

EXPERIMENTAL AND NUMERICAL STUDY OF SHOCK FRONT PERTURBATION DECAY

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ABSTRACT

Perturbations on shock fronts appear in many gas-dynamic configurations, including shock tubes, shock focusing problems, and reactive flows. As a perturbed shock propagates, its front progressively recovers a more planar shape, but the decay rate of finite-amplitude perturbations remains unclear.

The objective of this work is to quantify the decay rate of perturbations on inert shock fronts and to identify the physical mechanism responsible for this decay. Three complementary approaches are used: shock tube experiments, two-dimensional Euler simulations, and Geometrical Shock Dynamics calculations. In all three cases, the perturbation amplitude is measured using the Mach-number difference ΔM across the triple-point structure, separating the stronger Mach shock region from the weaker incident shock region.

The results show that ΔM decays approximately as $1/t$. This behavior is observed consistently in the experiments, Euler simulations, and GSD calculations. The measured decay is therefore closer to the nonlinear $1/t$ scaling discussed by Whitham than to the $t^{-3/2}$ or $t^{-1/2}$ decay laws previously reported from linear theory and early experimental interpretations.

The agreement between the three approaches shows that the decay is governed by nonlinear wave dynamics along the shock front. In particular, the Mach-number profile along the front can be interpreted as the superposition of two N-wave-like disturbances propagating in opposite directions. The $1/t$ decay of these N-waves provides a physical explanation for the observed scaling.

This work provides experimental, numerical, and theoretical evidence that finite-amplitude perturbations on inert shock fronts decay according to a nonlinear $1/t$ law.

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CHAPTER 1

INTRODUCTION

1.1 BACKGROUND

A shock wave is a propagating discontinuity in the flow properties of a gas, traveling at a speed faster than the local speed of sound. Across the shock, pressure, temperature, and density rise abruptly. The strength of a shock wave is characterized by its Mach number $M = u/c$, where u is the velocity of the shock and c is the speed of sound in the undisturbed gas ahead of it.

It is possible to obtain a perturbed shock by changing its shape for example. To look at how perturbed shocks behave, in the nonlinear theory we consider, it is possible to look at what happens when we have a concave portion of a planar shock wave. This situation is sketched in Figure 1.1. As the wavefront propagates down the converging rays, the shock speeds up as it gets stronger. The shock overshoots and the disturbance evens out as it spreads along the shock. This results in a non-uniform shock front in terms of Mach number, with a strong region and a weaker region. This stronger shock is called the Mach stem, and the weaker shock is called the incident shock, as shown in Figure 1.2. The intersection point of the Mach stem and the incident shock is the triple point. This triple point also intersects a third wave, the transverse wave, and a fourth wave, the shear layer.

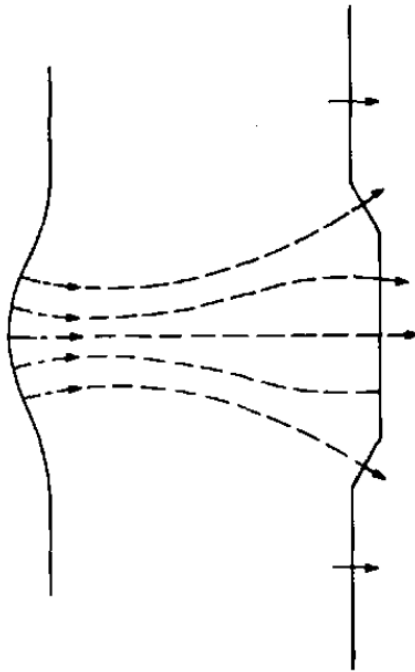


Figure 1.1: Sketch of a perturbed planar shock wave propagating through converging rays [1].

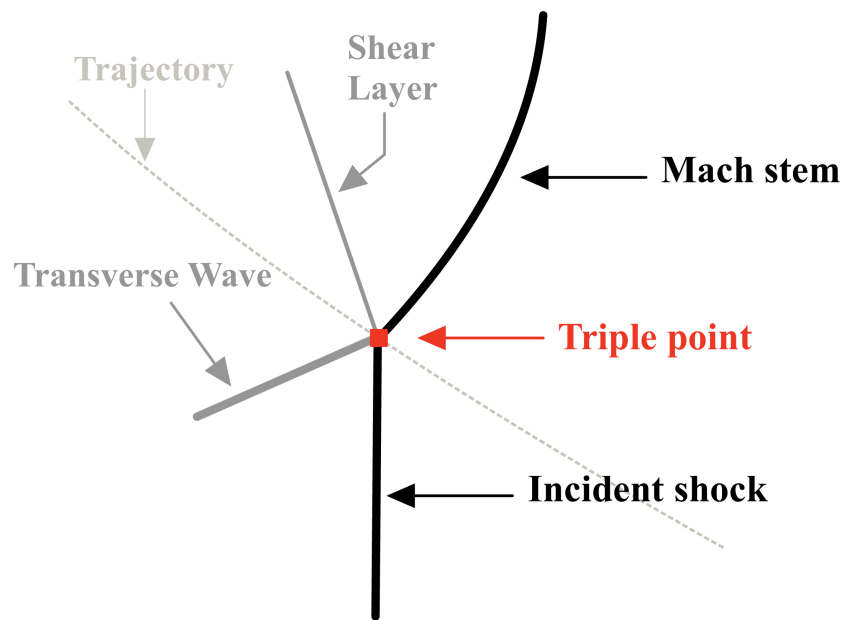


Figure 1.2: Sketch of the triple point structure, showing the Mach stem, the incident shock, the transverse wave and the shear layer.

1.2. PROBLEM DEFINITION

Therefore, what defines a perturbed shock is the presence of triple points on its front. As mentioned, across this triple point, there is a discontinuity in Mach number and in curvature. The differences in curvature and in strength decay as the shock propagates. The perturbations decay, meaning that the front recovers a planar structure with time, pointing out that this is a stable problem.

1.2 PROBLEM DEFINITION

As the perturbed shock propagates, the perturbations decay and the shock front progressively recovers a planar structure. The objective of this study is to quantify the decay rate of these perturbations and to identify the governing physical mechanisms responsible for it.

1.3 MOTIVATION

Perturbed shocks arise in many practical and fundamental applications. One important example is shock implosion problems, where a nearly planar shock is required in order to generate extremely high thermodynamic states. In such configurations, perturbations on the shock front can significantly affect the focusing process and reduce the ability to achieve the desired compression.

Another common situation in which perturbed shocks appear is the shock tube experiment. In practice, the diaphragm rupture is never perfectly instantaneous or uni-

1.4. LITERATURE REVIEW

form, which introduces disturbances on the leading shock front. Understanding how these perturbations evolve is therefore important for interpreting experimental results and for improving the understanding of shock dynamics in non-ideal conditions.

The lack of planarity of a shock front is also important from a combustion perspective. Shock perturbations can create regions with higher pressure and temperature that may act as hot-spots for chemical reactions. Such mechanisms are particularly relevant in processes involving deflagration-to-detonation transition (DDT). In reactive configurations, understanding this decay is also important because the perturbations may compete with amplification mechanisms associated with flame acceleration in shock-flame complexes when energy is released.

1.4 LITERATURE REVIEW

1.4.1 LINEAR THEORIES OF SHOCK-FRONT RELAXATION

The decay of perturbations on plane shock waves was first studied as a shock-wave stability problem. In this context, a shock is considered stable, since an initially distorted front progressively recovers a planar shape as it propagates. One of the earliest theoretical studies of this problem was developed by Freeman, who considered small perturbations imposed on a plane shock propagating in a channel [2, 3]. Freeman's analysis is based on a linearized description of the flow behind the shock. The disturbances are assumed to be small enough for the equations of the flow behind the shock wave to be linearized.

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In this framework, the disturbance does not grow indefinitely, but relaxes with time. For the asymptotic regime considered in the theory, Freeman predicted an oscillatory decay of the perturbation amplitude proportional to $t^{-3/2}$. This result became the main reference decay law for small-amplitude perturbations on plane shock waves. However, the theory also contains important limitations. Since it is linear, it does not describe the formation and subsequent motion of finite-amplitude structures such as triple points or Mach stems. Freeman also noted that the asymptotic behavior may be modified at large shock Mach numbers, where the decay can become slower and approach a $t^{-1/2}$ scaling. Therefore, although the $t^{-3/2}$ law is the main result of the linear theory for weaker shocks, it should not be viewed as a universal decay rate for all perturbed shocks. The limitations of the linear theory are especially important for the present work, where the perturbations are associated with finite Mach-number differences across the shock front. In this regime, the shock contains strong and weak regions separated by triple-point structures, and the problem becomes nonlinear. Freeman already recognized that a nonlinear description, such as the geometrical theory later developed by Whitham, would be required to describe this type of behavior. This provides a natural transition from the early linear theories of shock-front relaxation to nonlinear and geometrical descriptions of perturbed shock propagation.

1.4.2 EARLY EXPERIMENTAL MEASUREMENTS

Freeman's linear prediction was later tested experimentally by Lapworth in a shock tube configuration [4, 5]. In these experiments, a plane shock was perturbed by thin wedges

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mounted on the walls of a channel. Schlieren photographs were then taken at different downstream positions after the shock had passed the wedges, allowing the evolution of the shock shape to be reconstructed.

Because the undisturbed position of the plane shock was difficult to determine accurately, Lapworth introduced a quantity called the total perturbation. This quantity was defined as the distance between the most advanced and most delayed positions of the perturbed shock front in the direction of propagation. This provided a practical measure of the shock-front distortion that could be extracted directly from the schlieren images.

For shock Mach numbers $M_s = 1.41$ and $M_s = 1.60$, Lapworth found decay exponents of approximately $n = 1.45$ and $n = 1.65$, respectively. These values are close to Freeman's theoretical prediction of $n = 1.5$, and therefore provided qualitative experimental support for the $t^{-3/2}$ decay law. However, the measured perturbation amplitudes did not match the predictions of the linear theory, showing that the theory did not fully capture the experimental behavior.

These experiments also had important limitations. Shock tube experiments of this type were difficult to conduct, and the shock evolution could not be followed continuously in a single run. Instead, the shock shape was inferred from schlieren images taken at selected downstream positions. This allowed the geometry of the shock surface to be measured, but did not provide a direct measurement of the local shock velocity along the front. Therefore, the decay rate had to be inferred indirectly from the evolution of the shock shape.

Lapworth also noted that, at large downstream distances, the perturbations no longer

1.4. LITERATURE REVIEW

decayed regularly and the measurements became erratic. This behavior was attributed to flow separation and wake formation behind the wedges, which introduced additional disturbances that eventually affected the shock front. As a result, the experiments could only be compared with the theory over a limited range of propagation distances.

The interpretation of these early data was later revisited by Bowman [6]. Using a different analysis of the shock-front measurements, Bowman reported a slower decay rate, closer to $t^{-1/2}$. This disagreement is important because it shows that the reported decay law depends strongly on how the experimental data are interpreted. In other words, the same experimental data led to different decay rates when analyzed by different authors. Therefore, despite the early agreement between Freeman's theory and Lapworth's analysis, the experimental decay rate of shock-front perturbations remained unresolved.

1.4.3 NONLINEAR AND GEOMETRICAL DESCRIPTIONS

A different description of shock-front perturbations was proposed by Whitham through the theory of Geometrical Shock Dynamics [1, 7, 8]. This approach does not solve the full flow field behind the shock, but instead describes the evolution of the shock front itself. It is therefore particularly relevant when the main quantity of interest is the motion and deformation of the shock surface.

Whitham describes the motion of the shock using an orthogonal coordinate system (α, β) attached to the front, as sketched in Figure 1.3. The curves $\alpha = \text{const}$, with α proportional to time, are the successive positions of the front, while the curves $\beta = \text{const}$

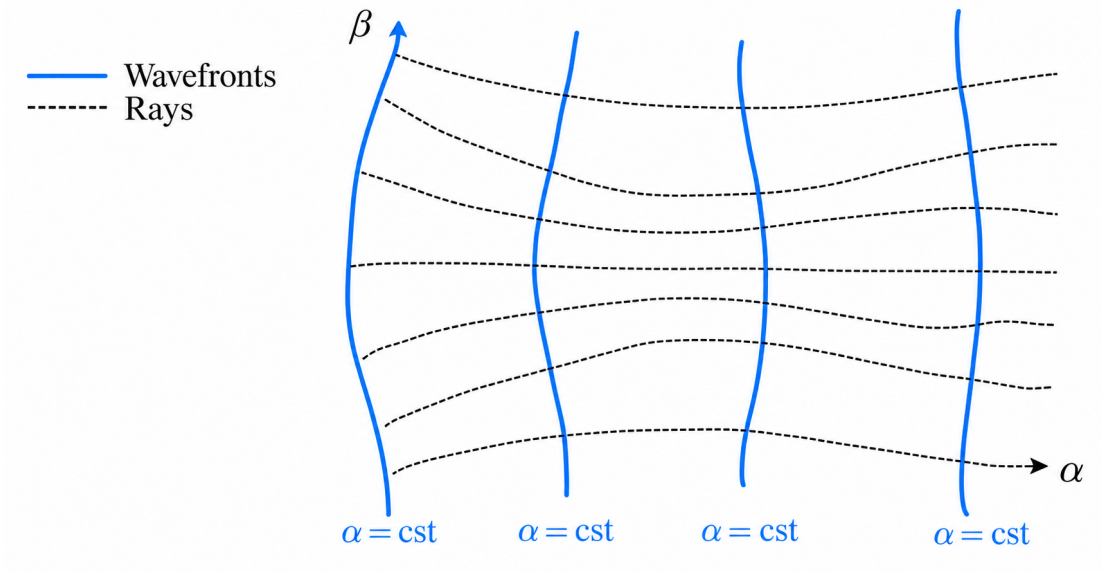


Figure 1.3: Sketch of the orthogonal coordinate system (α, β) used in Geometrical Shock Dynamics. Solid lines represent successive positions of the shock front ($\alpha = \text{cst}$), and dashed lines represent the rays normal to the front ($\beta = \text{cst}$).

are the rays normal to it. Progressing in α therefore corresponds to following the shock as it propagates, and progressing in β to moving along the front.

In this coordinate system, the local Mach number M , the local front inclination θ , and the local spacing A of the rays are related by a purely geometric identity, which follows directly from the construction itself. This identity becomes a closed, physical system once A is taken to be a function of M , in the same way that the Mach number of a shock moving down a channel is determined by the local channel area [8]. In characteristic form,

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the resulting system is

$$\left(\frac{\partial}{\partial\alpha} \pm c \frac{\partial}{\partial\beta}\right) \left(\theta \pm \int \frac{dM}{Ac}\right) = 0, \quad c(M) = \sqrt{-\frac{M}{AA'}}, \quad (1.1)$$

where $\theta \pm \int dM/(Ac)$ correspond to the Riemann invariants of the system, transported unchanged along characteristics, denoted $C^\pm$, given by $d\beta/d\alpha = \pm c(M)$ [8]. The propagation speed $c(M)$ depends on the local Mach number itself, and it is this dependence that makes the system nonlinear: since $c(M)$ increases with M , a wave carrying an increase in M steepens as it propagates, exactly like a compression wave in ordinary gas dynamics, and eventually breaks [8]. In GSD, the result of this breaking is a discontinuity in M and θ together, which Whitham calls a shock-shock.

To see more precisely what is transmitted along the front, Whitham considers the small-amplitude limit of this system, in which M remains close to a reference value M_0 . The equations then become linear,

$$\frac{\partial\theta}{\partial\beta} + \frac{1}{c_0^2 A_0} \frac{\partial M}{\partial\alpha} = 0, \quad \frac{\partial\theta}{\partial\alpha} + \frac{1}{A_0} \frac{\partial M}{\partial\beta} = 0, \quad (1.2)$$

where $c_0 = c(M_0)$ is now a constant. Differentiating the first equation with respect to α and the second with respect to β , and subtracting the two to eliminate θ , gives a wave equation for M ,

$$\frac{\partial^2 M}{\partial\alpha^2} = c_0^2 \frac{\partial^2 M}{\partial\beta^2}, \quad (1.3)$$

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whose general solution is

$$M = g_1(\beta - c_0\alpha) + g_2(\beta + c_0\alpha) \quad (1.4)$$

[8]. The same linear wave equation can be written for the shock inclination angle θ . In this linear limit, the disturbance travels along the front without distortion, in each direction, at the fixed speed c_0 . No shock-shock can form, since the propagation speed no longer depends on the amplitude of the disturbance.

Away from this small-amplitude limit, Whitham used the full nonlinear theory to obtain an explicit decay law for a periodically repeating disturbance: generated continuously along a sinusoidal wall, the disturbance settles into a regularly spaced train of shock-shocks whose amplitude decays as $1/t$, independently of its initial amplitude [8]. This result, however, describes a disturbance generated at a single boundary and travelling in essentially one direction.

Whitham also applied this framework to the case considered by Freeman, in which the shock is initially sinusoidal in shape rather than generated continuously by a wall. In this case, shock-shocks form and travel in each direction along the front, reflecting and interacting with each other, as in the configuration considered in the present work. Without derivation, Whitham argues that the results are essentially the same as for the wall problem, with the disturbance decaying like $1/t$ [8].

This $1/t$ decay law is important because it provides a nonlinear prediction that differs from the $t^{-3/2}$ scaling obtained from the linear theory. However, it was never directly

1.4. LITERATURE REVIEW

verified experimentally for finite-amplitude shock-front perturbations.

This leaves an important gap between the existing theories. Linear theory predicts a decay rate for small perturbations, while Whitham's nonlinear theory suggests a $1/t$ decay for finite-amplitude disturbances. The experimental evidence available in the literature does not clearly establish which decay law applies to shock front perturbations with triple-point structures.

1.4.4 NONLINEAR MODELLING OF WRINKLED SHOCK FRONTS

More recent studies have focused on the nonlinear dynamics of wrinkled shock fronts and on the formation of Mach stems and triple points. Clavin developed a weakly nonlinear analysis of shock-vortex interaction, leading to a nonlinear wave equation for the wrinkles of a strong shock front [9]. The solution of this model can develop singularities in the slope of the front in finite time, which corresponds to the formation of Mach stems. This model was later examined in more detail by Denet et al. [10], who compared its predictions with direct numerical simulations and preliminary shock tube experiments. Their work showed that the model captures important features of the nonlinear shock-front dynamics, including the formation of cusps and triple-point-like structures. A detailed numerical study of a related configuration was then presented by Lodato, Vervisch and Clavin [11], who considered the reflection of a shock from a sinusoidal wall. Their simulations showed the spontaneous formation of triple points on an initially smooth inert shock, followed by their transverse motion, interactions, and the formation of a cellular shock structure. These

1.5. STATEMENT OF WORK

studies are important for the present work because they show how triple-point dynamics can arise from the nonlinear evolution of the shock front itself. They also show that the later relaxation of the front is not related to viscous effects but results from the nonlinear dynamics of the triple points. However, the model equation used in these studies was derived in an asymptotic limit for strong shocks and weak perturbations. It provides valuable physical insight into the formation and motion of Mach stems, but it is not used here as a quantitative model for the decay rate.

1.5 STATEMENT OF WORK

In this study, the decay of shock-front perturbations is investigated using three complementary approaches: reactive shock tube experiments, Euler simulations, and Geometrical Shock Dynamics simulations. The objective is to measure how the perturbation amplitude evolves as the shock propagates.

The perturbation strength is quantified through the Mach-number jump across the triple point structure. More specifically, the Mach number is extracted on both sides of the discontinuity, corresponding to the stronger Mach stem region and the weaker incident shock region. The difference between these two values provides a measure of the local perturbation amplitude.

The evolution of this Mach-number difference is then followed in time, to determine the decay rate of the perturbation and to better understand the physical mechanisms

responsible for the observed decay.

1.6 OUTLINE OF THIS STUDY

The remainder of this thesis is organized as follows. Chapter 2 presents the experimental study. It describes the shock tube configuration, the generation of perturbed shock fronts, the imaging method, and the procedure used to extract the shock position and Mach number from the experimental data, before defining the perturbation amplitude ΔM and presenting its evolution for the three experimental datasets. Chapter 3 presents the Euler simulations used to reproduce the propagation and decay of perturbed shocks in a gas-dynamic framework. The numerical setup, initial conditions, and post-processing methods are described. Chapter 4 introduces the Geometrical Shock Dynamics (GSD) simulations. This chapter presents the simplified shock-dynamics model used to study the evolution of the shock front and to compare its predicted decay with the experimental and Euler results. Chapter 5 compares the results obtained from the experiments, Euler simulations, and GSD simulations, and proposes a physical explanation for the observed decay based on the propagation of N-waves along the shock front. Finally, Chapter 6 summarizes the main conclusions of this work and discusses its limitations.

CHAPTER 2

EXPERIMENTS

2.1 EXPERIMENTAL SETUP

To investigate the decay rate of perturbations on an inert shock front, shock tube experiments were conducted. A reactive shock tube was used, as this configuration naturally generates a strong shock wave decoupled from the reaction zone, which is relevant to the combustion applications motivating this study, as discussed in Section 1.3.

The experiment proceeded as follows. The reactive mixture is first ignited in the ignition section. In this section, a mesh grid is placed to promote turbulence and accelerate the flame. This allows the flame to transition into a detonation before entering the propagation section of the tube. The detonation then propagates through the second section until it reaches a converging-diverging obstacle. This obstacle quenches the detonation and separates the leading shock from the reacting flow behind it. As a result, a curved shock, decoupled from the reacting flow, is produced ahead of the flame, allowing the shock-front dynamics to be studied independently of the reaction zone. The strength of the transmitted shock depends on the blockage ratio of the obstacle and on the Chapman–Jouguet (CJ) detonation speed of the mixture [12]. In the present experiments, its Mach number is approximately 3. Downstream of the obstacle, the shock enters the observation section,

2.1. EXPERIMENTAL SETUP

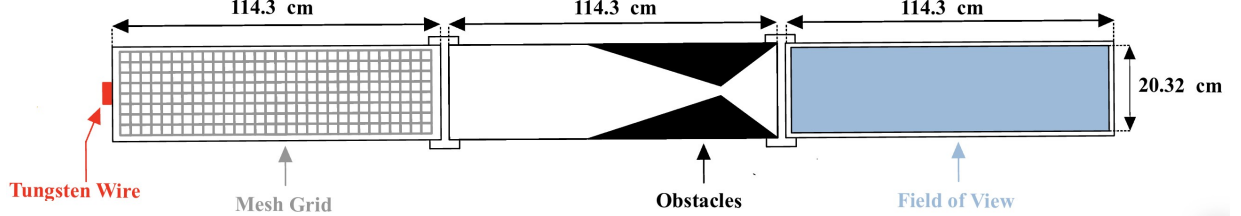


Figure 2.1: Setup of the reactive shock tube experiment.

where it is captured using high-speed imaging. Depending on the imaging technique used, the shock front is recorded at different downstream locations to allow the full evolution of the perturbations to be tracked.

The experiments were carried out in a narrow shock tube with a total length of 3.42 m. It consists of three sections of equal length (114 cm each [13]), as shown in Figure 2.1. The test section has a narrow rectangular geometry with a width of only 19 mm, which constrains the shock dynamics to be approximately two-dimensional. The field of view was located downstream of the region where the perturbed shock front was generated. It allowed the shock front to be tracked after its initial formation, while the perturbations were still clearly visible.

The experiments were conducted using a stoichiometric methane-oxygen mixture, $\text{CH}_4 + 2\text{O}_2$, at an initial pressure of 13.8 kPa and ambient temperature. The use of a detonation to generate the shock constrains the range of operating conditions: if the initial pressure is too high, the detonation transmits through the obstacle, while if it is too low, the detonation cannot be initiated. The experimental conditions were repeatable, with similar shock-front structures observed over multiple runs. Three independent datasets, referred

2.1. EXPERIMENTAL SETUP

to as A, B and C, were acquired under these conditions.

The shock front was visualized using two optical diagnostics: shadowgraph imaging and schlieren imaging. Shadowgraph imaging was performed using a Phantom V1210 camera at a frame rate of 39,167 fps. This setup captured the full field of view in a single run and was used to observe the overall shock dynamics. Schlieren imaging was performed using a Shimadzu HPV-X2 high-speed camera at a frame rate of 500,000 fps. Since the field of view of the Shimadzu camera covered only a portion of the observation section, each run was recorded at a different downstream location. Each dataset was then assembled by combining consecutive runs recorded at successive locations, allowing the evolution of the shock front to be followed over the entire observation section. Both optical setups used an arc lamp as the light source. The shadowgraph images were used for qualitative visualization of the shock dynamics, while the quantitative analysis presented in this work was performed using the schlieren images. For clarity, the intermediate results presented in the following sections are shown for a single dataset, the same procedure being applied to all three.

The triggering method was different for the two imaging techniques. For shadowgraph imaging, the ignition system and the Phantom camera were triggered simultaneously using a BNC signal, allowing the shock propagation to be recorded from the beginning of the experiment. For schlieren imaging, the Shimadzu HPV-X2 camera was triggered by pressure sensors located along the tube. These sensors detect the passage of the shock front and trigger the camera as the shock approaches the selected observation region.

2.2. OBSERVED SHOCK DYNAMICS

Table 2.1: Summary of experimental parameters.

Parameter	Value
Mixture	$\text{CH}_4 + 2\text{O}_2$
Initial pressure	13.8 kPa
Temperature	Ambient
Tube length	3.42 m
Channel width	19 mm
Schlieren frame rate	500,000 fps
Shadowgraph frame rate	39,167 fps

2.2 OBSERVED SHOCK DYNAMICS

The observed shock dynamics are presented in the images obtained from shadowgraph imaging in Figure 2.2 and schlieren imaging in Figure 2.4. The shadowgraph sequence provides the full field of view and gives an overview of the complete shock motion, while the schlieren sequence highlights the shock-front structures with greater clarity.

In the first shadowgraph frame shown in Figure 2.2 at $t = 16 \mu\text{s}$, the leading shock is strongly curved, and the first reflections appear near the top and bottom walls. These reflections lead to the formation of two triple points along the shock front. As shown in the annotated schlieren image in Figure 2.3, the triple points separate the Mach shocks from the incident shock. The flame is also visible behind the leading shock.

The following shadowgraph frames illustrate the evolution of these structures. At $t = 195 \mu\text{s}$, the two triple points are clearly visible and move along the shock front from the walls toward the centerline. At $t = 246 \mu\text{s}$, they have just interacted near the centerline and are reflected back toward the walls. By $t = 374 \mu\text{s}$, they continue to propagate towards

2.2. OBSERVED SHOCK DYNAMICS

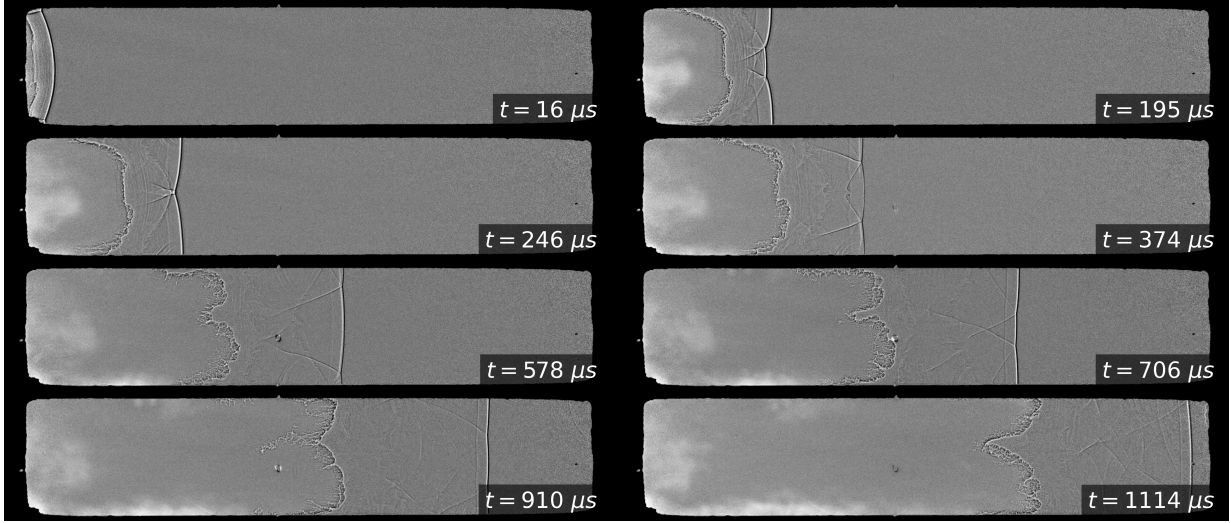


Figure 2.2: Shadowgraph sequence showing the evolution of the perturbed shock front propagating in a stoichiometric $\text{CH}_4 + 2\text{O}_2$ mixture at 13.8 kPa.

the walls. The following frames show the same repeated motion resulting from successive reflections between the walls and the centerline. Throughout the sequence, the shock front gradually becomes more planar. In the final frame at $t = 1114 \mu\text{s}$, the shock is nearly planar and the triple points are only weakly visible. The shadowgraph images also show that the flame follows the leading shock, while the distance between the shock and the flame increases as the shock propagates downstream.

The schlieren sequence in Figure 2.4 reveals the same dynamics with a clearer visualization of the shock-front structures. In the first schlieren frame at $t = 50 \mu\text{s}$, the curved shock and the two triple points are already visible. Then, on the next frame, the triple points move toward each other along the shock front, and at $t = 238 \mu\text{s}$ they interact near the centerline. In the frame at $t = 334 \mu\text{s}$, they have reflected and begin to travel back

2.2. OBSERVED SHOCK DYNAMICS

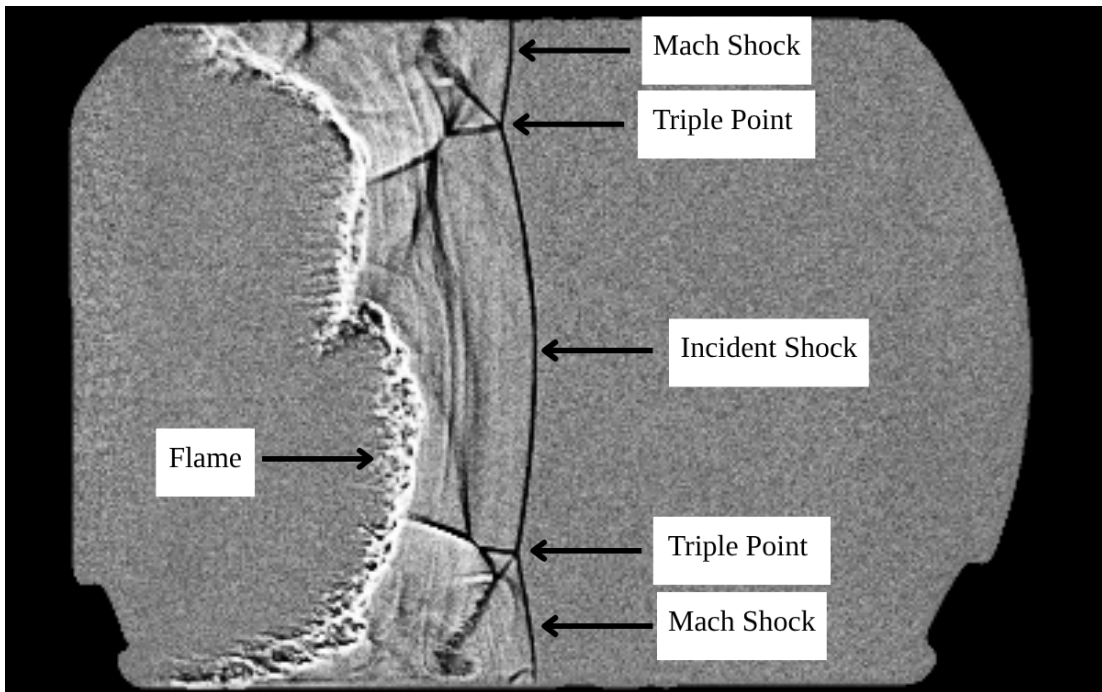


Figure 2.3: Annotated schlieren image at $t = 100 \mu s$ showing the triple point structure. The Mach stem, incident shock, and triple points are indicated.

2.3. SHOCK-FRONT EXTRACTION

toward the walls. At a later time, at $t = 532 \mu s$, the triple points are located close to the walls, while the subsequent frames at $t = 622 \mu s$, $770 \mu s$ and $972 \mu s$ show the continuation of this behavior. Throughout the sequence, the contrast between the strong and weak regions of the shock front progressively diminishes, indicating that the perturbations decay as the shock propagates downstream.

2.3 SHOCK-FRONT EXTRACTION

The shock front position was extracted from each schlieren frame using an intensity-based detection algorithm. For each row of pixels, the algorithm scans from right to left and identifies the first pixel darker than a prescribed threshold, giving the shock front position $x(y)$ at each vertical location. Outliers are then removed using a local median filter, and the resulting gaps are filled by interpolation. The extracted front is shown superimposed on a schlieren image in Figure 2.5.

2.4 MACH NUMBER RECONSTRUCTION

From the extracted shock front positions, a time-of-arrival map $t(x, y)$ is reconstructed. For each spatial position (x, y) , the time at which the shock front passed through that location is obtained by interpolating the detected front positions across frames. The resulting map is shown in Figure 2.6.

2.4. MACH NUMBER RECONSTRUCTION

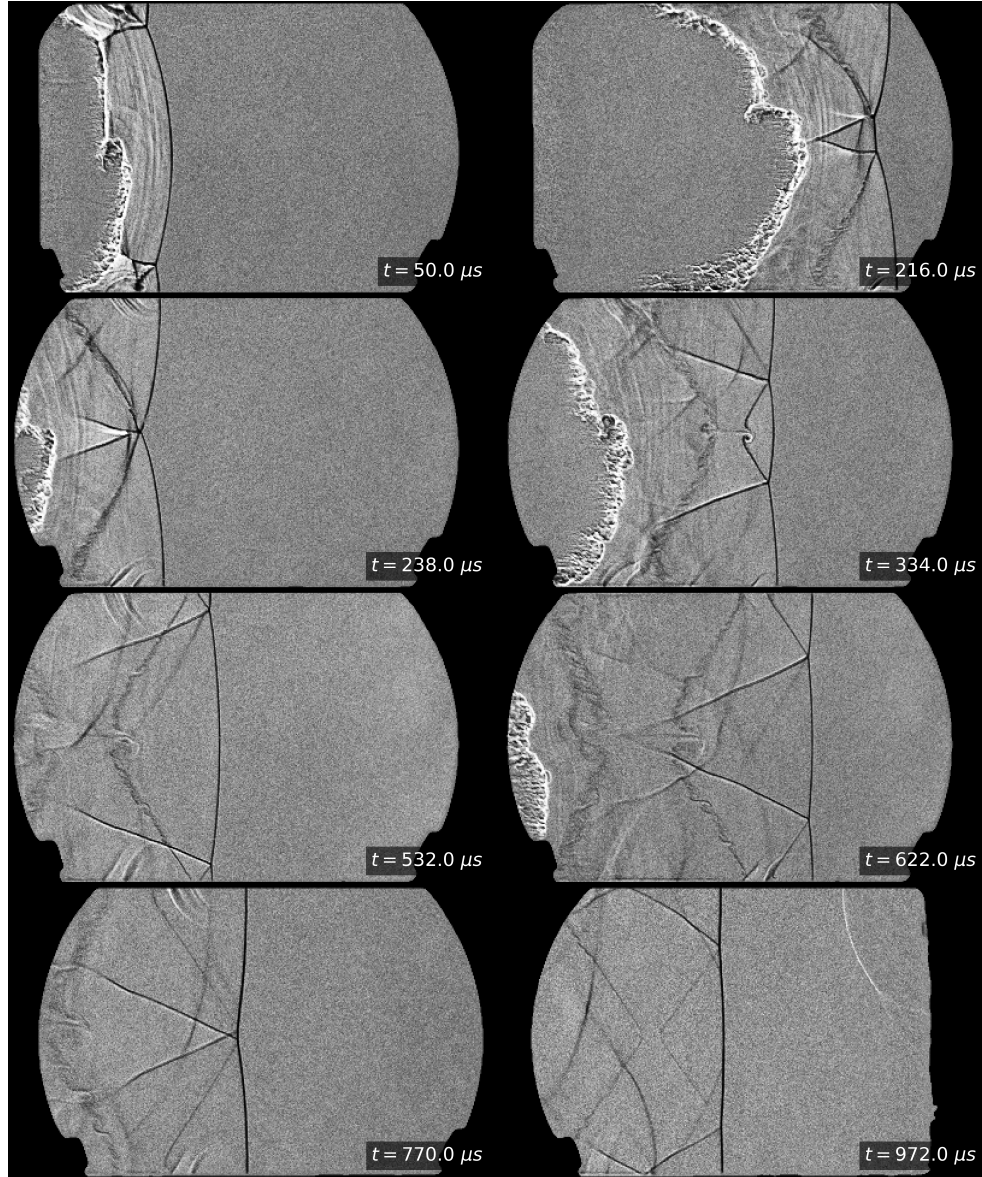


Figure 2.4: Schlieren sequence showing the decay of the triple point structure as the shock propagates downstream in a stoichiometric $\text{CH}_4 + 2\text{O}_2$ mixture at 13.8 kPa.

2.4. MACH NUMBER RECONSTRUCTION

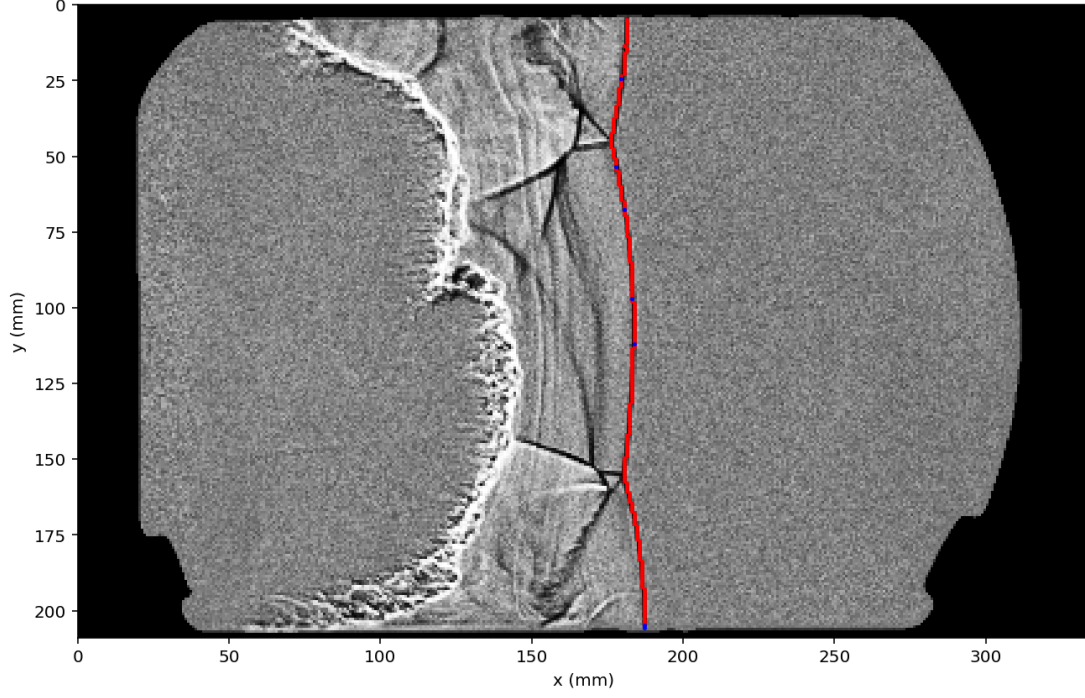


Figure 2.5: Detected shock front (red) superimposed on a schlieren image at $t = 128 \mu\text{s}$. The blue point indicates an interpolated value.

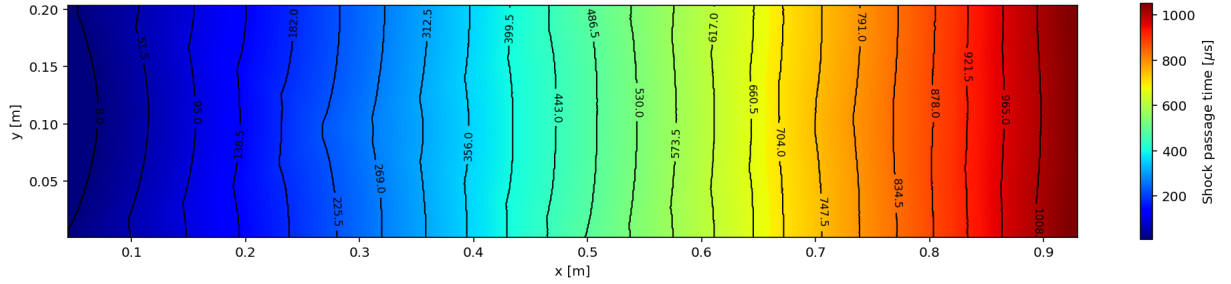


Figure 2.6: Time-of-arrival map $t(x, y)$ of the shock front.

2.4. MACH NUMBER RECONSTRUCTION

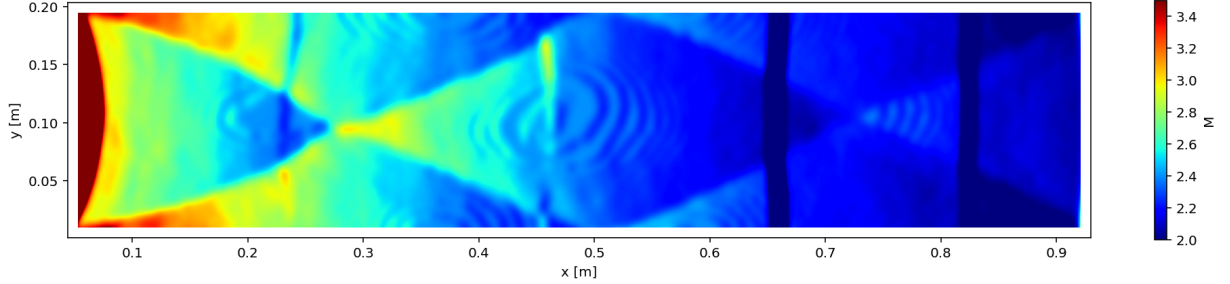


Figure 2.7: Reconstructed Mach number map $M(x, y)$ showing the strong (Mach stem) and weak (incident shock) regions along the shock front.

From this map, the local Mach number is reconstructed using the relation derived from Geometrical Shock Dynamics (introduced in Chapter 4),

$$M = \frac{1}{|\nabla \alpha|}, \quad \alpha = c_0 t, \quad (2.1)$$

where $c_0 = 352.4$ m/s is the speed of sound in the undisturbed gas mixture. A Gaussian filter is applied to the time-of-arrival map prior to differentiation to reduce noise, and the gradient is evaluated using a 21-point finite difference stencil [14]. The resulting Mach number map is shown in Figure 2.7, where two distinct regions are clearly visible along the shock front: a stronger region corresponding to the Mach stem, and a weaker region corresponding to the incident shock. The map also shows that the overall strength of the shock progressively decreases as it propagates downstream.

2.5 QUANTIFICATION OF THE PERTURBATION

The Mach number profile $M(y)$ is extracted along the shock front along contours of constant α . The shock-shocks are identified as the locations of maximum $|dM/dy|$ along the profile, and divide the shock front into two regions: an inner region, between the two shock-shocks, and an outer region, between the shock-shocks and the walls. One of these regions corresponds to the Mach stem and the other to the incident shock, and their roles are exchanged each time the triple points reflect from the walls or from the centerline. The perturbation is then quantified as the difference in mean Mach number between the stronger and the weaker region,

$$\Delta M = |\bar{M}_{\text{strong}} - \bar{M}_{\text{weak}}|, \quad (2.2)$$

where $\bar{M}_{\text{strong}}$ and $\bar{M}_{\text{weak}}$ are the mean Mach numbers in each region, excluding the immediate vicinity of the shock-shocks, where the profile is not representative of either region. ΔM therefore corresponds to the height of the step in Mach number across the shock-shocks, as illustrated in Figure 2.8. The evolution of $M(y)$ at several successive times is shown in Figure 2.9, where the progressive decrease of ΔM is clearly visible.

2.6 EXPERIMENTAL RESULTS

The evolution of ΔM as a function of the non-dimensional time $t^* = t c_s / H$ is shown in Figure 2.10 on a log-log scale for the three datasets, together with their mean, where

2.6. EXPERIMENTAL RESULTS

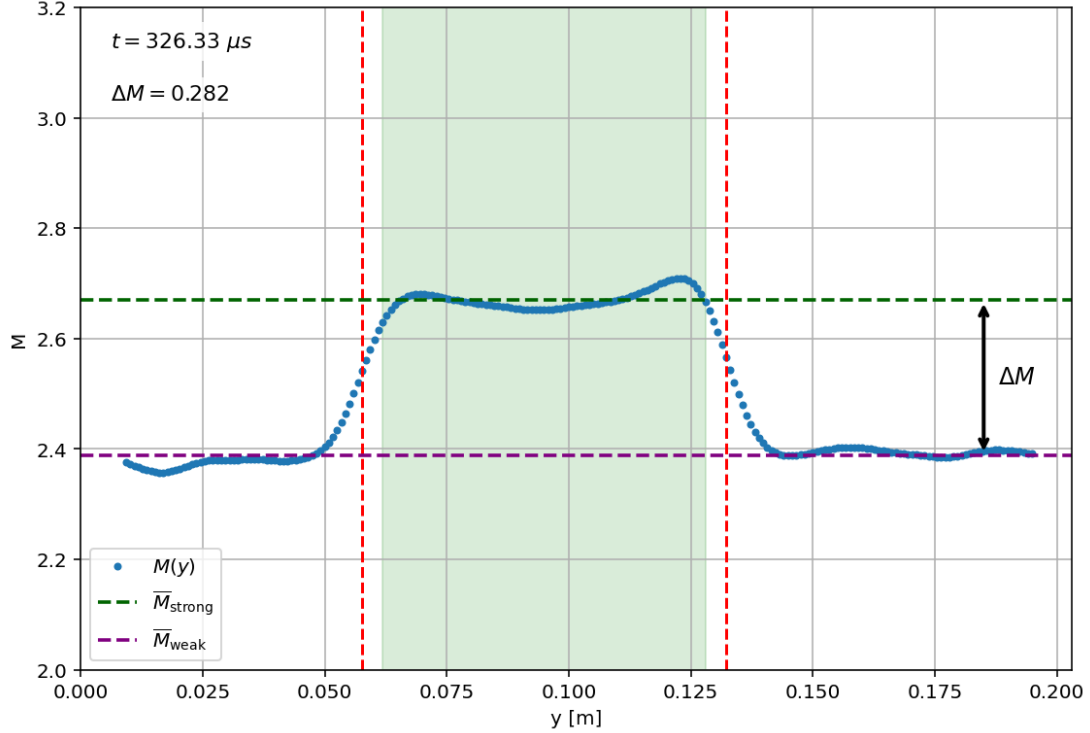


Figure 2.8: Mach number profile $M(y)$ at $t = 326.33 \mu s$ illustrating the definition of ΔM . The vertical dashed lines indicate the shock-shocks, and the shaded region the points between them used to compute the mean Mach number. The horizontal dashed lines indicate the mean Mach numbers of the strong and weak regions, $\bar{M}_{\text{strong}}$ and $\bar{M}_{\text{weak}}$.

2.6. EXPERIMENTAL RESULTS

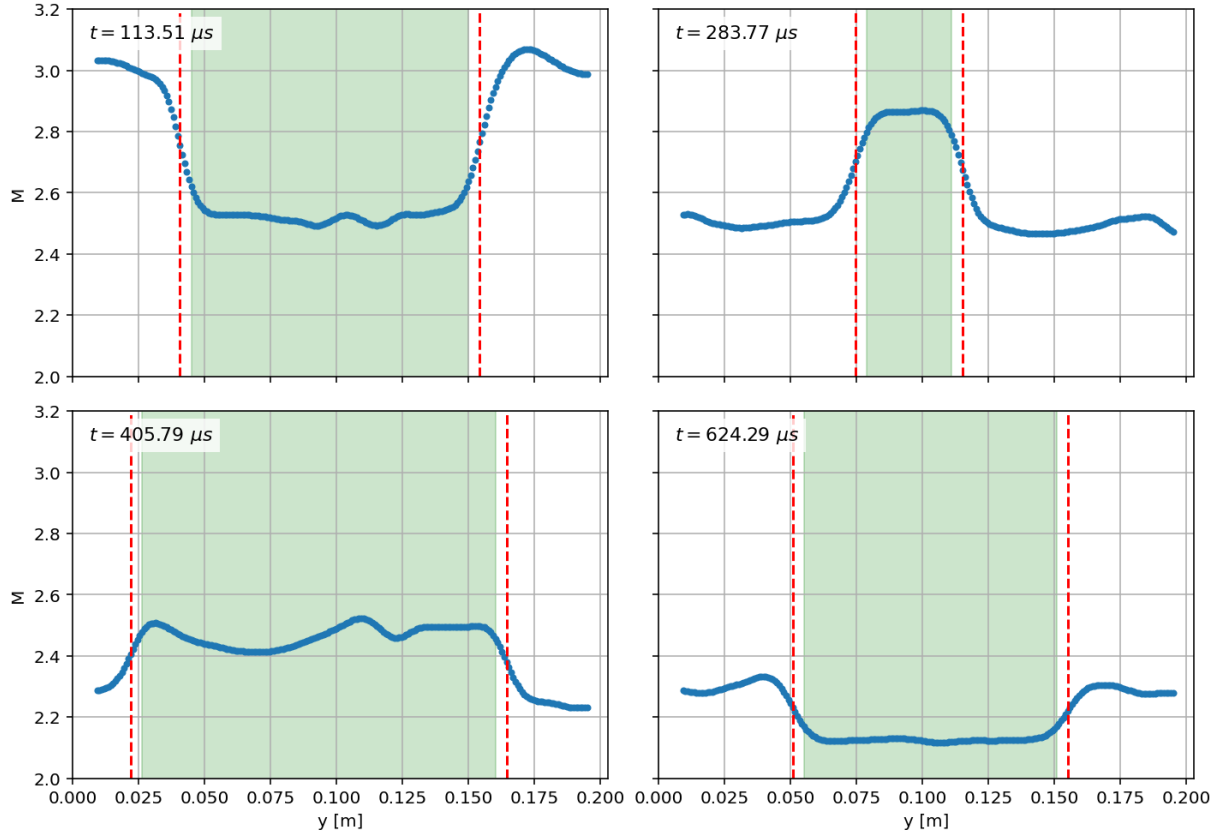


Figure 2.9: Mach number profiles $M(y)$ along the shock front at successive times. The shock-shocks are indicated by the vertical dashed lines, and the shaded regions indicate the points used to compute the mean Mach number between the shock-shocks.

2.6. EXPERIMENTAL RESULTS

H is the channel height and c_s the initial post-shock speed of sound. One unit of non-dimensional time therefore corresponds approximately to the time needed for a triple point to travel across the channel height H . The three datasets are in close agreement, confirming the repeatability of the experiments. A reference $1/t$ decay is included, and captures the behavior of the mean well over most of the data. However, in early times, the decay deviates from this trend. This is because the inner and outer shock dynamics are not yet fully coupled. It takes on the order of one to two reflections for information to propagate along the shock surface. After this transient phase, the perturbation follows the $1/t$ decay.

2.6. EXPERIMENTAL RESULTS

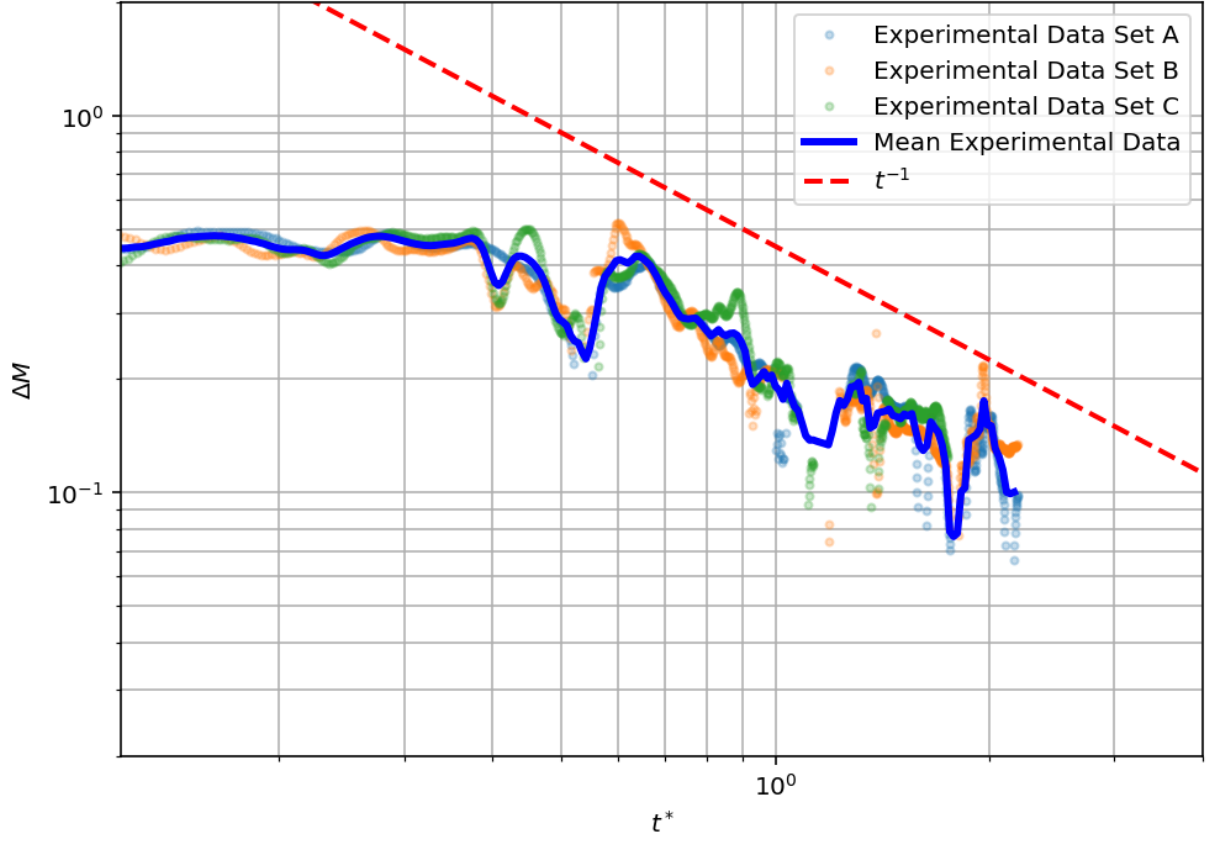


Figure 2.10: Evolution of ΔM as a function of non-dimensional time $t^* = t c_s / H$ on a log-log scale for the three experimental datasets (A, B and C) and their mean. Isolated outliers have been removed. The dashed line indicates a $1/t$ decay.

CHAPTER 3

EULER SIMULATIONS

3.1 NUMERICAL SETUP

Euler simulations were conducted to complement the experimental results and extend the observation range to longer propagation distances. The governing equations are the two-dimensional Euler equations for a compressible inviscid gas,

$$\frac{\partial \mathbf{U}}{\partial t} + \frac{\partial \mathbf{F}_x}{\partial x} + \frac{\partial \mathbf{F}_y}{\partial y} = 0, \quad (3.1)$$

where $\mathbf{U}$, $\mathbf{F}_x$, and $\mathbf{F}_y$ are the vectors of conserved variables and fluxes,

$$\mathbf{U} = \begin{pmatrix} \rho \\ \rho u \\ \rho v \\ \frac{p}{\gamma-1} + \frac{\rho(u^2+v^2)}{2} \end{pmatrix}, \quad \mathbf{F}_x = \begin{pmatrix} \rho u \\ \rho u^2 + p \\ \rho uv \\ u \left(\frac{\gamma p}{\gamma-1} + \frac{\rho(u^2+v^2)}{2} \right) \end{pmatrix}, \quad \mathbf{F}_y = \begin{pmatrix} \rho v \\ \rho uv \\ \rho v^2 + p \\ v \left(\frac{\gamma p}{\gamma-1} + \frac{\rho(u^2+v^2)}{2} \right) \end{pmatrix}. \quad (3.2)$$

In this work, the numerical simulations are carried out using a Discontinuous-Galerkin-Hancock scheme [15]. It provides third-order accuracy in both space and time while only requiring linear basis functions. The scheme is implemented in a large-scale parallel com-

3.1. NUMERICAL SETUP



Figure 3.1: Schematic of the computational domain. A planar shock of Mach number M_s propagates through a channel of height H toward a square obstacle of height $h = 0.4H$ placed at the bottom wall.

puting framework developed by Kaufmann and McDonald [16]. The HLL flux function is used to compute the intercell fluxes [17], while the Venkatakrishnan slope limiter is applied to improve the stability of the high-order solution [18].

The computational domain consists of a rectangular channel of height H , used as the reference length scale, and a total length of $17H$. A square obstacle of side $0.4H$ is placed at the bottom wall, occupying the region $x \in [0, 0.4H]$, $y \in [0, 0.4H]$, giving a blockage ratio of $h/H = 40\%$. A schematic of the computational domain is shown in Figure 3.1.

The domain is discretized with 500 grid points in the y -direction and 8500 grid points in the x -direction, corresponding to a uniform resolution of 500 points per unit length. All flow variables are non-dimensionalized using the channel height $H = 1$ as the reference length, and the upstream density $\rho_0 = 1$ and pressure $P_0 = 1$ as reference scales.

The boundary conditions are free in the x -direction and symmetrical in the y -direction. By symmetry, the computational domain represents half of the channel.

The simulation is initialized with a planar shock of Mach number $M_s = 2.5$ propagating

3.2. OBSERVED SHOCK DYNAMICS

Table 3.1: Summary of numerical parameters for the Euler simulations.

Parameter	Value
Shock Mach number	$M_s = 2.5$
Ratio of specific heats	$\gamma = 1.4$
Channel height	$H = 1$
Channel length	$17H$
Obstacle height	$h = 0.4H$
Blockage ratio	$h/H = 40\%$
Grid resolution	500 pts/unit
Grid points in y	500
Grid points in x	8500

in a perfect gas with $\gamma = 1.4$. The gas ahead of the shock is initially at rest. The post-shock pressure, density, and velocity are obtained from the normal-shock Rankine–Hugoniot relations for $M_s = 2.5$.

3.2 OBSERVED SHOCK DYNAMICS

The evolution of the shock front is illustrated in Figure 3.2, which shows the pressure field at several successive times.

At $t = 0.25$, a planar shock propagates to the right and approaches the obstacle. At $t = 0.55$, the shock has reached the obstacle and diffracts around it. A reflected wave is also visible, propagating back toward the left. At $t = 0.75$, the diffracted shock has reached the bottom wall, where it undergoes a regular reflection. By $t = 1.25$, this regular reflection has transitioned into a Mach reflection. A triple point forms at the intersection of the Mach stem, the incident shock, and the transverse wave, and begins to propagate

3.3. RESULTS

upward along the shock front. At $t = 2.3$, the triple point has reached the top wall and is reflected back downward.

The subsequent frames at $t = 2.9$, 4.3 , and 6.0 show the continuation of this behavior, with the triple point reflecting back and forth between the top and bottom walls. Throughout the sequence, the pressure difference between the strong region (Mach stem) and the weak region (incident shock) progressively decreases, and the curvature of the shock front diminishes. This indicates that the perturbation decays as the shock propagates downstream. Furthermore, since the gas is inviscid and non-reacting, the mean Mach number of the shock remains approximately constant throughout the propagation, unlike in the experiments. This allows the decay of the perturbation to be studied independently of any overall shock attenuation.

3.3 RESULTS

The local Mach number along the shock front is extracted from the pressure field using the normal-shock Rankine–Hugoniot relation,

$$M = \sqrt{1 + \frac{\gamma + 1}{2\gamma} \left(\frac{P_{\max}}{P_0} - 1 \right)}, \quad (3.3)$$

where $P_{\max}$ is the local peak pressure immediately behind the shock front and P_0 is the undisturbed pressure ahead of it. This gives the Mach number profile $M(y)$ along the shock front at each time, as shown in Figure 3.3.

3.3. RESULTS

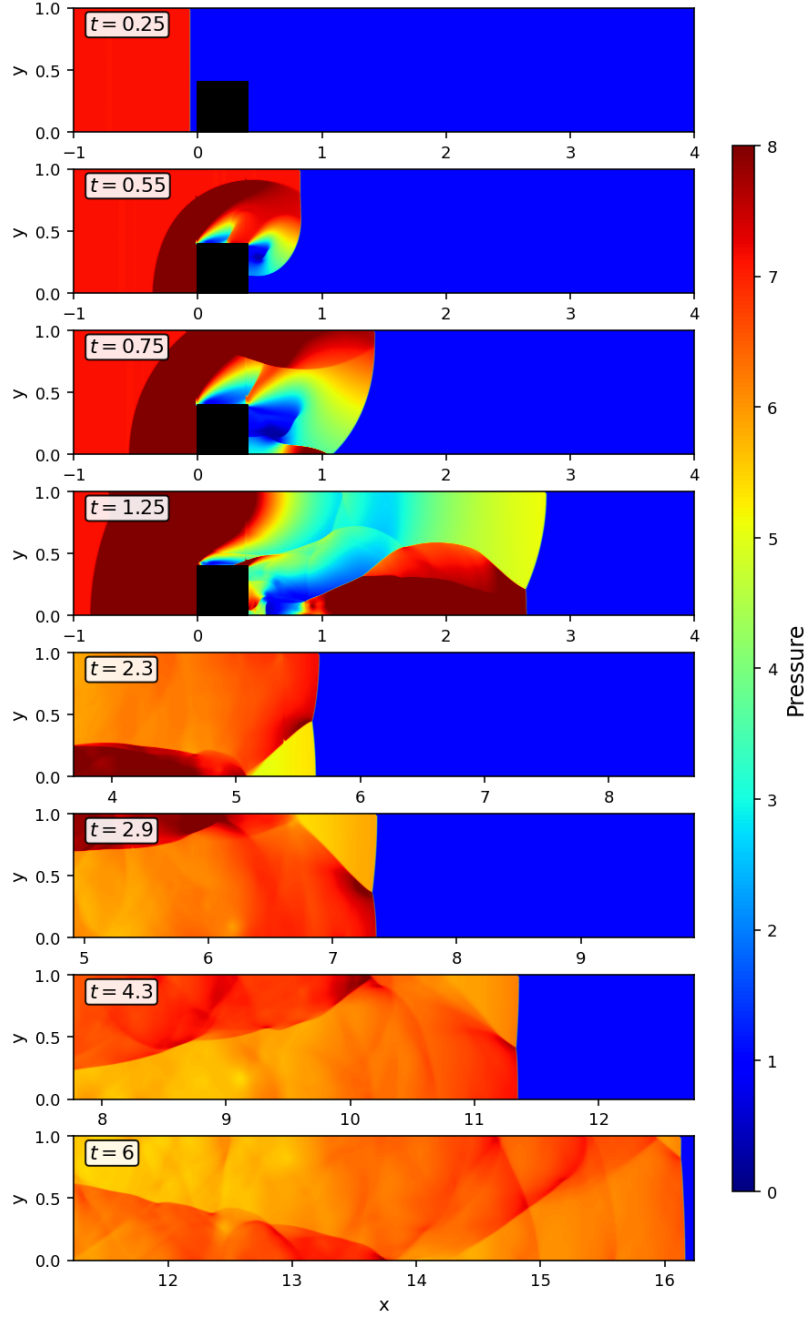


Figure 3.2: Pressure field at successive times showing the evolution of the shock front in the channel. The color scale represents the non-dimensionalized pressure $\bar{P} = P/P_0$. The shock propagates from left to right.

3.3. RESULTS

The shock-shock is identified as the location of maximum $|dM/dy|$ along the profile. Since a single triple point is present in the computational domain, the shock-shock separates the shock front into two regions, located between the shock-shock and the bottom wall, and between the shock-shock and the top wall. As in the experiments, one of these regions corresponds to the Mach stem and the other to the incident shock. The perturbation amplitude is then defined as in Section 2.5,

$$\Delta M = |\bar{M}_{\text{strong}} - \bar{M}_{\text{weak}}|, \quad (3.4)$$

where $\bar{M}_{\text{strong}}$ and $\bar{M}_{\text{weak}}$ are the mean Mach numbers in the strong and weak regions, respectively, again excluding the immediate vicinity of the shock-shock.

The evolution of ΔM as a function of the non-dimensional time $t^* = t c_s / (2H)$, defined as in Section 2.6 but based on the full channel height $2H$, is shown in Figure 3.4 on a log-log scale, with the origin of time taken at the appearance of the triple point. After an initial transient phase, the decay closely follows a scaling of $1/t$, consistent with the experimental observations.

3.3. RESULTS

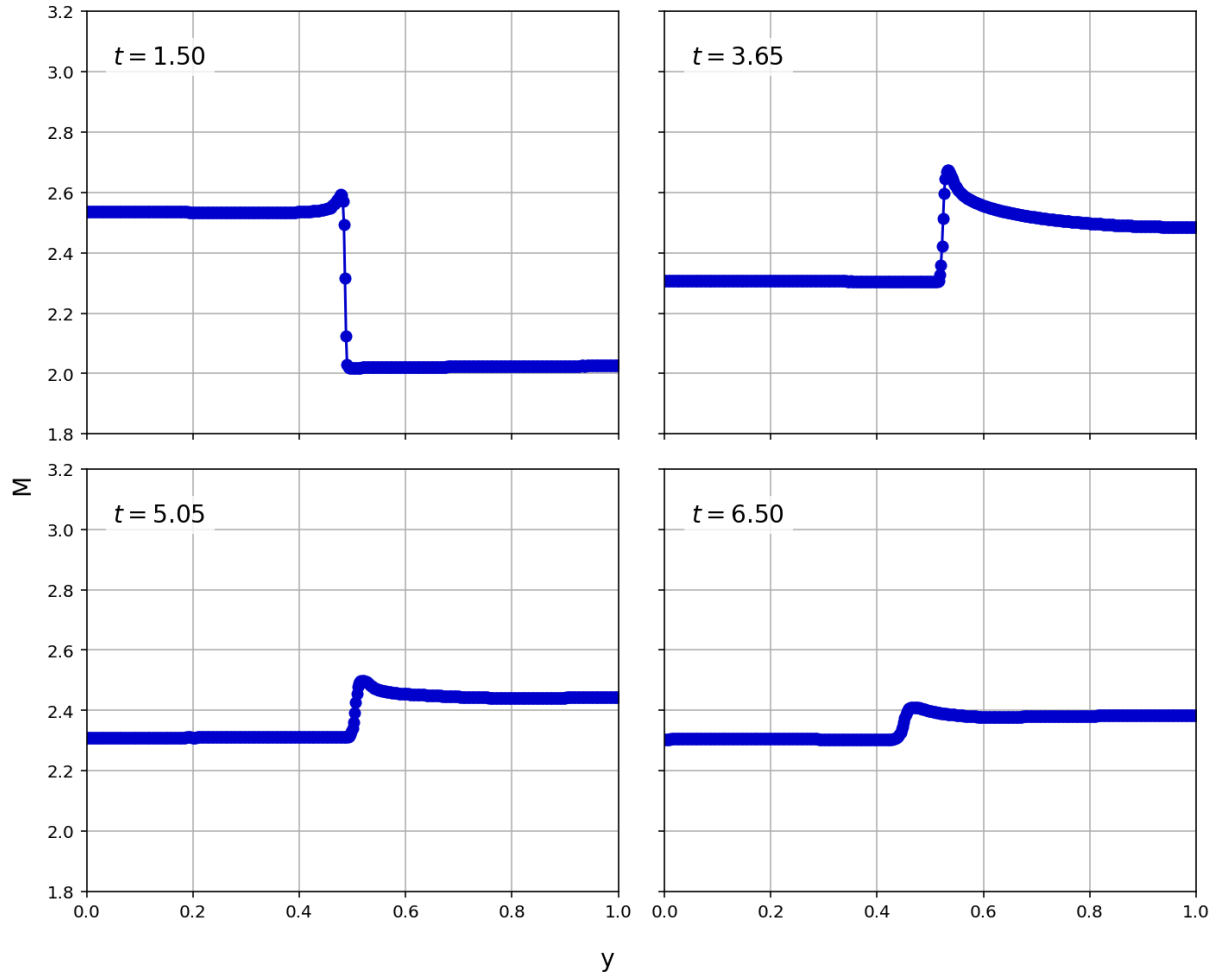


Figure 3.3: Mach number profiles $M(y)$ along the shock front at successive times.

3.3. RESULTS

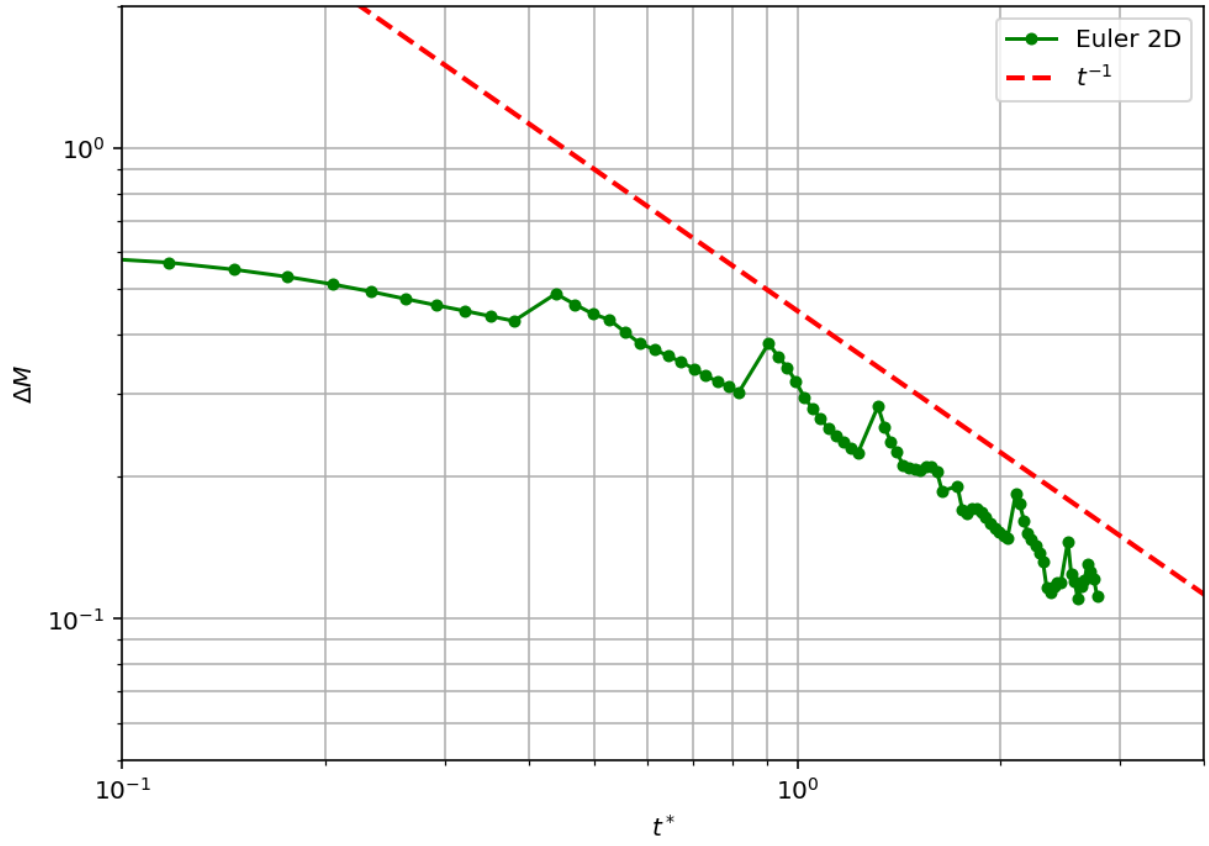


Figure 3.4: Evolution of ΔM as a function of non-dimensional time t^* on a log-log scale. A few isolated peaks caused by the reflections of the triple point have been removed. The dashed line indicates a $1/t$ decay.

CHAPTER 4

GEOMETRICAL SHOCK DYNAMICS

4.1 INTRODUCTION TO GSD

Geometrical Shock Dynamics (GSD) is an approximate method used to describe the motion of a shock front without solving the full flow field behind it. The method was developed by Whitham [1, 8] for problems of shock diffraction and stability. Its central assumption is that the propagation of the shock between two neighbouring rays can be treated as if the rays were solid walls, so that each ray tube behaves as a channel of varying area. The local Mach number is then related to the ray-tube area through the area–Mach number relation obtained by Chester and Chisnell for shocks propagating in channels [8]. Its theoretical background and its application to shock-front perturbations were discussed in Section 1.4.3. In contrast with Euler simulations, where the pressure, density, and velocity fields are resolved everywhere in the domain, GSD only follows the position and strength of the shock front. This makes the method much less expensive computationally.

The basic idea of GSD is that the shock propagates in the direction normal to itself, with a speed determined by the local geometry of the shock. In the present work, the shock position is described using the arrival-time function $\alpha = c_0 t$, where c_0 is the speed

4.2. NUMERICAL SETUP

of sound ahead of the shock. The governing equations of GSD are

$$\nabla \cdot \left(\frac{M}{A} \nabla \alpha \right) = 0, \quad M = \frac{1}{|\nabla \alpha|}, \quad (4.1)$$

where A is the ray-tube area. The system is closed by specifying A as a function of M ,

$$A = f(M). \quad (4.2)$$

Contours of constant α represent successive positions of the shock front, while variations of M describe changes in the local shock speed along the front.

Since the GSD equations are hyperbolic, gradients in Mach number along the shock front can steepen into discontinuities. These discontinuities are called shock-shocks. In the present problem, they play the same role as the triple-point structures observed in the experiments and Euler simulations: they separate stronger and weaker regions of the shock front. GSD is therefore well suited for studying the propagation and decay of shock-front perturbations.

4.2 NUMERICAL SETUP

The GSD simulations are based on a numerical implementation of the method proposed by Schwendeman [19]. The governing equations are those given in Equation (4.1), closed

4.2. NUMERICAL SETUP

by the area–Mach number relation used by Whitham [1, 8],

$$\frac{dA}{A} = -\frac{M\lambda(M)}{M^2 - 1} dM, \quad (4.3)$$

where $\lambda(M)$ is a slowly varying function of the shock strength. For $\gamma = 1.4$, λ varies only from 4 in the weak-shock limit to approximately 5.07 in the strong-shock limit [1]. In the strong-shock limit, Equation (4.3) reduces to a simple power law, which is used here for simplicity, following Schwendeman [19],

$$A = M^{-n}, \quad n = 5.0743 \quad \text{for} \quad \gamma = 1.4. \quad (4.4)$$

The governing equation is discretized on a uniform Cartesian grid using a finite difference scheme. Denoting $u = \partial\alpha/\partial x$ and $v = \partial\alpha/\partial y$, the discrete form of Equation (4.1) is

$$\frac{1}{\Delta x} (F_{i,j} u_{i,j} - F_{i-1,j} u_{i-1,j}) + \frac{1}{\Delta y} \left(G_{i,j+\frac{1}{2}} v_{i,j+\frac{1}{2}} - G_{i,j-\frac{1}{2}} v_{i,j-\frac{1}{2}} \right) = 0, \quad (4.5)$$

where $F = G = M/A$ and $M = (u^2 + v^2)^{-1/2}$. Upwind differencing is used in the x -direction, which is the general direction of shock propagation, while central differencing is used in the y -direction. To suppress numerical oscillations near shock-shocks, a numerical viscosity term is added in the y -direction, following [19]. For strong shocks, the viscosity parameter reduces to

$$\mu = \frac{n}{n+1} \approx 0.835 \quad \text{for} \quad n = 5.0743. \quad (4.6)$$

4.2. NUMERICAL SETUP

The computational domain is a rectangular channel of height $H = 1$ and length $22H$, with

$$x \in [0, 22H], \quad y \in [0, H].$$

The domain is discretized with a uniform resolution of 200 grid points per unit length. Wall boundary conditions, $\partial\alpha/\partial y = 0$, are applied at $y = 0$ and $y = H$.

The shock is initialized as a planar front with a non-uniform Mach number distribution prescribed along the front. Following Schwendeman's approach [19], the initial profile is given by

$$\frac{1}{M_0(\eta)} = \frac{1}{2} \left(\frac{M_c + M_w}{M_c M_w} \right) \left(1 + \frac{M_c - M_w}{M_c + M_w} \cos(2\pi\eta) \right), \quad \eta = \frac{y}{H}, \quad (4.7)$$

where $M_c = 2.5$ is the Mach number at the centerline and M_w is the Mach number near the walls. The wall Mach number M_w is determined from the area-Mach number relation, using the ray-tube areas A_c and A_w ,

$$M_w = M_c \left(\frac{A_c}{A_w} \right)^{1/n}, \quad (4.8)$$

where the ratio A_c/A_w is set by the channel geometry. This profile, shown in Figure 4.1, introduces a weaker central region and stronger wall regions. As the front propagates, we expect this non-uniformity to steepen and to form two shock-shocks along the shock front.

4.3. OBSERVED SHOCK DYNAMICS

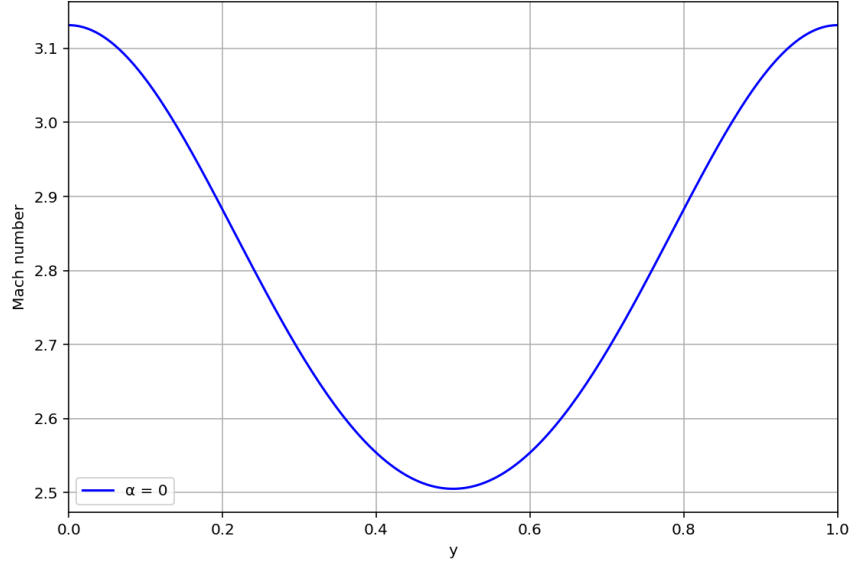


Figure 4.1: Initial Mach number profile $M_0(\eta)$ prescribed along the shock front at $x = 0$.

4.3 OBSERVED SHOCK DYNAMICS

The evolution of the shock front is shown in Figure 4.2. The black contours represent successive positions of the shock front at constant values of α , while the color indicates the local Mach number along the front.

Initially, the shock front has a non-uniform Mach number distribution, with a weaker region near the centerline and stronger regions near the walls. In GSD, this non-uniformity drives wave propagation along the shock surface. The low-Mach region near the centerline generates expansion waves that propagate toward the walls, while the high-Mach regions near the walls generate compression waves that propagate toward the centerline. As a result, the gradients in Mach number along the front steepen and form shock-shocks.

4.3. OBSERVED SHOCK DYNAMICS

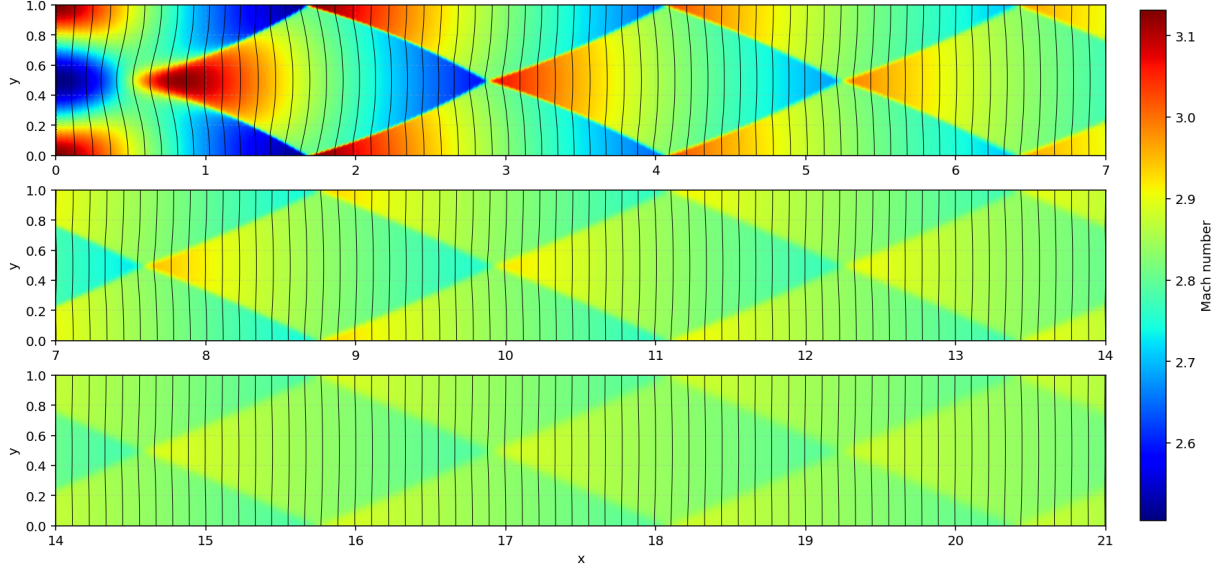


Figure 4.2: Mach number map showing the local shock strength as a function of position. Black contours represent successive positions of the shock front at constant values of α . The color indicates the local Mach number.

Once formed, the two shock-shocks propagate along the shock surface and reflect from the channel walls. During this process, the difference between the strong and weak regions progressively decreases. The shock front becomes more uniform, indicating that the initial perturbation decays as the shock propagates downstream. As in the Euler simulations, the mean Mach number of the shock remains approximately constant throughout the propagation.

4.4 ANALYSIS AND RESULTS

The Mach number profile $M(y)$ is extracted along contours of constant α from the GSD solution. As shown in Figure 4.3, the Mach number is nearly uniform within each of the regions separated by the shock-shocks, so that the profile consists of two almost flat levels connected by the shock-shocks. The mean Mach numbers $\bar{M}_{\text{strong}}$ and $\bar{M}_{\text{weak}}$ defined in Section 2.5 then reduce to the maximum and minimum values of M along the front, and the perturbation amplitude is simply

$$\Delta M = M_{\text{max}} - M_{\text{min}}, \quad (4.9)$$

where M_{max} and M_{min} are the maximum and minimum values of M along the shock front. This definition is equivalent to the one used for the experiments and Euler simulations, but does not require the shock-shocks to be located explicitly. It is not applicable to the experimental and Euler profiles, which exhibit fluctuations within each region as well as local overshoots near the shock-shocks (Figures 2.9 and 3.3), so that their extreme values are not representative of the mean levels.

The evolution of $M(y)$ at several successive times is shown in Figure 4.3. The profiles show that the difference between the strong and weak regions decreases as the shock propagates. This confirms the progressive decay of the initial perturbation.

The evolution of ΔM as a function of non-dimensional time, defined similarly using the channel height H , is shown in Figure 4.4 on a log-log scale. The decay closely follows

4.4. ANALYSIS AND RESULTS

a $1/t$ scaling after an initial transient phase.

Before post-processing, the evolution of ΔM exhibits narrow drops toward very small values, each occurring when the shock-shocks reflect from the walls or meet at the centerline. At these instants, the Mach number profile inverts, the strong and weak regions exchanging their roles. Since the shock-shocks have a finite numerical thickness, the profile momentarily passes through a state in which $M_{\max}$ and $M_{\min}$ are nearly equal, which artificially gives $\Delta M \approx 0$. These values are therefore not physical and were removed in post-processing.

4.4. ANALYSIS AND RESULTS

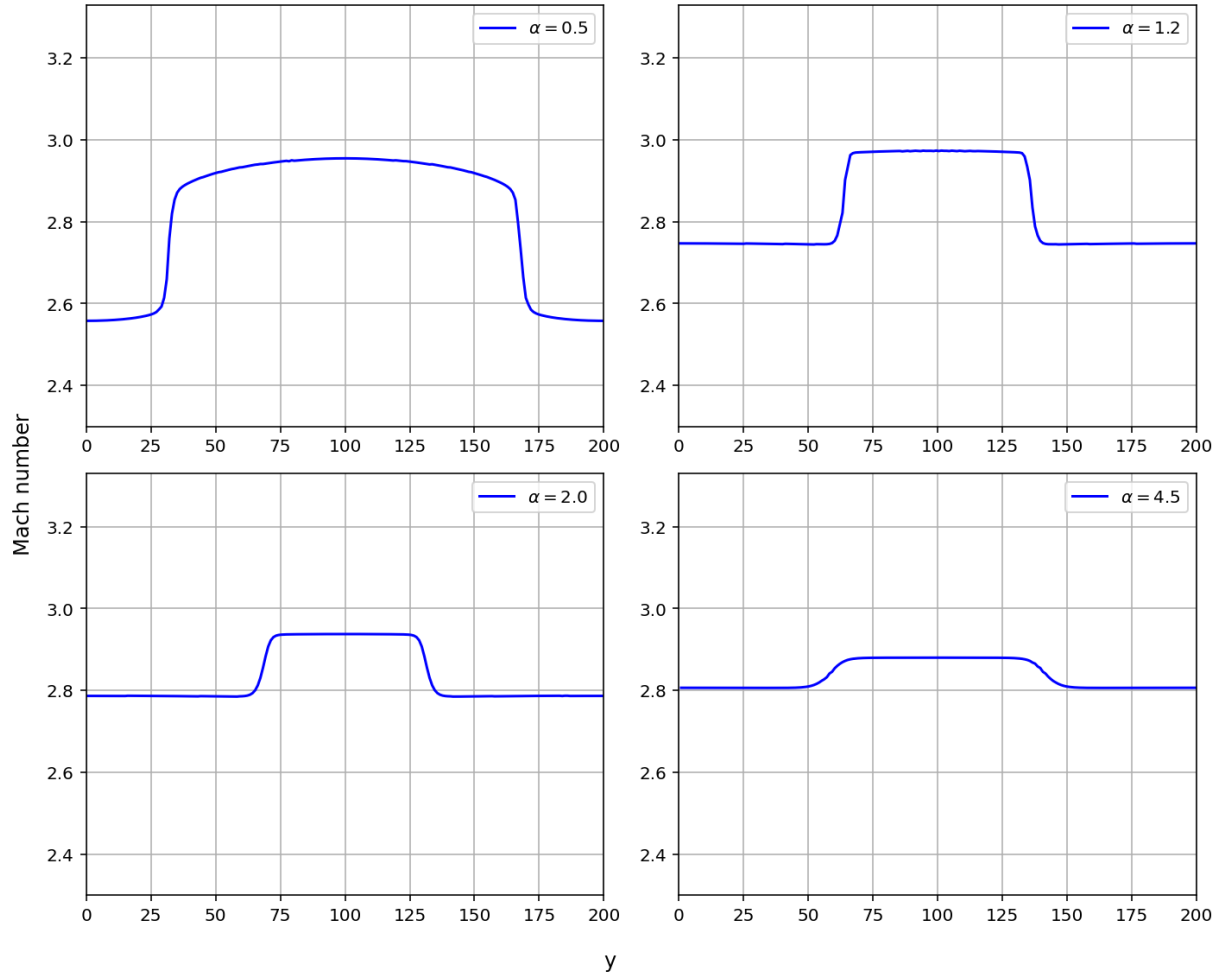


Figure 4.3: Mach number profiles $M(y)$ along the shock front at successive times.

4.4. ANALYSIS AND RESULTS

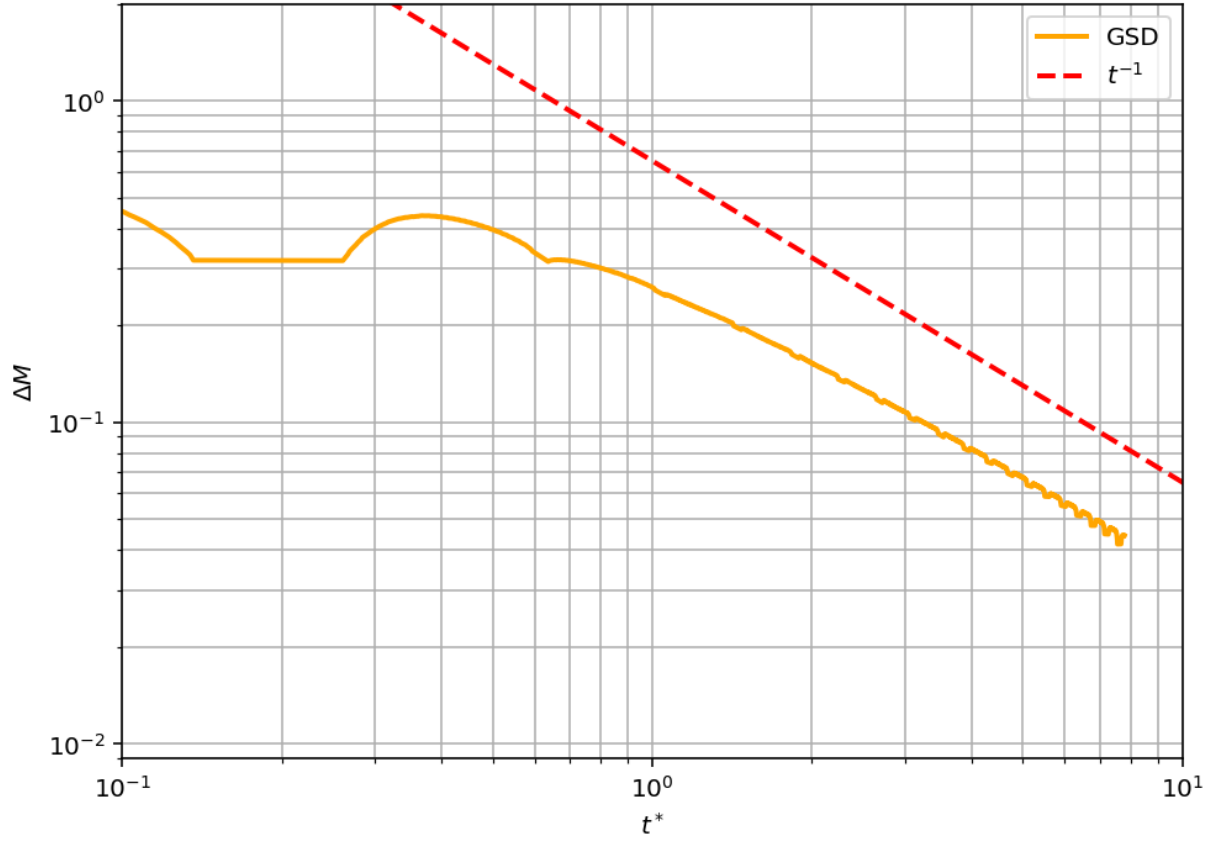


Figure 4.4: Evolution of ΔM as a function of non-dimensional time t^* on a log-log scale. Values affected by the numerical thickness of the shock-shocks during their reflections have been removed. The dashed line indicates a $1/t$ decay.

CHAPTER 5

DISCUSSION

5.1 COMPARISON OF DECAY RATES

The decay of ΔM obtained from the three approaches, namely shock tube experiments, Euler simulations, and GSD calculations, is compared in Figure 5.1. For the experiments, the mean of the three datasets presented in Figure 2.10 is shown. All results are plotted as a function of the non-dimensional time on a log-log scale. Three reference scalings are also shown: $t^{-1/2}$, t^{-1} , and $t^{-3/2}$, corresponding to decay rates previously reported in the literature.

After a brief initial transient, the three approaches show a decay close to t^{-1} . The reference scalings allow the present results to be compared with the decay laws proposed in previous studies. The $t^{-3/2}$ scaling predicted by Freeman [2, 3] and reported experimentally by Lapworth [5] decays faster than the decay observed here. The $t^{-1/2}$ scaling reported by Bowman [6] for the same experimental data decays more slowly. In contrast, the t^{-1} scaling discussed by Whitham [8] provides the closest agreement with the present experimental, Euler, and GSD results.

This agreement suggests that the decay observed in the present study is governed by

5.1. COMPARISON OF DECAY RATES

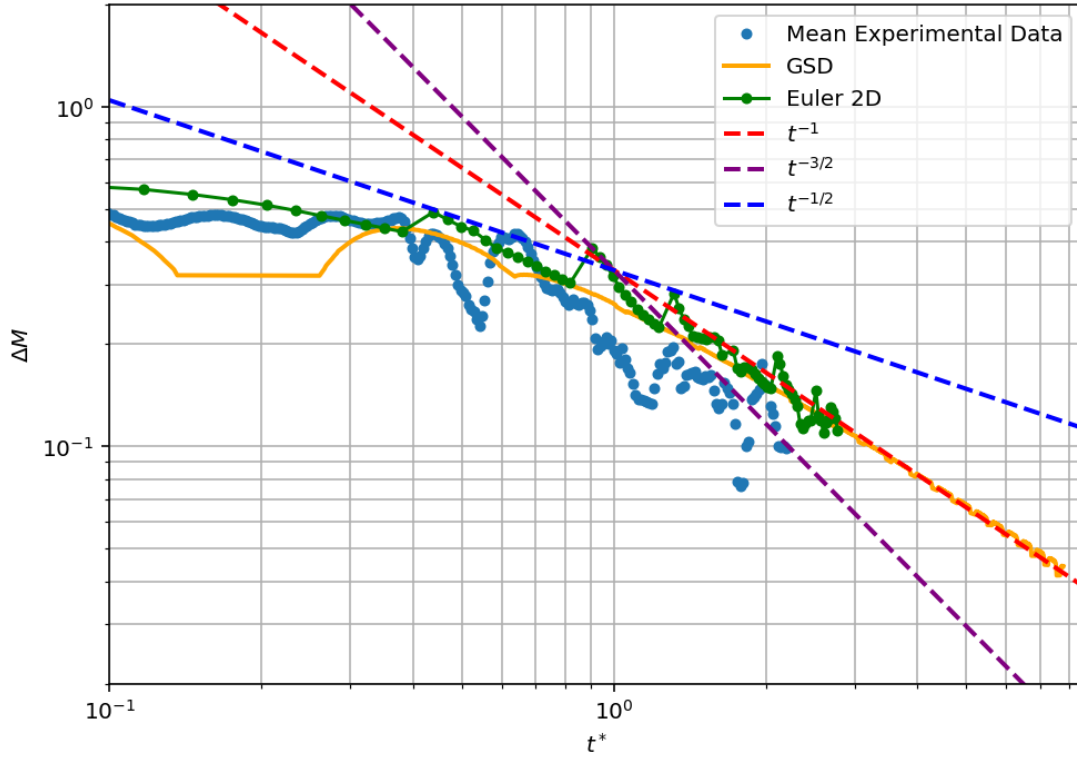


Figure 5.1: Comparison of ΔM as a function of non-dimensional time t^* from the shock tube experiments, Euler simulations, and GSD calculations, on a log-log scale. The reference scalings $t^{-3/2}$ (Freeman [2, 3], Lapworth [5]), t^{-1} (Whitham [8]), and $t^{-1/2}$ (Bowman [6]) correspond to decay rates previously reported in the literature.

the nonlinear dynamics of the perturbed shock front, rather than by the linear relaxation mechanism described by Freeman’s theory. The fact that the same scaling is obtained from the three different approaches also indicates that the observed decay is not specific to a particular experimental configuration or numerical model.

5.2 PHYSICAL ORIGIN OF THE DECAY

The good agreement between the GSD calculations and both the experiments and Euler simulations, shown in Figure 5.1, suggests that GSD captures the essential physics governing the decay of the perturbation. This agreement justifies using the GSD framework to seek a physical explanation for the observed $1/t$ scaling.

As discussed in Section 1.4.3, perturbations on the shock front propagate as nonlinear waves carrying the Riemann invariants involving θ and M along characteristics inclined at speed $\pm c(M)$. This is what allows the present problem to be conceptualized in terms of waves travelling along the shock surface, reflecting back and forth between the channel walls.

To understand the origin of the decay, it is useful to first consider a simpler problem studied by Whitham [8]: a planar shock of Mach number M_0 propagating along a periodic wall, as sketched in Figure 5.2. This problem is analogous to the present one: the shape of the wall perturbs the shock and creates discontinuities in both θ and M along the front, so that a difference ΔM appears between stronger and weaker regions of the shock, and this information travels along its surface. Since these disturbances are generated at the wall, they travel in one direction only, away from it. In this simple-wave approximation, θ is directly related to M , and the system reduces to a single nonlinear equation of inviscid Burgers type,

$$\frac{\partial M}{\partial \alpha} + c(M) \frac{\partial M}{\partial \beta} = 0, \quad (5.1)$$

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where $c(M)$ increases with M . This follows from the characteristic form of Equation (1.1). Since the disturbances propagate in the direction of increasing β only, all the C^- characteristics originate from the undisturbed part of the shock, so that the Riemann invariant $\theta - \int dM/(Ac)$ takes the same value everywhere, which provides a relation between θ and M valid throughout the disturbed region. Along each C^+ characteristic, $\theta + \int dM/(Ac)$ is also constant, so that θ and M are individually constant along it. Since c depends only on M , the C^+ characteristics are straight lines, $d\beta/d\alpha = c(M) = \text{const}$, each carrying a constant value of M [8].

Whitham considered a sinusoidal wall. Far from the wall, the expansion regions appear to originate from single points, so that the problem becomes equivalent to the piecewise-linear wall of Figure 5.2. At each convex corner, the wall turns away from the flow and generates an expansion fan: the characteristics diverge and M decreases smoothly. At each concave corner, the wall turns toward the flow and generates a compression. Since c increases with M , the stronger part of the compression travels faster, the characteristics converge and eventually cross, and a shock-shock forms. The periodic succession of shock-shocks and expansion fans gives the Mach number profile along the front a characteristic shape, with a sharp jump at each shock-shock followed by a linear variation through the expansion fan. Each period of this profile is an N-wave, whose wavelength is fixed by the period of the wall.

The decay of such an N-wave was analyzed by Wang et al. [20], as illustrated in Figure 5.3. Consider an N-wave of fixed wavelength Δ , in the frame of reference moving with the shock-shock. At time t , each half of the profile is a triangle of peak amplitude a (denoted

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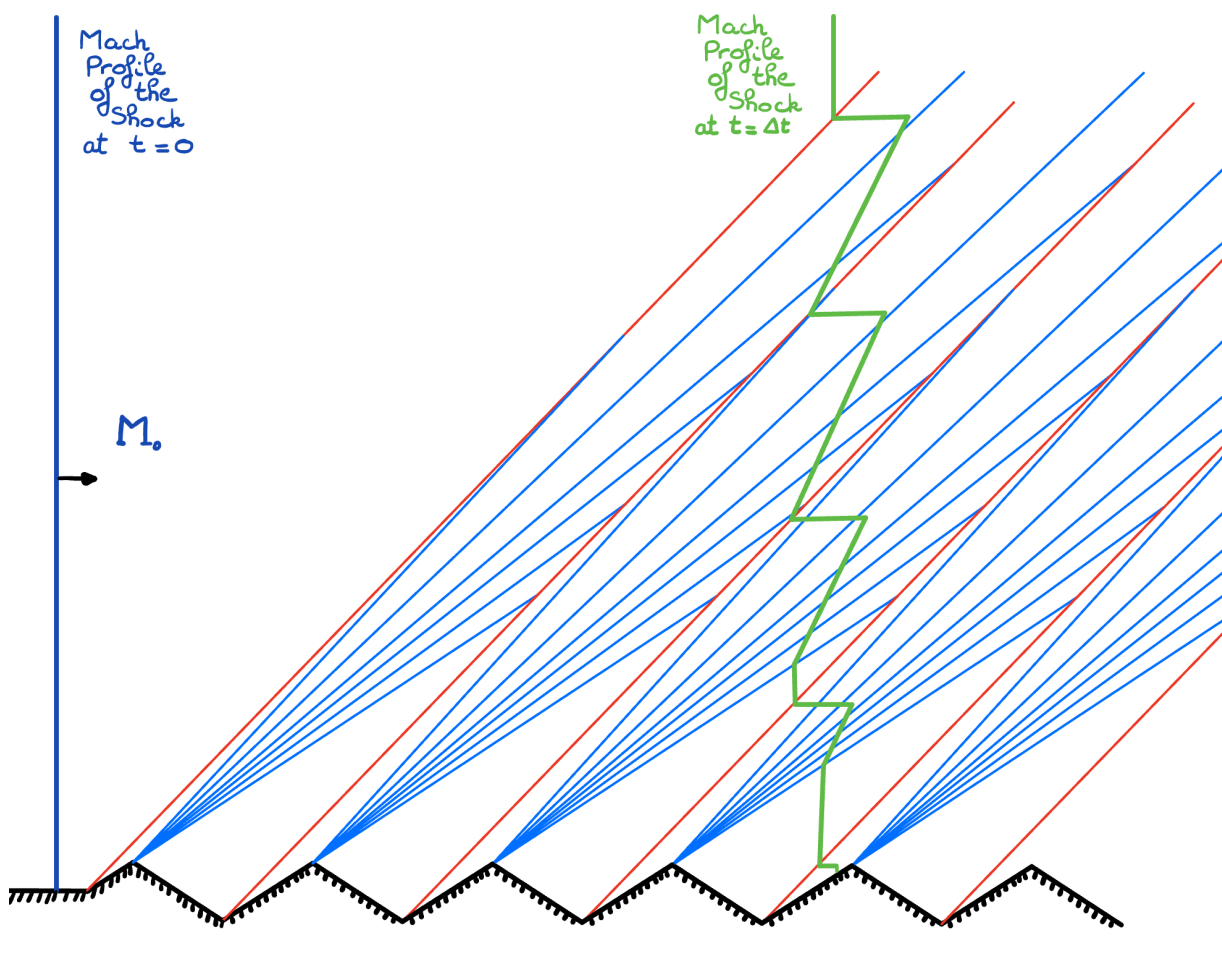


Figure 5.2: Sketch of a planar shock of Mach number M_0 propagating along a periodic piecewise-linear wall. Blue lines represent the characteristics of the expansion fans generated at the convex corners, and red lines the shock-shocks formed by the compressions at the concave corners. The green line shows the resulting Mach number profile along the shock at a later time.

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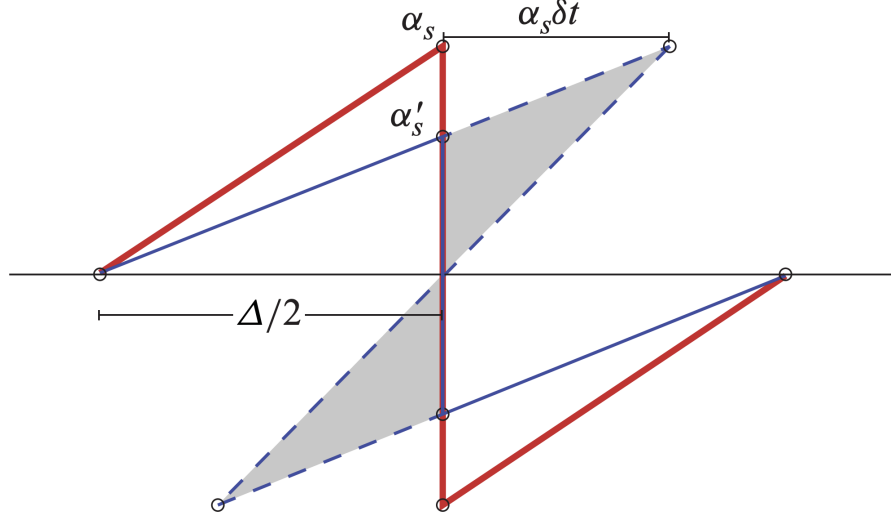


Figure 5.3: Decay of an N-wave of fixed wavelength Δ . The wave shape at time t evolves to a weaker form after a time increment δt . The equal-area construction determines the position of the fitted shock and shows that the peak amplitude decays as $1/t$. From [20].

α_s in the figure) and base $\Delta/2$. Since the evolution is governed by the inviscid Burgers equation, the same equation as Equation (5.1), each point of the profile is displaced over a time δt by a distance proportional to its own amplitude. This yields a multi-valued profile, shown by the dashed line, which is not physical. Instead, the shock-shock is fitted at the position given by the equal-area rule, and the peak amplitude decreases to a' . Matching the slopes of the profile before and after this increment gives a differential equation for the amplitude,

$$\frac{da}{dt} = -\frac{2a^2}{\Delta} \quad \Rightarrow \quad a = \frac{\Delta}{2t}, \quad (5.2)$$

so that the amplitude of an N-wave of fixed wavelength decays as $1/t$.

When the waves are weak, the superposition of two N-waves travelling in opposite di-

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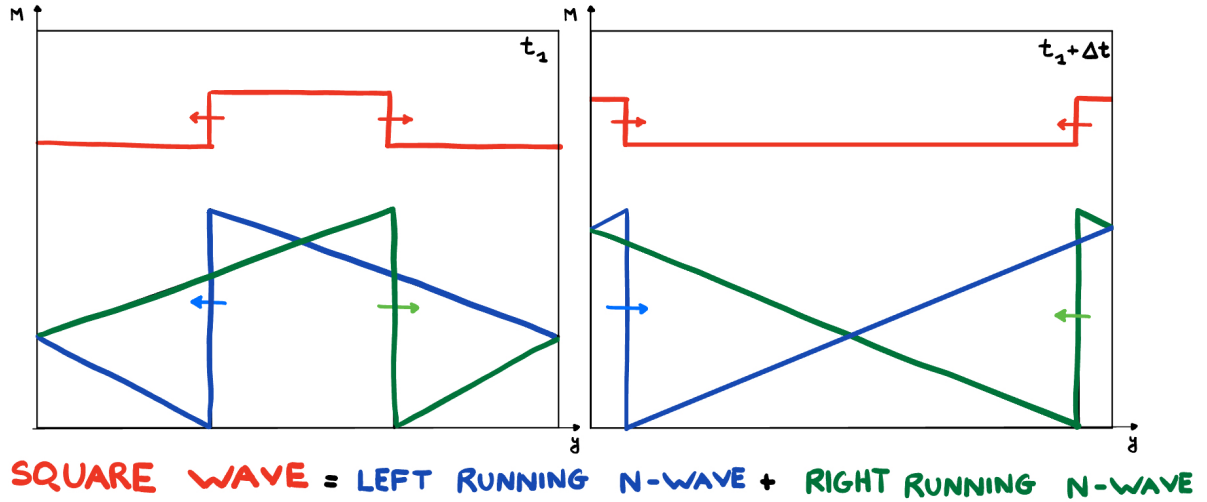


Figure 5.4: Sketch illustrating the decomposition of the square-wave Mach number profile into the superposition of two N-waves propagating in opposite directions along the shock front.

rections gives a square wave: a strong uniform region and a weak uniform region, connected by two sharp discontinuities, as illustrated in Figure 5.4. If these two N-waves are confined between two walls, they simply reflect back and forth, and their superposition maintains this square-wave structure at all times, with two shock-shocks travelling back and forth between the walls.

This is precisely the structure observed in the present problem. The Mach number profile $M(y)$ along the front, shown in Figure 4.3, consists at each instant of two regions, the Mach stem and the incident shock, each of nearly constant strength and connected by sharp discontinuities. It therefore corresponds to the superposition of two N-waves travelling in opposite directions between the channel walls. Since each of these N-waves decays as $1/t$, the same decay is expected for the square-wave structure, which provides a

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natural explanation for the $1/t$ scaling consistently observed across the three approaches presented in this work.

CHAPTER 6

CONCLUSION

This work investigated the decay of perturbations on an inert shock front using three complementary approaches: shock tube experiments, Euler simulations, and Geometrical Shock Dynamics calculations. In all three cases, the perturbation amplitude ΔM was found to decay as $1/t$ after a brief initial transient. The good agreement between the three approaches confirms that this decay law is robust and not specific to a particular experimental configuration or numerical model.

These results also allow the historical debate on the decay rate to be revisited. The $t^{-3/2}$ scaling predicted by Freeman's linear theory [2, 3] and the $t^{-1/2}$ scaling reported by Bowman [6] do not capture the observed behavior. The $1/t$ scaling proposed by Whitham [8] is instead in excellent agreement with the present results, and is now supported by quantitative experimental and numerical evidence for finite-amplitude perturbations in the nonlinear regime.

A physical explanation for this scaling was also identified. The Mach number profile along the shock front behaves as a superposition of two N-waves propagating in opposite directions and confined between the channel walls. Since the amplitude of an N-wave of fixed wavelength decays as $1/t$ [20], this provides a natural explanation for the observed decay law.

A limitation of the present study is that viscous effects were not accounted for. Solving the Navier–Stokes equations instead of the Euler equations would allow the influence of viscosity on the decay to be assessed. Indeed, viscous effects are expected to play a role in the limit of the perturbation wavelength approaching the shock thickness. It would be of interest to determine whether, in the far field, viscosity eventually drives the perturbation into the linear regime described by Freeman, where a $t^{-3/2}$ decay is predicted.

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