ANALYSIS OF THE GRAM-CHARLIER EXPANSION FOR THE EVALUATION OF LOSS OF LOAD PROBABILITY AND EXPECTED PRODUCTION COSTS FOR GENERATION PLANNING

by

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ABSTRACT

This thesis analyzes the Gram-Charlier expansion, for the evaluation of loss of load probability, capacity reserve margin, expected energy generation and production costing in generation expansion planning. The accuracy of the Gram-Charlier expansion method is studied in detail and guidelines are given of the applicability of this expansion for the above mentioned evaluations. The accuracy of the Gram-Charlier expansion depends upon such variables as the load shape, unit size, number of units, forced outage rate as well as number of terms used in the series. For the evaluation of loss of load probability several different systems are analysed with varying forced outage rates and unit sizes. For production costing calculations, system demand and load shape are varied. The accuracy of the results decreases with the decrease in forced outage rates and number of units. The results are always compared with those obtained from a conventional technique, based on a well known recursive algorithm and numerical convolution, which in all cases, proved to be computationally more expensive. This conventional technique is also discussed along with the Gram-Charlier series and its convergence. The comparison of the Gram-Charlier method and the conventional method is made

in terms of loss of load probability, capacity reserve margin required at a certain risk level, expected energy generation and production costing.

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TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

Chapter I

INTRODUCTION

Probabilistic methods are being used by utilities to plan the generation requirements and to predict the cost of generation. Installed capacity is required to meet the demand with an adequate level of continuity and quality of service. On the other hand, a larger reserve capacity, caused by system overexpansion makes electricity more expensive. The purpose of generating system planning is to generate electricity economically and reliably. Expected production costs comprise capital cost, maintenance cost and fuel cost. Installed reserve capacity affects the capital cost while the type of generating units and fuel requirements affect the operating cost which will be discussed in the later chapters.

Calabrese [1] introduced the loss of load probability (LOLP) approach which evaluates the probability of demand exceeding the available generating capacity. In this method a capacity outage table is constructed from the probability density function of capacity outage of units by a process of convolution. From the capacity outage table, the probability of outage capacity exceeding the reserve capacity directly gives the LOLP. In the conventional method using a recursive algorithm proposed by Billinton [2], convolution of probability density functions consisting of impulses at discrete capacity values may be approximately simulated by rounding the probability values at discrete capacity steps, which are integer multiples of some step size. Rounding the probability values may also be expressed as the interpolating of probability values at the capacities which are multiples of the selected step size.

Baleriaux et al. [3] suggested a sophisticated probabilistic model to simulate the production costs of a set of generating units meeting a demand. It was further improved by Booth [4], Sager et. al [5], and, Jenkins and Joy [6]. The method is based on the notion of an equivalent load [7]. The units are considered as hundred percent reliable capacity and a ficticious load whose probability density function is equal to the probability density function of outage capacity of the unit. The equivalent load may be viewed as the regular load augmented by the forced outage capacities of generating units. Since the effect of the random outage of the units is added into the load, essentially the load appears larger to the remaining units. The equivalent load is obtained by convolving the probability density function of the units outage capacity with the load distribution function.

In the economic operation of the generating system, the loading of a unit is determined from its incremental fuel cost. For the minimum total cost, cycling units are loaded at equal incremental costs. It is difficult to simulate the production costs with variable incremental fuel cost. Baleriaux and Booth [3,4] considered the unit as one block and calculated the average incremental fuel cost over the entire capacity of the unit. Joy [6], in the discussion of [4], suggested fragmenting the units in capacity blocks which need not occupy adjacent positions in the loading order. In the block representation of units, each unit is fragmented into several blocks of increasing marginal costs. A priority order of committment of the blocks is prepared in an ascending order of incremental fuel costs to better approximate the dispatch procedure of equalizing the incremental costs of the cycling units. The capacity blocks of the same unit need not be committed consecutively. However, the dependency of the unit's upper capacity block to the lower block must be taken into account. Essentially this implies that the higher capacity blocks can not be loaded unless the lower capacity blocks have already been loaded.

In the evaluation of LOLP as well as expected energy generation for production costing, repeated convolutions are required. In the construction of the capacity outage table for LOLP calculations using the recursive technique, if the unit sizes are not integral multiple of step size, interpolation is required. Each calculation may lead to digital truncation errors. The computational efforts increases considerably if the step size is decreased unless the capacities of the units correspond with this step size. This increase in computational time is further compounded for multistate representation of units. When units are divided into blocks to better simulate economic dispatch for production costing, the probability density function of the lower block must be deconvolved before proceeding to the upper blocks. This further complicates the computational procedure and increases the computational time considerably. The convolution of the load distribution and the probability density function of a unit is simulated numerically by using several discrete values of load function at several load values. If the discrete load values or the capacity of the unit is not an integral multiple of step size interpolation is required to evaluate each discrete point of the equivalent load curve. In addition, in the calculations of expected energy generation, numerical integration is needed. The area under the load curve is the total demand of energy. The expected energy generated by each capacity block is calculated from the equivalent load curve by numerical integration over the capacity of that block.

A computationally efficient method to obtain LOLP, expected energy generation for production costing has been reported by Schenk [8], Rau and Schenk [9], Rau et. al [10] and Stremel et. al [11]. The method, referred to as the Gram-Charlier series method, is based on the orthogonal expansion of a density function in terms of cumulants, the normal density function and its derivatives. The cumulants are easily derived from the moments as described in [12]. The cumulants exhibit the desired property that the sum of independent random variables is characterised by cumulants which are the sum of individual cumulants. Therefore the process of convolution is simulated simply by the addition of cumulants. Conversely, deconvolution may be obtained from the subtraction of cumulants. Thus, for a system of generating units, the process of convolution to construct a capacity outage table may be simulated by adding the appropriate cumulants of the random variables describing the unit's forced outage capacities. Similarly for the construction of the equivalent load distribution, the load function is convolved with the probability density function of capacity outage of each unit. This may simply be simulated by the addition of cumulants of the load to those of the unit's capacity outage. The effectiveness of the Gram-Charlier method is more apparent when units are represented as multistate units and where they are fragmented into blocks. The process of convolution or deconvolution is quite time consuming in the conventional method based on the recursive algorithm. In the Gram-Charlier method, the process of numerical integration has been replaced by the addition or subtraction of the appropriate cumulants.

In Chapter 2, probabilistic models of generating units, load and equivalent load are described. In Chapter 3, the evaluation of LOLP, expected energy generation for production costing procedures are discussed using the models described in Chapter 2.

The Gram-Charlier series, its convergence, the derivation of cumulants and some useful properties of those cumulants are discussed in Chapter 4. Chapter 5 is dedicated to applications of the Gram-Charlier method to the evaluation of LOLP, expected energy generation for production costing.

Chapter 6 describes the numerical results when the Gram-Charlier series is used for the evaluation of LOLP, expected energy generation and expected production costs. The accuracy of the method is also studied [13]. Conclusions and recommendations for further work are also discussed in this chapter.

Chapter II

PROBABILISTIC MODELS FOR GENERATION AND DEMAND

2.1 INTRODUCTION

The use of probabilistic methods for the evaluation of installed generation requirements may be one of the earliest industrial application of reliability techniques in engineering design[14]. Limitation of computation facilities severely restricted the numerical application of reliability procedures to the study of generating system adequacy in early times. With the advancement of computers and the work of many authors, namely; Lyman[15], Calabrese[1], Billinton[2], Baleriaux[3] and Booth[4], probabilistic methods became an integral part in power system planning.

The main concern of an electric utility is to supply power to its customers reliably and economically. The objective function is to minimize the cost while keeping an acceptable level of reliability. The problem can be dealt better with a suitable mathematical model based on probabilisitic methods. Not many years ago the capacity reserve margin was determined as a percent of the installed capacity. The complexity of the system has recently increased because of increased size and the addition of interconnection to neighbouring utilities.

In all sectors of Power System (i.e, generation, trammission and distribution) reliability indexes have been defined. Usually the reliability of each sector is defined separately to simplify the analysis. Recently, however, there has been a great deal of effort given to the evaluation of so called bulk power system which comprises both the generation and transmission system. In this analysis failure of the stations that affect the transmission network is usually accounted for by modifying the transmission line failure rates. Similarly for transformer failures. One of the best models for bulk power system reliability evaluation is PCAP [16].

The concept of LOLP [1] is an accepted index for computing the installed generating capacity requirements. Similarly in the distribution sector, the frequency and duration of the outages have become the important yardsticks. In this chapter the probabilistic models of unit capacity outage and demand are given. The concepts of equivalent or effective load is described in section 2.5.

2.2 BASIC TERMINOLGY

Before going into a more detailed analysis, the definitions of the terms very frequently used in this thesis are defined [3,8].

2.2.1 Outage

An outage describes the state of a component when it is not available to perform its intended function due to some events directly associated with that component. An outage may or may not cause an interruption of service to consumers depending upon system operation.

2.2.2 Forced Outage

A forced outage is an outage that results from emergency conditions directly associated with a component to be taken out of service immediately, either automatically or as soon as switching operation can be performed, or an outage caused by improper operation of equipment or human error.

2.2.3 Planned or Scheduled Outage

A scheduled outage is an outage that results when a component is taken out of the service at a selected time,usually for a purpose of construction, preventive maintenance or repair.

If it is possible to defer the outage when deferrment is desirable, the outage is called scheduled outage; otherwise the outage is a forced outage. For example, deferring an outage may be desirable to prevent the overload of facilities or an interruption of service to consumers.

2.2.4 Load Factor

Load Factor is the ratio of the average load over a designated period of time to the peak load occuring in that period. It can also be defined as the ratio of total area under the load curve over the considered time period to the product of the considered time period and peak load.

2.2.5 Capacity Factor

Capacity factor is the ratio of average load on a unit for the period of time considered to the capacity of the unit. It is also defined as the ratio of the actual energy generated by the unit in a designated period to its energy capability.

2.2.6 Available Capacity

Available capacity is defined as the installed capacity (IC) minus the forced and scheduled outage capacities (FOC & SOC). Thus

$$AC=IC-FOC-SOC$$

Since the forced outage capacity is a random variable, available capacity is also a random variable. The scheduled outage capacity is a deterministic quantity entirely in control of the system planner.

2.2.7 Reserve Capacity Margin

Reserve capacity margin (RM) is defined as the available capacity (AC) minus the peak load (PL). Note that loss of load occurs when the reserve capacity margin is negative.

$$RM=AC-PL$$

2.2.8 Reserve Capacity

Reserve capacity or simply Reserve is defined as the difference between installed capacity and the peak load (PL) during a specific period of time, thus

$$R=IC-PL$$

2.3 GENERATING UNIT MODEL

The reliability of the power system largely depends upon the reliability of the generating system. The units can be forced off-line due to technical problems. The availability or forced outage rate (FOR) of a unit is a reflection of the random process that takes place in including uncertainty in the evaluations. The simplest model of a generating unit is a Run-Fail-Repair-Run cycle [7] as shown in Fig. 1.

The system has two states. It tranfers from state 1 (the up state, corresponding to the unit being available) to state 2 (the down state, corresponding to non-availbility of the unit) and vice versa. For the ith cycle let

m_i = uptime

r_i = downtime

From the historical data of the availability and non-availability of the unit, an average uptime and downtime is defined as follows

$$\text{mean uptime} = m = \frac{1}{N} \sum_i m_i$$

$$\text{mean downtime} = r = \frac{1}{N} \sum_i r_i$$

$$\text{unit failure rate} = \lambda = \frac{1}{m}$$

$$\text{unit repair rate} = \mu = \frac{1}{r} .$$

where N= No. of times unit fail

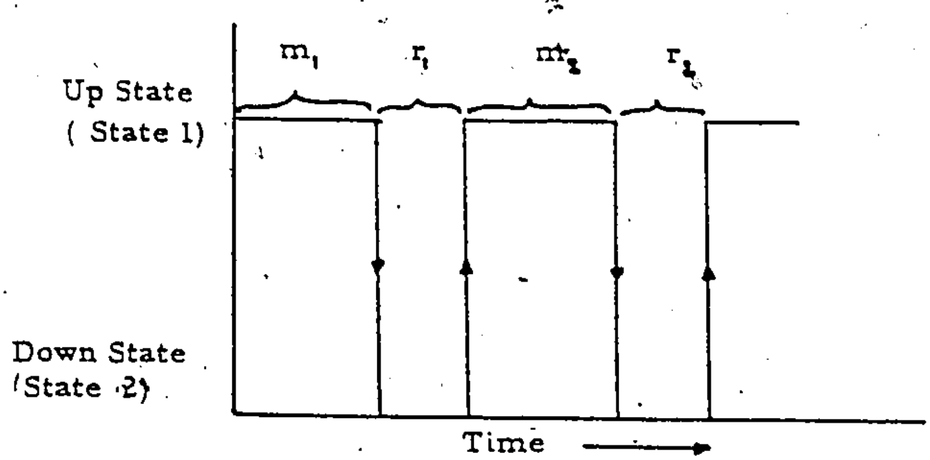

Figure 1: Random unit performance ignoring scheduled outages

The two state model for a unit may be represented as shown in Fig. 2.

This two state model of a unit may be described by a discrete state, continous transition Markov Process. This process is characterised by a lack of memory. The future of the process is completely defined by its immediate history.

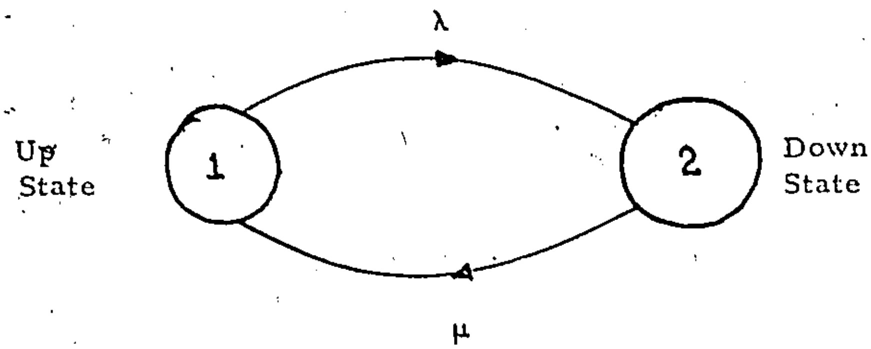

Figure 2: Two state representation of a unit

Define

$p_1(t)$ = Probability that the system is in up state at time t

$P_2(t)$ = Probability that the system is in down state at time t

Consider an incremental time interval Δt and assume that

(a) changes of state are possible at anytime

(b) the probability of two or more changes of state during this interval of time is negligible.

The probability of the system being in state 1 at time $t + \Delta t$ may be given by

$$P_1(t+\Delta t) = \left[P_1(t) - P_1(t)(\lambda\Delta t)\right] + P_2(t)(\mu\Delta t) \tag{2.1}$$

where

The term $P_1(t)(\lambda\Delta t)$ gives the probability of the system going into state 2 (down state), in time interval Δt, if the system were in state 1 (up state) at time t. Therefore, the term $\left[P_1(t)-P_1(t)(\lambda\Delta t\right]$ gives the probability of staying in state 1 in the time interval Δt. The term $P_2(t)(\mu\Delta t)$ gives the probability of the system going into state 1 in time interval Δt, if the system were in state 2 at time t.

Similarly the probability of the system being in state 2 at time $t+\Delta t$ may be given by

$$P_2(t+\Delta t) = \left[P_2(t) - P_2(t)(\mu\Delta t)\right] + P_1(t)(\lambda\Delta t) \tag{2.2}$$

$$\frac{P_1(t+\Delta t) - P_1(t)}{\Delta t} = -\lambda P_1(t) + \mu P_2(t) \tag{2.3}$$

$$\frac{P_2(t+\Delta t) - P_2(t)}{\Delta t} = \lambda P_1(t) - \mu P_2(t) \tag{2.4}$$

As $\Delta t \to 0$

$$\frac{d P_1(t)}{dt} = -\lambda P_1(t) + \mu P_2(t) \tag{2.5}$$

$$\frac{d P_2(t)}{dt} = \lambda P_1(t) - \mu P_2(t) \tag{2.6}$$

Since we are interested in the long term (steady state) probabilities of being either in state 1 or state 2, where P_1+P_2 must equal to unity, we may set the derivatives to zero and solve the resulting two equations for $P_1(\infty)$ and $P_2(\infty)$

$$P_1(\infty) = \frac{\mu}{\lambda+\mu} = p$$

$$P_2(\infty) = \frac{\lambda}{\lambda+\mu} = q$$

Therefore, the long term probabilities are given by

$$P(\text{up state}) = p = \frac{\mu}{\lambda+\mu} = \frac{m}{m+r}$$

$$P(\text{down state}) = q = \frac{\lambda}{\lambda+\mu} = \frac{r}{m+r}.$$

These two quantities are simply the unit availability and FOR, respectively. Clearly, $p+q=1$.

2.3.1 Forced Outage Rate (FOR)

The forced outage rate may be estimated by the relation

$$FOR = \frac{\text{forced outage hours (FOH)}}{\text{Service hours + forced outage hours}}$$

$$FOR = q$$

Utilities vary in the way they estimate this important parameter. For example, Ontario Hydro defines an equivalent forced outage rate (EFOR), which includes maintenance and derated hours, as well as an adjusted forced outage rate (AFOR), which includes maintenance and other factors.

2.3.2 Probability Density Function (PDF) for Forced Outage Capacity

For a unit of capacity C MW, FOF=q and availability p, the PDF of available and forced outage capacity are given in Fig. 3.

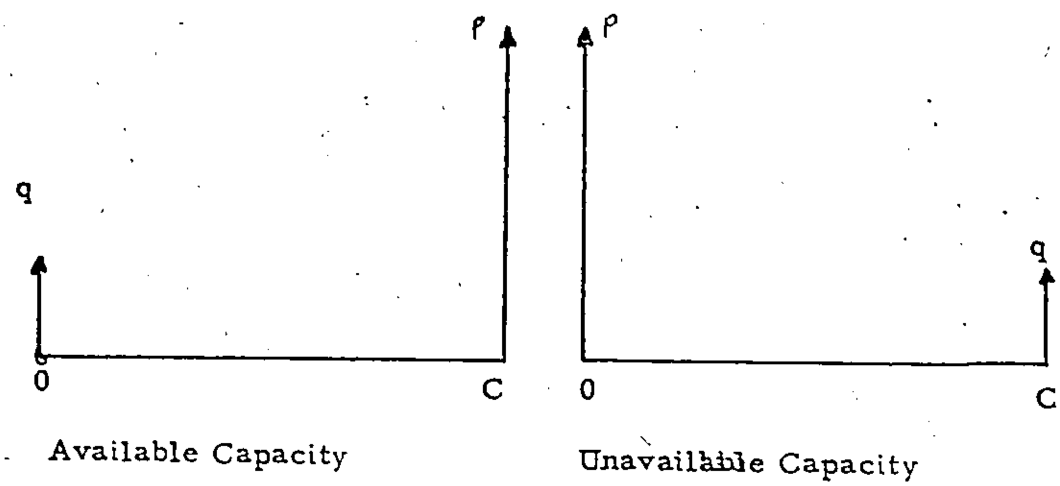

Figure 3: PDF of available and forced outage capacity

The PDF of forced outage capacity may be expressed as

$$f(x) = p\,\delta(x) + q\,\delta(x-C)$$

where $\delta(\cdot)$ is the well known impulse function (Dirac-delta) and X is a random variable representing capacity.

2.3.3 Multistate Representation of the PDF of Units

In the model considered before for generating units, the probability density function (PDF) for outage capacity of the unit consists of only two impulses; one at zero MW capacity with intensity p and other at the full capacity of the unit (=C) with intensity q. In other words, the probability of zero MW out of service is p, and C MW out of service is q.

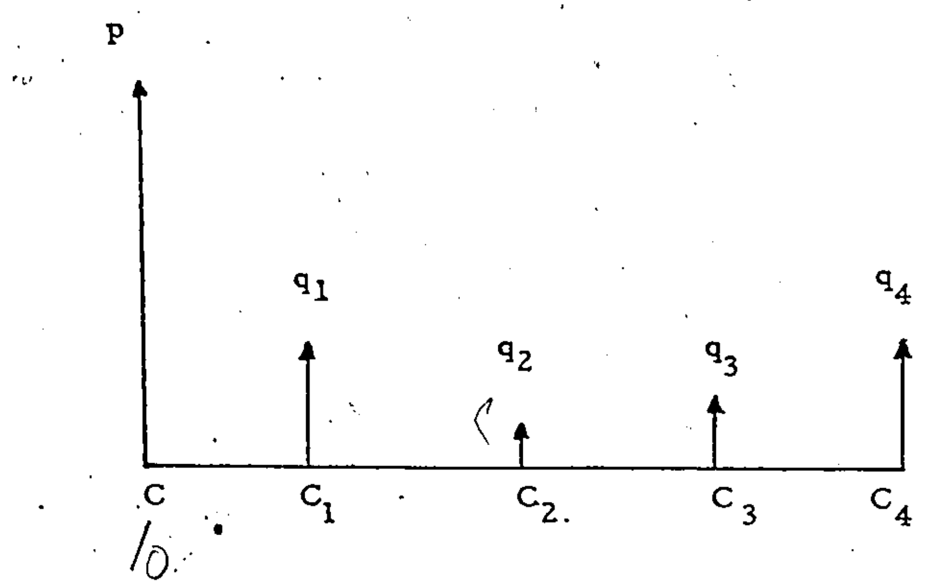

Figure 4: Multistate representation for outage PDF of a unit

This model is the simplest possible probabilistic model. In actual practice there may be more than two states, that is, a unit may have derated states also. In a derated state, a portion of the capacity may be out of service randomly due to technical reasons such as failure of one steam valve or failure of some pressure gauge of the boiler which may reduce the available capacity. A probability value can be assigned to the existence of that capacity state. Figure 4. shows the PDF of outage capacity when a unit has derated states and a full outage state. This may be written in the form

$$f(x) = p\,\delta(x) + q_1\,\delta(x-C_1) + q_2\,\delta(x-C_2) + q_3\,\delta(x-C_3) + q_4\,\delta(x-C_4)$$

2.4 LOAD MODELS

A probabilistic load model, suitable for production costing is described in this section. Historical data is collected and future demand is forecasted. If a recording of instantaneous demand were plotted for a particular historical day, a curve such as depicted in Fig. 5. might result. This curve is called load curve or demand curve.

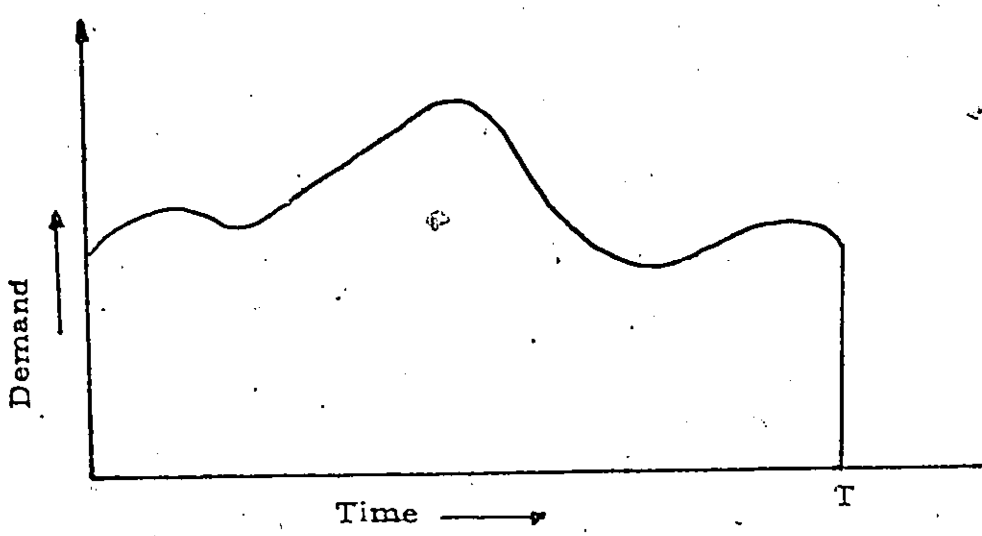

Figure 5: Load Curve

2.4.1 Load Duration Curve

The load duration curve in Fig. 6. is constructed from the load curve. It is obtained by determining the percentage of time for which the demand exceeded a particular level. The area under the load curve or load duration curve is the energy in MWh (GWh) consumed by the load. The minimum level of demand in the load duration curve is called the base load. The maximum load level of the system is called peak demand or peak load.

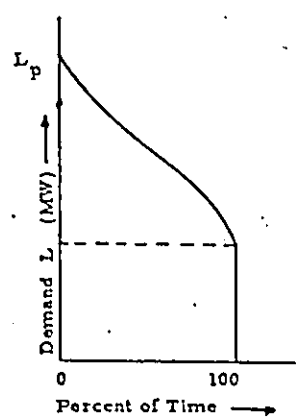

Figure 6: Load duration curve

2.4.2 Load Probability Distribution

The load probability distribution is obtained by interchanging the axes of the load duration curve (LDC) and normalizing the time as shown in Fig. 7. This curve can be interpreted as a probability distribution function. The y-axis gives the probability that the load exceeds the corresponding x-axis megawatt value.

The demand value L is a random variable and F(L) is its distribution function. The load probability distribution may be obtained for any interval of time from the load curve. The load probability distribution is also called the inverted load duration curve (ILDC).

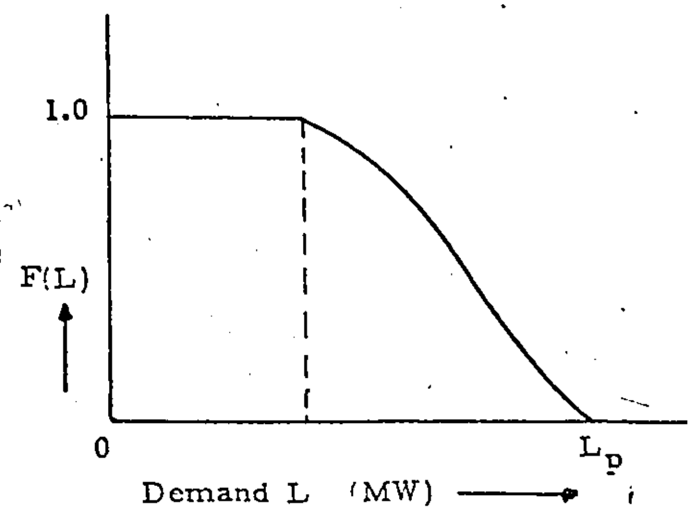

Figure 7: Load Probability distribution for a particular historical day

2.5 EQUIVALENT LOAD

Probabilistic models for generating units and load have been described in section 2.3 and 2.4 respectively. The two models may be combined to define the effective load of the system.

In the model for the system load, the load L is a random variable with probability distribution F(L) as shown in Fig. 7.

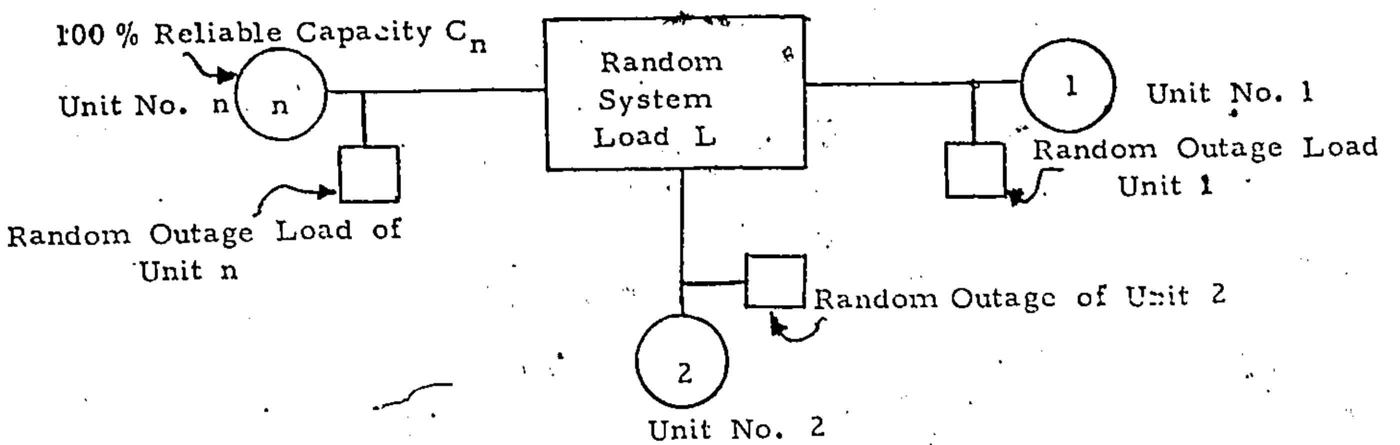

Figure 8: Equivalent Load Model of Units and Load

Similarly in the model for generating units, outage of a capacity X is also a random variable with PDF f(X). Let, there be for simplicity, only two states of the units (up and down states). Combining the two models Fig. 8 depicts the relationship between the system load and generating units where actual units have been replaced by ficticious perfectly reliable units and ficticious random loads, whose probability density functions are the outage capacity density functions of the units [7].

The equivalent load of the system when n units are committed is:

$$Le = L + \sum_{i=1}^{n} X_i$$

where Xi is random outage load. The installed capacity of the system is given by

$$IC = \sum_{i=1}^{n} C_i$$

Since L and X_i are independent random variables and Le is the sum of independent random variables, the distribution function of Le may be found out by convolution. The density function for the equivalent load ($Le=L+X_1$) with only the first unit may be given by

$$F^{(1)}(Le) = \int F(Le-x_1)\, f_1(x_1)\, d\, x_1$$

By successive convolutions, the distribution function of the effective load of the system by adding the effect of random outage of i units may be written as

$$F^{i}(Le) = \int F^{i-1}(Le-x_i)\, f_i(x_i)\, d\, x_i$$

$$F^{i}(Le) = F(L) \quad \text{for } i=0$$

For a unit with two states outage PDF is given by

$$f_i(x_i) = p_i\delta(x_i) + q_i\delta(x_i - C_i)$$

and the equivalent load is therefore,

$$F^{i}(Le) = \int_{x_i} F^{i-1}(Le - x_i)\left[p_i\delta(x_i) + q_i(x_i - C_i)\right] dx_i$$

$$= p_i F^{i-1}(Le) + q_i F^{i-1}(Le - C_i)$$

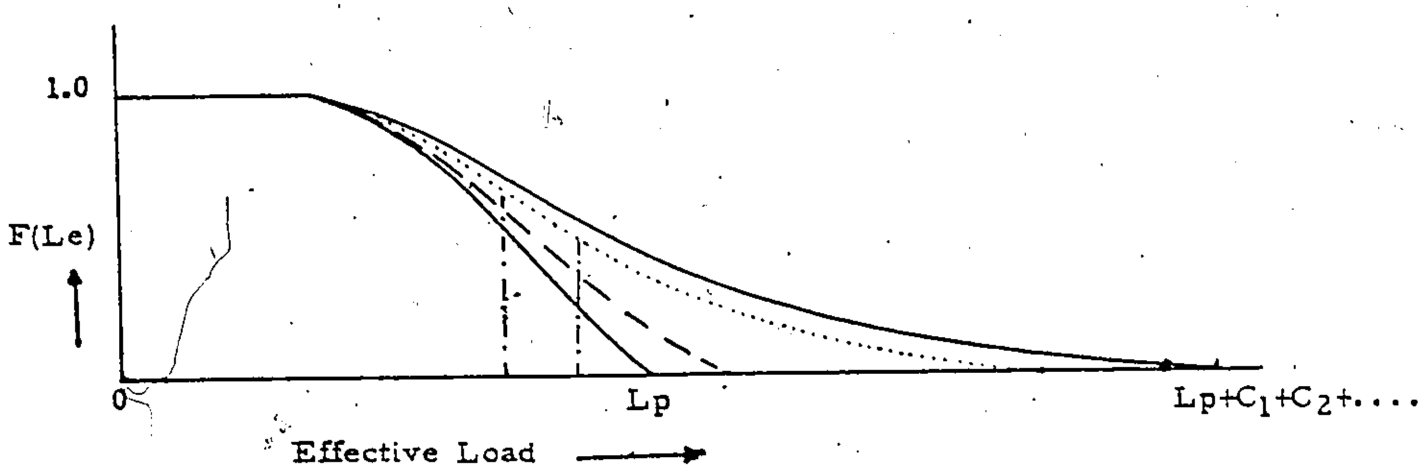

Figure 9: Equivalent Load Duration Curve

The effect on F(L) of accounting for the random outages of generating units is to inflate it. By adding this effect into the load the remaining units see a larger effective load as shown in Fig. 9. The curve obtained from the function F(L) is called the equivalent load duration curve (ELDC).

For any value of effective load, Fig.9 gives the probability of equalling or exceeding the specified value.

Chapter III

PROBABILISTIC METHODS IN GENERATION PLANNING

3.1 INTRODUCTION

In generation planning, uncertainty plays an important role. This uncertainty is reflected in the random failure of the generating units as well as in the variation in demand. Additionally, there is uncertainty in fuel prices and the general economic behaviour of the system.

The system must have enough reserve capacity to provide for the maintenance of the equipment, forced outages and load growth. In the past, the adequacy of both planned and installed capacity was measured in terms of percentage reserve [2]. Also a method using the largest capacity unit plus a fixed reserve is still in use. The important objection to the use of this criterion is the tendency to compare two totally different systems on the basis of their peak demand. If the two system have different load characteristics, they may need different reserve capacities. The difference of reserve capacity may be quite large with equal peak loads but different load characteristics and unit sizes. Therefore there is a need for an analytical approach to evaluate the reserve capacity requirements. The application of probabilistic methods provides an analytical basis for capacity planning which can be extended to cover interconnection, maintenance, expected energy generation and production costing. The method allows for a consistent evaluation of reliability indexes from which valid comparison may be made.

In the previous chapter, probabilistic models for load, unit available capacity and unit outage capacity were given. Also the concept of equivalent load has been explained. In this chapter, the applications of probabilistic methods in the evaluation of LOLP, expected energy generation and production costing for generation planning are presented. For a given load (forecasted demand) and unit data (unit sizes and FORs), expected energy generated by each unit in the future is evaluated to find out the expected production costs for expansion planning. The expected energy generated by the units depends upon the shape of the load duration curve, the incremental fuel cost of the unit and the FOR of the unit.

3.2 LOSS OF LOAD PROBABILITY

In the Loss of Load Probability approach, the system capacity outage is combined with the system load characteristics to give an expected risk of loss of load. It is a measure of the insufficiency of available generating capacity to meet the system demand. Loss of load probability can also be defined as the probability of demand exceeding the available capacity. There is not necessarily a loss of load whenever there is a capacity outage. As long as the capacity outage is less than the reserve capacity there is no loss of load.

From the capacity outage model of the units, a capacity outage probability table may be constructed. This table describes the probability density function for the capacity outage of the system. The outage probability density function comprises impulses of intensities equal to the probability of the outage of that capacity value at which the impulse is situated.

The outage of a unit is a random variable. In a system where a number of generating units is supplying a load, the capacity outage of the system is the sum of the capacity outages of all the units. Thus, the capacity outage of the entire system is also a random variable. The PDF of capacity outage of the system of n units can be derived analytically. Since the capacity outages of generating units are independent random variables the relation becomes quite easy. The PDF of the sum of two independent random variables can be given by convolution of the PDFs of the random variables.

A point can be made here regarding independence. If units belong to a plant, it is not proper to say that these units may fail independently of the other units in the same plant. A fire in the plant may force all units to fail. However, it is common practice to assume independence of all units.

The PDF of a random variable equal to the sum of several independent random variables can be given by repeated convolution. Since the outage of the unit capacity is assumed to be an independent random variable, the probability density function of the capacity outage of the system of n generating units is given by repeated convolution, simulating one unit at a time recursively. The capacity outage PDF has discrete values. A table can be constucted which gives discrete values of probability of a particular MW of capacity being out of service. The capacity outage PDF of a system is shown in Fig.10 with 12 discrete capacity states.

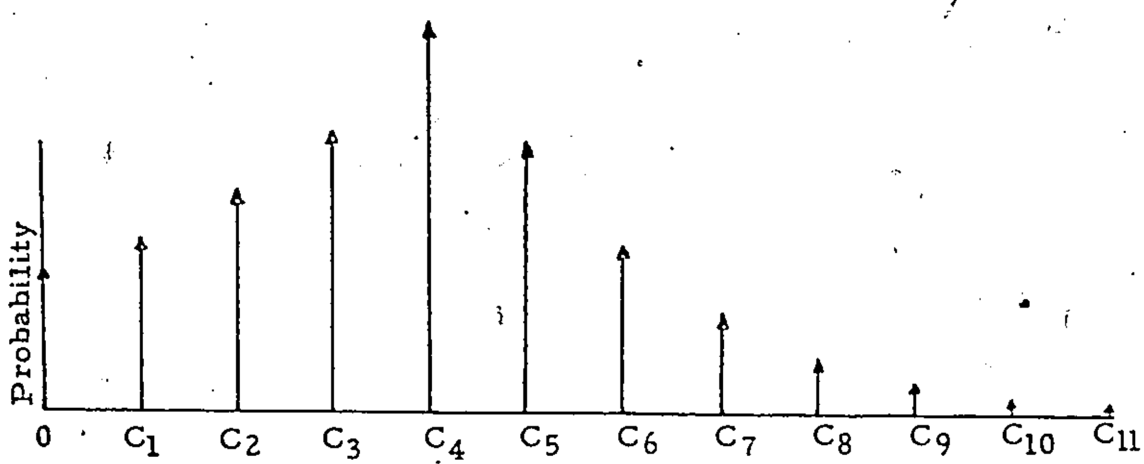

Figure 10: The Capacity Outage Table of a System

The capacity outage table is combined with the load duration curve to give expected loss of load. A typical system load capacity relationship is shown in Fig. 11.

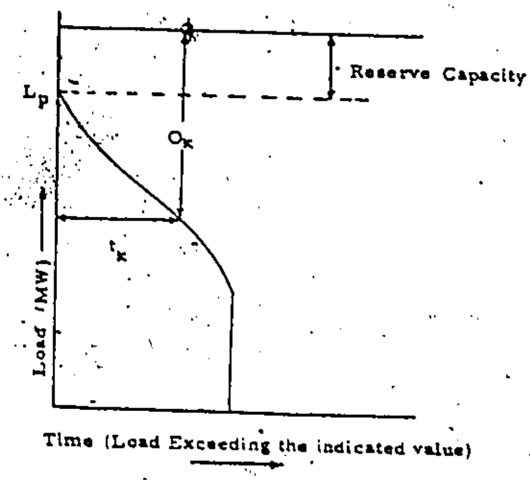

Figure 11: System load and Capacity

where O_K is a magnitude of the outage in the system capacity outage table at the kth impulse, p_K is the probability of an outage of capacity equal to Ok and t_K is the number of time units in the interval that an outage of magnitude O_K would cause loss of load.

A particular capacity outage will contribute to the expected load loss of the system by an amount equal to the product of the probability of existence of that particular outage and the number of time units in the study period that loss of load would occur if such a capacity outage were to exist. For outages less than reserve capacity, the contribution to load loss would be zero.

Suppose there are n number of impulses (probability of discrete values of capacity that can be out) in the capacity outage table at n discrete capacity outage values. The total load loss may be expressed mathematically by

$$E(t) = \sum_{k=1}^{n} P_k t_k$$

If the study period considered is in hours, the load loss would also be in hours. The period of study could be a year, month or a day, and correspondingly the loss of load will be in years, months or days respectively. This loss of load over a time period is usually referred to as loss of load expectation (LOLE).

There are three basic ways of applying the loss of load probability method to determine an annual level of system reliability.

1. Monthly basis considering maintenance
2. Annual basis neglecting maintenance
3. Worst period basis

In the monthly basis considering maintenance, peak load and load characteristic of a particular month are combined with the capacity outage PDF for that month. If the capacity has been taken out for maintenance, the capacity outage PDF is obtained from the rest of the capacity. If the capacity out for maintenance is not constant over the month, the month can be divided into several intervals having constant capacity taken out for maintenance. The load characteristics for the month is combined with the capacity outage PDF for each interval and then the LOLP is calculated by taking the average over the year.

In the annual approach neglecting maintenance, the annual forecasted peak is combined with the capacity outage probability density function to give an annual LOLP. Maintenance is neglected in this approach. A justification can be given for neglecting maintenance if the year can be divided into light load season and peak load season. The probability of load loss is very small in the light load season and maintenance can be performed in that period.

In the worst period approach, a worst month of the year is taken. Usually December is the worst month when annual peak occurs. The peak load and load characteristics of the worst month are combined with the capacity outage PDF. The year is considered to consist of 12 such worst months.

Many utilities do not use the load duration curve, rather, they use peak loads only. For example Ontario Hydro uses daily peak loads of the worst month of the year that is December. (If the outage is greater than reserve capacity, there is a loss of load.) By making a capacity outage table and calculating the daily LOLPs from daily peaks, the monthly LOLP is calculated by averaging over the month. This LOLP is considered to be the LOLP for the year.

A cumulative distribution table for the capacity outage is very useful in the evaluation of LOLP when daily peak loads are considered instead of a load duration curve. A cumulative distribution table can be constructed by integrating the density function from X to plus infinity, where X is the capacity outage random variable. The cumulative distribution function FC(X), which is defined as the probability of capacity being out more than or equal to the value X MW, may be written in the form

$$FC(X) = \int_{X}^{\infty} f(x)\, dx$$

where f(X) is density function for generating capacity outage

The cumulative distribution function, is a stepped curve and is shown in Fig. 12.

If RC is the reserve capacity, FC(RC) gives the probability of outage being greater than reserve capacity. Therefore FC(RC) is the LOLP of the system for a given peak.

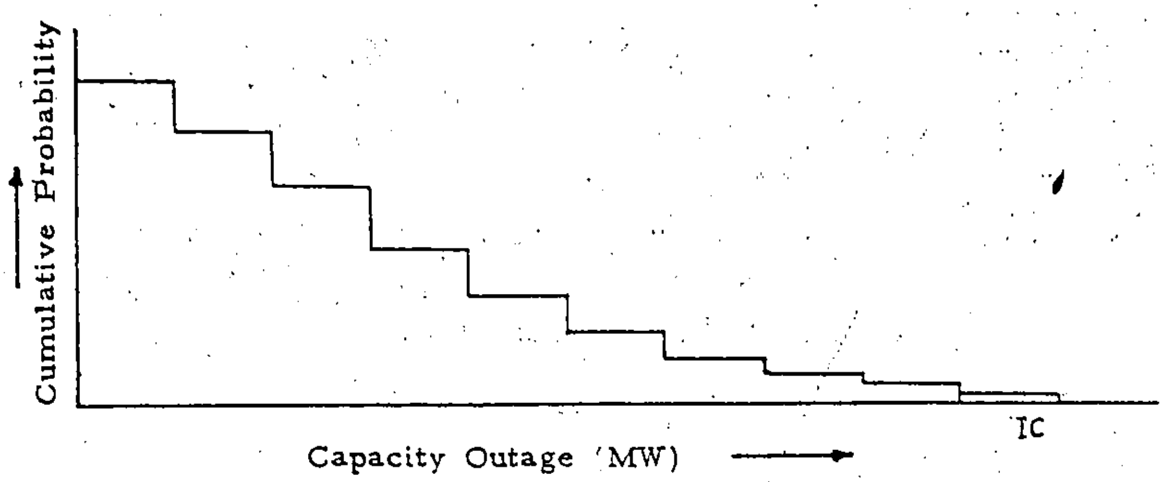

Figure 12: Cumulative Probability of Capacity Outage

3.3 EXPECTED ENERGY GENERATION

Another application of probabilistic methods in generation planning is to find out the expected energy generation by each unit as it is committed, according to a predetermined loading order, to meet the demand. Since the units are not 100% reliable, the exepected energy generated by a unit depends upon the forced outage rate of the unit, load shape and the priority of committment of the unit. The units with lowest marginal cost (incremental cost) are committed first[3,4] so that cheaper units generate maximum energy at cheaper cost. According to this order of committment of the units, the expected energy generated by each unit is calculated by simulating load duration curve and unit outages.

The load shape and the forced outage determine the energy generate by a particular unit. For a two state (up and down) model of the unit, let the probability of being available and unavailable of a unit i with capacity C_i be p_i and q_i respectively. Given a load probability distribution function F(L), the expected energy generated by first unit would be

$$E_1 = p_1 T \int_0^{C_1} F(L)\ dL$$

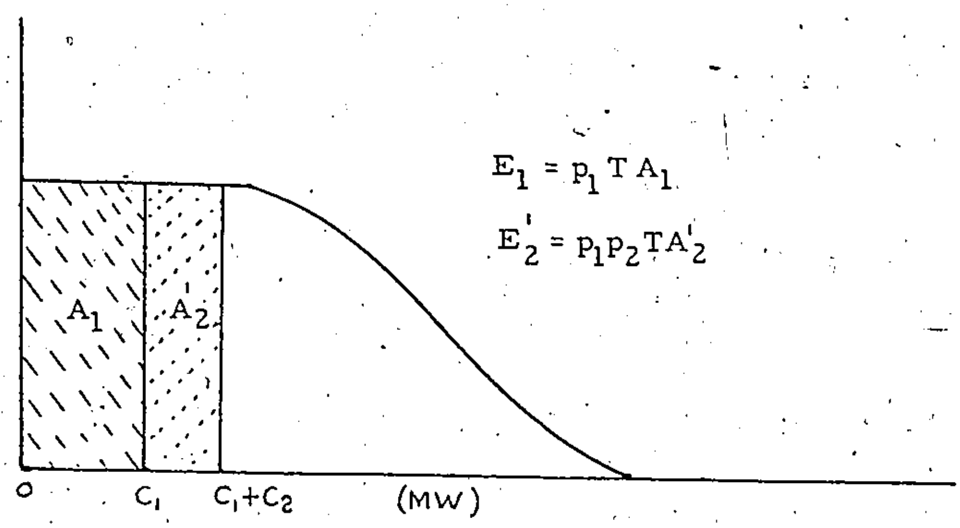

Figure 13: Energy Generated by Unit 1 and 2

where T is the time period over which the LDC is obtained. Figure 13 shows the expected energy generated by first unit. For the evaluation of the expected energy generated by second unit, there are two possibilities

(1) Unit no.1 has been committed as shown in Fig.13. The probability of its being in service is p_1. The expected energy generated by unit no.2 is given by

$$E_2' = T\, p_1\, p_2 \int_{C_1}^{C_1+C_2} F(L)\, dL$$

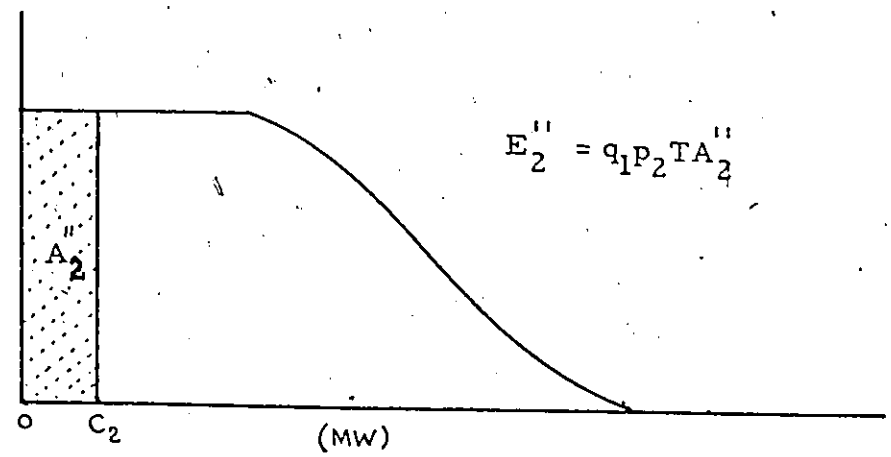

Figure 14: Energy Generated by Unit 2

(2) Unit no.1 has not been committed due to forced outage as shown in Fig.14. The probability of its being out of service is q_1. The expected energy generated by unit no.2 is given by

$$E_2'' = T q_1 p_2 \int_0^{C_2} F(L)\, dL$$

The total expected energy generated by unit no.2 is:

$$E_2 = E_2' + E_2'' = Tp_1p_2 \int_{C_1}^{C_1+C_2} F(L)\, dL + Tq_1p_2 \int_0^{C_2} F(L)\, dL$$

$$= Tp_2\left[p_1 \int_{C_1}^{C_1+C_2} F(L)\, dL + q_1 \int_0^{C_2} F(L)\, dL\right]$$

$$= Tp_2\left[p_1 \int_{C_1}^{C_1+C_2} F(L)\, dL + q_1 \int_{C_1}^{C_1+C_2} F(L-C_1)\, dL\right]$$

$$= Tp_2\left[\int_{C_1}^{C_1+C_2} \{p_1\, F(L) + q_1 F(L-C_1)\}\, dL\right]$$

$$= Tp_2 \int_{C_1}^{C_1+C_2} F^{(1)}(L)\, dL$$

where

$$F^{(1)}(L) = p_1 F(L) + q_1 F(L-C_1)$$

and $F^{(1)}(L)$ is defined as the equivalent load distribution function resulting by adding the effect of the random outage of the first unit. The same expression may be derived by convolving the outage density function of the first unit with the load probability distribution. By recursive methods, the equivalent load distribution function ($F^i(L)$) of the ith unit and the system load can be determined from the outage PDF of ith unit and the equivalent load distribution function ($F^{i-1}(L)$) of i-1 units and system load is given below

$$F^i(L) = p_i F^{i-1}(L) + q_i F^{i-1}(L-C_i)$$

The expected energy generated by ith unit is given by

$$E_i = p_i T \int_{IC_{i-1}}^{IC_i} F^{i-1}(L)\, dL$$

where $IC_i = \sum_{j=1}^{i} C_j$

3.3.1 Units with Multiblocks and Multistates

In the multiblock representations of a unit, the unit is divided into several blocks and each block is committed separately [23]. Each block may have different incremental cost. A priority list of the committment of the blocks is prepared in the ascending order of their incremental cost. At the same time it should never be overlooked that the upper block of a unit can not be committed before its lower blocks have been committed. Two consecutive blocks of a unit may not occupy adjacent positions in the priority list of their committment.

In a multistate representation, the unit capacity availability or unavailability models have more than two states. The PDF for multistate model has already been shown in Fig.4. A three block representation of a multistate unit having PDF of available and unavailable capacity is shown in Fig. 15 a and b. respectively.

The PDF of available and unavailable capacity for the first block is shown in Fig.16. Suppose the first block of this unit, of 150 MW, is committed at the ith priority, the expected energy generated by the first block may be given as

$$E_1 = T\left[0.05\int_{IC_{i-1}}^{IC_{i-1}+100} F^{i-1}(L)\,dL + 0.85\int_{IC_{i-1}}^{IC_{i-1}+150} F^{i-1}(L)\,dL\right]$$

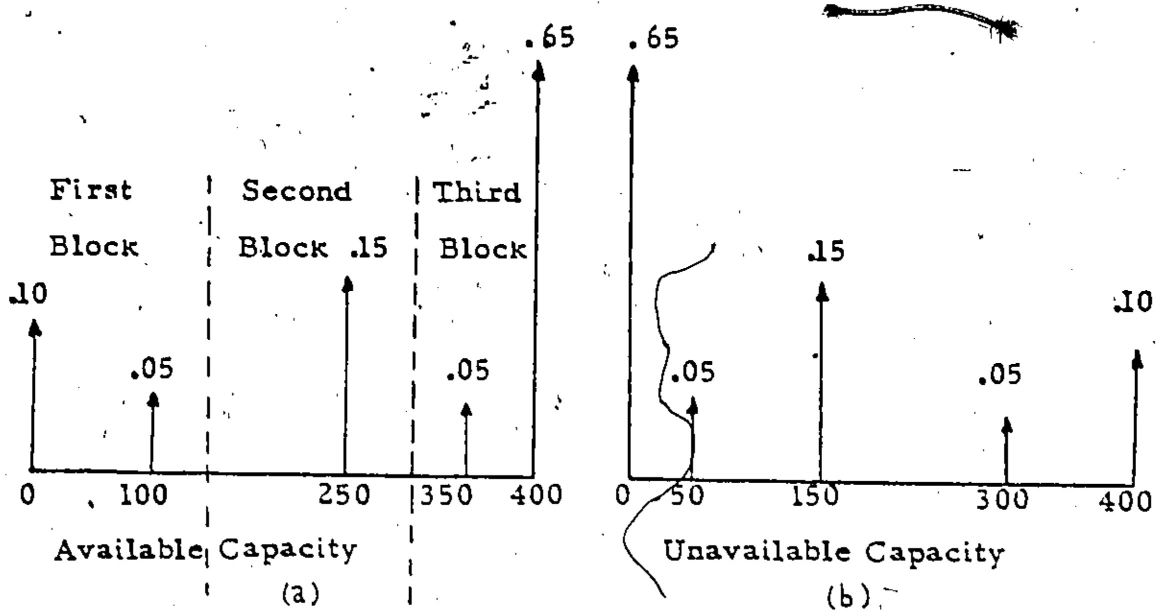

Figure 15 : The PDF of available and unavailable capacity of multistate units in multiblocks

After evaluating the expected energy generated by the first block, the effect of the outage of this block must be added into the equivalent load distribution function. This can be simulated by convolving the PDF of the outage of this block to the equivalent load distribution function $F^{i-1}(L)$. The PDF of available and unavailable capacity of the combined lower and middle block is shown in Fig. 17 a and b, respectively.

Before calculating the expected energy generated by the second block, the effect of the outage of the first block

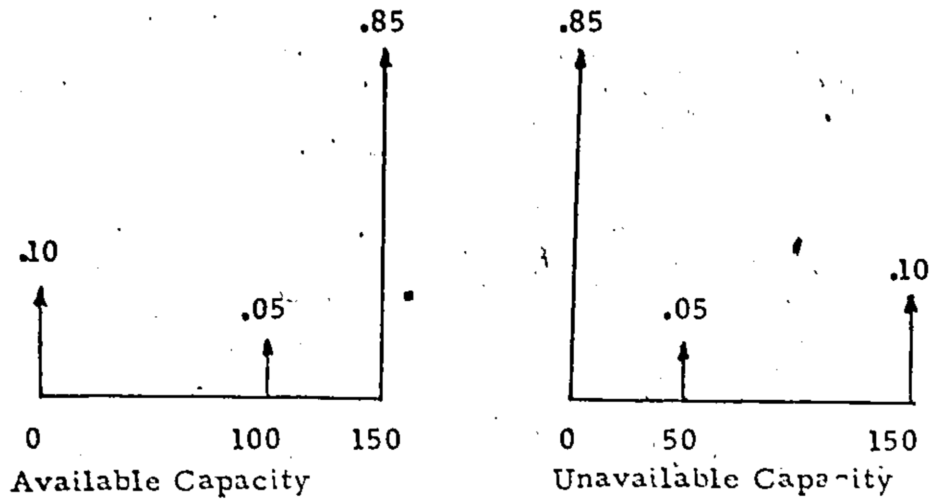

Figure 16 : The PDF of Available and Unavailable Capacity of the first Block

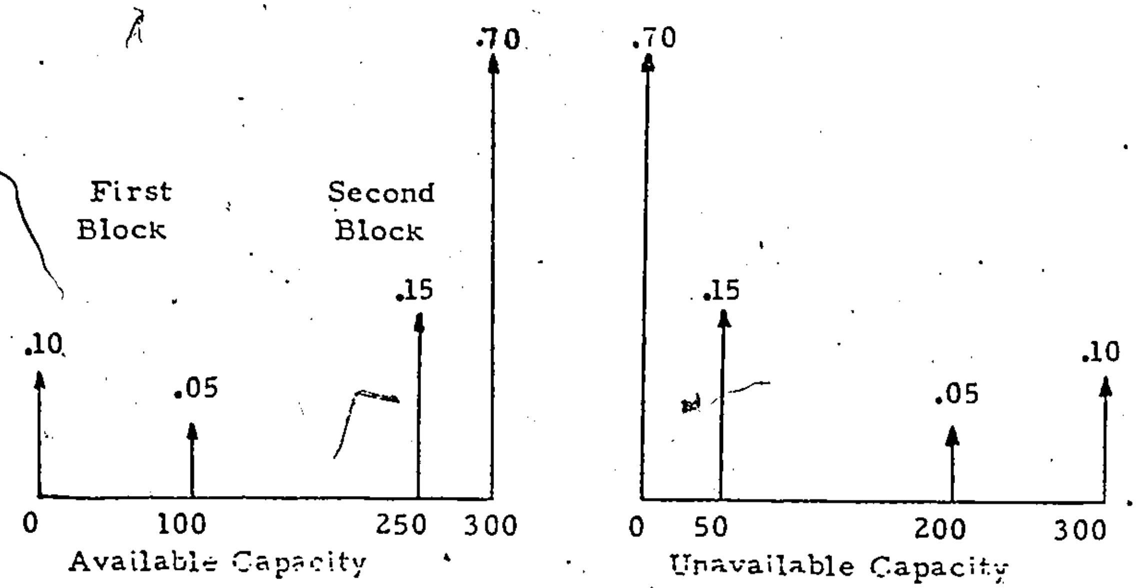

Figure 17 : The PDF of Available and Unavailable Capacity of Lower and Middle Blocks Combined

must be taken out from the equivalent load distribution function. This can be simulated by deconvolving (subtraction of cumulants in the Gram-Charlier method) the PDF of capacity outage of the first block with the equivalent load distribution function. Similarly, before calulating the expected energy generated by the third block, the effect of the first and second blocks combined must be taken out.

3.4 EXPECTED PRODUCTION COSTS

The expected production costs may be determined accurately only if a realistic load model is known for each future week in the study period and units are committed in a manner that reflects the actual operating procedures and conditions. Production costs of energy consists of fixed costs (operating and maintenance) and fuel costs. The operating and maintenance costs are hardly affected by the order of priority of committment of the units supplying a load. The energy cost may be expressed as (see Sullivan, [7])

$$EC = FC + OM$$

where FC is the fuel cost and OM is the Operation and Maintenance costs

Fuel costs of the system depend upon the type of fuel and the incremental heat rate. A typical input/output curve is shown in Fig. 18. The ordinate gives the amount of heat energy required in MBtu per hour to generate the corresponding MW power given on the abscissa. The incremental heat rate curve for a unit is obtained from the input/output curve by the relationship,

$$IHR_i = \frac{dIH_i}{dP_i} \quad \text{Btu / MWh}$$

where

IHR_i is the incremental heat rate, IH_i is the input heat in MBtu per hour and P_i is the power output in MW. Figure 19 shows a typical incremental heat rate curve,

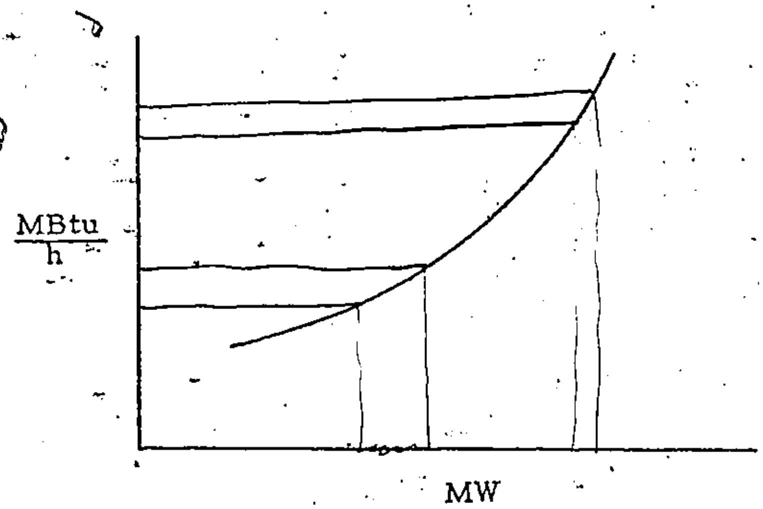

Figure 18:

The incremental fuel cost is calculated from the incremental heat rate. It is given by,

$$IFC_i = IHR_i \frac{\text{Unit Fuel Cost}}{\text{Heat Value}}$$

In economic dispatch, the units are loaded such that the incremental fuel cost is same for all the units [26]. In economic dispatch problems, the objective function is to minimize the fuel costs. Let the total fuel costs of the plant be F_T such that

$$F_T = \sum_{i=1}^{n} F_i \tag{3.1}$$

where

F_i is the fuel cost for unit i.

The total power output is constant and is given by

$$P_T = \sum_{i=1}^{n} P_i \tag{3.2}$$

where

P_i is the power generated by unit i.

To obtain a minimum F_T for a given P_T, it is required that $dF_T = 0$.

Since total fuel cost is dependent on the power output of each unit,

$$dF_T = \frac{\partial F_1}{\partial P_1} dP_1 + \frac{\partial F_2}{\partial P_2} dP_2 \text{ - - - - - - - - - - - } + \frac{\partial F_n}{\partial P_n} dP_n = 0 \qquad (3.3)$$

The total power P_T is constant. Therefore,

$$\sum_{i=0}^{n} dP_i = 0 \qquad (3.4)$$

multiplying Eqn. (3.4) by λ and substituting the resulting equation from Eqn (3.3), yields,

$$\sum_{i=0}^{n} \left(\frac{\partial F_i}{\partial P_i} - \lambda \right) dP_i = 0 \qquad (3.5)$$

This Eqn. (3.5) is satisfied if each term

$\left(\frac{\partial F_i}{\partial P_i} - \lambda \right) dP_i$ is zero. Therefore,

$$\frac{\partial F_i}{\partial P_i} = \lambda = IFC_i$$

and so all units must operate at the same incremental fuel cost λ for minimum total cost.

The incremental fuel cost ($IFC_i = \lambda$), is a variable quantity, is directly proporational to the incremental heat rate (IHR_i). To simulate the effect of variable IFC_i for production costing in generation expansion planning, the simplest procedure is to calculate the average or expected IFC_i ($E(IFC_i)$*) over the capacity of the unit. A more accurate but more complicated technique is to divide the unit into capacity blocks and then calculate the average IFC_i over the capacity of each block. A large number of blocks gives better accuracy at the expense of huge computational effort.

The fuel cost of unit i is given by,

$$FC_i = E_i E(IFC_i)$$

The total cost of the energy generated by unit i may be written as,

$$EC_i = FC_i + OM_i$$

---

*The letter E(.) will be used to denote statistical expectation of a random variable.

The expected production costs of the system may, therefore, be expressed as

$$\sum_{i=1}^{N} E_i E(IFC_i) + \sum_{i=1}^{N} OM_i$$

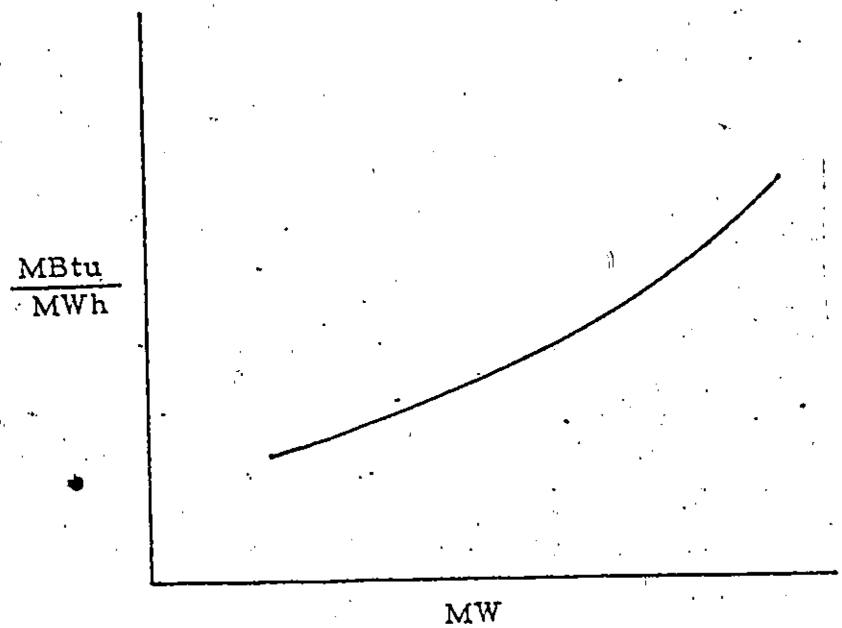

Figure 19 : A Typical Incremental Heat Rate Curve

Chapter IV

GRAM-CHARLIER EXPANSION

4.1 INTRODUCTION

Various kinds of series expansions and approximations are available [12] to represent a density function. In this chapter an expansion known as the Gram-Charlier series will be dicussed. The expansion uses the cumulants of the PDF to be approximated, by the normal PDF and its derivatives. Essentially, the normal PDF and its derivatives comprise a set of orthogonal polynomials. The cumulants, their important properties, the convergence of this series and the concept of skewness of a distribution are discussed in this chapter.

4.2 THE GRAM-CHARLIER EXPANSION

The Gram-Charlier series is an orthogonal expansion of the normal PDF and its derivatives [12]. Let

x = RV with PDF $f(x)$

μ = mean of x

σ = standard deviation

μ_r = rth central moment

Consider the standardized RV $z=\frac{x-\mu}{\sigma}$ and the corresponding density function $f(z)$. For the PDF $f(z)$ consider the expansion of the form

$$f(z) = C_0 N(z) + \frac{C_1}{1!} N^{(1)}(z) + \frac{C_2}{2!} N^{(2)}(z) + ---- \tag{4.1}$$

where

C_r are constant coefficients

and

$N^{(r)}(z)$ is the rth derivative of the normal PDF $N(z)$. The derivatives may be obtained from the relation,

$$N^{(r)}(z) = (-1)^r H_r(z)\ N(z) \tag{4.2}$$

where

$H_r(z)$ is Hermite polynomial of degree r. The Hermite polynomials are defined [17] by the relationship

$$\frac{d^r}{dz^r} e^{-\frac{z^2}{2}} = (-1)^r H_r(z) e^{-\frac{z^2}{2}} \tag{4.3}$$

The higher order Hermite polynomials may be obtained from the recursive formula;

$$H_{r+1}(z) = z\, H_r(z) - r\, H_{r-1}(z) \qquad (4.4)$$

As Hermite polynomials are a sequence of orthogonal polynomials, they satisfy the following relationship

$$\frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} H_m(z) H_n(z)\, e^{\frac{-z^2}{2}}\, dz = \begin{cases} n! & \text{for } m = n \\ 0 & \text{for } m \neq n \end{cases} \qquad (4.5)$$

Assuming that the series (4.1) may be integrated term by term, the coefficients may be obtained from the orthogonality condition of Eqn. (4.5).

Multiplying the terms of Eqn. (4.1) with $H_r(z)$ and integrating, and using the orthogonality relationship (4.5) one readily obtains

$$C_r = (-1)^r \int_{-\infty}^{\infty} H_r(z)\, f(z)\, dz \qquad (4.6)$$

Since $f(z)$ is the PDF of the standardized RV $z = \frac{x-\mu'}{\sigma}$, which has zero mean and unit standard deviation, its rth moment would be equal to $\frac{\mu_r}{\sigma^r}$.

Accordingly, it may be found that

$c_0 = 1$

$c_1 = c_2 = 0$

so that

$$f(Z) = N(z) + \frac{C_3}{3!} N^{(3)}(z) + \frac{C_4}{4!} N^{(4)}(z) + \cdots \tag{4.7}$$

For $r>2$, the coefficients C_r are given by (4.6), such that

$$C_3 = (-1)^3 \int_{-\infty}^{\infty} H_3(z)\, f(z)\, dz$$

$$= (-1)^3 \int_{-\infty}^{\infty} (z^3-3z)\, f(z)\, dz$$

$$= -\int_{-\infty}^{\infty} z^3 f(z)\, dz + 3 \int_{-\infty}^{\infty} z f(z)\, dz$$

$$= -\frac{\mu_3}{\sigma^3}$$

and

$$C_4 = (-1)^4 \int_{-\infty}^{\infty} H_4(z)\ f(z)\ dz$$

$$= \frac{\mu_4}{\sigma^4} - 6\frac{\mu_2}{\sigma^2} + 3$$

$$= \frac{\mu_4}{\sigma^4} - 6 + 3 = \frac{\mu_4}{\sigma^4} - 3$$

Similarly

$$C_5 = -\frac{\mu_5}{\sigma^5} + 10$$

$$C_6 = \frac{\mu_6}{\sigma^6} - 15\frac{\mu_4}{\sigma^4} + 30$$

Thus the Gram-Charlier series may be written in the form (up to the sixth derivative)

$$f(z) = N(z) - \frac{1}{3!}\frac{\mu_3}{\sigma^3}N^{(3)}(z) + \frac{1}{4!}\left(\frac{\mu_4}{\sigma^4} - 3\right)N^{(4)}(z) - \frac{1}{5!}\left(\frac{\mu_5}{\sigma^5} - 10\frac{\mu_3}{\sigma^3}\right)$$

$$N^{(5)}(z) + \frac{1}{6!}\left(\frac{\mu_6}{\sigma^6} - 15\frac{\mu_4}{\sigma^4} + 30\right)N^{(6)}(z) + ----- \qquad (4.8')$$

4.3 CUMULANTS AS THE COEFFICIENTS IN THE SERIES

The coefficients in the Gram-Charlier series may be written in terms of cumulants of the PDF. The cumulants $k_1, k_2, - - -, k_r, - -$ are defined by the identity

$$\exp\left\{k_1(it) + k_2 \frac{(it)^2}{2!} + \cdots\cdots + k_r \frac{(it)^2}{r!} + \cdots\cdots\right\}$$

$$= 1 + \mu_1' \frac{(it)}{1!} + \cdots + \mu_r' \frac{(it)^r}{r!} + \cdots\cdots$$

$$= \int_{-\infty}^{\infty} e^{itx} f(x)\, dx$$

$$= \phi(t) \qquad (4.9)$$

where

$\phi(t)$ is the characteristic function of the RV X, and μ_r' is the rth moment about the origin. From Eqn. (4.9)

$$\log \phi(t) = \sum_{r=1}^{\infty} \frac{k_r}{r!} (it)^r \qquad (4.10)$$

In order to deduce the relationship between the moments μ_r' and the cumulants k_r, identity (4.9) is used as follows,

$$1 + \sum_{r=1}^{\infty} \mu_r' \frac{(it)^r}{r!} = \exp \sum_{r=1}^{\infty} k_r \frac{(it)^r}{r!}$$

$$= \exp\{\frac{k_1(it)}{1!}\}\exp\{\frac{k_2(it)^2}{2!}\}\cdots\exp\{\frac{k_r(it)^r}{r!}\}$$

$$= \{1+\frac{k_1(it)}{1!} + \frac{k_1^2(it)^2}{2!} + \cdots\cdots\}$$

$$\{1+\frac{k_2(it)^2}{2!} + \frac{1}{2!}\left(\frac{k_2(it)^2}{2!}\right)^2 + \cdots\cdots\}$$

$$\cdots\cdots\cdots$$

$$\{1+\frac{k_r(it)^r}{r!} + \frac{1}{2!}\left(\frac{k_r(it)^r}{r!}\right)^2 + \cdots\cdots\} \quad (4.11)$$

Picking out the terms in the exponential expansion which, when multiplied together, give a power of $(it)^r$ One obtains in particular,

$$\mu_1' = k$$
$$\mu_2' = k_2 + k_1^2$$
$$\mu_3' = k_3 + 3k_2k_1 + k_1^3$$
$$\mu_4' = k_4 + 4k_3k_1 + 3k_2^2 + 6k_2k_1^2 + k_1^4$$

and conversely

$$k_1 = \mu_1'$$
$$k_2 = \mu_2' - \mu_1'^2$$
$$k_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3$$
$$k_4 = \mu_4' - 4\mu_3'\mu_1' - 3\mu_2'^2 + 12\mu_2'\mu_1'^2 - 6\mu_1'^4$$

The relation between central moments and cumulants is given by,

$$\mu_2 = k_2$$
$$\mu_3 = k_3$$
$$\mu_4 = k_4 + 3k_2$$

and conversely

$$k_2 = \mu_2$$
$$k_3 = \mu_3$$
$$k_4 = \mu_4 - 3\mu_2$$

The relation between moments and central moments and, central moments and cumulants are given in Appendix A.

The Gram-Charlier expansion of a PDF in terms of cumulants may be written as

$$f(z) = N(z) - \frac{1}{3!}\frac{k_3}{\sigma^3} N^{(3)}(z) + \frac{1}{4!}\frac{k_4}{\sigma^4} N^{(4)}(z) - \frac{1}{5!}\frac{k_5}{\sigma^5} N^{(5)}(z) +$$

$$\frac{1}{6!}\left(\frac{k_6}{\sigma^6} + 10\left(\frac{k_3}{\sigma^3}\right)^2\right)N^{(6)}(z) - \frac{1}{7!}\left(\frac{k_7}{\sigma^7} + 35\frac{k_3}{\sigma^3}\cdot\frac{k_4}{\sigma^4}\right)N^{(7)}(z)$$

$$+ \frac{1}{8!}\left(\frac{k_8}{\sigma^8} + 56\frac{k_3}{\sigma^3}\cdot\frac{k_5}{\sigma^5} + 35\left(\frac{k_4}{\sigma^4}\right)^2\right) N^{(8)}(z) ---- \quad (4.12)$$

This may also be expressed as

$$f(z) = N(z) - \frac{1}{3!} G_1 N^{(3)}(z) + \frac{1}{4!} G_2 N^{(4)}(z) - \frac{1}{5!} G_3 N^{(5)}(z)$$

$$+ \frac{1}{6!} (G_4 + 10 G_1^2) N^{(6)}(z) - \frac{1}{7!} (G_5 + 35 G_1 G_2) N^{(7)}(z)$$

$$+ \frac{1}{8!} (G_6 + 56 G_1 G_3 + 35 G_2^2) N^{(8)}(z) + ------ \quad (4.13)$$

where

$$G_{r-2} = \frac{k_r}{\sigma^r}$$

G_{r-2} may also be expressed as the rth cumulant of the PDF $f_Z(z)$.

There is no conceptual difficulty in evaluating more terms in this series.

4.4 CONVERGENCE OF THE SERIES

The convergence of the Gram-Charlier series has been discussed by Cramer [18]. The conditions under which this series converges are given in the following theorems,

(1) If f(x) is a function which has a continuous derivative such that

$$\int_{-\infty}^{\infty} \left(\frac{df}{dx}\right)^2 e^{\frac{1}{2}x^2} dx$$

converges and if f(x) tends to zero as |x| tends to infinity, then f(x) may be expressed as

$$f(x) = \sum_{r=o}^{\infty} \frac{C_r}{r!} N^{(r)}(x) \qquad (4.14)$$

where

$$C_r = \int_{-\infty}^{\infty} f(x)\, H_r(x)\, dx \tag{4.15}$$

This series is absolutely and uniformly convergent for $-\infty \leq x < \infty$.

(2) If f(x) is of bounded variation in every finite interval and if

$$\int_{-\infty}^{\infty} |f(x)|\, e^{\frac{1}{4}x^2}\, dx \tag{4.16}$$

exists, then the expansion of f(x) in series (4.14) converges everywhere to the sum $\frac{1}{2}(f(x+0)+f(x-0))$. This convergence is uniform in every finite interval of continuity.

From the statistical viewpoint, however, it is more important to know whether a few number of terms of the series gives a good approximation or not. Barton and Dennis [19] and also Draper and Tieney [20] discussed about the non-negative properties when the following terms in the series are used:

$$f(z) = N(z) - \frac{G_1}{3!} N^{(3)}(z) + \frac{G_2}{4!} N^{(4)}(z) \tag{4.17}$$

$$f(z) = N(z) - \frac{G_1}{3!} N^{(3)}(z) + \frac{G_2}{4!} N^{(4)}(z) - \frac{G_3}{5!} N^{(5)}(z) + \left(\frac{G_4+10G_1^2}{6!}\right) N^{(6)}(z) \tag{4.18}$$

It has been shown [19,20] that (4.17) or (4.18) do not give unimodal or non-negative values if the skewness and the kursosis coefficients $\beta_1 = \frac{\mu_3}{\sigma^3}$ and $\beta_2 = \frac{\mu_4}{\sigma^4}$ are not close enough to zero. It has also been shown that (4.18) has negative values if $|\beta_1| \geq 0.5$, and (4.17) has negative values if $|\beta_1| > 1.2$. Both have negative values if $\beta_2 > 4$.

Practically, this region of non-negative values is very small. In most of the cases [12] these negative values occur in the tails of a distribution where the probabiliy value is very small.

Cramer [17] discussed the expansion of the PDF f(x) of the sum of several indepedent RVs. Consider a RV X, which is the sum

$$X = X_1 + X_2 + X_3 + \cdots X_n$$

of n independent RVs. Also consider that all the RVs have the same distribution $f_1(x)$ with the mean μ_1, the standard deviation σ_1, and the rth cumulant k_r', so that $\mu = n\mu_1$, $\sigma = \sigma_1\sqrt{n}$.

An explicit expression for C_r in terms of k_r may be obtained from the identity [17],

$$\sum_{r=0}^{\infty} \frac{C_r}{r!}(-it)^r = \sum_{h=0}^{\infty} \frac{n^h}{h!}\left[\sum_{r=3}^{\infty} \frac{k_r'}{r!}\left(\frac{it}{\sqrt{n}}\right)^r\right]^h \qquad (4.19)$$

where

c_r is the rth coefficient of f(x) in the Gram-Charlier expansion.

Eqn. (4.19) shows that C_r is of the form

$$C_r = \frac{a_{r_1} n + a_{r_2} n^2 + --- + a_{r[r/3]} n^{[r/3]}}{n^{r/2}} \qquad (4.20)$$

where [r/3] denotes the greatest integer $\leq$ r/3, while the a_{rk} are polynomials in k_r' which are independent of n. Thus

$$C_r = O(n^{[r/3] - r/2}) \qquad (4.21)$$

The order of magnitude of the terms of the Gram-Charlier series (also called A-series) is not steadilily decreasing as r increases. This is clear from Eqn. (4.21).

For example $C_4 = O(n^{-\frac{5}{2}})$. $C_{12} = O(n^{-2})$

$C_{13} = O(n^{-\frac{5}{2}})$

The first few coefficients may be written as,

$$C_3 = \frac{k_3'}{n^{\frac{1}{2}}} \qquad C_5 = \frac{k_5'}{n^{\frac{3}{2}}}$$

$$C_4 = \frac{k_4'}{n} \qquad C_6 = \frac{k_6'}{n^2} + \frac{10\, k_3'}{n}$$

To take into account the partial sum of all the terms of the Gram-Charlier series involving a correction to N(x) of order $n^{-\frac{1}{2}}$ and n^{-1}, the following terms of the series are required,

$$f(x) = N(x) - \frac{G_1}{3!} N^{(3)}(x) + \frac{G_2}{4!} N^{(4)}(x) + \frac{10 G_1^2}{6!} N^{(6)}(x) \qquad (4.22)$$

which is called the Edgeworth form of the Gram-Charlier series. Another series similar to the Gram-Charlier series which gives a straightforward expansion in powers of n is discussed by Edgeworth [12].

The expansion of the function [17]

$$e^{\frac{t^2}{2}} \phi(z) = e^{n \sum_{r=3}^{\infty} \frac{k'_r}{r!} \left(\frac{it}{\sqrt{n}}\right)^r} \qquad (4.23)$$

in powers of t furnishes expressions of the coefficients C_r in the Gram-Charlier series. The same function (4.23) can however, be expanded in powers of $n^{-\frac{1}{2}}$,

$$e^{\frac{t^2}{2}} \phi(z) = e^{(it)^2 \sum_{1}^{\infty} \frac{k'_{r+2}}{(r+2)!} \left(\frac{it}{\sqrt{n}}\right)^r}$$

$$= \sum_{h=0}^{\infty} \frac{(it)^{2h}}{h!} \left[\sum_{r=1}^{\infty} \frac{k'_{r+2}}{(r+2)!} \left(\frac{it}{\sqrt{n}}\right)^r \right]^h \qquad (4.24)$$

After some development one obtains

$$\phi(t) = e^{-\frac{t^2}{2}} + \sum_{r=1}^{\infty} \frac{b_{r,r+2}\,(it)^{r+2} + \dots + b_{r,3r}\,(it)^{3r}}{n^{\frac{r}{2}}}\, e^{-\frac{t^2}{2}} \tag{4.25}$$

where

$b_{r,r+2k}$ is a polynomial in $k_3', \text{-------}, k_{r-k+3}'$ which is independent of n. By the integral relation

$$\int_{-\infty}^{\infty} e^{itx}\, N^{(r)}(x)\, dx = (-it)^r\, e^{-\frac{t^2}{2}} \tag{4.26}$$

one finds that (4.26) corresponds to the expansion in powers of $n^{-\frac{1}{2}}$, that is

$$f(x) = N(x) + \sum_{r=1}^{\infty} (-1)^r \left(\frac{b_{r,r+2}\, N^{(r+2)}(x) + \dots + b_{r,3r}\, N^{(3r)}(x)}{n^{\frac{r}{2}}} \right) \tag{4.27}$$

The first few terms of which are, writing all terms of a certain order with respect to n on the same line

$$\begin{aligned} f(x) &= N(x) \\ &- \frac{1}{3!} \frac{k_3'}{n^{\frac{1}{2}}} N^{(3)}(x) \\ &+ \frac{1}{4!} \cdot \frac{k_4'}{n} N^{(4)}(x) + \frac{10}{6!} \frac{k_3'^2}{n} N^{(6)}(x) \\ &- \frac{1}{5!} \cdot \frac{k_5'}{n^{\frac{3}{2}}} N^{(5)}(x) - \frac{35}{7!} \frac{k_3' k_4'}{n^{\frac{3}{2}}} N^{(7)}(x) - \frac{280}{9!} \frac{k_3'^3}{n^{\frac{3}{2}}} N^{(9)}(x) \end{aligned}$$

The first four terms of the series (4.28) are the same as (4.22), the Edgeworth form of the Gram-Charlier series. The Edgeworth series is, therefore, nothing more than a rearrangement of terms of the Gram-Charlier series.

The asymptotic properties of the Edgeworth series, in powers of $n^{-\frac{1}{2}}$ have been given by Cramer [24]. Let f(x) be an exact PDF of the sum of n independent RVs, all having same PDF $f_1(x)$ and $g_{rn}(x)$ be the Edgeworth series approximation of f(x) truncated after the C_r coefficient. Under fairly general conditions the error may be given by

$$R_{rn}(x) = \left| f(x) - g_{rn}(x) \right| < \frac{M}{n^{\frac{r-2}{2}}} \qquad (4.29)$$

where

M depends upon r and on the given PDF f(x), but it is independent of n and x.

for r = 3

$$R_{3n} = \left| f(x) - g_{3n}(x) \right| < \frac{M}{\sqrt{n}} \qquad (4.30)$$

It may be noted from here that if n is increased, the error in the approximation would be smaller.

TRUNCATION ERROR:

Springer [22] described an error analysis of the distribution function when it is approximated by series expansions. The analysis is based on Fourier theorems. Let F(x) be the true distribution function and G(x) be its Gram-Charlier series approximation over the interval (0,1).

The error may be given [22] by

$$S(x) = G(x) - F(x) = \sum_{n=1}^{\infty} d_n \sin (n\ \theta), \quad 0 \le x \le 1 \tag{4.31}$$

where

$$\theta = \cos^{-1}(2x-1)$$

$$d_n = (-1)^k \left(\frac{2}{\pi}\right) \sum_{j=0}^{k} \frac{(-4)^j\ (j+k-1)!}{(k-j)!\ (2j)!} (\mu'_{kf}-\mu'_{kg}) \tag{4.32}$$

In Eqn. (4.32) μ'_{kg} and μ'_{kf} are the kth moments of the approximating and exact PDFs g(x) and f(x) respectively.

IF Y is a RV over the interval $(0,\infty)$, a transformation of the type

$$X = e^{-Y} \qquad (4.33)$$

is required to give a RV X over the inverval (0,1).

Taking the first n terms of the Fourier series in Eqn. (4.31)

$$S(x) = \sum_{k=1}^{n} d_k \sin(k\theta) \qquad 0 \leq x \leq 1 \qquad (4.34)$$

This may also be written in the form

$$S(x) = 2 U_1 \sin\theta \qquad (4.35)$$

where U_1 is found from the recurrence relation

$$U_k = d_k + 2 (\cos\theta) U_{k+1} - U_{k+2} \qquad (4.36)$$

and $U_{n+2} = U_{n+1} = 0$

In the particular case when

$$\theta = \cos^{-1}(2x - 1)$$

the recurrence relation becomes

$$U_k = d_k + (4x - 2)\, U_{k+1} - U_{k+2} \qquad (4.37)$$

and the desired error approximation is then

$$S = -2U_1 \sqrt{x - x^2} \qquad (4.38)$$

The accuracy of the error depends upon the truncation of series, rounding of the Fourier coefficients and the method of evaluating the truncated series. The Fourier coefficients are evaluated until they are no longer significantly relative to the the desired accuracy. A very high degree of accuracy and a very large number of Fourier coefficients are required, which may be very time consuming process.

4.5 CONVOLUTION SIMULATED BY ADDITION OF CUMULANTS

The cumulants exhibit a very useful property that the sum of independent RVs is characterized by the cumulants which are the sum of individual cumulants. Therefore the process of convolution is simply the addition of cumulants. The PDF of a RV which is sum of a large number of independent RVs is obtained by the convolution of PDFs of independent RVS. The cumulants of the PDF obtained are the sum of the appropriate cumulants of the PDFs of independent RVs. Deconvolution may be conceived as subtraction of cumulants. Convolution as addition of cumulants is described in what follows,

Suppose X_1, X_2 ------X_N are N independent RVs with PDFs $f_1(x_1)$, $f_2(x_2)$ --------, $f_N(x_N)$ respectively. Further assume that $\phi_1(t)$, $\phi_2(t)$ $\phi_3(t)$ ------- $\phi_N(t)$ are characteristic functions of RVs X_1, X_2 -------X_N respectively. The sum of independent RVs, X its PDF f(x), and characteristic function $\phi(t)$ are given by

$$X = X_1 + X_2 + X_3 + \cdots + X_n + \cdots + X_N \tag{4.39}$$

$$f(x) = (f_1 * f_2 * \cdots * f_n * \cdots * f_N)(x) \tag{4.40}$$

$$\phi(t) = \phi_1(t) \cdot \phi_2(t) \cdot \cdots \cdot \phi_n(t) \cdot \cdots \cdot \phi_N(t) \tag{4.41}$$

From Eq. (4.9)

$$\phi_n(t) = e^{\sum_{r=1}^{\infty} k_{n,r} \frac{(It)^r}{r!}} \tag{4.42}$$

where $k_{n,r}$ is the rth cumulant of the RV X Therefore

$$\phi(t) = \prod_{1}^{N} e^{\sum_{r=1}^{\infty} k_{n,r} \frac{(it)^r}{r!}} \tag{4.43}$$

$$\phi(t) = e^{\sum_{n=1}^{N} \sum_{r=1}^{\infty} k_{n,r} \frac{(it)^r}{r!}} \tag{4.44}$$

Also

$$\phi(t) = e^{\sum_{r=1}^{\infty} k_r \frac{(it)^r}{r!}} \tag{4.45}$$

where k is the rth cumulant of the RV X. From (4.44) and (4.45)

$$e^{\sum_{r=1}^{\infty} k_r \frac{(it)^r}{r!}} = e^{\sum_{n=1}^{N} \sum_{r=1}^{\infty} k_{n,r} \frac{(it)^r}{r!}} \tag{4.45a}$$

which gives

$$k_r = \sum_{n=1}^{N} k_{n,r} \tag{4.46}$$

4.6 CUMULANTS AND TRANSFORMATION

Using a linear transformation of the kind

$$Y = aX+b$$

the cumulants associated with Y are found from:

$$e^{bit}\phi\,(at) = e^{\sum_{r=1}^{\infty} \frac{k'_r}{r!} (it)^r} \qquad (4.47)$$

$$\phi\,(at) = e^{-bit + \sum_{r=1}^{\infty} \frac{k'_r}{r!} (it)^r} \qquad (4.48)$$

where k'_r is rth cumulant of the RV Y. Also the characteristic function of the RV X is given by

$$\phi\,(t) = e^{\sum_{r=1}^{\infty} \frac{k'_r}{r!} (it)^r} \qquad (4.49)$$

comparing Eqn. (4.48) and (4.49)

$$k'_1 = a\,k_1 + b$$

$$k'_r = a^r k_r \qquad \text{for } r > 1$$

Therefore the effect of the change of origin is only on the first cumulant which is equal to the first moment.

4.7 SKEWNESS AND KURTOSIS OF A DISTRIBUTION

The first four cumulants of a distribution function describe significant characteristics of the distribution. The first cumulant is the mean (μ), whose standardized value is set to zero. The second cumulant is the variance (σ^2), whose standardized value is set to one. The third and fourth cumulants describe the skewness and kurtosis, respectively.

The standardized third cumulant is given by

$$G_1 = \frac{k_3}{\sigma^3} = E\left(\frac{(X-\mu)^3}{\sigma^3}\right) \qquad (4.50)$$

where

E(.) is the expected value.

G_1 is negative for a distribution skewed towards the left. It is zero for a symmetric distribution and positive for a distribution skewed towards the right. Fig. 20 shows the distribution skewed towards the left, symmetric and skewed towards the right in (a), (b) and (c) respectively.

The standardized fourth cumulant describing kurtosis is given by

$$G_2 = \frac{k_4}{\sigma^4} = E\left(\frac{(x-\mu)^4}{\sigma^4}\right) - 3 \qquad (4.51)$$

G_2 is zero for the Normal distribution. If G_2 is negative, the distribution is platykurtic. It has more pronounced "shoulders" and is flat topped as compared to the Normal. If G_2 is positive, the distribution is leptokurtic. A leptokurtic distribution has less pronounced "shoulders" and heavier tails than the Normal. It a has sharper peak. For the Normal distribution all the cumulants except the first and second are zero. Fig. 21 shows the kurtosis for three different distributions.

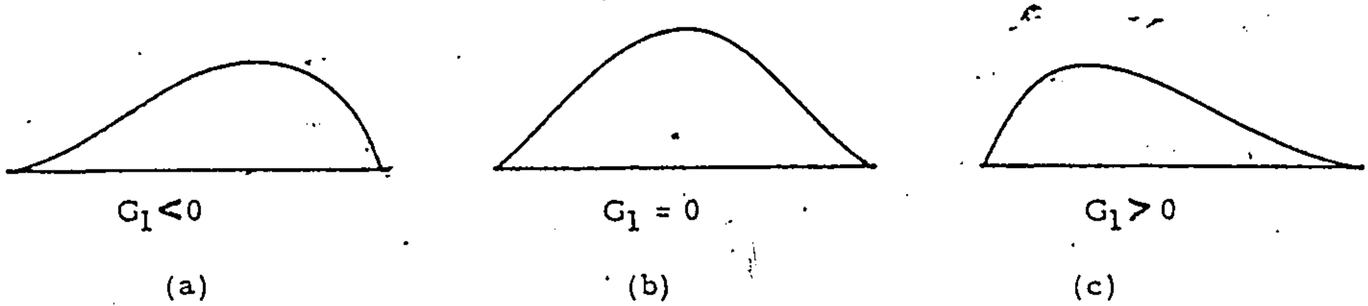

Figure 20: Skewness

Further, a measure of skewness may be obtained which is defined as

$$S = \frac{G_1\ (G_2 + 6)}{2(5G_2 - 6G_1^2 + 6)} \qquad 4.52)$$

This measure of skewness 'S' is called Pearson's coefficient of skewness. If the factor $|S| < 3$, the distribution may be reasonably approximated [13] by the Gram-Charlier expansion.

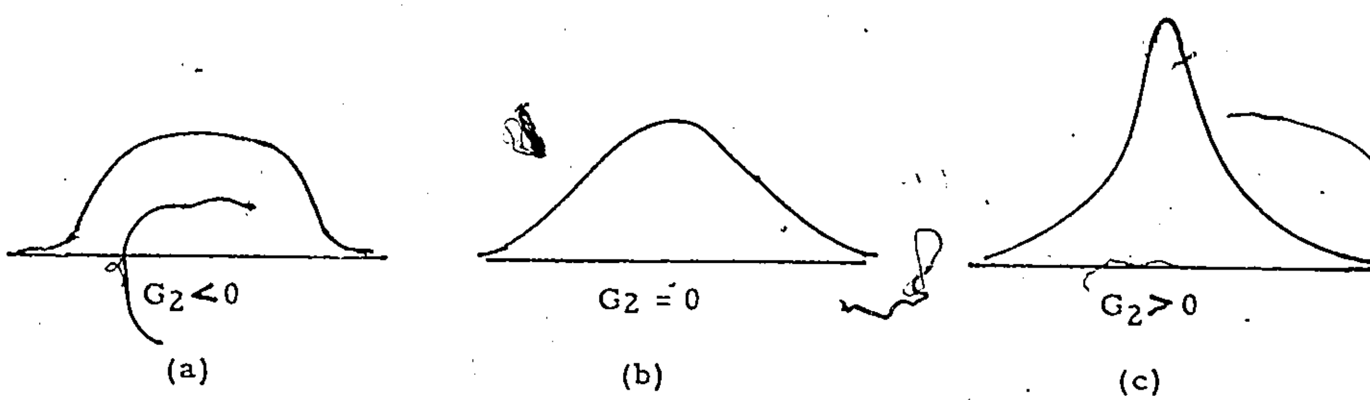

Figure 21: Kurtosis

Chapter V

APPLICATIONS OF THE GRAM-CHARLIER SERIES IN GENERATION PLANNING

5.1 INTRODUCTION

The Gram-Charlier series is very useful for the simulation of convolution in the evaluation of LOLP and expected energy generation for production costing [23] where repeated convolution and deconvolutions are required.

In the evaluation of LOLP, the PDF of system outage capacity is determined from the convolution of the PDFs of the capacity outage of all the units. As discussed in Chapter 4, the cumulants of a PDF obtained from the convolution of two or more PDFs are the sum of the appropriate cumulants of the convolved PDFs. Thus, the cumulants of the PDF of capacity outage of the generating system can be calculated by adding the appropriate cumulants of the PDFs of unit outage capacity.

In the evaluation of expected energy generation, the effect of the outage of a unit is added into the load distribution function to give the equivalent load distribution. The equivalent load distribution is simulated by convolving the unit outage capacity PDF with the load distribution. In the Gram-Charlier method, this is simulated by adding the cumulants of the load distribution and the units outage PDF to give the cumulants of the equivalent load distribution, which can be used with the Gram-Charlier expansion to represent the load distribution function.

5.2 EVALUATION OF LOLP BY THE GRAM-CHARLIER METHOD

Schenk [8] and Rau and Schenk [9] reported the use of the Gram-Charlier expansion in the evaluation of LOLP. Let there be n units in the generating system with PDFs of outage capacity $f_1(x_1)$, $f_2(x_2)$ ------ $f_n(x_n)$. The outage PDF of the system capacity is given by the convolution relation

$$f(x) = (f_1 * f_2 * f_3 * -.-.-.* f_n) (x) \tag{5.1}$$

where

X is the RV representing the outage of the system capacity.

The cumulants associated with the PDF $f_j(x_j)$ are $k_{j,r}$

where

the subscript r represents the rth cumulant. The cumulants of the entire generating system are obtained by using Equ (4.46) as follows,

$$k_r = \sum_{j=1}^{n} k_{j,r} \tag{5.2}$$

In Eqn. (5.2) the first cumulant k_1 is the mean of the RV describing the system outage capacity while the second cumulant k_2 is the variance so that,

$$k_1 = \mu$$

$$k_2 = \sigma^2$$

The third and fourth cumulants give an indication of the skewness and peakness of this distribution as was discussed previously.

The PDF $f_Z(z)$ of a standardized RV Z may be given by the Gram-Charlier expansion in terms of cumulants and normal PDF as follows,

$$f_Z(z) = N(z) - \frac{1}{3!}\frac{k_3}{\sigma^3}N^{(3)}(z) + \frac{1}{4!}\frac{k_4}{\sigma^4}N^{(4)}(z) - \frac{1}{5!}\frac{k_5}{\sigma^5}N^{(5)}(z)$$

$$+ \frac{1}{6!}\left(\frac{k_6}{\sigma^6} + \frac{10k_3^2}{\sigma^6}\right)N^{(6)}(z) + \text{--------} \qquad (5.3)$$

where

$$z = \frac{x-\mu}{\sigma}$$

The Gram-Charlier series method may be used to determine the LOLP which is the probability of the capacity outage of the system exceeding the reserve capacity (RC), as follows.

$$LOLP = \int_{RC}^{\infty} f(x)\, dx$$

or, equivalently

$$= \int_{\frac{RC-\mu}{\sigma}}^{\infty} f_z(z)\, dz \qquad (5.4)$$

From Eqns. (5.3) and (5.4) we have

$$LOLP = \int_{\frac{RC-\mu}{\sigma}}^{\infty} N(z)\, dz + K\left(\frac{RC-\mu}{\sigma}\right) \qquad (5.5)$$

where

$$K\left(\frac{RC-\mu}{\sigma}\right) = \frac{G_1}{3!} N^{(2)}\left(\frac{RC-\mu}{\sigma}\right) - \frac{G_2}{4!} N^{(3)}\left(\frac{RC-\mu}{\sigma}\right)$$

$$+ \frac{G_3}{5!} N^{(4)}\left(\frac{RC-\mu}{\sigma}\right) - \frac{G_4 + 10\, G_1^{\;2}}{6!} N^{(5)}\left(\frac{RC-\mu}{\sigma}\right) + \text{----} \qquad (5.6)$$

and

$$G_{r-2} = \frac{k_r}{\sigma^r} = \frac{\sum_{j=1}^{n} k_{j,r}}{\left(\sum_{j=1}^{n} \sigma_j^2\right)^{r/2}} \tag{5.7}$$

$$z_1 = \frac{RC-\mu}{\sigma}$$

$$LOLP = \int_{z_1}^{\infty} N(z)\,dz + K(z_1) \tag{5.8}$$

where

$$A(z_1) = \int_{z_1}^{\infty} N(z)\,dz \tag{5.9}$$

The term $A(z_1)$ in Eqn. (5.9) may be obtained from tables of the normal distribution or from a polynomial as shown later. This term gives the area under the normal PDF between $z = z_1$ and $z = \infty$ The entire table of values for different discrete values of Z can be stored into the computer memory. Other values of Z which are not in the table can be obtained by using linear interpolation.

The LOLP from the relation shown in Eqn. (5.8) may be obtained by hand calculators. The moments of the unit capacity outage PDF are calculated very easily because they comprise only discrete value of probability. The calculations of moments of generating unit capacity outage are given in Appendix A. The term (5.9) can also be evaluated by a well known rational approximation [24]. Consider the integral

$$Q(x) = \frac{1}{2\pi} \int_{x}^{\infty} e^{\frac{-t^2}{2}} \, dt \tag{5.10}$$

Let

$$R = (b_1 t + b_2 t^2 + b_3 t^3 + b_4 t^4 + b_5 t^5) \, e^{\frac{-x^2}{2}} + \varepsilon(x) \tag{5.11}$$

where $\varepsilon(x) < 7.5 \times 10^{-8}$

and $t = \frac{1}{1 + r|x|}$

$r = 0.2316419$

$b_1 = 0.319381530$

$b_2 = -0.356563782$

$b_3 = 1.781477937$

$b_4 = -1.821255978$

$b_5 = 1.330274429$

Then $Q(x) = \begin{cases} R & \text{for } x \geq 0 \\ 1-R & \text{for } x < 0 \end{cases}$ (5.12)

5.3 EVALUATION OF EXPECTED ENERGY GENERATION BY THE GRAM-CHARLIER EXPANSION METHOD

The calculations of expected energy generation are similar to those of LOLP. Schenk et al [23], Rau et al. [10] and Stremel et al.[11] have discussed this technique in detail. As is explained in section 3.3, the effect of the outage of a unit is added into the load distribution after calculating the expected energy generated by that unit. This effect is simulated by the convolution of the load distribution and the unit capacity outage PDF.

If it is assumed that the load probability distribution curve has a definite area and is not extended to negative loads (i.e the function F(L) = 0 for L < 0), the equivalent load probability distribution function $F^i(L)$, which is obtained after adding the effect of i units into load probability distribution, would look like curve ELC shown in Fig.22. The curve ELDC may be obtained when the load distribution is assumed to be extended to the negative direction of load L (i,e F(L) = 1 for L < 0). It may be noted that the equivalent load distribution curve with finite area (ELC) and the extended one (ELDC) have the same values for loads greater or equal to the total capacity of the convolved units as shown in curves ELC and ELDC respectively. The expected energy generated by the (i+1) th unit is calculated from the area under the equivalent load distribution curve between $\sum_{j=1}^{i-1} C_j$ and $\sum_{j=1}^{i} C_j$. Since the ELDC and the ELC are the same beyound $\sum_{j=1}^{i} C_j$, the expected energy generated by each unit calculated by both ways would be the same. The ordinate at convolved capacity gives the LOLP for that capacity.

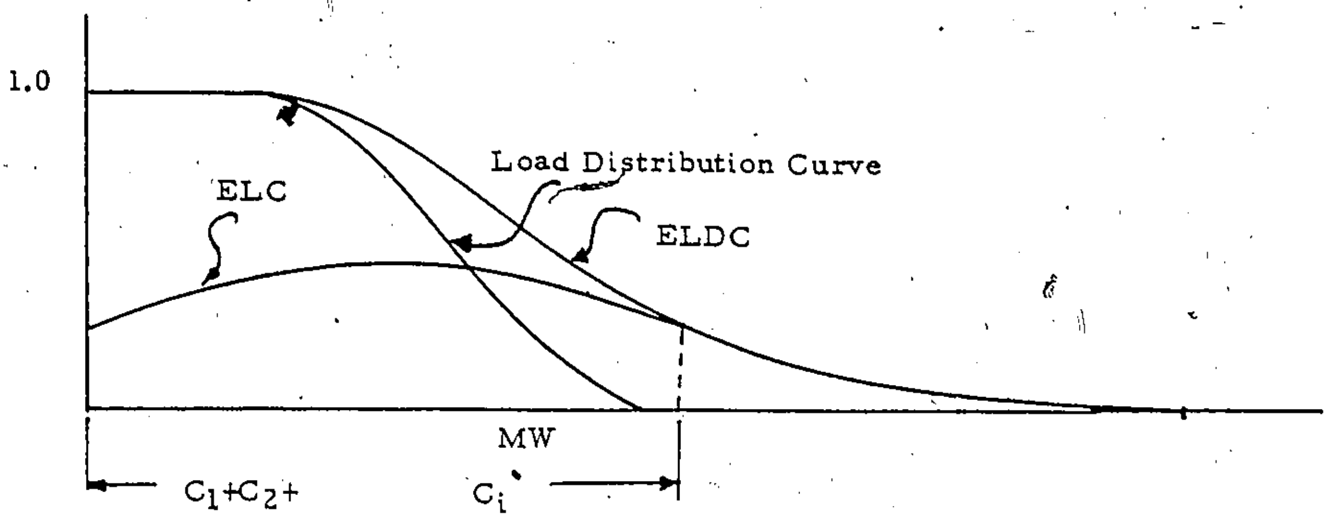

Figure 22:

Figure 23 gives a clearer picture when considering a definite area under the load curve. After adding the effect of the first unit into the load distribution, the ELC has the same area as that of the original load curve and the value of $F^{(1)}(0)$ is not unity ($F^{(1)}(0) \neq F(0)$). With the definite area concept the value of equivalent load distribution function $F^{(1)}(L)$ at L= 0 is obtained as follows

$$F^{(1)}(0) = p_1 F(0) + q_1 F(0-C_1)$$

$$= p$$

which is less than unity.

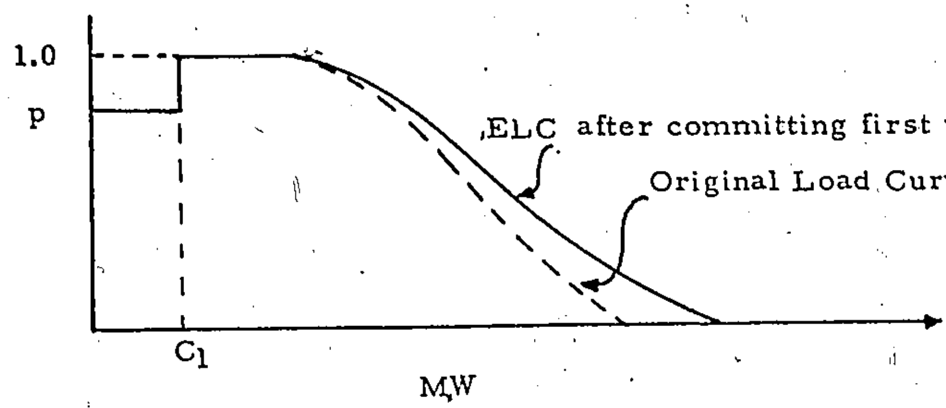

Figure 23:

To treat the load distribution function as a density function to simulate the convolution by the Gram-Charlier method, the area under load distribution must be made unity by compressing the y-axis such that

$$F_n(L) = \frac{1}{A} F(L) \tag{5.13}$$

where

$$A = \int_0^\infty F(L)\, dL \tag{5.4}$$

so that

$$\int_0^\infty F_n(L) = \int \frac{1}{A} F(L)\, dL$$

$$= \frac{1}{A} . A = 1 \quad (5.15)$$

$$F_n (L) = 0 \qquad \text{for } L < 0$$

The ELC function after convolving i units is given by

$$F^{i} (L) = \int F^{i-1} (L-x_i) \; f_i \; (x_i) \; dx \quad (5.16)$$

$$\frac{1}{A} F_n^{i} (L) = \frac{1}{A} \int F_n^{i-1} (L-x_i) \; f_i \; (x_i) \; dx_i \quad (5.17)$$

$$F_n^{i} (L) = \int F_n^{i-1} (L-x_i) \; f_i \; (x_i) \; dx_i \quad (5.18)$$

where $f_i (x_i)$ is the PDF of RV X_i

The expected energy generated by ith unit may be given by

$$E_i = \int_{IC_{i-1}}^{IC_i} F^{i-1} (L) \; dL \quad (5.19)$$

$$= A \int_{IC_{i-1}}^{IC_i} F_n^{i-1} (L) \; dL \quad (5.20)$$

$$\text{where} \quad IC_i = \sum_{j=1}^{i} C_j \quad (5.21)$$

The standardized form of $F_n^{i-1}(L)$ may be written as $F_Z^{i-1}(z)$ such that the standardized RV $Z = \frac{L-\mu}{\sigma}$, has zero mean and unity standard deviation where

μ is the mean and σ is the standard deviation of the RV L having the distribution function $F_n^{i-1}(L)$

The expected energy generated by unit i, when calculated from the equivalent distribution function $F_Z^{i-1}(Z)$ may be expressed as

$$E_i = A \int_{IC_{i-1}}^{IC_i} F_n^{i-1}(L)\, dL = A \int_{\frac{IC_{i-1}-\mu}{\sigma}}^{\frac{IC_i-\mu}{\sigma}} F_z^{i-1}(z)\, dz$$

$$= A \int_{z_{i-1}}^{z_i} F_z^{i-1}(z)\, dz \qquad (5.22)$$

where $z_i = \frac{IC_i-\mu}{\sigma}$

$F_Z^{i-1}(Z)$, in terms of the Gram-Charlier expansion, may be written as

$$F_z^{i-1}(z) = N(z) - \frac{G_1^{i-1}}{3!} N^{(3)}(z) + \frac{G_2^{i-1}}{4!} N^{(4)}(z) + \text{------} \qquad (5.23)$$

The expected energy generated by generating unit i may be given by

$$E_i = A \int_{z_{i-1}}^{z_i} F_z^{i-1}(z)\, dz = A\left(\int_{z_{i-1}}^{\infty} F_z^{i-1}(z)dz - \int_{z_i}^{\infty} F_z^{i-1}(z)dz\right) \tag{5.24}$$

From Eqns. (5.23) and (5.24)

$$E_i = A\left[\left\{\int_{z_i}^{\infty} N(z)\, dz + \frac{G_1^{i-1}}{3!} N^{(2)}(z_i) - \frac{G_2^{i-1}}{4!} N^{(3)}(z_i) + \cdots\right\}\right.$$

$$\left. - \left\{\int_{z_{i-1}}^{\infty} N(z)\, dz + \frac{G_1^{i-1}}{3!} N^{(2)}(z_{i-1}) - \frac{G_2^{i-1}}{4!} N^{(3)}(z_{i-1}) + \cdots\right\}\right] \tag{5.25}$$

where

$$G_{r-2}^{i-1} = \frac{k_r^L + \sum_{j=1}^{i-1} k_{j,r}}{\left(\sigma^2 + \sum_{j=1}^{i-1} \sigma_j^2\right)^{\frac{r}{2}}} \tag{5.26}$$

k_r^L is rth cumulant and σ is the standard deviation of the load distribution function F(L), k_{jr} is the rth cumulant of PDF of the capacity outage of jth unit and σ_j is its standard deviation.

The procedure for the evaluation of the moments of the load distribution function is given in Appendix B.

The multistate and multiblock representation of units can be handled the same way as described in subsection (3.3.1).

Steps in the Evaluation of Expected Energy Generation:

1. Calculate the moments of the standardized load distribution function (see Appendix B). Also calculate the moments of the capacity outage PDF of each unit.

2. From the moments, calculate the cumulants of the load distribution and unit capacity outage PDF by using the relation given in Appendix A.

3. Calculate the expected energy generated by ith unit in the priority list of committment, by evaluating G_r from (5.26), and using Eqns. (5.24) and (5.23).

5.4 PRODUCTION COSTS

The average incremental cost (marginal cost) is calculated for each block. Therefore the fuel costs may be directly calculated from the marginal cost and the expected energy generated by each block. The incremental cost has been discussed in section (3.4).

5.5 MERITS OF THE GRAM-CHARLIER EXPANSION METHOD

The cumulants of the load and unit capacity outage PDF are calculated only once. To simulate convolution or deconvolution only the addition or subtraction of cumulants is required. For the calculation of expected energy generated, a numerical integration is not required as is done in the Baleriaux-Booth [3,4] method. A lot of CPU time is saved in this method. This will be discussed in the next chapter.

Chapter VI

NUMERICAL RESULTS AND CONCLUSIONS

6.1 INTRODUCTION

The expansion of a density function in terms of a normal density function and its derivatives can be computationally useful only if a small number of terms of the series are required to give a good approximation. The knowledge of convergence properties of the series is of no use if a large number of terms are required. In the previous chapter, the application of the Gram-Charlier expansion for the evaluation of LOLP, expected energy generation and production costing are given. The conditions for convergence of this series is also discussed. In this chapter the accuracy of the Gram-Charlier series when used for the evaluation of LOLP, expected energy generation and production costing is studied. An attempt has been made to provide some guidelines in the application of the Gram-Charlier method. These guidelines are focused on the number of terms of the series, the coefficients G_1, G_2 and S, unit FOR, number of units and load shape. These guidelines are based on the numerical results obtained from the analysis of several systems of different kinds and sizes. The results are compared with the results obtained by recursive methods[3,4].

The basic Gram-Charlier expansions used are,

$$GCB = N(z) - \frac{G_1}{3!} N^{(3)}(z) + \frac{G_2}{4!} N^{(4)}(z) + \frac{10\, G_1^2}{6!} N^{(6)}(z)$$

$$GC4 = GCB - \frac{G_3}{5!} N^{(5)}(z) + \frac{G_4}{6!} N^{(6)}(z)$$

$$GC6 = GC4 - \frac{G_5 + 35.0 G_1 G_2}{7!} N^{(7)}(z) + \frac{G_6 + 56 G_1 G_3 + 35 G_1^2}{8!} N^{(8)}(z)$$

6.2 LOLP AND RESERVE CAPACITY MARGIN CALCULATIONS

Several different systems were analysed for the evaluation of LOLP and reserve capacity margin. A few of the results are shown in this section. The Gram-Charlier expansion fairs better in the evaluation of reserve capacity margin than the evaluation of LOLP. This can be seen from the fact that at a certain risk level, or LOLP value, the differences in reserve capacity are all quite reasonable. This is not entirely so for fixed values of reserve capacity. The LOLPs may differ by an order of magnitude. Figures 24, 25 and 26 show three different systems. The three different systems analysed are; IEEE Reliability Test System (IEEE RTS) [25], SASK System and OH System. The data for these systems is given in Appendix C. The IEEE RTS contains 32 units of different sizes and kinds. Similarly SASK System and OH System contain 39 units and 66 units respectively. Each figure contains four curves obtained by using different number of terms. In the analysis it is noticed that in most of the cases, series GC6 gives best result in the range of very small values of probability (10^{-4}) as can be seen from graphs in Figs. 24,25 and 26. The abscissa of the graphs gives the capacity outage and the ordinate gives the cumulative probability of the outage. The ordinate of each graph has been drawn to logarithimic scale.

A system with an installed capacity of 10000 MW was analysed in terms of its capacity reserve margins. The unit sizes are 50 MW, 100 MW, 200 MW, 500 MW, and 1000MW. Forced outage rates (FOR) are 0.01, 0.05, 0.10, 0.20. Tables 1 through 5 show the results. In Table 1, the unit size is 50 MW and the number of units is 200. The capacity reserve margins are evaluated at LOLPs of $1x10^{-4}$, $2x10^{-4}$, $4x10^{-4}$, $8x10^{-4}$ and 10^{-3}. The units are identical and the FORs considered for the four different cases are 0.01, 0.05, 0.10 and 0.20. The basic factors G_1 G_2 and S are shown. Additionally, the factors G_3, G_4, G_5 and G_6, are shown for comparison purposes.

In table 2, the unit size is 100 MW and the number of units is 100. Similarly, in Table 3, and Table 4 unit sizes are 200 MW, and 500 MW, respectively. In Table 5, the combination of the 41 units considered are; 3 units of 1000 MW each, 8 units of 500 MW each and 30 units of 100 MW each.

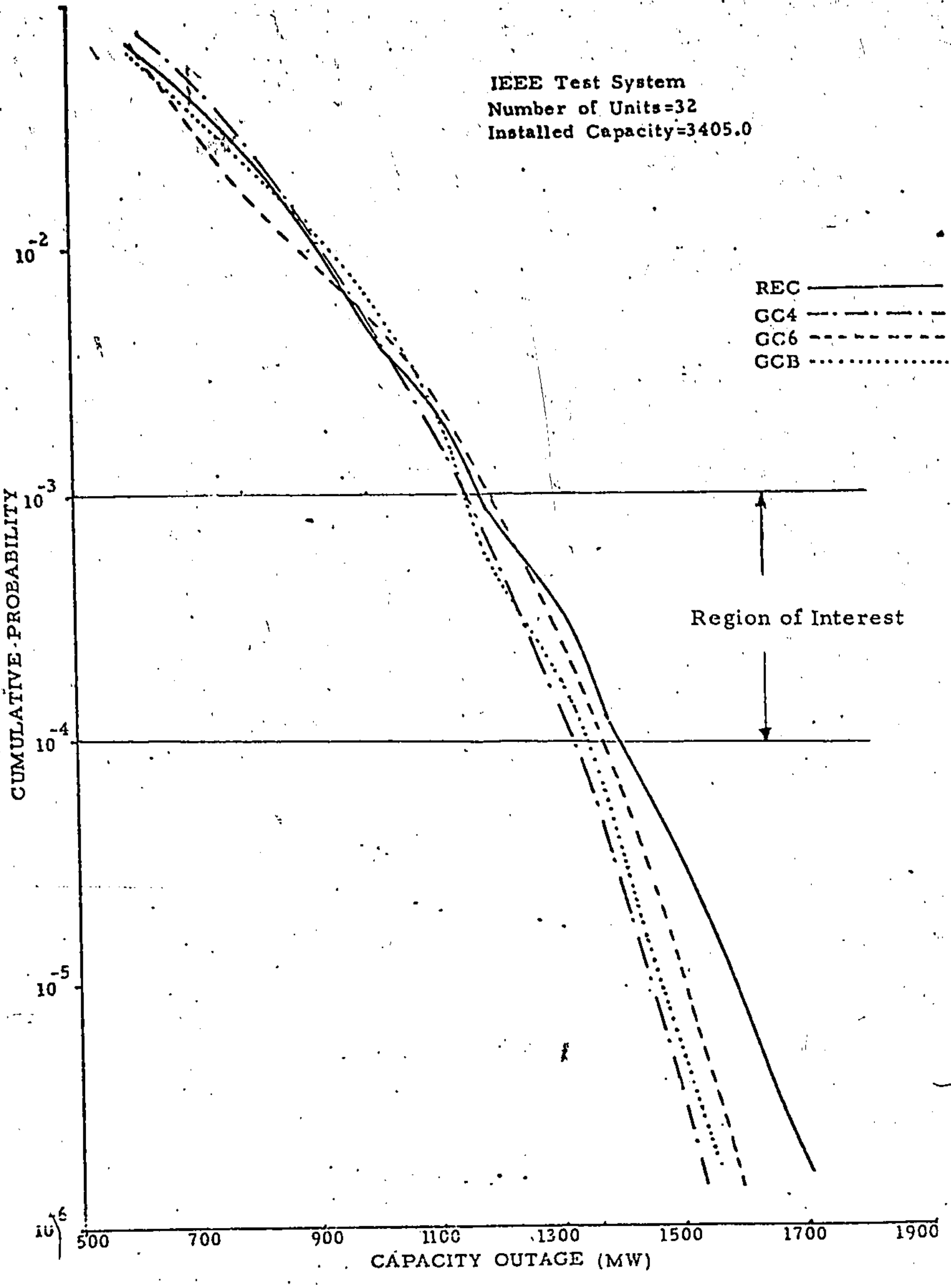

Figure 24. Analysis of LOLP for Reliability Test System.

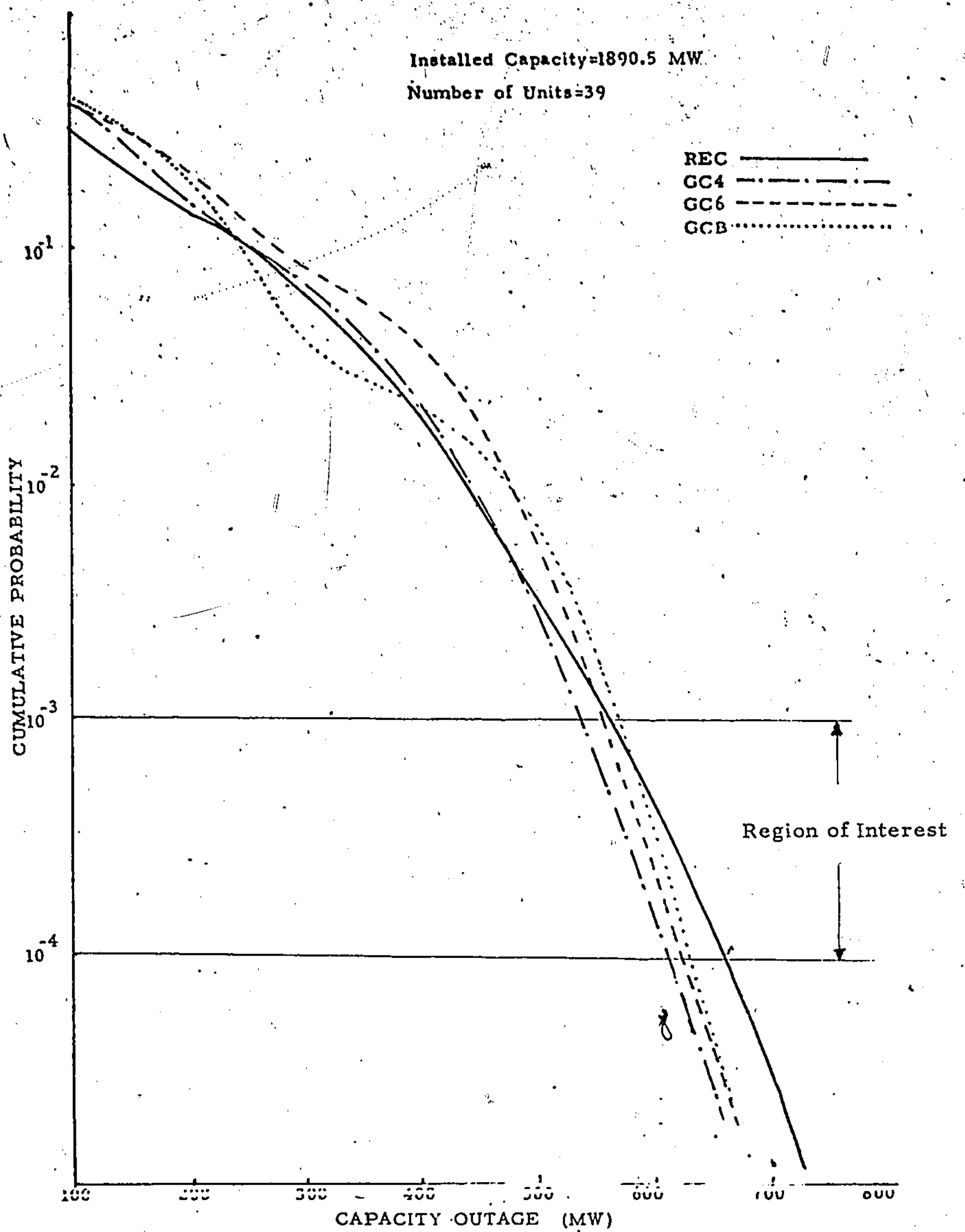

Figure 25. Analysis of LOLP for SASK System

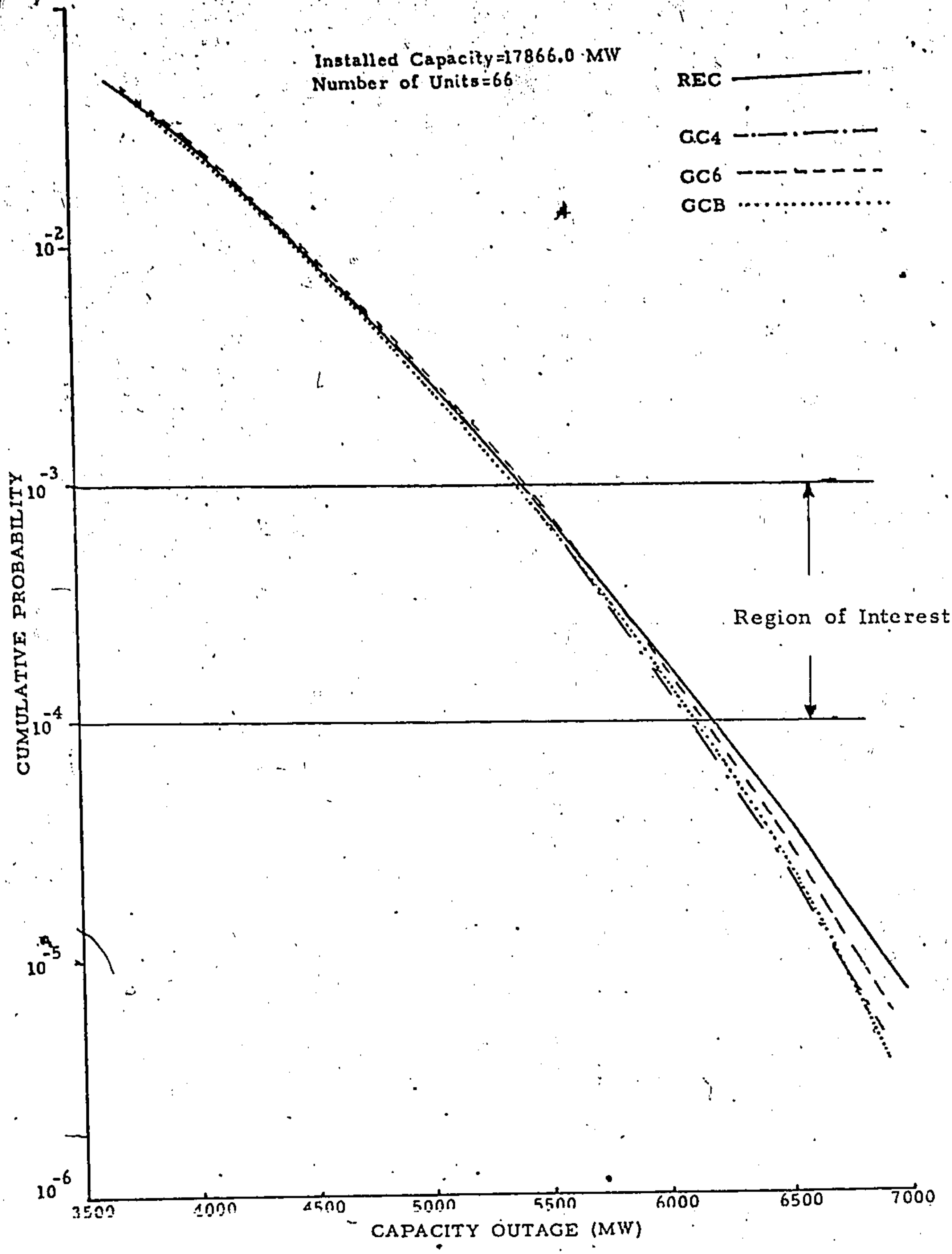

Figure 26. Analysis of LOLP for OH System

TABLE 1

Installed Capacity = 10000 MW
Number of Units = 200
Capacity of each Unit = 50 MW

LOLP $\times 10^{-4}$	Capacity Reserve Margin in MW											
	FOR=0.01			FOR=0.05			FOR=0.10			FOR=0.20		
	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF
1	486	482	0.8	1181	1182	0.1	1882	1893	0.6	3116	3136	0.6
2	471	453	3.9	1145	1148	0.3	1836	1846	0.6	3067	3080	0.4
4	441	438	0.7	1095	1114	1.7	1786	1798	0.7	2997	3022	0.8
8	395	413	4.3	1068	1076	0.7	1739	1748	0.5	2946	2959	0.4
10	392	402	2.5	1056	1062	0.6	1715	1729	0.7	2923	2939	0.6
G_1	0.696			0.292			0.188			0.106		
G_2	0.475			0.075			0.025			0.001		
G_3	0.309			-0.013			-0.001			-0.003		
G_4	0.182			-0.002			-0.002			-0.001		
G_5	0.078			-0.003			-0.001			0.		
G_6	-0.011			-0.002			0.			0.		
S	0.412			0.151			0.053			0.053		

TABLE 2

Installed Capacity = 10000 MW
Number of Units = 100
Capacity of each Unit = 100 MW

LOLP X10^-4	Capacity Reserve Margin in MW											
	FOR=0.01			FOR=0.05			FOR=0.10			FOR=0.20		
	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF
1	647	694	6.8	1488	1536	3.1	2278	2319	1.8	3599	3655	1.5
2	599	672	10.8	1443	1480	2.5	2204	2256	2.3	3523	3572	1.4
4	592	629	5.9	1384	1408	1.7	2143	2182	1.8	3441	3484	1.2
8	578	591	2.2	1325	1348	1.7	2068	2100	1.5	3352	3392	1.2
10	568	584	2.7	1298	1326	2.1	2042	2083	1.9	3318	3367	1.4
G_1	0.985			0.413			0.267			0.150		
G_2	0.950			0.050			0.051			0.002		
G_3	0.876			0.037			-0.003			-0.008		
G_4	0.729			-0.007			-0.009			-0.003		
G_5	0.443			-0.019			-0.005			-0.		
G_6	-0.090			-0.016			-0.001			-0.001		
S	0.694			0.222			0.129			0.076		

TABLE 3

Installed Capacity = 10000 MW
Number of Units = 50
Capacity of each Unit =200 MW

LOLP $\times 10^{-4}$	Capacity Reserve Margin in MW											
	FOR=0.01			FOR=0.05			FOR=0.10			FOR=0.20		
	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF
1	891	1067	16.5	1969	2090	5.8	2873	2974	3.4	4318	4405	2.0
2	876	992	11.7	1893	1985	4.6	2778	2880	3.5	4197	4310	2.6
4	844	965	12.5	1821	1919	5.1	2676	2768	3.3	4077	4173	2.3
8	797	908	12.2	1739	1796	3.2	2566	2657	3.4	3947	4043	2.4
10	791	881	10.2	1702	1779	4.3	2527	2601	2.8	3899	3993	2.3
G_1	1.393			0.584			0.377			0.212		
G_2	1.900			0.301			0.102			0.005		
G_3	2.479			0.105			-0.007			-0.024		
G_4	2.917			-0.027			-0.036			-0.011		
G_5	2.508			-0.107			-0.028			+0.002		
G_6	-0.722			-0.129			-0.004			0.006		
S	1.926			0.337			0.203			0.110		

TABLE 4

Installed Capacity = 10000 MW
Number of Units = 20
Capacity of each Unit =500 MW

LOLP	Capacity Reserve Margin in MW											
	FOR=0.01			FOR=0.05			FOR=0.10			FOR=0.20		
$\times 10^{-4}$	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF
1	1418	1970	28.0	2982	3388	11.9	4036	4442	9.1	5784	6000	3.6
2	1388	1918	27.6	2883	3219	10.4	3989	4303	7.3	5598	5893	5.0
4	1352	1814	25.5	2777	2984	6.9	3832	4022	4.7	5399	5676	4.9
8	1298	1606	20.0	2657	2873	7.5	3655	3902	6.3	5188	5441	4.6
10	1291	1500	13.9	2608	2848	8.4	3588	3850	6.8	5116	5391	5.1
G_1	2.202			0.923			0.596			0.335		
G_2	4.750			0.753			0.256			0.012		
G_3	9.802			0.418			-0.026			-0.096		
G_4	18.232			-0.171			-0.225			-0.071		
G_5	24.790			-1.062			-0.273			0.020		
G_6	-11.289			-2.023			-0.069			0.098		
S	16.650			0.670			0.371			0.187		

TABLE 5

Installed Capacity = 10000 MW
Number of Units = 41
3 Units of 1000 MW each
8 Units of 500 MW each
30 Units of 100 MW each

LOLP	Capacity Reserve Margin in MW											
	FOR=0.01			FOR=0.05			FOR=0.10			FOR=0.20		
$X10^{-4}$	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF
1	1556	2121	26.6	3143	3369	6.7	4391	4498	2.4	5927	6000	1.2
2	1500	2088	28.2	3052	3284	7.1	4258	4332	1.7	5762	5812	0.9
4	1484	1779	16.6	2955	3148	6.1	4093	4116	0.6	5578	5615	0.7
8	1451	1612	10.0	2848	2885	1.3	3921	3909	0.3	5375	5395	0.4
10	1434	1598	10.2	2807	2828	0.7	3861	3851	0.2	5302	5330	0.5
G_1	3.253			1.364			0.881			0.495		
G_2	11.848			1.877			0.637			0.031		
G_3	44.064			1.879			-0.119			-0.433		
G_4	153.080			-1.435			-1.886			-0.597		
G_5	396.252			-16.971			-4.365			0.322		
G_6	-346.964			-62.182			-2.140			3.005		
S	16.608			0.279			0.150			0.076		

As can be seen from the Tables 1 through 5, the factors G_1, G_2, and S, and the FOR dictate the applicability of the Gram-Charlier series method. It is clear that the accuracy of the method decreases with a decrease of FOR. It is also noticed that the accuracy decreases with the increase of factors G_1, G_2 and S. The conclusions of the usability of the series may be reached from G_1, G_2 and S only. However, the Tables also indicate the additional factors G_3, G_4, G_5 and G_6 which may give an indication of the range of these factors. From a look at the Tables, it is immediately noticed that when these factors G_i are very large, the results become less reliable. The capacity reserve margins obtained from the Gram-Charlier (GC) and the Recursive (REC) methods, and the absolute percentage difference between the GC method and the REC method is also shown in these Tables. It is assumed that the recursive method results are exact. Series GC6 is used in the above calculations. Note that in Tables 4 and 5 the factors S and G_i $i=1,6$ are all quite large. These values are a bit beyond the applicability of the Gram-Charlier series. As expected, therefore, the differences in reserve capacities may be as high as 28%.

The analyses of a few other systems are also shown here. The IEEE RTS data is given in Appendix C. Table 6 gives the capacity reserve margins of the IEEE Test System using the Gram-Charlier method and the recursive method at LOLPs

$2.56X10^{-4}$, $3.85X10^{-4}$ and $5.49X10^{-4}$. These LOLPs correspond to the loss of load expectation being 1 day in 15 years, 1 day in 10 years and 1 day in seven years, resepectively. The year is considered to consist of 260 days (week days of 52 weeks). The behaviour of the system is also studied when the FOR of each unit is halved or doubled. Its effect on the values of G_1, G_2 and S can be seen in Table 6. The OH System and the SASK System again analyzed with the above mentioned changes in FORs. The capacity reserve margins of both the systems for half and double their unit's FORs are also given in Tables 7 and 8 respectively.

The percent difference (%DIF) in the capacity reserve margin (CRM) is given by,

$$\%DIF = \frac{|CRM \text{ by GC method} - CRM \text{ by REC method}|}{CRM \text{ by REC method}} \times 100$$

As can be seen from Table 1 through 8, with large value of S, the % DIF in capacity reserve margin is also large. It has been observed [13] that if $|S|>3$, this absolute percent difference is large and the Gram-Charlier method may be beyond its range of applicability. S as well as the factors G_i, $i=1,6$ increase as the FOR is reduced, as expected.

TABLE 6

IEEE Reliability Test System
Installed Capacity = 3405 MW
Number of Units = 32

LOLP $\times 10^{-4}$	Capacity Reserve Margin in MW											
	0.5 X FOR			AVE FOR=.0613			2 X FOR			4 X FOR		
	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF
2.56	1008	1071	5.8	1312	1345	2.4	1656	1671	0.9	2086	2113	1.3
3.85	990	1016	2.6	1282	1307	1.9	1624	1632	0.5	2060	2079	0.9
5.49	970	995	2.5	1256	1259	0.2	1594	1596	0.1	2035	2048	0.6
G_1		1.825			1.164			0.635			0.156	
G_2		3.082			0.973			-0.021			-0.346	
G_3		3.288			-0.558			-0.911			-0.147	
G_4		-6.064			-5.063			-1.065			-0.676	
G_5		-56.465			-11.183			2.164			0.458	
G_6		-210.178			5.680			10.110			-2.950	
S		9.450			1.481			0.545			0.050	

TABLE 7

OH System
Installed Capacity = 17886.0 MW
Number of Units = 66

LOLP $\times 10^{-4}$	Capacity Reserve Margin in MW								
	0.25 X FOR			AVE FOR=.0894			2 X FOR		
	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF
2.56	2584	2642	2.19	5112	5164	1.0	7450	7510	0.8
3.85	2534	2570	1.4	4992	5033	0.8	7299	7362	0.8
5.49	2484	2507	0.9	4883	4914	0.6	7173	7228	0.7
G_1	1.207			0.530			0.300		
G_2	1.480			0.214			0.013		
G_3	1.656			-0.030			-0.077		
G_4	1.129			-0.204			-0.050		
G_5	-2.001			-0.236			0.036		
G_6	-12.629			0.041			0.099		
S	0.971			0.306			0.164		

TABLE 8

SASK System
Installed Capacity = 1890.0 MW
Number of Units = 39

LOLP $\times 10^{-4}$	Capacity Reserve Margin in MW											
	0.5 X FOR			AVE FOR=.0484			2 X FOR			4 X FOR		
	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF	GC	REC	%DIF
2.56	473	516	8.3	618	634	2.8	783	782	0.1	1010	991	1.9
3.85	465	501	7.1	605	611	0.9	769	764	0.6	991	974	1.7
5.49	458	491	6.7	593	594	0.2	747	747	0	973	959	1.4
G_1	2.438			1.605			0.961			0.421		
G_2	6.367			2.348			0.376			-0.522		
G_3	13.101			0.378			-2.133			-1.174		
G_4	-5.764			-17.962			-7.044			1.504		
G_5	-301.500			-89.579			-0.438			9.646		
G_6	-2259.274			-157.967			90.746			-3.988		
S	5.26			2.93			1.31			.497		

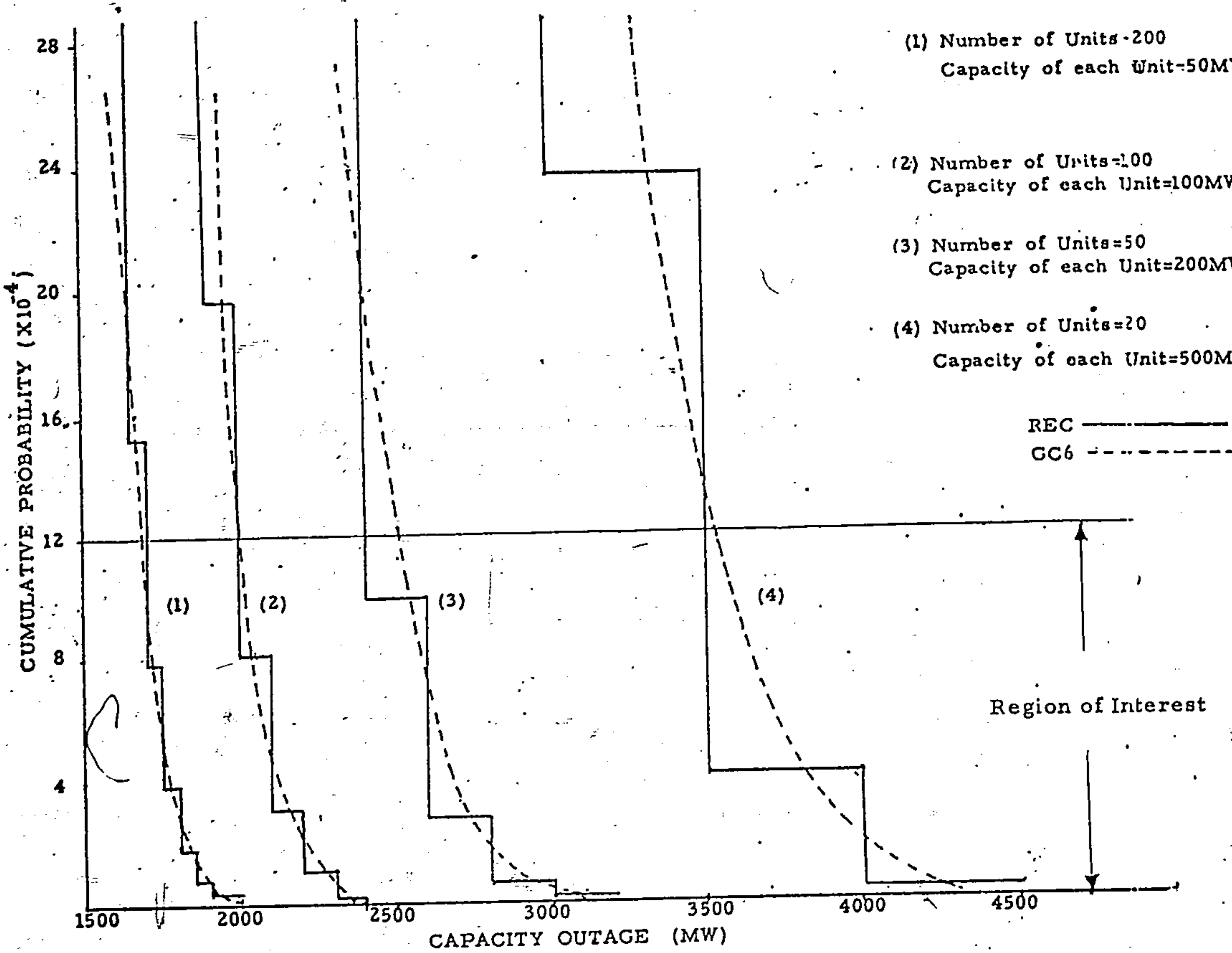

Figure 27. The Gram-Charlier Expansion Approximating Discontinuous Distributions.

As can be observed from Tables 1 through 8, for very small average FORs or for a small number of units, the percent difference in capacity reserve margins calculated from the GC and the REC techniques becomes large. Therefore, the GC method to evaluate LOLP is observed not to be a very accurate when the average FOR of the system is less than 0.02. In practical systems the average FOR has been observed to be always greater than this extreme value. The fact that larger number of units gives better results than those of smaller number of units when using the GC method can be explained from the Eqn.(4.21) where $C_r = O(n^{[\frac{r}{3}]-\frac{r}{2}})$. The error introduced when truncating the series depends upon n. If n is large in Eqn.(4.21) the neglected term would carry lesser weight.

In Tables 3 and 4, the percent difference is quite large even when the values of G_1, G_2 and S are small. This may be explained from Fig.27. In Tables 3 and 4, the unit size is 200 MW and 500 MW respectively. The cumulative probability curves as shown in Fig.27 have discontinuities. They increase at steps where the step size is the size of the unit. The Gram-Charlier expansion approximates them by a continuous curve. It must be noted too that the results obtained by the recursive method are themselves not 100% accurate. The accuracy depends upon the step size used in the calculations. A large step size gives less accurate results than

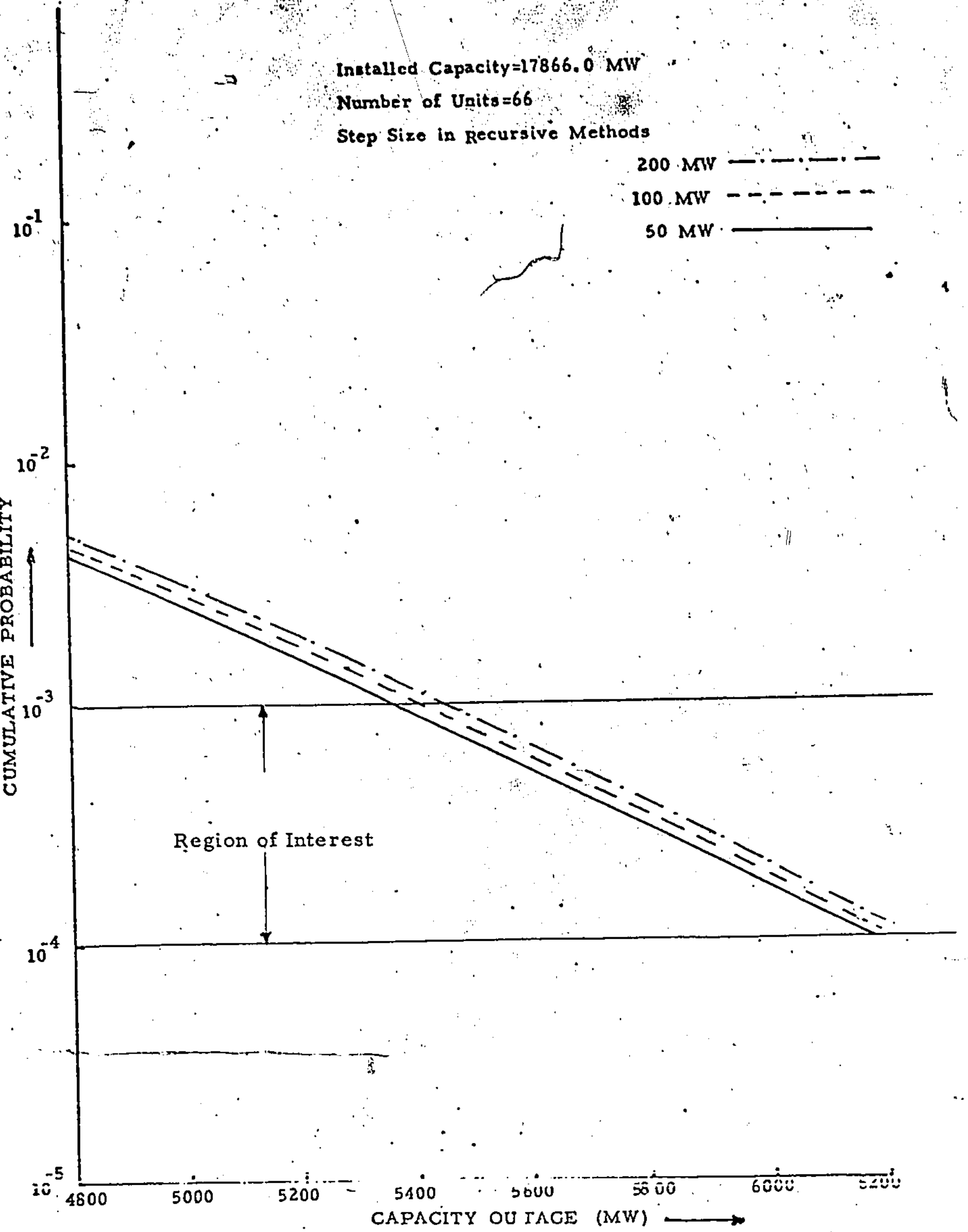

Figure 28. Effect of the Step Size on LOLP by REC Method.

those obtained with a smaller step size. Figure 28 shows the cumulative probability in a very small range of probability when different step sizes are used. The ordinate of the graph in Fig.28 has been scaled to a logarithimic scale. It is clear that the step size has a large effect on LOLPs calculations.

6.3 EXPECTED ENERGY GENERATION AND PRODUCTION COSTING CALCULATIONS

A system of installed capacity of 10000 MW was analysed. The system is comprised of 100 units of 100 MW each, with a FOR = 0.10. The incremental fuel cost is taken to be 10.0$/MWh, and it is assumed to be constant over the capacity of the unit. The units are not fragmented into blocks. Two load shapes are considered in this analysis as shown in Fig 29.(a) and (b)

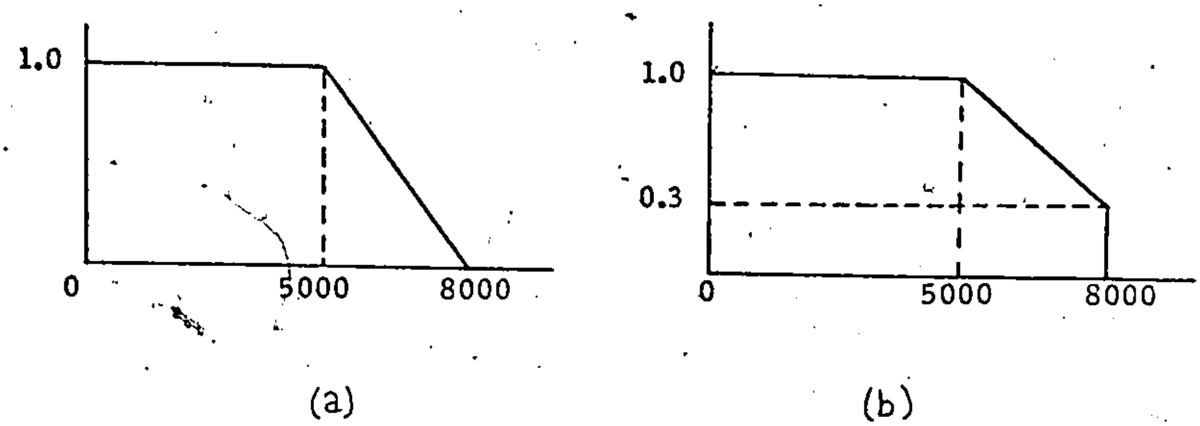

Figure 29: Load shapes of two different kinds of loads

In the load shape shown in Fig.29(a), the base load is 5000 MW and the peak load is 8000 MW. This load shape is very much similar to the load shape that occurs in actual practice. In Fig.29(b) the base load and peak loads are again 5000 MW and 8000 MW respectively but the peak load has been shaved off. The same peak load of 8000 MW is assumed in both the cases. The expected energy generated by the units and the production costs are given in Tables 9 and 10. The results obtained by the GC method are compared to those obtained by the REC method. The methods used to calculate the area under the LDC and the moments of LDC are given in Appendix B. For values of capacities up to the base load, the area under the ELDC are identical for both methods. In this region these areas are calculated directly and a numerical integration and the Gram-Charlier series is not utilized.

The solutions obtained by the GC method and the REC method are very close. An examination of Tables 9 and 10 reveals that the difference in expected production costs calculated from the GC and the REC methods is 0.2 M$ in load shape (a) and 0.36 M$ in load shape (b). In terms of a percentage of the total production costs, the difference is 0.28 and 0.51 percent for load shape (a) and (b) respectively. Note that some units have been lumped together to save space. However, sufficient detail is included for a fair comparison to be made.

TABLE 9

Production Costing for Load Shape 29(a)
Installed Capacity = 10000 MW
Time Period = 1000 Hours

UNIT NO.	CAP. MW	EXPECTED ENERGY GWH		PRODUCTION COST M$	
		GC	REC	GC	REC
1	100	90.00	90.00	0.9000	0.9000
2-49	100	90.00	90.00	0.9000	0.9000
50	100	90.00	90.00	0.9000	0.9000
51	100	86.23	89.99	0.8623	0.8994
52-60	9X100	764.90	774.62	7.6490	7.7490
61	100	76.08	76.38	0.7608	0.3638
62-70	9X100	575.90	569.91	5.7590	5.6991
71	100	49.68	49.50	0.4968	0.4950
72-80	9X100	312.69	324.06	3.1269	3.2406
81	100	21.12	22.50	0.2112	0.2250
82-90	9X100	86.55	91.93	0.8655	0.9193
91	100	3.14	1.69	0.0314	0.0169
92-100	9X100	6.84	2.35	0.0684	0.0235
TOTAL PRODUCTION COSTS				64.83	65.03

TABLE 10

Production Costing for Load Shape 29(b)
Installed Capacity = 10000 MW
Time Period = 1000 Hours

UNIT NO.	CAP. MW	EXPECTED ENERGY GWH		PRODUCTION COST M$	
		GC	REC	GC	REC
1	100	90.00	90.00	0.9000	0.9000
2-49	100	90.00	90.00	0.9000	0.9000
50	100	90.00	90.00	0.9000	0.9000
51	100	85.77	89.99	0.8577	0.8999
52-60	9X100	752.04	787.24	7.5204	7.8724
61	100	81.91	80.47	0.8191	0.8047
62-70	9X100	669.83	641.83	6.6983	6.4183
71	100	64.07	61.65	0.6407	0.6165
72-80	9X100	453.80	470.80	4.5346	4.7080
81	100	36.25	42.73	0.3625	0.4273
82-90	9X100	226.84	251.74	2.2684	2.5174
91	100	11.56	8.25	0.1156	0.0825
92-100	9X100	40.72	13.76	0.4072	0.1376
TOTAL PRODUCTION COSTS				69.22	69.58

From an analysis of the values obtained it may be said that the solution obtained from load shape (a) is better than those of load shape (b). The factors G_1, G_2 and S are larger for load shape (b). This decreases in accuracy may be accounted for by the fact that load shape (b) has additional discontinuities than load shape (a).

For the analysis of a more realistic system, the IEEE Reliability Test System is again considered. The load duration curve is constructed from the data given in [25]. The year is divided into four quarters. The load duration curves for all the four quarters are given in Appendix C. The load shape for the first quarter is shown in Fig. 30. The units are committed according to the economic dispatch strategy of increasing marginal costs. Tables 11 to 14 give the expected energy generation and production costs of all the four quarters by the GC method as well as the REC method. The expected energy generated by the GC method can be a little different from the actual energy under the load curve. It is therefore suggested that the total energy (expected energy generated and energy not served) should be made equal to the area under the LDC to better approximate the total production costs. The total energy calculatedby the GC method can be made equal to the energy under the LDC by multiplying the expected energy generated by each unit by the ratio of the total energy under the load curve minus the energy of the base units to the total energy by the GC method minus the energy of the base units. This adjustment makes the total energy by the GC method equal to the total energy under the LDC without changing the energy of the base units as shown in Fig.31. The above technique is used for the evaluation of expected energy generation and production costing in the four quarters for the IEEE RTS as shown in Tables 11 through 14. In Table 11, which gives the expected energy generation and production costs of the first quarter of IEEE Test System, the difference in the total production costs is only 0.05 M$. This is less than 0.2% of the total production costs. Similarly the difference in production costs in quarter 2nd, 3rd and fourth are only 0.06, 0.07 and 0.10 M$, respectively, as shown in Tables 12, 13 and 14. The difference in any case is not greater than 0.4 percent. In the IEEE RTS there are all kinds of units with marginal costs ranging from 5.59 $/MWh for the base units to 37.50 $/MWh for the peaking units. In the above system, all the six hydro units are combined together and are committed in one block. Before calculating the expected energy generated by each unit, a test is made in the computer program to fit in the Hydro unit to generate the given hydro energy. This test is done as each unit is committed to supply the load. In most of the cases a unit has to be fragmented into two capacity blocks to fit in the hydro unit to generate the given energy. A computer program is given at the end to cal-

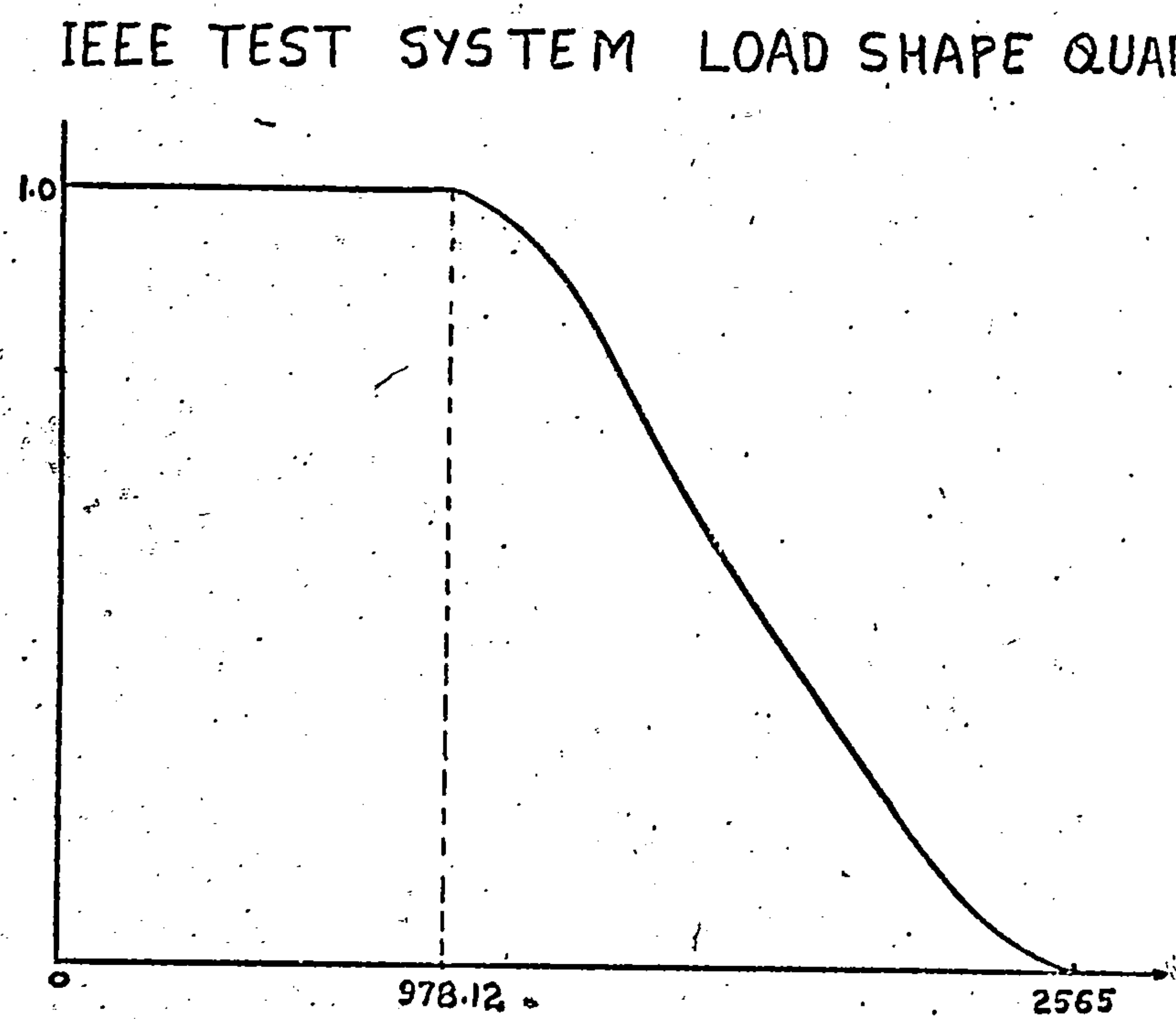

Figure 30. The Load Shape of the First Quarter of the IEEE RTS.

culate the expected energy generation and production costs by the GC method. The computer program automatically picks up a unit with minimum marginal cost. The hydro unit is also committed automatically in the given program.

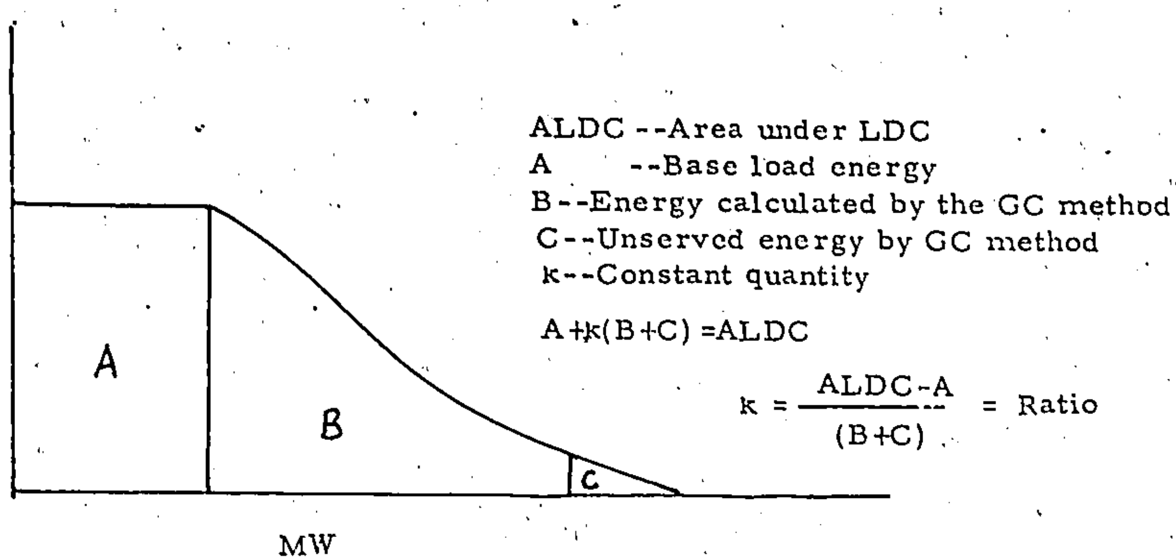

Figure 31: LDC

TABLE 11

Expected Energy Generation and Production Costs for First Quarter

Base Load = 978.12 MW
Peak Load = 2565 MW
Hydro Energy = 420 GWh
Hydro Capacity = 300 MW
Period = 2184 Hours

Loading Order	Unit No.	Block No.	Unit Size MW	Block Size MW	Expected Energy GWh GC	Expected Energy GWh REC	Production Costs M$ GC	Production Costs M$ REC
1	1	1	400	400	768.768	768.768	4.299	4.299
2	2	1	400	400	768.768	768.768	4.299	4.299
3	7	1	155	155	324.979	324.979	3.630	3.627
4	8	1	155	155	332.816	322.559	3.714	3.600
5	9	1	155	155	311.786	308.403	3.478	3.442
6	10	1	155	155	282.600	281.351	3.154	3.140
8	18	1	300	300	420.000	420.000	0	0
7,9	3	1,2	350	69,281*	343.277	342.382	3.913	3.903
10	14	1	76	76	46.716	51.567	0.695	0.767
11	15	1	76	76	38.301	43.274	0.570	0.644
12	16	1	76	76	30.828	33.910	0.459	0.505
13	17	1	76	76	24.332	25.421	0.362	0.378
14	4	1	197	197	38.397	38.801	0.763	0.771
15	5	1	197	197	18.169	18.308	0.361	0.364
16	6	1	197	197	6.235	6.619	0.123	0.131
17	11	1	100	100	1.245	1.526	0.027	0.034
18	12	1	100	100	0.603	0.829	0.013	0.018
19	13	1	100	100	0.661	0.421	0.014	0.009
20-24	23-27	1	5x12	5x12	0.275	0.144	0.008	0.004
25-28	19-22	1	4x20	4x20	0.240	0.105	0.009	0.004
				TOTAL	3757.970	3758.148	29.891	29.939

Expected Unserved Energy (GC) = 0.300 GWh
Expected Unserved Energy (REC) = 0.122 GWh
*69 MW block before hydro units; 281 MW after hydro units

TABLE 12

Expected Energy Generation and Production Costs for Second Quarter

Base Load	=	1001.74 MW
Peak Load	=	2565.00 MW
Hydro Energy	=	420 GWh
Hydro Capacity	=	300 MW
Period	=	2184 Hours

Loading Order	Unit No.	Block No.	Unit Size MW.	Block Size MW	Expected Energy GWh		Production Costs M$	
					GC	REC	GC	REC
1	1	1	400	400	768.768	768.768	4.299	4.299
2	2	1	400	400	768.768	769.768	4.299	4.299
3	3	1	155	400	324.979	324.979	3.627	3.627
4	8	1	155	400	330.616	323.984	3.690	3.616
5	9	1	155	400	315.973	312.626	3.526	3.489
6	10	1	155	400	289.984	284.603	3.237	3.176
8	18	1	300	300	420.000	420.000	0	0
7,9	3	1,2	350	121,229*	400.568	398.596	4.635	4.544
10	14	1	76	76	55.280	61.547	0.818	0.916
11	15	1	76	76	46.402	52.403	0.687	0.779
12	16	1	76	76	38.255	43.051	0.566	0.641
13	17	1	76	76	30.959	33.991	0.458	0.506
14	4	1	197	197	51.084	52.031	1.015	1.034
15	5	1	197	197	25.738	25.525	0.511	0.507
16	6	1	197	197	10.252	9.833	0.204	0.195
17	11	1	100	100	2.170	2.268	0.048	0.050
18	12	1	100	100	1.082	1.289	0.024	0.028
19	13	1	100	100	0.478	0.674	0.011	0.015
20-24	23-27	1	5x12	5x12	0.363	0.235	0.010	0.007
25-28	19-22	1	4x20	4x20	0.319	0.171	0.011	0.006
				TOTAL	3885.224	3885.556	31.676	31.734

Expected Unserved Energy (GC) = .539 GWh
Expected Unserved Energy (REC) = 0.207 GWn
*121 MW block before hydro units; 229 MW block after hydro units

TABLE 13

Expected Energy Generation and Production Costs for Third Quarter

Base Load = 965.62 MW
Peak Load = 2508.00 MW
Hydro Energy = 120 GWh
Hydro Capacity = 270 MW
Period = 2184 Hours

Loading Order	Unit No.	Block No.	Unit Size MW	Block Size MW	Expected Energy GWh		Production Costs M$	
					GC	REC	GC	REC
1	1	1	400	400	768.768	768.768	4.299	4.299
2	2	1	400	400	768.768	768.768	4.299	4.299
3	7	1	155	155	324.979	324.979	3.627	3.627
4	8	1	155	155	329.233	320.929	3.674	3.579
5	9	1	155	155	301.155	297.907	3.361	3.325
6	10	1	155	155	268.017	262.366	2.991	2.928
7	3	1	350	350	434.303	434.920	4.950	4.958
8	14	1	76	76	73.225	77.598	1.084	1.154
9	15	1	76	76	62.631	66.525	0.927	0.989
10,12	16	1,2	76	7,69*	52.490	55.350	0.776	0.823
11	18	1	270	270	120.000	120.000	0	0
13	17	1	76	76	15.985	20.914	0.237	0.311
14	4	1	197	197	26.508	26.095	0.527	0.518
15	5	1	197	197	9.844	10.563	0.196	0.210
16	6	1	197	197	2.966	3.730	0.059	0.074
17	11	1	100	100	1.018	0.783	0.022	0.017
18	12	1	100	100	0.648	0.407	0.014	0.009
19	13	1	100	100	0.398	0.203	0.009	0.004
20-24	23-27	1	5x12	5x12	0.160	0.067	0.005	0.002
25-28	19-22	1	4x20	8x20	0.130	0.048	0.005	0.002
				TOTAL	3560.717	3560.745	31.062	31.128

Expected Unserved Energy (GC) = 0.0800 GWh
Expected unserved Energy (REC) = 0.0520 GWh
*7 MW block before hydro units; 69 MW after hydro units

TABLE 14

Expected Energy Generation and Production Costs for Fourth Quarter

Base Load = 1005.91 MW
Peak Load = 2850.00 MW
Hydro Energy = 240 GWh
Hydro Capacity = 270 MW
Period = 2184 Hours

Loading Order	Unit No.	Block No.	Unit Size MW	Block Size MW	Expected Energy GWh		Production Costs M$	
					GC	REC	GC	REC
1	1	1	400	400	768.768	768.768	4.299	4.299
2	2	1	400	400	768.768	768.768	4.299	4.299
3	7	1	155	155	324.979	324.979	3.627	3.627
4	8	1	155	155	334.462	323.852	3.733	3.614
5	9	1	155	155	323.800	316.690	3.614	3.534
6	10	1	155	155	301.033	299.636	3.354	3.343
7	3	1	350	350	544.828	539.706	6.211	6.152
8	14	1	76	76	104.388	103.276	1.546	1.536
9	15	1	76	76	95.385	93.153	1.412	1.386
10-12	16	1,2	76	67.5,8.50*	77.541	80.581	1.148	1.199
11	18	1	270	270	240.000	240.000	0	0
13	17	1	76	76	44.707	50.413	0.662	0.750
14	4	1	197	197	79.803	89.809	1.586	1.784
15	5	1	197	197	45.484	48.759	0.904	0.968
16	6	1	197	197	23.550	23.928	0.468	0.475
17	11	1	100	100	6.459	6.372	0.143	0.141
18	12	1	100	100	3.870	3.792	0.085	0.084
19	13	1	100	100	2.181	2.225	0.048	0.049
20-24	23-27	1	5x12	5x12	0.800	0.861	0.023	0.025
25-28	19-22	1	4x20	4x20	0.641	0.693	0.023	0.026
			TOTAL		4087.600	4087.303	37.190	37.291

Expected Unserved Energy (GC) = 0.7269 GWh
Expected Unserved Energy (REC) = 1.0242 GWh
*67.5 MW block before hydro units; 8.50 MW block after hydro units.

TABLE 15

Expected Energy and Production Cost For First Quarter (Multi-Block Representation)

Loading Order	Unit No.	Block No.	Unit Size MW	Block Size MW	Expected Energy GWh GC	Expected Energy GWh REC	Production Costs M$ GC	Production Costs M$ REC
1	1	1	400	400	768.768	768.768	4.299	4.299
2	2	1	400	400	768.768	768.768	4.299	4.299
3	7	1	155	93	194.987	194.987	2.003	2.003
4	8	1	155	93	194.987	194.987	2.003	2.003
5	9	1	155	93	202.906	193.706	2.084	1.990
6	10	1	155	93	193.603	190.309	1.990	1.955
7	3	1	350	227.50	412.994	410.648	4.282	4.258
8	7	2	155	31	53.762	53.028	0.574	0.566
9	8	2	155	31	52.408	51.234	0.558	0.547
10	9	2	155	31	50.957	49.745	0.544	0.531
11,13	10	2a, 2b	155	3,28	39.220	34.956	0.419	0.373
12	18	1	300	300	420.000	420.000	0	0
14	3	2	350	52.50	49.306	49.591	0.536	0.539
15	7	3	155	31	29.036	29.171	0.324	0.325
16	8	3	155	31	27.387	27.697	0.305	0.309
17	9	3	155	31	25.785	26.329	0.288	0.294
18	10	3	155	31	24.160	25.071	0.270	0.280
19	3	3	350	70	44.702	48.105	0.510	0.548
20	14	1	76	28	24.468	26.915	0.323	0.356
21	14	2	76	22.80	13.613	15.010	0.167	0.184
22	15	1	76	38	21.007	23.637	0.278	0.312
23	15	2	76	22.80	11.594	13.121	0.142	0.161
24	16	1	76	38	17.810	20.030	0.235	0.265
25	16	2	76	22.80	9.758	10.855	0.120	0.133
26	17	1	76	38	14.905	16.269	0.197	0.215
27	17	2	76	22.80	8.116	8.884	0.100	0.107
28	14	3	76	15.20	5.103	5.375	0.076	0.080
29	15	3	76	15.20	4.835	5.064	0.072	0.075

TABLE 15 (cont'd)

Loading Order	Unit No.	Block No.	Unit Size MW	Block Size MW	Expected Energy GWh		Production Costs M$	
					GC	REC	GC	REC
30	16	3	76	15.20	4.607	4.747	0.069	0.071
31	17	3	76	15.20	4.386	4.467	0.065	0.066
32	4	1	197	118.20	26.386	26.483	0.521	0.523
33	5	1	197	118.20	17.201	17.683	0.340	0.349
34	6	1	197	118.20	10.378	10.906	0.205	0.215
35	11	1	100	55.00	3.135	3.289	0.062	0.065
36	12	1	100	55.00	2.306	2.376	0.046	0.047
37	13	1	100	55.00	1.658	1.746	0.033	0.034
38	4	2	197	39.40	0.821	0.899	0.019	0.020
39	4	3	197	39.40	0.619	0.714	0.012	0.014
40	5	2	197	39.40	0.494	0.604	0.011	0.013
41	5	3	197	39.40	0.371	0.478	0.007	0.009
42	6	2	197	39.40	0.293	0.393	0.007	0.009
43	6	3	197	39.40	0.384	0.300	0.008	0.006
44	11	2	100	25.00	0.225	0.167	0.005	0.004
45	12	2	100	25.00	0.202	0.141	0.004	0.003
46	13	2	100	25.00	0.182	0.119	0.004	0.003
47	11	3	100	20.00	0.128	0.080	0.003	0.002
48	12	3	100	20.00	0.117	0.070	0.003	0.002
49	13	3	100	20.00	0.109	0.061	0.002	0.001
50-54	23-27	1	12	12.00	0.275	0.143	0.008	0.004
55-58	19-22	1	20	20.00	0.240	0.105	0.009	0.004
				TOTAL	3757.975	3758.148	28.436	28.471

Expected Unserved Energy (GC) = 0.295 GWh
Expected Unserved Energy (REC) = 0.122 GWh

Table 15 shows the results for the first quarter when units are fragmented into blocks. Most of the units are divided into three capacity blocks. Each block has different incremental fuel cost depending upon the heat rate. The units are fragmented into blocks mainly to show the comparison of CPU time of both methods. As can be seen from Tables 11 & 15, the total production costs in the first quarter is less when units are fragmented than that of without fragmentation. This results from the fact that the cheaper units produce more energy when units are committed in blocks. The comparison of CPU times will be discussed in the next section.

In the calculation of expected energy generation by the GC method in Tables 9 through 15, series GC6 is utilized for probability values less than 10^{-3}. Between 10^{-3} and 10^{-2}, the series GC4 is utilized, and in the rest of the range, series GCB is used except in the base case when the expected energy generated is calculated directly by multiplying the capacity of the unit by time period and the avialability of the unit. The above arrangement was observed to provide the best combination, regarding computational efficiency and accuracy.

Table 16 gives the productions costs for different systems by the GC method and the REC method when different number of terms in the series are used. It can be seen that in all the other combinations, the difference in production costs is larger than that with the combination being used here.

The load shape affects the results much more than the type of the units. For example, for the fourth quarter, the factors G_i for the load distribution alone are G_1 = 0.3720, G_2 = -0.6998, G_3 = -0.4250, G_4 = 1.8539, G_5 = 7.7293, G_6 = -1.7596 and S = 0.0905. After adding up the cumulants of all the units' capacity outage PDF, the factors become; G_1 = 0.3549, G_2 = 0.5218, G_3 = -1.0258, G_4 = 1.2361, G_5 = 7.7293, G_6 = -1.7596 and S = 0.0996. Therefore there is a very slight change in these factors.

TABLE 16

Total Production Costs in M$					
System	GCB	GC4	GC6	COMB.*	REC
Load Shape 19(a)	64.86	63.72	65.32	64.83	64.99
Load Shape 19(b)	68.47	69.78	69.86	69.22	69.42
IEEE 1st Quarter Single Block	29.98	29.57	29.17	29.92	29.94
IEEE 2nd	31.86	31.67	31.27	31.77	31.73
IEEE 3rd	31.11	30.83	30.77	31.21	31.13
IEEE 4th	37.68	37.10	37.37	37.34	37.29
IEEE 1st quarter Multiblocks	28.68	27.97	27.88	28.45	28.47
IEEE 2nd	30.48	29.92	29.56	30.25	30.17
IEEE 3rd	29.76	29.22	29.18	29.61	29.71
IEEE 4th	35.95	35.07	34.75	35.67	35.81

COMB.* The following combination of the terms was used:
GC6 for Probability $<10^{-3}$
GC4 for $10^{-3} \leqslant$ Probability $\leqslant 10^{-2}$
GCB for probability $>10^{-2}$

6.4 COMPUTATIONAL EFFICIENCY

Computationally, the Gram-Charlier series method is very efficient. For the production costing of the IEEE RTS when units are not divided into blocks, the CPU time for the GC method is less than one second while it is more than 7 seconds for the REC method. The CPU time saving is more evident when units are fragmented into blocks. For the IEEE RTS, where all the units except peak and hydro are divided into three blocks, the CPU time for the GC method is less than 2 seconds as compared to 45 seconds by the REC method on IBM 360/65 computer. Undoubtedly, the GC method is very fast.

6.5 CONCLUSIONS

The main objective of this thesis is to investigate the Gram-Charlier expansion for the evaluation of LOLP, expected energy generation and production costing in generation planning. In the expansion planning studies, the evaluation of expected production costing by the Baleriaux-Booth method is a very time consuming technique. For optimum expansion planning, there may be several hundred configurations for a study period of 20 to 30 years. For each configuration, repeated calculations for integration and convolution are required. If the units are fragmented into blocks, the problem becomes more complex and computationally more inefficient.

The same problem may be simulated by the Gram-Charlier expansion method within an accuracy of 1.0 percent, which requires much less CPU time as compared to that of the Baleriaux-Booth method. Similarly in the evaluation of LOLP calculation a lot of CPU time may be saved by using the Gram-Charlier series method.

The accuracy of the Gram-Charlier expansion method employed for the evaluation of LOLP, capacity reserve margins and expected production costing has been studied. The accuracy of the method depends upon the variables such as the load shape, unit size, number of units as well as the forced outage rate. An attempt has been made to provide some guidelines in the applications of this method in the above mentioned evaluations. In the evaluation of LOLP and the capacity reserve margins, series GC6 gives the best results in the very small range of cumulative probability distribution. If the average forced outage is less than 0.02, the results obtained may not be satisfactory. Also if the factors S, G_1 and G_2 are very large the accuracy in the results may not be adequate for LOLP calculations. The Gram-Charlier method is very suitable for capacity reserve margins if $|S|<3.0$, G_2 <4.0 and G_1 <3.

In the evaluation of expected energy generation and production costing, it is noticed that for a very general load shape (load shape such as a IEEE RTS Load where a substantial portion of the load is base load and the cumulative load distribution function does not have very large discontinuities), the best results are obtained by using series GCB in the range of the cumulative probability $1-10^{-2}$, series GC4 in the range of cumulative probability $10^{-2}-10^{-3}$ and series GC6 in the range of cumulative probability values less that 10^{-3}. In the base case, the expected energy generated is calculated directly from the capacity and availability of the unit, and the time period.

It has also been suggested that the total energy (expected energy generated plus the energy not served) by the Gram-Charlier series method must be made equal to the energy under load curve by multiplying the expected energy generated by each unit, by the ratio of total energy by the Gram-Charlier method minus the base energy to the energy under load curve minus the base energy. By using this technique, the total production costs evaluated by the Gram-Charlier method is better approximated. It has been observed that the difference of total production costs evaluated by the recursive and the Gram-Charlier method was never more than 0.5%.

No doubt, the Gram-Charlier method is very fast and if used under the above mentioned guidelines gives reasonable good results. The Gram-Charlier method may be implemented into WASP (WIEN Automatic System Planning Package), an Optimal expansion planning program, which is considered to be computationally very expensive. The CPU time required to run the WASP may be substantially reduced by implementing the Gram-Charlier series method.

Appendix A

First 10 Hermite Polynomials are given as follows:

$$H_0 = 1$$

$$H_1 = x$$

$$H_2 = x^2-1$$

$$H_3 = x^3-3x$$

$$H_4 = x^4-6x^2+3$$

$$H_5 = x^5-10x^3+15x$$

$$H_6 = x^6-15x^4+45x^2-15$$

$$H_7 = x^7-21x^5+105x^3-105x$$

$$H_8 = x^8-28x^6-210x^4-420x^2+105$$

$$H_9 = x^9-36x^7+378x^5-1260x^3+945x$$

$$H_{10} = x^{10}-45x^8+630x^6-3150x^4+4725x^2-945$$

First 8 central moments in terms of moments about origin are written as:

$$\mu_1 = 0$$

$$\mu_2 = \mu'_2-\mu'^{2}_1$$

$$\mu_3 = \mu'_3-3\mu'_1\mu'_2+2\mu'^{3}_1$$

$$\mu_4 = \mu'_4-4\mu'_1\mu'_3+6\mu'^{2}_1\mu'^{1}_2-3\mu'^{4}_1$$

$$\mu_5 = \mu'_5-5\mu'_1\mu'_4+10\mu'^{2}_1\mu'_3-10\mu'^{3}_1\mu'_2+4\mu'^{5}_1$$

$$\mu_6 = \mu'_6-6\mu'_1\mu'_5+15\mu'^{2}_1\mu'_4-20\mu'^{3}_1\mu'_3+15\mu'^{4}_1\mu'_2-5\mu'^{6}_1$$

$$\mu_7 = \mu'_7-7\mu'_1\mu'_6+21\mu'^{2}_1\mu'_5-35\mu'^{3}_1\mu'_4+35\mu'^{4}_1\mu'_3+21\mu'^{5}_1\mu'_2+6\mu'^{7}_1$$

$$\mu_8 = \mu'_8-8\mu'_1\mu'_7+28\mu'^{2}_1\mu'_6-56\mu'^{3}_1\mu'_5+70\mu'^{4}_1\mu'_4-56\mu'^{5}_1\mu'_3+28\mu'^{6}_1-7\mu'^{8}_1$$

First 8 cumulents in terms of central moments are listed below:

$$k_2 = \mu_2$$

$$k_3 = \mu_3$$

$$k_4 = \mu_4 - 3\mu^2$$

$$k_5 = \mu_5 - 10\mu_3\mu_2$$

$$k_6 = \mu_6 - 15\mu_4\mu_2 - 10\,\mu_3^2 + 30\,\mu^2$$

$$k_7 = \mu_7 - 21\mu_5\mu_2 - 35\,\mu_4\mu_2 + 210\,\mu_3\mu_2{}^2$$

$$k_8 = \mu_8 - 28\mu_6\mu_2 - 56\mu_5\mu_3 - 35\mu_4{}^2 + 420\,\mu_4{}^2\mu_2{}^4 + 560\,\mu_3^2\mu_2 - 630\,\mu_2^4$$

The moments of the generating unit capacity outage with two states may be given by:

$$\mu_r' = c q^r$$

where

μ_r' is the rth moment

Appendix B

MOMENTS OF LDC

The area and the moments of the load duration curve (LDC) are calculated by using the intepolation techniques based on Newton's Form of the interpolating polynomial.

Let X_1, X_2 --- and X_n be the load points in the LDC and $f(x_1)$ ---- $f(X_n)$ be the cumulative probability values respectively. By making use of three points for interpolation, the interpolated value of load probability may written by [27]

$$p(x) = f(x_i) + f(x_i, x_{i+1})(x-x_i) + f(x_i, x_{i+1}, x_{i+2})(x-x_i)(x-x_{i+1})$$

where

$$f(x_i, x_{i+1}) = \frac{f(x_i) - f(x_{i+1})}{x_i - x_{i+1}} \qquad (\text{II-2})$$

and

$$f(x_i, x_{i+1}, x_{i+2}) = \frac{f(x_i, x_{i+1}) - f(x_{i+1}, x_{i+2})}{x_i - x_{i+2}} \qquad (\text{II-3})$$

The rth moment over the interval $X_i < X < X_{i+1}$ may be given by

$$m_{ri} = \int_{x_i}^{x_{i+1}} x^r p(x)\,dx = \int_{x_i}^{x_{i+1}} x^r f(x)\,dx + \int_{x_i}^{x_{i+1}} x^r (x-x_i)\, f(x_i, x_{i+1})\,dx$$

$$+ \int_{x_i}^{x_{i+1}} x^r (x-x_i)(x-x_{i+1})\, f(x_i, x_{i+1}, x_{i+2})\,dx \qquad \text{(II-4)}$$

The rth moment of the LDC may be given by

$$\mu_r = \sum_{i=1}^{n-1} m_{ri} \qquad \text{(II-5)}$$

The area under the LDC may be obtained from Eqns. (II-4) and (II-5) by inserting $r=0$.

From the moments, the cumulants may be calculated as is given in Appendix A.

Appendix C

IEEE Reliability Test System

The generating system data is given in Table C-1. The marginal costs are calculated by first obtaining the incremental heat rates and multiplying these by the fuel costs as given in Table C-2.

TABLE C-1

Generating Unit Data for IEEE Reliability Test System

Unit Size (MW)	No. of Units	Type	FOR	Marginal Costs ($/MWh) Single Block Representation	Three Blocks Repres. Lower	Middle	Upper
400	2	Nuclear	0.12	5.592	--	--	5.592
300	1	Coal	0.08	11.400	10.368	10.880	11.400
197	3	#6 Oil	0.05	19.971	19.757	22.563	19.872
155	4	Coal	0.04	11.160	10.272	10.680	11.160
100	3	#6 Oil	0.04	22.080	19.780	20.700	22.080
76	4	Coal	0.02	14.880	13.220	12.280	14.880
50	6	Hydro*	0.01	0.0	--	--	--
20	4	#2 Oil	0.10	37.500	--	--	37.500
12	5	#6 Oil	0.02	28.520	--	--	28.520

* Hydro Energy to be discharged

1st quarter = 420 GWh

2nd quarter = 420 GWh

3rd quarter = 120 GWh

4th quarter = 240 GWh

TABLE C-2

Fuel Type and Costs

Type	Cost $/ MBtu
#6 Oil	2.50
#2 Oil	3.00
Coal	1.20
Nuclear	0.60

The load models that were utilized in evaluation in four different quarters are given in Tables C-3, C-4, C-5 and C-6.

TABLE C-3

Load Model for the IEEE Reliability Test System Corresponding to the First Quarter

MW	Probability	MW	Probability	MW	Probability
978.12	1.000000	1518.48	0.630495	2055.78	0.238095
1054.54	0.985806	1579.54	0.591575	2117.86	0.200092
1120.51	0.970238	1643.34	0.546245	2186.01	0.161630
1176.57	0.938645	1679.76	0.516484	2231.55	0.125916
1240.09	0.892857	1733.53	0.472985	2280.60	0.088828
1291.90	0.858059	1777.63	0.432234	2332.44	0.053571
1343.80	0.817308	1822.18	0.395604	2378.19	0.029762
1390.76	0.766384	1865.95	0.358974	2452.25	0.009615
1432.70	0.715202	1923.41	0.315476	2539.35	0.000916
1474.02	0.672619	1989.07	0.272894	2565.00	0.000000

TABLE C-4

Load Model for IEEE Reliability Test System Corresponding to Second Quarter

MW	Probability	MW	Probability	MW	Probability
1001.74	1.000000	1538.62	0.652473	2145.16	0.250458
1063.38	0.994048	1598.78	0.615385	2199.32	0.208333
1128.60	0.980311	1655.16	0.581960	2248.34	0.167582
1186.68	0.952381	1711.69	0.544414	2304.98	0.122253
1243.06	0.909799	1757.20	0.507326	2349.31	0.086539
1289.73	0.873169	1800.29	0.464286	2400.38	0.048993
1333.80	0.836081	1844.39	0.430403	2462.40	0.017399
1374.02	0.798993	1901.92	0.395147	2527.95	0.004579
1407.67	0.760989	1968.00	0.362180	2565.00	0.000000
1450.51	0.721154	2031.48	0.324634		
1496.26	0.681319	2097.60	0.284799		

TABLE C-5

Load Model for IEEE Reliability Test System Corresponding to Third Quarter

MW	Probability	MW	Probability	MW	Probability
965.62	1.000000	1441.00	0.630495	1942.34	0.226648
1032.84	0.983517	1484.71	0.599661	1980.86	0.195513
1099.32	0.958333	1534.90	0.556777	2022.64	0.153388
1142.57	0.927198	1580.31	0.520604	2065.68	0.117216
1176.78	0.894231	1642.38	0.484890	2123.05	0.083791
1220.94	0.854396	1702.93	0.448718	2189.71	0.048535
1254.43	0.819597	1755.60	0.410714	2257.20	0.028846
1295.95	0.780220	1789.56	0.377289	2325.60	0.015568
1330.89	0.744963	1834.60	0.339744	2384.10	0.007326
1773.51	0.702381	1865.95	0.306319	2433.26	0.003663
1403.16	0.668956	1906.88	0.264194	2508.00	0.000000

TABLE C-6

Load Model for IEEE Reliability Test System Corresponding to Fourth Quarter

MW	Probability	MW	Probability	MW	Probability
1005.91	1.000000	1679.43	0.625458	2337.80	0.200092
1067.81	0.991758	1724.22	0.590202	2392.35	0.164835
1132.19	0.983059	1774.77	0.550366	2442.71	0.126374
1196.77	0.963370	1818.77	0.513278	2499.46	0.086996
1264.49	0.934066	1860.50	0.474817	2564.74	0.052656
1328.93	0.898810	1901.63	0.437271	2632.35	0.023810
1391.78	0.863553	1946.94	0.400183	2686.07	0.010531
1429.94	0.829670	2000.65	0.362180	2736.85	0.003663
1483.85	0.786630	2067.92	0.324634	2821.50	0.000916
1521.90	0.746337	2137.50	0.298993	2850.00	0.000000
1578.60	0.700550	2206.92	0.277015		
1626.28	0.661630	2276.08	0.240385		

TABLE C-7

SASK System Data
Installed Capacity = 1890 MW
Number of Units = 39

Capacity (MW)	Availability
62.0	0.9747
60.0	0.9859
142.0	0.9739
142.0	0.9723
142.0	0.9804
280.0	0.9550
62.0	0.9864
62.0	0.9844
95.0	0.9869
14.0	0.9868
24.0	0.9593
24.0	0.9973
31.0	0.9729
14.0	0.9710
19.0	0.9917
31.0	0.9922
19.0	0.9976
29.0	0.9923
22.0	0.9708
2.5	0.9793
2.5	0.9908
2.5	0.9713
10.0	0.9934
10.0	0.9970
15.0	0.9164
15.0	0.9725
15.0	0.9792
70.0	0.8800
34.5	0.9984
34.5	0.9946
34.5	0.9967
34.5	0.9984
34.5	0.9989
34.5	0.9980
39.0	0.9902
39.0	0.9531
63.0	0.9996
63.0	0.9998
63.0	0.9982

TABLE C-8

OH System
Installed Capacity = 17886 MW
Number of Units = 66

Capacity (MW)	No. of Units	Availability
500.00	4	0.91
200.00	1	0.92
22.00	1	0.92
500.00	4	0.92
25.00	18	0.85
100.00	4	0.92
200.00	4	0.91
66.00	4	0.91
100.00	1	0.92
500.00	14	0.92
300.00	8	0.91
750.00	3	0.88

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```
C     CALCULATION OF ENERGY BY THE GRAM-CHARLIER EXPANSION METHOD
      DIMENSION F(110),XVEC(110),AS(12),SYFI(10),C(100),
     1         P(30),Q(30),CBLK(30,4),CBK(30),FMT(18),ENERGY(100),
     1         IAA(30,4),ENRG(100),COST(100),YF(110),AIFC(30)
     1         ,XVC(12),XVB(12),TITLE(18)
      REAL M10,M20,M30,M40,M50,M60,M70,M80,IFC(30,6)/180*1000.0/,
     1         SUM1/0./,SUM2/0./,LOLP1,LOLP2,N(8),H(8),
     1         SCAP/0./,TENRGY/0./,TCOST/0./,
     1         M,IC(30),MIN,TC/0./,NZ
      INTEGER IFLAG/0/,FLAG/0/,CARRY/0/,LNB/0/
      COMMON M,VAR,SUM3,SUM4,SUM5,SUM6,SUM7,SUM8
      REAL M1(100),M2(100),M3(100),M4(100),M5(100),M6(100),M7(100),
     1         M8(100)
      DIMENSION CM1(100),CM2(100),CM3(100),CM4(100),CM5(100),CM6(100),
     1         CM7(100),CM8(100)
      DIMENSION CU3(100),CU4(100),CU5(100),CU6(100),CU7(100),CU8(100)
      VAR=0.
      M=0.
C
C     INPUT FOR THE TITLE
C
      READ(5,86)  (TITLE(KJ),KJ=1,18)
      WRITE(6,35)  (TITLE(KJ), KJ=1,18)
C
C     INPUT DATA FOR DEMAND ON THE SYSTEM
C
C     INPUT BASE LOAD,PEAK LOAD,NUMBER OF POINTS IN LDC CURVE
      READ(5,86) FMT
      READ(5,FMT) BASE,PEAK,NU
C
C     INPUT FOR X-AXIS OF LDC
      READ(5,86) FMT
      READ(5,FMT)  (XVEC(I),I=1,NU)
C
C     INPUT FOR Y-AXIS OF LDC
      READ(5,86)  FMT
      READ(5,FMT)  (F(I),I=1,NU)
C
C     INPUT FOR TIME PERIOD FOR ENERGY CALCULACTIONS
      READ(5,86)  FMT
      READ(5,FMT)  TIME
C     END OF INPUT FOR DEMAND
C
C     CALCULATIONS OF MOMENTS AND CUMULANTS OF LOAD
C
      DO 10 K=1,8
      SYFI(K)=BASE**(K+1)/(K+1.0)
   10 CONTINUE
      ALOAD=BASE
      AS(1)=10.0
      DO 12 J=2,11
      AS(J)=AS(J-1)*10.0
   12 CONTINUE
      F(NU+1)=0.
```

```
      XVEC(NU+1)=2.0*XVEC(NU)-XVEC(NU-1)
      NU=NU-1
      DO 20 I=1,NU
      B0=F(I)
      B1=(F(I)-F(I+1))/(XVEC(I)-XVEC(I+1))
      B12=(F(I+1)-F(I+2))/(XVEC(I+1)-XVEC(I+2))
      B2=(B1-B12)/(XVEC(I)-XVEC(I+2))
      AB1=B0-B1*XVEC(I)+B2*XVEC(I)*XVEC(I+1)
      AB2=B1-B2*(XVEC(I)+XVEC(I+1))
      AB3=B2
      XVC(1)=XVEC(I+1)/10.00
      XVB(1)=XVEC(I)/10.0
      DO 13 J=2,11
      XVC(J)=XVC(J-1)*XVC(1)
      XVB(J)=XVB(J-1)*XVB(1)
13    CONTINUE
      ALOAD=ALOAD+AB1*(XVC(1)-XVB(1))*AS(1)
     1        +AB2*(XVC(2)-XVB(2))*AS(2)/2.0
     1        +AB3*(XVC(3)-XVB(3))*AS(3)/3.0
      DO 14 K=1,8
      SYFI(K)=SYFI(K)+AB1*(XVC(K+1)-XVB(K+1))*AS(K+1)/(K+1.0)
     1          +AB2*(XVC(K+2)-XVB(K+2))*AS(K+2)/(K+2.0)
     1          +AB3*(XVC(K+3)-XVB(K+3))*AS(K+3)/(K+3.0)
14    CONTINUE
20    CONTINUE
      NU=NU+1
      M10=SYFI(1)/ALOAD
      M20=SYFI(2)/ALOAD
      M30=SYFI(3)/ALOAD
      M40=SYFI(4)/ALOAD
      M50=SYFI(5)/ALOAD
      M60=SYFI(6)/ALOAD
      M70=SYFI(7)/ALOAD
      M80=SYFI(8)/ALOAD
C.    THE CENTRAL MOMENTS OF LDC
      CM10=0.
      CM20=M20-M10**2
      CM30=M30-3.*M10*M20+2.*M10**3
      CM40=M40-4.*M10*M30+6.*M10**2*M20-3.*M10**4
      CM50=M50-5.*M10*M40+10.*M10**2*M30-10.*M10**3*M20
     1+4.*M10**5
      CM60=M60-6.*M10*M50+15.*M10**2*M40-20.*M10**3*M30
     1+15.*M10**4*M20-5.*M10**6
      CM70=M70-7.*M10*M60+21.*M10**2*M50-35.*M10**3*M40
     1+35.*M10**4*M30-21.*M10**5*M20+6.*M10**7
      CM80=M80-8.*M10*M70+28.*M10**2*M60-56.*M10**3*M50
     1+70.*M10**4*M40-56.*M10**5*M30+28*M10**6*M20
     2-7.*M10**8
      M=M+M10
      VAR=VAR+CM20
C     THE CUMULANTS ARE
      SUM3=CM30
      SUM4=CM40-3.*CM20**2
      SUM5=CM50-10.*CM30*CM20
```

```
      SUM6=CM60-15.*CM40*CM20-10.*CM30**2+30.*CM20**3
      SUM7=CM70-21.*CM50*CM20-35.*CM40*CM30
     1+210.*CM30*CM20**2
      SUM8=CM80-28.*CM60*CM20-56.*CM50*CM30-35.*CM40**2
     1+420.*CM40*CM20**2+560.*CM30**2*CM20-630.*CM20**4
      WRITE(6,58) ALOAD
      PRINT33
      PRINT36
      K=1
C
C     INPUT DATA FOR GENERATION SIDE
C
C     TOTAL NUMBER OF UNITS
      READ (5,86) FMT
      READ (5,FMT) NUM
C
C     INPUT NUMBER OF UNITS(OF ONE TYPE),CAPACITY,FOR,NUMBER OF BLOCKS
 85   READ (5,86) FMT
      READ (5,FMT) NIU,CAP,FOR,NB
C
C     BLOCK SIZE IN EACH UNIT
      READ (5,86) FMT
      READ (5,FMT) (CBK(J),J=1,NB)
C
C     INCREMENTAL FUEL COST OF EACH BLOCK
      READ (5,86) FMT
      READ(5,FMT) (AIFC(J),J=1,NB)
C     INPUT DATA ENDS
C
C     NUMBERING OF THE UNITS
C
      IF(LNB.LT.NB)LNB=NB
      KK=K+NIU-1
      DO 87 I=K,KK
      DO 87 J=1,NB
      Q(I)=FOR
      P(I)=1.0-FOR
      IFC(I,J)=AIFC(J)
      CBLK(I,J)=CBK(J)
      IC(I)=CAP
 87   CONTINUE
      K=KK+1
      IF(KK.LT.NUM) GOTO 85
C
C     HYDRO ENERGY INPUT
C
      READ(5,86)FMT
      READ(5,FMT) HYDRO
C
C     FIND OUT UNIT NUMBER OF HYDRO IN THE INPUT LIST
C
      DO 84 I=1,NUM
      IF(IFC(I,1).EQ.0.) GOTO 41
 84   CONTINUE
```

```
 41   IH=I
      HIFC=0.
      IFC(IH,1)=1000.0
 86   FORMAT(18A4)
C
      I=1
 78   IF(CARRY.EQ.1) GOTO 98
C
C     SELECTION OF MINIMUM MARGINAL COST BLOCK
      MIN=IFC(1,1)
      IU=1
      IB=1
      DO 88 IL=1,NUM
      DO 88 J=1,LNB
      IF(J.LE.1) GOTO 89
      IF(IFC(IL,J-1).LT.900) GOTO 88
 89   IF(MIN.LE.IFC(IL,J)) GOTO 88
      MIN=IFC(IL,J)
      IU=IL
      IB=J
 88   CONTINUE
      IF(MIN.GT.900.0) GOTO 100
C
C     TEST FOR FITTING HYDRO UNIT
C
      IF(SCAP.LT.BASE) GOTO 93
      IF(FLAG.EQ.1) GOTO 93
      ERROR=1.0
      XA=SCAP+CBLK(IU,IB)
      XB=XA+CBLK(IH,1)
      CALL UENRGY(XA,XB,ENGY)
      HYENG=ENGY*TIME*P(IH)*ALOAD/1000.0
      IF((HYENG-HYDRO).GE.0) GOTO 93
      ACA=SCAP
      ACB=SCAP+CBLK(IH,1)
      SSIZE=CBLK(IU,IB)/5.0
 99   DO 94 J=1,5
      FJ=J*SSIZE
      HA=ACA+FJ
      HB=ACB+FJ
      CALL UENRGY(HA,HB,ENGY)
      HYENG=ENGY*TIME*P(IH)*ALOAD/1000.0
      IF((HYENG-HYDRO).LE.(ERROR)) GOTO 95
 94   CONTINUE
 95   IF(ABS(HYENG-HYDRO).LE.ERROR) GOTO 96
      ACA=ACA+(J-1)*SSIZE
      ACB=ACB+(J-1)*SSIZE
      SSIZE=SSIZE/5.0
      GOTO 99
 96   LJJ=IB+1
      DO 76 J=LJJ,LNB
      LJ=(LNB-J)+LJJ
      IFC(IU,LJ+1)=IFC(IU,LJ)
      CBLK(IU,LJ+1)=CBLK(IU,LJ)
```

```
 76     CONTINUE
        CBLK(IU,IB+1)=CBLK(IU,IB)-(ACA+FJ-SCAP)
        CBLK(IU,IB)=ACA+FJ-SCAP
        IFC(IU,IB+1)=IFC(IU,IB)
        LNB=LNB+1
        CARRY=1
        GOTO 93
C
C       CALCULATION OF HYDRO ENERGY GENERATED
C
 98     XA=SCAP
        XB=XA+CBLK(IH,1)
        CALL UENRGY(XA,XB,ENGY)
        ENRG(I)=ENGY*TIME*P(IH)*ALOAD/1000.0
        COST(I)=0.
        CARRY=0
        FLAG=1
        IU=IH
        IB=1
        SCAP=SCAP+CBLK(IH,1)
        C(I)=CBLK(IH,1)
        GOTO 42
 93     IAA(IU,IB)=I
        IF(IB-1) 70,70,67
 67     IBM=IB-1
        M=M-M1(IAA(IU,IBM))
        VAR=VAR-CM2(IAA(IU,IBM))
        SUM3=SUM3-CU3(IAA(IU,IBM))
        SUM4=SUM4-CU4(IAA(IU,IBM))
        SUM5=SUM5-CU5(IAA(IU,IBM))
        SUM6=SUM6-CU6(IAA(IU,IBM))
        SUM7=SUM7-CU7(IAA(IU,IBM))
        SUM8=SUM8-CU8(IAA(IU,IBM))
        CSCAP=SCAP+CBLK(IU,IB)
        SSCAP=SCAP
        DO 72 J=1,IBM
        SSCAP=SSCAP-CBLK(IU,J)
 72     CONTINUE
        CBS=CSCAP-SSCAP
        IF(CSCAP.GT.BASE) GOTO 74
        IFLAG=IFLAG+1
        ENRGB=CBLK(IU,IB)*TIME*P(IU)/1000.0
        ENRG(I)=ENGPB
        GOTO 73
 70     CSCAP=SCAP+CBLK(IU,IB)
        CBS=CBLK(IU,IB)
        IF(CSCAP.GT.BASE) GOTO 74
        IFLAG=IFLAG+1
        ENRG(I)=CBLK(IU,IB)*TIME*P(IU)/1000.0
 73     COST(I)=ENRG(I)*IFC(IU,IB)/1000.0
        SCAP=CSCAP
        GOTO 97
C       CALCULATION OF ENERGY BY GRAM CHARLIER
 74     CAA=SCAP
```

```
      PARENG=0.
      IF(SCAP.GE.BASE) GOTO 61
      PSCAP=BASE-SCAP
      PARENG=PARENG+PSCAP*TIME*P(IU)
      CAA=BASE
 61   CBB=CSCAP
 68   CALL UENRGY(CAA,CBB,ENGY)
      ENRG(I)=(ENGY*TIME*ALOAD*P(IU)+PARENG)/1000.0
      COST(I)=ENRG(I)*IFC(IU,IB)/1000.0
      SCAP=CSCAP
 97   C(I)=CBS
 42   M1(I)=Q(IU)*C(I)
      M2(I)=M1(I)*C(I)
      M3(I)=M2(I)*C(I)
      M4(I)=M3(I)*C(I)
      M5(I)=M4(I)*C(I)
      M6(I)=M5(I)*C(I)
      M7(I)=M6(I)*C(I)
      M8(I)=M7(I)*C(I)
C     THE CENTRAL MOMENTS ARE
      CM1(I)=0.
      CM2(I)=M2(I)-M1(I)**2
      CM3(I)=M3(I)-3.*M1(I)*M2(I)+2.*M1(I)**3
      CM4(I)=M4(I)-4.*M1(I)*M3(I)+6.*M1(I)**2*M2(I)-3.*M1(I)**4
      CM5(I)=M5(I)-5.*M1(I)*M4(I)+10.*M1(I)**2*M3(I)-10.*M1(I)**3*M2(I)
     1+4.*M1(I)**5
      CM6(I)=M6(I)-6.*M1(I)*M5(I)+15.*M1(I)**2*M4(I)-20.*M1(I)**3*M3(I)
     1+15.*M1(I)**4*M2(I)-5.*M1(I)**6
      CM7(I)=M7(I)-7.*M1(I)*M6(I)+21.*M1(I)**2*M5(I)-35.*M1(I)**3*M4(I)
     1+35.*M1(I)**4*M3(I)-21.*M1(I)**5*M2(I)+6.*M1(I)**7
      CM8(I)=M8(I)-8.*M1(I)*M7(I)+28.*M1(I)**2*M6(I)-56.*M1(I)**3*M5(I)
     1+70.*M1(I)**4*M4(I)-56.*M1(I)**5*M3(I)+28*M1(I)**6*M2(I)
     2-7.*M1(I)**8
      M=M+M1(I)
      VAR=VAR+CM2(I)
C     THE CUMULANTS ARE
      CU3(I)=CM3(I)
      CU4(I)=CM4(I)-3.*CM2(I)**2
      CU5(I)=CM5(I)-10.*CM3(I)*CM2(I)
      CU6(I)=CM6(I)-15.*CM4(I)*CM2(I)-10.*CM3(I)**2+30.*CM2(I)**3
      CU7(I)=CM7(I)-21.*CM5(I)*CM2(I)-35.*CM4(I)*CM3(I)
     1+210.*CM3(I)*CM2(I)**2
      CU8(I)=CM8(I)-28.*CM6(I)*CM2(I)-56.*CM5(I)*CM3(I)-35.*CM4(I)**2
     1+420.*CM4(I)*CM2(I)**2+560.*CM3(I)**2*CM2(I)-630.*CM2(I)**4
      SUM3=SUM3+CU3(I)
      SUM4=SUM4+CU4(I)
      SUM5=SUM5+CU5(I)
      SUM6=SUM6+CU6(I)
      SUM7=SUM7+CU7(I)
      SUM8=SUM8+CU8(I)
      ENERGY(I)=ENRG(I)
      TENRGY=TENRGY+ENERGY(I)
      TCOST=TCOST+COST(I)
      WRITE(6,34) I,IU,IB,IC(IU),CBLK(IU,IB),IFC(IU,IB),ENERGY(I)
```

```
     1,COST(I)
      IFC(IU,IB)=1000.0
      I=I+1
      GOTO 78
100   NBLK=I-1
C
C     LOLP CALCULATIONS BY AVERAGING BETWEEN SCAP+-5
C
      ALOPM=SCAP-5.0
      ALOPP=SCAP+5.0
      CALL UENRGY(ALOPM,ALOPP,ENGY)
      LOLP1=ENGY*ALOAD/10.0
      WRITE(6,38) LOLP1
      XC=SCAP
      CALL PROBLY(XC,PROBB,PROB4,PROB6)
      PROB2=PROB6*ALOAD*TIME/1000.0
      WRITE(6,44)TENRGY
      WRITE(6,45)TCOST
      WRITE(6,46)PROB2
      TENRGY=TENRGY+PROB2
      ENLOAD=ALOAD*TIME/1000.0
      WRITE(6,49)ENLOAD
      DO 16 I=1,IFLAG
      TENRGY=TENRGY-ENERGY(IFLAG)
      ENLOAD=ENLOAD-ENERGY(IFLAG)
16    CONTINUE
      RATIO=ENLOAD/TENRGY
      NFLAG=IFLAG+1
      DO 17 I=NFLAG,NBLK
      COST(I)=COST(I)*RATIO
      ENERGY(I)=ENERGY(I)*RATIO
      WRITE(6,47) I,ENERGY(I),COST(I)
17    CONTINUE
      RCOST=0.0
      DO 18 I=1,NBLK
      RCOST=RCOST+COST(I)
18    CONTINUE
      WRITE(6,50)RCOST
      PROB2=PROB2*RATIO
      WRITE(6,52)PROB2
30    FORMAT(//5X,F10.5,F15.9)
31    FORMAT(/2X,'BLOCK SIZE OF THE UNITS'/)
32    FORMAT(/2X,'THE LOAD DURATION CURVE'/)
33    FORMAT(/5X,'LOADING',5X,'UNIT',6X,'BLOCK',6X,'UNIT SIZE',8X,
     1'BLOCK SIZE',11X,'IFC',14X,'EXPECTED ENERGY',8X,'COST')
34    FORMAT(5X,I5,5X,I5,5X,I5,5X,F12.2,5X,F12.2,5X,F15.6,5X,F15.4,
     15X,F9.6)
35    FORMAT(/2X,18A4)
36    FORMAT(6X,'ORDER',7X,'NO.',7X,'NO.',10X,'(MW)',14X,'(MW)',13X,
     1'$/MWH',15X,'(GWH)',16X,'(M$)'/)
37    FORMAT(//5X,'TOTAL CAPACITY=',F12.5,'MW'//)
38    FORMAT('0',5X,'LOLP BY AVERAGING=',F10.6/)
44    FORMAT(/5X,'TOTAL ENERGY GENERATED=',F10.4/)
45    FORMAT(/5X,'TOTAL COST OF ENERGY GENERATED=',F13.6/)
```

```
 46   FORMAT(/5X,'ENERGY NOT SERVED (GC METHOD)=',F10.5/)
 47   FORMAT(5X,I5,'ENERGY=',F15.5,5X,'COST=',F10.6)
 49   FORMAT(/5X,'ENERGY UNDER LDC=',F15.5)
 50   FORMAT(/5X,'COST AFTER EQUATING ENERGIES=$',F10.6)
 52   FORMAT(/5X,'UNSERVED ENERGY (AFTER EQUATING)=',F15.5)
 51   FORMAT(//' THE CAPACITIES OF THE UNITS ARE')
 53   FORMAT( //' THE FORCED OUTAGE RATE OF UNITS ARE')
 58   FORMAT(/5X,'AREA UNDER LDC CURVE (ALOAD)=',F12.4/)
 91   FORMAT(9F7.3)
 92   FORMAT(12F5.1)
      STOP
      END
C
C     SUBROUTINE TO CALCULATE ENEEGY BY (GC)
C
      SUBROUTINE UENRGY(CAA,CBB,ENGY)
      CALL PROBLY(CAA,PROBB,PROB4,PROB6)
      PROB1=PROBB
      PTEST=PROB1
      IF(PTEST.LT.0.01  .AND.  PTEST.GT.0.001 )PROB1=PROB4
      IF(PTEST.LE.0.00  )PROB1=PROB6
      CALL PROBLY(CBB,PROBB,PROB4,PROB6)
      PROB2=PROBB
      IF(PTEST.LT.0.01  .AND.  PTEST.GT.0.001 )PROB2=PROB4
      IF(PTEST.LE.0.001 )PROB2=PROB6
      ENGY=PROB1-PROB2
      RETURN
      END
C
C     SUBROUTINE TO CALCULATE PROBABILITY BY (GC)
C
      SUBROUTINE PROBLY(X,PROBB,PROB4,PROB6)
      REAL M,V,H(10),N(10),NZ
      COMMON M,VAR,SUM3,SUM4,SUM5,SUM6,SUM7,SUM8
      V=SQRT(VAR)
      G1=SUM3/V**3
      G2=SUM4/V**4
      G3=SUM5/V**5
      G4=SUM6/V**6
      G5=SUM7/V**7
      G6=SUM8/V**8
      Z=(X-M)/V
      NZ=EXP(-Z**2/2.0)/SQRT(2.0*3.1416)
      H(1)=Z
      H(2)=Z*H(1)-1.0
      DO 11 I=3,7
      H(I)=Z*H(I-1)-(I-1)*H(I-2)
 11   CONTINUE
      DO 12 I=1,7
      N(I)=(-1)**I*H(I)*NZ
 12   CONTINUE
      GT2=G1*N(2)/6.0-G2*N(3)/24.0
      GT3=G3*N(4)/120.0
      GT4A=G4*N(5)/720.0
```

```
      GT4B=10.0*(G1**2)*N(5)/720.0
      GT5A=G5*N(6)/5040.0
      GT5B=35.0*G1*G2*N(6)/5040.0
      GT6A=G6*N(7)/40320.0
      GT6B=(56.0*G1*G3+35.0*(G2**2))*N(7)/40320.0
      GT4=GT4A+GT4B
      GT5=GT5A+GT5B
      GT6=GT6A+GT6B
      SK=GT2-GT4B
      CALL AREA(Z,AR)
      PROBB=AR+SK
      PROB4=PROBB+GT3-GT4A
      PROB6=PROB4+GT5-GT6
      RETURN
      END
C
C     SUBROUTINE TO CALCULATE AREA UNDER NORMAL DISTRIBUTION
C
      SUBROUTINE AREA(Z,CURVE)
      DIMENSION A(191),EQ1(98),EQ2(93)
      EQUIVALENCE (A(1),EQ1(1)),(A(99),EQ2(1))
C
C     THE DATA FOR GAUSSIAN FUNCTION (ERROF FUNCTION)
      DATA EQ1/
     *.5000000,.5079783,.5159534,.5239221,.5318313,.5398278,.5477583,
     *.5556700,.5635594,.5714236,.5792597,.5870644,.5948347,.6025681,
     *.6102611,.6179114,.6255158,.6330717,.6405764,.6480272,.6554217,
     *.6627572,.6700314,.6772419,.6843863,.6914625,.6984681,.7054015,
     *.7122602,.7190427,.7257469,.7323711,.7389137,.7453731,.7517478,
     *.7580363,.7642375,.7703500,.7763727,.7823046,.7881415,.7938918,
     *.7995458,.8051054,.8105703,.8159398,.8212135,.8263912,.8314724,
     *.8364569,.8413447,.8461358,.8508301,.8554246,.8599239,.8643339,
     *.8686430,.8728568,.8769756,.8809997,.8849303,.8887675,.8925123,
     *.8961653,.8997273,.9031995,.9065825,.9098772,.9130850,.9162067,
     *.9192433,.9221961,.9250663,.9278550,.9305634,.9331928,.9357445,
     *.9382198,.9406201,.9429466,.9452007,.9473339,.9494973,.9515428,
     *.9535213,.9554344,.9572838,.9590704,.9607961,.9624619,.9640697,
     *.9656205,.9671159,.9685572,.9699458,.9712834,.9725711,.9738102/
      DATA EQ2/
     *.9750021,.9761482,.9772497,.9783062,.9793248,.9803002,.9812372,
     *.9821356,.9829969,.9838226,.9846137,.9853712,.9860966,.9867906,
     *.9874545,.9880893,.9886961,.9892759,.9898295,.9903541,.9908625,
     *.9913437,.9918025,.9922397,.9926564,.9930531,.9934309,.9937903,
     *.9941323,.9944572,.9947663,.9950598,.9953388,.9956034,.9958547,
     *.9960929,.9963188,.9965330,.9967359,.9969280,.9971099,.9972821,
     *.9974447,.9975988,.9977443,.9978818,.9980116,.9981342,.9982498,
     *.9983588,.9984618,.9985588,.9986501,.9988558,.9990323,.9991836,
     *.9993128,.9994228,.9995165,.9995959,.9996630,.9997197,.9997674,
     *.9998074,.9998409,.9998688,.9998922,.9999115,.9999276,.9999409,
     *.9999519,.9999609,.9999683,.9999744,.9999793,.9999833,.9999866,
     *.9999893,.9999915,.9999931,.9999946,.9999956,.9999965,.9999973,
     *.9999979,.9999983,.9999987,.9999989,.9999992,.9999993,.9999995,
     *.9999996,.9999996/
      IF(Z.GT.5.0)GOTO 26
```

```
      IF(Z.GT.3.0)GOTO 27
      IF(Z) 18,19,20
 19   CURVE=0.5
      RETURN
 20   Y=(Z/0.02)+1.0
      I=Y
      YC=Y-I
      CURVE=1.0-A(I)-(A(I+1)-A(I))*YC
      RETURN
 18   Z=-Z
      IF(Z.GT.3.0)Y=((Z-3.0)/0.05)+151.0
      IF(Z.LE.3.0)Y=(Z/.02)+1.0
      I=Y
      YC=Y-I
      CURVE=A(I)+(A(I+1)-A(I))*YC
      RETURN
 27   Y=(Z-3.0)/0.05+151
      I=Y
      YC=Y-I
      CURVE=1.0-A(I)-(A(I+1)-A(I))*YC
      RETURN
 26   CURVE=0.0
 1    RETURN
      END
//GO.SYSIN  DD  *
 IEEE TEST SYSTEM QUARTER 1 WITHOUT BLOCKS
(F8.2,F9.2,I4)
  978.12  2565.00   58
(10F8.2)
  978.12 1019.89 1054.54 1086.08 1120.51 1150.00 1176.57 1210.66 1240.09 1268.5
 1291.90 1320.21 1343.80 1369.37 1390.76 1418.23 1432.70 1450.13 1474.02 1494.0
 1518.48 1547.72 1579.54 1609.58 1643.24 1662.64 1679.76 1705.44 1733.53 1756.
 1777.63 1805.76 1822.18 1842.52 1865.95 1895.45 1923.41 1955.30 1989.07 2022.
 2055.78 2090.36 2117.86 2156.88 2186.01 2208.94 2231.55 2253.59 2280.60 2305.1
 2332.44 2361.60 2378.19 2407.68 2452.25 2488.56 2539.35 2565.00
(10F8.6)
1.000000 .994048 .985806 .979396 .970238 .956960 .938645 .913919 .892857 .87545
 .858059 .836996 .817308 .793040 .766484 .732601 .715202 .693223 .672619 .65155
 .630495 .612180 .591575 .570055 .546245 .531593 .516484 .496795 .472985 .45467
 .432234 .409799 .395604 .379579 .358974 .336081 .315476 .293040 .272894 .25732
 .238095 .224359 .200092 .183608 .161630 .143773 .125916 .106685 .088828 .06959
 .053571 .036172 .029762 .016941 .009615 .004121 .000916 .000000
(F8.2)
 2184.00
(I3)
 27
(I3,F8.2,F4.2,I3)
  2  400.00 .12   1
(F8.2)
  400.00
(F7.3)
  5.592
(I3,F8.2,F4.2,I3)
  1  350.00 .08   1
```

```
(F8.2)
 350.00
(F7.3)
 11.400
(I3,F8.2,F4.2,I3)
  3  197.00 .05   1
(F8.2)
  197.00
(F7.3)
 19.872
(I3,F8.2,F4.2,I3)
  4  155.00 .04   1
(F8.2)
  155.00
(F7.3)
 11.160
(I3,F8.2,F4.2,I3)
  3  100.00 .04   1
(F8.2)
  100.00
(F7.3)
 22.080
(I3,F8.2,F4.2,I3)
  4   76.00 .02   1
(F8.2)
   76.00
(F7.3)
 14.880
(I3,F8.2,F4.2,I3)
  1  300.00 .01   1
(F8.2)
  300.00
(F7.3)
  0.0
(I3,F8.2,F4.2,I3)
  4   20.00 .10   1
(F8.2)
   20.00
(F7.3)
 37.500
(I3,F8.2,F4.2,I3)
  5   12.00 .02   1
(F8.2)
   12.00
(F7.3)
 28.520
(F8.2)
  420.00
/*
```