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Optimization of ADCP and GPS Position and Velocity Measurements using a Kalman Filter

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Optimization of ADCP and GPS Position and Velocity Measurements using a Kalman Filter

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BEng, Dalhousie University, 1997
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Thesis submitted to the
Faculty of Graduate and Postdoctoral Studies
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ABSTRACT

This study optimized and evaluated a Kalman filter to improve measurements of boat velocity, boat position, apparent bedload velocity and water discharge using inputs of Stand Alone Global Positioning System (GPS) VTG (Doppler velocity) and GGA (triangulated position) and Acoustic Doppler Current Profiler (aDcp) bottom tracking (Doppler sonar). The filter significantly (α<0.05) improved mean velocity errors compared to Stand Alone GGA. Kalman filter velocity errors were lower than Stand Alone VTG velocity errors, but differences were rarely significant. Improvements were seen in mean position errors, although for low bedload transects, care was needed when using the Kalman velocity filter. The Kalman filter usually showed the lowest mean bedload biases, although biases were all very close ranging from -1.7 cm/s to 1.25 cm/s. The filter did not always show the lowest errors for discharge although it did show the least variability compared to each of the input signals.

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LIST OF SYMBOLS

CEP	circular error probable: radius of the circle which contains 50% of the data points	[L]
c_L	speed of light	[L T^{-1}]
c_S	speed of sound	[L T^{-1}]
D	absolute percentage difference for discharge	
dt	temporal derivative	[T]
E	radius of Earth along the semi-major axis = 6 378 137 m	[L]
f_d	received sound frequency	[T^{-1}]
f_R	frequency of satellite signal received at the boat antenna	[T^{-1}]
f_s	frequency of emitted sound	[T^{-1}]
f_T	frequency of satellite signal emitted	[T^{-1}]
H	system matrix	
K	Kalman gain	
KPX	Kalman position filter without prefiltering	
KPF	Kalman position filter with prefiltering	
KVX	Kalman velocity filter without prefiltering	
KVF	Kalman velocity filter with prefiltering	
L	boat location along transect path	[L]
Lt_1	latitude for current position	[°]
Lt_2	latitude for new position	[°]
P	error covariance matrix	
P_o^-	*a priori* estimate of initial system state error covariance matix	
Q	system state true variance matrix	
Q_D	river discharge	[L^3 T^{-1}]
R_E	radius of curvature in the prime vertical	[L]
R_N	radius of curvature in the meridian	[L]
S_E	east position for the system state	[L]
S_{Egga}	east GPS GGA boat position	[L]
S_N	north position for the system state	[L]
S_{Ngga}	north GPS GGA boat position	[L]
Sp	variance of the true boat position	[L^2]
U	water velocity vector	[L T^{-1}]
V_1	bottom track velocity in the port/starboard direction	[L T^{-1}]
V_2	bottom track velocity in the forward/aft direction	[L T^{-1}]

v_{diff}	given velocity minus RTK GGA velocity	[L T^{-1}]
V_E	east bottom track velocity in Earth coordinates	[L T^{-1}]
v_E	east velocity for the system state	[L T^{-1}]
v_{Ebt}	east bottom track boat velocity	[L T^{-1}]
v_{Egga}	east GGA velocity	[L T^{-1}]
$v_{EKalman}$	east Kalman velocity	[L T^{-1}]
v_{Ertk}	east RTK GGA velocity	[L T^{-1}]
$\overline{v}_{ERROR}$	mean boat velocity error	[L T^{-1}]
v_{Evtg}	east GPS VTG boat velocity	[L T^{-1}]
v_{given}	Given reference boat velocity	[L T^{-1}]
v_{mb}	bedload velocity	[L T^{-1}]
V_N	north bottom track velocity in Earth coordinates	[L T^{-1}]
v_N	north velocity for the system state	[L T^{-1}]
v_{Nbt}	north bottom track boat velocity	[L T^{-1}]
v_{Ngga}	north east GGA velocity	[L T^{-1}]
$v_{NKalman}$	north Kalman velocity	[L T^{-1}]
v_{Nrtk}	north RTK GGA velocity	[L T^{-1}]
v_{Nvtg}	north GPS VTG boat velocity	[L T^{-1}]
Vs	satellite velocity	[L T^{-1}]
$(Vs)_L$	component of satellite velocity in the line of sight of the boat	[L T^{-1}]
$(Vu)_L$	component of boat velocity in the line of sight of the satellite	[L T^{-1}]
$V_{w/b}$	velocity of the water relative to the boat	[L T^{-1}]
x	system state vector	
x_o^-	*a priori* estimate of initial system state	
Δx	change in distance in east/West direction	[L]
Δy	change in distance in north/south direction	[L]
z_k	measurement vector	
z_d	depth of the river	[L]

Greek Symbols

ϵ	ellipticity of the earth = 1/298.257223563	
$\epsilon_{position}$	mean position error	[L]
σ	standard deviation for error in the input stream	
σ_{bt}	BT east and north velocity error thresholds	[L T^{-1}]
σ_{diff}	standard deviation of velocity difference	[L T^{-1}]
σ_{Egga}	standard deviation of east GGA position errors	[L]
σ_{EMB}	standard deviation of east bottom tracking velocity errors	[L T^{-1}]
σ_{Evtg}	standard deviation of east VTG velocity errors	[L T^{-1}]
$(\sigma_{gga})_{position}$	GGA east and north position error thresholds	[L]
$(\sigma_{gga})_{velocity}$	GGA east and north velocity error thresholds	[L T^{-1}]

σ_{Ngga}	standard deviation of north GGA position errors	[L]
σ_{NMB}	standard deviation of east bottom tracking velocity errors	[L T^{-1}]
σ_{Nvtg}	standard deviation of east VTG velocity errors	[L T^{-1}]
σ_{vEgga}	standard deviation of east GGA velocity errors	[L T^{-1}]
σ_{vNgga}	standard deviation of north GGA velocity errors	[L T^{-1}]
σ_{vtg}	VTG east and north velocity error thresholds	[L T^{-1}]
θ_H	heading angle for the aDcp compass	[°]
θ_M	magnetic declination angle	[°]
Φ	system transform matrix	

CHAPTER 1

INTRODUCTION

An Acoustic Doppler Current Profiler (aDcp) is commonly deployed from a moving vessel to measure water velocities and river discharge. It is necessary to know the boat position and velocity in order to locate and resolve water velocities in earth coordinates, and this is commonly achieved using bottom tracking. Bottom tracking measures boat velocity by sending out an acoustic pulse at a known frequency into the water and calculating the change in frequency of the returned pulse reflected off the bottom of the river. Bottom tracking measurements however, can be biased if there is sediment movement at the bottom of the river, in which case Global Positioning System (GPS) is required to measure the reference boat velocity. On the other hand, GPS tends to make discharge estimates less accurate (Rennie and Rainville, 2006a) due to errors such as multipath at channel margins and changes in satellite configuration. It also varies in quality and may not always be available due to obstructions such as bridges and trees at channel margins. These errors and limitations in bottom tracking and GPS may lead to inaccurate aDcp measurements. To improve the reliability of aDcp measurements, it is highly desirable to obtain a continuous measurement of boat velocity that is not subject to missing data, high errors or bias.

A Kalman filter is a statistical tool commonly used to optimize parameters such as velocity, given noisy and often incomplete input measurements of that parameter and its variances. A Kalman filter, developed for use with an aDcp is being used in this study to optimize estimates of boat position and velocity given input measurements of GPS velocity, GPS position and bottom track velocity that may or may not be biased by sediment transport. These estimates are used in turn to calculate apparent bedload velocity and river discharge. Throughout this thesis, the term bedload refers to an apparent bedload velocity as measured with an aDcp and described by Rennie et. al. (2002).

Rennie and Rainville (2005) first tested the Kalman filter under non-moving bed conditions and observed it to improve estimates of both boat position and velocity compared to Wide Area Augmentation System (WAAS) GPS. They later assessed the performance of the filter under synthetic moving bed conditions (Rennie and Rainville, in review). The synthetic moving bed consisted of a moving bed vector and a random noise vector, and varied across the channel with peak intensity at the center of the channel. They found the Kalman filter was able to significantly improve velocity errors over WAAS GGA and it often showed an improvement in velocity errors compared to WAAS VTG. Rennie, Rainville and Kashyap (accepted) also tested the filter under synthetic moving-bed conditions and found the Kalman filter produced the most reliable bedload vectors and had the lowest mean single ping bedload velocity errors.

As previous studies, with the exception of Rennie, Rainville and Kashyap (accepted), which includes some preliminary results from this thesis, have evaluated the filter under non-moving bed and synthetic moving bed conditions, there is a need to assess its performance under real moving bed conditions. This study aims to test the filter under real moving bed conditions using the lowest quality non-differentially corrected Stand Alone GPS as input along with biased bottom tracking. This study also attempts for the first time to estimate on-the-fly estimates of errors in the GPS signals. Data have been collected by Environment Canada and University of Ottawa staff from various rivers in Alberta and British Columbia. Four variations of the Kalman filter will be tested and described in Chapter 4. The Kalman filter will be assessed for its ability to dynamically remove the bias from these bottom tracking signals and improve measurements of boat velocity, boat position, apparent bedload velocity and river discharge. The accuracies of the Kalman filter estimates will be determined by comparing them to high quality Real Time Kinematic (RTK) GPS measurements.

1.1 Objectives

The objectives of this study are:

1. To determine a constant set of optimum input parameters for the filter that may be recommended for use in the field when the quality of GPS and bottom tracking are unknown.
2. To assess the ability of the Kalman filter to improve measurements of boat velocity, boat position, apparent bedload velocity and river discharge under real moving bed conditions.

3. To assess the performance of the filter while using low quality non-differentially corrected GPS as an input signal.
4. To compare the performance of the filter with Stand Alone GPS and bottom tracking, in order to make recommendations on the best option to use for each measurement.

1.2 Thesis Outline

This thesis has been divided into the following Chapters described below.

Chapter 1, Introduction:

The Chapter outlines the problem and sets the objectives for the Thesis.

Chapter 2, Global Positioning System:

This Chapter describes Global Positioning System (GPS), and what GGA and VTG are. It defines common sources of errors for GPS and describes Stand Alone and Real Time Kinematic GPS. Precisions and accuracies for GPS are also covered.

Chapter 3, Acoustic Doppler Current Profiler:

This Chapter explains what an aDcp is, and errors associated with its measurements.

Chapter 4, Kalman Filter:

The Kalman filter is introduced here with a brief review of some of its applications. The mathematical theory and equations for the Kalman filter are also presented.

Chapter 5, Methodology:

Data used in this study are described in this Chapter, as well as filter inputs, and the four types of Kalman filters. Optimization of the filter using both a genetic algorithm and a heuristic approach are also explained. The performance statistics used to evaluate boat velocity, boat position, apparent bedload velocity and discharge are also presented.

Chapter 6, Results:

The main results for boat position, boat velocity, bedload velocity and discharge are summarized here for both high bedload and low bedload transects.

Chapter 7, Discussion:

This Chapter gives possible explanations and reasons for results that were obtained. Final choices for input parameters are also presented.

Chapter 8, Conclusions:

The conclusions summarize the main findings from the study.

Chapter 9, Recommendations:

This Chapter makes recommendations on the best signals to use for boat velocity, boat position, bedload velocity and discharge. Suggestions are also presented for additional studies or tests that may be completed to further validate or improve the current results.

CHAPTER 2

GLOBAL POSITIONING SYSTEM

Global Positioning System (GPS) is a space based satellite navigation system which provides accurate information on position, velocity and time anywhere in the world (Kaplan and Hegarty, 2006). It is owned by the U.S. Government and operated and maintained by the US Department of Defense (U.S. Department of Transportation and U.S. Department of Defense, 2005). The development of GPS began in the 1960s when the military, the National Aeronautics and Space Administration (NASA), and the Department of Transportation (DOT) expressed interest in using satellites for position determination (Kaplan, 1996). Since then it has developed into a fully operational navigation system which consists of at least 24 operational satellites and a ground/control monitoring segment which maintains the health of the system and updates the satellites with clock corrections.

Although several new GPS frequencies have been introduced in recent years, there are two main microwave frequencies being used by the GPS system today. The first is the L1 (1575.42 MHz) frequency, and the second is the L2 (1227.60 MHz) frequency. Also, there are two types of ranging codes (consisting of a series of binary digits) which are modulated on these signals, namely, course acquisition (C/A) code and long precision (P) code. The C/A code has a period of 1 ms, and repeats constantly. The P code is very long and repeats itself every 7 days (El-Rabbany, 2002). Selective availability (data scrambling) was originally used with the L1 C/A code for the purpose of reducing its accuracy compared to the L1 P code. Since selective availability was turned off in May of 2000, the accuracy of the L1 C/A code has increased (El-Rabbany, 2002; Tsui, 2005).

Differential correction can also increase the accuracy of GPS measurements. It works by using a network of ground based reference receivers at known locations which determine errors in satellite signals by calculating the difference in their known position and their position calculated from the satellite signal. This error is then broadcast back to the satellites and transmitted to the receivers which correct for this error. Some types of differentially corrected GPS (DGPS) include Coast Guard, Real Time Kinematic and Canadian Differential GPS (CDGPS).

2.1 GGA and VTG Velocities and Positions

GGA is a measure of GPS position and is stored in the NMEA $GPGGA string. This string contains information on 3 dimensional GPS position and time. VTG is GPS velocity, and for this study is the speed of the boat receiver antenna relative to the ground. It is stored in the $GPVTG string (RD Instruments, 2003).

2.1.1 GGA Position

GGA position is measured in terms of longitude and latitude and is determined by triangulation which is explained as follows. A GPS receiver determines the time it takes from when a signal is emitted from a satellite to when it arrives at the receiver. This time is then multiplied by the speed of light to determine the distance from the satellite to the receiver. A circle can be drawn on the earth with a circumference that depicts all points which are this distance away from the satellite. This is seen by Circle 1 in Fig. 2.1. To narrow down the possible positions of the receiver, a second satellite is needed which creates a second circle (Circle 2) of possible locations for the receiver. The receiver location is then narrowed down to the intersections of Circles 1 and 2, which are points A and B. The use of a third satellite will narrow down the position estimate to a single point (point B), determined by the intersections of Circles 1, 2 and 3. In order to accurately determine three dimensional position, an additional fourth satellite is needed. A fifth satellite is also recommended to correct for any timing offsets in the satellites (Kaplan and Hegarty, 2006).

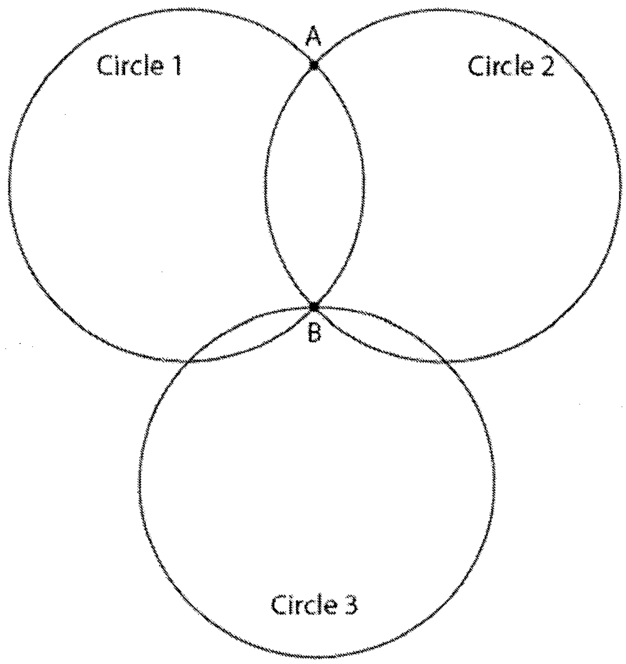

Figure 2.1 Determination of two-dimensional position.

2.1.2 GGA Velocity

GGA velocity is derived from GGA position. It is calculated from the change in distance between two successive ensembles, divided by the change in UTC time as given in the $GPGGA NMEA string. This method is fairly accurate as long as the velocity is assumed to be almost constant between successive ensembles.

2.1.3 VTG Velocity

VTG velocity is normally determined from the change in Doppler frequency of the satellite signal calculated at the receiver. This change in Doppler frequency is due to the relative motion of the satellite with respect to the user. The satellite moves at a known velocity (Vs) and emits a signal at a known frequency (f_T). The component of the user velocity in the line of sight of the satellite $(Vu)_L$ can be computed using Eq. 2.1.

$$f_R = f_T\left(1 - \frac{(Vs)_L - (Vu)_L}{c_L}\right) \tag{2.1}$$

Where f_R is the received frequency, $(V_S)_L$ is the velocity of the satellite in the line of site of the receiver (Fig. 2.2), and c_L is the speed of light (Kaplan and Hegarty, 2006). $(Vu)_L$ and $(V_S)_L$ are the components of the user and satellite velocities that contribute to the change in Doppler frequency. It should be noted that errors in VTG are independent of the type of differential correction being used. Furthermore, for the NovAtel OEM4 Propak receiver used in this study, RTK VTG is not Doppler based; rather, it is based on differences in RTK position estimates (NovAtel, Personal Communication).

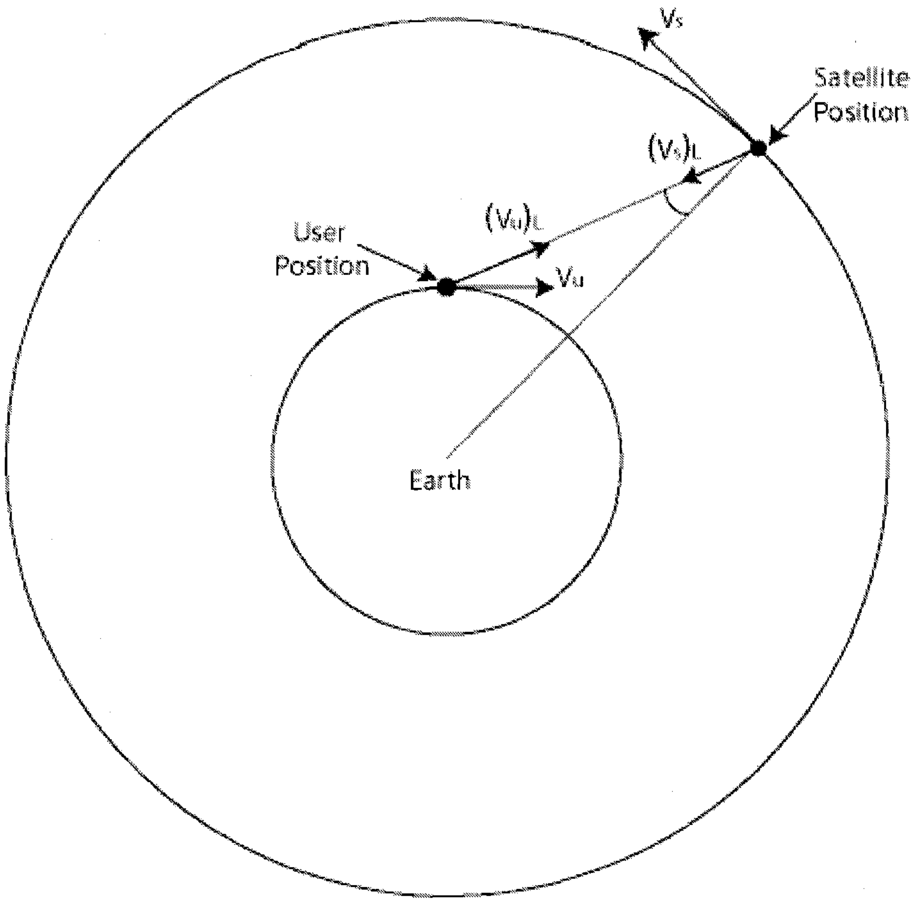

Figure 2.2 Satellite and user velocity vectors.

2.1.4 VTG Position

VTG position is determined by multiplying the VTG velocity for a given ensemble by the change in UTC time between the neighboring ensembles.

2.2 Sources of Errors

2.2.1 Ionospheric Delays

The Ionosphere is the part of the atmosphere located 70 to 1000 km above the earth. Within this region, GPS signals may refract due to interactions of the signal with free electrons causing errors of a few meters to tens of meters (Nayak, 2000). As the ionosphere is a dispersive medium, signals of different frequencies will refract at different degrees, causing different levels of delay. Ionospheric delays can be very large, but can be corrected with the use of dual frequency GPS.

2.2.2 Tropospheric Error

The Troposphere is the lowest portion of the earth's atmosphere. It starts at the earth's surface and extends upwards to the stratosphere. It contains almost all of the atmospheric water vapor, and is where weather occurs. In this portion of the atmosphere, the GPS signal may experience delay, attenuation and scintillation (Parkinson and Silker, 1996). Delays are mainly due the presence of gases such as O_2 and N_2 which have a high refraction index. Scintillation is caused by frequent changes in the refraction index due to turbulent mixing of air at different temperatures. Scintillation may cause outages or reduction in the accuracy of the signal (Parkinson and Silker, 1996). Atmospheric attenuation causes a degradation of the GPS signal due to absorption of the signal. For GPS signals atmospheric attenuation is mainly due to oxygen. Unlike the ionosphere, the troposphere is non-dispersive, meaning the wave propagation is not a function of frequency.

2.2.3 Multipath

Multipath occurs when the same GPS signal arrives at the receiver through multiple paths due to reflection from the water, trees or a man made structure such as a building or bridge. The arriving signals may interfere with each other leading to errors, or the delayed multipath signal may be mistaken for the actual signal.

Multipath is of great concern as it cannot be corrected through differential correction (Kaplan and Hegarty, 2006). It produces greater errors for the code phase signal than for the carrier phase signal (see below). This means that multipath will have a greater effect on Stand Alone GPS than on RTK GPS. Errors in range estimates for the code phase signal may be more than 100 meters, while errors in range for the carrier signal reach a maximum of 4.75 cm for L1 frequency and 6.11 cm for L2 frequency. The most effective way to reduce multipath errors is through the careful selection of a receiver. Some receivers are specially designed to mitigate the effects of multipath. The NovAtel OEM4 family of receivers has two special technologies called Pulse Aperture Correlator (PAC) and Vision Correlator to mitigate multipath effects (NovAtel, 2005).

2.2.4 Satellite Geometry

Satellite geometry refers to the location of satellites in relation to the user. Satellites which are more evenly and widely distributed in the visible sky will give a more accurate estimate of position than satellites which are bunched very close to each other (Kaplan and Hegarty, 2006). A number called the dilution of precision (DOP) indicates the quality of the satellite configuration, with a lower value indicating a better configuration. A similar measure called horizontal dilution of precision (HDOP) relates the effect of satellite geometry to horizontal position accuracy. Again, lower values indicate more accurate horizontal positions due to satellite geometry (El-Rabbany, 2002).

2.2.5 Clock Errors

Clock errors are due to offsets in GPS satellite and receiver clocks. These errors may be significant as satellite clock errors can reach up to 1 ms, which amounts to a 300 km position estimation error (Kaplan and Hegarty, 2006).

Satellite clock errors are determined by subtracting GPS system time from GPS satellite clock time. Although GPS satellites contain highly accurate atomic clocks, they are subject to clock errors which can range from 8.64 to 17.28 ns per day (El-Rabbany, 2002). GPS system time is determined by the ground control segment. It is highly accurate, and is maintained by a set of cesium atomic clocks between the Master Control Station (MSC) and the remote monitoring stations (Parkinson and Silker, 1996). It is considered to be coincident with UTC (Coordinated Universal System) Time, also known as Greenwich Mean Time (GMT). Although unlike UTC time, GPS system time does not include leap seconds which as of April, 2006, has amounted to a 14 second offset.

Similar to GPS Satellite clock errors, receiver clock errors are determined by subtracting GPS system time from GPS receiver clock time. Receiver clocks are normally inexpensive and tend to be less accurate than satellite clocks (El-Rabbany, 2002).

2.3 GPS Quality

2.3.1 Real Time Kinematic (RTK) GPS

Real Time Kinematic (RTK) GPS is a highly accurate type of differentially corrected

GPS (DGPS). It first became commercially available in 1993, and was developed by Trimble (Riley et. al., 2006). RTK differs from common DGPS in that it uses a local base station receiver at a known location to determine differential corrections, usually within 10 km of the rover receiver. The high accuracy of RTK GPS is due primarily to the fact that it uses the carrier GPS wave rather than the code wave.

The change in phase of the carrier phase signal can be determined by comparing the signal generated by the satellite to the replicated signal generated by the receiver. RTK is able to measure the fractional offset in the carrier wavelength to within an accuracy of 1%. For L1 and L2 frequencies, which have wavelengths of 19 and 24.4 cm, respectively, this is in the range of millimeters (El-Rabbany, 2002). There is still ambiguity, however, because this measurement only accounts for the fractional wavelength offset, it does not account for the number of full wavelengths between the satellite and receiver. This ambiguity can be resolved using techniques such as double differencing or wide lane searching (Kaplan and Hegarty, 2006). The use of the code phase signal with the carrier phase signal can also help in removing positioning ambiguities. However, if the code phase is affected by a great degree of multipath, estimations of ambiguities may be less accurate (NovAtel, 2005). RTK is also unique in its ability to work with either single or dual frequency GPS. Dual frequency RTK better removes integer ambiguities and ionospheric errors (NovAtel, 2005).

NovAtel has developed two types of real time kinematic products, RT-2 for dual frequency, and RT-20 for single frequency (NovAtel, 2005). RT-2 has a nominal accuracy of 2 cm (CEP) and RT-20 has a nominal accuracy of 20 cm (CEP) (NovAtel, 2005). CEP stands for circle error probable which is the radius of the circle which contains 50% of the data points. The accuracy of RT-2 and RT-20 increases as the baseline length (distance between base and rover receivers) and convergence time is decreased. Many errors, including ephemeris errors, ionospheric errors and tropospheric errors, increase with baseline length (NovAtel, 2005).

2.3.2 Stand Alone GPS

Stand Alone GPS is non-differentially corrected GPS. Although the accuracy of Stand Alone GPS has improved greatly since Selective Availability was turned off in May of 2000, it still has the largest amount of errors compared to all other types of GPS. Stand Alone GPS generally uses a single frequency code phase GPS signal, which is more greatly affected by ionospheric and multipath errors than RTK GPS. Satellite geometry can also be a large source of error for Stand Alone GPS.

2.4 Precision and Accuracy

Precision is a measure of the repeatability of a measurement. For instance, if a set of measurements were repeated for the same phenomena under the same conditions, precision measures how close each of these measurements is to the mean of these measurements. Accuracy, however, is a measure of bias and is thus indicative of how close a measurement is to a truth value (NovAtel, 2005). It is possible for a set of measurements to be highly precise, but yet inaccurate.

Measurements of GGA velocity depend on precision rather than accuracy, as they are calculated from relative changes in position divided by relative changes in time between successive ensembles. Similarly, for the scope of this thesis, we are concerned with relative position, or distance traveled from an initial starting point, rather than absolute positions. GGA relative positions also depend on precision rather than accuracy. GGA relative positions are calculated by summing the differences in GGA absolute positions. VTG velocity is a measure of instantaneous velocity for a given ensemble, and as such depends on accuracy. With the exception of RTK VTG, for the NovAtel OEM4 Propak receiver, the accuracy of VTG is normally independent of the type of differential correction method used (Rennie and Rainville, in review). VTG position is calculated from the instantaneous VTG velocity multiplied by the change in time between the two ensembles, so it is also affected more greatly by accuracy rather than precision.

In practice it is difficult to specify with certainty the precision or accuracy of a given measurement. It can depend on a number of factors including receiver quality, site location, baseline lengths, and atmospheric conditions. The following accuracies have been estimated for the NovAtel OEM4 Propak receiver (NovAtel, 2005). The accuracy of VTG velocity is estimated to be 3 cm/s RMS (root mean square error), the accuracy of RTK L1 GGA position is estimated at 20 cm CEP, while that for RTK L1L2 GGA position is 1 cm CEP. The position accuracy for Stand Alone GGA is 180 cm CEP. Rennie and Rainville (2006) estimated mean RTK GGA velocity precisions for a NovAtel Propak g2-db-d9-rtw receiver to be 2.6 cm/s for RTKL1L2 and 2.7 cm/s for RTKL1.

CHAPTER 3

ACOUSTIC DOPPLER CURRENT PROFILER

The Acoustic Doppler Current Profiler is a relatively recent device, with the first commercially available aDcp being developed in the mid 1970s (Gordon, 1996), and the first broadband aDcp developed in 1991 by RD Instruments (Simpson, 2001). They are often used to measure boat velocity and water velocities, which are in turn used to compute water discharge and bedload velocity (Rennie et. al., 2002). Boat velocities measured with an aDcp are usually more accurate than those measured with GPS, as long as there is no bias in bottom tracking due to sediment movement (Rennie et. al., 2002).

An aDcp is able to measure water velocity by sending out an acoustic pulse that is emitted from a transducer and reflected from backscatterers in the water. The velocity of the water relative to the boat ($V_{w/b}$) can then be calculated using Eq. 3.1 (Gordon, 1996).

$$f_d = 2f_s\left(\frac{V_{w/b}}{c_S}\right) \tag{3.1}$$

Where f_d is the Doppler shift frequency, f_s is the frequency of the emitted sound, and c_S is the speed of sound. Water velocities at different depths are determined by dividing each beam into a series of cells. The Doppler shift f_d can then be calculated at each cell, and these cells are then mapped into bins as shown in Fig. 3.1. Each bin stores an average velocity which represents the velocity at the center of the bin. This method is referred to as time gating (Simpson, 2001). Absolute water velocities in each bin can then be determined by correcting for the velocity of the boat which can be measured either through bottom tracking or GPS. Discharge measurements can then also be calculated by integrating the water velocities over the depth and length of the transect which will be discussed further in Chapter 5.

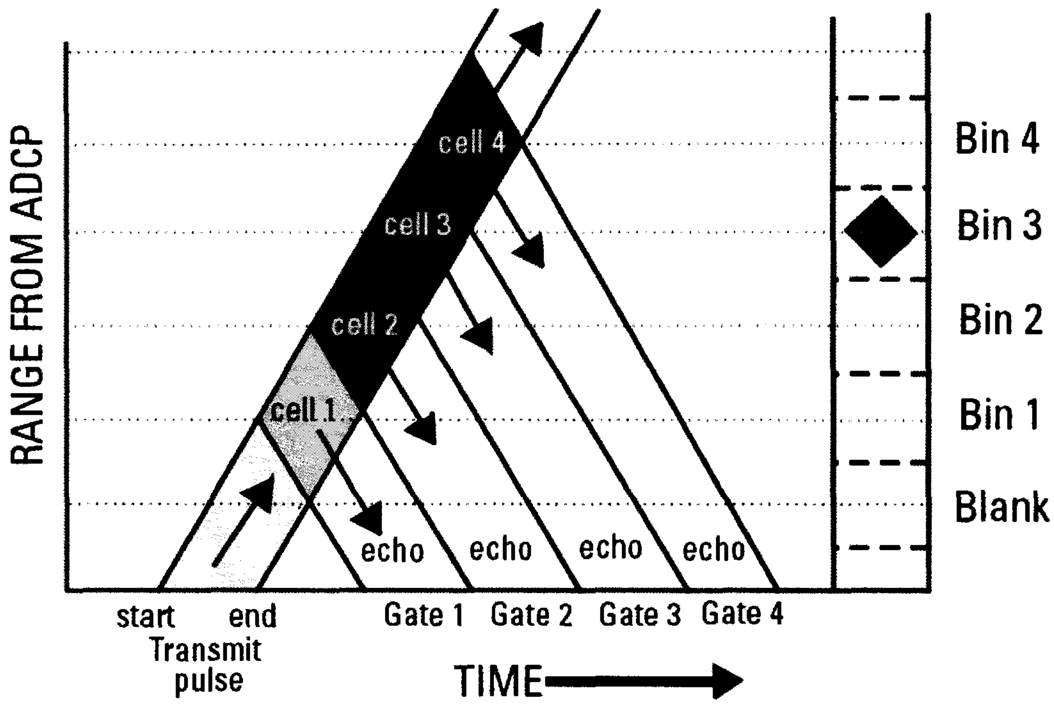

Figure 3.1 Time gating and bin mapping for the aDcp (Simpson, 2001).

Similar to water velocity measurements, bottom tracking measures the shift in Doppler frequency from an acoustic pulse that is reflected from the sediment bed. This can provide an accurate measure of boat velocity if the bed is stationary. More detailed information on aDcp measurements can be found in Gordon (1996) and Simpson (2001).

3.1 Sources of Errors

3.1.1 Instrument Noise

Instrument noise can be a significant source of errors for aDcp measurements, and Rennie and Millar (in press) found that it can dominate aDcp measurement errors. It can either be random in nature, or it may cause a bias in measurement. Random instrument errors may be due to imprecisions in estimates of the Doppler frequency shift, and can be influenced by aDcp type, sediment movement and pulse length (Rennie and Millar, in press). Broadband aDcps tend to be more accurate than narrow-band aDcps for a given frequency and bin size (Simpson, 2001). Also, sediment moving at different velocities along the bed will cause the bottom track signal to decorrelate leading to imprecision in the estimation of the frequency shift. Rennie and Millar (in press) found results which suggest instrument signal noise increases with increasing bedload transport rates. Also, longer pulses will usually produce fewer errors than shorter pulses since there are more correlation estimates to average (Rennie and Millar, in press).

Instrument noise bias errors may be due to speed of sound errors, beam angle errors, mean current speed and low signal/noise ratio. Speed of sound errors are caused by inaccurate calculations in the speed of sound due to poor estimates of water temperature and salinity. Although the temperature of the water is determined by a thermistor at the transducer, if the temperature becomes stratified with depth, errors in depth measurements as high as 7 percent may occur (Simpson, 2001). Large temperature gradients with depth are more common during summer months. Salinity values are entered by the user, and inaccurate estimates may introduce depth errors as high as 3 percent (Simpson, 2001). Beam angle errors are caused by inaccurate measurements of the beam angle during manufacturing. This angle is programmed into the aDcp memory and if inaccurate can lead to a velocity bias. Fortunately however, due to more stringent manufacturing processes, these types of errors currently tend to be low (Gordon, 1996).

3.1.2 Heading Errors/Magnetic Offsets

Heading errors may be caused by instability in the compass which can add random noise to the measurements, or by misalignment of the transducer with the compass. Misalignment of the compass is normally only a concern when an external compass is used. It is usually negligible when an internal compass is used (Gordon, 1996). The compass accuracy reported for Rio Grand Workhorse aDcp due to heading errors is ±2°.

Magnetic offsets are normally due to differences in magnetic north measured by the aDcp compass and true magnetic north, or the north geographic pole. This can be corrected for by using site-specific corrections for magnetic declination, which can be looked up in a chart, or by performing compass calibrations at the local site (RD Instruments, 2003).

3.1.3 Pitch/Roll

Pitch is due to rotations of the boat in the fore/aft directions, and roll is due to rotations of the boat in the starboard/port directions. These rotations may introduce small fluctuations in the component of the velocity along the beam axis for the transducer (Rennie, Millar and Church, 2002). This will in turn change the perceived shift in the

Doppler frequency and cause random errors in bottom tracking velocity. If the angle of pitch and roll is less than 5° then the errors for the horizontal components of the velocity may be small, although they can still be significant for the vertical components of the velocity. Also, errors due to pitch and roll may cause errors in depth measurements as the bin depths need to be remapped (Simpson, 2001).

3.1.4 GPS Errors

When bottom tracking is biased due to sediment movement, GPS is often used as the reference boat velocity for discharge, water velocity and bedload velocity measurements. Although aDcp instrument noise often dominates measurement errors (Rennie and Millar, in press), GPS errors also contribute to these errors. The errors generated by GPS depend on the differential correction method being used. As mentioned in Chapter 2, differentially corrected GPS is more accurate than Stand Alone GPS, with RTK GPS usually being the highest quality GPS. There has been little work done on the relative contributions that GPS correction methods make to discharge, water velocity and bedload velocity measurement errors. Rennie and Rainville (2006) assessed the performance of four different GPS correction methods. They found average velocity error magnitudes for RTKL1L2, RTK L1, Coast Guard, and Wide Area Augmentation System (WAAS) to be 2.6, 2.7, 8.7, and 9.7 m/s, respectively. Although the relative contribution of GPS errors to aDcp measurement errors will depend greatly on instrument noise, Rennie and Rainville (2006) found DGPS errors to increase water velocity measurement errors by up to 14%. Also, using bedload velocity error estimates determined by Rennie and Millar (in press) for two gravel-bed rivers, DGPS errors were found to increase bedload velocity errors by up to 11%. Discharge error was assessed as the maximum absolute percentage difference in bottom tracking discharge and DGPS discharge. Errors were the greatest for Wide Area Augmentation System (WAAS) at 3.1%.

CHAPTER 4

KALMAN FILTER

The Kalman filter was originally developed in 1960 by Rudolf Emil Kalman (Kalman, 1960). It is now considered to be one of the greatest discoveries in modern day control theory and is often used to estimate position and velocity of complex dynamic systems such as ships, spacecrafts and airplanes. It is a statistical tool that is able to estimate the optimum state of a system given a noisy and often incomplete set of input measurements and their variances. One of its first and most noteworthy applications was onboard the Apollo guidance system (Grewal and Angus, 2001).

The original Kalman filter was designed as a discrete linear model. It was discrete as it used a set of measurements sampled at discrete points in time. It was linear in that it modeled a system as a set of linear differential equations. A subsequent adaptation of the original filter was the Extended Kalman filter which was able to deal with nonlinear problems by linearizing characteristic equations about a particular point. Another adaptation, the Unscented Kalman filter, did not use linearized equations but instead modified the state matrix to model errors (El-Sheimy et. al., 2006).

4.1 Applications

As mentioned previously, the Kalman filter has commonly been used in navigation systems for ships, planes and spacecraft. Although the applications are too vast and numerous to mention, it has usually been used to fuse measurements obtained from different sensors to arrive at an optimum estimate. For example, it has been used to merge measurements from Inertial Navigation Units (INUs) and Global Positioning Systems (GPS) (Mohamed and Schwarz, 1999). An INU measures the acceleration of an object with a combination of gyroscopes and accelerometers. These acceleration measurements are integrated to determine position and velocity. Kalman filtering is usually effective in removing errors from these measurements, although its effectiveness is in part due to the type of Kalman filter being used and the accuracy of the INU (El-Sheimy et. al., 2006).

The Kalman filter has also shown great potential in the area of precision agriculture. One of the first applications was from a study by Hague and Tillett (1995) who used it for the navigation of a horticultural robot among crop rows. They used the Kalman filter to fuse positioning data from a vision sensing system with positioning data from an odometer obtained through dead reckoning. Dead reckoning works by advancing a known position by an incremental distance determined by multiplying an instantaneous velocity by a time increment. The filter allowed the vehicle to achieve a positioning accuracy of ±20 mm at a speed of up to 1.5 m/s. A later study by van Bergeijk et. al. (1998) integrated a Kalman filter with DGPS, a random velocity sensor, a wheel sensor, and an electronic compass for position determination. The filter was able to bridge gaps in position data when GPS was not available, and was able to achieve a position standard deviation of 0.71 m, compared to the GPS position standard deviation of 1.18 m. Oksanen et. al. (2005) used a Kalman filter with DGPS and an INU to improve positioning accuracy for an agricultural vehicle. Their findings showed that although the filter was able to remove noise from the input measurements, it was not able to remove biases.

Studies which have used a Kalman filter with an Acoustic Doppler Current Profiler (aDcps) and GPS has been very limited. Some work has been done in underwater navigation using a Doppler Velocity Log (DVL) and an INU (Lee et. al., 2005). Hydro-Québec has developed a submarine navigation system using an INU, DVL and a Bathymeter (Blain et. al., 2003). The Kalman filter was able to improve position estimates and fill in data gaps during DVL failures. Van Reenen (2004) developed a surface navigation system that utilized position measurements from an aDcp and GPS. The Kalman filter helped to fill in data gaps when GPS became unavailable, and maintain low position drifts. These studies however did not investigate improving boat velocity measurements, commonly used for the aDcp reference velocity, which are important in many aDcp measurements.

Other than the work done by van Reenan (2004), the only other known studies that have used a Kalman filter with an aDcp and GPS were done by Rennie and Rainville (2005; 2006b; in review). (Rennie, and Rainville (2005) conducted studies on the Gatineau River using unbiased bottom track and DGPS data, and found a Kalman filter was able to improve both the boat velocity and position estimates of DGPS under non-moving bed conditions. A later study by Rennie and Rainville (in review) assessed the ability of a Kalman filter to improve the reference GPS velocity used in aDcp measurements under moving bed conditions. Although bottom track data were originally unbiased, a synthetic moving bed was superimposed on the bottom track velocities. As the Kalman filter required unbiased bottom track data, this moving bed bias had to first be estimated, and then removed from the bottom tracking velocity before being used it in the filter. The study showed that the Kalman filter was able to significantly reduce mean velocity errors from both VTG and GGA data collected on the Gatineau River. For data collected from the Saint Mary's river, the Kalman filter mean velocity errors were always lower than VTG, although there were no significant differences between them. The Kalman filter mean velocity errors were always significantly lower than GGA. Rennie and Rainville (2006b) also found that for synthetic bedload, the Kalman filter best estimate of reference velocity significantly improved bedload estimates compared to VTG and GGA reference velocities. While these studies show the great potential of the Kalman filter to be used with aDcp measurements, they also bring to light the need to further assess and optimize the filter under real moving bed conditions.

As the Kalman filter being used in this thesis is the linear discrete model, technical descriptions in the remainder of this Chapter will be limited to this model, and how they pertain to the specific design of the filter being used in this thesis. This study is unique in that it tests and optimizes the filter under real moving bed conditions.

4.2 Technical Background

The objective of the Kalman filter is to predict the system state given a set of noisy input measurements and their error covariances. The Kalman filter predicts the *a priori* estimate for system state and error covariances for the next time step, and uses the Kalman gain and measurement matrix to update the *a priori* estimate to the Kalman best estimate of the system state. This process is repeated until the Kalman best estimate has been determined for all time steps. Two basic types of filters were developed, a position filter and a velocity filter, which are described in the following sections.

4.2.1 Position Filter

The position filter uses both position and velocity in the measurement matrix to predict the Kalman best estimates of position and velocity for the following system state vector (x).

$$x = \begin{bmatrix} S_E & S_N & v_E & v_N \end{bmatrix}^T \tag{4.1}$$

Where S_E and S_N are east and north positions, respectively, and v_E and v_N are east and north velocities, respectively. A known noisy measurement vector (z_k), assumed to have Gaussian white noise, is given at time k in Eq. 4.2.

$$z_k = \begin{bmatrix} S_{Egga,k} & S_{Ngga,k} & v_{Evtg,k} & v_{Nvtg,k} & v_{Ebt,k} & v_{Nbt,k} \end{bmatrix}^T \tag{4.2}$$

Where $S_{Egga,k}$ and $S_{Ngga,k}$ are the east and north GGA positions, respectively. $v_{Evtg,k}$ and $v_{\mathrm{Nvtg,k}}$ are east and north VTG velocities, respectively, and $v_{\mathrm{Ebt,k}}$ and $v_{\mathrm{Nbt,k}}$ are east and north bias-free bottom tracking velocities, respectively. The covariance matrix (R) of the measurement vector (z_k) is also used in determining the best estimates.

$$R = \begin{bmatrix} \sigma_{Egga}^{2} & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{Ngga}^{2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{Evtg}^{2} & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{Nvtg}^{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{EMB}^{2} & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{NMB}^{2} \end{bmatrix} \tag{4.3}$$

σ_{Egga} and σ_{Ngga} are the standard deviations of the east and north GGA position errors, respectively. σ_{Evtg} and σ_{Nvtg} are the standard deviations of the east and north VTG velocity errors, respectively, and σ_{EMB} and σ_{NMB} are the standard deviations of the east and north bottom track velocities under moving bed conditions, respectively. An estimate of the system state true variance (Q) is also used to compute the best estimate.

$$Q = \begin{bmatrix} \frac{Spdt^3}{3} & 0 & \frac{Spdt^2}{2} & 0 \\ 0 & \frac{Spdt^3}{3} & 0 & \frac{Spdt^2}{2} \\ \frac{Spdt^2}{2} & 0 & S_p dt & 0 \\ 0 & \frac{Spdt^2}{3} & 0 & Spdt \end{bmatrix} \tag{4.4}$$

Where Sp is the variance of the true boat position. A system matrix (H) is used to relate the measurement vector to the system state vector.

$$H = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.5}$$

The first step to determine the best estimate of the system state is to get an *a priori* estimate of the initial system state x_o^-, and its error covariance matrix P_o^-. Superscript dashes represent *a priori* estimates. The initial *a priori* estimates are the first values in the measurement matrix.

$$x_o^- = [S_{Egga}\ S_{Ngga}\ v_{Ebt}\ v_{Nbt}]^T \tag{4.6}$$

$$P_o^- = \begin{bmatrix} \sigma_{Egga}^2 & 0 & 0 & 0 \\ 0 & \sigma_{Ngga}^2 & 0 & 0 \\ 0 & 0 & \sigma_{Ebt}^2 & 0 \\ 0 & 0 & 0 & \sigma_{Nbt}^2 \end{bmatrix} \tag{4.7}$$

Next the Kalman Gain *(K)* is determined at time *k* through Eq. 4.8.

$$K_k = P_o^-\ H_k^T \left(H_k\ P_o^-\ H_k^T + R_k\right)^{-1} \tag{4.8}$$

Using the initial *a priori* estimate of the system state, the Kalman best estimate (x_k) of the system is obtained.

$$x_k = x_o^- + K_k\left(z_k - H_k x_o^-\right) \tag{4.9}$$

The best estimate of the error covariance matrix P_k is then determined.

$$P_k = (I - K_k H_k) P_o^- \tag{4.10}$$

The *a priori* estimates for the system state and error covariance for the next time steps are then calculated using Eqs. 4.11 and 4.12.

$$x_{k+1}^- = \Phi_k x_k \tag{4.11}$$

$$P_{k+1}^- = \Phi_k P_k \Phi_k^T + Q_k \tag{4.12}$$

Where Φ_k is called the system transform matrix. It converts the previous system state to the current system state, and is given by Eq. 4.13.

$$\phi_k = \begin{bmatrix} 1 & 0 & dt_k & 0 \\ 0 & 1 & 0 & dt_k \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.13}$$

Here, dt_k represents a temporal derivative. Eqs. 4.8 to 4.12 are repeated for all the records in the noisy dataset, and substituting x_k^- and P_k^- for x_o^- and P_o^-, respectively.

4.2.2 Velocity Filter

The velocity filter uses velocities only rather than positions in the measurement vector to determine best estimates of the system state. These velocities may be GGA, VTG and/or bottom tracking. The steps in calculating the Kalman best estimates of the system state are the same for both the position and the velocity filters. The differences between the filters lie in the values used in the *z*, *H*, and *R* matrices. If we assume that GGA, VTG and bottom tracking are present, the measurement vector (z) now replaces the GGA position values (S_{Egga} and S_{Ngga}) with GGA velocity values (v_{Egga} and v_{Ngga}) as in Eq. 4.14.

$$z = \begin{bmatrix} v_{Egga} & v_{Ngga} & v_{Evtg} & v_{Nvtg} & v_{Ebt} & v_{Nbt} \end{bmatrix}^T \tag{4.14}$$

The system matrix (H) used to relate the measurement vector to the system state vector now becomes:

$$H = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{4.15}$$

The covariance matrix (R) of the measurement vector (z_k) replaces σ_{Egga} and σ_{Ngga} with σ_{vEgga} and σ_{vNgga}, which are the standard deviations of the errors of the east and north GGA velocities respectively.

$$R = \begin{bmatrix} \sigma_{vEgga}{}^2 & 0 & 0 & 0 & 0 & 0 \\ 0 & \sigma_{vNgga}{}^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & \sigma_{Evtg}{}^2 & 0 & 0 & 0 \\ 0 & 0 & 0 & \sigma_{Nvtg}{}^2 & 0 & 0 \\ 0 & 0 & 0 & 0 & \sigma_{EMB}{}^2 & 0 \\ 0 & 0 & 0 & 0 & 0 & \sigma_{NMB}{}^2 \end{bmatrix} \tag{4.16}$$

CHAPTER 5

METHODOLOGY

The Kalman filter described in Chapter 4 was optimized to predict best values for boat velocity and position from varying quality input signals. Optimization involved determining the best Kalman filter input parameters to minimize errors in boat velocity and position. The predicted boat velocities were then used to calculate bedload velocity and discharge.

Data were collected by Environment Canada and University of Ottawa staff from various rivers in Alberta and British Columbia and are summarized in Appendix A. Each dataset in Table A.2 represents a transect for a particular river cross section. A total of 29 transects were used for optimization and 31 transects were used for testing. The input signals used for the filter were Stand Alone GGA and VTG collected at 10 Hz, and biased bottom tracking collected at approximately 2 Hz. GPS data were subsampled to match bottom track samples (see Rennie and Rainville, 2006a). The data streams often contained errors, and were noisy. Examples of errors included high bottom tracking vertical velocities and low beam correlations. These errors were prefiltered and removed. Data streams were also often incomplete and contained missing data when either GPS or bottom tracking became unavailable. The complete Matlab code for the Kalman filter program is given in Appendix I. The Kalman filter was designed to assess which data were missing, and make predictions regardless of which data were present. If no data were present dead reckoning was used, meaning that the filter only used previous values to determine the current value. Mean bedload velocities for the transects varied from 0.12 cm/s to 8.80 cm/s, which had to be estimated and removed from the bottom tracking signal before being used in the filter. Once moving bed was removed, the appropriate filter type was chosen based on which data were available, and the Kalman best estimate was determined. This process was repeated for each ensemble in the dataset. A diagram showing the main steps in the program flow is given in Fig. 5.1.

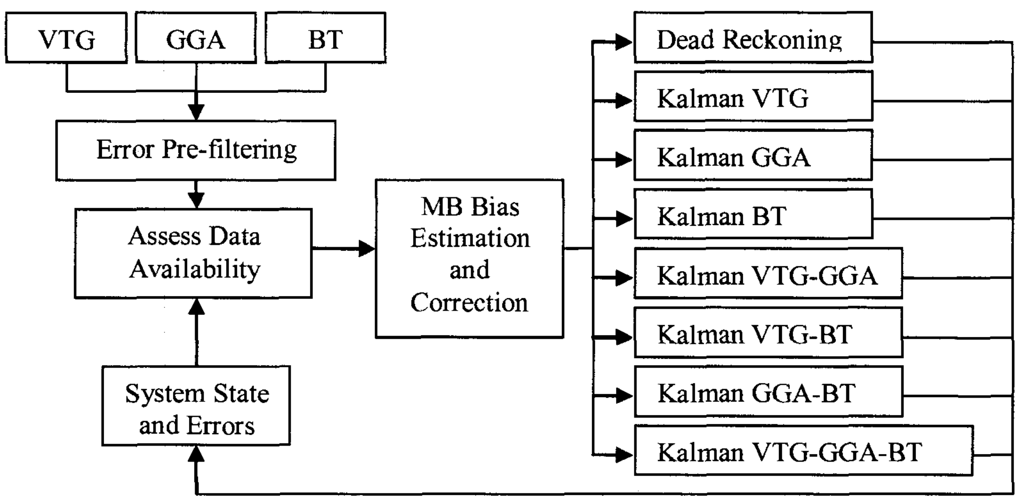

Figure 5.1 Kalman filter program flow diagram (Rennie and Rainville, in review).

RTK GGA was also collected simultaneously and used as a reference or truth value to verify the accuracy of the predicted Kalman filter values, as well as the Stand Alone GPS and bottom tracking values. With the exception of the data collected on the Fraser River, which was dual frequency RTKL1L2 (RTK 4), the RTK type for all transects was single frequency RTKL1 (RTK5). Bottom track data were collected with a Workhorse Rio Grand aDcp made by RD Instruments. GPS Stand Alone and RTK data were collected simultaneously with bottom tracking data using two NovAtel OEM4 Propak receivers. The GPS receivers were mounted parallel to each other in set ups similar to those shown in Figs. 5.2 and 5.3. NMEA $GPGGA and $GPVTG strings were collected from the receivers and matched to corresponding bottom tracking ensembles by RD Instruments software WinRiver.

Figure 5.2 Set up of aDcp and NovAtel receivers at Bow River, Alberta.

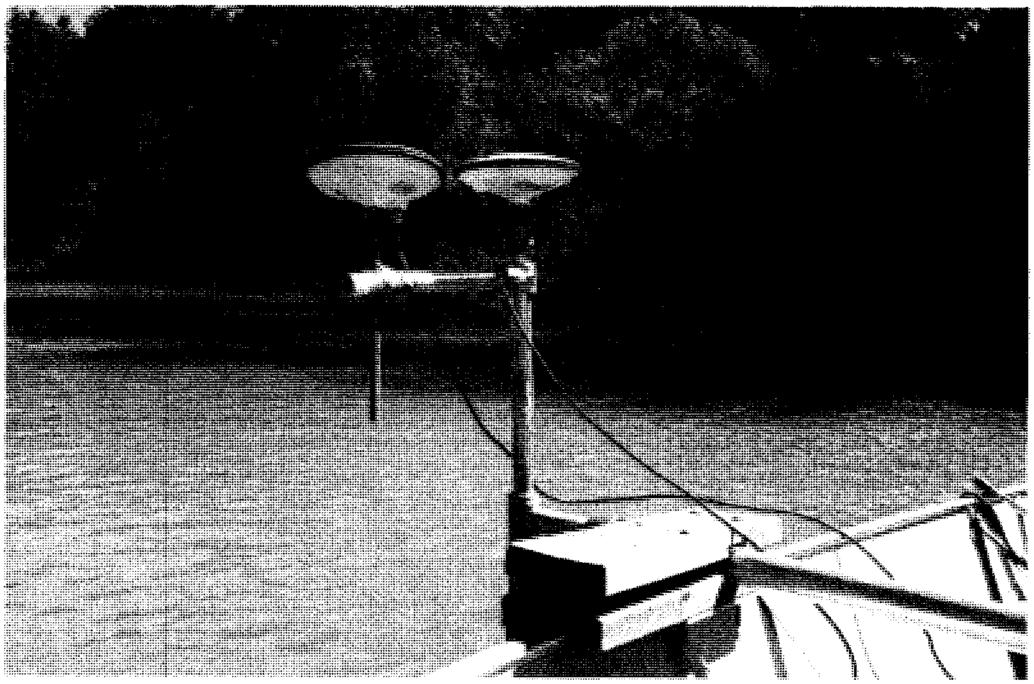

Figure 5.3 Set up of NovAtel receivers on the Fraser River, British Columbia.

This remainder of this Chapter describes the types of Kalman filters that were used, the various input parameters, the optimization of the input parameters and details on the methods used to calculate boat velocity, boat position, bedload velocity and discharge.

5.1 Kalman Filter Types

Four types of Kalman filters were developed and tested. It was determined by Rennie and Rainville (in review) that high errors in GGA horizontal velocities were associated with high GGA vertical velocities. Vertical velocities were determined as the change in the GPS antenna altitude divided by the change in time between ensembles. It was therefore of interest to know if removing Stand Alone GGA values with high errors would improve the Kalman filter performance. Stand Alone GGA was prefiltered by removing a value if the GGA vertical velocity exceeded a certain threshold limit. Four filter types were developed as defined in Table 5.1 based on whether Stand Alone GGA was prefiltered, and whether a position or velocity filter was being used. As described in the preceding chapter, the position filter used Stand Alone GGA position as input, whereas the velocity filter used Stand Alone GGA converted to velocity as input.

Table 5.1. Filter types

Filter Type	Symbol	Description
Position Filter without Prefiltering	KPX	Uses inputs of position and velocity to determine position and velocity. Stand Alone GGA is not prefiltered.
Position Filter with Prefiltering	KPF	Uses inputs of position and velocity to determine position and velocity. Stand Alone GGA is prefiltered.
Velocity Filter without Prefiltering	KVX	Uses inputs of velocity only to determine position and velocity. Stand Alone GGA is not prefiltered.
Velocity Filter with Prefiltering	KVF	Uses inputs of velocity only to determine position and velocity. Stand Alone GGA is prefiltered.

5.2 Adjustable Input Parameters

It was possible to improve the performance of the Kalman filter by adjusting the values of certain input parameters. Optimum values of these input parameters depended on the filter type being used, as well as the errors in the input signals. A description of each of the input parameters is given below.

Error Standard Deviation Thresholds: These are the minimum values for the standard deviations (σ) of the errors in the input data streams. A large value of σ for a given input stream forces the Kalman filter to de-emphasize the input stream. The user could input the minimum allowable value of σ for each input stream. Actual error standard deviations however, are evaluated on-the-fly and the σ value may be automatically increased accordingly. For example, the VTG east velocity error (σ_{Evtg}) is calculated according to the following Eq. 5.1.

$$\sigma_{Evtg} = \sqrt{\sigma_{EvtgVel}^{2} - \sigma_{v}^{2}} \tag{5.1}$$

Where $(\sigma_{EvtgVel})^2$ is the variance of the east component of the VTG velocity and $(\sigma_v)^2$ is the Kalman filter estimate of the variance of the true boat velocity, both of which are calculated from a number of previous ensembles as specified by the user. The on-the-fly errors for each of the input streams are calculated in a similar way. A list of the error standard deviation thresholds are given in Table 5.2.

Table 5.2. Error standard deviation thresholds that may be input by the user

Threshold	**Threshold Symbol**	**Units**
GGA east and north Position Error (σ_{Ngga}, σ_{Egga}) Thresholds	$(\sigma_{gga})_{position}$	m
GGA east and north Velocity Error (σ_{vNgga}, σ_{vEgga}) Thresholds	$(\sigma_{gga})_{velocity}$	m/s
VTG east and north Velocity Error (σ_{Nvtg}, σ_{Evtg}) Thresholds	σ_{vtg}	m/s
BT east and north Velocity Error (σ_{Nbt}, σ_{Ebt}) Thresholds	σ_{bt}	m/s

Number of Moving Bed Elements to Average (counts): This is the number of previous ensembles to average when calculating the moving bed bottom track velocity bias that is removed from the bottom tracking velocity before it is input into the Kalman filter. Moving bed, calculated as the difference in bottom tracking velocity and boat velocity, contains a large amount of noise. Calculating moving bed by averaging the single ping moving beds from a number of previous ensembles helps to reduce the noise, as can be seen in Fig. 5.4.

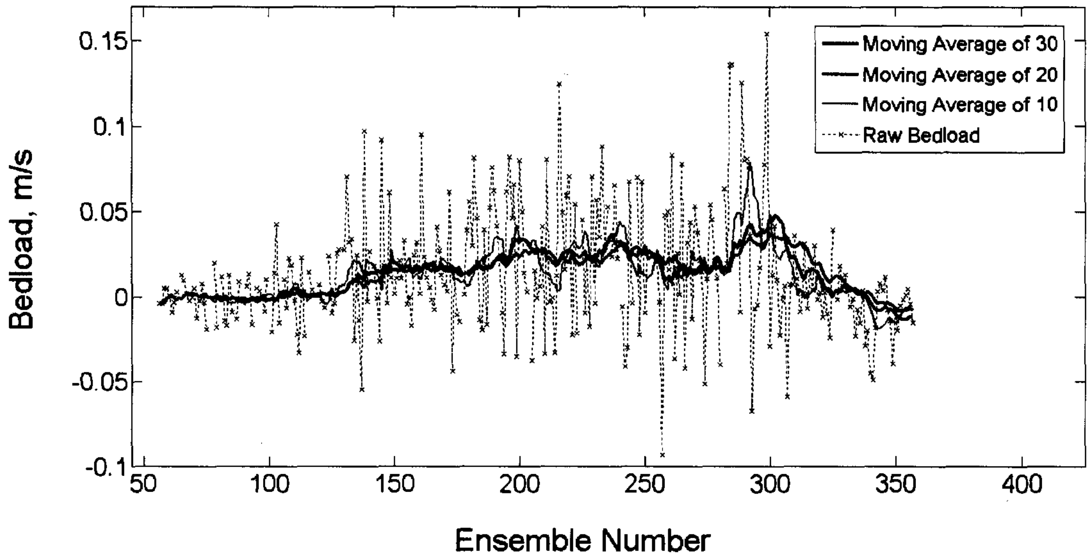

Figure 5.4 Example comparison of raw bedload and bedload calculated from moving averages.

Number of Bottom Tracking Elements to Average (counts):

This is the number of previous ensembles to utilize when calculating error standard deviations for bottom tracking.

Number of VTG Elements to Average (counts): This is the number of previous ensembles to utilize when calculating error standard deviations for VTG.

Number of GGA Elements to Average (counts): This is the number of previous ensembles to utilize when calculating error standard deviations for GGA.

Sp (m^2): This is the variance of the true boat position. This cannot be known explicitly, because all measurements of boat position include measurement error. The value of *Sp* can be estimated as mean($\Delta x^2+\Delta y^2$), where Δx is the change in east/west position and Δy is the change in the north/south position. Although it is possible for the user to input this value, an optimum value for most applications will be determined later in this Chapter.

GGA Vertical Velocity Threshold (m/s): This is the maximum allowable value for GGA vertical velocity error. If GGA prefiltering is active, and the calculated vertical velocity error is greater than this value, then both the corresponding GGA velocities and positions will be removed. The calculated vertical velocity error is the change in antenna altitude divided by the change in time between ensembles. The default threshold value has been set to 1 m/s.

Minimum Beam Correlation (counts): This is the minimum beam correlation allowed for each beam. It is set by default to 64. If any of the beams in a 4 beam solution has a correlation below this value, then the corresponding bottom tracking velocities will be removed.

Bottom Tracking Vertical Velocity Threshold (m/s): This is the maximum allowable bottom tracking vertical velocity. The default value is 0.10 m/s. It is assumed that if the vertical velocity is greater than this value, the boat may be experiencing high pitch and roll, which will affect bottom tracking measurements. If the bottom tracking vertical velocity for any particular ensemble is greater than this value, it will be removed.

Bottom Tracking Error Velocity Threshold (m/s): This is the maximum allowable bottom tracking error. It is set to a value of 0.10 m/s. If the bottom tracking velocity error is greater than this threshold, the corresponding bottom tracking velocities for that ensemble will be removed.

5.3 Bedload Estimation Prior to Filtering

Bedload velocity was estimated and removed from the bottom tracking velocity before it was input into the Kalman filter. As bedload at the channel margins was assumed to be low, bedload for the first ensemble was estimated to be 0.1 cm/s. Subsequent values of bedload were determined by taking a moving average of the Kalman filter estimates of bedloads from previous ensembles. The number of previous ensembles used in this calculation is defined in the variable 'mbAvg' by the user. If the number of previous ensembles is less than 'mbAvg', an average is taken from the first ensemble to the last ensemble prior to the current ensemble. This bedload is then subtracted from the bottom tracking velocity. A diagram depicting the process to calculate moving bed average is given in Fig. 5.5.

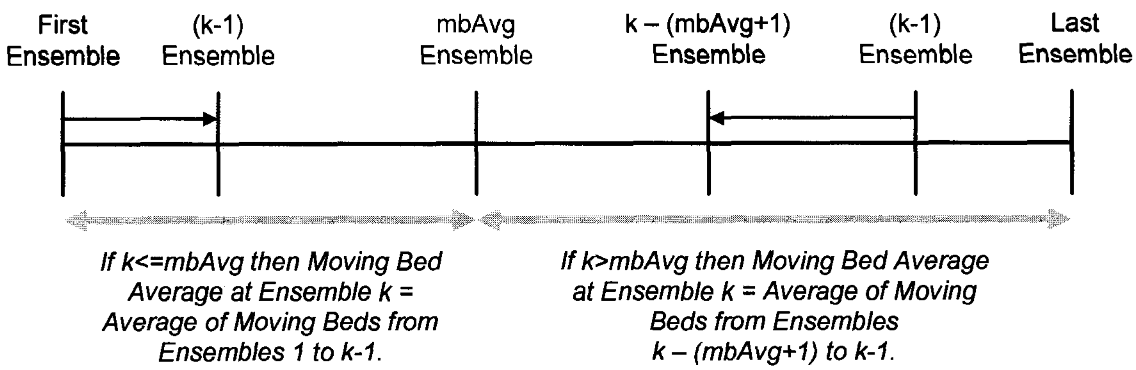

Figure 5.5 Calculation of moving bed average.

5.4 Optimization of the Kalman Filter

Optimizing the Kalman filter involved determining the best values for input parameters that would allow the filter to optimize estimates of boat velocity, boat position, bedload velocity and discharge. Values for input parameters were difficult to predict and could vary based on a number of factors. For example, error thresholds for bottom tracking could vary with the amount of moving bed, and errors for GPS could vary based on factors discussed in Chapter 2.

A genetic algorithm was initially used to determine the optimum input parameters that would minimize the mean boat velocity error. As minimizing the velocity error did not always minimize boat position, bedload velocity and discharge, the input parameters were later adjusted to determine a set of constant inputs that would be ideal for all measurements. The transects used in the optimization were divided into those with RTK mean bedloads greater than 0.025 m/s, and those with RTK mean bedloads greater than 0 m/s and less than 0.025 m/s. Transects used for moving bed tests and loop tests were not used in the optimization or testing.

5.4.1 Optimization using a Genetic Algorithm

A genetic algorithm developed by Houck et. al. (1995) was used in this study to determine optimum input values for different filter types and levels of moving bed. The input parameters optimized for the position filter were the error thresholds σ_{vtg}, σ_{bt} and $(\sigma_{gga})_{velocity}$, the number of moving bed and vtg/gga elements to average, and the variance of the true boat position (*Sp*). The value of the error threshold $(\sigma_{gga})_{position}$ was set to a constant of 1 m, which was based on the expected instrument performance. For the velocity filter, optimization was completed for all of the input parameters mentioned previously except $(\sigma_{gga})_{velocity}$ was used instead of $(\sigma_{gga})_{position}$. The search bounds used for each of the parameters are given in Table 5.3.

Table 5.3. Bounds for input parameters

Input Parameter	**Bound**
$(\sigma_{gga})_{velocity}$ σ_{vtg} σ_{bt}	0.005 to 0.2 m/s 0.01 to 0.2 m/s 0.01 to 0.2 m/s
Moving Bed Elements to Average	10 to 50
VTG/GGA Elements to Average	10 to 40
Sp	0.03 to 0.09 m^2

The population size used for the inputs was 50, and a maximum of 50 generations were used. The objective function was the Kalman filter velocity error, given in Eq. 5.2.

$$\sqrt{\left(v_{EKalman} - v_{ERtk}\right)^2 + \left(v_{NKalman} - v_{NRtk}\right)^2} \tag{5.2}$$

Where $v_{EKalman}$ and $v_{NKalman}$ are the east and north Kalman best estimates of velocity, and v_{Ertk} and v_{Nrtk} are the east are north RTK GGA velocities. The objective of the algorithm was to minimize this function. It should be noted that in estimating the on-the-fly error standard deviations (Eq. 5.1), the variance of the true boat velocity was estimated using RTK GGA velocity rather than the Kalman best estimate of velocity as was used in the actual filter. This is because RTK GGA velocity was considered a truth value with zero error, which would improve the accuracy of the optimization. The Kalman best estimate of velocity was used in the actual filter, since RTK would not normally be available in real situations. The algorithm was run with each of the four Kalman filters, KPX, KPF, KVX and KVF. It was found for the position filter that the algorithm was able to find stable values for the input parameters within the first 20 generations. However, on successive runs, values for σ_{vtg} and σ_{bt} thresholds would differ, although the threshold ratio of $(\sigma_{vtg}/\sigma_{bt})$ would remain constant. For the velocity filter, the genetic algorithm was run twice for each dataset. An initial run was completed on each transect to determine *Sp* and the number of moving bed and vtg/gga elements to average. Constants were determined by averaging these parameters over the transects in question. These constants were used in the second run to determine the error threshold values for VTG, GGA and BT velocities. The results of the optimization are given in Appendices B and C, and the optimum values determined for each filter based on moving bed levels are given in Table 5.4. From the values given in Table 5.4, a constant set of input parameters were determined, and are given in Table 5.5.

Table 5.4. Summary of results from optimization of input parameters

Filter Types	$\frac{\sigma_{vtg}}{\sigma_{bt}}$	(σ_{gga}) Position (m)	(σ_{gga}) Velocity (m/s)	*Sp* (m^2)	MB Elements to Average[a]	VTG / GGA Elements to Average[a]
For Mean RTK Bedload > 0.025 m/s						
KPX	0.5978	1	n/a	0.040	25	25
KPF	0.5885	1	n/a	0.040	25	25
KVX	0.5680	n/a	0.1709	0.040	25	25
KVF	0.5722	n/a	0.1500	0.040	25	25
For Mean RTK Bedload < 0.025 m/s						
KPX	1.1666	1	n/a	0.040	40	25
KPF	1.1847	1	n/a	0.040	40	25
KVX	1.2475	n/a	0.1179	0.040	40	25
KVF	1.3002	n/a	0.1265	0.040	40	25

a. Results for KVX and KVF filters were estimated from the first optimization. The actual mean values for *Sp* ranged from 0.035 to 0.045 m^2 so a constant value of 0.040 m^2 was chosen for all the filters. For the KVF Filter with RTK Bedload > 0.025 m/s, MB elements to average were closer to 30. As this was the only value that deviated slightly from the other filters at the same moving bed level, this value was estimated to be 25 to maintain a constant number for all filters.

Table 5.5. Final input parameters for the Kalman filter

Filter Types	σ_{vtg} (m/s)	σ_{bt} (m/s)	(σ_{gga}) Positio n (m)	(σ_{gga}) Velocity (m/s)	*Sp* (m^2)	MB Elements to Average	VTG / GGA Elements to Average
For Mean RTK Bedload > 0.025 m/s							
KPX KPF	0.0600	0.1000	1	n/a	0.040	25	25
KVX KVF	0.0360	0.0600	n/a	0.1500	0.040	25	25
For Mean RTK Bedload < 0.025 m/s							
KPX KPF	0.0600	0.0500	1	n/a	0.040	40	25
KVX KVF	0.0360	0.0300	n/a	0.1200	0.040	40	25

5.4.1 Adjustment of Parameters from the Genetic Algorithm

The input parameters determined from the genetic algorithm were adjusted using a heuristic approach in order to determine an optimum set of values that would optimize all of the measurements and not just boat velocity. These trials involved separating the number of VTG elements to average from GGA elements, adjusting the number of moving bed elements to average, and separating the number of moving bed elements to average to determine bottom tracking bias, from the number of bottom track elements to average to determine bottom tracking error. Also, for low bedload transects, separate trials were completed assuming moving bed was present (bias removed), and that moving bed was not present (no bias removal). The various trials that were completed are given in Tables 5.6 to 5.12. Two trials were run for high bedload transects, trials 1a and 2a, and five trials were run for low bedload transects, trials 1, 2, 3, 4 and 5.

Table 5.6. Parameters for Trial 1a, high bedload transects

INPUT PARAMETERS					
Minimum Thresholds		**Ensembles to Average in Estimating the Given Parameter**		**Other Parameters**	
σ_{bt}	0.15 m/s	σ_{bt}	40	*Sp*	0.04 m^2
σ_{vtg}	0.06 m/s	σ_{vtg}	20		
$(\sigma_{gga})_{Position}$	1 m	$(\sigma_{gga})_{Position/Velocity}$	5		
$(\sigma_{gga})_{Velocity}$	0.15 m/s	Moving Bed	5		

Table 5.7. Parameters for Trial 2a, high bedload transects

INPUT PARAMETERS					
Minimum Thresholds		**Ensembles to Average in Estimating the Given Parameter**		**Other Parameters**	
σ_{bt}	0.10 m/s	σ_{bt}	40	*Sp*	0.04 m^2
σ_{vtg}	0.06 m/s	σ_{vtg}	25		
$(\sigma_{gga})_{Position}$	1 m	$(\sigma_{gga})_{Position/Velocity}$	25		
$(\sigma_{gga})_{Velocity}$	0.15 m/s	Moving Bed	40		

Table 5.8. Parameters for Trial 1 for low bedload transects, assuming moving bed

INPUT PARAMETERS					
Minimum Thresholds		**Ensembles to Average in Estimating the Given Parameter**		**Other Parameters**	
σ_{bt}	0.05 m/s	σ_{bt}	40	*Sp*	0.04 m^2
σ_{vtg}	0.06 m/s	σ_{vtg}	25		
$(\sigma_{gga})_{Position}$	1 m	$(\sigma_{gga})_{Position/Velocity}$	25		
$(\sigma_{gga})_{Velocity}$	0.12 m/s	Moving Bed	40		

Table 5.9. Parameters for Trial 2 for low bedload transects, assuming moving bed

INPUT PARAMETERS					
Minimum Thresholds		**Ensembles to Average in Estimating the Given Parameter**		**Other Parameters**	
σ_{bt}	0.05 m/s	σ_{bt}	40	*Sp*	0.04 m^2
σ_{vtg}	0.06 m/s	σ_{vtg}	20		
$(\sigma_{gga})_{Position}$	1 m	$(\sigma_{gga})_{Position/Velocity}$	5		
$(\sigma_{gga})_{Velocity}$	0.12 m/s	Moving Bed	40		

Table 5.10. Parameters for Trial 3 for low bedload transects, assuming no moving bed

INPUT PARAMETERS					
Minimum Thresholds		**Ensembles to Average in Estimating the Given Parameter**		**Other Parameters**	
σ_{bt}	0.1	σ_{bt}	n/a	*Sp*	0.06
σ_{vtg}	0.1	σ_{vtg}	20		
$(\sigma_{gga})_{Position}$	2	$(\sigma_{gga})_{Position/Velocity}$	5		
$(\sigma_{gga})_{Velocity}$	1	Moving Bed	n/a		

Table 5.11. Parameters for Trial 4 for low bedload transects, assuming no moving bed

INPUT PARAMETERS					
Minimum Thresholds		**Ensembles to Average in Estimating the Given Parameter**		**Other Parameters**	
σ_{bt}	0.06	σ_{bt}	n/a	*Sp*	0.04
σ_{vtg}	0.06	σ_{vtg}	20		
$(\sigma_{gga})_{Position}$	1	$(\sigma_{gga})_{Position/Velocity}$	5		
$(\sigma_{gga})_{Velocity}$	0.15	Moving Bed	n/a		

Table 5.12. Parameters for Trial 5 for low bedload transects, assuming no moving bed

INPUT PARAMETERS					
Minimum Thresholds		**Ensembles to Average in Estimating the Given Parameter**		**Other Parameters**	
σ_{bt}	0.02	σ_{bt}	n/a	*Sp*	0.04
σ_{vtg}	0.06	σ_{vtg}	20		
$(\sigma_{gga})_{Position}$	1	$(\sigma_{gga})_{Position/Velocity}$	5		
$(\sigma_{gga})_{Velocity}$	0.15	Moving Bed	n/a		

5.5 Measurements and Performance Statistics

Calculations of boat velocity, boat position, bedload velocity and discharge were completed using measurements collected simultaneously from bottom tracking and Stand Alone GGA and VTG. The bottom tracking velocities used in the calculations were not corrected for bias due to sediment movement, as it was of interest to compare data collected from each source in its raw form. Errors and performance statistics were calculated using RTK GGA as a truth value.

5.5.1 Boat Velocity

Velocities from bottom tracking were collected from a four beam aDcp in instrument coordinates. Four velocity components (fore-aft, port-starboard, vertical, and error) were derived from the four beam velocities using a coordinate transformation matrix. A diagram showing the locations of the beams relative to the forward, aft, starboard and port directions is given in Fig. 5.6.

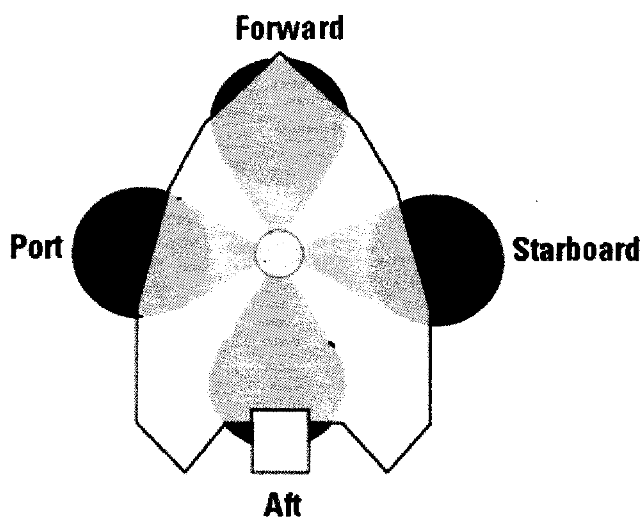

Figure 5.6 Locations of forward, aft, port and starboard locations (Simpson, 2001).

These velocity vectors were transformed into true east and north earth coordinates by rotating them by the aDcp compass heading and the magnetic declination angles for the site using Eqs. 5.3 and 5.4.

$$V_E = -V_1 \cos(\theta_H + \theta_M) + V_2 \sin(\theta_H + \theta_M) \tag{5.3}$$

$$V_N = V_2 \cos(\theta_H + \theta_M) + V_1 \sin(\theta_H + \theta_M) \tag{5.4}$$

Where V_1 is the velocity in the port/starboard direction, with port taken as positive, V_2 is the velocity in the forward/aft direction with forward taken as positive, θ_H is the heading angle for the aDcp compass in degrees, θ_M is the magnetic declination angle in degrees, and V_E and V_N are the true east and north velocities respectively. A diagram showing V_1 and V_2 relative to true east and north is given in Fig. 5.7. As bottom tracking measures the velocity of the sediment bed with respect to the boat, the east and north velocities were made negative to measure the velocity of the boat with respect to the sediment bed.

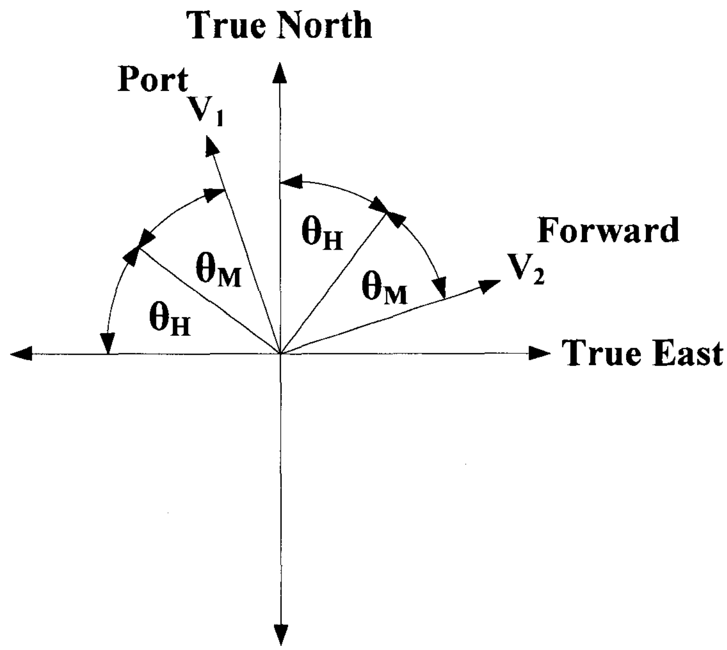

Figure 5.7 Forward and port velocities relative to true north and east.

GGA east and north velocities were derived from GGA east and north positions. The method to calculate GGA position will be described later in this Chapter. GGA velocities were calculated from the change in distance between two successive ensembles, divided by the change in UTC time given in the $GPGGA NMEA string.

VTG velocity was taken directly from the $GPVTG NMEA string. As this string did not contain a time stamp, it was matched to the GGA UTC time stamp in the GGA NMEA string immediately preceding it. The velocities given in the NMEA string were the resultant velocities, and were measured in km/h. They were rotated into true east and north components, and converted to m/s using the formulas 5.5 and 5.6.

$$V_{EVTG} = V_{VTG}\sin(\theta_{VTG})(1000/3600) \tag{5.5}$$

$$V_{NVTG} = V_{VTG} \cos(\theta_{VTG})(1000/3600) \quad (5.6)$$

Where V_{VTG} is the resultant VTG velocity in km/h, θ_{VTG} is the heading angle from true north in degrees, and V_{NVTG} and V_{NVTG} are the final north and east components of the VTG velocity in m/s.

5.5.1.2 Performance Statistics

The statistic used to determine the accuracy of the velocities for each of the input streams and the Kalman filter was the mean velocity error over one transect given in Eq. 5.7.

$$\overline{v_{ERROR}} = \overline{\sqrt{(v_{diff})^2_{east} + (v_{diff})^2_{north}}} \quad (5.7)$$

Where $v_{diff} = v_{given} - v_{rtk\,gga}$ and is calculated for the east and north components separately. The objective was to minimize the mean velocity error.

5.5.2 Boat Position

Boat Position was calculated as a relative position, or distance traveled from an initial starting point of zero, rather than an absolute position. East and north bottom tracking changes in positions were calculated by multiplying the bottom track boat velocity for a particular ensemble by the change in time between the chosen ensemble and its neighboring ensemble immediately following it. The changes in position were then summed to get the total distance traveled.

GGA positions were calculated from values stored in the GGA NMEA string. The horizontal positions in the $GPGGA string were recorded in terms of longitude and latitude. The distances between neighboring positions were calculated in meters by using the simplified ellipsoid technique as used in the WinRiver software by RD Instruments (Rennie and Rainville, 2006). This technique approximated the earth to have an ellipsoidal shape. There are two radii of interest in this technique. The radius of curvature in the prime verticle (R_E) is used to calculate the distance in the east/west direction and is given by Eq. 5.8.

$$R_E = \frac{E\pi}{180}\left(1 + \varepsilon \sin^2\left(\frac{\pi(Lt_1 + Lt_2)}{180(2)}\right)\right) \tag{5.8}$$

Where E is the radius of the Earth along the semi-major axis and equals 6 378 137 m. Lt_1 and Lt_2 are the latitudes in decimal degrees for the current and new positions, respectively. ε is the ellipticity and is equal to 1/298.257223563 (Tsui, J. B., 2005; Rennie and Rainville, 2006). The change in distance (Δx) in the east/west direction is calculated using Eq. 5.9.

$$\Delta x = R_E(Lg_2 - Lg_1)\cos\left(\frac{\pi(Lt_1 + Lt_2)}{180(2)}\right) \tag{5.9}$$

The second radius of interest is the radius of curvature in the meridian (R_N), and is given by Eq. 5.10.

$$R_N = \frac{E\pi}{180}\left(1 - 2\varepsilon + 3\varepsilon\sin^2\left(\frac{\pi(Lt_1 + Lt_2)}{180(2)}\right)\right) \qquad (5.10)$$

The change in distance (Δy) in the north/south direction is then calculated as this radius times the change in latitude between the two points, and is given by Eq. 5.11.

$$\Delta y = R_n(Lt_2 - Lt_1) \qquad (5.11)$$

The distance traveled can then be determined by summing the changes in distance between successive points in each direction.

VTG changes in position are determined by multiplying the VTG velocity for a given ensemble by the change in UTC time between given ensemble and the ensemble immediately following. The distance traveled is again determined by summing the changes in position.

It is very important to note that out of VTG, GGA and bottom tracking, GGA position normally produces the most accurate position trajectory. This is because GGA position is calculated from longitude and latitude. Estimates of position based on integration of velocities over time from VTG, bottom tracking and GGA velocities will have great inaccuracies when there is a high percentage of missing data, or when time measurements are inaccurate. In these cases missing data are interpolated, which can lead the trajectories to be over or under estimated.

5.5.2.1 Performance Statistics

The accuracies of the position estimates were determined by calculating the mean position error which is given in Eq. 5.12.

$$\varepsilon_{position} = \sqrt{\left(x_{RTK} - x_{given}\right)^2 + \left(y_{RTK} - y_{given}\right)^2} \tag{5.12}$$

Where x_{RTK} and y_{RTK} and y are east and north RTK positions, respectively, and x_{Given} and y_{Given} are the east and north given positions, respectively, measured in meters from an initial starting point of zero.

Distance Made Good (DMG, Course Made Good (CMG) and total Length were also calculated in Matlab. DMG is a measure of the straight line distance between the start and end points. CMG is the angle from the start position to the end position with respect to true north, and total Length is the total distance calculated along the trajectory. Biases and discrepancies for CMG, DMG and Length were also determined.

5.5.3 Bedload Velocity

Mean bedload velocity for a transect was calculated using Eqs. 5.14 to 5.16.

$$\left(v_{mb}\right)_{east} = \left(v_{Ebt}\right) - \left(v_{given}\right)_{east} \tag{5.14}$$

$$\left(v_{mb}\right)_{north} = \left(v_{Nbt}\right) - \left(v_{given}\right)_{north} \tag{5.15}$$

$$(v_{mb})_{mean} = \sqrt{\left((v_{mb})_{north}\right)^2 + \left((v_{mb})_{east}\right)^2} \quad (5.16)$$

Where v_{bt} is the bottom track velocity, v_{given} is either the GGA, VTG or Kalman filter velocity, and v_{mb} is the bedload velocity. The mean bedload is given in Eq. 5.16. It should be noted that mean transect bedload velocity given by Eq. 5.16 is a better indicator of bedload magnitude compared to calculating bedload on a single ping basis since GPS errors will tend to cancel out when mean bedloads are averaged over the east and north directions.

5.5.3.1 Performance Statistics

The bias and discrepancies of the mean bedload were calculated with respect to RTK GGA.

5.5.4. Discharge

Discharge across a transect was also calculated using the WinRiver software by merging data output from the filter into the original binary files and replacing the original velocity reference. The formula used to compute discharge by WinRiver is from Gordon (1989) and is given by the dot product in Eq. 5.17.

$$Q_D = \iint n \cdot U dz_d dL \quad (5.17)$$

Where Q_D is the measured discharge in m^3/s, n is the unit vector normal to the transect path at L, U is the water velocity vector in m/s, z_d is the depth in m of the river and L is a location along the transect path as shown in Fig. 5.8.

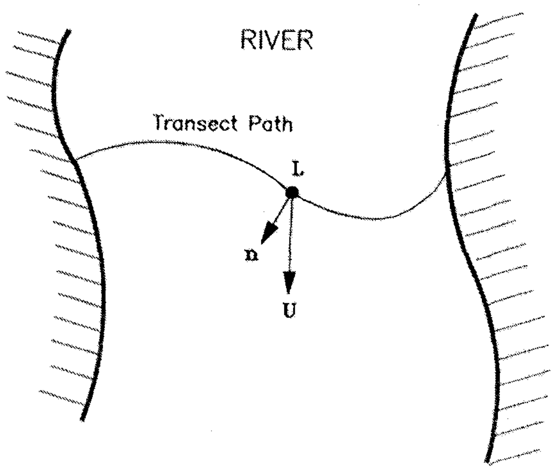

Figure 5.8 Transect path and vectors for calculating discharge (Gordon, 1989).

The discharge used in this study is the measured discharge computed by WinRiver. Measured discharge does not include some missing data from the surface, bottom and sides of the river. Missing data at the surface is the unmeasured area above the first depth cell. Missing data at the bottom is the unmeasured area due to side lobe interference for the aDcp and missing data at the sides is due to measurement limitations of the aDcp at low depths. These areas can be seen in Fig. 5.9.

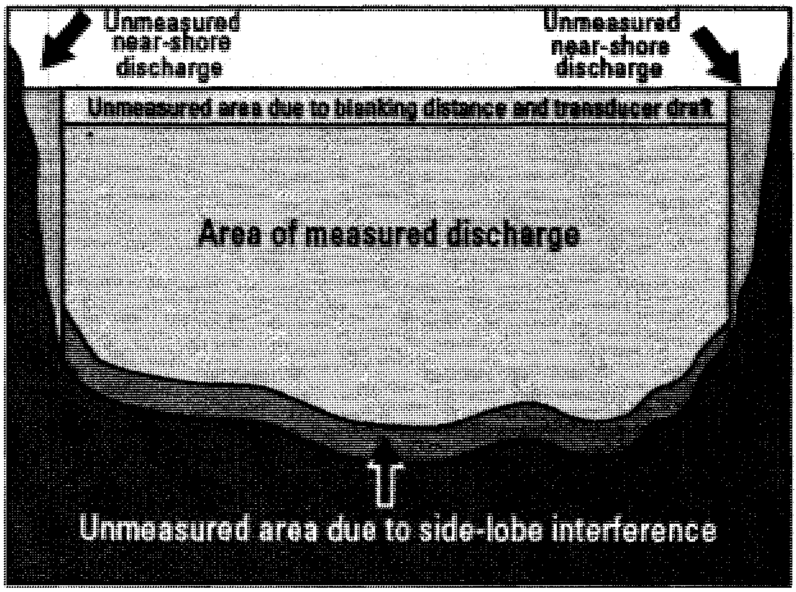

Figure 5.9 Measured and unmeasured areas for discharge calculations (Simpson, 2001).

5.5.4.1 Performance Statistics

Absolute percentage differences (*D*) in discharge calculations were computed according to Eq. 5.18.

$$D = \frac{\left|Q_{estimate} - Q_{known}\right|}{Q_{known}} \% \qquad (5.18)$$

Where Q_{known} is the measured RTK discharge, $Q_{estimate}$ is the discharge calculated using VTG, GGA, BT or the Kalman as the reference boat velocity. The expected error involved in the merging method was determined by calculating the absolute percentage differences between RTK GGA discharge calculated in WinRiver from the original binary file, and RTK GGA discharge calculated by merging the RTK GGA velocity back into the bottom track reference velocity. The error associated with the merging method was estimated to be 1.14 %.

5.6 Validation of Methods

Accuracy of Matlab scripts and routines used for calculating GPS and bottom tracking trajectories were validated by comparing results generated by these scripts to those generated by the Teledyne RD Instruments WinRiverII software. This version of Teledyne's software used improved methods for calculating GPS and bottom tracking values compared to its previous version WinRiverI. Figure 5.10 compares trajectories calculated by the Matlab scripts to those calculated by WinRiverII for bottom tracking, RTK GGA and RTK VTG. Both results appeared to be very close. These results were further checked by comparing values of CMG, DMG and Length for each of the trajectories (Table 5.13). Results were extremely close for GGA and VTG with differences in DMG, CMG and Length being less than or equal to 0.01 m, 0.03 °, and 0.02 m respectively. Bottom tracking showed slightly greater differences likely due to differences in how each program dealt with missing data, since the bottom track signal for this transect contained 30 missing elements.

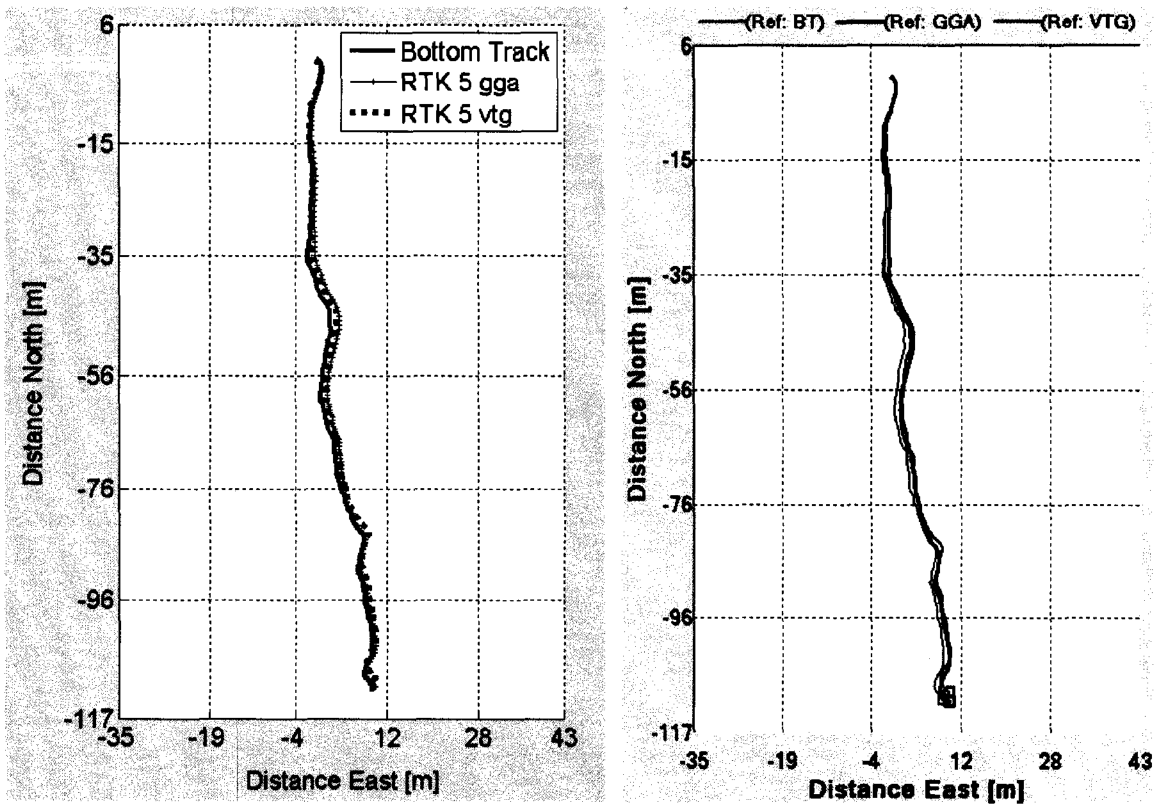

Figure 5.10 Plot of bottom track, GGA, and VTG trajectories calculated by Matlab scripts (Left) and by WinRiverII program (Right) for transect 05bm004_20060531002r at Bow River.

Table 5.13. Comparison of trajectories calculated by Matlab script and WinRiverII software for transect 05bm004_20060531002r

	Trajectory generated by Matlab script	Trajectory generated by WinRiverII software	Number of Missing Ensembles
RTK GGA Trajectory			
Distance Made Good (DMG) (m)	111.54	111.54	0
Course Made Good (CMG) (°)	174.90	174.88	
Total Length (m)	116.34	116.32	
RTK VTG Trajectory			
Distance Made Good (DMG) (m)	109.87	109.89	0
Course Made Good (CMG) (°)	174.78	174.75	
Total Length (m)	115.84	115.84	
Bottom Tracking Trajectory			
Distance Made Good (DMG) (m)	112.09	111.23	30
Course Made Good (CMG) (°)	175.17	175.43	
Total Length (m)	117.36	116.98	

A check was also done to validate the accuracy of methods used to merge GGA velocities back into the bottom reference velocity in the original binary file. This check was done using the data from the Gatineau River from Rennie and Rainville (2006a). Table 5.14 shows results obtained using the Matlab scripts in this project, compared to the discharge results obtained by Rennie and Rainville (2006a). It can be seen that discharge results were almost identical for most transects, with the greatest difference being 0.4 m^3/s for transect 7 with WAAS GGA.

Table 5.14. Comparison of discharge results for the Gatineau River obtained using GGA velocities calculated from Matlab scripts in this project to results obtained by Rennie and Rainville (2006a)

Transect	**Discharge Reported in Rennie and Rainville (2006a)**		**Discharge determined with GGA velocities from the Matlab Script**	
	RTK GGA (m^3/s)	**WAAS GGA (m^3/s)**	**RTK GGA (m^3/s)**	**WAAS GGA (m^3/s)**
1	626.7	618.5	626.7	618.5
2	639.7	638.7	639.7	638.7
3	629.9	627.2	629.9	627.1
4	636.8	638.5	636.8	638.6
5	635.6	618.4	635.6	618.4
6	640.4	630.3	640.4	629.2
7	619.0	606.4	619.1	606.0
8	631.8	633.1	631.8	633.0

CHAPTER 6

RESULTS

A complete set of Figures showing results for boat velocities, boat trajectories and bedload velocities for each transect are given in Appendix H.

6.1 Boat Velocity

Accuracies of Kalman velocity estimates were evaluated in terms of mean velocity errors. Data tables for trials 1a (high bedload) and 5 (low bedload) are given in Appendix D. It should be noted that for all results, the number of digits shown after the decimal places were usually greater than the number of significant digits in order to show very low differences in the results. Results indicated that the Kalman filter was able to improve boat velocity estimates compared to Stand Alone GGA, VTG and biased bottom tracking. Figure 6.1 shows an example of velocity estimates for a transect with high bedload. Large errors were seen for Stand Alone GGA velocities, which were to be expected, as these errors commonly occured due to multipath at channel margins. Bottom tracking showed a bias, and the Kalman (KPX) velocity fell somewhere between unbiased bottom tracking, VTG and GGA, being quite close to the truth value of RTK GGA.

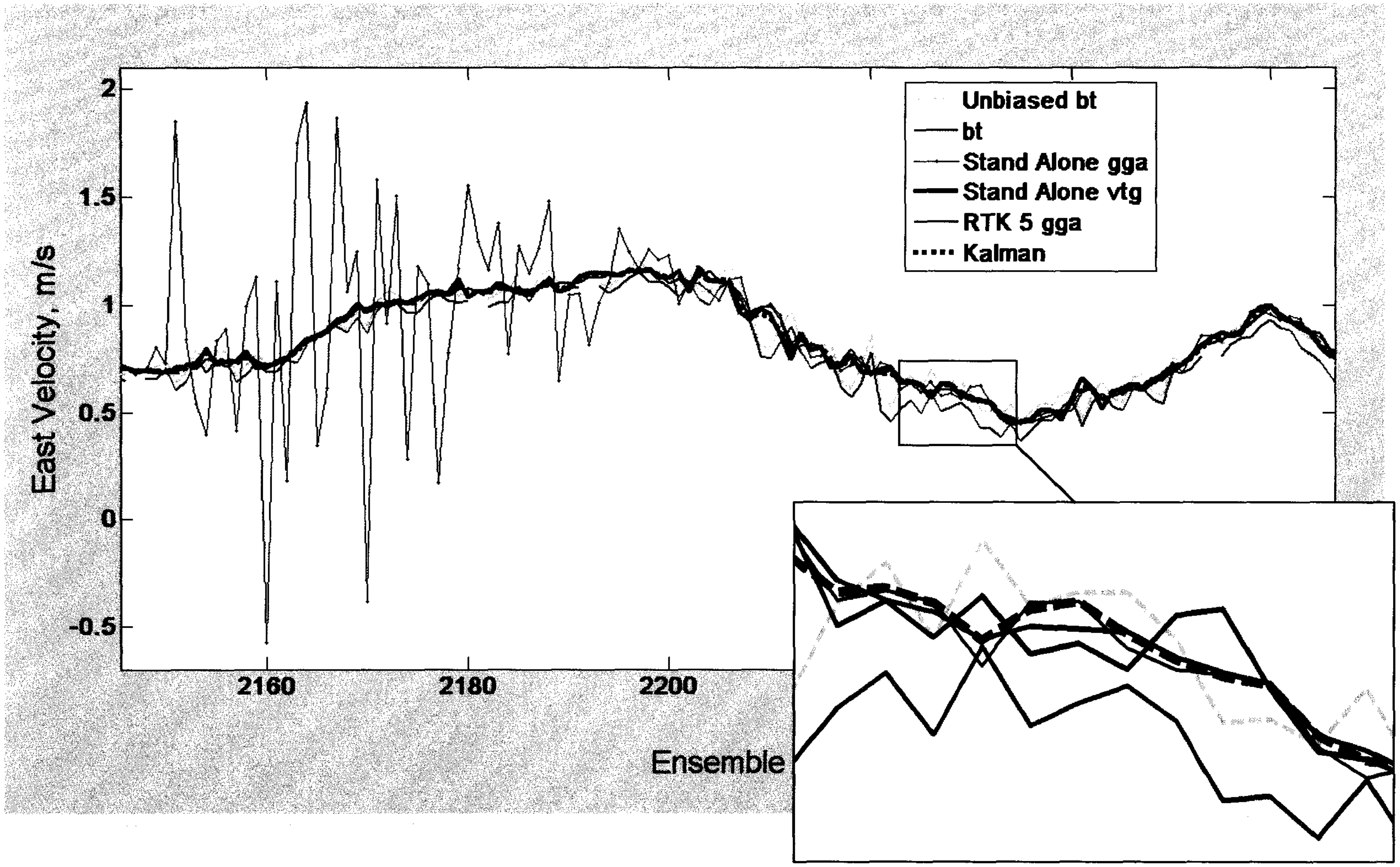

Figure 6.1 Example of boat velocity estimates for unbiased Bottom Tracking, biased Bottom Tracking (bt), Stand Alone GGA , Stand Alone VTG, RTK GGA and the Kalman (KPX) for transect 10ED001_20060705001r.

The Kalman filter produced a lower mean velocity error compared to each of Stand Alone VTG, GGAX, GGAF and BT in at least 10 out of 11 high bedload transects (Table 6.1). Complete results for ANOVA significance testing (α≤0.05) are given in Appendix E. Mean velocity errors were most frequently significantly different between the Kalman filter and Stand Alone GGAX (in at least 10 out of 11 transects). The Kalman filter also usually showed significantly lower errors than bottom tracking. The results for the Kalman filter were very close to VTG, which is reflected in the fact that the Kalman velocity error was only significantly lower than VTG in 2 out of 11 transects. There were no significant differences between the velocity errors between of the Kalman filter types. Figure 6.2 shows mean velocity errors for the high bedload transects.

Table 6.1. Summary of mean velocity errors for transects with bedload > 2.5 cm/s, Trial 1a

Kalman Filter	Given Reference Mean Velocity Error			
	VTG	**GGAX**	**GGAF**	**BT**
	Number of Transects where Kalman Mean Velocity Error is Lower than the Given Reference Mean Velocity Over Total Transects			
KPX	11/11	11/11	11/11	10/11
KPF	11/11	10/11	11/11	10/11
KVX	10/11	11/11	11/11	10/11
KVF	10/11	11/11	11/11	10/11
	Number of Transects where Kalman Mean Velocity Error is <u>Significantly</u> Lower than the Given Reference Mean Velocity Error			
KPX	2/11	11/11	5/11	8/11
KPF	2/11	10/11	5/11	8/11
KVX	2/11	11/11	6/11	8/11
KVF	2/11	11/11	6/11	8/11

For low bedload transects, the Kalman filter produced a lower velocity error compared to VTG in at least 19 of 20 transects, although it was only significantly lower in 8 of 20 transects (Table 6.2). The Kalman filter performed the best compared to Stand Alone GGAX, as it was significantly lower than GGAX in at least 15 of 20 transects. The Kalman errors were only significantly lower than bottom tracking in 8 out of 20 transects. Again, there were no significant differences found between the velocity errors for each of the Kalman filters. Figure 6.3 shows errors for each of the low bedload transects.

Table 6.2. Summary of mean velocity errors for transects with bedload < 2.5 cm/s, Trial 5

Kalman Filter	Given Reference Mean Velocity Error			
	VTG	GGAX	GGAF	BT
	Number of Transects where Kalman Mean Velocity is Lower than the Given Reference Mean Velocity Over Total Transects			
KPX	20/20	17/20	14/20	14/20
KPF	20/20	17/20	12/20	15/20
KVX	19/20	17/20	13/20	14/20
KVF	19/20	17/20	13/20	12/20
	Number of Transects where Kalman Mean Velocity Error is Significantly Lower than the Given Reference Mean Velocity Error			
KPX	8/20	15/20	13/20	8/20
KPF	8/20	16/20	11/20	8/20
KVX	8/20	16/20	12/20	8/20
KVF	8/20	16/20	12/20	8/20

Prefiltering of Stand Alone GGA before using each of the Kalman filters did not significantly improve velocity estimates. Prefiltered GGA, however, did show significantly lower errors compared to unfiltered GGA in 19 out of 31 transects.

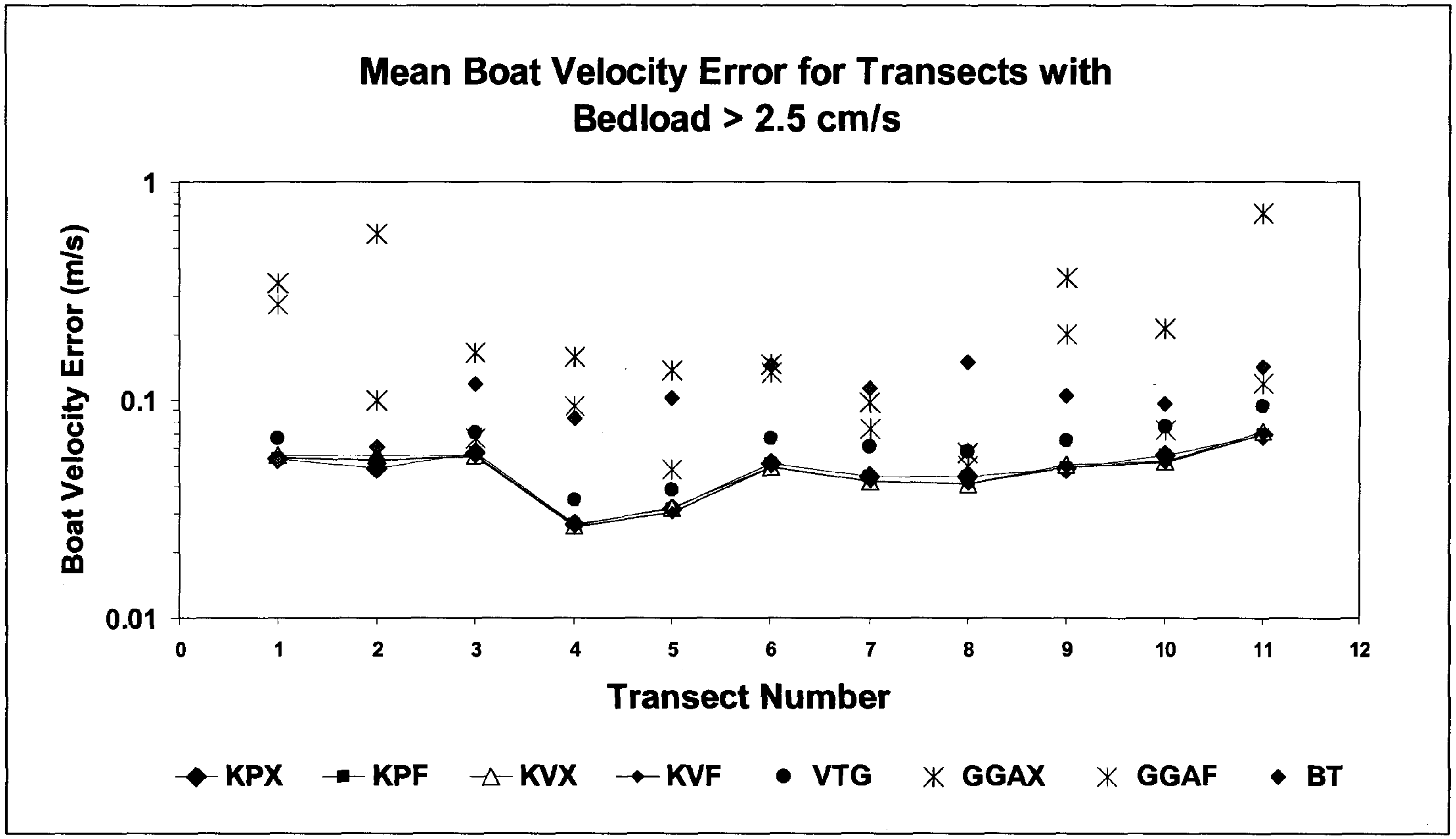

Figure 6.2 Boat velocity errors for transects with high bedload (> 2.5 cm/s), Trial 1a.

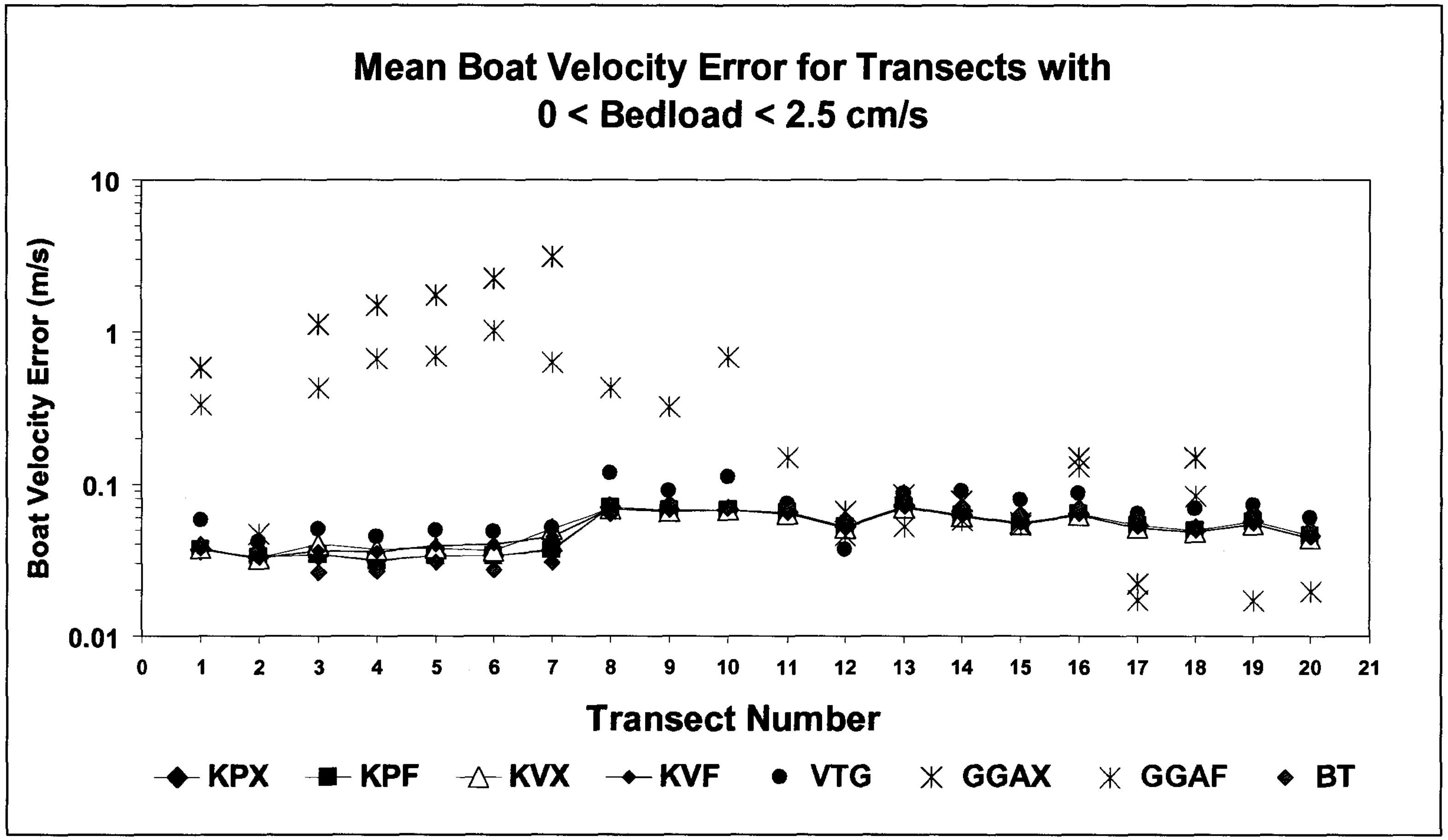

Figure 6.3 Boat velocity errors for transects with low bedload (< 2.5 cm/s), Trial 5.

6.2 Boat Position

Complete results for Mean Position Errors, Course Made Good (CMG), Distance Made Good (DMG) and Length are all given in Appendix F. Position results were determined using parameters from trial 1a (for high bedload transects) and trial 5 (for low bedload transects). The Kalman position filter was designed to primarily follow the path of GGA position, while the Kalman velocity filter was able to consider more fairly each of the input signals by assigning weightings to them. Figure 6.4 shows an example of the Kalman filter estimate of position using the KPX filter. It can be seen that although the filter is primarily following Stand Alone GGA, it is able to reduce the noise in the GGA signal. This is an example of a loop test that is used to determine if moving bed is present. Since all of the trajectories come back to their initial position, with the exception of bottom tracking, it can be concluded that moving bed is present and that the bottom track signal is biased.

The accuracies of position estimates were evaluated by considering only transects where Stand Alone GGA was not offset, as the Kalman position filter gave inaccurate results when it was offset. This was done in order to make recommendations on which filter to use under normal operating conditions. Stand Alone GGA was considered offset if its trajectory differed from both Stand Alone VTG and RTK GGA by a mean of at least 2.5 m in both the east and north directions. Transects that were offset and their mean offsets are given in Table 6.3. A total of 26 out of 31 transects that were not offset were therefore used to evaluate position. Figure 6.5 shows an example of position estimates for a transect where Stand Alone GGA is offset.

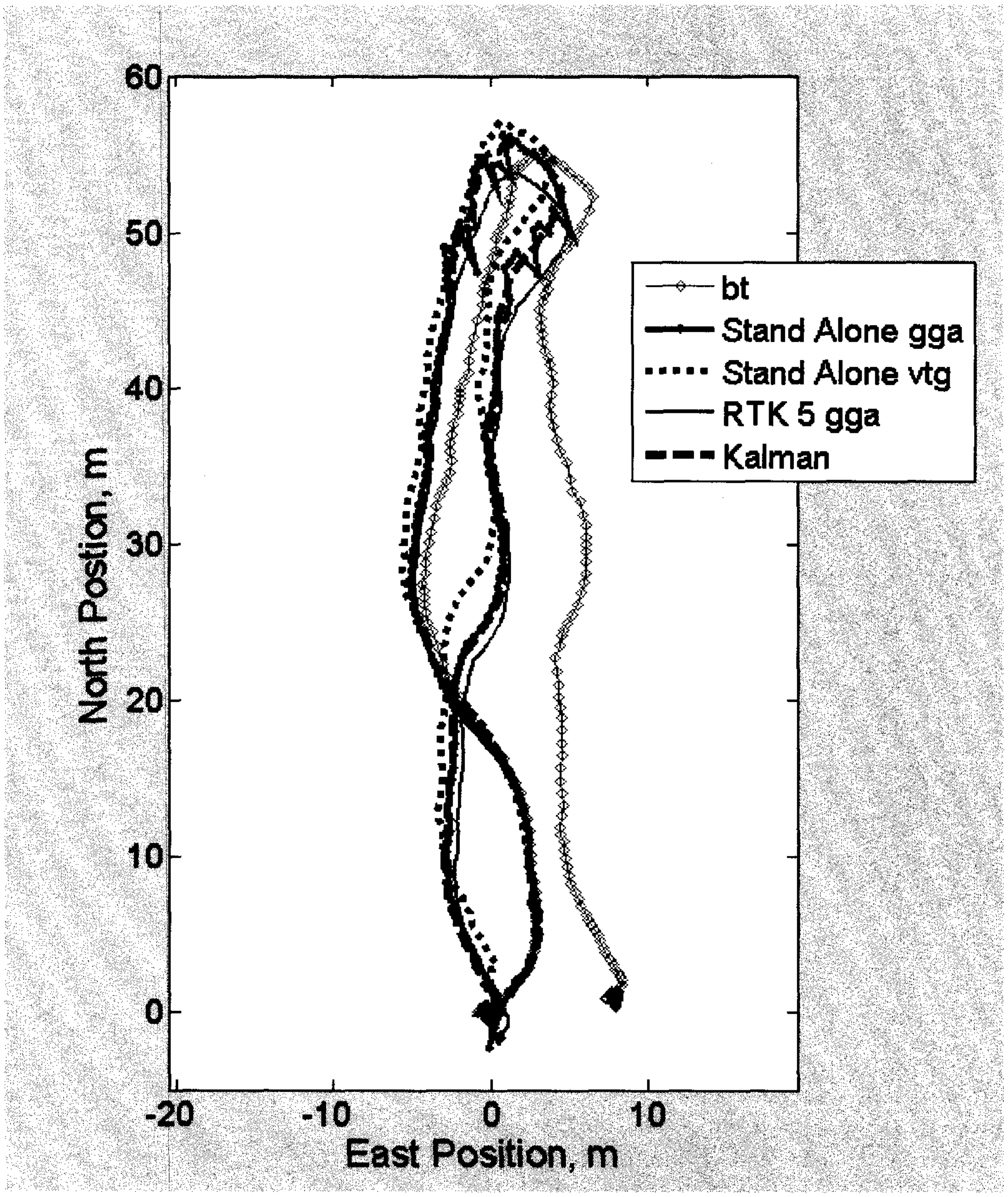

Figure 6.4 Example of KPX prediction of position for transect 07cd005_20060409005r which is a loop test.

Table 6.3. Transects considered to be offset and the amounts of their offset

Transect Name	Offset of East Stand Alone GGA from East RTK GGA (m)	Offset of North Stand Alone GGA from North RTK GGA (m)	Offset of North Stand Alone GGA from East Stand Alone VTG (m)	Offset of North Stand Alone GGA from North Stand Alone VTG (m)
07cd005_20060409002r	4.0655	-2.5293	4.0945	-3.5371
10EB001_20060623000r	7.1876	2.8663	15.5210	4.8134
07dd002_20060621005r	4.5178	8.5652	4.3821	7.4298
10EA003_20060623000r	9.6294	9.7987	10.6512	16.0044
10ED001_20060705000r	12.4306	-20.8884	18.0665	-30.1264

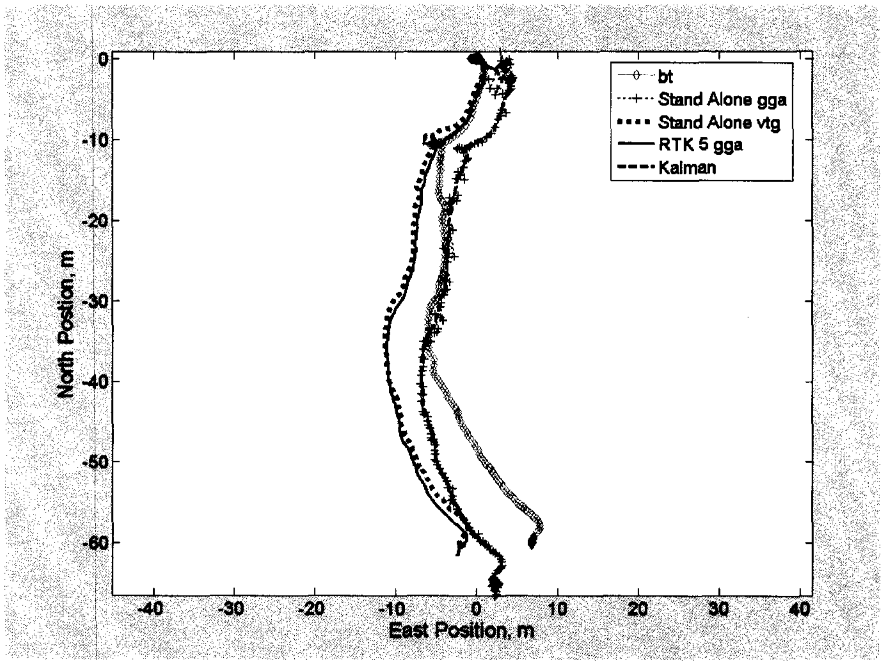

Figure 6.5 Transect 07cd005_20060409002r where Stand Alone GGA is offset.

6.2.1. Mean Boat Position

It was found that for transects with high bedloads (trial 1a) the Kalman filter gave the lowest mean position errors compared to Stand Alone VTG, GGAX, GGAF and BT in 6 out of 9 transects. It was also found that each Kalman filter always produced a lower mean position error compared to bottom tracking (Table 6.4). Although the Kalman filter mean position errors were usually lower than GGAX and GGAF position errors, they were not normally significantly lower. The Kalman filter showed a lower position error compared to Stand Alone VTG in 3 to 5 out of 9 transects, although significance was low. The results for low bedload transects (trial 5) are given in Table 6.5, with the Kalman filter showing the lowest position error compared to all other signals in 6 of 17 transects. The Kalman filter showed the best performance compared to bottom tracking, as Kalman errors were significantly lower than bottom tracking position in at least 14 of 17 transects. The results for Stand Alone VTG, GGAX and GGAF were all very similar. The Kalman Position filter gave lower mean position errors than the Kalman Velocity filter in at least 17 out of 26 transects (Table 6.6), with at least 8 out of 26 transects being significantly lower.

Table 6.4. Summary of mean position errors for transects with bedload > 2.5 cm/s, Trial 1a

Kalman Filters	Mean Position Error Measured with the given Reference			
	VTG	**GGAX**	**GGAF**	**BT**
	Number of Transects where Kalman Mean Position Error was Lower than the Given Reference Mean Position Error over Total Transects			
KPX	4/9	7/9	5/9	9/9
KPF	4/9	7/9	6/9	9/9
KVX	3/9	5/9	5/9	9/9
KVF	5/9	5/9	5/9	9/9
	Number of Transects where Kalman Mean Position Error was Significantly Lower than the Given Reference Mean Position Error over Total Transects			
KPX	2/9	0/9	0/9	9/9
KPF	2/9	0/9	0/9	9/9
KVX	0/9	2/9	2/9	9/9
KVF	0/9	2/9	2/9	9/9

Table 6.5. Summary of mean position errors for transects with bedload < 2.5 cm/s, Trial 5

Kalman Filters	Mean Position Error Measured with the Given Reference			
	VTG	GGAX	GGAF	BT
	Number of Transects where Kalman Mean Position Error is Lower than the Given Reference Mean Position Error over Total Transects			
KPX	10/17	12/17	12/17	15/17
KPF	9/17	13/17	11/17	17/17
KVX	4/17	5/17	5/17	16/17
KVF	7/17	5/17	5/17	17/17
	Number of Transects where Kalman Mean Position Error is Significantly Lower than the Given Reference Mean Position Error over Total Transects			
KPX	6/17	7/17	7/17	14/17
KPF	6/17	7/17	5/17	16/17
KVX	3/17	5/17	5/17	15/17
KVF	4/17	5/17	5/17	15/17

Table 6.6. Comparison of mean position errors for the Kalman position and velocity filters

Kalman Filters	Mean Position Error Measured with the Kalman Velocity Filters			
	KVX	KVF	KVX	KVF
	Number of Transects where Kalman Position Filter had a Lower Mean Position Error then the Kalman Velocity Filter over Total Transects		Number of Transects where Kalman Position Filter had a Significantly Lower Mean Position Error then the Kalman Velocity Filter over Total Transects	
KPX	17/26	18/26	8/26	11/26
KPF	17/26	18/26	10/26	11/26

6.2.2. Course Made Good

It was found that the Kalman filter produced the lowest Course Made Good (CMG) bias compared to all other signals in 4 out of 9 cases for high bedload transects, and 4 out of 17 cases for low bedload transects. Overall, the Kalman position filter showed the lowest variances for CMG bias (Table 6.7) when compared to all other estimates. It was found that prefiltering of GGA tended to decrease the accuracy of CMG estimates. It was also found that when using parameters from trial 5 for low bedload transects, the Kalman velocity filter CMG estimates tended to be biased due to biases in the bottom tracking signal as can be seen in Figs. 6.6 and 6.7.

Table 6.7. Course Made Good bias summary

CMG biases for transects with Bedload > 2.5 cm/s, Trial 1a, (degrees)								
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**GGAF**	**BT**
Mean	-0.03918	-0.03883	-0.05323	0.11305	0.05899	-0.02901	0.01224	0.78569
Median	-0.05855	-0.05875	-0.02515	0.09140	0.02700	-0.03905	0.09160	2.05340
St. Dev.	0.11820	0.11833	0.26793	0.19835	0.14997	0.19928	1.10919	5.76793
Variance	0.01397	0.01400	0.07179	0.03934	0.02249	0.03971	1.23030	33.26907
CMG biases for transects with Bedload < 2.5 cm/s, Trial 5, (degrees)								
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**GGAF**	**BT**
Mean	-0.34212	-0.18908	-0.70344	-0.88172	-0.16706	-0.20714	11.73054	-1.00279
Median	-0.11010	-0.11010	-0.51550	-0.51370	0.19260	-0.05720	0.15400	-0.50550
St. Dev.	1.38819	1.11590	2.06886	2.26697	1.43727	1.43944	44.36321	2.78215
Variance	1.92707	1.24523	4.28018	5.13917	2.06576	2.07199	1968.09446	7.74036

6.2.3 Distance Made Good

For transects with high moving bed, the Kalman filter produced the lowest bias for Distance Made Good (DMG) compared to all other signals in 9 out of 9 transects. For transects with low moving bed, the Kalman filter produced the lowest DMG bias in 10 out of 17 transects. Filtered Stand Alone GGA gave a greater mean, median, standard deviation and variance of the bias for all transects compared to unfiltered Stand Alone GGA (Table 6.8). Bottom tracking also showed great variance, although for transects with low bedloads the biases were usually lower compared to the other estimates. The Kalman position filters generally gave the lowest standard deviations and variances of the bias compared to all other estimates.

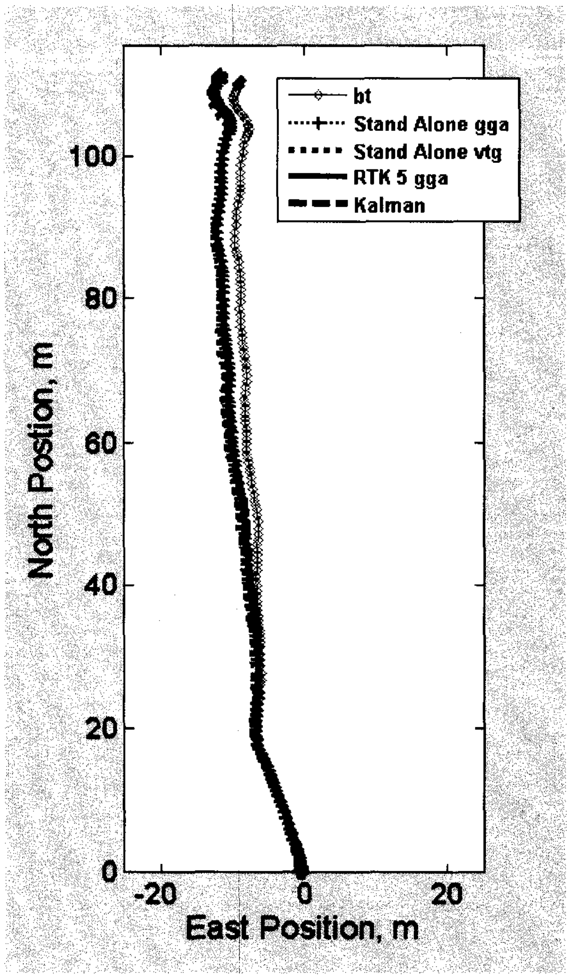

Figure 6.6 Results with KPX filter for transect 10ED001_20060705000r (low bedload).

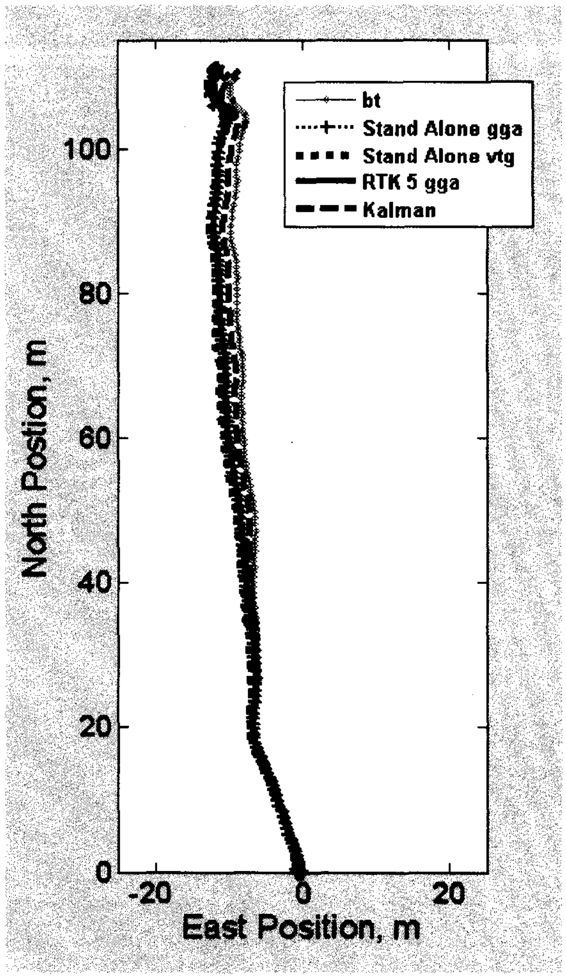

Figure 6.7 Results with KVX filter for transect 10ED001_20060705000r (low bedload)

Table 6.8. Distance Made Good bias summary

DMG biases for transects with Bedload > 2.5 cm/s, Trial 1a, (m)								
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**GGAF**	**BT**
Mean	-0.47485	-0.46980	-0.57471	-0.15146	-0.13509	0.20012	-1.53438	-3.81871
Median	-0.14932	-0.12549	-0.63362	-0.14097	-0.06801	-0.20277	0.61480	-2.62799
St. Dev.	0.93417	0.93394	1.35190	0.89361	1.08436	2.16454	5.72181	9.65913
Variance	0.87267	0.87224	1.82763	0.79854	1.17583	4.68524	32.73913	93.29884
DMG biases for transects with Bedload < 2.5 cm/s, Trial 5, (m)								
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**GGAF**	**BT**
Mean	0.41485	0.61373	-0.46463	-0.41979	-0.58354	0.58111	4.78238	0.03095
Median	0.31220	0.40780	-0.53080	-0.30400	-0.17010	0.58790	1.29850	0.00620
St. Dev.	0.85896	0.68635	1.18396	1.11500	1.16280	1.18534	13.25506	1.59745
Variance	0.73782	0.47107	1.40176	1.24322	1.35210	1.40504	175.69662	2.55183

6.2.4 Total Length

The Kalman filters and VTG all gave fairly close values for length biases, although the Kalman velocity filter showed less variation compared to the position filter, particularly for transects with low moving bed (Table 6.9). Bottom tracking gave reasonable estimates for low bedload transects, but showed great variation for high bedload transects. Stand Alone GGAX and GGAF both tended to greatly over predict length estimates. Overall, the Kalman filter gave the lowest length bias in 3 out of 9 transects with high moving bed, and in 7 out of 17 transects with low moving bed.

Table 6.9. Length bias summary

Length biases for Transects with Bedload > 2.5 cm/s, Trial 1a, (m)								
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**GGAF**	**BT**
Mean	-1.09426	-1.25343	-1.82719	-2.17433	-0.67576	65.22710	12.55776	0.87552
Median	-1.35970	-1.31480	-1.65840	-1.33590	-0.58160	25.46080	10.02000	-0.50630
St. Dev.	3.39733	2.88917	2.38914	2.67318	1.96899	109.56197	15.06786	11.54063
Variance	11.54188	8.34730	5.70798	7.14591	3.87694	12003.82596	227.04026	133.18603
Length biases for transects with Bedload < 2.5 cm/s, Trial 5, (m)								
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**GGAF**	**BT**
Mean	3.62551	2.51739	-0.45302	-0.45241	0.77846	90.84296	40.64788	0.56726
Median	1.12240	1.12240	-0.57840	-0.47270	0.66280	15.18980	11.14710	0.28640
St. Dev.	5.39741	3.33806	1.24310	1.22922	0.98091	138.20850	55.66417	1.55059
Variance	29.13203	11.14263	1.54530	1.51099	0.96219	19101.59050	3098.49980	2.40434

6.3 Apparent Bedload Velocity

Mean transect apparent bedload velocities were only estimated for transects with mean RTK bedloads > 2.5 cm/s (Table 6.10). Kalman filter bedload biases were often very close to VTG as can be seen in Table 6.11 and Fig. 6.8. The Kalman filter gave the lowest bedload biases in 9 out of 11 transects, although it was equal to VTG for 2 of these 9 transects. Stand Alone VTG gave the lowest bedload biases in 4 out of 11 transects. Bedload biases for all reference velocities tended to be fairly low, ranging from -1.7 cm/s to 1.25 cm/s, and there were no significant difference in the absolute mean biases between any of the Kalman filters and VTG, or between themselves. Unfiltered GGA did have a significantly different absolute mean bias compared to VTG, KPX and KPF. There were no obvious improvements in bedload estimates as a result of filtering GGA prior to using the Kalman filter, or between using filtered GGA versus unfiltered GGA. In fact, bedload estimates were sometimes worse when GGA was filtered. From Fig. 6.8 it can be seen there were few differences in the mean estimates from the position and velocity Kalman filters except for transects 2 and 10.

Table 6.10. Mean bedload for transects with RTK bedload >2.5 cm/s

MEAN BEDLOADS MEASURED WITH EACH REFERENCE VELOCITY (m/s)								
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**GGAF**	**RTK**
07cd005 _20060409002r	0.0284	0.0284	0.0267	0.0281	0.0292	0.0232	0.0289	0.0303
10EB001 _20060623000r	0.0293	0.0293	0.0354	0.0351	0.0306	0.0115	0.0231	0.0285
10EB001 _20060623003r	0.0919	0.0919	0.0920	0.0918	0.0918	0.0981	0.0939	0.0889
10ED001 _20060705001r	0.0583	0.0583	0.0579	0.0579	0.0584	0.0559	0.0591	0.0578
10ED001 _20060705002r	0.0257	0.0257	0.0258	0.0255	0.0258	0.0256	0.0246	0.0260
10ED002 _20060621000r	0.0615	0.0615	0.0628	0.0622	0.0615	0.0654	0.0644	0.0609
10ED002 _20060621001r	0.0568	0.0568	0.0560	0.0563	0.0566	0.0553	0.0590	0.0564
10ED002 _20060621002r	0.0658	0.0658	0.0658	0.0660	0.0665	0.0661	0.0687	0.0643
10ED002 _20060621003r	0.0610	0.0610	0.0624	0.0618	0.0611	0.0645	0.0708	0.0583
transect004 _ens1000r	0.0781	0.0780	0.0658	0.0828	0.0730	0.0564	0.0697	0.0729
transect003 _ens1000r	0.0342	0.0342	0.0337	0.0349	0.0340	0.0300	0.0425	0.0330

Table 6.11. Bedload biases for transects with RTK bedload >2.5 cm/s

BIASES FOR EACH REFERENCE VELOCITY (m/s)							
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**GGAF**
07cd005 _20060409002r	-0.0019	-0.0019	-0.0036	-0.0022	***-0.0011***	-0.0072	-0.0014
10EB001 _20060623000r	***0.0009***	***0.0009***	0.0069	0.0066	0.0022	-0.0170	-0.0053
10EB001 _20060623003r	0.0030	0.0030	0.0031	***0.0029***	***0.0029***	0.0092	0.0050
10ED001 _20060705001r	0.0005	0.0005	0.0002	***0.0001***	0.0007	-0.0018	0.0013
10ED001 _20060705002r	-0.0003	-0.0003	***-0.0002***	-0.0005	***-0.0002***	-0.0004	-0.0014
10ED002 _20060621000r	***0.0006***	***0.0006***	0.0019	0.0013	***0.0006***	0.0045	0.0035
10ED002 _20060621001r	0.0004	0.0004	-0.0004	***-0.0001***	0.0002	-0.0010	0.0026
10ED002 _20060621002r	***0.0015***	***0.0015***	0.0016	0.0017	0.0022	0.0018	0.0044
10ED002 _20060621003r	0.0028	***0.0027***	0.0041	0.0035	0.0028	0.0062	0.0125
transect004 _ens1000r	0.0052	0.0051	-0.0071	0.0100	***0.0001***	-0.0165	-0.0032
transect003 _ens1000r	0.0012	0.0012	***0.0007***	0.0019	0.0010	-0.0030	0.0095

Note: Values shown in bold italic represent the lowest bedload bias for each particular transect.

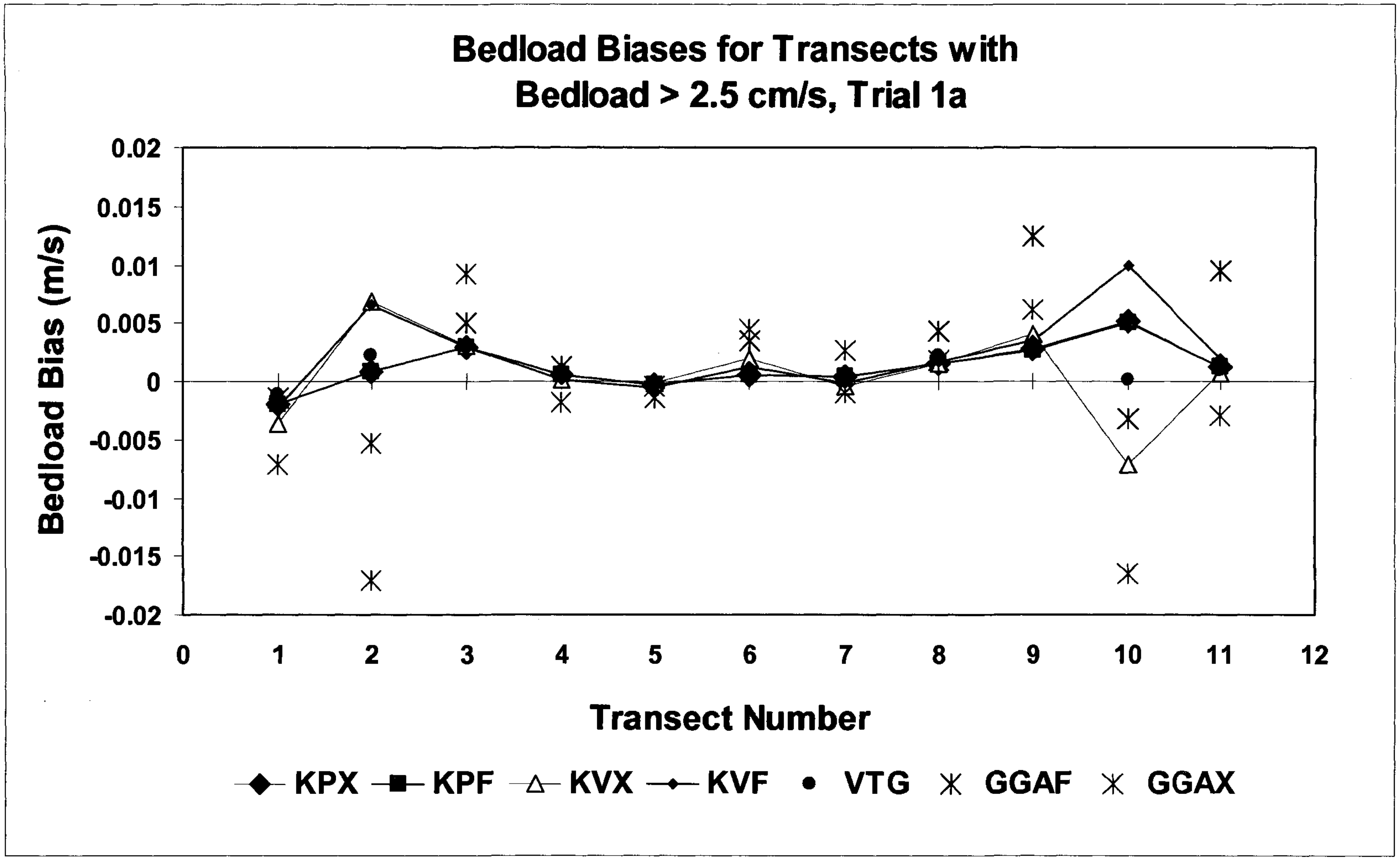

Figure 6.8 Bedload biases for high bedload transects, Trial 1a.

6.4 Discharge

Complete results for discharge are given in Appendix G. Discharge proved to be a difficult measurement to optimize. Initial results using parameters from the genetic algorithm (trials 2a and 1) did not always show the Kalman filter to give the best estimates of discharge. Both of these trials assumed that moving bed was present, and the methods in Chapter 5 were used to remove the bias from the bottom tracking signal. trial 2a for high bedload transects showed that the Kalman filter only produced the lowest discharge errors in 3 out of 11 transects. Similarly, trial 1 for low bedload transects showed the Kalman filter only produced the lowest discharge errors in 2 out of 20 transects. It was found for both trials that unbiased bottom tracking often gave a higher discharge error than biased bottom tracking, indicating the bias was likely not being properly removed.

It was later found that for transects with high bedload, lowering the number of moving bed elements to average (trial 1a) resulted in more accurate removal of bias from the bottom track signal, and improved discharge estimates for unbiased bottom tracking. Table G1 shows that for trial 1a discharge errors for unbiased bottom tracking were lower than biased bottom tracking in 10 out of 11 transects, which was an improvement over trial 2a. Lowering the number of GGA elements to average also helped to improve discharge estimates for high bedload transects. For trial 1a it was found that the Kalman filter gave the lowest discharge errors in 5 out of 11 transects, while Stand Alone VTG and GGA were each best in 3 out of 11 transects. Overall, discharge errors averaged over the 11 transects were not significantly different between each of the Kalman filters, or between the GPS inputs and each of the Kalman filters. For the discharge estimates, it was decided to use parameters in trial 1a rather than trial 2a for transects with high moving bed.

Removing the bias from bottom tracking for low bedload transects proved to be more difficult. Lowering the number of moving bed elements to average did not improve discharge estimates. Bedload for these transects often occurred in local areas. Figures 6.9 and 6.10 show examples of boat velocity estimates for low bedload transects using trial 1. Local areas of high bedload intensity were found to cause a bias in the unbiased bottom track signal later in the transect. However, for high bedload transects, the intensity of bedload was more evenly distributed along the transect. Fig. 6.11 shows an example of velocity estimates for a high bedload transect using trial 1a. It can be seen that the bias in the bottom track signal is being more adequately removed. Trial 5 also showed less variability in the Kalman filter discharge estimates compared to all other low bedload trials (Table 6.12). The Kalman filter also showed less variability compared to all other discharge estimates for trial 5 with the exception of bottom tracking, although no significant differences were found for mean discharge errors between each of the Kalman filters. In order to avoid the risk of introducing a bias in the unbiased bottom track signal, and subsequently the Kalman estimate, it was decided to use trial 5 for discharge estimates, which considered low bedload transects to have no moving bed.

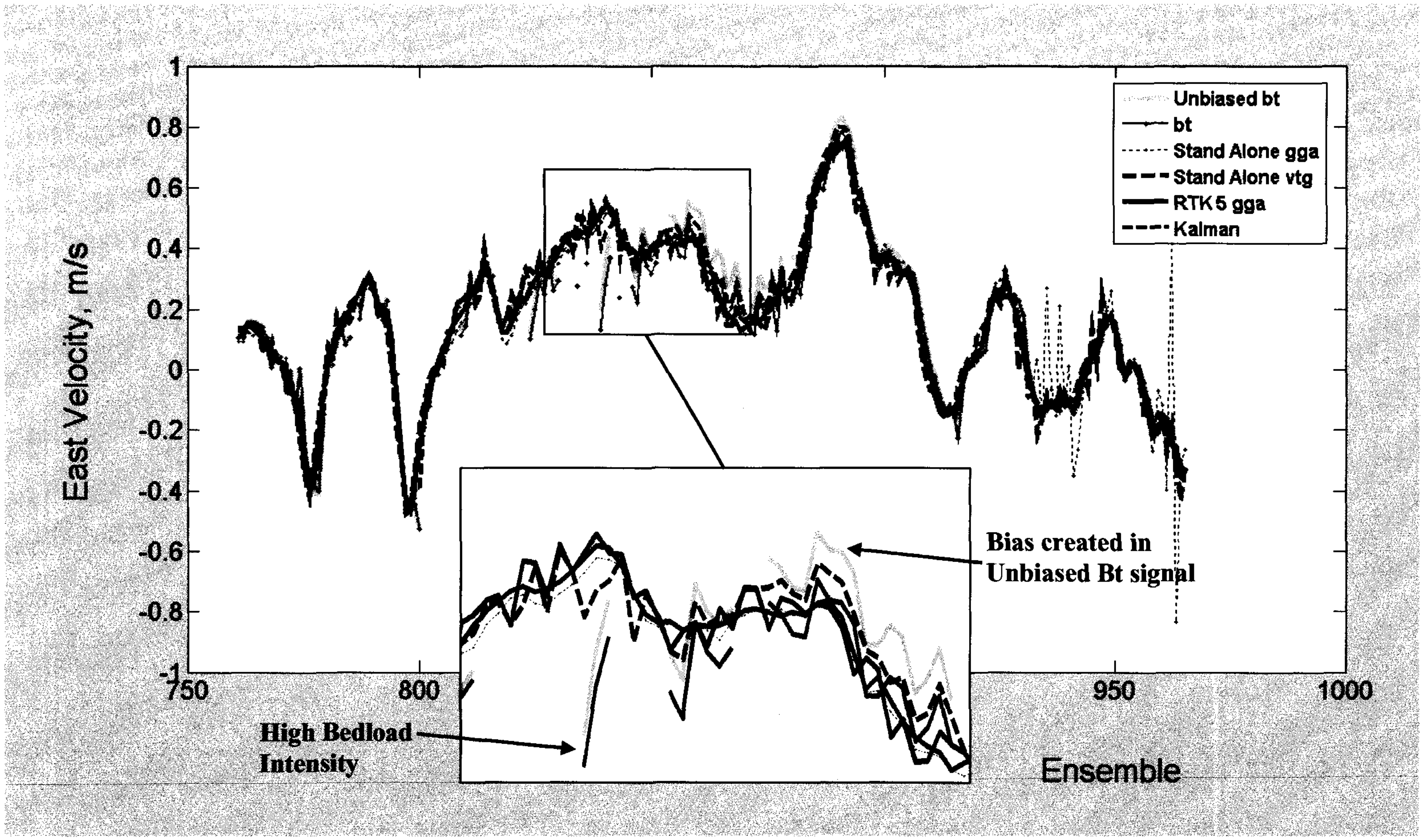

Figure 6.9 Velocity estimates for transect 10EA003_20060705001r, with low bedload (Trial 1).

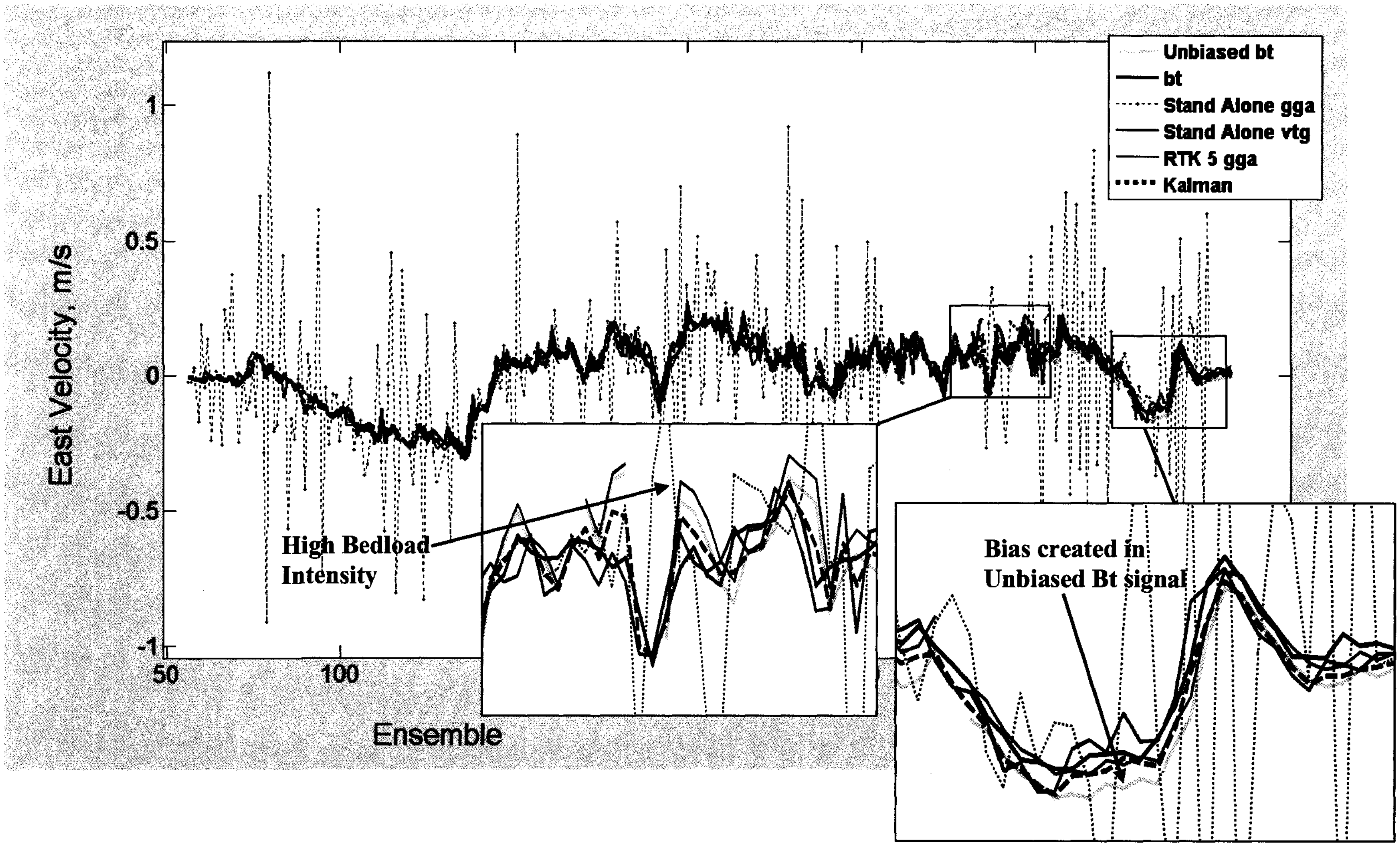

Figure 6.10 Velocity estimates for transect 07cd005_20060409001r, with low bedload (Trial 1).

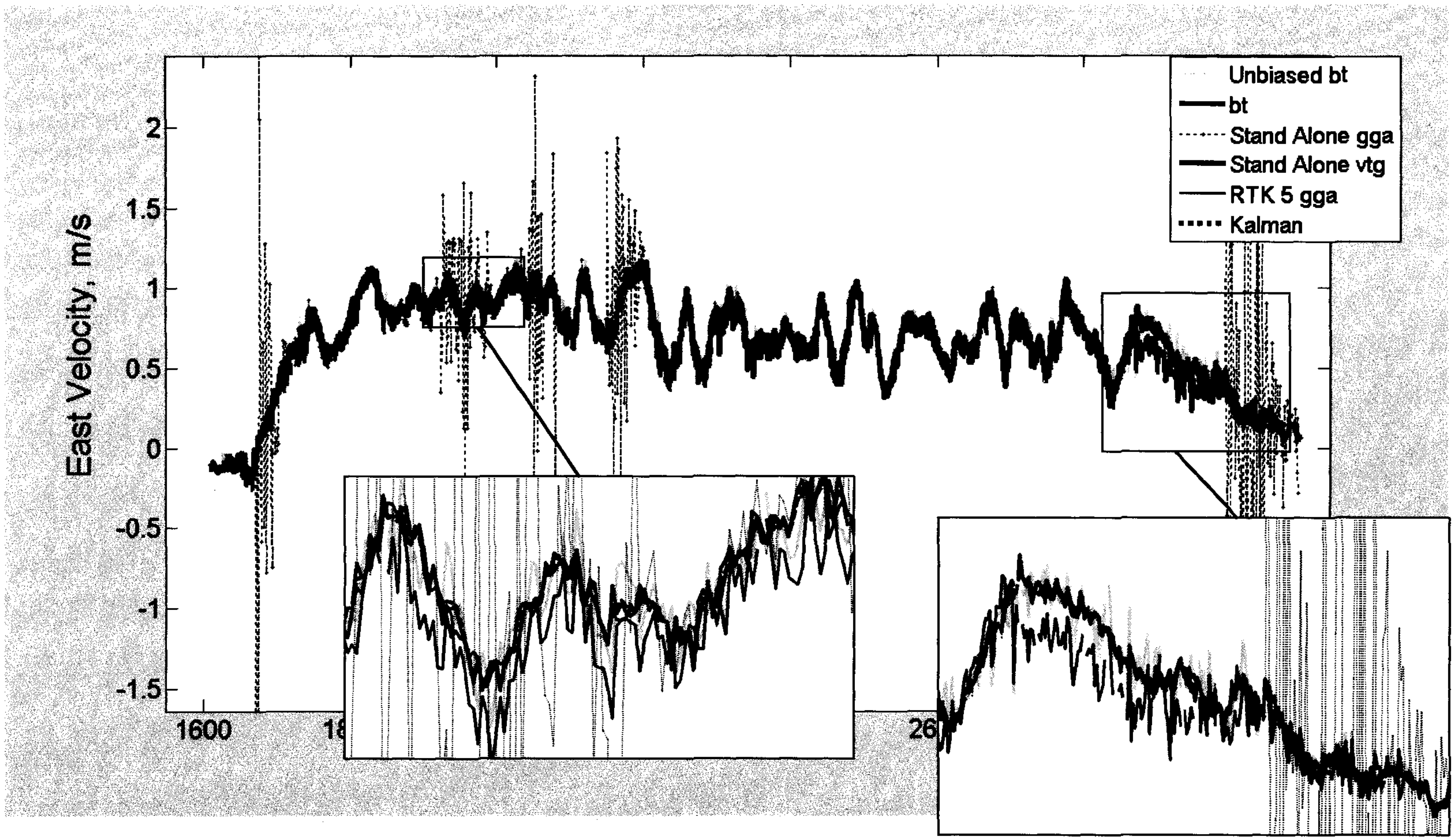

Figure 6.11 Transect 10ED001_20060705001r, with high bedload (Trial 1a).

Table 6.12. Summary of absolute percentage discharge errors for all trials

Trial 1							
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**BT**
Mean (%)	3.1862	3.1991	3.6146	2.8842	3.8709	21.7239	1.8615
Median (%)	2.4891	2.5070	1.7962	1.6066	1.1142	0.8353	1.7450
Standard Deviation (%)	4.1322	4.1743	4.6422	4.0471	6.2867	51.1990	1.0822
Variance	17.0754	17.4247	21.5500	16.3792	39.5229	2621.3367	1.1712
Range (%)	0.0644-15.4375	0.0644-15.7415	0.1191-15.5036	0.1191-16.9971	0.0593-18.8475	0.0341-173.6027	0.2032 3.9918
Trial 2							
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**BT**
Mean (%)	3.3291	3.3498	4.4363	3.6411	3.8709	21.7239	1.8615
Median (%)	2.4417	2.4509	2.1255	2.3202	1.1142	0.8353	1.7450
Standard Deviation (%)	4.2188	4.2354	5.4057	5.0596	6.2867	51.1990	1.0822
Variance	17.7986	17.9383	29.2215	25.5998	39.5229	2621.3367	1.1712
Range (%)	0.0681-15.6622	0.0681-15.7150	0.1339-16.8724	0.1339-21.8477	0.0593-18.8475	0.0341-173.6027	0.2032-3.9918
Trial 3							
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**BT**
Mean (%)	2.3601	2.3743	2.4111	2.2895	3.8709	21.7239	1.8615
Median (%)	1.0031	1.0031	1.0046	0.9941	1.1142	0.8353	1.7450
Standard Deviation (%)	3.5480	3.5668	3.5810	3.4930	6.2867	51.1990	1.0822
Variance	12.5882	12.7219	12.8233	12.2012	39.5229	2621.3367	1.1712
Range (%)	0.0272-12.3435	0.0251-12.3807	0.0231-12.5046	0.0258-11.7982	0.0593-18.8475	0.0341-173.6027	0.2032-3.9918

Trial 4							
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**BT**
Mean (%)	2.4812	2.4938	2.8164	2.9602	3.8709	21.7239	1.8615
Median (%)	1.1242	1.1245	1.3595	1.2016	1.1142	0.8353	1.7450
Standard Deviation (%)	3.6276	3.6250	3.7256	3.9904	6.2867	51.1990	1.0822
Variance	13.1593	13.1408	13.8805	15.9231	39.5229	2621.3367	1.1712
Range (%)	0.0699-12.4427	0.0699-12.4551	0.0502-13.6448	0.0499-13.8118	0.0593-18.8475	0.0341-173.6027	0.2032-3.9918
Trial 5							
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**BT**
Mean (%)	1.9870	1.9775	2.1600	2.3960	3.8709	21.7239	1.8615
Median (%)	1.1869	1.1905	1.2761	1.2840	1.1142	0.8353	1.7450
Standard Deviation (%)	2.3904	2.4307	2.5609	2.7898	6.2867	51.1990	1.0822
Variance	5.7138	5.9082	6.5583	7.7828	39.5229	2621.3367	1.1712
Range (%)	0.0245-9.6294	0.0258-9.8029	0.0870-10.9307	0.0870-9.2700	0.0593-18.8475	0.0341-173.6027	0.2032-3.9918
Trial 1a							
	KPX	**KPF**	**KVX**	**KVF**	**VTG**	**GGAX**	**BT**
Mean (%)	0.2838	0.3680	0.1545	0.2644	0.1628	0.6233	6.0477
Median (%)	0.1906	0.2134	0.1595	0.1931	0.1674	0.1342	6.4837
Standard Deviation (%)	0.3653	0.4430	0.0845	0.3329	0.0856	1.1681	2.8643
Variance	0.1334	0.1962	0.0071	0.1108	0.0073	1.3645	8.2043
Range (%)	0.0147-1.3124	0.0160-1.5243	0.0041-0.2943	0.0522-1.2405	0.0591-0.2932	0.0013-3.9588	0.0614-10.6635

Trial 2a

	KPX	KPF	KVX	KVF	VTG	GGAX	BT
Mean (%)	0.9892	0.9905	0.8608	0.9108	0.1628	0.6233	6.0477
Median (%)	0.3028	0.3029	0.2759	0.2809	0.1674	0.1342	6.4837
Standard Deviation (%)	2.0585	2.0570	1.7420	1.8135	0.0856	1.1681	2.8643
Variance	4.2375	4.2313	3.0346	3.2887	0.0073	1.3645	8.2043
Range (%)	0.0533-7.0742	0.0538-7.0704	0.0172-5.9890	0.0440-6.2151	0.0591-0.2932	0.0013-3.9588	0.0614-10.6635

CHAPTER 7

DISCUSSION

7.1 Boat Velocity

The Kalman filter seemed to work effectively in optimizing boat velocity for high bedload transects as it produced the lowest mean boat velocity errors in 10 out of 11 transects. Velocity errors for trials 1a and 2a were both very close, with Kalman mean boat velocity errors ranging from 0.0475 to 0.0485 m/s for trial 1a, and 0.0473 to 0.0511 m/s for trial 2a. Trial 1a however, was better able to remove the bias in the bottom track signal. This can be seen by observing the unbiased bottom track velocities in Figs. 7.1 and 7.2. There appears to be a lag in the unbiased bottom track velocities in Fig. 7.1 starting at ensemble 3200. Reducing the number of moving bed elements to average from 40 in trial 2a, to 5 in trial 1a helped the filter to more effectively remove the bottom track bias, and prevent a bias from showing up in the Kalman filter estimate.

For low bedload transects the Kalman velocity error was lower than Stand Alone GGAX and VTG in 17 to 20 of 20 transects. It was only lower than bottom track errors however, in 12 to 15 of 20 transects. This seemed reasonable as bottom tracking normally gives the most accurate boat velocities when there is no bedload transport. As mentioned in Chapter 6, and for the purpose of optimization, it was best to consider these transects

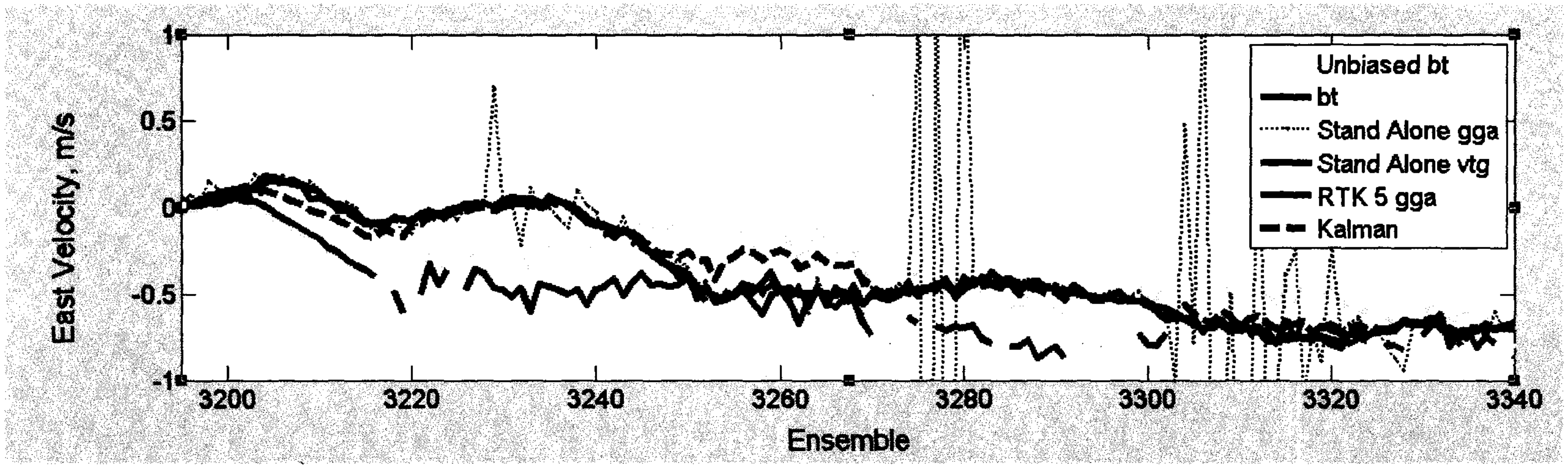

Figure 7.1 Velocity estimates for trial 2a, for transect 10ED001_20060705002r (high bedload).

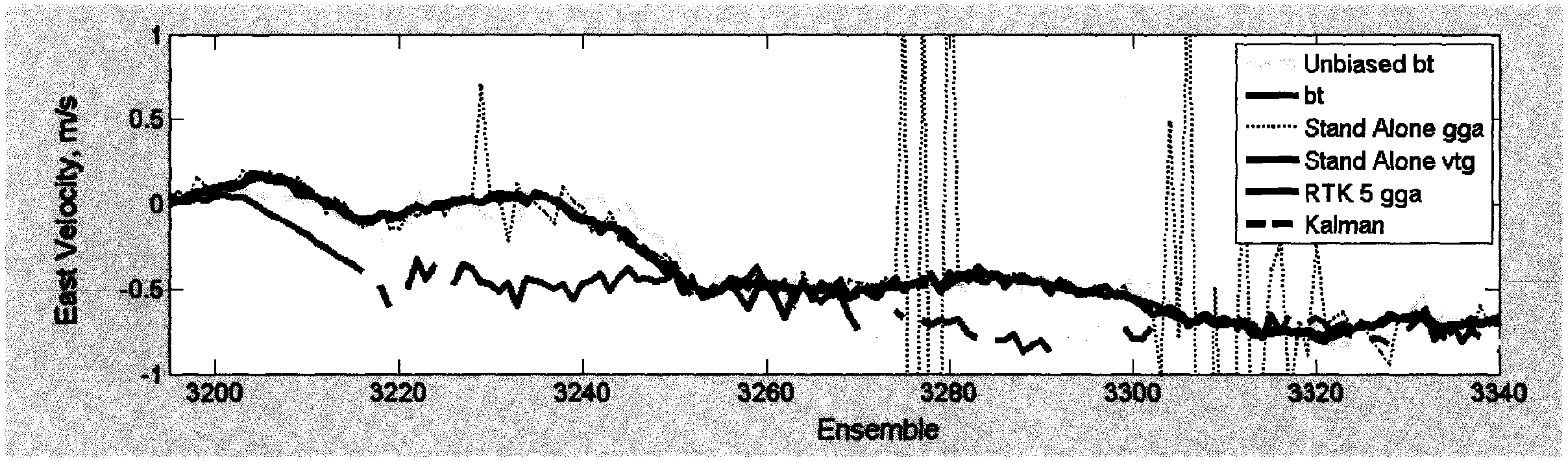

Figure 7.2 Velocity estimates for trial 1a, for transect 10ED001_20060705002r (high bedload).

as having no moving bed, as bedloads here were often localized and removing them introduced a risk of creating a bias in the Kalman estimate (Figs. 6.9 and 6.10).

For both high bedload and low bedload transects, the Kalman filter estimates were very close to Stand Alone VTG. This showed the Kalman filter's dependence on VTG, and that VTG was normally more reliable than GGA for velocity measurements. GGA errors were high due to the presence of multipath at the channel margins. Prefiltering of Stand Alone GGA did not improve the Kalman filter estimates. This suggested that the Kalman filter was able to effectively reduce errors in raw GGA without the need for prefiltering. This agrees with the results from Rennie and Rainville (in review), who found prefiltering WAAS GGA at levels ranging from 0.3 to 5.0 m/s did not improve Kalman filter estimates. Also, no significant differences were found between the errors for each of the Kalman filter types, suggesting they all performed equally well in optimizing boat velocity.

7.2 Boat Position

Of the measures that were used to evaluate boat position, Mean Position error was considered to give a better overall indication of position accuracy compared to CMG, DMG and Length, which could not individually represent boat position. As mentioned in Chapter 6, each of the position measures were calculated manually because merging GGA velocity values into the bottom track reference velocity could not accurately represent GGA position which was based on longitude and latitude.

7.2.1 Mean Position

The Kalman filter gave the lowest Mean Position errors in 12 out of 26 transects. Overall, the Kalman filter showed the greatest improvement when compared to bottom tracking for both high and low bedload transects. This was expected since even small biases in bottom track velocities caused large offsets in position because errors accumulated when velocities were integrated to calculate position (Figs. 7.3 and 7.4).

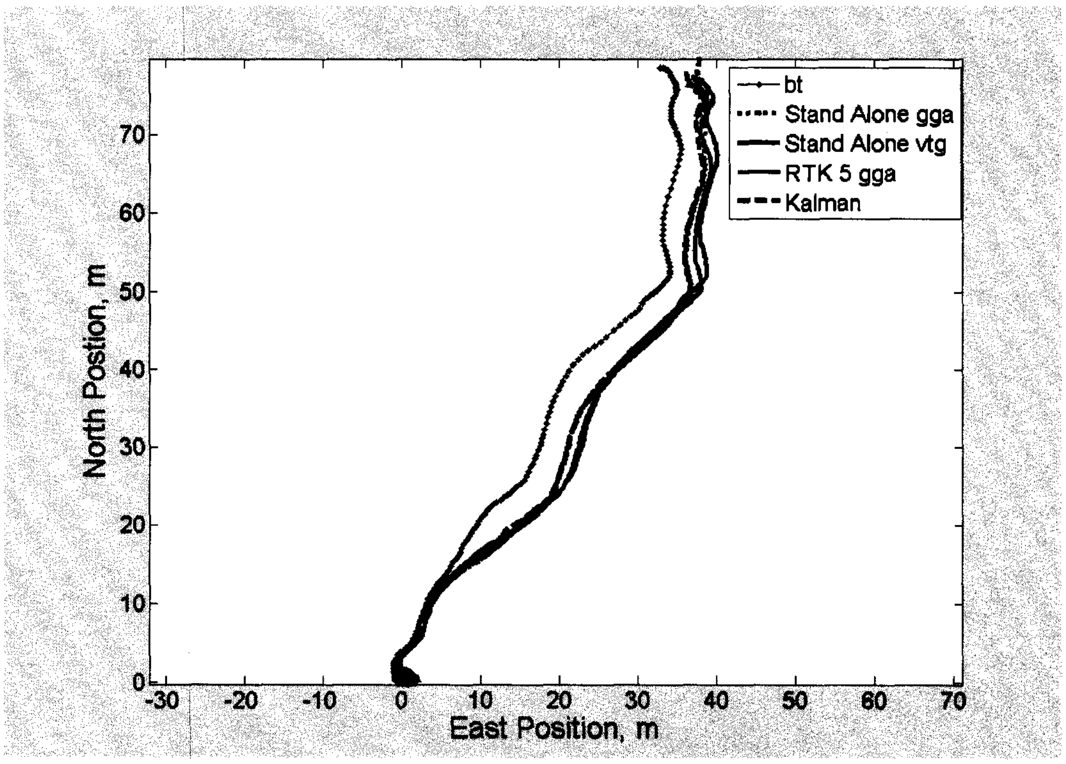

Figure 7.3 Trajectories for transect 10EA003_20060623005r with high bedload.

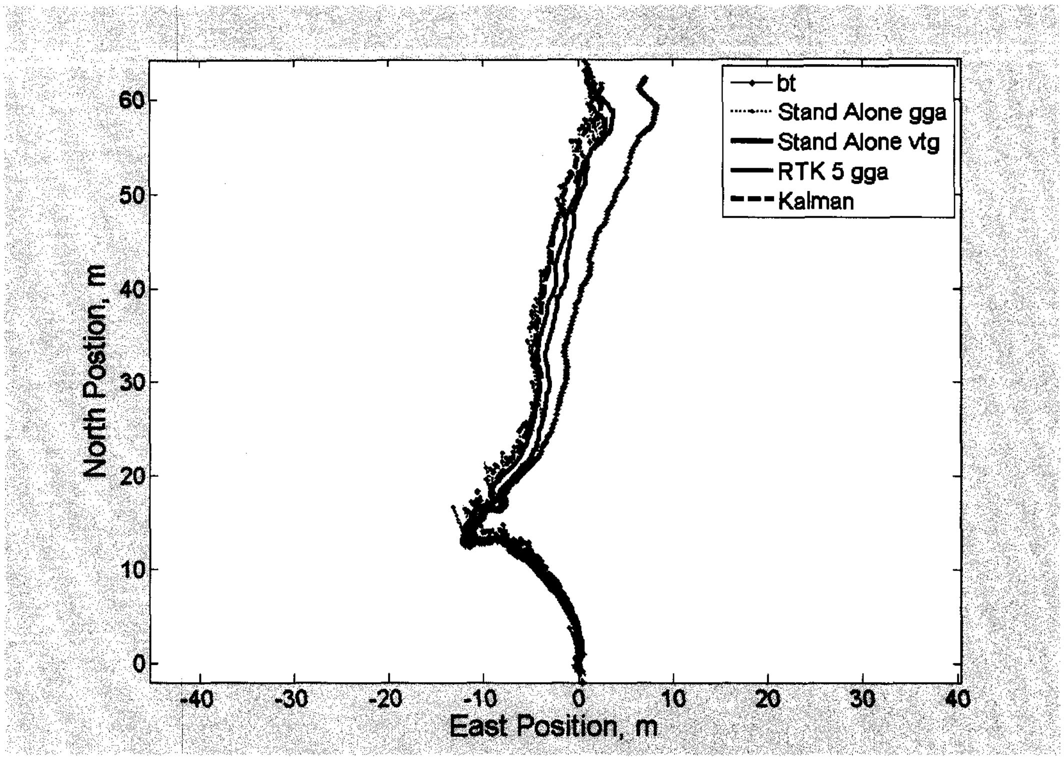

Figure 7.4 Trajectories for transect 07cd005_20060409001r with low bedload.

The Kalman position filter outperformed the velocity filter as its errors were lower in 65% of all transects. The position filter errors was also significantly lower in 30% of the transects. This was likely due to the dependence of the position filter on Stand Alone GGA, and the higher accuracy of GGA in estimating position since it is based on longitude and latitude, and is not affected by missing elements, unlike VTG which is derived from velocity.

7.2.2 Course Made Good

CMG was a good indicator to determine if a particular position estimate was offset. Results showed that the Kalman position filter usually gave lower mean biases for CMG compared to the Kalman velocity filter. This was expected as the position filter relied more heavily on GGA position. It was also found that unfiltered GGA often had higher CMG biases compared to filtered GGA. This is likely since CMG was calculated using only the initial start position and the final end position. These points may have been filtered in GGAX estimates due to the large errors associated with multipath at the beginning and end of the transect.

7.2.3 Distance Made Good

The fact that the Kalman filter was able to give the lowest DMG bias in 19 out of 27 transects indicated that the Kalman filter worked well to improve this measure. Biases were close for both the position and velocity filters. It was therefore difficult to determine which filter performed the best. However, standard deviations and variances of the mean biases were slightly higher for the velocity filter. This may have been due to the fact that missing data for the velocity trajectories were interpolated, which caused DMG estimates to be over or under predicted. Filtered GGA also showed large variances, likely caused by removal of points at the beginning and end of the transects.

7.2.4 Total Length

Biases in total length were the greatest for Stand Alone GGA (Table 6.9). This was expected as large errors in GGA were added to length estimates, causing GGA to greatly over predict length. It is likely that the great dependence of the position filter on Stand Alone GGA caused it to show greater variance in mean biases compared to the velocity filter. Variances for the position filter ranged from 8.35 to 29.13 m^2 while those for the velocity filter ranged from 0.96 to 7.15 m^2. Variances in bias were also high for bottom tracking estimates for high bedload transects.

7.3 Apparent Bedload Velocity

Mean apparent bedload velocity is an effective measure for estimating apparent bedload because it involves computing average apparent bedloads over a transect in both the east and north directions. This helps to eliminate noise originating from the GPS and bottom tracking signals. Single ping bedload errors do not necessarily give a good indication of errors in bedload measurements as Gaussian errors with a mean of zero do not cancel out. For the 11 transects tested, Kalman filter mean bedload estimates were very close to VTG which seemed logical since Kalman velocity estimates were also close to VTG. Figure 7.5 shows an example of the east bedload distribution for a transect where the VTG mean bedload bias was lower than the Kalman filter. However, the VTG and Kalman estimates appeared to be so close that differences in bedload estimates were almost indiscernible. Similarly, for the transect in Fig. 7.6, although the mean Kalman filter bedload bias was lower than VTG, both bedload estimates were still quite close. Overall biases for all signals were very low, ranging from -1.7 cm/s to 1.25 cm/s. Differences in biases may have been affected by higher errors associated with using single frequency L1 RTK 5 as opposed to dual frequency L1L2 RTK 4. With the exception of the Fraser River, RTK 5 was used as a truth value for all transects.

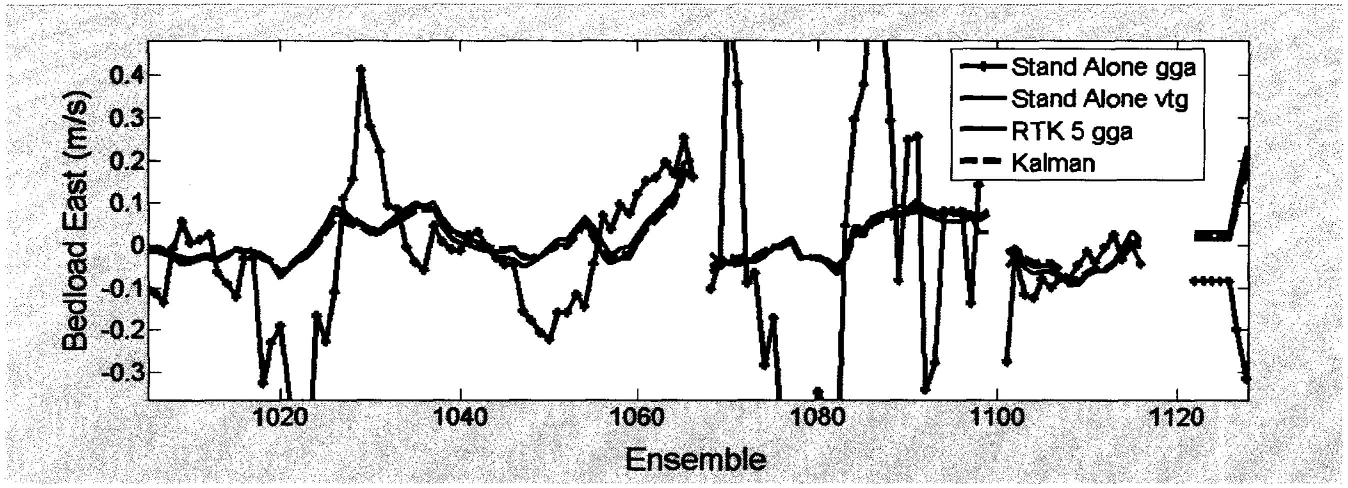

Figure 7.5 East bedload distribution for transect 10EB001_20060623000r.

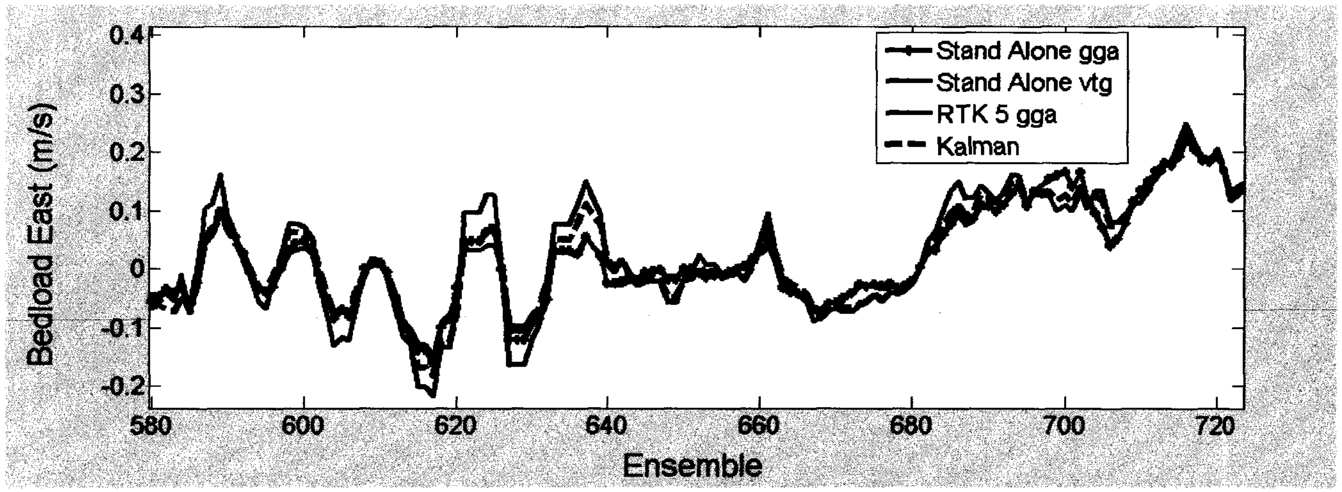

Figure 7.6 East bedload distribution for transect transect004_ens1000r.

7.4 Discharge

As mentioned in Section 7.1, it was found that although the parameters determined from the genetic algorithm (trials 2a and 1) were able to minimize mean velocity errors, they often introduced a bias in the Kalman filter estimate of velocity due to the imperfect removal of bias from the bottom track signal (Figs. 6.9 and 6.10). This caused discharge estimates from the Kalman filter to often be worse than from the other signals. Parameters therefore had to be optimized to reduce lag effects so that previous values had less of an impact on the Kalman filter estimate of velocity. A heuristic approach to determine input parameters proved to be adequate for optimizing discharge estimates. It is possible that the objective function for the genetic algorithm could have been modified to optimize measurements such as bedload. However, optimizing discharge with a genetic algorithm would prove to be challenging due to complexity involved in calculating discharge. It was determined heuristically that the best trials for optimizing discharge were trial 1a for high bedload transects and trial 5 for low bedload transects.

Nevertheless, discharge results showed great variation. Distributions of discharge errors were often skewed, so mean discharge errors calculated over all transects were often much greater than medians. For example, mean unfiltered GGA discharge error for trial 5 was 21.72 % while the median discharge error was 0.84 %. For trial 1a the mean GGA discharge error was 0.62 % while the median was 0.13 % (Table 6.12). In fact, for low bedload transects Stand Alone GGA had the highest mean and lowest median discharge errors compared to all other estimates. This is because GGA discharge was often very close to RTK discharge, but occasionally GGA displayed a large discharge error for a transect, which occurred when GGA velocity errors were not zero-mean (i.e. GGA displayed bias with trend). Distributions of discharge errors for low and high bedload transects can be seen in Figs. 7.7 and 7.8. These are box plots which give median values by a red line while individual values for discharge errors are given by a black dot. The figures show that for low bedload transects GGA had the lowest median discharge error. Mean GGA discharge error was higher due to a few points with high error.

Overall it can be concluded that for high bedload transects, bottom tracking produced significantly higher mean discharge errors compared to all other estimates (ANOVA significance testing with $\alpha \leq 0.05$). For low bedload transects, Stand Alone GGA produced significantly higher mean discharge errors compared to all other estimates.

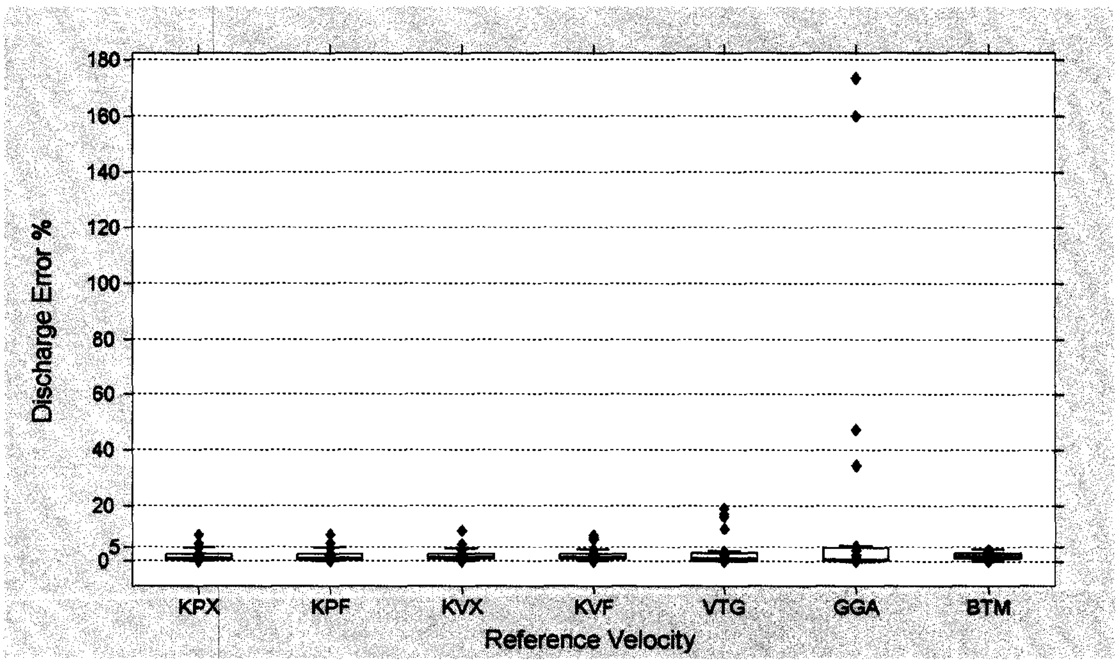

Figure 7.7 Box plots of discharge errors for low bedload transects where medians are given by a red line.

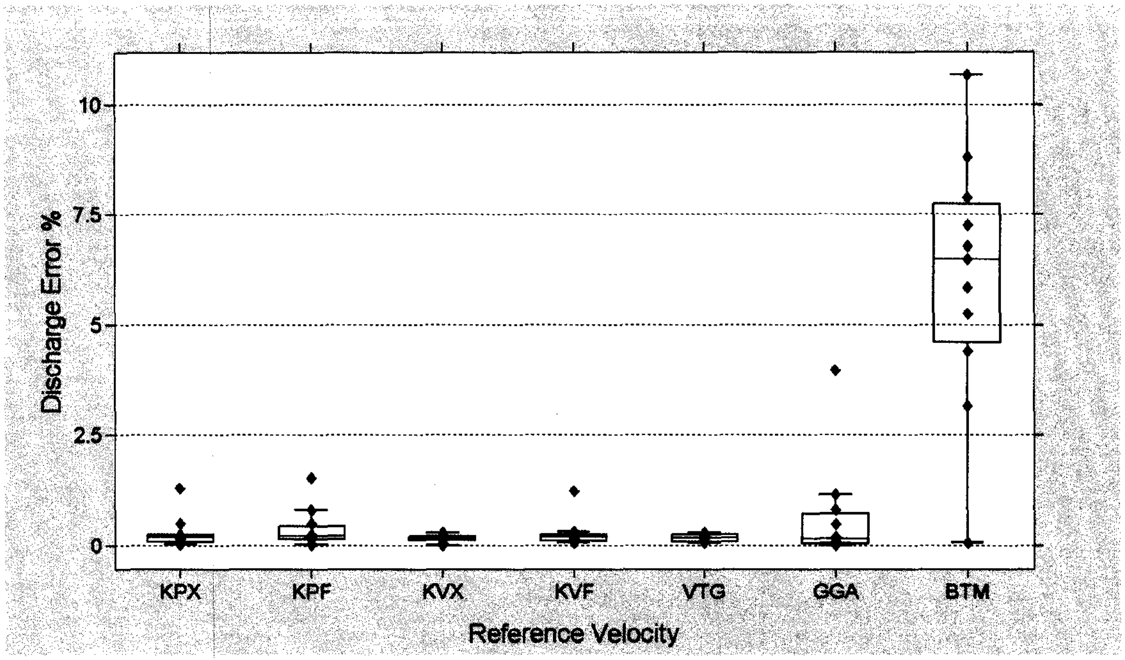

Figure 7.8 Boxplots of discharge errors for high bedload transects where medians are given by a red line.

However, discharge errors for the Kalman filter and VTG were often very close, and there were no significant differences found between the mean errors for the Kalman filter and VTG estimates for either high or low bedload transects. Also, there were no significant differences found for the errors between each of the Kalman filter types. The Kalman filter however did seem to show less variability compared to GPS for discharge estimates. For all 31 transects, the Kalman filter errors ranged from 0.0041 % to 10.93 % while VTG errors ranged from 0.059 % to 18.85 % and GGA ranged from 0.0013 % to 173.60 %. It therefore seemed that estimating discharge using the Kalman filter involved lower risk compared to GPS. This is important as the maximum error that is considered acceptable for a discharge measurement is 5%. In particular, the Kalman velocity filter appeared to provide relatively robust discharge estimates under high bedload conditions

(Figure 7.8). It should be acknowledged, however, that for some high bedload transects, VTG discharge estimates were more reliable than the Kalman position filter (Figure 7.8). This is likely due to the combination of the relatively large influence of poor Stand Alone GGA data and the fact that the procedure for removing bias from BT was imperfect.

It was interesting to find that even when unfiltered GGA mean velocity errors were higher than the Kalman errors, the discharge errors for GGA were often lower than the Kalman filter. This is readily explained if GGA errors are purely random and therefore contribute zero bias to discharge estimates (Rennie and Rainville 2006). However, the location of GGA errors in the transect may also be important. For example, for transect 10ED002_2006062100r the velocity errors for KPX and GGA were 0.051 m/s and 0.145 m/s, respectively, while the discharge errors were 0.19 % and 0.13 %, respectively. If we examine a plot of the single ping velocity errors along the transect (Fig. 7.9) and the depths along the same transect (Fig. 7.10) we can see that Stand Alone GGA velocity errors were often lower than the Kalman filter at the greatest depths. It should be noted that errors in depth measurements are indicated by straight vertical lines extending to 0. Velocity errors at greater depths have greater impact on discharge calculations than those at lower depths. This is because the boat velocity error is integrated over the entire depth, and thus velocity errors in deeper locations contribute to a larger volume of the measured discharge. A similar scenario can be seen in Figs. 7.11 and 7.12 for transect 10ED002_20060705002r. Here the Kalman filter velocity errors were lower than GGA velocity errors, while the Kalman filter discharge errors were higher.

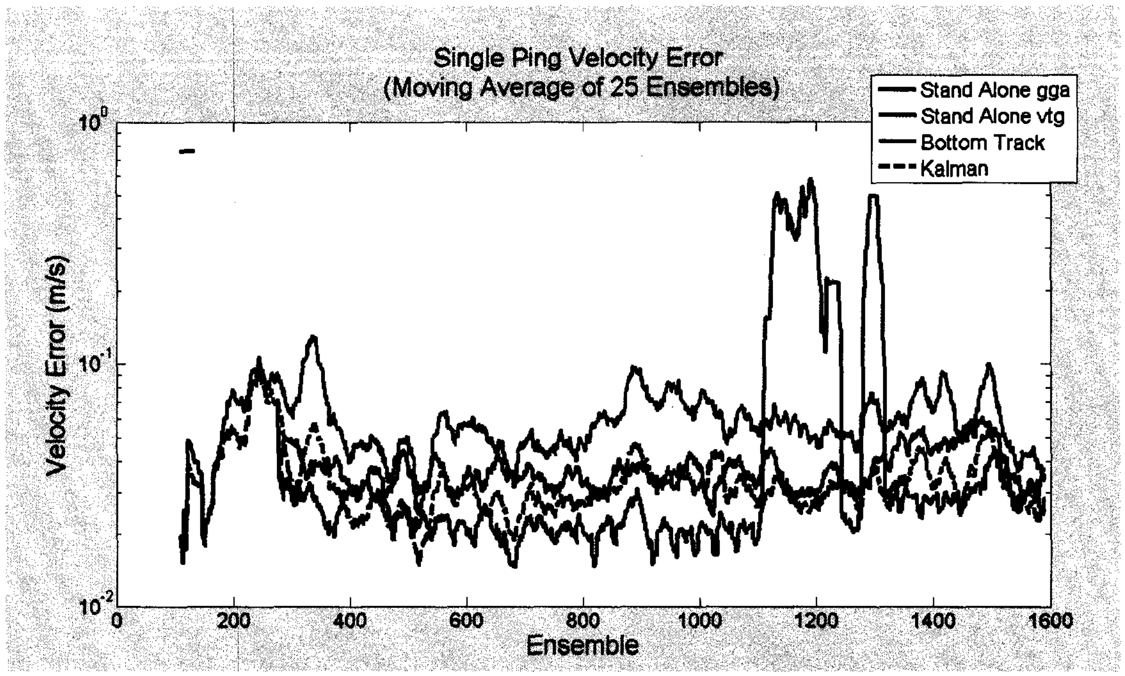

Figure 7.9 Single ping velocity errors for transect 10ED002_20060705000r.

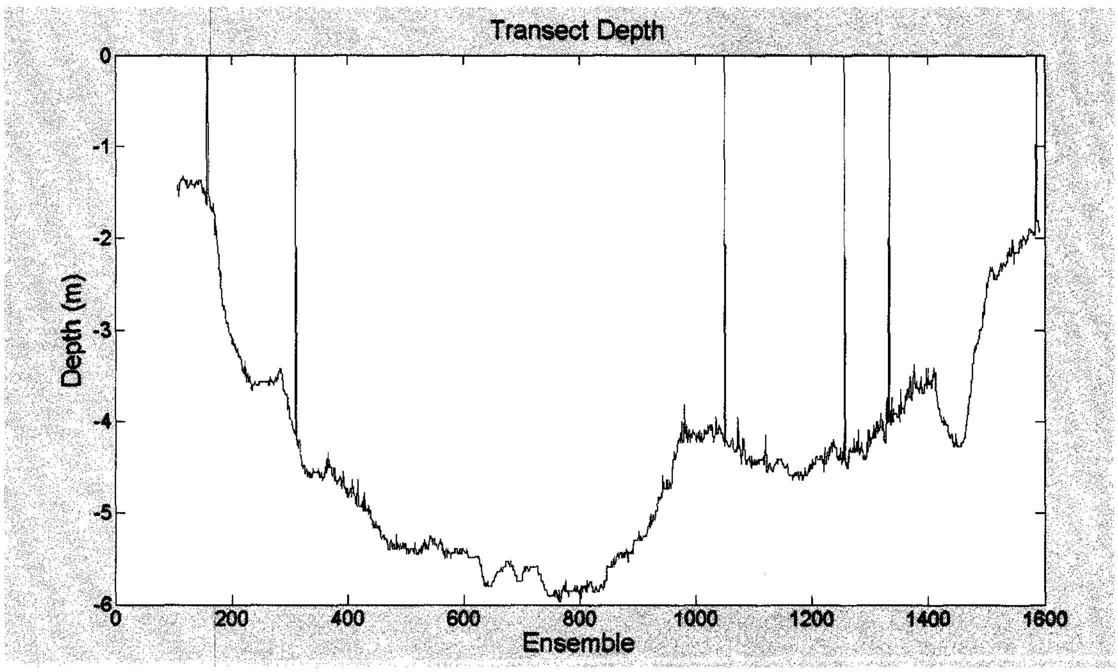

Figure 7.10 River depths for transect 10ED002_20060705000r.

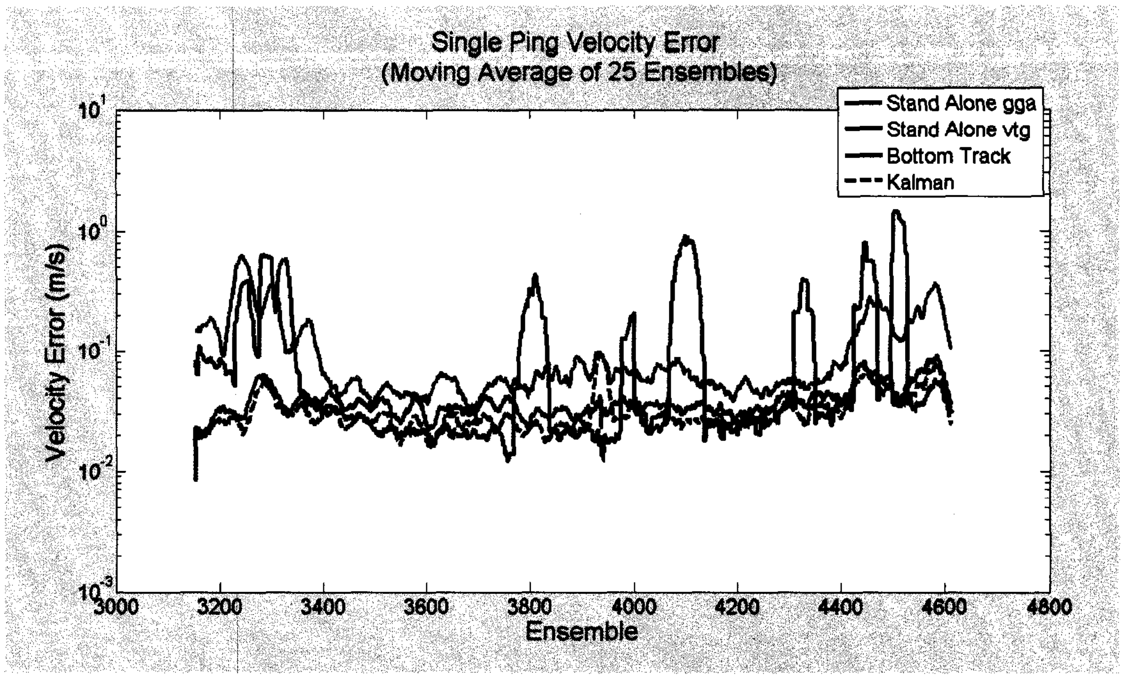

Figure 7.11 Single ping velocity errors for transect 10ED002_20060705002r.

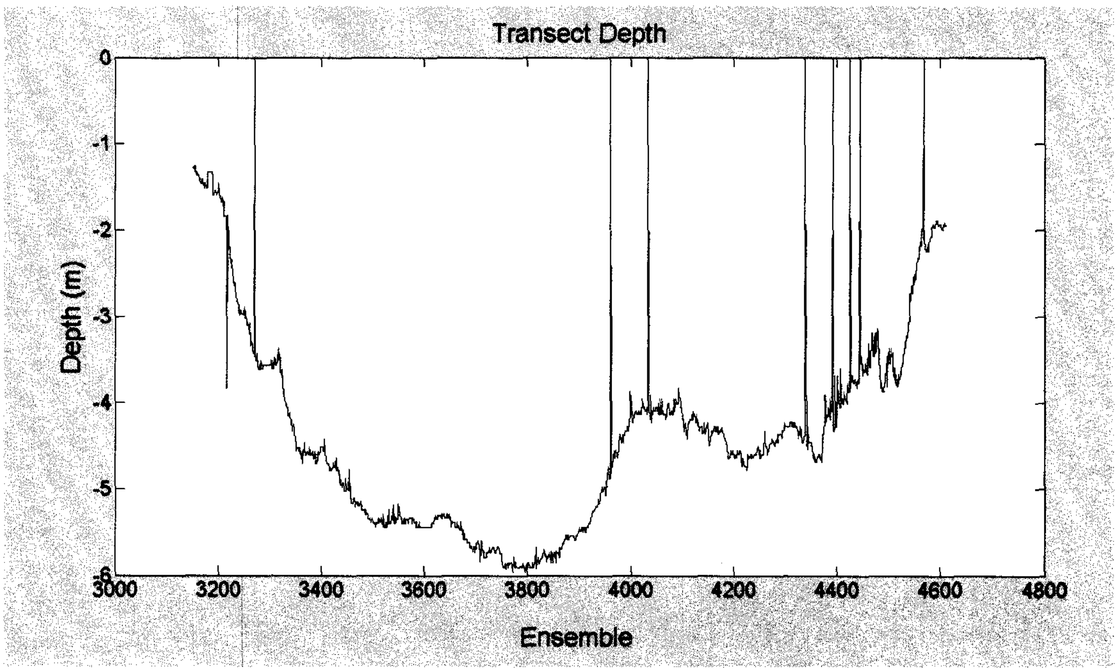

Figure 7.12 River depths for transect 10ED002_20060705002r.

CHAPTER 8

CONCLUSIONS

From the results of this study it was determined that the parameters from trial 1a worked best for transects with high bedload (> 2.5 cm/s) and that the parameters from trial 5 worked best for transects with low bedload (<2.5 cm/s). It was also best to consider low bed load transects as having no bedload in order to reduce the risk of introducing a bias in the Kalman filter estimate of velocity due to imperfect removal of the bias from the bottom tracking signal.

The Kalman filter was able to work effectively under moving bed conditions to improve estimates of boat velocity compared to Stand Alone GPS and bottom tracking. The Kalman filter was able to produce the lowest mean velocity errors in 10 out of 11 high bedload transects, and showed the greatest improvement compared to unfiltered Stand Alone GGA. For low bedload transects the Kalman filter produced the lowest velocity errors in 6 out of 20 transects. Bottom tracking often had low errors for these transects, and this was expected as it is normally most accurate when there is no sediment movement. No significant differences were found between estimates of mean boat velocity errors for the various Kalman filters.

The Kalman filter worked well to improve position estimates as it had the lowest mean position errors compared to all other signals in 6 out of 9 transects when mean bedloads were greater than 2.5 cm/s. The Kalman position filter gave lower mean position errors compared to the velocity filter in 65% of all transects and these errors were significantly lower in 30% of the transects.

The Kalman filter was able to produce the lowest biases in mean apparent bedload in 9 out of 11 high bedload transects. However, the biases for all reference velocities were generally quite low with biases ranging from -1.7 cm/s to 1.25 cm/s. Again, no significant differences were found between bedload estimates for each of the Kalman filter types.

Discharge estimates for all signals including the Kalman filter showed great variation, although the Kalman filter did show less variation compared to the other signals. Discharge errors for the Kalman filter ranged from 0.0041% to 10.93%. No significant differences were found between errors for the various Kalman filter types. Stand alone unfiltered GGA also often produced the most accurate discharge results although it showed high variance as its errors ranged from 0.0013% to 173.60%.

Overall, there were no significant differences found between results from each of the Kalman filter types for each of the measurements, with the exception of mean position estimates, when the position filter outperformed the velocity filter. It was also found that prefiltering of Stand Alone GGA prior to using the Kalman filter did not improve estimates made by the Kalman filter. In other words, the Kalman filter was able to effectively remove errors from Stand Alone GGA without the need for prefiltering. The

Kalman filter generally gave reliable results for all measurements and showed robustness in its ability to deal with missing data.

CHAPTER 9

RECOMMENDATIONS

The following recommendations have been made based on the results of this study.

1. The Kalman position or velocity filters should be used for estimating boat velocity under moving bed conditions as these filters produced the lowest velocity errors compared to Stand Alone GPS and bottom tracking in 10 out of 11 high bedload transects. For transects with negligible amounts of bedload, the Kalman filter may be used when bottom tracking is not available or to provide a continuous measurement when bottom tracking is lost during a measurement.
2. For position estimates it is recommended to use the Kalman position filter over the velocity filter. The position filter gave improved estimates compared to VTG and bottom tracking due to its dependence on GGA which is calculated from longitude and latitude rather than velocity. The Kalman position filter was also able to improve position estimates compared to Stand Alone GGA by removing noise in the Stand Alone GGA signal.
3. It situations where Stand Alone GGA is offset or of extremely poor quality, the Kalman velocity filter may be used to estimate position. However, as even small biases in bottom tracking may lead to large errors in position estimates, it is recommended that the bias first be removed when using the velocity filter. This may be done by using the parameters in trial 1.

4. Both the Kalman position and velocity filters are recommended to estimate mean bedload, although Stand Alone VTG produced equally good estimates of bedload. The advantage of the Kalman filter lies in its ability to deal with missing data.
5. It was found that Stand Alone GGA often had the lowest discharge errors even when mean boat velocity errors were the greatest. This was due to the fact that GGA errors were usually highest at channel margins where depths are the most shallow, and lowest at the channel center, where depths are the greatest. It may therefore be worthwhile to test the effect of varying the minimum threshold used for the standard deviation of GGA velocity error along the transect. Increasing this threshold at the channel margins and decreasing it at the channel center may help to improve the Kalman filter estimates of discharge.
6. Although the Kalman filter did not always give the best discharge estimates, it is recommended over Stand Alone GPS and bottom tracking as it showed the lowest variability in discharge estimates. Both the position and velocity filters performed equally well and discharge errors for the Kalman filters were all less than 11%. Also, mean discharge errors for the Kalman filters were below 2.5%.
7. It is recommended to test the filter with a greater number of high bedload transects to further validate the results of this study.

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APPENDIX A

DATA SUMMARY

Table A.1. Parameters for each river at the given gauging station[a]

STATION CODE	RIVER	MAX WIDTH (m)	AVERAGE DEPTH[b] (m)	DISCHARGE[b] (m^3/s)	LONGITUDE LATITUDE (deg min sec)	DATE
07CD005	Clearwater River	95	1.3879	87.5669	100° 55' 40" W 56° 39' 40" N	6/29/2000
07DA001	Athabasca River	444	0.9160	163.9770	111° 24' 0" W 56° 46' 50" N	12/18/2002
07DD002	Richardson River	24	1.1171	18.0950	111° 14' 22" W 58° 21' 36" N	10/17/2000
10EA003	Fort Simpson	110	1.5585	329.3399	125° 24' 39" W 61° 31' 48" N	7/12/2000
10EB001	Fort Simpson	173.5	3.8632	434.9088	125° 47' 50" W 61° 38' 9" N	6/16/1998
10ED001	Fort Simpson	450	7.5466	9502.9660	123° 28' 32" W 60° 14' 26" N	6/23/1999
10ED002	Fort Simpson	764	4.3597	5450.2860	121° 13' 41" W 61° 44' 34" N	6/10/1998
05BM002	Bow River at Carsland	140	0.8461	129.2331	113° 26' 31" W 50° 49' 26" N	7/24/2002
05BM004	Bow River at Bassano	170	0.7929	80.1535	112° 32' 20" W 50° 45' 0" N	10/17/2002

a. Data taken from MeasurementDB_2002 database developed by Environment Canada.
b. Measurements taken on the given date in column 6.

Table A.2. Summary of all transects

FILE NAME	MAGNETIC DECLINATION (degrees)	Number of Ensembles	Date of Measurement
Clearwater River			
07cd005_20060409001r	16.430	301	4/9/2006
07cd005_20060409002r	16.430	203	4/9/2006
07cd005_20060409005r (mb)[a]	16.430	255	4/9/2006
07cd005_20060409006r (loop)[a]	16.430	391	4/9/2006
Athabasca River			
07da001_20060508001r	16.780	412	5/8/2006
07da001_20060508003r	16.780	331	5/8/2006
07da001_20060508006r (loop)[a]	16.780	542	5/8/2006
07da001_20060508007r (mb)[a]	16.780	454	5/8/2006
Richardson River			
07dd002_20060621000r	17.150	154	6/21/2006
07dd002_20060621001r	17.150	164	6/21/2006
07dd002_20060621002r	17.150	165	6/21/2006
07dd002_20060621003r	17.150	188	6/21/2006
07dd002_20060621005r	17.150	167	6/21/2006
07dd002_20060621007r (mb)[a]	17.150	536	6/21/2006
Fort Simpson			
10EA003_20060623000r	24.830	118	6/23/2006
10EA003_20060623001r	24.830	177	6/23/2006
10EA003_20060623003r	24.830	174	6/23/2006
10EA003_20060623005r	24.830	206	6/23/2006
Fort Simpson			
10EB001_20060623000r	24.680	871	6/23/2006
10EB001_20060623003r	24.900	929	6/23/2006
Fort Simpson			
10ED001_20060705000r	23.600	1317	7/5/2006
10ED001_20060705001r	23.150	1492	7/5/2006
10ED001_20060705002r	23.280	1464	7/5/2006

Fort Simpson			
10ed002_20060621000r	23.93	1836	6/21/2006
10ed002_20060621001r	23.93	1699	6/21/2006
10ed002_20060621002r	23.93	1435	6/21/2006
10ed002_20060621003r	23.88	2032	6/21/2006
Bow River at Carsland			
05bm002_20060530000r (loop)[a]	15.816	534	5/30/2006
05bm002_20060530001r (loop)[a]	15.816	338	5/30/2006
05bm002_20060530002r	15.816	199	5/30/2006
05bm002_20060530003r	15.816	214	5/30/2006
05bm002_20060530004r	15.816	197	5/30/2006
05bm002_20060530005r	15.816	210	5/30/2006
Bow River at Bassano			
05bm004_20060531000r	15.383	204	5/31/2006
05bm004_20060531001r	15.383	204	5/31/2006
05bm004_20060531002r	15.383	197	5/31/2006
05bm004_20060531003r	15.383	208	5/31/2006
05bm004_20060531004r (mb)[a]	15.383	257	5/31/2006
05bm004_20060531005r (mb)[a]	15.383	553	5/31/2006
05bm004_20060531006r (loop)[a]	15.383	366	5/31/2006
Fraser River			
transect003_ens1000r	18.07	984	6/24/2006
transect004_ens1000r	18.08	1000	6/24/2006

a. loop stands for loop test, and mb stands for moving bed test.

APPENDIX B

FIRST OPTIMIZATION OF INPUT PARAMETERS FOR KVX AND KVF FILTERS

Table B.1. First optimization for KVX filter with mean RTK bedload > 0.025 m/s

Transect Name	σ_{bt} (m/s)	σ_{vtg} (m/s)	σ_{vtg}/σ_{bt}	(σ_{gga}) velocity (m/s)	MB Elements to Average	VTG Elements to Average	Sp (m^2)	Kalman Velocity Error (m/s)	VTG Velocity Error (m/s)	GGA Velocity Error (m/s)	BT Biased Velocity Error (m/s)	RTK Bedload (m/s)
07cd005_20060409002r	0.0651	0.0799	1.2273	0.1874	10	40	0.0376	0.0353	0.0665	0.3464	0.0530	0.0303
10EB001_20060623000r	0.1982	0.1162	0.5863	0.1756	30	30	0.0325	0.0378	0.0527	0.5801	0.0609	0.0285
10EB001_20060623003r	0.1952	0.1159	0.5938	0.1498	50	10	0.0317	0.0650	0.0860	0.1794	0.1262	0.0889
10ED001_20060705001r	0.1833	0.0731	0.3988	0.1621	40	20	0.0320	0.0252	0.0344	0.1583	0.0922	0.0578
10ED001_20060705002r	0.1911	0.0781	0.4087	0.1791	10	40	0.0304	0.0316	0.0386	0.1366	0.1017	0.0260
10ed002_20060621000r	0.1971	0.1302	0.6606	0.0953	20	30	0.0427	0.0472	0.0669	0.1452	0.1440	0.0609
10ed002_20060621001r	0.1995	0.0803	0.4025	0.1823	50	10	0.0311	0.0376	0.0614	0.0979	0.1139	0.0564
10ed002_20060621002r	0.1894	0.0782	0.4129	0.1695	10	30	0.0782	0.0419	0.0579	0.0570	0.1495	0.0643
10ed002_20060621003r	0.1956	0.0827	0.4228	0.1320	10	10	0.0307	0.0513	0.0681	0.3661	0.1091	0.0583
Averages:	0.1794	0.0927	0.5682	0.1592	26	24	0.0385					

Table B.2. First optimization for KVX filter with mean RTK bedload < 0.025 m/s

Transect Name	σ_{bt} (m/s)	σ_{vtg} (m/s)	σ_{vtg}/σ_{bt}	(σ_{gga}) velocity (m/s)	MB Elements to Average	VTG Elements to Average	Sp (m^2)	Kalman Velocity Error (m/s)	VTG Velocity Error (m/s)	GGA Velocity Error (m/s)	BT Biased Velocity Error (m/s)	RTK Bedload (m/s)
07dd002_20060621000r	0.0798	0.0278	0.3484	0.1858	10	30	0.044	0.0436	0.0505	1.1277	0.0265	0.0059
07dd002_20060621002r	0.0254	0.0104	0.4094	0.1406	30	20	0.075	0.0351	0.0497	1.7633	0.0309	0.0099
07dd002_20060621003r	0.0175	0.0311	1.7771	0.1844	30	40	0.033	0.0336	0.0489	2.2804	0.0274	0.0116
07dd002_20060621007r	0.0238	0.0119	0.5000	0.0411	40	40	0.031	0.0281	0.0458	2.1035	0.0294	0.0089
05bm002_20060530002r	0.0820	0.0726	0.8854	0.0175	30	10	0.045	0.0361	0.0871	0.0859	0.0811	0.0027
05bm002_20060530004r	0.0627	0.0827	1.3190	0.0320	40	10	0.037	0.0316	0.0794	0.0562	0.0632	0.0024
05bm002_20060530005r	0.1044	0.0927	0.8879	0.0559	40	10	0.042	0.0507	0.0870	0.1499	0.0700	0.0053
05bm004_20060531000r	0.1271	0.1900	1.4949	0.0211	50	30	0.051	0.0235	0.0634	0.0223	0.0599	0.0132
05bm004_20060531001r	0.0666	0.0895	1.3438	0.1354	30	40	0.030	0.0409	0.0686	0.1478	0.0529	0.0062
05bm004_20060531002r	0.1098	0.1809	1.6475	0.0155	40	40	0.041	0.0173	0.0724	0.0172	0.0645	0.0012
05bm004_20060531003r	0.0606	0.0720	1.1881	0.0133	50	10	0.038	0.0187	0.0596	0.0198	0.0509	0.0106
07cd005_20060409001r	0.0224	0.0301	1.3438	0.1223	30	40	0.035	0.0304	0.0578	0.5851	0.0398	0.0142
07da001_20060508001r	0.0656	0.0780	1.1890	0.1196	30	10	0.078	0.0228	0.0420	0.0469	0.0369	0.0140
07da001_20060508003r	0.0639	0.0711	1.1127	0.0953	50	10	0.030	0.0220	0.0428	0.0491	0.0381	0.0169
10EA003_20060623000r	0.0396	0.0790	1.9949	0.0793	40	20	0.031	0.0683	0.1180	0.4296	0.0641	0.0125
10EA003_20060623001r	0.0650	0.0601	0.9246	0.0304	50	30	0.074	0.0565	0.0902	0.3224	0.0742	0.0107
10EA003_20060623003r	0.0492	0.0301	0.6118	0.0621	50	30	0.032	0.0568	0.1116	0.6848	0.0701	0.0098
10EA003_20060623005r	0.0526	0.0585	1.1122	0.0223	50	30	0.040	0.0397	0.0750	0.1479	0.0720	0.0150
10ED001_20060705000r	0.1982	0.1162	0.5863	0.1756	30	30	0.033	0.0378	0.0527	0.5801	0.0609	0.0081
Averages:	0.0693	0.0729	1.0883	0.0816	37.89	25.26	0.043					

Table B.3. First optimization for KVF filter with mean RTK bedload > 0.025 m/s

Transect Name	σ_{bt} (m/s)	σ_{vtg} (m/s)	σ_{vtg}/σ_{bt}	(σ_{gga}) velocity (m/s)	MB Elements to Average	VTG Elements to Average	Sp (m^2)	Kalman Velocity Error (m/s)	VTG Velocity Error (m/s)	GGA Velocity Error (m/s)	BT Biased Velocity Error (m/s)	RTK Bedload (m/s)
07cd005_20060409002r	0.0515	0.0668	1.2971	0.1810	40	40	0.0396	0.0346	0.0665	0.2759	0.0530	0.0303
10EB001_20060623000r	0.1667	0.1136	0.6815	0.1904	30	30	0.0304	0.0361	0.0527	0.1003	0.0609	0.0285
10EB001_20060623003r	0.1905	0.1238	0.6499	0.1535	50	30	0.0304	0.0618	0.0850	0.0820	0.1248	0.0889
10ED001_20060705001r	0.1969	0.0887	0.4505	0.1904	50	10	0.0303	0.0238	0.0344	0.0941	0.0922	0.0578
10ED001_20060705002r	0.1916	0.0850	0.4436	0.1450	10	30	0.0342	0.0281	0.0386	0.0483	0.1017	0.0260
10ed002_20060621000r	0.1948	0.1319	0.6771	0.0941	30	30	0.0430	0.0468	0.0669	0.1337	0.1440	0.0609
10ed002_20060621001r	0.1984	0.0870	0.4385	0.1651	10	10	0.0308	0.0363	0.0614	0.0741	0.1139	0.0564
10ed002_20060621002r	0.1946	0.0790	0.4060	0.1142	20	30	0.0302	0.0357	0.0579	0.0497	0.1495	0.0643
10ed002_20060621003r	0.1986	0.0785	0.3953	0.1174	30	30	0.0302	0.0503	0.0681	0.2044	0.1091	0.0583
Averages:	0.1760	0.0949	0.6044	0.1501	30	27	0.0332					

Table B.4. First optimization for KVF filter with mean RTK bedload < 0.025 m/s

Transect Name	σ_{bt} (m/s)	σ_{vtg} (m/s)	σ_{vtg}/σ_{bt}	(σ_{gga}) velocity (m/s)	MB Elements to Average	VTG Elements to Average	Sp (m^2)	Kalman Velocity Error (m/s)	VTG Velocity Error (m/s)	GGA Velocity Error (m/s)	BT Biased Velocity Error (m/s)	RTK Bedload (m/s)
07dd002_20060621000r	0.0224	0.0114	0.5089	0.1663	40	40	0.033	0.0360	0.0505	0.4314	0.0265	0.0059
07dd002_20060621002r	0.0285	0.0134	0.4702	0.1575	30	20	0.032	0.0350	0.0497	0.7005	0.0309	0.0099
07dd002_20060621003r	0.0144	0.0245	1.7014	0.0615	50	30	0.030	0.0335	0.0489	1.0196	0.0274	0.0116
07dd002_20060621007r	0.0179	0.0228	1.2737	0.1328	30	30	0.045	0.0284	0.0458	1.3935	0.0294	0.0089
05bm002_20060530002r	0.0902	0.0827	0.9169	0.0117	50	10	0.037	0.0350	0.0871	0.0523	0.0811	0.0027
05bm002_20060530004r	0.0648	0.0843	1.3009	0.0318	30	10	0.059	0.0316	0.0794	0.0531	0.0632	0.0024
05bm002_20060530005r	0.0843	0.0896	1.0629	0.0434	40	10	0.046	0.0456	0.0870	0.1295	0.0700	0.0053
05bm004_20060531000r	0.0970	0.1843	1.9000	0.0107	50	30	0.056	0.0171	0.0634	0.0171	0.0599	0.0132
05bm004_20060531001r	0.0920	0.1073	1.1663	0.0745	30	10	0.035	0.0394	0.0686	0.0841	0.0529	0.0062
05bm004_20060531002r	0.1160	0.1919	1.6543	0.0076	40	30	0.047	0.0173	0.0724	0.0172	0.0645	0.0012
05bm004_20060531003r	0.0842	0.1022	1.2138	0.0133	30	30	0.065	0.0189	0.0596	0.0198	0.0509	0.0106
07cd005_20060409001r	0.0651	0.1874	2.8786	0.1256	30	40	0.054	0.0406	0.0578	0.3377	0.0398	0.0142
07da001_20060508001r	0.0656	0.0780	1.1890	0.1196	30	10	0.078	0.0228	0.0420	0.0469	0.0369	0.0140
07da001_20060508003r	0.0639	0.0711	1.1127	0.0953	50	10	0.030	0.0220	0.0428	0.0491	0.0381	0.0169
10EA003_20060623000r	0.0396	0.0790	1.9949	0.0793	40	20	0.031	0.0683	0.1180	0.4296	0.0641	0.0125
10EA003_20060623001r	0.0648	0.0588	0.9074	0.0266	40	30	0.078	0.0574	0.0902	0.3224	0.0742	0.0107
10EA003_20060623003r	0.0524	0.0628	1.1985	0.1749	50	20	0.030	0.0546	0.1116	0.6801	0.0701	0.0098
10EA003_20060623005r	0.0526	0.0585	1.1122	0.0223	50	30	0.040	0.0397	0.0750	0.1479	0.0720	0.0150
10ED001_20060705000r	0.1667	0.1136	0.6815	0.1904	30	30	0.030	0.0361	0.0527	0.1003	0.0609	0.0081
Averages:	0.0675	0.0855	1.2760	0.0813	38.95	23.16	0.045					

APPENDIX C

FINAL OPTIMIZATION OF INPUT PARAMETERS FOR ALL FILTER TYPES

Table C.1. Final parameter[a] values from optimization for KVX filter with mean RTK bedload > 0.025 m/s

Transect Name	σ_{bt} (m/s)	σ_{vtg} (m/s)	σ_{vtg}/σ_{bt}	(σ_{gga}) velocity (m/s)	Kalman Velocity Error (m/s)	VTG Velocity Error (m/s)	GGA Velocity Error (m/s)	BT Biased Velocity Error (m/s)	RTK Bedload (m/s)
07cd005_20060409002r	0.0568	0.0756	1.3310	0.1763	0.0358	0.0798	0.5926	0.3464	0.0303
10EB001_20060623000r	0.1513	0.1014	0.6702	0.1998	0.0376	0.0621	1.7988	0.5801	0.0285
10EB001_20060623003r	0.1993	0.1116	0.5600	0.1981	0.0667	0.3164	0.5845	0.1794	0.0889
10ED001_20060705001r	0.1833	0.0772	0.4212	0.1981	0.0255	0.0398	0.4503	0.1583	0.0578
10ED001_20060705002r	0.1922	0.0687	0.3574	0.1842	0.0310	0.0465	0.6441	0.1366	0.0260
10ed002_20060621000r	0.1736	0.1052	0.6060	0.0962	0.0260	0.0797	0.4030	0.1452	0.0609
10ed002_20060621001r	0.1915	0.0793	0.4141	0.2000	0.0254	0.0725	0.2162	0.0979	0.0564
10ed002_20060621002r	0.1988	0.0738	0.3712	0.1711	0.0272	0.0692	0.1222	0.0570	0.0643
10ed002_20060621003r	0.2000	0.0761	0.3805	0.1145	0.0283	0.1473	0.7426	0.3661	0.0583
Averages:	0.1719	0.0854	0.5680	0.1709					

a. Values for moving bed elements to average, VTG elements to average, and *Sp* were constant for all transects at 25, 25, and 0.040 m^2. respectively.

Table C.2. Final parameter[a] values from optimization for KVX filter with mean RTK bedload < 0.025 m/s

Transect Name	σ_{bt} (m/s)	σ_{vtg} (m/s)	σ_{vtg}/σ_{bt}	(σ_{gga}) velocity (m/s)	Kalman Velocity Error (m/s)	VTG Velocity Error (m/s)	GGA Velocity Error (m/s)	BT Biased Velocity Error (m/s)	RTK Bedload (m/s)
07dd002_20060621000r	0.0212	0.0278	1.3113	0.1999	0.0359	0.0628	1.6676	1.1277	0.0059
07dd002_20060621002r	0.0146	0.0379	2.5959	0.1737	0.0333	0.0611	3.0773	1.7633	0.0099
07dd002_20060621003r	0.0160	0.0381	2.3813	0.1841	0.0339	0.0603	4.9526	2.2804	0.0116
07dd002_20060621007r	0.0178	0.0223	1.2528	0.1396	0.0276	0.0583	4.1783	2.1035	0.0089
05bm002_20060530002r	0.0835	0.0763	0.9138	0.0210	0.0372	0.1071	0.2059	0.0859	0.0027
05bm002_20060530004r	0.0602	0.0774	1.2857	0.0398	0.0332	0.0941	0.1179	0.0562	0.0024
05bm002_20060530005r	0.1019	0.1053	1.0334	0.1980	0.0452	0.1031	0.2652	0.1499	0.0053
05bm004_20060531000r	0.1858	0.1814	0.9763	0.0480	0.0234	0.0729	0.0771	0.0223	0.0132
05bm004_20060531001r	0.0868	0.1091	1.2569	0.1969	0.0414	0.0806	0.2981	0.1478	0.0062
05bm004_20060531002r	0.1963	0.1797	0.9154	0.0391	0.0168	0.0834	0.0205	0.0172	0.0012
05bm004_20060531003r	0.0784	0.0587	0.7487	0.0110	0.0184	0.0700	0.0291	0.0198	0.0106
07cd005_20060409001r	0.0289	0.0375	1.2976	0.1944	0.0310	0.0712	0.9203	0.5851	0.0142
07da001_20060508001r	0.0468	0.0541	1.1560	0.0342	0.0219	0.0491	0.0976	0.0469	0.0140
07da001_20060508003r	0.1099	0.1152	1.0482	0.1920	0.0219	0.0493	0.1088	0.0491	0.0169
10EA003_20060623000r	0.0395	0.0778	1.9696	0.1995	0.0685	0.1377	1.0010	0.4296	0.0125
10EA003_20060623001r	0.0606	0.0594	0.9802	0.1005	0.0514	0.1103	0.6846	0.3224	0.0107
10EA003_20060623003r	0.0439	0.0488	1.1116	0.1077	0.0553	0.1340	1.0510	0.6848	0.0098
10EA003_20060623005r	0.0589	0.0558	0.9474	0.0371	0.0380	0.1025	0.4736	0.1479	0.0150
10ED001_20060705000r	0.1948	0.1015	0.5210	0.1235	0.0273	0.0429	0.2299	0.0667	0.0081
Averages:	0.0761	0.0771	1.2475	0.1179					

a. Values for moving bed elements to average, VTG elements to average, and *Sp* were constant for all transects at 40, 25, and 0.040 m^2. respectively.

Table C.3. Final parameter[a] values from optimization for KVF filter with mean RTK bedload > 0.025 m/s

Transect Name	σ_{bt} (m/s)	σ_{vtg} (m/s)	σ_{vtg}/σ_{bt}	(σ_{gga}) velocity (m/s)	Kalman Velocity Error (m/s)	VTG Velocity Error (m/s)	GGA Velocity Error (m/s)	BT Biased Velocity Error (m/s)	RTK Bedload (m/s)
07cd005_20060409002r	0.0648	0.0829	1.2793	0.1954	0.0348	0.0798	0.4353	0.2759	0.0303
10EB001_20060623000r	0.1749	0.1189	0.6798	0.2000	0.0358	0.0621	0.1806	0.1003	0.0285
10EB001_20060623003r	0.1959	0.1172	0.5983	0.1505	0.0623	0.3162	0.3437	0.0820	0.0889
10ED001_20060705001r	0.2000	0.0839	0.4195	0.1987	0.0243	0.0398	0.2428	0.0941	0.0578
10ED001_20060705002r	0.1954	0.0760	0.3889	0.1068	0.0255	0.0465	0.1057	0.0483	0.0260
10ed002_20060621000r	0.1762	0.1060	0.6016	0.0991	0.0260	0.0797	0.3616	0.1337	0.0609
10ed002_20060621001r	0.2000	0.0867	0.4335	0.1992	0.0243	0.0725	0.1390	0.0741	0.0564
10ed002_20060621002r	0.1996	0.0758	0.3798	0.1079	0.0255	0.0692	0.0927	0.0497	0.0643
10ed002_20060621003r	0.1992	0.0735	0.3690	0.0925	0.0277	0.1473	0.3677	0.2044	0.0583
Averages:	0.1784	0.0912	0.5722	0.1500					

a. Values for moving bed elements to average, VTG elements to average, and *Sp* were constant for all transects at 25, 25, and 0.040 m^2. respectively.

Table C.4. Final parameter[a] values from optimization for KVF filter with mean RTK bedload < 0.025 m/s

Transect Name	σ_{bt} (m/s)	σ_{vtg} (m/s)	σ_{vtg}/σ_{bt}	(σ_{gga}) velocity (m/s)	Kalman Velocity Error (m/s)	VTG Velocity Error (m/s)	GGA Velocity Error (m/s)	BT Biased Velocity Error (m/s)	RTK Bedload (m/s)
07dd002_20060621000r	0.0444	0.0959	2.1599	0.1982	0.0346	0.0628	0.6987	0.4314	0.0059
07dd002_20060621002r	0.0143	0.0379	2.6503	0.1732	0.0334	0.0611	1.0301	0.7005	0.0099
07dd002_20060621003r	0.0139	0.0346	2.4892	0.1890	0.0339	0.0603	1.8966	1.0196	0.0116
07dd002_20060621007r	0.0172	0.0231	1.3430	0.1906	0.0282	0.0583	2.7524	1.3935	0.0089
05bm002_20060530002r	0.0836	0.0869	1.0395	0.0247	0.0366	0.1071	0.0968	0.0523	0.0027
05bm002_20060530004r	0.0599	0.0774	1.2922	0.0381	0.0331	0.0941	0.1083	0.0531	0.0024
05bm002_20060530005r	0.1024	0.1065	1.0400	0.1960	0.0447	0.1031	0.2258	0.1295	0.0053
05bm004_20060531000r	0.1823	0.1472	0.8075	0.0274	0.0171	0.0729	0.0198	0.0171	0.0132
05bm004_20060531001r	0.1275	0.1445	1.1333	0.1984	0.0382	0.0806	0.1471	0.0841	0.0062
05bm004_20060531002r	0.1999	0.1882	0.9415	0.0405	0.0167	0.0834	0.0205	0.0172	0.0012
05bm004_20060531003r	0.0783	0.0586	0.7484	0.0110	0.0184	0.0700	0.0291	0.0198	0.0106
07cd005_20060409001r	0.0273	0.0355	1.3004	0.1993	0.0307	0.0712	0.5684	0.3377	0.0142
07da001_20060508001r	0.1262	0.1286	1.0190	0.1991	0.0215	0.0491	0.0976	0.0469	0.0140
07da001_20060508003r	0.1176	0.1215	1.0332	0.1983	0.0218	0.0493	0.1088	0.0491	0.0169
10EA003_20060623000r	0.0366	0.0778	2.1257	0.1461	0.0685	0.1377	1.0010	0.4296	0.0125
10EA003_20060623001r	0.0605	0.0594	0.9818	0.1000	0.0514	0.1103	0.6846	0.3224	0.0107
10EA003_20060623003r	0.0447	0.0496	1.1096	0.1095	0.0552	0.1340	1.0477	0.6801	0.0098
10EA003_20060623005r	0.0602	0.0572	0.9502	0.0381	0.0380	0.1025	0.4736	0.1479	0.0150
10ED001_20060705000r	0.1997	0.1078	0.5398	0.1267	0.0267	0.0429	0.1421	0.0462	0.0081
Averages:	0.0840	0.0862	1.3002	0.1265					

a. Values for moving bed elements to average, VTG elements to average, and *Sp* were constant for all transects at 40, 25, and 0.040 m^2. respectively.

Table C.5. Final parameter[a] values from optimization for KPX filter with mean RTK bedload > 0.025 m/s

Transect Name	σ_{bt} (m/s)	σ_{vtg} (m/s)	$\sigma_{vtg} / \sigma_{bt}$	MB Elements to Average	VTG Elements to Average	*Sp* (m^2)	Kalman Velocity Error (m/s)	VTG Velocity Error (m/s)	GGA Velocity Error (m/s)	BT Biased Velocity Error (m/s)	RTK Bedload (m/s)
07cd005_20060409002r	0.1325	0.1556	1.1743	20	30	0.0305	0.0665	0.3464	0.0530	0.2759	0.0303
10EB001_20060623000r	0.1905	0.1493	0.7837	30	20	0.0301	0.0527	0.5801	0.0609	0.1003	0.0285
10EB001_20060623003r	0.1952	0.1213	0.6214	50	30	0.0314	0.0860	0.1794	0.1262	0.0820	0.0889
10ED001_20060705001r	0.1997	0.1008	0.5048	20	40	0.0320	0.0614	0.0979	0.1139	0.0941	0.0578
10ED001_20060705002r	0.1985	0.0851	0.4287	10	20	0.0423	0.0579	0.0570	0.1495	0.0483	0.0260
10ed002_20060621000r	0.1938	0.0998	0.5150	20	30	0.0312	0.0669	0.1452	0.1440	0.1337	0.0609
10ed002_20060621001r	0.1997	0.0989	0.4952	20	20	0.0302	0.0614	0.0979	0.1139	0.0741	0.0564
10ed002_20060621002r	0.1946	0.0844	0.4337	20	30	0.0312	0.0579	0.0570	0.1495	0.0497	0.0643
10ed002_20060621003r	0.1934	0.0818	0.4230	10	10	0.0300	0.0681	0.3661	0.1091	0.2044	0.0583
Averages:	0.1887	0.1086	0.5978	22	26	0.0321					

a. Values for $(\sigma_{gga})_{position}$ were constant for all transects at 1 m.

Table C.6. Final parameter[a] values from optimization for KPX filter with mean RTK bedload < 0.025 m/s

Transect Name	σ_{bt} (m/s)	σ_{vtg} (m/s)	$\sigma_{vtg} / \sigma_{bt}$	MB Elements to Average	VTG Elements to Average	Sp (m^2)	Kalman Velocity Error (m/s)	VTG Velocity Error (m/s)	GGA Velocity Error (m/s)	BT Biased Velocity Error (m/s)	RTK Bedload (m/s)
07dd002_20060621000r	0.0731	0.1476	2.0192	40	30	0.030	0.0315	0.0505	1.1277	0.0265	0.0059
07dd002_20060621002r	0.0741	0.1118	1.5088	40	20	0.031	0.0322	0.0497	1.7633	0.0309	0.0099
07dd002_20060621003r	0.0571	0.1222	2.1401	40	40	0.033	0.0341	0.0489	2.2804	0.0274	0.0116
07dd002_20060621007r	0.0187	0.0231	1.2353	50	40	0.055	0.0276	0.0458	2.1035	0.0294	0.0089
05bm002_20060530002r	0.1794	0.1543	0.8601	50	30	0.047	0.0471	0.0871	0.0859	0.0811	0.0024
05bm002_20060530003r	0.1459	0.1557	1.0672	50	30	0.054	0.0460	0.0906	0.0780	0.0714	0.0029
05bm002_20060530004r	0.1396	0.1572	1.1261	50	30	0.090	0.0386	0.0794	0.0562	0.0632	0.0027
05bm002_20060530005r	0.1561	0.1639	1.0500	50	30	0.074	0.0448	0.0870	0.1499	0.0700	0.0053
05bm004_20060531000r	0.1894	0.1672	0.8828	50	30	0.031	0.0389	0.0634	0.0223	0.0599	0.0132
05bm004_20060531001r	0.1803	0.1922	1.0660	40	20	0.037	0.0387	0.0686	0.1478	0.0529	0.0062
05bm004_20060531002r	0.1995	0.1810	0.9073	40	10	0.035	0.0376	0.0724	0.0172	0.0645	0.0012
05bm004_20060531003r	0.1913	0.1870	0.9775	30	30	0.039	0.0335	0.0596	0.0198	0.0509	0.0106
07cd005_20060409001r	0.1229	0.1572	1.2791	40	20	0.030	0.0291	0.0578	0.5851	0.0398	0.0142
07da001_20060508001r	0.1760	0.1648	0.9364	50	30	0.035	0.0214	0.0420	0.0469	0.0369	0.0140
07da001_20060508003r	0.1965	0.1797	0.9145	40	10	0.045	0.0214	0.0428	0.0491	0.0381	0.0169
10EA003_20060623000r	0.0895	0.1588	1.7743	50	30	0.030	0.0658	0.1180	0.4296	0.0641	0.0125
10EA003_20060623001r	0.0652	0.0561	0.8604	50	30	0.042	0.0513	0.0902	0.3224	0.0742	0.0107
10EA003_20060623003r	0.0654	0.0954	1.4587	50	20	0.031	0.0550	0.1116	0.6848	0.0701	0.0098
10EA003_20060623005r	0.1960	0.1485	0.7577	10	20	0.033	0.0450	0.0750	0.1479	0.0720	0.0150
10ED001_20060705000r	0.1927	0.0985	0.5112	20	20	0.030	0.0453	0.0669	0.1452	0.1440	0.0081
Averages:	0.1354	0.1411	1.1666	42.00	26.00	0.042					

a. Values for $(\sigma_{gga})_{position}$ were constant for all transects at 1 m.

Table C.7. Final parameter[a] values from optimization for KPF filter with mean RTK Bedload > 0.025 m/s

Transect Name	σ_{bt} (m/s)	σ_{vtg} (m/s)	$\sigma_{vtg} / \sigma_{bt}$	MB Elements to Average	VTG Elements to Average	*Sp* (m^2)	Kalman Velocity Error (m/s)	VTG Velocity Error (m/s)	GGA Velocity Error (m/s)	BT Biased Velocity Error (m/s)	RTK Bedload (m/s)
07cd005_20060409002r	0.1550	0.1777	1.1465	10	40	0.0348	0.0328	0.0665	0.2759	0.0530	0.0303
10EB001_20060623000r	0.1956	0.1516	0.7751	30	20	0.0310	0.0332	0.0527	0.1003	0.0609	0.0285
10EB001_20060623003r	0.1955	0.1202	0.6148	50	10	0.0306	0.0642	0.0850	0.0820	0.1248	0.0889
10ED001_20060705001r	0.1955	0.1035	0.5294	20	10	0.0335	0.0369	0.0614	0.0741	0.1139	0.0578
10ED001_20060705002r	0.1976	0.0780	0.3947	20	20	0.0303	0.0400	0.0579	0.0497	0.1495	0.0260
10ed002_20060621000r	0.1997	0.1034	0.5178	30	40	0.0330	0.0454	0.0669	0.1337	0.1440	0.0609
10ed002_20060621001r	0.2000	0.1000	0.5000	40	20	0.0302	0.0368	0.0614	0.0741	0.1139	0.0564
10ed002_20060621002r	0.1984	0.0793	0.3997	20	30	0.0310	0.0400	0.0579	0.0497	0.1495	0.0643
10ed002_20060621003r	0.2000	0.0837	0.4185	10	30	0.0306	0.0455	0.0681	0.3661	0.1091	0.0583
Averages:	0.1930	0.1108	0.5885	26	24	0.0317					

a. Values for $(\sigma_{gga})_{position}$ were constant for all transects at 1 m.

Table C.8. Final parameter[a] values from optimization for KPF filter with mean RTK bedload < 0.025 m/s

Transect Name	σ_{bt} (m/s)	σ_{vtg} (m/s)	σ_{vtg}/σ_{bt}	MB Elements to Average	VTG Elements to Average	*Sp* (m^2)	Kalman Velocity Error (m/s)	VTG Velocity Error (m/s)	GGA Velocity Error (m/s)	BT Biased Velocity Error (m/s)	RTK Bedload (m/s)
07dd002_20060621000r	0.0927	0.1884	2.0324	40	30	0.031	0.0302	0.0505	0.4314	0.0265	0.0059
07dd002_20060621002r	0.0890	0.1327	1.4910	50	10	0.034	0.0309	0.0497	0.7005	0.0309	0.0099
07dd002_20060621003r	0.0736	0.1455	1.9769	30	20	0.030	0.0295	0.0489	1.0196	0.0274	0.0116
07dd002_20060621007r	0.0784	0.1356	1.7296	50	20	0.030	0.0264	0.0458	1.3935	0.0294	0.0089
05bm002_20060530002r	0.1794	0.1543	0.8601	50	30	0.047	0.0470	0.0871	0.0523	0.0811	0.0024
05bm002_20060530003r	0.1459	0.1557	1.0672	50	30	0.054	0.0460	0.0906	0.0574	0.0714	0.0029
05bm002_20060530004r	0.1396	0.1572	1.1261	50	30	0.090	0.0386	0.0794	0.0531	0.0632	0.0027
05bm002_20060530005r	0.1561	0.1639	1.0500	50	30	0.074	0.0449	0.0870	0.1295	0.0700	0.0053
05bm004_20060531000r	0.1961	0.1698	0.8659	50	20	0.031	0.0388	0.0634	0.0171	0.0599	0.0132
05bm004_20060531001r	0.1896	0.1957	1.0322	40	20	0.037	0.0386	0.0686	0.0841	0.0529	0.0062
05bm004_20060531002r	0.1942	0.1831	0.9428	40	30	0.038	0.0376	0.0724	0.0172	0.0645	0.0012
05bm004_20060531003r	0.1991	0.1977	0.9930	50	40	0.043	0.0333	0.0596	0.0198	0.0509	0.0106
07cd005_20060409001r	0.1356	0.1688	1.2448	40	30	0.030	0.0288	0.0578	0.3377	0.0398	0.0142
07da001_20060508001r	0.1760	0.1648	0.9364	50	30	0.035	0.0214	0.0420	0.0469	0.0369	0.0140
07da001_20060508003r	0.1965	0.1797	0.9145	40	10	0.045	0.0214	0.0428	0.0491	0.0381	0.0169
10EA003_20060623000r	0.0900	0.1594	1.7711	50	40	0.031	0.0659	0.1180	0.4296	0.0641	0.0125
10EA003_20060623001r	0.0706	0.0638	0.9037	40	30	0.030	0.0513	0.0902	0.3224	0.0742	0.0107
10EA003_20060623003r	0.0608	0.0902	1.4836	50	20	0.033	0.0551	0.1116	0.6801	0.0701	0.0098
10EA003_20060623005r	0.1960	0.1485	0.7577	10	20	0.033	0.0450	0.0750	0.1479	0.0720	0.0150
10ED001_20060705000r	0.1989	0.1025	0.5153	20	10	0.033	0.0454	0.0669	0.1337	0.1440	0.0081
Averages:	0.1429	0.1529	1.1847	42.50	25.00	0.040					

Values for $(\sigma_{gga})_{position}$ were constant for all transects at 1 m.

APPENDIX D

MEAN VELOCITY ERRORS

Table D.1. Mean velocity errors for transects with Bedload > 2.5 cm/s, Trial 1a

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT
07cd005_20060 409002r	**Velocity Error (m/s)**	0.05411	0.05405	0.05572	0.05432	0.06646	0.34645	0.27592	0.05297
10EB001_20060 623000r	**Velocity Error (m/s)**	0.04839	0.04840	0.05557	0.05327	0.05272	0.58008	0.10033	0.06086
10EB001_20060 623003r	**Velocity Error (m/s)**	0.05757	0.05758	0.05553	0.05497	0.07102	0.16534	0.06630	0.11915
10ED001_20060 705001r	**Velocity Error (m/s)**	0.02673	0.02674	0.02659	0.02623	0.03439	0.15826	0.09415	0.08294
10ED001_20060 705002r	**Velocity Error (m/s)**	0.03174	0.03174	0.03220	0.03037	0.03864	0.13656	0.04826	0.10235
10ED002_20060 621000r	**Velocity Error (m/s)**	0.05135	0.05135	0.04908	0.04924	0.06693	0.14529	0.13377	0.14406
10ED002_20060 621001r	**Velocity Error (m/s)**	0.04476	0.04476	0.04242	0.04236	0.06144	0.09794	0.07408	0.11390
10ED002_20060 621002r	**Velocity Error (m/s)**	0.04432	0.04432	0.04132	0.04121	0.05794	0.05699	0.04972	0.14945
10ED002_20060 621003r	**Velocity Error (m/s)**	0.04881	0.04882	0.05058	0.04919	0.06515	0.36342	0.20115	0.10524
transect003_ens 1000r	**Velocity Error (m/s)**	0.05599	0.05600	0.05249	0.05155	0.07529	0.21453	0.07295	0.09573
transect004_ens 1000r	**Velocity Error (m/s)**	0.06964	0.06963	0.07219	0.06986	0.09419	0.72140	0.11913	0.14166

Table D.2. Mean velocity errors for transects with Bedload < 2.5 cm/s, Trial 5

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT
07cd005_20060409001r	**Velocity Error (m/s)**	0.03694	0.03694	0.03792	0.03716	0.05785	0.58513	0.33772	0.03977
07DA001_20060508001r	**Velocity Error (m/s)**	0.03287	0.03287	0.03215	0.03215	0.04199	0.04689	0.04689	0.03686
07dd002_20060621000r	**Velocity Error (m/s)**	0.03459	0.03462	0.04039	0.03629	0.05053	1.12775	0.43142	0.02646
07dd002_20060621001r	**Velocity Error (m/s)**	0.03162	0.03140	0.03707	0.03553	0.04523	1.49812	0.67637	0.02693
07dd002_20060621002r	**Velocity Error (m/s)**	0.03376	0.03340	0.03798	0.03962	0.04969	1.76328	0.70050	0.03091
07dd002_20060621003r	**Velocity Error (m/s)**	0.03378	0.03362	0.03647	0.04007	0.04890	2.28043	1.01965	0.02737
07dd002_20060621005r	**Velocity Error (m/s)**	0.03631	0.03686	0.05045	0.04466	0.05152	3.12282	0.63360	0.03068
10EA003_20060623000r	**Velocity Error (m/s)**	0.06989	0.06989	0.06888	0.06888	0.11797	0.42962	0.42962	0.06408
10EA003_20060623001r	**Velocity Error (m/s)**	0.06735	0.06735	0.06670	0.06670	0.09020	0.32244	0.32244	0.07424
10EA003_20060623003r	**Velocity Error (m/s)**	0.06795	0.06795	0.06798	0.06799	0.11156	0.68479	0.68013	0.07005
10EA003_20060623005r	**Velocity Error (m/s)**	0.06543	0.06543	0.06358	0.06358	0.07500	0.14794	0.14794	0.07196
10ED001_20060705000r	**Velocity Error (m/s)**	0.05225	0.05225	0.05166	0.05159	0.03720	0.06669	0.04621	0.05862
05bm002_20060530002r	**Velocity Error (m/s)**	0.07181	0.07181	0.07020	0.06972	0.08705	0.08590	0.05232	0.08114
05bm002_20060530003r	**Velocity Error (m/s)**	0.06303	0.06303	0.06182	0.06192	0.09058	0.07799	0.05741	0.07142
05bm002_20060530004r	**Velocity Error (m/s)**	0.05556	0.05556	0.05429	0.05430	0.07937	0.05619	0.05313	0.06325
05bm002_20060530005r	**Velocity Error (m/s)**	0.06435	0.06434	0.06301	0.06257	0.08700	0.14987	0.12952	0.07001
05bm004_20060	**Velocity Error**	0.05358	0.05358	0.05181	0.05183	0.06344	0.02226	0.01714	0.05986

531000r	**(m/s)**								
05bm004_20060 531001r	**Velocity Error (m/s)**	0.04945	0.04945	0.04868	0.04823	0.06862	0.14781	0.08407	0.05292
05bm004_20060 531002r	**Velocity Error (m/s)**	0.05659	0.05659	0.05481	0.05481	0.07238	0.01720	0.01720	0.06445
05bm004_20060 531003r	**Velocity Error (m/s)**	0.04580	0.04580	0.04459	0.04459	0.05960	0.01981	0.01981	0.05090

APPENDIX E

ANOVA SIGNIFICANCE RESULTS FOR POSITION AND VELOCITY

Table E.1. ANOVA significance between the given signals for Mean Position errors

	KPX VTG	KPX GGAX	KPX GGAF	KPX BT	KPF VTG	KPF GGAX	KPF GGAF	KPF BT	KVX VTG	KVX GGAX	KVX GGAF	KVX BT	KVF VTG	KVF GGAX	KVF GGAF	KVF BT	KPX KPF	KVF KVX	KPX KVX	KPF KVX	KPX KVF	KPF KVF	GGAF VTG	GGAF BT	GGAF GGAX	GGAX VTG	GGAX BT	VTG BT
07cd005_20060409002r	y	n	n	n	y	n	n	n	n	y	y	y	n	y	y	y	n	n	y	y	y	y	y	n	n	y	n	y
10EB001_20060623000r	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	n	y	y	y	y	y	y	n	y	y	y
10EB001_20060623003r	y	n	n	y	y	n	n	y	n	y	y	y	n	y	y	y	n	n	y	y	y	y	y	y	n	y	y	y
10ED001_20060705001r	y	n	n	y	y	n	n	y	n	y	y	y	n	y	y	y	n	n	y	y	y	y	y	y	n	y	y	y
10ED001_20060705002r	y	n	n	y	y	n	n	y	n	y	y	y	n	y	y	y	n	n	y	y	y	y	y	y	n	y	y	y
10ED002_20060621000r	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	n	n	n	n	y	n	n	y	y
10ED002_20060621001r	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	n	n	n	n	y	n	n	y	y
10ED002_20060621002r	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	n	n	n	n	y	n	n	y	y
10ED002_20060621003r	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	n	n	n	n	y	n	n	y	y
transect003_ens1000r	y	n	n	y	y	n	n	y	y	y	y	y	y	y	y	y	n	n	y	y	y	y	y	y	n	y	y	y
transect004_ens1000r	n	n	n	y	n	n	n	y	y	y	y	y	y	y	y	y	n	y	y	y	y	y	n	y	n	n	y	y
05bm002_20060530002r	y	n	n	y	y	n	n	y	y	y	y	y	y	y	y	y	n	n	y	y	y	y	y	y	n	y	y	y
05bm002_20060530003r	n	n	n	y	n	n	n	y	n	n	n	y	y	y	y	y	n	n	n	n	y	y	y	y	n	y	y	y
05bm002_20060530004r	y	y	y	y	y	y	y	y	n	y	y	y	n	y	y	y	n	n	y	y	y	y	y	y	n	y	y	y
05bm002_20060530005r	y	n	n	y	y	n	n	y	y	y	y	y	y	y	y	y	n	n	n	n	n	n	y	y	n	y	y	y
05bm004_20060531000r	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	n	n	n	y	y	n	y	y	y
05bm004_20060531001r	y	n	n	y	y	n	n	y	y	y	y	y	y	y	y	y	n	n	y	y	y	y	y	y	n	y	y	y
05bm004_20060531002r	y	n	n	y	y	n	n	y	n	y	y	y	n	y	y	y	n	n	y	y	y	y	y	y	n	y	y	y
05bm004_20060531003r	y	n	n	y	y	n	n	y	n	y	y	y	n	y	y	y	n	n	n	y	y	y	y	y	n	y	y	y
07cd005_20060409001r	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	y	n	n	n	n	n	n	y	y	n	y	y	y
07DA001_20060508001r	y	n	n	y	y	n	n	y	y	y	y	y	y	y	y	y	n	n	n	y	y	y	y	y	n	y	y	y
07dd002_20060621000r	y	y	y	y	y	y	y	y	n	y	y	n	n	y	y	n	n	n	y	y	y	y	y	y	n	y	y	n
07dd002_20060621001r	n	y	y	n	y	y	n	y	n	y	y	n	n	y	y	n	y	n	n	y	y	y	y	y	n	y	y	n
07dd002_20060621002r	n	y	y	y	n	y	y	y	y	y	y	y	y	y	y	y	n	n	y	y	n	y	y	y	y	y	y	y
07dd002_20060621003r	n	y	y	n	n	y	y	n	n	y	y	y	n	y	y	y	n	n	n	n	n	n	y	n	y	y	y	y
07dd002_20060621005r	y	n	n	y	y	y	n	y	n	y	y	n	n	y	y	n	n	n	y	y	y	y	y	y	n	y	y	n
10EA003_20060623000r	y	n	n	y	y	y	n	y	n	y	y	n	n	y	y	n	n	n	y	y	y	y	y	y	n	y	y	n
10EA003_20060623001r	n	y	y	y	n	y	y	y	n	y	y	y	n	y	y	y	n	n	n	n	n	n	n	y	n	n	y	y
10EA003_20060623003r	y	y	y	y	y	y	y	y	n	y	y	y	n	y	y	y	n	n	y	y	y	y	y	n	n	y	n	y
10EA003_20060623005r	y	n	n	y	y	n	n	y	n	y	y	y	n	y	y	y	n	n	n	n	n	n	y	y	n	y	y	y
10ED001_20060705000r	y	y	y	y	y	y	y	y	y	y	y	n	y	y	y	n	n	n	y	y	y	y	y	y	n	y	y	y

Table E.2. ANOVA significance between the given signals for Mean Velocity errors

	KPX VTG	KPX GGAX	KPX GGAF	KPX BT	KPF VTG	KPF GGAX	KPF GGAF	KPF BT	KVX VTG	KVX GGAX	KVX GGAF	KVX BT	KVF VTG	KVF GGAX	KVF GGAF	KVF BT	KPX KPF	KVF KVX	KPX KVX	KPF KVX	KPX KVF	KPF KVF	GGAF VTG	GGAF BT	GGAF GGAX	GGAX VTG	GGAX BT	VTG BT
07cd005_20060409002r	n	y	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	n	n	n	n	n	y	y	y	y	y	n
10EB001_20060623000r	n	y	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	n	n	n	n	n	n	n	y	y	y	n
10EB001_20060623003r	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	n	n	n	n	n	n	y	y	y	y	y
10ED001_20060705001r	n	y	y	y	n	y	y	y	n	y	y	y	n	y	y	y	n	n	n	n	n	n	y	n	y	y	y	y
10ED001_20060705002r	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	n	n	n	n	n	n	y	y	y	y	y
10ED002_20060621000r	n	y	y	y	n	y	y	y	n	y	y	y	n	y	y	y	n	n	n	n	n	n	y	n	n	y	n	y
10ED002_20060621001r	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	n	n	n	n	n	y	y	y	y	y	y
10ED002_20060621002r	y	y	n	y	y	y	n	y	y	y	y	y	y	y	y	y	n	n	n	n	n	n	y	y	y	n	y	y
10ED002_20060621003r	n	y	y	y	n	y	y	y	n	y	y	y	n	y	y	y	n	n	n	n	n	n	y	y	y	y	y	y
transect003_ens1000r	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	y	n	n	n	n	n	n	n	n	y	y	y	n
transect004_ens1000r	n	y	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	n	n	n	n	n	n	n	y	y	y	n
05bm002_20060530002r	y	y	n	y	y	y	n	y	y	y	n	y	y	y	n	y	n	n	n	n	n	n	y	y	y	n	n	n
05bm002_20060530003r	y	y	n	y	y	y	n	y	y	y	n	y	y	y	n	y	n	n	n	n	n	n	y	n	n	n	n	n
05bm002_20060530004r	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	y	n	n	n	n	n	n	y	n	n	y	n	n
05bm002_20060530005r	y	y	y	n	y	y	y	n	y	y	y	n	y	y	y	n	n	n	n	n	n	n	y	y	n	y	y	n
05bm004_20060531000r	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	n	n	n	n	n	y	y	n	y	y	n
05bm004_20060531001r	n	y	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	n	n	n	n	n	n	n	y	y	y	n
05bm004_20060531002r	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	n	n	n	n	n	y	y	n	y	y	n
05bm004_20060531003r	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	n	n	n	n	n	y	y	n	y	y	y
07cd005_20060409001r	n	y	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	n	n	n	n	n	y	y	y	y	y	n
07DA001_20060508001r	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	y	n	n	n	n	n	n	n	y	n	n	y	n
07dd002_20060621000r	n	y	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	n	n	n	n	n	y	y	y	y	y	n
07dd002_20060621001r	n	y	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	n	n	n	n	n	y	y	y	y	y	n
07dd002_20060621002r	n	y	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	n	n	n	n	n	y	y	y	y	y	n
07dd002_20060621003r	n	y	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	n	n	n	n	n	y	y	y	y	y	n
07dd002_20060621005r	n	y	n	n	n	y	n	n	n	y	n	n	n	y	n	n	n	n	n	n	n	n	n	n	y	y	y	n
10EA003_20060623000r	n	n	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	n	n	n	n	n	y	y	n	y	y	n
10EA003_20060623001r	n	y	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	n	n	n	n	n	y	y	n	y	y	n
10EA003_20060623003r	n	y	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	n	n	n	n	n	y	y	n	y	y	n
10EA003_20060623005r	n	y	y	n	n	y	y	n	n	y	y	n	n	y	y	n	n	n	n	n	n	n	y	y	n	y	y	n
10ED001_20060705000r	n	y	y	y	n	y	y	y	n	y	y	y	n	y	y	y	n	n	n	n	n	n	n	y	y	y	n	y

APPENDIX F

RESULTS FOR MEAN POSITION ERROR, COURSE MADE GOOD (CMG), DISTANCE MADE GOOD (DMG) AND LENGTH

Table F.1. Mean Position Errors wrt RTK for transects with bedload > 2.5 cm/s, Trial 1a

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT
07cd005 _20060409002r	**Position Error (m)**	4.6541	4.6318	0.7889	1.0556	1.0452	4.8848	4.8945	4.8996
10EB001 _20060623000r	**Position Error (m)**	6.8791	6.8651	2.2487	2.2613	8.2136	7.7787	7.7550	2.9632
10EB001 _20060623003r	**Position Error (m)**	1.2637	1.2713	0.3954	0.4671	0.5962	1.2738	1.2536	16.7289
10ED001 _20060705001r	**Position Error (m)**	0.8766	0.8731	0.4793	0.4185	0.4408	0.9215	0.9137	12.0934
10ED001 _20060705002r	**Position Error (m)**	0.7130	0.7197	1.2245	1.2355	1.2005	0.7233	0.7232	10.9342
10ED002 _20060621000r	**Position Error (m)**	1.1057	1.1094	0.7004	0.6063	0.5396	1.1152	1.1171	22.2505
10ED002 _20060621001r	**Position Error (m)**	0.2384	0.2368	0.5749	0.5018	0.7671	0.2288	0.2254	62.9197
10ED002 _20060621002r	**Position Error (m)**	0.9454	0.9461	0.3899	0.3966	0.6039	0.9486	0.9560	18.5170
10ED002 _20060621003r	**Position Error (m)**	0.5526	0.5490	0.4600	0.3677	0.3913	0.6053	0.6006	72.9041
transect003 _ens1000r	**Position Error (m)**	0.3785	0.3707	1.8689	2.0452	0.8683	0.3693	0.3303	13.6597
transect004 _ens1000r	**Position Error (m)**	0.9254	0.8859	4.9644	3.5660	1.1348	1.0524	0.8972	21.3120

Table F.2. Course Made Good (CMG) estimations for transects with Bedload > 2.5 cm/s, Trial 1a

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT	RTK
07cd005 _20060409002r	**CMG (degrees)**	177.7803	177.7768	358.9995	358.5229	358.2350	177.8129	358.2570	173.3994	357.7613
	Bias (degrees)	-179.9809	-179.9845	1.2382	0.7616	0.4737	-179.9484	0.4957	-184.3619	
10EB001 _20060623000r	**CMG (degrees)**	284.5172	284.5732	284.5452	284.3965	284.2915	286.6085	283.9397	281.3930	284.4656
	Bias (degrees)	0.0515	0.1076	0.0795	-0.0691	-0.1741	2.1428	-0.5259	-3.0726	
10EB001 _20060623003r	**CMG (degrees)**	75.0821	75.0821	75.0409	75.0904	75.1654	75.1324	75.8675	65.5296	74.8626
	Bias (degrees)	0.2195	0.2195	0.1783	0.2278	0.3028	0.2698	1.0049	-9.3330	
10ED001 _20060705001r	**CMG (degrees)**	138.1142	138.1162	138.0465	138.0524	138.0301	138.2875	138.0063	141.2922	138.1017
	Bias (degrees)	0.0125	0.0145	-0.0552	-0.0493	-0.0716	0.1858	-0.0954	3.1905	
10ED001 _20060705002r	**CMG (degrees)**	221.5441	221.5437	221.7255	221.7697	221.6766	221.5820	221.7301	222.5019	221.5856
	Bias (degrees)	-0.0415	-0.0419	0.1400	0.1842	0.0911	-0.0036	0.1445	0.9163	
10ED002 _20060621000r	**CMG (degrees)**	246.7583	246.7583	246.7657	246.7928	246.8137	246.7823	247.1082	252.9437	246.8712
	Bias (degrees)	-0.1129	-0.1129	-0.1055	-0.0785	-0.0576	-0.0889	0.2370	6.0725	
10ED002 _20060621001r	**CMG (degrees)**	116.3377	116.3389	116.3807	116.3745	116.3750	116.3898	116.4146	123.1652	116.3759
	Bias (degrees)	-0.0382	-0.0370	0.0049	-0.0014	-0.0009	0.0140	0.0387	6.7893	
10ED002 _20060621002r	**CMG (degrees)**	243.0096	243.0096	243.0111	243.0090	242.9855	243.0106	243.0078	248.3303	243.0851
	Bias (degrees)	-0.0756	-0.0756	-0.0740	-0.0761	-0.0996	-0.0745	-0.0774	5.2452	
10ED002 _20060621003r	**CMG (degrees)**	117.0458	117.0460	117.0270	117.0416	117.0997	117.0284	116.7603	124.9287	117.2302
	Bias (degrees)	-0.1845	-0.1843	-0.2032	-0.1886	-0.1305	-0.2019	-0.4699	7.6985	
transect003 _ens1000r	**CMG (degrees)**	318.6113	318.6113	318.9229	318.9973	319.0317	318.4096	320.0516	317.4135	318.7789
	Bias (degrees)	-0.1676	-0.1676	0.1440	0.2184	0.2528	-0.3693	1.2726	-1.3654	

transect004 _ens1000r	**CMG (degrees)**	98.7792	98.7792	98.2305	99.3681	98.9437	98.7234	96.4618	93.6588	98.8888
	Bias (degrees)	-0.1096	-0.1096	-0.6583	0.4793	0.0549	-0.1654	-2.4270	-5.2299	

Table F.3. Distance Made Good (DMG) estimations for transects with Bedload > 2.5 cm/s, Trial 1a

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT	RTK
07cd005 _20060409002r	**DMG (m)**	64.9678	64.9663	60.7312	59.9701	60.1938	65.0755	54.1264	60.1555	61.3605
	Bias (m)	3.60728	3.60578	-0.62927	-1.39041	-1.16666	3.71499	-7.23411	-1.20500	
	Discrepancy	1.05879	1.05876	0.98974	0.97734	0.98099	1.06054	0.88210	0.98036	
10EB001 _20060623000r	**DMG (m)**	160.2757	160.3709	169.7646	169.8053	176.5615	180.7215	178.3768	168.0844	168.1637
	Bias (m)	-7.88808	-7.79279	1.60084	1.64156	8.39772	12.55775	10.21309	-0.07933	
	Discrepancy	0.95309	0.95366	1.00952	1.00976	1.04994	1.07468	1.06073	0.99953	
10EB001 _20060623003r	**DMG (m)**	167.1487	167.1487	167.8957	167.8908	167.8518	167.3967	166.2388	172.5829	168.6867
	Bias (m)	-1.53801	-1.53801	-0.79097	-0.79584	-0.83486	-1.28999	-2.44786	3.89619	
	Discrepancy	0.99088	0.99088	0.99531	0.99528	0.99505	0.99235	0.98549	1.02310	
10ED001 _20060705001r	**DMG (m)**	426.2970	426.3025	428.2883	428.1865	428.2595	428.5963	426.3400	427.5385	428.3275
	Bias (m)	-2.03045	-2.02496	-0.03917	-0.14097	-0.06801	0.26888	-1.98744	-0.78896	
	Discrepancy	0.99526	0.99527	0.99991	0.99967	0.99984	1.00063	0.99536	0.99816	
10ED001 _20060705002r	**DMG (m)**	427.3055	427.3203	429.2071	429.2479	429.4477	433.3661	430.9888	436.2780	427.7607
	Bias (m)	-0.45518	-0.44041	1.44640	1.48718	1.68704	5.60542	3.22808	8.51732	
	Discrepancy	0.99894	0.99897	1.00338	1.00348	1.00394	1.01310	1.00755	1.01991	
10ED002 _20060621000r	**DMG (m)**	705.5192	705.5192	705.3571	705.4991	705.8247	705.4866	708.7425	699.5486	705.1554
	Bias (m)	0.36381	0.36381	0.20167	0.34373	0.66935	0.33121	3.58715	-5.60674	
	Discrepancy	1.00052	1.00052	1.00029	1.00049	1.00095	1.00047	1.00509	0.99205	
10ED002 _20060621001r	**DMG (m)**	717.3367	717.3605	716.6490	717.0701	716.7881	717.2832	719.5586	714.8580	717.4860
	Bias (m)	-0.14932	-0.12549	-0.83698	-0.41584	-0.69787	-0.20277	2.07260	-2.62799	
	Discrepancy	0.99979	0.99983	0.99883	0.99942	0.99903	0.99972	1.00289	0.99634	
10ED002 _20060621002r	**DMG (m)**	717.1510	717.1508	716.7785	716.8216	717.1394	717.1752	717.2810	699.0027	716.6662
	Bias (m)	0.48481	0.48464	0.11230	0.15541	0.47324	0.50908	0.61480	-17.66349	

	Discrepancy	1.00068	1.00068	1.00016	1.00022	1.00066	1.00071	1.00086	0.97535	
10ED002 _20060621003r	**DMG (m)**	718.2177	718.2193	718.9086	717.9217	718.3346	718.0418	709.7694	724.3472	719.5422
	Bias (m)	-1.32452	-1.32294	-0.63362	-1.62051	-1.20767	-1.50040	-9.77285	4.80495	
	Discrepancy	0.99816	0.99816	0.99912	0.99775	0.99832	0.99791	0.98642	1.00668	
transect003 _ens1000r	**DMG (m)**	412.7587	412.7587	411.7026	412.0084	413.3261	412.1356	415.6006	393.0705	412.7956
	Bias (m)	-0.03698	-0.03698	-1.09308	-0.78721	0.53049	-0.66006	2.80499	-19.72517	
	Discrepancy	0.99991	0.99991	0.99735	0.99809	1.00129	0.99840	1.00680	0.95222	
transect004 _ens1000r	**DMG (m)**	428.0830	428.0830	424.1318	428.0818	425.9033	426.4105	415.7619	422.4963	427.6708
	Bias (m)	0.41218	0.41218	-3.53898	0.41094	-1.76756	-1.26031	-11.90889	-5.17454	
	Discrepancy	1.00096	1.00096	0.99172	1.00096	0.99587	0.99705	0.97215	0.98790	

Table F.4. Length estimations for transects with Bedload > 2.5 cm/s, Trial 1a

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT	RTK
07cd005 _20060409002r	**Length (m)**	81.1680	80.6986	75.0662	74.5154	75.7812	139.3217	119.1876	75.1932	74.9886
	Bias (m)	6.1794	5.7100	0.0776	-0.4732	0.7926	64.3331	44.1990	0.2046	
	Discrepancy	1.08240	1.07614	1.00103	0.99369	1.01057	1.85791	1.58941	1.00273	
10EB001 _20060623000r	**Length (m)**	169.7069	169.7226	172.5281	172.4224	179.2666	282.3300	184.1676	172.6557	171.5398
	Bias (m)	-1.8329	-1.8172	0.9883	0.8826	7.7268	110.7902	12.6278	1.1159	
	Discrepancy	0.98932	0.98941	1.00576	1.00515	1.04504	1.64586	1.07361	1.00650	
10EB001 _20060623003r	**Length (m)**	173.2881	173.5727	173.3763	173.3336	173.9332	195.2789	175.0764	176.5929	178.9874
	Bias (m)	-5.6993	-5.4147	-5.6112	-5.6539	-5.0542	16.2915	-3.9110	-2.3945	
	Discrepancy	0.96816	0.96975	0.96865	0.96841	0.97176	1.09102	0.97815	0.98662	
10ED001 _20060705001r	**Length (m)**	432.4324	432.3596	434.4299	434.2292	434.5271	464.5937	444.5609	434.0346	434.5409
	Bias (m)	-2.1085	-2.1813	-0.1111	-0.3118	-0.0138	30.0527	10.0200	-0.5063	
	Discrepancy	0.99515	0.99498	0.99974	0.99928	0.99997	1.06916	1.02306	0.99883	
10ED001 _20060705002r	**Length (m)**	432.1498	432.1615	433.9135	433.8630	434.2043	457.8440	437.3835	441.2929	432.3832
	Bias (m)	-0.2334	-0.2217	1.5304	1.4798	1.8212	25.4608	5.0004	8.9097	
	Discrepancy	0.99946	0.99949	1.00354	1.00342	1.00421	1.05888	1.01156	1.02061	
10ED002 _20060621000r	**Length (m)**	732.8406	732.8308	732.2863	732.3231	733.3013	753.0393	751.9504	749.1297	732.4449
	Bias (m)	0.3957	0.3858	-0.1586	-0.1218	0.8564	20.5944	19.5055	16.6847	
	Discrepancy	1.00054	1.00053	0.99978	0.99983	1.00117	1.02812	1.02663	1.02278	
10ED002 _20060621001r	**Length (m)**	737.7991	737.8440	737.5004	737.8229	738.5772	748.8112	745.2481	746.7135	739.1589
	Bias (m)	-1.3597	-1.3148	-1.6584	-1.3359	-0.5816	9.6524	6.0893	7.5547	
	Discrepancy	0.99816	0.99822	0.99776	0.99819	0.99921	1.01306	1.00824	1.01022	
10ED002 _20060621002r	**Length (m)**	726.3484	726.3459	725.8410	725.7788	726.5979	729.3971	727.2410	722.5579	725.9505
	Bias (m)	0.3980	0.3954	-0.1094	-0.1716	0.6475	3.4467	1.2905	-3.3925	

	Discrepancy	1.00055	1.00054	0.99985	0.99976	1.00089	1.00475	1.00178	0.99533	
10ED002 _20060621003r	**Length (m)**	742.2698	742.1281	743.0805	742.0084	743.6299	829.3836	762.3607	756.0635	745.0907
	Bias (m)	-2.8209	-2.9626	-2.0101	-3.0823	-1.4607	84.2930	17.2700	10.9729	
	Discrepancy	0.99621	0.99602	0.99730	0.99586	0.99804	1.11313	1.02318	1.01473	
transect003 _ens1000r	**Length (m)**	984.7352	984.9529	984.5385	984.4102	988.0009	1036.2031	999.2191	980.1689	989.1778
	Bias (m)	-4.4427	-4.2250	-4.6393	-4.7676	-1.1769	47.0252	10.0413	-9.0089	
	Discrepancy	0.99551	0.99573	0.99531	0.99518	0.99881	1.04754	1.01015	0.99089	
transect004 _ens1000r	**Length (m)**	817.4984	815.7339	807.7989	805.8720	810.3562	1161.7030	859.1896	790.5358	811.4759
	Bias (m)	6.0225	4.2580	-3.6770	-5.6039	-1.1197	350.2272	47.7138	-20.9401	
	Discrepancy	1.00742	1.00525	0.99547	0.99309	0.99862	1.43159	1.05880	0.97420	

Table F.5. Mean Position Errors wrt RTK for transects with Bedload < 2.5 cm/s, Trial 5

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT
07cd005_20060409001r	**Position Error (m)**	1.3185	1.3077	2.1856	2.0235	0.7999	1.4353	1.4317	2.3050
07DA001_20060508001r	**Position Error (m)**	0.1955	0.1955	3.2889	3.2889	2.5197	0.2087	0.2087	4.0663
07dd002_20060621000r	**Position Error (m)**	1.0988	1.0578	0.7480	0.3657	0.4186	1.8050	1.6650	0.4872
07dd002_20060621001r	**Position Error (m)**	1.2513	1.6701	0.7127	0.6357	0.9284	2.1687	1.9384	0.9944
07dd002_20060621002r	**Position Error (m)**	1.3772	1.4524	0.9051	1.2055	1.5274	2.4939	2.0679	3.6437
07dd002_20060621003r	**Position Error (m)**	0.6723	0.7846	0.6679	0.8033	0.5699	2.3119	1.6181	1.2249
07dd002_20060621005r	**Position Error (m)**	8.7137	8.3136	1.4270	0.8842	1.1436	9.8875	8.7365	1.0807
10EA003_20060623000r	**Position Error (m)**	9.4479	9.4479	3.1635	3.1635	5.8504	13.7586	13.7586	1.9536
10EA003_20060623001r	**Position Error (m)**	1.2036	1.2036	1.7177	1.7177	1.2770	1.4047	1.4047	3.1224
10EA003_20060623003r	**Position Error (m)**	0.9092	0.9173	1.0817	1.0816	0.5275	1.3862	1.3899	1.5110
10EA003_20060623005r	**Position Error (m)**	1.4390	1.4390	1.9240	1.9240	0.9309	1.4775	1.4775	3.1886
10ED001_20060705000r	**Position Error (m)**	21.6594	21.6594	3.2627	3.2670	10.6778	24.3087	24.3024	2.4705
05bm002_20060530002r	**Position Error (m)**	0.3719	0.3823	0.5120	0.5114	0.8935	0.2930	0.2795	1.3512
05bm002_20060530003r	**Position Error (m)**	0.3721	0.3700	1.2397	1.1389	0.8806	0.4140	0.4061	1.7218
05bm002_20060530004r	**Position Error (m)**	0.4720	0.4721	0.5424	0.5424	0.3452	0.5314	0.5312	0.9336
05bm002_20060530005r	**Position Error (m)**	0.4632	0.4601	0.8928	0.7969	0.8888	0.4587	0.4568	1.6325
05bm004_20060531000r	**Position Error (m)**	0.8809	0.8775	1.3287	1.3459	0.8019	0.9807	0.9789	1.7064
05bm004_20060531001r	**Position Error (m)**	0.2889	0.2858	0.8162	0.7448	1.1173	0.2873	0.2553	1.2083

05bm004_20060531002r	**Position Error (m)**	0.2629	0.2629	0.6196	0.6196	0.5060	0.1964	0.1964	1.0788
05bm004_20060531003r	**Position Error (m)**	0.1610	0.1610	0.7685	0.7685	0.8690	0.1697	0.1697	2.0059

Table F.6. Course Made Good (CMG) for transects with Bedload < 2.5 cm/s, Trial 5

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT	RTK
07cd005_200604 09001r	**CMG (degrees)**	1.4204	1.4336	5.8385	5.6250	2.4653	1.2666	11.0608	6.5131	2.0660
	Bias (degrees)	-0.6456	-0.6324	3.7725	3.5590	0.3993	-0.7994	8.9947	4.4471	
07DA001_20060 508001r	**CMG (degrees)**	281.9737	281.9737	282.9087	282.9087	281.6949	281.9886	281.9886	283.2195	282.0122
	Bias (degrees)	-0.0386	-0.0386	0.8964	0.8964	-0.3174	-0.0236	-0.0236	1.2073	
07dd002_200606 21000r	**CMG (degrees)**	19.5682	19.5443	15.4895	15.0460	15.4658	20.4427	26.8992	16.3027	16.8082
	Bias (degrees)	2.7600	2.7362	-1.3187	-1.7621	-1.3424	3.6345	10.0910	-0.5055	
07dd002_200606 21001r	**CMG (degrees)**	341.6121	342.4710	342.4365	341.2011	346.0722	343.4286	344.3357	337.6996	344.6212
	Bias (degrees)	-3.0090	-2.1501	-2.1847	-3.4201	1.4510	-1.1926	-0.2855	-6.9215	
07dd002_200606 21002r	**CMG (degrees)**	13.7154	14.0943	9.9735	9.3436	10.9217	13.1491	199.4950	16.1055	16.1623
	Bias (degrees)	-2.4468	-2.0680	-6.1888	-6.8186	-5.2405	-3.0132	183.3327	-0.0568	
07dd002_200606 21003r	**CMG (degrees)**	342.5771	343.9440	342.2264	341.7034	345.5358	342.6061	339.9764	338.5740	345.4547
	Bias (degrees)	-2.8776	-1.5107	-3.2283	-3.7513	0.0811	-2.8486	-5.4783	-6.8807	
07dd002_200606 21005r	**CMG (degrees)**	179.6058	176.8372	345.4746	341.6920	345.3989	25.2991	353.0136	338.0174	345.4746
	Bias (degrees)	-165.8688	-168.6375	0.0000	-3.7826	-0.0757	-320.1756	7.5390	-7.4573	
10EA003_20060 623000r	**CMG (degrees)**	338.7138	338.7138	334.3735	334.3735	336.1771	337.9970	337.9970	333.1003	335.2880
	Bias (degrees)	3.4258	3.4258	-0.9146	-0.9146	0.8890	2.7090	2.7090	-2.1878	
10EA003_20060 623001r	**CMG (degrees)**	19.6819	19.6819	18.3449	18.3449	19.4837	19.3031	19.3031	16.8477	18.9238
	Bias (degrees)	0.7581	0.7581	-0.5789	-0.5789	0.5600	0.3794	0.3794	-2.0761	
10EA003_20060 623003r	**CMG (degrees)**	336.3519	336.3519	335.0576	335.0595	335.7658	336.4802	337.6634	333.1661	335.5732
	Bias (degrees)	0.7787	0.7787	-0.5155	-0.5137	0.1926	0.9071	2.0902	-2.4071	
10EA003_20060 623005r	**CMG (degrees)**	25.1186	25.1186	23.9191	23.9191	25.5673	25.0550	25.0550	22.5274	25.4754
	Bias (degrees)	-0.3568	-0.3568	-1.5563	-1.5563	0.0919	-0.4204	-0.4204	-2.9480	

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT	RTK
10ED001_20060705000r	**CMG (degrees)**	222.0291	222.0291	221.8074	221.8083	221.0995	221.1179	221.1452	221.7957	221.3433
	Bias (degrees)	0.6858	0.6858	0.4641	0.4650	-0.2438	-0.2254	-0.1981	0.4524	
05bm002_20060530002r	**CMG (degrees)**	160.0725	160.0699	160.0396	160.0163	159.4135	160.0465	160.0337	160.0812	160.2590
	Bias (degrees)	-0.1865	-0.1891	-0.2194	-0.2426	-0.8454	-0.2125	-0.2252	-0.1778	
05bm002_20060530003r	**CMG (degrees)**	199.5244	199.5244	198.7307	198.7476	200.0028	199.7556	199.7885	198.8275	199.6345
	Bias (degrees)	-0.1101	-0.1101	-0.9038	-0.8869	0.3683	0.1211	0.1540	-0.8070	
05bm002_20060530004r	**CMG (degrees)**	159.6252	159.6355	159.8814	159.8827	160.1320	159.8151	160.1557	160.3444	159.8723
	Bias (degrees)	-0.2472	-0.2368	0.0091	0.0103	0.2597	-0.0572	0.2834	0.4721	
05bm002_20060530005r	**CMG (degrees)**	199.8336	199.8336	199.4934	199.4666	200.1389	200.0669	200.0813	199.8984	199.7132
	Bias (degrees)	0.1204	0.1204	-0.2198	-0.2466	0.4257	0.3537	0.3681	0.1852	
05bm004_20060531000r	**CMG (degrees)**	174.3412	174.3412	175.8386	175.8452	174.6610	174.2352	174.4496	175.8476	174.5050
	Bias (degrees)	-0.1638	-0.1638	1.3335	1.3401	0.1560	-0.2698	-0.0554	1.3426	
05bm004_20060531001r	**CMG (degrees)**	185.0375	185.0373	184.6934	184.7312	185.6693	185.1142	185.4082	184.2741	185.0800
	Bias (degrees)	-0.0425	-0.0427	-0.3866	-0.3488	0.5893	0.0343	0.3282	-0.8059	
05bm004_20060531002r	**CMG (degrees)**	174.8355	174.8355	175.0761	175.0761	175.0077	174.7924	174.7924	175.1679	174.8927
	Bias (degrees)	-0.0572	-0.0572	0.1834	0.1834	0.1150	-0.1003	-0.1003	0.2752	
05bm004_20060531003r	**CMG (degrees)**	185.8234	185.8234	185.0222	185.0222	186.0906	185.8610	185.8610	184.4843	185.8749
	Bias (degrees)	-0.0515	-0.0515	-0.8526	-0.8526	0.2158	-0.0138	-0.0138	-1.3906	

Table F.7. Distance Made Good (DMG) for transects with Bedload < 2.5 cm/s, Trial 5

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT	RTK
07cd005_200604 09001r	**DMG (m)**	62.6663	62.6201	62.9072	62.9535	61.9105	63.1585	39.9081	62.8373	60.7726
	Bias (m)	1.8936	1.8475	2.1346	2.1809	1.1378	2.3858	-20.8645	2.0647	
	Discrepancy	1.0312	1.0304	1.0351	1.0359	1.0187	1.0393	0.6567	1.0340	
07DA001_20060 508001r	**DMG (m)**	332.8912	332.8912	329.2402	329.2402	330.3446	333.6868	333.6868	329.7578	332.5790
	Bias (m)	0.3122	0.3122	-3.3388	-3.3388	-2.2344	1.1078	1.1078	-2.8212	
	Discrepancy	1.0009	1.0009	0.9900	0.9900	0.9933	1.0033	1.0033	0.9915	
07dd002_200606 21000r	**DMG (m)**	16.4306	16.5304	16.8909	15.9221	15.7461	17.7070	46.2205	16.8298	15.8980
	Bias (m)	0.5325	0.6323	0.9928	0.0241	-0.1520	1.8090	30.3224	0.9318	
	Discrepancy	1.0335	1.0398	1.0624	1.0015	0.9904	1.1138	2.9073	1.0586	
07dd002_200606 21001r	**DMG (m)**	17.3614	17.4679	14.8724	15.3282	14.1054	15.8327	33.5356	15.8534	15.8472
	Bias (m)	1.5142	1.6207	-0.9748	-0.5189	-1.7418	-0.0145	17.6885	0.0062	
	Discrepancy	1.0956	1.1023	0.9385	0.9673	0.8901	0.9991	2.1162	1.0004	
07dd002_200606 21002r	**DMG (m)**	14.9698	17.0298	15.4771	15.4681	13.6755	15.7777	22.4463	20.1356	15.7720
	Bias (m)	-0.8022	1.2578	-0.2949	-0.3040	-2.0965	0.0057	6.6742	4.3636	
	Discrepancy	0.9491	1.0797	0.9813	0.9807	0.8671	1.0004	1.4232	1.2767	
07dd002_200606 21003r	**DMG (m)**	14.9943	16.0676	15.3459	16.2055	15.1055	16.8029	54.2483	15.4804	16.4652
	Bias (m)	-1.4709	-0.3976	-1.1193	-0.2597	-1.3597	0.3377	37.7831	-0.9848	
	Discrepancy	0.9107	0.9759	0.9320	0.9842	0.9174	1.0205	3.2947	0.9402	
07dd002_200606 21005r	**DMG (m)**	7.8445	7.6243	14.0996	15.5790	13.8423	80.5831	18.6394	16.7530	15.7497
	Bias (m)	-7.9053	-8.1254	-1.6502	-0.1708	-1.9074	64.8333	2.8896	1.0033	
	Discrepancy	0.4981	0.4841	0.8952	0.9892	0.8789	5.1165	1.1835	1.0637	

10EA003_20060 623000r	**DMG (m)**	71.8606	71.8606	80.9031	80.9031	89.7503	107.3138	107.3138	83.2412	84.7751
	Bias (m)	-12.9145	-12.9145	-3.8720	-3.8720	4.9752	22.5387	22.5387	-1.5339	
	Discrepancy	0.8477	0.8477	0.9543	0.9543	1.0587	1.2659	1.2659	0.9819	
10EA003_20060 623001r	**DMG (m)**	80.3840	80.3840	80.4610	80.4610	77.5875	78.5470	78.5470	80.7944	80.4920
	Bias (m)	-0.1081	-0.1081	-0.0310	-0.0310	-2.9045	-1.9450	-1.9450	0.3024	
	Discrepancy	0.9987	0.9987	0.9996	0.9996	0.9639	0.9758	0.9758	1.0038	
10EA003_20060 623003r	**DMG (m)**	85.6983	85.6983	83.7787	83.7789	84.8031	87.5397	90.0505	84.9027	84.9455
	Bias (m)	0.7528	0.7528	-1.1668	-1.1667	-0.1424	2.5942	5.1050	-0.0428	
	Discrepancy	1.0089	1.0089	0.9863	0.9863	0.9983	1.0305	1.0601	0.9995	
10EA003_20060 623005r	**DMG (m)**	85.9005	85.9005	85.5219	85.5219	85.6047	85.8114	85.8114	85.1946	84.6844
	Bias (m)	1.2161	1.2161	0.8375	0.8375	0.9203	1.1270	1.1270	0.5102	
	Discrepancy	1.0144	1.0144	1.0099	1.0099	1.0109	1.0133	1.0133	1.0060	
10ED001_20060 705000r	**DMG (m)**	401.4560	401.4564	422.6071	422.5805	436.0183	440.3675	438.7597	426.7276	424.9586
	Bias (m)	-23.5026	-23.5022	-2.3515	-2.3781	11.0597	15.4089	13.8011	1.7690	
	Discrepancy	0.9447	0.9447	0.9945	0.9944	1.0260	1.0363	1.0325	1.0042	
05bm002_20060 530002r	**DMG (m)**	95.3290	95.3943	94.6327	94.7297	96.0304	96.0960	96.8066	94.2628	95.5081
	Bias (m)	-0.1791	-0.1138	-0.8754	-0.7784	0.5223	0.5879	1.2985	-1.2453	
	Discrepancy	0.9981	0.9988	0.9908	0.9919	1.0055	1.0062	1.0136	0.9870	
05bm002_20060 530003r	**DMG (m)**	98.2780	98.2780	96.4836	96.6026	97.0986	97.9971	99.5700	96.2273	96.7161
	Bias (m)	1.5619	1.5619	-0.2325	-0.1134	0.3825	1.2811	2.8540	-0.4887	
	Discrepancy	1.0161	1.0161	0.9976	0.9988	1.0040	1.0132	1.0295	0.9949	
05bm002_20060 530004r	**DMG (m)**	96.9983	97.0218	95.7329	95.7342	96.6759	97.6391	98.7941	95.8565	96.6140
	Bias (m)	0.3843	0.4078	-0.8810	-0.8798	0.0619	1.0251	2.1802	-0.7575	
	Discrepancy	1.0040	1.0042	0.9909	0.9909	1.0006	1.0106	1.0226	0.9922	

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT	RTK
05bm002_20060530005r	**DMG (m)**	97.3645	97.3645	95.9482	96.0540	97.0212	95.7581	93.9733	95.4415	97.2891
	Bias (m)	0.0755	0.0755	-1.3409	-1.2351	-0.2678	-1.5310	-3.3158	-1.8476	
	Discrepancy	1.0008	1.0008	0.9862	0.9873	0.9972	0.9843	0.9659	0.9810	
05bm004_20060531000r	**DMG (m)**	112.6409	112.6409	111.8714	111.8457	111.5160	112.5152	111.4083	112.4036	111.6861
	Bias (m)	0.9548	0.9548	0.1853	0.1596	-0.1701	0.8291	-0.2778	0.7175	
	Discrepancy	1.0085	1.0085	1.0017	1.0014	0.9985	1.0074	0.9975	1.0064	
05bm004_20060531001r	**DMG (m)**	111.6866	111.6853	110.9317	111.0125	111.2983	111.6496	112.9336	111.6421	111.4625
	Bias (m)	0.2241	0.2228	-0.5308	-0.4500	-0.1643	0.1871	1.4711	0.1796	
	Discrepancy	1.0020	1.0020	0.9952	0.9960	0.9985	1.0017	1.0132	1.0016	
05bm004_20060531002r	**DMG (m)**	111.6277	111.6277	111.4543	111.4543	111.2149	111.4992	111.4992	112.0893	111.5628
	Bias (m)	0.0650	0.0650	-0.1085	-0.1085	-0.3479	-0.0635	-0.0635	0.5265	
	Discrepancy	1.0006	1.0006	0.9990	0.9990	0.9969	0.9994	0.9994	1.0047	
05bm004_20060531003r	**DMG (m)**	112.0397	112.0397	110.7598	110.7598	110.5505	112.0693	112.0693	111.0255	111.9140
	Bias (m)	0.1257	0.1257	-1.1542	-1.1542	-1.3635	0.1553	0.1553	-0.8885	
	Discrepancy	1.0011	1.0011	0.9897	0.9897	0.9878	1.0014	1.0014	0.9921	

Table F8. Length estimations for transects with bedload < 2.5 cm/s, Trial 5

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT	RTK
07cd005_200604 09001r	**Length (m)**	80.9999	80.5413	78.3059	78.3988	78.6311	233.3908	142.5489	79.0570	75.8221
	Bias (m)	5.1778	4.7192	2.4838	2.5766	2.8090	157.5687	66.7268	3.2349	
	Discrepancy	1.0683	1.0622	1.0328	1.0340	1.0370	3.0781	1.8800	1.0427	
07DA001_20060 508001r	**Length (m)**	400.9686	400.9686	397.6775	397.6775	401.0811	403.6078	403.6078	399.3183	401.1871
	Bias (m)	-0.2185	-0.2185	-3.5096	-3.5096	-0.1060	2.4207	2.4207	-1.8688	
	Discrepancy	0.9995	0.9995	0.9913	0.9913	0.9997	1.0060	1.0060	0.9953	
07dd002_200606 21000r	**Length (m)**	22.5843	21.2664	17.8303	17.1442	17.9337	196.7393	80.4644	17.6941	16.9759
	Bias (m)	5.6083	4.2905	0.8543	0.1682	0.9577	179.7634	63.4884	0.7182	
	Discrepancy	1.3304	1.2527	1.0503	1.0099	1.0564	11.5893	4.7399	1.0423	
07dd002_200606 21001r	**Length (m)**	28.7090	22.8272	16.5202	16.5164	16.7487	278.1305	117.6382	16.5891	16.5876
	Bias (m)	12.1214	6.2396	-0.0674	-0.0712	0.1611	261.5430	101.0506	0.0015	
	Discrepancy	1.7308	1.3762	0.9959	0.9957	1.0097	16.7674	7.0920	1.0001	
07dd002_200606 21002r	**Length (m)**	31.6551	25.1897	17.4076	17.2416	16.2879	325.0184	124.7428	21.3687	16.7218
	Bias (m)	14.9333	8.4679	0.6858	0.5198	-0.4339	308.2966	108.0211	4.6469	
	Discrepancy	1.8930	1.5064	1.0410	1.0311	0.9741	19.4368	7.4599	1.2779	
07dd002_200606 21003r	**Length (m)**	32.8016	28.0875	16.4154	17.2110	17.3595	483.7801	217.0072	16.3739	16.9938
	Bias (m)	15.8079	11.0937	-0.5784	0.2172	0.3657	466.7863	200.0135	-0.6199	
	Discrepancy	1.9302	1.6528	0.9660	1.0128	1.0215	28.4681	12.7698	0.9635	
07dd002_200606 21005r	**Length (m)**	47.1723	38.3476	18.1946	17.8465	17.2560	603.4412	139.3080	17.4378	16.3099
	Bias (m)	30.8624	22.0377	1.8847	1.5366	0.9462	587.1313	122.9981	1.1280	
	Discrepancy	2.8922	2.3512	1.1156	1.0942	1.0580	36.9985	8.5413	1.0692	
10EA003_20060 623000r	**Length (m)**	95.6363	95.6363	90.3988	90.3988	95.0734	137.3351	137.3351	93.6902	93.6652
	Bias (m)	1.9711	1.9711	-3.2664	-3.2664	1.4082	43.6699	43.6699	0.0250	

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT	RTK
	Discrepancy	1.0210	1.0210	0.9651	0.9651	1.0150	1.4662	1.4662	1.0003	
10EA003_20060 623001r	**Length (m)**	92.2789	92.2789	91.8725	91.8725	91.9833	132.5795	132.5795	93.4827	92.9039
	Bias (m)	-0.6250	-0.6250	-1.0314	-1.0314	-0.9207	39.6756	39.6756	0.5788	
	Discrepancy	0.9933	0.9933	0.9889	0.9889	0.9901	1.4271	1.4271	1.0062	
10EA003_20060 623003r	**Length (m)**	94.9422	94.8949	93.4628	93.4625	95.2652	166.8628	167.5583	95.9250	94.6025
	Bias (m)	0.3397	0.2924	-1.1397	-1.1400	0.6628	72.2604	72.9559	1.3225	
	Discrepancy	1.0036	1.0031	0.9880	0.9879	1.0070	1.7638	1.7712	1.0140	
10EA003_20060 623005r	**Length (m)**	103.3373	103.3373	101.7890	101.7890	104.1288	117.9463	117.9463	102.2732	102.2149
	Bias (m)	1.1224	1.1224	-0.4259	-0.4259	1.9139	15.7314	15.7314	0.0583	
	Discrepancy	1.0110	1.0110	0.9958	0.9958	1.0187	1.1539	1.1539	1.0006	
10ED001_20060 705000r	**Length (m)**	446.4608	446.4599	431.5394	431.5174	444.1637	451.5817	448.2208	435.9751	433.9052
	Bias (m)	12.5556	12.5547	-2.3658	-2.3878	10.2585	17.6765	14.3156	2.0699	
	Discrepancy	1.0289	1.0289	0.9945	0.9945	1.0236	1.0407	1.0330	1.0048	
05bm002_20060 530002r	**Length (m)**	114.8789	114.9002	113.0406	113.0762	115.9277	119.9358	115.0326	113.8353	113.5489
	Bias (m)	1.3300	1.3513	-0.5083	-0.4727	2.3788	6.3869	1.4837	0.2864	
	Discrepancy	1.0117	1.0119	0.9955	0.9958	1.0209	1.0562	1.0131	1.0025	
05bm002_20060 530003r	**Length (m)**	113.4383	113.5391	111.3131	111.3565	113.5798	119.0481	114.9437	112.2196	112.1230
	Bias (m)	1.3153	1.4161	-0.8099	-0.7665	1.4568	6.9251	2.8207	0.0966	
	Discrepancy	1.0117	1.0126	0.9928	0.9932	1.0130	1.0618	1.0252	1.0009	
05bm002_20060 530004r	**Length (m)**	107.0258	107.0540	106.0677	106.0697	107.7195	109.5196	109.9509	106.8519	107.2577
	Bias (m)	-0.2320	-0.2037	-1.1900	-1.1880	0.4618	2.2619	2.6932	-0.4058	
	Discrepancy	0.9978	0.9981	0.9889	0.9889	1.0043	1.0211	1.0251	0.9962	

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT	RTK
05bm002_20060530005r	Length (m)	115.6846	115.6090	113.3131	113.1797	115.5285	129.7769	125.7342	113.3192	114.5871
	Bias (m)	1.0975	1.0219	-1.2740	-1.4074	0.9414	15.1898	11.1471	-1.2679	
	Discrepancy	1.0096	1.0089	0.9889	0.9877	1.0082	1.1326	1.0973	0.9889	
05bm004_20060531000r	Length (m)	118.0016	118.0288	116.6801	116.6596	117.2614	117.2290	116.1679	117.5866	116.1097
	Bias (m)	1.8919	1.9191	0.5705	0.5499	1.1517	1.1194	0.0583	1.4770	
	Discrepancy	1.0163	1.0165	1.0049	1.0047	1.0099	1.0096	1.0005	1.0127	
05bm004_20060531001r	Length (m)	120.7587	120.7037	119.2553	119.3066	120.5314	127.5483	121.8741	120.4758	119.9323
	Bias (m)	0.8264	0.7714	-0.6770	-0.6257	0.5991	7.6161	1.9419	0.5435	
	Discrepancy	1.0069	1.0064	0.9944	0.9948	1.0050	1.0635	1.0162	1.0045	
05bm004_20060531002r	Length (m)	116.8760	116.8760	116.1988	116.1988	117.3361	116.6266	116.6266	117.3569	116.3416
	Bias (m)	0.5343	0.5343	-0.1428	-0.1428	0.9945	0.2850	0.2850	1.0153	
	Discrepancy	1.0046	1.0046	0.9988	0.9988	1.0085	1.0024	1.0024	1.0087	
05bm004_20060531003r	Length (m)	115.9856	115.9856	114.4411	114.4411	115.2228	115.8826	115.8826	115.2085	115.3825
	Bias (m)	0.6030	0.6030	-0.9414	-0.9414	-0.1598	0.5001	0.5001	-0.1740	
	Discrepancy	1.0052	1.0052	0.9918	0.9918	0.9986	1.0043	1.0043	0.9985	

Table F.9. Mean Position Errors wrt RTK for transects with Bedload < 2.5 cm/s, Trial 1

Transect		KPX	KPF	KVX	KVF	VTG	GGAX	GGAF	BT
07cd005_20060409001r	**Position Error (m)**	1.3290	1.3095	1.6429	1.2347	0.7999	1.4353	1.4317	2.3050
07DA001_20060508001r	**Position Error (m)**	0.2041	0.2041	2.2530	2.2530	2.5197	0.2087	0.2087	4.0663
07dd002_20060621000r	**Position Error (m)**	1.1115	1.0828	2.2930	0.4168	0.4186	1.8050	1.6650	0.4872
07dd002_20060621001r	**Position Error (m)**	1.2073	1.5983	0.6104	0.5944	0.9284	2.1687	1.9384	0.9944
07dd002_20060621002r	**Position Error (m)**	1.4289	1.5100	1.3682	2.0088	1.5274	2.4939	2.0679	3.6437
07dd002_20060621003r	**Position Error (m)**	0.7600	0.8583	0.5165	1.1422	0.5699	2.3119	1.6181	1.2249
07dd002_20060621005r	**Position Error (m)**	8.8092	8.4421	6.2545	2.7249	1.1436	9.8875	8.7365	1.0807
10EA003_20060623000r	**Position Error (m)**	9.6004	9.6004	0.9880	0.9880	5.8504	13.7586	13.7586	1.9536
10EA003_20060623001r	**Position Error (m)**	1.1993	1.1993	1.5476	1.5476	1.2770	1.4047	1.4047	3.1224
10EA003_20060623003r	**Position Error (m)**	0.9665	0.9744	0.4878	0.5031	0.5275	1.3862	1.3899	1.5110
10EA003_20060623005r	**Position Error (m)**	1.2050	1.2050	1.0283	1.0283	0.9309	1.4775	1.4775	3.1886
10ED001_20060705000r	**Position Error (m)**	21.6065	21.6060	0.7096	0.6885	10.6778	24.3087	24.3024	2.4705
05bm002_20060530002r	**Position Error (m)**	0.3824	0.3970	1.3286	1.3516	0.8935	0.2930	0.2795	1.3512
05bm002_20060530003r	**Position Error (m)**	0.4315	0.4277	0.9330	1.0166	0.8806	0.4140	0.4061	1.7218
05bm002_20060530004r	**Position Error (m)**	0.5324	0.5327	0.8304	0.8305	0.3452	0.5314	0.5312	0.9336
05bm002_20060530005r	**Position Error (m)**	0.4355	0.4313	0.6100	0.7124	0.8888	0.4587	0.4568	1.6325
05bm004_20060531000r	**Position Error (m)**	0.9280	0.9244	0.3754	0.4732	0.8019	0.9807	0.9789	1.7064
05bm004_20060531001r	**Position Error (m)**	0.3298	0.3278	0.8357	1.0081	1.1173	0.2873	0.2553	1.2083

05bm004_20060531002r	**Position Error (m)**	0.2013	0.2013	0.3007	0.3007	0.5060	0.1964	0.1964	1.0788
05bm004_20060531003r	**Position Error (m)**	0.1833	0.1833	0.3130	0.3130	0.8690	0.1697	0.1697	2.0059

APPENDIX G

DISCHARGE RESULTS

Table G.1. Discharge Estimations for Transects with Bedload > 2.5 cm/s, Trial 1a

Transect		RTK	KPX	KPF	KVX	KVF	VTG	GGA	BT	UBT[a]
07cd005 _20060409002r	**Discharge (m^3/s)**	123.801	123.187	123.189	123.664	123.524	123.438	128.702	116.565	121.131
	Discharge Error (%)		0.4960	0.4943	0.1107	0.2237	0.2932	3.9588	5.8449	2.1567
10EB001 _20060623000r	**Discharge (m^3/s)**	829.130	827.321	827.332	827.86	826.634	827.684	835.782	792.656	815.748
	Discharge Error (%)		0.2182	0.2169	0.1532	0.3010	0.1744	0.8023	4.3991	1.6140
10EB001 _20060623003r	**Discharge (m^3/s)**	890.883	889.398	889.404	889.001	889.351	889.679	889.341	812.488	880.93
	Discharge Error (%)		0.1667	0.1660	0.2113	0.1720	0.1351	0.1731	8.7997	1.1172
10ED001 _20060705001r	**Discharge (m^3/s)**	2825.65	2829.46	2829.47	2830.16	2829.64	2827.46	2825.62	2736.68	2833.32
	Discharge Error (%)		0.1348	0.1349	0.1595	0.1411	0.0639	0.0013	3.1489	0.2714
10ED001 _20060705002r	**Discharge (m^3/s)**	2729.55	2722.83	2722.83	2723.91	2722.91	2722.51	2730.05	2731.22	2722.31
	Discharge Error (%)		0.2460	0.2461	0.2066	0.2433	0.2577	0.0183	0.0614	0.2652
10ED002 _20060621000r	**Discharge (m^3/s)**	11585.39	11607.47	11607.48	11610.28	11608.03	11592.34	11600.94	10834.22	11426.86
	Discharge Error (%)		0.1906	0.1907	0.2148	0.1955	0.0600	0.1342	6.4837	1.3683
10ED002 _20060621001r	**Discharge (m^3/s)**	11544.18	11551.82	11451.29	11547.58	11553.95	11551.00	11549.68	10940.46	11353.07
	Discharge Error (%)		0.0662	0.8046	0.0294	0.0847	0.0591	0.0477	5.2296	1.6554
10ED002 _20060621002r	**Discharge (m^3/s)**	11427.42	11454.29	11451.81	11449.08	11449.49	11446.55	11431.20	10527.52	11107.54
	Discharge Error (%)		0.2351	0.2134	0.1895	0.1931	0.1674	0.0330	7.8750	2.7993
10ED002 _20060621003r	**Discharge (m^3/s)**	11588.69	11583.91	11583.90	11603.28	11582.64	11576.00	11596.72	10748.26	11316.89
	Discharge Error (%)		0.0412	0.0413	0.1259	0.0522	0.1095	0.0693	7.2522	2.3454
transect003 _ens1000r	**Discharge (m^3/s)**	2143.59	2143.28	2143.25	2143.51	2142.28	2139.21	2168.32	1998.54	2104.95
	Discharge Error (%)		0.0147	0.0160	0.0041	0.0611	0.2047	1.1533	6.7669	1.8028

Transect		RTK	KPX	KPF	KVX	KVF	VTG	GGA	BT	UBT[a]
transect004_ens1000r	Discharge (m^3/s)	3096.06	3136.69	3143.25	3105.17	3134.47	3104.30	3110.47	2765.91	3164.29
	Discharge Error (%)		1.3124	1.5243	0.2943	1.2405	0.2660	0.4653	10.6635	2.2038

[a]UBT is unbiased bottom tracking. It is determined by subtracting bedload from the bottom track signal using the method described in section 5.3 to calculate bedload.

Table G.2. Discharge Estimations for Transects with Bedload > 2.5 cm/s, Trial 2a

Transect		RTK	KPX	KPF	KVX	KVF	VTG	GGA	BT	UBT
07cd005 _20060409002r	**Discharge (m^3/s)**	123.801	121.931	121.926	122.086	121.805	123.438	128.702	116.565	117.943
	Discharge Error (%)		1.5105	1.5145	1.3853	1.6123	0.2932	3.9588	5.8449	4.7318
10EB001 _20060623000r	**Discharge (m^3/s)**	829.130	827.125	827.130	829.561	827.811	827.684	835.782	792.656	823.214
	Discharge Error (%)		0.2418	0.2412	0.0520	0.1591	0.1744	0.8023	4.3991	0.7135
10EB001 _20060623003r	**Discharge (m^3/s)**	890.883	891.504	891.509	891.036	891.275	889.679	889.341	812.488	901.389
	Discharge Error (%)		0.0697	0.0703	0.0172	0.0440	0.1351	0.1731	8.7997	1.1793
10ED001 _20060705001r	**Discharge (m^3/s)**	2825.65	2829.179	2829.428	2830.367	2830.234	2827.46	2825.62	2736.68	2833.178
	Discharge Error (%)		0.1247	0.1336	0.1668	0.1621	0.0639	0.0013	3.1489	0.2663
10ED001 _20060705002r	**Discharge (m^3/s)**	2729.55	2721.855	2721.853	2721.853	2722.455	2722.51	2730.05	2731.22	2714.955
	Discharge Error (%)		0.3028	0.3029	0.3029	0.2809	0.2577	0.0183	0.0614	0.5556
10ED002 _20060621000r	**Discharge (m^3/s)**	11585.39	11613.96	11613.98	11617.18	11615.32	11592.34	11600.94	10834.22	11480.94
	Discharge Error (%)		0.2481	0.2482	0.2759	0.2598	0.0600	0.1342	6.4837	0.9001
10ED002 _20060621001r	**Discharge (m^3/s)**	11544.18	11580.31	11580.31	11575.48	11579.17	11551.00	11549.68	10940.46	11486.36
	Discharge Error (%)		0.3121	0.3121	0.2702	0.3022	0.0591	0.0477	5.2296	0.5017
10ED002 _20060621002r	**Discharge (m^3/s)**	11427.42	11466.20	11466.20	11462.89	11462.90	11446.55	11431.20	10527.52	11355.81
	Discharge Error (%)		0.3393	0.3393	0.3104	0.3104	0.1674	0.0330	7.8750	0.6267
10ED002 _20060621003r	**Discharge (m^3/s)**	11588.69	11594.87	11594.93	11603.70	11593.79	11576.00	11596.72	10748.26	11509.27
	Discharge Error (%)		0.0533	0.0538	0.1295	0.0440	0.1095	0.0693	7.2522	0.6853
transect003 _ens1000r	**Discharge (m^3/s)**	2143.59	2130.633	2130.53	2131.381	2130.112	2139.21	2168.32	1998.54	2105.586
	Discharge Error (%)		0.6046	0.6094	0.5697	0.6289	0.2047	1.1533	6.7669	1.7731
transect004 _ens1000r	**Discharge (m^3/s)**	3096.06	3315.082	3314.965	3281.483	3288.482	3104.30	3110.47	2765.91	3446.822
	Discharge Error (%)		7.0742	7.0704	5.9890	6.2151	0.2660	0.4653	10.6635	11.3293

Table G.3. Discharge Estimations for Transects with Low Bedload < 2.5 cm/s, Trial 1 (assuming moving bed)

Transect		RTK	KPX	KPF	KVX	KVF	VTG	GGA	BT	UBT[a]
07cd005 _20060409001r	**Discharge (m^3/s)**	123.016	126.078	126.100	126.857	126.674	125.448	129.697	122.766	127.778
	Discharge Error (%)		2.4891	2.5070	3.1224	2.9736	1.9770	5.4310	0.2032	3.8710
07da001 _20060508001r	**Discharge (m^3/s)**	387.155	386.262	386.262	386.365	386.365	386.513	389.223	377.238	384.115
	Discharge Error (%)		0.2307	0.2307	0.2041	0.2041	0.1658	0.5342	2.5615	0.7852
07dd002 _20060621000r	**Discharge (m^3/s)**	7.328	7.083	7.113	7.751	7.409	7.321	7.597	7.451	7.420
	Discharge Error (%)		3.3433	2.9340	5.7724	1.1053	0.0955	3.6709	1.6785	1.2555
07dd002 _20060621001r	**Discharge (m^3/s)**	7.394	6.504	6.516	6.520	6.705	6.539	9.928	7.282	7.076
	Discharge Error (%)		12.0368	11.8745	11.8204	9.3184	11.5634	34.2710	1.5147	4.3008
07dd002 _20060621002r	**Discharge (m^3/s)**	7.566	6.398	6.375	6.393	6.280	6.140	11.129	7.788	7.406
	Discharge Error (%)		15.4375	15.7415	15.5036	16.9971	18.8475	47.0923	2.9342	2.1147
07dd002 _20060621003r	**Discharge (m^3/s)**	8.069	7.303	7.294	7.269	7.425	6.791	22.077	7.850	8.298
	Discharge Error (%)		9.4931	9.6047	9.9145	7.9812	15.8384	173.6027	2.7141	2.8380
07dd002 _20060621005r	**Discharge (m^3/s)**	7.290	6.999	6.969	6.402	7.005	6.060	18.967	7.581	6.459
	Discharge Error (%)		3.9918	4.4033	12.1811	3.9095	16.8724	160.1783	3.9918	11.3992
10EA003 _20060623000r	**Discharge (m^3/s)**	224.758	218.217	218.217	220.721	220.721	217.451	223.297	217.106	219.727
	Discharge Error (%)		2.9102	2.9102	1.7962	1.7962	3.2511	0.6500	3.4046	2.2384
10EA003 _20060623001r	**Discharge (m^3/s)**	234.489	226.411	226.411	226.661	226.661	231.306	229.484	230.208	226.679
	Discharge Error (%)		3.4449	3.4449	3.3383	3.3383	1.3574	2.1344	1.8257	3.3306
10EA003 _20060623003r	**Discharge (m^3/s)**	226.756	228.186	228.184	230.382	230.399	226.923	221.887	219.713	224.813
	Discharge Error (%)		0.6306	0.6298	1.5991	1.6066	0.0736	2.1472	3.1060	0.8569
10EA003 _20060623005r	**Discharge (m^3/s)**	225.256	232.048	232.048	231.070	231.070	230.042	226.993	223.848	231.642
	Discharge Error (%)		3.0152	3.0152	2.5811	2.5811	2.1247	0.7711	0.6251	2.8350
10ED001	**Discharge (m^3/s)**	2663.041	2671.532	2671.535	2672.472	2672.235	2669.176	2665.057	2654.063	2676.619

_20060705000r	**Discharge Error (%)**		0.3188	0.3190	0.3541	0.3452	0.2304	0.0757	0.3371	0.5099
05bm002 _20060530002r	**Discharge (m^3/s)**	144.140	148.019	148.019	147.546	147.632	145.515	145.394	146.751	151.707
	Discharge Error (%)		2.6911	2.6911	2.3630	2.4226	0.9539	0.8700	1.8114	5.2498
05bm002 _20060530003r	**Discharge (m^3/s)**	145.575	147.441	147.442	147.400	147.437	145.282	147.250	148.264	148.682
	Discharge Error (%)		1.2818	1.2825	1.2536	1.2791	0.2013	1.1506	1.8472	2.1343
05bm002 _20060530004r	**Discharge (m^3/s)**	144.513	145.356	145.356	145.069	145.079	142.348	144.622	145.484	147.926
	Discharge Error (%)		0.5833	0.5833	0.3847	0.3917	1.4981	0.0754	0.6719	2.3617
05bm002 _20060530005r	**Discharge (m^3/s)**	147.136	151.147	151.150	149.953	150.399	146.563	148.314	148.351	150.980
	Discharge Error (%)		2.7260	2.7281	1.9146	2.2177	0.3894	0.8006	0.8258	2.6125
05bm004 _20060531000r	**Discharge (m^3/s)**	132.789	133.939	133.940	133.667	133.564	132.213	132.726	136.811	135.057
	Discharge Error (%)		0.8660	0.8668	0.6612	0.5836	0.4338	0.0474	3.0289	1.7080
05bm004 _20060531001r	**Discharge (m^3/s)**	137.414	138.299	138.290	137.945	138.462	137.703	138.474	135.627	139.553
	Discharge Error (%)		0.6440	0.6375	0.3864	0.7627	0.2103	0.7714	1.3004	1.5566
05bm004 _20060531002r	**Discharge (m^3/s)**	134.832	135.213	135.213	135.182	135.182	134.912	134.786	137.066	136.695
	Discharge Error (%)		0.2826	0.2826	0.2596	0.2596	0.0593	0.0341	1.6569	1.3817
05bm004 _20060531003r	**Discharge (m^3/s)**	135.191	135.104	135.104	135.030	135.030	133.468	134.961	133.581	136.375
	Discharge Error (%)		0.0644	0.0644	0.1191	0.1191	1.2745	0.1701	1.1909	0.8758

[a]UBT is unbiased bottom tracking. It is determined by subtracting bedload from the bottom track signal using the method described in section 5.3 to calculate bedload.

Table G.4. Discharge Estimations for Transects with Bedload < 2.5 cm/s, Trial 5, (assuming no moving bed)

Transect		RTK	KPX	KPF	KVX	KVF	VTG	GGA	BT
07cd005 _20060409001r	**Discharge (m^3/s)**	123.016	124.454	124.463	124.784	124.985	125.448	129.697	122.766
	Discharge Error (%)		1.1690	1.1763	1.4372	1.6006	1.9770	5.4310	0.2032
07da001 _20060508001r	**Discharge (m^3/s)**	387.155	379.010	379.010	379.014	379.014	386.513	389.223	377.238
	Discharge Error (%)		2.1038	2.1038	2.1028	2.1028	0.1658	0.5342	2.5615
07dd002 _20060621000r	**Discharge (m^3/s)**	7.328	7.120	7.137	7.249	7.250	7.321	7.597	7.451
	Discharge Error (%)		2.8384	2.6064	1.0781	1.0644	0.0955	3.6709	1.6785
07dd002 _20060621001r	**Discharge (m^3/s)**	7.394	6.924	6.925	6.958	7.102	6.539	9.928	7.282
	Discharge Error (%)		6.3565	6.3430	5.8967	3.9491	11.5634	34.2710	1.5147
07dd002 _20060621002r	**Discharge (m^3/s)**	7.566	7.222	7.213	7.237	6.983	6.140	11.129	7.788
	Discharge Error (%)		4.5467	4.6656	4.3484	7.7055	18.8475	47.0923	2.9342
07dd002 _20060621003r	**Discharge (m^3/s)**	8.069	7.292	7.278	7.187	7.321	6.791	22.077	7.850
	Discharge Error (%)		9.6294	9.8029	10.9307	9.2700	15.8384	173.6027	2.7141
07dd002 _20060621005r	**Discharge (m^3/s)**	7.290	7.323	7.301	7.574	7.905	6.060	18.967	7.581
	Discharge Error (%)		0.4527	0.1509	3.8957	8.4362	16.8724	160.1783	3.9918
10EA003 _20060623000r	**Discharge (m^3/s)**	224.758	218.961	218.961	218.415	218.415	217.451	223.297	217.106
	Discharge Error (%)		2.5792	2.5792	2.8221	2.8221	3.2511	0.6500	3.4046
10EA003 _20060623001r	**Discharge (m^3/s)**	234.489	231.664	231.664	231.685	231.685	231.306	229.484	230.208
	Discharge Error (%)		1.2047	1.2047	1.1958	1.1958	1.3574	2.1344	1.8257
10EA003 _20060623003r	**Discharge (m^3/s)**	226.756	224.914	224.913	223.897	223.897	226.923	221.887	219.713
	Discharge Error (%)		0.8123	0.8128	1.2608	1.2608	0.0736	2.1472	3.1060
10EA003 _20060623005r	**Discharge (m^3/s)**	225.256	224.900	224.900	224.805	224.805	230.042	226.993	223.848
	Discharge Error (%)		0.1580	0.1580	0.2002	0.2002	2.1247	0.7711	0.6251
10ED001	**Discharge (m^3/s)**	2663.041	2655.065	2655.064	2655.477	2655.300	2669.176	2665.057	2654.063

_20060705000r	**Discharge Error (%)**		0.2995	0.2995	0.2840	0.2907	0.2304	0.0757	0.3371
05bm002 _20060530002r	**Discharge (m^3/s)**	144.140	145.210	145.211	145.213	145.271	145.515	145.394	146.751
	Discharge Error (%)		0.7423	0.7430	0.7444	0.7847	0.9539	0.8700	1.8114
05bm002 _20060530003r	**Discharge (m^3/s)**	145.575	147.516	147.516	147.455	147.478	145.282	147.250	148.264
	Discharge Error (%)		1.3333	1.3333	1.2914	1.3072	0.2013	1.1506	1.8472
05bm002 _20060530004r	**Discharge (m^3/s)**	144.513	144.163	144.163	144.219	144.219	142.348	144.622	145.484
	Discharge Error (%)		0.2422	0.2422	0.2034	0.2034	1.4981	0.0754	0.6719
05bm002 _20060530005r	**Discharge (m^3/s)**	147.136	147.172	147.174	147.264	147.264	146.563	148.314	148.351
	Discharge Error (%)		0.0245	0.0258	0.0870	0.0870	0.3894	0.8006	0.8258
05bm004 _20060531000r	**Discharge (m^3/s)**	132.789	135.631	135.631	135.561	135.554	132.213	132.726	136.811
	Discharge Error (%)		2.1402	2.1402	2.0875	2.0823	0.4338	0.0474	3.0289
05bm004 _20060531001r	**Discharge (m^3/s)**	137.414	136.315	136.315	136.083	135.776	137.703	138.474	135.627
	Discharge Error (%)		0.7998	0.7998	0.9686	1.1920	0.2103	0.7714	1.3004
05bm004 _20060531002r	**Discharge (m^3/s)**	134.832	135.700	135.776	135.776	135.776	134.912	134.786	137.066
	Discharge Error (%)		0.6438	0.7001	0.7001	0.7001	0.0593	0.0341	1.6569
05bm004 _20060531003r	**Discharge (m^3/s)**	135.191	132.943	132.943	132.939	132.939	133.468	134.961	133.581
	Discharge Error (%)		1.6628	1.6628	1.6658	1.6658	1.2745	0.1701	1.1909