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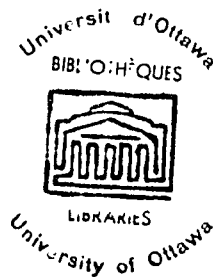
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SHORT TERM AXIAL LOAD - MOMENT
INTERACTION CHARACTERISTICS OF
REINFORCED CONCRETE BEAM - COLUMN
MEMBERS



by

RON ERNEST BROWN

A thesis submitted in partial fulfillment
of the requirements for the degree of
Master of Applied Science in the Faculty
of Graduate Studies of the University of
Ottawa, Ottawa, Canada.

October, 1969.

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SYNOPSIS

This thesis presents the test results from experimental work carried out on reinforced concrete beam-column members. The tests were designed to study the strength and deformational behaviour of these members under the combined effects of axial load and moment.

Fifty-five members were tested and these were subdivided into eleven series (S1-S11) of five specimens. The specimens in each series were identical in all respects.

Each member was 6' - 0" long and had cross-sectional dimensions of 6" x 8". The test span in each case was 5' - 6".

The percentage of longitudinal steel reinforcement was the only variable between the different series and it was varied between 1.04 and 6.58 percent. This range was chosen to lie within that allowed for column members by the Building Code Requirements for Reinforced Concrete (ACI 318-63).

Each member of a series was tested to failure under the action of axial load and bending moment. The members in each series were tested under different levels of axial load up to a maximum of 100 kips.

Continuous measurements of loads, strains and deflections were made throughout the tests. Concrete strains were measured over a gauge length of 10" at the central portion of the specimens using ten linear variable differential transformers. The transformers were mounted using 1" x 1" x $\frac{1}{2}$ " aluminum mountings which were cemented to the surface of the specimens. Deflections were measured at seven locations in the test span using deflection dial gauges. The results obtained were used to derive the axial load-moment, moment-curvature, and load-deflection be-

haviour of the specimens.

Presented also are "plastic design" theory expressions for reinforced concrete sections. These expressions were derived from the equilibrium and compatibility equations for the section and they enable rapid calculation of the axial load-moment interaction diagram and the ultimate curvature capacity of beam-column sections.

Satisfactory agreement was obtained between the theoretical and experimental results.

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CHAPTER I

INTRODUCTION

1.1 OBJECT

The object of this study was the determination of the interaction characteristics of reinforced concrete members subjected to axial load and bending. This was achieved by the determination of the axial load/moment and moment/curvature characteristics from short duration tests carried out on fifty-five reinforced concrete members. Theoretical design expressions have been developed which enable the rapid calculation of the ultimate curvature of beam-column members.

1.2 LIMIT DESIGN AND PLASTIC DESIGN

The limit design of structural concrete and the plastic design of structural steel are based on a recognition of the inelastic behaviour of structures at high loads and the resulting redistribution of internal moments and forces at various points within a structure. The two design methods differ in an important respect.

The plastic design theory for steel considers the structure at incipient collapse, i.e., the structure has become a mechanism by the formation of a sufficient number of plastic hinges. These hinges are assumed to be formed at discrete points in the members of the structure and they rotate at a moment of resistance equal to the ultimate moment capacity of the critical sections. Little attention has been paid to the magnitude of the post yield rotations as redistribution of moments proceed. It is known that the hinges can develop sufficient rotation to enable complete redistribution of moments without rupture.

Limit design of structural concrete, however, recognises that

concrete crushes at comparatively low strains and that the ductile capacity of a hinge could be exhausted before full redistribution of internal moments and forces is achieved. It is therefore of prime importance to consider the deformational capacity of hinging regions in any theory of limit design for structural concrete and that the capacity should be of a magnitude as to enable full plastic adaptability under load to become effective.

Due to the response of reinforced concrete to load; limit design requires a reasonable factor of safety against yielding of the critical sections at working loads. Thus, the magnitudes of the crack openings and deflections are kept within permissible limits.

Limit design of reinforced concrete involves a consideration of the following:

- (a) Limit Equilibrium
- (b) Rotational Compatibility
- (c) Structure Serviceability

Methods of limit design have been proposed by Baker (1)*, Sawyer (2), Cohn (3), Berwanger (4). These methods have emphasized the above three considerations to varying degrees.

A study of the rotational capacity of reinforced concrete sections is essential to the determination of the rotational compatibility condition at hinging regions throughout a structure.

1.3 COMPATIBILITY CONDITION

A general statement of the compatibility condition at plastic hinges can be expressed mathematically. Consider a statically indeterminate structure subjected to a loading system which can be defined by a

*The numbers in parenthesis refer to the list of references at Appendix A.

loading parameter F^* .

Let: F_i = Load corresponding to the formation
of the i th plastic hinge.

F_u = Structure collapse load corresponding
to the u th plastic hinge.

θ_i = Inelastic rotation at i th hinge for
 $F > F_i$

θ_{cs} = Rotation capacity of i th hinge.

Thus for $1 \leq i \leq u$, if $\theta_i \leq \theta_{cs}$, full redistribution of moments will occur at ultimate load F_u . If for at least one value of i , the effective rotation at plastic hinge i exceeds the rotation capacity of the section, i.e., $\theta_i > \theta_{cs}$, a local fracture occurs at i .

Thus, the inelastic rotation at any plastic hinge should be less than the available capacity for load levels up to the ultimate load.

1.4 ROTATION CAPACITY

The rotation capacity of a critical section in a reinforced concrete member may be expressed as the total rotation along a length L_p , of the member over which the moment varies from the yield moment M_y , to the ultimate moment, M_u . Cohn (3) has derived an expression for rotation capacity, θ_p , as follows: (See Fig. 1)**

$$\theta_p = \int_0^{L_p} (\phi_z - \phi_y) dz = A_\phi = L_p \phi_p \quad \dots\dots\dots(1.4.1)$$

where ϕ_z = Curvature at distance z from support.

* A list of the symbol used is given in Section 1.9.

** All figures are given in Appendix B.

ϕ_y = Curvature at first yield of tension reinforcement.

Equation (1.4.1) may be written as follows:

$$\theta_p = L_p' \phi_p \quad \dots\dots\dots(1.4.2)$$

where $L_p' = \beta L_p \quad \dots\dots\dots(1.4.3)$

Thus, the rotation capacity θ_p , can be evaluated by the area of plastic curvature A_ϕ . The curvature distribution factor β , is less than unity and depends among other factors, on the loading, the variation of flexural rigidity along the member and the longitudinal and transverse reinforcement (3).

Chan (5) has proposed that the available rotation θ_p may be given by the expression:

$$\theta_p = \beta L_p M_u \left(\frac{1}{(EI)_u} - \frac{1}{(EI)_e} \right) \quad \dots\dots\dots(1.4.4)$$

where $(EI)_u$ = flexural rigidity at ultimate moment

$(EI)_e$ = elastic flexural rigidity

The curvature distribution factor β , assumes the values of 0.305 for unbound sections and 0.2 for bound sections.

Mattock (6) has expressed the ratio of the total inelastic θ_{tu} over a distance s , to the inelastic rotation θ_u in length $\frac{d}{2}$ as follows:

$$\frac{\theta_{tu}}{\theta_u} = 1 + (1.14(s/d)^{\frac{1}{2}} - 1) (1 - (q-q') (d/16.2)^{\frac{1}{2}}/q_b) \dots\dots\dots(1.4.5)$$

and θ_u is given by

$$\theta_u = (\phi_u - \phi_y) \frac{M_u}{M_y} \frac{d}{2}$$

where A_b = area of tension reinforcement required to produce balance failure of beam section.

A_s = area of tension reinforcement

A'_s = area of compression reinforcement

b = breadth of section

d = distance from centroid of tension reinforcement
to compression edge of beam

f'_c = ultimate compression strength of concrete as
determined from 6" x 12" cylinders

f_y = yield stress of tension steel

f'_y = yield stress of compression steel

$$q = \frac{A_s}{bd} \frac{f_y}{f'_c}$$

$$q' = \frac{A'_s}{bd} \frac{f'_y}{f'_c}$$

$$q_b = -\frac{A_b}{bd} \frac{f_y}{f'_c}$$

s = distance from section of maximum moment to adjacent section of zero moment

ϕ_u = ultimate curvature

Other researchers have proposed expressions for the rotation capacity of hinging regions. The structure of these expressions, however, is similar to those given above in that they contain an expression for the ultimate plastic curvature associated with some empirically determined length which defines the zone of flexural yielding in the vicinity of the hinge.

The determination of the curvature relationship is therefore

of fundamental importance in the determination of the rotational capacity of hinging regions in reinforced concrete members.

1.5 MOMENT-CURVATURE RELATIONSHIP

Curvature is a measure of rotation per unit length and is given by the following expression:

$$\text{Curvature, } \phi = \frac{\text{concrete compressive strain at compression face of beam}}{\text{neutral axis depth}}$$

The moment-curvature relationship for a reinforced concrete section has three important stages as shown in Fig. 2. Stage 1 corresponds to narrow cracking of the member and this is accompanied by an abrupt change in its flexural rigidity. The second stage is evidenced by wide cracking and large deflection and the third stage is evidenced by crushing and spalling of the concrete at the critical sections.

Although considerable research has been carried out on the moment-curvature behaviour of critical sections undergoing pure flexure, there have been few experimental programs directed towards a quantitative description of sections which are subjected to combined stresses.

The strength and ductility of critical sections under the combined effects of flexural and direct compressive stresses is the subject of this study.

1.6 ASSUMPTIONS

- (a) At the start of yield of the tension reinforcement, the concrete compressive stress varies linearly from zero at the neutral axis to a maximum at the compression face of the beam.
- (b) The tensile strength of concrete may be neglected.
- (c) The stress-strain relationship for concrete is that proposed by Hognestad (7) and shown in Fig. 3.
- (d) The concrete displaced by the steel reinforcement may be neglected.

- (e) The stress-strain curve for the tension and compression reinforcing is assumed as a trapezoid similar to that shown in Fig. 4. The effects of strain hardening being neglected.
- (f) There is no slip between the reinforcing steel and the concrete.
- (g) There is a linear strain distribution across the section for all combinations of axial load and bending moments.

1.7 SCOPE

The principal aims of this work is a study of the interaction characteristics and the development of design expressions for the ultimate strength and curvature of reinforced concrete sections subjected to combined bending and axial load. A comparison of the theoretical with the experimental results obtained from tests is carried out for fifty-five reinforced beam-columns.

1.8 ACKNOWLEDGEMENTS

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1.9 NOTATION

The symbols used are listed below for convenience:

A_s	area of tension reinforcement
A'_s	area of compression reinforcement
A_ϕ	area of plastic curvatures
a	depth of stress block
b	breadth of beam
C_1	compressive force in concrete
C_2	compressive force in the compression reinforcement
d	distance from centroid of tension reinforcement to compression edge of beam
D	overall depth of section
e	concrete strain
e'	eccentricity of ultimate
e_{cy}	concrete strain at first yield of tension reinforcement
e_o	compressive strain in concrete corresponding to maximum stress
e_s	steel strain
e_{sh}	steel strain at commencement of strain-hardening

e_u	ultimate concrete strain
e_y	yield strain of steel reinforcement
E_c	static modulus of elasticity of concrete
E_d	resonance modulus of elasticity of concrete
$(EI)_e$	elastic flexural rigidity
E_s	modulus of elasticity of steel reinforcement in the elastic region
E_{sh}	modulus of strain-hardening
$(EI)_u$	flexural rigidity at ultimate moment
f_c	concrete compressive stress
f'_c	compressive strength of concrete as determined from cylinder tests
f_s	steel stress
$\bar{f}_u$	ultimate steel stress
f_y	assumed yield stress in tension reinforcement
f'_y	assumed yield stress in compression reinforcement
F	load on statically indeterminate structure
F_i	load on structure corresponding to formation of i th plastic hinge
F_u	collapse load of structure corresponding to formation of u th plastic hinge
k_1, k_2, k_3	stress block parameters
kd	depth for transformed area for doubly reinforced sections
k_n, k'_n, k_p	section parameters
M_b	moment at balanced failure
M_o	ultimate moment at zero axial load
M_{oy}	yield moment at zero axial load
M_u	ultimate moment
M_y	yield moment
n	modular ratio
n'	resonance frequency

P	axial load
P_b	axial load at balanced failure
P_o	axial load capacity
P_u	axial load at failure
p	A_s/bd
p'	A'_s/bd
p_t	$(A_s + A'_s)/bD$
q	pf_y/f'_c
q'	$p'f'_y/f'_c$
q_b	tension reinforcement factor for balanced ultimate strength conditions for beams
q'_u	$q - q'$
s	distance from section of maximum moment to adjacent section of zero moment
T	force in tension reinforcement
W	weight of prism
ϕ_p	plastic curvature
ϕ_u	ultimate curvature
ϕ_y	curvature at yield
μ	$\phi_u/\phi_y =$ rotation factor
θ_p	rotation capacity
θ_{tu}	inelastic rotation occurring between the section of maximum moment and an adjacent section of zero moment
θ_u	inelastic rotation at ultimate moment, occurring within a length $d/2$ to one side of the section of maximum moment

CHAPTER 2

ULTIMATE STRENGTH AND CURVATURE OF
REINFORCED CONCRETE BEAM-COLUMNS2.1 MEMBERS IN PURE FLEXURE

The member in pure flexure is treated as one subjected to bending and zero axial load.

Theoretical expressions for M_y , M_u , ϕ_y and ϕ_u for double reinforced concrete sections have been presented by Cohn (3), but are repeated here for convenience.

When the tension reinforcement just reaches the yield point, the following relations based on a semielastic design can be written (Fig. 5a):

$$k_y = (n^2 (p+p')^2 + 2n(p+k_n p'))^{\frac{1}{2}} - n(p+p') \dots (2.1.1)$$

$$e_{cy} = \frac{f_y}{E_s} \frac{k_y}{1 - k_y} \dots (2.1.2)$$

$$\phi_y = \frac{e_{cy}}{k_y d} \dots (2.1.3)$$

$$M_y = A_s f_y d \left(1 - \frac{k_y}{3}\right) + A'_s f_s d \frac{k_y}{(3-k_n)} \dots (2.1.4)$$

Referring to Fig. 5b, the general expression for the neutral axis at the ultimate moment is:

$$k_u = \frac{1}{k_1 k_3} \left(p \frac{f_y}{f'_c} - p \frac{f'_s}{f'_c} \right) \dots (2.1.5)$$

If $q \leq k_1 k_3 k_n$ then $k_u < k_n$, the section behaves as if reinforced in tension only; hence,

$$k_u = \frac{1}{k_1 k_3} \frac{p f_y}{f'_c}$$

$$\text{If } \frac{k_1 k_3}{2} \frac{(1 + k_n)}{(1 - \frac{p}{p'})} < q \leq k_1 k_3 \frac{1}{(1 - \frac{p}{p'})} \frac{e_u}{e_u + e_y} \quad \dots (2.1.6)$$

If $k_u > k_n$ and $f_s = f_y = f'_s$, both tension and compression reinforcement are at the yield state; hence,

$$k_u = \frac{1}{k_1 k_3} \left(\frac{p f_y}{f'_c} - \frac{p' f_y}{f'_c} \right) \quad \dots (2.1.7)$$

$$\text{or } k_u = \frac{1}{k_1 k_3} (q - q') \quad \dots (2.1.8)$$

If $q > \frac{1}{k_1 k_3} \frac{e_u}{(1 - \frac{p}{p'})} \frac{e_u}{e_u + e_y}$: This is the balanced condition

and the concrete crushes simultaneously with yield of compression and tension steel; hence,

$$k_u = \frac{e_u}{e_u + e_y} \quad \dots (2.1.9)$$

The ultimate curvature is given by:

$$\phi_u = \frac{e_u}{k_u d} \quad \dots (2.1.10)$$

The ultimate moment is expressed by:

$$M_u = k_1 k_3 f'_c k_u b d^2 (1 - k_2 k_u) + A'_s f'_s d (1 - k_n) \dots (2.1.11)$$

The condition in which the concrete crushes before yield of the tension reinforcement, i.e., an over-reinforced section, is not considered here.

It has been reported by Baker and Amarakone (8), however, that if the bending resistance of brittle sections is kept within safe limits, the ductility which results from long term strain effects will be safe and favourable to the distribution of bending moments in the structure.

Such sections are, however, not permitted by most building codes.

The values of k_u and hence the ultimate curvature and ductility factor of hinges subjected to pure flexure are seen therefore to depend essentially on a limitation of the steel percentages. Recent research by Mattock (6), Baker and Amarakone (8), Corley (9) and Roy and Sozen (10) has shown that if the concrete and other section properties remain constant, the curvature of the hinge will be increased in the presence of a shear force and also by the effects of helical or rectangular transverse reinforcement.

Although many researchers in their proposals for limit design have suggested that the strength of flexural hinges should be limited to the value of the yield moment, M_y , the current ACI building code (Building Code Requirements for Reinforced Concrete (ACI 318-63)), permits hinges to develop a design ultimate moment of $0.9 M_u$ where:

$$M_u = b d^2 f'_c q'_u (1 - 0.59 q'_u) + q' f'_c b d^2 (1 - k_n) \quad \dots (2.1.12)$$

and

$$q'_u = (p - p') \frac{f_y}{f_c} \quad \dots (2.1.13)$$

Sawyer (2) has recommended that the design bending resistance of purely flexural hinges, M_e , should be given by the relation $M_e = 0.85 M_u$, where M_e is the moment value at the intersection of the two lines of a bilinear approximation to the moment-curvature relationship and M_u is the experimental value of ultimate moment.

Berwanger (4) has suggested that the value of the yield moment given by the expression,

$$M_y = \frac{f_c}{2} k b d^2 (1 - \frac{k}{3}) + A'_s f'_s d (1 - k_n) \quad \dots (2.1.14)$$

is sufficiently conservative to be applied to all types of hinges and that the rotation factor, $\frac{\phi_u}{\phi_y}$, can be given by the relation;

$$\mu = \frac{\phi_u}{\phi_y} = \frac{1}{5q_y q'_u} \quad \dots (2.1.15)$$

2.2 HINGES UNDER COMBINED FLEXURAL AND DIRECT COMPRESSIVE STRESSES

While the ultimate strength of beam-columns and beam-column sections have been studied both experimentally and analytically, Hognestad, (7, 13), Broms and Viest, (11, 12), Pfrang, Seiss and Sozen, (14), it would appear that comparatively little work has been done to produce analytical or experimental verification of the rotation capacity of these members. Notable analytical work in this direction has been reported by Pfrang, Seiss and Sozen (14), Breen (15), Pfrang and Seiss (16).

The ultimate strength capacity and behavioural mode of beam-columns whose cross section failure is initiated by either yielding of the tension reinforcement or crushing of concrete can be predicted from their axial-load-moment interaction diagram Fig. 6. For axial loads greater than P_b , failure will be initiated by crushing of the concrete. This is called a compression failure. For axial loads less than P_b , yielding of the tension reinforcement precedes the crushing of the concrete and this is called a tension failure. Hinge failures in the compression mode can be subdivided into two groups. The first group is one for which the section has both tensile and compressive stresses at failure. The second is that group for which the section has compressive stresses only, and these typify hinges in the lower storeys of multi-storey frames where axial loads are high and moments are comparatively low.

From the above discussion it is seen that while for purely flexural hinges the failure mode and hence the rotational ductility can be controlled essentially by a limitation of the amount of reinforcement; ductility of hinges under the combined effects of bending moment and axial load can be controlled primarily by limiting the load on the hinge to a value less than the balanced load of the section.

Based on the above, it would seem that a number of interesting design techniques could be evolved, some of which have already received attention by a number of researchers. Some of these concepts are described briefly as follows:

(a) A design concept which limits the formation of plastic hinges to flexural members only. This could be achieved by a limitation of stiffness factors of beams and columns meeting a joint.

(b) A design concept which limits the amount of redistribution of moments by the definition of a redistribution factor which corresponds to the rotational capacity of the hinging regions. Such a method has been proposed by Cohn (5, 17) and has been limited to the solution of continuous beams and slabs.

(c) A design concept based upon the assumption that hinging regions can provide the rotation capacity required for the formation of a collapse mechanism and the subsequent design to ensure that rotational compatibility is satisfied at all hinges. Such a method has been proposed by Berwanger (4).

2.3 INTERACTION DIAGRAM

Although there has been considerable analytical and experimental study of the ultimate strength of beam-columns and beam-column sections, there has been comparatively little work done to produce

analytical or experimental verification of the rotation capacity of these members.

A qualitative description of the strength capacity of reinforced concrete hinge is given by an interaction diagram as shown in Fig. 6.

Points within the interaction envelope define combinations of axial load and moment which are possible without failure of the cross section. It is apparent, therefore, that points within the envelope represent a series of possible strain or curvature configurations under axial load and moment. The boundary of the envelope is the axial-load-moment relation at failure. The curve and its bounded region gives a qualitative description of the curvature. Thus for loads less than P_D , the segment BA, Fig. 6 represents increasing curvature and ductility and segment BD representing increasing brittleness and reduced curvatures.

In addition to the failure diagram envelope, it is also possible to establish an interaction diagram corresponding to yield in the tension reinforcement. This envelope is shown in dotted lines in Fig. 6. The author believes that the derivation of such a curve is important in the determination of a safe design moment in any limit design method. Sawyer (2) has suggested tentatively that a design moment, M_e , should be given by the relation $M_e = 0.85 M_u$. Berwanger (4) has suggested that the design moment should be made equal to M_{Oy} (Fig. 6).

It is important to note that the choice of values of design moment and curvature depends on a reasonable assumption of the different load combinations which are possible within the interaction diagram permissible space. This is due to the fact that design moments and curvatures based upon a given ultimate load may be exceeded under partial loading conditions. Although this factor is not of essential

concern to this thesis, it was thought that a constant axial load and a variable moment application might be in reasonable agreement with the effect of dead and live load acting in normal structures.

Reinforced concrete is not a linear elastic material and it is not possible to write continuous mathematical expressions relating axial load, moment and curvature over the full range of their variation. Most analytical approaches to the establishment of the interaction diagram and moment-curvature relationship for a section have adopted the following procedure:

- (a) Assume a strain configuration over the depth of the section.
- (b) Assume a stress-strain relation for concrete and steel reinforcement and therefore determine the stress distribution across the section.
- (c) Determine the resultant axial load and moment.
- (d) From the assumed strain configuration determine the curvature.

This approach is amenable to computer solution and the results of such studies have been published elsewhere, Roy and Sozen (10), Pfrang, Seiss and Sozen (14), Pfrang and Seiss (16). While such results are pertinent to a descriptive behaviour of reinforced concrete sections, the models chosen have not represented fully the effects of member cracking, triaxial stress and time-dependent deformations on the ultimate strength and curvature of reinforced concrete members.

2.4 MATHEMATICAL EXPRESSIONS FOR BEAM-COLUMNS

Theoretical expressions for the ultimate strength of beam-column sections have been derived by Hognestad (7, 13). Similar expressions are derived here and these expressions are based on the assumptions presented in section 1.6. It is assumed that there is

negligible error in neglecting the concrete displaced by the compression reinforcement.

(a) Tensile and Compression Hinges

Referring to Fig. 7 in Appendix B

$$C_1 = k_1 k_3 f'_c ab \quad \dots (2.4.1)$$

$$C_2 = \Lambda'_s f'_s \quad \dots (2.4.2)$$

$$T = \Lambda_s f_s \quad \dots (2.4.3)$$

Strain Compatibility:

$$\frac{a}{d} = \frac{e_u}{e_s + e_u} \quad \dots (2.4.4)$$

Curvature:

$$\phi_u = \frac{e_s + e_u}{d} \quad \dots (2.4.5)$$

Equilibrium of Forces:

$$P_u = k_1 k_3 f'_c ab + \Lambda'_s f'_s - \Lambda_s f_s \quad \dots (2.4.6)$$

$$\frac{P_u}{bd} = k_1 k_3 f'_c \left(\frac{a}{d}\right) + p' f'_s - p f_s \quad \dots (2.4.7)$$

Equilibrium of Moments about Plastic Centroid:

$$M_u = k_1 k_3 f'_c ab(d - k_p d - k_2 a) + \Lambda'_s f'_s(d - k_p d - k_n d) + \Lambda_s f_s k_p d. \quad \dots (2.4.8)$$

$$\frac{M_u}{bd} = k_1 k_3 f'_c \left(\frac{a}{d}\right) (d - k_p d - k_2 a) + p' f'_s(d - k_p d - k_n d) + p f_s k_p d. \quad \dots (2.4.9)$$

Let $S = k_1 k_3 f'_c$

$$e' = \frac{M_u}{bd} \times \frac{bd}{P_u}$$

Therefore:

$$e' \left(S \left(\frac{a}{d} \right) + p' f'_s - p f_s \right) = S \left(\frac{a}{d} \right) d(1 - k_p) - S \left(\frac{a}{d} \right)^2 k_2 d + p' f'_s d(1 - k_p - k_n) + p f_s k_p d \quad \dots (2.4.10)$$

$$S k_2 d \left(\frac{a}{d}\right)^2 + S(e' - d(1 - k_p)) \left(\frac{a}{d}\right) - (e' + k_p d)(pf_s - p'f'_s) - p'f'_s d(1 - k_n) = 0 \quad \dots (2.4.11)$$

Substituting Section Parameters in equation 2.4.11, (See Section Parameters in Fig. 8 in Appendix B)

$$Sk_2 d \left(\frac{a}{d}\right)^2 + S m \left(\frac{a}{d}\right) - t(pf_s - p'f'_s) - p'f'_s v = 0 \quad \dots (2.4.12)$$

Therefore:

$$Sk_2 d \left(\frac{a}{d}\right)^2 + S m \left(\frac{a}{d}\right) - pf_s t + p'f'_s (t - v) = 0 \quad \dots (2.4.13)$$

Let

$$c = t - v$$

$$x = \frac{1}{k_1 k_2 k_3}$$

Therefore:

$$\left(\frac{a}{d}\right)^2 + \frac{m}{k_2 d} \left(\frac{a}{d}\right) - \frac{x}{d} \left(\frac{pf_s t}{f'_c} - \frac{p'f'_s c}{f'_c}\right) = 0 \quad \dots (2.4.14)$$

Solving and Neglecting the Negative Root of equation 2.4.14

$$\left(\frac{a}{d}\right) = \frac{m}{2k_2 d} \left(-1 + \left(1 + \frac{4xk_2 d^2}{m^2} \left(\frac{pf_s t}{f'_c} - \frac{p'f'_s c}{f'_c}\right)\right)^{\frac{1}{2}}\right) \quad \dots (2.4.15)$$

Let

$$4xk_2^2 d = 4 \frac{k_2}{k_1 k_3} d = h$$

$$a = k_u d$$

Therefore:

$$k_u = \frac{m}{2k_2 d} \left(\left(1 + \frac{h}{m^2} \left(\frac{pf_s t}{f'_c} - \frac{p'f'_s c}{f'_c}\right)\right)^{\frac{1}{2}} - 1\right) \quad \dots (2.4.16)$$

From equations 2.4.4 and 2.4.5

$$e_s = e_u \left(\frac{1 - k_u}{k_u}\right)$$

$$\phi_u = \frac{e_u}{d} \left(1 + \frac{e_s}{e_u}\right)$$

Therefore:

$$\phi_u = \frac{e_u}{k_u d}$$

$$\phi_u = \frac{e_u}{d} \frac{1}{\frac{m}{2k_2 d} \left(\left(1 + \frac{h}{m^2} \left(\frac{p f_s t}{f'_c} - \frac{p' f'_s c}{f'_c} \right) \right)^{\frac{1}{2}} - 1 \right)} \quad \dots (2.4.17)$$

$$\phi_u = \frac{2k_2 e_u}{(m^2 + h \left(\frac{p f_s t}{f'_c} - \frac{p' f'_s c}{f'_c} \right)^{\frac{1}{2}} - m)} \quad \dots (2.4.18)$$

Case I: Balanced Failure

$$\begin{aligned} e_s &= e_y & f_s &= f_y \\ e'_s &\geq e_y & f'_s &= f_y \end{aligned}$$

Therefore:

$$\phi_u = \frac{2k_2 e_u}{((m^2 + h(qt - q'c))^{\frac{1}{2}} - m)} \quad \dots (2.4.19)$$

Case II: Tensile Failure

$$\begin{aligned} e_s &> e_y & f_s &= f_y \\ e'_s &> e_y & f'_s &= f_y \end{aligned}$$

Therefore:

$$\phi_u = \frac{2k_2 e_u}{((m^2 + h(qt - q'c))^{\frac{1}{2}} - m)} \quad \dots (2.4.20)$$

Case III: Compression Failure

$$\begin{aligned} e_s &< e_y & f_s &= E_s e_s \\ e'_s &\geq e_y & f'_s &= f_y \end{aligned}$$

Therefore:

$$\phi_u = \frac{2k_2 e_u}{((m^2 + h \left(\frac{p f_s t}{f'_c} - q'c \right))^{\frac{1}{2}} - m)} \quad \dots (2.4.21)$$

(b) Compressive Hinge

Referring to Fig. 9 in Appendix B

$$C_1 = k_1 k_3 f'_c b d (1 + k'_n) \quad \dots (2.4.22)$$

$$C_2 = A'_s f'_s \quad \dots (2.4.23)$$

$$T = A_s f_s \quad \dots (2.4.24)$$

Equilibrium of Forces:

$$P_u = k_1 k_3 f'_c b d (1 + k'_n) + A'_s f'_s + A_s f_s \quad \dots (2.4.25)$$

Let

$$k_1 k_3 f'_c = S$$

$$\frac{P_u}{b d} = S(1 + k'_n) + p' f'_s + p f_s \quad \dots (2.4.26)$$

Equilibrium of Moments about Plastic Centroid:

$$M_u = k_1 k_3 f'_c b d (1 + k'_n) (d - k_p d - k_2 a) + A'_s f'_s (d - k_p d - k_n d) - A_s f_s k_p d \quad \dots (2.4.27)$$

$$\frac{M_u}{b d} = S(1 + k'_n) (d - k_p d) - S(1 + k'_n) k_2 a + p' f'_s d (1 - k_p - k_n) - p f_s k_p d \quad \dots (2.4.28)$$

Let

$$e' = \frac{M_u}{b d} \times \frac{b d}{P_u}$$

$$e' (S(1 + k'_n) + p' f'_s + p f_s) = S(1 + k'_n) (d - k_p d) - S(1 + k'_n) k_2 a + p' f'_s d (1 - k_p - k_n) - p f_s k_p d \quad \dots (2.4.29)$$

Re-Arranging:

$$S(1 + k'_n) (e' - d(1 - k_p)) + (e' + k_p d) (p f_s + p' f'_s) - p' f'_s d (1 - k_n) = -S(1 + k'_n) k_2 a \quad \dots (2.4.30)$$

Substituting Section Parameters from Fig. 8 in Appendix B

$$S(1+k'_n)m + t(pf_s + p'f'_s) - p'f'_sv = -S(1+k'_n)k_2a \quad \dots (2.4.31)$$

Therefore:

$$a = - \left(\frac{1}{k_1 k_2 k_3 (1+k'_n)} \left(\frac{pf_s t}{f'_c} + \frac{p'f'_s c}{f'_c} \right) + \frac{m}{k_2} \right) \quad \dots (2.4.32)$$

But:

$$\phi_u = \frac{e_u}{a}$$

Let:

$$B = - \frac{1}{k_1 k_3 (1+k'_n)}$$

Therefore:

$$\phi_u = \frac{k_2 e_u}{B \left(\frac{pf_s t}{f'_c} + \frac{p'f'_s c}{f'_c} \right) - m} \quad \dots (2.4.33)$$

Case IV:

$$\begin{aligned} e_s &< e_y & f_s &= E_s e_s \\ e'_s &\geq e_y & f'_s &= f_y \end{aligned}$$

Therefore:

$$\phi_u = \frac{k_2 e_u}{B \left(\frac{pf_s t}{f'_c} + q'c \right) - m} \quad \dots (2.4.34)$$

Case V:

$$\begin{aligned} e_s &> e_y & f_s &= f_y \\ e'_s &> e_y & f'_s &= f_y \end{aligned}$$

Therefore:

$$\phi_u = \frac{k_2 e_u}{B(qt + q'c) - m} \quad \dots (2.4.35)$$

The mathematical expressions which have been presented above enable rapid calculation of the strength and deformational capacity of plastic hinges in reinforced concrete beam-columns subjected to short duration loads. The expressions have not been extended to include the strain-hardening effects of steel reinforcement. In general the expressions show that the ultimate curvature of a section will depend on the following:

$$\phi_u = R \text{ (ultimate concrete stress, ultimate concrete strain, stress block parameters at failure, yield stress of reinforcement, percentage of reinforcement, bending moment, axial load, section dimensions concrete cover to reinforcement).}$$

Curvature and ductility requirements may therefore be satisfied by judicious manipulations of these variables. The effects of some of these variables have already been studied analytically by Pfrang, Seiss and Sozen (14). Recent research on concrete ductility, Mattock (6), Corley (9), Roy and Sozen (10) have recorded that the presence of shear stress and the stressing effects of helical and rectangular ties have the effect of increasing the ultimate strain and hence the ultimate curvature. The stress block parameters at failure have been examined experimentally by Hognestad, Hanson and McHenry (18), and they have been found to be dependent on the ultimate concrete cylinder strength, f'_c (psi), as follows:

$$k_1 k_3 = \frac{3900 + 0.35 f'_c}{3200 + f'_c} \quad \dots (2.4.36)$$

and

$$\frac{k_2}{k_1 k_3} = \frac{1600 + 0.46 f'_c - 0.125 f'^2_c (10^{-4})}{3900 + 0.35 f'_c} \dots (2.4.37)$$

Many researchers have documented work on a so-called ultimate concrete strain, the most popular value used in research is that of $e_u = .0038$ as suggested by Hognestad (13).

The most important factor in the determination of the ultimate curvature, however, is the level of axial load and eccentricity. For loads below P_b ductile behaviour is assured, but in practical structures, it may prove uneconomical to ensure that a ductile condition can exist at the hinge. This limitation may also prove to be a drawback to proposed limit design techniques where a built-in ductility is required to satisfy a preassumed redistribution of stresses. In this case, the hinge has to be designed to satisfy two failure possibilities, one of strength and the other for rotation. This problem is simplified at the purely flexural hinge where ductility is enhanced by a simple limitation of steel reinforcement.

To date it would appear that there has been little interest in a yield interaction diagram. This diagram would describe the combinations of load and moment on a section which result in first yielding of the tension reinforcement. The effects of shrinkage and creep in concrete would affect the accurate determination of such a diagram. These factors render it difficult to determine a suitable concrete stress distribution at the yielding of beam-column hinges. A similar situation for purely flexural hinges is overcome by the use of a semielastic design method.

As a result of the above, there have not been many recommendations on the design strength of beam-column hinges. Sawyer (2) has proposed tentatively that the design moment, M_e , should be given by the relation $M_e = 0.85 M_u$. The value of M_e is not the elastic limit moment but the moment value at the intersection of the two lines of a bilinear approximation of the moment-curvature relationship. The value of M_u is the experimental value of the ultimate moment and not the somewhat lower theoretical value given by the ultimate strength theory.

Berwanger (4) has proposed a design moment equal to the yield moment of the section under zero axial load. This value would seem to be conservative (Fig. 6) and it may well offset the reduction in hinge strength due to time dependent effects and torsion. Kahn and Mattock (19) have reported that the presence of a plastic hinge has no effect on the ultimate shear strength of a reinforced concrete section.

The behaviour of beam sections which are subjected to combined torsion and bending moments depend on the relative amounts of web reinforcement, longitudinal reinforcement, and the arrangement of the longitudinal reinforcement. Cowan (20) has reported that for low steel stresses an unsymmetrically reinforced section with adequate transverse reinforcement will not be reduced in flexural strength in the presence of torsion.

The current ACI Building Code Requirements for Reinforced Concrete (ACI 318-63), limits the design moment and axial load for tied columns to 0.7 of their ultimate strength.

The above discussions are to be applied to ductile hinges only. These are hinges whose axial load is less than the balanced load of the section. While brittle hinges are to be avoided it might prove uneconomical in any method of analysis or design to ensure against this condition. However the present trend towards greater use of the moment-rotation relationship instead of the more fundamental moment-curvature relation may well show that brittle hinges can, with a safe assumption of strength provide the necessary rotation capacity to aid the stress redistribution process that may be required structurally.

CHAPTER 3

DESCRIPTION OF EXPERIMENTAL WORK

3.1 TEST PROGRAM

A total of fifty-five specimens were tested and these were subdivided into eleven series, (S1-S11), each having five identical specimens, (DRG. No. A-1216-30-S1 to DRG. No. A-1216-30-S11, Appendix F). The difference between the series was the ratio of the longitudinal steel reinforcement used in the specimens (Table 1). Series S1-S7 were reinforced unsymmetrically to simulate beams or unequally reinforced columns. Series S5 and S6 were similar in all respects except for the volume of transverse reinforcement that was used in the section. Series S8-S11 were symmetrically reinforced to typify the normal column section. The percentage of longitudinal steel reinforcement used in each member (Table 2) was chosen to lie within the range of maximum and minimum values specified for columns by the Building Code Requirements for Reinforced Concrete (ACI 318-63).

Each member of a series was axially loaded and brought to failure by the application of two-point loading system (Fig. 10a). The level of axial load chosen for each member was of a value which would result in tensile failure in some members and compression failure in others. The two-point transverse loading was chosen so as to eliminate shearing effects in the central portion of the member and thus provide a region of the member which was subjected to pure bending moment. Concrete strains in this region were measured using a layout of displacement transducers. The displacement transducers were linear variable differential transformers with a built-in 6 volt DC excited oscillator. The trans-

formers consisted of a coil assembly and core which when displaced linearly along the axis and within the bore of the coil assembly produced a voltage change in the output which is proportional to the displacement.

Deflections were measured at seven locations (Fig. 10a) throughout the test span using deflection dial gauges.

3.2 DESIGN OF TEST SPECIMEN

The typical dimensions of a test specimen are shown in Fig. 10. All specimens were rectangular and having a 6" x 8" cross section. The overall length of each was 6' - 0", the effective test span being 5' - 6". Although these dimensions were based on the loading capacity of the test equipment and the material strengths expected, they were also chosen to prevent buckling of the specimens under the test loads. These dimensions correspond to a $\frac{\text{span}}{\text{depth}}$ ratio of 9 and a $\frac{\text{span}}{\text{breadth}}$ ratio of 12. The load points were located at 21" from either end, this allowed a 30" long region of constant moment. Strains were measured over a centrally located 10" gauge length. This length was chosen to be slightly greater than the depth of the members and short enough to avoid the local effects of load distribution through the depth of the member.

A typical member of a series was designed to fail under load combinations which could be attained by the test equipment to be used. This was achieved by assuming approximate values of steel reinforcement yield stress, $f_y = 40,000$ psi, concrete compressive stress and strain, $f'_c = 3,000$ psi and $e_u = .003$ in./in. These values were used in a computer determination of axial load-moment interaction diagram for each series. This diagram gave a fair indication of the axial and transverse loads that would be required during the tests. The capacity of the horizontal and transverse hydraulic cylinders (Fig. 11) used in the test was 100 kips,

but it was not the intention of the test program to produce failures under axial loads only. It was decided to keep the axial load capacity of the sections at values which were not more than 300 percent of the maximum loading capacity of the test equipment. The amount of steel reinforcement used in the members is shown in Table 1 at Appendix C. Complete section details for each series are given in Table 2 in Appendix C.

3.3 FABRICATION AND MATERIALS

Due to the volume of concrete required and the need for uniform concrete in all beams, it was not possible to use the laboratory concrete mixing facilities at the University of Ottawa. The manufacture of the specimens were therefore done commercially.

The specimens were cast by a long line process, the total length of each line being approximately 300 feet. They were separated by square bulkheads which were built into the steel moulds. This produced the desired square ends to the specimens to enable a proper contact surface with the axial load device. (See Fig. 13).

The cement used was normal Portland cement and the concrete mix ingredients per cubic yard was as follows:

$\frac{3}{4}$ " Maximum size aggregate	1865 lbs.
Sand	1390 lbs.
Water	190 lbs.
Cement	450 lbs.
"Protex" dispersing agent	27 ozs.
"Protex" air entraining agent to yield	6%±1%
Moisture content of coarse aggregate	0%
Moisture content of sand	2%

Two batches of concrete mix were used in the manufacture of the specimens. The second batch was used in the manufacture of series S6, S8,

S9 and S11. The remaining series were cast from the first batch. The workability and air entrainment property of the batches are given in Table 3 in Appendix C.

The fine and coarse aggregates were subjected to the normal aggregate sieve analysis and the results are presented in Table 4 in Appendix C.

Eight different sizes, No. 3 - No. 9, of deformed steel bars were used as longitudinal and transverse reinforcement. The bars were of intermediate grade billet steel and tension tests were carried out on duplicate coupons of each bar. Strains over a 6" gauge length were recorded by the automatic recorder of the testing machine and were also measured using an extensometer. The results of these tests are presented in Table 5 in Appendix C.

Prior to the start of casting, all the steel moulds were inspected for the accurate layout of the cover to the steel reinforcement. All placed concrete was adequately vibrated and the top face of the specimens were trowelled to an even finish.

For each series of five concrete specimens, a total number of three 6" x 12" test cylinders were taken. These were taken at the beginning, middle and end of the concrete for each series. A similar sequence was applied to the 3" x 3" x 12" concrete prisms which were to be used in the determination of the resonance modulus of elasticity of the concrete.

All specimens, concrete cylinders and prisms inclusive, were allowed to stand for eight hours before steam curing was applied for twenty-four hours. They were then removed from the moulds, wire brushed and stored in curing pits for two days and then were finally transported to the laboratory for testing.

3.4 TEST APPARATUS

The basic description of both the axial and transverse loading used in this test is one which applies a load through a hydraulic jack and a subsequent weighing of the applied load using load cells.

Prior to this experimental program, the loading frame that was used had been designed for transverse load application only. The principal features of the frame were the reaction beams to which was bolted a 100 kip capacity hydraulic ram, a transverse loading beam and loading bed manufactured from I-beam sections. The hydraulic ram was operated by a hand-operated pump which was detachable from the ram. Transverse load was therefore transmitted by the loading beam to the test member by two knife edges (Fig. 11, Appendix 3) which were symmetrically arranged on the beam at 30 inches apart. The applied loads were weighed using two 50 kip capacity load cells as supports. The load cells were connected to a Gilmore Model 471-210 Load Indicator. This indicator had an indicator range of ± 100 kips and featured an accuracy of .02% of reading.

The loading frame was modified to accept the axial loading devices by extending the existing bed at either ends to accomodate two symmetrically placed abutments (Fig. 11). A 100 kip capacity hydraulic ram was bolted to one abutment and a load cell of similar capacity to the other abutment.

The hydraulic ram was connected to a hydraulic pump through a servovalve. The pump was controlled by a power and failsafe panel, and all these units, the load cell included, were connected through a Gilmore Model 431 Servo Amplifier. The function of the Servo Amplifier was to detect load losses from the load cell due to shortening of the specimen and subsequently to command the servovalve to effect a restoration of the preset load to the ram. The axial load once preset on the amplifier,

can be maintained at that level throughout the test. The load cell was also connected to the load indicator previously described. A schematic description of the load assembly is given at Fig. 12, Appendix B and Plates 3, 4, and 5 in Appendix E.

Concrete strains over a centrally located 10" gauge length were measured using ten direct current differential transducers. The transducers were activated by a six volt direct current power supply. The voltage output from the transducers were monitored through a switching unit to a voltage-to-frequency counter. A sketch of this electronic circuitry is shown at Fig. 12, Appendix B.

Deflections of the specimens were measured at seven locations in the test span (Fig. 10a, Appendix B). Deflection dial gauges were used for these measurements. A deflection dial gauge was also used to measure the lateral deflection of the central portion of the specimens.

3.5 TEST PROCEDURE

The test program entailed that each of the five specimens in a series would be tested at different levels of axial load. The first specimen, however, was tested without axial load. The transducers were attached to the specimens using specially designed mountings. These mountings consisted of two rectangular ($1" \times 1" \times \frac{1}{2}"$) aluminum blocks. The transducers and transducer cores were fitted into holes which were drilled into these mountings as shown at Plate 3, Appendix E. The mountings were cemented to the specimen using an epoxy adhesive and was allowed to stand for at least four hours before testing. The specimens were thereafter placed in their test position and carefully centered. For specimens tested under axial load it was necessary to use the specially designed endbearing devices (Fig. 13) to transmit load to the ends of the specimens.

The value of axial load was pre-set on the Servo Amplifier as described in section 3.4 and then gradually applied to the specimen. The transducers and deflection dial gauges were then set to their initial readings and the transverse load was applied slowly using the manual pump of the vertical hydraulic cylinder. For each increment of load, records were made of the outputs of the transducers, load cells and deflection dial gauges. The specimens were loaded to failure and the mode of failure was recorded for each specimen.

The concrete cylinders (6" x 12") were tested in a compression test machine (Plate 1), and the strains were measured and recorded for loads up to approximately half of the ultimate strength. The cylinders were brought to failure and the ultimate compressive loads were recorded. Deformation tests were carried out on two of the three cylinders tested for each series of specimen.

The concrete prisms, (3" x 3" x 12") were tested for the resonance modulus of elasticity using the SCT5 Electrodynamic Materials Tester (Plate 2). A total of four prisms were tested for each series of specimens. Both concrete cylinders and prisms were tested on the same day on which their respective beam specimens were tested.

CHAPTER 4

RESULTS AND CALCULATIONS

4.1 CONCRETE TEST PROPERTIES

The ultimate concrete stress, f'_c , the initial tangent modulus, E_c , and the resonance moduli of elasticity, E_d , are presented in Table 6 in Appendix C.

The ultimate compression load for each cylinder was divided by the cross sectional area and the average value of the ultimate compressive stress for the three cylinders was calculated for each series of specimens.

The initial tangent moduli of concrete, E_c , were the average values as obtained from two of the three cylinders mentioned above. The experimental results were fitted to a straight line using a curve fitting process and the slope or tangent modulus determined. The average values were compared with those calculated using Inge Lyse's empirical equation for the initial tangent modulus of elasticity.

$$E_c = 1,800,000 + 460 f'_c \quad \text{..... (4.1.1)}$$

This equation is reported (7) to give values which are ± 10 percent of the actual values. These values of moduli of elasticity are presented in Table 6 at Appendix C.

The resonance modulus for each series of specimens was determined from the test on four 3" x 3" x 12" concrete prisms. The values of resonance modulus were calculated using the formula (21).

$$E_d = KW (n')^2 \quad \text{..... (4.12)}$$

$$\text{where } K = \frac{0.01035 \times \text{length of specimen}}{\text{cross sectional area of specimen}} \quad \text{..... (4.13)}$$

The value of W and n' are respectively, the weight of the prisms and the resonance frequency as obtained from the electrodynamic tester. The

average value of four tests is reported for each series of specimens in Table 6 in Appendix C.

4.2 TENSION TESTS OF STEEL REINFORCEMENT

The results of the twelve tests carried out on the steel reinforcing bars are reported in Table 5 in Appendix C. The object was the determination of the modulus of elasticity, the yield stress, the ultimate stress and the overall tensile behaviour of the specimens. The latter is presented graphically in Fig. 15 and 16 in Appendix B for the No. 6 and No. 7 steel bars. The yield stress and strain was obtained from these curves and was calculated as the coordinates at the intersection of the elastic and strain hardening segments of the curve. The ultimate stress was obtained by dividing the ultimate load by the cross sectional area of the specimen.

The moduli of the elasticity reported in Table 5 in Appendix C are the average values from the tests on two coupons and these were obtained by applying a curve-fitting computer program to the pre-yield stress/strain results.

4.3 TEST RESULTS OF BEAM-COLUMNS

The results of loads, strains and deflections from the various tests were processed using computer programs as described below. A flow chart for each program is given in Appendix D.

The observed cracking patterns at failure for specimen series S6 and S10 are shown in Plates 6 and 7 in Appendix E.

Program I

This program was designed to check the data for possible mistakes in the recording of results and errors due to malfunctions of the recording instrument. The output of this program was the incremental increases in values of loads, strains and deflections. Details

of the significant errors are given and discussed in section 5.6.

Program II

The procedure of the test program did not permit the measurement of the strains produced by the axial load and such strains had to be obtained by calculation. Using the data presented in Table 6 in Appendix C, the Hognestad stress/strain relation for concrete and the properties of the member section this computer program was used to evaluate the strain distribution due to the prestressing axial load.

The second paragraph of the program was devoted to the determination of the load/deflection profile of the specimens for each increment of load. The profiles were evaluated from the seven dial gauge readings taken across the span. These readings included the central deflection of the test specimen.

The third paragraph was used to evaluate the strain at which cracking of the member was first visible. Further, the program evaluated from the transducer data set, the strain at the elevation of the various transducers (Fig. 10b). This was done by evaluating the strain measured by each transducer and presenting the strain at a particular elevation as the average value of complementary pairs of transducer readings. The average values of strains were then fitted to a straight line by a curve fitting process. All the transducer readings were used in the determination of the strain distribution for strains recorded before perceptible cracking, however, for other cases, only readings of transducers T1, T2, T3, T7, T8, T9 and T10 (Fig. 10b) were used in the fitting of the straight line.

The fourth paragraph of the program was devoted to the algebraic addition of the strains due to the prestressing axial load and the fitted straight line strain distribution mentioned above. The slope of the resultant strain profile and the depth of the neutral axis were then evaluated.

The fifth paragraph was employed in the evaluation of the total moment due to the dead weight of the member, deflection, eccentricity and the transverse load.

This process was repeated for each load increment and resulted in the load/moment, moment/curvature and load/deflection relations for the specimens tested. Graphical representations and a discussion of these relations will be presented in later articles of this thesis.

Program III

This program was used in the theoretical evaluation of the axial load/moment interaction diagram and the theoretical curvatures as expressed by Equations 2.4.18 and 2.4.33. The procedure involved the use of the values of concrete cylinder strength, yield stress of steel reinforcement, ultimate concrete strain and the elastic modulus of elasticity of the steel reinforcement. The ultimate concrete strain used was the average failure strain as obtained from the test results of the four axially loaded specimens of each series. An outline of the program is given below and a detailed flow diagram is given in Appendix D.

- (a) Calculate the balanced condition and determine P_b and M_b as well as the depth of neutral axis.
- (b) Calculate the section failure parameters as described in Fig. 8, Appendix B and calculate the ultimate curvature using the relevant theoretical expression. Calculate the ultimate curvature by dividing the ultimate concrete strain by the depth of neutral axis.
- (c) Reduce the depth of neutral axis incrementally from the balanced condition and calculate the resultant axial load and moment.
- (d) Repeat (b).

- (e) Increase the depth of neutral axis incrementally from the balanced condition and calculate the resultant axial load and moment.
- (f) Repeat (b).

The relations resulting from this program are represented graphically and are discussed in sections 5.4, 5.5, 5.6 and 5.7.

CHAPTER 5

DISCUSSION OF RESULTS AND CONCLUSIONS

5.1 RESULTS OF CONCRETE TESTS

The results of the concrete tests are presented in Table 6 in Appendix C.

Although it was desired to have concrete of equal strength in all test specimens, it was observed that there were variations in the values of the ultimate concrete compressive strengths. The lower values of the strengths of series S6, S8, S9 and S11 is due to the fact that these specimens were cast from a second batch of concrete. Variations in the strength of the other series were relatively small and were ascribed to the difference in the age of the specimens at the time of testing. In most cases the ultimate concrete compressive stress obtained from the tests exceeded their intended values.

The initial tangent moduli as determined from tests on the 6" x 12" cylinders were compared with the resonance moduli of elasticity. It was observed that in all instances the resonance modulus was higher than the corresponding initial tangent modulus. This observation is in general agreement with the results of previous research (Whitehurst (21), Sharma and Gupta (22), Stanton (23), Jones (24)) in this field. Whitehurst (21) has reported that the static modulus in compression was equivalent to 89 percent of the resonance modulus.

From Table 6 it was observed that the ratio of static modulus to resonance modulus varied between 0.74 and 0.93.

Sharma and Gupta (22) have established a relationship between static modulus of elasticity and the ratio of static to resonance moduli as follows:

$$\frac{E_c}{E_d} = 0.368 + 0.0871 E_c 10^{-6} \quad \text{..... (5.1.1)}$$

It was found (Fig. 14, Appendix B) that this expression could not find general application with all the results in Table 6. This may be due to the fact that the range of static moduli values over which the two tests were conducted were substantially different.

Jones (24) has reported that the following empirical relation was consistent with many results:

$$E_d = E_c + 1 \quad \text{..... (5.1.2)}$$

This relation could not be applied to most of the results in Table 6.

A comparison of the initial tangent modulus as obtained from the compression test and those obtained from Inge Lyse's equation is given in Table 6. It was observed that the ratio $\frac{E_c}{E_c}$ for all the tests were within the range of 0.82 - 1.01. This variation is in satisfactory agreement with the limitations of Inge Lyse's equation as stated in section 4.1.

5.2 RESULTS OF TENSION TESTS OF STEEL REINFORCEMENT

The results of the twelve tests carried out on the steel reinforcing bars are reported in Table 5 in Appendix C.

It was observed that the values of yield stress were in excess of the intended values and that a constant value of yield stress was not attained by all the steel specimens. Based on the minimum value of yield stress attained there was a maximum variation of approximately 18 percent in the values of yield stress.

The overall tensile behaviour of the No. 6 and No. 7 steel bars is shown in Figs. 16 and 17. In general it was found that the bars showed

sharp transition from the yield state into the strain hardening range.

5.3 INTERACTION CHARACTERISTICS

The graphical representation of the interaction characteristics for the eleven series of specimens are presented in Fig. 17 - Fig. 60 in Appendix B. The behaviour of each series is represented by four different curves as follows:

- (a) Axial Load-Moment Diagram
- (b) Moment-Curvature Diagram
- (c) Load-Deflection Diagram
- (d) Eccentricity-Ultimate Curvature Diagram

A summary of the results is also given in Table 7 in Appendix C.

5.4 AXIAL LOAD-MOMENT DIAGRAMS

The theoretical axial load-moment diagrams are presented as the first graph in each series of interaction characteristics in Figs. 17, 21, 25, 29, 33, 37, 41, 45, 49, 53 and 57. The curves were obtained from the computer output of Program III. The input data were the average values of concrete cylinder strengths and the average ultimate strains as obtained from the four axially loaded specimens of each series. The five experimental values of ultimate load and moment for each series are also presented on these graphs. These values showed satisfactory agreement with the theoretical values. In cases where there were variations between the theoretical curve and the experimental points it has been ascribed to the fact that the average values of concrete strengths were used as the input data for the theoretical curves. It is observed that experimental points on the graph which represented tensile failures showed greater deviations from the theoretical curve than points which represented compression failures. This has been ascribed to the assumptions at section 1.6 in which strain-hardening of the steel reinforcement was

neglected. The increase in stress in the tensile reinforcement due to strain hardening would increase the moment capacity of the section. The accuracy of the experimental values were also dependent on the accurate interpretation of the failure load. In many cases the test specimens were able to carry an increasing load after first crushing of concrete at the top of the specimens. Although care was taken in the determination of the load at first crushing, it is believed that this has contributed to the scatter in some of the results.

Each interaction diagram exhibited the usual branches which are characteristic of compression and tensile failures. This behaviour had good correspondnece with the type of failure exhibited by the test specimens.

Although great care was taken to keep the percentage of longitudinal reinforcement as the only variable between the different series, it was found that there were variations between the yield stress of the steel reinforcement and the ultimate concrete compressive stress for each series of specimens. This factor has tended in some cases to invalidate the comparative behaviour of the different series of specimens.

Referring to Figs. 45, 49 and Figs. 54, 58, it is observed that although the magnitude of balance moment, M_b , is increased as the percentage of reinforcement is increased the magnitude of the balance load, P_b , is essentially constant for each pair of symmetrically reinforced sections. From Table 2, Figs. 37 and 45, it is observed that the value of the balance moment is increased as q'_u is increased but that the balance load is decreased as q'_u is increased for unsymmetrically reinforced sections.

A study of all the curves show that the strength capacity of a cross section is increased if q'_u is increased (Table 2, Appendix C) and

that the moment corresponding to balanced failure is markedly increased by increases in the percentage of steel reinforcement.

5.5 MOMENT-CURVATURE DIAGRAMS

The experimental moment-curvature-thrust diagrams are presented at Figs. 18, 22, 26, 30, 34, 38, 42, 46, 50, 54 and 58. The curves are the plotted values of the output data of Program II. Five moment-curvature diagrams, one for each member of a series are presented on each graph. The values of axial load at which the moment-curvature relations were measured are indicated on each graph.

A typical curve, Fig. 18, shows that as the axial load is increased that the slope of the moment-curvature diagram is increased up to and above the balance load. This indicates that the rotational stiffness has increased over this range of loading. For loads which are much higher than the balance load (Fig. 38), the rate of increase of stiffness with axial load is much lower than that of lower loads. For series S8, Figs. 45 and 46, it is observed that the section has a balance load of 67.4 kips and that while there is a rapid change of slope between curves A and B (axial load 0.0 kip and 20.2 kips respectively), there is a lower rate of change of slope between curves B and D, the axial load being 20.2 kips and 60.4 kips respectively. It is noted that at higher axial loads, curve E, axial load 90.2 kips, the rate of change of stiffness becomes negative.

Referring to Table 7 and Fig. 38, it is observed that curves A, B, C and D, (Series S6), indicate very high ductility but that the ductility is decreasing rapidly as the level of axial load increases. It is also observed at Fig. 18, curves D and E that as the axial load (99.8 kips) increases above the balanced load of the section (65.4 kips) the reduction in moment capacity is greater than the reduction in ultimate

curvature and this causes an apparent increase in the ductility of the section. A comparison of curves B of Figs. 26 and 34, (Series S3 and S5) shows that the ultimate curvature of the sections under an axial load of approximately 20 kips are $.0012 \text{ in.}^{-1}$ and $.00086 \text{ in.}^{-1}$ respectively. The difference between these sections was principally the percentage of steel reinforcement (Table 2). It is therefore concluded that ductility of members is decreased by an increase in the amount of longitudinal reinforcement.

It was not possible to study closely the effects of the transverse steel reinforcement on the ultimate curvature as it was observed that the concrete strength of series S5 and S6 were markedly different.

5.6 LOAD-DEFLECTION DIAGRAMS

The load-deflection diagrams are presented at Figs. 19, 23, 27, 31, 35, 39, 43, 47, 51, 55 and 59. The conclusions drawn from these curves were similar to those included at section 5.3.

5.7 ECCENTRICITY-ULTIMATE CURVATURE DIAGRAMS

The theoretical curves of eccentricity/ultimate curvature are presented at Figs. 20, 24, 28, 32, 36, 40, 44, 48, 52, 56 and 60. The curves were obtained in the manner described in section 4.3. The values of eccentricity and ultimate curvature which were obtained from the tests are also presented on these curves. Although only four experimental values were available, the trend was in satisfactory agreement with the theoretical values. The scatter in some of the results has been ascribed to the sensitivity of the theoretical expressions to the value of limiting strain. The average values of ultimate strains as calculated from the test values of the four axially loaded specimens of each series were used in the evaluation of the theoretical expressions.

A study of the theoretical curves shows that the theoretical

expressions satisfy the descriptive behaviour at ultimate, of reinforced concrete beam-column sections.

Referring to Figs. 24 and 28 (Series S2 and S3), it was found that for approximately the same value of ultimate strain (.0033 in./in.) the former section showed higher values of ultimate curvature. Figs. 48, 52, 56 and 60 when compared with similar curves for unsymmetrically reinforced sections showed that the symmetrically reinforced concrete beam-columns possessed better ductile characteristics than unsymmetrically reinforced sections.

5.8 DISCUSSION OF EXPERIMENTAL ERRORS

The horizontal pins shown at Fig. 13 performed satisfactorily but it was noted that on application of the axial load the test specimen tended to move vertically due to rotation at the ball bearings. In some cases corrective measures were successful but in others, attempts were made to measure the new elevation of the test beam.

Although careful attention was paid to the determination of the load at first fracture of the concrete, it was not possible in all cases to achieve this goal. This may have resulted in slightly incorrect values of the failure loads.

In order to avoid damage to the linear variable differential transducers it was necessary to remove them before the initiation of concrete failure. This was necessary for cases in which compression failures were expected and this could introduce slight errors in the values of the recorded ultimate curvatures. The measurement of deflections were continuous throughout and gave a fair indication of the relative deformation of the specimens at failure.

The output from computer Program I showed in some instances that there was malfunctioning of some transducers during the tests. This

may have been due to slipping of the mountings or more likely due to sticking of the transducer core. Where this was observed it was necessary to modify computer Program II to omit the use of these readings and this may have resulted in a loss of accuracy in the final results.

5.9 CONCLUSIONS

From an examination of the results, the following conclusions may be made:

1. The magnitude of the balance point load is unaffected by the amount of longitudinal reinforcement if the section is equally reinforced in tension and compression.
2. The magnitude of the balance load is increased if the factor q_u' is decreased and is essentially constant for $q_u' = 0$.
3. The strength capacity of a given section is increased if q_u' is increasing.
4. The moment corresponding to the balance condition is markedly increased by increases in the longitudinal steel reinforcement.
5. The rotational stiffness of reinforced concrete beam-columns is markedly increased in the presence of axial load.
6. The rate of increase of stiffness with axial load is greater for loads less than P_b ; for higher magnitudes this rate is decreased and becomes negative for loads above certain load levels.
7. There are large amounts of ductility at low values of axial load, and the ductility is decreased as the

axial load increases.

8. For axial loads greater than the balance load, the reduction in moment capacity with increasing load is greater than the reduction in ultimate curvature and this causes an apparent increase in ductility.
9. Ductility of beam-column sections is decreased by increasing the amount of longitudinal reinforcement.
10. The theoretical expressions presented satisfy the descriptive behaviour at ultimate, of reinforced concrete beam-columns.
11. Symmetrically reinforced concrete beam-columns possess better ductile characteristics than unsymmetrically reinforced sections.

5.10 SUGGESTIONS FOR FURTHER RESEARCH

The study carried out in this thesis has verified a number of the important factors which influence the interaction characteristics of hinges formed at beam-column sections. However, a number of derived suggestions for further research seems to be apparent:

1. The theoretical expressions presented in this thesis require further experimental examination. Using the limited test population of this program the expressions seem to overestimate values of ultimate curvature at high values of eccentricity.
2. Studies leading to the development of yield interaction envelopes, would undoubtedly provide valuable information on the acceptable design moment of beam-column sections.
3. Experimental studies on full scale structures leading to a better knowledge of the behaviour of beam-column hinges.

4. The extension of this thesis to account for the strain-hardening effects of the steel reinforcement.

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APPENDIX B: FIGURES

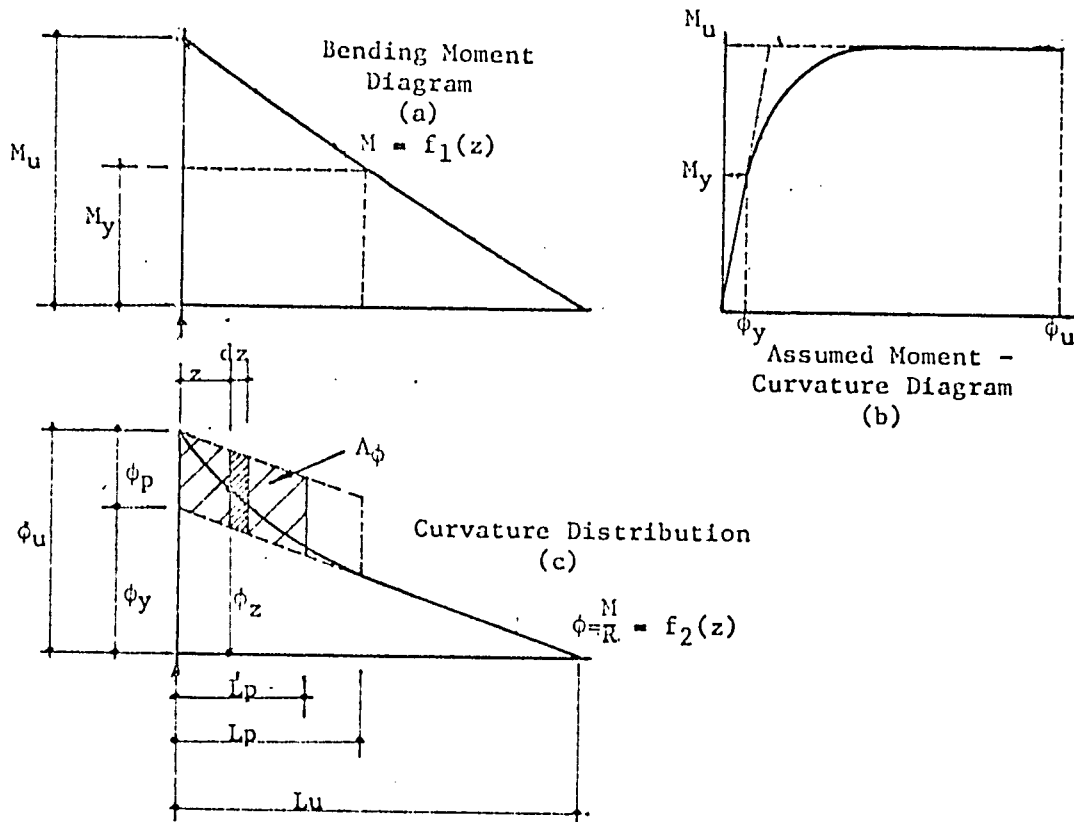


FIG. 1: DISTRIBUTION OF CURVATURE NEAR A SUPPORT

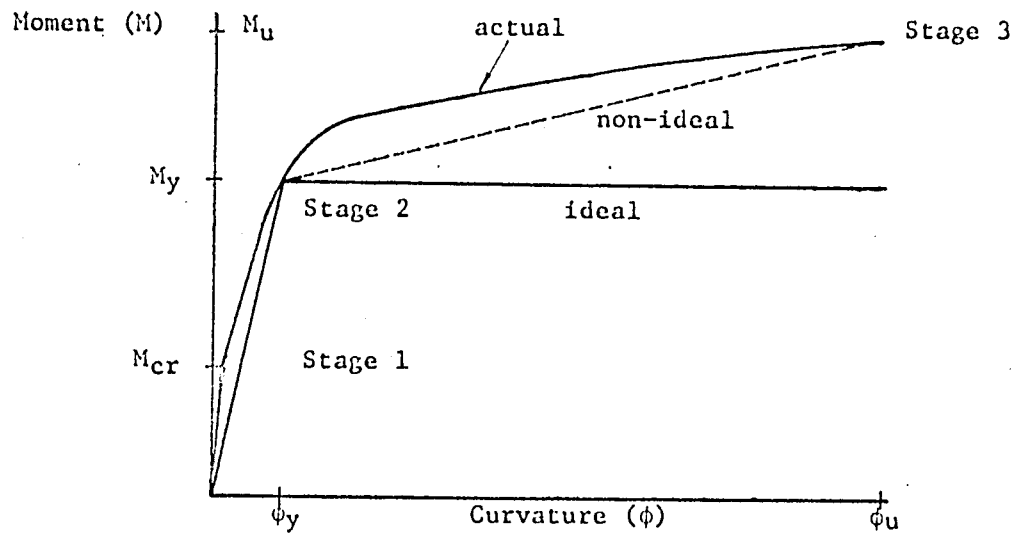


FIG. 2: MOMENT-CURVATURE RELATION

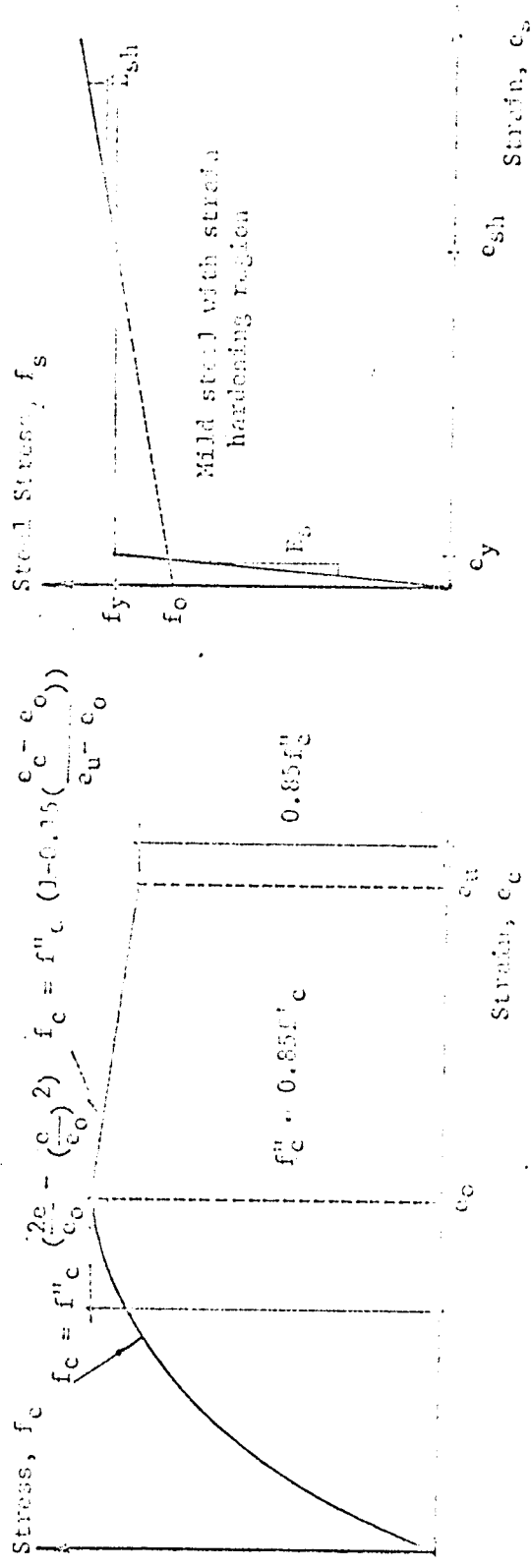


FIG. 3: ASSUMED STRESS STRAIN RELATIONSHIP FOR CONCRETE IN COMPRESSION. (HOGNESIAN)

FIG. 4: TYPICAL STEEL REINFORCEMENT CURVE

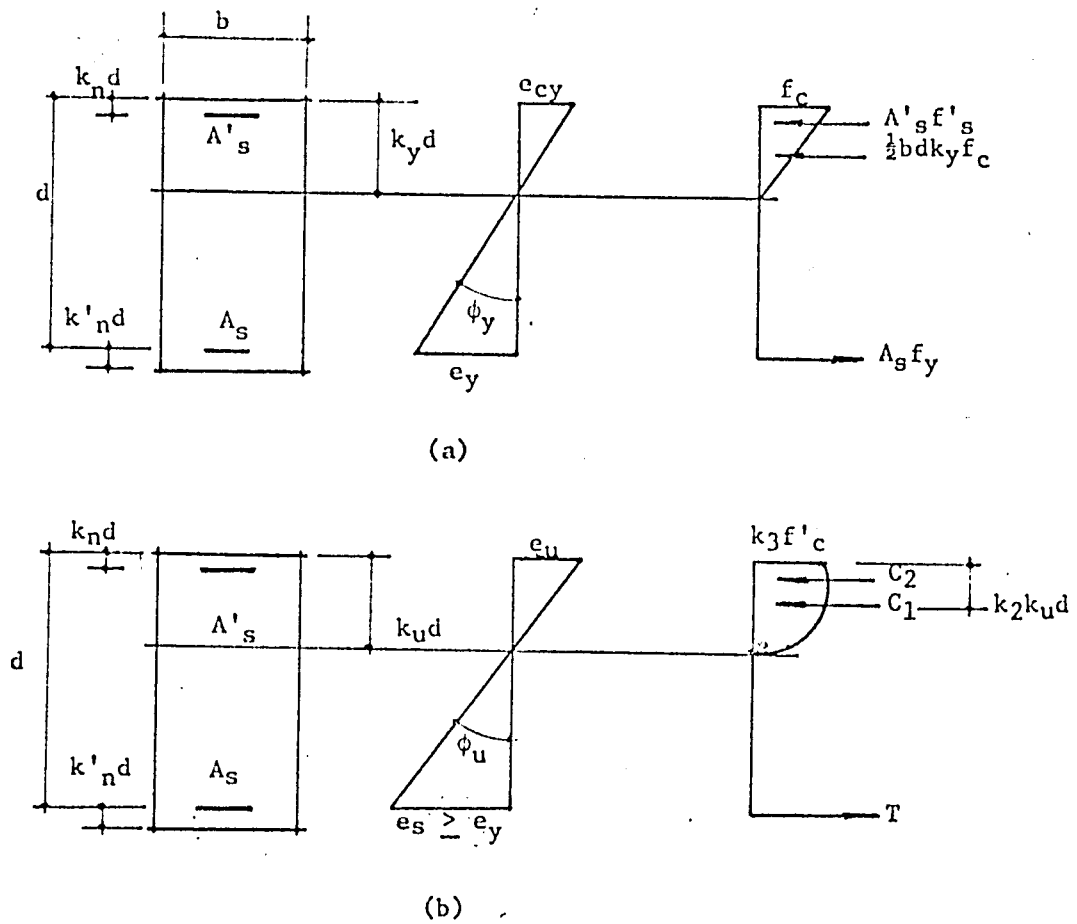


FIG. 5: STRESS AND STRAIN DISTRIBUTION AT YIELD
OF TENSION REINFORCEMENT
AND AT ULTIMATE

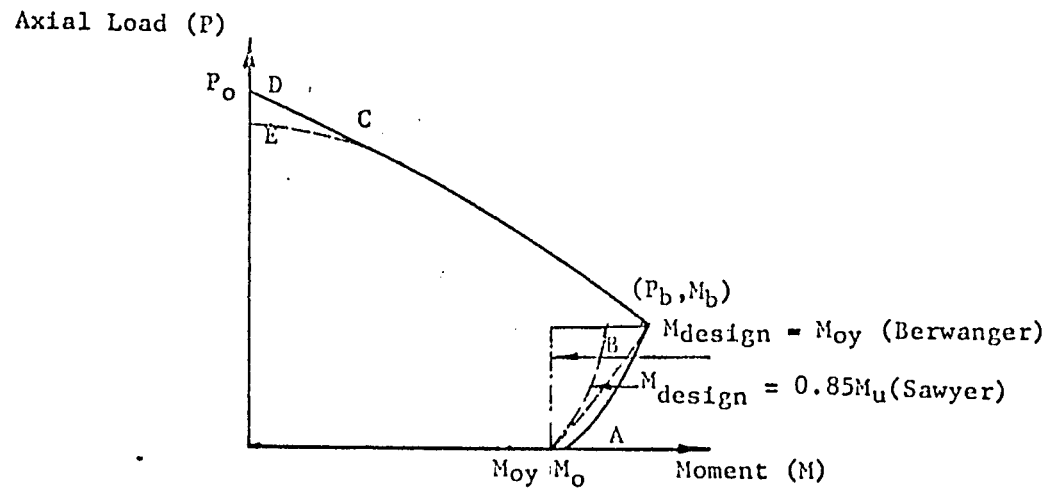


FIG. 6: AXIAL LOAD - MOMENT INTERACTION DIAGRAM

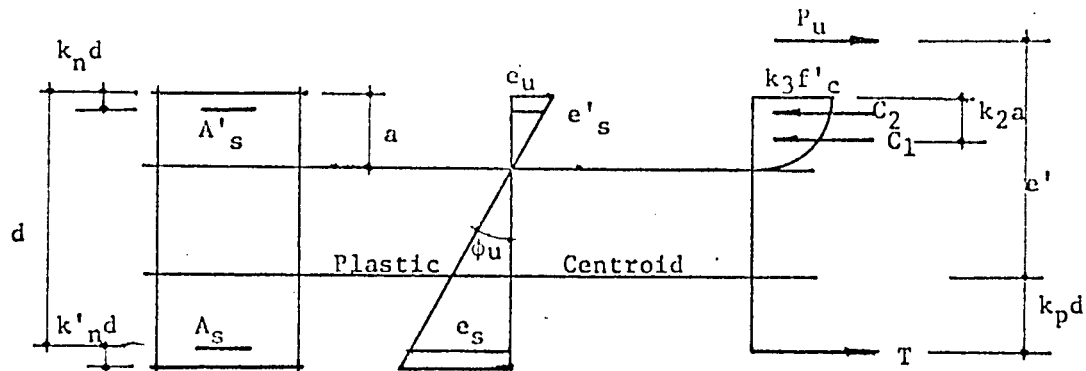


FIG. 7: STRESS AND STRAIN DISTRIBUTION FOR BEAM-COLUMN SECTION (TENSILE AND COMPRESSION HINGES)

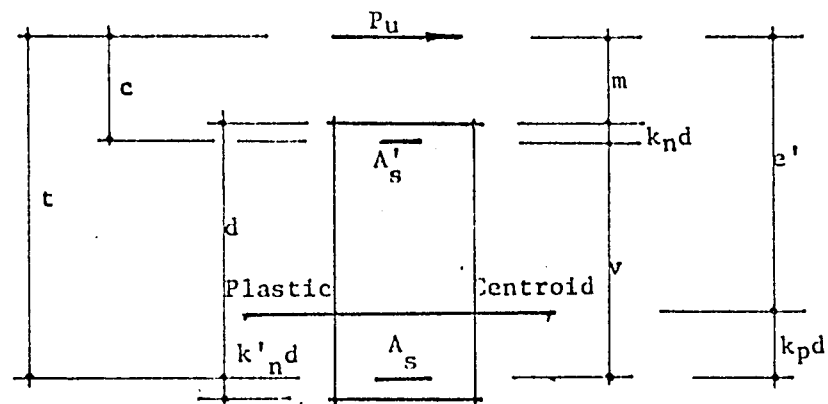


FIG. 8: SECTION FAILURE PARAMETERS FOR BEAM-COLUMN SECTIONS

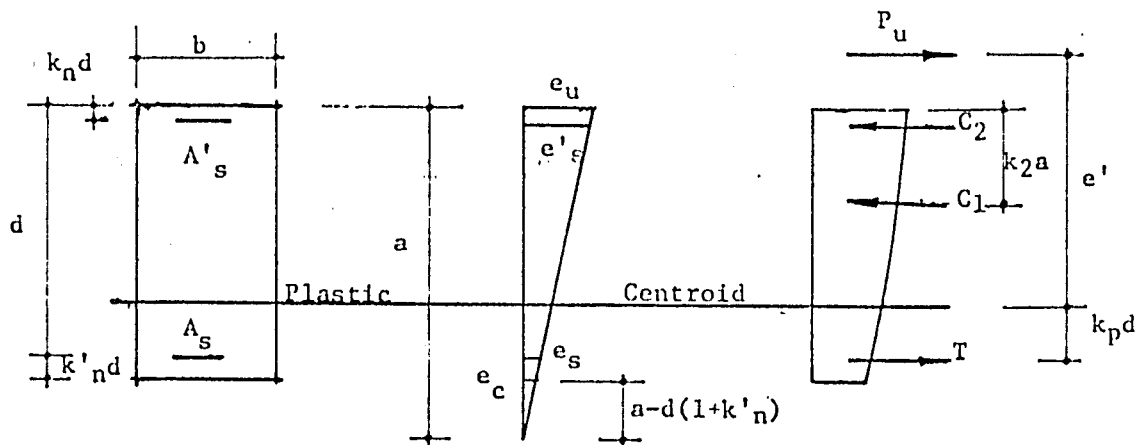
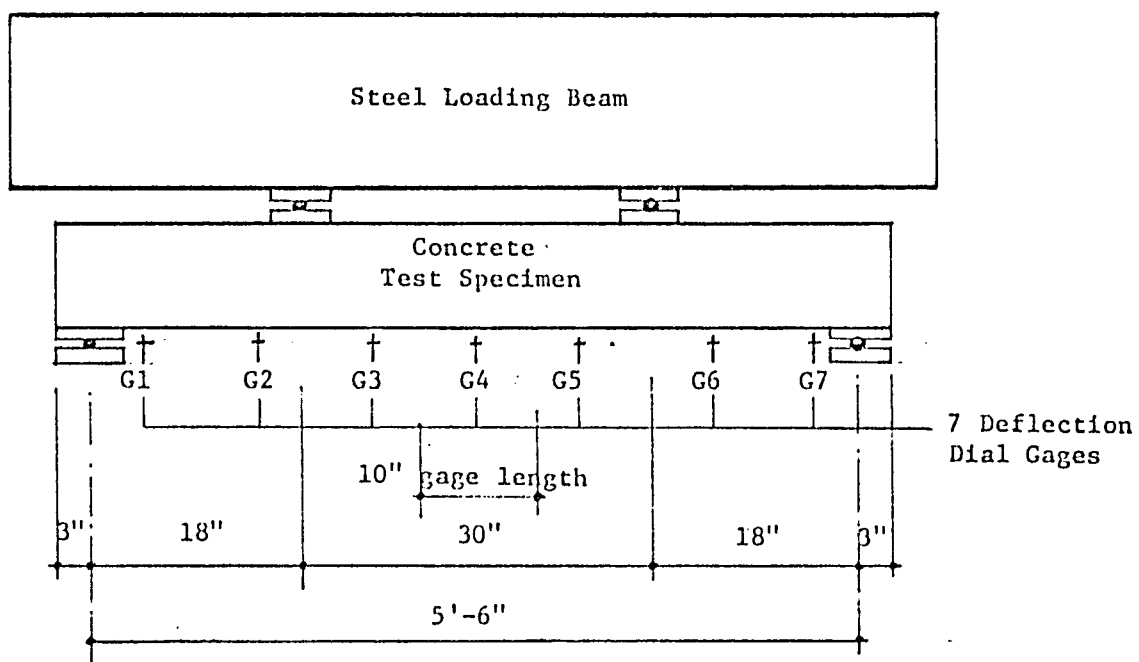
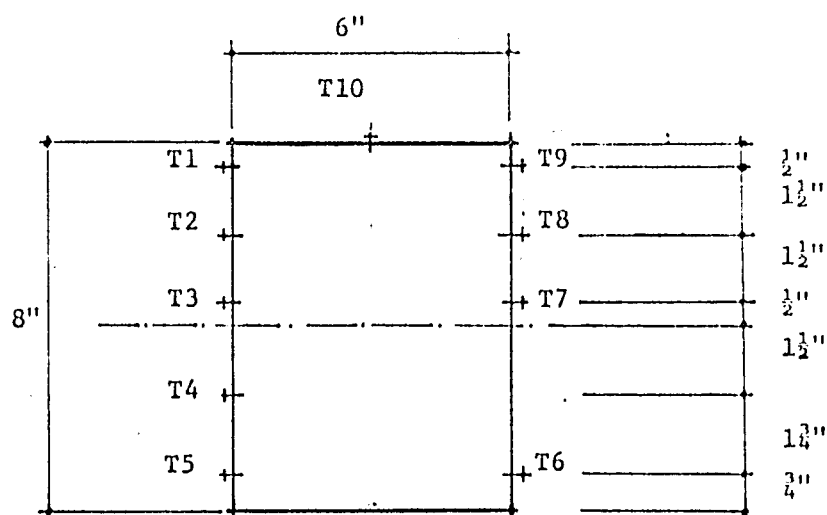


FIG. 9: STRESS AND STRAIN DISTRIBUTION AT ULTIMATE FOR BEAM-COLUMN SECTION (COMPRESSIVE HINGE)



(a)



(b)

FIG. 10: SCHEMATIC DRAWING - TRANSVERSE LOAD -
TEST DIMENSIONS AND TRANSDUCER LAYOUT

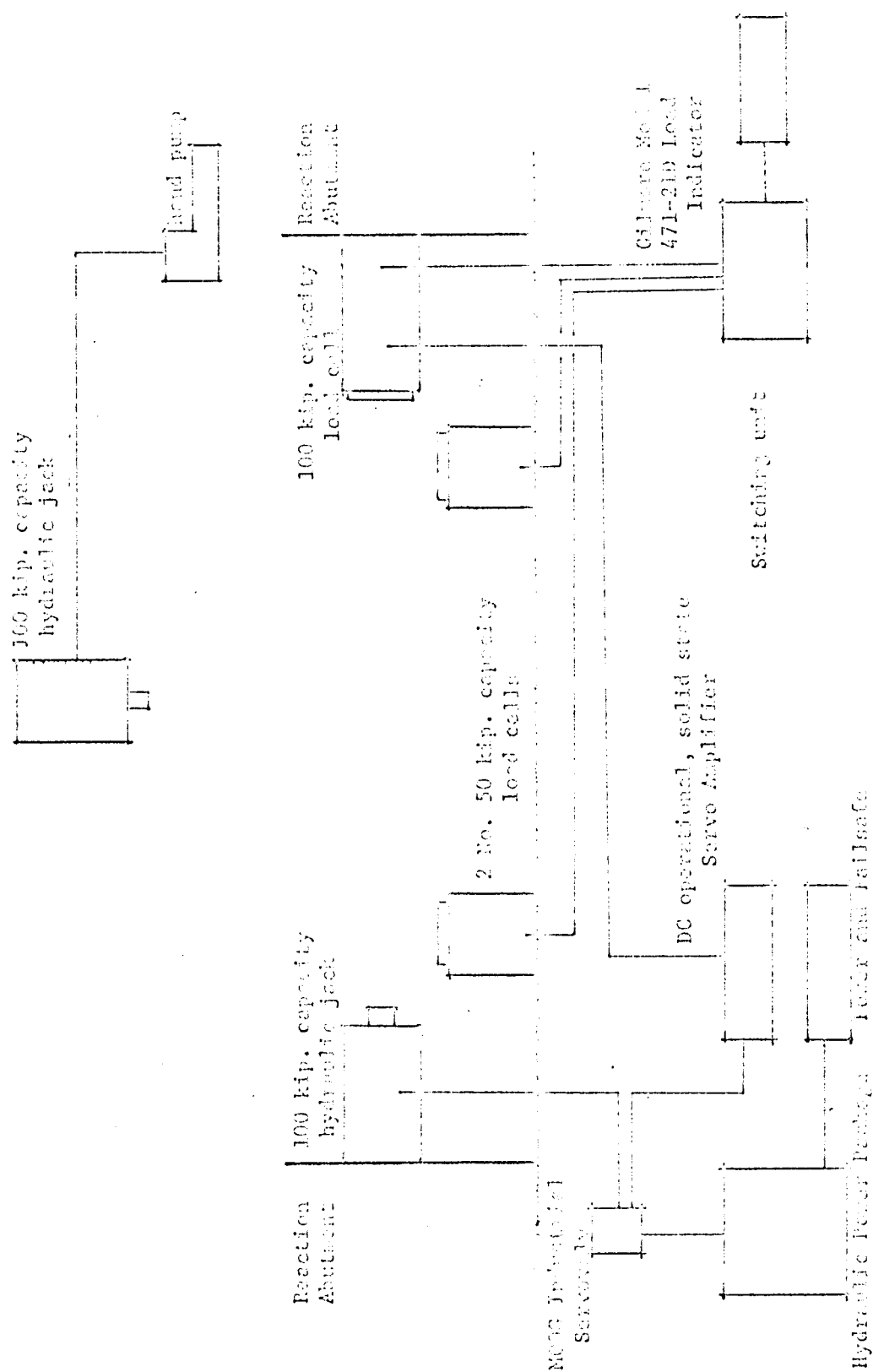


FIG. 11: SCHEMATIC LOADING AND LOAD MEASURING ELECTRONIC CIRCUITRY

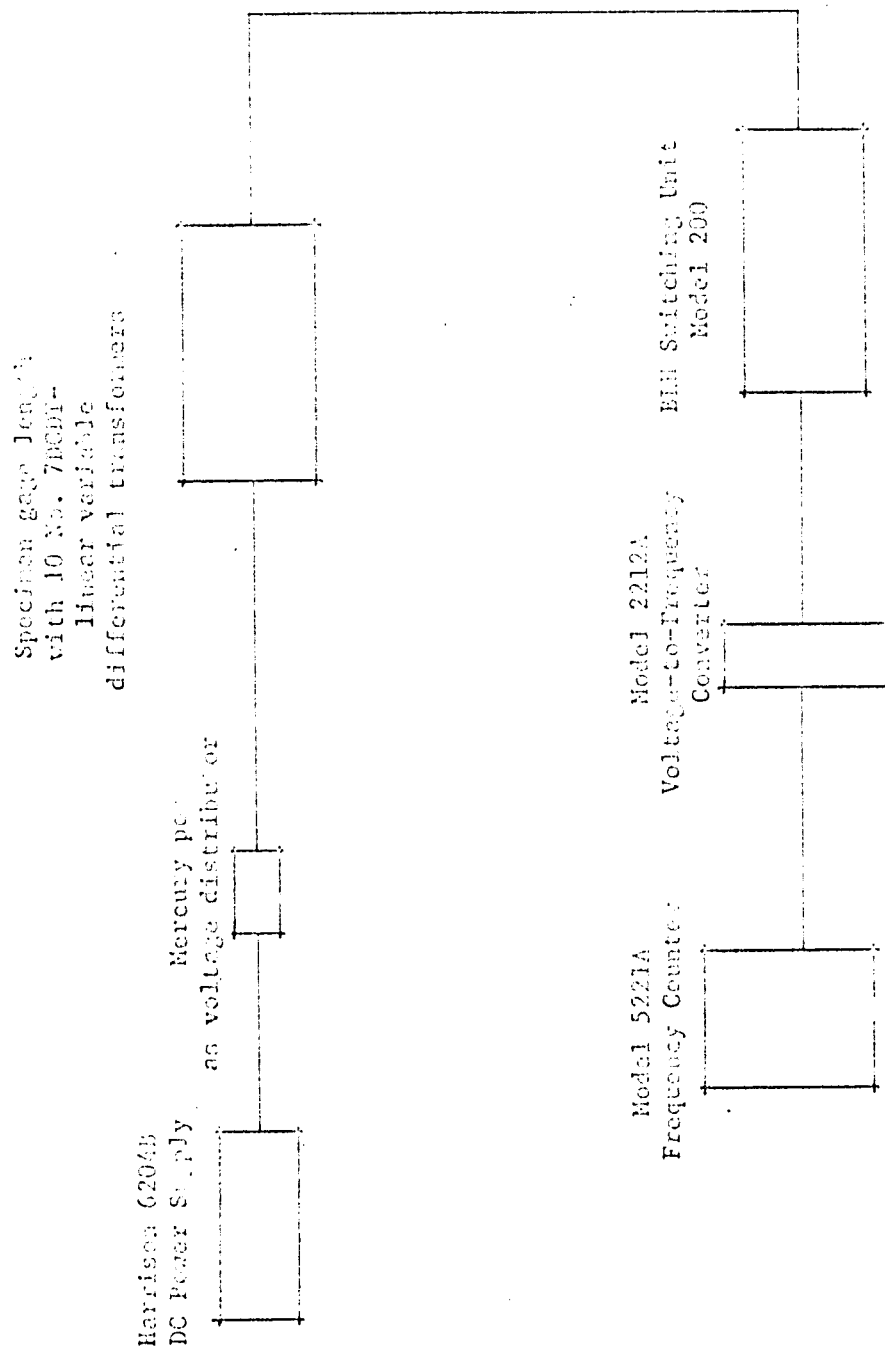


FIG. 12: SCHEMATIC LAYOUT OF STRAIN MEASURING
CIRCUITRY AND DEVICES

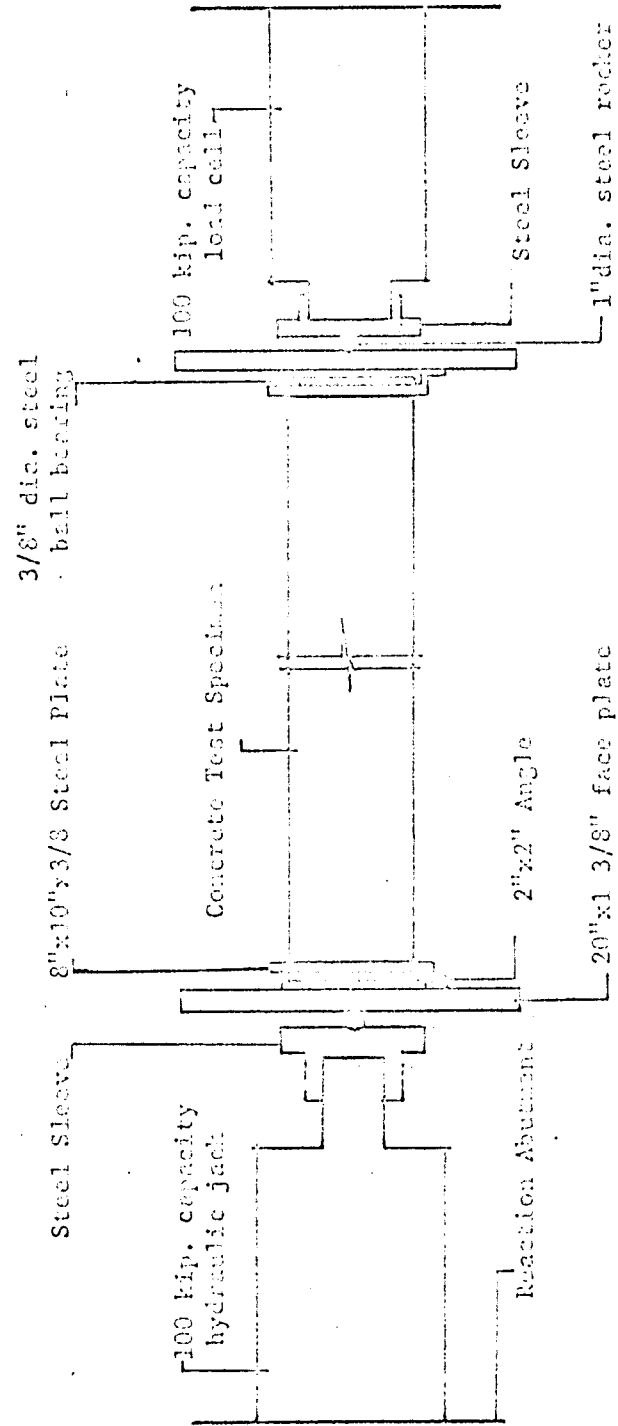


FIG. 13: SCHEMATIC DRAWING - AXIAL LOADING AND STRETCH
OF HORIZONTAL PIPE

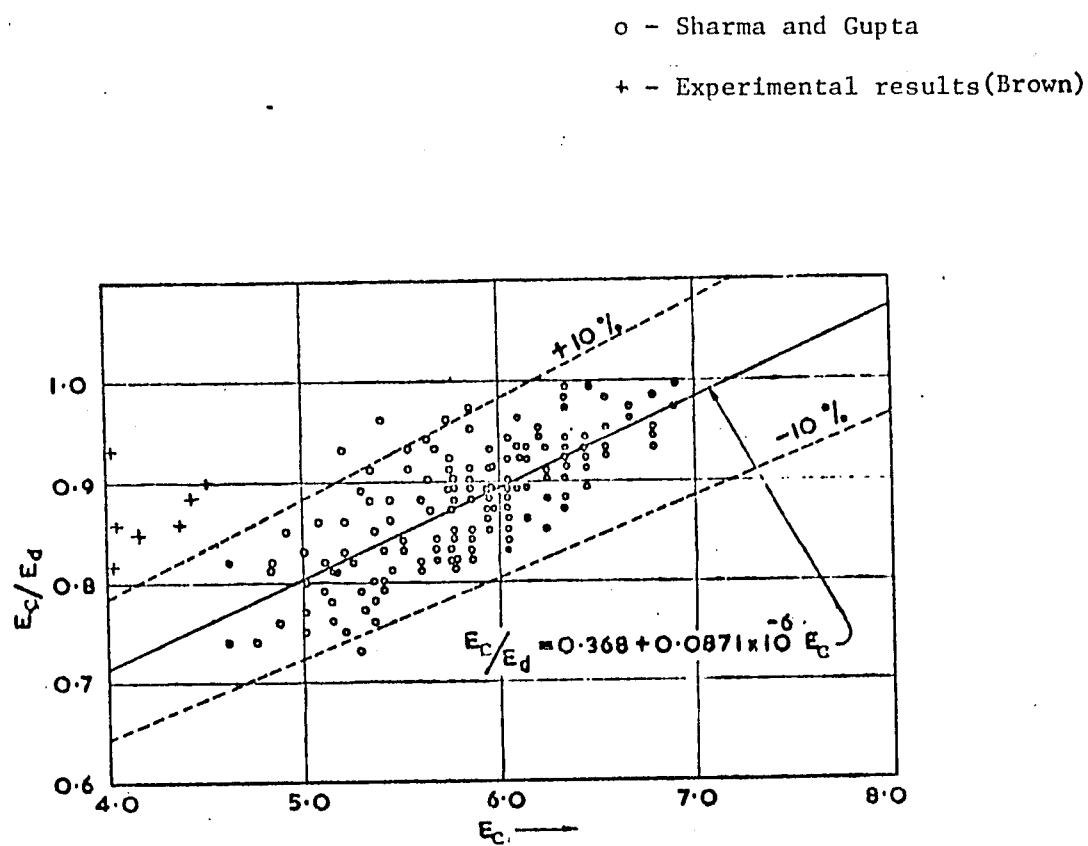


Fig. 14 Relationship between static modulus of elasticity and the ratio of static to resonance moduli. After Sharma and Gupta (22)

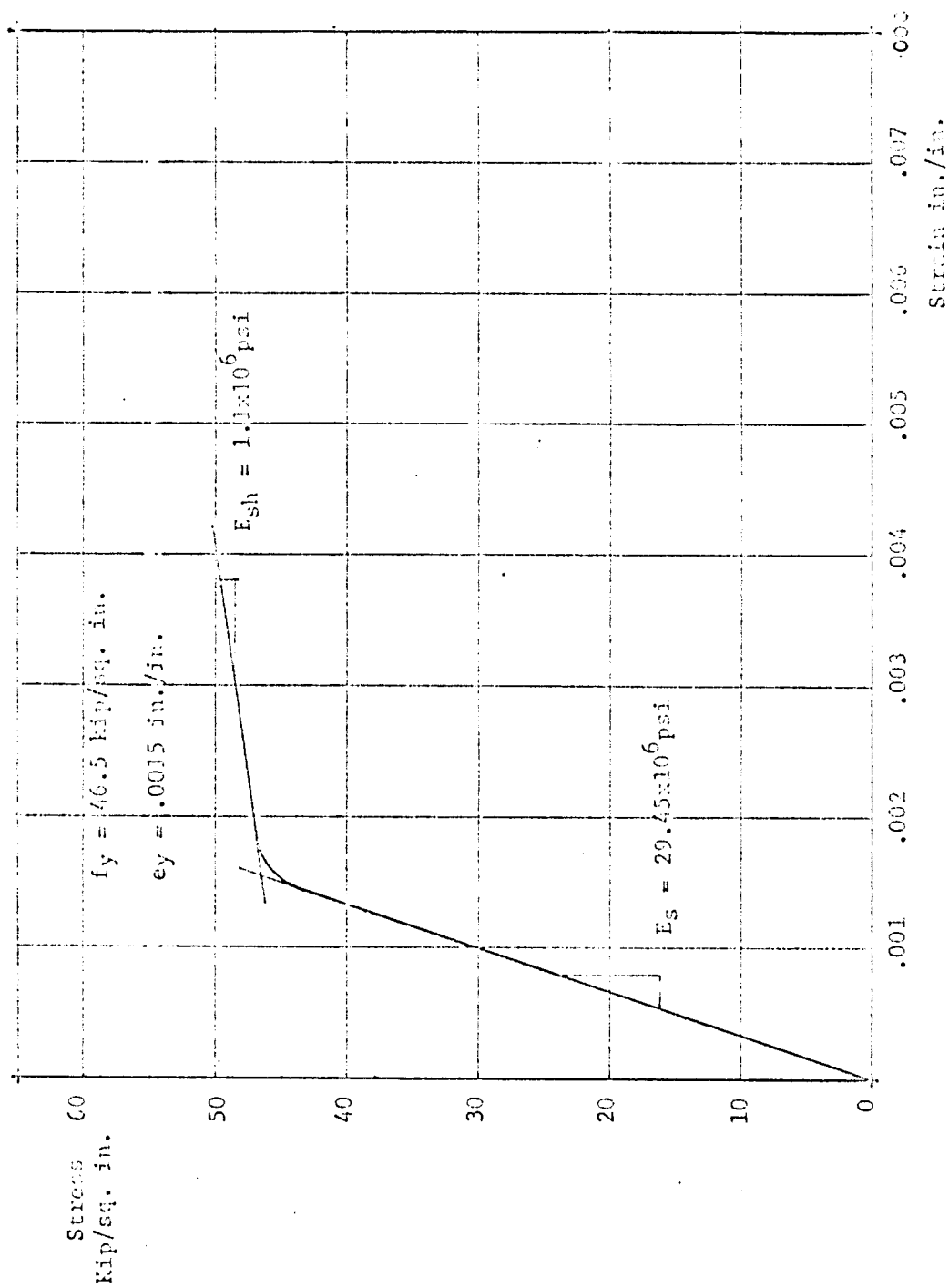


FIG. 15: STRESS-STRAIN RELATION FOR No. 6 MILD STEEL PER

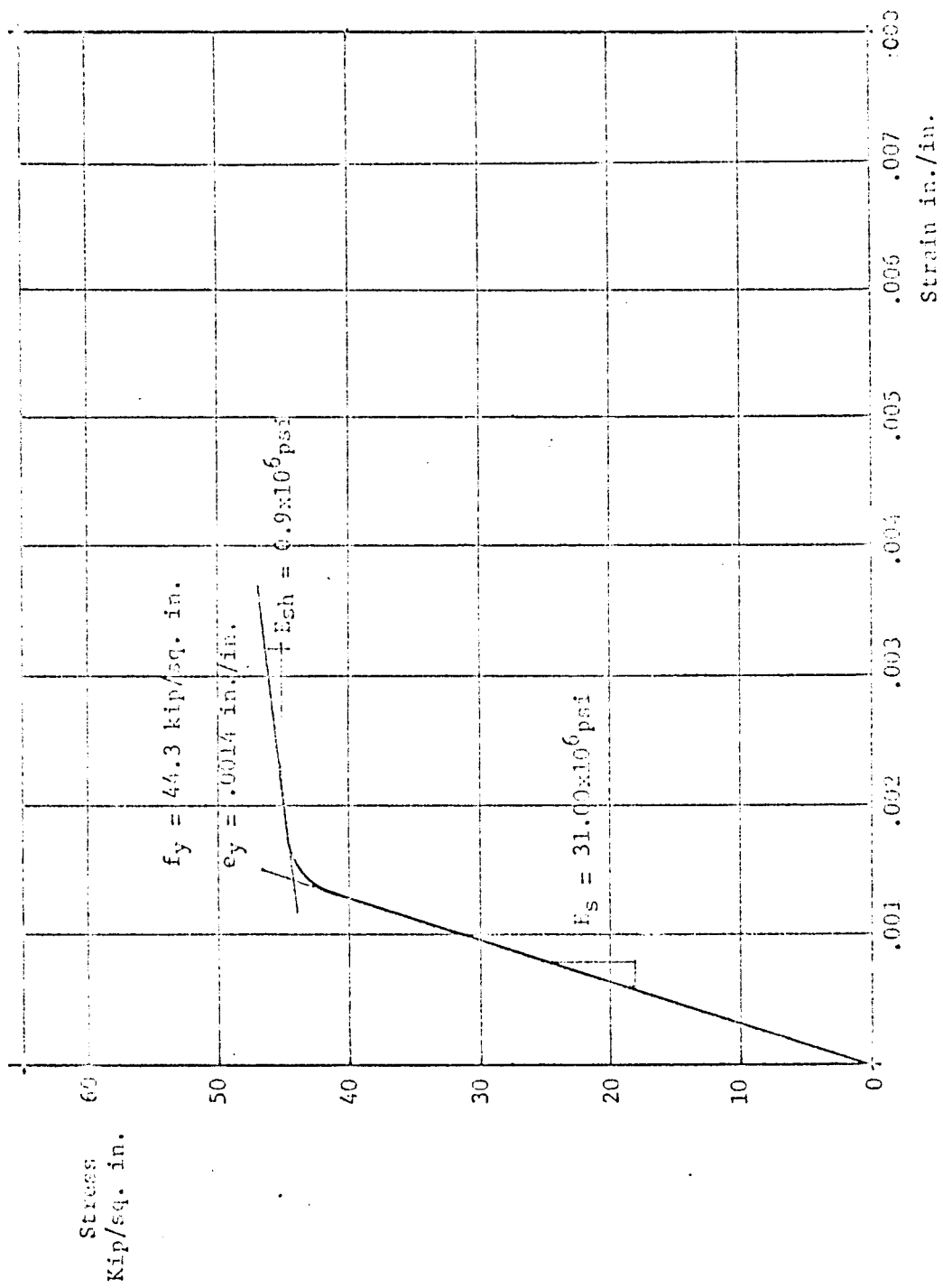


FIG. 16: STRESS-STRAIN RELATION FOR No 7 MILD STEEL BAR

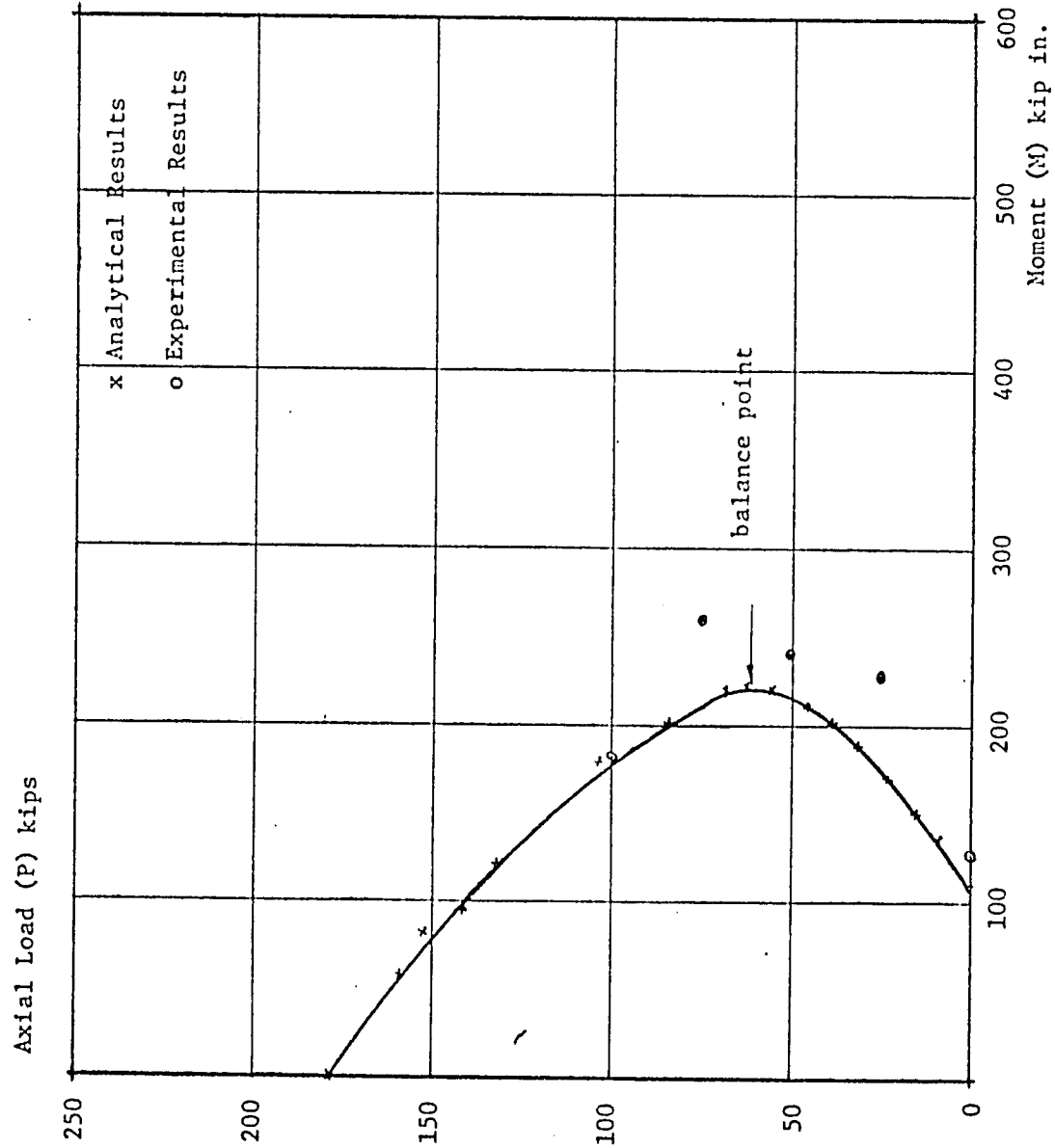


FIG. 17: AXIAL LOAD - MOMENT INTERACTION DIAGRAM - S1

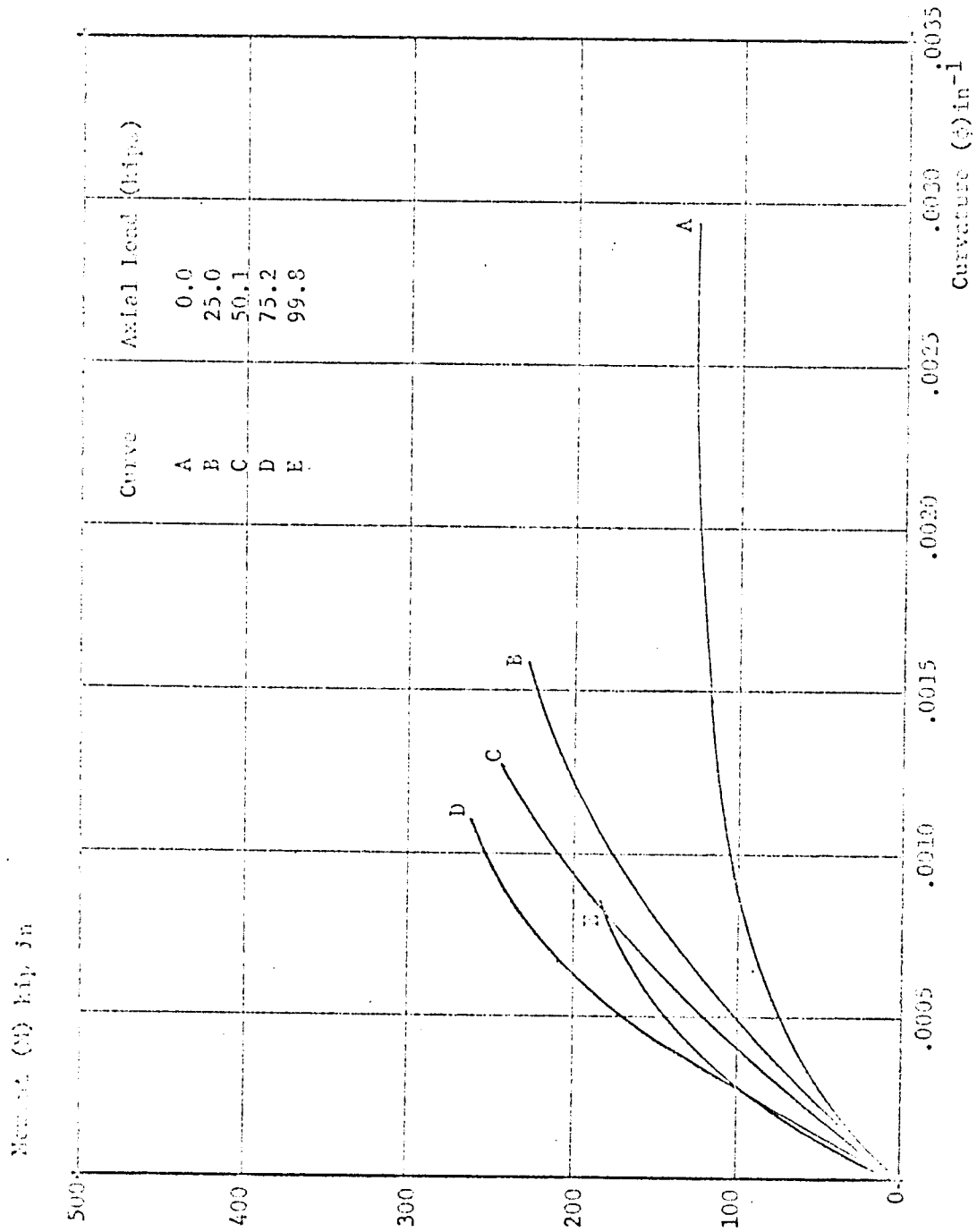


FIG. 18: NOURIT - CURVATURE DIAGRAM - S1

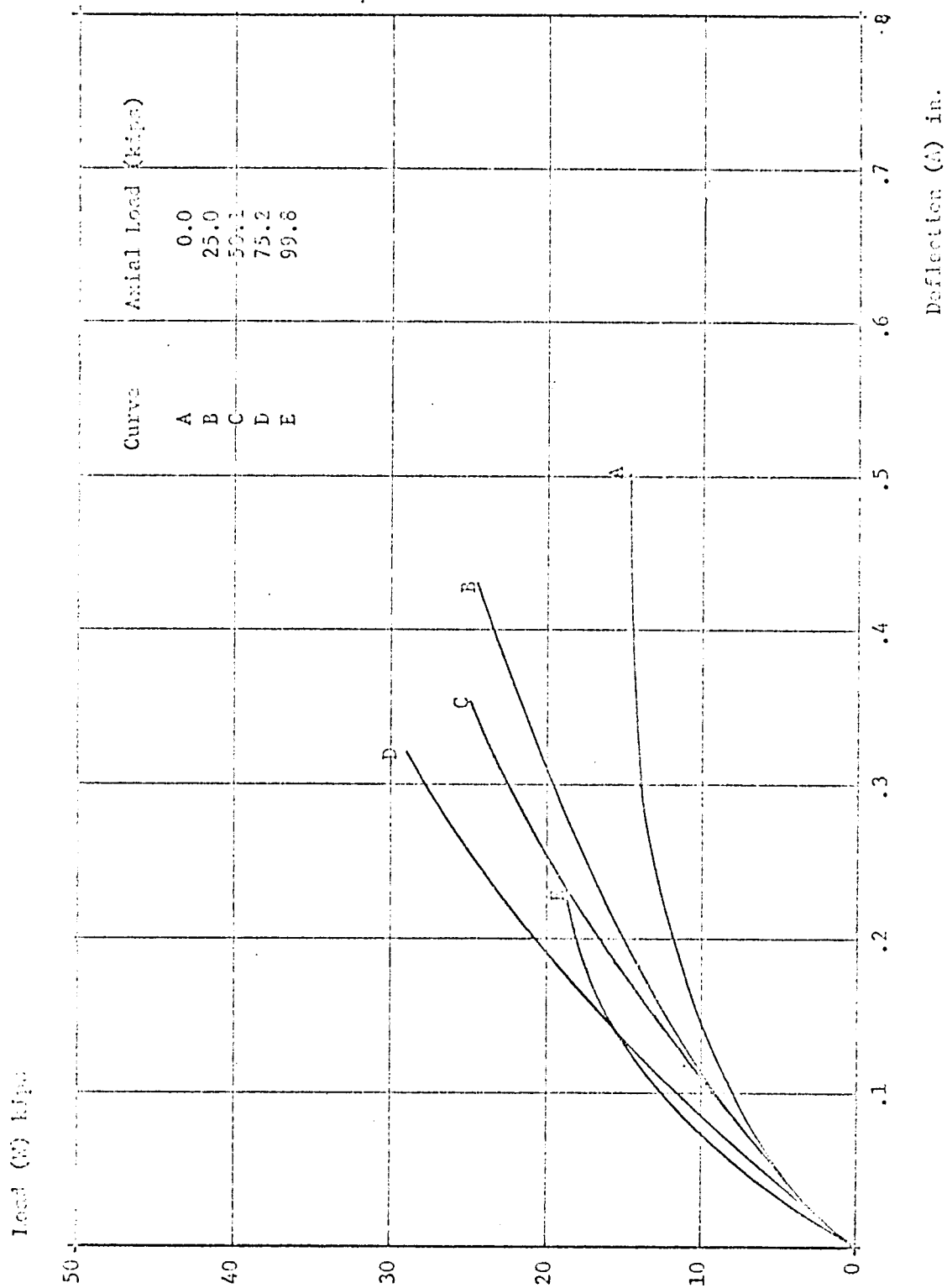


FIG. 19: LOAD - DEFLECTION DIAGRAM - S1

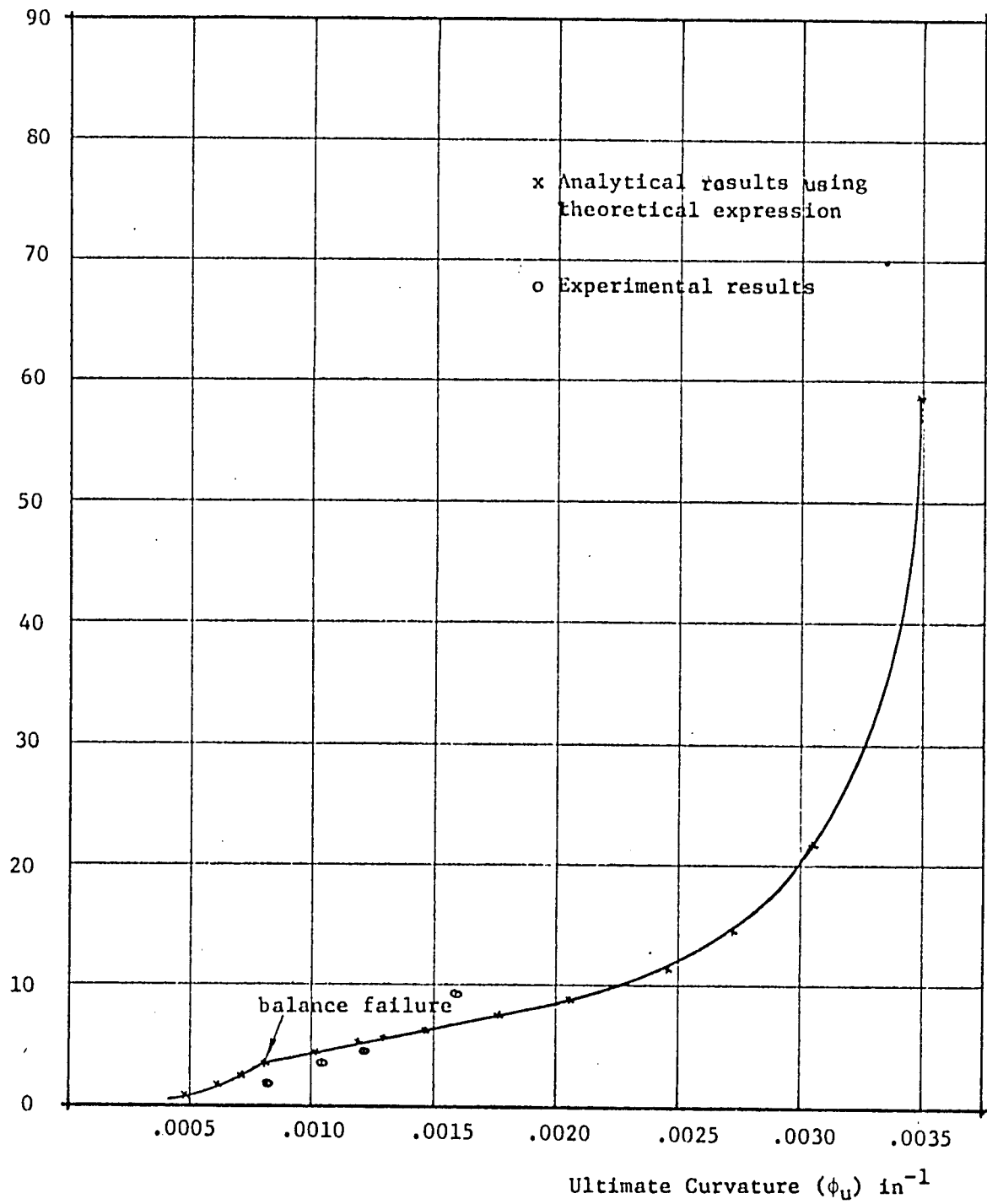
Eccentricity (e') in.

FIG. 20: ECCENTRICITY - ULTIMATE CURVATURE DIAGRAM - S1

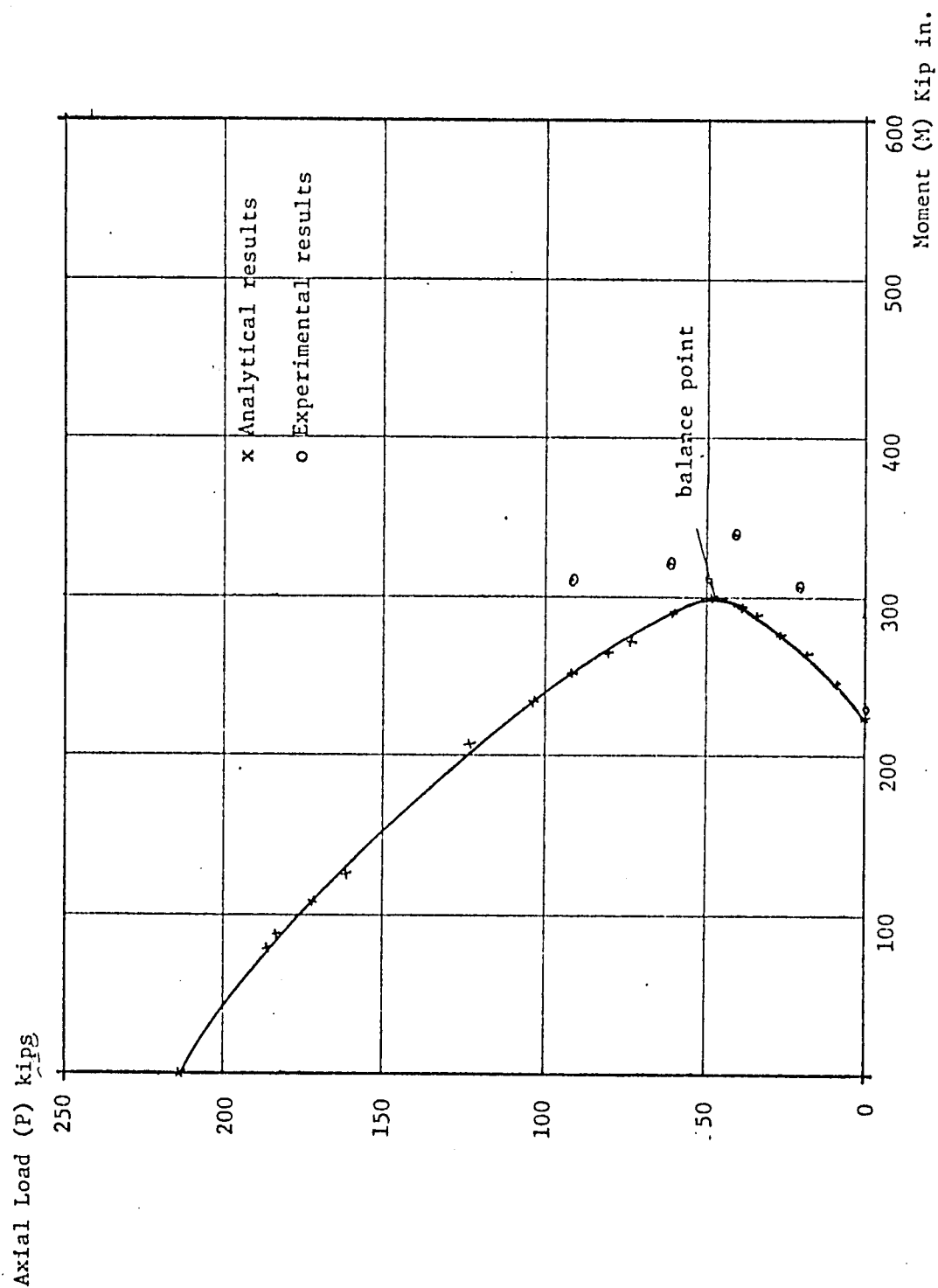


FIG. 21: AXIAL LOAD - MOMENT INTERACTION DIAGRAM - S2

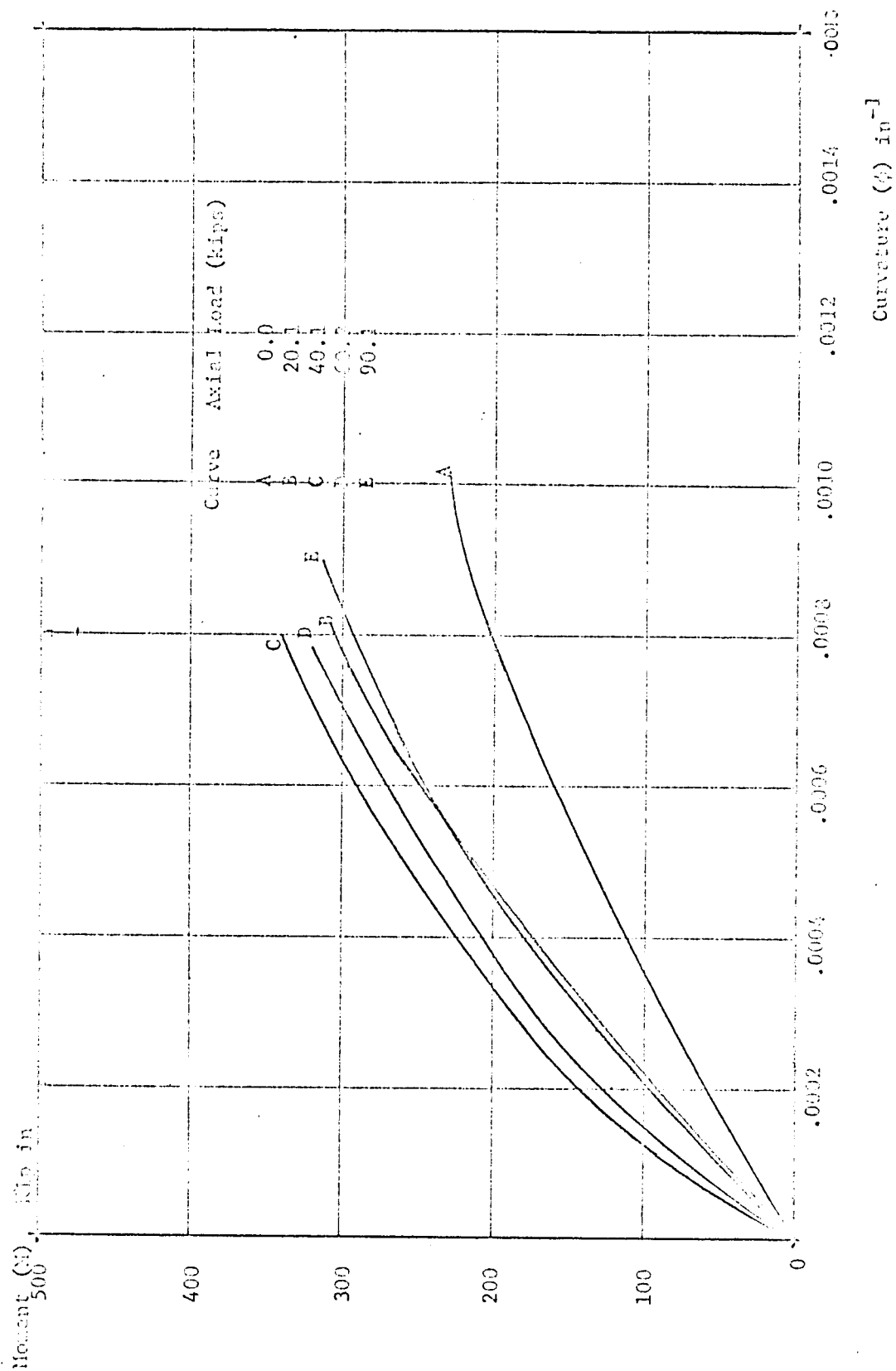


FIG. 22: MOMENT-CURVATURE DIAGRAM - S2

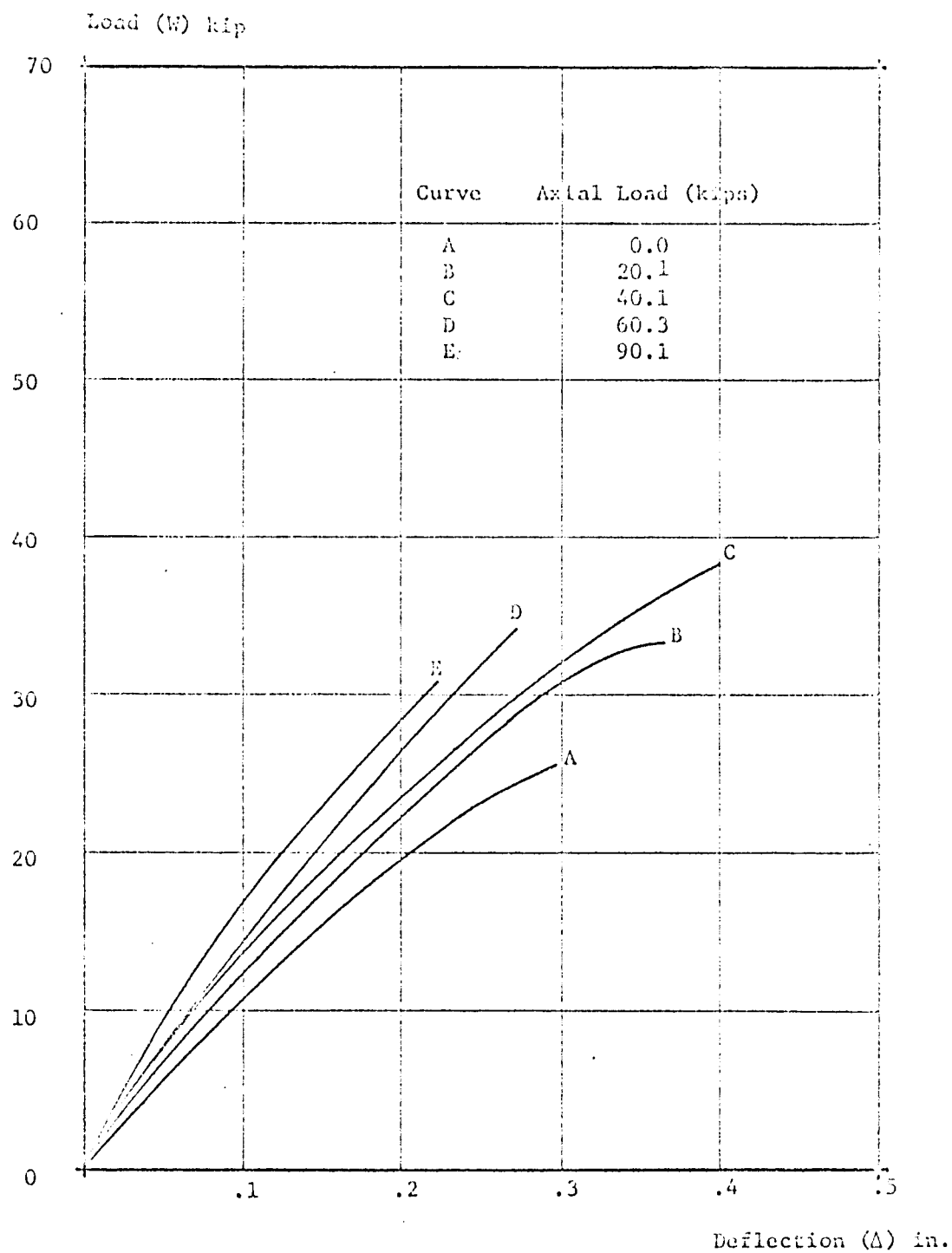


FIG. 23: LOAD-DEFLECTION DIAGRAM - S2

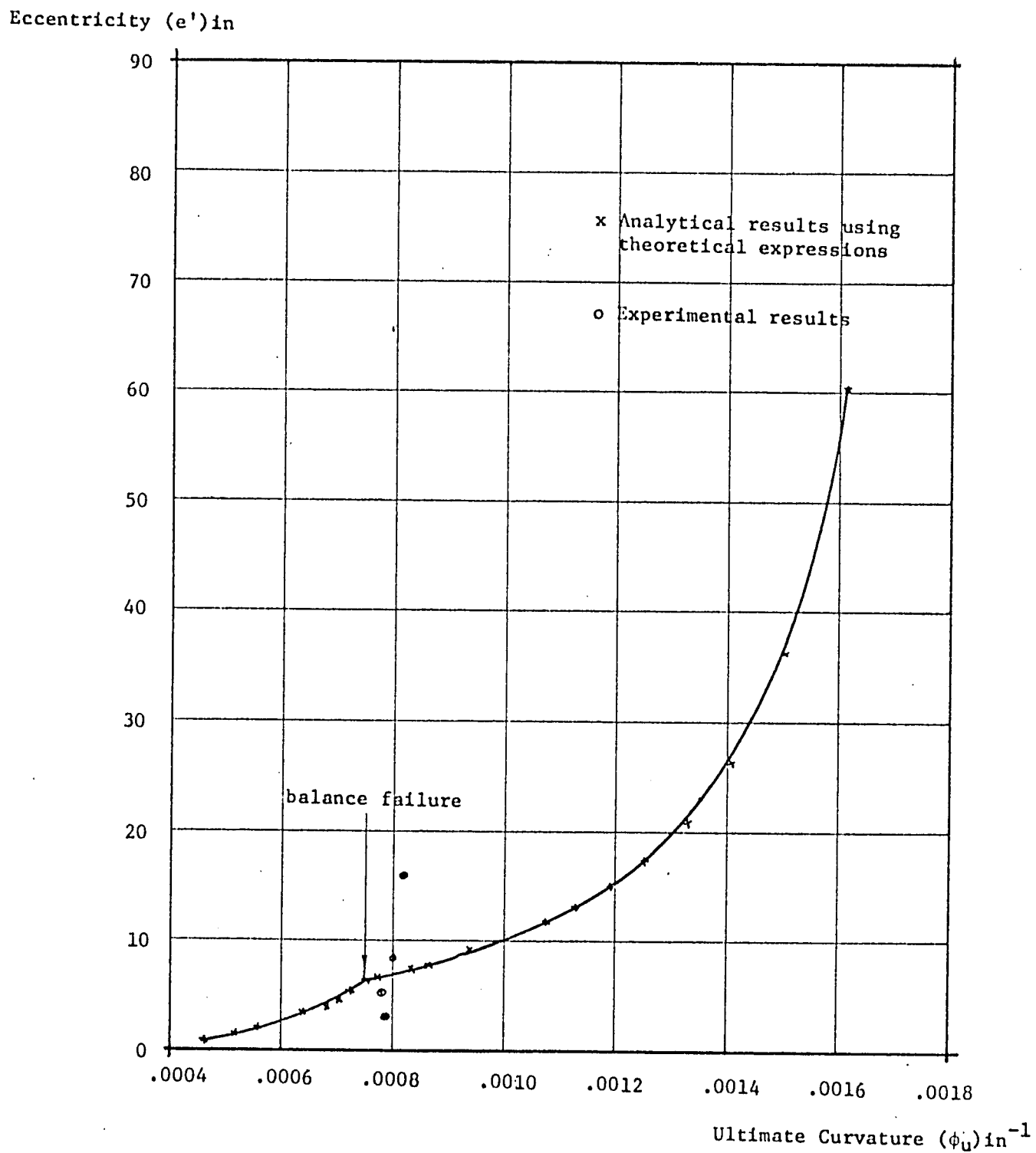


FIG. 24: ECCENTRICITY - ULTIMATE CURVATURE DIAGRAM - S2

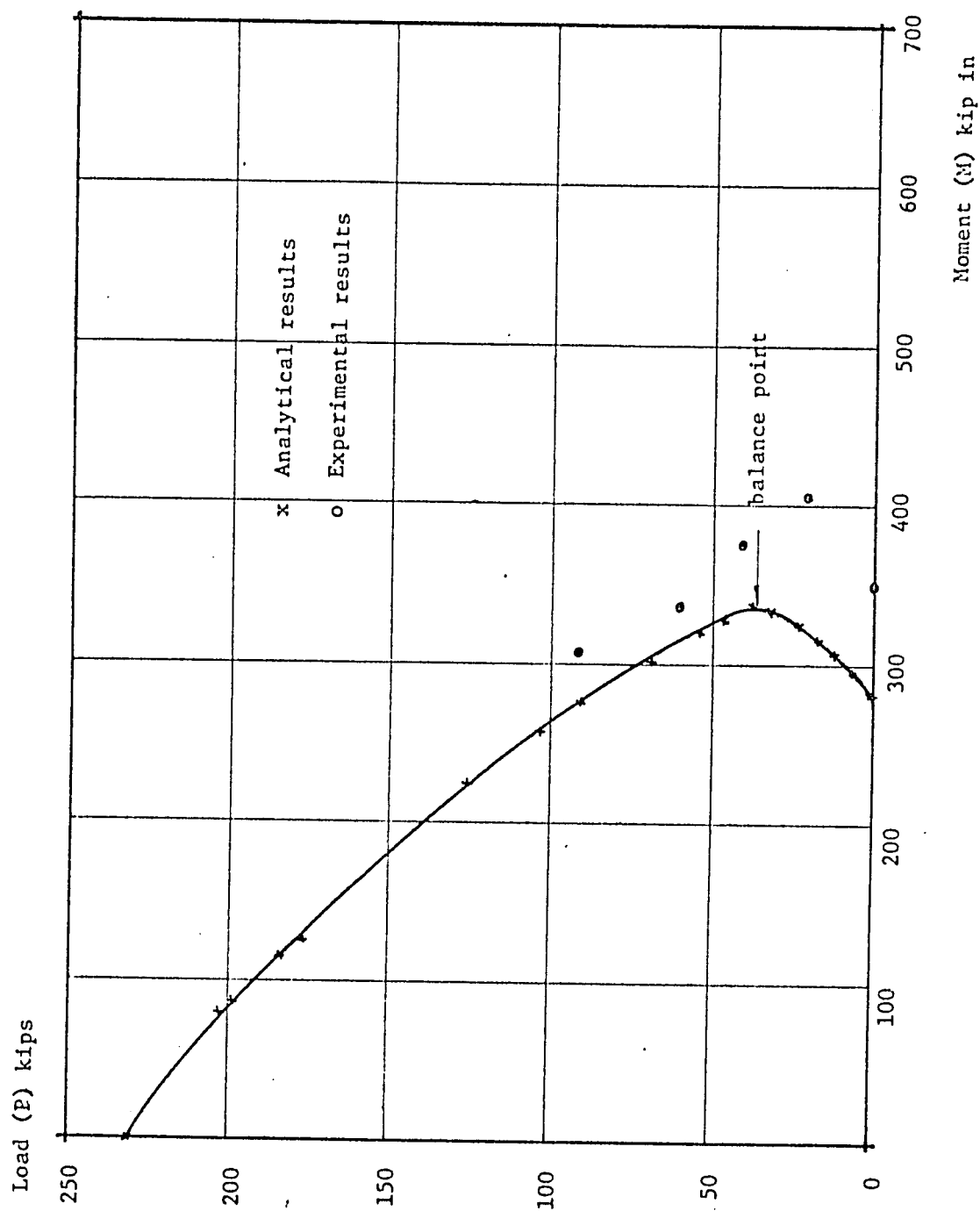


FIG. 25: AXIAL LOAD - MOMENT INTERACTION DIAGRAM - S3

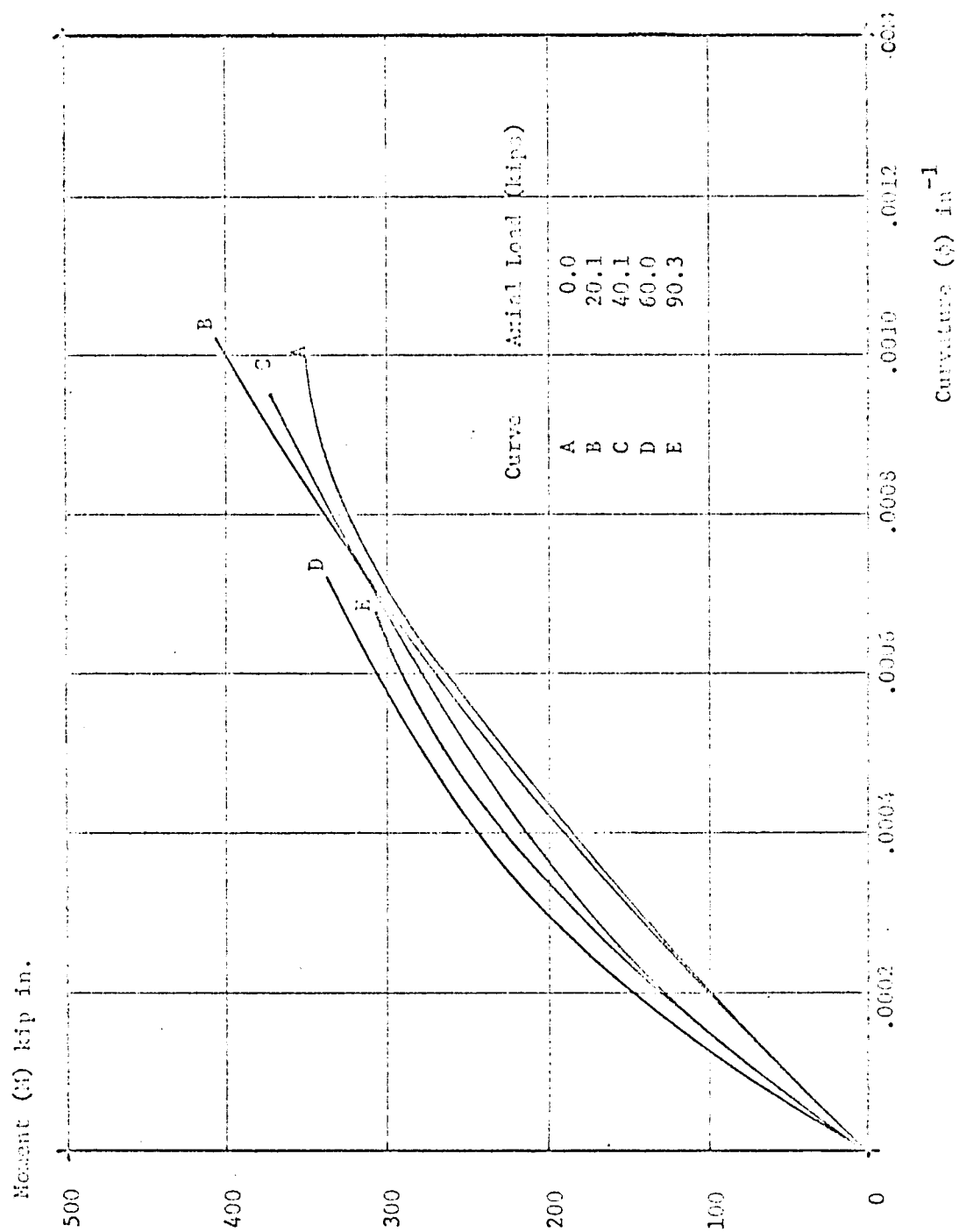


FIG. 26: MOMENT - CURVATURE DIAGRAM - S3

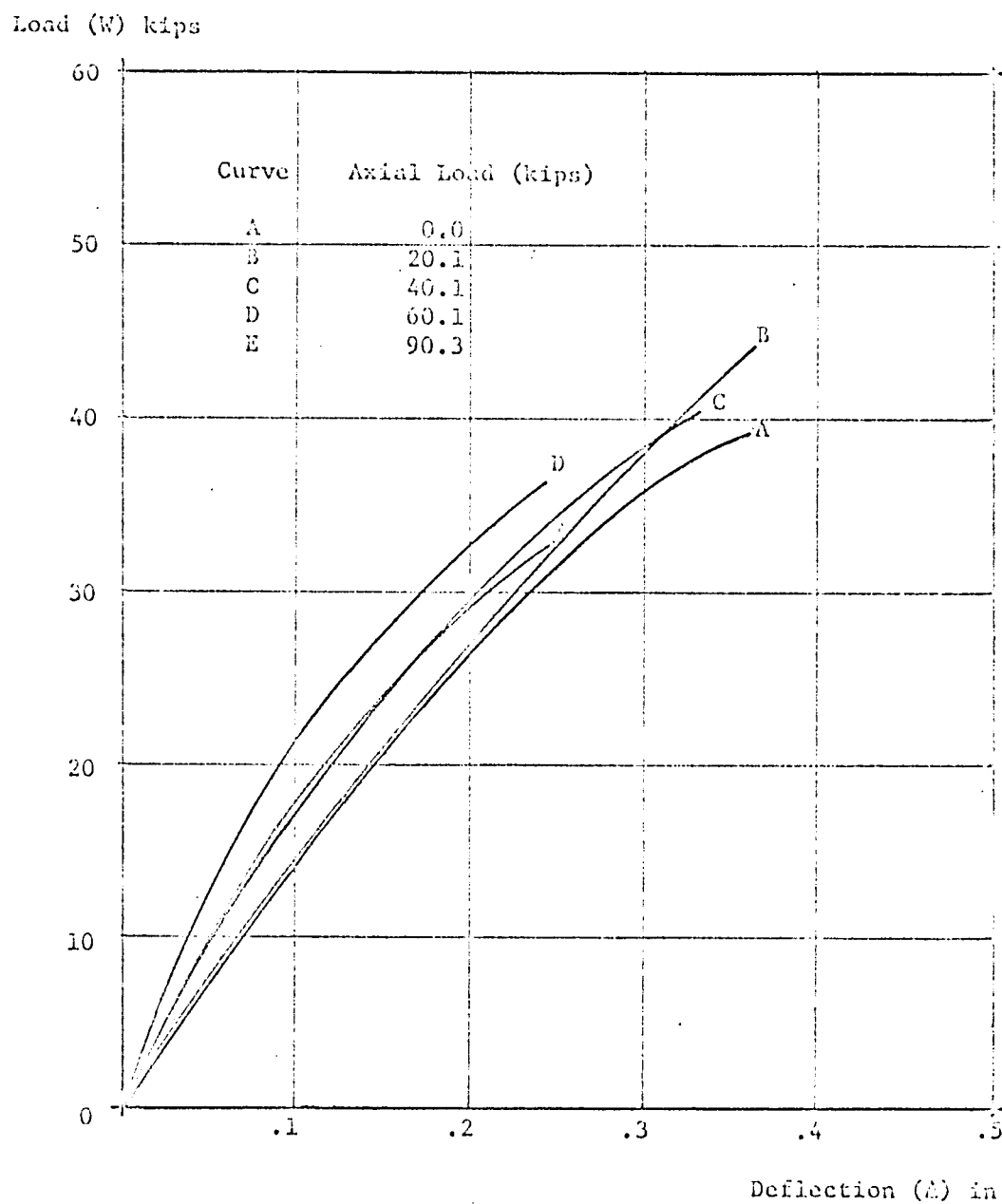


FIG. 27: LOAD - DEFLECTION DIAGRAM -S3

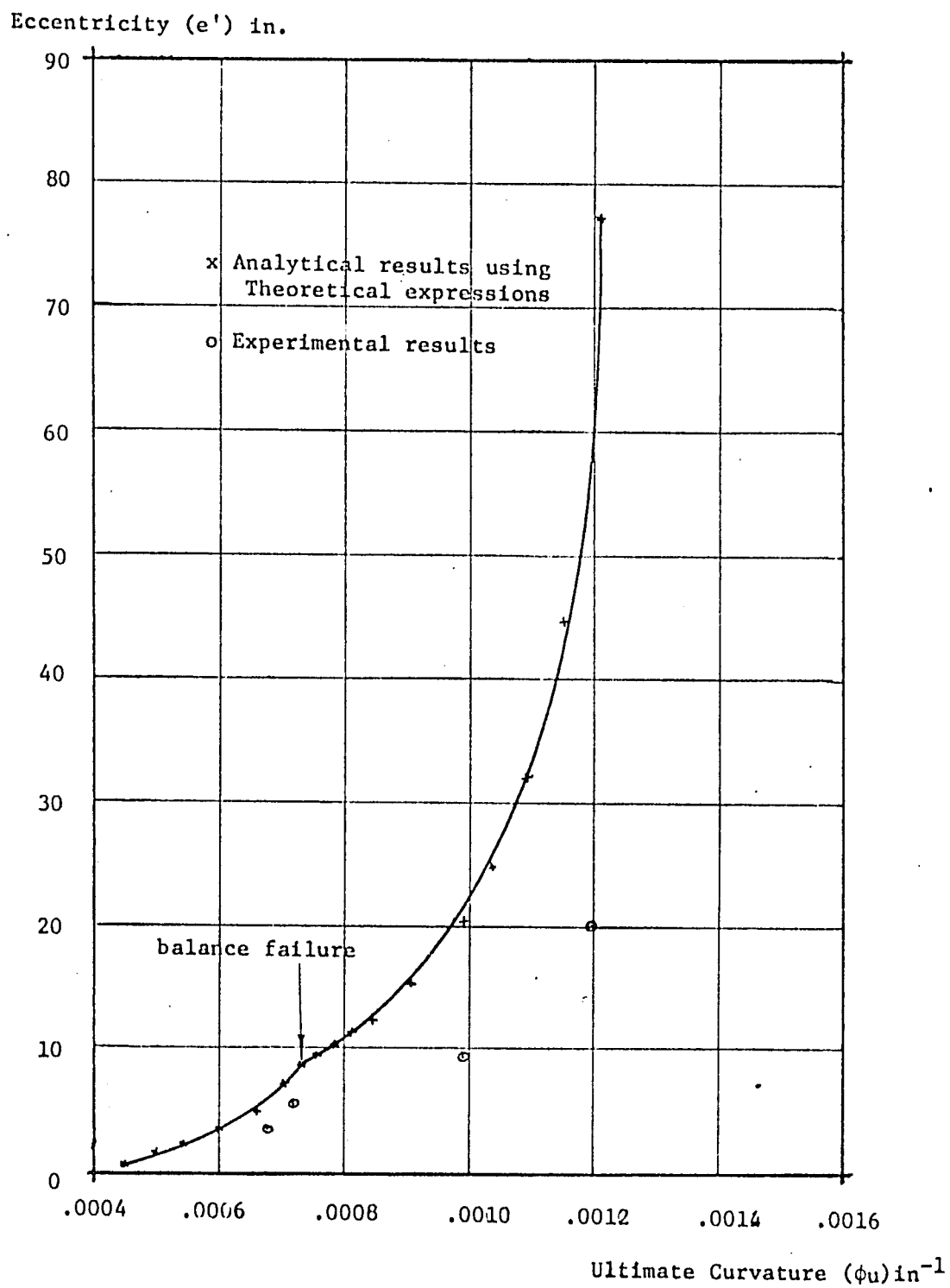


FIG. 28: ECCENTRICITY - ULTIMATE CURVATURE DIAGRAM - S3

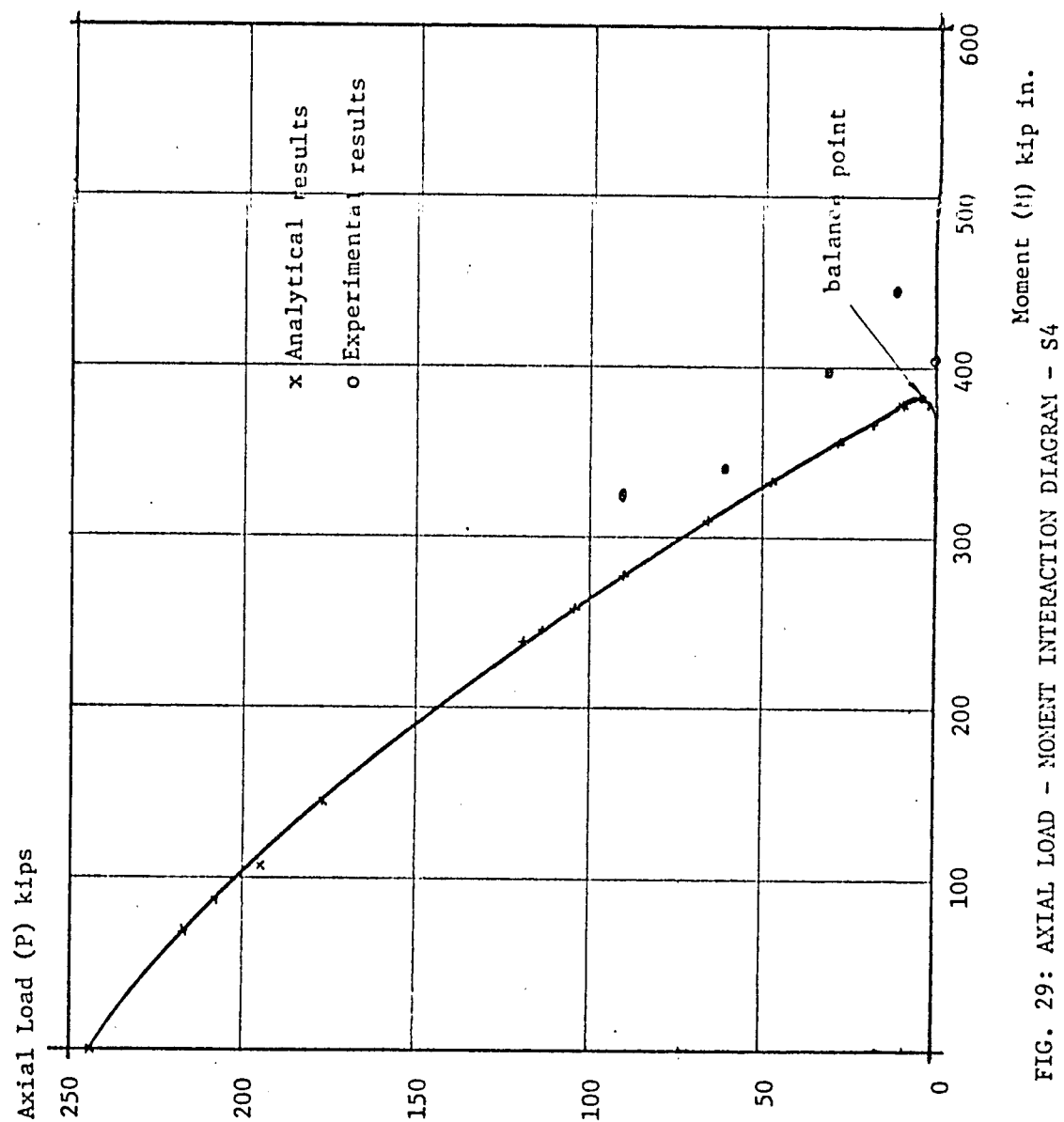


FIG. 29: AXIAL LOAD - MOMENT INTERACTION DIAGRAM - S4

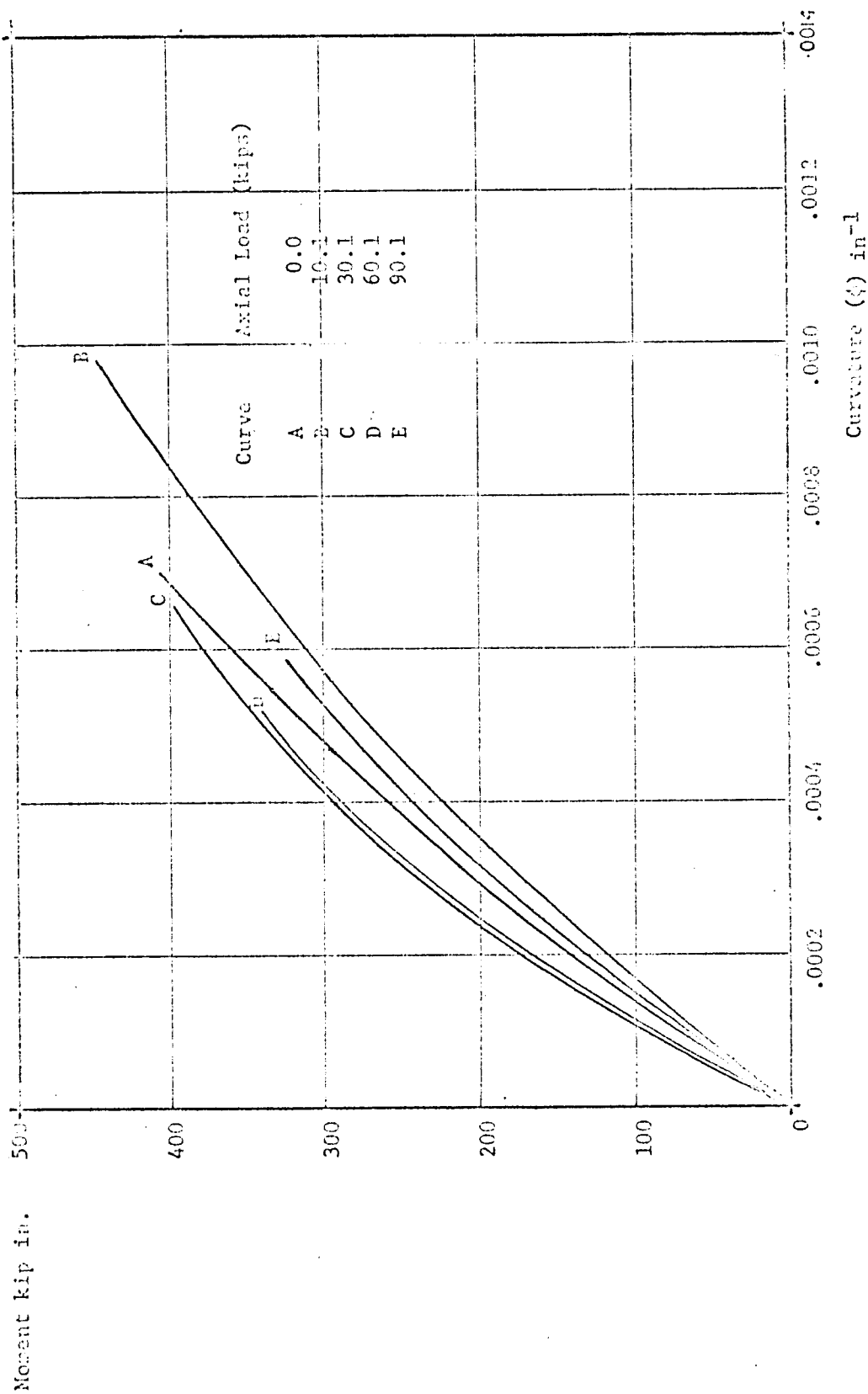


FIG. 30: NONCENT - CURVATURE DIAGRAM S4

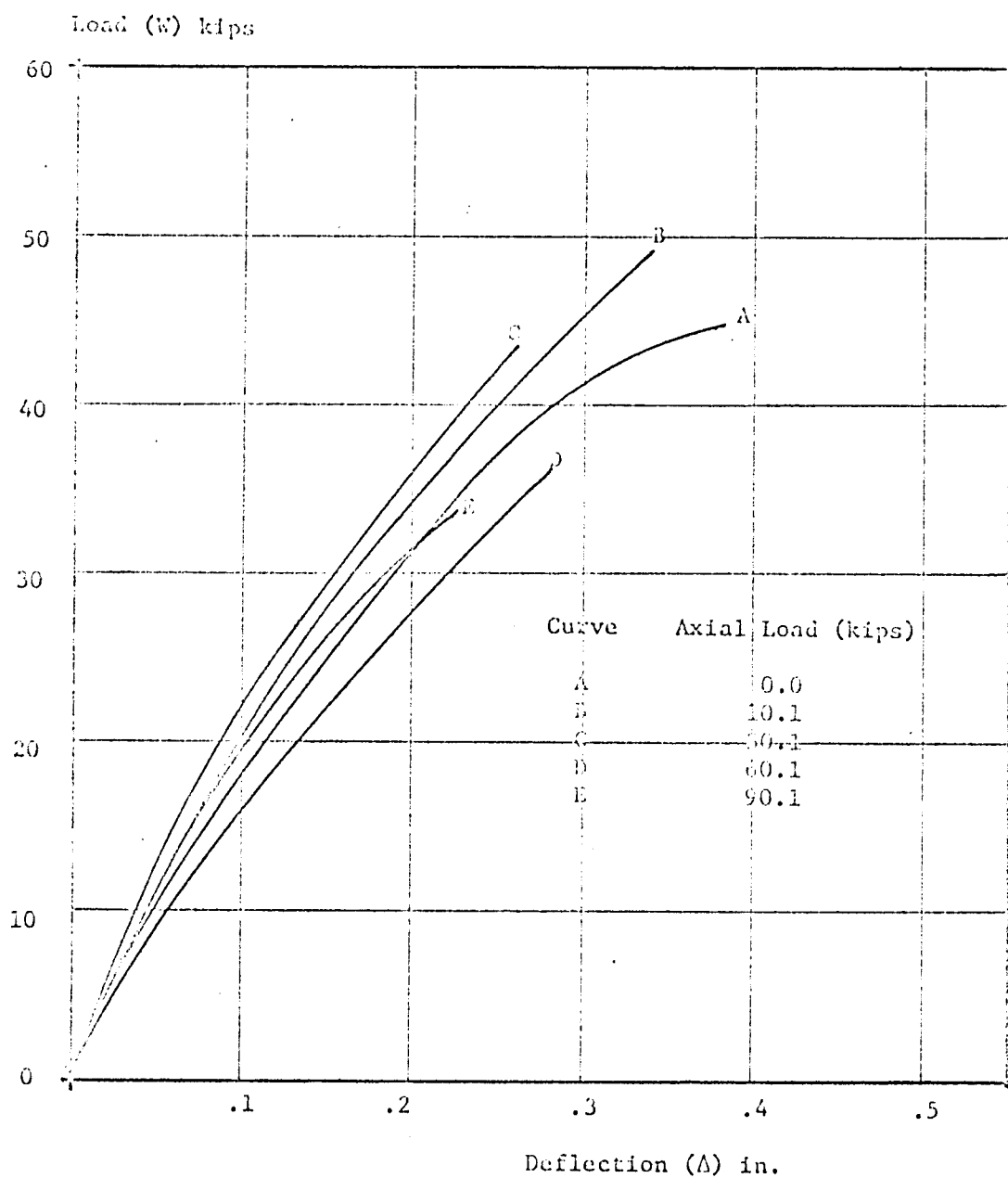


FIG. 31: LOAD - DEFLECTION DIAGRAM - S4

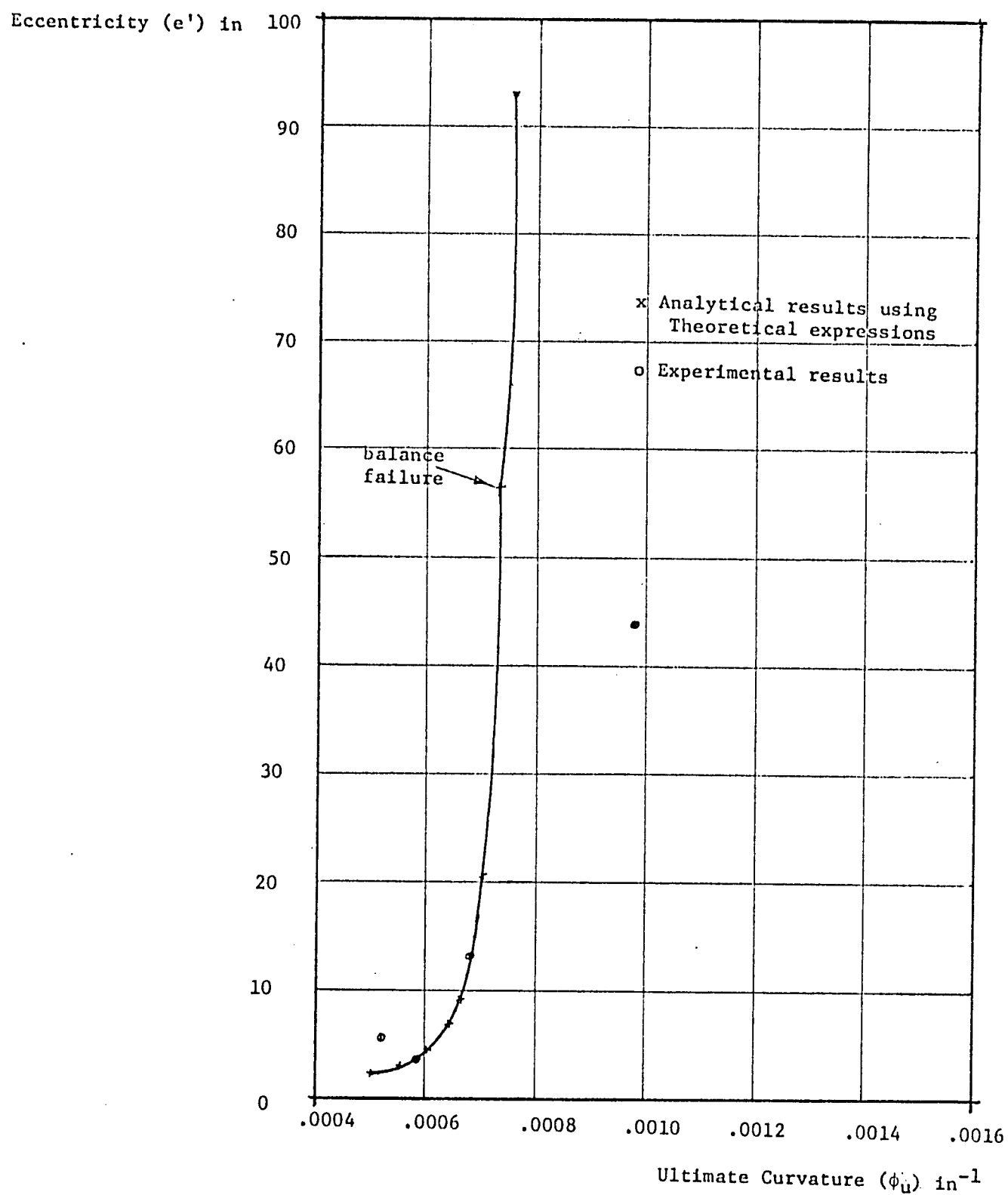
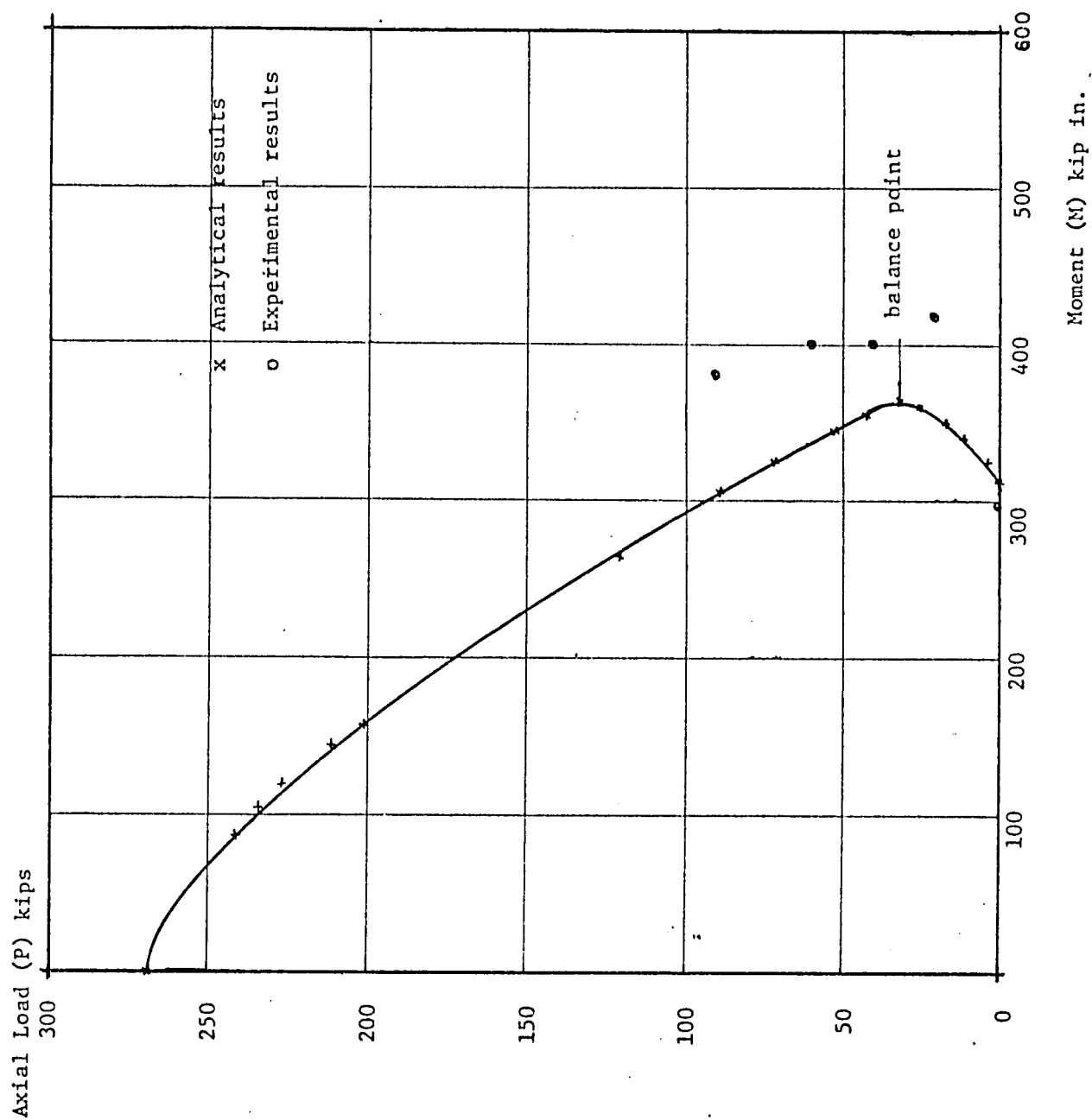


FIG. 32: ECCENTRICITY - ULTIMATE CURVATURE DIAGRAM - S4



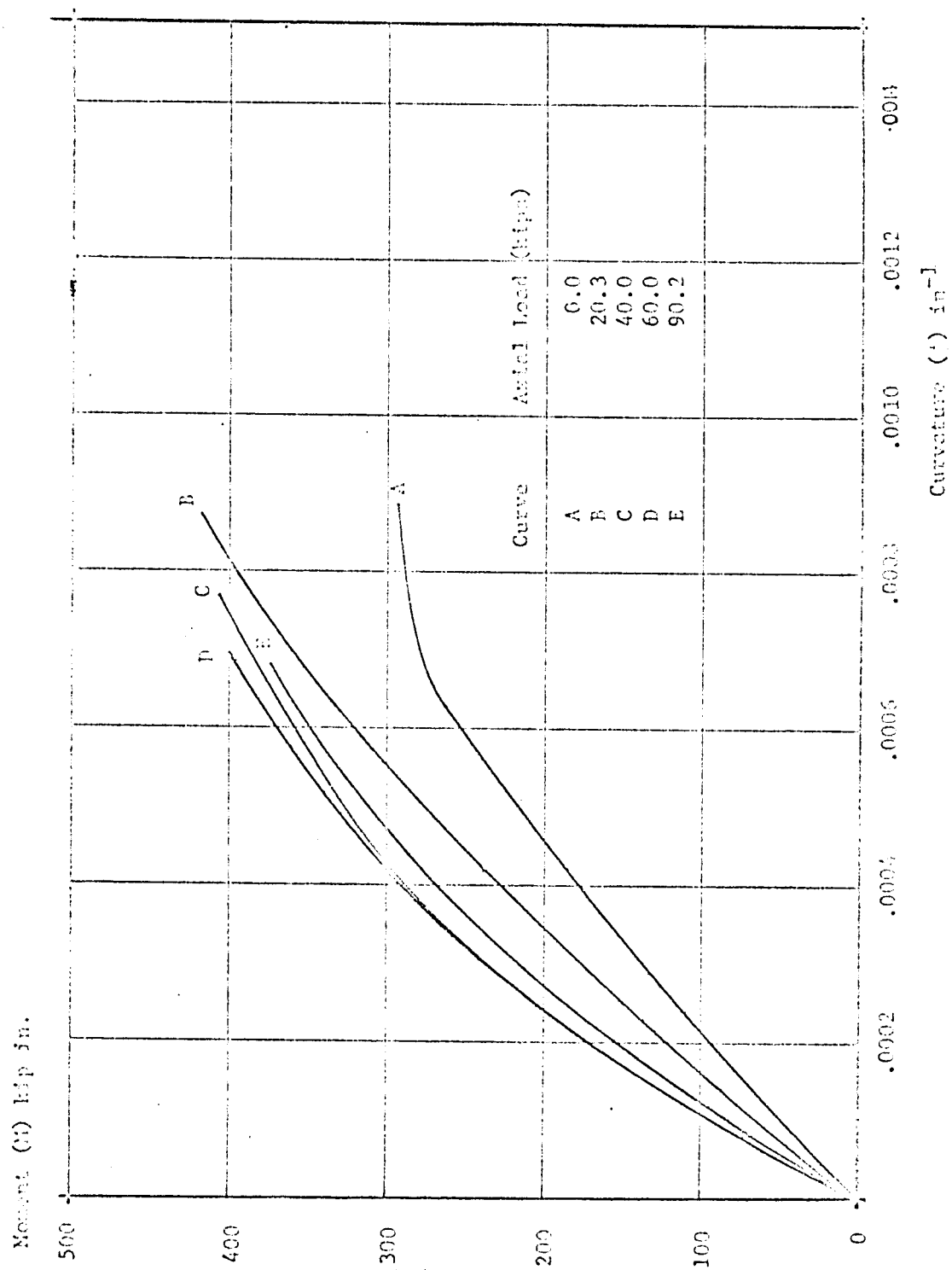


FIG. 34: MOMENT-CURVATURE DIAGRAM - S5

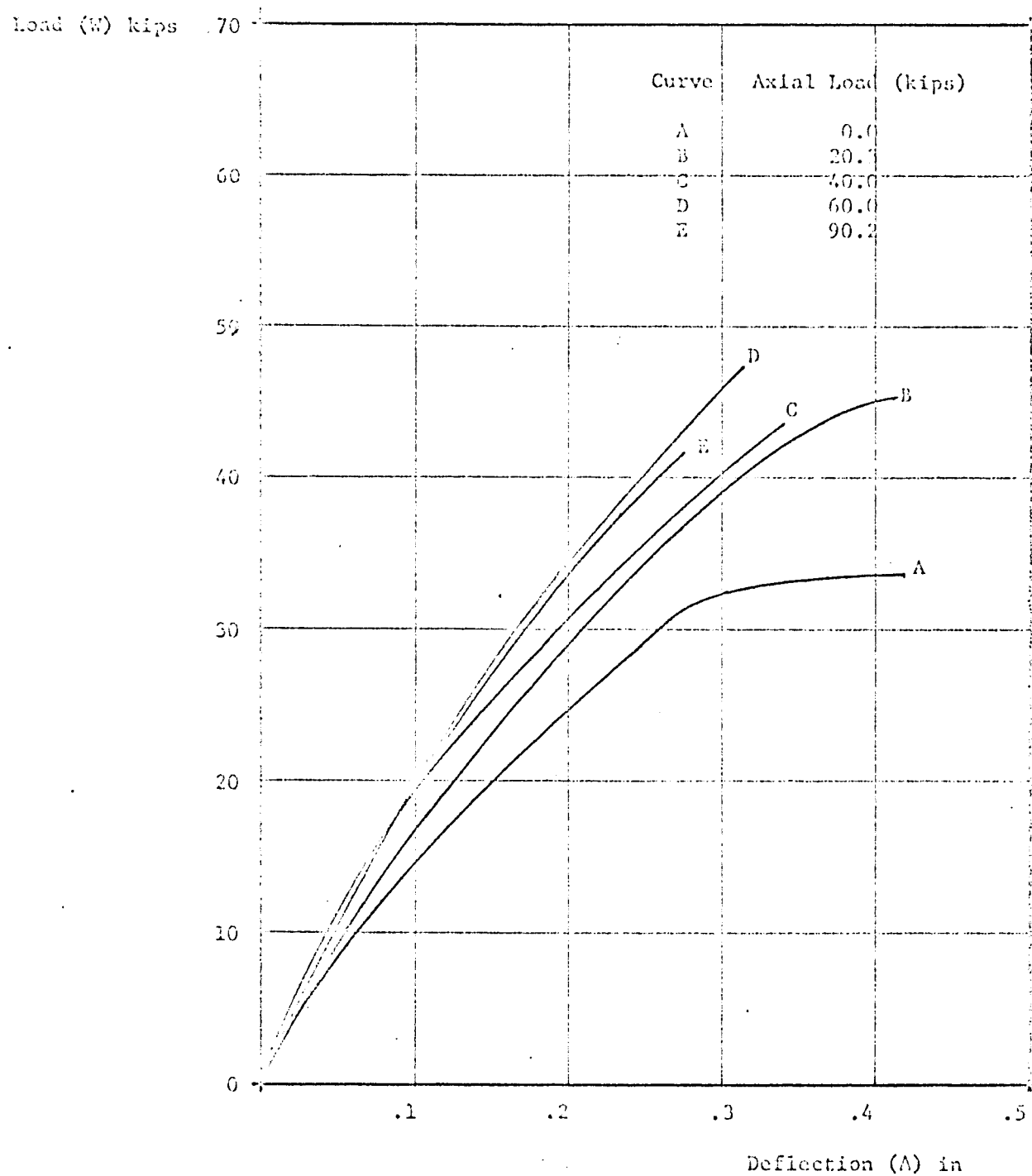


FIG. 35: LOAD - DEFLECTION DIAGRAM - S5

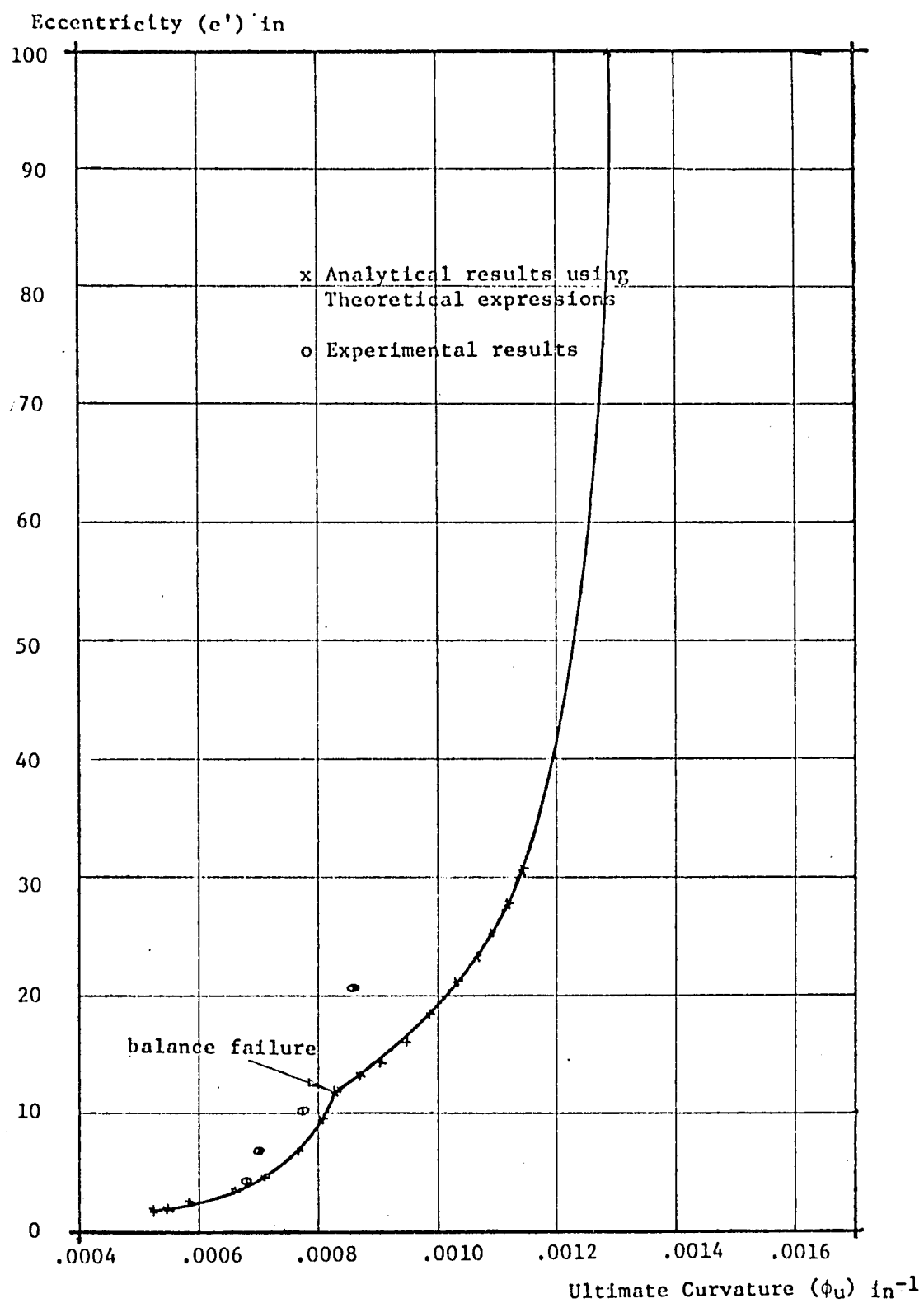


FIG. 36: Eccentricity - ULTIMATE CURVATURE DIAGRAM - S5

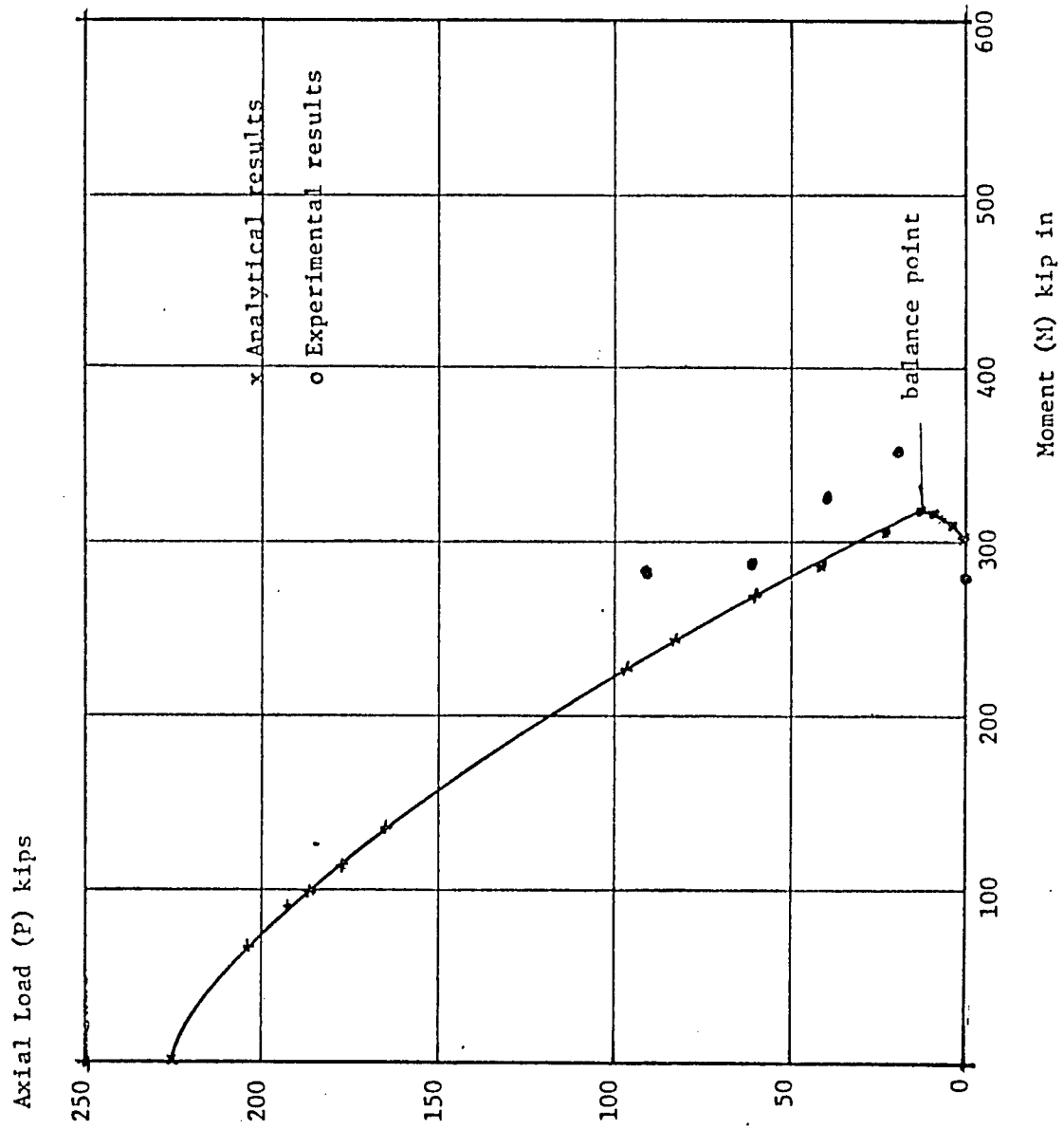


FIG. 37: AXIAL LOAD - MOMENT INTERACTION DIAGRAM - S6

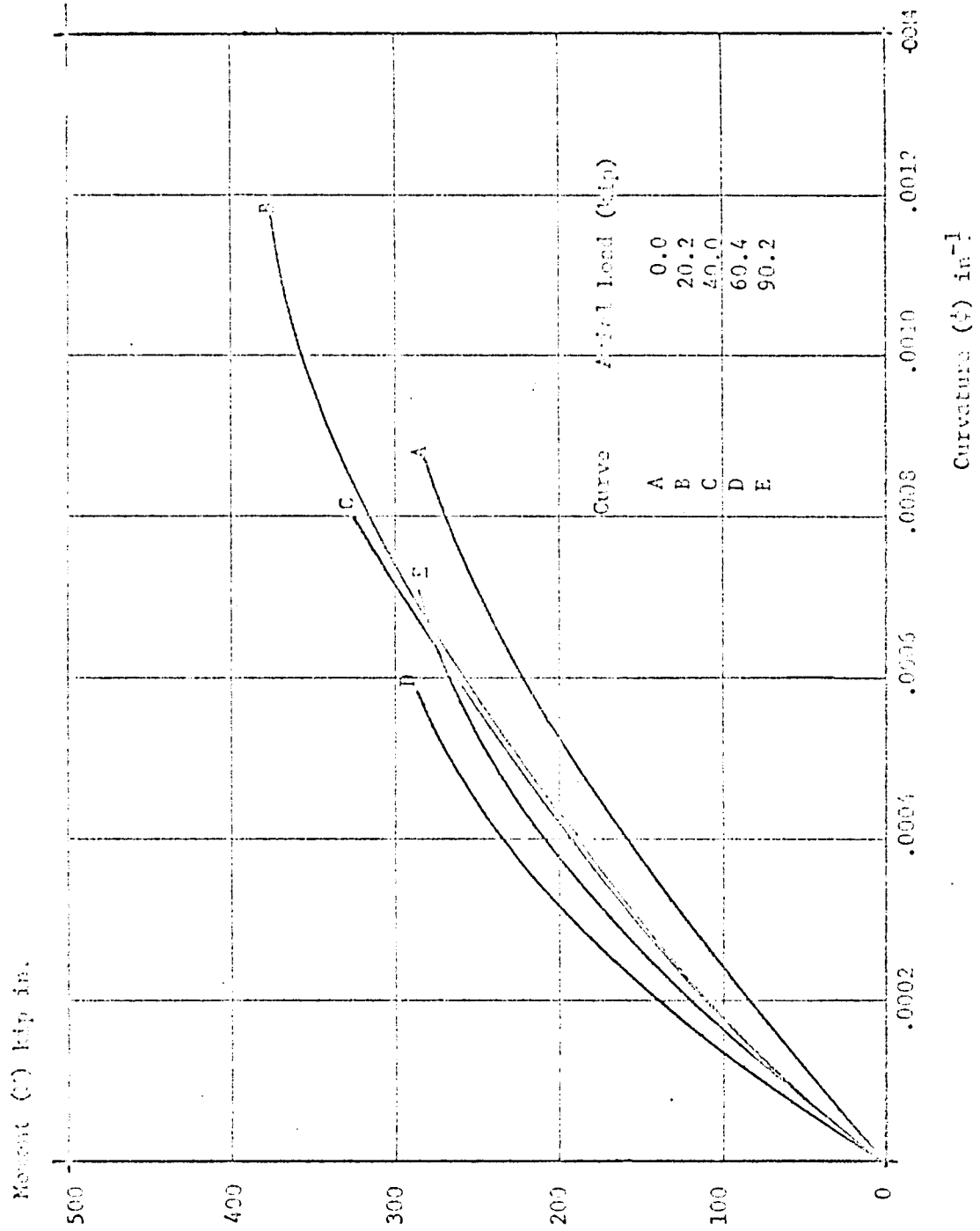


FIG. 30: MOMENT - CURVATURE DIAGRAM - S6

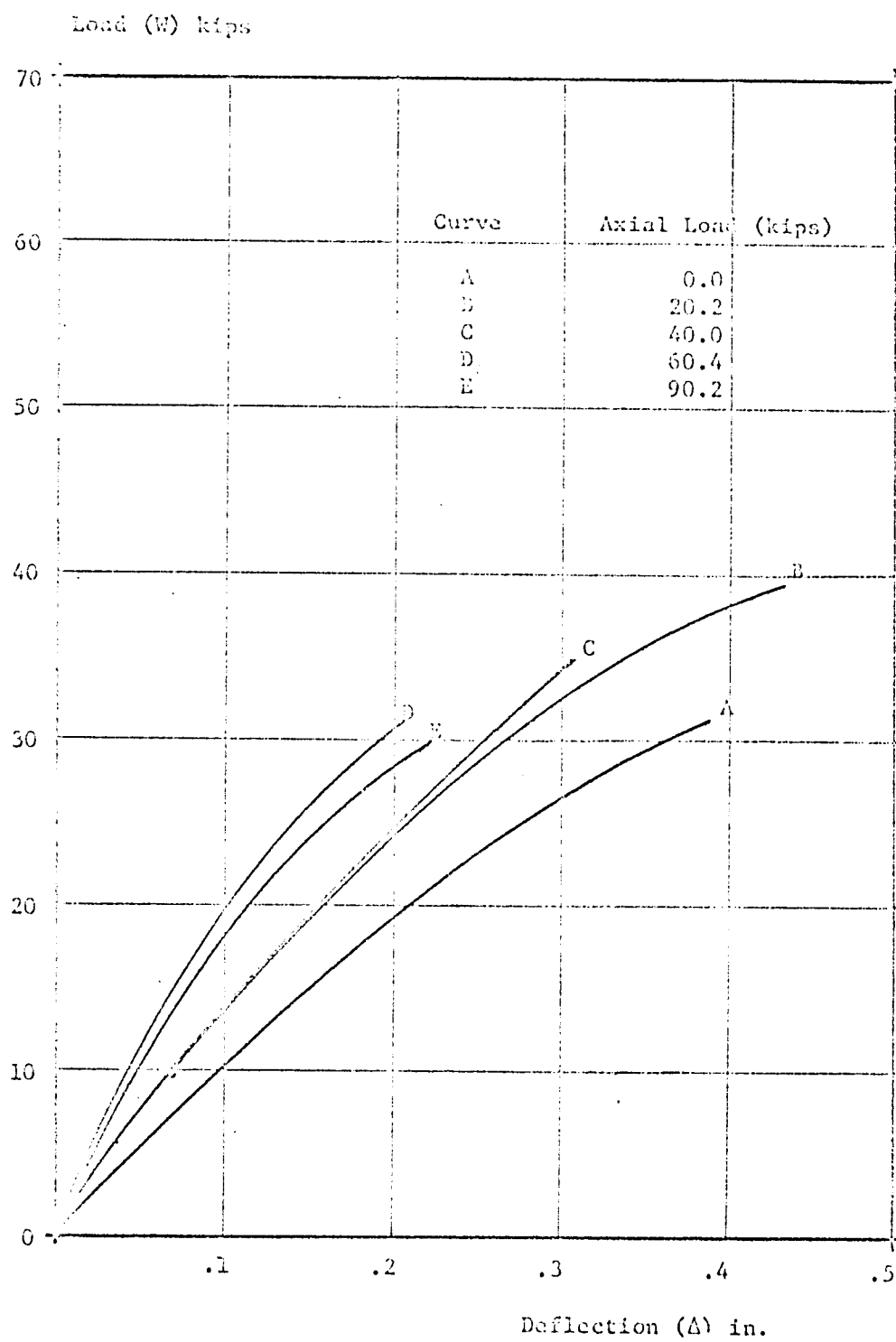


FIG. 39: LOAD - DEFLECTION DIAGRAM - S6

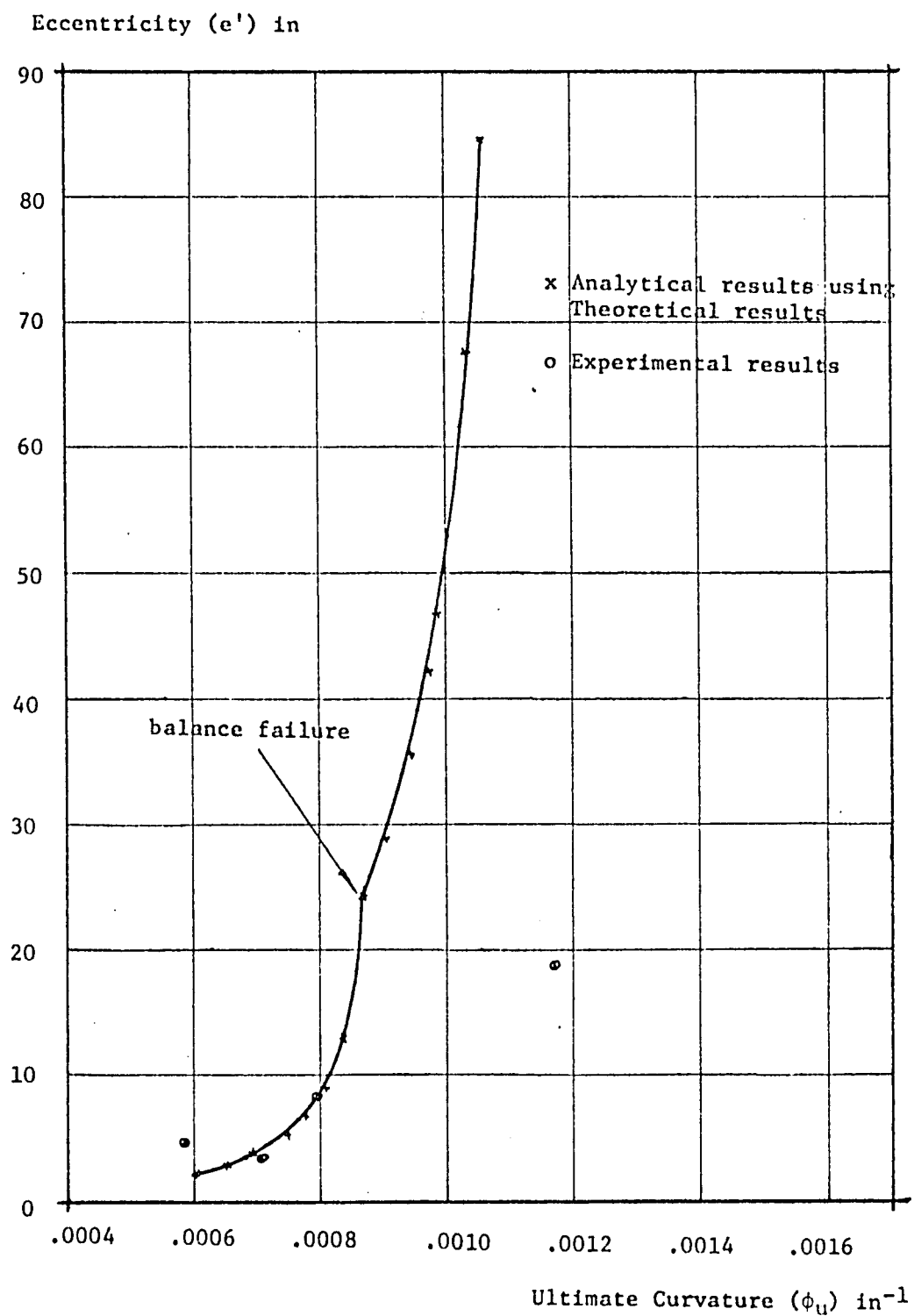


FIG. 40: Eccentricity - Ultimate Curvature Diagram - S6

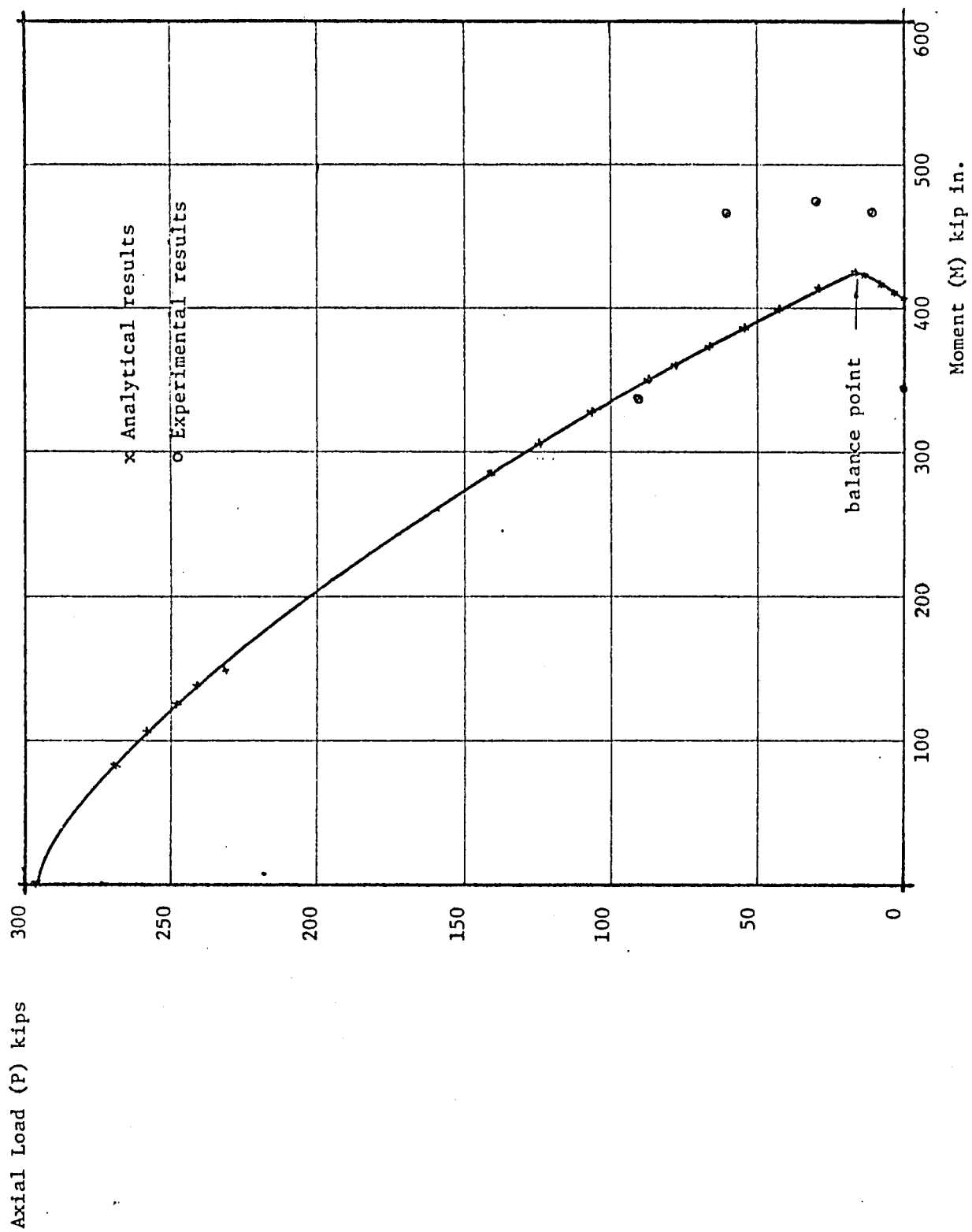


FIG. 41: AXIAL LOAD - MOMENT INTERACTION DIAGRAM - S7

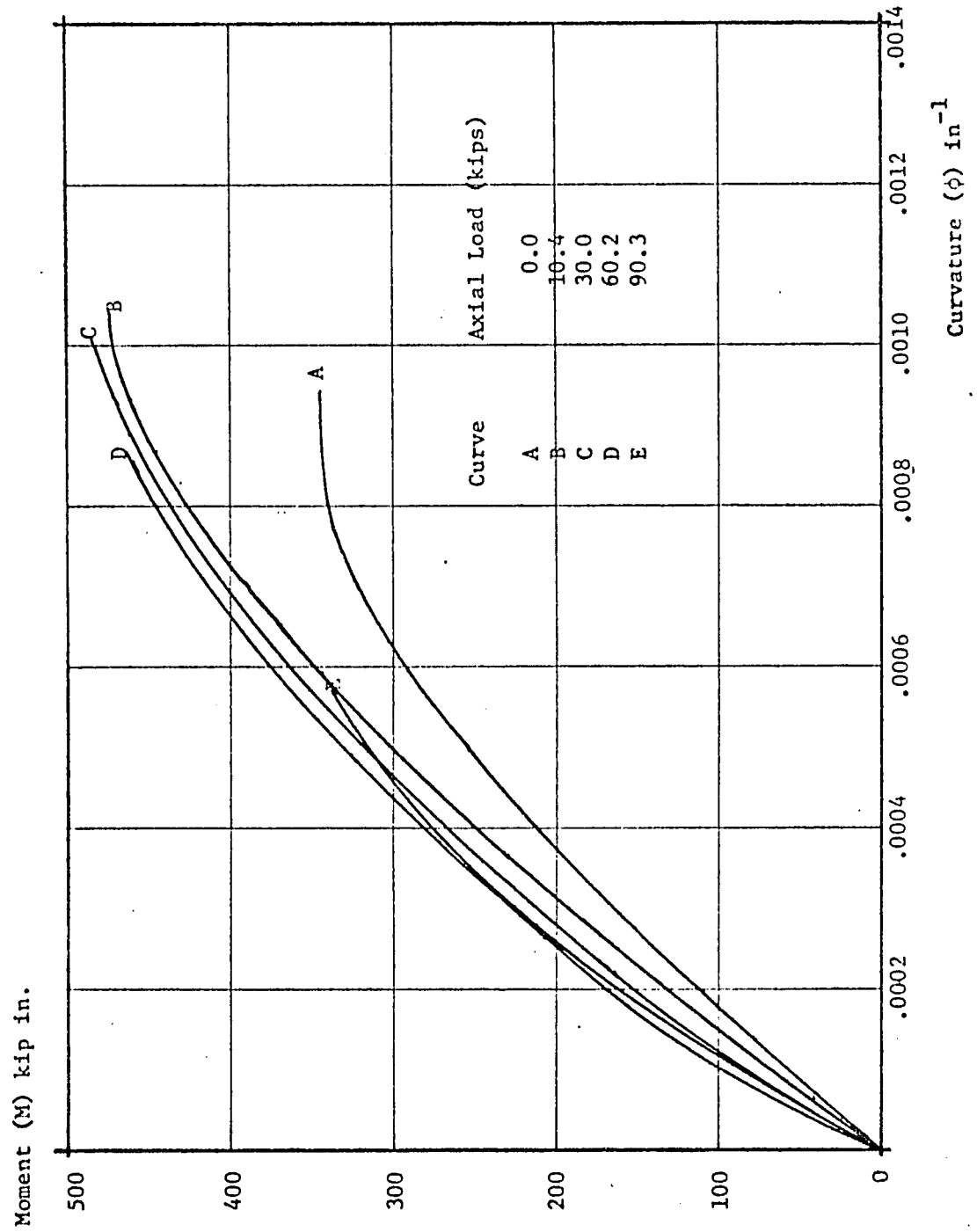


FIG. 42: MOMENT - CURVATURE DIAGRAM - S7

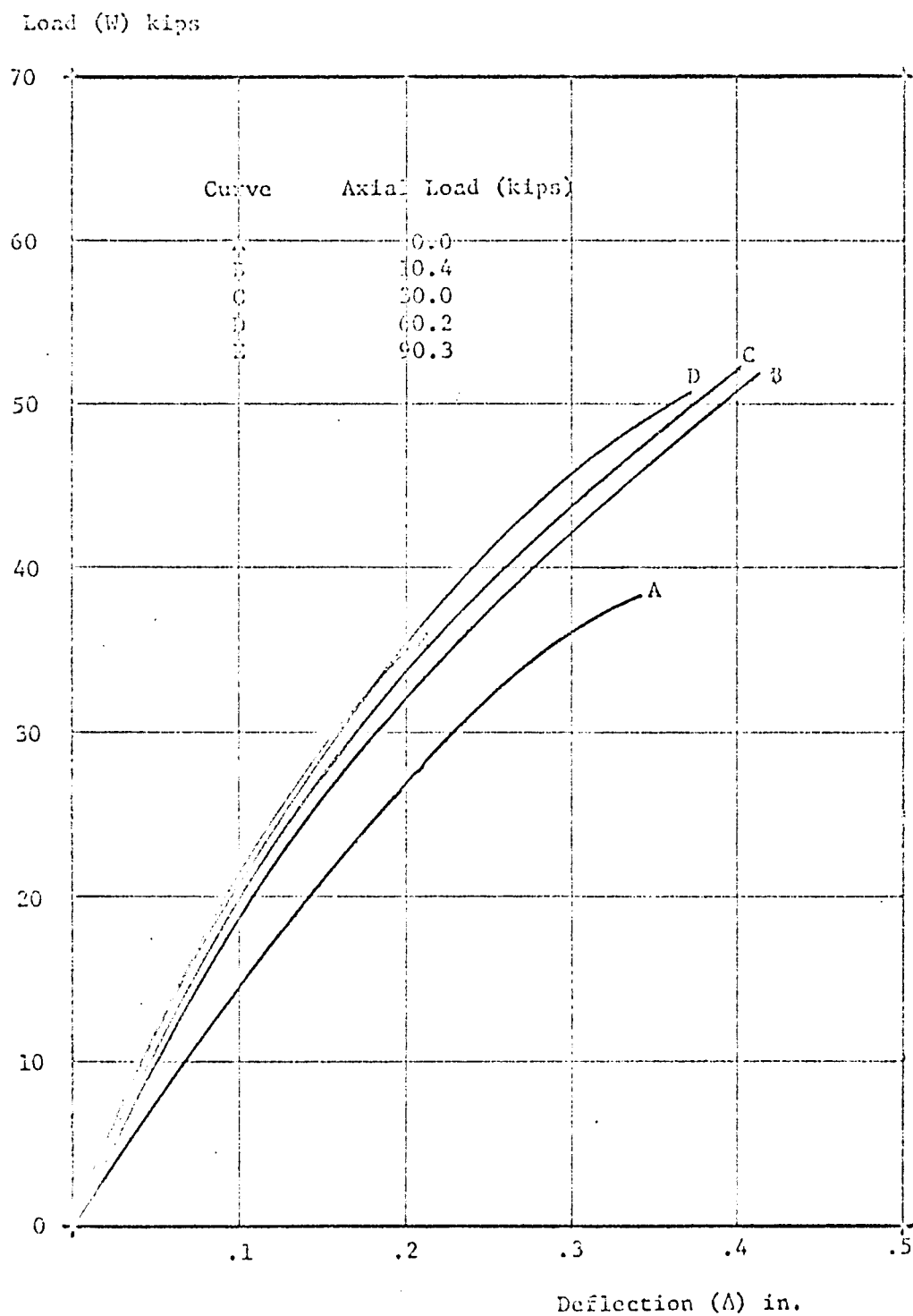


FIG. 43: LOAD - DEFLECTION DIAGRAM - S7

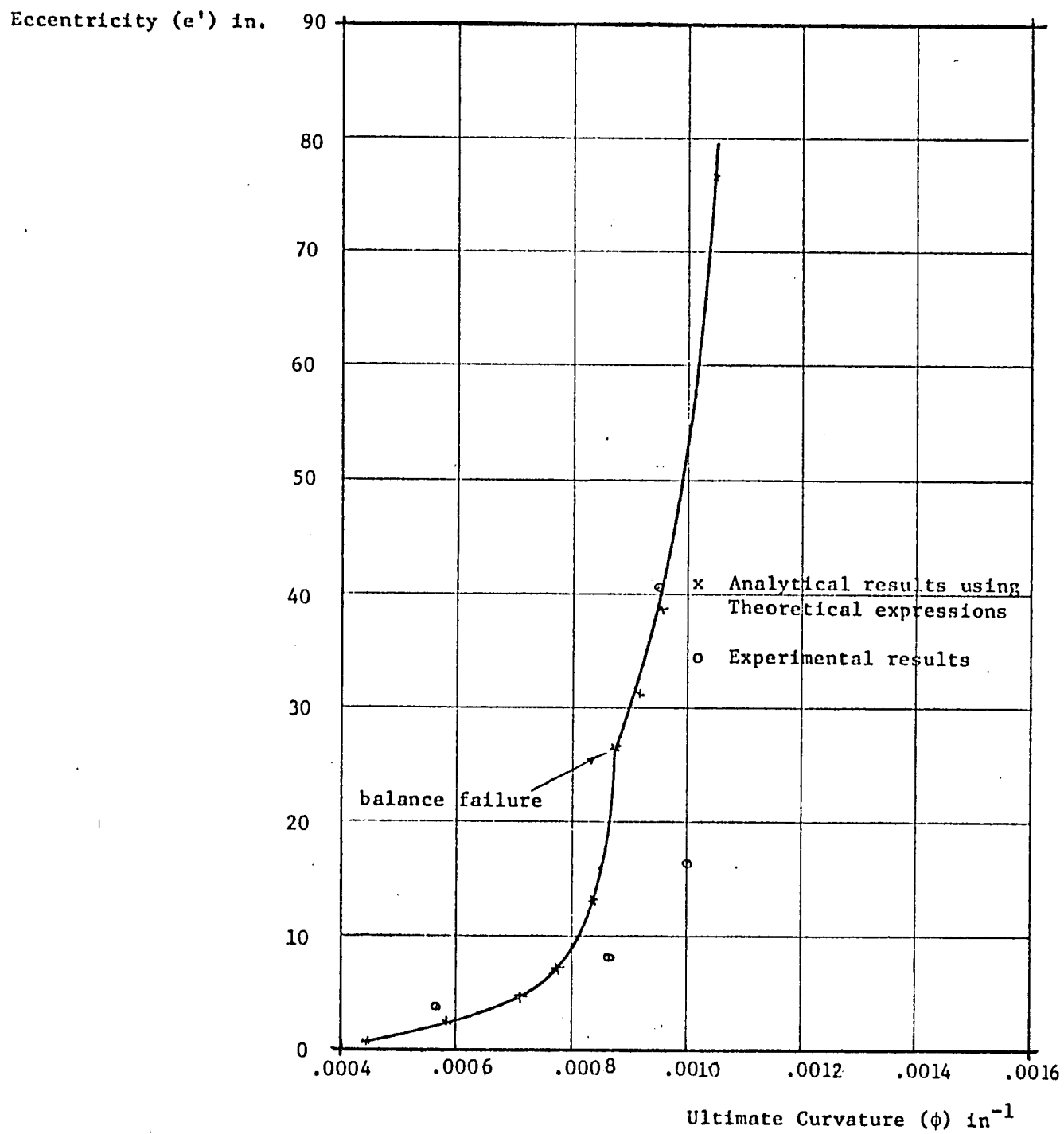


FIG. 44: ECCENTRICITY - ULTIMATE CURVATURE DIAGRAM - S7

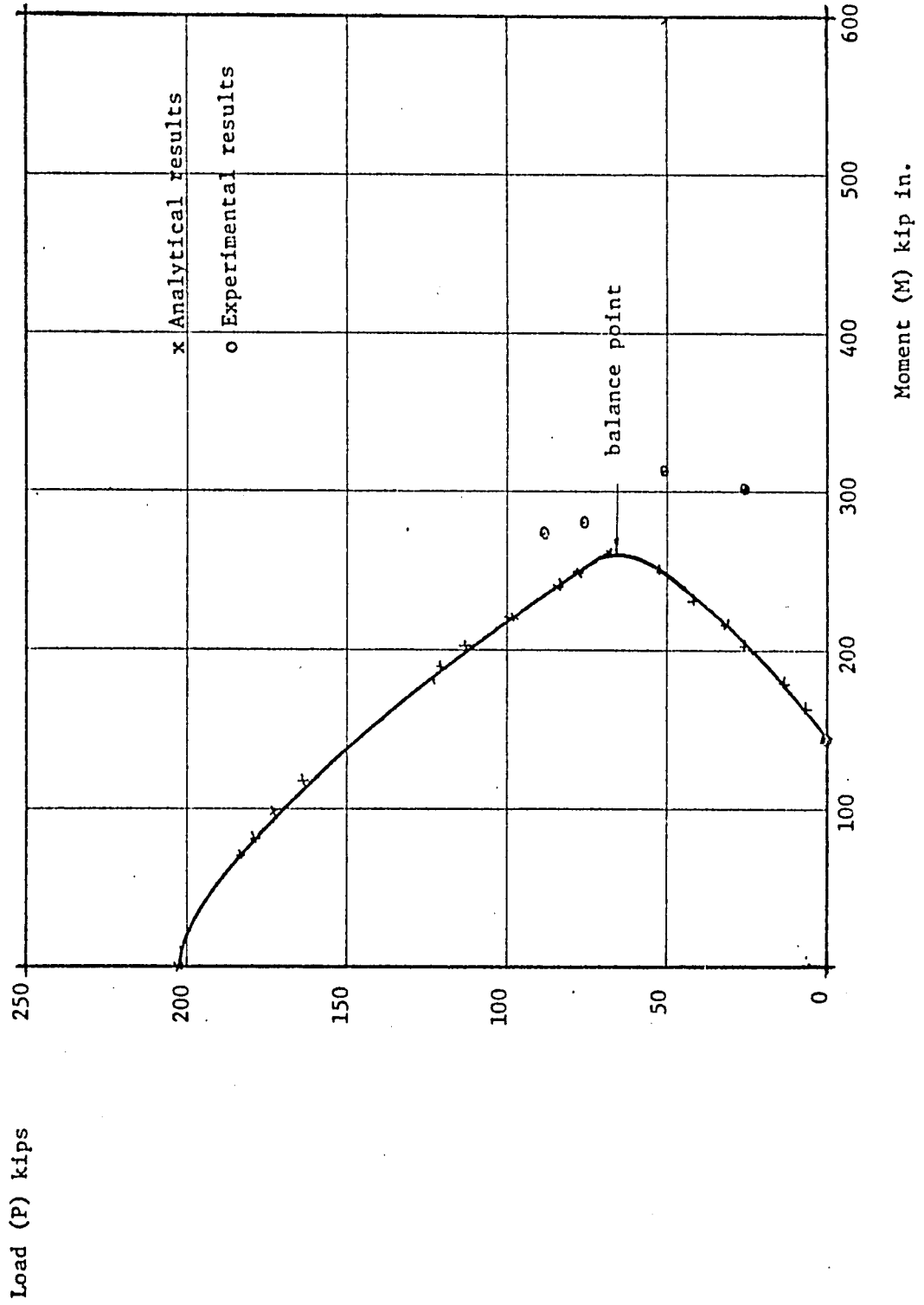


FIG. 45: AXIAL LOAD - MOMENT INTERACTION DIAGRAM - S8

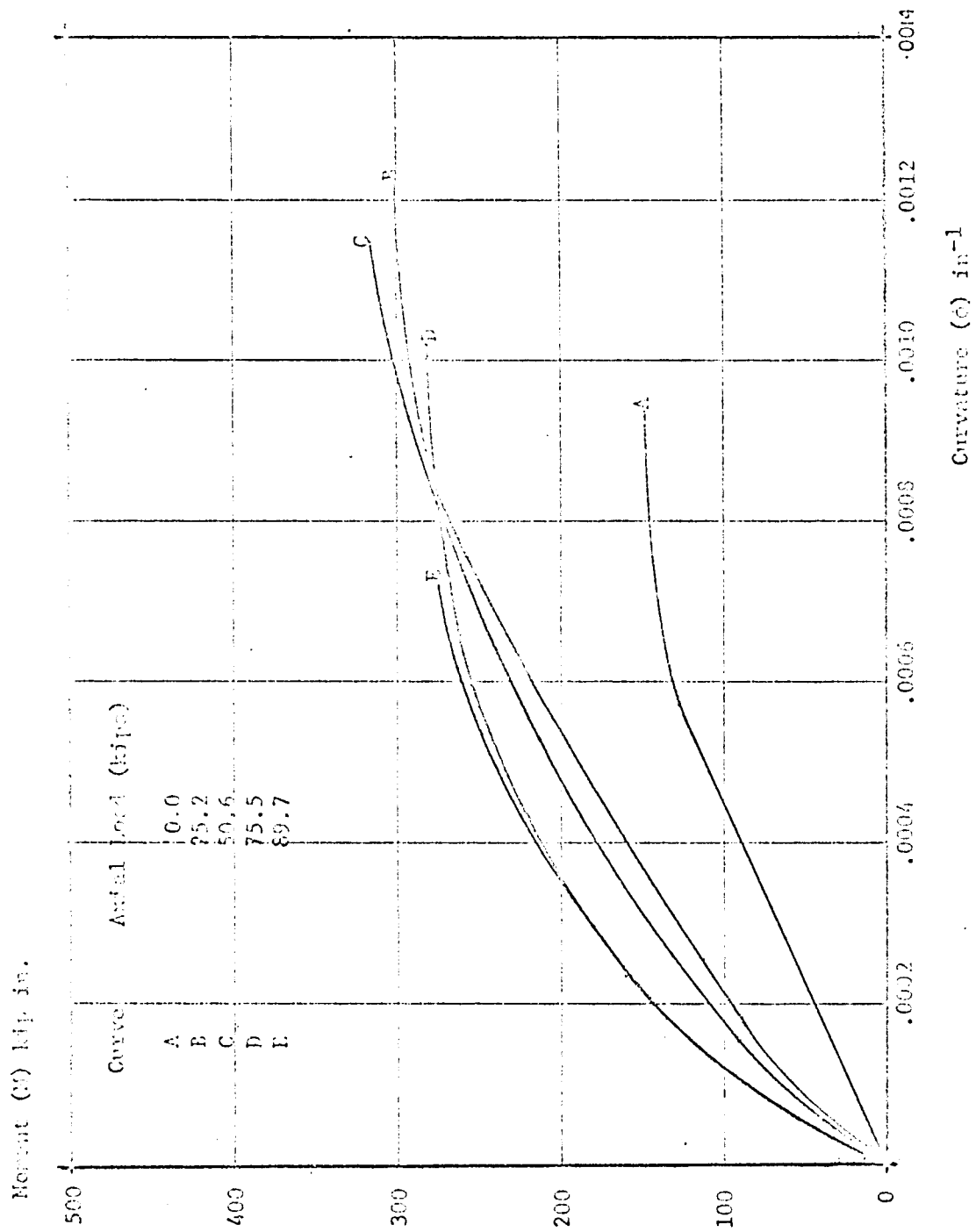


FIG. 46: MOMENT - CURVATURE DIAGRAM - S8

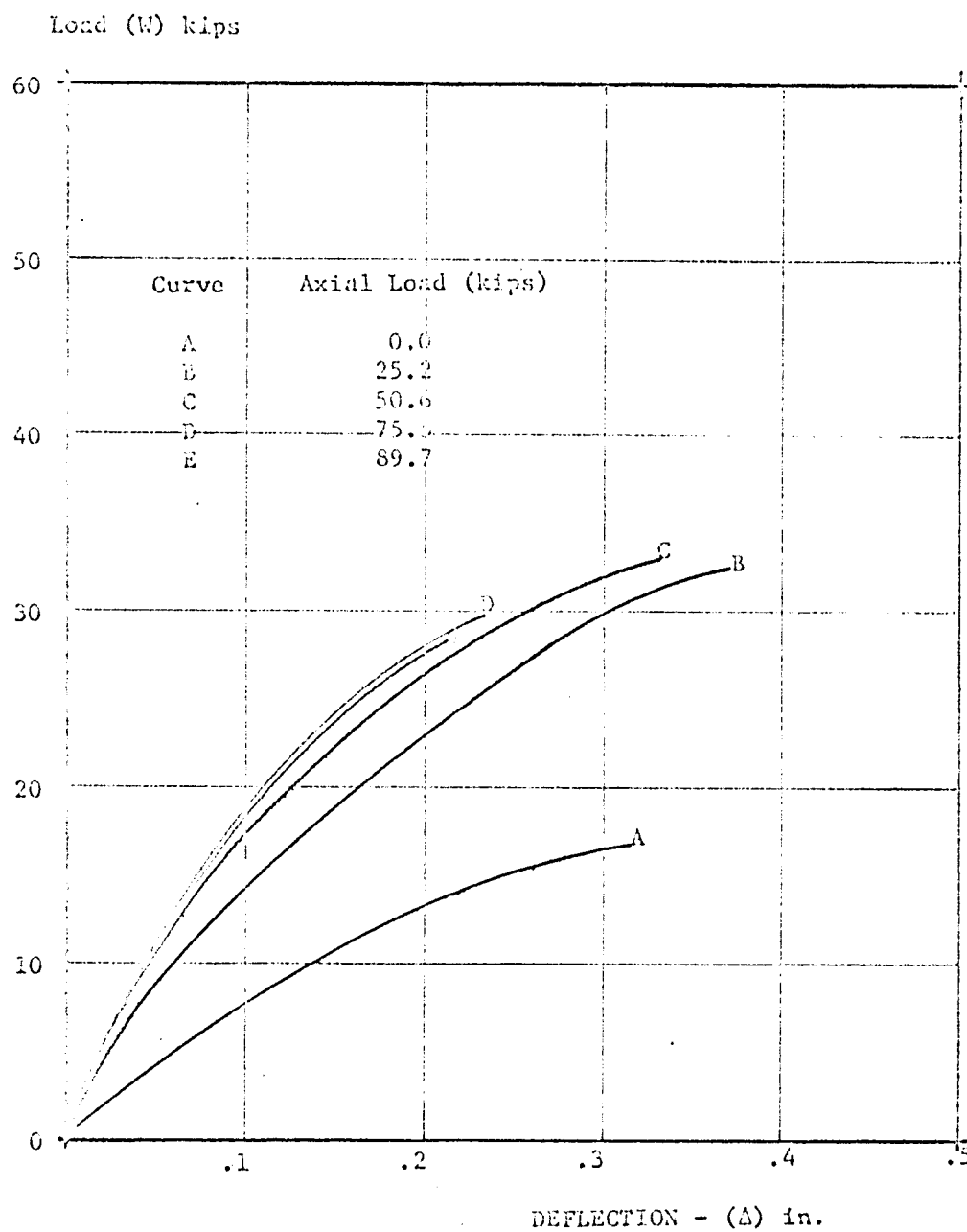


FIG. 47: LOAD - DEFLECTION DIAGRAM - S8

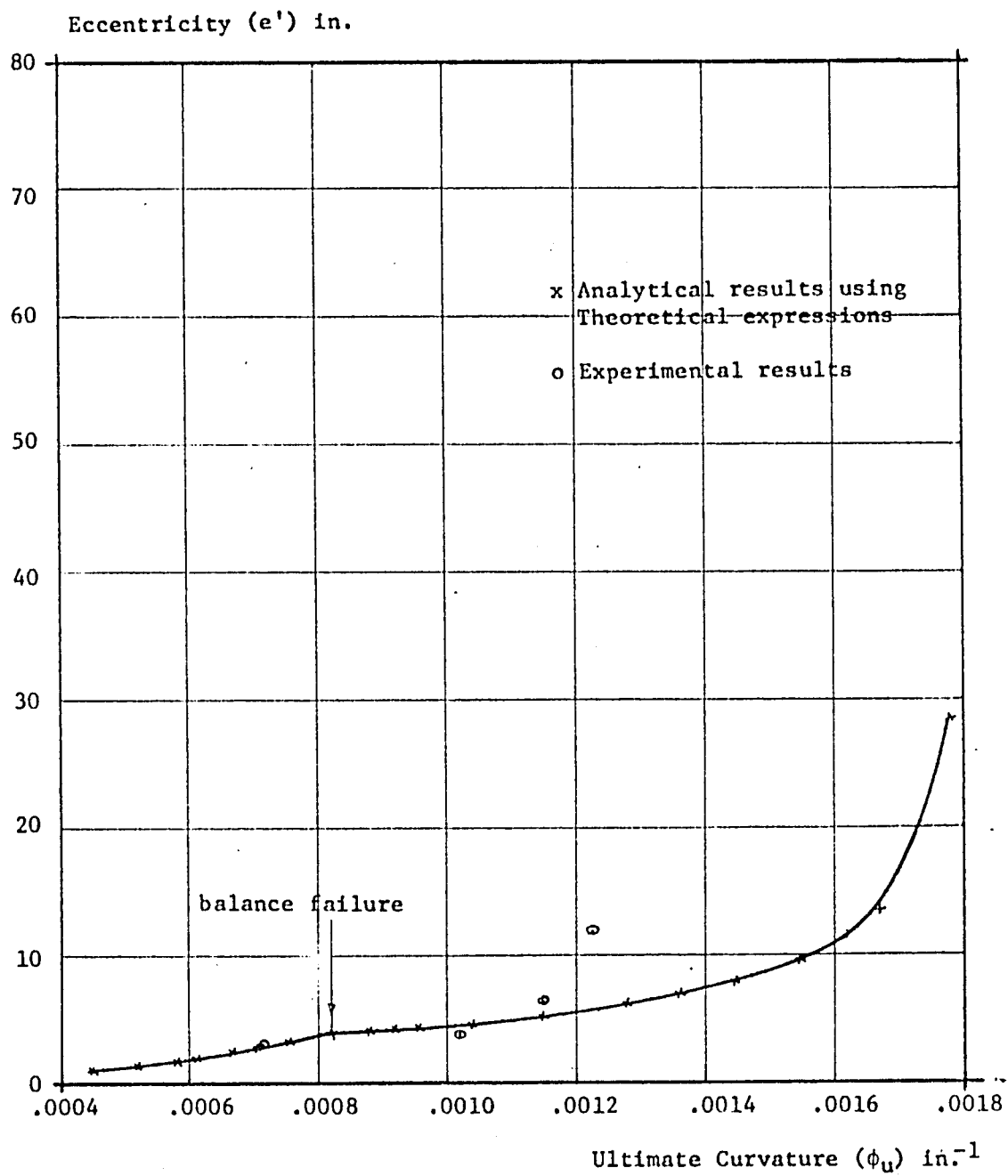


FIG. 48: ECCENTRICITY - ULTIMATE CURVATURE DIAGRAM - S8

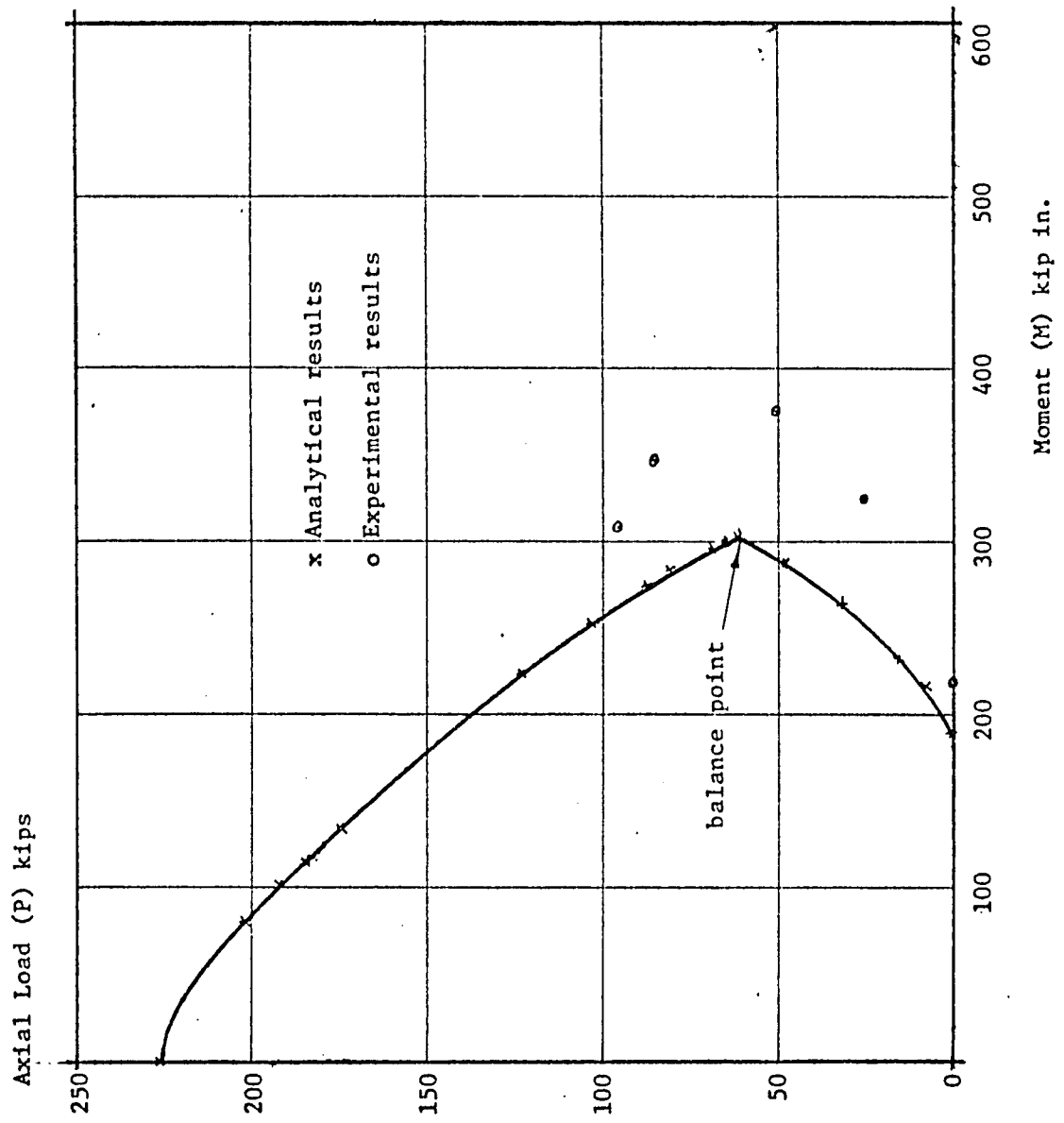


FIG. 49: AXIAL LOAD - MOMENT INTERACTION DIAGRAM - S9

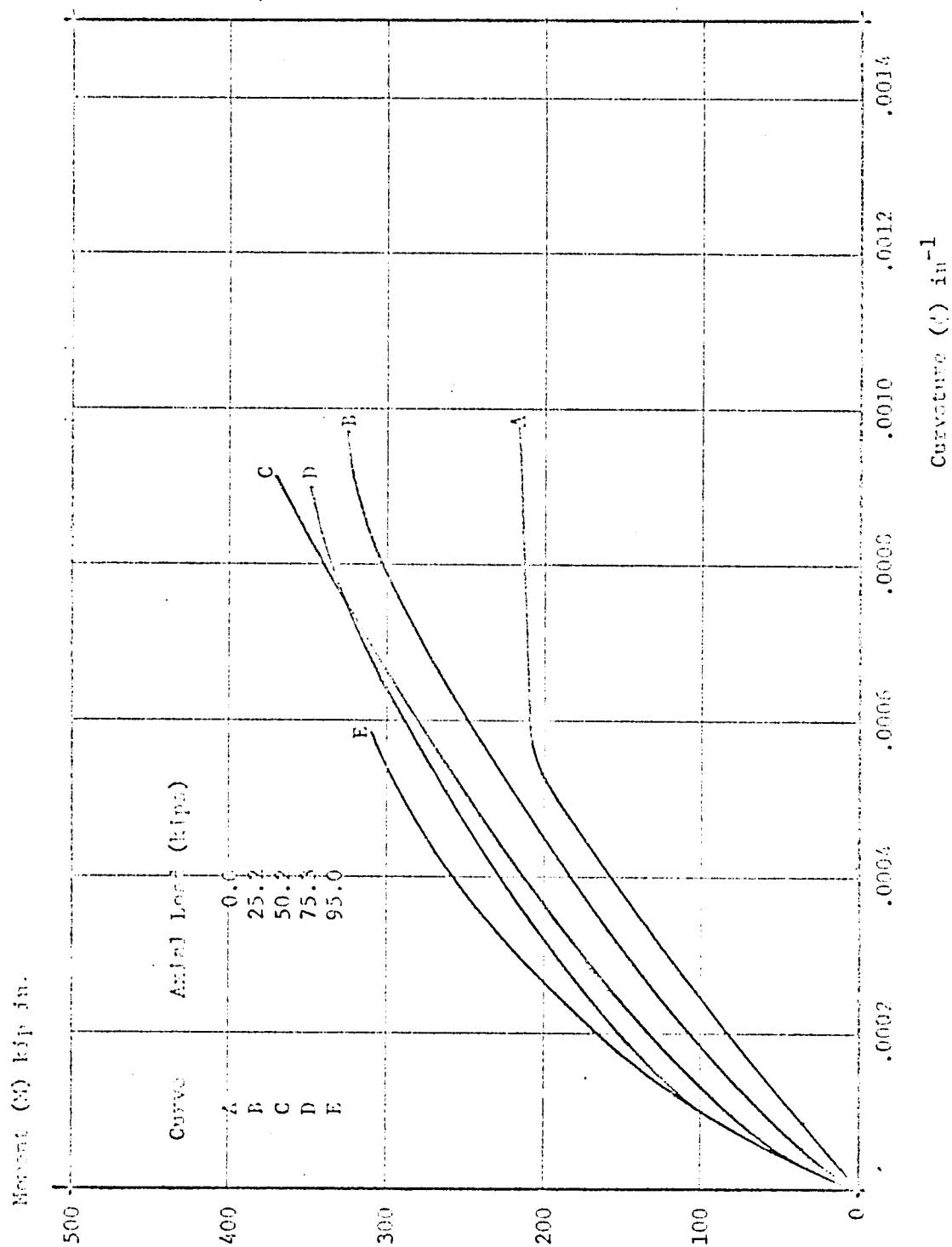


FIG. 50: MOMENT CURVATURE DIAGRAM - S9

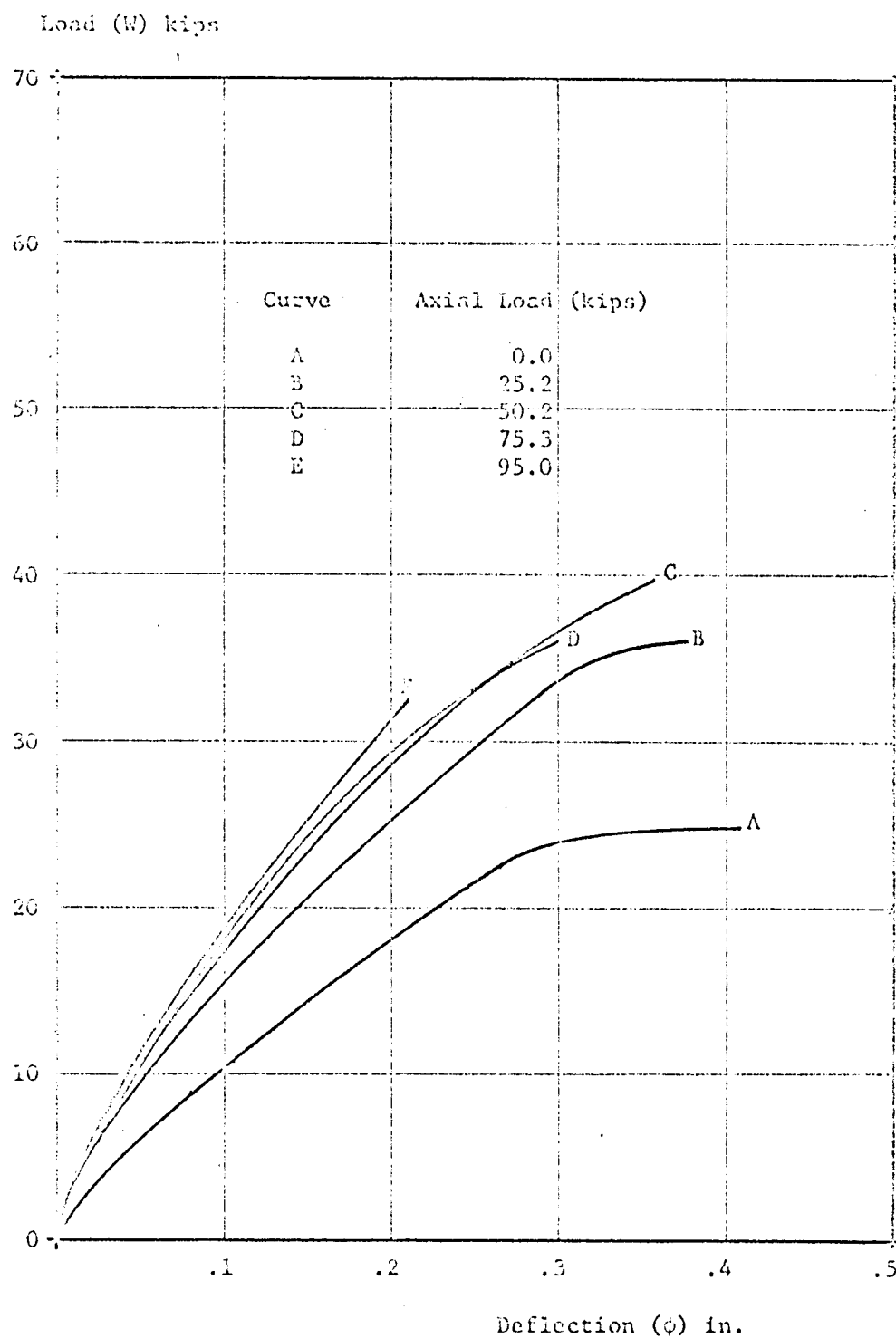


FIG. 51: LOAD - DEFLECTION DIAGRAM - S9

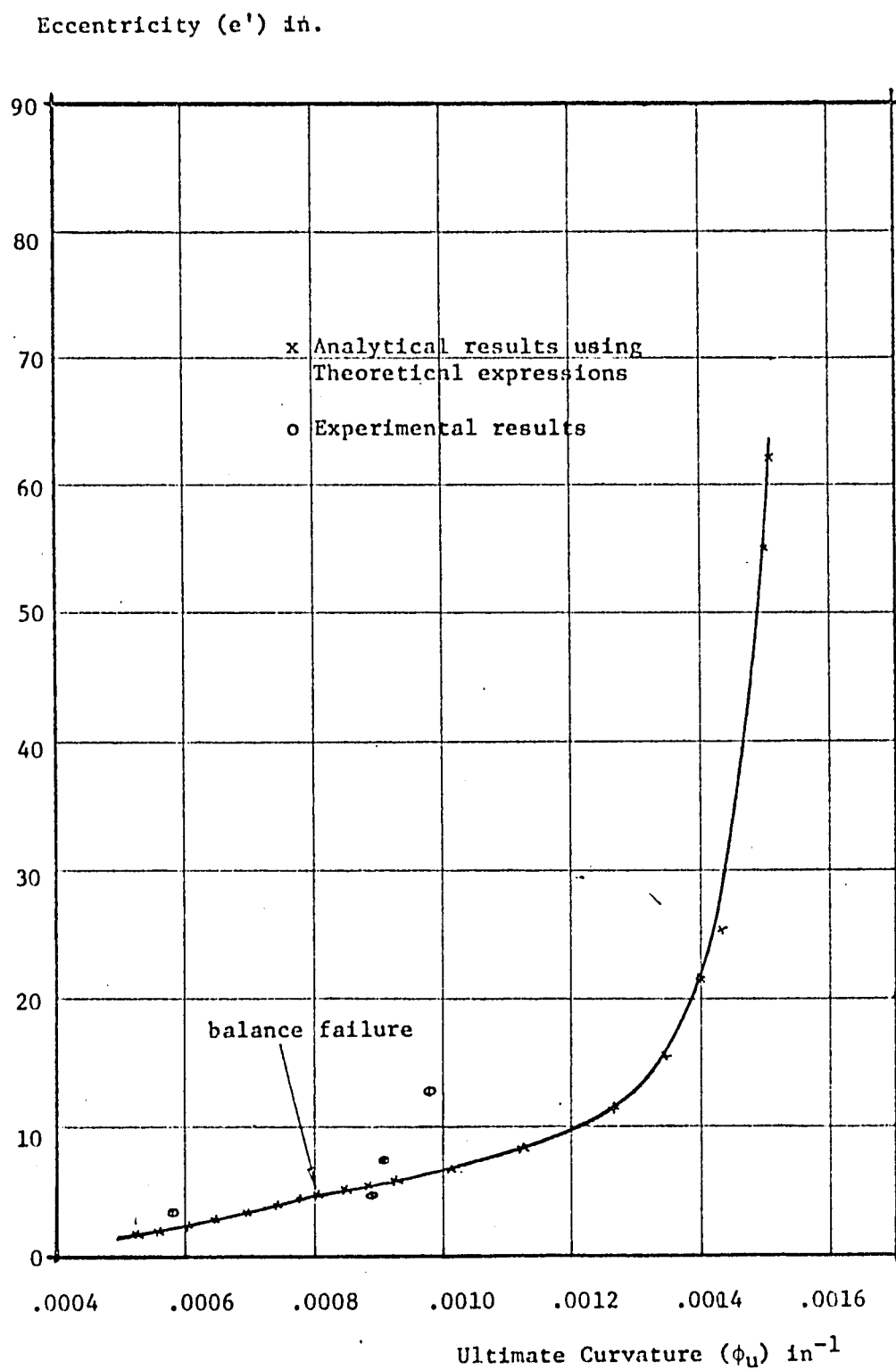


FIG. 52: ECCENTRICITY - ULTIMATE CURVATURE DIAGRAM - S9

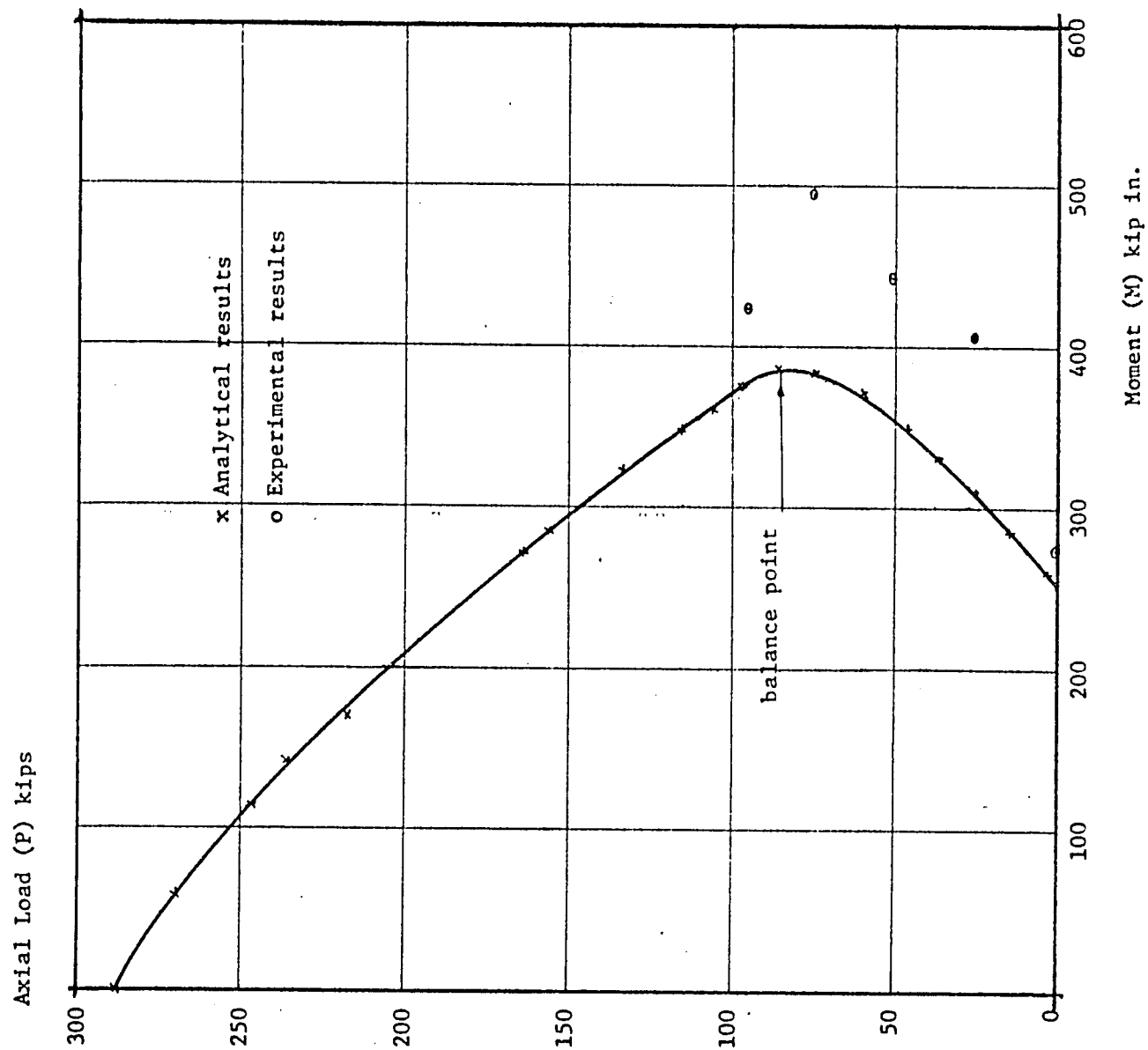


FIG. 53: AXIAL LOAD - MOMENT INTERACTION DIAGRAM - S10

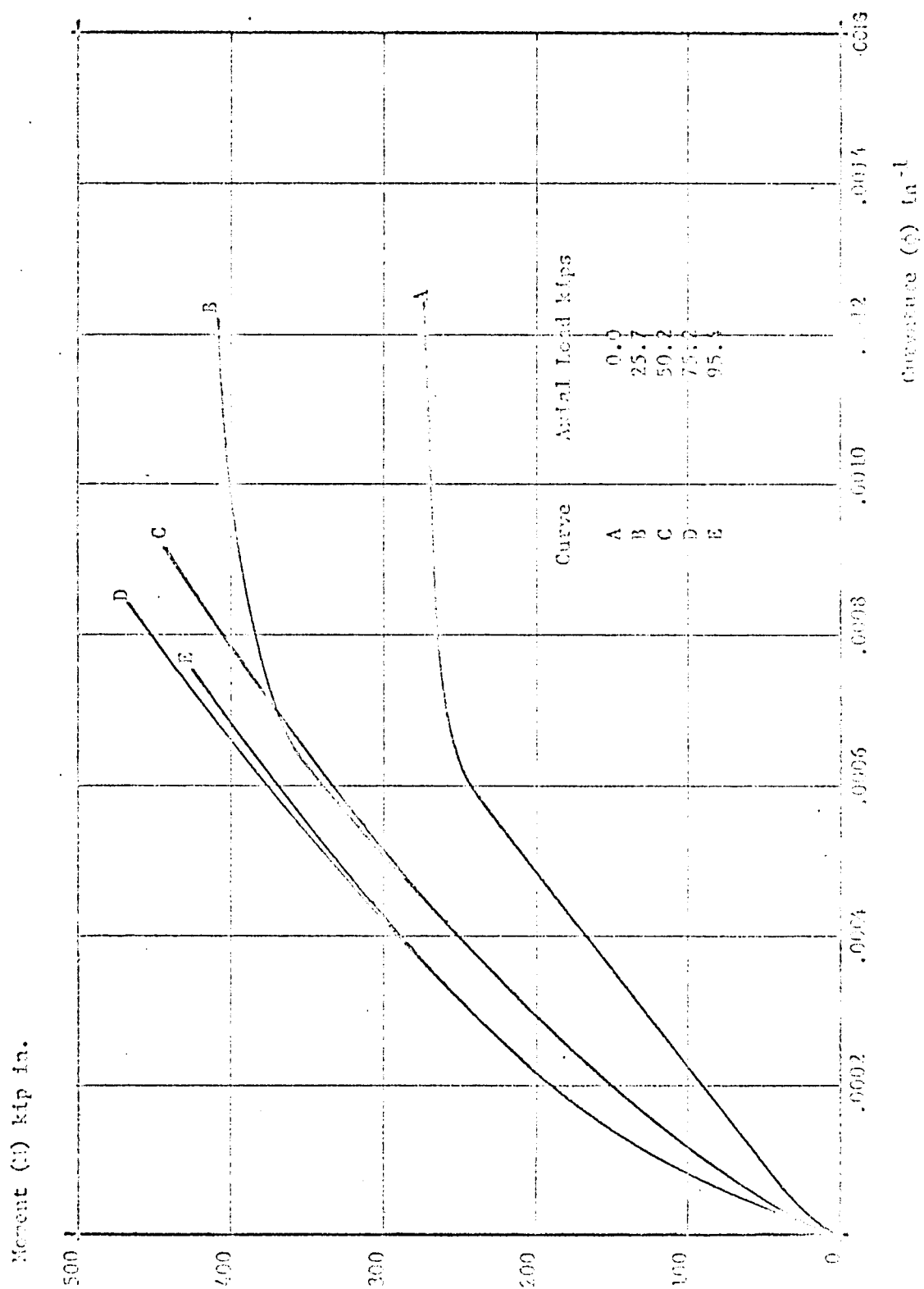


FIG. 54: MOMENT - CURVATURE DIAGRAM - S10

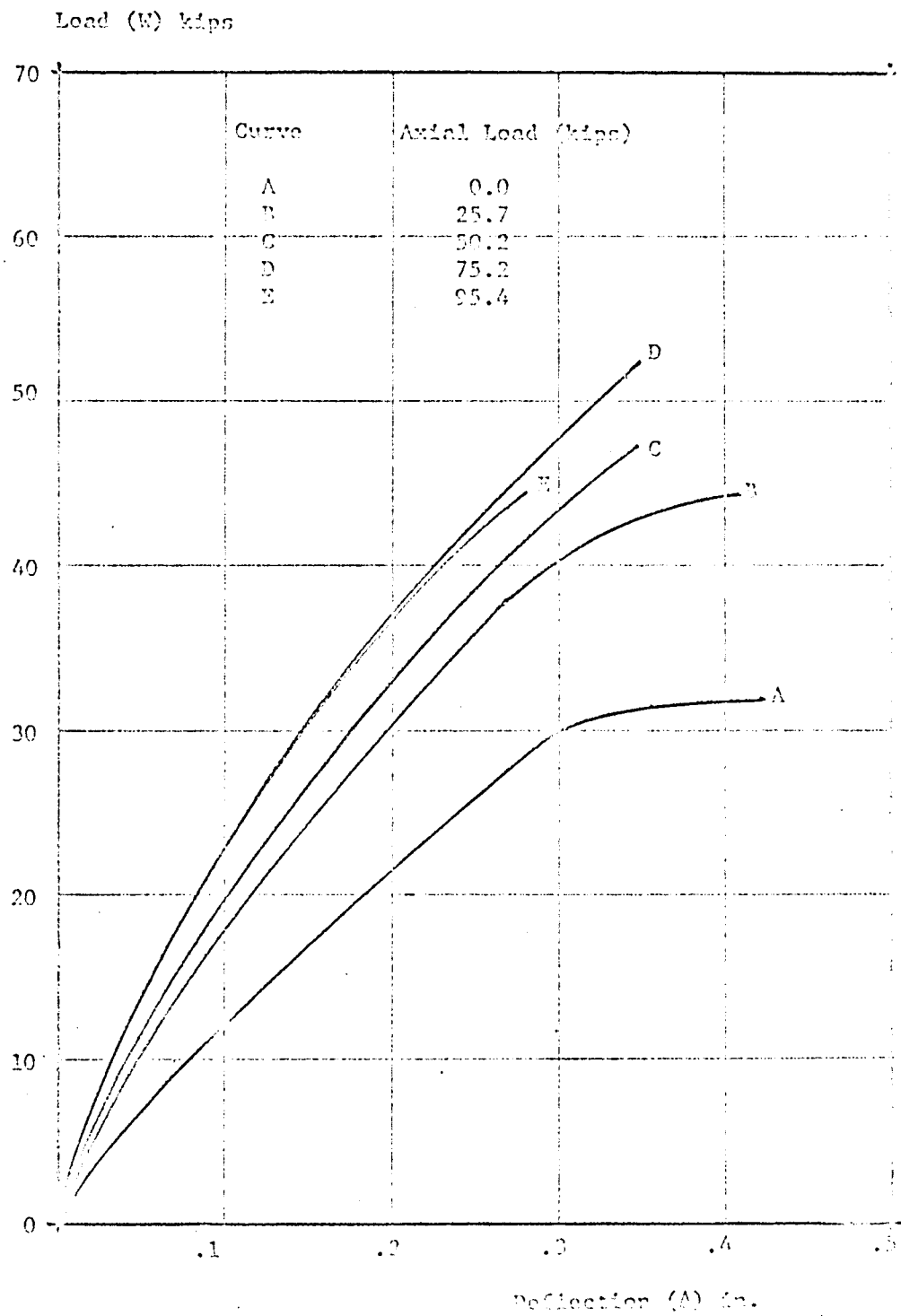


FIG. 55: LOAD - DEFLECTION DIAGRAM - S10

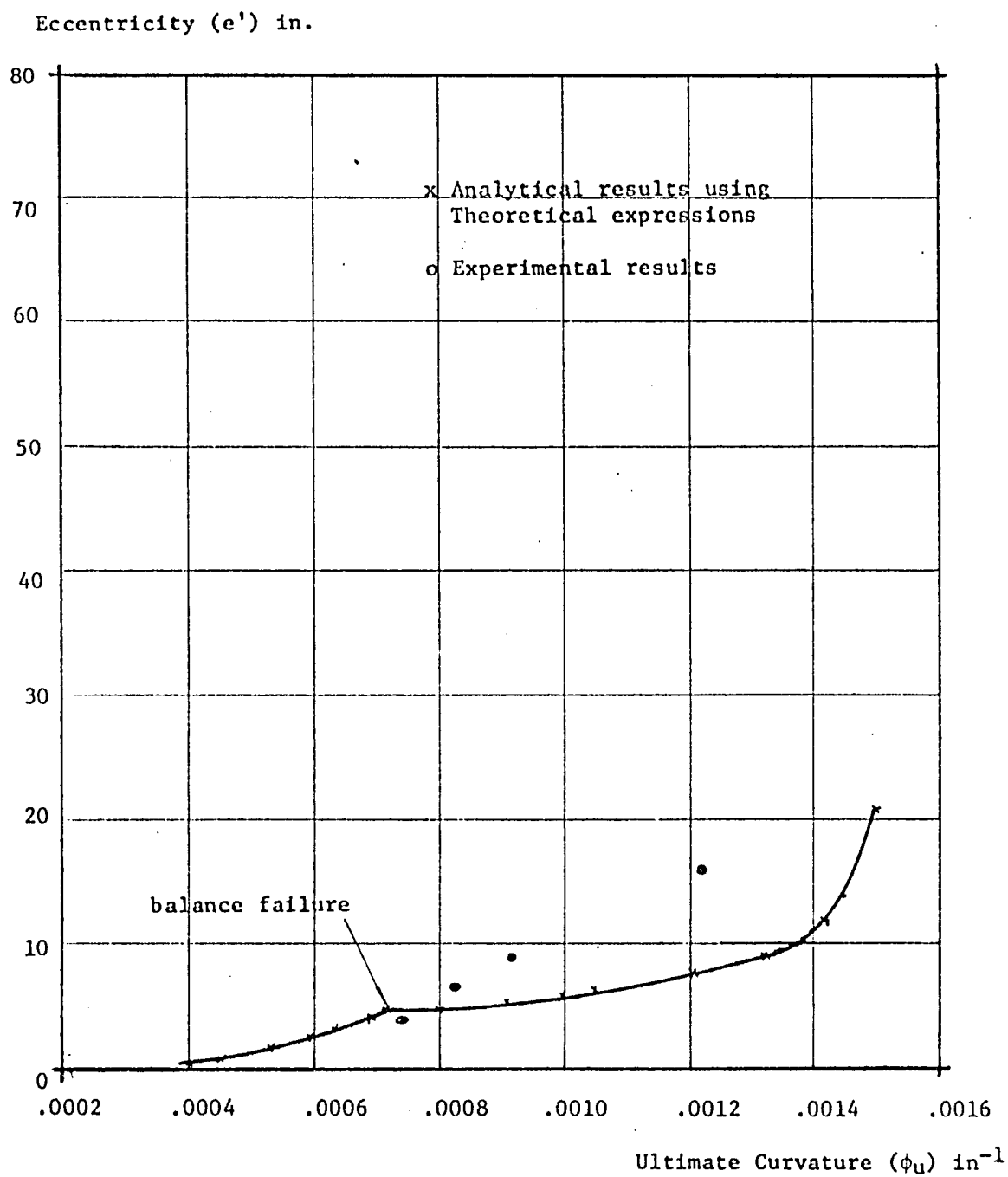


FIG. 56: ECCENTRICITY - ULTIMATE CURVATURE - S10

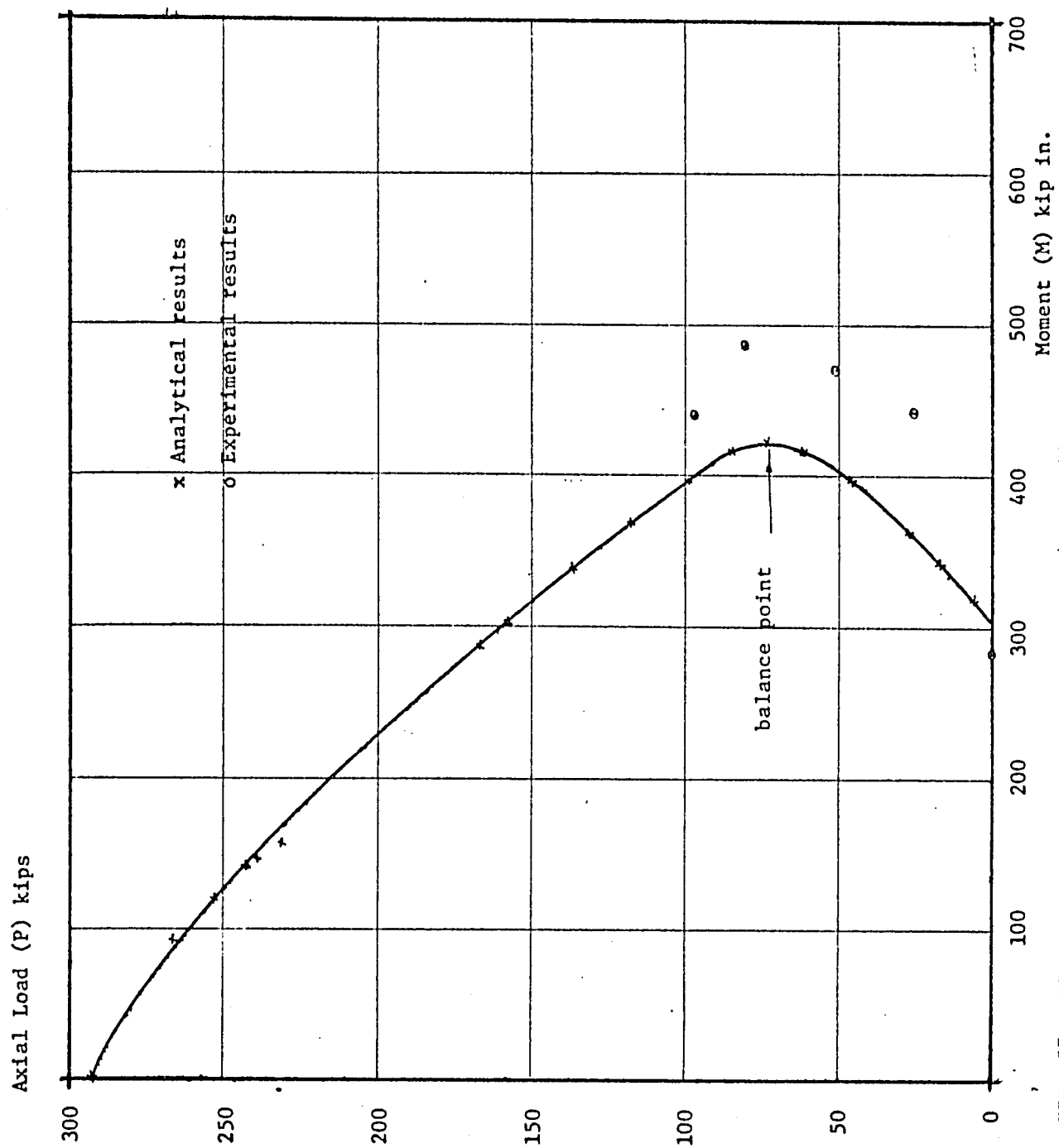


FIG. 57: AXIAL LOAD - MOMENT INTERACTION DIAGRAM - S11

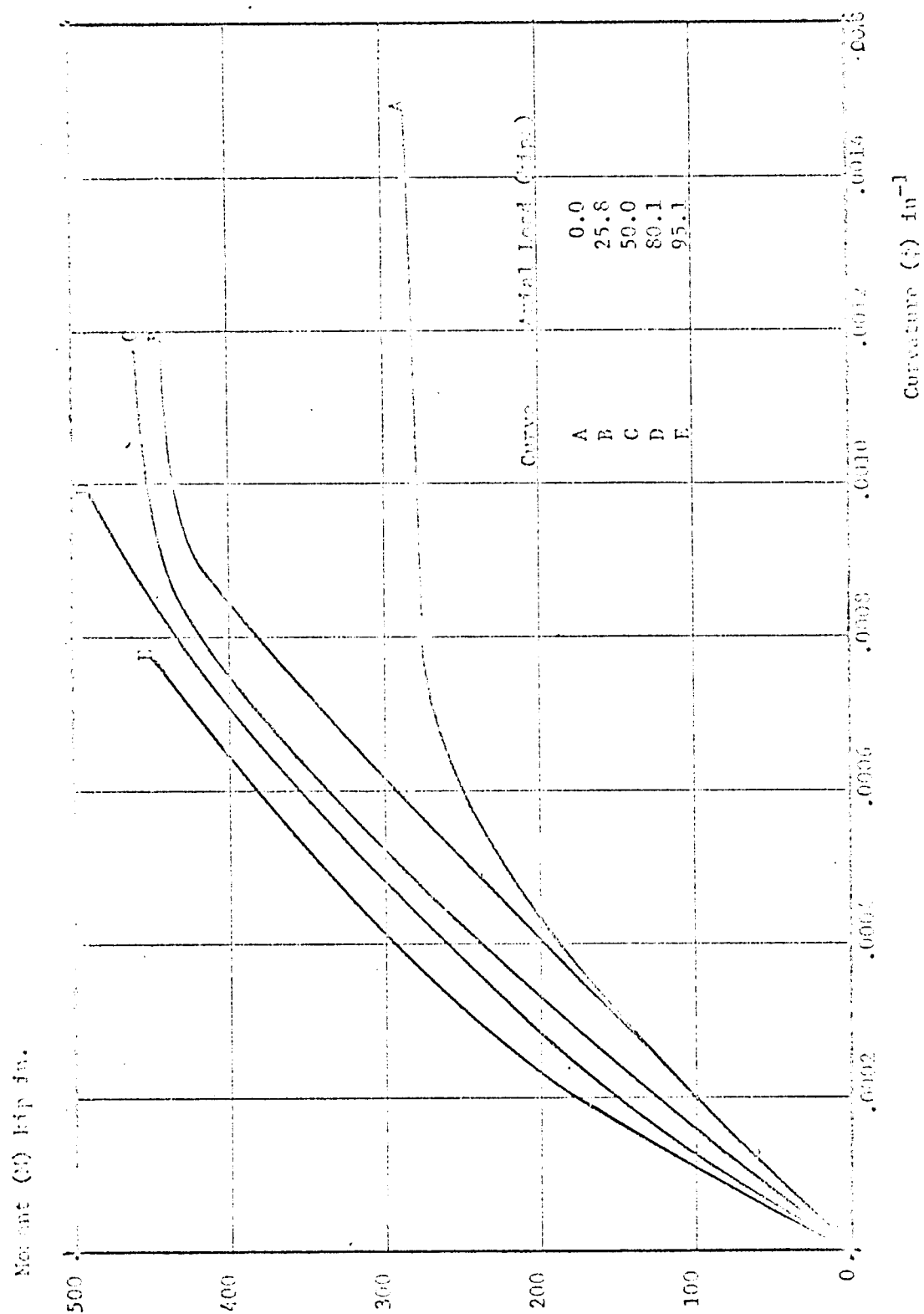


FIG. 58: MOMENT - CURVATURE DIAGRAM - S11

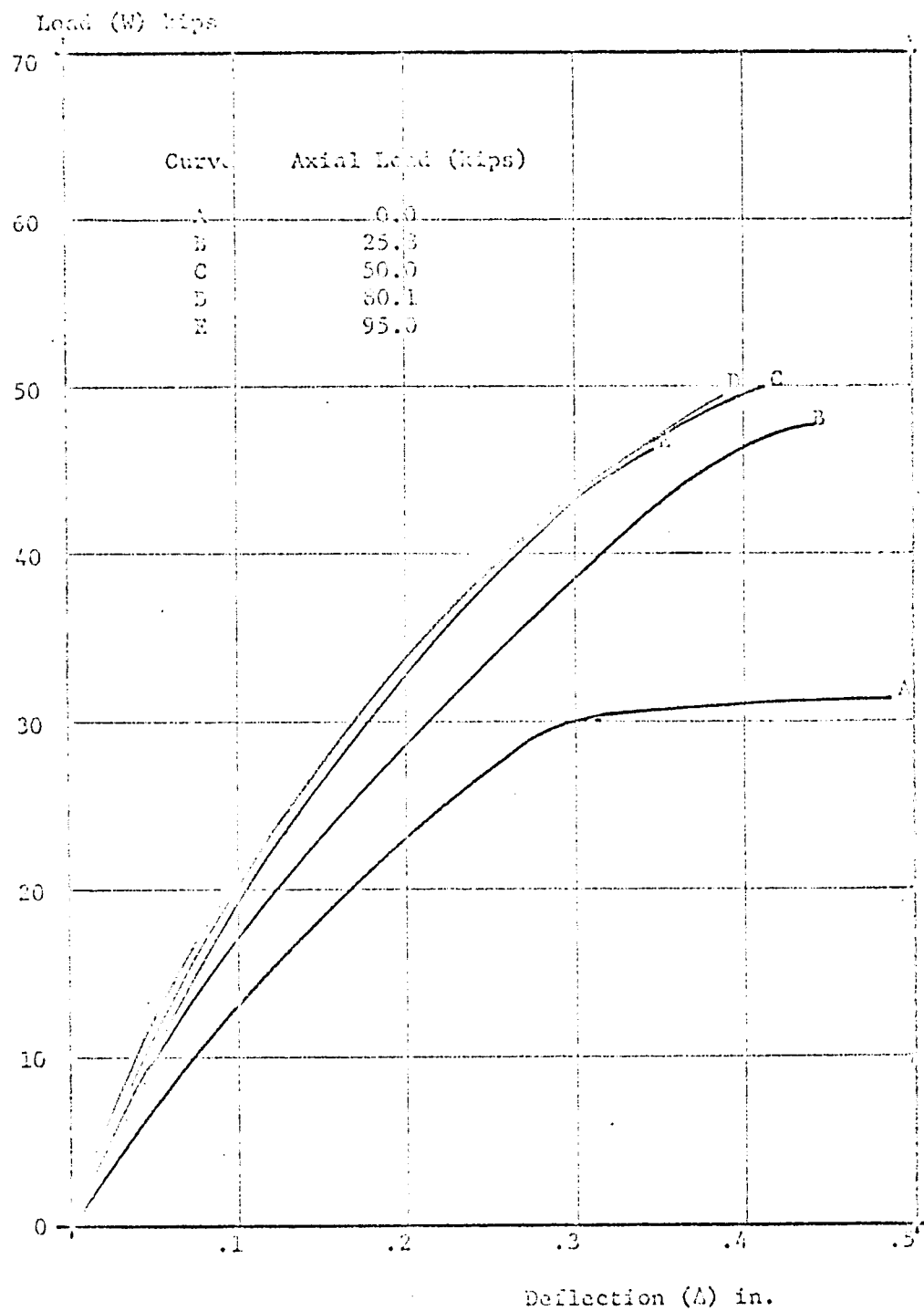


FIG. 59: LOAD - DEFLECTION DIAGRAM - S9

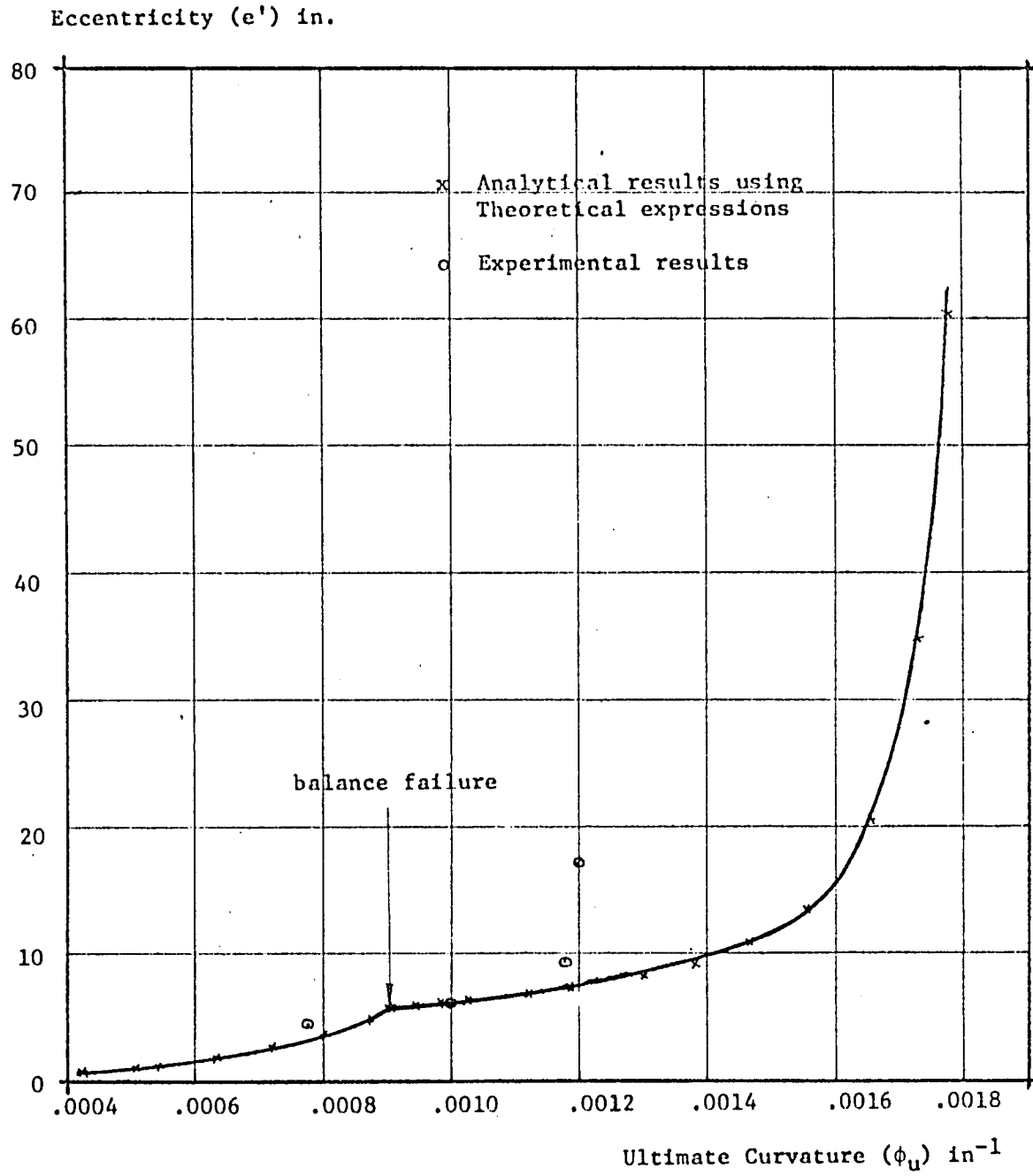
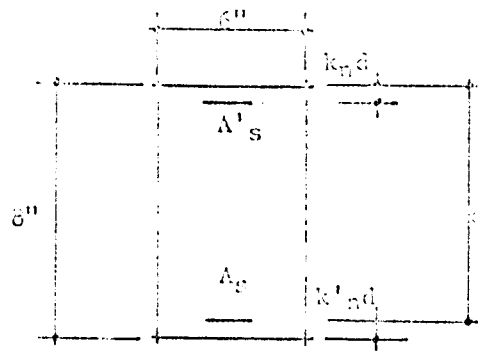


FIG. 60: ECCENTRICITY - ULTIMATE CURVATURE DIAGRAM - S11

APPENDIX C: TABLES OF EXPERIMENTAL RESULTS



Specimen Series	k_{nd} in	k'_{nd} in	d in	A_s in^2	A'_s in^2	f_y ksi	f'_y ksi	f'_c ksi
S1	1.0	1.50	6.50	0.40	0.10	44.2	43.0	3.86
S2	1.0	1.50	6.50	0.68	0.10	46.5	43.0	4.28
S3	1.0	1.50	6.50	1.20	0.10	43.9	43.0	4.39
S4	1.0	1.50	6.50	1.80	0.10	43.9	43.0	4.12
S5	1.5	2.25	5.75	1.58	0.62	42.0	46.2	4.42
S6	1.5	2.25	5.75	1.58	0.62	42.0	46.2	3.28
S7	1.5	2.25	5.75	2.00	0.88	49.8	46.5	4.22
S8	2.0	2.00	6.00	0.62	0.62	46.2	46.2	3.67
S9	2.0	2.00	6.00	0.88	0.63	46.5	46.5	3.68
S10	2.0	2.00	6.00	1.20	1.20	43.9	43.9	4.56
S11	2.0	2.00	6.00	1.58	1.58	42.0	42.0	4.21

TABLE 1: PROPERTIES OF TEST SPECIMENS

Speci- men Series	$k_n d$ in	$k'_n d$ in	d in	A_s in ²	A'_s in ²	p	p'	p-p'	$\frac{p_t - A_s + A'_s}{b d}$	q	q'	q'u=q-q'
S1	1.0	1.5	6.5	0.40	0.10	.0103	.0026	.0077	.0104	.1173	.0285	.0888
S2	1.0	1.5	6.5	0.88	0.10	.0226	.0026	.0200	.0204	.2451	.0257	.2193
S3	1.0	1.5	6.5	1.20	0.10	.0308	.0026	.0282	.0271	.3076	.0251	.2825
S4	1.0	1.5	6.5	1.80	0.10	.0462	.0026	.0436	.0396	.4910	.0267	.4642
S5	1.5	2.25	5.75	1.58	0.62	.0458	.0180	.0278	.0458	.4344	.1875	.2469
S6	1.5	2.25	5.75	1.58	0.62	.0458	.0180	.0278	.0458	.5858	.2529	.3329
S7	1.5	2.25	5.75	2.0	0.88	.0580	.0255	.0325	.0600	.7445	.2813	.4631
S8	2.0	2.0	6.0	0.62	0.62	.0172	.0172	0	.0258	.2163	.2166	0
S9	2.0	2.0	6.0	0.88	0.88	.0244	.0244	0	.0367	.3088	.3088	0
S10	2.0	2.0	6.0	1.20	1.20	.0333	.0333	0	.0500	.3203	.3203	0
S11	2.0	2.0	6.0	1.58	1.58	.0439	.0439	0	.0658	.4373	.4373	0

TABLE 2: DESIGN PROPERTIES OF TEST SPECIMENS

Batch	Specimen Series Name	Slump (in.)	Air Entrainment (%)
1	S1, S2, S3, S4, S5, S7, S10	3.25	5.75
2	S6, S8, S9, S11	2.5	6.0

TABLE 3: WORKABILITY AND AIR ENTRAINMENT OF CONCRETE

Aggregate	Percentage Retained on Sieve No.										Fineness Modulus	
	1"	2"	5/8"	3/8"	4	8	16	30	50	100	200	
Sand					3.2	16.7	30	44	69.5	90.0	97.7	2.53
Gravel		4	18	74	97						97.7	-

TABLE 4: SIEVE ANALYSIS OF AGGREGATES

Bar No.	Modulus of Elasticity, E_s psi x 10^6	Modulus of Elasticity, E_s psi x 10^6 (Avg.)	f_y ksi	$\bar{f}_y$ ksi	e_y 6" ga.	$\bar{e}_y$	Area Sq. In.	$\bar{f}_u$ ksi
2	31.93	31.93	43.0	43.0	.00135	.00135	.05	70.0
3	28.26 28.48	28.37	43.0 48.2	48.1	.00148 .00165	.00156	.11	84.2
4	31.13	31.13	44.2	44.2	.00136	.00136	.20	70.0
5	30.89 30.61	30.75	46.0 46.4	46.2	.00143 .00141	.00142	.31	71.0
6	29.44 30.91	30.17	46.5 46.6	46.5	.00155 .00144	.00149	.44	78.7
7	30.09 31.00	30.54	43.5 43.3	43.9	.00140 .00142	.00141	.60	75.3
8	30.82	30.82	42.0	42.0	.00135	.00135	.79	75.0
9	29.90	29.90	49.8	49.8	.00175	.00175	1.0	83.9

TABLE 5: TEST RESULTS OF STEEL REINFORCING BARS

TABLE 6: CONCRETE TEST PROPERTIES

Beam Series	Ult. Com- pressive Stress - Day of Test	f'c (Avg) Day of Test	Initial Modulus of Elasti- city, E _c 6" x 12" cylinders	Resonance Modulus of Elasti- city, E _d 3" x 3" x 12" prisms	$\bar{E}_c$ (Avg) $\bar{E}_d$ (Avg)	E _c = 1,800,000 +460 f'c	$\frac{E_c}{E_d}$	Beam age on Day of Test
	psi	psi	psi (x10 ⁶)	psi (x10 ⁶)	psi (x10 ⁶)	psi (x10 ⁶)	days	
S1	3713		3.84	4.59				
	3925	3867	3.82	4.68	3.84	4.65	.83	.93 12
	3961			4.74				
				4.58				
S2	4138			4.96				
	4314	4285	4.53	5.13	4.53	5.04	.90	.83 16
	4403			4.99				
				5.08				
S3	4403		4.58	5.07				
	4421	4391	4.18	5.03	4.39	5.08	.86	.87 18
	4350			5.16				
				5.05				
S4	4173		3.96	5.05				
	4138	4126	4.10	5.02	4.03	4.94	.82	.92 19
	4067			4.84				
				4.86				
S5	4597		4.09	5.05				
	4297	4427	4.40	5.03	4.25	4.95	.86	.90 21
	4385			4.85				
				4.86				

S6	3112	3283	3.26	4.42	3.28	4.45	3.31	.74	1.01	23
	3324		3.30	4.42						
	3413			4.49						
				4.46						
S7	4208		3.92	5.05	4.16	4.94	3.74	.85	.90	24
	4208	4220	4.39	5.02						
	4244			4.84						
				4.87						
S8	3625		3.55	4.52	3.43	4.42	3.49	.78	1.01	26
	3855	3678	3.41	4.34						
	3554			4.42						
				4.41						
S9	3607		3.43	4.52	3.55	4.42	3.49	.81	.99	27
	3731	3684	3.76	4.34						
	3713			4.42						
				4.40						
S10	4562		4.81	5.12	4.46	5.06	3.90	.88	.88	29
	4438	4568	4.12	5.07						
	4703			4.92						
				5.11						
S11	4261		4.00	4.31	4.05	4.34	3.73	.93	.92	30
	4067	4214	4.10	4.27						
	4314			4.37						
				4.40						

TABLE 6: CONCRETE TEST PROPERTIES (CONTD)

Specimen	Name	M_u	P_u	ϕ_u	e_u	$\bar{e}_u$	M_b	P_b
Series		(Test)	(Test)	(Test)	(Test)	(Avg.)	(calc.)	(calc.)
		kip in	kip	in ⁻¹	in/in	in/in	kip in	kip
S1	A	127.6	0.0	.00293	.0036			
	B	229.2	25.0	.00158	.0039			
	C	243.6	50.1	.00124	.0045	.0038	223.6	65.4
	D	261.6	75.2	.00105	.0038			
	E	182.4	99.8	.00082	.0031			
S2	A	230.0	0.0	.00102	.0033			
	B	309.4	20.1	.00082	.0032			
	C	340.0	40.0	.00080	.0034	.0034	301.5	48.1
	D	321.2	60.3	.00078	.0034			
	E	313.2	90.1	.00090	.0036			
S3	A	350.0	0.0	.00100	.0033			
	B	406.3	20.1	.00120	.0036			
	C	375.2	40.1	.00095	.0029	.0033	338.6	38.0
	D	338.0	60.0	.00073	.0031			
	E	308.1	90.3	.00068	.0034			
S4	A	404.0	0.0	.00052	.0032			
	B	444.4	10.1	.00098	.0036			
	C	397.9	30.1	.00068	.0030	.0033	379.7	6.7
	D	340.5	60.1	.00052	.0034			
	E	325.0	90.1	.00059	.0032			
S5	A	296.4	0.0	.00088	.0030			
	B	417.6	20.3	.00086	.0033			
	C	405.7	40.0	.00077	.0032	.0032	365.7	31.0
	D	401.7	60.0	.00070	.0033			
	E	382.2	90.3	.00068	.0029			
S6	A	281.3	0.0	.00087	.0029			
	B	377.8	20.2	.00117	.0047			
	C	324.2	40.0	.00080	.0032	.0034	319.8	13.1
	D	289.2	60.4	.00058	.0026			
	E	287.0	90.2	.00071	.0035			

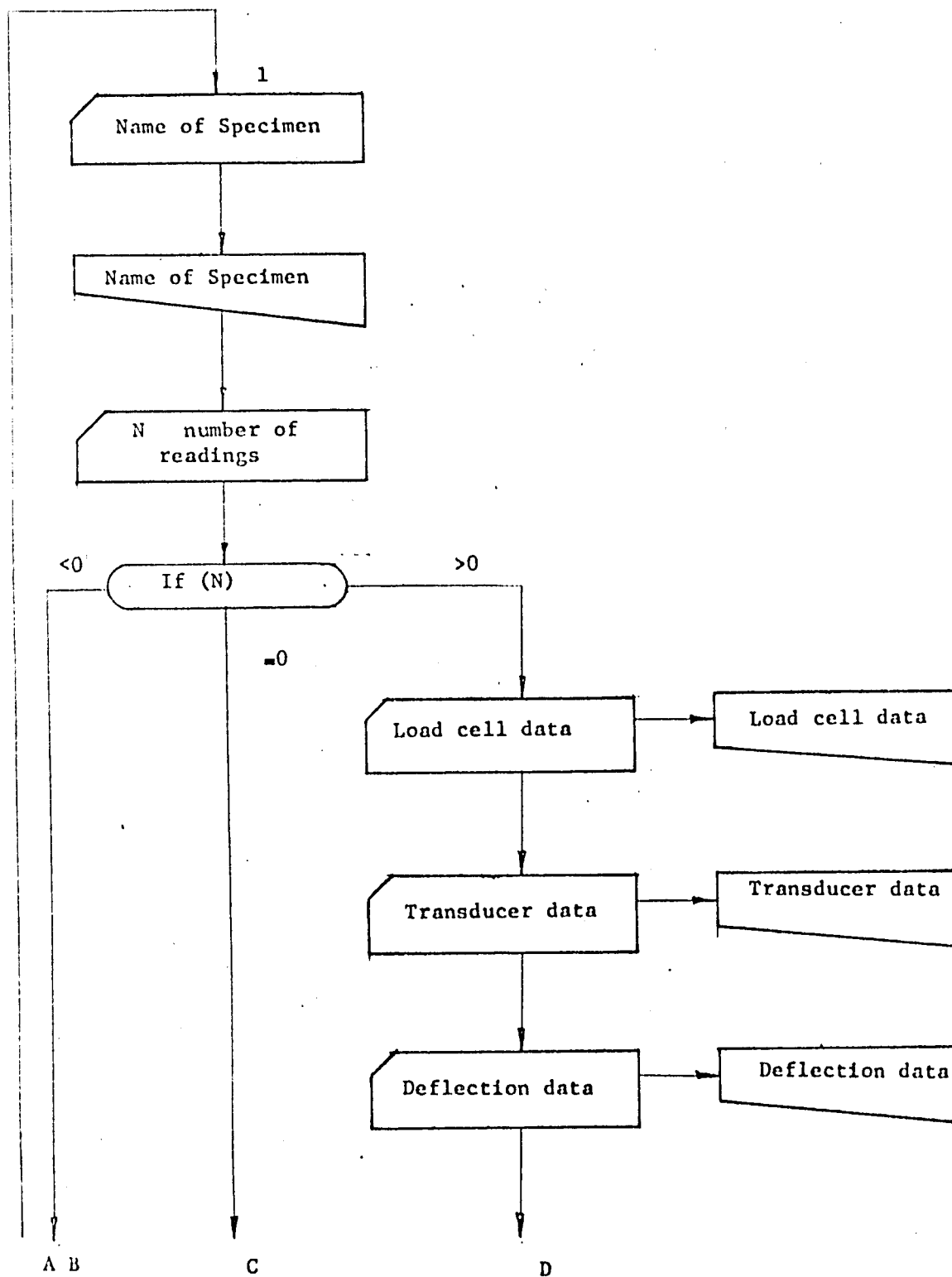
TABLE 7: SUMMARY OF TEST RESULTS

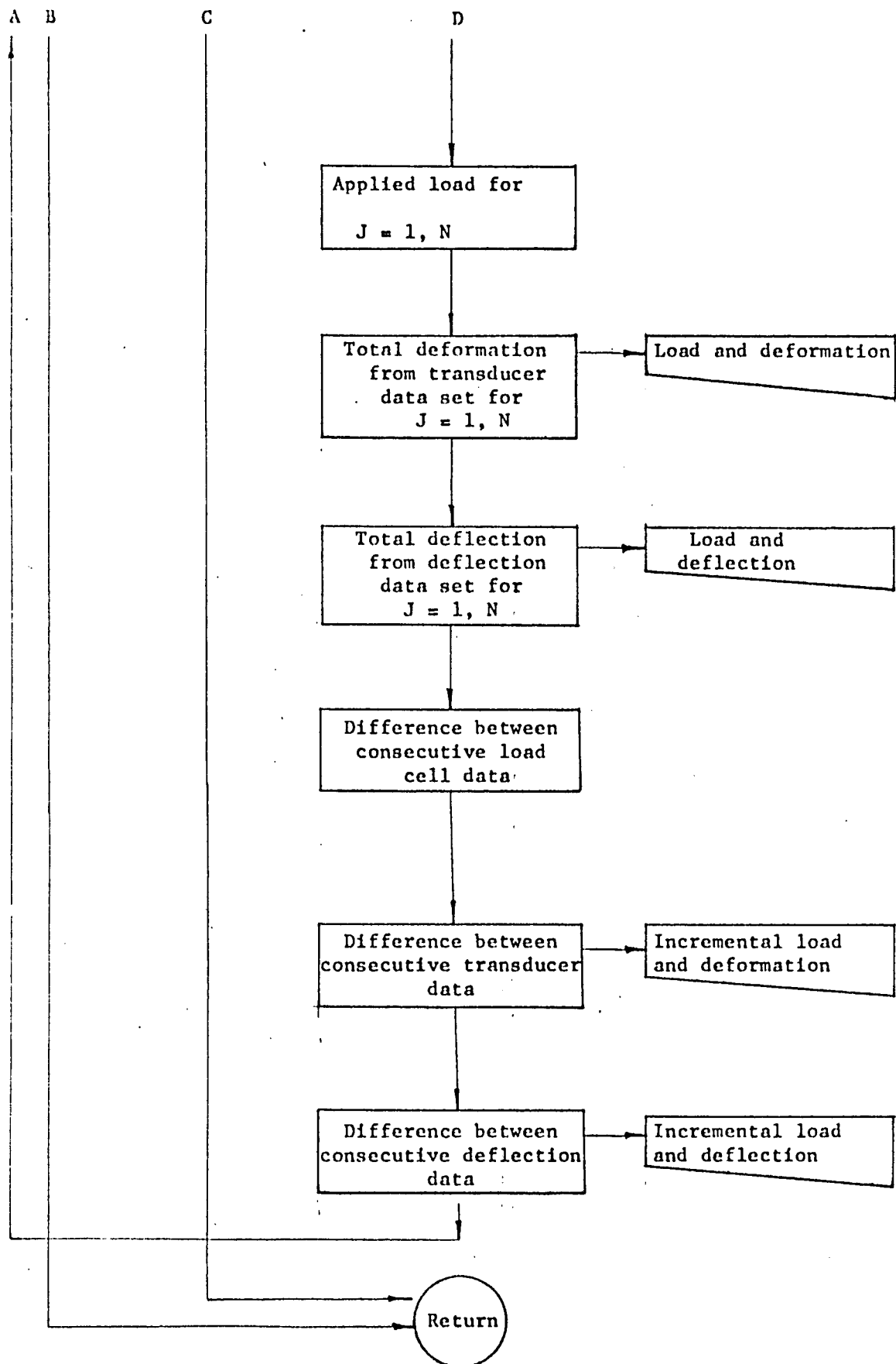
S7	A	343.8	0.0	.00094	.0032	.0033	444.7	11.1
	B	468.0	10.4	.00095	.0034			
	C	482.9	30.0	.00101	.0038			
	D	478.4	60.2	.00087	.0035			
	E	337.8	90.2	.00057	.0025			
S8	A	145.8	0.0	.00093	.0022	.0035	260.2	67.4
	B	301.9	25.2	.00123	.0035			
	C	315.1	50.5	.00115	.0043			
	D	281.3	75.5	.00102	.0042			
	E	274.7	89.6	.00072	.0033			
S9	A	217.8	0.0	.00097	.0023	.0032	305.5	61.8
	B	324.0	25.2	.00098	.0031			
	C	375.0	50.2	.00091	.0037			
	D	348.9	75.3	.00089	.0036			
	E	309.1	95.0	.00058	.0033			
S10	A	273.2	0.0	.00124	.0030	.0032	388.5	86.7
	B	408.3	25.6	.00122	.0034			
	C	443.4	50.2	.00091	.0033			
	D	496.8	75.2	.00117	.0032			
	E	425.0	95.4	.00074	.0031			
S11	A	283.3	0.0	.00148	.0036	.0038	425.1	74.5
	B	442.5	25.8	.00119	.0038			
	C	470.0	50.0	.00117	.0044			
	D	488.1	80.1	.00092	.0039			
	E	440.4	95.0	.00077	.0035			

TABLE 7: SUMMARY OF TEST RESULTS (CONTD)

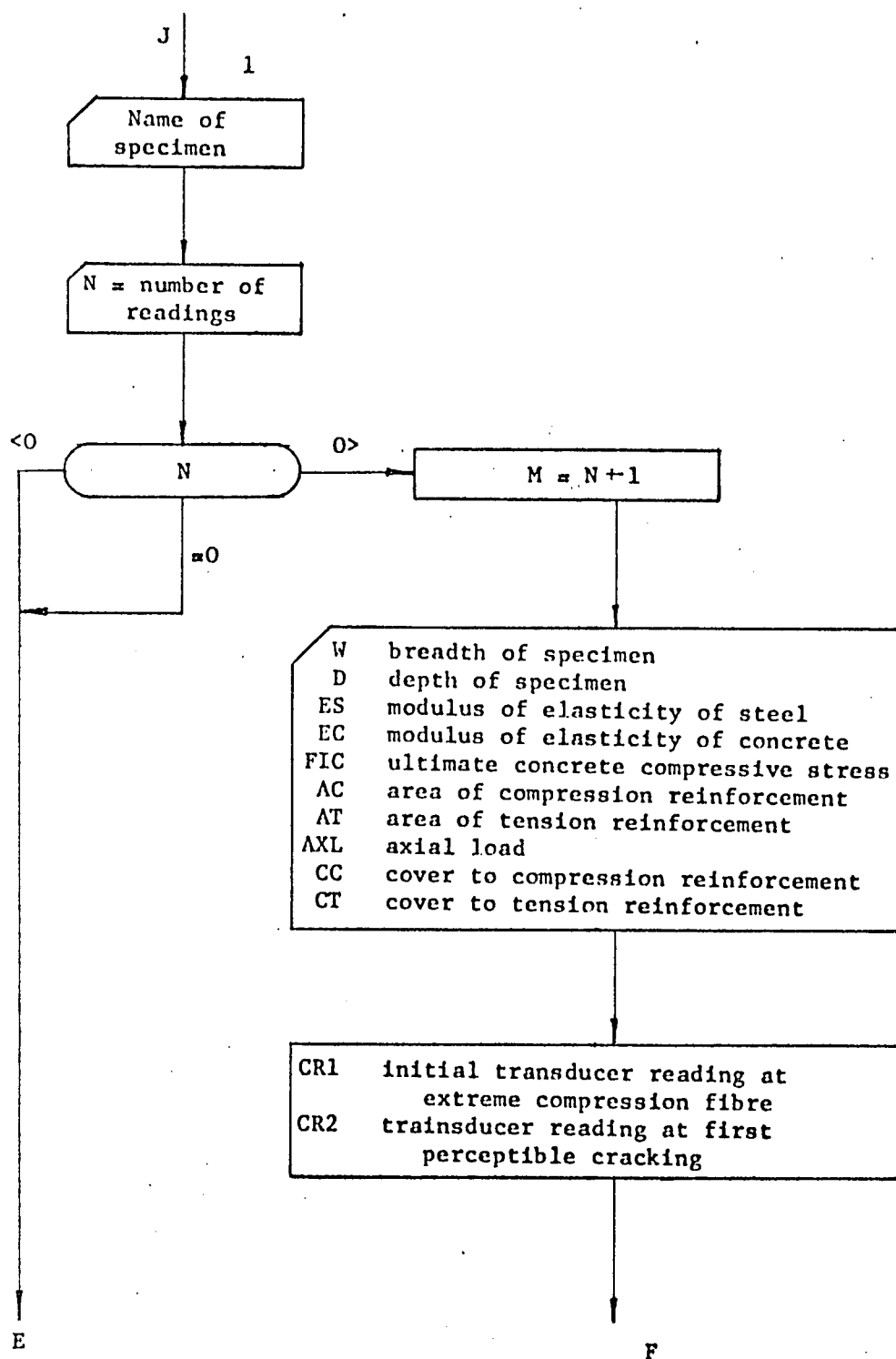
APPENDIX D: COMPUTER FLOW DIAGRAMS

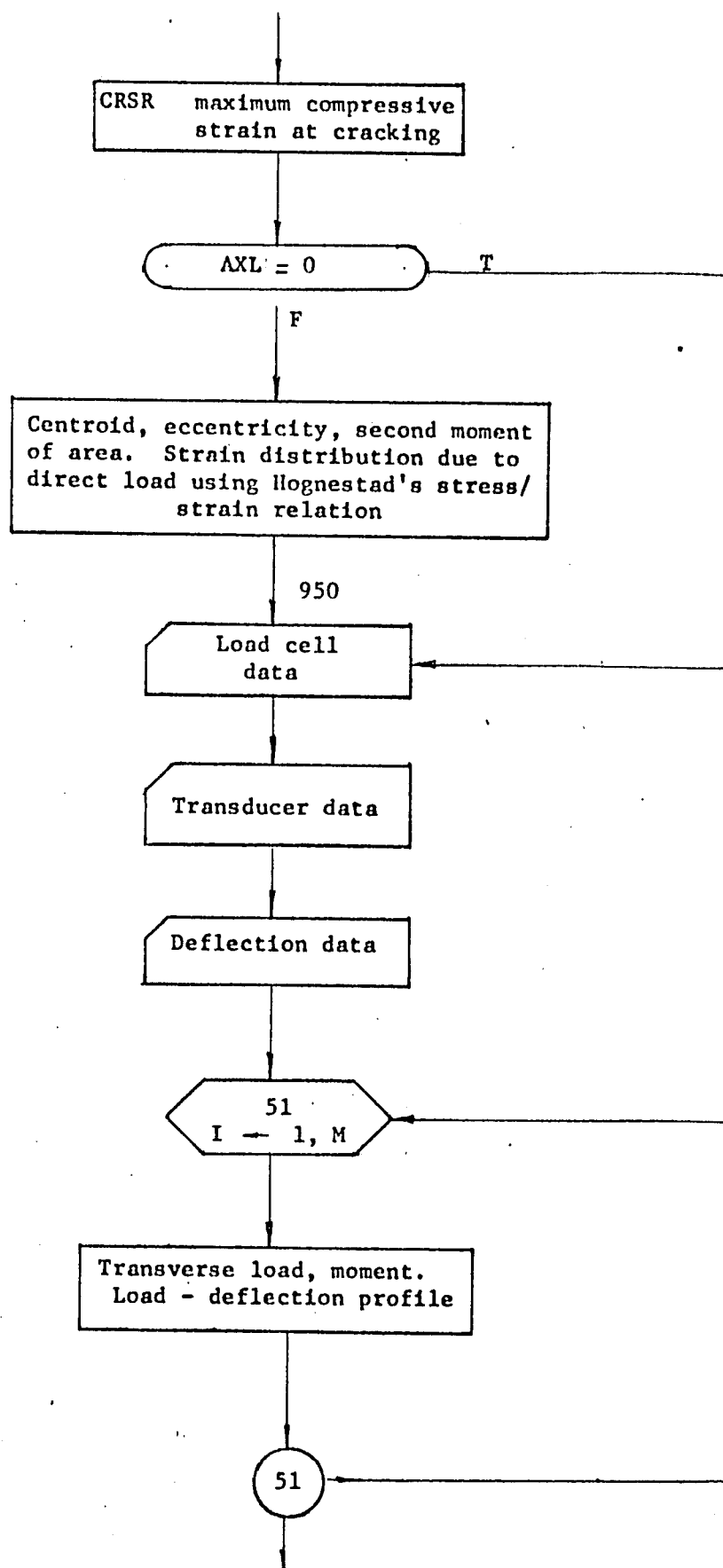
FLOW DIAGRAM - PROGRAM I

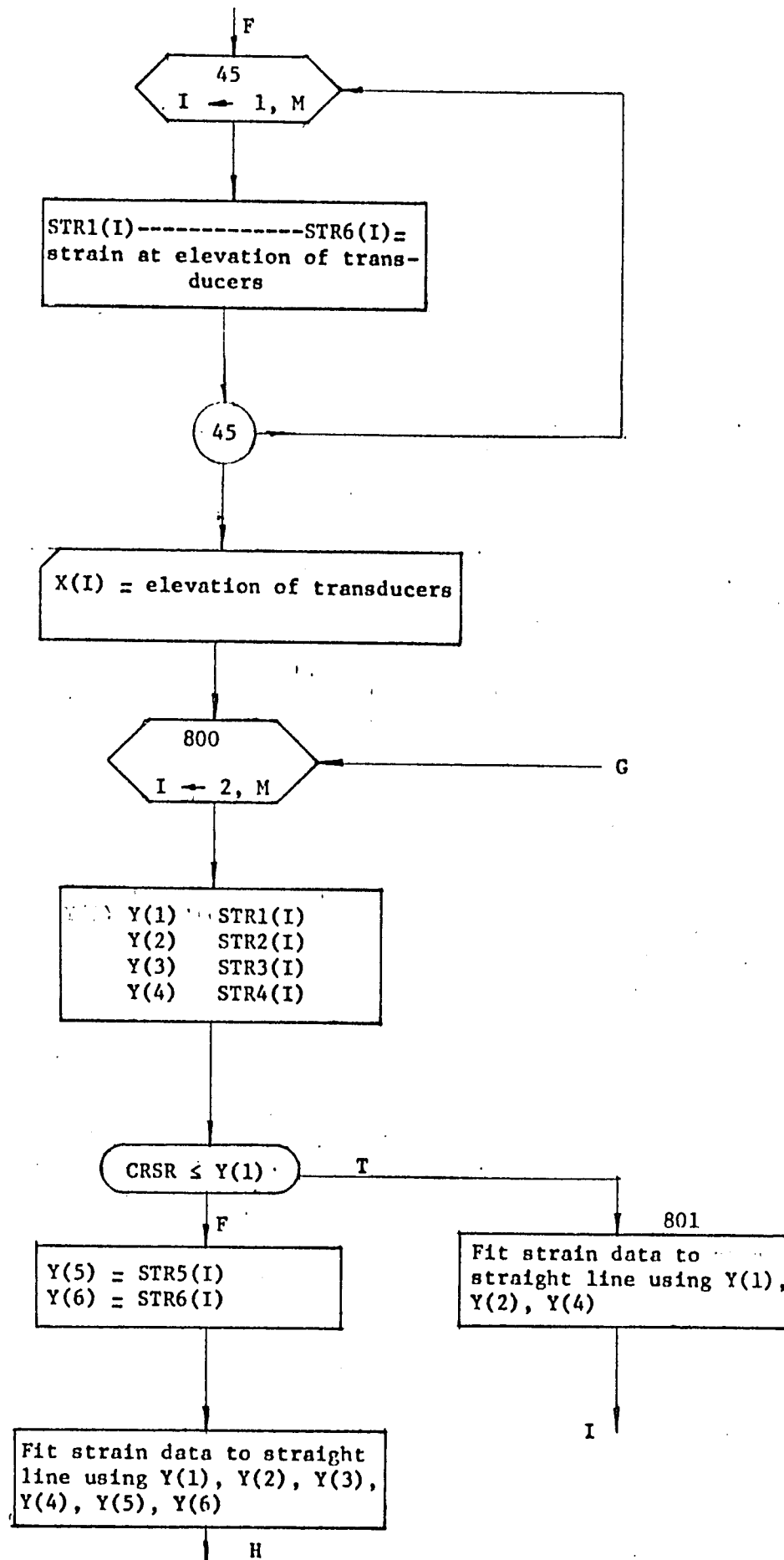


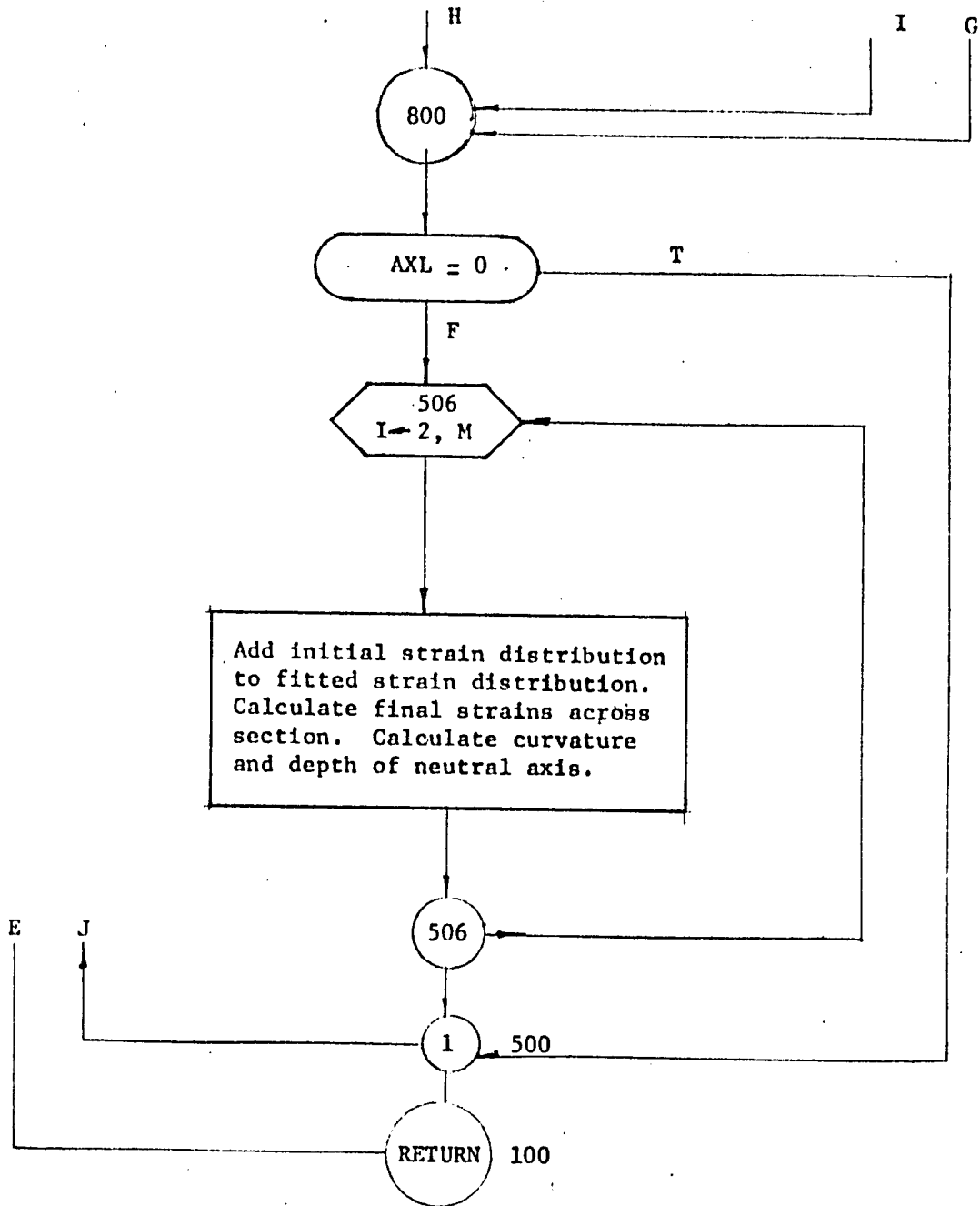


FLOW DIAGRAM - PROGRAM II

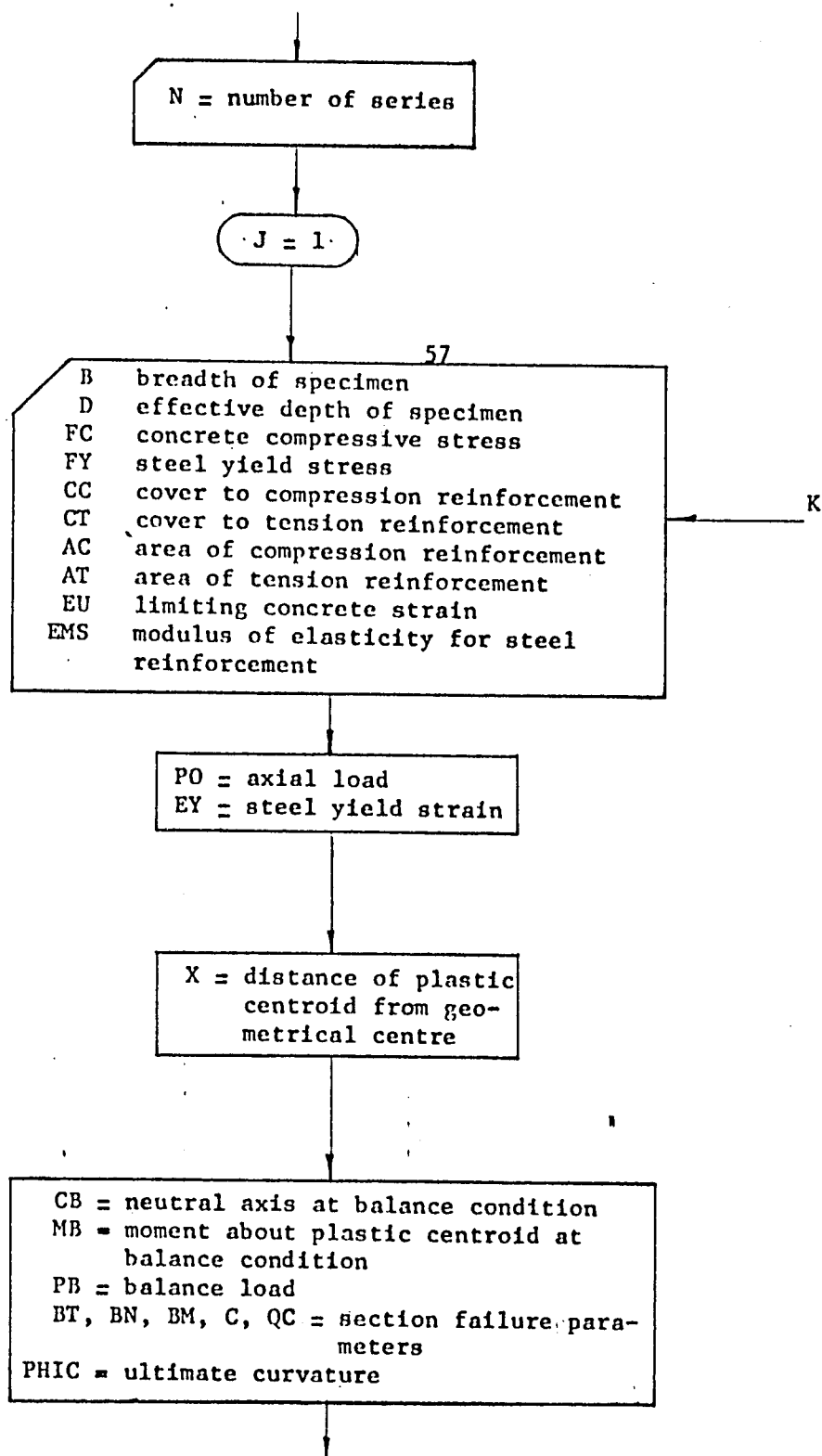


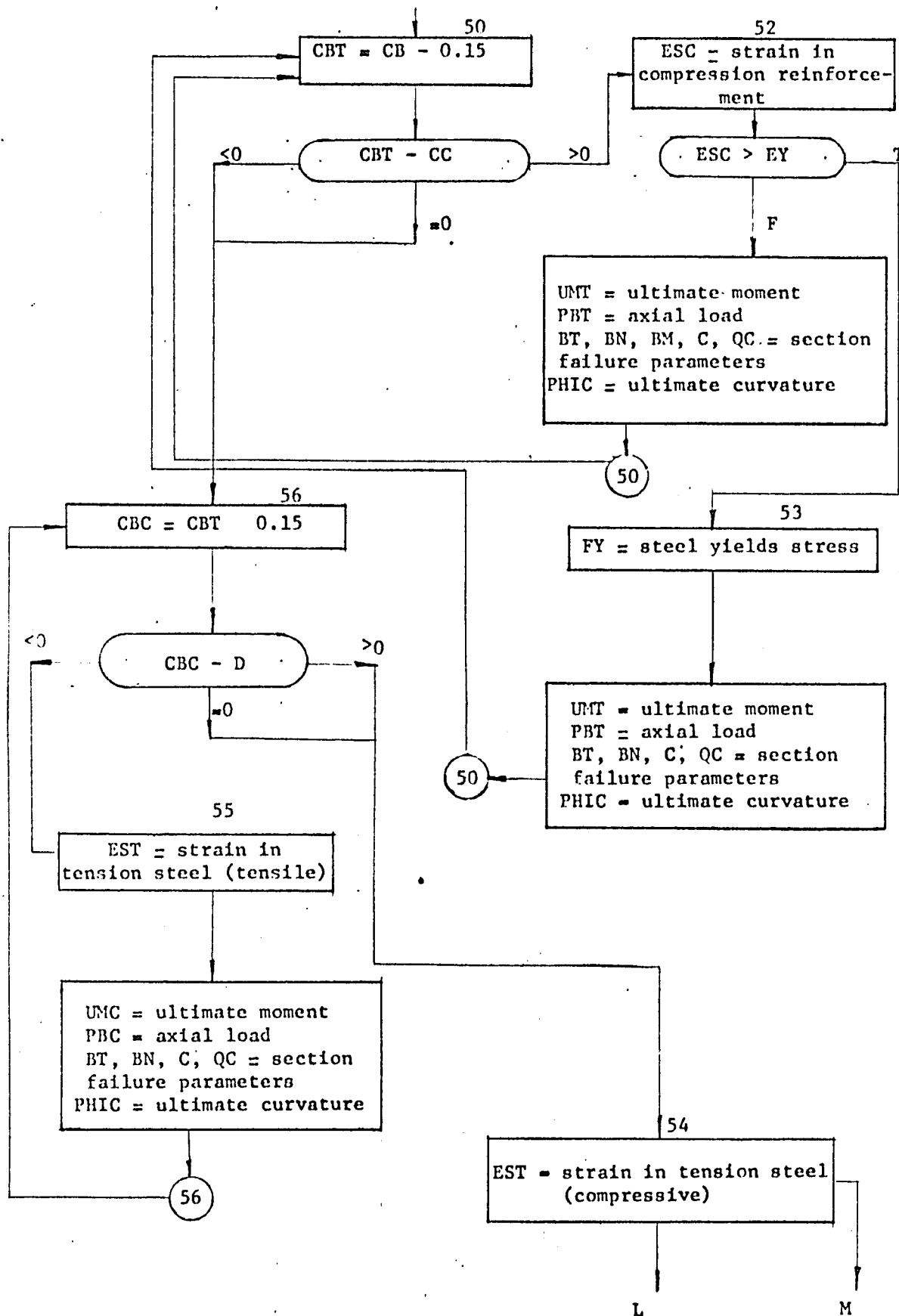


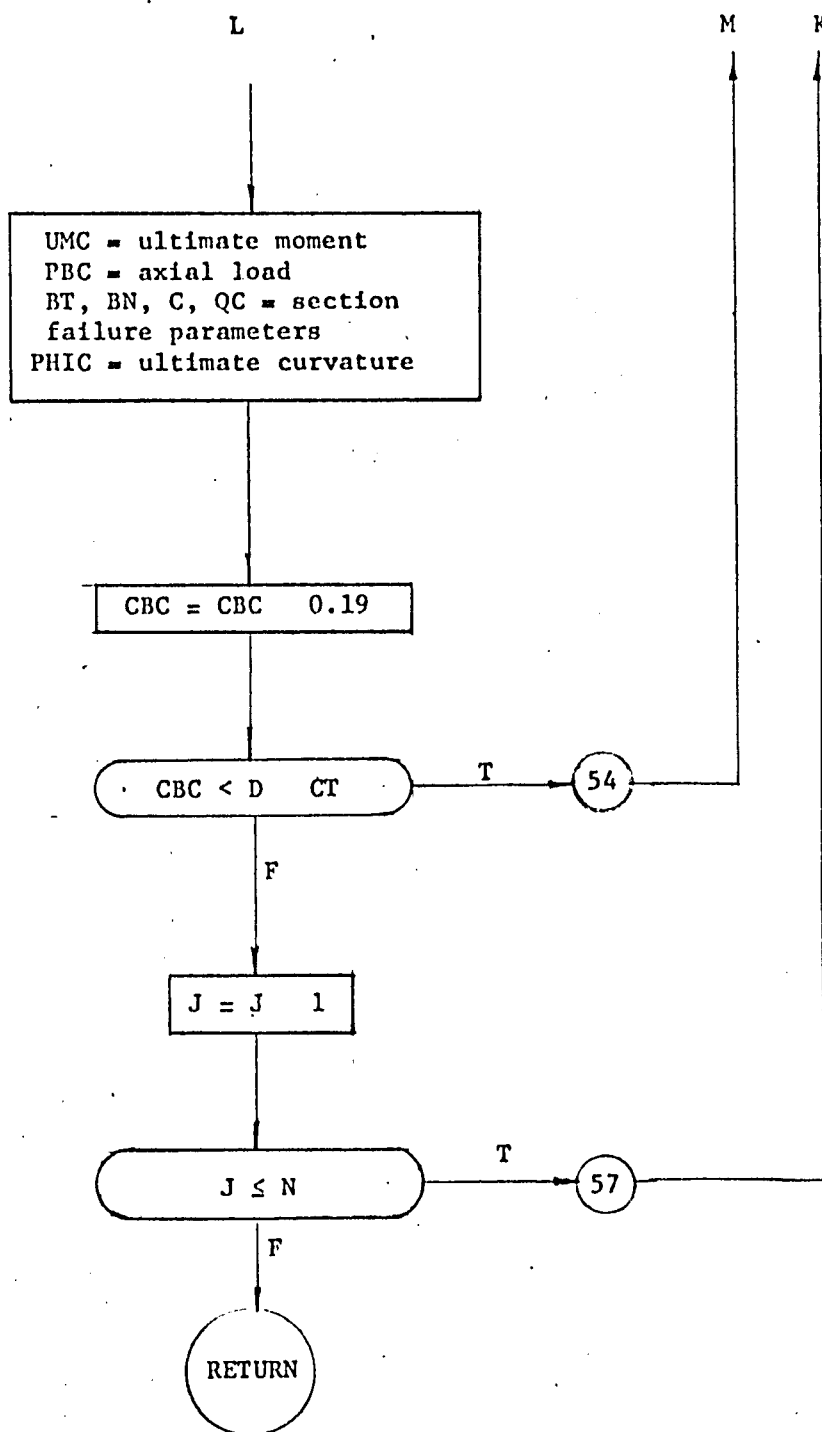




FLOW DIAGRAM - PROGRAM III







APPENDIX E : PHOTOGRAPHS

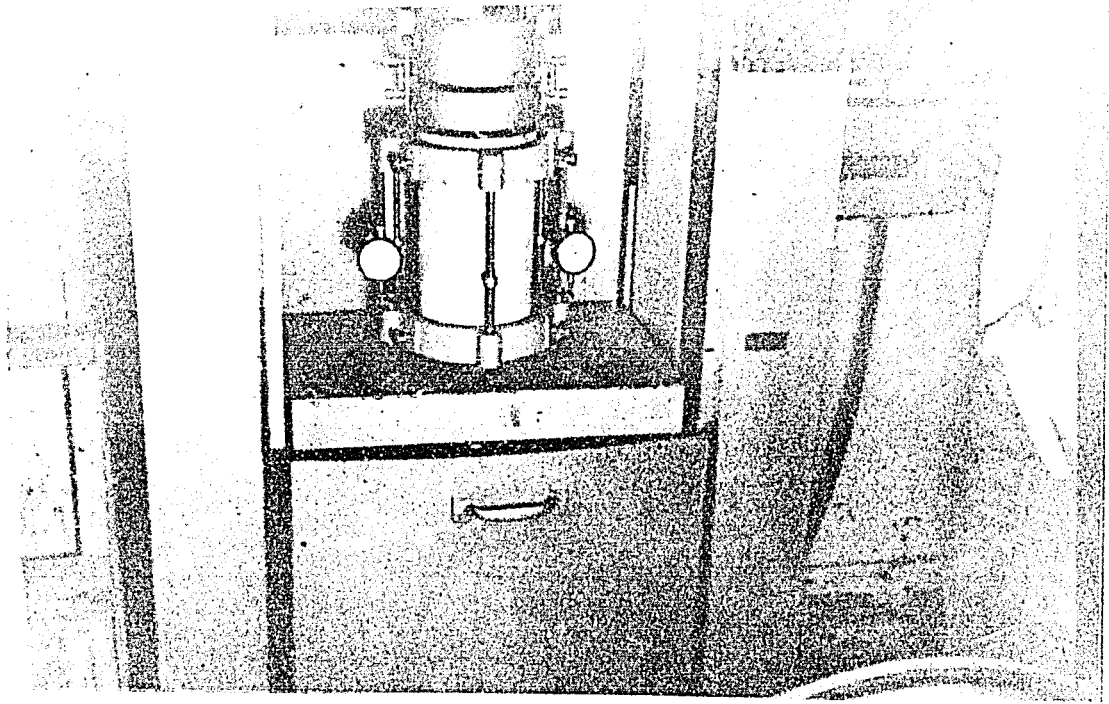


PLATE 1 – COMPRESSION TEST ON 6" x 12" CONCRETE CYLINDER.

PLATE 2 – ELECTRO DYNAMIC MATERIAL TESTER - RESONANCE
MODULUS OF ELASTICITY 3" x 3" x 12" PRISMS.

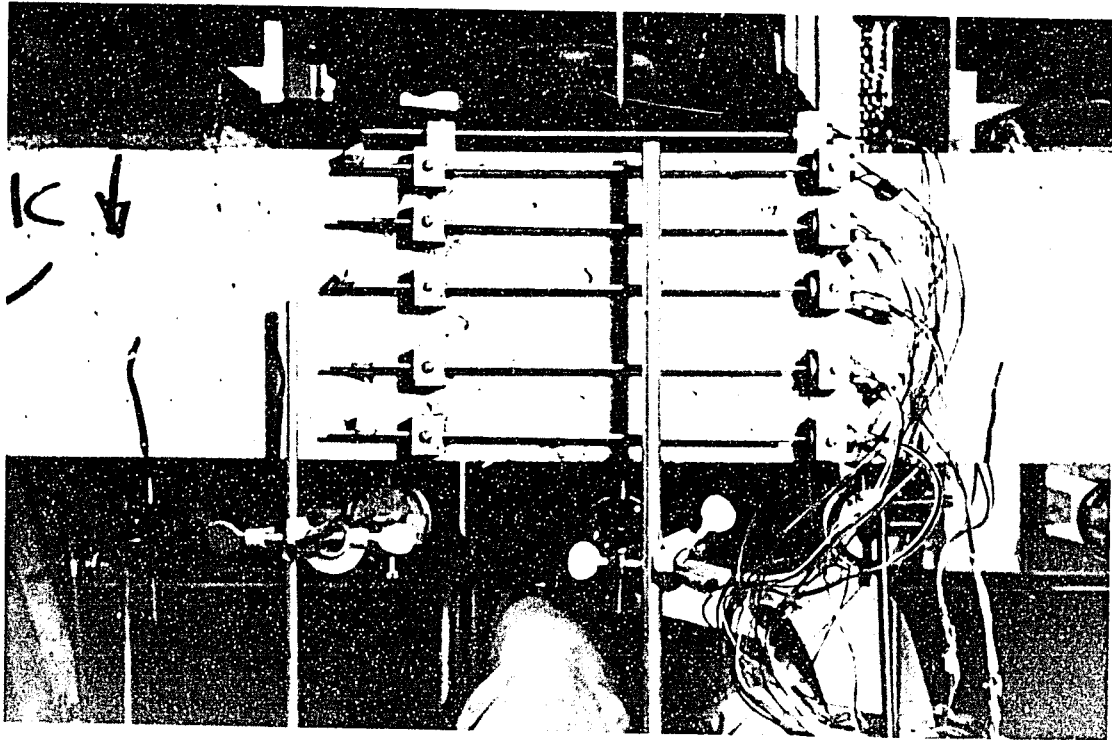
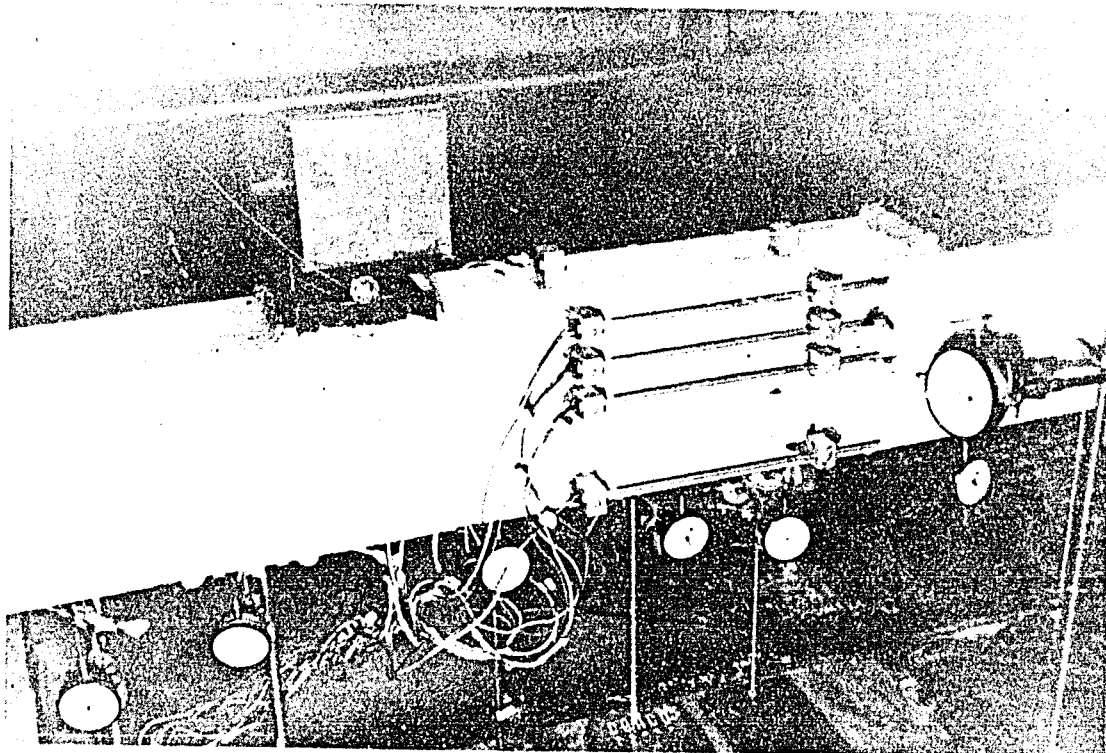


PLATE 3 - LAYOUT OF LINEAR VARIABLE DIFFERENTIAL TRANSFORMERS
AND DEFLECTION GAUGES.



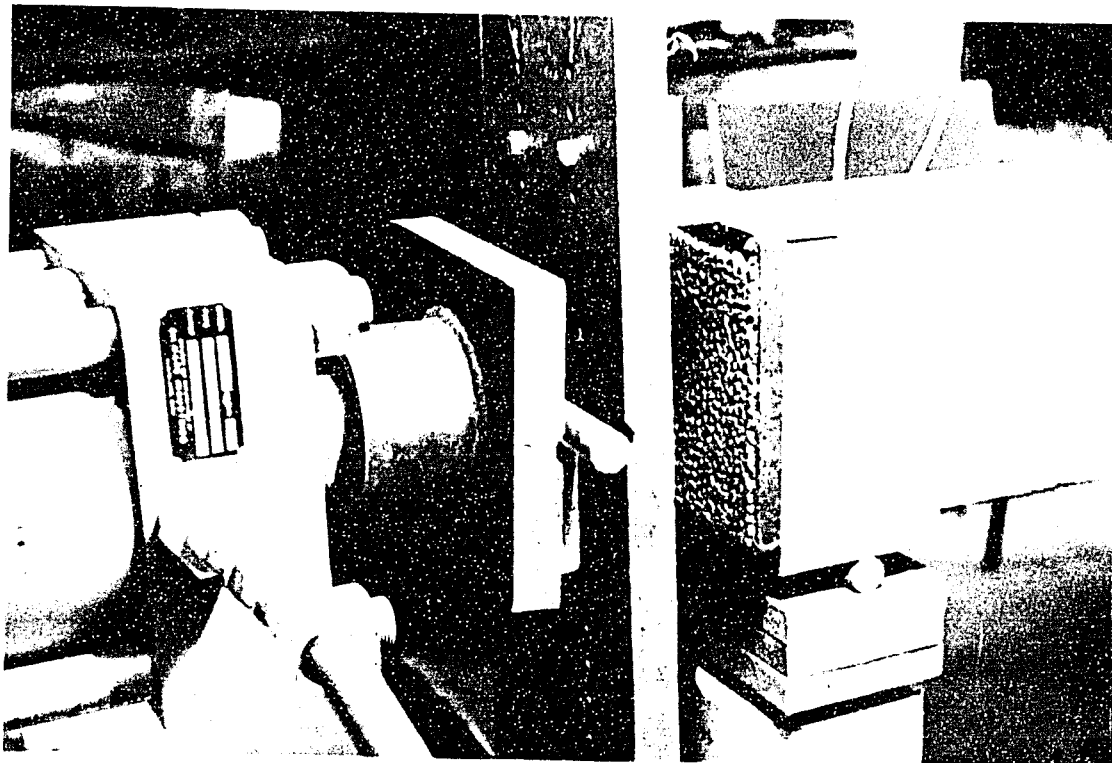


PLATE 4 — 100 KIP CAPACITY HYDRAULIC CYLINDER AND DETAILS OF VERTICAL AND HORIZONTAL PINS.

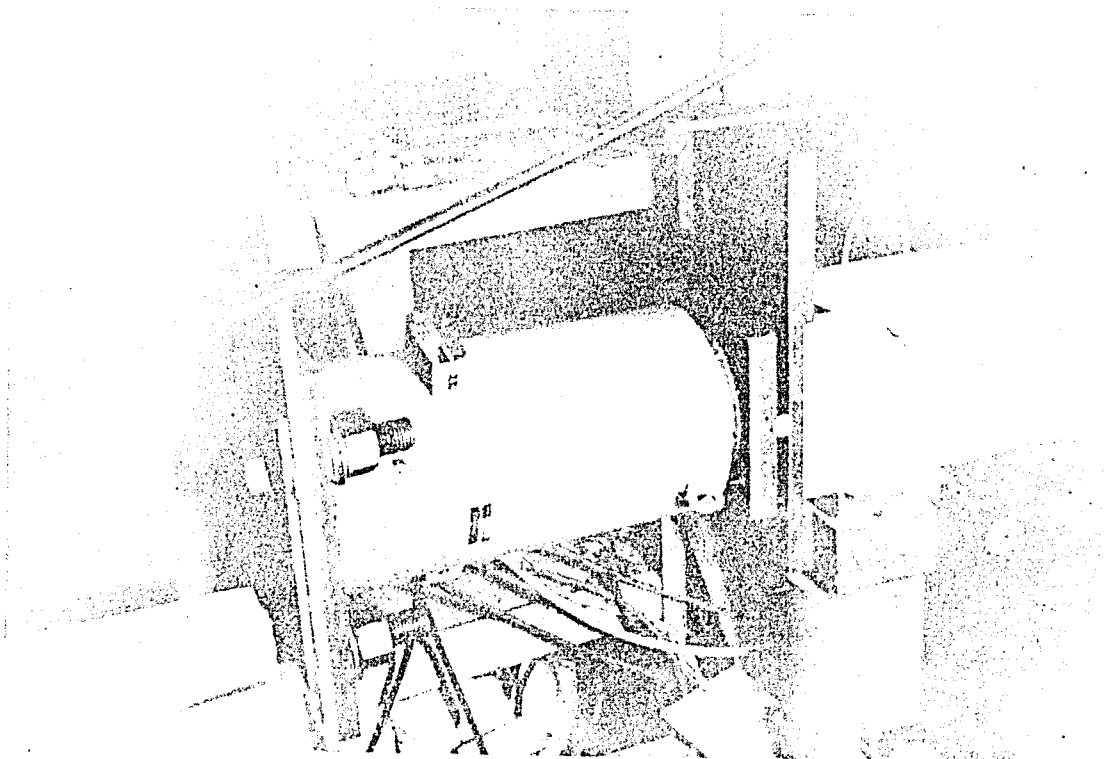


PLATE 5 — 100 KIP CAPACITY LOAD CELL.

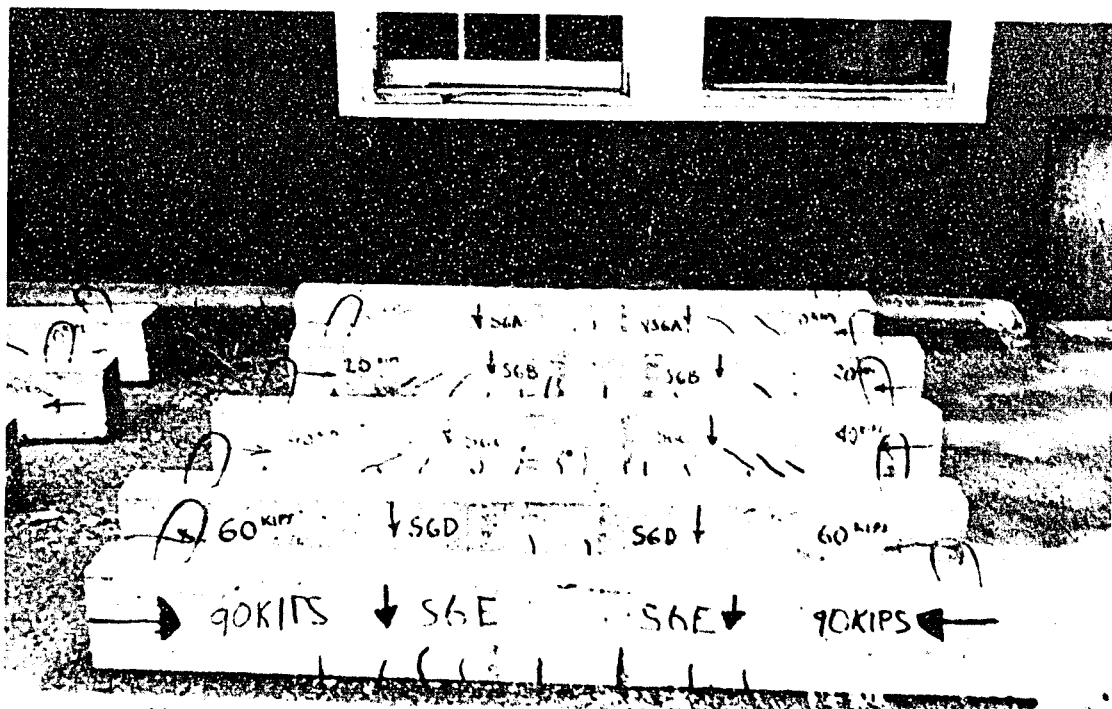


PLATE 6 - UNSYMMETRICALLY REINFORCED BEAM-COLUMN AFTER
TEST - SERIES S6.

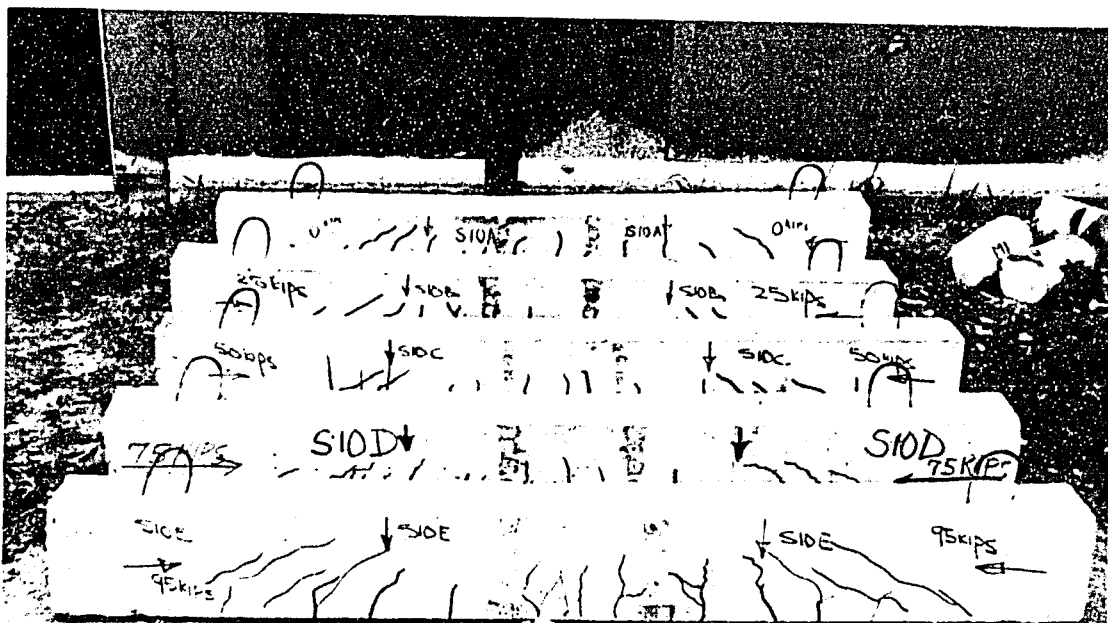


PLATE 7 - SYMMETRICALLY REINFORCED BEAM-COLUMN AFTER
TEST - SERIES S10.

APPENDIX F: FABRICATION DRAWINGS

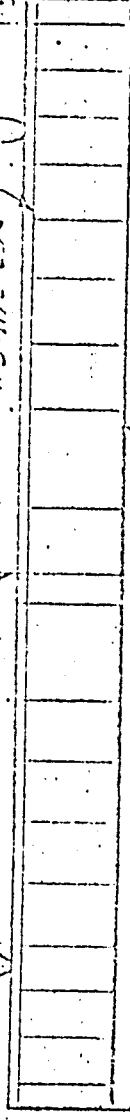
6'10"

5'-11 1/2" = 0.55 LENGTH

STAIRS 1/2" 20# 4 SPACES @ 4" = 11 1/4" 2 SPACES @ 4" = 8" 4 SPACES @ 4" = 16" 10 SPACES @ 4" = 40"

SECTION

2 1/4" 20# 10x200 x 5'-11 1/2" 10



STAIRS 1/2" 20#

2 1/4" 20# 10x200 x 5'-11 1/2" 10

ELEVATION

NOTES:

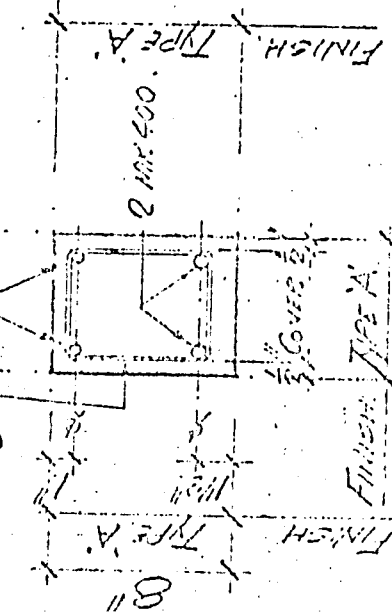
1. f.c. 5000 psi
2. f.c. 4000 psi

NOTE: BOTH ENDS TO HAVE FINISH TYPE 'A' INTO THEY MUST BE SAVED TO SIDES.

FINISH TYPE 'A'

STAIRS 1/2" 20# 10x200

2 1/4" 20# 10x200



Detail of 1/2" Stair
10x200 x 2'-11 1/2" 10
2 1/4" 20# 10x200

HOUSE DRAWING

5/8" 4"

Detail of 1/2" Stair
10x200 x 2'-11 1/2" 10
2 1/4" 20# 10x200

WILSON CONCRETE PROD. LTD.
BELLEVILLE ONTARIO

FOR UNIVERSITY OF GUYANA

DESCR. REINFORCED CONCRETE

TOLERANCES LENGTH 1/4" WIDTH 1/8" DEPTH 1/8"

DATE 15-JAN-88 ISSUE 1(1000) REV. A

DATE 15-JAN-88 ISSUE 2(1000) REV. A

DATE 15-JAN-88 ISSUE 2(1000) REV. A

DATE 15-JAN-88 ISSUE 2(1000) REV. A

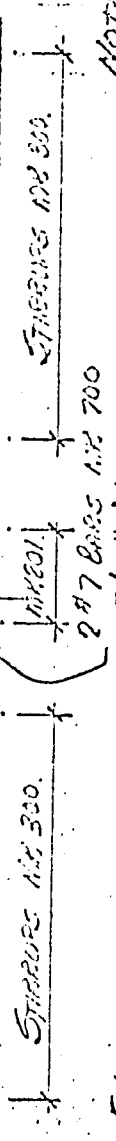
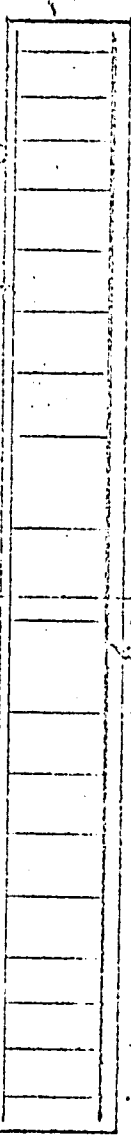
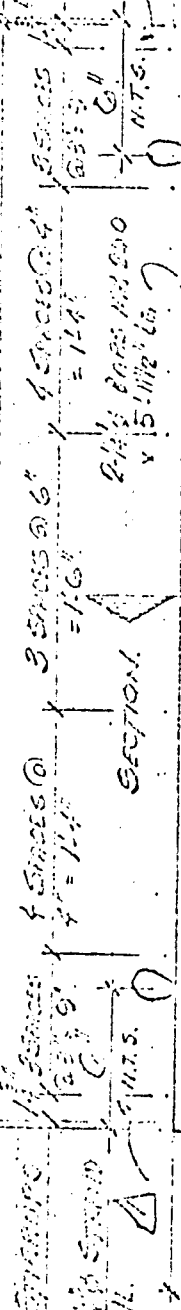
DATE 15-JAN-88 ISSUE 2(1000) REV. A

DATE 15-JAN-88 ISSUE 2(1000) REV. A

DATE 15-JAN-88 ISSUE 2(1000) REV. A

6'10"

5'-11 1/2" = OVER LENGTH



ELEVATION

NOTES:

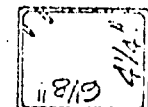
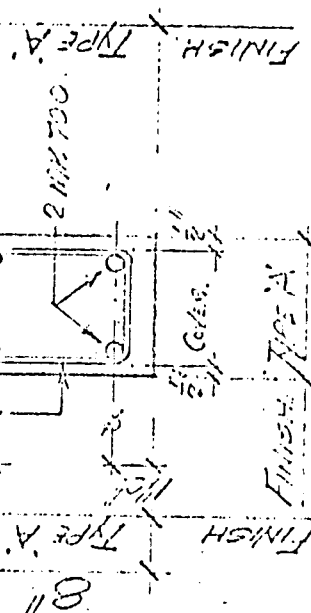
1. F.C. 3000 PSI
2. F.C. 10000 PSI

NOTE: BOTH ENDS TO HAVE FINISH TYPE 'A', AND THEY MUST BE SEVERED TO SIDES.

FINISH TYPE 'A'

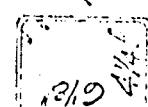


2 #7 BARS 11\"/>



11/5 DMS.

DETAIL OF TOP CORNER
2 #7 BARS 11\"/>



11/5 DMS.

DETAIL OF #3 STIRRUPS
11/5 DMS. 11\"/>

SECTION

WILSON CONCRETE PROD. LTD.
BELLEVILLE ONTARIO

JOB UNIVERSITY OF OTTAWA

DESCR. REINFORCED CONCRETE FLOOR

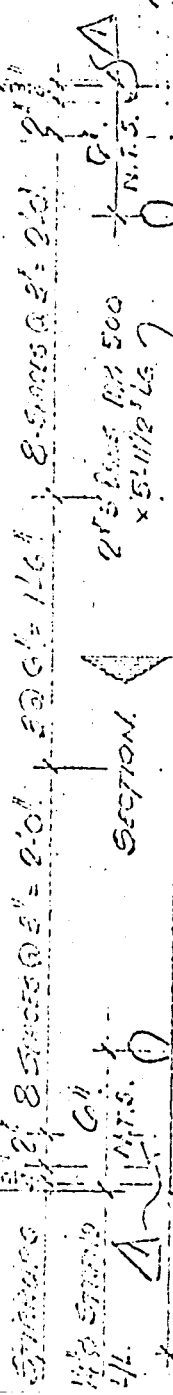
TOLERANCES
LENGTH $\pm 1/8"$ WIDTH $\pm 1/8"$ DEPTH $\pm 1/8"$

DATE	ISSUE	REV.	CHG.
10-20-66	#1 (500)		CH
12-20-66	#2 (200)	A	7

A-1216-30-S3

6'10"

5'-11 1/2" = OVER LENGTH



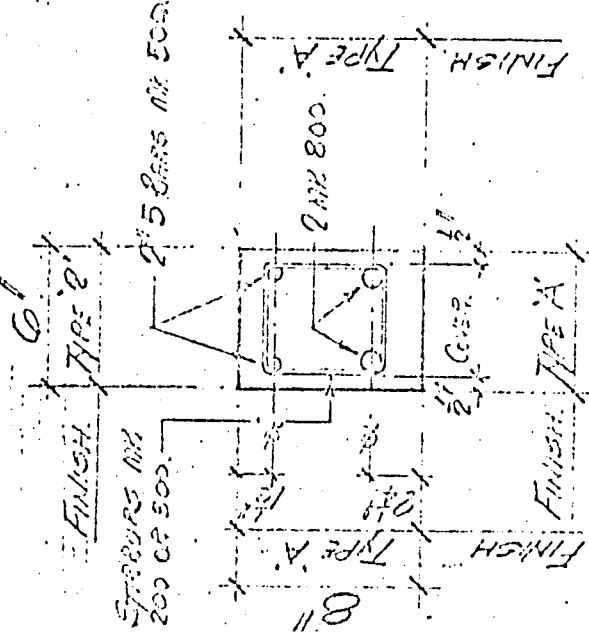
SECTION

NOTE: BOTH ENDS TO HAVE
FINISH TYPE A, AND
THEY MUST BE SECURED
TO SIDES.

ELEVATION

NOTES:

1. FC — 5000 PSI
2. FG — 4000 PSI



SECTION



DETAIL OF 5/8\"/>



1/2\"/>

DETAIL OF 5/8\"/>

WILSON CONCRETE PROD. LTD. BELLEVILLE ONTARIO	
JOB UNIVERSITY OF OTTAWA	
DESCR. REINFORCED CONCR. TEST LEAD	
TOLERANCES	+ 1/2" WIDTH - 1/2" DEPTH + 1/2"
DATE	ISSUE
10/11/68	#1 (200)
15/11/68	#2 (100)
PEN.	
CH. D.	
S5	A-1216-30-55

6'10"

5'11 1/2" = ONE LENGTH

23 SPACERS @ 2" - 5'0"

SECTION

2#5 BARS MAX 500
x 5'11 1/2" LG

6'10"

Notes: Both Ends To Have
Finish Type A, And
They Must Be Same
To Sides.

SPACERS MAX 500

2#8 BARS MAX 500
x 5'11 1/2" LG

Notes:

1. f.c
2. f.c

3000K
40000K

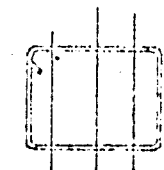
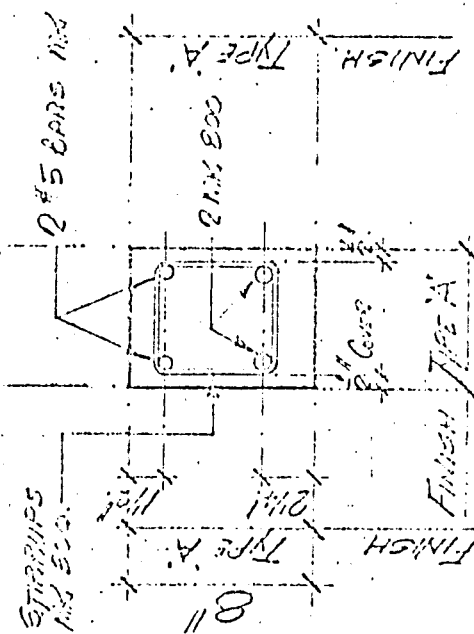
ELEVATION



Detail of #3 Spacing
MAX 500 x 1'11 1/2" LG
No THUS REQ.

2#5 BARS MAX 500

FINISH TYPE A



Detail of Spacing
MAX 500 x 1'11 1/2" LG
No THUS REQ.

SECTION

WILSON CONCRETE PROD. LTD.
BELLEVILLE
ONTARIO

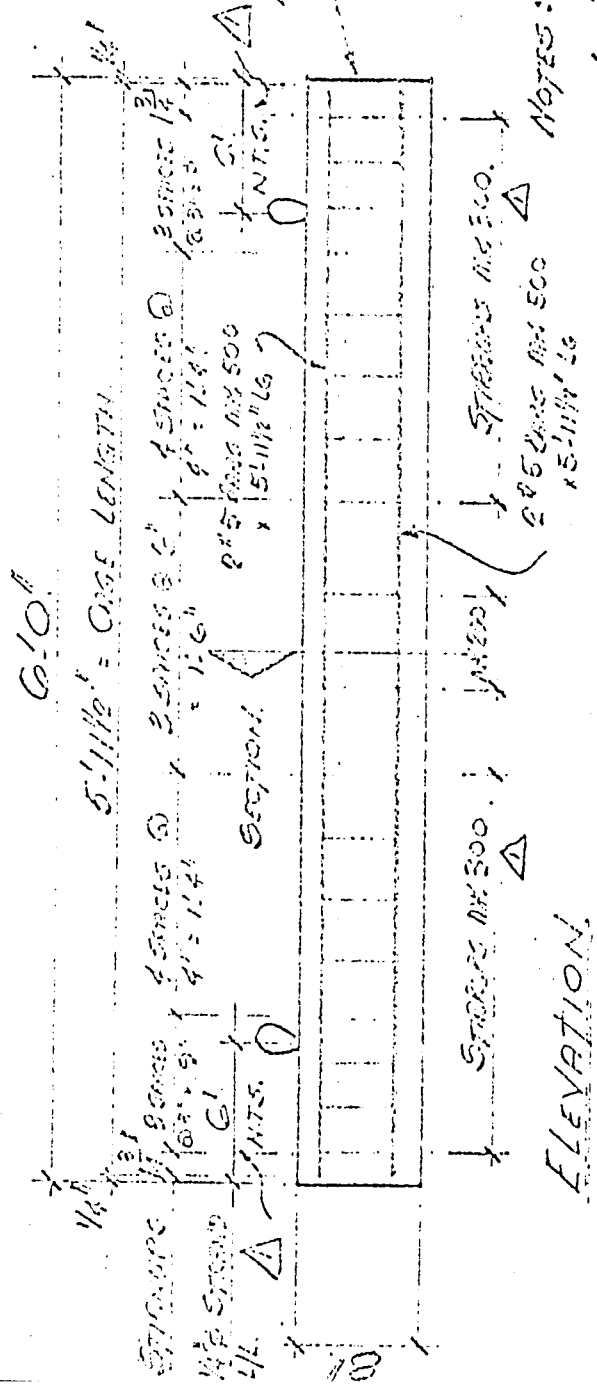
JOB: UNIVERSITY OF OTTAWA

DESCR: REINFORCED CONCRETE BEAM

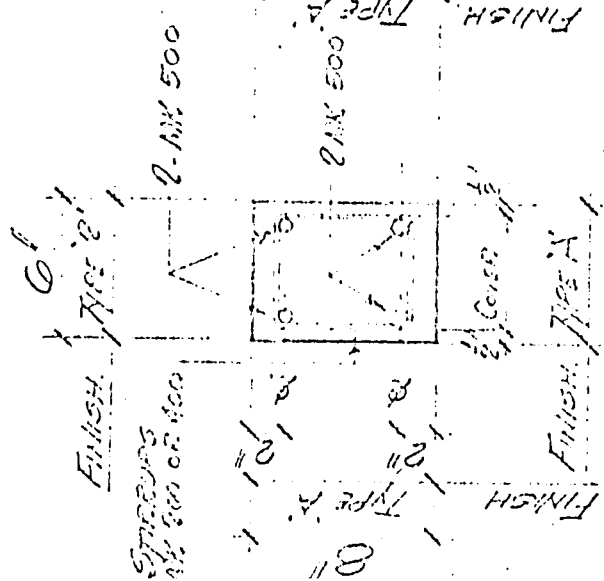
TOLERANCES:
LENGTH: $\pm 1/8"$ WIDTH: $\pm 1/8"$ DEPTH: $\pm 1/8"$

DATE	ISSUE	REV	CH'D
10/20/69	1 (P. 00)		
15/05/69	2 (P. 00)	A	

S6 A-1216-30-S6



ELEVATION



SECTION



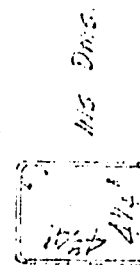
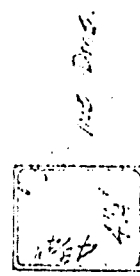
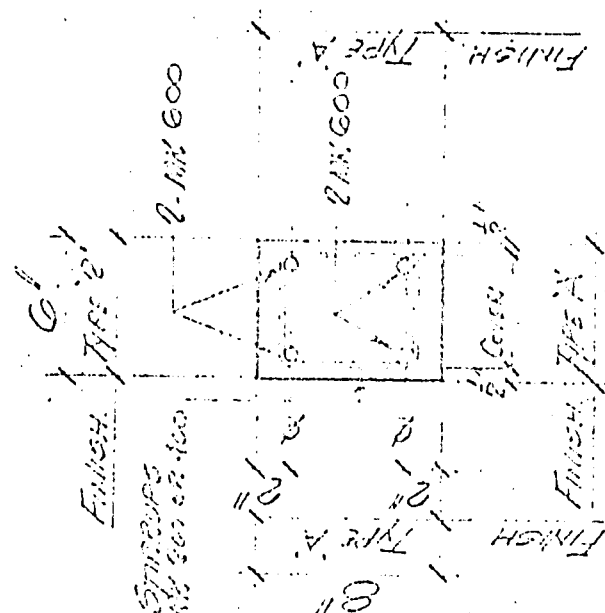
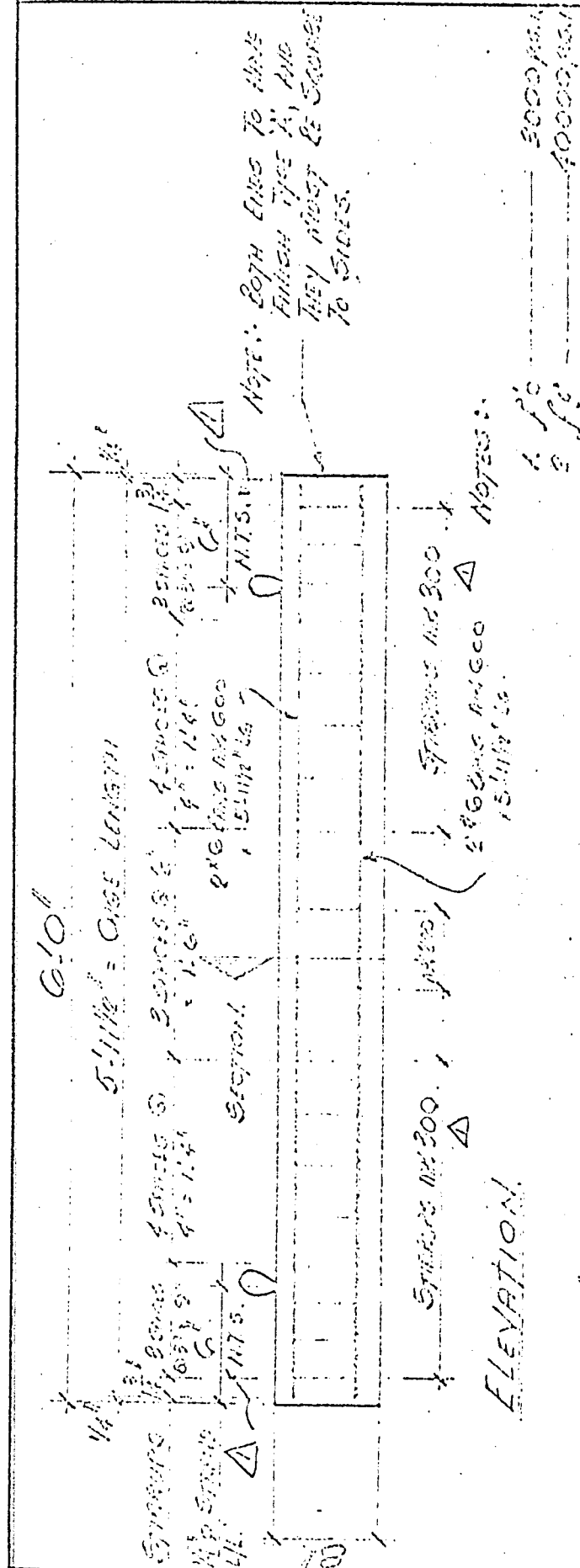
MS. DMS.



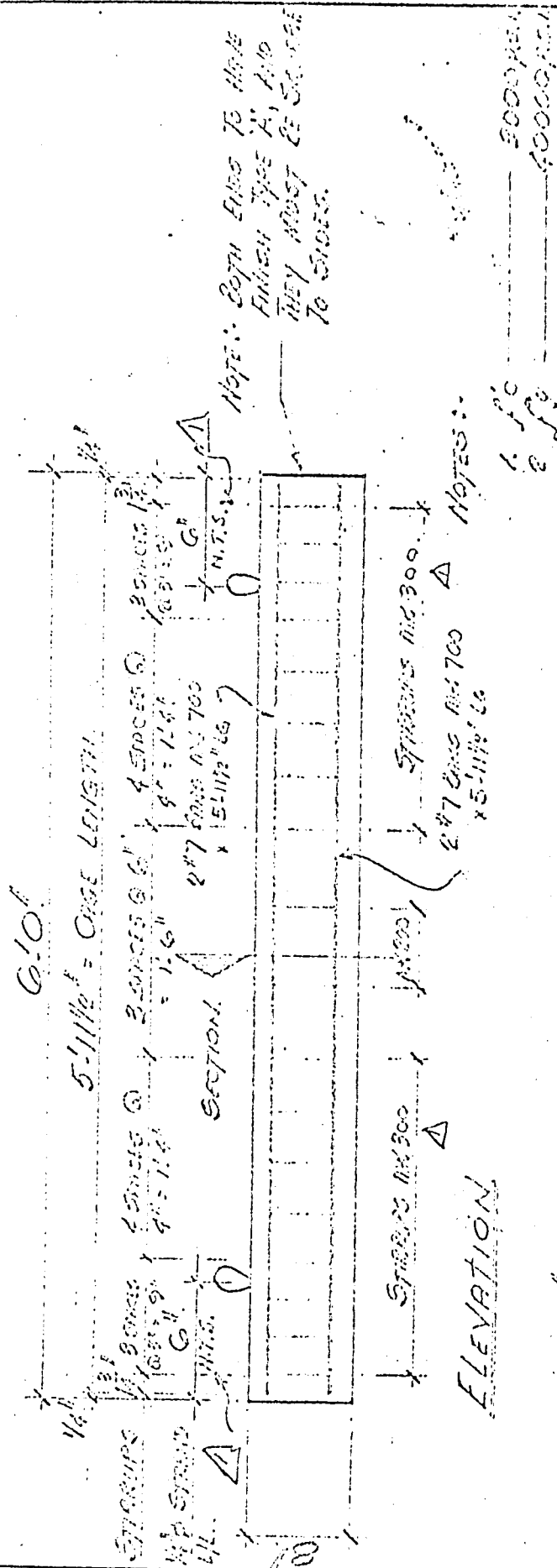
MS. DMS.

Detail of 1/4" x 1/2" x 1/2" spaced at 12" on center

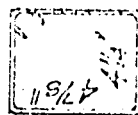
WILSON CONCRETE PROD. LTD. BELLEVILLE ONTARIO				JOB UNIVERSITY OF GUELPH			
DESIGN: DIMENSIONS CONC. TEST LEAD				TOLERANCES: LENGTH: $\pm \frac{1}{8}$ " WIDTH: $\pm \frac{1}{8}$ " DEPTH: $\pm \frac{1}{8}$ "			
DATE	ISSUE	REV	CHD	DATE	ISSUE	REV	CHD
10/20/15	11/1/15	---	---	10/20/15	11/1/15	---	---
15/ July 15	11/1/15	A	---	15/ July 15	11/1/15	A	---
S8	A-1216-30-S8						



WILSON CONCRETE PROD. LTD. BELLEVILLE ONTARIO		UNIVERSITY OF GUELPH		UNIVERSITY OF GUELPH	
J03		DESCR. REINFORCED CONC. JOIST LEARN		TOLERANCES	
LENGTH 44' 0"		WIDTH 4' 0"		THICKNESS 4" 0"	
DATE 19-04-69		ISSUE 1/1000		REV. ---	
19-04-69		1/1000		C.D. 1/1000	
S9		A-1216-30-S9		S9	

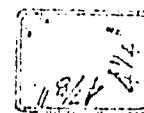


100



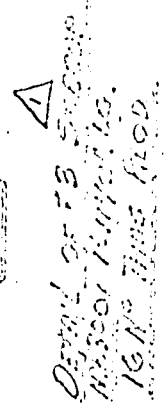
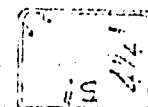
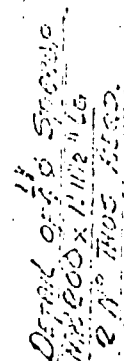
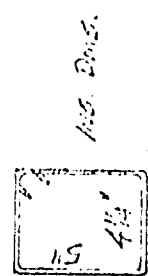
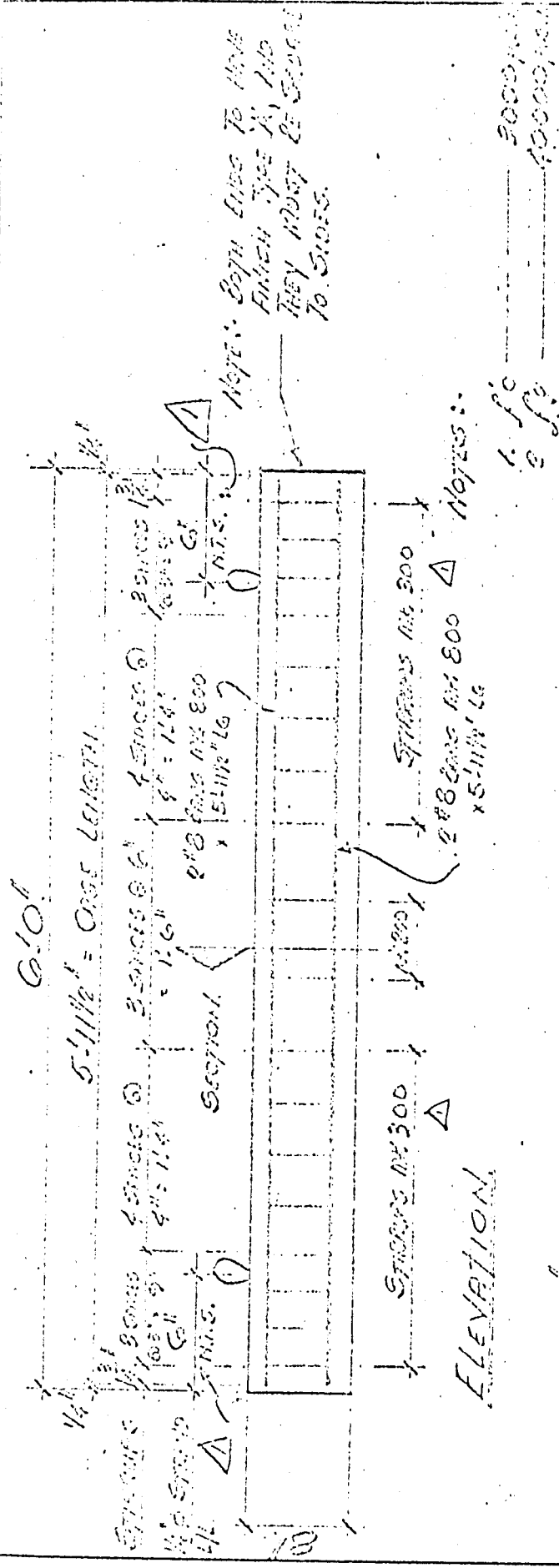
Detail of \$6 Stereo
\$1000 x 1000 ft.
2nd Floor Area

415. DMS.



Detail of 13 Spiders
in 1000 Pupae
16th Time Recd

WILSON CONCRETE PROD. LTD. BELLEVILLE ONTARIO	JOB	UNIVERSITY OF GUELPH				A-1216-30-S10	
DESCRIPTION	REINFORCED CONCRETE		TOLERANCES		DATE		S10
LENGTH	4 1/2'	WIDTH	4 1/2'	DEPTH	4 1/2'	12/2/55	15 July 55
ISSUE	REV	CH'D					
1	-	-					
2	-	-					



WILSON CONCRETE PROD. LTD. BELLEVILLE ONTARIO			
JOB	UNIVERSITY OF GUELPH		
ORDER	REINFORCED CONC. FOOTING		
TOLERANCES	+1/4" L -1/4" W -1/4" D		
LENGTH	10'-0"	WIDTH	1'-0"
DATE	10-10-72	ISSUE	1/1000
	5-10-72		1/1000
		REV	---
		CHECKED	---
S11	A-1216-30-S11		

APPENDIX G : BIBLIOGRAPHY

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