Essays on Monetary Policy Impacts on Wage Inequality

by

Koami Dzigbodi Amegble

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Department of Economics
Faculty of Social Sciences
University of Ottawa

Abstract

Chapter 1 —This paper examines the impact of non-systematic monetary policy on wage inequality in Canada, using two key measures: the wage Gini index, which captures overall wage dispersion, and the wage ratio, which reflects disparities between high- and low-skilled workers. We obtain a non-systematic monetary policy innovations measure from an estimated standard Taylor rule. To estimate cumulative impulse response functions over the short and medium terms, we employ a smooth local projection method. Controlling for lagged oil price returns, employment rate, and fiscal policy variables, we find no significant effects on the wage Gini coefficient. On the other hand, contractionary monetary policy shocks widen the skill-based wage gap (over the two-year horizon), and expansionary monetary policy shocks narrow it (over both the one- and two-year horizons). These findings highlight the nuanced role of monetary policy in shaping wage dynamics and equitable labour market outcomes.

Chapter 2 —This paper proposes a novel approach for comparing impulse response functions across groups of countries, regions, or monetary zones. Our test statistics are Wald-type statistics with a weighting matrix that accounts for cross-sectional dependency. Instead of using HAC correction for the time dependency, we propose a non-overlapping block bootstrap procedure for the test. We assess the properties of the test by simulation. Our main findings are: (1) our proposed test has good size and power properties in finite samples; (2) the χ^2 approximation does not suffice to control size; and (3) the test performs well for assessing short-, medium-, and long-term impacts, both non-cumulative and cumulative impacts.

Chapter 3 —This paper examines the heterogeneous effects of non-systematic monetary policy on wage inequality across Canadian provinces. Using two measures of wage inequality — the Gini coefficient and the wage ratio between high- and low-skilled workers — and a monetary policy innovation series derived from a Taylor rule framework, our smooth local projection based analysis reveals that the effects of non-systematic monetary policy on wage inequality vary significantly across regions. In particular, regional heterogeneity is more pronounced when we consider the entire wage distribution. Moreover, these effects are asymmetric: expansionary monetary policy shocks lead to more heterogeneous regional responses in wage inequality than contractionary monetary policy shocks.

Declaration of Authorship

I, Koami Dzigbodi Amegble, declare that this thesis titled, “Essays in Monetary Policy Shock and Wage Inequality” and the work presented in it are my own.

Chapter 1 of this thesis was done jointly with Maral Kichian. My contribution is equal to hers.

Chapter 2 of this thesis is solo authored by myself.

Chapter 3 of this thesis was done jointly with Maral Kichian. My contribution is equal to hers.

I confirm that:

- This work was done wholly or mainly while in candidature for a research degree at this University.
- Where any part of this thesis has previously been submitted for a degree or any other qualification at this University or any other institution, this has been clearly stated.
- Where I have consulted the published work of others, this is always clearly attributed.
- Where I have quoted from the work of others, the source is always given. With the exception of such quotations, this thesis is entirely my own work.
- I have acknowledged all main sources of help.
- Where the thesis is based on work done by myself jointly with others, I have made clear exactly what was done by others and what I have contributed myself.

Signed: ______________________________

Date: August 8, 2025

General Introduction

This dissertation, structured as three distinct chapters, investigates the impact of non-systematic monetary policy on wage inequality in Canada, with particular emphasis on their heterogeneous effects. The first chapter focuses on this issue at the national level. The second chapter introduces a novel impulse response function (IRF)-based statistical test to assess the heterogeneous effects across several observational units, employing a simulation-based approach. The third chapter applies the IRF-based test developed in Chapter 2 to explore regional heterogeneity in the effects of non-systematic monetary policy on wage inequality across Canadian provinces. Collectively, these chapters offer empirical insights and methodological contributions to the literature on the distributional consequences of non-systematic monetary policy in Canada. This dissertation contributes to several strands of the economic literature on monetary policy and wage inequality by underscoring the importance of accounting for regional disparities when evaluating the distributional effects of non-systematic monetary policy.

In the first chapter, my coauthor and I examine the impact of non-systematic monetary policy shocks on wage inequality in Canada. We first compute a monetary policy innovations measure using a standard Taylor rule following the strategy used in Samarina and Nguyen (2024). More specifically, we apply the instrumental regression method within the Taylor rule model to compute regression residuals, which serve as our innovation series. We then use monthly wage data from the Canadian Labour Force Survey (LFS) to construct two measures of wage inequality: the wage Gini coefficient and the wage ratio. We compute the wage ratio as in Dolado et al. (2021), which is the ratio of wages for high-skilled workers to those for low-skilled workers, where the skill level is determined based on the worker's education level. Finally, we incorporate the smooth local projection method developed by Barnichon and Brownlees (2019) into the local projection framework of Furceri et al. (2018) to analyze the monthly effects of non-systematic monetary policy on wage inequality in Canada, with a notable focus on possible asymmetric effects. We control for external influences, such as oil price and adjustments in the cost of living due to strikes and fiscal policy, to ensure the exogeneity of our innovation series with respect to the smooth local projection (SLP) equation disturbance. Given this exogeneity, the impacts of non-systematic monetary policy shocks on wage inequality can be obtained by examining the impacts of the residualized shock from the LP (or, the SLP in our case), where the latter is the residual from an OLS regression of our Taylor rule-derived innovations series on all the included control variables in the LP (or, SLP); (see Montiel Olea et al., 2025). These contributions collectively enhance our understanding of the complex interactions between macroeconomic policies and wage distribution.

We find that non-systematic monetary policy shocks have a significant effect on wage inequality over one- and two-year horizons, and that they affect the Gini wage coefficient and the wage ratio in distinct ways. Using our baseline specification, and whether contractionary or expansionary, they do not significantly affect the wage ratio, but they do affect (symmetrically) the Gini coefficient. However, after controlling for lagged oil price returns, employment rate, and fiscal policy, we find that significant effects are no longer observed on the wage Gini coefficient. On the other hand, contractionary monetary policy shocks do widen the skill-based wage gap (over the two-year horizon), and expansionary monetary policy shocks narrow it (over both the one- and two-year horizons).

In the second chapter, I develop a novel impulse response function-based statistical test to examine the heterogeneous effects of non-systematic monetary policy on wage inequality. The test is developed in the context of a system of local projections across multiple horizons and several cross-sectional units. Building on the insight of Engle and Mezrich (1996), I first begin by separating time-invariant parameters from dynamic ones in time series models, a technique further refined by Noureldin et al. (2014). Time-invariant parameters encapsulate the stable, underlying structures of the data — features that do not change over time — while dynamic parameters capture the evolving, time-dependent behaviours. To address the issue of invariant heteroskedasticity across observational units, I introduce a novel correction to the variance-covariance matrix of the OLS estimators. This adjustment not only accounts for heteroskedasticity but also incorporates time-invariant cross-sectional dependencies, thereby enhancing the model's robustness. I then use the proposed correction within a Wald-type statistic to test the homogeneity hypothesis. However, this correction alone does not suffice to standardize the statistic; to address this issue, I apply a non-overlapping block bootstrap, which allows for accounting for unmodeled time dependencies of a general form (Kang et al., 2021; Leśkow and Molenda, 2013; Härdle et al., 2003).

I assess the properties of the test by simulation, and I find that: (1) the proposed test has good size and power properties in finite samples; (2) the χ^2 approximation does not suffice to control size; and (3) the test performs well for assessing short-, medium-, and long-term impacts, both non-cumulative and cumulative.[1]

In the third chapter, my coauthor and I examine the possible heterogeneous impacts of non-systematic monetary policy on wage inequality across the Canadian provinces by using the IRF comparison-based test developed in chapter 2, and by employing the monetary innovation series constructed in chapter 1. More precisely, we obtain our innovations measure based on the Taylor rule, and we include it in a smooth local projection equation that describes wage inequality (Barnichon and Brownlees, 2019). As in Chapter 1, we ensure the exogeneity of our innovation series by controlling for various influences, including oil price returns, strike-related cost-of-living adjustments, labour market dynamics (employment rate), and fiscal policy. We then assess both non-cumulative and cumulative non-systematic monetary policy impacts, as in Chapter 1, by examining the impacts of the SLP residualized shocks. The cumulative IRFs are computed using the approach in Furceri et al. (2018) and Jordà et al. (2019). We also adapt our impulse response function comparison-based test to the context of the smooth local projection context to estimate non-cumulative IRFs (Jordà, 2005).

We find that the impact on the entire wage distribution exhibits greater regional heterogeneity across provinces, while the effects on high- to low-skilled workers, who represent particular portions of that distribution, are more uniform. We also find that the regional asymmetric effects depend on the type of monetary shock: expansionary monetary policy shocks lead to more heterogeneous regional responses in wage inequality than contractionary

[1] The test is implemented using the *lpregtestpkg* R package, which I developed. It is available upon request.

monetary policy shocks. Furthermore, our results likely indicate that structural characteristics may help explain regional divergence in wage inequality responses to monetary policy. Finally, cumulative IRFs more effectively reveal regional heterogeneity than their non-cumulative counterparts.

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To my Mom: ***Tossou Amivi***
because I owe it all to you.

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Chapter 1

Monetary Policy Shocks and Inequality in Canada

By Koami Amegble and Maral Kichian

Abstract

This paper examines the impact of non-systematic monetary policy on wage inequality in Canada, using two key measures: the wage Gini index, which captures overall wage dispersion, and the wage ratio, which reflects disparities between high- and low-skilled workers. A primary non-systematic monetary innovation series is constructed from an estimated standard Taylor rule. We then include this series in a smooth local projection (SLP) that describes wage inequality. Assuming that, conditional on the control variables in the SLP including lagged oil price returns, employment rate, and fiscal policy variables, our innovation series is exogenous with respect to the SLP disturbance term, we find no significant effects on the wage Gini coefficient. On the other hand, contractionary monetary policy shocks widen the skill-based wage gap (over the two-year horizon), and expansionary monetary policy shocks narrow it (over both the one- and two-year horizons). These findings highlight the nuanced role of non-systematic monetary policy in shaping wage dynamics and equitable labour market outcomes.

1.1 Introduction

Wage inequality has been a persistent and growing concern in many advanced economies, including in Canada (Johnson and Papageorgiou, 2020; Connors et al., 2020; Hoffmann et al., 2020). Various factors, such as technological, cultural, economic and financial, have been suggested as potential drivers (de Haan and Sturm, 2017; Antonelli and Gehringer, 2017; Wahiba and Dina, 2023). Monetary policy has received considerable attention in the more recent literature (Bilbiie, 2024; Afonso and Gil, 2024; Amberg et al., 2022). Central banks are not explicitly mandated to control for wage inequality. However, they can adversely affect wage inequality when using monetary policy to influence economic activity through stabilization mechanisms such as changes in interest rates, money supply, and credit availability.

This study investigates the effects of non-systematic monetary policy — defined as the unexpected or discretionary deviation from the Bank of Canada's policy framework (Christiano et al., 1999; Ramey, 2016) — on wage inequality in Canada. The Bank of Canada plays a crucial role in Canada's economic dynamics through its monetary policy framework, which includes inflation targeting and a flexible exchange rate. The literature (reviewed below) on the effects of non-systematic monetary policy and inequality in Canada remains scarce.

Identifying monetary policy shocks remains a challenge. Empirical studies examining the impact of monetary policy shocks on wage inequality in other advanced economies employ various identification methods. These include high-frequency identification (Faia and Pezone, 2024; Andreas Gulyas, 2024), standard Taylor rule and Structural Vector Autoregression (SVAR)-based identification (Samarina and Nguyen, 2024; Mumtaz and Theophilopoulou, 2017), and narrative identification (Villarreal and Kronick, 2020; Furceri et al., 2019; Coibion et al., 2017). For example, Faia and Pezone (2024) use high-frequency changes in European Central Bank (ECB) policy rates around policy announcement dates to identify monetary policy shocks and examine their effects on wage dispersion in Italy. They find that wage rigidity amplifies the impact of monetary policy on employment. McKay and Wolf (2023) apply the same identification strategy using U.S. data to investigate the distributional effects of monetary policy on wage inequality, including changes in labour income. They find that low-income households are better off during periods of expansionary monetary policy. Mumtaz and Theophilopoulou (2017) use an SVAR model with a sign restriction strategy to identify monetary policy shocks. They evaluate the impact of these shocks on several inequality measures, including the Gini coefficient and the cross-sectional standard deviation of gross wages, focusing on the UK Family Expenditure Survey (FES) from 1969 to 2012. They conclude that contractionary monetary policy shocks increase earnings disparities, leaving low-income households worse off.

Villarreal and Kronick (2020) use narrative-based monetary policy shock series constructed by Champagne and Sekkel (2018) to assess the impact of monetary policy on income inequality in Canada. They employ a local projection regression and two inequality measures: the gap between wages and gross operating surplus, and the wage Gini coefficient, where the latter is constructed using residuals from a regression of monthly wages on observable employee characteristics, including sex, age, education, geographic area, and industry. Their findings suggest that expansionary monetary policy shocks increase income inequality, whereas contractionary monetary policy shocks reduce it. Samarina and Nguyen (2024) examine the effect of monetary policy on income inequality in the euro area using several identification techniques, including a Taylor rule-based approach. Using the Gini coefficient as their measure of inequality, they find that expansionary monetary policy leads to a reduction in income inequality.

This paper contributes to the literature by revisiting the distributional effects of non-systematic monetary policy on wage inequality in Canada. We begin by obtaining monetary policy innovation series from a standard Taylor rule, following the strategy employed in Samarina and Nguyen (2024). Specifically, we apply the instrumental regression method within the Taylor rule framework to compute regression residuals, which serve as our measure of monetary policy innovations. We then use monthly wage data from the Canadian Labour Force Survey (LFS) to construct two measures of wage inequality: the wage Gini coefficient and the wage ratio. The wage ratio, computed as in Dolado et al. (2021), is defined as the ratio of high-skilled to low-skilled workers' wages, where skill level is determined based on educational attainment. This approach allows us to capture any heterogeneous effects of non-systematic monetary policy across different segments of the Canadian workforce.

Meanwhile, Furceri et al. (2018) employ a panel local projection method using data from 32 advanced and emerging economies, including Canada, and they find that contractionary monetary policy shocks increase income inequality. We follow their approach by applying a single-country version of their local projection framework to analyze the effects of non-systematic monetary policy shocks in Canada. This framework is one of two methods proposed in the literature to measure the cumulative impact of monetary policy shocks on a dependent variable within local projection models. More specifically, the first approach involves regressing the sum of future values of the response variable (y_t) over a horizon h, $\sum_{j=1}^{h} y_{t+j}$, on monetary policy shocks and past values of the response variable (Stock and Watson, 2018; Ramey and Zubairy, 2018). The second approach captures the cumulative impact by regressing the change in y_t over h periods ($y_{t+h} - y_{t-1}$) on monetary policy shocks and past changes in y_t as regressors (Furceri et al., 2018; Jordà et al., 2019). It is worth noting that the first approach introduces complexity in the error structure, which is difficult to model accurately using the heteroskedasticity and autocorrelation consistent (HAC) estimation method. On the other hand, the second approach is less problematic in this regard, as it evaluates the impact of shocks between the period of the shock and a specified horizon h. As such, the HAC method is more likely to account appropriately for the error structure. Accordingly, we adopt the second approach in this paper.

Villarreal and Kronick (2020), who also focus on Canada, use the monthly residual wage from a regression to construct a wage Gini coefficient. The regression controls for the impact of socioeconomic factors on wage inequality. However, the regression process does not ensure that residuals are non-negative; some will inevitably be negative. The residuals, therefore, need to be truncated to obtain a positive Gini value, resulting in an inequality measure that may not accurately represent some features of the original wage distribution[1]. In this paper, our Gini coefficient is constructed based on the monthly wage series using an approach consistent with the wage distribution.

The local projection (LP) method is considered an alternative to the vector autoregression (VAR) method because it is easy to estimate and it is flexible (Dufour and Renault, 1998; Jordà, 2005; Li et al., 2021). However, using LP with high-frequency data, such as daily, monthly, or data including seasonal patterns, introduces high variability in impulse response functions (IRFs). There are several methods to manage high variability in regression coefficients, including seasonal adjustment methods, lowess regression, and cubic B-spline smoothing. Regarding the smoothing method, Villarreal and Kronick (2020) use a lowess regression[2] method with a bandwidth of 0.15 to smooth seasonal variability in the residual Gini coefficient. They then use the smoothed residual Gini coefficient to assess the impact of monetary policy shocks on wage inequality. Pre-smoothing or seasonally adjusting a series may unintentionally remove relevant information or introduce bias. On the other hand, using a smoothing method in regression can enhance integrated adaptive modelling in more complex scenarios (Kramar and Alchakov, 2023; Barnichon and Matthes, 2018; Melaas et al., 2018; Schimek, 2000). Barnichon and Brownlees (2019) propose an optimal smoothing method that addresses high variability in IRFs through the smooth local projection (SLP) method[3]. Their method, based on Ridge regression, effectively reduces the large variability in IRFs observed with the standard local projection. Our Gini coefficient in this paper is thus constructed based on monthly wage series using an approach consistent with the wage distri-

[1]The residual Gini coefficient can accurately reflect the original residual wage distribution by applying a bijective transformation to the residual wage series. We will consider this approach in future research.

[2]Lowess regression is a non-parametric technique for fitting smooth curves to data

[3]High variability may result from factors like seasonal changes and economic cycles.

bution without employing any pre-smoothing techniques; however, smoothing is performed through SLP.

Our paper makes several significant contributions to the existing literature. First, we incorporate the smooth local projection method into the local projection framework of Furceri et al. (2018), specifically adapting it for a single-country analysis. Second, we apply this approach to analyze the monthly effects of non-systematic monetary policy on wage inequality in Canada, providing insight into the distributional consequences of monetary policy at a relatively more granular temporal level. Third, we further enrich the modelling framework by introducing possible asymmetry in our smooth local projection equation, building on the asymmetry approach in Ramey and Zubairy (2018) that they considered within the context of local projections. Fourth, we employ our proposed methodology to investigate whether contractionary monetary policy shocks affect wage distribution differently from expansionary monetary policy shocks.

Beyond methodological advancements, our paper makes some substantive contributions to the literature on monetary policy and inequality. Specifically, we show that monetary policy shocks influence wage inequality in Canada and we underscore the usefulness of fiscal policy in countering some of the negative impacts of non-systematic monetary policy. In addition, we examine possible asymmetric effects of non-systematic monetary policy. These contributions collectively enhance our understanding of the complex interactions between macroeconomic policies and wage distribution.

In what follows, and for presentation clarity, the series that we obtain from the estimation of the Taylor rule is denoted hereafter as our innovation series, whereas, when we refer our non-systematic monetary policy shock, this is the residualized shock of the associated local projection (or smooth local projection) equation, that is, the residual from the OLS regression of the innovation in question on all the included control variables in the LP (or, SLP).

We find that non-systematic monetary policy shocks have a significant effect on wage inequality over one- and two-year horizons, and that they affect the Gini wage coefficient and the wage ratio in distinct ways. Using our baseline specification, and whether contractionary or expansionary, they do not significantly affect the wage ratio, but they do affect (symmetrically) the Gini coefficient. However, after controlling for lagged oil price returns, changes in the employment rate and its lags, and fiscal policy factor and its lags, we find no significant effects on the wage Gini coefficient. On the other hand, contractionary monetary policy shocks widen the skill-based wage gap (over the two-year horizon), and expansionary monetary policy shocks narrow it (over both the one- and two-year horizons).

The rest of the paper is structured as follows: section 1.2 introduces the methodology of the paper. It addresses the wage inequality measures and the monetary policy innovation series used in the study. Section 1.3 comments on the results and proposes some mitigation policies to address the adverse effects of non-systematic monetary policy on wage inequality. Section 1.3 also presents the robustness checks. The final section offers some concluding remarks.

1.2 Methodology

1.2.1 Empirical Estimation Method: Smooth Local Projection Regression

We use the Barnichon and Brownlees (2019) smooth local projection (SLP) method to address the short- and longer-term effects of non-systematic monetary policy on wage inequality. Specifically, we apply the B-spline framework to the long-difference specification (Jordà and Taylor, 2025; Furceri et al., 2018) through the SLP regression. Therefore, our baseline model is specified as follows:

$$\begin{aligned} y_{t+h} - y_{t-1} &= \sum_{k=1}^{K} a_k B_k(h) + \sum_{k=1}^{K} b_k B_k(h)\epsilon_t + \sum_{l=1}^{p}\sum_{k=1}^{K} c_{lk} B_k(h)\Delta y_{t-l} + u_{t+h}, && (1.1)\\ h &= H_{min}, \ldots, H_{max}, && (1.2) \end{aligned}$$

where ϵ_t represents non-systematic monetary policy innovations, y_t is the log of wage inequality variable at time t, and $\Delta y_{t-l} = y_{t-l} - y_{t-l-1}$ represents the lagged change in this variable, h is the horizon of the projection, and p represents the maximum lag, and u_{t+h} is a prediction error term with variance $\sigma^2_{(h)}$. Further details on the definition of y_t and ϵ_t are given below. H_{min} and H_{max} define the beginning and end horizons of the shocks, respectively. $B_k(h)$, $k = 1, \ldots, K$, is a set of B-spline basis functions defined in $\mathbb{R}$ at horizon h, and $a_k, b_k,\ c_{lk},\ k = 1, \ldots, K$ are sets of scalar parameters. The B-spline basis function used by Barnichon and Brownlees (2019) is a cubic B-spline basis with equidistant knots ranging from $H_{min} - 2$ to $H_{max} - 1$. Cubic splines are piecewise polynomial functions of degree three that are used to fit the data. In addition, by modelling the change $y_{t+h} - y_{t-1}$ (see equation (1.1)) captures how the wage inequality variable changes over time due to the impact of non-systematic monetary policy. Therefore, (1.1) allows us to assess the effect of ϵ_t over the period from $t-1$ to $t+h$. Including the lags of Δy_{t-l} allows the regression to capture the dynamic adjustments of the wage inequality in response to the shocks (see Furceri et al., 2018;

Jordà et al., 2019). Statistically, we assume that, the conditional on the control variables, ϵ_t is uncorrelated with u_{t+h} (exogeneity conditional on controls; see Montiel Olea et al. (2025)).

Equation (1.1) is a non-linear relationship, as the right-hand side consists of a combination of cubic B-spline bases. Additional nonlinearity arises from seasonal patterns, which we approximate using the methodological framework of Barnichon and Brownlees (2019). The coefficients of (1.1) are estimated using the Ridge regression method (Hoerl and Kennard, 1970) with a penalized cubic B-spline. A shrinkage parameter, which is a penalty term, is added to the optimization process to avoid overfitting and to guarantee smoothness; see Barnichon and Brownlees (2019) and appendix 1-A.2 for more detail on how penalized cubic B-splines are used with Ridge regression.

A given projection coefficient (effect of a unit shock) at horizon h derived from the regression is expressed as a linear combination of b_k in the cubic B-spline bases, $B_k(h),\ k = 1, \ldots, K$, and it is defined as follows:

$$\beta_{(h)} = \sum_{k=1}^{K} b_k B_k(h), \tag{1.3}$$

where $b_k,\ k = 1, \ldots, K$ is a set of scalar parameters as defined in (1.1). The estimated impulse response coefficient (IRF) at horizon h is defined as :

$$IRF(h) = E\left(y_{t+h} \mid \Lambda_t, \epsilon_t = \delta\right) - E\left(y_{t+h} \mid \Lambda_t, \epsilon_t = 0\right) = \hat{\beta}_{(h)} \cdot \delta, \tag{1.4}$$

where Λ is a vector of control variables, and δ is the size of the shock which does not interfere with the estimation of $\beta_{(h)}$. Moreover, the IRF scales proportionally with the shock size δ, so that choosing a specific magnitude has direct implications for the interpretation of the impact of the shock (Barnichon and Brownlees, 2019).

Following standard practice in the literature (Ramey and Zubairy, 2018; Furceri et al., 2018; Jordà et al., 2019; Barnichon and Brownlees, 2019), we correct for heteroskedasticity and time dependence in the LP disturbances using a combination of lagged Δy_{t-l} terms and Newey-West standard errors. Using a robust standard errors correction allows for any measurement noise in the dependent variable, which goes into the error terms[4].

In addition to regression (1.1), our study aims to address a potential asymmetric impact of non-systematic monetary policy on wage inequality. To do so, we adapt the asymmetric regression approach of Ramey and Zubairy (2018) that they used within the LP framework

[4]The full development of the estimation method is reported in the appendix.

to our context, deriving its SLP version. Our proposed regression is as follows:

$$y_{t+h} - y_{t-1} = I_t X_t \phi^{+}_{(h)} + (1 - I_t)\, X_t \phi^{-}_{(h)} + u_{t+h},\ h = H_{min}, \ldots, H_{max}, \tag{1.5}$$

where

$$X_t \phi^{+}_{(h)} = \sum_{k=1}^{K} a^{+}_{k} B_k(h) + \sum_{k=1}^{K} b^{+}_{k} B_k(h)\epsilon_t + \sum_{l=1}^{p}\sum_{k=1}^{K} c^{+}_{lk} B_k(h)\Delta y_{t-l}, \tag{1.6}$$

$$X_t \phi^{-}_{(h)} = \sum_{k=1}^{K} a^{-}_{k} B_k(h) + \sum_{k=1}^{K} b^{-}_{k} B_k(h)\epsilon_t + \sum_{l=1}^{p}\sum_{k=1}^{K} c^{-}_{lk} B_k(h)\Delta y_{t-l}, \tag{1.7}$$

and I_t is the dummy variable that defines the positive shock, with $I_t = 1$ if $\epsilon_t > 0$ otherwise 0. a^{+}_{k}, b^{+}_{k} and $c^{+}_{lk}, k = 1, \ldots, K$ are the sets of scalar parameters for the positive shocks and a^{-}_{k}, b^{-}_{k} and $c^{-}_{lk}, k = 1, \ldots, K$ for the negative shocks.

1.2.2 Inequality Measure and Data

Several measures of inequality are commonly used in the literature to assess the impact of non-systematic monetary policy on wage (or income) inequality. In our analysis, however, we focus on the most widely used measures: the Gini coefficient and the wage ratio (Samarina and Nguyen, 2024; Furceri et al., 2018; Villarreal and Kronick, 2020). Accordingly, we construct these two measures of wage inequality using monthly data from the Canadian Labour Force Survey (LFS), covering the period from January 1997 to July 2023 (see Figure 1-1). The LFS is a monthly survey conducted by Statistics Canada to assess labour market conditions in Canada. It provides the data at the household level, which also has information of the province/territory of residence[5]. Thus, the LFS enables an analysis of the labour income distribution for those who are working. Monthly earnings series from the LFS are used to calculate the Gini index. The monthly earnings data are estimated based on the weekly work hours and the earnings. In addition, to account for the LFS sampling distribution, LFS data provide the sampling weights that are used to calculate the wage Gini coefficient:

$$G(w) = -\sum_{i=2}^{N} (q_{\omega,i} p_{w,i-1} - p_{w,i} q_{\omega,i-1}), \tag{1.8}$$

[5]LFS microdata can be found on ODESI scholar portal `http://odesi2.scholarsportal.info/webview/`

with

$$p_{w,i} = \frac{\sum_{j=1}^{i} \omega_{(j)} W_{(j)}}{\sum_{j=1}^{N} \omega_{(j)} W_{(j)}} \text{ , and } q_{\omega,i} = \frac{\sum_{j=1}^{i} \omega_{(j)}}{\sum_{j=1}^{N} \omega_{(j)}}$$

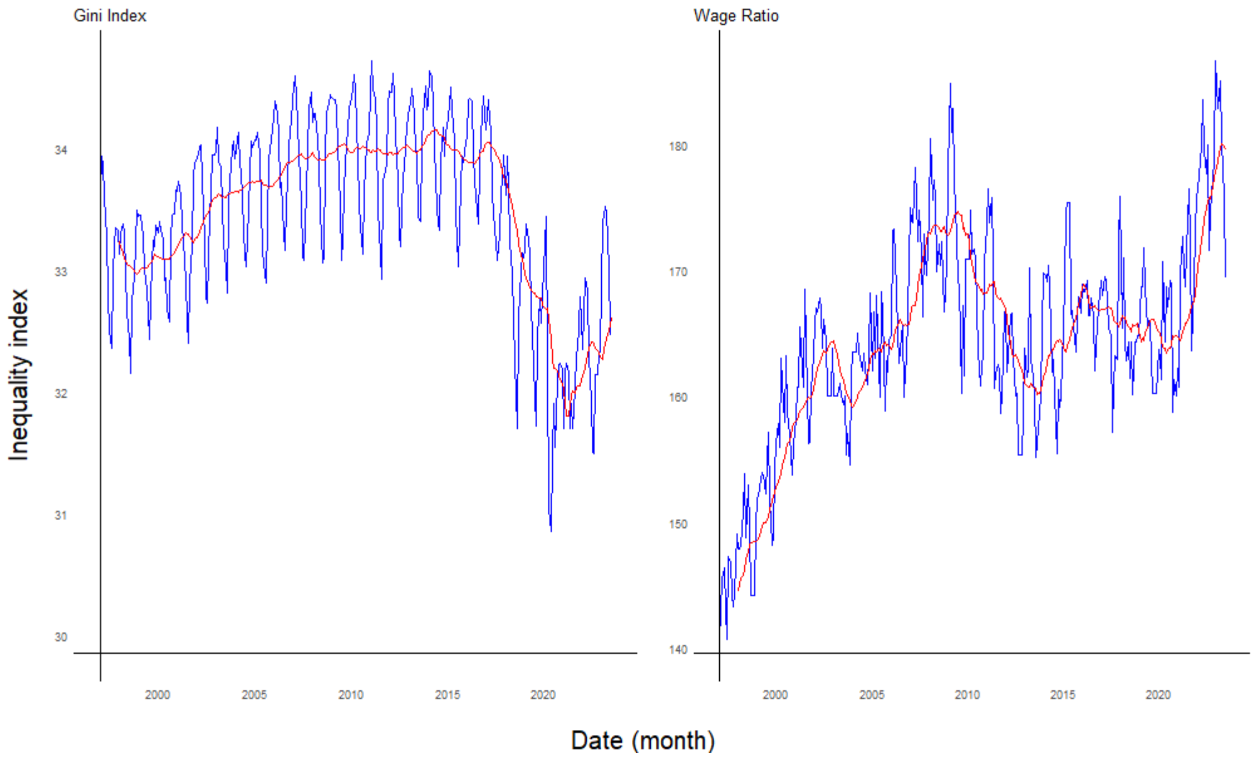
where $\omega_{(j)}$ defines the sampling weight associated to invididual j wage, $W_{(j)}$. Moreover, $W_{(j)}$ and $\omega_{(j)}$ denote the ordered versions of W_j and ω_j, respectively. The Gini coefficient takes a value between 0 and 1; perfect equality is characterized by 0, and full inequality corresponds to 1.

We also define an alternative measure of wage inequality: the wage ratio between high- and low-skilled workers, as defined in Dolado et al. (2021). Skill level is determined based on the education attainment. The low-skilled (LS) workers have a primary or secondary education, typically no more than 10 years of schooling, while high-skilled (HS) workers have more than 10 years of education. Therefore, the wage ratio ($R(w)_{\underline{q},\bar{q}}$) is defined as

$$R(w)_{\underline{q},\bar{q}} = \frac{W^{\bar{q}}}{W^{\underline{q}}}, \tag{1.9}$$

where $W^{\underline{q}}$ and $W^{\bar{q}}$ denote respectively the median wages of the low-skilled and the high-skilled worker groups. The data is again from LFS data. $R(w)_{\underline{q},\bar{q}}$ takes a value that is great or equal to 1; perfect equality is characterized by 1.

Figure 1-1: Wage Inequality Trends



Note: Figure 1-1 displays the Gini coefficient based on the low-skilled and high-skilled wage ratio. We construct them using the Canadian Labour Force Survey from the Odesi database (Ontario Council of University Libraries, 2023). The red line represents the 12 months moving average of the series.

Figure 1-1 shows that the overall inequality captured by the Gini coefficient increased before 2015 and decreased until 2020 when the COVID-19 pandemic occurred. Then, overall inequality rises again. The wage gap between high- and low-skilled workers in Canada shows significant changes between 2000 and 2023, reflecting a different angle on labour market dynamics and economic conditions. For example, the wage ratio increased steadily until the 2008 financial crisis, when it reached its peak. Subsequently, it declined between 2010 and 2014, probably due to post-crisis recovery, with an increase in the minimum wage or strengthened labour protection policies. Between 2015 and 2020, the wage ratio remained fairly stable before rising sharply during the COVID-19 pandemic, and the pandemic's effect on the wage ratio persisted until 2023.

1.2.3 Monetary Policy Shocks Series

Monetary policy shocks can be assessed through several approaches, including SVAR identification (Samarina and Nguyen, 2024; Mumtaz and Theophilopoulou, 2017), High-Frequency

Identification (HFI) (Faia and Pezone, 2024; Andreas Gulyas, 2024), the Narrative Approach (Champagne and Sekkel, 2018; Romer and Romer, 2004), and Taylor Rule identification (Samarina and Nguyen, 2024; Carvalho et al., 2021; Mavroeidis, 2010).

SVAR identification is widely used to capture macroeconomic interdependencies by employing flexible methods such as recursive structures, sign restrictions, or instrumental variables. However, it relies on strong assumptions, and the Cholesky ordering imposes restrictive constraints by assuming that monetary policy does not react to specific economic indicators within the same period (Wolf, 2020; Arias et al., 2019). HFI isolates exogenous monetary shocks by analyzing financial market reactions within narrow time windows around policy announcements. This small window-based aims to ensure that policy shocks are not contaminated by shocks from other sources. However, it requires high-quality financial data, which may not always be available, and it primarily captures short-term effects. Furthermore, market expectations can influence observed shocks, as financial markets react to more than changes in monetary policy (Nakamura and Steinsson, 2018).

The Narrative Approach identifies exogenous monetary policy shocks by analyzing historical documents, meeting minutes, and expert reports to pinpoint episodes where external factors influenced policy decisions. This method avoids reliance on structural assumptions but is constrained by the subjectivity of historical interpretation and the limited frequency of available records.

The Taylor Rule approach is an explanation of how central banks adjust interest rates in response to macroeconomic conditions such as inflation and output gaps. It recommends setting the interest rate based on specific current and expected conditions[6], notably including expected inflation, a measure of slack in real activity, and lags of the interest rate as regressors. Deviations from this regression are considered monetary policy shocks (Christiano et al., 1999; Ramey, 2016). This approach corrects for systematic policy behaviour by removing predictable reactions. Using ordinary least squares (OLS) regression to estimate this equation can introduce endogeneity bias (Carvalho et al., 2021), which instrumental variable methods can mitigate.

A monetary policy shock series based on the narrative approach proposed by Champagne and Sekkel (2018) is available for Canada. However, this series was not accessible to us beyond 2017. Therefore, we take advantage of the simplicity and flexibility of a Taylor rule strategy to construct an alternative shock series. [7]. We then include our innovation series in

[6]According to the rule, the change in the bank rate should be set in response to changes in price level or/and changes in real income (Taylor, 1993)

[7]As in Samarina and Nguyen (2024), we tested this series for Granger Causality (see appendix 1-A.3).

a smooth local projection framework. If, conditional on all the included variables in the SLP, this innovation series is uncorrelated with the SLP disturbance, then, as mentioned before, we can obtain the impacts of non-systematic monetary policy by studying the impacts of the residualized shock, where the latter is the residual from an OLS regression of our innovation series on all control variables in the SLP (see Montiel Olea et al., 2025).

Considering Clarida et al. (2000) framework of Taylor rule strategy, we can write the gap between the target rate for the nominal interest rate (i_t^*) and its target value (i^*) as a function of the gap between the expected inflation and its target level ($E(\pi_{t+1|t}) - \pi^*$) and output gap ($E(x_{t|t})$), that is,

$$i_t^* - i^* = \psi_\pi(E(\pi_{t+1|t}) - \pi^*) + \psi_x E(x_{t|t}), \tag{1.10}$$

where i^* is the level of the nominal interest when inflation and output are at equilibrium, and $E(\pi_{t+1|t})$ is the conditional expectation of inflation at time $t+1$ and $E(x_{t|t})$ is the conditional expectation of output growth at time t. Equation (1.10) features a forward-looking Taylor rule; that is, the bank rate responds to expected inflation to capture the price stability objective of the Bank of Canada.

According to Mavroeidis (2010), due to exogenous factors, the actual nominal interest rate i_t can unexpectedly deviate from the target rate i_t^* and adjust partially to its target. Especially, the following specification represents the actual nominal interest rate:

$$i_t = \rho_1 i_{t-1} + \rho_2 i_{t-2} + (1 - \rho) i_t^* + e_t, \tag{1.11}$$

where $\rho = \rho_1 + \rho_2$ can represent the interest rate smoothing parameter (Mavroeidis, 2010), and e_t is monetary policy innovation. Using (1.10) and (1.11), we have the following,

$$i_t = (1 - \rho)(i^* - \psi_\pi \pi^*) + \rho_1 i_{t-1} + \rho_2 i_{t-2} + (1 - \rho)\psi_\pi E(\pi_{t+1|t}) + (1 - \rho)\psi_x E(x_{t|t}) + e_t. \tag{1.12}$$

In the above equation, $i^* = \psi_\pi \pi^*$ reflects the longer-run relationship between the nominal interest and inflation. This relationship is relevant for inflation-targeting regimes, where the central bank adjusts the nominal interest rate to maintain a stable inflation rate. In addition, setting $\rho_2 = 0$ allows less persistent monetary policy (Woodford, 2003; Mavroeidis, 2010). In fact, $\rho_2 = 0$ matches the new Keynesian framework, where central banks adjust rates gradually in response to economic conditions. Given these two conditions and using the fact that $\rho = \rho_1$, we adopt the specification from Samarina and Nguyen (2024).

$$i_t = \rho i_{t-1} + \beta E(\pi_{t+1|t}) + \gamma E(x_{t|t}) + e_t, \tag{1.13}$$

where $\beta = (1-\rho)\psi_\pi$ and $\gamma = (1-\rho)\psi_x$. Since $E(x_{t|t}) = x_t$, and the realized inflation does not meet the expectation $(\pi_{t+1} - E(\pi_{t+1|t}) = d_t)$, we can rewrite (1.13) substituting the realized variables of inflation and output gap.

$$i_t = \rho i_{t-1} + \beta\pi_{t+1} + \gamma x_t + \epsilon_t, \tag{1.14}$$

where $\epsilon_t = e_t - \beta(\pi_{t+1} - E(\pi_{t+1|t}))$.

The monetary policy innovation can now be derived by estimating (1.14). It can be estimated using ordinary least square (OLS) regression, instrumental variables regression, or the Generalized Method of Moments (GMM). Using OLS to estimate (1.11) can lead to endogeneity bias[8]. Here, we use instrumental variables (IV) regression with the GMM approach to estimate our innovation. This estimate is defined as follows:

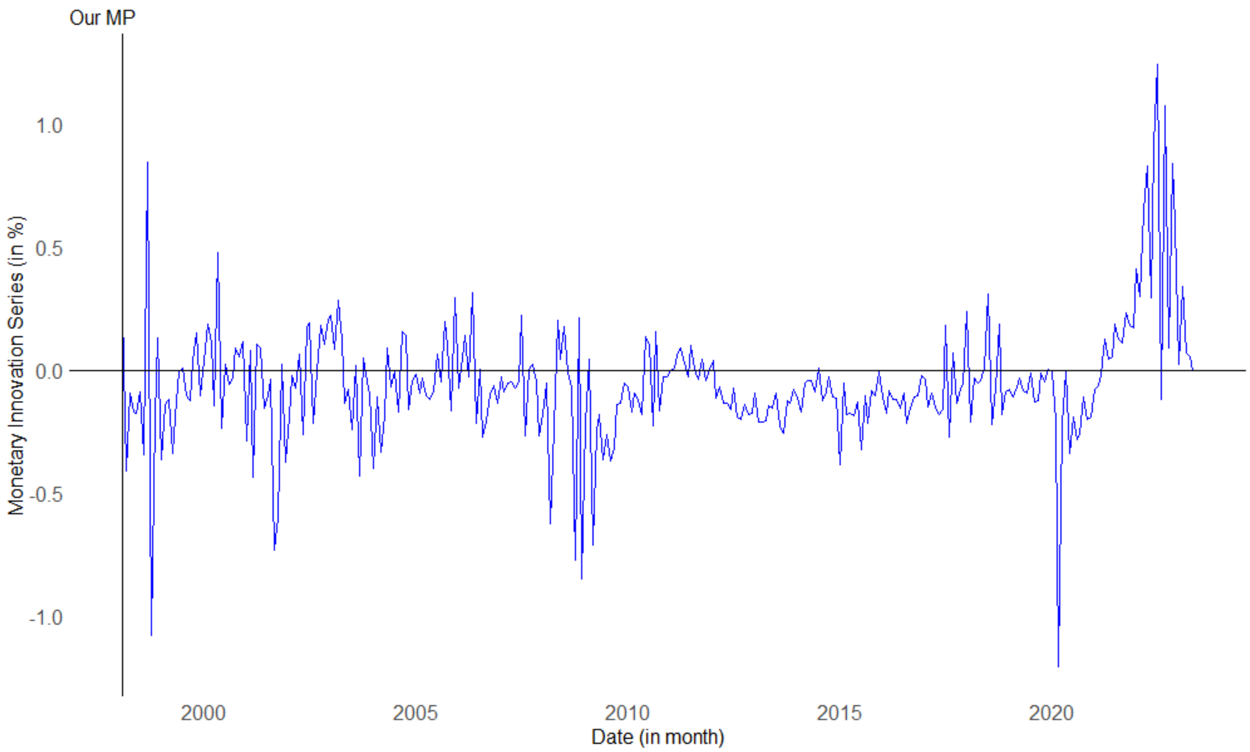
$$\hat{\epsilon}_t = i_t - \hat{\rho} i_{t-1} - \hat{\beta}\pi_{t+1} - \hat{\gamma} x_t. \tag{1.15}$$

Our instruments are 12 lags of the Canadian GDP growth rate, 12 lags of the monthly overnight rate, and 12 lags of Canadian inflation, as well as a constant. We include the constant in the regression to capture any deviation from the steady state. Our instruments also include the contemporaneous value of U.S. monthly GDP growth and its lags, which are assumed to be exogenous as Canada is a small economy that the U.S. economy can influence. To proxy U.S. monthly GDP growth, we use the Brave-Butters-Kelly real gross domestic product (BBKMGDP) series. This series, developed by Brave et al. (2019), measures real U.S. GDP growth monthly through several methodological approaches: mixed-frequency state-space modelling, dynamic factor models, and Kalman filtering and smoothing (see Brave et al., 2019, for more details). These approaches incorporate various data sources, including financial market data (stock prices, bond yields and commodity prices), real-time economic indicators (high-frequency data on employment, retail sales, industrial production, and housing market), and survey data (consumer confidence surveys, business confidence surveys, and labour market surveys). Thus, it provides updates more timely than traditional quarterly U.S. GDP measures. The monthly overnight rate and monthly inflation series are from the Bank of Canada database, and the monthly Canadian real GDP growth rate is calculated based on the monthly real GDP series from Statistics Canada. All the series are from January 1997 to July 2023. Figure 1-2 displays our series; it notably shows that several

[8]However, Carvalho et al. (2021) investigate whether the endogeneity bias in OLS estimation of the Taylor rules is economically significant. They find that endogeneity bias in OLS estimates of Taylor rules is quantitatively small, and OLS estimates are more precise than GMM estimates in practice.

important monetary shocks occurred during the Great Financial Recession and the COVID periods.

Figure 1-2: Our Monetary Policy Innovation Series



Note: Figure 1-2 displays our monetary policy innovation series from equation (1.14). The Augmented Dickey-Fuller test and Phillips-Perron unit root test indicate that this series is stationary.

While Figure 1-2 shows our innovation series, we remind that it is only when we include this series in our SLP framework, and that we assume that, conditional on the control variables, it is exogenous to the wage inequality disturbance, that we obtain our non-systematic monetary shock impacts.

However, before examining the impacts of non-systematic monetary shocks on wage inequality, we first compare their estimated impacts on key macroeconomic variables with those obtained when using instead the series of narrative-based monetary policy shocks of Champagne and Sekkel (2018). We describe how Champagne and Sekkel (2018) series are constructed later in this paper. We use SLP regression over our sample period ending in 2017 to estimate these impulse response functions, while the control variables are 12 lags of the response variables[9]. The findings are presented in Figures 1-3. Figures 1-3(a) and 1-3(b)

[9]

$$y_{t+h} = \beta_{(h)}\epsilon_t + z_t\gamma_{(h)} + u_{t+h},\ h = H_{min}, \ldots, H_{max}, \tag{1.16}$$

illustrate the impulse response functions on inflation and the economic growth rate. In contrast, Figures 1-3(c) and 1-3(d) depict their effects on the exchange rate and unemployment rate.

The results generally show similar effects on the selected macroeconomic variables. However, their statistical significances sometimes differ in some cases. Specifically, the two series have no statistically significant effect on the growth rate, but they lead to a significant increase in inflation in the short term. Additionally, both series result in a significant decrease in the exchange rate, measured as the monthly average U.S. dollar to Canadian dollar (CAD). However, the Champagne and Sekkel (2018) shock series has a persistent effect on the exchange rate. Importantly, only our series leads to a statistically significant decrease in the unemployment rate, highlighting its unique impact on labour market dynamics.

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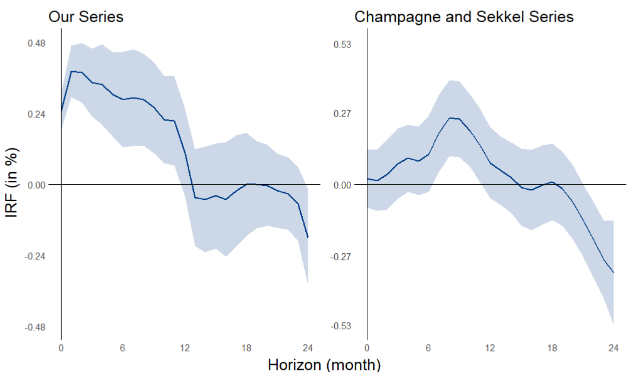
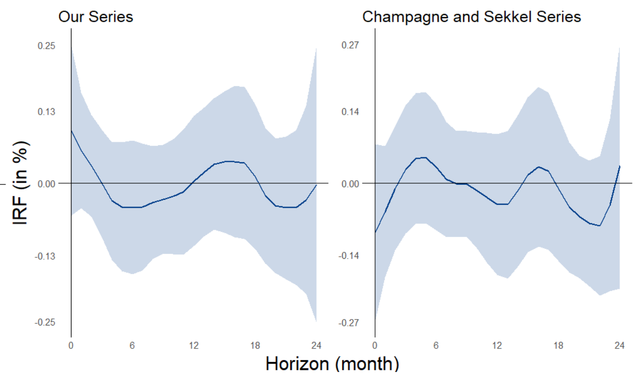
with $\beta_{(h)} = \sum_k^K b_k B_k(h)$, and where z_t contains the 12 lags of y_t, and u_{t+h} is the regression residuals. The unemployment and exchange rates are from CANSIM tables 14-10-0287 and 33-10-0163-01, respectively.

Figure 1-3: Impact of Our Series Versus Champagne and Sekkel (2018) Series: Selected Macroeconomic Variables

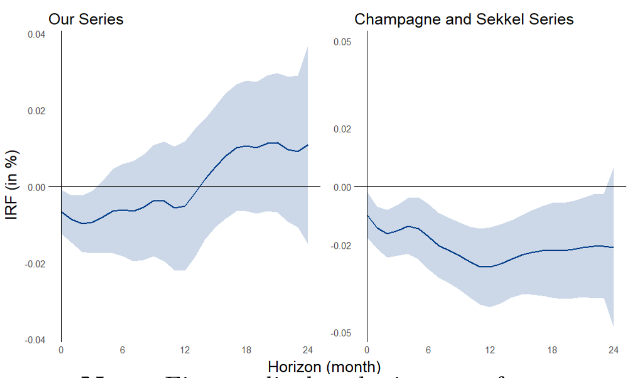
(a) Inflation.



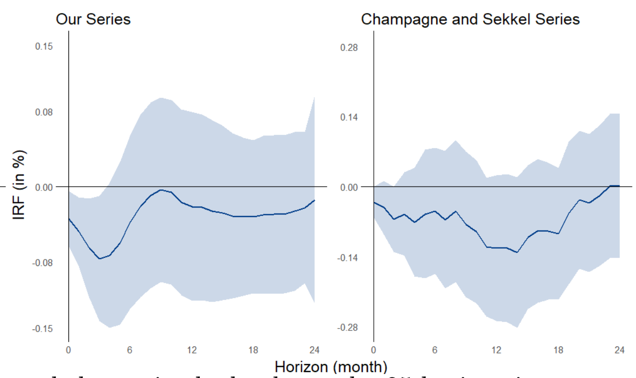
(b) Economic Growth rate.



(c) Exchange Rate.



(d) Unemployment Rate.



Note: Figures display the impact of an unexpected change in the bank rate by 25-basis-points on the unemployment rate, growth, inflation, and exchange rate, using our series vs. Champagne and Sekkel (2018) series. We plot 95% confidence intervals using heteroskedasticity-consistent standard errors of the IRFs. The impulse response coefficients are expressed in percentage terms.

1.3 Results

1.3.1 Baseline Results

The regression (1.1) is applied to examine the impact of non-systematic monetary policy on wage inequality. We use lags of the change in the defined inequality measure to control for serial correlation. This specification enables us to capture the short-term dynamics of non-systematic monetary policy. We choose the maximum lag (p) in alignment with the methodology of the LFS. In the LFS, households are selected on a rotation basis, remaining in the sample for six consecutive months before being replaced by new households. We account for this rotation scheme by including a six-month lag of the wage inequality measure as a control variable. The Gini coefficient and the wage ratio are not adjusted seasonally. The main purpose of the SLP method is to smooth out high fluctuations, including seasonal variations, in the impulse response coefficients. As in Barnichon and Brownlees (2019), we estimate (1.1) using Ridge regression, where the shrinkage parameter λ is selected using 20-fold cross-validation.

We first analyze the impact of an unexpected 25-basis-point increase in the overnight rate on wage inequality. We then focus on potential asymmetric effects of non-systematic monetary policy on wage inequality[10].

Effect of Non-Systematic Monetary Policy on Wage Inequality: Baseline Result

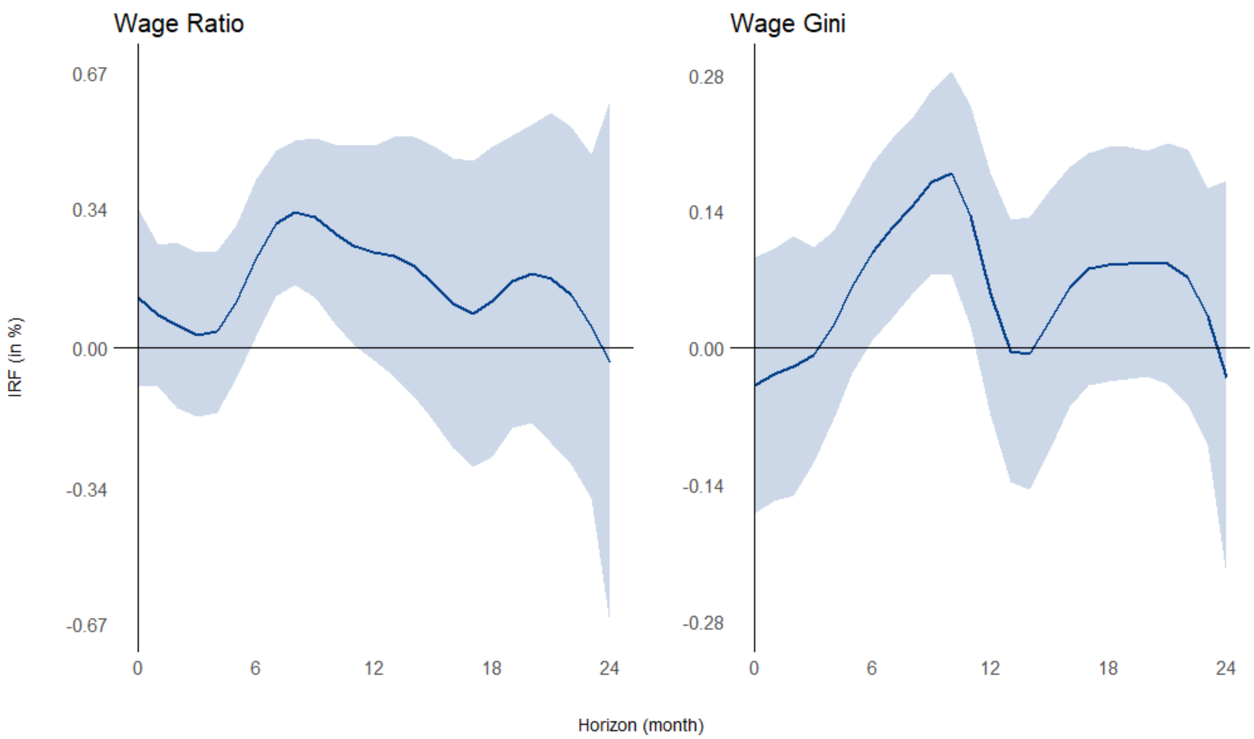
This section examines the effects of an unexpected 25-basis-point increase in the bank rate ($\epsilon_t - \epsilon_{t_0} = 0.25\%$) on wage inequality. Figure 1-4 presents the impulse response coefficients of this monetary policy shock on the wage Gini coefficient and the wage ratio. Statistical significance is assessed using a 95% confidence interval, represented by the shaded region in the figure.

The findings reveal that an unexpected 25-basis-point increase in the bank rate leads to a statistically significant increase in both the wage ratio and the Gini coefficient between the sixth and eleventh months after the shock. Specifically, the wage gap between high- and low-skilled workers increases by 0.219% in the sixth month and expands further to 0.249%

[10] There are several channels identified in the literature through which monetary policy shocks can impact income inequality, such as the income composition channel, financial market segmentation channel, portfolio channel, savings redistribution channel, and earnings heterogeneity channel (see Coibion et al., 2017; Amaral, 2017; Cloyne et al., 2019). Recently, (Dolado et al., 2021) also showed that expansionary monetary policy shocks increase high-skilled employment, enhancing the productivity of complementary capital, prompting a further increase in investment demand. This mechanism generates a multiplier effect, which they refer to as labour market frictions and capital-skilled complementarity. This study investigates the earnings heterogeneity channel, labour market frictions, and the capital-skilled complementarity channel.

in the eleventh month. Likewise, the Gini coefficient rises by 0.099% in the sixth month and reaches 0.136% in the eleventh month. However, this increase in wage inequality is temporary as the effect dissipates.

Figure 1-4: Impact of Our Monetary Policy Shocks on Wage Inequality Measures



Note: Figure 1-4 displays the impact of non-systematic monetary policy on the Gini coefficient and the wage rate at horizon h (25-basis-point). We plot 95% confidence intervals using heteroskedasticity-consistent standard errors of the IRFs. The impulse response coefficients are expressed in percentage terms.

Contractionary versus Expansionary Monetary Policy Shocks on Wage Inequality

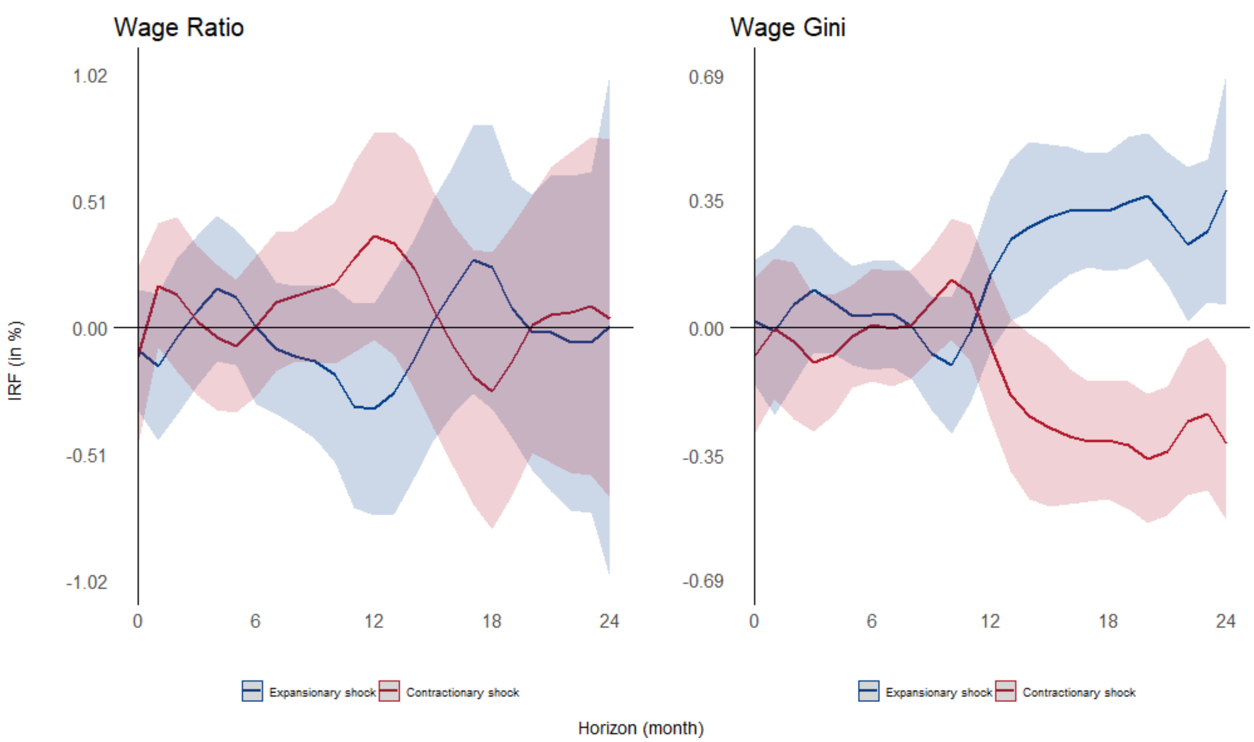
We examine whether the sign of the shock matters for wage inequality. A growing consensus in the literature suggests that contractionary monetary policy shocks have a greater impact on wage inequality than expansionary monetary policy shocks[11]. We revisit this potential asymmetric impact by estimating (1.5)-(1.7) as detailed in section 1.2.1. More precisely, we examine the impact of an unexpected 25-basis-point increase in bank rate (contractionary policy) as opposed to an unexpected 25-basis-point decrease in the bank rate (expansionary policy).

[11]See Mumtaz and Theophilopoulou (2017); Madeira and Salazar (2023); Andreas Gulyas (2024); Chang and Schorfheide (2024).

Figure 1-5 presents the baseline results. We find that neither contractionary nor expansionary monetary policy shocks significantly impact the wage ratio. However, their effects on the Gini coefficient are symmetric.

Contractionary monetary policy shocks lead to a statistically significant decline in the Gini coefficient, reducing it by 0.245% in the 14th month and by 0.319% in the 24th month following the shock. This indicates a decrease in overall wage inequality. Expansionary monetary policy shocks produce a symmetric effect, with the Gini coefficient increasing by 0.238% in the 13th month and continuing to increase, reaching a 0.3% increase in the 24th month. These findings suggest that non-systematic monetary policy shocks have a symmetric effect on overall wage inequality.

Figure 1-5: Asymmetric Effect of Non-Systematic Monetary Policy on Wage Inequality Measures



Note: Figure 1-5 displays the asymmetric effect of non-systematic monetary policy on the Gini coefficient and the wage ratio at horizon h (25 basis points). We plot 95% confidence intervals using heteroskedasticity-consistent standard errors of the IRFs. The impulse response coefficients are expressed in percentage terms.

1.3.2 Robustness Checks

This section examines the robustness of the results in two ways: (i) changes to the baseline specification and (ii) use of an alternative measure for the monetary policy shocks.

Controlling for Lagged Oil Price Returns in the Estimation Equation

A change in oil prices can play a crucial role in wage disparities. Boudarbat and Chernoff (2020) investigate the impact of oil price on wage inequality and find that fluctuations in oil prices lead to greater wage disparity in the regions that depend on oil production. Lechthaler and Mileva (2024) study the effect of trade liberalization and external shocks, including oil prices, on wage inequality. They find that oil price shocks can increase wage inequality by disproportionately affecting oil-dependent industries, leading to intersectoral wage disparities. In addition, Alp et al. (2023) investigate the effects of oil price fluctuations on inflation in advanced economies. They find that increases in oil prices significantly influence inflation, both directly through energy prices and indirectly through food and core prices (see also Bańbura et al., 2024). While the Bank of Canada (BoC) takes commodity shocks into account, it does so indirectly. The BoC primarily focuses on overall inflation and aims to maintain it within the 1-3% target band. In addition, the BoC measures inflation using 12-month change in the Consumer Price Index (CPI) but focuses on core inflation measures (which excludes the prices of energy and food) to filter out short-term volatility. Consequently, the BOC only reacts to commodity shocks to the extent that these end up affecting its operational target. In this respect, the impacts of the commodity shocks occur after a certain time.

We integrate commodity shocks into our estimation equation to account for potential effects from all of the above-mentioned channels. Therefore, we reestimate (1.1) by introducing 12 lags of the logarithmic change in monthly real oil prices as control variables, that is,

$$
\begin{aligned}
y_{t+h} - y_{t-1} = & \sum_{k=1}^{K} a_k B_k(h) + \sum_{k=1}^{K} b_k B_k(h)\epsilon_t + \sum_{l=1}^{p}\sum_{k=1}^{K} c_{lk} B_k(h)\Delta y_{t-l} + \\
& \sum_{l=1}^{p}\sum_{k=1}^{K} d_{lk} B_k(h)\Delta z_{t-l} + u_{t+h},
\end{aligned} \tag{1.17}
$$

$$
h = H_{min}, \dots, H_{max}, \tag{1.18}
$$

where z_t represents the log of monthly real oil prices, $\Delta z_{t-l} = z_{t-l} - z_{t-l-1}$ its change, and $d_{lk},\ k = 1, \dots, K$ is a set scalar parameter associated to Δz_{t-l}. All other parameters and variables remain the same as defined in (1.1). We use West Texas Intermediate (WTI) crude oil prices and the U.S. consumer price index from the FRED database to compute real oil price series. We also address possible asymmetric impacts by modifying (1.5) to control also for the oil price changes, that is,

$$y_{t+h} - y_{t-1} = I_t X_t \phi^{+}_{(h)} + (1 - I_t) X_t \phi^{-}_{(h)} + u_{t+h}, \tag{1.19}$$

$$h = H_{min}, \ldots, H_{max}, \tag{1.20}$$

where

$$X_t \phi^{+}_{(h)} = \sum_{k=1}^{K} a^{+}_{k} B_k(h) + \sum_{k=1}^{K} b^{+}_{k} B_k(h) \epsilon_t + \sum_{l=1}^{p} \sum_{k=1}^{K} c^{+}_{lk} B_k(h) \Delta y_{t-l} + \sum_{l=1}^{p} \sum_{k=1}^{K} d^{+}_{lk} B_k(h) \Delta z_{t-l}, \tag{1.21}$$

$$X_t \phi^{-}_{(h)} = \sum_{k=1}^{K} a^{-}_{k} B_k(h) + \sum_{k=1}^{K} b^{-}_{k} B_k(h) \epsilon_t + \sum_{l=1}^{p} \sum_{k=1}^{K} c^{-}_{lk} B_k(h) \Delta y_{t-l} + \sum_{l=1}^{p} \sum_{k=1}^{K} d^{-}_{lk} B_k(h) \Delta z_{t-l}, \tag{1.22}$$

where c^{+}_{lk} and c^{-}_{lk}, $k = 1, \ldots, K$ are sets of parameters associated to Δz_{t-l} in the two states (positive shocks and negative shocks). We estimate (1.17) and (1.19)-(1.22) using the same estimation method as used for (1.1) and (1.5)-(1.7). In the following sections, we first analyze the impact of non-systematic monetary policy on wage inequality when also controlling for oil price impacts and then analyze the asymmetric impact. Statistical significance is again assessed using a 95% confidence band.

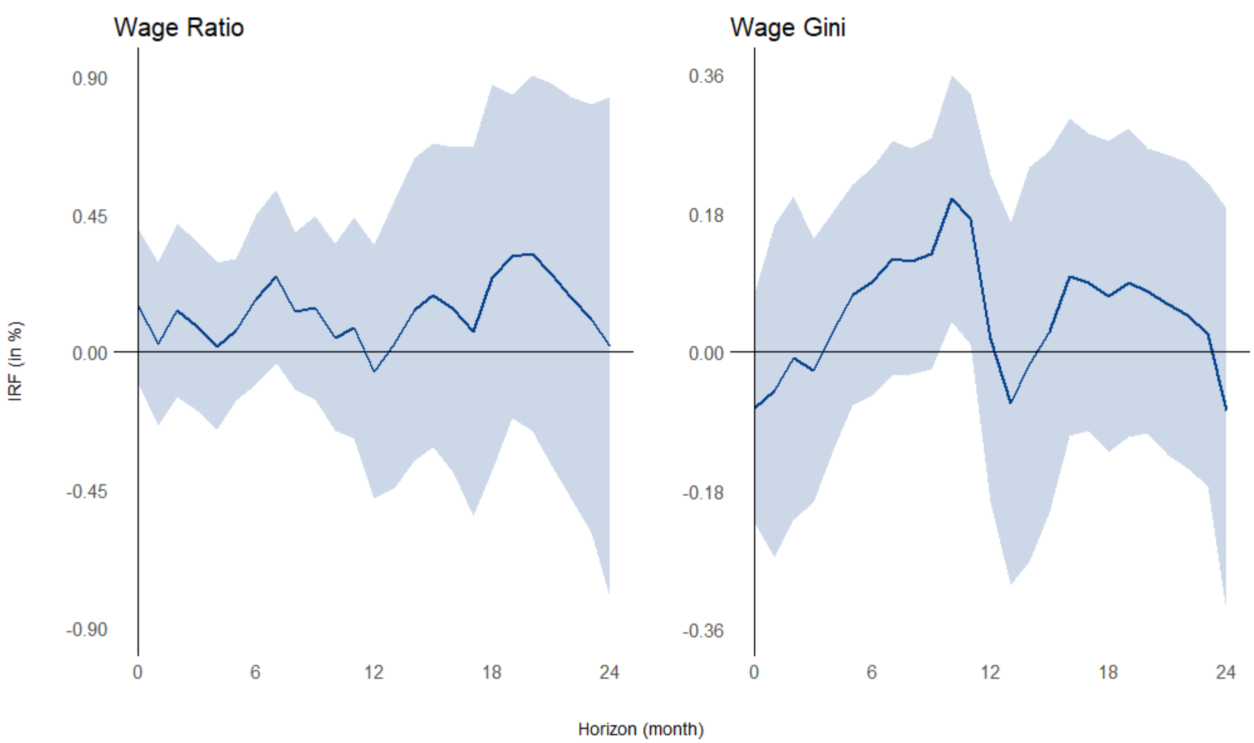
Effect of Non-Systematic Monetary Policy on Wage Inequality by Controlling for Lagged Oil Price Returns

Figure 1-6 illustrates the overall impact of non-systematic monetary policy on wage inequality, incorporating lagged oil price returns, and without differentiating the direction of the monetary policy shock. The results indicate that, under these conditions, an unexpected 25-basis-point increase in the bank rate leads to a statistically significant increase in the Gini coefficient in the 10th and 11th months, by 0.2% and 0.173%, respectively.

So, controlling for lagged oil price returns, wage inequality, as measured by the Gini coefficient, tends to increase approximately one year after the shock. Although the magnitude of this increase remains comparable to that observed in the baseline model, the timing of the impact changes. Given the significant influence of oil price changes on the Canadian economy, particularly in the western provinces (Alberta and Saskatchewan) as well as the eastern province of Newfoundland and Labrador, isolating the effects of oil price changes provides a clearer picture of the true impact of non-systematic monetary policy on wage distribution. This finding underscores the need for policymakers to consider redistributive measures to mitigate the adverse effects of such policy shocks on low-income groups.

Controlling for lagged oil price returns directly does not alter the response of the wage ratio to non-systematic monetary policy, and the impact on the wage ratio is not statistically significant.

Figure 1-6: Impact of Our Monetary Policy Shocks on Wage Inequality: Controlling for Lagged Oil Price Returns



Note: Figure 1-6 displays the global impact of non-systematic monetary policy on the Gini coefficient and the wage rate at horizon h (25-basis-point), controlling for lagged oil price returns. We plot 95% confidence intervals using heteroskedasticity-consistent standard errors of the IRFs. The impulse response coefficients are expressed in percentage terms.

Asymmetric Effect of Non-Systematic Monetary Policy on Wage Inequality by Controlling for Lagged Oil Price Returns

Figure 1-7 compares the effects of an expansionary monetary policy shock with those of a contractionary shock of the same magnitude. The findings indicate that a contractionary monetary policy shock leads to a 0.318% increase in the wage ratio in the first month following the shock. In a similar manner, an expansionary monetary policy shock results in a statistically significant decrease in the wage ratio in the first month (-0.309%). However, these effects are short-lived as the impact of contractionary and expansionary monetary policy shocks vanishes after the first month.

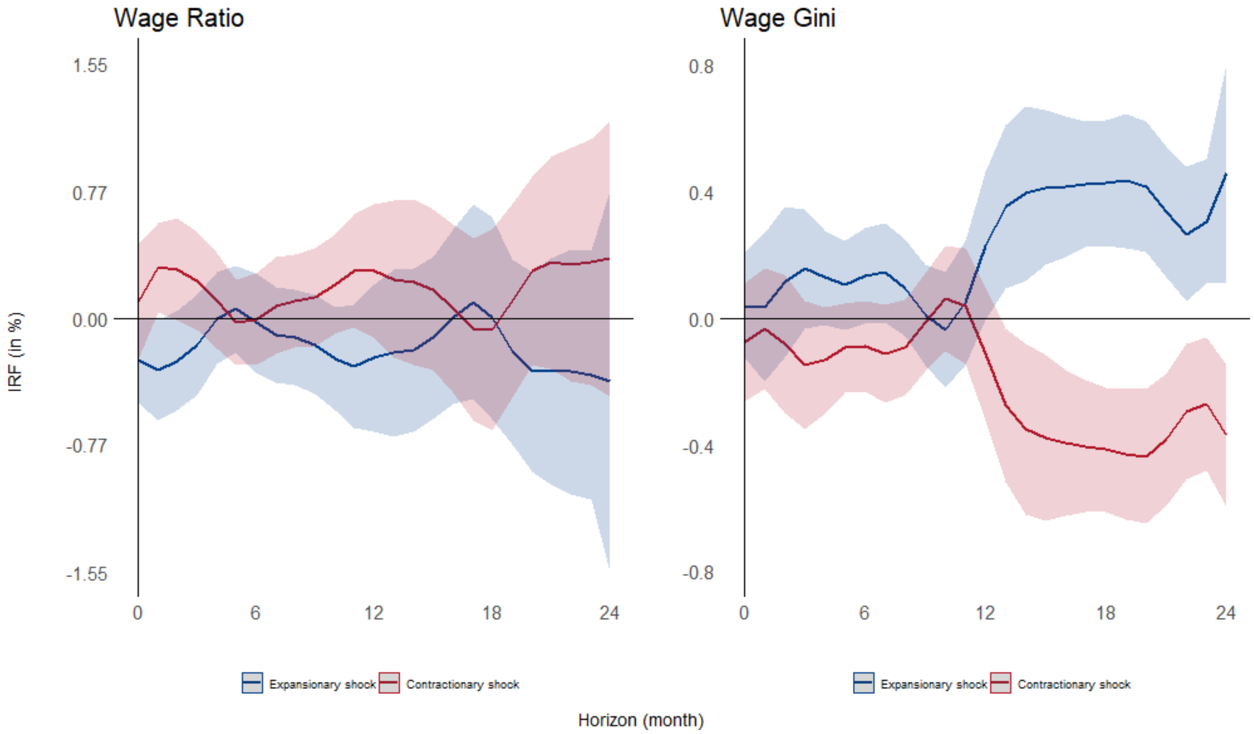
Contractionary and expansionary monetary policy shocks have a symmetric impact on the Gini coefficient. Specifically, an unexpected 25-basis-point increase in the policy rate has a statistically significant effect on the Gini coefficient, observable between the 13th and 24th months following the shock.

Under an expansionary monetary policy, the Gini coefficient increases by 0.353% in the 13th month after the shock, and it continues to rise, reaching 0.458% by the 24th month. On the other hand, following a contractionary monetary policy, the Gini coefficient decreases by 0.275% in the 13th month and further declines to 0.372% by the 24th month. These results indicate that non-systematic monetary policy has symmetric effects on wage inequality over the medium to longer term, although the magnitude of the responses differs slightly between the two policy stances.

The impact on the Gini coefficient aligns with the results observed in the baseline model, where lagged oil price returns are not controlled for. However, the magnitude of the effects differs. When controlling for lagged oil price returns, the decrease in the Gini coefficient is more pronounced. Furthermore, the wage ratio exhibits a short-term increase in this controlled setting, and a pattern is absent in the baseline regression.

Furthermore, the findings reveal that oil price changes play a crucial role in shaping the immediate labour market response to non-systematic monetary policy shocks. Additionally, given the significance of the energy sector in Canada, oil price fluctuations have a substantial impact on the economy, particularly influencing the energy sector due to the high volatility of oil prices (Kilian, 2009; Boehm and Plagborg-Møller, 2018). Specifically, when oil prices increase, employment and wages in the energy sector rise, potentially reducing overall wage inequality, as measured by the Gini coefficient. Consequently, this sectoral wage effect may overshadow the impact of contractionary monetary policy on wage inequality. Controlling for lagged oil price returns eliminates this sectoral effect, especially on the wage gap between high- and low-skilled workers.

Figure 1-7: Asymmetric Effect of Non-Systematic Monetary Policy on Wage Inequality: Controlling for Lagged Oil Price Returns



Note: Figure 1-7 displays the asymmetric effect of non-systematic monetary policy on the Gini coefficient and the wage ratio at horizon h (25-basis-point), controlling for lagged oil price returns. We plot 95% confidence intervals using heteroskedasticity-consistent standard errors of the IRFs. The impulse response coefficients are expressed in percentage terms.

Accounting for Employment Rate in the Estimation Equation

Since our wage inequality measures are derived from LFS data, it is crucial to account for the working-age population, as the survey methodology is designed for this demographic setting. One key concern in labour market analysis is that changes in employment dynamics can distort inequality measures if working-age population is not properly controlled. To address this, we include changes in the employment rate and its lags as control variables, allowing us to capture the proportion of individuals employed at a given time. This approach ensures that shifts in employment do not confound the assessment of wage disparities, but instead provide a clearer structural view of the labour market.

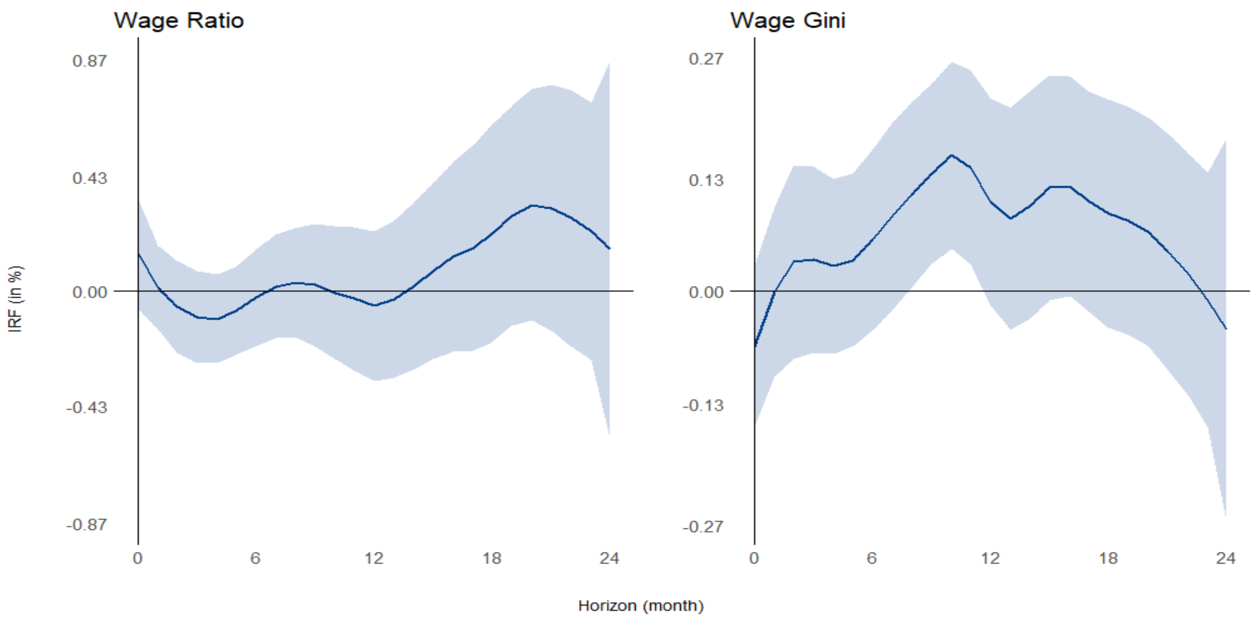
In addition, focusing on wage inequality among the employed (intensive margin) may overlook broader labour market effects, especially if a contractionary monetary policy shock leads to more layoffs among low-wage workers. Furthermore, unlike labour force participation — which fluctuates due to discouraged workers exiting the labour market or new entrants seeking jobs — the employment rate provides a more stable and reliable measure of actual job-holding. As Blanchard and Summers (1986) argue in their seminal work on hysteresis, employment trends tend to persist over time, meaning that past labour market conditions influence current employment dynamics. Similarly, Rothstein (2012) emphasizes the importance of distinguishing between participation and employment rates when evaluating the health of the labour market, noting that employment figures more accurately reflect job accessibility and security.

Incorporating these controls helps to mitigate potential biases from extensive margin adjustments and enhances the robustness of our wage inequality analysis. This ensures that the observed disparities reflect underlying economic and institutional factors rather than temporary shifts in labour supply.

Hence, in addition to the control variables for lagged oil price returns, we introduce changes in the employment rate and its lags as control variables and estimate regression (1.17) and (1.19)-(1.21).

Figure 1-8 displays the impact of non-systematic monetary policy, and Figure 1-9 shows the asymmetric effect of non-systematic monetary policy. For Figure 1-8, the results show that an unexpected 25 basis points increase in the policy rate does not statistically affect the wage ratio. However, the Gini coefficient increases between the 8th (0.111%) and the 11th month (0.143%) after the shock.

Figure 1-8: Impact of Our Series on Wage Inequality: Controlling for Oil Price Returns and Employment Rate



Note: Figure 1-8 displays the impact of non-systematic monetary policy on the Gini coefficient and the wage rate at horizon h (25-basis-point), controlling for lagged oil price returns, changes in the employment rate and its lags. We plot 95% confidence intervals using heteroskedasticity-consistent standard errors of the IRFs. The impulse response coefficients are expressed in percentage terms.

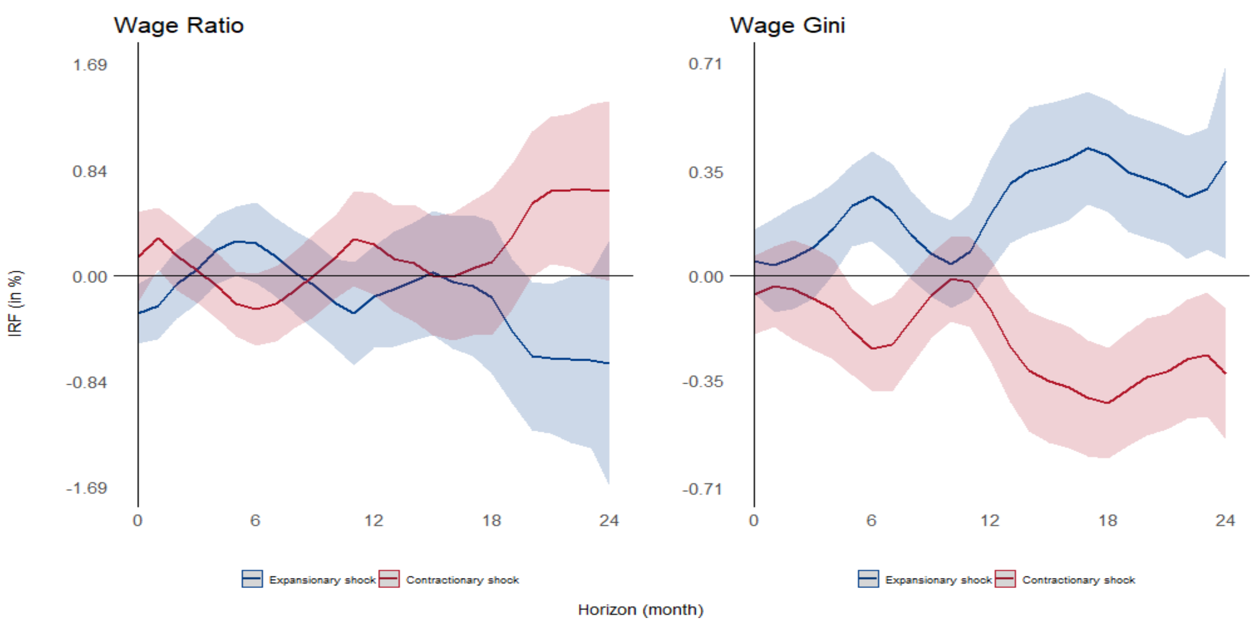
The results indicate that non-systematic monetary policy shocks have a statistically significant and symmetric effect on wage inequality. A contractionary monetary policy shock increases the wage ratio in the short and medium term, with an initial rise of 0.295% in the first month following the shock. This effect fades temporarily before reemerging in the 21st and 22nd months, reaching 0.676% and 0.678%, respectively. On the other hand, an expansionary monetary policy shock reduces the wage gap between high- and low-skilled workers. The gap narrows down by 0.306%, with the effect dissipating before reappearing between the 20th and 22nd months, leading to reductions of 0.642% and 0.673%, respectively.

Similarly, contractionary and expansionary monetary policy shocks exert symmetric effects on the wage Gini coefficient, with all significant impacts occurring in the short and medium terms. For example, a contractionary monetary policy shock reduces the Gini coefficient between the fifth (-0.187%) and seventh (-0.152%) months, with the decline fading before reemerging between the 13th (-0.239%) and 24th (-0.328%) months. Conversely, an expansionary monetary policy shock increases the Gini coefficient between the fourth (0.155%) and seventh (0.216%) months, and it further increases between the 12th (0.200%) and 24th

(0.382%) months.

These patterns remain robust when controlling for changes in the employment rate and its lags, on the one hand, and lagged oil price returns, on the other. The results confirm that non-systematic monetary policy shocks continue to exert statistically significant effects on wage inequality in the short and medium term. Specifically, while contractionary and expansionary monetary policy shocks have symmetric effects on the overall wage distribution — reflected in the Gini coefficient — they have opposite effects on the wage gap between high- and low-skilled workers. *Contractionary shocks tend to increase the wage ratio, whereas expansionary shocks reduce it, suggesting that the distributional effects of these policies differ across inequality measures.*

Figure 1-9: Asymmetric Effect of Non-Systematic Monetary Policy on Wage Inequality: Controlling for Employment Rate and Oil Price Returns



Note: Figure 1-9 displays the asymmetric effect of non-systematic monetary policy on the Gini coefficient and the wage ratio at horizon h (25-basis-point), controlling for lagged oil price returns, changes in the employment rate and its lags. We plot 95% confidence intervals using heteroskedasticity-consistent standard errors of the IRFs. The impulse response coefficients are expressed in percentage terms.

A Different Measure for Monetary Policy Shocks: Champagne and Sekkel (2018) Series

We use narrative-based monetary policy shock series constructed by Champagne and Sekkel (2018) for a robustness check. Champagne and Sekkel (2018) used a Romer and Romer (2004) approach to identify monetary policy shocks (ϵ_m) for Canada. Champagne and Sekkel (2018) use as (1.23) to construct the shock series:

$$
\begin{aligned}
\Delta i_m = {} & \alpha + \beta_1 i_{t-d14} + \sum_{h=1}^{3} \rho_h u_{t-h} + \sum_{j=-1}^{2} \gamma_j \hat{y}^f_{m,j} + \sum_{j=-1}^{2} \delta_j \pi^f_{m,j} \\
& + \sum_{j=-1}^{2} \theta_j \left(\hat{y}^f_{m,j} - \hat{y}^f_{m-1,j} \right) + \sum_{j=-1}^{2} \phi_j \left(\pi^f_{m,j} - \pi^f_{m-1,j} \right) \\
& + \beta_2 FFR_{t-d14} + \beta_3 ER_{t-d14} + \beta_4 \Delta FFR_{m-m_1} + \beta_5 \Delta ER_{m-m_1} + \epsilon_m,
\end{aligned}
\tag{1.23}
$$

where m is the Bank of Canada Governing Council meeting-by-meeting frequency, j is a quarter of the real-time data or forecast relative to the meeting date, $t-h$ and $t-d14$ represent the previous months and two weeks relative to the meeting date, respectively; parameters α, $\beta_1, \beta_1, \beta_2, \beta_3, \beta_4, \beta_5,\ \rho_h,\ h = 1, 2, 3,\ \gamma_j, \delta_j, \theta_j$ and $\phi_j,\ j = -1, \ldots, 2$ are regression coefficients. Therefore, (1.23) represents the regression of the change in the policy target rate Δi_m between two meetings on two-quarter-ahead Bank of Canada staff forecasts of real output growth ($\hat{y}^f_{m,j}$), inflation ($\pi^f_{m,j}$), and their nowcast and real-time one-quarter lag.

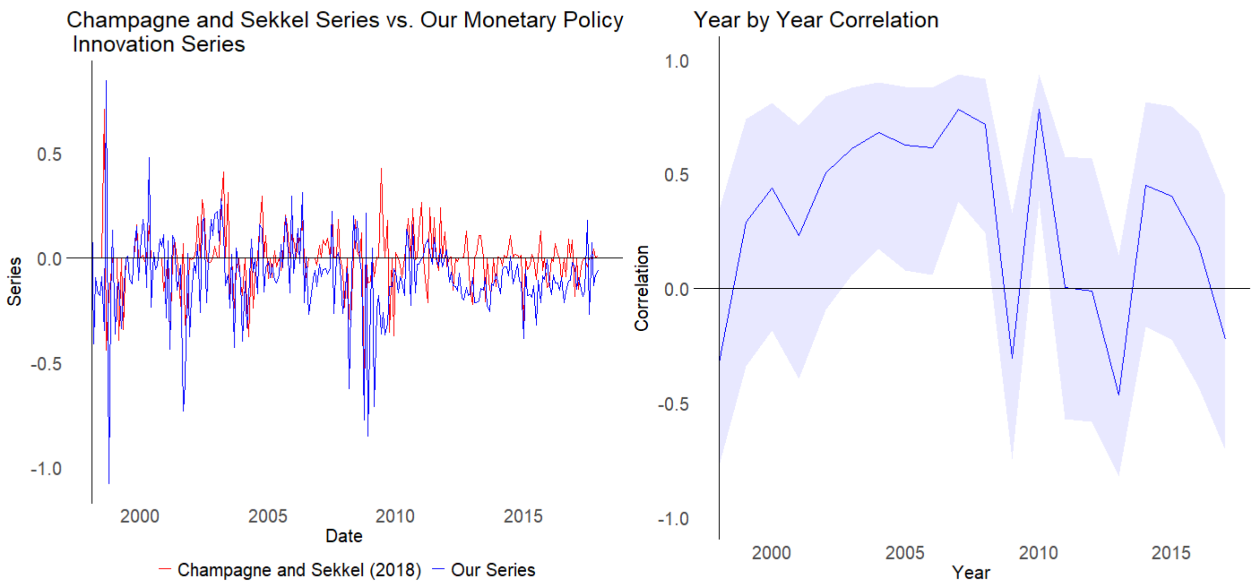
Champagne and Sekkel (2018) use a quarterly inflation rate series from 1973:Q1 to 2015:Q5, which combines different inflation proxies over this period. These proxies include total CPI inflation (from 1974:Q1 to 1979:Q4), core inflation (from 1980:Q1 to 2000:Q1), and CPIX inflation (from 2000:Q2 to 2015:Q3). Core inflation is defined as the CPI excluding food and energy, and also the effects of changes in indirect taxes. CPIX excludes the eight most volatile CPI components and adjusts the remaining components for the effects of indirect taxes. The gross output series is a quarterly dataset spanning from 1973:Q1 to 2015:Q5, calculated using the annualized quarterly real GDP growth rate (seasonally adjusted from 1982:Q2 to 2015:Q3) and the annualized quarterly real Gross National Expenditures (GNE) growth rates (seasonally adjusted from 1973:Q1 to 1982:Q1). For more details, see the annex of Champagne and Sekkel (2018). The regressors also include the revision of the forecasts relative to the previous round of forecasts ($\hat{y}^f_{m,j} - \hat{y}^f_{m-1,j}$ and $\pi^f_{m,j} - \pi^f_{m-1,j}$), the unemployment rate for the previous three months u_{t-h} and the policy target rate two weeks before the meeting. Champagne and Sekkel (2018) also include the levels and changes of the U.S. Federal Funds Rate (FFR_{t-d14}) and the USD/CAD dollar exchange rate (ER_{t-d14}) as control variables since Canada is a small open economy that can be influenced by the US economy.

The obtained series (ϵ_m) is exogenous to the Bank of Canada rate, inflation (CPI), gross domestic product (GDP), unemployment rate, and other macroeconomic indicators (*e.g.* export, import, industrial production), which makes it a suitable instrument to identify the impact of monetary shocks on income inequality. Furthermore, the series is monthly and covers January 1975 to October 2017. Therefore, we restrict our analysis from January 1997 to October 2017 in order to perform our estimation[12].

Figure 1-10 displays the Champagne and Sekkel (2018) series, our innovation series, and the year-over-year correlations between the two series. We observe a statistically significant positive correlation between the two series in two periods: between 2003 and 2008; and 2010 and 2011. The first period (2003-2008) corresponds to significant economic growth and global prosperity, driven by technological progress, globalization, and a commodity price boom. However, this period of prosperity abruptly ended with the global financial crisis (Blanchard, 2009; Bernanke, 2013). Despite the crisis, the Canadian economy performed well, benefiting from its status as a major commodity exporter and the relatively lower interest rates set by the Bank of Canada. These policies were designed to support economic growth and control inflation, enabling Canada to maintain financial stability despite the global financial crisis (Poloz, 2008; Bank of Canada, 2009; Cross, 2009; Harris, 2008). The second period(2010-2011) marks the post-crisis recovery, during which both the global economy and the Canadian economy rebounded. In many advanced economies, the recovery was supported by fiscal stimulus policies and accommodative monetary policies (Organisation for Economic Co-operation and Development, 2011; Bank of Canada, 2010).

[12]The initial series was available until 2015, but the authors kindly provided us with an extension of the series up to October 2017.

Figure 1-10: Champagne and Sekkel (2018) Shocks and our Monetary Policy Innovation Series



Note: Figure 1-10 displays Champagne and Sekkel (2018) series, it is correlated with our series in 95% confidence band.

We conduct robustness checks by estimating (1.1), (1.5), (1.17) and (1.19), and replacing our innovation series by the Champagne and Sekkel (2018) series, and we employ the same estimation methods. In the following sections, we first summarize the impacts obtained for the models without accounting for possible asymmetric effects and then analyze the asymmetric impact.

Impact of Champagne and Sekkel (2018) Monetary Policy Shock Series on Wage Inequality

Figure 1-11(a) illustrates the impact of non-systematic monetary policy from Champagne and Sekkel (2018) on wage inequality, without controlling for lagged oil price returns, changes in the employment rate and its lags. In contrast, Figure 1-11(b) presents the results when these factors are controlled for. To ensure accurate comparability, we also display the impact of our monetary policy (MP) innovation series in these figures, restricting it to the same period as the Champagne and Sekkel (2018) MP shock series (from 1997 to 2017).

The findings indicate that when oil price returns, changes in the employment rate and its lags are not controlled for, monetary policy shocks have a statistically significant impact on the wage Gini coefficient in the short and medium terms. Specifically, the wage Gini coefficient decreases from the first month (-0.22%) to the fourth month (-0.132%), and also decreases in the 13th (-0.211%) and the 16th (-0.159%) months after the shocks. The impact of these shocks on the wage Gini coefficient is similar to the effect of our non-systematic monetary policy shock in the short run (see Figure 1-11(a)).

The results for the wage ratio are similar: an unexpected 25-basis-point increase in the bank rate leads to a statistically significant decrease in the wage ratio, declining from the first month (-0.521%) after the shock to the 17th month (-0.628%).

Furthermore, when controlling for oil price returns, changes in the employment rate and its lags, the results indicate a statistically significant decrease in the wage ratio from the third month (-0.551%) to the sixtieth month (-0.611%).

These results indicate that the impact of non-systematic monetary policy on wage inequality is more pronounced when external factors, such as oil price returns, and the working-age population, are considered. However, the effect on the wage Gini coefficient is not statistically significant. On the other hand, our series temporarily reduces significantly the Gini coefficient between the 12th and 14th months. Furthermore, the results based on the wage ratio align with those of our non-systematic monetary policy shock analysis.

Figures 1-12(a) and 1-12(b) illustrate the asymmetric impact of the Champagne and Sekkel (2018) shock series on the Gini coefficient and the wage ratio. The results indicate distinct effects depending on whether the non-systematic monetary policy shocks are contractionary or expansionary and whether oil price returns, changes in the employment rate and its lags are controlled for.

In the first scenario, contractionary monetary policy shocks reduce the Gini coefficient in the short and medium term when the oil price returns, changes in the employment rate and its lags are not controlled for. The Gini coefficient declines significantly in the short term, from -0.354% in the first month to -0.269% in the fourth month, suggesting an initial reduction in wage inequality. It also decreases between the 14th and 19th months before fading. On the other hand, expansionary monetary policy shocks exhibit short- and medium-term effects, with the Gini coefficient increasing from 0.365% in the first month to 0.153% in the sixth month, and it further increases between the 14th and 19th months. These findings align with the results obtained when using our series.

Expansionary and contractionary monetary policy shocks have a symmetric effect on the wage ratio in the short and medium term. With a contractionary monetary policy shock, the wage ratio declines from the 3rd month (-0.635%) to the 18th month (-0.94%) and decreases further in the 22nd (-0.967%) and 23rd months (-1.333%). On the other hand, under an expansionary monetary policy shock, the wage ratio increases from the 3rd month (0.977%)

to the 17th month (1.072%), and this increase is statistically significant.

The findings when using the Champagne and Sekkel (2018) expansionary and contractionary monetary policy shocks align with the results obtained by using our series, though the timing of the effects differs. The second scenario, which controls for lagged oil price returns, and employment rate and its lags, yields similar results to the first scenario regarding the Gini coefficient and the wage ratio. Using our series in the context of the first scenario, the Gini coefficient decreases statistically significantly in the short and medium term. However, using the Champagne and Sekkel (2018) shock series, the Gini coefficient only decreases statistically significantly in the short term.

The impact on the wage ratio is the same with contractionary and expansionary monetary policy shocks across both scenarios. Using the two series yields a statistically significant decline in the wage ratio, indicating a narrowing wage gap between high-and low-skilled workers. However, with our series, the results show that both policy shocks increase the wage ratio in the short and medium term.

Figure 1-11: Impact of Champagne and Sekkel (2018) Series on Wage Inequality

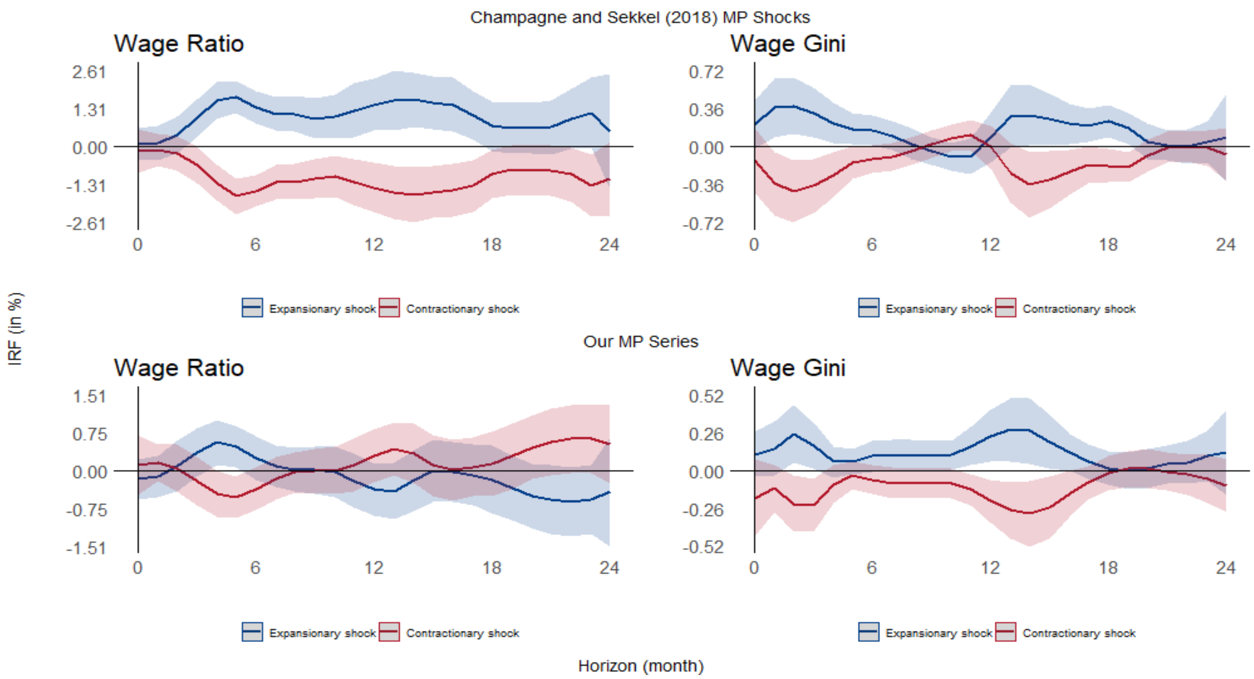
(a) Impact Without Controlling for Lagged Oil Price Returns and Employment Rate.

(b) Impact With Control for Lagged Oil Price Returns and Employment Rate.

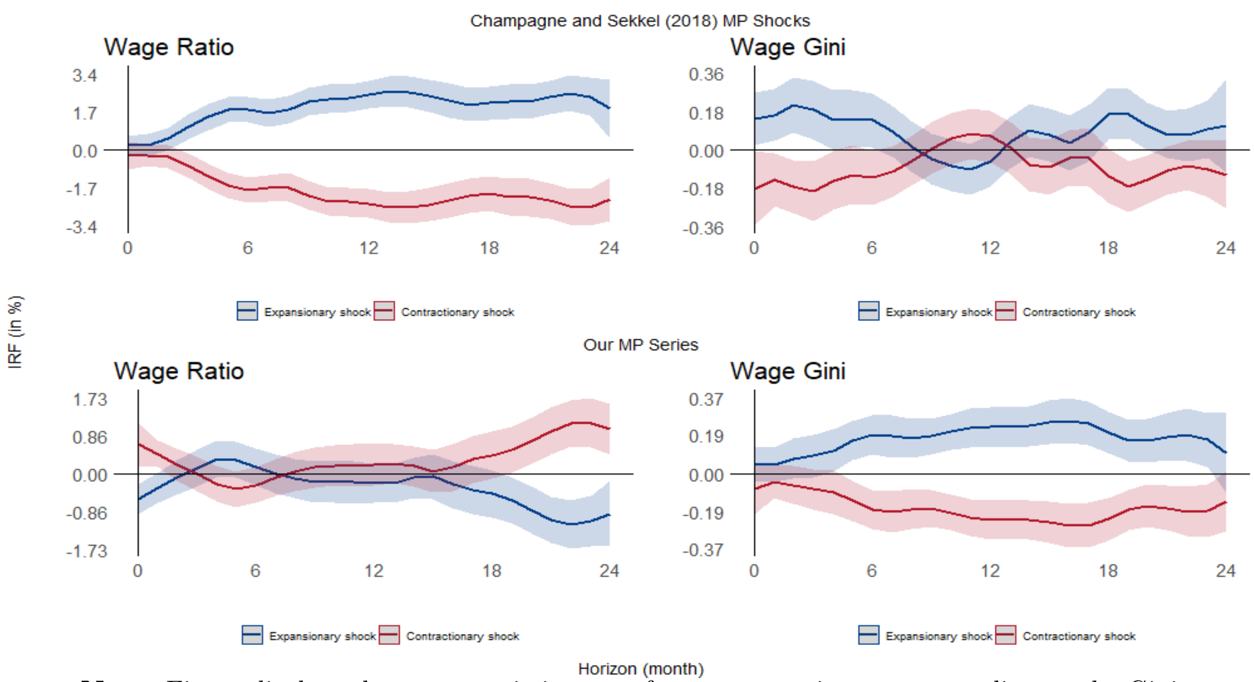
Note: Figure displays the impact of non-systematic monetary policy on the Gini coefficient and the wage ratio at horizon h (a 25-basis-point), controlling for lagged oil price returns, changes in the employment rate and its lags. The IRFs are estimated with the series range monthly from 1997 to 2017. We plot 95% confidence intervals using heteroskedasticity-consistent standard errors of the IRFs. The impulse response coefficients are expressed in percentage terms.

Figure 1-12: Asymmetric Impact of Champagne and Sekkel (2018) Monetary Policy Shock Series on Wage Inequality

(a) Impact without Controlling for Lagged Oil Price Returns and Employment Rate.



(b) Impact with Control for Lagged Oil Price Returns and Employment Rate.



Note: Figure displays the asymmetric impact of non-systematic monetary policy on the Gini coefficient and the wage ratio at horizon h (a 25-basis-point), controlling for lagged oil price returns, changes in the employment rate and its lags. The IRFs are estimated with the series range monthly from 1998 to 2017. We plot 95% confidence intervals using heteroskedasticity-consistent standard errors of the IRFs. The impulse response coefficients are expressed in percentage terms.

1.3.3 Discussion

Key Findings

Our analysis of the impact of non-systematic monetary policy on wage inequality reveals that these shocks have short- and medium-term effects on wage inequality. Specifically, our findings indicate that these monetary policy shocks tend to increase overall wage inequality in the short term. However, when we distinguish the effects by the direction of the shock, a more nuanced pattern emerges.

In the medium term, defined as periods beyond 12 months, a contractionary monetary policy shock leads to a significant decrease in the Gini coefficient while an expansionary monetary policy shock increases it. This suggests that, over time, with a contractionary monetary policy shock, the dispersion of wages narrows, potentially due to delayed labour market adjustments or redistributive effects driven by structural changes in employment and sectoral wage dynamics.

When we do not control for key macroeconomic factors, such as lagged oil price returns, changes in the employment rate and its lags, we find that non-systematic monetary policy shocks have no significant impact on the wage ratio. However, when these controls are introduced, the results change. In particular, an expansionary monetary policy shock reduces the wage gap between high-skilled and low-skilled workers in the medium term, with the most pronounced effects occurring between the 20th and 23rd months following the shock. This suggests that ignoring external economic forces may obscure the true distributional impacts of non-systematic monetary policy and highlights the importance of considering broader macroeconomic conditions in such analyses.

Our findings are consistent with those of Dolado et al. (2021), who emphasize the role of labour market frictions and capital-skill complementarity as critical mechanisms through which monetary policy influences wage structures. These factors create an additional channel that shapes wage inequality dynamics, as monetary policy interventions interact with firms' hiring decisions and investment in human capital.

Even after controlling for lagged oil price returns, changes in the employment rate and its lags, our results confirm that the Gini coefficient continues to decline over the medium term with contractionary monetary policy shock. This implies that while non-systematic monetary policy does affect the relative wage positions of different skill groups, the overall wage dispersion, at least as measured by the Gini coefficient, follows a downward trajectory.

Our findings show that non-systematic monetary policy shocks have heterogeneous effects on wage distribution. Although the direction of aggregate effects on inequality varies depending on the time horizon and the measure used, the key takeaway is that non-systematic monetary policy is not distributionally neutral over the two-year horizon considered. Its effects depend on labour market frictions, capital-skill complementarity, and external economic conditions, highlighting the complexity of its role in shaping wage inequality dynamics.

By comparing the results obtained with our series to those obtained with the Champagne and Sekkel (2018) shock series, we notice some divergences, particularly in the impacts on the wage ratio when controlling for lagged oil price returns and the labour market dynamics (changes in the employment rate and its lags).

Several factors can explain this significant difference. First, the two series use different strategies to assess non-systematic monetary policy shocks. The Champagne and Sekkel (2018) series employs a narrative-based approach, focusing on the change in the policy target rate between two meetings of the Governing Council, whereas our series is based on the Taylor rule. This difference in the methodology to obtain shock series can lead to variations in the interpretation and measurement of monetary policy shocks. Second, the Champagne and Sekkel (2018) series incorporates lagged changes in the USD/CAD exchange rate and federal funds rate as external control variables, potentially capturing international monetary influences.

On the other hand, our series emphasizes external demand conditions using the monthly U.S. GDP growth rate and its lags. These different external control variables can influence the observed impact on wage inequality. Third, the period covered by the two series is different. The Champagne and Sekkel (2018) series reflects the economic context from January 1997 to October 2017, encompassing both the pre- and post-financial crisis conditions of 2008 and the 2015–2016 oil crisis. By contrast, our series spans a more extended period, potentially capturing more recent economic events, including the COVID-19 pandemic and recent developments in monetary policy. These differences likely also contribute to the observed variations in the impacts on wage inequality and the Gini coefficient.

Mitigation of the Impact of Non-Systematic Monetary Policy on Wage Inequality

We now explore whether, and to what extent, public policies can mitigate the adverse effects of non-systematic monetary policy on wage inequality. The literature suggests that progressive taxation is an effective tool for reducing disparities in disposable income and narrowing the wage gap between high- and low-income earners (Bilbiie, 2024; Saez and Zucman, 2016). By redistributing income — through higher tax burdens on top earners and targeted social transfers — progressive tax systems can cushion households against the unequal effects of non-systematic monetary policy shocks. Canada operates a progressive income tax system alongside various federal and provincial social welfare programs, such as the Canada Child Benefit, Employment Insurance (EI), and social assistance initiatives. Moreover, tax policy varies across provinces and territories, with differences in income tax rates, corporate tax rates, capital gains taxation, and tax brackets. Since our wage inequality measures are based on gross earnings, they do not directly capture the redistributive effects of these fiscal instruments. It is therefore essential to control for relevant policy variables to better isolate the direct effect of non-systematic monetary policy shocks and assess whether the observed dynamics are shaped or attenuated by existing public policy frameworks.

To evaluate these mitigation effects, we incorporate public policy variables that are available monthly in the Statistics Canada Common Output Data Repository (CODR) (Statistics Canada, 2024a,c,b,d).[13] To capture fiscal policy effects, we derive a composite variable using principal component analysis (PCA), where the first principal component (considered as fiscal policy factor) represents changes in log of fiscal policy variables, including taxes on income, profits, and capital gains; payroll taxes; social benefits; employers' social contributions; and major wage settlements.

Consequently, we re-estimate (1.17) and (1.19)–(1.22) using monthly data and incorporating fiscal policy factor (from the PCA) and its lags in Δz_{t-l}, along with changes in the employment rate and its lags. To be clear, we continue to examine the impact of our series to evaluate fiscal policy mitigation. We assess both shorter- and longer-term impacts.

Figure 1-13 illustrates the asymmetric impact of non-systematic monetary policy on wage inequality, measured by the Gini coefficient and the wage ratio, within the framework of our SLP equation. The analysis controls for key macroeconomic factors, including lagged oil price returns; change in the employment rate and its lags; and the fiscal policy factor and its lags, ensuring that the estimated effects capture the net influence of monetary policy interventions.

The findings reveal that expansionary monetary policy shocks lead to a statistically significant increase in the Gini coefficient, beginning in the fourth month after the shock. This increase generally persists over time, with temporary reversals in the 10th, 11th, 22nd, 23rd,

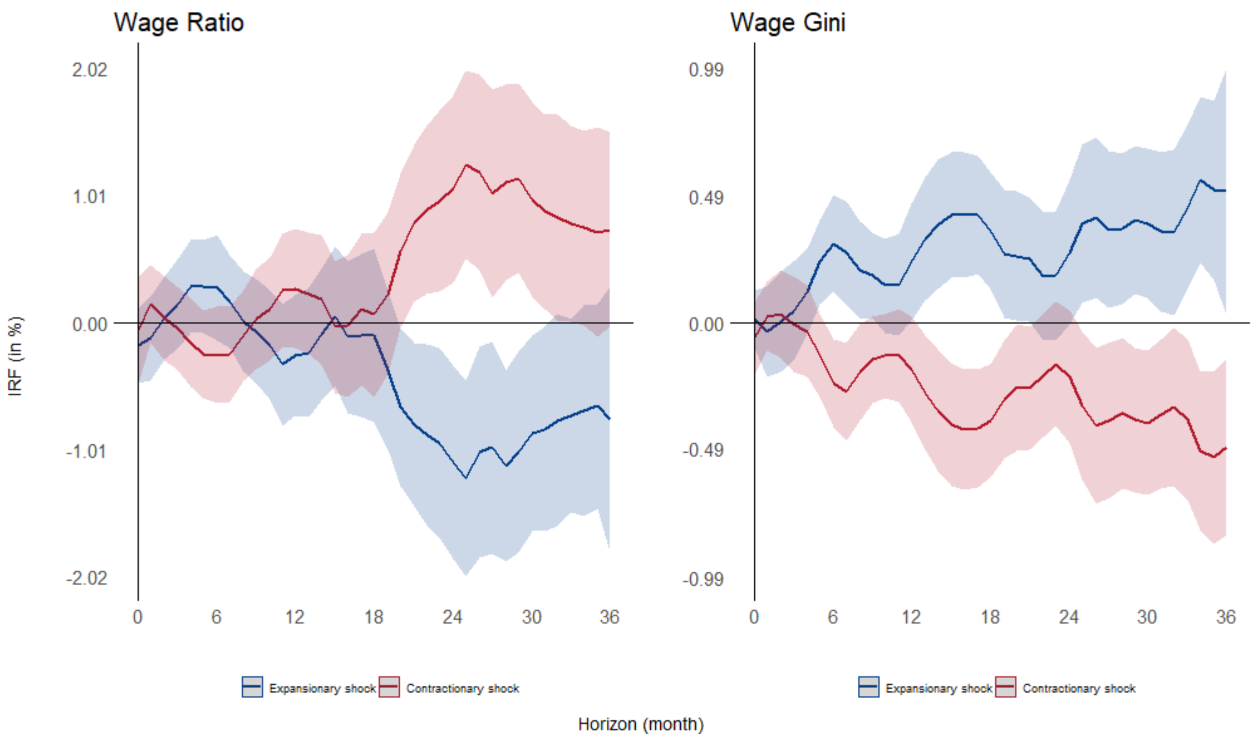
[13]We make use of several series to extract our factors. These include the Employment insurance benefit payments series; the Major wage settlements, number of employees series; the Wages, salaries and employers' social contributions series; and the Statement of government operations and balance sheet, government finance statistics series, which includes taxes on income, profits, and capital gains; taxes on payroll and workforce; and social benefits. These series are available quarterly, and we convert them into monthly series. We use the monthly consumer price index to transform the following into real terms: employment insurance benefit payments; wages, salaries, and employers' social contributions; taxes on income, profits, and capital gains; taxes on payroll and workforce; and social benefits.

and 24th months. The magnitude of the effect ranges from 0.236% in the fourth month to 0.509% by the 36th month. On the other hand, contractionary monetary policy shocks are associated with statistically significant reductions in the Gini coefficient, starting in the sixth month (-0.237%) and continuing through the 36th month (-0.486%), except for intermittent deviations between the ninth and 12th months and between the 22nd and 24th months. While both types of shocks affect wage inequality as measured by the Gini coefficient, their effects differ in magnitude and persistence. In particular, contractionary monetary policy shocks result in a more pronounced and sustained reduction in wage inequality over the medium to longer term, indicating a stronger influence on the overall wage distribution.

The analysis of the wage ratio further underscores the role of non-systematic monetary policy in shaping wage inequality. Controlling for fiscal policy factor and its lags, a contractionary monetary policy shock leads to a statistically significant increase in the wage ratio, beginning in the 21st month (0.792%) and persisting through the 33rd month (0.786%). This indicates that a contractionary monetary policy shock exacerbates wage disparities by widening the gap between high- and low-skilled workers in the longer term. Conversely, expansionary monetary policy shocks reduce the wage ratio, with statistically significant effects emerging in the 20th month (-0.674%) and continuing through the 31st month (-0.848%).

Overall, the results highlight the complex and multidimensional impact of non-systematic monetary policy on wage inequality. While contractionary and expansionary monetary policy shocks exhibit symmetric effects on the Gini coefficient — reducing or increasing overall wage inequality, respectively — their influence on the wage ratio indicates divergent outcomes for high- and low-skilled workers. Specifically, contractionary monetary policy shocks tend to widen the skill-based wage gap over the longer term, whereas expansionary monetary policy shocks help narrow it. These findings reveal the importance of fiscal policy in shaping distributional outcomes. By moderating the skill-based wage disparities amplified by non-systematic monetary policy, fiscal interventions — particularly those targeting low-income households — play a crucial role in promoting equity. Thus, addressing both the aggregate and distributional dimensions of wage inequality requires complementing monetary policy with well-designed fiscal interventions.

Figure 1-13: Asymmetric Effects of Non-Systematic Monetary Policy Shocks on Wage Inequality: Controlling for Oil Price Returns, for Employment Rate and for Fiscal Policy



Note: Figure 1-13 displays the asymmetric effect of non-systematic monetary policy on the Gini coefficient and the wage ratio at horizon h (25 basis points), controlling for lagged oil price returns, changes in the employment rate and its lags, fiscal policy factor and its lags. We plot 95% confidence intervals using heteroskedasticity-consistent standard errors of the IRFs. The impulse response coefficients are expressed in percentage terms.

An Evaluation of the Impacts in Dollar Terms

We assess the impact of non-systematic monetary policy on wage inequality in dollar terms using an asymmetric SLP regression that controls for lagged oil price returns, changes in the employment rate and its lags, and fiscal policy factor and its lags. Specifically, we measure the change in the monthly median wage of low-skilled workers between $t-1$ and $t+h$, where the horizon h is three years. The dollar impact on the monthly median wage of low-skilled workers is estimated, assuming that the monthly median wage of high-skilled workers remains fixed over three years. Although fixing high-skilled wages is a strong assumption, low-skilled wages are generally more responsive to monetary and business cycle shocks, while high-skilled wages tend to be more rigid to these shocks (see Dolado et al., 2021).

To estimate the change in the monthly median wage of low-skilled workers, we first use the estimated impulse response functions (IRF) from (1.17) and (1.19)-(1.22) to compute the growth rate of the wage rate. We then use the estimated growth rate to project the median wage of low-skilled workers at the horizon $t+h$, given the median wage of high-skilled workers at horizon $t-1$. Consequently, we evaluate the change in the median wage of low-skilled workers by comparing the projected median wage at $t+h$ with the median wage at $t-1$. For more intuitive and policy-relevant interpretation, we annualize the monthly median wage by multiplying it by 12, allowing us to analyze the impact on an annual rather than a monthly basis. In addition, we use the average monthly median wage for 2022 as a baseline for comparison. For example, let $\hat{\beta}^{+}(h)$ represent the estimated IRF for a contractionary monetary policy shock:

$$\begin{aligned}
\hat{\beta}^{+}_{(h)}(\delta) &= E\left(y_{t+h}-y_{t-1}|\Gamma_{t-1},\epsilon_t^{+}=\delta\right)-E\left(y_{t+h}-y_{t-1}|\Gamma_{t-1},\epsilon_t^{+}=0\right), && (1.24)\\
&= \left(y^{(1)}_{t+h}-y^{(1)}_{t-1}\right)-\left(y^{(0)}_{t+h}-y^{(0)}_{t-1}\right), && (1.25)\\
&= gr^{+(1)}_{t+h|t-1}-gr^{+(0)}_{t+h|t-1}, && (1.26)
\end{aligned}$$

where Γ_{t-1} is the set of information depending on Δy_{t-l} and $\Delta z_{t-l},\ l=1,\dots,p$. $y^{(1)}_{t+h}$. Here, $y^{(1)}_{t-1}$ is the logarithmic of the wage ratio after the shock in $t+h$ and $t-1$, respectively; $y^{(0)}_{t+h}$ and $y^{(0)}_{t-1}$ are the logarithmic of the wage ratio before the shock and δ is the size of the shock. Let $w^{q,(0)}_{t-1}$ be the median wage for low-skilled workers at $t-1$. The change in the wage of low-skilled workers due to the non-systematic monetary policy is defined as follows

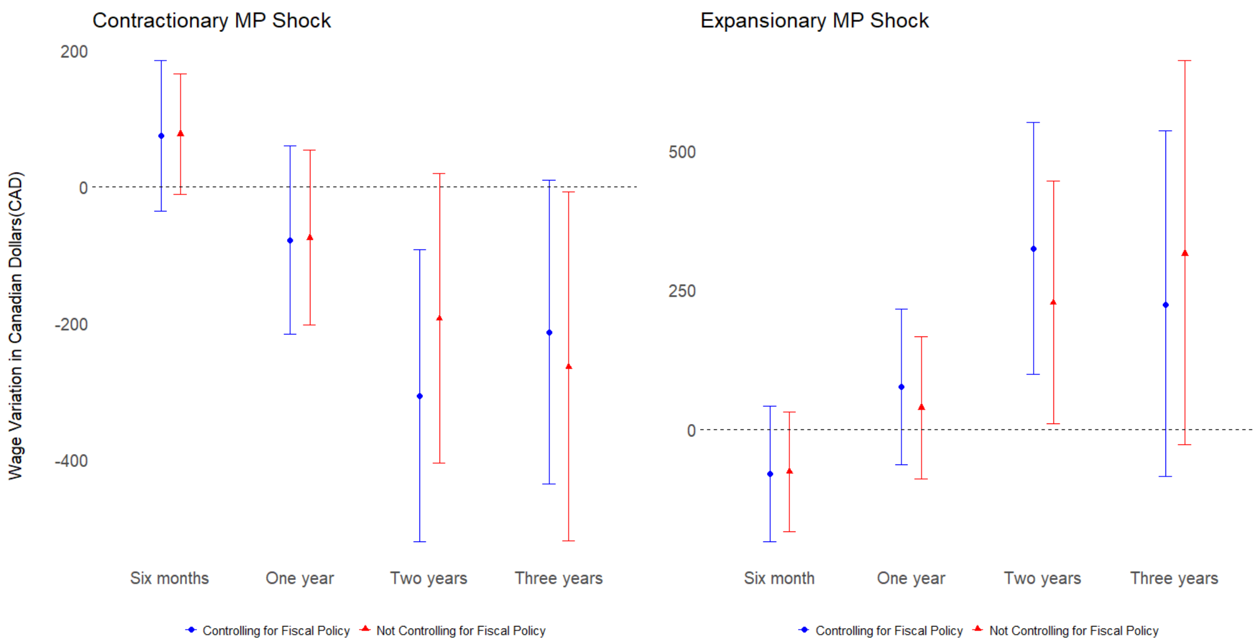
$$\Delta w^{q}_{t+h|t-1}=\left(e^{-\hat{\beta}^{+}_{(h)}}-1\right)w^{q,(0)}_{t-1}. \qquad (1.27)$$

Figure 1-14 illustrates the estimated medium-term effects of non-systematic monetary policy shocks on the median wage of low-skilled workers over different time horizons, such as six, twelve, twenty-four and 36 months, following a non-systematic monetary policy shock in 2022. The figure compares wage trajectories under expansionary and contractionary monetary policy shocks, with and without controlling for fiscal policy, enabling an assessment of the interaction between monetary and fiscal policy on low-skilled workers' wages.

We focus on periods in which the estimated effects of non-systematic monetary policy shocks are statistically significant. Comparing results with and without controls for fiscal policy factor and its lags highlights the important role that fiscal interventions play in shaping wage outcomes. Without controlling for fiscal policy factor and its lags, the estimated decline in the median annual wage of low-skilled workers following a contractionary monetary policy shock is CAD 264 over a three-year horizon. However, when fiscal measures are included in the specification, the estimated wage loss rises to CAD 307 after two years.

Interestingly, expansionary monetary policy shocks do not lead to wage increases in the medium term when fiscal policy is not considered. On the other hand, for this same shock, when fiscal policy is accounted for, wages increase by CAD 320 after two years, whereas without fiscal policy controls, the increase is CAD 225 over the same period.

Figure 1-14: Medium- and Longer-Term Effects of Our Monetary Policy Shocks on Low-Skilled Wages: Evaluated in Canadian Dollars (CAD)



Note: Figure 1-14 displays the medium-term effect of non-systematic monetary policy shocks on the median wage of low-skilled workers. The figure shows the gain or loss in wages over 6, 12, 24, and 36 months after the shock that is occured in 2022 (25 basis points). The blue and the red bands correspond to the limits of a 95 % confidence interval. We compare the impact before and after controlling for fiscal policy factor and its lags.

Across all scenarios, non-systematic monetary policy shocks exhibit minimal short-term effects (within the first year) on wages. The results show a symmetric but delayed impact of non-systematic monetary policy on the earnings of low-skilled workers.

These key findings provide important information. The Bank of Canada is not mandated to consider wage inequality when implementing monetary policy. However, public policies can mitigate the potential negative impacts of non-systematic monetary policy on wage inequality. We find that controlling for fiscal policy highlights their critical interaction with non-systematic monetary policy. Expansionary fiscal measures, such as direct transfers or public investments, can further support the reduction in wage inequality during contractionary monetary regimes. In contrast, expansionary fiscal policies could neutralize or even reverse these equity gains. Hence, the government can attenuate the adverse effects of non-systematic monetary policy through fiscal instruments such as income taxes, payroll taxes and social benefits.

In addition to tax policy, other public policies can help reduce wage inequality caused by non-systematic monetary policy. For example, the government can use minimum wage legislation to reduce wage inequality by increasing the minimum wage (Autor et al., 2016). Faia and Pezone (2024) show that when policymakers implement and improve minimum wage laws, workers at the bottom of the income scale receive wages that match basic living standards, reducing wage inequality. In Canada, each province and territory sets its own minimum wage and adjusts it periodically.

Implementing job training programs increases participants' earnings, reducing wage inequality (Smith and Sweetman, 2021). These programs are implemented at provincial and territorial levels in various forms. Collective bargaining can lead to higher wages and better working conditions for unionized workers compared to non-unionized workers (Farber, 2010; Card, 1996). Card et al. (2020) examine the changing relationship between unionization and wage inequality in Canada and the United States, and find that union reduces wage inequality more in the public sector than in the private sector. This suggests a channel that can mitigate the adverse impact of non-systematic monetary policy on wage inequality. Although Canada has collective bargaining and unions in provinces and territories, these policies vary significantly.

1.4 Concluding Remarks

Wage inequality remains a significant concern in many developed countries, including Canada. National and provincial policies aim to address and mitigate these inequalities, whereas monetary policy potentially plays a role in creating or exacerbating them. This paper delves into the impact of non-systematic monetary policy on wage inequality using two specific measures: the Gini coefficient and the wage ratio, the latter controlling for differences in skill levels. To accurately assess these effects, we obtain monetary policy innovation series from a standard Taylor rule, following the strategy employed in Samarina and Nguyen (2024), and we use SLP regression and the residualized shocks for obtaining impacts. We also examine the differential effects of contractionary and expansionary monetary policy shocks on these inequality measures, taking into account different economic and labour market dynamics.

The main findings of this paper are as follows:

- **Non-systematic monetary policy shocks have significant medium (one-year horizon)- and longer-term (two-year horizon) effects on wage inequality**. These persistent impacts underscore the need to consider distributional dynamics in the formulation of monetary policy.
- **Differentiated effects on the Gini coefficient and wage ratio**. Under the baseline specification, neither contractionary nor expansionary monetary policy shocks have a statistically significant impact on the wage ratio. However, both types of shocks exhibit symmetric effects on overall wage inequality, as measured by the Gini coefficient.
- **The role of external economic factors and labour market dynamics**. When controlling for lagged oil price returns and changes in the employment rate and its lags, contractionary and expansionary monetary policy shocks continue to exhibit symmetric effects on the overall wage distribution. However, their effects on the wage gap between high- and low-skilled workers diverge: contractionary monetary policy shocks tend to increase the wage ratio, while expansionary monetary policy shocks reduce it over the medium term.
- **Impact of jointly controlling for lagged oil price returns, changes in the employment rate and its lags, and fiscal policy factor and its lags**. Incorporating fiscal policy into the analysis offers further insights into the distributional consequences of non-systematic monetary policy shocks. When fiscal policy factor (derived from PCA) and its lags are controlled for, contractionary monetary policy shocks widen the skill-based wage gap over the longer term, whereas expansionary monetary

policy shocks help narrow it over the medium to longer term. These effects are notably more pronounced than in models without fiscal policy controls, highlighting the complex interaction between monetary and fiscal policy in shaping wage inequality outcomes.

Our findings make an important contribution to the literature on monetary policy and inequality. Indeed, the results highlight the complex role of non-systematic monetary policy in wage distribution and underscore the need for policymakers to consider broader economic conditions and complementary fiscal measures when designing policies to achieve equitable labour market outcomes. Fiscal policies, such as a progressive income tax system, minimum wage legislation, support for collective bargaining and unionization, and job training programs, could amplify the inequality-reducing effects of non-systematic monetary policy.

This study focuses on the impact of non-systematic monetary policy on wage inequality at the national level in Canada. However, given the different economic structures of the provinces, the impact of these shocks may vary regionally. Future research investigating the heterogeneous effects of non-systematic monetary policy shocks on wage inequality across provinces may suggest more targeted policy recommendations to federal and provincial policymakers, enabling the design of effective, region-specific interventions.

1-A Appendix

1-A.1 Use of AI in Research

In this study, OpenAI ChatGPT and Grammarly were used to check the grammar, catch typos, and enhance the writing to improve the clarity of the text.

1-A.2 Smooth Local Projection Estimation Method

To estimate (1.1), Barnichon and Brownlees (2019) use generalized ridge estimation, and the parameter estimate is given by[14]:

$$\hat{\theta} = \arg\min_{\theta} \left\{ \|\mathcal{Y} - \mathcal{X}\theta\|^2 + \lambda\theta' \mathbf{P}\theta \right\} = \left(\mathcal{X}'\mathcal{X} + \lambda \mathbf{P} \right)^{-1} \mathcal{X}'\mathcal{Y} \tag{1-A.1}$$

where $\mathcal{Y} = \mathcal{Y}_t$ for $t = 1, ..., T - H_{min}$ is a vector of $(y_{t+H_{min}} - y_{t-1}, ..., y_{min(T,t+H_{max})} - y_{t-1}{}')$, $\mathcal{X} = \mathcal{X}_t$ for $t = 1, ..., T - H_{min}$ is matrix defined by pooling horizontally the matrices $\mathcal{X}_{\alpha t}$, $\mathcal{X}_{\beta t}$, $\mathcal{X}_{\gamma_l t}$. $\mathcal{X}_{\alpha t}$ is $d_t \times K$ matrix with $\mathcal{X}_{\beta t(h,k)} = B_k(h)\Delta r_t$, and d_t is the dynamic length of $\mathcal{Y}_t$. Similarly, $\mathcal{X}_{\alpha t}$, and $\mathcal{X}_{\gamma_l t}$ are matrices which elements are respectively $B_k(h)$ and $B_k(h)\Delta y_{t-l}$. λ is a positive shrinkage parameter and $\mathbf{P}$ is a symmetric positive semi-definite penalty matrix. θ is a vector of B-splines parameters $(\{a_i, i = 1, ..., K\}, \{b_i, i = 1, ..., K\}, \{c_{ji}, i = 1, ..., K, j = 1, ..., p\})$. The solution of (1.25) is a standard least square regression when $\lambda = 0$, and when λ is very large, $\hat{\theta}$ is over-smooth. So, it is a problem of the trade-off between bias and variance. Barnichon and Brownlees (2019) select λ using k-fold cross validation. They derive the asymptotic variance of $\hat{\theta}$ using the Newey-West estimator:

$$\hat{V}(\hat{\theta}) = T \left[\sum_{t=1}^{T-H_{\min}} \mathcal{X}_t' \mathcal{X}_t + \lambda P \right]^{-1} \left[\hat{\Gamma}_0 + \sum_{l=1}^{L} w_l \left(\hat{\Gamma}_l + \hat{\Gamma}_l' \right) \right] \left[\sum_{t=1}^{T-H_{\min}} \mathcal{X}_t' \mathcal{X}_t + \lambda P \right]^{-1} \tag{1-A.2}$$

where $w_l = 1 - \frac{l}{1+L}$, $\hat{\Gamma}_l = \frac{1}{T} \sum_{l+1}^{T-H_{min}} \mathcal{X}_t' \hat{\mathcal{U}}_t \hat{\mathcal{U}}_{t-l}' \mathcal{X}_{t-l}$, and $\hat{\mathcal{U}}_t$ is regression residuals.

[14] In this paper, we are considering Barnichon and Brownlees (2019) notations.

1-A.3 Granger Causality Test

We have tested the innovation series for Granger-causality driven by reasonable predictors. That is, given the following regression

$$\epsilon_t = \sum_{j=1}^{K} \gamma_j \epsilon_{t-j} + \sum_{j=1}^{K} \alpha_j w_{t-j} + \mu_t, \tag{1-A.3}$$

We test the following hypothesis:

$$H_0 : \alpha_j = 0 \text{ for } j = 1, \ldots, K$$

where ϵ_t is a variable of interest, and w_t is a vector of predictor variables, which are lags of oil price returns, lags of the inequality measure, employment rate and fiscal policy factor (derived from PCA). In addition, the choice of $K = 24$ lags is motivated by Stock and Watson (2018). When our sample ends in 2021, we cannot reject the null hypothesis of no Granger causality at the 5% level. However, we do reject with the full sample. This presents the typical end-of-sample problem, whereby it is difficult to know whether this rejection is due to a structural change that occurred after 2021, or not. However, this is not an issue in the context of the residualized shock from Montiel Olea et al. (2025), given the econometric assumption that our innovation series is uncorrelated with the error term of the local projection, conditional on all control variables.

Table 1-A.1: Granger Causality Test for Our Innovation Series

	Gini coefficient			Wage ratio		
Time Sample	1997-2017	1997-2021	1997-2023	1997-2018	1997-2021	1997-2023
Test statistic	1.122	1.186	1.479	0.938	1.157	1.451
P-value	0.281	0.177	0.015	0.624	0.214	0.019

Note: Table 1-A.1 Displays the result from the Granger causality test of our innovation series using 24 lags with the two inequality measures.

Chapter 2

A Heterogeneity Test for Comparing Local Projection-Based Impulse Response Functions

By Koami Amegble

Abstract

This paper proposes a novel approach for comparing impulse response functions across groups of countries, regions, or monetary zones. Our test statistics are Wald-type statistics with a weighting matrix that accounts for cross-sectional dependency. Instead of using HAC correction for the time dependency, we propose a non-overlapping block bootstrap procedure for the test. We assess the properties of the test by simulation. Our main findings are: (1) our proposed test has good size and power properties in finite samples; (2) the χ^2 approximation does not suffice to control size; and (3) the test performs well for assessing short-, medium-, and long-term impacts, both non-cumulative and cumulative impacts.

2.1 Introduction

A widely used framework to estimate impulse responses in macro-economics relies on multi-horizon auto-regressions, or the so-called "local projections" (Dufour and Renault, 1998; Jordà, 2005; Olea and Plagborg-Møller, 2021; Plagborg-Møller and Wolf, 2021). These projections are linear regressions - over multiple horizons - where future outcomes are regressed on a current covariate,which serves as a shock indicator. In this framework, impulse responses are regression coefficients that can be estimated straightforwardly using: (i) least-squares univariately, that is, per horizon; or (ii) jointly, over a set of horizons (see Barnichon and Brownlees, 2019, for more details).

This paper examines a system of local projections across multiple horizons and several cross-sectional units. The latter may be countries, states or provinces that may be subject to a common shock. We also consider cumulative local projections following Stock and Watson (2018), where the regressand is the sum of future outcomes up to a given horizon. Our objective is to assess the homogeneity of responses to a common shock within the system, while imposing minimal restriction on cross-sectional and time series dependences.

This research question is policy-relevant, particularly when shock impacts spill over across cross-sectional units. Such spillovers arise when a common shock stems from shared economic or political structures, as in federations and monetary or trade regulatory zones. In this context, common shocks include monetary, fiscal, financial, and geopolitical disturbances, which can propagate both across and within multiple economic units.

Given the generality of these questions, it is important to specify the specification of cross-sectional dependence broad enough to capture potential spillovers, while ignoring it altogether is counterfactual. We are thus confronted with a system of regressions featuring a complex error structure across horizons and cross-sectional units, where no specific structure can be imposed on the disturbances that can be justified on reasonable economic grounds.

Indeed, despite the simplicity of local projections, their error terms are typically dependent on time. Inference can be conducted using standard serial correlation adjustments (Newey and West, 1987; Andrews, 1991; Andrews and Monahan, 1992; Robinson, 1998) or, alternatively, by lag augmentation (Olea and Plagborg-Møller, 2021). However, the finite sample performance of such adjustments is known to depend on the horizon and requires a large sample size. In the multi-horizon settings, complications arising from dimensionality can introduce severe distortions; see Dufour et al. (2013) and references therein. For the problem under consideration, the cross-sectional dimension increases the system's dimension even further, sometimes prohibitively (in the sense that estimates may not be invertible).

Our objective is to address such complications in a parsimonious way, that is, specifically to avoid degrees-of-freedom losses, as sample sizes that are typically available and relevant at the macro-economic frequency can be quite short.

To be specific our methodological objectives are as follows: (i) to build upon the validity of univariate least-squares estimation for each horizon and each cross-sectional unit; (ii) to avoid the usual corrections for serial dependence, following the results of Olea and Plagborg-Møller (2021); (iii) to build upon the efficiency prospects of joint (across horizons) estimation, following the results of Barnichon and Brownlees (2019); (iv) to avoid imposing any structure of the nature of cross-sectional dependence since economic theory does not provide guidance in this regard; (v) to capitalize on the successful application of bootstrap techniques for multiple hypothesis testing.

The dimensionality of the test problem can be alleviated through pair-wise comparisons of responses between unit pairs. However, such an approach requires size corrections due to multiple testing problems. A promising approach that can circumvent the cost of Bonferroni-type corrections involves bootstrap-based corrections; see *e.g.* Bergamelli et al. (2019), Khalaf et al. (2022), Dufour et al. (2015), and Harvey and Liu (2021). This method relies on carefully designed resampling schemes that replicate the dependence structure within the data at all levels. In this paper, we follow such an approach using a bootstrap-based Wald-type combined statistics. By Wald-type, we refer to a weighted sum of squares statistic, yet our proposed weight does not aim to fully standardize the criterion (to yield a χ^2 null distribution). As emphasized above, full standardization can be intractable or infeasible; instead, we provide a justification for the weights that we propose. Finally, we implement a customized bootstrap procedure to achieve the desired size correction, which we verify through simulation.

The justification for our proposed weighting matrix builds upon the so-called variance targeting approach of Engle and Mezrich (1996) and Noureldin et al. (2014). The key idea is to disentangle the estimation of time-invariant parameters from that of dynamic ones. To begin, we augment the considered regressions with lags. Next, ignoring time series dependences, we derive corrections for heteroskedasticity and time-invariant cross-sectional dependencies based on univariate least-squares residuals. The Wald statistic is computed with these corrections, ignoring further time series adjustments. To complete the procedure, we then use the non-overlapping block bootstrap method to correct for remaining dependencies (Politis and Romano, 1994; Davison and Hinkley, 1997; Härdle et al., 2003; Lahiri, 2003; Leśkow and Molenda, 2013; Kang et al., 2021).

We conduct a Monte Carlo simulation to assess the size and power of our proposed test.

Specifically, we simulate two models: one with cross-sectional dependency and one without. We consider the impulse response functions at horizons 1, 6, and 12 for simple impacts and cumulative responses. The main findings are: (1) our proposed test has good size and power properties in finite samples; (2) the χ^2 approximation does not suffice to control the size; and (3) the test effectively assesses short-, medium-, and long-term impacts for both non-cumulative and cumulative impulse response functions.

The rest of the paper is structured as follows: Section 2.2 provides the setup of our proposed comparison test and gives a brief overview of impulse response estimation with the local projection method. It also presents our bootstrap approach for implementing the test. Section 2.3 concerns our Monte Carlo simulation setup, and section 2.4 provides the simulation output, such as the size and power, along with comments and findings. The paper ends with concluding remarks.

2.2 Bootstrap Method for Impulse Response Function Comparison

We develop our proposed test for comparing impulse response functions by first introducing the local projection regression model and then incorporating our bootstrap approach. We aim to develop a test suitable for comparing impulse response coefficients when linear predictive (LP) models are estimated using univariate horizon-specific regressions. However, our inference procedure corrects for joint dependencies across both horizons and cross-sectional units.

2.2.1 Model Setup: Local Projections, and Tested Hypotheses

Local projections (LP) are multi-horizon autoregressions commonly used to estimate impulse response functions (Dufour and Renault, 1998; Jordà, 2005); see Plagborg-Møller and Wolf (2021) for a detailed derivation of linkages between Vector Autoregressive (VAR) based and LP-based estimations. We consider the LP system

$$y_{i,t+h} = \beta_{i(h)} x_{i,t} + \sum_{k=1}^{l} \gamma_{k,i(h)} z_{i,kt} + u_{i,t+h}, \quad t = l+1, ..., T-h, \tag{2.1}$$

$$i = 1, ..., m,\ h = h_0, ...,\ H, \tag{2.2}$$

where $y_{i,t+h}$, $x_{i,t}$ and $z_{i,kt}$ refer to observed variables at time t, horizon h and cross-sectional unit i (for example, a country, region or province), and $u_{i,t+h}$ is the regression disturbance.

Typically, the $z_{i,kt}$ variables include the lagged values[1] of $y_{i,t}$. m is fixed, and is not large relative to T. All time series are stationary, and disturbances can be heteroskedastic and dependent, over time, between horizons and cross-sectionally. No specific structure is assumed to model these dependencies, which can be quite complicated. The main contribution of this paper is to propose a tractable and reliable procedure that effectively accounts for these dependencies in inference.

In this context, the set of coefficients $\{\beta_{i(h)}, h = h_0, ..., \ H\}$ captures the impulse response function of $y_{i,t}$ with respect to $x_{i,t}$. The key assumption here is that the shock under consideration is observed through the indicator $x_{i,t}$, which is thus considered exogenous. In what follows, we set focus on the responses to a common shock, that is

$$x_{i,t} = x_t, \quad t = 1, ..., T. \tag{2.3}$$

While equation-by-equation ordinary least-squares (OLS) estimation is broadly considered for a given i (see Jordà, 2005, for more details), Barnichon and Brownlees (2019) emphasize the usefulness of joint estimation over the considered horizons. In this paper, we further consider the cross-sectional dimension of the system.

For presentation clarity, let us rewrite (2.1) in matrix form. To begin with, we introduce the stacked form

$$\boldsymbol{y}_{i(h)} = \boldsymbol{x}^{(h)}\beta_{i(h)} + \boldsymbol{z}_i^{(h)}\boldsymbol{\gamma}_{i(h)} + \boldsymbol{u}_{i(h)}, \quad h = h_0, ..., H, \quad i = 1, ..., m, \tag{2.4}$$

where $\boldsymbol{y}_{i(h)}$ is the $(T-h-l)\times 1$ vector of $y_{i,t+h}$ with $t = l+1, ..., T-h$, $\boldsymbol{z}_i^{(h)}$ is the $(T-h-l)\times l$ matrix of $z_{i,kt}$ with $k = 1, ..., l$ and $t = l+1, ..., T-h$, $\boldsymbol{x}^{(h)}$ is the $(T-h-l)\times 1$ vector of x_t with $t = l+1, ..., T-h$ and $\boldsymbol{u}_{i(h)}$ is the $(T-h-l)\times 1$ vector of $u_{i,t+h}$ with $t = l+1, ..., T-h$. We next further stack the regressions over the horizon, leading to

$$\mathcal{Y}_i = \mathcal{X}\boldsymbol{\beta}_i + \mathcal{Z}_i\boldsymbol{\gamma}_i + \mathcal{U}_i, \quad i = 1, ..., m, \tag{2.5}$$

$$N = K\left[T - l - \frac{H + h_0}{2}\right], \ K = H - h_0 + 1, \tag{2.6}$$

where $\mathcal{Y}_i$ is a $N\times 1$ vector that stacks the $\boldsymbol{y}_{i(h)}$ over $h = h_0, ..., H$, $\mathcal{U}_i$ is a $N\times 1$ vector that stacks the $\boldsymbol{u}_{i(h)}$ over $h = h_0, ..., H$, $\mathcal{X}$ is a $N\times K$ and $\mathcal{Z}_i$ is a $N\times Kl$ matrix such that $\mathcal{X}_i$ a bloc diagonal matrix of $\boldsymbol{x}^{(h)}$, $h = h_0, ..., H$, and $\mathcal{Z}_i$ a bloc diagonal matrix of $\boldsymbol{z}_i^{(h)}$, $h = h_0, ..., H$, conformably, $\boldsymbol{\beta}_i$ is a $K\times 1$ vector and $\boldsymbol{\gamma}_i$ is a $Kl\times 1$ vector.

[1] Note that $z_{i,kt}$ can also contain the lagged value of x_t; however, for simplicity, we assume that it only consists of the lagged value of $y_{i,t}$.

Our objective is to propose and validate a test for the null hypothesis

$$\boldsymbol{H}_0 : \boldsymbol{\beta}_i = \boldsymbol{\beta}_j, \ \forall i \neq j, \tag{2.7}$$

which can also be expressed in the context of (2.1) as:

$$\boldsymbol{H}_0 : \beta_{i(h)} = \beta_{j(h)} \ , \ \forall i \neq j \text{ and } \forall h. \tag{2.8}$$

$\boldsymbol{H}_0$ implies that the common shock has the same impact on the response variables across the cross-sectional units. In other words, our aim is to test whether the cross-sectional units respond homogeneously to shocks.

We also investigate the cumulative impact of the shocks on each outcome. As in Ramey and Zubairy (2018), we define a cumulative impulse response as the sum of the impulse responses over the K horizons[2] (see also Stock and Watson, 2018). This leads to the following regressions (the stacked form is similar to (2.4))

$$\sum_{h=h_0}^{H} \boldsymbol{y}_{i(h)} = \boldsymbol{x}^{(H)} \beta^c_{i(H)} + \boldsymbol{z}_i^{(H)} \boldsymbol{\gamma}^c_{i(H)} + \boldsymbol{u}_{i(H)}, \quad i = 1, \ldots, m, \tag{2.9}$$

where $\boldsymbol{x}^{(H)}$ is the $(T-H-l) \times 1$ vector of x_t with $t = l+1, \ldots, T-H$, $\boldsymbol{z}_i^{(H)}$ is the $(T-H-l) \times l$ matrix of $z_{i,kt}$ with $k = 1, \ldots, l$ and $t = l+1, \ldots, T-h$, and $\boldsymbol{u}_{i(H)}$ is the regression disturbance. $\beta^c_{i(H)}$ and $\boldsymbol{\gamma}^c_{i(H)}$ are cumulative impulse response functions associated to $\boldsymbol{x}^{(H)}$ and $\boldsymbol{z}_i^{(H)}$, respectively.

For convenience we express $\boldsymbol{H}_0$ in matrix form as

$$\boldsymbol{H}_0 : \boldsymbol{\Gamma} = \boldsymbol{0}, \text{ where } \boldsymbol{\Gamma} = \boldsymbol{R}\boldsymbol{\beta}, \tag{2.10}$$

[2]Our test procedure can also be applied to the long-difference specification (Furceri et al., 2018; Jordà et al., 2019; Jordà and Taylor, 2025), which captures the cumulative change in the impulse response functions:

$$\boldsymbol{y}_{i(H)} - \boldsymbol{y}_{i(-1)} = \boldsymbol{x}^{(H)} \beta^c_{i(H)} + \boldsymbol{z}_i^{(H)} \boldsymbol{\gamma}^c_{i(H)} + \boldsymbol{u}_{i(H)}, \quad i = 1, \ldots, m,$$

where $\boldsymbol{y}_{i(-1)}$ is the $(T-H-l) \times 1$ vector of $y_{i,t+h}$.

$\boldsymbol{\beta} = (\boldsymbol{\beta}_1, \ldots, \boldsymbol{\beta}_m)'$ a $mK \times 1$ vector and $\boldsymbol{R}$ is the $(m-1)K \times mK$ matrix

$$\boldsymbol{R} = \begin{bmatrix} \boldsymbol{I}_K & -\boldsymbol{I}_K & \mathbf{0}_K & . & . & . \\ \mathbf{0}_K & \boldsymbol{I}_K & -\boldsymbol{I}_K & . & . & . \\ . & . & . & . & . & . \\ . & . & . & \boldsymbol{I}_K & -\boldsymbol{I}_K & \mathbf{0}_K \\ . & . & . & \mathbf{0}_K & \boldsymbol{I}_K & -\boldsymbol{I}_K \end{bmatrix}. \tag{2.11}$$

2.2.2 Bootstrap-based Test for Impulse Response Function Comparison

Having defined the context, let us now present our proposed test of $\boldsymbol{H}_0$. Let

$$\hat{\boldsymbol{\beta}}_i = \left(\mathcal{X}_i' M_i \mathcal{X}_i\right)^{-1} \mathcal{X}_i' M_i \mathcal{Y}_i, \tag{2.12}$$

refer to the OLS estimators of $\boldsymbol{\beta}_i$, with

$$M_i = I_N - \mathcal{Z}_i \left(\mathcal{Z}_i' \mathcal{Z}_i\right)^{-1} \mathcal{Z}_i'. \tag{2.13}$$

Since there is no theoretical justification for imposing a specific structure on the disturbance distributions, the variance/covariance matrix of $\hat{\boldsymbol{\beta}}_i$ can be estimated using a heteroskedasticity autocorrelation consistent (HAC) approach. However, given the small sample sizes typical in low-frequency macroeconomic data, HAC corrections tend to perform poorly in multi-dimensional models (Dufour et al., 2013). In other words, using HAC corrections becomes unrealistic due to the loss of degree of freedom when using small sample sizes of data. To avoid these problems, we propose the following two-step approach.

Building on the insight of Engle and Mezrich (1996), our approach begins by separating time-invariant parameters from dynamic ones in time series models, a technique further refined by Noureldin et al. (2014). Time-invariant parameters encapsulate the stable, underlying structures of the data — features that remain constant over time — while dynamic parameters capture the evolving, time-dependent behaviors. In this study, the stable characteristics represent invariant heteroskedasticity across observational units, setting the stage for a deeper exploration of the data's structural dynamics.

To address this challenges, we introduce a novel correction to the variance-covariance matrix of the OLS estimators. This adjustment not only accounts for heteroskedasticity but also incorporates time-invariant cross-sectional dependencies, enhancing the robustness of our model. Specifically, the corrected variance-covariance matrix is given by:

$$\widehat{Var}\left(\hat{\boldsymbol{\beta}}_i\right) = \left(\mathcal{Q}_i^{'}\mathcal{Q}_i\right)^{-1}\sum_{t=1}^{N}\left(\mathcal{Q}_{i,t}^{'}\hat{\mathcal{V}}_{i,t}\right)\left(\hat{\mathcal{V}}_{i,t}^{'}\mathcal{Q}_{i,t}\right)\left(\mathcal{Q}_i^{'}\mathcal{Q}_i\right)^{-1}, \tag{2.14}$$

where $\hat{\mathcal{V}}_{i,t}$ and $\hat{\mathcal{Q}}_{i,t}$ are defined as follows

$$\widehat{\mathcal{V}}_i = M_i\widehat{\mathcal{U}}_i \tag{2.15}$$

$$\mathcal{Q}_i = M_i\mathcal{X} \tag{2.16}$$

with $\hat{\mathcal{U}}$ is the OLS residual in (2.4).

We use (2.14) within a Wald-type statistic to test the null hypothesis $\boldsymbol{H}_0$ (which we further define below). We, however, emphasize that this correction alone does not suffice to standardize the statistic, so the usual χ^2 approximation is not expected to hold. To address this issue, we apply a non-overlapping block bootstrap as a second step. This approach derives the p-value associated with the corrected Wald statistic, ensuring that we properly account for unmodeled time dependencies of a general form (Kang et al., 2021; Leśkow and Molenda, 2013; Härdle et al., 2003).

Given the matrix form of the null hypothesis as in (2.10), our proposed Wald-type statistic is:

$$T_w = \hat{\boldsymbol{\Gamma}}^{'}\boldsymbol{W}\hat{\boldsymbol{\Gamma}}, \tag{2.17}$$

where

$$\boldsymbol{W} = \left(\boldsymbol{R}\hat{\boldsymbol{\Sigma}}\boldsymbol{R}^{'}\right)^{-1}, \tag{2.18}$$

and based on (2.13) we set the conformable blocks of $\hat{\boldsymbol{\Sigma}}$ to:

$$\hat{\boldsymbol{\Sigma}}_{i,j} = \left(\mathcal{Q}_i^{'}\mathcal{Q}_i\right)^{-1}\sum_{t=1}^{N}\left(\mathcal{Q}_{i,t}^{'}\hat{\mathcal{V}}_{i,t}\right)\left(\hat{\mathcal{V}}_{j,t}^{'}\mathcal{Q}_{j,t}\right)\left(\mathcal{Q}_j^{'}\mathcal{Q}_j\right)^{-1}, \ \forall\, i,j = 1,\ldots,m. \tag{2.19}$$

As explained above, we apply a bootstrap procedure to this statistic to better approximate its distribution under the null hypothesis. Specifically, we implement a non-overlapping block bootstrap procedure for time series data (Härdle et al., 2003; Leśkow and Molenda, 2013). Our bootstrap approach is non-parametric in the sense that we do not resample under the null hypothesis; in this case, it is important to adjust the bootstrap statistic (see (2.21) below) so that the bootstrap OLS estimator $\hat{\boldsymbol{\Gamma}}^{(b)}$ is compared to its $\hat{\boldsymbol{\Gamma}}$ as derived from the observed sample (rather than to zero, the hypothesized value); see *e.g.* Dufour et al. (2019).

The following steps describe our proposed bootstrap procedure:

Step 1 Use the full sample and derive the OLS estimator $\hat{\boldsymbol{\beta}}_i$, for $i = 1, \ldots, m$ according to (2.12). With the estimate, compute the regression residuals $\hat{\mathcal{V}}_i$ following (2.15), and use it to derive the test statistics T_w according to (2.17).

Step 2 For $b = 1$ to B, generate $\hat{\mathcal{V}}_i^{(b)}$, $i = 1, \ldots, m$ by using simultaneous non-overlapping block bootstrap procedure[3]. The following procedure describes how to generate $\hat{\mathcal{V}}_i^{(b)}$:

Let us define $\hat{\mathcal{V}}$ be a $N \times m$ matrix such that column i is composed of $\hat{\mathcal{V}}_i$, for $i = 1, \ldots, m$. The goal is to create $N \times 1$ vector of indices by resampling among the row indices of $\hat{\mathcal{V}}$. Let $\mathcal{N} = \{\mathcal{N}_{h_0}, \ldots, \mathcal{N}_H\}$ when $\mathcal{N}_h$, $h = h_0, \ldots, H$ is a $(T-h-l) \times 1$ vector of indices, C be the number of non-overlapping blocks such that $N_h = A_h C$, where $N_h = T - h - l$. We first resample C block from $\mathcal{N}_h$, $h = h_0, \ldots, H$, and then combine them together in order to generate $\hat{\mathcal{V}}_i^{(b)}$. Given $\mathcal{A}_h = \{a^h_{(1)}, \ldots, a^h_{(C)}\}$ a set of indices which represent the order of each block, the j-block is created by performing the following steps:

1. Sample randomly with replacement a-th indice between A_h and N_h
2. The j-th block is defined as follows :

$$S^h_{(j)} = \{a^h_{(j)} - c, c = 0, \ldots, A_h - 1, \text{ and } a^h_{(j)} \in \mathcal{A}_h\},\ j = 1, \ldots, C \tag{2.20}$$

such that the length of $S^h_{(j)}$ is A_h.

Hence, the generated vector of indices of length N_h is comprised of $g_h^{(b)} = \{S^h_{(1)}, \ldots, S^h_{(C)}\}$, which is constructed by putting side by side each block according to the sampling order. Finally, $\hat{\mathcal{V}}$ is sorted according to $g^{(b)}$ to give $\hat{\mathcal{V}}^{(b)}$, where $g^{(b)} = \{g^{(b)}_{h_0}, \ldots, g^{(b)}_H\}$. Therefore, $\hat{\mathcal{V}}_i^{(b)}$ is given by column i of $\hat{\mathcal{V}}^{(b)}$. So with $\hat{\mathcal{V}}_i^{(b)}$, calculate $\hat{\mathcal{W}}_i^{(b)} = \mathcal{Q}_i \hat{\boldsymbol{\beta}}_i + \hat{\mathcal{V}}_i^{(b)}$.

Step 3 With $\left(\mathcal{W}_i^{(b)}, \mathcal{Q}_i\right)$ estimate $\hat{\boldsymbol{\beta}}_i^{(b)}$ according to (2.12). Using $\hat{\boldsymbol{\beta}}_i^{(b)}$ $i = 1, ..., m$ compute:

1. $\hat{\boldsymbol{\Sigma}}^{(b)}_{i,j}$ $\forall i$ and j and, $\boldsymbol{W}^{(b)}$,
2. $\hat{\boldsymbol{\Gamma}}^{(b)} = \boldsymbol{R}\hat{\boldsymbol{\beta}}^{(b)}$,
3. $T_w^{(b)}$ such that:

$$T_w^{(b)} = \left(\hat{\boldsymbol{\Gamma}}^{(b)} - \hat{\boldsymbol{\Gamma}}\right)' \boldsymbol{W}^{(b)} \left(\hat{\boldsymbol{\Gamma}}^{(b)} - \hat{\boldsymbol{\Gamma}}\right). \tag{2.21}$$

Step 4 With $T_w^{(b)}$, compute the p-value. We compute the bootstrap p-value as follows:

[3] We demean the regression residuals as follows : $\hat{\mathcal{V}}_i - \bar{\hat{\mathcal{V}}}_i$, where $\bar{\hat{\mathcal{V}}}_i$ is the average residuals.

$$p_b = \frac{1}{B}\sum_{b=1}^{B} \mathbf{1}\left(T_w \leq T_w^{(b)}\right). \tag{2.22}$$

We reject H_0 if $p_b < \alpha$, where α is the level of significance.

For the cumulative impact comparison test, we use the same approach. The null hypothesis for the cumulative impulse response function at horizon h is formulated as follows:

$$\boldsymbol{H}_{0,c} : \beta^c_{i(H)} = \beta^c_{j(H)},\ \forall i \neq j. \tag{2.23}$$

We can write (2.23) in matrix form, $\boldsymbol{H}_{0,c} : \mathbf{\Gamma}^c = \mathbf{\Lambda}\boldsymbol{\beta}$, with $\mathbf{\Lambda}$ a matrix of $(m-1) \times m$ such that

$$\mathbf{\Lambda} = \begin{bmatrix} 1 & -1 & 0 & . & . & . \\ 0 & 1 & -1 & . & . & . \\ . & . & . & . & . & . \\ . & . & . & 1 & -1 & 0 \\ . & . & . & 0 & 1 & -1 \end{bmatrix}. \tag{2.24}$$

the expression of test statistics:

$$T^c_w = \hat{\mathbf{\Gamma}}^{c\prime}\boldsymbol{W}^c\hat{\mathbf{\Gamma}}^c, \tag{2.25}$$

where,

$$\boldsymbol{W}^c = \left(\mathbf{\Lambda}\hat{\mathbf{\Sigma}}^c\mathbf{\Lambda}'\right)^{-1}, \tag{2.26}$$

where, $\hat{\mathbf{\Sigma}}^c$ is compute in the same way as in (2.19). The implementation of the test is similar to the non-cumulative impact one:

Step 1 Use the full sample and derive the OLS estimator $\hat{\boldsymbol{\beta}}_i$, for $i = 1, \ldots, m$ according to (2.5). Then, we generate non-cumulative regression residuals $\hat{\mathcal{V}}_i$ such that $\hat{\mathcal{V}}_i = \mathcal{W}_i - \mathcal{Q}_i\hat{\boldsymbol{\beta}}_i$, and use it to derive the test statistics T^c_w according to (2.20).

Step 2 This step is similar to step 2 of simple impact. Hence, we generate $\left(\mathcal{W}_i^{(b)}, \mathcal{Q}_i\right)$.

Step 3 With $\left(\mathcal{W}_i^{(b)}, \mathcal{Q}_i\right)$ estimate $\hat{\boldsymbol{\beta}}_i^{(b)}$ respect to (2.4). So with $\hat{\boldsymbol{\beta}}_i^{(b)}$, for $i = 1, \ldots, m$ compute:

1. $\boldsymbol{W}^{c,(b)}$,
2. $\hat{\mathbf{\Gamma}}^{c,(b)} = \mathbf{\Lambda}\hat{\boldsymbol{\beta}}^{(b)}$,

3. $T_w^{c,(b)}$ such that:

$$T_w^{(b)} = \left(\hat{\mathbf{\Gamma}}^{c,(b)} - \hat{\mathbf{\Gamma}}^{c}\right)' \boldsymbol{W}^{c,(b)} \left(\hat{\mathbf{\Gamma}}^{c,(b)} - \hat{\mathbf{\Gamma}}^{c}\right). \tag{2.27}$$

We reject $H_{0,c}$ if $p_b < \alpha$.

2.3 Monte Carlo Study

In this section, we examine the performance of our proposed a multi-hypothesis test by using a simulation study to assess its size and power.

2.3.1 Monte Carlo Design, Under the Null and Alternative Hypotheses

We consider the following data generation process (DGP):

$$y_{i,t} = \eta_i + \beta_i s_t + \gamma_i y_{i,t-1} + \epsilon_{i,t}\ ,\ t = 1, \ldots, T;\ i = 1, \ldots, m;\ y_{i,0} = 0. \tag{2.28}$$

The proposed DGP under the null hypothesis H_0, with m regions or countries, is defined such that $m = 3$, s_t are independent and identically distributed (iid) series shocks that follow $\mathcal{N}(0, 1)$ process, and the $\epsilon_{i,t}$ are iid given i. in addition, s_t and $\epsilon_{i,t}$ are independent. Equation (2.28) can be seen as a local projection regression model (Olea and Plagborg-Møller, 2021). In fact, for a given i, (2.28) can be written as follows:

$$y_{i,t+h} = a(\eta_i, \gamma_i, h) + b(\gamma_i, h) y_{i,t-1} + c(\gamma_i, h, \beta_i) s_t + u_{i,t+h}, \tag{2.29}$$

where:

$$\begin{aligned} u_{i,t+h} &= \sum_{l=0}^{h-1} \gamma_i^l \beta_i s_{t+h-l} + \sum_{l=0}^{h} \gamma_i^l \epsilon_{i,t+h-l}; && (2.30) \\ a(\eta_i, \gamma_i, h) &= \eta_i \frac{1 - \gamma_i^{h+1}}{1 - \gamma_i}; && (2.31) \\ b(\gamma_i, h) &= \gamma_i^{h+1}; && (2.32) \\ c(\gamma_i, h, \beta) &= \gamma_i^h \beta_i. && (2.33) \end{aligned}$$

Equation (2.29) represents the local projection regression form of (2.28). Notice we have also $E(u_{i,t+h} s_t) = 0$.

In (2.28), we set the parameters β_i, and γ_i, $i = 1, \ldots, m$ such that $\beta_1 = \beta_2 = \beta_3 = \beta$, $\gamma_1 = \gamma_2 = \gamma_3$, and η_i, $i = 1, \ldots, m$ can be different. Then, this setup allows for homogeneity in the lag coefficient γ_i and the shock coefficient β_i. Perfect homogeneity occurs when η_i, $i = 1, \ldots, m$ are the same. Here, the shock variable s_t is generated independently of the error terms ϵ_t outside the system.

Under H_0, the observational units respond homogenously to the shock variable. To account for dependency and non-dependency issues, we distinguish two scenarios. The first scenario incorporates dependency through the shock variable s_t. The shock series for the m observational units are the same. The second scenario incorporates dependency through the shock variable and the error terms. To illustrate, we compute the covariance between $y_{i,t}$ and $y_{j,t}$. Using (2.28), we can derive the moving average (MA) representation of $y_{i,t}$ as follows:

$$y_{i,t} = \frac{\eta_i}{1-\gamma_i} + \beta_i \sum_{l=0}^{\infty} \gamma_i^l s_{t-l} + \sum_{l=0}^{\infty} \gamma_i^l \epsilon_{i,t-l}, \ i = 1, \ldots, m. \tag{2.34}$$

With (2.34), we calculate the conditional expectations:

$$\begin{aligned} E\left[\left(y_{i,t} - \frac{\eta_i}{1-\gamma_i}\right)\left(y_{j,t} - \frac{\eta_j}{1-\gamma_j}\right)|G_t\right] &= E\left[\left(\beta_i \sum_{l=0}^{\infty} \gamma_i^l s_{t-l}\right)\left(\beta_j \sum_{l=0}^{\infty} \gamma_j^l s_{t-l}\right)|G_t\right] \\ + E\left[\left(\beta_i \sum_{l=0}^{\infty} \gamma_i^l s_{t-l}\right)\left(\sum_{l=0}^{\infty} \gamma_j^l \epsilon_{j,t-l}\right)|G_t\right] &+ E\left[\left(\beta_j \sum_{l=0}^{\infty} \gamma_j^l s_{t-l}\right)\left(\sum_{l=0}^{\infty} \gamma_i^l \epsilon_{i,t-l}\right)|G_t\right] + \\ E\left[\left(\sum_{l=0}^{\infty} \gamma_j^l \epsilon_{i,t-l}\right)\left(\sum_{l=0}^{\infty} \gamma_i^l \epsilon_{i,t-l}\right)|G_t\right], &\quad \text{where } G_t = \{s_t, s_{t-1}, \ldots, s_{t-k}, ...\}. \end{aligned} \tag{2.35}$$

The four expressions in (2.35) are as follows:

$$E\left[\left(\beta_i \sum_{l=0}^{\infty} \gamma_i^l s_{t-l}\right)\left(\beta_j \sum_{l=0}^{\infty} \gamma_j^l s_{t-l}\right)|G_t\right] = \left(\beta_i \sum_{l=0}^{\infty} \gamma_i^l s_{t-l}\right)\left(\beta_j \sum_{l=0}^{\infty} \gamma_j^l s_{t-l}\right), \tag{2.36}$$

$$E\left[\left(\beta_i \sum_{l=0}^{\infty} \gamma_i^l s_{t-l}\right)\left(\sum_{l=0}^{\infty} \gamma_j^l \epsilon_{j,t-l}\right)|G_t\right] = 0, \tag{2.37}$$

$$E\left[\left(\beta_j \sum_{l=0}^{\infty} \gamma_j^l s_{t-l}\right)\left(\sum_{l=0}^{\infty} \gamma_i^l \epsilon_{i,t-l}\right)|G_t\right] = 0, \tag{2.38}$$

and

$$E\left[\left(\sum_{l=0}^{\infty}\gamma_j^l\epsilon_{i,t-l}\right)\left(\sum_{l=0}^{\infty}\gamma_i^l\epsilon_{i,t-l}\right)|G_t\right]=\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}E\left[\epsilon_{i,t-l}\epsilon_{j,t-p}|G_t\right]. \tag{2.39}$$

(2.37) and (2.38) hold because of the exogeneity of the shock variable s_t, that is,

$$E\left[\epsilon_{i,t-l}|G_t\right]=0,\ \forall i. \tag{2.40}$$

For the first scenario, where the cross-sectional errors are independent, that is,

$$E\left[\epsilon_{i,t-l}\epsilon_{j,t-p}|G_t\right]=0,\ \forall p,l, \tag{2.41}$$

and the conditional expectation depends only on the shock variables. The unconditional covariance is [4]:

$$E\left[\left(y_{i,t}-\frac{\eta_i}{1-\gamma_i}\right)\left(y_{j,t}-\frac{\eta_j}{1-\gamma_j}\right)\right]=\frac{\sigma_s^2\beta_i\beta_j}{1-\gamma_i\gamma_j}. \tag{2.42}$$

Hence, (2.42) shows that the two observational units are dependent due to the same shocks. For the second scenario, where the model errors are dependent, that is,

$$E\left[\epsilon_{i,t-l}\epsilon_{j,t-p}|G_t\right]=\sigma_{l,p},\ \ \forall p,l, \tag{2.43}$$

the conditional covariance depends on the shock series and the covariance between the error terms of the two cross-sectional observations. Hence, the unconditional covariance is given by

$$E\left[\left(y_{i,t}-\frac{\eta_i}{1-\gamma_i}\right)\left(y_{j,t}-\frac{\eta_j}{1-\gamma_j}\right)\right]=\frac{\sigma_s^2\beta_i\beta_j}{1-\gamma_i\gamma_j}+\sum_{l=0}^{\infty}\sum_{p=0}^{\infty}\gamma_i^l\gamma_i^p\sigma_{lp}. \tag{2.44}$$

For the simulation purpose, and for the first scenario, the error terms are independent across the observed units, $\epsilon_{i,t}\sim\mathcal{N}\left(0,\sigma=1\right)$. In the second scenario, we set the variance-covariance matrix between $\epsilon_{i,t},\ i=1,2,3$ such that $var(\epsilon_{i,t})=\sigma_{ii}$ with $\sigma_{ii}=1,\ i=1,3$, and the covariances are $\sigma_{1,2}=0.2$, $\sigma_{1,3}=0.5$, and $\sigma_{2,3}=0.4$. Additionally, we address the size in the case of whether the DGPs have the same intercept (perfect homogeneity) or not.

To assess the power of the test, we define an alternative hypothesis where $\beta_i,\ i=1,2,3$ are different while the assumptions about γ_i, η_i and $\epsilon_{i,t}$ remain the same.

[4]Since $y_{i,t},\ i=1,\ldots,m,\ t=1,\ldots,T$ are stationary process we have $|\gamma_i\gamma_j|<1$.

2.3.2 Assessing the Size and Power of the Proposed Tests

To evaluate the size and power of the proposed test, we compute the probability of the rejection under H_0 or under H_a. Rejections will be based on the bootstrap p-value, as described above. Typically, we use M Monte Carlo simulations over B bootstrap replications. This approach is usually used to assess the statistics test's size and power (see Dufour et al., 2019; Li and Racine, 2013; MacKinnon, 2002).

The test size or type I error rate is the probability of rejecting the null hypothesis when it is true. We simulate data from the DGP under the null hypothesis and compute the test statistic in each case, along with its associated bootstrap p-value, using the algorithm described in Section 2.2.2. After numerous realizations of the DGP under H_0, we calculate the rejection rate of H_0; in other words, how frequently the process rejects the homogeneity assumption. This frequency represents the type I error. Let α denote the level of the test from which H_0 can be rejected, and $p_{b,k}$ the p-value from the bootstrap procedure under H_0, where k refers to the simulated sample. The empirical rejection of H_0 is computed using (2.45):

$$size\,(\alpha) = \frac{1}{M}\sum_{k=1}^{M} I\,(p_{b,k} < \alpha), \tag{2.45}$$

where $p_{b,k}$ is given by (2.22).

Given M number of Monte Carlo simulations, B bootstrap replications, m number of observational units ($m = 3$) and α the level of the test, we implement the following steps to evaluate the test's size:

(a) Simulate a sample $(y_{i,t}, s_{i,t}, \epsilon_{i,t})\,,\ i = 1, \ldots, m$, according to (2.28), with $\beta_1 = \beta_2 = \beta_3 = \beta$, and according to the cross-sectional dependence scenarios described in section 2.3.1;

(b) Derive the OLS estimator $\hat{\beta}_i,\ i = 1, \ldots, m$ according to equation (2.12), and compute $T_{w,k},\ k = 1, \ldots, M$;

(c) Given k, perform the bootstrap step described in section 2.2.2 to obtain the bootstrap distribution of $T^{(b)}_{w,k}$ for each simulation, and then derive the p-value $p_{b,k},\ k = 1, \ldots, M$ according to (2.22);

(d) Repeat steps (a), (b), and (c) to investigate the test's size according to (2.45).

The power of the test, or type II error rate, refers to the probability of correctly rejecting H_0 when it is false. To assess the power, we use the same algorithm described in section 2.2.2; the only change is that the DGP is generated under H_a. Therefore, we evaluate the test's power using (2.45).

More precisely, we assess the power of the test by performing the following steps:

(a) Simulate a sample $(y_{i,t}, s_{i,t}, \epsilon_{i,t})$ according to (2.28), with $\beta_1 \neq \beta_2 \neq \beta_3$, and according to the cross-sectional dependence scenarios described in section 2.2.2. Recall that $s_{i,t}$ and $\epsilon_{i,t}$ are the same process generated under H_0;

(b) Derive the OLS estimator $\hat{\beta}_i,\ i = 1, \ldots, m$ according to (2.12), and calculate $T^a_{w,k},\ k = 1, \ldots, M$;

(c) This step is similar to step (c) of the size-assessing algorithm. Refer to the algorithm above for size ;

(d) Repeat steps (a), (b), and (c) to investigate the test's power according to (2.45).

2.4 Simulation Results and comments

We assess size and power using $M = 1000$ Monte Carlo simulations and $B = 399$ bootstrap replications[5].

2.4.1 Size of Test

As the methodology section describes, we evaluate the test's size using the DGP given by (2.23). We estimate with and without cross-sectional DGPs, with sample sizes $T \in \{100, 300, 500\}$. We estimate impulse response functions for horizons $H \in \{1, 6, 12\}$ using the same shock variable for the observational unit models. For assessing the test size, we use various settings of IRF coefficient β. The size is assessed using various settings of the β: small shock ($\beta = 0.01$), moderate shock ($\beta = 0.25$) and persistence shock ($\beta = 0.9$). We also assess the size when the DGPs have the same intercept ($\eta_i = \eta = 0.1,\ i = 1, 2, 3$) and the DGP have different intercepts ($\eta_1 = 0.1, \eta_2 = 0.2, \eta_3 = 0.3$). We also assess the size in the context of non-cumulative and cumulative impacts.

To show the relevance of our bootstrap approach, we assess whether the test statistics deviate from the expected chi-square behavior, highlighting the necessity of the bootstrap correction. Figure 2-1 displays the distribution of test statistics under the correct model with $\beta = 0.25$ (homogeneity in terms of the impulse response functions) and DGPs with different intercepts ($\eta_1 = 0.1, \eta_2 = 0.2, \eta_3 = 0.3$).

[5]The test is implemented using R version 4.3.2. We use 24 cores to parallelize the simulation with the **doparallel** r package.

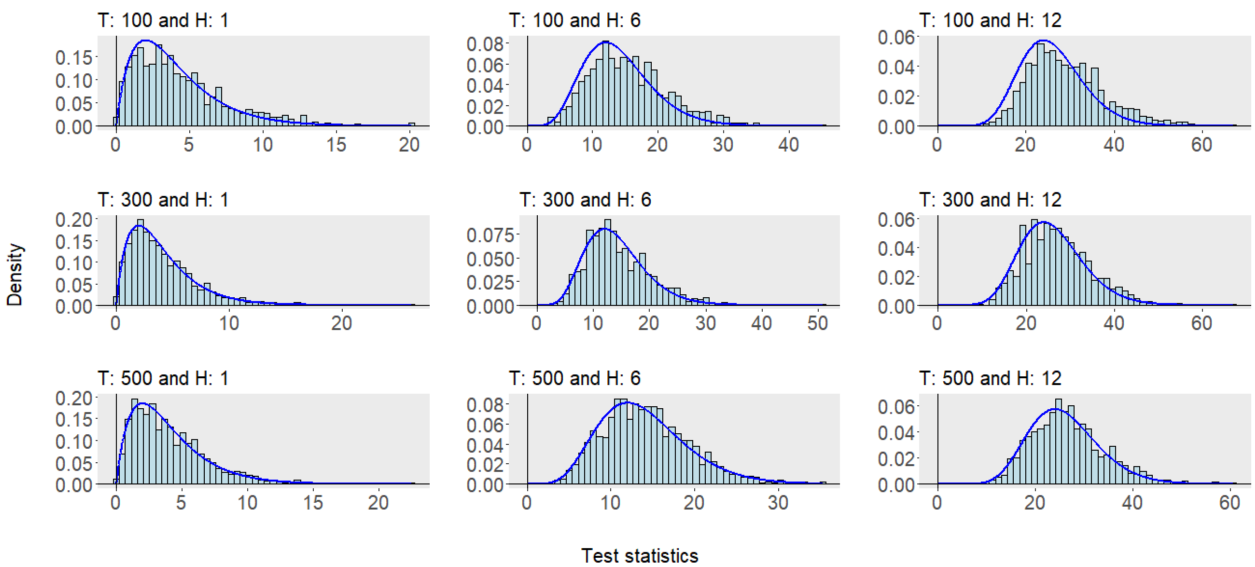
It displays a hypothetical χ_2 distribution of the statistics of the test, and it also displays the empirical empirical distribution of test statistics under H_0 (see the histograms in figure 2-1 and 2-2). Panels 2-1(a), 2-1(b) display the cross-sectional dependency and non-cross-sectional dependency histograms for $H = 1,\ 2,$ and 12. Each panel contains multiple histograms with different sample sizes $T \in \{100, 300, 500\}$. In addition, each plot compares the empirical histogram of the test statistics under the null hypothesis against the theoretical chi-square distribution (blue curve) with degrees of freedom corresponding to the respective H values ($df = 4,\ df = 14,\ df = 26$). In fact, according to our null hypothesis for the non-cumulative impact, the theoretical degrees of freedom of the test statistic should be $(m-1)K$. For example, for $H = 1$ and $m = 3$, we have $K = 2$, and the degrees of freedom are 4.

The empirical distributions (histograms) generally tend to follow the shape of the theoretical chi-square distribution (blue curve). However, there is a notable divergence between the two distributions. For example, when the sample sizes are small ($T = 100$), there is a notable deviation between the empirical and the theoretical distributions, particularly in the tails. Nevertheless, as T increases, the empirical distributions converge closely to the theoretical chi-square. In addition, as the horizon increases, there is a more pronounced deviation, with the empirical distribution showing more dispersion and heavier tails than the theoretical chi-square distribution. Comparing the results from the two models, cross-sectional vs. non-cross-sectional dependency, the analysis shows that under cross-sectional dependency, the empirical distribution deviates more from the theoretical distribution than in the non-cross-sectional case. Such deviation suggests that the limit theory of the test statistic may be less reliable when dependencies are present or the time horizon increases. In addition, with non-cross-sectional dependencies, even when the sample size is small, there is a significant deviation of the empirical distribution of the test statistic from its theoretical distribution, suggesting potential small-sample bias.

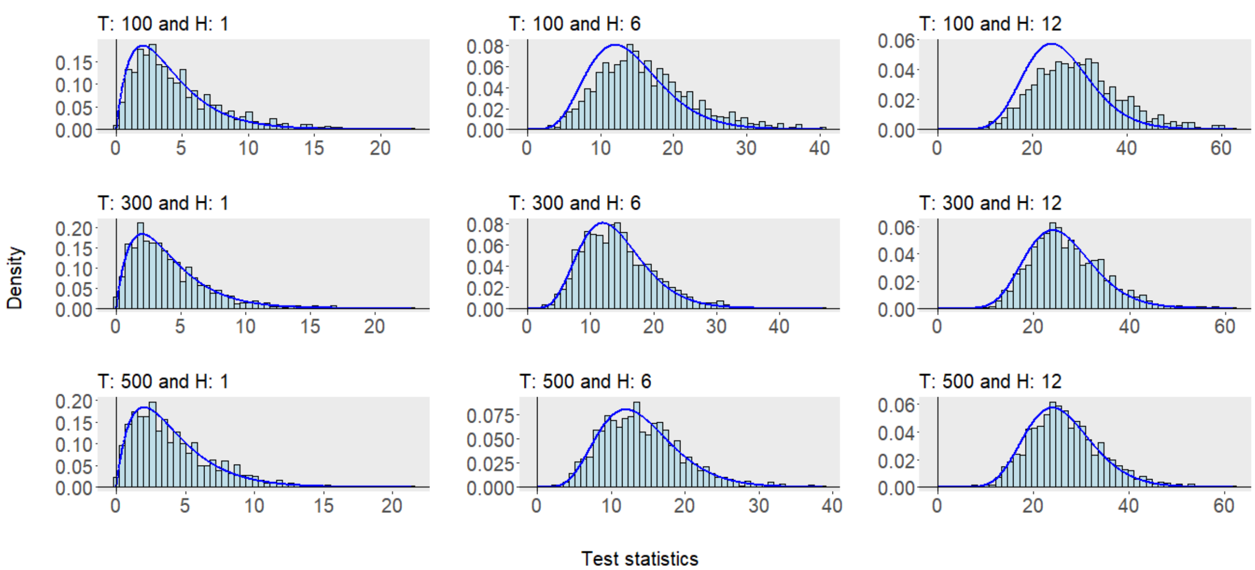
The results for the cumulative test statistics are displayed in Figure 2-2. In the case of cumulative impact, the theoretical degrees of freedom of the test statistics should be $m-1$; when $m = 3$, the degrees of freedom are 2. Comparing the two distributions, we observe that for the models with cross-sectional dependency, the empirical distribution of the test statistics tends to deviate more from the theoretical chi-square distribution. Additionally, as the horizon increases, the empirical distributions tend to exhibit heavy tails, resulting in more significant divergence. However, for the non-cross-sectional dependency model, the empirical distributions tend to align with the theoretical chi-square distribution. However, higher horizons lead to slight deviations from the theoretical distribution. Hence, using our bootstrap test approach, we will demonstrate below that we can better approximate the test statistic distributions to control the type I error rate.

Figure 2-1: Distribution of Test Statistics, Non-Cumulative Impact: Moderate Shocks ($\beta = 0.25$) with $\eta_1 = 0.1,\ \eta_2 = 0.2,$ and $\eta_3 = 0.3$

(a) Cross-Sectional Dependency



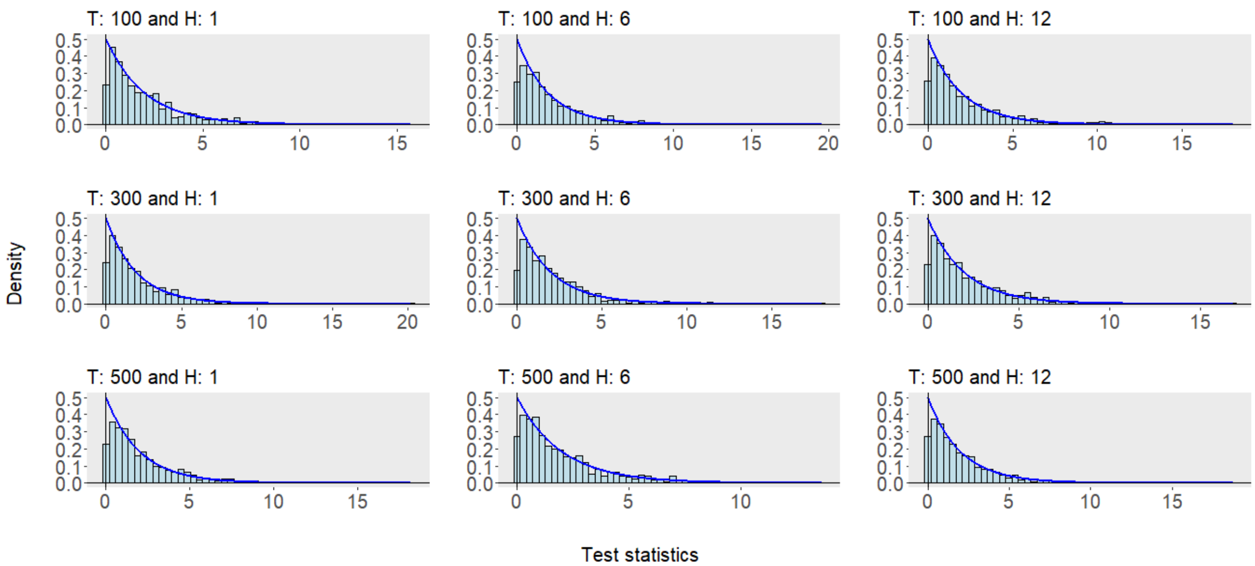
(b) Non Cross-Sectional Dependency



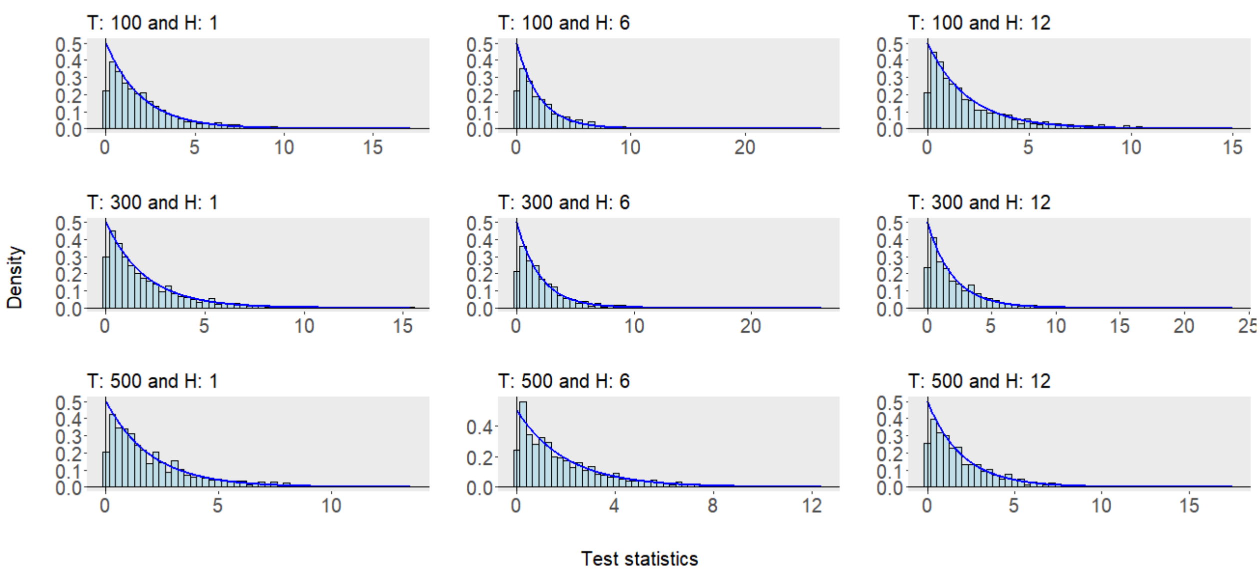
Note: Figure 2-1 Displays the histogram of test statistics for the cross-sectional and non-cross-sectional dependency models, with $H \in \{1, 6, 12\}$. The blue curve represents the theoretical chi-square distribution.

Figure 2-2: Distribution of Test Statistics, Cumulative Impact: Moderate Shocks ($\beta = 0.25$) with $\eta_1 = 0.1,\ \eta_2 = 0.2,$ and $\eta_3 = 0.3$

(a) Cross-Sectional Dependency



(b) Non Cross-Sectional Dependency



Note: Figure 2-2 Displays the histogram of test statistics for the cross-sectional and non-cross-sectional dependency models, with $H \in \{1, 6, 12\}$. The blue curve represents the theoretical chi-square distribution.

Table 2-1 presents the test size assessment results. The results concern both the size of cross-sectional dependency and non-cross-sectional dependency. On the left-hand side, the table displays the results for the case when the intercepts of DGP are not the same, and on the right-hand side, the results for the case when the intercepts are the same. The table also reports different scenarios of the shock impacts: small shock ($\beta = 0.01$), moderate shock ($\beta = 0.25$) and persistent shock ($\beta = 0.9$). The cumulative impact results are reported in Table 2-2. The size assessment results from Table 2-1 reveal that the size of the test under non-cross-sectional dependency is very close to the nominal level of 0.05 across all scenarios. The results are similar regardless of the true value of β (0.01, 0.25, 0.9) and the horizons H (1, 6, 12). Similar results are obtained for the cross-sectional dependency model when the size performance remains fairly stable and close to 0.05. The impact of the projection horizon on test size is minimal; the results indicate that the projection horizon has a negligible effect on test size, suggesting that the test remains robust across different horizons. In addition, as the sample size increases, the test maintains its size well under both dependency and no-dependency models.

Table 2-1: Size Based on $H_0: \ y_{i,t} = \eta_i + \beta_i s_t + \gamma_i y_{i,t-1} + \epsilon_{i,t}$ with $M = 1000, \ B = 399$

	Non Cumulative Impact											
	$\eta_1 = 0.1, \ \eta_2 = 0.2, \ \eta_3 = 0.3$						$\eta_1 = \eta_2 = \eta_3 = 0.1$					
	No dependency			Dependency			No dependency			Dependency		
	01	06	12	01	06	12	01	06	12	01	06	12
	$\beta_1 = \beta_2 = \beta_3 = 0.01$											
100	0.058	0.063	0.058	0.060	0.047	0.049	0.058	0.063	0.058	0.060	0.047	0.049
300	0.050	0.053	0.052	0.051	0.055	0.052	0.050	0.053	0.053	0.052	0.055	0.052
500	0.055	0.054	0.053	0.049	0.045	0.056	0.055	0.054	0.053	0.049	0.045	0.056
	$\beta_1 = \beta_2 = \beta_3 = 0.25$											
100	0.058	0.061	0.055	0.056	0.047	0.051	0.058	0.061	0.055	0.057	0.047	0.051
300	0.049	0.052	0.053	0.051	0.054	0.051	0.049	0.052	0.053	0.051	0.054	0.052
500	0.056	0.051	0.052	0.052	0.042	0.055	0.056	0.051	0.052	0.052	0.042	0.055
	$\beta_1 = \beta_2 = \beta_3 = 0.9$											
100	0.060	0.058	0.054	0.058	0.049	0.049	0.060	0.058	0.054	0.058	0.049	0.049
300	0.049	0.055	0.052	0.050	0.052	0.051	0.049	0.055	0.052	0.050	0.052	0.051
500	0.054	0.050	0.052	0.053	0.043	0.052	0.054	0.050	0.052	0.053	0.043	0.052

Note: Table 2-1 displays the size when using non-cross-sectional dependency with $\sigma_i = 1$ and $\sigma_{i,j} = 0$; cross-sectional dependency with $\sigma_i = 1$, $\sigma_{1,2} = 0.2$, $\sigma_{1,3} = 0.5$, and $\sigma_{2,3} = 0.4$ The sizes are assessed with $\alpha = 0.05$.

Concerning the cumulative impact results reported in table 2-2, the results show that for the non-cross-sectional model, the size remains close to the nominal level of 0.05 across all the projection horizons ($H = 1, 6, 12$ and sample size ($T = 100, 300, 500$). Similar results are obtained for the cross-section-dependency model, where the size generally remains close to the nominal level of 0.05. The deviations from the nominal level are minimal, revealing robust performance. The projection horizon has little effect on size performance, confirming stability across all specifications. As the sample sizes increase, the size tends to stabilize, particularly in the presence of cross-sectional dependency. The type of shocks (small, moderate, and persistent) does not affect the size performance across all specifications. Indeed, larger values of β (0.9) do not seem to introduce significant biases in size performance.

Table 2-2: Size Based on $H_0:\ y_{i,t} = \eta_i + \beta_i s_t + \gamma_i y_{i,t-1} + \epsilon_{i,t}$ with $M = 1000,\ B = 399$

	Cumulative Impact											
	$\eta_1 = 0.1,\ \eta_2 = 0.2,\ \eta_3 = 0.3$						$\eta_1 = \eta_2 = \eta_3 = 0.1$					
	No dependency			Dependency			No dependency			Dependency		
	01	06	12	01	06	12	01	06	12	01	06	12
	$\beta_1 = \beta_2 = \beta_3 = 0.01$											
100	0.051	0.047	0.055	0.044	0.048	0.048	0.051	0.047	0.055	0.044	0.048	0.048
300	0.047	0.059	0.059	0.044	0.048	0.051	0.047	0.059	0.059	0.044	0.048	0.051
500	0.070	0.057	0.058	0.059	0.055	0.056	0.070	0.057	0.058	0.059	0.055	0.056
	$\beta_1 = \beta_2 = \beta_3 = 0.25$											
100	0.051	0.046	0.052	0.046	0.046	0.051	0.051	0.046	0.052	0.046	0.046	0.051
300	0.049	0.059	0.058	0.045	0.047	0.047	0.049	0.059	0.058	0.045	0.047	0.046
500	0.070	0.055	0.059	0.061	0.056	0.057	0.070	0.055	0.059	0.061	0.056	0.057
	$\beta_1 = \beta_2 = \beta_3 = 0.9$											
100	0.051	0.047	0.055	0.048	0.043	0.054	0.051	0.047	0.055	0.048	0.044	0.054
300	0.048	0.063	0.058	0.045	0.044	0.043	0.048	0.063	0.058	0.045	0.044	0.043
500	0.069	0.058	0.060	0.061	0.058	0.055	0.069	0.058	0.060	0.061	0.058	0.055

Note: Table 2-2 displays the size when using non-cross-sectional dependency with $\sigma_i = 1$ and $\sigma_{i,j} = 0$; cross-sectional dependency with $\sigma_i = 1$, $\sigma_{1,2} = 0.2$, $\sigma_{1,3} = 0.5$, and $\sigma_{2,3} = 0.4$ The sizes are assessed with $\alpha = 0.05$.

Comparing the non-cumulative impact and cumulative impact results, we conclude our proposed test performs well in finite samples for both simple and cumulative effects, regardless of whether the model includes cross-sectional dependency or non-cross-sectional dependency, or the type of shocks. The probability of rejecting the null hypothesis is very low and close to the nominal value of 0.05. Furthermore, the test performs well for non-cumulative and cumulative impacts in the short, medium, and long runs.

2.4.2 Power of Test

Our alternative model exhibits a heterogeneous impact for impulse response functions, with coefficients of the shock variable allowed to differ across countries or regions. The number of

Monte Carlo simulations and bootstrap replications remains the same when evaluating the power of the test. We report the power for $T \in \{100, 300, 500\}$ and for significance level $\alpha = 0.05$ with total horizon $H \in \{1, 6, 12\}$.

The results in Table 2-3 provide insights into the power of the test for non-cumulative impulse response functions under varying conditions, including the presence of cross-sectional dependency. The results indicate that the test is powerful. The test frequently rejects the null hypothesis when it is not correctly specified. Larger sample sizes generally lead to higher power across all scenarios. Non-cross-sectional dependency significantly boosts power for all scenarios, particularly for small sample sizes and small parameter values. For example, with $\beta_1 = 0.01$, $\beta_2 = 0.1$, and $\beta_3 = 0.25$, and $T = 100$, the test's power increases from 0.233 for non-cross-sectional dependency to 0.453 for cross-sectional dependency model. This enhancement is more pronounced for small projection horizons ($H = 1$) and diminishes the power slightly as H increases. The test's power increases across all scenarios as the sample size increases, regardless of dependency or the magnitude of β. However, the power decreases as the projection horizon H increases, especially for small parameter values of β and for the non-cross-sectional dependency model.

Table 2-3: Power based on $H_0: \ y_{i,t} = \eta_i + \beta_i s_t + \gamma_i y_{i,t-1} + \epsilon_{i,t}$ with $M = 1000, \ B = 399$

	Non Cumulative Impact											
	$\eta_1 = 0.1, \ \eta_2 = 0.2, \ \eta_3 = 0.3$						$\eta_1 = \eta_2 = \eta_3 = 0.1$					
	No dependency			Dependency			No dependency			Dependency		
	01	06	12	01	06	12	01	06	12	01	06	12
	$\beta_1 = 0.01, \beta_2 = 0.1, \beta_3 = 0.25$											
100	0.233	0.150	0.110	0.453	0.231	0.159	0.233	0.150	0.110	0.453	0.230	0.159
300	0.626	0.389	0.267	0.936	0.785	0.658	0.626	0.390	0.267	0.936	0.785	0.658
500	0.870	0.656	0.524	0.996	0.972	0.919	0.870	0.656	0.524	0.996	0.972	0.919
	$\beta_1 = 0.25, \beta_2 = 0.35, \beta_3 = 0.50$											
100	0.251	0.155	0.120	0.465	0.248	0.168	0.251	0.155	0.120	0.464	0.248	0.168
300	0.658	0.425	0.293	0.952	0.812	0.691	0.657	0.425	0.293	0.952	0.811	0.691
500	0.889	0.698	0.562	0.997	0.978	0.939	0.889	0.698	0.562	0.997	0.978	0.939
	$\beta_1 = 0.5, \beta_2 = 0.7, \beta_3 = 0.9$											
100	0.569	0.337	0.235	0.876	0.622	0.446	0.569	0.337	0.234	0.876	0.621	0.446
300	0.981	0.879	0.776	1.000	0.998	0.987	0.981	0.879	0.776	1.000	0.998	0.987
500	1.000	0.992	0.971	1.000	1.000	1.000	1.000	0.992	0.971	1.000	1.000	1.000

Note: Table 2-3 displays the power when using non-cross-sectional dependency with $\sigma_i = 1$ and $\sigma_{i,j} = 0$; cross-sectional dependency with $\sigma_i = 1$, $\sigma_{1,2} = 0.2$, $\sigma_{1,3} = 0.5$, and $\sigma_{2,3} = 0.4$ The powers are assessed with $\alpha = 0.05$.

Table 2-4 reports the test's power for the cumulative impact. It shows the same output as in Table 2-3. The results indicate that the power increases with large sample sizes (T) and higher parameter values of β. For example, for non-cross-sectional dependency and for the small parameter values ($\beta_1 = 0.01,\ \beta_2 = 0.1,\ \beta_{0.25}$), the power is relatively low, especially for smaller sample sizes and longer projection horizons ($H = 12$). For the larger parameter values ($\beta_1 = 0.5,\ \beta_2 = 0.7,\ \beta_3 = 0.9$), the test's power tends to be 1 for the larger sample size ($T = 500$). The test's power is substantially enhanced for the cross-sectional dependency model, especially for small sample sizes ($T = 100$) and small parameter values ($\beta_1 = 0.01,\ \beta_2 = 0.1,\ \beta_{0.25}$). The improvement in power due to cross-sectional dependency is more pronounced for the shorter projection horizons ($H = 1$). However, the power of the test decreases across all scenarios as the projection horizon increases. The decrease in the test power as H increases is likely due to the increasing complexity of the regression residual structure in cumulative local projection regressions.

Table 2-4: Power based on $H_0:\ y_{i,t} = \eta_i + \beta_i s_t + \gamma_i y_{i,t-1} + \epsilon_{i,t}$ with $M = 1000,\ B = 399$

	Cumulative Impact											
	$\eta_1 = 0.1,\ \eta_2 = 0.2,\ \eta_3 = 0.3$						$\eta_1 = \eta_2 = \eta_3 = 0.1$					
	No dependency			Dependency			No dependency			Dependency		
	01	06	12	01	06	12	01	06	12	01	06	12
	$\beta_1 = 0.01, \beta_2 = 0.1, \beta_3 = 0.25$											
100	0.184	0.077	0.069	0.311	0.087	0.069	0.184	0.077	0.070	0.310	0.088	0.070
300	0.486	0.162	0.122	0.825	0.296	0.170	0.486	0.162	0.122	0.825	0.296	0.170
500	0.741	0.232	0.143	0.973	0.464	0.261	0.741	0.232	0.143	0.973	0.464	0.261
	$\beta_1 = 0.25, \beta_2 = 0.35, \beta_3 = 0.50$											
100	0.194	0.081	0.072	0.338	0.089	0.072	0.194	0.081	0.072	0.337	0.089	0.070
300	0.518	0.167	0.121	0.849	0.316	0.173	0.518	0.167	0.121	0.849	0.316	0.174
500	0.781	0.247	0.154	0.980	0.486	0.274	0.781	0.246	0.154	0.980	0.486	0.275
	$\beta_1 = 0.5, \beta_2 = 0.7, \beta_3 = 0.9$											
100	0.418	0.110	0.080	0.690	0.175	0.101	0.418	0.110	0.080	0.690	0.175	0.101
300	0.911	0.348	0.203	0.997	0.595	0.341	0.911	0.348	0.203	0.997	0.595	0.341
500	0.996	0.535	0.306	1.000	0.862	0.551	0.996	0.535	0.306	1.000	0.862	0.551

Note: Table 2-4 displays the power when using non-cross-sectional dependency with $\sigma_i = 1$ and $\sigma_{i,j} = 0$; cross-sectional dependency with $\sigma_i = 1$, $\sigma_{1,2} = 0.2$, $\sigma_{1,3} = 0.5$, and $\sigma_{2,3} = 0.4$ The powers are assessed with $\alpha = 0.05$.

2.4.3 Comments

The results of the test, in terms of size and power, are very informative. They show we can compare the impulse response functions across observational units, even when these models are estimated separately. Our proposed test statistics are thus able to account for cross-sectional dependency quite well when comparing these impulse response functions.

Our weighting matrix effectively controls for cross-sectional dependency regarding error terms and the presence of the shock variable across individual models. Our bootstrap method, utilizing the non-overlapping block bootstrap procedure, further corrects for serial dependency. Indeed, cross-sectional dependency significantly enhances the power of the test, particularly for small sample sizes and small projection horizons. This enhancement is attributed to our bootstrap method, which accounts for serial and cross-sectional dependency.

Our test has very good size and power properties regardless of the type of shocks (small, moderate or persistent), making it highly likely to detect heterogeneity in the model and suitable for a wide range of empirical applications. Moreover, our proposed test performs well over short horizons and has moderate power in long horizons, making it suitable for comparing the short- and medium-term impacts.

Our goal is not to derive the asymptotic properties of our proposed test; however, given our simulation findings on size and power, we can observe that it exhibits good properties in finite samples. Additionally, our simulation indicates that the limit distribution of our test statistics under the null hypothesis cannot be approximated by a χ^2 distribution. This finding validates our methodological choice: approximate the test statistics distribution under the null hypothesis by the bootstrap distribution following lag addition and cross-sectional corrections.

2.5 Concluding Remarks

Impulse response functions are a powerful tool for evaluating the impact of exogenous shocks on macroeconomic variables (e.g., gross domestic product, inflation, income, unemployment, inequality). When the shock indicator is the same, comparing IRFs across groups of countries, economic regions, or monetary zones can provide policymakers with crucial insights for designing effective interventions. This paper demonstrates that such cross-group IRF comparisons can be effectively conducted within a local projection framework, even when the models for individual countries or regions are estimated separately.

We propose a novel approach to facilitate these comparisons by developing a Wald-type test that incorporates a weighting matrix to account for cross-sectional dependency. To address serial dependencies in regression residuals, we implement a non-overlapping block bootstrap technique, which enhances the reliability of the test. Our analysis evaluates both non-cumulative and cumulative impacts, contrasting the test's performance in the presence and absence of cross-sectional dependency in error terms. Additionally, we assess the size and power of the test across different types of shocks (small, moderate and persistent).

The main finding of this paper can be summarized as follows:

1. The proposed test exhibits robust size and power properties in finite samples, ensuring its reliability across a range of empirical applications;
2. The χ^2 approximation is insufficient to maintain accurate size control, underscoring the need for our proposed modification;
3. The test performs well in assessing impacts across short-, medium-, and long-term impacts, both non-cumulative and cumulative impulse response functions.

While this paper focuses primarily on testing the heterogeneity of a common shock's impact across observed units, the proposed test can be easily adapted to compare IRFs between two specific units and across multiple horizons. Moreover, although our methodology was developed within the local projection framework, it can be extended to a vector autoregressive (VAR) context, broadening its applicability.

This study does not address the theoretical derivation of the limit distribution of the proposed test statistic, leaving this as an avenue for future research. A deeper exploration of the theoretical underpinnings of the test will further strengthen its utility and provide deeper insights into its performance in large samples.

In summary, our contribution offers a practical and flexible method for comparing IRFs, equipping policymakers and researchers with a valuable tool for evaluating the heterogeneous impacts of economic shocks across regions and units.

2-A Appendix

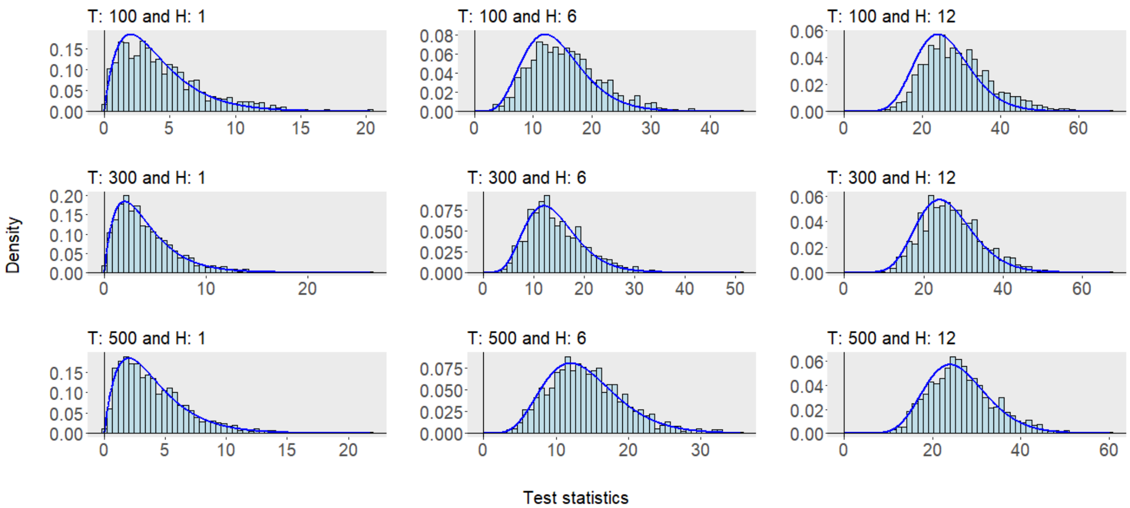
2-A.1 Use of AI in Research

In this study, OpenAI ChatGPT and Grammarly were used to check the grammar, catch typos, and enhance the writing to improve the clarity of the text.

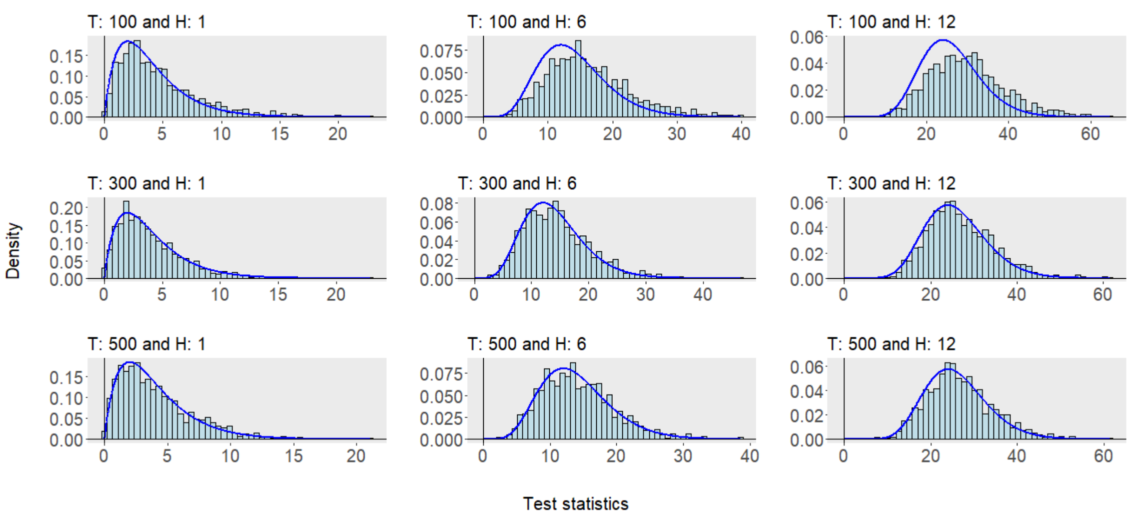
2-A.2 Distribution of Test Statistics Under Null Hypothesis for Different Types of Impact

Figure 2-A.1: Distribution of Test Statistics, Non-Cumulative Impact: Small Shocks ($\beta = 0.01$) with $\eta_1 = 0.1$, $\eta_2 = 0.2$, and $\eta_3 = 0.3$

(a) Cross-Sectional Dependency



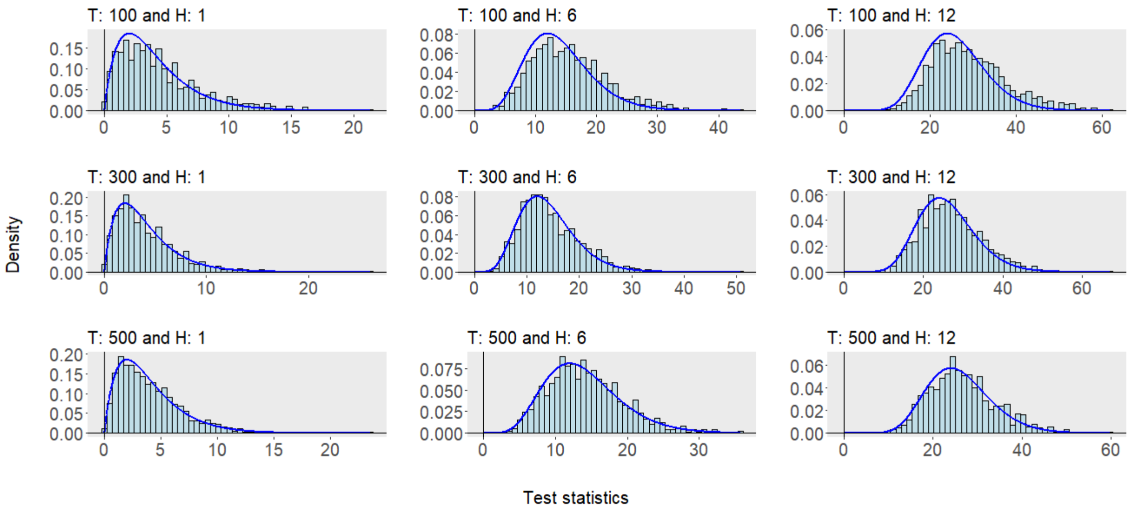
(b) Non Cross-Sectional Dependency



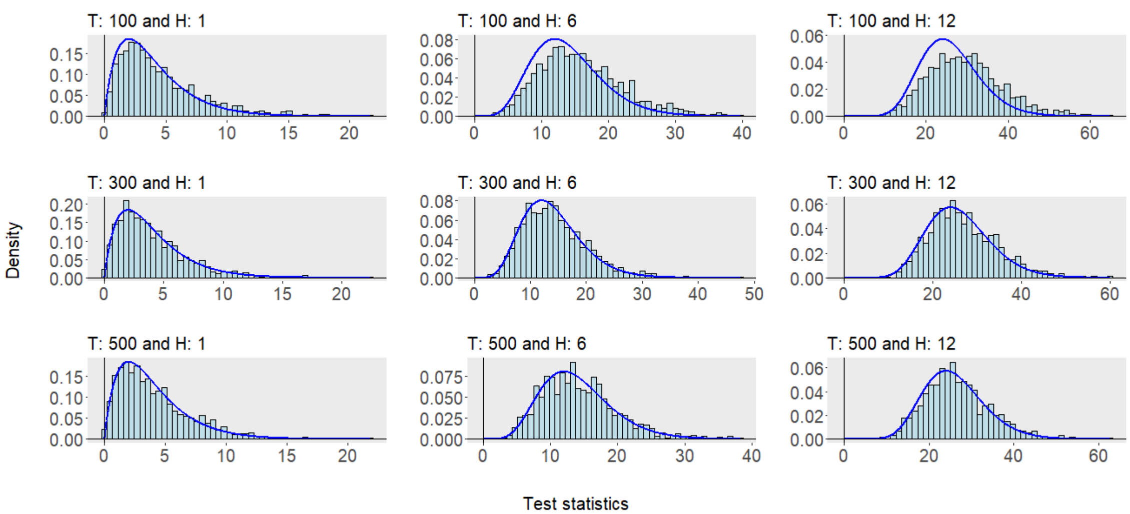
Note: Figure 2-A.1 displays the histogram of test statistics for the cross-sectional and non-cross-sectional dependency models, with $H \in \{1, 6, 12\}$. The blue curve represents the theoretical chi-square distribution.

Figure 2-A.2: Distribution of Test Statistics, Non-Cumulative Impact: Persistent Shocks ($\beta = 0.9$) with $\eta_1 = 0.1$, $\eta_2 = 0.2$, and $\eta_3 = 0.3$

(a) Cross-Sectional Dependency



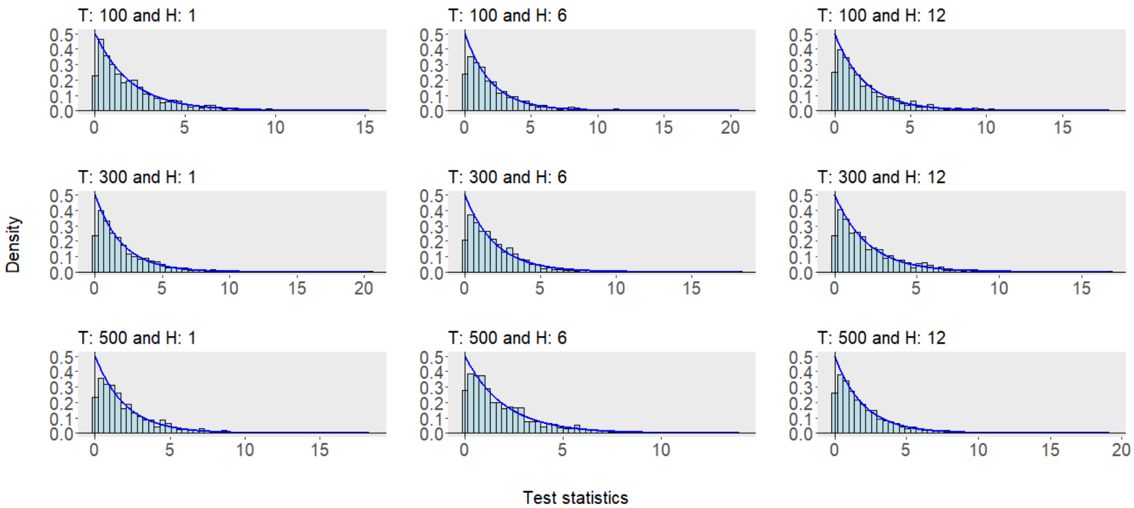
(b) Non Cross-Sectional Dependency



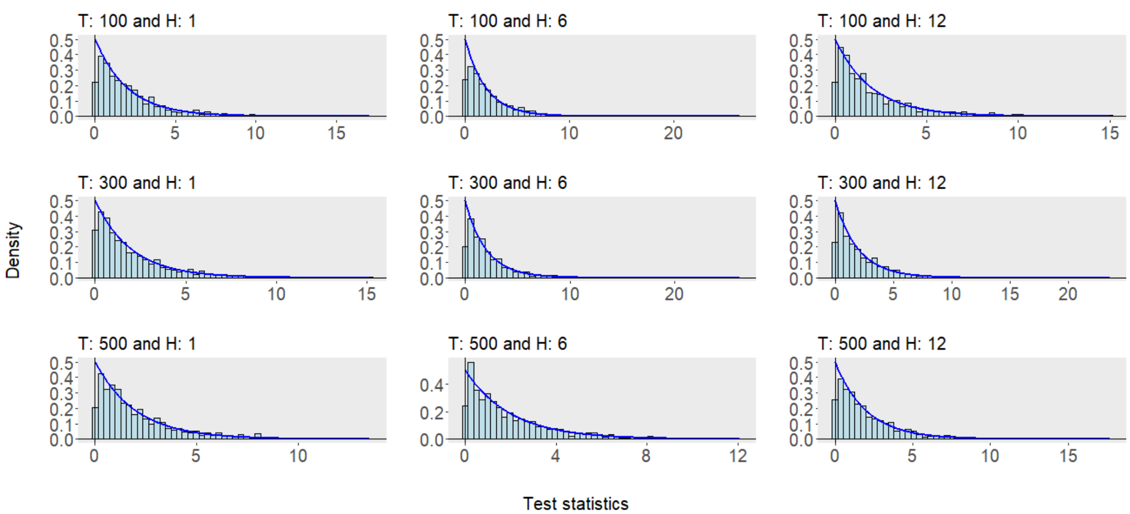
Note: Figure 2-A.2 Displays the histogram of test statistics for the cross-sectional and non-cross-sectional dependency models, with $H \in \{1, 6, 12\}$. The blue curve represents the theoretical chi-square distribution.

Figure 2-A.3: Distribution of Test Statistics, Cumulative Impact: Small Shocks ($\beta = 0.01$) with $\eta_1 = 0.1,\ \eta_2 = 0.2,$ and $\eta_3 = 0.3$

(a) Cross-Sectional Dependency

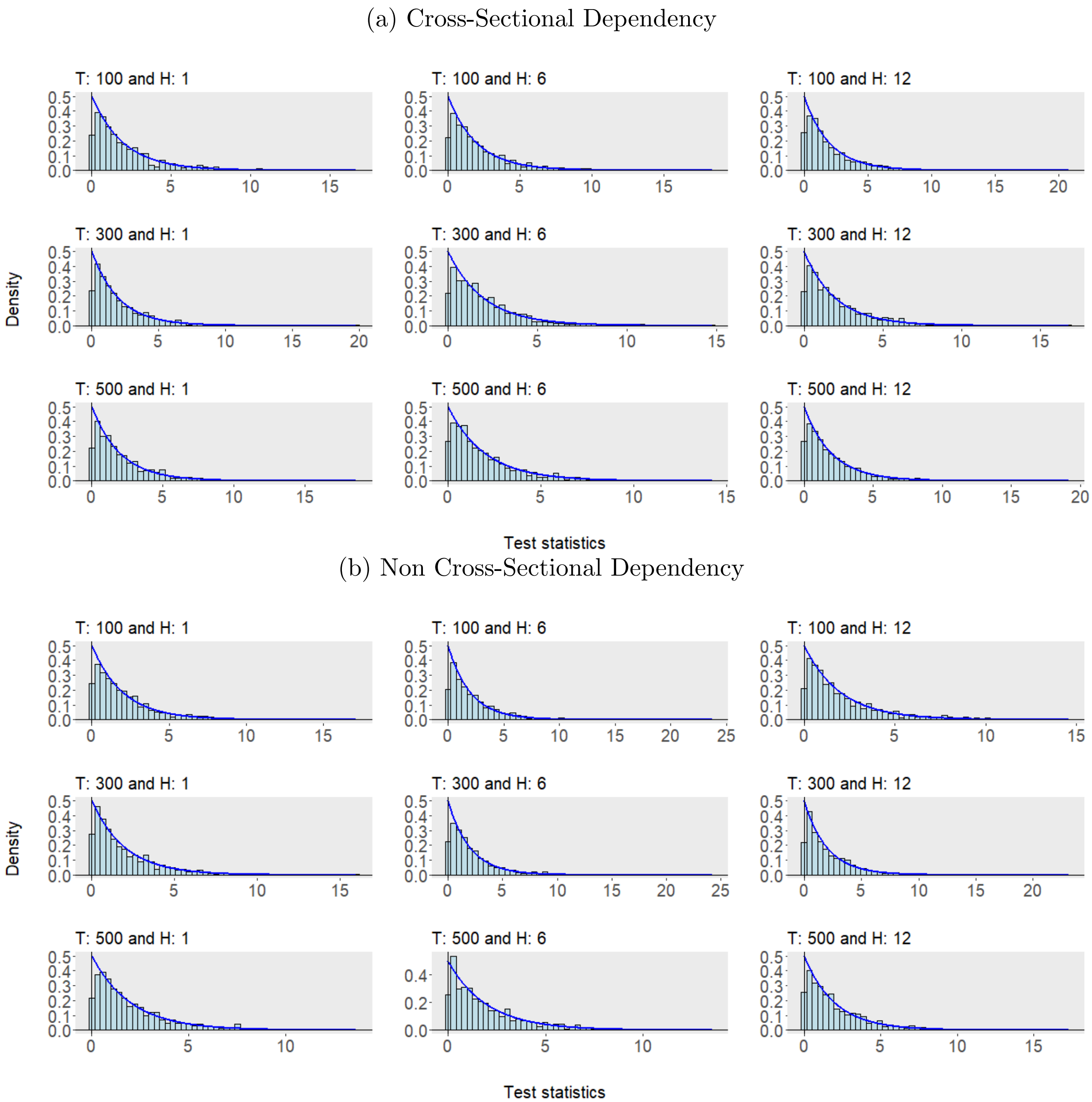


(b) Non Cross-Sectional Dependency



Note: Figure 2-A.3 Displays the histogram of test statistics for the cross-sectional and non-cross-sectional dependency models, with $H \in \{1, 6, 12\}$. The blue curve represents the theoretical chi-square distribution.

Figure 2-A.4: Distribution of Test Statistics, Cumulative Impact: Persistent Shocks ($\beta = 0.9$) with $\eta_1 = 0.1,\ \eta_2 = 0.2,$ and $\eta_3 = 0.3$



Note: Figure 2-A.4 Displays the histogram of test statistics for the cross-sectional and non-cross-sectional dependency models, with $H \in \{1, 6, 12\}$. The blue curve represents the theoretical chi-square distribution.

Chapter 3

The Heterogeneous Effect of Monetary Policy Shocks on Wage Inequality Across the Canadian Provinces

By Koami Amegble and Maral Kichian

Abstract

This paper examines the heterogeneous effects of non-systematic monetary policy on wage inequality across Canadian provinces. Using two measures of wage inequality — the Gini coefficient and the wage ratio between high- and low-skilled workers — and a monetary policy innovation series derived from a Taylor rule framework, our smooth local projection based analysis reveals that the effects of non-systematic monetary policy on wage inequality vary significantly across regions. In particular, regional heterogeneity is more pronounced when we consider the entire wage distribution. Moreover, these effects are asymmetric: expansionary monetary policy shocks lead to more heterogeneous regional responses in wage inequality than contractionary monetary policy shocks.

3.1 Introduction

It has been shown in the literature that monetary policy shocks significantly influence wage inequality (de Haan and Sturm, 2017; Antonelli and Gehringer, 2017; Wahiba and Dina, 2023; Juan-Francisco et al., 2019). This observation was also established for the case of Canada by Villarreal and Kronick (2020) and Furceri et al. (2018). Similarly, in chapter one of this thesis, we found that when controlling for lagged oil price returns, changes in the employment rate and its lags, and fiscal policy factor[1] and its lags, contractionary and expansionary monetary policy shocks have symmetric effects on the Gini coefficient — reducing or increasing overall wage inequality, respectively. However, their effects on the wage ratio diverge, indicating differential impacts on high- and low-skilled workers. Specifically, contractionary monetary policy shocks tend to widen the skill-based wage gap over the longer term, while expansionary monetary policy shocks help narrow it. However, chapter 1 did not analyze how wage inequality in the different Canadian provinces may respond differently to non-systematic monetary policy.

In this paper, we address this gap. Canada has ten provinces, each of them boasting a unique economic landscape[2]. Figure 3-1 presents the results of Principal Component Analysis (PCA) applied to the ten provinces to find common variation over 1997 to 2023 in their shares of gross domestic product (GDP) across different industries. From this, it is possible to discern four groups of provinces: Ontario and Quebec have economies primarily driven by manufacturing and industrial sectors. Alberta, Saskatchewan, and Newfoundland and Labrador primarily depend on resource extraction industries, including oil, gas, mining, and forestry. Nova Scotia, New Brunswick, and Prince Edward Island are characterized by traditional industries such as fisheries and tourism. The remaining two provinces, British Columbia and Manitoba, have relatively diversified economies. It is thus possible that non-systematic monetary policies may have varying impacts on the wage distributions in these different province groupings. Elements such as sectoral composition and labour market institutions can further influence wage disparities among provinces (Herr and Ruoff, 2024; Junicke et al., 2025).

Indeed, the literature has highlighted varied impacts of monetary policy shocks on income or wealth inequality across different countries governed by a unified monetary policy. Several studies focus on the Eurozone area and use a Panel Vector AutoRegression (PVAR) framework to analyze these effects, including Samarina and Nguyen (2024), Creel and El Herradi

[1]The fiscal policy factor is derived from Principal Component Analysis using fiscal policy variables. For more details, please see Chapter 1.

[2]The country also has three territories. However, due to lack of data, we will not treat these cases.

(2022) and Dossche et al. (2021). These works find that certain countries are affected differently than others, with the effects emerging either through the labour-market channel (a decrease in policy rate leads to an increase in wages, thus decreasing wage inequality) or via the income channel. The latter study further indicates that these variations in labour market structures and the extent of reliance on social safety nets also matter for the obtained effects.

We note that, using PVAR to estimate impulse response functions (IRFs) presents limitations for analyzing heterogeneity. The PVAR specifications employed by Samarina and Nguyen (2024), Creel and El Herradi (2022), and Dossche et al. (2021) rely on fixed-coefficient estimation, where impulse response functions (IRFs) are assumed to be identical across units. This specification implicitly assumes that all cross-sectional units share the same dynamic responses to shocks. This assumption of parameter homogeneity constrains the model's ability to capture unit-specific reactions to monetary policy shocks, thereby limiting its effectiveness in detecting heterogeneous effects. In addition, using pooled estimation without correcting for cross-sectional dependence can lead to biased estimates (Pesaran, 2006), particularly when the transmission of the monetary policy shocks varies across the region.

We proceed differently. Using the impulse response function (IRF) comparison-based test developed in chapter two of this thesis, we assess the varying impacts of non-systematic monetary policy across provinces. For this purpose, we consider the wage Gini coefficient and the wage ratio — defined as the wage ratio between high and low-skilled workers, where the skill level is based on the education level (see also Dolado et al., 2021) — and we employ monetary innovations measure based on the Taylor rule as in Chapter 1. Any obtained diversity in responses can thus shed light on the underlying regional economic dynamics in Canada.

We thus make several contributions to the existing literature. First, we conduct our analyses by including our monetary innovation series in a smooth local projection (SLP) regression (Barnichon and Brownlees, 2019). As chapter 1, we ensure the exogeneity of our innovation series by controlling for various influences, including oil price returns, strike-related cost-of-living adjustments, labour market dynamics (employment rate), and fiscal policy. We obtain non-cumulative and cumulative non-systematic monetary policy impacts, as in chapter 1, by examining the impacts of the SLP residualized shocks. We then apply the impulse response function comparison-based tests developed in chapter two of this thesis to document differences in monetary shock impacts across the Canadian provinces.

Our findings indicate that the impact of non-systematic monetary policy on the entire wage distribution exhibits greater regional heterogeneity across provinces, while the effects on high- to low-skilled workers, who represent particular portions of that distribution, are more uniform. We also find that the regional asymmetric effects depend on the type of monetary policy shock: expansionary monetary policy shocks lead to more heterogeneous regional responses in wage inequality than contractionary monetary policy shocks. Furthermore, our results likely indicate that structural characteristics may help explain regional divergence in wage inequality responses to non-systematic monetary policy. Finally, cumulative IRFs more effectively reveal regional heterogeneity than their non-cumulative counterparts.

The remainder of the paper is organized as follows: Section 3.2 outlines the methodology employed. Section 3.3 presents the data relevant to wage inequality, the monetary policy shock series, and the control variables incorporated in our analysis. In Section 3.4, we report the results from the application of our tests and we document some heterogeneity in non-systematic monetary policy shock impacts across provinces, for non-cumulative and for cumulative impulse responses, and over 1- and 2-year horizons. The study concludes with final remarks summarizing our findings.

Figure 3-1: Provinces Similarity and Dissimilarity Regarding Gross Domestic Product (GDP): Share in Total Industry

Note: The figures illustrate the Principal Component Analysis (PCA) results of gross domestic product share in total industry from 1997 to 2021. Dimensions 1 and 2, derived from PCA, account for most of the variability in GDP distribution across the industry, observed among provinces over time. The two dimensions explain 63.4% of the variability in the dataset. The Figure indicates that Alberta, Saskatchewan, and Newfoundland and Labrador are dominated by industries such as mining, quarrying, oil and gas extraction, and construction. In contrast, Ontario and Quebec are primarily driven by manufacturing, finance, insurance, real estate, leasing, and industries related to information, culture, and recreation. Nova Scotia, New Brunswick, and Prince Edward Island are mainly characterized by traditional sectors, including fishing, forestry, and agriculture. Manitoba and British Columbia are positioned near the center, indicating a more balanced distribution of GDP.

3.2 Methodology

3.2.1 Empirical Estimation Method

We use the smooth local projection (SLP) method proposed by Barnichon and Brownlees (2019) to estimate the effect of non-systemtic monetary policy on wage inequality. The regression setup is defined as follows:

$$
\begin{aligned}
y_{i,t+h} = & \sum_{k=1}^{K} a_{i,k} B_k(h) + \sum_{k=1}^{K} b_{i,k} B_k(h) \epsilon_t \\
& + \sum_{l=1}^{p} \sum_{k=1}^{K} c_{i,lk} B_k(h) y_{i,t-l} + \sum_{l=1}^{p} \sum_{k=1}^{K} d_{i,lk} B_k(h) z_{i,t-l} + u_{i,t+h}, \qquad (3.1) \\
i = & 1, \ldots, m, \quad h = H_{\min}, \ldots, H_{\max}, \qquad (3.2)
\end{aligned}
$$

where ϵ_t represents non-systematic monetary policy innovations, which is assumed to be common to all provinces. $y_{i,t}$ is the log of the wage inequality variable for province i at time t, and $y_{i,t-l}$ denotes its lags. $z_{i,t-l}$ contains the lags of various control variables. The projection horizon is denoted by h, and p represents the maximum lag order. $u_{i,t+h}$ is the error term with variance $\sigma^2_{i,(h)}$ for province i. Statistically, as in Chapter 1, we assume that conditional on the control variables, ϵ_t is uncorrelated with u_{t+h} (exogeneity conditional on controls; see Montiel Olea et al. (2025)).

The terms $H_{\min}$ and $H_{\max}$ define the minimum and maximum horizons for the projection, respectively. The functions $B_k(h)$, for $k = 1, \ldots, K$, denote a set of B-spline basis functions defined on $\mathbb{R}$ with respect to horizon h. The coefficients $a_{i,k}$, $b_{i,k}$, $c_{i,lk}$, and $d_{i,lk}$ are scalar parameters to be estimated.

Following Barnichon and Brownlees (2019), we use a cubic B-spline basis with equidistant knots ranging from $H_{\min} - 2$ to $H_{\max} - 1$. Cubic splines are piecewise polynomial functions of degree three, commonly used for flexible functional approximation of time-varying responses.

Prior researches show that oil price fluctuations can influence the labour market and contribute to wage disparities among workers (Bachmeier and Keen, 2023; Nusair, 2020; Koirala and Ma, 2020). Fiscal policy may also attenuate the effects of non-systematic monetary policy on wage inequality. In particular, progressive taxation has been shown to reduce disparities in disposable income, thereby narrowing the gap between high- and low-income earners (Bilbiie, 2024; Saez and Zucman, 2016). By leveraging increased tax revenue from higher-income individuals, governments can fund social programs that mitigate the adverse effects of non-systematic monetary policy.

Additionally, collective bargaining often leads to higher wages and better working conditions for unionized workers compared to their non-unionized counterparts (Farber, 2010; Card, 1996). In Chapter 1, we find that controlling for lagged oil price returns, changes in the employment rate and its lags, and fiscal policy factor (derived from PCA) and its lags, increases in the overnight interest rate generally reduce overall wage inequality in Canada. However, they also widen the wage gap between high- and low-skilled workers.

To comprehensively account for these factors at the provincial level, we include lags of oil price returns, changes in the employment rate and its lags, and for fiscal variables, we construct a composite indicator using PCA and use the largest obtained principal component[3] and its lags in our regressions. This indicator is based on changes in the log of variables, including social transfers, cost-of-living adjustments, payroll and workforce taxes, income, profit and capital gains taxes, employment insurance benefits, and employers' social contributions.

In addition to the baseline regression in equation (3.1), our study also investigates the potential asymmetric effects of non-systematic monetary policy on wage inequality. To this end, we adapt the asymmetric regression framework proposed by Ramey and Zubairy (2018), originally developed within the local projection (LP) context, to obtain its smooth local projection (SLP) counterpart. The proposed regression is as follows:

$$y_{i,t+h} = I_t X_{i,t} \phi^{+}_{i,(h)} + (1 - I_t) X_t \phi^{-}_{i,(h)} + u_{i,t+h}, \; h = H_{min}, ..., H_{max}, \tag{3.3}$$

where

$$\begin{aligned} X_{i,t}\phi^{+}_{i,(h)} &= \sum_{k=1}^{K} a^{+}_{i,k} B_k(h) + \sum_{k=1}^{K} b^{+}_{i,k} B_k(h) \epsilon_t \\ &\quad + \sum_{l=1}^{p} \sum_{k=1}^{K} c^{+}_{i,lk} B_k(h) y_{t-l} + \sum_{l=1}^{p} \sum_{k=1}^{K} d^{+}_{i,lk} B_k(h) z_{i,t-l}, \qquad (3.4) \\ X_{i,t}\phi^{-}_{i,(h)} &= \sum_{k=1}^{K} a^{-}_{i,k} B_k(h) + \sum_{k=1}^{K} b^{-}_{i,k} B_k(h) \epsilon_t \\ &\quad + \sum_{l=1}^{p} \sum_{k=1}^{K} c^{-}_{i,lk} B_k(h) y_{t-l} + \sum_{l=1}^{p} \sum_{k=1}^{K} d^{-}_{i,lk} B_k(h) z_{i,t-l}. \qquad (3.5) \end{aligned}$$

[3] As in Chapter 1, we call this largest principal component as fiscal policy factor.

In the above system I_t is the dummy variable that indicates the positive shock, with $I_t = 1$ if $\epsilon_t > 0$ otherwise 0, $a^{+}_{i,k}$, $b^{+}_{i,k}$ $c^{+}_{i,lk}$, and $d^{+}_{i,lk}, k = 1, \ldots, K$ are the sets of scalar parameters for the positive shocks, and $a^{-}_{i,k}$, $b^{-}_{i,k}$, $c^{-}_{i,lk}$, and $d^{-}_{i,lk}, k = 1, \ldots, K$ for the negative shocks.

We also investigate the medium (one year) and long-horizon (two years) asymmetric effects of non-systematic monetary policy in each province and evaluate whether contractionary and expansionary policies have similar effects on wage inequality. Hence, we estimate the cumulative version of (3.3) characterized by long-difference specification (Furceri et al., 2018; Jordà et al., 2019; Jordà and Taylor, 2025), where the cumulative effect over h periods is represented by $y_{t+h} - y_{t-1}$ in the regression equation, with y_{t+h} denoting the response variable at time $t + h$.

$$y_{i,t+h} - y_{i,t-1} = I_t X_{i,t} \phi^{c,+}_{i,(h)} + (1 - I_t) X_t \phi^{c,-}_{i,(h)} + e_{i,t+h}, \; H_{min} \leq h \leq H_{max}, \tag{3.6}$$

where $X_{i,t}\phi^{c,+}_{i,(h)}$ and $X_{i,t}\phi^{c,-}_{i,(h)}$ are defined as follows: where

$$\begin{aligned} X_{i,t}\phi^{c,+}_{i,(h)} = & \sum_{k=1}^{K} a^{c,+}_{i,k} B_k(h) + \sum_{k=1}^{K} b^{c,+}_{i,k} B_k(h)\epsilon_t + \\ & \sum_{l=1}^{p}\sum_{k=1}^{K} c^{c,+}_{i,lk} B_k(h)\Delta y_{t-l} + \sum_{l=1}^{p}\sum_{k=1}^{K} d^{c,+}_{i,lk} B_k(h) z_{i,t-l} \end{aligned} \tag{3.7}$$

$$\begin{aligned} X_{i,t}\phi^{c,-}_{i,(h)} = & \sum_{k=1}^{K} a^{c,-}_{i,k} B_k(h) + \sum_{k=1}^{K} b^{c,-}_{i,k} B_k(h)\epsilon_t + \\ & \sum_{l=1}^{p}\sum_{k=1}^{K} c^{c,-}_{i,lk} B_k(h)\Delta y_{t-l} + \sum_{l=1}^{p}\sum_{k=1}^{K} d^{c,-}_{i,lk} B_k(h) z_{i,t-l}, \end{aligned} \tag{3.8}$$

and $e_{i,t+h}$ is the error term. Coefficients $a^{c,+}_{i,k}$, $b^{c,+}_{i,k}$ $c^{c,+}_{i,lk}$, $d^{c,+}_{i,lk}$, $a^{c,-}_{i,k}$, $b^{c,-}_{i,k}$ $c^{c,-}_{i,lk}$, $d^{c,-}_{i,lk}, k = 1, ..., K$ are scalar parameters, representing the cumulative version scalar parameters.

3.2.2 Heterogeneity Tests

In chapter 2 of this thesis, we developed an impulse response function comparison-based test for assessing the heterogeneous response of wage inequality due to a common shock. In the current context, this common shock is the non-systematic monetary policy shock. Essentially, we seek to test the following hypothesis:

$$\boldsymbol{H}_0 : \beta_{i(h)} = \beta_{j(h)}, \ \ \forall i \neq j \text{ and } \forall h, \tag{3.9}$$

where $\beta_{i(h)}$ represents the impulse response coefficient for province i, and its expression is given by (3.10),

$$\beta_{i(h)} = \sum_{k=1}^{K} b_{i,k} B_k(h). \tag{3.10}$$

The hypothesis (3.10) addresses the heterogeneous response due to an increase in the policy rate without accounting for potential asymmetry from shocks of different sign. On the other hand, (3.11) below does allow for this effect. Hence,

$$\boldsymbol{H}_{0a} : \beta^+_{i(h)} = \beta^+_{j(h)} \text{ and } \beta^-_{i(h)} = \beta^-_{j(h)}, \ \ \forall i \neq j \text{ and } \forall h, \tag{3.11}$$

$$\boldsymbol{H}_{0b} : \beta^+_{i(h)} = \beta^+_{j(h)}, \ \ \forall i \neq j \text{ and } \forall h, \tag{3.12}$$

$$\boldsymbol{H}_{0c} : \beta^-_{i(h)} = \beta^-_{j(h)}, \ \ \forall i \neq j \text{ and } \forall h, \tag{3.13}$$

where $\beta^+_{i(h)}$ and $\beta^+_{i(h)}$ are IRFs for contractionary and expansionary monetary policy shocks, respectively. Hypothesis (3.11) states that the impacts of contractionary and expansionary policy shocks are exactly the same across the Canadian provinces, while (3.12) and (3.13), respectively, show the homogeneity in response to wage inequality after contractionary and expansionary monetary policy shocks. We rewrite (3.9), (3.11), (3.12) and (3.13) in matrix form using (3.10). The matrix form of (3.9) is given by (3.14).

$$\boldsymbol{H}_0 : \Gamma = \boldsymbol{C}\boldsymbol{b} = \boldsymbol{0}, \ \ \forall i \neq j \tag{3.14}$$

where $\boldsymbol{b}' = (b_1, \dots, b_m)$ is $Km \times 1$ vector and $\boldsymbol{C} = \boldsymbol{J} \bigotimes \boldsymbol{B}$ is $s \times q$ matrix with $s = (H_{max} - H_{min} - 1)(m-1)$, $q = mK$, and $\boldsymbol{B} = (B(H_{min}), \dots, B(H_{max}))$ is $(H_{max} - H_{min} + 1) \times K$ matrix and $\boldsymbol{J}$ is $(m-1) \times m$ such that

$$\boldsymbol{J} = \begin{bmatrix} 1 & -1 & 0 & . & . & . \\ 0 & 1 & -1 & . & . & . \\ . & . & . & . & . & . \\ . & . & . & 1 & -1 & 0 \\ . & . & . & 0 & 1 & -1 \end{bmatrix}. \tag{3.15}$$

For (3.11), the matrix $\tilde{\boldsymbol{C}}$ is a block diagonal matrix of $\boldsymbol{C}^{+}$ and $\boldsymbol{C}^{-}$, such that

$$\boldsymbol{H}_{0a} : \Gamma^{+} = \boldsymbol{C}^{+}\boldsymbol{b} = \mathbf{0} \text{ and } \Gamma^{-} = \boldsymbol{C}^{-}\boldsymbol{b} = \mathbf{0}, \quad \forall i \neq j, \tag{3.16}$$

where $\boldsymbol{C}^{+} = [\boldsymbol{C}, \mathbf{0}]$, $\boldsymbol{C}^{-} = [\mathbf{0}, \boldsymbol{C}]$, and $\boldsymbol{b}' = (\boldsymbol{b}^{+}, \boldsymbol{b}^{-})$. Equations (3.12) and (3.13) can be rewritten as follows:

$$\boldsymbol{H}_{0b} : \Gamma^{+} = \boldsymbol{C}^{+}\boldsymbol{b} = \mathbf{0}\ , \ \forall i \neq j \ \text{ and } \boldsymbol{H}_{0c} : \Gamma^{-} = \boldsymbol{C}^{-}\boldsymbol{b} = \mathbf{0}, \ \forall i \neq j. \tag{3.17}$$

Based on the insightful work of Engle and Mezrich (1996), our proposed test approach separates time-invariant parameters, such as the variance-covariance matrix, from dynamic ones in the regression to account for time-invariant heteroskedasticity[4]. It then addresses time dependency by employing a non-overlapping block bootstrap method. More precisely, the bootstrap samples are drawn from the unrestricted residuals of the full model, spanning the entire vector of horizons and the full set of units (here, all the provinces). All bootstrap tests conducted over the full set or a subset of horizons and provinces are thus simulated simultaneously. This approach alleviates concerns related to multiple inference from a non-parametric bootstrap perspective. The test statistic is a Wald-type statistic, shown in Chapter 2 to exhibit good power and size properties in finite samples through a simulation application[5].

3.3 Data

3.3.1 Inequality Data

As with the case where we examine the whole of Canada (see chapter one of this thesis), we construct two wage inequality measures: the Gini coefficient and the wage ratio using the Canadian monthly Labour Force Survey (LFS). The two equality measures range from January 1997 to July 2023 and they are computed for the ten Canadian provinces: Newfoundland and Labrador (NF), Prince Edward Island (PE), New Brunswick (NB), Nova Scotia (NS), Quebec (QC), Ontario (ON), Manitoba (MB), Saskatchewan (SK), Alberta (AB), and British Colombia (BC)[6]. Hence, using the monthly earning data, we calculate the wage Gini

[4]Engle and Mezrich work in a completely different univariate GARCH context, in which they propose a two-step estimation method. First, the time-invariant parameters (long-run variance) are estimated using a simple method. These estimates are then plugged into the objective function, which is subsequently optimized over the parameters that govern time variation (the GARCH components). The idea we build upon from Engle and Mezrich is to separate the treatment of the time-invariant components of the model from the time-varying ones.

[5]For more detail, please see the second chapter.

[6]As we do not have access to the territories' LFS data, we focus only on the ten Canadian provinces.

coefficient and wage ratio based on the following formula :

$$G(w) = -\sum_{i=2}^{N}(p_{\omega,i}p_{w,i-1} - p_{w,i}p_{\omega,i-1}), \tag{3.18}$$

$$R(w)_{\underline{q},\bar{q}} = \frac{W^{\bar{q}}}{W^{\underline{q}}}, \tag{3.19}$$

with

$$p_{w,i} = \frac{\sum_{j=1}^{i}\omega_{(j)}W_{(j)}}{\sum_{j=1}^{N}\omega_{(j)}W_{(j)}}\text{ , and } p_{\omega,i} = \frac{\sum_{j=1}^{i}\omega_{(j)}}{\sum_{j=1}^{N}\omega_{(j)}},$$

where $\omega_{(j)}$ defines the sampling weight associated to invididual j wage, $W_{(j)}$. In addition, $W_{(j)}$ and $\omega_{(j)}$ are ordered versions of W_j and ω_j. $W^{\underline{q}}$ and $W^{\bar{q}}$ denote respectively the median wages of the low-skilled and the high-skilled worker groups. The Gini coefficient takes a value between 0 and 1; perfect equality is characterized by 0, and ideal inequality is defined as 1. $R(w)_{\underline{q},\bar{q}}$ takes a value that is greater than or equal to 1; perfect equality is characterized by a value of 1.

3.3.2 Other Data for Control Variables

To adequately uncover the effect of non-systematic monetary policy on wage inequality, we incorporate a range of control variables, including fiscal policies, employment rates, and oil price returns in our regressions. Data on fiscal policy variables — such as social transfers, employment insurance benefits (for both self-employed and salaried workers), cost-of-living adjustments, payroll and workforce taxes, income taxes, profits and capital gains taxes, and employers' social contributions — are obtained from Statistics Canada's Common Output Database Repository (CODR) tables (Statistics Canada, 2024a,c,b,d).

In addition, monthly data on major wage settlements are collected at the provincial level, capturing the number of employees covered by cost-of-living adjustments. Moreover, monthly data on employment rates, benefit payments, and employer social contributions at the provincial level are also gathered. However, Government Financial Statistics (GFS) data at the national level on social transfers, payroll taxes, and income taxes are only available at quarterly frequency. To ensure temporal consistency, we transform these quarterly series into a monthly frequency by evenly distributing each quarterly value across the three months of the corresponding quarter, accounting for the consolidation of provincial and territorial government data.

To express monetary variables in real terms, we use the monthly Consumer Price Index (CPI) to deflate employment insurance benefit payments; wages, salaries, and employer social contributions, taxes on income, profits, and capital gains, taxes on payroll and workforce; and social benefits. Finally, we compute a real oil price series using the West Texas Intermediate (WTI) crude oil prices and the U.S. Consumer Price Index, sourced from the FRED database. Finally, to streamline our fiscal policy controls, we reduce the dimension of our seven variables by applying PCA, and using the obtained largest factor instead.

3.3.3 Monetary Policy Shock Series

As in Chapter 1, we obtain a monetary innovations measure based on the standard Taylor rule following the approach of Samarina and Nguyen (2024). The Taylor rule typically recommends setting the policy interest rate based on current and expected macroeconomic conditions — specifically, expected inflation, a measure of the output gap, and lags of the interest rate. We derive our monetary policy innovation series as the regression disturbance term from equation (3.20).

$$i_t = \rho i_{t-1} + \beta E(\pi_{t+1|t}) + \gamma E(x_{t|t}) + \epsilon_t, \tag{3.20}$$

where x_t is a measure of the output gap which we proxy by the monthly GDP growth (in our case, for Canadian data) like Samarina and Nguyen (2024), π_t is inflation, i_t is the monthly overnight rate, and the ϵ_t is the regression disturbance, which is our monetary policy innovation series (for more details, please see Chapter 1). This estimate is defined as follows,

$$\hat{\epsilon}_t = i_t - \hat{\rho} i_{t-1} - \hat{\beta} E(\pi_{t+1|t}) - \hat{\gamma} E(x_{t|t}). \tag{3.21}$$

$E(\pi_{t+1|t})$ is the conditional expectation of inflation at time $t+1$ and $E(x_{t|t})$ is conditional expectation of output growth at time t. Equation (3.20) features a forward-looking Taylor rule. That is, the bank rate responds to expected inflation to capture the price stability objective of the Bank of Canada.

To compute the monetary policy innovation series, we replace the expectations terms $E(\pi_{t+1|t})$ and $E(x_{t|t})$ in equation (3.14) with their realized counterparts, π_{t+1} and x_t, respectively. We estimate the regression using an instrumental variables (IV) approach with the Generalized Method of Moments (GMM), where the instruments include 12 lags each of Canadian GDP growth, the monthly overnight rate, and Canadian inflation. Additionally, we incorporate the contemporaneous value and lags of U.S. monthly GDP growth, which are treated as exogenous, given that Canada is a small open economy and thus more likely influenced by, rather than influencing, the U.S. economy.

To proxy U.S. monthly GDP growth, we use the Brave-Butters-Kelley Monthly GDP (BBKMGDP) series developed by Brave et al. (2019). This series provides a high-frequency measure of real U.S. GDP growth using a combination of mixed-frequency state-space models, dynamic factor models, and Kalman filtering and smoothing techniques (see Brave et al., 2019, for more details). It incorporates diverse sources of data, including financial market indicators (e.g., stock prices, bond yields, and commodity prices), real-time economic indicators (e.g., employment, retail sales, industrial production, and housing market data), and survey-based measures (e.g., consumer and business confidence, and labour market surveys). As a result, BBKMGDP offers more timely estimates of U.S. GDP than traditional quarterly measures.

The monthly overnight rate and inflation data are obtained from the Bank of Canada database. The Canadian real GDP growth rate is calculated from Statistics Canada's monthly real GDP series[7]. All series span the period from January 1997 to July 2023.

As explained in chapter 1, when we refer to our non-systematic monetary policy shock, this is the residualized shock of the smooth local projection equation (that is, the residual from the OLS regression of the innovation in (3.21) on all the included control variables in the SLP).

3.4 Test Results: Non-Systematic Monetary Policy Shock Impacts on Wage Inequality

3.4.1 Test Results and Interpretation

Regression equation (3.1) is estimated separately for each province. As mentioned above, we use the Gini coefficient and the wage ratio as our two measures of inequality, and we include our innovation series in a smooth local projection framework. Consistent with the specification in Chapter 1, we include six lags of the change in the inequality measure and twelve lags of the remaining control variables in our regression. Since both the Gini coefficient and the wage ratio are seasonally unadjusted, we rely on the smooth local projection (SLP) method to account for and smooth out seasonal fluctuations as well as other short-term variations in the estimated impulse response coefficients. Following Barnichon and Brownlees (2019), we estimate equation (3.1) using Ridge regression to improve estimation stability. As the inclusion of lags may account for serial correlation (Olea and Plagborg-Møller, 2021), we then account for heteroskedasticity by computing heteroskedasticity-consistent standard

[7]For further details, see Section 1.2.3 of Chapter One.

errors to construct confidence bands for the impulse response functions (IRFs).

Initially, we assess the impact of non-systematic monetary policy on wage inequality without distinguishing between expansionary and contractionary monetary policy shocks. Specifically, we analyze the effect of an unexpected 25-basis-point increase in the bank rate on wage inequality. We estimate non-cumulative and cumulative impulse responses. After estimating these responses for each province individually, we apply our heterogeneity tests across different provincial groupings and over two distinct horizons: the 1-year horizon (months 1 to 12) and the 2-year horizon (months 13 to 24).

More precisely, we examine both global and pairwise heterogeneity in the impact of non-systematic monetary policy on wage inequality. *Global heterogeneity* is obtained when the test rejects the null hypothesis of homogeneous responses of wage inequality across all provinces or within specific sub-groups. *Pairwise heterogeneity*, on the other hand, is detected when the test rejects the null of homogeneity between individual province pairs within a given subgroup.

For hypothesis testing, we consider significance levels of 0.05, 0.1, and 0.32. For pairwise comparisons, we apply the Bonferroni correction using a significance level of 0.32[8]. This setting aligns with the common practice of reporting 68% confidence bands for impulse response functions (IRFs)[9]. It is important to note that our test evaluates a *joint* null hypothesis over the full horizon, whereas standard confidence bands for IRFs typically provide *pointwise* coverage—valid at each horizon individually rather than simultaneously. Since IRFs are often interpreted as sequences over time, our interpretation maintains joint error control not only across horizons, but also across provinces. That said, we report all p-values so that readers may interpret test outcomes relative to any desired level of significance.

Table 3-1 summarizes the results of the above tests for the non-cumulative and cumulative impulse responses when they are applied to: (i) all 10 provinces taken together, (ii) the four groups of provinces, and (iii) the different pairs of provinces with each of these groupings (when there are more than 2 provinces in a group).

The results reveal significant provincial heterogeneity in the Gini coefficient's response to

[8]Assume we have m hypotheses, $H_1, \ldots, H_m$, each with its corresponding p-value $p_1, \ldots, p_m$. Given a test significance level α, the Bonferroni correction defines an adjusted significance level α^* as:

$$\alpha^* = \frac{\alpha}{m}, \tag{3.22}$$

such that the null hypothesis H_i is rejected if $p_i \leq \alpha^*$.

[9]See for example Anzuini and Rossi (2022); Graeber et al. (2024); Beckmann and Geiger (2025); Franta and Libich (2024); Najjar and Shapiro (2025); Charnavoki and Dolado (2014); Djogbenou (2024).

non-systematic monetary policy. When we consider all 10 provinces, the null hypothesis of homogeneous non-cumulative impulse responses is rejected at both the medium ($p = 0.043$) and the longer horizon ($p = 0.030$). For cumulative responses, heterogeneity is also found at the medium horizon ($p = 0.033$). Subgroup analyses show strong evidence of heterogeneity in the cumulative Gini responses within resource-rich provinces ($p = 0.043$), and weaker evidence in non-cumulative responses for resource-rich ($p = 0.296$) and diversified economies ($p = 0.145$) at the medium horizon. No significant heterogeneity is found within the Maritime or industrial hub provinces. At the pairwise level, the Gini response differs between Alberta and Saskatchewan for both non-cumulative ($p = 0.103$) and cumulative ($p = 0.015$) IRFs at the medium horizon.

On the other hand, using the wage ratio as an indicator of inequality yields more homogeneous responses. Across all provinces, the test does not reject homogeneity in non-cumulative IRFs. However, cumulative IRFs show weak evidence of heterogeneity at the medium horizon ($p = 0.113$). Within subgroups, the wage ratio responses in resource-rich provinces show heterogeneity in both non-cumulative ($p = 0.163$) and cumulative ($p = 0.035$) IRFs. No significant heterogeneity is found within diversified economies ($p = 0.206$). Pairwise comparisons reveal heterogeneity in wage ratio responses between Saskatchewan and Newfoundland for both non-cumulative ($p = 0.073$) and cumulative ($p = 0.053$) IRFs, and between Newfoundland and Alberta for cumulative IRFs ($p = 0.013$).

Following this, we examine the asymmetric effects of an unexpected interest rate change. We first focus on how contractionary monetary policy shock differentially affects low-skilled and high-skilled workers, as well as its broader implications for wage inequality across province groups, compared to the same size expansionary monetary policy shock. This analysis provides a clearer understanding of both the general and more nuanced distributional consequences of the interest rate change. Table 3-2 presents p-values for the non-cumulative impulse response functions, while Table 3-3 reports the corresponding p-values for the cumulative IRFs.

An expansionary monetary policy shock reveals statistically significant heterogeneity in the long-horizon non-cumulative response of the wage Gini coefficient across Canadian provinces ($p = 0.050$). However, the test does not reject the null hypothesis of a homogeneous cumulative response at the same horizon. Subgroup analysis shows significant heterogeneity in non-cumulative responses among the Maritime provinces ($p = 0.050$) and resource-rich provinces ($p = 0.025$) at the longer horizon, though this heterogeneity is less obvious at the medium horizon ($p = 0.303$). In cumulative terms, heterogeneity appears weaker at the long horizon ($p = 0.256$) but somewhat stronger at the medium horizon ($p = 0.063$).

Within diversified economies, cumulative responses exhibit weak heterogeneity at both horizons ($p = 0.318$ and $p = 0.288$). Pairwise comparisons underscore significant differences in long-horizon non-cumulative responses. Notably, disparities emerge between Prince Edward Island and New Brunswick ($p = 0.023$), Nova Scotia and Prince Edward Island ($p = 0.013$), and Newfoundland and Alberta ($p = 0.010$).

Table 3-1: Comparing IRFs Obtained from the Baseline (3.1) Specification for Different Province Groupings

	Wage Gini Coefficient				Wage Ratio			
	Medium Horizon		Long Horizon		Medium Horizon		Long Horizon	
	T_w	$p-value$	T_w	$p-value$	T_w	$p-value$	T_w	$p-value$
Non-Cumulative IRFs								
All 10 provinces	219.037	0.043^{*}	197.179	0.030^{*}	132.757	0.810	101.862	0.875
Maritime provinces (NS, NB, PE)	30.849	0.388	12.692	0.882	9.009	1.000	14.585	0.962
Ind.&Man. hub provinces (QC, ON)	6.891	0.634	2.863	0.902	1.713	0.992	5.903	0.702
Resource-rich provinces (AB, SK, NF)	37.025	0.296^{+}	27.847	0.378	40.744	0.163^{+}	21.167	0.524
Diversified economies (MB, BC)	24.412	0.145^{+}	8.428	0.599	6.130	0.927	6.306	0.747
NS, NB	3.612	0.972	8.866	0.622	3.405	0.997	6.337	0.920
PE, NB	22.230	0.165	1.865	0.992	6.058	0.992	7.535	0.842
NS, PE	21.165	0.158	6.506	0.712	5.149	0.972	7.860	0.817
SK, NF	19.620	0.276	8.826	0.657	27.022	0.073^{+}	13.450	0.331
SK, AB	26.505	0.103^{+}	20.053	0.140	18.845	0.198	6.778	0.729
NF, AB	9.789	0.687	11.194	0.514	20.079	0.241	8.502	0.596
Cumulative IRFs								
All 10 provinces	54.288	0.033^{*}	14.723	0.378	40.410	0.113^{+}	6.413	0.885
Maritime provinces (NS, NB, PE)	5.344	0.376	0.083	0.965	1.836	0.694	0.820	0.719
Ind.&Man. hub provinces (QC, ON)	0.173	0.774	0.692	0.476	1.458	0.459	0.003	0.952
Resource-rich provinces (AB, SK, NF)	16.116	0.043^{*}	1.497	0.569	20.396	0.035^{*}	3.252	0.293^{+}
Diversified economies (MB, BC)	0.173	0.792	0.056	0.852	4.616	0.206^{+}	0.020	0.897
NS, NB	1.915	0.406	0.050	0.860	1.836	0.459	0.672	0.471
PE, NB	4.823	0.160	0.059	0.842	0.425	0.672	0.003	0.970
NS, PE	1.470	0.461	0.004	0.950	0.246	0.729	0.248	0.639
SK, NF	1.986	0.373	0.580	0.489	11.887	0.053^{+}	1.616	0.246
SK, AB	16.082	0.015^{+}	1.231	0.358	0.261	0.759	0.215	0.669
NF, AB	6.484	0.120	0.034	0.907	18.723	0.013^{+}	2.792	0.135

Note: Table 3-1 presents the results of the IRF comparison test, implemented using 399 bootstrap replications. T_w denotes the test statistic, and the reported values are the associated p-values. Symbols indicating significance levels are as follows: (*) for 0.05, (†) for 0.10, and (+) for 0.32. For both overall and subgroup comparisons, significance levels of 0.05, 0.10, and 0.32 are used. However, for pairwise comparisons within subgroups, a Bonferroni correction is applied, based on a significance level of 0.32. For instance, a subgroup of three provinces yields three pairwise comparisons, resulting in an adjusted Bonferroni-corrected significance level of 0.106, and the symbol indicating significance level is (+).

On the other hand, contractionary monetary policy shocks exhibit overall weaker evidence of heterogeneity. The long-horizon non-cumulative IRFs show mild heterogeneity across provinces ($p = 0.316$), with similar patterns within subgroups: the Maritime provinces ($p =$

0.283) and resource-rich provinces ($p = 0.130$). However, cumulative IRFs detect weak heterogeneity within diversified economies at the long horizon ($p = 0.236$), and more pronounced heterogeneity within resource-rich provinces at the medium horizon ($p = 0.035$). Pairwise comparisons highlight notable differences in the resource-rich group: between Saskatchewan and Alberta at the medium horizon in both non-cumulative ($p = 0.075$) and cumulative IRFs ($p = 0.028$), and between Newfoundland and Alberta at the long horizon for non-cumulative ($p = 0.015$) and at the medium horizon for cumulative IRFs ($p = 0.025$).

Turning to the wage gap between high- and low-skilled workers, non-cumulative IRFs do not reveal significant heterogeneity in provincial responses to either expansionary or contractionary monetary policy shocks. Nonetheless, within resource-rich provinces, weak evidence of heterogeneity follows contractionary monetary policy shocks ($p = 0.313$). However, when cumulative IRFs are used, weak evidence of heterogeneity appears across provinces in response to both types of shocks. This is true at the medium horizon ($p = 0.160$ for expansionary and $p = 0.296$ for contractionary shocks) and at the long horizon ($p = 0.298$ and $p = 0.155$, respectively). Within the Maritime provinces, strong evidence of heterogeneity in cumulative responses to both expansionary and contractionary monetary policy shocks is observed at the long horizon ($p = 0.045$ and $p = 0.013$, respectively). These effects are mainly driven by differences between Nova Scotia and New Brunswick ($p = 0.018$ and $p = 0.008$, respectively). Within resource-rich provinces, cumulative IRFs indicate strong heterogeneity in the wage ratio at the medium horizon for both types of policy shocks ($p = 0.008$ and $p = 0.075$, respectively). The divergence between Newfoundland and Alberta significantly contributes to this heterogeneity ($p = 0.005$ and $p = 0.025$, respectively).

Table 3-2: Comparing IRFs Obtained from the Asymmetric Specification (3.3) for Different Province Groupings

	Wage Gini Coefficient				Wage Ratio			
	Medium Horizon		Long Horizon		Medium Horizon		Long Horizon	
	T_w	$p-value$	T_w	$p-value$	T_w	$p-value$	T_w	$p-value$
Expansionary Monetary Policy Shock								
All 10 provinces	85.733	0.729	125.648	$0.058^{\dagger}$	85.678	0.792	61.931	0.927
Maritime provinces (NS, NB, PE)	14.766	0.724	35.441	$0.050^{\dagger}$	15.550	0.682	16.493	0.464
Ind.&Man. hub provinces (QC, ON)	4.818	0.652	3.059	0.779	5.889	0.717	1.729	0.985
Resource-rich provinces (AB, SK, NF)	21.091	0.303^{+}	35.294	0.025^{*}	16.900	0.479	11.040	0.749
Diversified economies (MB, BC)	6.414	0.614	4.178	0.694	8.521	0.353	2.513	0.905
NS, NB	5.675	0.729	1.847	0.980	10.021	0.406	7.482	0.446
PE, NB	12.825	0.238	30.389	0.023^{+}	9.296	0.489	6.504	0.564
NS, PE	3.716	0.882	26.644	0.013^{+}	1.969	0.992	9.408	0.278
SK, NF	1.862	0.987	9.584	0.238	4.217	0.827	3.731	0.754
SK, AB	14.865	0.148	10.903	0.195	4.629	0.724	5.772	0.504
NF, AB	11.845	0.263	29.093	0.010^{+}	10.453	0.378	4.345	0.789
Contractionary Monetary Policy Shock								
All 10 provinces	211.878	0.702	396.470	0.316^{+}	265.732	0.459	218.545	0.930
Maritime provinces (NS, NB, PE)	26.517	0.642	79.469	0.283^{+}	26.958	0.827	21.807	0.872
Ind.&Man. hub provinces (QC, ON)	4.761	0.777	8.310	0.534	5.774	0.805	6.145	0.945
Resource-rich provinces (AB, SK, NF)	43.736	0.328	70.066	0.130^{+}	27.196	0.559	56.518	0.313^{+}
Diversified economies (MB, BC)	6.327	0.920	4.951	0.927	7.159	0.739	3.595	0.972
NS, NB	12.093	0.526	7.317	0.892	11.615	0.657	5.878	0.840
PE, NB	21.113	0.273	62.604	0.155	19.843	0.664	11.587	0.772
NS, PE	4.993	0.945	66.084	0.110	9.688	0.737	14.674	0.617
SK, NF	5.621	0.970	10.623	0.521	3.296	0.925	16.754	0.251
SK, AB	35.365	0.075^{+}	43.661	0.168	7.486	0.481	18.384	0.130
NF, AB	20.050	0.356	39.133	0.015^{+}	22.298	0.363	34.796	0.398

Note: Table 3-2 presents the results of the IRF comparison test, implemented using 399 bootstrap replications. T_w denotes the test statistic, and the reported values are the associated p-values. Symbols indicating significance levels are as follows: (*) for 0.05, (†) for 0.10, and (+) for 0.32. For both overall and subgroup comparisons, significance levels of 0.05, 0.10, and 0.32 are used. However, for pairwise comparisons within subgroups, a Bonferroni correction is applied, based on a significance level of 0.32. For instance, a subgroup of three provinces yields three pairwise comparisons, resulting in an adjusted Bonferroni-corrected significance level of 0.106, and the symbol indicating significance level is (+).

Table 3-3: Comparing Cumulative IRFs Obtained from the Asymmetric Specification (3.3) for Different Province Groupings

	Wage Gini Coefficient				Wage Ratio			
	Medium Horizon		Long Horizon		Medium Horizon		Long Horizon	
	T_w	$p-value$	T_w	$p-value$	T_w	$p-value$	T_w	$p-value$
Expansionary Monetary Policy Shock								
All 10 provinces	21.629	0.195^+	4.764	0.802	22.848	0.160^+	9.552	0.298^+
Maritime provinces (NS, NB, PE)	4.344	0.291^+	1.634	0.386	0.406	0.870	5.113	0.045^*
Ind.&Man. hub provinces (QC, ON)	0.044	0.850	0.087	0.722	0.083	0.830	0.045	0.815
Resource-rich provinces (AB, SK, NF)	8.235	$0.063^\dagger$	2.234	0.256^+	13.811	0.008^*	1.893	0.296
Diversified economies (MB, BC)	1.824	0.318^+	0.900	0.288^+	0.028	0.885	0.019	0.885
NS, NB	0.896	0.469	0.008	0.902	0.173	0.759	4.995	0.018^+
PE, NB	4.246	0.115	1.288	0.238	0.399	0.642	1.225	0.218
NS, PE	1.574	0.356	1.429	0.180	0.009	0.930	0.286	0.514
SK, NF	0.142	0.744	1.044	0.233	0.358	0.612	1.497	0.155
SK, AB	5.945	0.040^+	1.882	0.138	2.611	0.133	0.157	0.664
NF, AB	7.017	0.028^+	0.079	0.719	11.416	0.005^+	0.588	0.346
Contractionary Monetary Policy Shock								
All 10 provinces	86.941	0.449	163.707	0.707	118.505	0.296^+	378.018	0.155^+
Maritime provinces (NS, NB, PE)	16.785	0.361	37.092	0.419	3.288	0.820	195.648	0.013^*
Ind.&Man. hub provinces (QC, ON)	0.204	0.842	0.976	0.805	1.248	0.714	0.928	0.797
Resource-rich provinces (AB, SK, NF)	56.510	0.035^*	35.923	0.393	45.760	$0.070^\dagger$	38.681	0.333
Diversified economies (MB, BC)	3.624	0.496	31.199	0.236^+	0.780	0.724	0.208	0.912
NS, NB	8.874	0.268	1.753	0.742	2.360	0.579	180.371	0.008^+
PE, NB	15.830	0.168	30.845	0.251	3.085	0.551	25.451	0.266
NS, PE	5.896	0.406	36.949	0.178	0.818	0.739	49.966	0.125
SK, NF	0.071	0.915	18.490	0.341	3.492	0.441	35.689	0.148
SK, AB	40.581	0.028^+	35.644	0.170	14.214	0.135	9.954	0.486
NF, AB	41.008	0.025^+	0.569	0.850	42.163	0.025^+	12.850	0.351

Note: Table 3-3 presents the results of the IRF comparison test, implemented using 399 bootstrap replications. T_w denotes the test statistic, and the reported values are the associated p-values. Symbols indicating significance levels are as follows: (*) for 0.05, (†) for 0.10, and (+) for 0.32. For both overall and subgroup comparisons, significance levels of 0.05, 0.10, and 0.32 are used. However, for pairwise comparisons within subgroups, a Bonferroni correction is applied, based on a significance level of 0.32. For instance, a subgroup of three provinces yields three pairwise comparisons, resulting in an adjusted Bonferroni-corrected significance level of 0.106, and the symbol indicating significance level is (+).

3.4.2 Discussion on the Key Findings

From the results of IRF comparison-based tests, we can highlight five key findings: (1) greater regional heterogeneous within the entire distribution of wages; (2) less evidence of regional heterogeneity in the impact of non-systematic monetary policy on the wage gap between high- and low-skilled workers; (3) regional asymmetric effects depending on the type of monetary policy shock; (4) structural characteristics may help explain regional differences; and (5) cumulative IRFs can better reveal regional heterogeneity.

Greater regional heterogeneity within the entire distribution of wages. The Gini coefficient, a comprehensive measure of overall wage inequality, reflects broad dispersion across the wage distribution and reveals marked heterogeneity in provincial responses to non-systematic monetary policy. This heterogeneity is particularly pronounced at medium- and long-term horizons, underscoring that the central segment of the wage distribution — where middle-class workers tend to be concentrated — exhibits greater regional divergence than the wage gap between high- and low-skilled workers. These findings suggest that middle-income earners are especially susceptible to differences in local labour market conditions and to sectoral composition, which amplify the uneven impact of non-systematic monetary policy across provinces. For instance, Fortin et al. (2012) highlight that changes in unionization and sectoral shifts disproportionately affect middle-income earners in Canada. Moreover, our results are consistent with findings from several studies in Eurozone countries, where the middle of the distribution is found to be more sensitive to monetary policy, primarily due to greater reliance on labour income and higher exposure to sectoral shocks (Creel and El Herradi, 2022; Lenza and Slacalek, 2024).

Less evidence of regional heterogeneity in the impact of non-systematic monetary policy on the wage gap between high- and low-skilled workers. The test reveals that the wage ratio between high- and low-skilled workers exhibits relatively uniform responses to non-systematic monetary policy across provinces. This consistent pattern suggests that national-level factors — such as education policies and labour market institutions — may play a more significant role in shaping wage inequality at the extremes of the wage distribution. Supporting this, Dolado et al. (2021) highlight that skill-based wage differentials are more strongly influenced by global technological change than by regional non-systematic monetary policy. Furthermore, previous studies indicate that firms often adjust employment margins or working hours, rather than wage levels, in response to economic shocks—particularly in highly regulated or public sector-dominated labour markets (Mathä et al., 2021; Lucifora and Origo, 2025; Al Yussef et al., 2025).

Regional asymmetric effects on wage inequality depending on the type of monetary shock. According to the test results, expansionary monetary policy shocks lead to more pronounced regional disparities in wage inequality, particularly at longer horizons. On the other hand, contractionary monetary policy shocks tend to produce more uniform effects across provinces. This regional asymmetric effect is consistent with findings from Furceri et al. (2018), who show that expansionary monetary policy can exacerbate income inequality, especially in regions with differing capacities to absorb increased liquidity. In addition, provinces with stronger labour markets, more innovative and larger firms exhibit greater resilience and quicker recovery after non-systematic monetary policy (De Groot et al., 2023). Georgopoulos (2009) further supports this view, highlighting that provinces such as Ontario, Quebec, and British Columbia exhibit a higher capacity to absorb and transmit additional liquidity from expansionary monetary policy. On the other hand, provinces such as Newfoundland and Labrador, Prince Edward Island, Saskatchewan, and Alberta display a lower capacity to absorb and utilize increased liquidity effectively.

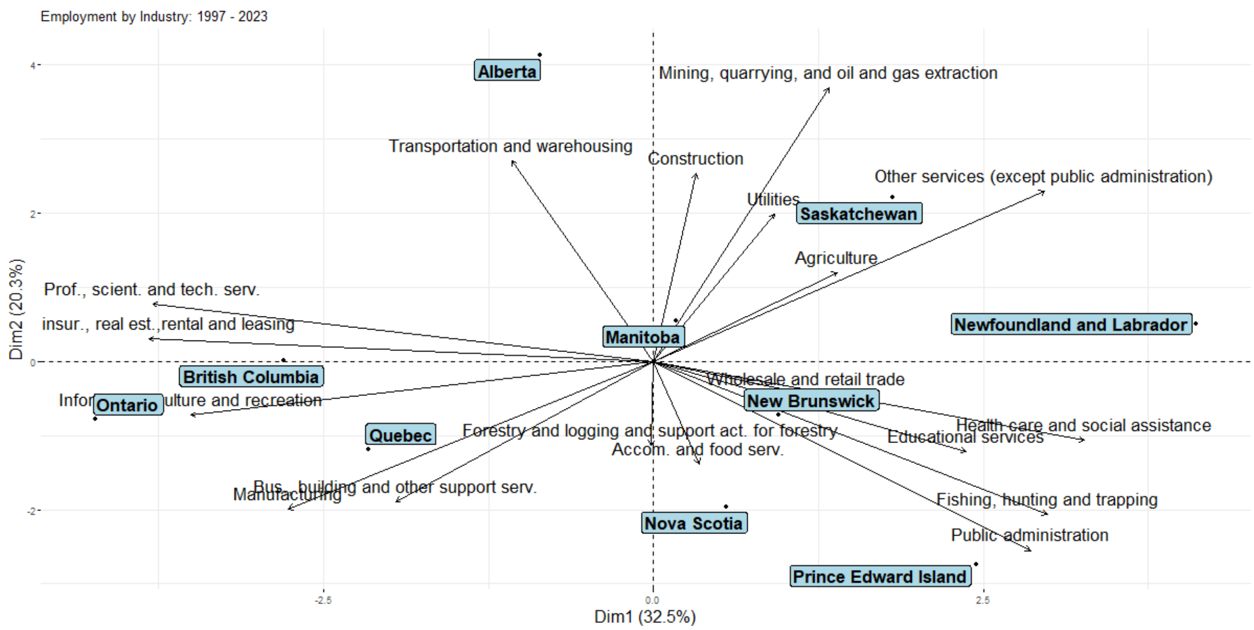
Structural characteristics may help explain regional differences. Regional divergences in responses to non-systematic monetary policy can be explained by structural and institutional differences. Factors such as union density, sectoral composition, public sector employment, and minimum wage policies influence how local labour markets respond to monetary policy (Nsiala, 2021). For instance, regions with higher levels of unionization may experience greater wage rigidity, which can affect the transmission of non-systematic monetary policy. Minton and Wheaton (2022) provide robust evidence that local labour markets do not respond uniformly to monetary policy due to these underlying structural and institutional characteristics.

To illustrate this, we analyse the similarities and differences among Canadian provinces in terms of employment patterns, which reflect local labour market structures. Figure 3-2 shows labour market composition and its differences across provinces. For example, Nova Scotia, Prince Edward Island, and New Brunswick are dominated by employment in fishing, hunting, and trapping, as well as public administration. On the other hand, Quebec and Ontario are more driven by manufacturing and business services, while Alberta's employment is concentrated in mining, quarrying, and oil and gas extraction. These differences in labour market structures may help explain some of the regional variation in how wage inequality responds to non-systematic monetary policy.

Moreover, variations in unionization levels, minimum wage regulations, and the size of the public sector across provinces contribute to heterogeneous wage adjustments (Fortin et al., 2012). Union-induced wage rigidity, in particular, can thus further amplify regional disparities in the transmission of monetary policy (Dellas and Tavlas, 2005).

Cumulative IRFs can better reveal regional heterogeneity. Cumulative impulse response functions provide a more comprehensive view of the long-horizon effects of non-systematic monetary policy on wage inequality. They are particularly effective in revealing persistent regional disparities that may not be apparent in non-cumulative analyses. For example, cumulative IRFs play a key role in revealing persistent differences in wage inequality responses between provinces like Nova Scotia and New Brunswick. De Groot et al. (2023) also underscores the utility of cumulative IRFs in capturing the prolonged impact of monetary policy on regional inequality.

Figure 3-2: Provinces Similarity and Dissimilarity Regarding Employment: Share in Total Industry



Note: The figures present the results of Principal Component Analysis (PCA) based on employment shares across industries from 1997 to 2023. Dimensions 1 and 2, extracted from the PCA, capture the majority of the variation in industry-level employment across provinces over time. These two dimensions explain 52.7% of the total variability in the dataset.

3.4.3 Some Illustrative Cases

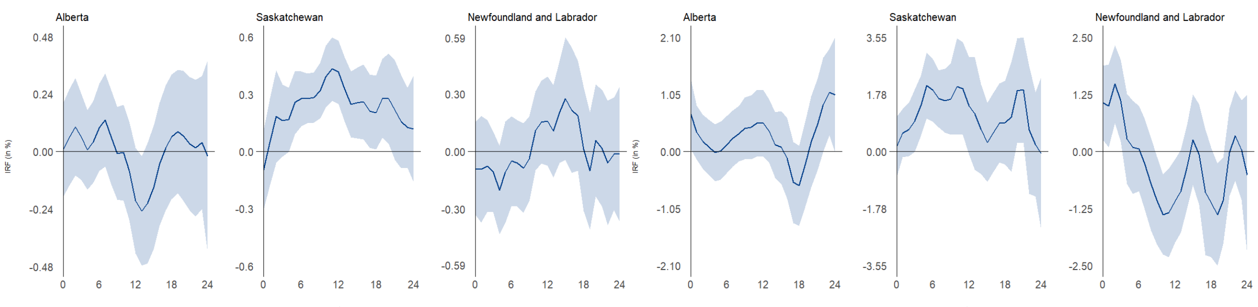
The IRF comparison-based test highlights key differences in the responses of the Gini coefficient and the wage ratio. In this section, we illustrate these findings in the case of the resource-rich provinces by examining their individual non-cumulative and cumulative IRFs. Figure 3-3 displays the IRFs of wage inequality to aggregate non-systematic monetary policy, including contractionary and expansionary monetary policy shocks. The effects are illustrated using an unexpected 25 basis point change in the bank rate, with statistically significant responses indicated within a 95% confidence interval, while the confidence interval is computed using heteroskedasticity-consistent standard errors.

We see many differences in the response of wage inequality to non-systematic monetary policy across Canada's resource-rich provinces — Alberta, Saskatchewan, and Newfoundland and Labrador. Following an unexpected 25-basis-point increase in the bank rate, Alberta experiences a short-lived reduction in the Gini coefficient, whereas Saskatchewan shows a more persistent rise in overall wage inequality. Newfoundland and Labrador, by contrast, displays statistically insignificant changes in the Gini coefficient, suggesting limited aggregate distributional effects. When examining the wage ratio, all three provinces show significant yet divergent reactions. Alberta and Saskatchewan exhibit both short-term and delayed increases in the wage ratio, with a more gradual and cyclical response in Saskatchewan. Newfoundland and Labrador, on the other hand, shows pronounced volatility, marked by sharp rises and subsequent declines, indicating unstable dynamics in skill-based wage inequality.

Figure 3-3: Non -Systematic Monetary Policy Shock Impacts on Wage Inequality in Resource-Rich Provinces

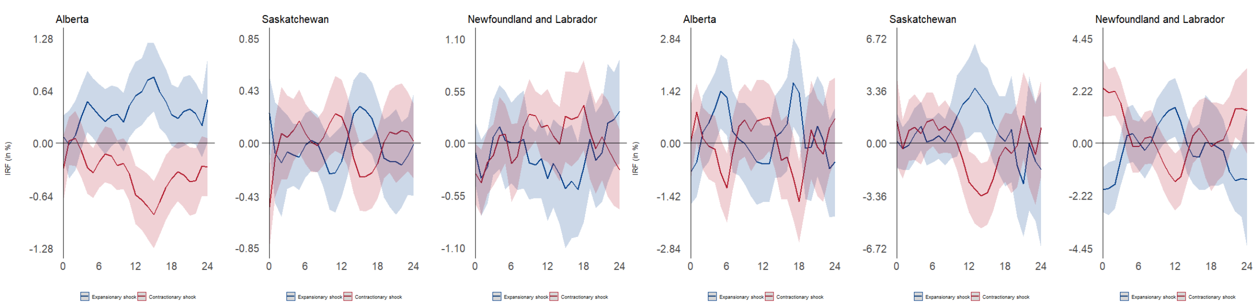
(a) Gini Coefficient (Aggregate Shock).

(b) Wage Ratio (Aggregate Shock).



(c) Gini Coefficient (Asymmetric Shock).

(d) Wage Ratio (Asymmetric Shock).



Note: The figures display the IRFs of an unexpected change in the bank rate by 25-basis-point on wage inequality. The blue (expansionary policy) and red (contractionary policy) bands indicate a 95% confidence interval, where standard errors are obtained using heteroskedasticity-consistent standard errors.

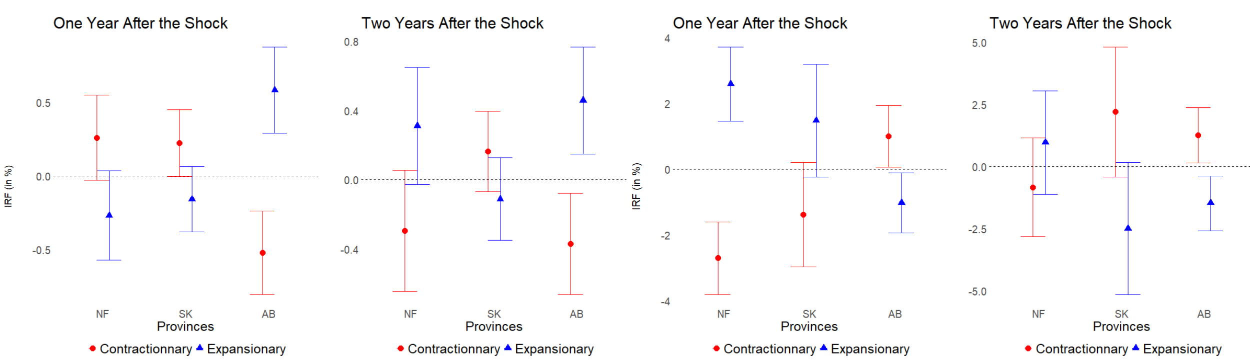
The findings also highlight symmetric effects depending on the stance of non-systematic monetary policy. Contractionary monetary policy shocks reduce overall wage inequality in Alberta and Newfoundland and Labrador, with more modest and short-lived declines observed in Saskatchewan. However, the response of the wage ratio exhibits greater variability: Alberta experiences an initial increase followed by subsequent declines, while Saskatchewan shows a persistent narrowing of the wage gap. In contrast, Newfoundland and Labrador displays considerable volatility, with an early rise in the wage ratio, a decline around the twelfth month, and a resurgence in the twenty-third month. These patterns suggest that contractionary monetary policy shocks may compress wages in certain provinces while generating temporary imbalances in others.

Expansionary monetary policy shocks, on the other hand, appear to have a more consistent and sustained compressing effect on wage inequality in Alberta and Saskatchewan. In Alberta, both the Gini coefficient and the wage ratio increase notably around the fifth and sixth months, followed by delayed convergence effects. Saskatchewan follows a similar trajectory, particularly in the wage ratio, which rises significantly in the medium term. In contrast, Newfoundland and Labrador exhibits an initial decline in the wage ratio following the expansionary monetary policy shocks, a partial correction in subsequent months, and a renewed decrease around the twenty-second month.

These results underscore the importance of local labour market structures and sectoral composition in shaping the regional transmission of non-systematic monetary policy, particularly in provinces with resource-dependent economies.

Figure 3-4: Cumulative Impact of Non-Systematic Monetary Policy Shock on Wage Inequality in Resource-Rich Provinces

(a) Gini Coefficient. (b) Wage Ratio.



Note: The figures display the cumulative IRFs of an unexpected change in the bank rate by 25-basis-point on wage inequality. The blue (expansionary policy) and red (contractionary policy) bands indicate a 95% confidence interval, where standard errors are obtained using heteroskedasticity-consistent standard errors.

We evaluate the cumulative effects of non-systematic monetary policy over one- and two-year horizons for the individual provinces. Figure 3-4 illustrates changes in the Gini coefficient and the wage ratio in response to both contractionary and expansionary monetary policy shocks. The results reveal that both types of shocks have a statistically significant cumulative effect on the Gini coefficient in Alberta over one and two years, while the effect is not statistically significant in Newfoundland and Labrador and Saskatchewan. On the other hand, the wage gap between high- and low-skilled workers is significantly affected by non-systematic monetary policy in Newfoundland and Labrador and Alberta, regardless of the direction of the shock. For instance, a contractionary monetary policy shock cumulatively reduces the wage gap in Newfoundland and Labrador over the medium horizon, whereas it increases the gap in Alberta over the medium and longer terms.

3.5 Dollar Value of Cumulative Monetary Policy Effects on Low-Skilled Workers' Wages

We make use of cumulative IRFs to quantify in dollar terms overall impacts following one, two and three years under contractionary and expansionary monetary policy shocks. Specifically, we begin by determining the average monthly median wage for year 2022 and then apply the IRF as the growth rate for the wage ratio (see below, notably footnote 10, for the details). We assume that the average median monthly wage for high-skilled workers remains unchanged after experiencing either contractionary or expansionary monetary policy shocks, and that all changes are reflected solely for low-skilled workers.

Although fixing high-skilled wages is a strong assumption, low-skilled wages are generally more responsive to monetary and business cycle shocks, while high-skilled wages tend to be more rigid (see Dolado et al., 2021). In this context, we calculate the post-shock average median monthly wage as follows:

$$\Delta w^{q}_{t+h|t-1} = \left(e^{-\hat{\beta}^{c}_{(h)}} - 1\right) w^{q,(0)}_{t-1}, \tag{3.23}$$

where $\hat{\beta}^{c}_{h}$ is the estimated cumulative IRF at horizon h, $w^{q}_{(0)}$, and $w^{q}_{(h)}$ are the lower-skilled wage before and after the shock, respectively[10]. To assess the potential effects of non-systematic monetary policy on the wages of low-skilled workers, we compute the average

[10] To illustrate, let $\hat{\beta}^{c,+}(h)$ represent the estimated cumulative IRF for a contractionary monetary policy shock:

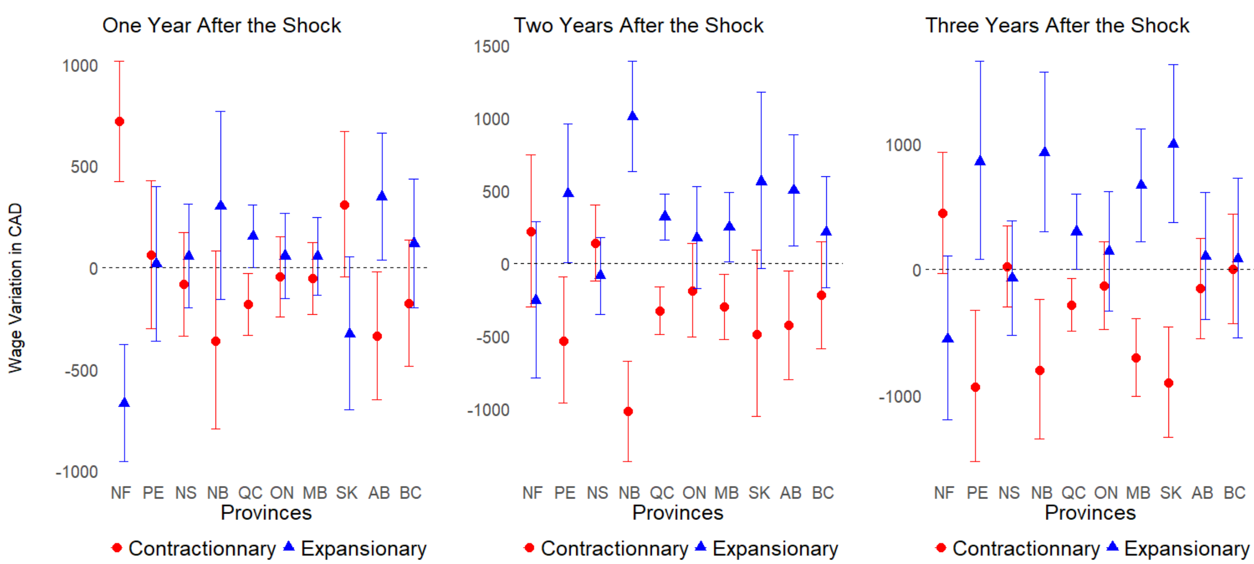
$$\begin{aligned} \hat{\beta}^{c,+}_{(h)}(\delta) &= E\left(y_{t+h} - y_{t-1}|\Gamma_{t-1}, \epsilon^{+}_{t} = \delta\right) - E\left(y_{t+h} - y_{t-1}|\Gamma_{t-1}, \epsilon^{+}_{t} = 0\right), && (3.24)\\ &= \left(y^{(1)}_{t+h} - y^{(1)}_{t-1}\right) - \left(y^{(0)}_{t+h} - y^{(0)}_{t-1}\right), && (3.25)\\ &= gr^{+(1)}_{t+h|t-1} - gr^{+(0)}_{t+h|t-1}, && (3.26) \end{aligned}$$

where Γ_{t-1} is the set of information depending on Δy_{t-l} and Δz_{t-l} $l = 1, ..., p$. $y^{(1)}_{t+h}$. Here, $y^{(1)}_{t-1}$ is the logarithmic of the wage ratio after the shock in $t+h$ and $t-1$, respectively; $y^{(0)}_{t+h}$ and $y^{(0)}_{t-1}$ are the logarithmic of the wage ratio before the shock and δ is the size of the shock. Let $w^{q,(0)}_{t-1}$ be the median wage for low-skilled workers at $t-1$.

annual median wage by multiplying the monthly median wage by 12. This transformation enables a more intuitive understanding of wage fluctuations over time, facilitating more straightforward comparisons across policy horizons. Figure 3-5 presents the projected impact of contractionary and expansionary monetary policy shocks on the annual median wage of low-skilled workers across Canadian provinces, with effects traced one, two, and three years after the shock.

The results suggest that the majority of provinces are not significantly affected by non-systematic monetary policy of either sign across all three horizon options. Among the provinces that do exhibit significant effects, only Newfoundland and Labrador experiences a net positive response to a non-systematic monetary policy shock. For example, in Quebec, a contractionary monetary policy shock reduces the average annual median wage of low-skilled workers by approximately CAD 325 after two years. Under an expansionary monetary policy shock, the wage increases by about CAD 317 after the same period, but under a contractionary monetary policy shock, the wage declines further to CAD 280 after three years. These findings suggest that expansionary monetary policy shocks provide short-term benefits for low-skilled workers, primarily by stimulating aggregate demand, which increases job opportunities and wage pressure in sectors where low-skilled labour is concentrated.

Figure 3-5: Cumulative Impacts on the Wage Ratio: Asymmetric Impact in Terms of Dollars



Note: The figures display the cumulative impact on the change in average median wage after an unexpected change in the bank rate by 25-basis-point on wage inequality. The blue (expansionary policy) and red (contractionary policy) bands indicate a 95% confidence interval, where standard errors are obtained using heteroskedasticity-consistent standard errors.

On the other hand, Ontario would not exhibit any statistically significant cumulative change in low-skilled workers' average annual median wage under either policy regime. This divergence is noteworthy because Quebec and Ontario share broadly similar economic structures. The result implies that similar macroeconomic structures do not necessarily imply uniform non-systematic monetary policy transmission, likely due to differences in labour market dynamics, industrial composition, and the provincial implementation of federal policies.

In resource-rich provinces, the responses of low-skilled workers' wages to non-systematic monetary policy are even more varied. In Newfoundland and Labrador, the average annual median wage increases by CAD 718 under a contractionary monetary policy shock but decreases by CAD 685 under an expansionary monetary policy shock after one year. In Saskatchewan, a contractionary monetary policy shock leads to a wage decline of CAD 902 after three years, whereas an expansionary monetary policy shock results in a wage increase of CAD 954 over the same period — indicating more persistent vulnerabilities under contractionary monetary policy regime. Similarly, in Alberta, the average annual median wage declines by CAD 336 after one year under a contractionary monetary policy shock, while it increases by CAD 345 under an expansionary monetary policy shock. The negative effects of contractionary monetary policy shocks in Alberta deepen over time, with the wage decline reaching CAD 426 after two years, while the positive impact of expansionary monetary policy grows to CAD 494 over the same period. These outcomes suggest a more complex and province-specific interaction between non-systematic monetary policy and labour market dynamics in energy-dependent economies.

Conversely, in the Maritime provinces — where economies are typically smaller and more reliant on small and medium-sized enterprises (SMEs) [11] — Contractionary monetary policy shocks have a larger impact on the wages of low-skilled workers. In Prince Edward Island, low-skilled workers experience declines in their average annual median wage of CAD 530 after two years and CAD 934 after three years. In New Brunswick, the corresponding decreases are CAD 1,015 and CAD 934. In contrast, expansionary monetary policy shocks result in wage increases of similar magnitude over the same periods, with gains of CAD 477 and CAD 860, respectively. These findings suggest that regions with limited economic diversification or lower firm-level resilience may be less effective at transmitting the benefits of expansionary monetary policy and more exposed to the adverse effects of contractionary monetary policy. This underscores the economic vulnerability of the Maritime provinces, particularly for low-skilled workers during periods of contractionary monetary policy.

[11] See Innovation, Science and Economic Development Canada (2024)

In Manitoba, contractionary and expansionary monetary policy shocks appear to produce broadly symmetric effects on the wages of low-skilled workers over the medium term. For example, under an expansionary monetary policy shock, the average annual median wage increases by CAD 248 after two years and by CAD 652 after three years. Under a contractionary monetary policy shock, the corresponding declines are CAD 297 and CAD 701, respectively. This persistent downward pressure under a contractionary monetary policy regime may reflect underlying structural weaknesses or limitations in the effectiveness of non-systematic monetary policy transmission, particularly in contractionary monetary policy regime.

In this projection scenario, the average annual median wage is estimated using 2022 as the base year. While the absolute wage levels would differ if an alternative base year were chosen, the overall trajectory of wage growth is expected to remain broadly consistent. It is important to highlight a key simplifying assumption underlying this projection: we assume that wages for high-skilled workers remain constant following the non-systematic monetary policy shock. This is a strong assumption, as it disregards the potential responsiveness of high-skilled wages to macroeconomic fluctuations and policy interventions. In practice, empirical evidence suggests that wage adjustments often vary significantly across skill levels due to differences in labour market frictions, bargaining power, and capital-skill complementarity. For instance, Dolado et al. (2021) show that high-skilled workers may still experience wage changes in response to monetary policy shocks, particularly in the presence of labour market frictions and complementarity between capital and skilled labour.

3.6 Concluding Remarks

This paper examines the heterogeneous effects of non-systematic monetary policy on wage inequality across Canadian provinces. While there is broad consensus that non-systematic monetary policy influences wage disparities, the extent and nature of these effects vary widely across regions due to differences in economic structures, industrial composition, and labour market dynamics. By employing key distributional metrics — such as the Gini coefficient and wage ratios — alongside a monetary policy innovation series derived from a Taylor rule framework, this paper provides a comprehensive analysis of regional heterogeneity in wage inequality responses to non-systematic monetary policy. The smooth local projection-based analysis controls for lagged oil price returns, changes in the employment rate and its lags, and fiscal policy factor (derived from PCA) and its lags to isolate the impact of contractionary and expansionary monetary policy shocks.

Our analysis yields several key insights into the heterogeneous effects of non-systematic monetary policy on wage inequality across Canadian provinces. These findings are summarized as follows:

(1) Greater regional heterogeneity in the impact of non-systematic monetary policy on the entire wage distribution, as opposed to portions of that distribution (see the next point): The Gini coefficient responds heterogeneously to non-systematic monetary policy across provinces at medium and long horizons;

(2) Less evidence of regional heterogeneity in the impact of non-systematic monetary policy on the wage gap between high- and low-skilled workers: Unlike the Gini coefficient, the wage gap between high- and low-skilled workers shows no strong evidence of heterogeneous responses across provinces, suggesting a more uniform effect of non-systematic monetary policy at the tails of the wage distribution;

(3) Regional asymmetric effects on wage inequality depending on the type of monetary shock: Heterogeneous regional responses are more evident following expansionary monetary policy shocks—especially over the longer term—while contractionary monetary policy shocks tend to produce more uniform wage inequality responses across regions;

(4) Structural characteristics may help explain regional differences: Regional differences in responses of wage inequality to non-systematic monetary policy shocks may stem from structural factors like union density, sectoral composition, public sector size, and minimum wage policies, which shape local labour market reactions to non-systematic monetary policy;

(5) Cumulative IRFs can better reveal regional heterogeneity: Cumulative impulse responses highlight clearer differences across certain provincial pairs that may not be as detectable with non-cumulative IRFs.

Our findings underscore the importance of incorporating regional heterogeneity into analysis of non-systematic monetary policy transmission and suggest the need for targeted or complementary regional policies to ensure equitable outcomes from monetary policy interventions. A one-size-fits-all approach to monetary policy may inadvertently exacerbate regional wage disparities, thereby necessitating supportive fiscal policies to mitigate adverse effects in more vulnerable provinces. For instance, policies such as workforce training, upskilling initiatives, and targeted labour market programs may help cushion the impact of monetary policy on wage inequality, particularly in economically disadvantaged regions.

3-A Appendix

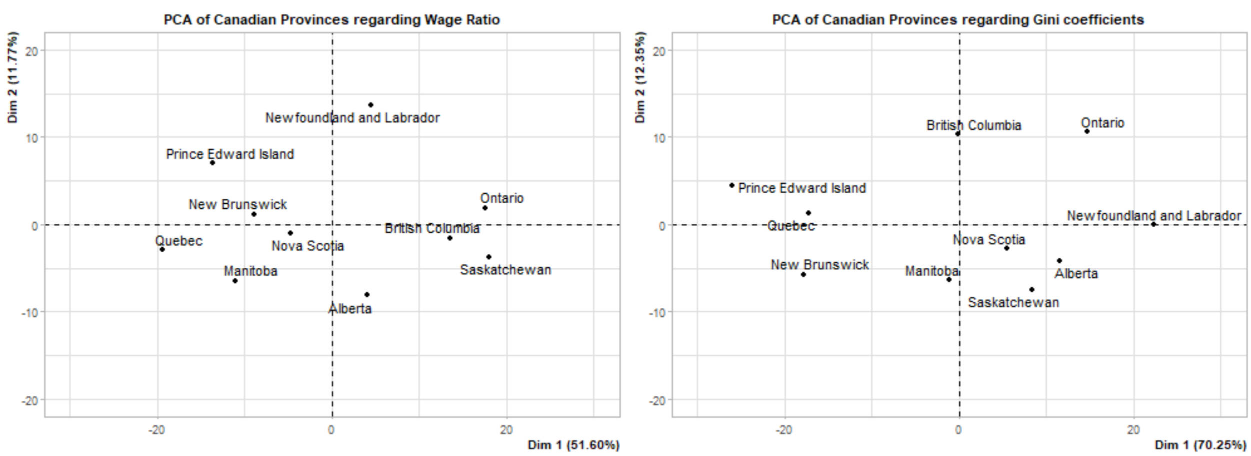
3-A.1 Use of AI in Research

In this study, OpenAI ChatGPT and Grammarly were used to check the grammar, catch typos, and enhance the writing to improve the clarity of the text.

3-A.2 Principal Component Analysis

Figure 3-A.1: Provinces Similiarity and Dissimilarity Regarding Wage Inequality

(a) Wage Ratio. (b) Gini Coefficient.



Note: The figures present the results of the Principal Component Analysis (PCA) based on monthly wage Gini coefficients and wage ratios from 1998 to 2023. Dimensions 1 and 2, extracted from the PCA, capture the majority of the variation in wage inequality across provinces over time. Specifically, for the wage Gini coefficient, these two dimensions explain 82.6% of the total variability, while for the wage ratio, they account for 63.37% of the variability in the dataset.

3-A.3 Descriptive Results of the Impacts of Non-Systematic Monetary Policy on Wage Inequality

We provide a full description on how non-systematic monetary policy impacts wage inequality across Canadian provinces.

3-A.4 Industrial and Manufacturing Hub Provinces

This analysis examines the effects of non-systematic monetary policy on wage inequality in Ontario and Quebec — Canada's key industrial and manufacturing provinces. Both provinces exhibit statistically significant increase in overall wage inequality, as measured by the Gini coefficient and the wage ratio, following an unexpected increase in the bank rate by 25-basis-point.

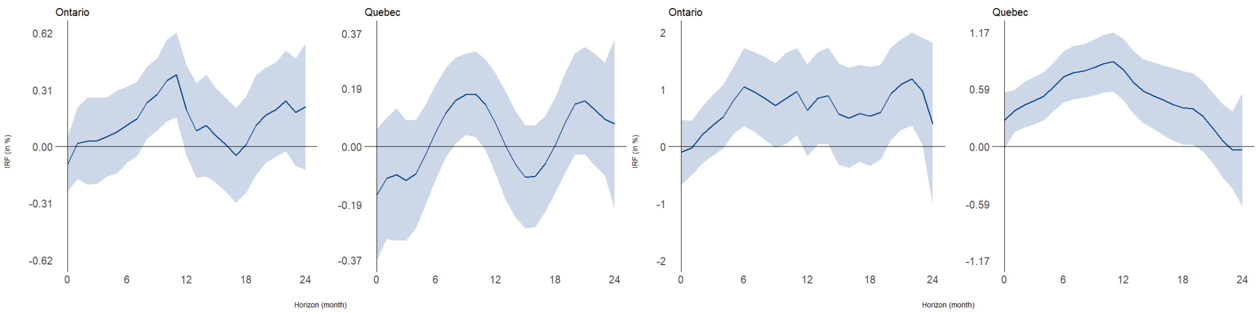
Under a contractionary monetary policy shock, Ontario experiences a significant increase in the wage ratio in the 12th month, with no significant change in the Gini coefficient. On the other hand, Quebec experiences an earlier and more sustained rise in the wage gap, alongside a temporary decline in the Gini coefficient in the 16th and 18th months.

Expansionary monetary policy shocks lead to a reduction in wage inequality, particularly as captured by the wage ratio. In Ontario, the wage gap narrows consistently, with the decline reaching 1.51% in the 23rd month. The Gini coefficient initially increases but later declines, suggesting a delayed equalizing effect. Quebec exhibits a similar trajectory, with the wage ratio decreasing significantly over the short and long horizons. However, unlike Ontario, Quebec experiences a mid-period rise in the Gini coefficient, indicating some temporary divergence in overall wage distribution.

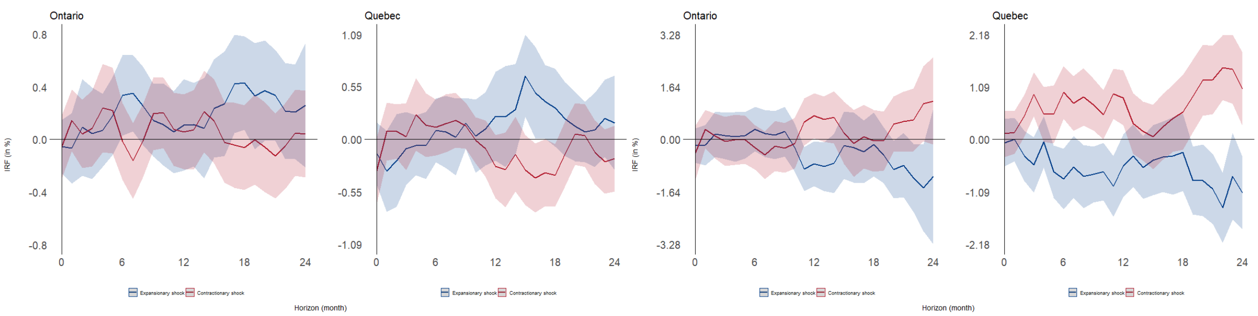
Overall, the findings indicate that non-systematic monetary policy exerts symmetric as well as region-specific effects on wage inequality. Differences in labour markets, industrial structure, and skill composition drive this heterogeneity, underscoring the need for regionally tailored policy assessments.

Figure 3-A.2: Impact of Non-Systematic Monetary Policy on Wage Inequality in Industrial and Manufacturing Hubs Provinces

(a) Gini Coefficient (Aggregate Shock). (b) Wage Ratio (Aggregate Shock).



(c) Gini Coefficient (Asymmetric Shock). (d) Wage Ratio (Asymmetric Shock).



Note: The figures display the IRFs an unexpected change in the bank rate by 25 basis points on wage inequality. The blue (expansionary policy) and red (contractionary policy) bands indicate a 95% confidence interval, where standard errors are obtained using heteroskedasticity-consistent standard errors.

3-A.5 Maritime Provinces

The Maritime Provinces — New Brunswick, Nova Scotia, and Prince Edward Island — exhibit distinct regional responses to non-systematic monetary policy, particularly in terms of wage inequality. Following an unexpected increase in the bank rate by 25-basis-point, the Gini coefficient generally declines, indicating a narrowing of overall wage inequality, while the wage ratio increases, reflecting a widening gap between high- and low-skilled workers. However, the timing and persistence of these effects vary across provinces.

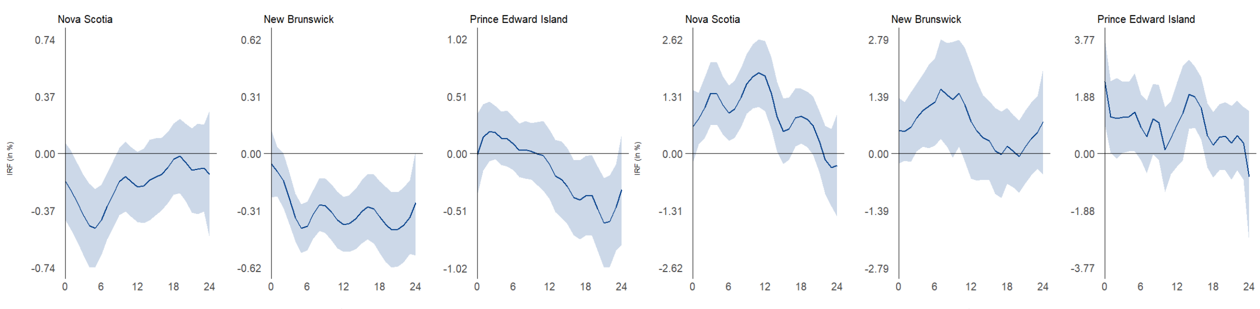
In Prince Edward Island, the Gini coefficient declines significantly from the 16th to 23rd month, while the wage ratio increases sharply immediately after the shock and resurfaces later, suggesting cyclical inequality dynamics. New Brunswick experiences a sustained decline in the Gini coefficient from the 3rd to 23rd month, with wage ratio increases in the short term. Nova Scotia shows a short-term drop in overall inequality and rising skill-based wage gaps that re-emerge later.

Under a contractionary monetary policy shock, the Gini coefficient declines significantly across all three Maritime provinces, indicating a reduction in overall wage inequality. However, the wage ratio displays heterogeneous responses: Nova Scotia and New Brunswick experience rising skill-based wage disparities, while Prince Edward Island shows a temporary decline followed by renewed increases.

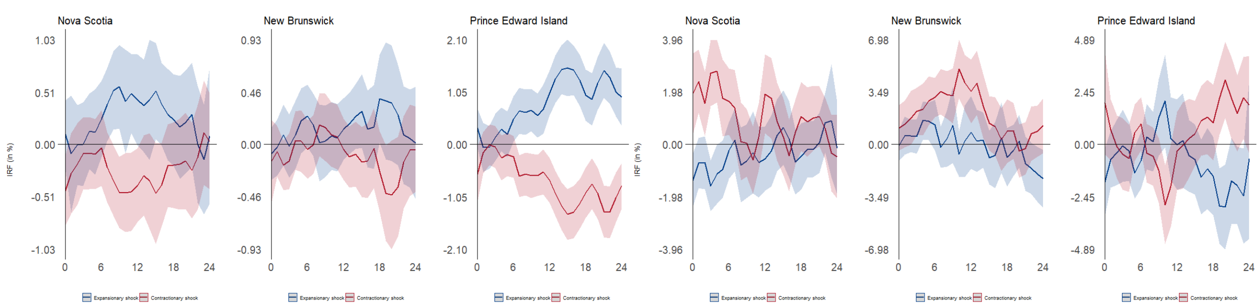
In contrast, an expansionary monetary policy shock results in a significant and sustained rise in the Gini coefficient across all provinces. The wage ratio, however, follows province-specific trajectories, with some regions exhibiting temporary increases and others showing delayed declines. These patterns highlight persistent and symmetric distributional effects, but with varying implications for skill-based inequality.

Figure 3-A.3: Impact of Non-Systematic Monetary Policy on Wage Inequality in Maritime Provinces

(a) Gini Coefficient (Aggregate Shock). (b) Wage Ratio (Aggregate Shock).



(c) Gini Coefficient (Asymmetric Shock). (d) Wage Ratio (Asymmetric Shock).



Note: The figures display the IRFs of an unexpected change in the bank rate by 25 basis points on wage inequality. The blue (expansionary policy) and red (contractionary policy) bands indicate a 95% confidence interval, where standard errors are obtained using heteroskedasticity-consistent standard errors.

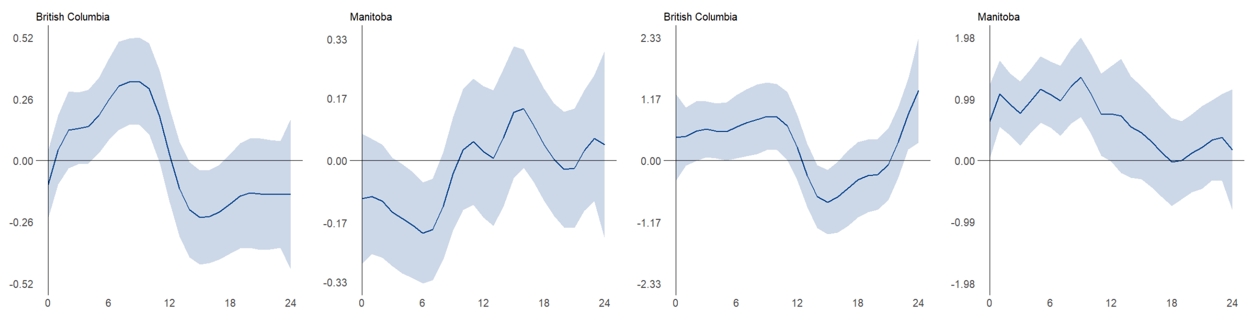
3-A.6 Diversified Economies Provinces

Manitoba and British Columbia, clustered as diversified economy provinces, exhibit distinct wage inequality responses to non-systematic monetary policy. An unexpected 25-basis-point rate increase leads to an immediate and sustained rise in the wage ratio in Manitoba, peaking at 0.74% by the 11th month. British Columbia shows a similar short term pattern, followed by a medium-term dip and renewed increases in the 23rd and 24th months.

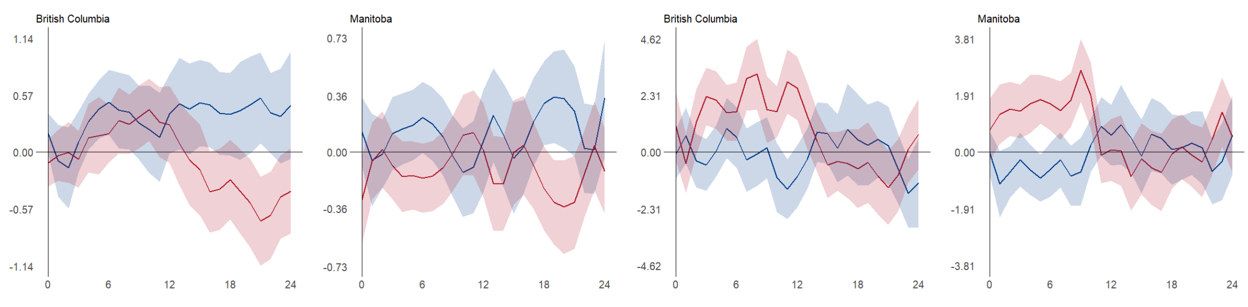
In British Columbia, the Gini coefficient initially increases but then declines significantly between the 14th and 17th months. In contrast, Manitoba experiences a short term reduction in overall wage inequality between the fifth and seventh months. Under expansionary monetary policy shocks, the Gini coefficient rises in both provinces, while the wage ratio declines—particularly in British Columbia, indicating a narrowing of skill-based wage gaps. Conversely, contractionary monetary policy shocks lead to rising wage ratios in both provinces during the short and medium term. While the Gini coefficient declines in Manitoba, it fluctuates in British Columbia, underscoring the heterogeneous and evolving effects of non-systematic monetary policy on wage inequality across regions and over time.

Figure 3-A.4: Impact of Non-Systematic Monetary Policy on Wage Inequality in Diversified Economy Provinces

(a) Gini Coefficient (Aggregate Shock). (b) Wage Ratio (Aggregate Shock).



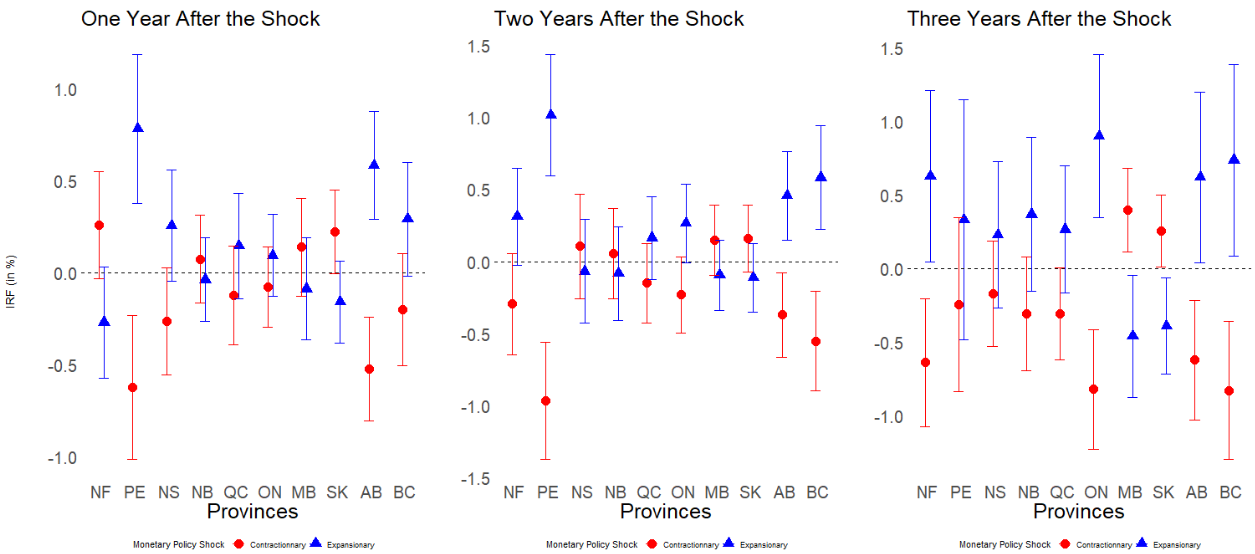
(c) Gini Coefficient (Asymmetric Shock). (d) Wage Ratio (Asymmetric Shock).



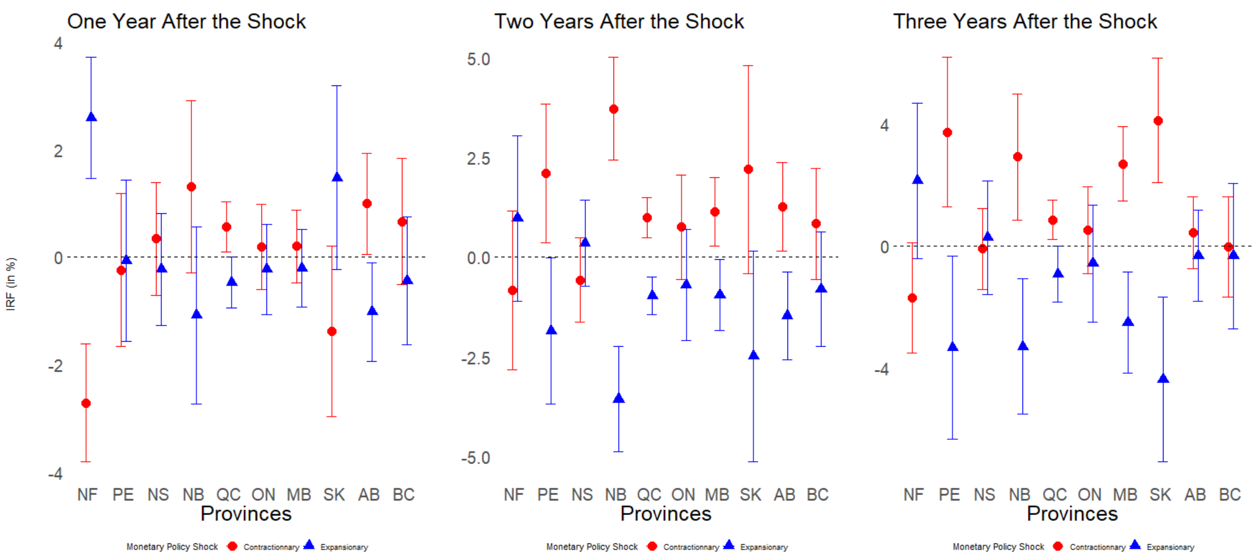
Note: The figures display the IRFs of an unexpected change in the bank rate by 25 basis points on wage inequality. The blue (expansionary policy) and red (contractionary policy) bands indicate a 95% confidence interval, where standard errors are obtained using heteroskedasticity-consistent standard errors.

Figure 3-A.5: Cumulative Impacts on The Wage Ratio: Asymmetric Impact

(a) Gini Coefficient.



(b) Wage Ratio.



Note: The figures display the cumulative impact on wage ratio after an unexpected change in the bank rate by 25 basis points on wage inequality. The blue (expansionary policy) and red (contractionary policy) bands indicate a 95% confidence interval, where standard errors are obtained using heteroskedasticity-consistent standard errors.

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