

Development of Drape Simulation Software and the Optimisation of Variable-Length Textiles

Nicholas Burnford

A thesis submitted to the Faculty of Graduate and Postdoctoral Studies in partial fulfilment
of the requirements for the Degree of

MASTER OF APPLIED SCIENCE

In Mechanical Engineering

Ottawa-Carleton Institute for Mechanical and Aerospace Engineering

University of Ottawa

Ottawa, Canada

March 2011

© Nicholas Burnford, Ottawa, Canada, 2011

Abstract

The group of manufacturing methods known as Liquid Composite Moulding (LCM) is becoming the industry standard for Polymer Matrix Composite (PMC) production. These processes are versatile and cost-effective, but they are extremely dependant on the availability of textile preforms offering good quality and consistency. Good quality preforms are realised through well-controlled fibre volume fractions, fibre orientations and thicknesses as well as the absence of defects such as out-of-plane deformations and inter-fibre gaps. Preform quality is largely determined by the draping operation, which may be modeled to better inform the design of PMC components. A draping simulation software was developed in this thesis, which can simulate the draping of textiles onto model surfaces using the kinematic draping algorithm.

In addition, the thesis presents a novel textile architecture where yarn spacing lengths in a textile may vary across the textile. These variable-length textiles can be custom-tailored for specific PMC applications, offering advantages over conventional constant-length textiles such as a larger surface area covered by a single piece of textile, lower values of in-plane shear, and controlled fibre orientations. The variable-length textiles can be optimised manually or using algorithms based on Monte Carlo methods which are implemented in the software.

The draping simulation software was validated by comparing laboratory trials with drape simulations, and results obtained using generic demonstrators and an industrial component; several optimisation results are presented, demonstrating the advantages associated with variable-length textiles over conventional, constant-length textiles.

Acknowledgements

I would like to thank all of my colleagues from our little room in the basement of Colonel By at the University of Ottawa with whom great times and cooperation were shared. The atmosphere of the office was greatly missed while writing this thesis.

Much appreciation goes out to my thesis supervisor François Robitaille for challenging my writing abilities and for the ongoing advice and guidance provided to me during the development of the software as well as the drafting of this thesis. Additional thanks go out to Reza Samadi who provided help in many aspects of the development of the software, and to James Cowan for assisting with validation of the software.

Table of Contents

Abstract	ii
Acknowledgements	iii
List of Figures	vii
List of Tables	xiv
Nomenclature	xv
1. Introduction	1
1.1. Liquid Moulding Processes	1
1.2. Preforming	5
1.3. Draping Simulation	8
1.4. Thesis Overview	13
1.4.1. Kinematic Simulation Software	14
1.4.2. Software Optimisation of Variable-Length Textiles	15
2. Literature Review	17
2.1. Reinforcements	17
2.1.1. Fibres	17
2.1.2. Yarns	21
2.1.3. 2D Textile Reinforcements	29
2.1.4. 2.5D and 3D Textile Reinforcements	37
2.2. In-Plane Shear	39
2.3. Effect of Fibre Orientation on Strength and Stiffness	44
2.4. Preforming Practice	48
2.5. Kinematic Draping	51
2.6. Pseudo Random Iterative Models for Optimisation	55
3. Draping Simulation Software	59
3.1. Software Overview	59
3.2. Model Geometry Specification and Format	61
3.3. Model Pre-processing	62
3.4. Draping Algorithm	67
3.4.1. Start Point of the Draping Operation	68
3.4.2. Creation of the Geodesic Lines	72
3.4.2.1. Initial Geodesic Direction Vectors	73
3.4.2.2. Intersection of Geodesic and Element Edge	75

3.4.2.3.	Subsequent Geodesic Direction Vectors	82
3.4.3.	Creating the Mesh	89
3.4.3.1.	Meshing the Geodesics	90
3.4.3.2.	Meshing Between Geodesics	97
3.4.3.2.1.	Intersection of the Two Spheres	99
3.4.3.2.2.	Definition of the Brick Space	102
3.4.3.2.3.	Intersection of the Circle Plane with an Element Plane	105
3.4.3.2.4.	Intersection of the Line of Intersection and Element Edge	108
3.4.3.2.5.	Selecting the Best Among Possible Solutions.....	115
3.4.3.3.	Sequence of the Meshing Operation.....	122
3.5.	Output of the Software	125
3.6.	Additional features	128
4.	Textile Optimisation	135
4.1.	Manual Optimisation	136
4.2.	Software Optimisation Algorithm.....	137
4.2.1.	Optimisation Parameters	138
4.2.2.	Parameter to Maximise	146
4.2.3.	Optimisation Type	152
5.	Results and Validation	156
5.1.	Constant Length Textiles: Validation of the Draping Software	156
5.1.1.	Validation on the Hemisphere	156
5.1.2.	Validation on the Aerospace Mould	163
5.2.	Variable-Length Textiles: Optimisation Results	169
5.2.1.	Surface Area Maximisation on the Hemisphere	169
5.2.2.	Surface Area Optimisation on the Aerospace Mould	175
5.2.3.	Fibre Orientation Optimisation on the Aerospace Mould.....	182
6.	Conclusion and Recommendations	188
	References	191
	Appendix A	195
	Format of the .model Files	195
	Format of the .dparam Files	196
	Appendix B	197
	Optimisation Parameters for Maximising the Surface Area on the Hemisphere.....	197
	Optimisation Parameters for Maximising the Surface Area on the Aerospace Mould.....	198

Optimisation Parameters for Modifying the Fibre Orientation on the Aerospace Mould	199
--	-----

List of Figures

Figure 1-1 The RTM Process [1]	3
Figure 1-2 The VARTM Process [2]	4
Figure 1-3 The Quickstep™ Process [5].....	4
Figure 1-4 Rolls Royce Prototype Jet Engine Nose Cone Preform [6]	6
Figure 1-5 Presence of In-Plane Shear on a Draped Textile.....	8
Figure 1-6 Surface Draped by a Kinematic Method [10]	9
Figure 1-7 Example of a Unit Cell Used in FE Simulation [11]	10
Figure 1-8 Elementary Pattern Used in FE Simulation [12].....	11
Figure 1-9 Mapping of Elementary Patterns to Finite Elements [12].....	11
Figure 1-10 Hybrid Kinematic Energy Drape [14].....	12
Figure 1-11 Variable-Length Textile Reinforcement	14
Figure 2-1 Deformation Modes of a Single Fibre.....	18
Figure 2-2 Representative Wavy Fibre Element [17]	19
Figure 2-3 Section of Bent Fibre	20
Figure 2-4 Fibre Waviness and Misalignment Within a Yarn [25]	22
Figure 2-5 Deformation Modes of a Single Yarn	23
Figure 2-6 Schematic of BES-FY [29]	24
Figure 2-7 Quarter Yarn for Second Moment of Inertia Calculations.....	26
Figure 2-8 Uncompacted and Compacted Textiles [32]	27
Figure 2-9 Cross Sections of Uncompacted and Compacted Textiles [32].....	27
Figure 2-10 Intra-Yarn Shear [34]	28
Figure 2-11 Intra-Yarn Shear Displacement Profile [33]	28

Figure 2-12 Various 2D Weaves [35]	30
Figure 2-13 Warp Knit Without and With Structural Yarns [35]	31
Figure 2-14 Chain Stitch Warp Knit [36]	31
Figure 2-15 Tricot Stitch Warp Knit [36]	32
Figure 2-16 Plain Weft Knit [37]	33
Figure 2-17 Different Stitching Patterns [38]	34
Figure 2-18 Braided Textile [39]	35
Figure 2-19 Deformation Modes C1 and C2 of a Textile [40]	35
Figure 2-20 Deformation Modes C3, C4 and C5 of a Textile	36
Figure 2-21 3D Weaving Process [44]	38
Figure 2-22 3D Braided Preforms [44]	39
Figure 2-23 In-Plane Shear of a Textile [35]	39
Figure 2-24 Effect of Textile Architecture on In-Plane Shear Behaviour [46]	40
Figure 2-25 Picture Frame Test [47]	41
Figure 2-26 Bias Extension Test [47]	42
Figure 2-27 Shear Force as a Function of Shear Angle [47]	43
Figure 2-28 Deformation by In-Plane Shearing of a Textile Reinforcement [49]	44
Figure 2-29 Off-Axis Stresses and Strains [51]	46
Figure 2-30 Q_{II} Versus Fibre Angle [51]	48
Figure 2-31 Nesting of Woven Textiles [53]	49
Figure 2-32 Lap Joints Within a Preform [54]	50
Figure 2-33 Symmetric Step Joint [55]	50
Figure 2-34 Early Drape Simulation Over a Sphere [9]	52

Figure 2-35 Determination of Warp and Weft Crossover Points [58].....	53
Figure 2-36 Effect of Constrained Yarns on Draping Simulation [7]	54
Figure 2-37 Effect of Start Location on Draping Simulation [58].....	54
Figure 3-1 Draping Software GUI	60
Figure 3-2 Meshed Solid Model Suitable for the Draping Software	62
Figure 3-3 Angle Between Vectors Normal to Neighbouring Elements	65
Figure 3-4 Algorithm to Determine Element Normal Vectors	66
Figure 3-5 Solid Model as Loaded in the Draping Software	67
Figure 3-6 Start Point Coordinate Dialog	68
Figure 3-7 2D Vectors on an Element Projection	69
Figure 3-8 Multiple Elements Intersecting the Selection Vector.....	71
Figure 3-9 Different Orientations of Geodesics at the Start Point.....	73
Figure 3-10 Determining the Initial Geodesic Direction Vector	74
Figure 3-11 Intersections Between Geodesic and Element Edges for the First Element	76
Figure 3-12 Algorithm for m Value Comparison	79
Figure 3-13 Intersection Between Geodesic and Element Edge Extension.....	80
Figure 3-14 Intersections Between Geodesic and Element Edges for Subsequent Elements.....	81
Figure 3-15 Change of Geodesic Direction Vector	83
Figure 3-16 Algorithm for Geodesic Generation (Part 1).....	87
Figure 3-17 Algorithm for Geodesic Generation (Part 2: Geodesic Loop)	88
Figure 3-18 Crossover Location Array	89
Figure 3-19 Arrays Used for Storing Yarn Spacings.....	90
Figure 3-20 Yarn Crossover Points on Geodesics	91

Figure 3-21 Location of Yarn Crossover Points	92
Figure 3-22 Geodesic Crossover Location When $l_m=l_t$	93
Figure 3-23 Geodesic Crossover Location When $l_m<l_t$	94
Figure 3-24 Geodesic Crossover Location When $l_m>l_t$	95
Figure 3-25 Algorithm to Mesh Geodesics.....	96
Figure 3-26 Location of Mesh Quadrants	97
Figure 3-27 Geometric Relations for Meshing	98
Figure 3-28 3D Representation of the Geometric Relations for Meshing	98
Figure 3-29 Circle of Intersection of Two Spheres	100
Figure 3-30 Determining the Centre and Radius of the Circle of Intersection	101
Figure 3-31 Definition of the Brick Space.....	103
Figure 3-32 Inclusion of Different Elements in the Brick Space.....	105
Figure 3-33 Intersection of the Circle Plane with an Element Plane	106
Figure 3-34 Lines of Intersection Which Do and Do Not Intersect an Element.....	108
Figure 3-35 Crossover Location Not Within the Bounds of an Element.....	109
Figure 3-36 One Intersection Between the Line of Intersection and the Element Edges	110
Figure 3-37 Intersection of the Circle of Intersection and the Line of Intersection	112
Figure 3-38 Possible Intersections Between a Sphere and a Line	113
Figure 3-39 Convex and Concave Drape Elements.....	116
Figure 3-40 Geometry for the Convexity Check	116
Figure 3-41 Limit Conditions for the Convexity Check.....	117
Figure 3-42 Angles in Constant-Length and Variable-Length Textiles	118
Figure 3-43 Determining Angles β_2 and β_4	119

Figure 3-44 Setting Yarn Spacings of Yarn Crossover that Could Not be Draped	120
Figure 3-45 Order for Finding Yarn Crossovers.....	122
Figure 3-46 Algorithm for Meshing Between Geodesics (Part 1).....	123
Figure 3-47 Algorithm for Meshing Between Geodesics (Part 2: Brick Space Loop).....	124
Figure 3-48 Simulated Mesh Displayed on Model	125
Figure 3-49 Colour Map of the In-Plane Shear Angle.....	126
Figure 3-50 Colour Map of the Fibre Volume Fraction	127
Figure 3-51 Colour Coded Yarn Spacings for Variable-Length Textiles.....	128
Figure 3-52 Model Draped with Multiple Meshes	129
Figure 3-53 Drape Simulation Limited by a Patch	130
Figure 3-54 Creation of a Patch Within the Software.....	131
Figure 3-55 Gravity Effect on the Mesh	132
Figure 3-56 Draping Simulation with Magnets	133
Figure 3-57 Variable-Length Textile Draped on a Flat Surface	134
Figure 4-1 Yarn Spacing of a Textile	135
Figure 4-2 Manual Optimisation Using the Draping Software.....	137
Figure 4-3 Algorithm for the Software Optimisation of a Variable-Length Textile	138
Figure 4-4 Optimisation Performed Without Maximum and Minimum Yarn Spacings	139
Figure 4-5 Optimisation Performed Without a Yarn Spacing Change Limit	140
Figure 4-6 Optimisations Performed with a Yarn Spacing Change Limited to 0.1 mm	141
Figure 4-7 Optimisations Performed with a Yarn Spacing Change Limited to 0.01 mm	142
Figure 4-8 Similarity Check for Neighbouring Yarn Spacings	143
Figure 4-9 Effect of the Minimum Similarity Parameter on the Results of the Optimisation....	144

Figure 4-10 Optimising for Surface Area	147
Figure 4-11 Optimising for the In-Plane Shear Angle.....	148
Figure 4-12 Optimising for the Product of In-Plane Shear Angle and Area	149
Figure 4-13 Parameter to Maximise Based on an Allowable Range for v_f	151
Figure 4-14 Optimising for Fibre Orientation	151
Figure 4-15 Non Optimised Textile and Initial Section of a Progressive Optimisation.....	153
Figure 4-16 Layer of Yarn Spacings Eligible for Modification in a Progressive Optimisation .	154
Figure 4-17 End Result of a Progressive Optimisation	155
Figure 5-1 Hemisphere Model Used for Software Validation.....	157
Figure 5-2 Hemisphere Mould Used for Software Validation.....	157
Figure 5-3 Comparison of Drape and Simulation (Hemisphere).....	158
Figure 5-4 Points of Reference on the Hemisphere Used for Calibrating	159
Figure 5-5 Laboratory Drape on the Hemisphere With Start Point and Geodesics [72]	160
Figure 5-6 One Quadrant Validation Test on the Hemisphere	160
Figure 5-7 Four Quadrant Validation Test on the Hemisphere [72].....	162
Figure 5-8 Aerospace Mould Used for Validation	163
Figure 5-9 Comparison of Drape and Simulation (Aerospace Mould).....	164
Figure 5-10 Points of Reference on the Mould Used for Calibrating.....	165
Figure 5-11 Four Quadrant Validation Test on the Aerospace Mould (View 1).....	166
Figure 5-12 Four Quadrant Validation Test on the Aerospace Mould (View 2).....	167
Figure 5-13 Visualisation of the Difference in Size Between the Model and Mould.....	168
Figure 5-14 Non-Optimised Textile Draped on the Hemisphere.....	170
Figure 5-15 Progressive Optimisation for the Hemisphere	171

Figure 5-16 Manual Optimisation for the Hemisphere.....	172
Figure 5-17 Optimised Textile for the Hemisphere as it Appears When Laid Flat.....	173
Figure 5-18 Pattern Used for Manufacturing the Hemisphere Textile	173
Figure 5-19 Pins Used for the Manufacture of Optimised Textiles.....	174
Figure 5-20 Manufactured Textile for the Hemisphere Optimised for Surface Area.....	175
Figure 5-21 Patch on the Aerospace Mould to Maximise Surface Area	175
Figure 5-22 Non-Optimised Textile on the Aerospace Mould	176
Figure 5-23 Result of a Progressive Optimisation on the Aerospace Mould	177
Figure 5-24 Manual Optimisation A of the Aerospace Mould.....	178
Figure 5-25 Manual Optimisation B of the Aerospace mould.....	178
Figure 5-26 Optimised Textile on the Aerospace Mould	179
Figure 5-27 Optimised Textile for the Aerospace Mould as it Appears When Laid Flat.....	180
Figure 5-28 Pattern Used for Manufacturing the Aerospace Textile.....	180
Figure 5-29 Manufactured Textile for the Aerospace Mould Optimised for Surface Area.....	181
Figure 5-30 Manufactured Textile Draped Onto the Aerospace Mould.....	182
Figure 5-31 Aerospace Mould With a Patch for Fibre Orientation Optimisation	183
Figure 5-32 Optimisation of the Fibre Orientation on the Aerospace Mould.....	184
Figure 5-33 Yarn Spacings of the Fibre Orientation Optimisation.....	184
Figure 5-34 Pattern Used for Manufacturing the Aerospace Mould Textile (Textile A)	185
Figure 5-35 Pattern Used for Manufacturing the Aerospace Mould Textile (Textile B)	186
Figure 5-36 Manufactured Textile for the Optimisation of the Fibre Orientation (Textile A)..	186
Figure 5-37 Manufactured Textile for the Optimisation of the Fibre Orientation (Textile B) ..	187
Figure A-0-1 Format of the .model File.....	195

List of Tables

Table 2-1 Indicative Properties of Fibres [15, 16].....	17
Table 2-2 Indicative Values for Young's Modulus and Poisson's Ratio of Unidirectional PMCs [51].....	45
Table 4-1 Comparison of the Parameters to Maximise	150
Table B-1 Optimisation Parameters for Maximising the Surface on the Hemisphere	197
Table B-2 Optimisation Parameters for Maximising the Surface on the Aerospace Mould	198
Table B-3 Optimisation Parameters for Modifying the Fibre Orientation	199

Nomenclature

Roman Letters	Units
A	Component in the Equation of a Plane
A_p	Surface Area of a Drape Element
a	Amplitude of Curved Fibre m
B	Component in the Equation of a Plane
b_{max}	Upper Threshold of the Brick Space
b_{min}	Lower Threshold of the Brick Space
$bound_1$	First Point of Intersection Between the Line of Intersection and an Element Edge
$bound_2$	Second Point of Intersection Between the Line of Intersection and an Element Edge
C	Component in the Equation of a Plane
$\vec{c}_n$	Vector Normal to the Plane of the Circle of Intersection
c_c	Centre of the Circle of Intersection
c_d	Component in the Equation of the Plane of the Circle of Intersection
c_r	Radius of the Circle of Intersection
c_1	Constant Used to Define the Equation of a Line of Intersection Between Two Planes
c_2	Constant Used to Define the Equation of a Line of Intersection Between Two Planes
D	Component in the Equation of a Plane
d_c	Distance Between Centroid of Fibre and Yarn mm
d_b	Distance Between Possible Solution and Bound
$\vec{d}_1$	Directional Vector 1 of Element
$\vec{d}_2$	Directional Vector 2 of Element

E	Young's Modulus	GPa
E_x	Young's Modulus Along the Fibre Direction	GPa
E_y	Young's Modulus Across the Fibre Direction	GPa
E_s	Shear Modulus	GPa
$\vec{e}_n$	Vector Normal to the Plane of an Element Within the Brick Space	
e_d	Component in the Equation of the Plane for an Element Within the Brick Space	
$fishnet[a][b]$	Array Containing the Yarn Crossover Points	
$\overrightarrow{geod1}$	Initial Direction Vector of Geodesic 1	
$\overrightarrow{geod1}_1$	Direction Vector of Geodesic 1 Before Element Edge	
$\overrightarrow{geod1}_2$	Direction Vector of Geodesic 1 After Element Edge	
$\overrightarrow{geod2}$	Initial Direction Vector of Geodesic 2	
$geod1_s$	Start Point of Vector $\overrightarrow{geod1}$	
$geod1_e$	End Point of Vector $\overrightarrow{geod1}$	
$geo[i][j]$	Array Containing the Geodesic Intersection Points	
I	2nd Moment of Inertia	m^4
I_d	2nd Moment of Inertia of a Dry Yarn	mm^4
I_y	2nd Moment of Inertia of a Yarn	mm^4
k	Variable Used in Solving for Two Lines	
L	Length of Period of a Curved Fibre	m
l	Length Between Adjacent Geodesic Intersection Points	
l_t	Length Between Yarn Crossover and Next Intersection Point	
l_{t-1}	Value of l_t for the Previous Iteration	

l_d	Deformed Length	m
l_o	Undeformed Length	m
l_m	Yarn Spacing	
l_{ma}	Yarn Spacing in the a direction	
l_{mb}	Yarn Spacing in the b direction	
M	Bending Moment	N·m
m_i	Variable Used in Solving for Two Lines	
m_{xy}	Variable Used in Solving for Two Lines	
m_{yz}	Variable Used in Solving for Two Lines	
m_{zx}	Variable Used in Solving for Two Lines	
n_1	Node 1 of Current Element	
n_2	Node 2 of Current Element	
n_3	Node 3 of Current Element	
n_a	First Node in Common Element Edge	
n_b	Second Node in Common Element Edge	
n_c	Node Not Part of Common Element Edge	
N_f	Number of Fibres Within a Yarn	
N_i	Number of Iterations Needed Before Checking for Steady State Condition	
N_m	Number of Yarn Crossovers Present Within the Mesh	
P_{avg}	Average Value of the Parameter to Maximise	
P_i	Current Value of the Parameter to Maximise	
p_A	Previous Yarn Crossover Point Along a	
p_a	Point Used to Determine Subsequent Geodesic Direction Vector	

p_B	Previous Yarn Crossover Point Along b	
p_b	Point Used to Determine Subsequent Geodesic Direction Vector	
p_d	Point Used to Determine Subsequent Geodesic Direction Vector	
p_e	Point of Intersection Between Geodesic and Element Edge	
p_i	Point of Intersection Between Geodesic and Element Edge	
p_l	Point on the Line of Intersection Between Two Planes	
p_O	Previous Yarn Crossover Point Along a b	
Q_{ij}	Off-Axis Stiffness Components	GPa
$Q_{xx}, Q_{xy}, Q_{yy}, Q_{ss}$	On-Axis Stiffness Components	GPa
q	Variable Used in Solving for Two Lines	
r	Variable Used in Solving for Two Lines	
r_a	Major Radius of an Ellipse	mm
r_b	Minor Radius of an Ellipse	mm
r_c	Radius of Curvature of Bent Fibre	m
r_f	Radius of Bent Fibre	m
s	Start Point of Drape	
ss	Steady State Value	
ss_c	Steady State Criteria	
t_r	Distance Between Bound and c_c	
$\overrightarrow{t_{n1}}$	Vector Normal to Triangle 1	
$\overrightarrow{t_{n2}}$	Vector Normal to Triangle 2	
u	Parametric Variable for the Line of Intersection Between Two Planes	
$\vec{v}$	Displacement Vector of a Sheared Yarn	m

v_i	Variable Used in Solving for Two Lines	
$\overrightarrow{vn_i}$	Vector Normal to the Plane of Element i	
$\overrightarrow{vn}$	Vector Normal to the Plane of the Current Element	
v_f	Fibre Volume Fraction	%
w_i	Variable Used in Solving for Two Lines	

Greek Letters

α_1, α_2	Angles Between Perpendicular Yarns for Constant-Length Textiles	rad
β	Ratio of Length of Period to Amplitude of a Fibre	
$\beta_1, \beta_2, \beta_3, \beta_4$	Angles Between Perpendicular Yarns for Variable-Length Textiles	rad
β_{shear}	In-Plane Shear Angle of a Draped Element	
ε	Strain	
ε_m	Threshold Used to Determine Closeness Between Two Components	
ε_x	Strain Along the Fibre Direction	
ε_y	Strain Across the Fibre Direction	
ε_s	Shear Strain	
$\varepsilon_1, \varepsilon_2, \varepsilon_6$	Off-Axis Strain Components	
θ_c	Angle Between the Planes of Two Triangles	rad
θ_e	Angle Between Neighbouring Elements	rad
θ_f	Fibre Orientation	rad
θ_g	Angle of Rotation for Geodesics	rad
θ_{min}	User-Defined Minimum In-Plane Shear Angle	rad
μ	Parametric Variable used in the Intersection of a Sphere and a Line	

ν_x	Major Poisson's Ratio	
ν_y	Minor Poisson's Ratio	
ρ	Radius of Curvature	m
σ_x	Stress Along the Fibre Direction	MPa
σ_y	Stress Across the Fibre Direction	MPa
σ_z	Shear Stress	MPa
$\sigma_1, \sigma_2, \sigma_6$	Off-Axis Stress Components	MPa
φ_1	First Angle Between Yarn and Specified Fibre Angle	rad
φ_2	Second Angle Between Yarn and Specified Fibre Angle	rad
φ	Value Used as Parameter to Maximise Based on φ_1 on and φ_2	rad

1. Introduction

The term composite identifies a class of engineered materials typically made from two constituent materials characterised by different physical and mechanical properties. The constituent materials can be polymers, metals or ceramics. Once combined, they offer performance advantages beyond those of the constituents. This thesis deals with polymer matrix composites (PMCs) made from carbon or glass fibres, encased in a polymer resin. More specifically the thesis discusses aspects of manufacturing PMCs where the reinforcement is a textile. These PMCs are widely used in the aerospace, automotive, nautical and recreational goods industries. Historically, the very first PMCs were manufactured using fibres that were pre-impregnated with partially cured, cooled resin in semi-products called prepregs. Due to the high costs associated with this material form, the industry is moving toward liquid moulding processes, which are described in this thesis.

1.1. Liquid Moulding Processes

The basic principle of liquid moulding processes for PMC production consists in creating a dry bed of structural fibres, known as a preform, the volume and shape of which correspond generally to the volume and shape of the PMC part to be made. The operation of creating this bed of dry fibres is commonly known as preforming. Once the dry fibres are in position within a mould, the preform is infused with a curable liquid polymer or resin and the fibres are impregnated in an operation called resin infusion. Many different liquid moulding processes exist including resin transfer moulding (RTM), RTM-light, vacuum assisted resin transfer

moulding (VARTM), resin film infusion (RFI), Seeman Composites Resin Infusion Molding Process (SCRIMP™) and Quickstep™ [1-4]. The RTM process, as shown in Figure 1-1, uses a mould made of at least two stiff parts, forming a closed cavity which has the shape and dimensions of the PMC component to be made. During part manufacture one half of the mould is covered or draped with pieces of dry textile reinforcement, introduced along a prescribed sequence of orientations and layering. Alternatively, the preforming can be carried out on a separate tool, with the preform transported in the actual mould afterwards. Once the preform is ready the mould is closed and resin is injected through carefully chosen injection ports; the thickness of the gap separating the mould parts ensures predictable component thickness. The orientation of the fibres and volume fraction throughout the component are largely determined when the preform is made, so precision in preform construction is of paramount importance. RTM-light is very similar to RTM with the exception that it uses one rigid mould half and one thin shell mould half made of composite material. This thin and relatively flexible mould half limits the resin injection pressures that can be used; resin flow in RTM-light is often driven by vacuum.

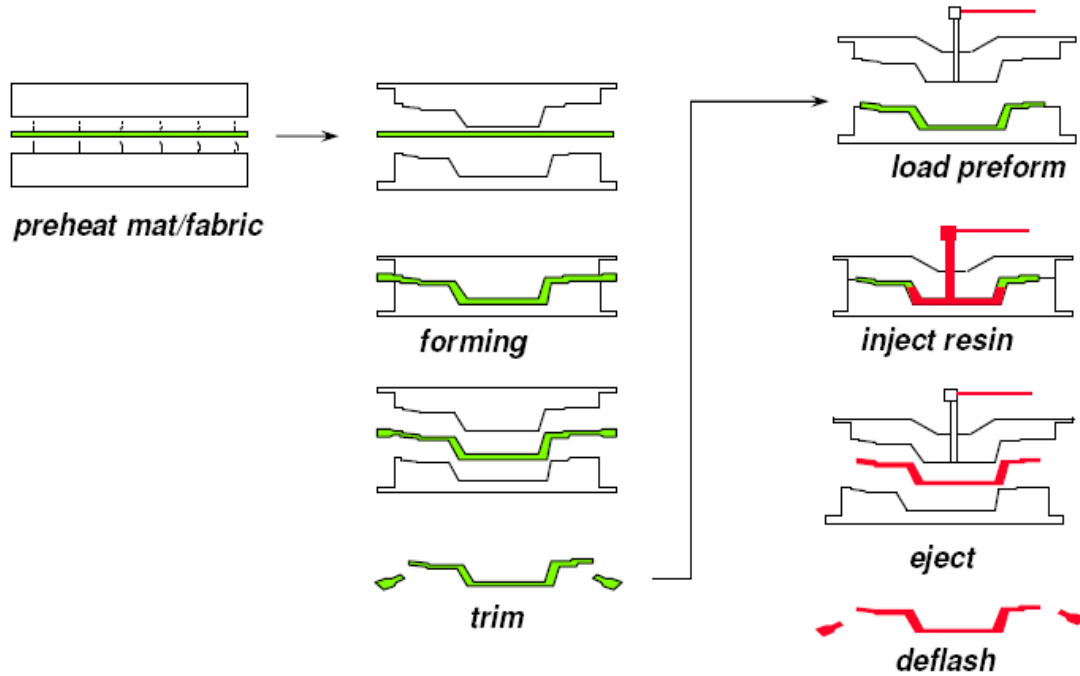


Figure 1-1 The RTM Process [1]

VARTM, shown in Figure 1-2, is a further evolution of RTM where the top mould is replaced by a series of breather cloths, a perforated film and an air-tight membrane which compresses the preform when a vacuum is drawn for resin infusion. Similarly to RTM, the preform construction largely determines how the fibres are oriented in the PMC component, while the compaction and vacuum level will have an effect on the final fibre volume fraction and PMC component thickness. RFI is very similar to VARTM with the difference of having a film of semi-cured resin incorporated in the layup, which will melt when heated and infuse into the textile reinforcement, primarily through the thickness. The resin film replaces the liquid resin which is drawn in by vacuum in the VARTM process. The patented SCRIMP™ process is also very similar to the VARTM process with the addition of a patented flow medium which assists the flow of resin [3]. While the process does offer advantages in terms of resin infusion over

VARTM, it is still largely dependent on the preform construction in order to produce high quality PMC components.

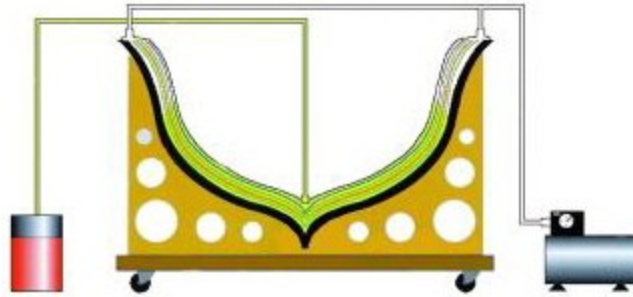


Figure 1-2 The VARTM Process [2]

The Quickstep™ process, shown in Figure 1-3, resembles both VARTM and RTM; while the preform is held within a flexible membrane much like in VARTM, a surrounding liquid can exert positive pressure on the preform much like the mould halves in RTM. However, unlike both RTM and VARTM, the curing process is activated by heat transferred via a heat transfer fluid (HTF) [4]; cure in RTM depends on heat transferred through the solid mould. Again, a high quality PMC component can only be manufactured by using a high quality preform.

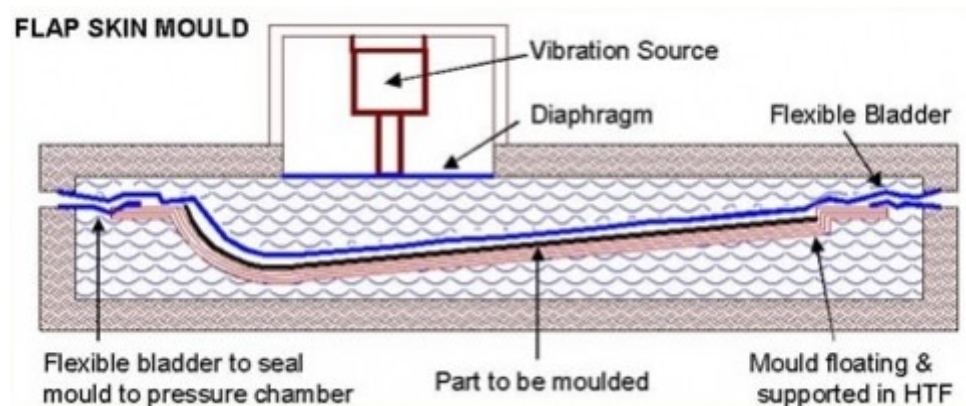


Figure 1-3 The Quickstep™ Process [5]

1.2. Preforming

The performance, integrity and reproducibility of PMC parts manufactured using any of the above liquid moulding processes are highly dependent on the quality of the preform. A preform used in manufacturing a prototype engine nose cone appears in Figure 1-4. In this thesis, high preform quality refers primarily to the absence, or the presence in a limited amount, of defects such as inter-yarn gaps and wrinkles in the textile reinforcement, and also to the consistency of the volume fraction v_f at different scales throughout the preform. The edges of a high quality preform should be devoid of fraying. Finally, preform quality refers to whether preform thickness and fibre orientation are as specified. The thickness and fibre orientation directly impact the mass and structural performance of the finished PMC component while any gaps and wrinkles may lead to major manufacturing problems as well as premature failure. The v_f is defined as the percentage of the total volume of the PMC part which is occupied by the reinforcement. Typical values of v_f range from 55% to 70% depending on the reinforcement and manufacturing process used. A PMC part with a low v_f will benefit fully from the mechanical properties of the reinforcement, but will be heavier due to excess resin. Conversely, a PMC part with a v_f which is too high may not be consolidated due to a lack of resin, resulting in voids and very poor structural properties. The fibre orientation is the direction along which fibres impart their strength and stiffness to the PMC part. Fibre orientations which diverge from design specifications or are out-of-plane will yield a PMC component lacking the desired design stiffness and/or strength. Lastly, the thickness of the finished PMC component is nearly identical to the thickness of the compacted preform; thus it is imperative that the preform thickness corresponds to the desired thickness of the finished PMC component.

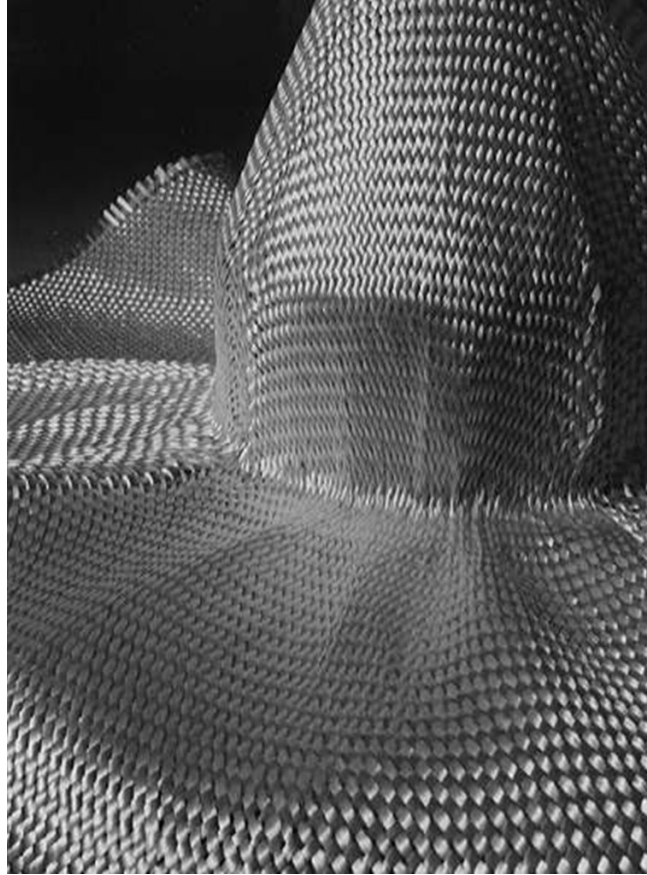


Figure 1-4 Rolls Royce Prototype Jet Engine Nose Cone Preform [6]

Textile reinforcements used in creating preforms generally come as rolled stock, which needs to be cut, draped and compacted upon preform manufacturing. Draping involves placing layered pieces of dry textile reinforcement onto the mould surface while compacting involves applying pressure, either positively or through vacuum, to the dry fibre bed in order to reduce its thickness and ensure a higher fibre volume fraction. The draping operation must be repeated to attain the desired preform thickness, while the compaction operation may be done in between draping operations or only once at the end. Both the draping and compaction of the textile reinforcement deform the textile. Possible deformation modes for the textile include fibre stretching, fibre straightening, interfibre slip, tow buckling and in-plane shear amongst others

[7]. Whilst these deformations enable the draping of high v_f preforms, they can also lead to defects and low preform quality if badly applied or controlled. Deformation modes will be further detailed in Chapter 2.

In-plane shear is especially important to the preforming process as it is the deformation mode that enables flat sheets of textile reinforcements to conform to doubly-curved surfaces, as seen in Figure 1-4. As such, it accounts for the vast majority of preform deformation [7]. The grid traced on the woven reinforcement in Figure 1-5 demonstrates in-plane-shear in a draped textile; squares in the grid deform into diamonds as the textile is sheared. In-plane shear is introduced during the draping operation; it is limitations on in-plane shear that dictate the surface area that can be covered by a textile reinforcement as it is draped. All textile reinforcements are limited in the amount of in-plane shear deformation that they can withstand before deforming out-of-plane through wrinkling and buckling. Out-of-plane deformations of dry reinforcements before infusion are particularly damaging to PMC parts because they may result in major difficulties for resin injection or major defects in the part upon infusion. In the RTM process, out-of-plane deformations could result in the inability to close the mould due to areas of increased thickness. The most critical consequence of out-of-plane deformation is the near complete loss of structural properties due to fibres which are no longer aligned with applied loads, whereas little or no load may be transferred to the fibres. To effectively avoid the above defects and component deficiencies, it is important to drape the textile correctly, which can be made possible with accurate predictive simulations of the draping operation.

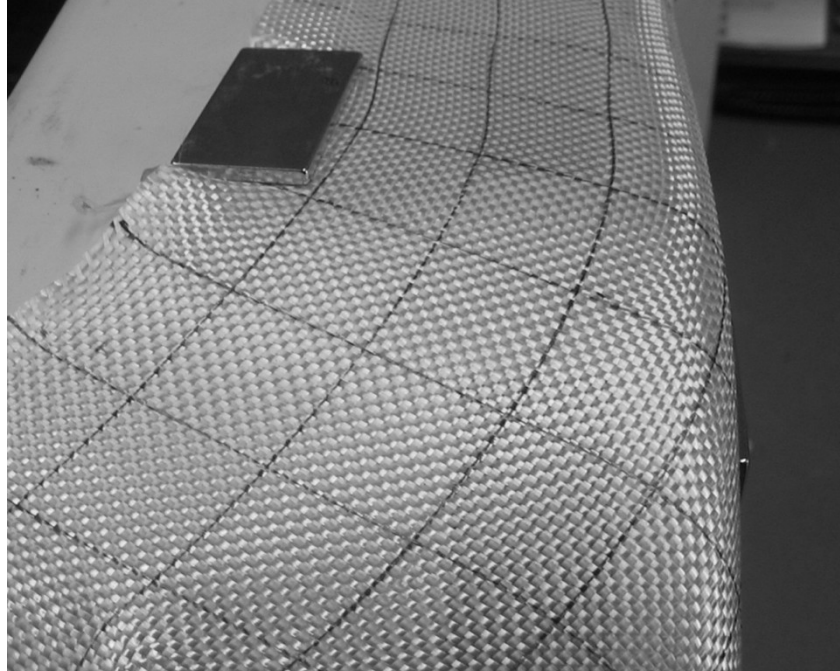


Figure 1-5 Presence of In-Plane Shear on a Draped Textile

1.3. Draping Simulation

The design of preforms and parts, as well as their manufacturing, can be engineered by simulating the draping operation using software to detect areas of high in-plane shear, or of low fibre volume fraction, and the presence of potential out-of-plane deformation zones. The idea of simulating the draping of a 2D textile reinforcement onto a double curvature surface defined in 3D has been pursued and methods have been elaborated towards this purpose, including kinematic algorithms, finite element (FE) simulations and hybrid kinematic energy methods.

The kinematic or geometric [8, 9] method is essentially a mapping algorithm which predicts the path of yarns on the curved surface from simple geometric assumptions. The main assumption made is that the textile reinforcement acts much like a fishnet; a square mesh made

of inextensible segments which correspond to the centrelines of the textile's yarns, running in two directions, which are initially orthogonal. The yarns may rotate relative to one another, but yarn stretching and slipping is not permitted: the fishnet may only deform by the rotation of segments around the yarn crossovers. The kinematic method is dependent on the initial creation of two orthogonal geodesics which cross each other and define much of the ensuing shape of the fishnet. A draped fishnet can be seen in Figure 1-6 where the segments have rotated about each other to enable the simulated textile to conform to the surface; the geodesics can be seen in bold on the same figure.

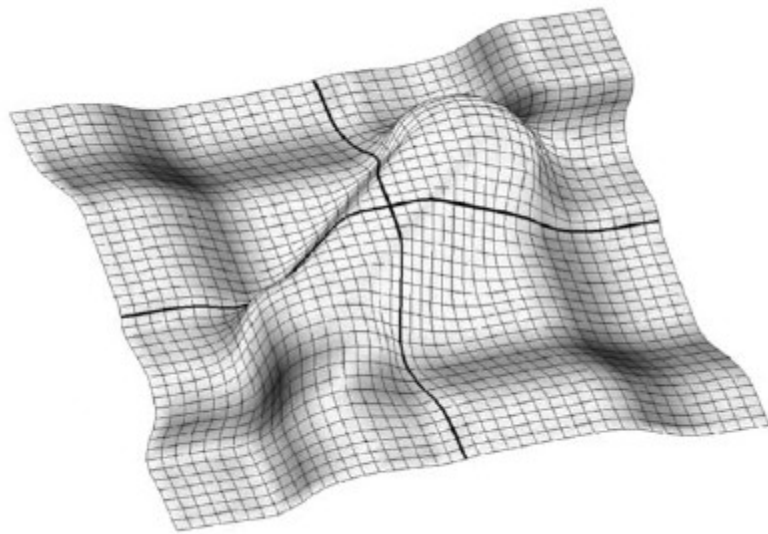


Figure 1-6 Surface Draped by a Kinematic Method [10]

It is also possible to simulate the draping of textile reinforcements using finite elements. Different methods have been established, including a simplified unit cell method which uses 1D bar elements as shear elements and tow elements [11], seen in Figure 1-7. The tow elements of the unit cell represent the yarns of the textile while the added shear elements represent the resistance to in-plane shearing of the textile, which makes it possible to find a minimum strain-

energy solution for the textile. The draping is simulated by finding the configuration in which the total strain-energy of the simulated textile is minimal. The undeformed unit cell used for this particular FE simulation can be seen below in Figure 1-7.

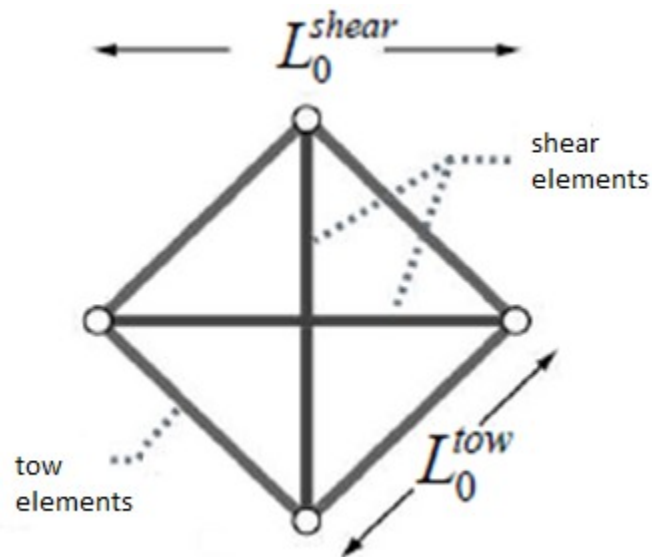


Figure 1-7 Example of a Unit Cell Used in FE Simulation [11]

Alternatively, FE simulations can be performed using an elementary pattern as seen in Figure 1-8 [12]. The elementary pattern represents a crossover of perpendicular yarns before deformation, which is repeated in order to create an isoparametric bilinear four-node shell element. Each node in the element has three degrees of freedom to account for the three components of the node's displacement. The directions of the yarns in the elementary pattern correspond to the reference coordinates of the element (ξ_1 and ξ_2), as seen in Figure 1-9. Such modelling can be used for simulating draping in a punch and die, where the punch and die are of the desired preform shape. Non-linear kinematics are used in order to calculate the deformation energy for each element, which must be minimised in order to determine the correct

configuration. Dedicated software such as PAM-FORM™ [13] enable such simulations to be done but the constitutive behaviour of the material must be fully characterised for those to be meaningful.

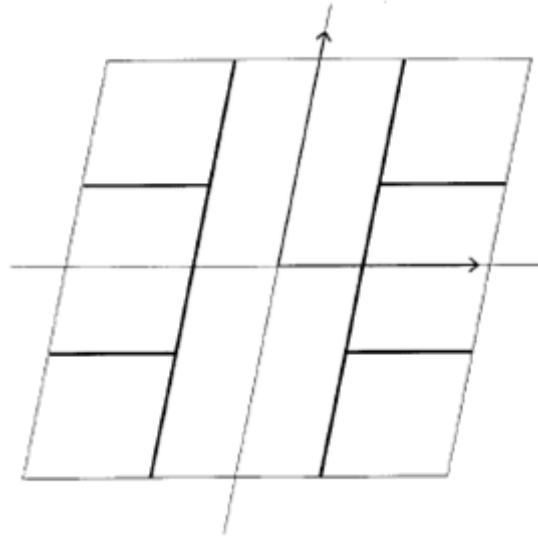


Figure 1-8 Elementary Pattern Used in FE Simulation [12]

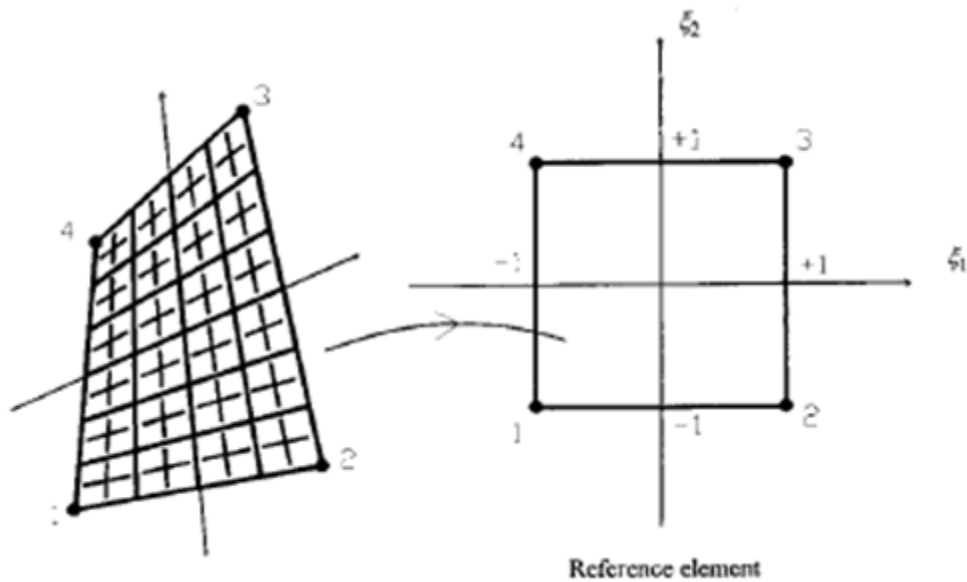


Figure 1-9 Mapping of Elementary Patterns to Finite Elements [12]

The hybrid kinematic energy method operates similarly to the kinematic method by mapping the textile to a pin-jointed mesh similar to a fishnet. The segments may rotate relative to each other in order for the mesh to conform to the model surface. However, instead of solving for geometric constraints only, the hybrid kinematic energy method solves for minimal shear deformation energy [14]. Unlike the purely kinematic method, this method does not rely on initially determined geodesics that extend to the boundaries of the surface to cover, even before coverage is actually started. Instead these basic lines are determined as coverage progresses, allowing the simulated textile to deform more freely. The direction of the fibres and the shear deformation of each of the fishnet's segments is defined by identifying the configuration in which the deformation energy is minimal. Additionally, the magnitude of the resistance to shear can be defined differently depending on the direction of the shear; this gives the method the ability to simulate a wider variety of textiles, including textiles which have a tendency to shear in one direction with more ease than in the other direction, such as stitched textiles or twill or satin weaves. This can be seen in Figure 1-10.

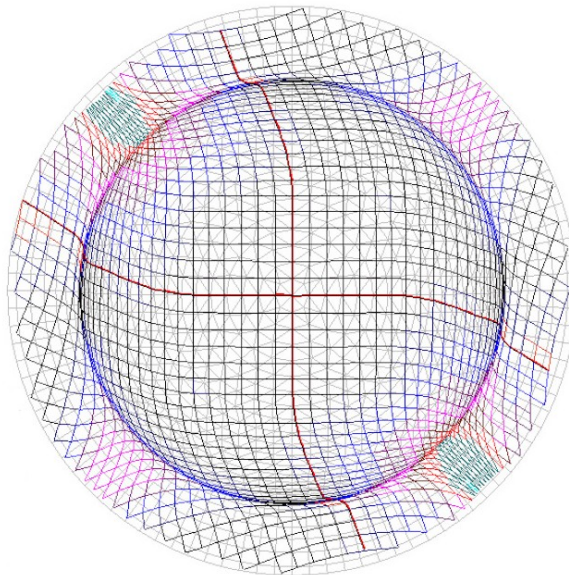


Figure 1-10 Hybrid Kinematic Energy Drape [14]

1.4. Thesis Overview

Simulation software can accurately predict and display how a textile reinforcement will deform when draped over a 3D surface. These predictions are used for better informing the design of preforms used in liquid moulding processes. This enables the creation of preforms which have significantly less defects such as out of plane deformations, and allows for the manufacturing of preforms with predictable thicknesses, fibre volume fractions and fibre orientations. However, the extent to which a specific textile can deform in order to conform to a surface is fixed and limited by the locking angle of that textile. Therefore there are limits to the surfaces and shapes that can be draped without defect. In conventional textiles the spacing between yarns and between yarn crossovers are constant for a given textile. In this thesis, a novel textile architecture is explored where the textile's yarn spacing is allowed to vary across the textile, as seen in Figure 1-11. This enables the creation of textile reinforcements which can be optimised for improved drapability onto a specific surface, and/or offer advantages over conventional textile reinforcements such as less in-plane shear or controlled fibre orientation for better mechanical properties of the preform. The optimisation of these variable-length textiles can be done manually or using software developed as part of this thesis; care must also be taken to ensure that the optimised textiles can be manufactured flat, hence the software probes in-plane shear and v_f of the optimised textiles both in the flat state before draping and in the draped state. It is also important to limit segment length difference from yarn to adjacent yarn, as large differences may lead to high curvature in yarns and impact the feasibility of manufacturing the textile. The physical apparatus to build such textiles was developed and manufactured in a parallel project; textiles manufactured using this apparatus are used for discussion and validation in this thesis.

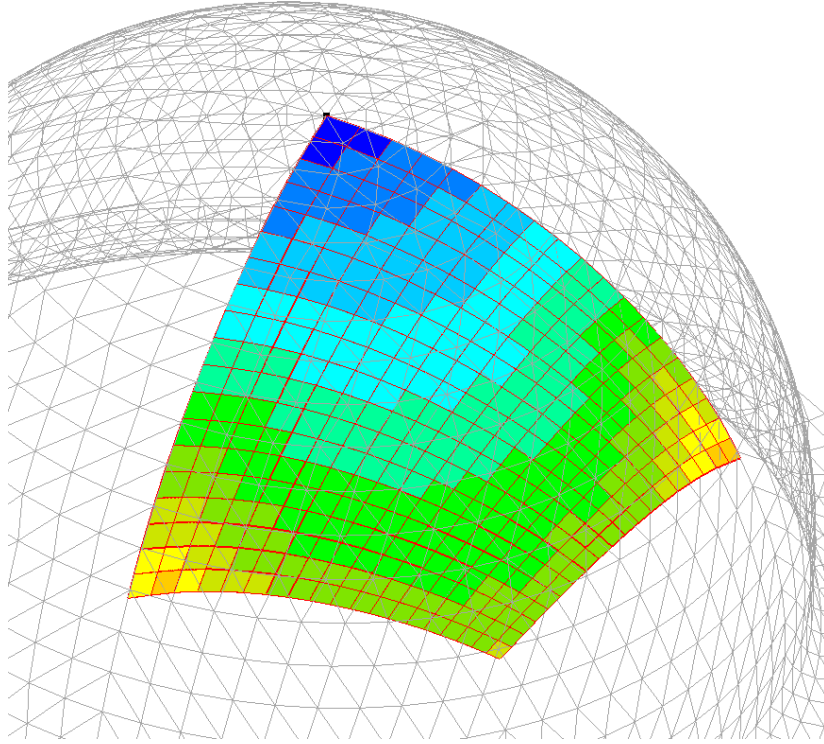


Figure 1-11 Variable-Length Textile Reinforcement

1.4.1. Kinematic Simulation Software

Software simulating the draping of 2D textile reinforcements onto arbitrary surfaces has been created using the kinematic method as its basis, as described in section 1.3. The kinematic method was chosen due to its simplicity in using only geometric assumptions. Due to the nature of variable-length textiles, mechanical properties such as in-plane shear stiffness may change across such textiles and more complex simulation methods may be needed eventually. The simulation software can accurately predict and display how textile reinforcements, conventional and optimised, will deform when draped over a 3D surface. The 3D surface, textile parameters and start location of the drape are entirely user-defined, allowing for much flexibility in scenarios that may be simulated. Additionally the software may drape areas limited as user-

defined patches. Simulation results appear in 3D renderings of the surface, over which the software creates a colour map of the magnitude of shear deformation and fibre volume fraction across the draped surface. The simulation capability, along with the colour maps, enable the designer and manufacturer of PMC parts to see potential defects and make modifications to the design before costly prototypes or trials are done. The software was validated by comparing simulations with laboratory trials performed with actual textile reinforcements.

1.4.2. Software Optimisation of Variable-Length Textiles

In addition to simulation, the software described in this thesis can be used for optimising textile reinforcements, with the aim of better draping a certain surface. The optimisation is accomplished by changing the lengths of the segments within the fishnet. Manual changes can be made via user input within the program, which will then update the simulation in real time to show the effect of the change. The software may also optimise the textile using automated algorithms which operate much like Monte Carlo algorithms, by relying on random changes and evaluating if these changes increase or decrease the chosen optimisation parameter or combination of parameters. These algorithms were tested both qualitatively and quantitatively by comparing non-optimised textiles with optimised textiles. Textiles optimised both manually and using the software were manufactured in a parallel effort, by Reza in order to determine how customised textiles behave beyond the simulation.

One objective of this thesis is to demonstrate how the operation of draping can be improved by customised variable-length textile reinforcements. Improvements to the draping

operation include less in-plane-shear, a larger surface area that can be draped with a single piece of textile, and altered fibre orientation. Chapter 1 consists of an introduction to the thesis as well as an insight into PMC manufacturing and its ramifications. Chapter 2 contains a literature review on textile reinforcements, their deformation modes, information about preforms and preforming and algorithms for simulation and optimisation. Chapter 3 discusses the algorithm used for the draping simulation while Chapter 4 discusses the algorithm used for the optimisation of the textiles. In Chapter 5, validation of the draping simulation is discussed as well as the results of optimised textiles. Lastly, Chapter 6 contains the conclusion, possible improvements to the software and recommendations for future work.

2. Literature Review

2.1. Reinforcements

The fibre reinforcement is the component that imparts stiffness and strength to PMCs. In liquid moulding processes, textile reinforcements are handled and draped during preforming, before any resin is added. Textile reinforcements can be separated in a series of hierarchical building blocks, the first of which being individual fibres. Aligned fibres are collated into yarns, and yarns are used in creating textile reinforcements. This can be achieved through one of several textile manufacturing processes such as weaving, braiding, knitting or others.

2.1.1. Fibres

Fibres are long cylindrical filaments which are typically made of materials such as glass or carbon. Most glass and carbon fibres used in PMC components have diameters ranging from 8 μm to 15 μm . There are many different glass and carbon fibres readily available, with different mechanical properties such as E-Glass and S-Glass fibres as well as polyacrylonitrile (PAN) based and pitch based carbon fibres. Indicative values are given in Table 2-1.

	E-Glass	S-Glass	PAN Carbon	Pitch Carbon
Young's Modulus (GPa)	72.4	86.2	260-360	415-830
Tensile Strength (MPa)	3448	4585	3650-4820	1720-2400
Density (g/cm^3)	2.58	2.48	1.76-1.78	2.00-2.13

Table 2-1 Indicative Properties of Fibres [15, 16]

During the preforming process the fibres undergo certain deformations. Deformation modes include axial stretching (A1), axial compression (A2) that may induce buckling, bending (A3) which may lead to failure if pursued to small radii, and torsion (A4). The deformation modes can be seen in Figure 2-1.

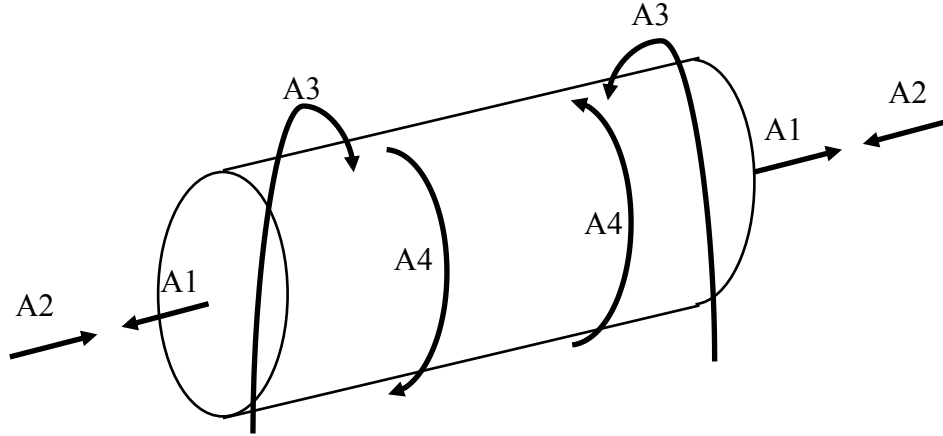


Figure 2-1 Deformation Modes of a Single Fibre

Most fibres used in PMC parts are very stiff, therefore these high modulus fibres need large tensile forces for significant axial deformations to occur. Large tensile forces are not present in typical preforming operations; thus deformation mode A1 is ignored in this work. Additionally, tensile forces will remove any inherent waviness [17] in the fibres, but given the small amplitude of the fibre waviness, the axial deformation that occurs due to fibre straightening can be neglected. For graphite fibres, the ratio β as given by equation (2.1) lies between 100-300 where L is the length of period and a is the amplitude of the curved fibre, as seen in Figure 2-2

$$\beta = \frac{L}{a} \quad (2.1)$$

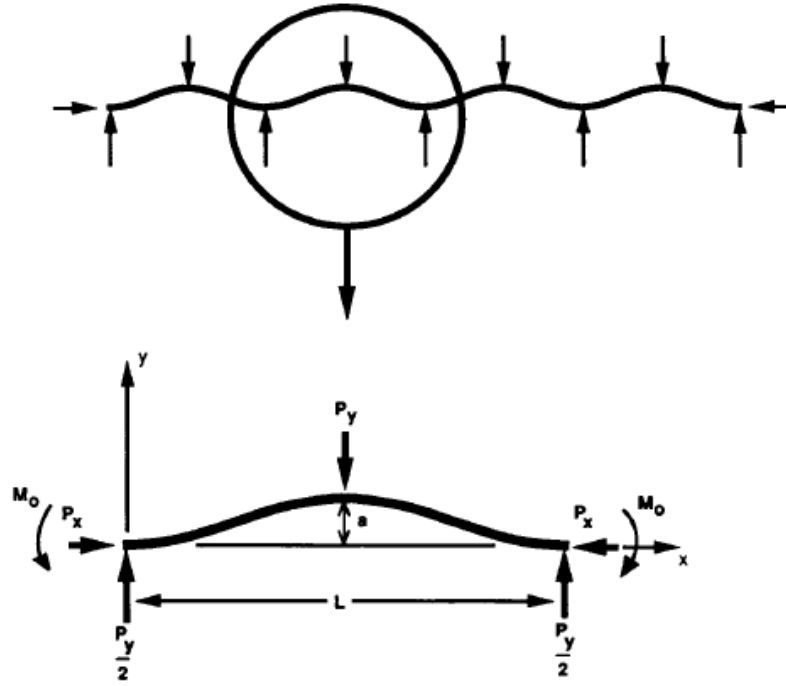


Figure 2-2 Representative Wavy Fibre Element [17]

Fibres may buckle under compression loading, due to their high modulus and small diameters; in this work buckling will be examined in a forthcoming section on yarns since compression loading and buckling on a single fibre is highly unlikely. Hence deformation mode A2 is not discussed further. Fibres used in PMCs are generally made of materials with a high elastic modulus. It is well established that high modulus materials have high bending rigidities, which may make them less apt to conforming to curved surfaces. The relation between the bending moment M , the Young's modulus E , the radius of curvature ρ and the 2nd moment of inertia I [18] is given by:

$$\frac{1}{\rho} = \frac{M}{EI} \quad (2.2)$$

It is possible to demonstrate that even if individual fibres have high bending rigidities and low strain to failure, the curvature radius needed to strain a fibre beyond its limit is small in comparison to typical radii found on moulds. Strain is defined as the relative increase in length of the fibre as shown in equation (2.3), where ε is the strain, l_d is the deformed length and l_o is the initial length [19].

$$\varepsilon = \frac{l_d - l_o}{l_o} = \frac{l_d}{l_o} - 1 \quad (2.3)$$

A diagram of a bent fibre is presented below in Figure 2-3 where r_f is the radius of the fibre and r_c is the radius of curvature of the fibre.

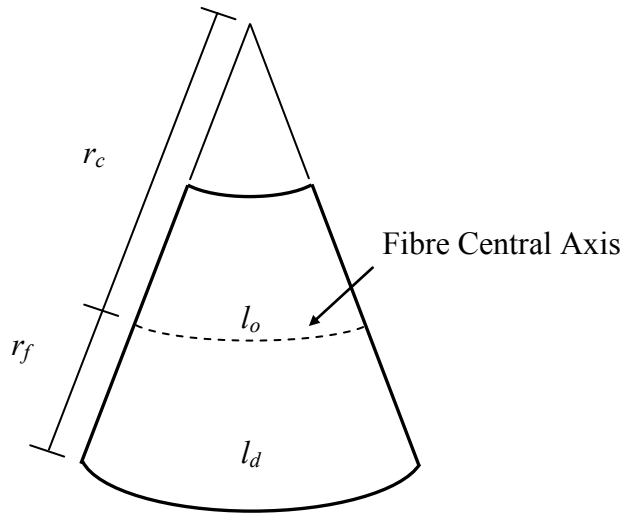


Figure 2-3 Section of Bent Fibre

Reducing the above diagram into triangles leads to the ratio:

$$\frac{l_d}{l_o} = \frac{r_c + r_f}{r_c} = 1 + \frac{r_f}{r_c} \quad (2.4)$$

Inserting equation (2.4) into equation (2.3) leads to equation (2.5) which can be used for calculating the maximum axial strain in the fibre based on the fibre and curvature radii. Using the maximum strain of a very stiff pitch based carbon fibre of 0.42 % [20] and a fibre diameter of 10 μm , we can determine the minimum curvature radius of the fibre to be 1.2 mm, for discussion purposes.

$$\varepsilon = \frac{r_f}{r_c} \quad (2.5)$$

The minimum bending radius of most fibres will be smaller than 1.2 mm due to the fact that most fibres have a maximum strain greater than that of pitch-based carbon fibres. For example, glass fibre can have a maximum strain as high as 4.88 % [21]. Due to the very small minimum bending radius of fibres compared to typical radii found in moulds, deformation mode A3 is not at risk of leading to fibre failure in typical preforming operations. Finally, because of the general absence of forces applying torsion moments to individual fibres, torsion A4 on single fibres is not discussed further.

While short unaligned fibres may be used in reinforcements such as random mats and sprayed fibre glass mats, fibres as discussed in this thesis are long, aligned and grouped in bundles called yarns.

2.1.2. Yarns

Yarns are bundles of aligned continuous fibres comprising a few hundred up to a few tens of thousands fibres; a typical yarn contains 12 000 fibres, '12 K' carbon yarns being prevalent in

reinforcement construction. Yarns are mostly used as the basis for the production of textile reinforcements, which are themselves used in preform production. One noticeable exception where yarns are used directly for making PMC parts is the process of filament winding. Yarns can be characterised by a mass per unit length (tex or Denier), a yarn volume fraction, and a cross section area, both of which may change along the length of a yarn. Whilst theoretically perfect square packing of fibres in a yarn leads to a v_f of 0.785 and hexagonal packing leads to a v_f of 0.906 [22], uncompacted yarns typically expand to have a yarn volume fraction of about 0.5-0.6 or less because of some degree of fibre waviness, misalignment [23] and intertwining, as seen in Figure 2-4. Due to waviness and misalignment of fibres, contacts between fibres within a yarn are more likely to be pinpoints, while linear contacts are also possible [24, 25].

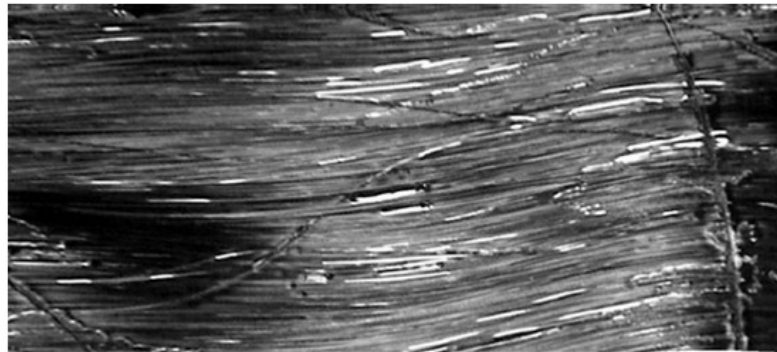


Figure 2-4 Fibre Waviness and Misalignment Within a Yarn [25]

During preforming, yarns are subjected to many of the same deformation modes as the individual fibres, such as axial stretching (B1), axial compression (B2), bending (B3) and torsion (B4). Additionally, yarns may deform by transverse compression (B5), transverse widening (B6) and intra-yarn shear (B7) as seen in Figure 2-5.

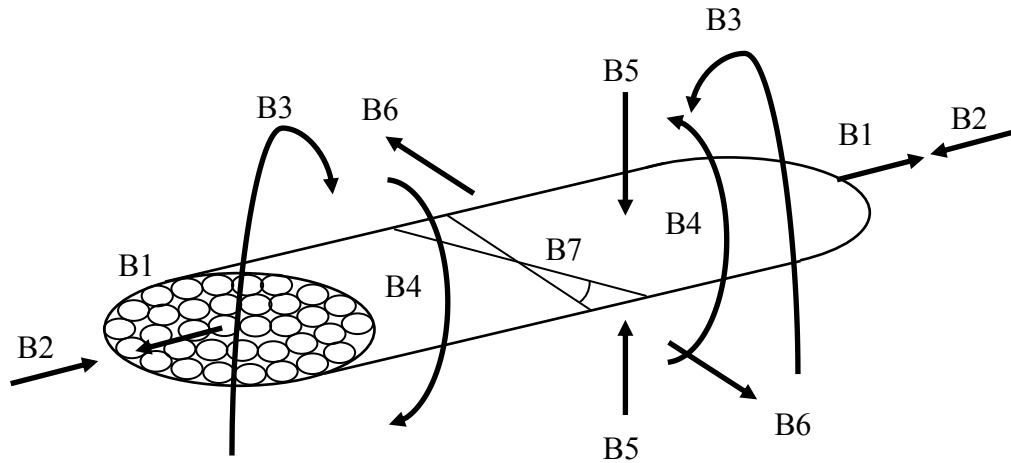


Figure 2-5 Deformation Modes of a Single Yarn

Similarly to individual fibres, the axial forces that arise while preforming will not cause noticeable axial deformations thus deformation mode B1 is ignored. The yarns may buckle when subjected to compressive loads applied along their length. Compressive loads, deformation mode B2, may occur as the textile is forced into a small mould or if in-plane shear reaches its limit. Single yarns will very rarely buckle; it is more likely that many yarns may buckle simultaneously resulting in textile buckling. As the yarns are weaved and assembled into a textile, and also during preform manufacturing, forces will act on the yarns that will result in yarn bending, deformation mode B3. Due to the fact that yarns are made of bundles of fibres they behave differently to continuous materials when subject to bending [26]. In addition, equation (2.2) does not hold for non-isotropic materials. It is for this reason that much research has been conducted on the bending and bending stiffness of single yarns. The earliest and simplest methods known as Fabric Assurance by Simple Testing (FAST) [27] and Kawabata Evaluation System for Fabrics (KES-F) [28] come from the clothing textiles industry, where simple tests were devised to determine, among other variables, the bending stiffness of a certain fibre or yarn. More in-depth measurement methods, which use multiple supports rather than the one fixed support of previous

methods, may provide better results for the bending rigidity of yarn. One of these methods is the Bending Evaluation System for Fabric and Yarn (BES-FY) [29] illustrated in Figure 2-6. Additionally, some researchers have attempted to use the yarn's geometric and mechanical variables or numerical approaches in order to determine their bending stiffness [30, 31].

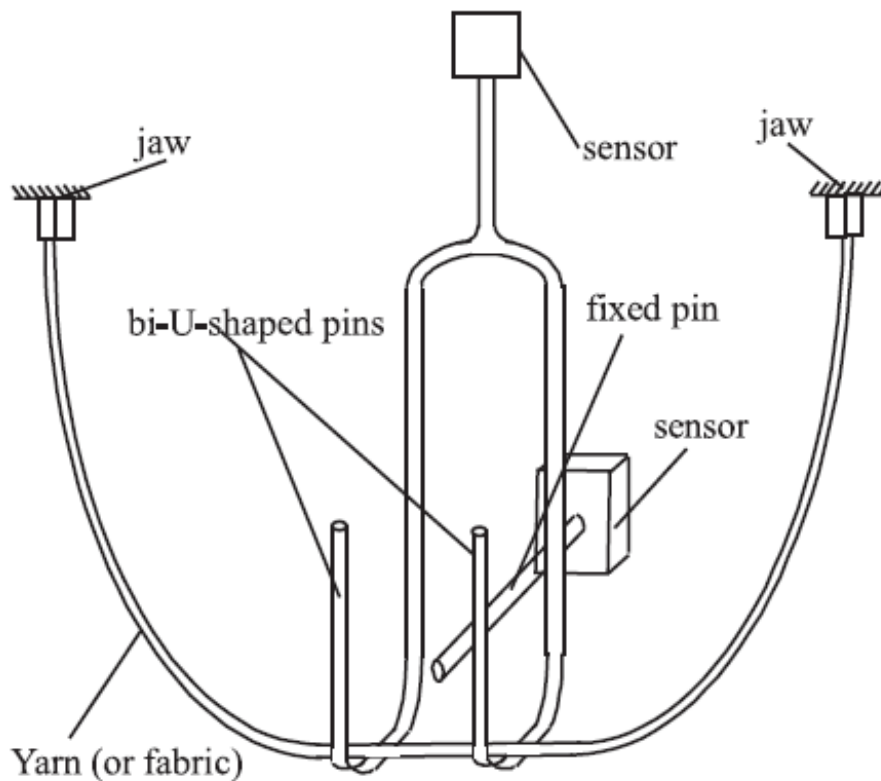


Figure 2-6 Schematic of BES-FY [29]

It is possible to estimate the bending stiffness of a dry yarn and to compare it to the bending stiffness of an impregnated yarn where the fibres are fixed in cured resin. The bending stiffness is the product of the Young's modulus E and the second moment of inertia I . Considering only the effect of the fibre reinforcement, the Young's modulus for both cases are identical and therefore only the second moments of inertia are compared. Assuming that the individual fibres within the dry yarn can slide past each other with no friction and no

entanglements, the second moment of inertia of the yarn is the sum of all the moments of inertia of individual fibres about their individual centroids. Using a fibre diameter of 10 μm and assuming that the fibre sections are perfectly circular, the second moment of inertia can be found as follows:

$$I_d = N_f \frac{\pi r_f^4}{4} \quad (2.6)$$

where N_f is the number of fibres within the yarn, assumed to be 12 000 for this calculation. The second moment of inertia for the dry yarn I_d is 5.89 E-06 mm^4 . In order to determine the second moment of inertia of a cured yarn the shape of the yarn must be determined. An ellipse containing 12 000 fibres was used as a simple representation of the yarn cross-section. The major and minor radii of the ellipse were chosen to produce a yarn that is wider than it is high with a desired cross section area. This area was found by multiplying the number of fibres by the fibre area, and adjusting for the fibre volume fraction. A square packed yarn and a v_f of 0.785 were chosen. The chosen values for r_a and r_b are 0.8 mm and 0.48 mm respectively. The cured yarn behaves as a single solid therefore the second moment of inertia of each fibre must be calculated with respect to the yarn's centroid and not the individual fibre centroids. This is accomplished using the distance between centroids, d_c , which varies depending on the fibre's location in the yarn, as seen in Figure 2-7.

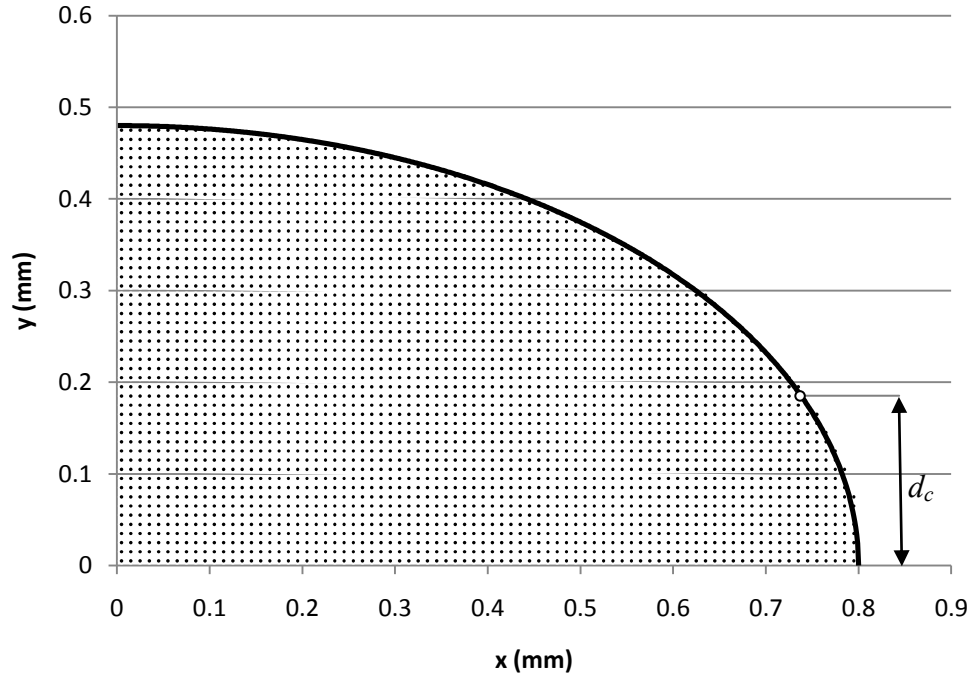


Figure 2-7 Quarter Yarn for Second Moment of Inertia Calculations

The equation used to determine the second moment of inertia of a cured yarn is

$$I_y = \sum \left(\frac{\pi r_f^4}{4} + \pi r_f^2 d_c^2 \right) \quad (2.7)$$

The result of equation (2.7) is 5.51 E-02 mm⁴, several orders of magnitude larger than the dry yarn.

During preforming and while the yarns are woven together to create a textile reinforcement, torsional forces are minimal if present at all, therefore deformation mode B4 is not discussed further. Transverse compression or compaction (B5) of the yarn results in the rearrangements of the fibres by lateral spreading (B6) leading to an increase in the v_f of the yarn

and the reduction of large voids between yarns [32]. Additional compaction, as the v_f increases, results in the flattening of wavy yarns and fibres. The yarns deform much like a bending beam supported at contact areas between the yarns [17]. The reorganisation of the yarns triggered by transverse compressive forces is essential in achieving high v_f and limiting the presence and size of large fibre-free voids which may become resin rich zones upon infusion. An uncompacted textile (A) is compared to a compacted textile (B) in Figure 2-8 and Figure 2-9. The large white areas seen in Figure 2-9 are resin rich areas, virtually eliminated in the compacted textile.

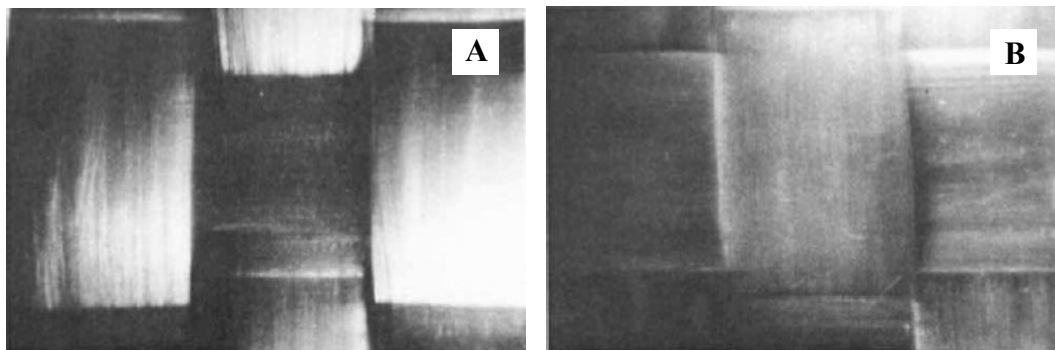


Figure 2-8 Uncompacted and Compacted Textiles [32]

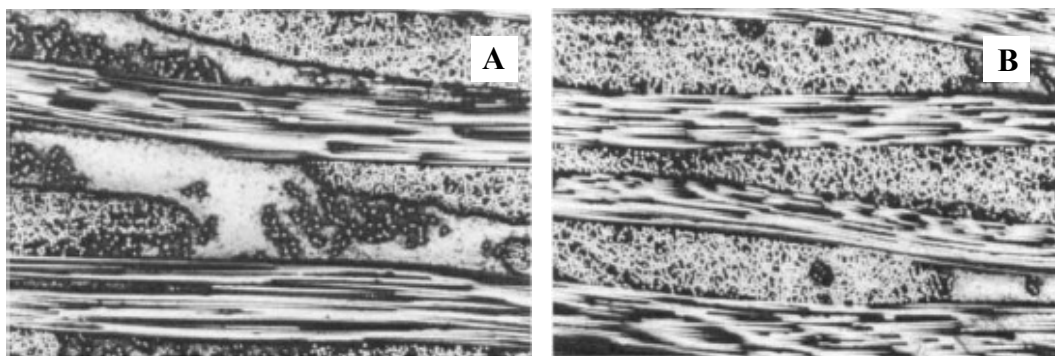


Figure 2-9 Cross Sections of Uncompacted and Compacted Textiles [32]

Finally, intra yarn shear (B7) is a deformation mode which occurs when a textile undergoes in-plane shear. As the angle between the yarns changes, the individual fibres within a yarn slide past each other [33, 34] effectively shearing the yarn as illustrated in Figure 2-10. The displacement vector $\vec{v}$ varies as a function of the distance from the centre of the yarn as seen in Figure 2-11. The magnitude of intra-yarn shear is much smaller than inter-yarn shear and has little real effect on the draping of the textile.

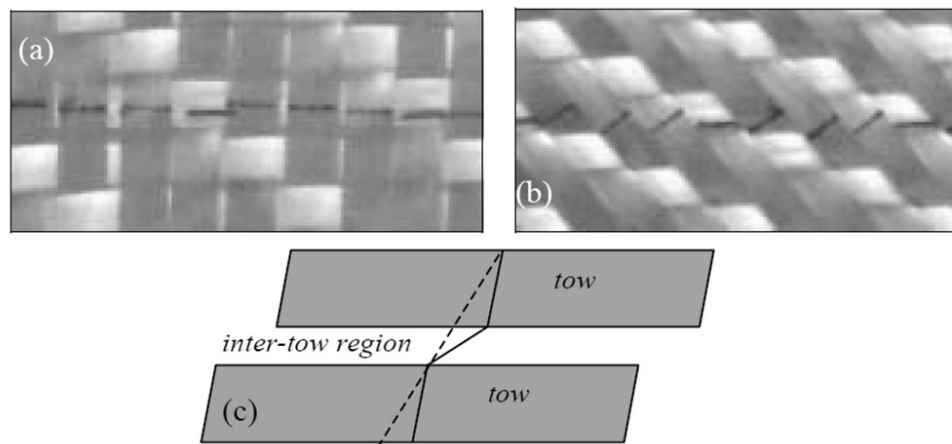


Figure 2-10 Intra-Yarn Shear [34]

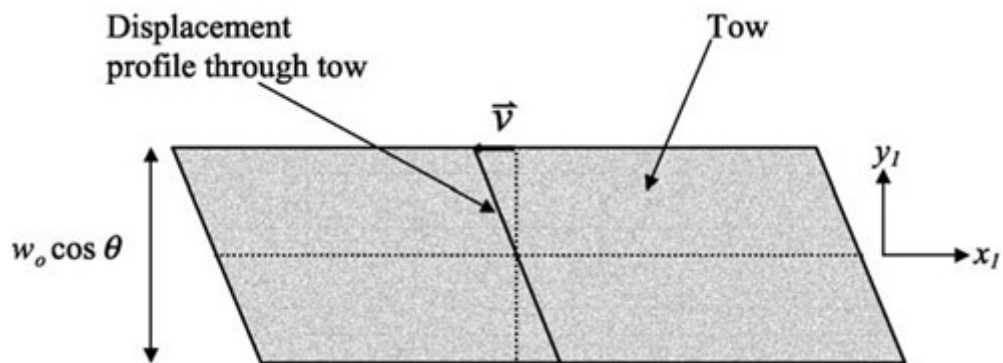


Figure 2-11 Intra-Yarn Shear Displacement Profile [33]

2.1.3. 2D Textile Reinforcements

When draping the preform, the textile architecture has a large effect on how the textile reinforcement will conform and shear on the surface. Textile reinforcements can be classified in different types according to their manufacturing process and structure, including weaves, knits, braids and stitched textile reinforcements. Such reinforcements are referred to as periodic fabrics and are generally supplied as rolls.

Typical 2D bidirectional weaves comprise of warp yarns which run the length of the weave, and weft yarns which cross and overlap the warp yarns in a manner described by the type of weave. The overlap of yarns creates crimp in the yarns as they are forced to bend around each other. Several different weave types exist including the plain weave, satin weave and twill weaves, shown below in Figure 2-12. Different weaves can have different warp and weft spacing defined as the distance between adjacent warp or weft yarns, and can have different linear densities for the warp and weft yarns. It is also possible to form triaxial weaves in applications where the textile reinforcement must be as close to isotropic as possible. Triaxial weaves behave very differently from regular 2D biaxial weaves and will not be discussed further in this thesis.

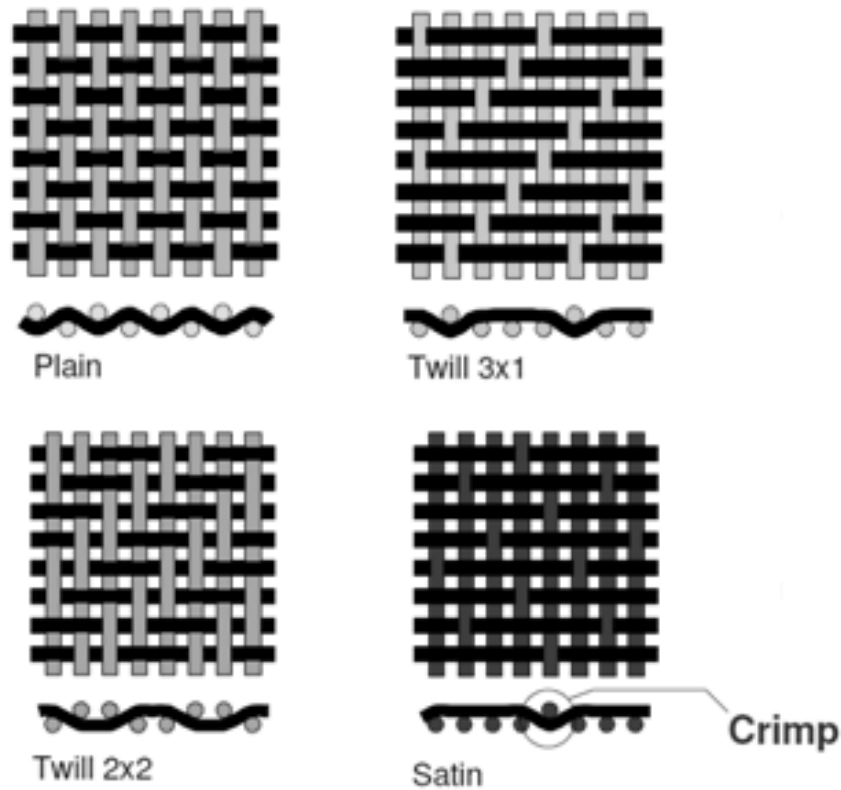


Figure 2-12 Various 2D Weaves [35]

Knits are separated into two distinct types known as warp knits and weft knits. Warp knits are further differentiated by the type of stitching pattern used, including for example the chain stitch and the tricot stitch as seen in Figure 2-13, Figure 2-14 and Figure 2-15. Furthermore, the tricot stitch can have many different configurations such as 1:1, 2:1 and others; the numbers in the configuration indicating the size of the offset along the warp and weft directions for subsequent stitches along a particular warp yarn. Warp knits are typically used as non-structural threads which interlace around straight load-bearing yarns following a specific pattern based on the type of stitching pattern used. Therefore, in typical carbon fibre reinforcements, the yarns of reinforcement fibres are held together by a light warp knit, which ideally does not pierce through the structural yarns. Such reinforcements are usually referred to

as non-crimp fabrics (NCFs) since structural yarns are not interlaced and therefore remain essentially flat. A 1:1 tricot stitch warp knit can be seen on the left of Figure 2-13 while the same warp knit assembling yarns can be seen on the right of the same figure.

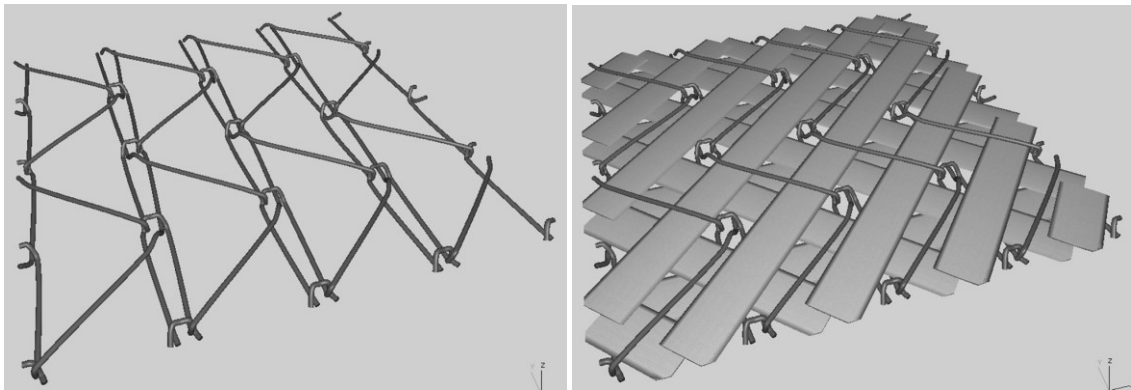


Figure 2-13 Warp Knit Without and With Structural Yarns [35]

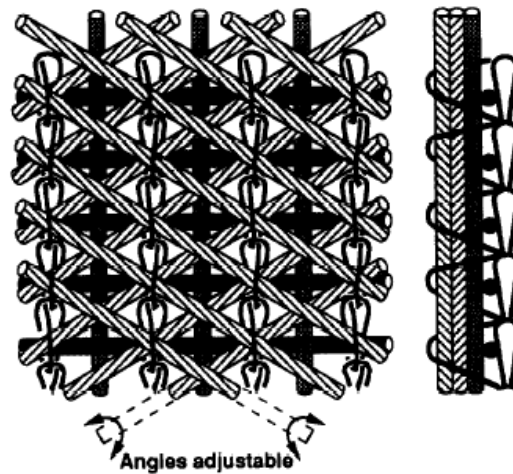


Figure 2-14 Chain Stitch Warp Knit [36]

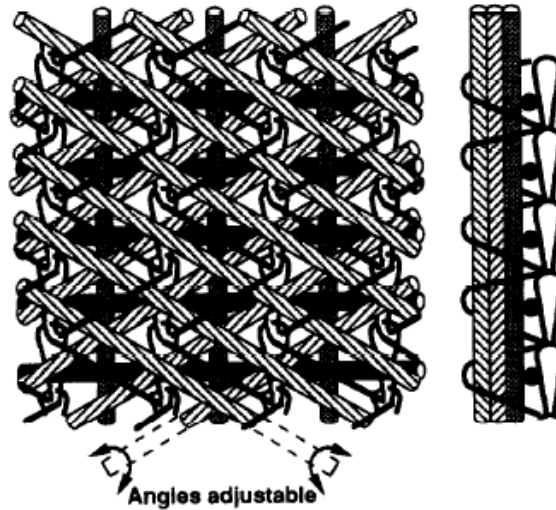


Figure 2-15 Tricot Stitch Warp Knit [36]

Weft knits do not use a distinct stitching thread, but instead consist of knitted yarns that interlock in a specific pattern. There is no warp yarn in weft knits as the weft yarns follow paths that align lengthwise as well as crosswise. A weft knit in its most basic form, called the plain weft knit or jersey knit, can be seen in Figure 2-16. Weft knits used as reinforcements typically lead to low ν_f PMC parts; hence they are only used in special circumstances and will not be discussed further in this thesis.



Figure 2-16 Plain Weft Knit [37]

Stitched textiles are much like warp knits in that structural weft and warp yarns do not intertwine and additional non-structural stitched threads hold these yarns in place. The stitched threads usually assemble the layers of textile reinforcement together with a lockstitch, which uses a top thread and bottom thread in the same manner typical of a common sewing machine. Stitched fabrics are also regarded as non-crimp fabrics. Various stitching patterns exist, as seen in Figure 2-17.

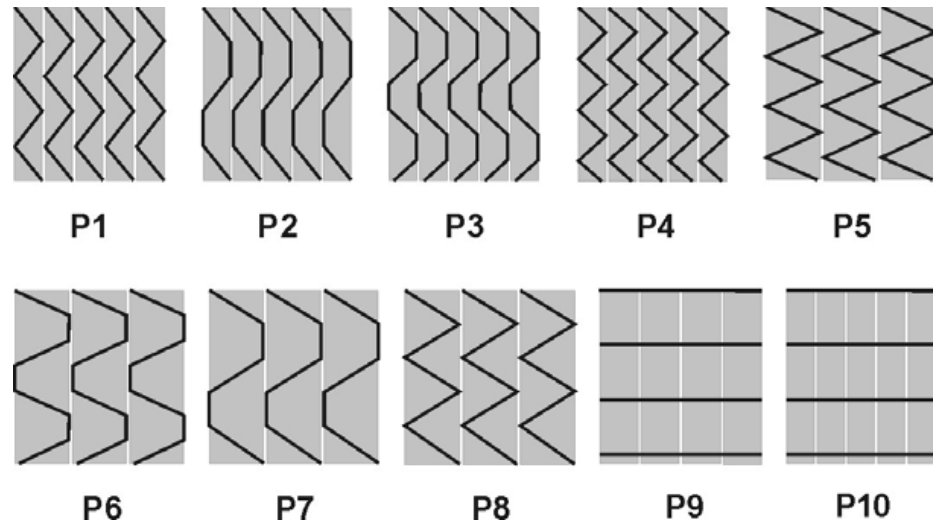


Figure 2-17 Different Stitching Patterns [38]

Braids are made of three or more yarns which intertwine in order to form a continuous tubular structure as seen in Figure 2-18. To create a preform using a braided textile reinforcement, the braided tubular section is usually slipped over a mould or core of the desired shape. The braid will deform and conform to the mould surface but it is not draped as such. Due to the multi-axial nature of braids, deformation modes are different to those found in draped weaves, knits or stitched textiles, hence braided reinforcements are not discussed further in this thesis.



Figure 2-18 Braided Textile [39]

Textile reinforcements are subject to forces during the preforming which cause them to deform. Deformation modes for 2D textile reinforcements include in-plane shear (C1), intraply yarn slip (C2), textile buckling (C3), bending (C4) and fibre stretching and straightening (C5) as seen in Figure 2-19 [40] and Figure 2-20.

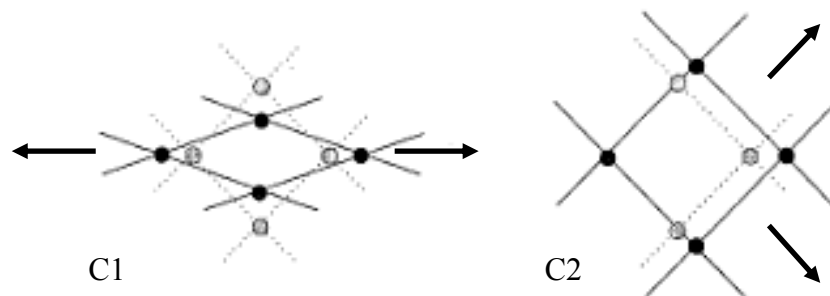


Figure 2-19 Deformation Modes C1 and C2 of a Textile [40]

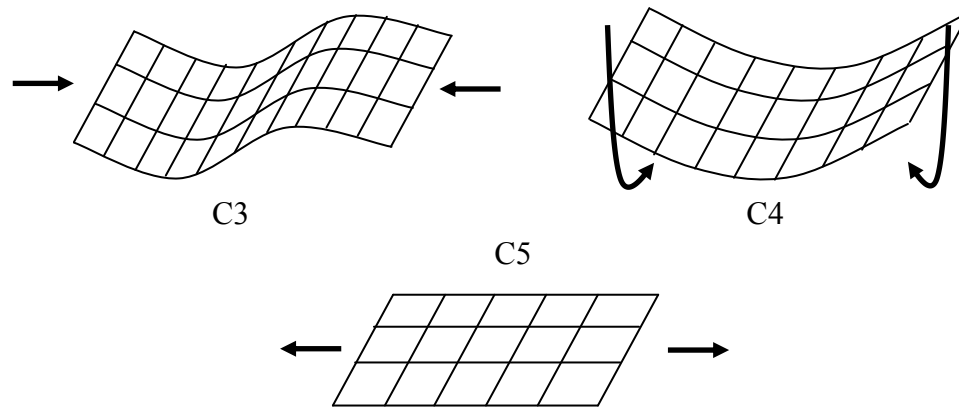


Figure 2-20 Deformation Modes C3, C4 and C5 of a Textile

In-plane shear (C1) is the predominant deformation mode which occurs during preforming. It is characterised by the rotation of yarns about crossover points. In-plane shear is discussed in detail a later section. Intraply yarn slip (C2) is defined as the lateral movement of the yarns at their crossover points. This undesirable deformation mode becomes more prevalent as the textile reaches its in-plane shearing limit. Textile buckling (C3) occurs when numerous yarns are subjected to in-plane compressive forces. These yarns then buckle jointly, which impart a wrinkled appearance to the textile. Bending of the textile reinforcement (C4) occurs when the textile reinforcement is draped over single and double curvature surfaces. Whilst the main deformation mode, in-plane shear, occurs when the textile reinforcement is draped over double curvature surfaces, bending can occur over single curvatures. Bending is not a concern for typical 2D textiles in most cases as curvature radii are generally large enough, hence it is not discussed further in this thesis. Textile stretching and straightening (C5) are negligible since the resistance of the textile to in-plane shear is several orders of magnitude smaller than the stiffness of the individual yarns.

2.1.4. 2.5D and 3D Textile Reinforcements

Constructing textile preforms for more complex or realistic parts featuring thick sections, integral flanges and/or assemblies of intersecting shells or other similar features can be made easier and more productive using 3D and/or near net shape textiles. Designing and manufacturing such textiles demands thorough consideration of the shape of the part, and eventual alterations to this shape aiming at better accommodating preform construction. Mentions of knitted reinforcements pertaining to net shape composite parts often refer to thick non-structural knits with in-laid load-bearing yarns [41]. These are essentially thicker versions (typically 10-12 layers) of 2D knits or stitched fabrics with in-laid load-bearing yarns, often labelled 2.5D fabrics in the literature. 2.5D textiles generally behave much like 2D textiles with the exception of greater bending rigidity due to the thickness of the textile.

3D weaving extends on the 2D process by integrating superimposed layers of yarns with diverse degrees of z-yarn interlacing [42] as shown in Figure 2-21. Z-yarns work as binders [43] or structural yarns within the 3D weave, giving the 3D textile some strength in the z direction and much better interlaminar resistance. 3D weaves are mostly made as thick slivers or wider sheets where machinery is available. As 3D weaves are mostly produced flat, their deformability through bending and shear is important to preforming. Due to the thickness of the 3D weave, bending rigidity is much higher than for 2D weaves requiring analysis of the minimum bend radius achievable while draping.

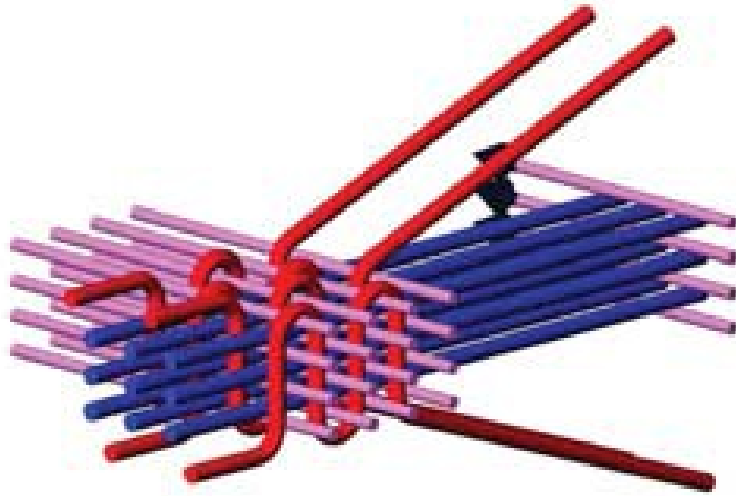


Figure 2-21 3D Weaving Process [44]

Applications of braiding as discussed in a previous section typically consist of relatively involved assemblies of simpler tubular or circular 2D braids used as profiles or on cores. On the other hand, 3D braiding enables stronger departures from structural properties typical of thin reinforcements via more extensive interlacing through the thickness of parts in order to create performs made to final shape in much more complex processes [45]. Figure 2-22 depicts 3D braided preforms in the shape of I-beams and other profiles. These preforms as well as other preforms created with 3D braids are not intended for draping and thus this thesis does not deal with 3D braided textiles.

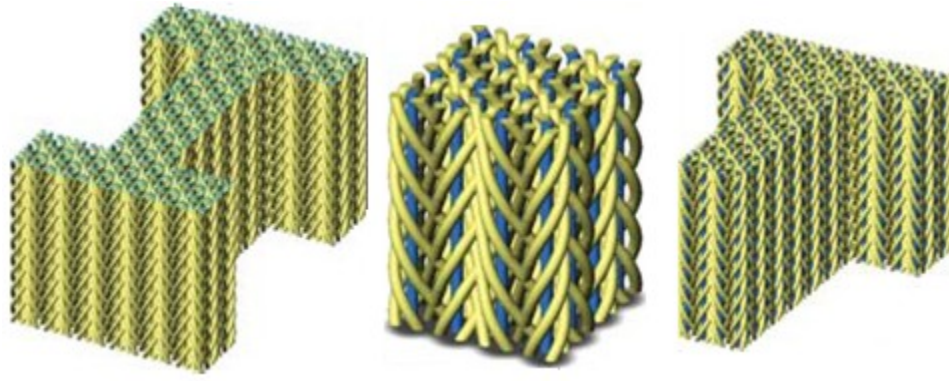


Figure 2-22 3D Braided Preforms [44]

2.2. *In-Plane Shear*

In-plane shear is the most important deformation mode occurring when textile reinforcements are forced to conform to a complex surface. For the purpose of this thesis, a complex surface is defined as any surface which is doubly curved, meaning that the surface curves along two perpendicular axes known as the principle curvature directions, at any given point. A surface with only single curvature will only bend the textile reinforcement and not cause any in-plane shear. Figure 2-23 shows models of an undeformed textile on the left and of the same textile after in-plane shear has taken place, on the right.

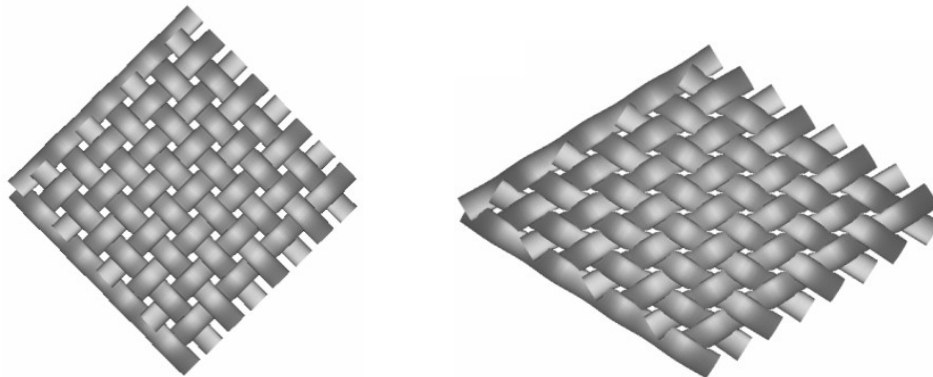


Figure 2-23 In-Plane Shear of a Textile [35]

The textile architecture has an impact on the in-plane shear behaviour of a textile reinforcement. Textile reinforcements such as plain weaves tend to deform in an equal manner in both orthogonal bisector directions while twill weaves, satin weaves and many knits will deform more easily in one direction than the other [46]. An example of this difference can be observed in Figure 2-24; the left-hand side shows a woven textile which shears equally in both directions while the right-hand side shows a textile which shears predominantly along a certain direction.

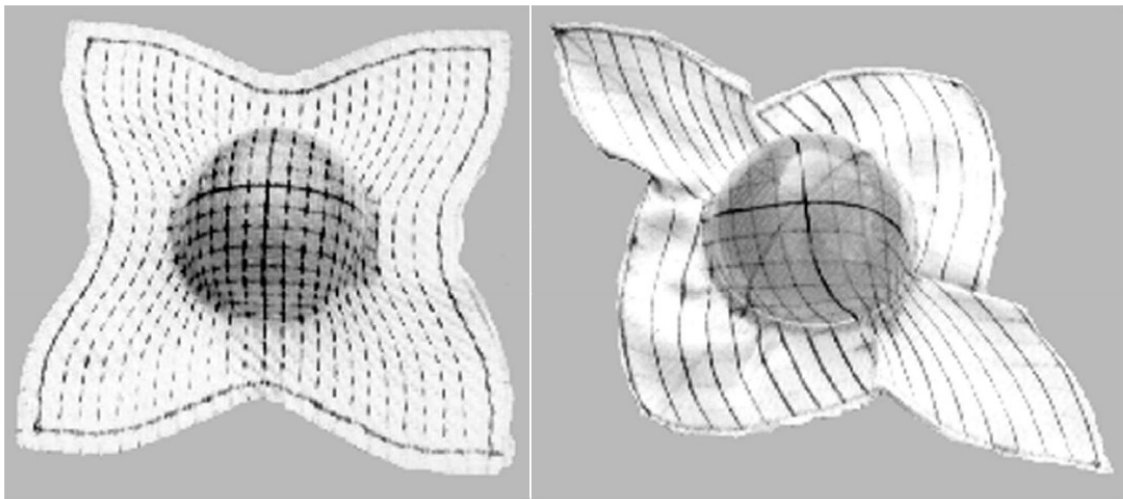


Figure 2-24 Effect of Textile Architecture on In-Plane Shear Behaviour [46]

A textile's resistance to in-plane shear can be measured in specially designed tests such as the picture frame or bias extension experiments. The picture frame test is a hinged square frame, as shown in Figure 2-25, in which a sample of the textile reinforcement is held. Tension from a tensile testing machine deforms the frame from a square into a rhombus, forcing the textile to shear. The force required to shear the textile is recorded as a function of the vertical displacement of the moving mounting of the picture frame. By the use of geometrical relations

the shear angle can be determined from the displacement and therefore a measurement of the shear force as function of shear angle is obtained.

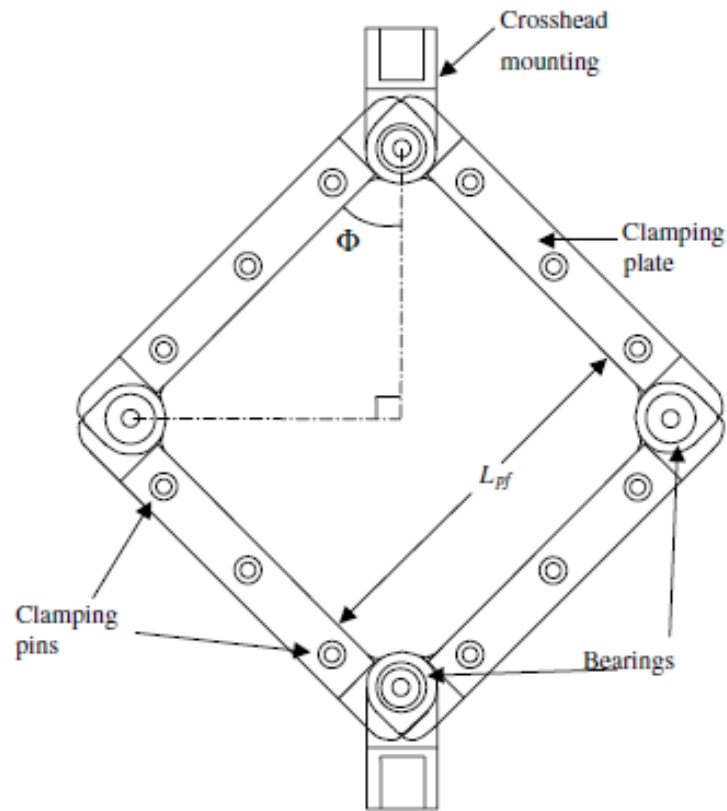


Figure 2-25 Picture Frame Test [47]

The bias extension test consists of applying tension to a rectangular piece of textile reinforcement as shown in Figure 2-26. Different zones of the textile will deform in different ways with pure in-plane shear taking place in the section labelled A. The force is measured as a function of the displacement. Since only section A deforms by pure in-plane shear certain mathematical manipulations are required to obtain the force required to shear area A and the shear angle in that zone of the textile [47]. It is also possible to use finite element analysis in order to correlate the relation between shear and shear angle [48].

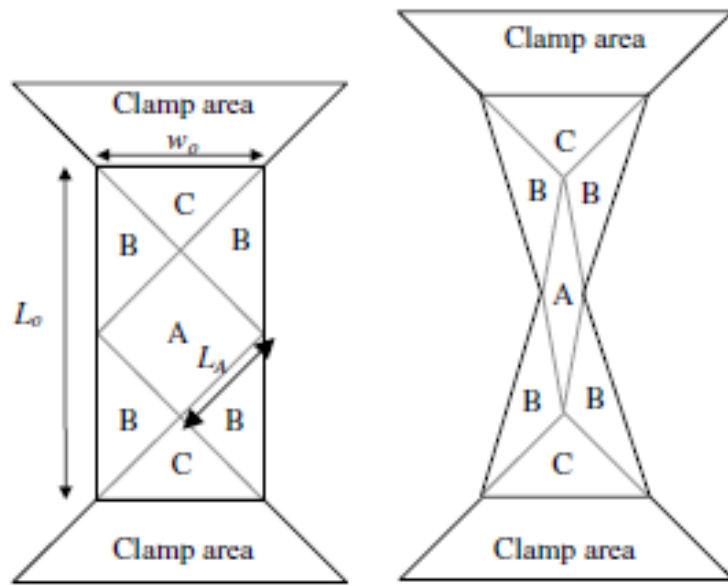


Figure 2-26 Bias Extension Test [47]

A graph showing the results of picture frame (symbols) and bias extension (lines) tests is shown in Figure 2-27. In this particular case the undeformed textile is considered to have a shear angle of 0° which progressively increases with in-plane shear. The convention used throughout this thesis considers an undeformed textile to have a shear angle of 90° in relation to the initially perpendicular yarns. As the textile is sheared, the in-plane shear angle decreases.

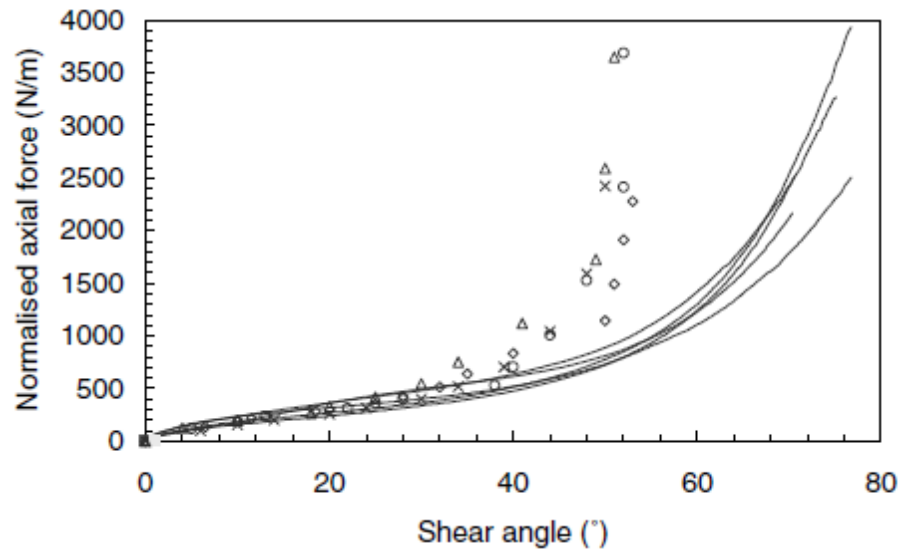


Figure 2-27 Shear Force as a Function of Shear Angle [47]

Textile reinforcements are limited in the amount of in-plane shear that they can withstand without wrinkling. This limit is based on the textile architecture and is measured as the maximum shear angle recorded before out-of-plane deformations occur. As the textile reinforcement is sheared, the yarns eventually contact each other and subsequent deformation will occur in the mode requiring less energy, allowing fibres to buckle out-of-plane. In a plain weave the ratio of the tow width to tow spacing essentially determines at which angle adjacent yarns contact [49]. The maximum angle is called the locking angle and is easily detected as it gives the textile reinforcement a wrinkled appearance [50]. The behaviour of the textile reinforcement when sheared can be seen in Figure 2-28.

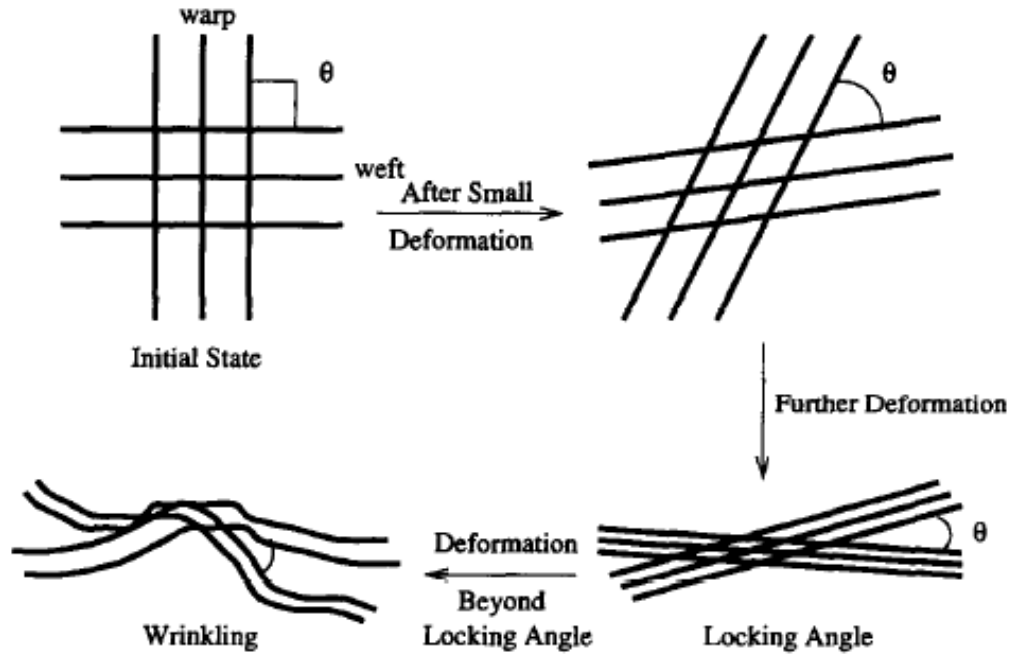


Figure 2-28 Deformation by In-Plane Shearing of a Textile Reinforcement [49]

2.3. Effect of Fibre Orientation on Strength and Stiffness

PMC components are orthotropic, meaning that the mechanical properties are not equal in every direction; therefore the fibre orientation in a PMC component is critical to performance. Mechanical properties such as strength and stiffness are much higher along the fibres, and weaker across the fibres. Taking the simplified case of a unidirectional orthotropic composite loaded on-axis, the in-plane strains are computed by the use of the following matrix [51]:

$$\begin{bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_s \end{bmatrix} = \begin{bmatrix} \frac{1}{E_x} & -\frac{\nu_y}{E_y} & 0 \\ -\frac{\nu_x}{E_x} & \frac{1}{E_y} & 0 \\ 0 & 0 & \frac{1}{E_s} \end{bmatrix} \cdot \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_s \end{bmatrix} \quad (2.8)$$

Values for Young's moduli E_x and E_y , and for Poisson's ratios ν_x and ν_y of the composite depend on the fibre used and of the ν_f . Values of material properties are available for most common fibres. Indicative values of Young's moduli and Poisson's ratios for unidirectional composites are given in Table 2-2 to demonstrate the large differences of magnitude along and across the fibre.

Material	E_x (GPa)	E_y (GPa)	ν_x	E_s (GPa)
Carbon/epoxy T300/5208	181	10.3	0.28	7.17
Carbon/epoxy B(4)/5505	204	18.5	0.23	5.59
Carbon/epoxy AS/3501	138	8.96	0.3	7.1
Glass/epoxy Scotchply 1002	38.6	8.27	0.26	4.14
Aramid/epoxy Kevlar 49	76	5.5	0.34	2.3

Table 2-2 Indicative Values for Young's Modulus and Poisson's Ratio of Unidirectional PMCs [51]

In the off-axis unidirectional case where loading axes are not parallel and/or perpendicular to the fibre orientations it is useful to determine the off-axis stiffness of the composite and analyse its off-axis strength. The nomenclature used for on-axis and off-axis components of stress and strain can be seen in Figure 2-29.

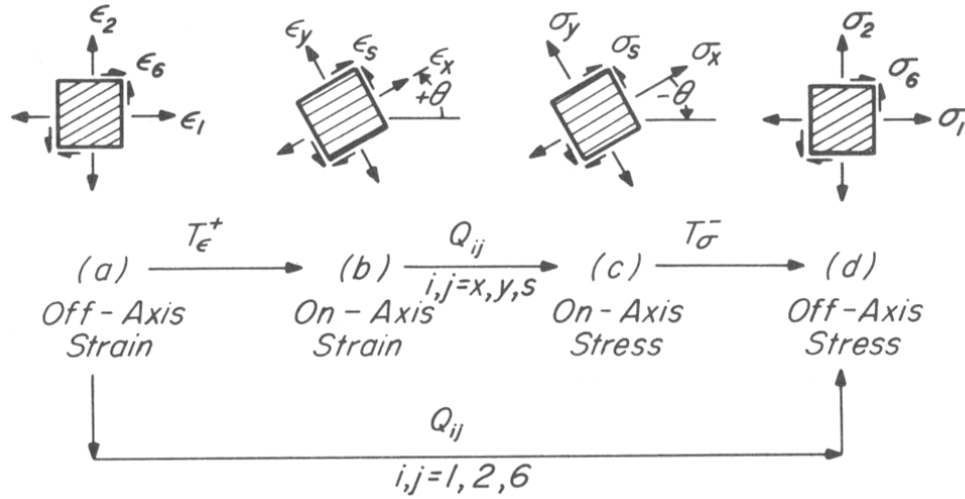


Figure 2-29 Off-Axis Stresses and Strains [51]

Stresses in an off-axis composite can be found using a matrix similar to equation (2.8) with the exception that coupling exists between each component of stress and each component of strain [51]:

$$\begin{bmatrix} \sigma_1 \\ \sigma_2 \\ \sigma_6 \end{bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & Q_{16} \\ Q_{21} & Q_{22} & Q_{26} \\ Q_{61} & Q_{62} & Q_{66} \end{bmatrix} \begin{bmatrix} \epsilon_1 \\ \epsilon_2 \\ \epsilon_6 \end{bmatrix} \quad (2.9)$$

Axial and shear couplings that are specific to orthotropic uniaxial composites loaded off-axis are present. The components of the stiffness matrix Q_{ij} , which vary with the angle θ (see Figure 2-29) are found using the following matrix:

$$\begin{bmatrix} Q_{11} \\ Q_{22} \\ Q_{12} \\ Q_{66} \\ Q_{16} \\ Q_{26} \end{bmatrix} = \begin{bmatrix} c_\theta^4 & s_\theta^4 & 2c_\theta^2 s_\theta^2 & 4c_\theta^2 s_\theta^2 \\ s_\theta^4 & c_\theta^4 & 2c_\theta^2 s_\theta^2 & 4c_\theta^2 s_\theta^2 \\ c_\theta^2 s_\theta^2 & c_\theta^2 s_\theta^2 & c_\theta^4 + s_\theta^4 & -4c_\theta^2 s_\theta^2 \\ c_\theta^2 s_\theta^2 & c_\theta^2 s_\theta^2 & -2c_\theta^2 s_\theta^2 & (c_\theta^2 - s_\theta^2)^2 \\ c_\theta^3 s_\theta & -c_\theta s_\theta^3 & c_\theta s_\theta^3 - c_\theta^3 s_\theta & 2(c_\theta s_\theta^3 - c_\theta^3 s_\theta) \\ c_\theta s_\theta^3 & -c_\theta^3 s_\theta & c_\theta^3 s_\theta - c_\theta s_\theta^3 & 2(c_\theta^3 s_\theta - c_\theta s_\theta^3) \end{bmatrix} \cdot \begin{bmatrix} Q_{xx} \\ Q_{yy} \\ Q_{xy} \\ Q_{ss} \end{bmatrix} \quad (2.10)$$

where c_θ and s_θ are $\cos(\theta)$ and $\sin(\theta)$ respectively. Q_{xx} , Q_{yy} , Q_{xy} and Q_{ss} are the on-axis stiffness components of the composite which are found using the following equations [51]:

$$Q_{xx} = (1 - \nu_x \nu_y)^{-1} E_x \quad (2.11)$$

$$Q_{yy} = (1 - \nu_x \nu_y)^{-1} E_y \quad (2.12)$$

$$Q_{xy} = (1 - \nu_x \nu_y)^{-1} \nu_y E_x = (1 - \nu_x \nu_y)^{-1} \nu_x E_y \quad (2.13)$$

$$Q_{ss} = E_s \quad (2.14)$$

The stiffness of a unidirectional composite decreases rapidly as the angle θ between the fibre axis and loading axis is increased as shown in Figure 2-30. Low stiffness of the PMC component and eventual failure may result should the fibre orientation deviate from the design specification as a result of draping operations that are badly engineered or implemented.

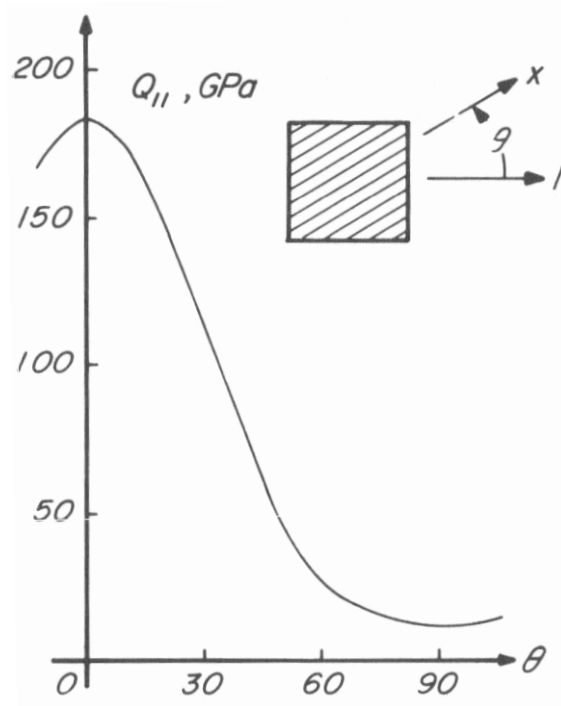


Figure 2-30 Q_{11} Versus Fibre Angle [51]

2.4. Preforming Practice

The preforming operation involves the deformation of the textile layers which are then superimposed to create a preform which should be a near net shape fibre bed comprised of only dry reinforcement, with possibly a small amount of binder to hold the textile layers in place. The process and the deformation modes encountered during preforming were discussed in previous sections, with the exception of nesting. Nesting occurs when multiple layers of textile reinforcement are compacted and tows from one layer penetrate between the tows of the adjacent layer. Nesting may reduce the relative motion between textile layers and enable the textile to be compacted more easily [52]. It should be noted that in theory textiles are periodic and are made of perfectly straight and equidistant yarns. Therefore, in theory they can be superimposed while

keeping this theoretical regularity with yarns from different layers perfectly stacked on top of each other, or else perfectly staggered as seen in Figure 2-31 [53]. In practice there is no such regularity in superimposed textile layers so nesting will be prevalent in real preforms and it will vary from point to point in the preform.

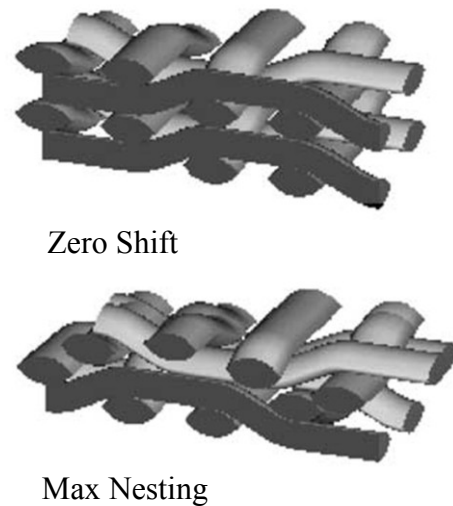


Figure 2-31 Nesting of Woven Textiles [53]

Another point of practical importance must be mentioned. It is not possible, using current textiles and draping methods, to successfully drape any preform with a single piece of textile, covering the surface of the mould entirely. Generally this can only be done in cases of small parts which are mostly flat or only gently curved. For this reason, multiple pieces of fabric are used within each layer, creating joints in the preform. Several types of joints exist such as lap joints, butt joints and step joints, Figure 2-32 and Figure 2-33. Only butt and step joints may be used for manufacturing processes such as RTM where the extra thickness of the lap joint makes it impossible in practice for the mould to close on the preform. While commonly used in VARTM applications, lap joints increase the local volume fraction of the preform at the joint

seems [54]. Joints may cause problems by enabling resin to racetrack in the gaps between the textile pieces [55] during the infusion process.

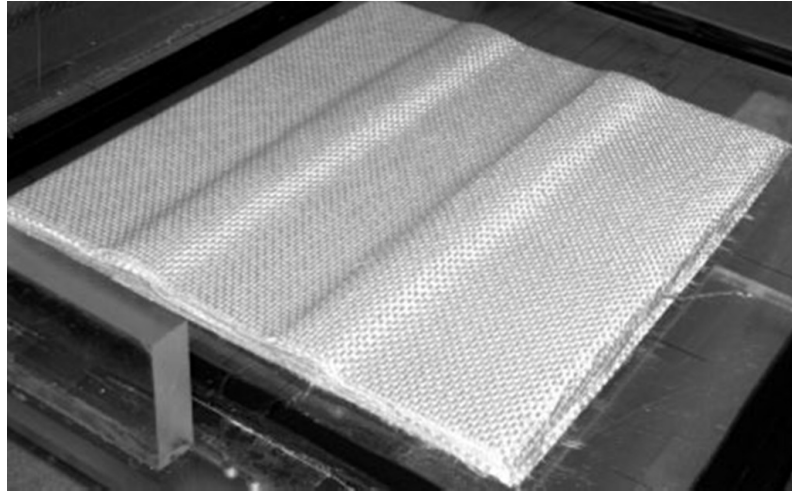


Figure 2-32 Lap Joints Within a Preform [54]

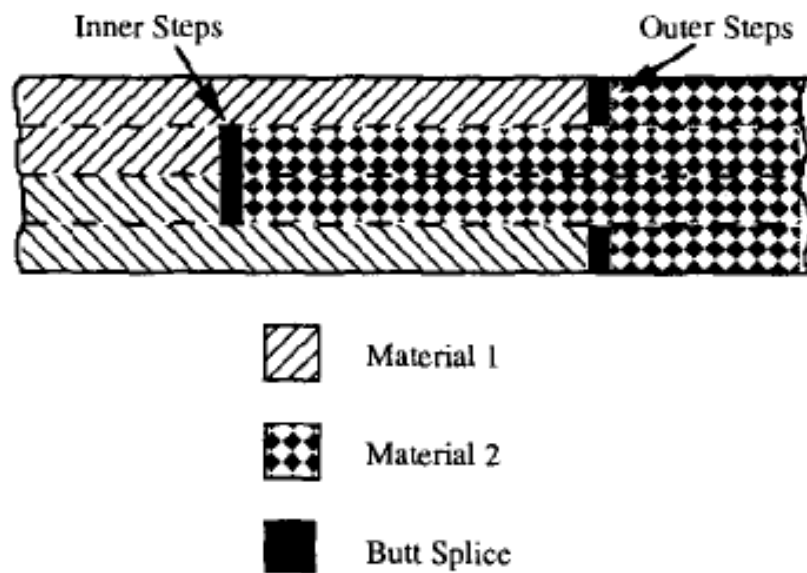


Figure 2-33 Symmetric Step Joint [55]

To temporarily affix multiple pieces of textile to the preform mould, adhesives commonly called tackifiers or binders are typically used. Tackifiers, often supplied as a powder, can also be applied between layers in multilayered textiles to simplify the handling of textile pieces in the creation of complex preforms [56]. While the practical advantages to the preforming operation are evident, the use of a tackifier may have adverse effects on the infusion operation. Preform permeability decreases and may become uneven as tackifier concentration is increased unless the tackifier is located within yarns [57]. The tackifier may also interfere with the resin on a chemical basis, therefore care must be taken to ensure compatibility between the tackifier and resin [57].

2.5. *Kinematic Draping*

The simulation of the draping operation can be performed using geometric principles, using a method called kinematic simulation algorithm. The algorithm is based on five assumptions as laid out by Mack and Taylor [8]:

- The fibres are inextensible
- Fibre crossovers (or nodes) act as pin-joints with no relative slip
- Fibre segments are straight between joints
- Uniform surface contact is achieved between yarns and the surface to be draped
- Fabric layers are infinitely thin

The result is similar to a fishnet and can be solved as long as a start point and certain constraints including initial draping orientations are selected by the user, on an exploratory basis, or through deduction. Early simulations were performed on spheres where the start point is the top of the sphere and the initial constraints are two yarns laid at 90° from each other, which emanate from the start point and continue to the end of the surface to be covered [9], Figure 2-34.

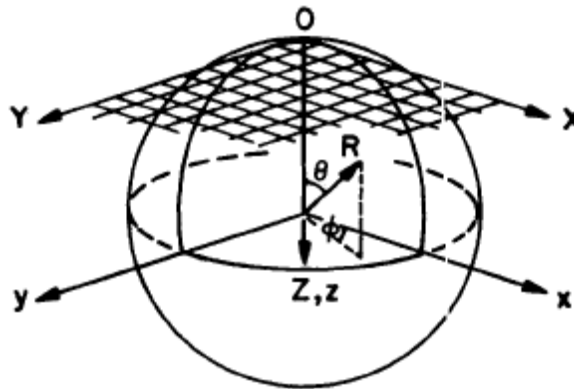


Figure 2-34 Early Drape Simulation Over a Sphere [9]

The process of modeling the draping kinematically involves calculating the location of the crossover points of weft and warp yarns. The crossover points must lie on the surface of the mould to be draped, at the intersection of two spheres. The two spheres are centered on the constrained warp and weft yarns, at a distance equal to the yarn segment length from the start point along each yarn, hence the spheres have a radius equal to the yarn segment length as seen in Figure 2-35.

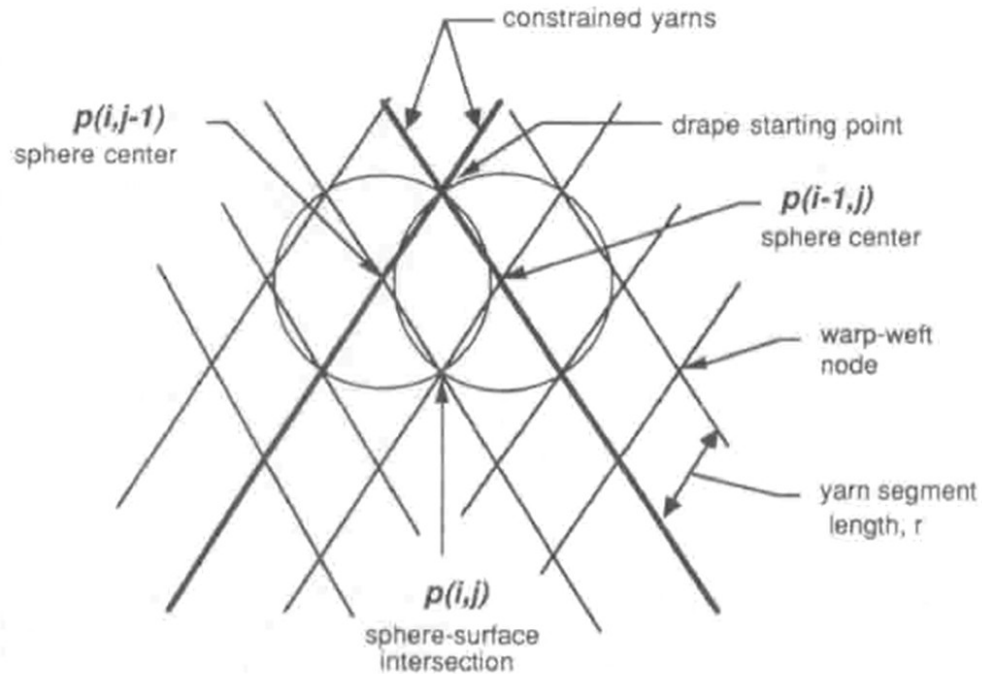


Figure 2-35 Determination of Warp and Weft Crossover Points [58]

Subsequent crossover points are positioned in succession based on previously determined cross-over points, in the same manner. It is possible to determine the location of any crossover point if and only if the location of the adjacent crossover points in the weft and warp direction are known. It is also possible to constrain the initial warp and weft yarns in different manners for the same start point and geometry, leading to different simulation results as seen in Figure 2-36 [7]. Similarly, the start point may be changed while the angle between constrained yarns and the geometry remain identical, leading to drastically different results as seen in Figure 2-37 [58].

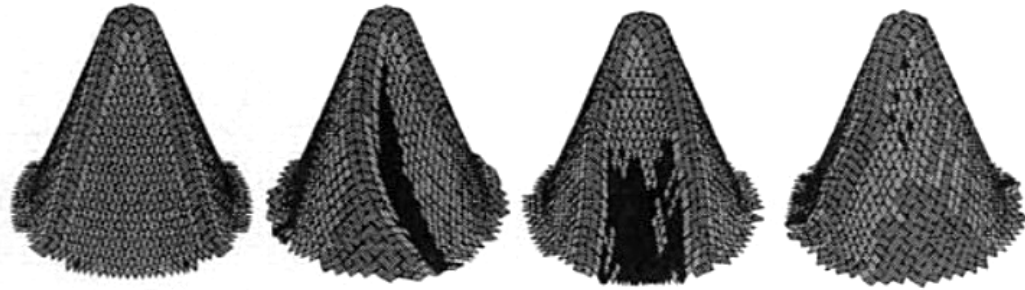


Figure 2-36 Effect of Constrained Yarns on Draping Simulation [7]

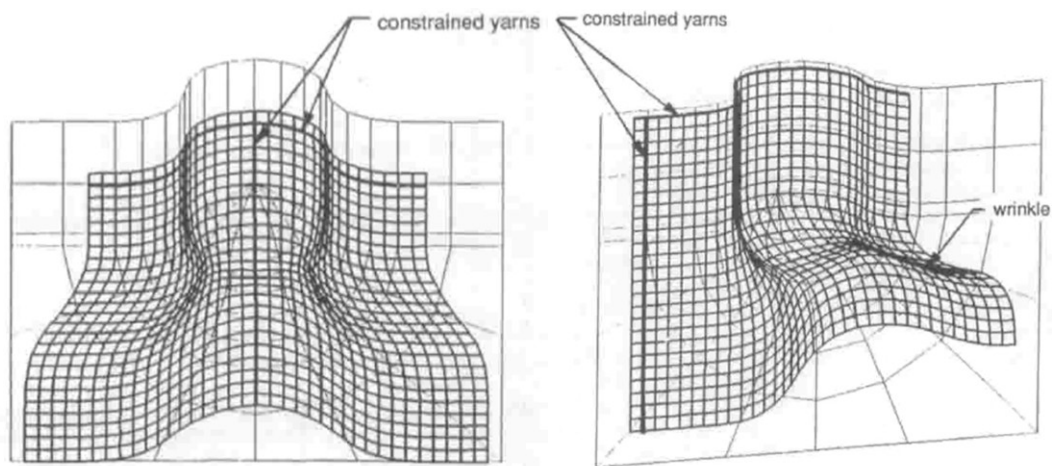


Figure 2-37 Effect of Start Location on Draping Simulation [58]

The constrained warp and weft yarns do not always follow easily determined straight lines; on complex geometries it is usual for the constrained yarns to behave as geodesics [59], defined as lines that follow non-slip paths on a curved surface. In order to fully simulate the entire draping process, including the manual operations done by a person, it is possible to perform simulations with modified geodesics that are further manipulated and modified [10].

The algorithm and mathematics for kinematic draping are described in Chapter 3 and included as the basis of the software developed in this thesis, prior to the modifications needed to simulate the draping of variable-length textiles.

2.6. Pseudo Random Iterative Models for Optimisation

Algorithms such as the Monte Carlo family of methods are used when deterministic methods are either unfeasible or would take unrealistic amounts of computing resources to achieve results. The Monte Carlo methods involve using statistical probabilities to solve large and complex problems related to physical or mathematical systems [60]. The methods can also be used for optimisation and are thus based on the random walk, a type of Markov Chain where successive random steps are taken. A Markov Chain is defined as a process where the next state depends only on the current state, with no consideration on how the present state was achieved. Monte Carlo methods include among others the evolution strategy [61], the Metropolis method [62] and simulated annealing [63]. All of the methods start at a specific configuration which is altered randomly, or mutated in the case of evolution strategy; at each iteration a decision process occurs to decide if the new or original configuration is kept.

The evolution strategy method mimics evolution in a natural environment such as mutations or changes to an individual's configuration, which produce offspring with a higher survival rate. Original configurations are called parents and new configurations are called mutants. The decision process used for deciding whether the mutant or parent is kept is based on the fitness level, with a high fitness level being desirable. The fitness level is generally based on

a property for which the system is optimised; in the case of draping the fitness level could be attributed to the surface area covered by a single piece of textile, for example. After each iteration, the fitness levels of the parent and mutant are compared and the individual with the best fitness level becomes the parent for the next iteration. The evolution strategy may produce one or several (λ) mutants per iteration, each strategy being denoted as (1+1)-ES and (1+ λ)-ES respectively. In cases where multiple mutants are generated it is possible to disregard the parents when following the decision process, in a strategy denoted as (1, λ)-ES. The evolution strategy method may also be expanded to include recombinations of multiple individuals as well as mutations of single individuals [61]. This requires that multiple parents are used for each iteration, usually the μ best individuals of the previous iteration. The strategies which involve multiple parents are denoted as (μ + λ)-ES and (μ , λ)-ES, in which the parents may or may not be selected as the parents of the next generation, respectively.

The evolution strategy method may be considered a greedy algorithm [64] since it always selects the individual with the highest fitness rating. Only in strategies such as (1, λ)-ES or (μ , λ)-ES may a mutant with a lower fitness level than the parent be chosen due to the fact that the parent is discarded. Because greedy algorithms always select the best configuration for the current iteration, it is possible to reach a local maximum which can keep the system from reaching a global maximum. At any given iteration it is impossible to know if a certain state change will be beneficial in the short term as well as the long term. The more complex Monte Carlo methods [62, 63] address this issue by allowing for configuration changes which are not beneficial in the current system state. In the Metropolis [62] method, the probability of choosing the new configuration depends on the likelihood ratio between the new and original

configurations. Should the new configuration state be more likely than the original configuration, the new configuration is chosen, but if the new configuration is less likely than the original configuration there is a probability that either the original or chosen configurations are chosen, based on the likelihood ratio. The likelihood ratio is dependent on the application and probability functions used. In the case presented by Metropolis [62] the likelihood ratio is based on the energy of the system. Should the energy decrease with the new configuration, the likelihood ratio is unity or above and the new configuration is chosen. Should the energy increase with the new configuration, the likelihood ratio is less than unity and the probability P of keeping the new configuration is based on the energy difference Δe between configurations, the temperature T of the system and the Boltzmann constant k_b :

$$P = e^{(\Delta e/k_b T)} \quad (2.15)$$

Simulated annealing [63] is a variant of the Metropolis method where the temperature of the system is reduced after each iteration following a cooling schedule. The cooling of the system effectively reduces the probability of keeping configurations which increase the energy of the system. Much like annealing in metals, the molecules are permitted to move freely at high temperatures and are less free as the temperature is reduced. Simulated annealing enables the optimisation to recover from local maxima when the temperature is high while increasingly restricting the probability of new higher energy configurations as the system converges to a solution.

While both the Metropolis and simulated annealing algorithms attempt to solve the problems associated with local maxima, it is very difficult to determine the proper probability

functions to be used for a specific problem such as the optimisation of draping textiles. The evolution strategy, especially (1+1)-ES, is much simpler to implement since only the fitness rating needs to be defined.

3. Draping Simulation Software

In order to predict and visualise the behaviour of textiles as they are deformed onto a geometric shape, software which can accurately and consistently predict the deformation of a draped textile reinforcement was written and validated. The software developed for this thesis is based on the kinematic approach outlined in Chapter 2.5, which was modified to enable the software to simulate both standard constant-length and variable-length textile reinforcements. The algorithm used for draping both constant and variable-length textiles is detailed in the following sections. The optimisation of variable-length textiles is outlined in Chapter 4, while the validation of the draping software is detailed in Chapter 5.

3.1. *Software Overview*

The software was written in C++ and uses the OpenGL [65] framework to render 3D visualisations of the simulation. A user interface framework called QT [66] was used for providing a graphical user interface (GUI), enabling ease of use. The addition of the GUI enables the software to be used as a tool for the design and manufacture of textiles as well as for research. Windows-based computers can execute the software following the installation of the Microsoft Visual C++ 2005 Redistributable Package [67]. The software may drape, optimise and show drape results in one window while any user inputs may be done via a series of menus, keyboard shortcuts, slider bars and mouse commands. An OpenGL window and some text boxes allow for the program to display 3D renderings of the drape as well as information such as

surface area covered and various optimisation data. The software's GUI as it appears when first opened is shown in Figure 3-1.

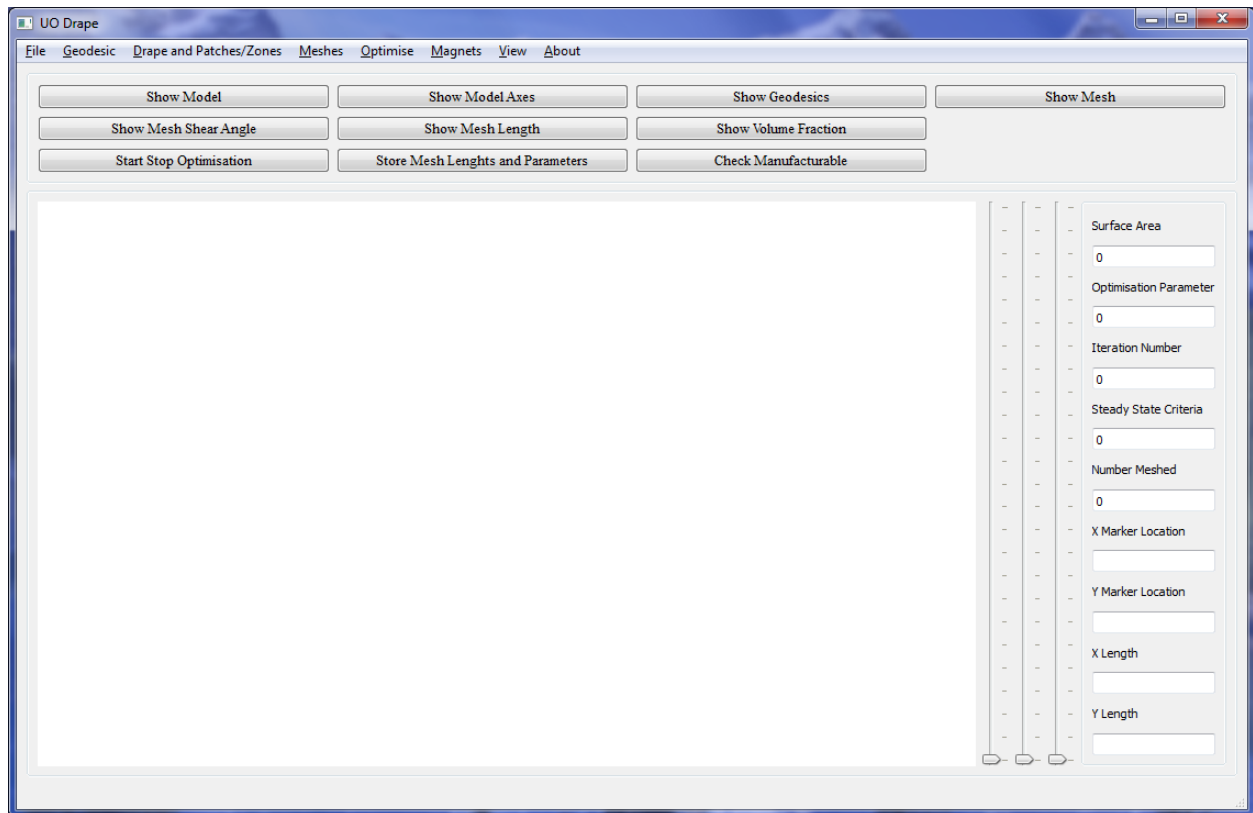


Figure 3-1 Draping Software GUI

Several steps of the draping process are performed individually within the software, outlined as follows:

1. Load a properly formatted model of the part to drape
2. Select the drape start coordinates
3. Toggle between available start elements (if more than 1 are available)
4. Input the desired geodesic orientation angle
5. Drape the model

Each step is dependent on the previous step, therefore every preceding step must be completed before a given step can be performed. However, it is possible to repeat a previously performed step, which will erase the data of subsequent steps. The details on how these steps are performed are explicated within this chapter. It is also possible to perform additional tasks such as loading and storing simulation data, creating and using patches or zones, creating multiple drape simulations on the same model, displaying quantitative drape results and adding virtual holder magnets. These tasks are detailed within this chapter.

3.2. Model Geometry Specification and Format

The software can simulate the draping operation on a wide variety of shapes derived from solid models created using software such as SolidWorks™. For the software to recognise these solid models they must be meshed using a pre-processor such as Gambit™. An example of a solid model which has been meshed using Gambit™ appears in Figure 3-2. The solid model is first stripped of its thickness so that only a shell remains. This shell is then covered with a continuous mesh made of three-noded triangular elements, which may be of varying sizes. The elements must be adequately sized to accurately follow the geometry and curvatures of the solid model. Performance reduction and issues within the algorithm may arise with poorly constructed meshes as detailed in the next section.

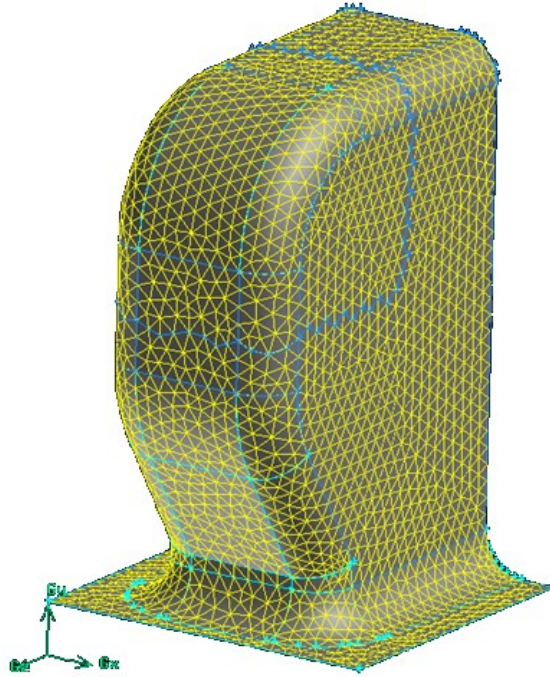


Figure 3-2 Meshed Solid Model Suitable for the Draping Software

Once a satisfactory mesh is created in separate software such as Gambit™, it must be saved under a specific format, details of which are found in Appendix A, which can be read by the draping software (.model). The number and coordinates of each node are specified under this format, as well as the number of elements and the identity of the three nodes that make up each element.

3.3. *Model Pre-processing*

When the model is loaded in the draping software, the vector normal to each element is calculated, normalised and stored within an array. This is done primarily to ensure that all normal vectors point inwards or outwards of the model. For example, vectors normal to the

elements of a sphere will all point either towards the inside or all towards the outside of the sphere, instead of a random mixture of inward and outward pointing vectors. Consistency of the normal vectors is important for the software to detect the orientation of the elements relative to gravity and for the software to be able to calculate the curvature of the model at any given point, for example. Storing the normal vectors within an array also removes redundant calculations as the normal vectors are used multiple times within the draping algorithm. To determine the normal vector of an element, two directional vectors $\vec{d}_1$ and $\vec{d}_2$ are needed. These are found by using two different combinations of the element nodes, equations (3.1) and (3.2). A cross product of the direction vectors yields the normal vector $\vec{vn}_i$, equation (3.3), where i is the number of the element.

$$\vec{d}_1 = \frac{n_1 - n_3}{\|n_1 - n_3\|} \quad (3.1)$$

$$\vec{d}_2 = \frac{n_1 - n_2}{\|n_1 - n_2\|} \quad (3.2)$$

$$\vec{vn}_i = \vec{d}_1 \times \vec{d}_2 \quad (3.3)$$

Once the normal vector for the first element is found, it is stored in the normal vector array and a flag indicating that the vector has been calculated for this element is set to 1 (true). Using this flag ensures that calculations are only performed once for each element, and removes the possibility of an infinite loop condition.

The next step of the algorithm consists in determining the normal vectors for neighbouring elements, defined as elements sharing two common nodes with an element. The

normal vector for each of these elements is found in the same manner as described above in equations (3.1) to (3.3). To determine if the vectors normal to neighbouring elements point towards the same side of the model surface, the angle θ_e between the normal vectors of the two elements is found:

$$\theta_e = \arccos(\overrightarrow{vn_1} \cdot \overrightarrow{vn_2}) \quad (3.4)$$

Any angle beyond 90° indicates that the normal vector of one element points inwards while the other points outwards. The orientation of the first element within the list of elements found in the .model file dictates the orientation of all the others, thus the direction of the neighbouring element's normal vectors are reversed if the angle between the normal vectors to the first and second element is over 90° . This development is based on the assumption that no vectors normal to neighbouring elements with the same inwards or outwards orientation may have an angle over 90° between them; or, that the absolute value of the angle between two neighbouring elements is always under 90° when 0° indicates that they lie in a common plane. For this assumption to be valid the mesh must be constructed with an adequate element size, or resolution. The element size must be sufficiently small to ensure that in curved areas the angle between neighbouring normal vectors with the same inwards or outwards orientation is always smaller than 90° . Figure 3-3 shows the cross sections of neighbouring elements with their normal vectors where the mesh is adequately sized for the curvature (left) and where the mesh is too rough for the curvature (right), allowing for a large angle between the normal vectors.

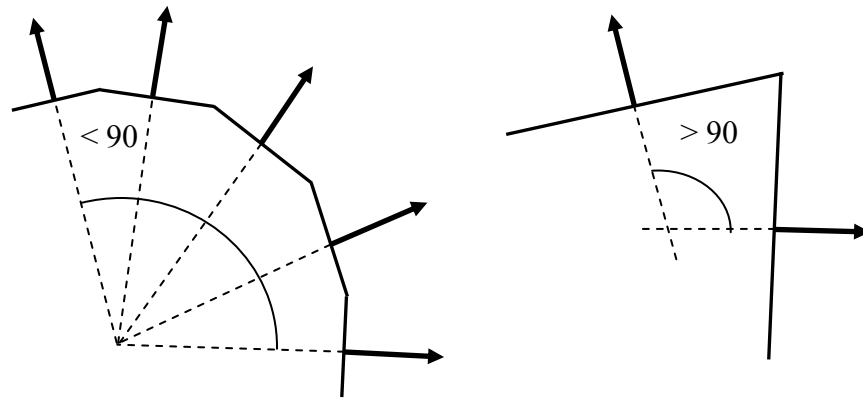


Figure 3-3 Angle Between Vectors Normal to Neighbouring Elements

It is possible to create meshes where the element density or size changes across the model. In flat areas devoid of curvature, the use of large elements is desired to limit their number while in areas of complex geometry and small curvature radii, the use of smaller elements is desired to avoid the condition seen on the right-hand side of Figure 3-3. Once the angles between the normal vectors of the first element and its neighbours are calculated and the orientation of these elements are made consistent, the normal vectors of the neighbouring elements are saved to the array and the flag indicating that the normal vector has been calculated is set to 1. The same process is repeated for each of the neighbours of the three neighbouring elements found above. The algorithm for populating the element normal vector array is described by the flowchart shown in Figure 3-4.

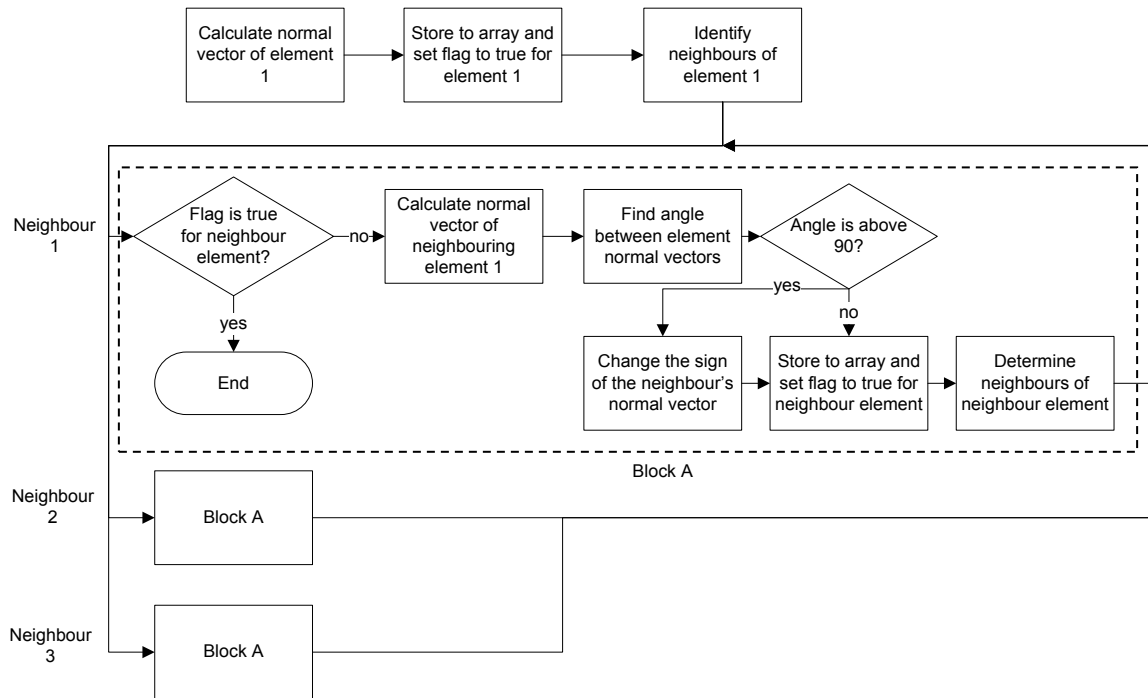


Figure 3-4 Algorithm to Determine Element Normal Vectors

The portion of the flowchart named block A is repeated for each of the neighbours. Due to the elements being three-noded, the maximum number of neighbours of any element is three, while elements found on the edge of the model may only have two neighbours, or even one. The algorithm will loop as new sets of neighbours are found until all flags indicating that the normal vector of the element has been calculated are set to 1 (true); the required continuity of the mesh ensures that all elements will be accounted for.

3.4. Draping Algorithm

The algorithm used for draping is based on the kinematic drape model discussed in section 2.5. The kinematic draping algorithm requires a surface which is supplied by the input model as described in section 3.2. A model of a hemisphere loaded in the software and ready to be draped can be seen in Figure 3-5. To drape the surface, a start location and some initial constraints must be defined, the methods used for which are detailed in this section along with the algorithm used for draping the surface of the model.

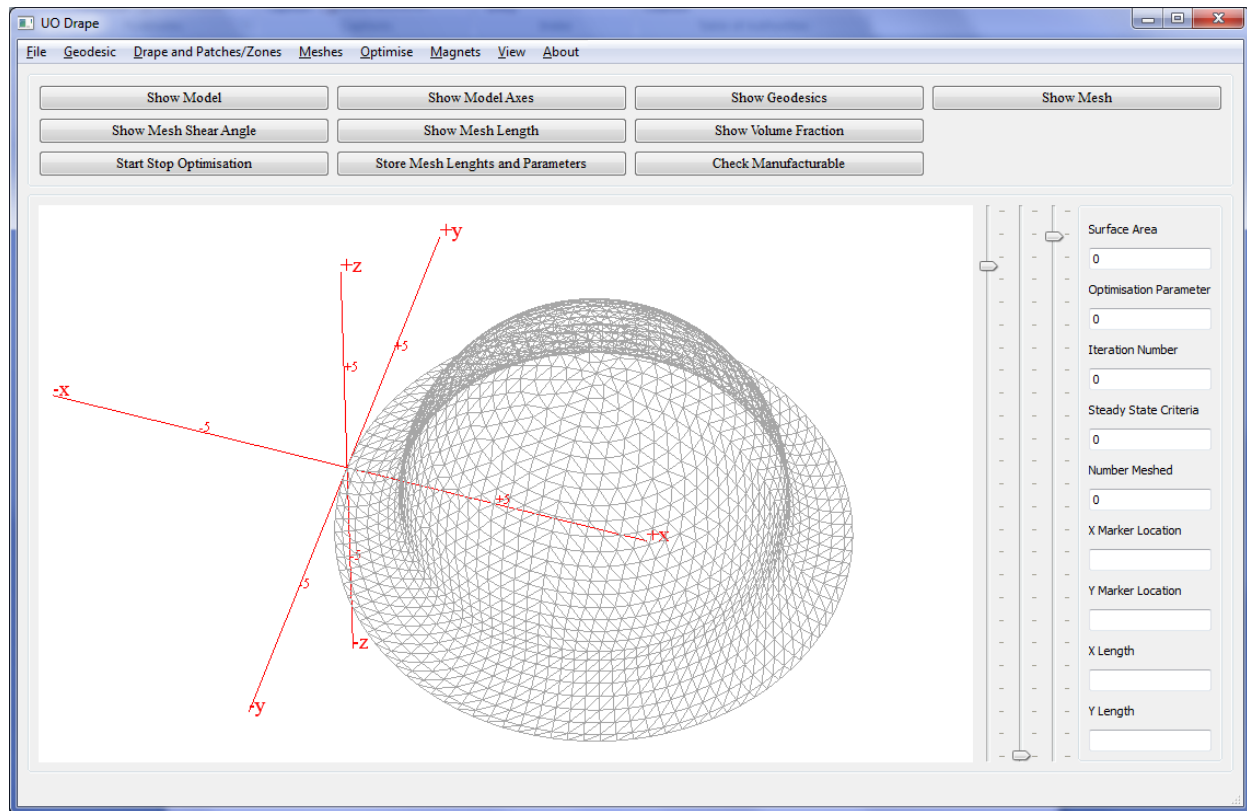


Figure 3-5 Solid Model as Loaded in the Draping Software

3.4.1. Start Point of the Draping Operation

The start point is selected arbitrarily by choosing one of the three orthogonal planes and selecting two coordinate values in this plane, in order to trace a vector perpendicular to the selected plane. This vector must intersect the model in either one or many points. The software can handle up to ten intersections of the vector and model; further intersections are unlikely and will not be shown. If the vector does not intersect any element the software will notify the user that the specified start point was not valid. The software will also notify the user if multiple intersections are found. The dialog used for specifying the start point of the drape can be seen in Figure 3-6, in this case the x - y plane was chosen.

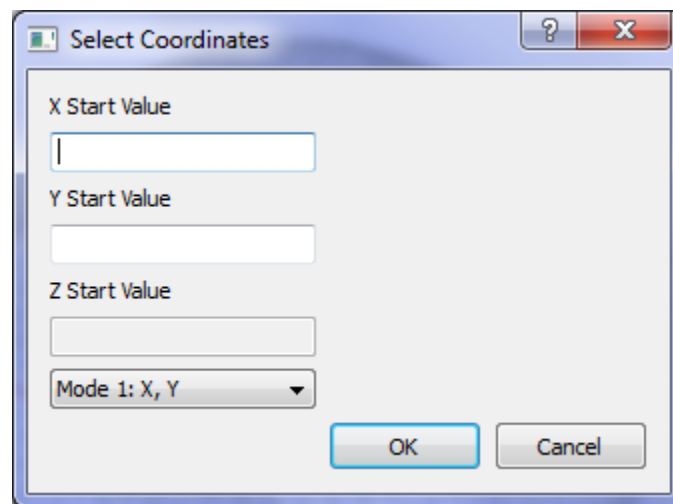


Figure 3-6 Start Point Coordinate Dialog

The first step taken by the software in determining the start point, based on the information given, is to determine where the intersections between the selection vector and the model occur. Since the element which contains the intersection is unknown initially, the software must iterate through all the elements to find possible intersection points. Each element is

projected onto the selected plane and six vectors are defined as seen in Figure 3-7. The left-hand side of the figure depicts an element for which the start point s is within the element. The right-hand side of the figure depicts an element for which the start point is outside of the element. The vectors are subscripted with a 2D to indicate that these calculations occur in 2D. When in the x - y plane, y - z plane or z - x plane, all z coordinates or x coordinates or y coordinates are ignored respectively.

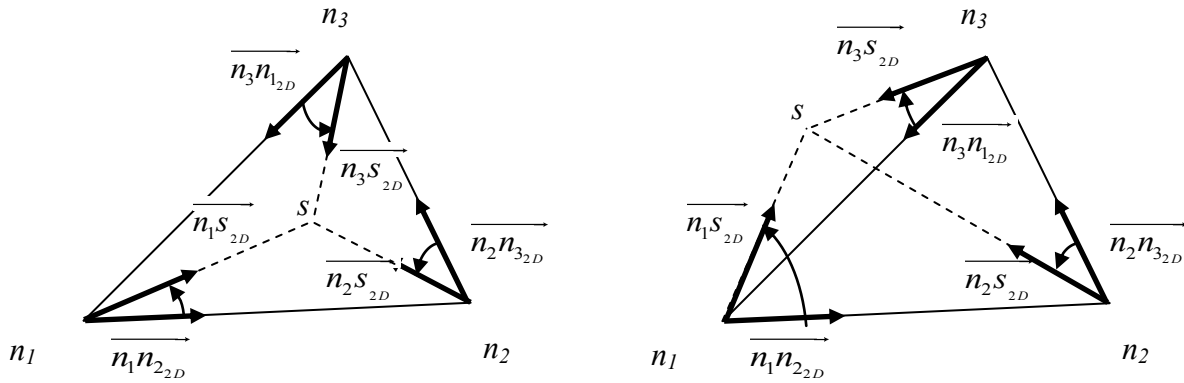


Figure 3-7 2D Vectors on an Element Projection

The equations used to find the six vectors in the x - y plane are:

$$\overrightarrow{n_1 n_{2D}} = [n_{2x} - n_{1x}, n_{2y} - n_{1y}] \quad (3.5)$$

$$\overrightarrow{n_2 n_{3D}} = [n_{3x} - n_{2x}, n_{3y} - n_{2y}] \quad (3.6)$$

$$\overrightarrow{n_3 n_{1D}} = [n_{1x} - n_{3x}, n_{1y} - n_{3y}] \quad (3.7)$$

$$\overrightarrow{n_1 s_{2D}} = [s_x - n_{1x}, s_y - n_{1y}] \quad (3.8)$$

$$\overrightarrow{n_2 s_{2D}} = [s_x - n_{2x}, s_y - n_{2y}] \quad (3.9)$$

$$\overrightarrow{n_3 s_{2D}} = [s_x - n_{3x}, s_y - n_{3y}] \quad (3.10)$$

With these six vectors, three different 2D cross products can be calculated. Vectors having the same origin are multiplied together: $\overrightarrow{n_1 n_{2D}} \times \overrightarrow{n_1 s_{2D}}$, $\overrightarrow{n_2 n_{3D}} \times \overrightarrow{n_2 s_{2D}}$ and $\overrightarrow{n_3 n_{1D}} \times \overrightarrow{n_3 s_{2D}}$. To determine if the start point is within the given element, the result of these three cross products is examined. If the point is placed such that the cross products yield three positive values or three negative values, point s must be within the element. If one cross product yields a result with a different sign than the other two, point s is outside of the element. The curved arrows in Figure 3-7 illustrate the right hand rule of the cross product. On the left-hand side all arrows are counter-clockwise indicating that the cross products are of the same sign. On the right-hand side two arrows are counter-clockwise and one is clockwise indicating a difference in the sign of the cross products. If any cross product has a result of zero this means that the two vectors are collinear and the start point coincides with one of the edges of the element. This solution is rejected as it can lead to an erroneous intersection. When an element is identified as holding an intersection point, it is stored and the total number of elements containing a possible solution is counted. If no intersections are found, the user is notified that the selected coordinates are not suitable and if multiple intersections are found, the user is notified of this. When multiple elements hold an intersection point, they are all highlighted in red as seen in Figure 3-8. The start point can then be chosen by toggling between the possibilities. A maximum of ten possibilities was chosen as it is very unlikely that a vector will intersect a model at more than ten points.

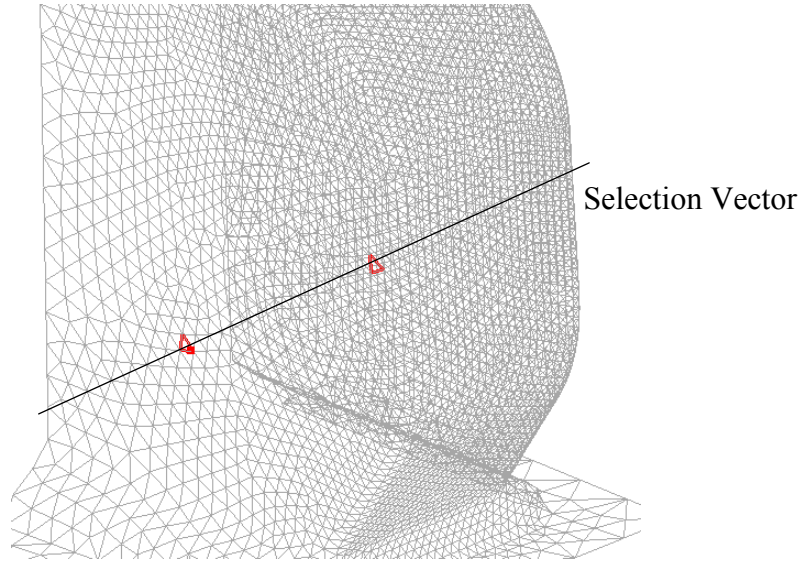


Figure 3-8 Multiple Elements Intersecting the Selection Vector

All previous operations were performed in 2D space, hence while the intersecting element and two coordinates are known, the third coordinate of the start point remains to be determined. The equation of the plane of the element in which the start point resides is needed:

$$Ax + By + Cz = D \quad (3.11)$$

The vector normal to the element was previously computed during pre-processing and it can be used to find values for A , B and C in equation (3.11) since the components of the vector appear directly in the equation of the plane:

$$\overrightarrow{vn} = (\hat{A}_i, \hat{B}_j, \hat{C}_k) \quad (3.12)$$

The last component in the plane's equation, D , can be found by computing the dot product of the plane's normal vector with the position vector of any point lying in the plane, say node 1 of the element for example.

$$D = An_{1x} + Bn_{1y} + Cn_{1z} \quad (3.13)$$

Substituting the two known components of the start point into equation (3.11) leads to the third component of the start point. The equation used in the case of the x - y plane is:

$$s_z = \frac{D - Bs_y - As_x}{C} \quad (3.14)$$

3.4.2. Creation of the Geodesic Lines

The kinematic draping model requires initial constraints to identify a unique solution for a given case. The initial constraints are two geodesics that represent one warp yarn and one weft yarn, both passing through the start point. Geodesics correspond to straight lines on a curved surface and describe the shortest path between two points on said surface. This is ideal to represent a constrained yarn on the preform as it is intuitive to place a yarn along a geodesic, and the yarn so positioned is less likely to move once in position. The geodesics cross at 90° from each other at the draping start point and the initial orientation of the geodesics in the selection plane can be modified by the user as seen in Figure 3-9, where the angle between the geodesics and axes of the selection plane is 0° on the left and 30° on the right. While the angle between the geodesics and the selection plane axes can change, the angle between the two geodesics is fixed at 90° .

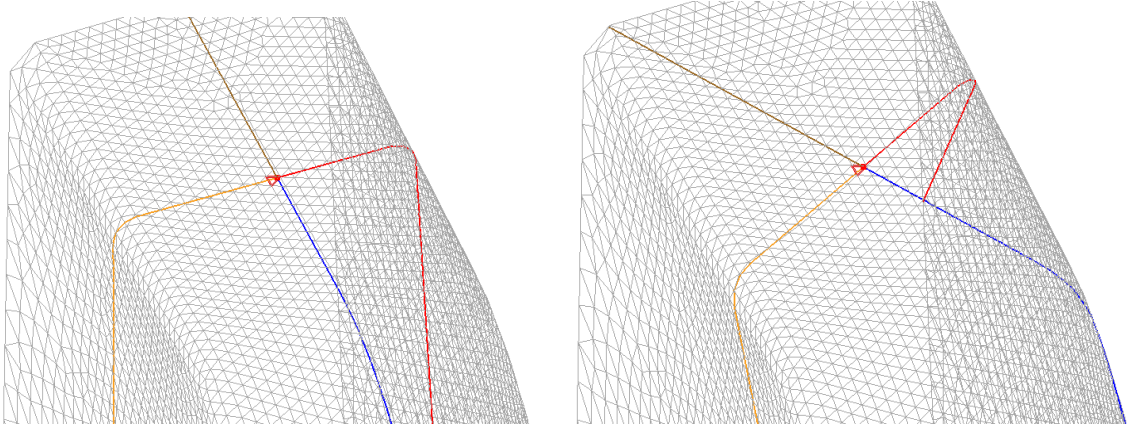


Figure 3-9 Different Orientations of Geodesics at the Start Point

3.4.2.1. Initial Geodesic Direction Vectors

Each geodesic is defined in the software as a series of segmented lines that extend within some of the model's elements. The default orientation of the geodesics is parallel to each of the reference axes forming the plane that was selected for finding the starting point of the drape, but this orientation may be modified by the user. Modifications to the default orientation are made once the default orientation has been established, which is dependent on the selected plane. If the x - y plane was chosen the orientation of the first segment is the y axis as projected on the starting element plane; similarly if the y - z plane was chosen then the orientation is the z axis as projected on the element plane, and if the z - x plane was chosen then the orientation is the x axis as projected on the element plane. The default initial direction vector for the first geodesic, $\overline{geod1}$, illustrated here for the case of an x - y plane selection, is as follows:

$$\overline{geod1} = (0, \hat{.}, \dots, \dots)_j : geod1_e - geod1_s \quad (3.15)$$

where $geod1_s$ and $geod1_e$ are the start and end points of the vector, respectively. The z component of $\overline{geod1}$ is unknown at this stage, it can be found using the start and end points of the vector as shown in Figure 3-10.

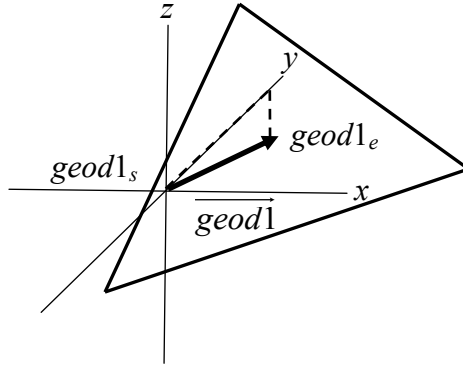


Figure 3-10 Determining the Initial Geodesic Direction Vector

The start point of the vector is known: it is the selected start point for the drape. The x (0) and y (1) components of the end point of the vector are known since the vector points along the y axis. The element plane on which the point resides is also known. Therefore, the same methodology used for finding the third coordinate of the drape start point in section 3.4.1 can be used to find the third coordinate of the end point of the vector. Using equation (3.15) with the known start and end points of the vector leads to a fully defined initial direction vector for the first geodesic for the default orientation. If an angle θ_g other than 0° is specified by the user, the vector must be subjected to a rotation about the vector normal to the element where the drape point resides. The rotation about the vector can easily be matched in a laboratory trial as the rotation is made about the tangent to the surface of the model. The following equation is used for rotating the vector:

$$\begin{bmatrix} geod1_x \\ geod1_y \\ geod1_z \end{bmatrix} = \begin{bmatrix} v_{nx}^2 + (1 - v_{nx}^2)c & v_{nx}v_{ny}(1 - c) - v_{nz}s & v_{nx}v_{nz}(1 - c) + v_{ny}s \\ v_{nx}v_{ny}(1 - c) + v_{nz}s & v_{ny}^2 + (1 - v_{ny}^2)c & v_{ny}v_{nz}(1 - c) - v_{nx}s \\ v_{nx}v_{nz}(1 - c) - v_{ny}s & v_{ny}v_{nz}(1 - c) + v_{nx}s & v_{nz}^2 + (1 - v_{nz}^2)c \end{bmatrix} \cdot \begin{bmatrix} geod1'_x \\ geod1'_y \\ geod1'_z \end{bmatrix} \quad (3.16)$$

where c , s and $geod1'$ are $\cos \theta_g$, $\sin \theta_g$ and the default orientation of the geodesic, respectively. The second geodesic is obtained by rotating the first geodesic by 90° about the vector normal to the element:

$$\begin{bmatrix} geod2_x \\ geod2_y \\ geod2_z \end{bmatrix} = R \begin{bmatrix} geod1_x \\ geod1_y \\ geod1_z \end{bmatrix} \quad (3.17)$$

where R is the rotation matrix as seen in equation (3.16) with $\theta_g = 90^\circ$.

3.4.2.2. *Intersection of Geodesic and Element Edge*

With the start point and initial direction of each geodesic known, it is now possible to determine the extremities of the geodesic segments in the first segment of each geodesic as shown in Figure 3-11. To simplify the way in which data is stored, the two geodesics are each split in two at the start point, now referred to as $geo[0][0]$, to form four geodesics that extend from that point and in one direction only. The intersection point coordinates are stored to an array $geo[i][j]$, where i refers to a specific geodesic and j refers to the sequential order from the

start of the geodesic at $j=0$. While the identity of the element edge that intersects the geodesic line segment is obvious to the human eye, the software must look for an intersection between every combination of geodesic directional vector and element edge. When all six geodesic and element edge combinations are examined, four intersection points $geo[0][1]$, $geo[1][1]$, $geo[2][1]$ and $geo[3][1]$ should exist.

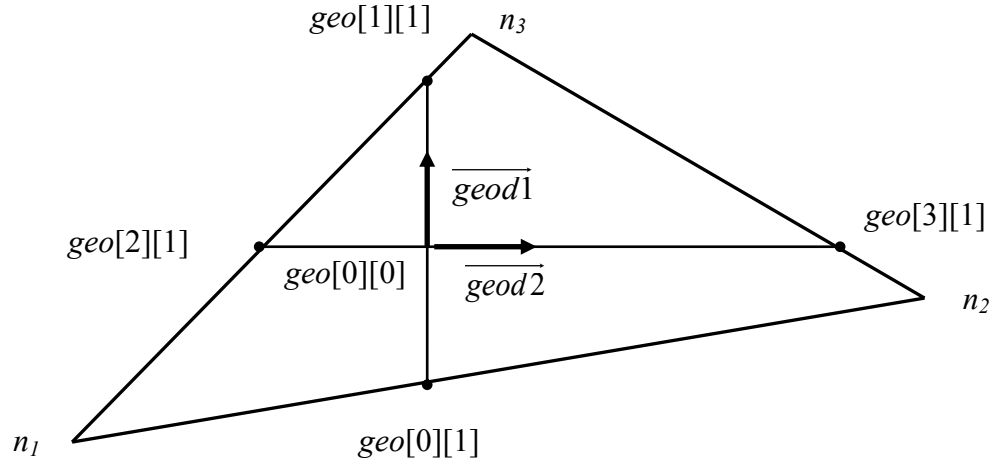


Figure 3-11 Intersections Between Geodesic and Element Edges for the First Element

The intersections are found by simultaneously solving the equations of the lines that represent a specific element edge and one of the geodesics:

$$\overrightarrow{n_1 n_2} \cdot m_i + n_1 = \overrightarrow{geod1} \cdot k + geo[0][0] \quad (3.18)$$

where m_i and k are unknown variables. Equation (3.18) can be expanded into three equations:

$$n_1 n_{2_x} \cdot m_i + n_{1_x} = geod1_x \cdot k + geo_x[0][0] \quad (3.19)$$

$$n_1 n_{2_y} \cdot m_i + n_{1_y} = geod1_y \cdot k + geo_y[0][0] \quad (3.20)$$

$$n_1 n_{2_z} \cdot m_i + n_{1_z} = geod1_z \cdot k + geo_z[0][0] \quad (3.21)$$

The system of equations can be solved in three different manners by combining equations (3.19) and (3.20), equations (3.20) and (3.21) or equations (3.19) and (3.21), and solving for m_i . In some cases the terms associated with the edge vector or the geodesic vector will equal 0 which may render one or many combinations unsolvable due to a division by 0. For this reason, the software must compute all three solutions. The three solutions for m_i are labelled m_{xy} , m_{yz} and m_{zx} , found with:

$$m_{xy} = \frac{geod1_x (n_1 n_{2_y} - geo_y[0][0]) + geod1_y (geo_x[0][0] - n_1 n_{2_x})}{geod1_y \cdot n_1 n_{2_x} - geod1_x \cdot n_1 n_{2_y}} \quad (3.22)$$

$$m_{yz} = \frac{geod1_y (n_1 n_{2_z} - geo_z[0][0]) + geod1_z (geo_y[0][0] - n_1 n_{2_y})}{geod1_z \cdot n_1 n_{2_y} - geod1_y \cdot n_1 n_{2_z}} \quad (3.23)$$

$$m_{zx} = \frac{geod1_x (n_1 n_{2_z} - geo_z[0][0]) + geod1_z (geo_x[0][0] - n_1 n_{2_x})}{geod1_z \cdot n_1 n_{2_x} - geod1_x \cdot n_1 n_{2_z}} \quad (3.24)$$

Additionally, due to peculiarities in floating point arithmetic [68] a series of checks must be performed before computing the solutions for m_i in order to avoid erroneous numbers. Since numbers are converted into binary form for computation and storage to memory it is possible for very small numbers to be significantly altered, which may cause large scale errors in computation; these errors usually arise when subtracting near identical numbers. The subtractions in the denominators of equations (3.22), (3.23) and (3.24) are susceptible to floating point errors therefore the components involved in the subtractions are checked:

$$\left| geod1_y \cdot n_1 n_{2_x} - geod1_x \cdot n_1 n_{2_y} \right| > \left| geod1_y \cdot n_1 n_{2_x} \cdot \varepsilon_m \right| \quad (3.25)$$

$$\left| geod1_z \cdot n_1 n_{2_y} - geod1_y \cdot n_1 n_{2_z} \right| > \left| geod1_z \cdot n_1 n_{2_y} \cdot \varepsilon_m \right| \quad (3.26)$$

$$\left| geod1_z \cdot n_1 n_{2_x} - geod1_x \cdot n_1 n_{2_z} \right| > \left| geod1_z \cdot n_1 n_{2_x} \cdot \varepsilon_m \right| \quad (3.27)$$

where ε_m is the threshold of closeness between the two components. The variable ε_m was set to 0.0001 by running the algorithm multiple times on different models with different parameters and choosing the most strict value that didn't produce false negatives. If equation (3.25) is true m_{xy} is computed, if equation (3.26) is true m_{yz} is computed and if equation (3.27) is true m_{zx} is computed. It is possible to have situations where multiple values for m_i are computed, which are not necessarily identical due to additional floating point arithmetic errors. In cases where only one m_i is found, this is used for further calculations. In cases where there are two m_i values the first value of m_i is taken. Should three m_i values be found the software compares for closeness to determine which pair of values are more alike, and the software then uses one of the values in the pair as the value for m_i . This is done so as to select the most precise value for m_i under the assumption that the more exact value is the one that is obtained multiple times. The algorithm used for comparing the three values is detailed in Figure 3-12 and uses an increasingly strict threshold to compare each pair until only one pair passes the comparison test or the maximum threshold is achieved, at which point the algorithm defaults to m_{xy} . When no m_i values can be found, the software is stopped to display the problem; this situation is unlikely in view of the precautions and steps taken before computing the values.

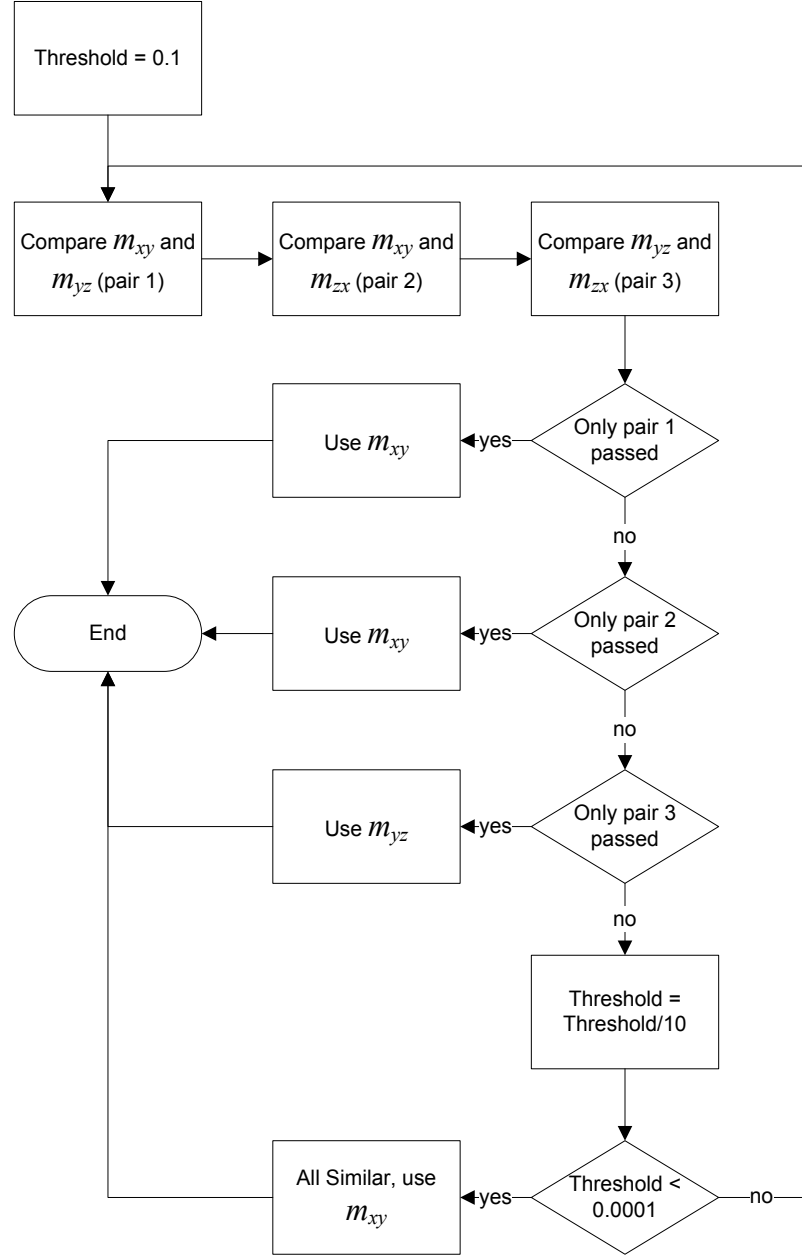


Figure 3-12 Algorithm for m Value Comparison

When a value of m_i is found which satisfies the intersection between two lines as outlined in equation (3.18), it is used to determine the coordinates of the point of intersection $geo[0][1]$ between the element edge and the geodesic:

$$geo[0][1] = n_1 + \overrightarrow{n_1 n_2} \cdot m_i \quad (3.28)$$

The intersection does not necessarily lie on the element edge as the algorithm rightly assumes infinite lines. In Figure 3-13 p_i is the intersection between geodesic $\overline{geod1}$ and the element edge $\overline{n_2n_3}$ but does not lie within the element edge boundaries.

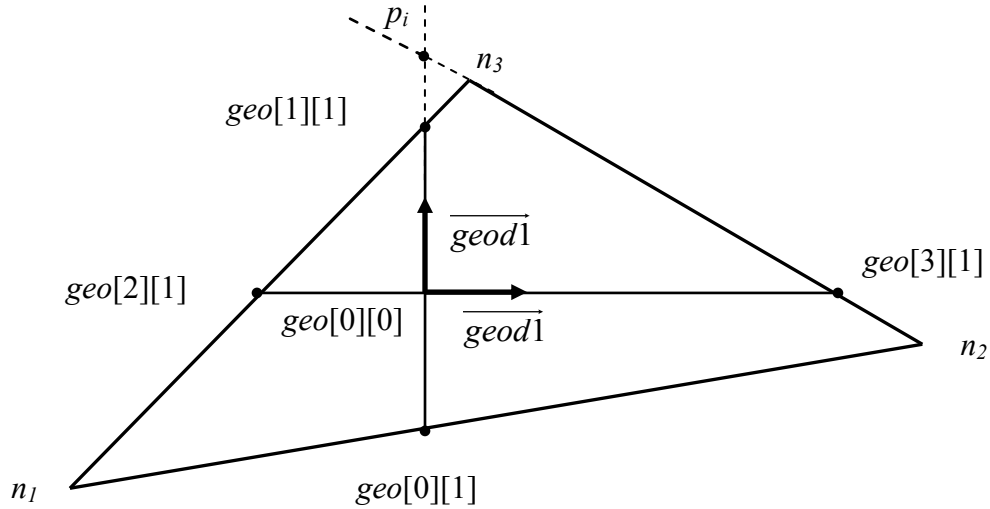


Figure 3-13 Intersection Between Geodesic and Element Edge Extension

To ensure that the intersection is valid and does lie within the element edge boundaries the value of m_i is inspected. Any value of m_i between 0 and 1 will result in an intersection which lies on the element edge. Any other value of m_i indicates an intersection which lies on an extension of the line of the element edge. Geodesic and element edge combinations which lead to intersections not within the element edge boundary are rejected.

The situations depicted in Figure 3-11 and Figure 3-13 relate to the element containing the drape start point. Subsequent elements along each geodesic will resemble what is depicted in Figure 3-14, in which only one geodesic lies within the element. The methodology is identical to

the one used for the element containing the drape start point with a few exceptions: only one geodesic is present, therefore only half of the geodesic to element edge combinations exists and the value of j used is the value of j for the last known intersection point along the geodesic. The software must also ensure not to use the value of $geo[1][j]$ as the subsequent intersection point as it will be the result of one of the geodesic and element edge combinations.

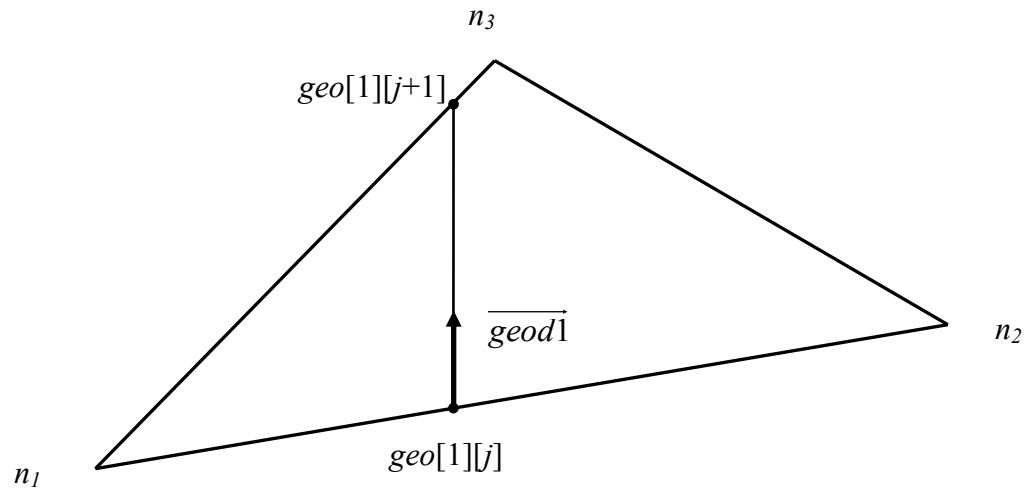


Figure 3-14 Intersections Between Geodesic and Element Edges for Subsequent Elements

The geodesic direction vector $\overrightarrow{geod1}$ used in determining subsequent intersection points is not necessarily identical to the initial geodesic direction vector. It is necessary to determine the direction vector for the subsequent geodesic segment after finding each intersection point, the method for which is detailed in the following section.

3.4.2.3. *Subsequent Geodesic Direction Vectors*

When a geodesic segment reaches an element edge at the geodesic intersection point $\text{geo}[i][j]$, it is necessary to determine on which element the next geodesic segment will lie. The element edge which was reached by the last known geodesic segment is used for determining the next element by finding which other element has the same element edge. The element edge is defined by the two nodes at its extremities, specific combinations of which are found in only two elements. The software must iterate through the element list to find the other element which has the same two nodes as the element on which the last known geodesic segment lies. If no other element contains the same node combination the software stops progressing this geodesic as it has reached the edge of the model. Once the next element has been identified, a new direction vector for the geodesic must be computed, based on the difference between the planes of the two elements. If the element planes in which the elements are defined are coplanar, the geodesic direction vector will not change; if the element planes are not coplanar the new direction vector will be such that should the element planes rotate and become coplanar, the direction vectors would be collinear. To determine the new geodesic direction vector $\overrightarrow{\text{geod}l_2}$ on element B the initial vector $\overrightarrow{\text{geod}l_1}$ on element A is reduced into two components parallel and perpendicular to the common edge $\overrightarrow{n_a n_b}$. The resulting components are then used for creating the new geodesic direction vector $\overrightarrow{\text{geod}l_2}$. The geometry involved in determining the new direction vector is shown in Figure 3-15.

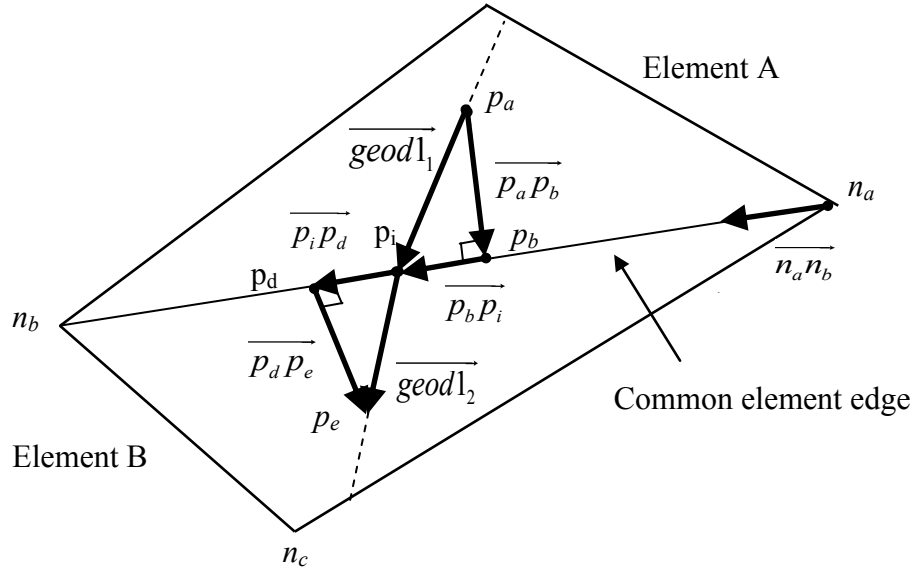


Figure 3-15 Change of Geodesic Direction Vector

Vectors $\overrightarrow{geodl_1}$ and $\overrightarrow{n_a n_b}$ are known unit vectors representing the geodesic direction vector on element A and the direction of the current common element edge vector, respectively. Point $geo[i][j]$, shown here as p_i to simplify vector notation, is the last known intersection point of the geodesic, which was previously determined and fully defined; it is also defined as the end point of $\overrightarrow{geodl_1}$. Vector $\overrightarrow{p_b p_i}$ is the projection of $\overrightarrow{geodl_1}$ onto the common element edge, for which the direction vector is $\overrightarrow{n_a n_b}$. Its direction is identical to the direction of $\overrightarrow{n_a n_b}$ but its magnitude $\|\overrightarrow{p_b p_i}\|$ must be found using the dot product:

$$\|\overrightarrow{p_b p_i}\| = \overrightarrow{geodl_1} \cdot \overrightarrow{n_a n_b} \quad (3.29)$$

$$\overrightarrow{p_b p_i} = \|\overrightarrow{p_b p_i}\| \cdot \overrightarrow{n_a n_b} \quad (3.30)$$

It is possible for $\overrightarrow{n_a n_b}$ to be oriented in the opposite direction as the one depicted in Figure 3-15 due to the fact that nodes n_a and n_b may be defined as pictured or with their positions reversed. In such cases the magnitude of projection $\|\overrightarrow{p_b p_i}\|$ will be negative but vector $\overrightarrow{p_b p_i}$ will remain as seen in Figure 3-15. Point p_b is the start point of $\overrightarrow{p_b p_i}$ and it is easily found:

$$p_b = p_i - \|\overrightarrow{p_b p_i}\| \cdot \overrightarrow{n_a n_b} \quad (3.31)$$

The projection of $\overrightarrow{geodl_1}$ perpendicular to the common element edge, $\overrightarrow{p_a p_b}$, is easily found once point p_a is defined. Point p_a is the start point of vector $\overrightarrow{geodl_1}$, determined by:

$$p_a = p_i - \overrightarrow{geodl_1} \quad (3.32)$$

$$\overrightarrow{p_a p_b} = p_b - p_a \quad (3.33)$$

The magnitude of $\overrightarrow{p_a p_b}$, $\|\overrightarrow{p_a p_b}\|$ is easily computed:

$$\|\overrightarrow{p_a p_b}\| = \sqrt{(p_{bx} - p_{ax})^2 + (p_{by} - p_{ay})^2 + (p_{bz} - p_{az})^2} \quad (3.34)$$

At this stage the initial geodesic direction vector $\overrightarrow{geodl_1}$ has been separated into two components, $\overrightarrow{p_b p_i}$ and $\overrightarrow{p_a p_b}$ with known directions and magnitudes. The new geodesic direction vector $\overrightarrow{geodl_2}$ is the sum of vectors $\overrightarrow{p_i p_d}$ and $\overrightarrow{p_d p_e}$ which have the same magnitude as vectors $\overrightarrow{p_b p_i}$ and $\overrightarrow{p_a p_b}$ respectively. Vector $\overrightarrow{p_i p_d}$ is also in the same direction as vector $\overrightarrow{p_b p_i}$:

$$\overrightarrow{p_i p_d} = \overrightarrow{p_b p_i} \quad (3.35)$$

The direction of vector $\overrightarrow{p_d p_e}$ must be perpendicular to the common element edge while being within the new element's plane, determined by a cross product of the edge vector and element B's normal vector, which was determined upon model pre-processing. Vector $\overrightarrow{p_d p_e}$ is found by multiplying the magnitude of vector $\overrightarrow{p_a p_b}$ with the unit vector obtained via the cross product:

$$\overrightarrow{p_d p_e} = \|\overrightarrow{p_a p_b}\| \frac{(\overrightarrow{v_n} \times \overrightarrow{n_a n_b})}{\|(\overrightarrow{v_n} \times \overrightarrow{n_a n_b})\|} \quad (3.36)$$

The orientation of vector $\overrightarrow{p_d p_e}$ is not necessarily as depicted in Figure 3-15 due to the different possible orientations of the common element edge $\overrightarrow{n_a n_b}$ and of the element plane normal vector $\overrightarrow{v_n}$, which will both affect the orientation of the result of the cross product following the right hand rule. The orientation of vector $\overrightarrow{p_d p_e}$ is checked by solving for the intersection of two lines:

$$\overrightarrow{p_d p_e} \cdot \overrightarrow{q} + n_c = \overrightarrow{n_a n_b} \cdot \overrightarrow{r} + n_a \quad (3.37)$$

where q and r are unknown variables to solve for, and n_c is the node within element B which is not part of the common element edge. The system can be expanded into a system of three equations and solved in a similar way as the system of equations formed by equations (3.19) to (3.24) seen in section 3.4.2.2. It is also necessary to take the same precautions in avoiding errors

caused by floating point arithmetic using equations similar to equations (3.25) to (3.27) using a threshold ε_m of 0.001 instead of 0.0001. Up to three different values of q may be found, the correct selection of which is detailed in section 3.4.2.2. and Figure 3-12. When a value for q is determined the sign of q is examined and the orientation of vector $\overrightarrow{p_d p_e}$ is reversed if q is positive; this ensures that vector $\overrightarrow{p_d p_e}$ is as shown in Figure 3-15. The new direction vector $\overrightarrow{geodl_2}$ can now be determined:

$$\overrightarrow{geodl_2} = \overrightarrow{p_i p_d} + \overrightarrow{p_d p_e} \quad (3.38)$$

The new direction vector is stored to memory and the next geodesic line segment direction vector can be determined following the same procedure as outlined in sections 3.4.2.2 and 3.4.2.3. A flowchart of the entire operation of creating the geodesics is shown in Figure 3-16 and Figure 3-17.

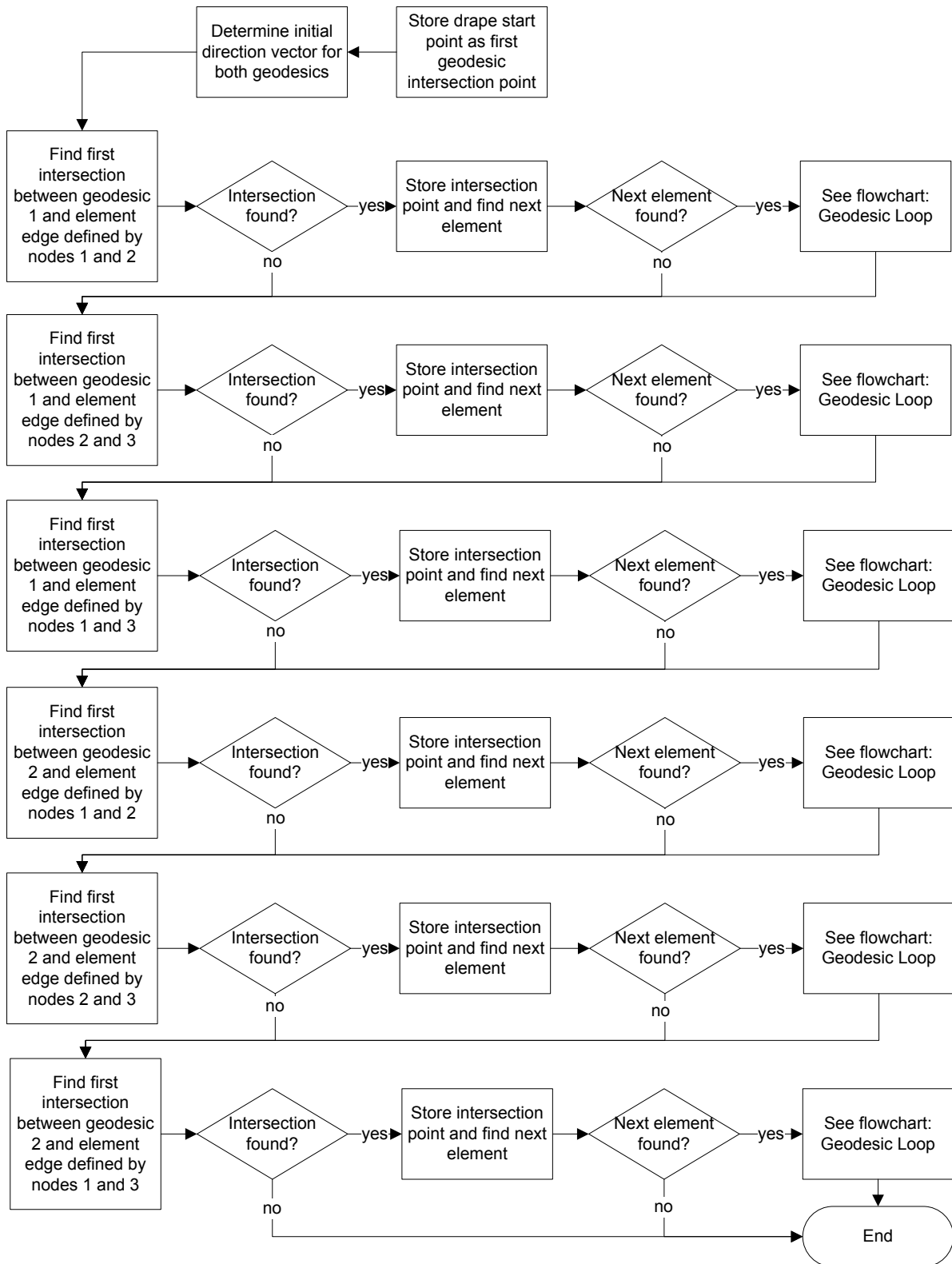


Figure 3-16 Algorithm for Geodesic Generation (Part 1)

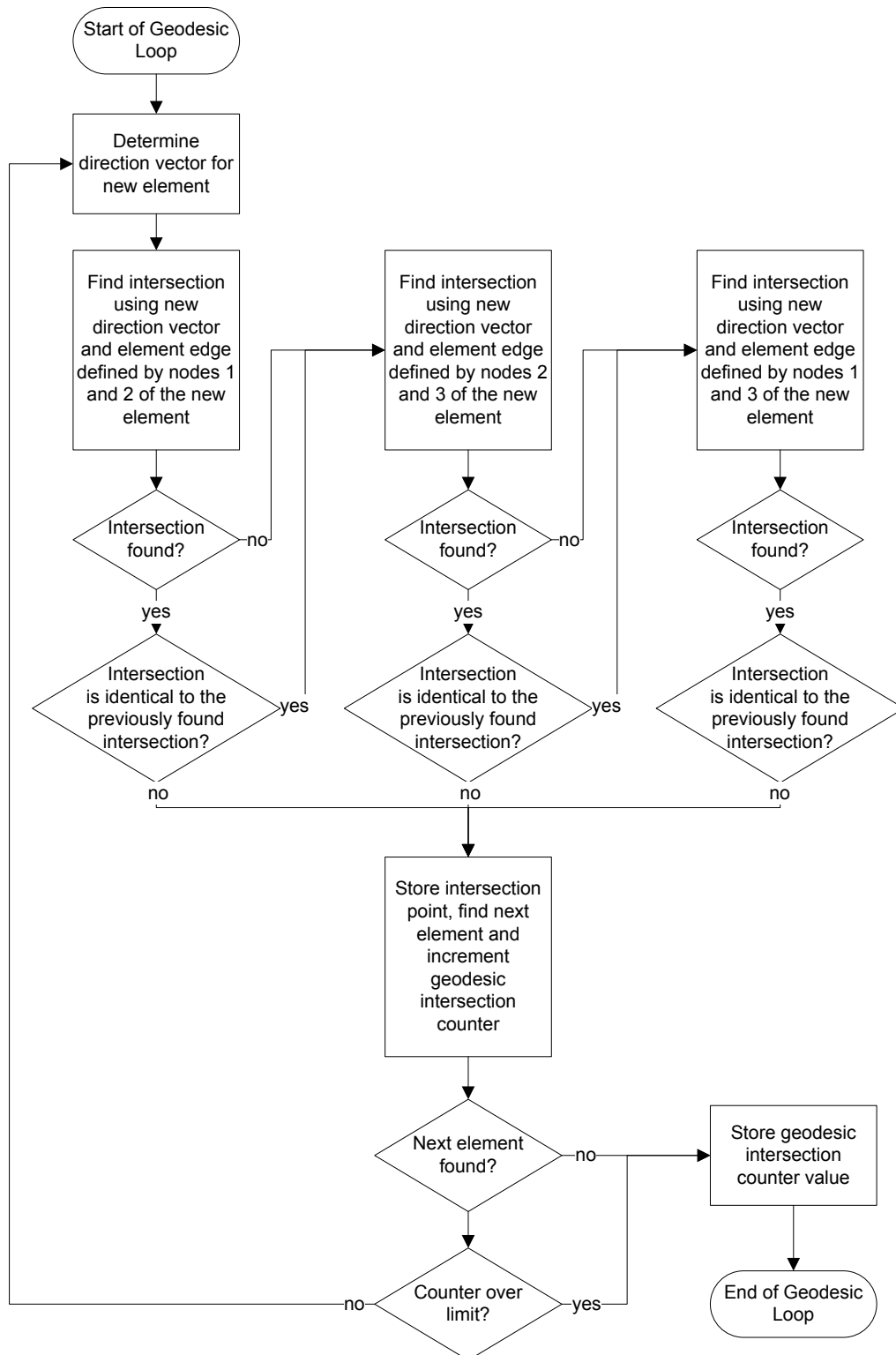


Figure 3-17 Algorithm for Geodesic Generation (Part 2: Geodesic Loop)

3.4.3. Creating the Mesh

With a start point and geodesics defined it is now possible for the software to create the mesh representing the fishnet that drapes the surface of the model. The mesh is a collection of yarn crossovers interconnected by lines that represent individual yarns of the textile reinforcement. The coordinates of the yarn crossovers are stored in an array $fishnet[a][b]$ where a and b are variables used for denoting the location of the crossovers seen as dots in Figure 3-18.

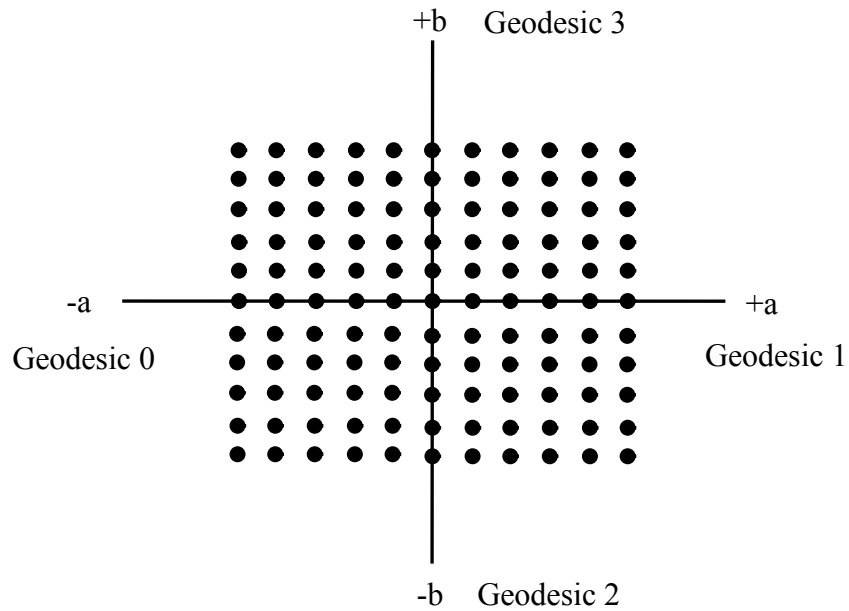


Figure 3-18 Crossover Location Array

Due to the fact that native C++ programming does not allow for negative indices in arrays, the centre of the array is set arbitrarily at $[250,250]$. Therefore, the bottom left of the array is at $[a,b] = [0,0]$ and the top right is at $[a,b] = [501,501]$. The distance between yarn crossovers l_m is a constant value for conventional constant-length textiles. However, this length is not constant when using variable-length textiles; in the latter case the software must also refer to

twin arrays containing the yarn spacings at each crossover. The twin arrays use the same references as the crossover coordinate array, a and b ; one array stores the l_m for the a direction while the other stores the l_m for the b direction. The appropriate value of l_m between yarn crossovers is stored to array using the indices of the yarn crossover furthest from the origin of the matrix [250][250] as shown in Figure 3-19. The arrows point from the yarn spacing to where it is stored in array; the left-hand side of Figure 3-19 represents the values of l_m in the a direction and the right-hand side of Figure 3-19 represents the values of l_m in the b direction.

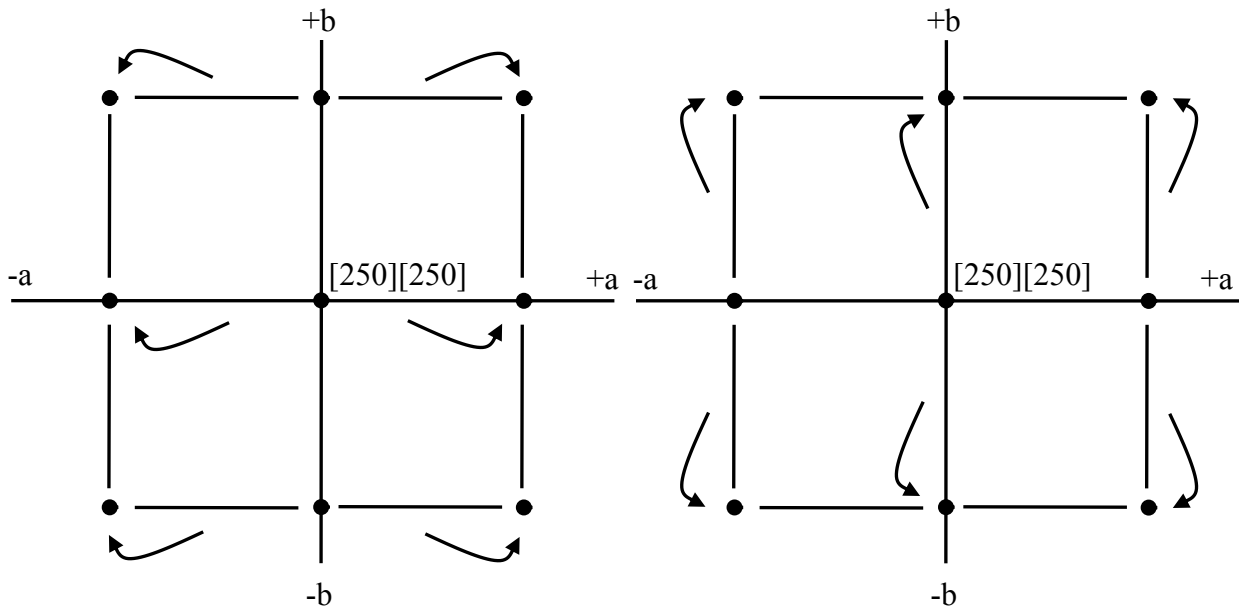


Figure 3-19 Arrays Used for Storing Yarn Spacings

3.4.3.1. Meshing the Geodesics

The first step in creating the mesh is to determine the location of the yarn crossovers on the geodesics, shown as red dots in Figure 3-20. It is important to note that in the example

given it appears that certain geodesics follow a series of element edges; this is a special case based on the model and initial conditions.

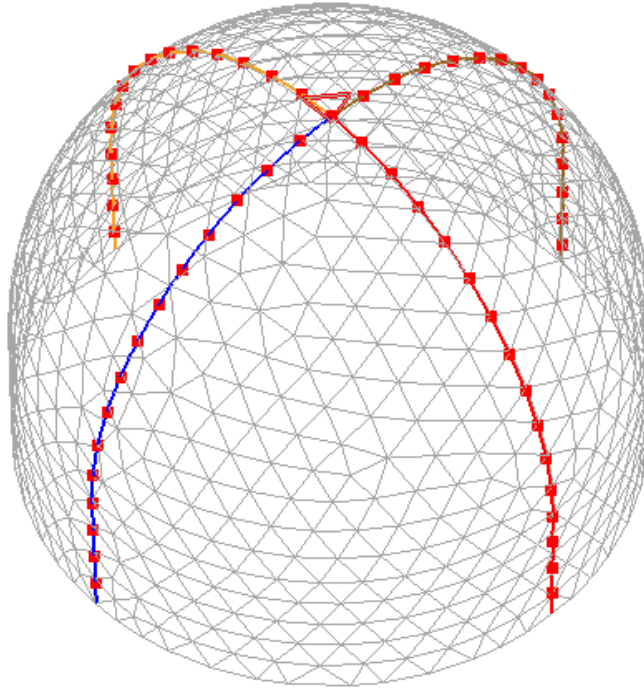


Figure 3-20 Yarn Crossover Points on Geodesics

Yarn crossover points on the geodesics do not normally coincide with the intersections between the geodesics and element edges; however, these intersections are used for calculating the location of the crossovers. The first yarn crossover point *fishnet*[250][250] is already known as it coincides with the start point of the drape. The subsequent crossover points are at a distance equal to l_m away from the start drape point, along a geodesic, as seen in Figure 3-21.

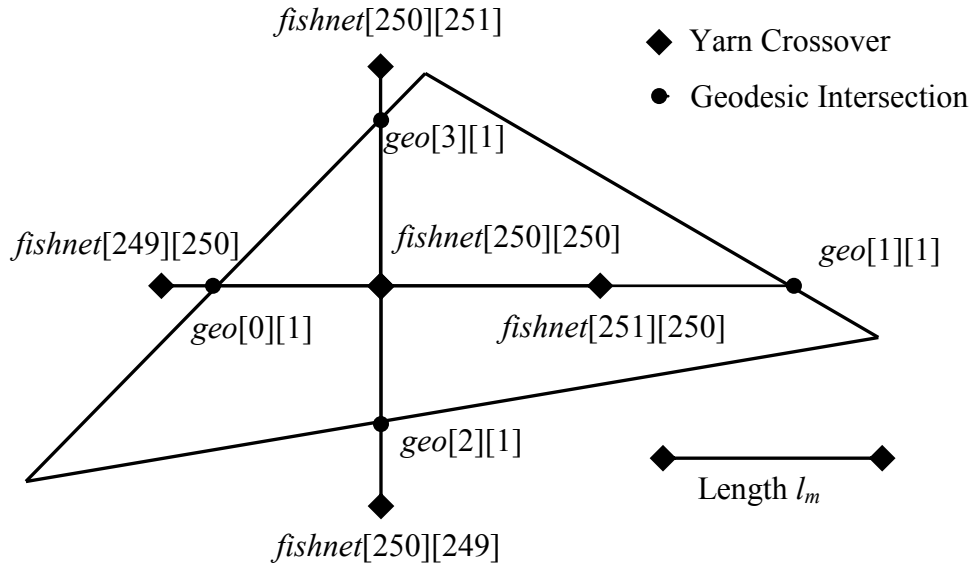


Figure 3-21 Location of Yarn Crossover Points

To find the first yarn crossover along one geodesic, the distance l between the drape start point $geo[i][0]$ and the first intersection point of the geodesic $geo[i][1]$ is calculated:

$$l = \|geo[i][0] - geo[i][1]\| \quad (3.39)$$

where i is any integer from 0 to 3 representing the geodesic. The total length l_t is the sum of the previous value of l_t , l_{t-1} , which is 0 when the previous crossover lies directly on a geodesic line segment intersection point or is the drape start point, and l :

$$l_t = l_{t-1} + l \quad (3.40)$$

The value of l_t may be equal (Figure 3-22), smaller (Figure 3-23) or larger (Figure 3-24) than the value of l_m .


$$fishnet[a][b+1] = geo[i][j+1] \quad (3.41)$$

If l_t is larger than l_m , Figure 3-23, the yarn crossover point will be on the geodesic segment between the two geodesic intersection points.

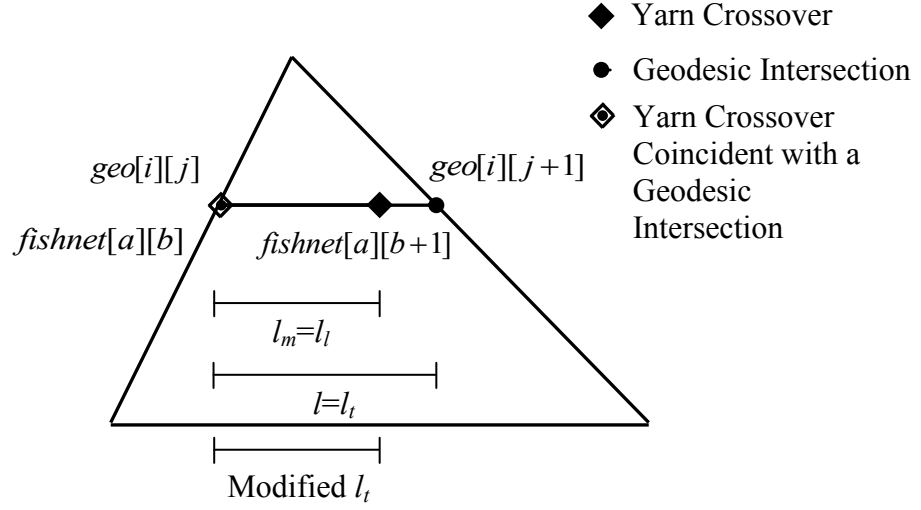


Figure 3-23 Geodesic Crossover Location When $l_m < l_t$

The unit direction vector of the geodesic line segment between the two intersections is found:

$$\vec{v}_g = \frac{o[i][j] - geo[i][j+1]}{\|geo[i][j] - geo[i][j+1]\|} \quad (3.42)$$

The crossover point is then calculated:

$$fishnet[a][b+1] = \vec{v}_g \cdot (l_m - (l_t - l)) + geo[i][j] \quad (3.43)$$

The flag indicating that the yarn crossover is known is set to 1 (true) and the value of l_t is modified as it will be used in equation (3.40) to find the value of l_t for the next crossover:

$$l_t = -(l_m - (l_t - l)) \quad (3.44)$$

As per equation (3.44) the length l_t becomes a negative value used for shortening the total length for the next crossover point so that it denotes only the length available to the next crossover point on the geodesic line segment. The relationship between l , l_t and l_{t-1} described by equation (3.40) can be seen in Figure 3-24 as the value of l is shortened by l_{t-1} to form the length of l_t . If l_t is larger than l_m the yarn crossover point does not lie on the geodesic line segment between intersections $geo[i][j]$ and $geo[i][j+1]$, as seen in Figure 3-24.

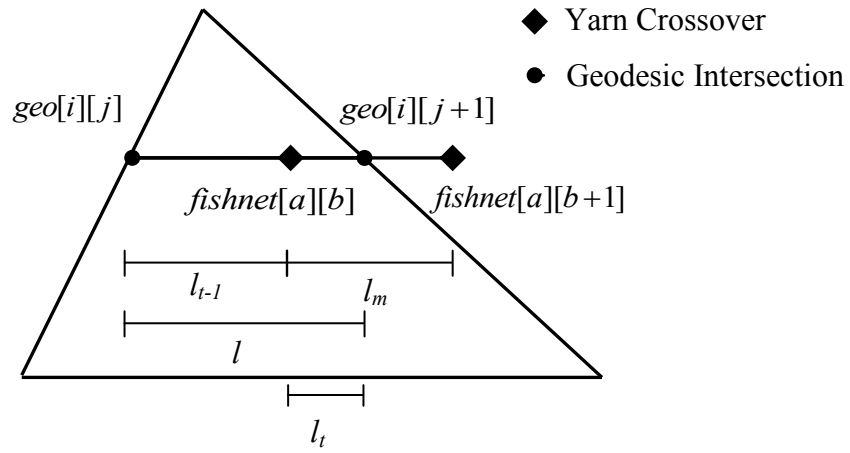


Figure 3-24 Geodesic Crossover Location When $l_m > l_t$

When this situation occurs j is incremented by 1, l_t is left unmodified and new l and l_t are found using equations (3.39) and (3.40). The complete algorithm is shown in Figure 3-25.

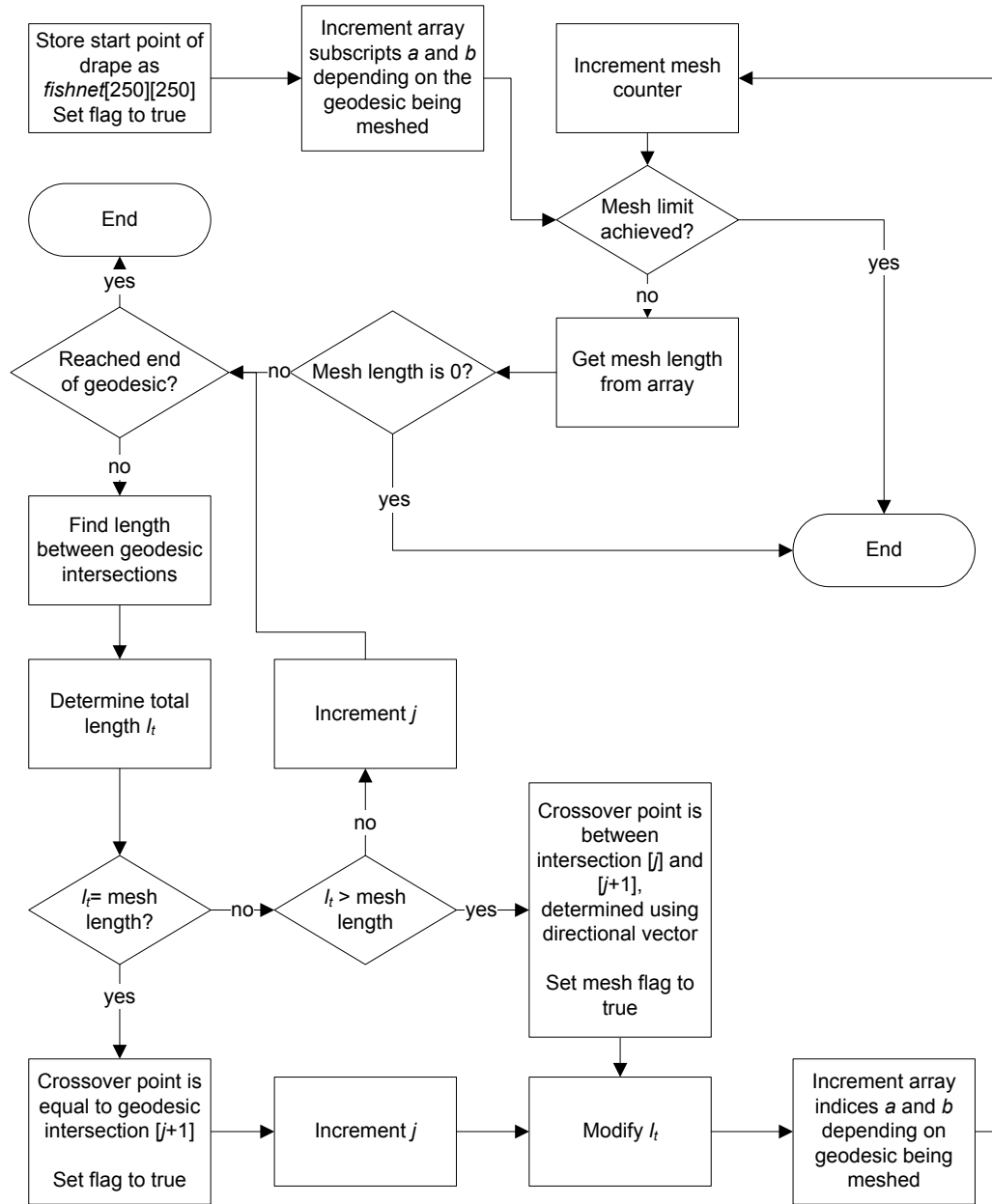


Figure 3-25 Algorithm to Mesh Geodesics

The algorithm continues until any one of four end conditions is met: 1) when the end of the geodesic is reached, determined when the value of j exceeds the number of intersections within the geodesic being meshed, determined in section 3.4.2.3; 2) when the user-defined

amount of yarn crossovers to mesh is reached; 3) when the limits of the *fishnet* array are reached; or 4) when a value for the yarn spacing is loaded from array as 0.

3.4.3.2. ***Meshing Between Geodesics***

Once all four geodesics are meshed it is possible to determine the remaining yarn crossovers, more specifically, crossovers of yarns which are not geodesics. The mesh is split into four quadrants as seen in Figure 3-26, which are fully independent from each other and can be meshed, or draped, separately.

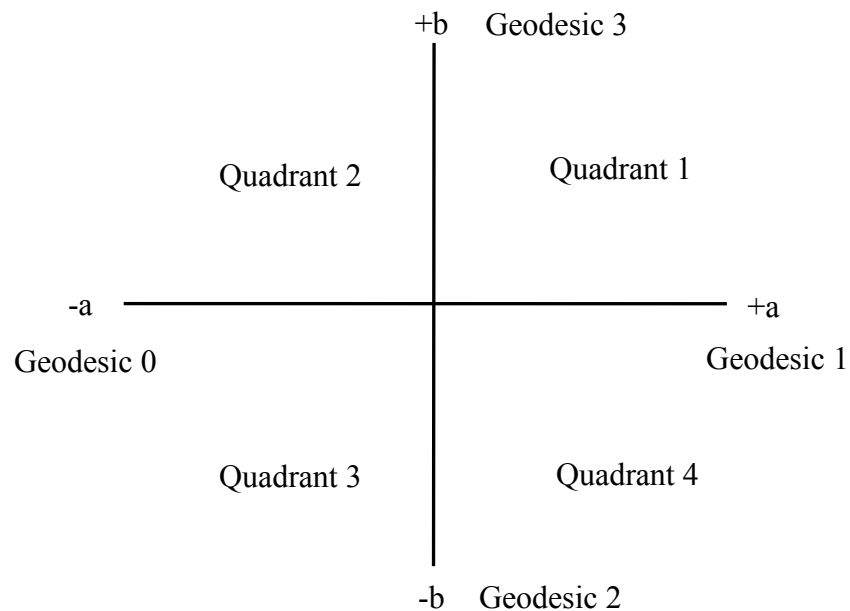


Figure 3-26 Location of Mesh Quadrants

The locations of yarn crossovers that are not on geodesics are dependent on the position of three yarn crossovers, two yarn spacings, and the geometry of the model being draped, as

depicted in Figure 3-27 and Figure 3-28. For the purpose of simplicity, only quadrant 1 will be discussed in detail since the algorithms for each quadrant are similar.

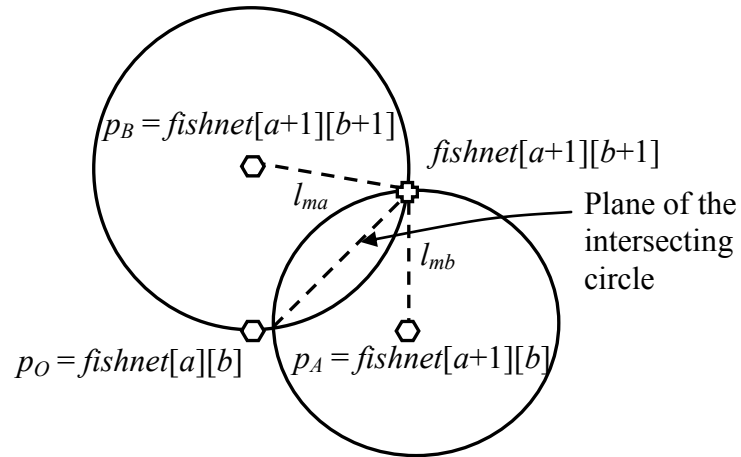


Figure 3-27 Geometric Relations for Meshing

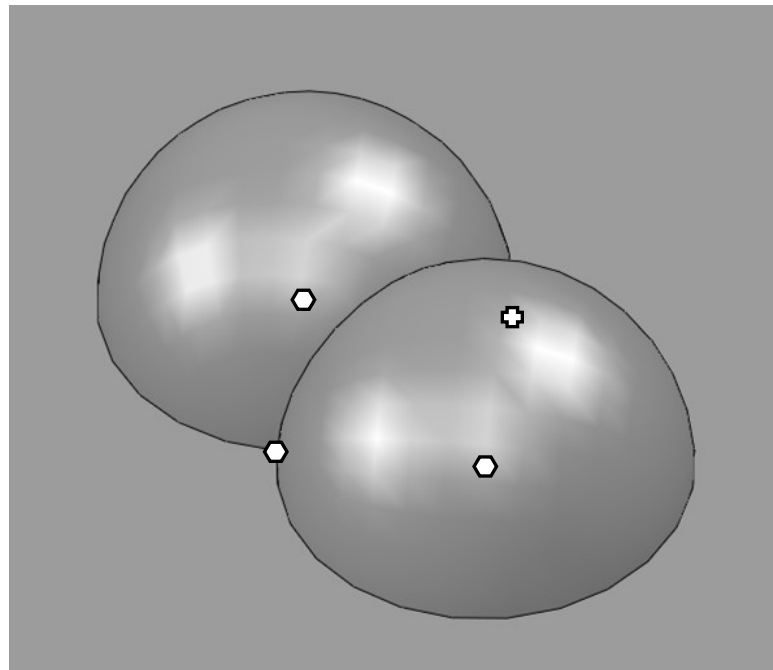


Figure 3-28 3D Representation of the Geometric Relations for Meshing

The three known yarn crossover points are shown as hexagons while the yarn crossover point to solve for is shown as a cross. If the yarn crossover to solve for is $fishnet[a+1][b+1]$ then the three known yarn crossovers required are $fishnet[a][b]$, $fishnet[a][b+1]$ and $fishnet[a+1][b]$, renamed p_o , p_b and p_a , respectively, for simplicity in notation. If one of the three crossovers is undetermined the flag indicating whether it has been solved is false and the location of the yarn crossover $fishnet[a+1][b+1]$ cannot be determined. When all three dependant yarn crossover points are defined the unknown crossover point is determined by finding the intersection point of two spheres, shown in Figure 3-28 and depicted as circles in Figure 3-27, and the model surface. There are two such possible points and only one is the correct location for the yarn crossover; for constant-length textiles the incorrect point is p_o .

3.4.3.2.1. Intersection of the Two Spheres

The intersection of the two spheres can be determined before including the surface into the equation. The intersection of two spheres is a circle, shown in Figure 3-29, and is found by solving for the two equations of the spheres in 3D space:

$$(x - p_{Ax})^2 + (y - p_{Ay})^2 + (z - p_{Az})^2 = l_{mb}^2 \quad (3.45)$$

$$(x - p_{Bx})^2 + (y - p_{By})^2 + (z - p_{Bz})^2 = l_{ma}^2 \quad (3.46)$$

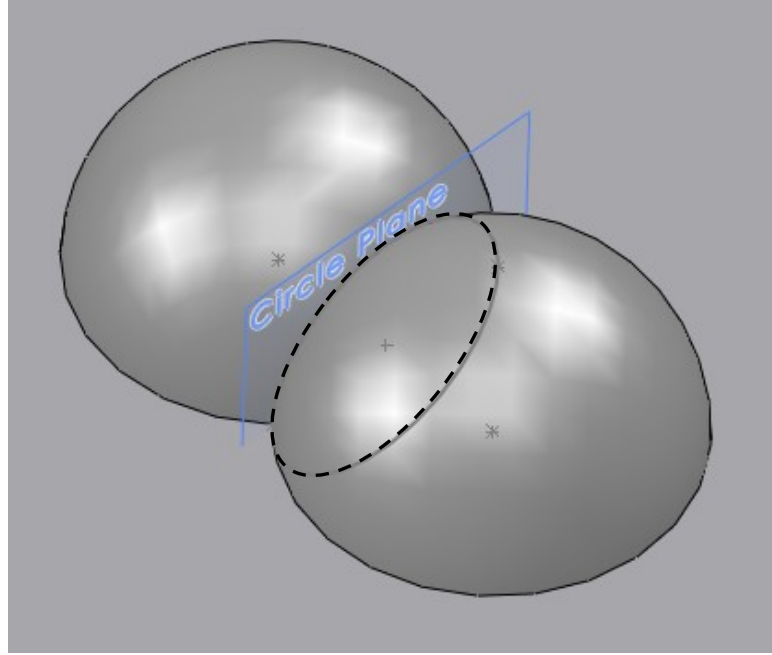


Figure 3-29 Circle of Intersection of Two Spheres

In order to accommodate the draping of both conventional and variable-length textiles, the yarn spacing l_m is separated into the a direction and b direction yarn spacings, l_{ma} and l_{mb} , respectively. Expanding both equations (3.45) and (3.46) and subtracting the expanded form of equation (3.46) from the expanded form of equation (3.45) leads to:

$$(2p_{Bx} - 2p_{Ax})x + (2p_{By} - 2p_{Ay})y + (2p_{Bz} - 2p_{Az})z = l_{mb}^2 - l_{ma}^2 + p_{Bx}^2 - p_{Ax}^2 + p_{By}^2 - p_{Ay}^2 + p_{Bz}^2 - p_{Az}^2 \quad (3.47)$$

which is the equation of the plane in which the circle of intersection between the two spheres is defined. For the remainder of this thesis, the circle of intersection between the two spheres will be referred to as: "the circle of intersection" and the plane of this circle will be referred to as: "the circle plane". From the definition of the equation of a plane:

$$c_{nx}x + c_{ny}y + c_{nz}z = c_d \quad (3.48)$$

the vector normal to the plane of the circle $\vec{c}_n$ can be found and made a unit vector:

$$\vec{c}_n = \frac{\vec{r}_B - \vec{p}_A}{\|\vec{p}_B - \vec{p}_A\|} \quad (3.49)$$

The right-hand side of equation (3.48), noted as c_d , must be divided by the magnitude of vector $\vec{c}_n$, as was done to the left-hand side of equation (3.48):

$$c_d = \frac{l_{mb}^2 - l_{ma}^2 + p_{Bx}^2 - p_{Ax}^2 + p_{By}^2 - p_{Ay}^2 + p_{Bz}^2 - p_{Az}^2}{\|\vec{p}_B - \vec{p}_A\|} \quad (3.50)$$

The centre and the radius of the circle of intersection still need to be determined. From Figure 3-30, the centre of the circle c_c is at a certain distance from point p_A along vector $\vec{c}_n$:

$$\vec{c}_c = \vec{p}_A + t\vec{c}_n \quad (3.51)$$

where t is a variable used to scale vector $\vec{c}_n$.

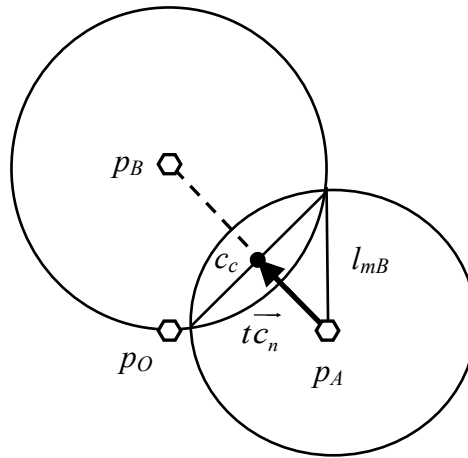


Figure 3-30 Determining the Centre and Radius of the Circle of Intersection

Substituting the components of c_c for the values of x , y and z in equation (3.48):

$$c_{nx}(p_{Ax} + t \cdot c_{nx}) + c_{ny}(p_{Ay} + t \cdot c_{ny}) + c_{nz}(p_{Az} + t \cdot c_{nz}) = c_d \quad (3.52)$$

and solving for t leads to:

$$t = \frac{c_d - c_{nx}p_{Ax} - c_{ny}p_{Ay} - c_{nz}p_{Az}}{c_{nx}^2 + c_{ny}^2 + c_{nz}^2} \quad (3.53)$$

The radius of the circle of intersection is determined by using a line between point p_A and one of the intersection points between the two spheres as seen in Figure 3-30. The length of this line is known, l_{mb} , and the resulting triangle can be solved using Pythagoras' theorem, thus the radius of the circle c_r is:

$$c_r = \sqrt{l_{mb}^2 - (t \cdot c_n)^2} \quad (3.54)$$

3.4.3.2.2. Definition of the Brick Space

It is now possible to determine the point of intersection between the circle of intersection of the two spheres and the model surface. This is done by finding the line of intersection between the circle plane and an element plane, referred to for the remainder of this thesis as: "the line of intersection". Since the sought coordinates of the yarn crossover are unknown at this stage, the element on which the crossover point lies is also unknown. The software must iterate through the elements of the model and determine the intersection of the circle with each element plane. Since the element list can be quite large, only a subset of elements which lie, in part or in whole, in a region called the brick space and defined around the circle of intersection are included in the

iteration. The use of the brick space decreases the time needed for the software to simulate the draping of a textile considerably. The brick space, shown in Figure 3-31, is a cube centered around the centre of the circle of intersection with length, width and height 10% larger than the radius of the circle, $1.1c_r$.

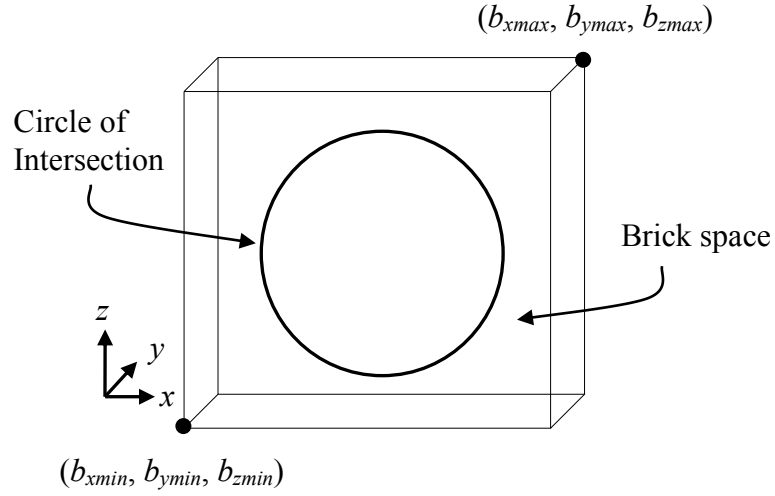


Figure 3-31 Definition of the Brick Space

The brick space is defined by maximum and minimum values for x , y and z , noted as b_{xmax} , b_{xmin} , b_{ymax} , b_{ymin} , b_{zmax} and b_{zmin} found with:

$$b_{xmax} = c_{cx} + 1.1c_r \quad (3.55)$$

$$b_{xmin} = c_{cx} - 1.1c_r \quad (3.56)$$

$$b_{ymax} = c_{cy} + 1.1c_r \quad (3.57)$$

$$b_{ymin} = c_{cy} - 1.1c_r \quad (3.58)$$

$$b_{zmax} = c_{cz} + 1.1c_r \quad (3.59)$$

$$b_{zmin} = c_{cz} - 1.1c_r \quad (3.60)$$

Any given element lies within the brick space if it fulfills a minimum of one requirement for each direction x , y and z . Nine requirements exist for each direction; it is not necessary that the same requirement is met for x , y and z . The requirements for the x direction are given by:

$$b_{x\min} \leq n_{1x} \leq b_{x\max} \quad (3.61)$$

$$b_{x\min} \leq n_{2x} \leq b_{x\max} \quad (3.62)$$

$$b_{x\min} \leq n_{3x} \leq b_{x\max} \quad (3.63)$$

$$n_{1x} \leq b_{x\min} \text{ and } n_{2x} \geq b_{x\max} \quad (3.64)$$

$$n_{2x} \leq b_{x\min} \text{ and } n_{1x} \geq b_{x\max} \quad (3.65)$$

$$n_{1x} \leq b_{x\min} \text{ and } n_{3x} \geq b_{x\max} \quad (3.66)$$

$$n_{3x} \leq b_{x\min} \text{ and } n_{1x} \geq b_{x\max} \quad (3.67)$$

$$n_{2x} \leq b_{x\min} \text{ and } n_{3x} \geq b_{x\max} \quad (3.68)$$

$$n_{3x} \leq b_{x\min} \text{ and } n_{2x} \geq b_{x\max} \quad (3.69)$$

The requirements for y and z are identical to the requirements for x but expressed in terms of y and z components respectively. Equations (3.61) to (3.63) ensure that elements represented in the left-hand side of Figure 3-32 are included in the brick space while equations (3.64) to (3.69) ensure that elements represented in the right-hand side of Figure 3-32 are included in the brick space.

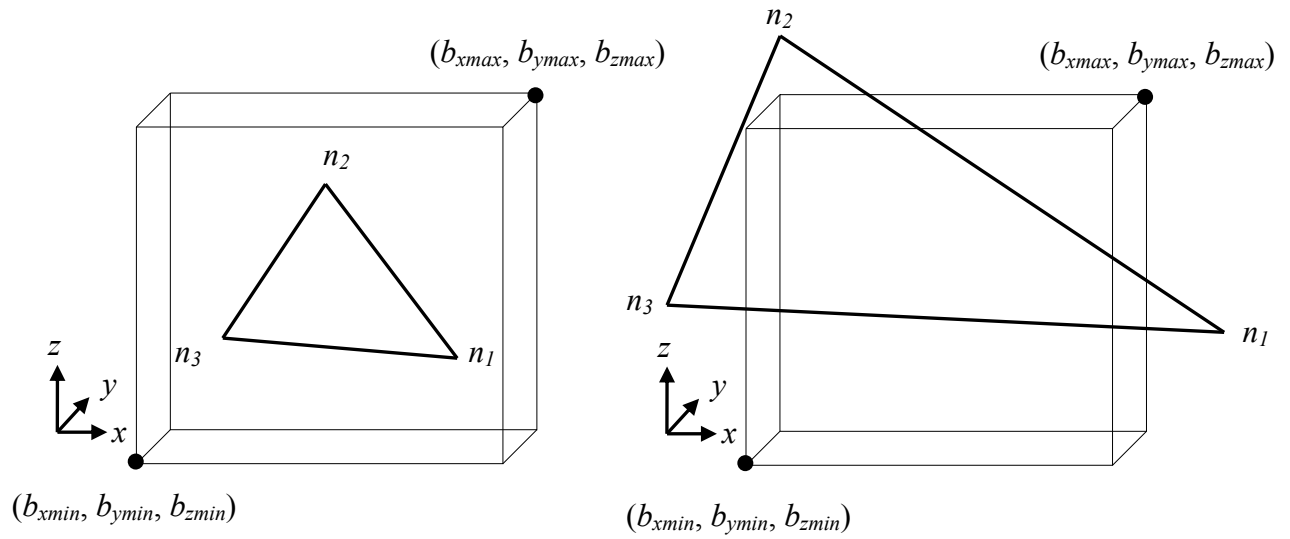


Figure 3-32 Inclusion of Different Elements in the Brick Space

3.4.3.2.3. *Intersection of the Circle Plane with an Element Plane*

Once the list of elements within the brick space is compiled it is possible to iterate through the list to find the line of intersection, shown as a dashed line in Figure 3-33, between the plane of an element and the plane of the circle of intersection. The element is represented by solid lines in Figure 3-33.

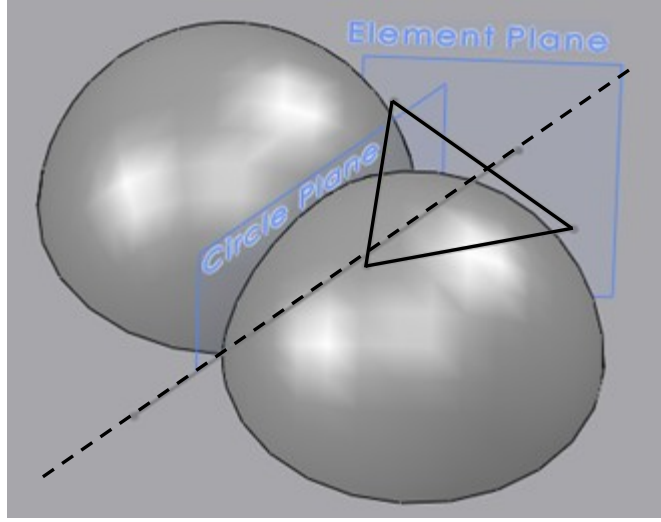


Figure 3-33 Intersection of the Circle Plane with an Element Plane

The equation of the element plane is:

$$e_{nx}x + e_{ny}y + e_{nz}z = e_d \quad (3.70)$$

where e_{nx} , e_{ny} and e_{nz} are the components of the vector normal to the element plane $\vec{e}_n$, determined earlier in pre-processing. The value of e_d is determined via the dot product of $\vec{e}_n$ and a point on the element plane, n_l :

$$e_d = e_{nx}n_{lx} + e_{ny}n_{ly} + e_{nz}n_{lz} \quad (3.71)$$

Finally, the intersection of the planes can be found as both equations are fully defined.

The intersection of the planes is a line defined by [69]:

$$(x, y, z) = c_1 \vec{c}_n + c_2 \vec{e}_n + u (\vec{c}_n \times \vec{e}_n) \quad (3.72)$$

where c_1 and c_2 are constants that need to be determined, and u is a parametric variable. Taking the dot product of equation (3.72) with the vectors $\vec{c}_n$ and $\vec{e}_n$ yields [69]:

$$\vec{c}_n(x, y, z) = c_1 \vec{c}_n \cdot \vec{c}_n + c_2 \vec{c}_n \cdot \vec{e}_n \quad (3.73)$$

$$\vec{e}_n(x, y, z) = c_1 \vec{c}_n \cdot \vec{e}_n + c_2 \vec{e}_n \cdot \vec{e}_n \quad (3.74)$$

The left-hand side of equations (3.73) and (3.74) is substituted by c_d and e_d , respectively. Solving for c_1 and c_2 yields [69]:

$$c_1 = \frac{c_d \vec{e}_n \cdot \vec{e}_n - e_d \vec{c}_n \cdot \vec{e}_n}{(\vec{c}_n \cdot \vec{c}_n)(\vec{e}_n \cdot \vec{e}_n) - (\vec{c}_n \cdot \vec{e}_n)^2} \quad (3.75)$$

$$c_2 = \frac{e_d \vec{c}_n \cdot \vec{c}_n - c_d \vec{c}_n \cdot \vec{e}_n}{(\vec{c}_n \cdot \vec{c}_n)(\vec{e}_n \cdot \vec{e}_n) - (\vec{c}_n \cdot \vec{e}_n)^2} \quad (3.76)$$

A point on the line can be found by substituting the values c_1 and c_2 into equation (3.72) and setting u as 1:

$$p_l = c_1 \vec{c}_n + c_2 \vec{e}_n + (\vec{c}_n \times \vec{e}_n) \quad (3.77)$$

This point, as well as the direction vector $\vec{c}_n \times \vec{e}_n$ of the line of intersection, are used to determine if the line intersects the element or passes next to it.

3.4.3.2.4. *Intersection of the Line of Intersection and Element Edge*

The element shown in Figure 3-33 lies in the path of the line of intersection between the circle plane and element plane but this is not always the case. The line of intersection may or may not intersect the element, as shown in Figure 3-34.

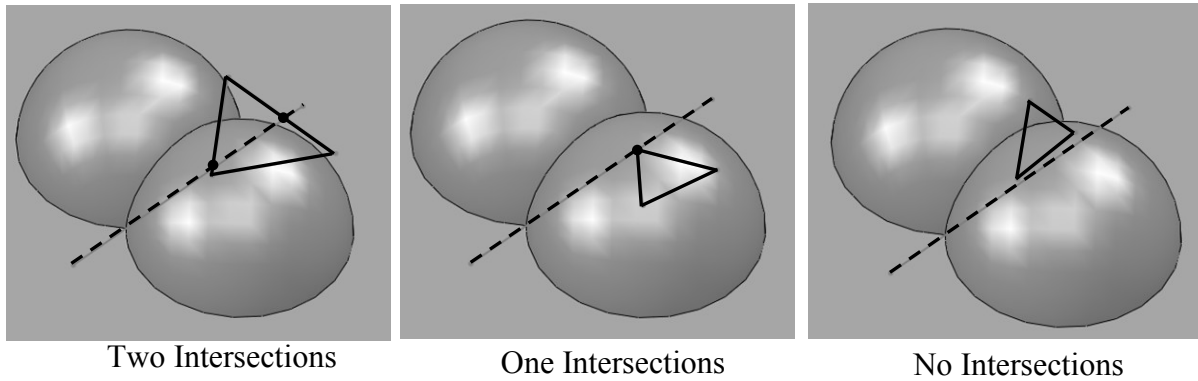


Figure 3-34 Lines of Intersection Which Do and Do Not Intersect an Element

To determine how many intersections exist the software must look for an intersection between the line of intersection and each of the three element edges, in a manner similar to what was done in section 3.4.2.2 by solving the equations of the two lines, using the edge between nodes n_1 and n_2 as an example:

$$\overrightarrow{n_1 + n_2} \cdot \vec{v}_i = p_l + (\overrightarrow{c_n} \times \overrightarrow{e_n}) \cdot \vec{w}_i \quad (3.78)$$

The same precautions are taken to limit errors caused by floating point arithmetic and divisions by zero as were taken in section 3.4.2.2. The value of v_i is found using the same algorithm detailed in Figure 3-12. If the value of v_i is between 0 and 1 inclusively the point of

intersection lies on the element edge; the point of intersection $bound_1$ between the line of intersection and the element edge between nodes n_1 and n_2 is given by:

$$bound_1 = n_1 + \overrightarrow{n_1 n_2} \cdot v_i \quad (3.79)$$

Should a second point of intersection be found, on a different element edge, it is labelled as $bound_2$. The intersections represent the boundaries on the element between which the coordinates for the yarn crossover point may be. However, the correct solution for the yarn crossover location will not necessarily be on or within the boundaries of an element for which boundary values were found, as seen in Figure 3-35.

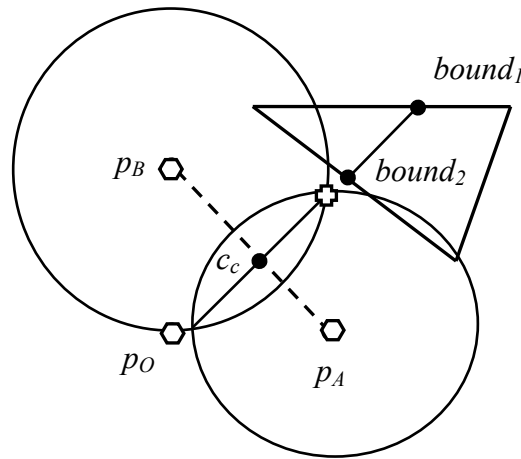


Figure 3-35 Crossover Location Not Within the Bounds of an Element

For this reason, a criterion must be developed for comparing the various possible solutions which arise from the intersections between the line of intersection and element edges determined during the iteration of the brick space. The criterion is dependent on the number of boundaries which was found for a given element and circle plane intersection.

No intersections found: when no intersections are found between the line of intersection and any of the element edges, the location of the yarn crossover does not lie on the given element and the software does not store the location of a possible solution for this element.

One intersections found: when only one point of intersection between the line of intersection and the element edges is found, the resulting possible solution for the location of the yarn crossover is the intersection itself, $bound_I$. However, this point of intersection, shown as a dot in Figure 3-36, is not necessarily the position of the true yarn crossover location, shown as a cross.

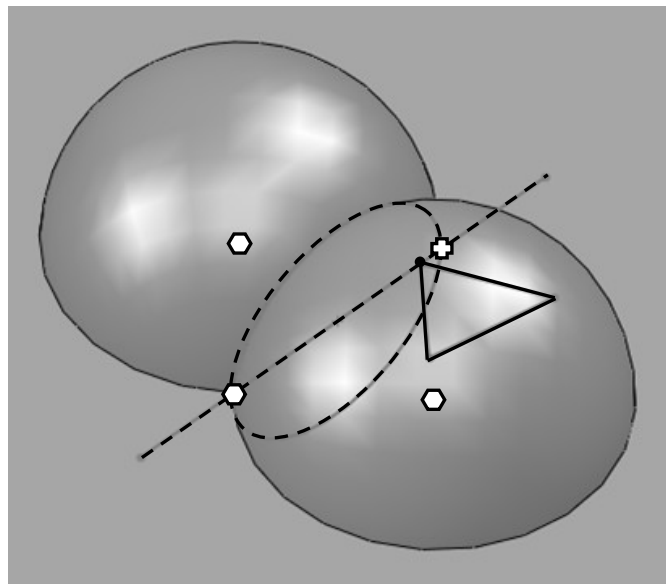


Figure 3-36 One Intersection Between the Line of Intersection and the Element Edges

The distance t_r between $bound_1$ and the centre c_c of the circle of intersection between the two spheres centered on p_A and p_B is used for determining the criterion used for comparing the possible solutions:

$$t_r = \|c_c - bound_1\| \quad (3.80)$$

To remove possible solutions which are not suitable for use as the yarn crossover location, the test radius t_r and the actual radius c_r must be within an allowable range:

$$\frac{|t_r - c_r|}{c_r} < 0.001 \quad (3.81)$$

The value of 0.001 was determined by running several simulations and selecting the strictest allowance which would not lead to the correct yarn crossover position being discarded. The result of the left-hand side of equation (3.81), used as the value for the criterion for determining the best solution, is stored along with the coordinates of the possible solution.

Two intersections found: when two intersections between the line of intersection and the element edges are found, up to two possible solutions for the yarn crossover location may exist for this element; these possible solutions do not lie at the location of the intersections, $bound_1$ and $bound_2$. The possible solutions, shown as lozenges in Figure 3-37, lie at the intersection(s) between the circle of intersection and the line defined by the two boundaries, which is identical to the line of intersection between the circle plane and the element plane.

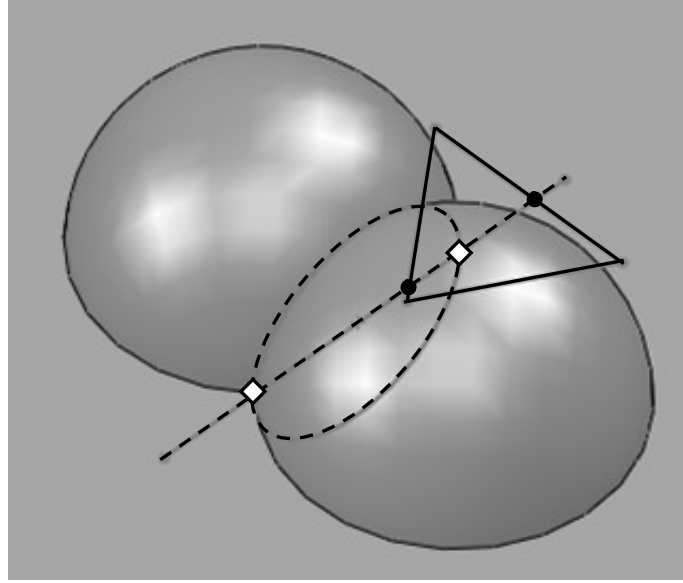


Figure 3-37 Intersection of the Circle of Intersection and the Line of Intersection

The parametric equation of the line defined by $bound_1$ and $bound_2$ is:

$$(x, y, z) = bound_1 + \mu(bound_2 - bound_1) \quad (3.82)$$

where μ is a parametric variable. The equation of a sphere with the same centre and radius as the circle of intersection is used:

$$(x - c_{cx})^2 + (y - c_{cy})^2 + (z - c_{cz})^2 = c_r^2 \quad (3.83)$$

A sphere with the same radius and centre as the circle of intersection is used instead of the circle of intersection. Using a sphere instead of a circle will lead to the same result as the circle and the line defined by the boundaries are coplanar, as seen in Figure 3-35 and Figure 3-37. Substituting equation (3.82) in equation (3.83) yields [70]:

$$a\mu^2 + b\mu + c = 0 \quad (3.84)$$

were a , b and c are found by [70]:

$$a = (bound_{2x} - bound_{1x})^2 + (bound_{2y} - bound_{1y})^2 + (bound_{2z} - bound_{1z})^2 \quad (3.85)$$

$$b = 2[(bound_{2x} - bound_{1x})(bound_{1x} - c_{cx}) + (bound_{2y} - bound_{1y})(bound_{1y} - c_{cy}) + (bound_{2z} - bound_{1z})(bound_{1z} - c_{cz})] \quad (3.86)$$

$$c = c_{cx}^2 + c_{cy}^2 + c_{cz}^2 + bound_{1x}^2 + bound_{1y}^2 + bound_{1z}^2 - 2(c_{cx}bound_{1x} + c_{cy}bound_{1y} + c_{cz}bound_{1z}) - c_r^2 \quad (3.87)$$

The determinant of equation (3.84) is used for determining whether the sphere and line intersect in 0, 1 or 2 points, as seen in Figure 3-38, which will yield 0, 1 or 2 possible solutions, respectively.

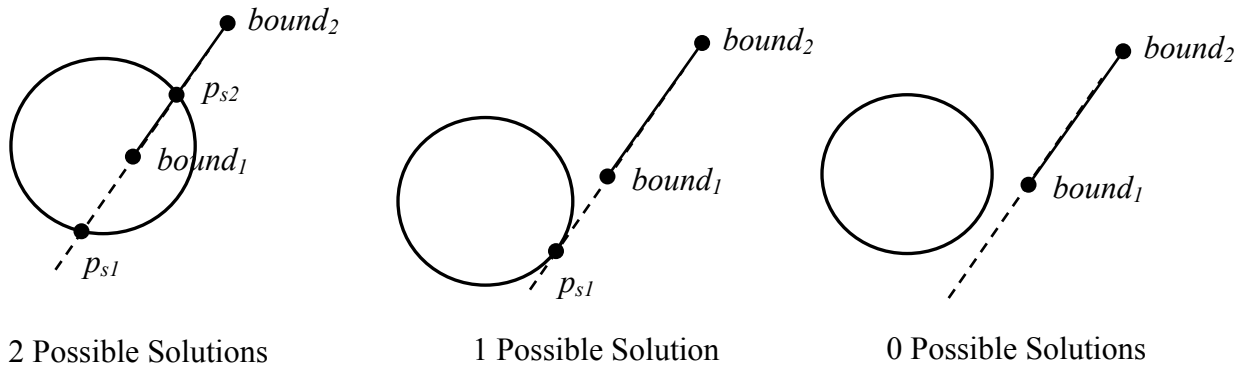


Figure 3-38 Possible Intersections Between a Sphere and a Line

The determinant is given by:

$$\det = b^2 - 4ac \quad (3.88)$$

If the value of the determinant is under 0, equal to 0 or above 0 there will be no, one or two possible solutions, respectively. When possible solutions exist their location is based on the value(s) of μ , found by:

$$\mu_{1,2} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} \quad (3.89)$$

When one possible solution exists, both values of μ are identical and the possible solution p_{sI} is found with:

$$p_{sI} = bound_1 + \mu(bound_2 - bound_1) \quad (3.90)$$

The distance between p_{sI} and the centre of the circle c_c is compared to c_r as in equations (3.80) and (3.81). If the distance between p_{sI} and c_c is not in the allowable range, the possible solution is discarded. If two possible solutions exist, two distinct values of μ exist, found by equation (3.89). The coordinates of the possible solution based on μ_1 are found by:

$$p_{sI} = bound_1 + \mu_1(bound_2 - bound_1) \quad (3.91)$$

Once again the distance between p_{sI} and c_c must be checked against the radius of the circle using equations (3.80) and (3.81). If the distance is in the allowable range p_{sI} is kept as a possible solution. It is possible that the value of p_{sI} lies outside of the element boundaries, such as p_{sI} at the left of Figure 3-38 or the left most possible solution (lozenge) in Figure 3-37. Possible solutions that do not lie within the boundaries do not necessarily lie on the surface of the model. Therefore, it is necessary to adjust the criterion used for determining the best solution for these possible solutions. The value of μ_1 and μ_2 determine if p_{sI} and p_{s2} are within the

boundaries or not, respectively. For values of μ_1 or μ_2 above 1, the distance from the bound d_b is (using μ_1 for demonstration):

$$d_b = \|[bound_1 + \mu_1(bound_2 - bound_1)] - bound_2\| \quad (3.92)$$

For values of μ_1 or μ_2 under 0, d_b is:

$$d_b = \|[bound_1 + \mu_1(bound_2 - bound_1)] - bound_1\| \quad (3.93)$$

For values of μ_1 or μ_2 between 0 and 1 inclusively, d_b is set to 0. The criterion used for determining the best solution is the sum of d_b and the result of the left-hand side of equation (3.81).

3.4.3.2.5. *Selecting the Best Among Possible Solutions*

Once the processes of finding the line of intersection between each element in the brick space and the circle plane, section 3.4.3.2.3, and of determining the intersection of this line of intersection and the elements edges, section 3.4.3.2.4, is completed, the software must determine the best solution from the list of possible solutions which was compiled in the previous sections. The best solution is determined by iterating through the list of possible solutions and selecting the possible solution which has the lowest value for the comparison criterion, and which does not create a concave drape element. It is possible, using variable-length textiles, that the possible solution for a yarn crossover point leads to a concave drape element as shown on the right-hand side of Figure 3-39. The drape element is defined as the polygon created by points p_O , p_A , p_B and the yarn crossover to be found.

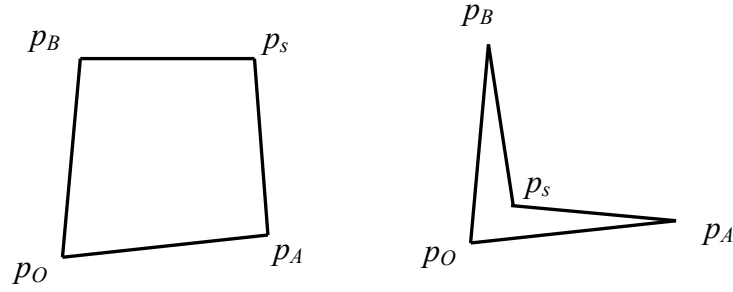


Figure 3-39 Convex and Concave Drape Elements

Concave drape elements are unacceptable for use in draping simulations as they would induce very large deformations within the yarns, deformations which are impossible for the yarns to conform to. The convexity check also ensures that the point p_O is not selected as the yarn crossover position, as it will be one of the possible solutions for constant-length textiles, and may be a possible solution for variable-length textiles. To determine if a drape element is concave or convex, it is separated into two triangles as shown in Figure 3-40.

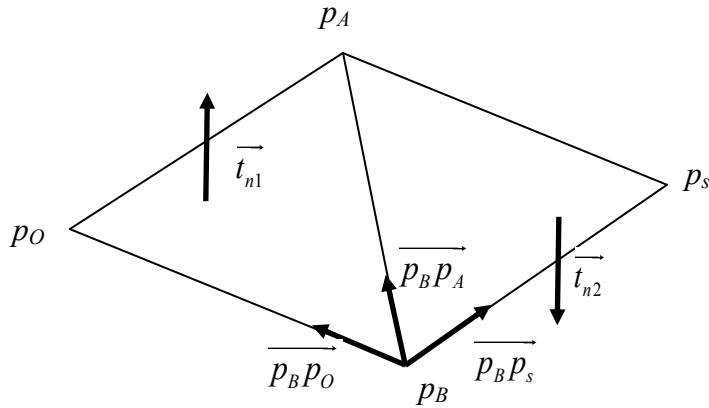


Figure 3-40 Geometry for the Convexity Check

The vectors $\vec{t}_{n1}$ and $\vec{t}_{n2}$ normal to the planes of the triangles are found with:

$$\vec{t}_{n1} = \vec{p_B p_A} \times \vec{p_B p_O} \quad (3.94)$$

$$\vec{t}_{n1} = \vec{p_B p_A} \times \vec{p_B p_s} \quad (3.95)$$

The vectors normal to the planes of the triangles point in opposite directions if the triangles are coplanar, yielding an angle of 180° between $\vec{t}_{n1}$ and $\vec{t}_{n2}$. This angle θ_c is determined by:

$$\theta_c = \arccos \left(\frac{\vec{t}_{n1} \cdot \vec{t}_{n2}}{\|\vec{t}_{n1}\| \cdot \|\vec{t}_{n2}\|} \right) \quad (3.96)$$

To limit computational errors within the software, the computation of the arccosine of any value below -0.98 is bypassed, the result of which is set to 180° . Similarly, the computation of the arccosine of any value above 0.98 is bypassed, the result of which is set to 0° . The computation of arccosine is bypassed for values nearing 1 and -1 due to the possibility of floating point arithmetic errors modifying a number such as 0.99 to 1.01. The arccosine of values equal to or above 1 and values equal to or below -1 is undefined, causing the software to issue an error. A drape element with a value of θ_c between 60° and 180° is considered convex; the two limit conditions are shown in Figure 3-41.

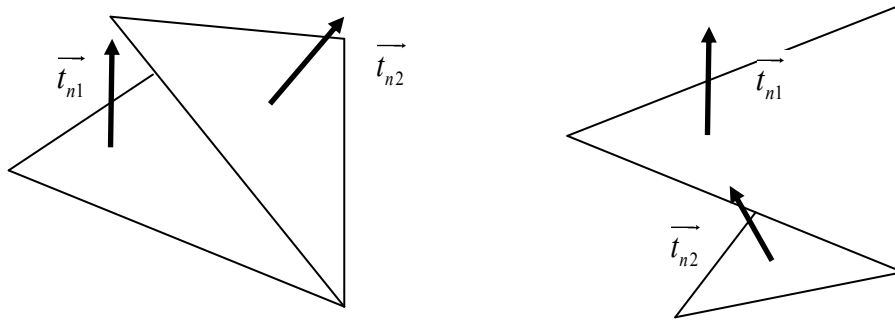


Figure 3-41 Limit Conditions for the Convexity Check

An angle of 60° was chosen as it represents a higher level of bend than that to which a textile would be realistically subjected.

In-plane shear angle of the best solution: once the best possible solution is found, the in-plane shear angle is determined so as to ensure that it is not below the user-defined minimum in-plane shear angle θ_{min} . If the in-plane shear angle is below the user-defined minimum this portion of the mesh cannot be draped, as out-of-plane deformations may occur since the textile has reached its locking angle as discussed in section 2.2. While the in-plane shear angle is simple to define and determine in constant-length textiles, the use of variable-length textiles complicates the matter. The left-hand side and right-hand side of Figure 3-42 show the various angles between yarn directions found in constant and variable-length textiles, respectively.

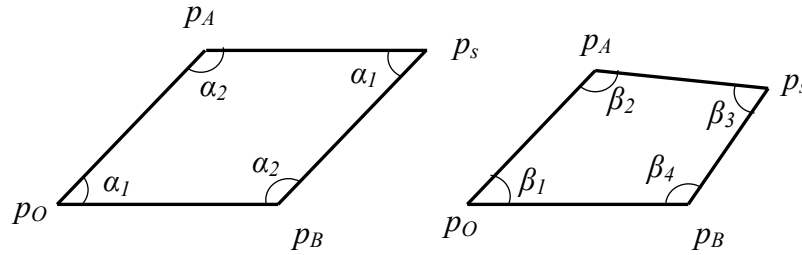


Figure 3-42 Angles in Constant-Length and Variable-Length Textiles

The angles in constant-length textiles are dependent on each other as follows:

$$\alpha_2 = 180 - \alpha_1 \quad (3.97)$$

therefore the in-plane shear angle is easily determined. The angles in variable-length textiles are independent of each other, the only restriction being that their sum must equal 360° . The method

proposed in this thesis to determine the in-plane shear angle β_{shear} of variable-length textiles is to take the average of all four angles. However, it is important to note that an angle of 80° involves the same amount of in-plane shear as an angle of 100° , therefore the average of the absolute values of the differences between the angles and 90° is taken:

$$\beta_{shear} = -\left(\frac{|90 - \beta_1| + |90 - \beta_2| + |90 - \beta_3| + |90 - \beta_4|}{4}\right) + 90 \quad (3.98)$$

to which 90° is added. The angles β_1 , β_2 , β_3 and β_4 are found using an equation similar to equation (3.96), using the vectors of the two lines which form the angle, for example vectors $\overrightarrow{p_O p_A}$ and $\overrightarrow{p_O p_B}$ to find β_1 . The average of the differences is made negative before adding 90° for β_{shear} to remain equal to or lower than 90° . As the software does not discriminate between constant and variable-length textiles, the value of β_{shear} is calculated in the same manner for constant-length textiles. Additionally, special care must be taken as the drape element may be bent as in Figure 3-41, which will greatly change the values of β_2 and β_4 . These values are calculated by taking the sum of the angles β_{2a} and β_{2b} , and β_{4a} and β_{4b} , as shown in Figure 3-43.

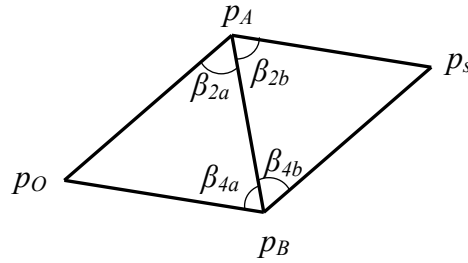


Figure 3-43 Determining Angles β_2 and β_4

While β_{shear} is used for determining the in-plane shear of the drape element, each angle, β_1 , β_2 , β_3 and β_4 , must be above the user specified angle θ_{min} for the software to accept the possible solution as the yarn crossover location. When a yarn crossover point satisfies every condition, it is stored to an array and the flag indicating that it has been meshed is set to true.

If the best possible solution does not satisfy the in-plane shear angle restrictions no yarn crossover point is stored; it is then concluded that this crossover cannot be draped with the current yarn spacings and the yarn spacings are modified to match the previous yarn crossover's yarn spacings. This is done to prevent large changes within the yarn spacings which result in portions of the mesh not to be draped. For example, the yarn crossover shown as an X in Figure 3-44 could not be draped as the in-plane shear angles exceed the limit, therefore the values of the yarn spacings for the lines labelled as 3 and 4 in Figure 3-44 are given the same values as the yarn spacings for the lines labelled 1 and 2, respectively. In some cases, this modification may enable the yarn crossover to be draped in a subsequent iteration or trial.

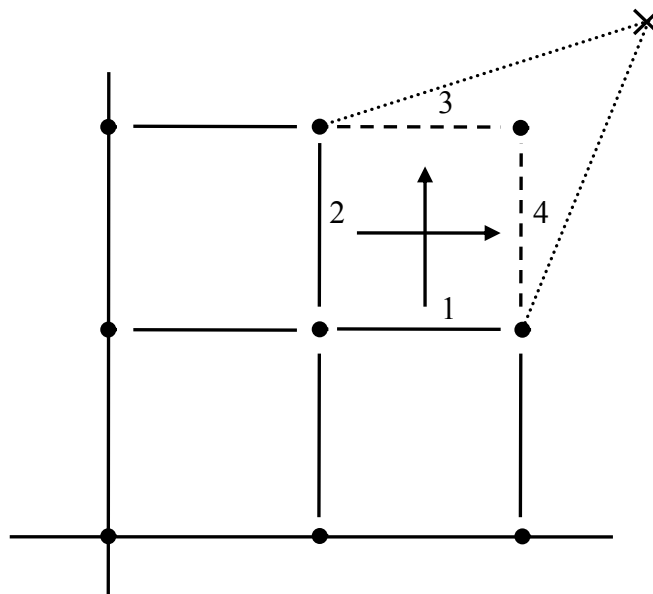


Figure 3-44 Setting Yarn Spacings of Yarn Crossover that Could Not be Draped

The process of changing the yarn spacings is transparent to the user and occurs almost exclusively in variable-length textiles during optimisation; simulations of constant-length textiles are unaffected.

Surface area of the drape element: when the location of a yarn crossover is determined, the surface area of the drape element is calculated for the software to display the total surface area covered by the textile after a draping simulation and for optimisation purposes, discussed in Chapter 4. The surface area of the drape element A_p is determined by the sum of the area of the two triangles seen in Figure 3-43, calculated using Heron's formula [71]:

$$A_p = \sqrt{s_1(s_1 - \|p_O - p_A\|)(s_1 - \|p_O - p_B\|)(s_1 - \|p_A - p_B\|)} \\ + \sqrt{s_2(s_2 - \|p_s - p_A\|)(s_2 - \|p_s - p_B\|)(s_2 - \|p_A - p_B\|)} \quad (3.99)$$

where s_1 and s_2 are determined by:

$$s_1 = \frac{\|p_O - p_A\| + \|p_O - p_B\| + \|p_A - p_B\|}{2} \quad (3.100)$$

and:

$$s_2 = \frac{\|p_s - p_A\| + \|p_s - p_B\| + \|p_A - p_B\|}{2} \quad (3.101)$$

3.4.3.3. *Sequence of the Meshing Operation*

The software follows a specific sequence for creating the mesh used for simulating the draping of a textile reinforcement. The geodesics are meshed first, section 3.4.3.1, followed by the area between the geodesics, section 3.4.3.2, in the order described in Figure 3-45.

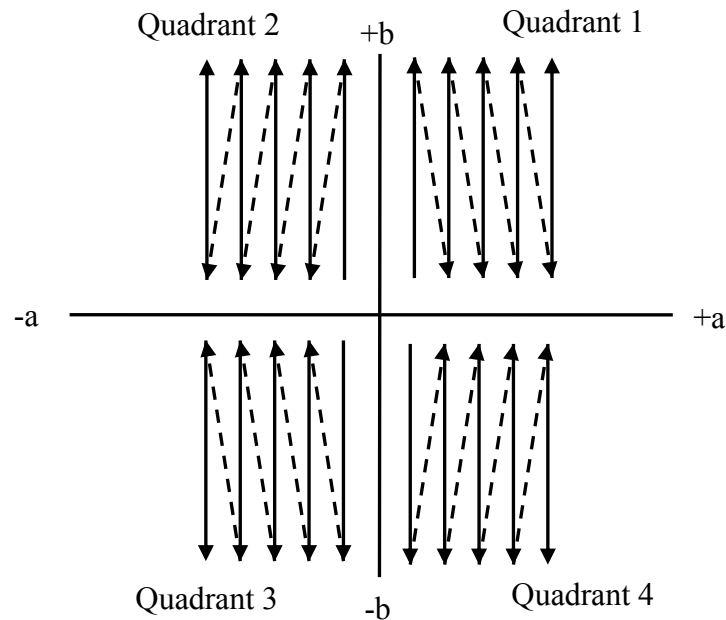


Figure 3-45 Order for Finding Yarn Crossovers

Each quadrant is draped separately as they are independent of each other due to the fixed geodesics. In addition, the pattern observed for each quadrant has no effect on the outcome of the draping simulation as long as the three dependant yarn crossovers needed to mesh a given crossover have been previously determined. The complete algorithm for finding the yarn crossover locations between the geodesics can be seen in Figure 3-46 and Figure 3-47.

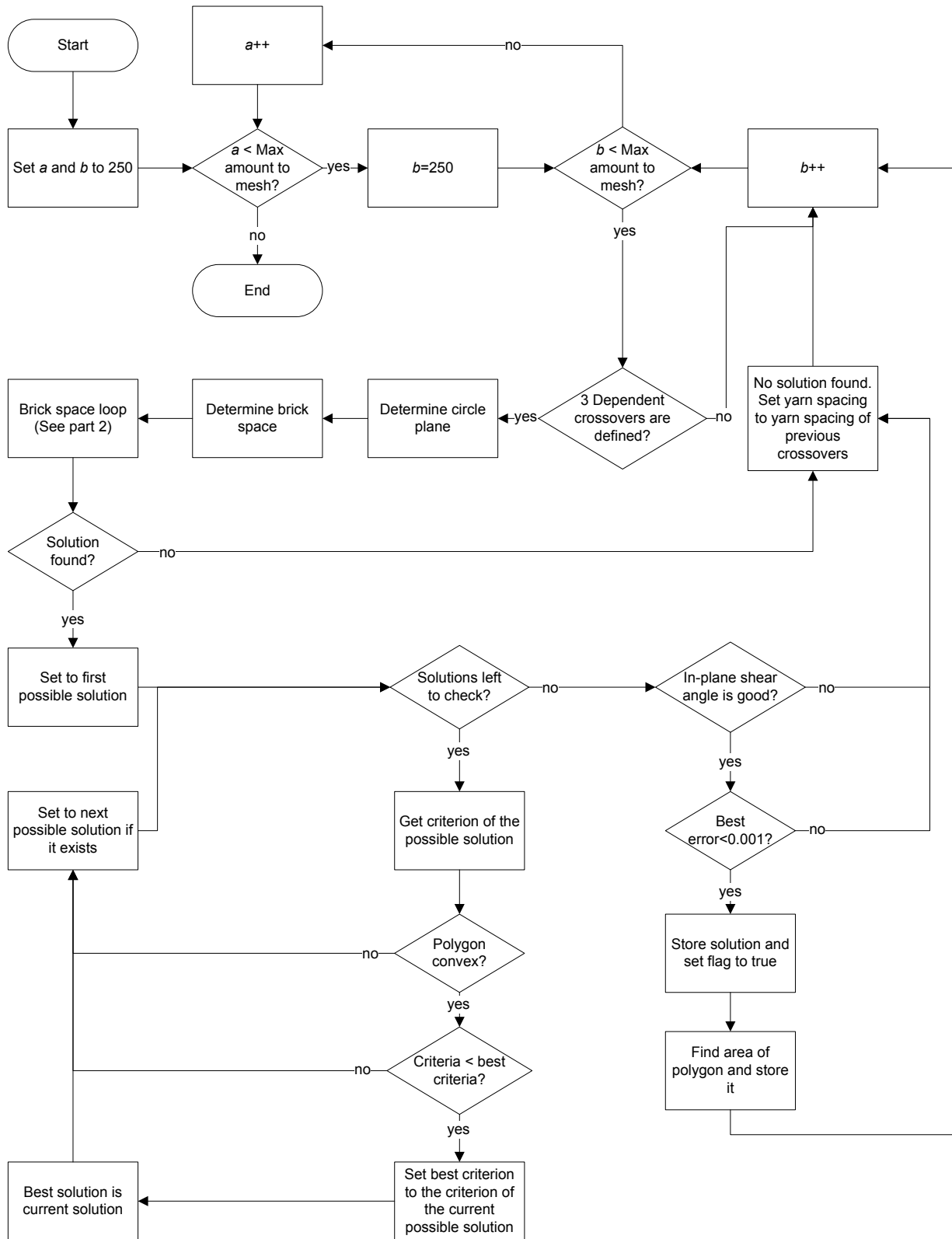


Figure 3-46 Algorithm for Meshing Between Geodesics (Part 1)

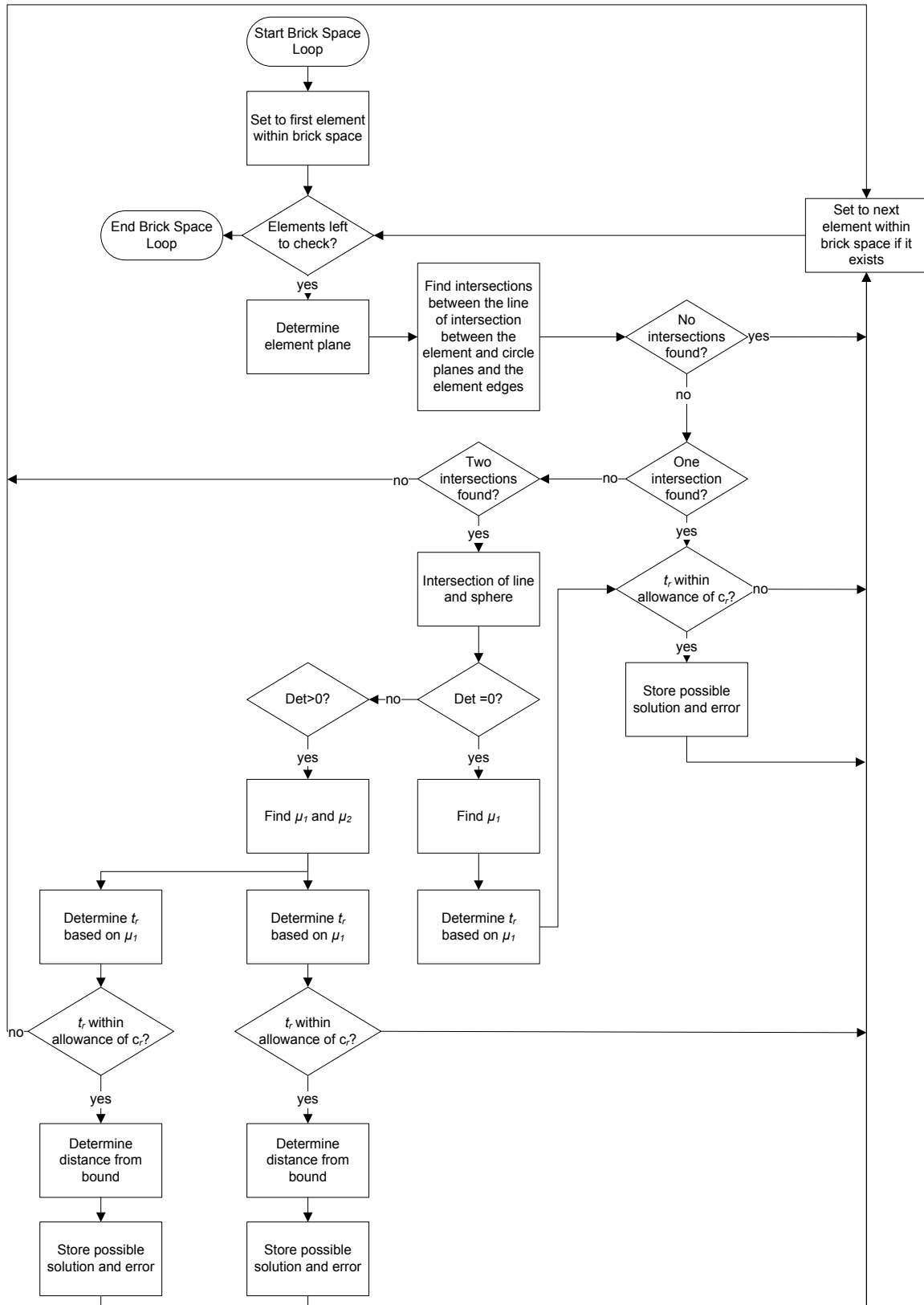


Figure 3-47 Algorithm for Meshing Between Geodesics (Part 2: Brick Space Loop)

The algorithm ends when the user-defined amount of yarn crossovers to mesh is reached.

3.5. *Output of the Software*

Once the draping simulation is complete, the model as well as the mesh representing the draped textile reinforcement can be viewed in the OpenGL window as seen in Figure 3-48.

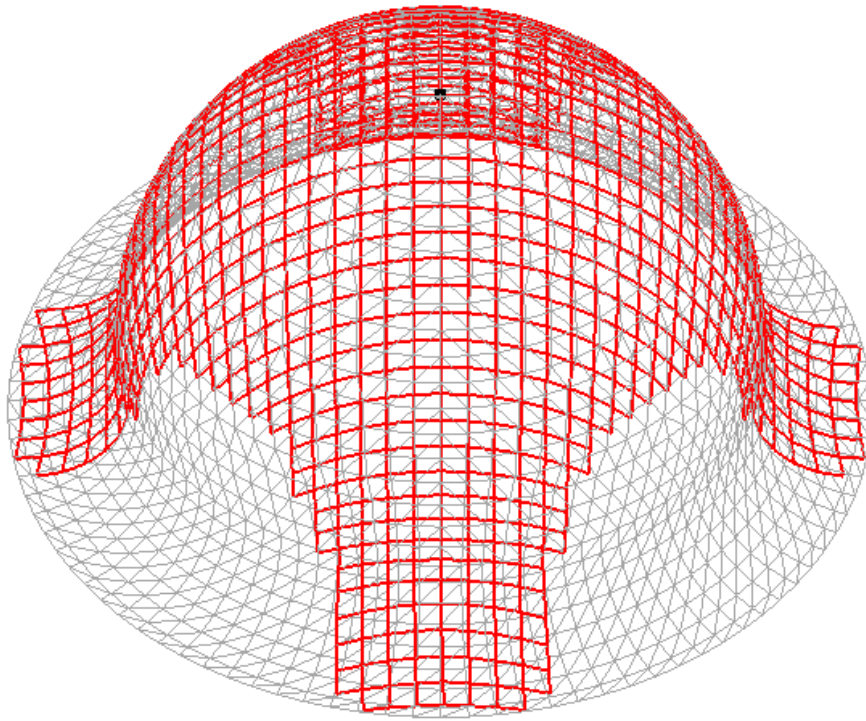


Figure 3-48 Simulated Mesh Displayed on Model

The simulation shown in Figure 3-48 was performed on a 6 cm radius sphere using a constant yarn spacing of 0.5 cm and θ_{min} of 60° . The software may overlay a colour map representing the distribution of the in-plane shear angle β_{shear} , Figure 3-49, or the fibre volume fraction v_f across the simulated textile, Figure 3-50.

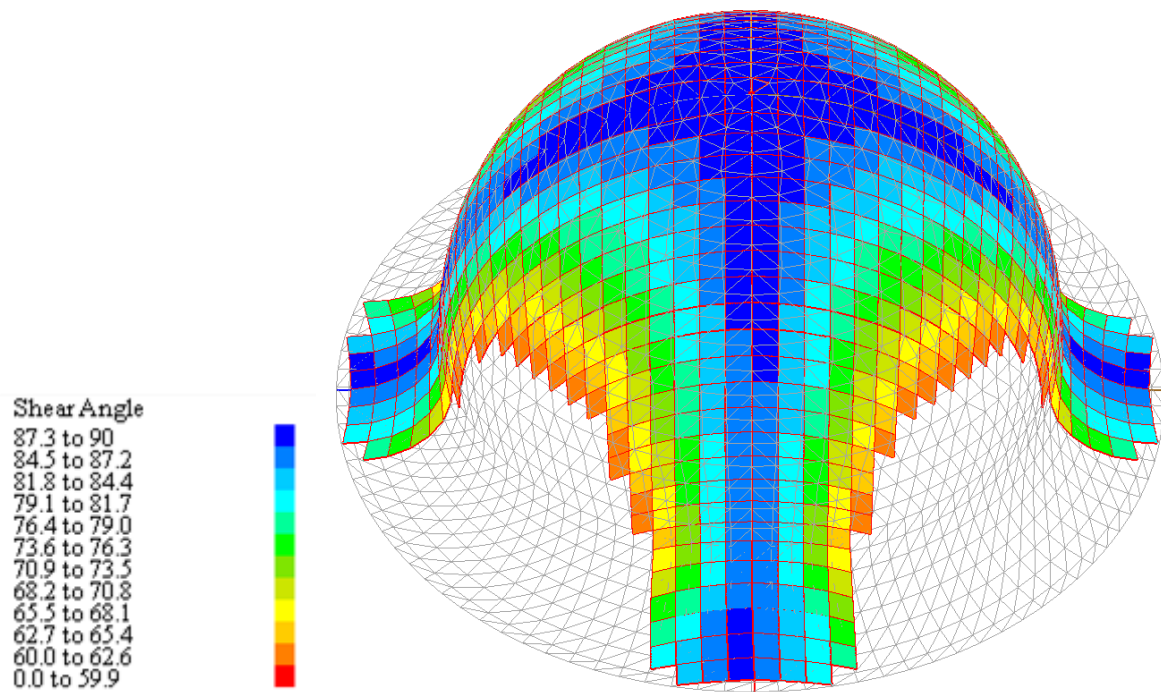


Figure 3-49 Colour Map of the In-Plane Shear Angle

The value attributed to each colour can be seen in the legend on the left of Figure 3-49. The upper and lower thresholds of the legend can be modified via the GUI to provide flexibility in the display. The gradient between the upper and lower thresholds is equally spaced and varies automatically based on the selected threshold values.

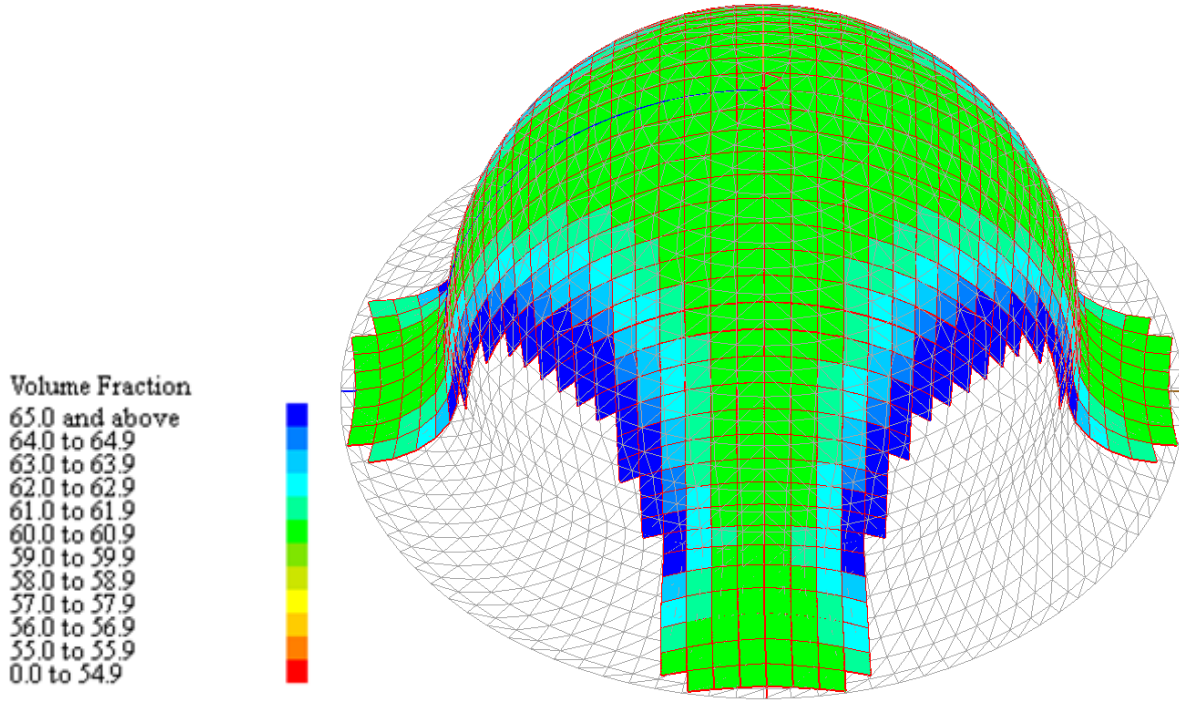


Figure 3-50 Colour Map of the Fibre Volume Fraction

Again in Figure 3-50, the legend for the v_f colour map is shown on the left and the thresholds can be modified via the GUI. The values of v_f are calculated based on the area occupied by each drape element, as calculating v_f based on in-plane shear angle may be inaccurate for variable-length textiles. A reference value for the volume fraction v_{fr} must be defined for a known reference area A_r via the GUI; in Figure 3-50 a v_{fr} of 0.6 was defined for an area A_r of 25 mm² corresponding, for this example, to an undeformed drape element with 5 mm spacing. The v_f for each individual drape element is found with:

$$v_f = \frac{A_r \cdot v_{fr}}{A_p} \quad (3.102)$$

where A_r is the area occupied by the drape element; constant thickness is assumed. Finally, yarn spacings can be colour-coded in cases of variable-length textiles as shown in Figure 3-51.

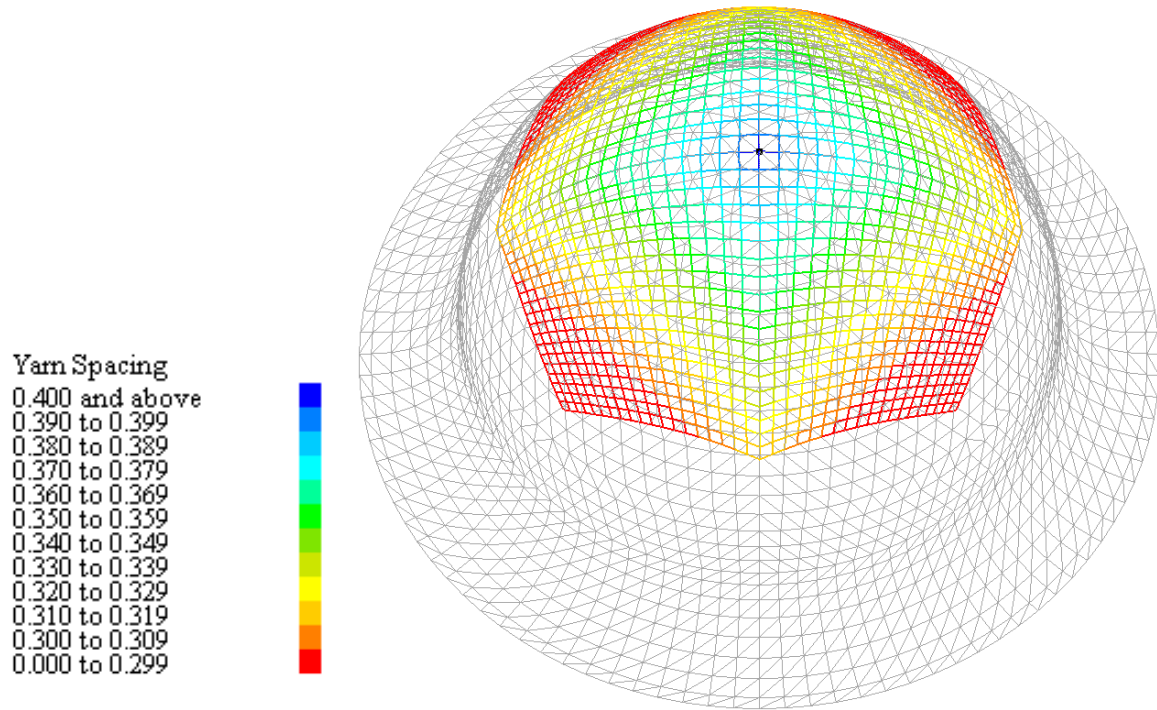


Figure 3-51 Colour Coded Yarn Spacings for Variable-Length Textiles

Similarly to the in-plane shear angle and volume fraction representations, the software provides a legend for yarn spacings with thresholds that can be modified via the GUI.

3.6. Additional features

While the simulation of the draping of textiles onto the surfaces of a model is the main purpose of the software, additional functionality was incorporated to further assist in the design of PMC components. Simulations may be saved at any stage of the draping process; from models with no start point defined to simulations on which draping and optimising has been completed. The save and load feature is available from the GUI and uses a specific file type .dparam detailed in Appendix A, which contains all the parameters used in creating a simulation. The yarn

spacings are stored in separate text files which are named in accordance with the name given to the .dparam file upon saving. It is also possible to create multiple meshes on the same model as shown in Figure 3-52, for simulating the draping of models that require multiple pieces of textile.

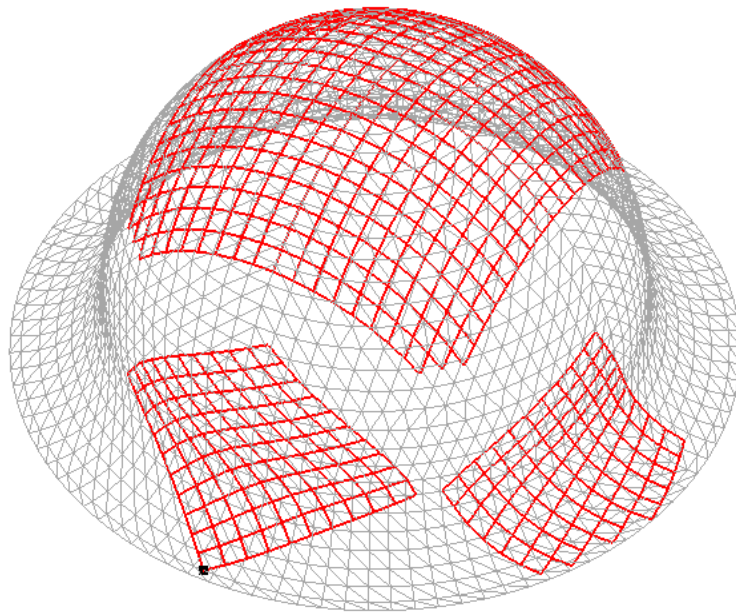


Figure 3-52 Model Draped with Multiple Meshes

In addition to multiple meshes, it is possible to specify limited zones to be draped on the model, referred to as patches, and limit the draping simulation to these patches as shown in Figure 3-53. When draping over a surface limited by a patch, elements outside of the patch cannot be selected as the next element for a geodesic nor included within the brick space, therefore the creation of geodesics and the creation of the mesh is effectively halted when the next element on which the geodesic or mesh would progress is not part of the patch.

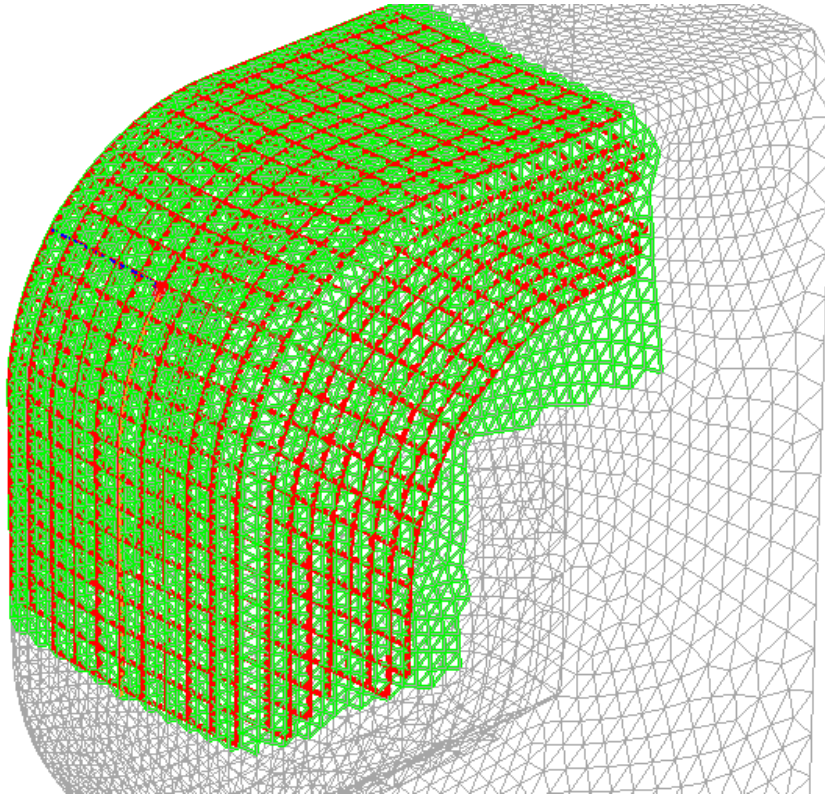


Figure 3-53 Drape Simulation Limited by a Patch

The elements within the patch can be highlighted in green as seen in Figure 3-53; this function can be toggled in the GUI. In cases where multiple patches are present, an active patch must be selected and is shown in vivid green; the active patch is the only patch considered when simulating the draping of a textile. Patches which are not active are shown in a darker shade of green. The patches may be created manually by creating a list of elements in the .model file. They may also be created via the GUI; when the software is in patch creation mode, elements not already in the patch being created and under the mouse cursor turn red as seen in Figure 3-54. At this point pressing the "enter" key adds the currently selected element to the patch list. Elements

within the patch are seen in blue and remain blue when under the mouse cursor. Pressing the "enter" key while over a blue element removes it from the patch being created.

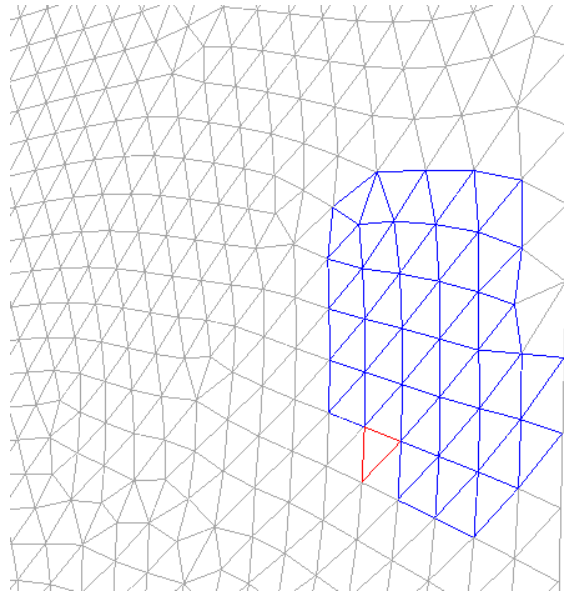


Figure 3-54 Creation of a Patch Within the Software

The patch creation process ends by saving a .txt file with a chosen name containing the numbers of the elements within the patch.

Draping with multiple pieces of textile requires that each piece be held in place during the manufacturing process; the software includes three features which help the PMC designer in planning for the manufacturing of the component. One of these features is the ability to display portions of the simulated drape that may not remain on the surface of the model due to gravity. Portions of the mesh which face downward relative to the gravity vector, defined using the GUI, are shown in red while other elements are shown in yellow as seen in Figure 3-55.

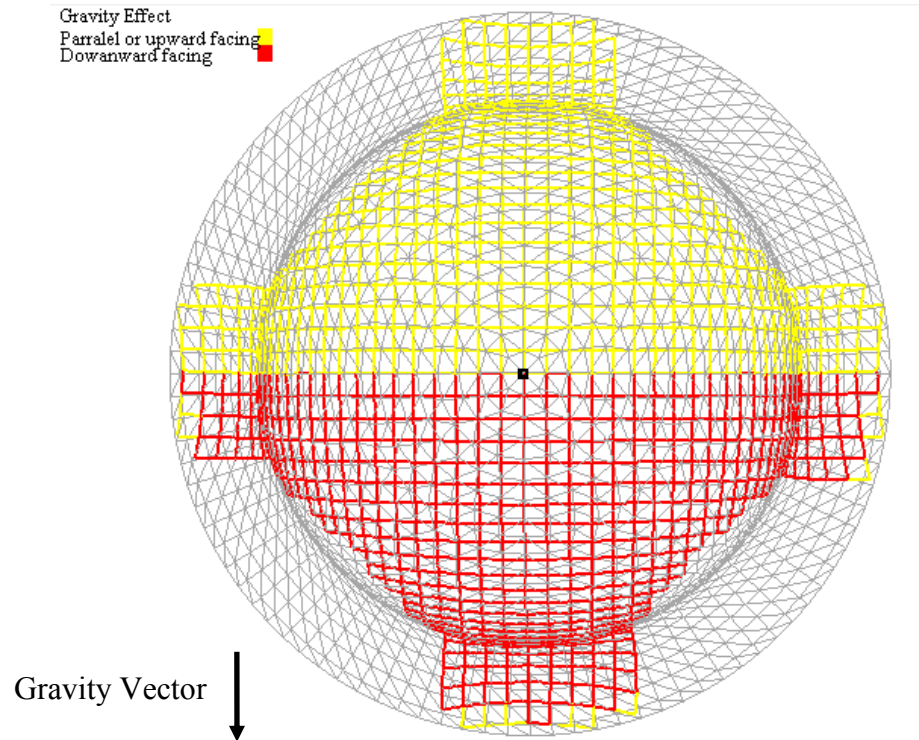


Figure 3-55 Gravity Effect on the Mesh

The second feature included for facilitating planning for manufacturing is the ability to add holder magnets as seen in Figure 3-56. In the software the magnets act only as visual cues for planning the manual layup of one or multiple pieces of textile reinforcement. The use of magnets as tools to temporarily affix textiles to a mould is detailed in an undergraduate thesis report written by James Cowan [72]. The functional restraining and facilitating of the draping provided by the magnets has not been implemented in the software at the time of writing.

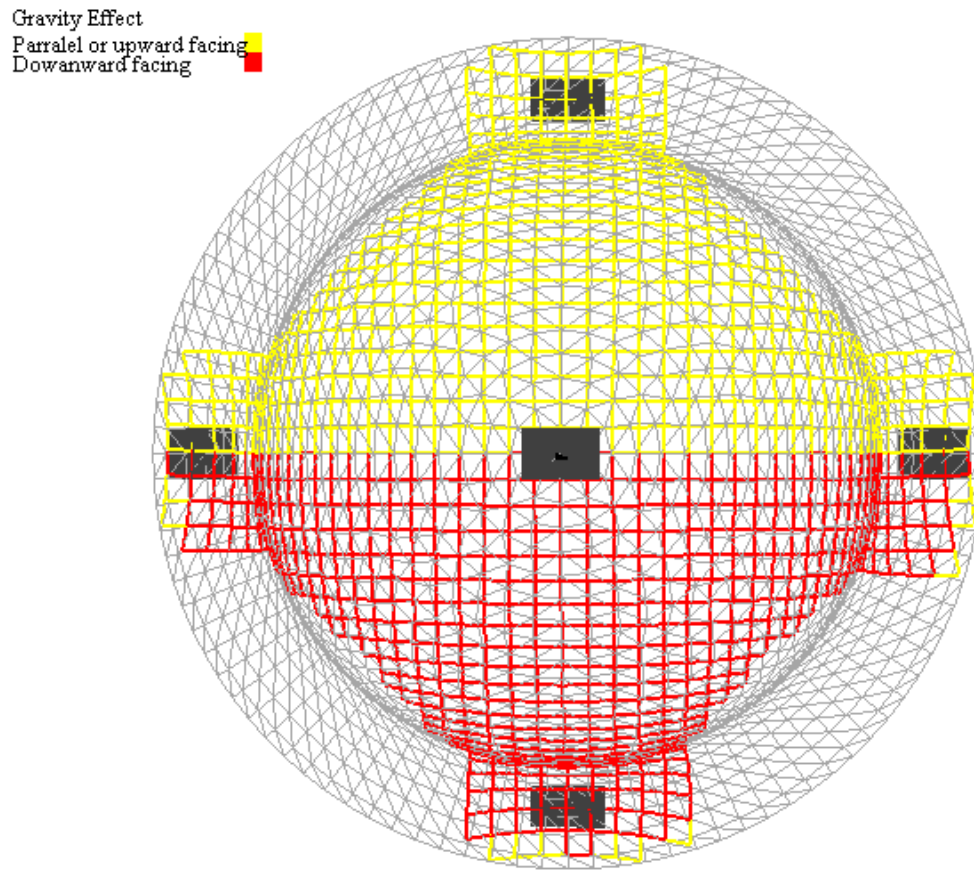


Figure 3-56 Draping Simulation with Magnets

The final feature incorporated into the software is a verification to ensure that the textile can be laid flat. While constant-length textiles can always be laid flat, it is essential to verify that variable-length textiles can exist in the flat state, as they are manufactured in the flat state. The question is not trivial as the geometry of the textile and the optimisation of the lengths are done in the draped state, on the geometry. Some variable-length textiles optimised in the draped state may not be feasible as flat textiles as the in-plane shear limit may be exceeded. To determine if a certain textile can be made flat, the yarn spacing data is used to simulate the draping of the textile onto a flat plate. Then, the software compares yarn crossovers from drape simulations on

the model and on the flat surface; yarn crossovers which do not exist in both simulations are discarded. Yarn crossover that do not appear in the simulation performed on the model cannot be draped, and those that do not appear in the simulation performed on the flat surface cannot be manufactured. In addition, the loading feature of the software discussed earlier can be used to load any saved simulation or optimisation on a flat surface so as to visualise the shape of the textile to be manufactured, before draping is performed. The net shape of the textile seen in Figure 3-51 is shown in Figure 3-57 as draped onto a flat surface. It is important to note that a large portion of the textile can no longer be seen as the in-plane shear angle of the missing sections is under the set limit of 60° , in this case.

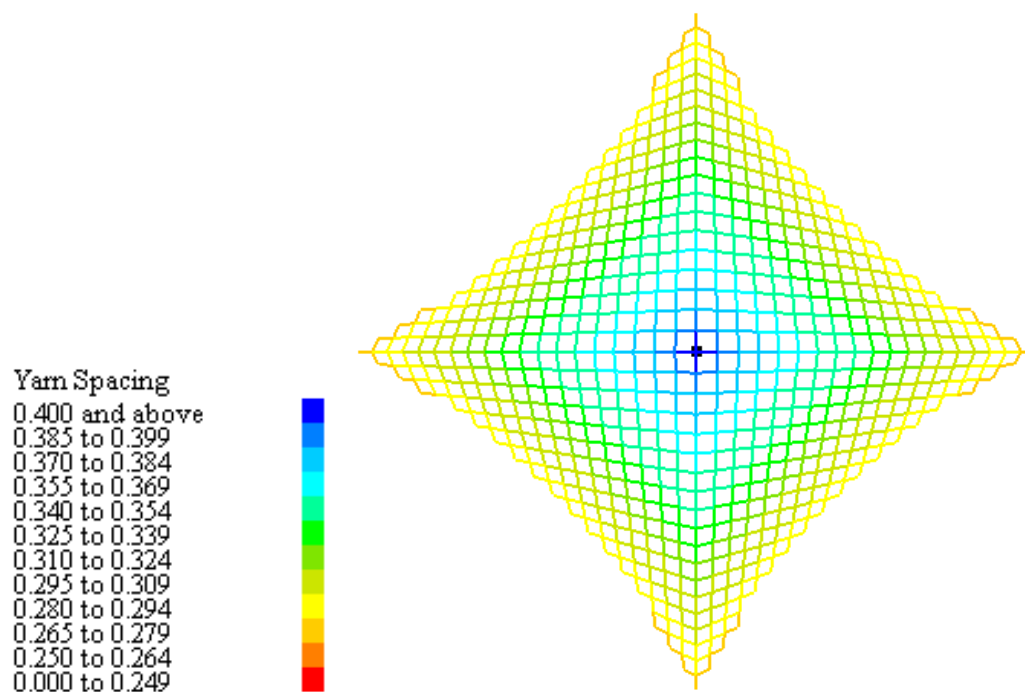


Figure 3-57 Variable-Length Textile Draped on a Flat Surface

4. Textile Optimisation

The objective behind variable-length textiles is to custom-tailor the yarn paths, defined in a the flat state, towards a specific application and geometry. The benefits include a greater surface area covered by a single piece of textile, less in-plane shear in the draped state, and fibre orientations that are potentially better aligned towards specific design requirements, say for load carrying for example. The benefits are quantified by the use of a parameter, defined in a later section, which needs to be maximised. The optimisation or improvement of the textile is achieved by varying the distance between adjacent yarns across the textile. The distance between adjacent yarns, referred to as the yarn spacing, is measured between the yarn crossovers as shown in Figure 4-1 where the yarns are represented by solid lines.

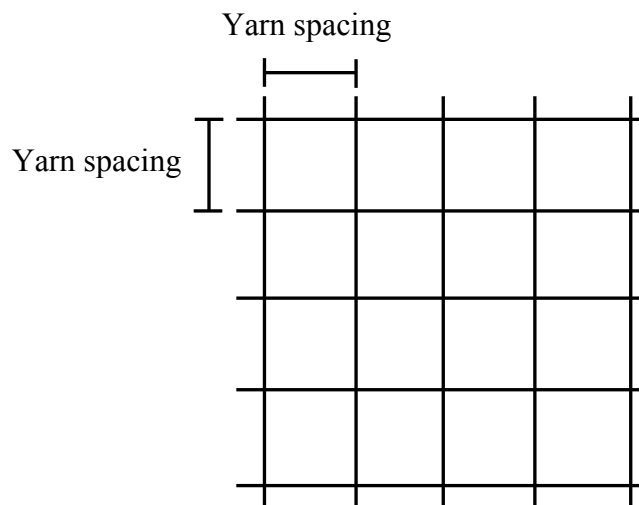


Figure 4-1 Yarn Spacing of a Textile

Changes to the yarn spacings can be made manually by the software, using optimisation algorithms. Several textile optimisations, presented in Chapter 5, have been performed within the scope of this thesis. Additionally, the machinery to manufacture variable-length textiles as well

as sample variable-length textiles were fabricated in a parallel research project. The manufactured textiles are discussed in Chapter 5. It is important to note that the current implementation of the software does not optimise the drape start point but relies on the user's ability to determine an acceptable start point for a given optimisation scenario. An algorithm that determines the optimal location of the drape start point could be implemented in the future.

4.1. *Manual Optimisation*

The manual optimisation of a textile reinforcement is accomplished by making changes to the yarn spacing array, described in section 3.4.3, directly in the draping software or through software such as Excel™. When using Excel™ to change values in the yarn spacing array it is possible to define formulas or relations between yarn spacings. However, it is necessary to load the modified yarn spacing array for the changes to be seen in the draping software. In contrast, modifications to the yarn spacing array made within the draping software are reflected instantly. When using the draping software to modify the yarn spacings, the user must move the marker seen as a black dot in Figure 4-2 to the desired location, and modify the yarn spacing. This method, albeit tedious, combined with the use of Excel™ based modifications can be used where automated optimisation fails to provide the desired results.

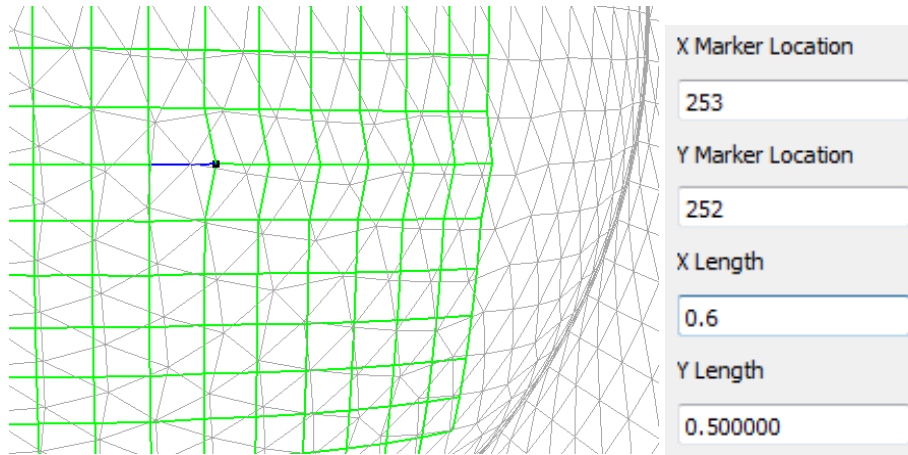


Figure 4-2 Manual Optimisation Using the Draping Software

4.2. *Software Optimisation Algorithm*

The draping software may be used for optimising a textile, relying on an algorithm based on the evolution strategy optimisation method and resembling the (1+1)-ES strategy described in section 2.6. The textile is modified by changing one value of the yarn spacing at a single location on the draped textile, effectively altering the configuration of the yarn spacings of the textile. This location, as well as the magnitude of the change to the yarn spacing, are based on a random number supplied by a pseudo-random number generator, ran2 [73]. Both the location of the yarn spacing and the magnitude of the change are limited by certain parameters discussed in the following section. The decision to keep or discard the modified configuration is based on the fitness levels of the original and modified configurations. The fitness level corresponds to the value of a parameter being maximised, discussed in section 4.2.2, which is determined upon draping the textile. Should the new, altered configuration result in an increase in the value of the parameter being maximised, the new configuration is kept and the original configuration discarded. Should the new configuration result in a decrease in the value of the parameter being

maximised, the original configuration is kept and the new configuration is discarded. The decision to keep or discard the new configuration completes one iteration of the optimisation process, for which the algorithm is shown in Figure 4-3.

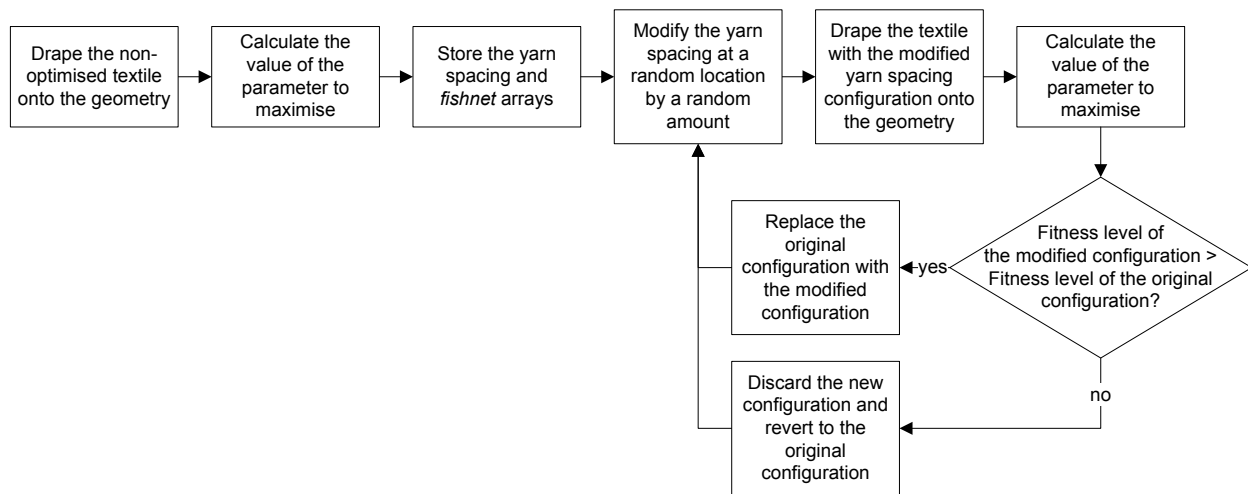


Figure 4-3 Algorithm for the Software Optimisation of a Variable-Length Textile

4.2.1. Optimisation Parameters

Several parameters need to be defined to ensure that optimisations are carried out efficiently and that their results lead to textiles that can be manufactured. The first parameters are the maximum and minimum allowable yarn spacings, which define the range of yarn spacings permitted for a particular optimisation. As the yarn spacing is increased the space available to each yarn increases, but this may reduce the fibre volume fraction. If the yarn spacing is decreased the fibre volume fraction will increase but the yarns may deform out-of-plane if forced to occupy a space that is too constrained. Ideally, the maximum and minimum values are relatively similar to the initial yarn spacing, defined as the yarn spacing of the non-optimised

constant-length textile on which the optimisation is performed. For example, if the initial yarn spacing is 4 mm, the maximum and minimum values could be 5 mm and 3 mm, respectively. The value of the initial yarn spacing is dependent on the width of the yarn and must be correctly sized to allow for a constant-length textile manufactured using the initial yarn spacing to have a proper value for the fibre volume fraction, say 60%. Failure to specify maximum and minimum yarn spacings may lead to textiles with very large yarn spacings in certain areas and very small yarn spacings in other areas, as shown in Figure 4-4. The process of identifying the acceptable values is done experimentally, making measurements on the actual yarns used to manufacture the textile to be draped.

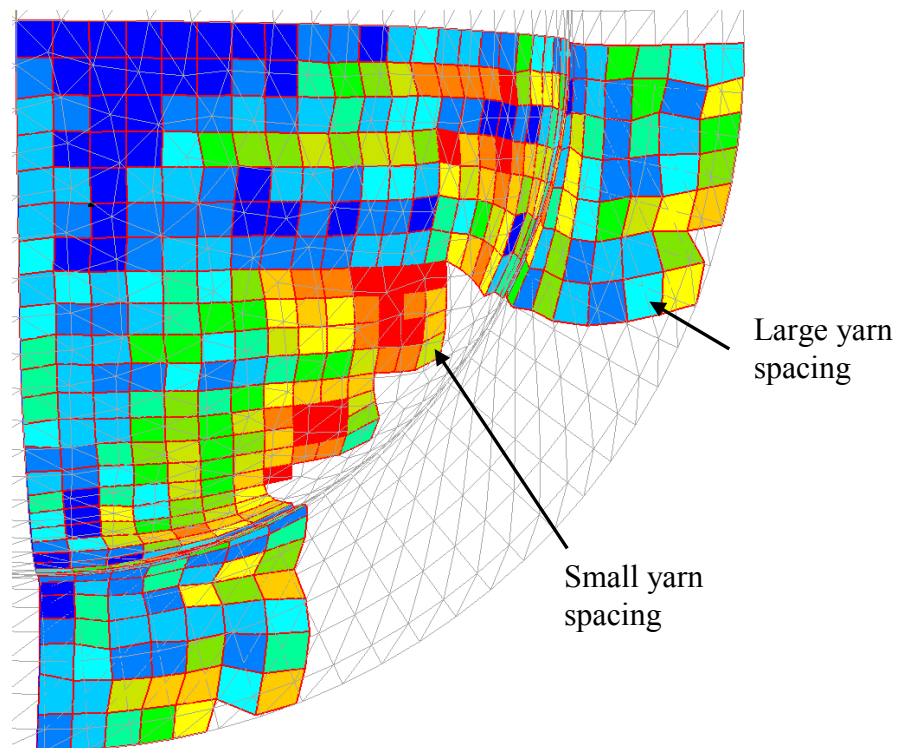


Figure 4-4 Optimisation Performed Without Maximum and Minimum Yarn Spacings

The next parameter is the maximum amount of change to the value of the yarn spacing which can occur in one iteration. This prevents the optimisation from making large modifications and provides for a higher level of consistency in yarn spacings over the surface of the textile. The maximum amount of change to the yarn spacing used in the optimisations discussed in Chapter 5 ranges from 1% to 3% of the initial yarn spacing. Smaller changes to the configuration of the yarn spacing may slow down the optimisation process. However, the benefits greatly outweigh the loss in optimisation speed. An example of a textile optimisation attempted without limiting the amount of yarn spacing change per iteration can be seen in Figure 4-5; the textile clearly is non-uniform and would not be suitable for use in PMC component manufacture as many yarns are crimped, causing the fibre orientation of the textile to vary greatly.

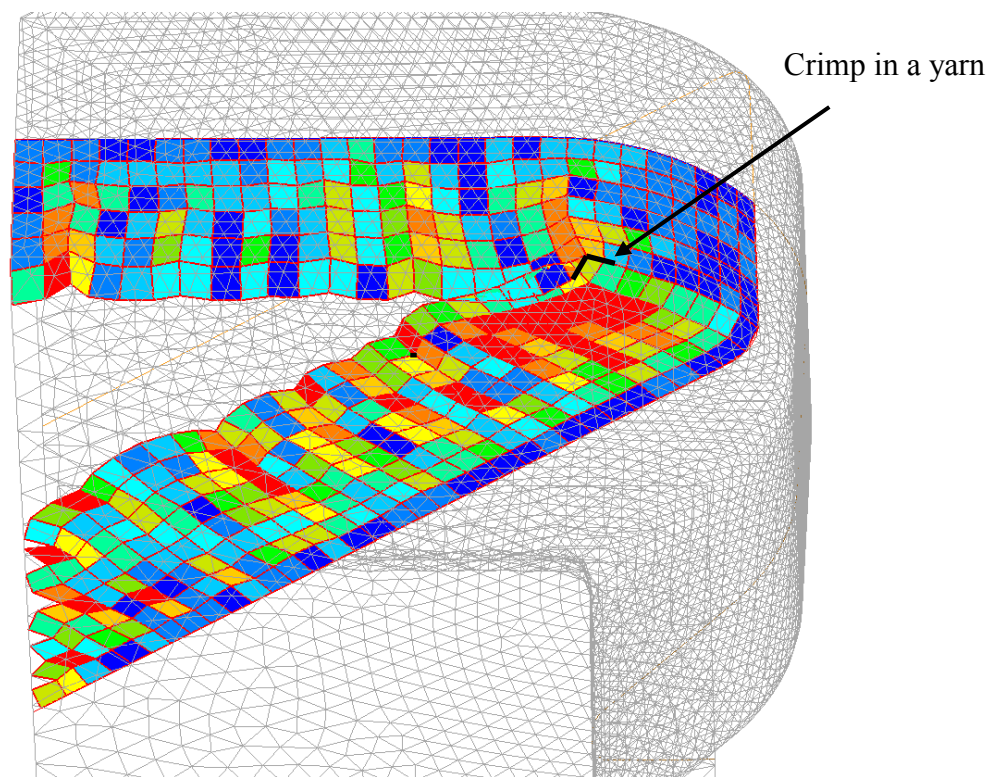


Figure 4-5 Optimisation Performed Without a Yarn Spacing Change Limit

To further illustrate the advantages that may be gained by limiting the amount of change to the yarn spacing which can occur in one iteration, two sets of optimisations were performed. The optimisations shown in Figure 4-6 were performed with a limit of 0.1 mm while the optimisations shown in Figure 4-7 were performed with a limit of 0.01 mm. All other parameters, such as the initial yarn spacing of 4 mm, are identical in both sets of optimisations

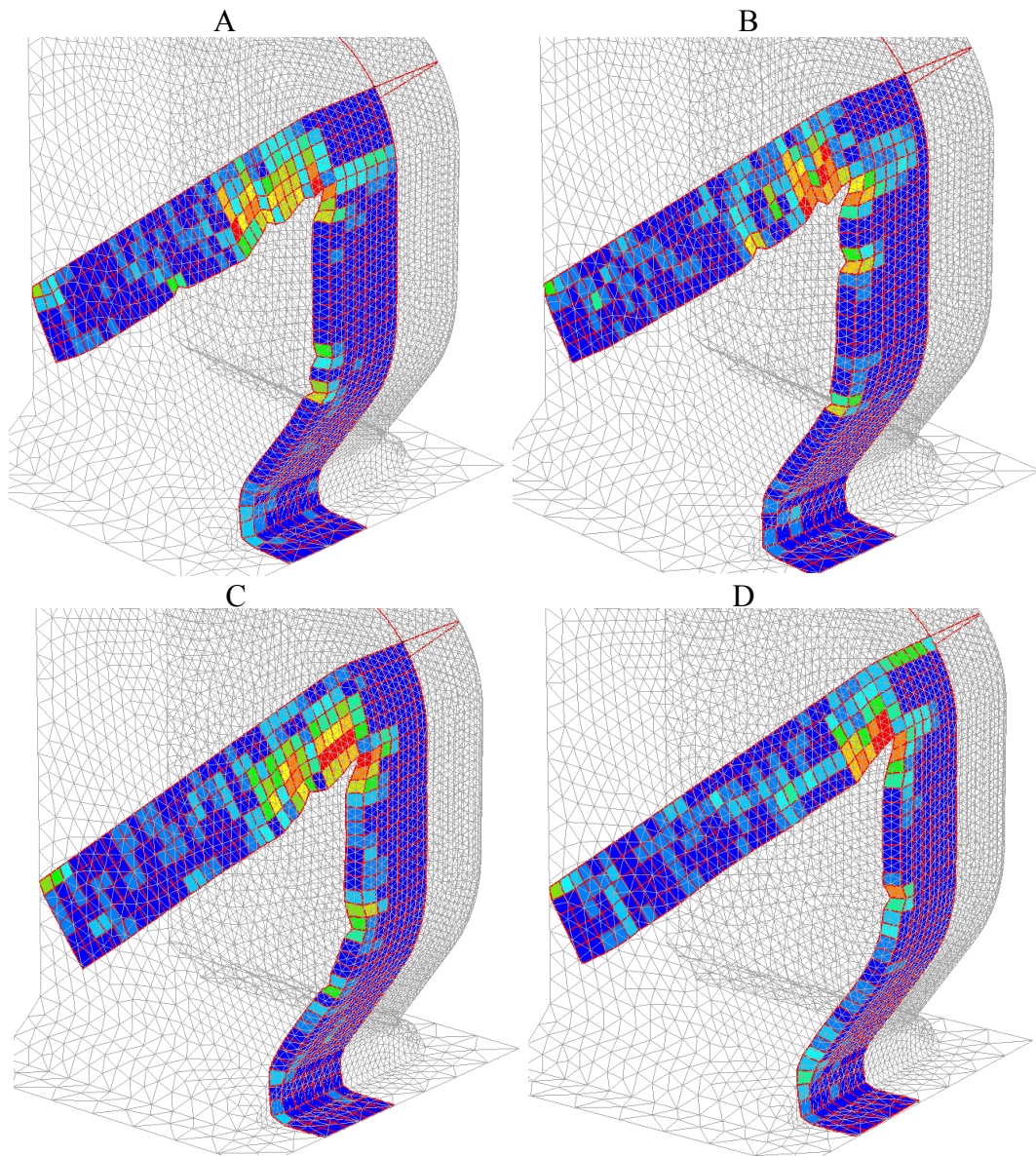


Figure 4-6 Optimisations Performed with a Yarn Spacing Change Limited to 0.1 mm

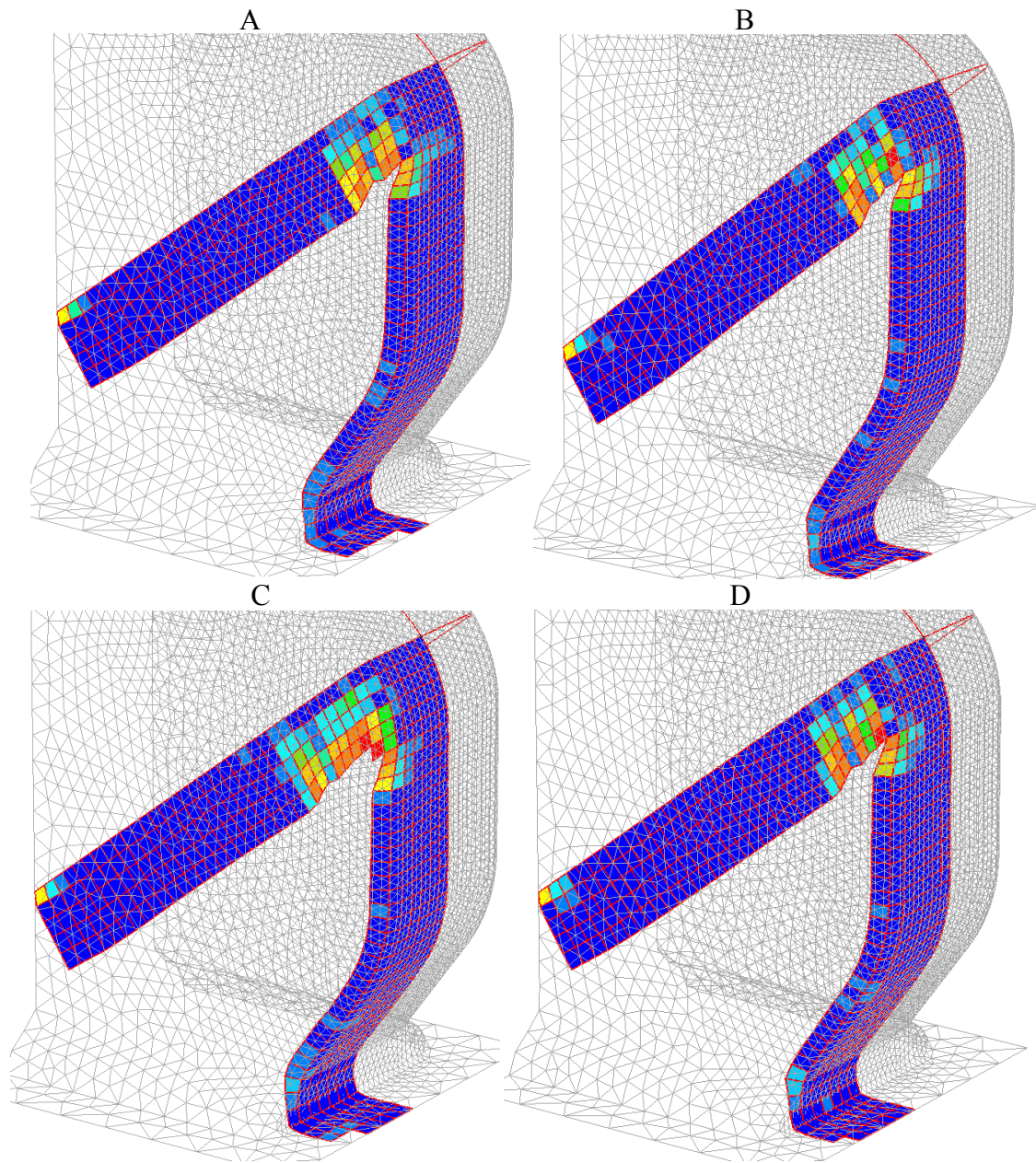


Figure 4-7 Optimisations Performed with a Yarn Spacing Change Limited to 0.01 mm

The optimisations performed with a limit of 0.01 mm are much more uniform than the optimisations performed with a limit of 0.1 mm. In addition, the average increase of the surface area obtained by optimising the textile is 22.67% for the optimisations with the 0.01 mm limit and 13.81% for the optimisations with the 0.1 mm limit. The standard deviation on the surface

area for the optimisations with limits set at 0.01 mm and 0.1 mm are 0.53 mm^2 and 4.29 mm^2 , respectively.

Another parameter used for limiting non-uniformity and abrupt changes in the draped variable-length textile is the minimum percentage of similarity required between neighbouring values of the yarn spacing along a given yarn. Additionally, a minimum percentage of similarity is required between neighbouring values of the yarn spacing on adjacent yarns. The yarn segments involved in the similarity check for yarn spacings in the a direction can be seen in Figure 4-8, where the yarn spacing shown as a dotted line must be within a certain percentage the yarn spacings shown as dashed lines.

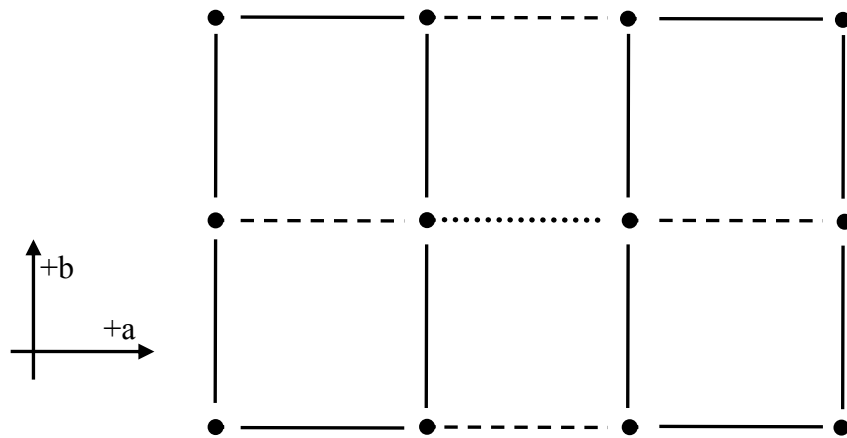


Figure 4-8 Similarity Check for Neighbouring Yarn Spacings

The value of the parameter for the minimum percentage of similarity is defined as a percentage below 100% where a specified value of say 95% indicates that neighbouring values of the yarn spacing must be similar to within a range between 95% and 105%. To illustrate the advantages of using a minimum similarity parameter near 100%, five different values of the

parameter were compared on four different optimisation scenarios. Two different models were used in this trial as well as two optimisation types, detailed in section 4.2.3, creating four distinct optimisation scenarios. Four optimisations were performed for each combination of the minimum similarity parameter and optimisation scenario, the average results of which can be seen in Figure 4-9. The value of the surface area shown in Figure 4-9 is the surface area of the portion of the textile which can be manufactured, following the removal of portions which cannot be manufactured as explained in section 3.6. It is clear that optimisations performed with a higher value for the minimum similarity parameter yield an optimised textile which covers a greater surface area.

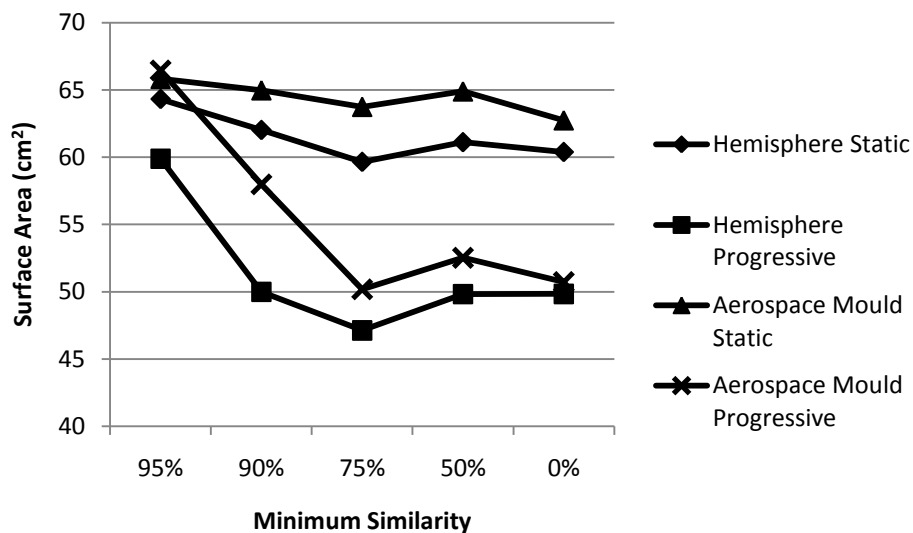


Figure 4-9 Effect of the Minimum Similarity Parameter on the Results of the Optimisation

The parameters defined above may effectively limit the textile from becoming non uniform and unsuitable for use in PMC manufacture. However, they do not guarantee that the textile will necessarily be manufacturable flat. When optimising, it is possible to select whether

or not the software will perform a verification at each iteration, to determine continuously which sections of the textile cannot be manufactured, using the method detailed in section 3.6. While valid changes to the textile configuration may be discarded as a result of a possible reduction in the value of the parameter to maximise stemming from the loss of yarn crossovers, the verification ensures that the results of the optimisation process may be used in practice.

The final optimisation parameter is the steady state criterion ss_c . This criterion is a threshold used for determining whether a given optimisation process has reached a steady state condition. Steady state checks only occur after the completion of a number of iterations N_i , the value of which is based on the amount of yarn crossovers N_m that can be successfully draped on the model. The value of N_i is the amount of iterations needed for the cumulative probability of each yarn spacing being selected for modification at least once to approach unity, and is determined using:

$$N_i = \sum_{i=1}^{2N_m} \frac{2N_m}{i} \quad (4.1)$$

where N_m is multiplied by two to account for yarn spacings in both fibre directions a and b .

When sufficient iterations have been completed the steady state check is performed with:

$$ss_i = \frac{P_i - P_{avg}}{P_i} \cdot 100 \quad (4.2)$$

where P_i is the current value of the parameter to maximise and P_{avg} is a moving average of the value P_i for the last N_i iterations. If ss_i is lower than the value of ss_c the optimisation process is

stopped as it is considered to have reached a steady state condition; the user is notified of this by the GUI.

4.2.2. Parameter to Maximise

The software may optimise a given textile for one parameter or for a combination of many parameters including the surface area, in-plane shear angle, fibre volume fraction and fibre orientation. The value of the selected parameter is calculated for each drape element forming the mesh simulating the draping of a textile; the values can be summed or averaged depending on the selected parameter to maximise. A major motivation for the optimisation of textiles is covering a larger surface area with a single piece of textile. Therefore, the first parameter to maximise is the surface area covered by the draped textile. As the optimisation progresses, only new configurations which lead to a larger area are retained. While simple, this method often leads to unchecked growth in certain portions of the mesh, resulting in shapes such as seen in Figure 4-10 where the textile has merely expanded to cover more area in the sections closer to the base of the hemisphere. It is important to note that optimisations which maximise the surface area are very dependent on the initial and maximum yarn spacings; if the initial and maximum yarn spacings are identical, the risk of having drape elements expand to cover a larger surface area is minimised or eliminated.

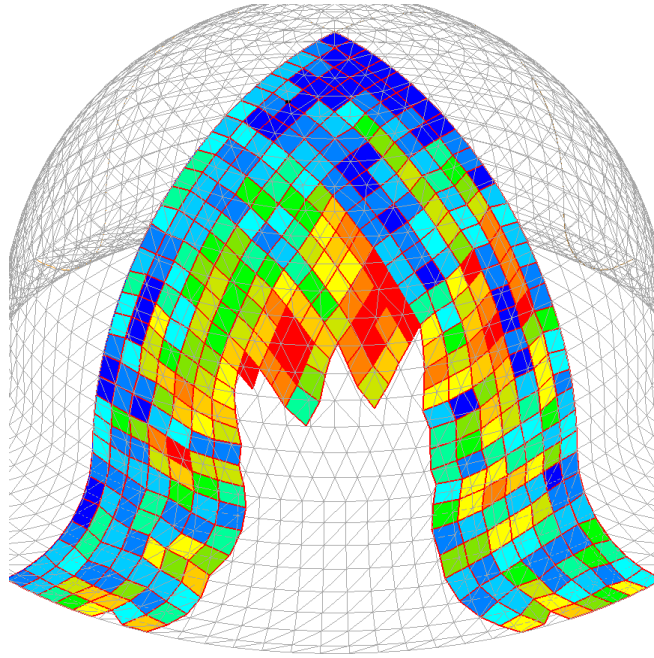


Figure 4-10 Optimising for Surface Area

A textile reinforcement cannot be draped over a large area on complex surfaces due to limits in the amount of deformation, namely in-plane shear, which can be applied to that textile before out-of-plane deformation occurs. For this reason, it is possible to optimise the textile for minimising in-plane shear in the textile. The simplest method consists in maximising the sum of in-plane shear angles for the drape elements within the draped textile. Due to the convention used in this thesis a high in-plane shear angle corresponds to low in-plane shear in the textile, as detailed in section 2.2. Therefore, maximising the sum of the in-plane shear angle leads to lower in-plane shear in the textile. It is important to note that additional drape elements will increase the value of the sum of in-plane shear angles; therefore, modifications which add new yarn crossovers are usually retained. However, optimisations done with the in-plane shear angle as a parameter tend to limit the addition of new yarn crossovers and concentrate on minimising in-plane shear in existing portions of the mesh, as seen in Figure 4-11.

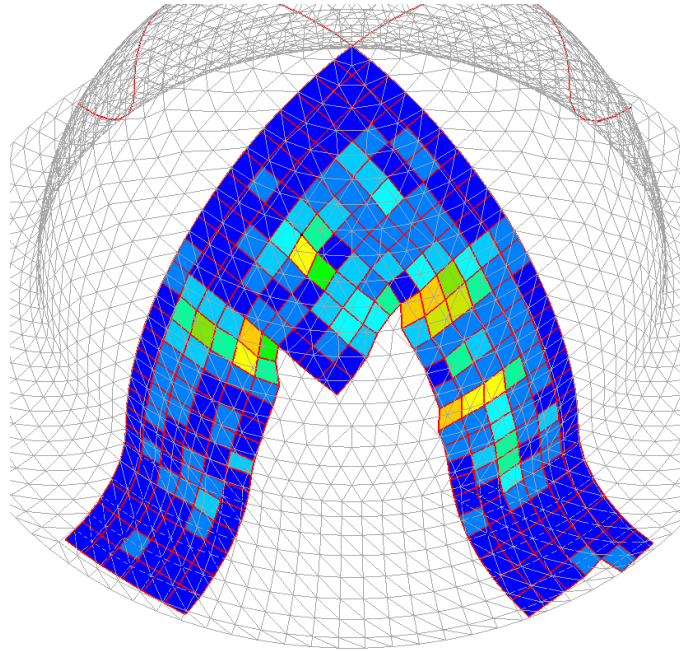


Figure 4-11 Optimising for the In-Plane Shear Angle

It is also possible to optimise the textile to minimise the average in-plane shear angle, which is not affected by the amount of yarn crossovers in the textile.

To minimise the issues associated with using either the surface area or the in-plane shear angle as the sole parameter to maximise, a combined parameter was established where the product of the in-plane shear angle and the surface area is calculated for each drape element and summed over all the drape elements. Maximising the product of the surface area and in-plane shear angle offers a compromise between the two methods, as seen in Figure 4-12. While the in-plane shear angle and the surface area do not share a common unit of measurement, the use of the product of the two values remains a useful quantity within the context of this thesis. Further investigation on the interaction between these two values including a dimensional analysis may be performed in the future.

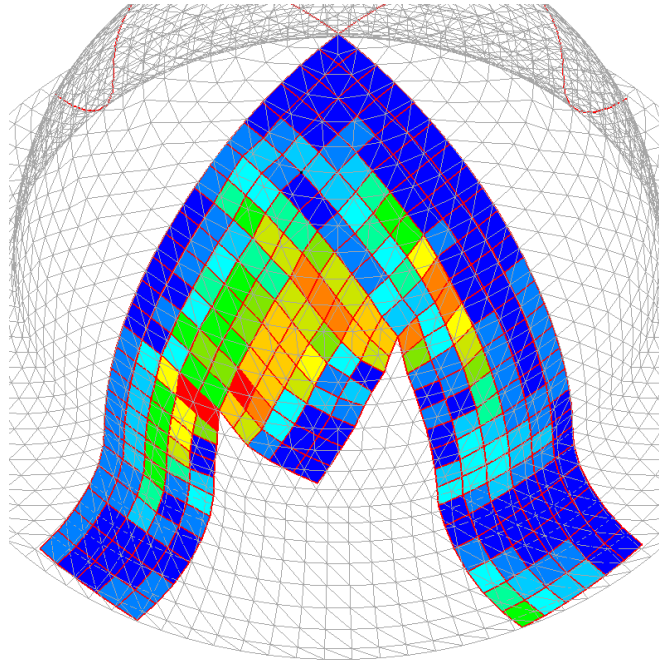


Figure 4-12 Optimising for the Product of In-Plane Shear Angle and Area

Alternatively, it is also possible to maximise the sum of in-plane shear and surface area instead of the product. However, this leads to results which are very similar to maximising in-plane shear angle only as the value of the surface area for each drape element is much smaller than the value of the in-plane shear angle. For example, if the yarn spacing can vary between 3 mm and 5 mm the surface area ranges from 9 mm² to 25 mm² while the in-plane shear angle ranges from 60° to 90°, using a 60° in-plane shear limit. Factors could be introduced in the equation determining the parameter to optimise so as to alter the priority given to each aspect of the drape.

A comparison of the parameters to maximise based on the in-plane shear angle and surface area can be seen in Table 4-1, where five optimisations were performed for each parameter to maximise. The same model and identical parameters, except for the parameter to

maximise, were used for the optimisations. The column describing the in-plane shear of the textile is based on visual inspection of the results; typical optimisations can be seen in Figure 4-10, Figure 4-11 and Figure 4-12 for optimisations with the surface area, in-plane shear angle and the product of in-plane shear angle and surface area as the parameter to maximise, respectively.

Parameter to Maximise	Average Area (cm ²)	Standard Deviation (cm ²)	In-Plane Shear
Surface Area	66.23	0.89	Most
In-Plane Shear Angle	66.74	3.93	Least
Surface Area * In-Plane Shear Angle	68.45	1.80	Average
Surface Area + In-Plane Shear Angle	67.07	2.33	Least

Table 4-1 Comparison of the Parameters to Maximise

It is also possible to introduce the fibre volume fraction within the definition of the parameter to maximise. In cases where a low variation of v_f over the surface is desired, assuming constant thickness, it is possible to minimise the standard deviation of v_f across the textile. Alternatively it is possible to optimise a textile with the goal of achieving values of v_f across the textile which lie between a maximum and a minimum. The parameter to maximise based on an allowable range of v_f is given a value of 1 for the ideal v_f , defined via the GUI, and a value of 0 for the maximum and minimum v_f , also defined via the GUI. For other values of v_f , the parameter to maximise is given a value based on linear interpolation between the ideal v_f and either the maximum or minimum value for v_f , as shown in Figure 4-13, where the relationship between v_f and the parameter to maximise is illustrated.

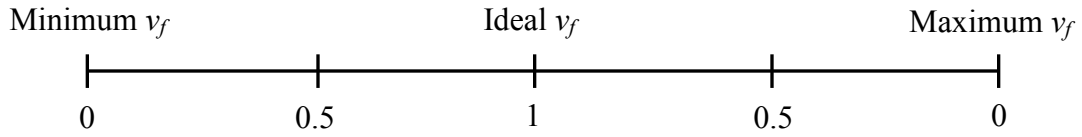


Figure 4-13 Parameter to Maximise Based on an Allowable Range for v_f

The final method of determining the parameter to maximise is based on fibre orientation. This optimisation is generally used in conjunction with a model that contains at least one patch for which a specific desired fibre orientation is defined, as seen in Figure 4-14.

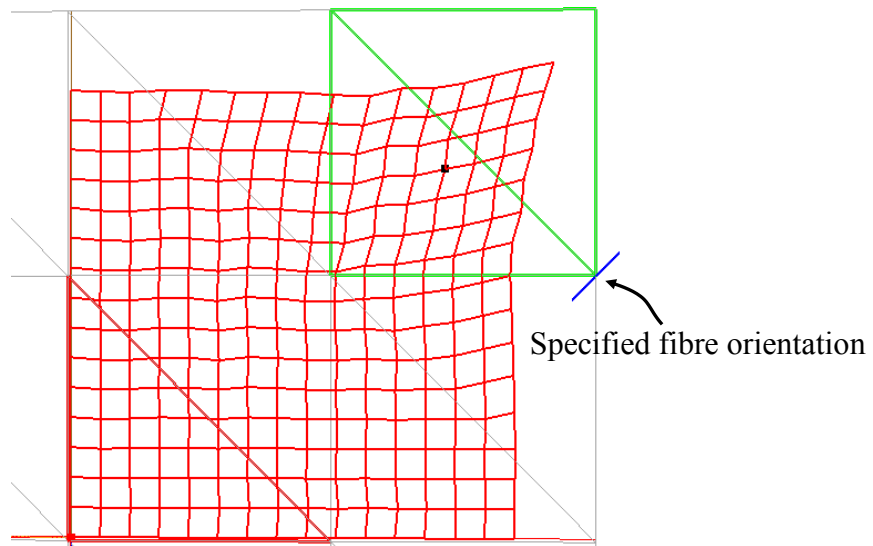


Figure 4-14 Optimising for Fibre Orientation

The specified fibre orientation is shown as a blue line in Figure 4-14, defined using the GUI. Two angles φ_1 and φ_2 can be found, the first between vector $\overrightarrow{p_A p_s}$ and the specified fibre orientation, and the second between vector $\overrightarrow{p_B p_s}$ and the specified fibre orientation. Both vectors $\overrightarrow{p_A p_s}$ and $\overrightarrow{p_B p_s}$ are as shown in Figure 3-40 while the equation used to find the value of the

angle is similar to equation (3.96). Once these angles are found, the sum of the absolute value of the difference between 90° and each angle is taken so as to obtain a value which is larger when the yarns and specified fibre orientation are aligned:

$$\varphi = |90 - \varphi_1| + |90 - \varphi_2| \quad (4.3)$$

This value of φ is used as the parameter to optimise. The behaviour resulting in such optimisation cases is the gradual alignment of yarns along the specified direction, within the area delimited by the patch, as shown in Figure 4-14.

4.2.3. Optimisation Type

The software is capable of performing textile reinforcement optimisations following two different schemes. The optimisation may be performed over the entire surface of the simulated textile or it may be limited to a specific section which grows as the optimisation progresses. The schemes are referred to as static and progressive optimisation, respectively. In static optimisation, all of the yarn spacings between draped yarn crossovers are eligible to be modified at any given time, while in progressive optimisation only a portion of the simulated textile is draped and only a portion of those yarn spacings between draped yarn crossovers can be modified. The initial section to be draped in a progressive optimisation is dependent on the initial size parameter, defined via the GUI. This parameter determines how many yarn crossovers can be draped along each geodesic. A non optimised constant-length textile and the initial section of a progressive optimisation with an initial size parameter of 5 can be seen on the left-hand side and the right-hand side of Figure 4-15, respectively. For the example given only one quadrant is

displayed; however, it is possible to perform a progressive optimisation on any number of quadrants.

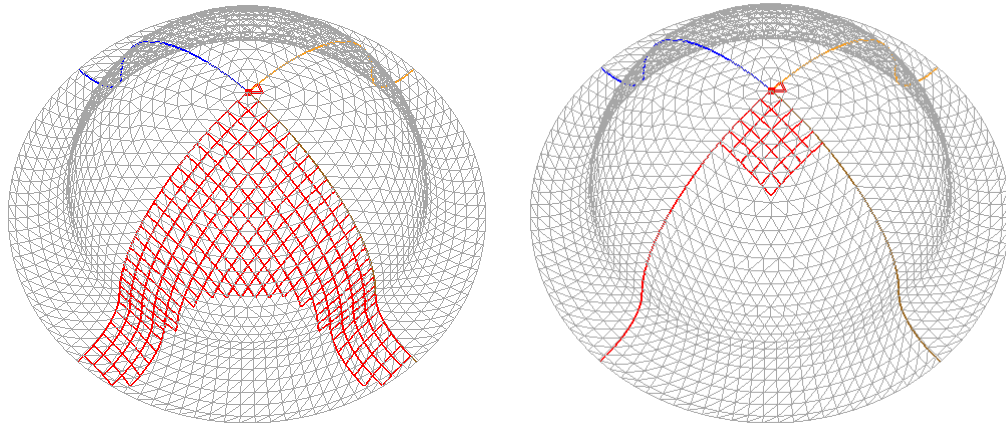


Figure 4-15 Non Optimised Textile and Initial Section of a Progressive Optimisation

The section of the textile for which changes to the yarn spacings are permitted is a layer of yarn spacings with a size counted in yarn crossovers from the outermost draped yarn crossover. This layer size is defined via the GUI and is shown in Figure 4-16, where the layer size is two, as shown by the two layers of yarn crossovers for which the yarn spacings have increased through optimisation to maximise the surface area.

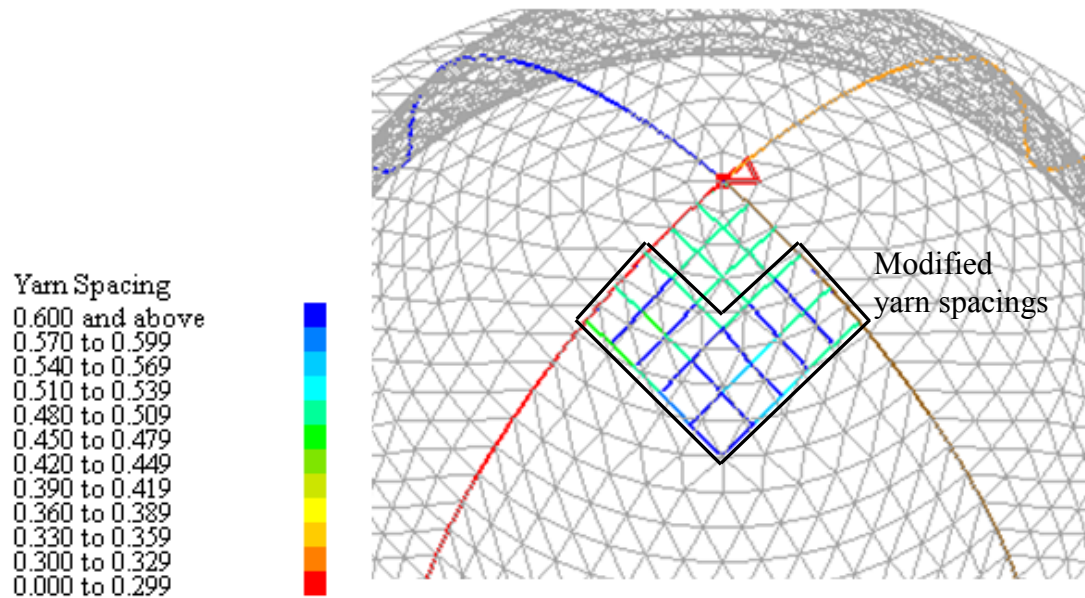


Figure 4-16 Layer of Yarn Spacings Eligible for Modification in a Progressive Optimisation

The size of the section of the textile being draped grows by one yarn crossover when the initial section being optimised reaches steady state. The section of yarn spacings eligible to be modified is also modified when the portion of the textile grows. The layer size does not increase in terms of depth but moves along with the outer edge of the draped textile. The end result of the optimisation is shown in Figure 4-17. The optimisation process ends when the final size of the draped textile is reached, defined via GUI along with the initial size parameter, or when the edge of the model or patch is reached. It is important to note that yarn spacings which were not eligible for modification in the initial section remain unchanged throughout the optimisation process.

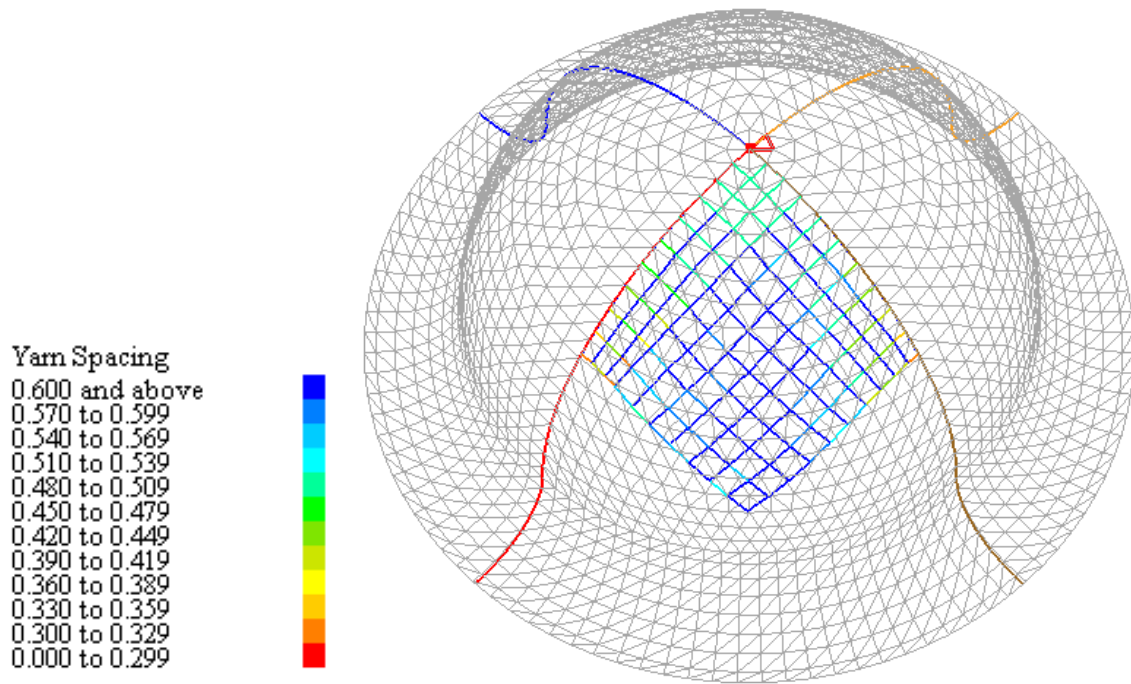


Figure 4-17 End Result of a Progressive Optimisation

To prevent sections of the textile from being locked at their original values, it is possible to set the layer size to a value equal to the initial size, or to define a layer size of zero which indicates that any yarn spacing between yarn crossovers that are currently draped are eligible for modification. To increase the speed of the progressive optimisation process, the yarn spacings of new layers added when the progressive optimisation reaches a steady state condition can be modified to match the values of the previous layer in a manner similar to what is described in Figure 3-44. This feature is optional and may be activated when defining the optimisation parameters.

5. Results and Validation

5.1. Constant Length Textiles: Validation of the Draping Software

The kinematic draping model used as the starting point of work presented in this thesis is widely used [7-10, 58, 59] and its validity is not questioned. However, the current implementation must be validated to ensure proper implementation. Furthermore, validation was also required regarding modifications made to enable the draping of variable-length textiles. In this chapter the draping software is validated by comparing textile reinforcements that were draped onto moulds with results obtained from the simulation of the draping on a model of the same mould. Validation of the software was performed with assistance from James Cowan, undergraduate student at the University of Ottawa in 2010, who produced a document detailing some aspects of this software validation [72]. Since the software does not account for variable resistance to in-plane shear, a plain woven fibreglass was used for validation work related to constant-length textiles. Validation of the software was performed on two different moulds, a hemisphere and a non-structural aerospace fairing.

5.1.1. Validation on the Hemisphere

A hemisphere was chosen as a first validation tool since it is doubly curved, thus forcing textiles draped onto it to deform by in-plane shear. The model of the hemisphere is shown in Figure 5-1 and the machined mould based on the model is shown in Figure 5-2. The hemisphere has a radius of 7.5 cm and is mounted on a cylinder of the same radius with a height of 4 cm.

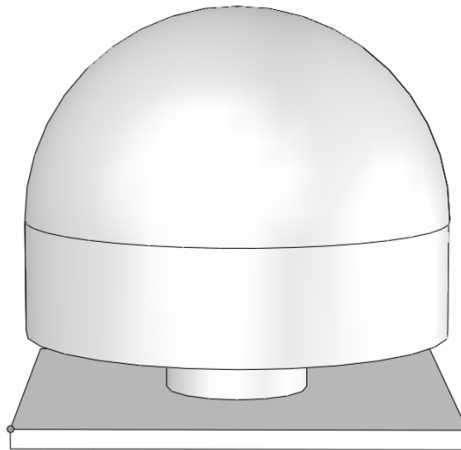


Figure 5-1 Hemisphere Model Used for Software Validation

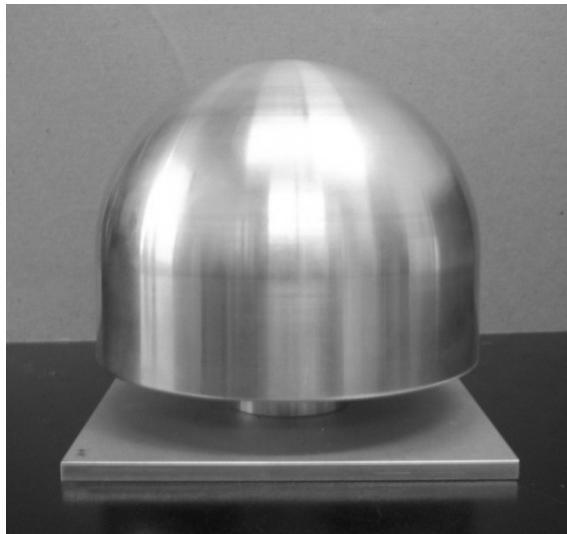


Figure 5-2 Hemisphere Mould Used for Software Validation

Initial validation may be performed by visual inspection of the draped textile and comparison with the simulation results. Examples of each appear on the left-hand side and the right-hand side of Figure 5-3, respectively.

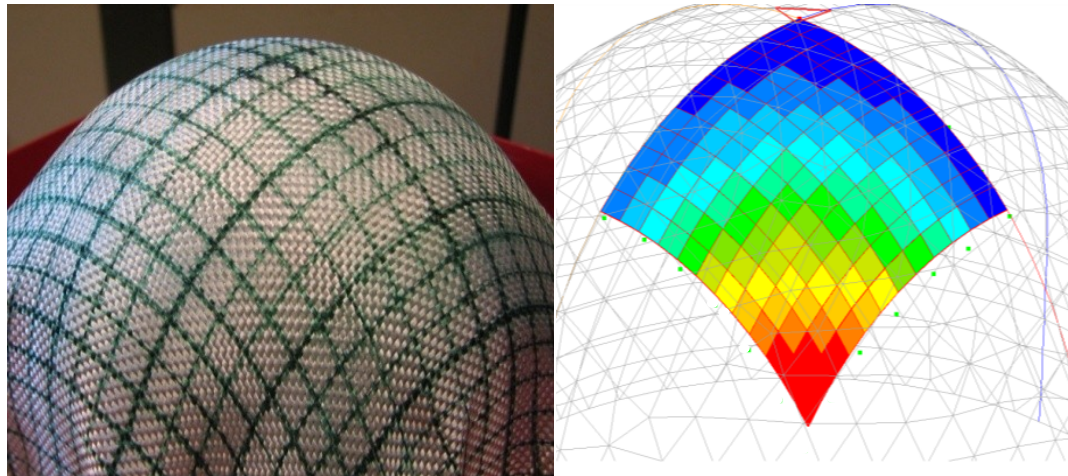


Figure 5-3 Comparison of Drape and Simulation (Hemisphere)

The lines seen on the draped fabric are aligned with the warp and weft yarns and were drawn at a 1 cm interval. The direction of the lines on the draped fabric appear to be very similar to the direction of the virtual yarns seen in the simulation. Additionally, the shape of the drape elements created by the intersecting lines are the same shape in the experiment and simulation. More accurate validation was done with the use of an Immersion MicroScribe G2X 3D coordinate measurement machine (CMM) which has a specified accuracy of 0.23 mm. The CMM is used for storing the 3D position of intersections of the lines drawn on the textile. The CMM must be calibrated using points of reference on the mould as seen in Figure 5-4, to ensure that the origin of the model used by the simulation is the same as the origin of the mould used for the laboratory drape.

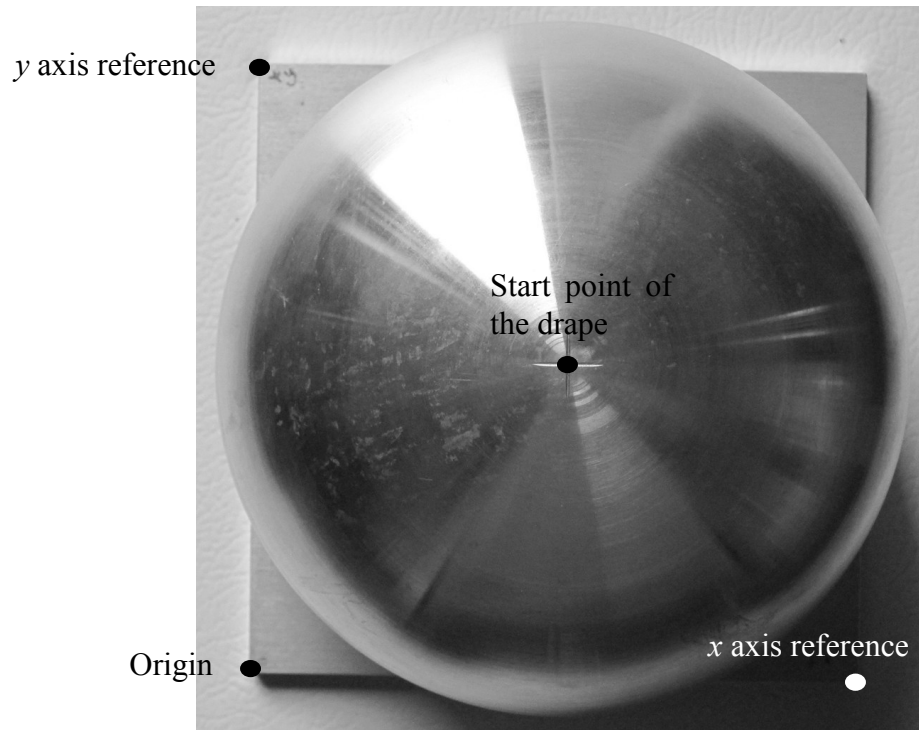


Figure 5-4 Points of Reference on the Hemisphere Used for Calibrating

The textile must be handled carefully to ensure that no deformation occurs to it before the draping process is undertaken. While draping, it is important to position the start point of the drape accurately on the appropriate marking and to lay yarns along the geodesics as intended. The experimental draping for which CMM data was collected is shown in Figure 5-5 where the arrows represent the geodesics, the intersection of which is the drape start point.



Figure 5-5 Laboratory Drape on the Hemisphere With Start Point and Geodesics [72]

The coordinates of the intersections of the lines drawn on the textile were stored to file; this file can then be loaded as green dots into the draping software and superimposed over the simulation results as seen in Figure 5-6.

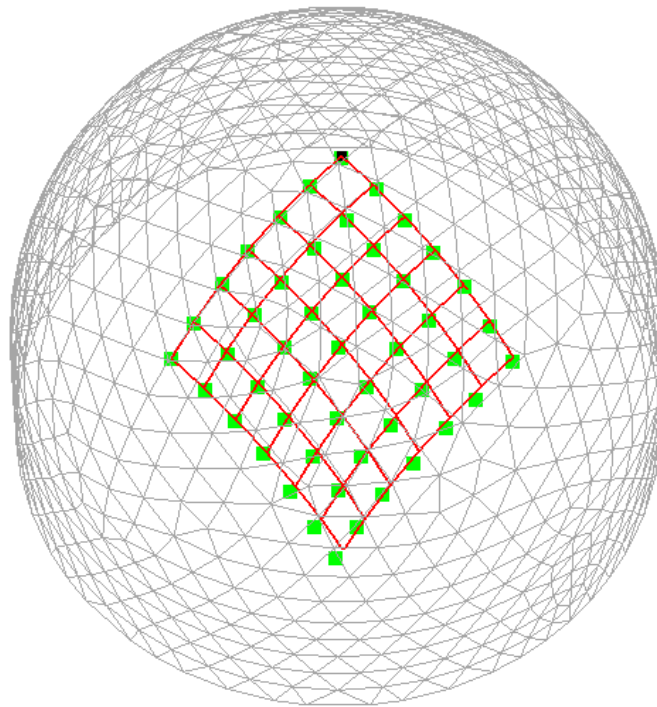


Figure 5-6 One Quadrant Validation Test on the Hemisphere

A certain amount of manipulation, in the order of a few millimetres, to the parameters of the draping simulation is needed to ensure proper alignment between the start points and geodesics of the experimental trial and simulation. The start point of the drape for the simulation, intended to be at $(x,y) = (7.5 \text{ cm}, 7.5 \text{ cm})$ was adjusted to $(x,y) = (7.399 \text{ cm}, 7.702 \text{ cm})$. The misalignment is caused by the practical difficulty of placing the textile perfectly onto the drape start point marking of the hemisphere. When superimposed, it is clear that the software predicts the behaviour of the textile accurately as it is draped over the surface. However, the accuracy of the simulation tends to decrease as the amount of in-plane shear increases, as seen at the bottom tip of the simulation results in Figure 5-6. As the in-plane shear of the textile increases, other deformation modes such as yarn slip occur which the software does not account for. Additionally, handling of the textile during the draping process is done by hand which may induce certain deformations that are not accounted for by the software. While the position of the CMM data does not align perfectly onto the results of the simulation, the direction of the yarns as well as the angles between intersecting yarns are almost identical; therefore the software did accurately simulate the draping of a textile onto the hemisphere. Consistency was verified by performing a validation trial over four quadrants of the hemisphere, using the same mould and model. The results of the four quadrant validation test superimposed onto the drape simulation, with start point $(x,y) = (7.360 \text{ cm}, 7.407 \text{ cm})$ and geodesic rotation of 5° , are shown in Figure 5-7.

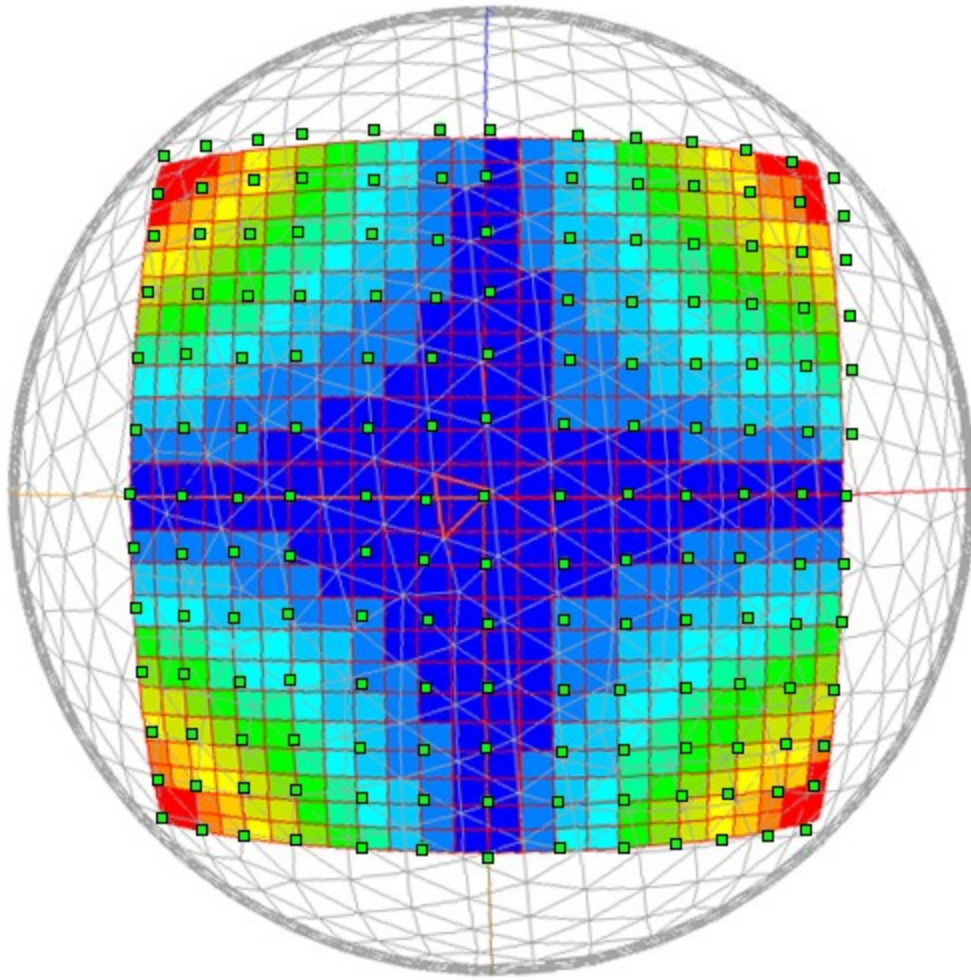


Figure 5-7 Four Quadrant Validation Test on the Hemisphere [72]

The results obtained from the two bottom quadrants correlate very well with the simulation while the two top quadrants show larger discrepancies between the experiment and simulation. The discrepancies in the top quadrants were attributed to likely movement of the draped textile during acquisition of the CMM data. The distances between the top geodesic and adjacent dots are inconsistent, pointing to the possibility of the textile having been moved inadvertently between the acquisition of the intersection locations on the geodesics, collected first, and the acquisition of the intersection locations between geodesics, collected last. Due to the satisfactory results obtained from both validation trials, this trial was not redone.

5.1.2. Validation on the Aerospace Mould

To further validate the draping software, an aerospace fairing mould, shown in Figure 5-8, was used for comparing results of experimental trials with predictions of the draping software.



Figure 5-8 Aerospace Mould Used for Validation

As with the hemisphere, quick visual inspection of the results shown in Figure 5-9 offers promising results.

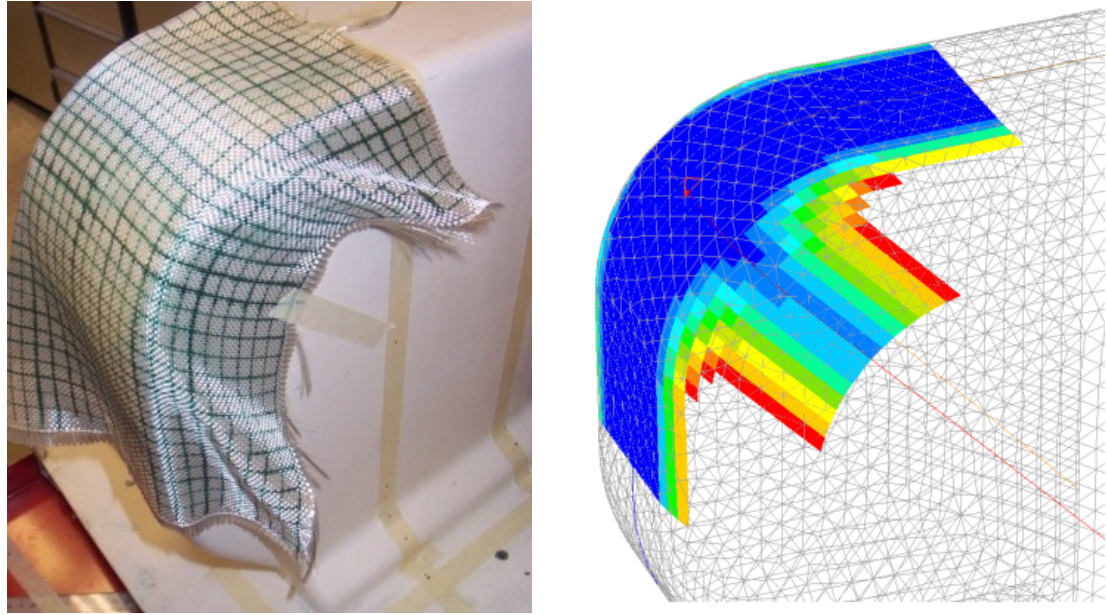


Figure 5-9 Comparison of Drape and Simulation (Aerospace Mould)

The simulation predicts that sections of the textile would exceed the in-plane shear limit, as shown by the absent drape elements on the simulation. When draping the textile onto the mould, the fabric deforms out-of-plane over the same sections as the locking angle of the textile has been exceeded. Again, a CMM was used to store the location of intersections of lines drawn onto the textile, which were then superimposed onto the simulation results generated by the software. The CMM was calibrated using points of reference on the mould surface shown in Figure 5-10 to ensure alignment of the origins and axes of the mould and model.

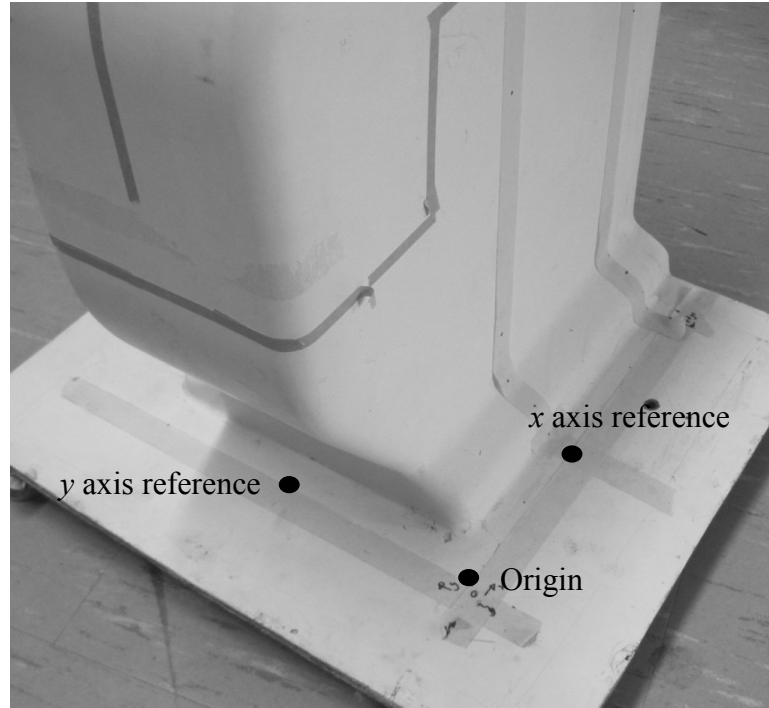


Figure 5-10 Points of Reference on the Mould Used for Calibrating

The draping simulation is centered at $(y,z) = (12.11 \text{ cm}, 31.02 \text{ cm})$, chosen to ensure the CMM data coincides with the simulation results. The textile used for the simulation has a yarn spacing of 1 cm and 15 yarn crossovers were meshed in each direction. The CMM data of the experimental drape superimposed on the simulation results is shown in Figure 5-11 and Figure 5-12. To achieve a simulation which predicts out-of-plane deformation on the top right corner of the mould seen in Figure 5-12, the in-plane shear limit of the simulation was set to 36° .

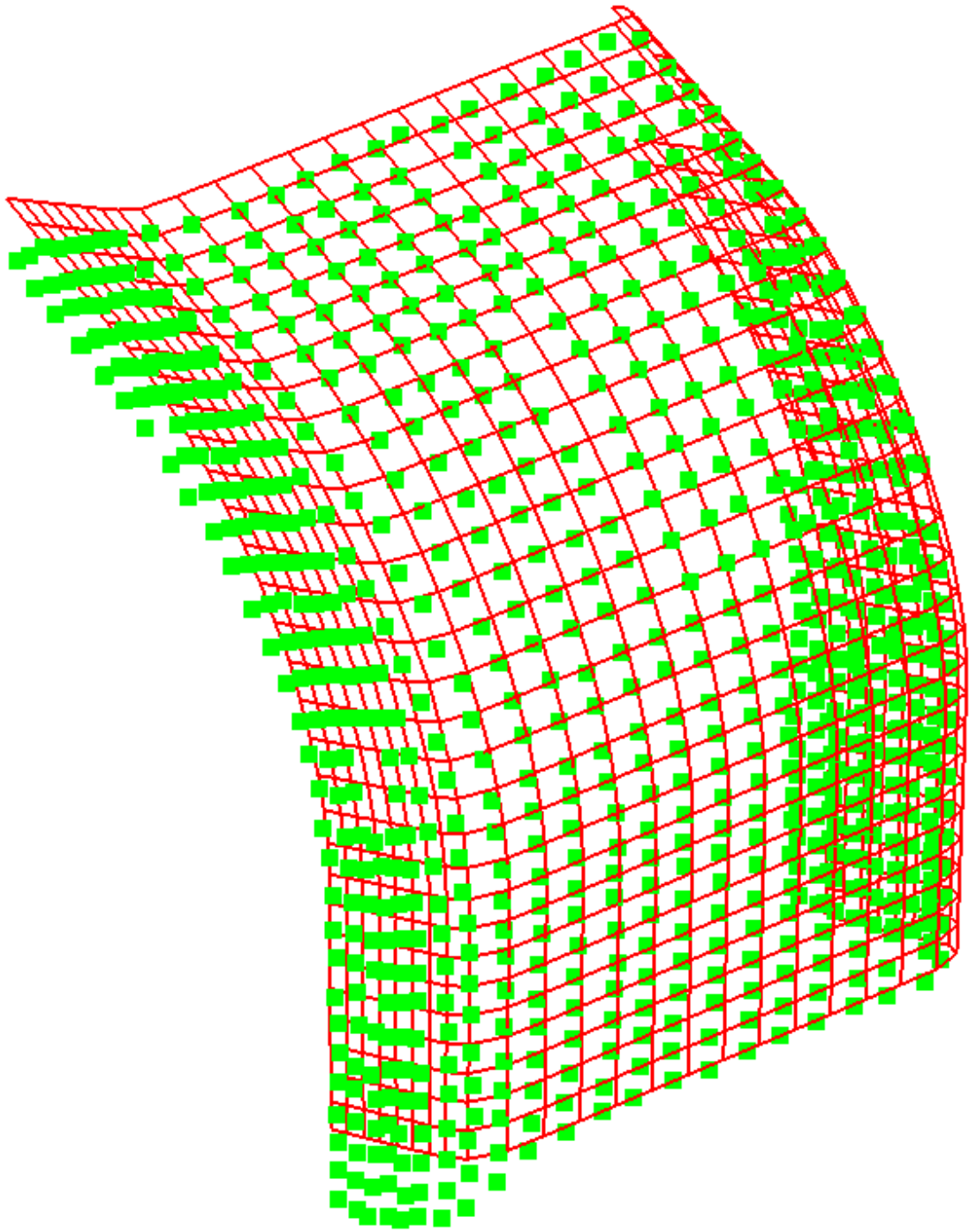


Figure 5-11 Four Quadrant Validation Test on the Aerospace Mould (View 1)

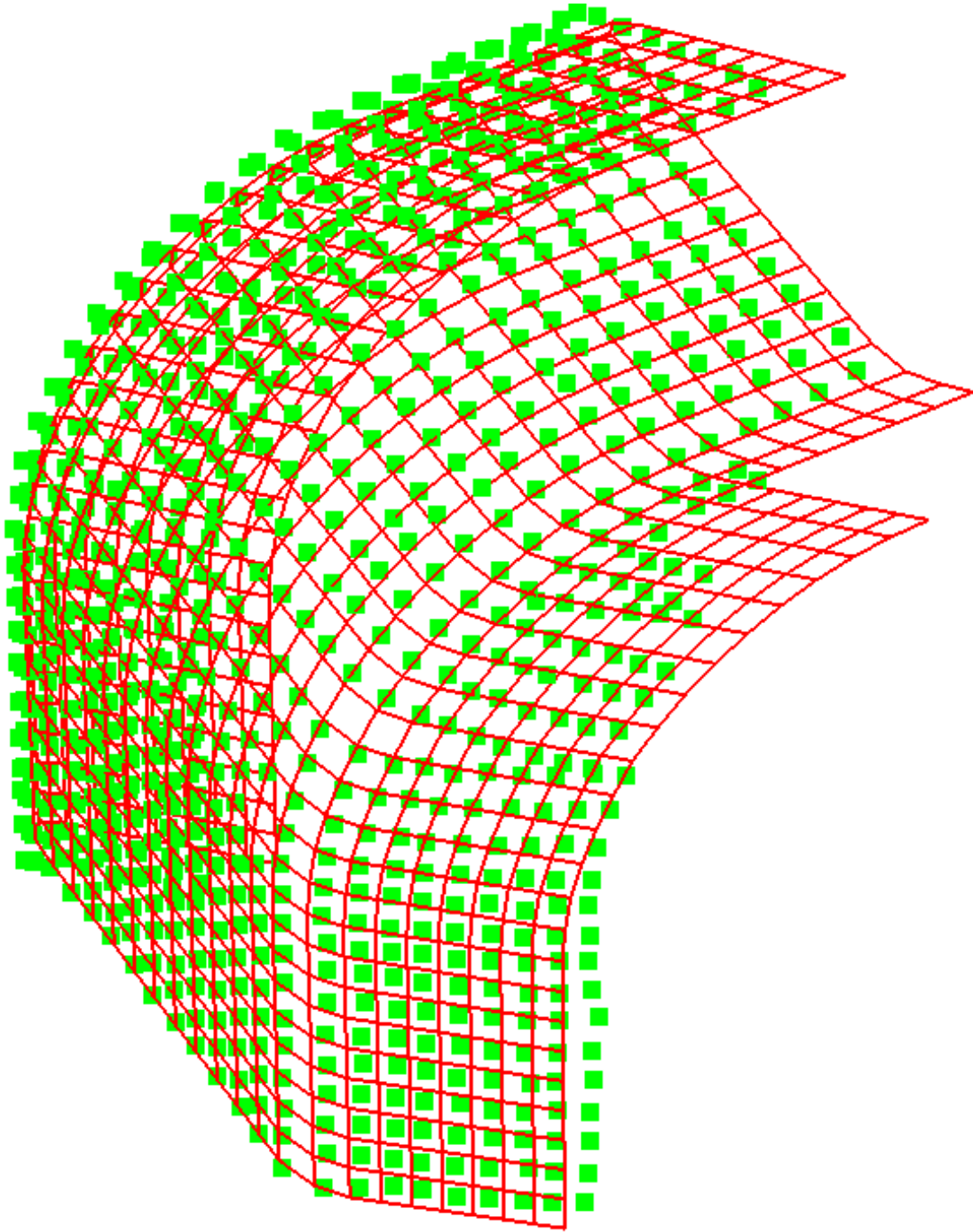


Figure 5-12 Four Quadrant Validation Test on the Aerospace Mould (View 2)

As with the validation performed on the hemisphere, the results of the experimental drape on the aerospace mould corresponds to those of the simulation in areas where in-plane shear is

low and diverges somewhat from the simulation where high in-plane shear is observed. Again, this is due to limitations in the CMM precision, human error and manipulations made to the textile while draping which may introduce additional deformations. Also, the model of the mould which was supplied is slightly smaller than the mould as the model actually corresponds to the part made using the solid mould; therefore the experimental draping extends beyond the simulation, as seen on the left in Figure 5-13. This may account for the greater discrepancies between the experimental draping and simulation which occur on the left-hand side of mould.

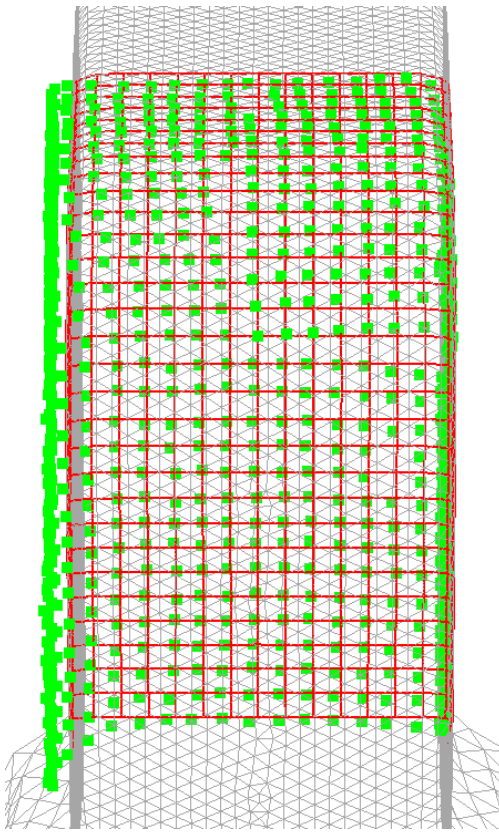


Figure 5-13 Visualisation of the Difference in Size Between the Model and Mould

5.2. *Variable-Length Textiles: Optimisation Results*

The optimisation process and software were used in designing several textiles to evaluate the effectiveness of the optimisation process. Optimisations performed aimed at maximising the surface area covered by a single piece of textile reinforcement or at modifying the fibre orientations to meet specific requirements. Both the models for the hemisphere and the aerospace mould were used in order to perform the optimisations.

5.2.1. *Surface Area Maximisation on the Hemisphere*

The goal of this optimisation was to maximise the surface area covered by a single piece of textile on the hemisphere described previously. The optimisation was performed on one quadrant of the hemisphere as all quadrants are identical; the configuration of yarn spacings found for one quadrant can be copied over to complete the hemisphere. The non-optimised constant-length textile with a yarn spacing of 4 mm leads to a draped surface of 91.0 cm² shown in Figure 5-14, before reaching the maximum value of in-plane shear set at 60°.

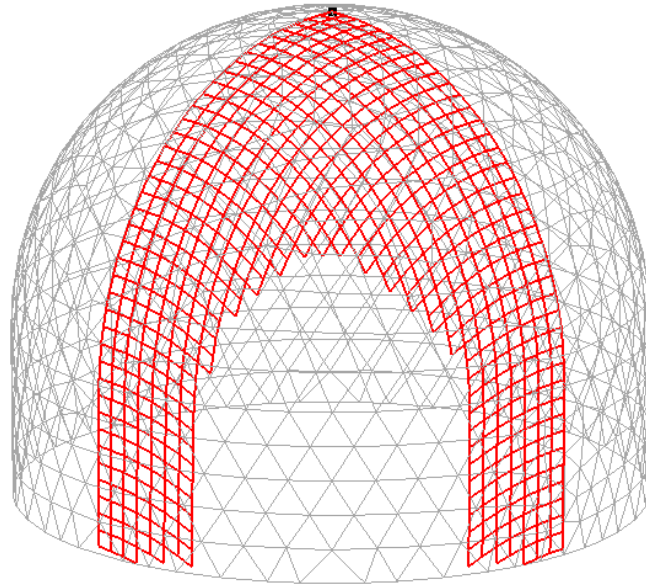


Figure 5-14 Non-Optimised Textile Draped on the Hemisphere

Several dozen attempts at optimising a textile for this model were done with the best results attained from a progressive drape, details of which can be found in Appendix B. The result of the progressive optimisation is shown in Figure 5-15, with an area of 108.4 cm^2 .

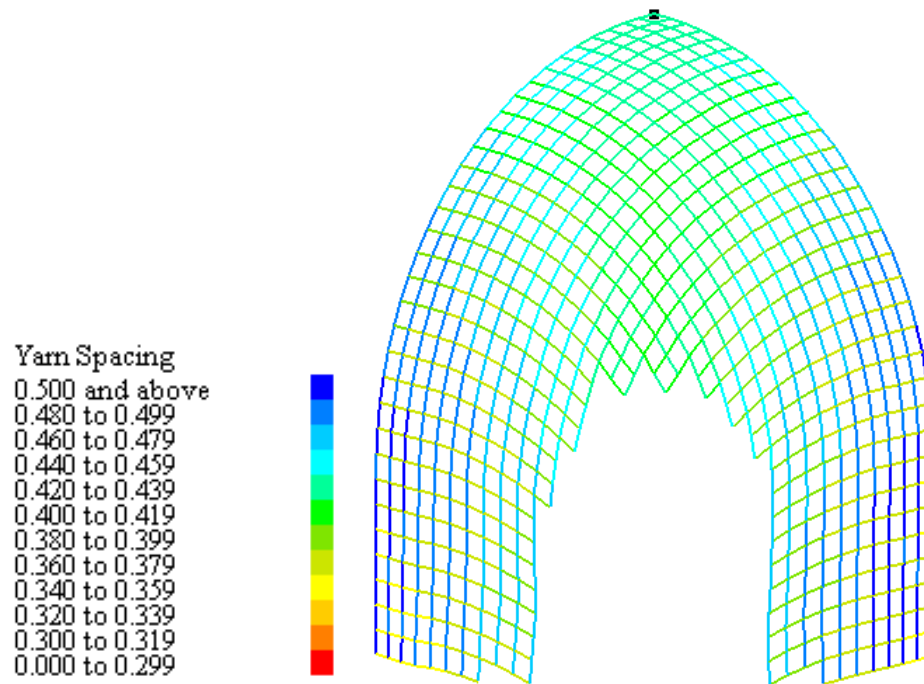


Figure 5-15 Progressive Optimisation for the Hemisphere

While a modest increase of 19% to the surface area was achieved, the progressive optimisation did not meet the objective of fully draping the hemisphere. A manual optimisation was performed with the goal of producing a textile that can drape lower along the hemisphere without reaching the maximum in-plane shear limit. The result of the manual optimisation is shown in Figure 5-16, the surface area of which is 103.3 cm².

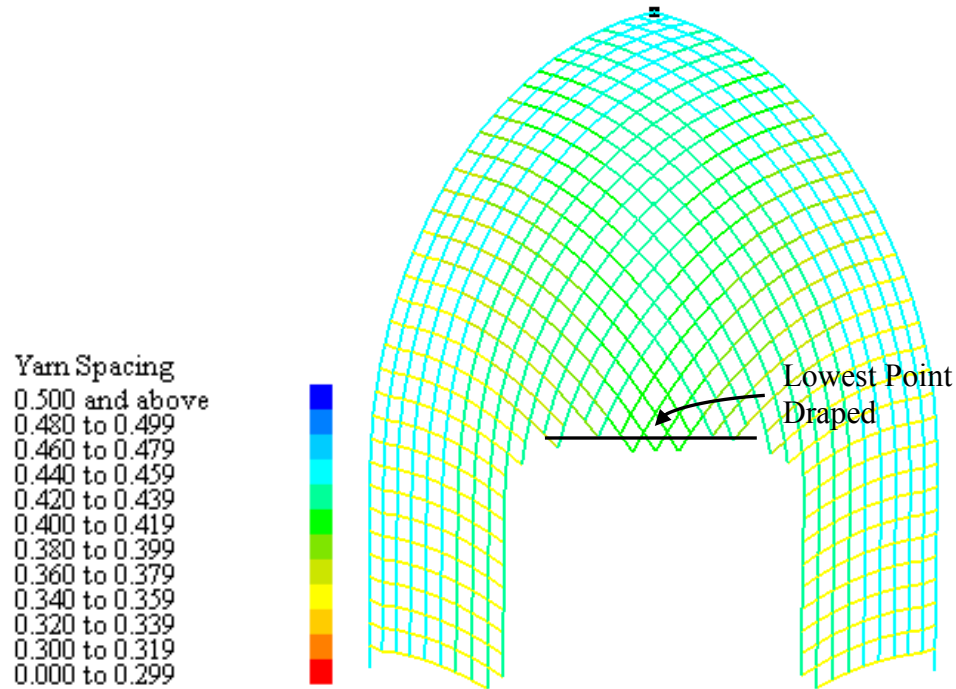


Figure 5-16 Manual Optimisation for the Hemisphere

The surface area covered through manual optimisation is smaller than the surface area covered by the progressive optimisation; however, the lowest point draped on the manually optimised textile is further away from the apex of the hemisphere than the lowest point draped on the textile resulting from the progressive optimisation. Depending on the situation, design and manufacturing options, either textile may be more desirable. Once the optimisation was done, the manually optimised textile was manufactured by Reza Samadi as part of a parallel research project. A pattern shown in Figure 5-18, based on the optimised textile as it appears when laid flat as shown in Figure 5-17, was used for aligning and stitching the textile.

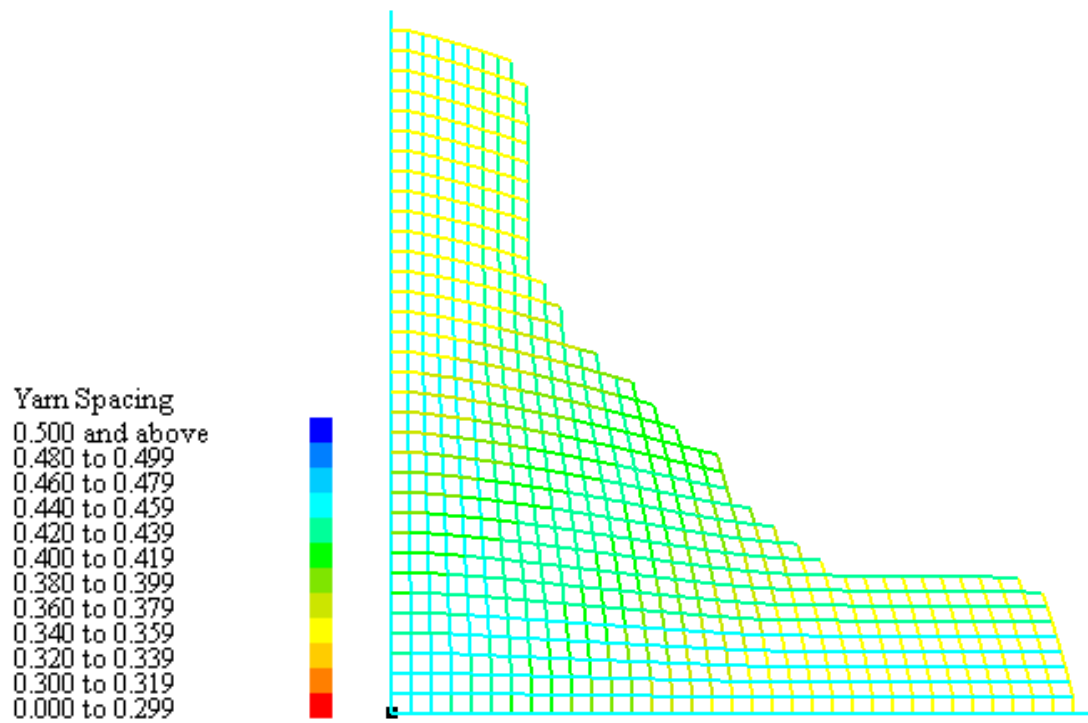


Figure 5-17 Optimised Textile for the Hemisphere as it Appears When Laid Flat

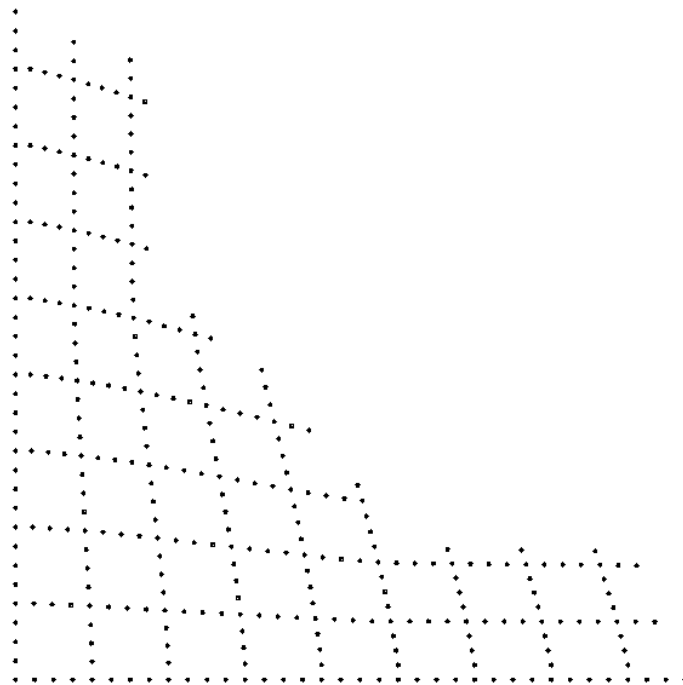


Figure 5-18 Pattern Used for Manufacturing the Hemisphere Textile

The textile was manufactured by laying yarns between pins, shown in Figure 5-19. The pins are placed at the location of the dots shown in Figure 5-18, onto a paper template which is removed once the textile is stitched.

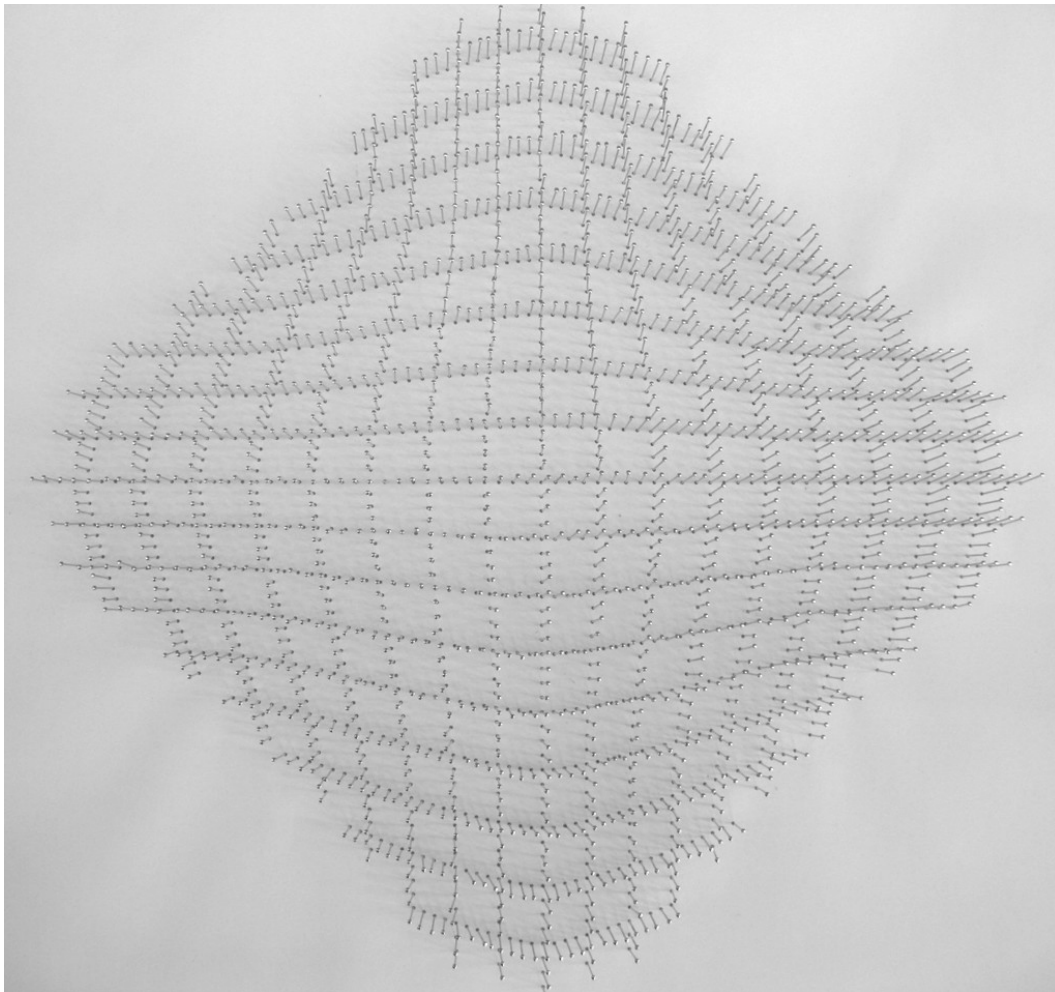


Figure 5-19 Pins Used for the Manufacture of Optimised Textiles

The manufactured textile appears in Figure 5-20 as draped over the hemisphere mould. It should be noted that the textile covers the hemisphere better than the non-optimised textiles did, proving the effectiveness of the optimisation process and demonstrating that variable-length textiles can in fact be manufactured.

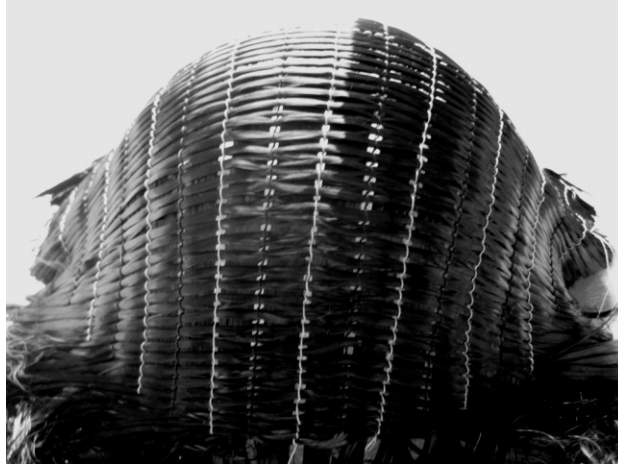


Figure 5-20 Manufactured Textile for the Hemisphere Optimised for Surface Area

5.2.2. Surface Area Optimisation on the Aerospace Mould

The goal of this optimisation was to maximise the surface area covered by a single piece of textile within a patch, visible in Figure 5-21, on the aerospace mould. Again, the maximum in-plane shear angle for the textile was set at 60° .

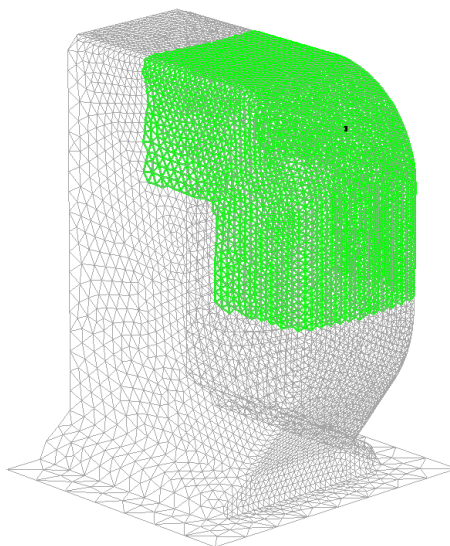


Figure 5-21 Patch on the Aerospace Mould to Maximise Surface Area

The surface area covered by the non-optimised constant-length textile as seen in Figure 5-22 is 674 cm².

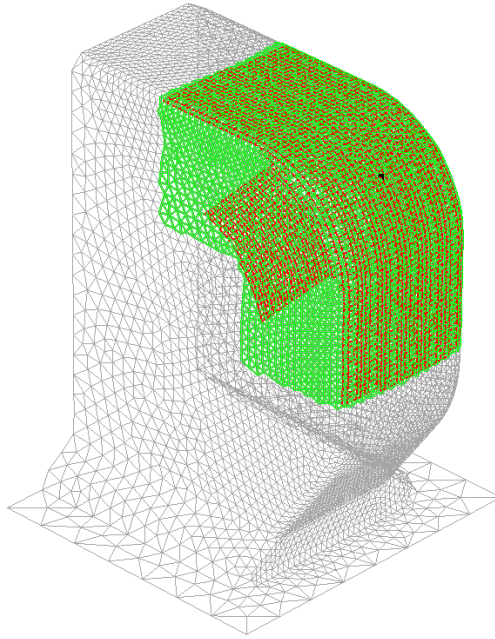


Figure 5-22 Non-Optimised Textile on the Aerospace Mould

Multiple optimisations using the software were attempted; as with the hemisphere a progressive drape was the most successful, the details of which are found in Appendix B. The result of the progressive drape is shown in Figure 5-23; only one quadrant was optimised to speed up optimisation.

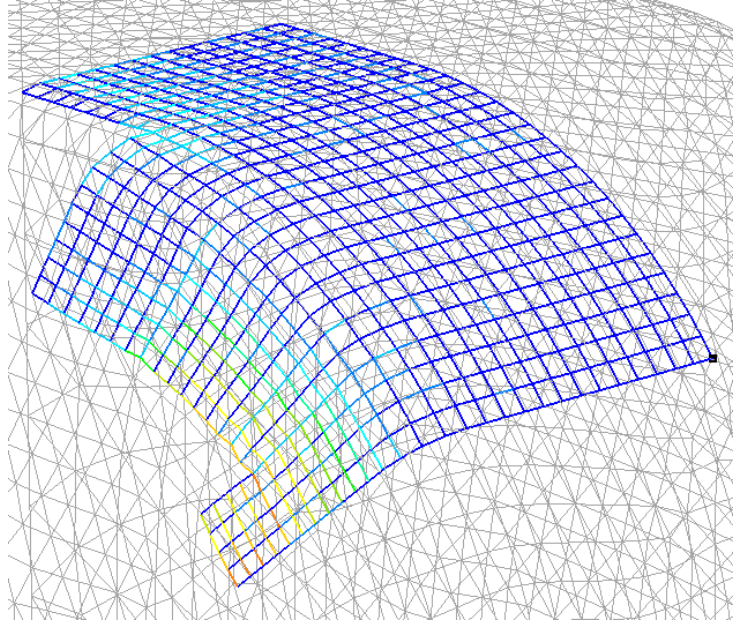


Figure 5-23 Result of a Progressive Optimisation on the Aerospace Mould

While the results of the optimisation are not ideal in the sense that the patch is not fully covered, the general pattern of the yarn spacings was used as a starting point to perform the manual optimisation of a textile that covers a greater surface area. The results of two different manual optimisations are shown in Figure 5-24 and Figure 5-25.

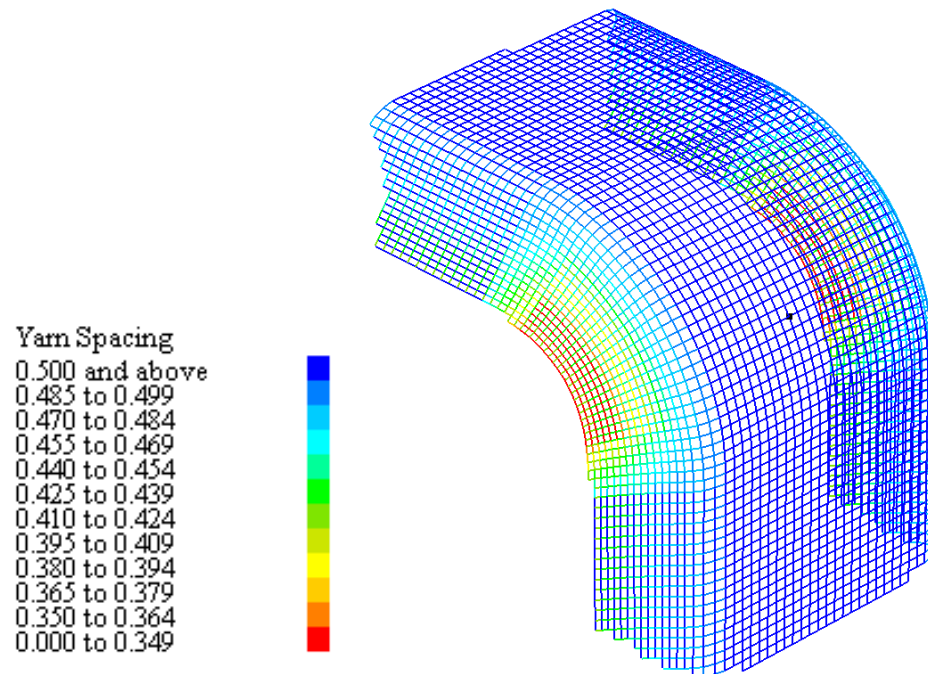


Figure 5-24 Manual Optimisation A of the Aerospace Mould

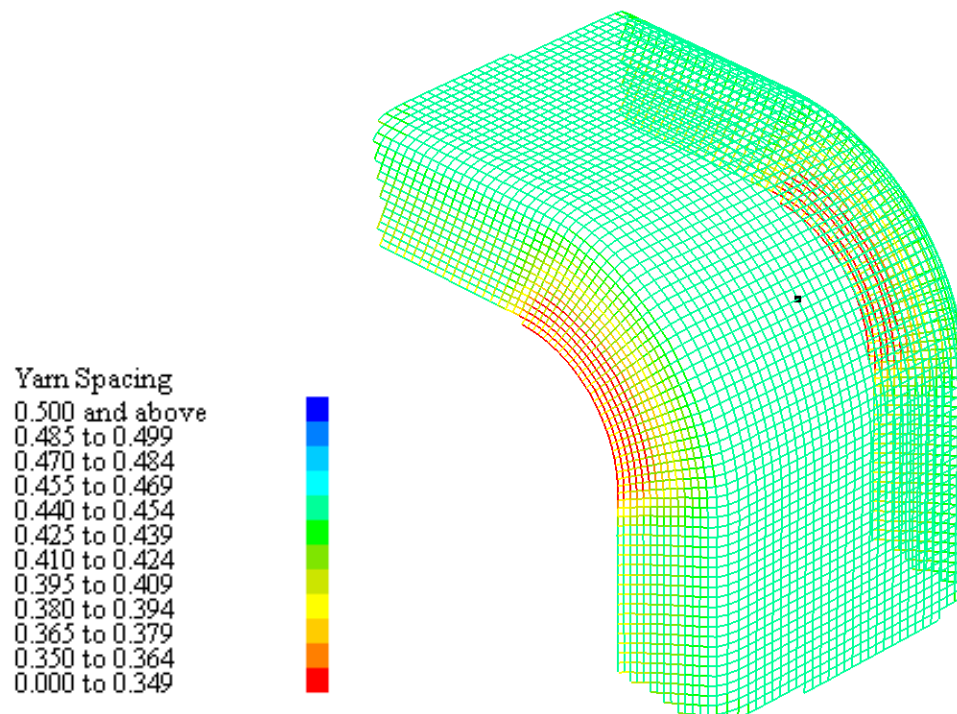


Figure 5-25 Manual Optimisation B of the Aerospace mould

The optimisations shown in Figure 5-24 and in Figure 5-25 were performed with maximum values for yarn spacing of 5 mm and 4.5 mm, respectively. This was done to observe the impact of changing the range of allowable yarn spacings on the optimisation. The optimisation performed with a maximum value of 4.5 mm achieved a surface area of 806 cm² while the optimisation performed with a maximum value of 5 mm achieved a surface area of 850 cm², both exceeding the surface area of 674 cm² covered by the non-optimised textile. In addition to the increase in surface area, the optimisation enables the textile to drape around the corner of the mould across the entirety of the patch. The optimised textile with a maximum yarn spacing of 5 mm as draped on the mould is shown in Figure 5-26.

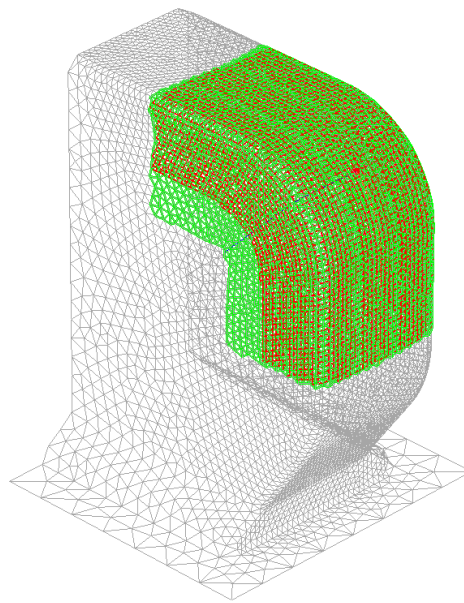


Figure 5-26 Optimised Textile on the Aerospace Mould

The optimised textile as seen when laid flat for manufacturing is shown in Figure 5-27, the pattern used for manufacturing is shown in Figure 5-28 and the manufactured textile based

on this optimisation can be seen in Figure 5-29; the textile was manufactured by Reza Samadi in a parallel research project.

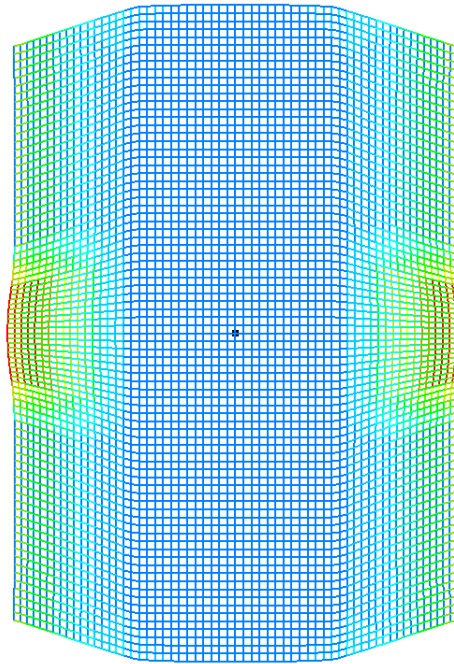


Figure 5-27 Optimised Textile for the Aerospace Mould as it Appears When Laid Flat

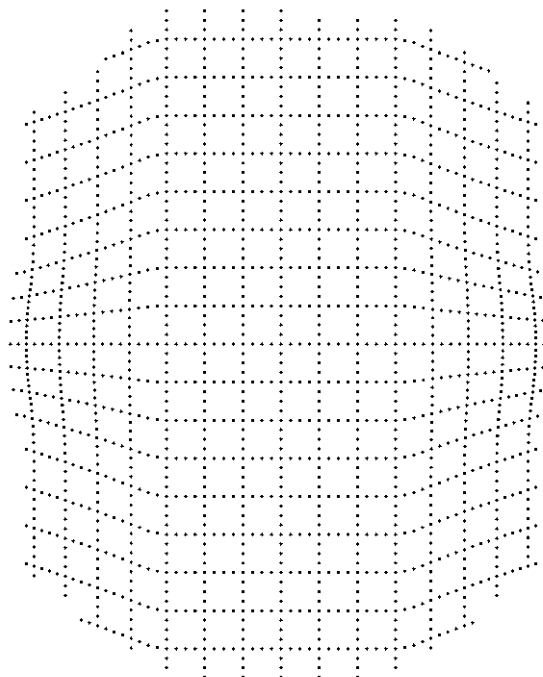


Figure 5-28 Pattern Used for Manufacturing the Aerospace Textile



Figure 5-29 Manufactured Textile for the Aerospace Mould Optimised for Surface Area

The manufactured optimised variable-length textile can be seen as draped on the aerospace mould in Figure 5-30. The manufactured textile behaves as predicted by the simulation and can successfully be used for draping the designated patch seen in Figure 5-21 without wrinkling.

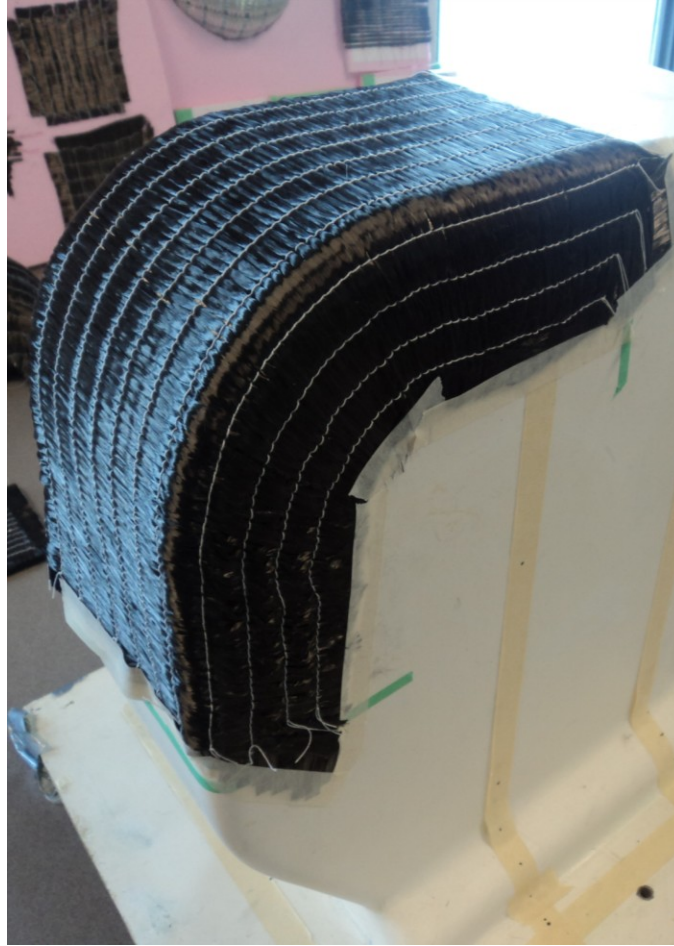


Figure 5-30 Manufactured Textile Draped Onto the Aerospace Mould

5.2.3. Fibre Orientation Optimisation on the Aerospace Mould

The third optimisation performed aimed at modifying the fibre orientation for a specific patch on the aerospace mould, as outlined in green in Figure 5-31. In some cases, design requirements exist such as the ability to carry loads acting in a specific direction. The specified fibre orientation for the patch, illustrated by the blue line in Figure 5-31, is not parallel to the initial orientation of the weft and warp yarns, shown as the solid black lines in Figure 5-31; therefore the fibre orientation must be modified within the textile.

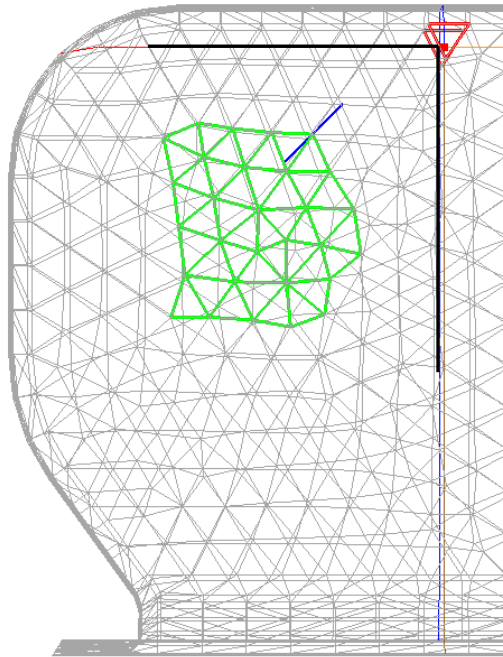


Figure 5-31 Aerospace Mould With a Patch for Fibre Orientation Optimisation

The optimisation which provided the best result was performed using a progressive optimisation with a parameter to maximise developed especially for this optimisation scenario. The parameter that was maximised is the sum of the in-plane shear angle and the angle φ defined in section 4.2.2. However, the value of the in-plane shear angle is set to 0° for drape elements within the patch to ensure that the modification of the fibre orientation of these drape elements is given absolute priority. Additionally, to ensure that the fibre orientation for sections of the textile that do not lie within the patch are not modified, the value of φ is set to 0° for drape element that do not lie within the patch. The optimisation parameters for the progressive optimisation are detailed in Appendix B and the result is shown in Figure 5-32 and Figure 5-33.

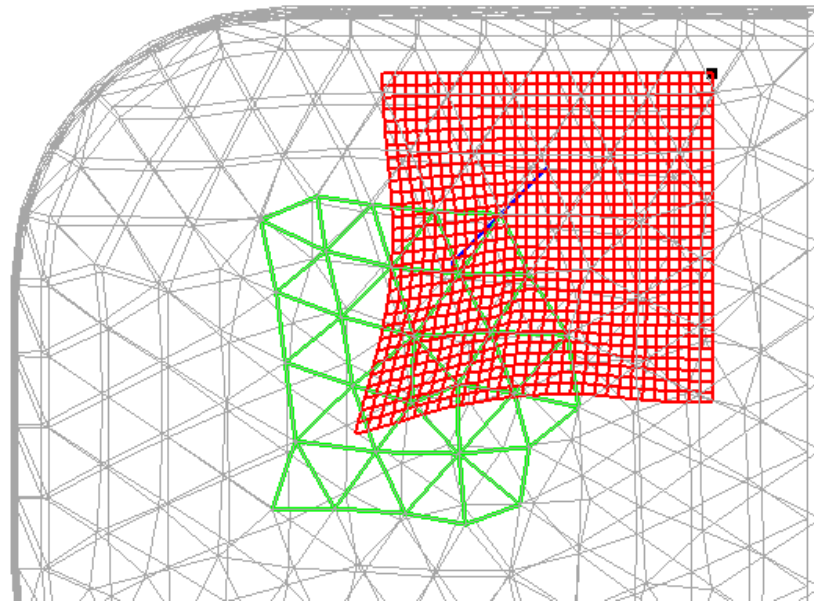


Figure 5-32 Optimisation of the Fibre Orientation on the Aerospace Mould

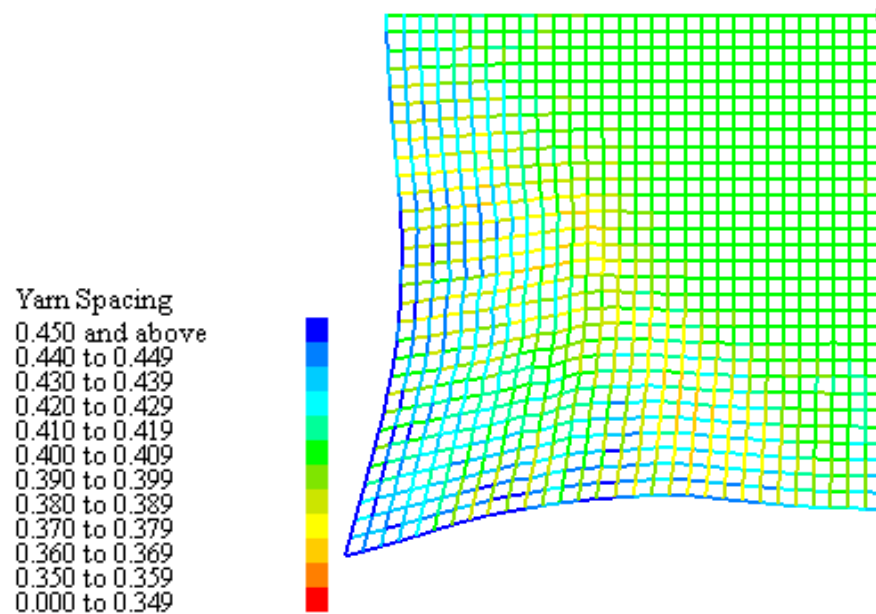


Figure 5-33 Yarn Spacings of the Fibre Orientation Optimisation

While the fibre orientations in the section of the textile that lies within the patch are not parallel to the specified fibre orientation, it is clear that they were modified towards this. Should

a load be applied along the specified orientation, the optimised variable-length textile would likely perform better than a non-optimised textile as the fibres are in better alignment with the applied load. Additionally, the section of the textile not within the patch is mostly unmodified, therefore the optimisation had minimal effect on in-plane shear angle and ν_f . Two textiles optimised of orientation were manufactured by Reza Samadi in a parallel research project. The patterns used for manufacturing these two textiles are shown in Figure 5-34 and Figure 5-35.

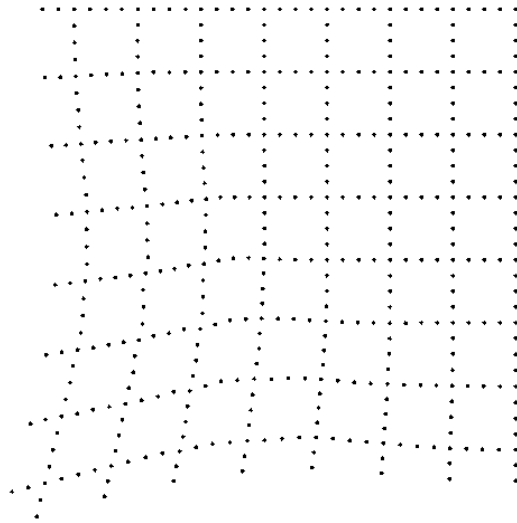


Figure 5-34 Pattern Used for Manufacturing the Aerospace Mould Textile (Textile A)

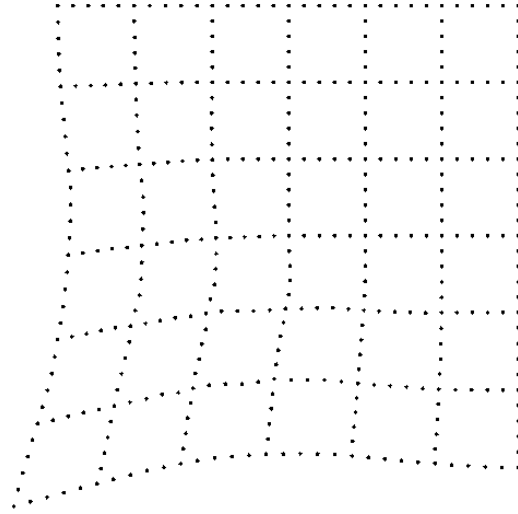


Figure 5-35 Pattern Used for Manufacturing the Aerospace Mould Textile (Textile B)

Textile A and Textile B were stitched using stitch lines at every 4 and 5 yarns, respectively. Textile A is shown in Figure 5-36 and Textile B is shown in Figure 5-37.

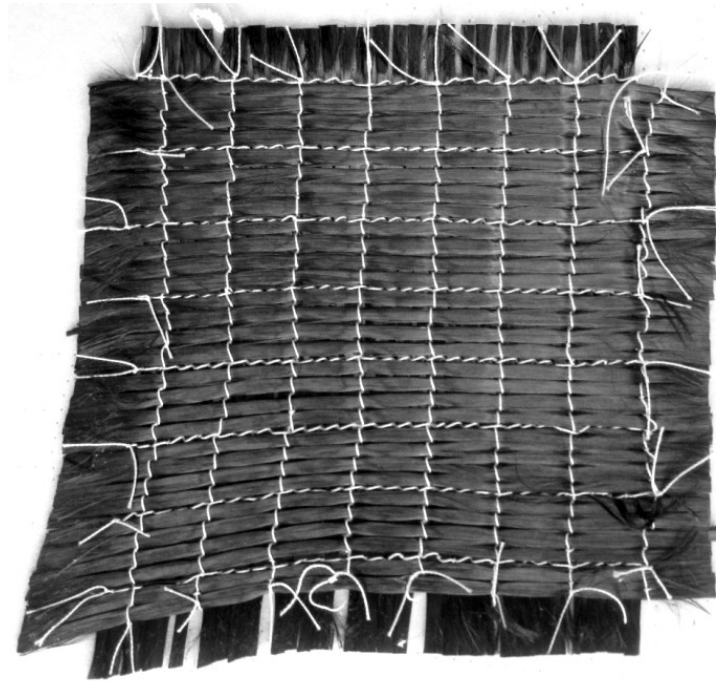


Figure 5-36 Manufactured Textile for the Optimisation of the Fibre Orientation (Textile A)



Figure 5-37 Manufactured Textile for the Optimisation of the Fibre Orientation (Textile B)

Textile A, with a stitch line at every 4 yarns appears to have kept it's intended shape better than Textile B, with a stitch line at every 5 yarns. This may be attributed to the number of stitch lines, indicating that more stitch lines are more effective at maintaining the shape of the textile.

6. Conclusion and Recommendations

The objective of this thesis was to implement a modified version of the kinematic draping algorithm to demonstrate how the operation of draping could be improved by customised, variable-length textile reinforcements. The kinematic algorithm uses geometric relations for predicting the deformation of the textile by in-plane shear during the draping operation; other deformation modes are ignored as in-plane shear is the dominant deformation mode. Additionally, the algorithm does not account for variations of the resistance to in-plane shear along different axes, which may be present in some textiles. Therefore, the software is only suitable for textiles whose resistance to in-plane shear is similar in all directions.

The software created for implementing the draping model uses a graphical user interface for providing an easy-to-use tool supporting the design and manufacture of PMC components. Geometric models can be loaded into the software and draped using many different scenarios. The results of drape simulations are displayed via 3D renderings while colour maps and text fields relay quantitative information about the drape simulation. The simulation was compared to laboratory tests, and results were very satisfactory. However, due to the inherent difficulties and imprecision involved with manually handling and draping textiles, and also to the small contribution to textile deformation by deformation modes not accounted for in the simulation, results are not perfect. While the imperfections are minor and do not detract from the benefits of the software, it may be advisable to develop more comprehensive draping simulation algorithms based on deformation energy or finite elements, as current models using deformation energy or finite elements are not equipped to drape variable-length textiles.

The optimisation of the variable-length textiles can be accomplished manually, via algorithms implemented within the draping software, or both. These algorithms have proven successful in determining optimised configurations for several optimisation scenarios. These scenarios involved maximising the surface area covered by a single textile on two different moulds, which was achieved, and modifying the fibre orientation in a specific section of a model, which was also completed successfully. In addition, textiles were manufactured in a parallel research project following the specifications obtained from optimisations. Further research and development into manufacturing methods for variable-length textiles is ongoing.

Some features intended for the software were left untested or uncompleted within the timeframe of the thesis. The implementation of two optimisation algorithms, simulated annealing and weighted optimisation, was initiated but these strategies require parameter identification that is complex and will require additional studies. An alternate method of optimising the textile, which modified the geodesics along with the yarn spacings, was also studied but was sidetracked due to success obtained by optimising the yarn spacings alone. This alternate method may prove useful in certain situations and warrants further investigation. An algorithm for quantifying the curvature of the model at any given point based on current methods of extrapolating curvature information from a mesh [74] was not completely implemented. This algorithm would be used for determining areas on the model where the radius of curvature may affect the textile's ability to conform to the surface of the model. This is especially useful in the case of thicker textiles, where the bending rigidity prevents the textile from conforming to small radii. Lastly, improvements to existing optimisation parameters or the definition of new optimisation parameters is needed to more clearly define the goal of the optimisation, or to include unused

parameters such as the volume fraction or possibly the curvature radius of the model on which the drape is performed.

The results obtained in this thesis suggest that the use of variable-length textiles can greatly improve LCM manufacturing processes. However, this new technology requires further investigation in both the optimisation and manufacturing process, as well as the possible effects that changes in the yarn spacings within a given textile may have on the mechanical properties of textile reinforcements.

References

1. Rudd, C.D. and A.C. Long, *Liquid Moulding Technologies*. 1997, Nottingham: Woodhead Publishing Limited.
2. Shoock, G., *Reinforced Plastics for Commercial Composites Source Book*. 1986, Metals Park: ASM.
3. Seemann Composites Inc. *SCRIMP*. 2010 [cited 2010; Available from: <http://seemanncomposites.com/scrimp.html>].
4. Quickstep Technologies Pty Ltd. *How Quickstep Works*. 2005 [cited 2010; Available from: <http://www.quickstep.com.au/what-is-quickstep/how-quickstep-works>].
5. Quickstep Technologies Pty Ltd. *Quickstep Original*. 2005 [cited 2010; Available from: <http://www.quickstep.com.au/patents-processes/quickstep-original>].
6. Long, A., *EFACS Final Report*. 2000, University of Nottingham.
7. Bergsma, O.K. *Computer Simulation of 3D forming processes of fabric reinforced plastics*. in *International Committee on Composite Materials 9*. 1993. Madrid.
8. Mack, C. and H.M. Taylor, *The fitting of woven cloth to surfaces*. Journal of the Textile Institute, 1956. **57**: p. 477-488.
9. Robertson, R.E., E.S. Hsiue, E.N. Sickafus, and G.S.Y. Yeh, *Fibre Rearrangements During the Molding of Continuous Fibre Composites. 1. Flat Cloth to a Hemisphere*. Polymer Composites, 1981. **2**(3): p. 126-131.
10. Hancock, S.G. and K.D. Potter, *The use of kinematic drape modelling to inform the hand lay-up of complex composite components using woven reinforcements*. Composites Part A: Applied Science and Manufacturing, 2006. **37**(3): p. 413-422.
11. Binkley, C. and S.B. Sharma. *A Membrane Force Based Fabric Drape Model With Rapid Convergence*. in *8th International Conference on Textile Composites*. 2006. Nottingham.
12. Boisse, P., M. Borr, K. Buet, and A. Cherouat, *Finite element simulations of textile composite forming including the biaxial fabric behaviour* Composites Part B: Engineering, 1997. **28**(4): p. 453-464.
13. ESI Group. *Composites & Plastics: PAM-FORM*. 2011 [cited 2011; Available from: <http://www.esi-group.com/products/composites-plastics/pam-form>].
14. Souter, B.J., A.C. Long, F. Robitaille, and C.D. Rudd. *Modelling the influence of reinforcement architecture on formability*. in *9th European Conference on Composite Materials*. 2000. Brighton.
15. Callister, W.D., *Materials Science And Engineering An Introduction*. 2003: John Wiley & Sons Inc.
16. MatWeb. *Material Property Data*. 2010 [cited 2010; Available from: <http://www.matweb.com/index.aspx>].
17. Cai, Z. and T.G. Gutowski, *3-D Deformation of a Lubricated Fiber Bundle*. Journal of Composite Materials, 1992. **26**(8): p. 1207-1237.
18. Shigley, J.E., C.R. Mischke, and R.G. Budynas, *Mechanical Engineering Design*. 2004, New York: McGraw-Hill.
19. Bazergui, A., T. Bui-Quoc, A. Biron, G. McIntyre, and C. Laberge, *Résistance Des Matériaux*. 2002, Montréal: Ecole Polytechnique de Montréal.
20. MatWeb. *Cytec Thornel® P-120S 2K Carbon Fiber, Pitch Precursor* 2010 [cited 2010; Available from:].

- <http://www.matweb.com/search/DataSheet.aspx?MatGUID=26b06184b56b45ea9053ae62c42cdf4>.
21. Dostal, C.A., *Engineered Materials Handbook: Composites, Volume I* 1987: CRC Press.
 22. Gutowski, T.G., Z. Cai, S. Bauer, D. Boucher, J. Kingery, and S. Wineman, *Consolidation Experiments for Laminate Composites*. Journal of Composite Materials, 1987. **21**(7): p. 650-669.
 23. Lee, B., K.H. Leong, and I. Herszberg, *Effect of Weaving on the Tensile Properties of Carbon Fibre Tows and Woven Composites*. Journal of Reinforced Plastics and Composites, 2001. **20**(8): p. 652-670.
 24. Taylor, D.S., *The Determination of Contact between the Constituents of Fibre Assemblies*. Journal of the Textile Institute, 1956. **47**: p. 141-146.
 25. Potter, K.D., B. Khan, M. Wisnom, T. Bell, and J. Stevens, *Variability, fibre waviness and misalignment in the determination of the properties of composite materials and structures* Composites Part A: Applied Science and Manufacturing, 2008. **39**(9): p. 1343-1354.
 26. Dolatabadi, M.K. and R. Kovar. *Contribution to Formulate Bending Rigidity of Tensioned Yarn*. in *9th International Conference on Textile Composites*. 2008. Newark.
 27. Ly, N.G., D.H. Tester, P. Buckenham, A.F. Rocznio, A.L. Adriaansen, F. Scaysbrook, and S.D. Jong, *Simple Instruments for Quality Control by Finishers and Tailors*. Textile Research Journal, 1991. **61**(7): p. 402-406.
 28. Minazio, P.G., *FAST-Fabric Assurance by Simple Testing*. International Journal of Clothing Science and Technology, 1995. **7**(23): p. 43-48.
 29. Yu, W. and Z. Du, *Determination of the Bending Characteristic Parameters of the Bending Evaluation System of Fabric and Yarn*. Textile Research Journal, 2006. **76**(9): p. 702-711.
 30. Park, J.W. and A.G. Oh, *Bending Rigidity of Yarns*. Textile Research Journal, 2006. **76**(6): p. 478-485.
 31. Šomodí, Ž., A. Hursa, and D. Rogale, *Numerical simulation of textile flexibility testing*. International Journal of Clothing Science and Technology, 2003. **15**(3): p. 276-283.
 32. Robitaille, F. and R. Gauvin, *Compaction of Textile Reinforcements for Composites Manufacturing. III: Reorganization of the Fiber Network*. Polymer Composites, 1999. **20**(1): p. 48-61.
 33. Harrison, P., M.J. Clifford, A.C. Long, and C.D. Rudd, *A constituent-based predictive approach to modelling the rheology of viscous textile composites* Composites Part A: Applied Science and Manufacturing, 2004. **35**(7-8): p. 915-931.
 34. Harrison, P., J. Wiggers, A.C. Long, and C.D. Rudd. *Constitutive Modelling Based on Meso and Micro Kinematics for Woven and Stitched Dry Fabrics*. in *International Committee on Composites 14*. 2003. San Diego.
 35. Robitaille, F., *MCG 4190 Selected topics I: Processing and performance of polymers & composites: introduction*. 2010, University of Ottawa: Ottawa. p. 20-32.
 36. Du, G.W. and F. Ko, *Analysis of multi-axial warp-knit preforms for composite reinforcement*. Composites Science and Technology, 1996. **56**(3): p. 253-260.
 37. Rudd, C.D., M.J. Owen, and V. Middleton, *Mechanical properties of weft knit glass fibre/polyester laminates* Composites Science and Technology, 1990. **39**(3): p. 261-277.

38. Sjögren, A., F. Edgren, and L.E. Aps. *Effects of stitch pattern on the mechanical properties of non-crimp fabric composites*. in *11th European conference on composite materials*. 2004. Rhodes.
39. A&P Technology. *Braid Basics*. 2010 [cited 2010; Available from: <http://www.braider.com/?a=24&b=basics>].
40. Kruckenberg, T.M. and R. Paton, eds. *Resin Transfer Moulding for Aerospace Structures*. 1998, Chapman & Hall: London.
41. Pamuk, G. and F. Ceken, *Manufacturing of weft-knitted fabric reinforced composite materials: A review*. *Materials and Manufacturing Processes*, 2008. **23**(7): p. 635-640.
42. Chiu, C.H. and C.C. Cheng, *Weaving method of 3D woven preforms for advanced composite materials*. *Textile Research Journal*, 2003. **73**(1): p. 37-41.
43. McIlhagger, R., J.P. Quinn, A.T. McIlhagger, S. Wilson, D. Simpson, and W. Wenger, *The influence of binder tow density on the mechanical properties of spatially reinforced composites. Part 1 - Impact resistance*. *Composites Part A: Applied Science and Manufacturing*, 2007. **38**(3): p. 795-801.
44. Wappat, K., *HYBRIDMAT 4: Advances in the manufacture of 3-D preform reinforcement for advanced structural composites in aerospace – a mission to the USA. Report of a DTI global watch mission*, U. Department of Trade and Industry, Editor. 2006.
45. Malkan, S.R. and F.K. Ko, *Three-dimensionally braided preform for a composite connecting rod. Concept development and demonstration*. *Journal of Reinforced Plastics and Composites*, 1991. **10**(2): p. 167-183.
46. Long, A.C., B.J. Souter, F. Robitaille, and C.D. Rudd, *Effects of fibre architecture on reinforcement fabric deformation*. *Plastics, Rubber and Composites*, 2002. **31**(2): p. 87-97.
47. Harrison, P., M.J. Clifford, and A.C. Long, *Shear characterisation of viscous woven textile composites: a comparison between picture frame and bias extension experiments*. *Composites Science and Technology*, 2003. **64**(10-11): p. 1453-1465.
48. Lin, H., M.J. Clifford, A.C. Long, and M. Sherburn, *Finite element modelling of fabric shear*. *Modelling and Simulation in Materials Science and Engineering*, 2009. **17**(1).
49. Prodromou, A.G. and J. Chen, *On the relationship between shear angle and wrinkling of textile composite preforms*. *Composites Part A: Applied Science and Manufacturing*, 1997. **28**(5): p. 491-503.
50. Smith, P., C.D. Rudd, and A.C. Long, *The effect of shear deformation on the processing and mechanical properties of aligned reinforcements*. *Composites Science and Technology*, 1996. **57**(3): p. 327-344.
51. Tsai, S.W. and H.T. Hahn, *Introduction to Composite Materials*. 1980, Lancaster: Technomic Publishing Company.
52. Robitaille, F., B.R. Clayton, A.C. Long, B.J. Souter, and C.D. Rudd, *Geometric modelling of industrial preforms: woven and braided textiles*. *Proceedings of the Institution of Mechanical Engineers, Part L: Journal of Materials: Design and Applications*, 1999. **213**(2): p. 69-83.
53. Lomov, S.V., I. Verpoest, T. Peeters, D. Roose, and M. Zako, *Nesting in textile laminates: geometrical modelling of the laminate*. *Composites Science and Technology*, 2003. **63**(7): p. 993-1007.
54. Samadi, R. and F. Robitaille, *Monitoring local fiber volume fraction in preform lap joints using Hall-effect sensors*. *Polymer Composites*, 2010. **31**(2): p. 314-320.

55. Steenkamer, D.A., D.J. Wilkins, and V.M. Karbhari, *The influence of preform joints on the processing of RTM composites*. Composites Manufacturing, 1995. **6**(1): p. 23-24.
56. Estrada, G., C. Vieux-Pernon, and S.G. Advani, *Experimental Characterization of the Influence of Tackifier Material on Preform Permeability*. Journal of Composite Materials, 2002. **36**(19): p. 2297-2310.
57. Shih, C.H. and L.J. Lee, *Tackification of Textile Fiber Preforms in Resin Transfer Molding*. Journal of Composite Materials, 2001. **35**(21): p. 1954-1981.
58. Van West, B.P., R.B. Pipes, and M. Keefe, *A Simulation of the Draping of Bidirectional Fabrics over Arbitrary Surfaces*. Journal of the Textile Institute, 1990. **81**(4): p. 448-460.
59. Trochu, F., A. Hammami, and Y. Benoit, *Prediction of fibre orientation and net shape definition of complex composite parts* Composites Part A: Applied Science and Manufacturing, 1996. **27**(4): p. 319-328.
60. Metropolis, N. and S. Ulam, *The Monte Carlo Method*. Journal of the American Statistical Association, 1949. **44**(247): p. 335-341.
61. Rechenberg, I., *Evolution Strategy: Optimisation of Technical Systems according to the Principles of Biological Evolution*. 1973, Stuttgart: Frommann-Holzboog.
62. Metropolis, N., A.W. Rosenbluth, M.N. Rosenbluth, and A.H. Teller, *Equation of State Calculations by Fast Computing Machines*. The Journal of Chemical Physics, 1953. **21**(6): p. 1087-1092.
63. Kirkpatrick, S., C.D. Gelatt, and M.P. Vecchi, *Optimization by Simulated Annealing*. Science, 1983. **220**(4598): p. 671-680.
64. Cormen, T.H., C.E. Leiserson, R.L. Rivest, and C. Stein, *Introduction to Algorithms*. 2009: MIT Press.
65. Khronos Group. *OpenGL - The Industry Standard for High Performance Graphics*. 2007 [cited 2010; Available from: <http://www.opengl.org/>].
66. Nokia Corporation. *Qt - A cross-platform application and UI framework*. 2010 [cited 2010; Available from: <http://qt.nokia.com/>].
67. Microsoft. *Microsoft Visual C++ 2005 Redistributable Package (x86)*. 2010 [cited 2010; Available from: <http://www.microsoft.com/downloads/en/details.aspx?familyid=32bc1bee-a3f9-4c13-9c99-220b62a191ee&displaylang=en>].
68. Goldberg, D., *What Every Computer Scientist Should Know About Floating-Point Arithmetic* ACM Computing Surveys, 1991. **23**(1).
69. Bourke, P. *The Intersection of Two Planes*. 2000 [cited 2010; Available from: <http://local.wasp.uwa.edu.au/~pbourke/geometry/planeplane/>].
70. Bourke, P. *Intersection of a Line and a Sphere (or Circle)*. 1992 [cited 2010; Available from: <http://local.wasp.uwa.edu.au/~pbourke/geometry/sphereline/>].
71. Weisstein, E. *MathWorld - Heron's Formula*. 2010 [cited 2010; Available from: <http://mathworld.wolfram.com/HéronsFormula.html>].
72. Cowan, J., *Manufacturing Device, Method and Trials for Preforms Made of Layered 2D Fabric Pieces*. 2010, University of Ottawa.
73. Press, W.H., S. Teukolsky, W. Vetterling, and B. Flannery, *Numerical Recipes: The Art of Scientific Computing*. 2007: Cambridge University Press.
74. Rusinkiewicz, S. *Estimating Curvatures and Their Derivatives on Triangle Meshes*. in *Second International Symposium on 3D Data Processing, Visualization and Transmission*. 2004. Thessaloniki.

Appendix A

Format of the .model Files

The values appearing in the .model file are separated by an indentation (tab key) and are described in Figure A-0-1.

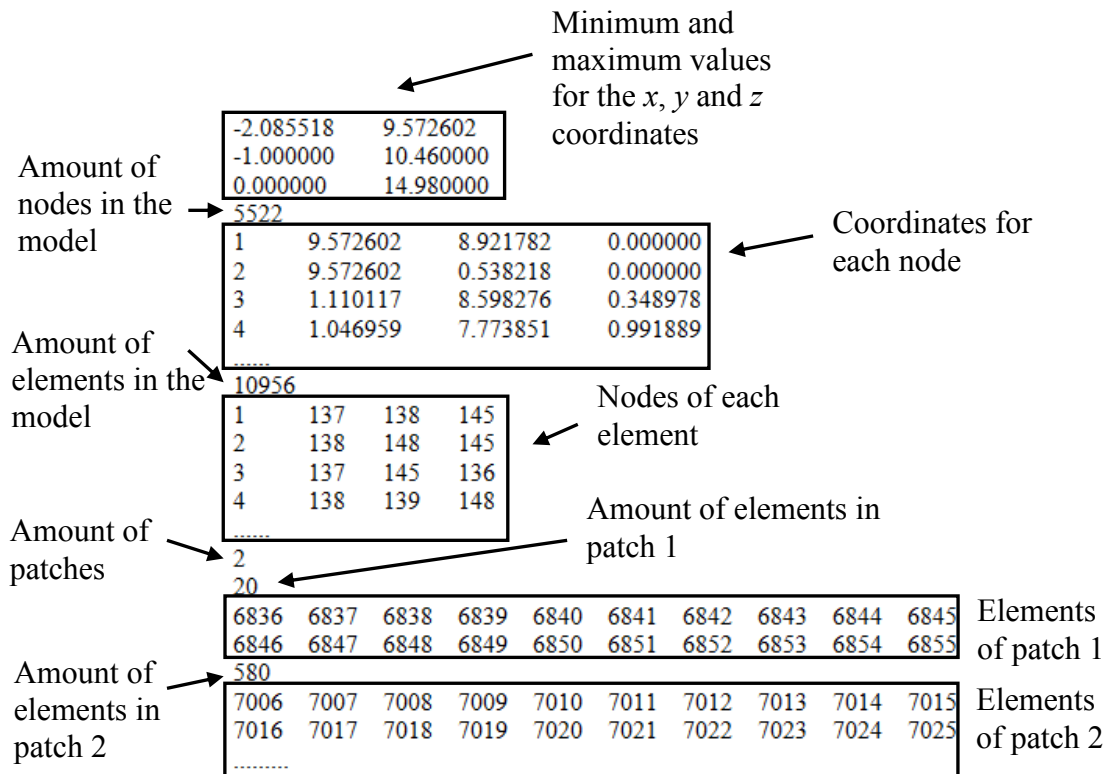


Figure A-0-1 Format of the .model File

Format of the .dparam Files

The values appearing in the .dparam file are as follows:

- Model name
- Mode for start coordinate selection: 0, 1 or 2
- x start coordinate
- y start coordinate
- z start coordinate
- Selected start triangle when many options exist: 0 to 9
- Geodesic angle
- Draping simulation performed on a patch: true (1) or false (0)
- Amount of yarn crossovers to drape along each geodesic
- Yarn Spacing
- Quadrant being draped: 1, 2, 3, 4 or 0 for all
- In-plane shear limit
- Steady state criteria
- Parameter to be maximised: area (0), in-plane shear angle (1), in-plane shear angle *area (2), average in-plane shear angle (3), v_f standard deviation (4), v_f range (5), specified orientation (6), specified orientation + in-plane shear angle (7)
- Optimisation type: length (0) or geodesic (1)
- Maximum yarn spacing
- Minimum yarn spacing
- Maximum yarn spacing change per iteration
- Optimisation type: static (0) or progressive (1)
- Initial size for progressive optimisation
- Final size for progressive optimisation
- Layer size for progressive optimisation
- Modify the new lines for progressive optimisation: yes (1) or no (0)
- Surface area covered by the non-optimised textile
- Value of the parameter to be maximised for the non-optimised textile
- Minimum percentage of similarity required
- Simulated annealing: true (1) or false (0)
- Parameter for simulated annealing 1
- Parameter for simulated annealing 2
- Weighted Random: true (1) or false (0)
- Parameter for weighted random 1
- Parameter for weighted random 2
- Parameter for weighted random 3
- Parameter for weighted random 4
- Optimal v_f
- Area for the optimal v_f
- Difference between optimal v_f and maximum/minimum

Appendix B

Optimisation Parameters for Maximising the Surface Area on the Hemisphere

Parameter	Value
Model File	Sphere Rig.model
Coordinate Mode	0
X Coordinate	7.51 cm
Y Coordinate	7.51 cm
Z Coordinate	13.50 cm
Geodesic Angle	0
Draped on Patch	No
Number of Yarn Crossovers to Drape Along Each Geodesic	34
Yarn Spacing	0.4 cm
Quadrant Draped	1
Maximum In-Plane Shear Angle	60°
Steady State Criteria	0.01
Parameter to Maximise	Angle * Area
Maximum Yarn Spacing	0.5 cm
Minimum Yarn Spacing	0.3 cm
Maximum Yarn Spacing Change Per Iteration	0.005 cm
Progressive Optimisation	Yes
Initial Size for Progressive Optimisation	10
Final Size for Progressive Optimisation	60
Layer Size for Progressive Optimisation	10
Modify Length of New Layers for Progressive Optimisation	Yes
Minimum Similarity Between Adjacent Yarn Spacings	99%
Check for Manufacturability	Yes

Table B-1 Optimisation Parameters for Maximising the Surface on the Hemisphere

Optimisation Parameters for Maximising the Surface Area on the Aerospace Mould

Parameter	Value
Model File	Large patch.model
Coordinate Mode	1
X Coordinate	-3.10 cm
Y Coordinate	12.01 cm
Z Coordinate	34.29 cm
Geodesic Angle	0
Draped on Patch	Yes
Number of Yarn Crossovers to Drape Along Each Geodesic	28
Yarn Spacing	0.45 cm
Quadrant Draped	2
Maximum In-Plane Shear Angle	60°
Steady State Criteria	0.01
Parameter to Maximise	Angle * Area
Maximum Yarn Spacing	0.45 cm
Minimum Yarn Spacing	0.35 cm
Maximum Yarn Spacing Change Per Iteration	0.01 cm
Progressive Optimisation	Yes
Initial Size for Progressive Optimisation	10
Final Size for Progressive Optimisation	50
Layer Size for Progressive Optimisation	10
Modify Length of New Layers for Progressive Optimisation	Yes
Minimum Similarity Between Adjacent Yarn Spacings	97%
Check for Manufacturability	Yes

Table B-2 Optimisation Parameters for Maximising the Surface on the Aerospace Mould

Optimisation Parameters for Modifying the Fibre Orientation on the Aerospace Mould

Parameter	Value
Model File	Roughmeshmolepatch2b .model
Coordinate Mode	2
X Coordinate	20.32 cm
Y Coordinate	4.98 cm
Z Coordinate	35.56 cm
Geodesic Angle	0
Draped on Patch	No
Number of Yarn Crossovers to Drape Along Each Geodesic	30
Yarn Spacing	0.40 cm
Quadrant Draped	1
Maximum In-Plane Shear Angle	60°
Steady State Criteria	0.01
Parameter to Maximise	Angle * Design Orientation
Maximum Yarn Spacing	0.45 cm
Minimum Yarn Spacing	0.35 cm
Maximum Yarn Spacing Change Per Iteration	0.01 cm
Progressive Optimisation	Yes
Initial Size for Progressive Optimisation	15
Final Size for Progressive Optimisation	30
Layer Size for Progressive Optimisation	10
Modify Length of New Layers for Progressive Optimisation	Yes
Minimum Similarity Between Adjacent Yarn Spacings	97%
Check for Manufacturability	Yes

Table B-3 Optimisation Parameters for Modifying the Fibre Orientation