An Adaptive, Multirate Method for Eliminating Intersymbol Interference in Non-Ideal Channel

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Abstract

In this thesis, an adaptive method for eliminating intersymbol interference (ISI) using multirate techniques, is presented. This is inspired by the work of Chevillat and Ungerboeck on the design of finite impulse response (FIR) 'multirate zero ISI filters' for data transmission over a band-limited ideal channel. Their analysis is extended to the case when the channel is non-ideal. It is shown that there are infinitely many multirate filters (dependent of the upsampling /downsampling factor M^c) that can eliminate ISI. As an illustration, two simple adaptive algorithms for computing the zero ISI filters are presented and applied to discrete multitone transmission (DMT) and discrete wavelet multitone transmission (DWMT) systems. It is found that even the minimum length, multirate zero ISI filters compare favourably (in terms of reduced complexity and performance) with other techniques, such as the cyclic prefix method.

Contents

List of Figures

List of Acronyms

PRFB	Perfect Reconstruction Filter Bank
TMUX	Transmultiplexer
DMT	DiscreteMultitone Transmission
DWMT	Discrete Wavelet Multitone Transmission
ISI	Intersymbol Interference
FIR	Finite Impulse Response
TDM	Time-Domain Multiplexed
FDM	Frequency-Domain Multiplexed
OFDM	Orthogonal Frequency Division Multiplexing
SNR	Singal-to-Noise Ratio
FEQ	Frequency-Domain Equalizer

• TEQ	Time-Domain Equalizer
• ICI	Inter-Channel Interference
• IBI	Inter-Block Interference
• ICBI	Inter-Channel-Inter-Block Interference
• CIR	Channel Impulse Response
• ADSL	Asymmetric Digital Subscriber Line
• AWG 26	American Wire Gauge No. 26

Chapter 1

Introduction

1.1 Preliminaries

A digital filter is simply a linear time-invariant operator. An **analysis filter bank** is a structure composed of filters that decompose a signal into a collection of subsignals. The important feature of these subsignals (called subband signals) is that they are sufficient to reconstruct the original signal by means of a **synthesis filter bank**. The analysis and synthesis filter banks together constitute a **perfect reconstruction (PR) filter bank.** Such PR filter banks are useful because the signal decomposition by a synthesis filter bank may help to emphasize some aspects of the original signal. Alternatively, the subband signals may be more convenient to work with than the original signal.

Filter banks have proven to be versatile in applications to communication systems. For instance, they have found applications in image and video compression. They have also been used in speech coding (using subband coding), denoising, and feature detection. There are several good references on the topic, some of them are listed in the bibliography (see for instance, [2], [3], [4], [5], [6]).

Here an important application of the dual system, namely **transmultiplexers,** will be considered. In filter banks, the synthesis filter bank follows the analysis filter bank. In transmultiplexers it is the other way around— it is the analysis filter bank that follows the synthesis filter bank. Transmultiplexers have found application in multicarrier modulation for high-speed data transmission over the twisted pair channel of a digital subscriber line.

1.2 Motivation

Multicarrier modulation [7] can be viewed as a class of orthogonal frequency modulations, or simply as a transmultiplexer. In **discrete multitone transmission (DMT)**, the synthesis and analysis filter banks are simply the inverse Fourier transform and the Fourier transform respectively [8]. Thus, digital data is passed through the synthesis fiter bank, i.e., an inverse Fourier transform is performed on each data block of length M. The output is then converted to an analog signal (so that it is suitable for transmission over channel) by the D/A converter. At the receiver end, the analog signal is converted back to the digital domain by the A/D converter. This digital signal is then passed through the analysis filter bank, i.e., a Fourier transform is performed on this digital signal in blocks of length M. In other words, the inverse Fourier transform is used for modulation while the Fourier transform is used for demodulation. Multicarrier modulation also includes a bit-loading algorithm that enables efficient data transmission by taking channel characteristics into account.

Instead of the Fourier transform and its inverse, it is possible to utilize other sets of PR filter banks. A reason for considering other filter banks is to investigate the possibility of achieving superior spectral containment without losing the PR property. Thus, M-band filter banks are used in **discrete wavelet multitone transmission (DWMT)** systems [9], [10]. It should be noted that for DWMT systems, the pulse waveforms for different symbols overlap in time.

The above discussion has been under the assumption that the channel is ideal and the noise can be neglected. The issues become more complicated when the channel is not ideal. A non-ideal channel introduces amplitude as well as phase distortion and the latter results in intersymbol interference (ISI) [11]. It is thus desirable to eliminate ISI so that the (unavoidable) detection errors are only due to additive noise.

In DMT, this is tackled by employing a **cyclic prefix** [8]. The cyclic prefix is inserted at the start of each transmission segment. This compensates for the high degree of spectral overlap among DMT subchannels, if the channel impulse response is shorter than the length of the cyclic prefix. However, employment of a cyclic prefix leads to significant overhead when the channel impulse response length is long relative to the block length. Of course, there are algorithms that adapt the taps of a receiver pre-detection equalization filter to shorten the channel impulse response. However, such systems suffer significant performance degradations due to non-ideal tap settings in pre-detection equalizers and also increase the complexity of the receiver.

The cyclic prefix solution is not relevant for DWMT due to overlapping nature of the M-band wavelet filter bank. There is a solution proposed for the the DWMT systems proposed by Sandberg and Tzannes [10]. It does not require pre-detection equalization, but the complexity of the post-detection equalization schemes is greater than that of DMT systems. It is thus desirable to develop adaptive algorithms to combat ISI in DWMT systems which are relatively simple to design and implement.

The motivation for the analysis in the thesis is as follows: an adaptive method for tackling the ISI problem is sought that does not significantly increase receiver complexity, perhaps by transferring most of the processing for ISI elimination to the transmitter.

1.3 Objectives

In this thesis, a novel adaptive method for eliminating ISI is presented. A multirate system that can be used to combat ISI for an arbitrary non-ideal channel that is applicable to a wide variety of systems, such as multicarrier systems like DMT and DWMT, is presented. This is based on the work of Chevillat and Ungerboeck [1] on the design of optimum, multirate FIR filters for data transmission over ideal band-limited channels. The transmitter and receiver filters are designed in the time domain directly. This is in contrast to the traditional designs (such as raised cosine filters) where the filters are designed in the frequency domain and then converted to time domain filters for digital implementation.

In the Chevillat-Ungerboeck multirate system, the data is first upsampled by some integer M^c, where $M^c > 1$. This upsampled data is convolved with a transmitter filter. After passing through an ideal channel, it is convolved with a receiver filter and then downsampled by M^c. The convolution of the transmitter and receiver filters, when downsampled by M^c, is required to satisfy the zero ISI condition. The authors go on to describe an algorithm to design such 'zero ISI filters' which also have excellent spectral containment properties.

The solution proposed in the thesis generalizes the above to the case when the channel is non-ideal. As above, the data is first upsampled by M^c. The condition that convolution of the transmitter and receiver filters and the channel satisfy the 'multirate zero ISI condition' is imposed. This enables design of zero ISI FIR filters for any $M^c > 1$. The minimal length of such multirate zero ISI filters for a given length of the channel impulse response is also stated and proved. As an illustration, two simple, adaptive algorithms for computing minimal length zero ISI filters. One algorithm requires the computation of only one column of the inverse of an $(M^c + 1) \times (M^c + 1)$ matrix formed from the channel taps when the channel impulse response is of length $(M^c)^2$. The other algorithm is applicable when $M^c = 2$ and the channel impulse response is of arbitrary length (c) and requires the computation of one column of inverse of a sparse $(c - 1) \times (c - 1)$ matrix. Similar algorithms can be written for arbitrary M^c and for non-minimal channel impulse response lengths. Note that if the channel is know precisely, this method guarantees zero ISI.

The method is quite general and has a wide area of applicability. In particular, it is shown that it can be used in multicarrier systems to combat ISI. If the channel is known precisely in DMT systems, there is no ISI and no cyclic prefix is needed. If the channel is known with less precision, the multirate system proposed in the thesis can reduce the complexity of the equalizers in the receiver, by reducing the ISI. It is shown that our technique can be applied to DWMT transmission—the overlapped nature of M-band filter banks is no obstacle and introduces no additional complications.

It must be emphasized that the proposed method yields a considerably simpler precoder design as compared to those proposed in the literature. The receiver design is simplified considerably and there is a large flexibility in the design of such filters. The increase in transmitter power depends on the channel, but is not found to be excessive in realistic examples.

1.4 Contents

In Chapter 2, the fundamentals of DMT and DWMT are reviewed and the multirate method of Chevillat and Ungerboeck for eliminating ISI is discussed. In Chapter 3, a generalization of the multirate solution to the case of a non-ideal channel is presented. As an illustration, two simple, adaptive algorithms for the design of the multirate zero ISI filters are also presented. In Chapter 4, the formalism is applied to the transmultiplexer problem, particularly to the DMT and DWMT systems. In Chapter 5 some simulation results for multicarrier systems using the zero ISI filters are presented and compared with the conventional DMT systems that employ a cyclic prefix. The results in Chapters 3, 4 and 5 are the original contributions in this thesis

Chapter 2

Orthogonal Frequency Division Multiplexing and Intersymbol Interference

2.1 Filter Banks and Transmultiplexers

It is often useful to resolve band-limited analog signals into frequency subband components using a set of (frequency-selective) filters that cover the frequency region. This is useful since one can, for instance, allocate the appropriate number of bits based on its relative importance (perceptual or energy-wise) when digitizing the analog signal. One then attempts to combine the resolved signals appropriately and transmit them after conversion to an analog signal using a digital-to-analog (D/A) converter. Upon reception, the received signal is converted back to a digital signal via an analog-to-digital (A/D) converter. The aim is to recover the input signal by appropriate digital signal processing, in the absence of channel distortions and noise and quantization errors. This leads us to the notion of a **perfect reconstruction (PR) filter bank.**

A PR filter bank consists of two parts: analysis and synthesis filter banks. The M-band analysis filter bank analyses a digital signal into M orthogonal subbands. These set of M signals (which are produced by filtering the signal with M filters) are critically downsampled (i.e., by M) so that the total filtered signal length is the same as that of the input signal. The set of analysis filters and the critical downsampling constitute the **analysis filter bank**. These analysed signals are then subjected to digital processing, such as quantization and appropriate bit allocation in the various frequency bands. In the **synthesis filter bank**, this output is first critically upsampled (i.e., by M) and then passed through M synthesis filters and then combined. The analysis and synthesis filters are designed so that (in the absence of any quantization error and noise) the signal is resynthesized perfectly; hence the name PR filter bank. It is illustated in Figure **??**.

There are several choices for the analysis and synthesis filters in PR filter banks. For instance, the analysis and synthesis filter banks could be chosen to be the Fourier transform and its inverse. Other popular alternatives are the filters in the lapped orthogonal

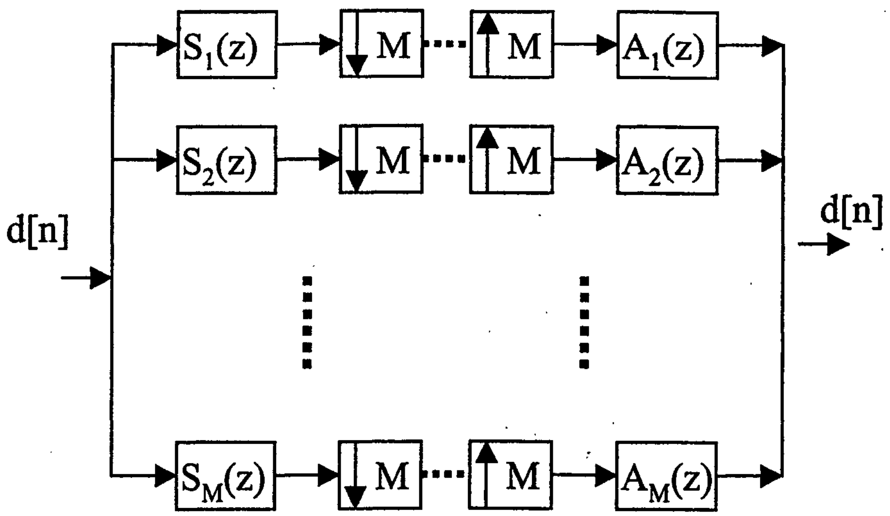

Figure 2-1: M-Band Perfect Reconstruction Filter Bank.

transforms and the cosine-modulated filter banks. In other words, an essential condition demanded of the (causal and FIR) analysis and synthesis filters is that they satisfy the perfect reconstruction condition. Further constraints are then imposed, such as good stopband attenuation.

A transmultiplexer filter bank (or simply a **transmultiplexer**) is the dual of a filter bank and is shown in Figure 2-2. In other words, in the transmultiplexer, the synthesis filter bank is followed by the analysis filter bank. The synthesis bank combines a collection of critically upsampled and filtered signals into a single signal at a higher rate and converts time-domain-multiplexed (TDM) signals into frequency domain multiplexed (FDM) signals. The analysis filter bank analyses this output using the analysis filters and recovers the input signal (in the absence of any errors) after critical downsampling.

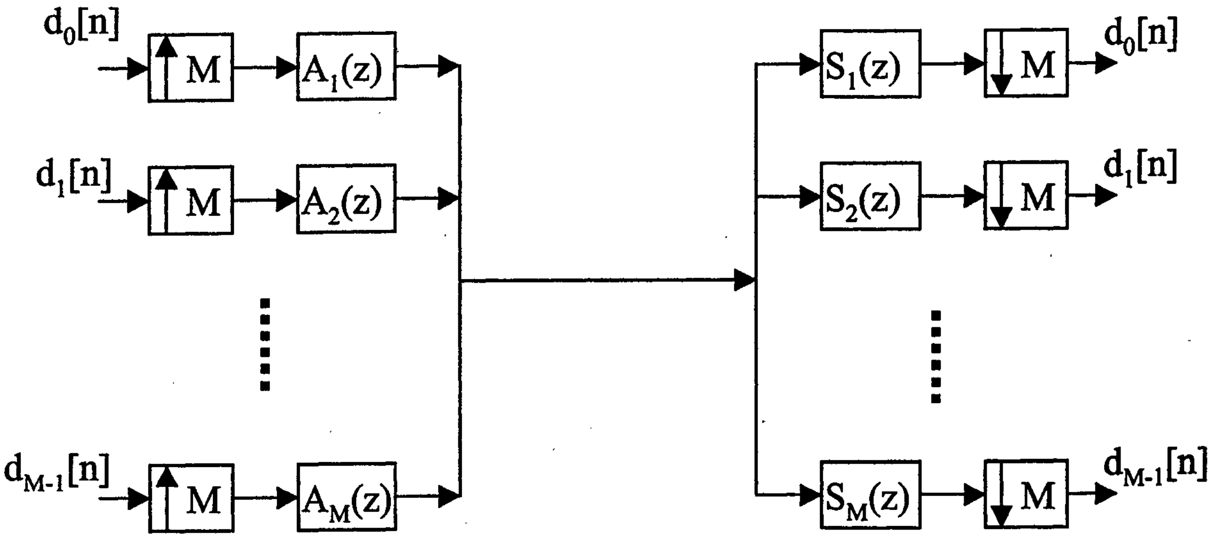

Figure 2-2: Transmultiplexer

2.2 Discrete Multitone Transmission (DMT)

2.2.1 Introduction

Multicarrier modulation provides an excellent solution in terms of efficient bandwidth utilization, approaching the capacity stated in the water-pouring theorem as the number of subchannels is increased [7]. In orthogonal frequency division multiplexing (OFDM), the transmission channel is partitioned into several subchannels, each with its own carrier. In other words, the transmultiplexer forms the core of multicarrier systems.

Another important feature in OFDM is a **bit allocation algorithm**. In OFDM a channel is divided into a set of parallel independent subchannels by the synthesis filter bank. The signal-to-noise ratio (SNR) of each subchannel is determined and then an appropriate number of bits is assigned to each subchannel to minimize the probability of error. An adaptive bit allocation algorithm optimizes the performance of an OFDM system.

Multicarrier modulation has important advantages over single-carrier systems. It has found use in high-speed copper wire communications. The following are some of the reasons for its usefulness.

- It provides an efficient way to access and distribute several multiplexed data streams.
- It has much better immunity to ISI than single carrier systems due to its longer symbol times.
- It also provides an effective way for combating narrowband interference; the data rate in the subchannel affected by the narrowband interference can be reduced appropriately.

In particular, in discrete multitone transmission (DMT), the orthogonal transformation used is the Fourier transform. Thus, the modulation and demodulation are performed by the inverse Fourier transform and the Fourier transform respectively. DMT has been chosen as the signalling standard for asymmetric digital subscriber lines (ADSL) [12].

2.2.2 Combating ISI: Cyclic Prefix and Precoder

The simple picture above has to be modified in real applications. This is because any non-ideal channel necessarily introduces ISI. Recall that ISI is a consequence of phase distortion by any non-ideal channel. In general, ISI is a measure of the interference in a particular symbol by preceding symbols. Thus, it follows that the longer the symbol duration (i.e., the longer the time span the symbol used during modulation), the less harmful are the effects of channel-induced ISI. Another way is to introduce a guard time; then the impulse response from one symbol in the subchannel dies out before the beginning of the next symbol. In DMT, the length of the symbol is determined by the size of the FFT. System latency and computational complexity limit the size of the FFT used in practice.

In DMT systems, the ISI is eliminated using a **cyclic prefix**. The subchannels then become independent. This is because in discrete time (and for finite number of subchannels N) the property that time domain convolution corresponds to Fourier transform multiplication if at least one of the input sequences convolved is periodic with period N. A cyclic prefix simply prefixes a time-domain block of samples (x_i, $i = 1, ...N$) of size N

by the last ν samples of the block, ν being the channel impulse response length. This makes it appear as if the data were periodic with period N so that

$$Y_i = H_i X_i \tag{2.1}$$

where H_i is the channel frequency response at subchannel i. Thus, data can be recovered using one-tap equalizers $1/H_i$ known as the frequency domain equalizers (FEQ).

However, the channel impulse response length can be large. Then the required overhead with respect to data rate is $\nu/(N+\nu)$ can be significant and leads to a significant loss in data rate. Increasing N does not help matters since this implies large memory requirements. Further, larger N leads to longer latency in processing. Hence, it is desirable to reduce overhead due to cyclic prefix, so that throughput is not decreased substantially.

One solution is to use a combination of short-length equalization and the DMT. The equalizer used is known as a time-domain equalizer (TEQ). This TEQ shortens the channel response so that the combined response of the channel and the TEQ taps is limited to being equal to or less than the length of the cyclic prefix. A DMT transceiver block diagram is shown in Figure 2-3.

There are several algorithms for the design of such (pre-detection adaptive receiver) TEQ filters to shorten the channel impulse response; see for instance, [13], [14], [15]. Un-

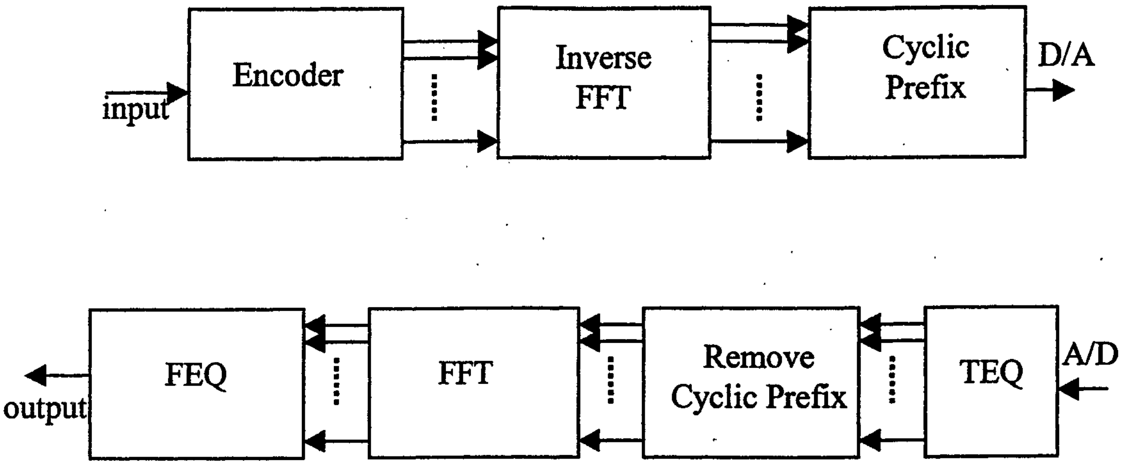

Figure 2-3: DMT transceiver block diagram.

fortunately, it has been shown that the performance of a DMT system can be degraded severely by intersymbol interference (ISI) when the taps of the channel-shortening equalization filter deviate from their ideal settings [10]. Also, the TEQ only partially equalizes the channel. This is because full equalization leads to noise enhancement at the receiver. Another problem is that the optimization criterion to obtain the TEQ taps is usually non-convex [16]. Such non-convex optimization problems are difficult to solve and are not guaranteed to reach the global minimum.

A method to alleviate this involves the use of a **precoder.** The proposed precoder for DMT by Cheong and Cioffi [17] eliminates the distortion due to insufficient cyclic prefix length by processing the signals at the transmitter so that the signals appear to be undistorted at the receiver with an increase in transmitted power. It uses a decision feedback equalizer (DFE) at the transmitter to tackle the IBI. The precoder does cause a power increase, but produces improvement over when nothing is done to mitigate the distortion.

Their paper presented the general formalism for the case when the length of the channel impulse response is less than block size of the DMT system (typically 512) so that the interblock interference is caused by only one of the preceding block. Even for this case, where a simple closed form solution is avaliable, several matrix multiplications and inversions are needed. In the more general case the channel impulse response is much larger so that a more complicated precoder will be required.

Furthermore, a time-domain equalizer is still required at the receiver. Further, since perfect equalization is undesirable due to noise enhancement, a cyclic prefix is also necessary. Although there is some performance gain in the use of the proposed DMT precoder by Cheong and Cioffi, there is also an increase in complexity due to the use of a DFE.

2.3 Discrete Wavelet Multitone Transmission (DWMT)

The inverse Fourier transform and the Fourier transform used in the transmultiplexer may be replaced with the synthesis and analysis filters of any perfect reconstruction filter bank system. Thus, in Discrete Wavelet Multitone Transmission (DWMT) systems the

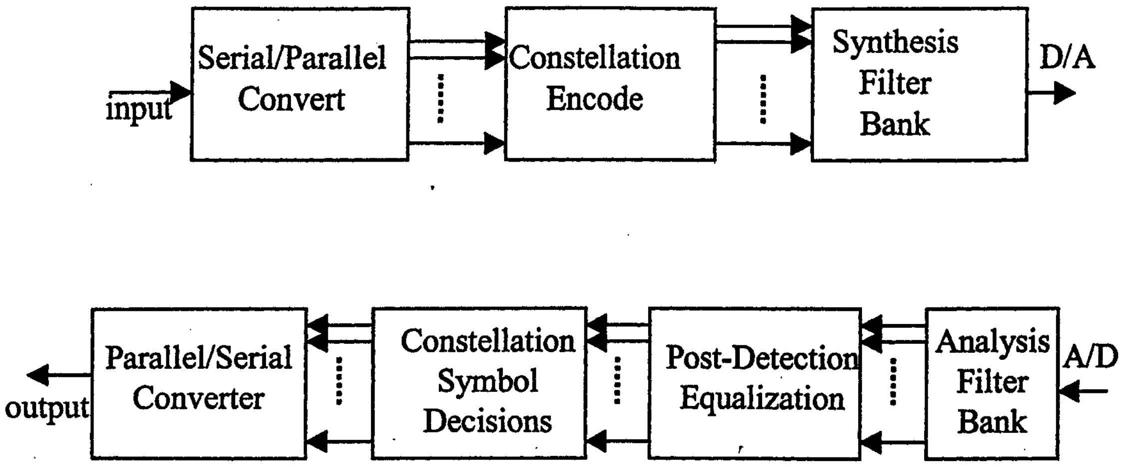

Figure 2-4: Block diagram of a DWMT transceiver.

synthesis and analysis filters of any PR M-band filter bank, such as the lapped orthogonal transforms [18], the cosine-modulated filter banks [6] or the M-band wavelet filter banks (see [19], [2]) are used. A block-level diagram for a DWMT transceiver is shown in Figure2-4.

Note that DMT schemes use rectangular pulses for data modulation. This is essential to maintain PR property of the set of subchannel waveforms (which follows from the orthogonality of the transform). However, a given subchannel has significant spectral overlap with a large number of its neighbouring subchannels, and subchannel isolation is retained only for channels which have virtually no distortion. The DWMT system achieves excellent spectral containment (by PR filter design) while maintaining perfect reconstruction at the expense of longer filters [10].

As discussed above, an alternative form of multicarrier transmission is overlapped discrete multitone or discrete wavelet multitone (DWMT) transmission. The orthonormal set of waveforms used are the generalization of inverse Fourier transform and the Fourier transform— the synthesis and analysis filter banks. The filter length in M-band filter banks is usually larger than the number of subchannels; their ratio is called the overlap factor g. In contrast, for the Fourier transform $g = 1$. Since $g > 1$ in a DWMT system, it provides longer symbol duration and far lower sidelobes (e.g., -25 dB for $g = 2$ M-band wavelet filter bank and -45 dB for a $g = 8$ cosine-modulated filter bank) than the DMT. In contrast, the adjacent channel sidelobe level in DMT is about -13 dB and there is no way to lower this sidelobe level using a Fourier transform. However, note that a symbol buffer of size gM must be maintained by the DWMT modulator.

There are three types of ISI (as defined in [10]): inter-channel interference (ICI), inter-block interference (IBI), and inter-channel-inter-block interference (ICBI). In ICI , the interfering symbols are from the same data block but different subchannels, while in IBI the interfering symbols are same subchannel but different blocks. Symbols contributing nonzero energy, but sharing neither block nor subchannel with a particular data symbol introduce ICBI. Thus, low sidelobes and long symbol duration are key to reducing ISI in multicarrier systems.

The DWMT system proposed (as in [9], [10]) potentially has several advantages over DMT systems.

- It incorporates careful pulse design to achieve excellent spectral containment, without sacrificing the Nyquist properties of the set of subchannel waveforms.

- It does not suffer significant performance degradations due to non-ideal tap settings in pre-detection equalizers; in fact no pre-detection equalizer or a cyclic prefix is needed.
- It is better able to lessen the effects of narrowband interference. This robustness with regard to narrowband interference and ISI can be directly attributed to the spectral containment properties of DWMT subchannels.

Unfortunately, the presence of non-ideal channel conditions make matters more difficult for DWMT systems [10]. The post-detection processing is significantly more complicated for DWMT than for DMT transmission. Note that the cyclic prefix solution cannot be employed in DWMT transmission due to the overlapped nature of the filter banks (i.e., $g > 1$).

In summary, attempts to reduce ISI without increasing the complexity of the receiver are important to study. An example is provided by the precoder for DMT with insufficient cyclic prefix proposed by Cheong and Cioffi. Any improvement in performance resulting with only moderate increase in transmitter power and receiver complexity is desirable. Similarly, there is a need for a simple and adaptive algorithm that can work for DWMT systems, without need for complicated post-detection equalizers. The hope is to capitalize on the strength of DWMT over DMT transmission: subchannel spectral containment and bandwidth efficiency.

2.4 Multirate Zero ISI Filters for Ideal Channel

The problem of combatting ISI arises when transmitting data over a band-limited channel. When the channel is ideal, the analog solutions are well-known. For instance, the data transmitted using raised-cosine pulses have no ISI. When the channel is non-ideal, the channel frequency response influences the choice of the transmitted pulse for zero ISI transmission.

However, zero ISI is maintained only approximately when raised cosine filters are implemented digitally using FIR filters. This is due to error introduced by truncation. It was shown by Chevillat and Ungerboeck that one can design optimum FIR transmitter and receiver filters (matched in order to maximize signal-to-noise ratio) that ensure that data transmitted over band-limited channels are ISI-free [1]. They use matched filters along with an upsampler and a downsampler. The filter design can be further optimized by maximizing the spectral concentration within the desired region. Their work was based on a paper by Halpern [20] .

The Chevillat-Ungerboeck multirate system for zero ISI transmission is illustrated in Figure 2-5. The input digital data is first upsampled by some integer M and then passed through a transmitter filter $F^c(z)$. Upon reception, it is passed through the receiver filter $G^c(z)$ (which is matched to $F^c(z)$) and then downsampled by M^c. The following theorem states the condition that the transceiver filters must satisfy for zero

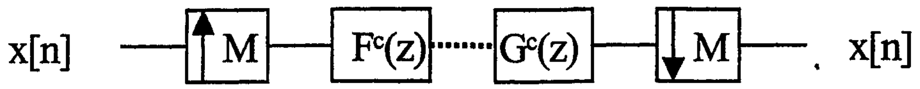

Figure 2-5: Multirate System of Chevillat and Ungerboeck [1]

ISI transmission in the absense of any channel distortion.

Theorem 1 *(Chevillat and Ungerboeck [1])If the transmitter and receiver filters are such that*

$$F^c(z)\,G^c(z)\,|_{\downarrow M^c} = z^{-L^c}. \tag{2.2}$$

for some L^c, then there is no ISI in the system illustrated in Figure 2-5.

Proof. Consider the case of data transmission system as shown in the figure 4. Then, zero ISI requires that the transmitter and receiver filters ($F^c(z)$ and $G^c(z)$ respectively) satisfy

$$X\left(z^{M^c}\right) F^c(z)\,G^c(z)\,|_{\downarrow M^c} = z^{-L^c} X(z). \tag{2.3}$$

This is the case if the transceiver filters satisfy

$$F^c(z)\,G^c(z)\,|_{\downarrow M^c} = z^{-L^c}. \tag{2.4}$$

■

This can be restated as a system of linear equations. That was how it was originally stated by Chevillat and Ungerboeck in [1]. However, they restricted their attention to the optimum (in SNR) case: matched transmitter and receiver filters. Then, the N-tap transmitter filter is

$$F^c(z) = \sum_{i=0}^{N-1} f_i z^{-i} \tag{2.5}$$

and is clocked at rate M^c/T. The matched receiver filter is then

$$G^c(z) = \sum_{i=0}^{N-1} f_i z^{-(N-1-i)} \tag{2.6}$$

and the zero ISI condition requires

$$\sum_{i=0}^{N-1} f_i f_{i+M^c l} = \mathbf{f}^T \mathbf{S}_l \mathbf{f} = \delta_l, \quad |l| \leq \lfloor (N-1)/M^c \rfloor. \tag{2.7}$$

Here $\mathbf{f}$ is the coefficient row vector corresponding to $F^c(z)$, δ_l is the Kronecker delta symbol, and $\lfloor x \rfloor$ is the integer part of x. The shift matrix ($\mathbf{S}_l$)elements are zero except when $s_{l,ik} = 1$ for $k - i = M^c l$.

Note that this condition is independent of the input data. In particular, the data may be digital, the discretized version of an analog signal or arbitrary complex numbers. For instance, this may be used in the transmultiplexer between the synthesis and analysis filter bank. However, it is not useful since it is derived for an ideal channel. When the channel is ideal, it is totally unnecessary since the PR filter banks already do the perfect reconstruction! What is needed is the ability to transmit data ISI-free over a dispersive channel. It would be useful to design a modification of the Chevillat and Ungerboeck system for ISI-free digital transmission over a non-ideal channel. In other words, a perfect reconstruction system between the synthesis and analysis filter banks in the presence of a non-ideal channel is sought. That is done in the next chapter: the multirate zero ISI formalism will be extended to the case of a non-ideal channel.

Several papers have been written based on this work; some recent papers are listed in the bibliography ([21], [22], [23]). However, all of them continue to assume that the channel is ideal. As mentioned earlier, that is not very helpful for applications such as multicarrier communication systems.

2.5 Summary

In this chapter, the salient features of OFDM were summarized briefly. In particular, multicarrier modulation schemes (such as DMT and DWMT) are based on the PR transmultiplexer. However, these systems are perfectly reconstructing only when the channel is ideal. Any realistic channel is non-ideal and time-varying. In order to recover the transmitted data precisely, additional processing is needed. Thus, in DMT systems a cyclic prefix is employed. This introduces a significant overhead when the channel impulse response length is comparable to the number of subchannels. There are algorithms for shortening the channel impulse response, but this increases the complexity of the receiver. No cyclic prefix is employed for the DWMT, but the receiver is very complex. Thus, it is desirable to reduce the complexity of the receiver, perhaps by transferring some of the complexity to the transmitter via a precoder.

A multirate system for ISI-free transmission proposed by Chevillat and Ungerboeck was briefly reviewed. The analysis was valid only for ideal channel, but it was shown to be superior to the conventional approach based on a continuous signal with zero ISI property, such as the raised-cosine filter. This is because there is no error due to truncation; zero ISI is guaranteed by the multirate zero ISI condition. In the next chapter, this is extended to the case when the channel is non-ideal.

Chapter 3

Multirate Zero ISI Filters for Arbitrary Non-Ideal Channel

In this chapter, the proposed multirate solution for eliminating ISI is presented. This solution is an extension to the case of an arbitrary channel of the multirate zero ISI filter system introduced by Chevillat and Ungerboeck. Methods of designing such multirate zero ISI filters adaptively and find the minimum length of such filters for a given channel impulse response length and upsampling/downsampling rate M^c. As an illustration, two simple algorithms are presented: one for the case the channel impulse response is of length $(M^c)^2$, and the other when the channel is of arbitrary length with $M^c = 2$.

Figure 3-1: A multirate system proposed for cancelling ISI.

3.1 Adaptive Zero ISI Filters for Non-Ideal Channel

Let us consider an extension of the multirate system discussed in the previous chapter, i.e., now a non-ideal channel is included. This is illustrated in Figure 3-1. It will be shown that such a system guarantees zero ISI (assuming the channel is known precisely) with FIR filters $F^c(z)$ and $G^c(z)$.

It is important to point out that multirate signal processing is very crucial in our solution in terms of the FIR filters. The upsampling of original data prior to transmission introduces redundancy. This redundant version of the data when filtered by any set of transmitter and receiver filters and the channel is clearly corrupted. However, it can be arranged such that the downsampled received data is identical to the original data. Then ISI is eliminated precisely. As emphasized, this is possible only because of the multirate signal processing (i.e., $M^c > 1$). This implies that the transmitter and receiver filters satisfy fewer equations. These can be solved with FIRs, although there will be a certain minimim length that depends on M^c. All this is stated more precisely in the following two theorems.

Theorem 2 *Consider the multirate system shown in Figure 3-1. If the transmitter and receiver filters are such that*

$$F^c(z)\,C(z)\,G^c(z)\,|_{\downarrow M^c} = z^{-L^c}\ ,i.e., \tag{3.1}$$

$$P(z)\,C(z)\,|_{\downarrow M^c} = z^{-L^c}, \tag{3.2}$$

where $P(z) = F^c(z)\,G^c(z)$, *then there is no ISI in the system.*

Proof. Consider the generalized multirate system illustrated above. In this case the zero ISI condition becomes

$$X\left(z^{M^c}\right) F^c(z)\,C(z)\,G^c(z)\,|_{\downarrow M^c} = z^{-L^c}X(z)\,. \tag{3.3}$$

Clearly, this can be ensured by demanding that

$$F^c(z)\,C(z)\,G^c(z)\,|_{\downarrow M^c} = z^{-L^c}. \tag{3.4}$$

This condition is also independent of $X(z)$, as is desired. This may be written in terms of the coefficients of the product filter $P(z) = F^c(z)\,G^c(z)$:

$$P(z)\,C(z)\,|_{\downarrow M^c} = z^{-L^c}. \tag{3.5}$$

■

This can also be restated as a system of linear equations. For now, it is left as it is and the matrix interpretation is demonstrated below in some examples. Results are stated in terms of the product filter $P(z)$ throughout, for simplicity. Further, results for $P(z)$ are also more general as they place no restrictions on the transmitter and receiver filters (except that $F^c(z)\,G^c(z) = P(z)$). Thus, for instance, we may take $G^c(z) = 1$, so that $F^c(z) = P(z)$.

It is useful to determine the minimum degree of $P(z)$ for there to be a solution to the multirate zero ISI condition. The following gives the minimum length the zero ISI filter $P(z)$ must have in order to be able to satisfy the zero ISI condition.

Theorem 3 *If $C(z)$ is of degree c in z^{-1}, then, a zero-ISI filter $P(z)$ must be of degree p, such that*

$$\lfloor \frac{c+p}{M^c} \rfloor \leq p \tag{3.6}$$

where $\lfloor x \rfloor$ is the greatest integer less than x.

Proof. Since $C(z)$ is of degree c, the impulse response of C is of length $c+1$. If $P(z)$ is of degree p then $C(z)\,P(z)$ is clearly of degree $c+p$. When this product is downsampled by M^c, then there remain only $\lfloor \frac{c+p}{M^c} \rfloor + 1$ terms. Then, the number of variables $(p+1)$ must equal or exceed the number of terms in the downsampled product, i.e.,

$$\lfloor \frac{c+p}{M^c} \rfloor + 1 \leq p+1, \text{ i.e.,}$$

$$\lfloor \frac{c+p}{M^c} \rfloor \leq p. \tag{3.7}$$

Hence proved. ■

Thus, the taps in the filter $P(z)$ can be found by means of a solution of a system of linear equations, the number of which depends on M^c and the channel length (L^c). The number of taps is required to be above a certain minimum specified above.

It is useful to state the conditions on the minimal length zero ISI filters. The simplest case is when the channel length is $(M^c)^2$.

Corollary 1 *When the channel impulse response is of length $(M^c)^2$ (i.e., $c = (M^c)^2 - 1$), the smallest p which solves the zero-ISI condition*

$$\lfloor \frac{(M^c)^2 - 1 + p}{M^c} \rfloor = p, \text{ i.e.,}$$

$$M^c + \lfloor \frac{p-1}{M^c} \rfloor = p \tag{3.8}$$

is $p = M^c$. This means that $(M^c + 1)$ taps are needed here.

The minimum value of M^c is 2. The following corollary gives the minimum length of a zero ISI filter for that case.

Corollary 2 *For the case when $M^c = 2$, the minimum degree of the zero ISI filter ($p_{\min}$) is $c-1$.*

Proof. This follows from Theorem 3 which states that p, the degree of any zero ISI filter must satisfy

$$\lfloor\frac{c+p}{2}\rfloor \leq p, \text{ i.e.,}$$
$$\lfloor\frac{c+p-2p}{2}\rfloor \leq 0,$$

where p has been subtracted from both sides. Thus, it must be that

$$\lfloor\frac{c-p_{\min}}{2}\rfloor = 0, \tag{3.9}$$

and the smallest value satisfying this equation is $p_{\min} = c-1$. Hence proved. ■

The results in the above two corollaries form the basis for the two algorithms introduced in the next section. Also, the following result for general M^c can be stated in the form of a corollary.

Corollary 3 *The minimum degree of the zero ISI filter ($p_{\min}$) for M^c is the integer in* *the series*

$$\frac{c-1}{M^c-1}, \frac{c-2}{M^c-1}, \dots \frac{c-(M^c-1)}{M^c-1} \tag{3.10}$$

Proof. The degree p of any zero ISI filter must satisfy

$$\lfloor \frac{c+p}{M^c} \rfloor \leq p, \text{ i.e.;}$$
$$\lfloor \frac{c+p-M^c p}{M^c} \rfloor \leq 0,$$

where p has been subtracted from both sides. Thus, it must be that

$$\lfloor \frac{c-(M^c-1)p_{\min}}{M^c} \rfloor = 0, \tag{3.11}$$

and the smallest value satisfying this equation is given by the one equation below that leads to an integer value for $p_{\min:}$

$$\begin{aligned} c-(M^c-1)p_{\min} &= 1 \\ c-(M^c-1)p_{\min} &= 2 \\ \vdots &= \vdots \\ c-(M^c-1)\,p_{\min} &= M^c-1. \end{aligned} \tag{3.12}$$

Hence proved. ■

Some examples that illustrate some of the features of the proposed method are presented below. Mostly, the minimal length zero ISI filters (for a given channel length and upsampling/downsampling factor) are discussed.

Example 1 *Consider the case of the channel modelled as* $C(z) = \sum_{i=0}^{8} c_i z^{-i}$. *For the case* $M^c = 2$, $p_{\min} = 8 - 1 = 7$,*i.e., the zero ISI filter is of filter length 8. The zero ISI condition becomes*

$$P(z)\,C(z)\,|_{\downarrow 2} = z^{-L^c}$$

for some integer L^c. *In this example,* L^c *can be 0, 1,... or 7. For instance, let* $L^c = 7$ *so that the zero ISI condition can be written as the following matrix equation*

$$\begin{pmatrix} c_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_2 & c_1 & c_0 & 0 & 0 & 0 & 0 & 0 \\ c_4 & c_3 & c_2 & c_1 & c_0 & 0 & 0 & 0 \\ c_6 & c_5 & c_4 & c_3 & c_2 & c_1 & c_0 & 0 \\ c_8 & c_7 & c_6 & c_5 & c_4 & c_3 & c_2 & c_1 \\ 0 & 0 & c_8 & c_7 & c_6 & c_5 & c_4 & c_3 \\ 0 & 0 & 0 & 0 & c_8 & c_7 & c_6 & c_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_8 & c_7 \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Then, the zero ISI filter taps are given by

$$\begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \\ p_6 \\ p_7 \end{pmatrix} = \begin{pmatrix} c_0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ c_2 & c_1 & c_0 & 0 & 0 & 0 & 0 & 0 \\ c_4 & c_3 & c_2 & c_1 & c_0 & 0 & 0 & 0 \\ c_6 & c_5 & c_4 & c_3 & c_2 & c_1 & c_0 & 0 \\ c_8 & c_7 & c_6 & c_5 & c_4 & c_3 & c_2 & c_1 \\ 0 & 0 & c_8 & c_7 & c_6 & c_5 & c_4 & c_3 \\ 0 & 0 & 0 & 0 & c_8 & c_7 & c_6 & c_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_8 & c_7 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Note that each column of the inverse matrix is a candidate minimal length multirate zero ISI filter, each with a different delay. In other words, the 1 *in the column matrix on the right could be in any row, the rest of the entries being zero. Although all choices of L^c are equivalent in the absence of additive noise, the choice of L^c can significantly affect performance in the presence of noise; this is discussed in the next section.*

Example 2 *Consider the choice $M^c = 3$, so that $p_{\min} = 3$. The zero ISI conditions read*

$$\begin{pmatrix} c_0 & 0 & 0 & 0 \\ c_3 & c_2 & c_1 & c_0 \\ c_6 & c_5 & c_4 & c_3 \\ 0 & 0 & c_7 & c_6 \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \tag{3.13}$$

(for $L^c = 3$ for simplicity) so that the minimal zero ISI filters are given by

$$\begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \end{pmatrix} = \begin{pmatrix} c_0 & 0 & 0 & 0 \\ c_3 & c_2 & c_1 & c_0 \\ c_6 & c_5 & c_4 & c_3 \\ 0 & 0 & c_7 & c_6 \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{3.14}$$

So the minimal filter lengths depend on M^c. Of course, longer filters could have been chosen. For instance, take $p = 4$ for $M^c = 3$ so that the zero ISI condition (again for $L^c = 3$) reads

$$\begin{pmatrix} c_0 & 0 & 0 & 0 & 0 \\ c_3 & c_2 & c_1 & c_0 & 0 \\ c_6 & c_5 & c_4 & c_3 & c_2 \\ 0 & 0 & c_7 & c_6 & c_5 \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \tag{3.15}$$

There are four equations for five unknowns and hence the zero ISI condition does not uniquely determine the zero ISI filter taps. This additional degree of freedom can be used for better filter design such as minimization of the energy of filter.

3.2 Additional Remarks

The following remarks clarify some aspects of the proposed multirate system.

- The zero ISI filters clearly depend on the channel and on M^c— the upsampling/downsampling rate. This is obvious from the above.
- There is no solution for a finite number of filter coefficients when $M^c = 1$. The infinite-tap solution is given by $C(z)P(z) = 1$, so that $P(z) = 1/C(z)$. This is the well-known zero-forcing equalizer.
- Contrast this with the case of linear equalizer with the peak distortion criterion. There the complete elimination of the ISI requires the use of an inverse filter to $C(z)$, i.e., $\frac{1}{C(z)}$ also known as a zero-forcing filter. Such an equalizer requires an infinite number of taps when implemented by a FIR filter. Any FIR filter with a finite number of taps will always have ISI. In our discussion above, ISI has been eliminated with a FIR filter with a finite number of taps. The key is multirate signal processing (the upsampling and the downsampling by M^c). This

results in filter coefficients satisfying fewer equations, which in turn are solvable using a finite number of taps, the minimum length of which is determined by upsampling/downsampling factor, as discussed in the previous section.

- The theorems give us a general way of designing zero-ISI filters. As noted above, M^c has to be greater than 1. The larger the value of M^c, the fewer are the conditions to be satisfied and smaller is the length of the minimal zero ISI filter. Thus, for a given length the larger upsampling/downsampling factor gives us greater freedom in the design of a zero ISI filter. This additional degree of freedom can be used to design filters with additional desirable properties.

- In order to maximize the signal-to-noise ratio in the presence of additive white gaussian noise, matched filters must be used (i.e., identical filter coefficients in transmitter and receiver filters but in time-reversed order with respect to each other [1]). Note that the product filter $P(z) = F^c(z)\,G^c(z)$ may be arbitrarily distributed between the transmitter and receiver filter. In particular, $P(z)$ may be chosen to be the transmitter (receiver) filter, with no receiver (transmitter) filter. The design of zero-ISI filters that are also matched are considerably harder to design than zero ISI filters which are not required to be matched transmitter and receiver filters. This is because zero ISI conditions become a set of simultaneous quadratic equations in several variables. If, on the other hand, the transmitter and receiver filters are not related, the zero ISI condition equations are only linear and hence much easier to solve.

- The relevant matrix has been assumed to be nonsingular. There are some cases when the relevant matrix of channel coefficients is singular for symmetry reasons (apart from some channel taps (ex. c_0) vanishing). For instance, if

$$c_i = a^i \tag{3.16}$$

the determinant of the relevant matrix vanishes so that the matrix is singular. An arbitrary, randomly generated channel (or a practical one such as a twisted-pair) did not lead to any singularity problems.

3.3 Discussion of Noise Effects

So far no other restrictions on the filters (other than the zero ISI condition)have been placed. It is desirable to maximize signal-to-noise ratio by using matched transmitter and receiver filters [1]. However, their design is more complicated as it involves the solution of simultaneous non-linear (quadratic) equations. This is because the receiver filter coefficients are the same as the transmitter filter coefficients, but in time reversed order, i.e., $f_i = g_{N-i}$, where N is the number of taps in the transmitter filter. This makes it harder to find the zero ISI filters, as then simultaneous quadratic equations need to be solved.

Recall that the condition for no intersymbol interference

$$P(z)\,C(z)\,|_{\downarrow M^c} = z^{-L^c} \tag{3.17}$$

can be satisfied exactly only when the channel (i.e., C(z)) is known precisely. Otherwise, there will be some intersymbol interference. In other words, let $\delta C(z)$ be the error in the channel taps, then ISI results because

$$P(z)\,\delta C(z)\,|_{\downarrow M^c} \neq 0. \tag{3.18}$$

Thus, the sensitivity of the filter coefficients to fluctuations in the channel are governed by the channel taps. Thus, instead of the multirate zero ISI condition one obtains

$$P(z)\,[C(z) + \delta C(z)]\,|_{\downarrow M^c} = z^{-L^c} + \Delta(z). \tag{3.19}$$

If the correct filter is $P(z) + \delta P(z)$ so that

$$(P(z) + \delta P(z))(C(z) + \delta C(z))|_{\downarrow M^c} = z^{-L^c}, \tag{3.20}$$

then

$$-\Delta(z) = \delta P(z)\left[C(z) + \delta C(z)\right]|_{\downarrow M^c}. \tag{3.21}$$

Thus, instead of $z^{-L^c}X(z)$ one gets $(z^{-L^c} + \Delta z)X(z)$ so that the error due to inaccurate determination of channel taps is $(\Delta z)X(z)$. Note that Δz is large if $\delta P(z)$ is large. Thus, it is desirable that $P(z)$ taps be as small as possible (i.e., energy of the filter be least) so that the fractional change is also small.

Apart from noise in channel taps, there is also the additive noise due to the channel. If the noise is Δz_{noise}, the zero ISI condition is modified to

$$\left(X\left(z^M\right)P(z)\,C(z) + \Delta z_{noise}\right)|_{\downarrow M^c}.$$

If the energy in the filter $P(z)\,C(z)$ are large compared to 1 (i.e., 0 dB), a lot more signal energy has to be used in transmission through channel.

So a good choice of $P(z)$ could be determined as follows: Determine the product $P_i(z)\,C(z)$ where $P_i(z)$ is the zero ISI filter corresponding to the choice of delay z^{-i} and form a vector from the filter coefficients in the product. In other words, perform a convolution of the channel with each of the candidate zero ISI filters. Ideally, this convolution should just be a delay, but that cannot be the case in general. Then, the filter whose convolution with the channel has the least norm is selected. Clearly, this is not a rigorous method of choosing the best filter from the set, but it is a simple and plausible best guess.

3.4 Two Simple Algorithms

In this section two simple algorithms are presented. These enable us to design zero ISI filters simply, given the channel taps. Both require the computation of one column of a matrix (of different dimensions in the two cases) formed from the channel taps. The first algorithm requires that the channel be of length $(M^c)^2$, where M^c is the upsampling/downsampling factor discussed above. The second algorithm is less restrictive as the only restriction is that $M^c = 2$; the channel can be of arbitrary length. One can write similar algorithms for arbitrary M^c and channel lengths, but it becomes more complicated to state and cumbersome to implement.

3.4.1 Channel Impulse Response length $= (M^c)^2$

It is best to begin with a simple example which will naturally suggest the general algorithm for this case.

Example 3 *Consider, for instance, the case when the length of the channel impulse* *response is 25. The zero ISI condition when M^c=5 is*

$$C(z)\,P(z)\,|_{\downarrow 5} = z^{-L^c}.$$

This can be rewritten in a matrix form as

$$\begin{pmatrix} c_0 & 0 & 0 & 0 & 0 & 0 \\ c_5 & c_4 & c_3 & c_2 & c_1 & c_0 \\ c_{10} & c_9 & c_8 & c_7 & c_6 & c_5 \\ c_{15} & c_{14} & c_{13} & c_{12} & c_{11} & c_{10} \\ c_{20} & c_{19} & c_{18} & c_{17} & c_{16} & c_{15} \\ 0 & c_{24} & c_{23} & c_{22} & c_{21} & c_{20} \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}, \qquad (3.22)$$

where $C(z) = \sum_{k=0}^{24} c_k z^{-k}$ *and* $P(z) = \sum_{k=0}^{5} p_k z^{-k}$. *Thus, the zero ISI filter* $P(z)$ *is given by*

$$\begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{pmatrix} = \begin{pmatrix} c_0 & 0 & 0 & 0 & 0 & 0 \\ c_5 & c_4 & c_3 & c_2 & c_1 & c_0 \\ c_{10} & c_9 & c_8 & c_7 & c_6 & c_5 \\ c_{15} & c_{14} & c_{13} & c_{12} & c_{11} & c_{10} \\ c_{20} & c_{19} & c_{18} & c_{17} & c_{16} & c_{15} \\ 0 & c_{24} & c_{23} & c_{22} & c_{21} & c_{20} \end{pmatrix}^{-1} \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}. \qquad (3.23)$$

Note that any column of the inverse of the matrix formed from the channel coefficients specified above satisfies the multirate zero ISI condition for a particular delay. As discussed in the previous example, for best performance in the presence of noise the inverse of the matrix has to be computed.

It is not hard to see that for the general case, a minimal-length zero ISI filter is given by any column of the matrix

$$\begin{pmatrix} c_0 & 0 & \cdots & \cdots & \cdots & 0 \\ c_{M^c} & c_{(M^c-1)} & \cdots & \cdots & \cdots & c_0 \\ c_{2M^c} & c_{(2M^c-1)} & \cdots & \cdots & \cdots & c_{M^c} \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ \vdots & \vdots & \ddots & \ddots & \ddots & \vdots \\ 0 & c_{(M^c)^2-1} & \cdots & \cdots & \cdots & c_{(M^c-1)M^c} \end{pmatrix}^{-1} . \tag{3.24}$$

This suggests a simple implementation of the proposed solution.

Algorithm 1 *Let the channel impulse response of length* `Mc×Mc` *be written as* `C(i)`*, where* `i=0,..Mc×Mc-1`*. Construct the* `(Mc+1)×(Mc+1)` *matrix* `CM` *as follows (written in pseudocode):*

- `CM(1,1)=C(1);`

```
• For i=2 to Mc
    For j=1 to Mc+1
    CM(i,Mc+1-j+1)=C((i-2)*Mc+j);
    End
  End

• For j=1 to Mc
    CM(Mc+1,Mc+1-j+1)=C((Mc-1)*Mc+j);
  End
```

- Rest of the entries in CM are zero.

- Compute a column of the inverse of the matrix CM to obtain the set of possible zero ISI filters. From among them, pick the filter whose convolution with channel is the minimum in Euclidean norm.

3.4.2 Case: $M^c = 2$

There are two cases for channel impulse response length: even or odd. From Corollary 2, it is known that the minimal length zero ISI filter is of degree $c - 1$. The algorithm for this case can be developed by studying a couple of examples where c is even or odd.

Example 4 *Consider a channel model of length 7 (degree $c = 6$), i.e., $C(z) = \sum_{i=0}^{6} c_i z^{-i}$ so that $p_{\min} = 5$. The minimal length multirate zero ISI condition then reads*

$$\left(\sum_{i=0}^{6} c_i z^{-i}\right)\left(\sum_{j=0}^{5} p_j z^{-j}\right) = z^{-L^c} \tag{3.25}$$

where L^c can be $0, ...,$ or 5. Choosing $L^c = 0$ for illustration, the above condition may be written in matrix form as

$$\begin{pmatrix} c_0 & 0 & 0 & 0 & 0 & 0 \\ c_2 & c_1 & c_0 & 0 & 0 & 0 \\ c_4 & c_3 & c_2 & c_1 & c_0 & 0 \\ c_6 & c_5 & c_4 & c_3 & c_2 & c_1 \\ 0 & 0 & c_6 & c_5 & c_4 & c_3 \\ 0 & 0 & 0 & 0 & c_6 & c_5 \end{pmatrix} \begin{pmatrix} p_0 \\ p_1 \\ p_2 \\ p_3 \\ p_4 \\ p_5 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}. \tag{3.26}$$

Clearly, any column of the matrix

$$\begin{pmatrix} c_0 & 0 & 0 & 0 & 0 & 0 \\ c_2 & c_1 & c_0 & 0 & 0 & 0 \\ c_4 & c_3 & c_2 & c_1 & c_0 & 0 \\ c_6 & c_5 & c_4 & c_3 & c_2 & c_1 \\ 0 & 0 & c_6 & c_5 & c_4 & c_3 \\ 0 & 0 & 0 & 0 & c_6 & c_5 \end{pmatrix}^{-1} \tag{3.27}$$

is a valid filter, each corresponding to a different delay.

Example 5 *Consider the case* $c = 5$*, so that* $C(z) = \sum_{i=0}^{5} c_i z^{-i}$ *and* $p_{\min} = 4$*. Then, as in the previous example, it follows that*

$$\left(\sum_{i=0}^{5} c_i z^{-i}\right)\left(\sum_{j=0}^{4} p_j z^{-j}\right) = z^{-L^c} \tag{3.28}$$

with $L^c = 0, \ldots,$ *or* 4*. The the minimal length zero ISI filters are given by any one column* *of the matrix*

$$\begin{pmatrix} c_0 & 0 & 0 & 0 & 0 \\ c_2 & c_1 & c_0 & 0 & 0 \\ c_4 & c_3 & c_2 & c_1 & c_0 \\ 0 & c_5 & c_4 & c_3 & c_2 \\ 0 & 0 & 0 & c_5 & c_4 \end{pmatrix}^{-1}, \tag{3.29}$$

depending on the delay.

An inspection of the columns in the matrix of channel coefficients that has to be inverted to get the minimal length multirate zero ISI filters suggests the solution for the general case. This is summarized in the following algorithm.

Algorithm 2 *The channel impulse response length (c) is arbitrary and* $\mathtt{M}^c = 2$.

`Channel Impulse Response Length (c) may be even or odd.`

1. <u>`Even c:`</u>

- `For i=1 to c/2`

```
 CM(i,1)=C(2*i-1);

 CM(i+1,2)=C(2*i);

End

• For j=3 to C

 when j is even

 CM(j/2+1 to j/2+c/2,j)=CM(2 to c/2+1,2);

 else (i.e., j is odd)

 CM((j-1)/2+1 to (j-1)/2+c/2,j)=CM(1 to c/2,1);

 End

End

• Rest of the entries in CM are zero.
```

1. Odd c:

```
• For i=1 to (c-1)/2+1
```

```
 CM(i,1)= C(2*i-1);
End

• For i=1 to (c-1)/2
 CM(i+1,2)= C(2*(i-1)+2);
End

• For j=3 to C
 when j is even
 CM(j/2+1 to j/2+(c-1)/2,j)=CM(2 to (c-1)/2+1,2);
 else (i.e., j is odd)
 CM((j-1)/2+1 to(j-1)/2+(c-1)/2+1,j)=CM(1 to (c-1)/2+1,1);
 End
End

• Rest of the entries in CM are zero.
```

- `Compute a column of the inverse of the matrix CM to obtain the set of possible zero ISI filters. From among them, pick the filter whose convolution with channel is the minimum in Euclidean norm.`

3.4.3 Comments on the the General Case

It is clear from the above that similar algorithms can be designed for minimal-length zero ISI filters for different M^c. However, they become more complicated to state for arbitrary channel impulse response lengths and arbitrary M^c. One can see this from the algorithm for the case $M^c = 2$: the even and odd channel impulse response length cases had to be considered separately. Likewise, for the general case, the cases of the channel impulse response length leaving a remainder of $0, 1, ...M^c - 1$, when divided by M^c, needs to be taken into consideration. Howeover, the case $M^c = 2$ is of most interest from a practical point of view, and illustrates the significant features of the multirate solution. Hence, the algorithm for the general case has not been written explicitly.

In this chapter, only the construction of the minimal length zero ISI filters were considered. However, there are clearly an infinite number of non-minimal length multirate zero ISI filters with lengths larger than a prescribed minimum p_{min}. Such non-minimal filters enable better filter designs. For instance, one can impose additional conditions such as the requirement that the energy of the filter be a minimum. Thus, superior multirate zero ISI filters can be designed once the restriction of minimal length is removed.

It is clear that the key to the proposed multirate system for combating ISI is the upsampling/downsampling. In particular, it is the upsampling of the data that enables perfect ISI cancellation using only FIR filters. The downsampling at the receiver implies that fewer conditions need to be satisfied for perfect reconstruction. Also, the proposed method is not a mere inversion of channel. This is because of the freedom in the choice of delay in the downsampling, which enables us to choose the best filter even for the minimal length case.

Chapter 4

Application to the Multicarrier Systems

In this chapter, the solution to eliminate ISI discussed in the previous chapter is applied to OFDM systems such as DMT and DWMT. A brief discussion of some of the difficulties of solutions based on single-rate signal processing is presented. It is then shown how those problems are addressed by the proposed multirate solution.

4.1 General Issues Regarding ISI

The transmultiplexer forms an integral part of the multicarrier transceiver. Recall that the perfect reconstruction condition is satisfied by the synthesis and analysis filters in the transmultiplexer. However, this is true only when the channel is ideal. A non-ideal channel introduces phase distortion which destroys the PR property. Hence, in practical situations, techniques are needed to compensate for this ISI, so that the signal is reconstructed perfectly in the absence of additive noise. Also, it is desirable that this not require a lot of transmitter power.

Suppose the taps of a channel model are known. It is natural to consider the design of the synthesis and analysis filters such that, together with the channel, the three form a PR filter bank. Note that the channel has to be a part of the filter design, because it is useless to have a synthesis–analysis PR system due to channel distortion. This is a very complicated problem in filter design. Also, a longer channel impulse response (CIR) length will require longer analysis and synthesis filters. Moreover, the channel is time-varying, so that different sets of analysis and synthesis filters would be needed to eliminate ISI, which is not feasible in real-time due to sheer complexity.

Another approach is to design synthesis filters such that after the data has passed through the synthesis filter bank, the resulting signal is a pulse that can be transmitted without ISI. It is conceivable that such filters can be designed, given the channel. Recall that if the transmitter and receiver filters ($G_{T,R}(f)$) satisfy the condition

$$G_T(f)C(f)G_R(f) = X_{rc}(f)\, e^{-j2\pi f t_0} \tag{4.1}$$

then ISI is eliminated. Here, $X_{rc}(f)$ is a zero ISI spectrum, such as the raised cosine spectrum with an arbitrary rolloff factor and t_0 is the delay. For additive white Gaussian noise, the optimum filters are the matched filters. This condition is naturally defined in frequency domain. Sampling the corresponding time-continuous signals $G_{R,T}(t)$, obtained by the inverse Fourier transform, results in the filter coefficients $g_{T,R}$. However, as in the case of raised cosine filters, truncation introduces error and zero ISI is never perfectly satisfied using FIR digital filters.

The discussion of the previous chapter provides us with a solution that accomplishes ISI-free data transmission without being dependent on the choice of the PR filter bank. The multirate zero ISI filter system is simply inserted between the synthesis and analysis filter bank. This resurrects the orthogonality property of FFT or DWT even in the presence of channel!

4.2 DMT Solution

Cyclic prefix and bit loading algorithm are the two key components in DMT transmission.

4.2.1 Cyclic Prefix

As discussed in Chapter 2, the orthogonal transformation employed by DMT transmission is the Fourier transform. DMT employs the rectangular pulse for data modulation which maintains the orthogonality of the transform. However, a rectangular pulse has sidelobe down to only -13 dB so that a given subchannel has significant overlap with a number of its neighbouring subchannels. Thus, subchannel isolation is retained only for channels which have virtually no distortion. Using nonrectangular pulses with DMT will destroy the perfect reconstruction property of the Fourier transform.

The high degree of spectral overlap among the DMT subchannels is compensated using a cyclic prefix that is inserted at the beginning of each transmission segment. If the channel impulse response is of length ν, the cyclic prefix of length at least ν is used. Then the effect of the channel, which is a linear convolution of the signals with the CIR, appears to be a circular convolution. This ensures the independence of the subchannels.

A potential problem in the employment of the cyclic prefix is the overhead with respect to the data rate $\nu/(N_c + \nu)$. On highly dispersive practical channels ν can be large. The data rate loss due to the overhead may be reduced by making N_c large, but this implies large memory requirement dominating the implementation complexity.

A technique to reduce cyclic prefix overhead is to employ equalization techniques. Hence, a TEQ is used to shorten the CIR duration, thus improving the bit rate. A

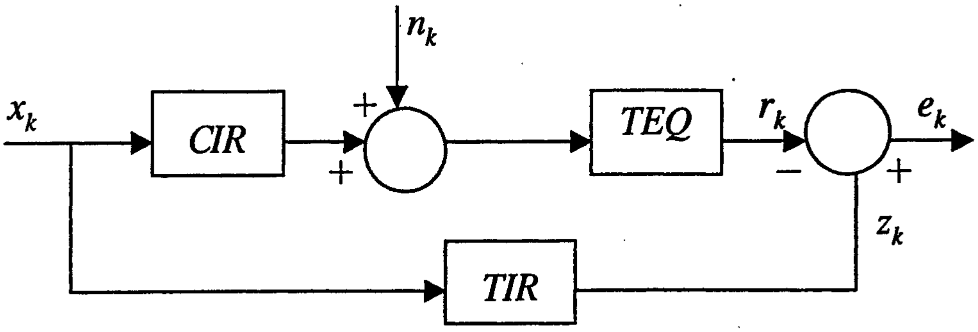

Figure 4-1: The TEQ: a block diagram.

block diagram of the TEQ is shown in Figure 4-1. There are several proposals for the design of the TEQ. For instance, a proposal is to linearly equalize the CIR to a much shorter target impulse response (TIR) by the minimum-mean-square-error constraint [13]. Specifically, the TEQ and TIR coefficients are chosen to minimize the mean square of the error sequence $e_k \equiv z_k - r_k$.

Another criterion is to maximize the geometric mean of the SNR's over all the sub-channels, referred to as the geometric SNR. It has been shown that it is optimum for the DMT system [16].

The major drawback is that the TEQ enhances the complexity of the receiver. Also, the various ways of obtaining the TEQ usually result in the optimization criterion that is non-convex [16]. Such non-convex optimization problems are difficult to solve and any solution obtained is not guaranteed to be the global minimum. Also note that perfect equalization is not desirable for the TEQ as it leads to the enhancement of noise.

4.2.2 Bit Loading

In considering the probability of error for a multicarrier system, the probabilities of error for each of the subchannels must be considered. The average of the probabilities of error for each of the subchannels is the probability of error for the system. Clearly, subchannels with largest probabilities of error would dominate in such an average. A bit loading algorithm attempts to make the probability of error the same for all the subchannels.

Consider a QAM constellation (centered at origin and with zero mean value) with $d_{\min}$ being the minimum distance between points in the constellation, and all points assumed to be equally likely. The energy of such a constellation is

$$\mathcal{E} = \frac{M-1}{6} d_{\min}^2 \tag{4.2}$$

per symbol. Strictly speaking, this is only true when b is an even integer, but is still an accurate approximation for most cases of interest (see further discussion in [24]). The above expression can be rewritten as

$$M = 1 + \frac{6\mathcal{E}}{d_{\min}^2}. \tag{4.3}$$

The probability of symbol error in QAM is approximately given by

$$P_e \leq 4Q\left[\frac{d_{\min}}{2\sigma}\right] = 4Q\left[\frac{d\,|C|}{2\sigma}\right] \tag{4.4}$$

where $|C|$ is the channel gain. For instance, for $(P_e/2)=10^{-7}$ it is required that

$$\left(\frac{d_{\min}}{2\sigma}\right)^2 = (14.5 + \gamma_m - \gamma_c)\mathrm{dB} \equiv 3\Gamma\ \mathrm{dB} \tag{4.5}$$

where γ_c and γ_m are the coding gain and margin respectively.

From the above, a simple bit loading algorithm may be written. For a given probability of error (ex., $P_e = 0.5 \times 10^{-7}$), the number of bits that can be carried by QAM is

$$b = \log_2(M) = \log_2\left(1 + \frac{SNR_c}{\Gamma}\right) \tag{4.6}$$

where the channel output SNR$_c$ is given by

$$SNR_c = \frac{\mathcal{E}\,|C|^2}{2\sigma^2}. \tag{4.7}$$

This analysis for QAM can be carried over to DMT transmission as the latter can be viewed as an aggregate of ISI-free QAM subchannels, thanks to the cyclic prefix that renders each subchannel independent. Then, for the ith subchannel, a simple bit loading algorithm is

$$b_i = \log_2 \left(1 + \frac{SNR_{ci}}{\Gamma} \right) \tag{4.8}$$

where the quantity SNR_{ci} is computed using

$$SNR_{ci} = \frac{|C_i|^2 \mathcal{E}_i}{2\sigma_i^2}. \tag{4.9}$$

The b_i are rounded off to the nearest number.

4.3 DWMT Solution

The DWMT system employs nonrectangular pulses with considerably lower sidelobes (for instance 45 dB down). Such pulses are also required to satisfy the PR condition and hence overlap in time. It is hoped that the high level of subchannel spectral containment would reduce ISI problems. Thus, it has been shown that DWMT systems are more robust with regard to ICI and narrowband channel disturbances than DMT [10]. This can be attributed to the high degree of spectral containment present in the DWMT subchannels.

However, there is ISI introduced by the channel that spoils the perfect reconstruction. Unlike the case of DMT transmission, cyclic prefix cannot be used to eliminate ISI. This is because of the overlapping nature of the transform; i.e., the analysis and synthesis filter lengths are longer than the number of bands in the filter bank. As discussed below, the equalizers needed are thus more complex than the 1-tap FEQ used in DMT systems.

Let s_i^m denote the symbol in symbol block i that is transmitted on subchannel m. For a fixed pair (m_1, i_1), the detector output $\theta_{i_1}^{m_1}$ has the form

$$\theta_{i_1}^{m_1} = \chi + \sum_i \sum_m \alpha_i^m s_i^m \tag{4.10}$$

where χ denotes the contribution due to noise, and α_i^m depend on the channel and the DWMT transceiver filters. Ideally, the only symbol with nonzero contribution in $\theta_{i_1}^{m_1}$ should be $s_{i_1}^{m_1}$. In DMT this is possible due to equalization and cyclic prefix. This is not possible in DWMT, with or without perfect channel shortening (recall that perfect shortening is not desirable as it enhances noise).

In order to address the effects of ISI in the outputs of the detector, the decision statistic for a given symbol $s_{i_1}^{m_1}$ is obtained by combining linearly the outputs of several detectors. The decision statistic $\overline{s_{i_1}^{m_1}}$ can then be written in the form

$$\overline{s_{i_1}^{m_1}} = a_{i_1}^{m_1} s_{i_1}^{m_1} + \sum_{i_2, m_2}{}' \, a_{i_2}^{m_2} s_{i_2}^{m_2} + w \tag{4.11}$$

where the primed double summation is over all pairs $(i_2, m_2) \neq (i_1, m_1)$. The first term in the decision statistic is from the symbol of interest, the second term is the ISI contribution and the final term is the contribution from channel noise. It is demanded that the signal-to-interference ration (SINR) in $\overline{s_{i_1}^{m_1}}$ be maximized, where SINR is given by

$$\gamma_{i_1}^{m_1} = \frac{\left(a_{i_1}^{m_1}\right)^2}{E\left[w^2\right] + \sum' \left(a_{i_2}^{m_2}\right)^2}. \tag{4.12}$$

From the above it follows that the process of obtaining the optimum post-detection combining weight vector that maximizes the SINR is quite complicated; details are presented in [10]. Moreover, these weights need to be computed for each subchannel, which becomes complicated for any realistic case where M is very large (ex., 512).

Some of the complexity can be reduced by employing a TEQ for pre-detection equalization. This reduces the channel impulse response length, which lessens the ISI. Consequently, the complexity of the post-detection combiner is reduced. Despite that, the computational processing for the detection of each symbol is high, and the receiver complexity is increased considerably.

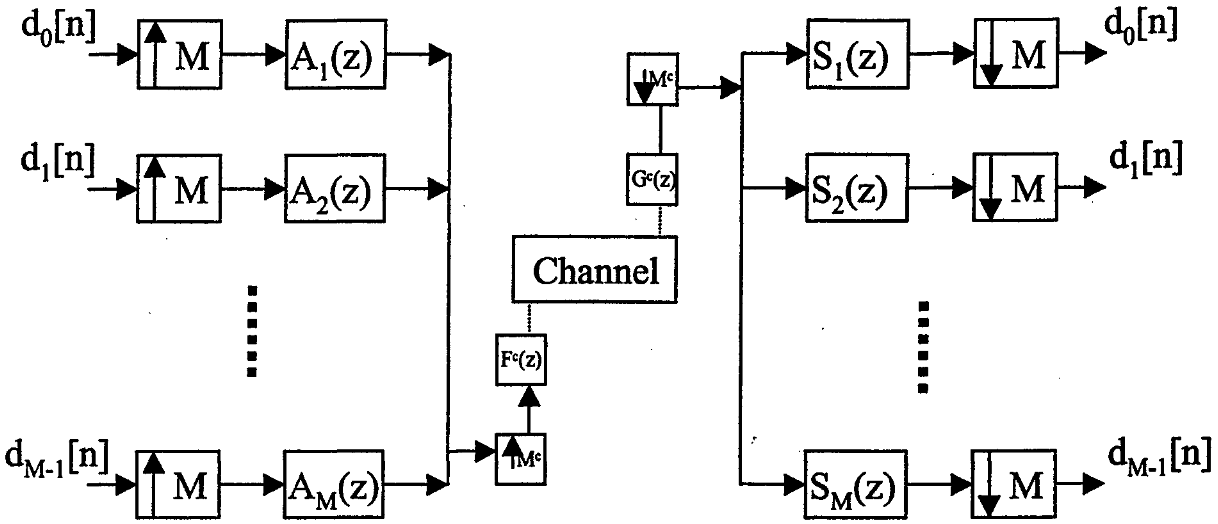

Figure 4-2: Perfect Transmultiplexer in the presence of channel.

4.4 Transmultiplexer Problem in the Presence of an Arbitrary Channel

Consider a data stream $d(n)$, with z-transform $D(z)$. In the perfect transmultiplexer (see Figure 2-2), the data stream first passes through the synthesis filter bank (upsampled by M and then convolved with M synthesis filters $F_k(z), k = 1,..,M$ and then summed) and the output is $X(z)$. In the absence of channel and noise, this output is passed through the analysis filter bank (i.e., $X(z)$ convolved with M analysis filters $G_k(z)$, and then downsampled by M) yielding the original data upto a delay, i.e, $z^{-I_l}D(z)$. In the presence of channel (and noise), this relationship is no longer valid due to ISI and attenuation. It is desirable to eliminate ISI. Here a solution based on the discussions in the previous chapter is considered: an upsampling and downsampling by some M^c (M^cgreater than 1, but otherwise arbitrary) and additional filters—transmitter and receiver filters ($F^c(z)$ and $G^c(z)$ respectively)— before and after the channel to combat ISI.

Consider the problem of a perfect transmultiplexer in the presence of a channel (modelled as $C(z)$) as shown in Figure 4-2. The output of the synthesis filters is $X(z)$ as above. This is upsampled by M^c and then convolved with the three filters (transmitter, channel and receiver) and then downsampled by M^c. Thus, the outputs at the various stages prior to the analysis filter bank are $X\left(z^{M^c}\right)$, $X\left(z^{M^c}\right)F^c(z)$, $X\left(z^{M^c}\right)F^c(z)C(z)$, $X\left(z^{M^c}\right)F^c(z)C(z)G^c(z)$ and $X\left(z^{M^c}\right)F^c(z)G^c(z)|_{\downarrow M^c}$ respectively. In order to ensure no ISI, it must be that

$$X\left(z^{M^c}\right)F^c(z)C(z)G^c(z)|_{\downarrow M^c} = z^{-L^c}S(z). \tag{4.13}$$

This can be ensured by demanding that

$$F^c(z)C(z)G^c(z)|_{\downarrow M^c} = z^{-L^c}. \tag{4.14}$$

This condition is independent of $S(z)$, as is desired, and hence the results obtained in the previous section can be applied immediately.

4.5 Modified Multicarrier Modulation Architecture

Recall that for the case $M^c = 2$, the filters of minimal length are of degree $(c - 1)$ and satisfy only the zero ISI condition. For simplicity, the modified multicarrier modulation architecture is presented below for $M^c = 2$; extension to the general case is straightforward.

1. The transceiver filters are the same as before: IFFT and synthesis filter banks for DMT and DWMT respectively.
2. Upsample by 2 the processed data (i.e., output of data passed through the synthesis filter bank).
3. Determine the CIR by means of a training sequence.
4. Then design a minimal length zero ISI filter; details discussed in the previous chapter.
5. Pass the upsampled data through this zero ISI filter prior to transmission.
6. Upon reception, the received signal is downsampled by 2. If the channel had been determined perfectly, the received data would be the transmitted data precisely.
7. This is then passed through the analysis filter bank, as before. The output is precisely the original data if the channel impulse response is known exactly. If the error is small, the simplest detection mechanism (finding the data closest in

Euclidean distance to the received signal) would still yield "perfect reconstruction" after passing through the analysis filter bank.

8. Repeat steps 3 and 4 frequently for time-varying channels.

4.6 Comments on the Multirate Solution

It is clear that the channel has to be estimated accurately to combat ISI by the multirate algorithm (or for other solutions employed in DMT and DWMT systems). Several schemes for channel identification are known. An easy method is to use a periodic training sequence with period slightly longer that the CIR [25]. The receiver then measures the channel output (averaged over several cycles) and then divides the DFT of the channel output by the DFT of the known training sequence. In other words, the channel estimate in the frequency domain is

$$C'_n = \frac{1}{L}\sum_{i=1}^{L}\frac{Y_{i,n}}{X_n} \tag{4.15}$$

where X_n is the nth element of the DFT of the input training sequence and $Y_{i,n}$ is the nth element of the DFT of the channel output on the ith cycle.

The CIR obtained from such a channel estimation scheme may be used as an input to the adaptive multirate zero ISI system. In order to avoid increasing receiver complexity, the multirate zero ISI filter is placed in the transmitter (i.e., $P(z) = F^c(z)$). From the CIR, the minimal length zero ISI filter may be designed; two simple algorithms were presented, particularly one for the case of $M^c = 2$.

As discussed earlier, the multirate system can be applied to the DMT system. Note that ISI elimination is possible without the use of a cyclic prefix or a TEQ. Further, there is no noise enhancement since the filtering is done prior to transmission. This would simplify the receiver structure. However, all this is true assuming that the channel is well known. So some residual ISI is still expected. But this suggests that the multirate zero ISI filter system can simplify the TEQ and the receiver significantly.

If the channel were known perfectly, the multirate zero ISI filter would eliminate ISI in DWMT systems as well. In particular, no complex post-detection equalization would be needed. This suggests that the inclusion of the multirate zero ISI filter would significantly reduce the computational burden of the optimal post-detection processing.

An expected problem in the solution proposed is the increase in transmitter power. It is expected that the multirate filter will increase the required transmitted power by about $20 \log \|P\|$, over the case of no such filter. However, the filter is more than just an amplifier, because there is also an multirate sampling involved which guarantees zero ISI in the absence of noise. It is possible that less power might be needed because the filter shapes the transmitted data to combat ISI. This is discussed further in the next chapter where some simulation results are presented.

Another problem is the size of the matrices that need to be inverted for determining the multirate zero ISI filters. For instance, it is of the order of the CIR when $M^c = 2$. The matrices do have some symmetries and are sparse, which could be exploited to design faster algorithms for inversion of such matrices.

However, it is clear that the precoder design is much simpler than those proposed in the literature. Usually, there are issues of rate of convergence of algorithm, stability issues in algorithms proposed in the literature; there are no such issues in the proposed system. The computational load is simply in the determination of a column of the inverse of a matrix formed from channel coefficients. The size of the relevant matrix decreases with increase in upsampling/downsampling factor.

Chapter 5

Simulations

5.1 Preliminaries

5.1.1 Cyclic Prefix and Multirate Zero ISI Filter Solutions

In the simulations, two ISI-elimination techniques for DMT transmission systems are studied. One is the cyclic prefix solution and the other is the proposed minimal length, multirate zero ISI filter solution with no cyclic prefix. These are incorporated in a DMT system with an FFT (and hence block) size of 512.

Results are presented for data transmission with and without a bit loading algorithm. When no bit loading algorithm is used, it implies that the standard DMT with cyclic prefix solution performance has been underestimated, but that is also the case for the multirate zero ISI filter solution.

This is also done because there is a significant difference between the two methods of ISI elimination: there is also an upsampling and downsampling in the proposed multirate system, which is absent in the cyclic prefix method. As discussed below, the optimum bit loading algorithm for the multirate system is not obvious due to the upsampling/downsampling.

A comparison with the standard DWMT receiver was not undertaken due to the complexity of the post-detection processing. This is also being investigated in detail elsewhere by N. Tressler at the BARLO group in the University of Ottawa. However, the performance of any system equipped with the same multirate zero ISI filters, be it a DWMT system or a DMT system, is the same under additive noise, when there is no bit loading algorithm, assuming model of the system to be as in Chapter 4. This is because the underlying ISI elimination method is the same and depends only on the channel and not on the particular PR transceiver filters. So results for the multirate system here reflect its performance in other PR transmultiplexer systems, when no bit loading is done.

For simplicity, only the case $M^c = 2$ is considered for the multirate zero ISI filters and results are presented for the case of binary signalling. An example is provided to show why higher values of M^c (for minimal length filters) may not offer any improvement over the $M^c = 2$ case.

5.1.2 SNR

The SNR is defined as the mean energy of the transmitted signal prior to transmission over channel divided by the noise variance. In particular, this means that when systems with the zero ISI filters are considered, the signal energy is calculated after the signal has passed through the transmitter filter (note that the multirate filter system chosen is such that there is no receiver filter, i.e., $P(z) = F^c(z)$). This takes into account the amplification produced by the transmitter filter. Thus, the SNR is calculated by calculating the total energy of the signal (prior to passing through channel) and then dividing it by the number of input data. Note in particular that an upsampling by 2 in the multirate zero ISI filter configuration (for $M^c = 2$) leads to a 3dB loss in SNR in such systems.

5.1.3 Bit Loading Algorithm

A simple bit loading algorithm was presented in the previous chapter (see also [24], [26]). This bit loading algorithm is for standard DMT system is given by

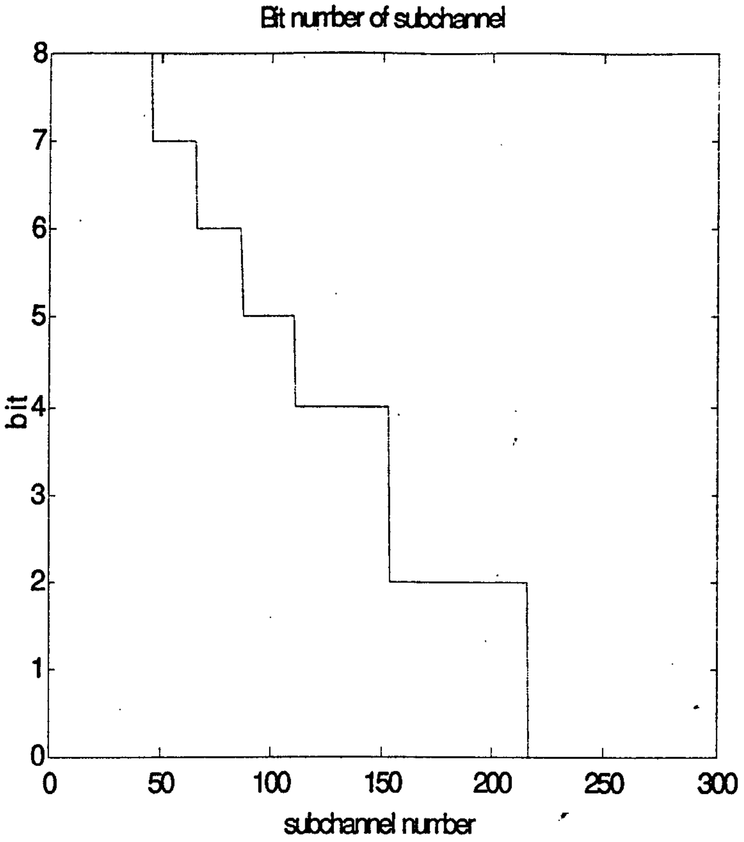

Figure 5-1: A sample bit allocation for the DMT subchannels, based on a bit loading algorithm.

$$b_i = \log_2\left(1 + \frac{SNR_{ci}}{\Gamma}\right). \tag{5.1}$$

As an example, the margin is set at 6 dB with no coding gain. The bit assignments are illustrated in Figure 5-1.

It should be pointed out that this bit loading algorithm is not optimal for the multirate system. This is because there is an upsampling (and a transmitter filter) prior to transmission through the channel. The upsampling by M^c results in a signal whose spectrum is an M^c-fold periodic repetition of the input signal spectrum. Thus, it is not obvious what the optimum bit loading is for this case. In fact, the above bit assignment led to a poorer performance of the multirate system as compared to when no bit loading was done. Thus, for simplicity, no bit loading algorithm is used for the multirate zero ISI filter system.

5.2 Choice of Filter

It is desirable that the multirate zero ISI filter (P) satisfy two conditions:

- The energy of the filter be 0 dB; i.e., $||P|| = 1$.
- The filter also satisfy the single rate zero ISI condition; i.e., $||PC|| = 1$.

Neither of those conditions can be satisfied for any realistic channel. The first condition cannot be satisfied because of channel attenuation. Some amplification is needed to compensate for channel attenuation.

The second condition can never be satisfied for any FIR $P(z)$ as the condition $P(z)\,C(z) = z^{-L^c}$ implies an overdetermined systems of equations. In other words, the best choice among the minimal length multirate solutions is the one 'closest' to satisfying

$$P(z)\,C(z) \approx z^{-L^c}. \tag{5.2}$$

Note that multirate zero ISI condition (which all multirate zero ISI filters satisfy) is much less restrictive since it only demands that

$$C(z)\,P(z)\,|_{\downarrow M^c} = z^{-L^c}. \tag{5.3}$$

The condition in Equation 5.2 cannot be valid in general for it leads to a system of linear equations with more equations than unknowns.

So it is reasonable to impose that condition that the filter be as close as possible to satisfying the two conditions. That is, the following two constraints can be imposed for better performance under additive noise:

- Minimize $||P||$.
- Minimize $||PC||$.

Recall that there is a minimal length multirate zero ISI filter $P_i(z)$ corresponding to a different delay. In that case, a criterion that is reasonable to adopt is that we choose the filter $P_r(z)$ among the set of all possible minimal length multirate zero ISI filter that minimizes $||P||$ and $||PC||$.

It is possible that different filters minimize $||P||$ and $||PC||$. Several criteria can be adopted to pick one among them such as,

- $\min_i(\alpha\,||P_iC|| + (1-\alpha)\,||P_i||)$, where $0 \leq \alpha \leq 1$.
- $\min_i\left(\sqrt{\alpha\,||P_iC||^2 + (1-\alpha)\,||P_i||^2}\right)$, with $0 \leq \alpha \leq 1$.

In our simulations, it was found that filters that satisfied one condition, say $\min ||P||$, are also close satisfying the other condition, $\min ||PC||$, and performed similarly under additive noise. This can be intuitively understood from the fact that filters with least energy are not likely to have very large values of $||PC||$, and vice versa.

The criterion adopted in the simulations is to choose the filter closest to satisfying the single-rate zero ISI condition ($\min_i ||P_iC||$). The reason for the choice is to choose the best filter from ISI point of view. It is expected that the algorithm will not perform very well when $||PC||$ is very large compared to the ideal $||PC|| = 1$. Further, for better performance, in terms of SNR for a given error probability, it is also expected that the multirate solution will not fare as well if $||P||$ is very large (for example, of order 10^2 or higher). This expectation is confirmed by experiments.

The above discussion immediately suggests that non-minimal length multirate zero ISI filters can further improve ISI-elimination capability. This is because the additional degrees of freedom can be used to find the filter that simultaneously minimizes $||PC||$ and $||P||$. In this thesis, only the minimal length case is discussed.

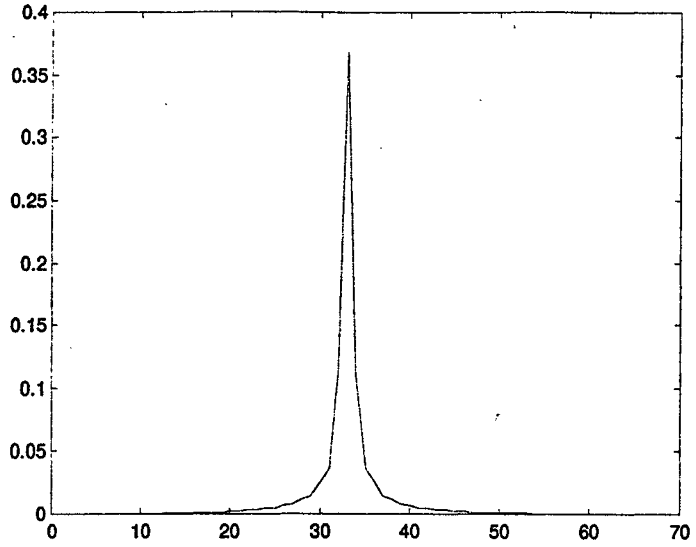

Figure 5-2: A linear phase FIR filter with 64 taps.

Finally, these filters are clearly not optimal; that would require matched transmitter and receiver filters. That is, identical filter coefficients occur in both filters, but in time reversed order with respect to each other [1]. Such filters, also satisfying the multirate zero ISI condition, are harder to design.

5.3 Example 1: A Linear Phase FIR Channel Model

In the first example, the channel is a linear phase FIR model [27]of the twisted pair channel for ADSL with sampling rate of 2.2 MHz and for a 1 km line. In Figure 5-2, the channel taps in this ADSL-inspired 64-tap model is plotted.

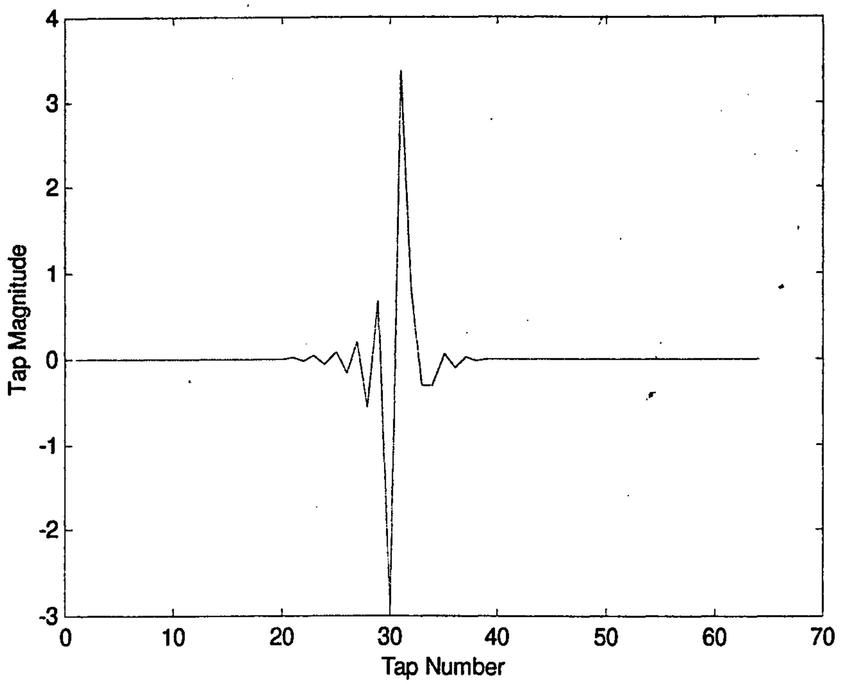

Figure 5-3: The corresponding minimal length multirate zero ISI filter with $L^c = 32$.

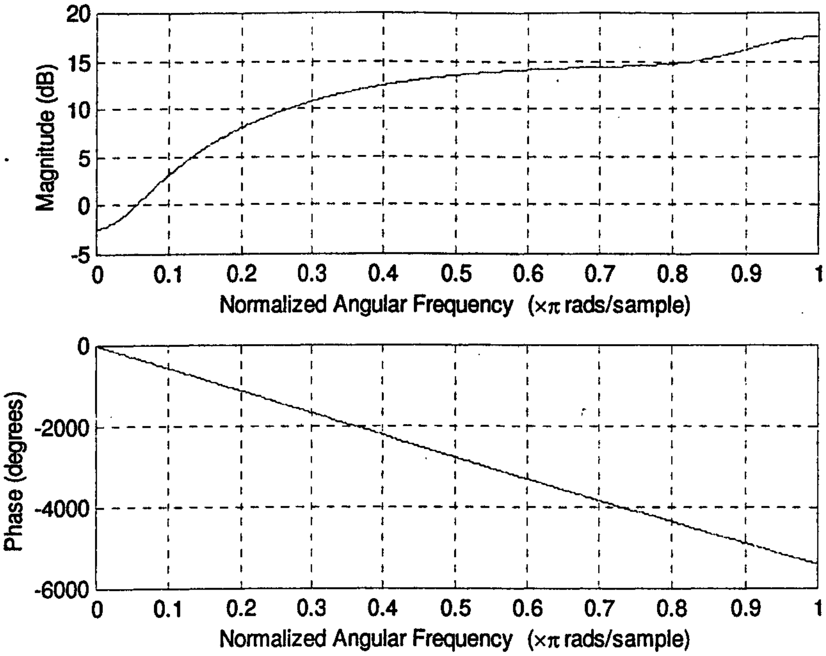

Figure 5-4: Magnitude and phase response of filter in Figure 5-3.

The corresponding (best choice) minimal-length multirate zero ISI taps are shown in Figure 5-3. The frequency response of the filter is shown in Figure 5-4. The filter also is linear phase. It has the characteristics of an emphasizer; it suppresses the low frequency components relative to the high-frequency part to undo the effects of channel attenuation. However, recall that there is also an upsampling/downsampling involved that is crucial for zero ISI.

In Figure 5-5 is shown the bit error rate (BER) for the two systems as a function of SNR. In the simulations, the power in the channel noise at the receiver is set to -30 dB. The results for cyclic prefix are marked by 'o' (without bit loading) and '$*$' (with

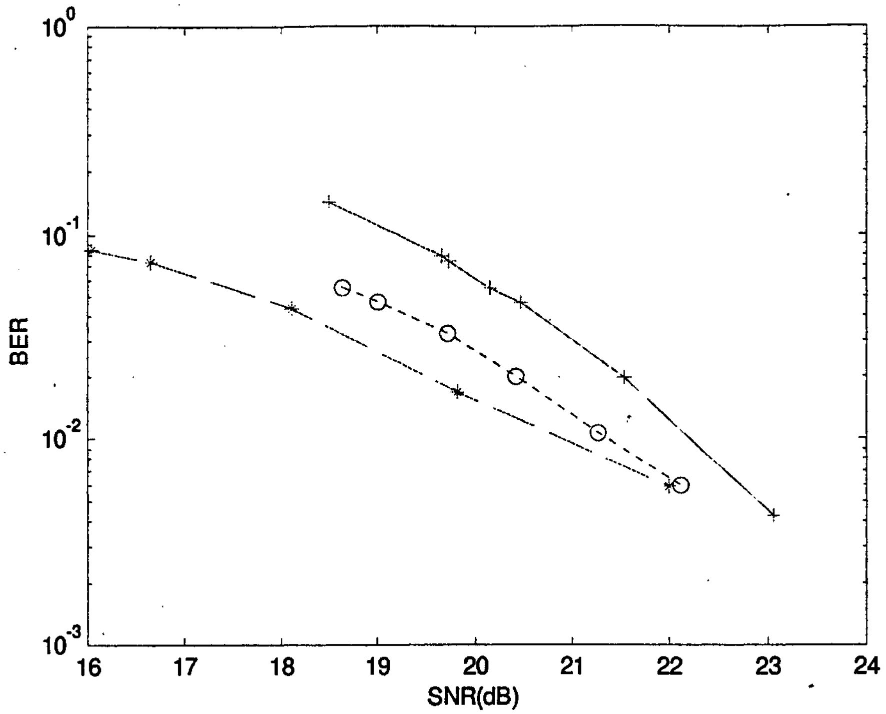

Figure 5-5: BER vs SNR for linear phase channel model ('+',zero ISI filter, '*' bit-loaded DMT, 'o' DMT without bit-loading).

bit loading), and those for the multirate filter by '+'. In this case, it is seen that the cyclic prefix solution outperforms the multirate solution at low SNRs. The use of a bit loading algorithm clearly improves the performance. However, at high SNRs (BER of order 10^{-2} or less), the performance of the multirate method is comparable to the cyclic prefix method with little increase in transmitter power requirements. It is found that $||P|| = 4.65$ and $||PC|| = 1.3$, which makes plausible the relatively good performance. Note that no cyclic prefix has been used when the multirate solution is implemented.

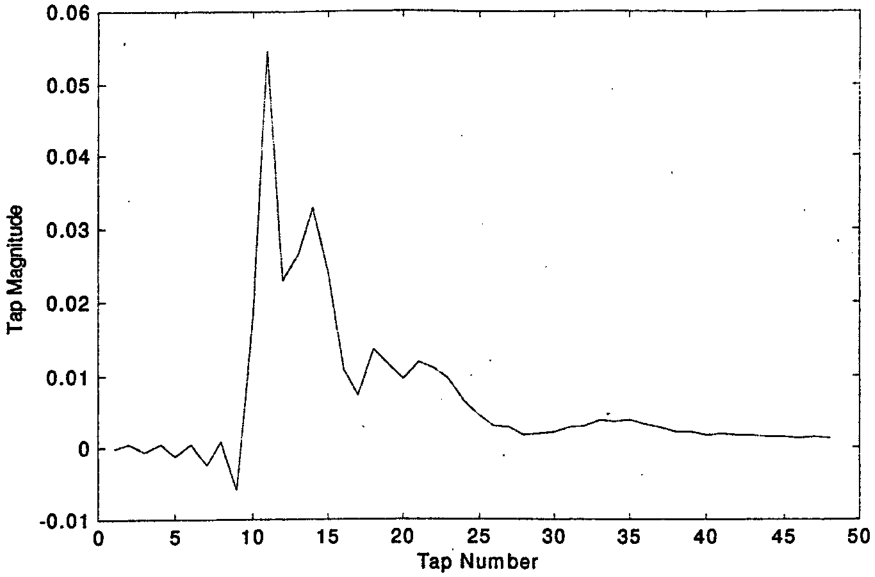

Figure 5-6: A model of a 1km ADSL channel with bridge taps.

5.4 Example 2: An ADSL Model with Bridged Taps

In our next example, the channel model is a more realistic ADSL channel model with bridge taps (see Figure 5-6) [28]. This models 300m and 130 m bridge tap at 800m and 900 m of a 1km AWG 26. The frequency response of channel is plotted in Figure 5-7.

The filter coefficients of the zero ISI filters for this channel are plotted in Figure 5-8. Note the relatively large variation in the magnitude of the filter coefficients. This follows from the large variation in the channel tap magnitudes. The frequency response of this zero ISI filter is in Figure 5-9.

In Figure 5-10 is plotted the probability of error vs the signal-to-noise ratio for DMT

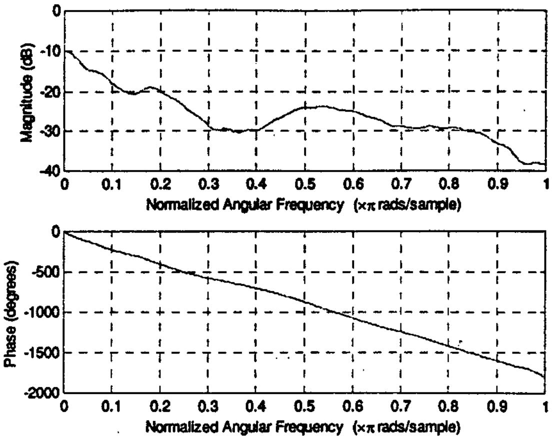

Figure 5-7: Magnitude and phase response of ADSL channel model in previous figure.

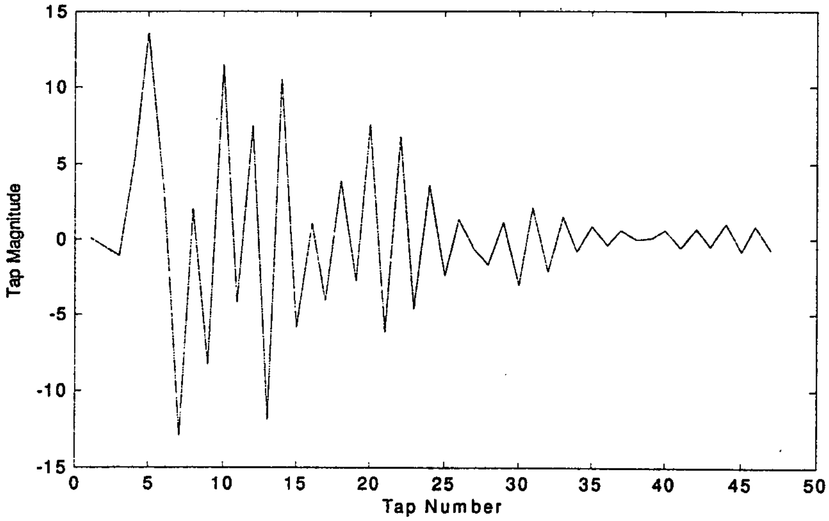

Figure 5-8: Minimal multirate zero ISI filter ($M^c = 2, L^c = 9$) for channel in Figure 5-6.

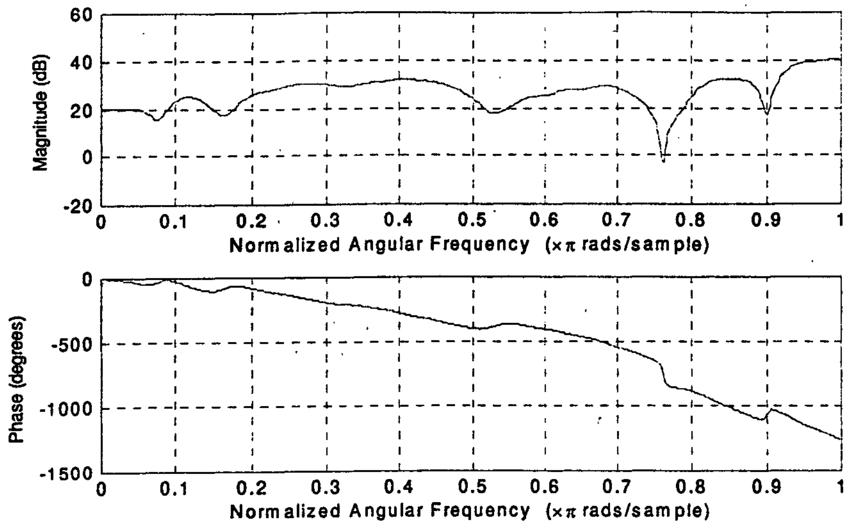

Figure 5-9: Frequency response of filter in Figure 5-8.

with cyclic prefix and DMT and DWMT with the zero ISI filter mentioned above. The performance of the multirate solution is comparable to the cyclic prefix solution with bit loading algorithm at lower BERs (of order 10^{-2} or less) for a slight increase (about 1 dB) in transmitter power. This can be understood from the fact that $||P|| \approx 35$ (also $||PC|| \approx 1.33$).

5.5 Example 3: Large $||P||$

As an example of where the multirate solution underperforms, consider the channel in Figure 5-11. For this channel, $||PC|| = 2.62$ and $||P|| \approx 145$. This is not expected to perform as well due to the relatively large value of $||P||$.

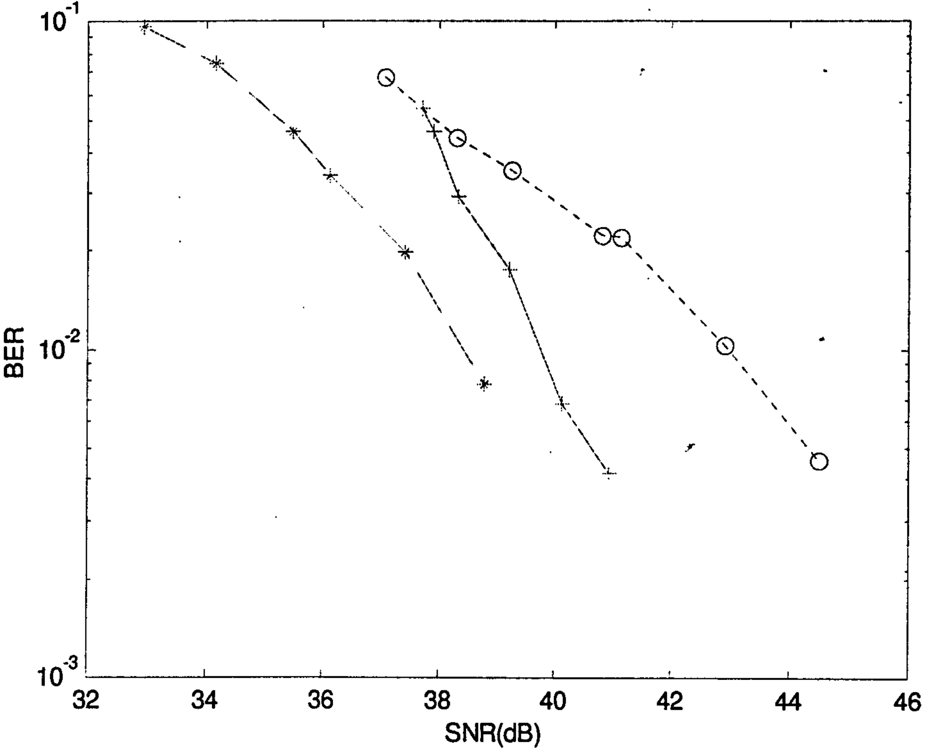

Figure 5-10: BER vs SNR ('+' zero ISI filter, '*' bit-loaded DMT, 'o' DMT without bit-loading).

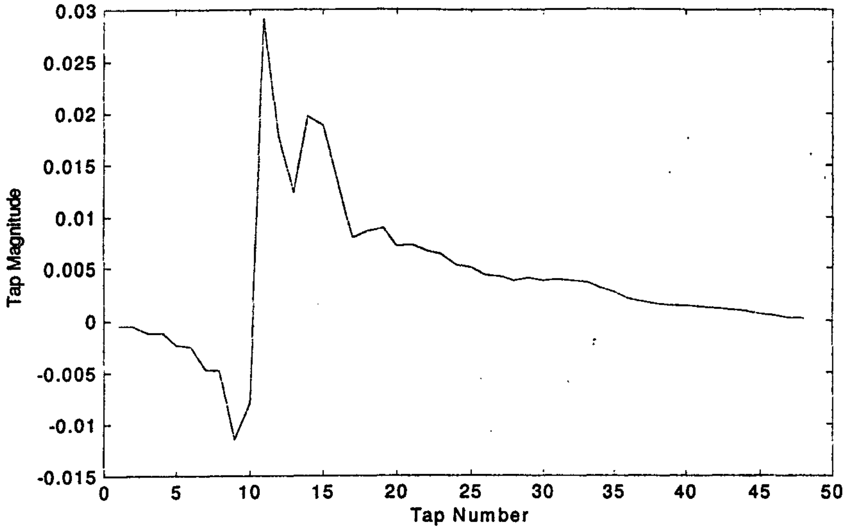

Figure 5-11: A channel model.

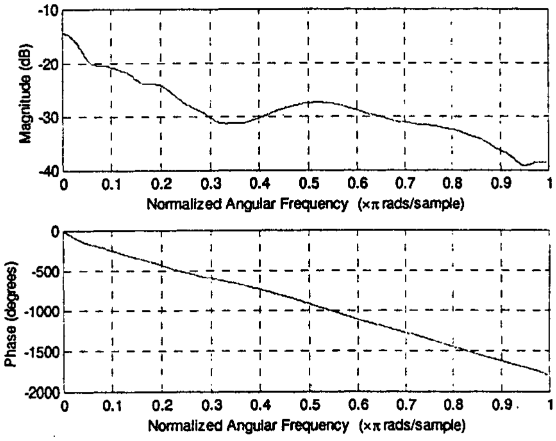

Figure 5-12: Frequency response of channel in previous figure.

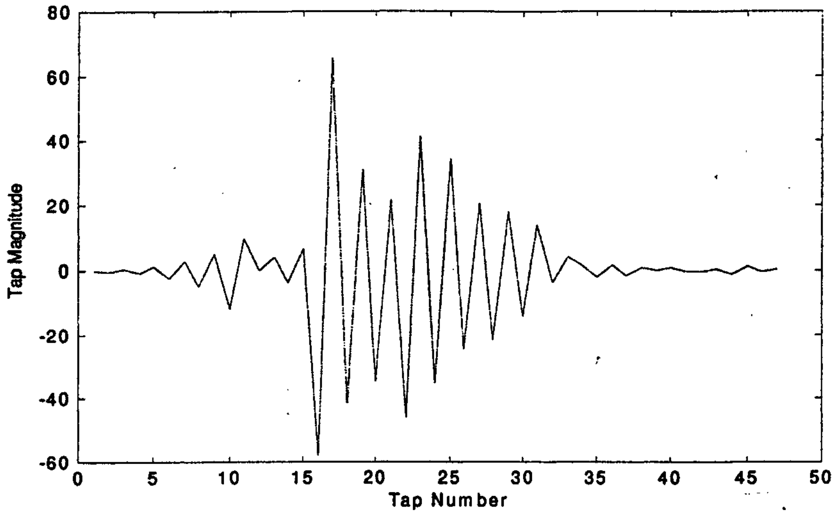

Figure 5-13: Minimal length multirate zero ISI filter for channel in Figure 5-11.

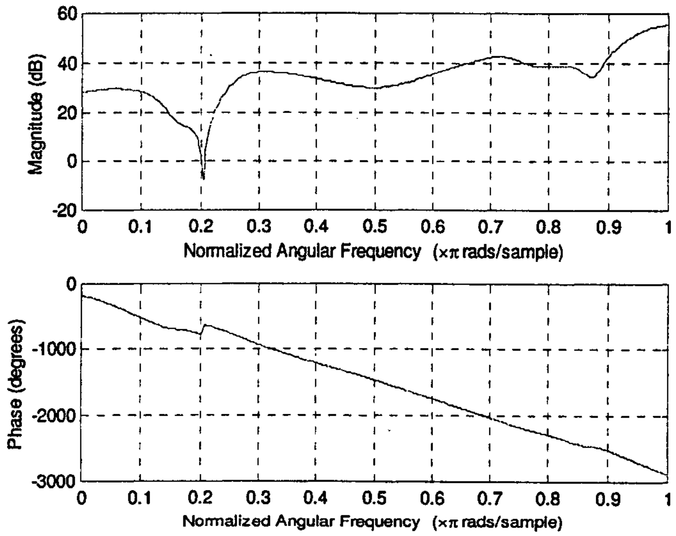

Figure 5-14: Frequency response of zero ISI filter in previous figure.

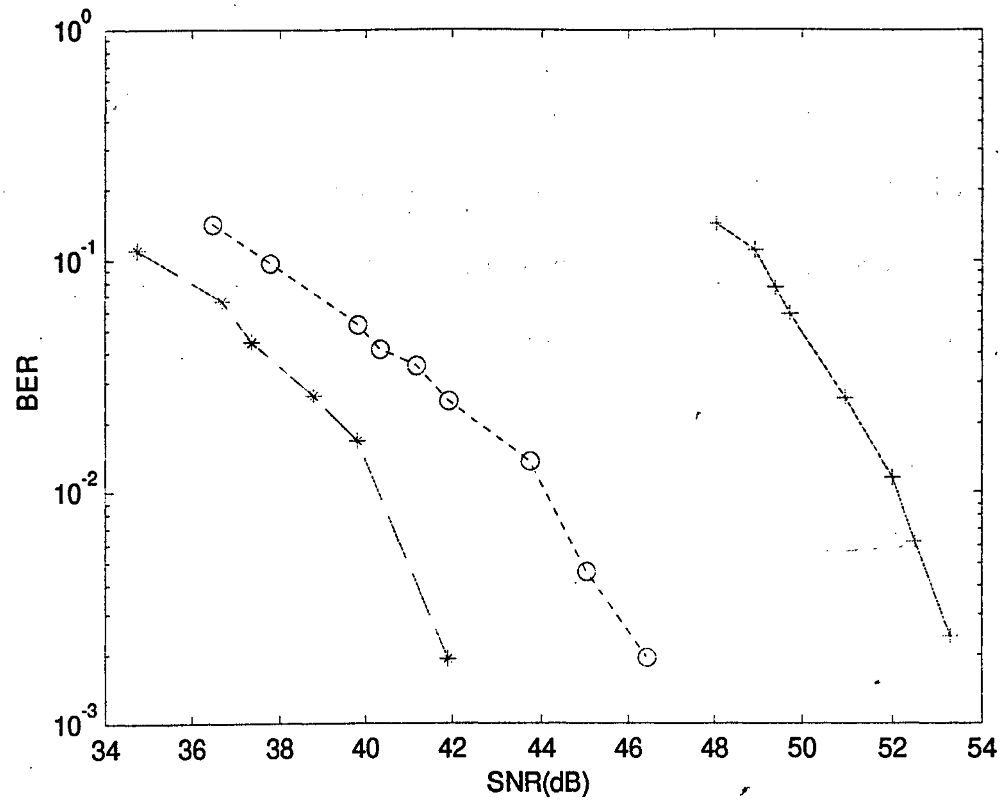

Figure 5-15: BER vs SNR ('+' zero ISI filter, '*' bit-loaded DMT, 'o' DMT without bit-loading).

The corresponding multirate zero ISI filter is shown in Figure 5-13. The plot of probability of error vs SNR confirms the expectation that the cyclic prefix solution (marked 'o') is much better than the minimal length multirate zero ISI filter for this channel. The increase in transmitter power needed for the multirate system at BER 10^{-2} is about 10 dB.

5.6 Example 4: Different Choices for M^c

There are an infinite number of possible multirate zero ISI filter systems that eliminate ISI for a given channel impulse response for any $M^c > 1$. In general, the only requirement for a possible zero ISI filter is that the filter length exceed a certain minimum, as specified in Theorem 3. Furthermore, there are several possible solutions even for the minimal length multirate zero ISI filter.

It is expected that a larger value for M^c will introduce a larger overhead. As an illustration, a channel of impulse length 25 is chosen. The values for M^c are taken to be 2 and 5. The channel model taps are shown in Figure 5-16. This channel is a linear phase FIR model of the ADSL channel. The magnitude and phase response of this channel is shown in Figure 5-16.

There are two different of minimal length, multirate zero ISI FIR filters. Figure 5-18 shows the taps for the case $M^c = 2$ and the taps for $M^c = 5$ case is shown in Figure 5-19. The values of $(||P||, ||PC||)$ for the two systems are $(1.14, 5.08)$ and $(1.22, 3.24)$ for $M^c = 2$ and $M^c = 5$ respectively. Note that the higher the sampling frequency, smaller is the length of the zero ISI filter, and less is the computational load in calculating it. The BER is plotted as a function of SNR for the two systems in Figure ??, and it is clear that the $M^c = 2$ system requires less transmitter power.

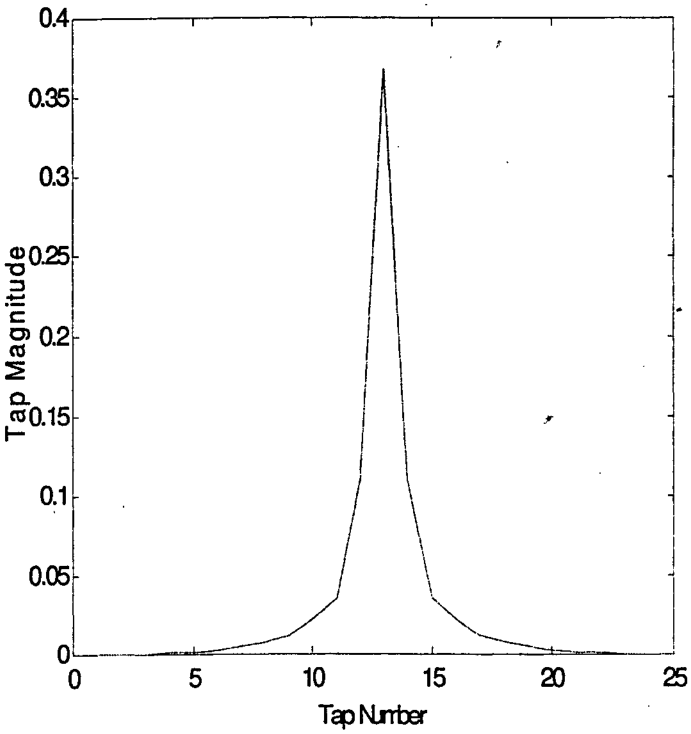

Figure 5-16: A sample channel with 25 taps.

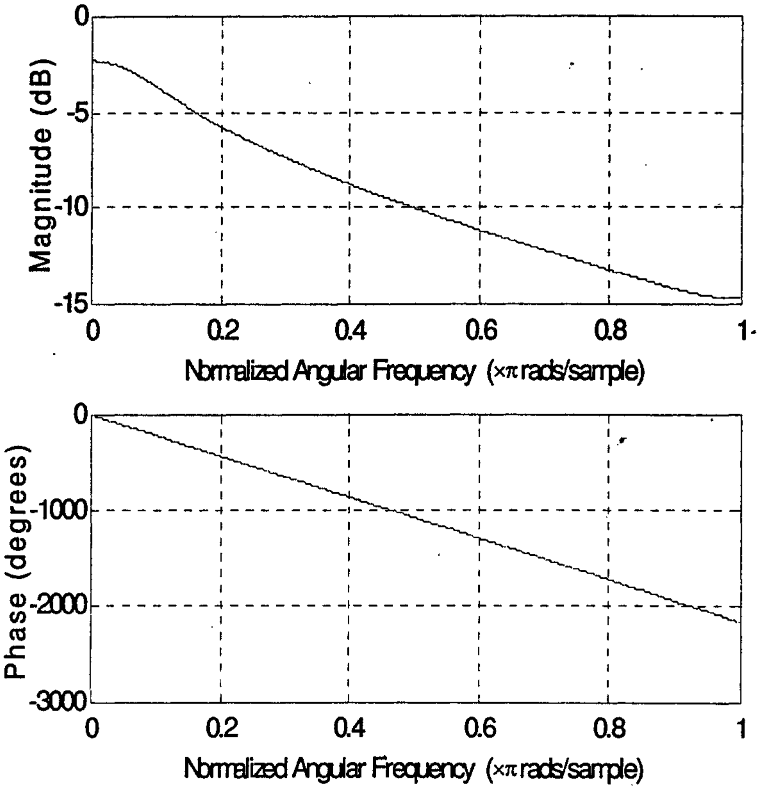

Figure 5-17: The magnitude and phase response of channel in Figure 5-16.

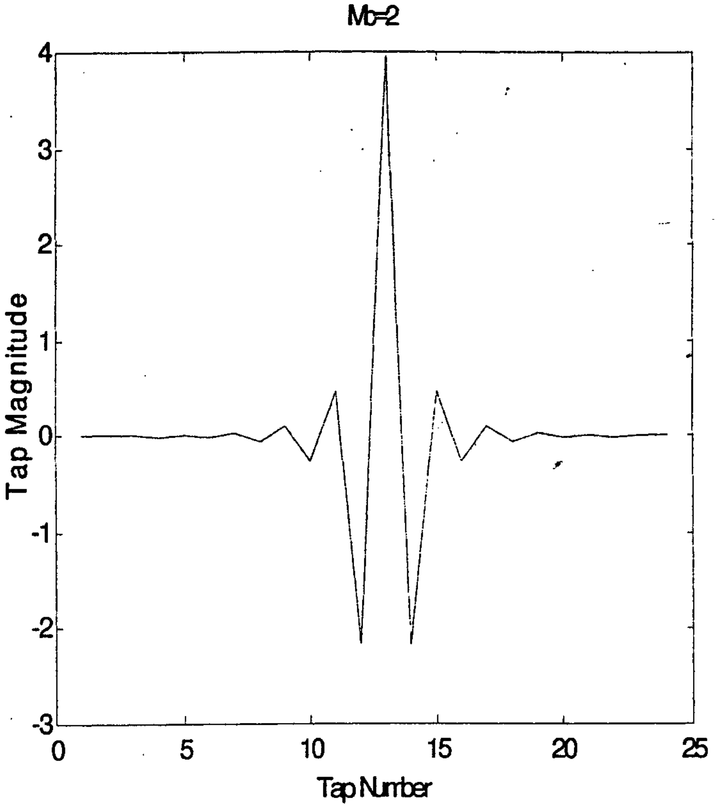

Figure 5-18: Taps of multirate zero ISI filter for the channel in Figure 5-16 and $M^c = 2$.

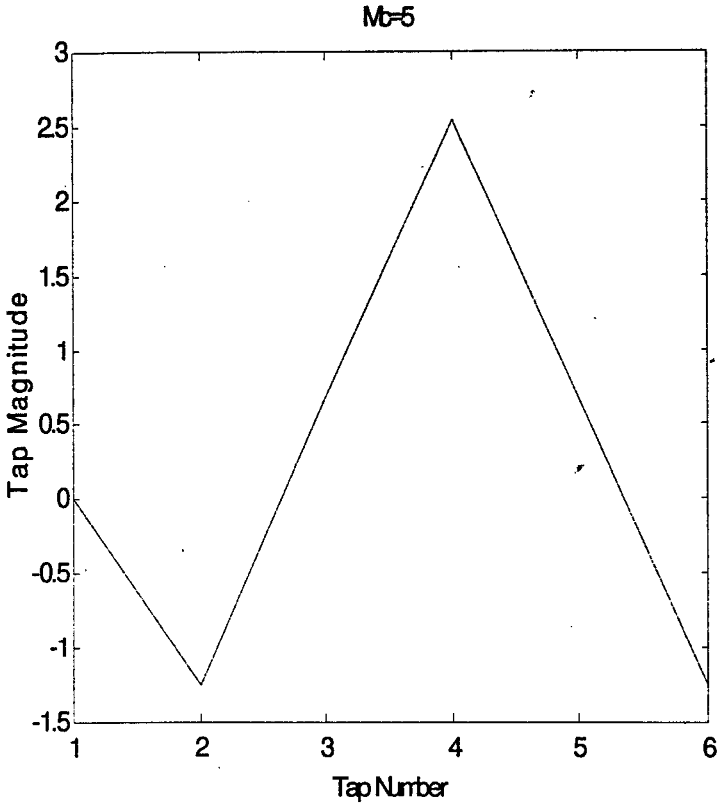

Figure 5-19: Taps for $M^c = 5$ zero ISI filter for channel in Figure 5-16.

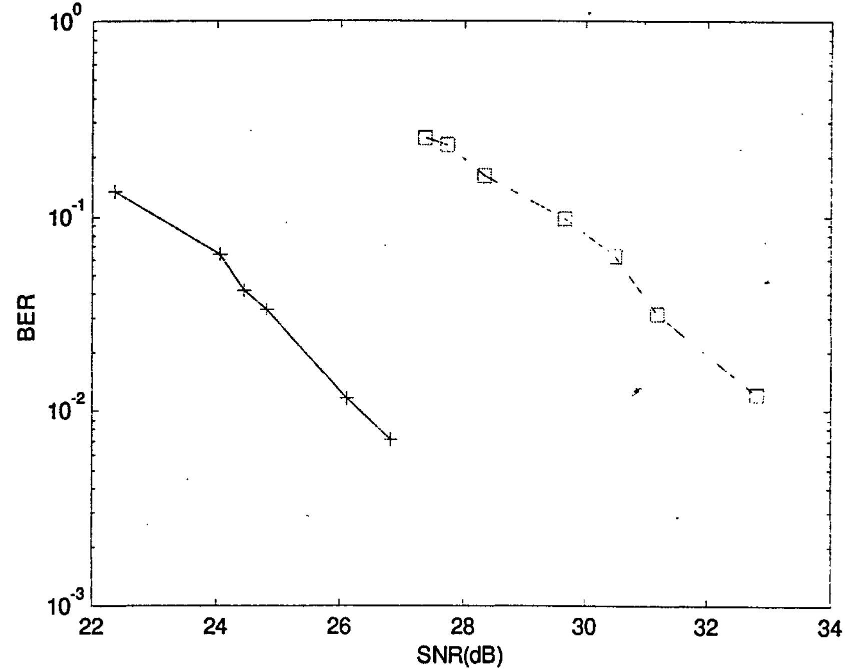

Figure 5-20: Comparison of BER vs SNR for $M^c = 2$ (solid) and $M^c = 5$ (dashed).

5.7 Additional Comments

It should be pointed out that the multirate solution assumes that the channel is known very well, but the CIR can be of arbitrary length. However, the longer the CIR, the larger is the size of matrix that needs to be inverted. The channel needs to be estimated accurately for the cyclic prefix solution (the FEQ, in particular), but it is also required that the CIR length not exceed the length of the cyclic prefix.

It was observed that the multirate zero ISI filter method of combatting ISI can perform as well as the cyclic prefix solution without any significant additional transmitter provided that $||PC|| \approx 1$ and $||P|| \approx$of order 10; for larger values, the multirate solution requires higher transmitter power. Also, it has been assumed that the matrix to be inverted is not singular, as discussed in the previous chapter. It was also found that the case $M^c = 2$ is likely to be of most interest due to its simplicity and lower transmitter power requirements.

Note that only the minimal length filters were studied. So there is no degree of freedom left over for better filter design (for instance, minimizing $||PC||$ and $||P||$). It is likely that better performance will be obtained from filters of non-minimal length. Neverthless, it is encouraging to note that even the minimal length filters perform quite well.

It should be emphasized that the proposed multirate method is much simpler than the conventional DMT systems that employ a TEQ. No cyclic prefix or precoder is required for the multirate system. There is some increase in required transmitter power, the precise amount depends on the channel. However, the receiver complexity is reduced considerably as no cyclic prefix, TEQ or FEQ are needed. The proposed system is also simpler than the precoder studied in the literature (for example, [17]).

Chapter 6

Conclusion

The problem of intersymbol interference arises when transmitting data over non-ideal channels. Such channels introduce amplitude and phase distortion. It is a classic problem in digital communications and there are some well-known solutions, such as those based on raised cosine filters. However, due to necessary truncation, zero ISI is maintained only approximately in such implementations.

A multirate system proposed by Chevillat and Ungerboeck is intrinsically digital and guarantees zero ISI in the absence of channel and noise. In this multirate system, the input data is first upsampled by some positive integer (M^c) and then filtered by an appropriately designed transmitter filter. Upon reception, the received sequence is filtered by the matched receiver filter and then downsampled. The M^c-downsampled convolution of the transmitter and receiver filters is required to satisfy the zero ISI condition, i.e., it satisfies the 'multirate zero ISI condition'.

Summary of Contributions

The work of Chevillat and Ungerboeck focussed on optimum transceiver FIR filters (i.e., matched transmitter and receiver filters) over an ideal channel. In this thesis, their work is extended to the case of an arbitrary, non-ideal channel. As before, the data is first upsampled by M^c and then filtered by the transmitter filter. The received data is then also filtered by the receiver filter, after it has passed through the channel. The convolution of the channel and the transceiver filters now satisfy the multirate zero ISI condition.

This analysis leads to the design of zero ISI filters for any $M^c>1$. Based on this, two simple adaptive algorithms for computing minimal length zero ISI filters were presented. A potentially useful algorithm is applicable for channel of arbitrary length (c) and $M^c = 2$ and requires the computation of the inverse of an $(c-1) \times (c-1)$ matrix formed from the channel taps. This gives one of the infinitely many possible non-minimal length zero ISI filters.

Intersymbol interference is a serious concern in multicarrier modulation systems such as the DMT and DWMT transmission systems. The problem is that channel distortion destroys the perfect reconstruction property of the perfect reconstruction filter bank, even in the absence of noise. In the proposed modified DMT/DWMT architecture, the multirate zero ISI filter system is placed between the synthesis and analysis filter banks of the transmultiplexer in DMT/DWMT system. In the absence of noise, perfect reconstruction is recovered because of the multirate zero ISI filters.

In the previous chapter, some simulation results were presented. It was found that that even the (non-optimum, but best choice) minimal length zero ISI filters can perform as well as the cyclic prefix method without any significant increase in transmitter power.

Future Research Issues

The work in the thesis can be extended in several ways. For instance, it would be interesting to investigate the performance of the proposed multirate system on other realistic channel models. Also, most of the experiments were done with $M^c = 2$. The issue of optimum M^c (perhaps, channel dependent?) needs to be explored in greater detail.

The results in the previous chapter were based on the minimal-length multirate zero ISI filters, which determined the filter uniquely for a given delay (L^c) and upsampling/downsampling (M^c). It is clear that non-minimal filters will yield better performance, since there will be some filter taps which will remain unspecified by the multirate zero ISI condition. These unspecified taps can then be used for better filter design, such as minimization of the energy of the multirate zero ISI filter. Methods for adaptive design of better non-minimal filters need to be studied.

The filters studied in the thesis were not optimum. As discussed in the thesis, design of optimum matched filters is harder, since it involves the solution of a system of quadratic equations. It is useful to investigate and see if the problem can be reformulated so that it is possible to design optimal multirate filters adaptively.

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