ON GUARANTEED ASYMPTOTIC STABILITY OF UNCERTAIN SYSTEMS WITH INCOMPLETE STATE INFORMATION

By

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ABSTRACT

In this thesis, we present a method for designing stabilizing feedback control laws for linear uncertain(unstable) systems with incomplete state information.In earlier papers [1-24] on this topic, it was assumed that the entire state vector was available. We have eliminated this restrictive assumption via use of Luenberger observer. By introducing a new matching condition,we are able to design a reduced order observer. It has been shown that the combined plant-observer system is asymptotically stable with respect to the zero state.

CONTENTS

CHAPTER 1. INTRODUCTION

1.1 Review

In automatic control literature, a control problem can be represented by a system of differential equations characterized by a finite number of parameters, and system uncertainty is represented by the uncertainty of those parameters. The problems of stability and control of systems with uncertain parameters have been treated in several categories according to different assumptions and approaches. The first is the stochastic approach where a prior probability law is imposed on the uncertain parameters. If the statistical property of the uncertain parameters cannot be assumed beforehand but can be identified in the course of evolution of the dynamic process, the idea of adaptive or learning control is used. The third is the sensitivity approach. This approach is based on the assumption that the parameter uncertainty is small so that first-order perturbation equations can be obtained from which a controller is designed to optimize certain measure of performance.

The stochastic approach needs the knowledge of a distribution. Adaptive control gives good performance, but the procedure is usually complicated and expensive. The sensitivity approach is limited mainly by the assumption of small perturbation.

In recent years, some new approaches have been developed for controlling the so-called uncertain dynamical system which lead to either asymptotic stability or ultimate boundedness of the state of the system(see for example Chang and Peng (1972); Leitmann (1978, 1979, 1981); Molander (1979); Gutman (1979); Virkely and Wood (1980); Corless and Leitmann (1981); Thorp and Barmish (1981); Barmish and Leitmann (1982), and their bibliographies [1-23]). Although these problems involve controlling a system with uncertain parameters,they differ from stochastic and adaptive control problems in two fundamental ways:

1. No a priori statistics is assumed for the uncertain parameters; instead, only a bound on the parameter variations is assumed to be given and the objective is to design a controller which satisfactorily regulates the system.

2.The uncertain parameters might be rapidly varying; Lebesgue measureability is only assumed.

Given such a situation with so-called matching conditions which are the keys in the previous papers, the objective is to construct a state feedback control law, either linear or nonlinear, which guarantees a prescribed system behavior. Here the term 'guaranteed performance' is used to mean that the resulting closed-loop system will have certain desirable properties(for example, asymptotic stability or ultimate boundedness).

The issue of linear versus nonlinear control was considered by [2] and [13]. It was shown that there are some cases when one might prefer nonlinear control over linear control[16]. But Barmish has shown[13] that a linear time-invariant feedback control will suffice for a large class of systems stabilized by a nonlinear control. In this thesis, we concentrate our attention on guaranteed stability of uncertain systems via linear control.

In the literature, dealing with the questions of ultimate boundedness and guaranteed stability, there is another fundamental question; namely, what are the sufficient conditions that the uncertain parameters must satisfy, to guarantee ultimate boundedness and stability. In all of the previous references, the success of the various control schemes proposed depends crucially on the satisfaction of additional a priori assumptions. These assumptions essentially restrict the locations and sizes of the uncertain parameters within the system matrices. In almost all the previous references, these restrictions have been appropriately referred to as matching conditions. It can be interpreted roughly as follows: A system is matched if the uncertainties with the system matrix A ($\Delta A(q(t))$) and input matrix B ($\Delta B(q(t))$) are in the range of B;at the same time, $\Delta A(q(t))$ is in the range of C. i.e., the uncertainties are "reachable" from input and output. The simplest example of such a system is obtained by considering a nth-order linear differential equation with coefficients which include uncertain parameters. In Thorp and Barmish[9], these matching conditions are somewhat generalized leading to a weaker requirement on the system structure. In Barmish and Leitmann[10], it is shown that ultimate boundedness is still possible as long as the matching conditions are not violated 'too severly'. This means that the matching condition is only sufficient for stabilizeability and not necessary. That is, there exist systems which fail to satisfy the matching condition and yet can be stabilized. Examples of such systems can be found in[11] and [22]. In Petersen's work [15], he considered the necessity of the matching condition under introducing a strengthened notion of stabilizeability refered to as structural stabilizeability via a nominally determined quadratic Lyapunov function. It is then shown that if a system is to have this stronger property, the matching condition must necessarily be satisfied.

This kind of deterministic design procedure for uncertain systems which are nominally time-invariant can be applied to many cases of interest. For instance, see the application to macroeconomics discussed in[8]. Recently, Ryan and Leitmann[24] developed this theory for application to robotic tracking. They used this class of feedback controllers to the tracking problem for a robotic manipulator with n controlled degrees of freedom and uncertain dynamics. In view of the kinematic definitions implicit in a 2n-dimensional state space description of the mechanical system, n of the state equations are free of uncertainty and, while uncertainty may be present in each of the remaining n state equations, a control component also appears therein; this implies that the system uncertainty is matched. As a consequence of the latter property, the tracking system, with arbitrary bounded uncertainty, is practically stabilizeable with respect to the proposed class of feedback controls, in the sense that given any feasible path to be tracked and an arbitrary small neighborhood Σ of the origin in the appropriate error space, there exists a control such that the tracking error for the feedback controlled uncertain system is ultimately bounded with respect to Σ.

There is also a variety of other results which can be used to design linear controllers for linear time-invariant systems with uncertainty [26-30]. All are characterized by a concern with a quadratic cost function which is specified by weighting matrices that are assumed given in the problem statement. In [27]and [28], with time-invariant uncertainty, the weighting on state in the cost function is modified in a search for better controls. In [26] and [27], instead of imposing matching conditions on the state matrix A(.), the disturbance is assumed to enter linearly, and restrictive conditions are imposed on the bounding sets. None of these approaches utilizes matching conditions, and none can be guaranteed to succeed. That is,given an arbitrary but bounded set for the disturbances,there is no guaranty that a linear feedback gain exists which stabilizes the system for all admissible uncertainties. Matrix norm inequalities can be obtained [20] which guarantees stability only if the uncertainty is sufficiently small. This is because they do not take advantage of system structure.

So far, most of those guaranteed controller designs are based on the assumption that the state variable of the system to be controlled is available for measurement. Namely, the feedback control is a function of the complete state vector. In many practical situations, however, only a few output quantities are available. That is, the entire state is not available for direct measurement, only part of the state is given by the system outputs. Thus either a suitable estimate approximating the unavailable states must be provided, or a new approach that directly accounts for the nonavailability of the complete state information must be devised. In their recent paper [25], Galimidi and Barmish presented a method for designing an output feedback controller leading to asymptotic stability of a given equilibrium point. They seek a controller which operates on the measured output vector. Generally, the lack of full state information leads to additional restriction on the class of systems for which a stabilizing controller can be found. In their paper, they show that some additional requirements must be met in order to guarantee the existence of a suitable output feedback controller. It is shown how the absence of full state information leads to a set of linear constraints on the entries of the matrix P used in the Lyapunov function $V(x) = x'Px$. That is, stabilizeability via output feedback can be guaranteed if stabilizeability via full state feedback can be established using a Lyapunov matrix P satisfying certain linear constraints. This formulation leads to a purely algebraic problem-the constrained Lyapunov problem (CLP). Given the matrices describing an uncertain linear system, a solution to the CLP, when it exists, can be used to design a stabilizing output feedback controller.

In this thesis, we adopt the first approach, whereby one develops and uses an approximate state vector. One of the problems discussed in this thesis is that of reconstructing the state vector from the available outputs. The system which performs the reconstruction is called an observer.

One possible method for obtaining the state vector is to simulate a model of the given system, drive the model with the same inputs as the original system, and use the state vector of the model as an approximation to the unknown state vector. In this method the dynamic behavior of the observer is identical to that of the system it observes. If initial conditions were not set properly or if there were slight disturbances, the model generally would not recover fast enough to provide an estimate suitable for control. Another approach is to differentiate the available outputs a number of times and then combine these derivatives appropriately to obtain the state vector. In this case, the estimate responds instantaneously to disturbance, but it is highly sensitive to a small quantity of additive noise.

It is important to design an observer which is a compromise between these two simple procedures. It is desirable that the dynamic elements of the observer be faster than those of the system it observes, but not so fast as to possess the undesirable characteristics of differentiators (which correspond to poles at ∞). Based on these ideas, Luenberger developed the observer theory. Although Kalman had done some work on this problem before, primarily for sampled-data systems, he had treated both the nonrandom problem and the problem of estimating the state when measurement of the output are corrupted by noise. Luenberger observer [31-38], which differs from the Kalman filter, has the inputs and available outputs of the system whose state is to be approximated as its inputs and has a state vector that is linearly related to the approximate state. The simplicity of its design and its resolution of the difficulty imposed by missing measurements make the observer an attractive general design component.

In the observer theory, two kinds of observer can be designed: identity observer(full-order observer) or reduced order observer. The identity observer although possessing an ample measure of simplicity also possesses a certain degree of redundancy. The redundancy stems from the fact that while the observer constructs an estimate of the entire state, part of the state as given by the system outputs are already available by direct measurement. This redundancy can be eliminated and an observer of lower dimension but still of arbitrary dynamics can be constructed. In this thesis, we will mainly construct a reduced order observer. The basic idea of a reduced-order observer is that if the output y(t) is of dimension r an observer of order n-r is constructed with state z(t) that approximates $\hat{C}x(t)$ for some $r \times n$ matrix $\hat{C}$. Hence, as more output variables are made available, the required order of the observer is decreased.

Once the observer has been introduced, an approximate state vector will be substituted for the unavailable state. This decomposes the control design problem into two phases. The first phase consists of designing the control assuming that the entire state vector is available. This can be done using standard procedures[1-14]. The second phase consists of constructing a system that produces an estimate of the unavailable states. The observer is a dynamic system whose characteristics are somewhat free to be determined by the designer, and it is through its introduction that dynamics enter the overall two-phase design procedure when the entire state is not available.

A primary consideration that arises when this philosophy is used is the problem that use of the estimated state vector, rather than the actual state vector called for by a control. Various criteria can be used to measure this deterioration. Especially, the most important consideration is the effect of an observer on the closed-loop stability properties of the system. It would be undesirable, for example, if an otherwise stable control design becomes unstable when it was realized by introduction of an observer. If, initially, the estimated state vector is equal to the actual state vector, i.e., $\bar{z}(0) = 0$, equality will be maintained for all future time. This important fact is due to what might be described as the complete uncontrollability of the observer from u(it is shown in Chapter 4). It implies that if there is initial equality between the state and its estimate the closed-loop system using the estimate behaves exactly like the closed-loop system using the actual state to obtain the control. However, generally the initial equality between the state and its estimate will not be obtained, thus stability properties of the complete system, including the observer, must be investigated. If a system were time invariant, then its asymptotic stability would follow immediately from the eigenvalues of the system. Since the system we consider is a time-varying system, a closer analysis of the stability of the system is required. Another issue shown in this thesis is that the observer we constructed, fortunately, does not disturb stability properties when they are introduced. It is shown that the resulting closed-loop system is asymptotically stable even when the linear uncertain system is originally unstable.

1.2 Organization of the Thesis

Following is the organization of the rest of the thesis.

In chapter 2, the uncertain dynamical system and its nominal time-invariant system described by the nth order vector-differential equation are given. Introducing the observeability of the nominal system and a new matching condition, basic assumptions for stabilizability are presented. Under the assumption that all the states of the system are available for direct measurement we present a theorem developed by Thorp and Barmish[9], it is shown that the system is asymptotically stable based on complete state information. Also steps for construction of feedback matrix K are given.

In chapter 3, a method for designing stabilizing feedback control laws for the uncertain systems with incomplete state information is presented. First, the closed-loop system with the observer is given. Then, we describe the procedure to construct the observer. Finally, it is shown that the overall closed-loop system is asymptotically stable.

In chapter 4, two examples and their simulation results are discussed. One is a two-dimensional system which contains only one uncertain parameter. Another is a three-dimensional system including two random parameters. For different random processes with different amplitudes, under different feedback gains, the simulation results are given.

Finally, in chapter 5, some concluding remarks and suggestions for further work are presented.

CHAPTER 2. GUARANTEED STABILITY BASED ON COMPLETE STATE INFORMATION

2.1 System and Assumptions

Consider the uncertain dynamical system described by the nth order vector-differential equation:

$$(\Sigma) \qquad \begin{cases} \dot{x}(t) = [A + \Delta A(q(t))]x(t) + [B + \Delta B(q(t))]u(t) \\ \\ y(t) = Cx(t) \qquad\qquad t \in [0, \infty) \end{cases}$$

With the nominal system being time-invariant

$$NOM(\Sigma) \qquad \begin{cases} \dot{x}(t) = Ax(t) + Bu(t), \\ \\ y(t) = Cx(t) \end{cases}$$

Where

A is a constant $n \times n$ matrix,

B is a constant $n \times m$ matrix,

C is a constant $r \times n$ matrix,

$\Delta A(q)$ is an $n \times n$ matrix,

$\Delta B(q)$ is an $n \times m$ matrix,

$x(t) \in R^n$ is the state,

$u(t) \in R^m$ is the control,

$y(t) \in R^r$ is the output,

and $\{q(t), t \geq 0\}$ is the uncertainty which is restricted to a prescribed bounded set $Q \subset R^k$,that is,$q(t) \in Q$ for all $t \geq 0$.

Before giving the assumptions, we need the following definitions.

Definition 2.1 :

The time-invariant system NOM(Σ) is observable if the rank of observability matrix $O = (C' \; A'C' \cdots (A')^{n-1}C')$ equals n.

Definition 2.2 :

The system NOM(Σ) is controllable if the rank of controllability matrix $C = (B \; AB \cdots A^{n-1}B)$ is n.

Definition 2.3 :

The control system NOM(Σ) is stabilizable if there exists an $m \times n$ matrix K such that A+BK is a stability matrix. (i.e., all eigenvalues of A+BK have negative real parts.)

It is well known that controllability implies stabilizability in the control theory of time-invariant linear system. The converse is not true.

In the sequel we use the following

Assumptions :

(A1) $\{A, B\}$ is stabilizeable.

(A2) $\{A, C\}$ is observable.

(A3) $\Delta A(.)$ and $\Delta B(.)$ are continuous on R^k.

(A4) The uncertainty $q(.) : [0, \infty) \rightarrow Q$ is Lebesgue measurable.

(A5) The set Q is compact(closed and bounded).

(A6) Matching conditions: There exist continuous matrix valued functions $D(.) : Q \rightarrow R^{m\times n}$,$E(.) : Q \rightarrow R^{n\times r}$ and $V(.) : Q \rightarrow R^{m\times m}$ such that

1) $\Delta A(q) = BD(q)$

2) $\Delta A(q) = E(q)C$

3) $\Delta B(q) = BV(q)$

4) $\| V(q) \| < 1 \qquad \forall q \in Q.$

where the norm of a matrix T is taken as

$$\| T \| \equiv (\lambda_{max}[T'T])^{1/2}$$

and $\lambda_{max}(M)$ denotes the largest eigenvalue of M.

(A7) $RankB = m \leq n$ and $rankC = r \leq n$.These assumptions are made without any loss of generality.

Remarks :

(i) In this thesis, we add assumptions(A2) and (A6 2)) which guarantee observanbility of the uncertain system (Σ), the other assumptions being those of the previous paper [9].

(ii) The matrices $\Delta A(q)$ and $\Delta B(q)$ might each depend on different components of q. That is,we might have q=(r,s) where $\Delta A(.)$ depends only on r and $\Delta B(.)$ on s.

2.2 Steps for Construction of Feedback Matrix K

In this chapter, we assume all the states of the system are available for direct measurement.

Under the assumptions of previous section, except(A2) and (A6 2)), Thorp and Barmish [9] constructed a purely linear feedback control which guarantees uniform asymptotic stability in the sense of Lyapunov. The following theorem is given in [9].

Theorem 2.1 : Consider the uncertain linear system

$$\dot{x}(t) = [A + \Delta A(q(t))]x(t) + [B + \Delta B(q(t))]u(t), \qquad (1)$$

with state feedback control

$$u(t) \equiv Kx(t). \qquad (2)$$

where K is constructed via steps 2.1 through 2.5. Then, for all admissible uncertainty q(.)(see (A5)), the origin x=0 is uniformly asymptotically stable.

The following steps generate the feedback gain K:

Step2.1: Construct a matrix K_0 such that $\bar{A} = A + BK_0$ is a stability matrix.This is possible by (A1). Let

$$V(x) = x'Px, \qquad P = P' > 0$$

be a Lyapunov function for the (stable) system $\dot{x} = \bar{A}x$.

Step2.2: Let T be any $n \times n$ nonsingular matrix such that TB has the block structure $TB = \begin{pmatrix} 0 \\ I_m \end{pmatrix}$($I_m$ denotes a unit matrix of dimension m).This is possible since rank B=m.

Step2.3: Form the matrices

$$\bar{F} = T\bar{A}T^{-1},$$

$$\Delta F(q) = T[\Delta A(q) + \Delta B(q)K_0]T^{-1},$$

$$G = TB,$$

$$\Delta G(q) = T\Delta B(q),$$

$$S = TP^{-1}T',$$

and the partition

$$M(q) \equiv [\bar{F} + \Delta F(q)]S + S[\bar{F}' + \Delta F'(q)]$$
$$= \begin{pmatrix} M_{11} & M_{12}(q) \\ M'_{12}(q) & M_{22}(q) \end{pmatrix}.$$

Due to the matching condition the partition block M_{11} is either empty or invertable and independent of $q \in Q$.

Step2.4: Select a real scalar $\gamma < 0$ such that [1]

$$\gamma < -\frac{|\max_{q\in Q} \lambda_{max}[M_{22}(q) - M'_{12}(q)M_{11}^{-1}M_{12}(q)]|}{2(1 - \max_{q\in Q} \| V(q) \|)}.$$

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1 When m=n,the M_{11} block is empty,and the $M_{22}(q) = M(q)$.For this case,the numerator is defined to be simply

$$\max_{q\in Q} \lambda_{max}[M_{22}(q)].$$

We note that the denominator is strictly positive,since any maximizer q* of $\| V(q) \|$ satisfies $\| V(q*) \| < 1$. The existence of such a maximizer is assured because V(.) is continuous and Q is compact.

Step2.5: The desired feedback matrix is now given by

$$K = K_0 + \gamma[0|I_m](T^{-1})'P = K_0 + \gamma B'P \equiv K_0 + K_1. \quad (3)$$

2.3 Proof for the Theorem

Proof : The theorem is established by introducing new state variables

$$\bar{x} \equiv Tx \quad and \quad \tilde{x} \equiv S^{-1}\bar{x}.$$

Since the transformation matrices **T** and S are independent of q(.), it suffices to establish uniform asymptotic stability in $\tilde{x}$-space and/or $\bar{x}$-space. A straightforward computation yields the transformed state equations

$$\dot{\bar{x}}(t) = [\bar{F} + \Delta F(q(t))]\bar{x}(t) + [G + \Delta G(q(t))]u_*(t), \tag{5}$$

where

$$u_*(t) = K_1 T^{-1}\bar{x}(t),$$

and

$$\dot{\tilde{x}}(t) = S^{-1}[\bar{F} + \Delta F(q(t))]S\tilde{x}(t) + S^{-1}[G + \Delta G(q(t))]u_*(t), \tag{6}$$

where

$$u_*(t) = K_1 T^{-1} S\tilde{x}(t).$$

To facilitate completion of the proof, we require a number of observations.

Observation 2.1.

The transformed matrices $\Delta F(.), G, \Delta G(.)$ in (5) also satisfy assumptions (A5) of the section 2.1; that is, by defining

$$D_{*}(q) \equiv [D(q) + V(q)K_0]T^{-1}, V_{*}(q) \equiv V(q),$$

it follows that

$$\Delta F(q) = GD_{*}(q), \quad \forall q \in Q, \tag{7}$$

$$\Delta G(q) = GV_{*}(q), \quad \forall q \in Q, \tag{8}$$

$$\| V_{*}(q) \| < 1, \quad \forall q \in Q. \tag{9}$$

Observation 2.2.

In light of Observation 2.1 and the special structure of (1), see Step2.2 and 2.3 the state equation (5) can be written in the partitioned form

$$\dot{\bar{x}}(t) = \begin{pmatrix} \bar{F}_{11} & \bar{F}_{12} \\ \bar{F}_{21} + \Delta F_{21}(q(t)) & \bar{F}_{22} + \Delta F_{22}(q(t)) \end{pmatrix} \bar{x}(t) + \begin{pmatrix} 0 \\ I_m + V(q(t)) \end{pmatrix} u_{*}(t).$$

The fact that the uncertainty q(.) does not appear in the (1,1)-partitions and (1,2)-partitions of $\bar{F} + \Delta F(.)$ is a consequence of the transformed matching conditions (7)-(9).

Observation 2.3.

The uncontrolled nominal $\bar{x}$-system

$$\dot{\bar{x}}(t) = \bar{F}\bar{x}(t)$$

has Lyapunov function

$$V_*(\bar{x}) \equiv \bar{x}'S^{-1}\bar{x};$$

recall that

$$S = TP^{-1}T'.$$

To establish this fact, we first note that S^{-1} is positive definite. Recall that **P** is positive definite and **T** is invertible. Next, we compute the Lyapunov derivative

$$\mathcal{L}_*(\bar{x}(t),t) \equiv gradV_*'(\bar{x}(t))\dot{\bar{x}}(t).$$

Then, given a fixed pair $(\bar{x},t)$, one can compute

$$\mathcal{L}_*(\bar{x},t) = \bar{x}'[(T^{-1})'(\bar{A}'P + P\bar{A})T^{-1}]\bar{x}.$$

Hence, the negative definiteness of $\bar{A}'P + P\bar{A}$ implies that $\mathcal{L}_*(\bar{x},t)$ is negative definite as well.

Observation 2.4.

Using Observation 2.3, it is easily shown that

$$\tilde{V}(\tilde{x}) \equiv \tilde{x}'S\tilde{x}$$

serves as a Lyapunov function 2 for the uncontrolled $\tilde{x}$-system

$$\dot{\tilde{x}}(t) = S^{-1}\bar{F}S\tilde{x}(t).$$

Observation 2.5.

By partitioning $\tilde{x}(t)$ into $\begin{pmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{pmatrix}$, we notice that the control $u_*(t)$ can be simplified. Namely,

$$u_*(t) = K_1 T^{-1} S\tilde{x}(t) = \gamma[0|I_m](T^{-1})' P T^{-1} S \begin{pmatrix} \tilde{x}_1(t) \\ \tilde{x}_2(t) \end{pmatrix} = \gamma\tilde{x}_2(t).$$

Returning to the proof of the theorem, we take $\tilde{V}(\tilde{x})$ as a Lyapunov candidate for the controlled $\tilde{x}$-system (6) with uncertainties entering through $\Delta F(.)$ and V(.). Using Observation 2.5, the Lyapunov derivative is computed to be

$$\begin{aligned}\tilde{\mathcal{L}}(\tilde{x}, t) &= \tilde{x}' S\{S^{-1}[\bar{F} + \Delta F(q(t))]S\tilde{x} + S^{-1}[G + \Delta G(q(t))]\gamma\tilde{x}_2(t)\} \\ &= \tilde{x}' M(q(t))\tilde{x} + \gamma\tilde{x}_2' H(q(t))\tilde{x}_2,\end{aligned}$$

where

$$H(q) \equiv 2I_m + V(q) + V'(q).$$

2 The negative definiteness of the Lyapunov derivative

$$\dot{\mathcal{L}}(\tilde{x}, t) = \tilde{x}'[FS + SF']\tilde{x}$$

is an easy consequence of the negative definiteness of the matrix

$$F'S^{-1} + S^{-1}F.$$

Using the definition of M(q) in conjunction with Observation 2.2, it follows that the partition M_{11} of M(q) is independent of q. Without loss of generality, we assume that this partition is nonempty. We also note that M_{11} is negative definite; that is, M_{11} is the (1,1)- partition of $\bar{F}S + S\bar{F}'$(see Observation 2.4 and its footnote). We now rewrite $\tilde{\mathcal{L}}(\tilde{x},t)$ as

$$\tilde{\mathcal{L}}(\tilde{x},t) = \tilde{x}' \begin{pmatrix} M_{11} & M_{12}(q(t)) \\ M_{12}(q(t)) & M_{22}(q(t)) + \gamma H(q(t)) \end{pmatrix} \tilde{x} \equiv \tilde{x}'\tilde{M}(q(t))\tilde{x}.$$

Consequently, to complete the proof of the theorem, it suffices to show that there is a constant $\tilde{\lambda} < 0$ such that

$$\tilde{x}'\tilde{M}(q)\tilde{x} \leq \tilde{\lambda}\tilde{x}'\tilde{x} \tag{10}$$

holds for all $\tilde{x} \in R^n$ and all $q \in Q$. To meet this end, we prove the following claim.

<u>*Claim 2.1.*</u> For each fixed $q \in Q$ the matrix $\tilde{M}(q)$ is negative definite.

To prove this claim, we first decompose $\tilde{M}(q)$ as

$$\tilde{M}(q) = \begin{pmatrix} I & 0 \\ M_{12}'(q)M_{11}^{-1} & I \end{pmatrix} \begin{pmatrix} M_{11} & 0 \\ 0 & M_{22}(q) + \gamma H(q) - M_{12}'(q)M_{11}^{-1}M_{12}(q) \end{pmatrix} \times \begin{pmatrix} I & M_{11}^{-1}M_{12}(q) \\ 0 & I \end{pmatrix}. \tag{11}$$

Recalling that M_{11} is already negative definite, it suffices to show that

$$\Theta(q) \equiv M_{22}(q) + \gamma H(q) - M_{12}'(q)M_{11}^{-1}M_{12}(q)$$

is negative definite. Let $\eta \in R^m$ be chosen arbitrarily, and note that

$$\eta'\Theta(q)\eta \leq \lambda_{max}[M_{22}(q) - M_{12}'(q)M_{11}^{-1}M_{12}(q)]\eta'\eta + 2\gamma\eta'\eta + 2\gamma\eta' V(q)\eta.$$

Consequently, it is appearent that the choice of γ given in Step **2.4** guarantees that

$$\eta'\Theta(q)\eta < 0.$$

Hence, $\Theta(q)$ is negative definite, and the claim is proven.

Returning to the proof of (10), we now select $q* \in Q$ such that

$$\max_{q \in Q} \lambda_{max}[\tilde{M}(q)] = \lambda_{max}[\tilde{M}(q*)],$$

and define

$$\tilde{\lambda} = \lambda_{max}[\tilde{M}(q)].$$

The existence of such a $\tilde{\lambda}$ follows from the continuity of $\tilde{M}(.)$ and the compactness of Q. The negativity of $\tilde{\lambda}$ follows immediately from the preceding claim. Finally, the satisfaction of (10) follows from the fact that

$$\tilde{x}'\tilde{M}(q)\tilde{x} \leq \lambda_{max}[\tilde{M}(q)]\tilde{x}'\tilde{x} \leq \tilde{\lambda}\tilde{x}'\tilde{x}, \quad \forall \tilde{x} \in R^n, \quad \forall q \in Q.$$

The proof of the theorem is now complete. So the closed-loop system

$$\begin{aligned}\dot{x}(t) &= [A + \Delta A(q(t))]x(t) + [B + \Delta B(q(t))]u(t) \\ &= [(A + \Delta A(q(t))) + (B + \Delta B(q(t)))K]x(t)\end{aligned}$$

is uniformly asymptotically stable.

Similar linear or nonlinear controller designs were done in previous papers [1-14]. However,if the states are not available for direct measurement,only the output vector y(t) is available,then the feedback control law (2) can not be implemented and other techniques must be found to realize it. Thus, either an output feedback controller leading to asymptotic stability under some additional conditions must be designed, or a suitable estimate approximating the unavailable states must be determined so that it can be substituted into the control law. In this thesis,Luenberger reduced order observer is used to estimate the missing states of the system and thereby a feedback controller, that guarantees an uniform asymptotic stability in the presence of uncertainty, can be designed.

CHAPTER 3. STABILITY WITH INCOMPLETE STATE INFORMATION USING OBSERVER

3.1 The Closed-Loop System with Observer

From(Σ), the output is $y(t)=Cx(t)$,and hence the state is only partially observed through this output. It suffices to estimate the missing components of the state variable and design a reduced order compensator [31].

Once a reduced order observer has been constructed which produces an estimate of the states or a linear transformation thereof,it is important to consider the effect induced by using this estimate in place of the true value called for by a control law. Especially,one is concerned with the effect of an observer on the closed-loop stability properties of the system. In this section,it will be shown that if a linear control law is realized with the observer,the resulting eigenvalues of the system are those of the observer itself and those that would be obtained if the control law was directly implemented. Thus an observer does not change the closed-loop eigenvalues of a design but merely adjoins its own eigenvalues.

Let us consider

$$(\Sigma)\quad \begin{cases} \dot{x}(t) = [A + \Delta A(q(t))]x(t) + [B + \Delta B(q(t))]u(t) \qquad \textit{for any } q \in Q, \\ \qquad = \tilde{A}(t)x(t) + \tilde{B}(t)u(t), \\ y(t) = Cx(t) \quad (\textit{output}) \qquad t \in [0, \infty). \end{cases}$$

Now if the control law cannot be realized directly because of missing states,we must construct an observer.That is,an observer of the form,

$$\dot{z}(t) = R(t)z(t) + S(t)y(t), \tag{12}$$

must be constructed such that the estimated state has the form

$$\hat{x}(t) = H_1(t)y(t) + H_2(t)z(t). \tag{13}$$

Then the control is given by

$$u(t) = K\hat{x}(t) = KH_1(t)y(t) + KH_2(t)z(t). \tag{14}$$

First,it is assumed that there is an $(n - r) \times n$ matrix $\hat{C}(t)$ such that $\begin{pmatrix} C \\ \hat{C}(t) \end{pmatrix}$ is nonsingular for all $t \in [0, \infty)$.

Then,we consider [3]

$$z(t) \cong \hat{C}(t)x(t), \tag{15}$$

[3] $\cong$ means equality as t large enough.

and hence

$$\begin{pmatrix} y(t) \\ z(t) \end{pmatrix} \cong \begin{pmatrix} C \\ \hat{C}(t) \end{pmatrix} x(t), \tag{16}$$

$$x(t) \cong \begin{pmatrix} C \\ \hat{C}(t) \end{pmatrix}^{-1} \begin{pmatrix} y(t) \\ z(t) \end{pmatrix}. \tag{17}$$

Therefore,we can consider $\begin{pmatrix} C \\ \hat{C}(t) \end{pmatrix}^{-1} \begin{pmatrix} y(t) \\ z(t) \end{pmatrix} = \hat{x}(t)$ as an estimate of the true state x(t) and use this estimate as the input to the feedback controller

$$u(t) = K\begin{pmatrix} C \\ \hat{C}(t) \end{pmatrix}^{-1} \begin{pmatrix} y(t) \\ z(t) \end{pmatrix} = KH_1(t)y(t) + KH_2(t)z(t),$$

where

$$(H_1(t) \quad H_2(t)) = \begin{pmatrix} C \\ \hat{C}(t) \end{pmatrix}^{-1}. \tag{18}$$

In this case,when (14) is substituted into (1),the state equation of the system reduces to

$$\dot{x}(t) = [\tilde{A}(t) + \tilde{B}(t)KH_1(t)C]x(t) + \tilde{B}(t)KH_2(t)z(t), \tag{19}$$

where K is given in (2).

From (15),(17),(19) and

$$\dot{z}(t) \cong \dot{\hat{C}}(t)x(t) + \hat{C}(t)\dot{x}(t). \tag{20}$$

We have

$$\begin{aligned} \dot{z}(t) \cong & \{\hat{C}(t)[\tilde{A}(t) + \tilde{B}(t)K] + \dot{\hat{C}}(t)\}H_2(t)z(t) \\ & + \{\hat{C}(t)[\tilde{A}(t) + \tilde{B}(t)K] + \dot{\hat{C}}(t)\}H_1(t)y(t). \end{aligned} \tag{21}$$

This is the observer equation,where

$$R(t) \equiv \{\hat{C}(t)[\tilde{A}(t) + \tilde{B}(t)K] + \dot{\hat{C}}(t)\}H_2(t), \tag{12a}$$

and

$$S(t) \equiv \{\hat{C}(t)[\tilde{A}(t) + \tilde{B}(t)K] + \dot{\hat{C}}(t)\}H_1(t). \tag{12b}$$

From this observer equation,we get its state z(t) by simulation which can be used to implement the control law (**14**).

Introducing the transformation

$$\bar{z}(t) = \hat{C}(t)x(t) - z(t), \tag{22}$$

the whole structure can be simplified.

From (**18**),

$$H_1(t)C + H_2(t)\hat{C}(t) = I, \tag{23}$$

and from (**12**),(**14**),(**15**)

$$\begin{aligned}\dot{\bar{z}}(t) &= \dot{\hat{C}}(t)x(t) + \hat{C}(t)\dot{x}(t) - \dot{z}(t) \\ &= \dot{\hat{C}}(t)H_2(t)\hat{C}(t)x(t) - \dot{\hat{C}}(t)H_2(t)z(t) + \hat{C}(t)\tilde{A}(t)H_2(t)\hat{C}(t)x(t) - \hat{C}(t)\tilde{A}(t)H_2(t)z(t).\end{aligned}$$

Therefore,

$$\begin{aligned}\dot{\bar{z}}(t) &= [\dot{\hat{C}}(t) + \hat{C}(t)\tilde{A}(t)]H_2(t)\bar{z}(t) \\ &= E(t)\bar{z}(t).\end{aligned} \tag{24}$$

From(19),

$$\dot{x}(t) = [\tilde{A}(t) + \tilde{B}(t)K]x(t) - \tilde{B}(t)KH_2(t)\bar{z}(t). \tag{25}$$

The effect of observer on the dynamics of the closed-loop system is examined by the transformation

$$\begin{pmatrix} x(t) \\ \bar{z}(t) \end{pmatrix} = \begin{pmatrix} I & 0 \\ \hat{C}(t) & -I \end{pmatrix} \begin{pmatrix} x(t) \\ z(t) \end{pmatrix}.$$

Using (24) and (25),we arrive at the following equation

$$\begin{pmatrix} \dot{x}(t) \\ \dot{\bar{z}}(t) \end{pmatrix} = \begin{pmatrix} \tilde{A}(t) + \tilde{B}(t)K & -\tilde{B}(t)KH_2(t) \\ 0 & [\dot{\hat{C}}(t) + \hat{C}(t)\tilde{A}(t)]H_2(t) \end{pmatrix} \begin{pmatrix} x(t) \\ \bar{z}(t) \end{pmatrix}. \tag{26}$$

The closed-loop stability of system (26) will be considered in section 3.3.

3.2 Construction of the Observer ($\hat{C}(t)$)

For y(t)=Cx(t),it is assumed here without loss of generality that the first r columns of C,with a possible renumbering of the states,are linearly independent.Thus,C may be partitioned as $C = (C_1|C_2)$,where C_1 is $r \times r$,nonsigular. In this case it can also be assumed,by introducing a change of coordinates, that the matrix C takes the form $\bar{C} = (I_r|0)$.Such a form will yield simpler design procedures.

And it will be necessary later to consider the partitioning

$$\tilde{A}(t) = \begin{pmatrix} \tilde{A}_{11}(t) & \tilde{A}_{12}(t) \\ \tilde{A}_{21}(t) & \tilde{A}_{22}(t) \end{pmatrix},$$

where $\tilde{A}_{11}(t)$ and$\tilde{A}_{22}(t)$ are $r \times r$ and $(n-r) \times (n-r)$,respectively.

Before the construction of observer,we wish to emphasize the following points:

1. In chapter 2,it was assumed $\Delta A(q) = E(q)C$ for $\forall q(t) \in Q$.After the transformation, matrix $\Delta\bar{A}(q) = T^{-1}\Delta A(q)T$ also satisfies this matching condition for $\bar{C} = (I_r|0) = CT$.

Let

$$\bar{E}(q) = T^{-1}E(q). \tag{27}$$

Then

$$\Delta\bar{A}(q) = E(q)\bar{C}. \tag{28}$$

2. From 1,if system (Σ) with output distribution matrix $C = (I_r|0)$ satisfies (28),then

$$\Delta A(q) = (\Delta A_r(q)|0). \tag{29}$$

Therefore,$\tilde{A}_{12}(t) = A_{12}, \tilde{A}_{22}(t) = A_{22}$,where A_{12}, A_{22} are the partitions of

$$A = \begin{pmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{pmatrix}.$$

3. If $\{A, C\}$ is observable pair,then so is $\{A_{22}, A_{12}\}$. This can be easily proved directly by using the definition of observablity.

Writing $\hat{C}(t)$ as

$$\hat{C}(t) = (L(t) \quad N(t)). \tag{30}$$

We have

$$\begin{pmatrix} C \\ \hat{C}(t) \end{pmatrix} = \begin{pmatrix} I_r & 0 \\ L(t) & N(t) \end{pmatrix}. \tag{31}$$

Since this is invertable,the $(n-r) \times (n-r)$ matrix N(t) is nonsigular for all $t \in [0, \infty), i.e. (N(t)\xi \;\; \xi) \geq \epsilon \mid \xi \mid^2, \forall t \geq 0$ for some $\epsilon > 0$.

Then

$$\begin{pmatrix} C \\ \hat{C}(t) \end{pmatrix}^{-1} = (H_1(t) \quad H_2(t)) = \begin{pmatrix} I_r & 0 \\ -N^{-1}(t)L(t) & N^{-1}(t) \end{pmatrix}. \tag{32}$$

Thus (24) becomes

$$\dot{\bar{z}}(t) = [\dot{\hat{C}}(t) + \hat{C}(t)\tilde{A}(t)]H_2(t)\bar{z}(t)$$

$$= \{(\dot{L}(t) \quad \dot{N}(t)) + (L(t)\tilde{A}_{11}(t) + N(t)\tilde{A}_{21}(t) \quad L(t)\tilde{A}_{12}(t) + N(t)\tilde{A}_{22}(t))\}H_2(t)\bar{z}(t)$$

$$= \{\dot{N}(t) + [N(t)\tilde{A}_{22}(t) + L(t)\tilde{A}_{12}(t)]\}N^{-1}(t)\bar{z}(t). \qquad (33)$$

Defining

$$E(t) \equiv \{\dot{N}(t) + [N(t)\tilde{A}_{22}(t) + L(t)\tilde{A}_{12}(t)]\}N^{-1}(t), \qquad (34)$$

where N(t) should be nonsingular for all $t \in [0,\infty)$,L(t) can be chosen arbitrary. If we choose N(t)=I,(34) reduces to

$$E(t) = \tilde{A}_{22}(t) + L(t)\tilde{A}_{12}(t) \quad t \in [0,\infty). \qquad (35)$$

This is convenient both theoretically and computationally.

In general,given a E(t),it is hard to find a solution for L(t) in(35). However,we are merely concerned with the question of stability of the system. That is,we must show $\bar{z}(t) \to 0$ as $t \to \infty$. It will be seen later that only the eigenvalues of E(t) are important in the design procedure.Since $\tilde{A}_{12}(t) = A_{12}, \tilde{A}_{22}(t) = A_{22}$ and $\{A_{22} \quad A_{12}\}$ is observable,(35) reduces to

$$E(t) = A_{22} + LA_{12}, \qquad (36)$$

and it is well known that there exists a matrix L such that $E = A_{22} + LA_{12}$ is a stability matrix.That is,by proper choice of L,all the eigenvalues of E are negtive:

$$\lambda_i\{E\} = \lambda_i\{A_{22} + LA_{12}\} < 0, \quad i = 1, 2 \cdots n - r \qquad (37)$$

In fact,L(t) can be chose to be a constant matrix L which defines E(t) by (37).Hence by (30)

$$\hat{C}(t) = \hat{C} = (L \quad I). \tag{38}$$

Thus we have constructed the observer completely.

For convenience of the reader we present below the necessary steps.

Steps For Construction of Stabilizer :

Step 3.1: For a given compact set Q,assuming complete state information, determine the feedback control law (2) (i.e,matrix $\bar{K}$) via steps 2.1-2.5.

Step 3.2: Determine a matrix L satisfying (37)(note that such a matrix exists) giving $\hat{C}$ by (38).

Step 3.3: Substitute $\hat{C}$,K into (12a) and (12b),and construct the observer via (12) for any $q \in Q$.

Step 3.4: For incomplete state information, the control law is now given by (14).

3.3 Stability with Incomplete State Information

In section 3.1,it was shown that an observer does not change the closed-loop eigenvalues but merely adjoins its own eigenvalues.In the time-varying system,however,a more detailed analysis of the stability of (26) is required since the eigenvalues of the time-varying system do not in general determine its stability.It will be shown that the introduction of an observer does not disturb stability properties. First,considering the second part of (26),i.e.,equation (24),

$$\dot{\bar{z}}(t) = E(t)\bar{z}(t),$$

it has been shown in the previous section that for a proper choice of L(t)=L constant,E(t) can be chosen to be a constant matrix having eigenvalues with negative real parts.Thus (24) is (uniformly) asymptotically stable and there exist positive constants c_1, c_2 such that

$$\| \Psi(t-\tau) \| \leq c_1 e^{-c_2(t-\tau)}, \qquad for \quad 0 \leq \tau < t < \infty \tag{39}$$

where $\Psi(t) = e^{Et}$ is the transition matrix corresponding to E.

Then

$$\bar{z}(t) = \Psi(t)\bar{z}(0) \tag{40}$$

Using (22) yields an explicit solution for the state of the controller:

$$z(t) = \hat{C}x(t) + e^{Et}[z(0) - \hat{C}x(0)], \tag{41}$$

The control u(t) is then found by inserting (13) and (41) into (2), yielding

$$u(t) = Kx(t) + KH_2(t)e^{Et}[z(0) - \hat{C}x(0)]. \tag{42}$$

Comparing (2) and (42), it is seen that the control law when only y(t) is measurable differs from the control when all states of the plant are measurable by the term

$$KH_2e^{Et}[z(0) - \hat{C}x(0)]. \tag{43}$$

Since $\Psi(t) = e^{Et}$ is nonsingular for all $t \geq 0, \bar{z}(t)$ will be identically zero for all $t \in [0, \infty)$ if and only if $z(0) = \hat{C}x(0)$.

On the other hand, from (39),(40)

$$\| \bar{z}(t) \| \leq c_1 e^{-c_2 t} \| \bar{z}(0) \|, \tag{44}$$

where

$$\bar{z}(0) = (\hat{C}(0)x(0) - z(0)). \tag{45}$$

From (26),the state x of the system is given by

$$x(t) = \Phi(t,0)x(0) - \int_0^t \Phi(t,\tau)\tilde{B}(\tau)KH_2(\tau)\bar{z}(\tau)d\tau. \tag{46}$$

Where $\Phi(t,\tau)$ is the transition matrix corresponding to $(\tilde{A}(t)+\tilde{B}(t)K)$ of system (4).Since system (4) is uniformly asymptotically stable, there exist $c_3, c_4 > 0$ such that[42]

$$\| \Phi(t,\tau) \| \leq c_3 e^{-c_4(t-\tau)}. \quad 0 \leq \tau < t < \infty \tag{47}$$

Since $\tilde{B}(t) = B + \Delta B(q(t)) = B(I + V(q(t)))$,where $\| V(q(t)) \| < 1$ for all $q \in Q$,and H(t)=constant,there exists a finite positive number h,such that

$$\| \tilde{B}(\tau) K H_2(\tau) \| \leq h < \infty \qquad t \in [0, \infty), \tag{48}$$

and therefore it follows from(34) that

$$\begin{aligned} \| x(t) \| &\leq c_3 e^{-c_4 t} \| x(0) \| + c_1 c_3 h \int_0^t e^{-c_4(t-\tau)} e^{-c_2 \tau} d\tau \| \bar{z}(0) \| \\ &= c_3 e^{-c_4 t} \| x(0) \| + c_1 c_3 h \frac{e^{-c_2 t} - e^{-c_4 t}}{c_4 - c_2} \| \bar{z}(0) \| . \end{aligned}$$

Thus $\| x(t) \| \rightarrow 0$ as $t \rightarrow \infty$ (even for $c_2 = c_4 > 0$) for all finite x(0),$\bar{z}(0)$. This means that (26) is uniformly asymptotically stable.

We can summarize the above result in the following theorem.

Theorem3.1 :

Under the assumptions (A1-A7) in chapter 2,given any compact set $Q \subset R^k$, there exists an observer for the system

$$(\Sigma) \qquad \begin{cases} \dot{x}(t) = [A + \Delta A(q(t))]x(t) + [B + \Delta B(q(t))]u(t) \\ \qquad = \tilde{A}(t)x(t) + \tilde{B}(t)u(t) \\ y(t) = Cx(t) \quad (output) \quad t \in [0, \infty) \end{cases}$$

such that the combined plant-observer system(26) is uniformly asymptotically stabie with respect to the zero state for any uncertainty $q(t) \in Q$.

Remark :

If the system is stabilized for a given compact set $Q \subset R^k$, then the system remains stable for any $Q' \subset Q$.Otherwise,one would require to redesign the stabilizer following the steps 3.1-3.4.

CHAPTER 4. TWO EXAMPLES WITH SIMULATION RESULTS

4.1 Example One

Let

$$(\Sigma) \qquad \begin{cases} \dot{x}_1(t) = x_1(t) + q(t)x_2(t) - u(t) \\ \dot{x}_2(t) = x_1(t) \\ y(t) = x_2(t) \end{cases}$$

here $A_0 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix}, B_0 = \begin{pmatrix} -1 \\ 0 \end{pmatrix}, C_0 = (0 \;\; 1), \Delta A(q) = \begin{pmatrix} 0 & q(t) \\ 0 & 0 \end{pmatrix}, \Delta B(q) =$ 0.

It can be easily shown that the system satisfies all the assumptions of chapter 2. First, we construct the controller assuming that the state is completely available.

From step2.1,we can find $K_0 = (4 \;\; 2)$ such that

$$\bar{A} = A_0 + B_0K_0 = \begin{pmatrix} 1 & 0 \\ 1 & 0 \end{pmatrix} + \begin{pmatrix} -1 \\ 0 \end{pmatrix}(4 \;\; 2) = \begin{pmatrix} -3 & -2 \\ 1 & 0 \end{pmatrix}$$

is a stable matrix while A_0 is not. Let $\Gamma = \begin{pmatrix} 1 & 0 \\ 0 & 4 \end{pmatrix}$, solving Lyapunov equation $\bar{A}'P + P\bar{A} = -\Gamma$, we got

$$P = \begin{pmatrix} 1/2 & 1 \\ 1 & 1/4 \end{pmatrix}, \quad P^{-1} = \begin{pmatrix} 4 & -1 \\ -1 & 1/2 \end{pmatrix}.$$

From step2.2, to make $TB_0 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$, we find $T = \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix}$.

From step 2.3, $\bar{F} = T\bar{A}T^{-1} = \begin{pmatrix} 1 & -1 \\ 6 & -4 \end{pmatrix}, \Delta F(q) = T\Delta A(q)T^{-1} = \begin{pmatrix} 0 & 0 \\ -q & 0 \end{pmatrix}, G = TB = \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \Delta G(q) = 0, S = TP^{-1}T' = \begin{pmatrix} 0.5 & 1.5 \\ 1.5 & 6.5 \end{pmatrix}$.

$$\begin{aligned} M(q) &= \begin{pmatrix} 1 & -1 \\ 6-q & -4 \end{pmatrix} \begin{pmatrix} 0.5 & 1.5 \\ 1.5 & 6.5 \end{pmatrix} + \begin{pmatrix} 0.5 & 1.5 \\ 1.5 & 6.5 \end{pmatrix} \begin{pmatrix} 1 & 6-q \\ -1 & -4 \end{pmatrix} \\ &= \begin{pmatrix} -2 & -0.5q-8 \\ -0.5q-8 & -3q-34 \end{pmatrix} \end{aligned}$$

From step 2.4, in order to find γ, such that

$$\begin{aligned} \gamma &< -0.5 \mid \max_{q\in Q}[-3q-34+0.5(0.5q+8)^2] \mid \\ &= -0.5 \mid \max_{q\in Q}[q-2+0.125q^2] \mid \end{aligned}$$

Depending on Q, we can find $\gamma = \gamma_Q < -0.5 \mid \max_{q\in Q}[q-2+0.125q^2] \mid$.

From step 2.5, the desired feedback matrix is given by

$$K = K_0 + K_1 = (4 \quad 2) + \gamma_Q(-1 \quad 0)\begin{pmatrix} 1/2 & 1 \\ 1 & 4 \end{pmatrix} = (4-0.5\gamma_Q \quad 2-\gamma_Q).$$

Then, changing(Σ) to standard form by $\bar{x} = Tx$:

$$\begin{cases} \dot{\bar{x}}_1(t) = \bar{x}_1(t) - \bar{x}_2(t) \\ \dot{\bar{x}}_2(t) = -q(t)\bar{x}_1(t) \\ y(t) = \bar{x}_1(t) \end{cases}$$

It is clear that $G(t) = A_{22}(t) + H(t)A_{12}(t) = -H(t)$.

We choose H(t)=5,therefore $\hat{C} = (5 \quad 1)$ and the closed-loop system is

$$\begin{pmatrix} \dot{\bar{x}}_1(t) \\ \dot{\bar{x}}_2(t) \\ \dot{\bar{z}}(t) \end{pmatrix} = \begin{pmatrix} 1 & -1 & 0 \\ 6-1.5\gamma_Q-q(t) & -4+0.5\gamma_Q & -4+0.5\gamma_Q \\ 0 & 0 & -5 \end{pmatrix} \begin{pmatrix} \bar{x}_1(t) \\ \bar{x}_2(t) \\ \bar{z}(t) \end{pmatrix}.$$

From the computer simulations, the result shown that the system is asymptotically stable for different random processes $q_1(t)$(GGPER), $q_2(t)$(GGUBS) and $q_3(t)$(GGNML)(generated by the computer) with values in compact set $Q_1 = [0 \;\; 3]$ and $Q_1' = [-1 \,.3]$ with $\gamma_{Q_1} = 2$. When the uncertainties take the values $Q_2 = [0 \;\; 4]$ and $Q_2' = [-1 \;\; 4]$ it is shown that redesigning γ_{Q_2} is required as the systems are not asymptotically stable with $\gamma_{Q_1} = 2$. From step **2.4**, with $\gamma_{Q_2} = 3$ asymptotic stability is achieved.

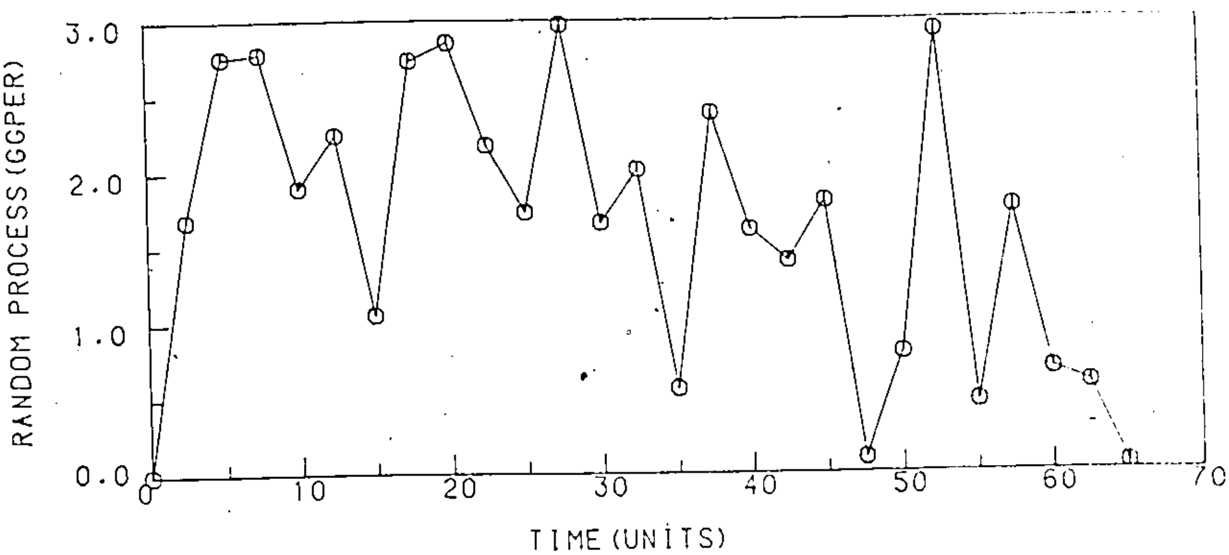

Fig.1. Random process $\{q_1(t), t \geq 0\}$ taking values from the compact set $Q_1 = [0, 3]$.

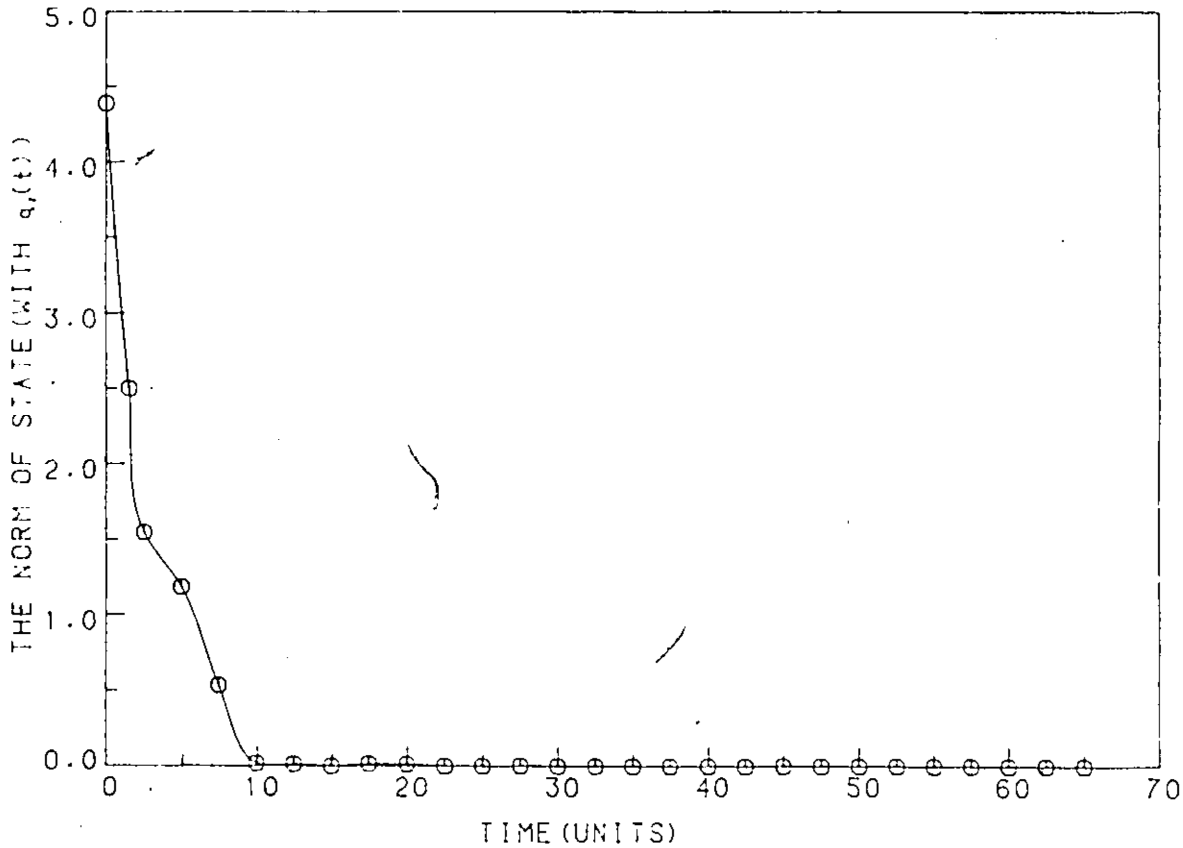

Fig.2. Stabilizing feedback gain with $\gamma_{Q_1} = 2$.

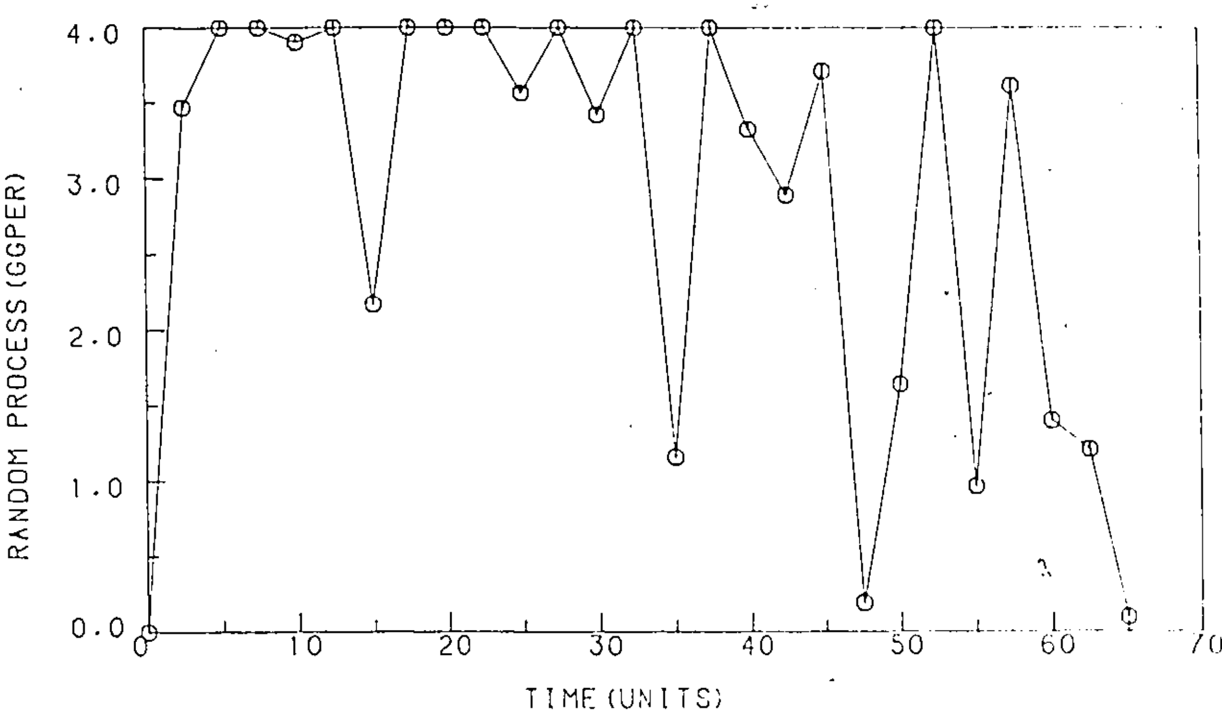

Fig.3. Random process $\{q_1(t)\}$ with values from the set $Q_2 = [0,4]$.

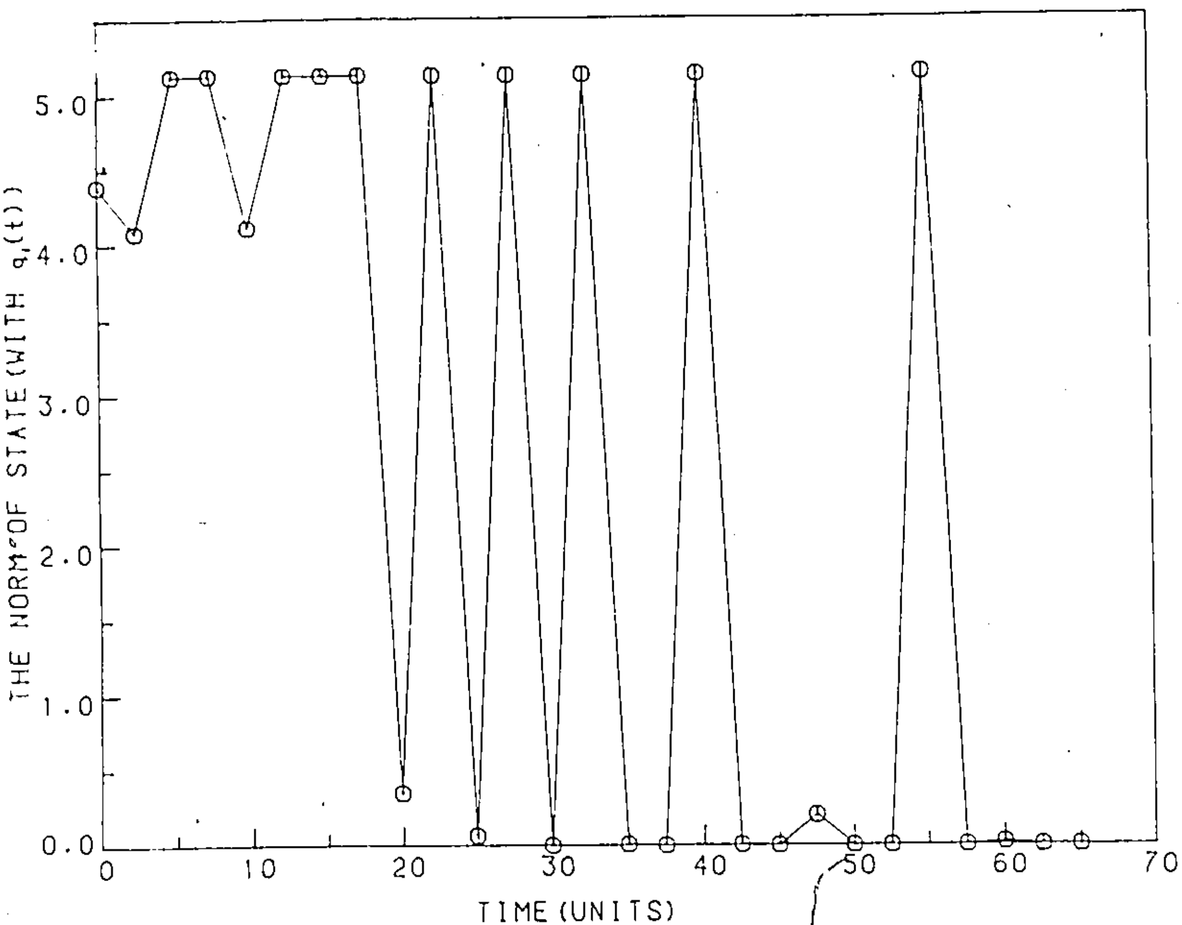

Fig.4. Same feedback gain with $\gamma_{Q_1} = 2$(not asymptotically stable).

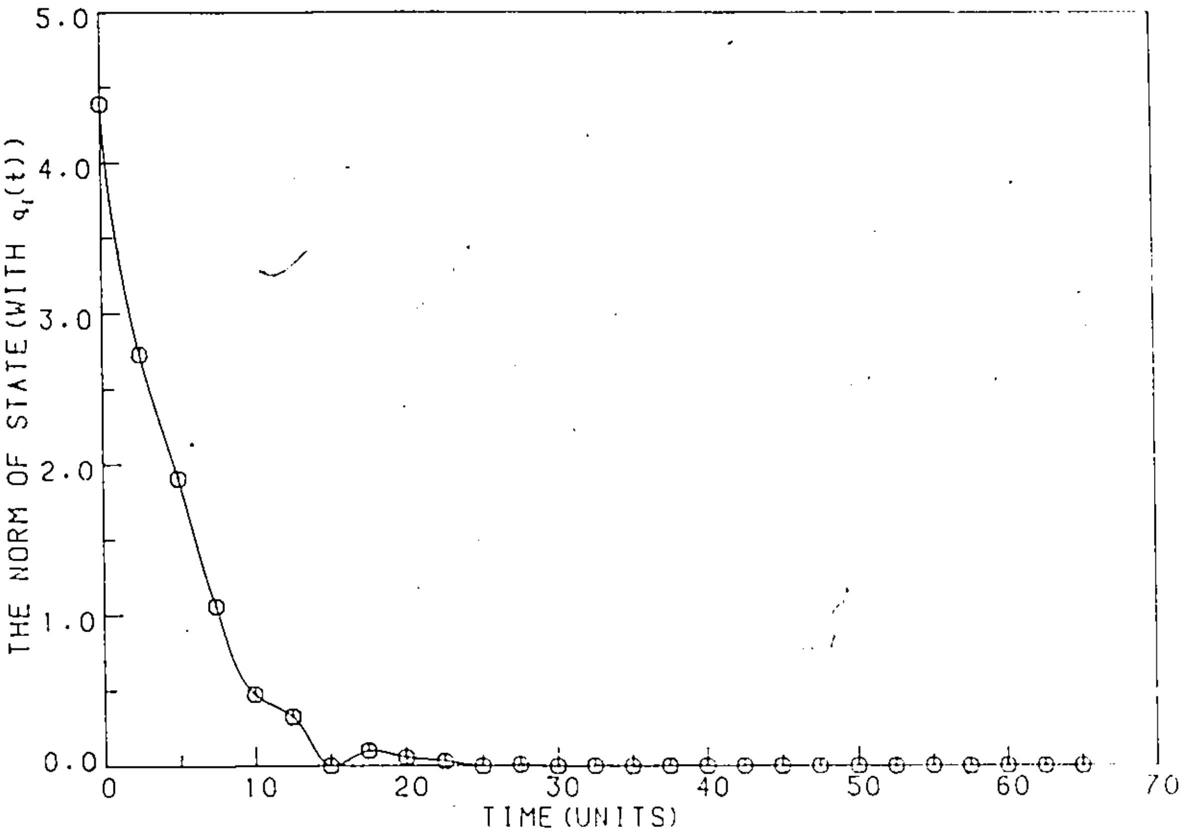

Fig.5. Stabilizing feedback gain with $\gamma_{Q_2} = 3$ (redesigned).

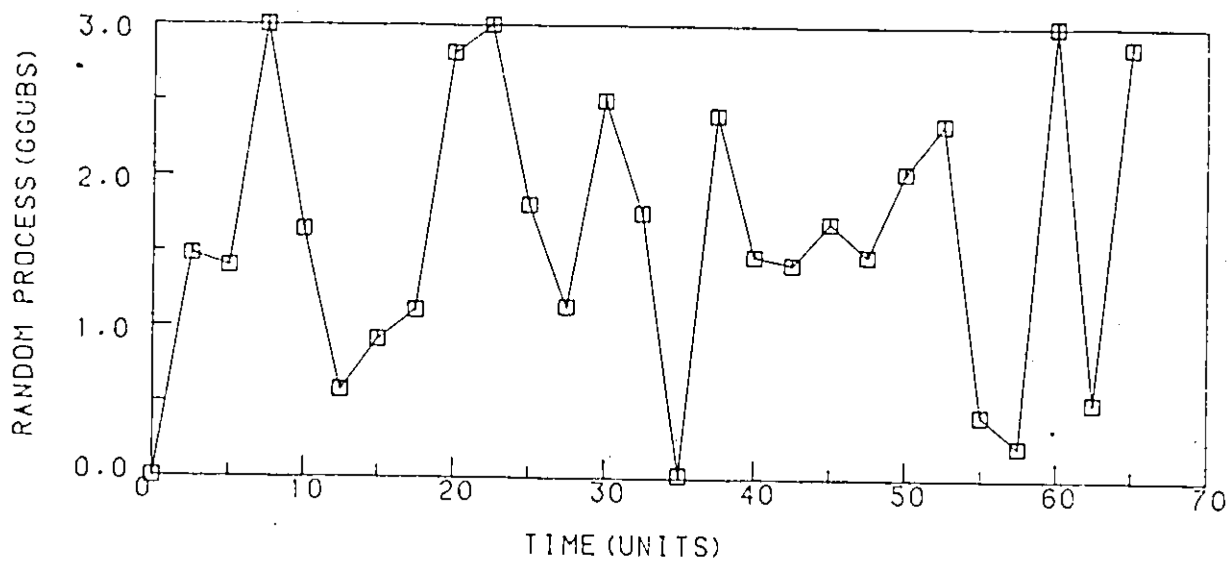

Fig.6. Random process $\{q_2(t), t \geq 0\}$ taking values from the compact set $Q_1 = [0,3]$.

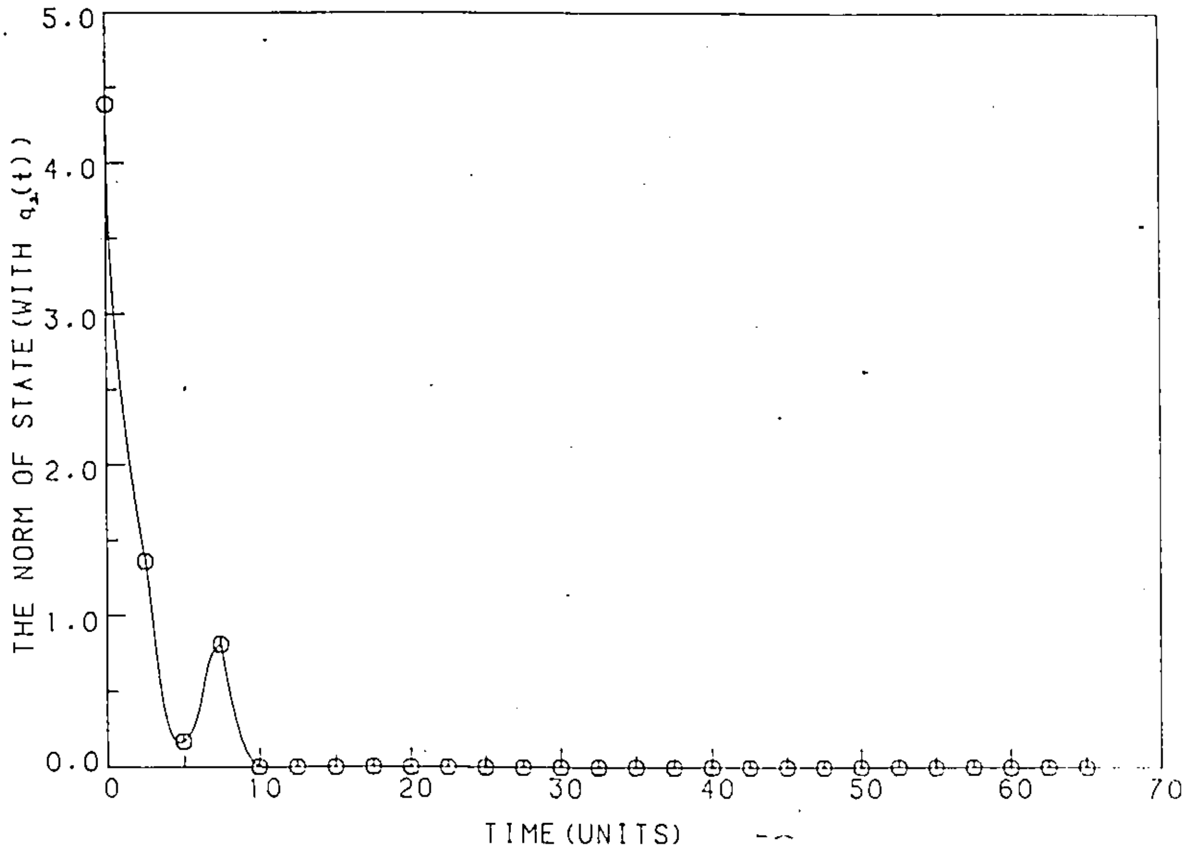

Fig.7. Stabilizing feedback gain with $\gamma_{Q_1} = 2$.

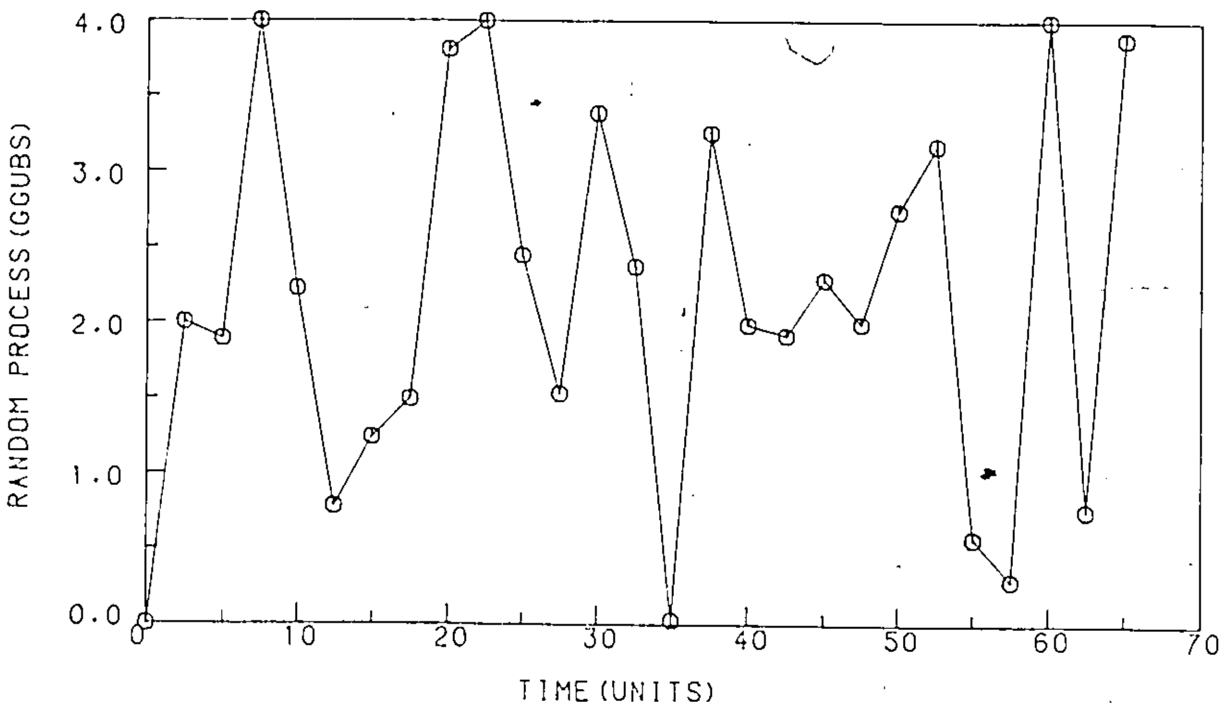

Fig.8. Random process $\{q_2(t)\}$ with values from the set $Q_2 = [0, 4]$.

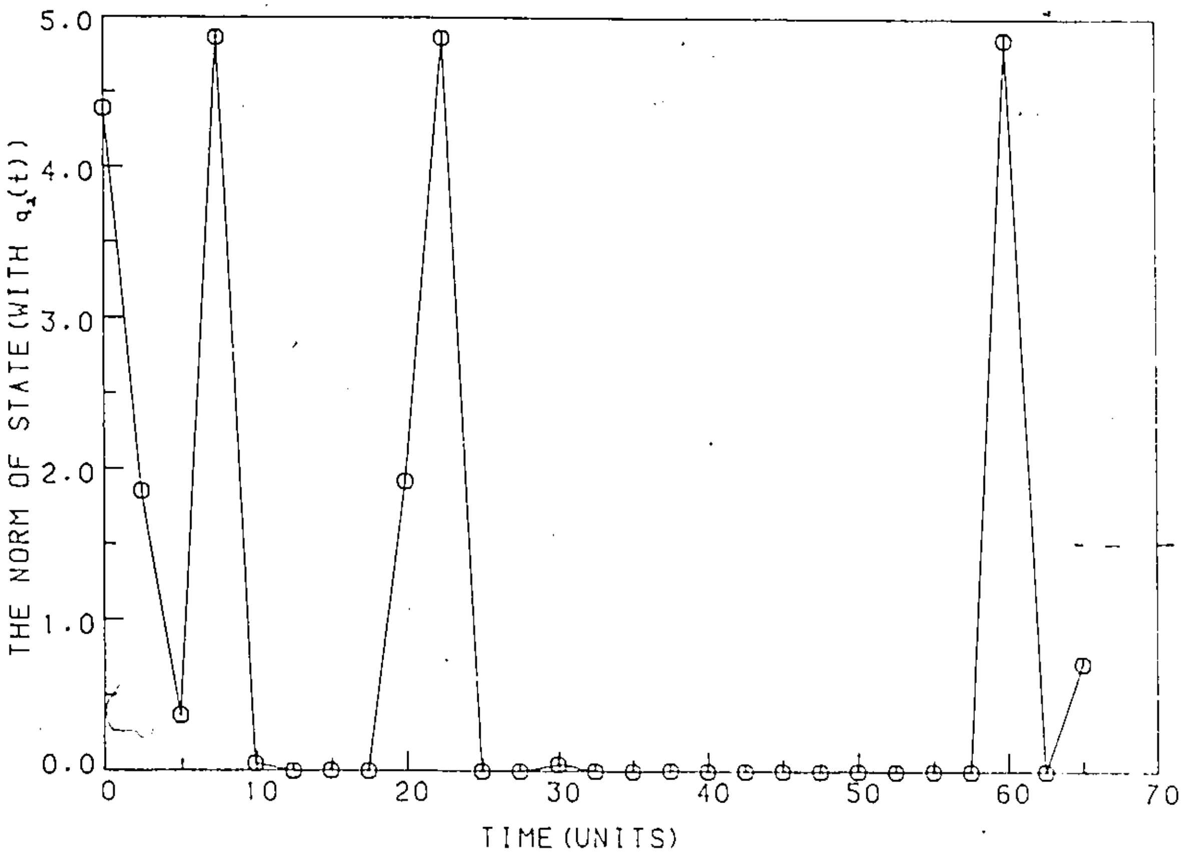

Fig.9. Same feedback gain with $\gamma_{Q_1} = 2$(not asymptotically stable).

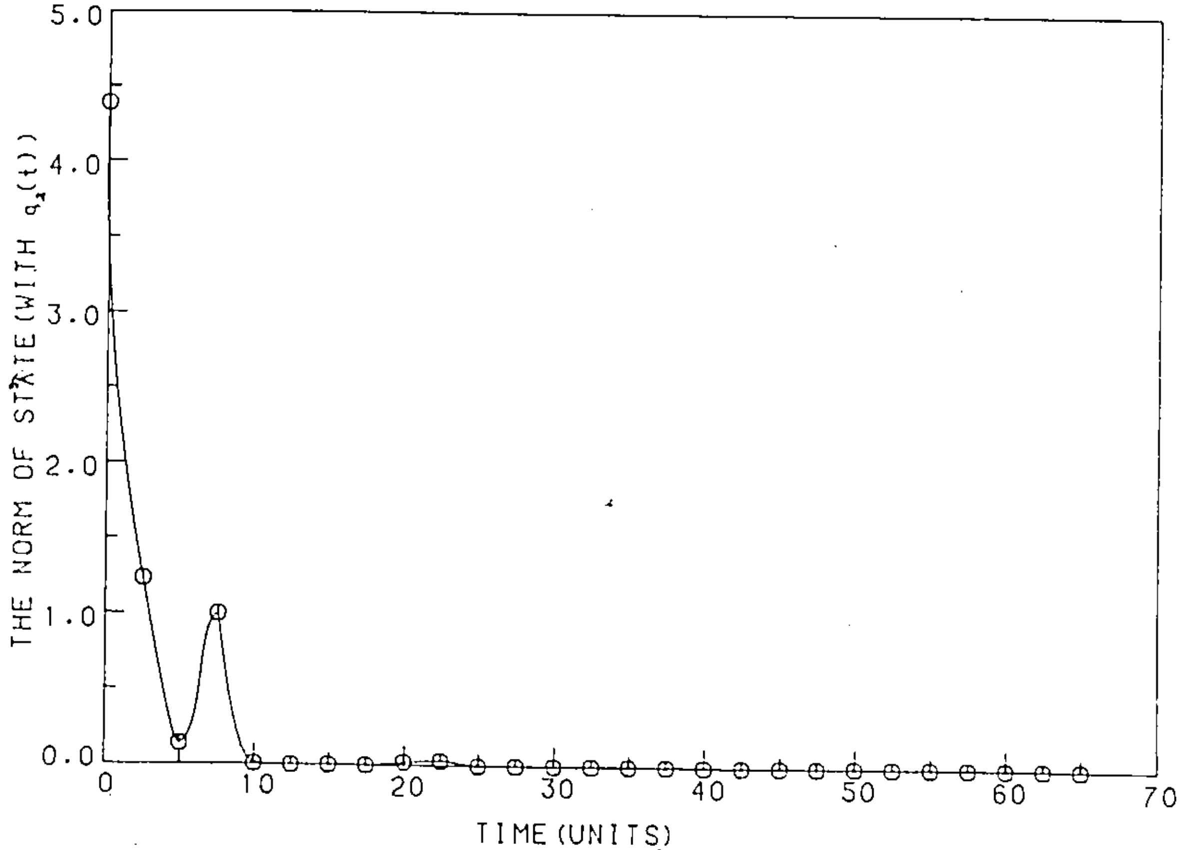

Fig.10. Stabilizing feedback gain with $\gamma_{Q_2} = 3$ (redesigned).

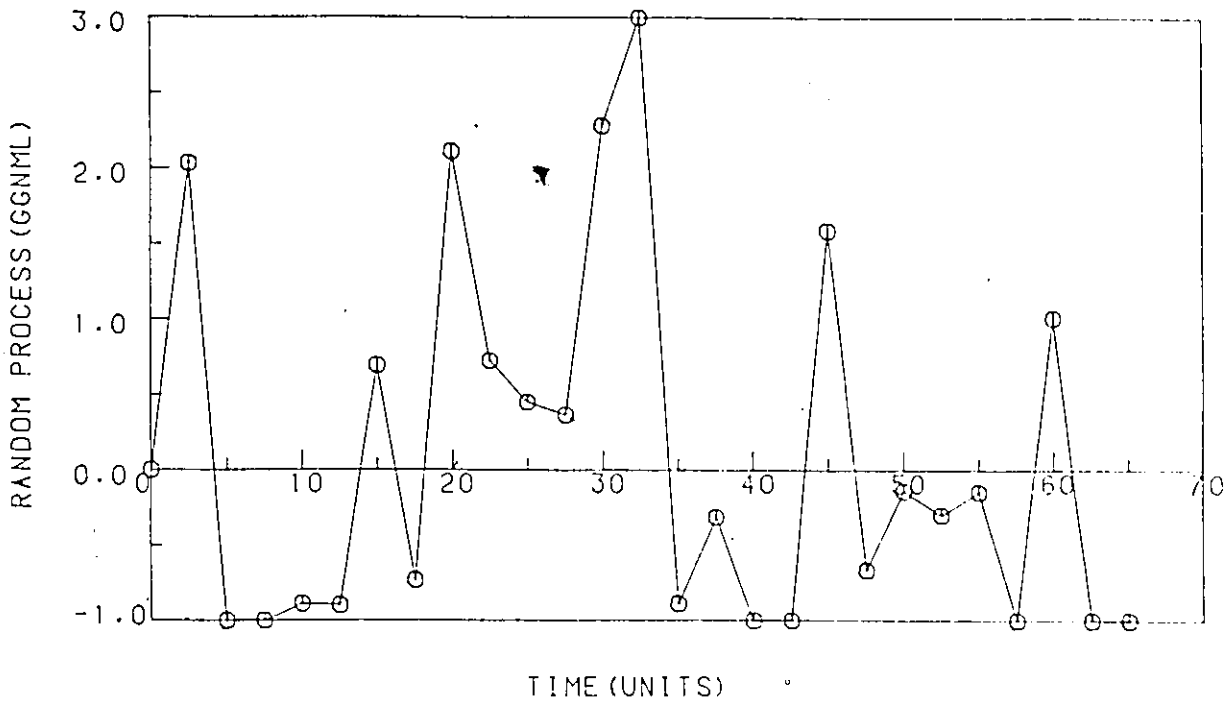

Fig.11. Random process $\{q_3(t), t \geq 0\}$ taking values from the compact set $Q_1' = [-1, 3]$.

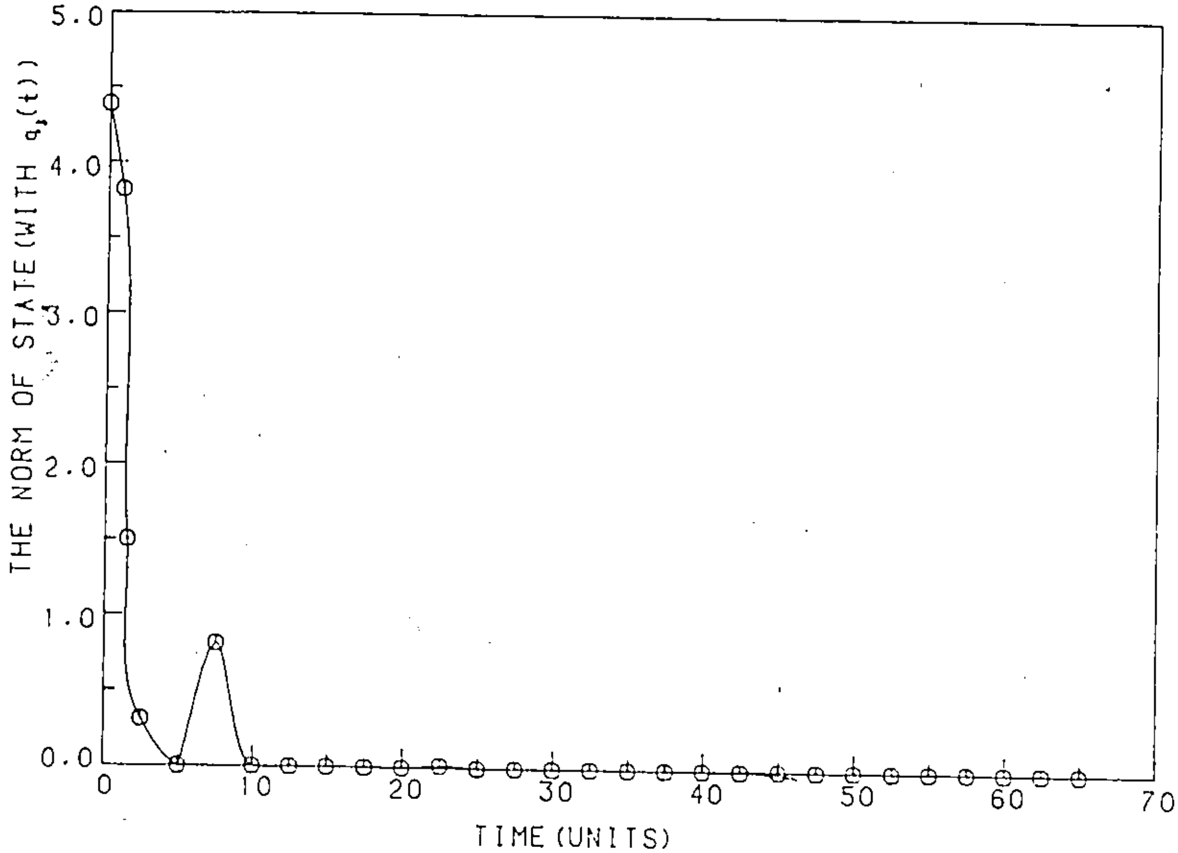

Fig.12. Stabilizing feedback gain with $\gamma_{Q_1'} = 2$.

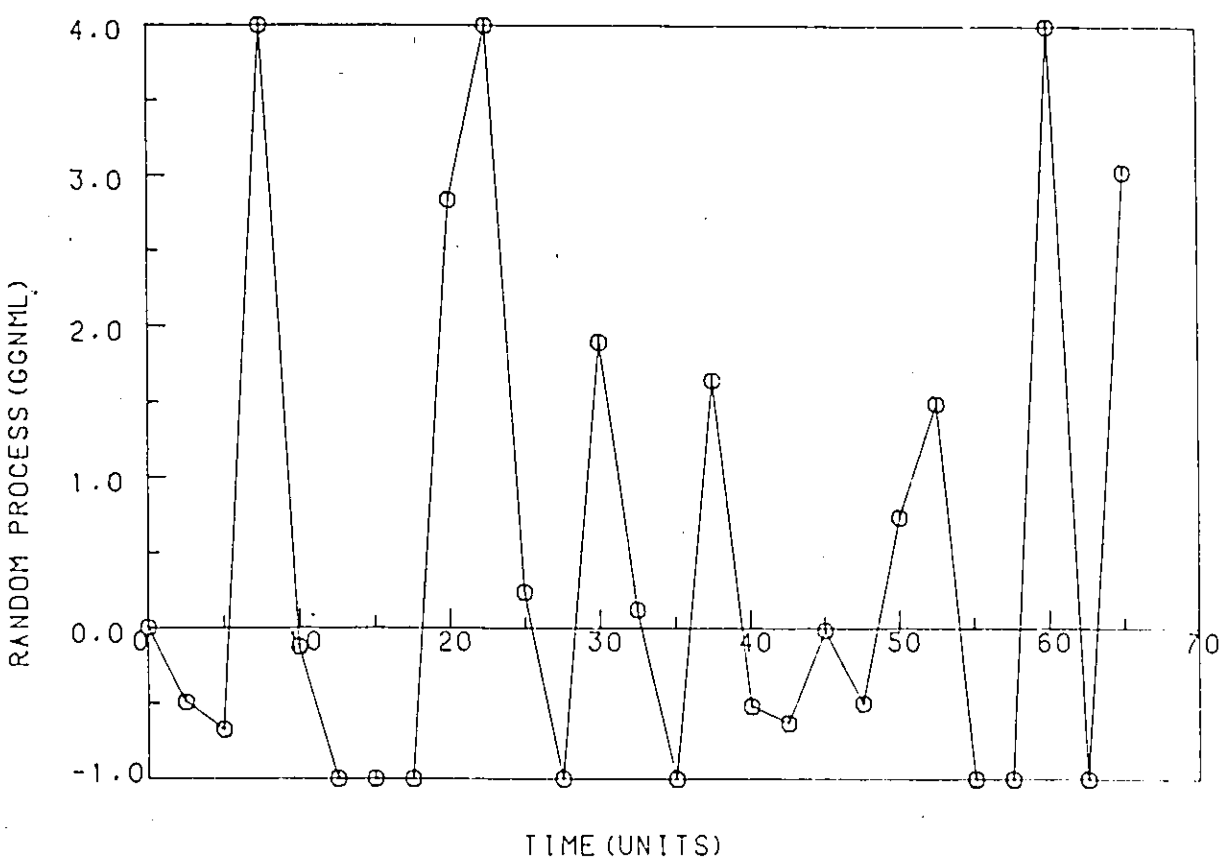

Fig.13. Random process $\{q_3(t)\}$ with values from the set Q'_2 [illegible] 1.4.

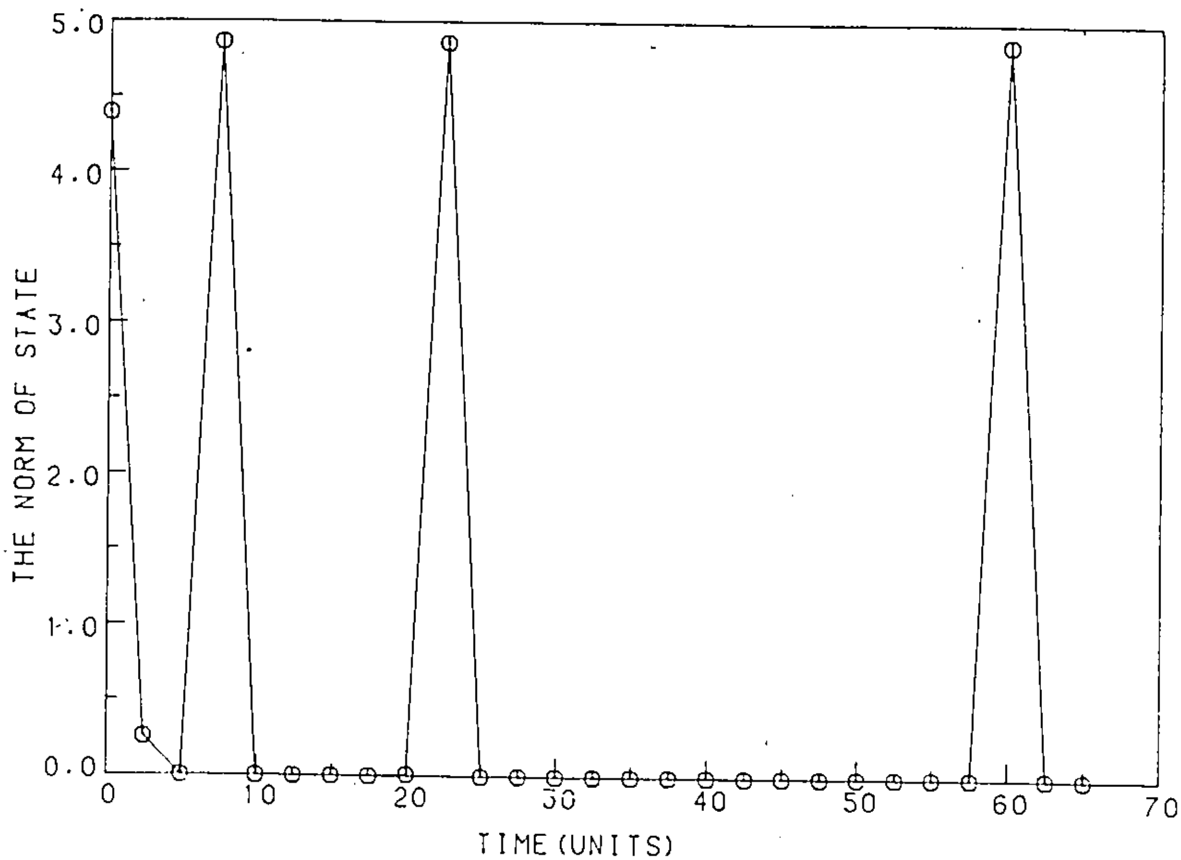

Fig.14. Same feedback gain with $\gamma_{Q_1'} = 2$(not asymptotically stable).

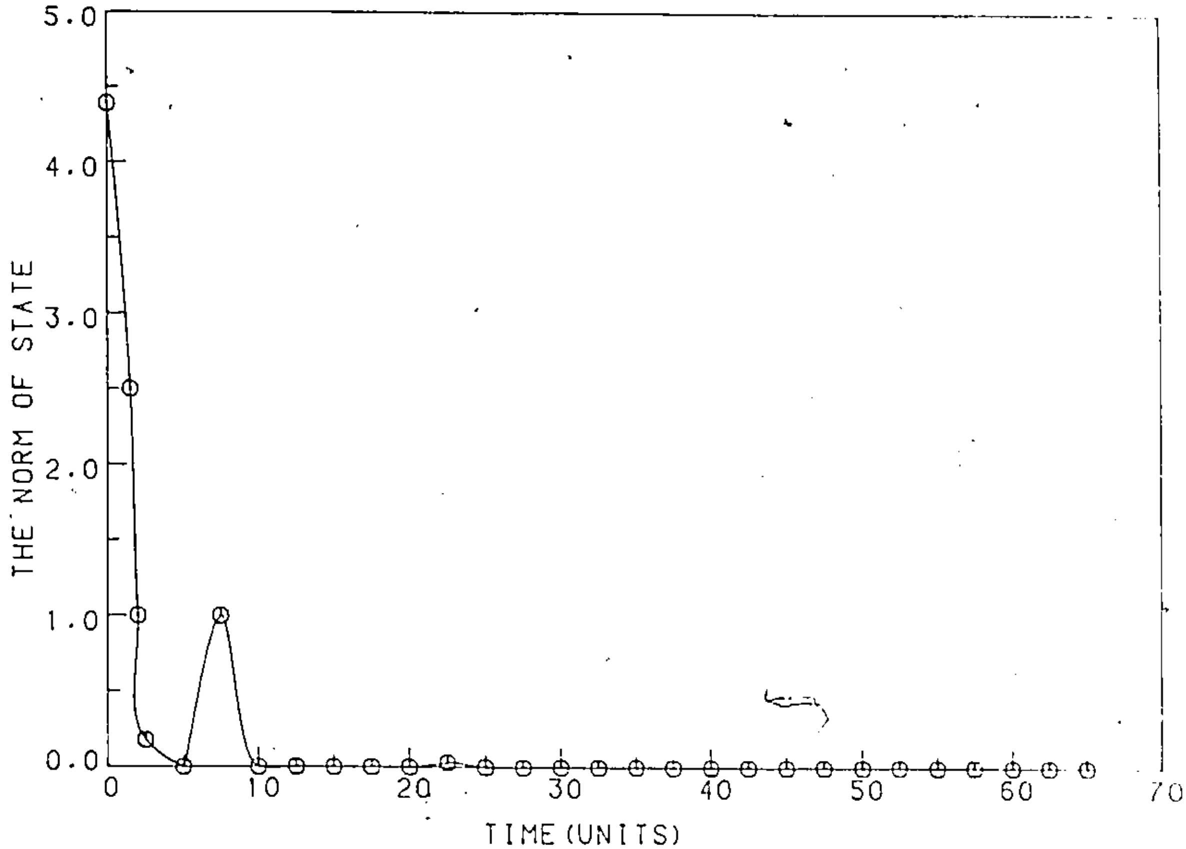

Fig.15. Stabilizing feedback gain with $\gamma_{Q_2'} = 3$ (redesigned).

4.2 Example Two

$$\begin{cases} \dot{x_1}(t) = x_2(t) \\ \dot{x_2}(t) = x_1(t) + x_3(t) \\ \dot{x_3}(t) = q_1(t)x_1(t) + q_2(t)x_2(t) + u(t) \\ y_1(t) = x_1(t) \\ y_2(t) = x_2(t) \end{cases}$$

here

$$A = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}, B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, C = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix},$$

$$\Delta A(q) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ q_1 & q_2 & 0 \end{pmatrix}, \Delta B(q) = 0.$$

$\{A, B\}$ is controllable, $\{A, C\}$ is observable, A is not stable. It can be easily shown that the system satisfies all the assumptions of chapter 2.

It is found from step2.1 that K_0=(-3 -3 -2) makes $\bar{A} = A + BK_0 = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ -3 & -3 & -2 \end{pmatrix}$ stable.

By solving Lyapunov equation

$$P\bar{A} + \bar{A}'P = -\Gamma,$$

where $\Gamma = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{pmatrix}$, we obtain

$$P = \begin{pmatrix} 16 & 14 & 5 \\ 14 & 14 & 5 \\ 5 & 5 & 3 \end{pmatrix}, \quad P^{-1} = \frac{1}{34} \begin{pmatrix} 17 & -17 & 0 \\ -17 & 23 & -10 \\ 0 & -10 & 28 \end{pmatrix}.$$

From step2.2, $T = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$ since $B = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$. Following

$$\bar{F} = T\bar{A}T^{-1} = \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ -3 & -3 & -2 \end{pmatrix}, \Delta F(q) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ q_1 & q_2 & 0 \end{pmatrix},$$

$$G = TB = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}, \Delta G(q) = 0, S = P^{-1} = \frac{1}{34} \begin{pmatrix} 17 & -17 & 0 \\ -17 & 23 & -10 \\ 0 & -10 & 28 \end{pmatrix},$$

$$M(q) = \frac{1}{34} \begin{pmatrix} -34 & 40 & 17(q_1 - q_2) - 10 \\ 40 & -54 & 30 - 17q_1 + 23q_2 \\ 17(q_1 - q_2) - 10 & 30 - 17q_1 + 23q_2 & -52 - 20q_2 \end{pmatrix}.$$

Depending on Q,it can be shown that

$$\gamma = \gamma_Q < -0.5 \mid \max_{q \in Q}\{-1.53 - 0.588q_2 + 0.288q_1^2 + 0.288q_2^2 - 0.424q_1q_2 + 0.169q_1 + 0.751q_2 + 1.496\} \mid$$

$$< -0.5 \mid \max_{q \in Q}\{0.288q_1^2 + 0.288q_2^2 - 0.424q_1q_2 + 0.169q_1 + 0.169q_2 - 0.03\} \mid,$$

and

$$\begin{aligned} K &= K_0 + \gamma_Q B'P \\ &= (-3 \ \ -3 \ \ -2) + \gamma_Q (0 \ 0 \ 1) \begin{pmatrix} 16 & 14 & 5 \\ 14 & 14 & 5 \\ 5 & 5 & 3 \end{pmatrix} \\ &= (-3 + 5\gamma_Q \ \ -3 + 5\gamma_Q \ \ -2 + 3\gamma_Q). \end{aligned}$$

Let $L = (L_1 \ L_2)$,it is clear that

$$E = A_{22} + LA_{12} = 0 + (L_1 \ L_2)\begin{pmatrix} 0 \\ 1 \end{pmatrix} = L_2,$$

L_1 can be chosen arbitrary, and $L_2 = -5$ is chosen.

Hence $\hat{C} = (1 \ -5 \ 1)$ and the closed-loop system is

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{z} \end{pmatrix} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 \\ q_1 - 3 + 5\gamma_Q & q_2 - 3 + 5\gamma_Q & -2 + 3\gamma_Q & -2 + 3\gamma_Q \\ 0 & 0 & 0 & -5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ z \end{pmatrix}.$$

The simulation results show that the system is asymptotically stable for different random processes $q_1^{(1)}(t), q_2^{(1)}(t)$ and $q_1^{(2)}(t), q_2^{(2)}(t)$ with values in compact set $Q_1 : \{0 \leq q_1 \leq 3, 0 \leq q_2 \leq 3\}$ and $\gamma_{Q_1} = -1.29$. As the uncertainty set increases to $Q_2 : \{0 \leq q_1 \leq 4, 0 \leq q_2 \leq 4\}$, $\gamma_{Q_2} = -2.29$ is required; otherwise, the system is not asymptotically stable with $\gamma_{Q_1} = -1.29$.

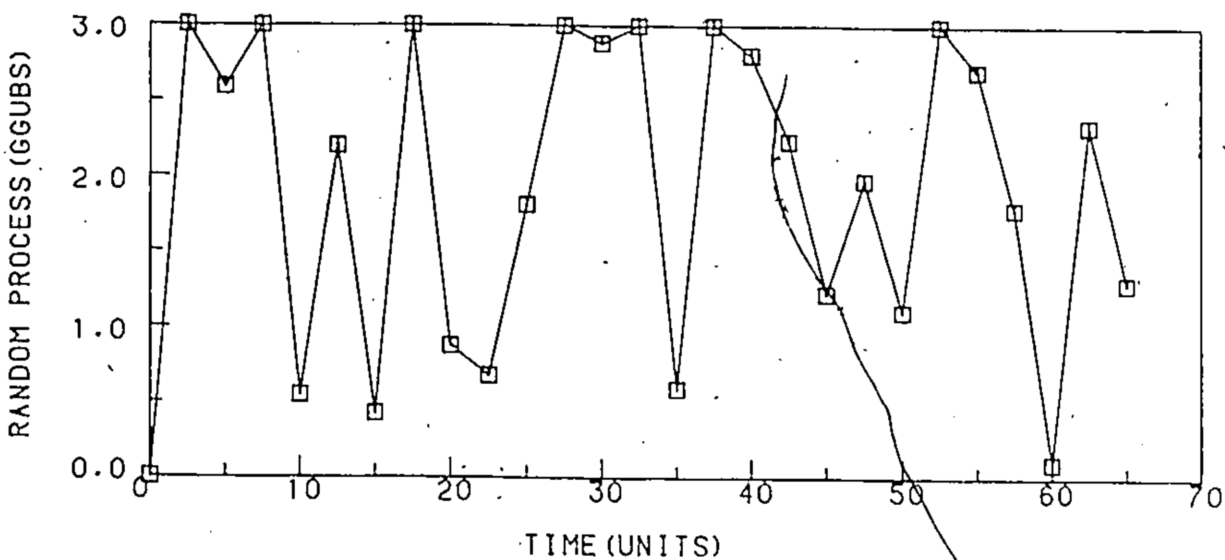

Fig.16. Random process $\{q_1^{(1)}(t), t \geq 0\}$ taking values from the compact set $Q_1 = [0,3]$.

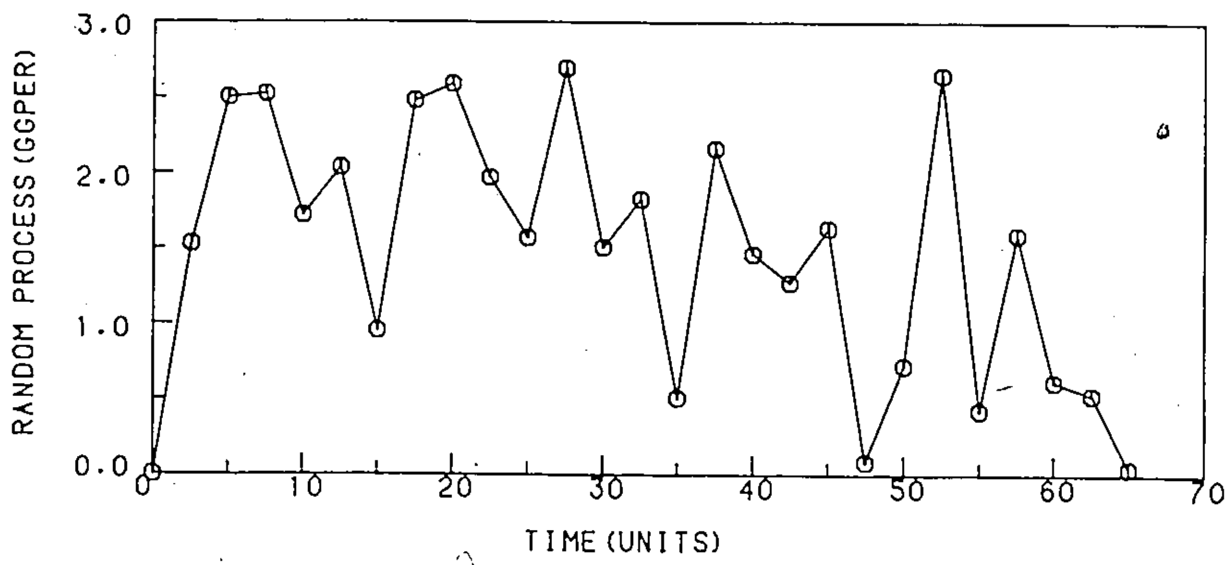

Fig.17. Random process $\{q_2^{(1)}(t), t \geq 0\}$ taking values from the compact set $Q_1 = [0, 3]$.

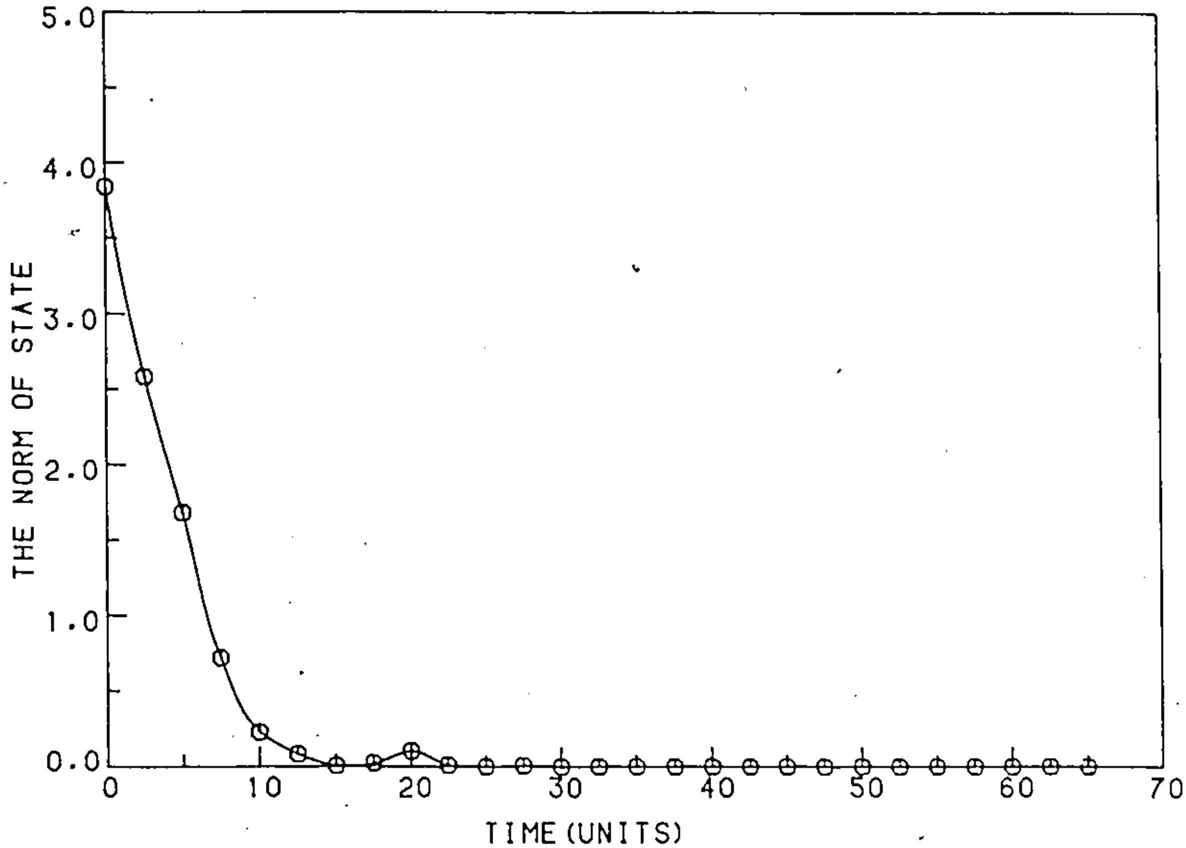

Fig.18. Stabilizing feedback gain with $\gamma_{Q_1} = -1.29$.

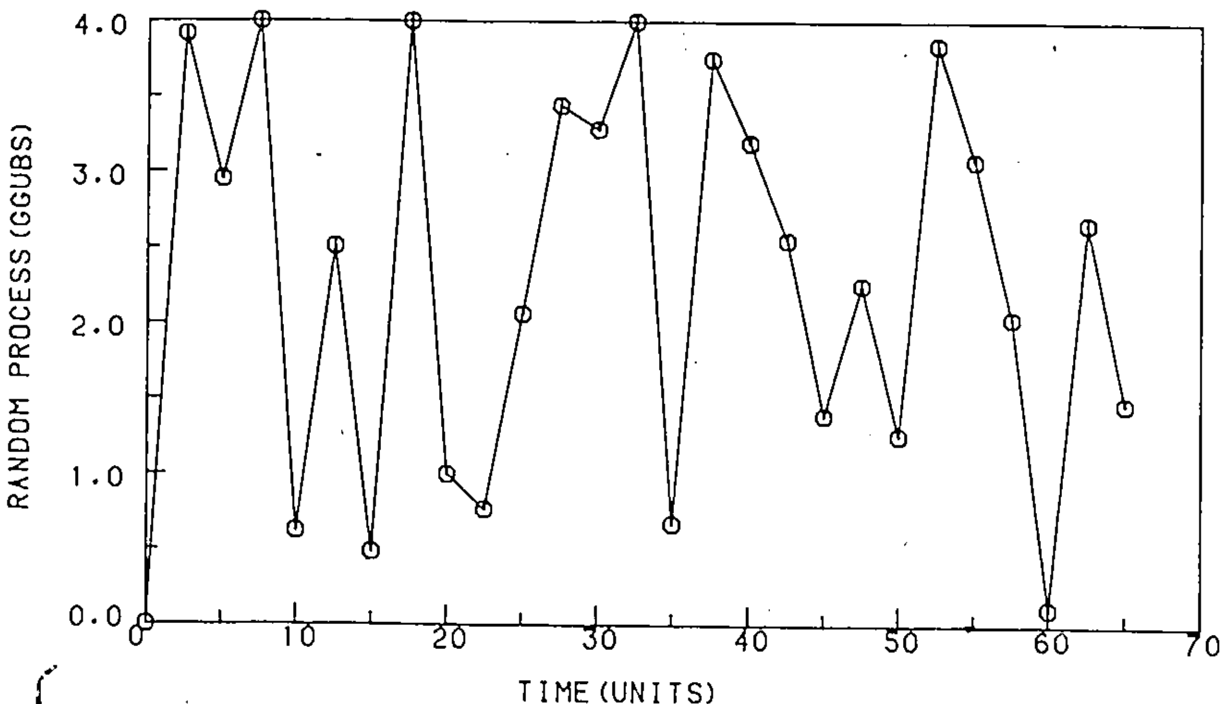

Fig.19. Random process $\{q_1^{(1)}(t)\}$ with values from the set $Q_2 = [0, 4]$.

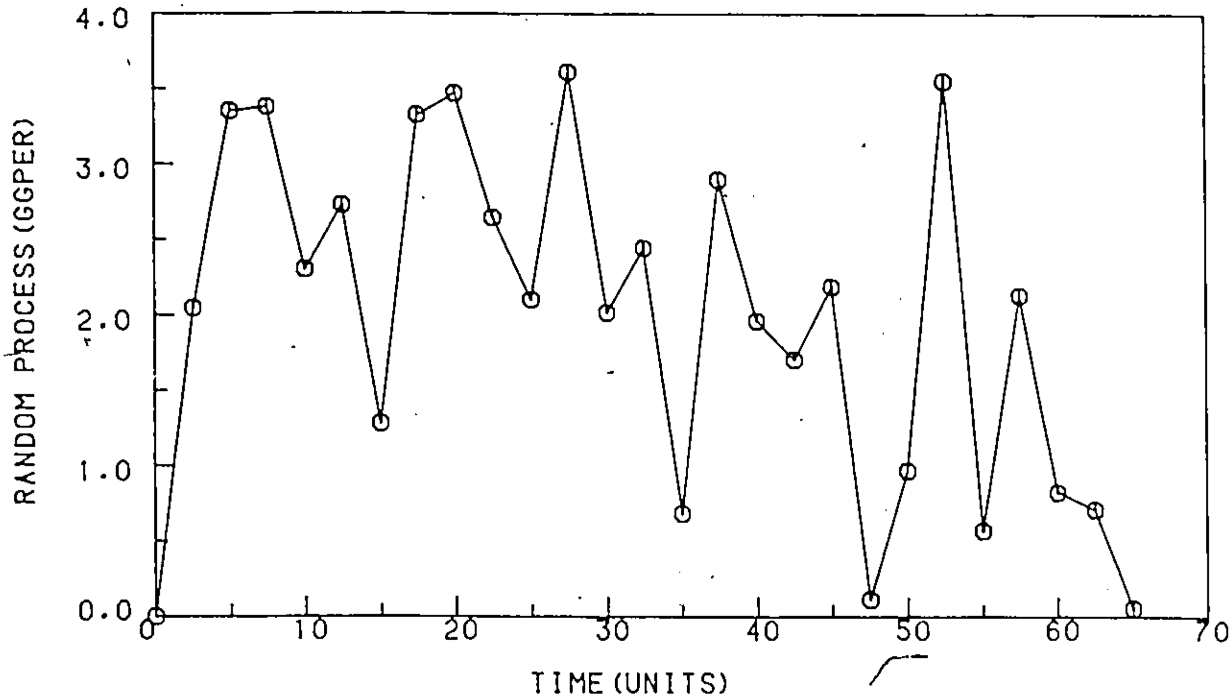

Fig.20. Random process $\{q_2^{(1)}(t)\}$ with values from the set $Q_2 = [0, 4]$.

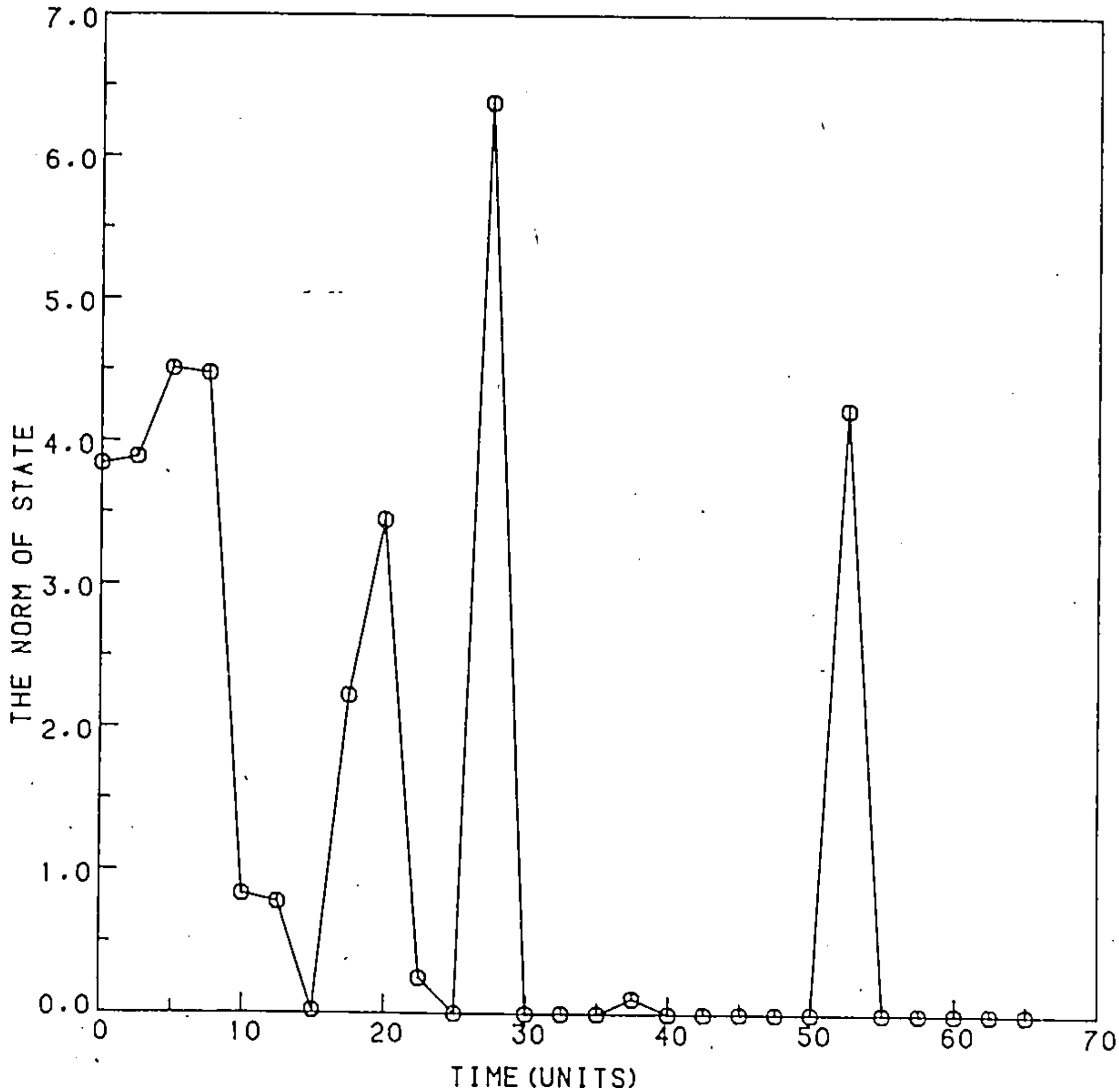

Fig.21. Same feedback gain with $\gamma_{Q_1} = -1.29$ (not asymptotically stable).

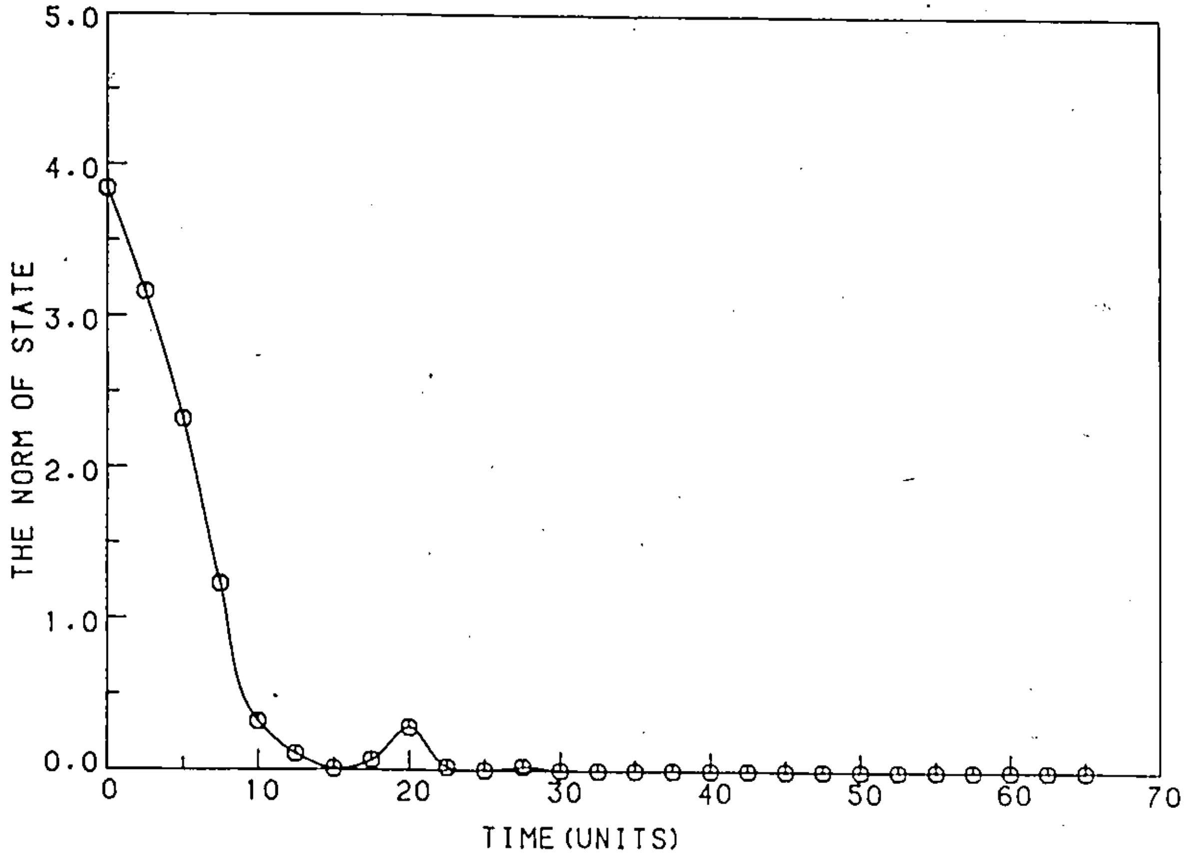

Fig.22. Stabilizing feedback gain with $\gamma_{Q_2} = -2.29$ (redesigned).

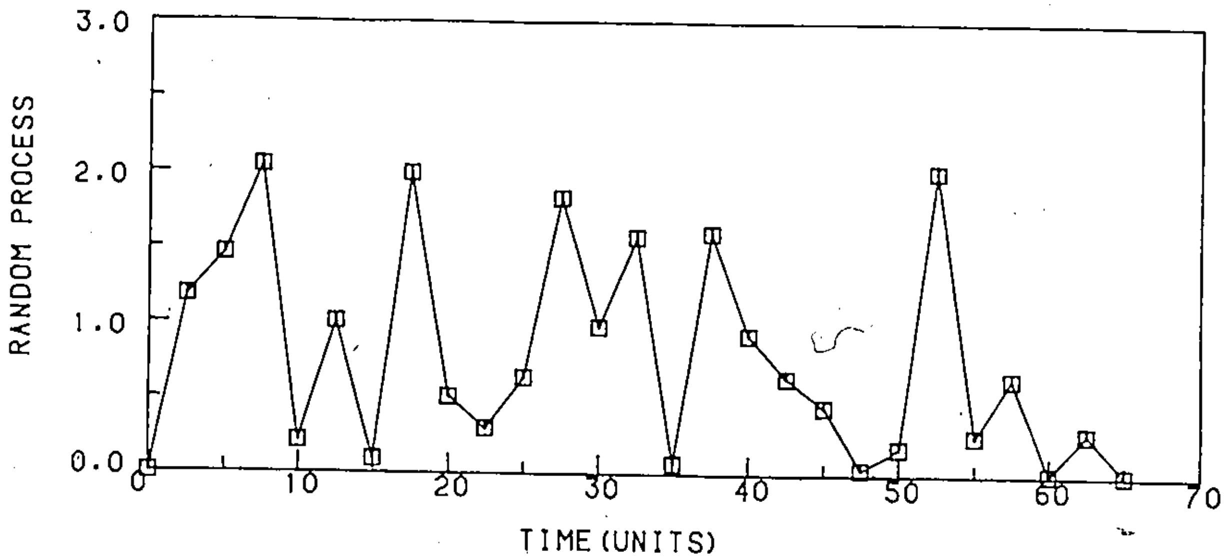

Fig.23. Random process $\{q_1^{(2)}(t), t \geq 0\}$ taking values from the compact set $Q_1 = [0, 3]$.

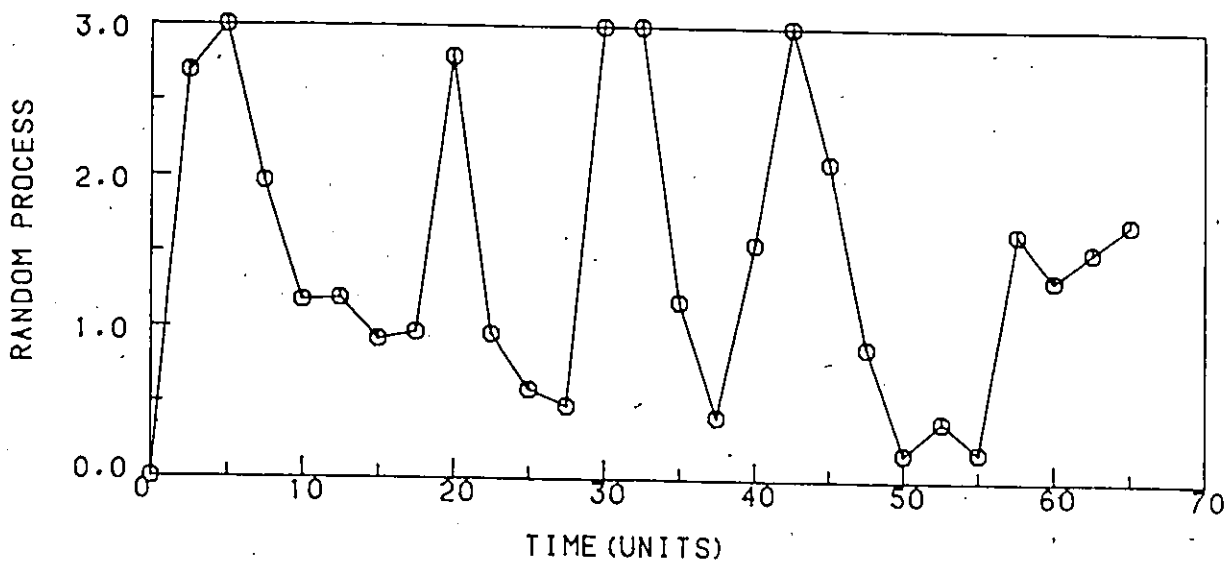

Fig.24. Random process $\{q_2^{(2)}(t), t \geq 0\}$ taking values from the compact set $Q_1 = [0, 3]$.

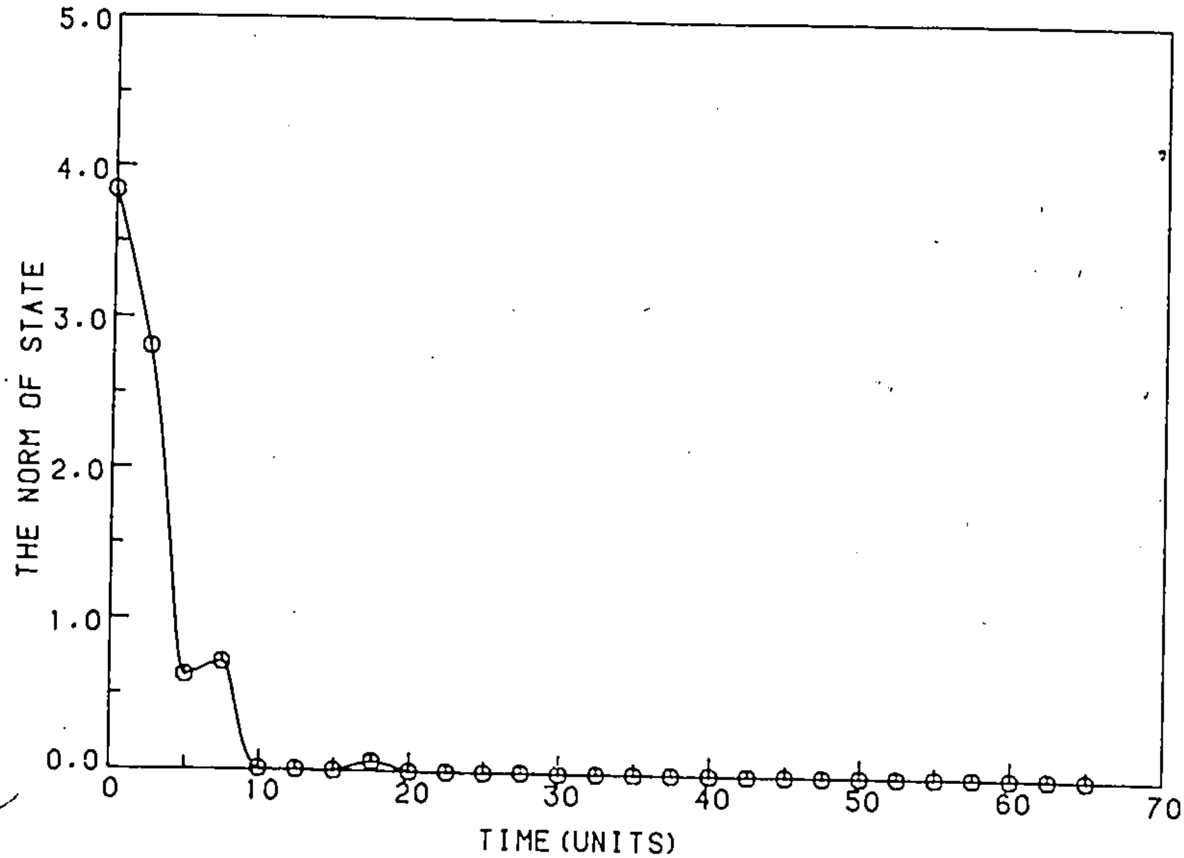

Fig.25. Stabilizing feedback gain with $\gamma_{Q_1} = -1.29$.

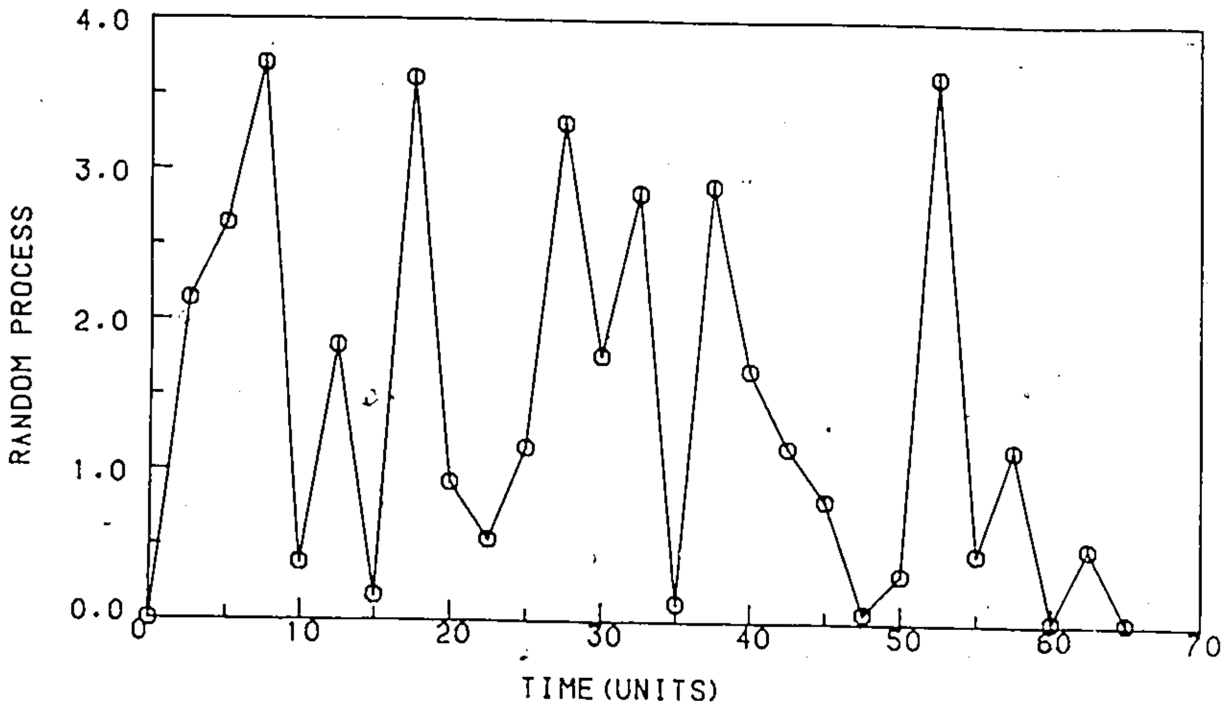

Fig.26. Random process $\{q_1^{(2)}(t)\}$ with values from the set $Q_2 = [0, 4]$.

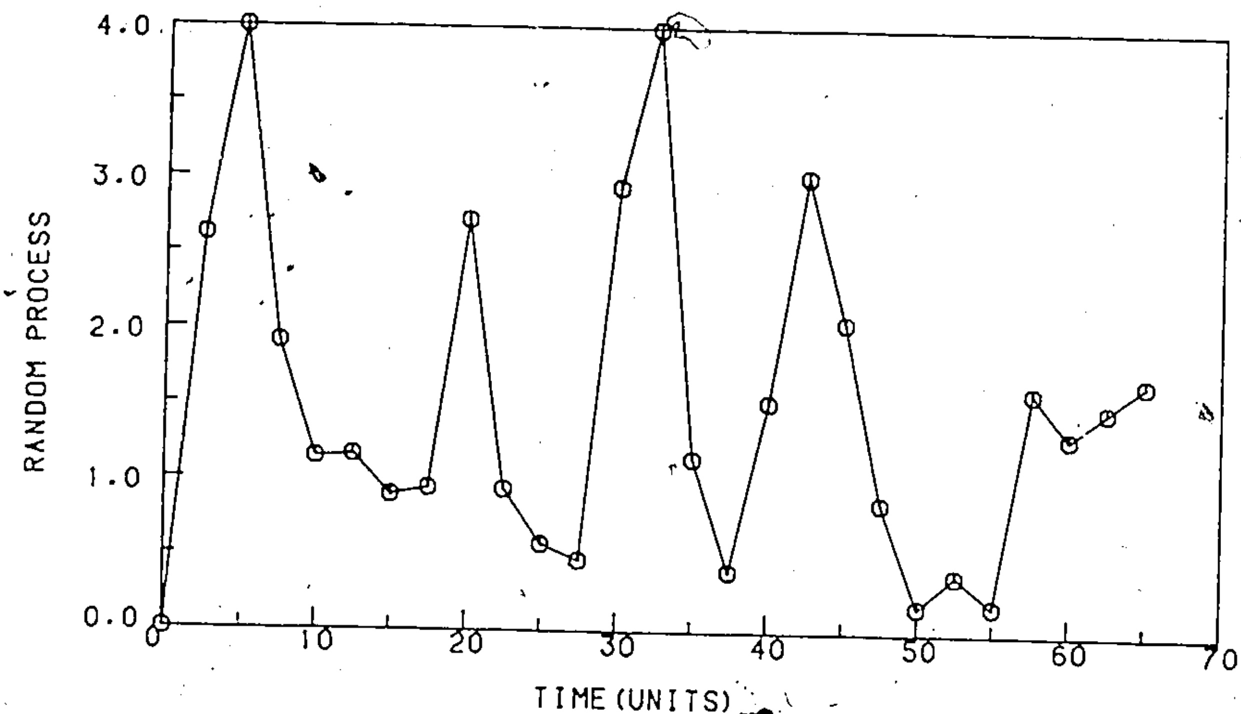

Fig.27. Random process $\{q_2^{(2)}(t)\}$ with values from the set $Q_2 = [0, 4]$.

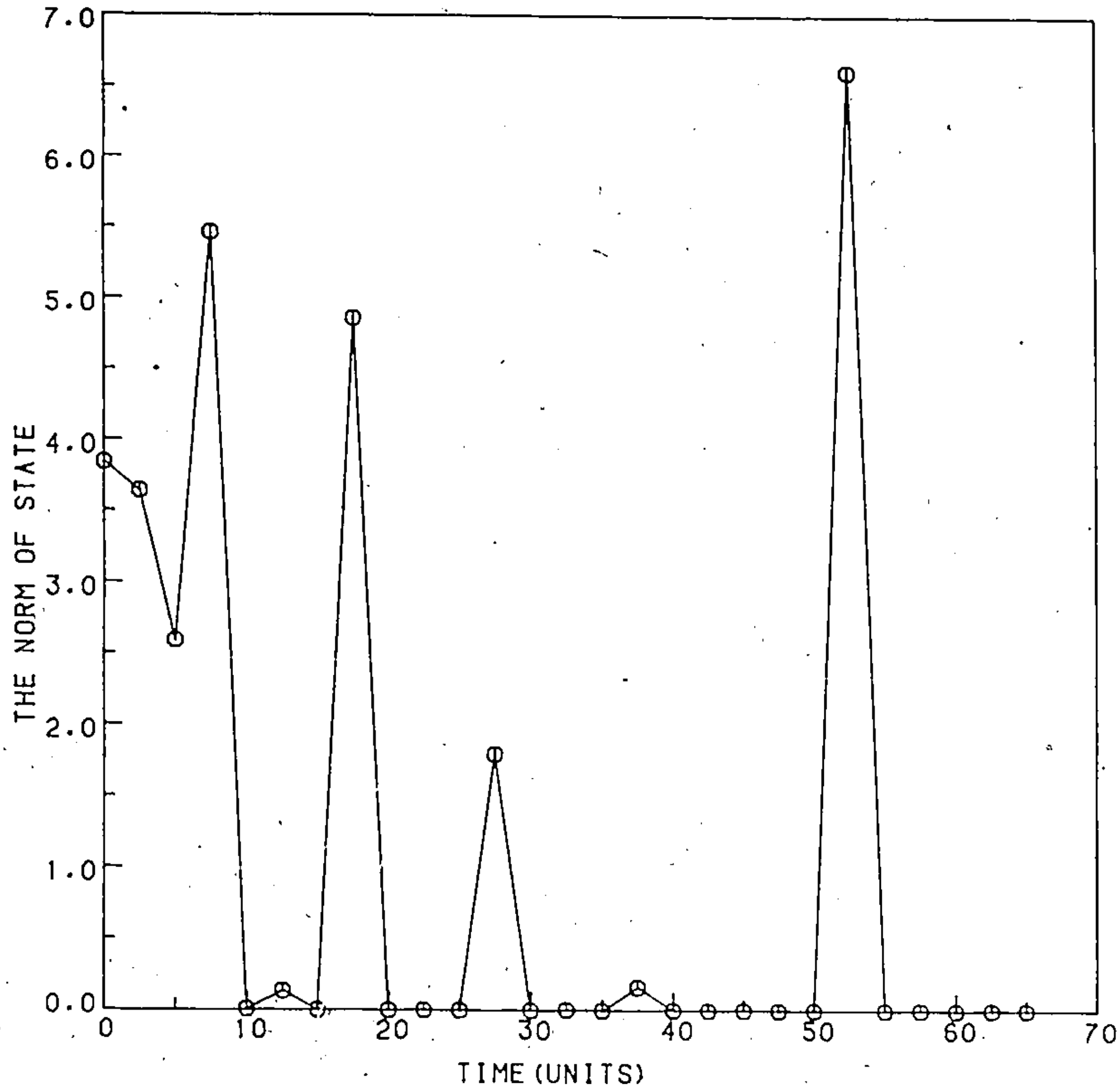

Fig.28. Same feedback gain with $\gamma_{Q_1} = -1.29$ (not asymptotically stable).

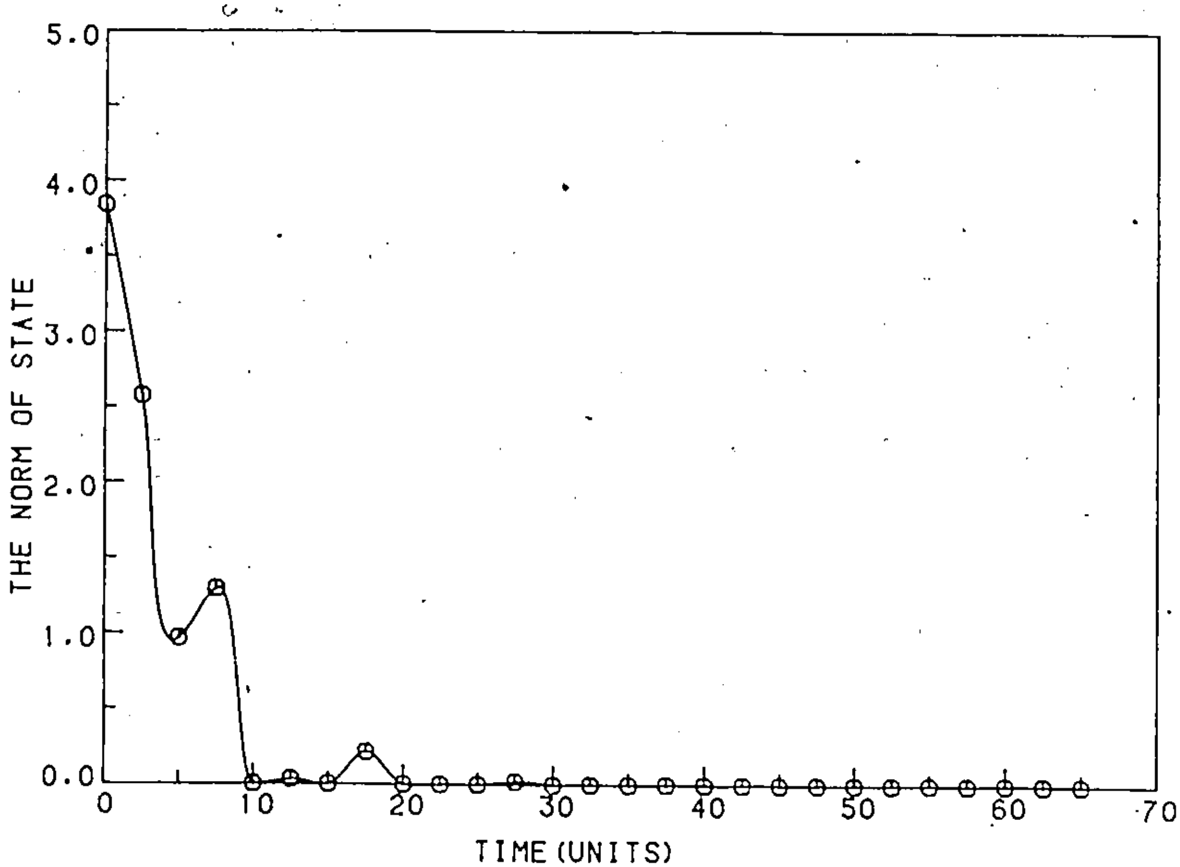

Fig.29. Stabilizing feedback gain with $\gamma_{Q_2} = -2.29$ (redesigned).

CHAPTER 5. SOME CONCLUDING REMARKS AND SUGGESTION FOR FUTURE WORK

In the literature[1-23], it is generally assumed that the state of uncertain system to be controlled is fully measurable. In most control situations, however, the state vector is not available for direct measurement. In this thesis, A technique for designing stabilizing linear feedback control laws for uncertain dynamical systems with incomplete state information has been developed. Our approach is based on Luenberger observer theory and allows uncertainties in both the plant and input(control) matrices. The design procedure enjoys the following attractive properties:

1. For any given compact set Q, assuming complete state information, the feedback control law is determined in several steps. The feedback law(feedback gain matrix) depends only on the intensity of the random process, that is, the size of the compact set Q, rather than the probabilistic laws that govern the process.

2. In chapter 2, the assumptions (A2) and (A6 2)) guarantee observability of the uncertain system[44],it is known that the controllability and observeability properties of a time-variable linear system may be fully determined from the system solution. However, the so-

lution to such uncertain system is generally not available in closed form. In his paper, L.M.Silverman examined the extent to which the various types of controllability and observability may be characterized in terms of the known system coefficient matrices. The criteria they developed for uniform observability and controllability which do not require calculation of the transition matrix has been successfully adopted in our thesis under assumptions (A2) and (A6 2)). It results in the applicability of the observer theory and simplicity of the design procedure (for $\hat{C}$).

3. For any $q \in Q$, by an observer equation, the control law is realized for the system with incomplete state information.

It would be desirable to relax the assumptions in chapter 2. These assumptions restrict the manner the uncertainty enter the system. It is interesting to determine the least restrictive conditions permitting the construction of stabilizer.

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