variations dans les mesures montrant la ressemblance de certains détails de chaque paires de stèles jumelles, mais aussi des imperfections à l'intérieur d'une même stèle telles la différence de longueur (258 mm, 265 mm) du pied gauche et du pied droit de la stèle 10. Bien que des erreurs, qui justifieraient certaines de ces irrégularités, ne peuvent être évitées lors des relevés de mesures, (erreurs dues à l'usure de la pierre, aux différentes méthodes employées pour mesurer, etc.), ces écarts témoignent, selon Sanders, de l'imprécision des sculpteurs dans l'exécution de leurs oeuvres. Il se rallie donc à Robertson en supposant l'utilisation de modèles, le manque de précision et la liberté du sculpteur quant à l'emplacement de certains détails. Robertson donne une information supplémentaire: "patterns may also have been used for heads with allowance for identifying individual portraits" (Sanders, 1977, p. 85). Parlant des "profile figures of room 2 at Tepantitla", Miller fait une remarque semblable: "a comparison of the heads (...) suggests an almost portrait like individuality for each figure" (Miller, 1973, p. 33).

Sanders est le seul auteur à relever l'emploi de patrons comme technique traditionnelle durant la période classique. Il est d'accord avec Andrews et Ronner pour supposer qu'un maître sculpteur était chargé de tracer l'esquisse de certains détails et que l'exécution finale de l'oeuvre était réalisée par des ouvriers. Andrews et

Ronner ajoutent "there were master artisans and perhaps even a non elite class of professionals who may have been part of a guild and sculpture was a full-time specialization in the Southern Maya Lowlands during the Classic Period" (Sanders, 1977, p. 85). La ressemblance des détails faciaux vérifiée à l'aide de mesures des stèles 10 et 11, et la disproportion de la tête de la stèle 11 par rapport au reste du corps, amèment Maler à conclure que la tête de la stèle 11 n'est que la copie de celle de stèle 10 et en outre, que ces deux stèles ont été exécutées par le même sculpteur (Sanders, 1977).

L'existence de modèles est discutable dans certains cas. Ainsi, le "floor painting in Portico 24 at Tetitla", (exemple rare à Teotihuacan de plancher ayant un motif, les autres étant normalement recouverts de peinture rouge ou de stuc est cité par Moore (Miller, 1970) comme exemple de possibilité d'emploi de stencils. Cependant, Miller n'est pas de cet avis: "this is the one case of repeating motifs where extenuating circumstances render the use of a pattern unlikely" (Miller, 1973, p. 34). Il est pourtant moins catégorique en ajoutant: "certainly the designs are simple enough so that use of a stencil would not have been necessary to keep the motifs fairly regular. On the other hand, the dimensions are close enough to each other to suggest some kind of planned measurement" (Miller, 1973,

p. 34). Le doute de Miller provient surtout de l'exceptionnalité de ce plancher: en fait, il explique les répétitions de ce motif, comme étant la conséquence d'un accident: pour couvrir une tache, l'artiste a dessiné une figure et ensuite répété le motif six fois afin d'obtenir une décoration agréable. Mais alors, pourquoi ne pas l'avoir recouvert tout simplement de peinture rouge?

L'utilisation de patrons est, dans d'autres cas, plus difficile à contester. Dans les "profile birds of structure 2 at zone 2", la forme des yeux, la longueur de l'oiseau et surtout le nombre de gouttes varient dans les 17 représentations de ce motif. Miller se justifie en disant: "I do not think that these variations are great enough to discount the possibility that a pattern was employed to block out the shape of the bird motifs. After the stencil was used, each bird was painted individually and each panel may even have been painted by a different artist" (Miller, 1973, p. 33).

Essayant de trouver une relation entre l'image et le monument sur lequel l'image est reproduite (sculptée ou peinte), Miller constate que l'espace est mesuré dans certaines murales, supposant outre l'utilisation de patrons, une planification de l'oeuvre. Ainsi, dans les "profile figures of room 2 at Tepantitla", les mesures dans la Table 1 indiquent que l'espace à peindre était mesuré: "otherwise we cannot account for the fact that the figures fit so well into the allotted space, that they are roughly the same size, and that they have similar spacing between them" (Miller, 1973, p. 33).

Table I

COMPARAISON DES MESURES DES
FIGURES DE ROOM 2 AT TEPANTITLA

Longueur	Murale 3	Murale 4
Longueur	72 cm 72 cm	82 cm 88 cm
Hauteur	76 cm 75 cm	78 cm 82 cm
Distance entre les figures	25 cm	21 cm
Distance entre une figure et l'extremité du paneau	30 cm	27 cm

Un cas particulièrement intéressant est la murale "circles of substructure 3 of zone 2". Cette murale consiste en une rangée de cercles peints en rouge, dont les contours n'ont pas été tracés en noir, au préalable, comme tel est le cas dans la plupart des murales de Teotihuacan, mais gravés à l'aide d'un instrument pointu ou compas (kind of farmer's compass). Les treize premiers cercles, gravés sur le stuc encore humide, sont de 32 cm de diamètre. Les deux derniers cercles sont sensiblement plus larges (33.5 cm et 34.5 cm); ceci s'explique si on considère que l'espace restant ne pouvait convenir trois cercles de 32 cm de diamètre et d'autre part, deux cercles de 32 cm auraient été trop espacés par rapport aux autres. Une fois les cercles gravés, vint l'application de la peinture qui ne respecte pas toujours les contours. Ainsi, les deux derniers cercles atteignent un diamètre de 35 cm et de 38 cm après l'application de la peinture. Mais cet ajustement n'est pas suffisant pour rendre le tableau uniforme. L'espace entre les deux cercles est de 11.5 cm alors qu'il est de 7.5 cm environ entre les autres cercles. Les soudaines différences de diamètres ainsi que de la longueur entre l'extrémité du panneau et le premier et dernier cercle, témoignent d'une certaine planification mais aussi du manque de précision dans l'exécution: "if the manner of painting the panel of the East Tablero of substructure 3 is here reconstructed correctly, it suggests that the painters used merely their eyes to a great extent in designing their paintings even if vague patterns were used to block out the basic design. The farmer's compass (if it was used in drawing the circles of structure 3) may be considered as stencil in this case" (Miller, 1973, p. 34).

Si Miller n'est pas certain de l'utilisation de compas lors de l'exécution de cette murale, doutant ainsi de la connaissance de cet instrument par les habitants de Teotihuacan, Guerra (1969), par contre, confirme cette connaissance chez les Aztèques en nous informant sur les différents instruments utilisés lors de la construction de monuments: "the plumb (temetzetepilolli), water level (atezcatl), ruler (tlahuahuanaloni), compasses (tlayolloanaloni), square (tlanacazanimi), trowel (tenextlasoloni), wedge (tlatlilli), and the lever (quamniztli)"(Guerra, 1969, p. 43). Il ajoute aussi l'emploi d'instruments en cuivre pour la sculpture du bois ainsi que d'une sorte de scie formée par des morceaux d'obsidienne placés entre deux bouts de bois, utilisée pour couper le bois.

Mais, il n'y a pas toujours une planification préalable à l'exécution d'une oeuvre. Comparons les murales 1 et 4 du "Portico 11 at Tetitla". Dans la murale 1, l'entrée de porte est construite après la murale alors que dans la murale 4, il n'y a pas de doute possible, l'application de la peinture vient après la construction de l'entrée de porte. Cependant les motifs répétés situés à côté de l'entrée de porte sont coupés. Miller propose deux explications: "perhaps there was only one pattern for a room or portico and where it did not fit, it was cut off" (Miller, 1973, p. 35) oú que la grandeur d'un motif était invariable et ce pour des raisons liées au symbolisme. Cette remarque rappelle celle faite par F. Hébert-Stevens (1972), à savoir que les rapports entre les différents graphismes signifiants des stèles de Chavin et leur structure de composition, sont étroitement liés.

L' importance d'une étude de symboles est soulignée par Rodriguez (1979) qui analysant les murales de Teotihuacan, insiste sur l'examination des couleurs. Se basant sur des sources ethnohistoriques pour relever la signification des couleurs, il écrit: "it is difficult to understand Mexican painting of the past without being conversant with its symbolism, because the colour combinations correspond more often to the magic-religious conventions than to physical laws of contrast and harmony" (Rodriguez, 1969, p. 16). Les lois régissant l'exécution d'une oeuvre semblent être selon Rodriguez plus de nature symbolique que sujettes à une composition stricte. L'artiste, transporté par le sujet de son oeuvre, portera son attention sur le choix des couleurs sans accorder d'importance aux proportions des figures et sans respecter les contours.

Notons aussi un détail frappant dans l'étude de Fettweis-Vienot (1980) sur les murales de Coba : une goutte de peinture bleue appliquée sur le rouge du chapiteau d'une colonne, divisant exactement cette surface en deux arties égales, chacune de 83.5 cm. Ceci révèle non seulement une planification de l'espace mais aussi, "une intention de symétrie, et permet de mieux comprendre l'organisation de la composition et des répétitions éventuelles d'éléments de part et d'autres de cet axe central." (una intencion de simetria y permite entender mejor la organizacion de la composicion, y repeticiones eventuales de una parte y otra de un eje central), (Fettweis-Vienot, 1980, p. 27-28). Fettweis-Vienot cite en outre, deux autres cas (Structure III-B del Grupo del Rey à Cacaxtla où le milieu d'une surface est indiqué exactement par un trait, ainsi que le cas où un carré (ou trait vertical) divise des linteaux en deux parties égales (porte intérieure du Castillo à Tulum, Structure 64 à Tancah, à Playa del Carmen, Structure B à San Miguel del Ruz, et à Paalmul -- voir Feittweis (1973). La présence de ces signes indiquant le milieu d'une surface laisse supposer une intention de symétrie, si l'on tient compte que les écrits mayas sont composés de couplets mis en parallèles, de chants et de contre-chants (Girard (1972) et Leon Portilla (1973)) : "Esta estructura binaria siendo generalizada en la literatura, hay muchas posibilidades de verla transparentarse igualmente en otra expresion artistica como la pintura mural", (cette structure binaire étant généralisée dans la littérature, une possibilité de la voir transparaître également sous une autre expression artistique comme la peinture murale , existe), (Fettweis-Vienot, 1980, p. 41). Aussi, lors de l'interprétation des murales de Coba, Fettweis-Vienot porte une attention spéciale aux proportions, aux couleurs et à la position dans l'ensemble, de chaque élément.

2. Analyse géométrique des stèles et des monuments

L'étude des proportions des monuments grecs ou égyptiens nous ont permis d'établir les connaissances géométriques et le système de mesure de ces anciennes civilisations. Afin de tirer des conclusions semblables, certains auteurs ont considéré les rapports proportionnels entre la hauteur et la largeur des monuments mayas, surtout des stèles. Sidrys (1978) classe les mesures de 806 stèles suivant les régions et les sous-régions mayas. Selon les résultats, il n'existe pas de grandeur standard pour les stèles. Cependant, si on exclut la région "Eastern Motagua" on note une certaine standardisation quant à la grandeur d'une stèle (Figure 1).

Clancy (1977) dans son étude des monuments sculptés de Tikal, veut tracer la voie à suivre pour arriver à établir le niveau de connaissance géométrique maya. Son but est de montrer que les Mayas de Tikal préétablissaient une structure de composition concernant les rapports proportionnels entre les images sculptées en relief et entre les images et le monument sur lequel elles sont sculptées. Cette structure de composition est formée par l'intersection et le recoupement de "triangles pythagoritiens" et de triangles isocèles. Les points et les lignes de cette composition sont reliés par des angles de 22°, 37°, 54°, 67° et 90° (angles semblables à ceux d'un triangle pythagoritien). Elle considère la base horizontale d'une stèle et ses deux extrémités à partir desquelles elle trace deux diagonales: une de 54° et l'autre de 67°. Les points A' et B' établis par l'intersection de ces quatre diagonales déterminent la droite centrale verticale. Elle trace ensuite une droite horizontale joignant les extrémités C de deux diagonales de 54°. La Figure 2 est le schéma de cette structure de composition de base. L'examen des différents détails d'une sculpture et les points critiques établis par la structure de composition, sont résumés dans la Table II. Les stèles du classique moyen témoignent d'une organization symétrique et d'une composition plus simple que celle des stèles du classique récent, où les relations d'angles et de triangles sont plus nombreuses et complexes.

Bien que consciente que dans l'étude des proportions, l'unité de mesure n'est qu'une unité relative, Clancy propose une unité de mesure égale à environ 7½" ou 19.2 cm. La Table III ci-dessous résume ses observations.

Table III

Grandeur des stèles		Largeur vs hauteur
Classique Moyen		3 par 8
Classique Récent		5 par 11
Description		Unités au-dessus de la base horizontale
mi-jambe	A'	3
taille		5
poitrine	B'	6
oeil ou boucle d'oreille	C'	≈ 7

Notons que la grandeur moyenne d'un homme donnée par Clancy, soit d'environ 8 à 9 unités, se rapproche de celle donnée par Sidrys.

L'étude de F. Hébert-Stevens sur les graphismes chaviniens présente un élément nouveau: l'explication de rapports d'association et de composition dans le domaine de l'art. Il ajoute cependant que la distinction de ces rapports est due à l'existence d'une trame orthogonale permettant un découpage et le décèlement d'une structure géométrique de la stèle: "association et composition sont étroitement liées dans la mesure où la multiplication des déterminatifs associatifs n'est possible que si elle s'organise sur une grille de composition. Les entités différentielles des deux stèles du portrait noir et blanc de Chavin ne sont lisibles que grâce à leur symétrie et à l'orthogonalité du graphisme" (Hébert-Stevens, 1972, p.). En effet, le découpage de cette stèle (Figure 3 et Figure 4) dévoile l'existence d'enveloppes de nature géométrique: "la tête et le massif hanches-jambes s'inscrivent dans des carrés égaux, les ailes dans un double carré, les ailerons de queue dans un rectangle $\sqrt{3}/2$" (Hébert-Stevens, 1972, p. 125) le centre de la stèle (torse) dans un rectangle doré "dont la hauteur est aussi dans un rapport doré avec le coté des carrés tête et jambes" (Hébert-Stevens 1972, p. 124). Hébert-Stevens remarque que ce découpage s'applique aux deux colonnes. Il est superposable si on considère uniquement la mise en place des graphismes (identiques dans les deux colonnes); cependant, certains détails (tels oeil circulaire → rectangulaire, machoire horizontale → verticale, etc.) varient d'une colonne à l'autre. L'identification de ces détails grâce au découpage, facilitera ensuite l'étude des rapports d'association et de composition: "un premier découpage permet donc d'isoler des entités graphiques commutables, marquées de déterminatifs particuliers donnant naissance à des rapports de similitudes et d'oppositions" (Hébert-Stevens, 1972, p. 124).

Les stèles de Chavin se prêtent plus facilement à un découpage et à une analyse géométrique que les stèles mayas, et ce à cause de leur symétrie bilatérale. L'étude effectuée par F. Hébert-Stevens met en valeur en plus de l'existence d'une trame orthogonale, des réseaux de 45° et de 30° et des rapports dorés. L'étude sur les stèles de Tikal est plus complexe; celles-ci ne présentent pas comme les stèles de Chavin, des grilles géométriques facilement décelables. Or Clancy soutient l'hypothèse d'une connaissance des triangles pythagoriciens chez les Mayas. Pour y parvenir, elle est obligée d'établir trois critères, grâce auxquels elle pourra établir une grille qu'elle proposera comme modèle applicable à toutes les stèles.

1. Relations d'angles ou de droites sont significatives si elles se répètent au moins dans trois monuments différents.

2. Relations d'angles semblables si elles ne diffèrent que par 2° au maximum.

3. Une structure suivant ces deux critères et s'appliquant à au moins 50% des monuments, sera considérée comme préétablie.

Même si ces critères devraient être améliorés afin d'arriver à des résultats concluants, l'approche de Clancy demeure une voie à suivre pour de prochaines recherches qui confirmeront peut-être son hypothèse quant à l'existence d'une structure de composition préétablie des stèles mayas ainsi que de leur connaissance géométrique. L'étude d'une structure géométrique de F. Hébert-Stevens, qui considère séparement chaque stèle de Chavin, semble plus appliquer une connaissance géométrique qu'en extraire. Si mathématiquement elle n'est pas concluante, elle présente en revanche une très bonne analyse sémiotique des graphismes.

Outre l'étude des structures géométriques des stèles de Chavin, F. Hébert-Stevens se propose d'examiner, par des méthodes semblables, les tracés géométriques des monuments mésoaméricains, sud-américains et asiatiques, mais dans un but plus précis: il veut montrer la connaissance géométrique de ces civilisations et ce dans l'optique d'une théorie si non diffusionniste, tout au moins comparative: "nous pensons en effet pouvoir montrer par des exemples (...) que les édifices précolombiens au Mexique comme au Pérou, sont composés d'après les tracés géométriques semblables à ceux qu'utilisaient les architectes de l'Antiquité et du Moyen Age: constatation surprenante car l'usage des tracés qui suppose une connaissance poussée de la géométrie, voir de la trigonométrie paraissait jusqu'alors s'être développé sur des aires limitées à l'Occident et à l'Inde où il était d'ailleurs entouré du plus grand secret" (Hébert-Stevens, 1972, p. 156).

Son étude démontre l'importance d'une structure de composition d'un édifice: elle assure l'agencement des parties et leurs dimensions. Des diagonales de 30° et 45°, des cercles et des carrés exinscrits et/ou inscrits, ainsi que des triangles équilatéraux donnent naissance à des réseaux géométriques démontrant la connaissance de $\sqrt{2}$, $\sqrt{3}$ et $\sqrt{5}$ ($\sqrt{2}$: longueur de la diagonale du carré de coté 1; $\sqrt{3}$: diagonale du triangle équilatéral de coté 1; $\sqrt{5}$: diagonale du double carré, c'est-à-dire, rectangle de largeur 1 et longueur 2). "Les architectes mayas connaissaient parfaitement les figures obtenues par la quatripartition du cercle et l'inscription des divers polygones réguliers" (Hébert-Stevens, 1972, p. 157). et "ne procédaient pas par addition de mesures mais par fractionnement, la plus grande dimension de l'édifice étant prise pour unité" (Hébert-Stevens, 1972, p. 157).

F. Hébert-Stevens procède par recollement de la coupe et de l'élévation d'un monument afin de démontrer comment le tracé géométrique détermine les positions des différents éléments architecturaux, tels les entrées de porte, les arêtes des murs, le faîtage de la toiture, les décors et moulures. Le Palais des Stucs à Acanceh (Figure 5 et Figure 6) décrit ci-dessous illustre la méthode de F. Hébert-Stevens. Chaque voûte du Palais des Stucs dont la coupe transversale de ses deux salles longitudinales est montrée dans la Figure 5, est inscrite dans un carré; les carrés qui en résultent forment un rectangle appelé double carré. Un cercle de diamètre égal à la longueur de ce rectangle et dont le centre est l'intersection des médiatrices, est ensuite tracé. Un carré (de coté) 1 est exinscrit à ce cercle: un cercle est par la suite exinscrit à ce carré. Un carré $\sqrt{2}$ est alors exinscrit à ce deuxième cercle. Les cotés horizontaux du carré 1, sont ensuite prolongés jusqu'à intersection avec les cotés verticaux du carré $\sqrt{2}$. De ces points d'intersection, quatre diagonales de 45° sont tirées, rencontrant le rectangle d'origine à des points marquant les nus extérieurs des murs latéraux et ceux du mur central. Quatre diagonales de 30° sont tirées des arêtes du carré 1, rencontrant le rectangle à des points marquant cette fois-ci les nus intérieurs des murs latéraux et ceux du mur central. Quatre diagonales de 45° sont alors tirées des arêtes du carré 1 et des points d'intersection de la base du rectangle d'origine avec le carré $\sqrt{2}$, se rencontrant au sommet des voûtes. Les quatorze points-clé déterminés par cette composition sont indiqués dans la Figure 5. Selon F. Hébert-Stevens, ce tracé résoud égalementment les problèmes de stabilité des voûtes, sans calculs et indépendamment de l'échelle.

Dans le plan du sol du Palais des Stucs (Figure 6), un cercle est tracé ayant comme centre l'intersection des médiatrices du rectangle (double carré) et comme rayon le coté d'un seul carré du rectangle. Un carré dont la diagonale représente l'axe longitudinal du palais, est ensuite tracé. F. Hébert-Stevens trace alors ses médiatrices, un réseau de diagonales de 30°, ainsi que les triangles équilatéraux correspondants. Cette composition définit les points importants du plan du sol.

L'analyse géométrique de l'art et de l'architecture soulève de réels problèmes. Le premier est causé par l'état actuel des oeuvres d'art, habituellement usés par l'érosion ou la décoloration, réduisant la précision des mesures. S'ajoutent ensuite les jugements subjectifs concernant les marges d'erreur permissibles. Un troisième problème majeur, déjà mentionné auparavant, est la difficulté de déterminer si une structure géométrique a été extraite de l'oeuvre étudiée ou si elle lui a été imposée. Ainsi, une assertion telle "les figures géométriques-circonférences, carrés, doubles carrés, triangles équilatéraux - faciles à tracer sur le terrain, ont d'abord servi aux techniques d'arpentage. Mais la géométrie en tant que système a fourni à l'architecte une ébauche abstraite, un médiateur sur lequel il pouvait élaborer son projet" (Hébert-Stevens, 1972, p. 158), laisse sous-entendre une connaissance géométrique (qui n'est pas toujours démontrée) appliquée à la construction de monuments. Il est donc impératif de considérer attentivement ces deux facteurs en examinant l'oeuvre de F. Hébert-Stevens.

Il est intéressant de noter que Arochi (1977) a publié une étude sur la structure géométrique du El Castillo à Chichen Itza. Non seulement les assertions quant aux connaissances géométriques extraites de El Castille sont discutables si l'on tient compte de l'état actuel de la pyramide, de la marge d'erreur permissible, mais il utilise un résultat de Reyes (cité dans Arochi, 1977): "le côté d'un octogone divise le diamètre d'un cercle exinscrit en rapport doré" (Arochi, 1977, traduction p. 57) qui peut être mathématiquement démontré comme étant faux. Cet exemple, ainsi que l'analyse de Harleston (Tomkins (1976)) sur Teotihuacan et celle de Diaz-Bolio (1975), sur certains motifs de Chichen-Itza (oeuvres citées uniquement car les résultats dont les preuves, si elles existent, sont inadéquates, ne peuvent être qualifiés autrement que d'extravagants) rappellent la mise en garde de ne pas supposer ou imposer des notions connues et utilisées aujourd'hui, attention que demande toute étude de monuments effectuée dans le but d'extraire une connaissance géométrique, mathématique, ou autre.

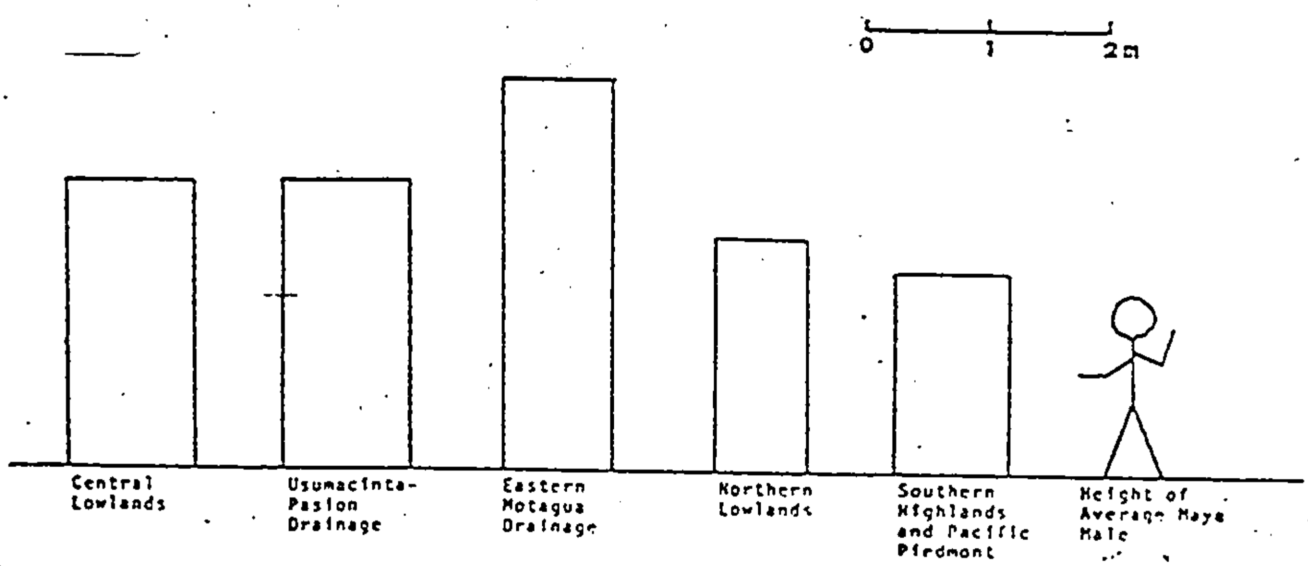


Figure 1 : Taille et forme des stèles mayas, selon cinq sous-régions différentes.

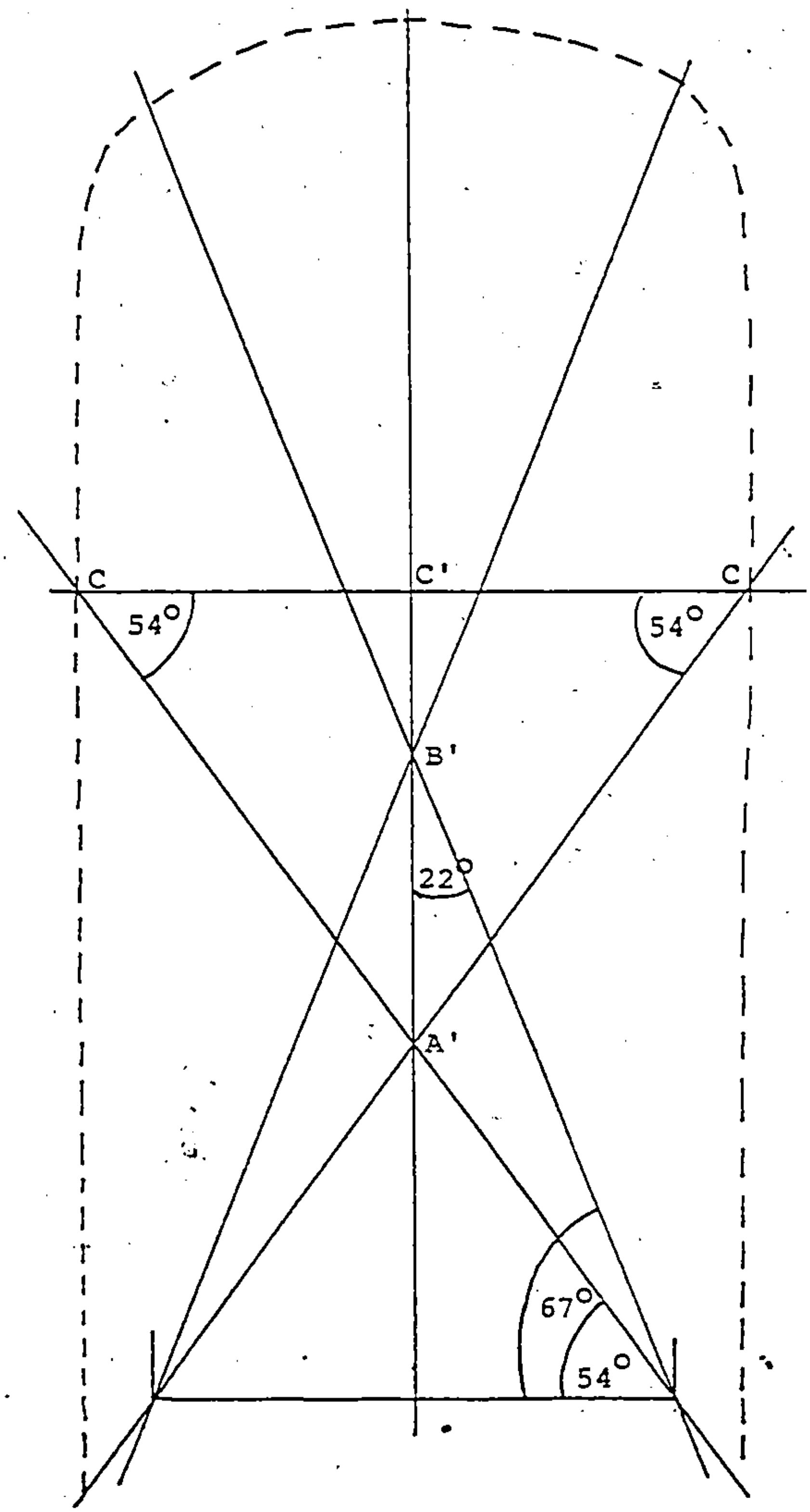


Figure 2a : Stucture de composition des stèles de Tikal.

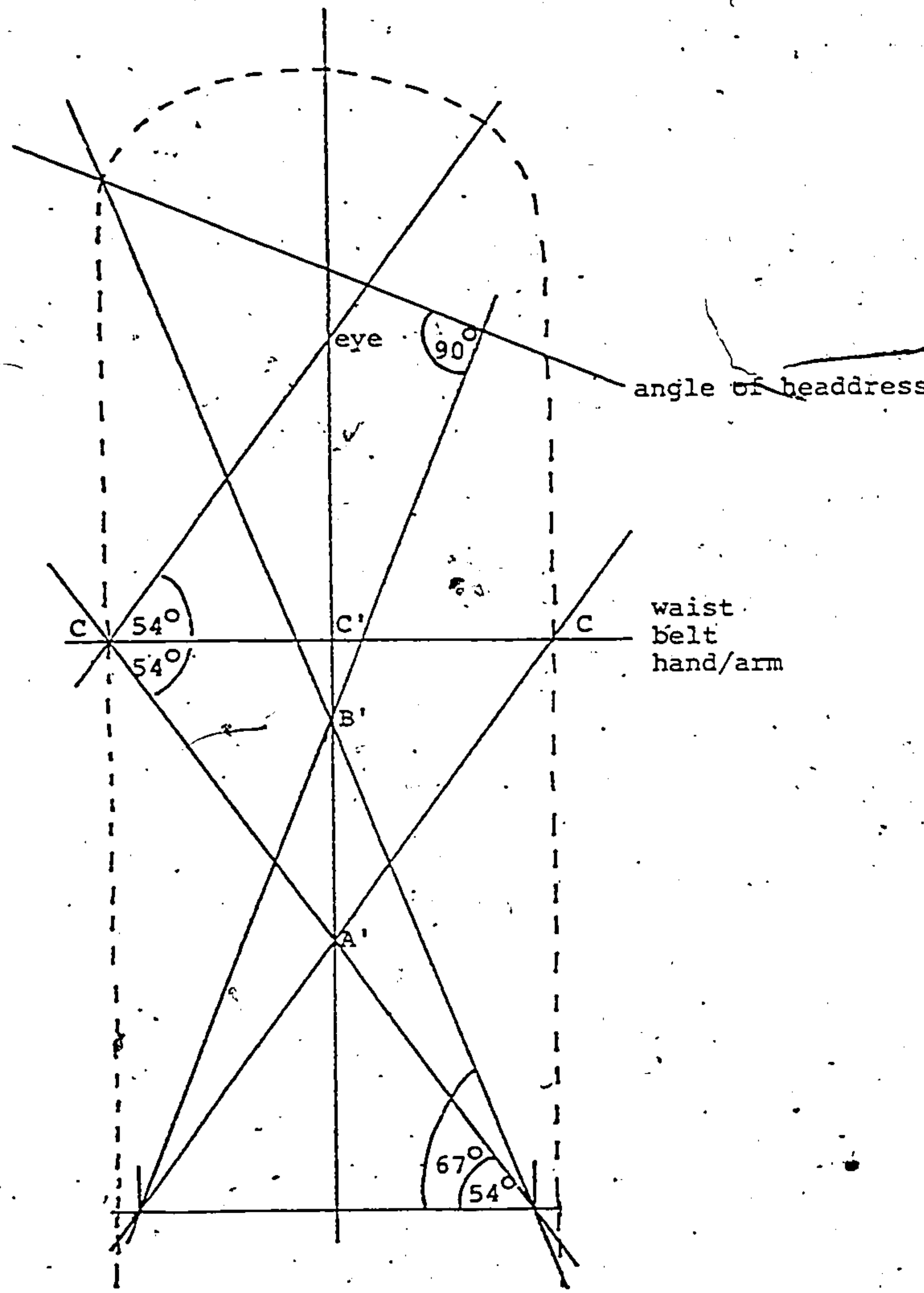


Figure 2b : Structure de composition des stèles de Tikal du Classique Moyen.

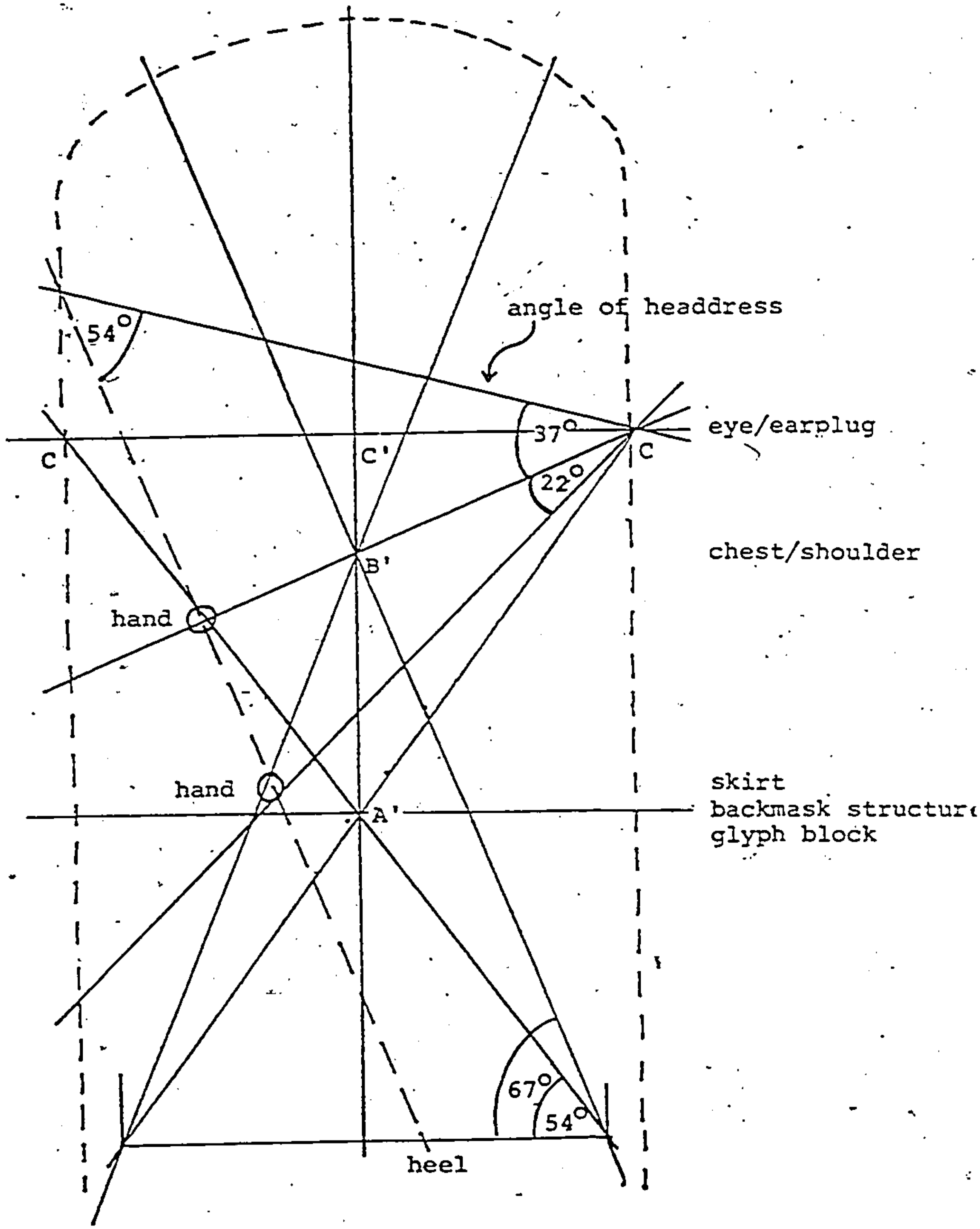


Figure 2c : Structure de composition des stèles de Tikal du Classique Récent.

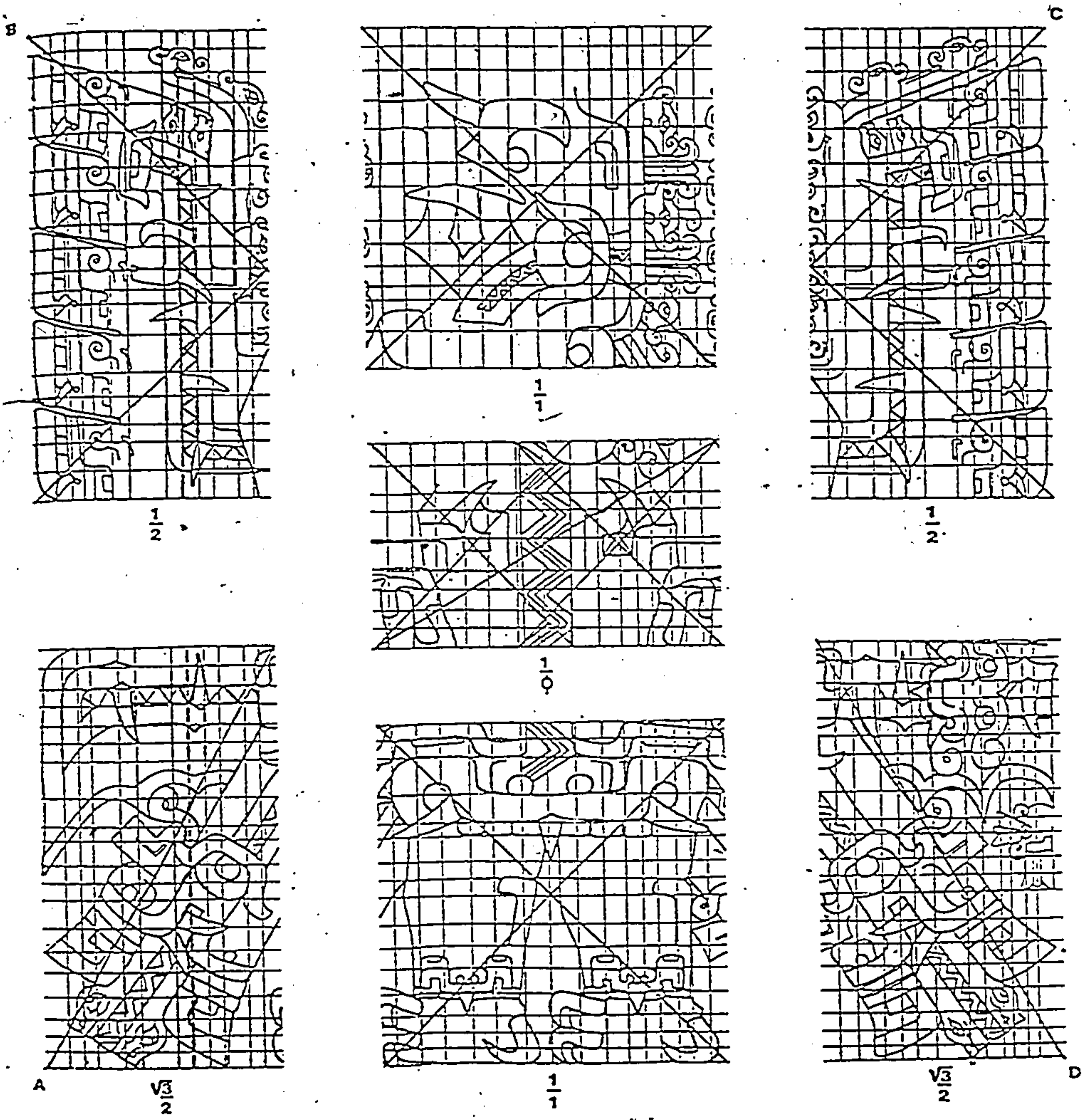


Figure 3 : Condor du portail noir et blanc de Chavin

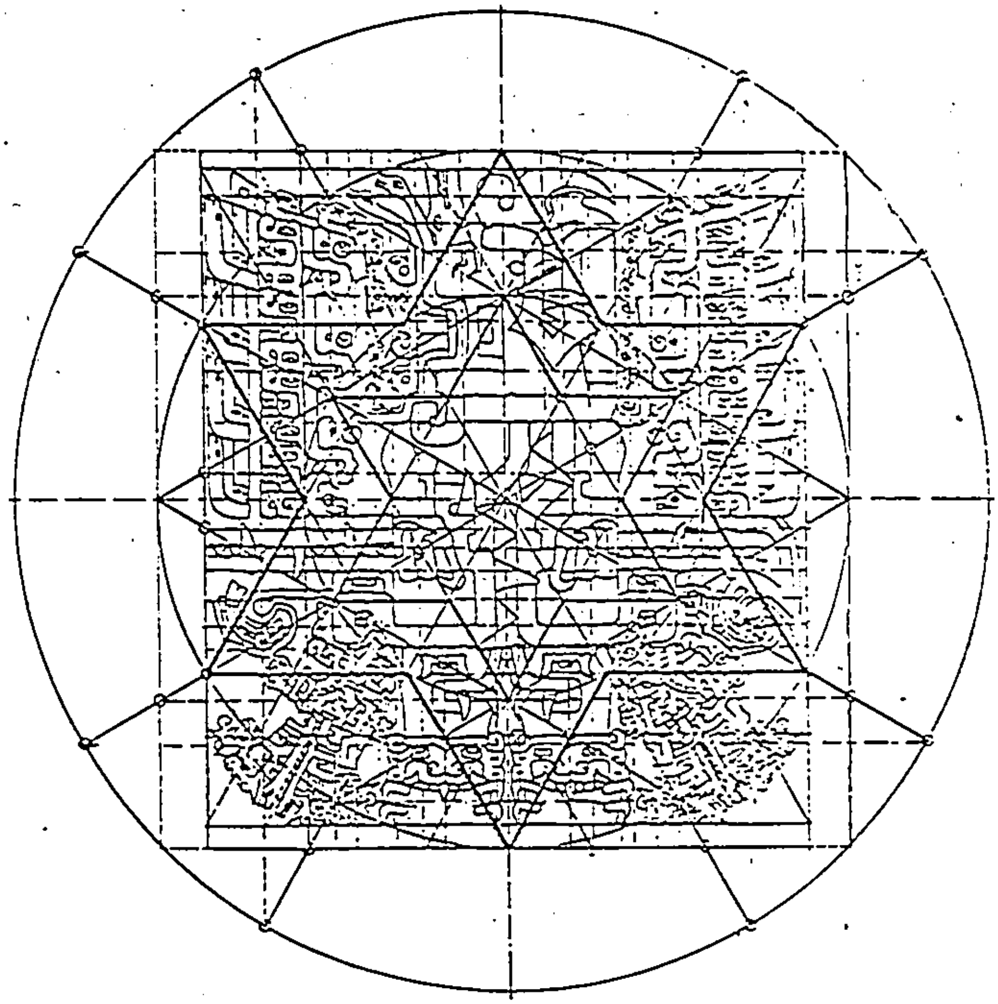


Figure 4. : Bas-relief de la colonne du portail noir et blanc de Chavin.

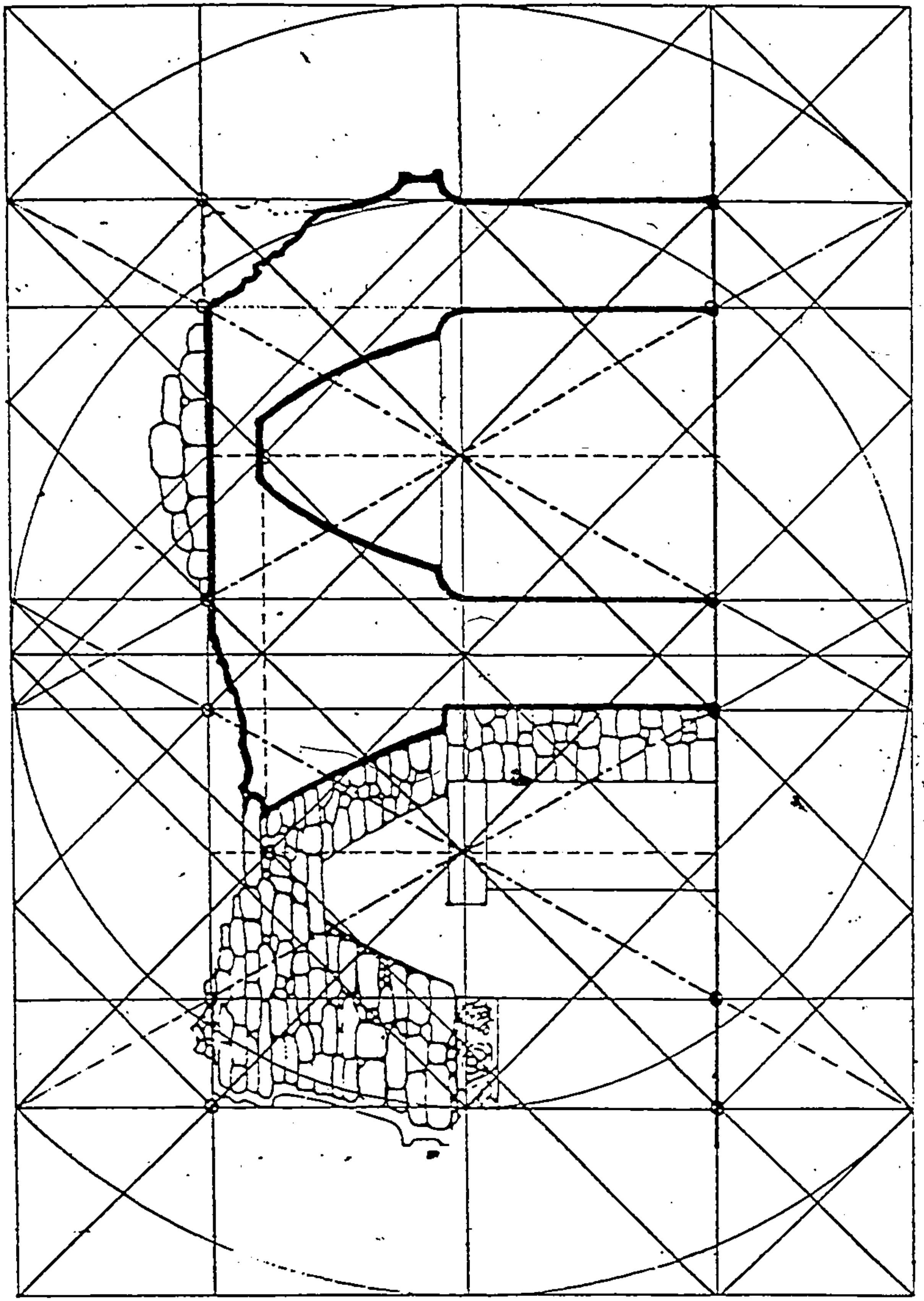


Figure 5 : Coupe transversale des deux salles longitudinales du Palais de Stucs d'Acanceh.

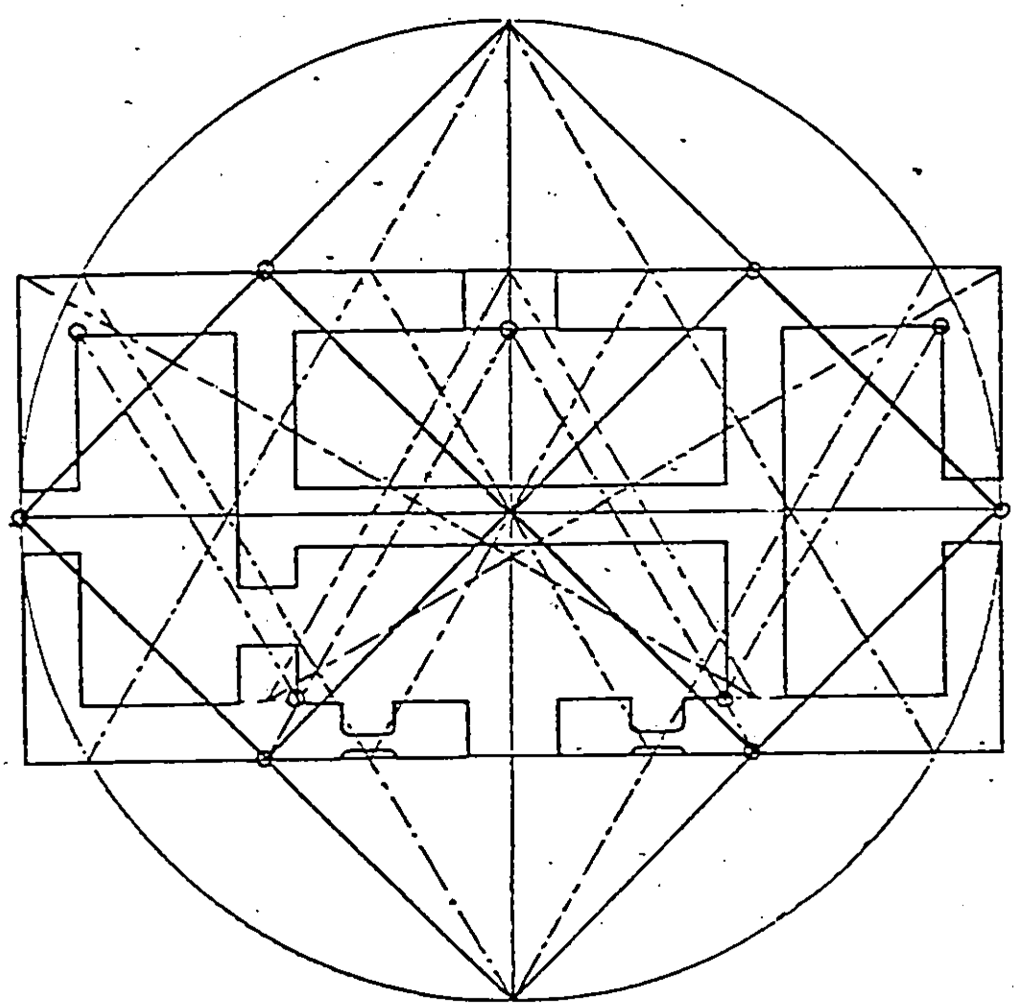


Figure 6 : Plan du sol du Palais des Stucs d'Acanceh.

Table II

Eléments de composition des stèles de Tikal

Motif Feature	Stela 9	3	7	27	10	30	16	21	5	20	22
Belt Hanging	A'	A'			A'					A'?	
Apron Line			A'	A'							
Skirt Line						A'		A'	A'		A'
Backmask							A'	A'	A'		A'
Belt Line	B'	B'			B'						
Waist		B'		B'	B'	B'	B'				
Hand				B'		B'			B'		
Chest/Shoulder				B'			B'	B'	B'	B'	B'
Center-Point of Stela	C'	B'	C'	B'	B'	B'	B'	-	-	-	-
Chest	C'			C'	C'						
Waist		C'	C'								
Earplug						C'	C'		C'	C'	C'
Hand	CC	CC									
Belt Line		CC	CC								
Chest Line				CC	CC						
Neck						CC	CC		CC		
Nose (plug)										CC	CC
Top of Backmask									CC	CC	CC

CHAPTER 3

GEOMETRICAL KNOWLEDGE EXTRACTED FROM SITE PLANS

Hartung opined that "the placement of structures on archaelogical maps reveals for a trained observer the possible existence of geometric forms, systems and even networks which are not always casual" (Hartung, 1977, p. 125). He believes that in the case of the Maya, there is "an excellent opportunity for the application of geometry on a large scale" (Hartung, 1977, p. 116). The analysis of the site plans discussed in this chapter will be augmented by different arguments derived from other disciplines such as astronomy and cosmology, in order to justify the geometrical notions displayed in these ceremonial centers.

Territorial organisation

Studies on the Lowland Classic Maya have revealed the existence of major and minor ceremonial centers and hamlets. Marcus (1973) pointed out the influence of the Maya quadripartite view of the universe on their own territorial structure. The Maya divided their universe into four regions, each associated with a world direction and color: East and red - South and yellow - North and white - West and black - (Thompson (1954) also suggests the color green for the center). Following this model, it has also been suggested that they designated four capital cities. Locational analysis and epigraphic data (emblem glyphs) have been used to determine what were, at a given time, the four major ceremonial centers considered by the Maya as occupying the four quadrants of the universe. These were not always the same four primary centers over the centuries. For example, Stela A of Copan indicates Copan, Tikal, Calakmul (this emblem glyph is not proved to represent Calakmul) and Palenque were the four capital cities in A.D. 731 while Stela 10 at Seibal names Seibal, Tikal, Calakmul (?), and Motul de San Juan as the four capitals in A.D. 849. The location of these capital cities do not correspond exactly to the world directions. Marcus mentions military alliances as a possible explanation for linking these four primary centers together.

A hexagonal lattice of almost equidistant minor ceremonial centers appears to surround each capital (Figure 1a). Marcus also quotes the following assumptions for the Central-Place theory from the "Die Zentraten arte in Suddeutschland" by W. Christaller (Zeiss, June 1933) for the determination of secondary centers:

i) uniform distribution of population and purchasing power;

ii) uniform terrain and resource distribution;

iii) equal transport facility in all directions;

iv) all central places performing the same functions and serving areas of the same size, the most economical spacing of such "service centers" would be equidistant resulting in hexagonal patterns or "lattices". [6]

~~She~~ believes that the number (5 to 8) and the size of these secondary centers depend more on factors such as population density, service functions, and routes of travel and transport, than on topography, good soil, water bodies and defensive necessities. Marcus notes however that this hexagonal structure is typical of areas without major rivers. Where major rivers exist, "ribbon-strip" settlements are found. The identification of these secondary center mentions only the major ceremonial center on which it is dependent (most probably linked to it by royal marriage alliances), whereas the four capital cities name each other with no restriction. Surveys may provide more evidence for the existence of hexagonal lattices of tertiary centers surrounding a secondary center, and also shifting villages and hamlets around these tertiary centers.

This territorial organization of the Lowland Classic Maya is summarized in Figure 1b. Only the eastern area is represented but the same pattern is meant to be repeated for the other capitals.

Geometric Evidence from Ceremonial Centers

Map interpretation of Mesoamerican ceremonial centers (Hartung, 1971) reveals the existence of precise right angles, triangles with a 30° angle, triangles with two 45° angles, isosceles triangles and equal distances. But in most cases, a closer examination using arguments such as the visibility between geometrically related buildings and dates of construction, would be necessary to justify the geometric forms encountered. The analysis of the site plan of Tikal (Hartung, 1977) offers a good example of displayed geometrical notions with explanations implying they were not coincidental (Figure 2).

There is an isosceles right triangle with right angle at Temple III formed by an East-West baseline from Temple I to Temple III and a South baseline from Temple III to Structure 5D-90 on the south side of the Plaza of the Seven Temples. The building construction time of Temple I and Temple III are around A.D. 700 and A.D. 810 respectively. Approximate facade orientations of Temple I and Temple III are 9° E/N and 18° E/N respectively. Equinoctial observation could have been made only before the construction of Temple III.

There is a right angle (89°57') at Temple I formed by the lines from the doorways of Temple IV to Temple I and from Temple I to Temple V. The distance from Temple I to Temple V is 270 m and from Temple I to Temple IV is 750 m. The construction times are approximately A.D. 700 for Temple V and A.D. 750 for Temple IV.

From Temple I and Temple V, two South-North lines connect to the twin pyramids of Complex 0 (built around A.D. 731) which in turn connect to the twin pyramids of Complex P by two parallel lines having approximately the same orientation as a corresponding axis of the North Acropolis.

There is an isosceles triangle formed by the equal distances from the doorway of Structure 5D-104 inside the South Acropolis to the doorways of Temple I and Temple III. Map measurements give 282 m and 283 m from 5D-104 to Temple I and Temple III respectively. Hartung (1977) suggests that Structure 5D-104 and Temple I already existed and that an East-West line was drawn from the doorway of Temple I; then the doorway of Temple III was placed at the point where a line from the doorway of Structure 5D-104 intersected the East-West line, forming an isosceles triangle with the base line from Temple I to Temple III.

In order to establish a geometrical motivation for Maya site planning, special attention has to be given to the date of construction of buildings, points of reference from which directions or measurements are taken, direct visibility, and finally the importance of field measurements. For example, in studying the map of Copan, Hartung (1977) emphasizes the importance of points of reference in examining the relative positions of the platform in front of the entrance to Temple 22 - the clear-cut masonry block in the stairway of Temple 11, the central market of the ball-court, Stela M and Altar 0 and the three markets in the East court of the Acropolis (Figure 3). He comments: "The placement of markers in open space is very suggestive for astronomical interpretations and seems to be at least due to geometric considerations" (Hartung, 1977, p. 124). Equal distances are indicated by the arcs in Figure 3. One set of distances runs from the central point of the masonry block of Temple 11, to the central point of the area in front of the doorway of Temple 22 (and not to the platform in front of Temple 22), to the central marker in the ball-court, and also to the central of the three markers in the East court of the Acropolis. The second set runs from the central point of the masonry block of Temple 11 to Stela M and Altar 0. (Note: The line joining

Stela M and Altar O is parallel to the transverse axis of the ball-court).

Arrangement of Structures in Ceremonial Centers

First impressions are somewhat misleading concerning an apparent lack of planning in the design of Mesoamerican ceremonial centers and cities. However, a number of scholars have noted the peculiar shapes of some constructions, structures having a different alignment than that of an annex and unusual orientations of buildings or of groups of buildings. These phenomena show to some extent an intention of town planning. Several hypothesis have been advanced to clarify the apparent disorder in the arrangement of constructions in ceremonial centers.

Astronomy

Written documents such as the Venus Table elaborated in the Dresden Codex, provide evidence of the knowledge and accuracy of astronomy in Mesoamerica. Astronomical symbols (cross-sticks) depicted in codices and on monuments suggest that celestial bodies were observed (Figure 4). Therefore, archaeologists have endeavoured to determine buildings which could have been used by the Maya as astronomical observatories and have examined the orientation of structures. Several buildings have been baptized "astronomical observatories" by authors. Among these structures, the Group E complex at Uaxactun, where the disposition of Building E VII and the three small buildings constructed on a single platform permits the determination of solstices and equinoxes, is considered to be an astronomical solar observatory (Hartung, 1975 and Aveni, 1980).

Aveni, Gibbs and Hartung (1975) exhibit several astronomical alignments that can be made from the Caracol tower at Chichen-Itza and conclude that it is also an observatory.

Results of field investigations with transits of the building orientation at 60 Mesoamerican sites are summarized in Aveni's polar diagram (Figure 5). This diagram reveals that many axial directions were distributed about 17° East of North (the 17° family), others slightly East of North, while only a few were oriented West of North.

Since any edifice can be found to be aligned to some astral body, the question is to prove the relevancy of such an alignment. In the analysis of the orientation of structures, religion and ethnohistorical factors justify in most cases the significance of a particular alignment. The sun and the moon have major importance in Maya cosmology and religion as demonstrated by Maya symbolism and mythology. The most important planetary body is Venus which was described in Maya folklore and observed scientifically (Venus calendar in Dresden Codex). Among stars, the Pleiades are often mentioned in Mesoamerican myths. Therefore, alignments to sun positions (zenith passages of the sun, solstices, equinoxes), moon positions (although no such alignments have been found yet), Venus (heliacal rising and setting), and the Pleiades, are not likely to be casual. Moreover, the positions of these celestial bodies could have served for the elaboration of the agricultural, ritualistic and civil calendars.

In some cases, even if there is no ethnohistorical source to justify an alignment, the astronomical arguments are considered significant, an interesting example being Mound J at Monte Alban (Aveni and Linsley, 1972).

Hartung (1975) presented some "logical archaeological-architectural bases" for an archaeoastronomical analysis of the sites and also indicated some techniques by which astronomical directions were marked. In some instances, lines were traced with paint on stone or on the stucco coating of stones. In this category can be found some "astronomical circles" or pecked crosses.

Aveni, Buckingham and Hartung (1978) made the following comment on pecked crosses : " These symbols may have been intended as astronomical orientational calendars or ritual games. Evidence is presented which implies that more than one and perhaps all of these functions were employed simultaneously, a view which is shown to be consistent with the cosmological attitude of the Pre-Columbian people" (Aveni, et al., 1978, p. 267).

Surfaces (vertical), corners of windows and doors, and horizontal and vertical tubes were also used to define directions for astronomical observations. Doorways of buildings (especially central doorways), landings on stairways, projections of the middle or end of stairways, and platforms and altars may also be points of observation if they are linked with specific points of reference such as (a) sculptured elements (stelae, altars, rock reliefs, etc.), (b) outstanding architectural elements of a building (its door or central doorway, its corners, or a characteristic detail), or (c) elements of the distant landscape either artificial or natural.

The elements so described are considered in archaeoastronomy to reflect an astronomical motivation for pre-Columbian site planning. Astronomical alignments are presented here for they may imply in certain cases, some knowledge of geometry. The following astronomical considerations of the Governor's Palace at Uxmal and the site plan of Teotihuacan are exhibited to verify the existence of deliberate right angles. A more detailed discussion on Teotihuacan is presented in order to illustrate how, sometimes, various hypotheses can be propesed to explain the placement of buildings in a site plan, and consequently to justify the geometrical notion displayed.

1. Uxmal

Structures at Uxmal are generally oriented 9° East of North. The primary exception is the Governor's Palace which is skewed about 20° from this axis. Viewed from the central doorway of this construction, a visual line perpendicular to its front face passes over the base of a half-fallen stone column, then over the center of a platform surmounted by a double headed jaguar, and continues to the ruins of Nohpat, about 6 kms distant. This orientation also marks an extreme rising position of Venus during the Uxmal building period around A.D. 750 (Aveni, 1975). The right angle so displayed is thus justified by astronomical considerations and the peculiar orientation of the Governor's Palace. The Venus alignment strongly argues for the non-coincidental nature of the right-angle since it is known that Venus had great importance in the Maya world (Figure 6.). It may also be noted that Lamb (1980) has argued that the Governor's Palace and some other buildings at Uxmal are associated with Kukulcan - Venus. This would support the importance of and the interest in this planet in the proposed astronomical orientation.

Aveni and Hartung (1982) examined the parallelism of the walls of the Governor's Palace. They found that the short walls are parallel within $1\frac{1}{4}^{\circ}$, but the longer walls (walls that also make a right angle when aligned with Nohpat and Venus) were almost perfectly parallel. In fact, they studied the parallelism of walls of other monuments at Uxmal, Chichen-Itza and Palenque. Table 1 summarizes the results: the data obtained from the Colgate University buildings are listed in order to compare and realize the alteration of rectangularity due to time as well as the contemporary tolerance in the construction of a right angle.

In their appendix, they describe a method to construct a right angle using very simple tools and basic knowledge of geometry. The first method uses the notion of the 3-4-5 right triangle of which the perimeter 12 can be divided by the two sides (Aveni & Hartung, 1982, p. 80).

1. Place two of the pegs at any distance apart. (A more precise right angle results when they are placed farther apart.) Designate this side to have a length of 3 units.

2. Then connect the pegs with a string four times to obtain a string of length equal to that of the perimeter, 12 units.

3. Divide the string into three equal parts and mark it at the end of the first part so as to determine the length of four units.

4. Connect the third peg to the string at the mark and the two ends of the string to one of the other pegs.

5. Loop the loose (8 unit) string around the remaining peg and pull at the peg marking four units until the string is straight between all three pegs.

6. When this is done, there is one and only one position in which to locate the unplaced peg. This is a right triangle with an angle we find consistently to differ from 90 degrees by less than 10 minutes at arc.

The second method uses the fact that a triangle whose longest side is the diameter of the circle circumscribed to it is a right triangle. Aveni and Hartung note that the Maya "would only need to draw a half circle with string at constant length, connected to the center point and then inscribe a triangle in the proper manner"(Aveni & Hartung, 1982, p. 80).

2. <u>Teotihuacan</u>

During the mapping project's work at Teotihuacan, Drewitt observed that the plan of this site presented a series of almost perpendicular axes determined by major streets such as the Street of the Dead and the East-West Avenue. In a first analysis which will be described in Chapter 4, Drewitt proposes a unit of measure of about 80.5 cm. He then superimposes a grid formed by North-South lines parallel to the Street of the Dead and by East-West lines parallel to the East-West Avenue. Since the angle between these two streets deviates from a 90° angle of about $1\frac{1}{4}^{\circ}$, the grid was then non-orthogonal. The grid was adjusted "to allow for a strip 40.25 meters wide (50 Teotihuacan units of measure) down the middle of the Street of the Dead and a strip just over 8 meters wide (10 Teotihuacan units of measure) along the East-West Avenue" (Drewitt, p. 4). Except for the first E/W lines and N/S lines from the intersection of the Street of the Dead and the East-West Avenue (starting point of the grid), the grid was constructed of intervals of 322 m (400 units). He noted then that all long streets, many ceremonial structures and precints, and even the deviation of the Rio San Juan by canals, follow this non-orthogonal grid. In addition, he describes other instances of deviation from a 90° angle of $1\frac{1}{4}^{\circ}$ occurring in Mesoamerica. Such examples are taken from Mounds A and B at Kaminaljuyu, structures and complexes at Monte Alban, the central Acropolis at Tikal, Structure B complex at Tula and Complex A at La Venta. He concludes that this skewed axial system "seems to have been a basic element in the layout of single buildings and of building complexes and in that respect formed an integral part of construction practices" (Drewitt, p. 9).

Noticing this deviation of about 1°, Aveni (1980) wonders whether the ancient architects of Teotihuacan were resisting the local topography for astronomical reasons. His explanation is based on astronomical hypotheses.

At Teotihuacan, Aveni (1977) observed that a line joining the pecked cross (Teo 6) at Cerro Gordo and the Viking Cross (Teo 1) near the Pyramid of the Sun, 7 kms distant, is oriented 16°30' and is perpendicular to a major baseline which is oriented 16°30' North of West. In addition, a near right angle is formed by the Street of the Dead, oriented 15°28' East of North, and a line joining the Viking Cross (Teo 1) and another pecked cross (Teo 5) at Cerro Colorado, 3 kms distant, oriented 15°21' North of West (Figure 7). The latter line is within 1° of an alignment with the Pleiades. Aveni notes : "While Sirius rose along the baseline viewed ~~West to~~ East, we found a more likely astronomical possibility in the Pleiades". He adds : "The Pleiades underwent heliacal rising on the same day as the first of the two annual passages of the sun across the zenith, a day of great importance in demarcating the seasons. The appearance of the Pleiades may have served to announce the beginning of this important day when the sun at high noon cast no shadows" (Aveni, 1977, p. 5).

The importance of the Pleiades in Mesoamerican cosmology is worth mentioning when examining an astronomical explanation for the orientation of Teotihuacan. On this, Aveni (1980) refers to Malmstrom (1978) for another hypothesis. The error of 1° can be explained by the relief and the poor distinction of the Pleiades at this latitude at the time of the construction of Teotihuacan. Concerning this orthogonality, Aveni writes: "The Teotihuacanos could have employed the baseline to determine the principal axis of the site. According to Dow, they may have constructed a very accurate right angle to lay out the Street of the Dead. Alternatively, the baseline may have served to transfer the basic axis of the ceremonial center, established earlier, to the outlying section of the city" (Aveni, 1977, p. 4).

The hypotheses of Aveni are based on astronomical alignments from pecked crosses, the interpretations of which have been given by Aveni, Hartung and Buckingham (1978). The relation between pecked crosses and astronomy is also discussed by Aveni, Hartung and Kelley (1982). Considering this relation, the right angle seems intentional. But it should be added here, as Drewitt points out (personal communication), that the visual line Teo 1 - Teo 5 does not follow the grid or any of the streets, walls of constructions or canals in the plan of Teotihuacan. Moreover, several other pecked crosses have been found in Teotihuacan. It would be worthwhile to check other astronomical alignments or other geometrical evidence in order to discuss the significance of their location in the map of Teotihuacan.

Chiu and Morrison (1980) describe, in a general manner, the way to construct a grid. Let equidistant points be marked on a segment AB by the use of long cords and from each point, arcs of fixed radius be drawn on each side of AB. The intersection of these arcs will determine points where passes a perpendicular line to AB. Similarly, equidistant points are marked on one of the perpendicular lines of AB (called CD) followed by the drawing of arcs on each side of CD which will determine then a series of perpendicular lines to CD and therefore parallel to AB (Figure 8a).

Aveni (1977) gave 16°30' East of North as the orientation of Teo 1, while Aveni (1980) gives 17° East of North. The paper of Chiu and Morrison, probably written before Aveni's publication in 1980, locates Teo 6 between 16.5° and 17° East of North. In order to explain the non-orthogonality of Teotihuacan, they seem to imply the necessity of having two perpendicular axes from which it is possible to construct a perfect rectangular grid by the method described above. They write : "if the grid axes were chosen without purpose, or in any way independent of external directions, the unobstructed valley floor would seem to leave no reason but human error for the small but real off-set of the grid axes from a right angle crossing" (Chiu and Morrison, 1980, p. S58). Since Teo 1- Teo 5 (non perpendicular to Teo 1 - Teo 6) cannot be aligned with any relief element, the only external element has to be astronomical. Not finding a suitable astronomical alignment for Teo 1 - Teo 6, although a possibility had been suggested by Aveni (1977 and 1980), Chiu and Morrison propose the following explanation.

If we draw a line AB aligned to a certain astronomical object and a perpendicular CD to AB, and then, if from a distant point E from AB we draw a line EF aligned to this same astronomical element, and the series of perpendiculars to EF, this series of lines will form, with the series of perpendiculars to CD, a perfect rectangular grid. The non-orthogonality of the grid results from the non-parallelism of the two lines aligned to the same event. Chiu and Morrison explain this non-parallelism by observing that "two sights of one and the same horizon event will lead to distinct but closely similar ground directions whenever they are taken from two stations with different elevations of the horizon, it is the inclination of the apparent path of the object to the horizon that accounts for the difference of the observed azimuth of the setting (or rising, as the case may be)" (Chiu and Morrison, 1980, p. S59).

The aim is thus to find an event whose alignment from Teo 6 would be almost parallel to Teo 1 - Teo 5. They eliminate the Pleiades by comparing the two alignments (Figure 8b): "The first alignment is not acceptable at a reasonable date, and the second one is less than striking with respect to any horizon figure" (Chiu and Morrison, 1980, p. S60). However, they note that the sun on August 12 sets behind Mount Xaplan at about 15 kms from Teo 6. If Teo 6 is located 300 m south of the peak of Cerro Gordo, the azimuths of sun and peak are both $236^{\circ}.25$. Along the line Teo 1 - Teo 5, the sunset on August 12 is at $280^{\circ}.6$ and the horizon has an elevation of $1^{\circ}.4$. Measurements on the spot itself are necessary in order to locate Teo 6 more precisely. However, according to the article by Chiu and Morrison, the sunset on August 12, seen from Teo 1 and Teo 6, toward Teo 5 and Mount Xaplan (respectively), would be the event that, owing to the difference of elevation, would explain the non-orthogonality of the plan of Teotihuacan. This argument is all the more important if we consider the debate on the "zero date" and the origin of the Mayan Calendar as well as the legend narrating that Teotihuacan was at the source of all creation.

Heyden (1975 and 1976) has presented a detailed description of a cave underneath the Pyramid of the Sun at Teotihuacan and an interesting analysis of numerous rites involving caves and earth deities. Heyden demonstrates the importance of caves and springs, the rituals involving water and earth deities, and notes that the glyph of Teotihuacan in Codex Xolotl represents two pyramids over a cave symbol. This cave is a natural formation of lava bubbles, exhibiting man-made modifications. Heyden's paper also contained a hypothesis for the planning of Teotihuacan, based on the ritualistic functions of caves and the position of the Pyramid of the Sun over the cave.

The entrance to the cave is on the western side of the Pyramid of the Sun below the stairway. The cave consist of a long tunnel ending in four chambers forming a cloverleaf almost below the center of the pyramid (Figure 9). Two additional chambers are situated on each side of the tunnel a little before the midway point. The cave contained a spring whose course dried up naturally or was changed artificially. Millon (1980) explains that this spring was in fact a flow of water redirected from outside the pyramid.

The cave was first used as a shrine and afterwards, the Pyramid of the Sun was built over it. Heyden believes that "the cave determined the site for the construction of a primitive place of worship and then for the pyramid itself" (Heyden, 1975, p. 139). Teotihuacanos first constructed the Pyramid of the Sun and then at approximately the same time, the Street of the Dead and the Pyramid of the Moon.

Heyden also reports Tobriner's interpretation of the direction of the Street of the Dead: "He suggests that the Teotihuacanos built the most important road in their city oriented toward Cerro Gordo because they considered this mountain to be one of the main sources of their water supply, and therefore associated it with the adoration of water gods" (Heyden, 1975, p. 143).

According to these authors, the site plan of Teotihuacan was based upon the worship of cave and water deities. In this interpretation of Teotihuacan planning, the baseline toward Cerro Colorado is not mentioned. If indeed the Street of the Dead was aligned to Cerro Gordo, the astronomical orientation of the Cerro Colorado baseline towards the Pleiades would seem to be coincidental.

Geomagnetism and Geomancy

Fuson (1969) examined the families and shifts in alignments of structures in the Maya area. He noticed that one family of alignments (namely the one of almost 7°) coincided with the magnetic declination in those regions. This magnetic declination varied between 6° to 8° East of North and was at 7°30' East of North for the central area of Guatemala and Yucatan in 1965. He then wondered whether "Maya structures varied orientation as the earth's magnetic field fluctuated" (Fuson, 1969, p. 504).

Carlson (1975) followed the same reasoning and came to a similar suggestion after noticing the family of 8° West of North orientation of the formative Olmec sites. This geomagnetic hypothesis is supported by discoveries made in the Maya region and Olmec territories. The evidence of Maya knowledge, since the Early Classic Period, of cinnabar and the pigment obtained from it, and also the liquid mercury found in some sites, prove that the Maya discovered liquid mercury and could have used it (with a rock containing magnetite) as a compass. Carlson (1975) also believed that the Olmecs could have used a geomagnetic pointer compass in place of a magnetized iron needle. He explained how the Olmecs might have used a lodestone bar (M-160 of San Lorenzo) as a floating pointer to orient their ceremonial centers.

The main difficulty with this hypothesis is the secular variation in time and location of the magnetic inclination and declination. A method to predict or extrapolate this magnetic change into the past has not yet been found. However, investigations in archaeomagnetism and paleomagnetism could prove fruitful. The determination of the secular variation might permit the geomagnetic hypothesis to explain shifting alignments as, for example, the change of orientation of structures in both Uaxactun and Copan toward a North-South axis around A.D. 700.

Maya architecture gives witness to the importance of cosmology and religion in Maya construction and supports the possibility of geomancy practice in Mesoamerica. Carlson (1977) gives a good definition of geomancy when describing the Feng-Shui geomancy: "It is a complex divinatory art of locating an auspicious site and then determining the proper orientation of the human construction so as to place man and his works in harmony and balance with the cosmic forces as perceived. This is a practice based on an ancient tradition of a dualistic, animistic, highly interactive cosmology employing a complex of physical environmental factors such as the cardinal directions, location of topographical features such as rivers and mountains, astronomical archetypes, and the geomagnetic field declination" (Carlson, 1977, p. 75). On this point Hartung (1977) writes: "The basis of Maya geometry seems to lie in geomancy" (Hartung, 1977, p. 125).

Considering the similarities between Mesoamerican and Chinese cosmologies enumerated by diffusionists such as Kelley, Moran and others, Carlson proposes a geomancy model as a general approach to understanding Mesoamerican site structure and orientation and especially as a model for the evaluation of the "geomantical alignment hypothesis".

This model (Carlson, 1976, p. 189, p. 201) is here summarized:

> explicit elements of the four quartered, layered cosmology in microcosm (examples: the Castillo at Chichen Itza or the funerary Temple I at Tikal);
>
> the shape of cyclical time should be manifest in both temporal numerology and ground-plan structure (Castillo at Chichen Itza);
>
> influence of the local "mystical geography", i.e., the location of "sacred places", caves, world mountains, etc. and the placement of ancestral bones. This aspect should manifest itself in kinship structure and terminology and in social organization and community structure;
>
> the use of geomagnetic compass and the partition of the circles of horizontal space and cyclical time into numerologically interesting integral units. (In examining orientations of structures, further studies should be made to look for "sacred ratios".)

The discussion of Klein (1980) concerning the symbolism of circular structure should also be mentioned. She reports that the Aztecs constructed 'round' buildings "at the center of square plan urban centers to mark the axis mundi as a place of disease, the sun's annual 'death' and descent into the underworld at the vernal equinox, and chaos in general" (Klein, 1980, p. 11). The Caracol of Chichen-Itza and its relative position with respect to the Sacred Cenote was proposed as a marker for the axis mundi.

Aveni and Hartung (1978) suggested that, "El Caracol de Chichen-Itza puede haber sido presisamente el instrumento astronomico usado para derivar los datos empiricos que aparecen en las tablas de Venus en el Codice de Dresden" (The Caracol of Chichen-Itza could have been precisely the astronomical instrument used to derive the empirical data that appear in the Venus Table of the Dresden Codex), (Aveni and Hartung, 1978, p. 4). However, the Caracol was constructed in the Post Classic while the Venus Table dates from the Classic.

They examined circular structures at Mayapan and Paalmul in order to find astronomical alignments similar to those encountered at Chichen-Itza. Considering the ruined state of these sites, only few similarities could be demonstrated, leaving unanswered the question of whether these centers served an astronomical purpose or whether they were simply imitations of Chichen-Itza.

The function of circular structures can be difficult to determine. An example in Puerto Rico presented by Aveni (1980) is a case where no astronomical or symbolic explanation can be given. Moreover. the location of this structure has no apparent relation with other surrounding sites. In addition, the study of Seidenberg (1981), describing the ritual origin of the circle and the square, presents an enumeration of evidence from myths related to the construction of the circle. His analysis, not restricted to Mesoamerica, also examines the relation between those geometrical forms and the social organisation.

The Orientation of Ball-Courts

Ball-courts are believed to be built according to both symbolic and urban criteria. The ball-game is seen by Seler (cited in Taladoire, 1979) as a "representation of the movements of the sun, either in its daily course (from East to West) or during the whole year (from North to South) : so a ball-court had to have either North-South or East-West orientation" (Taladoire, 1979, p. 12). Taladoire examined 619 ball-courts from which only 263 orientations are known, in order to determine the motivation underlying all ball-court emplacements. Only in the Maya Lowlands are the ball-courts oriented in the majority, towards a North-South direction. Only 64 out of 94 known ball-courts in this region have known orientation. Therefore, the conclusion that symbolic factors determined the North-South orientation of the Maya Lowlands ball-courts cannot yet be made.

Without excluding the possibility of a symbolic motivation for the orientation of some ball-courts, urban and astronomical functions (and in some cases topographic elements) seem to play a bigger role in the arrangement of ball-courts. This is the case in Tula and Chichen-Itza, where, as Aveni (1980) points out, the resemblance between Temple B at Tula and the Temple of the Warriors at Chichen-Itza (the Atlantean figures, the numerous columns, the sculpted feathered serpents) can be added to the fact that "not only are the principal ball-courts at both sites similarly located, but they also exhibit identical orientations"(Aveni, 1980, p. 240).

Effects of Light and Shadow

Since equinoxes and solstices are important days, special attention has been focused on light and shadow effects appearing on certain constructions at these times. If these effects are intentional, then the arrangements of the structures imply geometrical and astronomical knowledge.

Arochi (1977) believes that El Castillo at Chichen Itza was built in order to allow the light formation of seven isosceles triangles on the balustrade at summer solstice. The first triangle appears on top of the balustrade; as the sun moves, the other triangles are formed by the effect of light and shadow. The first triangle to appear is the last to disappear. Ethno-historical sources reveal that Kukulcan descended along the balustrade, took the gifts left by the people on the mouth of the serpent head ending the balustrade and ascended back to heaven. The formation of these isosceles triangles represents the descent and ascent of Kukulcan. When the seven triangles are formed, the light and shadow effect can be seen as the body of a serpent, the head being the serpent head carved at the end of the balustrade. In Arochi's opinion, the orientation of El Castillo is dictated by these symbolic and ritualistic factors. His interpretation of the formation of isosceles triangles may be questioned since half-truths and misinterpretations are found in Arochi's work.

Hartung (1977) also mentions the light formation of triangles on El Castillo and gives a second example at Tikal, where Temple II casts its shadow exactly on the stairway to Temple I.

Aveni (1980) describes two examples of astronomical hierophany in Palenque found by Schele (1977). One is from the square tower in the

Palace complex to the Temple of Inscriptions. On the winter soltice, the setting sun viewed from the tower, enters the Temple of Inscriptions at an angle "approximatively the same as that of the stairway leading to the tomb. The hierophany signifies the transfer of power of the deceased Pacal to his son, Lord Chan Bahlum" (Aveni, 1980, p. 284). The other hierophany occurs at the Temple of the Cross on the same date.

More investigations on manifestations of light and shadow effect should be made for they could be significant in cereminial center planning and imply some geometrical notions.

The Similarity of Constructions and the Transfer of Orientations

Teotihuacan is considered to be the source for the "17° family" of site orientations. Several other sites (Tenayuça, Tepozteco, Tula, etc.) built centuries later but within 100 kms of Teotihuacan maintained a similar orientation. Since the Pleiades no longer set along the Teotihuacan East-West axis, Aveni (1977) proposed the following interpretation: "The priest rulers of these new centers probably looked upon Teotihuacan as a holy city in ruins. In reference to the past, they may have planned their centers of worship with the same directional axes. The old alignment could have been transferred astronomically simply by sighting a substitute star which had replaced the Pleiades: by this time they would have set well to the north of the Teotihuacan East-West axis. Thus, the axis of Tula and the other members of the 17° group can be viewed as non-functional imitations of Teotihuacan" (Aveni, 1977, p. 7).

The significant astronomical orientation and the presence of the cave and pecked crosses in Teotihuacan do not exclude a geomagnetical explanation, according to Carlson (1977). In his opinion, geomagnetic technique could have been used for families and shifts in alignments. He suggests that if a geomagnetic compass played a part in the transfer of sacred orientations the instrument was used in a geomantic context.

The arrow shaped Building J at Monte Alban is similar to a contemporary (about 275 B.C.) structure, Building O, at Caballito Blanco situated about 50 kms East of Monte Alban. However, their orientations are different. The resemblance resides in the similar shape, their relative position with respect to other surrounding buildings, and to the fact that alignments from the front of the buildings are related to the sun while those from the arrow heads are directed toward bright stars. (See Aveni and Linsley (1972) for more details on the orientation of Mound J at Monte Alban and for a description of the orientation of Building O at

Caballito Blanco and its resemblance to Mound J.) The existence of this apparent non-functional imitation makes Aveni (1975 and 1980) wonder whether it was actually intended to serve an astronomical function or whether it was a mere imitation of Building J at Monte Alban. He decided upon the former hypothesis. Following a reasoning similar to Carlson's, one might speculate whether the differences in the orientations of these buildings are due to geomagnetical factors while the resemblances are due to the practice of geomancy.

A few examples of imitation in ceremonial center planning have been noted and an instance of structures having similar shapes but not similar alignments has been considered. The existence of centers which preserve the orientation of an "original" site or a similarity in shapes but not alignments raises interesting questions concerning the techniques used to transfer alignments and shapes of structure. Which arguments (astronomical, geomagnetical, geometrical, etc.) are the most suitable? Whatever motivated these practices, the very fact that geometrical elements such as right angles were preserved, constitutes a supplementary argument for asserting that Mesoamericans were aware of some geometrical principles.

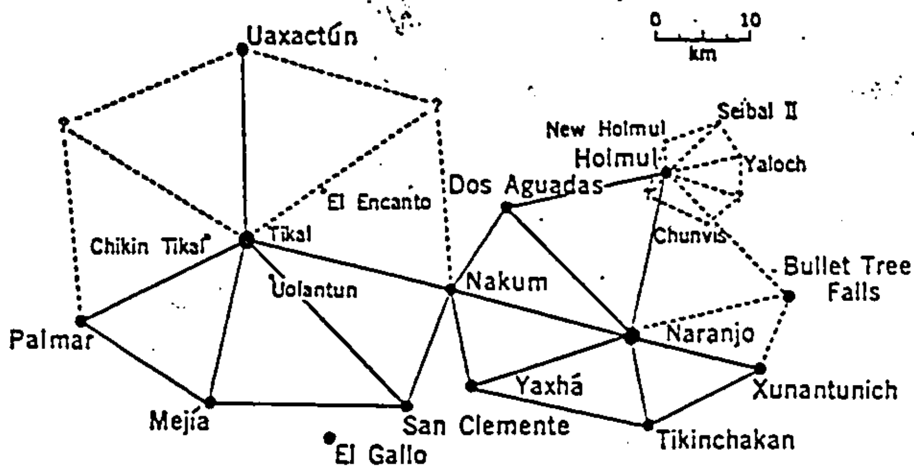


Figure 1a : Hexagonal lattices in the vicinity of Tikal.

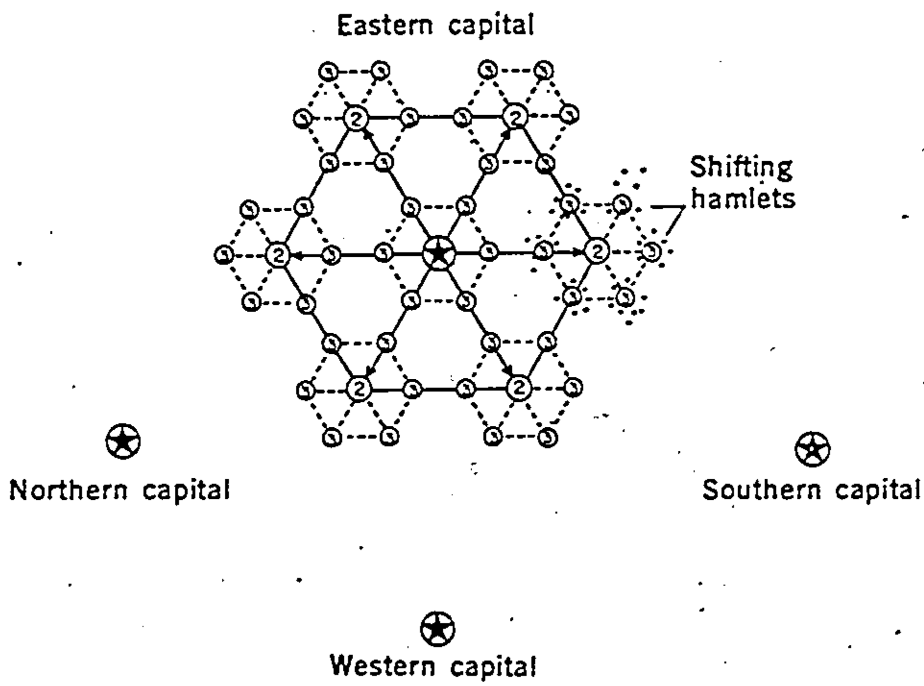


Figure 1b : Idealized diagram of the territorial organisation of the Lowland Classic Maya.

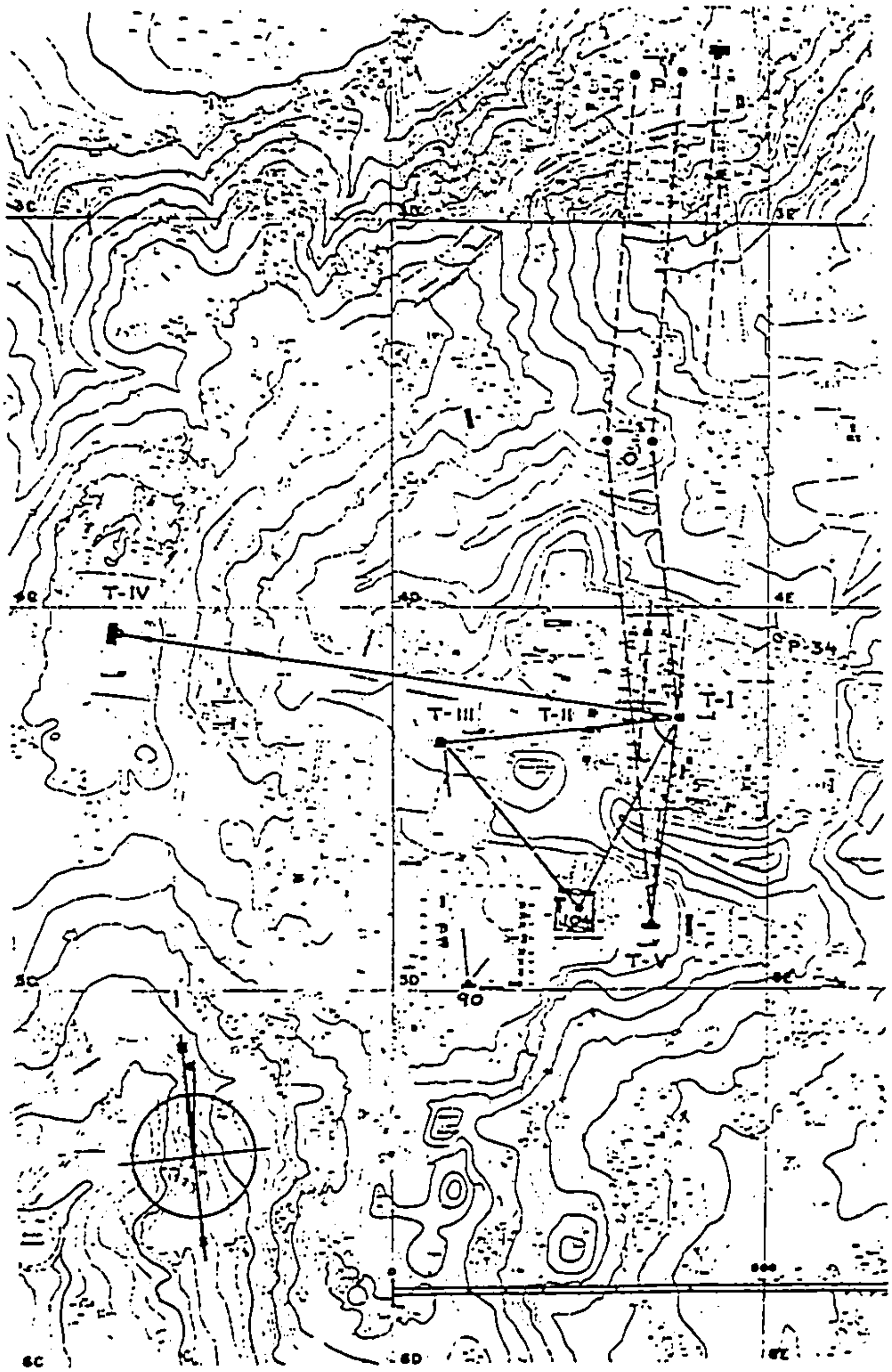


Figure 2 : Map of Tikal showing the geometrical relationships discussed in the text.

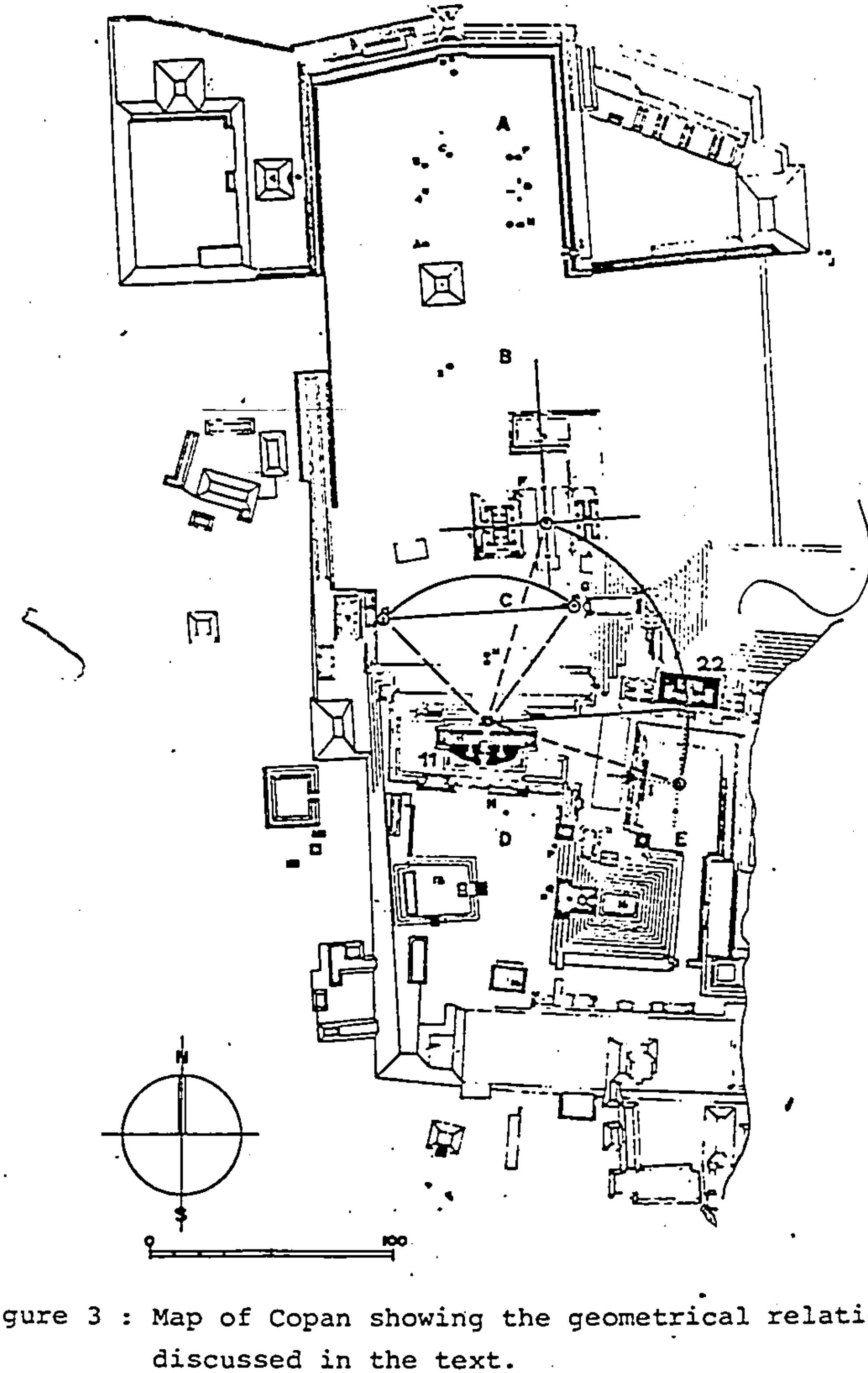


Figure 3 : Map of Copan showing the geometrical relationships discussed in the text.

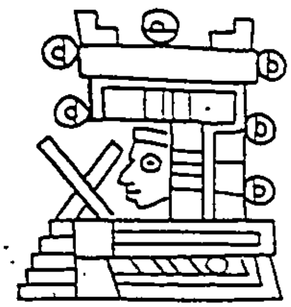


Figure 4 : Mixtec astronomer looking from the doorway of a temple through a pair of crossed sticks.

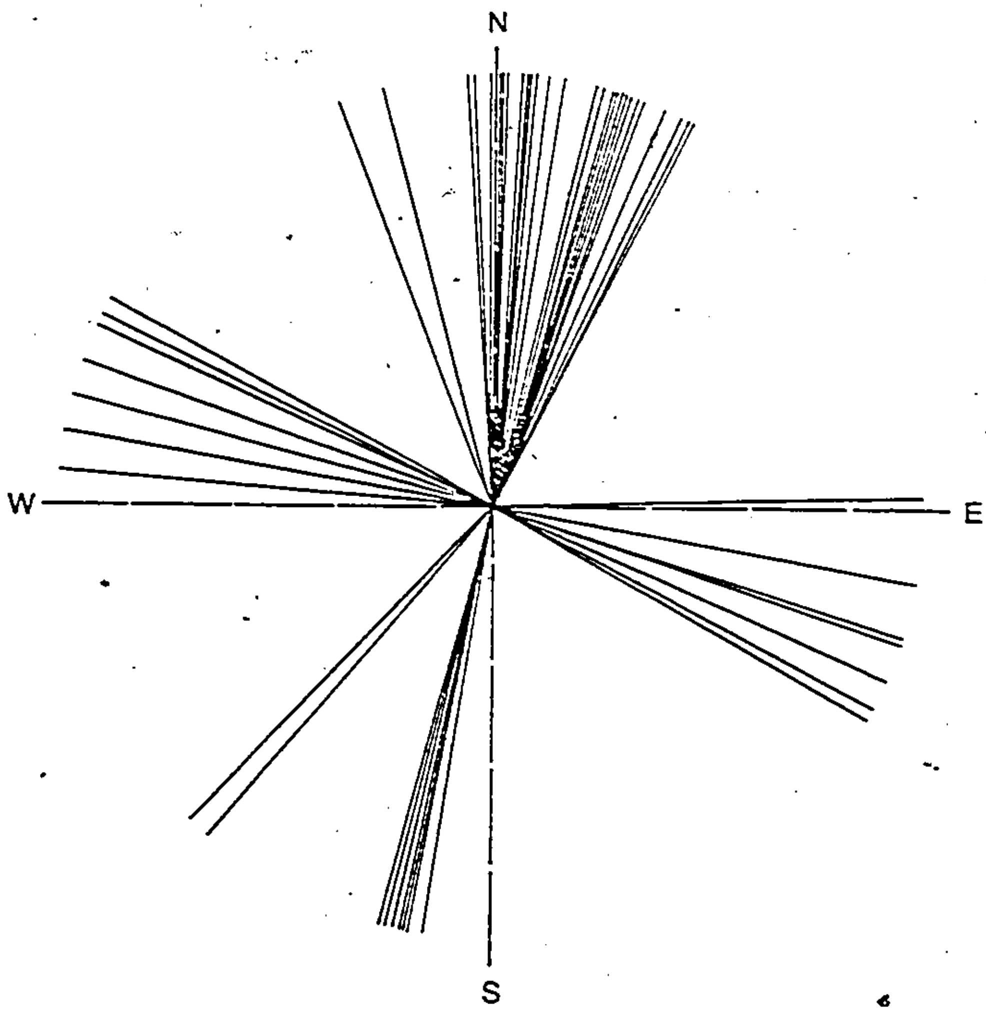


Figure 5 : Polar diagram showing the distribution of the axes of ceremonial centers in Mesoamerica.

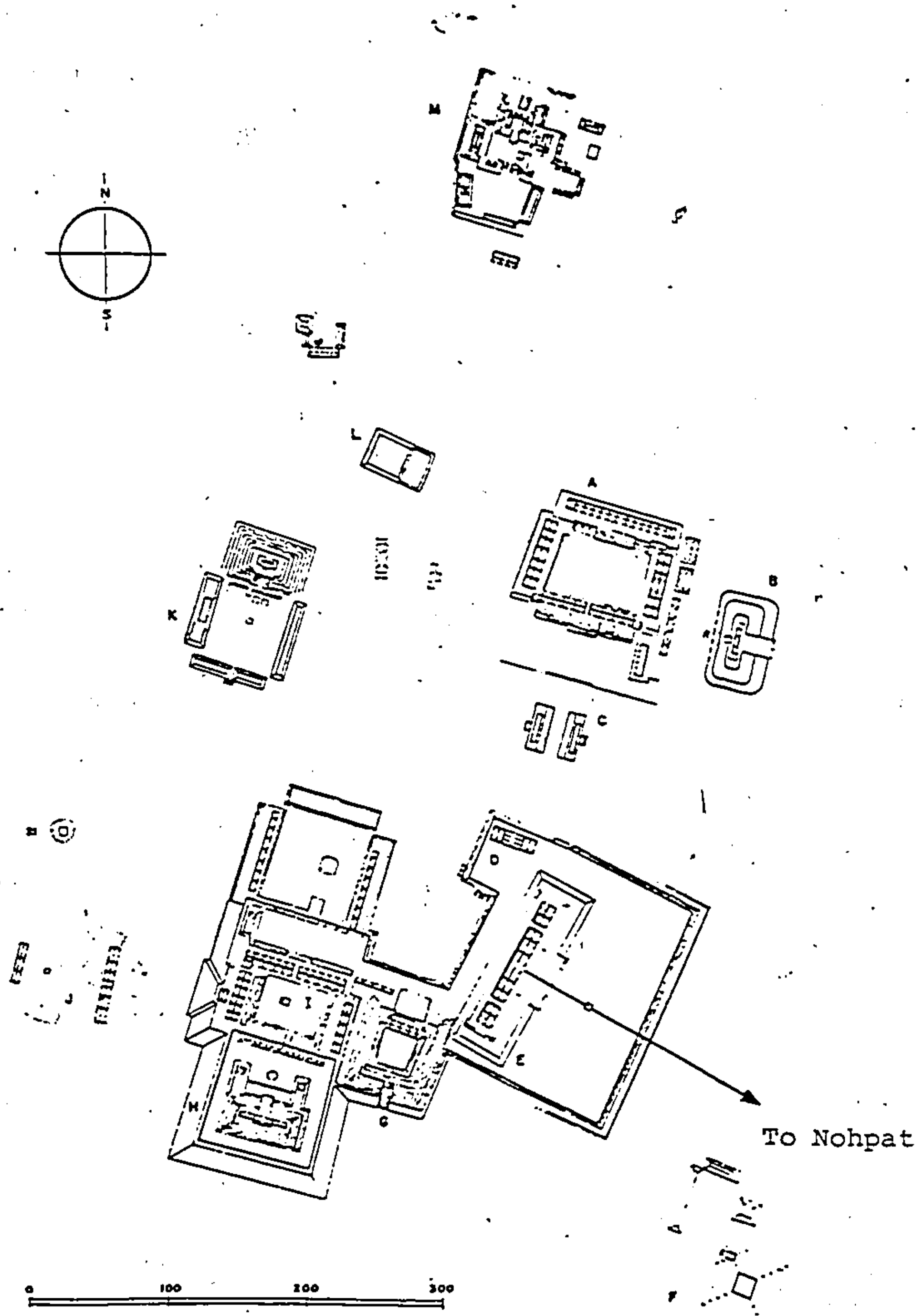


Figure 6 : Map of Uxmal with the peculiar orientation of the Palace of the Governor marked by an arrow.

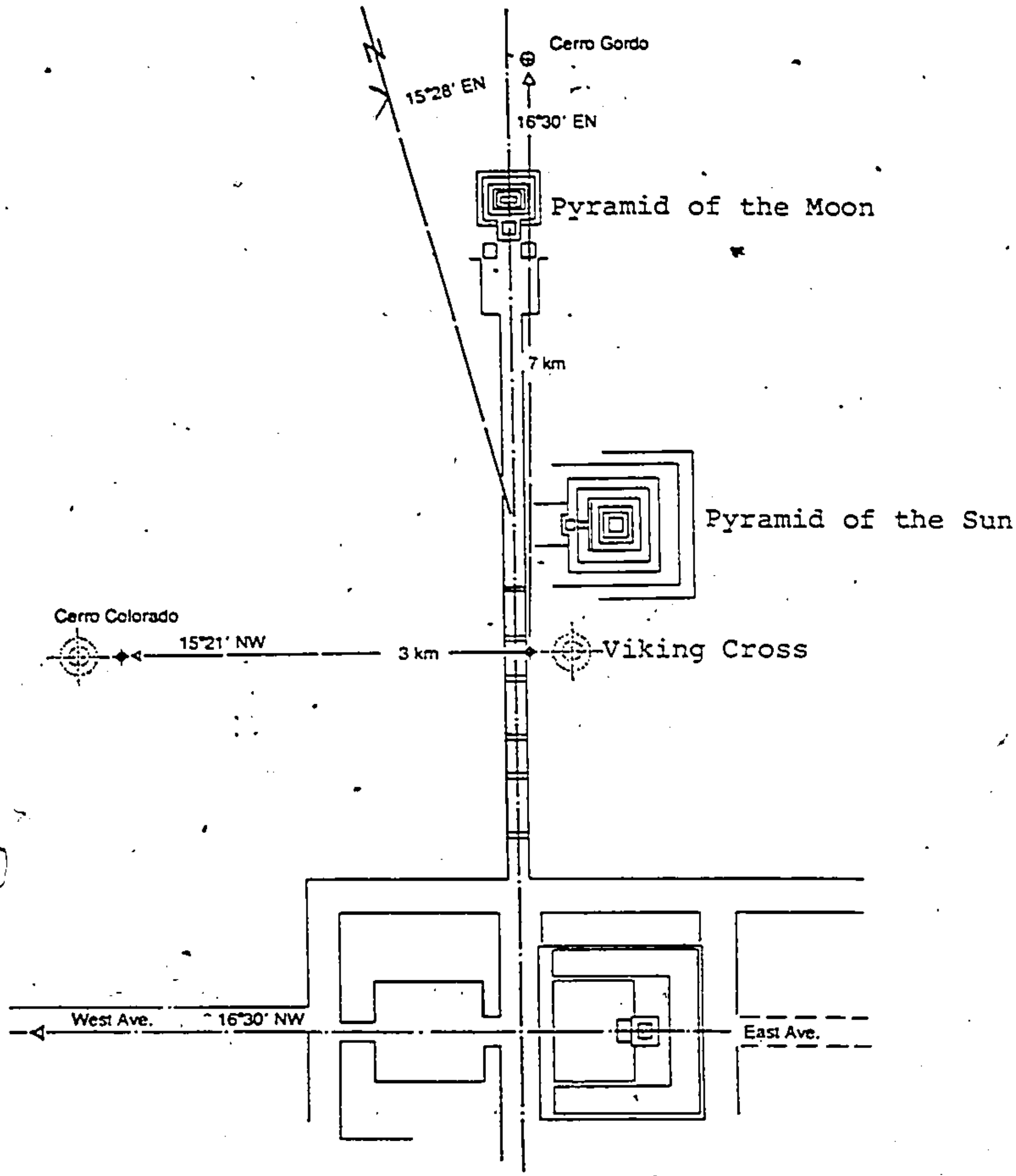


Figure 7 : Map of Teotihuacan showing the orientations discussed in the text.

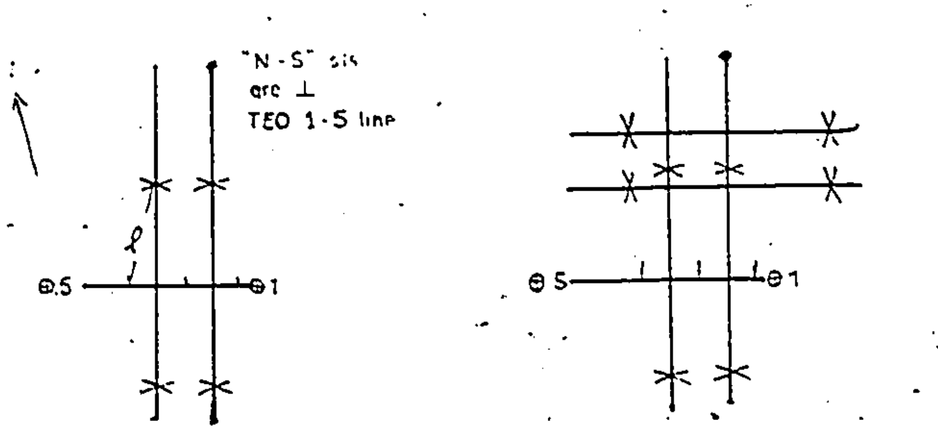


Figure 8a : Construction of a grid

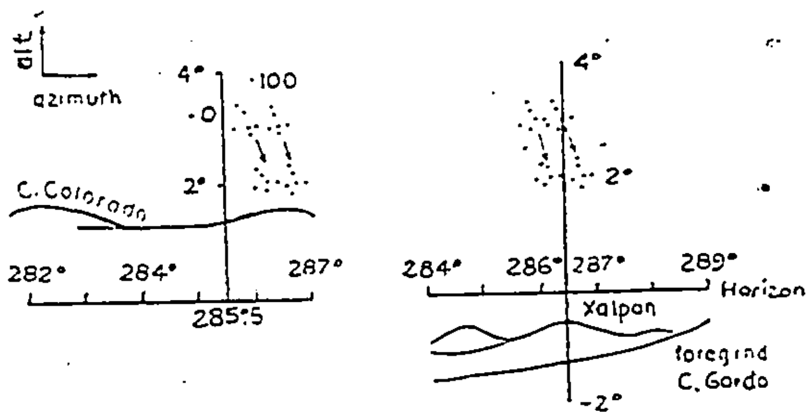


Figure 8b : The appearance of the setting Pleiades

Figure 8b : The appearance of the setting Pleiades

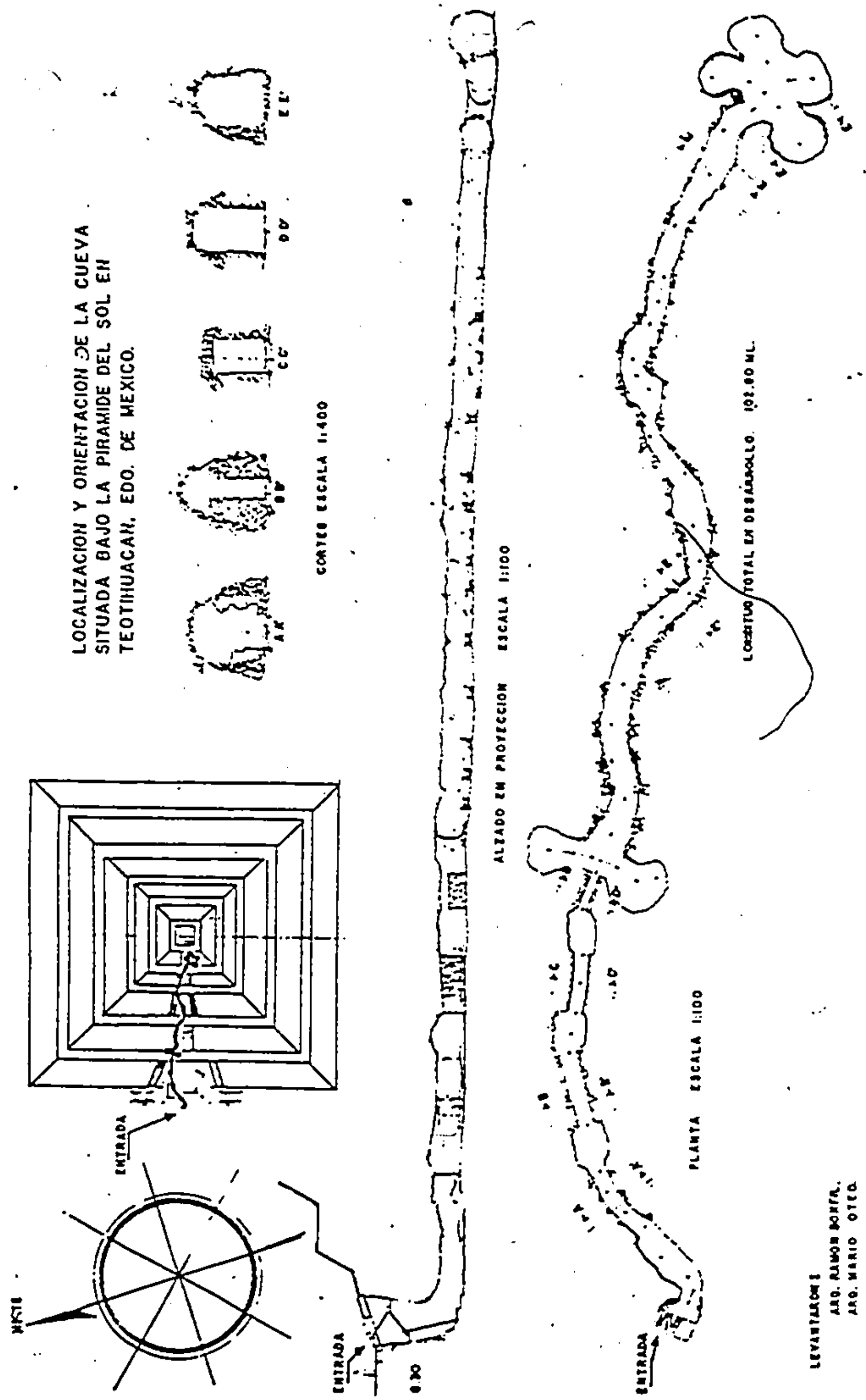


Figure 9 : The Cave underneath the Pyramid of the Sun.

Table 1

SUMMARY GEOMETRICAL DATA ON WALL ORIENTATION AND THE ANGLES AT CORNERS OF BUILDINGS

Ancient Sample			Average Deviation of Corners from Right Angles	Deviation from Parallel	
Site	Building	Epoch		Long Walls	Short Walls
Uxmal	Governor's Palace	A.D. 800	3°35'	< p.e.	1°16'
	House of Turtles	A.D. 800	0°24'	0°20'	0°11'
	Nunnery-North Building	A.D. 800	0°52'	< p.e.	1°44'
Chichén Itzá	Akab D'zib	A.D. 900?	1°01'	0°12'	2°03'
	El Castillo	A.D. 1000	0°23'	< p.e.* to 0°33'	
Palenque	Palace House E	A.D. 700?	1°12'	0°24'	< p.e.
	Palace House B	A.D. 700?	3°09'	0°23'	5°07'
	Temple of the Sun	A.D. 700?	2°35'	–	2°29'
	Temple of the Foliated Cross	A.D. 700?	2°00'	–	0°32'
Unweighted Average for Ancient Buildings			2°41'	0°15'	1°42'
Contemporary Sample					
Colgate University	West Hall	A.D. 1819	0°32'	0°35'	0°16'
	East Hall	A.D. 1850	0°20'	< p.e.	–
	Alumni Hall	A.D. 1859	< p.e.	< p.e.	0°16'
	Lathrop Hall	A.D. 1905	0°10'	–	-
	Hascall Hall	A.D. 1906	0°07'	< p.e.*	
	Memorial Chapel	A.D. 1916	0°35'	–	–
	Lawrence Hall	A.D. 1926	0°15'	–	0°21'
	McGregery Hall	A.D. 1930	0°10'	–	–
	Olin Hall	A.D. 1970	0°20'	< p.e.	1°02'
	Wynn Hall	A.D. 1979	< p.e.	–	< p.e.
Unweighted Average for Modern Buildings			0°16'	0°12'	0°22'

* Walls are of equal length

CHAPTER 4

UNITS OF MEASURE

In this chapter, a list of units of measure taken from ethnohistorical and linguistic sources will be given, followed by analyses of measurements taken from sites, plans or artifacts.

Ethnohistorical and Linguistic Sources

A study of possible units of measure utilized in Mesoamerica could not be undertaken if we did not have any evidence that the mere fact of measuring (which implies units of measure and methods of measuring), was indeed a preoccupation. Early native sources such as The Books of Chilam Balam and Popol Vuh, as well as the first Spanish texts on the Indians and their customs, bear witness to the indigenous concept of measurement and give in some instances, a value for a unit of measure. This notion of measuring is detected in the descriptions of how land was divided, found in the Book of Chilam Balam of Chumayel:

> "11 Ahau was the Katun when they carried [burdens] on their backs. Then the land-surveyor first came; this was Ah Ppizte who measured the leagues. Then there came the Chacte shrub for making the leagues with their walking sticks. Then he came [to] Uac-hab-nab to pull the weeds along the leagues, when Mizcit Ahau came to sweep clean the leagues, when the land-surveyor came. These were long leagues that he measures." (Roys, 1967, p. 65).

Evidence that measures were taken to divide land can also be found in the Popol-Vuh:

> "Great were the descriptions and the account of how all the sky and earth were formed, how it was formed and divided into four parts; how it was partitioned, and how the sky was divided; and the measuring-cord was brought, and it was stretched in the sky and over the earth, on the four angles, on the four corners, as was told by the Creator and the Maker, the Mother and the Father of life." (Goetz and Morley, 1950, p. 80).

Among the numerous translations of the Popol Vuh, one should also mention Edmonson's translation. He gives a translation of the passage describing the creation of the sky and the earth which is less mathematical but perhaps more accurate.

> "Great was its account
> And its description
> Of when there was finished
> The birth
> Of all the heaven
> And earth:
> The four creations
> The four humiliations
> The knowledge
> Of the four punishments
> The rope of tying together,
> The line of tying together,
> The womb of heaven,
> The womb of earth.
> Four creations,
> Four humiliations, it was told,
> By the Former
> And Shaper,
> The Mother
> And Father
> Of life
> And mankind." (Edmonson, 1971, Lines 63 to 74).

The difference arises from mis-interpretations concerning the meaning of certain words and other errors. With respect to one of this terms, Edmonson (1971: 7, n.63) writes : "All translators have read this as tzukuxit and rather loosely translated the passage having something to do with corners (tzuk), thus confounding the following ten lines. I read tzuquxit 'to cause to be born, be made to sprout, be nourished'". He therefore translates this term by 'creation'. Edmonson translates another term xukutaxit by 'humilations' when other authors give versions such as 'four sides' or 'quandrangulation'. A third term etaah could mean 'measure' (which is the common interpretation among several translations), but the translation 'knowledge' seems more suitable to Edmonson. Such differences exist in several translations of native writings such as the Book of Chilam Balam and the Popol Vuh, which warn us to be careful when using ethnohistorical sources.

Landa is more specific in that he indicates not only that land was measured but also the unit of measure. "Suelen, de costumbre, sembrar para cada casado con su mujer medida de 400 pies lo cual llaman hum uinic, medida con vara de 20 pies, 20 en ancho y 20 en largo", (Usually they sow for each household, with their wife, a measure of 400 feet which they call hum uinic, measured with a 'vara' of 20 feet, 20 wide and 20 long) (Landa, p. 40). A definition of uinic is given in the

Motul dictionary : "medida de tierra, para labrar, o labrada, de veinte kaanes o estadales..." (measure of land, to till, or tilled, of twenty kaans or 'estadales'...), (Motul dictionary, 1929,p.905). A Kaan is then defined as : "medida de un cordel con que los indios miden sus milpas, llamado mecate entre los espanoles.... tiene una medida de tres varas o cordeles de a tres brazas cada cordel, que hacen 36 (4 x 9) brazas de box. Hay otrá medida mayor que contiene otras tres varas o cordeles pero son de cuatro brazas cada uno, y de box tiene cada Kaan 48 (4 x 12) ...", (measure of a cord with which the Indians measure their milpas, called mecate among Spaniards ... it measures three varas or cords, each of length three brazas , which makes 36 (4 x 9) brazas de box. There is another larger measure which also equals three varas or strings but these are of 4 brazas each, and each Kaan has 48 (4 x 12) brazas de box....), (Motul dictionary,1929,p. 494). It is also written "... Kaan, mecate .. del nahualt mecatl; medida de 24 varas de Burgos de 838 mm cada mecate se subdivide en doce brazas de a dos varas de Burgos, o sean 20.112 metros lineales ...", (... Kaan, mecate ... from the nahualt mecatl; measure of 24 varas de Burgos of 838 mm; each mecate is divided into twelve brazas equal to two varas de Burgos, that is 20.112 meters ...), (Motul dictionary, 1929, p. 494). The Motul dictionary also states : "hun uinic in cal, veinte Kaanes o estadales tiene mi milpa : ca uinic, quarenta estadales, ox uinic, sesenta estadales", (hun uinic in cal, my 'milpa' measures twenty Kaans or ca uinic, forty 'estadales', ox uinic, sixty 'estadales' (Motul dictionary, 1929, p. 905), which indicates that a vigesimal system of units of measure had been in use in Mesoamerica.

A problem arises then, for right at the beginning of the Conquest, there has been confusion berween the Spanish vara (braza) and the Maya vara (braza). The value of the Spanish vara given by Gibson (1964) is approximately 84 cm (which agrees with the Motul dictionary): this vara was divided into 3 feet each of approximately 28 cm. Calderon (1971) says that tha Maya braza was called "zap" and was equal to 2 varas (Maya). His research for the value of the Maya vara begins by examining 1,128 measures taken from monuments of the most important sites from the Peten region and 973 measures taken from Chichen-Itza and surroundings. The first sample results in a unit of 4.56 cm and the second in a unit of 4.55 cm. He proposes then two hypotheses.

1. Because of the vigesimal Maya numerical system, he proposes a vigesimal system of measures: that is a Maya vara of 91.2 cm which would then be divided in 20 parts. Therefore, 1 zap is equal to 1.82 cm and 1 Kaan is 21.84 m = 24 varas.

2. Following a vigesimal system,

1 kaan = 20 varas = 18.2 m

which would be a value between the two definitions in the Motul dictionary, i.e., 1 kaan = 18 varas = 15.084 m

1 kaan = 24 varas = 20.112 m

After a second statistical analysis, he concludes that 1 kaan equals 24 varas (24 x 91 cm = 21.84 m), which, because of the resemblance between Spanish and Mayan varas became 20.11 m after the Conquest.

Calderon also explains one of the statements of Morley: "... en realidad, estas cuerdas de medir son,en el norte de Yucatan, de 21.50 m de largo. Los mayas dicen que los mecates tienen que ser un poco mas largos "por lo que se llevan los pajaros"", (... in fact, these measuring cords are 21.50 m long in the North of Yucatan. The Mayas say that the mecates have to be longer "for what the birds take" (quoted in Calderon, 1971, p. 82). This last remark becomes more clear if we suppose they added one braza (1.67 m) obtaining then 21.78 m which is almost the 21.84 m (Calderon write "lo que añadieron fue una vara (1.67 m)" (what they added was a vara (1.67)), which to be correct should be one braza instead of one vara .

According to a vigesimal system,

1 kalkaan = 20 kaans = 436.8 m

1 bakaan = 400 kaans = 8,736 m

Calderon cites the exaple of the Sacbe (white road) between Coba and Yaxuna (wide 1/2 kaan and long 8 kms) to be a possible example illustrating the larger units of measure.

Calderon calls the vara, Betan, a name derived from Be (camino-path) and tan (medio para realisar algo - means to realise something). He explains the semantics of kaan and betan : "mientras el kaan, medida agraria por excellencia, se asocia con kan (madera, cosecha, maiz, oro, etc.), el betan establece una conexion con la ingenieria y las comunicaciones donde la medida primigenia (el paso) es unidad de longitud, y, a la vez, el acto mismo de translacion y de comunicacion", (while the kaan, perfect agrarian unit, is associated with kan (wood, harvest, maize, gold, etc.), the betan establishes a connection between engineering and comunications where the elementary unit (the pacing) is the unit of measure, and, at the same time, the act itself of translation and comunication), (Calderon, 1971, p. 84). His statistical analysis gave a high incidence of multiples of 45.5 cm or 1/2 betan, which lead Calderon to propose : "Probablemente se refiere al medio betan el Diccionario de Motul cuando afirma que un kaan consta de 36 'brazas de box'", (Probably the Motul dictionary refers to the half-betan when reporting that one kaan equals 36 'brazas de box'), (Calderon, 1971,p.85)

Taking the system of measure to be vigesimal, he then proposes the following measure system.

1 bakaan	=	20 kalkaans	=	8,738 m
1 kalkaan	=	20 kaans	=	436.8 m
1 kaan	=	20 betans	=	21.84 m
1 betan	=	20 azabs	=	91 cm
1 chan			=	2.275 mm

where the names azab (mediana - median) and chan (pequeno - small), are given by Calderon to the fraction 4.55cm and 2.275 mm.

Another way of trying to find units of measure would be to look at what units are used today in Native Meso-america. It reflects the influence of the Spanish conquest in that the traditional distance measures have been more or less maintained. Through these we have a better understanding of their initial notion of measuring. I have listed the distance measures in three categories: distance measured using body measurements, different values for units of measure, and distance measured in terms of time or planting. These measures are taken from the Handbook of Middle America Indians.

a) Distance Measured Using Body Measurements

The Tzeltal

Nab - distance between the end of the thumb and the middle finger extended.

Chutub - distance between the end of the thumb and the index finger extended (equivalent to the jeme). It is worth noting that the jeme and the cuarta (distance between the thumb and little finger extended) are both units fractions of the vara but only the jeme was used by the Indians.

Shucubil - distance between the elbow to the end of the middle finger extended.

Jaub (braza) - distance between the ends of both extended arms.

Yaukobal (= meter) - distance from the armpit to the extended middle finger of the opposite hand.

The Yucatan

Sap (braza) - distance between the arms extended.

Chinaab (jeme) - distance between the thumb and the index extended.

Naab (cuarta) - distance between thumb and little finger (extended arm).

Kok (coto) - the measure covered by the 4 fingers of the closed fist (not counting the thumb).

Cuc (codo) - from elbow to the end of extended hand (almost a half vara).

Vara - distance between the middle of chest to the end of arm, with hand opened.

The Mixtec

Heights of human, inanimate and animal objects are indicated respectively by extending the right hand fore-fingers with the thumb at its base and the remaining fingers touching the upturned palm; by joining the fingers of the down-turned open palm; by pronation of the open hand to the diagonal. The span of the hand apparently restricted to use by diviners, is found in the more isolated areas of Guerrero and those fringing the Trigue.

Huichol and Cora

Finger width; stretched thumb tip to outstretched middle finger; nose to tip or middle finger at outstretched arm; double arm span; lengths are compared by means of a knotted rope or a notched bamboo.

From this enumeration, some units of measure seem to have a greater incidence: the distance between the thumb and finger index outstretched (jeme), the distance between the thumb and little finger extended (cuarta), the distance between the elbow to the end of middle finger extended, the distance between the ends of both extended arms (braza), and a variety of measures (approximating the vara) denoting the distance from the extended middle finger to the sternuum (to the opposite armpit or to the nose).

b) Different Values of Units of Measure

Popoloca

League and kms are used.

Chocho

Meter and league are used.

Mazatec

Metric system.

Totonac

Vara or yard.

Codo (elbow) = 1/2 vara.

Cuarta, *jeme* and standard Spanish units used likewise for fractions of vara.

The *garrocha* and *destajo* distance measures are here good examples of the evolution of a traditional measure influenced by the introduction of Spanish units of measure and the decimal metric system. The garrocha in the Papantla zone measures traditionally 12 cuartas each of 20 cm (= 2, 4 m) and the destajo is a plot of 50 x 50 garrochas (≈ 1.5 hectares). The garrocha is today reduced to 10 cuartas and therefore the destajo to almost 1 hectare.

Tarascan

The Spanish vara is equated with the meter and if the term hectare is known, it may be measured in paces rather than meters.

c) Distance Measured in Terms of Time or Planting

The Tzeltal

League is used and is equivalent to the distance a man can walk in one hour (≈ 5 km).

Yucatan

League (lub) - distance covered by walking one hour.

The auat (grito) - equivalent to a quater-league because of the supposition that it is the distance reached by a man's shout.

Zapotec

Land is measured by:

hectare (influence of metric system);
almud - amounts of seeds used to plant an area, which is the amount of land necessary for planting an almud of seeds;
tabla - used where land is scarce and population abundant or where land is fertile and population abundant and is usually 1 to 1.5 m wide and of varying length.

Mixtec

The metric system is used in more accessible areas. Distance is reckoned by reference to a "known" such as that between 2 points (fields, trees, villages, village and river), or may be reckoned in terms of elapsed "time"; forenoon, a "day" of daylight.

Chocho

Areas are traditionally measured by the quantity of maize planted.

Tarascan

Distances are expressed most commonly in terms of travel time, but also in leagues, units or kms. Land is measured by the amount of seed sown.

2 liters of maize plant 1 hectare.

5 liters = 1 medida.

20 medidas = 1 fanega or Yunta.

1 fanega equal the amount of land that can be planted in a season with a single yoke of oxen.

Huichal and Cora

day's journey - land area is measured by the number of measures of maize seed that could be planted on it.

More can be said about the possible units of measure used by the Aztecs. Gibson (1964) lists the different distribution of sizes of Tlatocatlallis which were divided into lots. These measures reflected the use of a unit as well as a vigesimal system. In the case of Tlalmitlis, only the list of measures taken from Huilzilopochco is given in varas; all the other lists of measures are in brazas (brazas de Burgos). Gibson notices that only the lists given in brazas indicate a possible vigesimal system. Drewitt (1969) reports that a unit of measure of about 80 cm was used and that a Tzontli, equal to 400 times the basic unit, was a unit used to measure land. Length was measured according to Guerra (1960) by means of a stick called Tlacaxilantli, equivalent to "the distance from the navel to the ground in a man, which is very close to a yard. Or three Spanish royal feet" (Guerra, 1960, p. 344). Although Guerra could not conclude on the establishment of a vigesimal system, he mentions the Tlalmecaltl or string used to measure greater distances, which was equal to twenty tlacaxilantli, and also the division of Tlacaxilantli in halves and in fifths.

Harvey and Williams (1981) cite the quahuitl reported by Fernando de Alva Ixtlilxochtl as the unit of measure in Texcoco. The quahuitl equal to 3 Spanish varas or 2.5 m. They make the following comments: "un certain nombre de nos informateurs se [sont] souvenus qu'un cordon représentant une unité de mesure était autrefois conservé à l'Hotel de ville (presidencia). Et jusqu'à une date récente également la corde était vendue par rouleaux de 2.5 m, confirmant ainsi l'estimation d'Ixtlilxochitl sur l'unité de mesure linéaire de Texcoco" (Harvey and Williams, 1981, p. 1080). A unit of area of 20 x 20 linear measures is also believed to be used: although aerial photographs showed that in Tepetlaoztoc, the 400 unit was equal to 0.25 hectare or $(20 \text{ quahuitl} \times 2.5 \text{ m})^2$.

Harvey and Williams (1981) made an important discovery deciphering the COdex Santa Maria Asuncion and the Codex Vergara. They interpret these codices as being a census of households, including the names of each head of family, the milcocoli (nahualt metaphor signifying disposition of fields) registering the lots of each family, and a third part the tlehuelmantli which they suppose to represent the same lots described in the milcocoli (or more precisely, to give the area of the lots in the milcocoli).

Table 1 summarizes their results. Since the tlahuemantli is not drawn in scale, the exact form of the lots is not precisely determined. The difference between the area calculated from the data given in the milcocoli and the data given in the tlehuemantli, is due to their assuming a right angle formed by side a and side b. In some cases, they calculated the area dividing the form into two right triangles: for the irregular forms, they calculated the area of the greater rectangle included in the lot and then added the area of the remaining triangles. In addition, the milcocoli and the tlehuemantli are sometimes written by different surveyors and at different times; therefore changes due to new divisions following an inheritance or a change of owner, will account for such discrepancies in the list.

This analysis of the codices reveals a possible method of area calculation based on arithmetic, but also implies the knowledge of some geometrical terms: "Le triangle rectangle était une forme appréciée pour les calculs. Néanmoins, les Aztèques savaient calculer la surface exacte d'autres triangles, comme nous en avons la preuve dans le folio 16V du codex d'Anuncion qui donne la superficie d'un champ presque triangulaire" (Harvey and Williams, 1981, p. 1080).

The analysis of Berlin (1969) on Tzeltal numeral classifiers is here summarized because it presents a new approach using linguistics, in examining the various possible units of mesure. Berlin defines the numeral classifier as "a series of predominantly monosyllabic, bimorphemic stems of the canonical shape CV(h)C" (Berlin, 1969, p. 20), occurring in quantifying expressions such as:

specific numeral + numeral classifier ± noun

As adjectives, a numeral classifier has the function to modify the noun. But the use of numeral classifier implies semantic considerations: "one has to select a context which allows for some limited range of meanings" (Berlin, 1969, p. 14). The absence of a noun after a numeral classifier emphasizes the fact that the qualitative features are sometimes implicit to the numeral classifier. As Berlin notes, "numeral classifiers are linguistic markers which indicate certain physical attributes which a set of objects, actions of events may manifest" (Berlin, 1969, p. 20). Berlin then classes the numeral classifier (which refer primarily to specific features of physical objects) in different semantic domains showing "(1) the gross semantic distinctions among classifiers and (2) the distributional pattern of classifiers vis-à-vis nouns within each set, i.e., contrastive, complementary or mixed" (Berlin, 1969, p. 43). The semantic domain to which units of measure belong is given in Table 2. Among the various numeral classifiers listed by the author, I chose to regroup those whose semantic features imply some geometrical notions (Table 3). Conscious that it is not consistent with Berlin's analysis, this regrouping is done uniquely for the purpose of the present study.

Different Results Obtained by Measurements of Artifacts

The analysis of Cramer (1938) in order to determine a Mayan unit of measure was based on the measures taken from the Nunnery at Uxmal. The sample was reduced to 286 measures since duplicates were discarded. He assumed that "any set of measurements of this sort would be almost certain to show a more frequent occurrence of multiples of the unit than of multiples of any other quantity not commensurable with the unit" (Cramer, 1938, p. 344), where he defines two numbers to be not commensurable if "few or none of their common multiple lie within the range of the recorded measurements" (Cramer, 1938, p. 344). He then describes two methods to find a unit of measure.

Method 1

Dividing the set of measures by 1, 2, 3 ..., he suspects that any unusual frequency or a number would imply a commensurability with the unit. According to Cramer, this method showed that "a remarkable number of entries, which were almost exactly whole numbers of feet or whole numbers of inches, and this in spite of the fact that fractions of an inch were not dropped in making the measurements of the ruins" (Cramer, 1980, p. 345).

Method 2

Let M be a set of numbers (where there is no predominance of multiples of any one quantity) that is then plotted on a X-axis.

Let us choose N and plot on the X-axis the numbers kN, k = 1,2,3 ...

Let us choose n smaller than N/2 and consider the intervals $I_k = (kN-n, kN + n)$ for all k. Since points are located at random on the axis, the number of points in I_k should be approximately (2n/N)M. Cramer defines then the predicted fraction of the M numbers as p = 2n/N.

Let now M be a set of numbers which would be predominantly multiples of some unit. Cramer expects "the plotted points to show a tendency to occur in regularly spaced clusters with centers at those points which represent the multiples of the unit which was used (Cramer, 1938, p. 345). Let us count the number m of points in each I_k (defined as previous). The actual fraction of the M numbers will then be given by P = m/M. Let R = P/p. If the value of R is close to 1, its corresponding value of N should be close to the unknown unit. In this case, Cramer notes that the value of R corresponding to N is roughly the same for multiples of N.

Cramer then applies this method on the set of measures taken from Uxmal. Taking n = 1/4 inch, assuming the errors on measuring to be less than n, he arrives at the following results.

N	3/4	1	1 1/2	1 3/4	2	2 1/4	2 1/2
R	0.97	1.29	1.14	0.85	1.39	0.94	0.93
N	2 3/4	3	3 1/4	3 1/2	3 3/4	4	4 1/2
R	1.00	1.48	0.98	0.78	1.13	1.33	1.19
N	5	6	7	8	9	10	12
R	1.19	1.58	0.98	1.28	1.57	1.26	1.68

These results show that for N=2 (and its multiples) as well as for N=3 (and its multiples), the corresponding values of R are approaching 1: but since 3 is commensurable with 2 and 3 is not a multiple, this was to be expected according to Cramer. He chose 3" to be the unit by noting several repetitions of 5'0" = 60" = 20 x 3", which could be significant considering that the Maya used a vigesimal number system. He also added: "the 3" unit seems more likely than 6", 9" or 12" units, first because of its more convenient size, and second because it would be rather hard to explain the very high value of R for N=3 on any other grounds" (Cramer, 1938, p. 347).

Clearly his justification of his choice for 3" as the unit, is not convincing. Whereas this method should not necessarily be dismissed, we cannot fail to note the degree of permeability the author relies upon and which is conveyed by the use of terms such "roughly", "expect", "likely", which seem to indicate an excessive tolerance of what can only be viewed as too wide a margin of error (assuming one could actually calculate it). Under such circumstances, the method cannot be said to be conclusive. It should also be noted that Cramer reduced his set of data because he was looking for a unit smaller than a foot. This method should be tested on a larger set of data (perhaps including duplicates) for it could give a better "large" approximation.

In his studies of the plan of Teotihuacan, Drewitt (1966 and unpublished manuscript) arrived at several results. A first set of measurements on the ceremonial part of Teotihuacan, as well as on the west side of the site and on the canals, revealed the existence of a possible unit approximately equal to 57 m. Drewitt (1966) noted: "Probablemente la medida de 57 metros fue un multiplo de una medida mas basica, pero en la planeacion de distintas regiones de la ciudad se usaba una medida aproximada de 57 metros o multiplos de ésta" (Drewitt, 1966, p. 86). In the unpublished manuscript, Drewitt observed that the distance separating major streets were equal to 320 m or 640 m. He then examined the widths of groups of staircases of ceremonial complexes and found several occurrances of 16 m and 8 m. Yayahuala, Zacuala Palace and Tetitla also showed numerous instances of squares of sides equal to 64 m (Millon, Drewitt and Cowgill, 1973). A unit of 80.5 cm was then chosen after noticing that 20 x 80.5 cm = 16.1 m, and that 20 x 16.1 m = 322 m. This unit of measure becomes even more significant when recalling that "an indigenous unit of linear measure close to the Spanish vara of 83.5 or 84 centimeters was in use and also a unit 400 times this basic unit or in round figures about 330 meters (Gibson, 1964, 257 f)" (Drewitt unpublished manuscript). In addition, Drewitt (1969) gives a list of other measurements of distances, buildings and complexes, all showing multiples of 80 cm.

Kendall (1974) proposes a statistical analysis of the data set obtained from Thom's measurements on megalithic sites. Although Kendall did not conclude to have found a definitive quantum (unit of measure), he points out the need for new survey of the circular circles which will produce a new set of data (possibly larger and more accurate). Nevertheless, an average value of q = 5.44 ft can be pointed out. The analysis of Kendall emphasizes the importance of the data-set for any statistical verification of the hypothesis of a quantum. Different values can be produced if the statistical analysis is performed on different portions of the data-set. The division of the data-set could be done for various reasons: in Kendall's paper, for example, he will study the measurements according to the shape of the object measured (accurate circles, less accurate circles, "eggs"), to geographical distinctions, or if a particular subset seems to demonstrate a significant "anomaly" with respect to the rest of the data-set.

Although his main concern was whether and not how measurements were made, Kendall does not exclude the possible use of whale-bone rod, stick or human pacing. Impressed by the fact that a trained guardsman has to maintain a pace of 30 inches ± 1/2 an inch, he modifies his simulation program and arrives at the following result.

Height = 13.589 $\quad$ q = 5.441 ft $\quad$ σ = 0.674 ft

If measurements were made by pacing the error would be greater than ± 2 inches (i.e., error for a trained man on rough grass). Thus when considering pacing as quantum, Kendall notes that "'pacing' is far too accurate or rather, 'pacing' may have been used, just as a whale-bone rod may have been used, but we shall never know, because the residual errors associated with the sizes of the stones completely swamps errors of the kind associated with good pacing" (Kendall, 1974, p. 258).

In addition, Kendall's paper offers a good discussion on the error on the quantum found by statistical method. He first notices that the error should be "(a) small relative to the value of the quantum ... and (b) small in the sense that it might well have arisen from (i) errors made by the constructors; (ii) errors made by the surveyors; (iii) errors associated with the finite sizes of the stones themselves; (iv) errors associated with the re-erection of fallen stones, etc." (Kendall, 1974, p. 233). Setting

$$X' = Mq + \sigma\varepsilon$$

where $q = 5.442$ and $\sigma = 1.507$ ft (which is at the order of the dimensions of a single stone and is found in several instances in Kendall analysis and "reflects the inherent uncertainty in the location of the starting or end point of the measurements" (Kendall, 1974, p. 257)), Kendall settles for a range of uncertainty of about 1 in. to be on the average value of the quantum. Kendall clarifies what is meant by a range of uncertainty: "it has nothing to do with how far one quantum may have varied from the next, as they were successively laid out along a radius or diameter, and it has nothing to do with how far one quantum-standard in the Northwest of Scotland may have differed from another in, say, Carnac". (Kendall, 1974, p. 260). This clarification is important because misinterpretations could follow from a misunderstanding of the term.

Table I

DATA FROM THE MILCOCOLI AND THE TLEHUELMANTLI

numéro de la maisonnée et du champ	superficie Milcocoli (quahuitls²)	superficie Tlahuelmantli (quahuitls²)	différence entre les 2 superficies (quahuitls²)	pourcentage de différence	longueur des côtés a, b, c, d
correspondance exacte = 9 champs (8 %)					
HH30-2	97	97	0	0	13, 7, 13, 8
HH4-5	140	140	0	0	14, 10, 14, 10
HH30-4	160	160	0	0	16, 10, 16, 10
HH47-3	180	180	0	0	20, 9, 20, 9
HH64-1	252	252	0	0	14, 18, 14, 18
HH60-1	300	300	0	0	20, 15, 20, 15
HH63-1	346	346	0	0	24, 16, 24, 13
HH31-1	375	375	0	0	15, 25, 15, 25
HH39-7	504	504	0	0	21, 24, 21, 24
superficies Milcocoli supérieures aux Tlahuelmantli = 29 champs (26 %)					
HH29-4	289	288	1	0	30, 10, 31, 9
HH25-1	462	461	1	0	37, 13, 33, 14
HH65-5	160	158	2	1	21, 5, 23, 10
HH16-2	203	200	3	1	25, 8, 26, 8
HH53-3	407	404	3	0	29, 15, 32, 12
HH43-2	529	526	3	0	30, 22, 23, 20
HH26-2	109	103	6	6	12, 7, 12, 12
HH4-8	237	231	6	3	14, 20, 10, 20
HH24-2	127	120	7	6	10, 14, 9, 13
HH44-3	230	223	7	3	15, 14, 17, 15
HH24-3	270	263	7	3	15, 18, 16, 17
HH2-3	347	340	7	2	37, 10, 36, 9
HH17-1	391	380	11	3	19, 13, 37, 24
HH64-2	410	399	11	3	16, 21, 20, 26
HH1-4	899	900	11	1	20, 37, 28, 38
HH26-1	379	365	14	4	20, 18, 20, 20
HH2-5	476	460	16	3	20, 21, 22, 25
HH43-3	180	160	20	11	25, 8, 22, 8
HH38-5	441	420	21	5	22, 17, 23, 23
HH15-2	217	188	29	13	15, 15, 15, 14
HH3-6	184	138	46	25	20, 10, 26, 8
HH58-5	263	207	56	21	29, 8, 27, 10
HH29-5	330	232	98	30	22, 15, 22, 15
HH23-2	309	208	101	33	23, 15, 23, 12
HH41-3	249	132	117	47	19, 15, 15, 15
HH38-7	610	470	140	23	27, 23, 25, 24
HH10-2	551	392	159	29	21, 23, 23, 28
HH2-9	435	252	183	42	24, 17, 26, 18
HH12-1	900	543	357	40	39, 28, 35, 22
superficies Tlahuelmantli supérieures aux Milcocoli = 75 champs (66 %)					
HH59-3	59	60	1	2	8, 8, 7, 8
HH20-3	107	108	1	1	8, 12, 10, 12
HH32-4	152	153	1	0	13, 12, 15, 10
HH41-2	193	194	1	0	19, 10, 18, 11

Table II

SEMANTIC DOMAIN

Semantic domain 38

nouns	numeral classifiers			
	/hoht/	/p'ihs/	/pohč/	/haw/
/te?/ 'tree'	×	×		×
/laso/ 'rope'	×	×		×
/č'ahan/ 'belt'				×
/k'abal/ 'hand'			×	
/nuhkul/ 'hide'			×	

— Distribution matrix: semantic domain 38

Domain meaning: semantic domain 38
'units of linear measure'

— Componential definitions of numeral classifiers: semantic domain 38

Number of Dimensions: 1

Definition of dimension		*Value*
'measurements' (A)	:	'thumb-to-middle-finger (a_1); hand-width (a_2): out-stretched-arm-width (a_3); non-absolute length measure with some object (a_4)

Definition of terms by components

/hoht/	:	a_1
/pohč/	:	a_2
/haw/	:	a_3
/p'ihs/	:	a_4

Table III

NUMERAL CLASSIFIERS

/b'ahk'/	'items of units of four hundred' < /?/. Semantic domain 61.
/b'al/	'cylindrical shapes made of thin flexible or solid objects' < bv /b'al/ (e.g., /-b'al/ 'to make cylindrical'). Semantic domain 11.
/čehp/	'enumeration in sets of 100's < ?. Semantic domain 61.
/hahp/	'wide [in relation to /kay/ Semantic domain 15] spaces between equidistantly spaced objects' < bv /hap/ (e.g., /-hap/ 'to make equidistant spaces between objects'). Semantic domain 15.
/haht/	'objects separated by tearing in vertical halves' < bv /hat/ (e.g., /-hat/ 'to tear object into vertical halves').
/haw/	'measure from tip of middle finger on each hand with arms outstretched in maximal position < tv /-haw*/ (e.g., /-hawa/ 'to measure by arms breadth'). Semantic domain 38.
/hehp/	'halves, length exceeding diameter of item' < bv /hep/ (e.g., /-hep/ 'to cut in half'). Semantic domain 26.
/hiht'/	'individual wraps of slender flexible objects in sequential lash-loops around two pieces of long, non-flexible objects at 90° angles to one another, as in fence making' < tv /-hit'/ 'to make lash-loops'. Semantic domain 1.
/hoht/	'distance between second finger (middle finger) and thumb when maximally extended' < tv /hot*/ (e.g., -hohti 'to measure between second finger and thumb'). Semantic domain 38.

/hoy/
'human group, standing, arranged in circle' < bv /hoy/ (e.g., /-hoy/ 'to arrange in circle'). Semantic domain 29.

/hum/
'human group, standing, arranged in circle with focus (in center of circle)' < pv/hum*/ (e.g., /-human/ 'to encircle it'). Semantic domain 29.

/kah/
'rows of objects, e.g., at right angles to columns' < bv /kah/ (e.g., /-kah/ 'to form into rows'). Semantic domain 31.

/keh/
'measure between knuckle of forefinger and thumb' < pv /keh*/ (e.g., /-kehan k'ab/' 'to measure between knuckle and thumb').

/k'aht/
'oblong, non-erect, solid objects, roughly rectangular' < bv /k'at/ (e.g., /-k'at/ /to form into semi-block shape'). Semantic domain 11.

/k'ol/
'spherical solid object, relatively larger than /wol ~ nol/ [Semantic domain 11]' < bv /k'ol/ (e.g., /-k'ol/ 'to form into ball').

/len/
'flattened, tapered edges, well-formed disk-shaped objects' < bv /len' (e.g., /-len/ 'to form into disk'). Semantic domain 11.

/mahk'/
'chunks of items with severed ends displaying 90° angles < bv /mak'/ (e.g., /-mak'/ 'to cut into chunks'). Semantic domain 26.

/mil/
'enumeration by sets of 1,000's < Spanish mil 'one-thousand'.

/pik/
'enumeration by 8,000 (occurs only with numeral {hun} 'one < ?.

/pohč/
'measure, across total width of hand' < ?. Semantic domain 38.

/p'ahs/
'chunks of items severed ends displaying non-90° angles < bv /p'as/ (e.g., /-p'as/ 'to cut at non-90° angles'). Semantic domain 26.

/p'al/
'blinks of eyes' < tv /-p'al/ 'to blink eyes'.

/p'ahš/ 'blows of cutting at slanting angles, e.g., not right angles' < bv /p'aš/ (e.g., /-p'aš/ 'to cut at slanting angles'). Semantic domain 80.

/p'ihs/ 'measure liquid or length, no specification as to absolute value' < tv /-p'is/ 'to measure'. Semantic domain 38.

/sehp/ 'flattened, 90° edges, well-formed disk-like objects' < bv /sep/ (e.g., /-sep/ 'to form into disk'). Semantic domain 11.

/šoht/ 'slender, flexible objects in nearly perfect circular coils' < bv /šot/ (e.g., /-šot/ 'to form into circular rolls'). Semantic domain 18.

/soht'/ 'chunks of items with ends displaying 90° angles, pieces in situ where severation took place' < bv /šot'/ (e.g., /-šot'/ 'to cut into chunks'). Semantic domain 26.

/tab'/ 'enumeration in sets of twenty (occurs only with {hun} 'one') < ?. Semantic Domaine 61.

/tel/ 'oblong, vertically erect, solid objects, tendency to parallel sides' < bv /tel/ (e.g., /-tel/ 'to put in vertical position'). Semantic domain 11.

/tohp'/ 'chunks, diameter equal to or greater than length of item' < bv /top'/ (e.g., -top' 'to cut in half'). Semantic domain 26.

/tuh¢'/ 'oblong, non-erect, solid objects, diminishing as tear-shape in one end' < bv /tu¢'/ (e.g., /-tu¢'/ 'to form into tear-shape (as certain tamales'). Semantic domain 11.

/wol/ 'spherical, solid object, relatively smaller than /k'ol/ [Semantic domaine 11]' < bv /wol/ (e.g., /-wol/ 'to form into sphere'). Semantic domain 11.

/wuy/ 'objects broken in half' < bv /wuy/ (e.g., /-wuy/ 'to break in half').

CHAPTER 5

SUMMARY AND COMMENTS

In Chapter 1, I examined the procedure used to analyze the geometrical patterns found on pottery, paintings and woven material. Among the analyses of regularized patterns, I described in detail Washburn's method, since her distinctions between basic and design units and between horizontal and vertical displacements are clearly indicated in her notation. This improves upon Loeb's and Shubnikov's system of classification. Equivalences between Washburn's, Loeb's and Shubnikov's classes are established and presented in tables. In the process of working with Washburn's classification, I discovered two new classes ($2\text{-}2_z 1_z 1$ and $2\text{-}2_z 1 2_z$), that do not appear in her own list and which I conclude should be included if her classification is to be complete.

Washburn's method is not only very clear and concise for the classification of geometrical designs, but also very useful for comparative studies, although it would be desireable that it be supplemented by other studies (political, social, symbolic, chemical analysis), before permitting anthropological conclusions concerning the interractions of tribes. On the other hand, Crowe's study of the geometrical patterns of the Bakuba and Benin, Alcina Franch's study of the "Pintaderas" of Mexico, and Field's study of Mexican stamp designs could have benefitted from Washburn's method.

In Chapter 2, it was noted that analyses of Teotihuacan paintings and of the twin stelae at Seibal demonstrated the possibility of a painting or sculpture technique making use of patterns or stencils. In addition, this first part of the chapter provides evidence that the space allotted to painting and carving was planned.

Researches into compositional structures are then presented in the second part of the chapter. Particular atten tion was given to Clancy's study of the Tikal Stelae and Hébert-Stevens' analyses of monuments and of Chavin Stelae.

Clancy's analysis proposes a compositional structure derived from the intersection and over-lapping of Pythagorean and isosceles triangles. Although her criteria should be revised, her analysis could prove to be of value in establishing the Mesoamerican knowledge of Pythagorean triangles.

In the light of the evidence presented in his study of the monuments and of Chavin Stelae, Hébert-Stevens seems to be imposing knowledge rather than extracting it. Moreover, studies such as those of Hébert-Stevens, Harleston and Diaz-Bolio, fail to take account of errors due to the present state of paintings and carved monuments. The effects of erosion or discoloration must be evaluated in establishing a tolerable margin of error for such investigations.

In Chapter 3, I discussed the evidence of geometrical concepts as revealed by Mesoamerican site plans. An examination of the map of Tikal showed the existence of isosceles triangles, right angles and parallel lines. In order to explain or justify the non-coincidental nature of the geometrical concepts displayed by the disposition of monuments, previous studies in the fieldsof archaeoastronomy, geomagnetism, ritual and symbolism, geomancy and hierophanies were presented.

Problems inherent in those studies are also considered. For instance, the possibility of determining one day the secular variation in time and location of the magnetic inclination will shed light on problems such as the similarity of orientation of buildings and the transfer of orientation. It will also provide supplementaty information to support the hypothesis of a geomantic model which, in turn, could be fruitful in further studies of a Mesoamerican geometrical knowledge.

I next focused on the literature dealing with the site plan of Teotihuacan to survey the numerous and diverse approaches used by investigators in different fields in examining orientations at the site. This demonstrates the complexity of the problem and shows the variety of proposed solutions found in the literature.

Chapter 4 attempted to study a specific geometrical concept, namely that of a unit of measure. I began by examining some ethnohistorical sources which confirmed the fact that land was measured in Pre-Columbian times. As well, I studied Harvey and Williams analysis of the Mexican codices which reveal the existence of a possible Mesoamerican method of area calculation based on arithmetic.

I next presented a list of the units of measure still in use today by native communities in the Mesoamerican region. Berlin's linguistic analysis on Tzeltal numeral classifiers was also considered as a useful source for investigating the nature of units of measure in Mesoamerica.

The second part of the chapter examined certain studies that propose precise units of measure. On this point, Drewitt's field survey of Teotihuacan, reveals the possible existence of a unit of measure equal to approximately 80.5 cm. In addition, Cramer gave a statistical study which proposed a unit of measure equal to 3". I found the latter study to be based on less convincing arguments.

Kendall's statistical analysis based on Thom's megalithic data, was also discussed. This raises important points that should be taken into account in future statistical approaches to the search for units of measure.

APPENDICE I

OBTENTION DES CLASSES $2\text{-}2_2 1_2 1$ ET $2\text{-}2_2 1 1_2$

Considérons la table des classes bi-dimensionnelles monochromes dont le nombre total se trouve augmenté dans la classification de D.K. Washburn vue sa distinction entre les déplacements horizontaux et verticaux. Afin d'établir la liste des classes bi-dimensionnelles contrechangées de D.K. Washburn, j'ai examiné une à une les possibilités de contrechange de chaque classe; contrechange à l'intérieur du module ainsi que contrechange entre deux modules, en prenant b comme figure de base et $\begin{matrix} d & b \\ q & p \end{matrix}$ comme module. Lors de cette verification de la liste de D.K. Washburn, j'ai obtenu deux classes supplémentaires (la classe $2\text{-}2_2 1_2 1$ et sa correspondance $2\text{-}2_2 1 1_2$) de la manière suivante.

Considérons la classe 2-211. Le procédé d'élimination commence en examinant les diverses possibilités de contrechange sur la rotation.

1. avec contrechange autour de l'axe des y (c'est-à-dire, un sous-indice 1 sous le 2 indiquant la rotation);
2. avec contrechange autour de l'axe des x (c'est-à-dire un sous-indice 2);

3. avec contrechange autour de l'axe de x et de l'axe y (c'est-à-dire un sous-indice 3);

4. sans contrechange autour de l'axe de x ni autour de l'axe des y (c'est-à-dire aucun sous-indice sous le 2 indiquant la rotation).

<u>1er cas</u>: Avec contrechange autour de l'axe des y.

(a) Sans contrechange sur la rotation à l'intérieur du module.

```
      d b  d b
      q p  q p
(a)
      d b  d b
      q p  q p
```

a_1 - Sans contrechange sur la réflexion horizontale à l'intérieur du module.

```
      d b  d b
      q p  q p
a1                  =>2-2₁1₂1
      d b  d b
      q p  q p
```

a_1: $\Rightarrow 2-2_1 1_2 1$

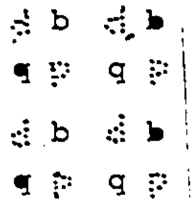
a_2 - Avec contrechange sur la réflexion horizontale entre modules.

```
      d b  d b
      q p  q p
a2                  =>2-2₁1²1²₂
      d b  d b
      q p  q p
```

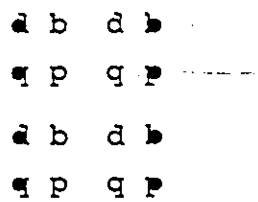
a_2: $\Rightarrow 2-2_1 1^2 1^2_2$

(b) Avec contrechange sur la rotation à l'intérieur du module.

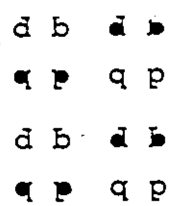
du module.

(b) 

b_1 - Sans contrechange sur la réflexion horizontale à l'intérieur du module.

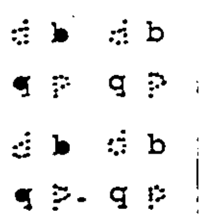
b_1  $\Rightarrow 2-2^2_1 1 1^2$

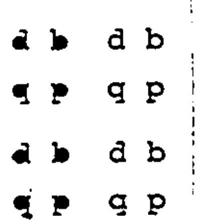
b_2 - Avec contrechange sur la réflexion horizontale entre modules.

b_2  $\Rightarrow 2-2^2_1 1^2_2 1_2$

Les trois autres cas peuvent être considérés de manière semblable.

<u>2ème cas:</u> Avec contrechange autour de l'axe des x.

(a) 

a_1  $\Rightarrow 2-2_2 1 1_2$

a_2

d b d b
q p q p
d b d b
q p q p

$\Rightarrow 2-2_2 1_2^2 1^2$

(b)

d b d b
q p q p
d b d b
q p q p

b_1

d b d b
q p q p
d b d b
q p q p

$\Rightarrow 2-2_2^2 1_2 1_2^2$

b_2

d b d b
q p q p
d b d b
q p q p

$\Rightarrow 2-2_2^2 1^2 1$

<u>3ème cas:</u> Avec contrechange autour de l'axe de x et de l'axe des y.

(a)

d b d b
q p q p
d b d b
q p q p

	d b **d b**		
	q p **q p**		
a_1	**d b** d b	$\Rightarrow 2\text{-}2_3 1_2 1_2$	
	q p q p		

	d b d **b**		
	q **p** **q** p		
a_2	d **b** **d** b	$\Rightarrow 2\text{-}2_3 1^2 1^2$	
	q p q **p**		

	d b d b		
	q p **q** p		
(b)	d b d b		
	q p **q** p		

	d b **d** b		
	q p **q** p		
b_1	**d** b **d** b	$\Rightarrow 2\text{-}2^2_3 1 1^2_2$	
	q p **q** p		

	d b d b		
	q p **q p**		
b_2	d b d b	$\Rightarrow 2\text{-}2^2_3 1^2_2 1$	
	q p **q p**		

<u>4ème cas:</u> sans contrechange autour de l'axe des x ni autour de l'axe des y.

(a)

d b d b
q p q p
d b d b
q p q p

a_1

d b d b
q p q p
d b d b
q p q p

$\Rightarrow 2-211$

a_2

d b d b
q p q p
d b d b
q p q p

$\Rightarrow 2-21_2^2 1_2^2$

(b)

d b d b
q p q p
d b d b
q p q p

b_1

d b d b
q p q p
d b d b
q p q p

$\Rightarrow 2-2^2 1_2 1^2$

b_2

d b d b
q p q p
d b d b
q p q p

$\Rightarrow 2-2^2 1^2 1_2$

La classification de D.K. Washburn est établie en tenant compte de la différence entre déplacements horizontaux et verticaux, mais aussi de la définition du module. Un fois la distinction du module faite, on peut alors noter le contrechange à l'intérieur du module et entre modules, et ainsi classer la motif étudié. Un changement de module peut entraîner une classification différente du motif. L'importance de la distinction du module est illustrée par l'exemple ci-dessous.

Considérons le motif suivant:

$$\begin{array}{cccccc}
d & \mathbf{b} & d & \mathbf{b} & d & \mathbf{b} \\
\mathbf{q} & p & \mathbf{q} & p & \mathbf{q} & p \\
\mathbf{d} & b & \mathbf{d} & b & \mathbf{d} & b \\
q & \mathbf{p} & q & \mathbf{p} & q & \mathbf{p} \\
d & \mathbf{b} & d & \mathbf{b} & d & \mathbf{b} \\
\mathbf{q} & p & \mathbf{q} & p & \mathbf{q} & p
\end{array}$$

Si $\begin{smallmatrix} d & b \\ q & p \end{smallmatrix}$ ou $\begin{smallmatrix} b & d \\ p & q \end{smallmatrix}$ est pris comme module, le motif appartient à la classe $2\text{-}2_1 1^2 1^2_2$. Par contre, si $\begin{smallmatrix} q & p \\ d & b \end{smallmatrix}$ ou $\begin{smallmatrix} p & q \\ b & d \end{smallmatrix}$ est pris comme module, le motif appartient alors à la classe $2\text{-}2^2_2 1_2 1^2_2$. Et ces deux classes $2\text{-}2_1 1^2 1^2_2$ et $2\text{-}2^2_2 1_2 1^2_2$ sont deux classes distinctes dans la classification de D.K. Washburn.

$$\begin{array}{cccccc}
\mathbf{d} & \mathbf{b} & \mathbf{d} & \mathbf{b} & \mathbf{d} & \mathbf{b} \\
\mathbf{q} & \mathbf{p} & \mathbf{q} & \mathbf{p} & \mathbf{q} & \mathbf{p} \\
d & b & d & b & d & b \\
q & p & q & p & q & p \\
\mathbf{d} & \mathbf{b} & \mathbf{d} & \mathbf{b} & \mathbf{d} & \mathbf{b} \\
\mathbf{q} & \mathbf{p} & \mathbf{q} & \mathbf{p} & \mathbf{q} & \mathbf{p}
\end{array}$$

Considérons maintenant le motif ci-dessus appartenant à la classe $2\text{-}2_1 1_2 1$, si $\begin{matrix} d & b \\ q & p \end{matrix}$ ou $\begin{matrix} b & d \\ p & q \end{matrix}$ est pris comme module, ou appartenant à la classe $2\text{-}2_2^2 1^2 1$, si $\begin{matrix} q & p \\ d & b \end{matrix}$ ou $\begin{matrix} p & q \\ b & d \end{matrix}$ est pris comme module.

Pour être cohérent avec les définitions de D.K.Washburn et son système de notation, la classe $2\text{-}2_1 1_2 1$ devrait être différente de la classe $2\text{-}2_2^2 1^2 1$. Par un argument semblable, la classe $2\text{-}2_2 1 1_2$ devrait être différente de la classe $2\text{-}2_2^2 1 1^2$ et donc s'ajouter avec sa correspondante $2\text{-}2_1 1_2 1$ aux 94 classes de D.K.Washburn.

BIBLIOGRAPHY

Alcina Franch, José (1958). Las "Pintaderas" mejicanas y sus relaciones. Consejo superior de investigaciones cientificas. Instituto "Gonzalo Fernandex de Oviedo", Madrid.

Arochi, Luis E. (1977). La piramide de Kukulcan, su simbolismo solar. Editorial Orion, Mexico.

Aveni, A.F. (1975). Possible Astronomical Orientation in Ancient Mesoamerica. In Archaeoastronomy in Pre-Columbian America, ed. by A.F. Aveni, University of Texas Press, Austin: pp. 163-190.

____________ (1977). Concepts of Positional Astronomy Employed in Ancient Mesoamerican Architecture. In Native American Astronomy, ed. by A.F. Aveni, University of Texas Press, Austin: pp. 3-19.

____________ (1980). Skywatchers of Ancient Mexico. University of Texas Press, Austin and London.

____________ (1981). Archaeoastronomy in the Maya Région: A Review of the Past Decade. Archaeoastronomy No. 3, J.H.A. xii: pp. S1-S16.

Aveni, A.F. and Gibbs, S. (1976). On the Orientation of Precolumbian Buildings in Central Mexico. American Antiquity 41: pp. 510-517.

Aveni, A.F., Gibbs, S.L. and Hartung, H. (1975). The Caracol Tower at Chichen Itza: An Ancient Astronomical Observatory? Science 188: pp. 977-985.

Aveni, A.F. and Hartung, H. (1978). Los observatorios astronomicos en Chichen Itza, Mayapan y Paalmul. Boletin de la Escuela de Ciencias Autiopologicos de la Universidad de Yucatan, Vol. 6, No. 32: pp. 2-13.

________________________ (1981). The Observation of the Sun at the Time of Passage through the Zenith in Mesoamerica. Archaeoastronomy, No. 3, J.H.A. xii, pp. S52-S70.

________________________ (1982). Precision in the Layout of Maya Architecture. Annals of the New York Academy of Sciences: pp. 63-80.

A. Aveni, B. Buckingham and H. Hartung (1978). The Pecked Cross Symbol in Ancient Mesoamerica. Science 202: pp. 267-279.

Aveni, A., Hartung, H. and Kelley, J.C. (1982). Alta Vista (Chalchihuites), Astronomical Implications of a Mesoamerican Ceremonial Outpost at the Tropic of Cancer. American Antiquity, Vol. 47, No. 2: pp. 316-335.

Aveni, A. and Linsley, R.M. (1972). Mound J, monte Alban: Possible Astronomical Orientation. American Antiquity 37: pp. 528-531.

Berlin, Brent (1968). Tzeltal Numeral Classifiers. Mouton - The Hague - Paris.

Calderon, Hector M. (1971). La ciencia matematica de los Mayas, Mexico.

Carlson, John B. (1975). Lodestone Compass: Chinese of Olmec Primacy? Science 189: pp. 753-760.

Carlson, John (1976). A Geomantic Model for the Interpretation of Mesoamerican Sites: An Essay in Cross-Cultural Comparison. In Mesoamerican Sites and World-Views, ed. by E.P. Benson Dumbarton Oaks, Washington, D.C.: pp. 143-211.

____________ (1977). The Case for Geomagnetic Alignments of Precolumbian Mesoamerican Sites - The Maya. Katunob, Vol. X, No. 2, University of Northern Colorado, Greeley: pp. 67-88.

Chiu, B.C. and Morrison, Philip (1980). Astronomical origin of the Offset Grid at Teotihuacan. Archaeoastronomy, No. 2, J.H.A. xi: pp. S55-S64.

Clancy, Flora (1977). Relief Carved Monuments at Tikal: A Possible Geometry. Unpublished manuscript, Colgate University.

Coxeter, H.S.M. and Moser, W.O.J. (1957). Generators and Relations for Discrete Groups. Ergebmisse der mathematik und ihrer Grenzgebiete 14, Berlin (Springer).

Coxeter, H.S.M. (1961). Introduction to Geometry. John Wiley and Sons, New York.

Cramer, G.F. (1938). Determination of a Mayan Unit of Linear Measurement. America Mathematical Monthly, Vol. 45: pp. 344-347.

Crowe, D.W. (1971). The Geometry of African Art. I. The Bakuba Art. Journal of Geometry 1: pp. 169-182.

___________ (1975). The Geometry of African Art. II. A Catalog of Benin Patterns. Historia Mathematica 2: pp. 253-271.

Diaz-Bolio, José. (1975). La Geometria de los Mayas et el arte crotalico.

Drewitt, Bruce (1966). Planeacion en la antiqua ciudad de Teotihuacan. Onceava Mesa Redonda, Mexico: pp. 79-94.

___________ (1969). Data Bearing on Urban Planning at Teotihuacan. AAA Meeting.

___________ (19). Measurement Units and Building Axes at Teotihuacan. Unpublished manuscript, University of Toronto.

Edmonson, Munro S. (1971). The Book of Counsel, The Popol Vuh of the Qviche Maya of Guatemala. Middle American Research Institute, publication 35. Tulane University, New Orleans.

Fettweis, M. (1973). Prospections archaeologiques sur la cote du Quintana Roo, Mexique 1970. Contributions à l'étude de l'architecture et de la peinture murale. Thèse présentée à l'Université de Louvain, Belgique.

Fettweis-Vienot, Martine (1980). Las pinturas murales de Coba. Boletin de la escuela de Ciencias Antropologicas de la Universidad de Yucatan, No. 40, pp. 1- .

Field, Frederick W. (1974). Pre-Hispanic Mexican Stamp Designs. Dover Publications, Inc., New York.

Fuson, Robert H. (1969). The Orientation of Mayan Ceremonial Centers. Annals of the Association of the American Geographers 3: pp. 494-511.

Gergely, Tomas E. (1980). Critique sur "La piramide de Kuculcan, su simbolismo solar", de Luis E. Arochi. Archaeoastronomy, Vol. III, No. 2, pp. 39-40.

Gibson, Charles (1964). The Aztecs Under Spanish Rule: A History of the Indians of the Valley of Mexico, 1519-1810. Standord, California: Stanford University Press.

Girard, Raphël (1972). Le Popol-vuh: histoire culturelle des maya-quichés. Payot, Paris.

Goetz, D. and Morley, S.G. (1950). Popul Vuh: the Sacred Book of the Ancient Quiche Maya. University of Oklahoma Press.

Guerra, Francisco (1960). Weights and Measures in Pre-Columbian America. Journal of the History of Medicine and Allied Science, Vol. 15: pp. 342-344.

______________ (1969). Aztec Science and Technology. History of Science, Vol. 8: pp. 32-52.

Hartung, Horst (1971). Die zeremonialzentren der Maya: Ein beigrag zür untersuchung der planungsprinzipen. Graz: Akad. Druck-u Verlag.

______________ (1975). A Scheme of Probable Astronomical Projections in Mesoamerican Architecture. In Archaeoastronomy in Pre-Columbian America, ed. by A.F. Aveni, University of Texas Press, Austin: pp. 191-204.

______________ (1977). Ancient Maya Architecture and Planning: Possibilities and Limitations for Astronomical Studies. In Native American Astronomy, ed. by A.F. Aveni, University of Texas Press, Austin: pp. 111-129.

Harvey, H.R. and Williams, B. (1981). L'arithmétrique aztêque. La Recherche, Vol. 12, No. 126: pp. 1068-1081.

Hébert-Stevens, François (1972). L'art ancien de l'Amérique du Sud. Arthaud, Paris.

Heyden, Doris (1975). An Interpretation of the Cave Underneath the Pyramid of the Sun in Teotihuacan, Mexico. American Antiquity. 40: pp. 131-146.

______________ (1976). Caves, Gods and Myths: World-View and Planning in Teotihuacan. In Mesoamerican Sites and World-Views, ed. by E.P. Benson Dumbarton Oaks, Washington, D.C.: pp. 1-35.

Kendall, D.G. (1974). Hunting quanta. Philosophical Transactions of the Royal Society, London, 276: pp. 231-266.

Klein, C.F. (1980). Indigenous American Architecture: The Symbolism of Circular Structures. SAH Annual Meeting, Madison, Wisconsin: pp. 11-12.

Lamb, Weldon (1980). The Sun, Moon and Venus at Uxmal. American Antiquity, Vol. 45, No. 1: pp. 79-86.

Loeb, Arthur L. (1971). Color and Symmetry. Wiley-Interscience, New York.

Leon Portilla, Miguel (1973). Time and Reality in the Thought of the Maya. Beacon Press, Boston.

Malmstrom, V.H. (1973). Origins of the Mesoamerican 260-Day Calendar. Science 181: pp. 939-940.

Marcus, Joyce (1975). Territorial Organisation of the Lowland Classic Maya. Science 180: pp. 911-916.

Martinez Hernandez, Juan (1929). Diccionario de Motul, Maya espanol atribuido a Fray Antonio de Ciudad Real y Arte de lengua Maya por Fray Juan Coronel. Tailenes de la compania tipografica Yucateca, S.A., Merida, Mexico.

Míller, Arthur G. (1973). The Mural Paintings of Teotihuacan. Dumbarton Oaks, Trustees for Harvard University, Washington, D.C.

Millon, René (1980). Handbook of Middle American Indians.

Rodriguez, A. (1969). A History of Mexican Mural Painting. G.P. Putnam's Sons, New York.

Rouse, Irving (1960). The Classification of Artifacts in Archaeology. American Antiquity, Vol. 25, No. 3: pp. 313-322.

Roys, Ralph L. (1967). The Book of Chilam Balam of Chumayel. University of Oklahoma Press, Norman.

Sanders, Frank J. (1977). The "twin stelae" of Seibal. American Antiquity, Vol. 42, No. 1: pp. 78-86.

Schele, L. (1977). Palenque: The House of the Dying Sun. In Native American Astronomy, ed. by A.F. Aveni, Austin University of Texas Press: pp. 42-56.

Seidenberg, A. (1981). Ritual Origin of Circle and Square. Archive for the History of Exact Sciences 25, No. 4: pp. 270-327.

Shepard, Anna O. (1948). The Symmetry of Abstract Design, with Special Reference to Ceramic Decoration. Carnegie Institution of Washington, Contributions to American Anthropology and History, Publication 574.

________________ (1974). Ceramics for the Archaeologist. Carnegie Institution of Washington, Publication 609, Washington, D.C.

Shubnikov, A.V. and Belov, N.V. (1964). Colored Symmetry. Ed. by T. Holster, New York, MacMillan.

Shubnikov, A.V. and Koptsik, V.A. (1974). Symmetry in Science and Art. Ed. by David Harker, New York, Plenum Press.

Sidrys, Raymond (1978). Megalithic Architecture and Sculpture of the Ancient Maya. In Papers on the Economy and Architecture of the Ancient Maya, ed. by R. Sidrys: pp. 155-183, Monograph 8, Institute of Archaeology, University of California, Los Angeles.

Taladoire, Eric (1979). Orientation of Ball Courts in Mesoamerica. Archaeoastronomy, Vol. II, No. 4: pp. 12-13.

Thompson, J. Eric S. (1970). The Rise and Fall of Maya Civilization. Norman, University of Oklahoma Press.

Tompkins, P. (1976). Mysteries of the Mexican Pyramids. Harper and Row.

Washburn, Dorothy K. (1977). A Symmetry Analysis of Upper Gila Area Ceramic Design. Peabody Museum of Archaeology and Ethnology, Harvard University, Vol. 68.