CLUSTER MODEL CALCULATIONS
ON Li$_6$

by

Valluri Sree Ram

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Department of Physics,
Faculty of Science and Engineering,
University of Ottawa,
OTTAWA, Canada

1970

ProQuest LLC
789 East Eisenhower Parkway
P.O. Box 1346
Ann Arbor, MI 48106-1346

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ABSTRACT

The radiative transition probabilities from the 3^+ state at 2.18 Mev and the 0^+ state at 3.56 Mev to the 1^+ ground state are calculated using cluster model wave functions. In this model, the 3^+ and the 1^+ states are assumed to have an alpha-deuteron cluster structure and the 0^+ state an alpha-deuteron cluster structure in which the deuteron cluster is excited to the lowest $T = 1$, $S = 0$ state.

The results obtained were in reasonable agreement with the experimental values, thus confirming a good degree of accuracy of the cluster model wave functions for Li^6.

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INTRODUCTION

The cluster model has had considerable success in describing the low-lying energy levels of Li^6. Pearlstein, Tang and Wildermuth$^{(1)}$ have attained good results in calculating the energy levels of this nucleus which have the highest space symmetry, i.e., the first six levels of Li^6.

The Li^6 nucleus is a very interesting nucleus both from the theoretical and experimental points of view. It is a stable nucleus (as opposed to the lighter He^5 or Li^5) containing on the one hand sufficiently many nucleons to exhibit many general features of nuclear phenomena and on the other hand sufficiently few that detailed calculations can be carried out on it.$^{(2)}$.

The radiative transitions from the excited 3^+ and 0^+ states to the 1^+ ground state have been observed experimentally. Hence the radiative level widths are known by means of the formula:

$$\Delta E . \quad \Delta t \sim \hbar \tag{1-1}$$

Δt = (Transition Probability per unit time)$^{-1}$

In this work, by the use of the cluster model, the radiative transition probabilities from the excited 3^+ and 0^+ states to the 1^+ ground state have been calculated. The radiative level widths are then compared with the experimental values.

The γ decay transition probabilities between two nuclear states depend sensitively on the wave functions of these states. Because of this dependence, significant information regarding nuclear wave functions can be obtained from a comparison of experimental γ decay transition probabilities with theoretical values calculated on basis of the cluster model. The comparison gives an idea of how good the cluster model is for the particular states involved in the transitions and helps in understanding the structure of the Li^6 nucleus.

In the cluster model, different correlations between the nucleons are favoured in different nuclear states. The low energy states of Li^6 are described by the α particle cluster-deuteron cluster structure.(3) In the 3^+ and 1^+ states, the clusters have no internal excitation; in the 0^+ state the deuteron is excited to its ${}^1S_0$ state.

In the single particle picture, if we are not to violate the Pauli principle, the low energy states of Li^6 must have four nucleons in the 1s shell and two nucleons in the 1p shell. Therefore the oscillator cluster functions must describe states with at least two oscillator quanta of energy, i.e., 2 Mev of energy above the zero-point energy, because otherwise the cluster wave function will vanish under antisymmetrization.

Since the clusters have no internal excitation in the cluster model, this energy appears in the relative oscillations of the two clusters. It is seen from the diagram that with two quanta of excitation, a 2s or a 1d oscillation is possible. The orbital angular momentum can thus be $\ell = 0$ or $\ell = 2$;the former corresponds to the 1^+ ground state and the latter to three excited states associated with total angular momentum $J = 3, 2, 1$, resulting from the addition of the orbital angular momentum $\ell = 2$ and the deuteron cluster spin $s = 1$ in the triplet state ($T = 0$). Spin-orbit interactions give the sequence shown in Figure 2.(4).

The 0^+ state at 3.56 Mev is assumed to be composed of an unexcited α particle cluster and a deuteron cluster excited to the lowest $T = 1$, $s = 0$ state. Since $L = 0$ and $S = 0$, $J = 0$. The 0^+ state lies lower in energy than the 2^+ state;it will be several Mev above the $J = 1$, $T = 0$ ground state since the singlet state of the deuteron cluster has a higher energy than the triplet state.

The spatial wave function describing the states are of the form:

$$\psi = \phi(\alpha) \; \phi(d) \; \chi(R_\alpha - R_d) \; W(R_{cm}, R) \tag{1-2}$$ (5)

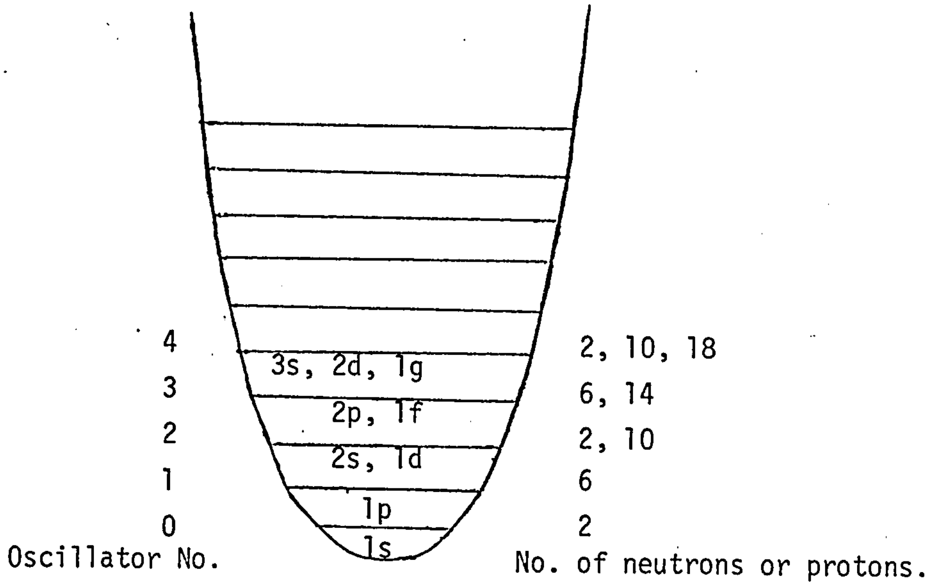

Figure 1: NOTATION IN THE OSCILLATOR MODEL

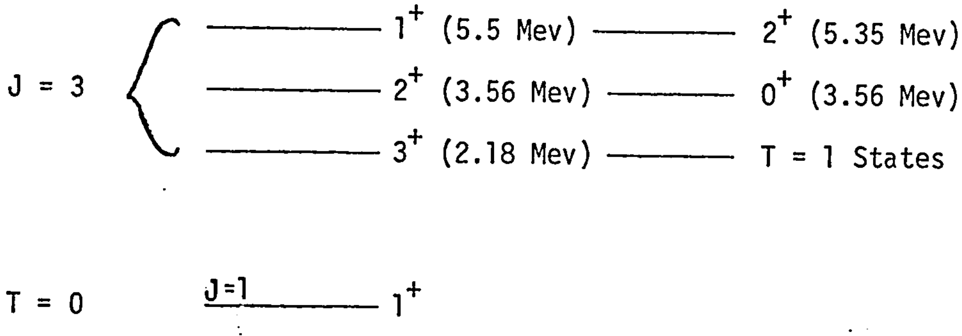

Figure 2: LEVEL SCHEME FOR Li^6

The wave function of the ground (1^+) state before antisymmetrization is given by (6):

$$(1\text{-}3)\quad \psi_{(1^+)} = N_{1^+}\, e^{-\frac{\alpha}{2}\sum\limits_{i=1}^{4}\bar{r}_1^2}\, e^{-\frac{\bar{\alpha}}{2}\sum\limits_{j=5}^{6}\bar{r}_j^2}\, e^{-2/3\beta R^2}\, e^{-(\mu R_{cm}^2 + QR_{cm}.R)}$$

$$\times\ R^2\, Y_0^0 \times \zeta(1234;56);$$

that of the 3^+ state is given by:

$$(1\text{-}4)\quad \psi_{(3^+)} = N_{3^+}\, e^{-\frac{\alpha}{2}\sum\limits_{i=1}^{4}\bar{r}_i^2}\, e^{-\frac{\bar{\alpha}}{2}\sum\limits_{j=5}^{6}\bar{r}_j^2}\, e^{-(2/3\ \beta R^2 + \mu R^2_{cm} + QR_{cm}.R)}$$

$$\times\ R^2\, Y_2^M \times \zeta(1234;56)\ ;$$

while that of the 0^+ state is given by;

$$(1\text{-}5)\quad \psi_{(0^+)} = N_{0^+}\, e^{-\frac{\alpha}{2}\sum\limits_{i=1}^{4}\bar{r}_1^2}\, e^{-\frac{\bar{\alpha}}{2}\sum\limits_{j=5}^{6}\bar{r}_j^2}\, e^{-(2/3\beta R^2 + \mu R^2_{cm} + QR_{cm}.R)}$$

$$\times\ R^2\, Y_0^0 \times \zeta(1234;\ 56)$$

The variational parameters chosen were those determined in the energy calculations of Tang, Pearlstein and Wildermuth; these are shown in Table 1.

The relative coordinates $\vec{\bar{r}}_i$ and $\vec{\bar{r}}_j$ in the alpha particle and deuteron clusters are defined by $\vec{\bar{r}}_i = \vec{r}_i - \vec{R}_\alpha$, $\vec{\bar{r}}_j = \vec{r}_j - \vec{R}_d$, where

$$\vec{R}_\alpha = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4}{4}\ ,\qquad R_d = \frac{\vec{r}_5 + \vec{r}_6}{2}$$

$$\vec{R} = \vec{R}_\alpha - \vec{R}_d\ ,\qquad \vec{R}_{cm} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4 + \vec{r}_5 + \vec{r}_6}{6}$$

r_1, r_2, r_3, and r_4 are the spatial co ordinates of the particles 1, 2, 3, 4 in the alpha particle cluster and r_5 and r_6 are the co ordinates of the particles 5, 6 in the deuteron cluster.

In our notation, subscripts 1, 3, 5 refer to neutrons and 2, 4, 6 to protons. For computational purposes the variational parameters have been redefined as

$$X = \frac{\beta}{\alpha} \quad , \quad Y = \frac{\kappa}{\alpha} \quad Z = \frac{\bar{\alpha}}{\alpha}$$

where $\kappa = 4.16 \times 10^{25}$ cm^{-2} (8) ,

$$\mu = 2\alpha + \bar{\alpha} \quad \text{and} \quad Q = \frac{4}{3}(\alpha - \bar{\alpha})^{(9)}$$

TABLE 1

CALCULATED ENERGIES AND ASSOCIATED VARIATIONAL PARAMETERS FOR THE FIRST SIX LEVELS IN Li^6

	TERMS	X	Y	Z	Calculated energies in Mev	Experimental energies in Mev
T =0	$^3S_1$	0.76	0.96	1.52	-30.00	-30.00
	$^3D_3$	0.86	0.96	1.51	-27.00	-27.82
	$^3D_2$	0.71	0.96	1.33	-24.70	-26.44
	$^3D_1$	0.52	0.96	1.16	-23.50	-24.50
T =1	$^1S_0$	0.75	0.96	0.98	-24.40	-26.44
	$^1D_2$	0.68	0.96	0.80	-21.70	-24.65

CHAPTER 2

Antisymmetrization of the cluster model wave functions

In this chapter the cluster model wave functions are antisymmetrized. The method of antisymmetrization used here differs from that of Wildermuth and Kannelopoulos10) in that we antisymmetrize the proton and neutron wave functions separately. This way we need not include the isospin in the wave functions, and since the electric quadrupole operator $E_{2\mu}$ and the magnetic dipole operator $M_{1\mu}$ are symmetric separately with respect to exchange of neutrons and protons, the calculation of the matrix elements is greatly simplified(11). The electric quadrupole operator $E_{2\mu}$ and the magnetic dipole operator $M_{1\mu}$ arise in the radiative transitions from the excited 3^+ and 0^+ states to the 1^+ ground state. A more detailed account of these operators is given in chapters 3 and 4.

The antisymmetrization operator A will be written as (II-1).

$A = A_n A_p$,(12) where $A_n = \frac{1}{3!} \sum_{n=1,3,5} (-1)^\pi P_n$ and

$$A_p = \frac{1}{3!} \sum_{p=2,4,6} (-1)^\pi P_p ,$$

P is a permutation operator and $(-1)^\pi$ the parity of the permutation. This method of antisymmetrization is not new; it has been used by Tran Duc Hoang(11) in calculations on He^5. It has not been applied to the Li^6 nucleus before.

The antisymmetrized wave functions are then given by

$$\phi = NA\,\psi = NA\{(1234;56)\,\chi\}$$

where χ is the spin wave function, N is a normalization factor and (1234;56) denotes the spatial wave function of the state considered.

The normalization constants are determined as follows:

$$(11\text{-}2) \qquad \langle\phi|\phi\rangle = N^2 \langle\psi|A^+ A|\psi\rangle$$
$$= N^2 \langle\psi|A\,|\psi\rangle = 1 \qquad \text{(A is idempotent and Hermitian).}$$

Subscripts i and f refer to initial and final states.

The spin configuration for the alpha-particle cluster can be represented as $\alpha_1\ \alpha_2\ \beta_3\ \beta_4$ where 1 and 3 are neutrons and 2 and 4 are protons. The deuteron cluster has spin S = 1 or 0. For the S = 1 state with $M_S = 1$, the spin wave function will then be $\alpha_1\ \alpha_2\ \beta_3\ \beta_4\ \alpha_5\ \alpha_6$. When $M_S = -1$, the spin wave function is $\alpha_1\ \alpha_2\ \beta_3\ \beta_4\ \beta_5\ \beta_6$ and $M_S = 0$ is represented by $\alpha_1\ \alpha_2\ \beta_3\ \beta_4 \cdot \frac{1}{\sqrt{2}}(\alpha_5\ \beta_6 + \beta_5\ \alpha_6)$.

The antisymmetrized wave function for a state with $M_S = 1$ is:

$$(11\text{-}3) \qquad \phi = NA\psi = NA\,\{(1234;56)\,\alpha_1\alpha_2\beta_3\beta_4\alpha_5\alpha_6\}$$

Now,

$$A = A_nA_p = \frac{1}{3!} \sum_{n=1,3,5} (-1)^\pi\ P_n \cdot \frac{1}{3!} \sum_{P=2,4,6} (-1)^\pi\ P_p$$

so that:

(11-4) $$\phi = \frac{N}{36}\{(\alpha_1\beta_3 - \beta_1\alpha_3)(\alpha_2\beta_4 - \beta_2\alpha_4)\alpha_5\alpha_6 \ (1234;56)$$

$$- (\alpha_1\beta_3 - \beta_1\alpha_3)(\alpha_2\beta_6 - \beta_2\alpha_6)\alpha_4\alpha_5 \ (1236;45) + (\alpha_1\beta_3 - \beta_1\alpha_3)(\alpha_4\beta_6 - \beta_4\alpha_6)$$

$$\alpha_2\alpha_5 \ (1634;25) - (\alpha_4\beta_2 - \beta_4\alpha_2)(\beta_1\alpha_5 - \alpha_1\beta_5)\alpha_3\alpha_6 \ (1254;36)$$

$$+ (\beta_2\alpha_6 - \alpha_2\beta_6)(\beta_1\alpha_5 - \alpha_1\beta_5)\alpha_3\alpha_4 \ (1256;34) - (\beta_4\alpha_6 - \alpha_4\beta_6)$$

$$(\beta_1\alpha_5 - \alpha_1\beta_5)\alpha_2\alpha_3 \ (1546;23) + (\alpha_4\beta_2 - \beta_4\alpha_2)(\beta_3\alpha_5 - \alpha_3\beta_5)\alpha_1\alpha_6$$

$$(5234;16) - (\beta_2\alpha_6 - \alpha_2\beta_6)(\beta_3\alpha_5 - \alpha_3\beta_5)\alpha_1\alpha_4 \ (5236;14)$$

$$+ (\beta_4\alpha_6 - \alpha_4\beta_6)(\beta_3\alpha_5 - \alpha_3\beta_5)\alpha_1\alpha_2 \ (3456;12)\}$$

For the state with $M_S = -1$, the spin wave function is $\alpha_1\alpha_2\beta_3\beta_4\beta_5\beta_6$, so that:

(11-5) $$\phi = NA\psi = NA\{(1234;56)\,\alpha_1\alpha_2\beta_3\beta_4\beta_5\beta_6\}$$

$$= \frac{N}{36}\{(\alpha_1\beta_3 - \beta_1\alpha_3)(\alpha_2\beta_4 - \beta_2\alpha_4)\beta_5\beta_6 \ (1234;56) - (\alpha_1\beta_3 - \beta_1\alpha_3)$$

$$(\beta_4\alpha_6 - \alpha_4\beta_6)\beta_2\beta_5 \ (1634;25) + (\alpha_1\beta_3 - \beta_1\alpha_3)(\alpha_6\beta_2 - \beta_6\alpha_2)\beta_4\beta_5$$

$$(1236;54) - (\beta_3\alpha_5 - \alpha_3\beta_5)(\alpha_2\beta_4 - \beta_2\alpha_4)\beta_1\beta_6 \ (5234;16)$$

$$+ (\beta_3\alpha_5 - \alpha_3\beta_5) (\beta_4\alpha_6 - \alpha_4\beta_6) \beta_1\beta_2 (3456;12) - (\beta_3\alpha_5 - \alpha_3\beta_5)$$

$$(\alpha_6\beta_2 - \beta_2\alpha_6) \beta_1\beta_4 \ (5623;14) + (\alpha_5\beta_1 - \beta_5\alpha_1) (\alpha_2\beta_4 - \beta_2\alpha_4) \ \beta_3\beta_6$$

$$(1254;36) - (\alpha_5\beta_1 - \beta_1\alpha_5) (\beta_4\alpha_6 - \alpha_4\beta_6) \beta_2\beta_3 (1546;23)$$

$$+ (\alpha_5\beta_1 - \alpha_1\beta_5) (\alpha_6\beta_2 - \alpha_2\beta_6) \beta_3\beta_4 \ (1256;34)$$

for $M_S = 0$, the spin wave function is $\alpha_1\alpha_2\beta_3\beta_4 \cdot \frac{1}{\sqrt{2}} (\alpha_5\beta_6 + \beta_5\alpha_6)$.

Therefore,

$$(11\text{-}6) \quad \phi = NA\psi = \frac{N}{\sqrt{2}} (\alpha_1\beta_3 - \beta_1\alpha_3) (\alpha_2\beta_4 - \beta_2\alpha_4) \alpha_5\beta_6$$

$$(1234;56) - (\alpha_1\beta_3 - \beta_1\alpha_3) (\beta_4\alpha_6 - \alpha_4\beta_6) \beta_2\alpha_5 \ (1634;25)$$

$$+ (\alpha_1\beta_3 - \beta_1\alpha_3) (\alpha_6\beta_2 - \beta_2\alpha_6) \beta_4\alpha_5 \ (1623;45) - (\beta_3\alpha_5 - \alpha_3\beta_5) \ \times$$

$$(\alpha_2\beta_4 - \beta_2\alpha_4) \alpha_1\beta_6 \ (5234;16) + (\beta_4\alpha_6 - \alpha_4\beta_6) (\beta_3\alpha_5 - \alpha_3\beta_5) \alpha_1\beta_2$$

$$(3456;12) - (\beta_3\alpha_5 - \alpha_3\beta_5) (\alpha_6\beta_2 - \beta_2\alpha_6) \alpha_1\beta_4 \ (5623;14)$$

$$+ (\beta_1\alpha_5 - \alpha_1\beta_5) (\alpha_2\beta_4 - \beta_2\alpha_4) \ \alpha_3\beta_6 (1254;36) - (\alpha_5\beta_1 - \alpha_1\beta_5)$$

$$\times \ (\beta_4\alpha_6 - \alpha_4\beta_6) \ \alpha_3\beta_2 \ (1546;23) + (\alpha_5\beta_1 - \beta_1\alpha_5) (\alpha_6\beta_2 - \alpha_2\beta_6) \ \alpha_3\beta_4$$

$$(1256;34) + (\alpha_1\beta_3 - \beta_1\alpha_3)\ (\alpha_2\beta_4 - \beta_2\alpha_4)\ \beta_5\alpha_6\ (1234;56)$$

$$- (\alpha_1\beta_3 - \beta_1\alpha_3)\ (\beta_4\alpha_6 - \alpha_4\beta_6)\ \alpha_2\beta_5\ (1634;25) + (\alpha_1\beta_3 - \beta_1\alpha_3)$$

$$\times\ (\alpha_6\beta_2 - \alpha_2\beta_6)\ \alpha_4\beta_5\ (1236;45) - (\beta_3\alpha_5 - \alpha_3\beta_5)\ (\alpha_2\beta_4 - \beta_2\alpha_4)$$

$$\beta_1\alpha_6\ (5234;16) + (\beta_3\alpha_5 - \alpha_3\beta_5)\ (\beta_4\alpha_6 - \alpha_4\beta_6)\ \beta_1\alpha_2\ (3456;12)$$

$$- (\beta_3\alpha_5 - \alpha_3\beta_5)\ (\alpha_6\beta_2 - \beta_6\alpha_2)\ \beta_1\alpha_4\ (5236;14) + (\alpha_5\beta_1 - \beta_5\alpha_1)$$

$$\times\ (\alpha_2\beta_4 - \beta_2\alpha_4)\ \beta_3\alpha_6\ (1254;36) - (\alpha_5\beta_1 - \beta_1\alpha_5)\ (\beta_4\alpha_6 - \alpha_4\beta_6)$$

$$\alpha_2\beta_3\ (1546;23) + (\alpha_5\beta_1 - \beta_1\alpha_5)\ (\alpha_6\beta_2 - \alpha_2\beta_6)\ \beta_3\alpha_4\ (1256;34)$$

Therefore,

$$(11\text{-}7) \qquad N_i^{\,2} < A\psi_i \mid A\psi_i > \ = \ N_i^{\,2} < A\psi_i \mid \psi_i >$$

$$= \frac{N_i^{\,2}}{36} \ <(1234;56)_i - 2\,(5214;36)_i + (3456;12)_i \mid (1234;56)_i>$$

$$= \ 1$$

$$(11\text{-}8) \qquad N_i^{\,2} = \frac{36}{<(1234;56)_i - 2(1254;36)_i + (3456;12)_i \mid (1234;56)_i >}$$

The calculation of N_f is similar

$$N_f^{\,2} = \frac{36}{\langle (1234;56)_f - 2(1254;36)_f + (3456;12)_f | (1234;56)_f \rangle} \tag{11-9}$$

In this chapter the method of antisymmetrization of wave functions has been shown and a method of calculation of the normalization constants has been given. The numerical values of the normalization constants for the 3^+, 1^+ and 0^+ states are given in Appendices 1 and 2.

In the next chapter, an expression for the radiative transition probability from the 3^+ excited state to the 1^+ ground state is derived. Integrals of the form $\langle A\phi_f|\theta|\phi_i\rangle$ where θ is an electric quadrupole operator are dealt with.

CHAPTER 3

THE RADIATIVE TRANSITION PROBABILITY FROM THE 3^+ EXCITED STATE TO THE 1^+ GROUND STATE

MATRIX ELEMENTS AND SELECTION RULES

The transition probability from a state i to a state f if (λ,μ) denotes the angular momentum and its Z component of the emitted photon of energy $h\omega$, is given by[13]:

$$(111\text{-}1) \qquad T_{if}(\sigma\lambda) = \frac{8\pi\,(\lambda+1)}{\lambda\,[(2\lambda+1)!!]^2} \cdot \frac{k^{2\lambda+1}}{\hbar} \cdot B(\sigma\lambda)$$

where σ = E or M stands for the electric or magnetic mode of decay and $B(\sigma\lambda)$ is the reduced matrix element

$$B(\sigma\lambda\,,\,J_i \rightarrow J_f) = \frac{1}{2J_i+1} \sum_{M_i M_f} |<f\,|\,\theta_{\lambda\mu}|i>|^2$$

$\theta_{\lambda\mu}$ stands for the electric or magnetic multipole operator $Q_{\lambda\mu}$ or $M_{\lambda\mu}$. In the case of the transition from the 3^+ to the 1^+ state, λ takes the following values:[14]

$$(111\text{-}2) \qquad |J_i + J_f| > \lambda > |J_i - J_f| \;;\; \lambda = 4, 3, 2.$$

Of these, $\lambda = 2$ makes the biggest contribution; the contributions from $\lambda = 3$ and 4 are negligible,

The parity of $Q_{\lambda\mu}$ is that of Y^{μ}_{λ}, namely $(-1)^{\lambda}$. In the case of the state with $\lambda = 2$, it is given by $(-1)^2 = 1$. The product of the parities of initial and final states is positive and hence the transition takes place principally by the emission of electric quadrupole photons.

In this case then $\theta_{\lambda\mu} = Q_{\lambda\mu} = Q_{2\mu}$

$$Q_{2\mu} = \sum_i e_i \bar{r}_i^2 \overset{\mu}{\overset{*}{Y}}_2 (\omega_i) - i \frac{\mu_o k}{3} \sum_i g_{si} \vec{\sigma}_i \times \vec{r}_i \cdot \nabla (r^2 \overset{\mu}{\overset{*}{Y}}_2)_i \tag{111-3}$$

$$T_{if} \ (E2) = \frac{24\pi}{2 \times (5 \times 3)^2} \times \frac{k^5}{\hbar c} \times B(E2) \tag{111-4}$$

$$B(E2) = \frac{1}{7} \sum_{M_i M_f} | < f \ | \ Q_{2\mu} | i> |^2 \tag{111-5}$$

In the case of $\lambda = 2$, μ takes all values from -2 to + 2. The selection rule applicable here is $\mu = M_f - M_i$, where in the 1^+ state M_f can vary from -1 to +1 and in the 3^+ state M_i can vary from -3 to +3. Also $\mu = M_f - M_i$, and can take values from -2 to +2.

The different magnetic sub states of the excited 3^+ state can be expressed in terms of sums of products of wave functions of the form $\{ |L \ M_L> \ |S \ M_S> \times$ clebseh Gordon coefficient$\}$.

These are given by:

$$|3^+ \ 3 > = |2 \ 2 > | 1 \ 1 > \tag{111-6}$$

$$|3^+ \ -3 > = |2 \ -2 > | 1 \ -1 >$$

$$|3^+ \ 1 > = \sqrt{\frac{2}{5}} \ |20> | 11 > + \frac{1}{\sqrt{15}} | 22 > | 1 - 1> + \sqrt{\frac{8}{15}} | 21> |10 >$$

$$|3^+ \; 0> = \frac{1}{\sqrt{5}} |2-1>|11> + \sqrt{\frac{3}{5}} |20>|10> + \frac{1}{\sqrt{5}} |2+1>|1-1>$$

$$|3^+ \; -1> = \frac{1}{\sqrt{15}} |2-2>|11> + \sqrt{\frac{8}{15}} |2-1>|10> + \sqrt{\frac{2}{5}} |20>|1-1>$$

$$|3^+ \; 2> = \frac{1}{\sqrt{3}} |22>|10> + \sqrt{\frac{2}{3}} |21>|11>$$

$$|3^+ \; -2> = \frac{1}{\sqrt{3}} |2-2>|10> + \sqrt{\frac{2}{3}} |2-1>|1-1>$$

$$|1-1|> = \alpha_1\alpha_2\beta_3\beta_4\beta_5\beta_6$$

$$|1+1|> = \alpha_1\alpha_2\beta_3\beta_4\alpha_5\alpha_6$$

$$|10> = \alpha_1\alpha_2\beta_3\beta_4 \times \frac{1}{\sqrt{2}} (\alpha_5\beta_6 + \beta_5\alpha_6)$$

The second term of $Q_{2\mu}$ is of the form $- i \frac{\mu_0 k}{3} \sum_i (g_{si} (\vec{\sigma}_i \times \vec{r}_i) . \nabla(r^2 \overset{\mu}{Y_2^*})_i)$

μ_0 = nuclear magneton = 0.5050×10^{-23} erg gauss^{-1}, $K = \frac{2.18 \text{ MeV}}{\hbar c} = 10^{11} (\text{cm})^{-1}$ * (1∘1

The matrix element of the second term of the electric quadrupole operator $Q_{2\mu}$ is small compared with the first term.

Calculation of B(E2)

Using $Q_{2\mu} = \sum_i e_i \bar{r}_i^2 \overset{\mu}{Y_2^*} (\omega_i)$, we now calculate B(E2):

TABLE 2

POSSIBLE VALUES OF M_{Li}, M_{si}, M_{Lf}, M_{sf}

M_i	M_{Li}	M_{si}	M_f	M_{Lf}	M_{sf}
-3	-2	-1	-1	0	-1
-2	-2; -1	0; -1	0	0	0
-1	-2; -1; 0	1; 0; -1	1	0	1
0	0; 1; -1	0; -1; 1			
1	1; 2; 0	0; -1; 1			
2	1; 2	1; 0			
3	2	1			

TABLE 3

ALLOWED VALUES OF M_i and M_f

μ	M_f	M_i
-2	-1; 0; 1	1; 2; 3
-1	-1; 0; 1	0; 1; 2
0	0; 1; -1	0; 1; -1
1	1; -1; 0	0; -2; -1
2	-1; 0; 1	-3; -2; -1

$$
\begin{aligned}
\text{(III-7)}\quad B(E2) &= \frac{1}{7} \sum_{M_i M_f} |\langle f \,|\, Q_{2\mu} \,|\, i \rangle|^2 \\
&= \frac{1}{7} \Big[|\langle \phi_f(1) \,|\, Q_{22} \,|\, \phi_i(-1) \rangle|^2 + |\langle (\phi_f(-1) \,|\, Q_{22} \,|\, \phi_i(-3) \rangle|^2 \\
&+ |\langle \phi_f(0) \,|\, Q_{22} \,|\, \phi_i(-2) \rangle|^2 + |\langle \phi_f(-1) \,|\, Q_{2-2} \,|\, \phi_i(1) \rangle|^2 \\
&+ |\langle \phi_f(0) \,|\, Q_{2-2} \,|\, \phi_i(2) \rangle|^2 + |\langle \phi_f(1) \,|\, Q_{2-2} \,|\, \phi_i(3) \rangle|^2 \\
&+ |\langle \phi_f(0) \,|\, Q_{20} \,|\, \phi_i(0) \rangle|^2 + |\langle \phi_f(1) \,|\, Q_{20} \,|\, \phi_i(1) \rangle|^2 \\
&+ |\langle \phi_f(-1) \,|\, Q_{20} \,|\, \phi_i(-1) \rangle|^2 + |\langle \phi_f(0) \,|\, Q_{21} \,|\, \phi_i(-1) \rangle|^2 \\
&+ |\langle \phi_f(1) \,|\, Q_{21} \,|\, \phi_i(0) \rangle|^2 + |\langle \phi_f(-1) \,|\, Q_{21} \,|\, \phi_i(-2) \rangle|^2 \\
&+ |\langle \phi_f(0) \,|\, Q_{2-1} \,|\, \phi_i(1) \rangle|^2 + |\langle \phi_f(-1) \,|\, Q_{2-1} \,|\, \phi_i(0) \rangle|^2 \\
&+ |\langle \phi_f(1) \,|\, Q_{2-1} \,|\, \phi_i(2) \rangle|^2 \Big]
\end{aligned}
$$

$\phi_i(M_i)$ and $\phi_f(M_f)$ are respectively the antisymmetrized wave functions of the initial and final states. Since $Q_{2\mu}$ is symmetric under exchange of protons and neutrons separately, A_n and A_p commute with $Q_{2\mu}$; therefore

(III-8) $\langle\phi_f\,(M_f)\,|Q_{2\mu}\,|\phi_i(M_i)\rangle \;=\; N_f N_i\,\langle\psi_f(M_f)\,|\,A^+A\,Q_{2\mu}|\,\psi_i(M_i)\rangle$.

But A is Hermitian and idempotent ($A^+A = A^2=A=A^+$), so that:

(III-9) $N_f N_i\,\langle\psi_f(M_f)|\,A^+A\,Q_{2\mu}\,|\psi_i(M_i)\rangle$ becomes

$$= N_f N_i\,\langle\psi_f(M_f)|\,A^+Q_{2\mu}\,|\psi_i(M_i)\rangle \;=\; N_i\,\langle\phi_f(M_f)|\,Q_{2\mu}\,|\psi_i(M_i)\rangle$$

$(\;A^+ = A)$.

We notice that this is a much simpler expression to calculate than

$$\langle\phi_f(M_f)|\,Q_{2\mu}\,|\phi_i(M_i)\rangle\ .$$

This explains why separate antisymmetrization over protons and neutrons is preferred.

For example

(III-10) $\phi_f(1) = N_f A\,\{(1234;\,56)_f\;\;\alpha_1\alpha_2\beta_3\beta_4\alpha_5\alpha_6\,\}$

$$= N_f A\{\,e^{-\alpha/2\sum_{i=1}^{4}\bar{r}_i^2}\,e^{-\bar{\alpha}/2\sum_{j=5}^{6}\bar{r}_j^2}\,e^{-(2/3\beta R^2\,+\,\mu R^2_{cm}\,+\,QR_{cm}R)}\times R^2Y_o^o\times\alpha_1\alpha_2\beta_3\beta_4\alpha_5\alpha_6\}$$

we can write: (1234;56), as follows:

(III-11) $$(1234;\,5\,6)_i \;=\; e^{-\alpha/2\sum_{i=1}^{4}\bar{r}_i^2}\;e^{-\bar{\alpha}/2\sum_{j=5}^{6}\bar{r}_j^2}\;e^{-(2/3\beta R^2\;+\;\mu R^2_{cm}+\,QR_{cm}R)}$$

$$\times\;\;R^2Y_2^M$$

We have already expressed the different magnetic substates of the 3^+ state in terms of sums of products of spin and orbital wave functions.

The state with $M_i = -1$ can be expressed as

(111-12) $\psi_i(-1) = \frac{1}{\sqrt{15}} \mid 2-2>_i \, |11> \; + \sqrt{\frac{8}{15}} \mid 2-1>_i \, |10> \; + \sqrt{\frac{2}{5}} \, |20>_i \, |1-1>$

Therefore:

(111-13) $N_i <\phi_f(1) \mid Q_{22} \mid \psi_i(-1) > \; = \; N_f N_i < A \, \{(1234;56)_f$

$\times \, \alpha_1\alpha_2\beta_3\beta_4\alpha_5\alpha_6 \mid Q_{22} \mid \{ \frac{1}{\sqrt{15}} \mid 2-2>_i \mid 11> \; + \sqrt{\frac{8}{15}} \mid 2-1>_i \mid 10>$

$+ \sqrt{\frac{2}{5}} \mid 20>_i \mid 1-1>\}>$

$= \frac{N_f N_i}{36} < \{(1234;56)_f - (1634;25)_f - (5234;16)_f + (3456;12)_f\} \mid Q_{22} \mid \frac{1}{\sqrt{15}} \, |2-2>$

(111-14) $\mid 2-2>_i \; = \; e^{-\alpha/2} \sum_{i=1}^{4} \bar{r}_i^2 \; e^{-\bar{\alpha}/2} \sum_{j=5}^{6} \bar{r}_j^2 \; e^{-(2/3\beta R^2 + \mu R^2_{cm} + QR_{cm} \cdot R)}$

$\times \, R^2 Y_2^{-2}$

Also $\mu = 2$ can be formed with $M_f = 0$ and $M_i = -2$.

(111-15) $N_i < \phi_f(0) \mid Q_{22} \mid \psi_i(-2)>$

$= \; N_f N_i < \; A \, \{(1234;56)_f \; \frac{1}{\sqrt{2}} \, (\alpha_1\alpha_2\beta_3\beta_4 \; (\alpha_5\beta_6 + \beta_5\alpha_6)\}$

$|Q_{22} \mid \{ \frac{1}{\sqrt{3}} \mid 2-2> \mid 10> + \sqrt{\frac{2}{3}} \; |2-1> \, |1-1>\}>$

We have expressed $|3^{+}-2>$; in terms of L and S wave functions.

Therefore

(111-16) $$N_i < \phi_f(0) \mid Q_{22} \mid \psi_i(-2)>$$

$$= \frac{N_i N_f}{36} < \frac{1}{2} \{ (1234;56)_f - (1236;54)_f - (5234;16)_f + (5623;14)$$

$$+ (1234;56)_f - (1634;25)_f - (1254;36) + (1546;23)_f \} \mid Q_{22} \mid \frac{1}{\sqrt{3}} \mid 2 - 2>_i>$$

We know that

(111-17) $$<(1236;54)_f \mid Q_{22} \mid (2 - 2)_i>$$

$$= <(1634;25)_f \mid Q_{22} \mid (2 - 2)_i> \; ; \; <(1254;36)_f \mid Q_{22} \mid (2 - 2)_i>$$

$$= <(5234;16)_f \mid Q_{22} \mid (2 - 2)_i> \; ; \; <(1546;23)_f \mid Q_{22} \mid (2 - 2)_i>$$

$$= <(5623;14)_f \mid Q_{22} \mid (2 - 2)_i> = <(3456;12)_f \mid Q_{22} \mid (2 - 2)_i>$$

$$= <(1256;34)_f \mid Q_{22} \mid (2 - 2)_i>.$$

This is valid because in an integral any two variables of integration can be exchanged without changing the value of the integral, provided that the integration domains of these variables are the same, as they are in the above calculations.

Hence $N_i < \phi_f(0) \mid Q_{22} \mid \psi_i(-2)>$

$$= \frac{N_i N_f}{36} <\{ (1234;56)_f - (1236;54)_f - (1254;36)_f + (5623;14)_f \mid Q_{22} \mid \frac{1}{\sqrt{3}} \mid 2\text{-}2$$

We have considered now two possibilities for the formation of $\mu = 2$ this value of μ can also arise when $M_f = -1$, and $M_i = -3$:

(111-19) $$N_i < \phi_f(-1) \mid Q_{22} \mid \psi_i(-3)>$$

$$= \frac{N_i N_f}{36} <\{(1234;56)_f - (1236;54)_f - (1254;36)_f + (1256;34)_f\} \mid Q_{22} \mid (2 - 2)_i>$$

We have used the relation that $|3^{+} - 3>_i = |2 - 2>_i \; |1 - 1>$.

These three possibilities for $\mu = 2$ exhaust all possible cases.

(111-20) Now $N_i^2 < \phi_f(1) \mid Q_{22} \mid \psi_i(-1)>|^2 + N_i^2 \, |<\phi_f(0)| \; Q_{22}| \; \psi_i(-2)>|^2$

$+ N_i^2 \, |<\phi_f(-1) \mid Q_{22}| \; \psi_i(-3)>|^2$ becomes

$$= \frac{N_f^2 \; N_i^2}{(36)^2} \times \left(\frac{1}{15} + \frac{1}{3} + 1\right) \times|<\{(1234;\, 56)_f - (1236;\, 54)_f - (1254;\, 36)_f$$

$$+ (1256;\, 34)_f\}| \quad Q_{22} \mid \quad (2 - 2)_i >|^2$$

$$= \frac{N_f^2 \; N_i^2}{(36)^2} \times \frac{7}{5} \times | \, I_1|^2$$ where

$$I_1 = <\{ (1234;\, 56)_f - (1236;54)_f - (1254;\, 36)_f + (1256;\, 34)_f\} \mid Q_{22} \mid (2 - 2)_i>$$

In an exactly similar way the $\mu = -2$ matrix element becomes:

(111-21)

$$= N_i^2|< \phi_f(-1) \mid Q_{2\,-\,2}| \;\; \psi_i(1)|^2 + N_i^2|<\phi_f(0)| \; Q_{2\,-\,2}| \; \phi_i(2)>|^2$$

$$+ \; N_i^2|< \phi_f(1) \mid Q_{2\,-\,2}| \; \phi_i(3)>|^2$$

$$= \frac{7}{5} \times \frac{N_f^2 \, N_i^2}{(36)^2} \times | < (1234;56)_f - (1236;54)_f - (1254;36)_f + (1256;34)_f$$

$$|Q_{2\,-\,2} \mid \; (2 + 2)_i> |^2 = \frac{7}{5} \times \frac{N_f^2 \; N_i^2}{(36)^2} \times | \, I_2|^2$$

where $I_2 = \; <\{ (1234;56)_f - (1236;54)_f - (1254;36)_f + (1256;34)_f \}$

$$| \; Q_{2\,-\,2} \; | \; (2 + 2)_i >$$

The case $\mu = 0$ can be formed when $M_f = M_i = 1$. The 3^+ state with $M_i = 1$ can be written as:

(111-22) $$|3^+ 1\rangle = \sqrt{\frac{2}{5}}\ |20\rangle\ |11\rangle + \frac{1}{\sqrt{15}}\ |22\rangle\ |1\ -1\rangle + \sqrt{\frac{8}{15}}\ |21\rangle\ |10\rangle$$

$$\phi_f(1) = N_f A\ \{ (1234;56)_f \quad \alpha_1\alpha_2\beta_3\beta_4\alpha_5\alpha_6 \}$$

$$N_i^2\ |\langle \phi_f(1)\ |\ Q_{20}|\ \psi_i(1)\rangle\ |^2$$

$$= \frac{N_f^2\ N_i^2}{(36)^2}\ |\ \langle\{(1234;56)_f - (1634;25)_f - (5234;16)_f + (3456;12)_f\}\ |\ Q_{20}|\ \sqrt{\frac{2}{5}}\ \ |\ 20_i\rangle\ |^2$$

$\mu = 0$ also results when $M_f = M_i = -1$

The $|3^+ -1\rangle$ state can be written as:

(111-23) $$|3^+ -1\rangle = \frac{1}{\sqrt{15}}\ |2\ -2\rangle\ |11\rangle + \sqrt{\frac{8}{15}}\ |\ 2\ -1\rangle\ |10\rangle + \sqrt{\frac{2}{5}}\ |\ 20\rangle\ |1\ -1\rangle$$

$$N_i^2\ |\ \langle\phi_f(-1)\ \ |\ Q_{20}\ |\ \psi_i(-1)\ \rangle\ |^2$$

$$= \frac{N_f^2\ N_i^2}{(36)^2}\ |\ \langle\ \{(1234;56)_f - (1236;54)_f - (1254;36)_f + (1256;34)_f\}\ |Q_{20}|\ \sqrt{\frac{2}{5}}\quad (20)_i\rangle|^2$$

The only case left which gives $\mu = 0$ is when $M_f = M_i = 0$

Since

$$|3^+ 0\rangle = \frac{1}{\sqrt{5}}\ |\ 21\rangle\ |1\ -1\rangle + \sqrt{\frac{3}{5}}\ |\ 20\rangle\ |10\rangle + \frac{1}{\sqrt{5}}\ |\ 2\ -1\rangle\ |11\rangle$$

we have: (III-24)

$$N_i^2\ \ |\langle\phi_f(0)\ \ |\ Q_{20}\ |\ \psi_i(0)\rangle\ |^2$$

$$= \frac{N_f^2\ N_i^2}{(36)^2}\ |\ \langle\{(1234;56)_f - (1236;54)_f - (1254;36)_f + (5623;14)_f\}\ |\ Q_{20}\ \ |\ \sqrt{\frac{3}{5}}\ \ |\ \ 20\ \rangle_i\ |^2$$

Therefore $N_i^2 \; | < \phi_f(0) | \; Q_{20} \; | \; \psi_i(0) > |^2$

$$= \frac{N_f^2 \; N_i^2}{(36)^2} \times \frac{3}{5} \, |< (1234;56)_f - (1236;54)_f - (1254;36)_f + (5623;14)_f \; | \; Q_{20} \; | \; (20)_i >|^2$$

Since we have considered all three possibilities for μ = 0, we can write;

$$(111\text{-}25) \quad N_i^2 \, [|<\phi_f(1) \mid Q_{20} \, |\psi_i(1)>|^2 + |< \phi_f(-1) \mid Q_{20} | \; \psi_i(-1)> |^2 + |< \phi_f(0) \mid Q_{20} | \; \psi_i(0)> |^2]$$

$$= \frac{N_f^2 \; N_i^2}{(36)^2} \cdot \frac{7}{5} \cdot | < (1234;56)_f - (1634;25)_f - (5234;16)_f + (3456;12)_f\} \mid Q_{20} \mid (20)_i >|^2$$

$$= \frac{N_f^2 \; N_i^2}{(36)^2} \cdot \frac{7}{5} \cdot |I_3|^2$$

where $I_3 = <(1234;56)_f - (1634;25)_f - (5234;16)_f + (3456;12)_f \; |Q_{20}| (20)_i>$

We have now considered the cases where μ = 2, - 2, and 0.

We shall now derive an expression for Q_{21} and $Q_{2\,-\,1}$

With $M_f = 0$ and $M_i = -1$, we can have μ = 1.

$$(111\text{-}26) \quad N_i^2 \, |< \; \phi_f(0) \mid Q_{21} | \; \psi_i(-1)> |^2$$

$$= \frac{N_i^2 \; N_f^2}{(36)^2} \times |<\{ (1234;56)_f - (1236;54)_f - (5234;16)_f + (1256;34)_f \} \mid Q_{21} \mid \sqrt{\frac{8}{15}} \quad (2 - 1)_i > |^2$$

We also have the case with $M_f = 1$ and $M_i = 0$.

Then, $N_i^2 \,|< \phi_f(1) \,|\, Q_{21} \,|\, \psi_i(0)> |^2$ becomes:

(111-27)

$$= \frac{N_f^2 \quad N_i^2}{(36)^2} \quad x \quad |<\{(1234;56)_f - (1236;54)_f - (5234;16)_f + (1256;34)_f$$

$$|\, Q_{21} | \quad \frac{1}{\sqrt{5}} \quad |\; 2 - 1>>|^2$$

When $M_f = -1$ and $M_i = -2$, we have:

(111-28)

$$N_i^2 |< \phi_f(-1) \,|\, Q_{21} | \quad \phi_i(-2)>|^2$$

$$= \frac{N_f^2 \quad N_i^2}{(36)^2} \quad x|<\{(1234;56)_f - (1236;54)_f - (5234;16)_f + (1256;34)_f$$

$$|\, Q_{21} \,| \sqrt{\frac{2}{3}} \quad (2 - 1)> |^2$$

Now we can write the Q_{21} matrix element as:

(111-29)

$$|< \phi_f(M_f) \mid Q_{21} \mid \phi_i(M_i) \mid^2$$

$$= |<\phi_f(0) \mid Q_{21} \mid \phi_i(-1)> |^2 + |< \phi_f(1) \mid Q_{21} \mid \phi_i(0) >|^2$$

$$+ |<\phi_f(-1) \mid Q_{21} \mid \phi_i(-2)>|^2$$

$$= \frac{7}{5} \quad \frac{N_f^2 \quad N_i^2}{(36)^2} \quad |<\{(1234;56)_f - (1236;54)_f - (1254;36)_f + (1256;36)_f\}$$

$$| \; Q_{21} \mid (2 - 1)>|^2$$

$$= \frac{7}{5} \quad \frac{N_f^2 \quad N_i^2}{(36)^2} \quad . \mid I_4 \mid^2$$

In exactly the same way:

(111-30)

$$|<\phi_f(0)| \; Q_{2-1} \mid \phi_i(1)> |^2 + |<\phi_f(-1) \mid Q_{2-1} \mid \phi_i \; (0)>|^2$$

$$+|<\phi_f(1) \mid Q_{2-1}| \; \phi_i(2)> |^2$$

$$= \frac{7}{5} \quad \frac{N_f^2 \quad N_i^2}{(36)^2} \; . |<\{(1234;56)_f - (1236;54)_f - (1254;36)_f + (1256;34)_f\}$$

$$| \; Q_{2-1}| \; (21)_i>|^2$$

$$= \frac{7}{5} \quad \frac{N_f^2 \quad N_i^2}{(36)^2} \quad . \mid I_5 \mid^2$$

We can therefore express B(E2) as:

(111-31)

$$B(E2) = \frac{1}{7} \cdot \frac{7}{5} \cdot \frac{N_f^2 \; N_i^2}{(36)^2} \cdot [\, |I_1|^2 + |I_2|^2 + |I_3|^2 + |I_4|^2 + |I_5|^2$$

Therefore the transition probability can be calculated since k can be calculated from the measured energy of this transition.

In Appendix 1 it has been proved that I_k , where k can take values from 1 to 5, is independent of k.

(111-32) Therefore $I_1 = I_2 = I_3 = I_4 = I_5$

B(E2) can be written as:

(111-33) $$B(E2) = \frac{1}{5} \times \frac{N_f^2 \; N_i^2}{(36)^2} \times 5|I_1|^2 = \frac{N_f^2 \; N_i^2}{(36)^2} \times |I_1|^2$$

CHAPTER 4

THE RADIATIVE TRANSITION PROBABILITY FROM THE 0^+ STATE TO THE 1^+ STATE

An expression for the radiative transition probability from the 0^+ excited state at 3.56 Mev to the 1^+ ground state will now be derived. For this transition,

$$J_i = 0 \;\; ; \; J_f = 1 \; ; \;\; \lambda = 1 = |J_i - J_f| .$$

The product of the parities of the initial and final states is positive; we therefore have an $M_{1\mu}$ transition in this case (parity is given by $-(-1)^1 = 1$). $\mu = M_f - M_i$ and can take values -1, 0 and 1.

The expression for $T_{if}\,(\sigma\lambda)$ has already been given in chapter 3.$^{13)}$. In the present case $\theta_{\lambda\mu}$ stands for the magnetic multipole operator $M_{1\mu}$:

$$\text{(1V-1)} \quad M_{1\mu} = \mu_o \sum_i (g_{si} . \vec{s}_i + g_{\ell i} \; \vec{\ell}_i) \; . \; \nabla(rY_1^{\mu *} \;)_i$$

μ_o is the nuclear magneton and the g's are the spin and orbital gyromagnetic ratios of the proton and neutron.

$$\text{(1V-2)} \quad M_{11} = \mu_o \sum_i (g_{si} \;\; \vec{s}_i \; + \; g_{\ell i} \;\; \vec{\ell}_i \;) \; . \; \nabla(rY_1^{\mu *} \;)_i$$

$$= \; - \; \mu_o \; \times \sqrt{\frac{3}{8\pi}} \;\; \sum_i (g_{si} . \; \vec{s}_i + g_{\ell i} \; \vec{\ell}_i) \; . \; \nabla(r \sin\theta \; e^{-i\phi} \;)$$

$$= \; - \; \mu_o \; \times \sqrt{\frac{3}{8\pi}} \;\; \sum_i (g_{si} \;\; \vec{s}_i + g_{\ell i} \; \vec{\ell}_i) \; . \;\; (\vec{i} - i\vec{j})$$

$$= -\mu_o \times \sqrt{\frac{3}{8\pi}} \sum_i g_{si} (s_{xi} - i\, s_{yi}) + g_{\ell i} (\ell_{xi} - i\, \ell_{yi})$$

$$= -\mu_o \times \sqrt{\frac{3}{8\pi}} \times \sum_i (g_{si}\, s_{i-} + g_{\ell i}\, \ell_{i-})$$

In obtaining the final form for M_{11} we have made use of the definition

$s_- = s_x - is_y$ and $\ell_- = \ell_x - i\, \ell_y$

In a similar way it is found that:

(1V-3) $$M_{1\,-1} = -\mu_o \times \sqrt{\frac{3}{8\pi}} \sum_i (g_{si}\, s_{i+} + g_{\ell i}\, \ell_{i+})$$

and

$$M_{10} = \mu_o \times \sqrt{\frac{3}{4\pi}} \sum_i (g_{si}\, s_{Zi} + g_{\ell i}\, \ell_{Zi})$$

Again, particles 1, 3, 5 are neutrons and 2, 4, 6 are protons.

The orbital gyromagnetic ratio is one for protons and zero for neutrons, the spin gyromagnetic ratios are $g_s = 5.5856$ for protons $= g_{sp}$ and $g_n = -3.8263$ for neutrons $= g_{np}$

The expression for B(M1) is

(1V-4) $$B(M1) = \frac{1}{2\,.\,0 + 1} \sum_{M_i M_f} |< f \,|\, \theta_{1\mu} |i> |^2$$

$$= |< f \,|\, M_{11} \,|i>|^2 + |<f \,|M_{1\,-1}|i>|^2 + |< f|M_{10} \,|\, i >|^2$$

Then $<f|\ M_{11}\ |i>$ becomes:

$$(1V\text{-}5)\quad < f\ |\ M_{11}|i> \ = \ N_f <A\ \{\ (1234;56)_f\ \alpha_1\alpha_2\beta_3\beta_4\alpha_5\alpha_6|M_{11}|$$

$$\times\ N_i(1234;56)_i\ \frac{1}{\sqrt{2}}\ \alpha_1\alpha_2\beta_3\beta_4\ (\alpha_5\beta_6 - \beta_5\alpha_6)$$

$$(M_f - M_i = 1;\ \ M_{sf} = 1,\ M_{\ell f} = 0);\ M_{i\ell} = 0;\ \ M_{is} = 0)$$

Therefore $<f|\ M_{11}|i>$ becomes:

$$(1V\text{-}6)\quad <f\ |\ M_{11}\ |\ i> \ = \ N_fN_i< A\ (1234;56)_f\ \ \alpha_1\alpha_2\beta_3\beta_4\alpha_5\alpha_6|\{\ -\ \mu_0\ \times \sqrt{\frac{3}{8\pi}}$$

$$\times\ \sum_i\ (g_{si}\ s_{-i} + g_{\ell i}\ \ \ell_{-i})\}|\ (1234;56)_i$$

$$\times\ \frac{1}{\sqrt{2}}\ \ \alpha_1\alpha_2\beta_3\beta_4\ \ (\alpha_5\beta_6\ -\ \beta_5\alpha_6)>$$

The wave function (1234;56); is of the form:

$$(1V\text{-}7)\quad (1234;56)_i = e^{-\alpha/2}\ \sum_{i=1}^{4} \bar{r}_i^{2}\ \ e^{-\bar{\alpha}/2}\ \sum_{j=5}^{6} \bar{r}_j^{2}\ e^{-(2/3\beta R^2 + \mu R_{cm}^2 + QR_{cm}\cdot R)}$$

$$\times\ R^2Y_0^0$$

M_{11} can therefore be written in the form:

$$(1V\text{-}8)\quad M_{11}\ = -\mu \times \sqrt{\frac{3}{8\ \pi}} \times\ \{(g_{s1}\ s_{-1} + g_{\ell 1}\ \ell_{-1}) + (g_{s2}\ s_{-2} + g_{\ell 2}\ \ \ell_{-2})$$

$$+\ \ (g_{s3}\ \ s_{-3} + g_{\ell 3}\ \ \ell_{-3}) + (g_{s4}\ s_{-4} +\ \ g_{\ell 4}\ \ell_{-4}) + (g_{s5}\ s_{-5} + g_{\ell 5}\ \ \ell_{-5})$$

$$+\ \ (g_{\ell 6}\ \ell_{-6} +\ \ g_{s6}\ s_{-6})\ \}$$

We have already stated that the orbital factor g_ℓ is zero for neutrons. This makes $g_{\ell 1} = g_{\ell 3} = g_{\ell 5} = 0$, and since the orbital factor is unity for protons we have $g_{\ell 2} = g_{\ell 4} = g_{\ell 6} = 1$. also $g_{s1} = g_{s3} = g_{s5} = -3.8263$ and $g_{s2} = g_{s4} = g_{s6} = 5.5856$15)

We shall now allow M_{11} to operate on the initial 0^+ wave functions:

(1V-9)
$$[(g_{s1}\, s_{-1} + g_{s3}\, s_{-3} + g_{s5}\, s_{-5}) + (g_{s2}\, s_{-2} + g_{s4}\, s_{-4} + g_{s6}\, s_{-6})$$
$$\times \frac{1}{\sqrt{2}}\, \alpha_1 \alpha_2 \beta_3 \beta_4 \, (\alpha_5 \beta_6 - \beta_5 \alpha_6)$$
$$= \frac{g_{s1}}{\sqrt{2}} (\beta_1 \alpha_2 \beta_3 \beta_4 \, (\alpha_5 \beta_6 - \beta_5 \alpha_6) + \alpha_1 \alpha_2 \beta_3 \beta_4 \beta_5 \beta_6))$$
$$+ \frac{g_{s2}}{\sqrt{2}} (\alpha_1 \beta_2 \beta_3 \beta_4 \, (\alpha_5 \beta_6 - \beta_5 \alpha_6) + \alpha_1 \alpha_2 \beta_3 \beta_4 \beta_5 \beta_6))$$

The action of ℓ_- on Y_0^0 produces zero because $\ell = 0$. The action of ℓ_+ and ℓ_z on Y_0^0 similarly gives zero.

Now $<f|\, M_{11} |i>$ becomes:

(1V-10)
$$<f|\, M_{11} |\, i> = -N_i \mu_o \times \sqrt{\frac{3}{8\pi}} <N_f\, A\, (1234;56)_f\, \alpha_1 \alpha_2 \beta_3 \beta_4 \alpha_5 \alpha_6 \,|\, \frac{g_{s1}}{\sqrt{2}}$$
$$\{ \beta_1 \alpha_2 \beta_3 \beta_4 \, (\alpha_5 \beta_6 - \beta_5 \alpha_6) + \alpha_1 \alpha_2 \beta_3 \beta_4 \beta_5 \beta_6 \}$$
$$+ \frac{g_{s2}}{\sqrt{2}} \{ \alpha_1 \beta_2 \beta_3 \beta_4 \, (\alpha_5 \beta_6 - \beta_5 \alpha_6) - \alpha_1 \alpha_2 \beta_3 \beta_4 \beta_5 \beta_6 \}$$
$$\cdot |\, (1234;56)_i >$$

When the final-state wave function is antisymmetrized and its scalar product with the 0^+ wave function is taken, there results:

(1V-11) $$\langle f | M_{11} | i \rangle = -\mu_0 \sqrt{\frac{3}{8\pi}} \; \frac{N_f N_i}{36} \times \langle \frac{g_{s1}}{\sqrt{2}} \{ (1234;56)_f - (1236;54)_f - (5234;16)_f + (5623;14)_f \} + \frac{g_{s2}}{\sqrt{2}} \{ (1634;25)_f - (1546;23)_f - (1234;56)_f + (1254;36)_f \} \, | (1234;56)_i \rangle$$

$$= -\mu_0 \times \sqrt{\frac{3}{8\pi}} \times \frac{N_f N_i}{36} \times \frac{g_{s1} - g_{s2}}{\sqrt{2}}$$

$$\times \langle (1234;56)_f - (1236;54)_f - (5234;16)_f + (5623;14)_f \, | (1234;56)_i \rangle$$

In a similar manner $\langle f | M_{1-1} | i \rangle$ is calculated and gives:

(1V-12) $$\langle f | M_{1\,-1} | i \rangle = -\mu_0 \sqrt{\frac{3}{8\pi}} \; \frac{N_f N_i}{36} \times \frac{-g_{s2} + g_{s1}}{\sqrt{2}}$$

$$\times \langle (1234;56)_f - (1236;54)_f - (5234;16)_f + (5623;14)_f \, / \, (1234;56)_i \rangle$$

The action of $[g_{s1} (s_{Z1} + s_{Z3} + s_{Z5}) + g_{s2}(s_{Z2} + s_{Z4} + s_{Z6})]$ on

$$\frac{1}{\sqrt{2}} \alpha_1 \alpha_2 \beta_3 \beta_4 (\alpha_5 \beta_6 - \beta_5 \alpha_6)$$

gives:

(1V-13) $$\frac{1}{\sqrt{2}} [\frac{g_{s1}}{2} \{ (\alpha_1 \alpha_2 \beta_3 \beta_4 (\alpha_5 \beta_6 - \beta_5 \alpha_6) - \alpha_1 \alpha_2 \beta_3 \beta_4 (\alpha_5 \beta_6 - \beta_5 \alpha_6)$$

$$+ \alpha_1 \alpha_2 \beta_3 \beta_4 (\alpha_5 \beta_6 + \beta_5 \alpha_6) \} + \frac{g_{s2}}{2} \{ (\alpha_1 \alpha_2 \beta_3 \beta_4 (\alpha_5 \beta_6 - \beta_5 \alpha_6)$$

$$- \alpha_1\alpha_2\beta_3\beta_4 \; (\alpha_5\beta_6 - \beta_5\alpha_6) + \alpha_1\alpha_2\beta_3\beta_4 \; (\alpha_5\beta_6 + \beta_5\alpha_6) \}]$$

Therefore:

(1V-14) $$<f \, |M_{10}| \, i> = N_f N_i < A \, \{(1234;56)_f \; \frac{\alpha_1\alpha_2\beta_3\beta_4 \; (\alpha_5\beta_6 + \beta_5\alpha_6)}{\sqrt{2}}$$

$$\left| \sqrt{\frac{3}{4\pi}} \; \frac{\mu_o}{\sqrt{2}} \times (- \frac{g_{s1}}{2} \quad (\alpha_1\alpha_2\beta_3\beta_4 \; (\alpha_5\beta_6 + \beta_5\alpha_6) \right.$$

$$+ \frac{g_{s2}}{2} \; \alpha_1\alpha_2\beta_3\beta_4 \; (\alpha_5\beta_6 + \beta_5\alpha_6) \; | \; (1234;56)_i >$$

$$= \frac{N_f N_i}{36} \times \mu_o \times \sqrt{\frac{3}{4\pi}} \times \frac{1}{4} \; (-g_{s1} + g_{s2}) \times 2$$

$$\times \{< (1234;56)_f - (1236;54)_f - (5234;16)_f + (5623;14)_f\} | (1234;56)_i >$$

$$= - \frac{N_f N_i}{36} \times \mu_o \times \sqrt{\frac{3}{8\pi}} \times \frac{g_{s1} - g_{s2}}{\sqrt{2}}$$

$$\times <\{ (1234;56)_f - (1236;54)_f - (5234;16)_f + (5623;14)_f\} | (1234;56)_i> .$$

(1V-15) We observe that $< f \, | \, M_{11} | \, i > = < f \, | M_{1\,-1} \, | \, i > \; = <f \, | \, M_{10} \, | \, i >$

Therefore $<f| \, M_{1\mu} | i>$ is independent of μ.

Since $< f \, |M_{1\mu}|i>$ is independent of μ:

(1V-16) $$B(M1) = 3 \; | <f \, | \, M_{11} \, | \, i>|^2$$

The M1 transition probability is therefore:

$$(1V\text{-}17) \qquad T(M1) = \frac{8\pi \times 2}{(3.1)^2} \times \frac{k^3}{\hbar} \times 3 \times |\langle f \mid M_{11} \mid i\rangle|^2$$

In this chapter, we have derived an expression for the M1 transition probability. It is seen that $\langle f \mid M_{1\mu} \mid i\rangle$ is independent of μ.

CHAPTER 5

NUMERICAL CALCULATIONS AND CONCLUSIONS

The following expressions were derived in chapters 3 and 4 for the probability of transitions from the 3^+ and the 0^+ states to the 1^+ state respectively.

$$T(E2) = \frac{8\pi \times 3}{450} \cdot e^2 \cdot \frac{k^5}{\hbar} \times B(E2)$$

where $$B(E2) = \frac{1}{2J_i+1} \sum_{M_i M_f} |\langle f | Q_{2\mu} | i \rangle|^2$$

and $$T(M1) = \frac{8\pi \times 2}{9} \times \frac{k^3}{\hbar} \cdot B(M1)$$

where $$B(M1) = \frac{1}{2J_i+1} \sum_{M_i} \sum_{M_f} |\langle f | M_{1\mu} | i \rangle|^2$$

$$= 3 \, |\langle f | M_{11} | i \rangle|^2$$

We shall now calculate T(E2) and T(M1) numerically using the numerical values of the width parameters given by Pearlstein, Tang and Wildermuth$^{7)}$.

The integrals I_1, I_2, I_3, I_4, and I_5 must first be evaluated. These are all the same according to Appendix 1.

We have already seen in chapter 3 that

$$I_1 = \langle \{(1234;56)_f - (1236;54)_f - (1254;36)_f + (1256;34)_f\} \mid Q_{22} \mid (2-2)_i \rangle$$

(V-1) $<(1234;56)_f \;|\; Q_{22} \;|\; (2-2)_i>$

$$= \left|\frac{1}{c_1}\right|^3 \times \left(\frac{1}{\alpha} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\alpha}} \times 4\pi\right) \times \left(\frac{3}{4\alpha} \times \frac{1}{2} \times \sqrt{\frac{3\pi}{2\alpha}} \times 4\pi\right)$$

$$\times \left(\frac{2}{3\alpha} \times \frac{1}{2} \times \sqrt{\frac{4\pi}{3\alpha}} \times 4\pi\right) \times \left(\frac{1}{\bar{\alpha}} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\bar{\alpha}}} \times 4\pi\right)$$

$$\times \frac{1}{\sqrt{4\pi}} \times \frac{7.5.3.1}{\{2(A_{11} + A_{11}')\}^4} \times \frac{1}{2} \times \sqrt{\frac{\pi}{A_{11} + A_{11}'}}$$

$$\times \frac{4\pi \times 1}{2(A_{12} + A_{12}')} \times \frac{1}{2} \times \sqrt{\frac{\pi}{A_{12} + A_{12}'}} \qquad \text{(Appendix 1)}$$

$$<(1234;56)_f \;|\; Q_{22} \;|\; (2-2)_i> \;=\; 0.8221 \times 10^8 \quad (\text{fm})^{24}$$

(V-2) $<(1236;54)_f \;|\; Q_{22} \;|\; (2-2)_i>$

$$= \left|\frac{16}{9c_2}\right|^3 \times \left(\frac{1}{\alpha} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\alpha}} \times 4\pi\right) \times \left(\frac{3}{4\alpha} \times \frac{1}{2} \times \sqrt{\frac{3\pi}{2\alpha}} \times 4\pi\right)$$

$$\times \left(\frac{1}{2} \Big/ \left(\frac{\alpha}{3} + \frac{\bar{\alpha}}{2}\right) \times \frac{1}{2} \times \sqrt{\frac{\pi}{\frac{\alpha}{3} + \frac{\bar{\alpha}}{2}}} \times 4\pi\right) \times \left(\frac{1}{4U} \times \frac{1}{2} \times \sqrt{\frac{\pi}{2U}} \times 4\pi\right)$$

$$\times \frac{4\pi}{c_2^2} \left(E_{14}^{12} + E_{11}^{12} + E_{16}^{12}\right) \times \Big[\Big(\frac{5.3.1}{(2B_{11}')^3} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_{11}'}}$$

$$\times \frac{3.1}{(2B_{12}')^2} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_{12}'}} + \frac{7.5.3.1}{(2B_{11}')^4} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_{11}'}} \times \frac{1}{2B_{12}'} \times \frac{1}{2}$$

$$\times \sqrt{\frac{\pi}{B_{12}'}})] \qquad \text{(Appendix 1.....)}$$

The numerical values of the parameters in the expressions for $< (1234;56)_f \mid Q_{22} \mid (2\ -2)_i >$ and $<(1236;54)_f \mid Q_{22} \mid (2 - 2)_i>$ are listed in Tables 1 and 2.

We find that $< (1236;54)_f \mid Q_{22} \mid (2 - 2)_i> = 0.1566 \times 10^8 \ (\text{fm})^{24}$

We shall now calculate the expression for $< (1254;36)_f \mid Q_{22} \mid (2\ -2)_i>$ and $<(1256;34)_f \mid Q_{22} \mid (2 - 2)_i>$.

$$\text{(V-3)} \quad < (1254;36)_f \mid Q_{22} \mid (2 - 2)_i > = \left|\frac{16}{9c_2}\right|^3 \times \left(\frac{1}{\alpha} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\alpha}} \times 4\pi\right)$$

$$\times \left(\frac{3}{4\alpha} \times \frac{1}{2} \times \sqrt{\frac{3\pi}{2\alpha}} \times 4\pi\right) \times \left(\frac{1}{2} \Big/ \left(\frac{\alpha}{3} + \frac{\bar{\alpha}}{2}\right) \times \frac{1}{2} \times \sqrt{\frac{\pi}{\frac{\alpha}{3} + \frac{\bar{\alpha}}{2}}} \times 4\pi\right)$$

$$\times \left(\frac{1}{4U} \times \frac{1}{2} \times \sqrt{\frac{\pi}{2U}} \times 4\pi\right) \times \sqrt{4\pi} \times 2\left(\frac{E_{11}^{\ 2} + E_{13}^{\ 2}}{c_2^2}\right)$$

$$\times \left[\frac{5.3.1}{(2B_{11}')^3} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_{11}'}} \times \frac{3.1}{(2B_{12}')^2} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_{12}'}} \right.$$

$$\left. + \frac{7.5.3.1}{(2B_{11}')^4} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{B_{11}'}} \cdot \frac{1}{2B_{12}'} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{B_{12}'}} \right]$$

TABLE 1

NUMERICAL VALUES OF PARAMETERS

FOR CALCULATION OF $<(1234;56)_f \mid Q_{22} \mid (2 - 2)_i>$

ALPHA = 0.4333

$\overline{\text{ALPHA}}$ = 0.6543

$A_{11} + A_{11}'$ = 0.4394

$A_{12} + A_{12}'$ = 0.0285

U = 1.5200

Q = -0.2933

TABLE 2

NUMERICAL VALUES OF PARAMETERS

FOR CALCULATION OF $<(1236;54)_f \mid Q_{22} \mid (2 - 2)_i>$

ALPHA = 0.4333

$\overline{\text{ALPHA}}$ = 0.6543

D_1 = 0.3603

D_2 = -0.9709

E_{11}' = 0.3766

E_{14}' = -0.6453

E_{16}' = 0.2524

B_{11}' = 0.5670

B_{12}' = 0.6024

c_2 = -3.0600

U = 1.5200

(This expression is derived in Appendix 1).

Also we have (V-4): $< (1256;34)_f \mid Q_{22} \mid (2-2)_i >$

$$= \left| \frac{8}{3c_3} \right|^3 \times \left(\frac{1}{\alpha} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\alpha}} \times 4\pi\right) \times \left(\frac{1}{2}\Big/(\alpha + \bar{\alpha}) \times \frac{1}{2} \times \sqrt{\frac{\pi}{\alpha + \bar{\alpha}}} \times 4\pi\right)$$

$$\times \left(\frac{1}{2}\Big/\left(\frac{\alpha + \bar{\alpha}}{4}\right) \times \frac{1}{2} \times \sqrt{\frac{\pi}{\frac{\alpha + \bar{\alpha}}{4}}} \times 4\pi\right) \times \left(\frac{1}{4U} \times \frac{1}{2} \times \sqrt{\frac{\pi}{2U}} \times 4\pi\right)$$

$$\times \sqrt{4\pi} \times \left(\frac{D_{11}^{12} + D_{11}^{2} + D_{12}^{2}}{c_3^2}\right) \times \left[\frac{5.3.1}{(2c_{11}')^3} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{c_{11}'}} \cdot \frac{3.1}{(2Y_2')^2} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{Y_2'}} \right.$$

$$\left. + \frac{7.5.3.1}{(2c_{11}')^4} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{c_{11}'}} \cdot \frac{1}{2Y_2'} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{2Y_2'}} \right]$$

(From Appendix 1....)

The numerical values of the parameters in the expressions $< (1254;36)_f \mid Q_{22} \mid (2-2)_i >$ and $<(1256;34)_f \mid Q_{22} \mid (2-2)_i>$ are listed in Tables 3 and 4.

Using the values of the parameters given in Table 3, we find

TABLE 3

NUMERICAL VALUES OF PARAMETERS

FOR CALCULATION OF $\langle(1254;36)_f \mid Q_{22} \mid (2 - 2)_i\rangle$

ALPHA = 0.4333

$\overline{\text{ALPHA}}$ = 0.6543

U = 1.5200

c_2 = -3.0600

E_{11} = 0.3744

E_{13} = -0.6453

B_{11}' = 0.5670

B_{12}' = 0.6024

TABLE 4

NUMERICAL VALUES OF PARAMETERS
FOR CALCULATION OF $\langle (1256;34)_f \mid Q_{22} \mid (2-2)_i \rangle$

ALPHA = 0.4333

$\overline{\text{ALPHA}}$ = 0.6543

c_3 = 5.7220

D_{11} = 0.5501

D_{11}' = 0.0197

D_{12} = 0.4444

c_{11}' = 0.6727

Y_2' = 0.0211

U = 1.5200

$$<(1254;36)_f \mid Q_{22} \mid (2-2)_i> \; = \; 0.1790 \times 10^8 \quad (\mathrm{fm})^{24}$$

and

$$<(1256;34)_f \mid Q_{22} \mid (2-2)_i> \; = \; 0.0664 \times 10^8 \quad (\mathrm{fm})^{24}$$

Hence we find I_1 to be:

$$I_1 = <(1234;56)_f - (1236;54)_f - (1254;36)_f + (1256;34)_f \mid Q_{22} \mid (2-2)_i> \tag{V-5}$$

$$= (0.8244 - 0.1790 - 0.1566 + 0.0664) \times 10^8$$

$$= 0.5552 \times 10^8 \quad (\mathrm{fm})^{24}$$

We can therefore write B(E2) as:

$$B(E2) = \frac{N_f^{\,2}\, N_i^{\,2}}{(36)^2} \times |\, I_1 \,|^2 \tag{V-6}$$

$$= \left(\frac{6}{\sqrt{0.3960 \times 10^8}}\right)^2 \times \left(\frac{6}{\sqrt{0.1840 \times 10^8}}\right)^2 \times \frac{1}{(36)^2} \times |I_1|^2$$

$$= \frac{1}{0.3960 \times 0.1840 \times 10^{16}} \times (0.5552)^2 \times 10^{16} = 4.36 \quad (\mathrm{fm})^4$$

Since B(E2) is known, and $k = \frac{E}{\hbar c} = \frac{2.18 \text{ Mev}}{\hbar c}$, we find:

$$T(E2) = 26.5 \times 10^{10}/\mathrm{sec.} \tag{V-7}$$

In order to calculate B(M1), we have to evaluate $< f \mid M_{11} \mid i>$

In Chapter 4 we derived an expression for $<f \mid M_{11} \mid i >$ which is:

$$= - \mu_o \times \sqrt{\frac{3}{8\pi}} \times \frac{N_f N_i}{36} \times \frac{g_{s1} - g_{s2}}{\sqrt{2}} \times < (1234;56)_f - (1236;54)_f$$

$$- (5234;16)_f + (5623;14)_f \mid (1234;56)>$$

$$|< f \mid M_{11} \mid i>|^2 = \mu_o^2 \times \frac{3}{8\pi} \times \frac{1}{2} \times \frac{N_f^2 N_i^2}{(36)^2} \times (g_{s1} - g_{s2})^2$$

$$\times \; |< (1234;56)_f - (1236;54)_f - (1254;36)_f + (5623;14)_f \mid (1234;56)_i >|^2$$

Using the result of Appendix 2 we write:

$$\text{(V-8)} \quad < (1234;56)_f \mid (1234;56)_i > = \frac{32.\ \pi^7 .\ \sqrt{16}\ .\ 2}{c_1^3 \times (\alpha)\ 4.5\ .(\ \bar{\alpha} + \bar{\alpha}^{'})^{1.5}} \ .$$

$$15\pi \ . \ \frac{1}{(2.(A_{22})^{3.5}\ .\ (A_{23})^{1.5}\ )}$$

where

$$A_{22} = \frac{4\beta}{3} + \frac{U + U^{'}}{c_1^2} - \frac{Q + Q^{'}}{c_1} \ ; \quad A_{23} = \frac{U + U^{'}}{c_1^2}$$

Therefore

$$\text{(V-9)} \quad <(1234;\ 56)_f \mid (1234;56)_i > \ = \ 0.8757 \times 10^8 \ (\text{fm})^{22}$$

TABLE 5

NUMERICAL VALUES OF PARAMETERS FOR THE DIFFERENT INTEGRALS

$<(1234;56)_f \mid (1234;56)_i>$ $<(1254;36)_f \mid (1234;56)_i>$ $<(3456;12)_f \mid (1234;56)_i>$

and

$<(1236;54)_f \mid (1234;56)_i>$

ALPHA	0.4333	ALPHA	0.4333	ALPHA	0.4333
$\overline{\text{ALPHA}}$	0.6543	$\overline{\text{ALPHA}}$	0.6543	$\overline{\text{ALPHA}}$	0.6543
$\overline{\text{ALPHA}}'$	0.4246	$\overline{\text{ALPHA}}'$	0.4246	$\overline{\text{ALPHA}}''$	0.4246
c_1	-19.8673	c_2	-3.2358	c_3	5.3871
A_{22}	0.4319	U	1.5200	U	1.5200
A_{23}	0.0071	U'	1.2913	U'	1.2913
		B_{22}	0.5498	c_{22}	0.6771
		X_{22}	0.0522	c_{23}	0.0238

Also we find $<(1254;36)_f \, |(1234;56)_i>$:

$$(V\text{-}10) \quad <(1254;36)_f \, |(1234;56)_i> \; = \frac{256 \, . \, 32. \, \pi^7 \, . \, \sqrt{(1.5) \, . \, (2)} \, . \, 2}{243 \, . \, (c_2)^5 \, . \, (\alpha)^3 . \, (\frac{\bar{\alpha} + \bar{\alpha}'}{4} + \frac{\alpha}{3})^{1.5} \, (U + U')^{1.5}}$$

$$. \; \frac{\pi}{2\pi} \; [\; \frac{9\pi}{(2B_{22})^{2.5} \, . \, (2X_{22})^{2.5}} \; + \; \frac{15\pi}{(2B_{22})^{3.5} \, . \, (2X_{22})^{1.5}} \;]$$

(Appendix 2)

We calculate $<(1254;36)_f \, | \, (1234;56)_i>$ to be:

$$(V\text{-}11) \quad <(1254;36)_f \, | \, (1234;56)_i> \; = \; 0.3548 \times 10^8 \quad (fm)^{22}$$

$$(V\text{-}12) \quad <(1236;54)_f \, |(1234;56)_i> \; = \; 0.3548 \times 10^8 \quad (fm)^{22}$$

In a similar way:

$$(V\text{-}13) \quad <(3456;12)_f \, |(1234;56)_i> \; = \; <(5623;14)_f \, | \, (1234;56)_i>$$

This is found to be:

(V-14)

$$\langle (3456;12)_f \mid (1234;56)_i \rangle = \frac{512 \cdot \pi^6 \cdot 2\sqrt{2}}{(27\, c^5_3 \times (\alpha)^{1.5} \times (\alpha + \bar{\alpha})^{1.5} \quad \left(\frac{\alpha + \bar{\alpha}^{1}}{4}\right)^{1.5}}$$

$$\frac{}{(U + U')^{1.5} \quad)} \cdot 2\pi$$

$$\times \left[\frac{9\pi}{(2c_{22})^{2.5} \cdot (2c_{23})^{2.5}} + \frac{15\pi}{(2c_{22})^{3.5} \cdot (2c_{23})^{1.5}} \right]$$

(From Appendix 2)

$$\langle (5623;14)_f \mid (1234;56)_i \rangle = 0.1323 \times 10^8 \quad (\mathrm{fm})^{22}$$

So we have:

(V-15)

$$<\{(1234;56)_f - (1236;54)_f - (1254;36)_f + (5623;14)_f\} \mid (1234;56)_i>$$

$$= (0.8757 - 2 \times 0.3548 + 0.1323) \times 10^8 \ (\mathrm{fm})^{22}$$

$$= 0.2984 \times 10^8 \qquad (\mathrm{fm})^{22}$$

We can now calculate T(M1). using $\mu_0 = 0.5050 \times 10^{-23}$ erg gauss^{-1},

$$k = E/\hbar c = \frac{3.56 \text{ Mev}}{\hbar c} ,$$

$$N_f^2 N_i^2 = \frac{(36)^2}{0.3960 \times 0.3500 \times 10^{16}} , \quad g_{s1} = 5.5856,$$

$$g_{s2} = -3.8263$$

We find:

(V-16) $$T(M1) = 8.1 \times 10^{15} \ / \ \text{sec.}$$

The level width Γ_γ for electric quadrupole photon emission is obtained from the transition probability T(E2):

$$\text{(V-17)} \qquad \Delta t = \frac{1}{T(E2)} = \frac{1}{26.5 \times 10^{10}} \quad \text{secs.}$$

$\Delta E \,.\, \Delta t \sim \hbar$ (From the uncertainty principle)

$$\text{(V-18)} \qquad \Delta E = \frac{10^{-27} \times 1.05 \times 26.5 \times 10^{10}}{1.6 \times 10^{-12}} \text{ e.v.} = 1.75 \times 10^{-4} \text{ e.v.}$$

The level width Γ_γ for magnetic dipole gamma emission is obtained similarly:

$$\text{(V-19)} \qquad \Delta t = \frac{1}{8.1 \times 10^{15}} \quad \text{secs.}$$

$$\text{(V-20)} \qquad \Delta E = \frac{1.05 \times 10^{-27}}{1.6 \times 10^{-12}} \times 8.1 \times 10^{15} \text{ e.v} = 5.32 \text{ e.v.}$$

The calculated radiative widths of the 2.18 Mev (3^+) and 3.56 Mev (0^+) states can now be compared with the experimental results.

E_x	J_n	Γ_γ	Reference	Γ_γ from cluster model
2.18	3^+	$4^{+3}_{-1.5} \times 10^{-4}$ e.v.	8) Ba 60 n	1.75×10^{-4} e. v.
		3×10^{-5} e.v.	13) Da 59 a	
3.56	0^+	6.2 ± 0.6 e.v.	8) Ba 60 n	5.32 e.v.
		4.7 ± 0.9 e.v.	9) Ba 63 f	
		9 ± 2 e.v.	10) Be 63 i	
		$9.1^{+2.0}_{-1.5}$ e.v.	11) Co 59 i	
		$8.8^{+1.9}_{-1.3}$ e.v.	12) Sk 63	

Calculations of the radiative widths have also been done by omission of the term $R_{cm}.R$ in the wave functions.

The wave functions now become:

$$\psi_{(1^+)} = N_{1^+}\, e^{-\frac{\alpha}{2}\sum\limits_{i=1}^{4}\bar{r}_i^2}\, e^{-\frac{\bar{\alpha}}{2}\sum\limits_{j=5}^{6}\bar{r}_j^2}\, e^{-\frac{2}{3}\beta R^2}\, e^{-\mu R^2_{cm}}\,.R^2 Y_0^0\ \zeta(1234;56)$$

$$\psi_{3^+} = N_{3^+}\, e^{-\frac{\alpha}{2}\sum\limits_{i=1}^{4}\bar{r}_i^2}\, e^{-\frac{\bar{\alpha}}{2}\sum\limits_{j=5}^{6}\bar{r}_j^2}\, e^{-\frac{2}{3}\beta R^2}\, e^{-\mu R^2_{cm}}\,.R^2 Y_2^M\ \zeta(1234;56)$$

$$\psi_{0^+} = N_{0^+}\, e^{-\frac{\alpha}{2}\sum\limits_{i=1}^{4}\bar{r}_i^2}\, e^{-\frac{\bar{\alpha}}{2}\sum\limits_{j=5}^{6}\bar{r}_j^2}\, e^{-\frac{2}{3}\beta R^2}\, e^{-\mu R^2_{cm}}\,.R^2 Y_0^0\ \zeta(1234;56)$$

The radiative widths were found to be 1.2 times the radiative widths which were calculated with the inclusion of the cross term $R_{cm}.R$.

We see that the radiative widths obtained by use of the cluster model are in reasonable agreement with the results of Barber et al.(8) These results were obtained by measuring the electron energy spectra resulting from the scattering of 40 Mev primary electrons for the purpose of studying nuclear excitations. A target of Li^6 was employed and the scattering angles were 132^o and 160^o.

In addition to the elastic peaks all spectra showed peaks corresponding to excitations of the target nucleus into well-defined energy states. Peaks corresponding to known levels in Li^6 at 2.18 and 3.56 Mev were measured and analysed by a virtual photon theory to give values of $4\,^{+3}_{-1.5} \times 10^{-4}$ e.v. and 6.2 ± 0.6 e.v. for the respective radiative widths to the ground state.

SINGLE-PARTICLE SHELL MODEL CALCULATION

A calculation of the transition probability based on the single-particle model is carried out in order to show that the cluster model gives an improved description of the Li^6 nucleus.

The transition probability for radiative decay by an electric multipole, for a single particle, is of the form:

$$\text{(V-21)} \qquad T_E(L) = \frac{4.4\ (L + 1)}{L\ (L\ \ + 1)!!)^2} \cdot \left(\frac{3}{L + 3}\right)^2 \cdot \left(\frac{E}{197\ \text{Mev}}\right)^{2L + 1} \cdot$$

$$\cdot\ S(j_i, L, j_f)$$

$$\cdot\ (R \text{ in fm})^{2L} \times 10^{21}/\text{sec.}$$

$L = 2$ (orbital angular momentum)

$E = 2.18$ Mev, $R = 3.5$ fm

$S =$ Statistical factor (of the order of unity)

$$\text{(V-22)} \qquad T_E(2) = 5 \times 10^{12}/\text{sec.}$$

The transition probability for radiative decay by a magnetic dipole, for a single-particle, is of the form:

$$\text{(V-23)} \qquad T_{sp}(ML) = \frac{0.19 \times (L + 1)}{(L)\ [(2L + 1)!!]^2} \cdot \left(\frac{3}{L + 3}\right)^2 \cdot \left(\mu_p - \frac{1}{2}\right)^2 \cdot$$

$$\cdot \left(\frac{E}{197\ \text{Mev}}\right)^3 \cdot (R)^{2L - 2} \cdot S(j_i, L, j_f) \cdot 10^{21}/\text{sec.}$$

$\mu_p = 2.79$, $E = 3.56$ Mev, $R^0 = 1$

S is a statistical factor of order unity

Therefore T_{Sp} (M1) = 1.4 X 10^{17}/secs.

These values are about 18 and 15 times larger than the transition probability obtained using the cluster model. The fact that the cluster model transition probabilities T(E2) and T(M1) are smaller than the single-particle shell model values is due partly to the correlation built into the cluster model wave functions, and partly to the fact that the single-particle wave functions are not antisymmetrized. Calculations using the unantisymmetrized cluster model wave functions give T(E2) = 240 X 10^{10}/sec and T(M1) = 27 X 10^{15}/sec which are factors of about 9 and 3 larger than the value obtained using antisymmetrized cluster model wave functions. The use of the cluster model has then lead to calculations which are in fair agreement with the values obtained by experiment. It is also seen from the values obtained by use of the single-particle shell model that this description of the Li^6 nucleus deviates appreciably from the correct description.

From the use of the cluster model we find that the description of Li^6 as composed of an alpha particle cluster and a deuteron cluster gives values of the radiative transition probabilities which are compatible with experiment.

With the help of the Ritz variational method it has been shown that a generalized alpha particle-deuteron cluster wave function yields a better description of the Li^6 nucleus than a generalized triton-He^3 cluster wave function. We find from experiment that break-up of the Li^6 ground state into a free alpha particle and deuteron requires only 1.5 Mev, whereas break-up into a free triton plus He^3 requires some 15 Mev. This result shows the importance of cluster correlations in the nucleus.

From the calculations it is apparent that the transition probabilities depend sensitively on the wave functions of the states involved in the transitions.

We can conclude that the cluster model gives quite a reasonable description of the Li^6 nucleus.

APPENDIX A

METHOD OF CALCULATION OF THE INTEGRAL I_K

We have derived an expression for B(E2) in Chapter 3. In this chapter, I_1, I_2, I_3, I_4 and I_5 will be calculated.

(A-1) $$I_1 = < \frac{N_i N_f}{36} (1234;56)_f - (1236;54)_f - (1254;36)_f + (1256;34)_f$$

$$|Q_{22}| \; (2\text{-}2)_i >$$

Expressions for N_i and N_f have been calculated

(A-2) $$N_i^2 = \frac{36}{<(1234;56)_i - 2(1254;36)_i + (3456;12)_i \; |(1234;56)_i>}$$

(A-3) $$N_f^2 = \frac{36}{<(1234;56)_f - 2(1254;36)_f + (3456;12)_f \; |(1234;56)_f>}$$

(A-4) $$(1234;56)_i = e^{-\frac{\alpha}{2} \sum_{i=1}^{4} \bar{r}_i^2} e^{-\frac{\bar{\alpha}}{2} \sum_{j=5}^{6} \bar{r}_j^2} e^{-(2/3\beta R^2 + \mu R^2{}_{cm} + QR_{cm}.R)}$$

$$.\; R^2 Y_2^M$$

(A-5) $$\vec{\bar{r}}_i = \vec{r}_i - \vec{R}_\alpha , \quad \vec{\bar{r}}_j = \vec{r}_j - \vec{R}_d ,$$

$$\vec{R}_\alpha = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4}{4} , \quad R_d = \frac{\vec{r}_5 + \vec{r}_6}{2} ,$$

$$\vec{R} = \vec{R}_\alpha - \vec{R}_d$$

(A-6) $$(1234;56)_i = e^{-\frac{\alpha}{2}\{\frac{3}{4}(r_i^2 + r_2^2 + r_3^2 + r_4^2) - \frac{1}{2}(r_1 \cdot r_2}$$

$$^{+ r_1 \cdot r_3 + r_1 \cdot r_4 + r_2 \cdot r_3 + r_2 \cdot r_4 + r_3 \cdot r_4)\}}$$

$$e^{-\frac{\bar{\alpha}}{4}(r_5^2 + r_6^2 - 2\, r_5 \cdot r_6)}$$

$$e^{-(\frac{2}{3}\beta R^2 + UR_{cm}^2 + QR_{cm} \cdot R)}$$

$$\cdot \quad R^2 Y_2^M$$

(A-7) $$<(1234;56)_i \quad |(1234;56)_i>$$

$$= \int e^{-\alpha\{\frac{3}{4}(r_1^2 + r_2^2 + r_3^2 + r_4^2) - \frac{1}{2}(r_1 \cdot r_2 + r_1 \cdot r_3}$$

$$^{+ \quad r_1 \cdot r_4 + r_2 \cdot r_3 + r_2 \cdot r_4 + r_3 \cdot r_4)\}}$$

$$e^{-\frac{\bar{\alpha}}{2}(r_5 + r_6 - 2\, r_5 \cdot r_6)} \quad e^{-\frac{4}{3}\beta R^2}$$

$$e^{-2(UR^2{}_{cm} + QR_{cm} \cdot R)} \quad R^2 Y_2{}^M \times R^2 Y_2{}^{M^*}$$

$$d\vec{r}_1 \;\; d\vec{r}_2 \;\; ---------- \;\; d\vec{r}_6$$

The following linear transformations from the set of variables $(\vec{r}_1 ---- \vec{r}_6)$ to the new variables $(\vec{\rho}_1 ---- \vec{\rho}_6)$ will remove the cross terms in the exponential.

(A-8) $\vec{\rho}_1 = \vec{R}, \quad \vec{\rho}_2 = \vec{R} + c_1 \vec{R}_{cm}, \quad \vec{\rho}_3 = \vec{r}_5 - \vec{r}_6,$

$\vec{\rho}_4 = \vec{r}_4 - \frac{1}{3}(\vec{r}_1 + \vec{r}_2 + \vec{r}_3), \quad \vec{\rho}_5 = \vec{r}_3 - \frac{1}{2}(\vec{r}_1 + \vec{r}_2),$

$\vec{\rho}_6 = \vec{r}_1 - \vec{r}_2.$

The value of $c_1 = \frac{2\mu}{Q}$

(A-9) $<(1234;56)_i \,|(1234;56)_i>$ now becomes:

$$= |J|^3 \int [e^{-\alpha(\frac{\rho_6^2}{2} + \frac{2}{3}\rho_5^2 + \frac{3}{4}\rho_4^2)} e^{-\frac{\bar{\alpha}}{2}\rho_3^2}$$

$$e^{-(A_1\rho_1{}^2 + A_2\rho_2{}^2)} \cdot \rho_1^2 Y_2^{*M} \times \rho_1^2 Y_2^M d\vec{\rho}_1 \text{---} d\vec{\rho}_6]$$

$$A_1 = \frac{4\beta}{3} + \frac{2\mu}{c_1^2} - \frac{2Q}{c_1} \qquad A_2 = \frac{2\mu}{c_1^2}$$

(A-10) $$J = \frac{\partial\ (\vec{r}_1, \vec{r}_2, \vec{r}_3, \vec{r}_4, \vec{r}_5, \vec{r}_6)}{\partial\ (\vec{\rho}_1, \vec{\rho}_2, \vec{\rho}_3, \vec{\rho}_4, \vec{\rho}_5, \vec{\rho}_6)} = \frac{1}{c_1}$$

(A-11) $$<(1234;56)_i \mid (1234;56)_i >$$

$$= R1 = \left| \frac{1}{c1} \right|^3 \int \left[e^{-\alpha\left(\frac{\rho_6^2}{2} + \frac{2}{3} \rho_5^2 + \frac{3}{4} \rho_4^2 \right)} e^{-\frac{\bar{\alpha}}{2} \rho_3^2} \right.$$

$$\left. e^{-(A_1 \rho_1^2 + A_2 \rho_2^2)} \rho_1^4 Y_2^{M^*} (\Omega_1) Y_2^M (\Omega_1) d\vec{\rho}_1 \text{----------} d\vec{\rho}_6 \right]$$

$$= \left| \frac{1}{c1} \right|^3 x \int e^{-\frac{\alpha \rho_6^2}{2}} \rho_6^2 d\rho_6 d\Omega_6 \int e^{-\frac{2\alpha}{3} \rho_5^2} \rho_5^2 d\rho_5 d\Omega_5$$

$$\int e^{-\alpha \times \frac{3}{4} \rho_4^2} \rho_4^2 d\rho_4 d\Omega_4 \int e^{-\bar{\alpha}/2 \rho_3^2} \rho_3^2 d\rho_3 d\Omega_3$$

$$x \int e^{-A_1 \rho_1^2} \rho_1^4 \rho_1^2 d\rho_1 Y_2^{M*} (\Omega_1) Y_2^M (\Omega_1) d\Omega_1$$

$$x \int e^{-A_2 \rho_2^2} \rho_2^2 d\rho_2 d\Omega_2$$

(A-12) $$< (1234;56)_i \mid (1234;56)_i >$$

$$= \left| \frac{1}{c1} \right|^3 x \left(\frac{1}{\alpha} x \frac{1}{2} x \sqrt{\frac{2\pi}{\alpha}} x 4\pi \right) x \left(\frac{3}{4\alpha} x \frac{1}{2} x \sqrt{\frac{3\pi}{2\alpha}} x 4\pi \right)$$

$$x \left(\frac{2}{3\alpha} x \frac{1}{2} x \sqrt{\frac{4\pi}{3\alpha}} x 4\pi \right) x \left(\frac{1}{\bar{\alpha}} x \frac{1}{2} x \sqrt{\frac{2\pi}{\bar{\alpha}}} x 4\pi \right)$$

$$\times \quad \frac{5.3.1}{(2A_1)^3} \times \sqrt{\frac{1}{2}} \times \sqrt{\frac{\pi}{A_1}} \times \frac{1}{2A_2} \times \frac{1}{2} \times \sqrt{\frac{\pi}{A_2}} \times 4\pi$$

$$\left(\int Y_2^{M*} \; (\Omega_1) \quad Y_2^{M} \; (\Omega_1) \; d\Omega_1 \; = \; 1 \right)$$

$$R1 \; = \; 0.4392 \times 10^{8} \; (\mathrm{fm})^{22}$$

(A-13) $\quad R2 \; = \; <(1254;36)_i \; | \; (1234;56)_i> \quad =$

$$\int [\, e^{-\frac{\alpha}{2} \{ \frac{3}{4} (r_1^2 + r_2^2 + r_5^2 + r_4^2) - \frac{1}{2} (r_1 \cdot r_2 + r_1 \cdot r_5 + r_1 \cdot r_4 + r_2 \cdot r_5}$$

$$+ \; r_2 \cdot r_4 + r_5 \cdot r_4) \} \; e^{-2/3 \; \beta R'^2} \quad e^{-(UR^2{}_{cm} + QR_{cm} . R')}$$

$$e^{-\frac{\bar{\alpha}}{4}} \; (r_3^2 + r_6^2 - 2r_3 . r_6) \; \times R'^2 \; Y_2^{M*} \; (R') \; \times \; e^{-\frac{\alpha}{2}} \{ \; \frac{3}{4} (r_1^2 + r_2^2 + r_3^2$$

$$+ \; r_4^2) - \frac{1}{2} (r_1 \cdot r_2 + r_1 \cdot r_3 + r_1 \cdot r_4 + r_2 \cdot r_3 + r_2 \cdot r_4 + r_3 \cdot r_4)\}$$

$$e^{-2/3 \; \beta R^2} \qquad e^{-(UR^2{}_{cm} + QR_{cm} . R)} \qquad e^{-\frac{\bar{\alpha}}{4}} \; (r_5^2 + r_6^2 \; - \; 2r_5 \cdot r_6)$$

$$\times \; R^2 Y_2^M \; (R) \qquad d\vec{r}_1 \; \text{---------------} \; d\vec{r}_6]$$

The linear transformations are:

(A-14) $$\vec{R} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4}{4} - \frac{\vec{r}_5 + \vec{r}_6}{2} = \vec{\rho}_1,$$

$$\vec{R}' = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_5 + \vec{r}_4}{4} - \frac{\vec{r}_3 + \vec{r}_6}{2} = \vec{\rho}_2,$$

$$\vec{R} + \vec{R}_{cm} = \vec{\rho}_3 , \qquad \vec{r}_5 - \vec{r}_6 = \vec{\rho}_4,$$

$$\vec{r}_4 - \frac{1}{2}(\vec{r}_1 + \vec{r}_2) = \vec{\rho}_5, \quad \vec{r}_1 - \vec{r}_2 = \vec{\rho}_6.$$

(A-15) $$\vec{\rho}_3 - \vec{\rho}_1 = \vec{R}_{cm} , \qquad \vec{\rho}_2 + \vec{\rho}_3 - \vec{\rho}_1 = \vec{R}' + \vec{R}_{cm},$$

$$\vec{r}_3 - \vec{r}_5 = \frac{4}{3}(\vec{\rho}_1 - \vec{\rho}_2), \vec{r}_5 - \vec{r}_6 = \vec{\rho}_4,$$

$$\vec{r}_3 - \vec{r}_6 = \frac{4}{3}(\vec{\rho}_1 - \vec{\rho}_2) + \vec{\rho}_4.$$

The integral becomes:

(A-16) $$<(1254;36)_i \,|(1234;56)_i>$$

$$= \left|\frac{16}{9}\right|^3 \times \int [\, e^{-\frac{\alpha}{2}\left(\frac{\rho_6^2}{2} + \frac{2}{3}\rho_5^2 \right.}$$

$$\left. + \frac{3}{4}\left(\frac{-8\vec{\rho}_1 - 4\vec{\rho}_2 + 6\vec{\rho}_4}{9}\right)^2 \right. \cdot e^{-\frac{\bar{\alpha}}{4}\left(\frac{4}{3}(\vec{\rho}_1 - \vec{\rho}_2) + \vec{\rho}_4\right)^2}$$

$$e^{-(\frac{2}{3}\beta R'^2 + UR_{cm}^2 + QR_{cm}\cdot R')} \qquad \rho_2^2 Y_2^{M*}(\Omega_2)$$

$$\cdot\ e^{-\frac{\alpha}{2}\left(\frac{\rho_6^2}{2} + \frac{2}{3}\rho_5^2 + \frac{3}{4}\left(\frac{4\vec{\rho}_1 - 16\vec{\rho}_2 + 6\vec{\rho}_4}{9}\right)^2\right)}$$

$$e^{-\frac{\alpha}{2}\rho_4^2} \qquad e^{-(2/3\ \beta R^2 + UR_{cm}^2 + QR_{cm}\cdot R)}$$

$$\cdot\ \rho_1^2 Y_2^M(\Omega_1) \qquad d\vec{\rho}_1 \text{ ---- } d\vec{\rho}_6$$

Now we have transformations:

(A-17) $\vec{\zeta}_1 = \vec{\rho}_1$, $\vec{\zeta}_2 = \vec{\rho}_2$, $\vec{\zeta}_3 = \vec{\rho}_3$, $\vec{\zeta}_4 = \vec{\rho}_4 + d_1\vec{\zeta}_1 + d_2\vec{\zeta}_2$,
$\vec{\zeta}_5 = \vec{\rho}_5$ and $\vec{\zeta}_6 = \vec{\rho}_6$.

This process eliminates the cross terms

$\vec{\rho}_1 \cdot \vec{\rho}_4$ and $\vec{\rho}_2 \cdot \vec{\rho}_4$

and the transformations:

(A-18) $\vec{\sigma}_1 = \vec{\zeta}_1$, $\vec{\sigma}_2 = \vec{\zeta}_2$, $\vec{\sigma}_3 = \vec{\zeta}_3 + e_1\vec{\sigma}_1 + e_2\vec{\sigma}_2$, $\vec{\sigma}_4 = \vec{\zeta}_4$,
$\vec{\sigma}_5 = \vec{\zeta}_5$ and $\vec{\sigma}_6 = \vec{\zeta}_6$ eliminate cross terms $\vec{R}_{cm}\cdot\vec{R}$ and $\vec{R}_{cm}\cdot\vec{R}'$.

Finally the transformations:

(A-19) $\vec{\eta}_1 = \vec{\sigma}_1$, $\vec{\eta}_2 = \vec{\eta}_1 + c_2\vec{\sigma}_2$, $\vec{\eta}_3 = \vec{\sigma}_3$, $\vec{\eta}_4 = \vec{\sigma}_4$,

$\vec{\eta}_5 = \vec{\sigma}_5$ and $\vec{\eta}_6 = \vec{\sigma}_6$ eliminate the cross term $\vec{\sigma}_1 \cdot \vec{\sigma}_2$.

Therefore the integral becomes:

$$\text{(A-20)} \qquad R2 = \left| \frac{16}{9c_2} \right|^3 \int e^{-\frac{\alpha}{2}\eta_6^2} \eta_6^2 \, d\eta_6 \, d\Omega_6$$

$$\int e^{-\frac{2\alpha}{3}\eta_5^2} \eta_5^2 \, d\eta_5 \, d\Omega_5 \int e^{-2U\eta_3^2} \eta_3^2 \, d\eta_3 \, d\Omega_3$$

$$\int e^{-\eta_4^2 \left(\frac{\bar{\alpha}}{2} + \frac{\alpha}{3} \right)} \eta_4^2 \, d\eta_4 \, d\Omega_4$$

$$\times \left[\int e^{-\eta_1^2 B_1} \eta_1^2 \, Y_2^K (\Omega_1) \, e^{-\eta_2^2 X_2} \times \left(\frac{\eta_1^2 + \eta_2^2 - 2\vec{\eta}_1 \cdot \vec{\eta}_2}{c_2^2} \right) \right.$$

$$\left. \times \; Y_2^{K*} (\Omega) \times \eta_1^2 \, d\eta_1 \, d\Omega_1 \cdot \eta_2^2 \, d\eta_2 \, d\Omega_2 \right]$$

because the resultant Jacobian

$$J = \frac{\partial(\vec{r}_1, \vec{r}_2, \text{----} \vec{r}_6)}{\partial(\vec{\eta}_1, \vec{\eta}_2, \text{----} \vec{\eta}_6)} = \frac{16}{9c_2}$$

(A-21)
$$\int \left[e^{-\eta_1^2 B_1} \; e^{-\eta_2^2 X_2} \; x \; \eta_1^2 Y_2^K(\Omega_1) \cdot \frac{(\eta_2^2 + \eta_1^2 - 2\eta_1 \cdot \eta_2)}{c_2^2} Y_2^{*K}(\Omega) \right.$$

$$\left. \cdot \; \eta_1^2 \; d\eta_1 \; d\Omega_1 \; \eta_2^2 \; d\eta_2 \; d\Omega_2 \right]$$

$$= \int e^{-\eta_1^2 B_1} \; e^{-\eta_2^2 X_2} \cdot \eta_1^2 \cdot \frac{\eta_1^2}{c_2^2} \cdot \eta_1^2 \, d\eta_1 \;\; \eta_2^2 \, d\eta_2 \;\; d\Omega_2$$

$$= \frac{4\pi}{c_2^2} \int e^{-\eta_1^2 B_1} \; e^{-\eta_2^2 X_2} \; \eta_1^6 \, d\eta_1 \; \eta_2^2 \; d\eta_2$$

$$= \frac{4\pi}{c_2^2} \int e^{-\eta_1^2 B_1} \; \eta_1^6 \, d\eta_1 \int e^{-\eta_2^2 X_2} \; \eta_2^2 \, d\eta_2$$

$$= \frac{4\pi}{c_2^2} \cdot \frac{5.3.1}{(2B_1)^3} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{B_1}} \cdot \frac{1}{2x_2} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{x_2}}$$

(A-22)
$$B_1 = \frac{10\,\alpha}{27} + \frac{\alpha}{3} d_1^2 + \frac{2\alpha}{9} \; d_1 + \frac{4\bar{\alpha}}{9} - \frac{2d_1}{3} \; \bar{\alpha} + 2Ue_1^2 + 2U + 4Ue_1$$

$$+ \; \frac{2\beta}{3} + B_2 \mid c_2^2 - B_3 \mid c_2 - Qe_1 - Q + \frac{\bar{\alpha}}{2} d_1^2 \, ,$$

$$B_2 = \frac{34\alpha}{27} + \frac{\alpha}{3} d_2^2 + \frac{10\alpha}{9} \; d_2 + \frac{4\bar{\alpha}}{9} + \frac{2d_2}{3} \; \bar{\alpha} + \frac{2\beta}{3}$$

$$+ \ Ue_2^2 - Qe_2 + Ue_2^2 + \frac{\bar{\alpha}}{2} \ ,$$

$$B_3 = \frac{2\alpha}{3} d_1 d_2 - \frac{8\alpha}{27} + \frac{10\alpha}{9} d_1 + \frac{2\alpha}{9} d_2 - \frac{8\bar{\alpha}}{9} - \frac{2d_2}{3} \quad \bar{\alpha}$$

$$+ \frac{2d_1}{3} \ \bar{\alpha} + 4Ue_1 e_2 + 4Ue_2 + d_1 d_2 \bar{\alpha} - Qe_1 - Q - Qe_2 .$$

where $$d_1 = \frac{\frac{2\bar{\alpha}}{3} - \frac{2\alpha}{9}}{\frac{2\alpha}{3} + \bar{\alpha}} , \qquad d_2 = \frac{\left(\frac{2\bar{\alpha}}{3} + \frac{10\alpha}{9} \right)}{\frac{2\alpha}{3} + \bar{\alpha}} \ ,$$

$$e_1 = \frac{Q - 4U}{4U} \ , \qquad e_2 = \frac{Q}{4U}$$

$$c_2 = \frac{2B_2}{B_3} \quad \text{and} \qquad x_2 = \frac{B_2}{c_2^2}$$

(A-23) $$R2 = \left| \frac{16}{9c_2} \right|^3 \cdot \left(\frac{1}{\alpha} \cdot \frac{1}{2} \cdot \sqrt{\frac{2\pi}{\alpha}} \cdot 4\pi \right) \cdot \left(\frac{3}{4\alpha} \cdot \frac{1}{2} \sqrt{\frac{3\pi}{2\alpha}} : 4\pi \right)$$

$$\cdot \left(\frac{1}{2\left(\frac{\alpha}{3} + \frac{\bar{\alpha}}{2} \right)} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{\frac{\alpha}{3} + \frac{\bar{\alpha}}{2}}} \cdot 4\pi \right) \cdot \left(\frac{1}{4U} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{2U}} \times 4\pi \right)$$

$$\cdot \frac{4\pi}{c_2^2} \cdot \left[\frac{5.3.1}{(2B_1)^3} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{B_1}} \cdot \frac{1}{2x_2} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{x_2}} \right]$$

$$\therefore R2 = 0.1691 \times 10^8 \ (fm)^{22}$$

(A-24) $$R3 = \langle (3456;12)_i \mid (1234;56)_i \rangle$$

$$= \int \Big[e^{-\frac{\alpha}{2}\{ \frac{3}{4} (r_3^2 + r_4^2 + r_5^2 + r_6^2) - \frac{1}{2} (r_3 \cdot r_4 + r_3 \cdot r_5}$$

$$+ \quad {}^{r_3 \cdot r_6 + r_4 \cdot r_5 + r_4 \cdot r_6 + r_5 \cdot r_6)\}}$$

$$e^{-\bar{\alpha}/4(r_1^2 + r_2^2 - 2\, r_1 \cdot r_2)} \quad e^{-(2/3\beta R^2 + UR_{cm}^2 + QR_{cm} \cdot R')}$$

$$\times \; R'^2 \; Y_2^K$$

$$\times \quad e^{-\frac{\alpha}{2}\{ 3/4 (r_1^2 + r_2^2 + r_3^2 + r_4^2) - \frac{1}{2} (r_1 \cdot r_2 + r_1 \cdot r_3}$$

$$+ \quad {}^{r_1 \cdot r_4 + r_2 \cdot r_3 + r_2 \cdot r_4 + r_3 \cdot r_4)} \quad e^{-\frac{\bar{\alpha}}{4} (r_5^2 + r_6^2 - 2r_5 \cdot r_6)}$$

$$e^{-(2/3\, \beta R^2 + UR_{cm}^2 + Q\, R_{cm} \cdot R)} \quad R^2\, Y_2^K$$

$$\times \; d\vec{r}_1 \; d\vec{r}_2 \text{ -------- } d\vec{r}_6 \; \Big]$$

The following successive transformations remove the cross terms in the exponential.

(A-25) $$\vec{\rho}_1 \doteq \vec{R} = \frac{\vec{r}_1 + \vec{r}_2 + \vec{r}_3 + \vec{r}_4}{4} - \frac{\vec{r}_5 + \vec{r}_6}{2} ,$$

$$\vec{\rho}_2 = \vec{R}' = \frac{\vec{r}_3 + \vec{r}_4 + \vec{r}_5 + \vec{r}_6}{4} - \frac{\vec{r}_1 + \vec{r}_2}{2}$$

$$\vec{\rho}_3 = \vec{R} + \vec{R}_{cm}, \quad \vec{\rho}_4 = \vec{r}_5 - \vec{r}_6 ,$$

$$\vec{r}_2 - \frac{1}{2} (\vec{r}_3 + \vec{r}_4) = \vec{\rho}_5 \text{ and } \vec{\rho}_6 = \vec{r}_3 - \vec{r}_4$$

(A-26)
$$\vec{\varepsilon}_1 = \vec{\rho}_1, \quad \vec{\varepsilon}_2 = \vec{\rho}_2, \quad \vec{\varepsilon}_3 = \vec{\rho}_3, \quad \vec{\varepsilon}_4 = \vec{\rho}_4,$$

$$\vec{\varepsilon}_5 = \vec{\rho}_5 + d_1\vec{\varepsilon}_1 + d_2\vec{\varepsilon}_2 \text{ and } \vec{\varepsilon}_6 = \vec{\rho}_6$$

(A-27)
$$\vec{\sigma}_1 = \vec{\varepsilon}_1, \quad \vec{\sigma}_2 = \vec{\varepsilon}_2, \quad \vec{\sigma}_3 = \vec{\varepsilon}_3 + e_1\vec{\varepsilon}_1 + e_2\vec{\varepsilon}_2,$$

$$\vec{\sigma}_4 = \vec{\varepsilon}_4, \quad \vec{\sigma}_5 = \vec{\varepsilon}_5 \quad \text{and} \quad \vec{\sigma}_6 = \vec{\varepsilon}_6$$

(A-28)
$$\vec{\eta}_1 = \vec{\sigma}_1, \quad \vec{\eta}_2 = \vec{\eta}_1 + c_3\vec{\sigma}_2, \quad \vec{\eta}_3 = \vec{\sigma}_3, \quad \vec{\eta}_4 = \vec{\sigma}_4$$

$$\vec{\eta}_5 = \vec{\sigma}_5 \quad \text{and} \quad \vec{\eta}_6 = \vec{\sigma}_6$$

(A-29) R3 =

$$\left| \frac{8}{3c_3} \right|^3 \times \int e^{-\frac{\alpha}{2} \eta_6^2} \eta_6^2 \, d\eta_6 \, d\Omega_6 \int e^{-\eta_5^2(\bar{\alpha} + \alpha)_2} \eta_5^2 d\eta_5 \, d\Omega_5$$

$$\cdot \int e^{-\eta_4^2 \left(\frac{\alpha}{4} + \frac{\bar{\alpha}}{4}\right)} \eta_4^2 d\eta_4 d\Omega_4 \int e^{-2U\eta_3^2} \eta_3^2 d\eta_3 d\Omega_3$$

$$\cdot \int [e^{-\eta_1^2 c_{11}} \quad e^{-\eta_2^2 Y_2} \eta_1^2 Y_2^{K} (\Omega_1) \; . \; \frac{(\eta_2^2 + \eta_1^2 - 2\eta_1 \cdot \eta_2)}{c_3^2}$$

$$\cdot \quad Y_2^{K^*}(\Omega) \; . \; \eta_1^2 d\eta_1 \; d\Omega_1 \; \eta_2^2 d\eta_2 \; d\Omega_2]$$

$$c_{11} = \frac{8\alpha}{9} + \frac{4\bar{\alpha}}{9} + \bar{\alpha} d_1^2 - \frac{4d_1}{3} \bar{\alpha} + 2Ue_1^2 + \frac{2\beta}{3} + 2U - Q$$

$$+ \quad 4Ue_1 - Qe_1 + \alpha d_1^2 + \frac{2\alpha}{3} - \frac{4\alpha}{3} d_1 + \frac{Y_2}{c_3^2} - \frac{\left(\frac{16\alpha}{27} - \frac{Q^2}{4U}\right)}{c_3}$$

where
$$Y_2 = \frac{c_{22}}{c_3^2} = [\frac{2\alpha}{9} + \frac{16\bar{\alpha}}{9} + \bar{\alpha} \, d_2^2 - \frac{8\bar{\alpha}}{3} d_2 + \frac{2\beta}{3}$$

$$+ \quad 2Ue_2^2 - Qe_2 + \bar{\alpha} \, d_2^2 + \frac{8\alpha}{3} - \frac{8\alpha}{3} d_2] / c_3^2$$

and
$$c_3 = \frac{2c_{22}}{\frac{16\alpha}{27} - \frac{Q^2}{4U}}$$

(A-30) The Jacobian J = $\dfrac{\partial(\vec{r}_1 \text{ ---------- } \vec{r}_6)}{\partial(\vec{\eta}_1 \text{ ---------- } \vec{\eta}_6)} = \dfrac{8}{3c_3}$

(A-31) $$R3 = \left| \frac{8}{3c_3} \right|^3 \cdot \left(\frac{1}{\alpha} \cdot \frac{1}{2} \cdot \sqrt{\frac{2\pi}{\alpha}} \cdot 4\pi \right) \cdot \left(\frac{1}{2(\alpha + \bar{\alpha})} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{\alpha + \bar{\alpha}}} \cdot 4\pi \right)$$

$$\cdot \left(\frac{1}{2 \left(\frac{\alpha}{4} + \frac{\bar{\alpha}}{4} \right)} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{\frac{\alpha + \bar{\alpha}}{4}}} \cdot 4\pi \right) \cdot \left(\frac{1}{4U} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{2U}} \cdot 4\pi \right)$$

$$\cdot \frac{4\pi}{c_3^2} \cdot \left[\frac{5.3.1}{(2c_{11})^3} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{c_{11}}} \cdot \frac{1}{2Y_2} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{Y_2}} \right]$$

$$R3 = 0.0831 \times 10^8 \ (\mathrm{fm})^{22}$$

The normalization constant N_i becomes therefore $N_i^2 =$

$$\frac{36}{R1 - 2R2 + R3} = \frac{36}{0.1841 \times 10^8} \quad (\mathrm{fm})^{-22}$$

Now we shall calculate N_f

(A-32) $$<(1234;56)_f \ | \ 1234;56)_f >$$

$$= \left| \frac{1}{c_1} \right|^3 \times \int e^{-\frac{\alpha}{2} \rho_6^2} \rho_6^2 \, d\rho_6 \, d\Omega_6 \int e^{-\alpha \times \frac{2}{3} \rho_5^2} \rho_5^2 \, d\rho_5 \, d\Omega_5$$

$$\int e^{-\alpha \times 3/4 \rho_4^2} \rho_4^2 \, d\rho_4 \, d\Omega_4 \quad \int e^{-\bar{\alpha}/2 \, \rho_3^2} \rho_3^2 \, d\rho_3 \, d\Omega_3$$

$$x \int e^{-A_1' \rho_1^2} \rho_1^4 \rho_1^2 \, d\rho_1 \, Y_o^o \, Y_o^o \, d\Omega_1 \int e^{-A_2' \rho_2^2} \rho_2^2 \, d\rho_2 \, d\Omega_2$$

$$A_1' = \frac{4\beta'}{3} + \frac{2U}{c_1^2} - \frac{2Q}{c_1} \qquad A_2' = \frac{2U}{c_1^2}$$

(A-33) $$R_{11} = \left| \frac{1}{c_1} \right|^3 . \left(\frac{1}{\alpha} . \frac{1}{2} . \sqrt{\frac{2\pi}{\alpha}} . 4\pi\right) . \left(\frac{3}{4\alpha} . \frac{1}{2} . \sqrt{\frac{3\pi}{2\alpha}} . 4\pi\right)$$

$$. \left(\frac{2}{3\alpha} . \frac{1}{2} . \sqrt{\frac{4\pi}{3\alpha}} . 4\pi \right) . \left(\frac{1}{\bar{\alpha}} . \frac{1}{2} . \sqrt{\frac{2\pi}{\bar{\alpha}}} . 4\pi \right)$$

$$.. \left(\frac{5.3.1}{(2A_1')^3} . \frac{1}{2} . \sqrt{\frac{\pi}{A_1'}} \right) . \left(\frac{1}{2A_2'} . \frac{1}{2} . \sqrt{\frac{\pi}{A_2'}} . 4\pi \right)$$

$$R_{11} = 0.6963 \times 10^8 \, (\mathrm{fm})^{22}$$

(A-34) $$R_{22} = <(1254;36)_f \, |(1234;56)_f>$$

$$= \left| \frac{16}{9c_2} \right|^3 . \int e^{-\frac{\alpha}{2}\eta_6^2} \eta_6^2 \, d\eta_6 \, d\Omega_6 \int e^{-\frac{2\alpha}{3}\eta_5^2} \eta_5^2 \, d\eta_5 \, d\Omega_5$$

$$. \int e^{-2U\eta_3^2} \eta_3^2 \, d\eta_3 \, d\Omega_3 \int e^{-\eta_4^2\left(\frac{\alpha}{3} + \frac{\bar{\alpha}}{2}\right)} \eta_4^2 \, d\eta_4 \, d\Omega_4$$

$$\cdot \int [e^{-\eta_1^2 B_1'} \eta_1^2 Y_o^o \; e^{-\eta_2^2 X_2'} \cdot \frac{\eta_2^2 + \eta_1^2 - 2\eta_1 \cdot \eta_2}{c_2^2} \; Y_o^o$$

$$\cdot \eta_1^2 \, d\eta_1 \, d\Omega_1 \; \eta_2^2 \, d\eta_2 \, d\Omega_2 \,]$$

We know that:

$$\int [e^{-\eta_1^2 B_1'} \; e^{-\eta_2^2 X_2'} \; \eta_1^2 Y_o^o \cdot \frac{\eta_2^2 + \eta_1^2 - 2\eta_1 \cdot \eta_2}{c_2^2} Y_o^o \tag{A-35}$$

$$\cdot \eta_1^2 \, d\eta_1 \, d\Omega_1 \; \eta_2^2 \, d\eta_2 \, d\Omega_2]$$

$$= \int [e^{-\eta_1^2 B_1'} \; e^{-\eta_2^2 X_2'} \cdot \frac{1}{4\pi} \times 4\pi \times 4\pi \times \frac{\eta_1^2 + \eta_2^2}{c_2^2}$$

$$\cdot \eta_1^2 \; \eta_1^2 \, d\eta_1 \; \eta_2^2 \, d\eta_2 \,]$$

$$= \frac{4\pi}{c_2^2} \cdot \int [e^{-\eta_1^2 B_1'} \; e^{-\eta_2^2 X_2'} \cdot \eta_1^2 \, (\eta_1^2 + \eta_2^2)$$

$$\cdot \quad \eta_1^2 \, d\eta_1 \; \eta_2^2 \, d\eta_2]$$

$$= \frac{4\pi}{c_2^2} \cdot \Big[\frac{3.1}{(2B_1')^2} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{B_1'}} \cdot \frac{3.1}{(2\,x_2')^2} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{x_2'}}$$

$$+ \frac{5.3.1}{(2B_1')^3} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{B_1'}} \cdot \frac{1}{2\,x_2'} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{x_2'}} \Big]$$

(A-36) $$R_{22} = \left| \frac{16}{9c_2} \right|^3 \cdot \left(\frac{1}{\alpha} \cdot \frac{1}{2} \cdot \sqrt{\frac{2\pi}{\alpha}} \cdot 4\pi \right)$$

$$\cdot \left(\frac{3}{4\alpha} \cdot \frac{1}{2} \cdot \sqrt{\frac{3\pi}{2\alpha}} \cdot 4\pi \right) \cdot \left(\frac{1}{2\,(\frac{\alpha}{3} + \frac{\bar{\alpha}}{2})} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{\frac{\alpha}{3} + \frac{\bar{\alpha}}{2}}} \cdot 4\pi \right)$$

$$\cdot \left(\frac{1}{4U} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{2U}} \cdot 4\pi \right) \cdot \frac{4\pi}{c_2^2} \cdot \Big[\Big\{ \frac{3.1}{(2B_1')^2} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{B_1'}}$$

$$\times \frac{3.1}{(2\,x_2')^2} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{x_2'}} + \frac{5.3.1.}{(2B_1')^3} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{B_1'}} \cdot \frac{1}{2\,x_2'} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{x_2'}} \Big\} \Big]$$

$$= 0.2033 \times 10^8 \quad (\mathrm{fm})^{22}$$

(A-37) Similarly R_{33} is :

$$R_{33} = \left| \frac{8}{3c_3} \right|^3 . \left(\frac{1}{\alpha} . \frac{1}{2} . \sqrt{\frac{2\pi}{\alpha}} . 4\pi \right)$$

$$. \left(\frac{1}{2(\alpha + \bar{\alpha})} . \frac{1}{2} . \sqrt{\frac{\pi}{\alpha + \bar{\alpha}}} . 4\pi \right) . \left(\frac{1}{2\left(\frac{\alpha + \bar{\alpha}}{4}\right)} . \frac{1}{2} . \sqrt{\frac{\pi}{\frac{\alpha + \bar{\alpha}}{4}}} . 4\pi \right)$$

$$. \left(\frac{1}{4U} . \frac{1}{2} . \sqrt{\frac{\pi}{2U}} . 4\pi \right) . \frac{4\pi}{c_3^2} . \left[\frac{3.1}{(2c'_{11})^2} . \frac{1}{2} . \sqrt{\frac{\pi}{c'_{11}}} \right.$$

$$\left. . \frac{3.1}{(2Y'_2)^2} . \frac{1}{2} . \sqrt{\frac{\pi}{Y'_2}} + \frac{5.3.1}{(2c'_{11})^3} . \frac{1}{2} . \sqrt{\frac{\pi}{c'_{11}}} . \frac{1}{2Y'_2} . \frac{1}{2} . \sqrt{\frac{\pi}{Y'_2}} \right]$$

$$R_{33} = 0.1209 \times 10^8 \ (\text{fm})^{22}.$$

From (A-1)

$$I_1 = \frac{N_i N_f}{36} < (1234;56)_f - (1236;54)_f - (1254;36)_f + (1256;34)_f$$

$$|Q_{22}| \ (2 - 2)_i >$$

N_i and N_f are known $N_i^2 = \dfrac{36}{0.1841 \times 10^8}$, $(\text{fm})^{-22}$

$$N_f^2 = \frac{36}{0.3926 \times 10^8} \quad (\text{fm})^{-22}$$

(A-38) $$\langle (1234;56)_f \mid Q_{2K} \mid (2K)_i \rangle$$

$$= \left|\frac{1}{c_1}\right|^3 \int [e^{-\frac{\alpha}{2}\left(\frac{\rho 6^2}{2} + \frac{2}{3}\rho_5^2 + \frac{3}{4}\rho_4^2\right)} e^{-\frac{\bar{\alpha}}{4}\rho_3^2} e^{-A_{11}\rho_1^2}$$

$$e^{-A_{12}\rho_2^2} \rho_1^2 Y_0^0 \cdot Q_{2K} \; e^{-\frac{\alpha}{2}\left(\frac{\rho 6^2}{2} + \frac{2}{3}\rho_5^2 + \frac{3}{4}\rho_4^2\right)}$$

$$e^{-\frac{\bar{\alpha}}{4}\rho_3^2} e^{-A'_{11}\rho_1^2} e^{-A'_{12}\rho_2^2} \rho_1^2 Y_2^K(\Omega_1)\, d\vec{\rho}_1 \text{------} d\vec{\rho}_6]$$

where

$$A_{11} = \frac{2\beta}{3} + \frac{U}{c_1^2} - \frac{Q}{c_1}, \quad A_{12} = \frac{U}{c_1^2}$$

$$A'_{11} = \frac{2\beta'}{3} + \frac{U}{c_1^2} - \frac{Q}{c_1}, \quad A'_{12} = \frac{U}{c_1^2}$$

$$Q_{2K} = r_2^{-2}\, Y_2^{K*}(\Omega_2) + r_4^{-2}\, Y_2^{*K}(\Omega_4) + r_6^{-2}\, \overset{*}{Y}{}_2^{K}(\Omega_6),$$

$$\vec{\bar{r}}_2 = \vec{r}_2 - \vec{R}_{cm} = \frac{\vec{\rho}_1}{3} - \frac{\vec{\rho}_4}{4} - \frac{\vec{\rho}_5}{3} - \frac{\vec{\rho}_6}{2},$$

$$\bar{\vec{r}}_4 = \vec{r}_4 - \vec{R}_{cm} = \frac{\vec{\rho}_1}{3} + \frac{3\vec{\rho}_4}{4} ,$$

and

$$\bar{\vec{r}}_6 = \vec{r}_6 - \vec{R}_{cm} = - \frac{2\vec{\rho}_1}{3} - \frac{\vec{\rho}_3}{2}$$

since we know that $\int Y_\ell^{*M} (\Omega_i) \, Y_\ell^{M} (\Omega j) \, d\Omega_i \, d\Omega_j$

$= 0$, we can omit terms containing $\vec{\rho}_2$,

$\vec{\rho}_3$, $\vec{\rho}_4$, $\vec{\rho}_5$ and $\vec{\rho}_6$ in $\bar{\vec{r}}_2$, $\bar{\vec{r}}_4$ and $\bar{\vec{r}}_6$.

(A-39) $$r_2^{-2} \, Y_2^{K*} \, (\Omega_2) + \bar{r}_4^2 \, Y_2^{*K} \, (\Omega_4) + \bar{r}_6^2 \, Y_2^{*K} \, (\Omega_6)$$

$$= \frac{\rho_1^2}{9} \, Y_2^{*K}(\Omega_1) + \frac{\rho_1^2}{9} \, Y_2^{*K} \, (\Omega_1) + \frac{4\rho_1^2}{9} \, Y_2^{*K} \, (\Omega_1)$$

$$= \frac{6\rho_1^2}{9} \, Y_2^{*K} \, (\Omega_1)$$

(A-40) $<(1234;56)_f \quad |Q_{2K}| \; (2K)_i> \quad = \quad VI$

$$= \left|\frac{1}{c_1}\right|^3 \int [\; e^{-\alpha(\frac{\rho_6^2}{2} + \frac{2}{3}\rho_5^2 + \frac{3}{4}\rho_4^2)} \; e^{-\bar{\alpha}/2 \; \rho_3^2} \; e^{-(A_{11} + A'_{11}) \; \rho_1^2}$$

$$\cdot \; e^{-(A_{12} + A'_{12}) \; \rho_2^2} \rho_1^2 \; Y_0^0 \; \cdot \; \frac{6\rho_1^2}{9} \; Y_2^{*K} \; (\Omega_1) \rho_1^2 Y_2{}^K \; (\Omega_1)$$

$$d\vec{\rho}_1 \; \text{----------} \; d\vec{\rho}_6] =$$

$$= \left| \frac{1}{c_1} \right|^3 \int e^{-\alpha \frac{\rho_6^2}{2}} \rho_6^2 \; d\rho_6 \; d\Omega_6 \int e^{-\frac{2\alpha}{3}\rho_5^2} \rho_5^2 \; d\rho_5 \; d\Omega_5$$

$$\cdot \int e^{-3\alpha/4 \; \rho_4^2} \rho_4^2 \; d\rho_4 \; d\Omega_4 \quad \int e^{-\bar{\alpha}/2\rho_3^2} \rho_3^2 \; d\rho_3 \; d\Omega_3$$

$$\int e^{-(A_{11} + A'_{11}) \; \rho_1^2} \; \cdot \frac{1}{\sqrt{4\pi}} \cdot \; \frac{6}{9} \; \cdot \rho_1^6 \; Y_2^{*K} \; (\Omega_1) \; Y_2^K \; (\Omega_1)$$

$$\cdot \; \rho_1^2 \; d\rho_1 \quad d\Omega_1 \; \cdot \; \int e^{-(A_{12} + A'_{12}) \; \rho_2^2} \rho_2^2 \; d\rho_2 \; \cdot \; 4\pi$$

$$= \left|\frac{1}{c_1}\right|^3 \cdot \left(\frac{1}{\alpha} \cdot \frac{1}{2} \cdot \sqrt{\frac{2\pi}{\alpha}} \cdot 4\pi\right) \cdot \left(\frac{3}{4\alpha} \cdot \frac{1}{2} \cdot \sqrt{\frac{3\pi}{2\alpha}} \cdot 4\pi\right)$$

$$\times \left(\frac{2}{3\alpha} \times \frac{1}{2} \times \sqrt{\frac{4\pi}{3\alpha}} \times 4\pi\right) \times \left(\frac{1}{\alpha} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\bar{\alpha}}} \times 4\pi\right)$$

$$\cdot \frac{1}{\sqrt{4\pi}} \cdot \frac{6}{9} \cdot \frac{7.5.3.1}{\{2\,(A_{11}+A'_{11})\}^4} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{A_{11}+A'_{11}}} \cdot 4\pi$$

$$\cdot \frac{1}{2(A_{12}+A'_{12})} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{(A_{12}+A'_{12})}} = 0.8244 \text{ X } 10^8 \text{ (fm)}^{24}$$

We find that the value of this integral is independent of K.

(A-41) $U2 = \langle (1254;36)_f \mid Q_{2K} \mid (2K)_i \rangle$

$$= \left|\frac{16}{9c_2}\right|^3 \cdot \int e^{-\frac{\alpha}{2}\eta_6^2}\, \eta_6^2\, d\eta_6\, d\Omega_6 \int e^{-2\alpha/3\eta_5^2}\, \eta_5^2\, d\eta_5\, d\Omega_5$$

$$\cdot \int e^{-2U\eta_3^2} \eta_3^2 \, d\eta_3 \, d\Omega_3 \int e^{-\eta_4^2 \left(\frac{\bar{\alpha}}{2} + \alpha/3 \right)} \eta_4^2 \, d\eta_4 \, d\Omega_4$$

$$\cdot \int e^{-\eta_1^2 \beta_1} \; e^{-\eta_2^2 X \; 2} \cdot \frac{\eta_2^2 + \eta_1^2 - 2\eta_1 \cdot \eta_2}{c_2^2} \; Y_0^0 \cdot Q_{2K} \cdot \eta_1^2 Y_2^K \, d\vec{\eta}_1 \, d\vec{\eta}_2$$

where

$$B_1 = \frac{10\alpha}{27} + \frac{\alpha}{3} \; d_1^2 + \frac{2\alpha}{9} \; d_1 + \frac{4\bar{\alpha}}{9} - \frac{2d_1}{3} \; \bar{\alpha} + 2Ue_1^2 + 2U$$

$$+ \; 4Ue_1 + \frac{2\beta'}{3} + \frac{B2}{c_2^2} - \frac{B3}{c_2} - Qe_1 - Q + \frac{\bar{\alpha}}{2} \; d_1^2 \; ,$$

$$X_2 = B_2/c_2^2 = \frac{1}{c_2^2} \cdot \left[\frac{34\alpha}{27} + \frac{\alpha}{3} \; d_2^2 + \frac{10\alpha}{9} d_2 + \frac{4\bar{\alpha}}{9} \right.$$

$$\left. + \; \frac{2d_2}{3} \; \bar{\alpha} + \frac{2\beta}{3} + 2Ue_2^2 - Qe_2 + \frac{\bar{\alpha}}{2} \; d_2^2 \right] \; ,$$

$$B_3 = \frac{2\alpha}{3} d_1 \, d_2 - \frac{8\alpha}{27} + \frac{10\alpha}{9} \, d_1 + \frac{2\alpha}{9} d_2 - \frac{8\bar{\alpha}}{9} - \frac{2\bar{\alpha}}{3} \, d_2$$

$$+ \frac{2\bar{\alpha}}{3} \, d_1 + 4Ue_1e_2 + 4Ue_2 - Qe_1 - Q - Qe_2 + d_1 \, d_2 \, \bar{\alpha}$$

and $c_2 = \dfrac{2B_2}{B_3}$.

(A-42) $$U2 = \left| \frac{16}{9c_2} \right|^3 \cdot \left(\frac{1}{\alpha} \cdot \frac{1}{2} \cdot \sqrt{\frac{2\pi}{\alpha}} \cdot 4\pi\right) \cdot \left(\frac{3}{4\alpha} \cdot \frac{1}{2} \cdot \sqrt{\frac{3\pi}{2\alpha}} \cdot 4\pi\right)$$

$$\cdot \left(\frac{1}{2(\frac{\alpha}{3} + \frac{\bar{\alpha}}{2})} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{\frac{\alpha}{3} + \frac{\bar{\alpha}}{2}}} \cdot 4\pi\right) \cdot \left(\frac{1}{4U} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{2U}} \cdot 4\pi\right)$$

$$\cdot \int [e^{-\eta_1^2 B'_{11}} \, e^{-\eta_2^2 B'_{12}} \cdot \frac{\eta_2^2 + \eta_1^2 - 2\eta_1 \cdot \eta_2}{c_2^2} \, Y_0^0 \cdot Q_{2K}$$

$$\cdot \quad \eta_1^2 \, Y_2^{K} \, (\Omega_1) \quad d\vec{\eta}_1 \quad d\vec{\eta}_2 \,],$$

where

$$Q_{2K} = r_2^{-2}\ Y_2^{K*}\ (\Omega_2) + r_4^{-2}\ Y_2^{*K}\ (\Omega_4) + r_6^{-2}\ Y_2^{*K}\ (\Omega_6)\ ,$$

$$\vec{\bar{r}}_2 = \frac{2\vec{\rho}_1}{9} + \frac{4\vec{\rho}_2}{9} - \frac{\vec{\rho}_4}{6} - \frac{\vec{\rho}_5}{3} + \frac{\vec{\rho}_6}{2}\ ,$$

$$\vec{\bar{r}}_4 = \frac{2\vec{\rho}_1}{9} + \frac{4\vec{\rho}_2}{9} - \frac{\vec{\rho}_4}{6} + \frac{2\vec{\rho}_5}{3}\ ,$$

$$\vec{\bar{r}}_6 = -\frac{2\vec{\rho}_1}{3} - \frac{\vec{\rho}_4}{2},$$

$$\vec{\bar{r}}_2 = \frac{2\vec{\eta}_1}{9} + \frac{4}{9}\ \left(\frac{\vec{\eta}_2 - \vec{\eta}_1}{c_2} \right) - \frac{1}{6}\ (\vec{\eta}_4 - d_1\ \vec{\eta}_1 - d_2\ (\frac{\vec{\eta}_2 - \vec{\eta}_1}{c_2})) - \frac{\vec{\eta}_5}{3} + \frac{\vec{\eta}_6}{2}$$

$$= \vec{\eta}_1\ \left(\frac{2}{9} - \frac{4}{9c_2} + \frac{d_1}{6} - \frac{d_2}{6c_2} \right) + \vec{\eta}_2\ \left(\frac{4}{9c_2} + \frac{d_2}{6c_2} \right) - \frac{\vec{\eta}_4}{6} - \frac{\vec{\eta}_5}{3} + \frac{\vec{\eta}_6}{2}\ ,$$

$$\vec{\bar{r}}_4 = \vec{\eta}_1\ \left(\frac{2}{9} - \frac{4}{9c_2} + \frac{d_1}{6} - \frac{d_2}{6c_2} \right) + \vec{\eta}_2\ \left(\frac{4}{9c_2} + \frac{d_2}{6c_2} \right) \frac{-\vec{\eta}_4}{6} + \frac{2\vec{\eta}_5}{3}\ ,$$

$$\vec{\bar{r}}_6 = -\frac{2}{3}\ \vec{\eta}_1 - \frac{1}{2}\ (\ \vec{\eta}_4 - d_1\ \vec{\eta}_1 - d_2\ (\ \frac{\vec{\eta}_2 - \vec{\eta}_1}{c_2})\ )$$

$$= \vec{\eta}_1 \left(- \frac{2}{3} + \frac{d_1}{2} - \frac{d_2}{2c_2}\right) + \vec{\eta}_2 \frac{d_2}{2c_2} - \frac{\vec{\eta}_4}{2} ,$$

(A-43) Let $E_{11} = \frac{2}{9} - \frac{4}{9c_2} + \frac{d_1}{6} - \frac{d_2}{6c_2}$,

$$E_{12} = \frac{4}{9c_2} + \frac{d_2}{6c_2} ,$$

and $E_{13} = \frac{-2}{3} + \frac{d_1}{2} - \frac{d_2}{2c_2}$

(A-44)

$$\vec{\bar{r}}_2 = E_{11} \vec{\eta}_1 + E_{12} \vec{\eta}_2 \quad \frac{\vec{\eta}_4}{6} - \frac{\vec{\eta}_5}{3} + \frac{\vec{\eta}_6}{2} ,$$

$$\vec{\bar{r}}_4 = E_{11} \vec{\eta}_1 + E_{12} \vec{\eta}_2 - \frac{\vec{\eta}_4}{6} + \frac{2\vec{\eta}_5}{3} ,$$

$$\vec{\bar{r}}_6 = E_{13} \vec{\eta}_1 + \vec{\eta}_2 \frac{d_2}{2c_2} - \frac{\vec{\eta}_4}{2} .$$

Since $\int Y_\ell^{*M} (\Omega_i) \, Y_\ell^M (\Omega_j) \, d\Omega_i \, d\Omega_j = 0$

we can just write:

(A-45) $$Q_{2K} = E_{11}^2 \eta_1^2 \overset{*}{Y}{}_2^K (\Omega_1) + E_{11}^2 \eta_1^2 \overset{*}{Y}{}_2^K (\Omega_1) + E_{13}^2 \eta_1^2 Y_2^{*K} (\Omega_1)$$

$$= (E_{11}^{2} + E_{11}^{2} + E_{13}^{2}\, \eta_1^{2}\, \overset{*}{Y}{}_2^{K}\, (\Omega_1)$$

(A-46) $$U2 = \left| \frac{16}{9c_2} \right|^3 . \left(\frac{1}{\alpha} . \frac{1}{2} . \sqrt{\frac{2\pi}{\alpha}} . 4\pi\right) . \left(\frac{3}{4\alpha} . \frac{1}{2} . \sqrt{\frac{3\pi}{2\alpha}} . 4\pi\right)$$

$$. \left(\frac{1}{2(\frac{\alpha}{3} + \frac{\bar{\alpha}}{2})} . \frac{1}{2} . \sqrt{\frac{\pi}{\frac{\alpha}{3} + \frac{\bar{\alpha}}{2}}} . 4\pi\right) . \left(\frac{1}{4U} . \frac{1}{2} . \sqrt{\frac{\pi}{2U}} . 4\pi\right)$$

$$: \int \Big[e^{-\eta_1^2 B'_{11}} \quad e^{-\eta_2^2 B'_{12}} \quad . \frac{\eta_2^2 + \eta_1^2 - 2\eta_1 . \eta_2}{c_2^2} \quad . \frac{1}{\sqrt{4\pi}}$$

$$. (E_{11}^{2} + E_{11}^{2} + E_{13}^{2}\,)\, \eta_1^{2}\, \overset{*}{Y}{}_2^{K}\, (\Omega_1) \;.\; \eta_1^{2}\, Y_2^{K}(\Omega_1)\; \eta_1^2 d\eta_1 d\Omega_1 \eta_2^2 d\eta_2 d\Omega_2 \Big]$$

we can write:

(A-47) $$\int \Big[e^{-\eta_1^2 B'_{11}}\, e^{-\eta_2^2 B'_{12}} \; . \frac{\eta_2^2 + \eta_1^2 - 2\eta_1 . \eta_2}{c_2^2} \; . \frac{1}{\sqrt{4\pi}} \; . (E_{11}^{2} + E_{11}^{2} + E_{13}^{2})$$

$$. \,\eta_1^4\, Y_2^{*K}\, (\Omega_1)\; Y_2^{K}(\Omega_1)\; \eta_1^2 d\eta_1 \; d\Omega_1 \; \eta_2^2 d\eta_2 d\Omega_2 \Big]$$

$$= \int [e^{-\eta_1^2 B'_{11}} \quad e^{-\eta_2^2 B'_{12}} \cdot \frac{1}{\sqrt{4\pi}} \cdot \frac{\eta_2^2 + \eta_1^2}{c_2^2} \cdot (2E_{11}^2 + E_{13}^2)\ \eta_1^4$$

$$\cdot\ \eta_1^2\ d\eta_1\ \eta_2^2\ d\eta_2 d\Omega_2$$

$$= \int [e^{-\eta_1^2 B'_{11}} \quad e^{-\eta_2^2 B'_{12}} \cdot \frac{1}{\sqrt{4\pi}} \cdot \frac{\eta_2^2 + \eta_1^2}{c_2{}^2} \quad \cdot (2E_{11}^2 + E_{13}^2)$$

$$\cdot \quad 4\pi \quad \cdot\ \eta_1^6\ d\eta_1\ \eta_2^2\ d\eta_2]$$

$$= \int [e^{-\eta_1^2 B'_{11}} \quad e^{-\eta_2^2 B'_{12}} \quad \cdot \frac{4\pi}{\sqrt{4\pi}} \cdot \frac{(2E_{11}^2 + E_{13}^2)}{c_2^2} \quad \cdot \quad \{$$

$$\eta_2^4\ \eta_1^6\ d\eta_1\ d\eta_2 + \eta_1^8\ \eta_2^2\ d\eta_1\ d\eta_2\}\ ]$$

$$= \sqrt{4\pi} \ \cdot \ \frac{(2E_{11}^2 + E_{13}^2)}{c_2^2} \cdot [\int \{e^{-\eta_1^2\ B'_{11}}\ e^{\eta_2^2\ B'_{12}}\ \ \eta_2^4\ \eta_1^6$$

$$d\eta_1\ d\eta_2\ \} + \int e^{-\eta_1^2 B'_{11}} \quad e^{-\eta_2^2 B'_{12}}\ \eta_1^8\ \eta_2^2\ d\eta_1\ d\eta_2]$$

$$= \sqrt{4\pi} \cdot \left(\frac{2E_{11}^{2} + E_{13}^{2}}{c_2^{2}}\right). \left[\frac{5.3.1}{(2B'_{11})^{3}} \cdot \frac{1}{2} . \sqrt{\frac{\pi}{B'_{11}}} \cdot \frac{3.1}{(2B'_{12})^{2}}\right.$$

$$\left. . \frac{1}{2} . \sqrt{\frac{\pi}{B'_{12}}} + \frac{7.5.3.1}{(2B'_{11})^{4}} . \frac{1}{2} . \sqrt{\frac{\pi}{B'_{11}}} . \frac{1}{2B'_{12}} . \frac{1}{2} . \sqrt{\frac{\pi}{B'_{12}}} \right]$$

$$= \quad 0.1790 \text{ X } 10^{8} \ (\text{fm})^{24}$$

(A-48) $U3 = \langle (1236;54)_f \mid Q_{2K} \mid (2K)_i \rangle$

$$= \left|\frac{16}{9c_2}\right|^{3} . \int e^{-\frac{\alpha}{2}\eta_6^2} \eta_6^2 \, d\eta_6 \, d\Omega_6 \int e^{-\frac{2\alpha}{3}\eta_5^2} \eta_5^2 \, d\eta_5 \, d\Omega_5$$

$$. \int e^{-2U\eta_3^2} \eta_3^2 \, d\eta_3 \, d\Omega_3 \int e^{-\eta_4^2\left(\frac{\bar{\alpha}}{2} + \frac{\alpha}{3}\right)} \eta_4^2 \, d\eta_4 \, d\Omega_4$$

$$. \left[\int e^{-\eta_1^2 B'_{11}} \; e^{-\eta_2^2 B'_{12}} . \frac{\eta_2^2 + \eta_1^2 - 2\eta_1 . \eta_2}{c_2^2} . \frac{1}{\sqrt{4\pi}} . Q_{2K}\right.$$

$$\left. . \ \eta_1^2 Y_2^K (\Omega_1) \ d\vec{\eta}_1 \ d\vec{\eta}_2 \right]$$

where $Q_{2K} = r_2^{-2} \overset{*}{Y}{}_2^K (\Omega_2) + r_4^{-2} \overset{*}{Y}{}_2^K (\Omega_4) + r_6^{-2} \overset{*}{Y}{}_2^K (\Omega_6)$,

$$\vec{\bar{r}}_2 = \frac{2\vec{\rho}_1}{9} + \frac{4\vec{\rho}_2}{9} + \frac{\vec{\rho}_4}{6} - \frac{\vec{\rho}_5}{3} - \frac{\vec{\rho}_6}{2},$$

$$\vec{\bar{r}}_4 = \frac{2\vec{\rho}_1}{3} - \frac{4\vec{\rho}_2}{3} - \frac{\vec{\rho}_4}{2},$$

and $$\vec{\bar{r}}_6 = \frac{-2\vec{\rho}_1}{3} - \frac{\vec{\rho}_4}{2},$$

(A-49) $$\vec{\bar{r}}_2 = \frac{2\vec{\eta}_1}{9} + \frac{4}{9}\left(\frac{\vec{\eta}_2 - \vec{\eta}_1}{c_2}\right) + \frac{1}{6}\left(\vec{\eta}_4 - d_1\vec{\eta}_1 - d_2\left(\frac{\vec{\eta}_2 - \vec{\eta}_1}{c_2}\right)\right) - \frac{\vec{\eta}_5}{3} - \frac{\vec{\eta}_6}{2}$$

$$= \vec{\eta}_1\left(\frac{2}{9} - \frac{4}{9c_2} - \frac{d_1}{6} + \frac{d_2}{6c_2}\right) + \vec{\eta}_2\left(\frac{4}{9c_2} - \frac{d_2}{6c_2}\right) + \frac{\vec{\eta}_4}{6} - \frac{\vec{\eta}_5}{3} - \frac{\vec{\eta}_6}{2},$$

$$\vec{\bar{r}}_4 = \frac{2\vec{\eta}_1}{3} - \frac{4}{3}\left(\frac{\vec{\eta}_2 - \vec{\eta}_1}{c_2}\right) - \frac{1}{2}\left(\vec{\eta}_4 - d_1\vec{\eta}_1 - d_2\left(\frac{\vec{\eta}_2 - \vec{\eta}_1}{c_2}\right)\right)$$

$$= \vec{\eta}_1\left(\frac{2}{3} + \frac{4}{3c_2} + \frac{d_1}{2} - \frac{d_2}{2c_2}\right) + \vec{\eta}_2\left(\frac{-4}{3c_2} + \frac{d_2}{2c_2}\right) - \frac{\vec{\eta}_4}{2},$$

and $$\vec{\bar{r}}_6 = \frac{-2\vec{\eta}_1}{3} - \frac{1}{2}\left(\vec{\eta}_4 - d_1\vec{\eta}_1 - d_2\left(\frac{\vec{\eta}_2 - \vec{\eta}_1}{c_2}\right)\right)$$

$$= \vec{\eta}_1\left(-\frac{2}{3} + \frac{d_1}{2} - \frac{d_2}{2c_2}\right) + \vec{\eta}_2\frac{d_2}{2c_2} - \frac{\vec{\eta}_4}{2}$$

(A-50) $$\vec{\bar{r}}_2 = E_{11}\vec{\eta}_1 + E'_{12}\vec{\eta}_2 + \frac{\vec{\eta}_4}{6} - \frac{\vec{\eta}_5}{3} - \frac{\vec{\eta}_6}{2},$$

$$\vec{\bar{r}}_4 = E'_{14}\vec{\eta}_1 + E'_{15}\vec{\eta}_2 - \frac{\vec{\eta}_4}{2},$$

and $$\vec{\bar{r}}_6 = E'_{16}\vec{\eta}_1 + \frac{d_2}{2c_2}\vec{\eta}_2 - \frac{\vec{\eta}_4}{2}$$

Since $\int Y_\ell^{*M}(\Omega_i)\, Y_\ell^{M}(\Omega_j)\, d\Omega_i\, d\Omega_j = 0$ we have

(A-51) $$Q_{2K} = (E'^{2}_{11} + E'^{2}_{14} + E'^{2}_{16})\, \eta_1^2\, Y_2^{*K}(\Omega_i)$$

Therefore U3 becomes:

(A-52) $$U3 = \left|\frac{16}{9c_2}\right|^3 \cdot \left(\frac{1}{\alpha} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\alpha}} \times 4\pi\right)$$

$$\times \left(\frac{3}{4\alpha} \times \frac{1}{2} \times \sqrt{\frac{3\pi}{2\alpha}} \times 4\pi\right) \times \left(\frac{1}{2(\frac{\alpha}{3} + \frac{\bar{\alpha}}{2})} \times \frac{1}{2} \times \sqrt{\frac{\pi}{\frac{\alpha}{3} + \frac{\bar{\alpha}}{2}}} \times 4\pi\right)$$

$$\times \left(\frac{1}{4U} \times \frac{1}{2} \times \sqrt{\frac{\pi}{2U}} \times 4\pi\right) \times \sqrt{4\pi} \times \frac{(E_{11}^{'2} + E_{14}^{'2} + E_{16}^{'2})}{c_2^2}$$

$$\times \left[\frac{5.3.1}{(2B_{11}^{'})^3} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_{11}^{'}}} \times \frac{3.1}{(2B_{12}^{'})^2} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_{12}^{'}}}\right.$$

$$\left. + \frac{7.5.3.1}{(2B_{11}^{'})^4} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_{11}^{'}}} \times \frac{1}{2B_{12}^{'}} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_{12}^{'}}} \right]$$

$$= 0.1566 \text{ X } 10^8 \text{ (fm)}^{24}$$

(A-53) $U4 = \langle(1256;34)_f \; |Q_{2K}| \; (2K)_i\rangle$

$$= \left|\frac{8}{3c_3}\right|^3 \int e^{-\frac{\alpha}{2}\eta_6^2} \eta_6^2 \, d\eta_6 \, d\Omega_6 \int e^{-\eta_5^2(\alpha + \bar{\alpha})} \eta_5^2 \, d\eta_5 d\Omega_5$$

$$\int e^{-\eta_4^2 \left(\frac{\alpha + \bar{\alpha}}{4} \right)} \eta_4^2 \, d\eta_4 \, d\Omega_4 \quad \int e^{-\eta_3^2 \, 2U} \eta_3^2 \, d\eta_3 \, d\Omega_3$$

$$\times \Big[\int e^{-c'_{11} \eta_1^2} \, e^{-\gamma'_2 \eta_2^2} \eta_1^2 \, Y_2^K (\Omega_1) \times \frac{1}{\sqrt{4\pi}}$$

$$\times \frac{\eta_2^2 + \eta_1^2 - 2\eta_1 \cdot \eta_2}{c_3^2} \times Q_{2K} \times \eta_1^2 \, d\eta_1 \, d\Omega_1 \, \eta_2^2 \, d\eta_2 \, d\Omega_2 \Big]$$

where

$$Q_{2K} = \bar{r}_2^{2} \, Y_2^{*K} (\Omega_2) + \bar{r}_4^2 \, Y_2^{*K} (\Omega_4) + \bar{r}_6^2 \, Y_2^{*K} (\Omega_6) \; ,$$

$$\vec{\bar{r}}_2 = \frac{2}{3} (\vec{\rho}_1 + \vec{\rho}_2) - \frac{\vec{\rho}_6}{2} \, , \quad \vec{\bar{r}}_4 = \frac{2}{3} (\vec{\rho}_1 + \vec{\rho}_2) + \vec{\rho}_5 ,$$

and $\vec{\bar{r}}_6 = - \frac{2\vec{\rho}_1}{3} - \frac{\vec{\rho}_4}{2}$

(A-54) $$\vec{\bar{r}}_2 = \frac{2\vec{\eta}_1}{3} + \frac{2}{3} \left(\frac{\vec{\eta}_2 - \vec{\eta}_1}{c_3} \right) - \frac{\vec{\eta}_6}{2} \, , \quad \vec{\bar{r}}_4 = \frac{2\vec{\eta}_1}{3} + \frac{2}{3} \left(\frac{\vec{\eta}_2 - \vec{\eta}_1}{c_3} \right) + \vec{\eta}_5$$

$$\vec{\bar{r}}_6 = - \frac{2\vec{\rho}_1}{3} - \frac{\vec{\rho}_4}{2}$$

and $$\frac{\vec{\rho}_4}{2} = \frac{\vec{\eta}_4 - d_1 \eta_1 - d_2 \left(\frac{\vec{\eta}_2 - \vec{\eta}_1}{c_3} \right)}{2}$$

(A-55) $$\vec{\bar{r}}_2 = D_{11} \vec{n}_1 + \frac{2}{3c_3} \vec{n}_2 - \frac{\vec{n}_6}{2} ,$$

$$\vec{\bar{r}}_4 = D_{11}' \vec{n}_1 + \frac{2}{3c_3} \vec{n}_2 + \vec{n}_5 ,$$

and $$\vec{\bar{r}}_6 = D_{12} \vec{n}_1 + \frac{d_2}{2c_3} n_2 - \frac{\vec{n}_4}{2}$$

(A-56) $$\bar{r}_2^2 \, Y_2^{*K} (\Omega_2) + \bar{r}_4^2 \, Y_2^{*K} (\Omega_4) + \bar{r}_6^2 \, Y_2^{*K} (\Omega_6)$$

$$= (D_{11}^2 + D_{11}'^2 + D_{12}^2) \, n_1^2 \, Y_2^{*K} (\Omega_1)$$ because

$$\int Y_\ell^{M*} (\Omega_i) \; Y_\ell^M (\Omega_j) \; d\Omega_i \, d\Omega_j = 0$$

(A-57) $$U4 = \left|\frac{8}{3c_3}\right|^3 \times \left(\frac{1}{\alpha} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\alpha}} \times 4\pi\right) \times \left(\frac{1}{2(\alpha + \bar{\alpha})} \times \frac{1}{2} \times \sqrt{\frac{\pi}{\alpha + \bar{\alpha}}} \times 4\pi\right)$$

$$\times \left(\frac{1}{2 \left(\frac{\alpha + \bar{\alpha}}{4}\right)} \times \frac{1}{2} \times \sqrt{\frac{\pi}{\frac{\alpha + \bar{\alpha}}{4}}} \times 4\pi\right) \times \left(\frac{1}{4U} \times \frac{1}{2} \times \sqrt{\frac{\pi}{2U}} \times 4\pi\right)$$

$$\times \sqrt{4\pi} \quad \times \quad \frac{(D'^{2}_{11} + D^{2}_{11} + D^{2}_{12})}{c_3^2} \times \left[\frac{5.3.1}{(2c'_{11})^3} \times \frac{1}{2} \times \sqrt{\frac{\pi}{c'_{11}}} \right.$$

$$\left. \times \quad \frac{3.1}{(2Y'_2)^2} \quad \times \frac{1}{2} \times \sqrt{\frac{\pi}{Y'_2}} + \frac{7.5.3.1}{(2c'_{11})^4} \times \frac{1}{2} \times \sqrt{\frac{\pi}{c'_{11}}} \times \frac{1}{2Y'_2} \times \frac{1}{2} \times \sqrt{\frac{\pi}{Y'_2}} \right]$$

$$= \quad 0.0664 \times 10^8 \quad (\text{fm})^{24}$$

We see that U1, U2, U3 and U4 are independent of K. Therefore U1 -U2- U3 + U4 is independent of K. This only means that I_K is also independent of K.

(A-58) $$|I_1|^2 + |I_2|^2 + |I_3|^2 + |I_4|^2 + |I_5|^2 = 5|\,I_1|^2$$

(A-59) $$B(E2) = \frac{1}{5} \times \frac{N_f^{\,2} N_i^{\,2}}{(36)^2} \times 5\,|I_1|^2 = \frac{N_f^{\,2} N_i^{\,2}}{(36)^2} \times |I_1|^2$$

where

$$I_1 = \frac{N_f N_i}{36} \quad <(1234;56)_f - (1236;54)_f - (1254;36)_f + (1256;34)_f$$

$$|Q_{22}| \quad (2\text{-}2)_i>$$

A brief theory of the reduction of the Quadratic forms in the exponential in the various integrals to sums of square terms is given in (25).

APPENDIX B

METHOD OF CALCULATION FOR $<f|\,M_{11}|\,i>$

(B-1) $$< f|\,M_{11}|\,i > = -\mu_0 \times \sqrt{\frac{3}{8\pi}} \times \frac{N_f N_i}{36} \times \frac{g_{s1} - g_{s2}}{\sqrt{2}}$$

$$\times <\{(1234;56)_f - (1236;54)_f - (1254;36)_f + (5623;14)_f\}|\,(1234;56)_i>$$

(B-2) $$N_f^{\,2} \times <\{(1234;56)_f - 2(1254;36)_f + (3456;12)_f\}|\,(1234;56)_f> = 36$$

(B-3) $$N_i^{\,2} \times <\{(1234;56)_i - 2(1254;36)_i + (3456;12)_i\}|\,(1234;56)_i> = 36$$

The normalization constant of the 1^+ state has already been calculated in the previous appendix.

We shall calculate the normalization constant of the 0^+ state.

(B-4) $$< (1234;56)_i \;|(1234;56)_i> = \left(\frac{1}{c1}\right)^3 \int [e^{-\alpha} \left(\frac{\rho_6^{\,2}}{2} + \frac{2}{3}\rho_5^{\,2} + \frac{3}{4}\rho_4^{\,2}\right)$$

$$e^{-\bar{\alpha}'/2\rho_3^{\,2}} \; e^{-A_1 \rho_1^2} \; e^{-A_2 \rho_2^2}$$

$$\times \rho_1^{\,4} \; Y_0^{\,0} \; Y_0^{\,0} \; d\vec{\rho}_1 - \ldots\ldots\ldots d\vec{\rho}_6 \,]$$

$\bar{\alpha}'$ is different from $\bar{\alpha}$; this is the only parameter which is different from that of the 1^+ state. The other two parameters α and β are the same in the 0^+ and 1^+ states.

(B-5) $$A_1 = \frac{4\beta}{3} + \frac{U'}{c_1^2} + \frac{U'}{c_1^2} - \frac{Q'}{c_1} - \frac{Q'}{c_1} \ , \quad A_2 = \frac{U'}{c_1^2} + \frac{U'}{c_1^2}$$

and

$$c_1 = \frac{2(U' + U')}{(Q' + Q')} = \frac{2U'}{Q'}$$

(B-6) $$\langle (1234;56)_i \mid (1234;56)_i \rangle$$

$$= \left|\frac{1}{c_1}\right|^3 \times \left(\frac{1}{\alpha_1} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\alpha_1}} \times 4\pi\right) \times \left(\frac{3}{4\alpha_1} \times \frac{1}{2} \times \sqrt{\frac{3\pi}{2\alpha_1}} \times 4\pi\right)$$

$$\times \left(\frac{2}{3\alpha_1} \times \frac{1}{2} \times \sqrt{\frac{4\pi}{3\alpha_1}} \times 4\pi\right) \times \left(\frac{1}{\bar{\alpha}'} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\bar{\alpha}'}} \times 4\pi\right)$$

$$\times 4\pi \times \frac{5.3.1}{(2A_1)^3} \times \frac{1}{2} \times \sqrt{\frac{\pi}{A_1}} \times \frac{1}{2A_2} \times \frac{1}{2} \times \sqrt{\frac{\pi}{A_2}}$$

$$= 0.1346 \times 10^9 \ (\mathrm{fm})^{22}$$

$$(B\text{-}7) \qquad <(1254;36)_i \Big| (1234;56)_i >$$

$$= \left| \frac{16}{9c_2} \right|^3 \times \int e^{-\alpha_1/2\, n^2}\; n_6^2\, dn_6\, d\Omega_6 \int e^{-2\alpha_1/3\, n_5^2}\; n_5^2\, dn_5\, d\Omega_5$$

$$\int e^{-2.U'\, n_3^2}\; n_3^2\, dn_3\, d\Omega_3 \int e^{-(\bar{\alpha}'/2 + \alpha_1/3)\, n_4^2}\; n_4^2\, dn_4\, d\Omega_4$$

$$\int e^{-n_1^2\, B_1'}\; n_1^2\, Y_0^0\; e^{-n_2^2\, X_2'} \times \left(\frac{n_2^2 + n_1^2 - 2\vec{n}_1 \cdot \vec{n}_2}{c_2^2}\right) Y_0^0$$

$$\times\; n_1^2\, dn_1\, d\Omega_1 \,.\, n_2^2\, dn_2\, d\Omega_2$$

$$= \frac{16}{|9c_2|^3} \times \left[\left(\frac{1}{\alpha_1} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\alpha_1}} \times 4\pi \right) \times \left(\frac{3}{4\alpha_1} \times \frac{1}{2} \times \sqrt{\frac{3\pi}{2\alpha_1}} \times 4\pi \right) \right.$$

$$\times \left(\frac{1}{2} \Big/ \left(\frac{\alpha_1}{3} + \frac{\bar{\alpha}'}{2} \right) \times \frac{1}{2} \times \sqrt{\frac{\pi}{\frac{\alpha_1}{3} + \frac{\bar{\alpha}'}{2}}} \times 4\pi \right)$$

$$\times \left(\frac{1}{4U'} \times \frac{1}{2} \sqrt{\frac{\pi}{2U'}} \times 4\pi \right)$$

$$\times \frac{4\pi}{c_2^2} \times \left[\frac{5.3.1}{(2B_1')^3} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_1'}} \times \frac{1}{2X_2'} \times \frac{1}{2} \times \sqrt{\frac{\pi}{X_2'}} \right.$$

$$\left. + \frac{3.1}{(2B_1')^2} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_1'}} \times \frac{3.1}{(2X_2')^2} \times \frac{1}{2} \times \sqrt{\frac{\pi}{X_2'}} \right]$$

where

$$B_1' = \frac{10\alpha_1}{27} + \frac{\alpha_1}{3} d_1^2 + \frac{2\alpha_1}{9} d_1 + \frac{4\bar{\alpha}'}{9} - \frac{2d_1}{3} \bar{\alpha}' + U' e_1$$

$$+ U' + 2U' e_1 + \frac{2\beta}{3} + U' e_1^2 + U' + 2U' e_1$$

$$+ \frac{B_2'}{c_2^2} - \frac{B_3'}{c_2} - Q' e_1 - Q' + \frac{\bar{\alpha}'}{2} d_1^2 \quad ,$$

$$B_2' = \frac{34\,\alpha_1}{27} + \frac{\alpha_1}{3} d_2^2 + \frac{10\,\alpha_1}{9} d_2 + \frac{4\bar{\alpha}'}{9} + \frac{2d_2}{3} \bar{\alpha}' + \frac{2\beta}{3}$$

$$+ U' e_2^2 - Q' e_2 + U' e_2^2 + \frac{\bar{\alpha}'}{2} d_2^2 \quad ,$$

$$B_3' = \frac{2\,\alpha_1}{3} d_1 d_2 - \frac{8\,\alpha_1}{27} + \frac{10\,\alpha_1}{9} d_1 + \frac{2\,\alpha_1}{9} d_2 - \frac{8\,\bar{\alpha}'}{9}$$

$$- \frac{2d_2}{3} \bar{\alpha}' + \frac{2d_1}{3} \bar{\alpha}' + 4U' e_1 e_2 + 4U' e_2 + d_1 d_2 \bar{\alpha}'$$

$$- Q' e_1 - Q' - Q' e_2 \quad ,$$

$$d_1 = \frac{\dfrac{2\bar{\alpha}'}{3} - \dfrac{2\alpha_1}{9}}{\dfrac{2\alpha_1}{3} + \bar{\alpha}'} \quad , \quad d_2 = -\left(\frac{\dfrac{2\bar{\alpha}'}{3} + \dfrac{10\alpha_1}{9}}{\bar{\alpha}' + \dfrac{2\,\alpha_1}{3}} \right) \quad ,$$

$$e_1 = \frac{Q' - 4U'}{4U'} \quad , \quad e_2 = \frac{Q'}{4U'} \quad ,$$

$$c_2 = \frac{2B'}{B_3'} \quad \text{and} \quad X_2' = \frac{B_2'}{c_2^2}$$

$$<(1254;36)_i \; | \; (1234;56)_i> \;=\; 0.6355 \times 10^8 \;(\mathrm{fm})^{22}$$

(B-8) $<(3456;12)_i \;|\; (1234;56)_i>$

$$= \left|\frac{8}{3c_3}\right|^3 \int e^{\alpha_1/2} \; n_6^2 \; n_6^2 \; dn_6 \, d\Omega_6 \int e^{-n_5^2 (\bar{\alpha}' + \alpha_1)} \; n_5^2 \; dn_5 \, d\Omega_5$$

$$\int e^{-n_4^2 (\alpha_1/6 + \bar{\alpha}'/4)} \; n_4^2 \; dn_4 \, d\Omega_4 \int e^{-n_3^2 \times 2U'} \; n_3^2 \; dn_3 \; d\Omega_3$$

$$\int e^{-n_1^2 c_{11}'} \; e^{-n_2^2 Y_2'} \; n_1^2 \, Y^0_0 \times \left(\frac{n_2^2 + n_1^2 - 2\vec{n}_1 \cdot \vec{n}_2}{c_3^2} \right) Y^0_0$$

$$\times \; d\vec{n}_1 \; d\vec{n}_2 \quad .$$

where

$$c_{11}' = \frac{8\alpha_1}{9} + \frac{4\bar{\alpha}'}{9} + \bar{\alpha}' d_1^2 - \frac{4d_1}{3} \bar{\alpha}' + 2U' e_1^2 + \frac{2\beta}{3}$$

$$+ 2U' + 4U' e_1 - Q'e_1 - Q' + \alpha_1 d_1^2 + \frac{2\alpha_1}{3} - \frac{4\alpha_1}{3} d_1$$

$$+ \frac{Y_2'}{c_3^2} - \left(\frac{\frac{16\alpha_1}{27} - \frac{O'^2}{4U'}}{c_3}\right) ,$$

$$Y_2' = \frac{c_{22}'}{c_3^2} = \frac{1}{c_3^2} \left[\frac{2\alpha_1}{9} + \frac{16\bar{\alpha}'}{9} + \bar{\alpha}' d_2^2 - \frac{8\bar{\alpha}'}{3} d_2 + \frac{2\beta}{3}\right.$$

$$\left. + 2U' e_2^2 - Q' e_2 + \alpha_1 d_2^2 + \frac{8\alpha_1}{3} - \frac{8\alpha_1}{3} d_2 \right]$$

and

$$c_3 = \frac{2c_{22}'}{\frac{16\alpha_1}{27} - \frac{(O')^2}{4U'}}$$

(B-9) $\langle (3456;12)_i \mid (1234;56)_i \rangle$

$$= \left| \frac{8}{3c_3} \right|^3 \times \left[\left(\frac{1}{\alpha_1} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\alpha_1}} \times 4\pi \right) \times \left(\frac{1}{2(\bar{\alpha}' + \alpha_1)} \times \frac{1}{2} \times \sqrt{\frac{\pi}{\alpha_1 + \bar{\alpha}'}} \times 4\pi \right) \right.$$

$$\times \left(\frac{1}{2\left(\frac{\alpha_1}{6} + \frac{\bar{\alpha}'}{4}\right)} \times \frac{1}{2} \times \sqrt{\frac{\pi}{\frac{\alpha_1}{6} + \frac{\bar{\alpha}'}{4}}} \times 4\pi \right) \times \left(\frac{1}{4U'} \times \frac{1}{2} \times \sqrt{\frac{\pi}{2U'}} \times 4\pi \right)$$

$$\times \frac{4\pi}{c_3^2} \times \left(\frac{5.3.1}{(2c_{11}')^3} \times \frac{1}{2} \times \sqrt{\frac{\pi}{c_{11}'}} \times \frac{1}{2Y_2'} \times \frac{1}{2} \times \sqrt{\frac{\pi}{Y_2'}} \right.$$

$$\left. + \frac{3.1}{(2c_{11})^2} \times \frac{1}{2} \times \sqrt{\frac{\pi}{c_{11}'}} \times \frac{3.1}{(2Y_2')^2} \times \frac{1}{2} \times \sqrt{\frac{\pi}{Y_2'}} \right)$$

$$\langle (3456;12)_i \mid (1234;56)_i \rangle = 0.2747 \times 10^8 \ (\text{fm})^{22}$$

$$N_{0^+}^2 = \frac{36}{0.3497 \times 10^8} \ (\text{fm})^{-22}$$

(B-10) $\langle (1234;56)_f \mid (1234;56)_i \rangle$

$$= \left| \frac{1}{c_1} \right|^3 \int \left[e^{-\frac{\alpha_1}{2}\left(\frac{\rho_6^2}{2} + \frac{2}{3}\rho_5^2 + \frac{3}{4}\rho_4^2\right)} \quad e^{-\frac{\bar{\alpha}}{4}\rho_3^2} \right.$$

$$e^{-\rho_1^2 \left(\frac{2\beta}{3} + \frac{U}{c_1^2} - \frac{Q}{c_1} \right)} \quad e^{-\rho_2^2 \frac{U}{c_1^2}} \quad \cdot \quad \rho_1^2 \; Y_0^0$$

$$\times \; e^{-\frac{\alpha_1}{2} \left(\frac{\rho_6^2}{2} + \frac{2}{3} \rho_5^2 + \frac{3}{4} \rho_4^2 \right)} \quad e^{-\frac{\bar{\alpha}'}{4} \rho_3^2} \quad e^{-\rho_1^2 \left(\frac{2\beta}{3} + \frac{U'}{c_1^2} - \frac{Q'}{c_1} \right)]}$$

$$e^{-\rho_2^2 \cdot \frac{U'}{c_1^2}} \quad \rho_1^2 \; Y_0^0 \quad \cdot \quad d\vec{\rho}_1 \text{---------} d\vec{\rho}_6$$

$$c_1 = \frac{2(U + U')}{Q + Q'}$$

$$Q = \frac{4}{3} \times (\alpha - \bar{\alpha}) \; ,$$

$$Q' = \frac{4}{3} \times (\alpha - \bar{\alpha}') \; ,$$

$$U = 2\alpha + \bar{\alpha} \; ,$$

and $U' = 2\alpha + \bar{\alpha}'$

(B-11) $\langle (1234;56)_f \mid (1234;56)_i \rangle$

$$= \left| \frac{1}{c_1} \right|^3 \int e^{-\alpha_1 \left(\frac{\rho_6^2}{2} + \frac{2}{3} \rho_5^2 + \frac{3}{4} \rho_4^2 \right)} \quad e^{-\frac{\bar{\alpha} + \bar{\alpha}'}{4} \rho_3^2}$$

$$e^{-\rho_1^2\left(\frac{4\beta}{3}+\frac{U+U'}{c_1^2}-\frac{Q+Q'}{c_1}\right)}\quad e^{-\rho_2^2\left(\frac{U}{c_1^2}+\frac{U'}{c_1^2}\right)}$$

$$\times \quad \rho_1^4 \; Y_0^0 \; Y_0^0 \; d\vec{\rho}_1 \; \text{----------} d\vec{\rho}_6$$

$$= \left|\frac{1}{c_1}\right|^3 \times \left[\left(\frac{1}{\alpha_1}\times\frac{1}{2}\times\sqrt{\frac{2\alpha}{\alpha_1}}\times 4\pi\right)\right.$$

$$\times\left(\frac{3}{4\,\alpha_1}\times\frac{1}{2}\times\sqrt{\frac{3\,\pi}{2\,\alpha_1}}\times 4\pi\right)\times\left(\frac{2}{3\alpha_1}\times\frac{1}{2}\times\sqrt{\frac{4\,\pi}{3\alpha_1}}\times 4\pi\right)$$

$$\times\left(\frac{2}{\bar{\alpha}+\bar{\alpha}'}\times\frac{1}{2}\times\sqrt{\frac{4\,\pi}{\bar{\alpha}+\bar{\alpha}'}}\times 4\,\pi\right)$$

$$\times\frac{5.3.1}{(A_{22})^3}\times\frac{1}{2}\times\sqrt{\frac{\pi}{A_{22}}}\times\frac{1}{4\pi}\times 4\pi\times\frac{1}{2A_{23}}\times\frac{1}{2}\times\sqrt{\frac{\pi}{A_{23}}}$$

$$\left.\times\; 4\,\pi\right] \;=\; 0.8757\times 10^{8}\;(\text{fm})^{22}$$

$$A_{22} = \frac{4\beta}{3}+\frac{U+U'}{c_1^2}-\frac{Q+Q'}{c_1}\,,\quad\text{and}$$

$$A_{23} = \frac{U+U'}{c_1^2}\;;$$

(B-12) $<(1254;36)_f \mid (1234;56)_i>$

$$= \left|\frac{16}{9c_2}\right|^3 \int e^{-\alpha_1/2\, n_6^2}\, n_6^2\, dn_6\, d\Omega_6 \int e^{-2\alpha_1/3\, n_5^2}\, n_5^2\, dn_5\, d\Omega_5$$

$$\int e^{-(U+U')\, n_3^2}\, n_3^2\, dn_3\, d\Omega_3 \int e^{-\left(\frac{\bar{\alpha}+\bar{\alpha}'}{4}\right) n_4^2 - \frac{\alpha_1}{3} n_4^2}\, n_4^2\, dn_4\, d\Omega_4$$

$$\int e^{-n_1^2 B_{22}}\, n_1^2\, Y_0^0\, e^{-n_2^2 X_{22}}\, \frac{(n_2^2 + n_1^2 - 2n_1.n_2)}{c_2^2}\, Y_0^0$$

$$d\vec{n}_1 \quad d\vec{n}_2 \ .$$

$$B_{22} = \frac{10\,\alpha_1}{27} + \frac{d_1^{\,2}\alpha_1}{3} + \frac{2d_1\alpha_1}{9} + \frac{4\bar{\alpha}}{9} - \frac{2d_1\,\bar{\alpha}}{3} + \frac{2\ x\ (U+U')}{2}$$

$$x \quad e_1^{\,2} + U + U'$$

$$+ 2(U+U')\ .\ e_1 + \frac{2\beta}{3} + X_{22} - \frac{B_{23}}{c_2} - Q'\, e_1 - Q' + \frac{d_1^{\,2}}{4}\,(\bar{\alpha} + \bar{\alpha}') \ ,$$

$$X_{22} = \frac{B_{24}}{c_2^{\,2}} = \frac{1}{c_2^{\,2}} \{ \frac{34\alpha_1}{27} + \frac{d_2^{\,2}\,\alpha_1}{3} + \frac{10 d_2 \alpha_1}{9}$$

$$+ \frac{4\,\bar{\alpha}}{9} + \frac{2d_2\,\bar{\alpha}}{3} + \frac{2\beta}{3} + (U + U') \,.\, e_2^{\,2} - Qe_2$$

$$+ d_2^{\,2} \left(\frac{\bar{\alpha}}{4} + \frac{\bar{\alpha}'}{4} \right) \}$$

$$B_{23} = \frac{2\alpha_1 d_1 d_2}{3} - \frac{8\alpha_1}{27} + \frac{10\alpha_1 d_1}{9} + \frac{2\alpha_1\, d_2}{9}$$

$$- \frac{8\,\bar{\alpha}}{9} - \frac{2d_2\,\bar{\alpha}}{3} + \frac{2d_1\,\bar{\alpha}}{3} + 2Ue_1^{\,2} + 2U'e_1^{\,2} + 2(U + U')e_2$$

$$- Qe_1 - Q - Q'e_2 + \frac{d_1 d_2}{2} (\bar{\alpha} + \bar{\alpha}')$$

and

$$c_2 = \frac{2B_{24}}{B_{23}}$$

(B-13) $< (1254;36)_f \mid (1234;56)_i >$

$$= \left| \frac{16}{9c_2} \right|^3 \times \left[\left(\frac{1}{\alpha_1} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\alpha_1}} \times 4\pi \right) \times \left(\frac{3}{4\alpha_1} \right.\right.$$

$$\left. \times \frac{1}{2} \times \sqrt{\frac{3\pi}{2\alpha_1}} \times 4\pi \right) \times \left(\frac{1}{2(U + U')} \times \frac{1}{2} \times \sqrt{\frac{\pi}{U + U'}} \right.$$

$$\left. \times 4\pi \right) \times \left(\frac{1}{2\left[\frac{(\bar{\alpha} + \bar{\alpha}')}{4} + \frac{\alpha_1}{3} \right]} \right) \times \frac{1}{2}$$

$$\left. \times \sqrt{\frac{\pi}{\left(\frac{\bar{\alpha} + \bar{\alpha}'}{4} + \frac{\alpha_1}{3} \right)}} \times 4\pi \right)$$

$$\times \frac{4\pi}{c_2^2} \cdot \left[\frac{3.1}{(2B_{22})^2} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_{22}}} \times \frac{3.1}{(2X_{22})^2} \right.$$

$$\times \frac{1}{2} \times \sqrt{\frac{\pi}{X_{22}}} + \frac{5.3.1}{(2B_{22})^3} \times \frac{1}{2} \times \sqrt{\frac{\pi}{B_{22}}}$$

$$\left. \times \frac{1}{2X_{22}} \times \frac{1}{2} \times \sqrt{\frac{\pi}{X_{22}}} \right]$$

$$= 0.3548 \times 10^8 \ (\text{fm})^{22} = \ <(1236;54)_f \mid (1234;56)_i >$$

(B-14) $<(3456;12)_f \mid (1234;56)_i >$

$$= \left| \frac{8}{3c_3} \right|^3 \int e^{-\frac{\alpha}{2} \eta_6^2} \eta_6^2 \, d\eta_6 \, d\Omega_6 \int e^{-\eta_5^2 (\bar{\alpha} + \alpha)} \eta_5^2 \, d\eta_5 \, d\Omega_5$$

$$\int e^{-\eta_4^2 \left(\frac{\bar{\alpha} + \bar{\alpha}'}{4} \right)} \eta_4^2 \, d\eta_4 \, d\Omega_4 \int e^{-\eta_3^2 (U + U')} \eta_3^2 \, d\eta_3 \, d\Omega_3$$

$$\int e^{-\eta_1^2 c_{22}} e^{-\eta_2^2 c_{23}} \eta_1^2 Y_0^0 \left(\frac{\eta_2^2 + \eta_1^2 - 2\vec{\eta}_1 \cdot \vec{\eta}_2}{c_3^2} \right) Y_0^0 \, d\vec{\eta}_1 \, d\vec{\eta}_2$$

$$c_{22} = \frac{8\alpha}{9} + \frac{4\bar{\alpha}}{9} + \bar{\alpha} d_1^2 - \frac{4d_1}{3} \bar{\alpha} + Ue_1^2 + U'e_1^2 + \frac{2\beta}{3}$$

$$+ U + U' + 2Ue_1 + 2U'e_1 - Q'e_1 - Q' + \alpha d_1^2 + \frac{2\alpha}{3} - \frac{4\alpha}{3} d_1$$

$$+ \frac{c_{23}}{c_3^2} - \left(\frac{16\alpha}{27} - \frac{\frac{QQ'}{2(U + U')}}{c_3} \right)$$

$$c_{23} = \frac{2\alpha}{9} + \frac{16\bar{\alpha}}{9} + \bar{\alpha} d_2^2 - \frac{8\bar{\alpha}}{3} d_2 + \frac{2\beta}{3} + U'e_2^2 + Ue_2^2$$

$$- Qe_2 + \alpha d_2^2 + \frac{8\alpha}{3} - \frac{8\alpha}{3} d_2$$

and $$c_3 = \frac{2c_{22}}{\left(\frac{16\,\alpha}{27} - \frac{Q\,Q'}{2(U + U')}\right)}$$

(B-15) $< (3456;12)_f \;|\; (1234;56)_i >$

$$= \left|\frac{8}{3c_3}\right|^3 \left[\left(\frac{1}{\alpha} \times \frac{1}{2} \times \sqrt{\frac{2\pi}{\alpha}} \times 4\pi\right) \times \left(\frac{1}{2(\bar{\alpha} + \alpha)}\right.\right.$$

$$\times \frac{1}{2} \times \sqrt{\frac{\pi}{\alpha + \bar{\alpha}}} \times 4\pi\Big) \times \left(\frac{1}{2\left(\frac{\alpha + \bar{\alpha}'}{4}\right)}\right) \times \frac{1}{2}$$

$$\times \sqrt{\frac{\pi}{\frac{\alpha + \bar{\alpha}'}{4}}} \times 4\pi\Big) \times \left(\frac{1}{2(U + U')}\right) \times \frac{1}{2} \times \sqrt{\frac{\pi}{U + U'}}$$

$$\times 4\pi\Big) \times \frac{4\pi}{c_3^{\,2}}\Big] \times \Big[\frac{3.1}{(2c_{22})^2} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{c_{22}}}$$

$$\times \frac{3.1}{(2c_{23})^2} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{c_{23}}} + \frac{5.3.1}{(2c_{22})^3} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{c_{22}}}$$

$$\times \frac{1}{2c_{23}} \cdot \frac{1}{2} \cdot \sqrt{\frac{\pi}{c_{23}}}\Big] = 0.1323 \times 10^8 \quad (\mathrm{fm})^{22}$$

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VITA

Name: Sree Ram Valluri

Born: Madras, India, June 1948

Education: Primary: St. Thomas High School, Madras, 1953-1956
Secondary: Little Flower High School, Hyderabad, 1956-1962
Universities: B.Sc. New Science College, Osmania University, 1962-1966
M.Sc. Indian Institute of Technology, Kanpur, 1966-1968
University of Ottawa, 1968-1970.