

Design Relationships for Maximum Local Scour Depth for Bridge Pier Groups

Mehrdad Nazariha

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Department of Civil Engineering
Faculty of Engineering
University of Ottawa
Ottawa, Canada K1N 6N5

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In the name of God the Compassionate and the Merciful

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Abstract

An investigation of local scour at pier groups has been conducted with the aim of developing design recommendations and scour depth prediction equations for the bridge pier groups. A comprehensive series of steady uniform flow experiments was performed on groups of 2, 3, 4, and 6 model piers. All tests were conducted under *clear-water* scour conditions.

Parameters governing local scour depth around bridge piers include: flow characteristics, sediment properties, and pier size and shape. The effects of these parameters on the maximum local scour depth for a single pier (D_s), have been thoroughly investigated in the past. However, when design criteria developed for single piers are applied directly to pier groups, this approach ignores the following important *group* effects: 1) *sheltering*, 2) *reinforcement*, and 3) *horse-shoe vortex (HSV) compression*. Furthermore, the group effects will alter with changes in pier spacing, S , and flow angle of attack, β . **This study, which investigates the local scouring process for the aforementioned groups of model bridge piers, and presents design relationships for predicting the corresponding maximum local scour depths, is an attempt to address this limitation in current design practice.**

Initially the local scour was examined for five different single pier diameters (0.0127 m, 0.018 m, 0.025 m, 0.033 m, and 0.05 m). Having established single pier relationships, groups of 2, 3, 4, and 6 equally spaced piers were examined, while altering alignment to the flow and separation distances. For piers in line with the flow direction, downstream piers are *sheltered* by the upstream ones resulting in reducing scour depth around downstream piers. This effect diminishes with the increasing β if there is only one row of piers i.e., 2 and 3 pier groups. In the case of 2 rows of piers aligned to the flow direction (i.e., 4- and 6-pier groups), there is a specific range of β in which downstream piers are exposed to the flow. As β increases, depending on pier spacing, the piers of one row may shelter the downstream piers of the other row. The *reinforcement* effect is significant only for small ratios of pier spacing/ pier diameter (S/d) when $\beta \leq 10^\circ$. In these ranges of S/d and β ,

the front piers are scoured more than that of a single pier. Moreover, the scour depth of downstream piers, which are less than that of a single pier, increases with increasing S/d . The *compressed (HSV) arms* affect the local scour depth significantly if piers are staggered and $S/d \leq 5$. In this range of S/d , the HSV arms are intensified, resulting in a deeper scour hole. However, this effect also becomes insignificant with increasing S/d .

The data obtained for each pier group were analysed, using a *multiple regression model*, to develop local scour depth prediction equations for the front, middle, and rear piers. The *regression model* includes two important dimensionless parameters (β and S/d) associated with group effects, as regressors. The proposed prediction equations estimated the observed data and similar-type published data reasonably well, within marginal errors. From a general practical view-point, the number of prediction equations was reduced to two equations for all pier groups: one for the front piers and the second one for the downstream piers (i.e. combined middle and rear piers).

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List of Symbols and Abbreviations

A_f = characteristic projected area of the particle normal to the direction of flow.

A_p = projected area.

a^* = half width of a pier.

a_1^* = distance between points G and C (Fig. 3-5).

a , b , and c = parameters of prediction equations for different pier group arrangements.

a_1 , b_1 , c_1 , a_2 , b_2 , c_2 , a_3 , b_3 , and c_3 = parameters of prediction equations.

B = coefficient as function of the roughness.

$C = (X'X)^{-1}$.

C_L = lift coefficient.

C_1 = form coefficient (if particle is spherical in shape $C_1 = \pi/6$).

C_3 = form coefficient related to the effective surface area of the particle in the direction of the lift force.

$C_4 = C_3 k^2 / 2$.

D_g = scour depth of any pier in a group.

D_n = representative sediment size.

D_s = scour depth \ depth of scour at the pier at time t .

D_{sc} = maximum equilibrium depth of local scour.

D_{SF} = front pier scour depth.

D_{s_x} = local scour depth in any pier group.

D_{sm} = minimum scour depth.

$D_{sm(1-2)}$ = minimum scour depth between piers number 1 and 2.

D_{SR} = rear or downstream scour depth.

D_{s7} (scour depth after 7 hours).

D_{s24} (the scour depth after 24 hours).

$D_{15.9}$ = particle size for which 15.9% of the sediment mixture is finer.

D_{50} = particle size for which 50% of the sediment mixture is finer.

$D_{84.1}$ = particle size for which 84.1% of the sediment mixture is finer ,

d = pier width/ pier diameter,

d' = projected width of the pier,

dA = an infinitesimal element area of the surface ABCD,

$e_i = y_i - \hat{y}$,

F_d = drag force,

F_g = submerged weight of the particle,

F_L = lift force,

Fr = Froude number,

Fr_1 = Froude of number of the approach flow upstream of the pier,

Fr_c = Froude number using critical velocity for initiation of motion,

F_1 = first front pier for group piers,

F_3 and F_4 = second front pier for groups of 4 and 6 Piers respectively,

G = point of support of the particle,

g = gravitational acceleration,

$H = X(X'X)^{-1}$ X' = hat matrix,

I = hydraulic gradient,

$\bar{i}$, $\bar{j}$, and $\bar{k}$ = unit vectors in the x, y, and y directions respectively,

K_d = sediment size factor,

K_f = flow intensity factor,

K_s = pier shape factor/ height of grain for equivalent sand roughness,

K_y = flow depth factor,

K_p = pier alignment factor,

K_g = sediment gradation factor,

K_1 = correction for pier nose shape/ a multiplier which is independent of time,

K_2 = correction for angle of attack of flow,

k = constant,

$k_1 = C_1 C_4 / C_1$.

$k_2 = C_2 / C_1$.

L = pier length,
 M_2 = first middle pier for pier groups,
 M_5 = second middle pier for 6 pier groups,
 N_s = sediment number,
 N_{sc1} = sediment number at which local scour begins,
 N_{sc2} = sediment number at which continuous sediment transport occurs,
 n = number of observations,
 p = number of regressor parameters,
 Q = discharge of water,
 Q_s = discharge of sediment (volume of grains),
 q_{s1} = outgoing sediment,
 q_{s2} = oncoming sediment,
 q_t = discharge of sediment/m width (t = total load),
 R_2 = rear pier for 2 and 4 pier groups,
 R_3 = rear pier for 3 and 6 pier groups,
 R_6 = second rear pier for 6 pier groups,
 Re = Reynolds number,
 r_c = core size,
 r = the distance between two vortices,
 S = pier spacing,
 $S(\alpha)$ = least square function,
 S_0 = slope of the sediment bed or channel,
 T = time for local scour to reach equilibrium depth,
 t = time at which the local scour depth is measured,
 $t_{\eta, n-p}$ = one-sided t-distribution statistic corresponding to an exceedence probability η and $n-p$ degrees of freedom,
 U = depth-average flow velocity,
 U_c = critical velocity,
 U_{*c} = critical shear velocity,

U_* = shear velocity,

U_∞ = undisturbed approach velocity,

u = mean flow velocity,

u , v , and w = velocity components in x , y , and z directions respectively,

u = mean velocity,

u_* = shear velocity,

$\vec{v}$ = velocity vector,

w = channel width,

x = independent variable or regressor/ the pier center line spacing/ the longitudinal distance,

$x' = [1, x_1, x_2, \dots, x_k]$ = transpose of values corresponding to the observed values y_i ,

$(X'X)^{-1} X'X = I$ = unit matrix,

X_0^* = distance from the pier to face AB,

Y_0 = flow depth,

y = dependable variable or response variable\ distance from the wall,

y_{01} = uniform flow depth upstream,

y_{02} = uniform flow depth downstream,

$\hat{y}$ = estimated value for y ,

observations y_i ,

α_j = regressor coefficients,

$\alpha^* = (m-1)/m$,

β = flow angle of attack,

Γ = circulation,

$\delta(x)$ = boundary layer thickness at point x ,

γ^* = surface vorticity at separation,

γ = specific weight of water,

γ_s = specific weight of particle,

$\gamma'_s = \gamma_s - \gamma$,

$\Delta = (\rho_s - \rho)/\rho$,

ϵ = error associated with the model,

η = confidence level,

η , ζ , and ξ = vorticity components in x, y, and z directions respectively,

θ = angle of repose of the submerged sediment,

$\hat{\theta}'_{F_1}$, and $\hat{\theta}'_{\dots}$ = transpose of the vector of regression model parameters,

κ = Von Karman coefficient (0.4),

ν = kinematic viscosity of fluid,

ρ = density of water,

ρ_s = sediment density,

$\sigma_g = (D_{84.1}/D_{15.9})^{0.5}$ or D_{85}/D_{50} ,

σ^2 = residual mean square error (estimated model error),

τ_0 = sediment shear stress,

τ_c = critical shear stress,

ϕ = bed slope,

$\bar{\Omega}$ = vorticity vector,

ω = angular velocity

ω^* = fall velocity of particles,

Abbreviations:

FHWA= (U.S.) Federal Highway Administration

HSV= Horse-Shoe Vortex

$$D'_{F_1} = \frac{D_{SF_1}}{D_s}$$
$$D'_{M_2} = \frac{D_{SM_2}}{D_s}$$
$$D'_{R_3} = \frac{D_{SR_3}}{D_s}$$

$$D'_{F_1} = \frac{D_{SF_1}}{D_s}$$

$$D'_{M_s} = \frac{D_{SM_s}}{D_s}$$

$$D'_{R_o} = \frac{D_{SR_o}}{D_s}$$

Chapter 1

Introduction

1-1 Statement of the Problem

In a study of bridge failures in the United States, Brice and Blodgett (1978) reported that damage to bridges crossing waterways from major regional floods in 1964 and 1972 amounted to about U.S. \$100 million per event. During the Spring floods of 1987, 17 bridges in New York and New England were damaged or destroyed by scour. In 1985, 73 bridges were destroyed by floods in Pennsylvania, Virginia, and West Virginia, (Richardson et al. 1991). In most cases, failures of these bridges were due to local scouring in the vicinity of their support structures, i.e., at the bridge piers and/or abutments.

Assuming the riverbed in the vicinity of a bridge crossing is mobile (i.e., comprised of loose alluvial materials), during flood events the scouring process will normally include: (i) *general* scour, (ii) *contraction* scour, and (iii) *local* scour. *General scour* occurs when the boundary shear stress, exerted by water flowing over the loose boundary materials, exceeds the *critical (threshold)* values required to initiate movement of the individual particles that make up the surficial layers of sediment. Thus, general scour occurs when this *critical condition* is exceeded and whether the bridge is present or not. *Contraction* and *local* scour occur in the presence of bridge piers and/or when the bridge embankments encroach on the channel flows. The former is largely caused by the increased velocities produced by the sudden reduction in the cross-sectional area of flow and the latter is produced by large-scale turbulence *shed* and *horseshoe* vortices produced by the fluid-structure interaction.

For an unprotected columnar-shaped bridge pier installed in an alluvial channel, the principal dimensions of the corresponding local scour hole are a function of: (i) *flow* characteristics, including: approach velocity, depth and flow direction; (ii) *sediment properties*, including: representative grain size, gradation and density; and (iii) *bridge pier characteristics*, including: size, shape and spacing. However, for design purposes the anticipated maximum local scour depth (D_s) is usually related to the *shape* and *width* of the proposed pier structures through coefficients reflecting the design conditions, (Neill, 1964).

Since the pier's foundation structure (*footing*) is generally wider than the pier shaft and, since (D_s) has been found to depend largely on the maximum width of that portion of the structure exposed to the flow, it is crucial that the footing remain unexposed throughout the local erosion process.

Conventional methods of avoiding or reducing the severity of local scour include the use of

(i) *riprap* around the base of bridge piers (Graziano et al. 1990), (ii) *collars* and *plates* (Odgaard and Wang 1987), (iii) *grout mats* and *bags* (Fortherby 1993), (iv) *concrete blocks* (McCorquodale et al. 1993), and (v) *false piles* (Paice and Hey 1993).

Although scour around bridge piers could be better estimated if the three-dimensional horse-shoe vortex structure were simulated analytically, most formulas for predicting local scour depth were developed empirically by Laursen (1958), Chitale (1962), Ahmad (1962), Laursen (1963), Shen et al. (1969), Breusers et al. (1977), and Melville and Sutherland (1988) among others. Because of the complex interaction of the three-dimensional turbulent flow field around the pier with the erodible stream bed, a unique analytical approach has not been obtained thus far.

Only recently have investigators used an analytical approach to solve some parts of the problem. Applying the Reynolds average Navier-Stokes equations Olsen and Melaaen (1993) and Cabrales (1993) found contradictory results in predicting local scour depth around a cylindrical bridge pier. Although the former found encouraging results in simulating the three-dimensional scour pattern for a relatively small scour hole, the latter found a discrepancy between the experimental results and the results obtained from the mathematical model in determining the normalized bed shear stress around the cylinder. Yanmaz and Altinbilk (1991) proposed a semi-empirical time-dependent analysis of local scour depths using the sediment continuity equation for the scour hole around bridge piers. The results of laboratory experiments were utilized to obtain the values of coefficients needed for

such an analysis. Other researchers developed empirical formulas which are more dependent on experimental data than on an analytical approach, (Johnson (1992), and Kothyari et al. (1992a,b)).

There are a substantial number of bridge piers that have exposed pier groups (Jones et al. 1991). Some preliminary studies of scour around pier groups have been done but none give a complete treatment of the problem. A study by FHWA (U.S. Federal Highway Administration)¹ determined that scour for a group of piers would be no worse than scour at a solid pier with dimensions equivalent to the individual piers touching one another. Hannah (1978) studied local scour at pier groups with steady uniform flow under *clear-water* conditions using one sediment size. He proposed modifying coefficients based on Neill's relationship developed for the case of a single pier (Neill 1964). He showed that scour for a pier group was less than 20% greater than scour at a single pier when the spacing exceeded five pier diameters.

Raudkivi and Sutherland (1981) studied local scour at pier groups in straight channels. They found that the scour depths at rear piers were greater than the same at front piers for angles of attack equal to or more than 45° . Nouh (1986) investigated experimentally the characteristics of flow under clear-water conditions at pier groups in a meandering channel. He found that pier group transversal spacing, longitudinal spacing, and relative pier group size (with respect to the channel width) affect significantly the scour depth at the piers. Nouh found that the maximum scour depth at a pier group in a meandering channel might reach a value of 2.20 times that for a single pier in a straight channel and that scour depths at piers in the outer side of his meandering channel were generally larger than

¹ See Jones et al. (1991)

that at piers in the inner side of the channel.

1-2 Research Needs

In the past various research have attempted to develop design criteria using the following different approaches: (i) the *empirical* approach, (ii) the *analytical* approach, and (iii) the *semi-empirical* approach. Because of the complex nature of the problem (highly 3-dimensional fluid-structure-sediment interaction), the large majority of past contributions adopted the *empirical* approach, based on field and/or laboratory (physical model) studies. Moreover, again to reduce the level of complexity, most *laboratory* studies of the past developed their D_s -relationships for different *single-pier* scenarios (i.e., single piers having different shapes and alignments to the flow field). Accordingly, when criteria developed for *single* piers are applied directly to pier or pier *groups*, this approach ignores the possibly important *group* effect on D_s .

This limitation of current practice regarding estimation of D_s for bridge pier *groups* was recently addressed in a summary of priority research needs on the subject identified by the U.S. Federal Highway Administration (Jones et al. 1991). It was emphasised that . . . "*there is very little information on the effects of spacing between rows especially when the rows are staggered such that there are no clear flow paths through a pier group.*" Also it was noted that there is considerable controversy about what happens with pier scour when piers are close enough for the scour holes to overlap. For example, do the horseshoe vortices somehow combine to create a much stronger erosive force around 2 piers or do the piers scour like single obstructions and the only effect from the spacing

is the removal of the peak middle bed between the piers where the scour holes overlap?. "...A closer study of pier-pier group interrelationships is required varying the pier groups with which to compare the pier scour. Different pier groups need to be tested, i.e. 4 and 6 piers in rectangular grouping. The effects of pier diameter variation needs to be examined. More two-pier tests through all angles of attack are required to better define the sheltering, wake and transverse zones, i.e. for separation other than 5 diameter, . . . " Hannah (1978).

In summary, the current practice regarding estimation of maximum local scour for a pier group (D_g) is to modify, through various correction factors, those expressions established for a single pier (D_s). Since these factors are empirically derived and usually site-specific, there is clearly a need for further research on the subject of local scour around bridge pier groups. The principal goals of this research will be to identify those parameters that strongly influence the group effect and to develop empirical relationships for the front, middle, and rear piers in different grouping arrangements.

1-3 Objective and Scope of the Study

The main objective of this study was to investigate experimentally, the effects of pier spacing and flow angle of attack on local scour around bridge pier groups and also to develop local scour depth prediction equations for the bridge pier groups. The experimental program of this study comprised the following:

(A) an initial program of tests on various *single-pier* arrangements, to compare the accuracies of several traditional scour-predicting formulae under different flow conditions

(B) The main experimental program of our study was to examine the very important aspect of the influence of the flow angle of attack, β , of the approach flow and pier spacing, S , on D_g . The principal goal of the study was to develop, based on the results of the experimental program and the related theoretical work, empirical relationships for predicting maximum local scour around bridge pier groups D_g , under uniform *clear-water* flow conditions.

In this study, some tests for the case of a single pier were carried out to verify which pier scour prediction formulas give reasonable results. Then, tests on pier groups, were performed under *clear-water* conditions. Also, the results were compared with available data. In the second stage of this study, experiments carried out on pier groups with different pier grouping arrangements (2, 4, and 6 piers). Different pier spacings and flow angles of attack of the approach flows were applied. Emphasis was placed on the impacts of group effects like spacing, *reinforcement*, and *sheltering*. In some instances, tests were repeated at least twice if the trend of data for a specific case was significantly different from the others. In these cases the average value was considered in the analysis.

1-4 Study Program

The following conditions and parameters were tested to establish appropriate criteria for pier

or pier groups as a bridge *support* structure:

1) The hydraulic conditions applied in this study included:

- *steady state* approach flow conditions, and
- *clear-water* scour conditions,

2) The bridge pier arrangements tested included:

- *single* piers,
- *groups* of piers,

3) Also investigated were the effects of the following parameters and conditions on

local scour:

- *spacing* between piers in a group, and
- *patterns* of pier groups (different flow angle of attack),

4) The following measurements were carried out:

- flow depth,
- rate of water discharge,
- flow velocity,
- depth and dimensions of the scour hole.

1-5 Evaluation of the Results

An initial series of tests was carried out on a single pier to check the performance of various pier scour equations which have been developed in the past. Formulas for single-pier arrangements

included those of Shen et al I, and II (1969)¹, Laursen (1963), Jain and Fisher (1980), and Colorado State University (CSU)².

The main experimental program, which investigated the local scouring phenomenon around bridge pier groups, furnished the data necessary for the development of the related empirical scouring relationships. In this part of the program, experiments employing a set range of uniform flow rates and depths in the flume's test section were repeated for different pier grouping arrangements and alignments to the approach flow. Related dimensional analysis identified the most important parameters governing the problem. It is anticipated that included in these parameters were pier spacing and flow angle of attack of the approach flow, which in turn govern *sheltering* and *reinforcing* effects and the *compressed horseshoe* and *shed* vortex mechanisms.

The *reinforcing* effect is related to the spacing between piers. The scouring potential of the *shed vortex* is a function of the intensity of its convection speed and of the distance between the path and the affected pier. Therefore, both spacing and angles of attack determine the degree of this effect on the local scour. For a particular angle of attack the compressed *horseshoe vortex* pattern is dependent on the pier spacing.

In short, in formulating an expression for estimating maximum depth of scour around pier groups, two important parameters to consider are *pier spacing* and *flow angle of attack* of the

¹ See Jones (1984)

² See Richardson et al. (1991), pp. 52

approach flow.

1-6 Experimental Equipment

The experimental program was performed in a 7.7^m long × 0.515^m wide × 0.6^m deep aluminum flume. The flow was supplied by the laboratory pumping system. At the downstream end of the flume an adjustable gate was used to control the water surface profile. Two screens installed upstream of the flow measurement device eliminate large-scale turbulence effects of the entering flow. Model pier(s) was/were fixed and mounted vertically on the flume bed. Sediment with $D_{50}=0.49$ mm was used as the test bed materials.

Discharge was measured using a 90° V-Notch weir and flow depths were measured with a vernier point gauge. Maximum local scour depth was measured using a thin wire probe. The velocity of approaching flow was measured using a Pitot tube. To find the velocity profile in the test section the velocity was measured using a mini-Propeller flow meter.

1-7 Thesis Description

Chapter 2 deals with literature review of the past contributions to this study. A review of the following subjects is included: 1- local scour around the bridge piers and its mechanism, and 2- parameters influencing local scour.

Chapter 3 covers the theoretical background of this study in which the following subjects are

discussed: 1- erosion of cohesionless sediments, 2- an analytical consideration of the behaviour of the flow in the plane of symmetry ahead of a cylinder, 3- dimensional analysis, and 4- multiple linear regression model.

In *chapter 4*, the experimental equipment and procedure will be discussed.

Chapter 5 is devoted to the results and discussions including: 1- preliminary tests on single piers, 2- the effects of pier spacing and flow angles of attack on local scour depth for 2, 3, 4, and 6-pier groups, and local scour depth prediction equations developed for pier groups.

Chapter 6 presents conclusions and recommendations for future research.

References are presented in *chapter 7*

Finally, appendix is presented at the end of the thesis

Chapter 2

Literature Review

2-1 Design Methodology

In design methodology, both hydrologic and hydraulic aspects should be considered. Bridges crossing streams should be designed to withstand the effects of scour from a *superflood* (a 100-year or 500-year flood) with little risk of failing. This requires careful evaluation of the hydraulic, structural, and geotechnical aspects of bridge foundation design. Hydraulic studies of proposed bridge sites are a necessary part of bridge design. These studies should address both the sizing of the bridge waterway opening and the design of the foundations to resist scour. It is always cost-effective to provide a foundation that will not fail, even from a very large flood event or superflood.

2-2 Scour at Bridge Crossings

When an alluvial stream is partially obstructed by a bridge pier, the flow pattern in the channel around the pier is significantly changed. The pier produces an adverse pressure gradient just upstream from the pier. The boundary layer upstream of the pier undergoes a three-dimensional separation. The shear stress distribution around the pier is drastically changed due to the formation of a horseshoe vortex, resulting in scour around the pier, which, in turn, changes the flow pattern and shear, (Kothyari (1992a)).

Three types of scour may occur at bridge sites (Raudkivi 1986):

- 1) general scour of the stream, which is a response to the flow regime, would occur whether a bridge is built or not;
- 2) contraction scour, caused by contraction of the waterway by the bridge and its approaches; and
- 3) local scour, due to the interference with flow by piers and abutments. The local scour is in addition to numbers 1 and 2 above.

"Generally, scour depths from local scour are much larger than the other two; often by a factor of five to ten. But, if there are major changes in stream conditions, such as a large dam built upstream or downstream of the bridge structure, long-term bed elevation changes can be the larger element in the total scour," (Simons and Senturk (1992)).

2-2-1 General Scour

General scour occurs when the capacity of a river to carry sediment during flood flows or as the consequence of various man-made alternations is increased. An example of the latter is mining of sand and gravel in rivers and on flood plains. In this case the bed of the river is lowered during floods. To estimate the mean velocity and the cross-sectional area for a given discharge, the elevation and the slope of the water surface should be known in advance. Knowing the velocity and cross-sectional area, the average elevation of the bottom and the amount of scour can be estimated. An assessment of long-term stream bed elevation changes should be made using the principles of river mechanics. Such an assessment requires the consideration of all influences upon the bridge crossing; i.e., runoff from the watershed to the stream (hydrology), the sediment delivery to the channel (erosion), the sediment transport capacity of the stream (hydraulics) and the response of the stream to these factors (geomorphology and river mechanics).

To organize such an assessment, the following three-level fluvial system approach can be used:

- 1) a qualitative determination based on general geomorphic and river mechanics relationships;
- 2) an engineering geomorphic analysis using established qualitative and quantitative relationships to estimate the probable behaviour of the stream system to various scenarios of future conditions; and
- 3) physical process computer modelling using mathematical models such as BRI-STARS (Molinas, 1993), WSPRO (Mueller, 1993) and the U.S. Corps of Engineers' HEC 6¹ to make predictions of

¹ See Richardson et al. (1991)

quantitative changes in stream bed elevation due to changes in the stream and watershed.

In short, general scour at a bridge crossing is related to the flow and sediment-transport processes of the river being crossed. Evaluation of this scour requires modelling of the river channel for water and sediment routing.

2-2-2 Contraction Scour

Contraction scour occurs when the flow area of a stream decreases in the flow direction. The more the flow area is reduced, the higher the average velocity and bed shear stress through the contraction. Hence, there is an increase in erosive action in the contraction with more bed material being removed from the contracted reach than is transported into the reach. This increase in transport of the bed material from the reach lowers the natural bed elevation. As the bed elevation is lowered, the flow area increases and an *equilibrium* condition is reached; i.e., the quantity of bed material that is transported into the contracted reach is equal to that removed from the reach.

The increase in depth at a long contraction can be computed from the equations of motion and continuity for sediment and water. For the water motion the Chezy equation can be used and for the sediment transport a simple relationship to the average velocity (for definitions, see Fig. 2-1).

$$\text{water: } Q = \text{constant} = Q_c + Q_r$$

$$\text{sediment : } Q_s = \text{constant} = Bq_t = B(\alpha U^m)$$

in which; q_t = discharge of sediment/m width (t = total load), U = depth-average flow velocity,

Q = discharge of water, Q_s = discharge of sediment (volume of grains), Q_f = discharge of water in the floodplain, and Q_c = discharge of water in the main channel (Fig. 2-1). Neglecting floodplain flow, one finds:

$$\frac{y_{02}}{y_{01}} = \left(\frac{B_1}{B_2} \right)^{\alpha^*} \quad (2-1)$$

where $\alpha^* = (m-1)/m$ or with $m = 3$ to 5 , $\alpha^* = 0.67$ to 0.8 , y_{01} = uniform flow depth upstream, and y_{02} = uniform flow depth downstream. The value of $\alpha^* = 2/3$ corresponds to the value predicted by the regime method.

In the case of overbank flow with discharge Q_f , equation 2-1 becomes

$$\frac{y_{02}}{y_{01}} = \left(\frac{B_1}{B_2} \right)^{\alpha^*} \frac{Q}{(Q - Q_f)} \quad (2-2)$$

Experimental data to confirm the foregoing analysis are scarce. Sharma (1972) concluded from field data that the scour depth between bridge piers (excluding local scour effects) is reasonably predicted by the regime method. Norman (1975) concluded from field data that either Straub's¹, or Laursen's¹ approach gives good results. However, other investigators debate about the value of y_{02} .

¹ See Breusers and Raudkivi (1991)

2-2-3 Local Scour around Bridge Piers

Most researchers have investigated local scour around a single pier and have neglected the effect of groups of piers to simplify the problem [Chabert and Engeldinger (1956), Laursen and Toch (1956), Laursen (1963 and 1984), Carstens (1966), Shen et al. (1966), Gole and Chitale (1967), Shen et al. (1969), Melville and Raudkivi (1977), Jain and Fisher (1980), Qadar (1981), Raudkivi and Sutherland (1981), Raudkivi and Ettema (1983), Melville (1984, 1992), Raudkivi (1986), Melville and Sutherland (1988), Chiew (1989), Johnson and McCuen (1991), Johnson (1992), Qadar and Ansari (1991), Richardson et al. (1991), Santoro et al. (1991), Yanmaz and Altinbilk (1991), Barbe et al. (1992), Donguang et al. (1993), Fischer (1994), Glenn (1994), Landers et al. (1994), Dey et al. (1995), and Johnson and Ayyub (1996)]. A comprehensive literature review was made by Breusers et al. (1977). They gave a "state of the art" report on the subject of local scour around cylindrical piers. The report was restricted to the following conditions:

- cylindrical piers (all shapes),
- non-cohesive granular bed material, and
- one-way current (no tidal influence and waves).

They presented a critical review of the literature on model and field data and the empirical data were compared with theory. The final result was recommended design criteria together with suggested alternatives for scour protection. Because of its importance the Breusers et al's. report has been cited by many investigators.

Shen et al. (1966) investigated the variation of shear stress immediately upstream from a pier

in its scour hole using integration of the Navier-Stokes equation of motion in the boundary layer of the secondary flow. They related the equations of motion and the continuity equation. However, they neglected the effect of the momentum of the incoming flow, since the region of application was sufficiently close to the cylinder. They assumed that; 1) the flow and vortex system are steady, 2) there is no shear at the upper edge of the boundary layer, 3) the two-dimensional up coming flow obeys the potential flow theory, 4) the boundary-layer thickness in the radial and tangential directions are equal, and 5) a no slip condition exists at the bed and other simplifying assumptions. They concluded that the theory behind their analysis suffers in that it does not account for separation of the flow boundary layer. Also, they found that the pressure distribution in the vicinity of maximum scour (given by the potential theory) was not realized in physical situations.

Shen et al. (1966) also presented an analytical consideration of the behaviour of the flow in the flow in the plane of symmetry (stagnation plane) ahead of a cylinder of infinite extent situated normal to a steady incompressible shear flow. Using an imaginary control volume in the neighbourhood of a circular cylinder, they derived the basic integral equation about the control volume. They applied boundary conditions to the control volume and used simplified assumptions to govern the steady state equations of motion. However, they concluded that “ unfortunately except for a few simplified flows, there appears to be no simple solution to the problem. It is possible that they will have to be dealt with in an empirical manner.”

Most studies on the subject of local scour were carried out using physical models, [Chabert and Engeldinger (1956), Shen et al (1966, 1969), Dietz (1973), Melville (1975, 1984, 1992),

Breusers et al. (1967, 1977), Harter (1978), Nouh (1986), and Raudkivi (1986)]. Many different formulas for the maximum scour depth were developed on the basis of these and similar studies.

2-2-3-1 The Mechanism of Local Scour

A strong pressure field is induced by the presence of a bridge pier, in a flow field so that the flow in front of the structure separates from the bed and rolls up to form the aforementioned scouring vortex. The *horseshoe* system of vortices which develop around the pier has long been recognized by investigators as the prime agent in the local scouring process, Keutner (1932), Tison (1940), Posey (1949 and 1974), Laursen and Toch (1956), Neill (1964), Bata (1960), Roper (1967), Highway Research Board (1970), Melville (1975), and Shen et al. (1966).

Bata (1960), Moore and Masch (1965) and Tison (1937) attempted to find the magnitude of secondary currents near the front face of a pier. They all agreed that the secondary currents, created by the difference of stagnation pressures, caused scour but they were not able to correlate the magnitude of secondary currents with maximum scour depth. Tarapore (1962) assumed a particular velocity distribution in the scour hole to determine the maximum scour depth. However, his assumption of the velocity distribution in the scour hole did not appear to be realistic.

A cylindrical pier which is positioned vertically in a wide stream is shown in Fig. 2-2. The variation of the approach flow velocity with the depth is also shown in the figure. The bed of the stream at the base of the pier is assumed to have small irregularities, but is essentially level initially.

The stream flow around the pier in plan view will be essentially as shown in Fig. 2-3 for locations far from the bed. As the streamlines converge around the upstream half of the pier, the stream pattern is altered by the presence of the pier. A stagnation point is developed somewhere in the flow depth along the line bb' in Fig. 2-2. Above this point, there is a stagnation line and below it there is a secondary flow. The position of the stagnation point depends on the velocity distribution and the shape of the pier, however, Shen et al. (1966) claimed that the stagnation point occurs approximately at $3/4$ depth above the bed.

Since kinetic energy of the flow is converted fully or partially to potential energy or head, a rise in the water surface occurs at line bb' and part of the way to line cc' as shown in Fig. 2-2. The magnitude or rise in the water surface is dependent on the undisturbed approach velocity, the three-dimensional velocity distribution and shape of the pier. The rise in water surface can be predicted reasonably well from two-dimensional *ideal* fluid theory.

2-2-3-2 The Horse-Shoe Vortex (HSV)

The HSV feature has a significant effect on local scour around piers. Although it is not the initial cause for scouring, it develops as the result of separation of flow at the upstream rim of the scour hole. "The horseshoe vortex extends downstream, past the sides of the pier, for a few pier diameters before losing its identity and becoming part of general turbulence. The horseshoe vortex also pushes the maximum downflow velocity within the scour hole closer to the pier", (Raudkivi, 1986). The HSV system which develops at the base of the pier is an important feature of the flow

around the upstream half of cylindrical bridge piers. Moreover, the velocity component downward on the front face of the cylinder and a boundary-layer separation upstream of the structure are other characteristic features of the flow (Fig. 2-4). The vortex scour mechanism at bridge piers was studied by Qadar (1981). He analysed the cause of the scouring process itself. He illustrated the basic mechanism of the vortex scouring ahead of a cylindrical pier placed in a flow field and aligned with the flow (Fig. 2-5a). Figure 2-5b shows a vertical section along the pier's longitudinal axis. It shows how the approaching flow separates and rolls up to form the scouring vortex ABCDA. The vortex develops a scour hole in the sand bed in front of the pier and in the process ultimately sinks into the hole, causing gradual expansion in its size. In the ultimate equilibrium condition the position GHJKG is occupied by the vortex.

In addition to a horseshoe vortex around the base of a bridge pier, the flow pattern (Fig. 2-4) has some other components; namely:

- 1) downflow in front of the pier;
- 2) cast-off vortices and a wake; and
- 3) a bow wave.

The approach flow velocity, in the vertical plane of symmetry in the direction of flow, decreases from the free surface downward to zero at the bed. The stagnation pressure, $\rho u^2/2$ (ρ = density of water, and u = mean flow velocity), decreases as well. In addition, the stagnation pressure causes sideward acceleration of the flow past the cylinder. The separation of flow at the sides of the cylinder creates the wake with the cast-off vortices at the interfaces to the main stream. The cast-off vortices are translated downstream with the flow and interact with the horse-shoe vortex at the bed causing it to

oscillate laterally and vertically. A bow wave develops at the surface with rotation in the opposite sense to that in the horse-shoe vortex.

2-4 *Clear-water and Live-bed Scour Conditions*

Once the concept of this local scouring mechanism is understood, the next step is to determine the size of the scouring vortex. The shape, and the width of the pier, along with the angle of attack of the current affect the size of the vortex. As the maximum depth of scour is a function of the strength of the vortex which develops ahead of the pier, the depth of the local scour can be estimated in the next step. Two different approaches exist for estimating the depth of local scour. One estimates scour around piers in a *clear-water* condition and the other considers a *live-bed* scour condition.

When the shear velocity U_* or the velocity U exceeds about half the critical or the threshold value for movement of the sediment, local scour starts. "The clear-water scour occurs if the velocity U is less than the critical velocity U_c . At velocities greater than the threshold value, sediment is carried into the scour hole from upstream and the equilibrium scour depth is reached when the rates of supply and removal of sediment balance. This is the live-bed scour condition," (Raudkivi, 1986).

During a flood event, bridges over streams with coarse bed material are often subject to clear-water scour at low discharges, live-bed scour at the higher discharges and then clear-water scour on the falling stages. As indicated in Figs. 2-6a&b, clear-water scour reaches its maximum over a longer period than live-bed scour. This is because clear-water scour occurs mainly in coarse bed material

streams. In fact clear-water scour may not reach a maximum value until after several floods. Also, maximum clear-water scour is about 10 percent greater than the maximum live-bed scour

Since the differences between clear-water and live-bed scour conditions were not clearly distinguished by many investigators, the proposed prediction equations gave widely varying scour depths¹. Laursen (1952) suggested the following relations in the scouring process. He defined the rate of the local scour in volume per unit time as follows:

$$\frac{dQ_s}{dt} = q_{s1} - q_{s2} \quad (2-3)$$

in which; q_{s1} = outgoing sediment, q_{s2} = oncoming sediment. Three possibilities may happen as follows:

1- no scour	if	$q_{s1} = q_{s2} = 0$
2- clear-water scour	if	$0 = q_{s2} \ll q_{s1}$ and $q_{s1} > 0$
3- live-bed scour	if	$q_{s1} \geq q_{s2} > 0$

Carstens (1966) defined a *sediment number* as an indication of the conditions necessary to initiate clear-water scour as follows:

$$N_s = \frac{U}{\sqrt{(s-1)gD_n}} \quad (2-4)$$

in which; N_s = sediment number, U = mean velocity of the undisturbed flow, s = sediment specific density, g = gravitational acceleration, and D_n = representative sediment size. He defined N_{sc} as the

¹ See § 2-6-1 "Pier scour equations"

sediment number at which local scour begins and N_{sc2} as the value when continuous sediment transport occurs. Carstens also stated that “ for irrotational flow, the velocity at the sides of the pier is twice the approach velocity. Therefore, a reasonable approximation is that $N_{sc1} = N_{sc2}/2$.” The region of a localized scour is restricted to the range of sediment number between 1.12 and 2.24, which corresponds to N_{sc1} and N_{sc2} , respectively.

Chabert and Engeldinger (1956) identified two types of scour: *clear-water scour* and *live-bed scour*, according to the condition of sediment transport in the approach flow. Raudkivi and Ettema (1983) considered clear-water scour at cylindrical piers, while Melville (1984) and Kothyari et al. (1992b) considered live-bed scour at bridge piers of different shape.

For the most part, previous studies of local scouring around bridge piers have considered the case of a single pier installed in a sand-bed channel and subject to *clear-water*, steady (uniform) flow conditions.

2-5 Parameters Influencing Local Scour

The parameters influencing local scour may be classified into three categories:

1) streamflow characteristics, such as velocity of the approach flow, and flow depth; 2) bed material characteristics, such as sediment size, density, and distribution, and cohesion of the material; and 3) bridge pier characteristics, such as dimensions, shape, and surface condition of the pier, and any protection system.

2-5-1 The Influence of Flow Velocity

The depth of the local scour hole is closely related to the undisturbed approach flow velocity. It is evident from published literature that under *clear-water* scour conditions, the local scour depth increases almost linearly from a value of the ratio of shear velocity, U_* , to the critical shear velocity, U_{*c} of about 0.5 ($\frac{U}{U_{*c}} = 0.5$) to that of unity, reaching a maximum at the critical velocity. Among available data which clearly show this trend, are those of Chabert and Engeldinger (1956), Maza Alvarez and Sanchez Bribiesca (1966) and Ettema (1980). The data obtained from the experiments carried out by Ettema are shown in Fig. 2-7.

Chabert and Engeldinger (1956) studied the influences of flow velocity on the local scour. They found two different regimes: 1) for velocities at or below the threshold velocity of movement of the bed material, scour depth approaches a limit asymptotically (Fig. 2-8); for a larger velocity, scour depth fluctuates due to the periodic dumping of material in the scour hole by moving dunes.

2-5-2 The Influence of Water Depth, Grain Size, and Pier Diameter

Chabert and Engeldinger (1956) using grain sizes $D_{50} = 0.26, 0.52, 1.5$ and 3.0 mm, pier diameter $d = 2.5$ to 30 cm, and water depth $Y_0 = 0.1$ to 0.35 , found that the grain size has only a small influence on the local scour depth and for the water depth/pier diameter ratios larger than one, the influence of water depth is also negligible.

Ettema (1980), Chiew (1984) and others showed that scour depth increases with flow depth

up to a limiting value of the flow depth ratio Y_0/d (Y_0 = flow depth and d = pier width), beyond which there is no influence of flow depth. The limit expressed by most researchers ranges from $Y_0/d = 1$ to 3. For clear-water scour conditions, Bonasoundas (1973) concluded that the effects of flow depth become insignificant for $Y_0/d > 1$ to 3. Chiew (1984) compared his data with Ettema (1980) and Chee's (1982) data under clear-water condition to investigate the effects of flow depth on local scouring. They claimed that the scour depth is independent of Y_0/d when $Y_0/d > 4$ and $d/D_{50} > 20$ (D_{50} = grain size, see Fig. 2-9).

Shen et al. (1966) found a negligible influence of grain size for $D_{50} < 0.5$ mm. Laursen and Toch (1956) found the same influence of grain size for $D_{50} = 0.5$ to 5 mm. On the other hand, Bonasoundas (1973) and Ettema (1976) found some increase of scour depth for $d = 0.6$ to 4 mm. Baker (1986) found that although scour depths are reduced dramatically as σ_g increases, at very high flows ($U/U_c > 4.0$), scour depths become almost independent of σ_g .

Raudkivi and Ettema (1983) investigated the effect of sediment gradation on clear water scour. Their results indicated that the value of the equilibrium scour depth, $D_{se}(\sigma_g)$ [$\sigma_g = (D_{84.1}/D_{15.9})^{0.5}$], for nonuniform alluvial sediments can be estimated in terms of the geometric standard deviation, σ_g of the sediment as follows:

$$\frac{D_{se}(\sigma_g)}{d} = K_\sigma \frac{D_{se}}{d} \quad (2-5)$$

The parameter K_σ is a function which depends on σ_g as shown in Fig. 2-10.

Ettema (1980) using six pier sizes and sediment sizes from $D_{50}=0.24$ mm to 7.80 mm (fig 2-11), showed that the maximum value of clear-water $(D_{*c}/d)_{max}$ is unaffected by particle size as long as the value of d/D_{50} is 20 to 25. For smaller values, the grains are large compared to the groove excavated by the downflow, and the erosion process is impeded. The experimental results in fig 2-11 were divided into four groups:

1. $d/D_{50} \geq 130$; the sediment is fine relative to pier diameter.
2. $130 > d/D_{50} \geq 30$; the sediment is of an intermediate size.
3. $30 > d/D_{50} \geq 8$; the sediment is coarse relative to pier diameter and relative to the downflow.
4. $d/D_{50} < 8$; the stones are so large that the erosion phase does not develop.

A distinct trend is apparent for nonripple forming sediments. For ripple forming sediments the scour depth tends to be lower, except when $\sigma_g \approx 1.5$, and the results are more scattered.

When $U_*/U_{*c} < 0.8$ (U_* = shear velocity and U_{*c} = critical shear velocity), the formation of an armour layer one grain thick on the surface of non-uniform ripple-forming sands prevents the growth of ripples and the scour develops as in a non-ripple-forming sediment.

2-5-3 The Effect of Flow Angle of Attack

For piers other than cylindrical, the depth of local scour depends on the alignment of the pier

with the flow. The local scour depth is related to the projected width of the pier, and this width increases rapidly with the angle of attack of the flow. With an increasing angle of attack, the location of maximum scour depth moves along the exposed side of the pier from the front to the rear end of the pier. Laursen and Toch (1956) published a graph of the multiplying factor for the scour depth K_p , as a function of the angle of attack, β , and the ratio of pier length to width, L/d , Fig. 2-12.

Mostafa et al. (1993) introduced a skewness factor K_p to account for the influence of pier skewness on the local scouring as follows (Fig. 2-13):

$$K_p = \left[\frac{L}{d} \sin \beta + \cos \beta \right]^{0.68} \quad (2-6)$$

The above expression fitted well the Laursen and Toch's (1956) curves over the data range $2 \leq L/d \leq 16$. Further, they recommended that the local scour depth at skewed bridge piers be estimated from an equation which is based on the projected width of a pier. The equation which accounts for the effect of pier skewness is:

$$D_{s_{\max}} = 2.4 K_s K_p d \quad (2-7)$$

in which; $D_{s_{\max}}$ = maximum equilibrium depth of local scour, and K_s = pier shape factor.

Ansari and Qadar (1994) proposed an expression for estimating the ultimate depth of local scour at bridge piers, including the projected width of the pier (d') similar to that of Mostafa's et al. (1993) expression, as follows:

$$d' = d \cos \beta + L \sin \beta \quad (2-8)$$

They attempted to correlate scour depth directly to the projected width of a pier. They proposed the following equations to estimate the ultimate scour depth, D_s , due to a bridge pier:

$$D_s = 0.86(d')^3; \quad d' < 2.2m \quad (2-9)$$

$$D_s = 3.6(d')^{0.4}; \quad d' > 2.2m \quad (2-10)$$

The above projected width cannot be used in a pier group directly. Unlike a solid single pier whose projected width varies linearly with β , in the case of a pier group the projected width changes non-linearly with pier spacing. Thus, equation 2-6 or 2-8 cannot be applied to a pier group. The projected width of each pier of a pier group contributes to the total projected width differently for any β .

2-5-4 The Effects of Shape of the Pier

According to Breusers et al. (1977) pier shapes can be classified into two categories:

1) blunt-nosed pier, where a strong horse-shoe vortex system and thus the maximum scour depth occurs at the pier nose. The upstream pier shape should have a strong influence on the scour depth, and the length of the pier and downstream pier shape should have a minimum effect if the blunt-nosed pier is aligned with the flow.

2) sharp-nosed pier, where the horse-shoe vortex system is very weak and the maximum scour

depth occurs near the downstream end. For long piers under an angle of attack the point of maximum scour depths shifts toward the downstream end of the pier. The effect of pier shape on scour depth is significant, and many researchers have studied this effect, including: Flamant (1900), Rehbock (1921), Yarnell and Nagler (1931), Keutner (1932), Tison (1940), Ishihara (1942), Schneible (1951), Laursen and Toch (1956), Chabert and Engeldinger (1956), Romita (1960), Knezevic (1960), Varzeliotis (1960), Larras (1960), Paintal and Garde (1965), Shen and Schneider (1970), Ayala and Grandon (1994), and Parola et al. (1996).

Durand-Claye (1873) investigated the effect of the shape of piers on scour depth. He compared the results of the scour depth for the case of a square-nosed, a round-nosed and a triangular-nosed rectangular pier. For the same approach flows the first one produced the maximum scour depth, whereas the triangular one gave the smallest scour depth. Tison (1940) investigated the influence of shape velocity profile and other parameters. He identified the curvature of the flow at the upstream side of the piers as the main cause of secondary vertical currents and local scour. Chabert and Engeldinger (1956) studied the influence of the pier shape and β . Fig. 2-14 shows that for $\beta=0^\circ$, scour depth may be minimized by streamlining the pier, but this advantage disappears for $\beta>10^\circ$.

An exception is formed by the system of 2 circular piers at a spacing of 3 pier diameters, which shows only a minor influence of β (the influence of spacing regarding pier groups will be discussed later). They also studied the influence of the pier shape on local scour, for more details see Breusers et al. (1977).

2-6 Local Scour Around a Single Pier

The prediction of local scour holes that develop around bridge piers plays an important role in their design. There are three different approaches to predict the depth of scour around bridge piers: 1) the empirical approach, 2) the analytical approach, and 3) the semi-empirical approach. The University of Colorado in the United States, the University of Delft in Holland, the University of New Zealand, the University of Auckland, and the University of Canterbury in New Zealand have conducted many systematic investigations concerning local scour around bridge piers. Most of these studies produced empirical relationships based on laboratory data and also some field data. In subsection 2-3-1 several empirical scour prediction formulas will be presented for the case of a single pier.

Few studies have adopted the semi-empirical or analytical approaches to the problem. Based on the application of the sediment continuity equation, Yanmaz and Altinbilk (1991) developed a semi-empirical method, to determine the time variation of clear water local scour depth around single cylindrical and square bridge piers. Olsen and Melaaen (1993) used a finite-volume model to solve the 3D non-transient Navier-Stokes equations. A numerical model simulated the 3D scour pattern around a cylindrical pier positioned in a movable-bed that was composed of plastic particles with a relative density of 1.04. The K- ϵ turbulence model was used to solve the Reynolds-stress terms. The numerical model was used to reproduce the geometry of a scour hole as measured on a physical model. The scour depth after 12 hours was measured and compared with the computed values. They found a good agreement between the results of the numerical and physical models. However, since

they neglected the transient terms in their numerical model, they could not verify the validity of the numerical model for all cases of scour around obstacles. Nor was it verified that the model would calculate maximum depth in the scour hole correctly. They used one of the most powerful computers (CRAY-XMP) in their analyses, but were only able to simulate a relatively small scour hole in their numerical model.

Cabral (1993) applied a $K-\epsilon$ model to investigate numerically the surface shear stress distribution in the vicinity of a circular cylinder mounted normally to a rigid flat plate. He derived the governing equations from the Reynolds averaged Navier-Stokes equations with an eddy viscosity approximation. He compared his results with those of Melville and Raudkivi (1977). Using their data, Cabral found that the experimental results were not well reproduced by the mathematical model. He concluded that some characteristics of the $K-\epsilon$ model contributed to discrepancies observed between the experimental and mathematical results. For example, such a model uses an isotropic eddy-viscosity formulation for the Reynolds-stress tensor, whereas the turbulence of 3D curved flows is anisotropic. Moreover, the experimental data show significant discrepancies which suggest the inadequacy of the $K-\epsilon$ model to reproduce bed shear stress distributions reported by Melville and Raudkivi (1977).

Physical models and mathematical models are applied on the basis of the empirical or analytical approaches respectively. Physical models produce empirical relationships, based on laboratory or field data. For the most part, these empirical relationships are *site specific* and require some modification (coefficients) before they can be applied elsewhere. On the other hand,

mathematical models can be applied to predict local velocities, the eddy-viscosities and the bed shear-stress to facilitate calculations of bed-load and suspended-load transport. The ultimate goal in developing mathematical models is to simulate the development of the local scour hole in time.

2-6-1 Pier Scour Equations

Many researchers have developed scour depth equations using different approaches to the problem. Each scour depth prediction formula gives different estimations and in some cases the differences are very significant. Ahmed et al. (1993) measured river bed scours near bridge piers and abutments on Alberta rivers. They compared the measured values with computed values estimated by different equations. They found that while a few formulas underestimated scour depth, most conventional formulas over-predicted the expected scour, some by as much as 500% (Fig. 2-15).

Live-bed scour formulas for predicting local scour depth around a single bridge pier include those of:

Laursen (1958)¹:

$$\frac{d}{Y_0} = \left(5.5 \left[\left(\frac{1}{11.5} \right) \left(\frac{D_s}{Y_0} \right) + 1 \right]^{1.70} - 1 \right) \quad (2-11)$$

and Ahmad (1962):

¹ See Jones (1984)

$$\frac{D_s}{d} = \frac{Y_0}{d}(4.77Fr^{\frac{2}{3}}-1) \quad (2-12)$$

Clear-water scour formulas include those of :

Chitale (1962):

$$\frac{D_s}{Y_0} = -5.49Fr^2 + 6.65Fr - 0.51 \quad (2-13)$$

where Fr = Froude number,

and Laursen (1963):

$$\frac{d}{Y_0} = \frac{5.5 \frac{D_s}{Y_0} \left[\left(\frac{1}{11.5} \right) \left(\frac{D_s}{Y_0} \right) + 1 \right]^{\frac{7}{6}}}{\left[\left(\frac{\tau_0}{\tau_c} \right)^{\frac{1}{2}} - 1 \right]} \quad (2-14)$$

in which; τ_0 = sediment shear stress, τ_c = critical shear stress. Shen et al. (1966)¹ gave a relation for D_{sc} as:

¹ See Breusers et al. (1977)

$$\frac{D_{se}}{Y_0} = 2 \left[Fr^2 \left(\frac{d}{Y_0} \right)^3 \right]^{0.215} ; \quad Fr = \frac{\bar{U}}{\sqrt{gY_0}} \quad (2-15)$$

in which; $\bar{U}$ = mean flow velocity and D_{se} = maximum scour depth.

For a circular pier equation (2-15) can be approximated by:

$$\frac{D_{se}}{d} = 1.35 \left(\frac{Y_0}{d} \right)^{0.3} \quad (2-16)$$

Shen et al. (1969) also derived the following relation, which is known as *Shen I*:

$$D_{se} = 0.000223 Re^{0.619} \quad (SI-units) \quad (2-17)$$

in which Re = Reynolds number. Veiga Da Cunha (1970) considered Re number an unsuitable parameter to characterise the scour depth, since it varies with temperature. To avoid measuring the temperature, Shen et al. (1969) gave the following (modified) equation, so called *Shen II*:

$$\frac{D_{se}}{d} = 3.4 Fr^{\frac{2}{3}} \left(\frac{Y_0}{d} \right)^{\frac{1}{3}} \quad (2-18)$$

The Colorado State University's (CSU) equation¹ takes the form:

¹ See Richardson et al. (1991)

$$\frac{D_s}{Y_0} = 2.0K_1K_2\left(\frac{d}{Y_0}\right)^{0.65} Fr_1^{0.43} \quad (2-19)$$

in which; K_1 = correction for pier nose shape, K_2 = correction for angle of attack of flow, D_s = scour depth, and Fr_1 = Froude number of the approach flow upstream of the pier.

For *live-bed* scour, Jain and Fisher (1980) proposed the following relation:

$$\frac{D_s}{d} = 2.0(Fr - Fr_c)^{0.25} \left(\frac{Y_0}{d}\right)^{0.5} \quad (2-20)$$

and for *clear-water* scour:

$$\frac{D_s}{d} = 1.85(Fr_c)^{0.25} \left(\frac{Y_0}{d}\right)^{0.3} \quad (2-21)$$

in which Fr_c = Froude number using critical velocity for initiation of motion.

Breusers et al. (1977) found the scouring depth as a function of eight parameters as follows:

$$D_s = f_1(\rho, \nu, g, D_{50}, \rho_s, Y_0, \bar{U}, d) \quad (2-22)$$

Using the theorem of Vaschy-Buckingham and replacing some parameters they obtained:

$$\frac{D_s}{d} = f_2\left(\frac{U_* D_{50}}{\nu}, \frac{U_*^2}{\Delta g D_{50}}, \Delta, \frac{Y_0}{d}, \frac{D_{50}}{d}\right) \quad (2-23)$$

where $\Delta = (\rho_s - \rho) / \rho$ is the relative submerged density and $U_* = (g Y_0 I)^{1/2}$ in which; ρ = fluid density; ρ_s = sediment density, I = hydraulic gradient; D_{50} = the representative sediment size and ν = kinematic viscosity of fluid. They simplified equation (2-23) as follows:

$$\frac{D_s}{d} = f\left(\frac{U_*}{U_{*c}}, \frac{Y_0}{d}, \frac{D_{50}}{d}\right) \text{ or } \frac{D_{se}}{d} = f\left(\frac{Y_0}{d}, \frac{D_{50}}{d}\right) \quad (2-24)$$

The term $\frac{U_*^2}{\Delta g D_{50}}$ is empirically related to $\frac{U_{*c} D_{50}}{\nu}$ for the initiation of motion, where Δ is constant. Equation 2-24 implies that the scour depth D_s will depend mainly on the ratio of the mean velocity to the mean critical velocity and the relative values of the grain size, the flow depth and the pier diameter. Finally, Breusers et al. (1977) suggested the following relations for practical applications:

$$\frac{D_s}{d} = f_1\left(\frac{\bar{U}}{\bar{U}_c}\right) \left[2 \tanh\left(\frac{Y_0}{d}\right) \right] f_2(shape) f_3\left(\beta, \frac{L}{d}\right) \quad (2-25)$$

$$f_1\left(\frac{\bar{U}}{\bar{U}_c}\right) = 0 \quad \text{for} \quad \frac{\bar{U}}{\bar{U}_c} \leq 0.5 \quad (2-26)$$

$$f_1\left(\frac{\bar{U}}{\bar{U}_c}\right) = \left(2 \frac{\bar{U}}{\bar{U}_c} - 1\right) \quad \text{for} \quad 0.5 \leq \frac{\bar{U}}{\bar{U}_c} \leq 1.0 \quad (2-27)$$

$$\text{and} \quad f_1\left(\frac{\bar{U}}{U_c}\right) = 1 \quad \text{for} \quad \frac{\bar{U}}{U_c} \geq 1.0 \quad (2-28)$$

where $f_2(\text{shape}) = 1.0$ for circular and rounded piers

= 0.75 for stream-lined shapes

= 1.3 for rectangular piers

$f_3(\beta, L/d)^1$.

Simons and Senturk (1992) recommended the CSU equation and also compared this formula using the graphical comparison of Jones (1984) with some of the commonly used equations (Fig. 2-16). Based on laboratory data Melville and Sutherland (1988) introduced the following relation for estimating the equilibrium scour depth:

$$\frac{D_{se}}{d} = K_f K_y K_d K_g K_s K_\beta \quad (2-29)$$

where the K's are the following expressions describing the influence of each parameter on local scour depth: K_f = flow intensity, K_y = flow depth, K_d = sediment size, K_g = sediment gradation, K_s = pier shape, and K_β = pier alignment, respectively. The largest possible local scour depth was found to be as much as 2.4. Multiplying factors, which have the effect of reducing this value, can be applied where clear-water scour conditions exist (K_f), the flow depth is relatively shallow (K_y), and the sediment is relatively large (K_d). In the case of noncylindrical piers, two additional multiplying factors are applied

¹ See Fig. 2-10.

(K_s , and K_B). The proposed method was found to be generally conservative, because it was based on envelope curves to data [including previous data obtained by Breusers et al. 1977, Shen et al. (1971), Laursen and Toch (1956), and Hanco (1971)]. This method also generally predicts smaller scour depths than the previous methods because it takes into account the sediment grading.

2-7 Local Scour Around Pier Groups

The mechanism of scouring for a pier, which is part of a group of piers, is the same as for a single pier, i.e. the HSV is the prime agent causing the local scour around each pier. In addition to the single pier parameters, group effects are important if piers are not spaced far enough from each other.

Depending on pier spacing, the HSV that develops around each pier may interact. In the case of pier groups the following parameters should be considered in estimating the depth of the local scour: sheltering effects, the flow angle of attack, and the HSV developed around each pier. Only a few investigators have researched the *group* effect in a movable bed, [Dietz (1973), Hannah (1978), Raudkivi and Sutherland (1981), Elliott and Baker (1985), and Nough (1986)]. In general, these investigations produced correcting factors which may be applied to formulas developed for a single pier.

2-7-1 Movable Bed

Laursen and Toch (1956) produced a graph of coefficients to adjust aligned flow scour, to

allow for β for rectangular piers of different length/breadth ratios (Fig. 2-12). They found that D_s grows rapidly as the pier becomes non-aligned to the flow and the L/d ratio increases.

Dietz (1973) conducted a series of experiments examining the effects of S and β on D_s for two piers. He concluded that:

- (i) scour at the front pier is independent of pier spacing,
- (ii) two independent scour holes form at a spacing of greater than 4 diameters,
- (iii) at 25 diameters the downstream scour hole is still less than that at the front pier.

In stage 2 of his study, Dietz altered β and concluded:

- (i) scour depths for a twin pier (2 piers touching) increase with angle of attack,
- (ii) for $\beta \leq 30^\circ$ and S of between 0.5 and 1 diameter, a common scour hole deeper than that of twin piers is present. For $\beta > 30^\circ$ the hole is shallower,
- (iii) independent scour holes form between 4 and 6 diameters spacing and this is independent of β .
- (iv) for $\beta < 15^\circ$ the depth of scour is less sensitive to non-aligned flow if S is between 2 and 4 diameters.

Hannah (1978) studied local scour at groups of cylindrical piers with steady uniform flow and clear-water conditions. A series of tests was first performed on single piers and for a pier length:width ratio, 6:1 with semi-circular ends to provide a basis against which scour at pier groups could be evaluated. Pier groups of various S and with different β were investigated for one flow condition. For the sediment used in all tests, $D_{50}=0.75$ mm and $\sigma_g=1.32$, where $\sigma_g=D_{85}/D_{50}$. Four of the mechanisms which affect scour at pier groups and are not present in scouring at a single pier are:

- i) reinforcing: which causes increased scour depth at the front pier and is a consequence of the dynamic equilibrium, which exists in the base of a scour hole when stable

conditions have been reached. The scour hole formed around a downstream pier aids removal of the sediment from around the piers. Equilibrium in a scour hole is dynamic in nature. Bed material is observed to be continually lifted from the base of the scour hole by the HSV. However, material does not have sufficient energy to be carried clear of the scour hole and hence, it drops back into the scour hole. With a downstream pier placed so that the scour holes overlap, two effects which aid scouring at the upstream pier are observed. Both effects reduce the energy required by the active bed material (in the scour hole) to be carried clear of the scour hole. The first is a decrease in exit slope of the scour hole and the second effect is to lower the bed level behind the scour hole (Fig. 2-17a). As S and β are increased, reinforcing diminishes.

ii) sheltering: the presence of an upstream pier can cause a reduction of the effective approach velocity for downstream piers. A second form of sheltering occurs if material scoured from the upstream piers is deposited on the bed in front of the downstream piers. Flow is then deflected upward from the bed and around the downstream pier. The strength and thus the effectiveness of the HSV at this pier is thereby reduced. As pier separation increases, the velocity deficit in the wake of the upstream pier disappears and the sheltering effect becomes negligible. This reduction decreases the effect of the HSV and thereby reduces scour at downstream piers. Timonoff (1929) found that, "if a downstream pier is placed on the same axis as the upstream pier, then in its immediate proximity there is a definite sheltering effect."

iii) shed vortices: vortices shed from an upstream pier are convected downstream, following paths as illustrated in Fig. 2-17a. Should a second pier be placed close to one of these

paths, the vortices, by virtue of their velocity and pressure distributions, assist in lifting material from the scour hole. The scouring potential of the shed vortex is a function of the intensity of its convection speed and of the distance between the path and the affected pier. This effect, therefore, decreases more rapidly for piers in line with the flow than for those at β , which place downstream piers on the paths traced by vortices shed by upstream piers.

iv) “compressed” HSV: when piers are placed transverse to the flow, each will have, except at very close spacing, its own HSV. The interaction between piers in a group intensifies the reinforcing and compressed HSV effects (Fig. 2-17b). Each pier has its own streamline. For example, when cylindrical piers are spaced close together the flow velocity around piers will be increased.

Laursen and Toch (1956), Dietz (1973), and Elliott and Baker (1985), also studied the effect of pier spacing on scour around bridge piers in a movable bed. Elliott and Baker found that the scouring process will be affected by two processes: (1) the HSV's around the base of the piers will interact with each other, and (2) the flow will be accelerated by the contraction produced by the piers. Moreover, their results showed that the effect of pier spacing on scour depth can be examined by using the factor $f_s(x/d)$, where x = the pier centre line spacing, and d = pier diameter. Several later works have modified the above factor. However, equations developed in these later studies were derived only for clear-water scour, one set of pier geometries, one value of Y_0/d , and one sediment type.

Nouh (1986) investigated experimentally the characteristics of flow (including velocities,

turbulence intensities, and related boundary shear stresses) and clear-water scour at pier groups in a meandering channel. Nohh's study was an extension to the works of Hannah (1978) and Raudkivi and Sutherland (1981), who investigated the influence of several parameters. These parameters were: the location of the pier group with respect to the channel boundaries, the transversal pier spacing, the longitudinal pier spacing, and the relative pier group size with respect to the channel width. A group of 4 piers consisting of 2 lines in parallel was placed at different locations in the meandering test section of the model channel (Fig. 2-18).

Nohh (1986) found that the levels of turbulence intensity inside his pier group were relatively lower than that outside the group. Also, the normalised boundary shear stresses and related clear-water scour around a pier group show significant variation with the location in his meandering channel. The maximum scour depths were found to be larger at the front piers than at the rear piers, and at the outer piers than at the inner piers.

Nohh (1986) performed another series of tests to study the effect of the transversal pier spacing on maximum clear-water scour depths at the piers. The pier diameter was changed in each test run to provide a constant relative pier group size with respect to the channel width throughout the tests. It was found that the scour depths of the piers decrease as the transversal pier spacing increases. This was supported by the maximum scour depth observed for a single pier in the meandering channel which was larger than that observed at the pier in the long straight channel reach.

With relatively small transversal pier spacing, the observed scour depths at rear piers were

larger than that observed at the front piers. Raudkivi and Sutherland (1981) found their results in accord with the above-mentioned results. They found that the scour depths at rear piers were very sensitive to β , and for $\beta > 45^\circ$ these scour depths were greater than the same at front piers.

Jones (1989) investigated the effects of pier spacing on local scour depth in a movable bed. The test included groups of piers equally spaced laterally and longitudinally at $1.5 \times \text{dia}$, $2.0 \times \text{dia}$, and $3.0 \times \text{dia}$ centerline to centerline with the same outside dimensions of the pier groups in each case as illustrated in Fig. 2-19. A corresponding *equivalent rectangle* was tested for each pier group where the width of the *equivalent rectangle* was the cumulative projected width of the piers that obstructed flow in the lateral direction. He found that in every case the scour for the pier group was less than the scour for the *equivalent rectangle*. Therefore, he suggested that pier groups that project above the stream bed can be analysed conservatively by representing them as a single pier width equal to the projected area of the piers ignoring the clear space between piers.

2-7-2 Fixed Bed

Chambarel et al. (1991) investigated flow past one and two circular cylinders for $Re \leq 500$. The tests were conducted in a wind tunnel with a rigid bed. Cylinders were aligned to the flow with different S/d . They found that vortex shedding always occurs behind the downstream cylinder. However, for the upstream cylinder, vortex shedding appeared only when the intervals of the two cylinders exceeded a certain value ($S/d=3$). A discontinuous change of the vortex shedding frequency and the flow pattern was found to be associated with the changed in the upstream base pressure. The

presence of the downstream cylinder tended to decrease the drag and to increase the lift. This was because the downstream cylinder was in the wake of the upstream one, and the vortex shedding was strongly influenced and synchronized by the upstream cylinder.

Chan and Lam (1993) simulated numerically the flow around two cylinders in various arrangements (tandem, side by side, and staggered) in subcritical Re. They proposed a grid free method for high Re whereas the traditionally grid methods could give a satisfactory result only in low Re. Among the grid free methods, they chose vortex methods to simulate the vorticity of the flow field. Using the 2D-Navier-Stokes equation, they presented the vorticity transport equation as follows:

$$\frac{\partial \omega}{\partial t} + U \cdot \nabla \omega = \frac{1}{Re} \nabla^2 \omega \quad (2-30)$$

From equation 2-30, instead of the transportation of momentum, the flow development can be regarded as the results of vorticity generation $\frac{\partial \omega}{\partial t}$, vorticity convection $(U \cdot \nabla) \omega$, and diffusion of vorticity $\frac{1}{Re} \nabla^2 \omega$, where ω = angular velocity. For the vorticity generation, vorticity elements called *vortices* of strength $1/2 \gamma^* \Delta t$ are shed from the selected separating points of every cylinder at each individual time step (γ^* is the surface vorticity at separation).

Chan and Lam (1993), using the 2-D Navier-Stokes equation, also showed that when two vortices come close together each of the vortices is assigned a core size. They proposed the following relation for induced tangential velocity V_0 when two vortices come close together:

$$V_{\theta} = \frac{\Gamma}{2\pi r} \quad (r \geq r_c), \quad \text{and} \quad V_{\theta} = \frac{\Gamma r}{2\pi r_c^2} \quad (r < r_c) \quad (2-31)$$

in which; r_c = core size, r = the distance between two vortices. The core size can be found by solving the diffusion equation of vorticity in polar coordinates:

$$\frac{\partial \omega}{\partial t} = \nu \left(\frac{\partial^2 \omega}{\partial r^2} + \frac{1}{r} \frac{\partial \omega}{\partial r} \right) \quad (2-32)$$

Therefore, a Gaussian vorticity distribution can be obtained

$$\omega(r,t) = \frac{\Gamma_0}{2\pi \nu t} e^{-\frac{r^2}{4\nu t}} \quad (2-33)$$

and hence the circumferential velocity is given by

$$V_{\theta}(r,t) = \frac{\Gamma}{2\pi r} \left[1 - e^{-\frac{r^2}{4\nu t}} \right] \quad (2-34)$$

The core size is defined as the maximum value of V_{θ} due to the vortex. Therefore, set

$$\frac{dV_{\theta}}{dr} \Big|_{r=r_c} = 0 \quad (2-35)$$

After differentiation and some mathematical manipulation, we have

$$\left(1 + \frac{2r_c^2}{4\nu t} \right) e^{-\frac{r_c^2}{4\nu t}} = 1 \quad (2-36)$$

By substitution of $r_c = 2.24 \sqrt{U\ell}$ into equation 2-36, the condition can be satisfied. Moreover, the velocity distributions around the cylinders can be calculated easily by using,

$$\Gamma_n = \sum_{m=1}^N \Gamma_m L(m,n) + \Gamma_\infty + \sum_{m=1}^M \gamma(S_m) K(S_m, S_n) \Delta S_m \quad (2-37)$$

where $L(S_m, S_n)$ and $K(m, n)$ are the coupling coefficients derived from the mutual induction of vorticity by Biot-Savart's Theorem and the detailed calculation of these coupling coefficients were given by Lam (1987) and Lewis (1981).

For the side by side arrangement, Chan and Lam (1993) compared the result of their simulations with Bearman and Wadcock's (1973) experimental results. They measured the base pressure simultaneously on both cylinders and found that both cylinders experienced different base pressure in the range $1.1 \leq S/d \leq 2.3$. Chan and Lam (1993) also found that a biasing flow phenomenon occurred at $S/d=1.4$ as shown in Fig. 2-20a & b. They observed the formation of a wide wake and a narrow wake behind the cylinders. For $S/d=4$, the proximity interference decreased and the flow biasing effect was lost (Fig. 2-21a & b). They claimed that the reattachment of the shear layers from an upstream cylinder onto the downstream cylinder occurs at $S/d=3.5$. For $S/d < 3.5$, a downstream roll up of the shear layers of the upstream cylinder occurred as shown in Fig. 2-22a&b.

For the staggered arrangement, Chan and Lam (1993) conducted several tests on two cylinders using small S/d . From the simulated flow field, they found that the wake of the upstream cylinder was often *squeezed* by the downstream cylinder as shown in Fig. 2-23a & b and Fig. 2-24a

& b. Due to the narrow wake flow pattern, the drag of the upstream cylinder was reduced. The mean drag coefficient of the upstream cylinder for $\beta=30^\circ$ was less than that for $\beta=50^\circ$. However, they found that due to the wide wake formation, in general, the drag of the downstream cylinder was greater than the upstream one.

Finally, for both cases ($\beta=30^\circ$ and $\beta=50^\circ$), there was a great difference for the values of the lift coefficients. At the 50° inclination, the lift coefficient of the downstream cylinders was far more negative than the case of 30° . The reason was that the gap flow between both cylinders was intensified in the case of 50° , but for 30° inclination, the gap flow was blocked.

In short, Chan and Lam (1993), using the free grid method, simulated numerically the flow around two cylinders in various arrangements (tandem, side by side, and staggered). In a side by side arrangement it was found that a wide wake and a narrow wake were formed behind the cylinders. For $S/d < 4$, the proximity interference decreased. In the case of a tandem arrangement, the reattachment of the shear layers, from the upstream cylinder onto the downstream cylinder, occurred at $S/d=3.5$. The downstream cylinder was shielded from the upstream cylinder for $S/d < 3.5$. However, for $S/d > 3.5$, a roll up of the shear layers of the upstream cylinder occurred. For the staggered arrangement it was found that the wake of the upstream cylinder was often *squeezed* by the downstream cylinder. In addition, the drag of the upstream cylinder reduced and became less than that of the downstream one.

2-8 Summary and Conclusions

Most failures of bridges crossing rivers are caused by the local scouring phenomenon. Investigators generally conclude that the prime agent in the local scouring process is the HSV produced by the interaction of the river's flow with the bridge support structures (piers). To design a safe bridge crossing, it is necessary to locate pier footings deeper than the maximum anticipated depth of local scour. On the other hand, placing bridge pier foundations overly deep, is unattractive from an economic viewpoint.

Since there is no unique solution to this problem, establishing an optimum design depth is difficult. In the absence of a singular theoretical approach, an empirical approach is utilized to solve some parts of this complex problem. Physical models based on laboratory or field data give empirical relations between depth of the scour and the other parameters, such as: flow, sediment, and bridge pier characteristics.

Local scour around single piers has been investigated extensively, while studies of pier groups are rare. The studies about scouring around bridge pier groups can be categorized based on the two main bed conditions into: 1) movable bed and 2) fixed bed.

The majority of the works in the past have been performed for 2 pier groups (i.e., Laursen and Toch (1956), and Dietz (1973)). Hannah (1978) conducted his experiments for groups of 2, 3, 4, and 6 piers, however, he did not examine the effect of flow angle of attack variations on groups of 3, 4,

and 6 piers. In addition, even for the case of groups of 2 piers, he investigated the effect of different β on local scour depth only for one value of S/d . Nough (1986) did his experimental work for groups of 4 piers with a movable bed. However, he used a meandering channel instead of a straight channel to investigate the effects of location of the pier group with respect to the channel boundaries, the transversal pier spacing, the longitudinal pier spacing, and relative pier group size with respect to the channel width.

Bearman and Wadcock (1973), Chambarel et al. (1991), and Chan and Lam (1993) investigated flow past one and two circular cylinders with a fixed bed. Although their investigations gave an insight into the flow pattern formed around a group of 2 piers, they did not notice the effect of sediment movement in interaction with flow characteristics.

Lastly, no corresponding relationships have been developed specifically for bridge pier groups. The traditional approach to estimate local scour depths for pier groups is to apply certain coefficients to formulas developed for the case of a single pier. This research, which investigates the local scouring phenomenon associated with groups of piers, addresses this particular limitation in current design practice.

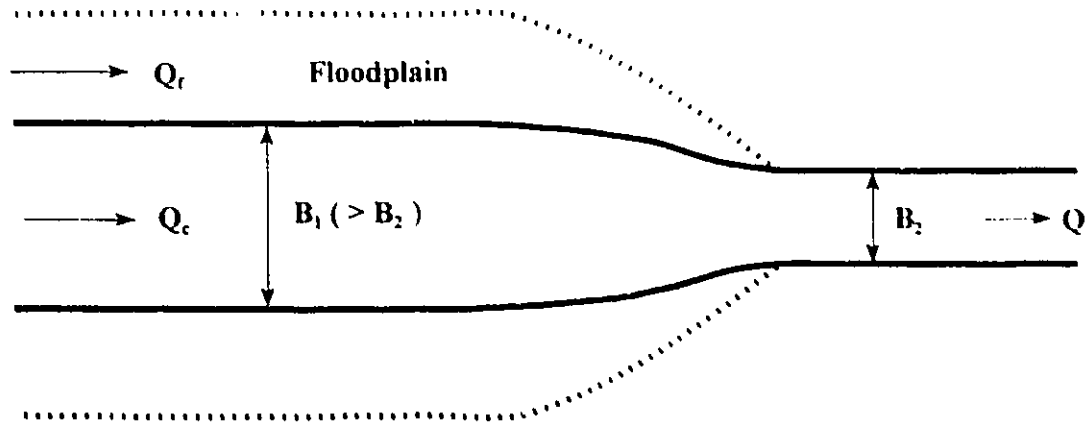


Fig. 2-1: Definition sketch for a channel contraction.

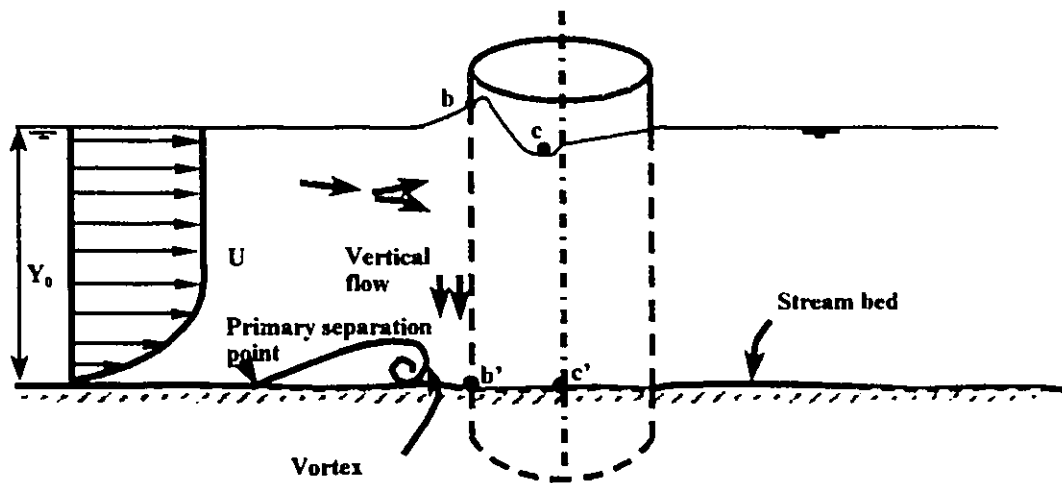


Fig. 2-2: Elevation view of the flow field in the vicinity of a cylindrical pier in a plane bed.

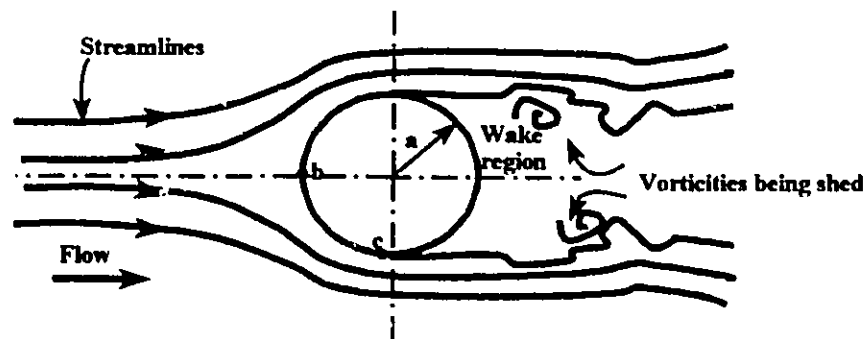


Fig. 2-3: Plan view of flow distribution around a cylindrical pier

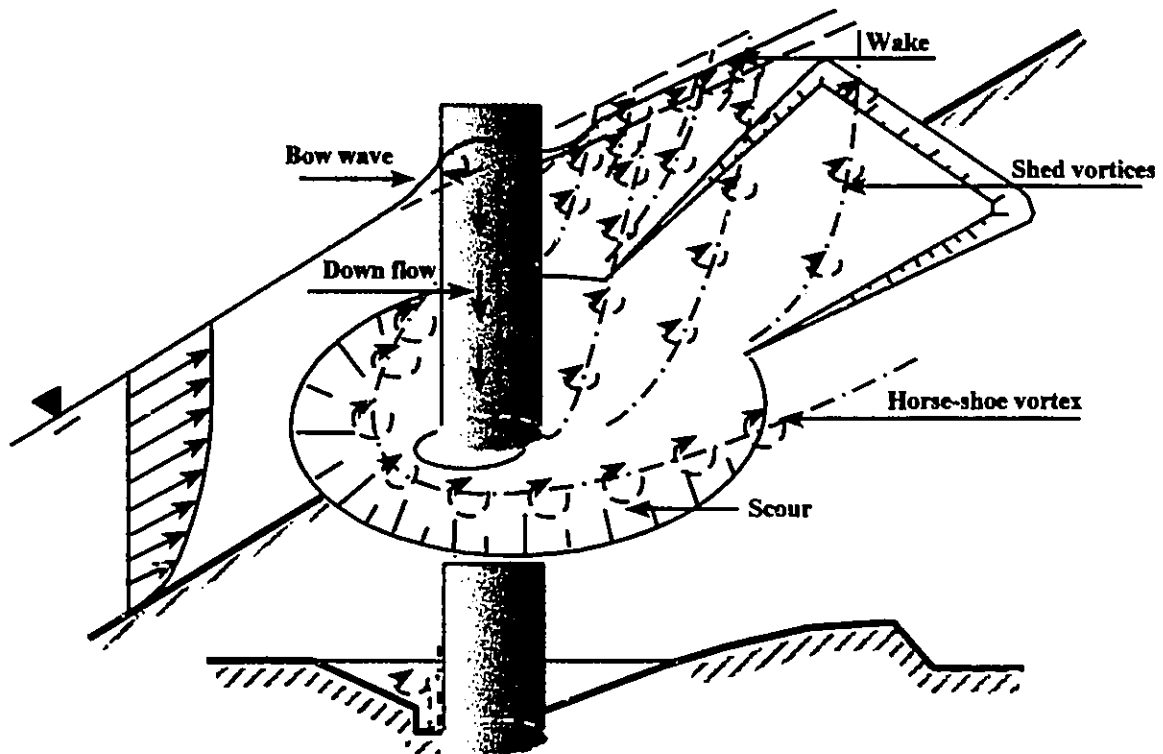
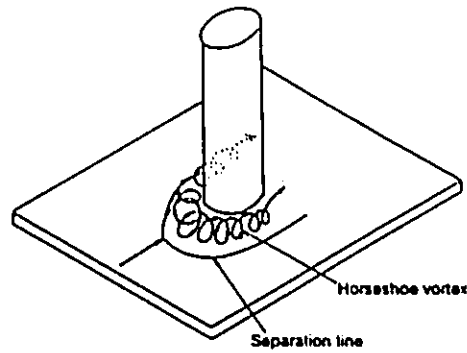
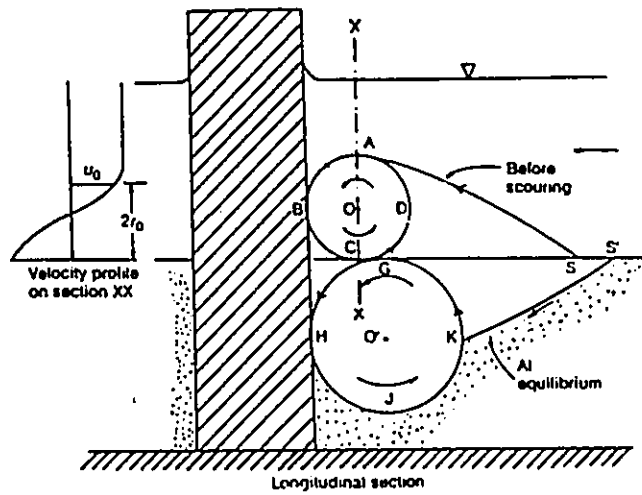


Fig. 2-4: Diagrammatic flow pattern at cylindrical pier



(a)



(b)

Fig. 2-5: a) Development of the scouring vortex, b) definition diagram (after Qadar 1981)

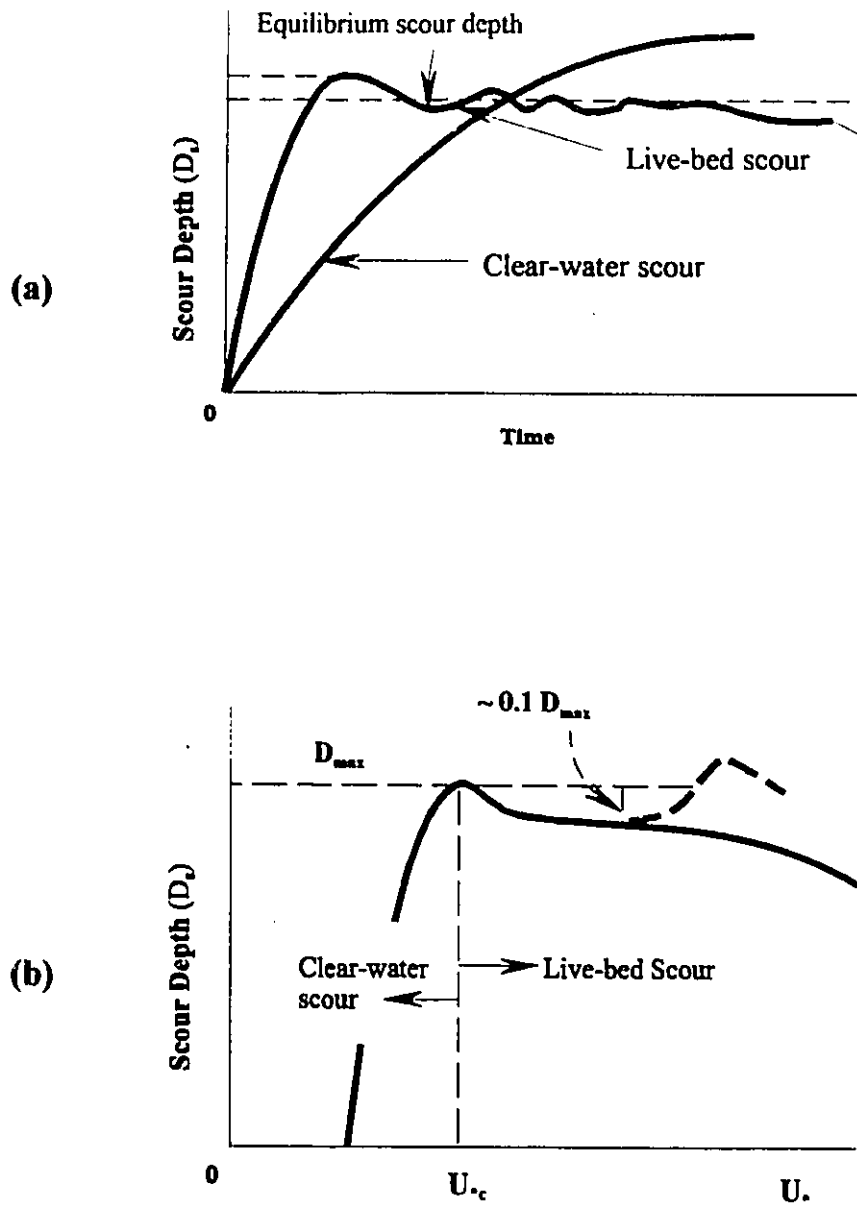


Fig. 2-6: Scour depth for given pier and sediment Size; a) as function of time; b) as function of velocity, (after Raudkivi 1986)

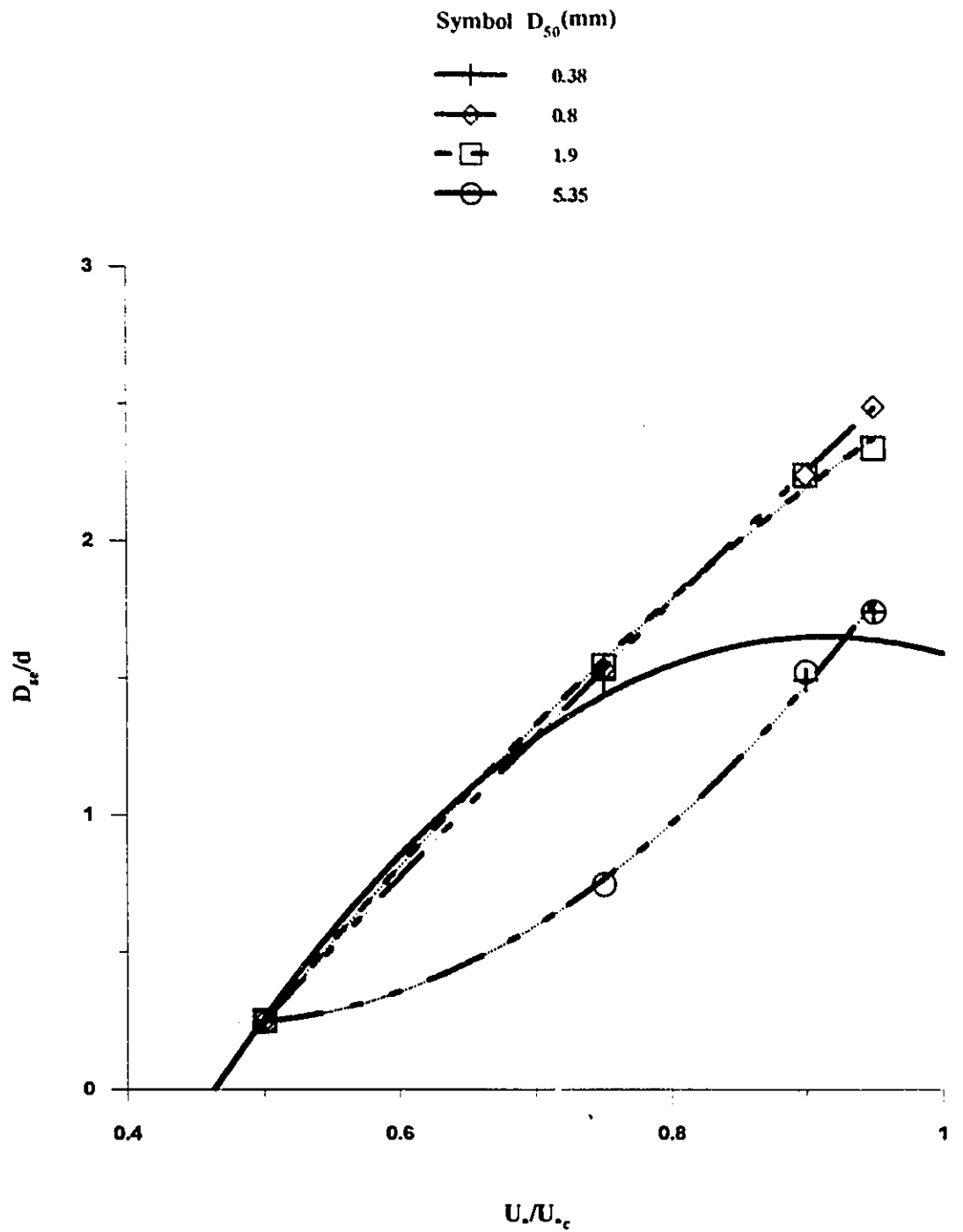


**Fig. 2-7: Equilibrium clear-water scour depth versus $U./U_{*c}$
(after Ettema 1980)**

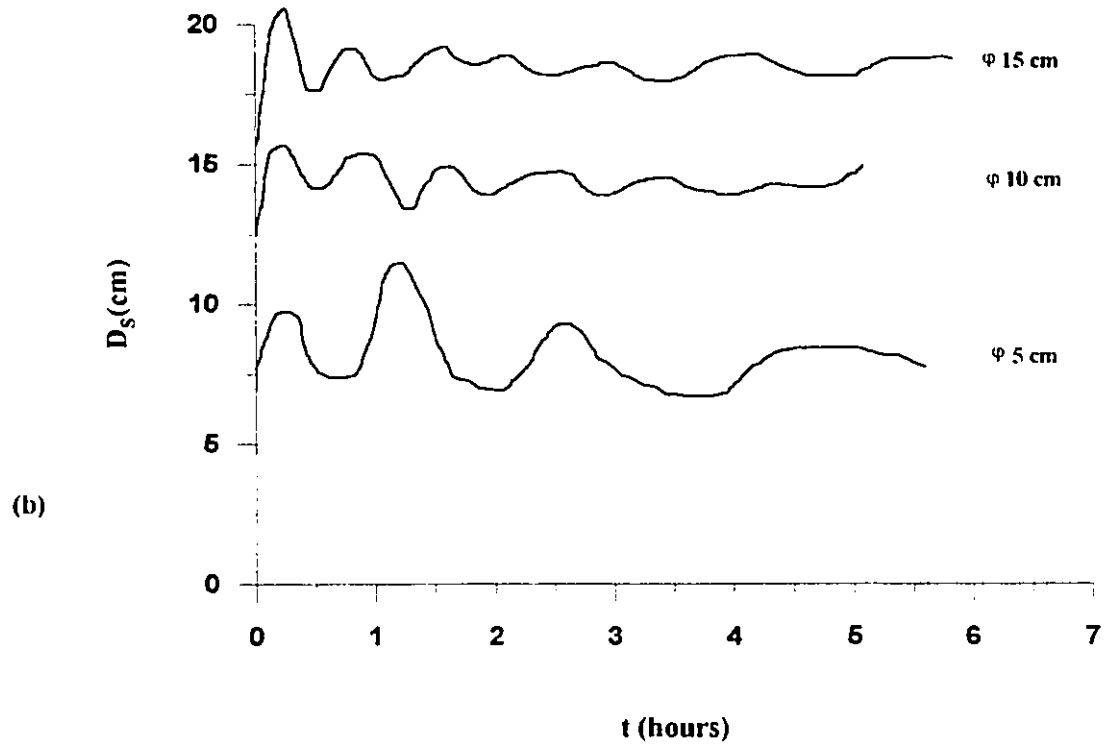
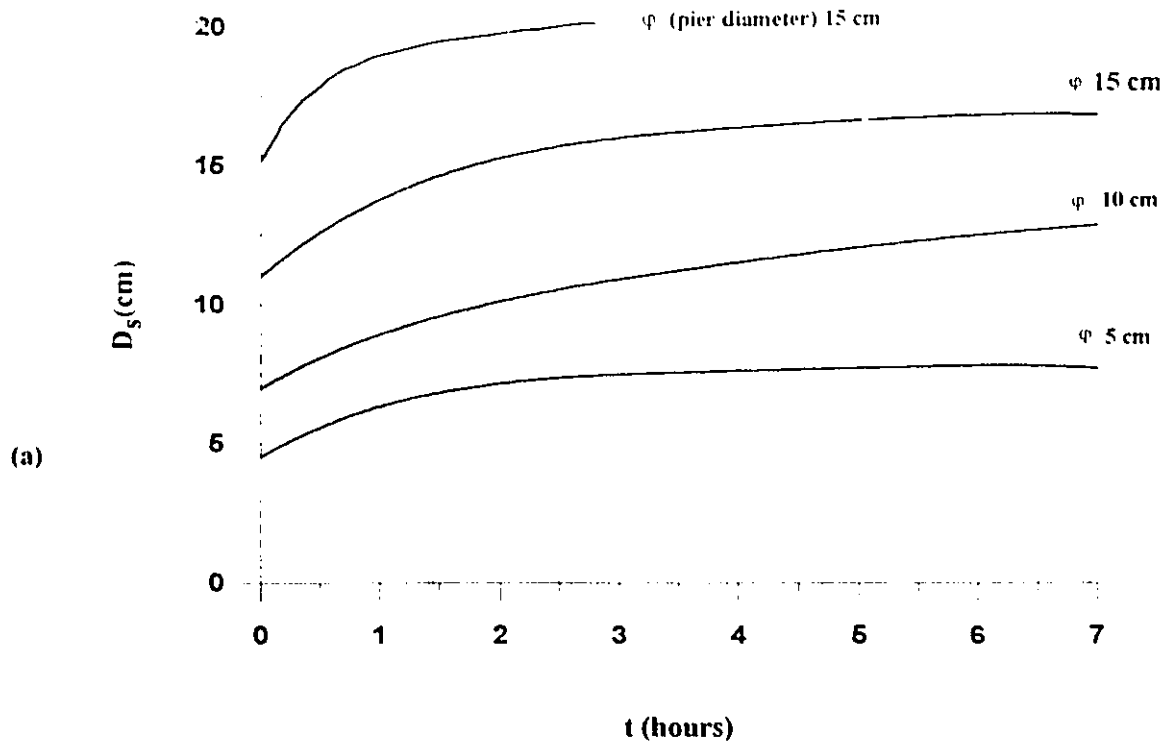


Fig. 2-8: a) Scour as a function of time ($U < U_c$), b) scour as a function of time ($U > U_c$) (after Chabert and Engeldinger 1956)

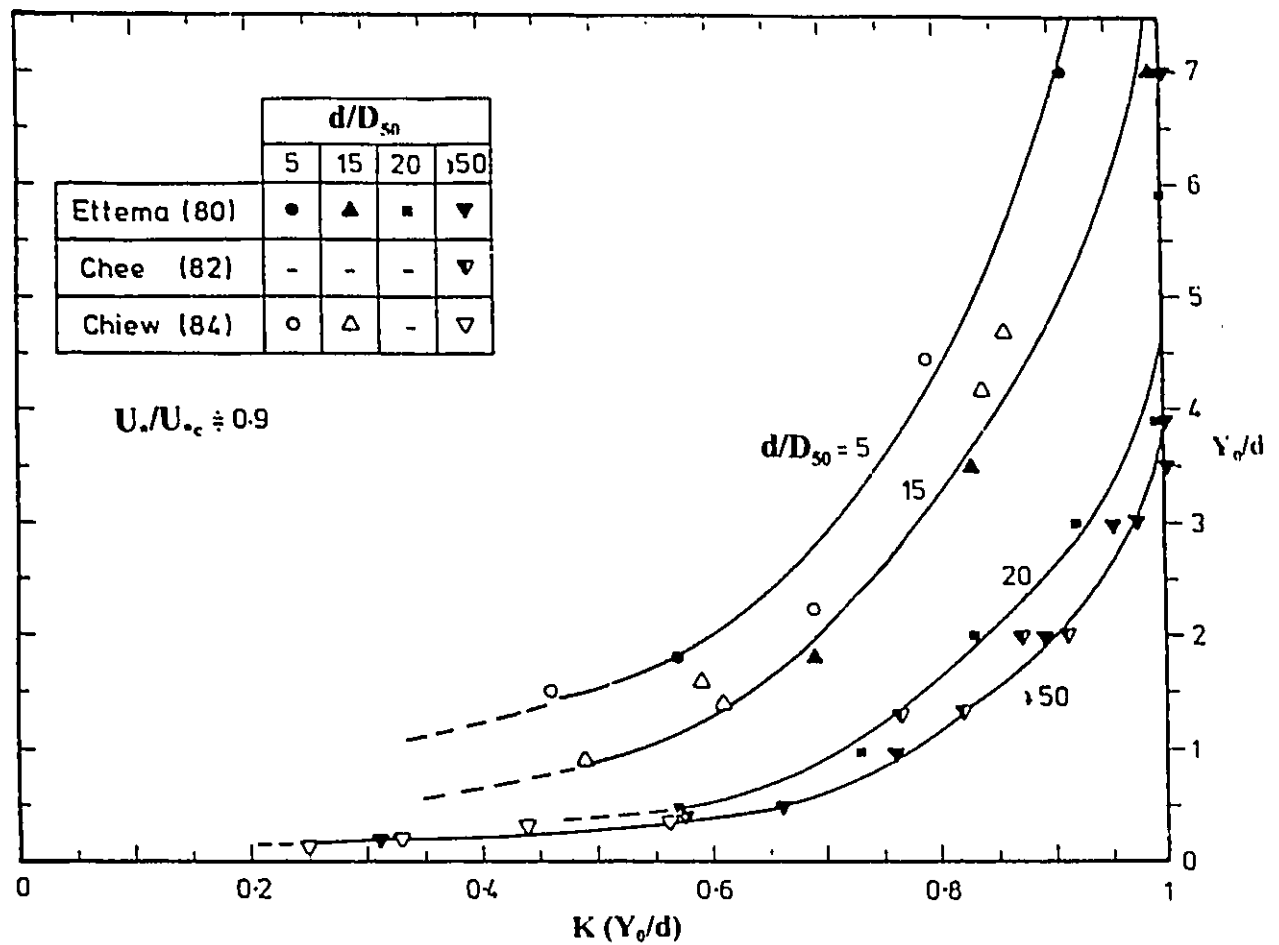


Fig. 2-9: Equilibrium scour depth versus flow depth, Y_0/d , with relative pier size d/D_{50} as parameter. 1) Ettema (1980), 2) Chee (1982), and 3) Chiew (1984)

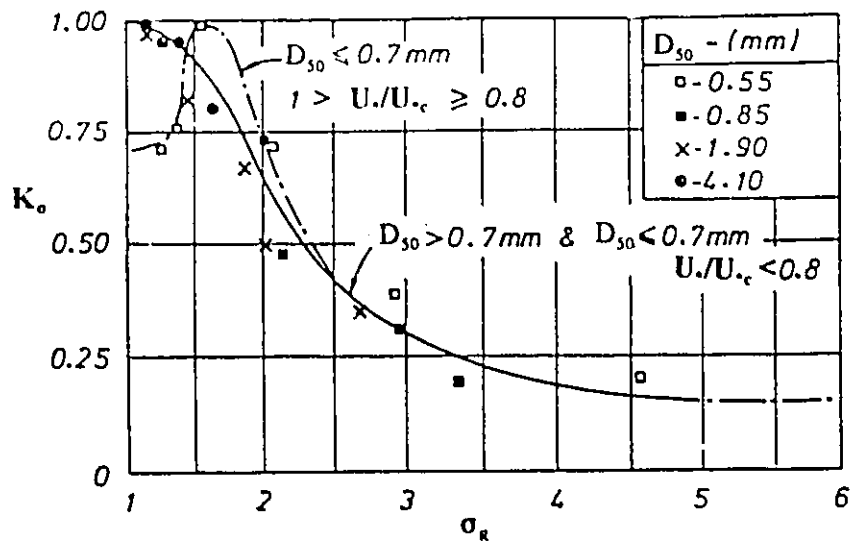


Fig. 2-10: Coefficient K_o as a function of the geometric deviation of the particle size distribution (after Ettema 1980)

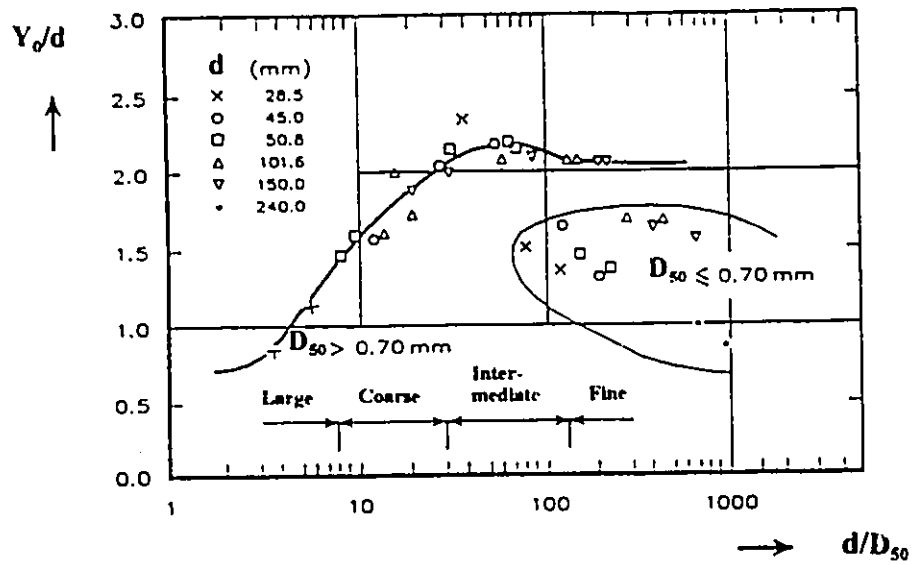


Fig. 2-11: Equilibrium clear-water scour depth versus d/D_{50} ; $U./U_c = 0.9$ (after Ettema 1980)

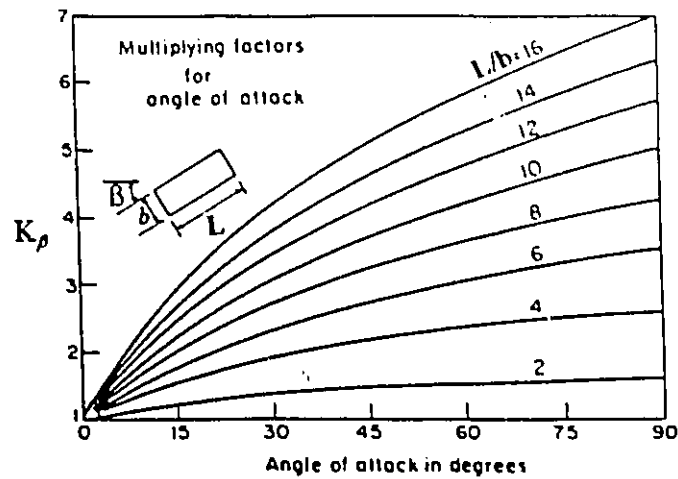


Fig.2-12: Alignment factor K_p for piers not aligned with flow (after Laursen and Toch 1956)

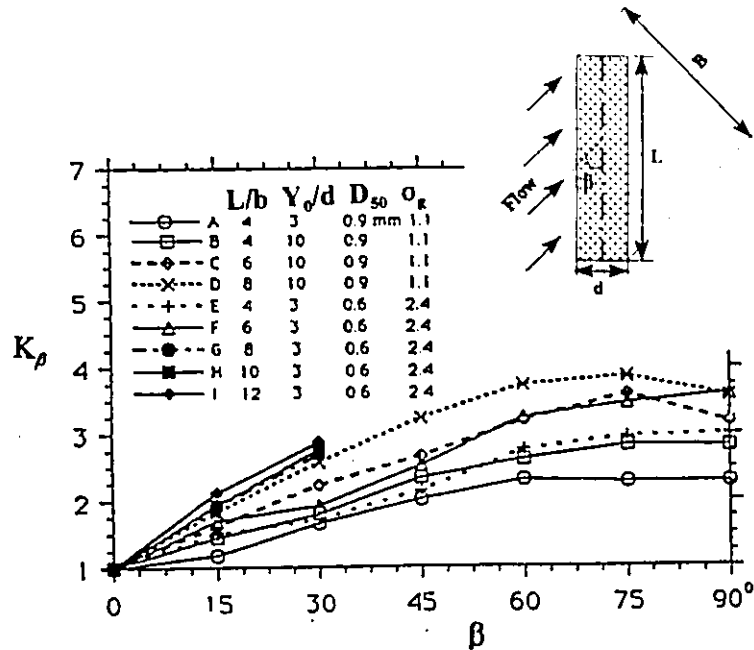


Fig.2-13: Measured values of K_p for rectangular piers (after Mostafa et al. 1993)

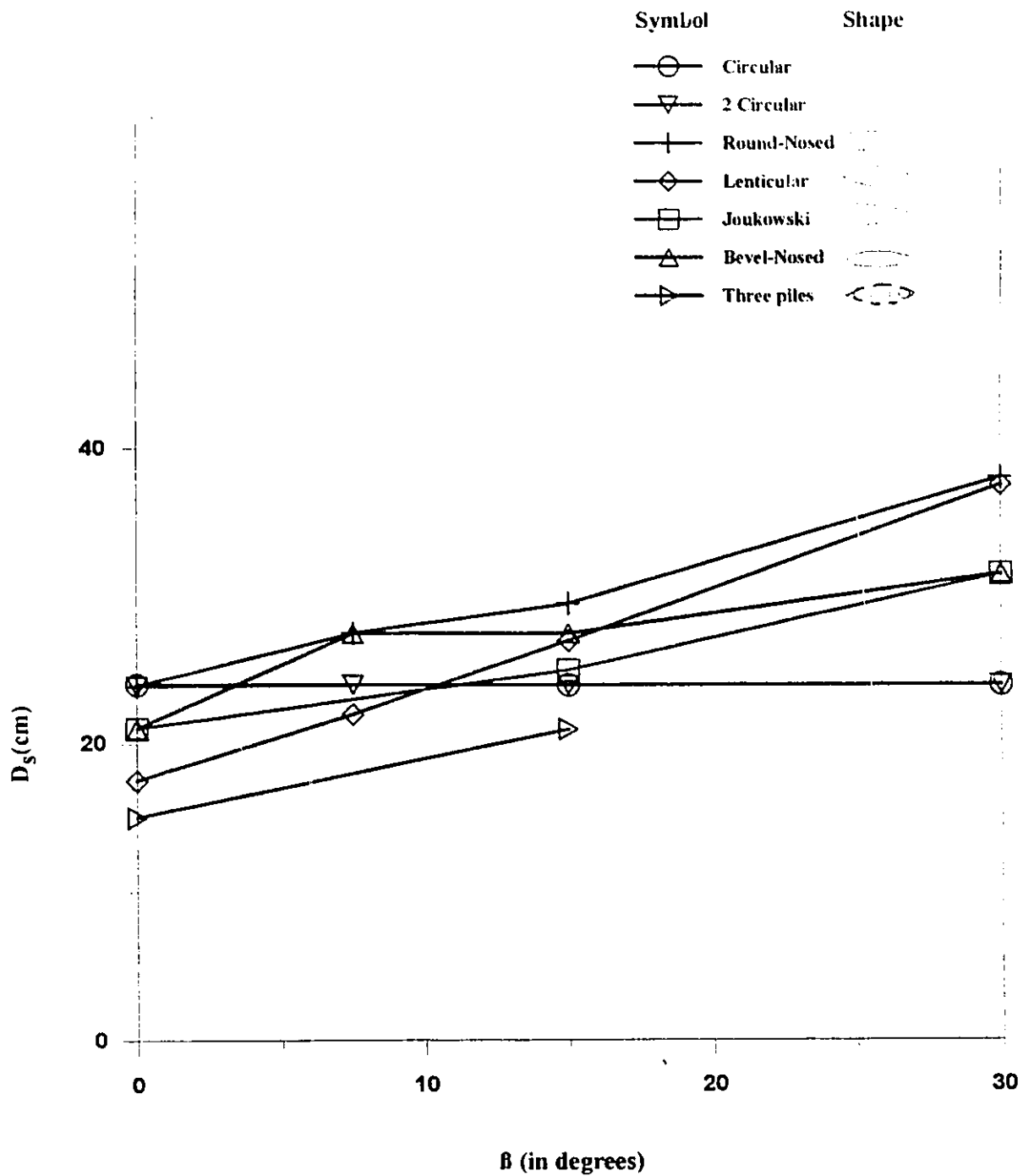


Fig. 2-14: Influence of pier shape and flow angle of attack on scour depth (after Chabert and Engeldinger 1956)

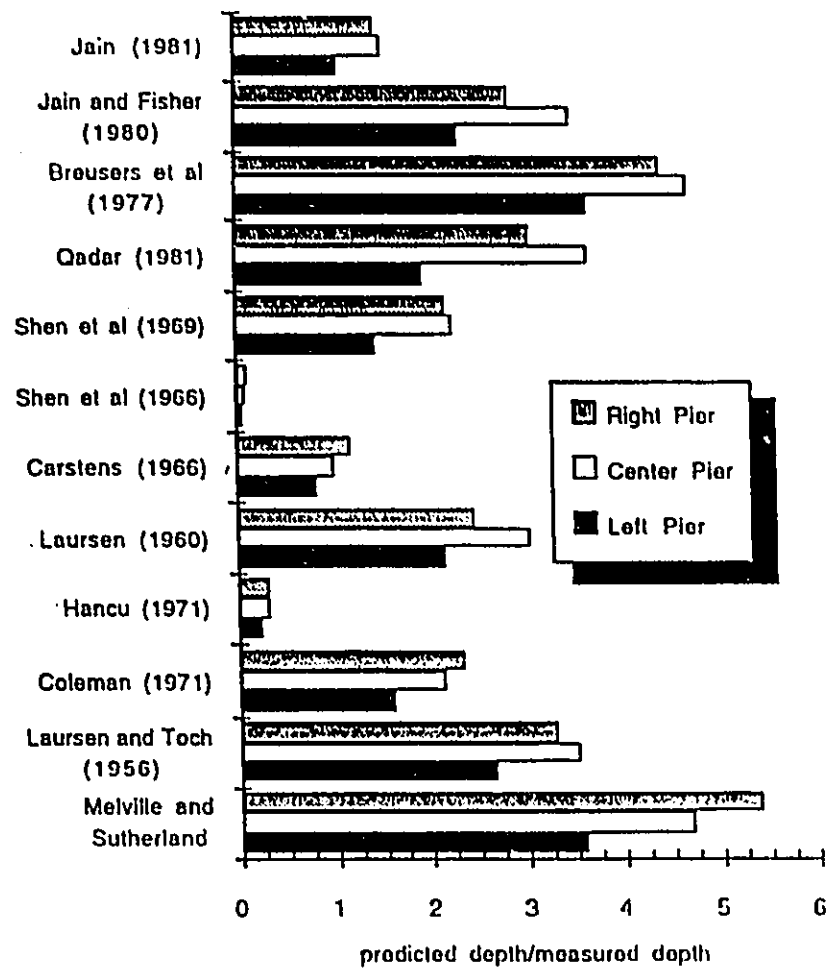


Fig.2-15: Comparison of predicted scour depths (after Ahmed et al. 1993)

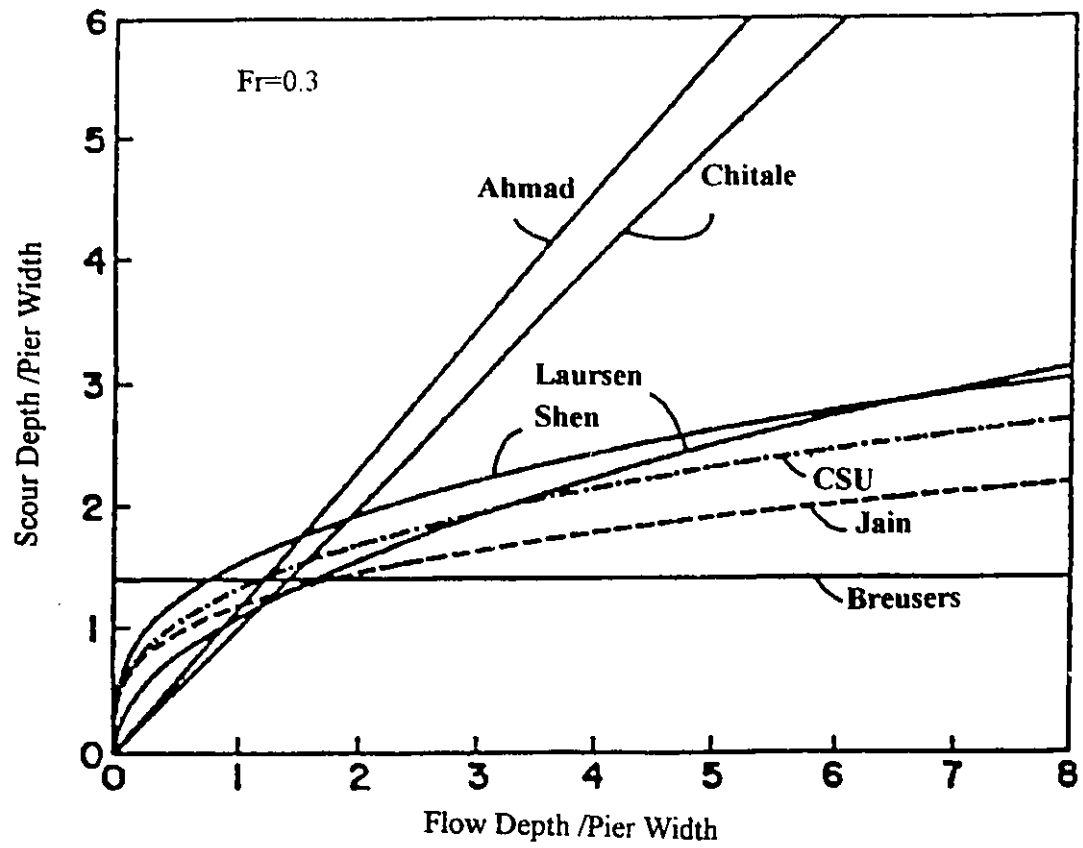
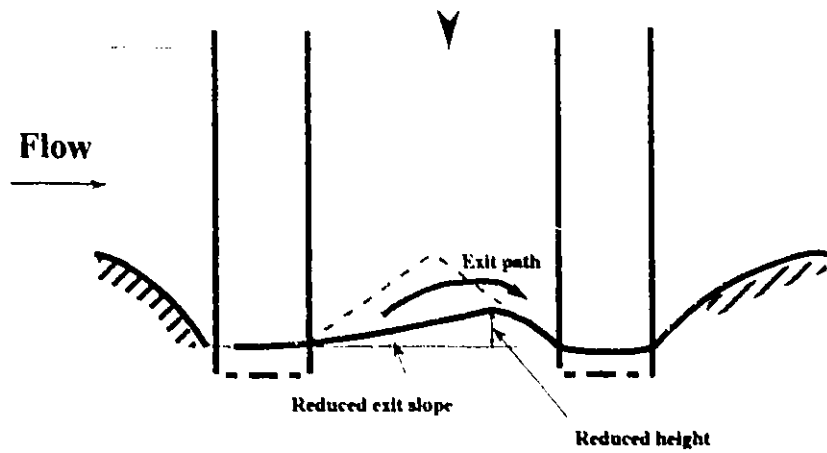
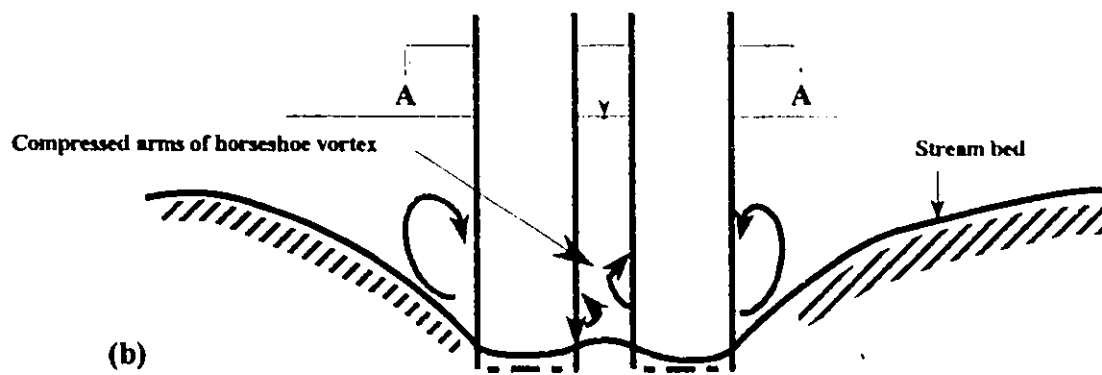
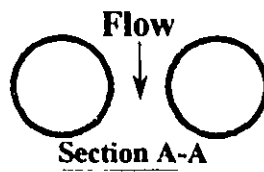


Fig.2-16: Comparison of scour formulas for variable depth ratios (after Jones 1984)



(a)



(b)

Transverse piers

Fig. 2-17: a) Reinforcement effect, b) compressed arms of HSV effect

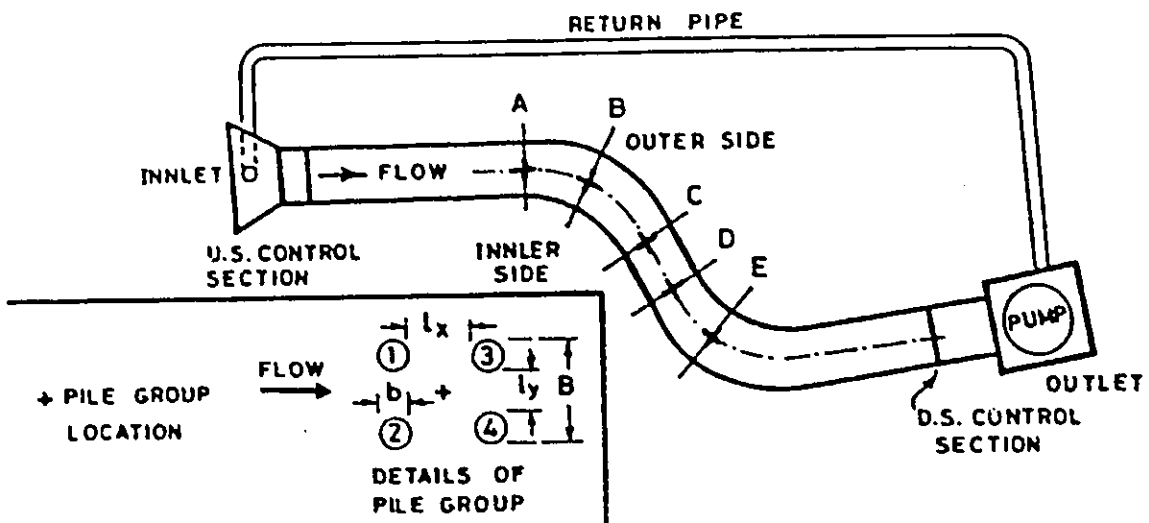


Fig. 2-18: Schematic diagram of Nough's physical model (after Nough 1986)

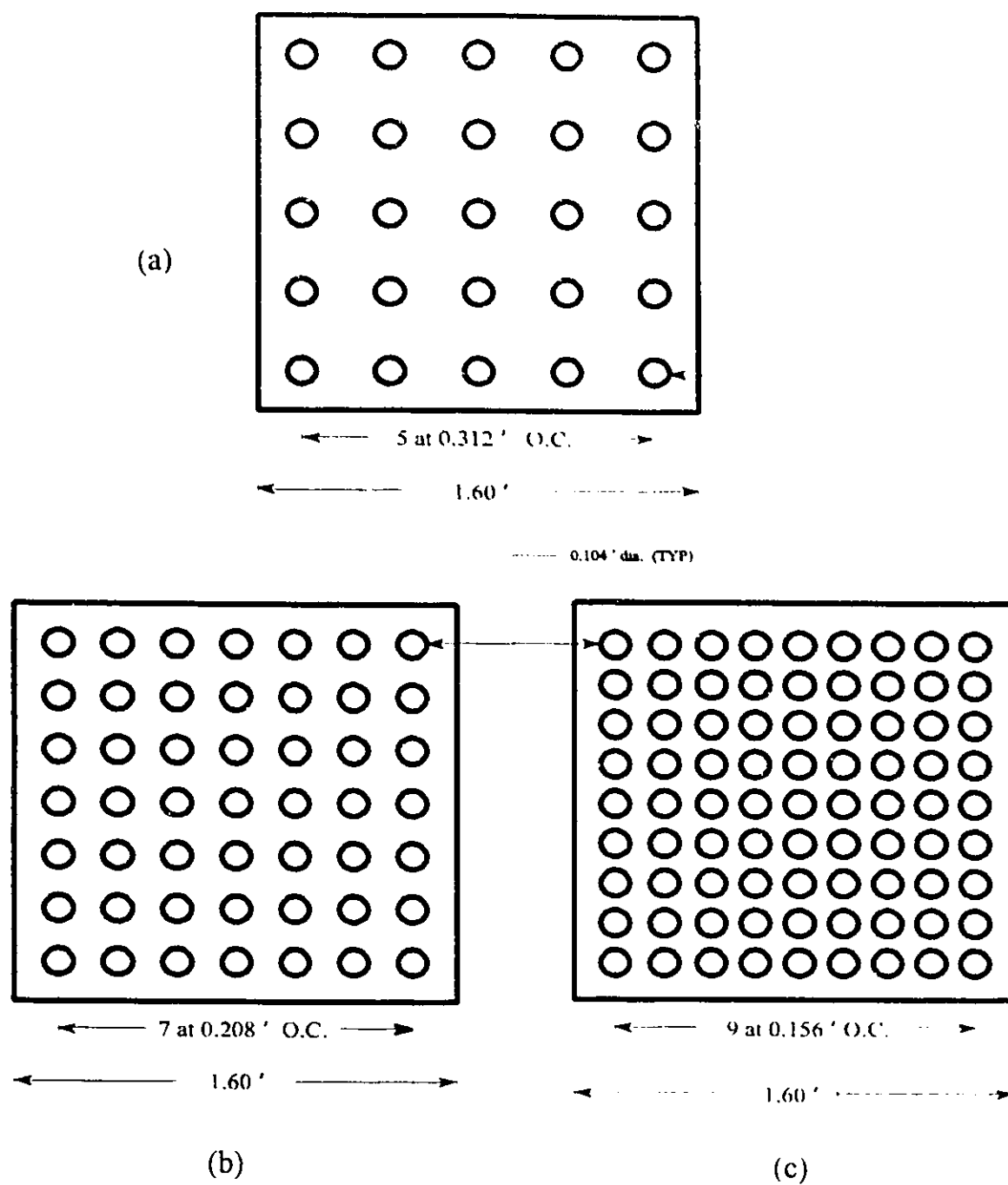


Fig. 2-19: Pile group patterns tested for scour; a) pile group spaced at 3.0 dia. O.C., (center to center), b) pile group spaced at 2.0 dia. O.C., and c) pile group spaced at 1.5 dia. O.C. (after Jones 1989)

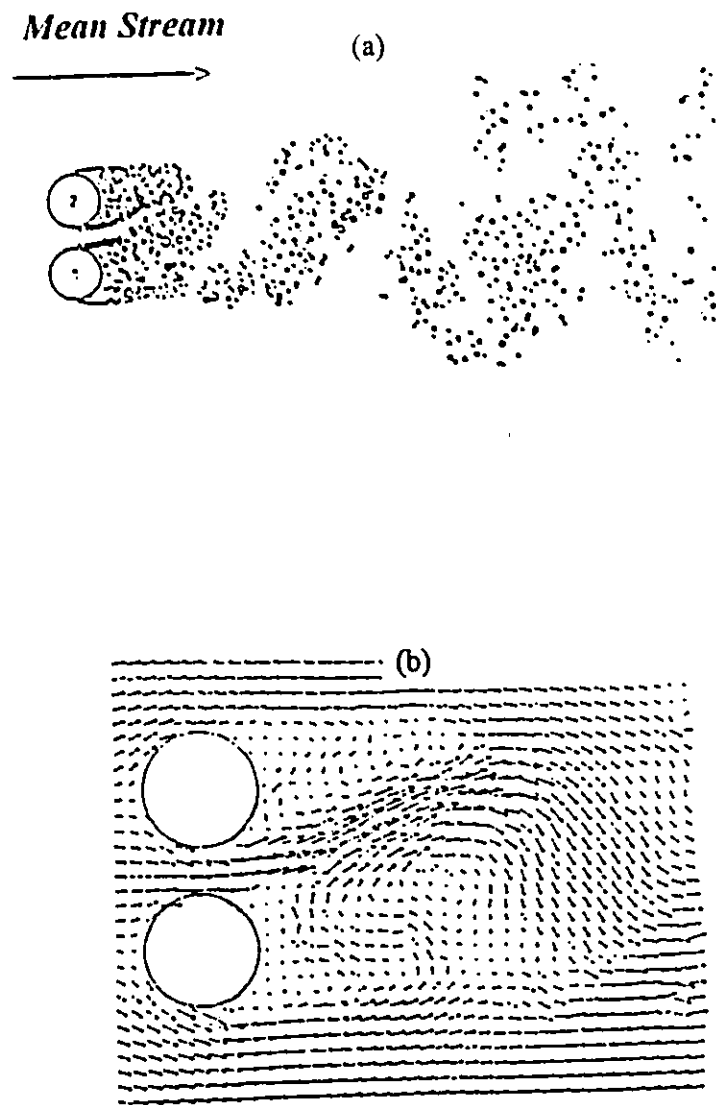


Fig. 2-20: Flow around two cylinders, side by side at $S/d = 1.4$, a) vorticity distribution, b) velocity vectors (after Chan and Lam 1993)

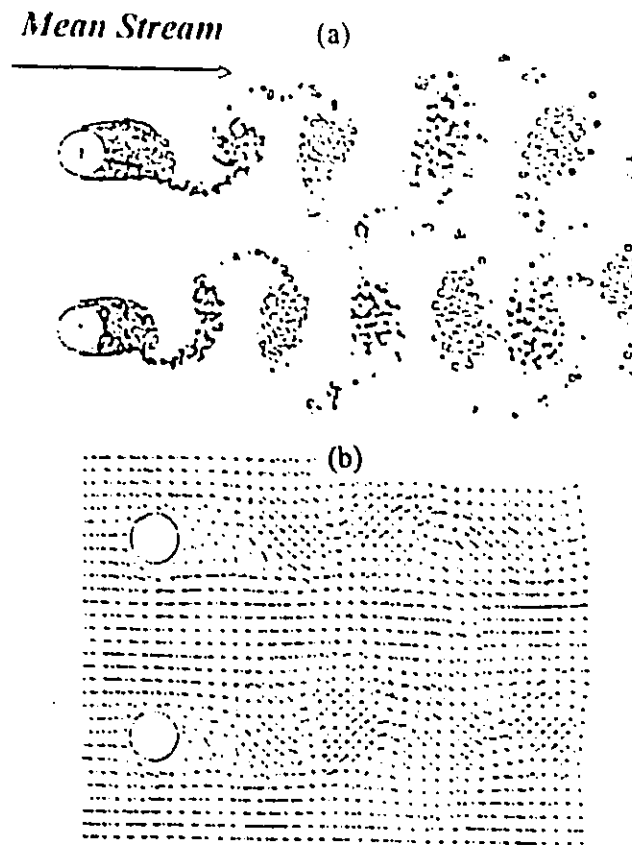


Fig. 2-21: Flow around two cylinders, Side by Side at $S/d = 4.0$, a) vorticity distribution, b) velocity vectors (after Chan and Lam 1993)

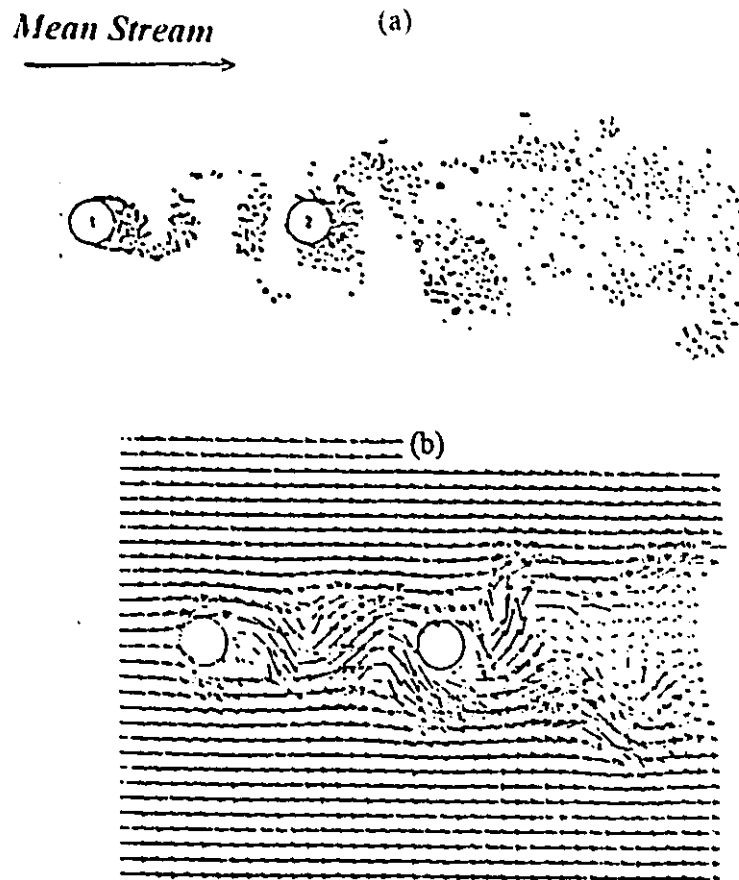


Fig. 2-22: Flow around two cylinders in tandem at $S/d = 5.0$, a) vorticity distribution, b) velocity vectors (after Chan and Lam 1993)

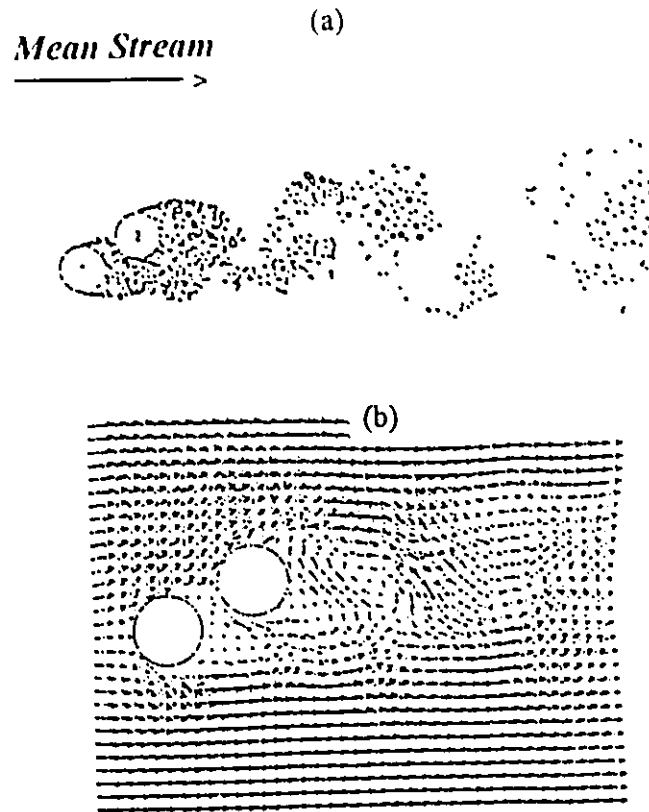


Fig. 2-23: Flow around two staggered cylinders ($\beta=30^\circ$) at $S/d=1.4$. a) vorticity distribution, b) velocity vectors (after Chan and Lam 1993)

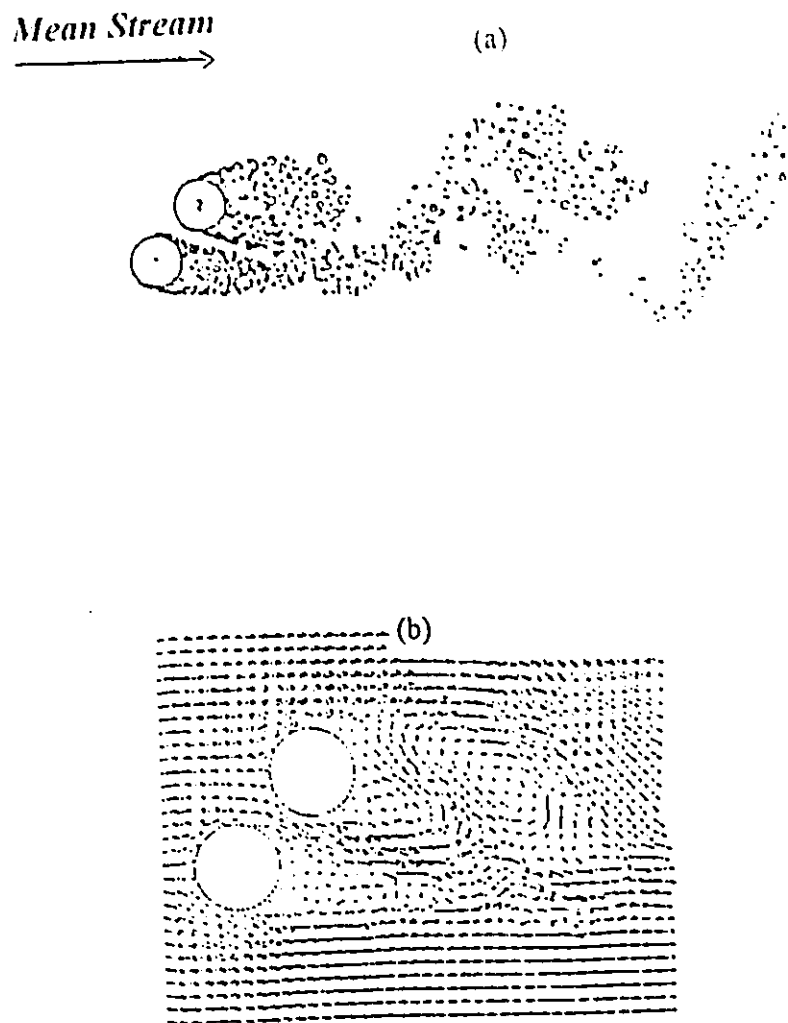


Fig. 2-24: Flow around two staggered cylinders ($\beta=50^\circ$) at $S/d=1.4$. a) vorticity distribution, b) velocity vectors (after Chan and Lam 1993)

Chapter 3

Theoretical Considerations

3-1 Beginning of Sediment Motion

Under clear-water scour conditions, sediments are removed from the scour hole formed around the piers. Sediment transports starts when the boundary shear stress at the bed level exceeds a critical value, τ_c . The *zero transport* condition, which is varying on the condition where transport begins, is generally referred to as the *incipient* or *threshold* condition. The stability analysis of a single particle exposed to the flow gives insight into the sediment transport process in general. The forces

acting on a single particle exposed to flowing water can be classified into two main groups, i.e. disturbing forces and resisting forces. The disturbing forces are applied by the flow and they consist of drag and lift forces. The resisting forces are the submerged weight of the particle and friction by material contacts. The force system acting on an exposed particle is illustrated in Fig. 3-1, in which F_g = submerged weight of the particle, F_L = lift force, F_d = drag force, g = gravitational acceleration, D_{50} = representative sediment size, γ = specific weight of water, γ_s = specific weight of particle, ϕ = bed slope, C_l = form coefficient (if particle is spherical in shape $C_l = \pi/6$), and G = point of support of the particle. The condition of static equilibrium requires balance of forces and moments, i.e.

$$\begin{aligned}\Sigma(\text{disturbing forces}) &= \Sigma(\text{resisting forces}) \text{ and} \\ \Sigma(\text{moments of disturbing forces}) &= \Sigma(\text{moments of resisting forces})\end{aligned}$$

The above principles form the base of most stability analyses carried out by different investigators, such as Vanoni (1964), Shields (1936), and Tison (1953).

White (1940) studied the equilibrium of a particle in laminar flow and defined the critical shear stress as follows:

$$\tau_c = 0.18(\gamma_s - \gamma)D_n \tan\theta \quad (3-1)$$

in which; θ = angle of repose of the submerged sediment and D_n = representative sediment size. For the case of turbulent flow the drag force is as follows:

$$F_d = \tau_0 C_d D_n^2 \quad (3-2)$$

where τ_0 = bed shear stress. Assuming fully turbulent flow $a_1 = a_2$ (Fig. 3-1), and $\tau_c = K(\gamma_s - \gamma)D_n \tan \theta$ with $k = C_1/C_2^{1/2}$ or

$$\frac{\tau_c}{(\gamma_s - \gamma)D_n} = k \tan \theta \quad (3-3)$$

The left-hand side of equation 3-3 is known as Shields' parameter, which is the ratio of drag force to gravitational force. The Shields' parameter is a type of Froude number that is related to grain size and shear velocity. The threshold condition for the beginning of particle motion depends on w , Y_0 , D_n , g , ρ_s , ρ , v , and U_* . A dimensional analysis yields:

$$\frac{\tau_c}{(\gamma_s - \gamma)D_n} = \frac{\rho U_{*c}^2}{\gamma'_s D_n} = f \left(\frac{D_n}{w}, \frac{D_n}{Y_0}, \frac{\rho_s}{\rho}, \frac{U_{*c} D_n}{v} \right) \quad (3-4)$$

in which, $\gamma'_s = \gamma_s - \gamma$, w = channel width, ρ_s = density of the particle, ρ = density of water, and U_{*c} = shear velocity at the threshold condition. The influence of $\frac{D_n}{w}$ and $\frac{D_n}{Y_0}$ on particle equilibrium for fine particles can be neglected. If ρ_s is constant, the influence of $\frac{\rho_s}{\rho}$ can be included in the coefficient in the final equation. Then equation 3-4 may be simplified as:

$$\frac{\rho U_{*c}^2}{\gamma'_s D_n} = f \left(\frac{U_{*c} D_n}{v} \right) \quad (3-5)$$

The left term of this equation is the Froude number expressed in terms of the shear velocity and the second term is the *particle* Reynolds number of the particle. Equation 3-5 generally defines incipient motion.

¹ C_2 is a form coefficient defining the effective surface

Many experiments have been conducted to develop an explicit solution of equation 3-5. Shields (1936) gave a graphical presentation as shown in Fig 3-2 (Shields' diagram) This graph is used extensively in determining the threshold condition as that corresponding to zero bed transport varying on bed transport.

3-2-1 The Lift Force

The magnitude of the lift force is determined by: 1) the local mean velocity and turbulence velocity fluctuations, and 2) a fluctuating hydrostatic pressure that varies in magnitude over the surface of the particle. The total lift force is obtained by adding the vertical component of the turbulent flow to the lift resulting from the inequality of hydrostatic pressure acting on the particle. This force can be expressed as follows:

$$F_L = C_L A_L \rho \frac{U^2}{2} \quad (3-6)$$

in which; C_L = lift coefficient, U = mean velocity, and A_L = characteristic projected area of the particle normal to the direction of flow. F_L has been determined experimentally by many researchers, including: Cheng (1969), Coleman (1969 and 1972), and Aksoy (1973).

Once the lift force is known, it can be introduced in equilibrium studies of submerged solid particles subject to turbulent flow. Therefore, the equilibrium equation for a completely developed turbulent flow is given by:

$$a_1' C_1 (\gamma_s - \gamma) D_n^3 \sin(\theta - \phi) - a_1' C_L C_3 D_n^2 \frac{l'^2 \sin(\theta)}{2} = a_1' C_2 \tau_0 D_n^2 \cos(\theta) \quad (3-7)$$

in which, a_1' = distance between points G and C (Fig. 3-3), and C_3 = form coefficient related to the effective surface area of the particle in the direction of the lift force.

The shear velocity is proportional to the velocity near the bed

$$\tau = \rho U_*^2 \quad (3-8)$$

Assuming

$$U = k U_* \quad (3-9)$$

For critical conditions, equation 3-7 yields with equation 3-8 and 3-9

$$C_1 (\gamma_s - \gamma) D_n^3 \sin(\theta - \phi) - C_L C_4 \tau_c \sin(\theta) = C_2 \tau_c \cos(\theta) \quad (3-10)$$

in which, $C_4 = C_3 k^2 / 2$ and $k = \text{constant}$. Solving for τ_c

$$\tau_c = \frac{C_1 (\gamma_s - \gamma) D_n^3 \sin(\theta - \phi)}{C_L C_4 \sin \theta + C_2 \cos \theta} \quad (3-11)$$

For a horizontal bed $\phi = 0$ and

$$\tau_c = \frac{C_1(\gamma_s - \gamma)D_n}{C_1C_4 + C_2cm\theta} = \frac{(\gamma_s - \gamma)D_n}{\left(\frac{C_1C_4}{C_1}\right) + \frac{C_2}{C_1}cm\theta} \quad (3-12)$$

or

$$\frac{\tau_c}{(\gamma_s - \gamma)D_n} = \frac{1}{\left(\frac{C_1C_4}{C_1}\right) + \frac{C_2}{C_1}cm\theta} \quad (3-13)$$

and

$$\frac{\tau_c}{(\gamma_s - \gamma)D_n} = \frac{1}{k_1 + k_2cm\theta} \quad (3-14)$$

in which; $k_1 = C_1C_4/C_1$ and $k_2 = C_2/C_1$.

For $C_1 > 0$, the value of τ_c is smaller and when $C_1 = 0$, equation 3-13 reduces to equation 3-14.

3-2 Analysis of Flow in the Plane of Symmetry (stagnation plane) ahead of a Cylinder.

Local scouring around bridge piers is caused by the complex fluid-structure-sediment interaction. The size of the HSV that forms around a pier, enlarges and expands with time as the scour hole becomes larger. The strength of the HSV can be theoretically estimated and is related to the local scour depth in front of the pier. The complex behaviour of the flow around a cylindrical pier

is described in this section

Assume a cylindrical pier of infinite extent situated normal to a steady incompressible shear flow over a surface of continuous curvature of very a large radius. The coordinate system is oriented as shown in Fig. 3-3. Thus, the basic equations for the vorticity in the flow field are:

$$\bar{\Omega} = \bar{\nabla} \times \bar{v} \quad (3-15)$$

$$\bar{\Omega} = \eta \bar{i} + \zeta \bar{j} + \xi \bar{k} \quad (3-16)$$

$$\bar{v} = u \bar{i} + v \bar{j} + w \bar{k} \quad (3-17)$$

$$\bar{\nabla} = \frac{\partial}{\partial x} \bar{i} + \frac{\partial}{\partial y} \bar{j} + \frac{\partial}{\partial z} \bar{k} \quad (3-18)$$

in which: $\bar{\Omega}$ = vorticity vector, $\bar{v}$ = velocity vector, η , ζ , and ξ = vorticity components in x, y, and z directions respectively, u, v, and w = velocity components in x, y, and z directions respectively, and $\bar{i}$, $\bar{j}$, and $\bar{k}$ = unit vectors in the x, y, and y directions respectively. The concept of circulation can be defined as:

$$\Gamma = \oint \bar{v} \times d\bar{s} \quad (3-19)$$

Equations 3-15 through 3-19 will provide the basic relationships for the analysis of the secondary phenomena occurring in front of piers in shear flows. One of the most important of these secondary phenomena which forms at the base of bridge piers is the horseshoe vortex system (HSV).

The rotational velocity, and consequently the energy of these vortices, is relatively high. In similar vortices, tangential velocities at the core on the order of the free stream velocity, have been noted by Titchener and Taylor (1956). Moreover, Shen et al. (1966) claimed that the secondary flow, and hence scour, is dependent upon the behaviour of the flow at the boundaries. This boundary behaviour is influenced strongly by the size and strength of the HSV.

Because of the continuity of the vortex filaments forming the core of the HSV, knowledge of the size and the strength of the core on the stagnation plane gives information about the core along its entire length. Consider the control volume ABCD located in the stagnation plane of a cylindrical pier and oriented as shown in Fig. 3-3. By definition, the circulation Γ about ABCD is

$$\Gamma = \oint \bar{v} \cdot d\bar{s} = \int_{ABCD} \bar{\Omega} \cdot d\bar{A} \quad (3-20)$$

in which; dA = an infinitesimal element area of the surface ABCD. By applying the *no-slip* requirement to the flow at the faces AD and CD and specifying that AB be placed far enough upstream so that the only component of the flow at that point is in the x-direction, then, equation 3-20 can be expressed as:

$$\int_B^C u dx = \int_{ABCD} \bar{\Omega} \cdot d\bar{A} \quad (3-21)$$

The above simplification is possible despite the shape of the bed. Therefore, equation 3-21 is valid even for beds with large scour holes.

The left-hand side of equation 3-21 can be evaluated provided a suitable expression for u along the face BC can be obtained. If it is assumed that BC is outside the region of velocity variation with y (the shear layer) and far from the free surface, $u(x)$ can be accurately predicted from perfect fluid theory. If " a^* " is the half width of the pier, then $u(x)$ for a cylindrical pier can be expressed as:

$$u(x) = U_{\infty} \left(1 - \frac{a^{*2}}{(a^* + x)^2} \right) \quad (3-22)$$

Substituting equation 3-22 into equation 3-21 and integrating the expression gives:

$$\int_{ABCD} \bar{\Omega} \cdot d\bar{A} = U_{\infty} X_0^* - \frac{a^* X_0^*}{a^* + X_0^*} U_{\infty} \quad (3-23)$$

in which; X_0^* = distance from the pier to face AB, and U_{∞} = undisturbed approach velocity. It is important to note that the initial term of equation 3-23 is the circulation which would exist about a hypothetical control volume ABCD if the pier was not present. Therefore,

$$\Delta \Gamma_{(due\ to\ presence\ of\ pier)} = - \frac{a^* X_0^*}{a^* + X_0^*} U_{\infty} \quad (3-24)$$

If the original restriction on the location of AB is strengthened to specify that $X_0^* \gg a^*$, equation 3-24 becomes, in the limit:

$$\Delta \Gamma_{(due\ to\ presence\ of\ pier)} \approx -a^* U_{\infty} \quad (3-25)$$

The HSV can be considered to possess a core which rotates as a rigid body. The tangential velocity in the core is, of course, proportional to the radius and the angular velocity constant at a value ω . Such behaviour occurs frequently in nature. Assuming that $\Delta\Gamma$ is proportional to the strength of the core, then

$$(\omega A)_{core} \propto (a \cdot U_m) \quad (3-26)$$

Dividing both sides by the kinematic viscosity and using the pier width

$$\left(\frac{\omega A}{\nu}\right)_{core} \propto \left(\frac{2a \cdot U_m}{\nu}\right) = Re \quad (3-27)$$

Therefore, the non-dimensional vortex strength of the HSV was the hypothesis presented above. If this hypothesis is true, then we see from equation 3-27 that the maximum scour depth should be a function of Re number with other factors being constant.

$$(D_s)_{max} \propto Re \quad (3-28)$$

When there is general sediment movement into the scour hole, the maximum scour depth occurs when a balance is reached between the rate of sediment transport into the hole and the transport capability of the vortex.

Equation 3-25 indicates that the presence of the pier actually decreases the amount of vorticity present in ABCD. The concentration of vorticity in the core of the HSV implies that the pier acts as a vorticity concentrating device. It also implies the existence of finite regions of zero or greatly

reduced vorticity within the control volume. This implication is proved by observations of dead water regions in the flow.

Moreover, the decrease in vorticity due to the presence of the pier also serves qualitatively to indicate that the distribution and not the magnitude of the vorticity is the important factor in secondary flow phenomena. Therefore, excluding protective structures, reductions in the adverse effects of secondary flow phenomena can only be accomplished through redistribution of the vorticity. In this case, this means that the formation of the HSV system would have to be prevented or at least the strength reduced.

3-3 Dimensional Analysis

Many practical engineering problems in fluid mechanics cannot be solved by analytical procedures alone. In these instances engineers must also rely on interpretation and judgement, based on experimental observations and experience respectively. Dimensional analysis is powerful tool in formulating problems which defy analytical solution and must be solved experimentally. The methods of dimensional analysis are based on Fourier's *principle of dimensional homogeneity* (1882)¹, which states that an equation expressing a physical relationship between quantities must be dimensionally homogeneous. This principle affords the means of ascertaining the forms of physical equations from knowledge of the relevant variables and their dimensions.

¹ See Street et al. (1996)

Elsewhere, it has been shown that the local scour depth for a single cylindrical bridge pier is a function of the following parameters:

$$D_s = \Phi(\rho, \nu, g, D_{50}, \rho_s, Y_0, \bar{U}, d) \quad (3-29)$$

applying dimensional analysis one obtains:

$$\frac{D_s}{d} = \Phi\left(\frac{U_* D_{50}}{\nu}, \frac{Y_0}{d}, \frac{D_{50}}{d}, \frac{U_*^2}{\Delta g D_{50}}\right) \quad \text{or} \quad \frac{D_s}{d} = \Phi\left(\frac{U_*}{U_{*c}}, \frac{Y_0}{d}, \frac{D_{50}}{D}\right) \quad (3-30)$$

The influence of these dimensionless parameters on local scour around a single bridge pier have been studied extensively by others, including Laursen and Toch (1956), Chabert and Engeldinger (1956), Neill (1965), Hanco (1971), Breusers et al. (1977), and Richardson et al. (1991). However, in the case of pier groups, pier spacing and flow angle of attack are considered to be more important parameters. Representing pier spacing and flow angle of attack by S and β respectively, the maximum scour depth for a pier group may be expressed as:

$$D_g = \Phi(\rho, \rho_s, \nu, g, D_{50}, Y_0, \bar{U}, d, S, \beta) \quad (3-31)$$

applying dimensional analysis, this becomes:

$$D_g = \Phi\left(\frac{U_*}{U_{*c}}, \frac{Y_0}{d}, \frac{D_{50}}{d}, \beta, \frac{S}{d}\right) \quad (3-32)$$

For the conditions applied in this study, the first three dimensionless quantities can be shown

to be less important than the last two. Accordingly, the following basic relationship is proposed for local scour associated with a group of cylindrical bridge piers:

$$\frac{D_s}{D_s} = a \beta^b \left(\frac{S}{d} \right)^c \quad (3-34)$$

Parameters a, b, and c for different pier group arrangements can be found using a linear multiple regression model as described in the following section.

3-4 Application of Multiple Linear Regression¹ on Bridge Pier Group Data

Logarithmic transformation of variables provides a linear relation. The logarithm of the ratio of $\frac{D_s}{D_s} = D'_s$ is regressed against a combination of the two independent variables, i.e., β and S/d .

A general form of the prediction equation written in terms of untransformed terms is:

$$\frac{\hat{D}_s}{D_s} = a \beta^b (S/d)^c \quad (3-63)$$

in which $\hat{D}_s$ = scour depth for any pier in a group, $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3]$, $a = \exp(\hat{\theta}_1)$, $b = \hat{\theta}_2$, and $c = \hat{\theta}_3$.

¹ See Appendix I

3-4-1 Predicting Scour Depths

Normalized scour depths predicted by equation 3-63 are the expected response to a specified set of regressors, and are not known with certainty. However, a local scour depth that has a small probability of being exceeded needs to be used for designing pier foundations to insure against the failure of a bridge. A probability statement about the distribution of $\ln(\frac{D_s}{D_s})$ provides a means of evaluating appropriate design values.

The transpose of the vector of predictor variables at a particular site is:

$$x_0 = [1, \ln(\beta), \ln(\frac{S}{D})] \quad (3-64)$$

A 100(1- η) percent upper confidence limit for $\ln(\frac{D_s}{D_s})$ is computed as follows (Montgomery and Peck 1982):

$$\ln(\frac{\hat{D}_s}{D_s}) + t_{\eta, n-p} \sqrt{\text{Var}(X_0 \theta)} \quad (3-65)$$

Where $\ln(\frac{\hat{D}_s}{D_s}) = X_0' \hat{\theta}$ is the predicted value of $\ln(\frac{D_s}{D_s})$. The values of $\hat{\theta}'$ are the transpose of the vector of regression model parameters, $t_{\eta, n-p}$ is the one-sided t-distribution statistic corresponding to an exceedence probability η and $n-p$ degrees of freedom.

The estimated variance of prediction is:

$$\text{Var}(x_0 \hat{\theta}) = \hat{\sigma}^2 [1 + x_0' (X'X)^{-1} x_0] \quad (3-66)$$

in which, σ^2 = the residual mean square error (estimated model error).

One-sided t-distribution statistics for $n-p$, degrees of freedom and several confidence levels are given in Table 5-2-2. An upper estimate of $\ln(\frac{D_s}{D_c})$ is obtained from equation 3-65 by selecting an appropriate confidence level, and obtaining the corresponding t statistic. Use of an upper limit of scour depth for design of pier foundations will provide a factor of safety needed to insure against failure of the structure.

3-5 Summary

The sediment removal from the scour hole, formed around a cylindrical model bridge pier, was studied using stability analysis for a single particle exposed to the flow. A threshold condition was defined based on Shields parameter for the beginning of particle motion. The behavior of the flow in the stagnation plane of a cylindrical pier was studied using the control volume method. It was emphasized that the HSV is one of the most important secondary phenomena which forms at the base of cylindrical piers. The HSV was considered to possess a core which rotates as a rigid body. The tangential velocity in the core is proportional to the radius and the angular velocity constant at a value ω . A non-dimensional vortex strength of the HSV was found to be dependent on R_c . The maximum scour depth was found to be a function of HSV as well. From equation 3-25, it was concluded that the presence of the pier decreases the amount of vorticity present in the stagnation plane of a cylindrical pier. In addition, the distribution and not the magnitude of the vorticity was the important factor in secondary flow phenomena.

Using dimensional analysis, the maximum scour depth was found to be dependent on several parameters, among them: pier spacing, S , and flow angle of attack, β , which are important in the case of pier groups. The dimensionless ratio, $\frac{D_g}{D_s}$, was also suggested to be a function of β and S/d (equation 3-34). Unknown parameters a , b , and c for different pier group arrangements can be found using a linear multiple regression model. The value of D_{s0} is obtained using any appropriate equation, developed for a single pier.

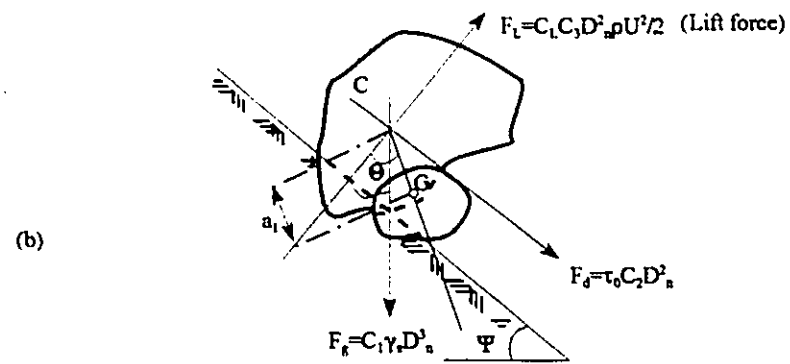
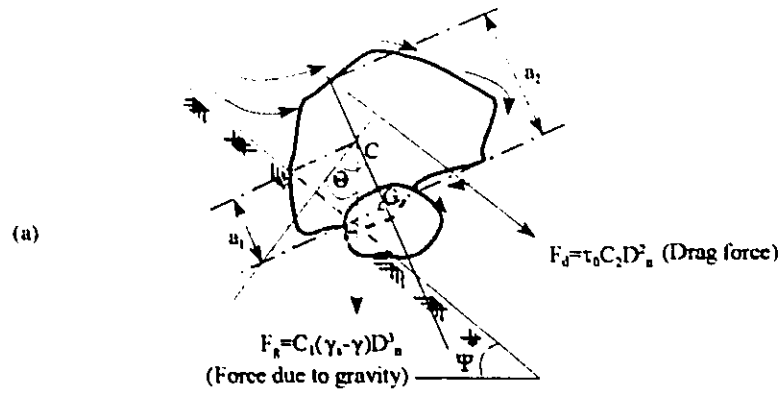


Fig. 3-1: Flow around a particle at rest; a) laminar flow; b) turbulent flow

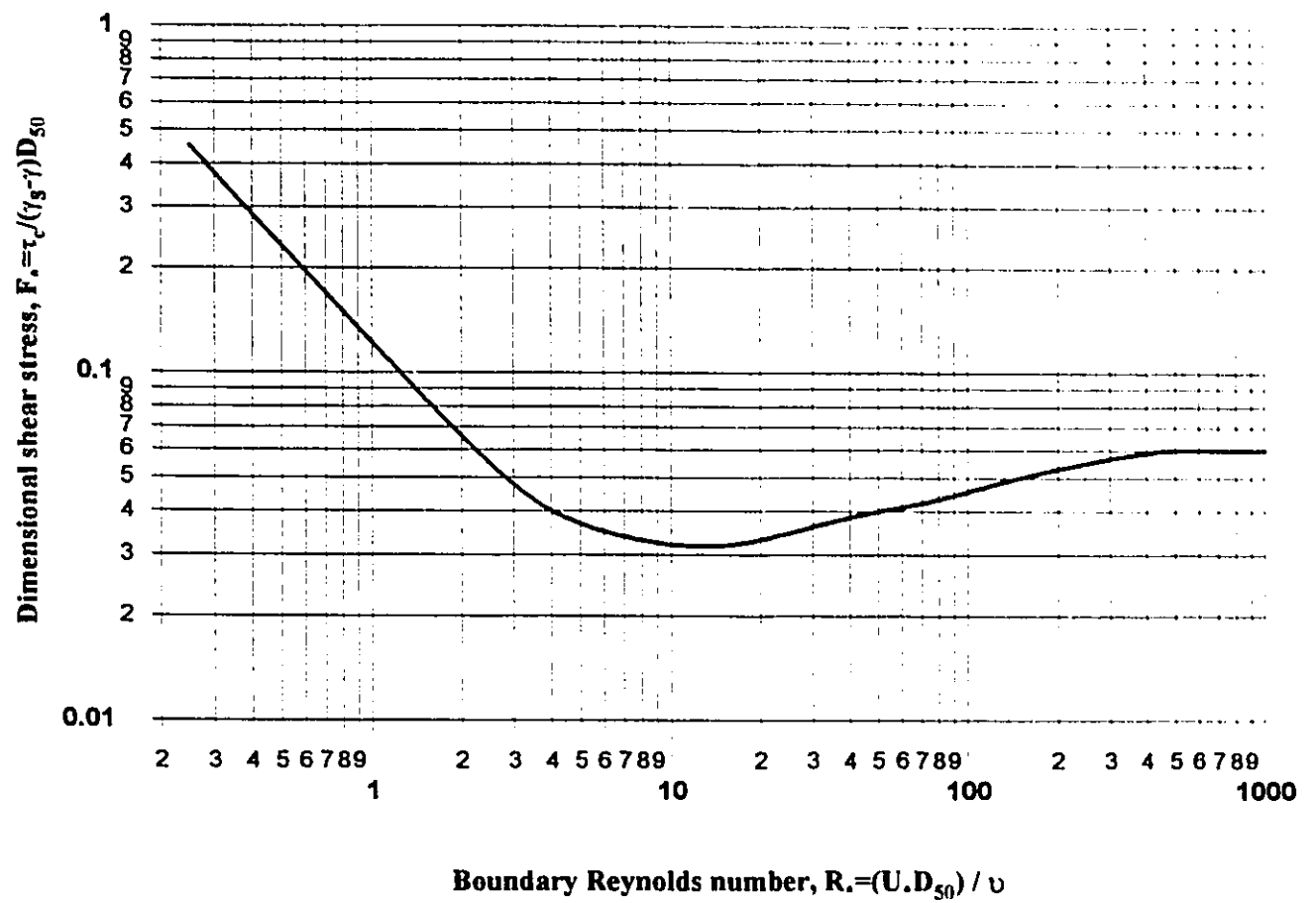


Fig. 3-2: Shields' (1936) diagram; dimensionless critical shear stress

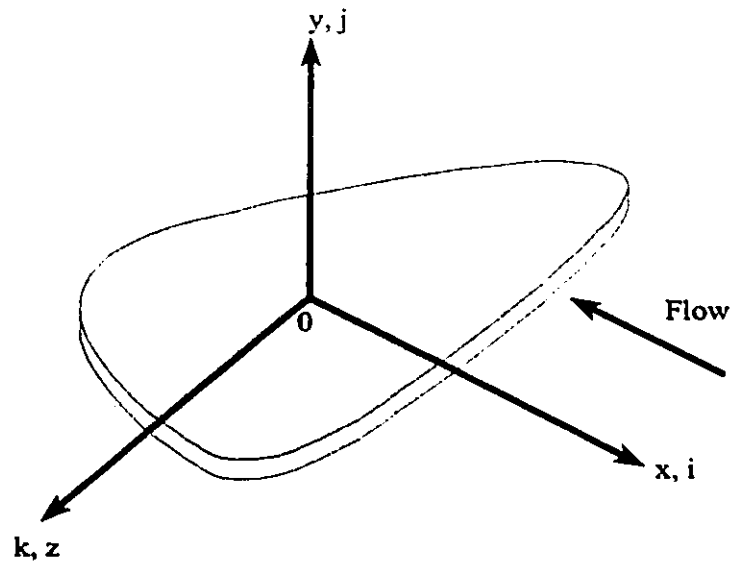


Fig. 3-3: Coordinate system orientation and flow direction

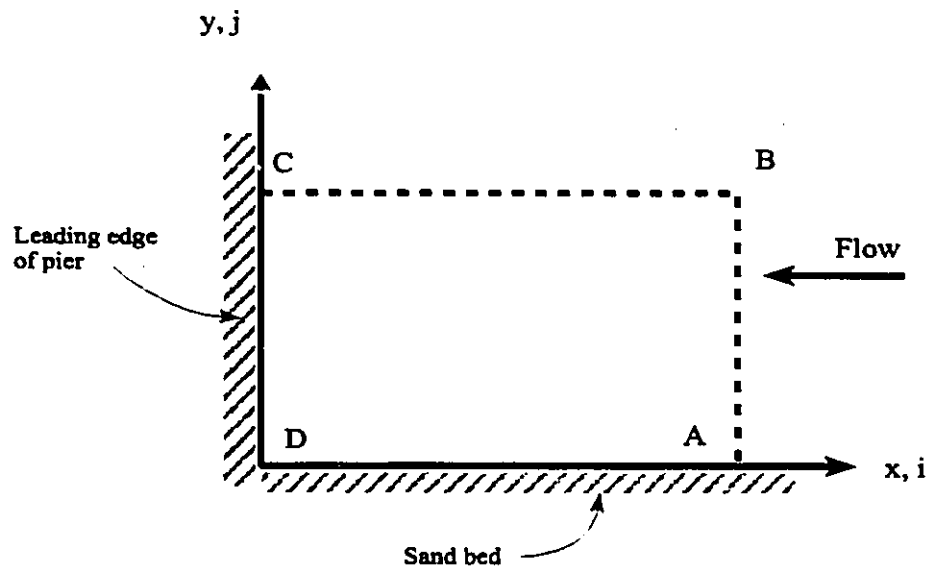


Fig. 3-4: Control volume in the stagnation plane

Chapter 4

Experimental Equipment and Procedures

4-1 The Flume Test Section

The scour experiments were performed in a 7.7 m long $\times$ 0.5 m wide $\times$ 0.6 m deep aluminum flume having a sand-bed test section which was 3m long $\times$ 0.2 m deep (Plate 4-1). An entry ramp, artificially faced with a layer of sand similar to the sand contained in the test section was installed upstream of the test section to provide a gradual transition to the test section. Both entry and exit section of the test section were found to scour locally due to *entry* turbulence and high local velocities which developed at the time of flume draining. To prevent this local scour, fixed plates, with bed material glued to them, were placed in the initial and final 30.0 mm of the test section. Also, a sand trap was installed downstream of the test section to collect scoured sediments before they entered the

reservoir tank (Figs 4-1 and 4-2a & b) The slope of the flume for all tests was set at $S_0 = 0.0006$. The small slope was used so that a wide range of sub-threshold velocities could be utilized in the testing program

4-2 The Sediment Test Bed

The test section, was composed of uniformly graded *play* sand. The *play* sand was selected following sieve analyses on a variety of sand samples from different sources. The result of the sieve analysis on the selected sand is shown in Fig. 4-3. A measure of the uniformity of the sand can be obtained by estimating the geometric standard deviation defined as follows:

$$\sigma_g = \left(\frac{D_{85}}{D_{16}} \right)^{0.5} \quad (4-1)$$

Referring to Fig. 4-3, the values of D_{16} and D_{85} are obtained as 0.33 and 0.66, respectively. Therefore, using equation 4-1, σ_g was found to be 1.4. This standard deviation value is less than 1.5, which is the limiting value for the uniform sediments (Chiew 1984). Kwan (1988) also claimed that, “At low values of σ_g (<1.5) no armouring occurs and the ripples which generate some sediment transport replenish the scour hole and so produce a lower scour depth. At higher values ($1.5 < \sigma_g < 1.7$), some weak armouring occurs but this only stabilises the bed and reduces ripple formation and sediment transport such that a larger scour depth is produced. Even at higher values of σ_g (>1.7) (non uniform region) armouring is significant on the bed and within the scour hole, the ripple sand behaves like non-ripple sand and the scour depth is decreased accordingly”.

On the other hand, a review of the published experimental data indicated that a bed sediment with $\sigma_g < 1.5$ may be considered as being virtually of a uniform particle size. However, σ_g for the bed sediment used in the experiments is within the 1.5 limit and thus it can be considered as being uniform bed sediment. Shield's diagram (Simons and Senturk 1997) was used to calculate the critical shear velocity U_{*c} ($=0.018$ m/s) for $D_{s0}=0.49$ mm and U_{*c} is used to compute the critical shear velocity, U_{*c} . Also, to plot the log-law profile at any cross section, it is necessary to find U_{*c} . In addition, the ratio $\frac{U}{U_{*c}}$, which is an indication of the local scour condition, can then be calculated.

The required slope of the test bed was achieved by moving a levelling screw, installed on top of flume walls, across the test section. The slope of the test bed was set to $S_0=0.0006$. The specific gravity of the sediment was $\rho_s=2.65$ t/m³, with an angle of repose $\phi=32^\circ$.

4-3 Velocity and Discharge Measurements

The flow depths applied in this study were selected to be more than 5 cm and diameters of the model piers was 1.27 cm. Since $5 \div 1.27 \approx 4$ and $d/D_{s0}=26$, as discussed earlier under subsection 2-5-2, according to Chiew (1984), Chee (1982) and Ettema (1980), there is no influence of water depth on the local scouring.

The water supply was provided by the laboratory pumping system and flow through the flume was re-circulated. Two screens were installed upstream of the flow measurement device to eliminate large scale turbulence in the entering flow. Discharges were measured using a 90° V-Notch weir.

After flowing over the weir, the water passed through screens (perforated steel plates) before entering the test section. Flow depth and sediment scour profiles were measured using a vernier point gauge suspended from an instrument carriage which travelled on parallel steel rails bolted to the top of the flume walls. The downstream end of the flume has an adjustable gate, which was used to control the water surface profile through the test section. The flow depths at the upstream and downstream ends of the test section were kept the same, within an error of ± 1 mm.

The velocity of the approach flow was measured using a 4-mm diameter Pitot-static tube (Plate 4-2) and a miniature propeller flow meter (Plate 4-3). The mini-propeller meter was the model 403 developed by Nixon instrumentation LTD, England, which was designed to record water flow velocities between 2.5 and 150 cm/s.

The sensing probe is a measuring head joined by a slim tube to the plug and socket which connects to an analogue indicator (model 401). The measuring head comprises a five bladed PVC rotor mounted on a hard stainless steel spindle, itself terminating in fine burnished conical pivots which run in jewels mounted in a shrouded frame. Minimum frictional resistance is thus ensured. An insulated gold wire contained within the support tube terminates 0.1 mm from the rotor blade tips. When the rotor is revolved by the rotor blades the gold wire tip slightly varies the measurable impedance between the tip and the tube. This variation is used to modulate a 15-K Hz carrier signal, generated within the indicating instrument, which in turn is applied to the electronic detector circuits. Following amplification and filtering out of the carrier frequency a square wave signal is obtained. In the analogue indicator, this is used to drive a diode pump integrator, hence obtaining a current

signal proportional to the rate at which the rotor turns.

4-4 Local Scour Hole Measurements

Engels (1929), Timonoff (1929), Keutner (1932), Tison (1937 and 1940), Inglis et al. (1939), Yarnell and Nagler (1931), Kain (1961), and Moore and Masch (1965) found, for both prototype and model tests, that the maximum scour depth occurs near the upstream nose of the pier. This observation was also confirmed in our study which measured scour depth on the plane of symmetry at a distance 2-5 mm upstream from single model piers throughout the tests. In the case of pier groups, minimum scour depths, D_{sm} , were measured at locations between any pair of piers. The positive or negative value of minimum scour depth is an indication of either scouring or deposition of sediments between any pair of piers in line. In addition, a separate scour hole is formed around the piers when $D_{sm} \leq 0$.

The dimensions of scour holes around single piers or groups of piers were measured in X and Y directions as shown in Fig. 4-4. The parameter w , is the width of the deposited materials. The distances between each pier and the perimeter of its scour hole (r_1 , r_2 , r_3 , and r_4) were measured in different directions. For the pier groups, in some cases the parameters may not be measurable if a separate hole is not formed, due to the small pier spacing. The length and width of the scour hole are X and Y, respectively.

Minimum scour depths were measured between the piers in groups of 2, 3, 4, and 6 piers.

Fig. 4-5 shows the plan and side view of 4- and 6-PGs. Measurements for 2- and 3-PGs are the same as for 4- and 6-PGs, respectively [the parameter $D_{sm(1-2)}$, stands for the minimum scour depth between piers number 1 and 2. A negative value for $D_{sm(1-2)}$, means deposition of sediments downstream of pier number 1, at the point of measurement].

4-5 Shape of Scour Hole

The general shape of a scour hole formed around a circular pier follows the shape of a frustum of an inverted cone. The angle the scour hole makes with the horizontal is roughly equal to the angle of repose. Plate (4-4) illustrates typical scour hole shapes observed for the circular cylindrical piers of our study. Other investigators, including Shen et al. (1966), reported similar observations in their studies.

4-6 Test Procedure

After setting the required test bed slope, the model piers were installed vertically into the sand bed. For pier groups, the required number of holes were made with different spacings on a plexiglass template (Fig. 4-6). Then, the model piers were inserted through the holes at the desired spacing. To align the template for a specific flow angle of attack, the vernier point gauge was traced along the line of the desired angle in respect to the line crossing the centers of piers.

Before supplying water to the flume, the downstream tailgate was raised so that initial depth of the flow in the flume would exceed the design flow depth. Water was then introduced to the flume

at a very slow rate (so as not to disturb the sand bed). The tailgate was then carefully lowered (and the flow rate slowly increased) until the design conditions were achieved. At this point in time, the test was started; the time between the initial lowering of the tailgate and establishing the design condition took about a minute for each test. Local scouring would start a few seconds before the design depth was achieved. At the end of the test period the tailgate was raised again until scouring ceased. The flow supply to the flume was then stopped by closing the control valve.

Maximum scour depth was measured by vernier point gauge. For the tests which were monitored in time, the point gauge was inserted into the flow upstream of the pier at a distance 2 to 5 mm from the structure. For the remaining tests, measurements were generally taken at the end of the test and after the system was drained. However, measurements were occasionally taken before and after draining the water to check that there was no significant difference between the two readings.

4-7 Calibration of the Flume

The case of turbulent pipe flow has been thoroughly investigated in the past and the results arrived at are also important in regards to flow in open channels. Velocity distributions at cross-sections along an open channel vary with distance from the initial cross-section. In sections close to the entrance the velocity distribution is nearly uniform. Further downstream the velocity distribution changes, due to the influence of friction, until a fully-developed velocity profile is attained at a given cross-section and remains constant thereafter.

The main objective of the flume calibration test was to verify the condition of fully-developed turbulent flow in the test section. The turbulent boundary layer thickness along a smooth flat plate is given as, (Schlichting (1979)):

$$\delta(x)=0.37x\left(\frac{Ux}{\nu}\right)^{-0.2} \quad (4-2)$$

in which; x = the longitudinal distance, U = free stream velocity, and ν = kinematic viscosity ($=1.1 \times 10^{-6}$ m²/s). The free velocity at section $x=2.0$ m into the test section was measured at 0.28 m/s. Substituting these values of x and U in equation 3-1, the boundary layer thickness at this point in the test section becomes $\delta(x)=0.054$ m, which is equal to the mean flow depth $Y_0=0.054$ m. Thus, the turbulent boundary layer extends across the entire flow depth over the last 1.0 m length of the test section.

Several additional experiments were carried out to observe the development of the boundary layer through the test section. The velocity profiles were plotted for different cross sections, starting from the upstream (entrance) section and extending to the downstream (exit) section. The test section was divided into seven sections, including the area where model pier groups were to be installed.

The velocity distribution at each cross section was measured using the miniature propeller flow meter. The data acquisition system was a 486-computer and a software program designed for this purpose. The program analysed the signals received and processed them as velocities at each cross section in terms of coordinates X and Z . Using a spreadsheet, the processed data were utilized

to plot the development of the boundary layer, see Fig. 4-7. The distances between sections 1, 3, 5, 6, 7, 8, and 9 and the entrance of the test section were 0.5, 0.9, 1.2, 1.5, 1.8, 2.1, and 2.4 m respectively. Fig. 4-7 shows that, except at section 1 (which was very close to the test section entrance), the turbulent boundary layer extends across the entire flow depth.

Schlichting (1968) presents the logarithmic law for the velocity distribution as:

$$\frac{u}{u_*} = \frac{1}{\kappa} \ln \frac{y}{K_s} + B \quad (4-3)$$

in which; u = mean velocity, u_* = shear velocity, y = distance from the wall, K_s = height of grain for equivalent sand roughness, B = a function of the roughness Reynolds number $\frac{u_* K_s}{\nu}$, and κ = Von Karman's coefficient (0.4). The value of B for the present study was found equal to 8. The following equation fitted well through the data obtained for test section 6.

$$\frac{u}{u_*} = 5.75 \log \left(\frac{y}{K_s} \right) + 8 \quad (4-4)$$

The velocity profiles for section 6, in which most of the tests were carried out, is depicted in Fig. 4-8.

4-8 Summary:

The effect of water depth was minimised to a large extent in this study by selecting water

depth such that $Y_0/d > 4$ and $d/D_{50} > 20$. The effect of sediment size was considered insignificant, since the ratio of the pier diameter to the grain size is greater than 20-25, (i.e. $d/D_{50} > 20-25$). Accepting this limiting value (and considering the median grain size as 0.48 mm.), the smallest pier diameter could be as low as 1 cm. Given that the flume width was 0.515 m and groups of up to six pier were the limiting factor in choosing pier diameter, cylindrical piers having diameter $d = 0.0127$ m were used in the present study ($d/D_{50} = 26.5$ or $12.7 \div 0.48 = 26.5$), which is above the limiting value recommended by Ettema (1980). However, if the flow depth and/or sediment size exceed the accepted limiting values, their effects on local scour are documented. To account for their effects on local scour one must apply appropriate correction factors to the observed data accordingly.

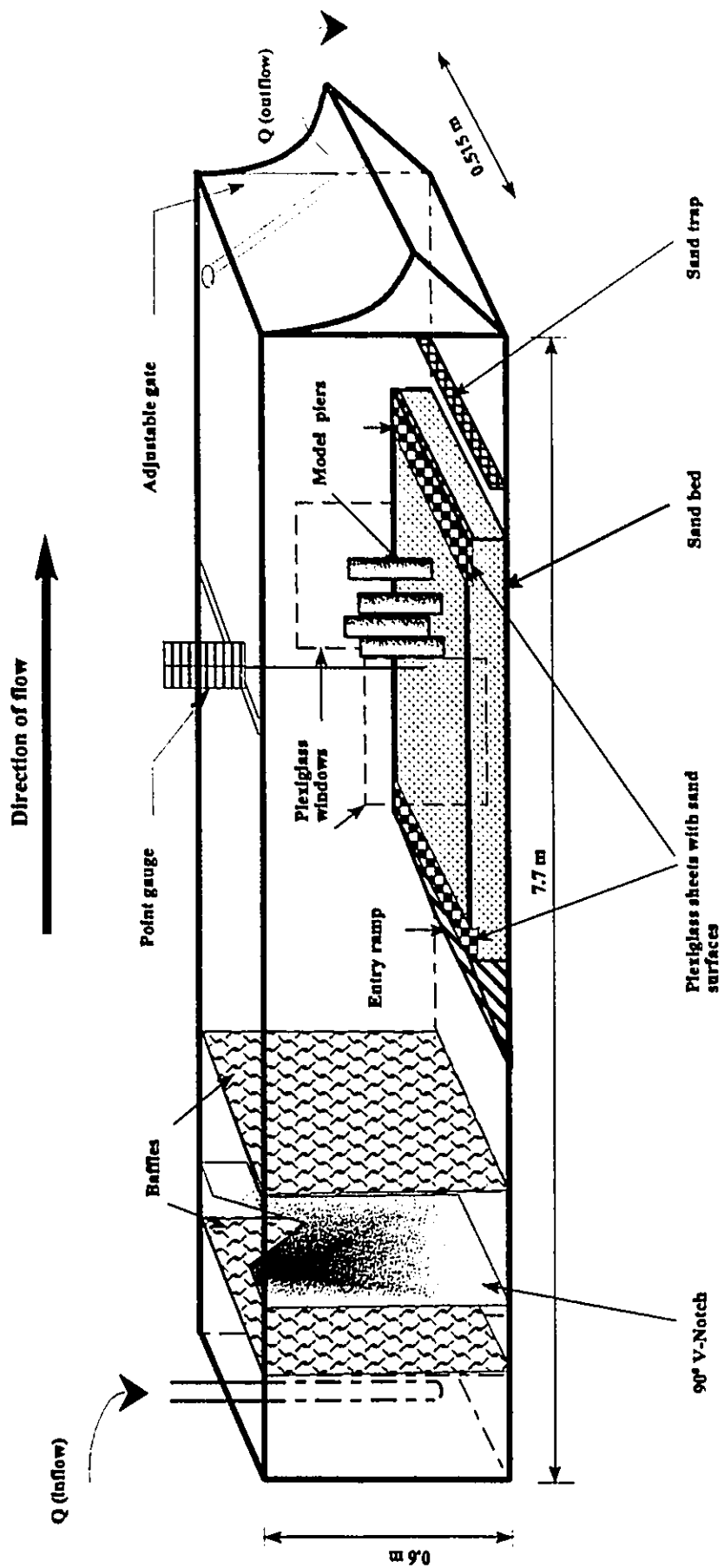


Fig. 4-1: Schematic layout of the laboratory flume

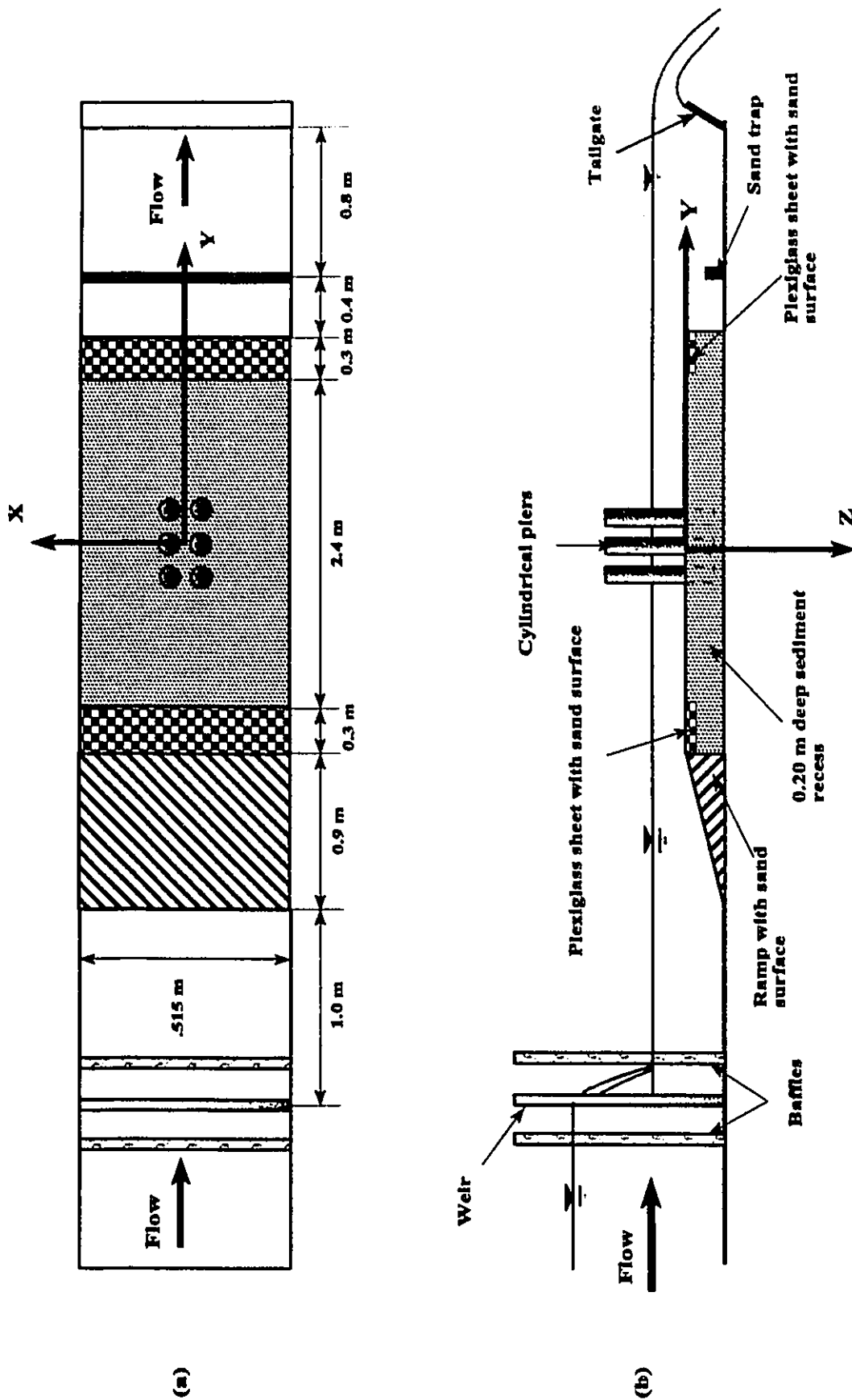


Fig. 4-2: Experimental setup; a) plan view, b) side view, (not to scale)

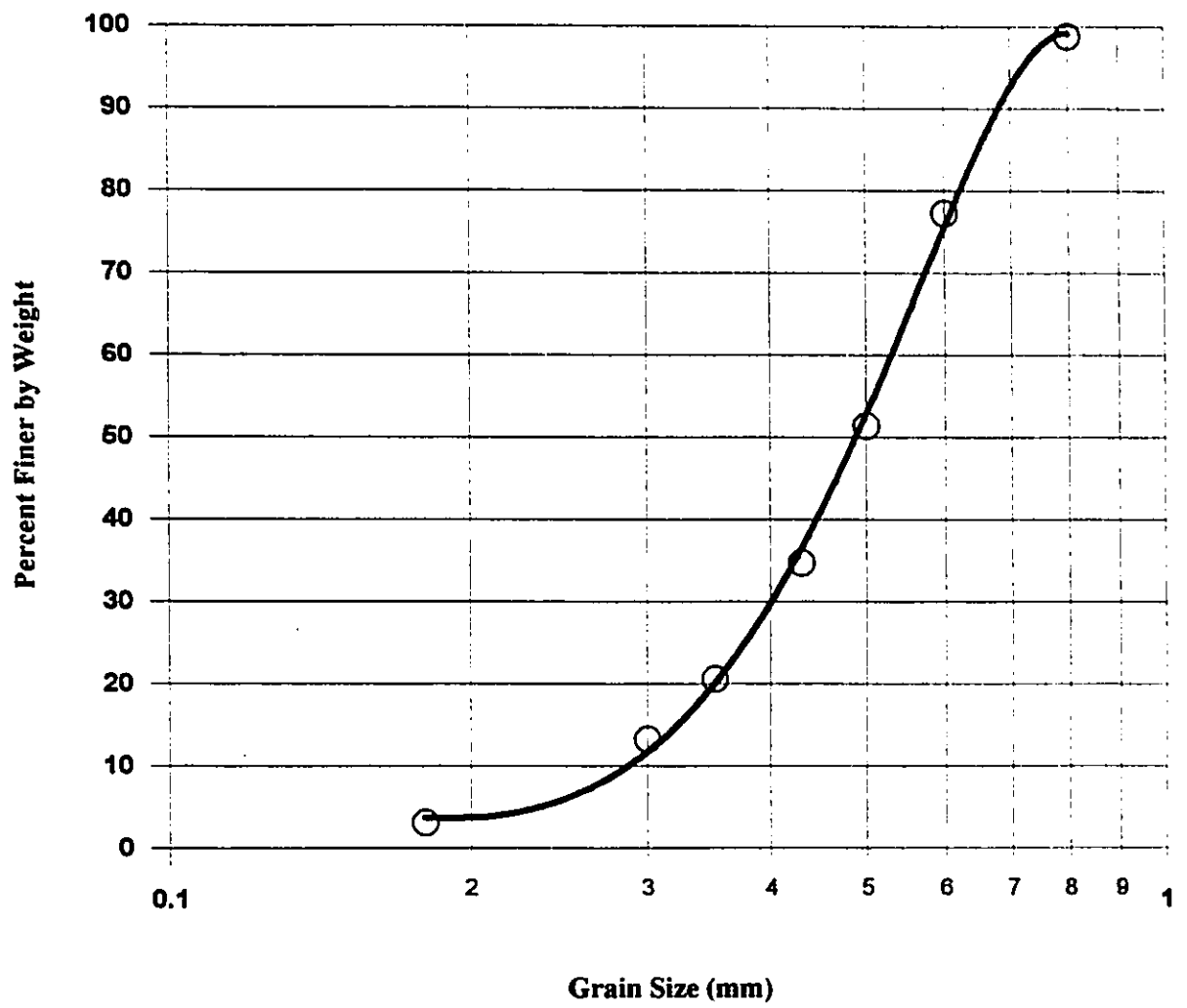


Fig. 4-3: Sieve analysis of the Play-Sand

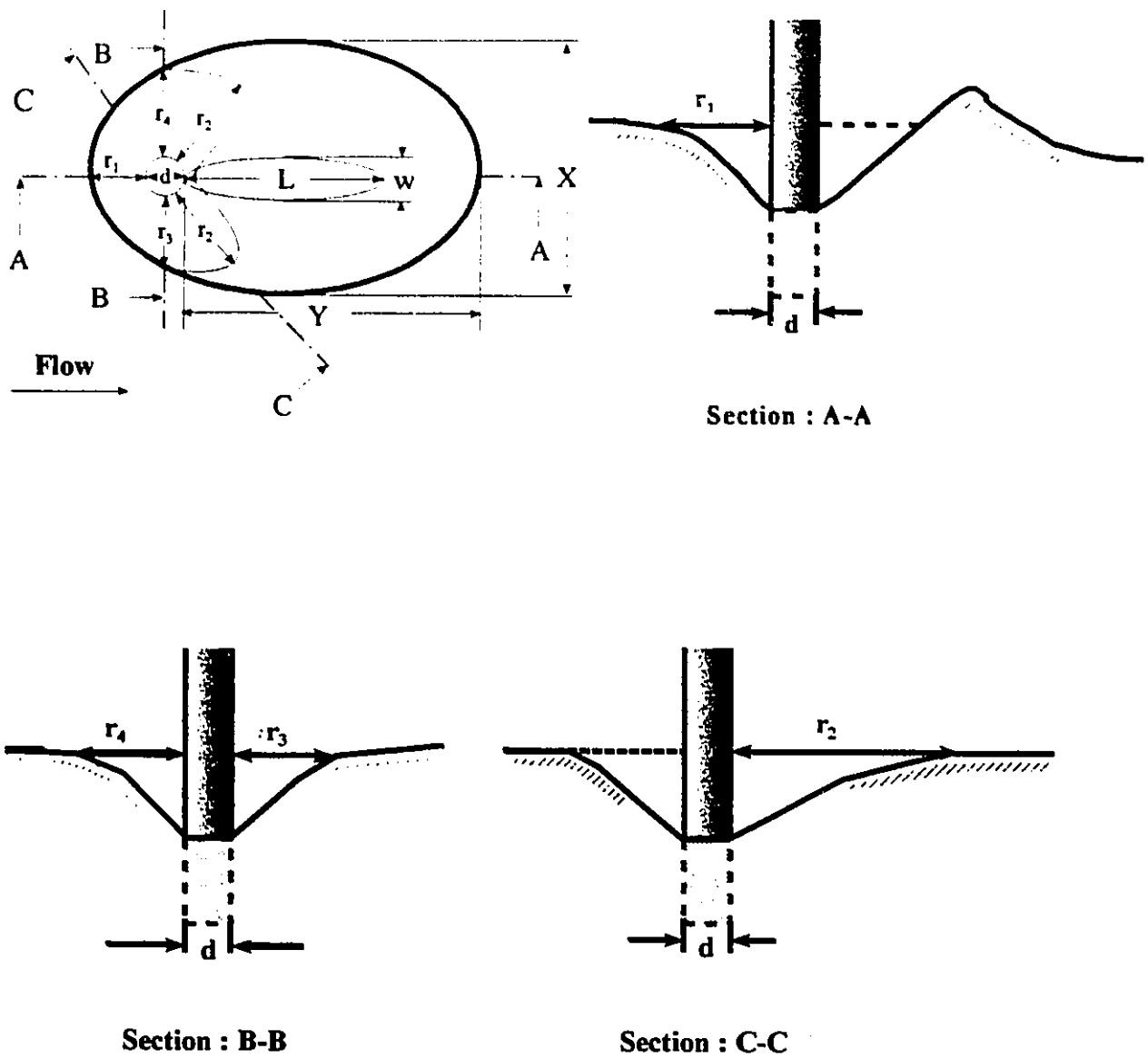


Fig 4-4: Schematic definition for scour hole measurements

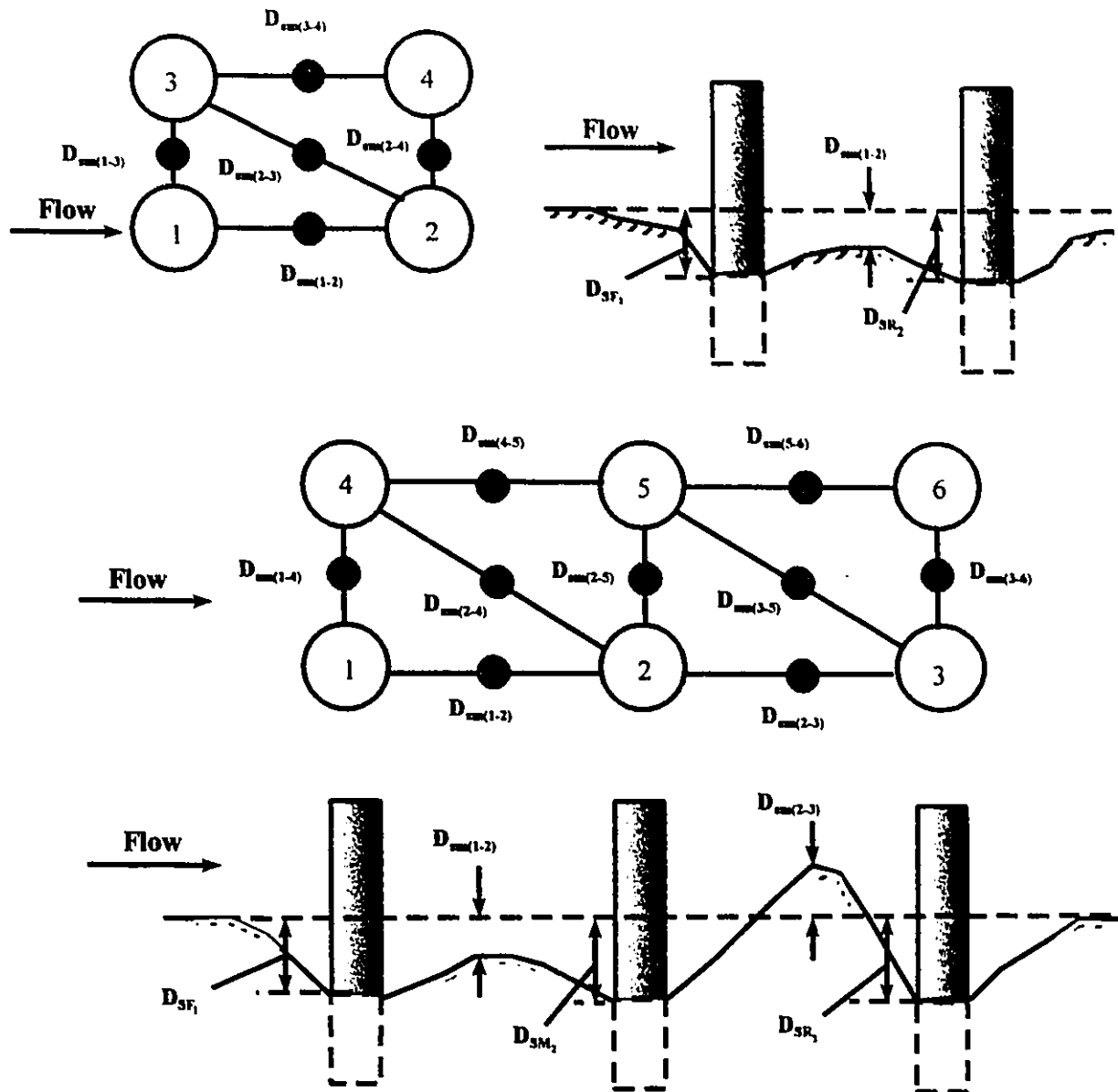


Fig. 4-5: Schematic definition for local scour depth measurements; plan view and side view for 4- and 6-PGs

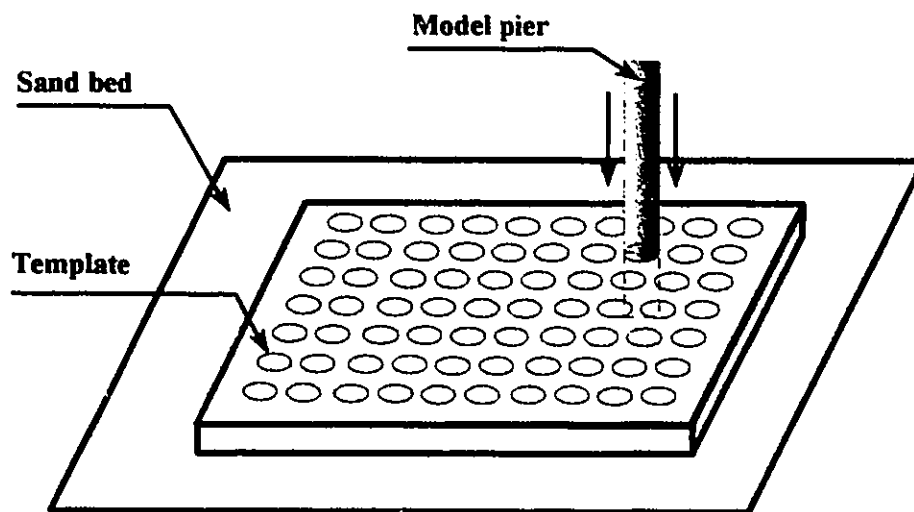


Fig. 4-6: Template system used for model pier installations

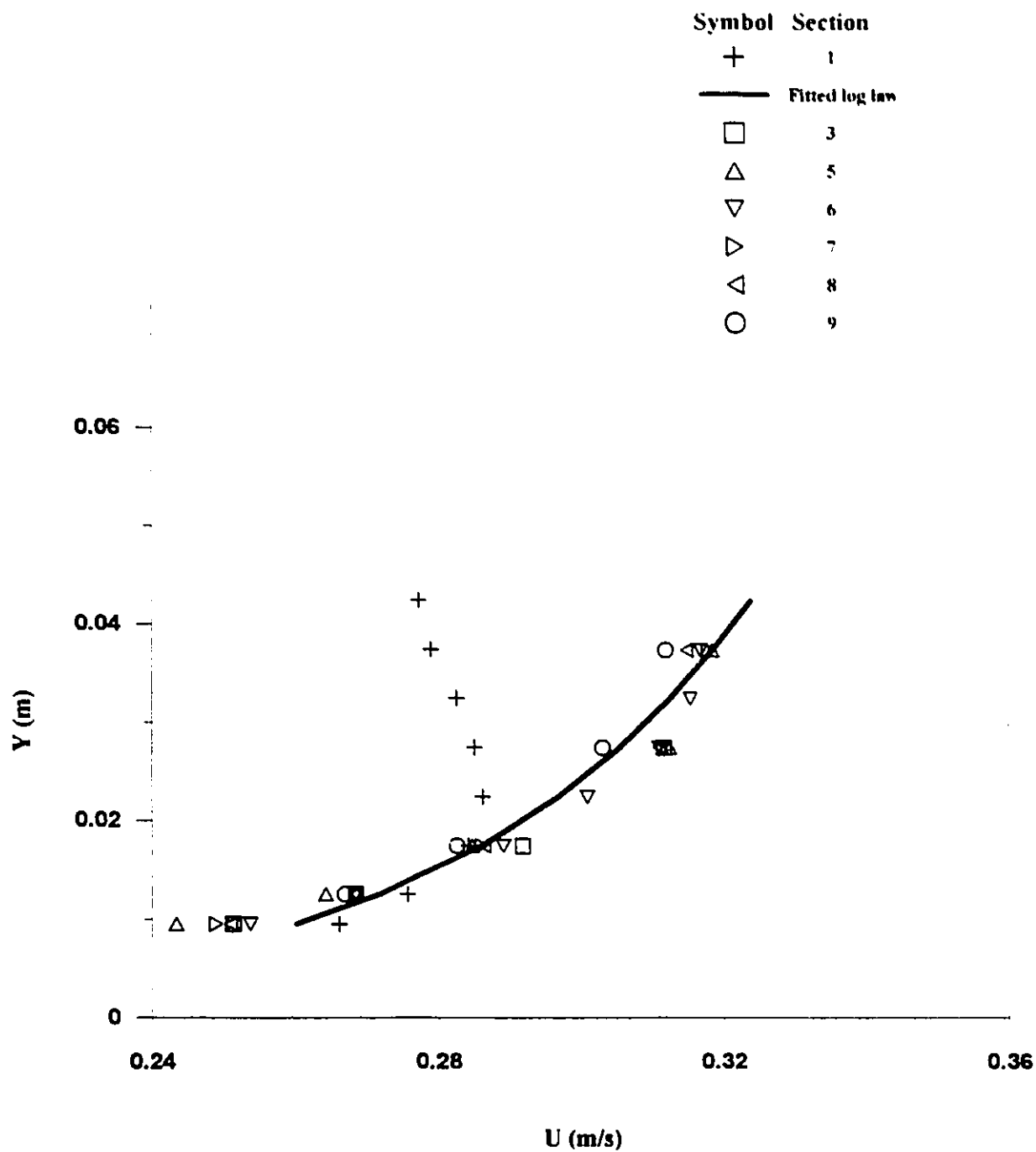


Fig. 4-7 : Comparison among velocity profiles in different cross sections along and at the center of the channel (flow depth versus velocity)

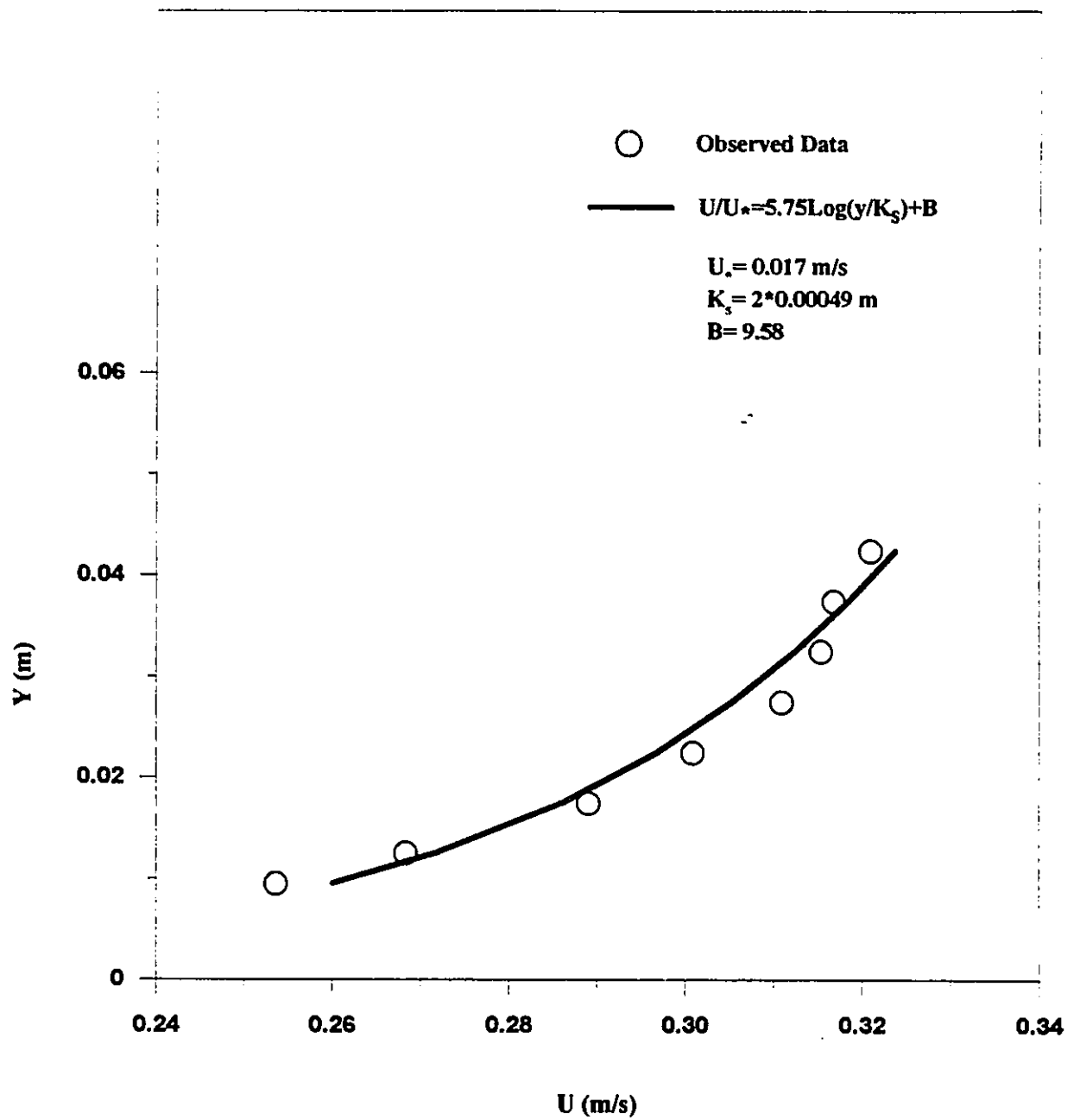


Fig. 4-8: Velocity profiles of undisturbed flows

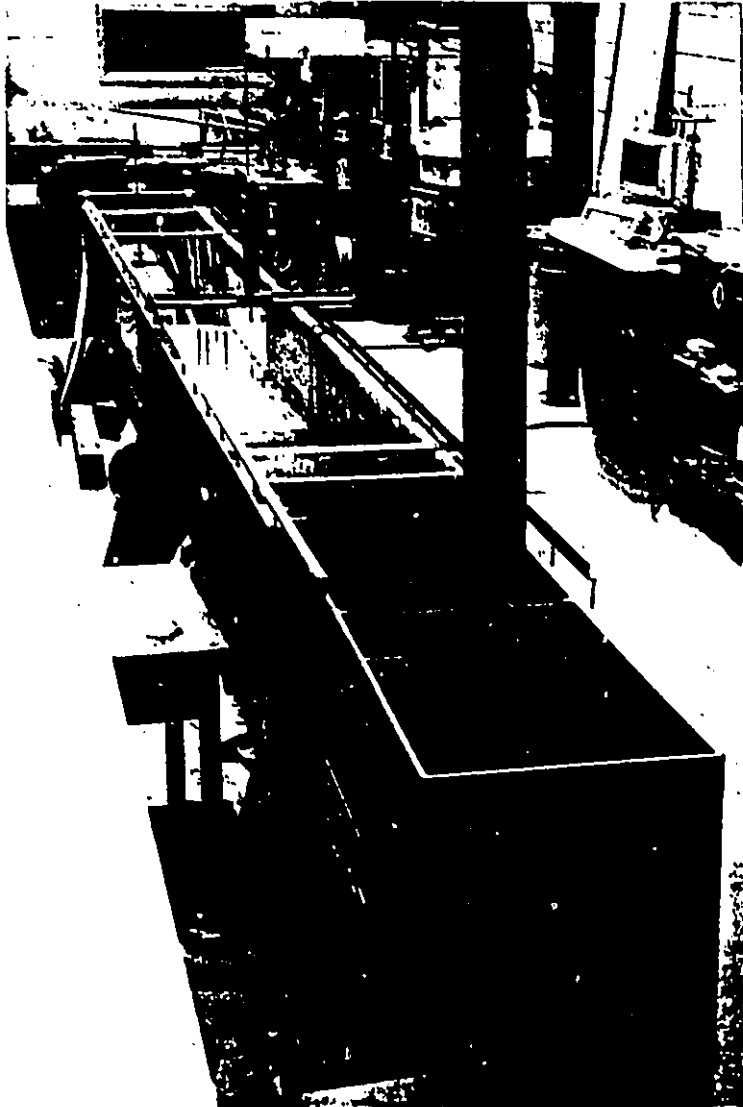


Plate 4-1: Laboratory flume

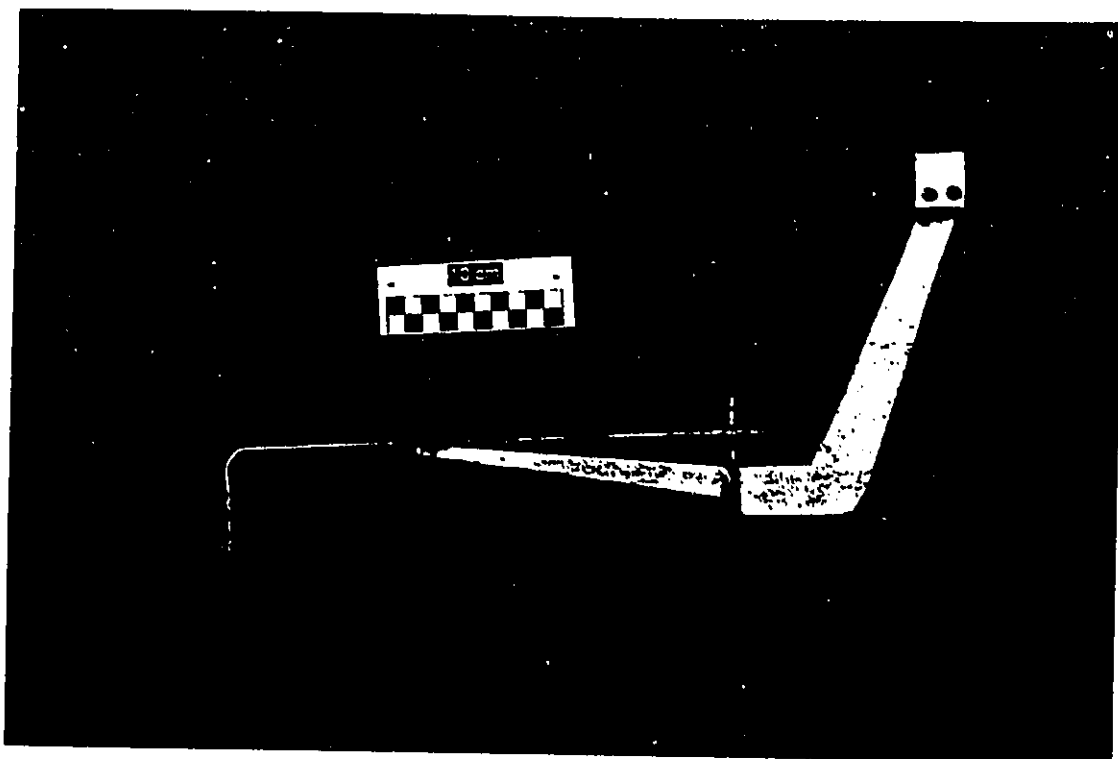


Plate 4-2: Pitot-static tube assembly

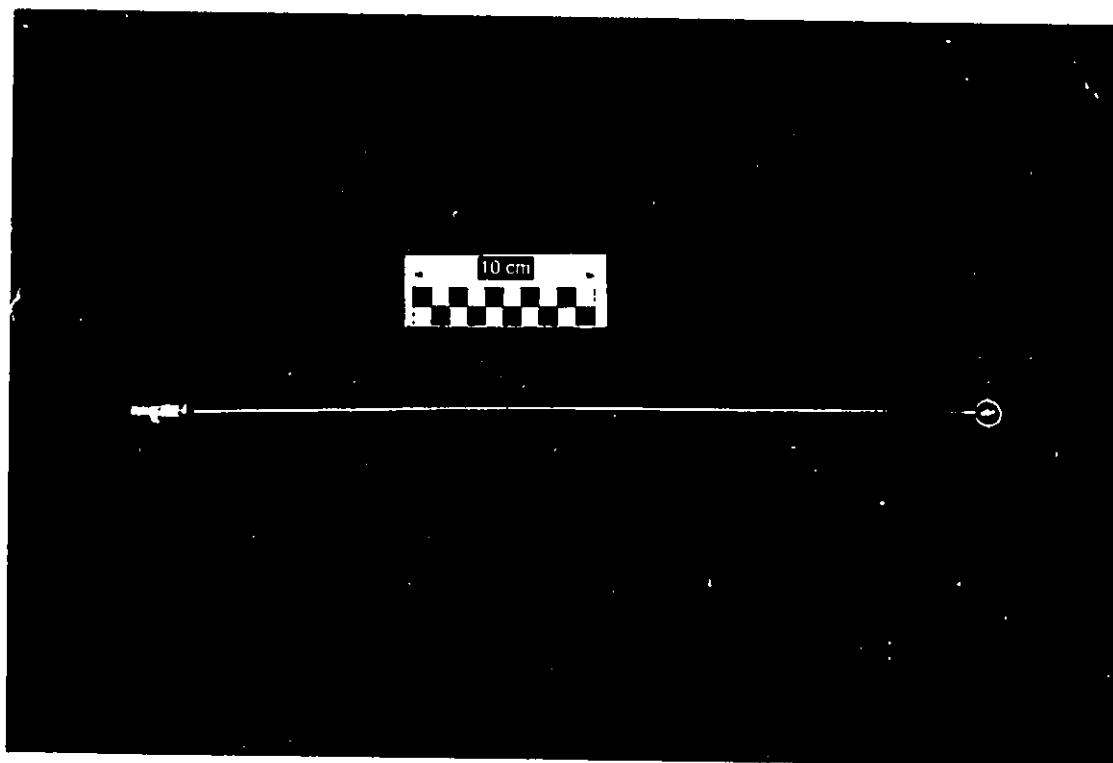


Plate 4-3: Mini-propeller flow meter

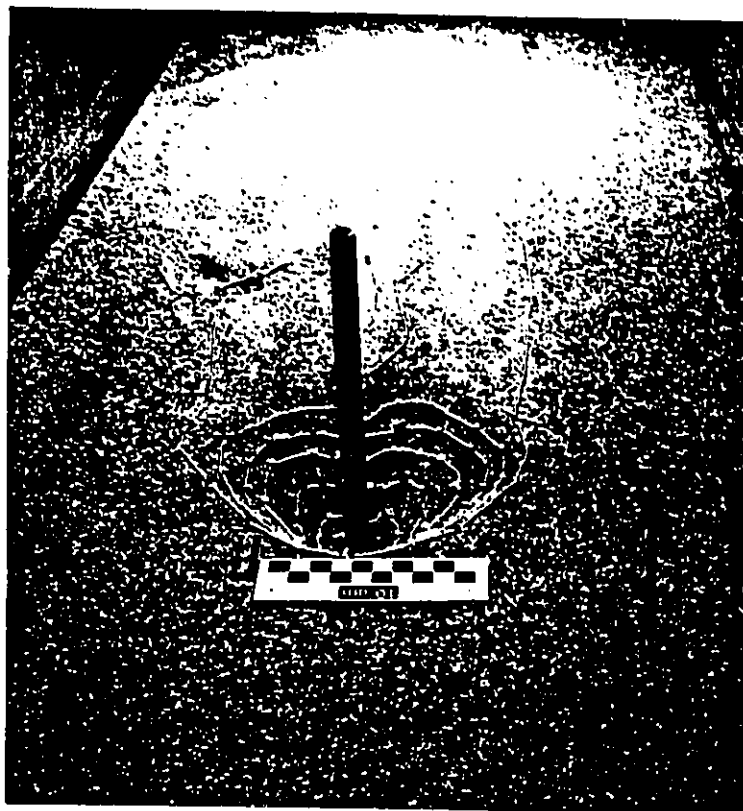


Plate 4-4: Typical scour hole shape for a circular cylindrical pier

Chapter 5

Results and Discussions

5-1 Local Scouring around a Single Pier

Local scouring around a single bridge pier has been thoroughly investigated in the past. However, since local scour depths in pier groups will normally be compared to that of a single pier of the same type, a few tests on single model piers were necessary to establish a basis for comparison. An initial series of tests was performed on six single-piers having diameters 0.0127, 0.018, 0.025, 0.033, 0.04, and 0.05 m under *clear-water* scouring conditions.

Development of the local scour profiles with time is shown in Figs. 5-1-1, 5-1-2, and 5-1-3. The tests were continued until the difference between two measurements at the same location was less than 1 mm over more than a one hour period. The influence of pier diameter on local scour depth was investigated for three flow conditions ($\frac{U}{U_{*c}} = 0.76, 0.96, 0.98$). Due to the size restriction of the flume, the smallest pier diameter which met the aforementioned modelling requirements for grain size and flow depth was accepted for the pier group tests. Next, a long-duration test (90 hours) was carried out for $d=0.0127$ m (Plate 5-1-1). Temporal variation of the local scouring process was predicted satisfactorily by Islam's (1986) relationship. Since the equilibrium scour depth was established for this test, different scour depth formulas were applied to find which one gave the closest agreement with the observed value. The Shen et al.'s (1966) equation (I) produced the closest agreement. Based on similar tests performed on different-sized single piers, a 2-hour test duration time was selected for the tests involving pier groups.

5-1-1 Temporal Variation of Local Scour Depth

Development of local scouring over time for different-sized single model piers is shown in Fig. 5-1-1 for $Y_0=0.054$ m. It is seen that the most sediment removal occurs during the first half hour, after which the rate of scouring progressively slows down. More than 83% of the total scouring occurs within the first two hours. Maximum local scour depth depends on the pier diameter; (the larger the pier diameter, the deeper the local scour depth). The same trends were observed for $Y_0=0.06$ and 0.065 m, shown in Figs. 5-1-2 and 5-1-3, respectively. All three figures show that under the same flow conditions, local scour approaches the equilibrium scour depth sooner for the smaller

pier diameters than that for the larger ones. In addition, Table 5-1-2 also shows that the ratio of D_{s2}/d and D_{s7}/d are greater for the smaller pier diameter.

Because of the size restrictions associated with the flume, a pier of diameter 0.0127 m was chosen for the individual model piers making up the pier groups. From the tests performed for the different-sized single piers it was observed that the size of the composite scour hole was greater for the larger pier diameters. For piers with $d \geq 0.018$ m, placed in a group, scour holes expanded and became large relative to channel width. To avoid a wall effect impacting on the local scouring process it was essential that the smallest possible pier diameter was used. Based on the limitations dictated by the flow depth and grain size (previously discussed in subsection 2-5-2), a pier diameter $d=0.0127$ m was deemed suitable for the main experimental program.

5-1-2 Time Evolution of Local Scour Depth

Since a 0.0127 m diameter pier was chosen for the model pier groups, a long-period test was performed for a single pier until the equilibrium scour depth was achieved. The maximum local scour depth was $D_s=31.5$ mm after 90 hours. In the past, logarithmic, trigonometric and exponential relations have been proposed to describe the variation of local scour depth with time. Sarma (1967) presented the following logarithmic relation :

$$t = K_1 \log \left(\frac{1 + \frac{D_s}{D_{se}}}{1 - \frac{D_s}{D_{se}}} \right) \quad (5-1-1)$$

in which D_s = depth of scour at the pier at time t , D_{se} = the equilibrium scour depth attained at $t \rightarrow \infty$ and K_1 = a multiplier which is independent of time. He also proposed the trigonometric relation:

$$\frac{D_s}{D_{se}} = \frac{2}{\pi} \tan^{-1} \left[a_1 \left(\frac{\omega^* t}{Y_0} \right)^{a_2} \right] \quad (5-1-2)$$

in which ω^* = fall velocity of the sediment grain, Y_0 = depth of flow and a_1 and a_2 are constants. Liu et al. (1961) proposed the exponential relation:

$$\frac{D_s}{D_{se}} = 1 - e^{-k_2 t} \quad (5-1-3)$$

in which K_2 = constant. Shen et al. (1965) developed a graphical relationship between d_t/d_{se} , t/T and the Froude number. Franzetti et al. (1982) suggested an exponential function to interpolate his experimental values of long time duration as follows:

$$\frac{D_s}{D_{se}} = 1 - \exp \left[-0.028 \left(\frac{tU}{d} \right)^{\frac{1}{3}} \right] \quad (5-1-4)$$

According to Li et al. (1986) the temporal scour rate varies in different periods. In principle it may be divided into three periods, but main erosion only occurs in the second scour period, which is thought to be the most significant one concerning practical scour problems.

Li et al. (1986) modified Franzetti's equation as follows:

$$\frac{D_s}{D_{se}} = \frac{\frac{tU}{d}}{K_s + \frac{tU}{d}} \quad (5-1-5)$$

in which; $K_s = 0.064 + \zeta^{0.41}$ and $\zeta = \left[(\gamma_s - \gamma) \left(\frac{d}{D_{50}} \right) \left(\frac{Ud}{v} \right) \right]^2 \left(\frac{U}{U_c} \right)^{-10}$.

Islam et al. (1986) proposed a nonexponential form of equation as follows:

$$\frac{D_s}{D_{se}} = \left[\sin \left(\frac{\pi t}{2T} \right) \right]^m \quad (5-1-6)$$

where

$$m = 0.135 \left(\frac{d}{D_{50}} \right)^{0.087} \left(\frac{Y_0}{d} \right)^{0.25}$$

All of the above equations indicate that the scour depth increases with time asymptotically to its equilibrium value. Temporal variation of the scour depth observed in our study for a single pier, estimated by the Franzetti (1982), Li et al. (1986), and Islam et al's (1986) equations, is shown in Fig. 5-1-4. Comparison among the different equations reveals that Islam et al's (1986) expression is the best estimator for the observed data. Plate (5-1-1) illustrates the formation of local scouring around this single pier.

5-1-3 Equilibrium Scour Depth Formulas

The performances of various pier scour prediction equations developed in the past were checked using data obtained for the single model pier over a long time period. Formulas for single

pier scour, included those of Laursen and Toch (1956), Shen et al., I, and II (1969), Chitale (1962), Jain and Fisher (1980), and the Colorado State University (CSU, Richardson et al. (1991)). Table 5-1-1 shows the results for the estimation of the equilibrium scour depth using the above-mentioned formulas. The equilibrium scour depth was best estimated using the Jain and Fischer (1980), Shen et al. I (1966) and CSU's equations. The Laursen and Toch (1956), Jain and Fisher (1980), and CSU's equations underpredicted the local scour depth. Chitale's (1962) equation on the other hand, overpredicted local scour depth significantly. Jones et al. (1995), using a wide range of field data, concluded that Shen et al's. (1966) equations give the best results with $\frac{\hat{D}_s}{D_s} = \lambda = 1.14$ for $0.2 \leq Fr \leq 0.5$ which are the most common ranges for natural streams ($\hat{D}_s$ = estimated scour depth).

5-1-4 The Test Duration Time Period

The local scour hole grows with time under the action of the HSV feature. Dargahi (1990) measured scour profiles under *clear-water* scour conditions and found that after 2 hours, approximately 60% of the final scour depth was reached. The vortices increased in diameter and the rate of scouring decreased with time, (after 12 hours, the scouring rate reduced significantly). The time history of scour depth was also measured at a fixed point upstream of a cylinder at a distance of 5 mm along the line of symmetry (Fig. 5-1-5).

Knight (1975) conducted his experiments under *clear-water* scour conditions. He continued his tests for a sufficient period until the equilibrium scour depth was reached. The duration of the runs varied from 10 to 120 minutes, the majority lasting about 60 minutes. For clear-water scour

conditions, Yanmaz and Altinbilk (1991) found that approximately 87% and 95% of the final scour depths were achieved after 2 and 4 hours, respectively. Dietz (1973) conducted a series of experiments examining both spacing and flow angles of attack for two piles. The tests were conducted for a duration of 2 hours. Ettema (1976) investigated the influence of bed material gradation on local scour. He found that D_{s24} (the scour depth after 24 hours) was 90% D_{sc} (equilibrium scour depth) and D_{s7} (scour depth after 7 hours) was 88% D_{s24} . He emphasised that the scouring was still occurring after 24 hours but that further scour profile development was very slow. Hannah (1978), using Ettema's finding, applied a correction factor of 1.25 to D_{s7} in order to establish the equilibrium scour depth.

In the present study, 14 tests were conducted to examine the variation of scour depth with time and the ratio between a 2-hour duration and other durations. The graphs for different pier diameters and hydraulic conditions are plotted in Figs. 5-1-1 to 5-1-3 and a summary of the results, which agree with the finding of Yanmaz and Altinbilk (1991), is tabulated in Table 5-1-2. They found that 87% and 95% of the final scour depths were achieved after 2 and 4 hours, respectively, and these are within the range of the same duration, as shown in Table 5-1-2 (83-92% and 91-95%). Durations of the tests in Table 5-1-2 are different (in order to have the same base for comparison the different ratios were presented in that Table). It is now possible to find a correction factor for the 2-hour duration ($\approx 90\%$ of the 7-hour duration). Hannah (1978) applied a correction factor of 1.25 for a 7-hour duration. According to this formulation, the correction factor for the present case is $1.25 \div 0.9 = 1.39 \approx 1.4$. The factor 1.4 was applied to all 2-hour duration tests to obtain the equivalent equilibrium scour depth.

5-1-5 Conclusions:

A series of tests was carried out on six single piers having different diameters (0.0127, 0.018, 0.025, 0.033, 0.04, and 0.05 m) under *clear-water* scour conditions. Development of local scouring with time showed that the larger the pier diameter the larger the local scour hole. The smallest pier diameter ($d=0.0127$ m) was selected for the present study, based on the size restrictions of the study flume and the requirements regarding grain size and flow depth.

Temporal variation of local scouring for $d=0.0127$ m, was accurately predicted with Islam et al.'s (1986) relationship (equation 5-1-3). The equilibrium scour depth was achieved after a long test (90 hours). This test also facilitated comparison among the different scour-depth formulas. In this instance Shen et al.'s (1966) equation (I) was found to be the best estimator of equilibrium scour depth.

The duration of scouring experiments was chosen based on a review of similar-type studies (chapter 2), and the results of our preliminary series of tests. A 2-hour test duration was selected for the present study. A factor of 1.4 was applied to observed local scour depths after a period of 2 hours to determine the equivalent equilibrium scour depth.

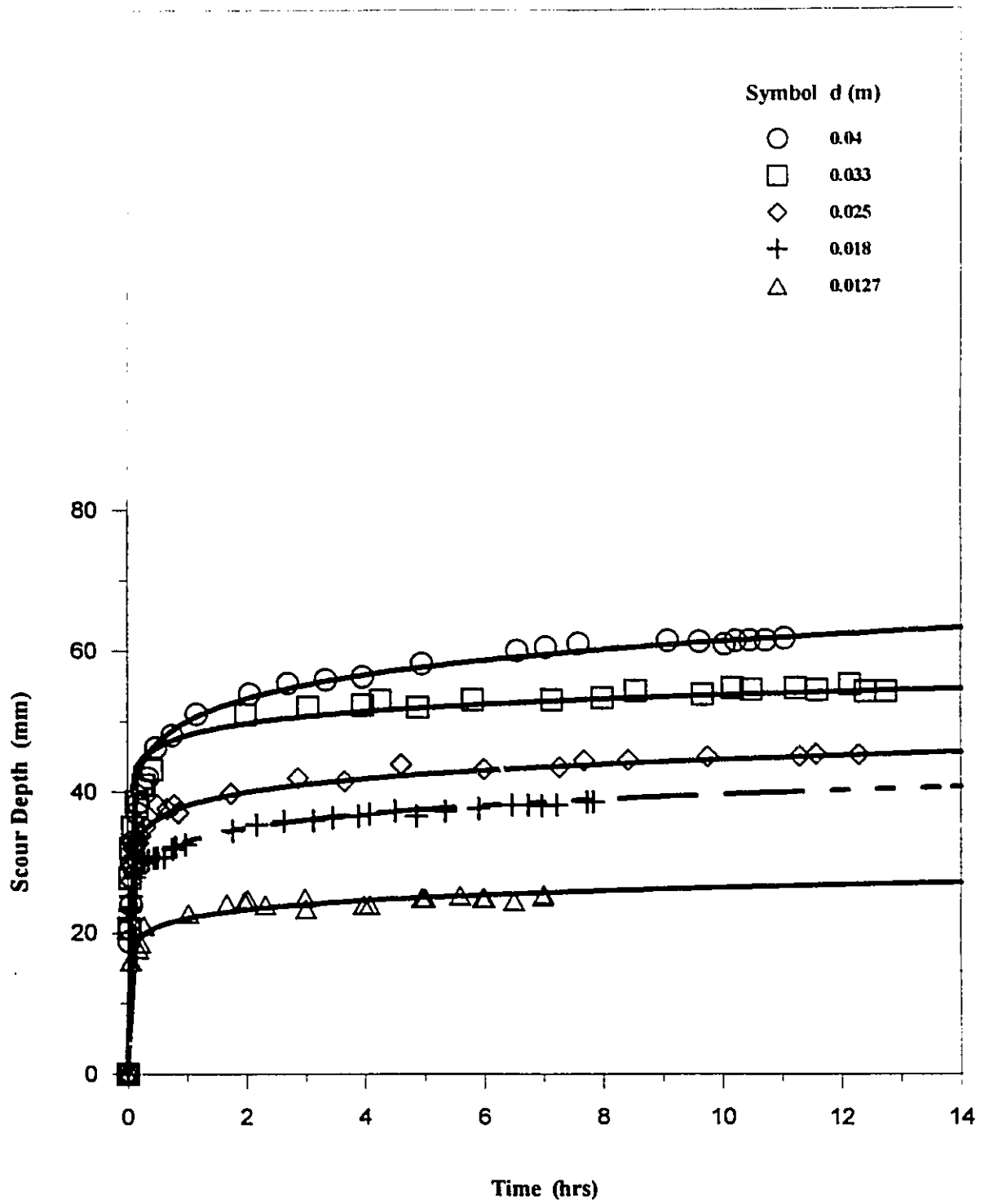


Fig. 5-1-1: Temporal variation of local scour depth; $Y_s = 0.053$ m

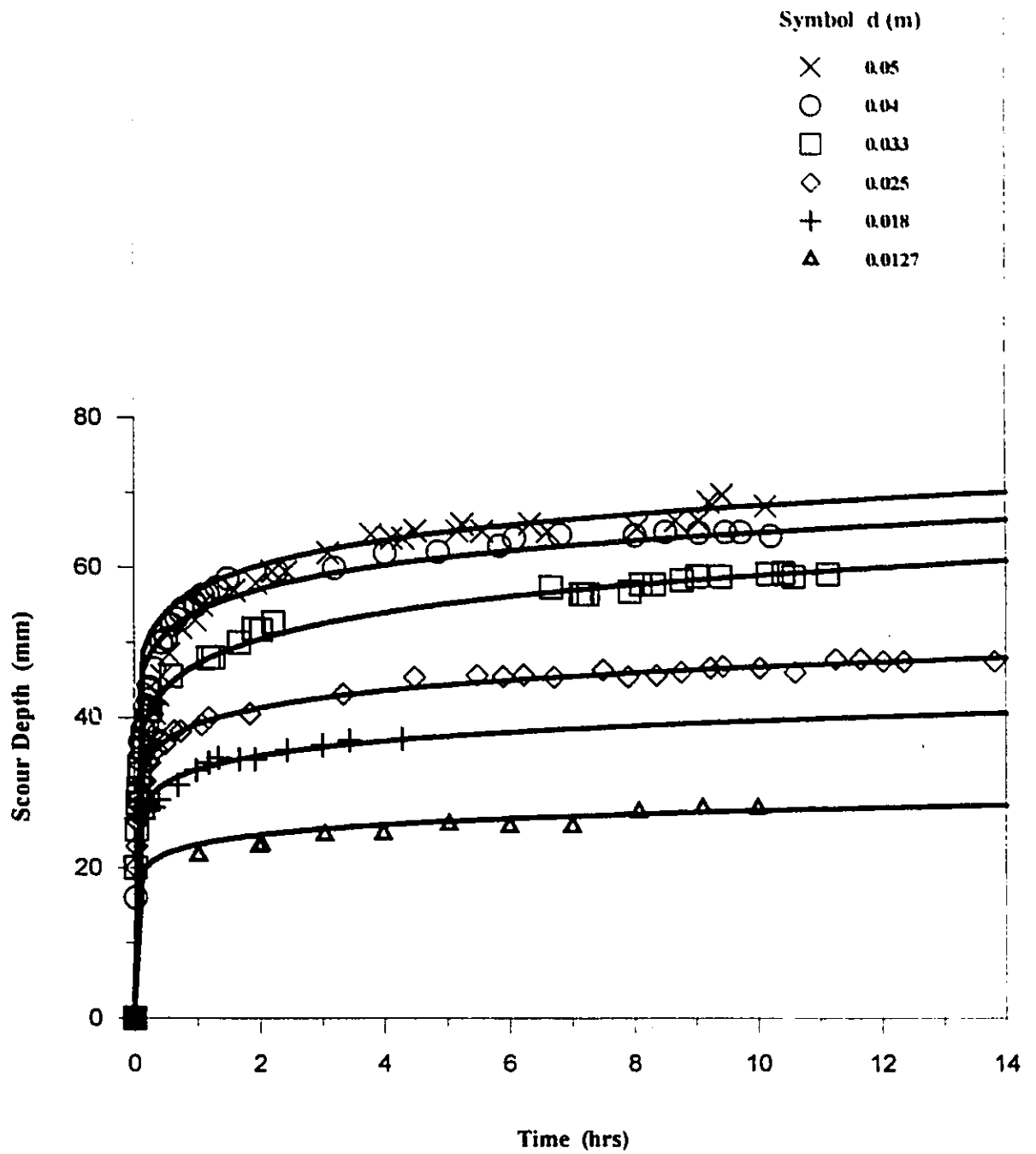


Fig. 5-1-2: Temporal variation of local scour depth; $Y_s = 0.06$ m

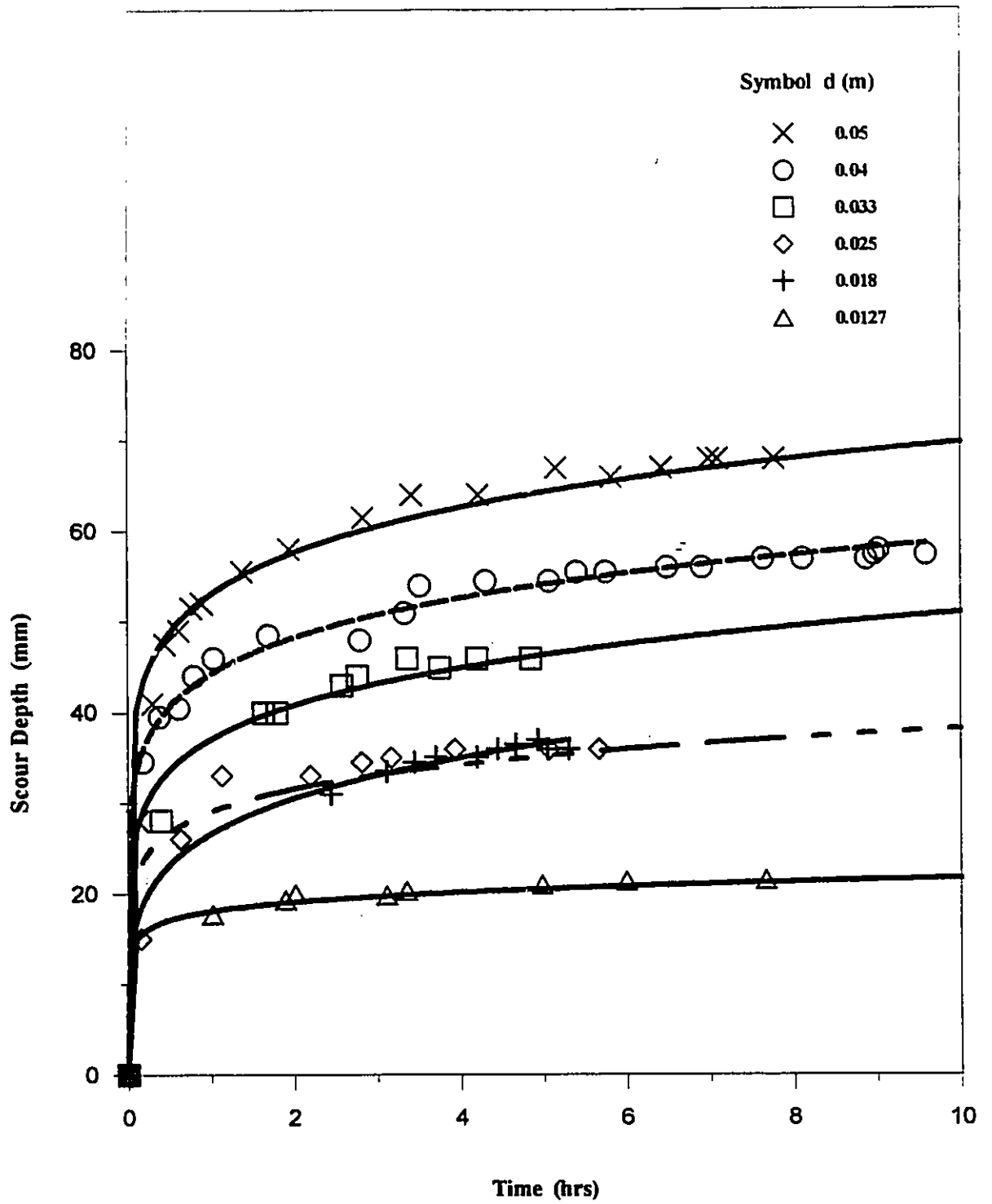


Fig. 5-1-3: Temporal variation of local scour depth; $Y_0 = 0.065$ m

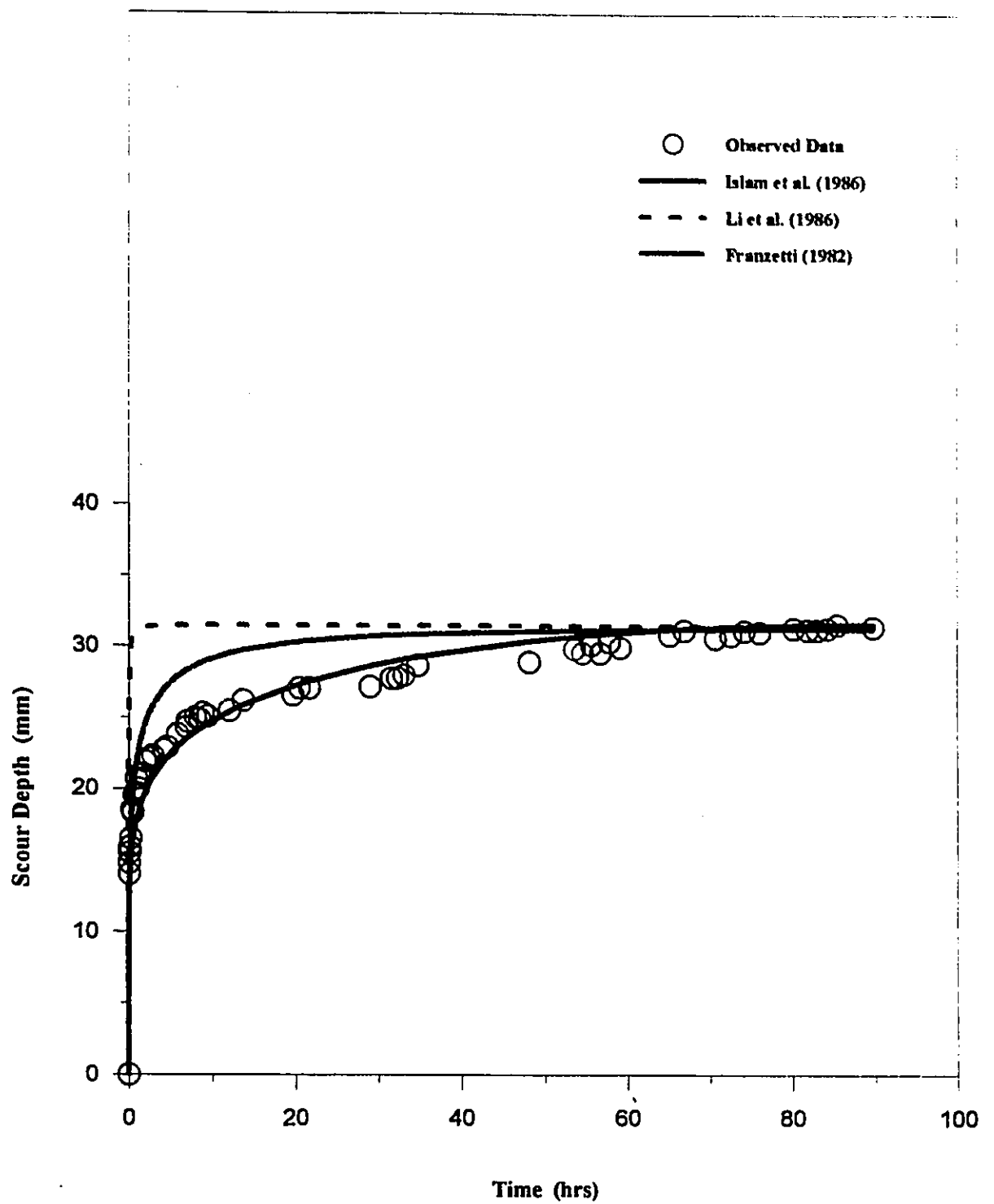


figure 5-1-4: Temporal variation of local scour depth; $Y_s = 0.054$ m; $d = 0.0127$ m



Plate 5-1-1: Scouring pattern around a single pier (90 hours duration); $d=0.0127$ m

Table 5-1-1: Comparison among different local scour prediction equations for a single pier

Investigator(s)	Prediction Equations	λ^*
Shen et al. (1966) (I)	$D_s = 0.000223 R_c^{0.619}$	1.05
" " (II)	$\frac{D_s}{d} = 3.4 Fr^{\frac{2}{3}} \left(\frac{Y_0}{d} \right)^{\frac{1}{3}}$	1.17
Laursen and Toch (1956)	$D_s = 1.35 d^{0.7} Y_0^{0.3}$	0.83
Chitale (1962)	$\frac{D_s}{Y_0} = -5.49 Fr^2 + 6.65 Fr - 0.51$	2.12
Jain and Fisher (1980)	$\frac{D_s}{d} = 1.85 Fr_c^{0.25} \left(\frac{Y_0}{d} \right)^{0.3}$	0.89
CSU (1991)	$\frac{D_s}{Y_0} = 2.0 K_1 K_2 \left(\frac{d}{Y_0} \right)^{0.65} Fr^{0.43}$	0.86

$$\lambda^* = \frac{\hat{D}_s}{D_s} = \text{predicted value/observed value}$$

Table 5-1-2: Time dependent scour depth ratios

Run No.	d (m)	Y0 (m)	V (m/s)	Dse (mm)	Ds2/Dse	Ds2/d	Ds2/Ds7	Ds7/d	Dse/d	Duration Time (hrs)
1	0.018	0.065	0.20	36.0	0.83	1.67	NA	NA	2.00	(5,18')
2	0.025	0.065	0.20	36.0	0.92	1.32	NA	NA	1.44	(5,40')
3	0.033	0.065	0.20	46.0	0.89	1.24	NA	NA	1.39	(4,51)
4	0.040	0.065	0.20	58.0	0.83	1.20	0.86	1.40	1.45	(9,35')
5	0.050	0.065	0.20	69.0	0.84	1.16	0.85	1.36	1.38	(8,19')
6	0.050	0.072	0.21	70.0	0.83	1.16	0.88	1.32	1.40	(10)
8	0.025	0.058	0.24	47.7	0.85	1.62	0.90	1.80	1.91	(13,49')
9	0.033	0.060	0.23	59.4	0.87	1.57	0.91	1.73	1.80	(11,8')
10	0.040	0.060	0.20	65.0	0.91	1.48	0.92	1.61	1.63	(10,31')
11	0.018	0.053	0.22	38.8	0.91	1.96	0.92	2.12	2.16	(8,8')
48	0.025	0.054	0.22	37.5	0.97	1.46	NA	NA	1.50	(8,8')
12	0.025	0.053	0.22	45.5	0.89	1.61	NA	NA	1.82	(12,7')
13	0.033	0.053	0.22	55.4	0.92	1.55	0.95	1.62	1.68	(12,45')
14	0.040	0.053	0.22	62.6	0.86	1.35	0.89	1.52	1.57	(11,43')
28	0.013	0.054	0.22	25.5	0.97	1.95	0.97	2.01	2.01	(7)
30	0.013	0.054	0.22	28.0	0.82	1.81	0.90	2.01	2.20	(10)
40	0.013	0.054	0.22	23.5	1.00	1.85	NA	NA	1.85	(2)
41	0.013	0.054	0.22	24.0	1.00	1.89	NA	NA	1.89	(2)
42	0.013	0.054	0.22	25.4	1.00	2.00	NA	NA	2.00	(2)
63	0.013	0.054	0.22	24.1	1.00	1.90	NA	NA	1.90	(2)
70	0.013	0.054	0.22	25.0	1.00	1.97	NA	NA	1.97	(2)
Single	0.013	0.054	0.22	31.5	0.76	1.89	0.97	1.94	2.48	(89,42')

d= pier diameter

Y0= depth of the water

V= mean velocity

Ds2= scour depth after 2 hours

Ds7= scour depth after 7 hours

Dse= final scour depth

(hrs)= duration of the test

NA= not available

5-2 Local Scouring around 2-Pier Groups

Local scour associated with a group of 2 piers (2-PG) is presented in this section. [Since a 4-pier group (4-PG) is comprised of two 2-PGs, the study of local scour around a 2-PG is helpful in analysing scour around a 4-PG]. A total of sixty-nine tests were performed for this analysis, using two different pier diameters ($d=0.0127$ and 0.018 m). The plexiglass cylindrical piers in any grouping, however, had the same diameter. Maximum local scour associated with each model pier was measured at a distance 2 to 5 mm upstream of the pier on the plane of symmetry. To impose the clear-water scour condition, the ratio of $\frac{U}{U_{*c}}$ was kept constant and less than unity. Also, the longitudinal slope of the flume was set at 0.0006 throughout the tests. Altering S/d and β at the same time for each pier group, requires a large number of tests. To reduce the number of tests required at this stage, large flow angles of attack (that produced the most severe scouring) were found for $S/d=5$. Initially, maximum local scour depth was measured for the range $0 \leq \beta \leq 90^\circ$, however, only large β were considered in subsequent tests of the tests using different S/d . The application of a linear regression model to the compiled data, along with logarithmic transformations of β and S/d , led to the development of equations to predict maximum scour depth. Two different equations were developed to predict the local scour depths associated with the front and rear piers.

5-2-1 The Effects of Different β on Local Scour Depth

The effect of varying β on local scour depth was investigated for each S/d considered. For $S/d = 1$ (i.e. two piers touching each other), since the scour depth varied linearly with β , only one extra test ($\beta=90^\circ$), additional to the tests mentioned in Table 5-2-1, was needed to cover the range:

$0 \leq \beta \leq 90^\circ$. The linear increase of the local scour depth (D_s) observed in the tests was due to the linear increase of the projected area (A_p) of the piers with increasing β (Fig. 5-2-1). The touching front and rear piers behaved as a single pier, the diameter (d) of which equalled the width of the projected area of the attached two piers. The greater the β , the larger were A_p and the D_s , consequently, the maximum scour depth occurred for $\beta = 90^\circ$, since it produces the largest A_p . A small difference in D_s was observed between the 2-PG arrangement ($d=0.0127$ m) and the single pier case ($d=0.025$ m). For the 2-PG, D_{SF} or D_{SR} was 46 mm while for a single pier it was 42 mm. The small difference between D_s and D_{SF} or D_{SR} was due to their different geometries. The formation of the local scour hole around the 2-PG is illustrated in Plate 5-2-1.

In the next series of tests, the effect of varying β on the local scour depth was studied for $S/d=2$. An increase in β , reduces the difference between the scour depth associated with the front and the rear piers (Fig. 5-2-2). This is because the rear pier is increasingly less sheltered by the front pier as β increases, which results in a larger scour depth for the rear pier. Plate 5-2-2 illustrates the scouring pattern produced around the piers. Also, generally larger scour hole was formed for $S/d=2$ compared with $S/d=1$.

The effect of varying β on the local scour depth was investigated for $S/d=3$. In addition to the tests included in Table 5-2-1, ten further tests were conducted with $d=0.018$ m and using $\beta=0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ, 50^\circ, 60^\circ, 70^\circ, 80^\circ$, and 90° to study the effect of varying pier diameter on the local scouring process. The general trend of the data was similar to that observed for $S/d=2$, but with lower scour depth ratios ($\frac{D_{SF_1}}{D_s} = D'_{F_1}$ and $\frac{D_{SR_2}}{D_s} = D'_{R_2}$, Fig. 5-2-3). For $S/d \leq 4$, both piers share a

common scour hole, since HSV that are generated around each pier interact with each other. Since this interaction becomes weaker with increasing S/d , a reduced scour depth is achieved. Also, for $\beta > 30^\circ$ it can be seen that the scour depth of the front pier is smaller than that of the rear pier. The reason is that, for $\beta > 30^\circ$, the rear pier is completely exposed to the flow and so the sheltering effect is lost. Moreover, the rear pier lies in the downstream portion of the scour hole formed around the front pier, consequently sediments around the rear pier are removed more easily.

In the next series of tests the effect of varying β on the local scour depth was investigated for $S/d=5$ (Fig. 5-2-4). In addition to those tests mentioned in Table 5-2-1, three further tests were performed with $\beta=60^\circ$, 80° , and 90° . The reason for selecting $S/d=5$ and applying a wider range of β ($0^\circ \leq \beta \leq 90^\circ$) was to facilitate comparison between our data with those of other investigators. In this case, an independent scour hole was formed around each pier in this case. As shown by the results of former cases ($S/d=2$ and 3) for $\beta \leq 30^\circ$, D'_{F_1} is larger than D'_{R_2} because of the sheltering which affects the rear pier. By increasing β the sheltering effect decreases, therefore the local scour depth of the rear pier increases more than that for the front pier. Both the front and rear piers produce essentially the same scour depth at $\beta=90^\circ$.

Hannah (1978) suggests that, for $10^\circ \leq \beta \leq 60^\circ$, the scour is influenced only by the sheltering and shed wake vortex mechanisms. However, the sheltering effect diminishes as β increases. Also, Laursen and Toch (1956) observed that the downstream pier becomes subject to a relatively higher approach velocity than the upstream pier. They attributed this higher velocity to contraction of the flow between the two piers. The higher velocity causes a deeper scour to form downstream, however

the local scour depth of the front pier does not change significantly with β . The results of our study are compared with those of Hannah (1978). Fig. 5-2-5 confirms the same trend of data in both studies. Moreover, D'_{F_1} and D'_{R_2} are very close to Hannah's (1978) report values.

The effect of varying β on the local scour depth was investigated for $S/d = 7$ (Fig. 5-2-6). (An additional test which is not listed in Table 5-2-1 was performed using $\beta = 90^\circ$). The decreasing trend of the data observed for D'_{F_1} , in the range $0 \leq \beta \leq 20^\circ$, is due to the reinforcement effect which decreases with increasing β . For $\beta > 20^\circ$, D'_{F_1} approaches unity. On the other hand, since R_2 becomes exposed to the flow for $\beta > 20^\circ$, D'_{R_2} shows an increasing trend that approaches unity.

In the next sets of experiments, the effect of varying β was studied for $S/d = 11$. Fig. 5-2-7 shows that the variation of D'_{F_1} is practically independent of β . This is because of the large S/d which results in a minimal reinforcement effect associated with R_2 . However, F_1 still shelters R_2 at $\beta = 0^\circ$ resulting in a shallower scour depth in front of the R_2 . Since the sheltering effect reduces with increasing β , D'_{R_2} approaches unity with increasing S/d .

5-2-2 Effects of Varying S/d on Local Scour Depth

The effect of different S/d on the local scour depth was investigated for a wide range of β . In addition to tests mentioned thus far, eight supplementary tests were performed using $\beta = 0^\circ$ for $S/d = 12, 13, 14, 15, 16, 18, 20$, and 40 (Fig. 5-2-10). Hannah (1978) and Dietz (1973) conducted two series of tests for aligned piers ($\beta = 0^\circ$). The results of this study were compared with those of Hannah

and Dietz for front pier scouring (Fig. 5-2-11).

For the case $\beta=0^\circ$, Dietz's (1973) data show a smaller scour depth than that of both the present and Hannah's study. Although the reinforcement effect associated with the front pier was commented on by Dietz, this effect was found to be more significant in the present study and also in Hannah's study (Fig. 5-2-11). The deeper the scour around the rear pier, the stronger the reinforcement effect is on the front pier. The scour hole formed around the rear pier eases sediment removal from around the front pier by lowering D_{SM} ¹ (Fig. 5-2-12). It was observed that the reinforcement effect was insignificant for $S/d > 10$. Measurements showed that the separation line starting from the side of the pier developed downstream with $\beta \approx 10^\circ$. In the absence of the reinforcement effect (for $S/d \geq 10$) approaches unity.

The observed in this study are compared with those values reported by Hannah and Dietz. Their data do not show the same trend as that of the present study for $\beta > 0^\circ$. The present study showed a minimum value for in the range $2 \leq S/d \leq 5$. Thereafter, the scour depth increases and reaches a maximum at $S/d=6$. Hannah's and Dietz's results do not show any decrease in D'_{R_2} in the range $1 \leq S/d \leq 6$.

There are two explanations for the observed significant decrease in D'_{R_2} in the range $1 \leq S/d \leq 6$. First, the scour depth in front of the rear pier will be reduced because the rear pier lies in the lower portion of the scour hole formed around the front pier. The position of the rear pier,

¹ D_{SM} = the maximum scour depth between two piers

relative to the front pier, is shown in Plates 5-2-3 and 5-2-4. As long as the rear pier is located within the scour hole of the front pier, the scour depth of the rear pier is less than that of the front pier. Second, when S becomes larger than R_2 (see Fig. 4-4) D'_{R_2} is the smallest in the range of $2.5 < S/d < 5$. By further increase in S/d , D'_{R_2} increases and reaches its maximum at $S/d = 6$. For $S/d \geq 5$, a separate scour hole is formed around the rear pier. Although the rear pier is still sheltered by the front pier, the sediments do not have a chance to deposit in front of the rear pier to reduce its scour depth. The distance between two piers is such that the wake vortices of the front pier and the vortex systems of the rear pier interact with each other. Because of the presence of these vortices, sediments cannot deposit around the rear pier. This interaction will be reduced by increasing S/d . On the other hand, the sheltering effect is still a dominant agent in reducing the scour depth of the rear pier.

A second minimum was observed for D'_{R_2} in the range $11 \leq S/d \leq 40$. For $S/d > 13$, D'_{R_2} increases until it approaches unity. This is because the sheltering effect is reduced with increasing S/d . Such a variation in the scour depth was not reported in the literature for $1 \leq S/d \leq 13$. Plates 5-2-3, 4, and 5 illustrate the development of scour holes around the piers. It can be seen that a common scour hole is formed around both the front and the rear piers for $S/d = 3$ and 4. On the other hand, a separate scour hole is formed for each pier at $S/d = 16$.

The sheltering effect was investigated using 4 different β . Based on the tests performed for $S/d = 5$, only crucial angles of attack ($\beta = 35^\circ, 40^\circ, 45^\circ, 50^\circ$) were applied at this stage of the study.

The results for $\beta = 35^\circ$ (Fig. 5-2-13) showed that D'_{R_2} and D'_{F_1} decreased sharply in the

range $1 \leq S/d \leq 2$. A smoother decline was observed as S/d was increased. For $S/d > 2$, D'_{R_2} was less dependent on S/d . The trend of data associated with D'_{F_1} is different from that of D'_{R_2} for $2 < S/d \leq 11$ in that was greater than D'_{F_1} . A decreasing trend of data was observed for D'_{F_1} in the range $2 \leq S/d < 7$ and an increasing trend in the range $7 \leq S/d \leq 11$. The decreasing trend is due to the reinforcement effect, which is less significant for larger S/d .

In the next set of tests, variations of D'_{F_1} and D'_{R_2} for different S/d were studied for $\beta = 40^\circ$. Similar to the case for $\beta = 35^\circ$ different trends of data were observed for D'_{F_1} and D'_{R_2} . A sharp decrease in D'_{F_1} and D'_{R_2} was evident for $S/d < 2$ (Fig. 5-2-14). In the range $7 \leq S/d \leq 15$, D'_{F_1} was less dependent on S/d than for the range $1 \leq S/d \leq 7$. However, in the range $5 \leq S/d \leq 15$, D'_{R_2} appears to be independent of S/d . Moreover, similar to the previous case ($\beta = 35^\circ$), D'_{R_2} was always greater than D'_{F_1} .

For $\beta = 45^\circ$, the trend of data in the range of $1 \leq S/d \leq 2$ is different than that in the range $2 < S/d \leq 7$. A sharp decrease in both D'_{F_1} and D'_{R_2} was observed for the range $1 \leq S/d \leq 2$ (Fig. 5-2-15). The results of the present study were compared with Hannah's (1978) results. The same trends of data are observed for both studies in that D'_{F_1} and D'_{R_2} eventually approaches unity. The results for $\beta = 50^\circ$, were similar to those for $\beta = 45^\circ$ (Fig. 5-2-16).

The effect of varying S/d on the local scour depth was investigated for $\beta = 90^\circ$. Since both the front and the rear piers are equally exposed to the flow, the scour depths in front of them are identical (Fig. 5-2-17). D'_{F_1} and D'_{R_2} were found to be constant and independent of S/d for $S/d > 4$,

however, a sharp decrease in scour depth was observed in the range $1 \leq S/d \leq 1.5$. The rate of decrease in both D'_{F_1} and D'_{R_2} was smooth in the range $1.5 \leq S/d \leq 4$.

5-2-3 Multiple Regression Analysis Model

Seventy measurements of the local scour depth gathered from the present study were analysed using multiple regression analyses to obtain local scour prediction equations for the front and rear piers. The regression analysis provides a probability statement about the distribution of the prediction, from which an upper confidence limit of the depth of local scour can be determined. An upper confidence limit having an appropriate exceedence probability can be used for design of bridge pier foundations to provide a desired factor of safety.

5-2-3-1 Analysis of Data

The local scour depth for a 2-PG is a function of S/d and β . The functional relation for the normalized local scour depth is as follows:

$$\frac{D_{S_i}}{D_s} = f \left(\beta, \frac{S}{D} \right) \quad (5-2-1)$$

where D_{S_i} is local scour depth in a group of piers (i.e., D_{SF_1} or D_{SR_2}). Multiple linear regression was used to develop prediction equations for normalized depth of the local scour, based on the proposed functional relation in equation (5-2-1).

Logarithmic transformation of all variables was found to provide the best linear relations. The logarithm of the ratio of $\frac{D_{s_i}}{D_s} = D'_{s_i}$ was regressed against a combination of the two independent variables, i.e., β and S/d . A general form of the prediction equation written in terms of untransformed values is as follows:

$$\frac{\hat{D}_{s_i}}{D_s} = a\beta^b(S/D)^c \quad (5-2-2)$$

in which $\hat{D}_{s_i}$ is the scour depth for any pier in a group, $\hat{\theta} = [\hat{\theta}_1, \hat{\theta}_2, \hat{\theta}_3]$, $a = \exp(\hat{\theta}_1)$, $b = \hat{\theta}_2$, and $c = \hat{\theta}_3$.

5-2-3-2 Predicting Scour Depths

Normalized scour depths predicted by equation 5-2-5 are the expected response to a specified set of regressors, and are not known with certainty. However, a local scour depth that has a small probability of being exceeded needs to be used for designing pier foundations to insure against the failure of a bridge. A probability statement about the distribution of $\ln(\frac{D_{s_i}}{D_s})$ provides a means of evaluating appropriate design values.

The transpose of the vector of predictor variables at a particular site is as follows:

$$x_0 = [1, \ln(\beta), \ln(\frac{S}{D})] \quad (5-2-3)$$

A 100(1- η) percent upper confidence limit for $\ln(\frac{D_{s_i}}{D_s})$ is computed as (Montgomery and

Peck 1982):

$$\ln\left(\frac{\hat{D}_{s2}}{D_s}\right) + t_{\eta, n-p} \sqrt{\text{Var}(X_0 \hat{\theta})} \quad (5-2-4)$$

Where $\ln\left(\frac{\hat{D}_{s2}}{D_s}\right) = X_0' \hat{\theta}$ is the predicted value of $\ln\left(\frac{D_{s2}}{D_s}\right)$. The values of $\hat{\theta}'_{F_1}$ and $\hat{\theta}'_{R_2}$ computed for a 2-PG are:

$$\hat{\theta}'_{F_1} = [0.22, -0.01, -0.1] \quad (5-2-5)$$

$$\hat{\theta}'_{R_2} = [-0.18, -0.08, -0.1] \quad (5-2-6)$$

The values of $\hat{\theta}'_{F_1}$ and $\hat{\theta}'_{R_2}$ are the transpose of the vector of regression model parameters, $t_{\eta, n-p}$ is the one-sided t-distribution statistic corresponding to an exceedence probability η and $n-p$ degrees of freedom.

The estimated variance of prediction is:

$$\text{Var}(x_0 \hat{\theta}) = \hat{\sigma}^2 [1 + x_0' (X'X)^{-1} x_0] \quad (5-2-7)$$

in which; σ^2 = the residual mean square error (estimated model error). For the 2-variable regression model, using thirty-six measurements, $\sigma_{F_1}^2 = 0.03$, $\sigma_{R_2}^2 = 0.3$. Among a total of seventy tests only data obtained for piers having $d = 0.0127$ m were used to find $\sigma_{F_1}^2$ and $\sigma_{R_2}^2$. In addition, since $S/d=1$ is not used in practice, its related scouring ratio was excluded from the data. It was observed

from the experiments that the linear increase of D_{F_1} and D_{R_1} with β was not the case for $S/d > 1$. However, a multiple regression model which was developed based on the thirty-six tests, was applied to all data in Figs. 5-2-20 and 5-2-21.

One-sided t-distribution statistics for $n-p=70-3=67$, degrees of freedom and several confidence levels are given in Table 5-2-2. An upper estimate of $\ln(\frac{D_{s_g}}{D_s})$ is obtained from equation 5-2-7 by selecting an appropriate confidence level (95 percent), and obtaining the corresponding t statistic from Table 5-2-2. Use of an upper limit of scour depth for design of pier foundations will provide a factor of safety needed to insure against failure of the structure.

5-2-3-3 Prediction Equation for a 2-PG

Parameters a, b, and c obtained for a 2-PG were substituted in equation 5-2-2. Therefore, equation 5-2-2 for the front and rear piers can be expressed as:

$$\frac{\hat{D}_{SF_1}}{D_s} = 1.25\beta^{-0.01} \left(\frac{S}{D}\right)^{-0.1} \quad (5-2-8)$$

$$\text{and} \quad \frac{\hat{D}_{SR_2}}{D_s} = 0.92\beta^{0.08} \left(\frac{S}{D}\right)^{-0.1} \quad (5-2-9)$$

Using equations (5-2-8 and 5-2-9) the values of $\frac{D_{SF_1}}{D_s}$ and $\frac{D_{SR_2}}{D_s}$ were computed for all

the data. These values are plotted against the observed values in Figs. 5-2-20 and 5-2-21. A large majority of the data fall within $\pm 10\%$ and $\pm 15\%$ of the predictions from the relationship proposed in this study, for the front and rear piers, respectively. Applying the F-test for the abovementioned prediction equations (and those which will be presented in the following sections for the cases of 3-, 4-, and 6-PGs) was justified, since it was determined that a linear relationship exists between the response $\ln(\frac{D_s}{D_s})$ and the regressors ($\ln\beta$ and $\ln(S/d)$).

One data point from the present study and two data points from Hannah's (1978) data were found as outliers in Figs. 5-2-20 and 5-2-21. The reason for being outliers is that all data were considered in these figures included different pier diameters (0.018 and 0.033 m), and even data for $S/d=1$.

The value of D_s may be obtained using any of the empirical equations for a single pier. Equations 5-2-11 and 5-2-12 represent the line of perfect agreement in Figs. 5-2-20 and 5-2-21, respectively. Comparison between observed and computed values using the line of perfect agreement has been used by many investigators to evaluate the local scour depth equations, including: Kothyari et al. (1992b), Laursen (1984), Li et al. (1986), Melville and Sutherland (1988), Islam et al. (1986), Qadar (1981), and Sturm and Janjua (1994).

5-2-4 Summary and Conclusions:

The effect of varying β on the local scour depth for a 2-PG was investigated for $1 \leq S/d \leq 11$.

Also the effect of varying S/d on the local scour depth was investigated for $S/d = 3, 5, 7$, and 11 (Fig. 5-2-7 and 5-2-8). In this series of tests different pier diameters were used. Although the pier diameters were different, the trend of data was almost identical for $d=12.7, 18$ and 33 mm. Compared to the rear pier, the local scour depth for the front pier was less influenced by changes in β (Fig. 5-2-8). The front pier scouring showed different trends of data from those of the rear pier. However, the same general trend of data was observed for all front piers. On the other hand, the rear pier scour depth showed that it was influenced significantly by changing β ($>35^\circ$). The present study and Hannah's (1978) study show different trends of data for $\beta \leq 35^\circ$ from those for $\beta > 35^\circ$ (Fig. 5-2-9). It can be concluded that the different pier diameters did not affect the scour depth significantly for both the front and rear piers.

The effect of varying S/d on the local scour depth for a 2-PG was investigated for a wide range of β . The results of the present study were compared with similar observations by other researchers. A good agreement was noted between the results of the present study and Hannah's (1978) study. The data trends for different β ($\beta=0^\circ, 35^\circ, 40^\circ, 45^\circ, 50^\circ$, and 90°) are compared in Figs. 5-2-18 and 19. All cases show the same trend of data for D'_{F_1} , except when $\beta=0^\circ$ (Fig. 5-2-18). In this case for $S/d=1$, the front pier scour depth is equal to that of a single pier. Moreover, an increasing trend was observed for $1 < S/d \leq 4$. In the range $4 \leq S/d \leq 20$, there is no significant difference between D'_{F_1} and D'_{R_2} when $\beta=0^\circ$ or $\beta=35^\circ$. It was evident that when $S/d > 4$ D'_{F_1} was independent of S/d for all β . The changes in D'_{R_2} for $\beta=0^\circ$ was significantly different from those for $\beta > 0^\circ$. It is emphasized that the sheltering effect plays an important role in reducing D'_{R_2} (Fig. 5-2-19). The same trend of data was observed for D'_{R_2} when $\beta > 35^\circ$. The results show that both D'_{F_1}

and D'_{R_1} are not influenced by changes in S/d and are equal to unity when $S/d \geq 4$.

Multiple regression analysis was used to analyse data obtained for a 2-PG. The functional relation for normalized local scour depth is expressed as equation 5-2-1. In addition, equation 5-2-2 was used to show the general form for any prediction equation. Using logarithmic transformation, variables β and S/d were transformed to $\ln(\beta)$ and $\ln(S/d)$, respectively. Therefore, the transpose of the vector of predictor variables at a particular site may be expressed using equation 5-2-3.

The values of the transpose of the vector of regression model parameters for the front and rear piers were substituted in equation 5-2-5. The prediction equations for a 2-PG are:

$$\frac{\hat{D}_{SF_1}}{D_s} = 1.25\beta^{-0.01} \left(\frac{S}{D} \right)^{-0.1} \quad (5-2-8)$$

$$\text{and} \quad \frac{\hat{D}_{SR_2}}{D_s} = 0.92\beta^{0.08} \left(\frac{S}{D} \right)^{-0.1} \quad (5-2-9)$$

Using these equations, $\frac{D_{SF_1}}{D_s}$ and $\frac{D_{SR_2}}{D_s}$ were computed and plotted against the observed values in Figs. 5-2-20 and 5-2-21. Perfect lines of agreement were plotted based on the proposed prediction equations for the front and rear piers. A good agreement was found between the computed and measured values. The majority of the data fell within $\pm 10\%$ and $\pm 15\%$ of the predictions from the relationship proposed in this study for the front and rear pier, respectively.

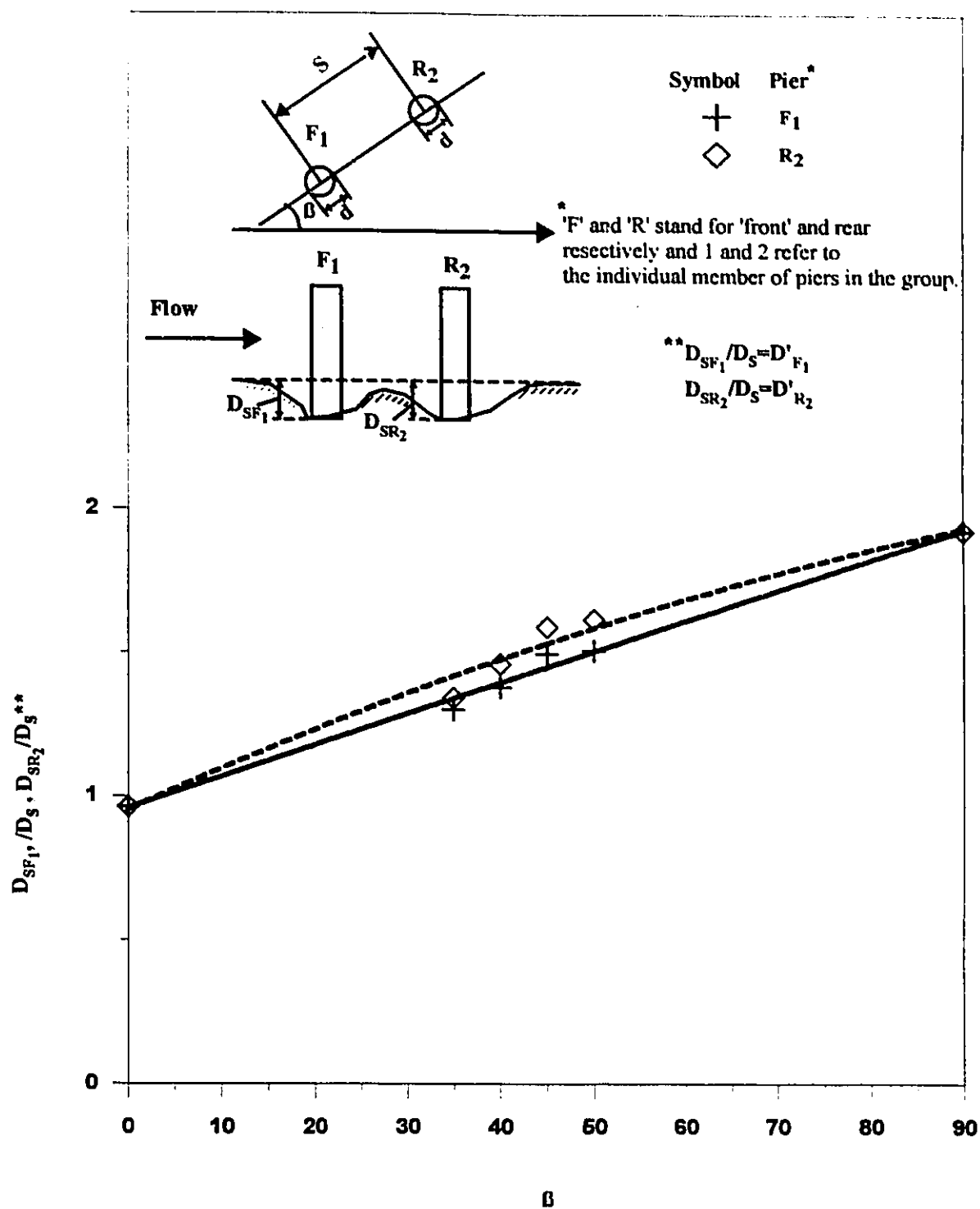


Fig. 5-2-1: Scour depth ratios versus flow angle of attack; groups of 2 piers; ($S/d=1$)

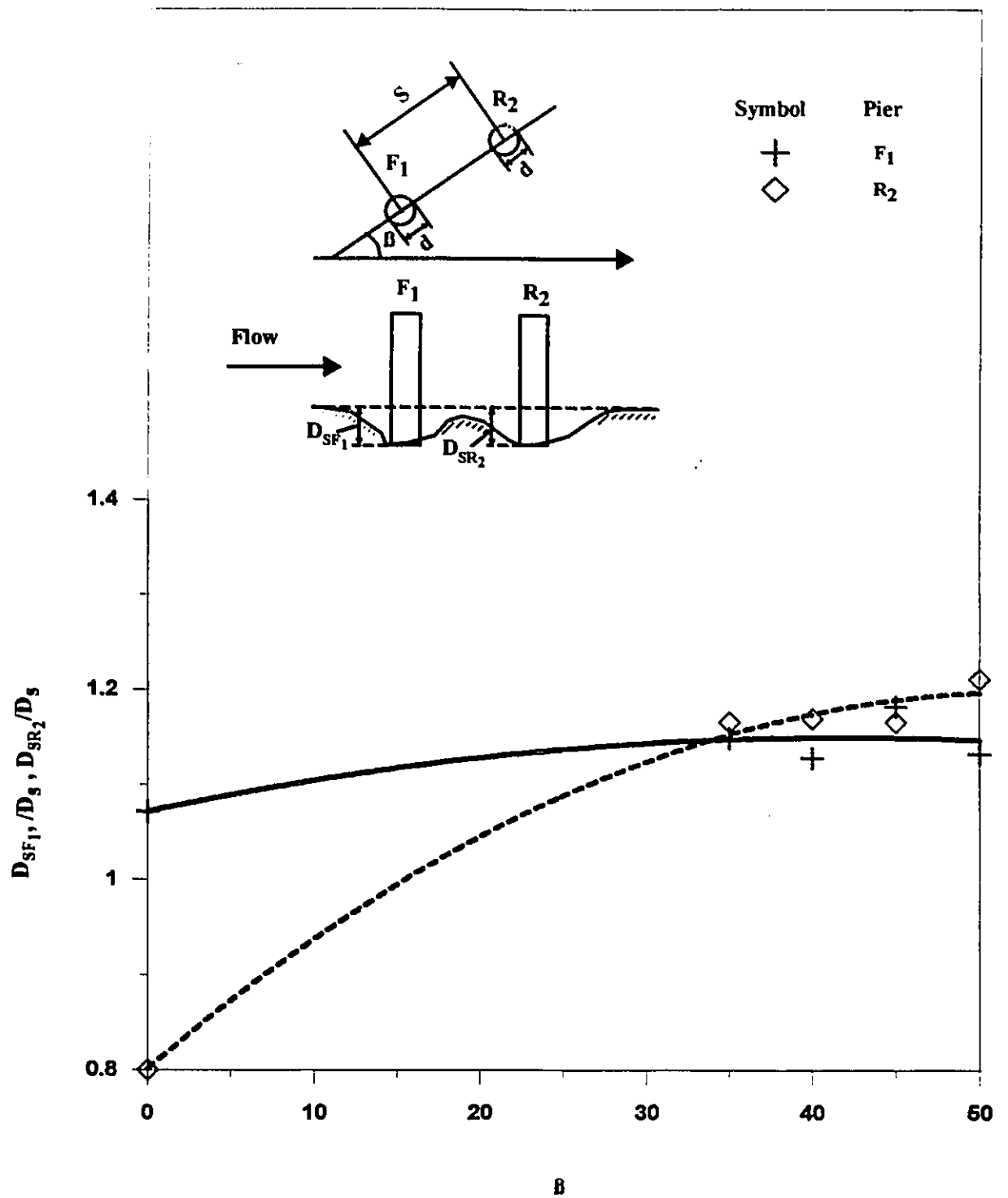


Fig. 5-2-2: Scour depth ratios versus flow angle of attack; groups of 2 piers; ($S/d=2$)

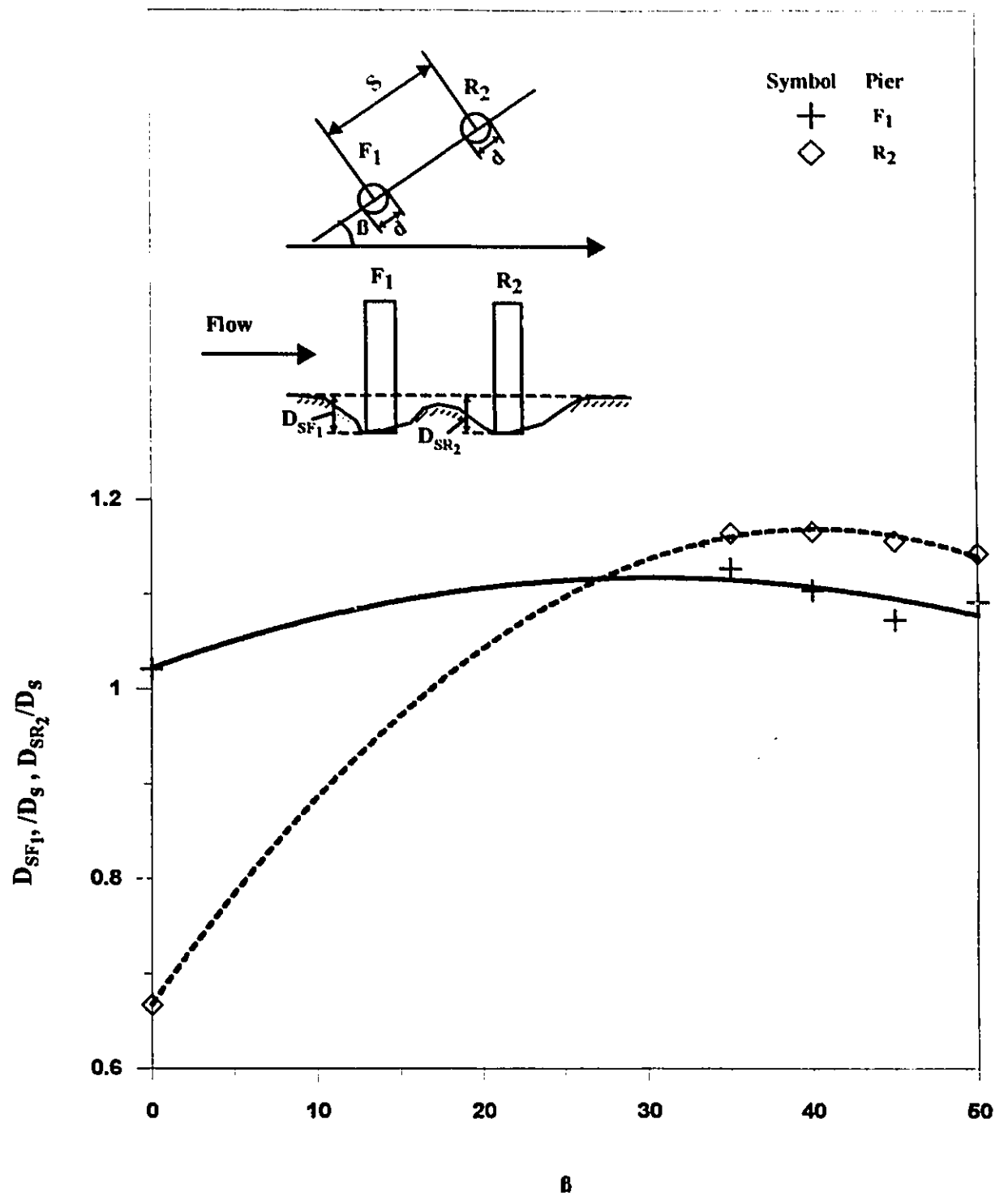


Fig. 5-2-3: Scour depth ratios versus flow angle of attack; groups of 2 piers; ($S/d=3$)

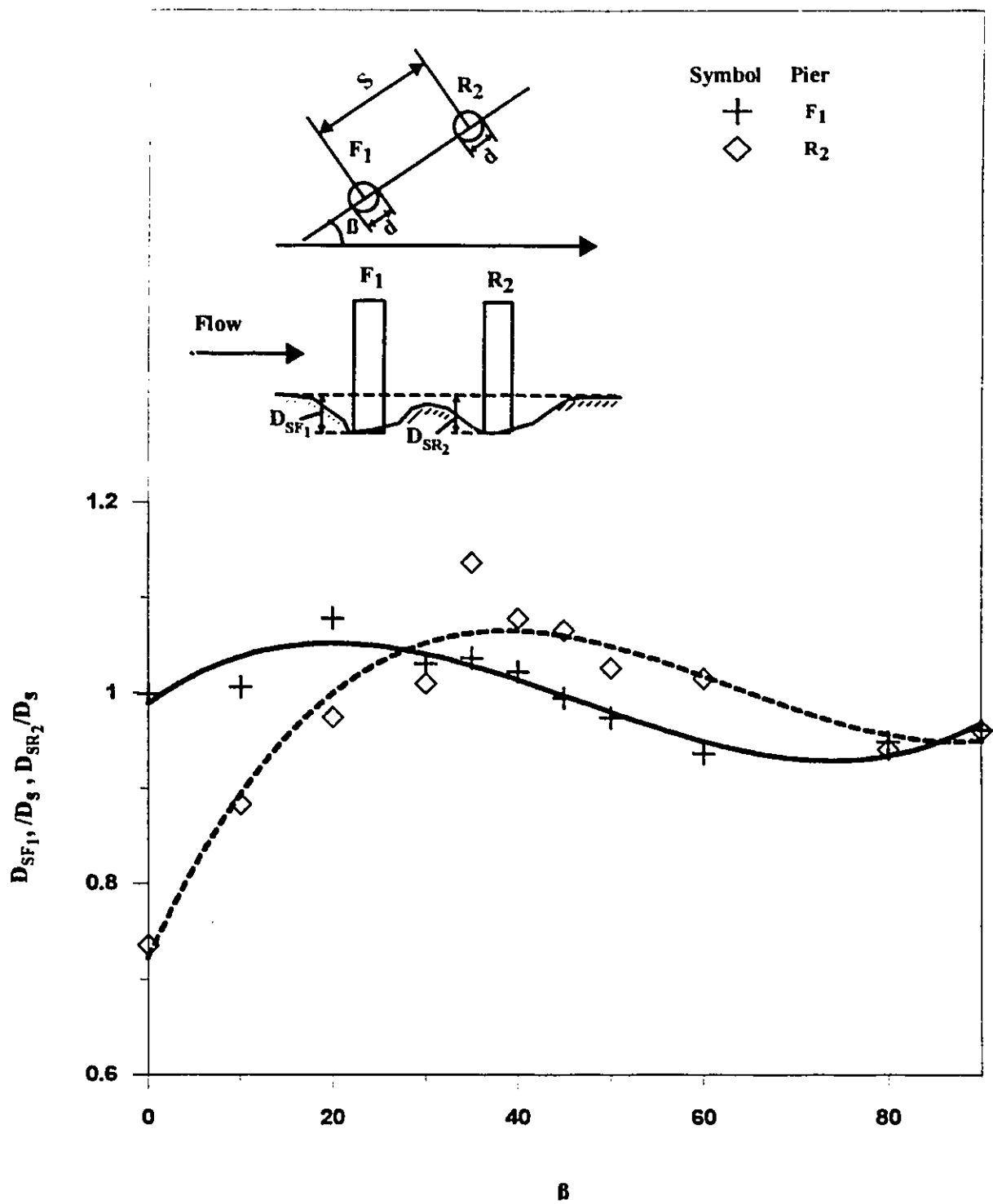


Fig. 5-2-4: Scour depth ratios versus flow angle of attack; groups of 2 piers; ($S/d=5$)

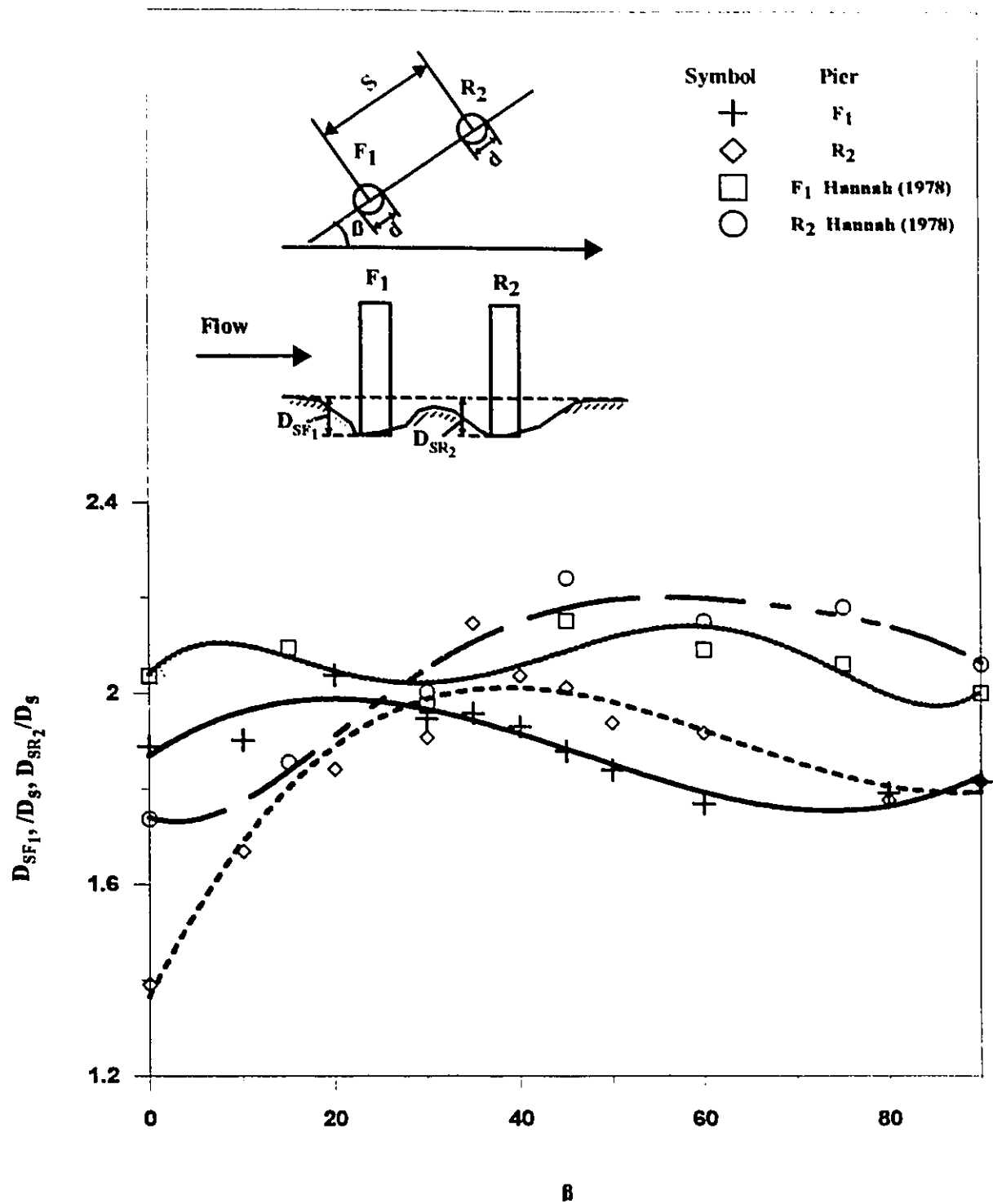


Fig. 5-2-5: Comparison of scour depth ratios versus flow angle of attack; groups of 2 piers; ($S/d=5$)

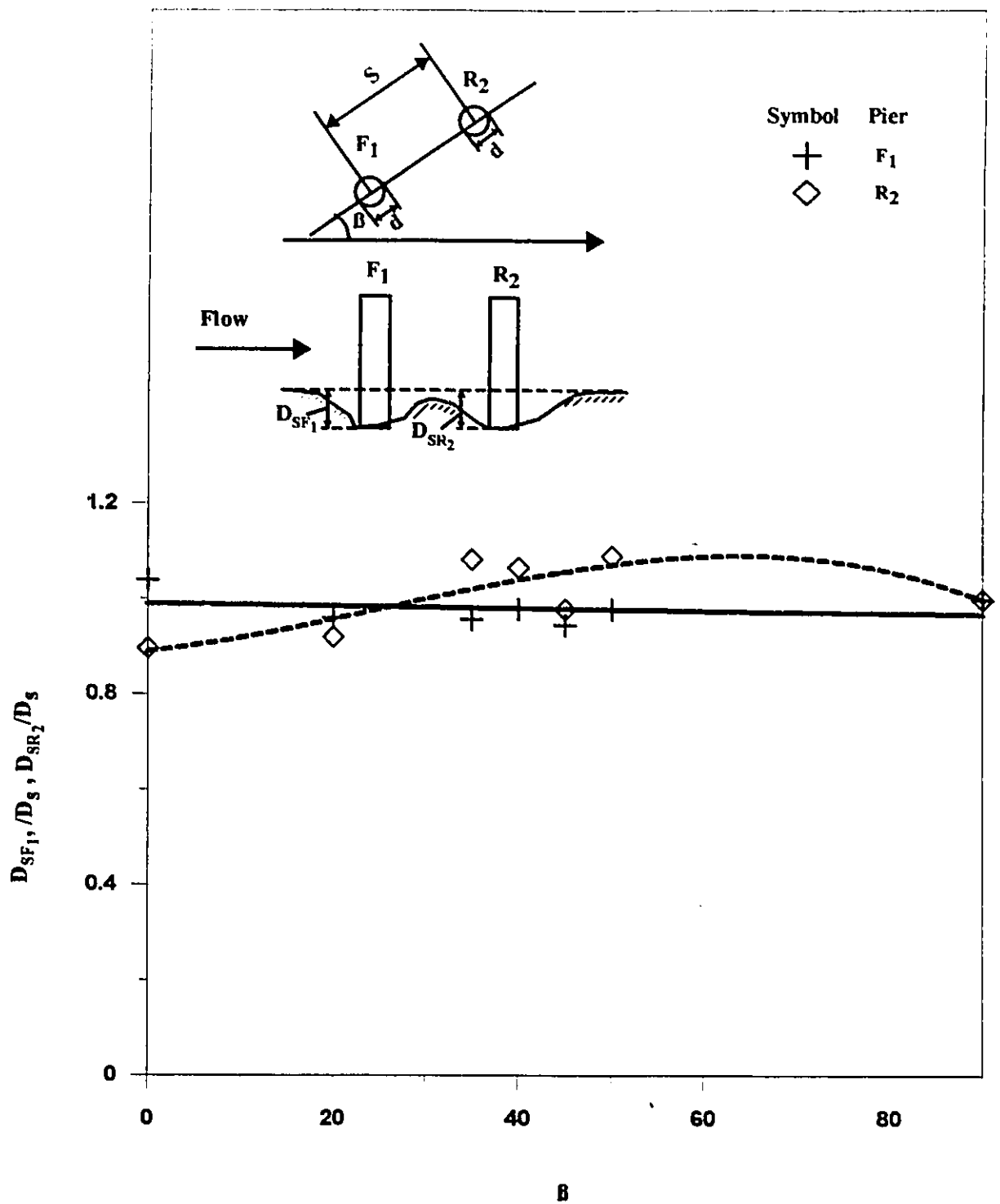


Fig. 5-2-6: Scour depth ratios versus flow angle of attack; groups of 2 piers; ($S/d=7$)

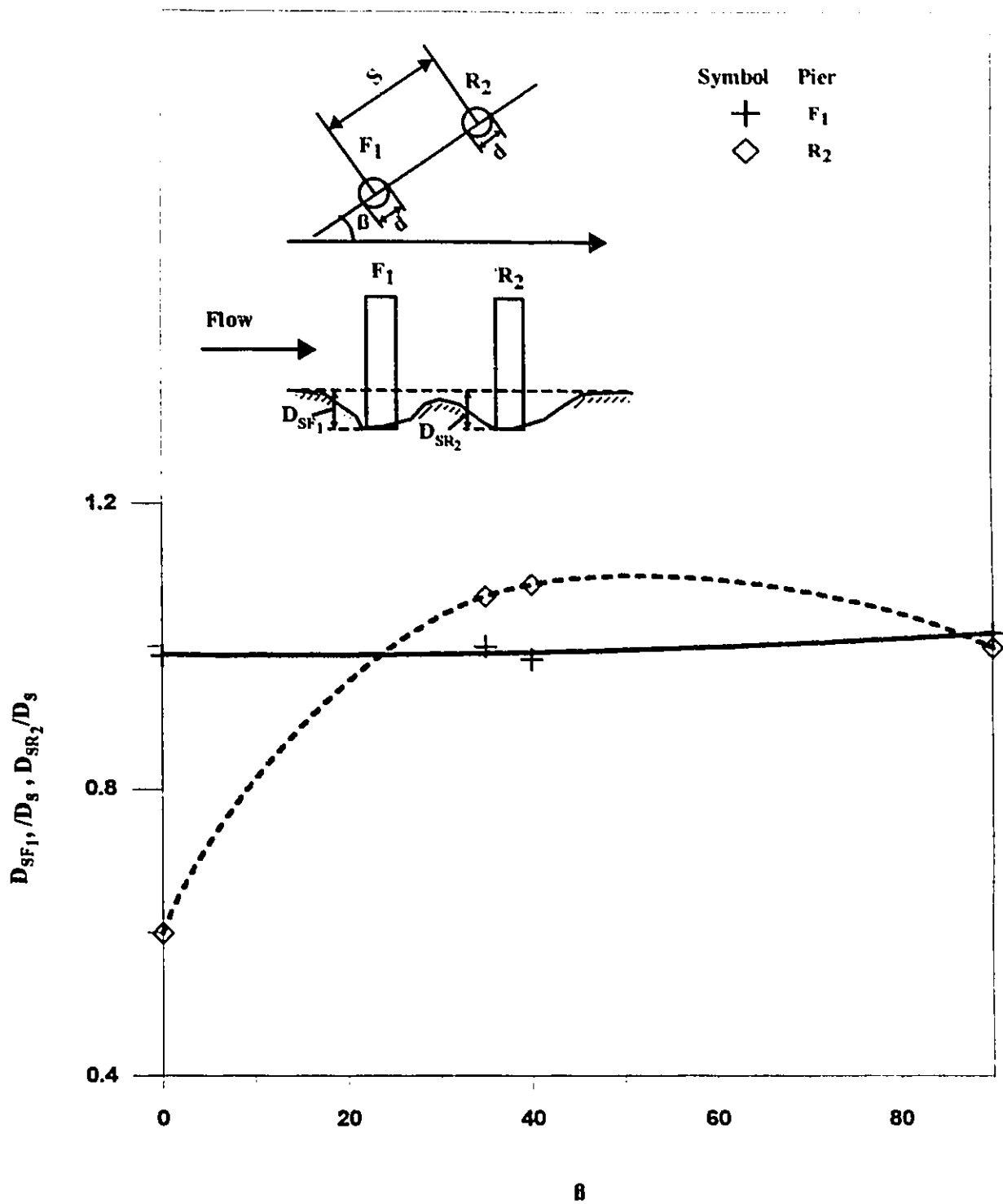


Fig. 5-2-7: Scour depth ratios versus flow angle of attack; group of 2 piers; ($S/d=11$)

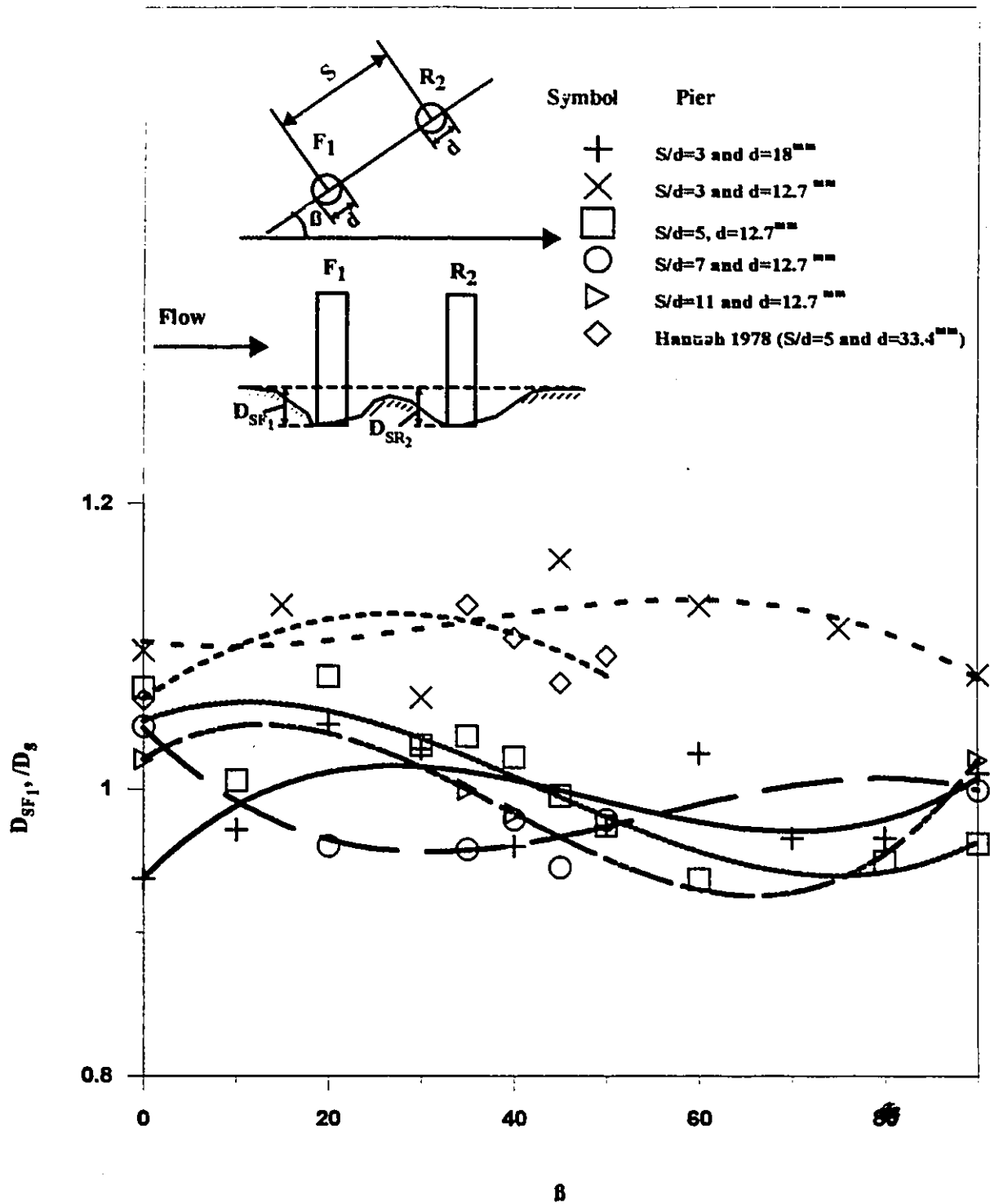


Fig. 5-2-8: Front pier scour depth ratios versus flow angle of attack groups of 2 piers; ($S/d=3, 5, 7$ and 11)

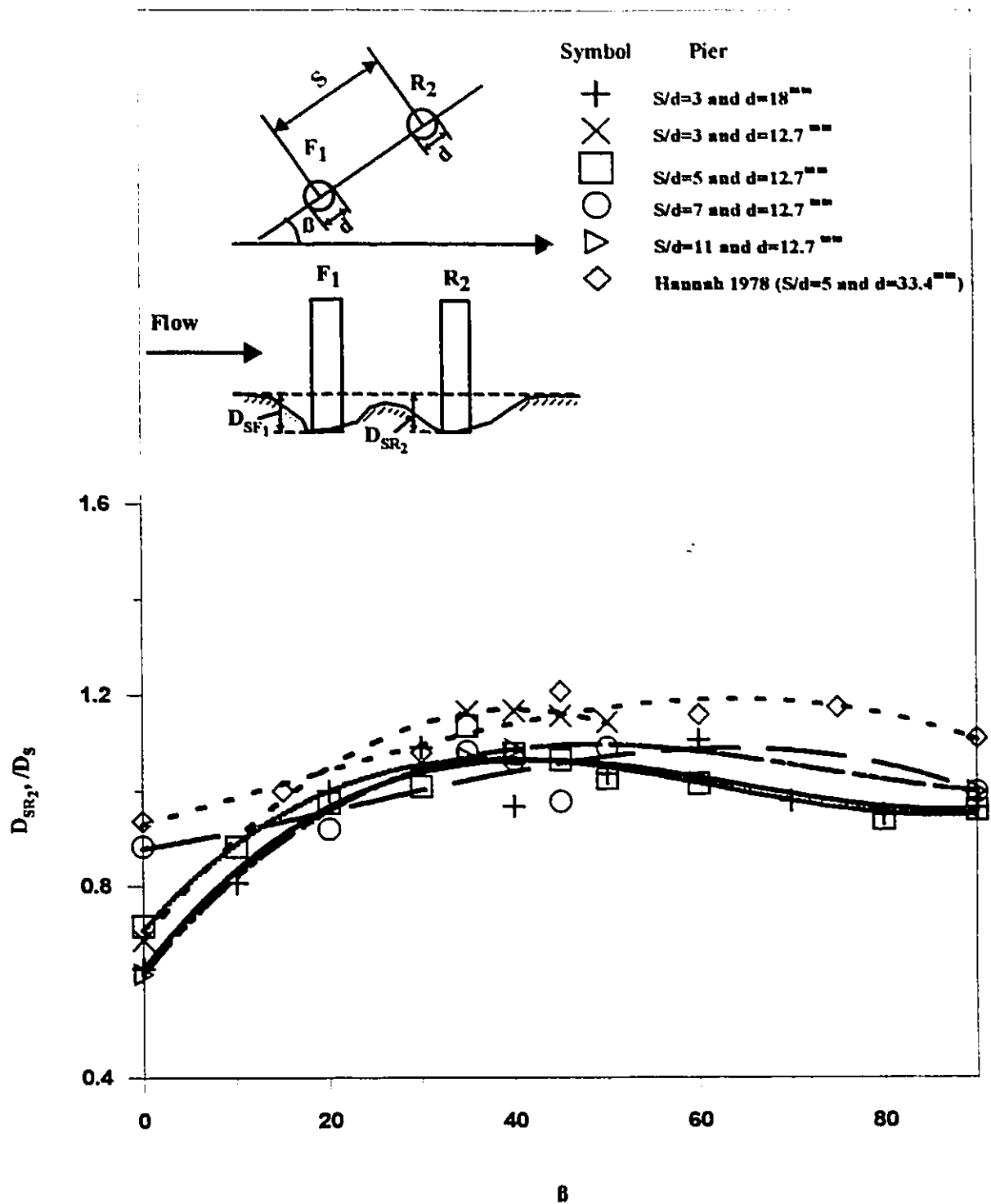


Fig. 5-2-9: Rear pier scour depth ratios versus flow angle of attack groups of 2 piers; ($S/d=3$, 5, 7, and 11)

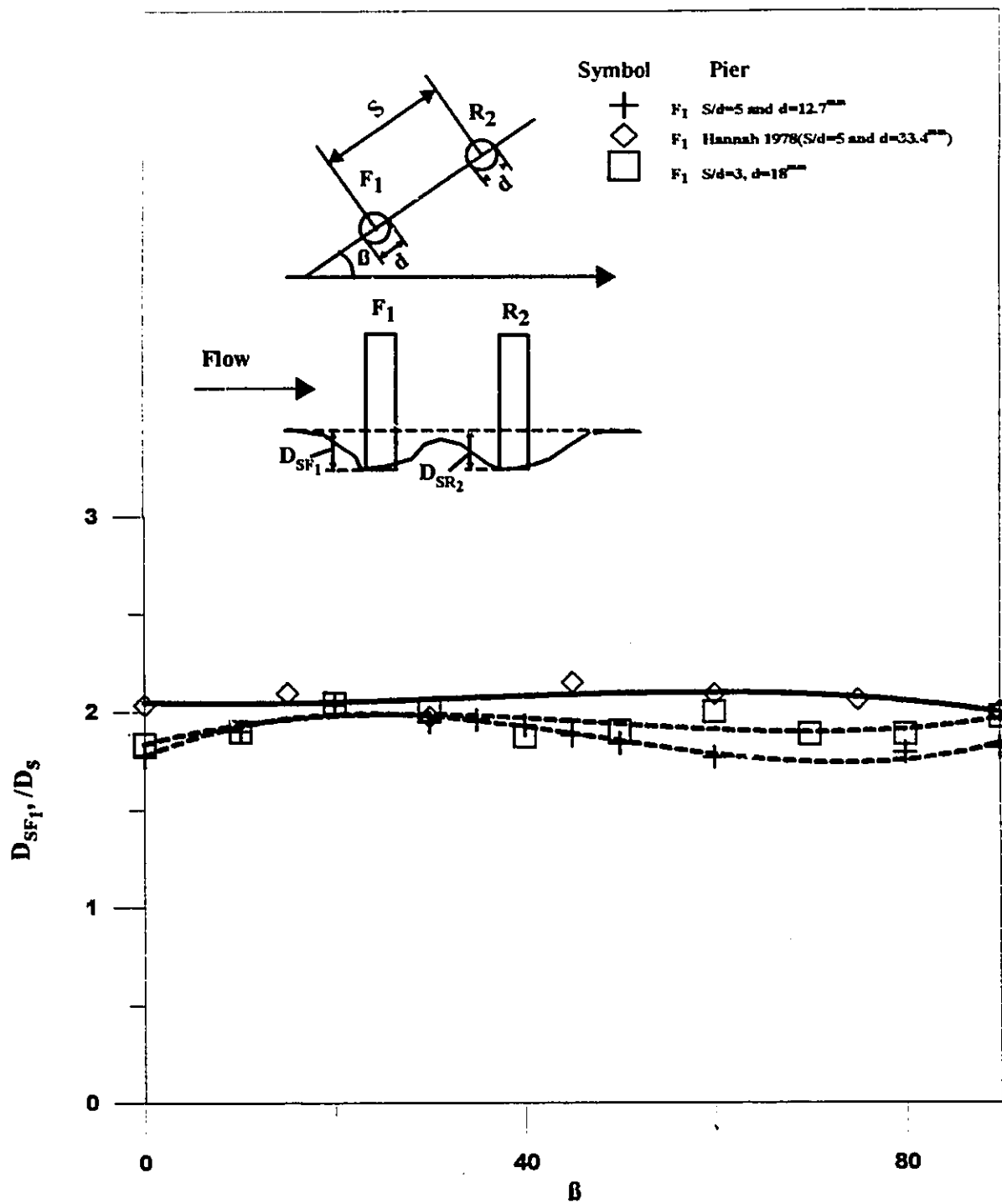


Fig. 5-2-10: Scour depth ratios versus flow angle of attack; groups of 2 piers; (S/d=3 and 5)

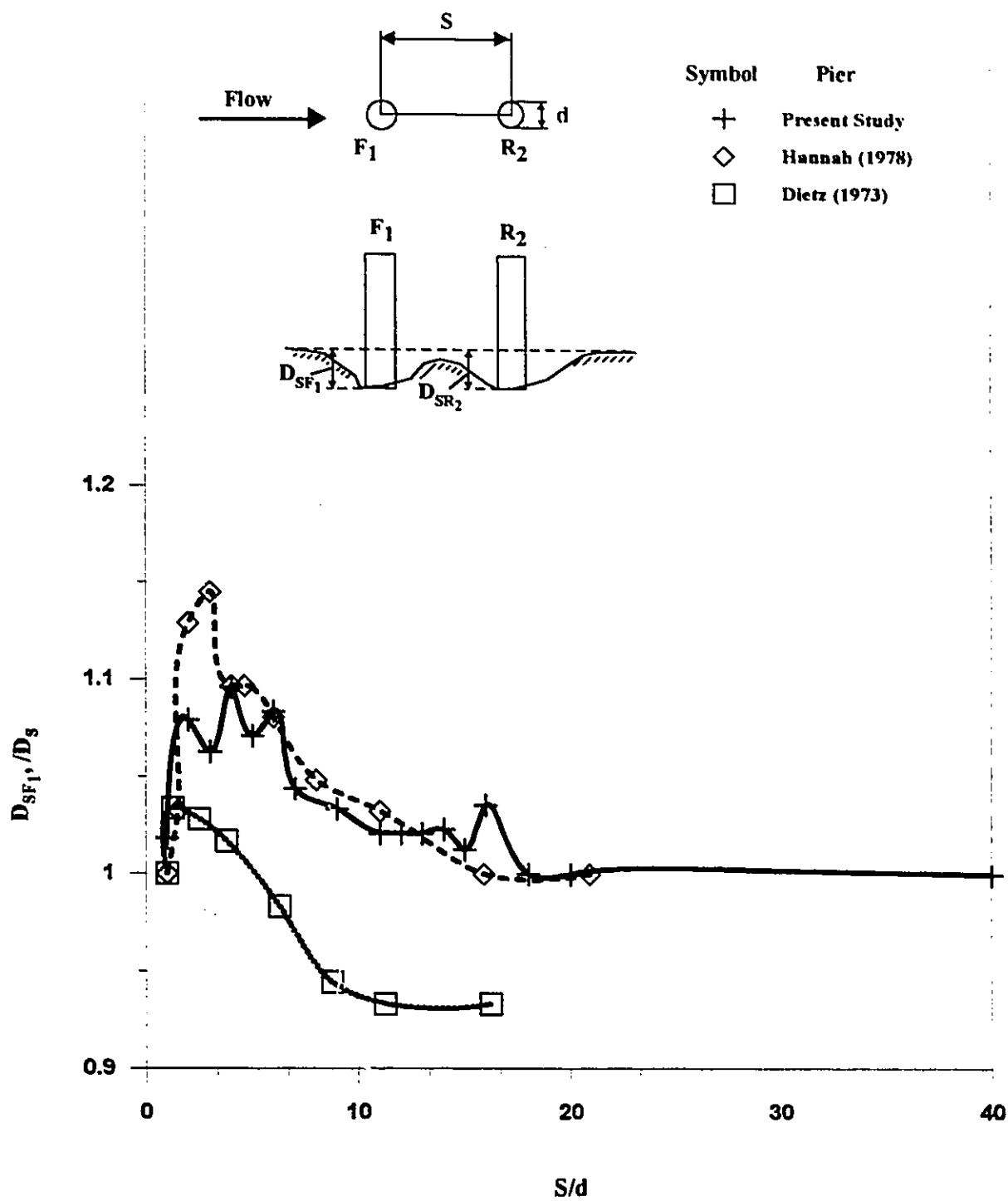


Fig. 5-2-11: Scour depth ratios versus pier spacing/ pier diameter groups of 2 piers; ($\beta=0^\circ$)

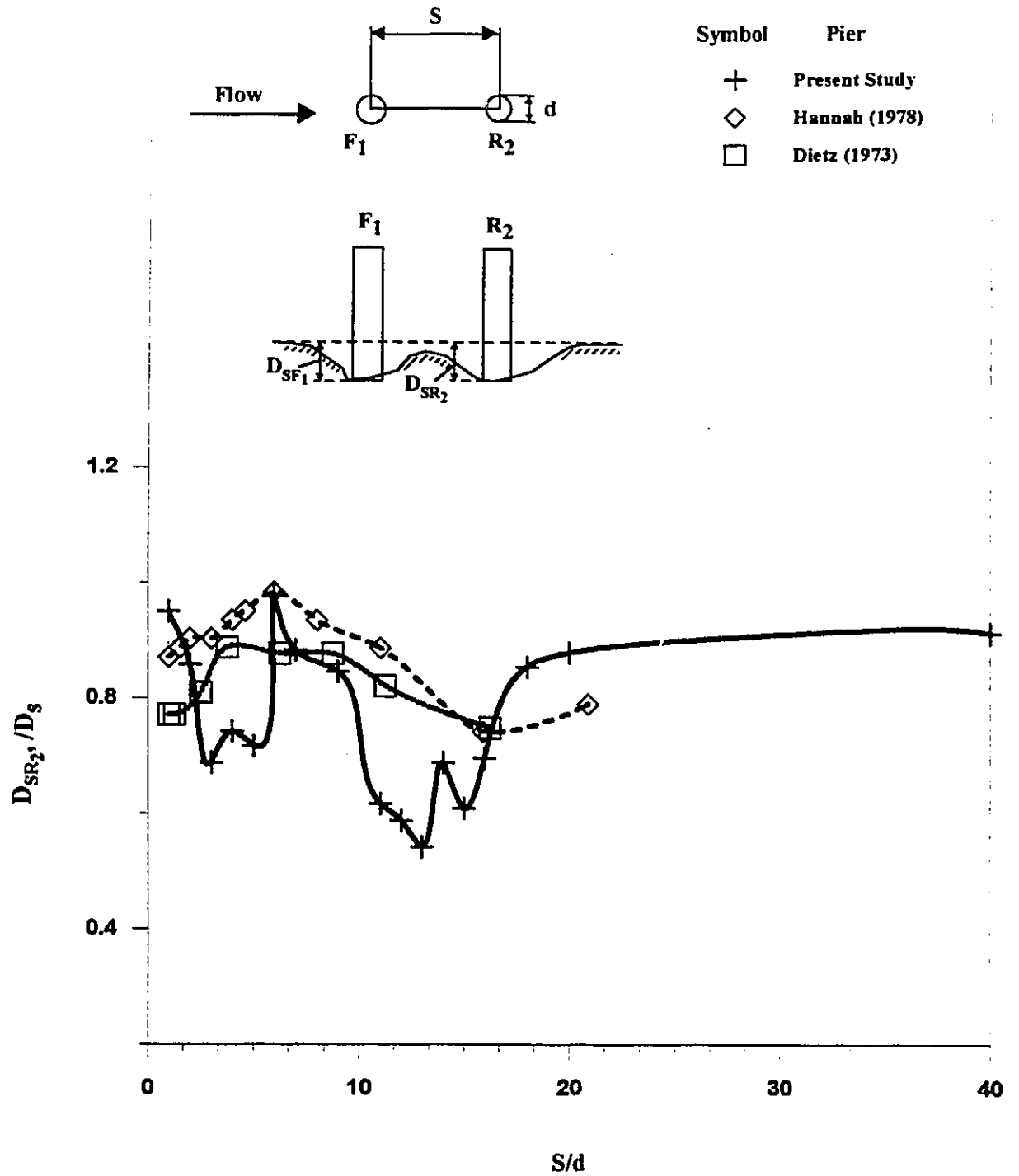


Fig. 5-2-12: Scour depth ratios versus pier spacing/ pier diameter groups of 2 piers; ($B=0^\circ$)

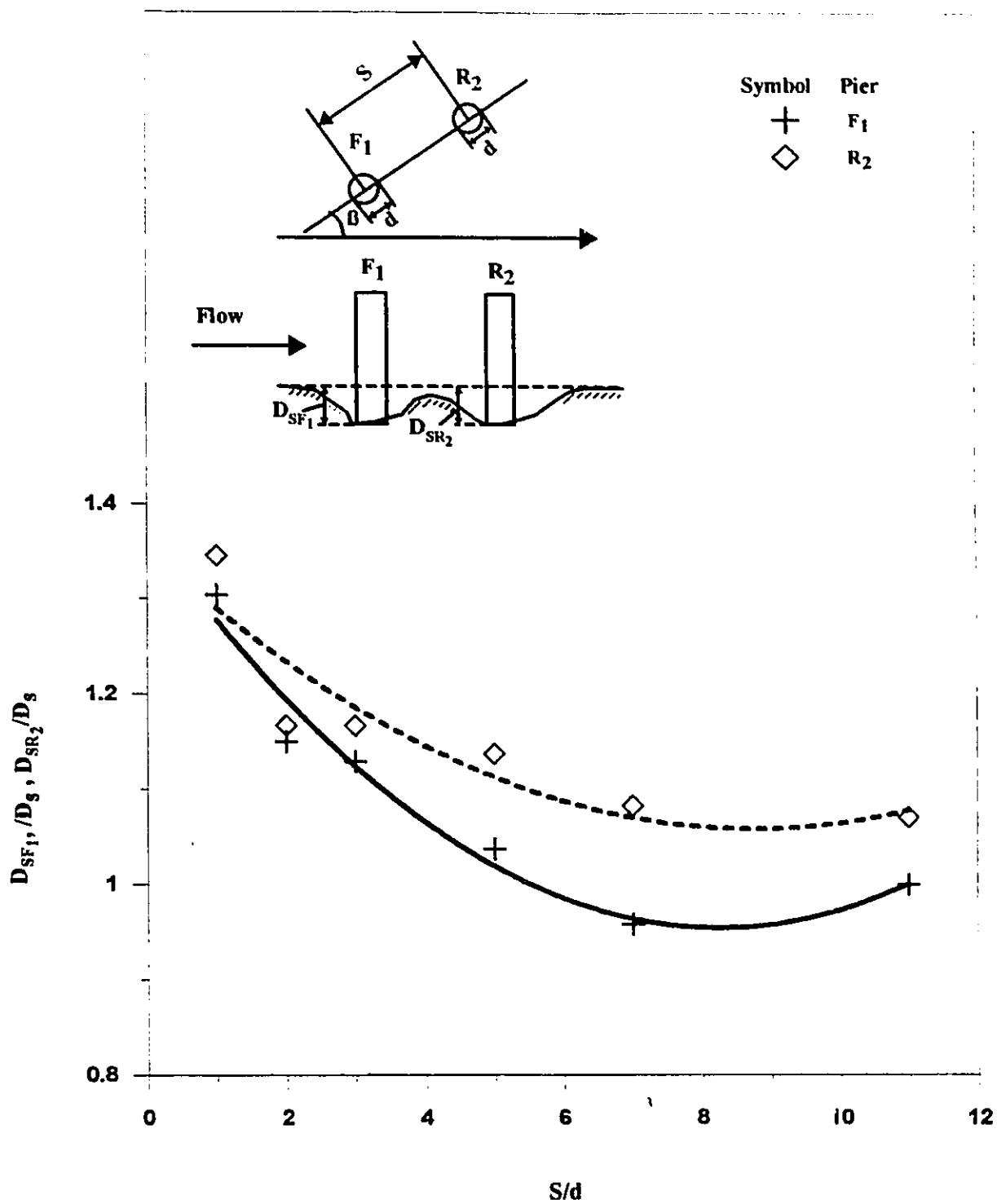


Fig. 5-2-13: Scour depth ratios versus pier spacing/ pier diameter groups of 2 piers; ($\beta=35^\circ$)

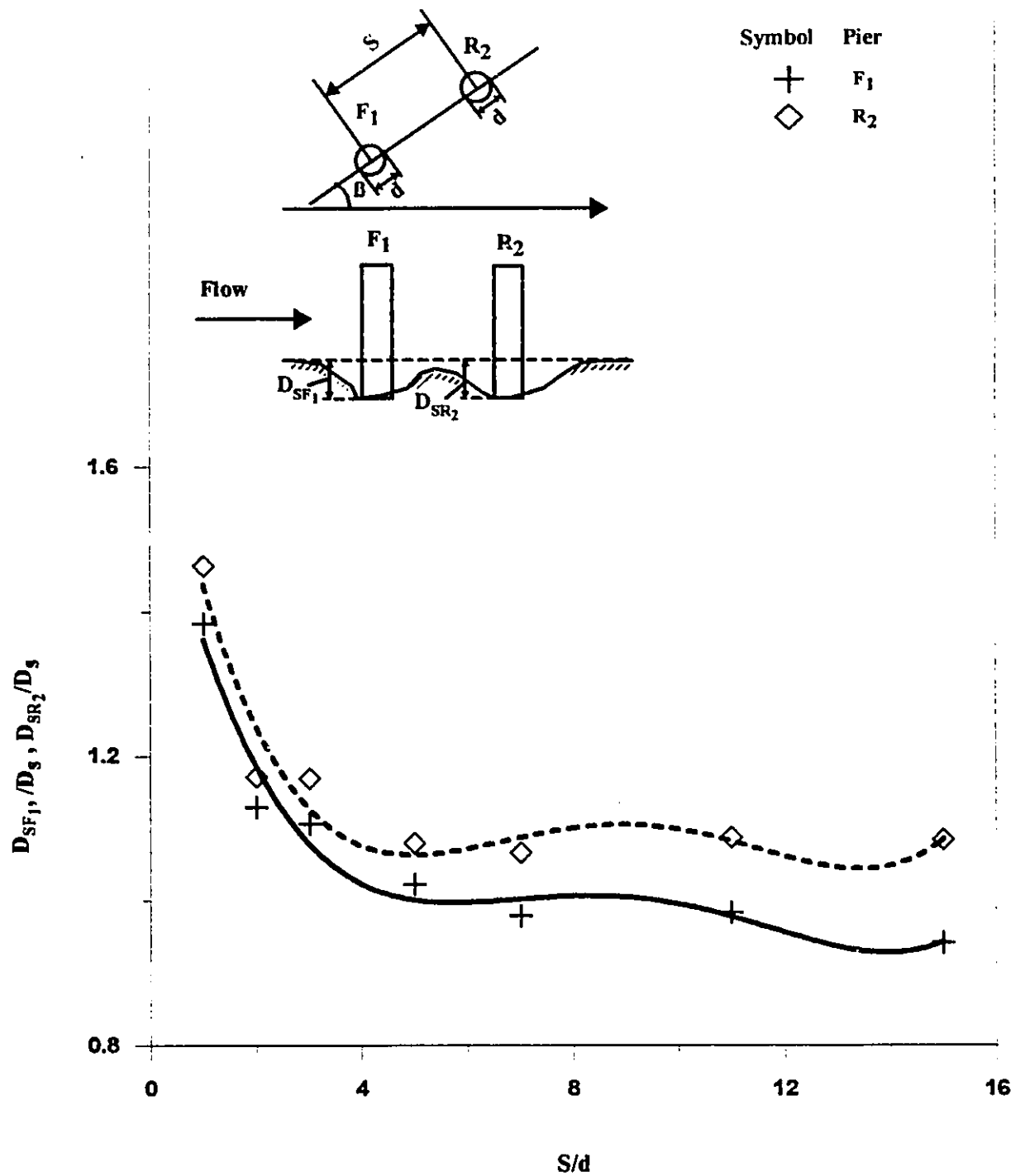


Fig. 5-2-14: Scour depth ratios versus pier spacing/ pier diameter groups of 2 piers; ($\beta=40^\circ$)

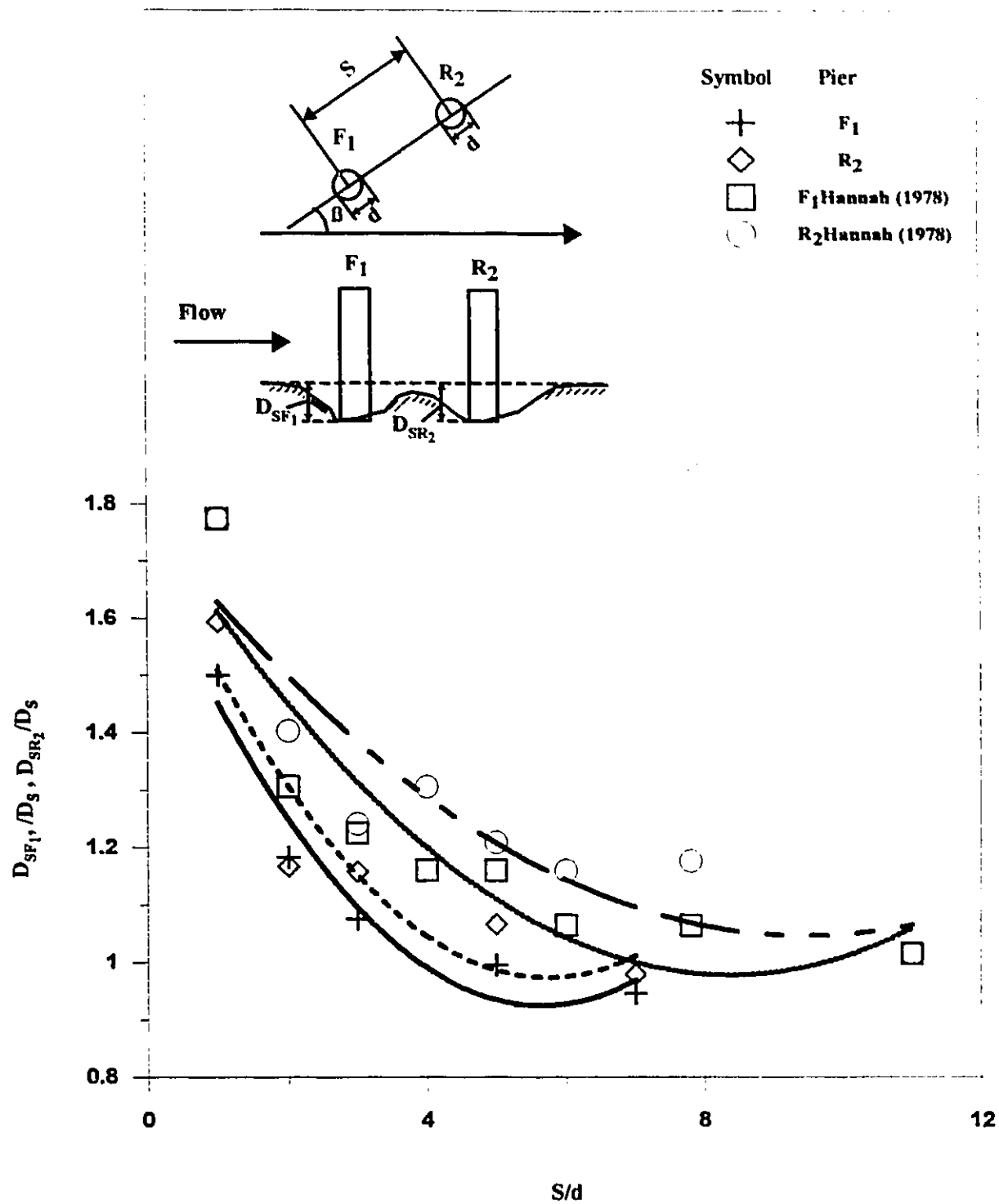


Fig. 5-2-15: Scour depth ratios versus pier spacing/ pier diameter groups of 2 piers; ($\beta=45^\circ$)

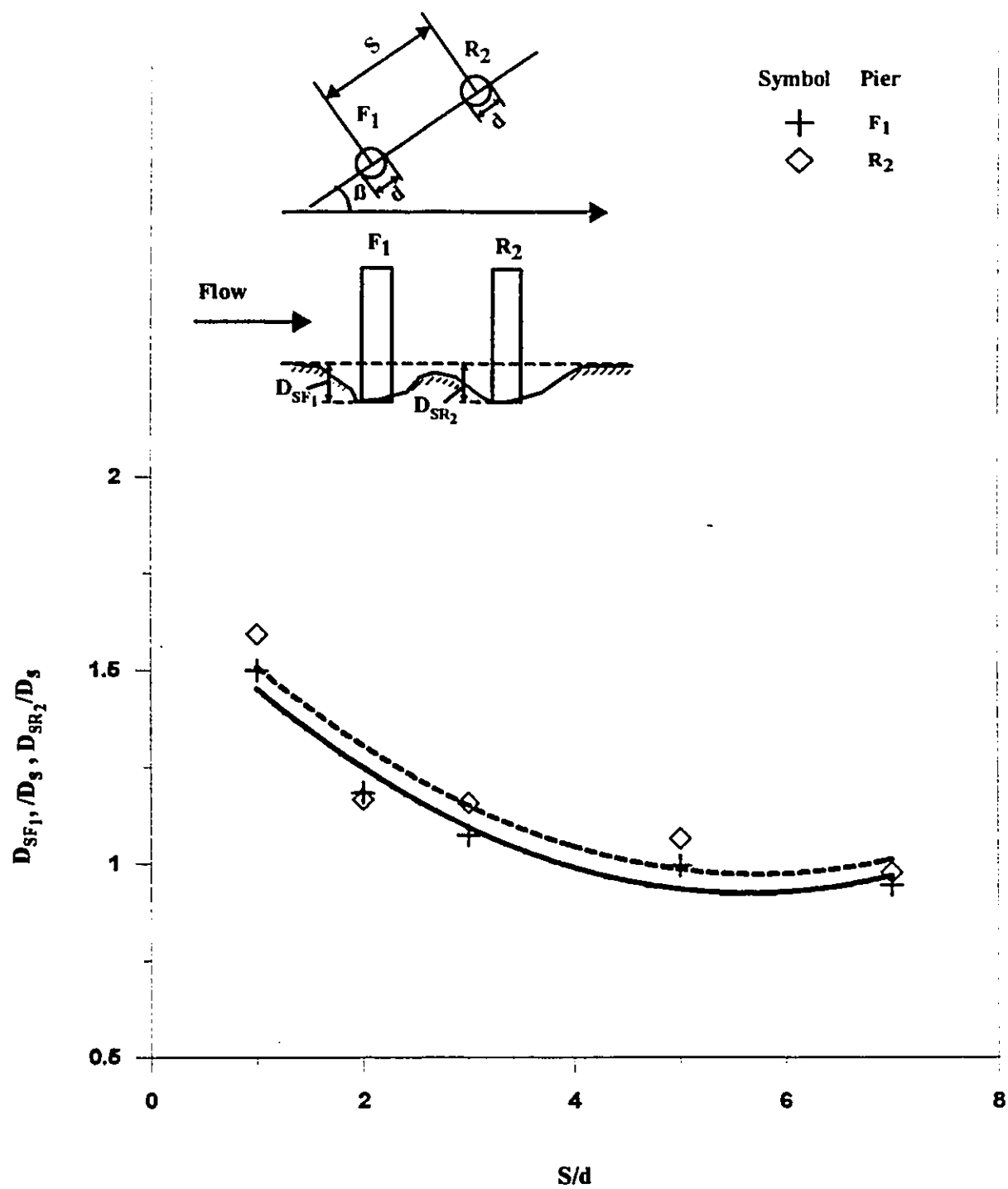


Fig. 5-2-16: Scour depth ratios versus flow angle of attack groups of 2 piers; ($\theta=45^\circ$)

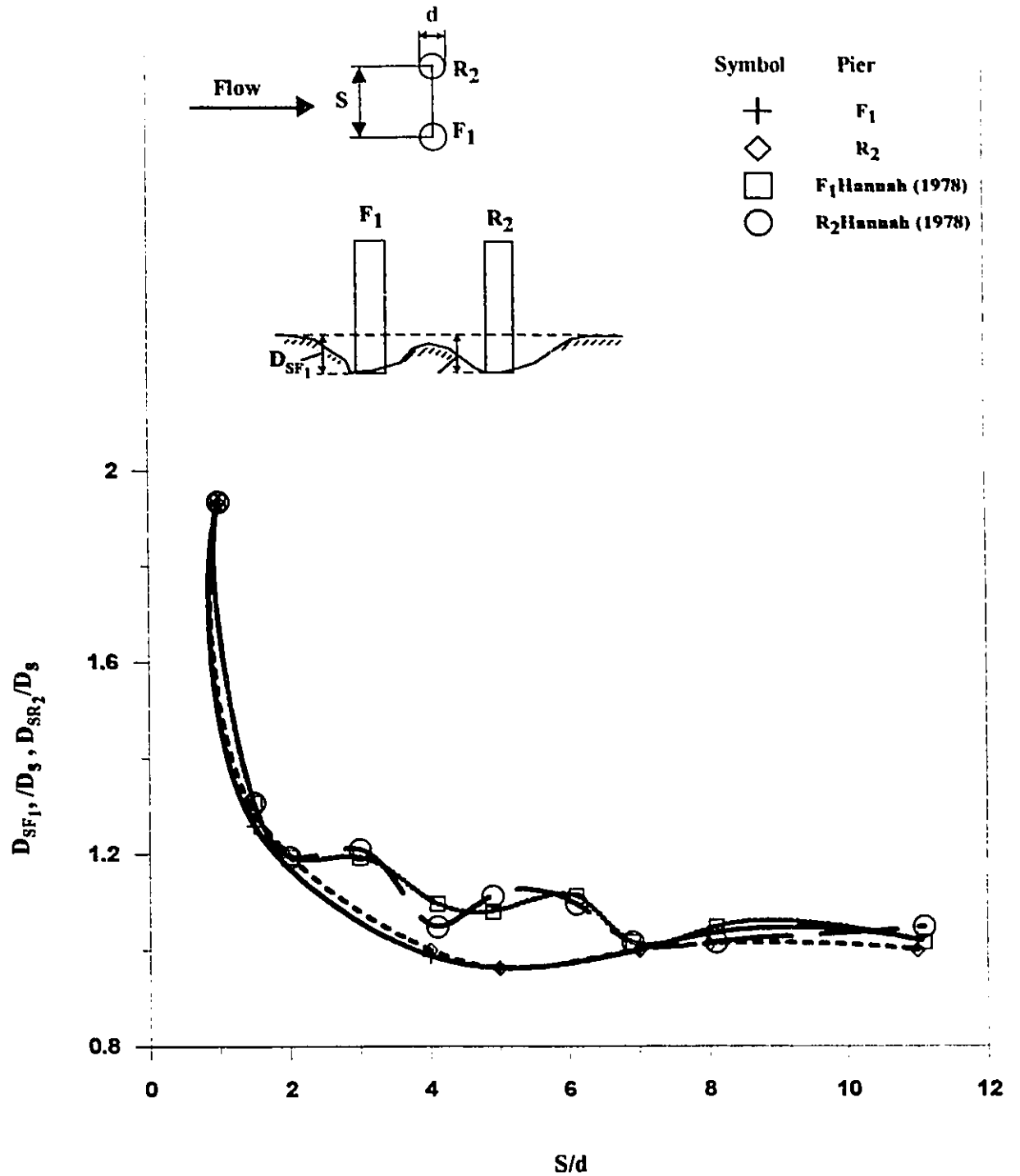


Fig. 5-2-17: Scour depth ratios versus pier spacing/ pier diameter groups of 2 piers; ($\beta=90^\circ$)

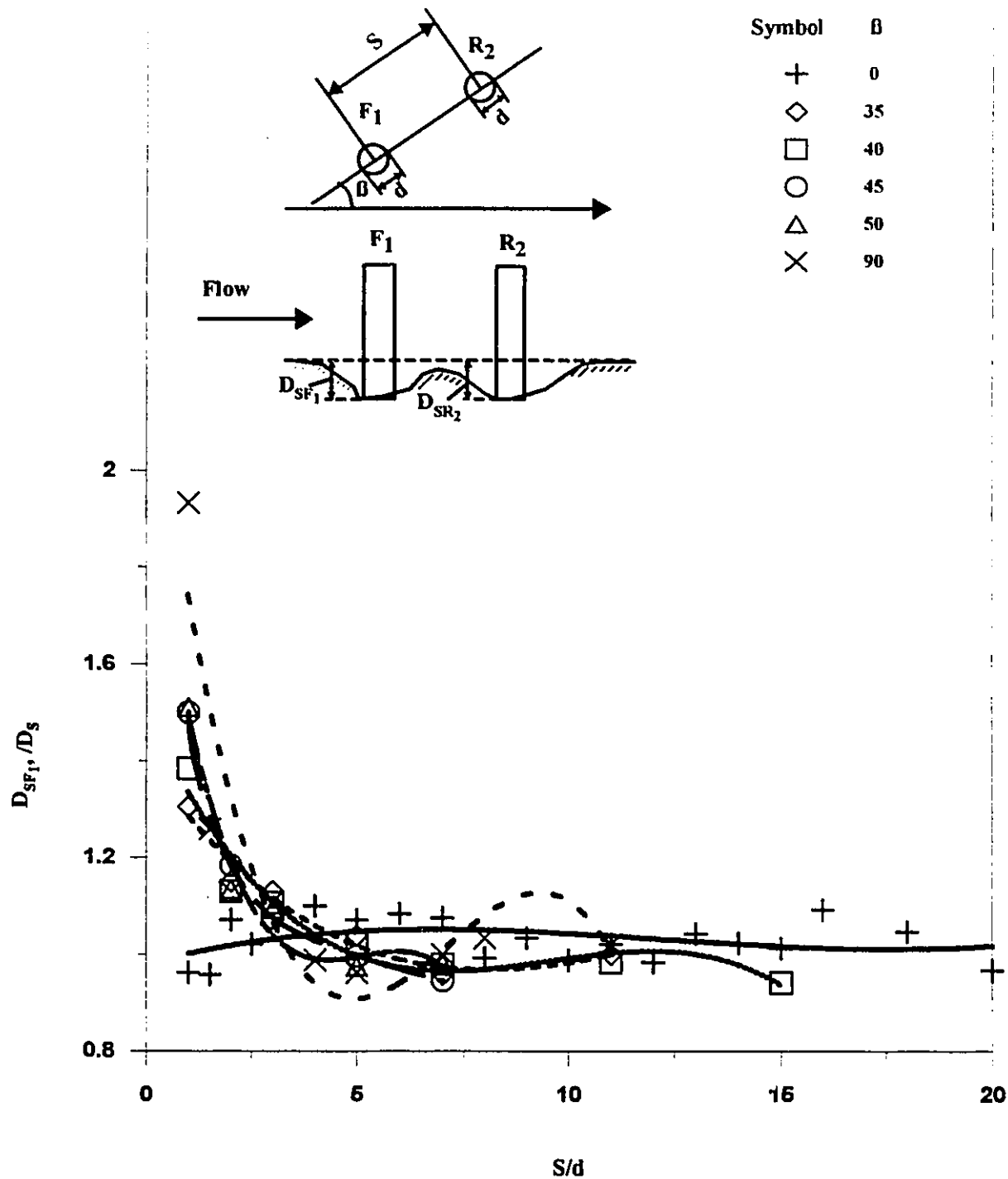


Fig. 5-2-18: Front pier scour depth ratios versus pier spacing/ pier diameter groups of 2 piers; ($\beta=0^\circ, 35^\circ, 40^\circ, 45^\circ, 50^\circ$, and 90°)

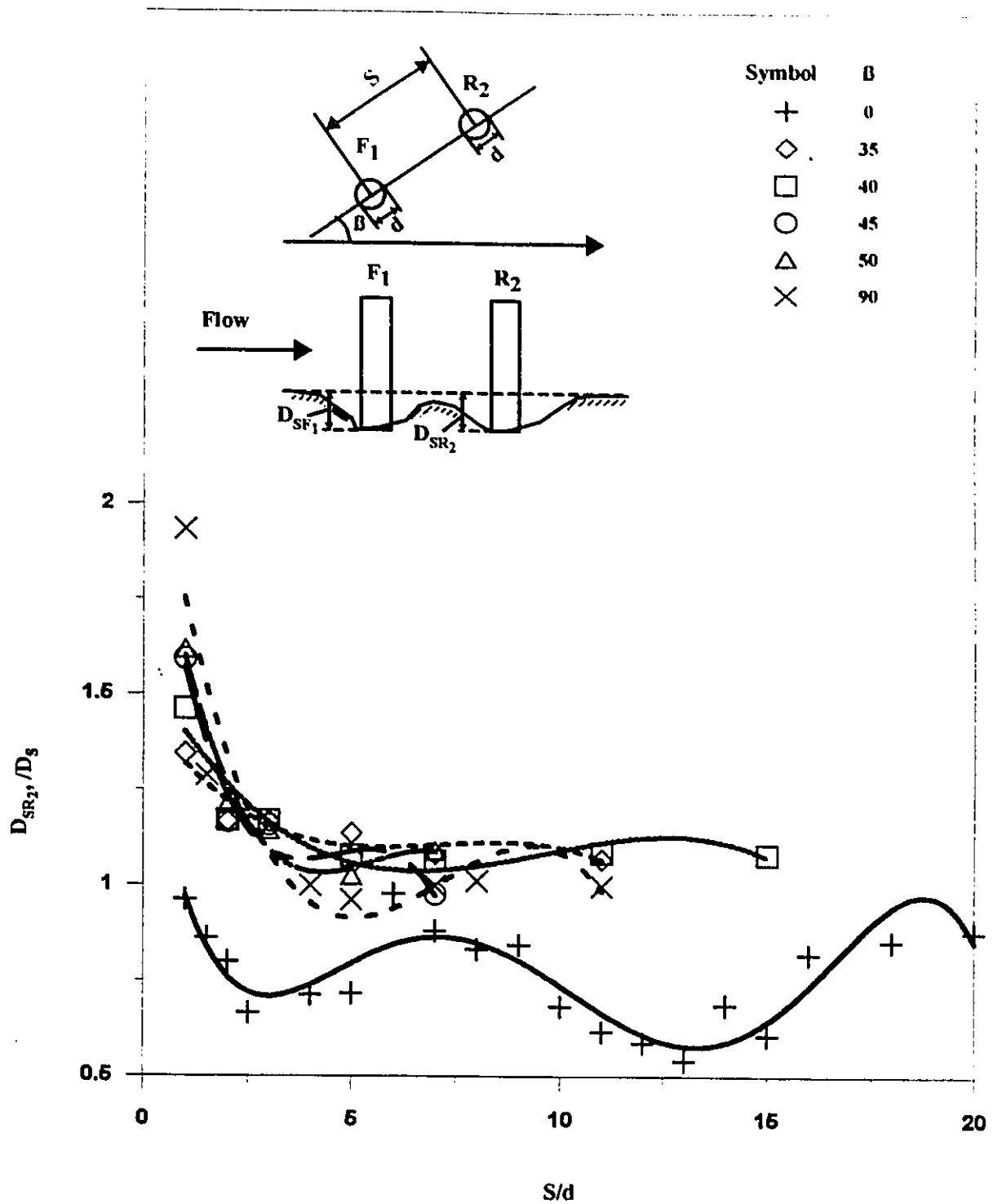


Fig. 5-2-19: Rear pier scour depth ratios versus pier spacing/ pier diameter groups of 2 Piers; ($\beta=0^0, 35^0, 40^0, 45^0, 50^0$, and 90^0)

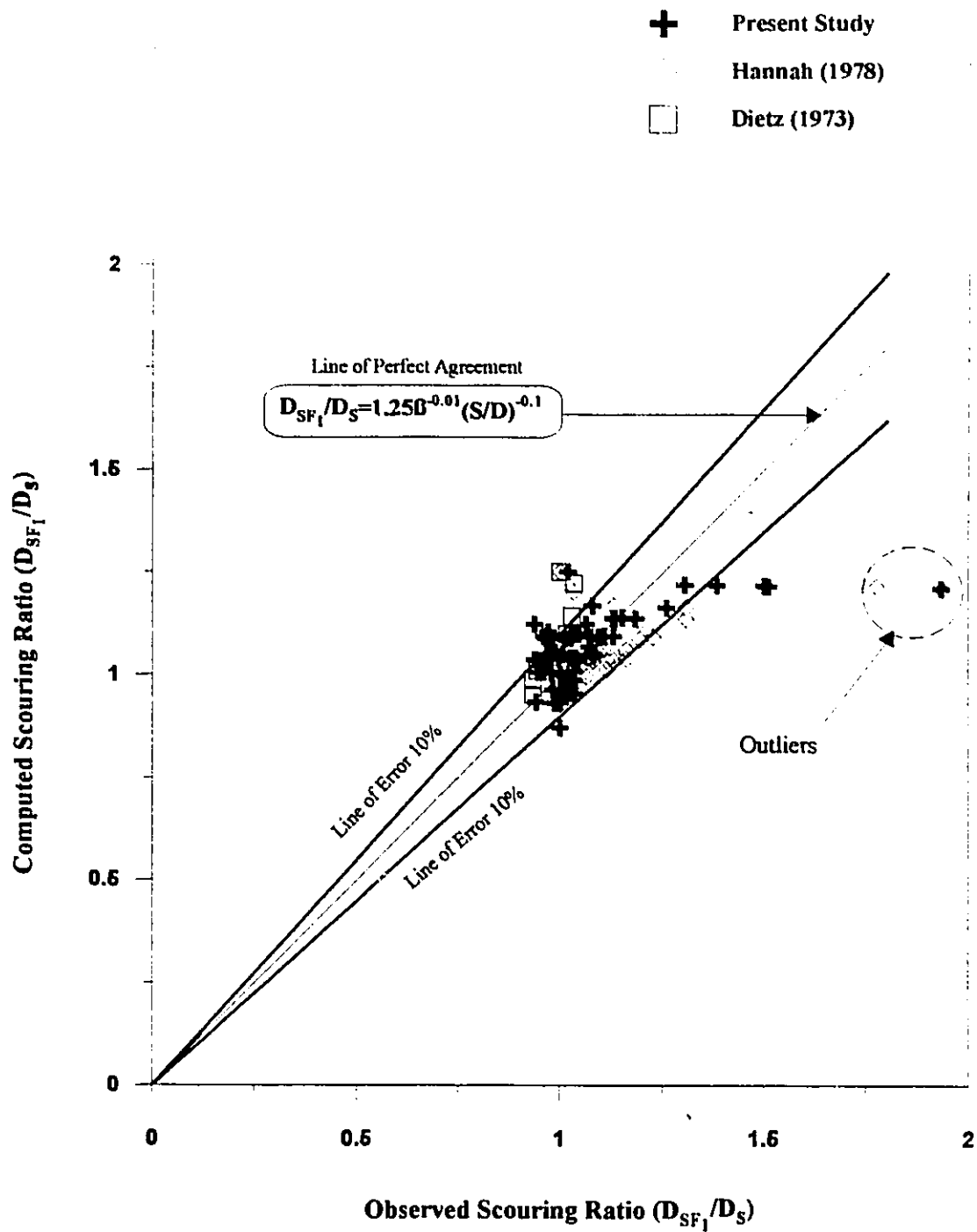


Fig. 5-2-20 : Comparison between computed and measured front pier (F_1) scouring ratios; groups of 2 piers

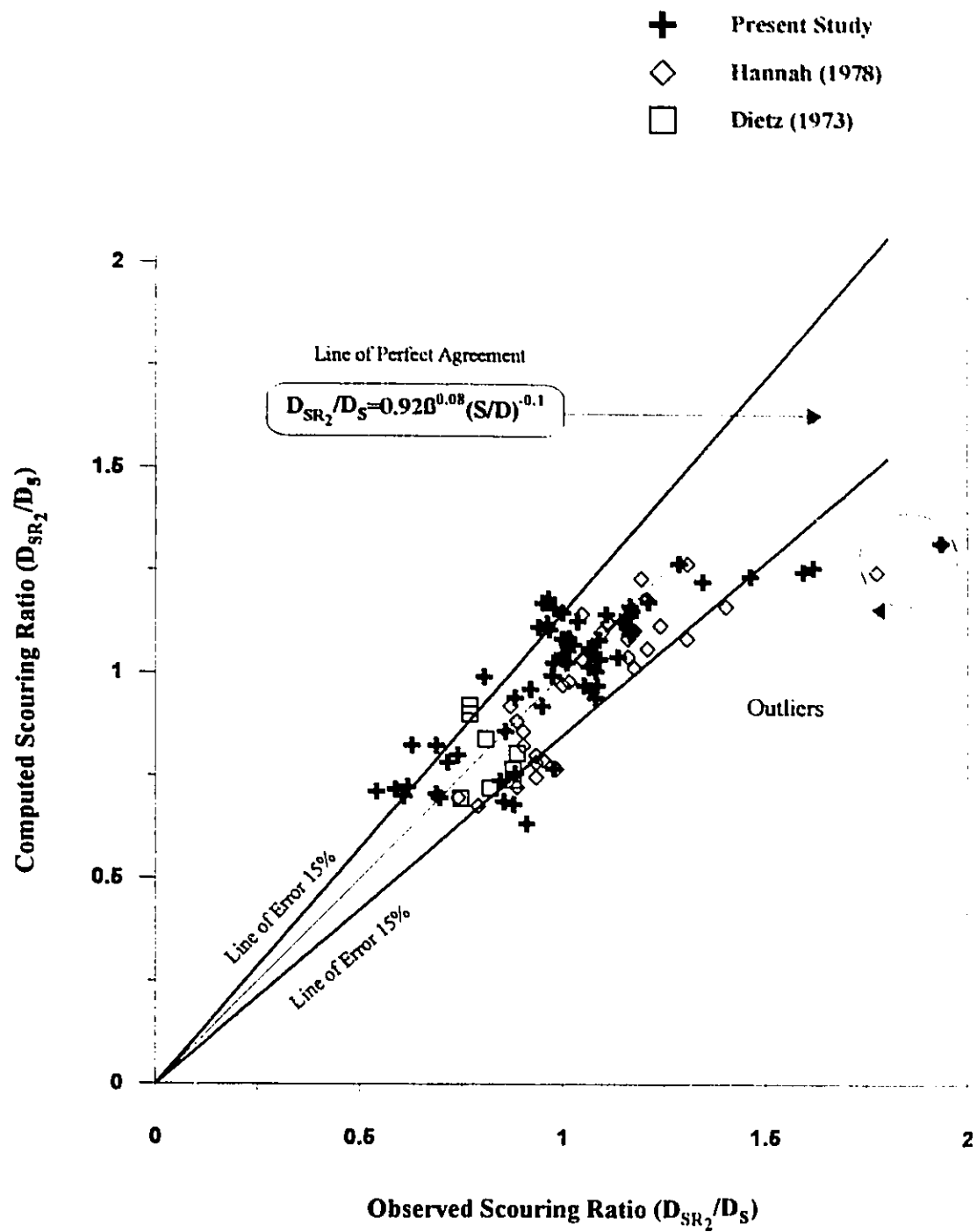


Fig. 5-2-21: Comparison between computed and measured rear pier (R_2) scouring ratios; groups of 2 piers

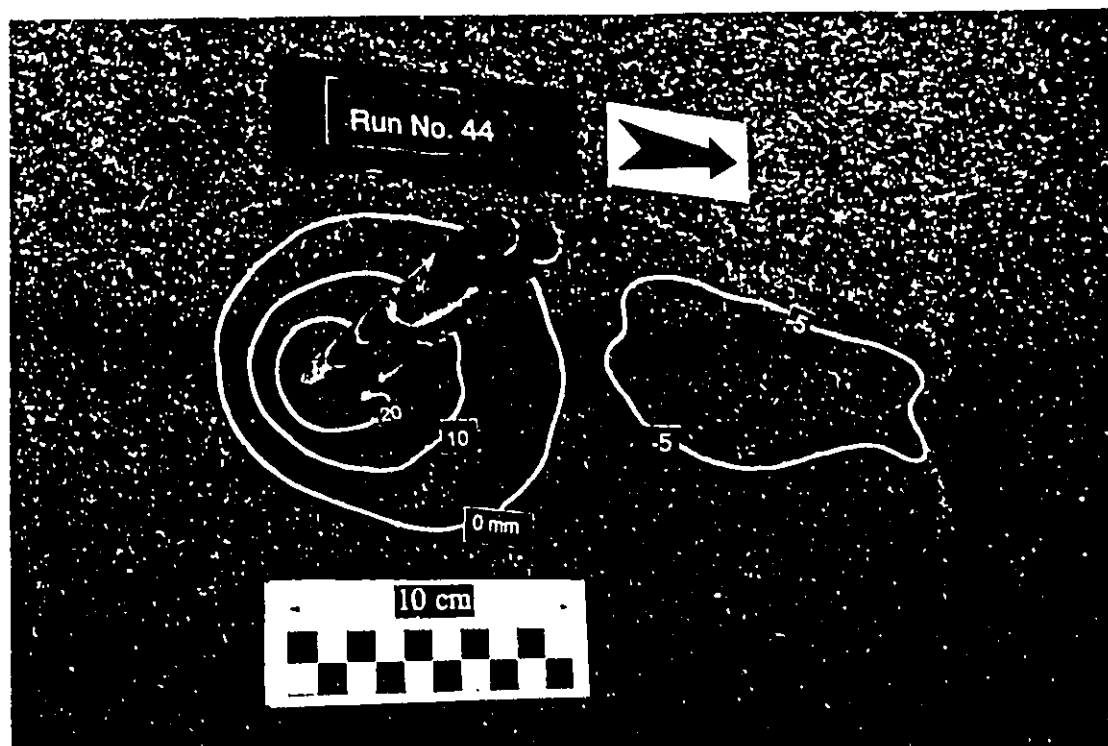


Plate 5-2-1: A group of 2 piers (touching); $S/d=1$ and $\beta=0^\circ$

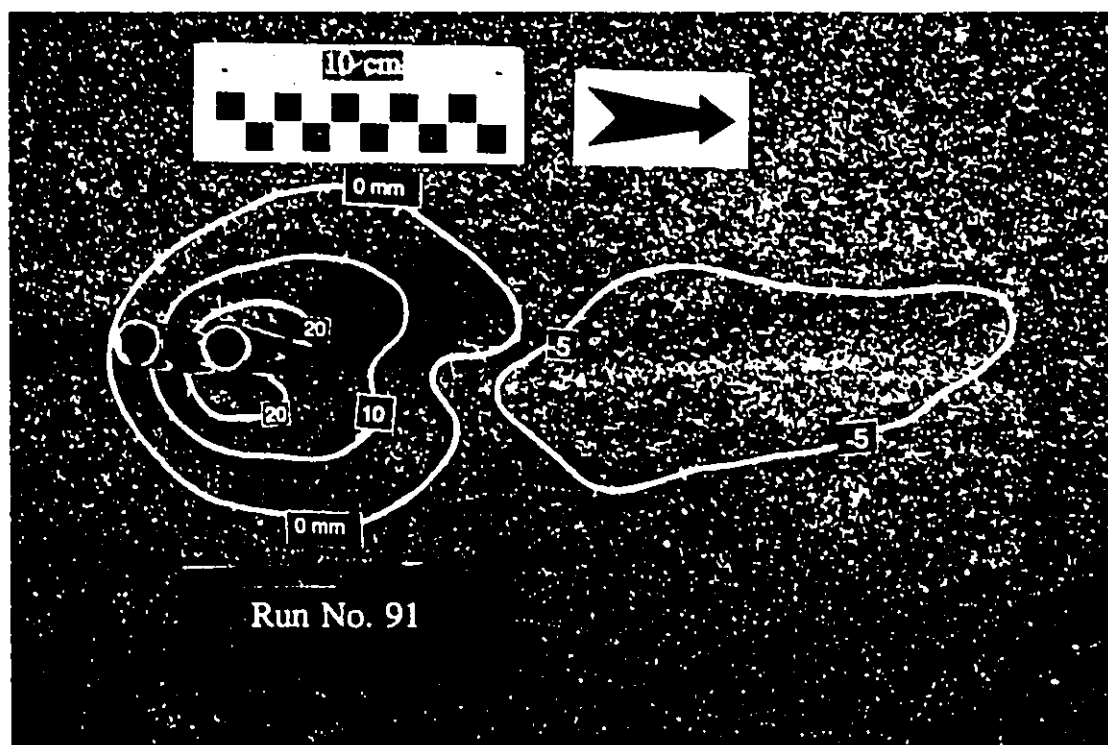


Plate 5-2-2: A group of 2 piers; $S/d=2$ and $\beta=0^\circ$

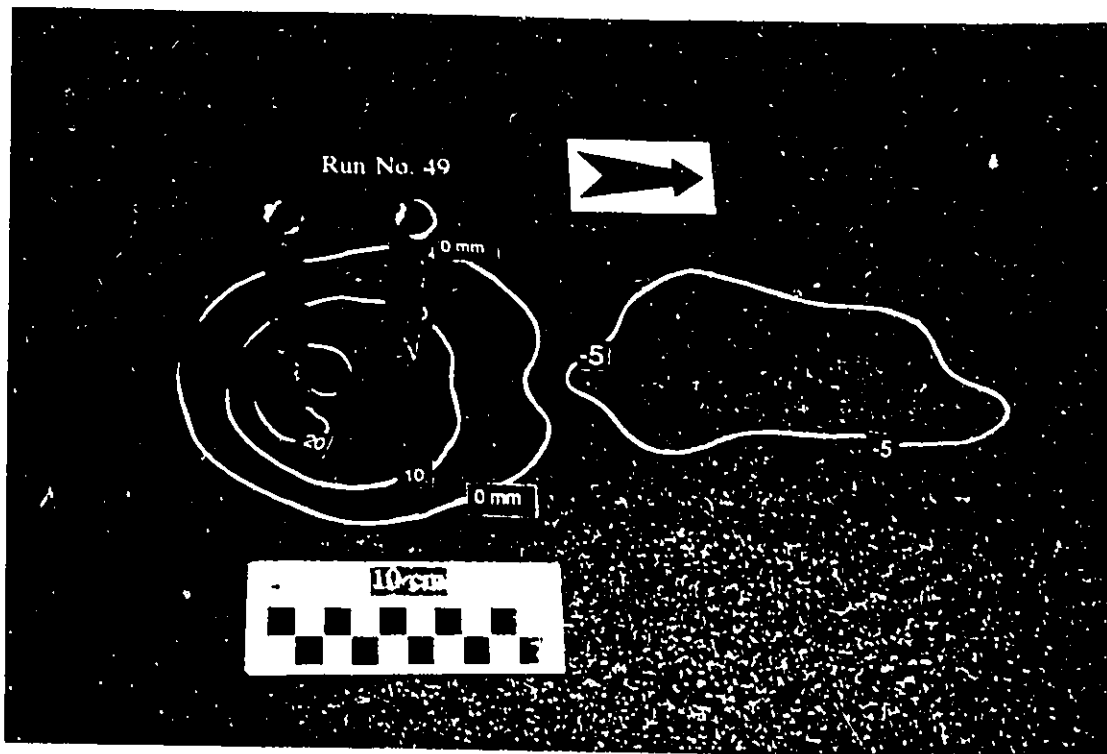


Plate 5-2-3: A group of 2 piers; $S/d=3$ and $\beta=0^\circ$

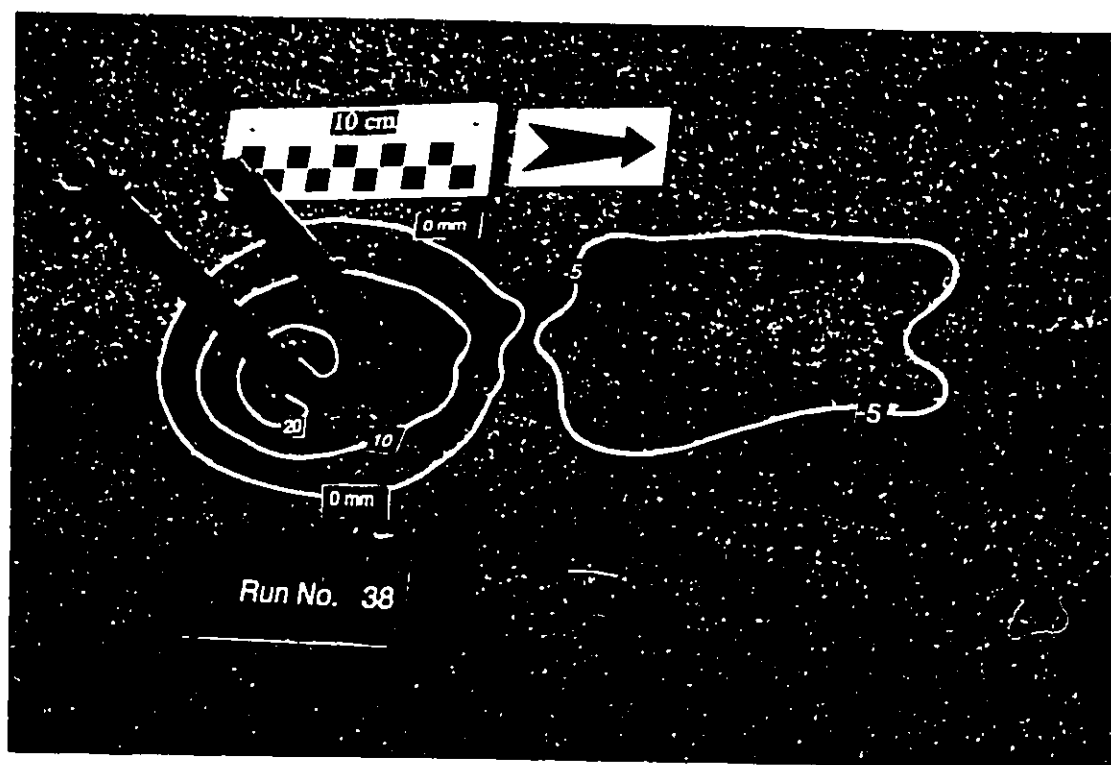


Plate 5-2-4: A group of 2 piers; $S/d=4$ and $\beta=0^\circ$



Plate 5-2-5: A group of 2 piers; $S/d=16$ and $\beta=0^\circ$

Table 5-2-1: Tests Performed for 2-PGs

β	S/d					
	1	2	3	5	7	11
0°	*	*	*	*	*	*
10°			*	*	*	
20°			*	*	*	
30°			*	*		
35°	*	*	*	*	*	*
40°	*	*	*	*	*	*
45°	*	*	*	*	*	
50°	*	*	*	*	*	

* Test performed

Table 5-2-2: One-sided t-distribution for 2-PGs (67 degrees of freedom)

Confidence level (percent)	One-sided t-distribution (Statistics)
60.0	0.254
75.0	0.679
90.0	1.296
95.0	1.670
97.5	2.000
99.0	2.390
99.5	2.660

5-3 Local Scouring around 3-Pier Groups

At this stage of our study, local scour depths were measured around 3-pier groups (3-PG). Thirty-one tests were carried out in this phase of the experimental program. The effects of sheltering and reinforcement on local scour depth were investigated for different β and S/d . The observed scour depth at each pier in the 3-PG was normalized using the corresponding single pier scour depth, D_s , i.e., $\frac{D_{SF_1}}{D_s} = D'_{F_1}$, $\frac{D_{SM_2}}{D_s} = D'_{M_2}$ and $\frac{D_{SR_3}}{D_s} = D'_{R_3}$ and the results were plotted against β . Applying multiple regression analysis to the log-transformed data suggests three prediction equations for the front, middle, and rear piers of a 3-PG.

5-3-1 The Effects of Varying β on Local Scour Depth

In light of the results of those tests performed on the 2-PG, S/d ratios of 2, 4, 6, and 8 were adopted for the 3-PG tests. The observed scour depths corresponding to different β for $S/d=2$ are shown in Fig. 5-3-1. As was the case for the 2-PGs, for $\beta \leq 10^\circ$, bed materials around the front pier of a 3-PG were scoured more than those around the downstream piers. Generally, deeper scour depths were produced by increasing β . Measurements showed that the size of scour holes formed around the middle and rear piers became larger with increasing β . Moreover, it seems that the upper limit of β that affects the size of the scour hole is 30° , beyond which no significant changes in the size of the scour hole around the rear pier were observed.

Although D'_{F_1} , D'_{M_2} , and D'_{R_3} showed increasing trends with increasing β for $S/d=2$, they exhibited different rates of increases. Moreover, for $\beta > 30^\circ$ the middle pier produced a scour hole

deeper than the one formed around the rear pier. There are two reasons for these different trends. First, the middle pier shelters the rear pier, resulting in a shallower scour hole around the rear pier. Second, the scour hole around the middle pier is affected by the reinforcement effect (due to the presence of the rear pier), which in turn helps removal of the sediments around the middle pier. This explains why for $\beta > 30^\circ$ the middle pier produces a scour hole deeper than the one formed around the rear pier. The maximum scouring ratio occurred in the range $30^\circ \leq \beta \leq 50^\circ$ for the rear pier, while it occurred at $\beta = 50^\circ$ for the middle pier. No variation in the scour depth of the front pier was observed for different values of β . The mild slope of the scouring-ratio for F_1 would become a horizontal line if the scour depth in front of M_2 and R_3 were not increased by changing β . Because of a larger scour hole around M_2 and R_3 , the sediments around F_1 can be removed more easily, thereby producing a deeper scour hole. For $\beta > 10^\circ$, the scour hole for each of the piers in a 3-PG is deeper than that of a single pier, i.e., $\frac{D_{SF_1}}{D_s}, \frac{D_{SM_2}}{D_s}, \text{ and } \frac{D_{SR_3}}{D_s} > 1$. Plates 5-3-1 to 5-3-4 show the scour holes formed around the piers in a 3-PG ($S/d=2$) for different β .

In the next series of tests, S/d was increased to 4 and different β were applied. The results show that D'_{F_1} is practically independent of β (Fig. 5-3-2) which is not the case for $S/d=2$ (Fig. 5-3-1). Since the reinforcement effect at $S/d=4$ is not as significant as for $S/d=2$ the D'_{F_1} data show a different trend for $S/d=4$, due to the fact that F_1 is now less affected by the downstream piers. In addition, an independent scour hole forms around F_1 . Moreover, scour hole depths around downstream piers increased with increasing β . The larger β means that the middle and rear piers are less sheltered by the front pier which results in more sediment removal and generates a deeper scour hole around of each of the piers in the group.

Scour around the middle and rear piers exhibited similar trends for $\beta \leq 20^\circ$. In this range, the middle and rear scour holes showed increasing trends with increasing β (Fig. 5-3-2). The results show that D'_{R_1} reaches a maximum value at $\beta=20^\circ$ beyond which it shows a decreasing trend approaching unity. The results show that, by increasing β , more sediments were deposited downstream of M_2 , resulting in a shallower scour hole around R_3 , thus, D'_{R_1} approached unity at a smaller S/d than that of D'_{M_2} . However, D'_{F_1} , D'_{M_2} , and D'_{R_3} are identical for $\beta > 50^\circ$. Plates 5-3-5, 5-3-6, and 5-3-7 show respectively the scour holes that formed around the 3-PG for $S/d=4$ and for $\beta=10^\circ$, 20° , and 30° . The larger the β , the more severe the scouring is. The volume of the sediments deposited downstream of the piers increases with increasing β . The increase in A_p (because of the increasing of β) is again responsible for a larger scour hole around piers.

The effect of varying β on the local scour depth was investigated for $S/d=6$. As was the case for $S/d=4$, D'_{F_1} was independent of β (Fig. 5-3-3). However, both D'_{F_1} and D'_{R_1} reached the same maximum value at $\beta=30^\circ$ and 20° , respectively. Furthermore, D'_{F_1} , D'_{M_2} , and D'_{R_3} were less than those observed for $S/d=4$. The larger S/d here was responsible for the decrease in those ratios. The trend of D'_{M_2} and D'_{R_3} were identical in the range $0^\circ \leq \beta \leq 50^\circ$.

The next series of tests were carried out for $S/d=8$. The trends of data were similar to the case for $S/d=6$, however, the range of variation of D'_{F_1} , D'_{M_2} , and D'_{R_3} was limited between 0.71 and 1.26, while it was limited between 0.82 and 1.38 for $S/d=6$. Moreover, the maximum value of D'_{F_1} , D'_{M_2} , and D'_{R_3} here were close to unity, i.e. each pier behaved like a single pier for large S/d . The data showed that D'_{F_1} was independent of β . In addition, it was noticed

that D'_{M_2} and D'_{R_1} were smaller than D'_{F_1} for $\beta < 10^\circ$. The scour depths in front of all three piers were the same at $\beta = 10^\circ$. The greater the β , the deeper D'_{M_2} and D'_{R_1} were. For $10^\circ < \beta < 50^\circ$, D'_{M_2} and D'_{R_1} were larger than D'_{F_1} . It can be seen that D'_{M_2} and D'_{R_1} reach maximum values at $\beta = 35^\circ$ and eventually approach the value D'_{F_1} at $\beta = 50^\circ$ (Fig. 5-3-4). The formation of a completely separate scour hole around each pier was more evident here.

5-3-2 Effects of Varying S/d on Local Scour Depth

Results show that, for a 3-PG aligned to the flow, the scour depth ratio is highest for the front pier and is lowest for the rear pier (Fig. 5-3-5). This is due to the sheltering effect that is more significant for the rear pier than for the middle pier (both the front and middle piers shelter the rear pier). The maxima D'_{M_2} and D'_{R_3} occur within $3 \leq S/d \leq 5$, which is in accordance with the results for a 2-PG (Fig. 5-2-13). Scour holes formed around 3-PGs for $S/d = 1, 3$, and 5 are shown in Figs. 5-3-14, 5-3-15, and 5-3-16 respectively. In the case of touching piers ($S/d = 1$), a common scour hole is produced around both piers (Plate 5-3-14). By increasing S/d , a separate scour hole begins to form around each pier. The separate scour hole is more well defined for $S/d = 5$ than for $S/d = 3$ (Plates 5-3-15 and 5-3-16). The only available data in the literature for local scour studies associated with a 3-PG are those presented by Hannah (1978) for piers aligned to the flow. He performed six tests with $1 \leq S/d \leq 6$. The results presented in our study show a good agreement with those of Hannah, particularly for the front and the middle piers (Fig. 5-3-6).

Besides the tests mentioned in Table 5-3-1, three supplementary tests were performed (using

S/d=1, 3, and 5) to study the combined effects of varying S/d and β on local scour. Results for $\beta=0^\circ$ show different data trends for D'_{F_1} , D'_{M_2} , and D'_{R_3} from those observed for $\beta=10^\circ$. For $\beta=10^\circ$, D'_{F_1} shows a decreasing trend approaching unity (Fig. 5-3-7). Maxima D'_{M_2} and D'_{R_3} occur at S/d = 4 and 6, respectively. Since M_2 and R_3 are not aligned to the flow, they are less affected by the sheltering associated with an upstream pier. The results show that D'_{M_2} increases from 1.08 to 1.27, while D'_{R_3} increases from 0.97 to 1.38.

The data trends for D'_{F_1} , D'_{M_2} , and D'_{R_3} for $\beta=20^\circ$ (Fig. 5-3-8) are similar to those of the previous case ($\beta=10^\circ$). However, D'_{M_2} and D'_{R_3} are greater than D'_{F_1} for $\beta=20^\circ$. The reason is that there is less sheltering associated with M_2 and R_3 . In addition, maxima D'_{M_2} and D'_{R_3} occur at S/d=4 and converge to the same value with increasing S/d. Nevertheless, in general D'_{R_3} is greater than D'_{M_2} . For S/d ≤ 7 , the sheltering effect does not change since β remains constant. On the other hand, the reinforcement effect decreases with S/d.

The effect of varying S/d on the local scour depth for $\beta=30^\circ$ is shown in Fig. 5-3-9. Like the previous case, the data trend for D'_{F_1} decreases with S/d, but the variation in D'_{F_1} is less dependent on S/d. The reason is that with increasing β there are more gaps between the piers for any S/d. In the range $2 \leq S/d \leq 5$, D'_{R_3} is larger than D'_{M_2} . Therefore, because the flow around M_2 will be less disturbed by the presence of R_3 for further increase in S/d, both D'_{M_2} and D'_{R_3} approach the same value for S/d > 5.

In the next series of tests the effect of varying S/d on the local scour depth was investigated using $\beta=40^\circ$. It seems that the decreasing trend observed for D'_{F_1} (Fig. 5-3-10) can be attributed to the presence of the HSV arms, which is significant for smaller S/d . The results show that the differences among D'_{F_1} , D'_{M_2} , and D'_{R_3} are not as significant as that observed in the former case. Nevertheless, D'_{R_3} is much closer to D'_{F_1} here than for the $\beta=30^\circ$ case. In addition, D'_{M_2} is greater than both D'_{F_1} and D'_{R_3} for $S/d < 8$.

In the next set of experiments the effect of varying S/d on local scour depth was investigated for $\beta=45^\circ$ (Fig. 5-3-11). The results show a similar trend of data with the previous case ($\beta=40^\circ$). As for $\beta=40^\circ$, D'_{F_1} , D'_{M_2} , and D'_{R_3} showed a decreasing trend with increasing S/d . Also, a sharp decrease in D'_{R_3} was observed, with a minimum occurring at $S/d=4$. The results for $\beta=50^\circ$ were similar to case for $\beta=45^\circ$ (Fig. 5-8-12).

5-3-4 Multiple Regression Analysis

Thirty-one measurements of local scour depths gathered from the present study were analysed using multiple regression analyses to obtain local scour prediction equations for the front, middle and rear piers. The regression analysis provides a probability statement about the distribution of the prediction, from which an upper confidence limit of local scour depth can be determined.

5-3-4-1 Analysis of Data

For the hydraulic conditions adopted in this study local scour depth parameters for any arrangement of pier groupings are primarily function of β and S/d . The functional relations for the normalized local scour depths are

$$\frac{D_{SF_1}}{D_s} = f_1 \left\{ \beta, \frac{S}{d} \right\} \quad (5-3-1)$$

$$\frac{D_{SM_2}}{D_s} = f_2 \left\{ \beta, \frac{S}{d} \right\} \quad (5-3-2)$$

$$\frac{D_{SR_3}}{D_s} = f_3 \left\{ \beta, \frac{S}{d} \right\} \quad (5-3-3)$$

Multiple linear regression was used to develop prediction equations for a 3-PG, based on the proposed functional relations in equations 5-3-1, 5-3-2, and 5-3-3. Using the multiple regression model, as given in the earlier matrix notation (equations 5-2-2, 5-2-3, and 5-2-4), a vector of fitted values corresponding to a vector of observed values is computed. Then the “hat” matrix $H_{31 \times 31}$

$(X(X'X)^{-1}X')$, which is necessary in the regression analysis, can be computed, using equation 5-2-4.

Logarithmic transformation of all variables was found to provide the best linear relations. The logarithms of the ratios $\frac{D_{SF_1}}{D_s}$, $\frac{D_{SM_2}}{D_s}$ and $\frac{D_{SR_3}}{D_s}$ were regressed against a combination of the two independent variables β and S/d .

The general forms of the prediction equations are

$$\frac{D_{SF_1}}{D_s} = a_1 \beta^{b_1} (S/d)^{c_1} \quad (5-3-4)$$

$$\frac{D_{SM_2}}{D_s} = a_2 \beta^{b_2} (S/d)^{c_2} \quad (5-3-5)$$

$$\frac{D_{SR_3}}{D_s} = a_3 \beta^{b_3} (S/d)^{c_3} \quad (5-3-6)$$

in which: $a_1 = \exp(\hat{\theta}'_{F(1)})$, $b_1 = \hat{\theta}'_{F(2)}$, $c_1 = \hat{\theta}'_{F(3)}$, $a_2 = \exp(\hat{\theta}'_{M(1)})$, $b_2 = \hat{\theta}'_{M(2)}$, $c_2 = \hat{\theta}'_{M(3)}$,

$a_3 = \exp(\hat{\theta}'_{R(1)})$, $b_3 = \hat{\theta}'_{R(2)}$, $c_3 = \hat{\theta}'_{R(3)}$, $\hat{\theta}'_{F_1} = [\hat{\theta}'_{F_1(1)}, \hat{\theta}'_{F_1(2)}, \hat{\theta}'_{F_1(3)}]$, $\hat{\theta}'_{M_2} = [\hat{\theta}'_{M_2(1)}, \hat{\theta}'_{M_2(2)}, \hat{\theta}'_{M_2(3)}]$,

and $\hat{\theta}'_{R_3} = [\hat{\theta}'_{R_3(1)}, \hat{\theta}'_{R_3(2)}, \hat{\theta}'_{R_3(3)}]$.

5-3-4-2 Predicting Scour Depths

Normalized scour depths predicted by equations 5-3-4, 5-3-5, and 5-3-6 are the expected response to a specified set of regressors, and are not known with certainty. Using a procedure similar to the case for the 2-PG, a probability statement about the distribution of $\ln(\frac{D_{SF_1}}{D_s})$, $\ln(\frac{D_{SM_2}}{D_s})$, and $\ln(\frac{D_{SR_3}}{D_s})$ provides a means of evaluating appropriate design value. Using the transpose of the vector of predictor variables, as given in equation 5-2-6, several 100(1- η) percent upper confidence limits for the logarithm of normalized scour depths can be computed. Equation 5-2-2 is used to compute the upper confidence limit using $\frac{D_{SF_1}}{D_s}$, $\frac{D_{SM_2}}{D_s}$ and $\frac{D_{SR_3}}{D_s}$ as substitutions to $\frac{D_s}{D_s}$. The

values of $\hat{\theta}'_{F_1}$, $\hat{\theta}'_{M_2}$, and $\hat{\theta}'_{R_3}$ computed for a 3-PG are

$$\hat{\theta}'_{F_1} = [0.19, 0.03, -0.09] \quad (5-3-7)$$

$$\hat{\theta}'_{M_2} = [0.04, 0.09, -0.07] \quad (5-3-8)$$

$$\hat{\theta}'_{R_3} = [-0.06, 0.13, -0.09] \quad (5-3-9)$$

The variance of prediction can be estimated using equation 5-2-7 in which σ^2 for the front, middle, and rear piers is substituted by $\hat{\sigma}_{F_1}^2$, $\hat{\sigma}_{M_2}^2$, and $\hat{\sigma}_{R_3}^2$. For the 2-variable regression model, using thirty-one measurements, the variances of predictions are $\hat{\sigma}_{F_1}^2 = 0.25$, $\hat{\sigma}_{M_2}^2 = 0.52$, and $\hat{\sigma}_{R_3}^2 = 1.53$. One-sided t-distribution statistics for $n-p=31-3=28$, degrees of freedom and several confidence levels are given in Table 5-3-2. Upper estimates of $\ln\left(\frac{D_{SF_1}}{D_s}\right)$, $\ln\left(\frac{D_{SM_2}}{D_s}\right)$, and $\ln\left(\frac{D_{SR_3}}{D_s}\right)$ are obtained from equation 5-2-7. With the inclusion of an appropriate factor of safety, the upper limit of scour depth can be used to design pier foundations.

5-3-4-3 Prediction Equation for a 3-PG

Parameters a_1 , b_1 , c_1 , a_2 , b_2 , c_2 , a_3 , b_3 , and c_3 obtained for a 3-PG were substituted in equation 5-2-5. Hence, for the front, middle, and rear piers of a 3-PG, equation 5-2-5 can be expressed as

$$\frac{\hat{D}_{SF_1}}{D_s} = 1.21\beta^{0.03} \left(\frac{S}{d} \right)^{0.09} \quad (5-3-10)$$

$$\frac{\hat{D}_{SM_2}}{D_s} = 1.04\beta^{0.09} \left(\frac{S}{d} \right)^{0.07} \quad (5-3-11)$$

$$\frac{\hat{D}_{SR_3}}{D_s} = 0.94\beta^{0.13} \left(\frac{S}{d} \right)^{-0.09} \quad (5-3-12)$$

Equations 5-3-10, 11 and 12 are used to predict $\frac{D_{SF_1}}{D_s}$, $\frac{D_{SM_2}}{D_s}$ and $\frac{D_{SR_3}}{D_s}$. The predicted values are plotted against observed values in Figs. 5-3-16, 5-3-17, and 5-3-18. Deviations from the line of perfect agreement in Fig. 5-3-16 show how well equation 5-3-10 predicts the local scour depth for a 3-PG. Most data fall within $\pm 10\%$ of the predictions from the relationship proposed in this study for the front pier. Equations 5-3-11 and 5-3-12 represent the line of perfect agreement in Figs. 5-3-17 and 5-3-18, respectively. These figures show that most data fall within $\pm 10\%$ and $\pm 20\%$ of the predicted values for the middle and rear piers, respectively. The results of the proposed equations are applicable to the 3-PGs, irrespective of the applied scour prediction equations for a single pier. The value of D_s may be obtained using any appropriate empirical equation. Thus, the designer might choose any of the available scour prediction equation that suits his/her situation most, to predict D_s .

5-3-5 Conclusions:

Upon comparing the 3-PG data presented in Figs. 5-3-1 ($S/d=2$), 5-3-2 ($S/d=4$), 5-3-3 ($S/d=6$), and 5-3-4 ($S/d=8$), the difference between D'_{M_2} and D'_{R_1} appears to vary with S/d . For any β , the middle pier provides less sheltering for the rear pier as S/d increases. In view of this, there is a difference Figs. 5-3-1 through 5-3-4 due to reduction in the sheltering effect. Furthermore, maximum values for D'_{F_1} , D'_{M_2} , and D'_{R_1} occur close to $\beta=40^\circ$. It can be seen for both the middle and the rear piers that the maximum scour depth ratios occur at different β , e.g., the maxima D'_{M_2} occur at $\beta=40^\circ$ and 30° for $S/d=2$ and 4, respectively (Figs. 5-3-2 and 5-3-3). Moreover, for the rear pier the peak ratio of scouring occurs at $\beta=30^\circ$ and 20° for $S/d=2$ and 4, respectively. In addition, for $S/d=2$ maxima D'_{M_2} and D'_{R_1} are greater than for $S/d>4$.

In the next series of experiments the effects of varying the pier spacing on the local scour depth were examined for different β . In general, the sheltering and reinforcement effects were observed to decrease with increasing S/d . It was observed that, with increasing S/d , D'_{F_1} , D'_{M_2} , and D'_{R_1} approach unity (Figs. 5-3-5 to 5-3-12). The results show that, for a 3-PG aligned to the flow ($\beta=0^\circ$), the local scour depth ratio is highest for the front pier and is lowest for the rear pier. The results of these experiments were compared with Hannah's (1978) data and good agreement was found between the two studies.

As β was increased, M_2 and R_3 became more exposed to the flow, which resulted in

greater D'_{F_1} , D'_{M_2} , and D'_{R_3} (Figs. 5-3-5 to 5-3-11). For $\beta > 30^\circ$, the downstream piers were exposed to the flow, thus, the values of D'_{F_1} , D'_{M_2} , and D'_{R_3} increased (Figs. 5-3-12 to 5-3-28). The results showed that a larger β was not necessarily associated with a larger scour depth. In the range $10^\circ \leq \beta \leq 30^\circ$ all model piers are exposed to the flow, however, the ratio of D'_{M_2} is smaller than the ratio of D'_{R_3} (Figs. 5-3-9 to 5-3-11). Moreover, the local scouring is more severe for R_3 .

For $\beta \geq 30^\circ$ and $2 < S/d < 6$, D'_{M_2} is greater than D'_{R_3} . In these ranges of β and S/d , M_2 was affected by the HSV's arms associated with F_1 and R_3 . This resulted in a deeper scour hole around M_2 than occurred around the other piers. In addition, it can be seen in Figs. 5-3-7 to 5-3-9 that D'_{F_1} is the smallest value. Figs. 5-3-10, 5-3-11, and 5-3-12 show that the scour depth associated with F_1 and R_3 approach the same scour depth when $\beta > 40^\circ$ and $S/d > 4$.

The general form of the prediction equation written in terms of untransformed values was found for each pier, using the transpose of the vector of the regression model. The values $\hat{\theta}'_{F_1}$, $\hat{\theta}'_{M_2}$, and $\hat{\theta}'_{R_3}$ are computed for a 3-PG, using equations 5-3-7, 5-3-8, and 5-3-9. Then, by substituting the values $\hat{\theta}'_{F_1}$, $\hat{\theta}'_{M_2}$, and $\hat{\theta}'_{R_3}$ in equation 5-2-5, the prediction equations (5-3-10, 5-3-11, and 5-3-12) for the front, middle, and rear piers were found. Using equations 5-3-10, 5-3-11, and 5-3-12, the values $\frac{D_{SF_1}}{D_s}$, $\frac{D_{SM_2}}{D_s}$ and $\frac{D_{SR_3}}{D_s}$ were computed and plotted against the observed values in Figs. 5-3-16, 5-3-17, and 5-3-18. For the vast majority of the data, the prediction equations were successful within an acceptable error band in predicting the local scour depth for a 3-PG.

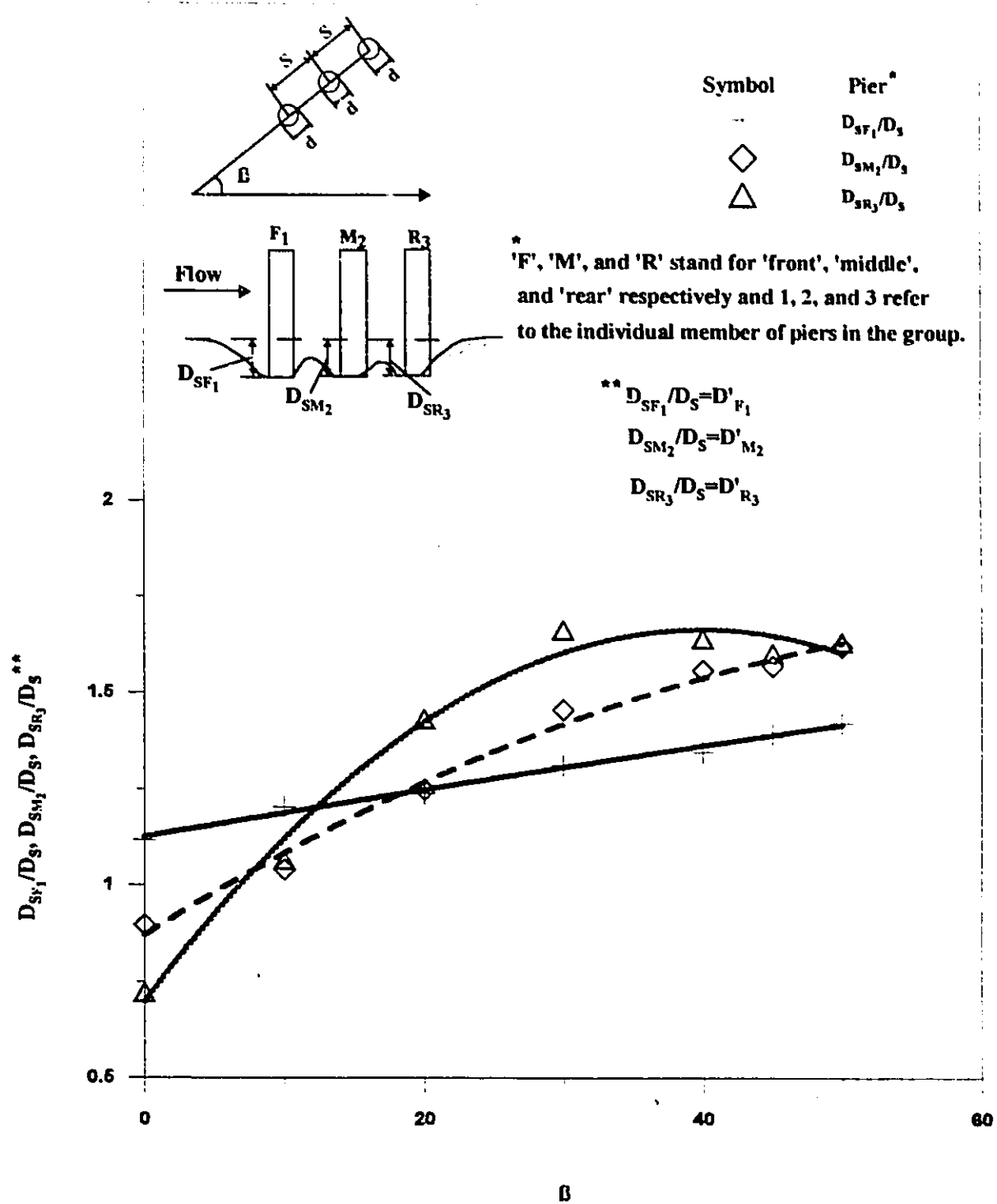


Fig. 5-3-1: Scour depth ratios versus flow angle of attack groups of 3 piers; ($S/d=2$)

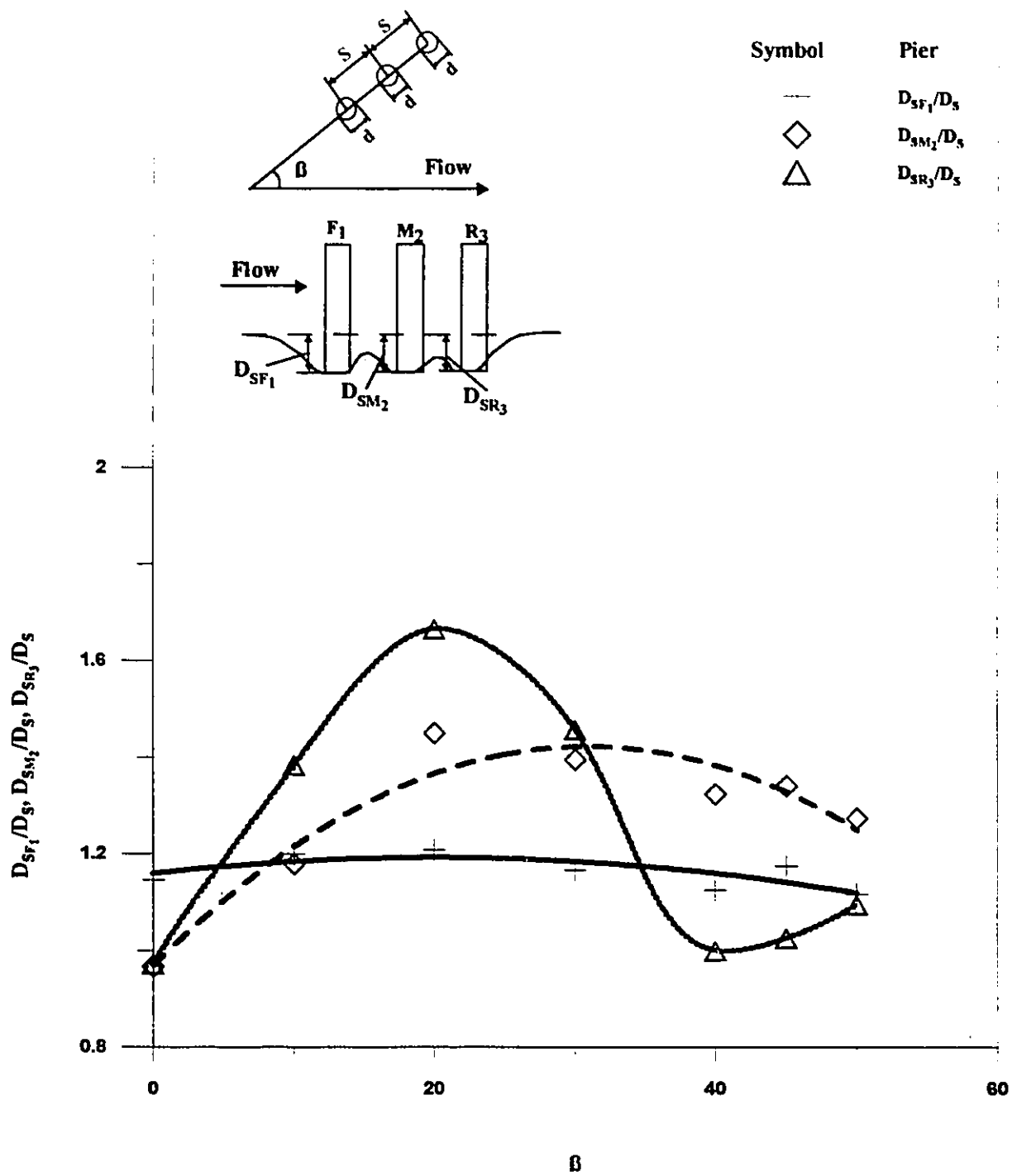


Fig. 5-3-2: Scour depth ratios versus flow angle of attack groups of 3 piers; ($S/d=4$)

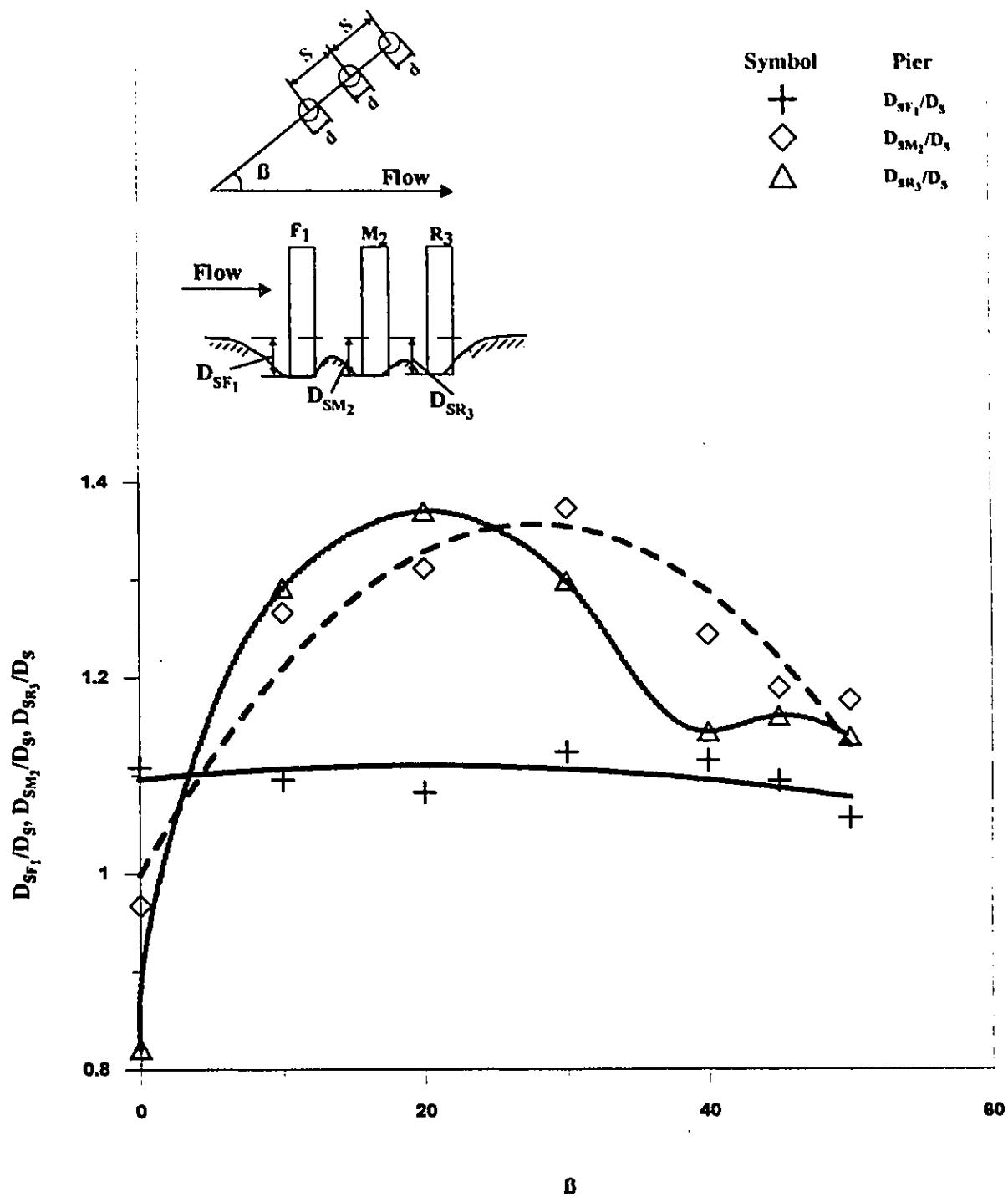


Fig. 5-3-3: Scour depth ratios versus angle of attack groups of 3 piers; ($S/d=6$)

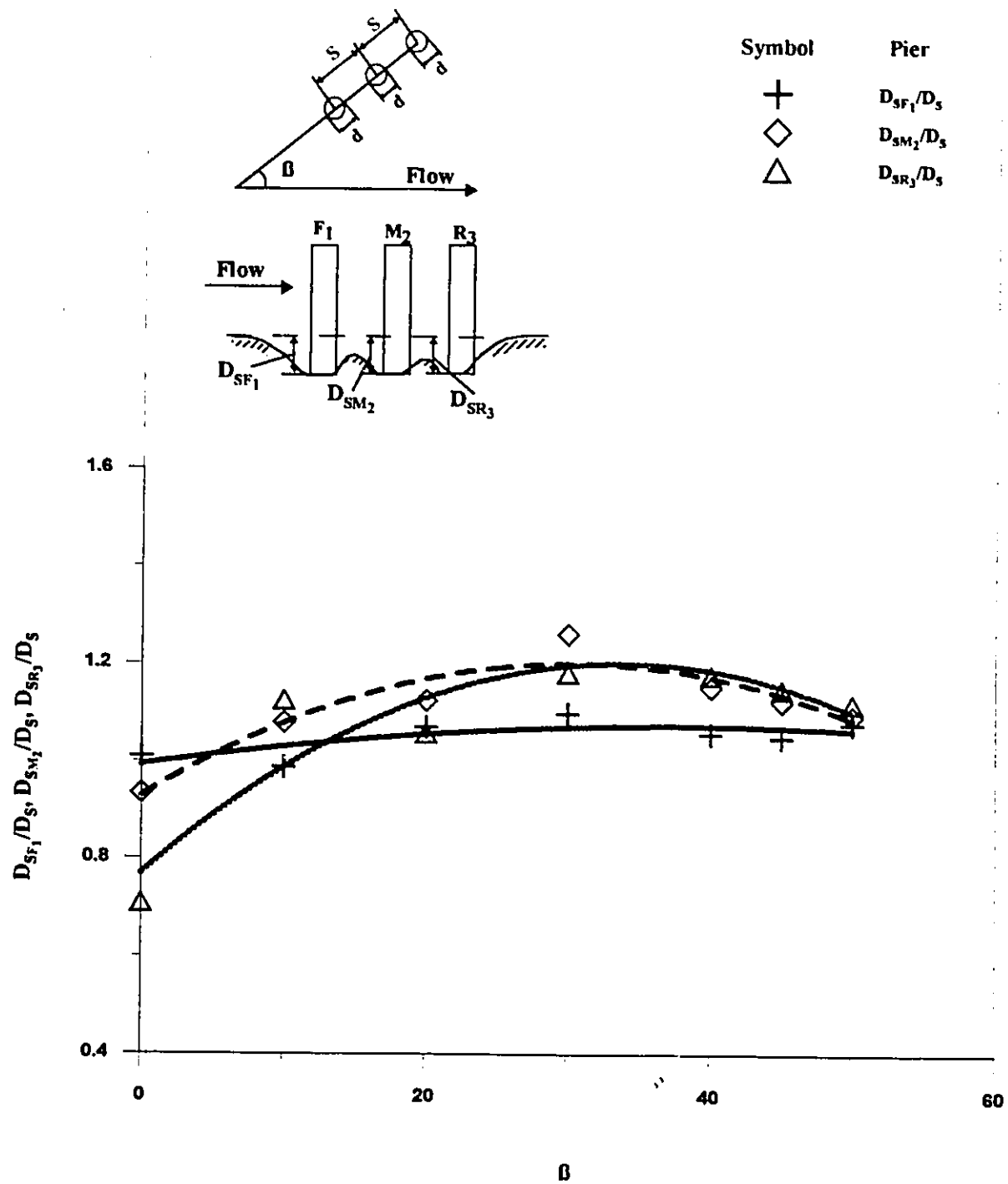


Fig. 5-3-4: Scour depth ratios versus flow angle of attack groups of 3 piers; ($S/d=8$)

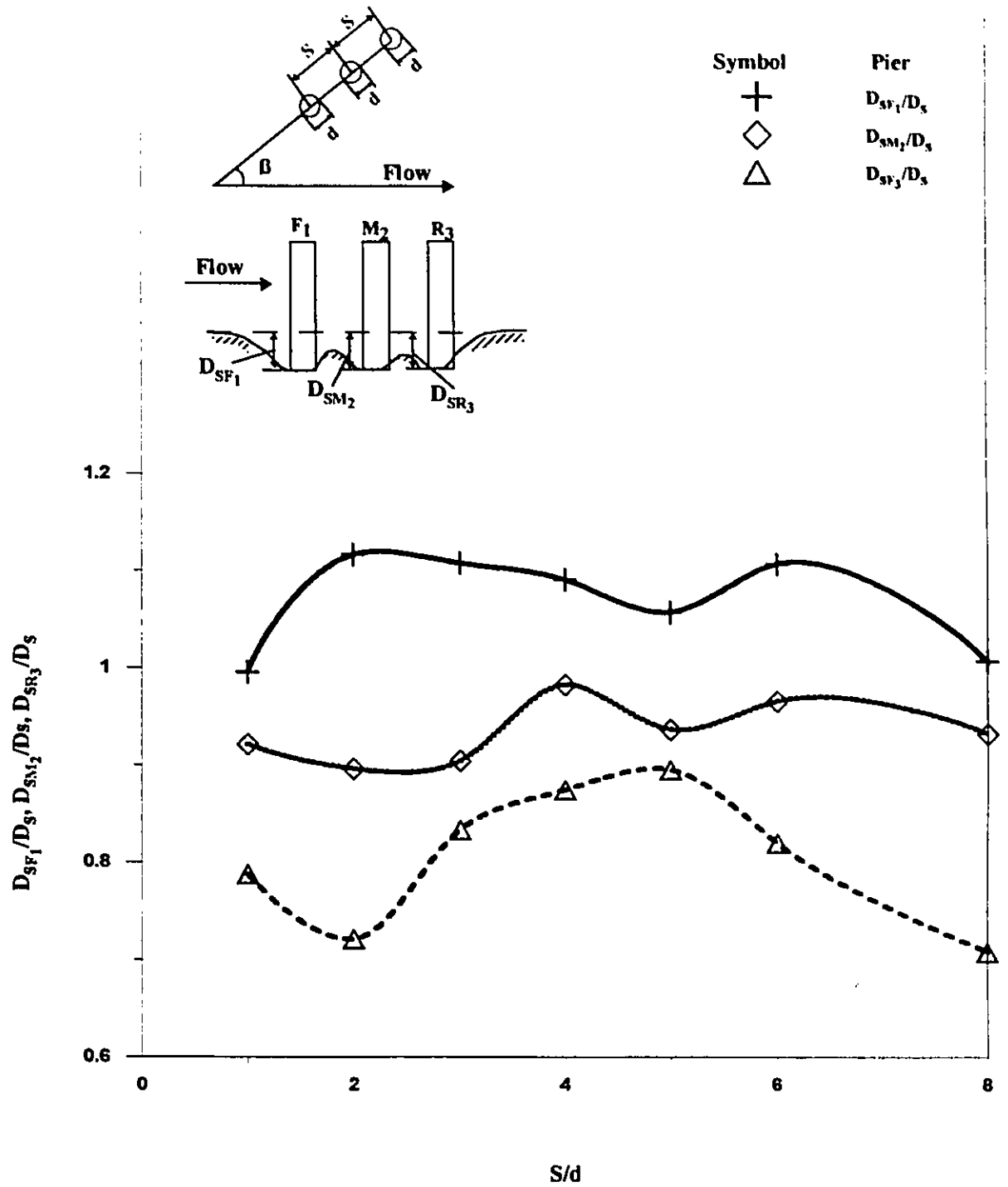


Fig. 5-3-5: Scour depth ratios versus separation groups of 3 piers; ($\beta=0^\circ$)

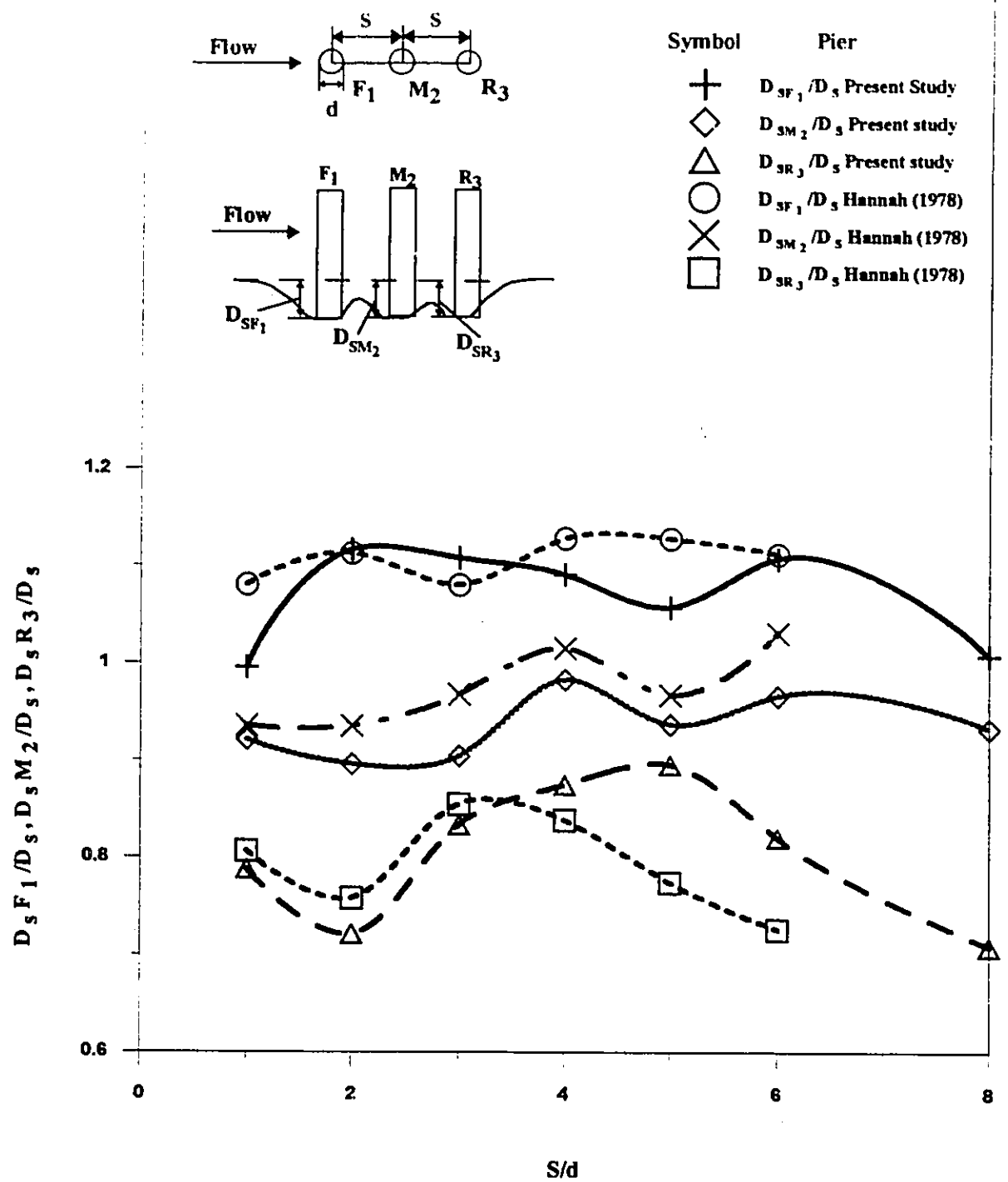


Fig. 5-3-6: Comparison between scour depth ratios groups of 3 piers; ($\beta=0^\circ$)

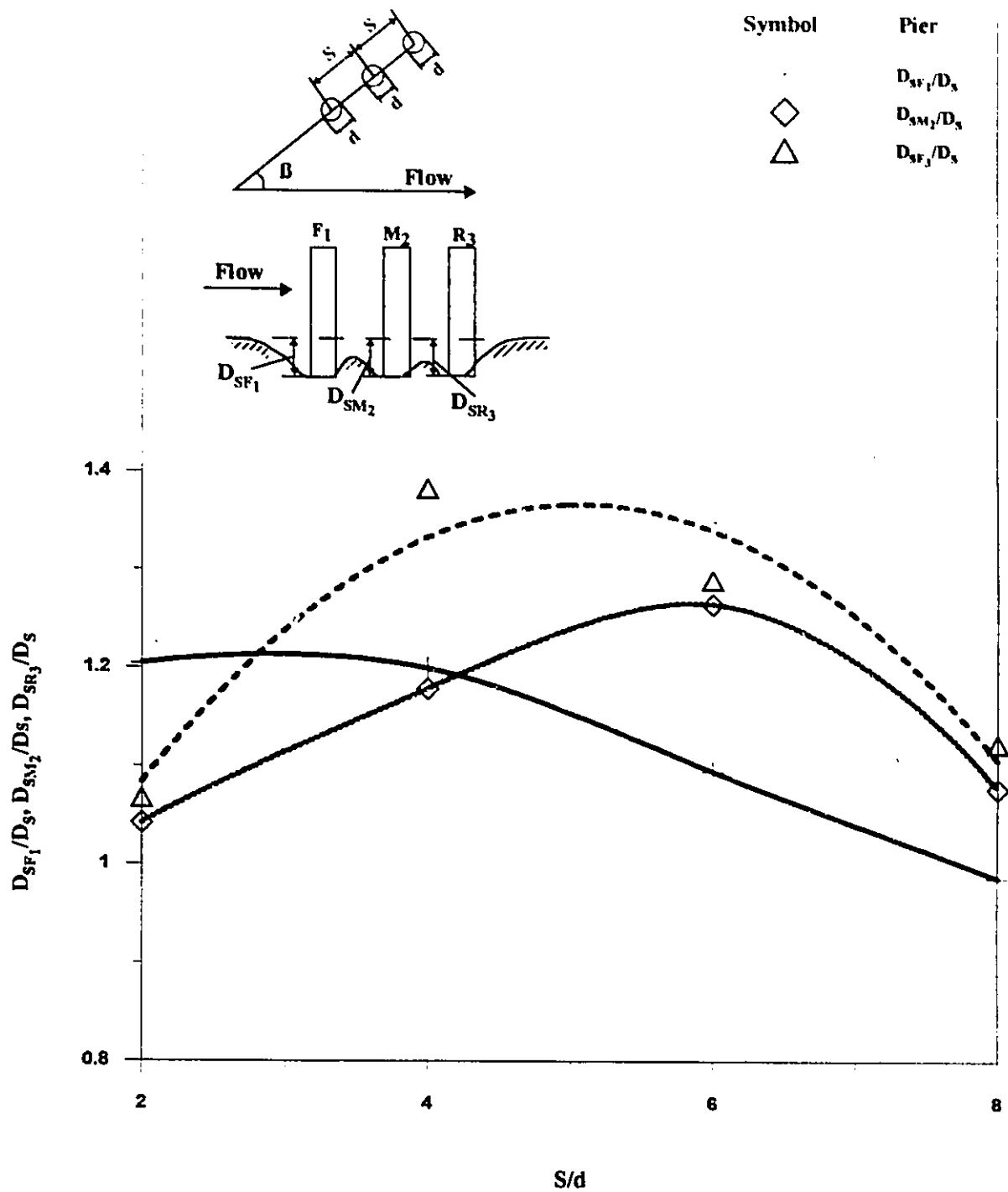


Fig. 5-3-7: Scour depth ratios versus separation groups of 3 piers; ($\beta=10^\circ$)

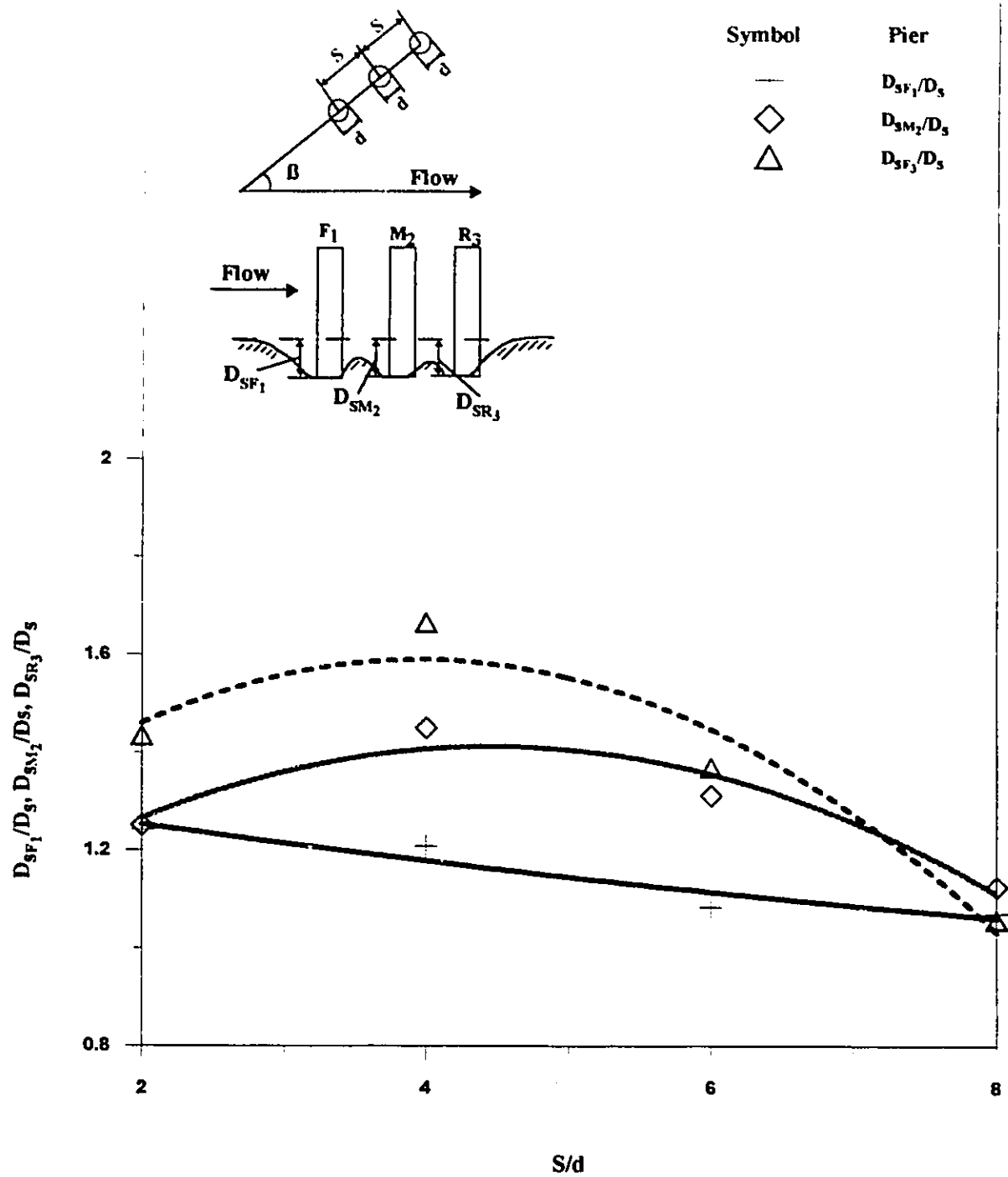


Fig. 5-3-8: Scour depth ratios versus separation groups of 3 piers; ($\beta=20^\circ$)

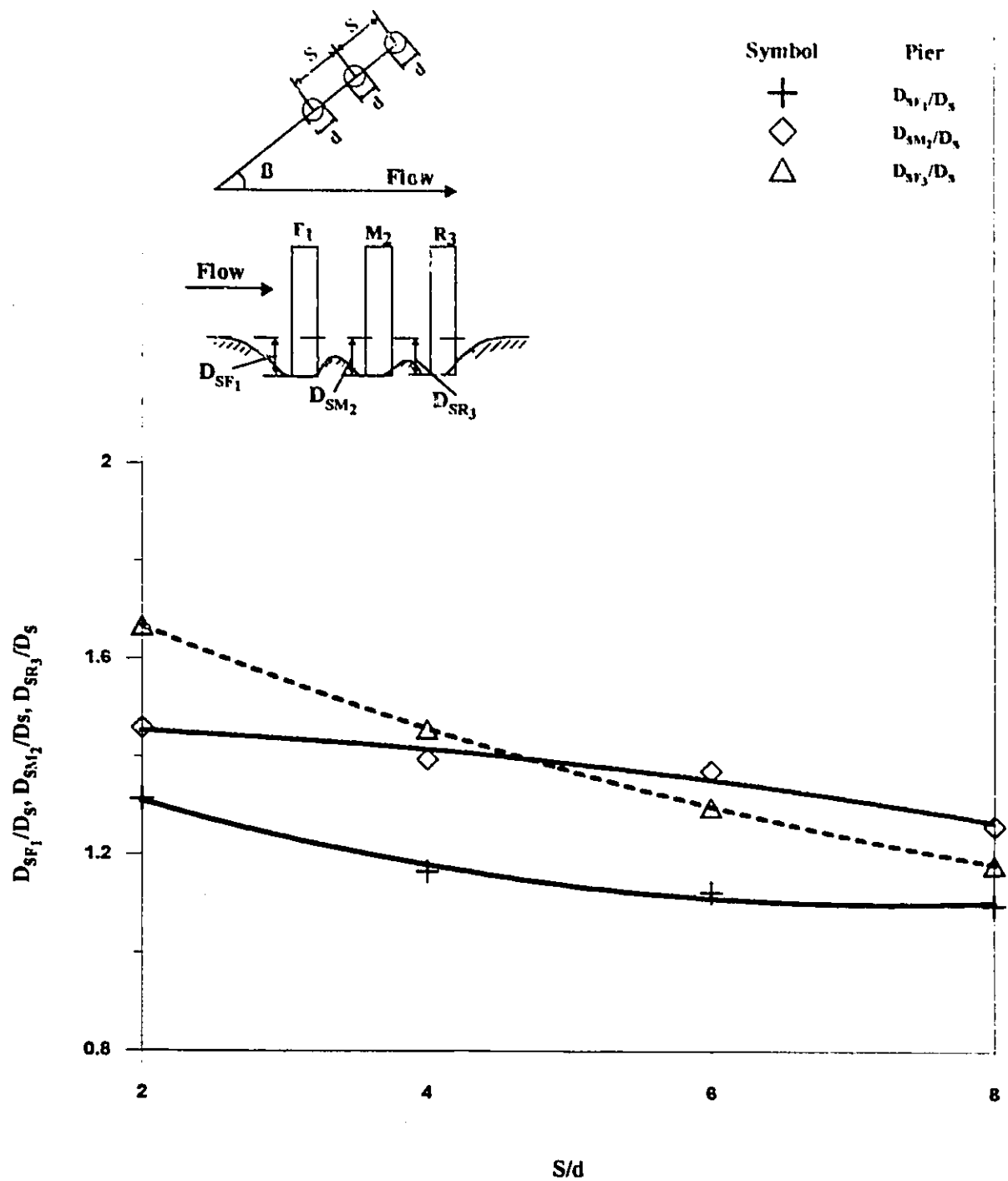


Fig. 5-3-9: Scour depth ratios versus separation groups of 3 piers; ($\beta=30^\circ$)

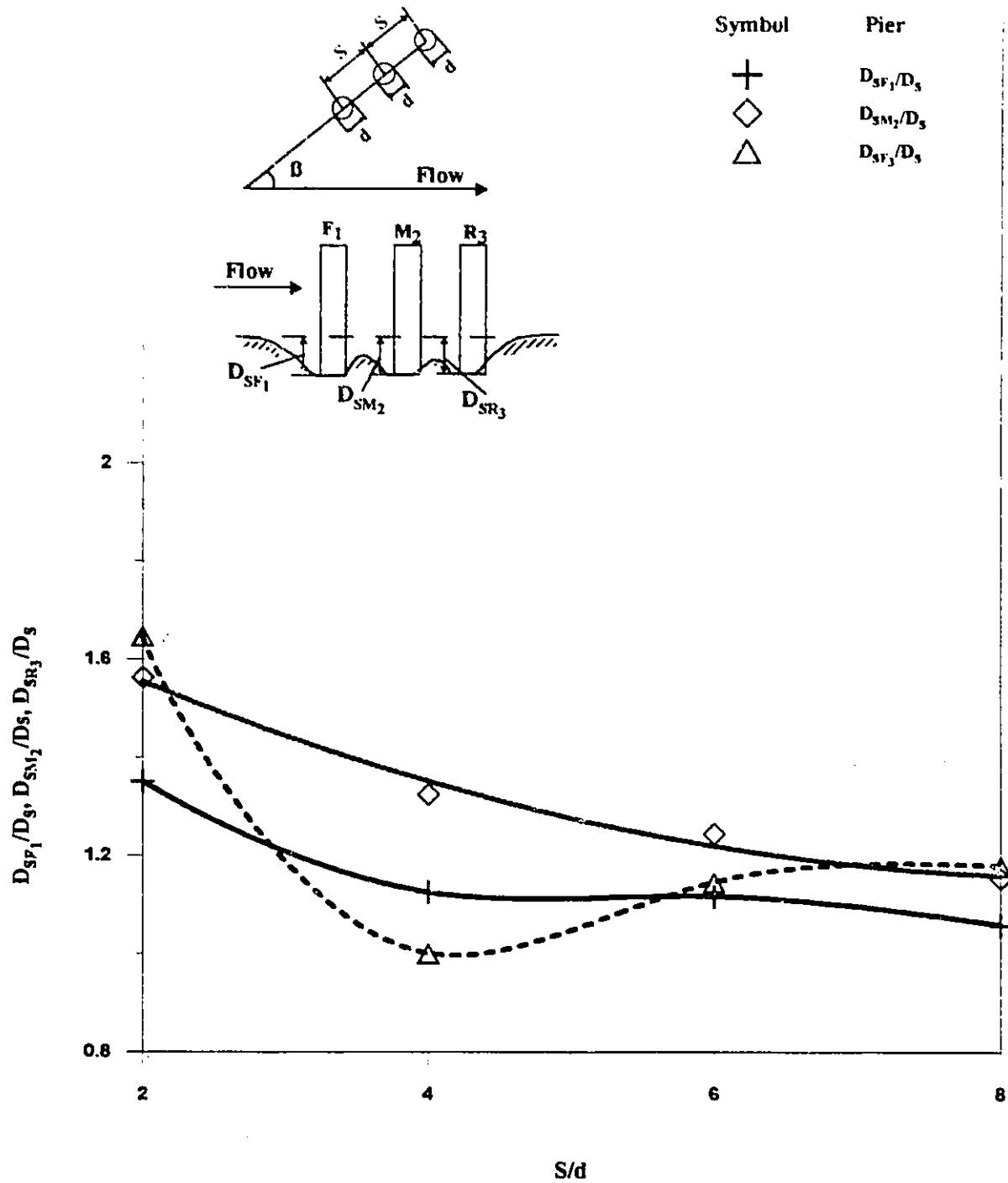


Fig. 5-3-10: Scour depth ratios versus separation groups of 3 piers; ($\beta=40^\circ$)

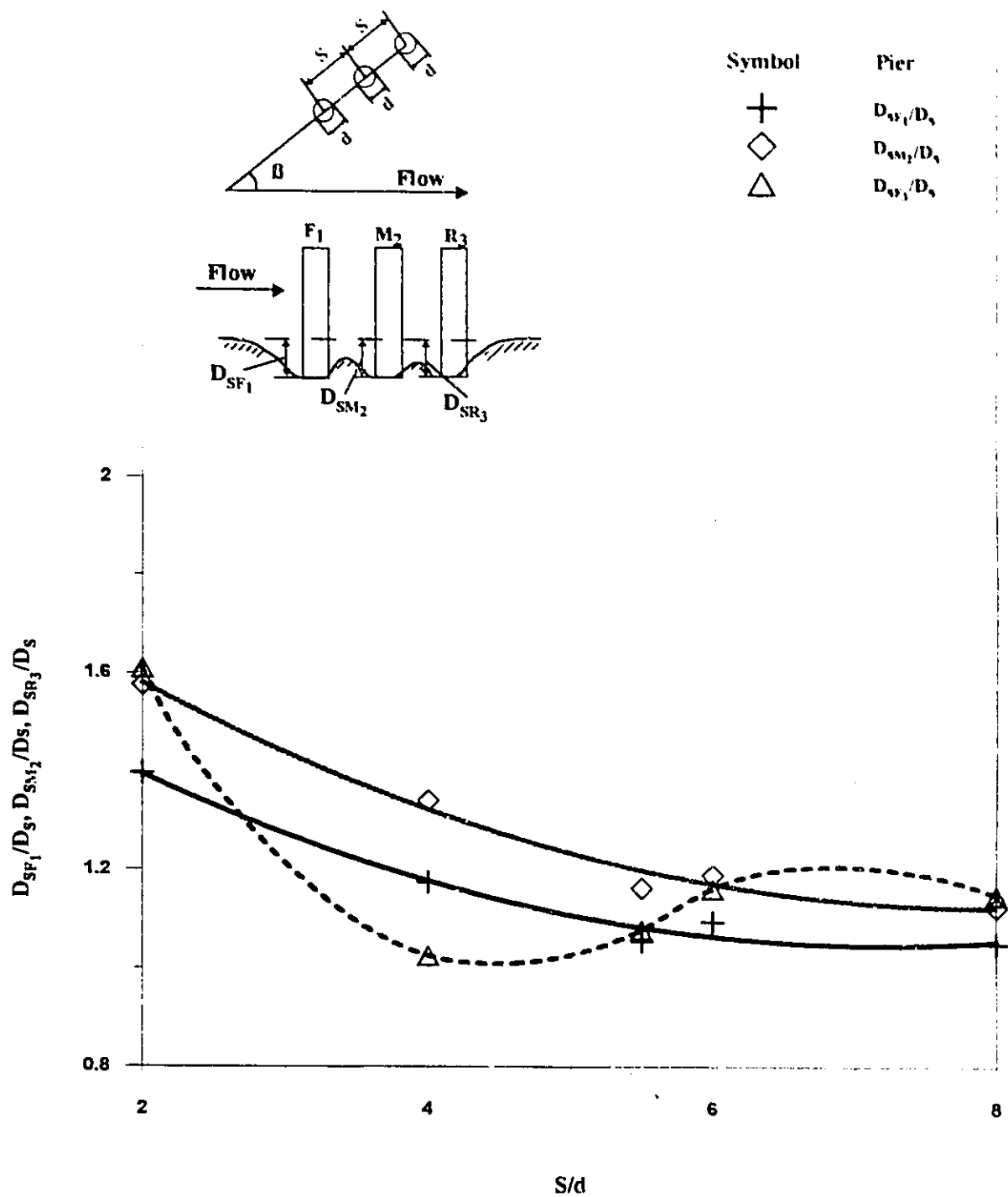


Fig. 5-3-11: Scour depth ratios versus separation groups of 3 piers; ($\theta=45^\circ$)

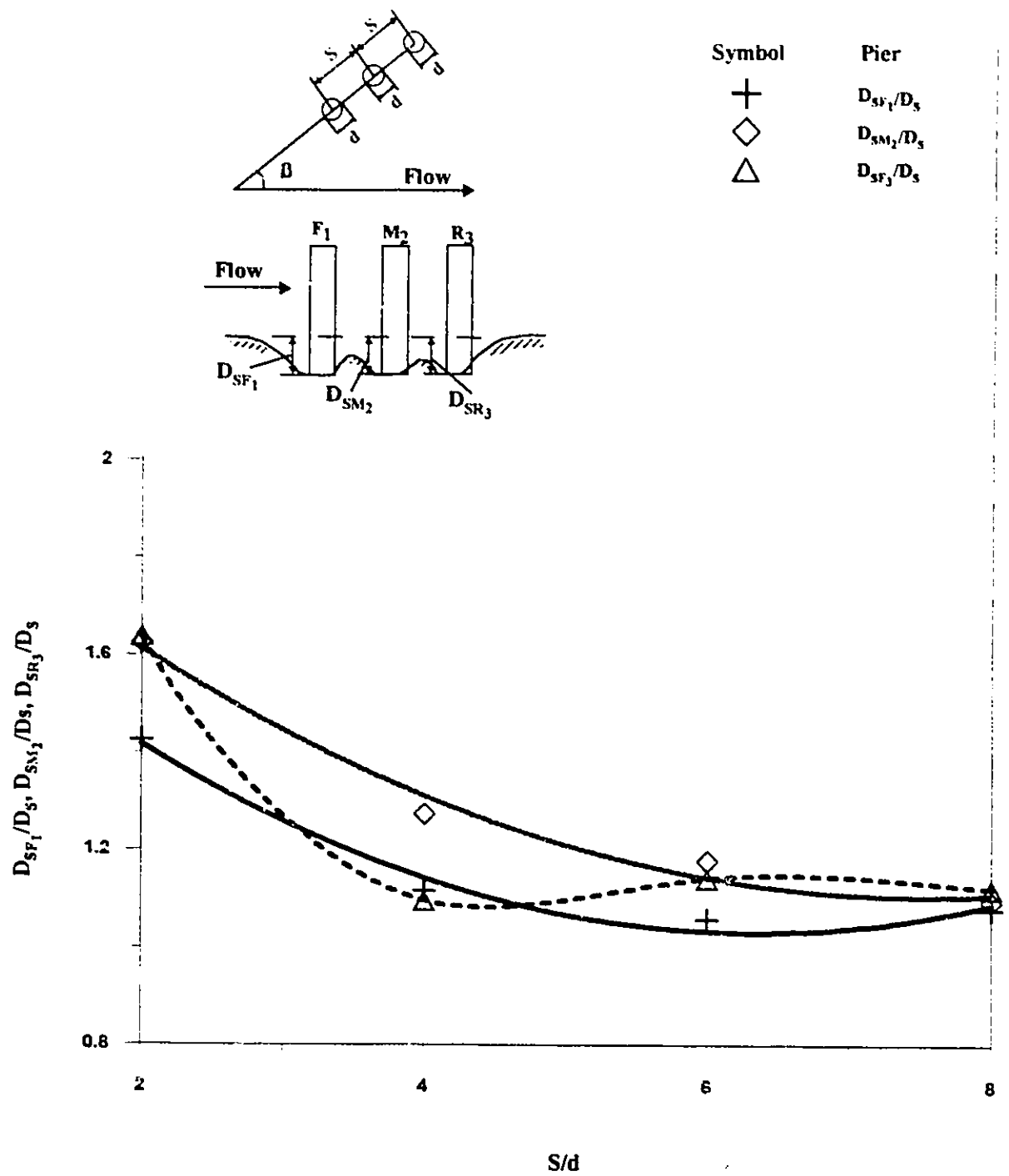


Fig. 5-3-12: Scour depth ratios versus separation groups of 3 piers; ($\beta=50^\circ$)

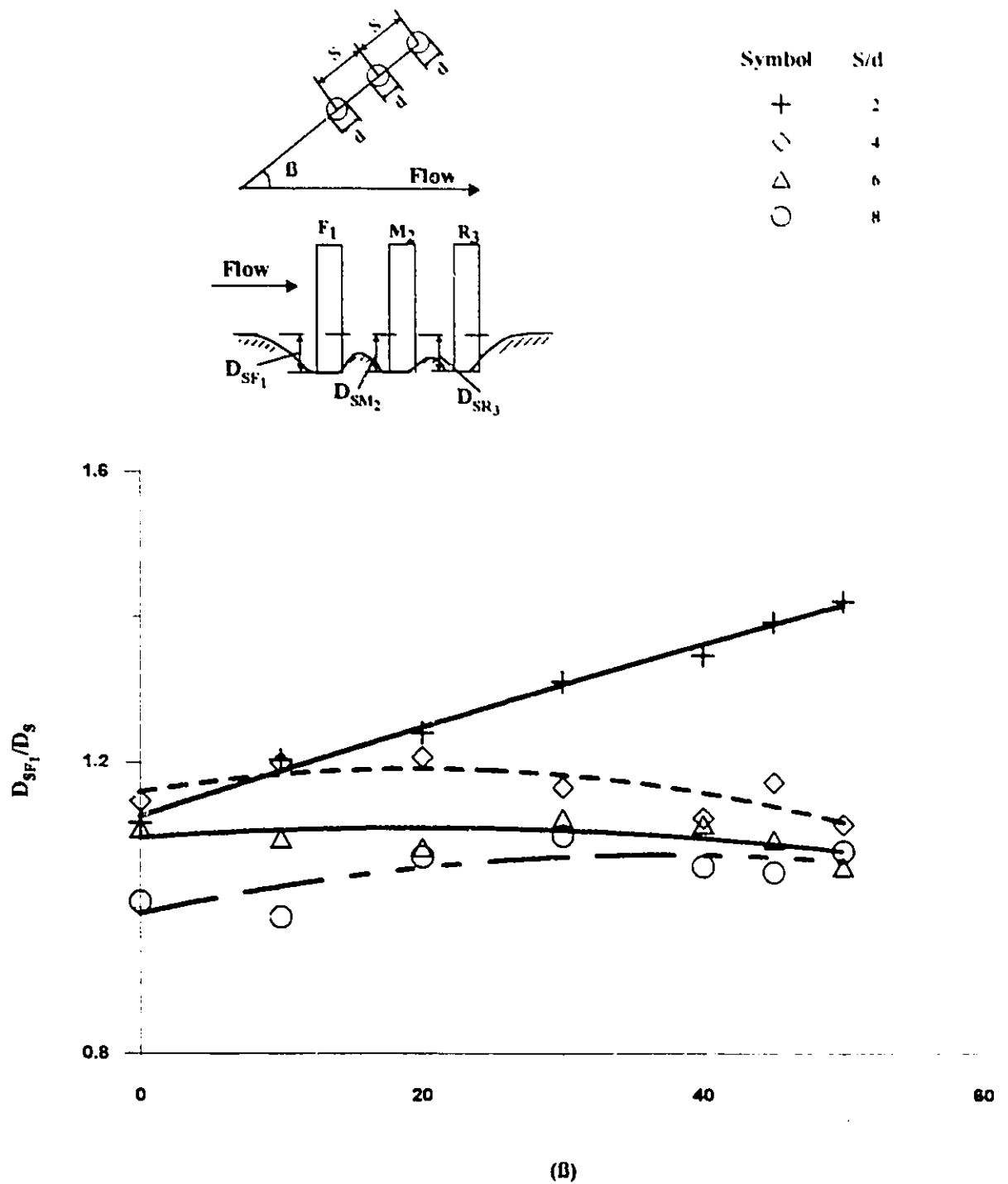


Fig. 5-3-13: Front pier scour depth ratios versus flow angle of attack for different S/d ; groups of 3 piers

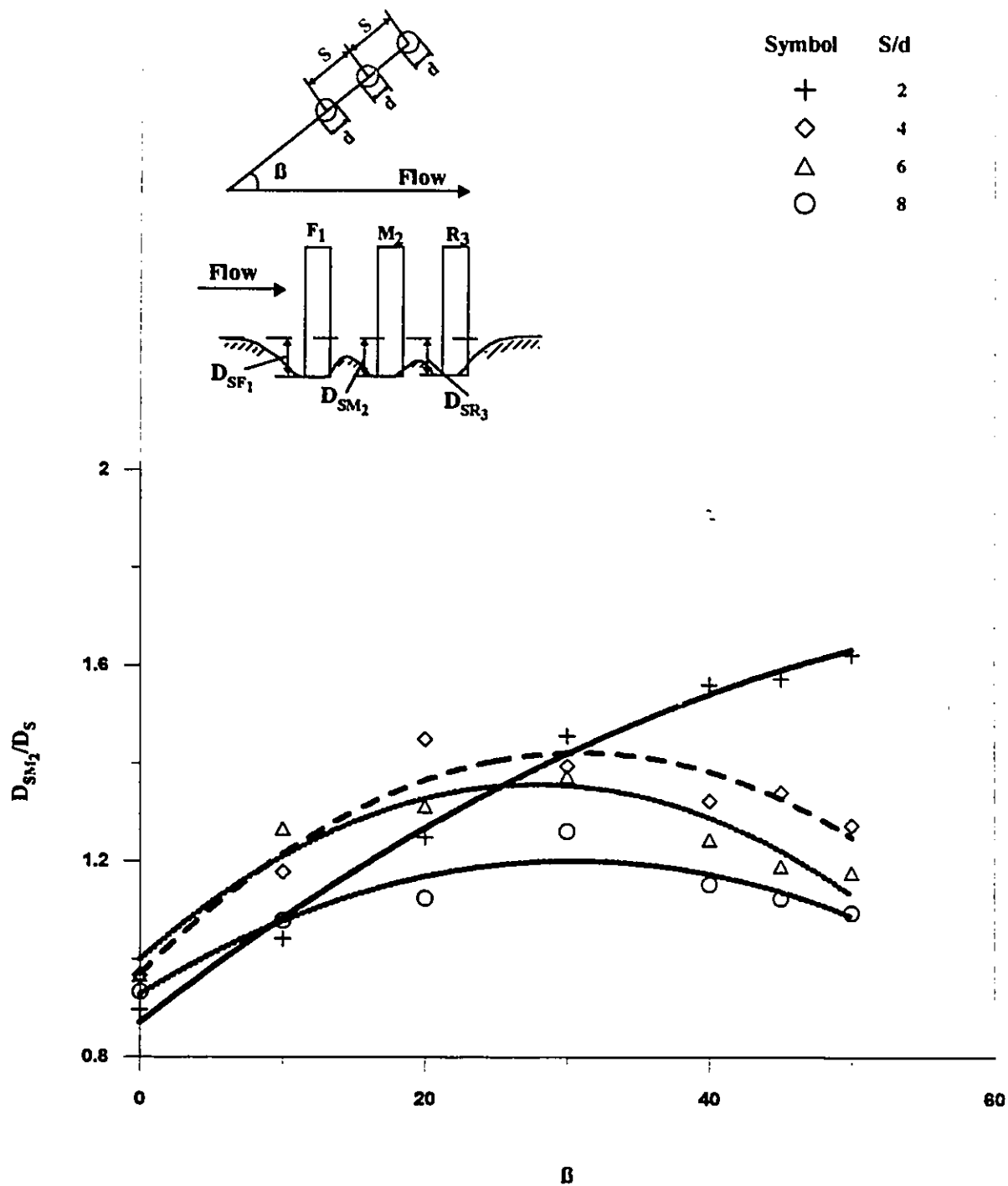


Fig. 5-3-14: Middle pier scour depth ratios versus angle of attack for different S/d ; groups of 3 piers

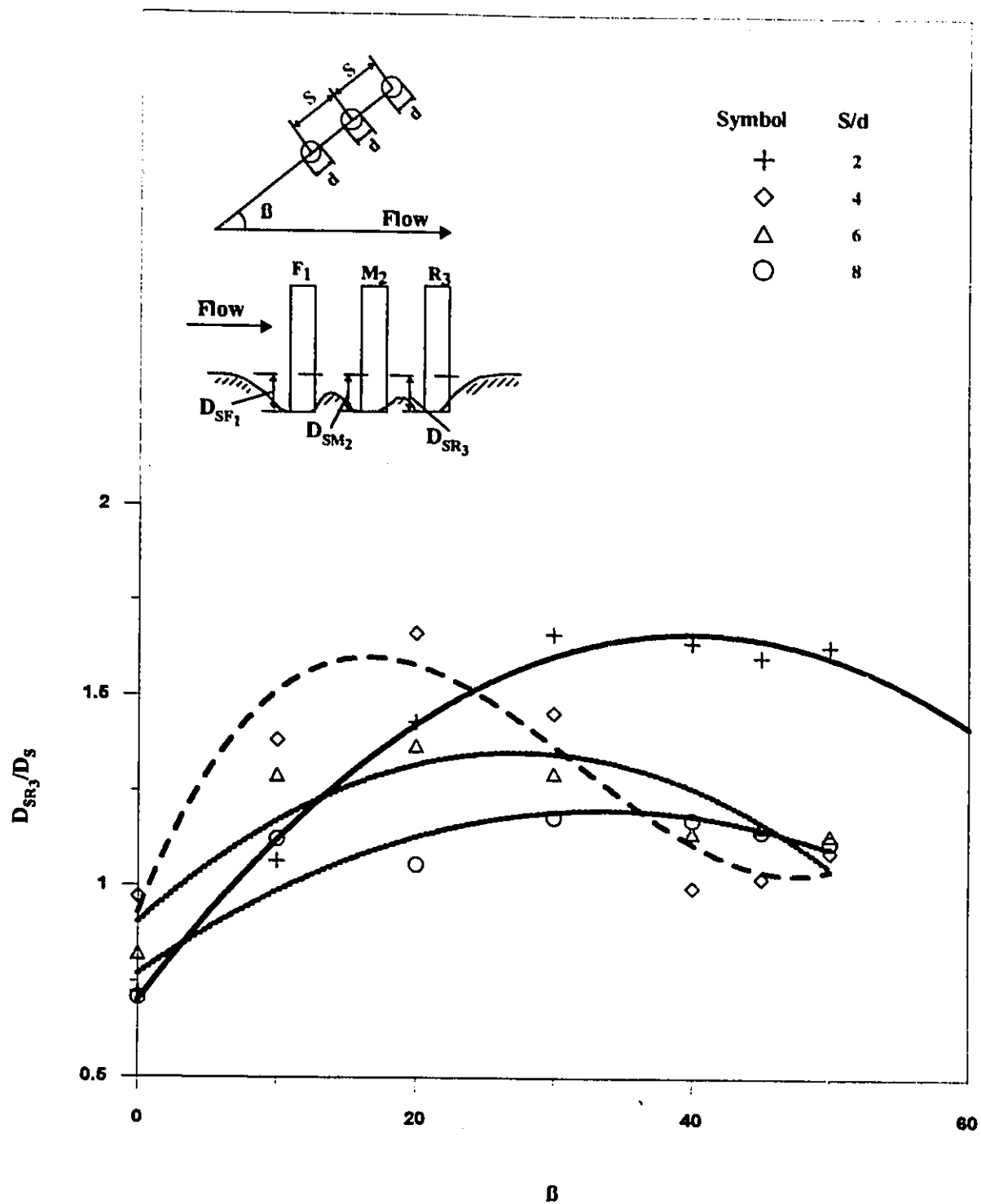


Fig. 5-3-15: Rear pier scour depth ratios versus flow angle of attack for different S/d ; groups of 3 piers

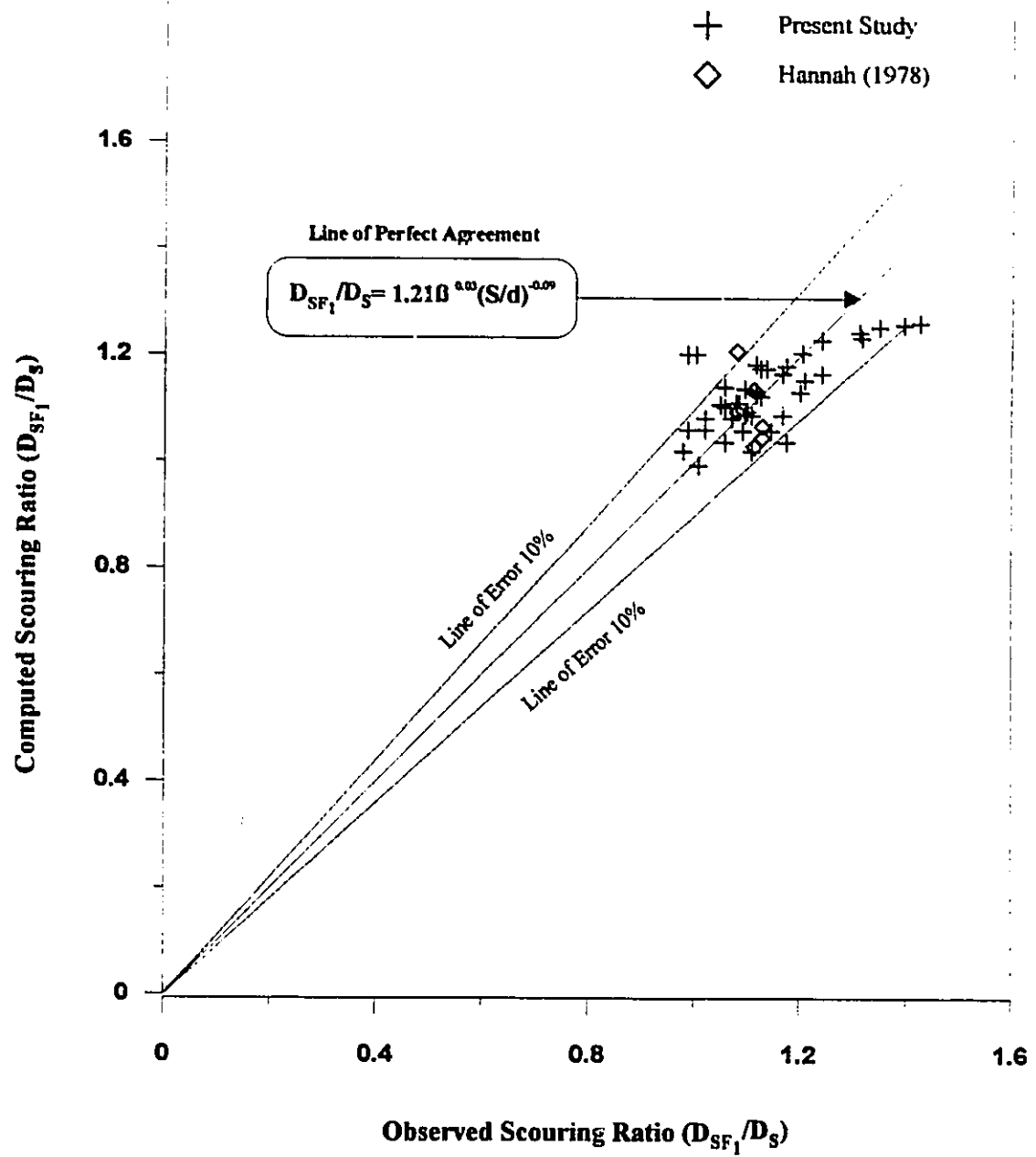


Fig. 5-3-16: Comparison between computed and measured front pier (F_1) scouring ratios; a group of 3 piers

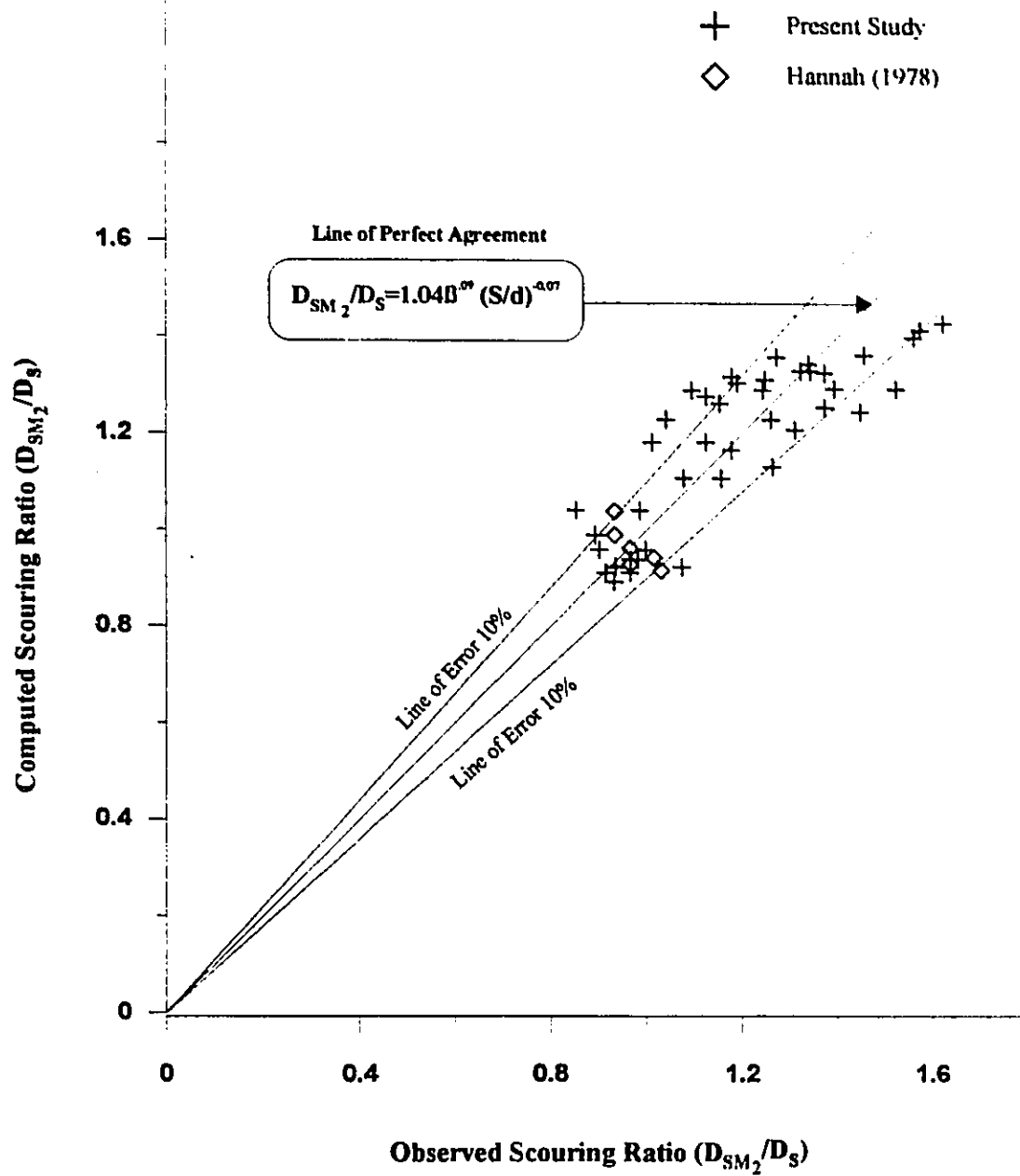


Fig. 5-3-17: Comparison between computed and measured middle pier (M_2) scouring ratios; a group of 3 piers

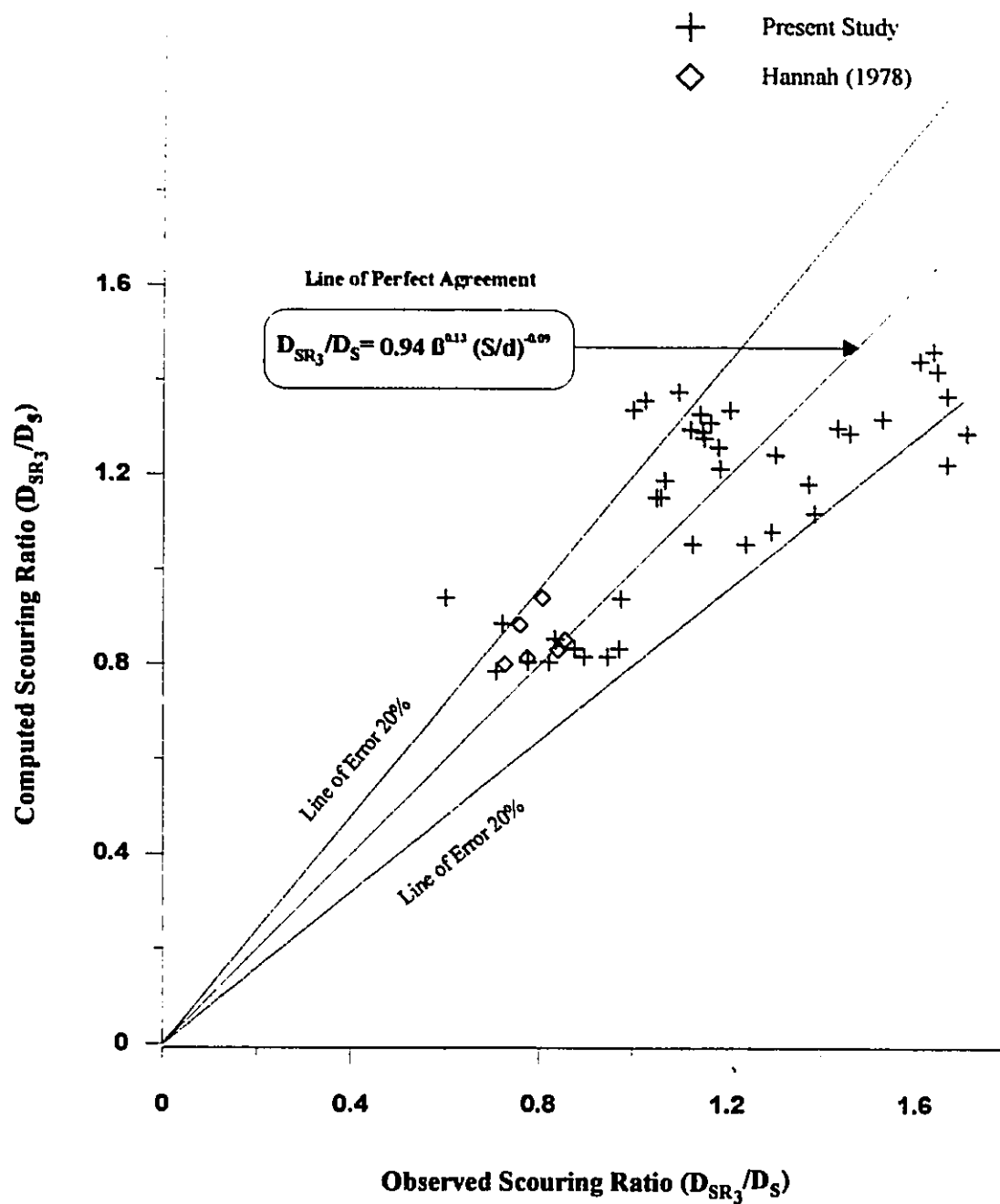


Fig. 5-3-18: Comparison between computed and measured rear pier (R_3) scouring ratios; a group of 3 piers

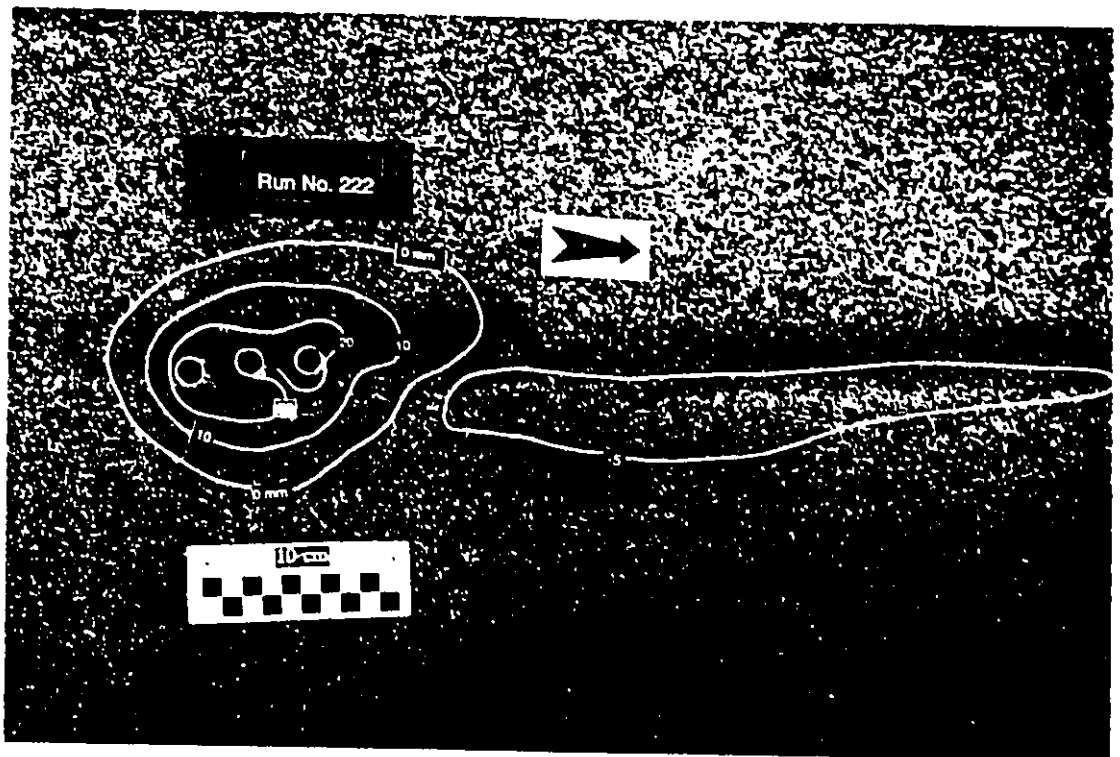
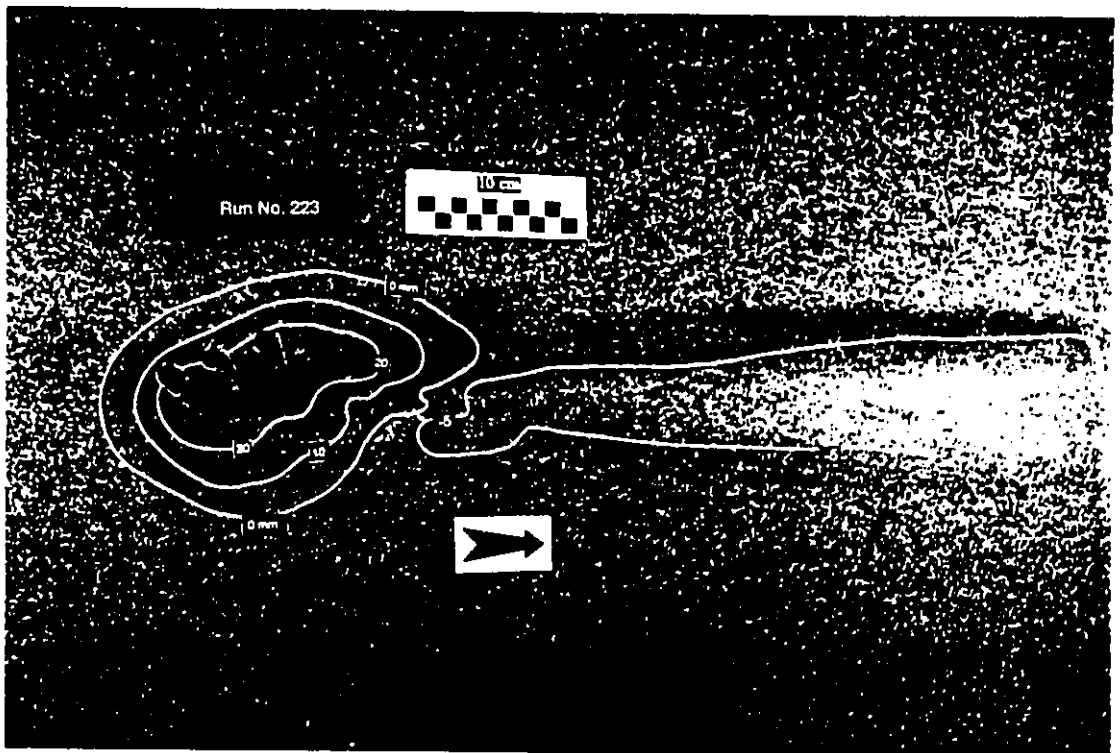


Plate 5-3-1: A group of 3 piers; $S/d=2$ and $\beta=10^\circ$



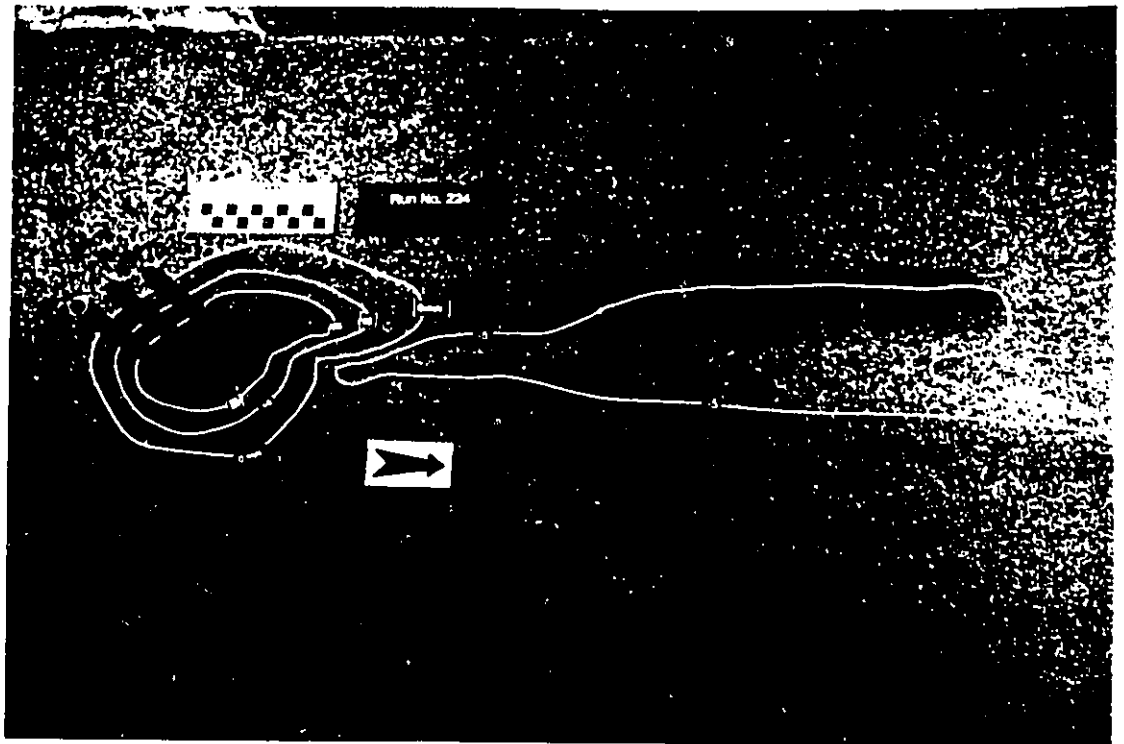


Plate 5-3-3: A group of 3 piers; $S/d=2$ and $\beta=30^\circ$

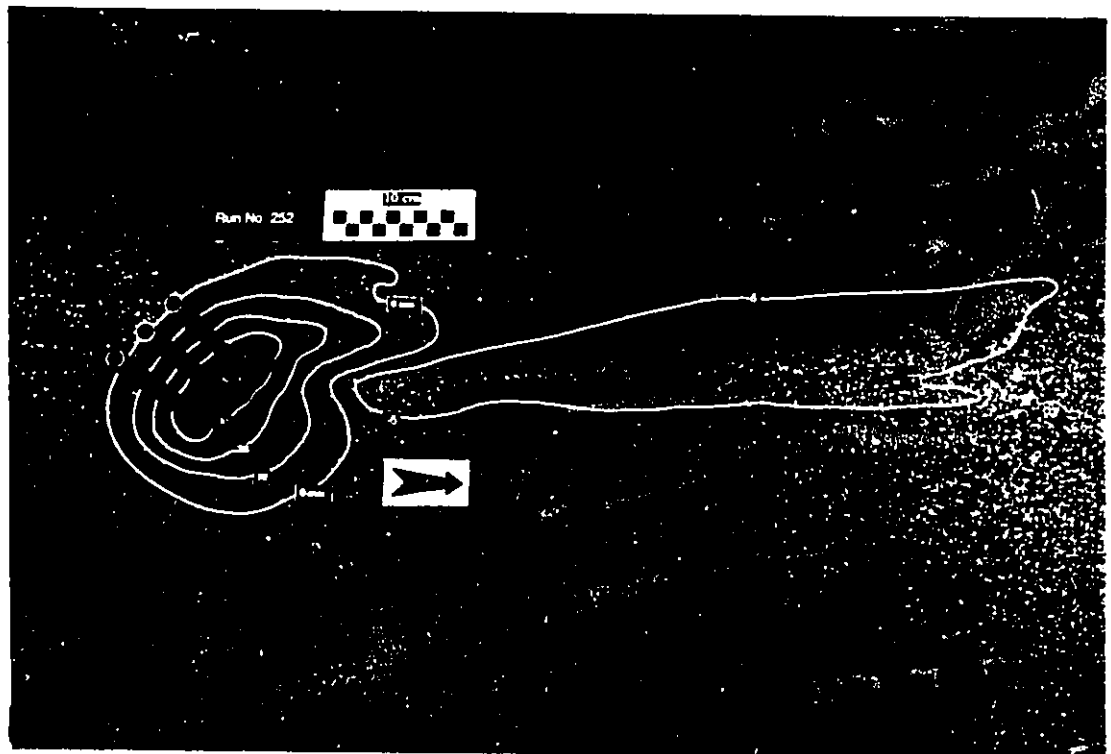


Plate 5-3-4: A group of 3 piers; $S/d=2$ and $\beta=50^\circ$

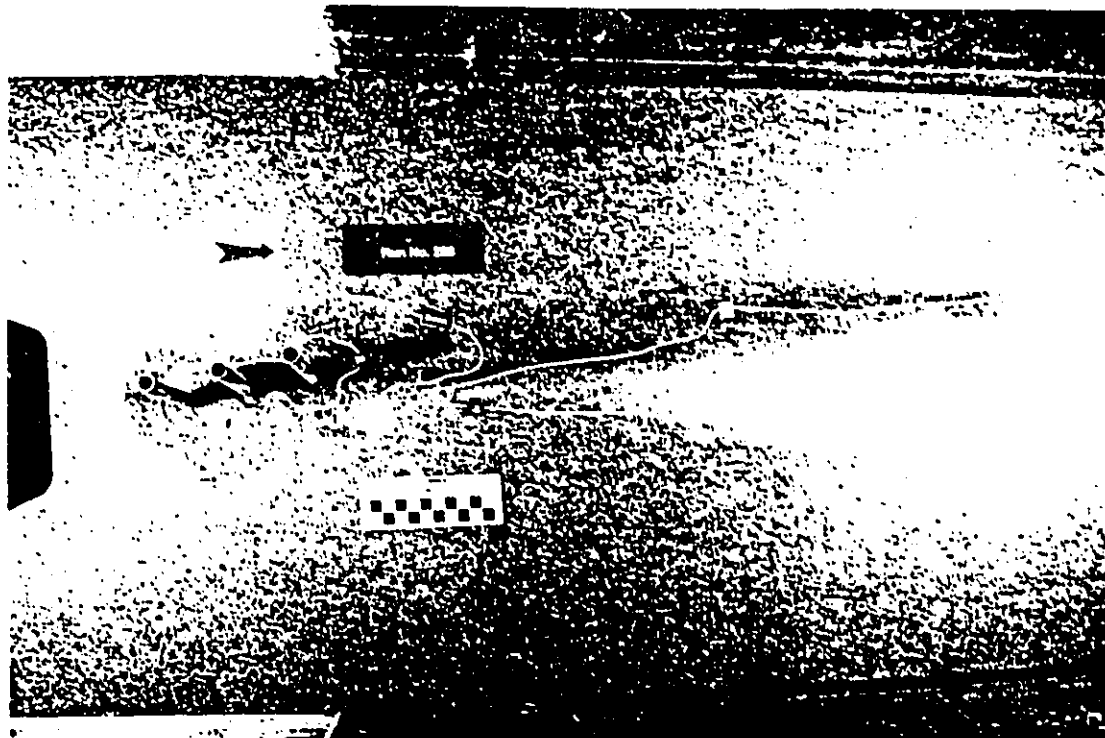


Plate 5-3-5: A group of 3 piers; $S/d=4$ and $\beta=10^\circ$

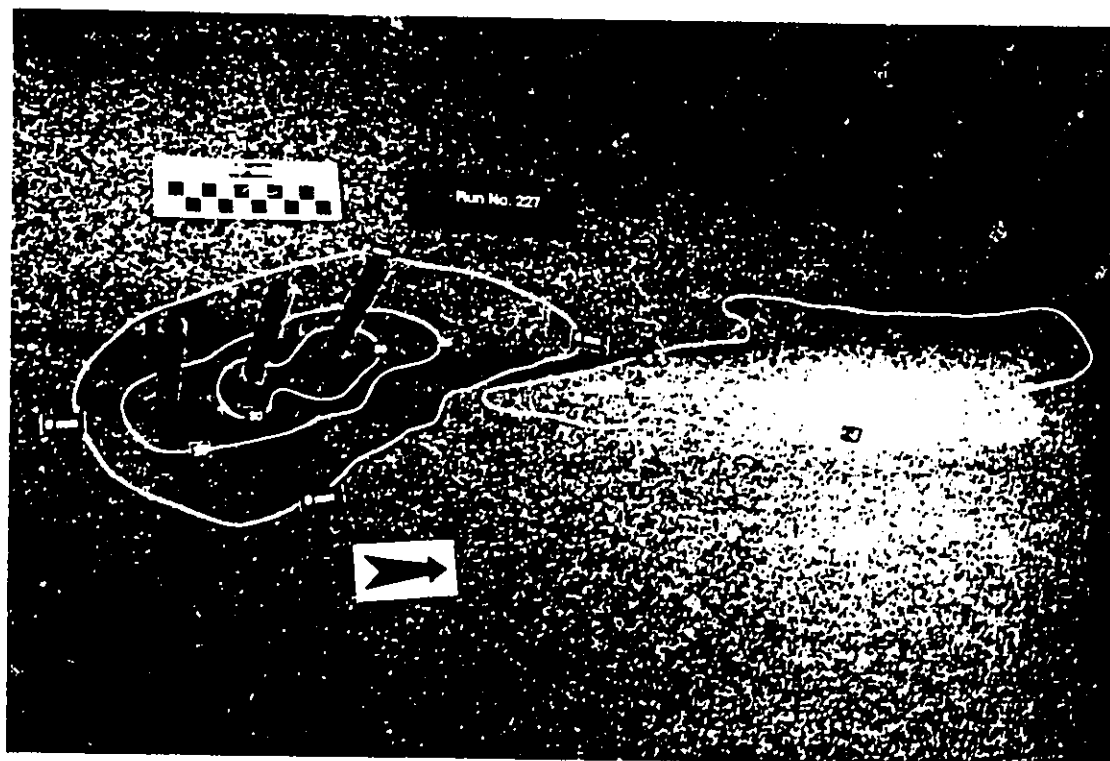


Plate 5-3-6: A group of 3 piers; $S/d=4$ and $\beta=20^\circ$

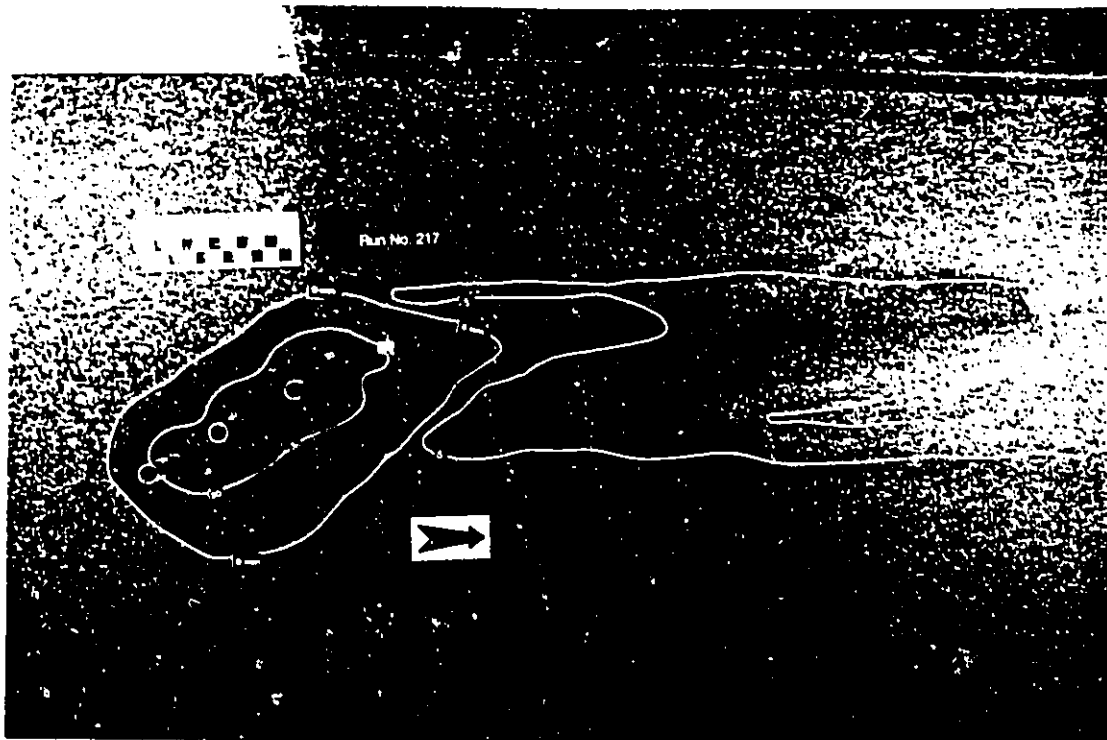


Plate 5-3-7: A group of 3 piers; $S/d=4$ and $\beta=30^\circ$

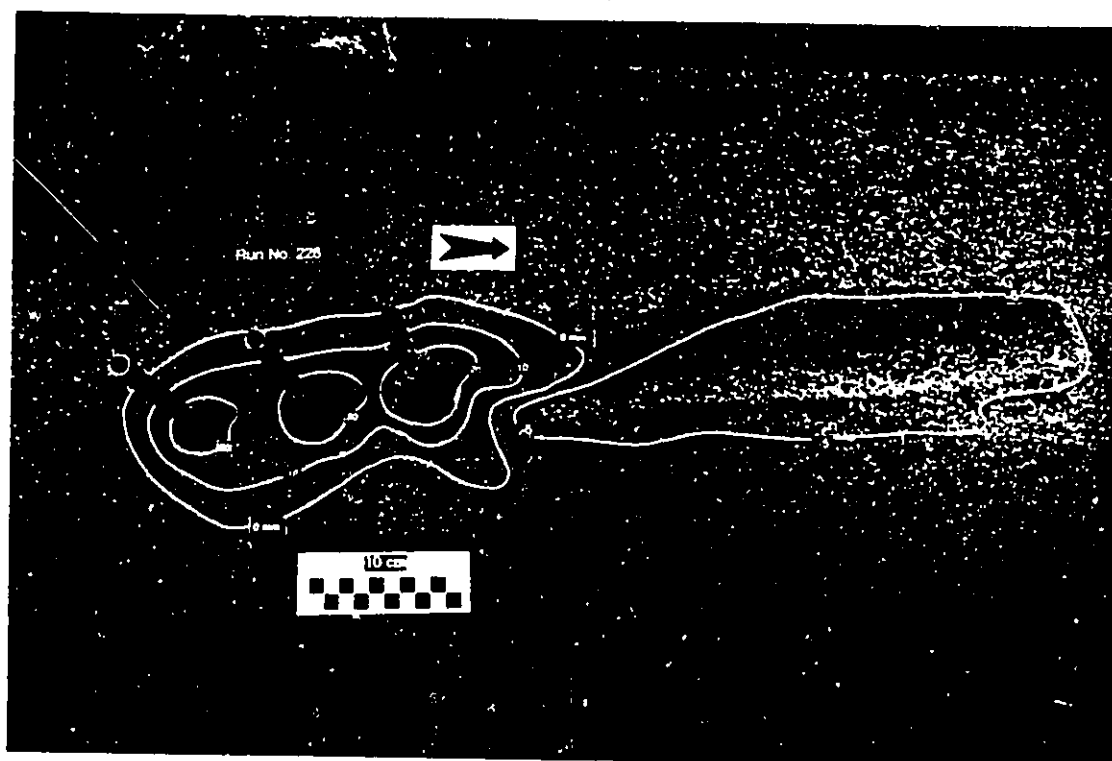


Plate 5-3-8: A group of 3 piers; $S/d=6$ and $\beta=10^\circ$



Plate 5-3-9: A group of 3 piers; $S/d=6$ and $\beta=20^\circ$

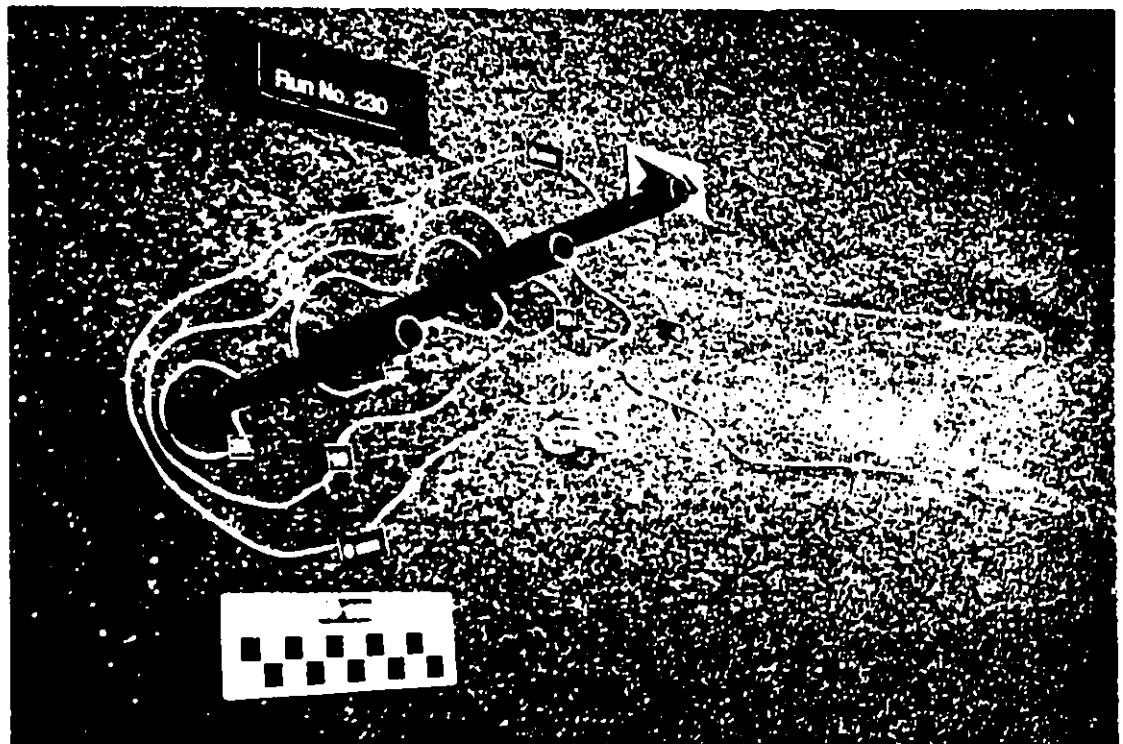


Plate 5-3-10: A group of 3 piers; $S/d=6$ and $\beta=30^\circ$

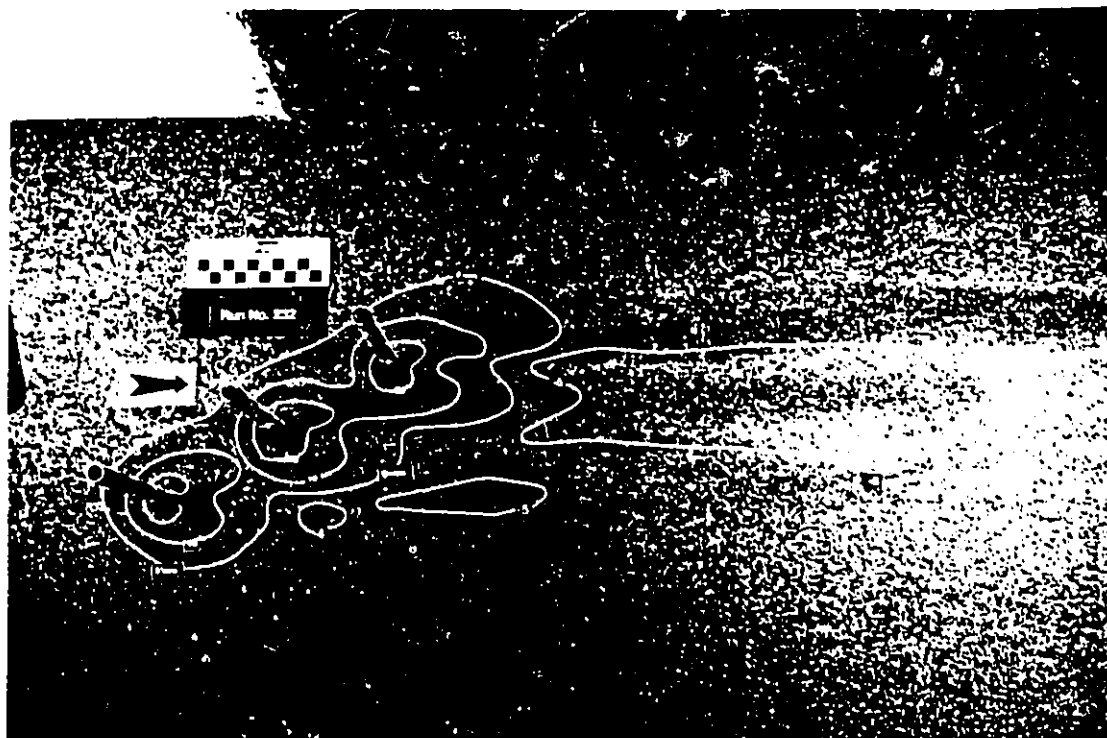


Plate 5-3-11: A group of 3 piers; $S/d=8$ and $\beta=10^\circ$

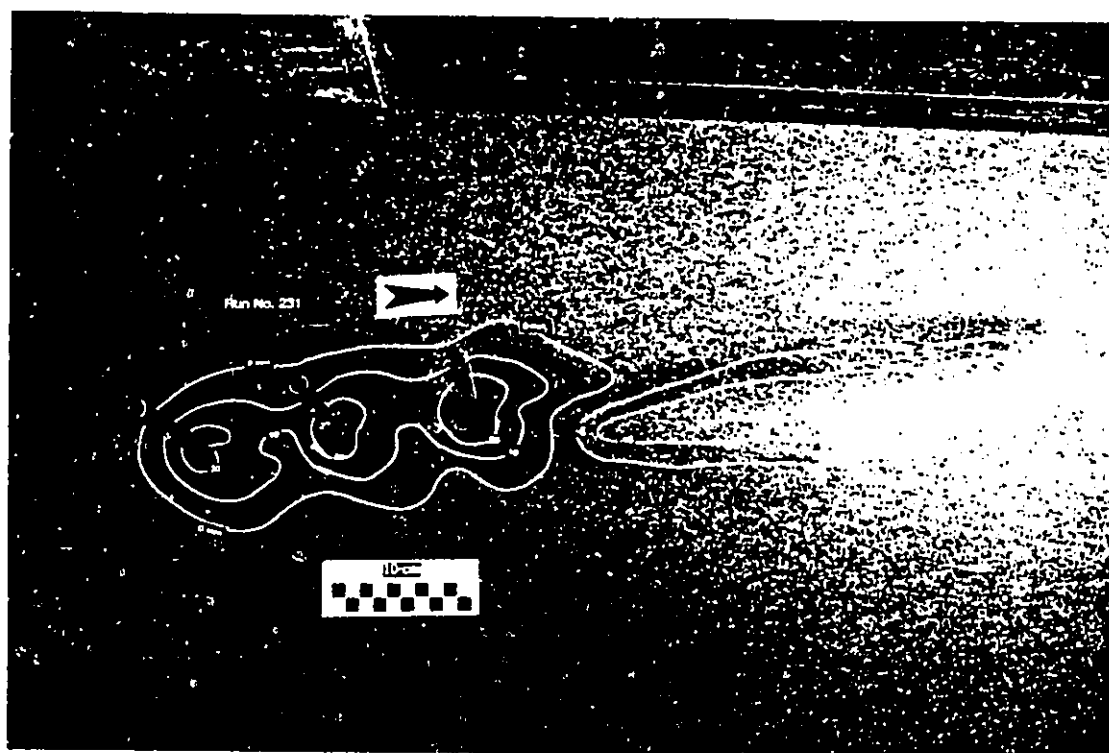


Plate 5-3-12: A group of 3 piers; $S/d=8$ and $\beta=30^\circ$

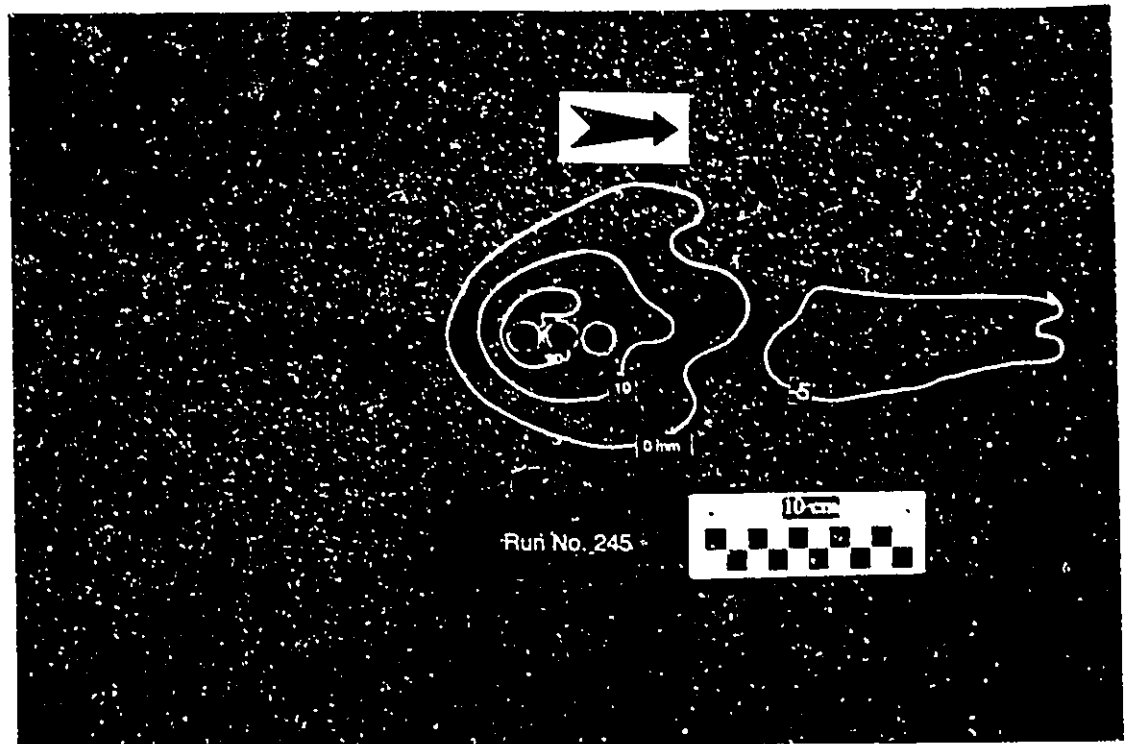


Plate 5-3-13: A group of 3 piers; $S/d=1$ and $\beta=0^\circ$

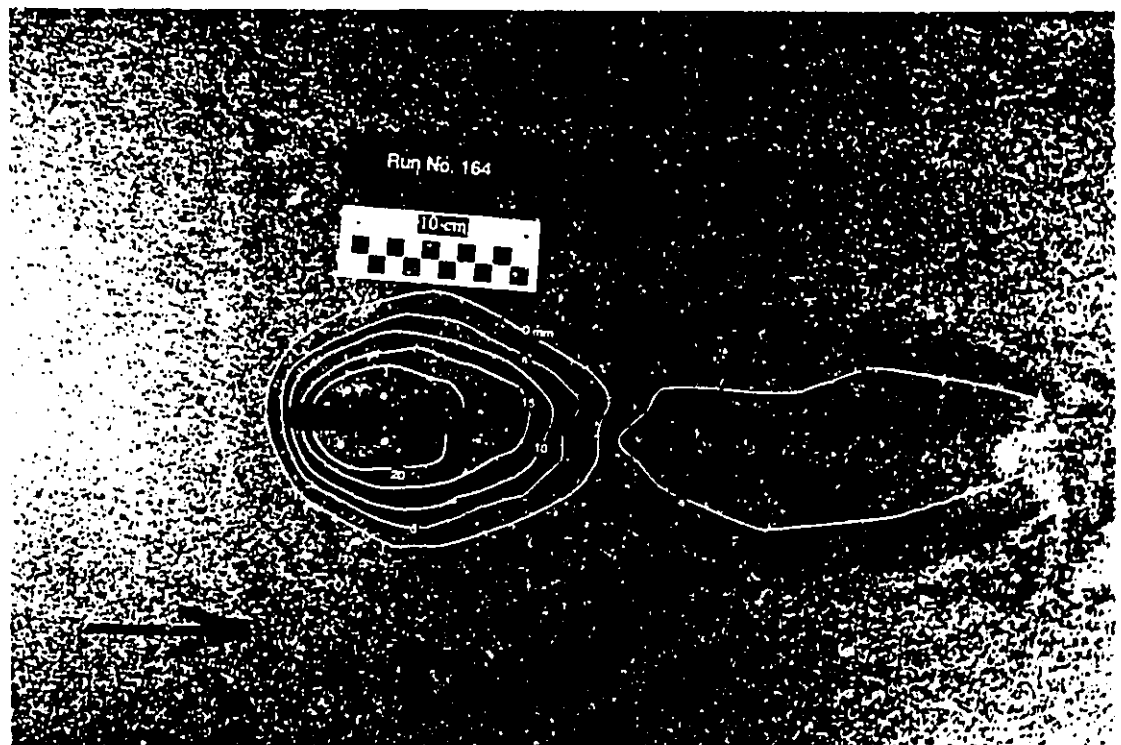


Plate 5-3-14: A group of 3 piers; $S/d=3$ and $\beta=0^\circ$

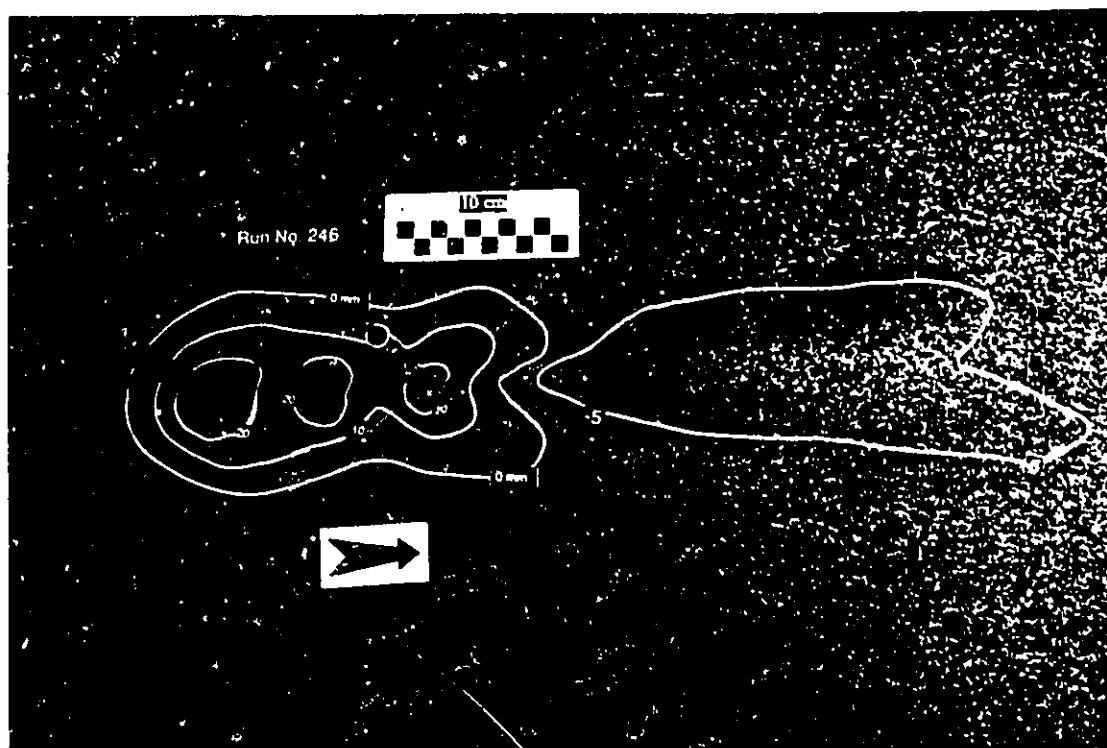


Plate 5-3-15: A group of 3 piers; $S/d=5$ and $\beta=0^\circ$

Table 5-3-1: Tests Performed for 3-PGs

β	S/d			
	2	4	6	8
0°	*	*	*	*
10°	*	*	*	*
20°	*	*	*	*
30°	*	*	*	*
35°	*	*	*	*
40°	*	*	*	*
45°	*	*	*	*
50°	*	*	*	*

* Test performed

Table 5-3-2: One-sided t-distribution for 3-PGs (28 degrees of freedom)

Confidence level (percent)	One-sided t-distribution (Statistics)
60.0	0.256
75.0	0.683
90.0	1.313
95.0	1.701
97.5	2.048
99.0	2.467
99.5	2.763

5-4 Local Scouring around 4-Pier Groups

Local scouring in the vicinity of a 4-PG was investigated at this phase of our experimental program. Table 5-4-1 identifies the β and S/d used in these experiments. The effects of sheltering, reinforcing, and compressed arms of HSV on local scour depth was investigated. Presented data include the normalized scour depth ratios ($\frac{D_{SF_1}}{D_s}=D'_{F_1}$, $\frac{D_{SR_2}}{D_s}=D'_{R_2}$, $\frac{D_{SF_3}}{D_s}=D'_{F_3}$, and $\frac{D_{SR_4}}{D_s}=D'_{R_4}$) plotted against β or S/d.

Thirty-one data points gathered from this stage of the study were analysed using multiple regression analyses. Using logarithm transformation, prediction equations were developed to find the normalized scour depths (D'_{F_1} , D'_{R_2} , D'_{F_3} , and D'_{R_4}). Four prediction equations were used to compute the local scour depth in front of F_1 , R_2 , F_3 , and R_4 . Good agreements were found between the computed and observed scour depths.

5-4-1 Effects of Varying β on Local Scour Depth

The effect of varying β on the local scour depth was investigated for S/d=2, 4, 6, and 8. The functional relationships between D'_{F_1} , D'_{R_2} , D'_{F_3} , D'_{R_4} and β are plotted in Figs. 5-4-1 through 5-4-20. The results for S/d=2 (Fig. 5-4-1) show an increasing trend of data for all piers in the 4-PG. This Fig. also shows that the scour depths in front of the piers in a group are always larger than that of a single pier. Since at the time of the pier installations β was set in the counter clockwise direction, F_3 was not sheltered by other piers for the range of β applied in the tests. Therefore, the scour depth in front of F_3 was nearly independent of β . On the other hand, Fig. 5-4-1 shows that the scour depth

around R_2 is increased in two steps. First, R_2 is sheltered by F_1 in the ranges $0^\circ \leq \beta \leq 10^\circ$ and $35^\circ \leq \beta \leq 45^\circ$. Second, it is exposed more to the flow in the range $10^\circ \leq \beta \leq 35^\circ$. Because of the presence of F_3 , the sheltering effect is not as significant as the effect of the arms of the HSV associated with R_2 . This is why D'_{R_2} does not decrease, although R_2 is sheltered by F_3 at $\beta=45^\circ$. For $S/d=2$, individual scour holes were not established around each pier; instead one scour hole was formed around all four piers. Plate 5-4-1 illustrates the nature of the local scouring around a 4-PG.

For $S/d=3$, D'_{F_1} , D'_{R_2} , and D'_{R_4} show an increasing trend with increasing β . Nevertheless, the variation of D'_{F_3} is relatively independent of β (Fig. 5-4-2). In this case, because of a larger S/d , as compared with the previous case, the sheltering effect exceeded that of the compressed arms of the HSV for the downstream piers. Therefore, since R_2 is sheltered by F_3 for $\beta > 30^\circ$, the variation of D'_{R_2} remains stable in this range. However, in this case, D'_{F_1} , D'_{R_2} , D'_{F_3} , and D'_{R_4} are smaller than their corresponding values for $S/d=2$. Because of a larger S/d , the arms of HSV are weaker which results in a shallower scour hole. The formation of the local scouring around the piers is shown in plate 5-4-2 for $\beta=0^\circ$ and $S/d=3$.

The next series of tests ($S/d=4$) investigated the effect of varying β on the local scour depth. Similar to the previous S/d cases, D'_{F_3} was found to be independent of β . The effect of sheltering on R_2 can be observed in Fig. 5-4-3. This figure shows a peak value of D'_{R_2} , followed by a sudden decrease. The peak value corresponds to the situation in which R_2 was exposed to the flow; while the sudden decrease is associated with the sheltering effect, which in turn is a result of the presence of the upstream pier (F_3). The development of the scour hole around R_2 shows that the value

for D'_{R_1} is smallest in the ranges $0^\circ \leq \beta \leq 10^\circ$ and $30^\circ \leq \beta \leq 40^\circ$, because R_2 is sheltered in these ranges by F_1 and F_3 , respectively. However, D'_{R_2} is largest in the range $20^\circ \leq \beta \leq 30^\circ$, because R_2 is completely exposed to the flow. At $\beta=45^\circ$, both F_1 and R_4 reach the same scour depth because they were equally exposed to the flow.

Next, the effect of different β on local scour depth was investigated for $S/d=6$. Similar to the previous cases, variation of D'_{F_3} was independent of β . The sheltering effect associated with R_2 was evident for the ranges of $0^\circ \leq \beta \leq 10^\circ$ and $30^\circ \leq \beta \leq 45^\circ$. In these ranges of β , R_2 was sheltered by F_1 and F_3 respectively. The maximum scour depth for R_2 occurred at $\beta=20^\circ$. The maximum scour depths associated with F_1 , R_2 , F_3 , and R_4 were smaller here than those for $S/d=4$ (Fig. 5-4-4). The reason is that compressed arms of the HSV became weaker with increasing S/d . The scouring pattern formed around the piers is illustrated in Plate 5-4-3 for $\beta=0^\circ$ and $S/d=6$.

In the final sets of experiments, the effect of different β on local scour depth was investigated for $S/d=8$ (Fig. 5-4-5). At this S/d value, an individual scour hole formed around each pier. Since, in this instance, S/d was greater than the previous cases, the maximum scour depths associated with the piers were less than those observed for $S/d=2$ to 6.

5-4-2 Effects of Different S/d on Local Scour Depth

In this phase of our study, β was kept constant within each experiment while S/d varied. The effect of the changing S/d on local scour depth was investigated after which the depths of scour

around the piers were compared with each other.

Using $\beta=0^\circ$, the effect of varying S/d on local scour depth was investigated. The results (Fig. 5-4-6) show a decreasing trend of data for all piers in the 4-PG. It is observed that $D'_{F_1}, D'_{R_2}, D'_{F_3}$, and D'_{R_4} approach unity with increasing S/d . Thus, the scour depths in front of all piers reach that of a single pier at $S/d=8$. The Fig. shows that the scour depth varies with S/d in the range $1 \leq S/d \leq 4$ more rapidly than in the range $4 < S/d \leq 8$. The results show that the effect of reinforcement is more significant in the range $1 \leq S/d \leq 4$ than that for $S/d > 4$. Also, since R_2 and R_4 are sheltered by F_1 and F_3 , respectively, D'_{R_2} and D'_{R_4} are smaller than D'_{F_1} and D'_{F_3} . For this special case in which piers are aligned to the flow and are spaced equally in respect to the walls of the channel, F_1 and R_2 are under the same flow conditions as are F_3 and R_4 . Therefore, it is expected that D'_{R_2} and D'_{R_4} are identical, as are D'_{F_1} and D'_{F_3} . The results for $\beta=0^\circ$, are compared with available data found in the literature. The results of Hannah (1978) and those of the present study show a good agreement (Fig. 5-4-7).

In the next set of tests, the effects of the varying of S/d on local scour depth was investigated for $\beta=10^\circ$. The results show a decreasing trend that is more significant in the range $2 \leq S/d \leq 4$ than that in the range $4 \leq S/d \leq 8$ (Fig. 5-4-8). The scour depths in front of grouped piers approach that of a single pier with increasing S/d . The front piers show a larger scour depth than the rear piers in the range $2 \leq S/d \leq 6$, because the reinforcement (associated with the rear piers) affects the front piers.

For $\beta=20^\circ$, D'_{F_1} , D'_{F_3} , and D'_{R_4} show a decreasing trend approaching unity, whereas

D'_{R_2} shows less response to the changing of S/d . In this case, R_2 was exposed to the flow while in the previous case it was sheltered by F_1 . Therefore, the overall scour depth is greater in this case than the former case. In addition, D'_{R_2} is greater than other normalized scour depths. The greater scour depth results from the specific flow pattern and pier alignments, that cause a situation in which the sediments are removed more easily from around R_2 .

The effect of different S/d on the local scour depth was investigated for $\beta=30^\circ$. The scour depth in front of the piers shows a decreasing trend that approaches the scour depth of a single pier (Fig. 5-4-10). The most significant change in the scour depth is associated with F_1 . In the next set of tests, the effect of different S/d on the local scour depth was investigated for $\beta=40^\circ$. As in the previous cases, a decreasing trend of data is observed for all normalized scour depths (Fig. 5-4-11). Here, also with increasing S/d , D'_{F_1} , D'_{R_2} , D'_{F_3} and D'_{R_4} approach unity. It can be seen once again that D'_{R_2} fluctuates the most. This fluctuation is due to the location of R_2 in respect to the other piers.

The final set of tests investigated the effect of varying S/d on the local scour depth for $\beta=45^\circ$. The pier arrangement for this β is such that R_2 is aligned with F_3 . This implies that the former is completely sheltered by the latter. On the other hand, F_1 and R_4 are positioned symmetrically which means that both are equally exposed to the flow. Furthermore, the values of D'_{F_1} and D'_{R_4} are identical for $S/d \leq 4$ (Fig. 5-4-12).

5-4-3 Comparison between a 4-PG and a 2-PG

It was mentioned earlier that a 4-PG can be treated as two 2-PGs. Here we will compare the results of both groups of piers. Figs. 5-4-21 and 5-4-22 show that the scour depth in a 4-PG is always greater than that of a 2-PG for $S/d=2$ and 3. This implies that the presence of an extra row of piers beside a 2-PG intensifies the strength of the HSV causing more removal of sediments from around the piers. We may also consider a 4-PG equivalent to two 2-PGs with $\beta=90^\circ$, namely $(F_1 - F_3)$ and $(R_2 - R_4)$. The effect of the presence of the second pier group (R_2 and R_4) is shown in Fig. 5-4-23. It is observed that the local scour depth increases dramatically with the presence of the second pier group. This implies the important effect of reinforcement, which is more significant for the smaller S/d . For $S/d>8$, both the 2 and 4-PGs approach the local scour depth of a single pier.

In the next set of experiments both pier groups were aligned to the flow. For the purpose of comparison, Hannah's (1978) data are also included in Fig. 5-2-24. This figure shows that the scour depth for a 4-PG is always greater than that for a 2-PG in the range $2 \leq S/d \leq 4$. Nevertheless, both pier groups approach the scour depth of a single pier for $S/d>4$. Local scour depths in front of the piers in a 2-PG or a 4-PG that are misaligned to the flow, are compared in Figs. 5-4-25 to 5-4-26. The local scour depths associated with a 4-PG are always greater than those of a 2-PG for $\beta=40^\circ$ and 45° in the range $2 \leq S/d \leq 8$. Once again, as was shown earlier in Fig. 5-4-23, beyond $S/d=8$ both the 2 and 4-PGs reach the scour depth of a single pier.

5-4-5 Multiple Regression Analysis

5-4-5-1 Analysis of Data

Thirty-one measurements of local scour depths gathered from this study were analysed using multiple regression analyses to obtain local scour prediction equations for the front and rear piers. As was mentioned for the 2- and 3-pier groups, local scour depth for a 4-PG is a function of β and S/d for any arrangement of pier groupings. The functional relations for the normalized local scour depth are:

$$\frac{D_{SF_1}}{D_s} = f_1 \left\{ \beta, \frac{S}{d} \right\} \quad (5-4-1)$$

$$\frac{D_{SR_2}}{D_s} = f_2 \left\{ \beta, \frac{S}{d} \right\} \quad (5-4-2)$$

$$\frac{D_{SF_3}}{D_s} = f_3 \left\{ \beta, \frac{S}{d} \right\} \quad (5-4-3)$$

$$\text{and} \quad \frac{D_{SR_4}}{D_s} = f_4 \left\{ \beta, \frac{S}{d} \right\} \quad (5-4-4)$$

A multiple linear regression was used to develop prediction equations for the 4-PG, based on the proposed functional relations in equations 5-4-1, 5-4-2, 5-4-3, and 5-4-4. Using the multiple regression model, as given in the earlier matrix notation (equations 5-2-2, 5-2-3, and 5-2-4), the

vector of fitted values corresponding to a vector of observed values is computed. Then, the "hat" matrix $H_{31 \times 31} = (X(X'X)^{-1}X')$ which is necessary in regression analysis can be computed for the 4-PG, using equation 5-2-4. Logarithmic transformation of all variables was found to provide the best linear relations. The logarithms of $\frac{D_{SF_1}}{D_s}$, $\frac{D_{SR_2}}{D_s}$, $\frac{D_{SF_3}}{D_s}$, and $\frac{D_{SR_4}}{D_s}$ were regressed against a combination of the two independent variables β and S/d .

The general forms of the prediction equations are:

$$\frac{D_{SF_1}}{D_s} = a_1 \beta^{b_1} \left(\frac{S}{d} \right)^{c_1} \quad (5-4-5)$$

$$\frac{D_{SR_2}}{D_s} = a_2 \beta^{b_2} \left(\frac{S}{d} \right)^{c_2} \quad (5-4-6)$$

$$\frac{D_{SF_3}}{D_s} = a_3 \beta^{b_3} \left(\frac{S}{d} \right)^{c_3} \quad (5-4-7)$$

$$\text{and} \quad \frac{D_{SR_4}}{D_s} = a_4 \beta^{b_4} \left(\frac{S}{d} \right)^{c_4} \quad (5-4-8)$$

in which; $a_1 = \exp(\hat{\theta}'_{F_1(1)})$, $b_1 = \hat{\theta}'_{F_1(2)}$, $c_1 = \hat{\theta}'_{F_1(3)}$, $a_2 = \exp(\hat{\theta}'_{R_2(1)})$, $b_2 = \hat{\theta}'_{R_2(2)}$, $c_2 = \hat{\theta}'_{R_2(3)}$, $a_3 = \exp(\hat{\theta}'_{R_3(1)})$, $b_3 = \hat{\theta}'_{R_3(2)}$, $c_3 = \hat{\theta}'_{R_3(3)}$, $a_4 = \exp(\hat{\theta}'_{R_4(1)})$, $b_4 = \hat{\theta}'_{R_4(2)}$, $c_4 = \hat{\theta}'_{R_4(3)}$, $\hat{\theta}'_{F_1} = [\hat{\theta}'_{F_1(1)}, \hat{\theta}'_{F_1(2)}, \hat{\theta}'_{F_1(3)}]$, $\hat{\theta}'_{R_2} = [\hat{\theta}'_{R_2(1)}, \hat{\theta}'_{R_2(2)}, \hat{\theta}'_{R_2(3)}]$, $\hat{\theta}'_{F_3} = [\hat{\theta}'_{F_3(1)}, \hat{\theta}'_{F_3(2)}, \hat{\theta}'_{F_3(3)}]$, and $\hat{\theta}'_{R_4} = [\hat{\theta}'_{R_4(1)}, \hat{\theta}'_{R_4(2)}, \hat{\theta}'_{R_4(3)}]$.

5-4-4-2 Predicting Scour Depths

Normalized scour depths predicted by equations 5-4-5, 5-4-6, 5-4-7, and 5-4-8 are the expected response to a specified set of regressors, and are not known with certainty. Using a procedure similar to the case for the 2-PG, a probability statement about the distribution of $\ln(\frac{D_{SF_1}}{D_s})$, $\ln(\frac{D_{SR_2}}{D_s})$, $\ln(\frac{D_{SF_3}}{D_s})$, and $\ln(\frac{D_{SR_4}}{D_s})$ provides a means of evaluating appropriate design values. Using the transpose of the vector of predictor variables, as given in equation 5-2-6, several $100(1-\alpha)$ percent upper confidence limits for the logarithm of normalized scour depths can be computed. Equation 5-2-7 is used to compute the upper confidence limit, using $\frac{D_{SF_1}}{D_s}$, $\frac{D_{SR_2}}{D_s}$, $\frac{D_{SF_3}}{D_s}$, and $\frac{D_{SR_4}}{D_s}$ as substitutions to $\frac{D_{S_s}}{D_s}$. The values of $\hat{\theta}'_{F_1}$, $\hat{\theta}'_{R_2}$, $\hat{\theta}'_{F_3}$, and $\hat{\theta}'_{R_4}$ computed for a 3-PG are indicated in equations 5-4-9 through 5-4-12:

$$\hat{\theta}'_{F_1}=[0.63,0.01,-0.32] \quad (5-4-9)$$

$$\hat{\theta}'_{R_2}=[0.38,0.06,-0.23] \quad (5-4-10)$$

$$\hat{\theta}'_{F_3}=[0.54,0.01,-0.24] \quad (5-4-11)$$

$$\text{and} \quad \hat{\theta}'_{R_4}=[0.37,0.06,-0.23] \quad (5-4-12)$$

The variance of prediction can be estimated using equation 5-2-10 in which σ^2 for the front, middle, and rear piers is substituted by $\hat{\sigma}_{F_1}^2$, $\hat{\sigma}_{R_2}^2$, $\hat{\sigma}_{F_3}^2$, and $\hat{\sigma}_{R_4}^2$. For the 2-variable regression model, using thirty-one measurements, the variances of predictions are $\hat{\sigma}_{F_1}^2 = 0.24$, $\hat{\sigma}_{R_2}^2 = 0.42$, $\hat{\sigma}_{F_3}^2 = 0.1$, and $\hat{\sigma}_{R_4}^2 = 0.29$. One-sided t-distribution statistics for $n-p=31-3=28$, degrees of freedom and several confidence levels are given in Table 5-4-2. Upper estimates of $\ln\left(\frac{D_{SF_1}}{D_s}\right)$, $\ln\left(\frac{D_{SR_2}}{D_s}\right)$, $\ln\left(\frac{D_{SF_3}}{D_s}\right)$, and $\ln\left(\frac{D_{SR_4}}{D_s}\right)$ are obtained from equation 5-2-7. With the inclusion of an appropriate factor of safety, the upper limit of scour depth can be used to design pier foundations.

5-4-4-3 Prediction Equation for a 4-PG

Parameters a_1 , b_1 , c_1 , a_2 , b_2 , c_2 , a_3 , b_3 , c_3 , a_4 , b_4 , and c_4 obtained for a 4-PG were substituted in equation 5-2-5. Hence, for the front and rear piers of a 4-PG, equation 5-2-5 can be expressed as:

$$\frac{\hat{D}_{SF_1}}{D_s} = 1.88\beta^{0.01} \left(\frac{S}{d} \right)^{-0.32} \quad (5-4-13)$$

$$\frac{\hat{D}_{SR_2}}{D_s} = 1.46\beta^{0.06} \left(\frac{S}{d} \right)^{-0.23} \quad (5-4-14)$$

$$\frac{\hat{D}_{SF_3}}{D_s} = 1.72\beta^{0.01} \left(\frac{S}{d} \right)^{-0.24} \quad (5-4-15)$$

Equations 5-4-13, 5-4-14, 5-4-15, and 5-4-16 are used to predict $\frac{D_{SF_1}}{D_s}$, $\frac{D_{SR_2}}{D_s}$, $\frac{D_{SF_3}}{D_s}$, and $\frac{D_{SR_4}}{D_s}$. The predicted values are plotted against observed values in Figs. 5-4-27, 5-4-28, 5-4-29, and 5-4-30. Most data fall within $\pm 10\%$, $\pm 10\%$, $\pm 5\%$, and $\pm 10\%$ of the predictions from the relationship proposed in this study, for the front and rear piers, respectively. The value of D_s may be obtained using any appropriate empirical equations for a single pier. Equations 5-4-13, 5-4-14, 5-4-15 and 5-4-16 represent the line of perfect agreement in Figs. 5-4-27, 5-4-28, 5-4-29, and 5-4-30 respectively.

$$\frac{\hat{D}_{SR_4}}{D_s} = 1.45\beta^{0.06} \left(\frac{S}{d} \right)^{-0.23} \quad (5-4-16)$$

Equations 5-4-14 and 5-4-16 are almost the same, therefore, the local scour depth in front of the rear piers can be estimated conservatively using equation 5-4-14. Coefficients of equations 5-4-13 and 5-4-15 are not as close as those obtained for the rear pier. Nevertheless, the local scour depth in front of the front piers can be estimated conservatively using equation 5-4-13.

5-4-5 Conclusions:

The variation of scour depth against β , for each pier in a 4-PG was investigated using different S/d . The results for $\beta=0^\circ$, 10° , 20° , 30° , 40° , and 45° showed the same decreasing trend of data, approaching unity, with increasing S/d for D'_{F_1} , D'_{R_2} , D'_{F_3} , and D'_{R_4} (Figs. 5-4-1 to 5-4-12). This implies that by further increasing S/d , the sheltering effect is more significant than that of the arms

of the HSV. Fig. 5-4-13 depicts the effect of increasing S/d on D'_{F_1} . According to the results, a larger scour depth was observed for the smaller S/d . The same trend of data was observed for R_2 , F_3 , and R_4 (Figs. 5-4-14 to 5-4-16). However, D'_{F_1} and D'_{F_3} were significantly dependent on β . Moreover, it can be seen that R_2 is sheltered by F_1 in the range $0^\circ \leq \beta \leq 10^\circ$. In addition, a sharp increase in D'_{R_2} is observed, resulting from the absence of the sheltering effect in the range $10 < \beta \leq 20^\circ$. The trend of data for R_4 is similar to those observed for R_2 . Figs. 5-4-13 to 5-4-16 show that D'_{F_1} , D'_{R_2} , D'_{F_3} , and D'_{R_4} are within the range of between 1 and 1.5.

The effects of different β on local scour depths was investigated for $S/d=2, 4, 6$, and 8. Generally, an increasing trend of data was observed for all piers in the group, except F_3 . It was evident that D'_{F_3} was independent of β . Moreover, it was shown that the of arms of HSV were weaker for a larger S/d while the sheltering effect was stronger for the smaller β . The smaller the S/d , the larger the peak value was for D'_{F_1} , D'_{R_2} , D'_{F_3} , and D'_{R_4} .

Comparison between 2 and 4-PGs reveals the fact that a deeper scour hole is expected in a 4-PG. In addition, a 4-PG can be considered as two 2-PGs which are close together. The closer the piers, the deeper the local scour depth is. Moreover, by increasing S/d , the difference between the local scour depth in 2 and 4-PGs is reduced. For $S/d > 8$, both the 2 and 4-PGs reach the scour depth of a single pier. The reinforcement effect plays a pivotal role in all cases in the range $2 \leq S/d \leq 4$.

The general form of the prediction equation written in terms of untransformed values was found for each pier, using the transpose of the vector of the regression model. The values of $\hat{\theta}'_{F_1}$, $\hat{\theta}'_{R_2}$,

, $\hat{\theta}'_{F_1}$, and $\hat{\theta}'_{R_1}$ computed for a 4-PG were found using equations 5-4-9, 5-4-10, and 5-4-12. By substituting the values of $\hat{\theta}'_{F_1}$, $\hat{\theta}'_{R_2}$, $\hat{\theta}'_{F_3}$, and $\hat{\theta}'_{R_4}$ in equation 5-2-5, the prediction equations for the front and rear piers were found using equations 5-4-13, 5-4-14, 5-4-15, and 5-4-16. Using equations 5-4-13, 14, 15, and 16, the values of $\frac{D_{SF_1}}{D_s}$, $\frac{D_{SR_2}}{D_s}$, $\frac{D_{SF_3}}{D_s}$, and $\frac{D_{SR_4}}{D_s}$ were computed and plotted against the observed values in Figs. 5-4-27, 5-4-28, 5-4-29 and 5-4-30. The prediction equations were found successful in predicting the local scour depth for a 4-PG within acceptable errors including vast majority of the data. It was concluded that equations 5-4-13 and 5-4-14 can be used to estimate the scour depth of the front and rear piers, respectively. Using these two equations give conservative results.

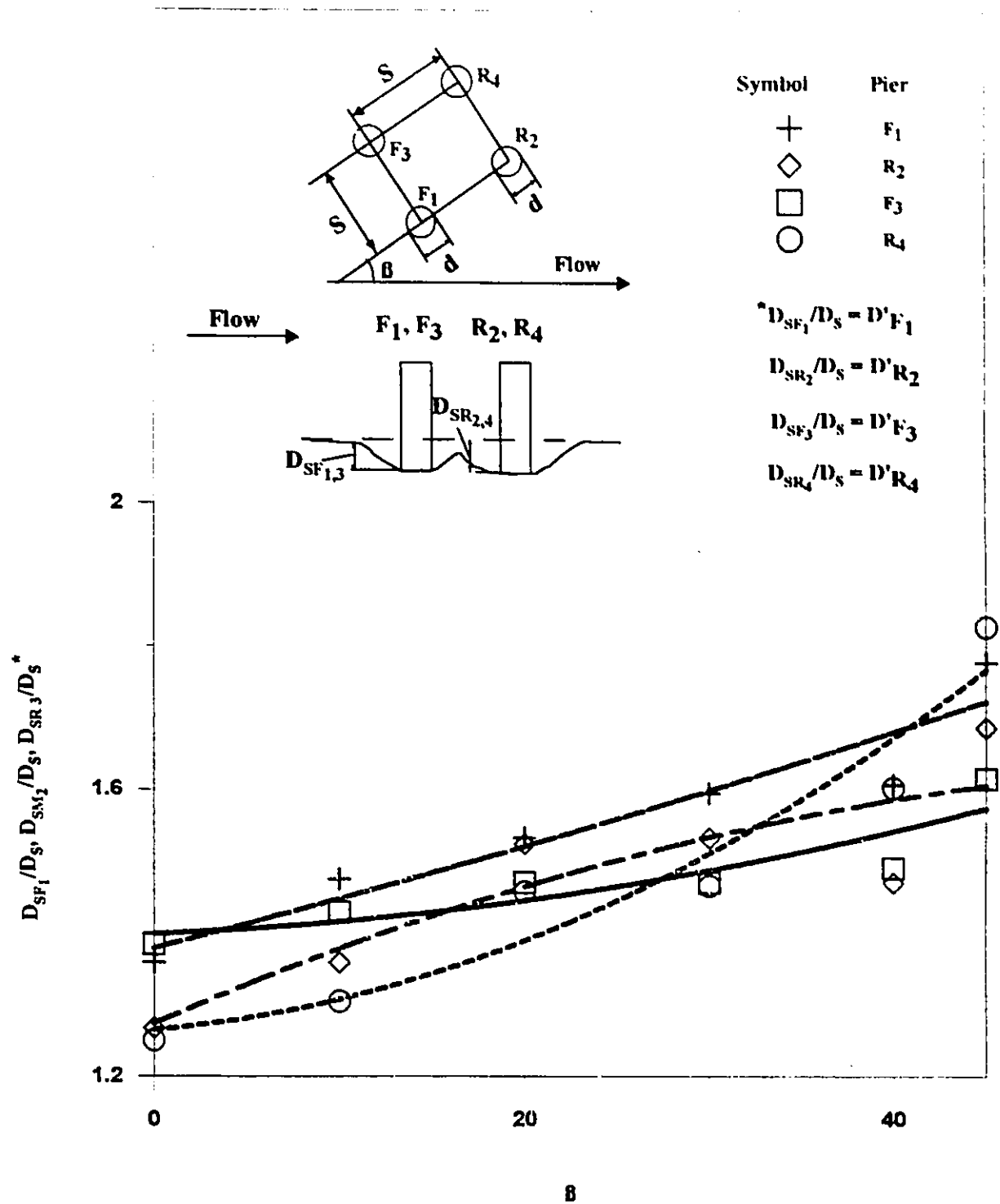


Fig. 5-4-1: Scour depth ratios versus flow angle of attack groups of 4 piers; ($S/d=2$)

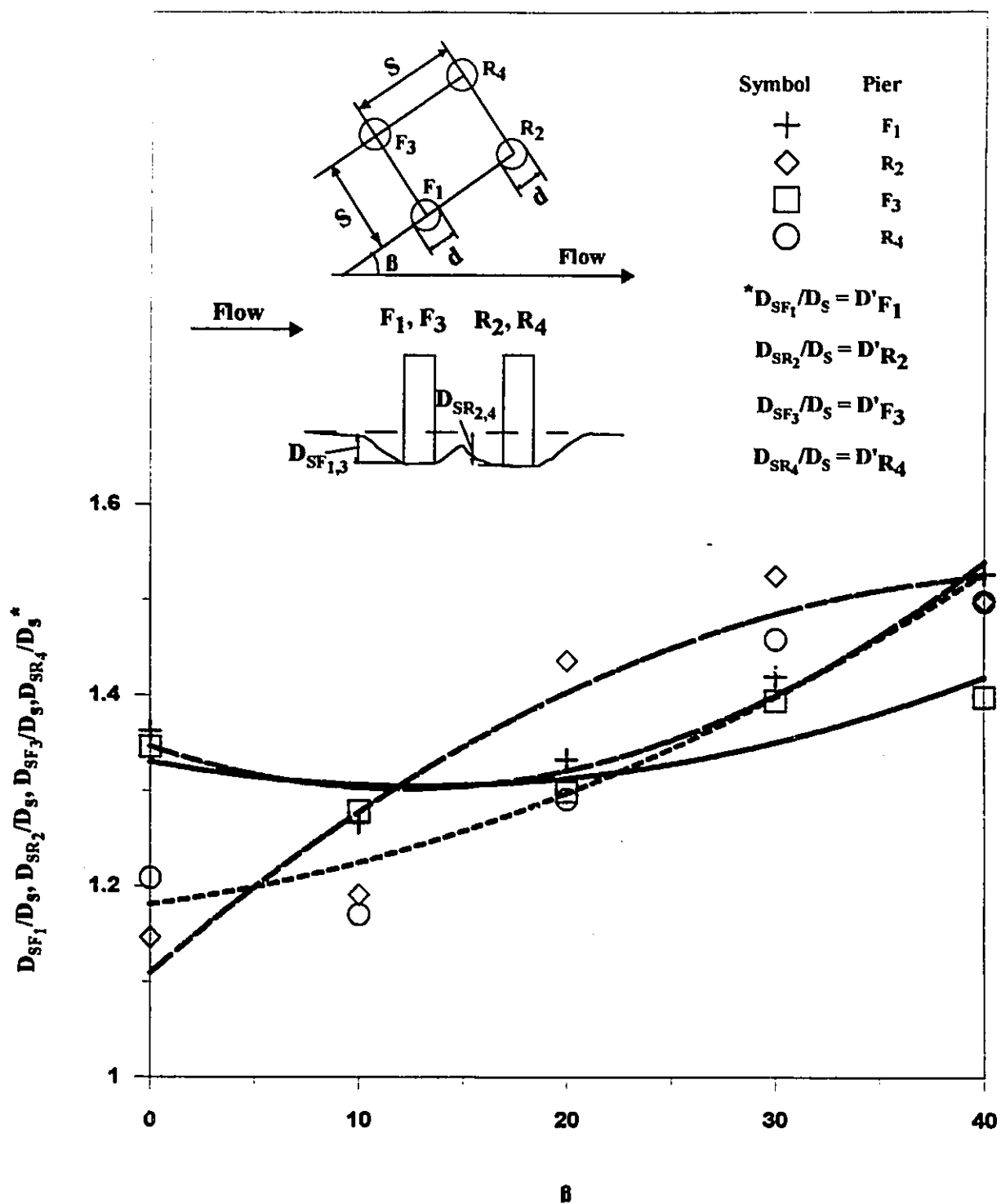


Fig. 5-4-2: Scour depth ratios versus flow angle of attack groups of 4 piers; ($S/d=3$)

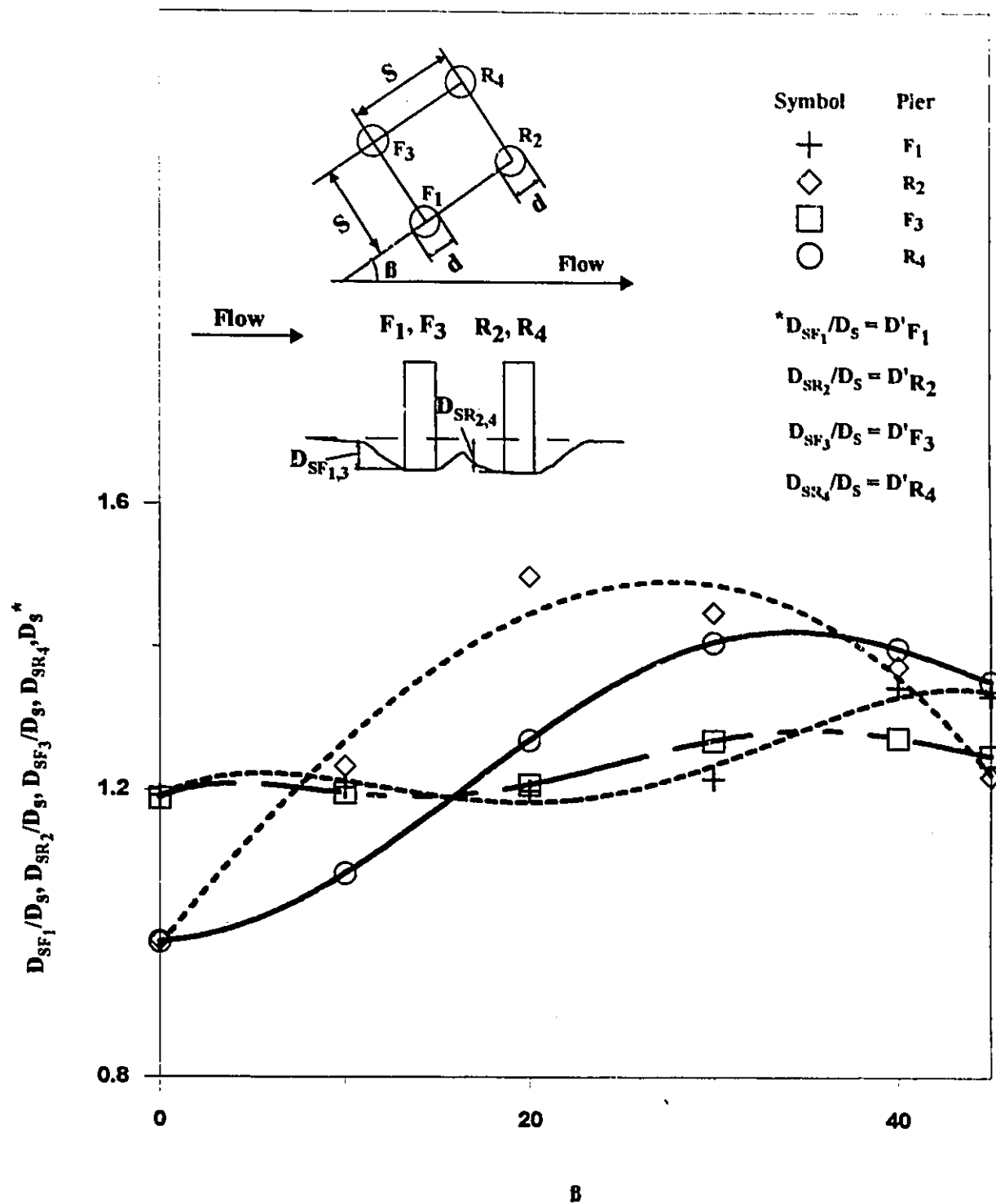


Fig. 5-4-3: Scour depth ratios versus flow angle of attack groups of 4 pier; ($S/d=4$)

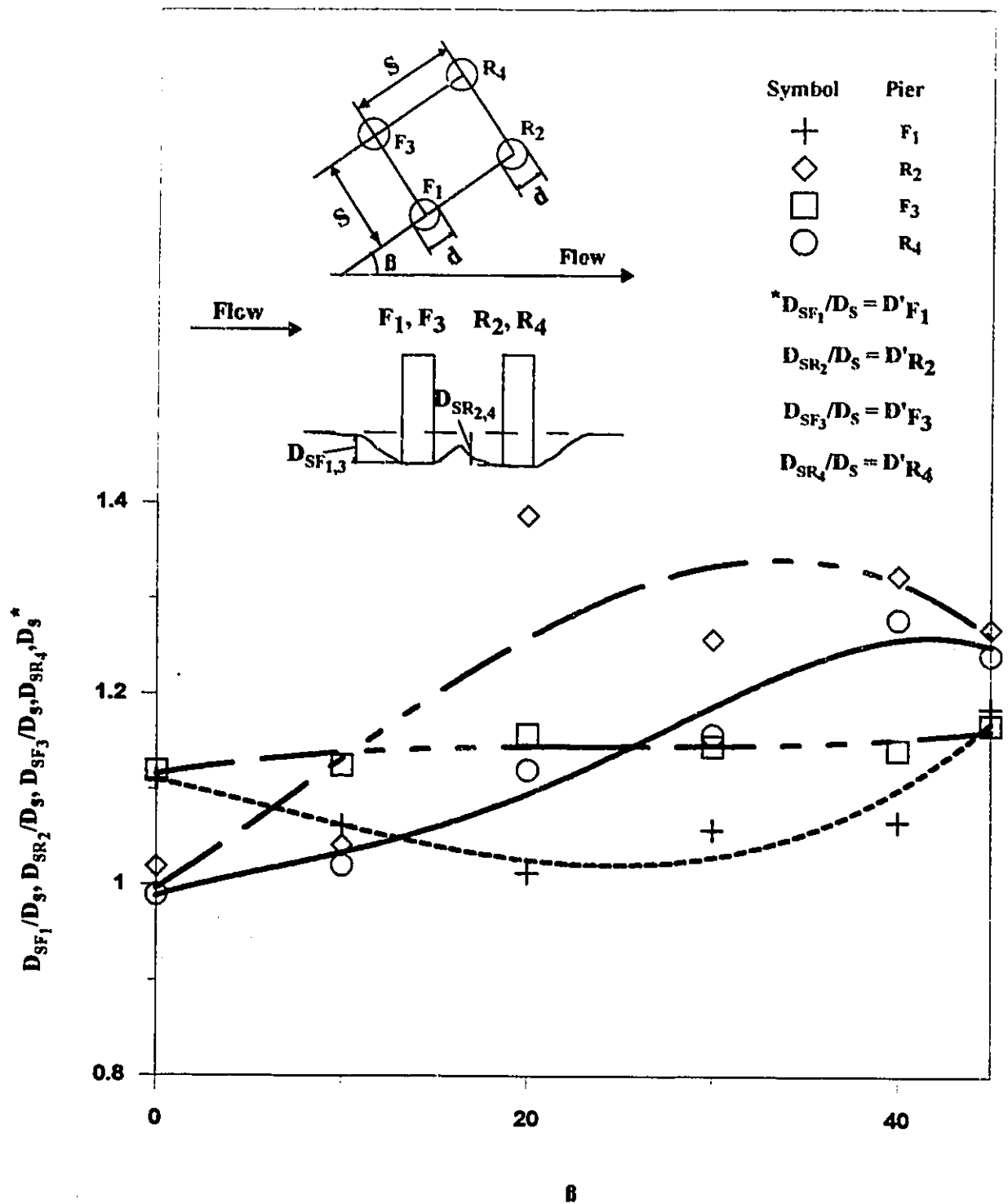


Fig. 5-4-4: Scour depth ratios versus flow angle of attack groups of 4 piers; ($S/d=6$)

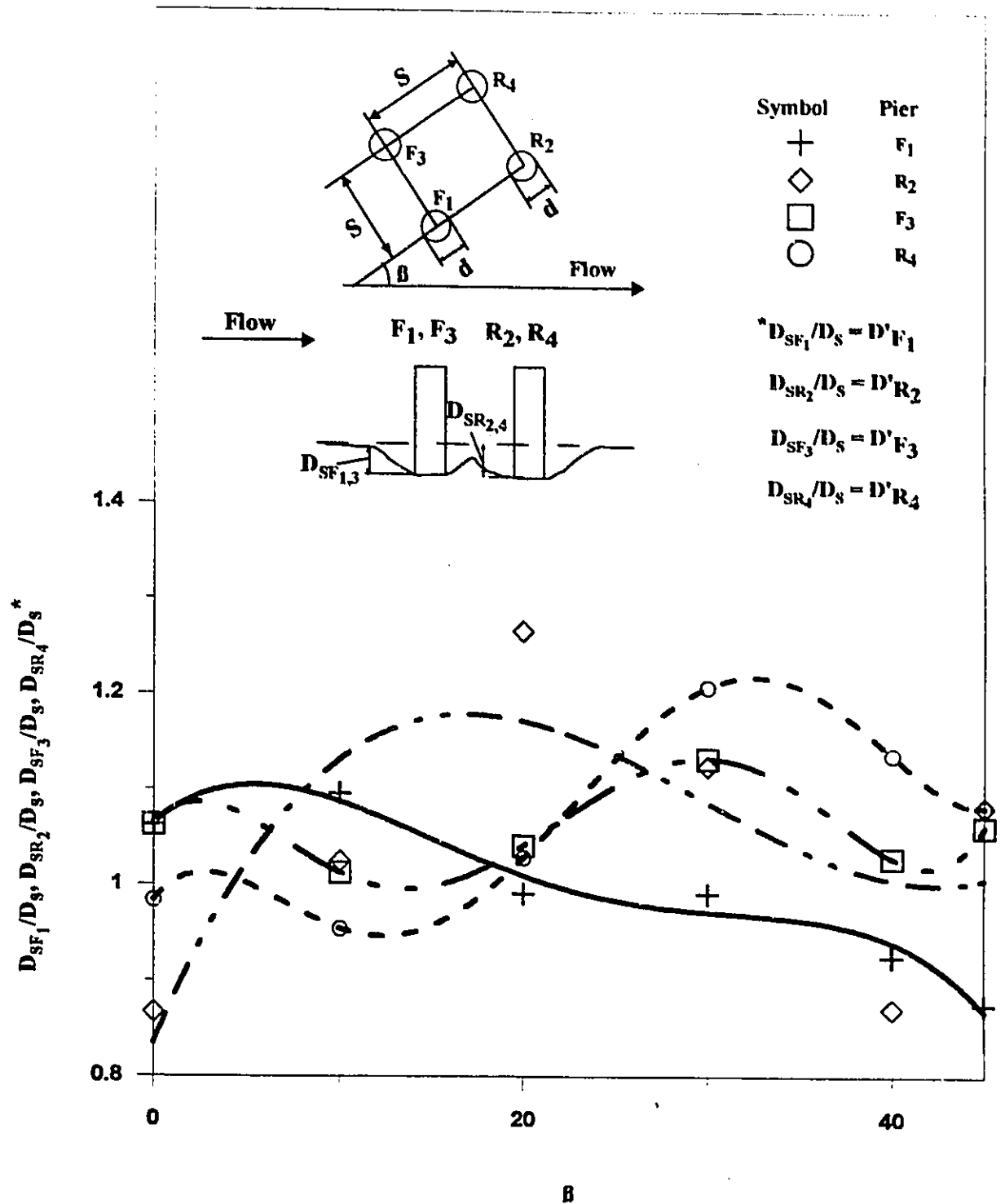


Fig. 5-4-5: Scour depth ratios versus flow angle of attack groups of 4 piers; ($S/d=8$)

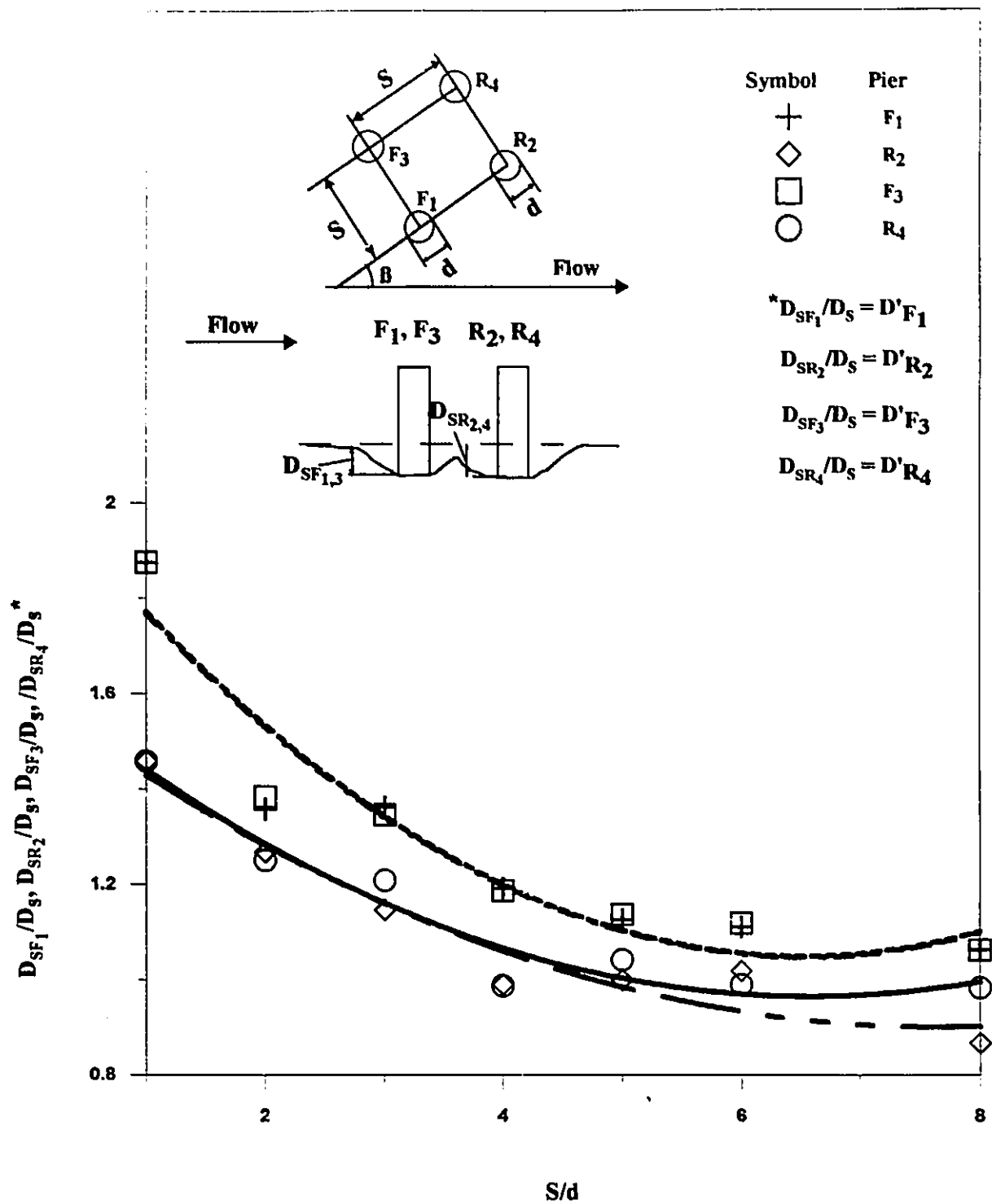


Fig. 5-4-6: Scour depth ratios versus pier spacing/ pier diameter groups of 4 piers; ($\beta=0^\circ$)

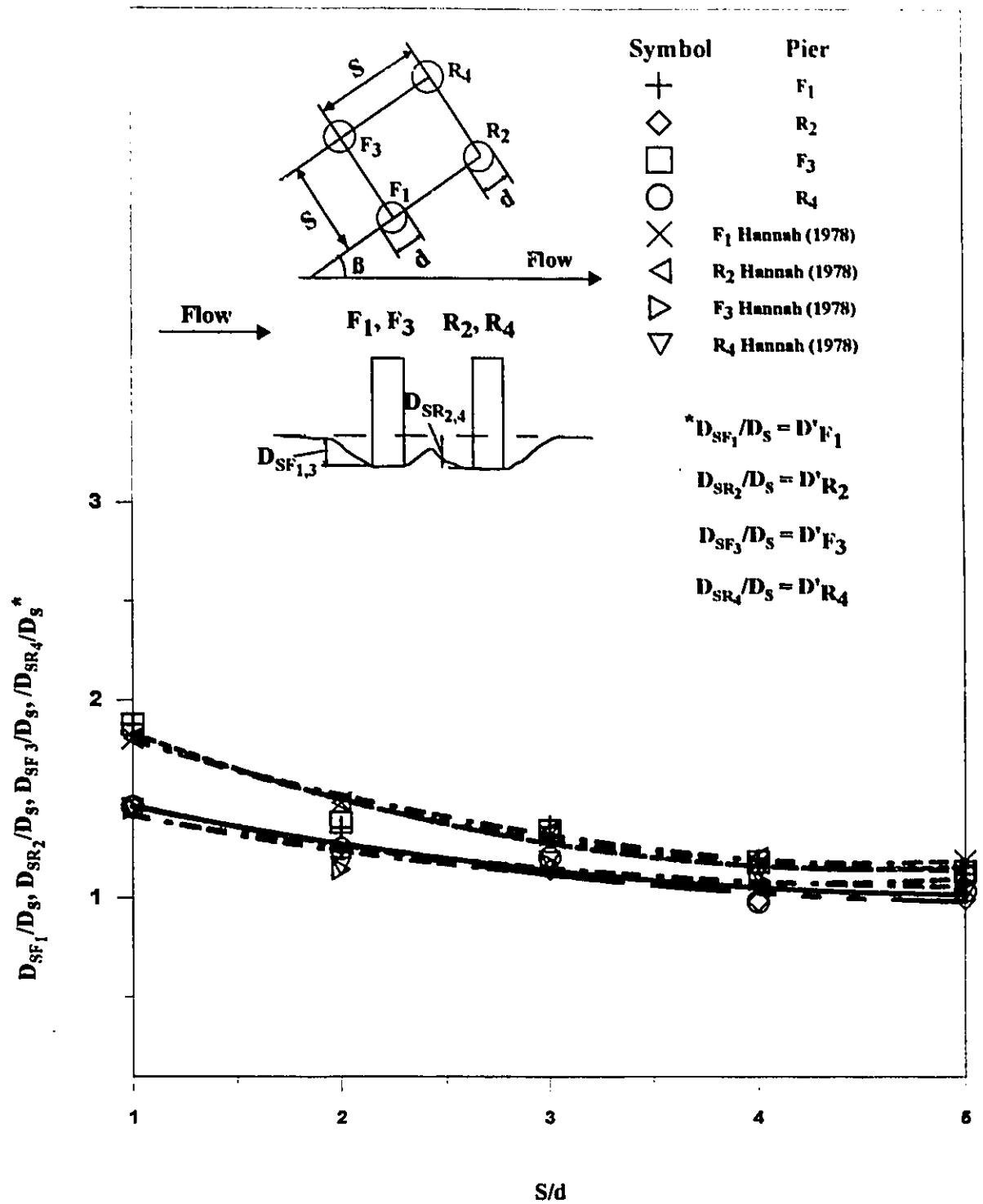


Fig. 5-4-7: Comparison of scour depth ratios versus pier spacing/ pier diameter; groups of 4 piers; ($\beta=0^\circ$)

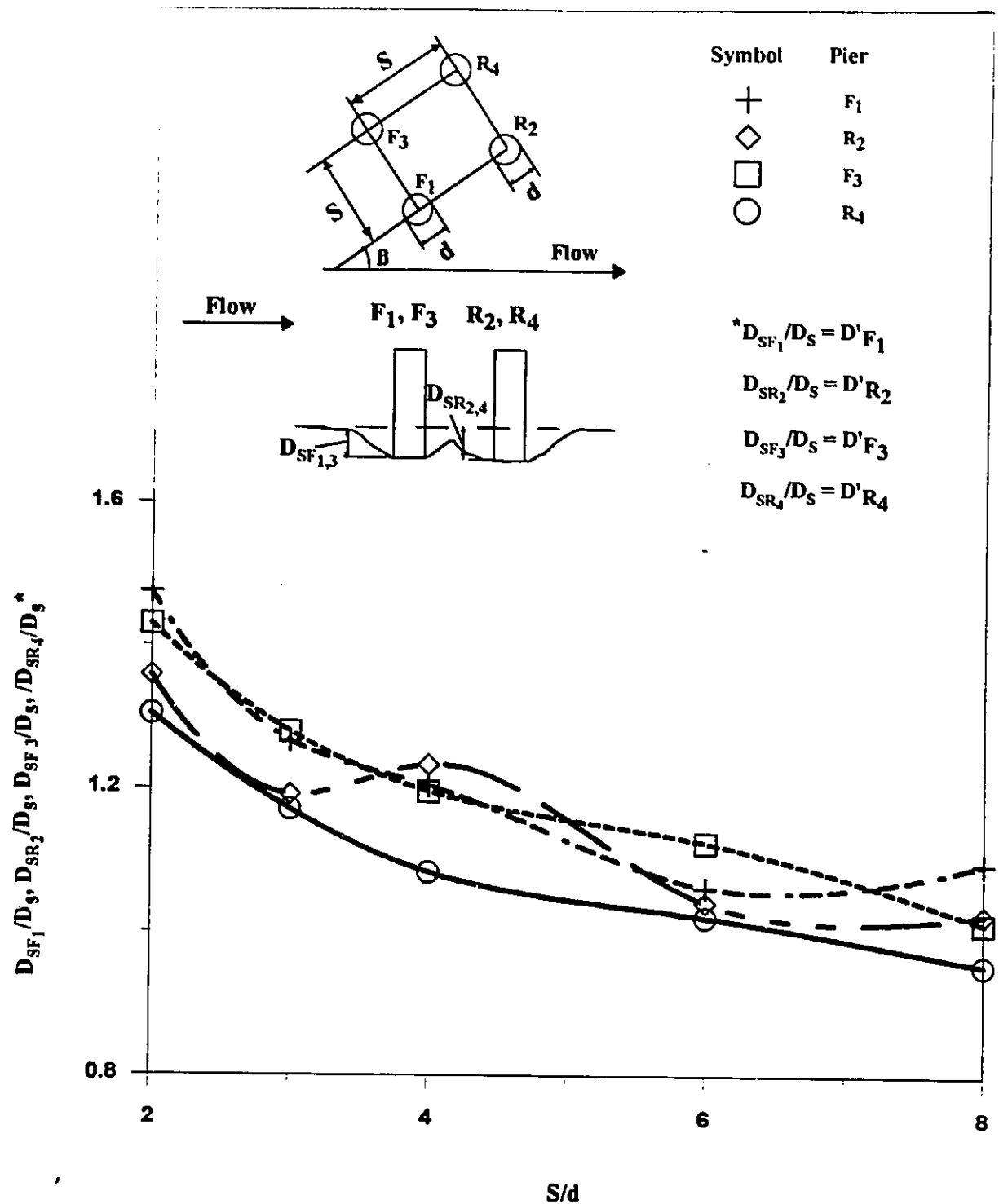


Fig. 5-4-8: Scour depth ratios versus pier spacing/ pier diameter groups of 4 piers; ($\beta=10^5$)

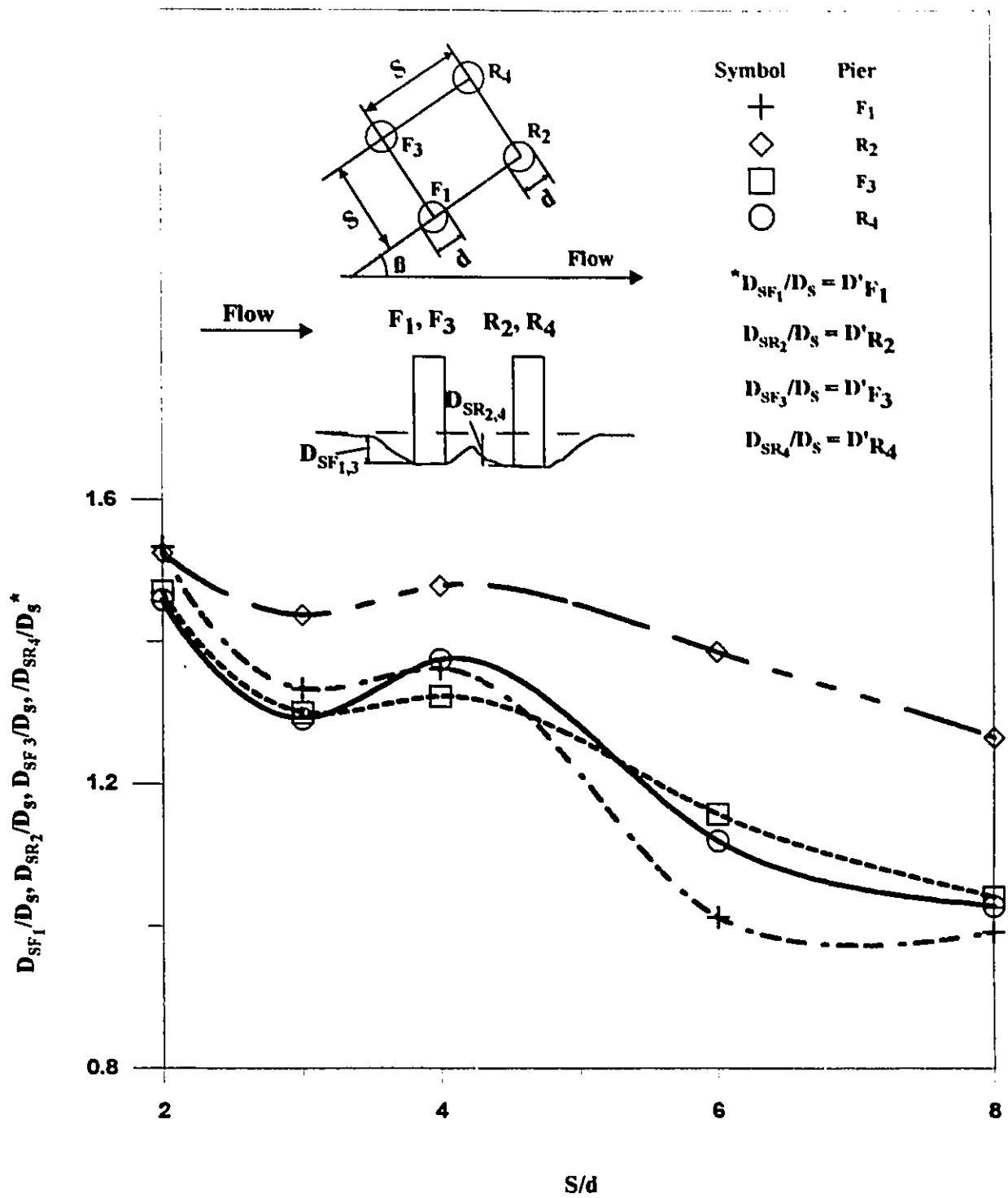


Fig. 5-4-9: Scour depth ratios versus pier spacing/ pier diameter groups of 4 piers; ($\beta=20^\circ$)

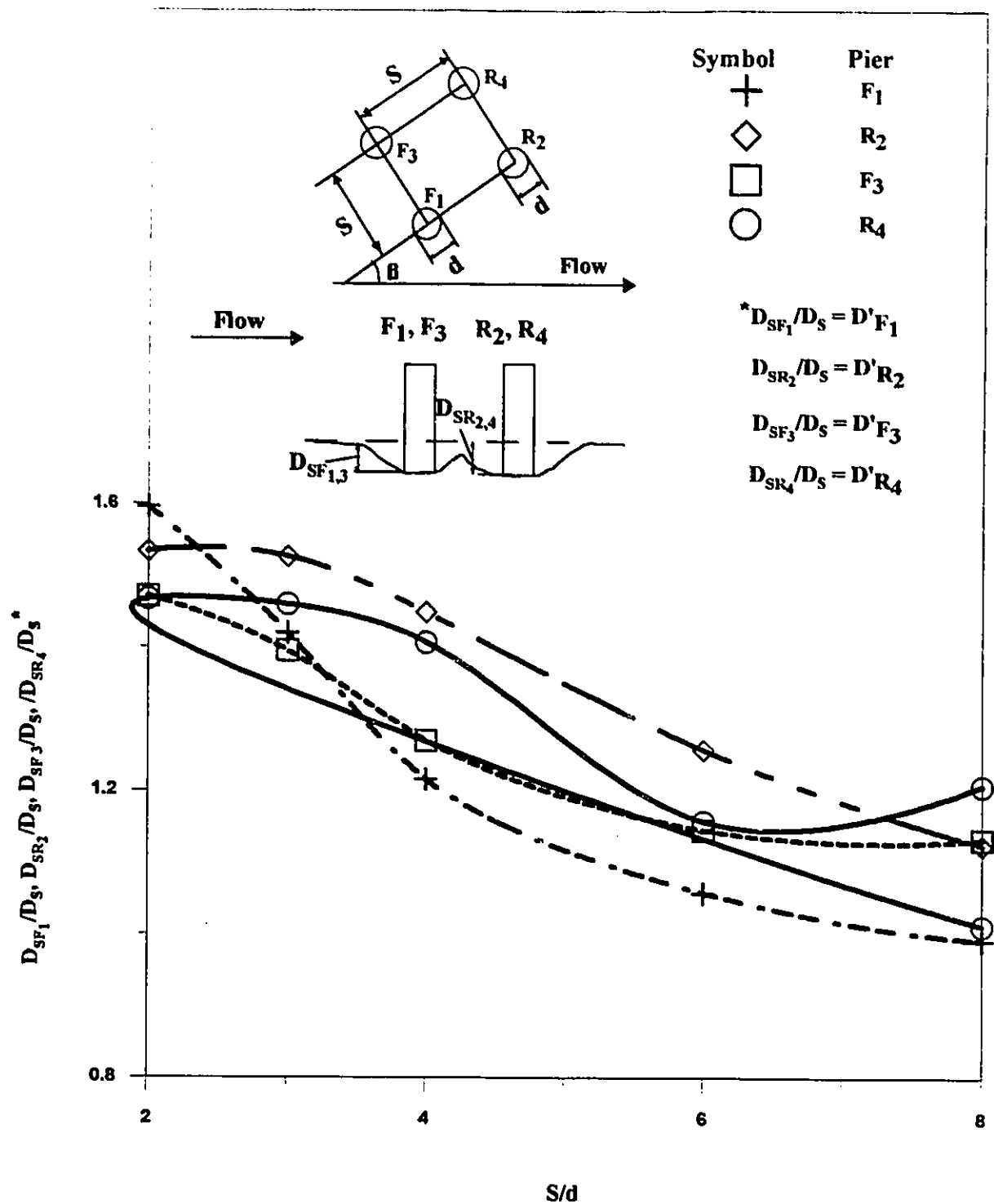


Fig. 5-4-10: Scour depth ratios versus pier spacing/ pier diameter groups of 4 piers; ($\theta=30^\circ$)

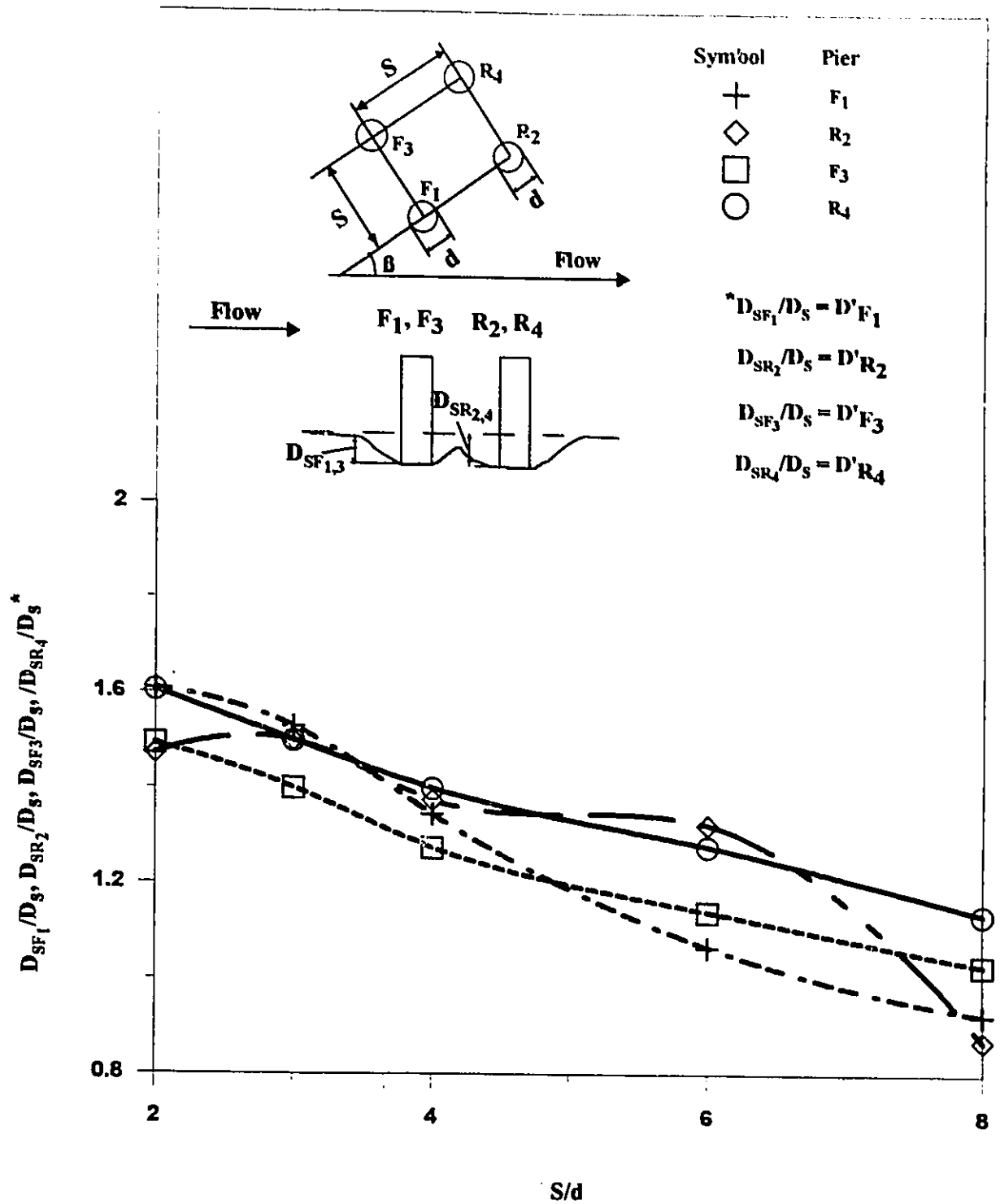


Fig. 5-4-11: Scour depth ratios versus pier spacing/pier diameter groups of 4 piers; ($\beta=40^\circ$)

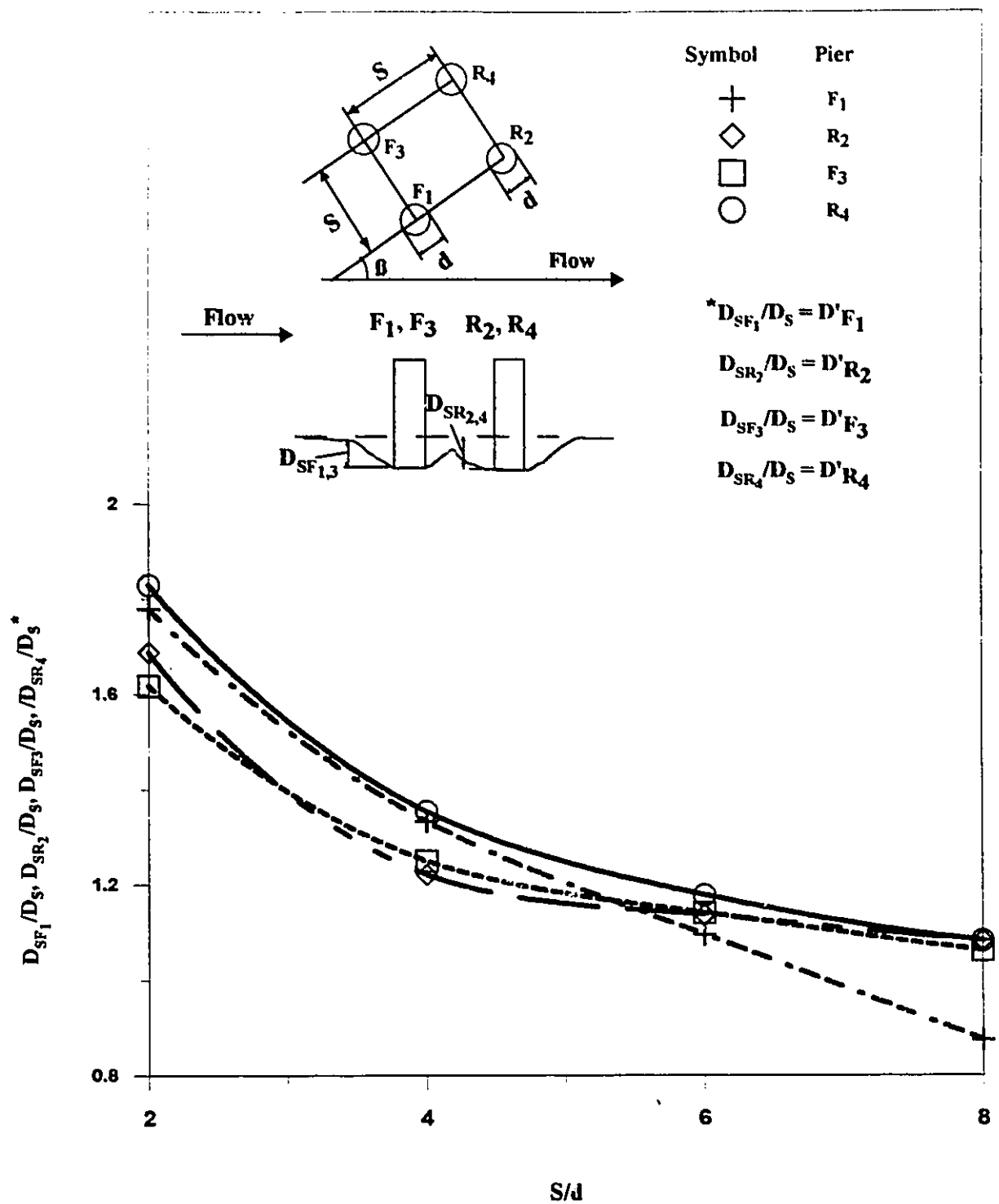


Fig. 5-4-12: Scour depth ratios versus pier spacing/pier diameter groups of 4 piers; ($\beta=45^\circ$)

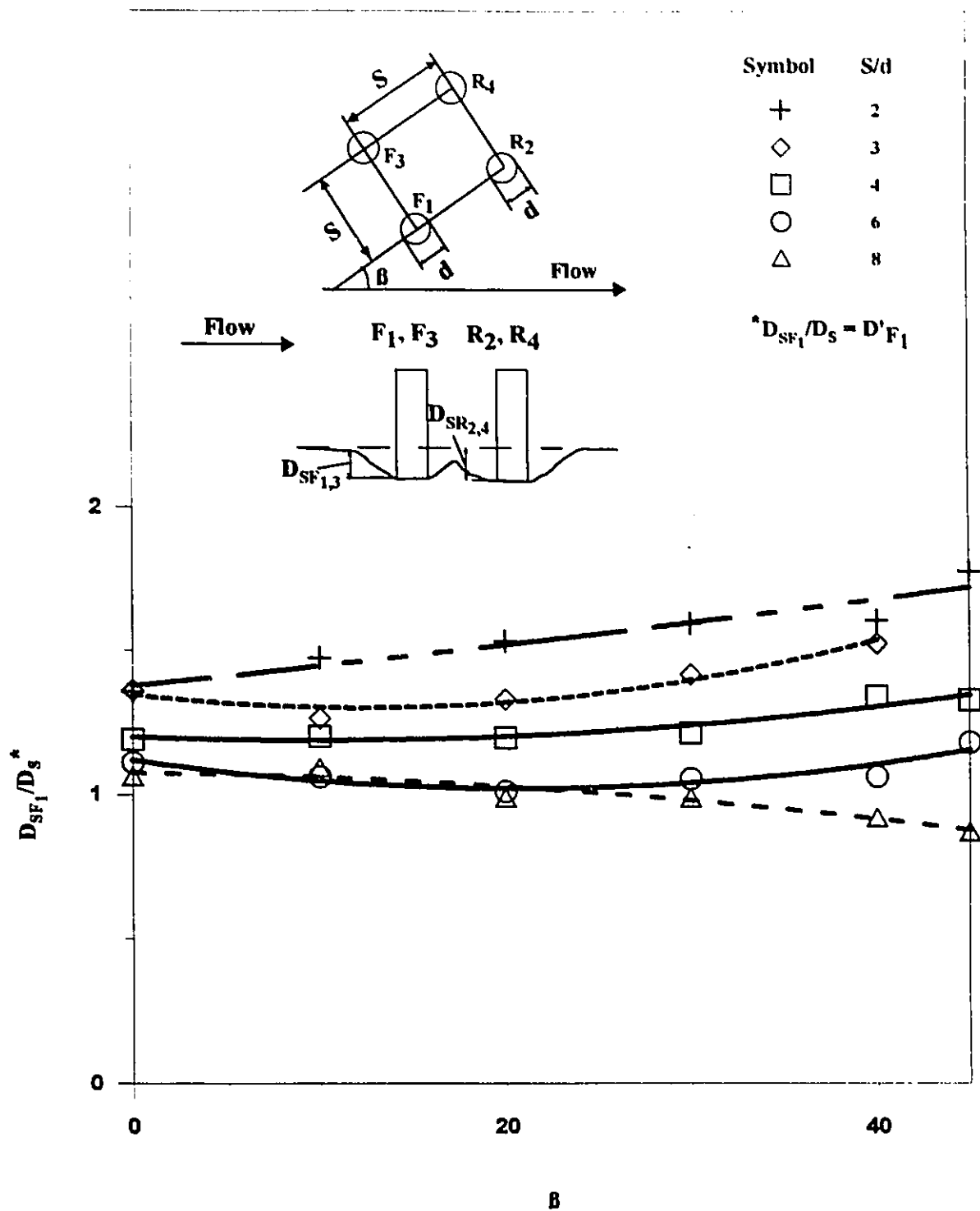


Fig. 5-4-13: First front pier scour depth ratios versus flow angle of attack; for different S/d ; groups of 4 Piers

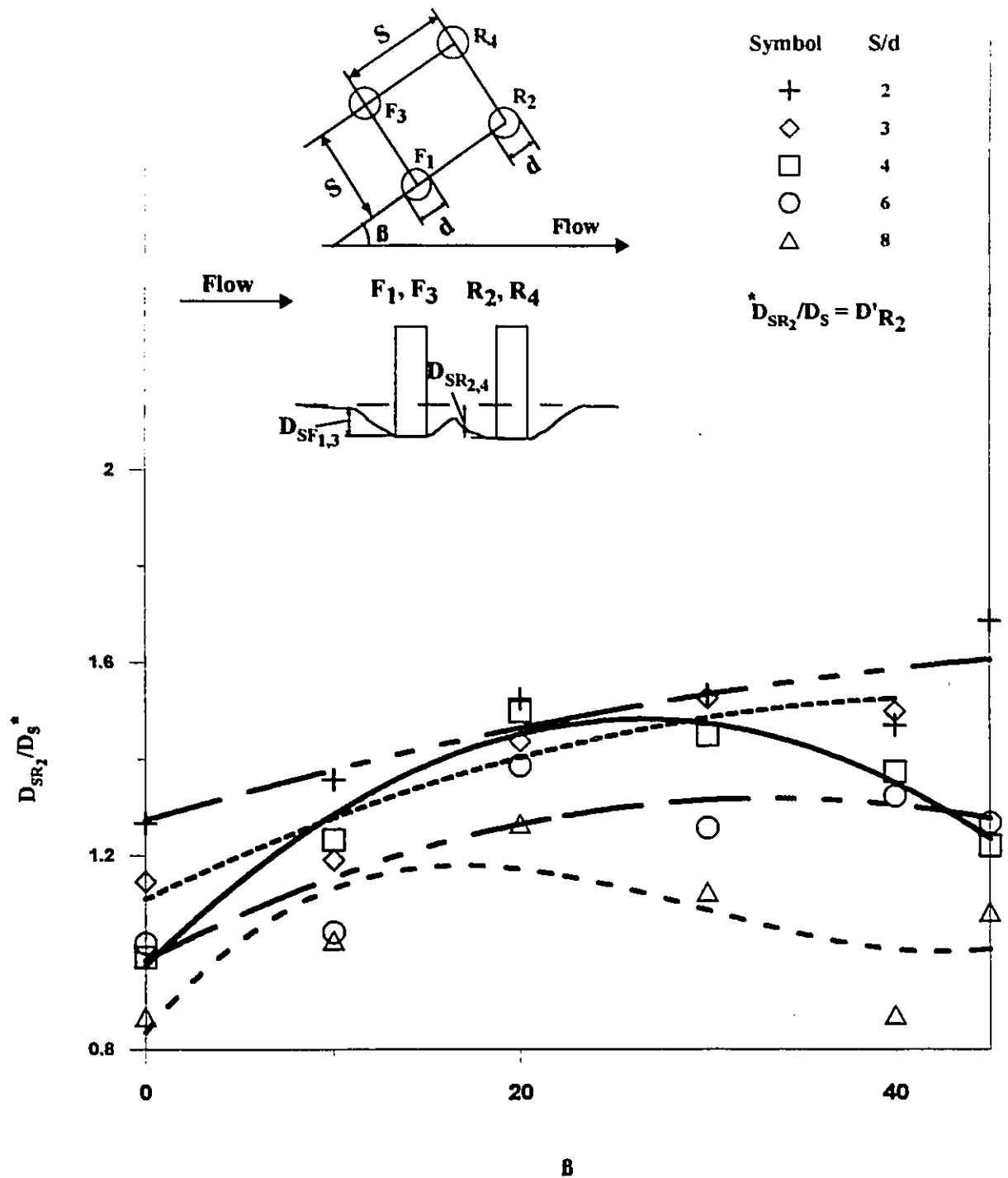


Fig. 5-4-14: First rear pier scour depth ratios versus angle of attack; for different S/d; groups of 4 piers

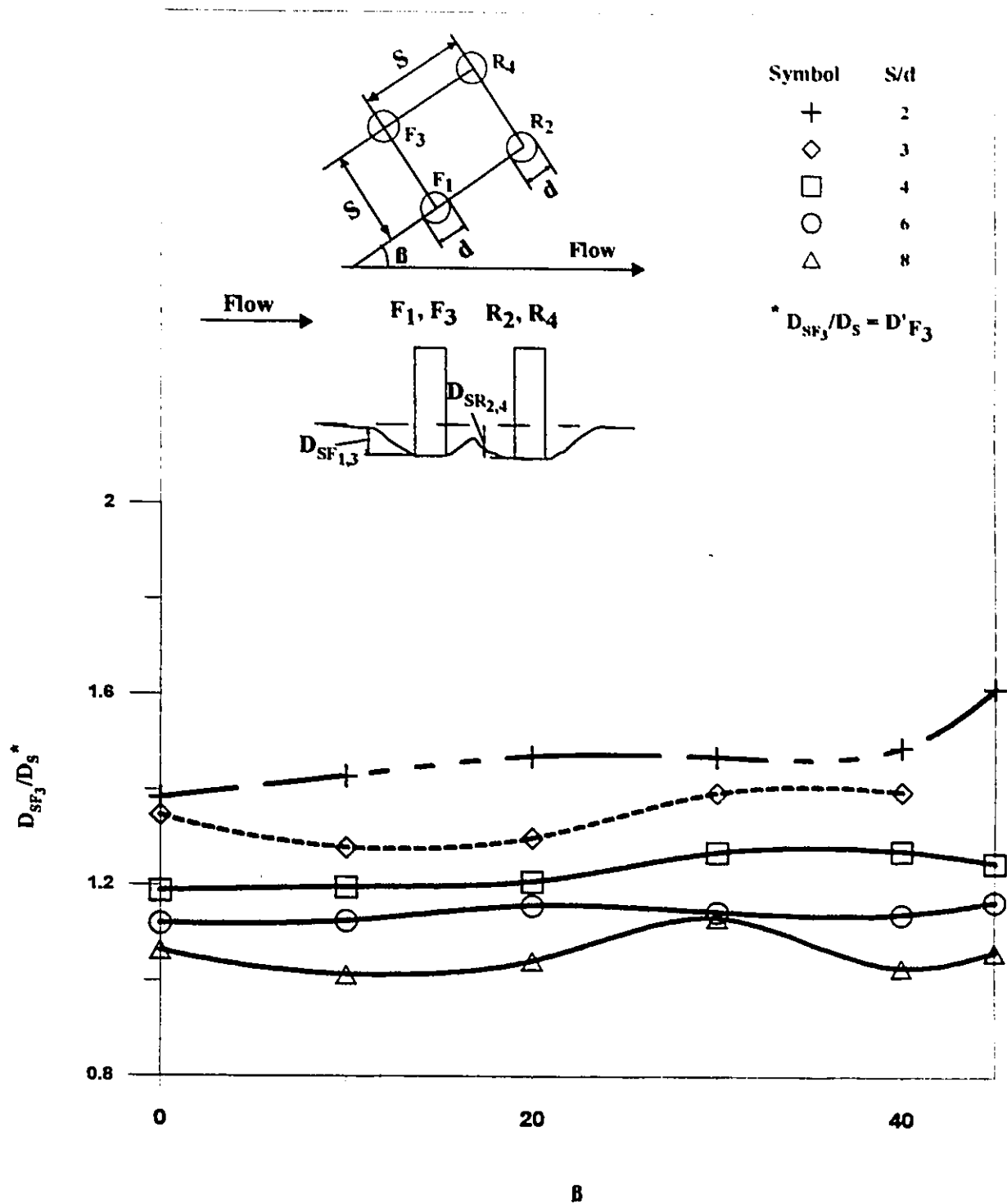


Fig. 5-4-15: Second front pier scour depth ratios versus flow angle of attack; for different S/d ; groups of 4 piers

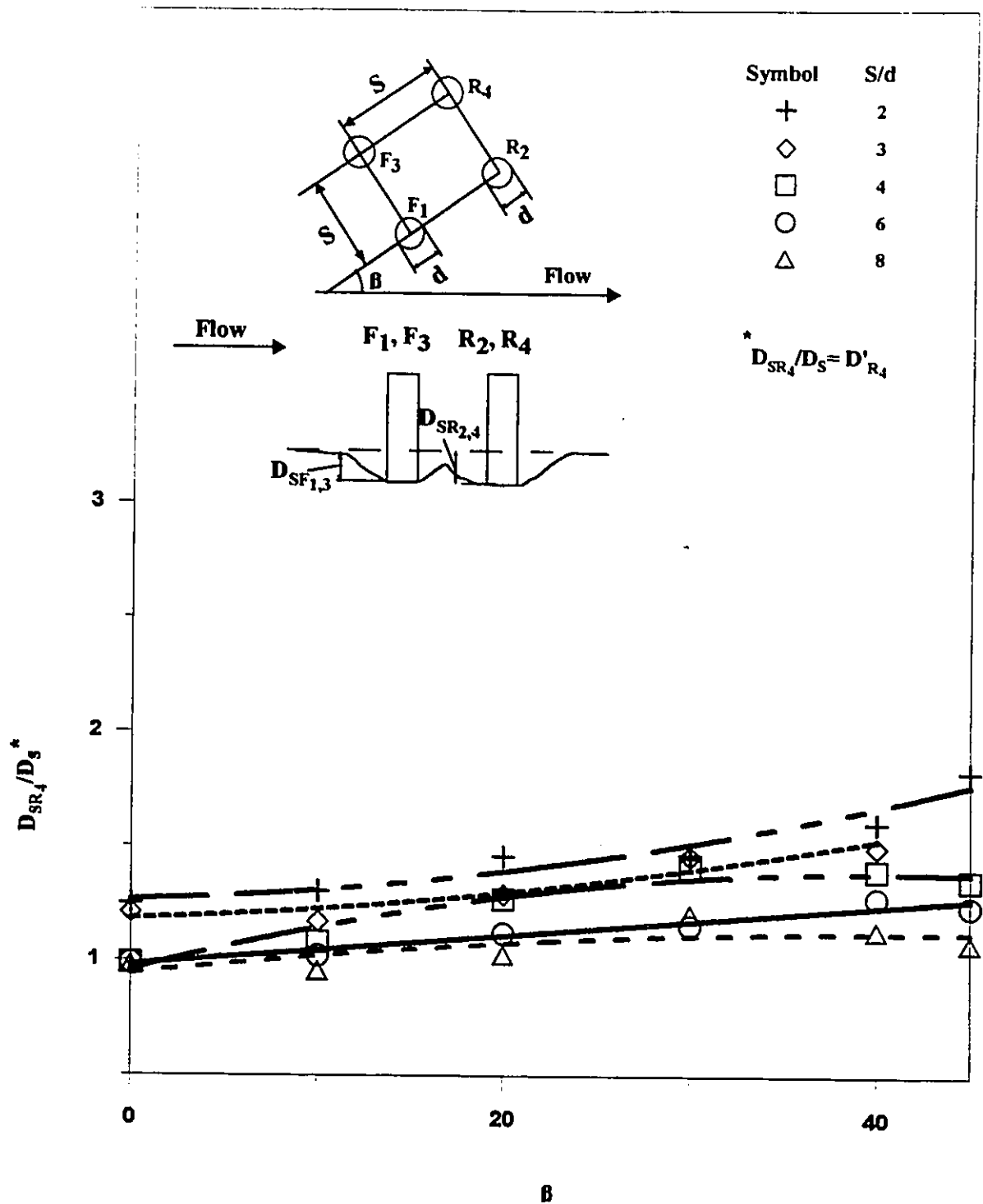
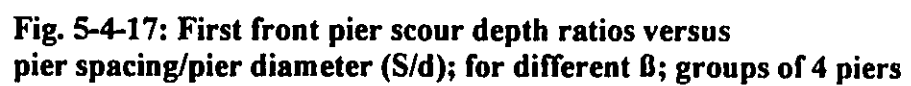


Fig. 5-4-16: Second rear pier scour depth ratios versus flow angle of attack; for different S/d; groups of 4 piers



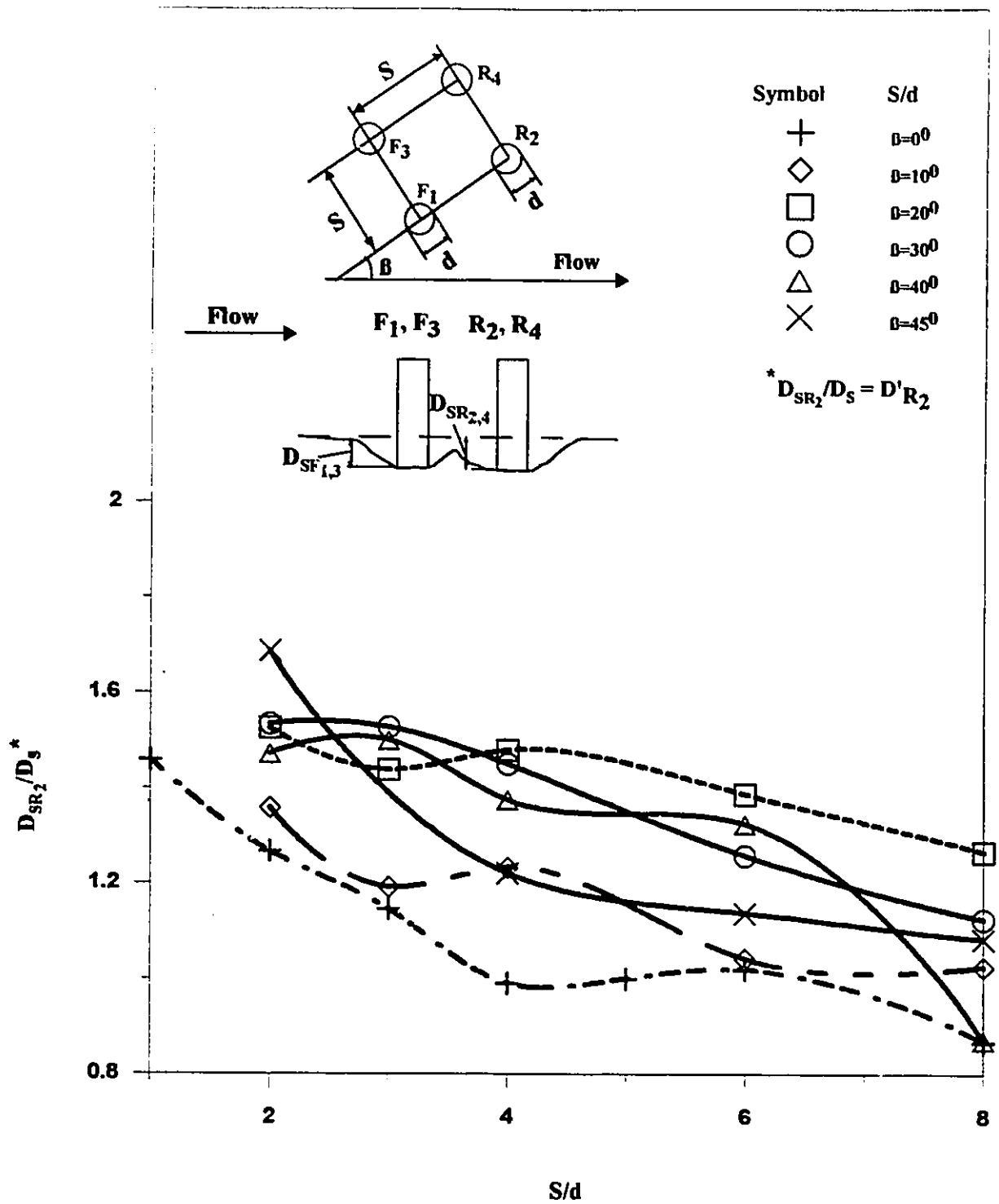


Fig. 5-4-18: First rear pier scour depth ratios versus pier spacing/ pier diameter; for different β ; groups of 4 piers

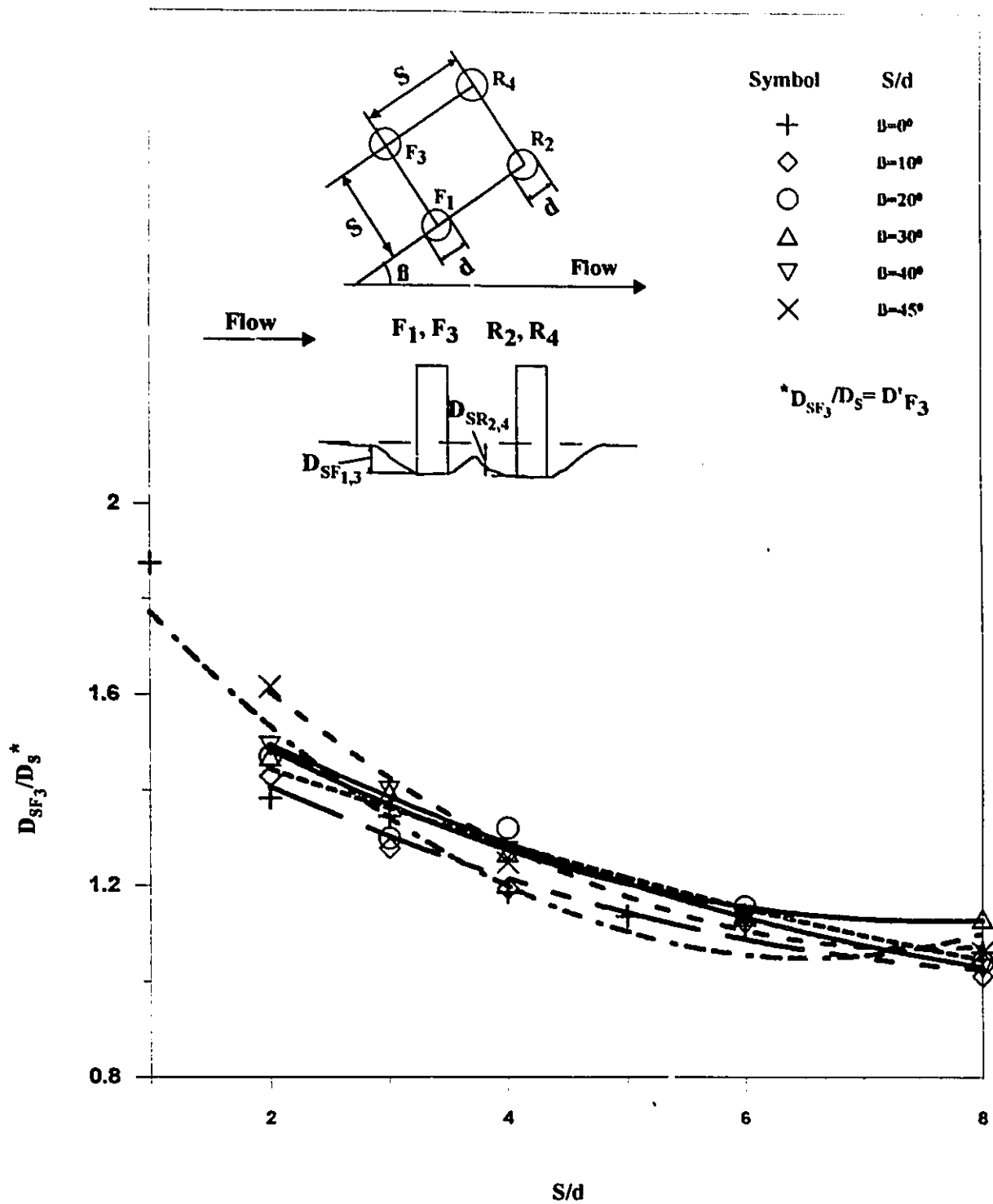


Fig. 5-4-19: Second front pier scour depth ratios versus pier spacing/ pier diameter; for different β ; groups of 4 piers

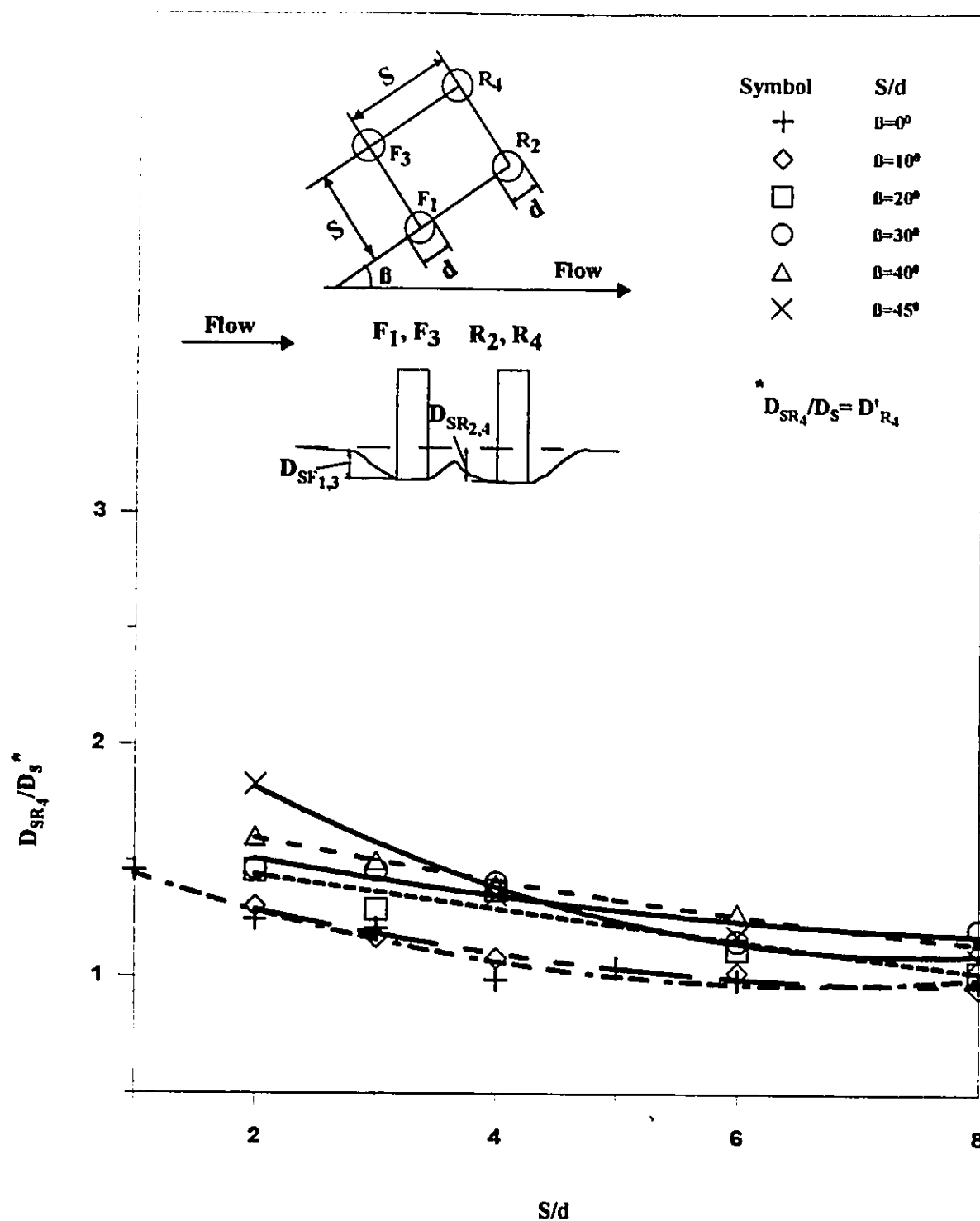


Fig. 5-4-20: Second rear pier scour depth ratios versus pier spacing/ pier diameter; ; for different β ; groups of 4 piers

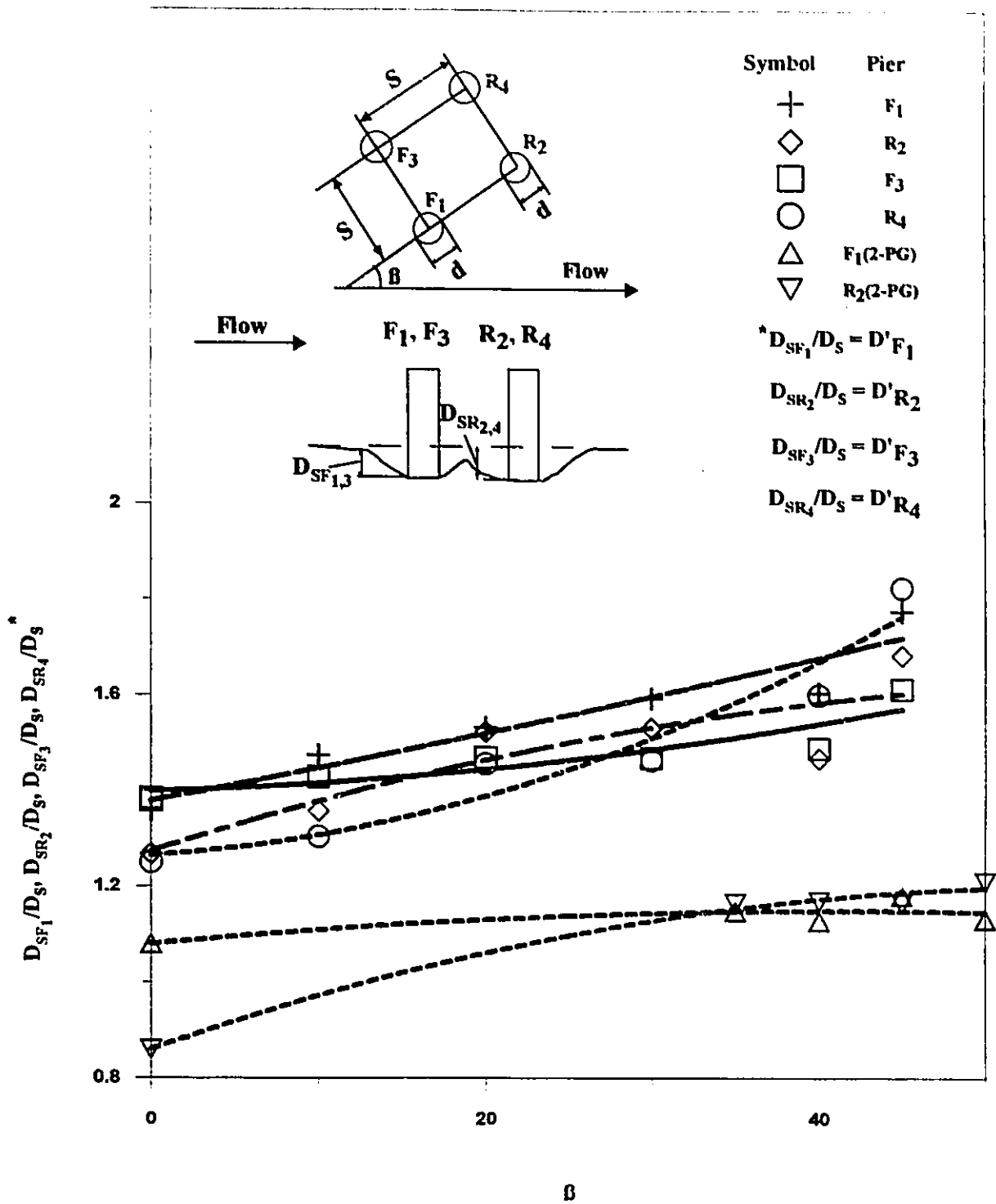


Fig. 5-4-21: Comparison of scour depth ratios versus flow angle of attack; groups of 4 piers; (S/d=2)

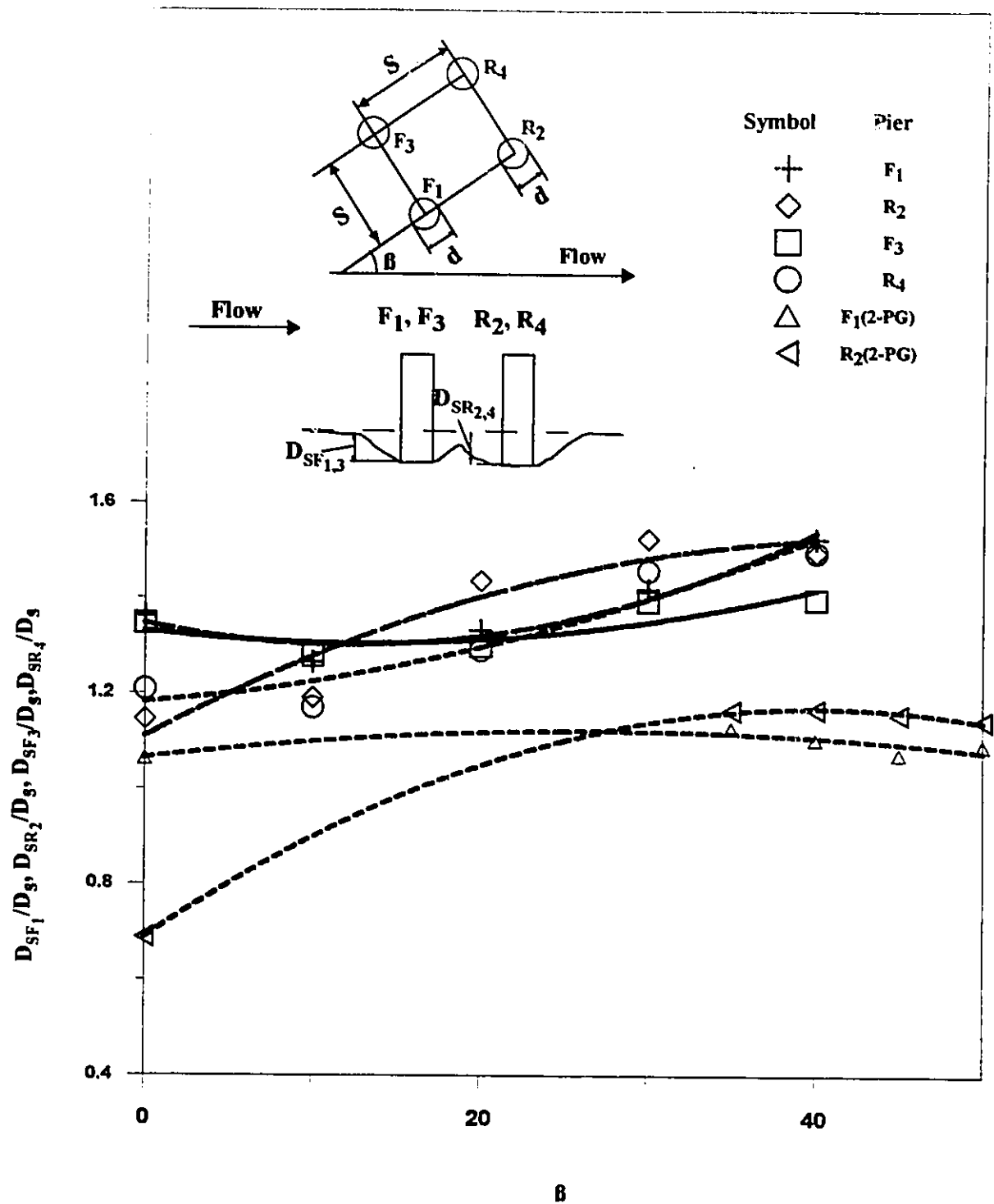


Fig. 5-4-22: Comparison of scour depth ratios versus flow angle of attack; groups of 2 and 4 piers; ($S/d=3$)

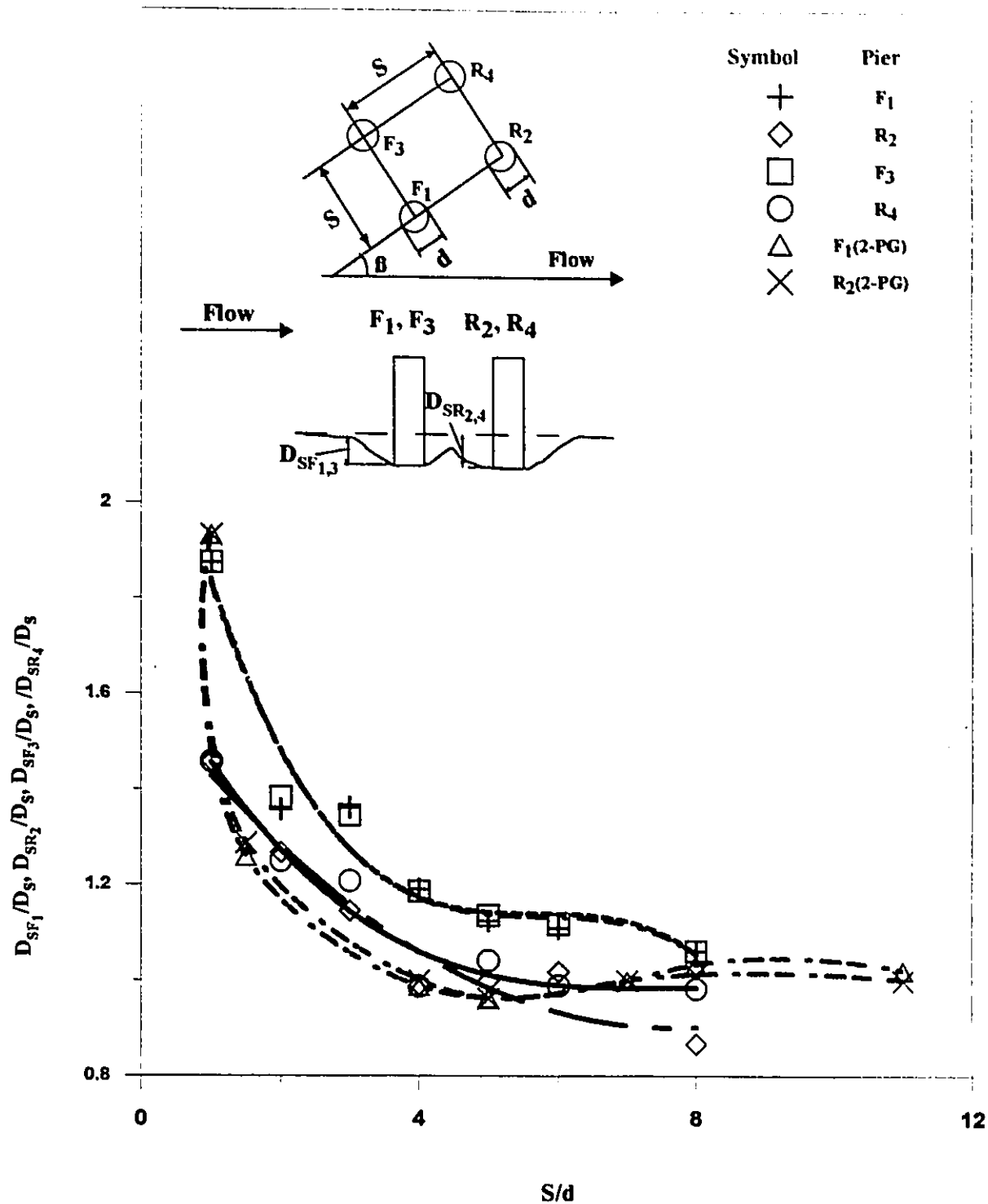


Fig. 5-4-23: Comparison of scour depth ratios versus pier spacing/ pier diameter groups of 2 and 4 piers; ($\beta=90^\circ$ and 0° , respectively)

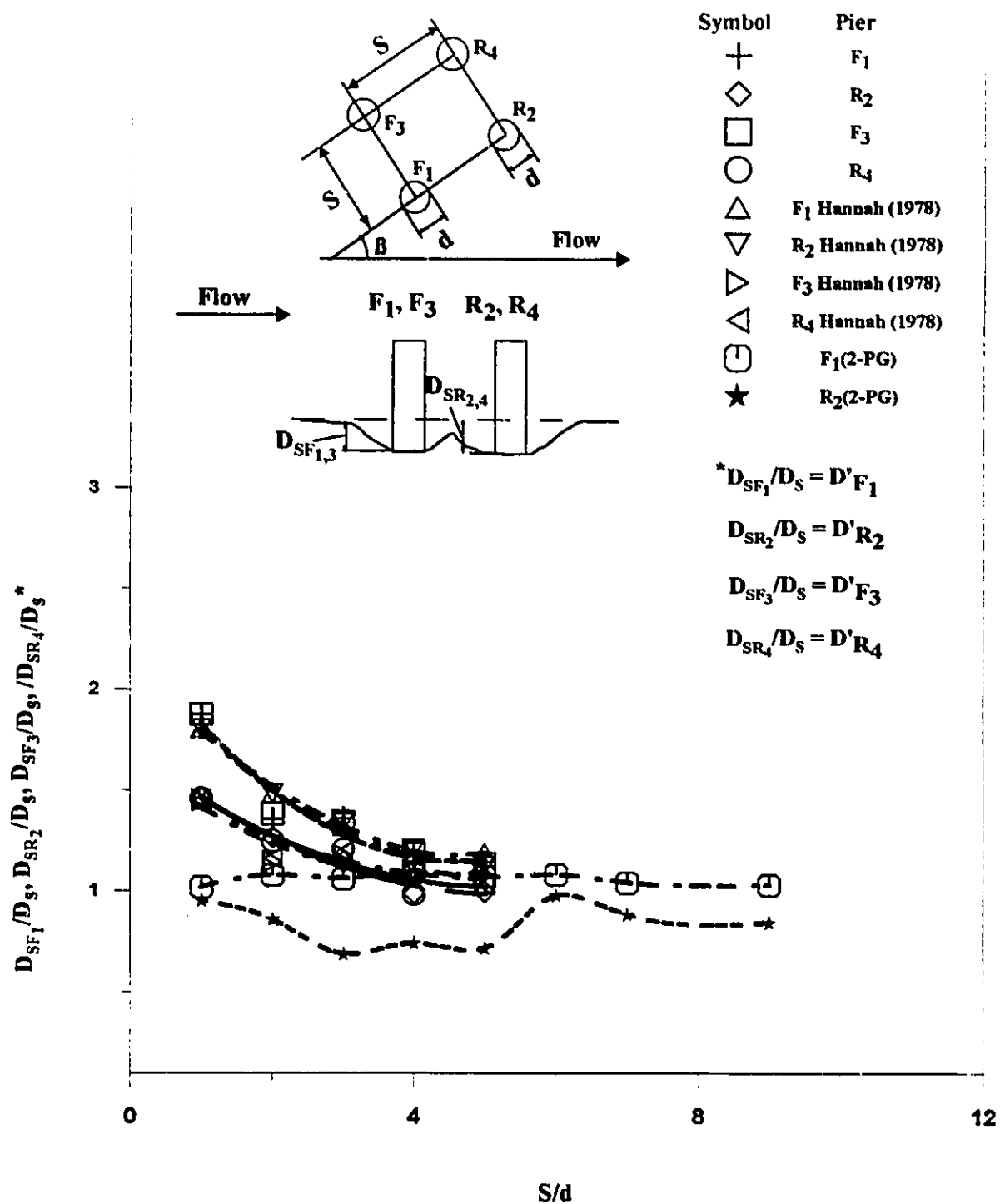


Fig. 5-4-24: Comparison of scour depth ratios versus pier spacing/ pier diameter; groups of 2 and 4 piers; ($\beta=0^\circ$)

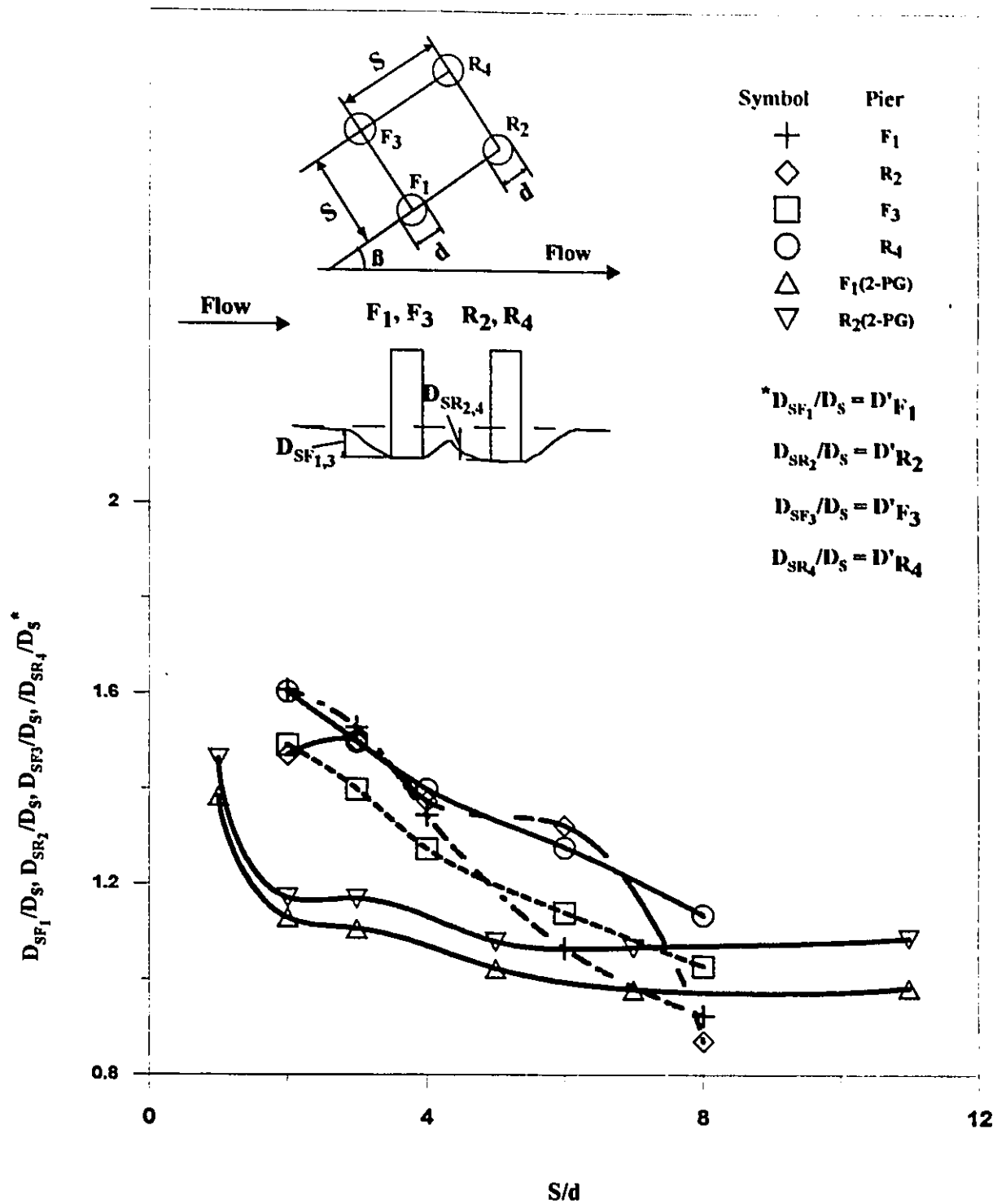


Fig. 5-4-25: Comparison of scour depth ratios versus pier spacing / pier diameter; groups of 2 and 4 piers; ($\beta=40^\circ$)

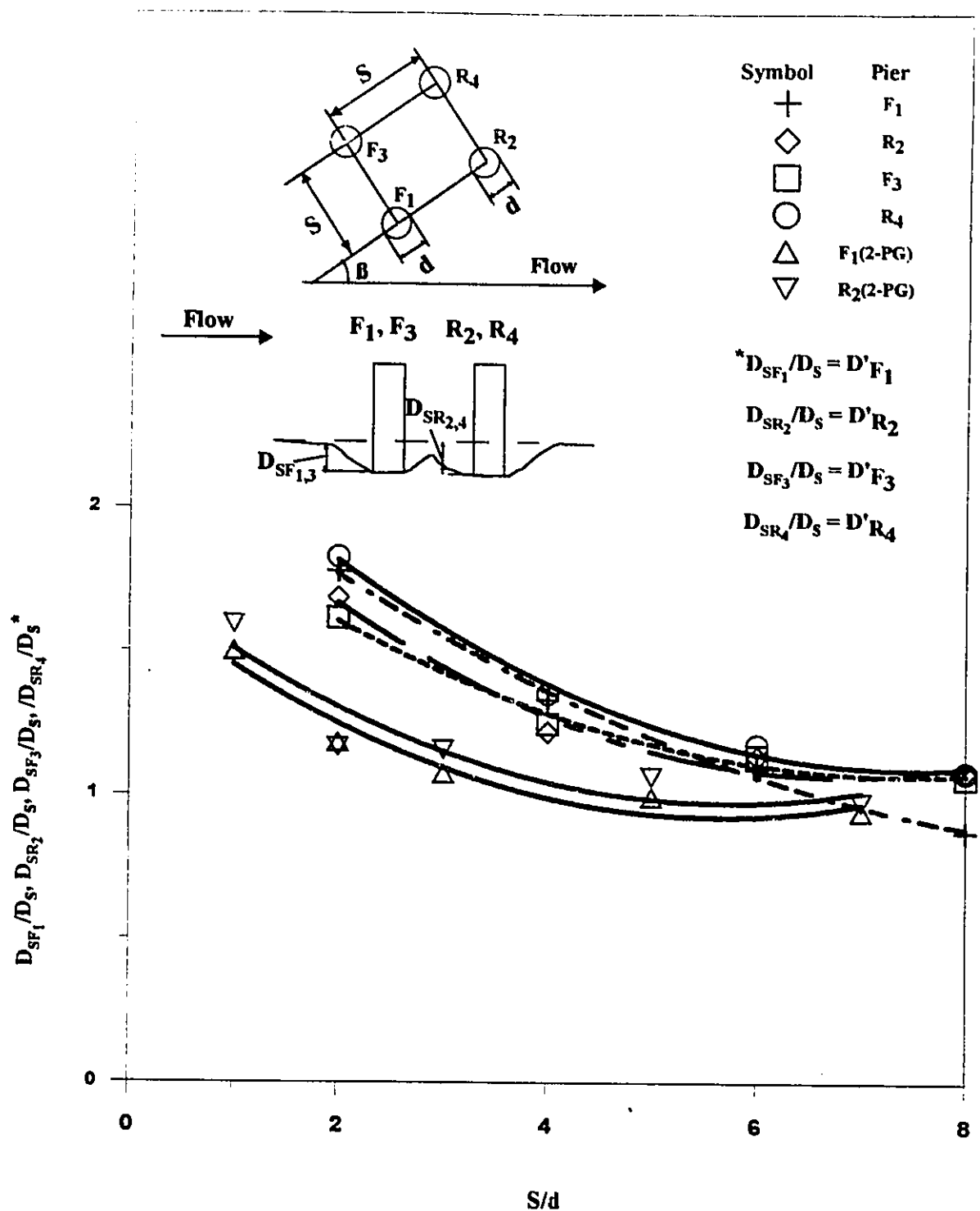


Fig. 5-4-26: Comparison of scour depth ratios versus pier spacing/ pier diameter; groups of 2 and 4 piers; ($\beta=45^\circ$)

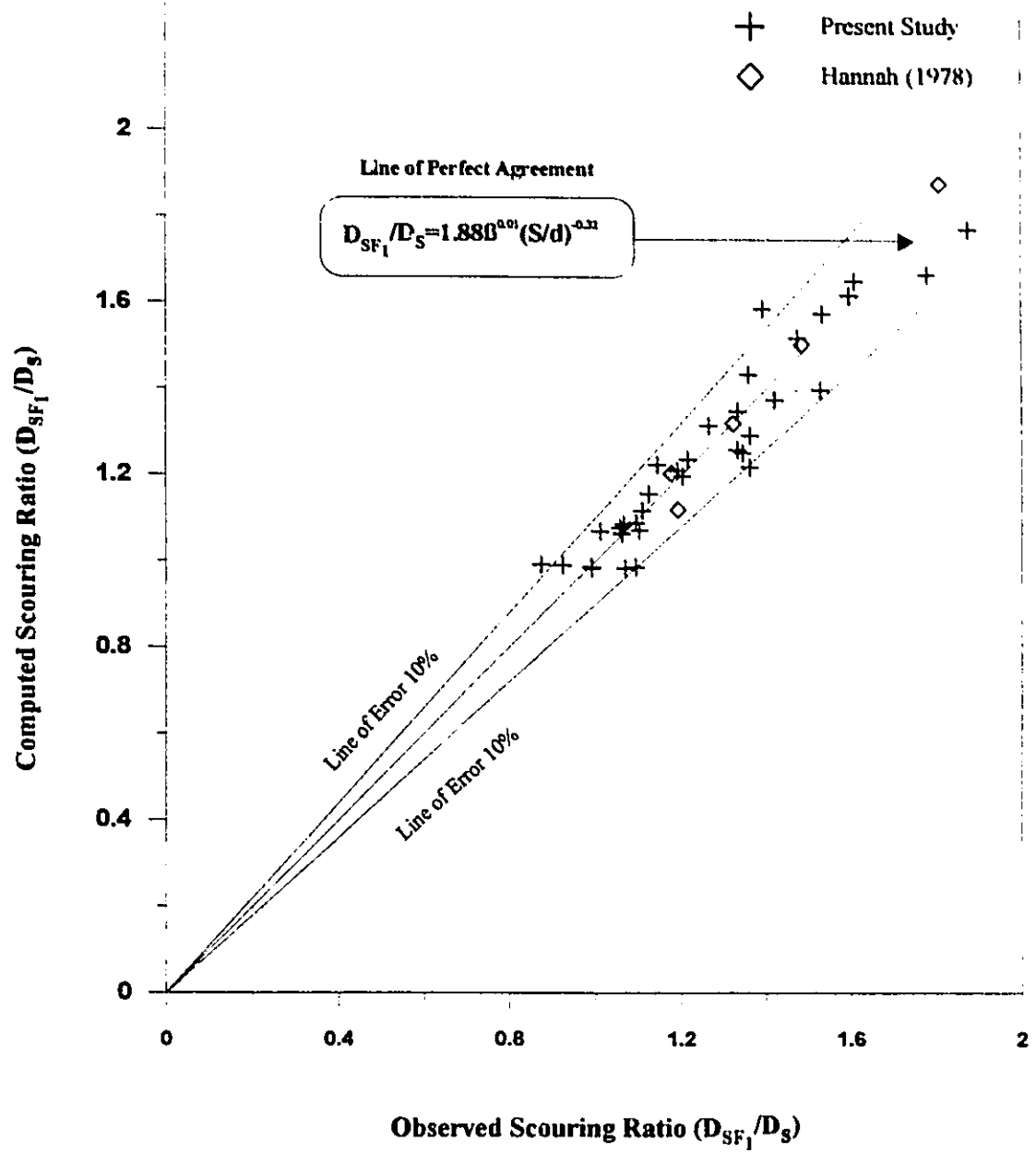


Fig. 5-4-27 : Comparison between computed and measured front pier (F_1) scouring ratios; a group of 4 piers

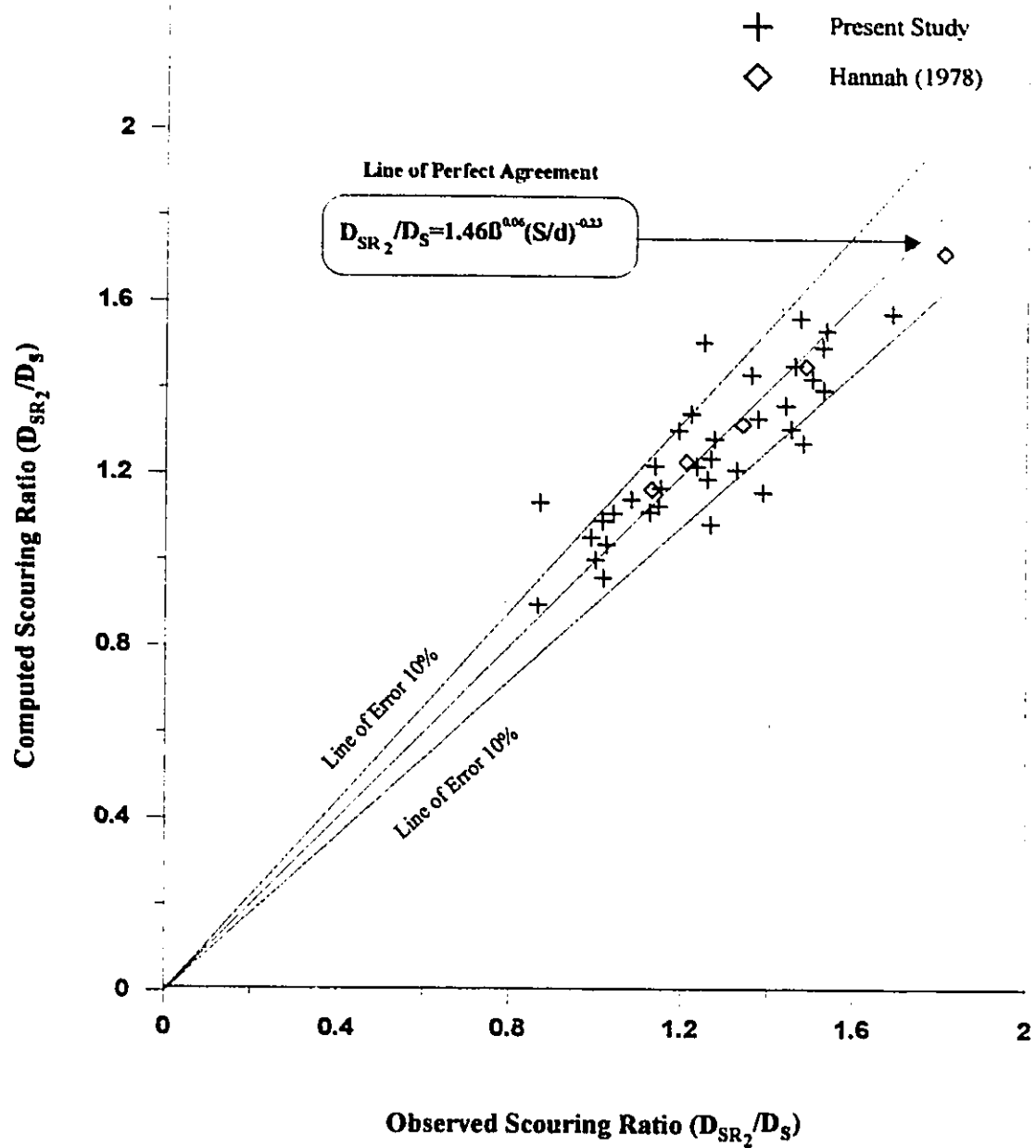


Fig. 5-4-28 : Comparison between computed and measured rear pier (R_2) scouring ratios; a group of 4 piers

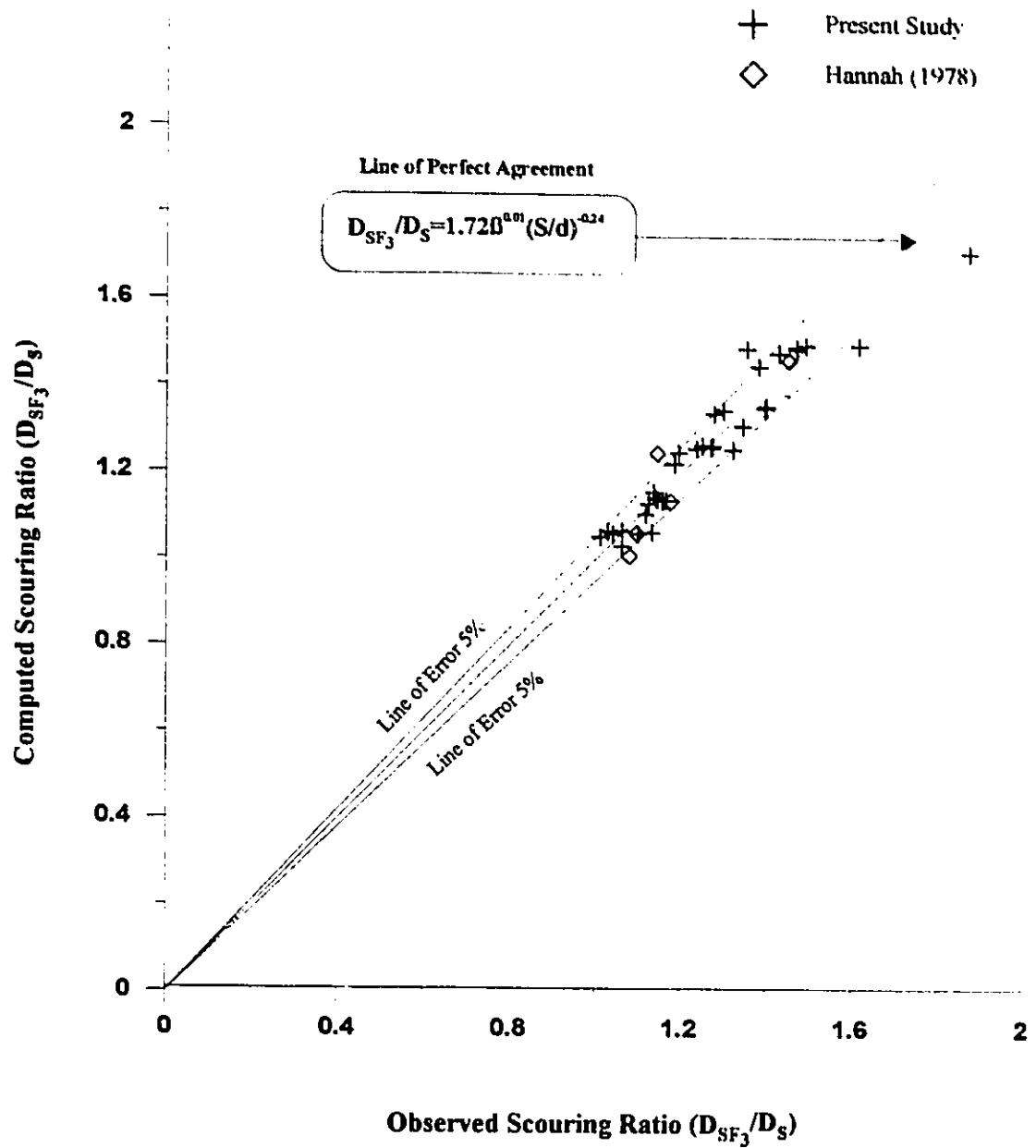


Fig. 5-4-29 : Comparison between computed and measured front pier (F_3) scouring ratios; a group of 4 piers

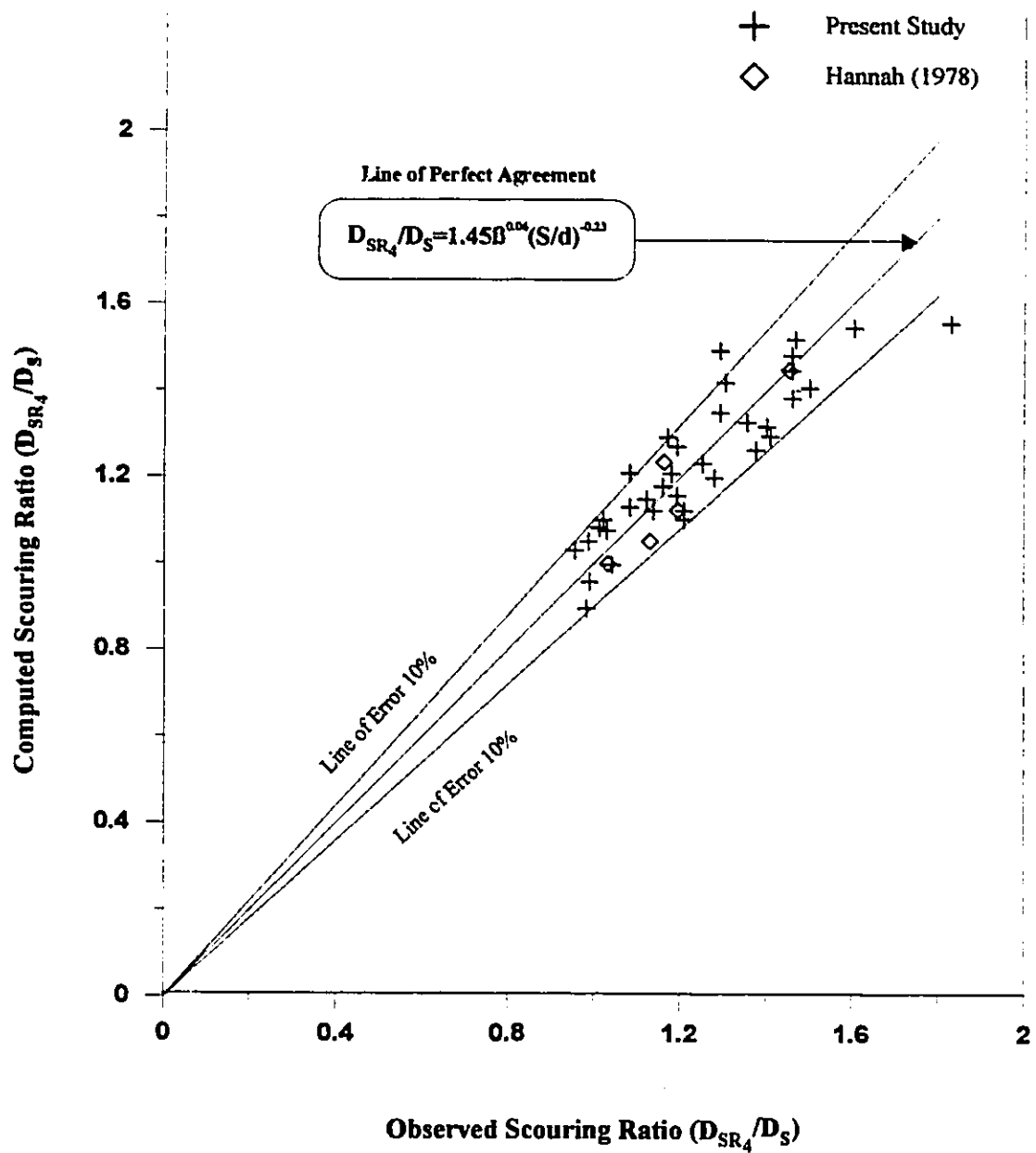


Fig. 5-4-30 : Comparison between computed and measured rear pier (R_4) scouring ratios; a group of 4 piers

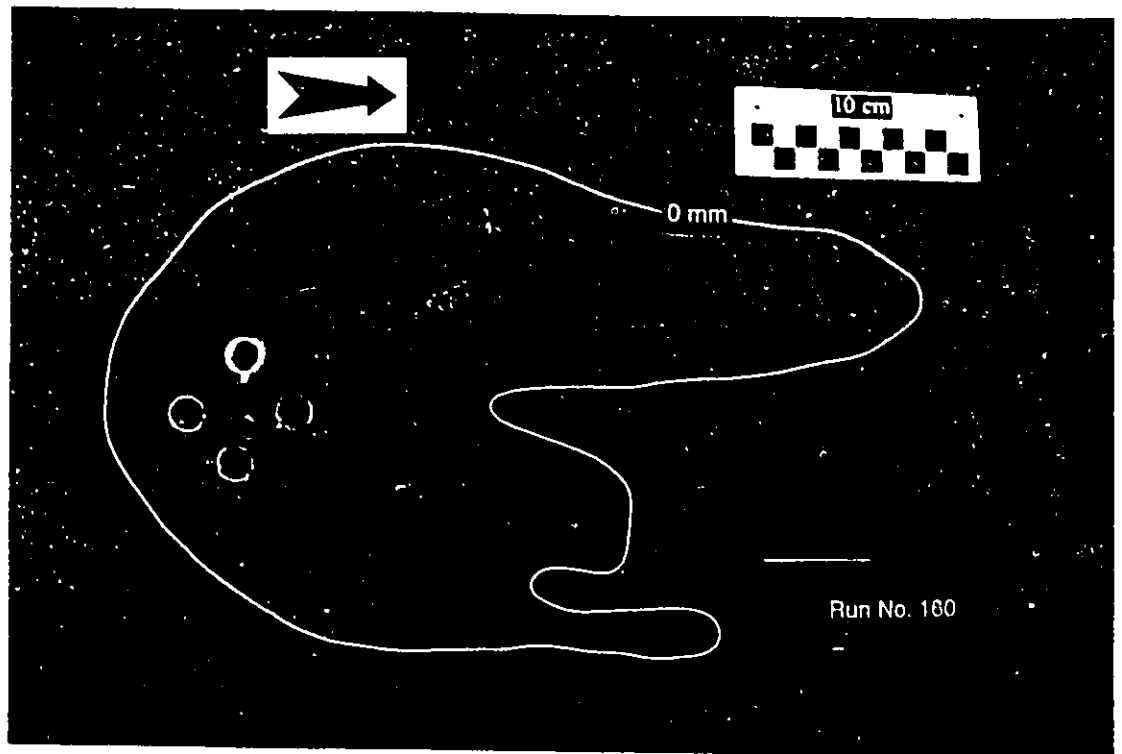


Plate 5-4-1: A group of 4 piers; $S/d=2$ and $\beta=45^\circ$

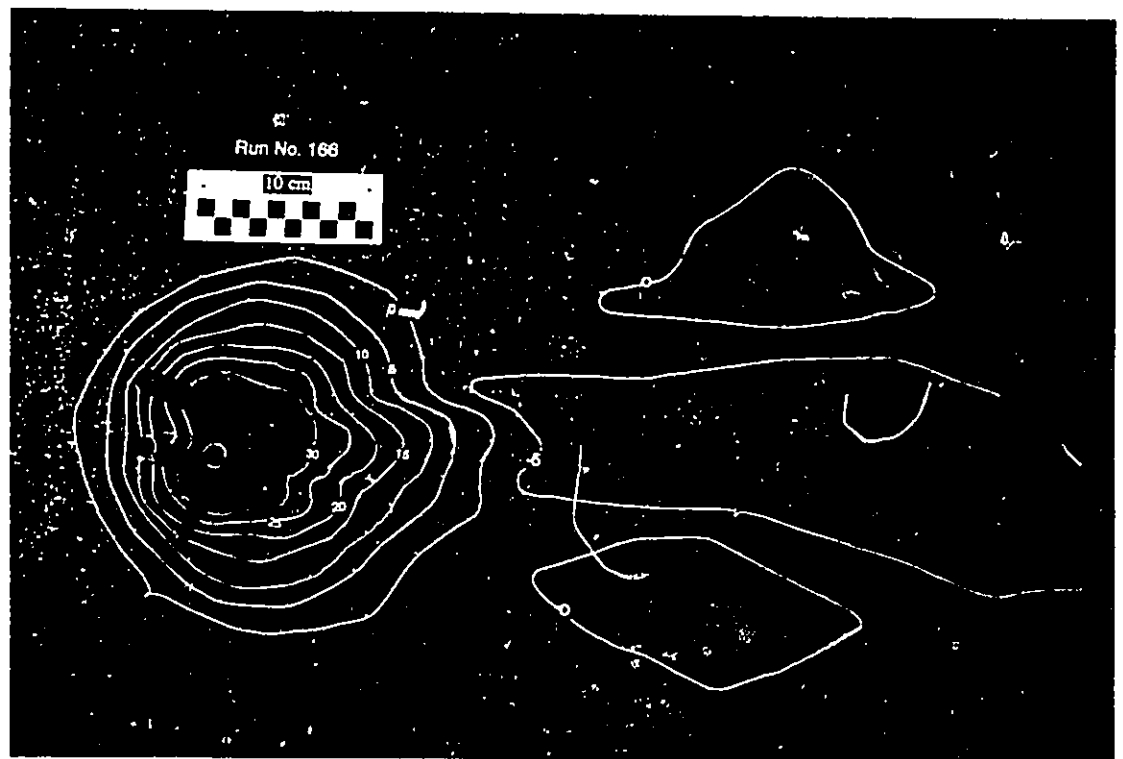


Plate 5-4-2: A group of 4 piers; $S/d=3$ and $\beta=0^\circ$

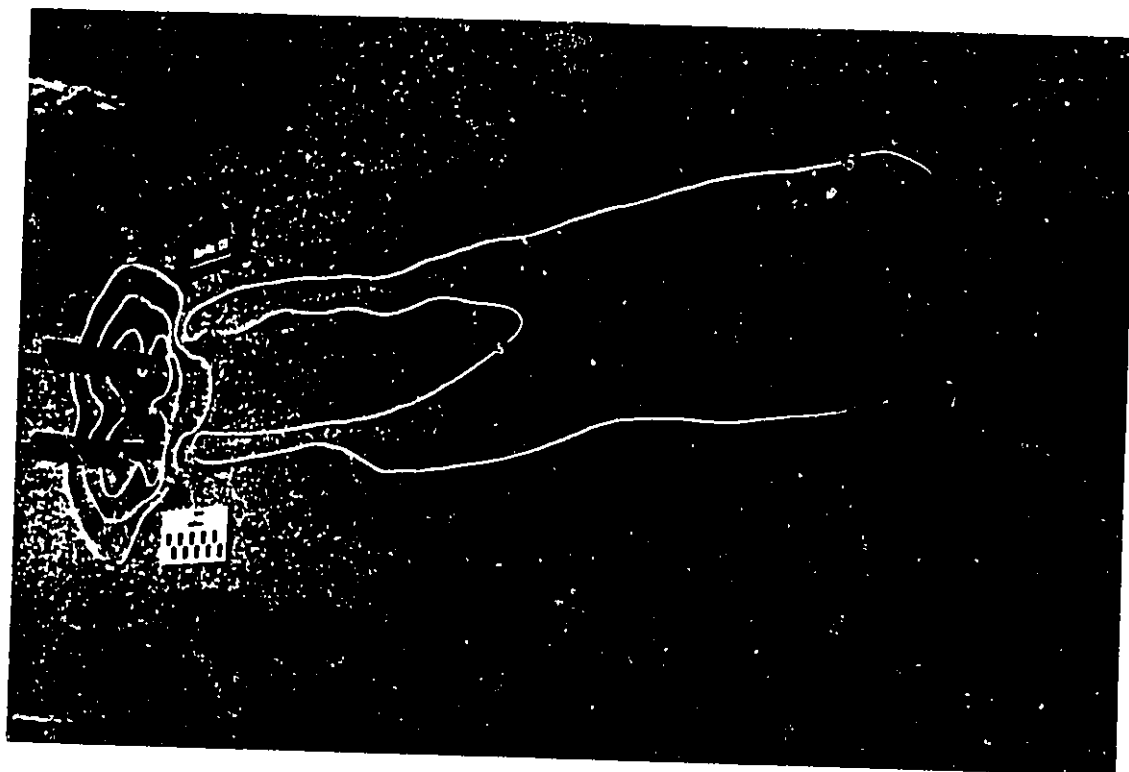


Plate 5-4-3: A group of 4 piers; $S/d=6$ and $\beta=0^\circ$

Table 5-4-1: Tests Performed for 4-PGs

β	S/d				
	2	3	4	6	8
0°	*	*	*	*	*
10°	*	*	*	*	*
20°	*	*	*	*	*
30°	*	*	*	*	*
40°	*	*	*	*	*
45°	*	*	*	*	*

* Test performed

Table 5-4-2 One-sided t-distribution for 4-PGs (28 degrees of freedom)

Confidence level (percent)	One-sided t-distribution (Statistics)
60.0	0.256
75.0	0.683
90.0	1.313
95.0	1.701
97.5	2.048
99.0	2.467
99.5	2.763

5-5 Local Scouring around 6-PGs

Local scouring in the vicinity of 6-PGs was explored at this phase of our study. The effects of sheltering and reinforcement on the local scour depths were investigated for $0^\circ \leq \beta \leq 45^\circ$, in 10° intervals, and $2 \leq S/d \leq 8$, in intervals of 2 (Table 5-5-1). For the first series of tests, S/d was kept constant while β was varied. In the subsequent series of tests, β was kept constant while S/d was varied. The data obtained in this phase of the experimental program were analysed using a multiple regression model. Different local scour depth prediction equations were developed for the front, middle, and rear piers using logarithmic transformation of normalized local scour depth ratios ($\frac{D_{SF_1}}{D_s}$, $\frac{D_{SM_2}}{D_s}$, $\frac{D_{SR_3}}{D_s}$, $\frac{D_{SF_4}}{D_s}$, $\frac{D_{SM_5}}{D_s}$, and $\frac{D_{SR_6}}{D_s}$). Good agreements were found between computed and observed normalized scour depth ratios.

5-5-1 Effects of Different β on Local Scour Depth

In this phase of the experimental program, for any S/d ($= 2, 3, 4, 6$, and 8) β varied between 0° and 45° . In the first sets of experiments on a 6-PG, the effects of varying β on local scour depth were studied for $S/d=2$. The results show (Fig. 5-5-1) an increasing trend of data with increasing β for all local scour depth ratios ($\frac{D_{SF_1}}{D_s}=D'_{F_1}$, $\frac{D_{SM_2}}{D_s}=D'_{M_2}$, $\frac{D_{SR_3}}{D_s}=D'_{R_3}$, $\frac{D_{SF_4}}{D_s}=D'_{F_4}$, $\frac{D_{SM_5}}{D_s}=D'_{M_5}$, and $\frac{D_{SR_6}}{D_s}=D'_{R_6}$). With increasing β , A_p increases causing a deeper scour hole around the piers. For this S/d , the result for a 6-PG is comparable with a single rectangular-shaped pier whose dimensions are 'a' and 'b' where $a=2 \times d$ (normal to the flow direction) and $b=3 \times d$ (parallel with the flow direction). Moreover, according to the data of Chabert and Engeldinger (1956), Hannah (1978), and Vittal et. al., (1994), the scour depth produced by a 6-PG would always be less than a single solid

pier having the same dimensions.

The scour depth in front of the piers increases with β except for D'_{F_4} , whose variation is independent of β . Considering that β increases in an anti-clockwise direction, F_4 is the only pier which is not sheltered by any other pier in the range $0^\circ \leq \beta \leq 45^\circ$. On the other hand, M_2 and R_3 , which are sheltered by F_1 and F_3 , have the smallest local scour depth among the piers. Piers downstream of F_1 and F_4 show a significant increase in their associated local scour depth starting at $\beta=10^\circ$. This is because they are exposed to the flow in the range $10^\circ \leq \beta \leq 30^\circ$. Plate 5-5-1 illustrates the formation of local scouring around a 6-PG with $S/d=2$ and $\beta=30^\circ$.

In the next series of tests, β was varied for $S/d=3$. The results show a similar trend of data with those of $S/d=2$ in that D'_{F_1} and D'_{F_4} showed almost no change to the variation of β (Fig. 5-5-2). Nevertheless, overall local scour depth in this case was less than that for $S/d=2$. The larger S/d is responsible for this difference. It was observed that D'_{M_2} , D'_{R_3} , D'_{M_3} and D'_{R_6} increased significantly with β , because their associated piers were exposed to the flow. However, their maximum values were reduced for $S/d=3$. The smaller local scour depth is due to the weakening of compressed arms of the HSV associated with increasing S/d . For example, the maximum scour depth associated with D'_{R_6} was found to be 1.8, while the corresponding value for $S/d=2$ was found to be 2.2. Furthermore, D'_{F_1} and D'_{F_4} showed almost no change with the variation of β .

In the next series of tests the effects of different β on local scour depths was investigated for $S/d=4$. Although D'_{F_1} and D'_{F_4} were once again found to be independent of the variation with β , on

the other hand the scour depths associated with the downstream piers showed increasing trends with increasing β (Fig. 5-5-3). A peak value was observed for D'_{R_6} at $\beta=30^\circ$, beyond which the scour depth decreased. The increasing trend of the first limb of the curve, which was due to a reduction of the sheltering effect with increasing β , was diminishing accordingly for D'_{R_6} . Such a trend was observed for D'_{M_5} as well, although its peak occurred at a different flow angle of attack ($\beta=40^\circ$).

The effects of different β on local scour depth ratios were investigated for $S/d=6$. Similar to the previous cases, D'_{F_4} was independent of β (Fig. 5-5-4). However, local scour depths around downstream piers showed a steep rise between $\beta=0^\circ$ and $\beta=20^\circ$ followed by more or less scour depth ratios. In the next set of experiments, the effects of varying β on local scour depth were investigated for $S/d=8$. Similar to the previous cases, variation of β did not affect the values of D'_{F_1} and D'_{F_4} whereas other scouring ratios, approaching a maximum at $\beta=20^\circ$, showed fluctuating trends (Fig. 5-5-5). The general trend of data shows that all local scouring ratios approach unity beyond $S/d=8$. The formation of local scouring holes around piers in a 6-PG is illustrated in Plate 5-5-2.

5-5-2 Effects of Varying S/d on Local Scour Depth

In this series of tests, β was kept constant in a set of experiments in which S/d varied. This procedure was repeated for $\beta=0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ$, and 45° . The range of S/d was between 2 and 8. However, two additional tests were carried out for $\beta=0^\circ$ using $S/d=1$ and 10 to investigate the effect of varying S/d on the local scour depth and to compare the results with available data. Results show (Fig. 5-5-6) the same trend of data for D'_{F_1} , D'_{M_2} , D'_{R_3} , D'_{F_4} , D'_{M_5} , and D'_{R_6} . According to

this figure, a sharp decrease is observed for all scouring ratios in the range $1 \leq S/d \leq 4$. Variations of these normalized scour depth ratios become smoother as S/d increases.

The reinforcement effect for a 6-PG was not as strong as it was for a 2 or 3-PG. Since for $S/d=1$, the projected width of a 6-PG is twice the diameter of a single pier (Fig. 5-5-25), the scour depth for $S/d=1$ is not equal to that of a single pier. However, a row of piers aligned to the flow, i.e., a 2 or 3-PG, has the same projected width as a single pier for $S/d=1$. The scour depth for a 6-PG in the case of $S/d=1$, was found to be 40 mm which is close to that of a single pier ($D_s=42$ mm) having a diameter twice as much as each pier of a 6-PG. The scour depths of rear piers are the smallest, whereas scour depths associated with front piers are the largest. The difference among the normalized scour depths ratios decreases with S/d . By further increase in S/d the scour depths of the piers approach the scour depth of a single pier.

The results of the present study are compared with Hannah's (1978) data. He conducted four tests for $\beta=0^\circ$ in the range $1 \leq S/d \leq 4$ (Figs. 5-5-7 and 5-5-8). His published data show a significant decrease at $S/d=2$, while the corresponding graphs based on the data, do not show such a decrease. In fact, the point of question ($S/d=2$) was deleted from the graph. Nevertheless, using a wider range of S/d , our study does not show such a decrease at $S/d=2$. Apart from this difference, a generally good agreement was observed between the results of the two studies. The local scour holes formed around the piers in a 6-PG are illustrated in Plates 5-5-3 and 5-5-4.

The effect of the varying S/d on local scour depth was investigated for $\beta=10^\circ$. The same trend

of data, similar to the case for $\beta = 0^\circ$, was observed for D'_{F_1} , D'_{M_2} , and D'_{F_4} (Fig. 5-5-9). On the other hand, D'_{R_1} , D'_{M_3} , and D'_{R_6} showed different data trends from those observed for $\beta = 0^\circ$. Both D'_{M_1} and D'_{R_6} followed almost the same trend while the value of D'_{R_3} showed a significant increase starting from $S/d=4$. Different trends of data between the two cases ($\beta = 0^\circ$, and 10°) are due to the fact that piers downstream of F_1 and F_4 , will be exposed to the flow when $\beta > 10^\circ$.

In the next series of tests the effect of varying S/d on the local scour depth was investigated for $\beta = 20^\circ$. The same trend of data was observed for all normalized scour depth ratios except for D'_{F_4} . The results show (Fig. 5-5-10) that the scour depth associated with F_4 is independent of the S/d . As well, this independency was observed for $\beta = 30^\circ$, 40° , and 45° (Figs. 5-5-11, 5-5-12, and 5-5-13). The reason is that for $\beta \geq 20^\circ$, D'_{F_4} will be less affected by downstream piers for all cases. Similar to the previous cases, the scour depths associated with the front piers for $\beta = 30^\circ$, 40° , and 45° , decreased significantly in the range $1 \leq S/d \leq 4$. In addition, variation of all scour depth ratios, except for that of D'_{R_3} , was independent of S/d in all cases. The deposition of sediments from upstream, caused a significant decrease in the scour depth in front of R_6 . Plates 5-5-5 to 5-5-11 illustrate the formation of local scouring around piers in a 6-PG.

5-5-3 Comparison between Results of 3- and 6 -PGs for Different S/d

It was mentioned earlier that a 6-PG may be regarded as being equivalent to two 3-PGs combined. In the following exercise, the local scour data for a 6-PG and a 3-PG extended to a 6-PG will be compared for $\beta = 0^\circ$, 10° , 20° , 30° , 40° , and 45° , and $S/d = 1, 2, 3, 4, 6, 8$, and 10 .

To investigate the effects of adding a row of 3 piers (F_1 , M_2 , and R_3) to an existing 3-PG, the results of 3 and 6-PGs were compared using $\beta=0^\circ$ and different S/d (Fig. 5-5-14). A significant difference was observed for $S/d=1$. At this S/d , the projected width of a 3-PG is equal to that of a single pier while for a 6-PG the projected width is twice as much as that of a single pier of that same group. Therefore, it is expected that a 6-PG will produce a larger scour hole than a 3-PG for $S/d=1$. However, this difference becomes less significant as S/d increases. The data also shows that scour depths for both pier groups approach the scour depth of a single pier with increasing S/d . The same conclusion applies to the second row of the 6-PG including F_4 , M_5 , and R_6 .

The effect of adding a row of 3 piers to a 3-PG was investigated for $\beta=10^\circ$ and different S/d . As was the case for $\beta=0^\circ$, the scour hole around a 6-PG is larger than that of a 3-PG. However, the difference between the two pier groups is not as significant as in the previous case. Moreover, with increasing S/d , D'_{F_1} , D'_{M_2} , D'_{R_3} , D'_{F_4} , D'_{M_5} , and D'_{R_6} approach unity.

The effect of adding an extra row of piers (3-PG) to a 3-PG was investigated using different S/d and $\beta=20^\circ$ (fig.5-5-16). Although the difference between the results of the two pier groups (3-PG and 6-PG) has been reduced, as compared with the previous cases, it is still significant for $S/d=1$. The reason for this reduction is that the projected width of a 3-PG, misaligned to the flow, is not half of that of a 6-PG while it was the case for $\beta=0^\circ$. In addition, by increasing β the scour depths associated with both rows of a 6-PG are no longer identical. Therefore, all six scour depth ratios (D'_{F_1} , D'_{M_2} , D'_{R_3} , D'_{F_4} , D'_{M_5} , and D'_{R_6}), were considered for this case for the purpose of comparison.

Using data gathered for a 3-PG and a 6-PG, the effect of adding 3 piers to a 3-PG on local scour depth was investigated for $\beta=30^\circ$, 40° , and 45° . The results show less of a difference between the scour depths of two pier groups, as compared with the previous case (Figs. 5-5-17, 5-5-18 and 19). However, it was observed that a 6-PG produced a larger scour hole than that of a 3-PG, although the local scour depths in both pier groups approached the scour depth of a single pier for $S/d>8$.

5-5-4 Comparison between the Results of 3- and 6-PGs for Different β

At this phase of our study the results of a 6-PG will be compared with a 3-PG. Since β is not constant and the results of the first row of the 6-PG (F_1 , M_2 and R_3) are different from the second row (F_3 , M_5 and R_6), all normalized scour depth ratios associated with a 6-PG are included in the comparison. The effect of varying β on local scour depth is shown in Fig. 5-5-20 for $S/d=2$. Both 3- and 6-PGs show the same trend of data. While the scour depth for a 6-PG is generally larger than that of a 3-PG, the difference between the scour depths for these two pier groups are small. However, the presence of the second row of piers, intensifies the strength of the arms of the HSV's, resulting in a larger scour depth in front of the piers in a 6-PG.

For $S/d=4$, the results (Fig. 5-5-21) show that the difference between the two cases is not significant in the range $0 \leq \beta \leq 20^\circ$. In this range of β , both data show the same trend. However, for $\beta > 20^\circ$ the normalized scour depth ratios associated with a 6-PG are larger than those of a 3-PG. This implies that the presence of the second row of piers aids sediment removal from the vicinity of the

piers resulting in deeper scour depths.

The results for $S/d=6$ show that downstream piers for both cases reach a peak value at $\beta=20^\circ$, while for $\beta>20^\circ$ they show different trends of data (Fig. 5-5-22). The former approaches the scour depth of a single pier whereas the latter does not decrease. On the other hand, D'_{F_1} and D'_{F_4} for both pier groups are almost independent of β . For $S/d=8$, it is seen, by the comparing the results of 3 and 6-PGs (Fig. 5-5-23), that the presence of an extra row of piers does not change the normalized scour depth ratios significantly. Since S/d is relatively large compared with the previous case, each pier has an independent scour hole. Therefore, there is less interaction between the arms of HSV's associated with each of the piers in a 6-PG.

5-5-5 Comparison between Local Scour Depths of a 2- and 6-PG

From another point of view, one 6-PG can be viewed as being equivalent to three 2-PGs. For $\beta=0^\circ$, F_1 and F_4 can be assumed to be piers of a 2-PG with $\beta=90^\circ$. Therefore, one can compare the results of this 2-PG with those of a 6-PG for $\beta=0^\circ$. Downstream piers (M_2 , M_5 , R_3 , and R_6) can be assumed as those extra rows of piers which intensify the arms of HSV's and caused increased more removal of sediments from around F_1 and F_4 . The results show (Fig. 5-5-24) the same trend for both 2 and 6-PGs in that they approach unity for $S/d \geq 8$. However, because of the presence of the extra piers downstream, the scour depth of a 2-PG is less than that of a 6-PG for a wider range of S/d .

5-5-6 Multiple Regression Analysis

Thirty-one measurements of local scour depths gathered from the present study were analyzed using multiple regression analyses to obtain local scour prediction equations for the front, middle and rear piers. The regression analysis provides a probability statement about the distribution of the prediction, from which an upper confidence limit of local scour depth can be determined.

5-5-6-1 Analysis of Data

Clearly, local scour depth parameters for any arrangement of pier groupings are primarily function of β and S/d . The functional relations for the normalized local scour depths are:

$$\frac{D_{SF_1}}{D_s} = f_1 \left\{ \beta, \frac{S}{d} \right\} \quad (5-5-1)$$

$$\frac{D_{SM_2}}{D_s} = f_2 \left\{ \beta, \frac{S}{d} \right\} \quad (5-5-2)$$

$$\frac{D_{SR_3}}{D_s} = f_3 \left\{ \beta, \frac{S}{d} \right\} \quad (5-5-3)$$

$$\frac{D_{SF_4}}{D_s} = f_4 \left\{ \beta, \frac{S}{d} \right\} \quad (5-5-4)$$

$$\frac{D_{SM_1}}{D_s} = f_5 \left\{ \beta, \frac{S}{d} \right\} \quad (5-5-5)$$

$$\text{and} \quad \frac{D_{SR_6}}{D_s} = f_6 \left\{ \beta, \frac{S}{d} \right\} \quad (5-5-6)$$

Multiple linear regression was used to develop prediction equations for the 6-PG, based on the proposed functional relations in equations 5-5-1, 5-5-2, 5-5-3, 5-5-4, 5-5-5, and 5-5-6. Using the multiple regression model, was given in the earlier matrix notation (equations 5-2-2, 5-2-3, and 5-2-4), the vector of fitted values corresponding to a vector of observed values is computed. Then, the “hat” matrix $H_{(31 \times 31)}$ which is necessary in regression analysis can be computed for the 6-PG, using equation 5-2-4. Logarithmic transformation of all variables was found to provide the best linear relations. The logarithm of the ratios of $\frac{D_{SF_1}}{D_s}$, $\frac{D_{SM_2}}{D_s}$, $\frac{D_{SR_3}}{D_s}$, $\frac{D_{SF_4}}{D_s}$, $\frac{D_{SM_5}}{D_s}$, and $\frac{D_{SR_6}}{D_s}$ were regressed against combination of the two independent variables β and S/d .

The general forms of the prediction equations are:

$$\frac{D_{SF_1}}{D_s} = a_1 \beta^{b_1} \left(\frac{S}{d} \right)^{c_1} \quad (5-5-7)$$

$$\frac{D_{SM_2}}{D_s} = a_2 \beta^{b_2} \left(\frac{S}{d} \right)^{c_2} \quad (5-5-8)$$

$$\frac{D_{SR_1}}{D_s} = a_3 \beta^{b_3} \left(\frac{S}{d} \right)^{c_3} \quad (5-5-9)$$

$$\frac{D_{SF_4}}{D_s} = a_4 \beta^{b_4} \left(\frac{S}{d} \right)^{c_4} \quad (5-5-10)$$

$$\frac{D_{SM_5}}{D_s} = a_5 \beta^{b_5} \left(\frac{S}{d} \right)^{c_5} \quad (5-5-11)$$

$$\frac{D_{SR_6}}{D_s} = a_6 \beta^{b_6} \left(\frac{S}{d} \right)^{c_6} \quad (5-5-12)$$

in which ; $a_1 = \exp(\hat{\theta}'_{F_1(1)})$, $b_1 = \hat{\theta}'_{F_1(2)}$, $c_1 = \hat{\theta}'_{F_1(3)}$, $a_2 = \exp(\hat{\theta}'_{M_2(1)})$, $b_2 = \hat{\theta}'_{M_2(2)}$, $c_2 = \hat{\theta}'_{M_2(3)}$,
 $a_3 = \exp(\hat{\theta}'_{R_3(1)})$, $b_3 = \hat{\theta}'_{R_3(2)}$, $c_3 = \hat{\theta}'_{R_3(3)}$, $a_4 = \exp(\hat{\theta}'_{F_4(1)})$, $b_4 = \hat{\theta}'_{F_4(2)}$, $c_4 = \hat{\theta}'_{F_4(3)}$, $a_5 = \exp(\hat{\theta}'_{M_5(1)})$,
 $b_5 = \hat{\theta}'_{M_5(2)}$, $c_5 = \hat{\theta}'_{M_5(3)}$, $a_6 = \exp(\hat{\theta}'_{R_6(1)})$, $b_6 = \hat{\theta}'_{R_6(2)}$, $c_6 = \hat{\theta}'_{R_6(3)}$, $\hat{\theta}'_{F_1} = [\hat{\theta}'_{F_1(1)}, \hat{\theta}'_{F_1(2)}, \hat{\theta}'_{F_1(3)}]$,
 $\hat{\theta}'_{M_2} = [\hat{\theta}'_{M_2(1)}, \hat{\theta}'_{M_2(2)}, \hat{\theta}'_{M_2(3)}]$, $\hat{\theta}'_{R_3} = [\hat{\theta}'_{R_3(1)}, \hat{\theta}'_{R_3(2)}, \hat{\theta}'_{R_3(3)}]$, $\hat{\theta}'_{F_4} = [\hat{\theta}'_{F_4(1)}, \hat{\theta}'_{F_4(2)}, \hat{\theta}'_{F_4(3)}]$,
 $\hat{\theta}'_{M_5} = [\hat{\theta}'_{M_5(1)}, \hat{\theta}'_{M_5(2)}, \hat{\theta}'_{M_5(3)}]$, and $\hat{\theta}'_{R_6} = [\hat{\theta}'_{R_6(1)}, \hat{\theta}'_{R_6(2)}, \hat{\theta}'_{R_6(3)}]$.

5-5-6-2 Predicting Scour Depths

Normalized scour depths predicted by equations 5-5-7, 5-5-8, 5-5-9, 5-5-10, 5-5-11, and 5-5-12 are the expected response to a specified set of regressors, and are not known with certainty. Using a procedure similar to the case for the 2-PG, a probability statement about the distribution of provides a means of evaluating appropriate design values. Using the transpose of the vector of predictor variables, as given in equation 5-2-6, several 100(1- α) percent upper confidence limits for the logarithm of normalized scour depths can be computed. Equation 5-2-7 is used to compute the upper confidence limit, using $\frac{D_{SF_1}}{D_s}, \frac{D_{SM_2}}{D_s}, \frac{D_{SR_3}}{D_s}, \frac{D_{SF_4}}{D_s}, \frac{D_{SM_5}}{D_s}$, and $\frac{D_{SR_6}}{D_s}$ as substitutions to $\frac{D_{S_i}}{D_s}$. The values of $\hat{\theta}'_{F_1}, \hat{\theta}'_{M_2}, \hat{\theta}'_{R_3}, \hat{\theta}'_{F_4}, \hat{\theta}'_{M_5}$, and $\hat{\theta}'_{R_6}$ computed for a 6-PG are:

$$\hat{\theta}'_{F_1}=[0.54,0.02,-0.20] \quad (5-5-13)$$

$$\hat{\theta}'_{M_2}=[0.50,0.05,-0.20] \quad (5-5-14)$$

$$\hat{\theta}'_{R_3}=[0.34,0.04,-0.15] \quad (5-5-15)$$

$$\hat{\theta}'_{F_4}=[0.53,0.01,-0.2] \quad (5-5-16)$$

$$\hat{\theta}'_{M_1} = [0.45, 0.05, -0.19] \quad (5-5-17)$$

$$\text{and } \hat{\theta}'_{R_6} = [0.39, 0.08, -0.2] \quad (5-5-18)$$

The variance of prediction can be estimated using equation 5-2-10 in which σ^2 for the front, middle, and rear piers is substituted by $\hat{\sigma}_{F_1}^2, \hat{\sigma}_{M_2}^2, \hat{\sigma}_{R_3}^2, \hat{\sigma}_{F_4}^2, \hat{\sigma}_{M_5}^2$, and $\hat{\sigma}_{R_6}^2$. For the 2-variable regression model, using thirty-one measurements, the variances of predictions are $\hat{\sigma}_{F_1}^2 = 0.39, \hat{\sigma}_{M_2}^2 = 0.65, \hat{\sigma}_{R_3}^2 = 0.69, \hat{\sigma}_{F_4}^2 = 0.24, \hat{\sigma}_{M_5}^2 = 0.63$, and $\hat{\sigma}_{R_6}^2 = 0.84$. One-sided t-distribution statistics for $n-p=31-3=28$, degrees of freedom and several confidence levels are given in Table 5-3-2. Upper estimates of $\ln(\frac{D_{SF_1}}{D_s}), \ln(\frac{D_{SM_2}}{D_s}), \ln(\frac{D_{SR_3}}{D_s}), \ln(\frac{D_{SF_4}}{D_s}), \ln(\frac{D_{SM_5}}{D_s})$, and $\ln(\frac{D_{SR_6}}{D_s})$ obtained from equation 5-2-7. With the inclusion of an appropriate factor of safety, the upper limit of scour depth can be used to design pier foundations.

5-5-6-3 Prediction Equation for a 6-PG

Parameters $a_1, b_1, c_1, a_2, b_2, c_2, a_3, b_3, c_3, a_4, b_4, c_4, a_5, b_5, c_5, a_6, b_6, c_6$ obtained for a 6-PG were substituted in equation 5-2-5. Hence, for the front, middle, and rear piers for a 6-PG equation 5-2-5 can be expressed as:

$$\frac{D_{SF_1}}{D_s} = 1.72 \beta^{0.02} \left(\frac{S}{d} \right)^{-0.20} \quad (5-5-19)$$

$$\frac{\hat{D}_{SM_2}}{D_s} = 1.65\beta^{0.05} \left(\frac{S}{d} \right)^{-0.20} \quad (5-5-20)$$

$$\frac{\hat{D}_{SR_3}}{D_s} = 1.4\beta^{0.04} \left(\frac{S}{d} \right)^{-0.15} \quad (5-5-21)$$

$$\frac{\hat{D}_{SF_4}}{D_s} = 1.7\beta^{0.01} \left(\frac{S}{d} \right)^{-0.20} \quad (5-5-22)$$

$$\frac{\hat{D}_{SM_5}}{D_s} = 1.57\beta^{0.05} \left(\frac{S}{d} \right)^{-0.19} \quad (5-5-23)$$

$$\text{and} \quad \frac{\hat{D}_{SR_6}}{D_s} = 1.48\beta^{0.08} \left(\frac{S}{d} \right)^{-0.20} \quad (5-5-24)$$

Equations 5-5-19 to 5-5-24, are used to predict the values of $\frac{D_{SF_1}}{D_s}, \frac{D_{SM_2}}{D_s}, \frac{D_{SR_3}}{D_s}, \frac{D_{SF_4}}{D_s}, \frac{D_{SM_5}}{D_s}$, and $\frac{D_{SR_6}}{D_s}$. Predicted values are plotted against observed values in Figs. 5-5-25, 5-5-26, , 5-5-27, 5-5-28, 5-5-29, and 5-5-30. Most data fall within $\pm 10\%$, $\pm 10\%$, $\pm 15\%$, $\pm 10\%$, $\pm 10\%$, and $\pm 10\%$, of the predictions from the relationship proposed in this study, for $F_1, M_2, R_3, F_4, M_5, R_6$ respectively. The value of D_s may be obtained using any appropriate empirical

equations for a single pier. Equations 5-5-19 to 5-5-24 represent the line of perfect agreement in Figs 5-5-25, 5-5-26, 5-5-27, 5-5-28, 5-5-29, and 5-5-30, respectively.

The number of prediction equations can be reduced by combining equations which have the same or close coefficients. Equation 5-5-19 can be used instead of 5-5-22 conservatively to predict the front pier local scour depths. Also, equation 5-5-20 can be used for the middle pier local scour depths. However, for the rear local scour depth prediction equations coefficients of equations 5-5-21 and 5-5-24 are not close enough, so we have to choose parameters 'a', 'b', and 'c' conservatively to develop prediction equations applicable to both rear piers. Those coefficients which give greater scour depth were included in the following equation:

$$\frac{\hat{D}_{SR_3 \text{ or } 6}}{D_s} = 1.48\beta^{0.08} \left(\frac{S}{d} \right)^{-0.15} \quad (5-5-25)$$

Using this equation, $\frac{D_{SR_3}}{D_s}$, and $\frac{D_{SR_6}}{D_s}$ were recalculated and plotted against the observed values in Fig. 5-5-31. Most data fall within +18% and -10% of the predictions from the relationship proposed here for the rear pier.

5-5-7 Conclusions:

The effect of varying β ($=0^\circ, 10^\circ, 20^\circ, 30^\circ, 40^\circ, 45^\circ$) on the local scour depth was investigated for $S/d = 2, 3, 4, 6$, or 8 for a 6-PG. It was found that by increasing β the sheltering effect diminishes, which results in a smaller local scour depth. However, because of the geometry of a 6-PG, by

changing β downstream piers might be sheltered by upstream piers in particular cases. Therefore, there is always a particular range of β in which sheltering is not significant. It was observed that in the range $10^\circ \leq \beta \leq 30^\circ$ sheltering is insignificant. In addition, it was found that the variations of D'_{F_1} and D'_{F_4} were independent of the variation of β .

Our experiments showed that interactions, between the arms of HSVs associated with each pier, were significant for $S/d \leq 4$ (Fig. 5-5-14). The front pier scour depths were independent of S/d when $S/d > 4$, however, with increasing S/d , all local scour depths ratios approached unity. In addition, for $\beta > 10^\circ$ the scour depths of the piers reach the scour depth of a single pier at a smaller S/d than that of the former case, because the sheltering effect diminishes for the downstream piers. The data obtained from this study were compared with those of Hannah (1978), and a good agreement was observed between the two set of data.

At this stage of the study a comparison was made between the results of 3- and 6-PGs using different S/d . A 6-PG was assumed to be equivalent to two 3-PGs combined, whose spacings altered from $S/d=2$ to 8. Generally, scour depths associated with a 6-PG are always greater than those of a 3-PG. However, with increasing S/d the scour depths associated with the 3 and 6-PGs, approach the scour depth of a single pier.

The effect of adding a row of 3-PG to an existing 3-PG was investigated using different β . The presence of the second row of piers evidently intensifies the arms of HSV resulting in a deeper scour hole around the piers in a 6-PG. It was observed that the difference between the scour depths

for a 6-PG and for a 3-PG was insignificant in the range $0^\circ \leq \beta \leq 20^\circ$. In addition, the local scour depths for both pier groups reached a maximum at $\beta=20^\circ$ when $S/d=6$. In general, a 6-PG produces a larger general scour hole than a 3-PG, especially for a smaller S/d and a larger β .

Assuming one 6-PG consists of three 2-PGs, the effect of the presence of two additional 2-PGs beside an initial 2-PG was investigated using different S/d for $\beta=0^\circ$. Both 2 and 6 pier groups showed the same trend of data. However, because of the presence of the two additional pier groups (downstream piers of a 6-PG), which intensify the arms of the HSV's, the scour depth of a 2-PG was found to be always less than that of a 6-PG.

The general form of the prediction equation written in terms of untransformed values was found for each pier, using the transpose of the vector of the regression model. The values $\hat{\theta}'_{F_1}$, $\hat{\theta}'_{M_2}$, $\hat{\theta}'_{R_3}$, $\hat{\theta}'_{F_4}$, $\hat{\theta}'_{M_5}$, and $\hat{\theta}'_{R_6}$ computed for a 6-PG were found using equations 5-5-13 to 5-5-18. By substituting the values of $\hat{\theta}'_{F_1}$, $\hat{\theta}'_{M_2}$, $\hat{\theta}'_{R_3}$, $\hat{\theta}'_{F_4}$, $\hat{\theta}'_{M_5}$, and $\hat{\theta}'_{R_6}$ in equation 5-2-5, the prediction equations for the front, middle, and rear piers were found using equations 5-5-19, 5-5-20, 5-5-21, 5-5-22, 5-5-23, and 5-5-24. Thus, the values of $\frac{D_{SF_1}}{D_s}$, $\frac{D_{SM_2}}{D_s}$, $\frac{D_{SR_3}}{D_s}$, $\frac{D_{SF_4}}{D_s}$, $\frac{D_{SM_5}}{D_s}$, and $\frac{D_{SR_6}}{D_s}$ were computed and plotted against the observed values in Figs. 5-5-25, 5-5-26, 5-5-27, 5-5-28, 5-5-29, and 5-5-30. The prediction equations were found successful in predicting the local scour depth for a 6-PG within acceptable errors.

Equations developed for predicting local scour depths were combined and reduced to three expressions. Coefficients found for the front piers were close to each other, and this was the case

also for the middle piers. Thus, equations 5-5-19 and 5-5-20 were found to be good estimators of the local scour depths for the front and middle piers, respectively. However, the coefficients with the rear pier local scour depth prediction equations were not close enough to choose one over the others. Therefore, to predict local scour depths conservatively, maxima of coefficients from equations 5-5-21 and 5-5-24 were selected. Finally, equation 5-5-25 was proposed as an alternative to those equations.

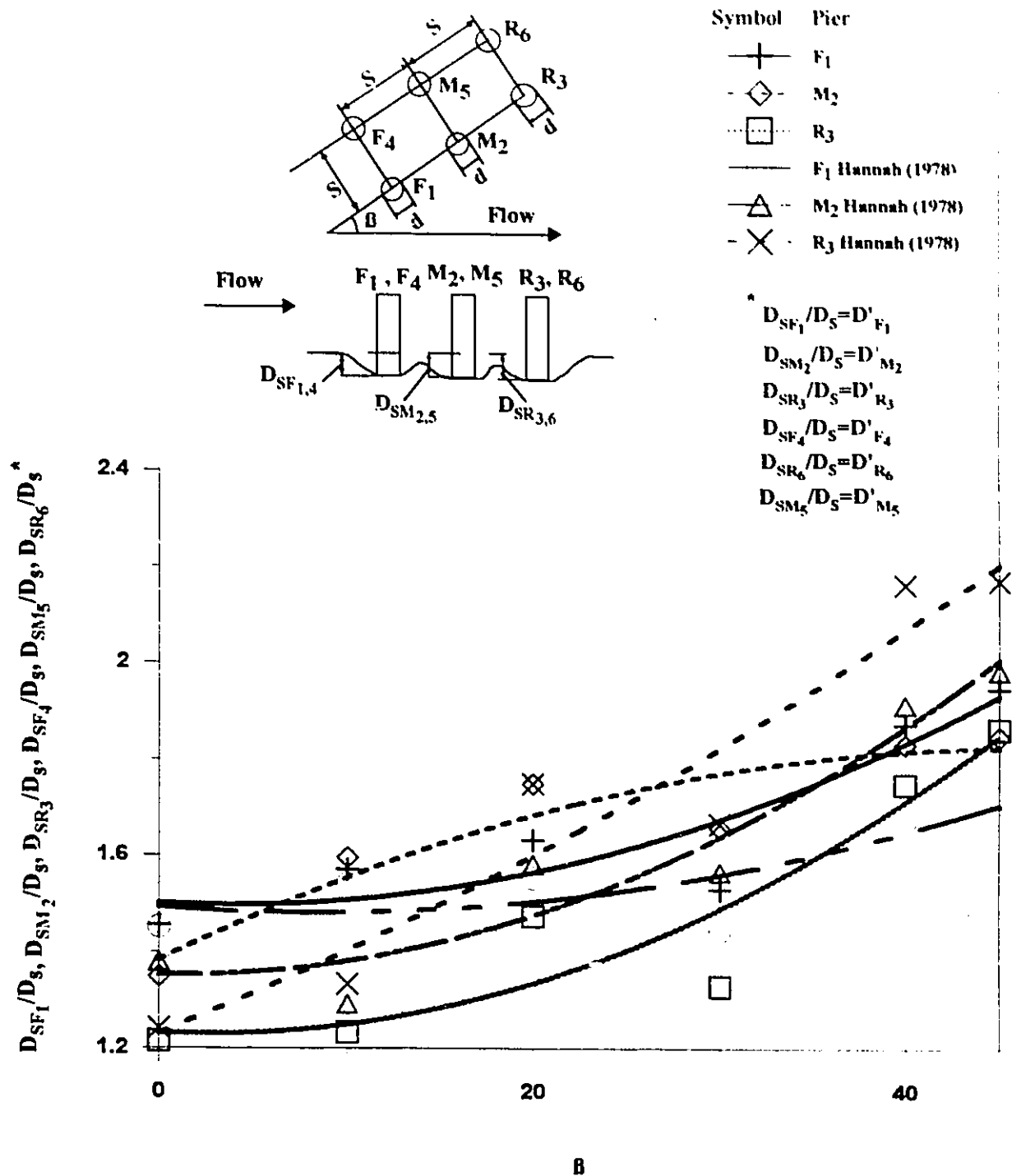
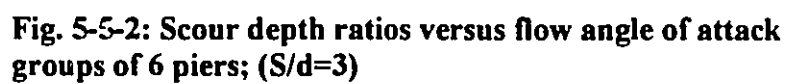


Fig. 5-5-1: Scour depth ratios versus flow angle of attack groups of 6 piers; ($S/d=2$)



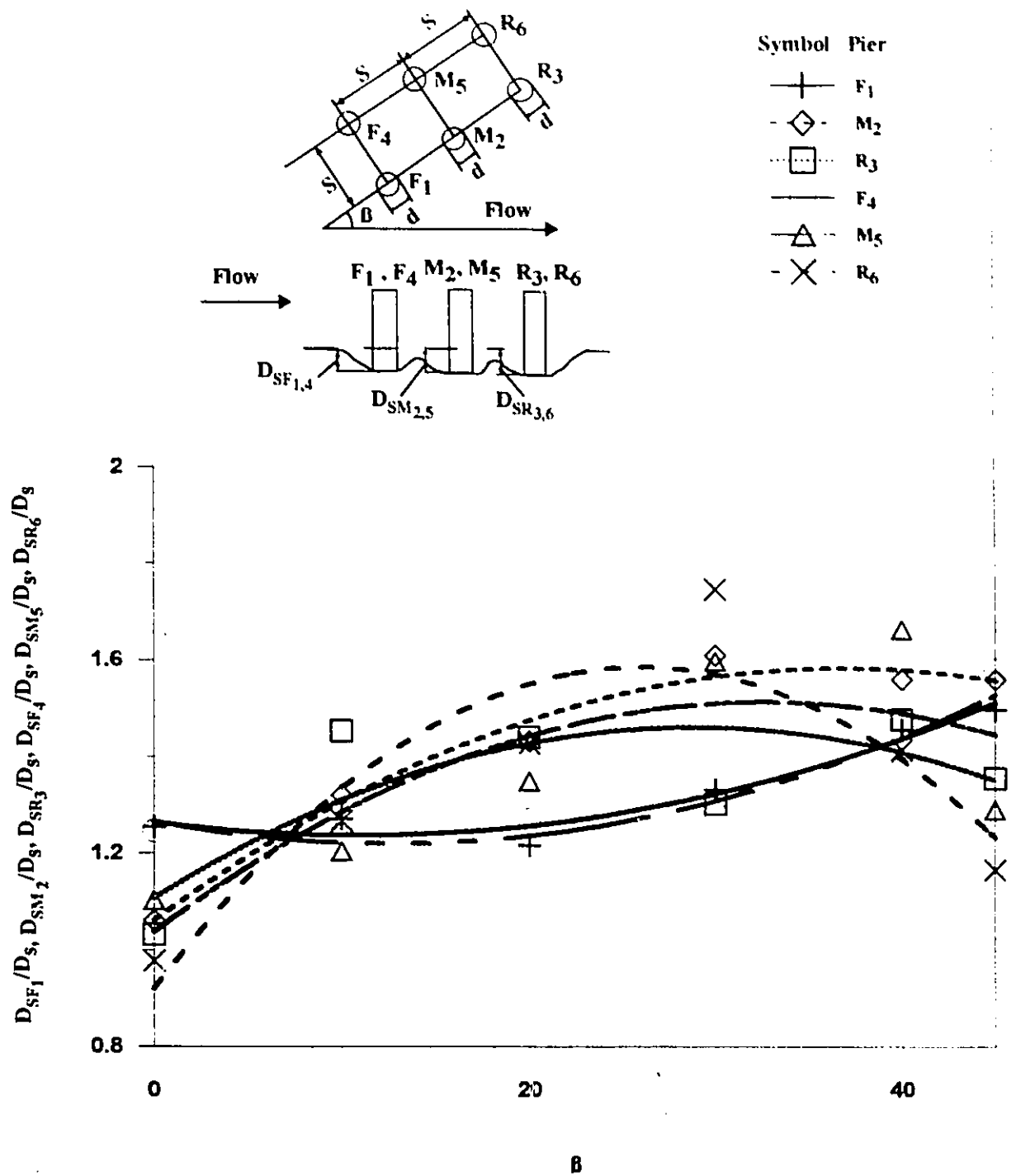


Fig. S-5-3: Scour depth ratios versus flow angle of attack groups of 6 piers; ($S/d=4$)

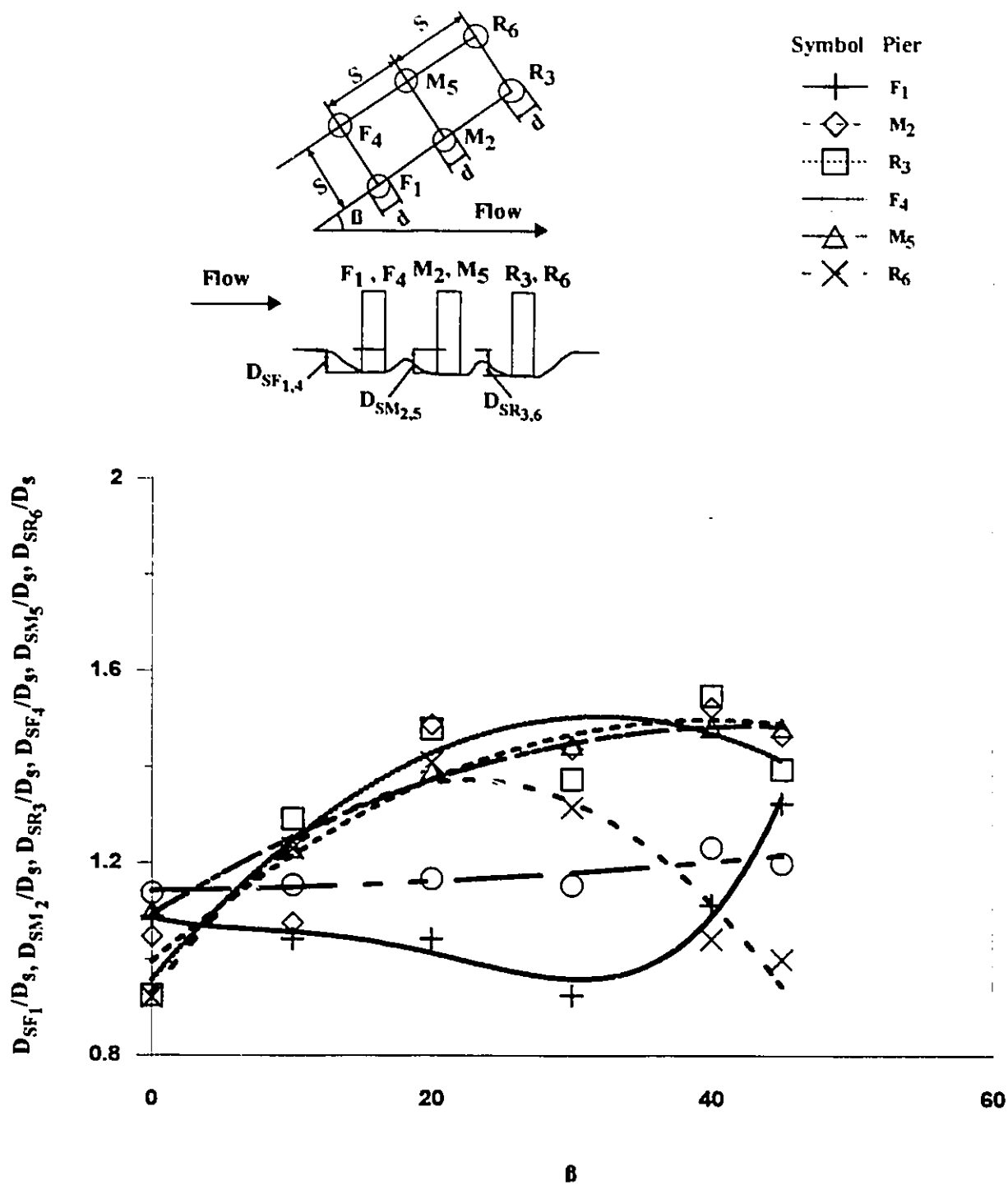


Fig. 5-5-4: Scour depth ratios versus flow angle of attack groups of 6 piers; ($S/d=6$)

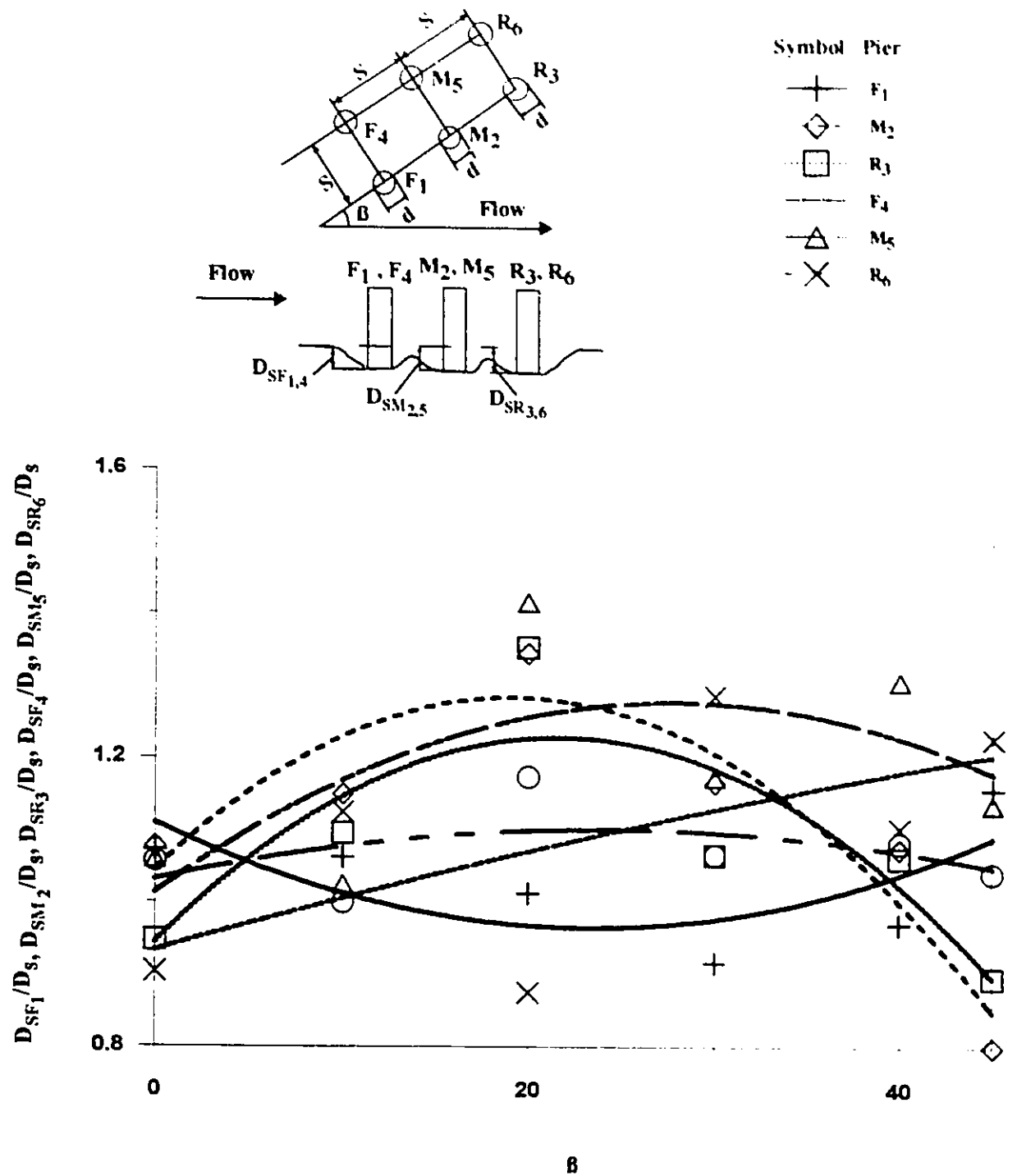


Fig. 5-5-5: Scour depth ratios versus flow angle of attack groups of 6 piers; ($S/d=8$)

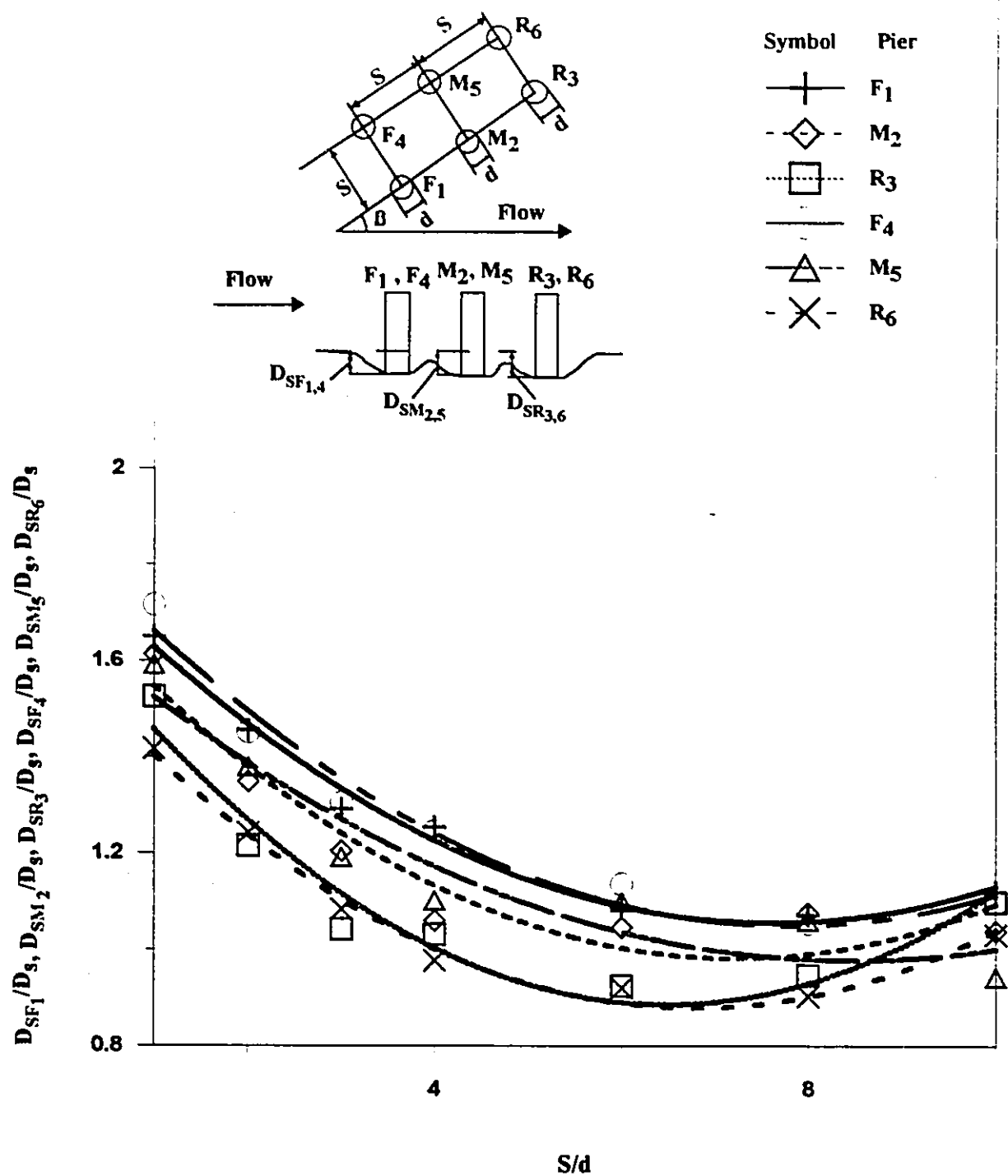


Fig. 5-5-6: Scour depth ratios versus pier spacing/pier diameter groups of 6 piers; ($\beta=0^\circ$)

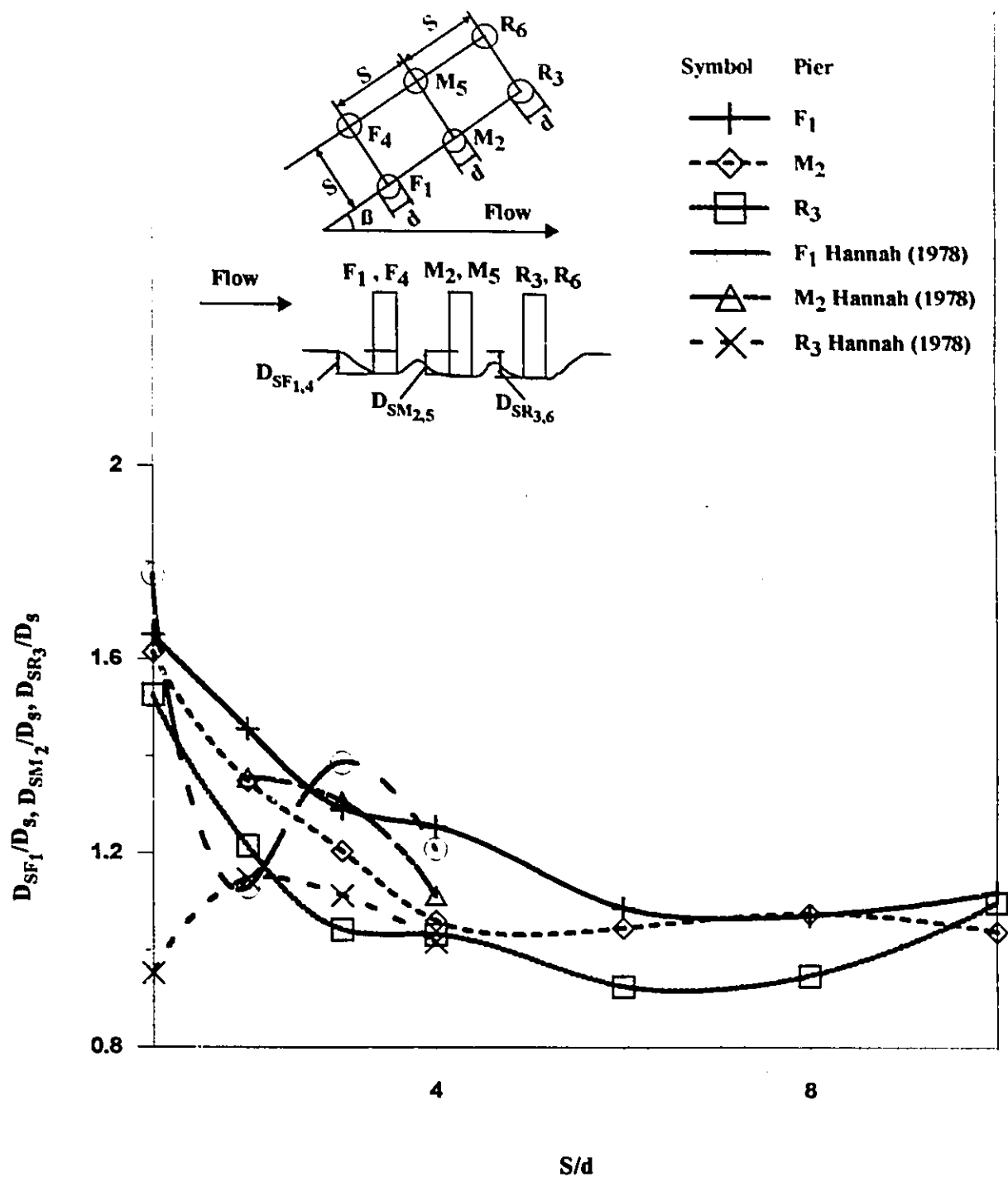


Fig. 5-5-7: Scour depth ratios versus pier spacing/pier diameter groups of 6 piers; first row; ($\beta=0^\circ$)

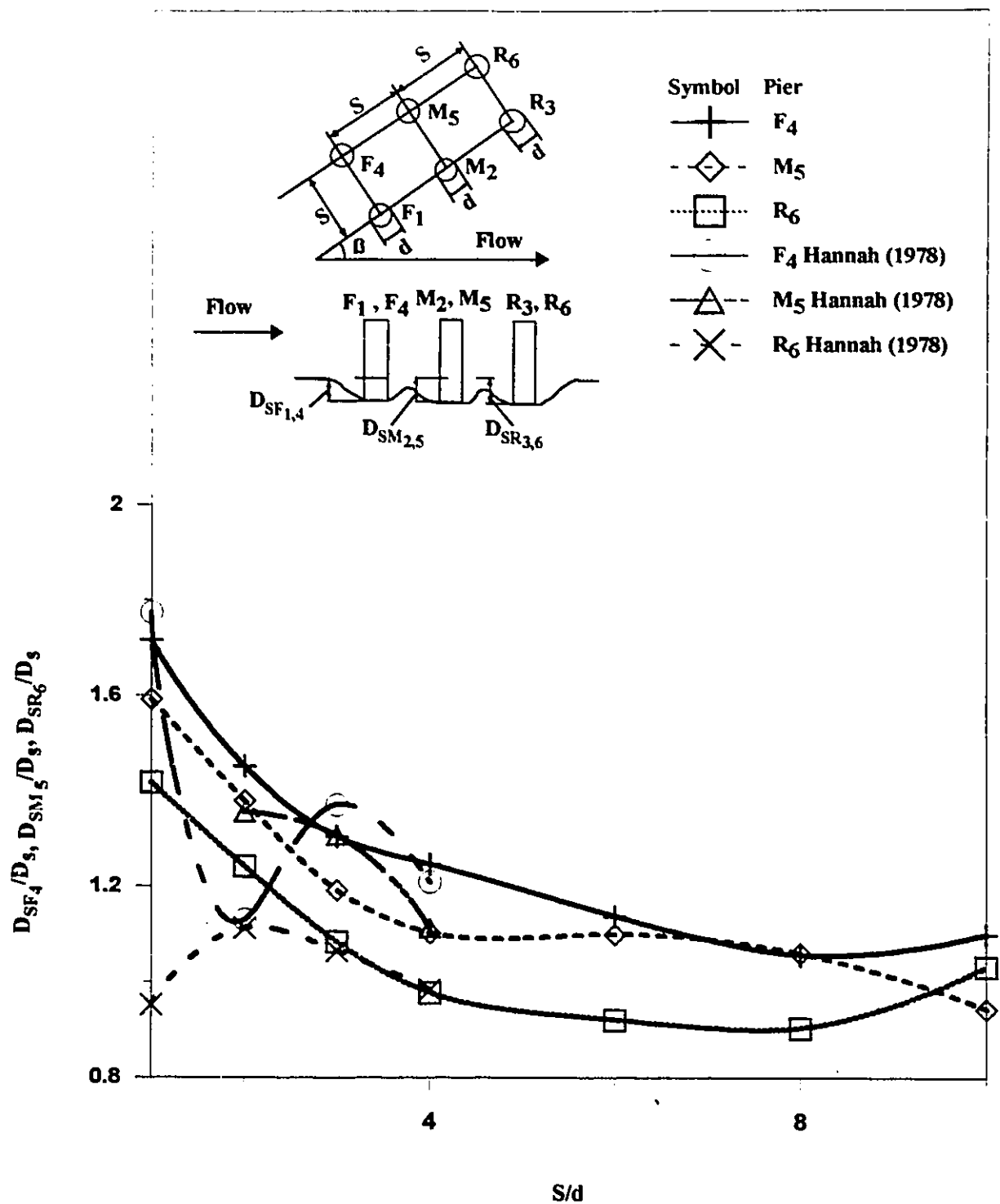


Fig. 5-5-8: Scour depth ratios versus pier spacing/pier diameter groups of 6 piers; second row; ($\beta=0^\circ$)

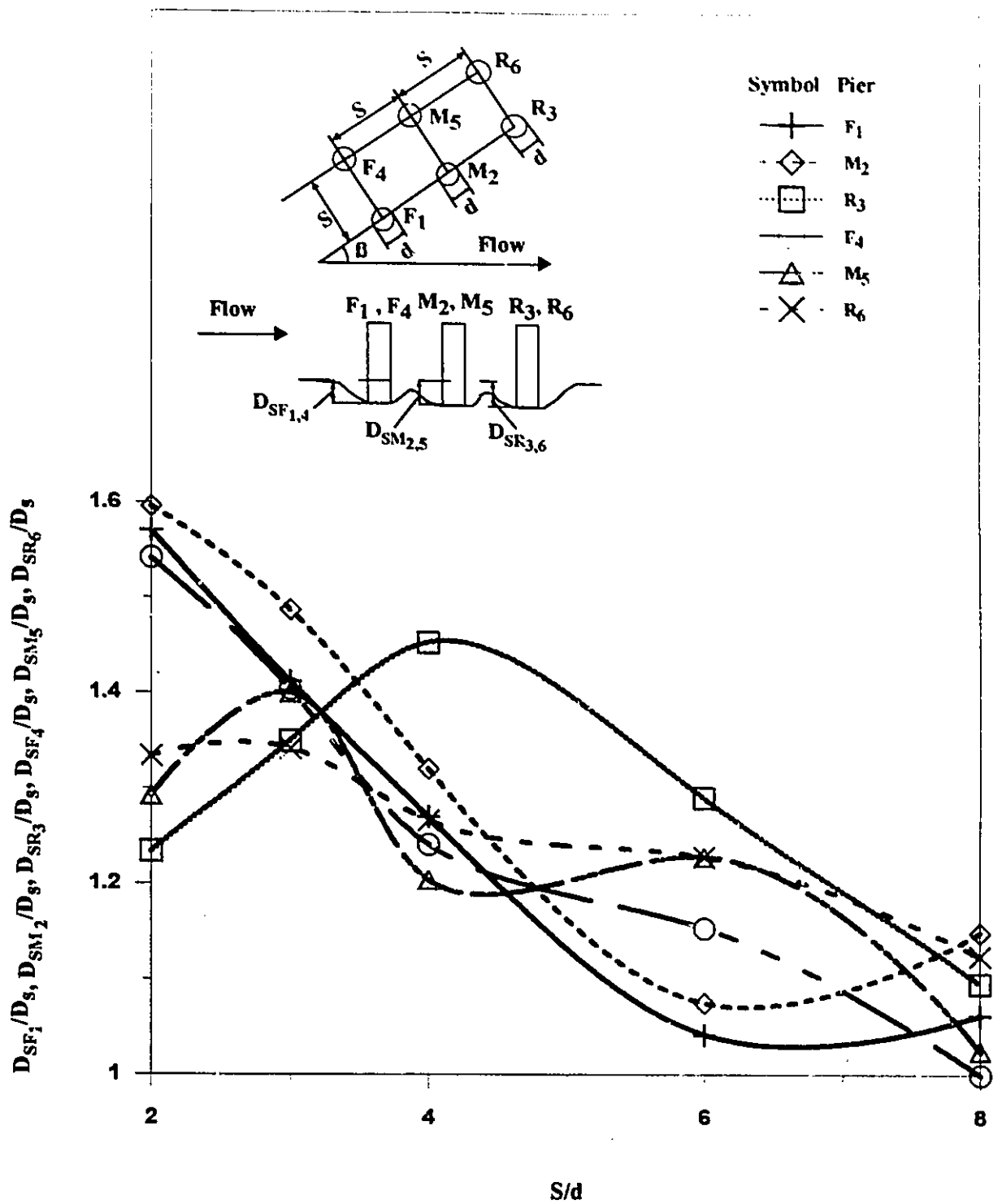


Fig. 5-5-9: Scour depth ratios versus pier spacing/pier diameter groups of 6 piers; ($\beta=10^\circ$)

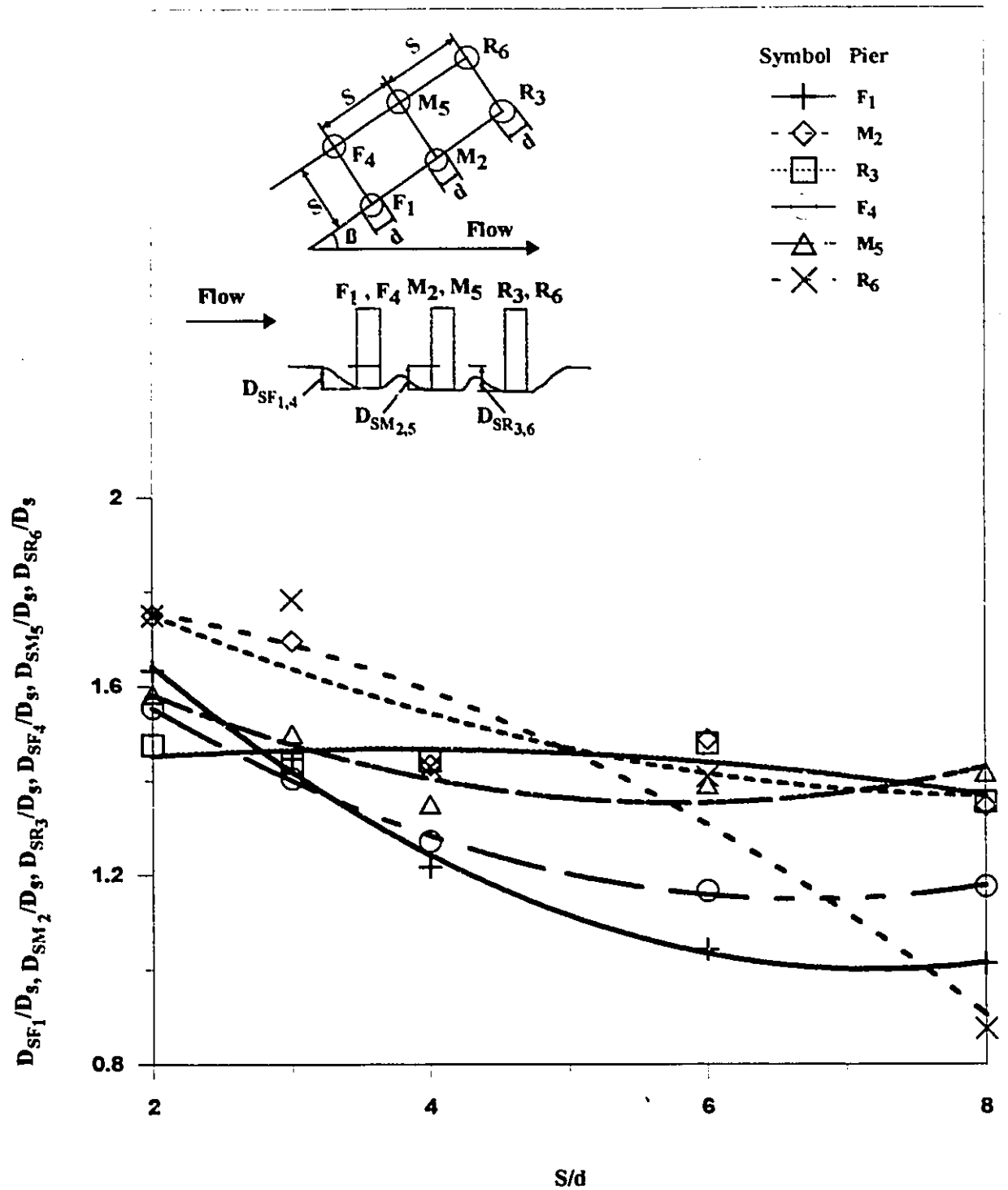


Fig. 5-5-10: Scour depth ratios versus pier spacing/pier diameter groups of 6 piers; ($\beta=20^\circ$)

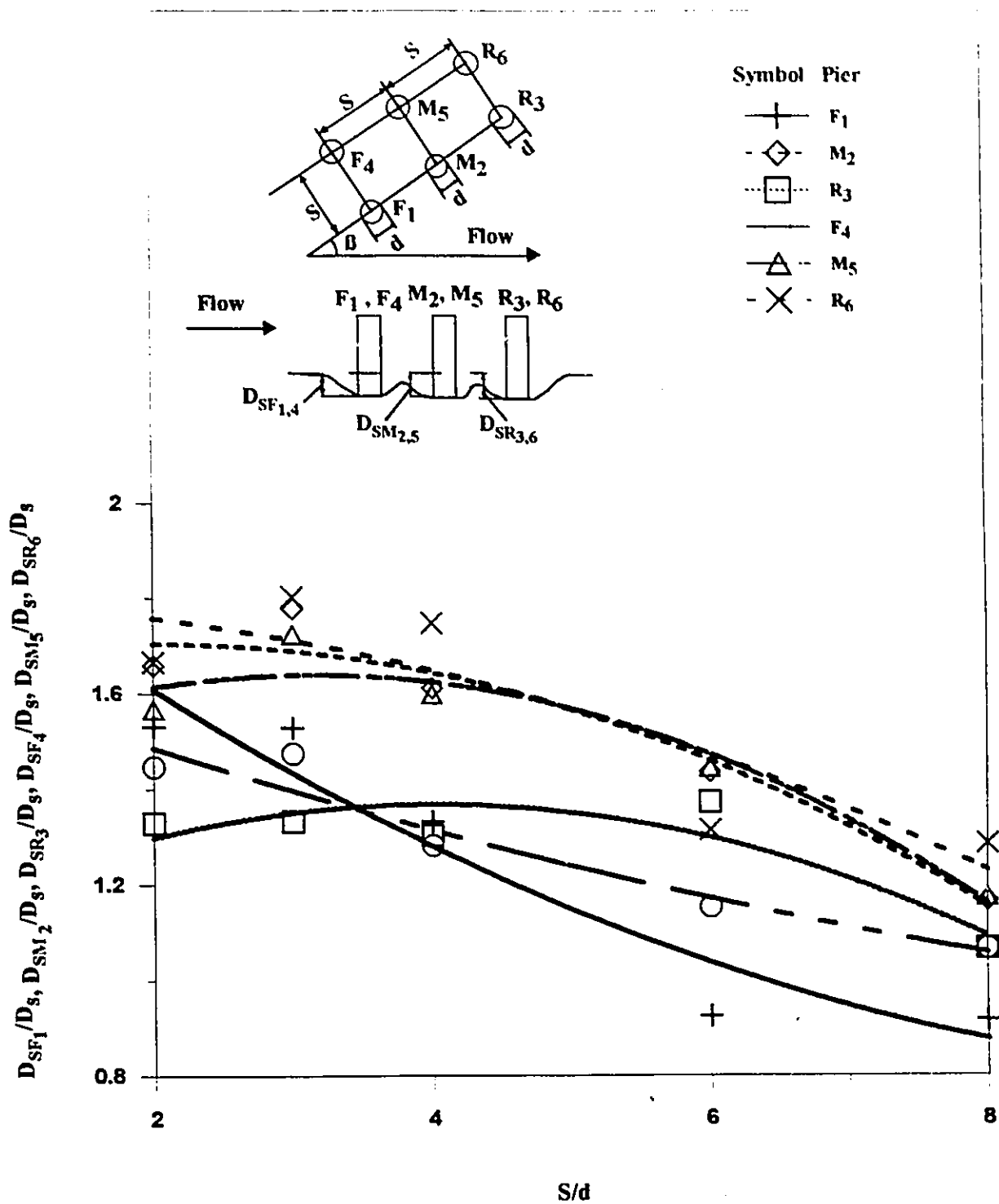


Fig. 5-5-11: Scour depth ratios versus pier spacing/pier diameter groups of 6 piers; ($\beta=30^\circ$)

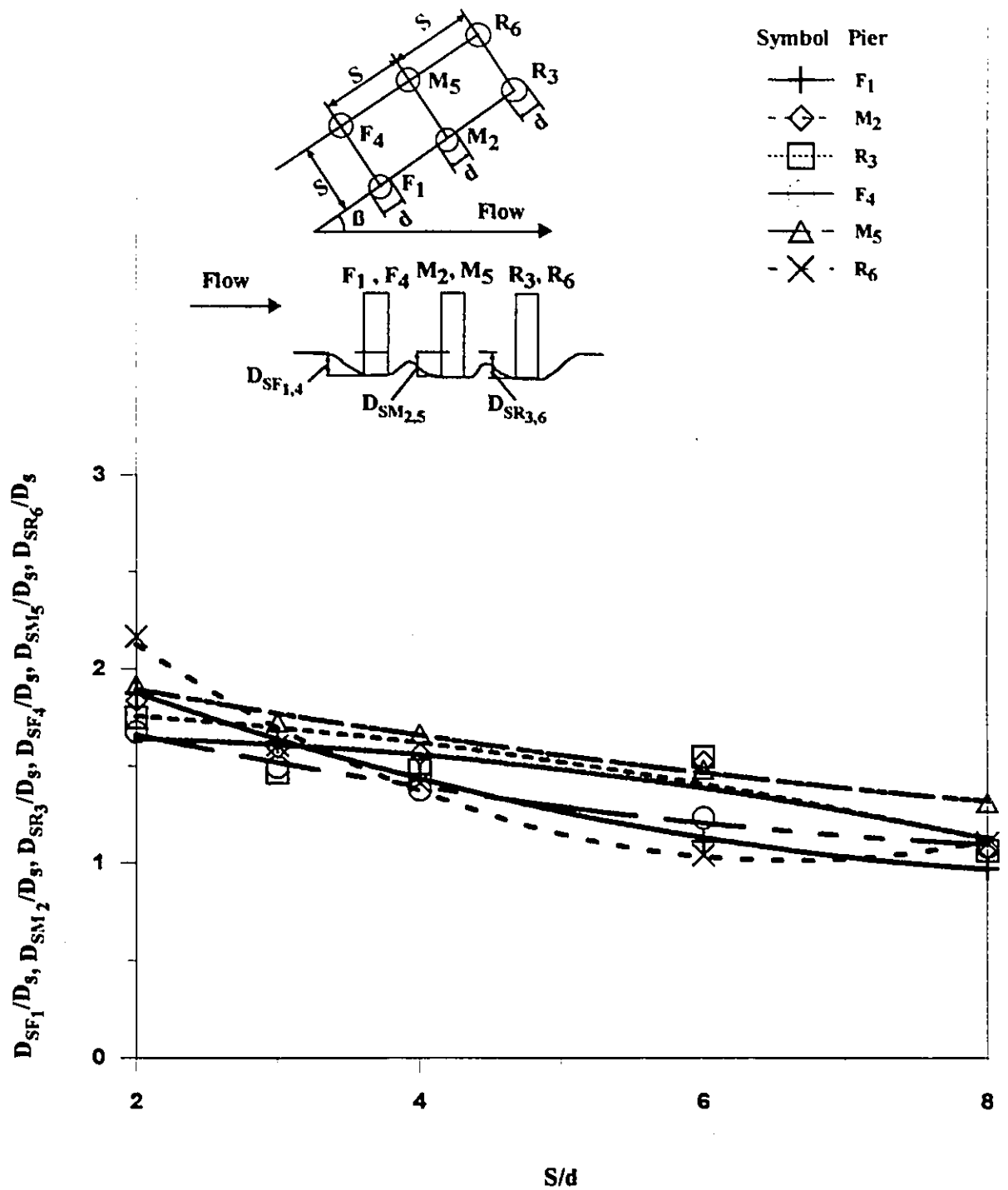


Fig. 5-5-12: Scour depth ratios versus pier spacing/pier diameter groups of 6 piers; ($\beta=40^\circ$)

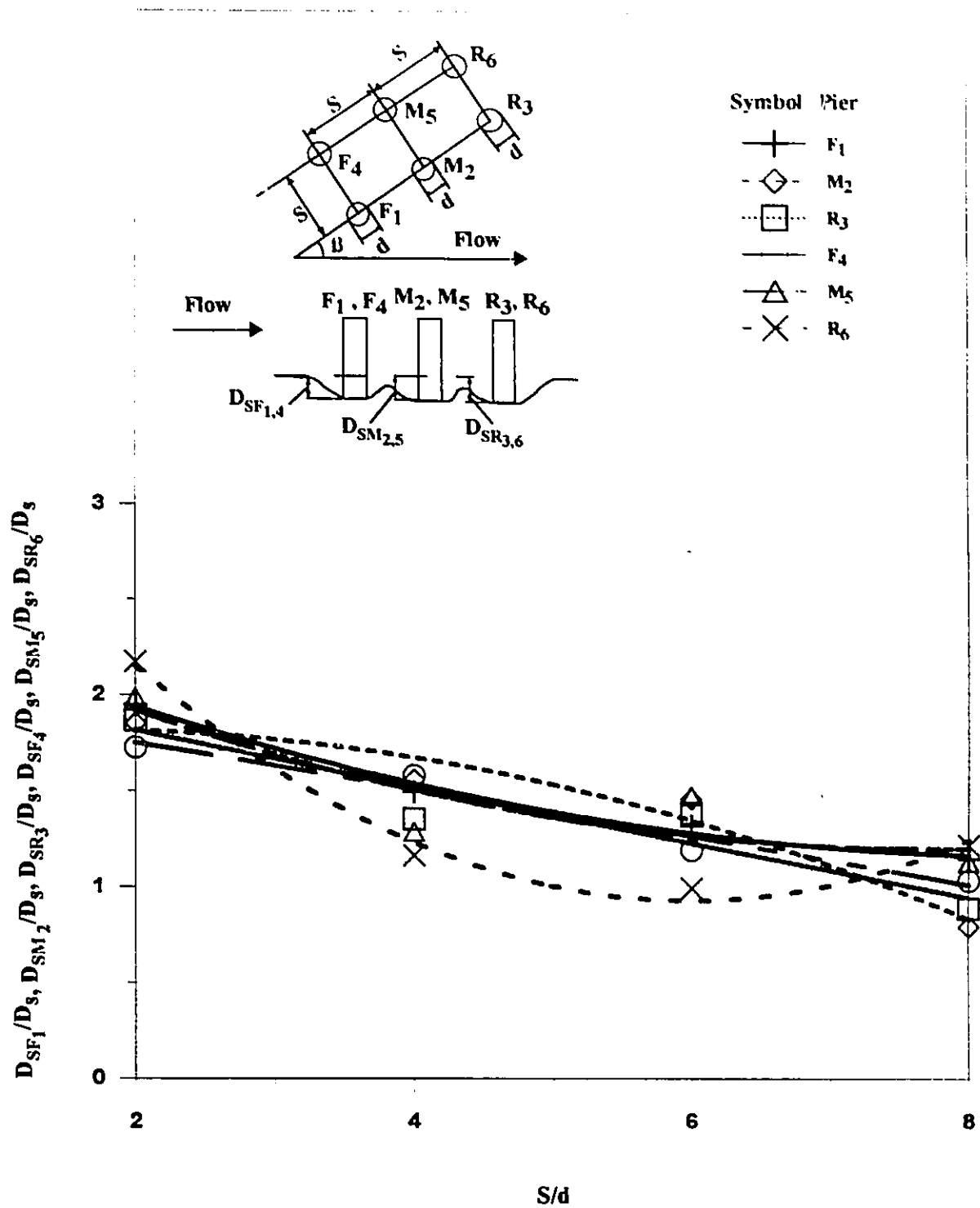


Fig. 5-5-13: Scour depth ratios versus pier spacing/pier diameter groups of 6 piers; ($\beta=45^\circ$)

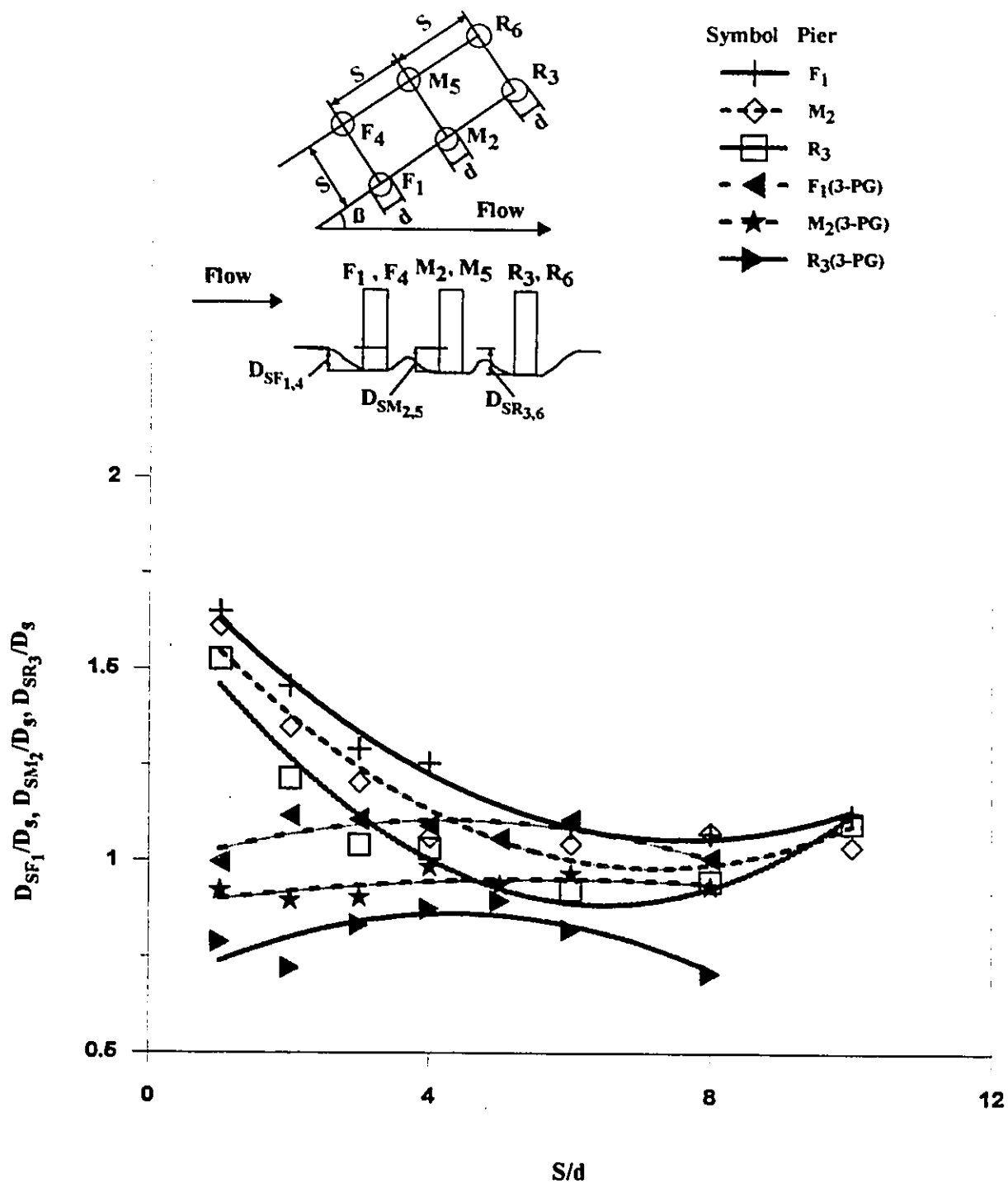


Fig. 5-5-14: Scour depth ratios versus pier spacing/pier diameter groups of 3 and 6 piers; ($\beta=0^\circ$)

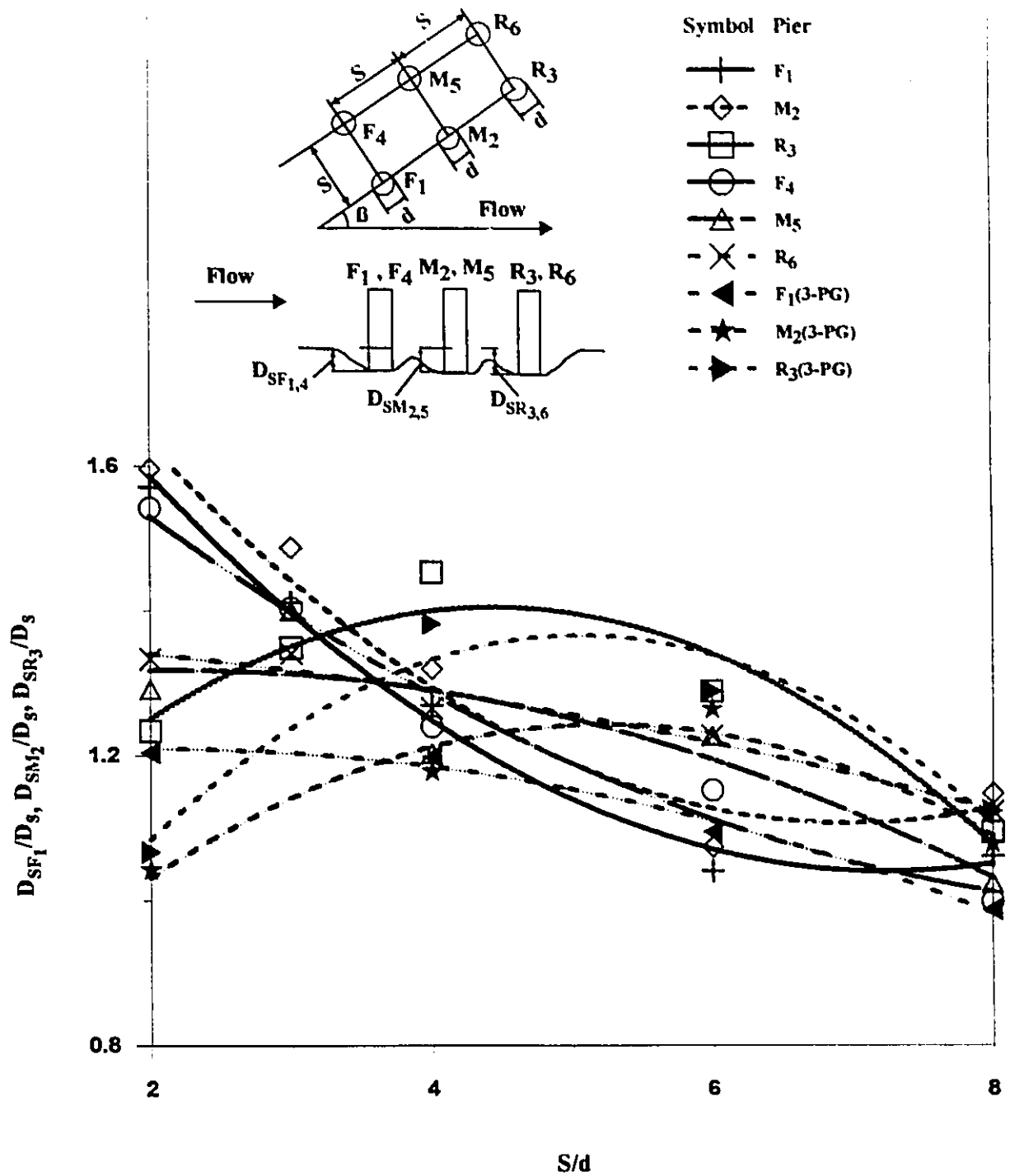


Fig. 5-5-15: Scour depth ratios versus pier spacing/pier diameter groups of 3 and 6 piers; ($\beta=10^\circ$)

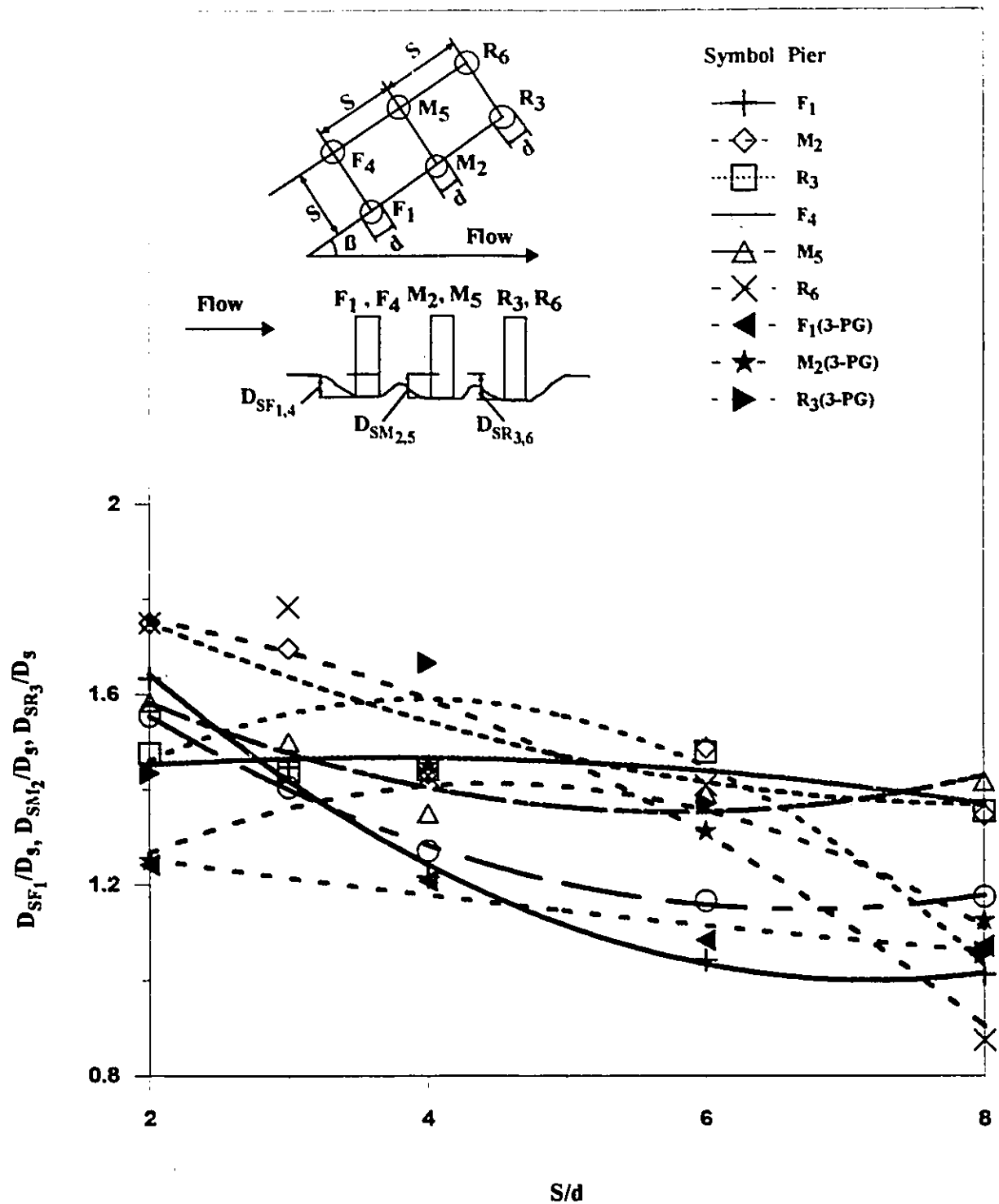


Fig. 5-5-16: Scour depth ratios versus pier spacing/pier diameter groups of 3 and 6 piers; ($\beta=20^\circ$)

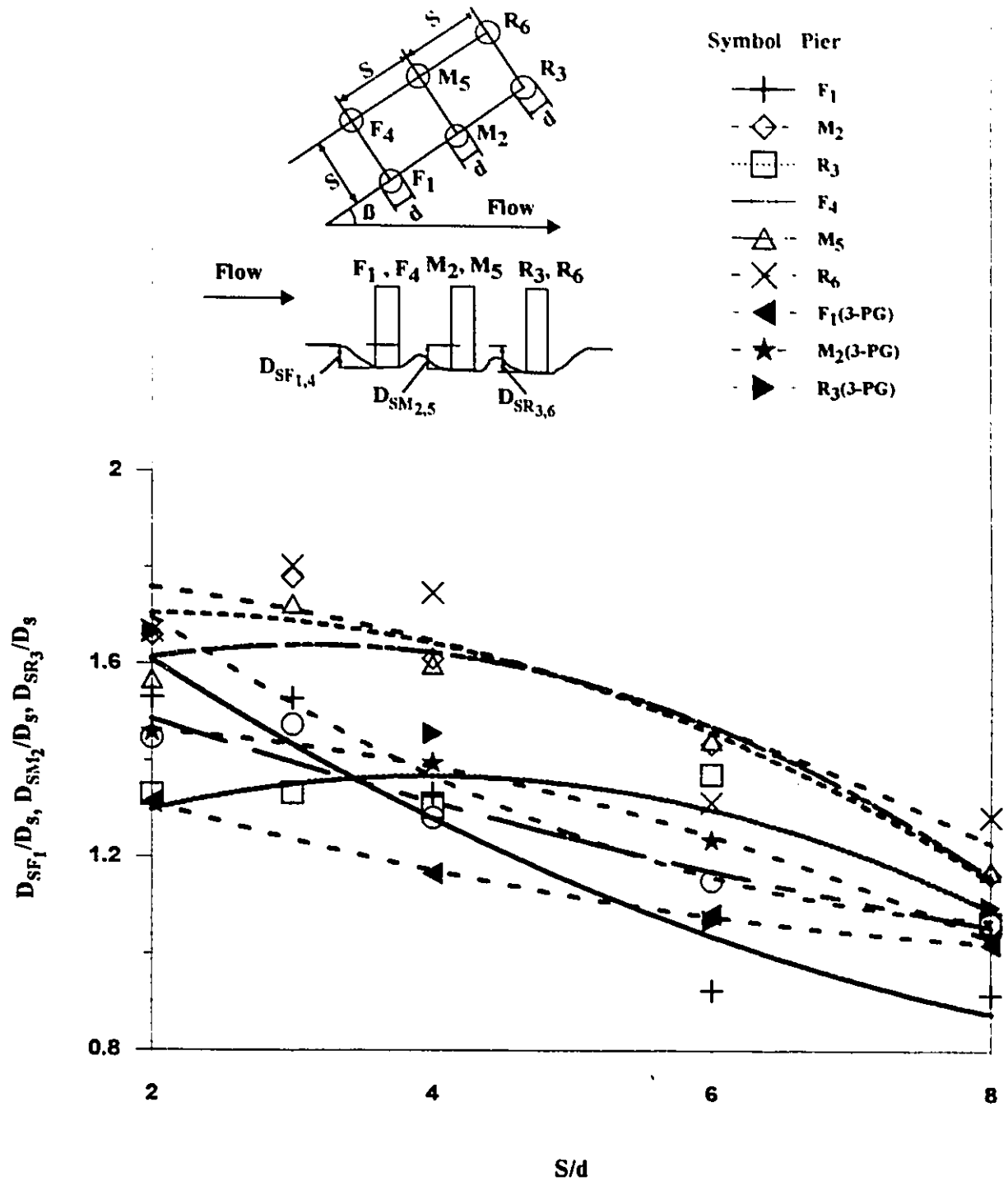


Fig. 5-5-17: Scour depth ratios versus pier spacing/pier diameter groups of 3 and 6 piers; ($\beta=30^\circ$)

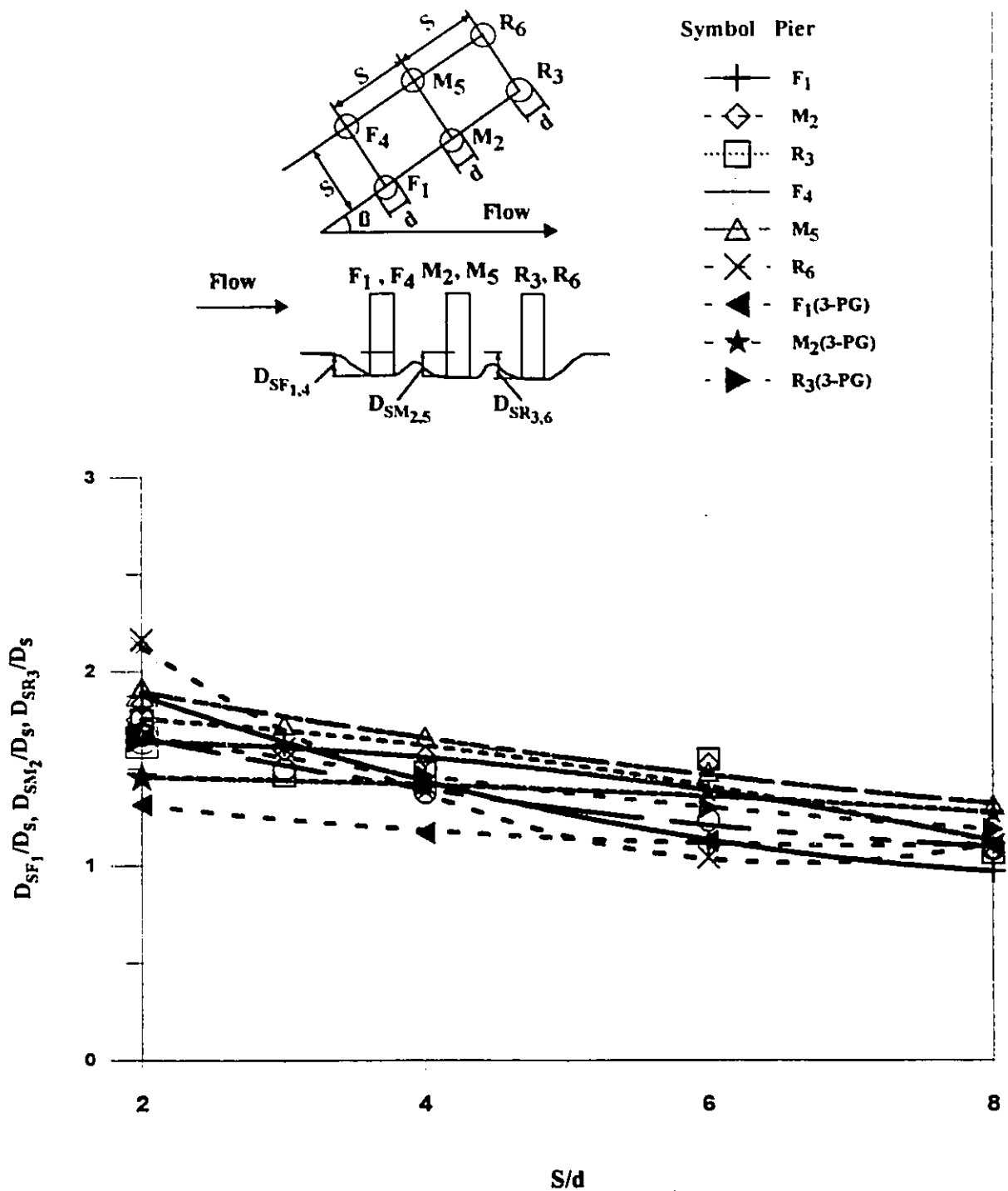


Fig. 5-5-18: Scour depth ratios versus pier spacing/pier diameter groups of 3 and 6 piers; ($\theta=40^\circ$)

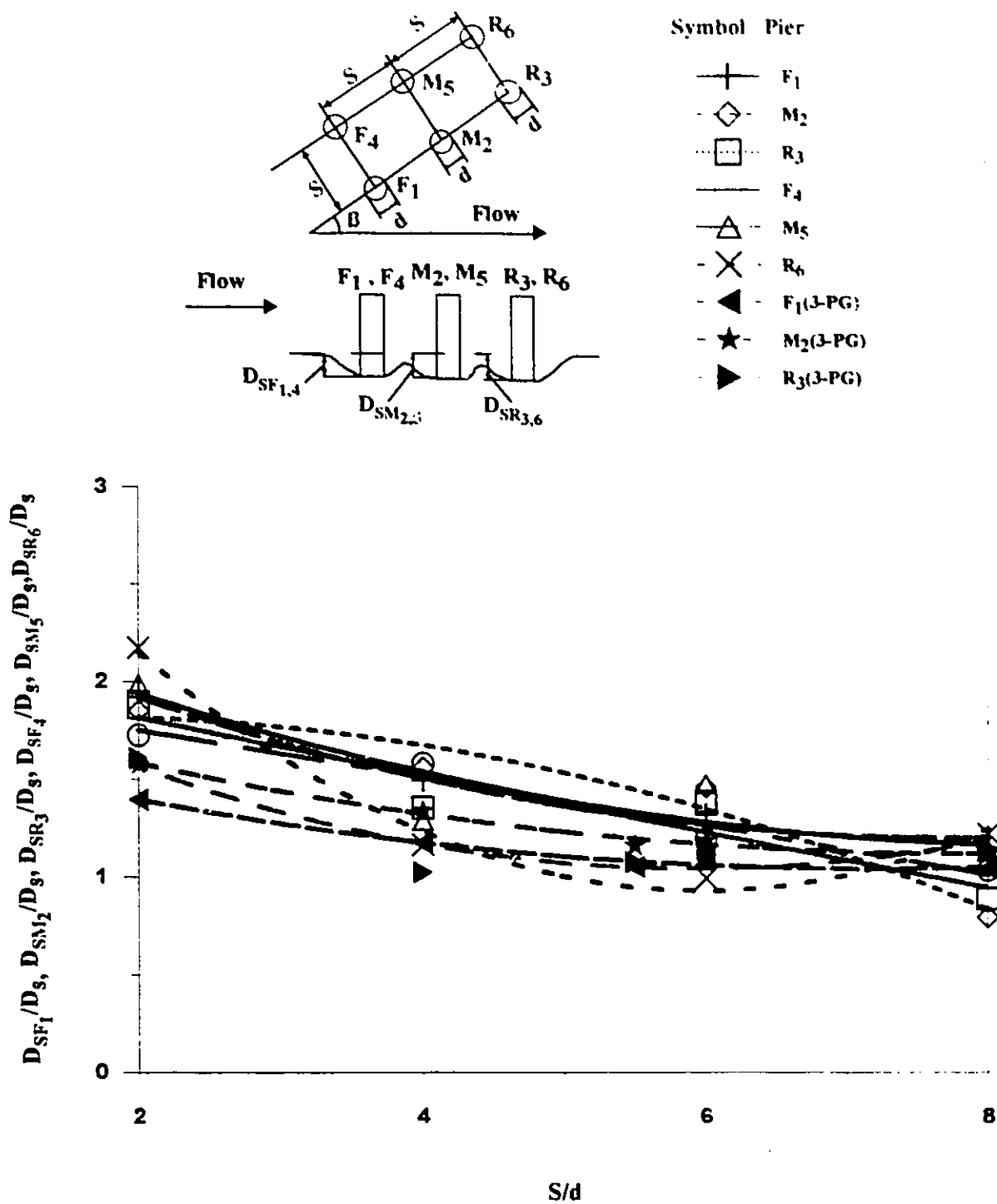


Fig. 5-5-19: Scour depth ratios versus pier spacing/pier diameter groups of 3 and 6 piers; ($\beta=45^\circ$)

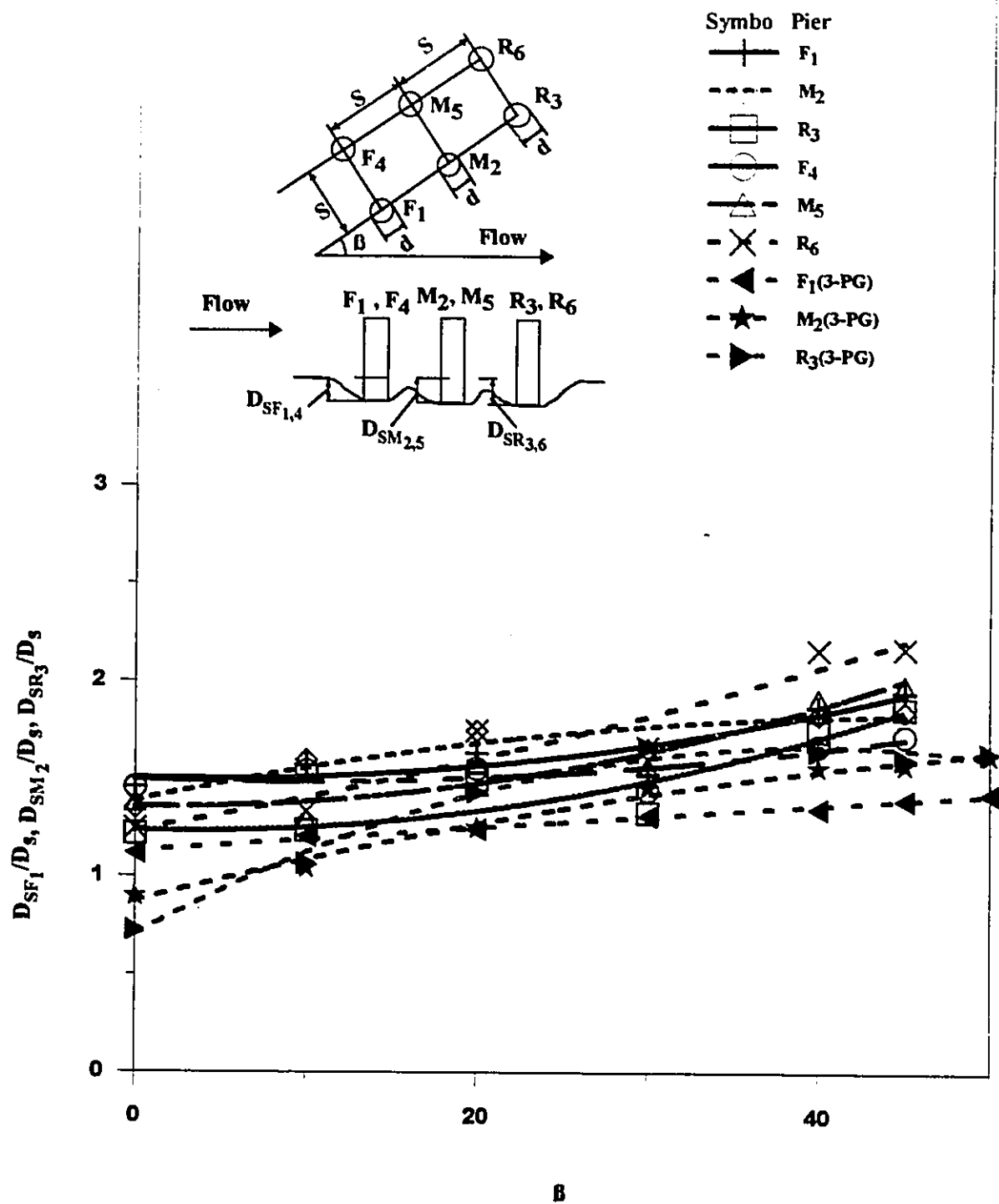


figure 5-5-20: Scour depth ratios versus flow angle of attack groups of 3 and 6 piers; ($S/d=2$)

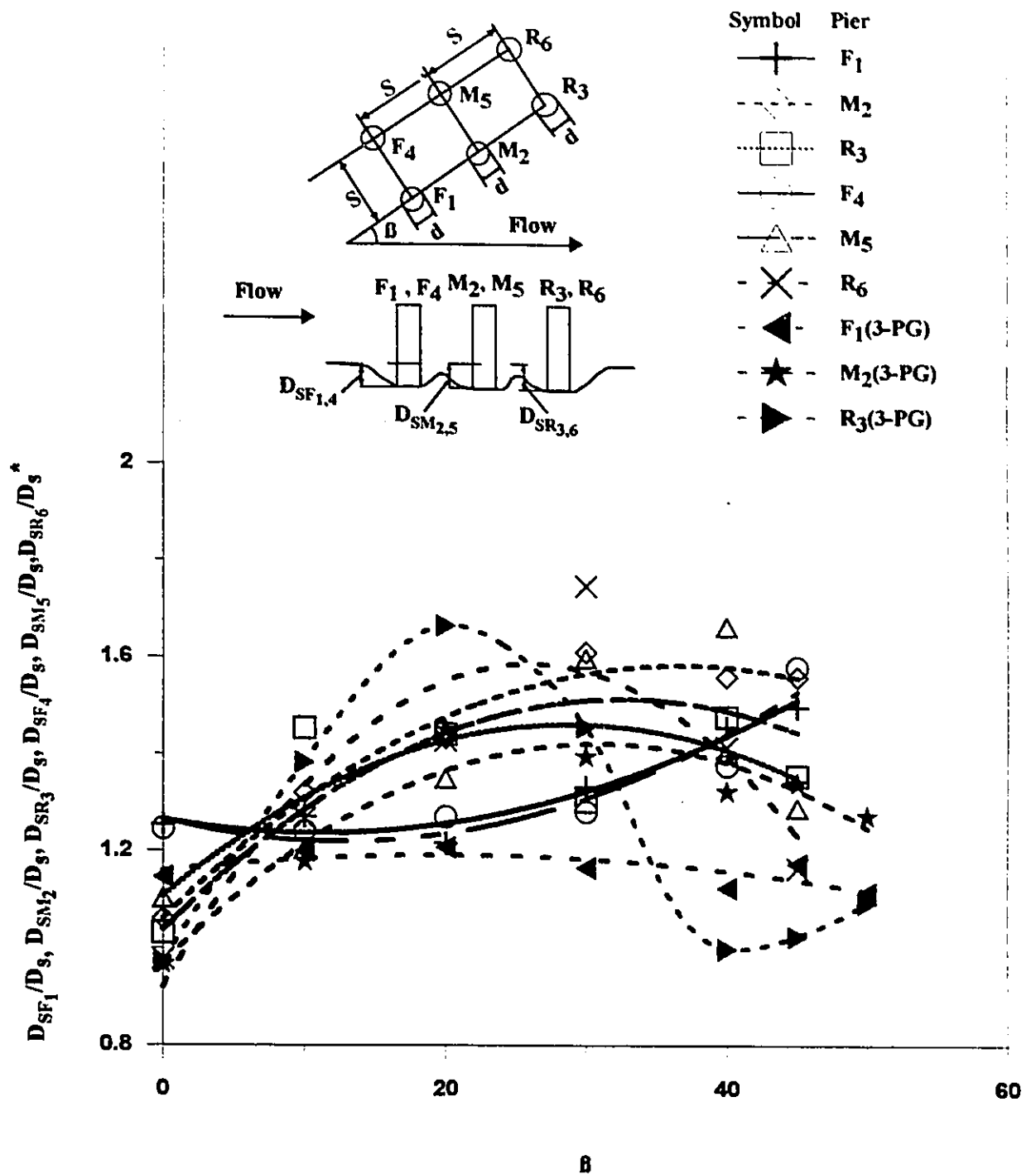


Fig. 5-5-21: Scour depth ratios versus flow angle of attack groups of 3 and 6 piers; ($S/d=4$)

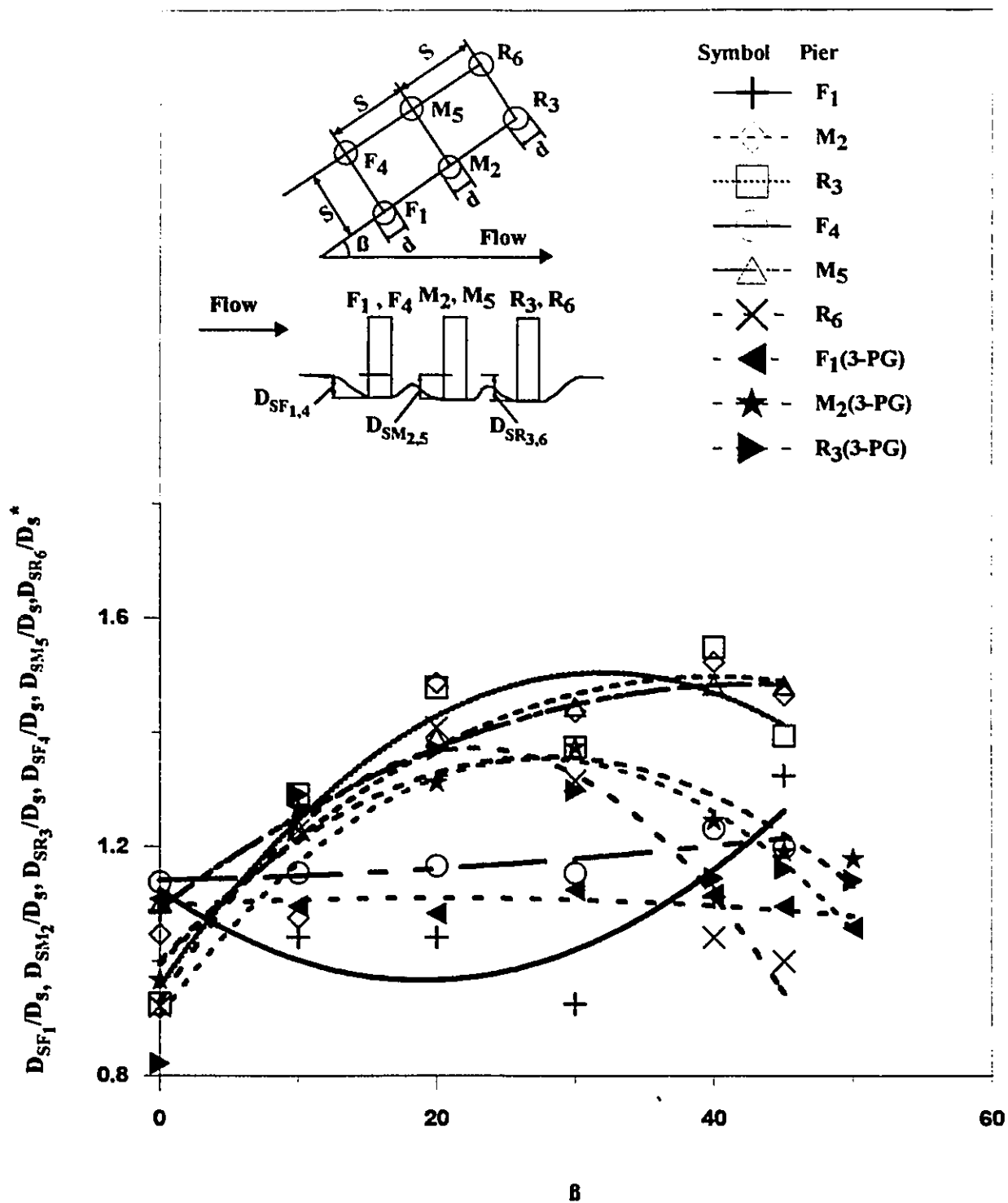


Fig. 5-5-22: Scour depth ratios versus flow angle of attack groups of 3 and 6 piers; ($S/d=6$)

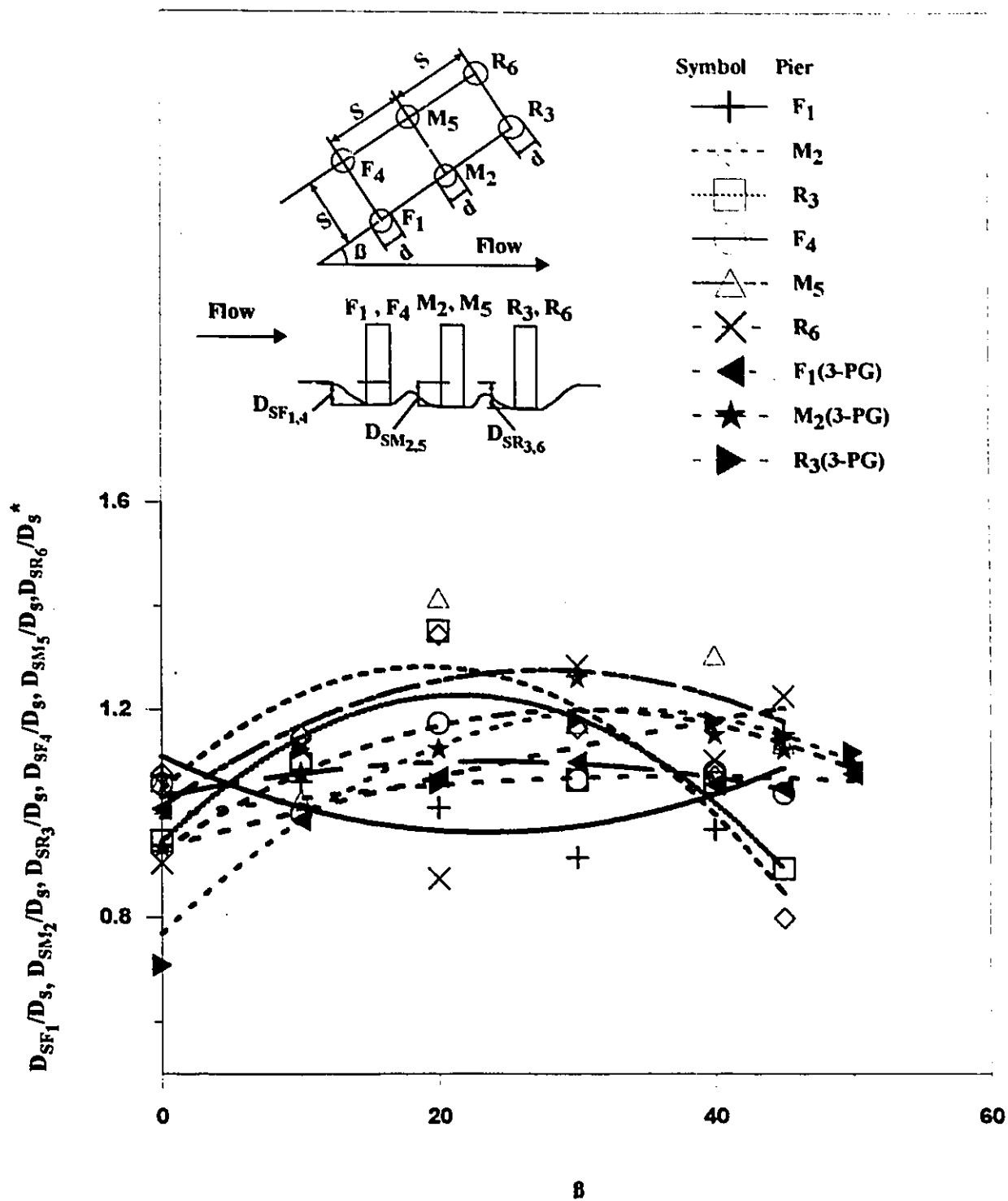


Fig. 5-5-23: Scour depth ratios versus flow angle of attack groups of 3 and 6 piers; ($S/d=8$)

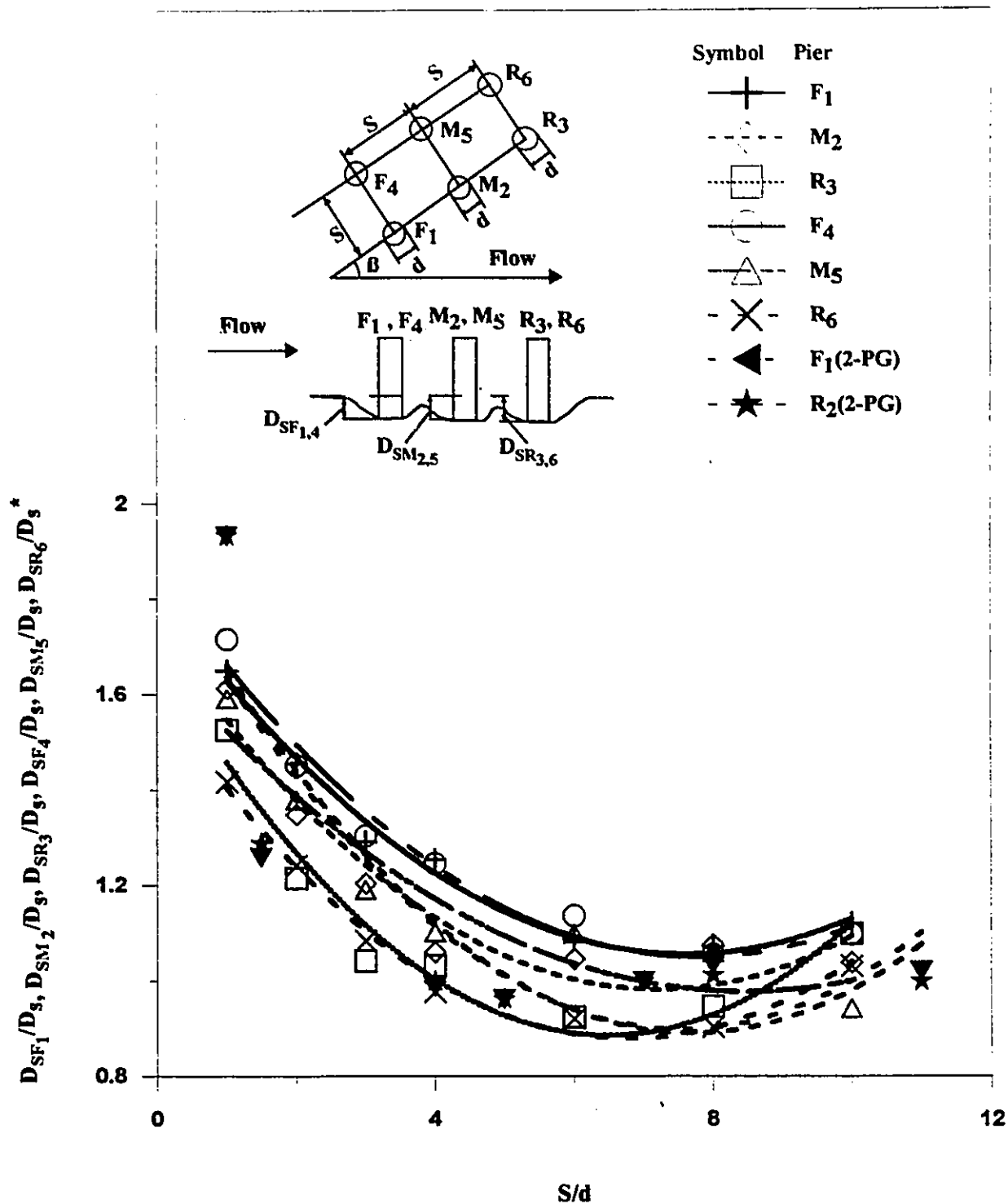


Fig. 5-5-24: Scour depth ratios versus pier spacing/pier diameter groups of 2 and 6 piers; ($\beta=90^\circ$ and 0° , respectively)

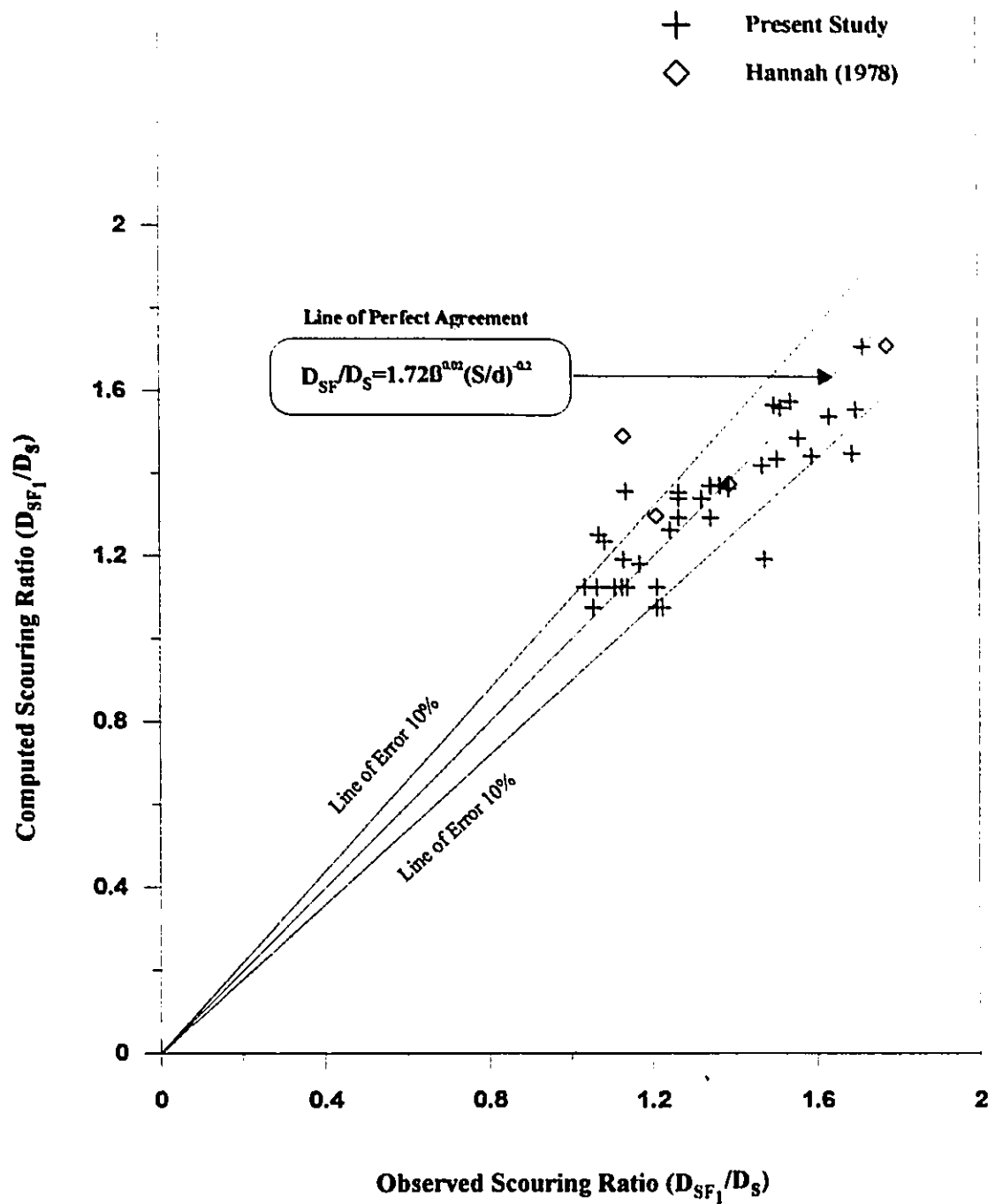
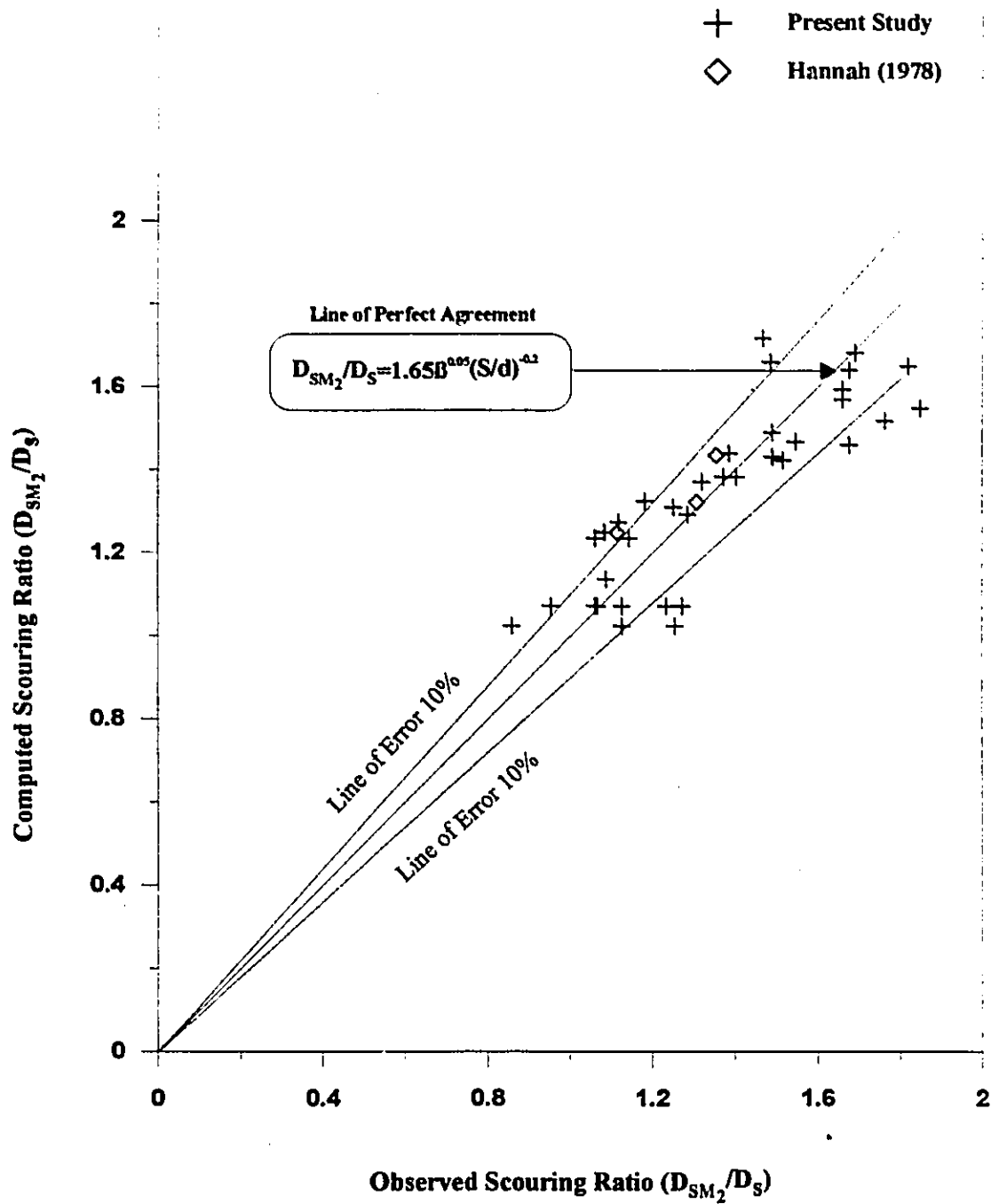


Fig. 5-5-25 : Comparison between computed and measured front pier (F_1) scouring ratios; groups of 6 piers



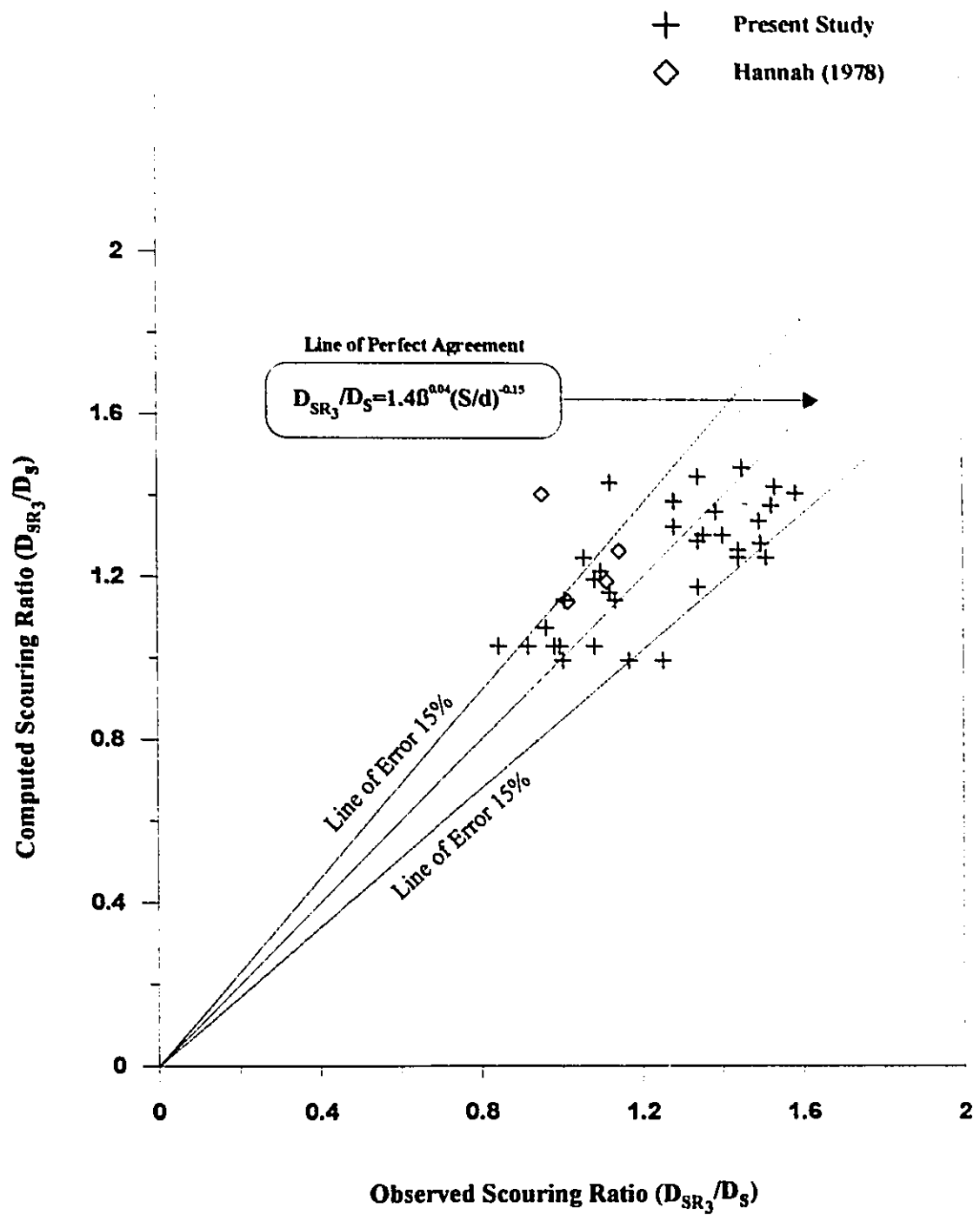


Fig. 5-5-27 : Comparison between computed and measured rear pier (R_3) scouring ratios; groups of 6 piers

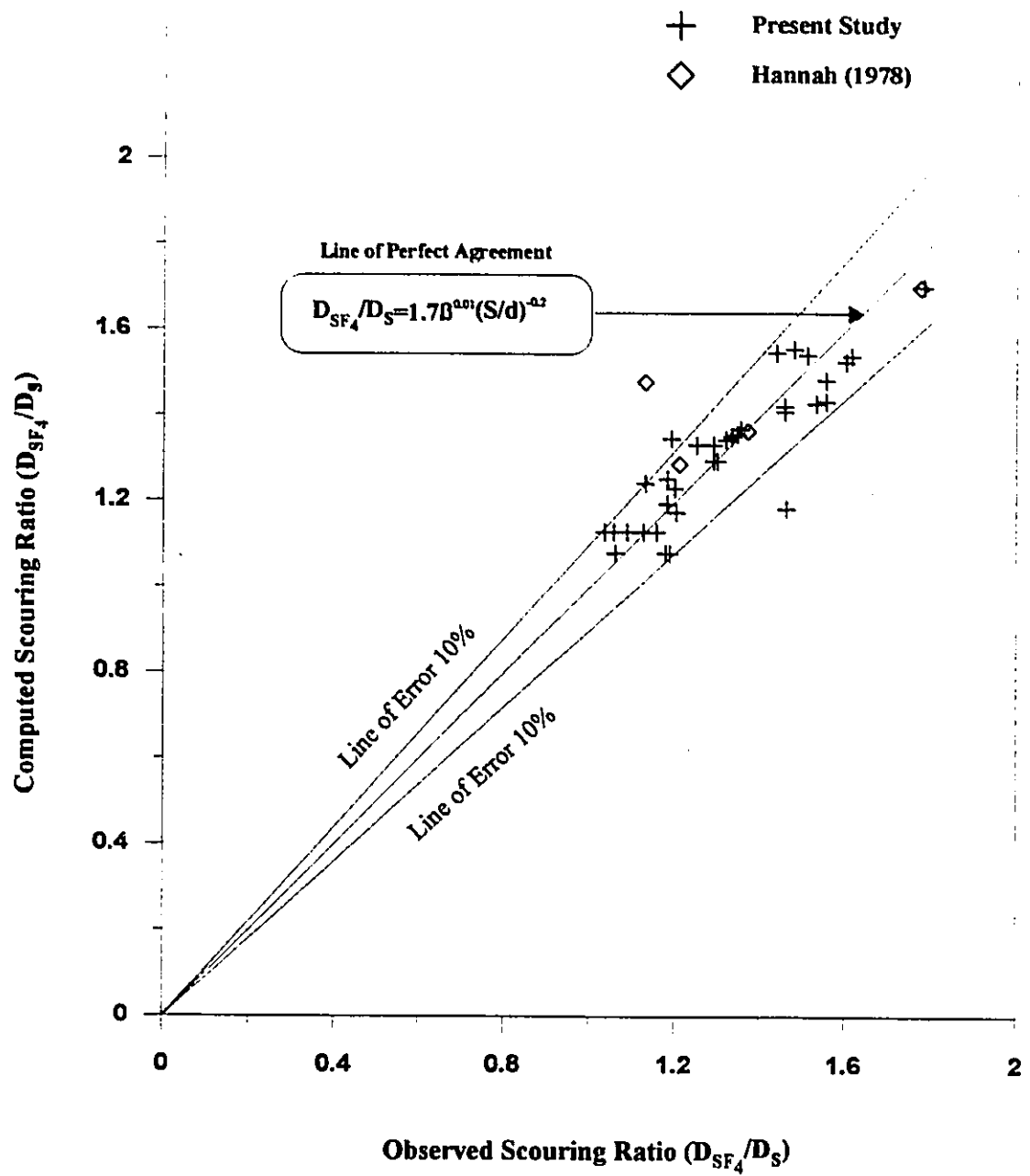


Fig. 5-5-28 : Comparison between computed and measured front pier (F_4) scouring ratios; groups of 6 piers

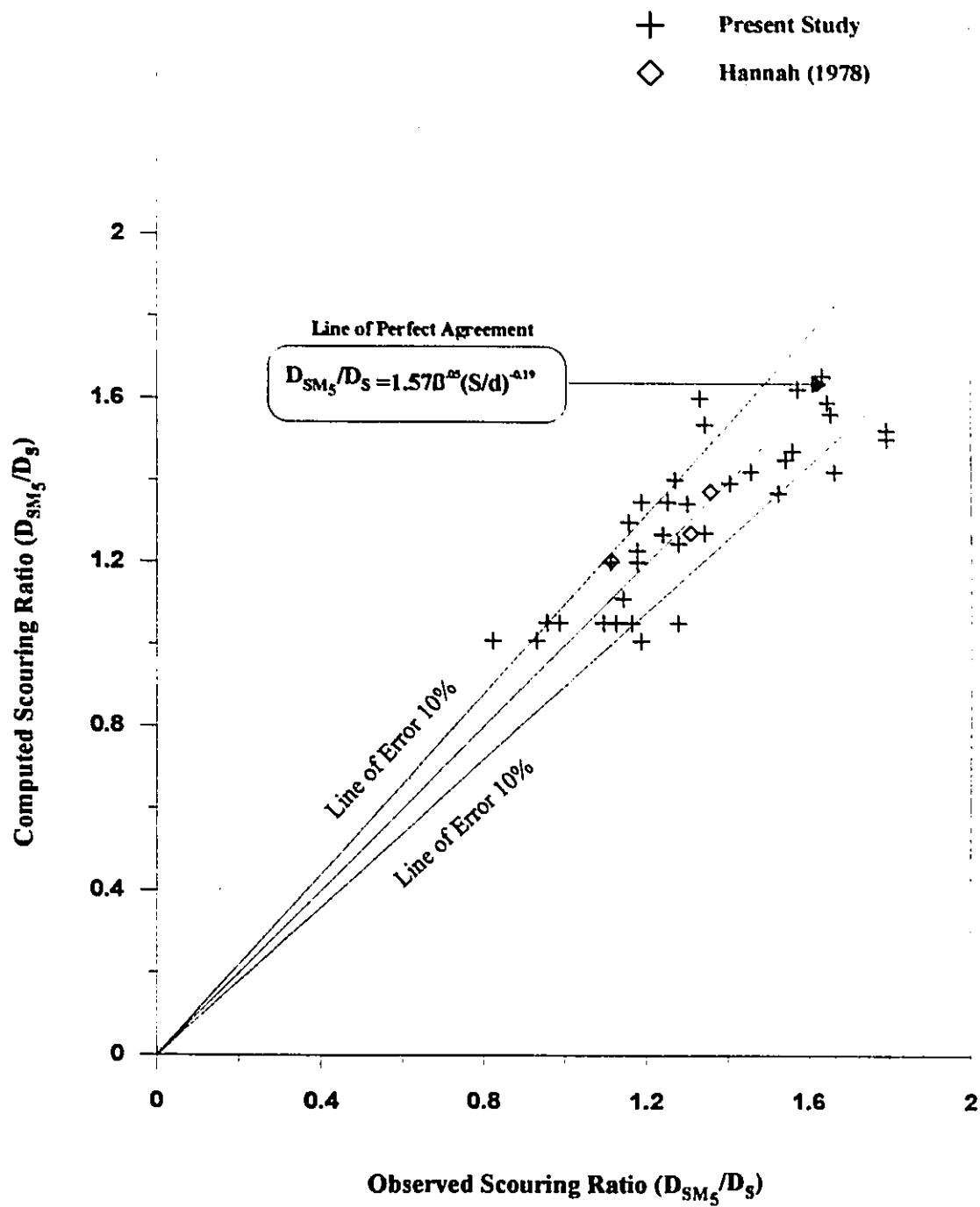


Fig. 5-5-29 : Comparison between computed and measured middle pier (M_5) scouring ratios; groups of 6 piers

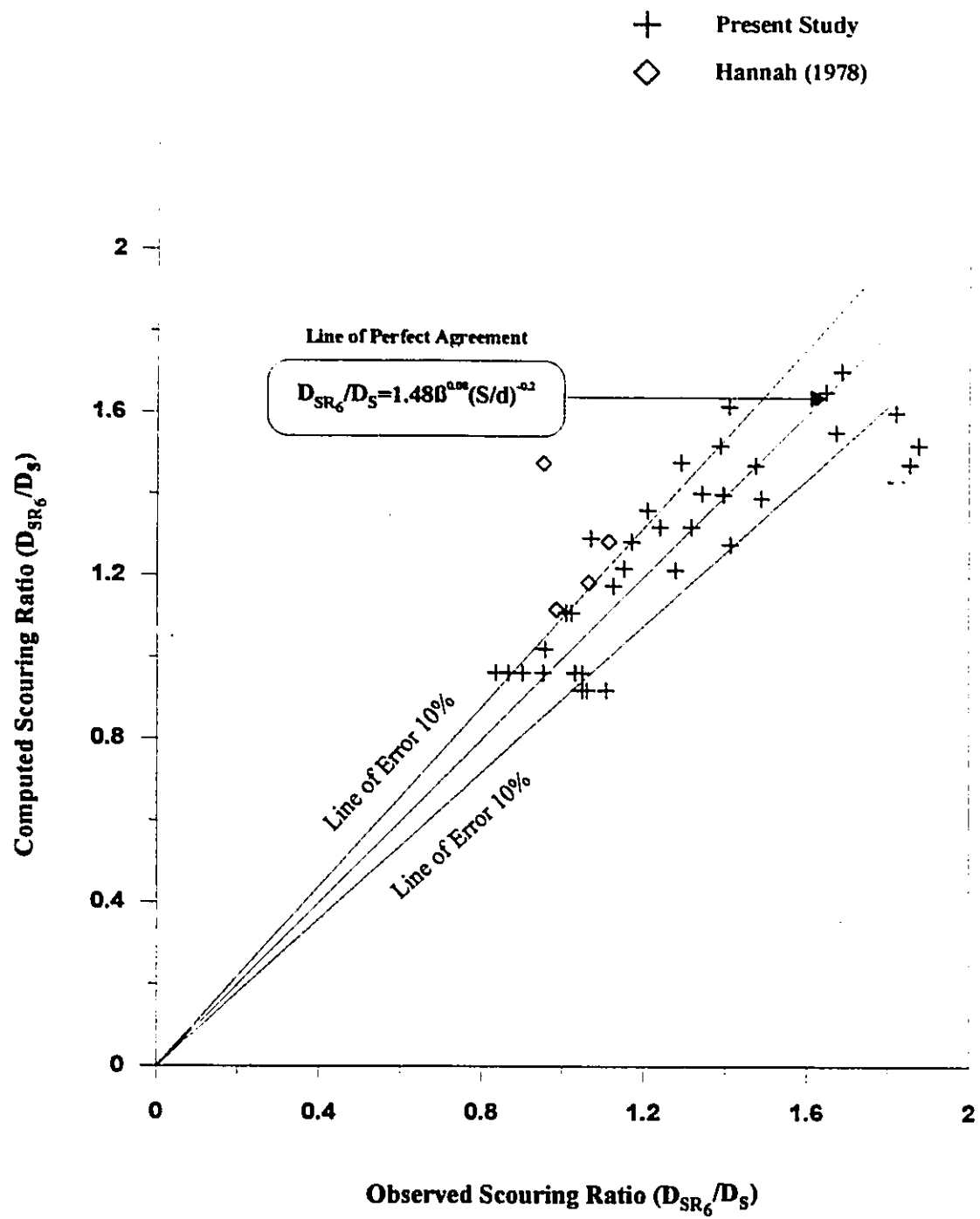


Fig. 5-5-30 : Comparison between computed and measured rear pier (R_6) scouring ratios; groups of 6 piers

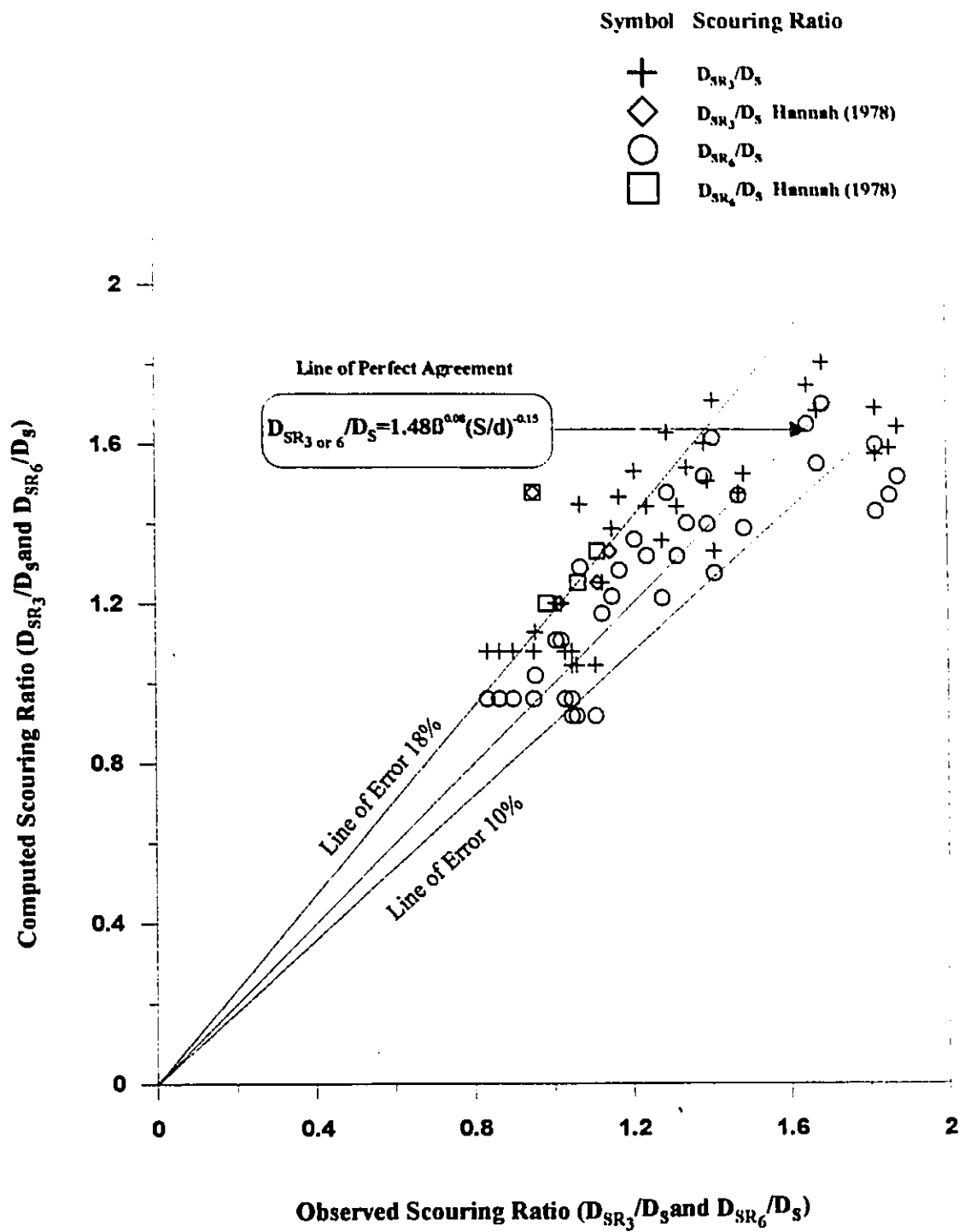


Fig. 5-5-31 : Comparison between computed and measured rear pier (R_3 and R_6) scouring ratios; groups of 6 piers

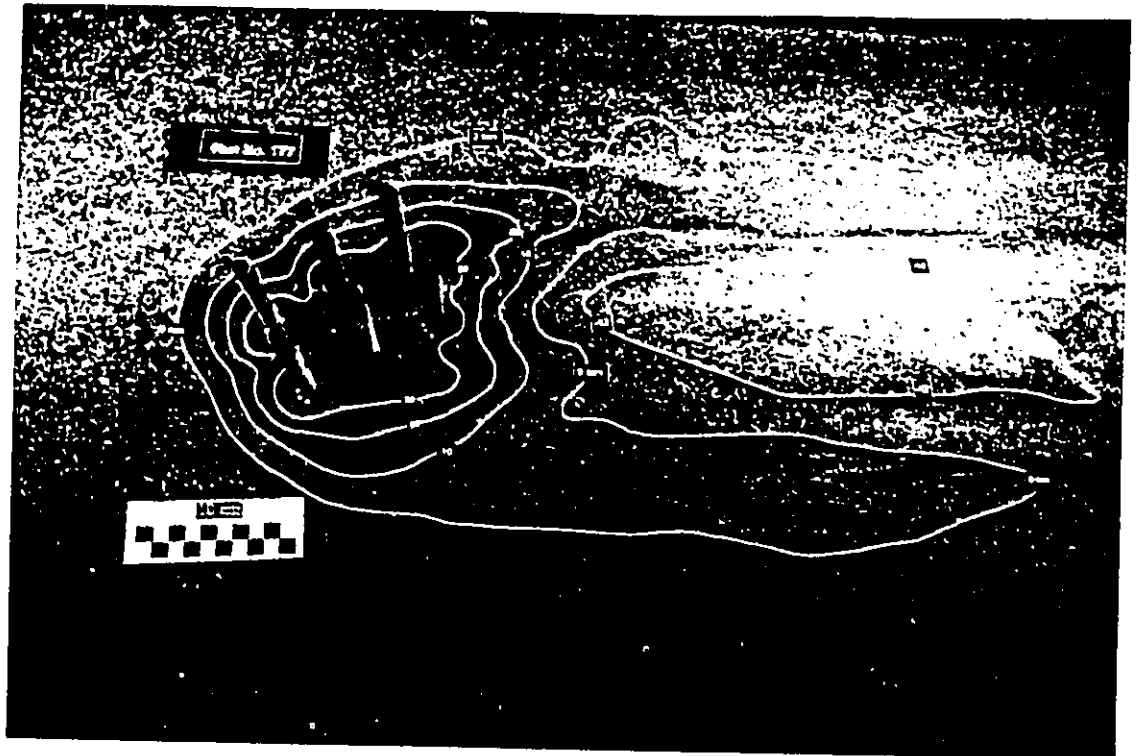


Plate 5-5-1: A group of 6 piers; $S/d=2$ and $\beta=30^\circ$

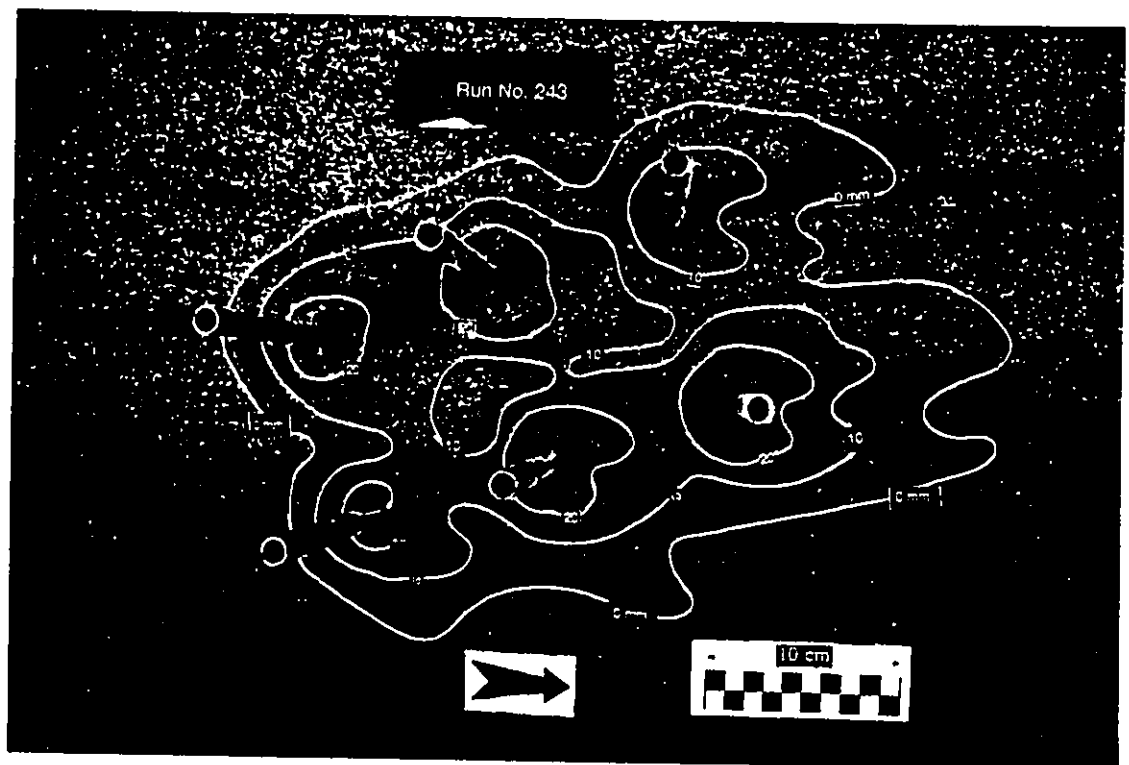


Plate 5-5-2: A group of 6 piers; $S/d=8$ and $\beta=20^\circ$



Plate 5-5-3: A group of 6 piers; $S/d=10$ and $\beta=0^\circ$

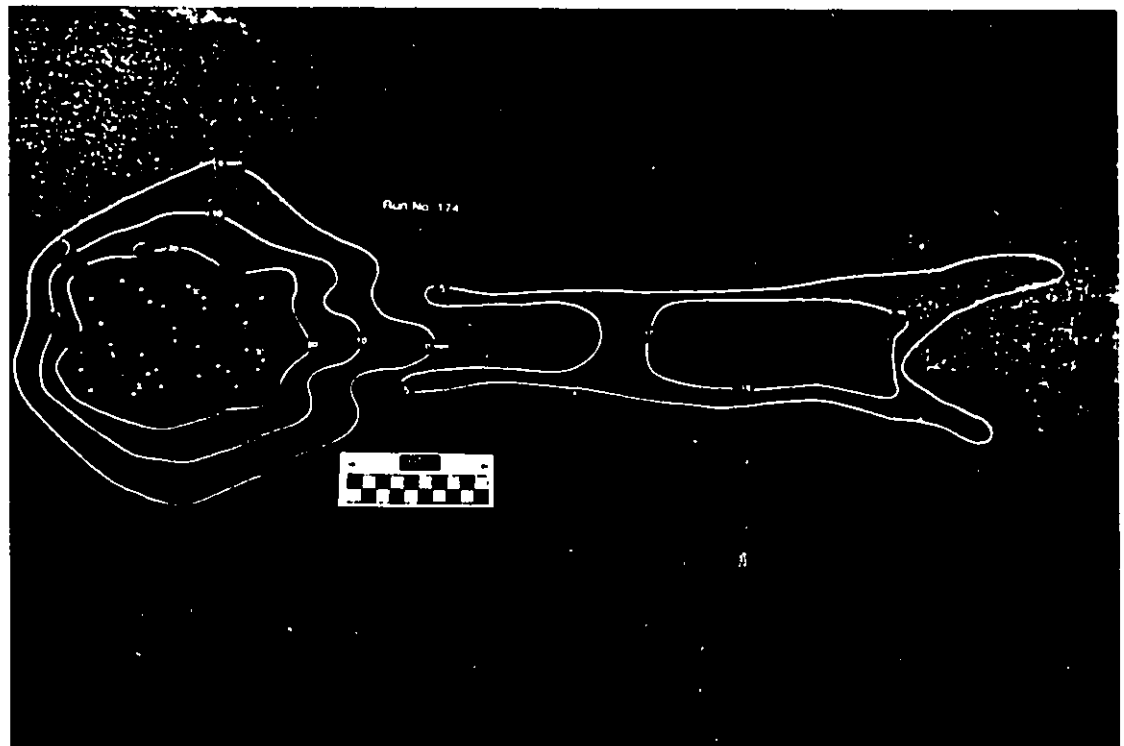


Plate 5-5-4: A group of 6 piers; $S/d=4$ and $\beta=0^\circ$



Plate 5-5-5: A group of 6 piers; $S/d=2$ and $\beta=40^\circ$

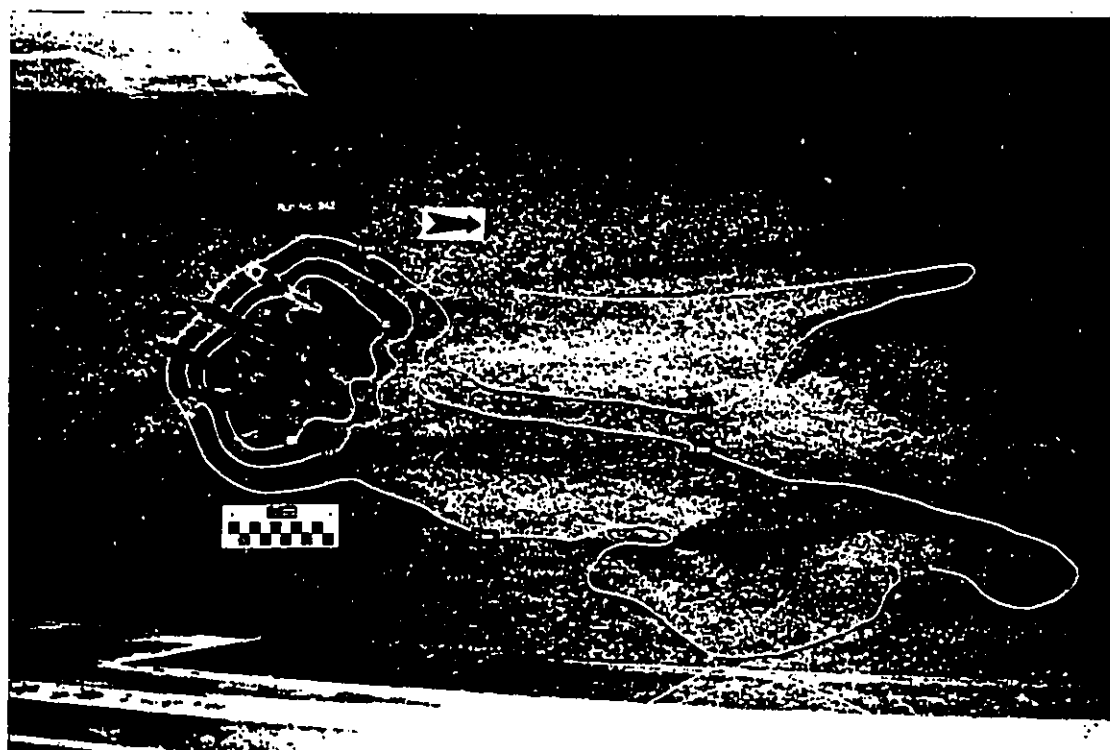


Plate 5-5-6: A group of 6 piers; $S/d=4$ and $\beta=40^\circ$

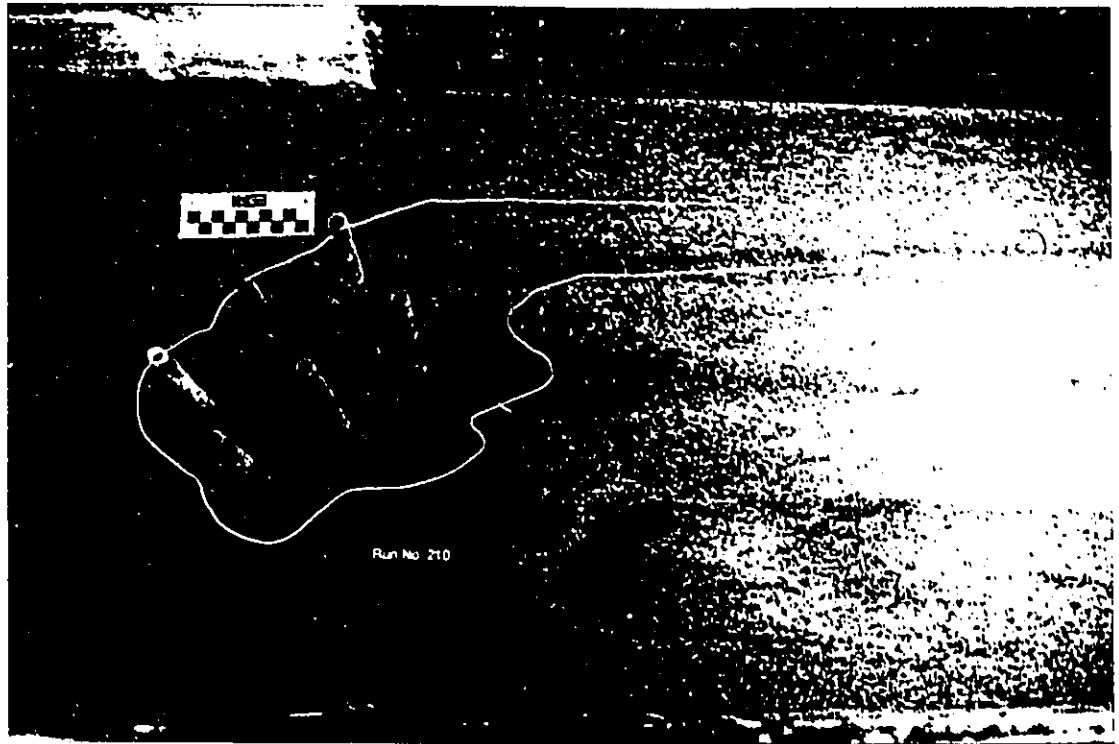


Plate 5-5-7: A group of 6 piers; $S/d=6$ and $\beta=40^\circ$

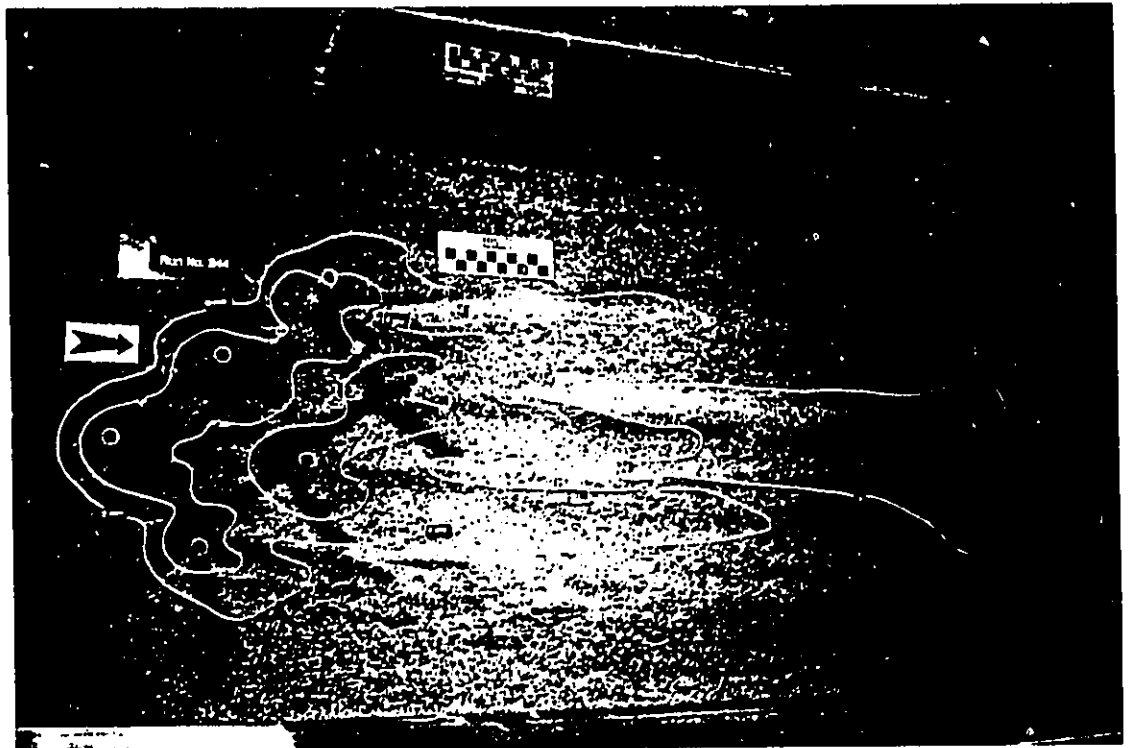


Plate 5-5-8: A group of 6 piers; $S/d=8$ and $\beta=40^\circ$

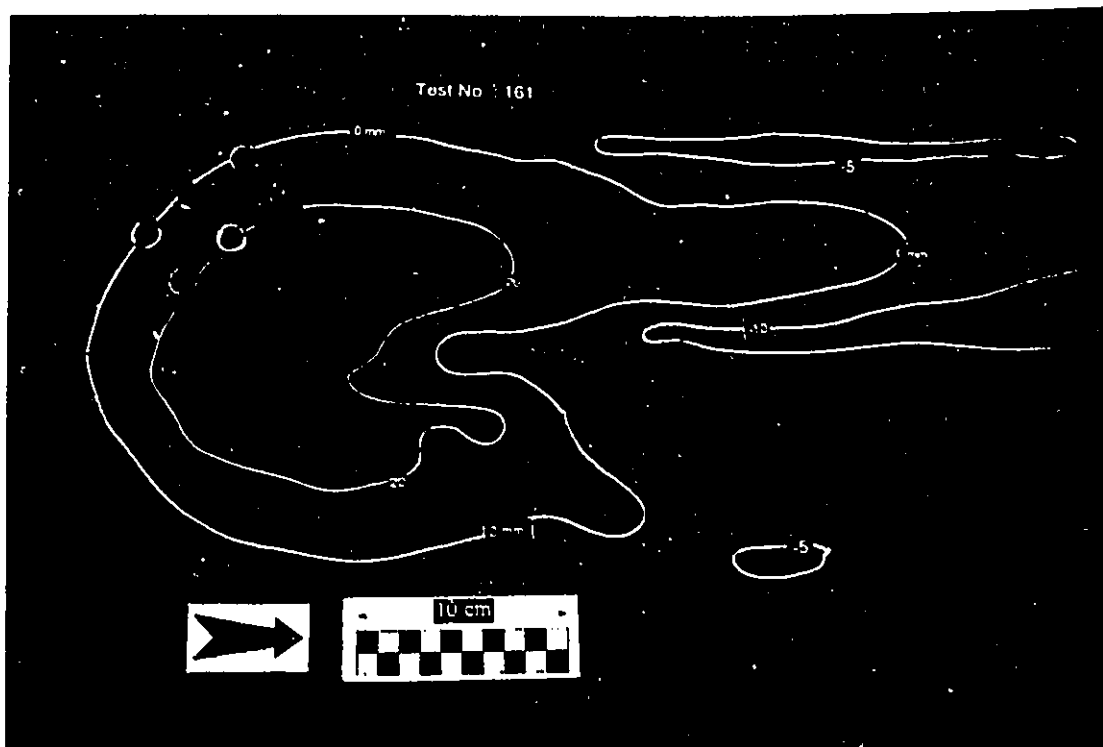


Plate 5-5-9: A group of 6 piers; $S/d=2$ and $\beta=45^\circ$

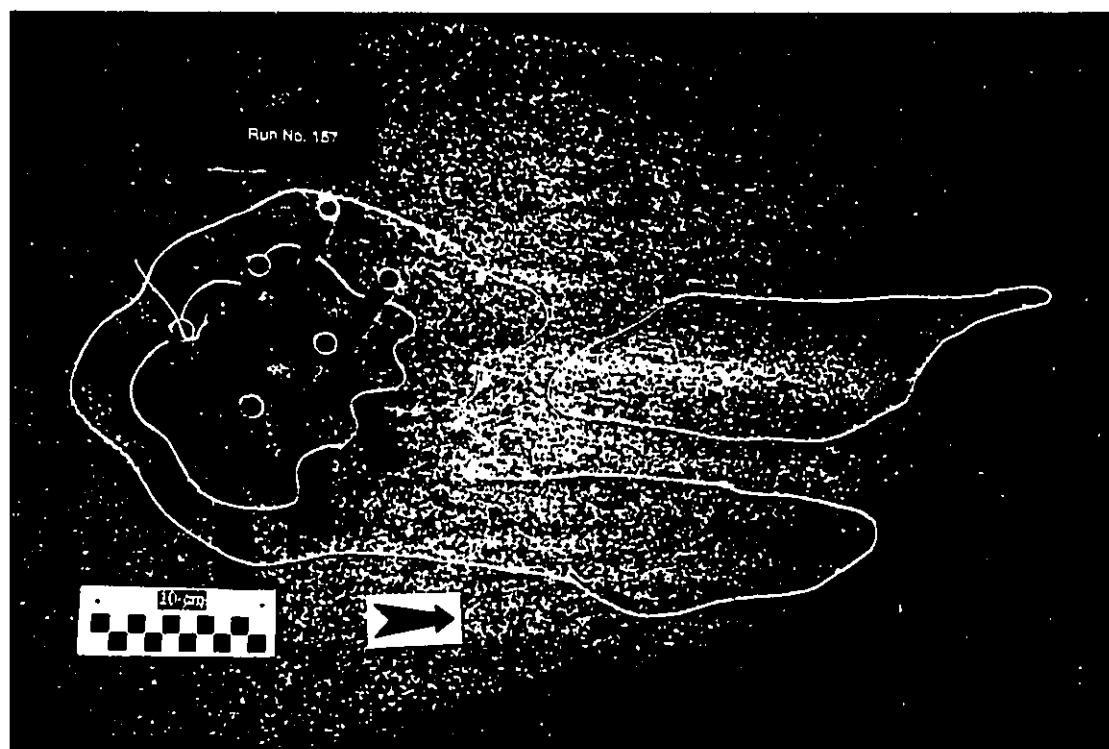


Plate 5-5-10: A group of 6 piers; $S/d=4$ and $\beta=45^\circ$

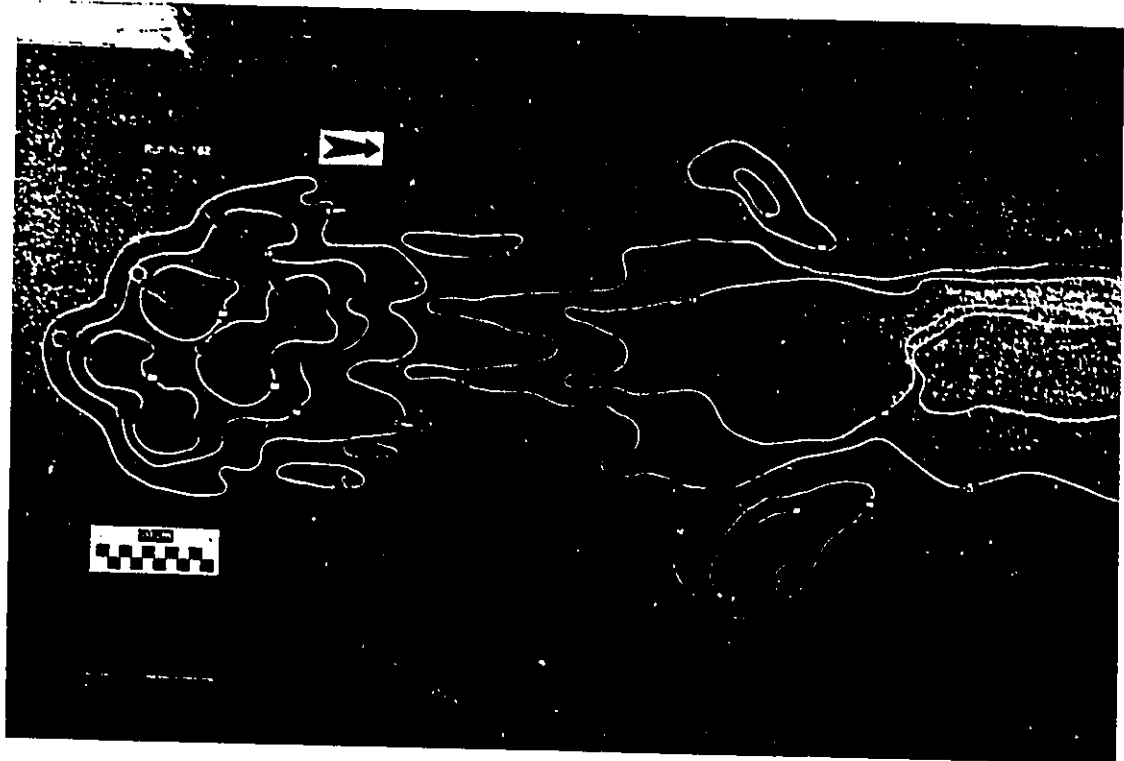


Plate 5-5-11: A group of 6 piers; $S/d=6$ and $\beta=45^\circ$

Table 5-5-1: Tests Performed for 6-PGs

β	S/d				
	2	3	4	6	8
0°	*	*	*	*	*
10°	*	*	*	*	*
20°	*	*	*	*	*
30°	*	*	*	*	*
40°	*	*	*	*	*
45°	*	*	*	*	*

* Test performed

Table 5-5-2: One-sided t-distribution for 6-PGs (28 degrees of freedom)

Confidence level (percent)	One-sided t-distribution (Statistics)
60.0	0.256
75.0	0.683
90.0	1.313
95.0	1.701
97.5	2.048
99.0	2.467
99.5	2.763

Chapter 6

Summary, Conclusions, and Recommendations for Future Research

6-1 Summary

All experiments, either for single or group piers were carried out for *clear-water* scour and *steady state* flow conditions. A few tests were carried out on single piers to discover which predicting equation most accurately estimated the local scour depth for the present study conditions.

Shen et al.'s (1966) expression 2-17, estimated the equilibrium local scour depth in front of the single model pier reasonably well (within a $\pm 5\%$ error margin).

The prime factors governing the local scouring process associated with groups of bridge piers are: **pier spacing, reinforcement, sheltering, and HSV compression**. The combined effects of two or more of these factors in a particular pier grouping arrangement is termed the **group effect**.

Previous laboratory studies on this topic, including those of Chabert and Engeldinger (1956), Dietz (1973), and Hannah (1978), have mainly considered 2-pier groups (2-PG's). Although Hannah (1978) also performed limited tests on groups of 3, 4, and 6 model bridge piers, he considered only approach flows that were parallel to the aligned rows of piers. In our research, the **group effect** was investigated for groups of 2, 3, 4, and 6 circular-shaped model bridge piers. The different flow angles of attack (β), defined the maximum local scour depth achieved under **clear-water-scour** conditions.

For the 2- and 3-PGs cases, for $S/d \leq 5$ and $\beta = 0$, **pier spacing, sheltering and reinforcement** effects were dominant factors in the local scouring process. With increasing β , the **sheltering and reinforcement** effects became weaker, however the **HSV compression** phenomenon increased with increasing β , and was significant at $\beta = 90^\circ$. For $S/d > 5$, the group effects diminished for the case $\beta = 0$.

In the case of 3-PGs, as β was varied, maximum D'_{F_1} occurred at $\beta = 40^\circ$ and 30° for $S/d = 2$ and 4 respectively. The maximum scouring for the rear pier (D'_{R_3}) occurred at $\beta = 30^\circ$ and 20° for $S/d = 2$ and 4 respectively. For $\beta > 20^\circ$, the downstream piers were fully exposed to the flow and,

in the absence of the **sheltering** effect by the front and middle piers, D'_{F_1} was found to be almost independent of β . In general, for the case of the 3-PG, the sheltering and reinforcement effects were observed to decrease with increasing S/d . Therefore, D'_{F_1} , D'_{M_2} , and D'_{R_3} approached unity with increasing S/d . The results show that for a 3-PG aligned to the flow ($\beta=0^\circ$) and different S/d , the local scour depth ratio is highest for the front pier and is lowest for the rear pier.

For the 4- and 6-PG cases, the **sheltering** effect diminished with increasing β . However, there is a particular range of β ($10^\circ \leq \beta \leq 30^\circ$) in which the maximum local scour depth associated with downstream piers reaches a maximum in the absence of the **sheltering** effect by upstream piers. The **HSV compression** which was significant in the case of 2- and 3-PGs for $\beta=90^\circ$, is a dominant factor in local scouring for 4- and 6-PGs for all β when $S/d \leq 4$. In the later cases, when two rows of piers come close together, the arms of HSV are intensified by the two adjacent (side) piers resulting in a deeper scour hole for the whole range of β .

When comparing the results of 3- and 6-PGs, using different S/d , it was observed that scour depths associated with a 6-PG were always greater than those of a 3-PG for $S/d < 8$. However, with increasing S/d , the scour depths associated with them approach the scour depth of a single pier. The presence of the second row of piers similar to the case of 4-PGs intensifies the arms of HSV resulting in a deeper scour hole around the piers in the 6-PG.

In all cases (2, 3, 4, and 6-PGs), the projected area is also an important parameter in determining the local scour depth when $\beta > 0^\circ$. The projected area of the piers increases with

increasing β resulting in a deeper scour hole. In addition, it was observed that the difference between the scour depth for a 6-PG and a 3-PG was insignificant in the range $0^\circ \leq \beta \leq 20^\circ$. The reason is that in the case of 6-PG, a larger projected area will be exposed to the flow because of the larger numbers of piers in a 6-PG than in a 3-PG.

Data obtained for each pier group were analysed using a **multiple regression model**. Using a logarithmic transformation of data, prediction equations were developed to find the normalized scour depths. Since behaviours of the front, middle, and rear piers were different from each other based on observed data, a different prediction equation was developed for each pier (Table 6-1). However, there was an insignificant difference between prediction equations for the rear piers, as was the case for the middle and front piers. Therefore, for the 4- and 6-PGs, 2 and 3 prediction equations were developed respectively. Moreover, it is more practical to use only two types of prediction equations: 1) for the front piers, 2) for the downstream piers (including middle and rear piers). A conservative prediction equation can be developed based on the largest values of a , b , and c among all prediction equations ($\frac{D_{sg}}{D_s} = a(\beta)^b \left(\frac{S}{d} \right)^c$), where D_{sg} = scour depth around any pier in a group. In addition, the effect of β on local scour depth of front piers is negligible and therefore can be eliminated from the relation. Accordingly, the following equations can be applied for the front (D_{SF}) and for the downstream piers (D_{SR}), respectively:

$$\frac{D_{SF}}{D_s} = 1.9 \left(\frac{S}{d} \right)^{-0.1} \quad (6-1)$$

and

$$\frac{D_{SR}}{D_s} = 1.65(\beta)^{0.13} \left(\frac{S}{d} \right)^{-0.07} \quad (6-2)$$

Note that a logarithmic transformation of data was employed in the regression model, thus, β cannot be zero. For piers aligned to the flow, β must be assumed to be 1 which is practically zero.

6-2 Main Conclusions

Based on results obtained in this study, using different β and S/d , the following conclusions can be drawn:

- i) $\beta > 10^0$ should be avoided. The scour depth for the downstream piers increases in this case as the sheltering effect diminishes.
- ii) To prevent severe local scouring, as a result of compressed HSV and/or reinforcement effects, $S/d < 8$ should be avoided.

However, if these two cases cannot be avoided, the local scour depth estimated for a single bridge pier should be increased by 70% for design purposes. Also, for the above cases a more accurate estimation of local scour depth can be obtained through equations 6-1 and 6-2 for the front and downstream piers, respectively. Finally, one can always reduce local scouring around the bridge piers by using various methods, such as riprap, collars, and grout mats (see the introduction section of chapter 1).

6-3 Future research

Future research into the local scouring process associated with bridge pier groups should include:

1) *Laboratory investigations*

i) The laboratory investigations performed in this research should be repeated *for live-bed scour* conditions. [This would require a modified test facility containing an ancillary *sediment supply* device to feed sediments into the flume test section at rates equal to the general scouring rates].

ii) Similar investigations should be performed using non-uniform sediment test beds to investigate the effect of *armouring* on the local scouring process for pier groups.

iii) In many instances the cross sections of rivers containing groups of bridge piers are several compound in shape. Further research is required to investigate the *related lateral momentum transfer* effect on the local scouring process.

iv) While it would introduce an additional level of complexity to the problem, the local scouring process should be investigated under unsteady flow conditions to study the *temporal* effects associated with the progression of flood waves.

v) Pier groups located in small bridge openings tend to collect debris, which reduces the conveyance area. Future research should investigate the impact of debris lodgings on the local scouring process.

2) Field data

The large amount of data accumulated from laboratory studies of local scour at a pier must

be correlated with data obtained from case histories studied in the field. However, a number of problems are inherent in the use of field data. In most instances, a greater number of parameters probably influence the development of local scour in the field compared to the controlled conditions of the laboratory. Care must be taken in assigning scour depth to the influence of the relevant parameters. It is often difficult to discern the influence of certain parameters from others. Scour depths recorded for pier failures in the field are useful only insofar as they give the order of magnitude of the scour depth, and represent the depth of scour at which the pier failed rather than the maximum depth of scour which could develop.

One might argue that field data, collected for bridge scour, will generate more questions than it will answer. Field data frequently lends itself to subjective interpretation. Just as designers have questions about combining the different scour components, researchers question about how to separate field measurements into components, so that data can be generalized to have transferability to other sites.

Table 6-1: Local scour depth prediction equations

Pier group type	Prediction equations for		
	Front pier	Middle pier	Rear pier
2-PG	$\frac{D_{SF_1}}{D_s} = 1.25\beta^{-0.01} \left(\frac{S}{d} \right)^{-0.1}$		$\frac{D_{SR_2}}{D_s} = 0.92\beta^{0.08} \left(\frac{S}{d} \right)^{-0.1}$
3-PG	$\frac{D_{SF_1}}{D_s} = 1.21\beta^{0.03} \left(\frac{S}{d} \right)^{-0.09}$	$\frac{D_{SM_2}}{D_s} = 1.04\beta^{0.08} \left(\frac{S}{d} \right)^{-0.07}$	$\frac{D_{SR_2}}{D_s} = 0.94\beta^{0.13} \left(\frac{S}{d} \right)^{-0.09}$
4-PG	$\frac{D_{SF_1}}{D_s} = 1.88\beta^{0.01} \left(\frac{S}{d} \right)^{-0.32}$		$\frac{D_{SR_2}}{D_s} = 1.46\beta^{0.08} \left(\frac{S}{d} \right)^{-0.23}$
	$\frac{D_{SF_2}}{D_s} = 1.72\beta^{0.01} \left(\frac{S}{d} \right)^{-0.24}$		$\frac{D_{SR_4}}{D_s} = 1.45\beta^{0.08} \left(\frac{S}{d} \right)^{-0.23}$
6-PG	$\frac{D_{SF_1}}{D_s} = 1.72\beta^{0.02} \left(\frac{S}{d} \right)^{-0.2}$	$\frac{D_{SM_2}}{D_s} = 1.65\beta^{0.08} \left(\frac{S}{d} \right)^{-0.2}$	$\frac{D_{SR_3}}{D_s} = 1.4\beta^{0.08} \left(\frac{S}{d} \right)^{-0.15}$
	$\frac{D_{SF_2}}{D_s} = 1.7\beta^{0.01} \left(\frac{S}{d} \right)^{-0.2}$	$\frac{D_{SM_2}}{D_s} = 1.57\beta^{0.08} \left(\frac{S}{d} \right)^{-0.19}$	$\frac{D_{SR_4}}{D_s} = 1.48\beta^{0.08} \left(\frac{S}{d} \right)^{-0.2}$

Chapter 7

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Appendix:

Multiple Linear Regression models

A regression model that involves more than one regressor variable is called a *multiple regression* model. A multiple regression model may be expressed as:

$$y = \alpha_0 + \alpha_1 x_1 + \dots + \alpha_k x_k + \epsilon \quad (1)$$

The above relation is called a multiple linear regression model with k regressors. The parameters α_j , $j=0, 1, \dots, k$ are called the regression coefficients. The parameter α_j represents the expected change in the response y per unit change in x_j ($i \neq j$) are held constant. Multiple linear regression models are often used as approximating functions. That is, the true functional relationship between y and $x_1, x_2, \dots, x_k$ is unknown, but over certain ranges of the regressor variables the linear regression model is an adequate approximation.

Estimation of the Model Parameters

Least Squares Estimation of the Regression Coefficients

The method of least squares is used to estimate the regression coefficients in equation 1. Suppose that $n > k$ observations are available, and let y_i denote the i th observed response and x_{ij} denote the i th observation or level of regressor x_j . The data will appear as in Table A. We assume that the error term ϵ in the model has $E(\epsilon)=0$, $V(\epsilon)=\sigma^2$, and that the errors are uncorrelated. Thus, the sample model corresponding to equation 1 can be expressed as follows:

$$\begin{aligned}
y_o &= \alpha_0 + \alpha_1 x_{1i} + \alpha_2 x_{2i} + \dots + \alpha_k x_{ki} + \epsilon_i \\
&= \alpha_0 + \sum_{j=1}^k \alpha_j x_{ij} + \epsilon_i, \quad i=1,2,\dots,n
\end{aligned} \tag{2}$$

Hence, the least squares function is:

$$\begin{aligned}
S(\alpha_0, \alpha_1, \dots, \alpha_k) &= \sum_{i=1}^n \epsilon_i^2 \\
&= \sum_{i=1}^n \left(y_i - \alpha_0 - \sum_{j=1}^k \alpha_j x_{ij} \right)^2
\end{aligned} \tag{3}$$

The function S is to be minimized with respect to $\alpha_0, \alpha_1, \dots, \alpha_k$. The least squares estimators of $\alpha_0, \alpha_1, \dots, \alpha_k$ must satisfy

$$\frac{\partial S}{\partial \alpha_0} \Big|_{\hat{\alpha}_0, \hat{\alpha}_1, \dots, \hat{\alpha}_k} = -2 \sum_{i=1}^n \left(y_i - \hat{\alpha}_0 - \sum_{j=1}^k \hat{\alpha}_j x_{ij} \right) = 0 \tag{4}$$

and

$$\frac{\partial S}{\partial \alpha_j} \Big|_{\hat{\alpha}_0, \hat{\alpha}_1, \dots, \hat{\alpha}_k} = -2 \sum_{i=1}^n \left(y_i - \hat{\alpha}_0 - \sum_{j=1}^k \hat{\alpha}_j x_{ij} \right) x_{ij} = 0, \quad j=1,2,\dots,k \tag{5}$$

Simplifying equation 5, we obtain the least squares normal equations as follows:

$$\begin{aligned}
n\hat{\alpha}_0 + \hat{\alpha}_1 \sum_{i=1}^n x_{i1} + \hat{\alpha}_2 \sum_{i=1}^n x_{i2} + \dots + \hat{\alpha}_k \sum_{i=1}^n x_{ik} &= \sum_{i=1}^n y_i \\
\hat{\alpha}_0 \sum_{i=1}^n x_{i1} + \hat{\alpha}_1 \sum_{i=1}^n x_{i2}^2 + \hat{\alpha}_2 \sum_{i=1}^n x_{i1}x_{i2} + \dots + \hat{\alpha}_k \sum_{i=1}^n x_{i1}x_{ik} &= \sum_{i=1}^n x_{i1}y_i \\
&\dots\dots\dots \\
\hat{\alpha}_0 \sum_{i=1}^n x_{ik} + \hat{\alpha}_1 \sum_{i=1}^n x_{ik}x_{i1} + \hat{\alpha}_2 \sum_{i=1}^n x_{ik}x_{i2} + \dots + \hat{\alpha}_k \sum_{i=1}^n x_{ik}^2 &= \sum_{i=1}^n x_{ik}y_i
\end{aligned} \tag{6}$$

Note that there are $p=k+1$ normal equations, one for each of the unknown regression coefficients.

The solution to the normal equations will be the least squares estimators, $\hat{\alpha}_0, \hat{\alpha}_1, \dots, \hat{\alpha}_k$.

These equations can be displayed in a more convenient form using matrix notations. The model in terms of the observations (equations 1), may be written as:

$$y = \alpha x + \epsilon \tag{6}$$

where

$$y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_k \end{bmatrix}, \quad X = \begin{bmatrix} 1 & x_{11} & x_{12} & \dots & x_{1k} \\ 1 & x_{21} & x_{22} & \dots & x_{2k} \\ \dots\dots\dots \\ 1 & x_{n1} & x_{n2} & \dots & x_{nk} \end{bmatrix} \tag{7}$$

$$\alpha = \begin{bmatrix} \alpha_0 \\ \alpha_1 \\ \vdots \\ \alpha_k \end{bmatrix}, \quad \epsilon = \begin{bmatrix} \epsilon_0 \\ \epsilon_1 \\ \vdots \\ \epsilon_k \end{bmatrix}$$

In general, y is an $(n \times 1)$ vector of the observations, X is an $(n \times p)$ matrix of the levels of the regressor variables, α is a $(p \times 1)$ vector of the regression coefficients, and ϵ is an $(n \times 1)$ vector of random errors.

The vector of least squares estimators, $\hat{\alpha}$, which minimizes the following relation must be found.

$$S(\alpha) = \sum_{i=1}^n \epsilon_i^2 = \epsilon' \epsilon = (y - X\alpha)'(y - X\alpha) \quad (8)$$

Note that $S(\alpha)$ may be expressed as follows:

$$\begin{aligned} S(\alpha) &= y'y - \alpha'X'y - y'X\alpha + \alpha'X'X\alpha \\ &= y'y - 2\alpha'X'y + \alpha'X'X\alpha \end{aligned} \quad (9)$$

Since $\alpha'X'y$ is a (1×1) matrix, or a scalar, and its transpose $(\alpha'X'y)' = y'X\alpha$ is the same scalar. The least squares estimators must satisfy the following relationship:

$$\left. \frac{\partial S}{\partial \alpha} \right|_{\hat{\alpha}} = -2X'y + 2X'X\hat{\alpha} = 0 \quad (10)$$

which simplifies to

$$X'X\alpha = X'y \quad (11)$$

Equation 11 are the least squares normal equations. They are identical to 5. To solve the normal equations multiply both sides of equation 11 by the inverse of $X'X$. Thus, the least squares estimator of α is:

$$\hat{\alpha} = (X'X)^{-1} X'y \quad (12)$$

provided that $(X'X)^{-1}$ exists. The $(X'X)^{-1}$ matrix will always exist if the regressors are linearly independent; that is, if no column of the X matrix is a linear combination of the other columns.

It is easy to see that the matrix form of the normal equations 11 is identical to the scalar form 5.

Writing out 11 in detail, we obtain:

$$\begin{bmatrix} n & \sum_{i=1}^n x_{i1} & \sum_{i=1}^n x_{i2} & \sum_{i=1}^n x_{ik} \\ \sum_{i=1}^n x_{i1} & \sum_{i=1}^n x_{i1}^2 & \sum_{i=1}^n x_{i1}x_{i2} & \sum_{i=1}^n x_{i1}x_{ik} \\ \sum_{i=1}^n x_{i2} & \sum_{i=1}^n x_{i2}^2 & \sum_{i=1}^n x_{i2}x_{i1} & \sum_{i=1}^n x_{i2}x_{ik} \\ \vdots & \vdots & \vdots & \vdots \\ \sum_{i=1}^n x_{ik} & \sum_{i=1}^n x_{ik}x_{i1} & \sum_{i=1}^n x_{ik}x_{i2} & \sum_{i=1}^n x_{ik}^2 \end{bmatrix} \begin{bmatrix} \hat{\alpha}_0 \\ \hat{\alpha}_1 \\ \vdots \\ \hat{\alpha}_k \end{bmatrix} = \begin{bmatrix} \sum_{i=1}^n y_i \\ \sum_{i=1}^n x_{i1}y_i \\ \vdots \\ \sum_{i=1}^n x_{ik}y_i \end{bmatrix} \quad (13)$$

If the indicated matrix multiplication is performed, the scalar form of the normal equations 5 is obtained. In this display it is easy to see that $X'X$ is a $(p \times p)$ symmetric matrix and $X'y$ is a $(p \times 1)$

column vector. Note the special structure of the $X'X$ matrix. The diagonal elements of $X'X$ are the sums of square of the elements in the columns of X , and the off-diagonal elements are the sums of cross-products of the elements in the columns of X . Furthermore, note that the elements of $X'y$ are the sums of cross-products of the columns of X and the observations y_i .

The fitted regression model corresponding to the levels of the regressor variables $x' = [1, x_1, x_2, \dots, x_k]$ is:

$$\hat{y} = x' \hat{\alpha} = \hat{\alpha}_0 + \sum_{j=1}^k \hat{\alpha}_j x_j \quad (14)$$

The vector of fitted values $\hat{y}_i$ corresponding to the observed values y_i is:

$$\begin{aligned} \hat{y} &= X\hat{\alpha} \\ &= X(X'X)^{-1}X'y \\ &= Hy \end{aligned} \quad (15)$$

The $n \times n$ matrix $H = X(X'X)^{-1}X'$ is usually called the *hat* matrix, because it maps the vector of observed values into a vector of fitted values. The hat matrix and its properties play an important role on regression analysis. The difference between the observed value y_i and the corresponding fitted value $\hat{y}_i$ is the residual $e_i = y_i - \hat{y}_i$.

There are several other ways to express the vector of residuals e that will prove useful, including:

$$e = y - X\hat{\alpha} \quad (16a)$$

$$=(I-H)y \quad (16b)$$

$$=y-Hy \quad (16c)$$

Properties of the Least Squares Estimators

The statistical properties of the least squares estimator $\hat{\alpha}$ may be expressed as:

$$\begin{aligned} E(\hat{\alpha}) &= E[(X'X)^{-1}X'y] \\ &= E[(X'X)^{-1}X'(X\alpha + \epsilon)] \\ &= E[(X'X)^{-1}X'X\alpha + (X'X)^{-1}X'\epsilon] \\ &= \alpha \end{aligned} \quad (17)$$

Since $E(\epsilon)=0$ and $(X'X)^{-1}X'X=I$, then $\hat{\alpha}$ is an unbiased estimator of α . The variance property of $\hat{\alpha}$ is expressed by the covariance matrix

$$Cov(\hat{\alpha}) = E\{[\hat{\alpha} - E(\hat{\alpha})][\hat{\alpha} - E(\hat{\alpha})]'\} \quad (18)$$

which is a $(p \times p)$ symmetric matrix whose j th diagonal element is the variance of $\hat{\alpha}_j$ and whose ij th off-diagonal element is the covariance between $\hat{\alpha}_i$ and $\hat{\alpha}_j$. The covariance matrix of $\hat{\alpha}$ is

$$Cov(\hat{\alpha}) = \sigma^2(X'X)^{-1} \quad (19)$$

Therefore, if we let $C = (X'X)^{-1}$, the variance of $\hat{\alpha}_j$ is $\sigma^2 C_{jj}$ and the covariance between $\hat{\alpha}_i$ and $\hat{\alpha}_j$ is $\sigma^2 C_{ij}$. The least squares estimator $\hat{\alpha}$ is the best linear unbiased estimator of α (the Gauss-

Markov theorem, Montgomery and Peck 1982). If we further assume that the errors ϵ_i are normally distributed, then $\hat{\alpha}$ is also the maximum likelihood estimator of α . The maximum likelihood estimator is the minimum variance unbiased estimator of α .

3-4-1-3 Estimation of σ^2

An estimator of σ^2 can be developed from the residual sum of squares as:

$$\begin{aligned} SS_E &= \sum_{i=1}^n (y_i - \hat{y}_i)^2 \\ &= \sum_{i=1}^n e_i^2 \\ &= e'e \end{aligned} \quad (20)$$

Substituting $e = y - X\hat{\alpha}$, we have

$$\begin{aligned} SS_E &= (y - X\hat{\alpha})'(y - X\hat{\alpha}) \\ &= y'y - \hat{\alpha}'X'y - y'X\hat{\alpha} + \hat{\alpha}'X'X\hat{\alpha} \\ &= y'y - 2\hat{\alpha}'X'y + \hat{\alpha}'X'X\hat{\alpha} \end{aligned} \quad (21)$$

Since $X'X\hat{\alpha} = X'y$, this last equation becomes

$$SS_E = y'y - \alpha'X'y \quad (22)$$

The residual sum of squares has $n-p$ degrees of freedom associated with it since p parameters are estimated in the regression model. The residual mean square is

$$MS_E = \frac{SS_E}{n-p} \quad (23)$$

We can show that the expected value of MS_E is σ^2 , so an unbiased estimator of σ^2 is given by

$$\hat{\sigma}^2 = MS_E \quad (24)$$

This estimator of σ^2 is used to find confidence interval for the regression coefficient $\hat{\alpha}_j$.

Confidence Interval on the Regression Coefficients

To construct confidence interval estimates for the regression coefficients $\hat{\alpha}_j$, we must assume that the errors ϵ_i are normally and independently distributed with mean zero and variance σ^2 . Therefore, the observations y_i are normally and independently distributed with mean $\alpha_0 + \sum_{j=1}^k \alpha_j x_{ij}$ and variance σ^2 . Since the least squares estimator $\hat{\alpha}$ is a linear combination of the observations, it follows that $\hat{\alpha}$ is normally distributed with mean vector α and covariance matrix $\sigma^2(X'X)^{-1}$. This implies that the marginal distribution of any regression coefficient $\hat{\alpha}_j$ is normal with mean α_j and variance $\sigma^2 C_{jj}$, where C_{jj} is the j th diagonal element of the $(X'X)^{-1}$ matrix. Consequently, each of the statistics

$$\frac{\hat{\alpha}_j - \alpha_j}{\sqrt{\hat{\sigma}^2 C_{jj}}}, \quad j=0,1,\dots,k \quad (25)$$

is distributed as t with $n-p$ degrees of freedom, where $\hat{\sigma}^2$ is the estimate of the error variance obtained from equation 24. Therefore, a $100(1-\eta)$ percent confidence interval for the regression

coefficient $\alpha_j, j=0, 1, \dots, k$ is

$$\hat{\alpha}_j - t_{\eta/2, n-p} \sqrt{\hat{\sigma}^2 C_{jj}} \leq \alpha_j \leq \hat{\alpha}_j + t_{\eta/2, n-p} \sqrt{\hat{\sigma}^2 C_{jj}} \quad (26)$$

We usually call the quantity

$$se(\hat{\alpha}_j) = \sqrt{\hat{\sigma}^2 C_{jj}} \quad (27)$$

the standard error of the regression coefficient $\hat{\alpha}_j$.

Table A: Data for multiple linear regression

Observation	i	y	x_1	x_2	...	x_k
	1	y_1	x_{11}	x_{12}	...	x_{1k}
	2	y_2	x_{21}	x_{22}	...	x_{2k}
	...	...	...	...	...	...
	n	y_n	x_{n1}	x_{n2}	...	x_{nk}