

UNIVERSITY OF OTTAWA

DOCTORAL THESIS

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Three Essays on Government  
Spending and Real Economic  
Activity

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## **Declaration of Authorship**

I, Sicong Ma, declare that this thesis titled "Three Essays on Government Spending and Real Economic Activity" and the work presented in it are my own.

Signed: Sicong MA

Date: February 2023

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# Abstract

Referring to the results of most empirical studies, an increase in expenditures on public capital leads to an appreciation of the real exchange rate. This result contradicts the findings in theoretical papers. In the latter studies, the real exchange rate depreciates, following a rise in public capital spending. The first chapter examines the puzzle related to the impact on the real exchange rate of government spending on physical capital. We have developed a theoretical model to sort out the puzzle, showing that the real exchange rate can move in both directions following an increase in public spending on capital. In contrast to some existing theoretical studies, we depart from an executive focus on the impact of government spending on capital on the demand side of the economy. Considering the impact of public capital on the supply side of the economy, we developed a small-open New Keynesian Dynamic Stochastic General Equilibrium model with tradable and non-tradable goods, real and nominal rigidities, and productive government spending. The latter affects the productivity of private factors aside from impacting the demand side of the economy. We explored the role played by the output elasticity of public capital on the dynamics of the real exchange rate. By being able to focus on both the supply and demand sides of government spending on public capital, we can shed light on its net effects on the dynamic response of the real exchange rate. Specifically, we assessed the responses of the real exchange rate and other real variables following innovations in government spending on public capital, and ran some sensitivity analyses on the magnitude of the output elasticity of public capital and of other key parameters of the model.

The second chapter explores the dynamic effects of government spending on private consumption and other macroeconomic variables. Responding to a rise in government spending, both neoclassical and New-Keynesian models predict that private consumption will decrease. However, empirical analyses find a rise, no change, or a fall in private consumption. To address the results' inconsistencies, this paper distinguishes public consumption from public investment because public capital is productive. Since the traditional structural vector autoregressive (SVAR) models neglect the forward-looking behaviours of economic agents, we

built SVAR models with rational expectations motivated by a New-Keynesian model. We found a rise in public investment in response to a shock to public consumption, and identified ‘deep’ structural parameters (not subject to the Lucas critique) via full information maximum likelihood estimation. Our findings suggest that an increase in public consumption associated with a positive response to public investment can result in an increase in private consumption. Blocking the response of public investment to a rise in public consumption leads to a decrease in private consumption.

Some studies have suggested that when the composition of government expenditures (consumption vs. investment) is suboptimal, a shift from public investment to consumption, or from public consumption to investment, may boost growth. The third chapter uses autoregressive distributed lag (ARDL) models to investigate, for the United States, the impact of the composition of government expenditures on real GDP growth. It is apparent that increasing the ratio of total public consumption to total public investment has a positive effect on GDP growth. In a disaggregated analysis including different government levels, we find that increasing the public consumption-to-investment ratio at the federal level does not affect GDP growth, indicating that the federal public spending composition is optimal. In contrast, an increase in the ratio at the state and local level has a positive impact on GDP growth. This suggests that the ratio of public consumption to investment is less than optimal at the state and local levels. It follows that state and local governments can reduce public investment to finance public consumption in order to promote GDP growth, while keeping government expenditures constant. The ARDL results still hold up in a simple linear model with Ordinary Least Squares (OLS) estimation. Our findings offer an alternative explanation for the observation in some studies that the government prioritizes public consumption over public investment, despite the high productivity of public investment found in several studies.

# General Introduction

This dissertation consists of three chapters on government spending and real economic activities. In all three chapters, theoretical and empirical models of government expenditures on consumption and investment are considered. This research provides alternative explanations for several key puzzles and contributes to multiple branches of the literature.

We begin chapter one with a puzzle concerning the relationship between government expenditures and the real exchange rate. The purpose of the first chapter is to address the apparent discrepancy between theoretical and empirical findings about the relationship between productive government spending and the real exchange rate. We hypothesize that the net effect on the relative price of traded to non-traded goods, and consequently the real exchange rate, is contingent on the output elasticity of public capital. In a small open economy with tradable and non-tradable goods, if the output elasticity is high (low), the real exchange rate depreciates (appreciates) because the supply (demand) effect outweighs the demand (supply) effect.

In addition to the real exchange rate puzzle, the literature on government expenditures also poses a puzzle concerning the relationship between government spending and private consumption. Chapter two differentiates between public consumption and public investment. By placing public consumption before public investment in a vector of variables, a structural vector autoregressive model that incorporates public consumption and public investment can examine how public investment responds to an increase in public consumption. Due to the wealth benefits of the related increase in public investment, rational households increase their private consumption in response to a rise in public consumption. Therefore, our paper shows that a shock to public consumption associated with an increase in public investment can explain why an increase in government spending produces a rise in private consumption. This chapter proposes that a theoretical examination of non-productive government expenditures should include productive expenditures in order to reconcile the theoretical forecast of private

consumption with empirical findings. This study provides an alternative explanation for the private consumption puzzle induced by an increase in non-productive government spending. If the supply-side effect of a boost in public investment overcomes the demand-side effect of rising public consumption and investment in reaction to a shock to public consumption, private consumption increases. If, however, the demand-side effect outweighs the supply-side effect, then private consumption declines in reaction to the shock. In addition, there is no change in private consumption if the demand-side effect is equal to the supply-side effect.

The third chapter examines the effect of government expenditure composition on economic growth. The literature suggests that there is disagreement with the a priori assumption that public investment is more productive than public consumption. When a government misallocates expenditures in favour of public consumption over public investment, public investment could be more productive than public consumption; hence, raising the share of total expenditures committed to public investment enhances GDP growth. Moreover, if the government misallocates funds in favour of public investment, public consumption may be more productive than public investment. Thus, increasing the proportion of total expenditures committed to public investment would hamper economic growth. To illustrate the potential benefits of GDP growth in a single nation, we selected the United States as a case study because the recent literature inadequately discussed the relationship between U.S. GDP growth and the composition of government expenditures. In this study, we examined the effect of a change in the ratio of public consumption to investment on the economic growth rate in the United States, while holding government spending constant. For the purpose of analyzing U.S. GDP growth, quarterly time series data are examined. Existence of unit roots and long-run relationships can lead to spurious regressions and inappropriate models in time-series analysis. We utilized the autoregressive distributed lag (ARDL) model to take advantage of the ARDL model's capacity to accommodate a mix of stationary ( $I(0)$ ) and first-order non-stationary ( $I(1)$ ) variables. This paper begins with the ARDL model, examining the relationship between GDP growth and the ratio of total public consumption to investment at the national level. Studies on different government levels motivate us to use expenditure data from fed-

eral, state, and local levels in the ARDL model, due to fiscal decentralization and the actions that should be carried out by each level of government. If an increase in the ratio of public consumption to investment increases GDP growth, the ratio is below its optimal level. Therefore, increasing public consumption while decreasing public investment can promote economic growth. If the effect of the ratio change is negative, the optimal level has been exceeded - and the government should reduce public consumption to fund public investment. Lastly, if GDP growth has a negligible response to a shift from public investment to consumption, the ratio of public consumption to investment is optimal for maximizing GDP growth; therefore, no shift between the two components is necessary. In addition, the breakdown of total government expenditures into federal, state, and local levels can indicate whether a shift in spending should be carried out by the federal government or by state and local governments.

# Chapter 1

## Output Elasticity of Public Capital and Real Exchange Rate Dynamics

### 1.1 INTRODUCTION

Referring to the Mundell-Fleming Model developed by Mundell (1963) and Fleming (1962) a positive shock to government spending leads to an increase in output and the interest rates. Ultimately, this gives rise to an appreciation of the nominal exchange rate. In the presence of price stickiness, the real exchange rate appreciates because of the increase in government spending. In the same manner, in the context of a small open economy with tradable and non-tradable goods, a rise in government spending (which is intensive in non-tradable goods) increases the price of non-tradable goods and leads to an appreciation of the CPI real exchange rate.

Still, empirical studies show that a positive shock to government spending leads instead to a depreciation of the real exchange rate. This finding has been obtained in several papers, including Corsetti and Müller (2006), Enders *et al.* (2011), Kim and Roubini (2008), Monacelli and Perotti (2006, 2010), Ravn *et al.* (2007). This contradiction between theoretical and empirical studies is known in the literature as the “real exchange rate puzzle” - addressed in the literature through an improvement of the theoretical models. Ravn *et al.* (2007) and Ravn *et al.* (2012) include deep habits in household behaviour, while Monacelli and Perotti (2006) use non-separable utility in consumption and leisure.

In this paper, we plan to address another puzzle related to the link between public

spending and the real exchange rate. Another trend in the literature has recently emerged with the re-examination of the link between public spending and the real exchange rate. The papers in that strand have assessed the distinctive effects on the real exchange rate of non-productive and productive government expenditures. The rationale for the distinction between the two types of government expenditures is that while both types of expenditures have a direct impact on the demand side of the economy, productive expenditures can additionally affect the supply side of the economy. As productive expenditures affect the level of public capital stock, they have an impact on the productivity of private factors of production (capital and labour). Basu and Kollmann (2013) indicate that, associated with highly elastic labour supply, government purchases that can increase the domestic productivity of private factors can generate a depreciation of the real exchange rate. In addition to the simple static two-country model, Iwata (2013) shows an example of Japan in an open-economy DSGE model with productive public capital, and finds that the real exchange rate depreciates in response to a rise in productive public spending.

This result is at odds with the empirical findings in Bénétrix and Lane (2013), which find an appreciation of the real exchange rate in response to an increase in public investment. The latter two studies rely, respectively, on a structural vector autoregressive (VAR) model for Ireland and a VAR model for the Euro area. In the same vein, the empirical results found by Y. Chen and Liu (2018) in a study using a structural VAR model for the Chinese economy suggest that a positive shock to public investment leads to an appreciation of the real exchange rate. It follows that, based on the existing theoretical models, an increase in public investment leads to a depreciation of the real exchange rate, while empirical models point to the opposite - i.e., an appreciation.

The objective of this paper is to address the inconsistency between the theoretical and empirical results on the connection between productive government expenditures and the real exchange rate. We claim that, from a theoretical perspective, an increase in public investment can lead to either an appreciation or a depreciation of the real exchange rate. Hence, the

apparent contradiction between the theoretical and empirical results on the link between productive government expenditures and the real exchange rate vanishes.

In an economy that produces both tradable and non-tradable goods, the real exchange rate, as suggested by Monacelli and Perotti (2010), depends on the terms of trade and the relative price of tradable to non-tradable goods. The response of the real exchange rate to an increase in productive government spending hinges upon its effects on the relative price of tradable to non-tradable goods. Through its impact on aggregate demand, the increase in productive government expenditures raises the price of non-tradable goods relative to tradable goods because government expenditures are intensive in services (non-tradable). Yet, from the supply side, the increase in productive expenditures puts downward pressure on the price of non-tradable goods. The latter effect stems from the improvement in the productivity of private factors brought about by the rise in the level of public capital. As a result, the net effect of productive government spending on the relative price of tradable to non-tradable goods is undetermined. It could lead to either an appreciation or a depreciation of the real exchange rate. For instance, non-tradable goods may become more expensive than the tradable goods following an innovation in the productive expenditures when the demand-side dominates the supply-side effect. The reverse may occur when the supply-side dominates the demand-side effect.

We conjecture that the net effect on the relative price of tradable to non-tradable goods, and hence on the real exchange rate, depends on the output elasticity of public capital. If the output elasticity is high (low), the real exchange rate depreciates (appreciates) because the supply (demand) dominates the demand (supply) effect.

This paper analyzes the impact of the output elasticity of public capital on the dynamics of the real exchange rate in a small open economy. We have developed a small-open New-Keynesian dynamic stochastic general equilibrium (DSGE) model with tradable and non-tradable goods, and nominal and real rigidities in the presence of productive public capital.

The remainder of the paper is as follows. Section 1.2 provides a review of the literature, while we present the specification of the small-open New-Keynesian DSGE model in Section 1.3. Section 1.4 presents the equilibrium conditions of the small open economy and the log-linearized version of the DSGE model. We discuss the data and calibration issues in Section 1.5. We perform the sensitivity analysis and discuss the simulation results in Section 1.6. The last section concludes.

## 1.2 LITERATURE REVIEW

As we stated in the Introduction, both non-productive and productive government expenditures lead to the real exchange rate puzzle. VAR studies summarized by Table 1 present responses of real exchange rate and other variables to different shocks to government expenditures. In terms of non-productive government expenditures, an appreciation of the real exchange rate generated by a positive shock to public consumption in the Mundell-Fleming Model contradicts the depreciation of the real exchange rate found in empirical studies.

To reconcile the theoretical prediction with the real depreciation observed in the data, Ravn *et al.* (2007) and Ravn *et al.* (2012) have introduced deep-habits in household preferences in the context of a two-country model. These papers show that domestic firms tend to have lower markups relative to foreign markups, because of an increase in domestic aggregate demand. Thus, the domestic economy becomes less expensive than the foreign economy, so the real exchange rate depreciates. Considering the international risk-sharing condition that ties the ratio of marginal utilities of consumption across countries to the real exchange rate, Monacelli and Perotti (2006) introduce Linnemann (2006)'s non-separable utility between consumption and leisure. They build a small-open New-Keynesian model with endogenous labour supply and nominal price rigidity. In their model, private consumption can positively react to an increase in public consumption because of the complementarity between consumption and labour supply. Consequently, because of the risk-sharing condition, the real exchange rate depreciates following an increase in private consumption.

Under the international risk sharing condition, private consumption is positively associated with the real exchange rate. Backus and Smith (1993) were the first to notice the puzzle related to the private consumption - real exchange rate anomaly. The correlation between the standard deviation of relative private consumption and the standard deviation of bilateral real exchange rate is negative. The correlation between the mean of relative private consumption and the mean of bilateral real exchange rate is negative but not statistically significant. Therefore, Backus and Smith (1993) conclude that, within eight OECD countries, there is no evidence on the positive relationship between the growth rates of relative private consumption and the growth rates of bilateral real exchange rate. More recent VAR studies conducted by Ravn *et al.* (2012), Corsetti, Meier, *et al.* (2012) and Monacelli and Perotti (2010) find a positive relationship between private consumption and the real exchange rate across countries, which is subject to government spending shocks.

In addition to a small open economy with one production sector, Monacelli and Perotti (2010) reveal in a VAR study with tradable and non-tradable goods, not only the real exchange rate puzzle, but also new puzzles of two components of the real exchange rate. The two components depend, respectively, on the terms of trade for the tradable sector and the relative price of tradable to non-tradable goods. Empirically, they find that a positive shock to public spending leads to a depreciation of the real exchange rate and a decrease in the price of non-tradable goods. Moreover, the terms of trade initially appreciate and then depreciate in the future.

Following the seminal work of Aschauer (1989b), Barro (1990) and Baxter and King (1993) several papers have analyzed the implications of including public capital in the representation of firms' technology of production. Moreover, Basu and Kollmann (2013) consider that the public capital stock potentially raises private-sector labour productivity in a simple static two-country model and demonstrate that a lower (higher) degree of externality of public capital to labour productivity could generate a real appreciation (depreciation). In addition to the above-mentioned static model, Di Giorgio *et al.* (2018) consider the public capital exter-

nality to labour productivity in a two-country overlapping generation (OLG) model. They find that the real exchange rate depreciates in response to productive government expenditures.

To the best of our knowledge, there is no study of the impact of productive government expenditures on the dynamics of the real exchange rate within the New Keynesian DSGE framework. In our model, productive public capital not only raises labour productivity but also the productivity of private capital in both home-tradable and non-tradable sectors. Thus, we refer the public capital externality as the output elasticity of public capital in the rest of this paper. Our DSGE model with price rigidities and investment adjustment costs finds that the threshold (0.1) found by Basu and Kollmann (2013) causes real exchange rate appreciation, and the threshold found by our DSGE model is greater than 0.1.

In contrast to the high values of the public capital externality (0.64 - 0.96) implied by Di Giorgio *et al.* (2018), this paper will consider plausible values of the output elasticity of public capital - ranging from a minimum of 0.04 to a maximum of 0.29. The values are estimated by Bom and Ligthart (2014) in a meta-analysis that has corrected for linear publication bias. Chatterjee and Mursagulov (2016) find an appreciation of the real exchange rate in the short run, irrespective of the value of the output elasticity of public capital. With the value above the threshold, our paper achieves a hump-shaped response of the real exchange rate consistent with Di Giorgio *et al.* (2018). However, with the value below the threshold, we obtain an appreciation of the real exchange rate in the short run, and, in the propagation of the public-investment shock, the hump-shaped response of the real exchange rate indicates a future real depreciation. As a result, within the range of low output elasticity of public capital, our paper can find either depreciation or appreciation in the short run. Even though the output elasticity of public capital is very low, the dynamics of the real exchange rate indicates a depreciation following a shock to public capital. Chatterjee and Mursagulov (2016) suggest that the output elasticity of public capital matters for the real exchange rate responses in the long run. Our paper finds that it also matters in the short run.

In Section 1.3, we illustrate our small-open New-Keynesian DSGE model with traded

and non-traded goods in the presence of productive public capital and of nominal and real rigidities. In the two-sector model, the public investment has a demand-side effect only on the non-tradable sector but a symmetric supply-side effect on the two sectors; therefore, we will discuss the transmission mechanism induced by the public investment shock at both aggregate and sectoral levels.

**Table 1:** Effects of Government Spending Shocks in Empirical Studies

Study	Country	Sample	Model	Identification	Shocks	Y	C	I	RER	CA/TB
Blanchard and Perotti (2002)	US	1947Q1-1997Q4	Structural VAR	Recursive	Spending	+	+	-		
Kim and Roubini (2008)	US	1973Q1-2004Q1	Structural VAR	Recursive	Budget deficit	+		-	Dep	+
Monacelli and Perotti (2006)	US					+	+	-	Dep	=
	UK					+	+	+	Dep	-
	AUS		Structural VAR			+	+	+	Dep	-
	CAN	1975Q1-2006Q2		Recursive	Consumption	+	+	+	Dep	-
Ravn <i>et al.</i> (2012)	US UK AUS CAN	1975Q1-2005Q4	Panel Structural VAR	Recursive	Consumption	+	+		Dep	-
Consetti, Meier, <i>et al.</i> (2012)	US	1983Q1-2007Q4	VAR	Recursive Anticipated shocks	Spending	+	+	-	Dep	=
Bénétrix and Lane (2013)	11 EMU	Annually 1970-2008	Panel VAR	Recursive	Spending	+			App	
					Investment	+			App	
					Consumption wage non-wage	+	=		App	
Enders <i>et al.</i> (2011)	US	1975Q1-2005Q4	VAR	Sign restrictions	Spending	+	=	-	Dep	
Iwata (2013)	Japan	1973Q1-1998Q4	Structural VAR	Sign restrictions	Consumption	+	+	=	Dep	-
					Investment	+	+	=	Dep	-
					Consumption	+	+	=	Dep	-
Y. Chen and Liu (2018)	China	1995Q1-2015Q2	Structural VAR	Sign restrictions	Investment	+	+	=	Dep	-
					Consumption	+	+	-	App	+

## 1.3 A SMALL OPEN ECONOMY MODEL

We have built a New-Keynesian DSGE model with tradable and non-tradable goods in the context of a small open economy. Economic agents in the economy are a representative household with separable preferences the same as Galí and Monacelli (2005), firms producing home-tradable goods, firms producing non-tradable goods, a central government, a central bank, and the rest of world.

### 1.3.1 Households

The objective of the representative household in this small open economy is to maximize its expected lifetime utility

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t, N_t) \quad (1.1)$$

where  $N_t$  denotes hours of labour and  $C_t$  is a composite index of consumption; and subject to a sequence of budget constraints

$$\begin{aligned} \int_0^1 P_{H,t}(j)(C_{H,t}(j) + I_{H,t}(j))dj + \int_0^1 \int_0^1 P_{i,t}(j)(C_{i,t}(j) + I_{i,t}(j))djdi \\ + \int_0^1 P_{N,t}(j)(C_{N,t}(j) + I_{N,t}(j))dj + E_t\{\nu_{t,t+1}D_{t+1}\} \\ \leq D_t + W_t N_t + Z_t K_t + T_t \end{aligned} \quad (1.2)$$

for  $t = 0, 1, 2, \dots$ , where  $P_{H,t}(j)$  and  $P_{i,t}(j)$  respectively denote the price of differentiated home-tradable goods and the price of differentiated foreign-tradable goods produced by country  $i$  (expressed in domestic currency).  $D_{t+1}$  denotes the nominal payment in the period  $t+1$  to the representative household that holds a portfolio which includes shares in firms at the end of the period  $t$ .  $W_t$  is the nominal wage,  $Z_t$  is the nominal rental cost of private capital, and  $T_t$  measures lump-sum transfers/taxes. All of the above variables are expressed in units of the domestic currency.  $\nu_{t,t+1}$  is the price of the portfolio held at the end of period  $t$ .<sup>1</sup> Following Galí and Monacelli (2005), we assume that households have access to a complete set

<sup>1</sup> $\nu_{t,t+1}$  is the stochastic discount factor in equilibrium

of contingent claims, traded internationally, and neither the utility function nor the budget constraint includes money that plays only a role of a unit of account.

The composite index of consumption,  $C_t$ , defined by

$$C_t = \left[ \mu^{\frac{1}{\eta}} (C_{T,t})^{\frac{\eta-1}{\eta}} + (1 - \mu)^{\frac{1}{\eta}} (C_{N,t})^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}} \quad (1.3)$$

where  $C_{N,t}$  is a composite index of private consumption on differentiated non-tradable goods described by the CES function

$$C_{N,t} = \left( \int_0^1 C_{N,t}(j)^{\frac{\epsilon_N-1}{\epsilon_N}} dj \right)^{\frac{\epsilon_N}{\epsilon_N-1}}$$

where  $j \in [0, 1]$  denotes the good variety.  $C_{T,t}$  is a composite index of private consumption on home-tradable and foreign-tradable (importable) goods defined as

$$C_{T,t} = \left[ (1 - \alpha)^{\frac{1}{\eta_T}} (C_{H,t})^{\frac{\eta_T-1}{\eta_T}} + \alpha^{\frac{1}{\eta_T}} (C_{F,t})^{\frac{\eta_T-1}{\eta_T}} \right]^{\frac{\eta_T}{\eta_T-1}} \quad (1.4)$$

where  $C_{H,t}$  is an index of private consumption on differentiated home-tradable goods defined by the CES function

$$C_{H,t} = \left( \int_0^1 C_{H,t}(j)^{\frac{\epsilon_T-1}{\epsilon_T}} dj \right)^{\frac{\epsilon_T}{\epsilon_T-1}}$$

and  $C_{F,t}$  is an index of foreign-tradable goods from different countries given by

$$C_{F,t} = \left( \int_0^1 (C_{i,t})^{\frac{\eta_T-1}{\eta_T}} di \right)^{\frac{\eta_T}{\eta_T-1}}$$

where  $C_{i,t}$  is an index of consumption on differentiated foreign-tradable goods imported from the country  $i$  and consumed by domestic households. It is defined by an analogous CES function:

$$C_{i,t} = \left( \int_0^1 C_{i,t}(j)^{\frac{\epsilon_T-1}{\epsilon_T}} dj \right)^{\frac{\epsilon_T}{\epsilon_T-1}}.$$

Parameters  $\epsilon_T > 1$  and  $\epsilon_N > 1$  respectively denote the elasticity of substitution between

varieties that are produced within any given country. Parameter  $\eta$  measures the elasticity of substitution between tradable and non-tradable goods. Parameter  $\mu \in [0, 1]$  is a ratio of private consumption on tradable goods to aggregate consumption. Parameter  $\eta_T$  denotes the substitutability between tradable goods produced in different countries. Parameter  $\alpha \in [0, 1]$  measures a natural index of openness that is inversely related to the degree of home bias in preferences.

The optimal allocations of consumption and investment for each sectoral good can be explained by the demand functions:

$$\begin{aligned} C_{N,t}(j) &= \left( \frac{P_{N,t}(j)}{P_{N,t}} \right)^{-\epsilon_N} C_{N,t} \\ I_{N,t}(j) &= \left( \frac{P_{N,t}(j)}{P_{N,t}} \right)^{-\epsilon_N} I_{N,t} \end{aligned} \tag{1.5}$$

$$\begin{aligned} C_{H,t}(j) &= \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon_T} C_{H,t} \\ I_{H,t}(j) &= \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon_T} I_{H,t} \end{aligned} \tag{1.6}$$

$$\begin{aligned} C_{i,t}(j) &= \left( \frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\epsilon_T} C_{i,t} \\ I_{i,t}(j) &= \left( \frac{P_{i,t}(j)}{P_{i,t}} \right)^{-\epsilon_T} I_{i,t} \end{aligned} \tag{1.7}$$

for all  $i, j \in [0, 1]$ , where  $P_{N,t} = \left( \int_0^1 P_{N,t}(j)^{1-\epsilon_N} dj \right)^{\frac{1}{1-\epsilon_N}}$  is the price index of non-tradable goods.  $P_{H,t} = \left( \int_0^1 P_{H,t}(j)^{1-\epsilon_T} dj \right)^{\frac{1}{1-\epsilon_T}}$  is the price index of home-tradable goods and  $P_{i,t} = \left( \int_0^1 P_{i,t}(j)^{1-\epsilon_T} dj \right)^{\frac{1}{1-\epsilon_T}}$  is the price index of foreign-tradable goods produced by the country  $i$

(expressed in domestic currency). It follows from (1.5), (1.6) and (1.7) that:

$$\begin{aligned}\int_0^1 P_{N,t}(j)(C_{N,t}(j) + I_{N,t}(j))dj &= P_{N,t}(C_{N,t} + I_{N,t}) \\ \int_0^1 P_{H,t}(j)(C_{H,t}(j) + I_{H,t}(j))dj &= P_{H,t}(C_{H,t} + I_{H,t}) \\ \int_0^1 P_{i,t}(j)(C_{i,t}(j) + I_{i,t}(j))dj &= P_{i,t}(C_{i,t} + I_{i,t})\end{aligned}$$

Moreover, the optimal allocations of private expenditures on foreign-tradable goods imported from the country  $i$  is:

$$\begin{aligned}C_{i,t} &= \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\eta_T} C_{F,t} \\ I_{i,t} &= \left(\frac{P_{i,t}}{P_{F,t}}\right)^{-\eta_T} I_{F,t}\end{aligned}\tag{1.8}$$

for all  $i \in [0, 1]$ , and where  $P_{F,t} = \left(\int_0^1 P_{i,t}^{1-\eta_T} di\right)^{\frac{1}{1-\eta_T}}$  is the price index of foreign-tradable goods denominated in domestic currency. Equation (1.8) indicates:

$$\int_0^1 P_{i,t}(C_{i,t} + I_{i,t})di = P_{F,t}(C_{F,t} + I_{F,t})$$

In addition, the optimal allocations of private consumption and investment on home-tradable and foreign-tradable goods are:

$$\begin{aligned}C_{H,t} &= (1 - \alpha) \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta_T} C_{T,t} \\ I_{H,t} &= (1 - \alpha) \left(\frac{P_{H,t}}{P_{T,t}}\right)^{-\eta_T} I_{T,t}\end{aligned}\tag{1.9}$$

$$\begin{aligned}C_{F,t} &= \alpha \left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\eta_T} C_{T,t} \\ I_{F,t} &= \alpha \left(\frac{P_{F,t}}{P_{T,t}}\right)^{-\eta_T} I_{T,t}\end{aligned}\tag{1.10}$$

where the price index of traded goods is given by:

$$P_{T,t} = \left[ (1 - \alpha)(P_{H,t})^{1-\eta_T} + \alpha(P_{F,t})^{1-\eta_T} \right]^{\frac{1}{1-\eta_T}}$$

and  $P_{H,t}(C_{H,t} + I_{H,t}) + P_{F,t}(C_{F,t} + I_{F,t}) = P_{T,t}(C_{T,t} + I_{T,t})$ .

Finally, the optimal allocations of private expenditures on tradable and non-tradable goods can be described as:

$$\begin{aligned} C_{T,t} &= \mu \left( \frac{P_{T,t}}{P_t} \right)^{-\eta} C_t \\ I_{T,t} &= \mu \left( \frac{P_{T,t}}{P_t} \right)^{-\eta} I_t \end{aligned} \tag{1.11}$$

$$\begin{aligned} C_{N,t} &= (1 - \mu) \left( \frac{P_{N,t}}{P_t} \right)^{-\eta} C_t \\ I_{N,t} &= (1 - \mu) \left( \frac{P_{N,t}}{P_t} \right)^{-\eta} I_t \end{aligned} \tag{1.12}$$

where  $P_t = \left[ \mu(P_{T,t})^{1-\eta} + (1 - \mu)(P_{N,t})^{1-\eta} \right]^{\frac{1}{1-\eta}}$ .

As a result, the total expenditures of domestic households on consumption and investment are  $P_{T,t}(C_{T,t} + I_{T,t}) + P_{N,t}(C_{N,t} + I_{N,t}) = P_t(C_t + I_t)$ . The period budget constraint can be rewritten as:

$$C_t + I_t + \frac{E_t\{\nu_{t,t+1}D_{t+1}\}}{P_t} \leq \frac{D_t}{P_t} + \frac{W_t N_t}{P_t} + \frac{Z_t K_t}{P_t} + \frac{T_t}{P_t}. \tag{1.13}$$

Following Galí and Monacelli (2005) and Corsetti, Meier, *et al.* (2012), we assume that the period separable utility takes the following form:

$$U(C, N) = \begin{cases} \frac{C^{1-\sigma}-1}{1-\sigma} - \zeta \left( \frac{N^{1+\varphi}}{1+\varphi} \right) & \text{for } \sigma \neq 1 \\ \log C - \zeta \left( \frac{N^{1+\varphi}}{1+\varphi} \right) & \text{for } \sigma = 1 \end{cases} \tag{1.14}$$

where  $\sigma \geq 0$  is a relative risk aversion coefficient determining the curvature of the utility of consumption and  $\varphi \geq 0$  is called as an inverse of Frisch elasticity of labour determining a marginal disutility in respect of labour supply.  $\zeta$  denotes the sensitivity of disutility in response to labour supply.

Under adjustment costs, the stock of private capital varies from one period to the next as a result of private investment,  $I_t$ , according to the following motion equation given by Monacelli and Perotti (2006):

$$K_{t+1} = (1 - \delta)K_t + \left[1 - \Psi\left(\frac{I_t}{K_t}\right)\right] I_t \quad (1.15)$$

where  $\delta$  is a depreciation rate of private capital stock.  $\Psi\left(\frac{I_t}{K_t}\right)$  is an increasing and convex function, in which  $\Psi'\left(\frac{I_t}{K_t}\right) > 0$  and  $\Psi''\left(\frac{I_t}{K_t}\right) > 0$ . In the steady state,  $I/K = \delta$ ,  $\Psi\left(\frac{I}{K}\right) = \Psi'\left(\frac{I}{K}\right) = 0$  and  $\Psi''\left(\frac{I}{K}\right) = \xi > 0$ . This is consistent with the assumption of Smets and Wouters (2003) and Christiano *et al.* (2005) that the adjustment costs only depend on the second-order derivative.

Let us define  $\chi_t$  and  $\chi_t \Lambda_{q,t}$  as the Lagrange multipliers on constraints (1.13) and (1.15) respectively. Solving the household's problem, which is to maximize its lifetime expected utility, and subject to the sequence of budget constraints and private capital accumulation, we obtain the following optimality conditions:

$$\zeta N_t^\varphi C_t^\sigma = \frac{W_t}{P_t} \quad (1.16)$$

which is a standard intratemporal optimality condition.

Taking conditional expectations into consideration, we can obtain a conventional stochastic Euler equation:

$$\beta R_t E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left( \frac{P_t}{P_{t+1}} \right) \right\} = 1 \quad (1.17)$$

where  $R_t = \frac{1}{E_t\{\nu_{t,t+1}\}}$  is the expected gross return on a riskless one-period discount bond paying

off one unit of domestic currency in  $t + 1$  (with  $E_t\{\nu_{t,t+1}\}$  being its price).

The optimality condition for private investment is given by:

$$\Lambda_{q,t} \left[ 1 - \Psi \left( \frac{I_t}{K_t} \right) - \Psi' \left( \frac{I_t}{K_t} \right) \frac{I_t}{K_t} \right] = 1 \quad (1.18)$$

where  $\Lambda_{q,t}$  is defined as Tobin's  $q$  that is the real shadow price of an additional unit of private investment. Equation (1.18) indicates that a rise in the real shadow price of private investment is associated with the investment-capital ratio. As a result, private investment rises with the real shadow price when the private capital stock is predetermined.

Efficiency also requires

$$\Lambda_{q,t} = \beta E_t \left\{ \left( \frac{C_{t+1}}{C_t} \right)^{-\sigma} \left[ \frac{Z_{t+1}}{P_{t+1}} + \Lambda_{q,t+1} \left( (1 - \delta) + \Psi' \left( \frac{I_{t+1}}{K_{t+1}} \right) \left( \frac{I_{t+1}}{K_{t+1}} \right)^2 \right) \right] \right\} \quad (1.19)$$

Equation (1.19) is a typical asset-price condition derived from an intertemporal Euler equation on the private capital, which relates the current real shadow price to the future expected marginal product of private capital and the future expected movement of the shadow price.

### 1.3.2 Firms

The small open economy has two sectors that produce home-traded and non-traded goods. We assume that labour and private capital are mobile between the two sectors.

#### Home-tradable sector:

A typical firm  $j \in [0, 1]$  produces differentiated home-tradable goods  $j$  with Cobb-Douglas production function:

$$Y_{H,t}(j) = A_{H,t} K_{H,t}(j)^{\psi_H} N_{H,t}(j)^{1-\psi_H} \quad (1.20)$$

where  $A_{H,t}$  is a home-tradable sector production technology parameter. The technology is common to every firm producing differentiated home-tradable goods and depends on the initial

level of technology parameter,  $A_{H,0}$ , and public capital stock,  $K_{G,t}$ , that is given by

$$A_{H,t} = A_{H,0}(K_{G,t})^{\psi_G} \quad (1.21)$$

where  $\psi_G$  denotes the output elasticity of public capital.

An index for aggregate home-tradable output can be described as the Dixit-Stiglitz aggregator (Dixit and Stiglitz, 1977):

$$Y_{H,t} = \left( \int_0^1 Y_{H,t}(j)^{\frac{\epsilon_T-1}{\epsilon_T}} dj \right)^{\frac{\epsilon_T}{\epsilon_T-1}} \quad (1.22)$$

where  $\epsilon_T > 1$  denotes elasticity of substitution between these differentiated home-tradable goods. The optimal allocation of differentiated home-tradable goods is  $Y_{H,t}(j) = \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon_T} Y_{H,t}$ .

Efficiency conditions for the typical firm  $j$  can be achieved by minimizing the real total cost,  $TC_{H,t}(j) = \left( \frac{W_t}{P_{H,t}} \right) N_{H,t}(j) + \left( \frac{Z_t}{P_{H,t}} \right) K_{H,t}(j)$ . Minimization of the aggregate real total cost is:

$$TC_{H,t} = \left( \frac{W_t}{P_{H,t}} \right) \int_0^1 N_{H,t}(j) dj + \left( \frac{Z_t}{P_{H,t}} \right) \int_0^1 K_{H,t}(j) dj \quad (1.23)$$

and subject to Equation (1.22).

The efficiency conditions are given by:

$$\frac{W_t}{P_{H,t}} = MC_{H,t}(1 - \psi_H) \left( \frac{Y_{H,t}(j)}{N_{H,t}(j)} \right)$$

$$\frac{Z_t}{P_{H,t}} = MC_{H,t}\psi_H \left( \frac{Y_{H,t}(j)}{K_{H,t}(j)} \right)$$

where  $MC_{H,t}$  denotes the real marginal cost for the home-tradable sector, which is common

to all firms. Thus, we can adjust the above conditions by erasing  $j$ .

$$\frac{W_t}{P_{H,t}} = MC_{H,t}(1 - \psi_H) \left( \frac{Y_{H,t}}{N_{H,t}} \right) \quad (1.24)$$

$$\frac{Z_t}{P_{H,t}} = MC_{H,t}\psi_H \left( \frac{Y_{H,t}}{K_{H,t}} \right) \quad (1.25)$$

The real marginal cost can be given by:

$$MC_{H,t} = \frac{1}{A_{H,t}} \left( \frac{W_t}{P_{H,t}(1 - \psi_H)} \right)^{1-\psi_H} \left( \frac{Z_t}{P_{H,t}\psi_H} \right)^{\psi_H} \quad (1.26)$$

According to the Calvo (1983) pricing rule, the typical firm  $j$  has a probability  $\theta_H \in (0, 1)$  that keeps its price fixed in the next period; as a result,  $1 - \theta_H$  is the probability that optimally resetting its price. The pricing problem of the firm is:

$$Max \quad \sum_{k=0}^{\infty} (\beta\theta_H)^k E_t \left\{ \widetilde{P}_{H,t}(j) Y_{H,t+k}(j) - NTC_{H,t+k}(j) \right\} \quad (1.27)$$

where  $\widetilde{P}_{H,t}(j)$  is the optimal price reset by the firm  $j$  in the period  $t$ , and  $NTC_{H,t}(j) \equiv P_{H,t}(j)TC_{H,t}(j)$  denotes the nominal total costs.

The optimal price can be achieved by substituting the optimal allocation of  $Y_{H,t}(j)$  into Equation (1.27).

$$Max \quad E_t \sum_{k=0}^{\infty} (\beta\theta_H)^k \left[ \widetilde{P}_{H,t}(j) \left( \frac{\widetilde{P}_{H,t}(j)}{P_{H,t+k}} \right)^{-\epsilon_T} Y_{H,t+k} - P_{H,t+k}(j) \left( \frac{\widetilde{P}_{H,t}(j)}{P_{H,t+k}} \right)^{-\epsilon_T} Y_{H,t+k} MC_{H,t+k}(j) \right] \quad (1.28)$$

Thus, the optimal price-setting strategy for the typical firm resetting its price in period  $t$  can be presented by the rule:

$$\widetilde{P}_{H,t}(j) = \left( \frac{\epsilon_T}{\epsilon_T - 1} \right) \sum_{k=0}^{\infty} (\beta\theta_H)^k E_t \left\{ P_{H,t+k}(j) MC_{H,t+k}(j) \right\}. \quad (1.29)$$

The price index of differentiated home-tradeable goods can be obtained as:

$$P_{H,t} = \left[ \int_0^{\theta_H} (P_{H,t-1})^{1-\epsilon_T} dj + \int_{\theta_H}^1 (\widetilde{P}_{H,t})^{1-\epsilon_T} dj \right]^{\frac{1}{1-\epsilon_T}}$$

$$P_{H,t} = \left[ \theta_H (P_{H,t-1})^{1-\epsilon_T} + (1 - \theta_H) (\widetilde{P}_{H,t})^{1-\epsilon_T} \right]^{\frac{1}{1-\epsilon_T}} \quad (1.30)$$

Note that there is a continuum of firms, and the group that can reset their price (and the group that cannot) are chosen randomly, regardless of when each firm last reset its price. It follows that the distribution of prices among differentiated firms does not change between periods.

### Non-tradable sector:

We assume that firms from the non-tradable sector set prices in a staggered fashion that is same as the home-tradeable sector.

$$\widetilde{P}_{N,t}(j) = \left( \frac{\epsilon_N}{\epsilon_N - 1} \right) \sum_{k=0}^{\infty} (\beta \theta_N)^k E_t \left\{ P_{N,t+k}(j) MC_{N,t+k}(j) \right\}. \quad (1.31)$$

$$P_{N,t} = \left[ \theta_N (P_{N,t-1})^{1-\epsilon_N} + (1 - \theta_N) (\widetilde{P}_{N,t})^{1-\epsilon_N} \right]^{\frac{1}{1-\epsilon_N}} \quad (1.32)$$

The production pattern, also, is identical to the home-tradeable sector and the technology depends on the public capital stock. A typical firm  $j \in [0, 1]$  produces differentiated non-tradeable goods  $j$  with Cobb-Douglas production function:

$$Y_{N,t}(j) = A_{N,t} K_{N,t}(j)^{\psi_N} N_{N,t}(j)^{1-\psi_N} \quad (1.33)$$

The technology is common to every firm producing differentiated non-tradeable goods and depends on the initial level of technology,  $A_{H,0}$ , and public capital stock,  $K_{G,t}$ , that is given

by

$$A_{N,t} = A_{N,0}(K_{G,t})^{\psi_G} \quad (1.34)$$

Here we lay out the efficiency conditions for the non-tradable sector.

$$\frac{W_t}{P_{N,t}} = MC_{N,t}(1 - \psi_N) \left( \frac{Y_{N,t}}{N_{N,t}} \right) \quad (1.35)$$

$$\frac{Z_t}{P_{N,t}} = MC_{N,t}\psi_N \left( \frac{Y_{N,t}}{K_{N,t}} \right) \quad (1.36)$$

$$MC_{N,t} = \frac{1}{A_{N,t}} \left( \frac{W_t}{P_{N,t}(1 - \psi_N)} \right)^{1 - \psi_N} \left( \frac{Z_t}{P_{N,t}\psi_N} \right)^{\psi_N} \quad (1.37)$$

### 1.3.3 Government

There is one central government in the small open economy. The government collects a lump-sum tax on household income to finance public spending - including both public consumption,  $G_t$ , and investment,  $I_t^G$ .

$$G_t + I_{G,t} = \frac{T_t}{P_t}$$

We assume that both public consumption and investment are composite of differentiated non-tradable goods. Moreover, public investment contributes to the accumulation of public capital stock according to the following motion equation:

$$K_{G,t+1} = (1 - \delta_G)K_{G,t} + I_{G,t} \quad (1.38)$$

where  $\delta_G$  is a depreciation rate on the public capital.

### 1.3.4 Inflation, the Real Exchange Rate and the Terms of Trade

Before moving to the analysis of equilibrium, we need to introduce several identities and assumptions. We start by defining the effective terms of trade,  $S_t \equiv \frac{P_{F,t}}{P_{H,t}}$ . The law of one price holds for the foreign-tradable goods,  $P_{F,t} = NER_t P_{F,t}^*$ , where  $NER_t$  is the nominal exchange

rate (the price of one unit of foreign currency expressed in domestic currency).

We define an effective real exchange rate as  $RER_t = NER_t P_t^* / P_t$ . Following the method generated by Monacelli and Perotti (2010) and holding the law of one price, the real exchange rate depends on the terms of trade and the relative price of traded to non-traded goods<sup>2</sup>:

$$RER_t = \left( \frac{S_t}{g(S_t)} \right) \times \left( \frac{Q_t/h(Q_t)}{Q_t^*/h^*(Q_t^*)} \right) \quad (1.39)$$

where  $Q_t^*$  has been assumed as constant for simplicity.

$$\frac{P_{T,t}}{P_{H,t}} = [(1 - \alpha) + \alpha(S_t)^{1-\eta_T}]^{\frac{1}{1-\eta_T}} \equiv g(S_t) \quad (1.40)$$

and the relative price of traded to non-traded goods is defined as

$$Q_t \equiv \frac{P_{T,t}}{P_{N,t}} = \frac{P_{T,t}/P_{H,t}}{P_{N,t}/P_{H,t}} = g(S_t)Q_{H,t} \quad (1.41)$$

where  $Q_{H,t} = \frac{P_{H,t}}{P_{N,t}}$  is the relative price of home- to non-traded goods.

$$\frac{P_t}{P_{N,t}} = [\mu Q_t^{1-\eta} + (1 - \mu)]^{\frac{1}{1-\eta}} \equiv h(Q_t) \quad (1.42)$$

## 1.4 EQUILIBRIUM

This section discusses the market equilibrium conditions that suggest that in each period all markets must clear, i.e., demand must be equal to supply in the markets of private capital, labour, the home-tradable goods, and the non-tradable goods.

<sup>2</sup>A rise in the real exchange rate indicates a real depreciation.

### 1.4.1 Factor Markets

In the private capital market, total private capital demands in home-tradable and non-tradable sectors are equal to the predetermined private capital stock in the period  $t$ :

$$K_t = K_{H,t} + K_{N,t} \quad (1.43)$$

In the labour market, the household labour supply is equal to total labour demand in home-tradable and non-tradable sectors in the period  $t$ :

$$N_t = N_{H,t} + N_{N,t} \quad (1.44)$$

### 1.4.2 Aggregate Demand for Home-tradable Goods and Trade Balance

The home-tradable output is equal to the sum of private consumption and investment on home-tradable goods and world-demand for home-tradable goods (exports):

$$\begin{aligned} Y_{H,t}(j) &= C_{H,t}(j) + I_{H,t}(j) + X_{H,t}(j) \\ &= \left( \frac{P_{H,t}(j)}{P_{H,t}} \right)^{-\epsilon_T} (C_{H,t} + I_{H,t} + X_{H,t}) \end{aligned} \quad (1.45)$$

Plugging Equation (1.45) into the definition of aggregate home-tradable output  $Y_{H,t} \equiv \left[ \int_0^1 Y_{H,t}(j)^{\frac{\epsilon_T-1}{\epsilon_T}} \right]^{\frac{\epsilon_T}{\epsilon_T-1}}$  leads to:

$$\begin{aligned} Y_{H,t} &= C_{H,t} + I_{H,t} + X_{H,t} \\ &= (1 - \alpha) \left( \frac{P_{H,t}}{P_{T,t}} \right)^{-\eta_T} (C_{T,t} + I_{T,t}) + \left( \frac{P_{H,t}}{NER_t P_{F,t}^*} \right)^{-\eta_T} X_{F,t} \\ &= (1 - \alpha) \left( \frac{P_{H,t}}{P_{T,t}} \right)^{-\eta_T} (C_{T,t} + I_{T,t}) + \alpha \left( \frac{P_{H,t}}{NER_t P_{F,t}^*} \right)^{-\eta_T} \left( \frac{P_{F,t}^*}{P_{T,t}^*} \right)^{-\eta_T} X_{T,t} \\ &= \left( \frac{P_{H,t}}{P_{T,t}} \right)^{-\eta_T} (C_{T,t} + I_{T,t}) \left[ (1 - \alpha) + \alpha \left( \frac{S_t}{g(S_t)} \right)^{\eta_T - \frac{1}{\sigma}} \right] \end{aligned} \quad (1.46)$$

where  $X_{H,t}$  is the foreign demand for home-tradable goods,  $X_{F,t}$  denotes aggregate demand for foreign-tradable goods in the world and  $X_{T,t}$  is the aggregate demand for tradable goods in the world. We recall that  $(\frac{S_t}{g(S_t)})$  is a component of the real exchange rate in Equation (1.39).

Let  $NX_t$  denote net exports in terms of home-tradable output, which is expressed as a fraction of steady-state home-tradable output,  $Y_H$ .

$$NX_t \equiv \frac{1}{Y_H} \left[ Y_{H,t} - \left( \frac{P_{T,t}}{P_{H,t}} \right) (C_{T,t} + I_{T,t}) \right] \quad (1.47)$$

### 1.4.3 Aggregate Demand for Non-tradable Goods

In the non-tradable goods market, output is equal to a sum of demands for non-tradable goods for private consumption and investment, and public consumption and investment:

$$\begin{aligned} Y_{N,t}(j) &= C_{N,t}(j) + I_{N,t}(j) + G_t(j) + I_{G,t}(j) \\ &= \left( \frac{P_{N,t}(j)}{P_{N,t}} \right)^{-\epsilon_N} (C_{N,t} + I_{N,t} + G_t + I_{G,t}) \end{aligned} \quad (1.48)$$

Plugging (1.48) into the definition of aggregate output of non-tradable sector,  $Y_{N,t} \equiv \left[ \int_0^1 Y_{N,t}(j)^{\frac{\epsilon_N-1}{\epsilon_N}} \right]^{\frac{\epsilon_N}{\epsilon_N-1}}$ , leads to:

$$Y_{N,t} = C_{N,t} + I_{N,t} + G_t + I_{G,t} \quad (1.49)$$

Besides, the total output can be defined as the sum of home-tradable output and non-tradable output:

$$Y_t = Y_{H,t} + Y_{N,t} \quad (1.50)$$

### 1.4.4 Log-Linearized DSGE Model

We layout the model in the log-linearized form.  $v_t \equiv \frac{V_t - V}{V}$  denotes a percentage deviation from a zero-inflation steady state.<sup>3</sup>

We assume that the central bank follows a Taylor Rule with targeting at inflation stability and economic growth.

$$r_t = \bar{r} + \phi_\pi \pi_t + \phi_y y_t \quad (1.51)$$

where  $\bar{r}$  denotes a target for the nominal interest rate, and parameters  $\phi_\pi$  and  $\phi_y$  are sensitivities of the basic interest rate in response to the CPI inflation and aggregate output. Log-linearizing the CPI formula yields  $p_t = \mu(p_{H,t} + \alpha s_t) + (1 - \mu)p_{N,t}$ ; thereby, the CPI inflation,  $\pi_t$ , is given by

$$\pi_t = \mu[\pi_{H,t} + \alpha \Delta s_t] + (1 - \mu)\pi_{N,t} \quad (1.52)$$

where  $\Delta s_t = s_t - s_{t-1}$ ,  $\pi_{H,t} = p_{H,t} - p_{H,t-1}$  is the inflation of home-traded goods and  $\pi_{N,t} = p_{N,t} - p_{N,t-1}$  is the inflation of non-traded goods. The terms of trade can be approximated (up to first order) by the log-linear expression

$$s_t = p_{F,t} - p_{H,t}. \quad (1.53)$$

Total output,  $y_t$ , can be derived by log-linearizing Equation (1.50)

$$y_t = \gamma_{Hy} y_{H,t} + (1 - \gamma_{Hy}) y_{N,t} \quad (1.54)$$

where  $\gamma_{Hy}$  represents the share of home-tradable output in the steady-state total output.

In terms of the representative household, a first-order approximation of Equations (1.15)-(1.19) around the deterministic steady state leads to

$$k_{t+1} = (1 - \delta)k_t + \delta i_t \quad (1.55)$$

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<sup>3</sup> $V$  is the steady state of variable  $V_t$

$$w_t - p_t = \sigma c_t + \varphi n_t \quad (1.56)$$

$$c_t = E_t\{c_{t+1}\} - \frac{1}{\sigma}(r_t - E_t\{\pi_{t+1}\}) \quad (1.57)$$

$$\lambda_{q,t} = \xi \delta^2 (i_t - k_t) \quad (1.58)$$

$$\lambda_{q,t} = [1 - \beta(1 - \delta)] E_t\{z_{t+1} - p_{t+1}\} + \beta E_t\{\lambda_{q,t+1}\} - (r_t - E_t\{\pi_{t+1}\}) \quad (1.59)$$

where  $\xi = \Psi''\left(\frac{I}{K}\right) > 0$ . In addition,  $rr_t = r_t - E_t\{\pi_{t+1}\}$  is the CPI-based real interest rate and  $z_t^r = z_t - p_t$  is the CPI-based real rental cost of private capital. Equation (1.58) indicates that a rise in the real shadow price of private investment is associated with the investment-capital ratio. As a result, private investment rises with the real shadow price when the private capital stock is predetermined. In the equation (1.59), Tobin's  $q$  depends on the expected future movements of the marginal product of capital (positively) and of the real interest rate (negatively).

In the home-tradable sector, log-linearized forms (1.24), (1.25), (1.26) and (1.21) around the steady state are given by:

$$w_t - p_{H,t} = mc_{H,t} + y_{H,t} - n_{H,t} \quad (1.60)$$

$$w_t - p_t + p_t - p_{H,t} = mc_{H,t} + y_{H,t} - n_{H,t}$$

$$w_t - p_t + \alpha s_t - (1 - \mu)q_t = mc_{H,t} + y_{H,t} - n_{H,t}$$

$$z_t - p_{H,t} = mc_{H,t} + y_{H,t} - k_{H,t} \quad (1.61)$$

$$z_t - p_t + \alpha s_t - (1 - \mu)q_t = mc_{H,t} + y_{H,t} - k_{H,t}$$

$$mc_{H,t} = -a_{H,t} + (1 - \psi_H)(w_t - p_{H,t}) + \psi_H(z_t - p_{H,t}) \quad (1.62)$$

$$= -a_{H,t} + (1 - \psi_H)(w_t - p_t + \alpha s_t - (1 - \mu)q_t) + \psi_H(z_t - p_t + \alpha s_t - (1 - \mu)q_t)$$

$$a_{H,t} = \psi_G k_{G,t} \quad (1.63)$$

Plugging the home-tradable Calvo price setting (Equation (1.29)) into Equation (1.30), the log-linearized form of New Keynesian Phillips curve for the home-tradable sector parallels the expression from a closed economy.

$$\pi_{H,t} = \beta E_t \{ \pi_{H,t+1} \} + \kappa_H mc_{H,t} \quad (1.64)$$

where  $\kappa_H \equiv \frac{(1-\beta\theta_H)(1-\theta_H)}{\theta_H}$ . The New Keynesian Phillips curve states that the inflation positively depends on the expected inflation in the next period and the real marginal cost.

In the non-tradable sector, log-linearization of Equations (1.35)-(1.34) leads to:

$$w_t - p_{N,t} = mc_{N,t} + y_{N,t} - n_{N,t} \quad (1.65)$$

$$w_t - p_t + \mu q_t = mc_{N,t} + y_{N,t} - n_{N,t}$$

$$z_t - p_{N,t} = mc_{N,t} + y_{N,t} - k_{N,t} \quad (1.66)$$

$$z_t - p_t + \mu q_t = mc_{N,t} + y_{N,t} - k_{N,t}$$

$$mc_{N,t} = -a_{N,t} + (1 - \psi_N)(w_t - p_t + \mu q_t) + \psi_N(z_t - p_t + \mu q_t) \quad (1.67)$$

$$a_{N,t} = \psi_G k_{G,t} \quad (1.68)$$

Log-linearized form of New Keynesian Phillips curve for non-tradable sector, under the Calvo price setting, is given by:

$$\pi_{N,t} = \beta E_t \{ \pi_{N,t+1} \} + \kappa_N mc_{N,t}. \quad (1.69)$$

where  $\kappa_N \equiv \frac{(1-\beta\theta_N)(1-\theta_N)}{\theta_N}$ .

The log-linearized form of the public capital accumulation is given by:

$$k_{G,t+1} = (1 - \delta_G)k_{G,t} + \delta_G i_{G,t} \quad (1.70)$$

We assume that public investment,  $i_{G,t}$ , is an exogenous variable that follows an  $AR(1)$  process:

$$i_{G,t} = \rho_{IG}i_{G,t-1} + \varepsilon_{IG,t} \quad (1.71)$$

where  $\rho_{IG}$  denotes a persistence of public investment in response to the public investment shock  $\varepsilon_{IG,t} \sim (0, \sigma_{IG})$ .

In terms of market clearing condition of private capital, the log-linearization of Equation (1.43) leads to:

$$k_t = \gamma_{Hk}k_{H,t} + (1 - \gamma_{Hk})k_{N,t} \quad (1.72)$$

where  $\gamma_{Hk}$  denotes the share of home-tradable private capital demand in the steady-state total private capital stock.

Under the labour market clearing condition, the log-linearization of Equation (1.44) leads to:

$$n_t = \gamma_{Hn}n_{H,t} + (1 - \gamma_{Hn})n_{N,t} \quad (1.73)$$

where  $\gamma_{Hn}$  is a share of home-tradable labour demand in the steady-state of total labour supply.

In the home-tradable goods market, log-linearizing Equation (1.46) around a zero-inflation steady state in which net exports are zero, yields:

$$y_{H,t} = \gamma_c c_{T,t} + (1 - \gamma_c) i_{T,t} + \left( \frac{\alpha\omega}{\sigma} \right) s_t \quad (1.74)$$

where  $\gamma_c$  is the share of consumption on traded goods in the steady-state home-traded output, and  $\omega \equiv \sigma\eta_T + (1 - \alpha)(\sigma\eta_T - 1)$ .<sup>4</sup>

In terms of net exports, around the zero-inflation steady state in which net exports are

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<sup>4</sup>See Appendix of Monacelli and Perotti (2006) for an explicit derivation

zero, log-linearizing Equation (1.47) leads to:

$$nx_t = y_{H,t} - \gamma_c c_{T,t} - (1 - \gamma_c) i_{T,t} - \alpha s_t$$

which combines with Equation (1.74) implies:

$$nx_t = \alpha \left( \frac{\omega}{\sigma} - 1 \right) s_t \quad (1.75)$$

where a depreciation of the terms of trade leads to an improvement on the net exports.<sup>5</sup>

Log-linearization of the optimality conditions of private consumption and investment on the traded goods leads to:

$$\begin{aligned} c_{T,t} &= -\eta(p_{T,t} - p_t) + c_t \\ &= -\eta(1 - \mu)q_t + c_t \end{aligned} \quad (1.76)$$

$$\begin{aligned} i_{T,t} &= -\eta(p_{T,t} - p_t) + i_t \\ &= -\eta(1 - \mu)q_t + i_t \end{aligned} \quad (1.77)$$

In the non-tradable goods market, using log-linearization on Equation (1.49), we can obtain:

$$y_{N,t} = \gamma_{Nc} c_{N,t} + (1 - \gamma_{Nc} - \gamma_g - \gamma_{ig}) i_{N,t} + \gamma_g g_t + \gamma_{ig} i_{G,t} \quad (1.78)$$

where  $\gamma_{Nc}$  is the share of consumption on non-tradable goods in the steady-state non-tradable output,  $\gamma_g$  is the share of public consumption in the steady-state non-tradable output, and  $\gamma_{ig}$  is the share of public investment in the steady-state non-tradable output.

We assume that public consumption,  $g_t$ , is an exogenous variable and follows an  $AR(1)$  process:

$$g_t = \rho_G g_{t-1} + \varepsilon_{G,t} \quad (1.79)$$

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<sup>5</sup>A rise in the terms of trade indicates a real depreciation.

where  $\rho_G$  denotes a persistence of public investment in response to the public consumption shock  $\varepsilon_{G,t} \sim (0, \sigma_G)$ , which is independent from the public investment shock.

Log-linearization of the optimality conditions of private consumption and investment on the non-tradable goods leads to:

$$\begin{aligned} c_{N,t} &= -\eta(p_{N,t} - p_t) + c_t \\ &= \eta\mu q_t + c_t \end{aligned} \tag{1.80}$$

$$\begin{aligned} i_{N,t} &= -\eta(p_{N,t} - p_t) + i_t \\ &= \eta\mu q_t + i_t \end{aligned} \tag{1.81}$$

Log-linearization of Equations (1.39) and (1.41) can be shown as follows:

$$rer_t = (1 - \alpha)s_t + (1 - \mu)q_t \tag{1.82}$$

$$q_t = \alpha s_t + q_{H,t} \tag{1.83}$$

Hence, movements in the real exchange rate proportionally depend on the terms of trade and the relative price of tradable to non-tradable goods (whose movements proportionally depend on the terms of trade and the relative price of home-tradable to non-tradable goods).

Under the assumption of complete asset markets, we simply recall a risk-sharing condition presented by Galí and Monacelli (2005) and Monacelli and Perotti (2006) and Monacelli and Perotti (2010):

$$c_t = y_t^* + \sigma^{-1}(rer_t) \tag{1.84}$$

where  $y_t^*$  denotes the percentage deviation from the steady-state value of the world output, which is an external variable. Thus, the assumption of complete asset markets at the international level leads to a co-movement, linking private consumption with world output and the real exchange rate. Under the assumption of a small open economy, the public investment

shock cannot affect the world output.

## 1.5 A NUMERICAL ANALYSIS FOR THE SMALL OPEN ECONOMY

In this section, we analyze the macroeconomic implications of public investment on the real exchange rate, under the assumption of the various values of the output elasticity of public capital ( $\psi_G$ ). We consider three levels (low, threshold and high) of output elasticity. Our baseline model considers a threshold. We will decrease to a low level and increase to a high level to see the impulse responses of the real exchange rate and other variables to a positive shock to public investment. Bom and Ligthart (2014) conduct a meta-analysis and correct for linear publication bias to estimate the output elasticity of public capital - ranging from a minimum of 0.04 in short-run models to a maximum of 0.29 in long-run models. Thereby, we consider 0.232 as a threshold. Then, we decrease the output elasticity down to 0.04, considered to be low, and increase it to 0.29, which is considered to be a high level.

### 1.5.1 Calibration

We state the main assumptions based on the recent literature for our New-Keynesian model. We set the risk aversion coefficient  $\sigma = 1$  and the inverse of Frisch elasticity of labour  $\varphi = 3$  (implying a labour supply elasticity of  $\frac{1}{3}$ ), in a way that is consistent with Galí and Monacelli (2005) and Monacelli and Perotti (2006). To imply a riskless annual return of about 4% in the steady state, we set  $\beta = 0.96$ . Due to the widely accepted assumption on the quarterly depreciation rate of physical capital being 0.025, we assume that the annual depreciation rates of private and public capital  $\delta = \delta_G = 0.1$ . We set the sensitivity of the basic interest rate in response to the CPI inflation,  $\phi_\pi = 1.5$ , and the sensitivity to the total output,  $\phi_y = 0.5$ , which are indicated by Gali (2008).

The elasticity of substitution between tradable and non-tradable goods  $\eta$  is equal to 0.5

following Cova *et al.* (2011), because tradable goods cannot be easily substituted for non-tradable goods. In a way that is consistent with Monacelli and Perotti (2006), the elasticity of substitution between home-tradable and foreign-tradable goods  $\eta_T$  is equal to 1.5.

We assume  $\theta_H = \theta_N = 0.75$  so that the price stickiness is same in the home-tradable and non-tradable sectors, which is consistent with Rebei and Ortega (2005). The elasticity of substitution between differentiated home-tradable goods,  $\epsilon_T$ , is equal to 6 and the elasticity of substitution between differentiated non-tradable goods,  $\epsilon_N$ , is equal to 4.33 as Cova *et al.* (2011) set steady-state home-tradable markup,  $MK_H = 1.2$ , and steady-state non-tradable markup,  $MK_N = 1.3$ . Higher non-tradable markup is supported by the findings of Amador and Soares (2017) who demonstrate that the non-tradable sector presents a weaker intensity of competition than the home-tradable sector. The capital share in home-tradable value-added,  $\psi_H$ , is equal to 0.6 and the share in the non-tradable value-added,  $\psi_N$ , is equal to 0.55 in a way that is consistent with Cova *et al.* (2011) because the home-tradable production is considered to be more capital intensive than the non-tradable production.

In order to calibrate shares of sectoral variables in the aggregate variables, we have collected annual data from the National Bureau of Statistics of China during periods between 2005 to 2017 to consider the average values as the steady-state values. We calibrate parameters for China as a small-open economy that does not have strong market power in the international market. In the literature, a small-open economy is a country that is a price-taker in the international market. Accordingly, several papers in the literature have used the small-open economy assumption for China in DSGE models. These papers include Zheng and Guo (2013), who investigated monetary policy and macroeconomic fluctuations in China. Another study in the same vein is found in Y. Chen and Liu (2018) who use a small-open New-Keynesian DSGE model to study the impact of government consumption and investment shocks on the real exchange rate in China with one production sector. As is common in the literature, in the absence of estimates of behavioural parameters for a given country, one relies on those estimated for another similar country. Even though China is not yet an advanced economy,

it shares some characteristics with those countries in that group. China has been a WTO member and a participant of the G20 Summit in order to integrate into the world economy. Estimates of these parameters fall within relatively narrow ranges, and the values the paper employs are comparable to those used in existing DSGE studies for the Chinese economy.

The parameter  $\gamma_{Hy} = 0.73$  is the ratio of steady-state home-tradable output to the steady-state total output, and then  $1 - \gamma_{Hy} = 0.27$  is the ratio of steady-state non-tradable output to the steady-state total output. Therefore, we can calibrate the steady-state private capital demand in the home-tradable production  $K_H = \psi_H \left( \frac{\gamma_{Hy}}{MK_H} \right)$ , and the steady-state labour demand in the home-tradable production  $N_H = (1 - \psi_H) \left( \frac{\gamma_{Hy}}{MK_H} \right)$ . In a same way, the steady-state private capital demand and labour demand in the non-tradable production can be given by  $K_N = \psi_N \left( \frac{\gamma_{Ny}}{MK_N} \right)$  and  $N_N = (1 - \psi_N) \left( \frac{\gamma_{Ny}}{MK_N} \right)$  respectively. Hence, the share of steady-state home-tradable capital demand in the steady-state total capital stock,  $\gamma_{Hk} = \frac{K_H}{K} = 0.76$ , can be calibrated, and the share of steady-state home-tradable labour demand in the steady-state total labour supply,  $\gamma_{Hn} = \frac{N_H}{N}$ , is 0.70.

From the dataset, we have the ratio of steady-state private consumption to the steady-state total output  $\frac{C}{Y} = 0.38$ . As Cova *et al.* (2011) offer the share of steady-state domestic consumption ( $C_D = C_H + C_N$ ) in the steady-state total consumption ( $\frac{C_D}{C} = 0.60$ ) and the share of steady-state consumption on home-tradable goods in the steady-state domestic consumption ( $\frac{C_H}{C_D} = 0.51$ ), we can calculate the ratio of steady-state consumption on home-tradable goods to the steady-state total output ( $\frac{C_H}{Y} = 0.12$ ) and the ratio of steady-state consumption on foreign-tradable goods to the steady-state total output ( $\frac{C_F}{Y} = 0.15$ ). As a result, the share of steady-state consumption on tradable goods in the steady-state home-tradable output ( $\gamma_c \equiv \frac{C_T}{Y_H}$ ) is equal to 0.37, and the share of steady-state consumption on non-tradable goods in the steady-state non-tradable output ( $\gamma_{Nc} \equiv \frac{C_N}{Y_N}$ ) is equal to 0.41. The degree of openness is  $\alpha = 0.56$  and the share of steady-state consumption on tradable goods in the total consumption is  $\mu = 0.71$ . The openness of China is close to that of Japan. Moreover, China's composition of government

expenditures is close to Japan's.<sup>6</sup>

If public capital accumulated by public investment goes into the production functions in the two sectors and improves the productivity of the two private production factors (labour and private capital), the public capital is productive. Therefore, we collected data for government expenditures on science and technology and transportation, defining “public investment” as a sum of government expenditures on science and technology and those on transportation. The ratio of steady-state public investment to the steady-state total output ( $\frac{I_G}{Y}$ ) is equal to 0.03; therefore, the share of steady-state public investment in the steady-state non-tradable output,  $\gamma_{ig} = \frac{I_G}{Y_N}$ , is equal to 0.11. The share of public consumption in the total output,  $\frac{G}{Y}$ , is 0.08. The ratio of the steady-state public consumption to the steady-state non-tradable output,  $\gamma_g = \frac{G}{Y_N}$ , is 0.30.

Empirically, based on the study of Devarajan *et al.* (1996) for developed and developing countries, if a rise in the share of one type of government expenditures in total expenditures significantly and positively affects GDP growth, the type of government expenditures is productive; otherwise, the type of government expenditures is non-productive. Due to the shortage of the sample periods from 2005 to 2017, we are not able to implement the time-series analysis for China. Correlation between the share of public investment in total government expenditures and GDP growth is positive: 0.104 - however, the correlation between the share of public consumption in total government expenditures and GDP growth is negative. Thus, public capital from government cumulative expenditures on science and technology and on transportation is productive.

The assumption that a rise in public investment is a sectoral shock in the non-tradable sector is based on a tradition that government spending is considered to be intensive in services (non-tradable goods). The data on government expenditures from the National Bureau of Statistics of China does not categorize by goods and services but by main items. Intuitively,

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<sup>6</sup>Using Japanese data from World Data Bank from 2005 to 2017, we do the calibration for Japan and show the results in Appendix A.

expenditure on science and technology is non-tradable. To rationalize the non-tradable assumption on the expenditure on transportation, we consider the tradition of how the Chinese government processes infrastructure and transportation projects. To complete such projects, such as building roads, railroads or bridges, the common method used by the government is to call for bids. The winner(s) of the bids can be either the state-owned enterprise(s) or private enterprise(s). Therefore, the government purchases the services of the enterprise(s).

Assuming that public investment focuses on non-tradable goods, a change in public investment affects the relative price of tradable to non-tradable goods from both demand and supply sides. Suppose that the demand-side effect dominates the supply-side effect. The relative price of tradable to non-tradable goods falls. More expensive non-tradable goods attract more demand for tradable goods and raise the price of home-tradable goods; thus, the terms of trade appreciate. Eventually, we can observe an appreciation of the real exchange rate. The conclusion would not be affected even though we assume that public investment induces demand-side effects on both tradable and non-tradable sectors. If the demand-side effect dominates the supply-side effect, the terms of trade appreciates and the relative price falls; such that, the real exchange rate appreciates.

In Section 1.6, we resort to numerical simulations. Under the Taylor Rule of the central bank, we vary the output elasticity of public capital to see the different impulse responses of the real exchange rate and other macroeconomic variables to a 1% rise in public investment. As in Basu and Kollmann (2013), we rely on the estimated range of the output elasticity of public capital provided by Bom and Ligthart (2014) from a meta-regression analysis. These estimated elasticities vary between 0.04 and 0.29. In a benchmark simulation called “threshold”, we find the value of the output elasticity,  $\psi_G = 0.232$ . In two other simulations called “Low” and “High”, we set the output elasticity of public capital to, respectively, the lower and upper bounds of the interval, which are 0.04 and 0.29.

**Table 2:** Ratios of Real Variables to Total Output in Steady State

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Name of Variables	Notations	Ratios	Sources
Home-Tradable Output	$Y_{H,ss}$	0.73	National Bureau of Statistics of China
Non-Tradable Output	$Y_{N,ss}$	0.27	National Bureau of Statistics of China
Private Consumption	$C_{ss}$	0.38	National Bureau of Statistics of China
Public Consumption	$G_{ss}$	0.08	National Bureau of Statistics of China
Public Investment	$I_{G,ss}$	0.03	National Bureau of Statistics of China

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**Table 3:** Parameters for Calibration

Parameter	Value	Description
$\sigma$	1.00	Risk aversion
$\beta$	0.96	Discount factor
$\delta$	0.10	Private capital depreciation rate
$\delta_G$	0.10	Public capital depreciation rate
$\varphi$	3.00	Inverse Frisch elasticity
$\phi_\pi$	1.50	Feedback Taylor rule inflation
$\phi_y$	0.50	Feedback Taylor rule total output
$\theta_H$	0.75	Home-tradable Calvo parameter
$\theta_N$	0.75	Non-tradable Calvo parameter
$\epsilon_T$	6.00	Elasticity of substitution between tradable goods
$\epsilon_N$	4.33	Elasticity of substitution between non-tradable goods
$\eta$	0.50	Elasticity of substitution between tradable and non-tradable goods
$\eta_T$	1.50	Elasticity of substitution between home-tradable and importable goods
$\psi_H$	0.60	Capital share in home-tradable production function
$\psi_N$	0.55	Capital share in non-tradable production function
$\gamma_c$	0.37	Share of tradable consumption in home-tradable output
$\gamma_{Nc}$	0.41	Share of non-tradable consumption in non-tradable output
$\gamma_{Hy}$	0.73	Share of home-tradable output in total output
$\alpha$	0.56	Openness
$\gamma_{IG}$	0.11	Share of public investment in non-tradable output
$\mu$	0.71	Share of tradable consumption in aggregate consumption
$\gamma_g$	0.30	Share of public consumption in non-tradable output
$\gamma_{Hk}$	0.76	Share of home-tradable capital demand in total capital stock
$\gamma_{Hn}$	0.70	Share of home-tradable labour demand in total labour supply
$\xi$	6.00	$\Phi''\left(\frac{I}{K}\right)$
$\psi_G$	[0.04,0.165,0.29]	Output elasticity of public capital
$\rho_G$	0.85	Autocorrelation public consumption
$\rho_{IG}$	0.85	Autocorrelation public investment
$\rho_y$	0.86	Autocorrelation world output

## 1.6 Sensitivity Analysis

In this section, we find a threshold ( $\psi_G = 0.232$ ) of output elasticity of public capital for which the real exchange rate has no response to a shock to public investment. We vary the values of output elasticity of public capital and compare impulse responses under two values that are “Low” ( $\psi_G = 0.04$ ) and “High” ( $\psi_G = 0.29$ ). We recall that government capital spending is a composite of differentiated non-tradable goods. We also discuss the sensitivity of the calibrated parameters and show the simulated responses.

It is well known that since the true value of the output elasticity of public capital in China or elsewhere is fixed, a change in government spending on public capital cannot lead at the same time to an appreciation and a depreciation of the real exchange rate in that country. Only one direction can be observed. Our purpose in the paper is to perform a counterfactual analysis on the impact of higher and lower values of the output elasticity on real exchange rate dynamics. Even though we do not have actual estimates of the range of that elasticity for China, we use some plausible values found in the literature to illustrate our main argument, which links the change in the real exchange rate to the magnitude of the output elasticity of public capital.

### 1.6.1 Benchmark

As shown in Figure 1, a 1% rise in public spending on capital generates a hump-shaped response of public capital because of the law of motion of public capital. At the same time, the increase in the public capital stock raises the productivity of private factors in both home-tradable and non-tradable sectors. Equations (1.63) and (1.68) decide how much the public capital contributes to the sectoral productivity of private factors. In addition to the supply-side effect, the public shock raises the aggregate demand for non-tradable goods so the demands for labour and private capital in the non-tradable sector increase, which is a demand-side effect.

When  $\psi_G = 0.232$ , for the non-tradable sector shown in Figure 2, the relative price of

tradable to non-tradable goods does not change. This is because the supply-side effect of public shock offsets the demand-side effect. The increase in the demand for non-tradable goods in turn increases the demand for labour and private capital. Consequently, real wage and real rental costs increase, due to the price rigidities of home-tradable and non-tradable goods. Therefore, the non-tradable output rises. As the public capital continues to increase, the supply-side effect of the shock becomes stronger, but the demand-side effect becomes weaker. We find that the relative price of tradable to non-tradable goods increases in the second period and has a humped-shaped response due to the law of motion of public capital.

In the home-tradable sector, the terms of trade initially do not change in response to the public shock. It eventually increases, following a hump-shaped pattern. The reason is that, as shown in Figure 2, demand for private capital decreases, while demand for labour increases in that sector. Ultimately, output falls in that sector, since the reduced supply of tradable goods matches the demand for tradable goods. Because of the Taylor-Rule policy targeting inflation implemented by the central bank, the nominal interest rate rises. The fall in the home-tradable output is also supported by an increase in the real interest rate. With a rise in the public capital, home-tradable output increases in the future and hence the future price of home-tradable goods falls. Consequently, future nominal and real interest rates decrease.

Taylor-Rule monetary policy is a representative monetary policy carried out in China. Zhang (2009) mentions that Law of People's Bank of China states that the objective of monetary policy is to maintain price stability and promote economic growth, and points out that the Taylor Rule can accurately measure the operational level of China's monetary policy. Thus, the interest rate reacts to inflation and output gaps.

Because the relative price of tradable to non-tradable goods and the terms of trade are components of the real exchange rate, shown in Equation (1.82), the real exchange rate initially has no response and then depreciates in the future. Also, net exports have a hump-shaped response with no initial response and then improvement in the future. Through the risk-sharing condition, there exists a co-movement between private consumption and the real

exchange rate. Therefore, we observe that private consumption does not change initially. When the future real exchange rate depreciates, future private consumption increases. The increase in the real wage leads to a rise in the labour supply. A decrease in the total demand for private capital as a result of the increase in the real rental cost causes a fall in private investment.

### 1.6.2 Output Elasticity of Public Capital

#### **Low:**

When  $\psi_G = 0.04$ , the lower output elasticity of public capital leads to an appreciation of the real exchange rate and the negative responses of the relative price of tradable to non-tradable goods. The negative response of the terms of trade causes net exports to deteriorate. This is because the demand-side effect of public shock dominates the supply-side effect. Moreover, it mitigates these negative responses in the future. Recall that the reduction of real exchange rate is the real appreciation. Due to its low output elasticity, non-tradable output has less persistence in returning to the steady state. Because of the reduction of home-tradable demand for labour and private capital, home-tradable output initially has a more negative response, which is supported by a rise in the real interest rate. The smaller increase in the real interest rate is because of the smaller decrease in the expected inflation.

As the low output elasticity leads to a smaller increase in future home-tradable output, the negative response of the future price of home-tradable goods is smaller. The decrease in future real interest rate is smaller because the smaller fall in future inflation causes a smaller decrease in the nominal interest rate under the inflation-targeting Taylor Rule. Later in Period 6, the real exchange rate depreciates - and both the terms of trade and the relative price of tradable to non-tradable goods rise. Our findings not only support Chatterjee and Mursagulov (2016) that the output elasticity of public capital matters in the long run, but also reveal that it matters in the short run.

The decrease in private consumption is explained by the negative wealth effect of government spending on capital, which raises the shadow value of wealth by implying an increase in future taxes. The hump-shaped response of private consumption is indicated by a rise in current real interest rates and a fall in future real interest rates. In other words, the public shock reduces current consumption because an increase in household income (including the real wage and rental cost) cannot offset the negative wealth effect of government capital spending. Also, the rise in the real interest rate forces households to substitute current for future consumption. This supports the co-movement between private consumption and the real exchange rate. The smaller decrease in the total demand for private capital leads to a less negative response of private investment.

With the low output elasticity of public capital, the appreciation of the real exchange rate in our DSGE model is consistent with the structural VAR result for China with one production sector found by Y. Chen and Liu (2018). We both find that the increase in public investment crowds out private investment. In addition, the magnitude of the reduction of private consumption is different from the one in Y. Chen and Liu's (2018) VAR study. Still, the hump-shaped response of private consumption is consistent with Y. Chen and Liu's (2018) finding. Due to the appreciation of the real exchange rate, we find the deterioration of net exports; however, Y. Chen and Liu (2018) show an improvement in net exports even though the real exchange rate appreciates. Also, the hump-shaped response of total output is consistent with Y. Chen and Liu (2018). The appreciation of the real exchange rate associated with the increase in total output is consistent with Bénétrix and Lane's (2013) panel VAR study of EMU countries.

To estimate the empirical value of the output elasticity of public capital, Feng and Wu (2018) dealt with the reverse causality from output to public expenditures on infrastructure capital by using the newly increased fixed assets (NIFA). Due to the time-to-build feature of public capital, NIFA is less likely to be affected by the current output, and the reverse causality has been mitigated. The estimate of the output elasticity is 0.037 - it is very close to

the low output elasticity, 0.04, used in our DSGE model. Also, the low output elasticity causes the appreciation of the real exchange rate, which is consistent with the VAR result found by Y. Chen and Liu (2018). As a result, 0.04 is an applicable value of the output elasticity of public capital to China.

**High:**

When  $\psi_G = 0.29$ , the public shock causes the relative price of tradable to non-tradable goods to increase. The fall in the price of non-tradable goods indicates that the supply-side effect dominates the demand-side effect in the non-tradable sector. Due to the public capital accumulation, Figure 2 shows a more positive response of the future relative price of tradable to non-tradable goods. The non-tradable output has a higher persistence returning to the steady state.

The more positive response of labour demand and the less negative response of private-capital demand indicate the less negative response of current home-tradable output, and hence the real interest rate increases. The larger decrease in the expected inflation rate causes the real interest rate to have a more positive response. Figure 2 demonstrates a rise in the terms of trade instead of a fall. This is because more expensive tradable goods than non-tradable goods lead to a reduction in demand for tradable goods - causing the demand for home-tradable goods to decrease. Therefore, the decrease in the demand for home-tradable goods puts downward pressure on the price of home-tradable goods.

The dynamic effect of public shock combined with the high output elasticity of public capital leads to a more aggressive increase in the future home-tradable output and hence, a fall in future real interest rate. The larger decrease in the future real interest rate is due to the larger decrease in the future nominal interest rate associated with the larger decrease in future inflation. The high output elasticity causes the terms of trade to increase, the real exchange rate to depreciate, and net exports to improve. As the innovation to public spending dies out, the stronger supply-side effect leads to more depreciation of the real exchange rate, a larger

increase in the terms of trade and more improvement on net exports in the future.

Through the risk-sharing condition, both current and future consumption increases with the depreciation of the real exchange rate. This is because increases in the household income are high enough to cover the increase in the shadow value of wealth by implying a rise in future taxes. The larger increase in the real interest rate supports the rise in current and future consumption. The larger decrease in the future real interest rate supports the more obvious hump-shaped response of private consumption. The shock to public capital brings about a more negative response of private investment as a result of the more reduction of private-capital demand.

With the high output elasticity of public capital, our DSGE model finds a depreciation of the real exchange rate in response to a rise in productive public spending. This theoretical result is consistent with many VAR studies for developed countries, such as the US found by Corsetti, Meier, *et al.* (2012) and Enders *et al.* (2011) and Japan found by Iwata (2013). The crowding-out effect of public investment on private investment is consistent with Corsetti, Meier, *et al.* (2012) and Enders *et al.* (2011); however, Iwata (2013) finds that the response of private investment is negligible. The crowding-in effect on private consumption is consistent with Corsetti, Meier, *et al.* (2012) and Iwata (2013); however, Enders *et al.* (2011) find a negligible response of private consumption.

### 1.6.3 Openness

To better understand the dynamic movement of the real exchange rate, compared with the benchmark of our DSGE model, we vary openness ( $\alpha$ ) and elasticity of substitution between tradable and non-tradable goods ( $\eta$ ) respectively. In the benchmark, the openness parameter ( $\alpha$ ) is 0.56, which means private consumption and private investment are not home-biased in tradable goods. When the output elasticity of public capital is the threshold, the real exchange rate, terms of trade and relative price of tradable to non-tradable goods do not respond to a rise in government expenditures on public capital because the supply-side effect offsets the

demand-side effect.

Higher openness indicates that private consumption and investment have a higher proportion of importable goods in their basket of tradable goods. When we increase the openness to 0.72, in Figure 3 we barely observe an appreciation or a depreciation of the real exchange rate in period 1 in response to a rise in public investment. The terms of trade and the relative price of tradable to non-tradable goods as the components of the real exchange rate do not initially respond to the shock to public capital. Due to the international risk sharing condition, private consumption has no initial response associated with the real exchange rate. During the process of public capital accumulation and the diminishing demand-side effect of the public shock, the supply-side effect dominates the demand-side effect. After period 2, the terms of trade and the relative price of tradable to non-tradable increase; such that, the real exchange rate depreciates. The magnitude of the increase in the relative price is greater than that in the benchmark. Because an increase in public capital improves the productivity of private factor, there is downward pressure on the non-tradable price. Less expensive non-tradable goods cause a shift from tradable goods to non-tradable goods. The consequential effect on the home-tradable is less if the openness is higher; thus, the reduction of tradable price is lower than that in the benchmark. With higher openness, the magnitude of the increase in the relative price is greater than that in the benchmark. For the terms of trade, the magnitude of depreciation is less than that in the benchmark. The proportion of private spending on tradable goods is major in total private spending. The change of the terms of trade has a leading effect on the real exchange rate. Therefore, the magnitude of the rise in the real exchange rate is less than that of the benchmark, and the increase in private consumption is less than that of the benchmark at all periods.

We have reduced the openness to 0.4; thus, private consumption and investment are home-biased in tradable goods. In response to a rise in government expenditures on public capital, because of the balance of demand-side and supply-side effects, the relative price of tradable to non-tradable goods, the terms of trade and the real exchange rate do not change.

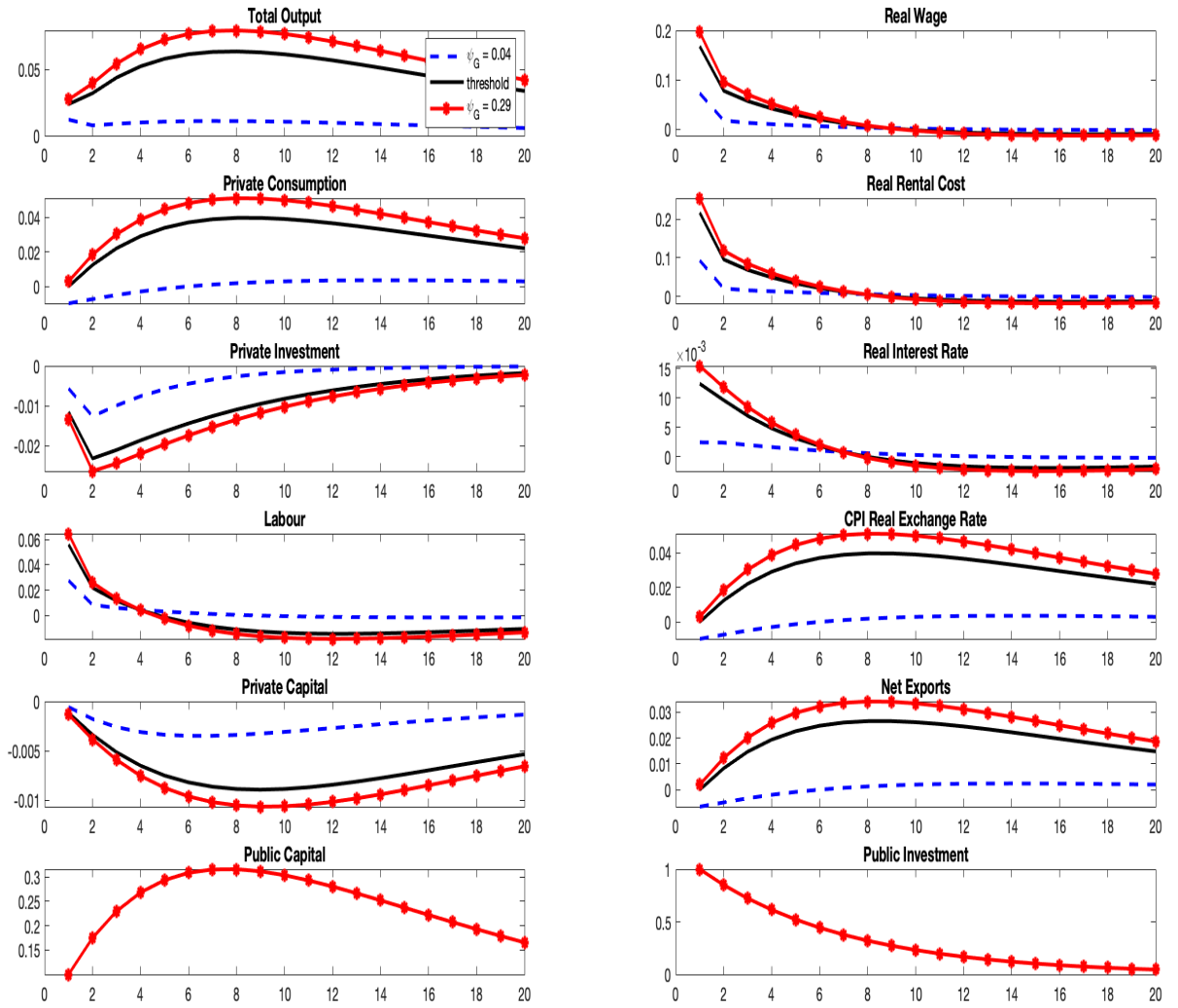
Then, the magnitude of the increase in the relative price is less than that in the benchmark, because the shifting effect on the home-tradable price is greater than that of the benchmark. The depreciation of the terms of trade is greater than that in the benchmark. Consequently, the magnitude of the real exchange rate depreciation is greater than the real depreciation of the benchmark.

#### **1.6.4 The Elasticity of Substitution between Tradable and Non-tradable Goods**

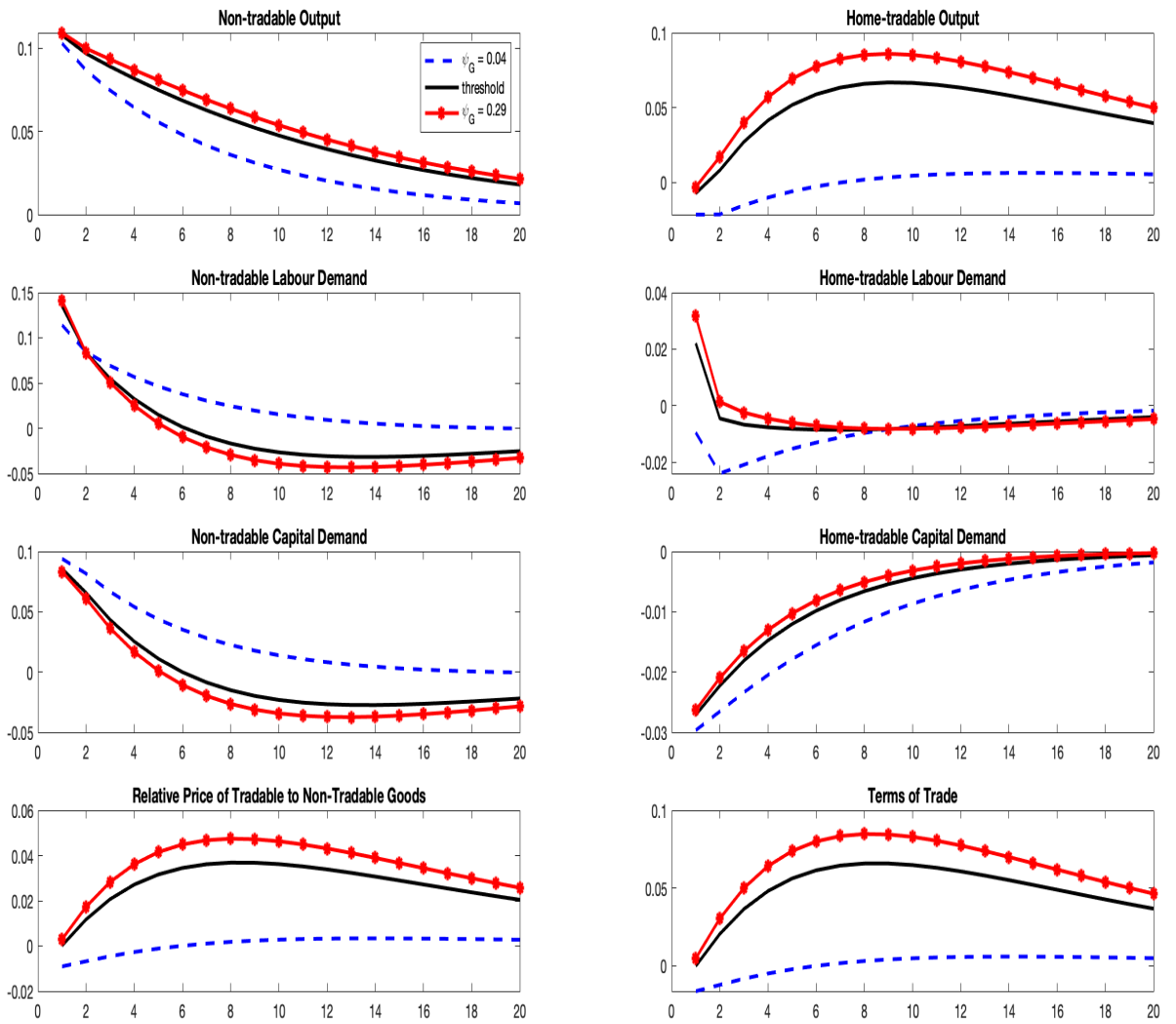
In the benchmark, the elasticity of substitution between tradable and non-tradable goods ( $\eta$ ) is 0.5. Households cannot easily substitute tradable goods for non-tradable goods. When we increase the elasticity of substitution to 1, the households can easily substitute tradable goods for non-tradable goods. As shown in Figure 4, the increase in the elasticity of substitution does not cause changes in the initial and dynamic responses of the real exchange rate, the terms of trade, or the relative price of tradable to non-tradable goods. A decline in the elasticity of substitution to 0.1 causes very difficult substitution between tradable and non-tradable goods. This decline has no effect on the real exchange rate, the terms of trade, or the relative price of tradable to non-tradable goods in all periods. Consequently, the elasticity of substitution between tradable and non-tradable goods is irrelevant to the real exchange rate in both the short and long runs.

In a small-open economy with incomplete asset markets, the positive association between the real exchange rate and private consumption from the risk-sharing condition disappears. Instead, in a model with a debt-elastic interest-rate premium and sticky prices, Bouakez and Eyquem (2015) explain that the real exchange rate is constrained by the real interest rate differential. If we maintain the assumption that government investment only focuses on non-tradable goods, an increase in government spending only raises domestic demand rather than demand for foreign goods; therefore, public investment only affects prices through the demand and supply sides but does not affect the current account. Our conclusion regarding the real

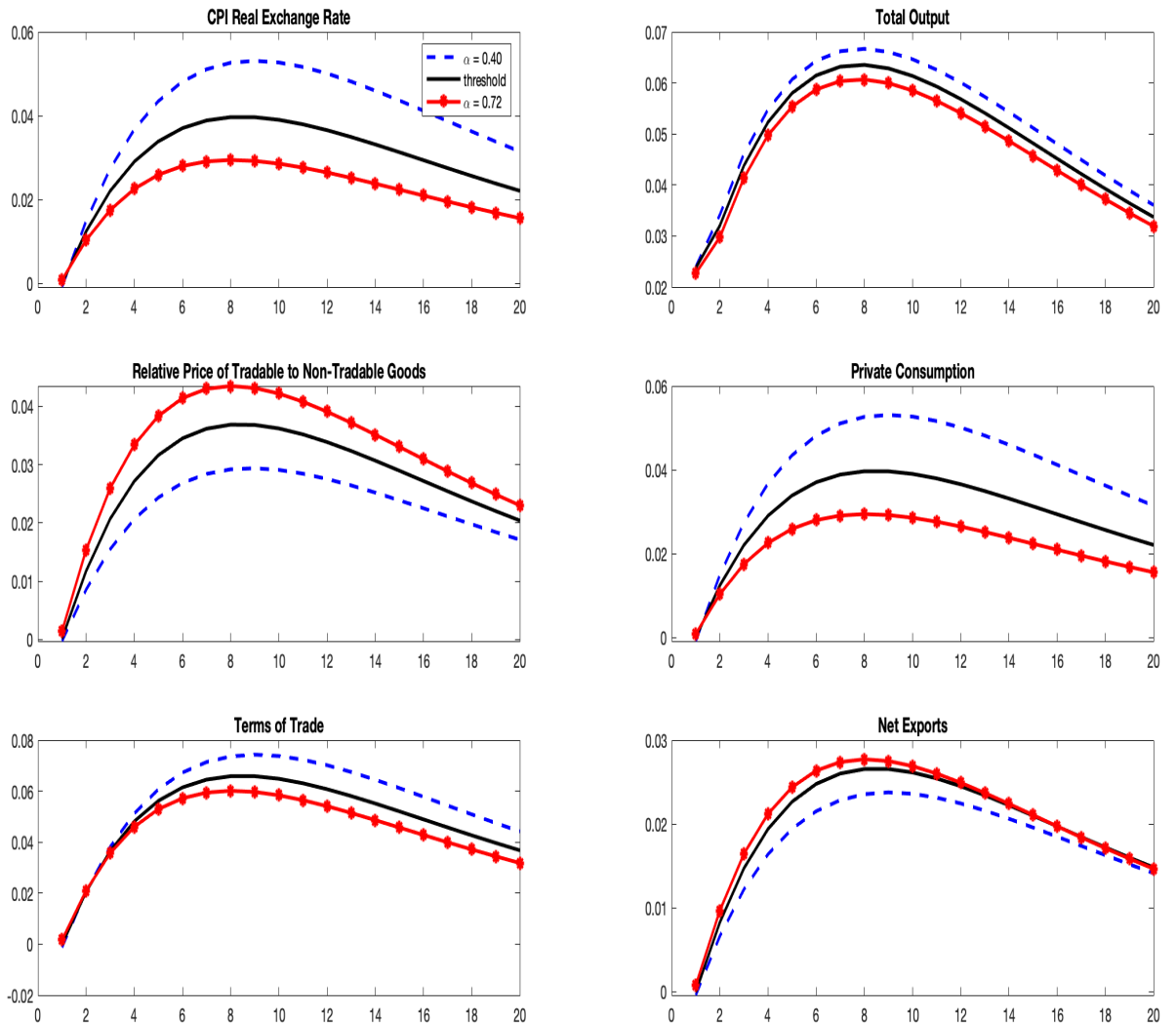
exchange rate will continue to depend on the output elasticity of public capital.



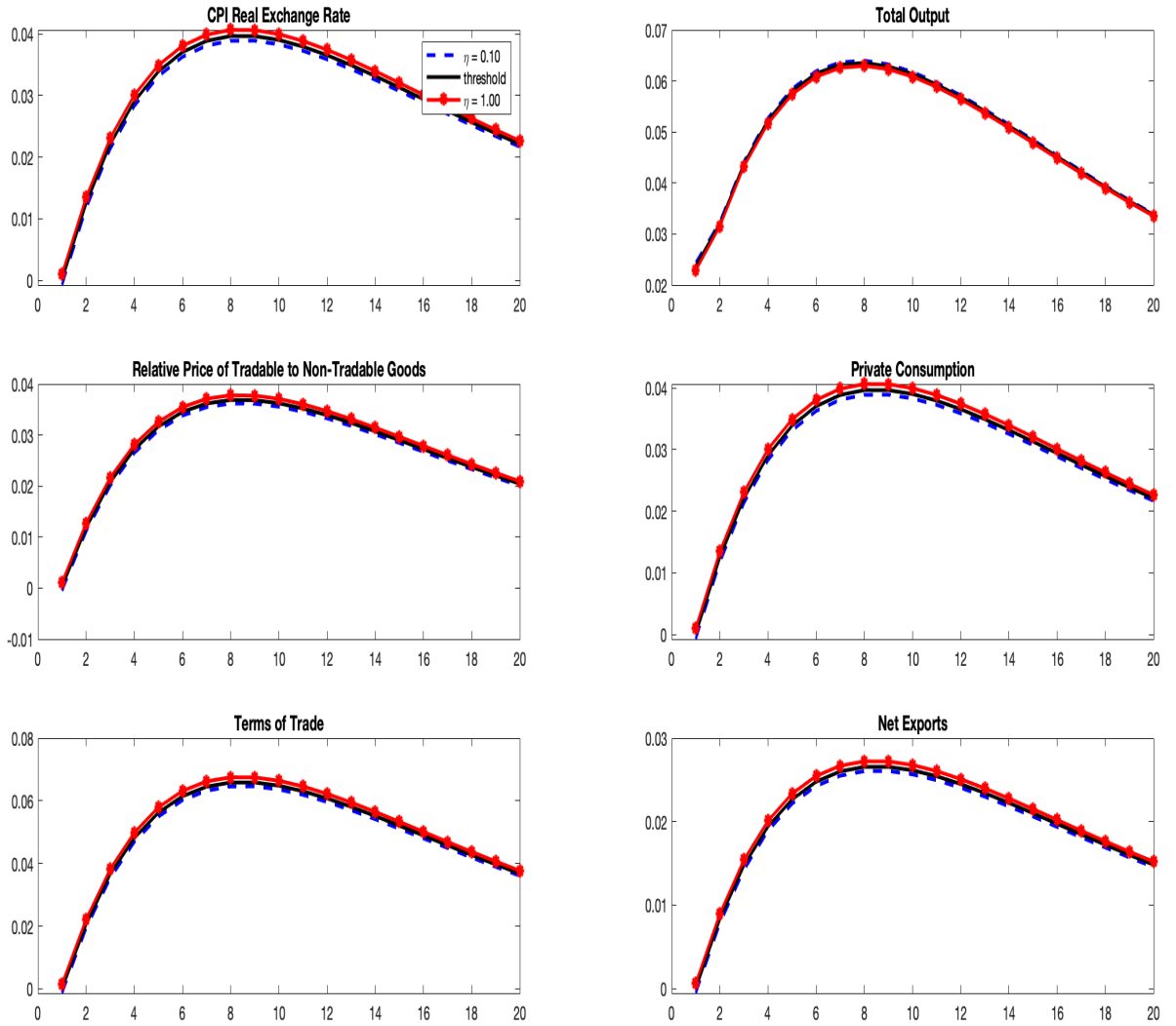
**Figure 1:** Impulse Responses to a 1% Rise in Public Investment



**Figure 2:** Impulse Responses of Home-tradable to Non-tradable Sector to a 1% Rise in Public Investment



**Figure 3:** Impulse Responses to a 1% Rise in Public Investment



**Figure 4:** Impulse Responses to a 1% Rise in Public Investment

## 1.7 CONCLUSION

In this chapter, we developed a theoretical model to deal with the puzzle related to the contradiction between theoretical and empirical papers on the relationship between government spending on physical capital and the real exchange rate. We developed a small-open New Keynesian DSGE model with traded and non-traded goods, real and nominal rigidities, and productive government spending.

In contrast to some existing theoretical models that focus only on the demand side effects of the increase in government spending, we have been able to capture the supply-side impact of government spending on physical capital. In our setting, public capital has a positive impact on the productivity of private factors and, hence, on the marginal cost of production and on price increases in the model with nominal rigidities. Still, public spending on capital has some demand-side effects on the dynamics of the relative price of traded to non-traded goods.

Through our ability to capture both the supply and demand sides of government spending on public capital, we can shed light on its net effects on the dynamics of some key variables in the economy. We sorted out the puzzle by showing that innovations to public spending on capital can lead to either an appreciation or a depreciation of the real exchange rate. The magnitude of the output elasticity of public capital is a key parameter driving the results. Our results suggest that the output elasticity of public capital is a determinant of the dynamic response of the real exchange rate to changes in the public capital stock.

Our simulation results suggest that the real exchange rate appreciates (depreciates) with low (high) values of the output elasticity of public capital. Thus, the seeming contradiction between theoretical and empirical studies on the link between the two variables disappears in light of our findings. Thus, the appreciation of the real exchange rate following an increase in government spending on capital is no longer incompatible with theoretical expectations.

Finally, it is worth noting that our results, with the low value of output elasticity of public capital, are in line with those found in Basu and Kollmann (2013). Using small values

## CHAPTER 1

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for the output elasticity of public capital, they found that the real exchange rate appreciated following an increase in government spending on public capital.

## Chapter 2

# Government Spending Shocks in A New-Keynesian Structural VAR Model

### 2.1 INTRODUCTION

There is no consensus yet in the theoretical and empirical literature on the effect of government spending on private consumption. This contradiction of private consumption between theoretical prediction and empirical analysis has been highlighted by Monacelli and Perotti (2006) and Monacelli and Perotti (2010). From a theoretical perspective, referring to a New-Keynesian model with price stickiness and separable utility, a rise in government spending increases output and causes the negative wealth effect by implying a rise in net taxes; the latter leads to a fall in private consumption. In contrast, their empirical studies employ quarterly data to separately estimate the effects of government spending in Australia, Canada, the United Kingdom, and the United States in Structural Vector Autoregressive (SVAR) models and find that output and private consumption rise in response to an increase in government spending, including both public consumption and investment. This observation is referred to as the co-movement of private consumption with output, and government spending has a crowding-in effect on private consumption.

More empirical studies support their empirical results. For example, Ravn *et al.* (2012) employ quarterly panel data of a sample of the aforementioned four countries; while Beetsma and Giuliodori (2011) employ annual panel data of a sample of the 14 European Union members. However, Mountford and Uhlig (2009) and Enders *et al.* (2011) employ U.S. quarterly data and suggest that private consumption does not change in response to the rise in gov-

ernment spending. In contrast to the increase in private consumption, Coenen and Straub (2005) employ annual data in Euro nations and find a negative response of private consumption to government spending shock. Consequently, we have seen that empirical studies on government spending have generated three different responses of private consumption to government spending shock. Not only does a contradiction exist between theoretical and empirical literature, but also the empirical studies have not found a consensus.

More recent empirical literature distinguishes shocks to public consumption and public investment, where public investment can improve productivity. Linnemann and Schabert (2006) indicate that, in a New-Keynesian model, a rise in productive government expenditure can raise private consumption because the production costs reduce such that inflation falls. Basu and Kollmann (2013) also suggest that government purchases can improve the productivity of private firms. Based on the suggestion of productive expenditure on public capital, Iwata (2013) employs quarterly data of Japan and finds a positive response of private consumption to a rise in public investment. Y. Chen and Liu (2018) find that public investment crowds in private consumption by employing quarterly data from China. These empirical results are consistent with the theoretical prediction.

Moreover, Iwata (2013) finds that an increase in public consumption leads private consumption to rise, and Y. Chen and Liu (2018) suggest that public consumption at least does not hurt private consumption. As a result, the non-negative changes in private consumption in response to a rise in public consumption still contradict the theoretical prediction that non-productive government expenditure crowds out private consumption. Even though more recently, government spending has been decomposed into public consumption and public investment, recent papers, such as Iwata (2013) and Y. Chen and Liu (2018), assume that public consumption and public investment are uncorrelated, and they separately analyze effects of either public consumption or public investment.

To address the contradiction, this chapter distinguishes between public consumption and public investment in the SVAR models. We use U.S. quarterly data from 1960Q1 to 2019Q4.

Building upon Blanchard and Perotti (2002), this paper first includes government spending. Then, we change government spending to public consumption. Finally, the SVAR model includes both public consumption and investment. The latter model is capable of answering our concern with how public investment responds to a rise in public consumption, such that we order public consumption first before public investment in the vector of variables.

In recent literature, Ramey (2016) points out that the SVAR models have two main foresight problems of private agents and policymakers. Since Keating (1990) created a method to impose restrictions of rational expectations on SVAR models, Leu (2011) employs Keating's (1990) method to include forward-looking behaviours of households, firms and the central bank in the SVAR model - motivated by a basic New-Keynesian model. To dispel Ramey's (2016) doubt on SVAR models, this paper develops an NK-SVAR model to include the forward-looking rational expectations. The advantage of the SVAR model with rational expectations has been emphasized by Keating (1990) that estimates of the structural parameters are invariant to policy changes, and not subject to the Lucas and Sargent's (1981) critique.

Our results suggest that a positive shock to public consumption leads to an increase in public investment. In addition, private consumption increases in response to the rise in public consumption following the increases in net taxes and the nominal interest rates. When we isolate the productive effect of public investment by considering only public consumption, an increase in public consumption leads to a decrease in private consumption following increases in net taxes and the nominal interest rates.

Our analysis of the positive shock to public consumption associated with the positive response of public investment solves the original contradiction. Rational households increase private consumption in response to the rise in public consumption because of the wealth effects of the associated increase in public investment. Therefore, our paper suggests that a shock to public consumption associated with a rise in public investment can explain why a rise in government spending causes an increase in private consumption.

This chapter sorts out the inconsistency of the private-consumption responses to a rise in public consumption among theoretical and empirical literature by combining public investment with public consumption. We suggest that theoretical analysis of non-productive government expenditure should include productive government expenditure to reconcile the theoretical prediction of private consumption with the empirical results. This paper gives an alternative explanation for the private consumption puzzle induced by a rise in non-productive government expenditure. In response to a shock to public consumption, if the supply-side effect of a rise in public investment dominates the demand-side effect of increases in public consumption and investment, private consumption increases. Alternatively, if the demand-side effect dominates the supply-side effect, private consumption decreases in response to the shock. In addition, private consumption does not change if the demand-side effect equals the supply-side effect.

The remainder of the paper is as follows. Section 2.2 gives a literature review. In Section 2.3, we derive the New Keynesian SVAR models. We discuss the data and the empirical results in Section 2.4. The last section concludes.

## **2.2 LITERATURE REVIEW**

In this section of the literature review, we present empirical studies on the effects of government spending on private consumption in SVAR models. To defend the Keynesian view of the effects of government spending - particularly on private consumption - that has been attacked by the neoclassical approach, Blanchard and Perotti (2002) use the SVAR approach to identify a shock to government spending defined as public consumption plus public investment. The SVAR approach has been used for monetary policy, but they believe that the approach is more appropriate for studies of fiscal policy because the fiscal shocks are exogenous with respect to output, and decision and implementation lags in fiscal policy cause little or no discretionary response to economic activities. They find that a positive government spending shock has positive effects on both output and private consumption.

Following Blanchard and Perotti (2002), Monacelli and Perotti (2006) extended the SVAR model for an open economy and found increases in output and private consumption in response to a rise in government spending. However, their small-open New Keynesian model had an opposite effect of government spending on private consumption. They highlight that there exists a contradiction between the New Keynesian model and the SVAR model. Later on, Ravn *et al.* (2012) employ a panel SVAR model of four industrial countries and find that a rise in government spending increases output and private consumption. To reconcile the extent of the theoretical prediction with the empirical results, Galí, López-Salido, *et al.* (2007) use the New Keynesian model including the rule-of-thumb household, and Ravn *et al.* (2012) suggest the deep-habit mechanism. Besides, Corsetti, Meier, *et al.* (2012) observe an initial increase in government spending but it decreases over time, and they show that the expected reversals of government spending by households can lead to a rise in private consumption in response to government spending shock.

From the supply side, government spending can improve the productivity of private production factors. In a New-Keynesian model, Linnemann and Schabert (2006) present that a rise in productive government expenditure improves the productivity of domestic private firms and can raise private consumption. Based on the idea of productive government expenditure, Iwata (2013) distinguishes non-productive government expenditure (public consumption) from productive government expenditure (public investment); the paper finds that a rise in public investment increases output and private consumption. In addition, a rise in public consumption increases private consumption. Y. Chen and Liu (2018) find that a shock to public investment has a positive effect on output and private consumption; however, a rise in public consumption increases output but has no effect on private consumption. Even though the empirical response of private consumption to the rise in public investment is consistent with the theoretical predictions in consideration of the productive feature of public investment, the increase in private consumption in response to the increase in public consumption contradicts the negative response of private consumption in neoclassical and New Keynesian models.

To address the contradiction of private consumption in response to a public consumption shock, Ramey (2016) points out that two main foresight problems of private agents and policymakers cause a serious flaw in the SVAR models. As shown in the previous SVAR papers, forward-looking behaviours of households, firms, and the central bank have not been considered in the SVAR models. Keating (1990) developed a method for introducing rational expectations into SVAR models. Leu (2011) employed Keating's (1990) method to identify a SVAR model under rational expectations motivated by a basic New-Keynesian model. Keating (1990) emphasizes that estimates of the structural parameters are invariant to policy changes. In other words, these parameter estimates are constant in response to changes in policy. Referring to Lucas and Sargent (1981), these parameters are called 'deep' structural parameters and not subject to the Lucas critique. Also, Leu (2011) contributes a viable way to the current New-Keynesian literature by identifying rational-expectation SVAR models from the New-Keynesian model.

However, the SVAR model with rational expectations has not been developed for fiscal policy studies and this method can dispel Ramey's (2016) doubts concerning SVAR models. As a result, we follow Keating (1990) to impose the rational-expectation restrictions (motivated by the New-Keynesian model) in the NK-SVAR models that we will develop later. The NK-SVAR models will account for the contemporaneous relationship between the reduced-form residuals and structural disturbances from the New-Keynesian model. The method suggests the use of the Full Information Maximum Likelihood Method through the maximization of the concentrated log-likelihood function subject to non-linear constraints.

### **2.3 NEW KEYNESIAN STRUCTURAL VAR Model**

To appropriately study the effects of government spending on private consumption, we developed a SVAR model to include rational-expectation restrictions imposed on the forward-looking variables from a New Keynesian model. Our model builds upon the contributions of Blanchard and Perotti (2002) in the literature on the SVAR model including government

spending, net taxes, real GDP and private consumption. Because our SVAR model imposes the rational-expectation restrictions on private consumption, we call our model New Keynesian SVAR (NK-SVAR) model - the name also used by Leu (2011).

In our NK-SVAR model, we firstly use government spending including both public consumption and investment defined by Blanchard and Perotti (2002) to estimate the effects of government spending on private consumption. After that, we only include public consumption to see the different response of private consumption to a rise in public consumption from government spending. Finally, we include both public consumption and public investment in the NK-SVAR model by ordering public consumption first and then public investment.

### 2.3.1 Baseline SVAR Model

Blanchard and Perotti (2002) include government spending ( $g$ ) that is a sum of public consumption ( $gc$ ) and public investment ( $gz$ ), net taxes ( $t$ ), real GDP ( $y$ ) and private consumption ( $c$ ). Blanchard and Perotti's (2002) SVAR model is written as:

$$\begin{aligned}
 u_{g,t} &= \alpha_{gy}u_{y,t} + e_{g,t} + c_{gt}e_{t,t} \\
 u_{t,t} &= \alpha_{ty}u_{y,t} + c_{tg}e_{g,t} + e_{t,t} \\
 u_{y,t} &= \gamma_{yg}u_{g,t} + \gamma_{yt}u_{t,t} + e_{y,t} \\
 u_{c,t} &= \gamma_{cg}u_{g,t} + \gamma_{ct}u_{t,t} + \gamma_{cy}u_{y,t} + e_{c,t}
 \end{aligned} \tag{2.1}$$

where  $u_{i,t}$  represents reduced-form residuals, and  $e_{i,t}$  represents structural disturbances. In order to study the effects of government spending, Blanchard and Perotti (2002) assume that government spending decisions come first, so that  $c_{gt} = 0$  and government spending does not respond to net taxes contemporaneously. Moreover, due to the quarterly data, they impose an exclusion restriction on  $\alpha_{gy}$  so government spending does not respond to output. Consequently, the reduced-form residual of government spending can be described as  $u_{g,t} = e_{g,t}$ , and the

SVAR model can be written as:

$$\begin{aligned}u_{g,t} &= e_{g,t} \\u_{t,t} &= \alpha_{ty}u_{y,t} + c_{tg}u_{g,t} + e_{t,t} \\u_{y,t} &= \gamma_{yg}u_{g,t} + \gamma_{yt}u_{t,t} + e_{y,t} \\u_{c,t} &= \gamma_{cg}u_{g,t} + \gamma_{ct}u_{t,t} + \gamma_{cy}u_{y,t} + e_{c,t}\end{aligned}\tag{2.2}$$

As a result,  $c_{tg}$  is  $u_{t,t}$ 's automatic feedback from the government spending shock, and  $c_{gt}$  is  $u_{g,t}$ 's automatic feedback from a shock to net taxes. We call  $c_{gt}$  and  $c_{tg}$  the correlation parameters between the reduced-form residuals of government spending and net taxes. Ramey (2011) explains that a Cholesky decomposition, in which government spending is ordered before other variables, is identical to Blanchard and Perotti's (2002) identification of a shock to government spending, because government spending does not respond to output and net taxes contemporaneously.

In the next sub-section, we will introduce the forward-looking behaviour of private consumption based on the New Keynesian model. Because of the forward-looking behaviour of private consumption, the four-variable SVAR model needs to be extended to include inflation and the nominal interest rates. The structural equations of inflation, the nominal interest rates and private consumption will follow the New Keynesian model.

### 2.3.2 NK-SVAR Model: Government Spending

In the log-linearized New Keynesian model, the representative household is a rational and forward-looking agent. The Euler equation indicates the rational and forward-looking behaviour of the household's decision on private consumption:

$$c_t = E_t c_{t+1} - \frac{1}{\sigma}(r_t - E_t \pi_{t+1}) + e_{c,t}\tag{2.3}$$

where  $e_{c,t}$  represents a structural shock to private consumption.  $\pi$  denotes inflation,  $r$  denotes the nominal interest rates, and  $\sigma \geq 0$  is a relative risk aversion coefficient determining the curvature of the utility of consumption.

The New Keynesian Phillips curve states that inflation positively depends on the expected inflation in the next period and the deviation of the current output from its steady state. Thus, the New Keynesian Phillips curve is written as:

$$\pi_t = \beta E_t \{\pi_{t+1}\} + \kappa y_t + e_{\pi,t} \quad (2.4)$$

where  $e_{\pi,t}$  denotes a shock to aggregate supply, and  $\beta$  is a discount factor.<sup>1</sup>

For the nominal interest rate, we include a forward-looking monetary policy rule. We assume that the central bank responds to the expected future deviation of inflation and contemporaneous variation of output.

$$r_t = \bar{r} + \pi_t + \phi_\pi E_t \pi_{t+1} + \phi_y y_t + e_{r,t} \quad (2.5)$$

where  $e_{r,t}$  represents a shock to monetary policy. As indicated by Leu (2011), the nominal interest rates respond to a rise in inflation and/or output such that the real interest rate ( $r_t - \pi_t$ ) sufficiently increases to stabilize the movements in inflation and output.

For the log-linearized New Keynesian model, we apply Keating's (1990) identification scheme used in Leu (2011) to convert the contemporaneous structural system into an innovative representation that comprises structural disturbances and reduced-form residuals. The conversion requires subtracting from each variable the expectation at  $t-1$  ( $E_{t-1}$ ) of this variable conditioned on all available previous information, such as  $x_t - E_{t-1}x_t = u_{x,t}$ , where  $u_{x,t}$  denotes the reduced-form residual of the variable.

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<sup>1</sup>When the production function for each firm is constant return to scale,  $\kappa \equiv \frac{(1-\beta\theta)(1-\theta)}{\theta}$ , where  $\theta \in (0, 1)$  is a probability that a firm keeps its price fixed in the next period and  $\epsilon > 1$  denotes elasticity of substitution between differentiated goods.

Subtracting  $E_{t-1}c_t$  from private consumption,  $c_t$ , in the Euler Equation (2.3) yields:

$$u_{c,t} = (E_t c_{t+1} - E_{t-1} c_{t+1}) - \frac{1}{\sigma} u_{r,t} + \frac{1}{\sigma} (E_t \pi_{t+1} - E_{t-1} \pi_{t+1}) + e_{c,t} \quad (2.6)$$

where  $c_t - E_{t-1}c_t = u_{c,t}$  and  $r_t - E_{t-1}r_t = u_{r,t}$ . For the New Keynesian Phillips curve, subtracting  $E_{t-1}\pi_t$  from inflation,  $\pi_t$ , in equation (2.4) yields:

$$u_{\pi,t} = \beta(E_t \pi_{t+1} - E_{t-1} \pi_{t+1}) + \kappa u_{y,t} + e_{\pi,t} \quad (2.7)$$

where  $y_t - E_{t-1}y_t = u_{y,t}$ . Subtracting  $E_{t-1}r_t$  from  $r_t$  in equation (2.5) of monetary policy rule yields:

$$u_{r,t} = u_{\pi,t} + \phi_{\pi}(E_t \pi_{t+1} - E_{t-1} \pi_{t+1}) + \phi_y u_{y,t} + e_{r,t} \quad (2.8)$$

where  $\pi_t - E_{t-1}\pi_t = u_{\pi,t}$ .

The structural disturbance in Equation (2.6) is related to the revision processes of private consumption,  $E_t c_{t+1} - E_{t-1} c_{t+1}$ , and inflation,  $E_t \pi_{t+1} - E_{t-1} \pi_{t+1}$ . The NK-SVAR model includes the forward-looking behaviour on private consumption of households through the forecasting terms of inflation and private consumption. The structural disturbance in Equation (2.7) is related to  $E_t \pi_{t+1} - E_{t-1} \pi_{t+1}$ . The SVAR model includes the forward-looking behaviour on inflation of private firms through the forecasting term of inflation. Finally, the structural disturbance in Equation (2.8) is related to  $E_t \pi_{t+1} - E_{t-1} \pi_{t+1}$ . The SVAR model includes the forward-looking behaviour on the nominal interest rates of the central bank through the forecasting term of inflation. Therefore, in Equations (2.6), (2.7) and (2.8),  $E_t c_{t+1} - E_{t-1} c_{t+1}$  and  $E_t \pi_{t+1} - E_{t-1} \pi_{t+1}$  need to be calculated. We can first write the reduced-form VAR:

$$x_t = A_1 x_{t-1} + A_2 x_{t-2} + \dots + A_p x_{t-p} + u_t \quad (2.9)$$

where  $p$  denotes the lag order of endogenous variables,  $n = 6$  is the number of endogenous variables,  $x$  represents the  $n \times 1$  vector of variables  $(g, t, y, \pi, r, c)'$ ,  $u_t$  represents the  $n \times 1$

vector of reduced-form residuals, and  $A_i$  is the  $n \times n$  matrix of the VAR coefficients.<sup>2</sup> Then we can write the reduced-form VAR in a stacked form:

$$X_t = AX_{t-1} + Qu_t \quad (2.10)$$

or equivalently

$$\begin{pmatrix} x_t \\ x_{t-1} \\ x_{t-2} \\ \vdots \\ x_{t-p+1} \end{pmatrix} = \begin{pmatrix} A_1 & A_2 & \cdots & \cdots & A_p \\ I_n & 0_n & \cdots & \cdots & 0_n \\ 0_n & I_n & 0_n & \cdots & 0_n \\ \vdots & \ddots & \ddots & \ddots & \vdots \\ 0_n & \cdots & 0_n & I_n & 0_n \end{pmatrix} \begin{pmatrix} x_{t-1} \\ x_{t-2} \\ x_{t-3} \\ \vdots \\ x_{t-p} \end{pmatrix} + \begin{pmatrix} I_n \\ 0_n \\ 0_n \\ \vdots \\ 0_n \end{pmatrix} u_t \quad (2.11)$$

Due to the unforecastable residual  $u_t$ , the  $j$ -step conditional expectation of Equation (2.10) is:

$$\begin{aligned} E_t X_{t+1} &= AX_t \\ E_t X_{t+2} &= AE_t X_{t+1} = A^2 X_t \\ &\vdots \\ E_t X_{t+j} &= A^j X_t \end{aligned} \quad (2.12)$$

The rational expectations of agents are from the two forecasting terms,  $E_t c_{t+1} - E_{t-1} c_{t+1}$  and  $E_t \pi_{t+1} - E_{t-1} \pi_{t+1}$ . We can derive each term described as a function of the sum of reduced-form residuals of endogenous variables, such that the forward-looking behaviours based on the contemporaneously available information of government, households, private firms, and the central bank can be included in the system of reduced-form residuals and structural distur-

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<sup>2</sup>Keating (1990) presents that deterministic variables cannot cause a revision in expectations, so they are omitted from the reduced-form VAR without loss of generality.

bances. To do so, we define two  $1 \times np$  vectors to locate the forecasted variables.

$$\begin{aligned} h'_\pi &= (0, 0, 0, 1, 0, 0, \dots, 0) \quad \text{for inflation} \\ h'_c &= (0, 0, 0, 0, 0, 1, 0, \dots, 0) \quad \text{for private consumption} \end{aligned} \tag{2.13}$$

The expected future values of inflation and private consumption one period ahead,  $j = 1$ , can be derived by pre-multiplying (2.12) by  $h$  vectors respectively:

$$\begin{aligned} E_t \pi_{t+1} &= h'_\pi A X_t \\ E_t c_{t+1} &= h'_c A X_t \end{aligned} \tag{2.14}$$

Thus, subtracting the expected value at  $t - 1$  from Equation (2.14) yields:

$$\begin{aligned} E_t \pi_{t+1} - E_{t-1} \pi_{t+1} &= h'_\pi A (X_t - E_{t-1} X_t) = h'_\pi A Q u_t \\ &= a_{41} u_{g,t} + a_{42} u_{t,t} + a_{43} u_{y,t} + a_{44} u_{\pi,t} + a_{45} u_{r,t} + a_{46} u_{c,t} \\ E_t c_{t+1} - E_{t-1} c_{t+1} &= h'_c A (X_t - E_{t-1} X_t) = h'_c A Q u_t \\ &= a_{61} u_{g,t} + a_{62} u_{t,t} + a_{63} u_{y,t} + a_{64} u_{\pi,t} + a_{65} u_{r,t} + a_{66} u_{c,t} \end{aligned} \tag{2.15}$$

where  $a_{ij}$  denotes the elements of  $A_1$  in Equation (2.11).

Due to the forward-looking behaviours of private firms and households, we have two additional endogenous variables that are inflation and the nominal interest rates. Perotti (2004a) who extends Blanchard and Perotti's (2002) model to include the two additional variables has the same of six variables required by our NK-SVAR model. As a result, this NK-SVAR model follows Perotti (2004a) to conduct the structure of government spending, net taxes and output; meanwhile, it inserts Equation (2.15) into equations (2.6)-(2.8). Thus, the contemporaneous system of reduced-form residuals and structural disturbances is:

$$u_{g,t} - c_{gt} u_{t,t} - \alpha_{gy} u_{y,t} - \alpha_{g\pi} u_{\pi,t} - \alpha_{gr} u_{r,t} = e_{g,t} \tag{2.16}$$

$$-c_{tg}u_{g,t} + u_{t,t} - \alpha_{ty}u_{y,t} - \alpha_{t\pi}u_{\pi,t} - \alpha_{tr}u_{r,t} = e_{t,t} \quad (2.17)$$

$$-\gamma_{yg}u_{g,t} - \gamma_{yt}u_{t,t} + u_{y,t} = e_{y,t} \quad (2.18)$$

$$u_{\pi,t} - \beta h'_{\pi} A Q u_t - \kappa u_{y,t} = e_{\pi_H,t} \quad (2.19)$$

$$u_{r,t} - u_{\pi,t} - \phi_{\pi} h'_{\pi} A Q u_t - \phi_y u_{y,t} = e_{r,t} \quad (2.20)$$

$$u_{c,t} - h'_c A Q u_t + \frac{1}{\sigma} u_{r,t} - \frac{1}{\sigma} h'_{\pi} A Q u_t = e_{c,t} \quad (2.21)$$

where  $e_g$  denotes a shock to government spending. Government spending and net taxes do not contemporaneously correspond to private consumption. Forward-looking behaviour embedded into equations (2.21), (2.19) and (2.20) suggests non-linear restrictions across the coefficients of each contemporaneous structural equations. Households, firms, and the central bank are assumed to incorporate all relevant innovations in forecasting future expected values of endogenous variables. Depending on the differences between expectation at  $t$  and  $t - 1$  and factoring the reduced-form residuals, we can write the NK-SVAR model as  $B_0 u_t = e_t$  where  $B_0$  is shown in the Appendix B.<sup>3</sup>

In this NK-SVAR model, 21 unknown parameters need to be estimated. They are 10 structural parameters ( $c_{gt}, \alpha_{gy}, \alpha_{g\pi}, \alpha_{gr}, c_{tg}, \alpha_{ty}, \alpha_{t\pi}, \alpha_{tr}, \gamma_{yg}, \gamma_{yt}$ ), 5 ‘deep’ structural parameters ( $\beta, \kappa, \phi_{\pi}, \phi_y, \frac{1}{\sigma}$ ) and 6 variances of the structural disturbances ( $\sigma_g^2, \sigma_t^2, \sigma_y^2, \sigma_{\pi}^2, \sigma_r^2, \sigma_c^2$ ). Because there are  $n(n+1)/2 = 21$  distinct moments in the variance-covariance matrix of reduced-form residuals, these 21 parameters can be identified. To uniquely solve for all  $n^2 = 36$  elements of  $B_0$ , we need additional  $n^2 - n(n-1)/2 = 21$  equations.  $B_0$  has 26 restrictions that are 3 unity restrictions on the main diagonal, 5 exclusion restrictions and 18 rational-expectation restrictions imposed on the elements from row 4 to row 6 in  $B_0$ . As a result, this NK-SVAR model is overidentified with 5 overidentifying restrictions.

<sup>3</sup>Due to the assumption for the quarterly data made by both Blanchard and Perotti (2002) and Perotti (2004a) that government spending does not respond to output within a quarter,  $\alpha_{gy} = 0$  and the NK-SVAR model is shown in the Appendix B.

### 2.3.3 NK-SVAR Model: Public Consumption Only

When we switch from government spending to public consumption, we only need to change the notation from government spending  $g$  to public consumption  $gc$ . Thus, the difference between the expectations at  $t$  and  $t - 1$  can be re-written as:

$$\begin{aligned} E_t \pi_{t+1} - E_{t-1} \pi_{t+1} &= h'_\pi A (X_t - E_{t-1} X_t) = h'_\pi A Q u_t \\ &= a_{41} u_{gc,t} + a_{42} u_{t,t} + a_{43} u_{y,t} + a_{44} u_{\pi,t} + a_{45} u_{r,t} + a_{46} u_{c,t} \end{aligned} \quad (2.22)$$

$$\begin{aligned} E_t c_{t+1} - E_{t-1} c_{t+1} &= h'_c A (X_t - E_{t-1} X_t) = h'_c A Q u_t \\ &= a_{61} u_{gc,t} + a_{62} u_{t,t} + a_{63} u_{y,t} + a_{64} u_{\pi,t} + a_{65} u_{r,t} + a_{66} u_{c,t} \end{aligned}$$

As a result, the contemporaneous system of reduced-form residuals and structural disturbances is:

$$u_{gc,t} - c_{gct} u_{t,t} - \alpha_{gcy} u_{y,t} - \alpha_{gc\pi} u_{\pi,t} - \alpha_{gcr} u_{r,t} = e_{gc,t} \quad (2.23)$$

$$-c_{tgc} u_{gc,t} + u_{t,t} - \alpha_{ty} u_{y,t} - \alpha_{t\pi} u_{\pi,t} - \alpha_{tr} u_{r,t} = e_{t,t} \quad (2.24)$$

$$-\gamma_{ygc} u_{gc,t} - \gamma_{yt} u_{t,t} + u_{y,t} = e_{y,t} \quad (2.25)$$

$$u_{\pi,t} - \beta h'_\pi A Q u_t - \kappa u_{y,t} = e_{\pi H,t} \quad (2.26)$$

$$u_{r,t} - u_{\pi,t} - \phi_\pi h'_\pi A Q u_t - \phi_y u_{y,t} = e_{r,t} \quad (2.27)$$

$$u_{c,t} - h'_c A Q u_t + \frac{1}{\sigma} u_{r,t} - \frac{1}{\sigma} h'_\pi A Q u_t = e_{c,t} \quad (2.28)$$

where  $e_{gc,t}$  denotes a shock to public consumption. Public consumption and net taxes do not contemporaneously correspond to private consumption. This NK-SVAR model is  $B_0 u_t = e_t$  where  $B_0$  is shown in the Appendix B.<sup>4</sup> This NK-SVAR model has the same overidentification issue, with 5 overidentifying restrictions, due to the rational-expectation restrictions.

<sup>4</sup> $B_0$  matrix is shown in the Appendix B when  $\alpha_{gy} = 0$ .

### 2.3.4 NK-SVAR Model: Public Consumption and Public Investment

We decompose government spending into public consumption and public investment. So, we extend the NK-SVAR model that only includes public consumption by adding public investment. There are seven endogenous variables,  $(gc, gz, t, y, \pi, r, c)'$  represented by  $x$  in Equation (2.9) where  $gz$  denotes public investment. Since  $n = 7$ , and inflation and private consumption become the fifth and seventh variables in the VAR model, we need to redefine the two  $1 \times np$  vectors to locate the forecasted variables:

$$h'_\pi = (0, 0, 0, 0, 1, 0, 0, \dots, 0) \quad \text{for inflation} \quad (2.29)$$

$$h'_c = (0, 0, 0, 0, 0, 0, 1, 0, \dots, 0) \quad \text{for private consumption}$$

As a result, the contemporaneous system of reduced-form residuals and structural disturbances is:

$$u_{gc,t} - C_{gcz}u_{gz,t} - C_{gct}u_{t,t} - \alpha_{gcy}u_{y,t} - \alpha_{gc\pi}u_{\pi,t} - \alpha_{gcr}u_{r,t} = e_{gc,t} \quad (2.30)$$

$$-C_{gzc}u_{gc,t} + u_{gz,t} - C_{gzt}u_{t,t} - \alpha_{gzy}u_{y,t} - \alpha_{gz\pi}u_{\pi,t} - \alpha_{g zr}u_{r,t} = e_{gz,t} \quad (2.31)$$

$$-C_{tgc}u_{gc,t} - C_{tgz}u_{gz,t} + u_{t,t} - \alpha_{ty}u_{y,t} - \alpha_{t\pi}u_{\pi,t} - \alpha_{tr}u_{r,t} = e_{t,t} \quad (2.32)$$

$$-\gamma_{ygc}u_{gc,t} - \gamma_{ygz}u_{gz,t} - \gamma_{yt}u_{t,t} + u_{y,t} = e_{y,t} \quad (2.33)$$

$$u_{\pi,t} - \beta h'_\pi A Q u_t - \kappa u_{y,t} = e_{\pi H,t} \quad (2.34)$$

$$u_{r,t} - u_{\pi,t} - \phi_\pi h'_\pi A Q u_t - \phi_y u_{y,t} = e_{r,t} \quad (2.35)$$

$$u_{c,t} - h'_c A Q u_t + \frac{1}{\sigma} u_{r,t} - \frac{1}{\sigma} h'_\pi A Q u_t = e_{c,t} \quad (2.36)$$

where  $e_{gz,t}$  denotes a shock to public investment. Public consumption, public investment and net taxes do not contemporaneously correspond to private consumption. In this NK-SVAR model,  $B_0 u_t = e_t$  where  $B_0$  is shown in the Appendix B.

$B_0$  has 30 unknown parameters, but there are  $n(n+1)/2 = 28$  distinct moments in the variance-covariance matrix of reduced-form residuals. Consequently, we impose two exclusion restrictions on  $\alpha_{gcy}$  and  $\alpha_{gzy}$ . As discussed by Blanchard and Perotti (2002), government spending decisions do not respond to output within a quarter. Therefore,  $B_0$  has 28 unknown parameters that can be identified. To uniquely solve for all  $n^2 = 49$  elements of  $B_0$ , we need additional  $n^2 - n(n-1)/2 = 28$  equations.  $B_0$  has 33 restrictions - the 4 unity restrictions on the main diagonal, 8 exclusion restrictions, and 21 rational-expectation restrictions imposed on the elements from row 5 to row 7 in  $B_0$ . As a result, 5 overidentifying restrictions result in that the NK-SVAR model is overidentified.

### 2.3.5 NK-SVAR Model Estimation

We write the  $n$ -dimensional SVAR( $p$ ) model as:

$$B_0x_t = \mu + B_1x_{t-1} + B_2x_{t-2} + \cdots + B_px_{t-p} + e_t \quad (2.37)$$

where  $x_t$  is a vector of endogenous variables.  $B_i$  is the coefficient matrix for the endogenous variables with lag order  $p$ .  $e_t$  is a vector of structural disturbances, and  $e_t \sim (0, \Sigma_e)$  is assumed to be Gaussian white noise.

Pre-multiplying Equation (2.37) by the inverse of the contemporaneous matrix  $B_0^{-1}$ , the corresponding reduced-form VAR( $p$ ) is:

$$x_t = A_0 + A_1x_{t-1} + A_2x_{t-2} + \cdots + A_px_{t-p} + u_t \quad (2.38)$$

where  $A_0 = B_0^{-1}\mu$  and  $A_i = B_0^{-1}B_i$ ,  $i = 1, \dots, p$ , and  $u_t = B_0^{-1}e_t \sim (0, \Sigma_u)$ . Hence, from the relation  $\Sigma_e = B_0\Sigma_uB_0'$  and the assumption of a diagonal matrix of  $\Sigma_e$ , we have  $n(n-1)/2$  independent equations that are off-diagonal zero elements of  $B_0\Sigma_uB_0'$ .

Then, we follow the two-step estimation employed by Leu (2011). Step one estimates the

reduced-form VAR specified by OLS. In the second step, the estimates of the reduced-form coefficient contained in  $\widehat{A}$ , rational-expectation restrictions, and unity and zero restrictions are imposed on  $B_0$ ; then, the SVAR (2.37) can be estimated by using Full Information Maximum Likelihood (FIML), with the assumption of normality in the structural disturbances. The structural parameters are obtained by maximizing the concentrated log-likelihood function from Kilian and Lütkepohl (2017):

$$\log l(B_0, \Sigma_e) = \text{constant} + \frac{T}{2} \log(\det(B_0)^2) - \frac{T}{2} \log(\det(\Sigma_e)) - \frac{T}{2} \text{tr}(B_0' \Sigma_e^{-1} B_0 \widetilde{\Sigma}_u) \quad (2.39)$$

with respect to  $B_0$  and  $\Sigma_e$ , subject to the identifying restrictions, where  $\widetilde{\Sigma}_u = \frac{1}{T} \sum_{t=1}^T \widehat{u}_t \widehat{u}_t'$  and  $\widehat{u}_t = x_t - \widehat{A}X_t$  is the LS residual.

Due to the overidentifying restrictions, the corresponding estimator for  $\Sigma_u$ :

$$\widetilde{\Sigma}_u^r := \widetilde{B}_0^{-1} \widetilde{\Sigma}_e \widetilde{B}_0'^{-1} \quad (2.40)$$

differs from  $\widetilde{\Sigma}_u$ . Consequently, we use the LR statistic:

$$\lambda_{LR} = T(\log|\widetilde{\Sigma}_u^r| - \log|\widetilde{\Sigma}_u|) \quad (2.41)$$

to check the overidentifying restrictions. Under the null hypothesis that the restricted (structural) model comes from the same asymptotic distribution of the unrestricted (reduced-form) model, the LR statistic has an asymptotic  $\chi^2$ -distribution with degrees of freedom equal to the number of overidentifying restrictions.

## 2.4 DATA AND EMPIRICAL RESULTS

We estimated the New-Keynesian SVAR model with the U.S. quarterly data (1947Q1 - 2020Q4) from the National Income and Product Accounts (NIPA) of the Bureau of Economic Analysis (BEA). The data for total government spending is a sum of public consumption and

public investment. Data for net taxes were calculated by following Perotti's (2004a) method:

$$ntx = taxr + contsi + transr - transper - subsidies$$

where net taxes ( $ntx$ ) is equal to the sum of current tax receipts ( $taxr$ ), contributions for government social insurance ( $contsi$ ), current transfer receipts ( $transr$ ) minus the sum of current transfer to persons ( $transper$ ) and subsidies ( $subsidies$ ). In addition to government spending, another component of GDP is private consumption that is the sum of nondurable goods and services consumption. Real terms of government spending in public consumption and public investment, net taxes, GDP, and private consumption are calculated through an implicit price deflator for GDP (seasonally adjusted, 2012 = 100).

We measured data for government spending, public consumption, public investment, GDP and private consumption as the differences of logarithm in real and per capita terms between  $t$  and  $t - 1$ . Inflation was calculated by taking the log-difference of the GDP deflator.

The long-term nominal interest rate in percent from OECD, Long-Term Government Bond Yields: 10-year: Main (Including Benchmark) for the United States retrieved from FRED, Federal Reserve Bank of St. Louis. Using the long-term nominal interest rate follows Perotti (2004a), because the long-term interest rate has a more significant effect on private consumption than the short-term interest rate.

In the next sub-sections, we will firstly show the impulse response functions under the recursive identification (Cholesky decomposition) by ordering government spending first. Then, we will present the estimates of 'deep' structural parameters and other structural parameters through FIML estimations and impulse response functions in the NK-SVAR model - including government spending and other five variables. After that, we will replace government spending with public consumption to show the structural estimates and impulse responses. Finally, in the NK-SVAR model including public consumption, public investment and other five variables, we will present the structural estimates and impulse response functions.

To obtain the impulse response functions, VARs include a linear time trend and four lags. The confidence intervals of impulse response functions are only 68% bands based on bootstrap standard errors. Ramey (2011) has discussed the 68% bands that have no theoretical justification but are common in government spending studies. As a result, we will also show the impulse response functions with 95% bands in the Appendix B.<sup>5</sup>

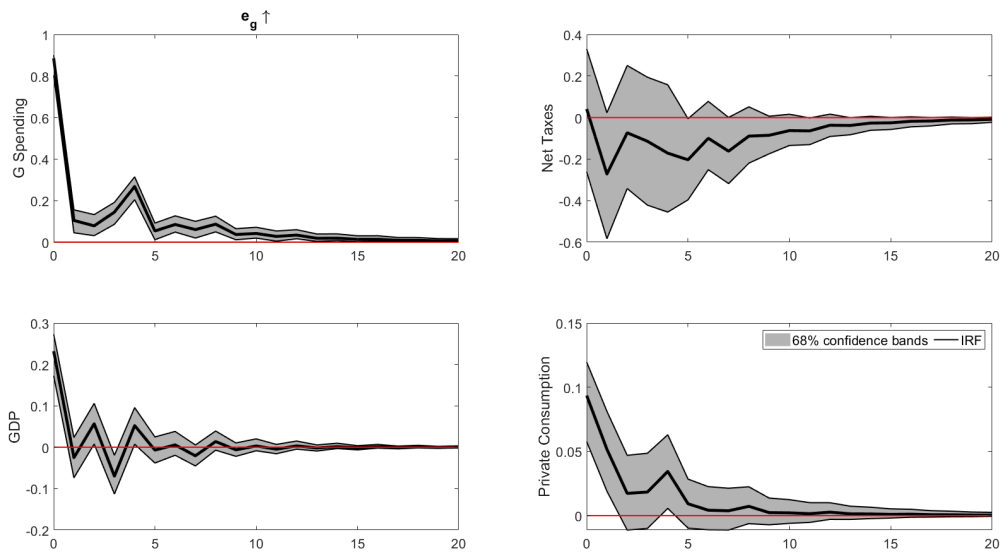
### 2.4.1 Impulse Response Functions: Cholesky Decomposition

Under the recursive identification of Cholesky decomposition, Figure 5 shows the impulse responses of government spending, net taxes, real GDP, and private consumption to a shock to government spending. The increase in government spending increases real GDP and private consumption. This finding is consistent with Blanchard and Perotti (2002), even though our data has more periods (1960Q1-2019Q4) than Blanchard and Perotti (2002) (1960Q1-1997Q4). Moreover, the increase in government spending does not lead to a statistically significant increase in net taxes.

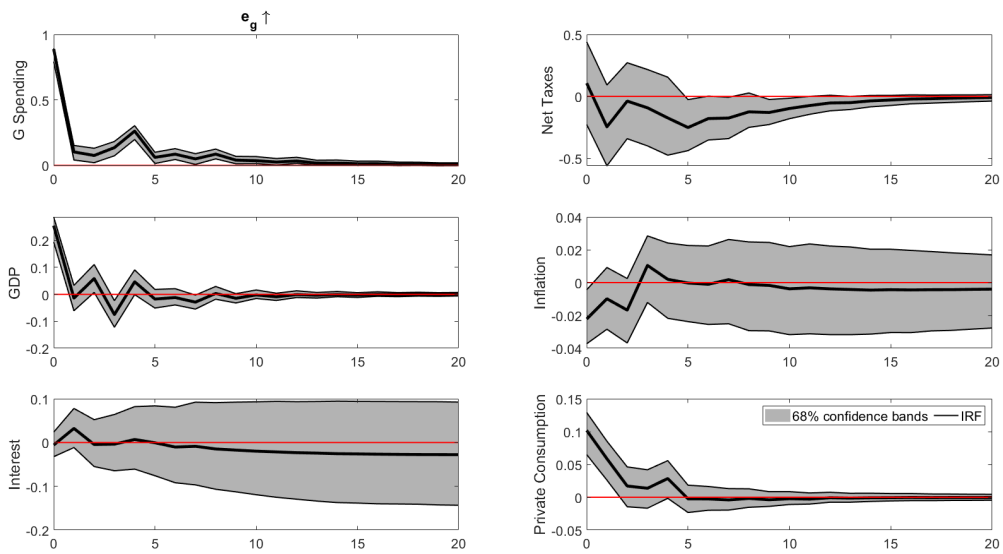
Based on the four-variable SVAR model, we follow Perotti (2004a) to extend the SVAR model to include inflation and the long-term nominal interest rate. Figure 6 presents the impulse response functions in the six-variable SVAR model identified by Cholesky decomposition. The increase in government spending raises real GDP and private consumption. The increases of the two variables are the same as the findings of Perotti (2004a). In addition, the change of the long-term nominal interest rate is not statistically significant, and the variation is consistent with the results found by Perotti (2004a).

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<sup>5</sup>To achieve the bootstrap confidence interval bands, we firstly estimate VAR(4) with the original sample and store the OLS estimates  $\hat{A} = (X'X)^{-1}X'Y$  where  $Y = XA + U$  and the vector of residuals  $\hat{U} = (\hat{u}_5 \dots \hat{u}_{240})$ . Following Runkle (1987)'s bootstrap method, we compute 1000 draws for the impulse response functions. Then, we obtain bias-corrected estimates  $A^*$  through correcting the bias of the OLS estimates that is  $bias = \bar{A}^b - \hat{A}$ , where  $\bar{A}^b$  denotes the mean across 1000 bootstrap replications. So, the bias-corrected estimates are  $A^* = \hat{A} - bias$ . We can use  $A^*$  to generate 2000 new bootstrap samples. Thus, we obtain 2000 different impulse response functions. Finally, we take  $\alpha$ th and  $(1 - \alpha)$ th percentiles to create the bootstrap confidence interval bands.



**Figure 5:** Impulse responses to a one-standard-deviation shock to government spending: four variables



**Figure 6:** Impulse responses to a one-standard-deviation shock to government spending: six variables

Recursively identified SVAR models find a rise in private consumption in response to a shock to government spending. This is a contradiction between theoretical and empirical studies in the current literature. Because the recursively identified SVAR models have no

theoretical foundations, we build a New-Keynesian SVAR model to include forward-looking behaviours of households, firms, and the central bank. Thus, in the NK-SVAR model, rational-expectation restrictions are imposed on the equations of the New Keynesian Phillips curve, the Taylor-Rule monetary policy and the dynamic Euler equation.

### 2.4.2 Effects of Government Spending in NK-SVAR Models

This sub-section presents values of structural parameters estimated by the FIML estimation and impulse response functions to a government spending shock.

Columns two and three in Table 4 show the estimates of structural parameters and bootstrap standard errors in the NK-SVAR without an exclusion restriction on  $\alpha_{gy}$ . Columns four and five show the estimates in the NK-SVAR model by imposing the exclusion restriction on  $\alpha_{gy}$ . The initial values of ‘deep’ parameters are taken from Bekaert *et al.* (2010). Some structural parameters initially start at the estimates provided by Blanchard and Perotti (2002), and the rest are chosen arbitrarily.

Estimates of these parameters identified by the NK-SVAR model with  $\alpha_{gy} \neq 0$  have consistent signs with the estimates identified by the NK-SVAR model with  $\alpha_{gy} = 0$ . The estimates of  $\beta$  are not statistically significant, and the values are obviously smaller than the theoretical values, 0.99, used for quarterly data. As suggested by Leu (2011) for Australia, the  $\beta$  estimates indicate backward-looking inflation expectation in the U.S. firms. This indication is supported by Fuhrer (1997) who reveals the empirically insignificant expectations of future prices. In terms of the New Keynesian Phillips curve parameter, the sizes of  $\kappa$  under the two NK-SVAR identifications are greater than Bekaert *et al.*'s (2010) estimates, but they are not statistically significant. However, the insignificance of  $\kappa$  does not indicate the unimportance of output gap because our NK-SVAR models do not use output gap in the New Keynesian Phillips curve.<sup>6</sup> In addition to the NKPC, the Taylor-Rule monetary policy does not hold

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<sup>6</sup>We followed Leu (2011) to use the output gap generated by applying the HP filter on the U.S. real GDP in the NKPC. However, the shock to net taxes cannot generate decreases in private consumption, which is counterfactual.

when we use the long-term interest rate rather than the short-term interest rate used by Leu (2011). From the estimates of ‘deep’ parameters, the parameter of risk aversion,  $\sigma$ , has a statistically significant estimate at 1% level.<sup>7</sup> The significance of  $\sigma$  supports Perotti’s (2004a) argument that the long-term interest rate is a determinant of private consumption and is more important than the short-term interest rate.

Structural parameters in the NK-SVAR models can be identified without further exclusion restrictions because we only have five ‘deep’ parameters from the New Keynesian model. This is the advantage of introducing rational-expectation restrictions. In the SVAR model of Perotti (2004a), either  $c_{gt}$  or  $c_{tg}$  must be zero to meet the necessary condition of exact identification.<sup>8</sup> In our NK-SVAR model, these two parameters can be simultaneously identified, but the estimates are not statistically significant. The signs and significance of the two parameters are consistent with Blanchard and Perotti (2002). Moreover, Perotti (2004a) imposes exclusion restrictions on the automatic responses of government spending to inflation and interest rates,  $\alpha_{g\pi}$  and  $\alpha_{gr}$ , and on the automatic response of net taxes to interest rates,  $\alpha_{tr}$ . The two estimates of  $\alpha_{g\pi}$  identified in the NK-SVAR models are negative, but they are not statistically significant from zero. For the parameter  $\alpha_{gr}$ , the estimate from the model with  $\alpha_{gy} \neq 0$  is negative and not statistically significant; in addition, the estimate from the model with  $\alpha_{gy} = 0$  is negative but statistically significant at 5% level. The negative and significant  $\alpha_{gr}$  means that government spending negatively responds to a rise in interest rates; therefore, the U.S. government is willing to cooperate with the Federal Reserve. The estimates of  $\alpha_{tr}$  identified in the two NK-SVAR models are tiny and not statistically significant from zero.

Even though Perotti (2004a) does not impose an exclusion restriction on  $\alpha_{ty}$ , he uses the value computed by Blanchard and Perotti (2002) through institutional information about taxes and transfers. Taking the advantage of the NK-SVAR models,  $\alpha_{ty}$  can be estimated by

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<sup>7</sup>The estimate of  $\sigma$  has a reasonable value in the range of empirical estimates between 1 and 7 shown by Y. Chen and Liu (2018)

<sup>8</sup>From Blanchard and Perotti (2002),  $c_{gt} = 0$  indicates that government spending comes first and government spending does not respond to net taxes;  $c_{tg} = 0$  indicates net taxes come first and net taxes do not respond to government spending.

the FIML method. The  $\alpha_{ty}$  estimate is not statistically significant in the NK-SVAR model with  $\alpha_{gy} \neq 0$ , but is statistically significant at 10% level in the model with  $\alpha_{gy} = 0$ . The positive  $\alpha_{ty}$  is consistent with the value computed by Blanchard and Perotti (2002). The automatic response of real GDP to government spending,  $\gamma_{yg}$ , is positive and greater than one but not statistically significant from zero. Moreover, the automatic response of real GDP to net taxes,  $\gamma_{yt}$ , is negative but not statistically significant from zero.

**Table 4:** Estimates of Structural Parameters: Government Spending

Parameter	NK-SVAR: $\alpha_{gy} \neq 0$		NK-SVAR: $\alpha_{gy} = 0$	
	Estimates	Bootstrap standard error	Estimates	Bootstrap standard error
$\beta$	0.3257	(0.8933)	0.4097	(1.0319)
$\kappa$	0.0697	(0.0849)	0.0776	(0.0867)
$\phi_{\pi}$	1.8303	(2.2188)	2.2507	(1.4691)
$\phi_y$	0.1458	(0.1608)	0.1376	(0.2429)
$\sigma$	3.7276	(0.2862)	2.3033	(0.2946)
$c_{gt}$	-0.0583	(0.0396)	-0.0544	(0.0427)
$\alpha_{gy}$	0.0010	(0.0064)		
$\alpha_{g\pi}$	-0.2095	(0.7302)	-0.8796	(0.5440)
$\alpha_{gr}$	-0.6672	(0.6306)	-1.2221	(0.7110)
$c_{tg}$	-0.2394	(0.2276)	-0.2049	(0.1439)
$\alpha_{ty}$	2.5027	(1.6098)	2.8002	(1.5244)
$\alpha_{t\pi}$	1.0262	(1.2734)	0.4760	(1.2665)
$\alpha_{tr}$	0.0002	(0.0081)	0.0018	(0.0037)
$\gamma_{yg}$	1.3195	(0.9461)	1.1360	(1.1476)
$\gamma_{yt}$	-1.1346	(0.7394)	-0.9101	(0.6398)
$\sigma_g$	0.5629	(0.3432)	0.3169	(0.6516)
$\sigma_t$	1.0503	(0.3657)	1.1032	(0.4065)
$\sigma_y$	0.5634	(0.6065)	0.3174	(0.4223)
$\sigma_{\pi}$	0.1136	(0.3041)	0.0129	(0.2733)
$\sigma_r$	0.8896	(0.2725)	0.7913	(0.3355)
$\sigma_c$	0.1536	(0.2760)	0.0236	(0.5958)

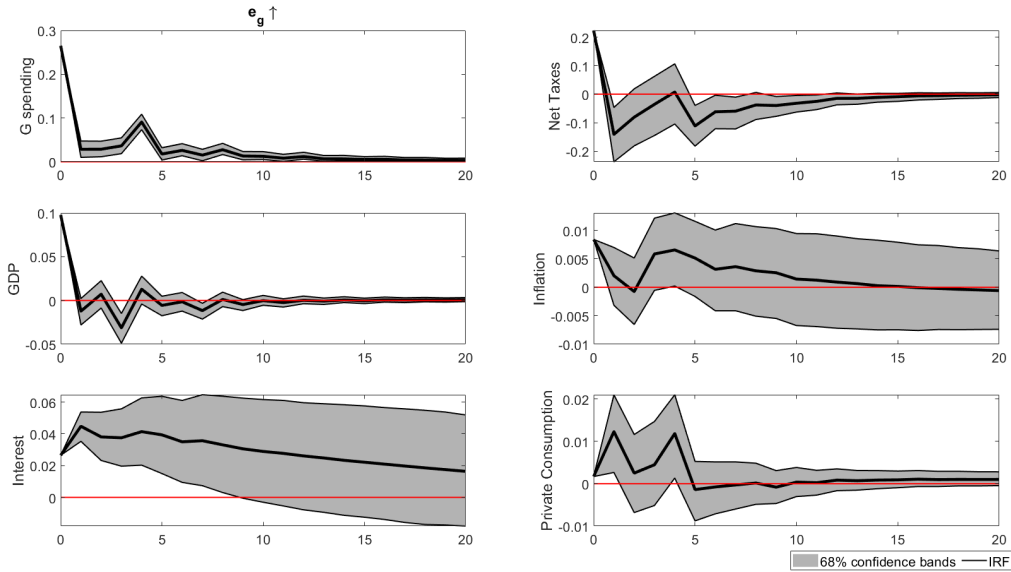
Due to the statistically insignificant estimate of  $\alpha_{gy}$ , we present the impulse response functions by following the assumption that government spending does not contemporaneously respond to real GDP.<sup>9</sup> Figure 7 presents impulse responses of the six variables to government

<sup>9</sup>Impulse response functions of the NK-SVAR model with  $\alpha_{gy} \neq 0$  are shown in the Appendix B. The qualitative responses are indifferent from those in the NK-SVAR model with  $\alpha_{gy} = 0$ .

spending shock. A positive shock to government spending increases both government spending and net taxes. The increase in government spending leads to increases in real GDP and private consumption. In a DSGE model estimated by Bayesian technique, Zubairy (2014) finds positive responses of real GDP and private consumption to government spending shock. Rich (2022) studies the effects of government spending on the U.S. output in a Bayesian VAR model, and finds that the positive responses of real GDP to a spending shock in all periods. Our paper shows that GDP growth initially increases, then falls below in the third period, and then converges to a steady state in the long run.

An increase in government spending also pushes inflation and the long-term nominal interest rate upward. As a result, an increase in private consumption indicates that the income effect is greater than the substitution effect induced by the increase in the interest rate. The impulse responses of government spending and real GDP are indifferent to the impulse responses from the Cholesky decomposition. However, net taxes and interest rates have stronger responses to government spending shock than the responses found by Cholesky decomposition. Also, the NK-SVAR model finds a rise in inflation in response to the spending shock, but Cholesky decomposition finds a reduction of inflation. Consequently, the increase in private consumption found by the NK-SVAR model is less than the increase found by Cholesky decomposition. Therefore, the dynamic IS curve should be included in the SVAR model to study the response of private consumption.

In this NK-SVAR model, government spending includes public consumption and investment. In addition, public consumption is considered as an increase in aggregate demand; however, public investment can increase the demand and improve production; thus it is productive. Next, our NK-SVAR model will only include public consumption to see how a rise in public consumption affects private consumption.



**Figure 7:** Impulse responses to a one-standard-deviation shock to government spending with  $\alpha_{gy} = 0$

### 2.4.3 Effects of Public Consumption in NK-SVAR Models

As we distinguish public consumption from public investment, we only consider public consumption in the NK-SVAR model without public investment. Table 5 presents the structural parameter estimates identified by both NK-SVAR models with  $\alpha_{gy} \neq 0$  and  $\alpha_{gy} = 0$ . In terms of ‘deep’ parameters, the estimate of parameter  $\kappa$  identified by the NK-SVAR model with  $\alpha_{gy} \neq 0$  is not statistically significant, but that identified by the NK-SVAR model with  $\alpha_{gy} = 0$  is statistically significant at 10% level. The estimate of  $\phi_y$  identified by the NK-SVAR model with  $\alpha_{gy} \neq 0$  is statistically significant at 5% level, and the estimate identified by the model with  $\alpha_{gy} = 0$  is statistically significant at 10% level.

We present the estimates of structural parameters that become statistically significant when we consider public consumption only. These estimates have the same signs as those that consider government spending. The NK-SVAR model with  $\alpha_{gy} = 0$  finds a statistically significant estimate of  $\alpha_{gc\pi}$  at 5% level. The estimate of  $\alpha_{gcr}$  from the NK-SVAR model with  $\alpha_{gy} = 0$  is statistically significant at 5% level. The estimates of  $\alpha_{ty}$  from both NK-SVAR models are statistically significant at 5% level. Also, the estimates of  $\gamma_{yt}$  identified by both

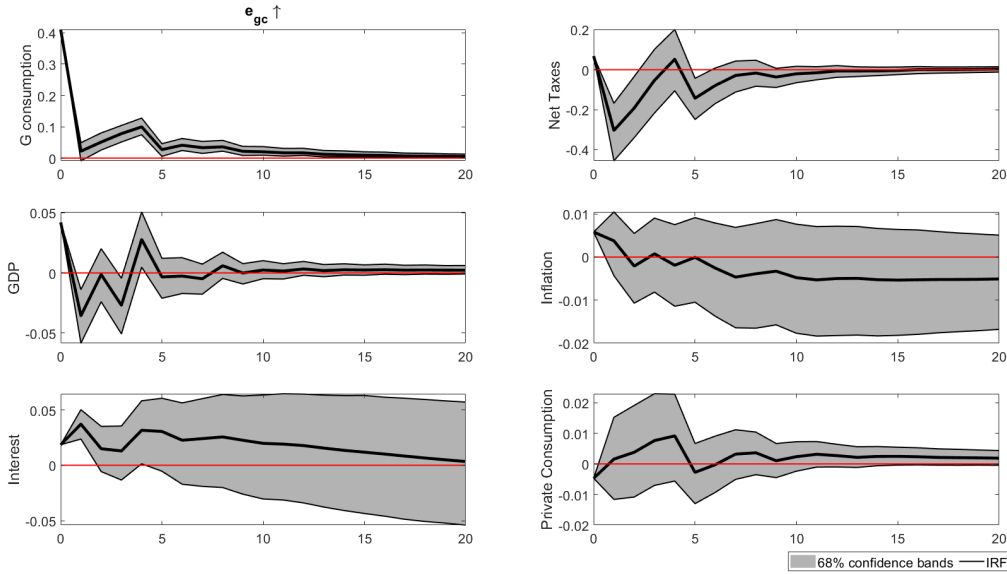
NK-SVAR models are statistically significant from zero at 5% level. In terms of  $\gamma_{ygc}$ , the estimates are smaller than 0, and not statistically significant.

**Table 5:** Estimates of Structural Parameters: Public Consumption Only

Parameter	NK-SVAR: $\alpha_{gy} \neq 0$		NK-SVAR: $\alpha_{gy} = 0$	
	Estimates	Bootstrap standard error	Estimates	Bootstrap standard error
$\beta$	0.3186	(1.0055)	0.4868	(0.8408)
$\kappa$	0.0765	(0.0599)	0.0943	(0.0550)
$\phi_\pi$	1.7740	(1.3285)	1.3070	(1.3552)
$\phi_y$	0.2246	(0.1138)	0.1845	(0.1115)
$\sigma$	2.7452	(0.2832)	2.3506	(0.2724)
$c_{gct}$	-0.0347	(0.0455)	-0.0160	(0.0387)
$\alpha_{gcy}$	0.0011	(0.0039)		
$\alpha_{gc\pi}$	-0.2905	(0.5302)	-1.4274	(0.6109)
$\alpha_{gcr}$	-0.5285	(0.6330)	-1.4230	(0.5853)
$c_{tgc}$	-0.2440	(0.1799)	-0.1868	(0.1433)
$\alpha_{ty}$	3.0306	(1.5351)	3.1950	(1.4269)
$\alpha_{t\pi}$	1.2805	(1.2532)	1.8628	(1.2884)
$\alpha_{tr}$	0.0014	(0.0048)	-0.0017	(0.0041)
$\gamma_{ygc}$	0.4406	(0.8385)	0.2887	(0.9075)
$\gamma_{yt}$	-1.3116	(0.6828)	-1.1231	(0.6271)
$\sigma_{gc}$	0.9105	(0.3431)	0.4465	(0.3667)
$\sigma_t$	0.2784	(0.3815)	1.2839	(0.3845)
$\sigma_y$	0.6701	(0.4153)	0.8173	(0.4103)
$\sigma_\pi$	0.0141	(0.2993)	0.0141	(0.2808)
$\sigma_r$	0.0400	(0.2932)	0.0316	(0.3165)
$\sigma_c$	0.2789	(0.2985)	0.4613	(0.3353)

With the assumption of  $\alpha_{gy} = 0$ , the NK-SVAR model obtains more statistically significant parameter estimates. Therefore, we present the impulse response functions identified by the NK-SVAR model with  $\alpha_{gy} = 0$ . Of course, we will show the impulse response functions identified by the model with  $\alpha_{gy} \neq 0$  in the Appendix B. Under both models, the qualitative responses are indifferent. With a shock to public consumption, the increase in public consumption raises real GDP but decreases private consumption, while the increase in aggregate demand induced by government purchases increases inflation and the interest rate. The estimate of  $\gamma_{ygc}$  that is less than one indicates that the real GDP increases by a smaller volume than the increasing volume in public consumption - such that increases in net taxes, inflation,

and the interest rate cause the reduction of private consumption. Therefore, with public consumption only, the NK-SVAR model imposing rational-expectation restrictions on households can find the negative response of private consumption that is predicted by the New Keynesian models.



**Figure 8:** Impulse responses to a one-standard-deviation shock to public consumption with  $\alpha_{gy} = 0$

### 2.4.4 Effects of Public Consumption and Investment in the NK-SVAR Model

To distinguish public investment from public consumption, we decompose government spending into the two expenditures in the NK-SVAR model. Because the number of unknown parameters is more than the distinct moments, we must impose exclusion restrictions on  $\alpha_{gcy}$  and  $\alpha_{gzy}$ . The two restrictions follow Blanchard and Perotti’s (2002) assumption that government spending does not respond to output within a quarter. Also, the statistically insignificant estimates of  $\alpha_{gy}$  and  $\alpha_{gcy}$  from the above identifications support this assumption.

For the estimates of ‘deep’ parameters,  $\beta$  does not differ much from the values found by previous NK-SVAR models with different spending variables. The estimate of  $\kappa$  is statistically significant from zero at 10% level, which is the same as the estimate identified by the model

only considering public consumption. Again, the estimate of  $\sigma$  is statistically significant from zero, which is consistent across the NK-SVAR models with three different spending variables.

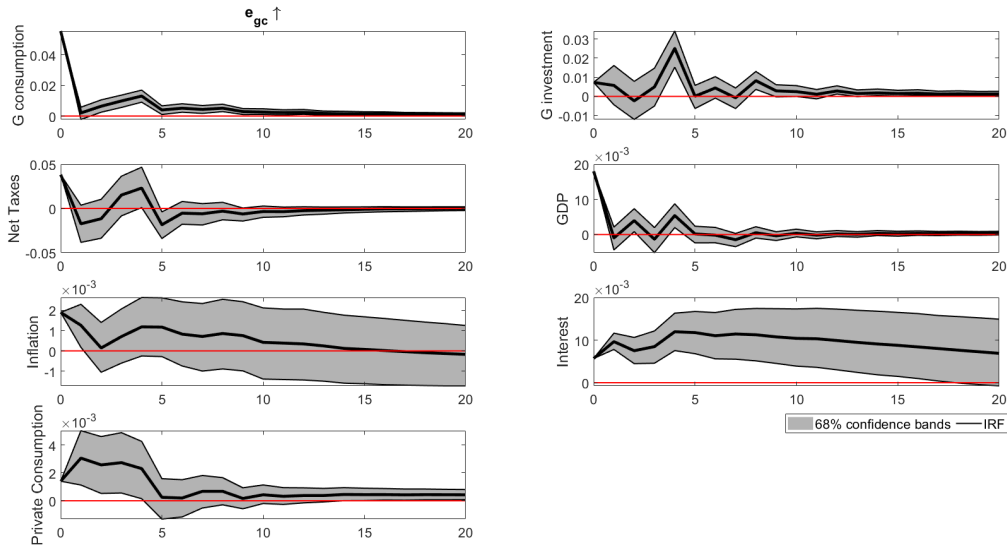
Important estimates of the structural parameters are the automatic responses of real GDP to different fiscal variables ( $\gamma_{ygc}$ ,  $\gamma_{ygz}$  and  $\gamma_{yt}$ ). The estimate of  $\gamma_{ygc}$  is smaller than one and not statistically significant from zero. This indicates that public consumption does not contribute to real GDP proportionally. Also, the insignificant  $\gamma_{ygc}$  implies that, if taxes increase, an increase in public consumption can decrease private consumption, because public consumption does not increase household income. The NK-SVAR model finds a statistically significant  $\gamma_{ygz}$  at 1% level, and the estimate is greater than one. The finding supports the theoretical assumption that public investment is more productive than public consumption to increase real GDP. As a result, we suggest that associated with a rise in public investment, an increase in public consumption probably increases private consumption. The estimate of  $\gamma_{yt}$  is negative and statistically significant at 1% level. Therefore, the net taxes negatively affect real GDP, which is consistent with the theoretical prediction.

We present impulse response functions to public consumption shock. Figure 9 presents the impulse responses of variables to a rise in public consumption. When a positive shock leads to a rise in public consumption, public investment has a positive response to the increase. In other words, we can say that increases in both public consumption and investment imply a rise in total government spending. There is no doubt that real GDP and net taxes rise. Even though the increase in net taxes mitigates the demand effect of the increase in public consumption, the positive shock to public consumption associated with the increased public investment causes an increase in private consumption. Because the automatic response of real GDP to public investment,  $\gamma_{ygz} = 2.1235$ , is greater than one, the increase in public consumption associated with the rise in net taxes leads to the increase in private consumption rather than reduction. In the NK-SVAR model with government spending, the automatic response of real GDP to government spending,  $\gamma_{ygg} = 1.1360$ , is greater than one, and private consumption increases in response to the rise in government spending. In terms of public

**Table 6:** Estimates of Structural Parameters: Public Consumption and Investment

Parameter	Estimates	Bootstrap standard error
$\beta$	0.3446	(0.6272)
$\kappa$	0.0811	(0.0445)
$\phi_\pi$	1.5473	(1.0177)
$\phi_y$	0.1042	(0.1050)
$\sigma$	2.6585	(0.2194)
$c_{gcz}$	0.0260	(0.0164)
$c_{gct}$	-0.0451	(0.0338)
$\alpha_{gc\pi}$	-0.0226	(0.0149)
$\alpha_{gcr}$	-0.8409	(0.5341)
$c_{gzc}$	0.2014	(0.4107)
$c_{gzt}$	-0.0587	(0.0349)
$\alpha_{gz\pi}$	-0.4997	(0.3746)
$\alpha_{g zr}$	-0.1239	(0.0454)
$c_{tgc}$	-0.2378	(0.1347)
$c_{tgz}$	-0.2063	(0.1380)
$\alpha_{ty}$	2.9069	(1.3735)
$\alpha_{t\pi}$	0.2657	(1.1839)
$\alpha_{tr}$	0.0070	(0.0033)
$\gamma_{ygc}$	0.8660	(0.6616)
$\gamma_{ygz}$	2.1235	(0.7340)
$\gamma_{yt}$	-1.1946	(0.4463)
$\sigma_{gc}$	0.0616	(0.3436)
$\sigma_{gz}$	0.5712	(0.3626)
$\sigma_t$	1.0094	(0.3857)
$\sigma_y$	0.4544	(0.1464)
$\sigma_\pi$	0.0141	(0.3718)
$\sigma_r$	0.2460	(0.0463)
$\sigma_c$	0.0245	(0.1683)

consumption only, the automatic response of real GDP to public consumption,  $\gamma_{ygc} = 0.2887$ , that is less than one finds a reduction of private consumption. Therefore, the automatic response of real GDP to fiscal variables is an important factor in determining the response of private consumption.



**Figure 9:** Impulse responses to a one-standard-deviation shock to public consumption

## 2.5 CONCLUSION

In this chapter, we analyzed the effects of shocks to different government expenditures using an empirically estimated New-Keynesian SVAR model, in which households, firms, and the central bank have rational and forward-looking expectations. We firstly developed an NK-SVAR model with government spending. Then we switched government spending to public consumption. Finally, we included both public consumption and public investment in the NK-SVAR model. Then, we derived NK-SVAR models with the contemporaneous relationship between the reduced-form residuals and structural disturbances. We imposed rational-expectation restrictions on the NK-SVAR model for identification by following Keating’s (1990) method. We estimated NK-SVAR models through the FIML method by maximizing the concentrated log-likelihood function subject to the non-linear constraints. Our results suggest that the estimates of structural parameters are invariant to different policy shocks. Therefore, our study is not subject to the Lucas critique.

We firstly analyzed the effects of a shock to government spending. Private consumption increases in response to a rise in government spending. The response of private consumption

is qualitatively indifferent from the response found by Cholesky decomposition. Since the total government spending includes public consumption and public investment, to isolate the productive effects of public investment, public consumption is the only fiscal expenditure in the second NK-SVAR model. In response to the increase in public consumption triggering an increase in net taxes, private consumption decreases. In order to find why a shock to government spending increases private consumption, we decomposed government spending into public consumption and public investment. The third NK-SVAR model reveals that an increase in public consumption leads to increases in both public investment and private consumption, even though net taxes rise.

Of course, with the evidence in the second NK-SVAR model with only public consumption, the finding that the rise in public consumption associated with the rise in public investment increases private consumption confirms that the increase in private consumption is caused by public investment instead of public consumption. In terms of the responses of private consumption to different fiscal expenditures, the automatic response of real GDP to government spending is greater than one, and the response to public investment is greater than government spending; however, the automatic response to public consumption is less than one. As a result, the automatic response of real GDP to fiscal expenditures determines the responses of private consumption.

Our study has addressed the contradiction of the private-consumption response to a public consumption shock in previous theoretical and empirical studies. To isolate the productive effects of public investment as a component of government spending, the empirical response of private consumption is consistent with the theoretical predictions. If a rise in public consumption increases public investment, rational households increase private consumption in response to the shock to public consumption. Therefore, our paper suggests that a study of non-productive government expenditure should combine with productive government expenditure, since the response of public investment to a rise in public consumption and the automatic response of output to public investment are both critical to the response of private

consumption.

Recent studies have provided another argument related to the effectiveness of government spending in boosting output. Our NK-SVAR results favour that public investment is more effective for the growth of output than public consumption, because of the productive nature of the public investment. For the argument that public consumption is more effective than public investment, our paper shows the opposite perspective, based on the value of the response of real GDP to public investment that is greater than two and the value for public consumption that is less than one.

Theoretical studies in fiscal policies in neoclassical and New-Keynesian models consider a change in government spending as an external shock; thus, government spending does not have a linear equation like the Taylor-Rule monetary policy, and the forward-looking behaviour has not been applied to government spending. Due to the fact that this is our first attempt using Keating's (1990) method to examine the effects of government spending, there is still much room for discussion regarding how to incorporate government spending into the NK-SVAR model with rational expectations. In future works, it would be also interesting to use survey data for expected next-period inflation and expected next-period private consumption.

# Chapter 3

## The Composition of Government Expenditures and Economic Growth: Evidence from the United States

### 3.1 INTRODUCTION

In this chapter, we are interested in understanding the effect on economic growth of the composition of government expenditures in the United States. Government spending is a vital component of sustainable economic expansion. Several studies, including Blanchard and Perotti (2002) and Ramey (2016), support the idea of the existence of a positive relationship between government spending and output. In a seminal study, Aschauer (1989b) contrasts productive government expenditures (public investment) with non-productive government expenditures (public consumption) - explaining that productive government expenditures have a more significant stimulative effect on economic activity.

As government spending is productive, Romer (2012) suggests that the government should not cut public capital spending because of fiscal consolidation. Scandizzo and Pierleoni (2020) provide an interesting review of both theoretical and empirical research, which show that public investment increases output through both demand and supply effects. In an empirical study, Perotti (2004b) finds in five OECD countries that, contrary to the characteristics of productive government spending, public investment is not more effective than public consumption in boosting output.

Previously, Mittnik and Neumann (2001) have discussed the possibility that public investment could be less productive than public consumption under the assumption of constant government expenditures. They introduce the ratio of public consumption to investment as a key explanatory variable for the output response, which means that a reduction in public investment finances an increase in public consumption. They explain why an increase in public investment has a negligible effect on output. In other words, excessive public investment relative to public consumption suggests that an increase in public investment has minimal impact on output. Even though public investment positively affects output, this does not necessarily imply that public investment contributes to economic growth more effectively than public consumption. In addition, the increase in public investment financed by a decrease in public consumption may hurt output due to diminishing returns to scale. They suggest that the effects of both public investment and public consumption on output are proportional to their relative sizes. If the ratio is above (below) its optimal level, a shift from public consumption to public investment has a positive (negative) effect on output.

In terms of output growth, in an endogenous growth model, Barro (1990) shows how public investment affects the rate of economic growth. He shows that the economic growth rate depends on the share of output used for public investment. If the share of public investment in output is below (or above) the optimal amount, it can aid (or hinder) output growth. Therefore, Devarajan *et al.* (1996) disagree with the prior studies' a priori assumption that public investment is more effective at accelerating economic growth than public consumption. They enable the data to reveal the more efficient components of government expenditures, with the proportion of each component in total government expenditures serving as the primary explanatory variable. When a government misallocates expenditures in favour of public consumption over public investment, public investment is more productive than public consumption - and increasing the proportion of total expenditures allocated to public investment increases GDP growth. Moreover, public consumption may be more productive than public investment if the government misallocates funds in favour of public investment. Simply

increasing the proportion of total expenditures allocated to public investment would hinder GDP growth.

Kneller *et al.* (1999), for a panel of 22 OECD countries, find that non-productive expenditure does not enhance growth, but productive expenditure does. This is an important result that mirrors the one in Devarajan *et al.* (1996), who find that increasing the share of public investment in total expenditures can positively impact GDP growth because developed countries misallocate public consumption over public investment. Conversely, in developing countries, they find that raising the share of public consumption in total expenditures is more productive than increasing the share of public investment, because developing countries have misallocated funds in favour of public investment.

By holding government spending constant in six developed countries, Mittnik and Neumann (2001) find that output depends on a ratio of public consumption to investment. If the country has a lower (higher) ratio than optimal, output can benefit from a shift from public investment (consumption) to consumption (investment). If a country's ratio is close to optimal, the output response to a spending shift could be negligible. Moreover, Devarajan *et al.* (1996) explain that GDP growth in a panel of 43 developing countries and a panel of 21 high-income countries depends on the share of each component in total government expenditures. To discuss the potential gain of GDP growth in a specific country, we use the United States as evidence because recent literature inadequately discusses the relationship between U.S. GDP growth and the composition of government expenditures. In this study, holding government spending constant, we examine the effect of a change in the ratio of public consumption to investment on the economic growth rate in the United States.

To study U.S. GDP growth, we analyze the U.S. quarterly time-series data. In a time-series analysis, the existence of unit roots and long-run relationships (cointegration) between variables can lead to spurious regressions and inappropriate models. Philips (2018) proposes using the Autoregressive Distributed Lag (ARDL) Model associated with the ARDL-bounds testing developed by Pesaran *et al.* (2001). Philips (2018) shows guidance in constructing the

ARDL model depending on the test results. We rely on the ARDL model to capitalize on the ARDL model's ability to accommodate a mixture of stationary ( $I(0)$ ) and first-order nonstationary ( $I(1)$ ) variables. To overcome the autocorrelation in residuals, the ARDL model allows incorporating lags of specific variables. This paper starts with the ARDL model, investigating the relationship between GDP growth and the ratio of total public consumption to investment at the national level.

Olson (1969) proposes a study on the different levels of government because economic theory provides the decision on the governmental action, but it does not indicate whether the function should be performed by a federal, state, or local government. In a study about fiscal decentralization and economic growth in the U.S. economy, Xie *et al.* (1999) decompose the consolidated government expenditures into federal, state and local levels. The previous studies motivate us to use expenditure data from different levels of government (federal, state and local) in the ARDL model.

If the effect of a rise in the public consumption-to-investment ratio on GDP growth is positive, the ratio is below the optimal level. It ensues that increasing public consumption by decreasing public investment would be beneficial for economic growth. If the effect is negative, the ratio exceeds the optimal level and the government should reduce public consumption to fund public investment. Finally, if GDP growth has a negligible response to a shift from public investment to consumption, the ratio of public consumption to investment is optimal for maximizing GDP growth; thus, no shift will be required between the two components. Additionally, the decomposition of total government expenditures into federal, state, and local levels can indicate whether a spending shift should be performed by the federal government or state and local governments.

The structure of the rest of the paper is as follows. The second section reviews the research on the effectiveness of productive and non-productive government expenditures. The third section describes the sources of the data. The fourth section describes the model specification in light of the unit-root tests, estimation results, and robustness. The last section concludes.

## 3.2 LITERATURE REVIEW

Government spending is an indispensable variable for the study of fiscal policy. Most studies examine the dynamic effects of government spending on economic activities, and government spending is generally regarded as an exogenous variable. The well-known paper by Blanchard and Perotti (2002) examines the effects of government spending on output. Ramey (2011) examines the impact of war on the anticipated increase in military expenditures. In addition to the aggregate demand effect of government spending, Aschauer (1989b) and Aschauer (1989a) highlight the supply-side effect of productive government spending (public investment) that has a positive output elasticity of public capital. Unlike public consumption, public investment has both demand and supply effects on output.

Several studies, following Aschauer (1989b) and Aschauer (1989a), have examined the impact of public investment on economic growth. If public capital stock is small, Ismihan and Ozkan (2011) argue that increasing public debt to finance public investment is a good strategy for stimulating economic growth because the productivity of additional public investment is high. C. Chen *et al.* (2017) conclude that the ratio of public investment to gross domestic product influences the effectiveness of public investment. After a certain point (the threshold), the effect of the ratio of public investment to GDP on output growth rate becomes negative. Additionally, C. Chen *et al.* (2017) demonstrate that exceeding the threshold of the public debt-to-GDP ratio has adverse effects on the growth rate. According to these findings, there are optimal ratios of both public investment to GDP and public debt to GDP for maximizing economic growth. Either exceeding the optimal ratio of public investment or that of public debt negatively impacts economic growth.

Kamiguchi and Tamai (2019) depict an inverted U-shaped relationship between the ratio of public debt to GDP and the growth rate of output. The right side of the curve suggests that the growth rate decreases as the public debt-to-GDP ratio increases. Devarajan *et al.* (1996) discussed the possibility of a nonlinear relationship between GDP growth and the

proportion of productive government expenditures within total government expenditures. A positive relationship exists when the share is low due to increasing returns to scale; a negative relationship exists when the share is high, due to decreasing returns to scale.

The effectiveness of public investment versus public consumption in increasing output or economic growth has been the subject of a heated debate in recent literature. There is no conclusive evidence that public investment is more effective than public consumption. Kneller *et al.* (1999) demonstrate that public consumption takes precedence over public investment in developed nations. Devarajan *et al.* (1996) demonstrate that developing nations favour public investment over public consumption. The latter study finds that an increase in the proportion of public investment to total government expenditures harms GDP growth, while an increase in the proportion of public consumption has a positive effect. They explain the situation in developing countries as a result of the misallocation of government funds toward public investment rather than public consumption. Consequently, excessive public investment hinders economic expansion. Devarajan *et al.* (1996) show that public consumption is not productive in developed countries because government spending is misallocated toward public consumption instead of public investment.

Based on the research of Devarajan *et al.* (1996), Mittnik and Neumann (2001) use quarterly data from developed countries and draw on the research of Devarajan *et al.* (1996) to conclude that a shift from public consumption to investment may or may not increase output, depending on the ratio of public consumption to investment. Thus, the ratio of public consumption to investment was taken into account. They present different output results in various countries. When a shift from public consumption to public investment has a negligible effect on output, the budget composition of government expenditures is optimal. A rise in output caused by a shift from public consumption to public investment suggests that public investment is more efficient than public consumption. A decrease in output caused by a shift from public consumption to public investment suggests that public investment is less efficient than public consumption.

In accordance with Devarajan *et al.* (1996), the proportion of public investment to total government expenditures and the proportion of public consumption to total government expenditures determine the response of GDP growth, and can serve as a guideline for allocating government expenditures. Mittnik and Neumann (2001) demonstrate that the ratio of public consumption to investment determines the output response to a change in the ratio of public consumption to public investment. Current studies examine the impact of the composition of government expenditure on GDP growth. Taking the U.S. economy as an example, if we observe a positive response of GDP growth to an increase in the relative size of public consumption to public investment, a shift from public investment to public consumption would be beneficial to economic growth.

Not only does current literature study government expenditures in general, but it also pays attention to the expenditures of state and local governments. Studies on state and local government expenditures explore the relationship between fiscal decentralization and economic development. Oates (1993) points out that the degree of fiscal decentralization in developed countries is higher than the degree in developing countries, suggesting that developing countries have higher potential gains from fiscal decentralization. Xie *et al.* (1999) analyze the efficiency of fiscal decentralization for the growth rate of the U.S. economy to address the fiscal debates on the allocation of expenditures among three levels of governments in the U.S. economy. We extend our analysis by considering not only the federal government but also the lower levels of government, which are the state and local governments.

Our study shows that the ratio of public consumption to investment has an impact on the effectiveness of government expenditures. The study offers an explanation for the claim that public consumption is more productive than public investment to boost growth.

### 3.3 DATA SOURCES

This chapter uses quarterly data in the United States from 1972Q1 to 2021Q4. We examine data from the United States after 1972, because President Nixon unilaterally ended the U.S. dollar's convertibility to gold. Shortly after that, in 1973, the Bretton Woods system ended. Therefore, this paper focuses on the post-Bretton U.S. economy.

We collected data for nominal GDP, implicit price deflators for GDP, total government expenditures, federal government consumption expenditures, gross federal government investment, state and local government consumption expenditures, gross state and local government investment, gross private domestic non-residential investment, and personal consumption expenditures on durable goods from the Bureau of Economic Analysis' National Income and Product Accounts (NIPA). The Federal Reserve Bank of St. Louis provides data on population and federal public debt as a percentage of GDP. The quarterly data are seasonally adjusted. National defense spending is part of the total government spending, but it is not part of the federal government's consumption or gross investments.

We compute the real GDP through the following formula:  $\text{real GDP} = \frac{\text{Nominal GDP}}{\text{GDP Deflator} \times 100}$ . Per-capita real GDP is the real GDP divided by population. According to Devarajan *et al.* (1996), we measure economic growth using the five-quarter forward-moving average of per-capita real GDP growth.

Federal government consumption expenditures are referred to as "federal public consumption," and federal gross government investment is referred to as "federal public investment." State and local government consumption expenditures are referred to as "state and local public consumption," and state and local gross government investment is referred to as "state and local public investment." Total public consumption is the sum of federal, state, and local public consumption, and total public investment is the sum of state and local public investment. Private investment is measured as the sum of gross private domestic non-residential investment and personal consumption expenditures on durable goods.

To study the effects of the relative size of public consumption to investment on economic growth, we measure the relative size of total public consumption to investment - using total public consumption as a percentage of total public investment ( $\frac{\text{total public consumption}}{\text{total public investment}} \times 100$ ), the relative size of federal public consumption to investment - using federal public consumption as a percentage of federal public investment, and the relative size of local and state public consumption to investment - using local and state public consumption as a percentage of local and state public investment.

According to Devarajan *et al.* (1996) and the assumption of a fixed size of government expenditures made by Mittnik and Neumann (2001), we control the effects of the size of total government expenditures on economic growth. Consequently, we measure the size of total government expenditures - using total government expenditures as a percentage of GDP. Also, we measure the size of private investment - using private investment as a percentage of GDP. According to C. Chen *et al.* (2017), the public debt-to-GDP ratio is a determinant of economic growth; thus, we measure the size of public debt by using public debt as a percentage of GDP.

### 3.4 EMPIRICAL ANALYSIS

To investigate how the relative size of total public consumption to investment affects economic growth in the U.S. economy, Equation (3.1) shows a linear model:

$$GRGDP = f(GC/GI, GSP/GDP, PD/GDP) \quad (3.1)$$

where the left-hand side variable,  $GRGDP$ , denotes the five-quarter forward-moving average of per-capita real GDP growth. As suggested by Devarajan *et al.* (1996), the selection of a five-quarter forward-lag structure can account for the endogeneity of GDP growth and expenditures, as well as the possibility of reverse causality.  $GRGDP$  can eliminate short-term fluctuations induced by a shift between public consumption and investment. On the

right-hand side,  $GC/GI$  denotes the relative size of total public consumption to investment,  $GSP/GDP$  denotes total government expenditures as a percentage of GDP, and  $PD/GDP$  denotes public debt as a percentage of GDP.

Because both the federal government and state and local governments manage the U.S. economy, not only do we study the growth effects of the relative size of total public consumption to investment, but we also have an interest in investigating the growth effects of the relative size of federal public consumption to investment and the relative size of state and local public consumption to investment. As a result, we specify an additional model, including expenditures of the federal government and state and local governments. The second model studies the growth effects of the relative size of federal public consumption to investment ( $FGC/FGI$ ), and the relative size of state and local public consumption to investment ( $SLGC/SLGI$ ) as below:

$$GRGDP = f(FGC/FGI, SLGC/SLGI, GSP/GDP, PD/GDP) \quad (3.2)$$

For the two models, we control the effects of the size of total government expenditures and the size of public debt. Due to the assumption of constant government expenditures, we add the size of public debt to each model to see whether the signs of the coefficients on the relative sizes and significance can be affected or not. We use the size of public debt to control the potential effect on real GDP growth, because Reinhart and Rogoff (2010) find that a high public debt-to-GDP ratio (90% and above) considerably lowers growth outcomes.

However, Herndon *et al.* (2014) blame the inaccurate relationship between public debt and GDP growth on coding errors and inappropriate weighting of summary statistics. They find that a rise in the public debt-to-GDP ratio increases GDP growth even though the ratio is above 90%. Nevertheless, Eberhardt and Presbitero (2015) find a negative relationship between public debt and economic growth across countries, but the threshold is not common to each country.

Following several empirical studies on government expenditures, such as Blanchard and Perotti (2002), Ramey (2011), and Corsetti, Meier, *et al.* (2012), we use quarterly data for government spending variables. Blanchard and Perotti (2002) explain why quarterly data is essential to identify fiscal shocks. The use of quarterly data can eliminate the direct effect of economic activity on fiscal policy, because the conduct of fiscal policy takes more than a quarter to learn about the unexpected event, pass the policy through the legislature, and finally implement it. Annual data cannot guarantee this evidence, because fiscal policy can respond to shocks to GDP within a year.

### 3.4.1 Unit Root Tests

Before we start regression analysis, we must subject these time series to unit-root tests. If dependent and independent variables are  $I(1)$ , we must apply the Johansen cointegration test. If the dependent variable is  $I(1)$  and independent variables are not higher than  $I(1)$ , we need to apply Pesaran *et al.*'s (2001) bounds cointegration test. Unit-root tests aim to determine whether a time-series variable is a random walk with a drift, has a deterministic trend, or both. As a result, we perform the Augmented Dickey-Fuller (ADF) unit-root tests for a random walk with a drift and a deterministic trend. After we find the unit roots for the specific variables, we will perform the unit-root tests with structural breaks.

#### **Augmented Dickey-Fuller (ADF) test:**

We start at the ADF unit-root test with the null hypothesis that a time-series variable is a random walk with a drift. Table 7 presents the ADF test results. We start at the lag order optimally chosen by Akaike Information Criterion (AIC) with the maximum lag order of eight. Following Benati (2014), because of the sensitivity of the unit-root testing results to the specified lag order, we consider the higher lag orders, up to four, if the AIC suggests that the lag order is lower than four.

For the five-quarter forward-moving average of per capita real GDP growth ( $GRGDP$ ),

the ADF unit-root test with four lags rejects the null hypothesis of a random walk with a drift. Therefore, the ADF unit-root test suggests that  $GRGDP$  is  $I(0)$ . Regarding the proportion of total public consumption in investment ( $GC/GI$ ), the ADF unit-root tests with three and four lags reject the null hypothesis of a random walk with a drift; thus, the ADF unit-root test suggests that  $GC/GI$  is  $I(0)$ . The ADF tests for the proportion of federal public consumption in investment ( $FGC/FGI$ ) with three lags and four lags cannot reject the null hypothesis of a random walk with a drift, but for the first difference, the ADF tests reject the null hypothesis. Therefore, the ADF test suggests that  $FGC/FGI$  is  $I(1)$ . When we perform the ADF test with four lags for the proportion of state and local public consumption in investment ( $SLGC/SLGI$ ), we reject the null hypothesis of a random walk with a drift. Thus, the ADF test suggests that  $SLGC/SLGI$  is  $I(0)$ . With four lags, the ADF test suggests that total government expenditures as a percentage of GDP ( $GSP/GDP$ ) is  $I(1)$ . The results of the ADF test with two, three and four lags for a random walk with a drift suggest that public debt as a percentage of GDP ( $PD/GDP$ ) is  $I(1)$ . Therefore, we need to perform the ADF test on a model with both a drift and a time trend. As a result, the ADF unit-root test of a random walk with a drift indicates that  $GRGDP$ ,  $GC/GI$ ,  $SLGC/SLGI$  and  $PI/GDP$  are stationary in a time-series model with a constant. In addition, the ADF test indicates that  $FGC/FGI$ ,  $GSP/GDP$  and  $PD/GDP$  are nonstationary in a model with a constant. Then, we perform the ADF unit-root test of a deterministic trend.

Table 8 presents the ADF unit-root test results with the null hypothesis that a time-series variable has a deterministic trend. The ADF test suggests that  $GRGDP$  is stationary in a model with a constant and a time trend. The ADF test for other variables in levels cannot reject the null hypothesis of a deterministic trend, but the test for these variables in first difference can reject the null hypothesis. Based on the above ADF unit-root tests, we can draw a conclusion for the time-series variables.  $GRGDP$  is stationary. All independent variables have a deterministic trend.  $FGC/FGI$ ,  $GSP/GDP$  and  $PD/GDP$  are random walks with a drift and have a deterministic trend. As a result, the unit-root results guide the

construction of a time-series regression model to take the first difference on the independent variables.

### **Unit-root tests with structural breaks:**

Perron (1989) points out that structural breaks may cause the conventional unit-root tests to find unit roots in time-series variables. Consequently, the variables could be stationary if the unit-root tests allow structural breaks. Due to the previous unit-root tests, we know that  $GRGDP$  is  $I(0)$ ; thus, we skip the unit-root test with structural breaks for  $GRGDP$ . As all independent variables exhibit a deterministic trend, the unit-root test that allows for structural breaks in trends is crucial. Because  $FGC/FGI$ ,  $GSP/GDP$  and  $PD/GDP$  are random walks with a drift and have a deterministic trend, we need to perform a unit-root test that allows for structural breaks in intercept and trend. To satisfy the conditions on structural breaks, we perform Zivot and Andrews's (2002) unit-root test to explore the evidence of the unit root by allowing the breaks in intercept, trend, or both. Table 9 presents the unit-root test results.

By allowing the break in trend, Zivot and Andrews's (2002) unit-root test cannot reject the null hypothesis that  $GC/GI$  is nonstationary. Therefore,  $GC/GI$  still has a unit root, even though we allow structural breaks in trend. When we allow for structural breaks in both intercept and trend, Zivot and Andrews's (2002) test cannot reject the null hypothesis that  $FGC/FGI$  is nonstationary. Zivot and Andrews's (2002) test, which allows for structural breaks in trend, shows that  $SLGC/SLGI$  is nonstationary. The Zivot and Andrews's (2002) test that allows for structural breaks in intercept and trend cannot reject the null hypothesis that  $GSP/GDP$  is nonstationary. For  $PI/GDP$ , the null hypothesis cannot be rejected by Zivot and Andrews's (2002) test that allows for structural breaks in trend. Finally, Zivot and Andrews's (2002) test allowing for structural breaks in trend suggests that  $PD/GDP$  is nonstationary. Therefore, Zivot and Andrews's (2002) test suggests that the unit-root results of these time-series variables are not due to structural breaks.

### 3.4.2 Autoregressive Distributed Lag (ARDL) Model

Based on the unit-root test results, some variables have stochastic and deterministic trends, and some have only a deterministic trend. Only  $GRGDP$  is stationary. Also, Zivot and Andrews's (2002) unit-root test allowing for structural breaks suggests that the independent variables are nonstationary when the test allows structural breaks to happen in intercept, trend, or both. As a result, we have both  $I(0)$  and  $I(1)$  time-series variables.

Shrestha and Bhatta (2018) present a guideline for employing an autoregressive distributed lag (ARDL) model according to the mix of  $I(0)$  and  $I(1)$  variables. We will show the ARDL model specification and estimation results. For the baseline model, we include  $GRGDP$  as a dependent variable, and  $D.GC/GI$ ,  $D.GSP/GDP$  and  $D.PD/GDP$  as independent variables. Taking the first difference on the independent variables aims to remove the random walk and deterministic trend. Therefore, the ARDL( $p, q, q, q$ ) Model 1.1 is as below:

$$\begin{aligned}
 GRGDP_{(t+1,t+5)} = c_0 + \sum_{i=1}^p \phi_i GRGDP_{(t+1-i,t+5-i)} + \sum_{j=0}^q \alpha_j D.GC/GI_{t-j} \\
 + \sum_{j=0}^q \beta_{1j} D.GSP/GDP_{t-j} + \sum_{j=0}^q \beta_{2j} D.PD/GDP_{t-j} + u_t
 \end{aligned} \tag{3.3}$$

where  $c_0$  denotes a constant term.  $GRGDP_{(t+1,t+5)}$  denotes the dependent variable, which is the five-quarter forward-moving averages of per capita real GDP growth, and  $\phi_i$  denotes the parameters of the past values of the dependent variable,  $i = 1, 2, \dots, p$ .  $D.GC/GI_t$  denotes the growth of relative size of total public consumption to public investment (proportion of public consumption in public investment in first difference) at time  $t$ , and  $\alpha_j, j = 0, 1, 2, \dots, q$ , denotes the parameters on the current and past values of  $D.GC/GI_{t-j}$ .  $D.GSP/GDP_t$  is the first difference of total government expenditures as a percentage of GDP at time  $t$ , and  $\beta_{1j}$  denotes the parameters on the current and past values of  $D.GSP/GDP_{t-j}$ .  $D.PD/GDP_t$  is the first difference of public debt as a percentage of GDP at time  $t$ , and  $\beta_{2j}$  denotes the parameters on the current and past values of  $D.PD/GDP_{t-j}$ .  $u_t \sim N(0, \sigma^2)$  denotes the regression residuals in the ARDL model.

Based on the baseline model addressing the relationship between economic growth and the relative size of total public consumption and investment, ARDL Model 1.2 introduces the first difference of private investment as a percentage of GDP,  $D.PI/GDP$ , to control its contribution to GDP growth. The ARDL( $p, q, q, q, q$ ) Model 1.2 is described below:

$$\begin{aligned}
GRGDP_{(t+1,t+5)} = & c_0 + \sum_{i=1}^p \phi_i GRGDP_{(t+1-i,t+5-i)} + \sum_{j=0}^q \alpha_j D.GC/GI_{t-j} \\
& + \sum_{j=0}^q \beta_{1j} D.GSP/GDP_{t-j} + \sum_{j=0}^q \beta_{2j} D.PD/GDP_{t-j} + \sum_{j=0}^q \beta_{3j} D.PI/GDP_{t-j} + u_t
\end{aligned} \tag{3.4}$$

where  $D.PI/GDP_t$  denotes growth of private investment as a percentage of GDP at time  $t$ , and  $\beta_{3j}$  represents the parameters on current and past values of  $D.PI/GDP_{t-j}$ .

In ARDL Model 2.1, Equation (3.5), we decompose total government expenditures into federal, state and local government expenditures. As a result, we have four government-expenditure variables: federal public consumption, federal public investment, state and local public consumption, and state and local public investment. This model investigates the relationships between economic growth and the relative size of federal public consumption to investment and between economic growth and the relative size of state and local public consumption to investment. Including the two new variables, the ARDL( $p, q, q, q, q$ ) Model 2.1 is presented below:

$$\begin{aligned}
GRGDP_{(t+1,t+5)} = & c_0 + \sum_{i=1}^p \phi_i GRGDP_{(t+1-i,t+5-i)} + \sum_{j=0}^q \alpha_{1j} D.FGC/FGI_{t-j} \\
& + \sum_{j=0}^q \alpha_{2j} D.SLGC/SLGI_{t-j} + \sum_{j=0}^q \beta_{1j} D.GSP/GDP_{t-j} \\
& + \sum_{j=0}^q \beta_{2j} D.PD/GDP_{t-j} + u_t
\end{aligned} \tag{3.5}$$

where  $D.FGC/FGI_t$  is the first difference of federal public consumption as a percentage of federal public investment at time  $t$ , and  $\alpha_{1j}$  represents the parameters on the current and

past values of  $D.FGC/FGI_{t-j}$ .  $D.SLGC/SLGI_{t-j}$  denotes the first difference of state and local public consumption as a percentage of public investment at time  $t$ , and  $\alpha_{2j}$  denotes the parameters on the current and past values of  $D.SLGC/SLGI_{t-j}$ .

Model 2.2 is extended by adding growth of private investment as a percentage of GDP. The ARDL( $p, q, q, q, q, q$ ) Model 2.2 is presented below:

$$\begin{aligned}
 GRGDP_{(t+1,t+5)} = & c_0 + \sum_{i=1}^p \phi_i GRGDP_{(t+1-i,t+5-i)} + \sum_{j=0}^q \alpha_{1j} D.FGC/FGI_{t-j} \\
 & + \sum_{j=0}^q \alpha_{2j} D.SLGC/SLGI_{t-j} + \sum_{j=0}^q \beta_{1j} D.GSP/GDP_{t-j} \\
 & + \sum_{j=0}^q \beta_{2j} D.PD/GDP_{t-j} + \sum_{j=0}^q \beta_{3j} D.PI/GDP_{t-j} + u_t
 \end{aligned} \tag{3.6}$$

We will show the ARDL estimation results and present post-estimation tests for autocorrelation, heteroskedasticity and model stability in the next section.

### 3.4.3 Estimation Analysis of ARDL models

#### Baseline ARDL model - general government:

First, we perform the ARDL(4, 3, 0, 1) Model 1.1, including an unrestricted constant and the optimal lag length suggested by AIC. We perform Bartlett's periodogram-based test for the white noise on the error terms in the ARDL(4, 3, 0, 1) Model 1.1. Bartlett's test cannot reject the null hypothesis that data come from a white process of uncorrelated random variables with a constant mean and constant variance. White's test for homoskedasticity against unrestricted forms of heteroskedasticity cannot reject the null hypothesis of homoskedasticity. Both the Breusch-Godfrey LM test and Durbin's alternative test cannot reject the null hypothesis of no serial correlation for autocorrelations with up to four lags.<sup>1</sup>

<sup>1</sup>To make sure that five-period forward-moving average is sufficient, we use either seven-period or ten-period forward-moving averages. The results have the same signs on coefficients and have close estimates to the results of the five-period average.

We test the parameter stability and structural breaks due to a change in regression coefficients over time, following the ARDL(4, 3, 0, 1) regression. We show the test results by performing the CUSUM tests from the recursive residuals introduced by Brown *et al.* (1975) and OLS residuals introduced by Ploberger and Krämer (1992). Both the CUSUM tests for recursive and OLS residuals have the null hypothesis that all parameters are stable over time. Ploberger and Krämer (1992) show that the CUSUM test based on recursive residuals can more effectively detect parameter instability occurring early in the sample than the CUSUM test based on OLS residuals. In contrast to the CUSUM test based on recursive residuals, the CUSUM test based on OLS residuals can better detect parameter instability for breaks occurring later in the sample. The null hypothesis of stable coefficients cannot be rejected by the CUSUM test based on recursive residuals, or the CUSUM test based on OLS residuals. As shown in Table 12, the two test statistics are smaller than the critical values at the 10% significance level.

In Table 10, the column for Model 1.1 shows the estimation results in the ARDL(4, 3, 0, 1) Model. Table 10 shows the current effects on the five-quarter forward-moving average of per capita real GDP growth. The entire estimation results are presented in Table C.1 in the Appendix C. From the estimation of the ARDL(4, 3, 0, 1) Model, we find that growth of the relative size of total public consumption to public investment,  $D.GC/GI_t$ , has a positive effect on GDP growth. The statistically significant and positive coefficient on  $D.GC/GI_t$  indicates that the current increase in total public consumption financed by a current reduction in public investment can prompt the five-quarter forward-moving average of GDP growth. Because the value of the coefficient on  $D.GC/GI_t$  is close to zero, the size of the coefficient has no economic importance for GDP growth.

Under the assumption of constant government spending, Mittnik and Neumann (2001) demonstrate that output has a negligible response to a shift from public consumption to public investment if the composition of government expenditures is optimal. As a result, the significant and positive coefficient in the ARDL(4, 3, 0, 1) Model 1.1 indicates that the relative

size of total public consumption to public investment is not optimal to maximize the average GDP growth in the next five quarters. This finding suggests that the government's budget should favour public consumption and reduce public investment. In other words, Devarajan *et al.* (1996) can explain this situation where public investment is too excessive to boost GDP growth. Consequently, a rise in public investment financed by a reduction in public consumption could lead to a decrease in GDP growth.

For other regressors in the ARDL(4, 3, 0, 1) Model 1.1, the statistically significant and positive coefficient on the first difference of total government expenditures as a percentage of GDP shows that government expenditures are a successful tool to stimulate economic growth. We also find a statistically significant and positive coefficient on growth of public debt as a percentage of GDP. Therefore, a rise in the size of public debt can increase the average GDP growth in the next five quarters.

Blackburne and Frank (2007) present the general formula to calculate the long-run parameter; thus, the long-run parameter of  $D.GC/GI$  is:

$$\alpha_{LR} = \frac{\sum_{j=0}^q \alpha_j}{1 - \sum_{i=1}^p \phi_i} \quad (3.7)$$

The long-run coefficients are shown in Table 11. The coefficient on the relative size of total public consumption to investment is positive and statistically significant at 10%; however, the size has no economic importance. Growth of government expenditures as a percentage of GDP has a positive and statistically significant effect on GDP growth at 10%. The effect of public debt as a percentage of GDP is not statistically significant.

The positive short-run effect and negligible long-run effect of the public debt-to-GDP ratio on economic growth differs from Reinhart and Rogoff's (2010) finding that a high level of public debt is negatively associated with growth. However, Herndon *et al.* (2014) find evidence supporting the positive relationship by addressing the flaws in Reinhart and Rogoff (2010), such as the selective exclusion of available data, coding errors, and inappropriate weighting of

summary statistics. Kourtellos *et al.* (2013) explain the different effects of higher public debt on economic growth in multiple regimes. The latter study shows that the negative growth effect of increasing public debt only occurs in less-democracy countries. Thus, in the U.S., a rise in public debt has only a short-run effect.

In Table 10, the column for Model 1.2 shows the estimation results and post-estimation tests of the ARDL(4, 3, 0, 1, 2) Model, by adding a growth of private investment as a percentage of GDP. Bartlett's periodogram-based test for the white noise on the error terms in the ARDL(4, 3, 0, 1, 2) Model 1.2 cannot reject the null hypothesis that data come from a white process of uncorrelated random variables having a constant mean and a constant variance. White's test for homoskedasticity against unrestricted forms of heteroskedasticity cannot reject the null hypothesis of homoskedasticity. For autocorrelation up to four lags, the Breusch-Godfrey LM test and Durbin's alternative test cannot reject the null hypothesis of no serial correlation. The null hypothesis of stable coefficients cannot be rejected by both the CUSUM test based on recursive residuals and the CUSUM test based on OLS residuals. As shown in Table 13, the two test statistics are smaller than the critical values at the 10% significance level.

The introduction of growth of private investment as a percentage of GDP does not change signs and significance of the coefficients on  $D.GC/GI_t$ ,  $D.GSP/GDP_t$  and  $D.PD/GDP_t$ . The coefficient on  $D.PI/GDP_t$  is not statistically significant and negative. The model results show that an increase in total public consumption through a reduction of total public investment at time  $t$  can increase the five-quarter forward-moving average GDP growth. As a result, the budget composition of total government expenditure is not optimal for maximizing GDP growth. However, the size of the coefficient of  $D.GC/GI_t$  has no economic importance.

Using the Blackburne and Frank's (2007) formula, we can compute the long-run coefficients. The long-run coefficients are shown in Table 11. The long-run coefficient of  $D.GC/GI$  is positive and statistically significant, but not economically important. The long-run effect of growth of government expenditures as a percentage of GDP is positive and statistically

significant, and the coefficient is greater than 1. One percent increase in growth of government expenditures as a percentage of GDP can increase average GDP growth by 3.02%. The coefficients of  $D.PD/GDP$  and  $D.PI/GDP$  are not statistically significant.

The U.S. economy is governed by both the federal government and state and local governments. At this point, we are unable to make any recommendations as to which government should adjust the budget composition of government expenditures. As a result, we will decompose total public consumption and investment into federal public consumption and investment, and state and local public consumption and investment. Then, we can explore the more complicated relationship between GDP growth and the relative sizes of public consumption to public investment for different levels of government.

#### **Extended ARDL Model - federal government and state and local government:**

The column of Model 2.1 in Table 14 shows the estimation and post-estimation tests for the ARDL(4, 0, 3, 0, 1) Model. This ARDL estimation explores the relationships between the five-quarter forward-moving average of GDP growth and the first difference in the proportion of federal public consumption in public investment, and between GDP growth and growth of the proportion of state and local public consumption in public investment. Bartlett's periodogram-based test for the white noise on the error terms in the ARDL(4, 0, 3, 0, 1) Model 2.1 cannot reject the null hypothesis that data come from a white process of uncorrelated random variables having a constant mean and a constant variance. White's test for homoskedasticity against unrestricted forms of heteroskedasticity cannot reject the null hypothesis of homoskedasticity. For autocorrelation up to four lags, the Breusch-Godfrey LM test and Durbin's alternative test cannot reject the null hypothesis of no serial correlation. The null hypothesis of stable coefficients cannot be rejected by both the CUSUM test based on recursive residuals and the CUSUM test based on OLS residuals. As suggested by the results in Table 14, the two test statistics are smaller than the critical values at the 10% significance level.

The coefficient on the first difference of total government expenditures as a percentage

of GDP at time  $t$  is statistically significant and positive, which is indifferent from the baseline ARDL(4, 3, 0, 1) Model, including total public consumption and investment. The coefficient on growth of public debt as a percentage of GDP at time  $t$  is positive and statistically significant at 1% level. The  $D.GSP/GDP_t$  coefficient in Model 2.1 becomes higher than that in the baseline model, but the value of the  $D.PD/GDP_t$  coefficient becomes lower than that in the baseline model.

The first difference in federal public consumption as a percentage of public investment does not have a statistically significant effect on the five-quarter forward-moving average GDP growth at time  $t$ . The ARDL(4, 0, 3, 0, 1) Model finds that a marginal change in the proportion of federal public consumption in public investment has no current effect on GDP growth at time  $t$ . The coefficient on the proportion of state and local public consumption in public investment is statistically significant and positive at time  $t$ . This observation indicates that growth of  $SLGC/SLGI$  at time  $t$  has a statistically significant and positive effect on GDP growth; however, the magnitude of the  $D.SLGC/SLGI_t$  coefficient is not economically significant.

Long-run coefficients can be computed through Blackburne and Frank's (2007) formula. Table 15 shows the long-run coefficients of the ARDL(4, 0, 3, 0, 1) Model. The long-run coefficient of growth of the proportion of federal public consumption in investment is negative and not statistically significant. In the long run, growth of proportion of state and local public consumption in investment positively and significantly affects GDP growth at 10%. One percentage-point rise in growth of the proportion of state and local public consumption in investment increases GDP growth by 0.08 percentage points. Therefore, we note that a change in the proportion of federal public consumption in public investment has a negligible effect on GDP growth. Consequently, due to the study of Mitnik and Neumann (2001), under the assumption of a constant government spending, the relative size of federal public consumption to public investment is optimal to maximize the average GDP growth in the long-run. At the 10% confidence level, the positive long-run coefficient of  $D.SLGC/SLGI$  suggests that the budget composition of state and local government expenditures is not optimal, and a shift

from state and local public investment to public consumption can increase the long-run GDP growth.

In Table 14, the column from Model 2.2 presents the estimation results and post-estimation tests for the ARDL(4, 0, 3, 0, 1, 2) Model. It adds private investment as a percentage of GDP in first difference,  $D.PI/GDP$ , from Model 2.1. Bartlett's periodogram-based test for the white noise on the error terms in the ARDL(4, 0, 3, 0, 1, 2) Model 2.2 cannot reject the null hypothesis that data come from a white process of uncorrelated random variables having a constant mean and a constant variance. White's test for homoskedasticity against unrestricted forms of heteroskedasticity cannot reject the null hypothesis of homoskedasticity. For autocorrelation up to four lags, the Breusch-Godfrey LM test and Durbin's alternative test cannot reject the null hypothesis of no serial correlation. Both the CUSUM test based on recursive residuals and the CUSUM test based on OLS residuals cannot reject the null hypothesis of stable coefficients because, in Table 17, the two test statistics are smaller than the critical values at the 10% significance level.

The  $D.SLGC/SLGI_t$  coefficient is statistically significant and positive, and the coefficient has a slightly lower value than that in Model 2.1. Growth of private investment as a percentage of GDP does not significantly affect the five-quarter forward-moving average GDP growth at time  $t$ . In terms of long-run effects, Table 15 shows a positive and statistically significant coefficient of  $D.SLGC/SLGI$ . Growth of government expenditures as a percentage of GDP has a long-run and significantly positive effect on GDP growth at 5%. Model 2.2 backs up the results of Model 2.1, in which state and local governments can shift from public investment to public consumption for boosting GDP growth. This finding indicates that state and local governments do not have an optimal ratio of public consumption to investment, and a rise in public investment financed by a reduction in public consumption would restrain the average GDP growth in the long-run.

Gechert's (2015) meta-analysis finds that public-investment multipliers are greater than public-consumption multipliers. Under the assumption of constant government spending, the

reason why public investment is more productive than public consumption is that the ratio of public consumption to investment is above the optimal ratio. Therefore, our results do not contradict Gechert's (2015) findings.

### 3.4.4 Robustness

We review the robustness of the results in the ARDL Models to ensure that the relationships between the five-quarter forward-moving average GDP growth and the relative size of public consumption to public investment found in the ARDL Models are not a statistical coincidence. Therefore, we conduct simple linear regression models including a dependent variable, a lagged dependent variable, and independent variables. The first simple linear model has the same time-series variables as the baseline Model 1.1, as shown below:

$$\begin{aligned} GRGDP_{(t+1,t+5)} = a_0 + \alpha GRGDP_{(t,t+4)} \\ + \beta_1 D.GSP/GDP_t + \beta_2 D.PD/GDP_t + \beta_3 D.GC/GI_t + e_t \end{aligned} \tag{3.8}$$

where  $a_0$  denotes a constant and  $\alpha$  denotes a parameter for the lagged value of the five-quarter forward-moving average GDP growth.  $\beta_i$  represents the parameters on the current values of regressors.  $e_t$  is the i.i.d residuals, so we need to test serial correlation for the regression disturbances.

In Table 18, the column of Model 3.1 shows the OLS estimation results and post-estimation tests for Equation (3.8). Bartlett's periodogram-based test for the white noise on the error terms in Model 3.1 cannot reject the null hypothesis that data come from a white process of uncorrelated random variables having a constant mean and a constant variance. White's test for homoskedasticity against unrestricted forms of heteroskedasticity rejects the null hypothesis of homoskedasticity. For autocorrelation up to two lags, the Breusch-Godfrey LM test and Durbin's alternative test cannot reject the null hypothesis of no serial correlation at 5% significance level.

OLS estimation in Model 3.1 finds a statistically significant and positive coefficient on  $D.GC/GI_t$ , and the coefficient size has no economic significance. In Model 3.1, the coefficients of  $D.GSP/GDP_t$  and  $D.PD/GDP_t$  are statistically significant and positive. The results in Model 3.1 are consistent with those of the ARDL(4, 3, 0, 1) Model 1.1.

Adding growth of private investment as a percentage of GDP based on Model 3.1 generates Model 3.2:

$$\begin{aligned} GRGDP_{(t+1,t+5)} = & a_0 + \alpha GRGDP_{(t,t+4)} \\ & + \beta_1 D.GSP/GDP_t + \beta_2 D.PD/GDP_t + \beta_3 D.GC/GI_t + \beta_4 D.PI/GDP + e_t \end{aligned} \tag{3.9}$$

In Table 18, the column of Model 3.2 shows the OLS estimation results and post-estimation tests for Equation (3.9). Bartlett's periodogram-based test for the white noise on the error terms in Model 3.2 cannot reject the null hypothesis that data come from a white process of uncorrelated random variables having a constant mean and a constant variance. White's test for homoskedasticity against unrestricted forms of heteroskedasticity cannot reject the null hypothesis of homoskedasticity at 5% significance level. Both the Breusch-Godfrey LM test and Durbin's alternative test cannot reject the null hypothesis of no serial correlation for autocorrelations with up to two lags.

In Model 3.2, the statistically significant but economically insignificant coefficient on  $D.GC/GI_t$  is consistent with that of ARDL(4, 3, 0, 1, 2) Model 1.2. Model 3.2 supports our finding in the ARDL Model 1.2. In Model 3.2, the coefficients of  $D.GSP/GDP_t$  and  $D.PD/GDP_t$  are statistically significant and positive, and the coefficient of  $D.PI/GDP_t$  is not statistically significant. They are consistent with the findings of the ARDL Model 1.2.

To investigate the robustness of the results in the ARDL(4, 0, 3, 0, 1) Model 2.1, we decompose total public consumption into federal public consumption and state and local public consumption, and decompose total public investment into federal public investment and state

and local public investment. We perform the OLS estimation on a linear model, shown below:

$$\begin{aligned} GRGDP_{(t+1,t+5)} = & a_0 + \alpha GRGDP_{(t,t+4)} + \beta_1 D.GSP/GDP_t \\ & + \beta_2 D.PD/GDP_t + \beta_3 D.FGC/FGI_t + \beta_4 D.SLGC/SLGI + e_t \end{aligned} \quad (3.10)$$

In Table 19, the column of Model 4.1 shows the OLS estimation results and post-estimation tests for Equation (3.10).

Bartlett's periodogram-based test for the white noise on the error terms in Model 4.1 cannot reject the null hypothesis that data come from a white process of uncorrelated random variables with a constant mean and a constant variance. White's test for homoskedasticity against unrestricted forms of heteroskedasticity cannot reject the null hypothesis of homoskedasticity at 5% significance level. At 5% significance level, neither the Breusch-Godfrey LM test nor Durbin's alternative test can reject the null hypothesis of no serial correlation when there is autocorrelation with up to two lags.

OLS estimation in Model 4.1 finds a statistically significant and positive coefficient on  $D.SLGC/SLGI_t$ , and the value is lower than that found by the ARDL(4, 0, 3, 0, 1) Model 2.1. Model 4.1 supports our finding in the ARDL Model 2.1 that the budget composition of state and local government expenditures is not optimal for maximizing the average GDP growth in the next five quarters, and a shift from state and local public investment to public consumption can increase GDP growth. The coefficient of  $D.FGC/FGI_t$  is not statistically significant. In Model 4.1, the coefficients of  $D.GSP/GDP_t$  and  $D.PD/GDP_t$  are statistically significant and positive, which are consistent with the ARDL Model 2.1.

From Model 4.1, we introduce private investment as a percentage of GDP in first difference to construct Model 4.2, shown below:

$$\begin{aligned} GRGDP_{(t+1,t+5)} = & a_0 + \alpha GRGDP_{(t,t+4)} + \beta_1 D.GSP/GDP_t + \beta_2 D.PD/GDP_t \\ & + \beta_3 D.FGC/FGI_t + \beta_4 D.SLGC/SLGI + \beta_5 D.PI/GDP + e_t \end{aligned} \quad (3.11)$$

In Table 19, the column for Model 4.2 shows the OLS estimation results and post-estimation tests for Equation (3.11). Bartlett's periodogram-based test for the white noise on the error terms in Model 4.2 cannot reject the null hypothesis that data comes from a white process of uncorrelated random variables having a constant mean and a constant variance. White's test for homoskedasticity against unrestricted forms of heteroskedasticity cannot reject the null hypothesis of homoskedasticity. For autocorrelation with up to two lags, the Breusch-Godfrey LM test and Durbin's alternative test cannot reject the null hypothesis of no serial correlation at 5% significance level.

In Model 4.2, the statistically significant and positive coefficient on  $D.SLGC/SLGI_t$  supports our finding in the ARDL Model 2.2. Therefore, to prompt the five-quarter forward-moving average GDP growth, we suggest that state and local government could reduce public investment to finance a rise in public consumption. In Model 4.2, the coefficients of  $D.GSP/GDP$  and  $D.PD/GDP$  are statistically significant and positive. Coefficients on  $D.FGC/FGI$  and  $D.PI/GDP$  are not statistically significant. The results in Model 4.2 are consistent with the findings in the ARDL Model 2.2.

### 3.5 CONCLUSION

In this study, the relationship between real GDP growth and the ratio of public consumption to investment has been investigated with ARDL models using quarterly time-series data. We have separated general government expenditures into two categories: federal government expenditures and state and local government expenditures. In the baseline ARDL model, the effect of a shift from total public investment to public consumption on GDP growth is examined first. The extended ARDL model includes both the ratio of federal public consumption to public investment and the ratio of state and local public consumption to public investment.

In the baseline model, we found that a shift from total public investment to public consumption had a positive impact on GDP growth. Therefore, the budget composition of

total government expenditures is not optimal for GDP growth maximization. The ratio of total public consumption to public investment is below the optimal level. Reducing the ratio of total public investment to total public consumption can stimulate economic growth. The results of the baseline ARDL model are maintained when we look at a simple linear model with lagged GDP growth as an independent variable.

In the extended version of the ARDL model in which government expenditures are differentiated by levels of government, the ratio of federal public consumption to investment has no effect on GDP growth. However, we have found a positive response of GDP growth to a shift from government consumption to investment at the state and local levels. This shows that the ratio of public consumption to investment at the state and local levels is not optimal. State and local governments can boost GDP growth by transferring spending from public investment to public consumption. Finally, since a change in the ratio of state and local public consumption to investment has the greatest effect on GDP growth, future studies could use panel data from state and local governments in the United States to investigate the relationship between GDP growth and the ratio of public consumption to investment.

**Table 7:** Augmented Dicky-Fuller Unit-Root Test - Random Walk with a Drift

Variable	Lags (AIC)	Test statistics	Critical value 5%	Null of non-stationarity
Level				
GRGDP	4	-6.099	-1.653	Reject
GC/GI	3	-1.669	-1.635	Reject
	4	-1.701	-1.635	Reject
FGC/FGI	3	-0.605	-1.653	Not reject
	4	-0.628	-1.653	Not reject
SLGC/SLGI	4	-2.565	-1.653	Reject
GSP/GDP	4	-1.639	-1.653	Not reject
PI/GDP	4	-3.235	-1.653	Not reject
PD/GDP	2	1.030	-1.653	Not reject
	3	0.736	-1.653	Not reject
	4	0.749	-1.653	Not reject
First difference				
D.FGC/FGI	3	-8.423	-1.635	Reject
	4	-6.935	-1.635	Reject
D.GSP/GDP	4	-5.459	-1.653	Reject
D.PD/GDP	2	-6.931	-1.653	Reject
	3	-6.168	-1.653	Reject
	4	-6.188	-1.653	Reject

**Notation:** GRGDP: five-quarter forward-moving average of per capita real GDP growth; GC/GI: proportion of total public consumption in investment; FGC/FGI: proportion of federal public consumption in investment; SLGC/SLGI: proportion of state and local public consumption in investment; GSP/GDP: proportion of total government spending in GDP; PI/GDP: proportion of private investment in GDP; PD/GDP: proportion of public debt in GDP. D.variable denotes the first difference of the variable.

**Table 8:** Augmented Dicky-Fuller Unit-Root Test-Deterministic Trend

Variable	Lags (AIC)	Test statistics	Critical value 5%	Null of non-stationarity
Level				
GRGDP	4	-6.139	-3.437	Reject
GC/GI	3	-3.178	-3.437	Not reject
	4	-3.261	-3.437	Not reject
FGC/FGI	3	-2.160	-3.437	Not reject
	4	-2.121	-3.437	Not reject
SLGC/SLGI	4	-3.137	-3.437	Not reject
GSP/GDP	4	-2.918	-3.437	Not reject
PI/GDP	4	-3.358	-3.437	Not reject
PD/GDP	2	-1.428	-3.437	Not reject
	3	-1.679	-3.437	Not reject
	4	-1.634	-3.437	Not reject
First difference				
D.GC/GI	3	-8.061	-3.437	Reject
	4	-6.015	-3.437	Reject
D.FGC/FGI	3	-8.420	-3.437	Reject
	4	-6.939	-3.437	Reject
D.SLGC/SLGI	4	-5.919	-3.437	Reject
D.GSP/GDP	4	-5.425	-3.437	Reject
D.PI/GDP	4	-5.240	-3.437	Reject
D.PD/GDP	2	-7.095	-3.437	Reject
	3	-6.343	-3.437	Reject
	4	-6.405	-3.437	Reject

**Notation:** GRGDP: five-year forward moving average of per capita real GDP growth; GC/GI: proportion of total public consumption in investment; FGC/FGI: proportion of federal public consumption in investment; SLGC/SLGI: proportion of state and local public consumption in investment; GSP/GDP: proportion of total government spending in GDP; PI/GDP: proportion of private investment in GDP; PD/GDP: proportion of public debt in GDP. D.variable denotes the first difference of the variable.

**Table 9:** Zivot and Andrews's (2002) unit root test by allowing structural breaks

Variable	Break	t-statistics	Critical value 5%	Null of non-stationarity
GC/GI	Intercept	-3.907	-4.80	Not reject
	Trend	-3.567	-4.42	Not reject
	Both	-3.862	-5.08	Not reject
FGC/FGI	Intercept	-3.927	-4.80	Not reject
	Trend	-2.779	-4.42	Not reject
	Both	-3.865	-5.80	Not reject
SLGC/SLGI	Intercept	-3.616	-4.80	Not reject
	Trend	-2.973	-4.42	Not reject
	Both	-3.458	-5.08	Not reject
GSP/GDP	Intercept	-3.570	-4.80	Not reject
	Trend	-2.958	-4.42	Not reject
	Both	-3.769	-5.08	Not reject
PI/GDP	Intercept	-3.668	-4.80	Not reject
	Trend	-3.410	-4.42	Not reject
	Both	-4.114	-5.08	Not reject
PD/GDP	Intercept	-3.620	-4.80	Not reject
	Trend	-2.970	-4.42	Not reject
	Both	-3.796	-5.08	Not reject

**Notation:** GRGDP: five-year forward moving average of per capita real GDP growth; GC/GI: proportion of total public consumption in investment; FGC/FGI: proportion of federal public consumption in investment; SLGC/SLGI: proportion of state and local public consumption in investment; GSP/GDP: proportion of total government spending in GDP; PI/GDP: proportion of private investment in GDP; PD/GDP: proportion of public debt in GDP.  $\Delta$  variable denotes the first difference of the variable.

**Table 10:** ARDL Model (current effect): General Government

GRGDP	Model 1.1: ARDL(4,3,0,1)		Model 1.2: ARDL(4,3,0,1,2)	
	Coefficient	p-value	Coefficient	p-value
D.GC/GI	0.0001 (0.00002)	0.020	0.0001 (0.00002)	0.016
D.GSP/GDP	0.4069 (0.1003)	0.000	0.4201 (0.1029)	0.000
D.PD/GDP	0.0293 (0.0103)	0.005	0.0330 (0.0144)	0.002
D.PI/GDP			-0.0100 (0.0637)	0.876
Observation	195		195	
Adjusted $R^2$	0.7956		0.7958	
Postestimation Tests				
	Statistics	p-value	Statistics	p-value
Cumulative periodogram white-noise	Bartlett's: 0.4495	0.9876	Bartlett's: 0.5816	0.8876
White's test	Chi2: 90.44	0.1404	Chi2: 117.53	0.5210
Breusch-Godfrey LM test for autocorrelation	lag(1): 0.022	0.8861	lag(1): 0.742	0.3902
	lag(2) 0.943	0.3915	lag(2) 0.923	0.3992
	lag(3) 0.915	0.4347	lag(3) 0.862	0.4621
	lag(4) 0.687	0.6020	lag(4) 0.646	0.6302
Durbin's alternative test for autocorrelation	lag(1): 0.019	0.8899	lag(1): 0.684	0.4094
	lag(2) 0.883	0.4151	lag(2) 0.851	0.4289
	lag(3) 0.857	0.4646	lag(3) 0.793	0.4994
	lag(4) 0.639	0.6351	lag(4) 0.591	0.6694

**Notation:** GRGDP: five-quarter forward moving average of per capita real GDP growth; D.GC/GI: first difference of proportion of total public consumption in investment; D.GSP/GDP: first difference of proportion of total government spending in GDP; D.PD/GDP: first difference of proportion of public debt in GDP; D.PI/GDP: first difference of proportion of private investment in GDP.

**Table 11:** ARDL Model (long-run effect): General Government

GRGDP	Model 1.1: ARDL(4,3,0,1)		Model 1.2: ARDL(4,3,0,1,2)	
	Coefficient	p-value	Coefficient	p-value
D.GC/GI	0.0010 (0.0006)	0.092	0.0009 (0.0005)	0.070
D.GSP/GDP	3.3293 (1.7984)	0.064	3.0237 (1.5145)	0.046
D.PD/GDP	0.1786 (0.1371)	0.193	0.1854 (0.1300)	0.154
D.PI/GDP			0.9832 (0.7371)	0.182

**Notation:** GRGDP: five-quarter forward moving average of per capita real GDP growth; D.GC/GI: first difference of proportion of total public consumption in investment; D.GSP/GDP: first difference of proportion of total government spending in GDP; D.PD/GDP: first difference of proportion of public debt in GDP; D.PI/GDP: first difference of proportion of private investment in GDP.

**Table 12:** CUSUM Tests for Parameter Instability of ARDL(4,0,3,1) Model 1.1

Type	Test statistic	Critical values		
		1%	5%	10%
Recursive	0.5277	1.1430	0.9479	0.8449
OLS	0.7941	1.6276	1.3581	1.2238

**Table 13:** CUSUM Tests for Parameter Instability of ARDL(4,3,0,1,2) Model 1.2

Type	Test statistic	Critical values		
		1%	5%	10%
Recursive	0.5710	1.1430	0.9479	0.8499
OLS	0.7040	1.6276	1.3581	1.2238

**Table 14:** ARDL Model (current effect): Federal Government and State and Local Governments

GRGDP	Model 2.1: ARDL(4,0,3,0,1)		Model 2.2: ARDL(4,0,3,0,1,2)	
	Coefficient	p-value	Coefficient	p-value
D.FGC/FGI	-0.0023 (0.0022)	0.292	-0.0026 (0.0022)	0.252
D.SLGC/SLGI	0.0044 (0.0015)	0.005	0.0046 (0.0016)	0.003
D.GSP/GDP	0.4937 (0.1103)	0.000	0.5189 (0.1142)	0.000
D.PD/GDP	0.0303 (0.0101)	0.003	0.0313 (0.0104)	0.003
D.PI/GDP			0.0023 (0.0636)	0.971
Observation	195		195	
Adjusted $R^2$	0.7987		0.7993	
Postestimation Tests				
	Statistics	p-value	Statistics	p-value
Cumulative periodogram white-noise	Bartlett's: 0.5816	0.8876	Bartlett's: 0.3940	0.9978
White's test	Chi2: 117.53	0.5210	Chi2: 131.55	0.5680
Breusch-Godfrey LM test for autocorrelation	lag(1): 0.742	0.3902	lag(1): 0.052	0.8194
	lag(2) 0.923	0.3992	lag(2) 1.133	0.3243
	lag(3) 0.862	0.4621	lag(3) 0.782	0.5053
	lag(4) 0.646	0.6302	lag(4) 0.646	0.6301
Durbin's alternative test for autocorrelation	lag(1): 0.684	0.4094	lag(1): 0.048	0.8273
	lag(2) 0.851	0.4289	lag(2) 1.041	0.3554
	lag(3) 0.793	0.4994	lag(3) 0.715	0.5445
	lag(4) 0.591	0.6694	lag(4) 0.588	0.6718

**Notation:** GRGDP: five-quarter forward moving average of per capita real GDP growth; D.FGC/FGI: first difference of proportion of federal public consumption in investment; D.SLGC/SLGI: first difference of proportion of state and local public consumption in investment; D.GSP/GDP: first difference of proportion of total government spending in GDP; D.PD/GDP: first difference of proportion of public debt in GDP; D.PI/GDP: first difference of proportion of private investment in GDP.

**Table 15:** ARDL Model (long-run effect): Federal Government and State and Local Governments

GRGDP	Model 2.1: ARDL(4,0,3,0,1)		Model 2.2: ARDL(4,0,3,0,1,2)	
	Coefficient	p-value	Coefficient	p-value
D.FGC/FGI	-0.0200 (0.0211)	0.344	-0.0186 (0.0177)	0.292
D.SLGC/SLGI	0.0751 (0.0438)	0.087	0.0686 (0.0366)	0.061
D.GSP/GDP	4.2016 (2.2465)	0.061	3.7516 (1.7944)	0.037
D.PD/GDP	0.1768 (0.1418)	0.213	0.1750 (0.1273)	0.169
D.PI/GDP			1.1592 (0.7521)	0.123

**Notation:** GRGDP: five-quarter forward moving average of per capita real GDP growth; D.FGC/FGI: first difference of proportion of federal public consumption in investment; D.SLGC/SLGI: first difference of proportion of state and local public consumption in investment; D.GSP/GDP: first difference of proportion of total government spending in GDP; D.PD/GDP: first difference of proportion of public debt in GDP; D.PI/GDP: first difference of proportion of private investment in GDP.

**Table 16:** CUSUM Tests for Parameter Instability of ARDL(4,0,3,0,1) Model 2.1

Type	Test statistic	Critical values		
		1%	5%	10%
Recursive	0.5710	1.1430	0.9479	0.8449
OLS	0.7040	1.6276	1.3581	1.2238

**Table 17:** CUSUM Tests for Parameter Instability of ARDL(4,0,3,0,1,2) Model 2.2

Type	Test statistic	Critical values		
		1%	5%	10%
Recursive	0.5886	1.1430	0.9479	0.8449
OLS	0.6957	1.6276	1.3581	1.2238

**Table 18:** Robustness Check: General Government

GRGDP	Model 3.1		Model 3.2	
	Coefficient	p-value	Coefficient	p-value
L.GRGDP	0.9379 (0.0372)	0.000	0.9411 (0.0375)	0.000
D.GC/GI	0.0000 (0.00002)	0.044	0.0000 (0.00002)	0.055
D.GSP/GDP	0.4484 (0.0930)	0.000	0.4313 (0.0959)	0.000
D.PD/GDP	0.0368 (0.0102)	0.000	0.0384 (0.0104)	0.000
D.PI/GDP			-0.0448 (0.0599)	0.455
Observation	198		198	
Adjusted $R^2$	0.7810		0.7805	
Postestimation Tests				
	Statistics	p-value	Statistics	p-value
Cumulative periodogram white-noise	Bartlett's: 0.9317	0.3504	Bartlett's: 0.8936	0.4016
White's test	Chi2: 28.10	0.0138	Chi2: 30.87	0.0570
Breusch-Godfrey LM test for autocorrelation	lag(1): 0.142	0.7070	lag(1): 0.082	0.7752
	lag(2) 2.412	0.0923	lag(2) 2.330	0.1000
	lag(3) 3.541	0.0157	lag(3) 3.555	0.0154
	lag(4) 2.989	0.0201	lag(4) 3.066	0.0178
Durbin's alternative test for autocorrelation	lag(1): 0.138	0.7112	lag(1): 0.079	0.7791
	lag(2) 2.385	0.0948	lag(2) 2.290	0.1040
	lag(3) 3.590	0.0147	lag(3) 3.587	0.0148
	lag(4) 3.037	0.0186	lag(4) 3.103	0.0167

**Notation:** GRGDP: five-quarter forward moving average of per capita real GDP growth; D.GC/GI: first difference of proportion of total public consumption in investment; D.GSP/GDP: first difference of proportion of total government spending in GDP; D.PD/GDP: first difference proportion of public debt in GDP; D.PI/GDP: first difference of proportion of private investment in GDP.

**Table 19:** Robustness Check: Federal Government and State and Local Government

GRGDP	Model 4.1		Model 4.2	
	Coefficient	p-value	Coefficient	p-value
L.GRGDP	0.9431 (0.0372)	0.000	0.9450 (0.0375)	0.000
D.FGC/FGI	-0.0022 (0.0022)	0.328	-0.0020 (0.0022)	0.364
D.SLGC/SLGI	0.0035 (0.0015)	0.020	0.0034 (0.0015)	0.027
D.GSP/GDP	0.5144 (0.1017)	0.000	0.4994 (0.1063)	0.000
D.PD/GDP	0.0364 (0.0102)	0.000	0.0375 (0.0105)	0.000
D.PI/GDP			-0.0301 (0.0606)	0.620
Observation	198		198	
Adjusted $R^2$	0.7821		0.7812	
Postestimation Tests				
	Statistics	p-value	Statistics	p-value
Cumulative periodogram white-noise	Bartlett's: 0.8986	0.3946	Bartlett's: 0.8762	0.4264
White's test	Chi2: 30.65	0.0600	Chi2: 33.19	0.1910
Breusch-Godfrey LM test for autocorrelation	lag(1): 0.083	0.7732	lag(1): 0.056	0.8133
	lag(2) 2.586	0.0780	lag(2) 2.550	0.0808
	lag(3) 3.639	0.0138	lag(3) 3.679	0.0131
	lag(4) 3.193	0.0145	lag(4) 3.261	0.0130
Durbin's alternative test for autocorrelation	lag(1): 0.080	0.7771	lag(1): 0.054	0.8170
	lag(2) 2.548	0.0809	lag(2) 2.499	0.0849
	lag(3) 3.677	0.0132	lag(3) 3.699	0.0128
	lag(4) 3.241	0.0134	lag(4) 3.297	0.0122

**Notation:** GRGDP: five-quarter forward moving average of per capita real GDP growth; D.FGC/FGI: first difference of proportion of federal public consumption in investment; D.SLGC/SLGI: first difference of proportion of state and local public consumption in investment; D.GSP/GDP: first difference of proportion of total government spending in GDP; D.PD/GDP: first difference of proportion of public debt in GDP; D.PI/GDP: first difference of proportion of private investment in GDP.

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# Appendix A

**Table A.1:** Ratios of Real Variables to Total Output in Steady State for Japan

Name of Variables	Notations	Ratios	Sources
Home-Tradable Output	$Y_{H,ss}$	0.30	World Data Bank
Non-Tradable Output	$Y_{N,ss}$	0.70	World Data Bank
Private Consumption	$C_{ss}$	0.56	World Data Bank
Private Investment	$I_{ss}$	0.18	World Data Bank
Public Consumption	$G_{ss}$	0.19	World Data Bank
Public Investment	$I_{G,ss}$	0.07	World Data Bank

**Table A.2:** Parameters for Calibration for Japan

Parameter	Value	Description
$\gamma_c$	0.77	Share of tradable consumption in home-tradable output
$\gamma_{Nc}$	0.47	Share of non-tradable consumption in non-tradable output
$\gamma_{Hy}$	0.30	Share of home-tradable output in total output
$\alpha$	0.54	Openness
$\gamma_{IG}$	0.11	Share of public investment in non-tradable output
$\mu$	0.41	Share of tradable consumption in aggregate consumption
$\gamma_g$	0.27	Share of public consumption in non-tradable output

# Appendix B

## B.1 NK-SVAR Models

### B.1.1 Government Spending

$$B_0 = \begin{pmatrix} 1 & -c_{gt} & -\alpha_{gy} & -\alpha_{g\pi} & -\alpha_{gr} & 0 \\ -c_{tg} & 1 & -\alpha_{ty} & -\alpha_{t\pi} & -\alpha_{tr} & 0 \\ -\gamma_{yg} & -\gamma_{yt} & 1 & 0 & 0 & 0 \\ -\beta a_{41} & -\beta a_{42} & -(\beta a_{43} + \kappa) & 1 - \beta a_{44} & -\beta a_{45} & -\beta a_{46} \\ -\phi_{\pi} a_{41} & -\phi_{\pi} a_{42} & -(\phi_{\pi} a_{43} + \phi_y) & -(1 + \phi_{\pi} a_{44}) & 1 - \phi_{\pi} a_{45} & -\phi_{\pi} a_{46} \\ -(a_{61} + \frac{a_{41}}{\sigma}) & -(a_{62} + \frac{a_{42}}{\sigma}) & -(a_{63} + \frac{a_{43}}{\sigma}) & -(a_{64} + \frac{a_{44}}{\sigma}) & -(a_{65} - \frac{1-a_{45}}{\sigma}) & 1 - a_{66} - \frac{a_{46}}{\sigma} \end{pmatrix}$$

When we impose an exclusion restriction  $\alpha_{gy} = 0$  in the NK-SVAR model, the matrix  $B_0$  can be written as:

$$B_0 = \begin{pmatrix} 1 & -c_{gt} & 0 & -\alpha_{g\pi} & -\alpha_{gr} & 0 \\ -c_{tg} & 1 & -\alpha_{ty} & -\alpha_{t\pi} & -\alpha_{tr} & 0 \\ -\gamma_{yg} & -\gamma_{yt} & 1 & 0 & 0 & 0 \\ -\beta a_{41} & -\beta a_{42} & -(\beta a_{43} + \kappa) & 1 - \beta a_{44} & -\beta a_{45} & -\beta a_{46} \\ -\phi_\pi a_{41} & -\phi_\pi a_{42} & -(\phi_\pi a_{43} + \phi_y) & -(1 + \phi_\pi a_{44}) & 1 - \phi_\pi a_{45} & -\phi_\pi a_{46} \\ -\left(a_{61} + \frac{a_{41}}{\sigma}\right) & -\left(a_{62} + \frac{a_{42}}{\sigma}\right) & -\left(a_{63} + \frac{a_{43}}{\sigma}\right) & -\left(a_{64} + \frac{a_{44}}{\sigma}\right) & -\left(a_{65} - \frac{1-a_{45}}{\sigma}\right) & 1 - a_{66} - \frac{a_{46}}{\sigma} \end{pmatrix}$$

In this NK-SVAR model, 20 unknown parameters need to be estimated - they include 9 structural parameters ( $c_{gt}, \alpha_{g\pi}, \alpha_{gr}, c_{tg}, \alpha_{ty}, \alpha_{t\pi}, \alpha_{tr}, \gamma_{yg}, \gamma_{yt}$ ), 5 ‘deep’ structural parameters ( $\beta, \kappa, \phi_\pi, \phi_y, \frac{1}{\sigma}$ ) and 6 variances of the structural disturbances ( $\sigma_g^2, \sigma_t^2, \sigma_y^2, \sigma_\pi^2, \sigma_r^2, \sigma_c^2$ ). Because there are  $n(n+1)/2 = 21$  distinct moments in the variance-covariance matrix of reduced-form residuals, these 20 parameters can be identified. To uniquely solve for all  $n^2 = 36$  elements of

$B_0$ , we need additional  $n^2 - n(n - 1)/2 = 21$  equations.  $B_0$  has 27 restrictions that are the 3 unity restrictions on the main diagonal, 6 exclusion restrictions and 18 rational expectations restrictions imposed on the elements from row 4 to row 6 in  $B_0$ . As a result, this NK-SVAR model is overidentified with 6 overidentifying restrictions.

### B.1.2 Public Consumption Only

$$B_0 = \begin{pmatrix} 1 & -c_{gct} & -\alpha_{gcy} & -\alpha_{gcp} & -\alpha_{gcr} & 0 \\ -c_{tgc} & 1 & -\alpha_{ty} & -\alpha_{tp} & -\alpha_{tr} & 0 \\ -\gamma_{gpc} & -\gamma_{yit} & 1 & 0 & 0 & 0 \\ -\beta a_{41} & -\beta a_{42} & -(\beta a_{43} + \kappa) & 1 - \beta a_{44} & -\beta a_{45} & -\beta a_{46} \\ -\phi_{\pi} a_{41} & -\phi_{\pi} a_{42} & -(\phi_{\pi} a_{43} + \phi_y) & -(1 + \phi_{\pi} a_{44}) & 1 - \phi_{\pi} a_{45} & -\phi_{\pi} a_{46} \\ -\left(a_{61} + \frac{a_{41}}{\sigma}\right) & -\left(a_{62} + \frac{a_{42}}{\sigma}\right) & -\left(a_{63} + \frac{a_{43}}{\sigma}\right) & -\left(a_{64} + \frac{a_{44}}{\sigma}\right) & -\left(a_{65} - \frac{1 - a_{45}}{\sigma}\right) & 1 - a_{66} - \frac{a_{46}}{\sigma} \end{pmatrix}$$

When we impose an exclusion restriction  $\alpha_{gy} = 0$  in the NK-SVAR model, the matrix  $B_0$  can be written as:

$$B_0 = \begin{pmatrix} 1 & -c_{gd} & 0 & -\alpha_{gcp} & -\alpha_{gcr} & 0 \\ -c_{tgc} & 1 & -\alpha_{ty} & -\alpha_{t\pi} & -\alpha_{tr} & 0 \\ -\gamma_{gce} & -\gamma_{gt} & 1 & 0 & 0 & 0 \\ -\beta_{a41} & -\beta_{a42} & -(\beta_{a43} + \kappa) & 1 - \beta_{a44} & -\beta_{a45} & -\beta_{a46} \\ -\phi_{\pi} a_{41} & -\phi_{\pi} a_{42} & -(\phi_{\pi} a_{43} + \phi_y) & -(1 + \phi_{\pi} a_{44}) & 1 - \phi_{\pi} a_{45} & -\phi_{\pi} a_{46} \\ -\left(a_{61} + \frac{a_{41}}{\sigma}\right) & -\left(a_{62} + \frac{a_{42}}{\sigma}\right) & -\left(a_{63} + \frac{a_{43}}{\sigma}\right) & -\left(a_{64} + \frac{a_{44}}{\sigma}\right) & -\left(a_{65} - \frac{1-a_{45}}{\sigma}\right) & 1 - a_{66} - \frac{a_{46}}{\sigma} \end{pmatrix}$$

This NK-SVAR model has the same overidentification issue with six overidentifying restrictions as the above model including government spending.

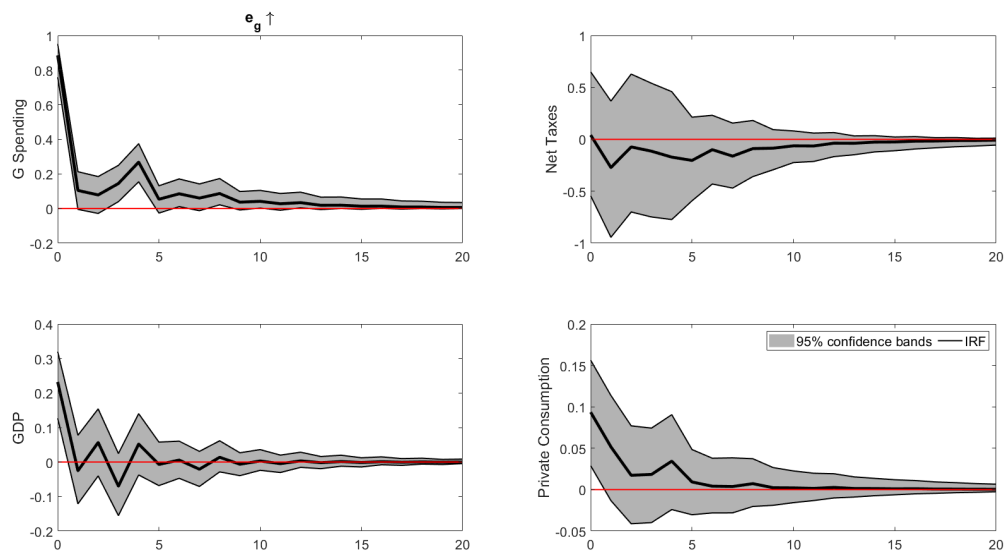
### B.1.3 Public Consumption and Public Investment

$$B_0 = \begin{pmatrix} 1 & -C_{gcz} & -C_{gct} & -\alpha_{gcy} & -\alpha_{gc\pi} & -\alpha_{gcr} & 0 \\ -C_{gzc} & 1 & -C_{gzt} & -\alpha_{gzy} & -\alpha_{gz\pi} & -\alpha_{gzr} & 0 \\ -C_{tgc} & -C_{tgz} & 1 & -\alpha_{ty} & -\alpha_{t\pi} & -\alpha_{tr} & 0 \\ -\gamma_{ygc} & -\gamma_{ygz} & -\gamma_{yt} & 1 & 0 & 0 & 0 \\ -\beta a_{51} & -\beta a_{52} & -\beta a_{53} & -(\beta a_{54} + \kappa) & 1 - \beta a_{55} & -\beta a_{56} & -\beta a_{57} \\ -\phi_{\pi} a_{51} & -\phi_{\pi} a_{52} & -\phi_{\pi} a_{53} & -(\phi_{\pi} a_{54} + \phi_y) & -(1 + \phi_{\pi} a_{55}) & 1 - \phi_{\pi} a_{56} & -\phi_{\pi} a_{57} \\ -(a_{71} + \frac{a_{51}}{\sigma}) & -(a_{72} + \frac{a_{52}}{\sigma}) & -(a_{73} + \frac{a_{53}}{\sigma}) & -(a_{74} + \frac{a_{54}}{\sigma}) & -(a_{75} + \frac{a_{55}}{\sigma}) & -(a_{76} - \frac{1 - a_{56}}{\sigma}) & 1 - a_{77} - \frac{a_{57}}{\sigma} \end{pmatrix}$$

## B.2 Impulse Response Functions

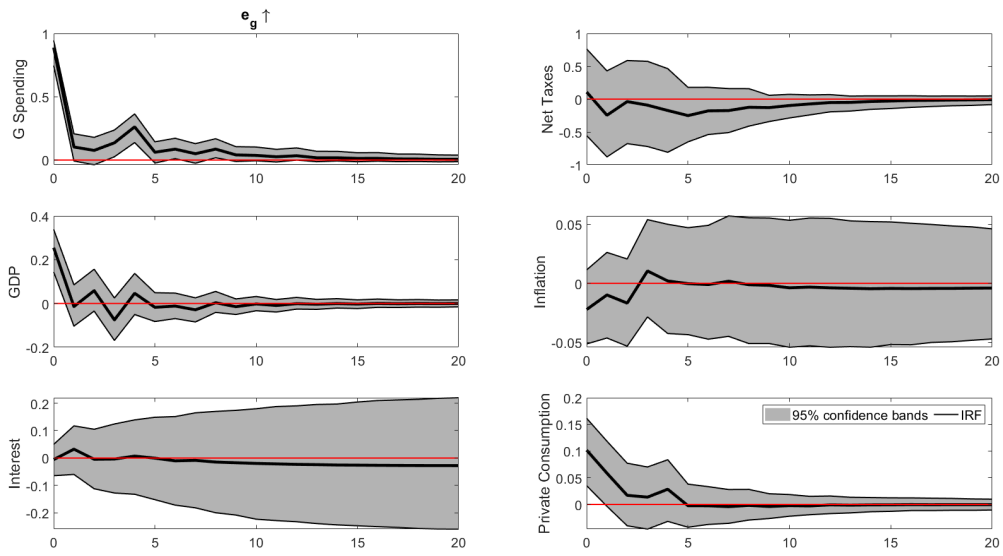
### B.2.1 Cholesky Decomposition

Figure (B.1) shows the impulse responses of government spending, net taxes, real GDP and private consumption to a shock to government spending with 95% confidence interval bands.



**Figure B.1:** Impulse responses to a one-standard-deviation shock to government spending: four variables

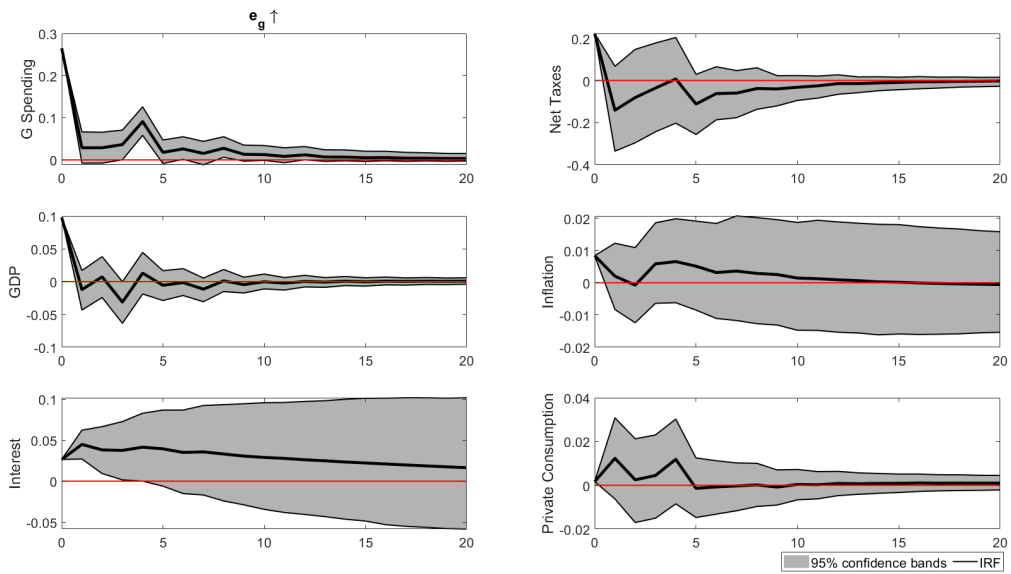
Figure (B.2) presents the impulse response functions in the six-variable SVAR model identified by Cholesky decomposition with 95% confidence interval bands.



**Figure B.2:** Impulse responses to a one-standard-deviation shock to government spending: six variables

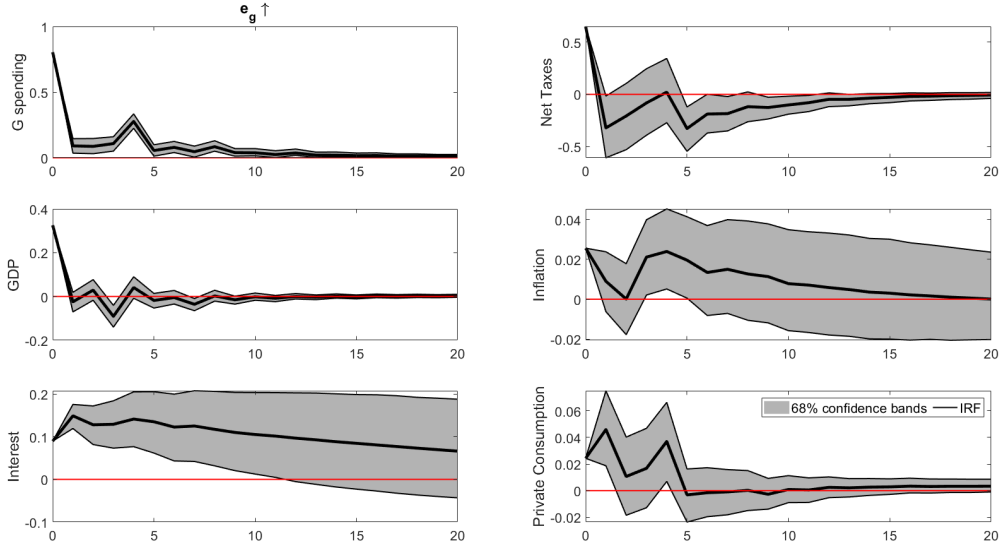
### B.2.2 Government Spending in NK-SVAR Models

Figure (B.3) shows the impulse response functions to a government spending shock with 95% confidence interval bands in the NK-SVAR model with  $\alpha_{gy} = 0$ .



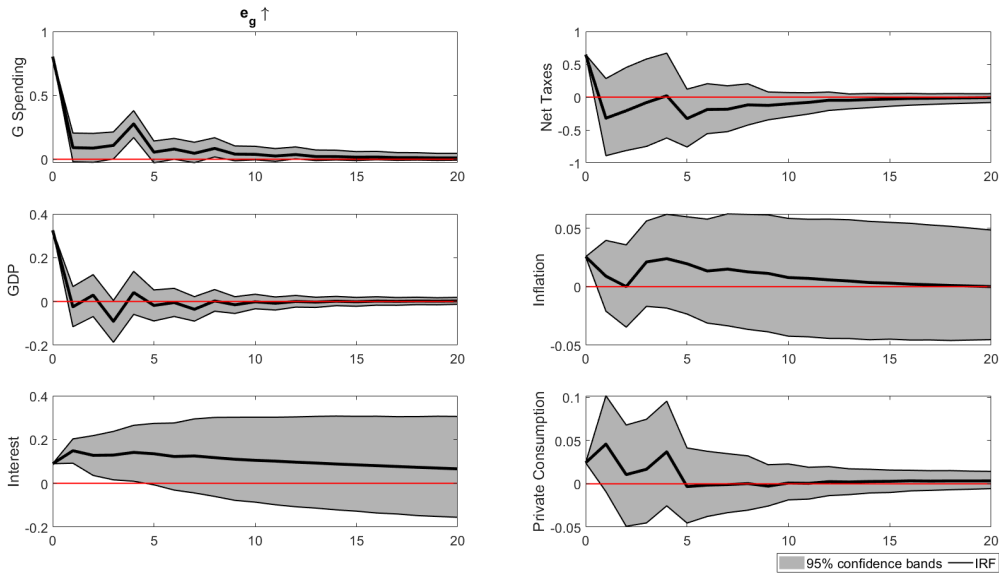
**Figure B.3:** Impulse responses to a one-standard-deviation shock to government spending with  $\alpha_{gy} = 0$

Figure (B.4) shows the impulse response functions to a government spending shock with 68% confidence interval bands identified by the NK-SVAR models with  $\alpha_{gy} \neq 0$ .



**Figure B.4:** Impulse responses to a one-standard-deviation shock to government spending with  $\alpha_{gy} \neq 0$

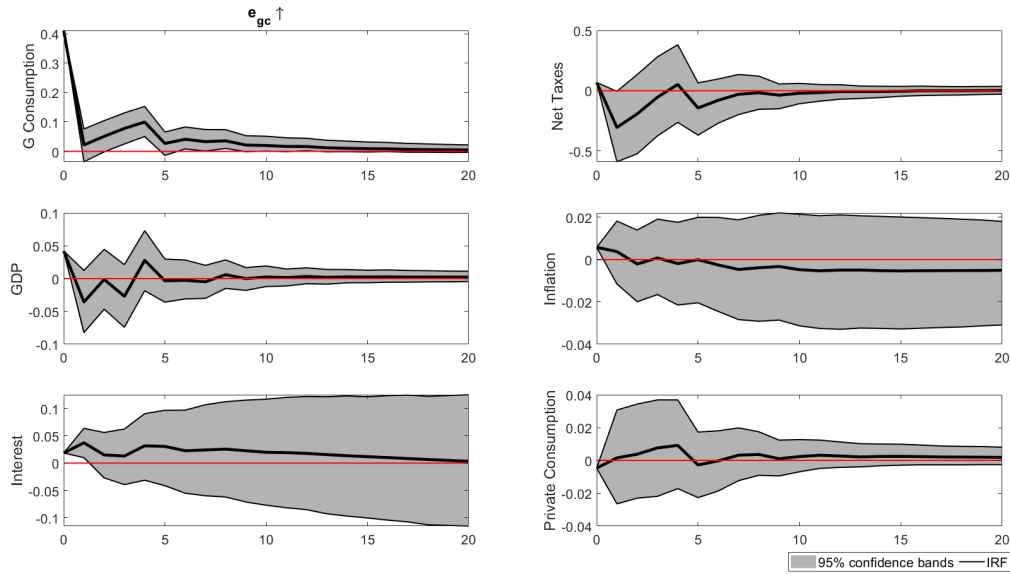
Figure (B.5) shows the impulse response functions to a government spending shock with 95% confidence interval bands identified by the NK-SVAR models with  $\alpha_{gy} \neq 0$ .



**Figure B.5:** Impulse responses to a one-standard-deviation shock to government spending with  $\alpha_{gy} \neq 0$

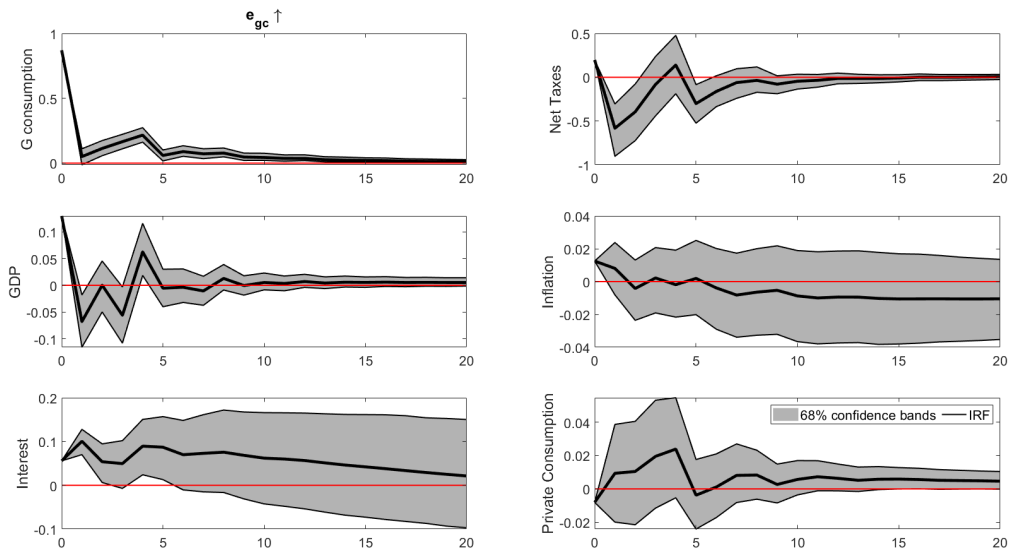
### B.2.3 Public Consumption in NK-SVAR Models

Figure (B.6) shows the impulse response functions to a public-consumption shock with 95% confidence interval bands in the NK-SVAR model with  $\alpha_{gy} = 0$ .



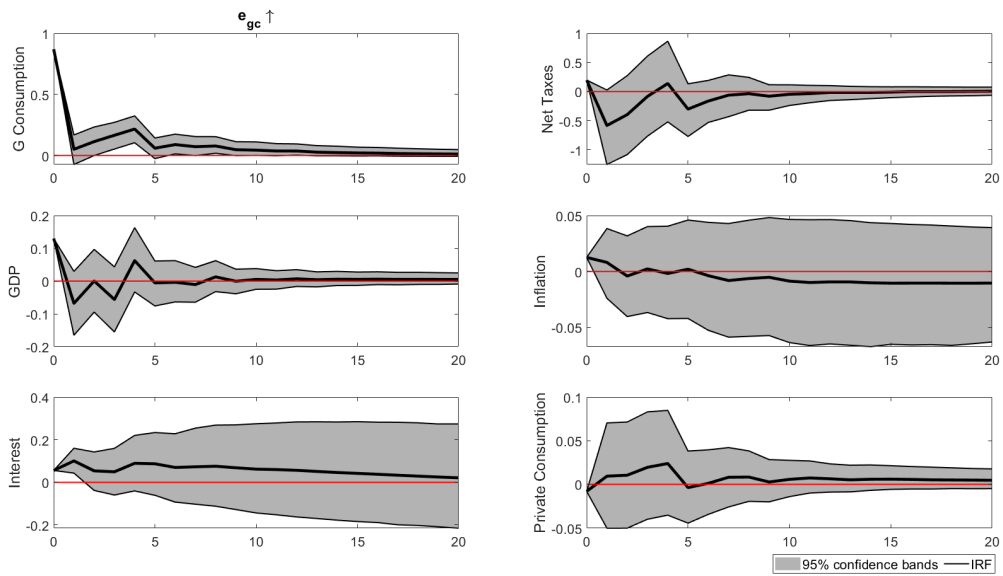
**Figure B.6:** Impulse responses to a one-standard-deviation shock to government spending with  $\alpha_{gy} = 0$

Figure (B.7) shows the impulse response functions to a public-consumption shock with 68% confidence interval bands identified by the NK-SVAR models with  $\alpha_{gy} \neq 0$ .



**Figure B.7:** Impulse responses to a one-standard-deviation shock to government spending with  $\alpha_{gy} \neq 0$

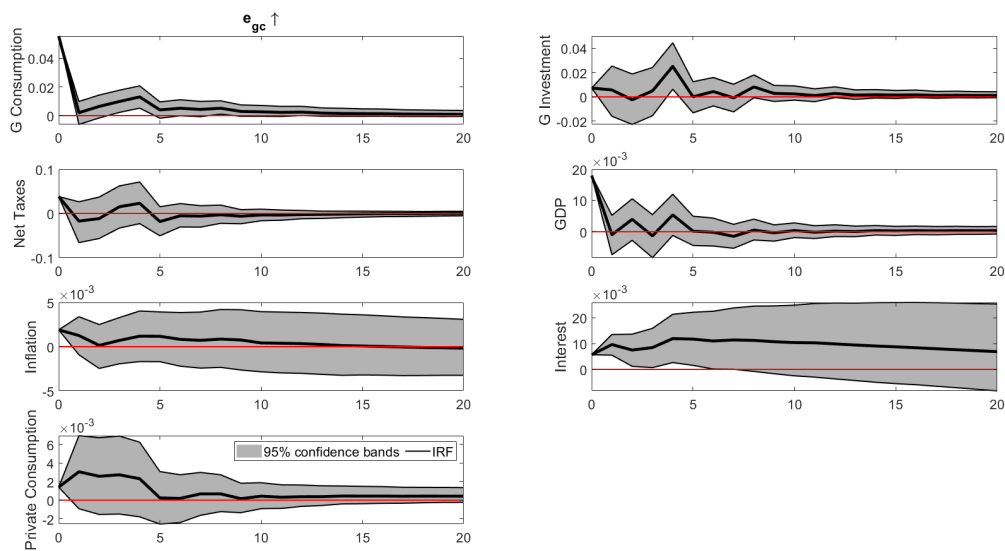
Figure (B.8) shows the impulse response functions to a public-consumption shock with 95% confidence interval bands identified by the NK-SVAR models with  $\alpha_{gy} \neq 0$ .



**Figure B.8:** Impulse responses to a one-standard-deviation shock to government spending with  $\alpha_{gy} \neq 0$

### B.2.4 Public Consumption and Investment in NK-SVAR Models

Figure (B.9) shows the impulse response functions to a public-consumption shock with 95% confidence interval bands in the NK-SVAR model.



**Figure B.9:** Impulse responses to a one-standard-deviation shock to public consumption

# Appendix C

**Table C.1:** ARDL Model: General Government

GRGDP	Model 1.1: ARDL(4,3,0,1)		Model 1.2: ARDL(4,3,0,1,2)	
	Coefficient	p-value	Coefficient	p-value
L.GRGDP	0.9643	0.000	0.9699	0.000
L2.GRGDP	0.0500	0.568	0.0494	0.573
L3.GRGDP	-0.0446	0.575	-0.0395	0.625
L4.GRGDP	-0.0919	0.137	-0.1540	0.071
D.GC/GI	0.0001	0.020	0.0001	0.016
L.(D.GC/GI)	-0.0000	0.557	-0.0000	0.600
L2.(D.GC/GI)	0.0000	0.163	0.0000	0.124
L3.(D.GC/GI)	0.0001	0.028	0.0001	0.022
D.GSP/GDP	0.4069	0.000	0.4201	0.000
D.PD/GDP	0.0315	0.002	0.0330	0.002
L.(D.PD/GDP)	-0.0097	0.330	-0.0073	0.474
D.PI/GDP			-0.0100	0.876
L.(D.PI/GDP)			0.0469	0.465
L2.PI/GDP			0.0997	0.102
Constant	0.0503	0.062	0.0538	0.058

**Notation:** GRGDP: five-quarter forward moving average of per capita real GDP growth; D.GC/GI: first difference of proportion of total public consumption in investment; D.GSP/GDP: first difference of proportion of total government spending in GDP; D.PD/GDP: first difference of proportion of public debt in GDP; PI/GDP: first difference of proportion of private investment in GDP.  $L\#$ .variable denotes the lag order of the variable.

**Table C.2:** ARDL Model: Federal Government and State and Local Governments

GRGDP	Model 2.1: ARDL(4,0,3,0,1)		Model 2.2: ARDL(4,0,3,0,1,2)	
	Coefficient	p-value	Coefficient	p-value
L.GRGDP	0.9454	0.000	0.9510	0.000
L2.GRGDP	0.0754	0.384	0.0765	0.377
L3.GRGDP	-0.0188	0.813	-0.0142	0.861
L4.GRGDP	-0.1194	0.053	-0.1517	0.021
D.FGC/FGI	-0.0023	0.292	-0.0026	0.252
D.SLGC/SLGI	0.0044	0.005	0.0046	0.003
L.(D.SLGC/SLGI)	-0.0011	0.4541	-0.0010	0.492
L2.(D.SLGC/SLGI)	0.0017	0.260	0.0019	0.201
L3.(D.SLGC/SLGI)	0.0039	0.010	0.0040	0.008
D.GSP/GDP	0.4937	0.000	0.5189	0.000
D.PD/GDP	0.0303	0.003	0.0313	0.003
L.(D.PD/GDP)	-0.0095	0.336	-0.0071	0.482
PI/GDP			0.0023	0.971
L.PI/GDP			0.0557	0.384
L2.PI/GDP			0.1023	0.090
Constant	0.0532	0.046	0.0590	0.036

**Notation:** GRGDP: five-quarter forward moving average of per capita real GDP growth; D.FGC/FGI: first difference of proportion of federal public consumption in investment; D.SLGC/SLGI: first difference of proportion of state and local public consumption in investment; D.GSP/GDP: first difference of proportion of total government spending in GDP; D.PD/GDP: first difference of proportion of public debt in GDP; D.PI/GDP: first difference of proportion of private investment in GDP.  $L\#$ .variable denotes the lag order of the variable.