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Optimum Population Distribution Described by Dynamic Models and Controlled by Immigration and Job Creation

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Dedication

To my parents, my husband Hanming, for their extraordinary encouragements and support.

Abstract

In this thesis, dynamic mathematical models are constructed to describe the population distribution in Canada based on the model in previous work by Ahmed and Rahim [1].

Numerical results demonstrate that the model population is in close agreement with the actual population. This indicates that the presented model can be used as a valuable tool for describing the dynamics of population distribution. We also demonstrate that by using modern Systems and Optimal Control theory [2], it is possible to formulate optimum immigration and job creation strategies while maintaining population level close to certain pre-specified targets.

An optimization algorithm [2] is then developed based on dynamic programming and gradient algorithm approach. Unknown parameters such as birth rate, death rates and transition rates are estimated and identified. The system model obtained by using the identified parameters is then augmented by adding a fourth equation describing the dynamics of unemployment rate. This model is then used to formulate a control problem with immigration and job creation rates being the decision (control) variables. Using optimal control theory, optimum immigration and job creation policies are determined. Results are illustrated by numerical simulation and they are found to be very encouraging.

Contents

List of Figures

List of Tables

Acronyms

DP	Dynamic Programming
DPE	Dynamic Programming Equation
GA	Gradient Algorithm
IDP	Iterative Dynamic Programming
G_1	Age Group G1 (age 0-14)
G_2	Age Group G2 (age 15-64)
G_3	Age Group G3 (age 65 and above)
ODE	Ordinary differential equation
AC	Absolutely continue

Symbols

x_1	Population of age group G1
x_2	population of age group G2
x_3	population of age group G3
x_4	Unemployed population in age group G2
$\dot{x_1}$	Growth rate of population of age group G1
$\dot{x_2}$	Growth rate of population of age group G2
$\dot{x_3}$	Growth rate of population of age group G3
$\dot{x_4}$	Growth rate of unemployed population of age group G2
$x_1(0)$	Initial population(condition) of age group 1
$x_2(0)$	Initial population(condition) of age group 2
$x_3(0)$	Initial population(condition) of age group 3
$x_4(0)$	Initial unemployed population(condition) of age group 2
b	Birth rate due to population of age group G2
d_1	Child mortality rate
d_2	The mortality of age group 2
d_3	The mortality of age group 3
τ_{12}	Transition rate from age group G1 to age group2
τ_{23}	Transition rate from age group G2 to age group3
e_1	Child emigration rate
e_2	Age group G2 population emigration rate

e_3	Age group G3 population emigration rate
i_1	Child immigration rate
i_2	Age group G2 population immigration rate
i_3	Age group G3 population immigration rate
x_0	Initial condition
$p1$	The fraction of accompanying children
$p2$	The fraction of accompanying seniors
u_1	Age group 2 population immigration rate
u_2	Job creation rate for age group 2
x_M	The upper boundary value of the target population
x_m	The lower boundary value of the target population
u_M	The upper limit of age group G2 population immigration rate
p	Percentage of labour force in the age group G2

Chapter 1

Introduction

1.1 Motivation

Since demography was introduced by the work of the English merchant John Graunt (1620-1674) [3], demographic research has focused on different areas.

Following Afred Lotka's work, stable population has been one of the most important research areas in demography. Multi-age models with constant vital rates over time [4] and multi-state stable population models can be regarded as good examples in this area [5]. Both of the models mentioned above use the fixed rates. There are limits to the use of fixed rate models because those models can not efficiently describe dynamic behavior. For instance, usually migration rates, birth rate, death rates, transition rates from one age category to the next, labour force participation rate and job creation rate are variable from time to time, even within short periods. Currently, there is not much work on any demographic models that can describe changing behavior or the relationship between current population and current rates composition. The mathematical model built in this thesis is based on the model presented in paper [1]. Birth rate, death rates, transition rates, emigration rates, immigration rates and job creation rate are variables.

Following the variable rates policy, dynamic multi-state population models with variable demographic rates were built, which recognize more than one living state [6]. Because the dynamic multi-state model is highly flexible, it can describe any observed population's behavior at any time.

Early work to model population using dynamic optimization was done by Arthur and McNicoll in 1977 [7]. They tried to create an age-dependent population policy theory by applying integral-equation control techniques and integrating an aged-structured population model with a vintage capital model. By employing fertility and savings as control variables, Arthur and McNicoll analyzed the relationship of social discounting and welfare trade-offs. The greatest innovation of this model is that decision variables and a performance function were introduced into demographic models.

Optimal control of immigration and saving rates were also analyzed in paper [8]. In this paper Feichtinger, Prskawetz and Veliov expanded the Arthur-McNicoll model and added immigration as another control variable. They presented an approach for dynamic optimization in population economics. The optimal control model that was built was based on age-structured differential equations for the dynamics of both the population and the capital. The maximum principle newly derived in paper [9][10] was applied into age-structured systems.

In addition, there are some papers investigating optimal control of population dynamics. In paper [11][12] optimal birth control of age-dependent models were studied. Optimal control of illicit drug, violence and social welfare was analyzed in paper [13].

Since immigration is one of the most efficient control instruments for many countries, immigration in Canada will be considered as a control variable in this thesis. According to current practice in Canada, each year the target level of immigration, for the following year, is proposed. The obvious question is: how is the target level of immigration determined? In general, this level is influenced by changes in manpower requirements and other socio-economic and political conditions of the country. If there are not enough job positions to match the level of immigration, this may cause many social and political problems. Currently, immigration and unemployment in Canada is an interesting and important topic [14]. Some papers have been published investigating the immigration and UI system [15] as well as immigration and the rate of growth of population and labour force [16]. This thesis will consider job creation rate as another control variable. Both immigration and job creation can be adjusted by the Canadian government as required to change the dynamics of population distribution in Canada.

Actually, in optimal control of population, the original population model was the descriptive model which was expanded by performance function and decision variables such as fertility, immigration, job creation, savings and welfare [17]. In general, the decision makers are assumed and are the control variables. The system develops from the given initial condition. Taking a certain time period, some of these developments are more likely to be accepted by the decision makers, while some are less likely to be accepted. But how should the decision-makers affect the system development over time? The mathematical foundation of such decision-processes is the optimal control theory.

It has been noticed that most demographic research has concentrated on fertility and birth function and from it population change can be predicted. This is fundamental and it is required in modeling any population dynamics. In contrast, this thesis will focus on constructing dynamic models for population distribution in Canada by following the systems and control theoretic approach as developed in [1].

In general, population changes with time because of birth, death, immigration and emigration etc. If the total population is divided into several age groups, there is a continuous process of transition from one age group to the next. To capture the temporal variation of population in each group one must build a dynamic model. Such a model can be constructed if basic parameters like birth rate, death rates, and transition rates from one age group to the next are available. Very often these parameters are not readily available; only the population data in each individual age group is available for a given period of time. Based on this information, estimation and identification of unknown parameters are explored using the mathematical model mentioned above. The methodology proposed in this thesis may be useful for the Government of Canada to formulate optimum immigration and job creation policies.

1.2 Objective

The main objective of this thesis is to demonstrate that by using modern Systems and Optimal Control theory, it is possible to formulate optimum immigration and job creation strategies while maintaining the population level close to certain pre-specified levels. It is reasonable to think that immigration should be tied to the demand for manpower, the availability of jobs and the ability to create new jobs.

Based on the previous paragraph, the following objective will be the focus of my thesis:

(1) The construction of a dynamic model describing the population distribution in Canada for the three age groups. It will be described later;

(2) The simulation of a system of ODE based on the available parameters, like birth rate, death rates, transition rates, immigration rates and emigration rates;

(3) The identification of the unknown parameters by minimizing the identification er-

ror(defined later) when the parameters are not available;

(4) The optimization of the immigration policy (rate) by minimizing an objective functional called the (cost function);

(5) The optimization of immigration and job creation rates by minimizing a similar cost function (described later).

1.3 Thesis Organization

In this thesis, demographic science is first described in chapter 2. Dynamic models for demography are given in chapter 3. Population distribution by groups, which is applied to evaluate the dynamic population model, is introduced in chapter 4. Optimization methodologies are defined in chapter 5. In chapter 6, the unknown parameters are identified based on the optimal control strategy and gradient algorithm. Optimal immigration policy is discussed in chapter 7. Immigration policy versus job creation rate, which is based on the optimal control principle and gradient algorithm, is displayed in chapter 8.

Chapter 2

Demographic Science

Since its emergence, demography has successfully developed in many different directions such as stable population model, dynamic population model and dynamic optimization population model. In this chapter we will first give the definition of demography, then the historical background of demography will be introduced.

2.1 Definition of Demography

Demography is the study of the characteristics of human populations, such as size, growth, density, distribution, and vital statistics [18].

2.2 Historical Review

Demography was born through the work of the English merchant John Graunt (1620-1674)[19]. In that work, he analysed London's "Bill of Mortality," which included the production of a sort of life table, "the life table was developed to express probabilities pertaining to individual persons [20]," however the calculation method used by Graunt was rather obscure. Another important development in the late seventeenth century included the work of Edmund Halley(1693), Astronomer Royal, who was famous for his research on comet. He calculated the first life table for the city of Breslau (now Wroclaw in Polish Silesia), which was based on actual numbers of deaths by age [21].

In 1760, Euler invented the famous concept of Stable Population. Approximately by 150 years later, Lotka, who is generally regarded as the father of Stable Population Theory, published his first paper about Stable Population. In the late eighteenth-century, significant achievement in demographic research was done by T.R. Malthus, who proposed the very first demographic model in 1798. As he stated: "It has been frequently observed that though we cannot hope to reach perfection in any thing, yet that it must be advantageous to us to place before our eyes the most perfect model," Malthus was usually considered "the founder of modern demography."

As previously mentioned, in the eighteenth-century, there were several important developments in demographic research. In the early nineteenth, Milne's (1815) great contribution to demography was the formulation of the conventional calculation and the presentation of the life table, which is the same concept accepted today. After that, from 1841, William Farr, the famous Victorian Registrar-General of England, started to produce the decennial series of English life tables, which have been used in demographic research up to the present.

After 1900 the greatest accomplishment in demographic research was due to Alfred Lotka, who developed the mathematical Stable Population in his three papers [22][23][24]. In these papers he indicated that if the schedules of age-specific mortality and fertility was kept constant, the fluctuation tendency of population would be close to the true value of population. Along this line, a predictable path to the final stage with the fixed age-structure was constructed. Finally, Lotka and Louis Dublin added the concept of "intrinsic" (or "true") rates into the theory [25]. Another twentieth century development in demographic research is Hajnal's Singulate Mean Age at Marriage (1953)[26], which is applied to indirectly estimate the mean age of marriage. Here the prefix "singulate" refers to the computational method of arriving at the mean age of marriage.

Excluding Lotka's Stable Population, the first widely used demographic models were the models of life tables, which were published by the United Nations (1955,1956). From this base, Coale and Demeny (1966) constructed Princeton regional model life tables and developed the mortality models [27].

Since the 1970's, the main development in demographic research has been building model. A good example of this is the multi-regional demographic modeling proposed by Rogers in 1975 [28]. Now, demography has become a discipline in the field of science as Achille Guillard envisioned in 1855.

Chapter 3

Dynamic Models for Demography

As Bartholomew noted: "The model is an abstraction of the real word in which the relevant relations between the real elements are replaced by similar relation between mathematical entities." Demographers have constructed different kind of mathematical models to describe population's characteristics in the real word.

3.1 Fundamental Population Models

3.1.1 Malthus' model

Malthus' model was proposed in 1798 by the Englishman, Thomas R. Malthus. By observing human populations, he conjectured that *"Populations appeared to increase by a fixed proportion over a given period of time, and that, in the absence of constraints, this proportion is not affected by the size of the population."*

Motivation for Malthus' Model:
To develop a mathematical representation of Malthus' model, we use the above hypothesis as our governing principle.

Notation:

- $t_0, t_1, t_2, \cdots, t_N$: discrete times at which the population is determined; all t_i are equally spaced with time step $h = t_{i+1} - t_i$ for all i;
- $P_0, P_1, P_2, \cdots, P_N$: populations at times $t_0, t_1, t_2, \cdots, t_N$, respectively;
- b and d: birth and death rates, respectively (with units consistent with those used to measure time);
- $$c = b - d \qquad \text{is the effective growth rate.}$$

Mathematical Equation:

$$(P_{i+1} - P_i)/P_i = c \times h$$

or

$$P_{i+1} = P_i + c \times h \times P_i$$

for $i = 0, 1, \cdots, N-1$. The initial population, P_0, is given at the initial time t_0[29].

3.1.2 The Lotka Equation

The greatest single contribution to population theory has been that of A.J.Lotka, contained in a series of papers extending from 1907 to 1948. Lotka emphasized the continuous model in development of the theory.

In the Lotka Theory, the unknown function is $B(t)$, the number of births at time t. Let us consider only women, and female born from these women. The data are the observed probability of surviving to age a, denoted l_a, and the chance $m_a da$ of bearing a child between the ages a and $a+da$. Of the births $B(t-a)$ at time $t-a$ the proportion l_a will survive to time t on this deterministic model, and of these $m_a da$ will themselves bear children during the interval of age da. We can find the total number of births at time t by the fundamental Lotka equation shown as below:

$$B(t) = \int_{\alpha}^{\beta} B(t-a) l_a m_a dx, \tag{3.1}$$

where ages α and β are the lower and upper limits of reproduction. In this form of the equation, we suppose that the birth and aging process has been going on for a long time. If the process started recently, we need an additional term, say $G(t)$, to provide for the births at time t occurring to the women alive at the beginning time [20].

3.2 System Theoretical Models for Population Distribution

Models mentioned above can not be directly used to model population distribution by categorical distribution. In this section, we will use system theoretical models for population distribution.

System theoretical models built in this thesis are based on the previous work by Ahmed and Rahim [1]. In [1], they presented a set of (dynamic) mathematical models representing the population distribution in general. The dynamic models presented in my work are modified according to the actual population grouping and characterization used by Statistics Canada [30]. This is then used to describe the population variation (dynamics) in Canada.

Statistics Canada divides the Canadian population into three age groups. The group $G1$ consists of all children younger than 15 years old. The population at time t in this group is represented by $x_1(t)$. Group G2 consists of the population between the age of 15 to 64, and we denote the population at time t in this age group by $x_2(t)$. The population over age 65 is grouped as G3, and the population count in this age group is represented by $x_3(t)$.

The reason why the population is partitioned into these different age groups is that many Government programs (for example; child care, education, health, old age pension, unemployment insurance, welfare etc.) and the cost of administering those programmes and services are strongly dependent on the population distribution.

3.2.1 Canadian Population Characteristics

"Canada's population was about 3.7 million in 1871 shortly after the confederation. Most Canadians lived and worked on the family farm, and the country was young with more than one-third of the population under the age of 15 [31]."

Today, Canada is a nation of 31,559,186 people based on the April 2003 census [32]. "The majority of Canadians live in cities. The birth rate has declined, and less than one-fifth of the population is under 15 years old. Each year, approximately 200,000 new Canadians arrive from around the world [31]."

"These shifting demographic patterns are reflected in many aspects of Canadian life: jobs, family arrangements, housing, education, health care, religion, language, leisure time, travel patterns and cultural pursuits [31]." Several characteristics are considered in this thesis:

- Transition Rate;
- Birth Rate;
- Death Rate;
- Immigration Rate;

- Emigration Rate;
- Employed;
- Labour force;
- Unemployment rate.

Transition Rate

Transition rate is the ratio of the number of people who will transit to the next age group to total population of the present age group. In this thesis, weekly transition rate τ_{12} and τ_{23} are defined as:

$$\tau_{12} = \frac{\text{Number of 14 years old people}}{\text{Total population of age group G1}} \times 100\%,$$

$$\tau_{23} = \frac{\text{Number of 64 years old people}}{\text{Total population of age group G2}} \times 100\%.$$

Birth Rate

In this thesis, we assume that there is no baby born by population in age groups G1 and G3. The weekly birth rate caused by population in age group G2 is defined as:

$$b = \frac{\text{Number of births}}{\text{Total population of age group G2}} \times 100\%.$$

Death Rate

Death rate is the ratio of deaths to total population over a specified period of time. In this thesis, we define the weekly death rates for three age groups by:

$$d_1 = \frac{\text{Number of deaths in age group G1}}{\text{Total population of age group G1}} \times 100\%,$$

$$d_2 = \frac{\text{Number of deaths in age group G2}}{\text{Total population of age group G2}} \times 100\%,$$

$$d_3 = \frac{\text{Number of deaths in age group G3}}{\text{Total population of age group G3}} \times 100\%.$$

Immigration Rate

Immigration population in Canada refers to "people who are, or have been, landed immigrants in Canada. A landed immigrant is a person who has been granted the right to live in Canada permanently by immigration authorities. Some immigrants have resided in Canada for a number of years, while others are recent arrivals. Most immigrants are born outside Canada, but a small number were born in Canada [30]."

The ratio of total immigrants to Canada to total population in a specified community or area over a specified period of time is the immigration rate. In this thesis, we define weekly immigration rates for three age groups by:

$$i_1 = \frac{\text{Number of immigrants in age group G1}}{\text{Total population of age group G1}} \times 100\%,$$

$$i_2 = \frac{\text{Number of immigrants in age group G2}}{\text{Total population of age group G2}} \times 100\%,$$

$$i_3 = \frac{\text{Number of immigrants in age group G3}}{\text{Total population of age group G3}} \times 100\%.$$

Emigration Rate

Emigration rate is the ratio of total emigrants from Canada to total population in a specified community or area over a specified period of time. In this thesis, we define weekly emigration rates for three age groups by:

$$e_1 = \frac{\text{Number of emigrants in age group G1}}{\text{Total population of age group G1}} \times 100\%,$$

$$e_2 = \frac{\text{Number of emigrants in age group G2}}{\text{Total population of age group G2}} \times 100\%,$$

$$e_3 = \frac{\text{Number of emigrants in age group G3}}{\text{Total population of age group G3}} \times 100\%.$$

Employed

The employed people are "persons 15 years of age and over, excluding institutional residents, who, during the week (Sunday to Saturday) prior to Census Day: did any work at all for pay or in self-employment; or were absent from their job or business for the entire week because of vacation, illness, a labour dispute at their place of work or other reasons [30]."

Labour Force

"The labour force consists of the number of people aged 15 and over who are employed (that is, those who currently have jobs) and unemployed (that is, those who do not have jobs but who are actively looking for work) [32]." In this thesis labour force only is considered in the age group G2.

Unemployment Rate

"The unemployment rate is the percentage of the labour force that actively seeks work but is unable to find work at a given time. Discouraged workers-persons who are not seeking work because they believe the prospects of finding it are extremely poor-are not counted as unemployed or as part of the labour force [32]."

$$\text{Unemployment rate} = \frac{\text{Number of umemployed people}}{\text{Number of people in the labour force}} \times 100\%.$$

3.2.2 Population Dynamics

Population in Canada is always changing with time because of these events like birth, death, immigration, emigration and transition from one age group to the next one. This behavior is described in dynamic mathematical models built below.

Yearly data on birth, death, immigration, emigration and population of all the three age groups for the period 1972 to 2002 was obtained from Statistics Canada [30]. This data was then converted to weekly values, which is shown in the following section. Here, τ_{12} stands for the weekly transition rate from G1 to G2 and τ_{23} from G2 to G3. The coefficient b stands for the weekly birth rate, d_1, d_2, d_3 stand for weekly mortality rates, i_1, i_2, i_3 stand for weekly immigration rates, and e_1, e_2, e_3 weekly emigration rates for the groups G1, G2, and G3, respectively. Firstly, we will give some characteristics of Canadian population, which will be applied in our demographic models.

3.2.3 Mathematical Model

The growth rate of population of group G1 is given by

$$\dot{x_1} = bx_2 - \tau_{12}x_1 - d_1x_1 - e_1x_1 + i_1x_1 \equiv f_1 \tag{3.2}$$

where b denotes the birth rate due to population of age group G2. We assume that the contribution to birth rate due to population below age 15 and above age 65 is negligible. The parameter τ_{12} denotes the maturity rate or transition rate from age group G1 to age group G2, d_1 denotes the child mortality rate, e_1 and i_1 denote the child emigration and immigration rates respectively. Similarly the growth rate of G2 can be written as

$$\dot{x_2} = \tau_{12}x_1 - \tau_{23}x_2 - d_2x_2 - e_2x_2 + i_2x_2 \equiv f_2 \tag{3.3}$$

where the parameter τ_{23} denotes the transition rate from age group G2 to age group G3, d_2 denotes the mortality rate of the age group G2, e_2 and i_2 are the emigration and immigration rates respectively. The growth rate of G3 can be written as

$$\dot{x_3} = \tau_{23}x_2 - d_3x_3 - e_3x_3 + i_3x_3 \equiv f_3 \tag{3.4}$$

where the parameters d_3, e_3, i_3 are, respectively, the mortality rate, emigration rate, and immigration rate of population in age group G3. We can write the above population model in the following Canonical form:

$$\dot{x} = f(x(t)), t \geq 0. \tag{3.5}$$

where

$$x \equiv (x_1, x_2, x_3)' \tag{3.6}$$

denotes the population vector in the three age groups. The vector function

$$f \equiv (f_1, f_2, f_3)' \tag{3.7}$$

denotes the vector field representing the population growth rates.

Chapter 4

Population Distribution by Groups

In this chapter, we will follow the mathematical models and the three age group policy proposed in section 3.2.

4.1 Simulation Results Based on Dynamic Models and Available Parameters

The solid lines in Figure 4.1, 4.2 and 4.3 represent the model population for the three age groups, which are based on the dynamic models constructed in subsection 3.2.3 and the weekly rates mentioned in section 3.2.

4.2 Census Data Obtained from Statistics Canada

In Figure 4.1, 4.2 and 4.3, the dashed lines display the real population for three age groups obtained from the census data mentioned previously.

4.3 Comparison and Conclusion

Comparison of the actual population with the model population using the infinitesimal rates computed from the data obtained from Statistics Canada is shown in Figures 4.1-4.3.

Based on the assumptions mentioned above, we conclude from the simulation results that the model population is fairly close to the actual population though it falls short of an exact match. The discrimination is possibly due to the presence of nonlinearity in the true population model and numerical errors in computing the rates from the raw data. Using neural networks, it is possible to construct more accurate nonlinear models. We leave it for future investigation.

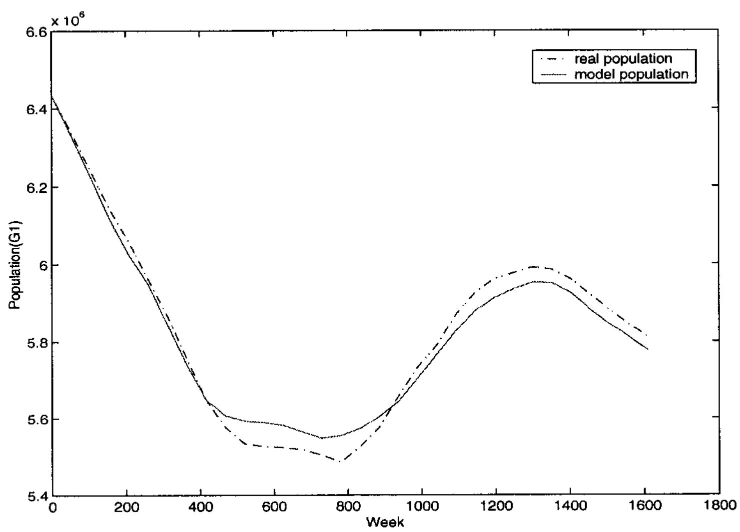

Figure 4.1: Population of G1

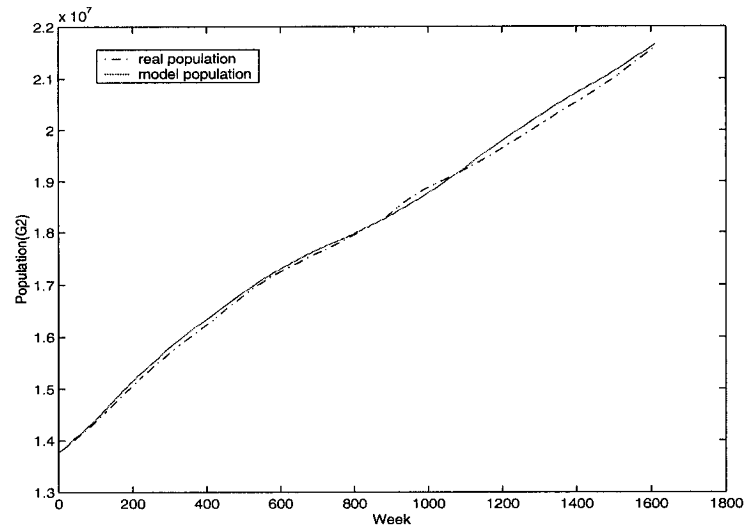

Figure 4.2: Population of G2

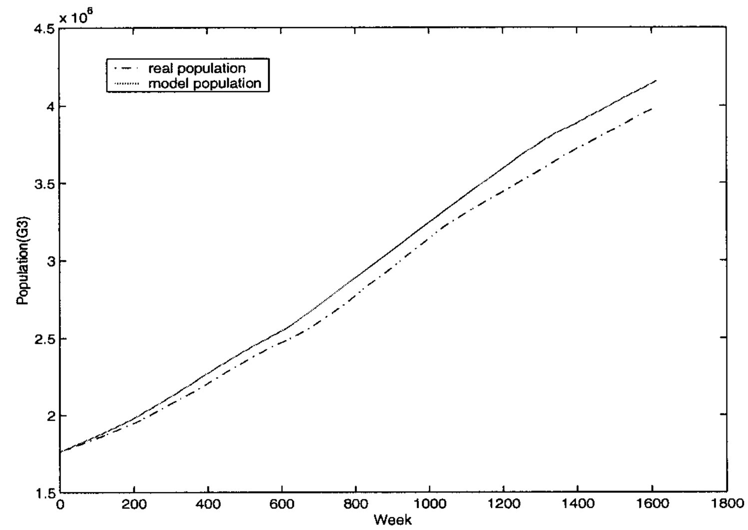

Figure 4.3: Population of G3

Chapter 5

Optimization Methodologies

The optimization problem is "a computational problem whose goal is to find the best of all possible solutions. More formally, find a solution in the feasible region which has the minimum (or maximum) value of the objective function [33]." In Chapter 6, 7 and 8, optimization methodologies will be employed in parameter identification, optimum immigration rate and job creation rate.

5.1 Dynamic Programming Principle

Mathematical way of thinking about it is to look at what you should do at the end, if you get to that stage. So you think about the best decision with the last potential goal (which you must choose) and then the second to last and so on. This way of tackling the problem backwards is Dynamic programming.

The word Programming in the name has nothing to do with writing computer programs. Mathematicians use the word to describe a set of rules which anyone can follow to solve a problem. They do not have to be written in a computer language [34].

Dynamic programming is an algorithmic technique in which an optimization problem is solved by considering subproblem solutions rather than recomputing them [33]. The essence of dynamic programming is Richard Bellman's Principle of Optimality that was first introduced in 1957 [35]. This principle, without rigorously defining the terms, is intuitive:

"An optimal policy has the property that whatever the initial state and the initial decisions are, the remaining decisions must constitute an optimal policy with regard to the state resulting from the first decision."

The overall problem can be divided into sub-problems without violating the optimal policy. Dynamic programming can make the complicated problem easier to solve. A complicated problem can be seen as a "dynamic" problem which is composed of time dependent stages. The state variable is defined in such a way that it can describe the process at a specific stage. Given the state of the process at the beginning of a stage, a decision, which is known as u (the control variable), is made. That gives a return at the stage and transform the process to the ending stage. The decision u must be optimal such that the overall return of all stages from the current stage to the ending stage must be the minimum or maximum according to the requirements of the objective function.

Considering a system governed by a state equation with $X(k)$ denoting the state variable at time stage k and $u(k)$ the control variable or the decision variable, we formulate the following system of equations:

$$\begin{cases} X(k+1) \equiv F(k, X(k), u(k)), \quad k = 0, 1, 2, \cdots, N-1 \\ X(0) = X^0 \end{cases} \tag{5.1}$$

where $X(k) \in R^N$ and $u(k) \in R^N$.

The controls belong to a specified set $\mathcal{U}_{ad}$ called the class of admissible controls. The cost function is defined as:

$$J(u) \equiv \sum_{k=0}^{N-1} \ell(k, X(k), u(k)) + W(N, X(N)) \tag{5.2}$$

where the summation represents the running cost and the last term determines the terminal cost. The objective is to find a control policy from the admissible class $\mathcal{U}_{ad}$ that minimizes the cost functional J. We formulate this as a dynamic programming problem. Given the state $X(r)$ at time r, we may define

$$J(r, X(r), u) \equiv \sum_{k=r}^{N-1} \ell(k, X(k), u(k)) + W(N, X(N)) \tag{5.3}$$

as the cost of operating the system from time r starting from state $X(r)$ and using the control policy $u \in \mathcal{U}_{ad}$, where $\mathcal{U}_{ad}$ is a closed bounded subset of R^m. We need to find a control policy u^* such that $J(u^*) \leq J(u)$. Let $I = 0, 1, 2, \cdots, N$ and $r \in I$. We define the value function:

$$V(r, X(r)) \equiv inf\{J(r, X(r), u), u \in \mathcal{U}_{ad}\} \tag{5.4}$$

where $V(r, X(r))$ gives the best performance at time r. Clearly this gives the minimal cost to run the system starting from state $X(r)$ at time r until the end of the whole process N.

To solve the above equations, several methods have been developed by scientists: Lagrange multipliers, tabular computation, quadratic programming algorithm [35][36] and IDP(Iterative Dynamic Programming)[37]. These methods have been used to shorten the computation time required by the optimization process.

The gradient algorithm is widely used in optimal control. It will also be used in our work.

5.2 Pontryagin's Minimum Principle

Consider the system

$$\dot{x} = f(t, x, u), \qquad x(t) \in R^s, \qquad t \in [t_1, t_2] = I,$$

with the cost functional

$$J(u) = \int_{t_1}^{t_2} \ell(t, x(t), u(t))dt. \tag{5.5}$$

The problem is to find a control $u \in \mathcal{U}$ that transfers the system starting at time t_1 from a smooth manifold $M_1(\subset R^s)$ of dimension $\leq s$ to a smooth manifold $M_2(\subset R^s)$ of dimension $\leq s$ at time t_2 (free) while minimizing the cost functional (5.5). Define the Hamiltonian $H(t, x, \psi, u)$ as

$$H(t, x, \psi, u) = (f(t, x, u), \psi) + \ell(t, x, u). \tag{5.6}$$

Here

$$(f(t, x, u), \psi) = f(t, x, u)\psi'$$

In order for the pair (x, u) to be optimal, it is necessary that there exists $\psi \in AC(I, R^s)$ such that [38]

(a)

$$H(t, x(t), \psi(t), u(t)) \leq H(t, x(t), \psi(t), v) \text{ for all } v \in Ua.e. \text{ on } I,$$

(b)

$$\begin{cases} \dot{x} = H_\psi, & x(0) = x^0 \\ \dot{\psi} = -H_x, & \psi(t_2) = 0 \end{cases} \tag{5.7}$$

(c)

$$\begin{cases} M_1 = \{x^0\} \\ M_2 = R^s \end{cases} \tag{5.8}$$

5.3 Gradient Method

Pioneered by H. J. Kelly, the gradient method consists of updating the controls using the gradient vector. In a way such that successive iterates produce maximum reduction of the cost. To illustrate it, we consider the following control problem:

$$minimize \quad \left\{ J(u) = \Phi(x(T)) + \int_0^T \ell(t, x(t), u(t))dt \right\} \tag{5.9}$$

subject to the dynamic constraint

$$\begin{cases} \dot{x} = f(t, x(t), u(t)), \quad u \in \mathcal{U} \\ x(0) = x^0 \end{cases} \tag{5.10}$$

Define the Hamiltonian $H(t, x, \psi, u)$ by

$$H(t, x, \psi, u) = \ell(t, x(t), u(t)) + (f(t, x(t), u(t)), \psi(t)) \tag{5.11}$$

Then it follows from Pontryagin's Minimum Principle that the optimal control u^* and the corresponding pair $\{x^*, \psi^*\}$ satisfy the system equation (5.8), the adjoint equation

$$\begin{cases} \dot{\psi} = -\ell_x(t, x(t), u(t)) - f_x'(t, x(t), u(t))\psi \\ \psi(T) = (\frac{\partial}{\partial x}\phi)(x(T)) \end{cases} \tag{5.12}$$

and the inequality

$$H(t, x^*(t), \psi^*(t), u^*(t)) \leq H(t, x^*(t), \psi^*(t), u(t)), \quad u \in \mathcal{U} \tag{5.13}$$

where $x^* \equiv x(u^*)$ and $\psi^* \equiv \psi(u^*)$. Using the above necessary conditions, we can devise an iterative algorithm for computing the optimal control u^*. We note that although the necessary conditions (5.10)-(5.13) are based on rather general assumptions on the functions f and ℓ, numerical evaluation of u^* calls for somewhat stronger conditions. The gradient algorithm is represented by the following steps:

Step 1 Guess $u_1 \in \mathcal{U}$ and set $n = 1$.
Step 2 Solve the initial-value problem (5.10) with $u(t) = u_n(t)$ to get $x_n(t) = x_n(u_n)(t)$.
Step 3 Using the data x_n and u_n, solve the adjoint system (5.12) to obtain ψ_n.
Step 4 Compute the gradient vector

$$g_n(t) = \frac{\partial H}{\partial u}(t, x_n(t), \psi_n(t), u_n(t)) \tag{5.14}$$

Step 5
(a) If $g_n(t) \neq 0$, modify $u_n(t)$ to $u_{n+1}(t) = u_n(t) - \epsilon g_n(t)$ by choosing $\epsilon > 0$ sufficiently small so that $u_{n+1}(t) \in \mathcal{U}$ and $J(u_{n+1}) \leq J(u_n)$. A stopping criterion may be used at this stage. If $|J(u_{n+1}) - J(u_n)| \leq \delta$ for some small $\delta > 0$, then stop; otherwise set $u_n = u_{n+1}$, $n = n + 1$, and go to step 2.
(b) If $g_n(t) = 0$ at the nth stage, then u_n is a (local) minimizing element of $J(u)$ [2].

Chapter 6

Parameter Identification

In chapter 4, the parameters, like the birth rate, death rates and transition rates, were assumed to be given or computed approximately from available population data. But in some cases, the actual parameters mentioned above may not be available however the population data for a particular period of time is available. In that situation, the methodology of system identification provides an effective tool for estimating these unknown parameters [39]. They can be determined also by an alternative approach provided by optimal control theory [2] and this is what we use here. Firstly, the methodology of system identification, which is the basic theory applied to identify unknown parameters in the following sections, is defined.

6.1 System Identification

The identification problem can be understood as an optimization problem. A number of unknown parameters are introduced in the differential equations which are expected to describe the evolution of the physical system. A fundamental problem in system modeling is determination of these unknown parameters so that the corresponding response of the model equation approximates as closely as possible the actual response of the physical system [2]. Consider the following model equation

$$M: \begin{cases} \dot{x} = f(t, x, \alpha), & t \in [0, T] \equiv I \\ x(0) = x^0 & given, \end{cases} \tag{6.1}$$

where $x(t) \in R^n$ is the state vector and α, taking values in a closed bounded subset $\mathcal{P}$ of R^m, is the parameter vector which is unknown. The function $f : I \times R^n \times R^m \longrightarrow R^n$ is known, except for the vector α. It is assumed that the response of the physical system, which M is desired to represent, is given in the form of data $y(t), t \in I$. Then the problem is to find a parameter $\alpha^* \in \mathcal{P} \subset R^m$ that minimizes the identification error

$$J(\alpha) = \frac{1}{2} \int_0^T |x(t, \alpha) - y(t)|^2 dt, \tag{6.2}$$

i.e.$J(\alpha^*) \leq J(\alpha)$ for all $\alpha \in \mathcal{P} \subset R^m$, where $x(t, \alpha)$ is the response of the model equation M corresponding to the parameter α and the initial condition x^0.

Based on the above discussion, it is clear that a very natural approach for solving the identification problem is to consider it as a control problem. We can obtain a set of necessary conditions for identification of the unknown parameters.
Define the Hamiltonian $H(t, x, \psi, \alpha)$ by

$$H(t, x, \psi, \alpha) = (f(t, x, \alpha), \psi) + \frac{1}{2}|x(t, \alpha) - y(t)|^2 \tag{6.3}$$

Then by the necessary conditions of optimality, it follows that the optimal parameter α^* and the corresponding pair x^*, ψ^* satisfy the following set of equations:
State equation

$$\begin{cases} \dot{x} = f(t, x, \alpha) \\ x(0) = x^0 \end{cases} \tag{6.4}$$

Costate equation

$$\begin{cases} \dot{\psi} = -f_x'(t,x,\alpha)\psi + (y(t) - x(t)) \\ \psi(T) = 0 \end{cases} \tag{6.5}$$

and the inequality

$$H(t, x^*(t), \psi^*(t), \alpha^*) \leq H(t, x^*(t), \psi^*(t), \alpha) \tag{6.6}$$

for all $\alpha \in \mathcal{P}$.

For numerical computation of the optimal parameters α^*, we can use the gradient method described in section 5.3.

6.2 Problem Formulation and its Solution

As mentioned in the previous section, we formulate the identification problem as an optimal control problem. The unknown parameters are found by minimizing the error between the model response and actual data. Population in the three age groups are known for a period of time, say $I \equiv [0, T]$, including the weekly emigration and immigration rates. We need to estimate the weekly birth and death rates for the three age groups and the transition rates from G1 to G2 and from G2 to G3. These six parameters, which are functions of time, are to be identified (estimated). The state equation, as described before, is given by

$$\begin{cases} \dot{x_1} = \alpha_3 x_2 - \alpha_1 x_1 - \alpha_4 x_1 - e_1 x_1 + i_1 x_1 \\ \dot{x_2} = \alpha_1 x_1 - \alpha_2 x_2 - \alpha_5 x_2 - e_2 x_2 + i_2 x_2 \\ \dot{x_3} = \alpha_2 x_2 - \alpha_6 x_3 - e_3 x_3 + i_3 x_3, \end{cases} \tag{6.7}$$

with the unknown parameters denoted by $\alpha \equiv \{\alpha_1, \cdots, \alpha_6\}$. The parameters α_1, α_2 denote the transition rates from G1 to G2 and from G2 to G3 respectively. The parameter α_3 denotes the birth rate in group G1, and the three parameters α_4, α_5, and α_6 denote the death rates in groups G1,G2, and G3 respectively. The initial population (condition) of the three age groups in the year 1972 is given by:

$$\begin{cases} x_1(0) = 6338787 \\ x_2(0) = 14073277 \\ x_3(0) = 1807496. \end{cases} \tag{6.8}$$

We choose the cost function as being the identification error,

$$J(\alpha) = \frac{1}{2}\int_0^T \| x(t,\alpha) - y(t) \|^2 \, dt \tag{6.9}$$

where $x(t, \alpha)$ denotes the population vector obtained by solving the model equation (6.7) corresponding to any arbitrary choice of the parameter α. The vector,

$$y(t) \equiv (y_1(t), y_2(t), y_3(t))', \tag{6.10}$$

denotes the actual population obtained from census data for the period $[0, T]$. The objective is to find the vector α that minimizes the error $J(\alpha)$. We use optimal control theory (Pontryagin Minimum Principle) to determine this. According to this theory one defines the Hamiltonian $H(t, x, \psi, \alpha)$ as

$$H(t, x, \psi, \alpha) = (f(t, x, \alpha), \psi) + \frac{1}{2} \| x(t, \alpha) - y(t) \|^2 \tag{6.11}$$

where the adjoint state (co-state) ψ (also known as Lagrange multiplier) is given by the solution of the adjoint system,

$$\dot{\psi} = -H_x, \tag{6.12}$$

written explicitly as,

$$\begin{cases} \dot{\psi}_1 = \alpha_1\psi_1 + \alpha_4\psi_1 + e_1\psi_1 - i_1\psi_1 - \alpha_1\psi_2 + y_1 - x_1 \\ \dot{\psi}_2 = -\alpha_3\psi_1 + \alpha_2\psi_2 + \alpha_5\psi_2 + e_2\psi_2 - i_2\psi_2 - \alpha_2\psi_3 + y_2 - x_2 \\ \dot{\psi}_3 = \alpha_6\psi_3 + e_3\psi_3 - i_3\psi_3 + y_3 - x_3 \end{cases}, \tag{6.13}$$

subject to the terminal condition given by $\psi(T) = 0$; that is,

$$\psi_1(T) = \psi_2(T) = \psi_3(T) = 0. \tag{6.14}$$

The gradient vector $\partial H/\partial\alpha \equiv g(\alpha)$ is given by

$$g(\alpha) \equiv g(\alpha_1, \alpha_2, \alpha_3, \alpha_4, \alpha_5, \alpha_6) = \begin{cases} x_1(\psi_2 - \psi_1) \\ x_2(\psi_3 - \psi_2) \\ x_2\psi_1 \\ -x_1\psi_1 \\ -x_2\psi_2 \\ -x_3\psi_3. \end{cases} \tag{6.15}$$

Using the state and co-state equations and the gradient as defined above, one can write an algorithm for numerical solution of the optimization problem.

6.3 Simulation Results Based on Identified Parameters

For the three age groups, Figures 6.1-6.3 show the actual and model population, the later based on identified parameters (birth rate, death rates and transition rates). Because the two curves overlap, it appears that there is no difference between the actual and the model population. In fact the maximum error over the time period ($31 years$) is about ± 327 which is considered to be an excellent match.

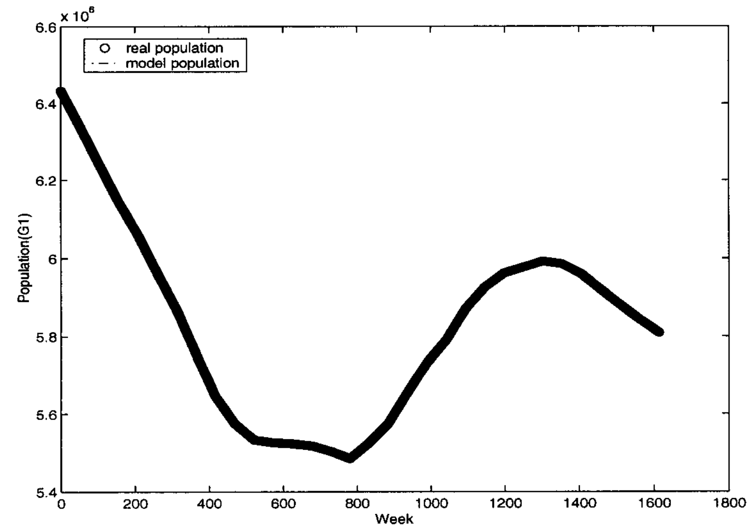

Figure 6.1: Population of G1

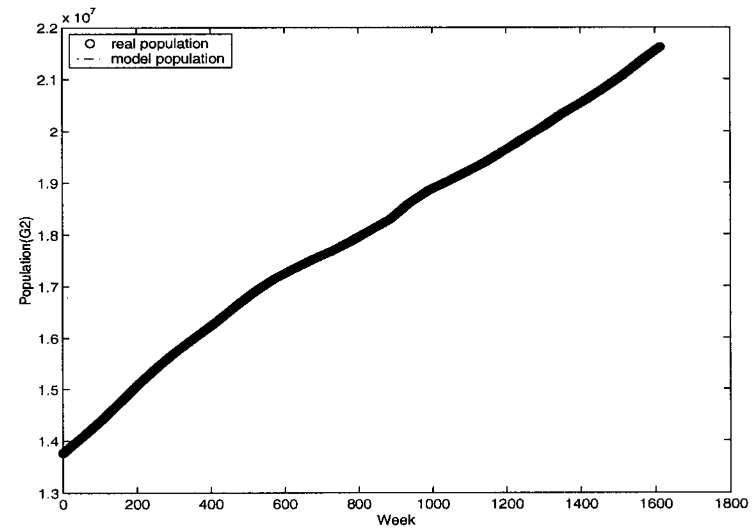

Figure 6.2: Population of G2

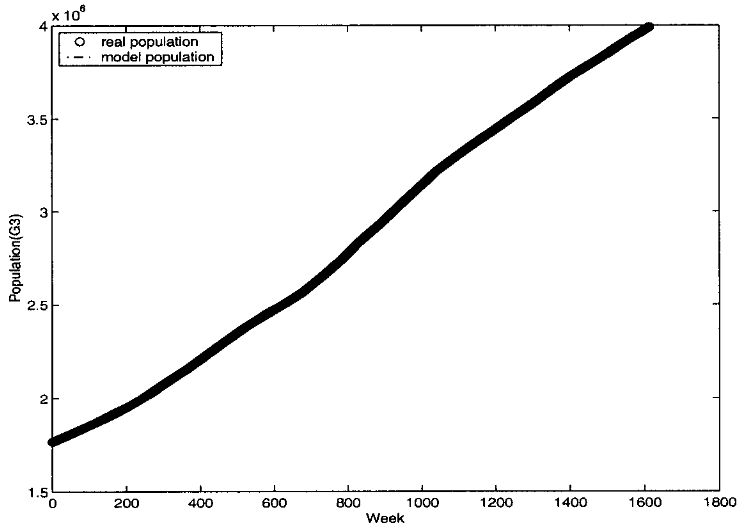

Figure 6.3: Population of G3

6.4 Comparison of Results Based on Available and Identified Parameters

The simulation results based on actual parameters are shown in Figures 4.1-4.3, and those based on identified parameters are displayed in Figures 6.1-6.3. It is clear that the model population based on identified parameters matches the real population much better.

Chapter 7

Optimum Immigration Policy

There has been a long history of immigration in Canada. The first period of immigration was in the last decade of the nineteenth-century. Since then many people have immigrated to Canada. The Canadian population has a high percentage of foreign-born people. In order to avoid population decline and maintain a level of labour force to sustain economic growth [40], Canada is looking for more immigrants. The Canadian government considers immigration as a priority.

Some developing countries like China and India are over populated. This causes many social problems in areas like education, employment,transportation and environment. In order to pursue better living standards, people from these developing countries are eager to migrate to developed countries. On the other hand, some developed countries, like Canada, Australia and New Zealand accept immigrants from other countries to prevent their own population decline and maintain a labour force for economic growth.

Since immigration is one of the most efficient instruments of population growth for many countries [8], immigration in Canada will be considered as a control variable in this thesis.

Canada has some very clear immigration policies. In the fall of each year, the immigration level for the coming year is set by the Canadian Government. Using systems and control theory as proposed here one can determine the optimum immigration levels subject to any number of constraints that may be applicable. This is formulated in the following section.

7.1 Problem Formulation and its Solution

Our objective is to maintain the total population above a minimum level and not to exceed far above a maximum level. Ideally the immigration policy should be one that maintains the population variation within the specified band determined by the minimum and the maximum levels. This is to be achieved by adopting an appropriate immigration rate (policy) for the age group G2. This appropriate immigration rate is obtained by minimizing a cost functional which penalizes when the population level is outside the boundary of the target described above.

The weekly birth and death rates; the transition rates from group G1 to group G2, and group G2 to group G3; the emigration rates; and the lower and upper boundaries of the target population are given. The immigration rate u of the age group G2 (adult population) is the control variable to be determined.

The state equation, including immigration rate as the control variable, is given by

$$\begin{cases} \dot{x_1} = bx_2 - \tau_{12}x_1 - d_1x_1 - e_1x_1 + p_1ux_2 \\ \dot{x_2} = \tau_{12}x_1 - \tau_{23}x_2 - d_2x_2 - e_2x_2 + ux_2 \\ \dot{x_3} = \tau_{23}x_2 - d_3x_3 - e_3x_3 + p_2ux_2 \end{cases} \tag{7.1}$$

where u denotes the (weekly) immigration rate. The parameters p_1 and p_2 are nonnegfractions determining the accompanying children and seniors. Here, $p_1 = 0.31$ and $p_2 = 0.054$ are obtained by using the mean of actual values reported by Statistics Canada. The range of the control variable is defined by the interval $0 \leq u \leq u_M = 5 \times 10^{-4}$, where the upper limit means that immigration rate must not exceed 0.05 percent of the adult population. This limit can be fixed by the immigration department on the basis of other national concerns. The initial population of the three age groups in the year 1976 is given by:

$$\begin{cases} x_1(0) = 5959921 \\ x_2(0) = 15467175 \\ x_3(0) = 2022697. \end{cases} \tag{7.2}$$

In view of the objective described above, the cost function is defined as follows:

$$\begin{aligned} J(u) &= \int_0^T (\lambda_1(x_M - x_1 - x_2 - x_3)^2 I_1((x_1 + x_2 + x_3) > x_M) \\ &\qquad + \lambda_2(x_m - x_1 - x_2 - x_3)^2 I_2((x_1 + x_2 + x_3) < x_m))dt \\ &= \int_0^T \ell(t, x, u)dt, \end{aligned} \tag{7.3}$$

for convenience of explanation, we define $\ell_1 = \lambda_1(x_M - x_1 - x_2 - x_3)^2$, $\ell_2 = \lambda_2(x_m - x_1 - x_2 - x_3)^2$. so $\ell(t, x, u)$ can be represented as $\ell = \ell_1 I_1 + \ell_2 I_2$ where $I_1(\zeta)$ and $I_2(\zeta)$ are indicator functions given by

$$I_1(\zeta) = \begin{cases} 0 & \zeta \leq x_M \\ 1 & \zeta > x_M \end{cases} \tag{7.4}$$

$$I_2(\zeta) = \begin{cases} 0 & \zeta \geq x_m \\ 1 & \zeta < x_m. \end{cases} \tag{7.5}$$

The symbols x_m, x_M denote the lower and upper boundaries of the target population $[x_m, x_M]$, λ_1 and λ_2 are the weights assigned to deviations from the boundaries of the target set. They are chosen as $\lambda_1 = 0.5 \times 10^{19}$ and $\lambda_2 = 0.5 \times 10^{20}$. We choose $\lambda_2 > \lambda_1$ to emphasize the relative importance of maintaining the population above the lower limit. Again, the Hamiltonian $H(t, x, \psi, u)$ is given by

$$H(t, x, \psi, u) = (f(t, x(t), u(t)), \psi(t)) + \ell(t, x(t), u(t)) \tag{7.6}$$

where the adjoint system is given by

$$\dot{\psi} = -H_x \tag{7.7}$$

it can be represented in three different cases

$$\dot{\psi} = \begin{cases} -f_x'\psi - \ell_{1x} & (x_1 + x_2 + x_3) > x_M \\ -f_x'\psi - \ell_{2x} & (x_1 + x_2 + x_3) < x_m \\ -f_x'\psi & x_m \leq (x_1 + x_2 + x_3) \leq x_M, \end{cases} \tag{7.8}$$

or equivalently,

$$\begin{cases} \dot{\psi}_1 = \tau_{12}\psi_1 + d_1\psi_1 + e_1\psi_1 - \tau_{12}\psi_2 + \beta \\ \dot{\psi}_2 = -b\psi_1 - p_1u\psi_1 + \tau_{23}\psi_2 + d_2\psi_2 + e_2\psi_2 - u\psi_2 - \tau_{23}\psi_3 - p_2u\psi_3 + \beta \\ \dot{\psi}_3 = d_3\psi_3 + e_3\psi_3 + \beta, \end{cases} \tag{7.9}$$

where, for convenience of presentation, we have introduced the function β as given by

$$\begin{aligned} \beta &= 2\lambda_1(x_M - x_1 - x_2 - x_3)I_1((x_1 + x_2 + x_3) > x_M) \\ &\quad +2\lambda_2(x_m - x_1 - x_2 - x_3)I_2((x_1 + x_2 + x_3) < x_m) \end{aligned} \tag{7.10}$$

and the terminal co-state $\psi(T) = 0$ as for equation (7.8). In this case the gradient vector is given by

$$g(u) = p_1x_2\psi_1 + x_2\psi_2 + p_2x_2\psi_3. \tag{7.11}$$

Using the state and co-state equations along with the gradient as defined above, we use the optimization methodologies indicated in chapter 5 to determine optimum immigration policies.

7.2 Numerical Analysis and Simulation Result

7.2.1 Without Specified Target (Fig.7.1-7.3)

In this subsection, we show numerical results on population growth corresponding to actual immigration and emigration data (Fig.7.1) obtained from Statistics Canada. No target population was specified. Figure 7.2 shows the actual population dynamics in Canada from 1976 to 2002 as reported by Statistics Canada. Figure 7.3, describes the corresponding model population based on identified birth, death, and transition rates and given immigration and emigration rates as shown in Fig.7.1A-7.1B. The results are quite close. Clearly Fig.7.3 shows the natural population dynamics without any external intervention (that is without any other control). Thus according to current immigration policy, the Canadian population would monotonically increase with time.

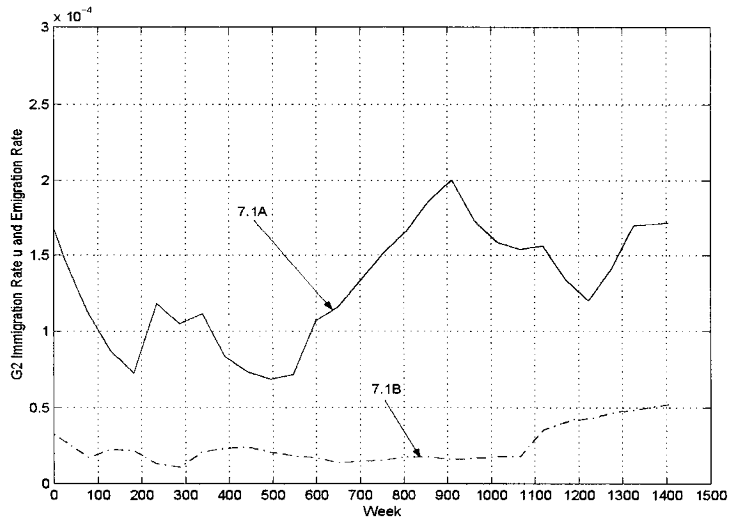

Figure 7.1: Immigration and Emigration Rates of G2 (Statistics Canada)

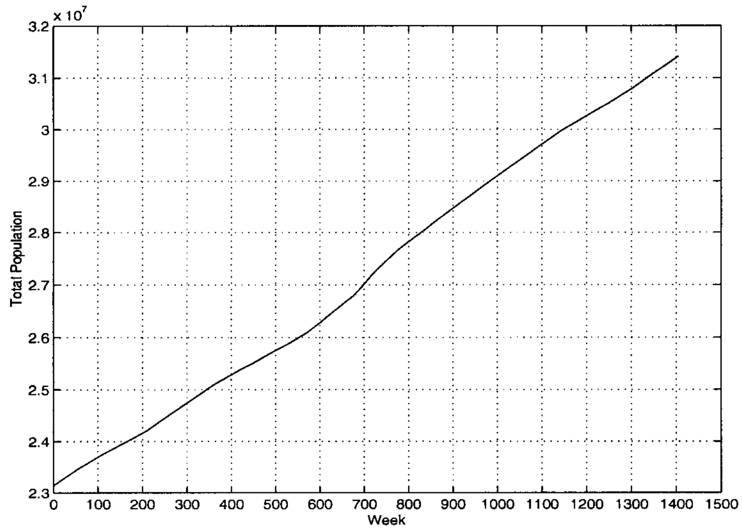

Figure 7.2: Total Population (Statistics Canada)

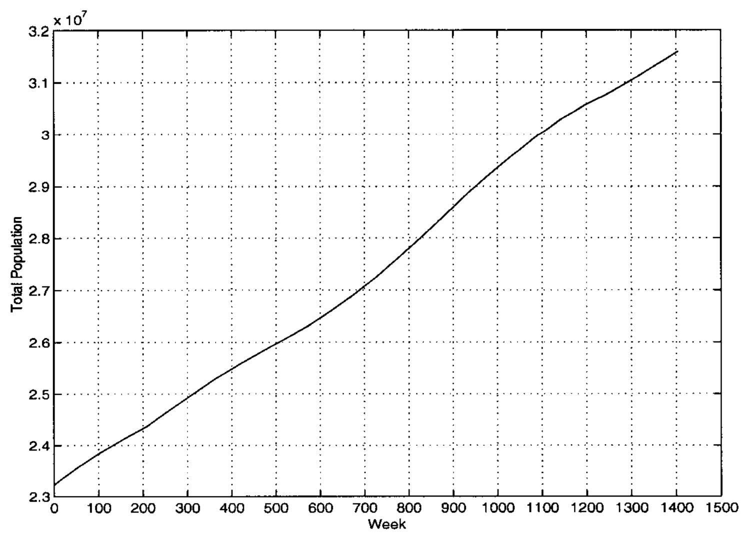

Figure 7.3: Total Population (Model)

7.2.2 With Specified Target (Fig.7.4-7.12)

In this subsection, we present numerical results corresponding to population targets as described in section 7.1. Using the system (7.1) with immigration rate u as the control variable and (7.3) as the objective functional, and using the optimization procedure as described in section 7.1, we obtain the optimum immigration policy. The results corresponding to the optimum immigration policy are displayed in Figures 7.4-7.12.

(A) Constant Target (Fig.7.4-Fig.7.8)

The gradients corresponding to the targets $T_1 \equiv [x_m, x_M] = [2.8 \times 10^7, 2.9 \times 10^7]$ and $T_2 \equiv [x_m, x_M] = [2.9 \times 10^7, 3 \times 10^7]$ are indicated in Figure 7.4 and 7.5 respectively. The optimum control (or immigration) policies corresponding to the targets T_1 and T_2 are shown in Figures 7.6 and 7.7. The curves 7.6B and 7.7B are the expanded versions of 7.6A and 7.7A respectively showing the detailed transition from high to low immigration rates. Note that the immigration rate corresponding to target T_2 remains at its maximum admissible rate for a longer period of time compared to that of target T_1 because the lower and the upper boundaries of T_2 are above those of T_1. It appears from this result that optimum immigration policy is to keep the immigration rate (actual number $= rate \times x_2$) at its highest admissible level (constant) during the early period of the planning horizon and then rapidly reduce to zero. In control theory this kind of phenomenon is known as bang-bang control.

The graphs of Figure 7.8 show the corresponding population growth for target sets T_1 and T_2. The cost functional (7.3) is designed so as to reach the desired targets. The solid curve represents population growth corresponding to the target T_1 and the dotted curve corresponds to T_2. It is clear from this result that population increases much more rapidly during the first 5-6 years. This is because the target is far from the initial population and the error is large which encourages maximum immigration rate for reaching the lower boundary as quickly as possible. The speed of approach is dependent on the maximum admissible immigration rate u_M. Once the total population exceeds the lower boundary, the growth slows down and the optimum immigration policy seems to maintain the population in the neighborhood of the target set. In fact once the population exceeds the upper boundary, the optimum policy seems to pull it down by cutting down immigration rate.

Comparing the population growth (Fig.7.8)corresponding to optimum policies with those of (Fig.7.2 and Fig.7.3) corresponding to the official immigration policy, it is clear that the total population can be well regulated and steered to any specified target.

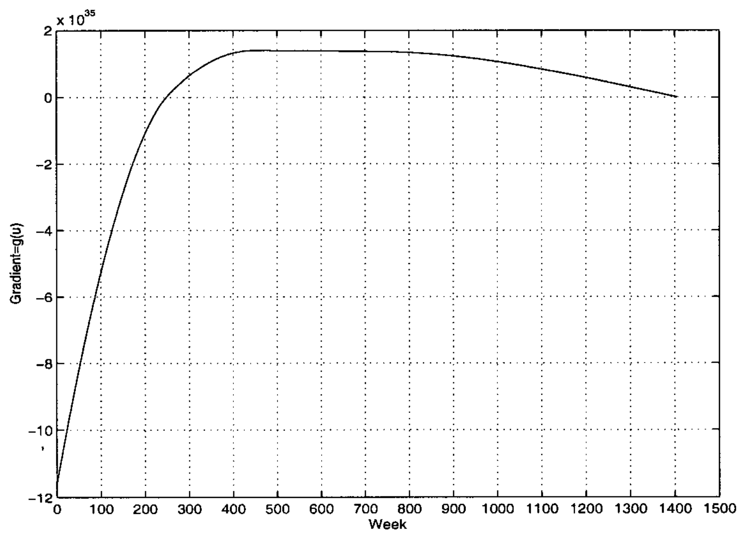

Figure 7.4: Gradient (Target T_1)

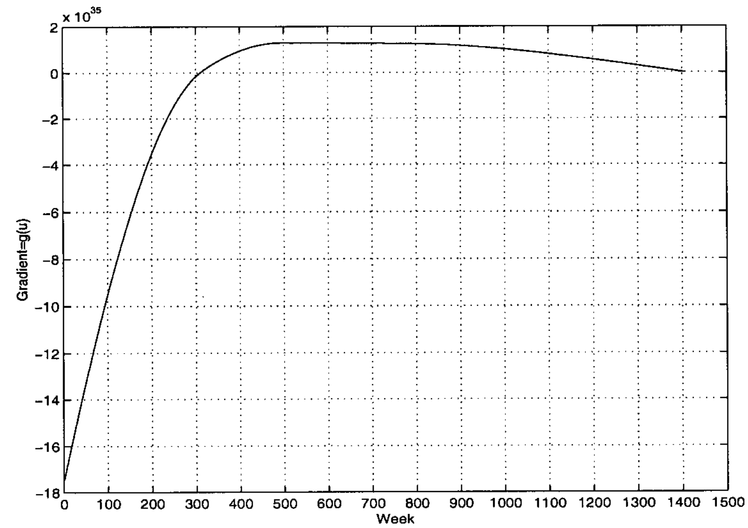

Figure 7.5: Gradient(Target T_2)

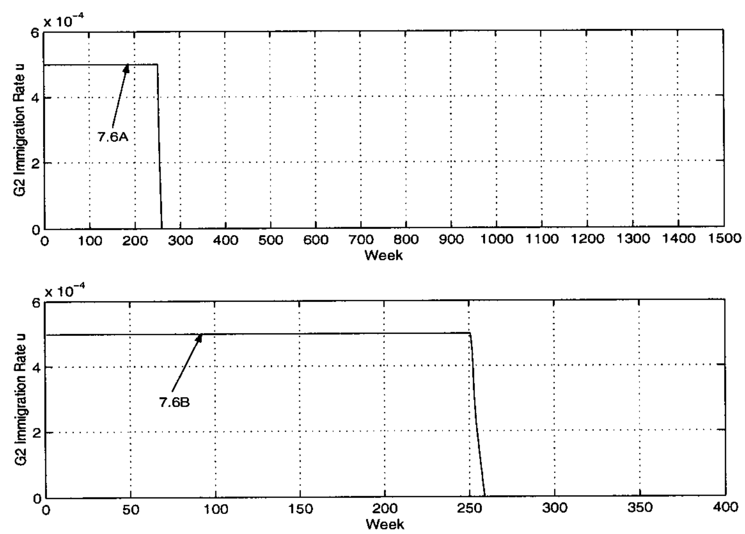

Figure 7.6: Optimum Immigration Rate of G2 (Target T_1)

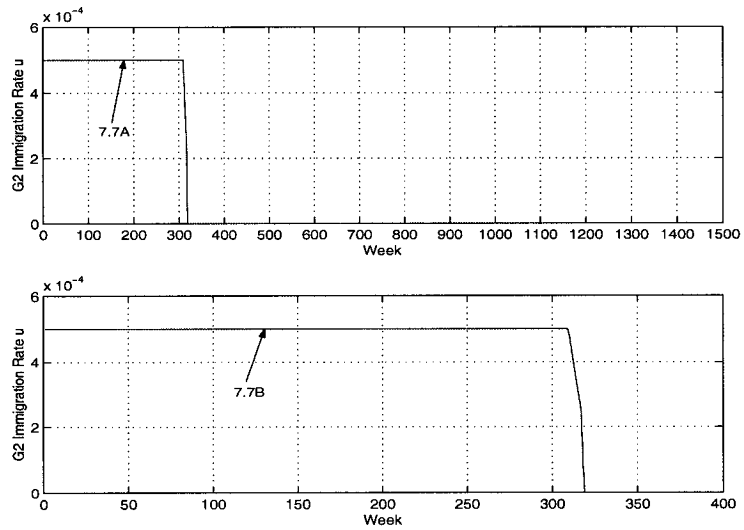

Figure 7.7: Optimum Immigration Rate of G2 (Target T_2)

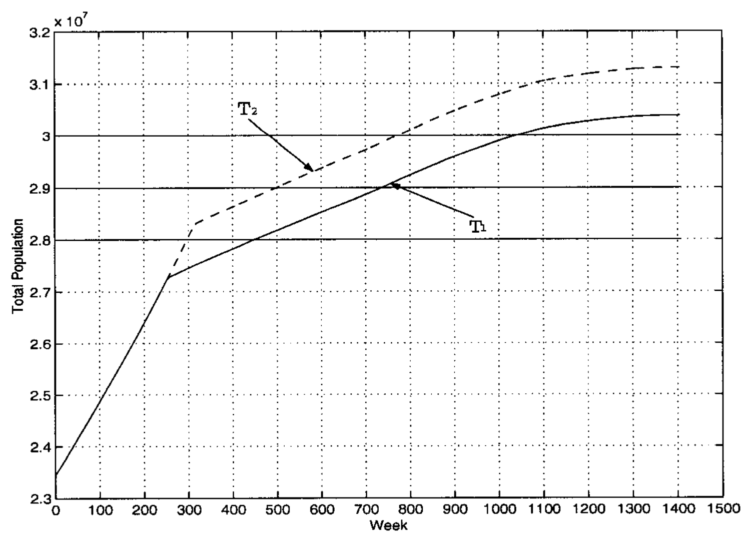

Figure 7.8: Total Population with Optimized Immigration Rate (Constant Target)

(B) Variable Target (Fig.7.9-Fig.7.12)

If it is required to reach the desired population level smoothly (over several years), the target set can be modified by use of a pair of smooth curves describing the upper and the lower boundaries starting around the initial population see Fig.7.10. For illustration, we chose the variable target given by $T_3(t) \equiv [x_m(t), x_M(t)]$ where

$$x_m(t) = 1.23 \times 10^7(1 - e^{-0.0007t}) + 2.30 \times 10^7$$
$$x_M(t) = 1.23 \times 10^7(1 - e^{-0.0007t}) + 2.35 \times 10^7.$$

The results are shown in Fig.7.9-7.10. Fig.7.9 shows the optimal control (immigration) policy. It is clear from the graph that it is smooth (not bang-bang as in the case of constant target) and the corresponding population grows smoothly and remains confined in the target set.

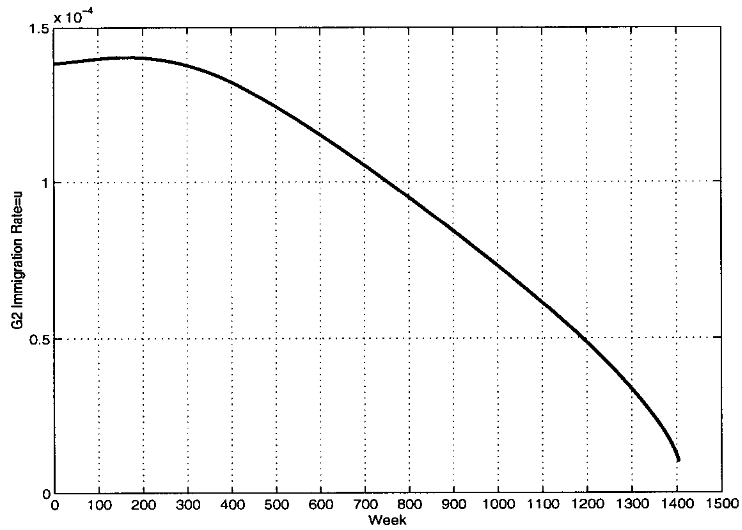

Figure 7.9: Optimum Immigration Rate of G2 ($T_3(t)$)

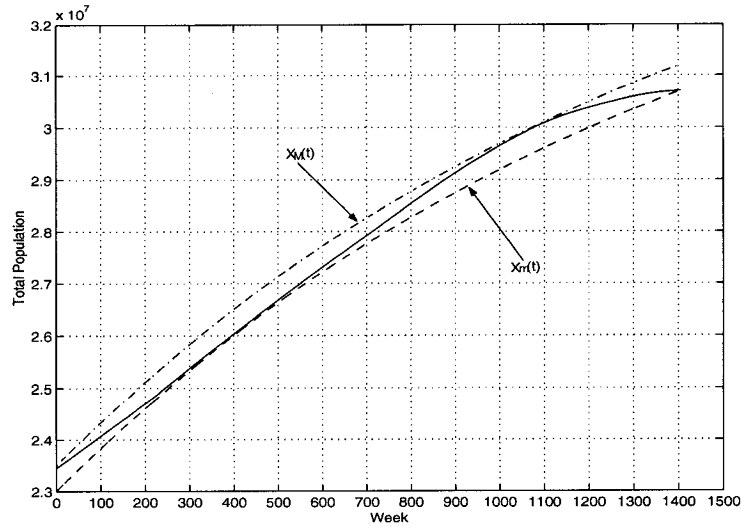

Figure 7.10: Total Population with Optimized Immigration Rate ($T_3(t)$)

Now, the variable target is defined by $T_4(t) \equiv [x_m(t), x_M(t)]$ where

$$x_m(t) = 1.23 \times 10^7(1 - e^{-0.0007t}) + 2.40 \times 10^7$$

$$x_M(t) = 1.23 \times 10^7(1 - e^{-0.0007t}) + 2.50 \times 10^7.$$

The results are shown in Fig.7.11 and 7.12. The optimal immigration rate is displayed in Figure 7.11. The immigration rate remains at its maximum admissible rate u_M at the beginning of simulation, then it gradually decreases compared to Figure 7.6 and 7.7. As can be seen, this is not bang bang control. The corresponding population growth is indicated in Figure 7.12. As shown in Figure 7.12, the initial value of total population is below the lower target $x_m(t)$.

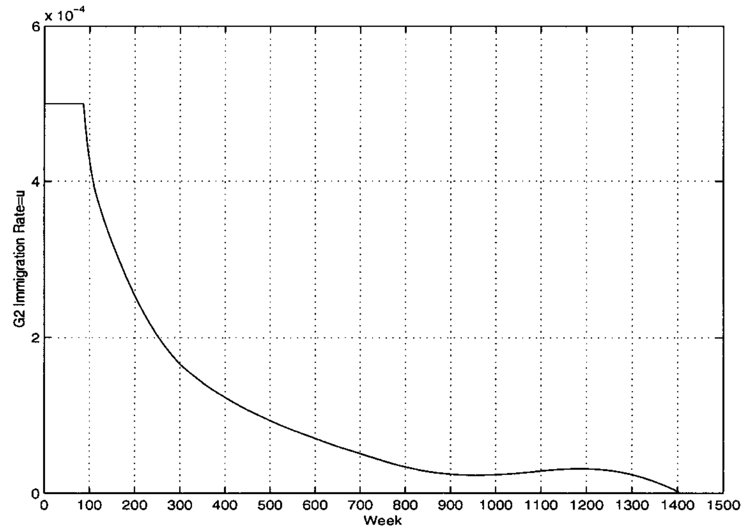

Figure 7.11: Optimum Immigration Rate of G2($T_4(t)$)

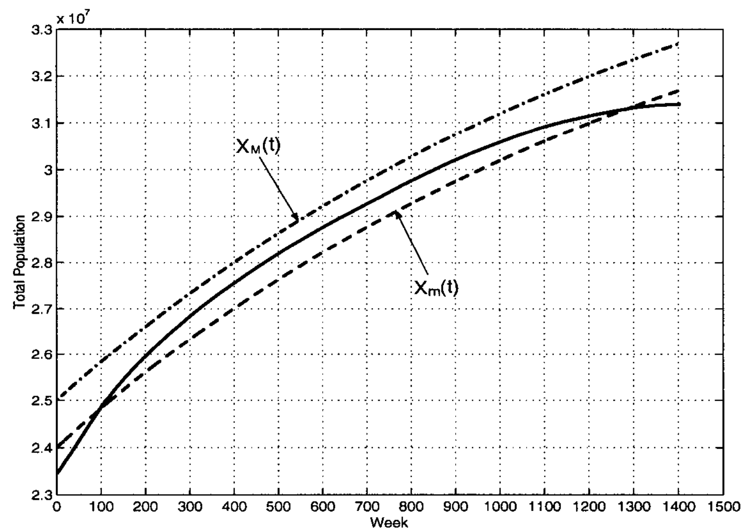

Figure 7.12: Total Population with Optimized Immigration Rate ($T_4(t)$)

We conclude from these results that control theory provides a promising tool for determining the optimum immigration policy seeking specified targets. In fact one can add to the cost function as many factors as one desires to reflect the concerns of the society and use the optimization methodology proposed here to determine the optimum policy. This technique can be used by the Department of Immigration as an intelligent tool for determining the optimum immigration policy.

We must mention that the numerical results presented here are applicable only for the years 1976-2002. For current and future years one must use the corresponding data for simulation and optimization and derive the optimum policies.

Chapter 8

Immigration Policy Versus Job Creation Rate

Besides having enough immigration to maintain the population within the predefined boundary values, keeping the labour force at a certain level to allow for economic development is also considered.

Although many factors may affect the immigration policy, the unemployment rate is one of the most important [41][42][43]. The question is what should be the intake rate (immigration rate) so as to satisfy the manpower demand and at the same time keep the unemployment rate low. In addition to humanitarian factors, it is reasonable to tie the immigration rate with availability of jobs and job creation rate. Otherwise one can expect many social and political problems.

8.1 Modified System Model and Problem Formulation

The system model (7.1) of the previous section does not include the unemployment factor. To introduce this factor, we note that the growth or decline of unemployment rate is proportional to the growth or decline of population of age group G2 (the employable population) and the job growth rate. For this purpose we introduce a fourth state variable x_4,the unemployed population, and modify the model (7.1) to the following one,

$$\begin{cases} \dot{x_1} = bx_2 - \tau_{12}x_1 - d_1x_1 - e_1x_1 + p_1u_1x_2 \\ \dot{x_2} = \tau_{12}x_1 - \tau_{23}x_2 - d_2x_2 - e_2x_2 + u_1x_2 \\ \dot{x_3} = \tau_{23}x_2 - d_3x_3 - e_3x_3 + p_2u_1x_2 \\ \dot{x_4} = \big(\tau_{12}x_1 - \tau_{23}x_2 - d_2x_2 - e_2x_2 + u_1x_2\big) - u_2x_4 \end{cases} \tag{8.1}$$

where $\dot{x_4}$ is the growth rate of the unemployed population(considering only second age group G2). This is given by the growth rate (rise/decline) of the population of group G2 minus the job creation (or loss) rate u_2x_4. The range of the decision variables $\{u_1, u_2\}$ are given by $0 = a_1 \leq u_1 \leq b_1$, $0 = a_2 \leq u_2 \leq b_2$. The initial condition is denoted by

$$x(0) = (x_{01}, x_{02}, x_{03}, x_{04})', \tag{8.2}$$

where the value of x_{04} gives the number of people initially unemployed. The cost functional (7.3) is modified as follows:

$$\begin{aligned} J(u) \quad = \quad & \int_0^T \{\lambda_1(x_M - x_1 - x_2 - x_3)^2 I_1((x_1 + x_2 + x_3) > x_M) \\ & + \lambda_2(x_m - x_1 - x_2 - x_3)^2 I_2((x_1 + x_2 + x_3) < x_m) \\ & + \lambda_3(x_4)^2 + \lambda_4(u_1x_2)^2 + \lambda_5(u_2x_4)^2\}dt, \end{aligned} \tag{8.3}$$

where λ_1, λ_2, λ_3, λ_4 and λ_5 are the weights assigned to each of the factors. The first two terms are identical to those of the cost functional (7.3). The third term represents the cost of unemployment (measured in terms of loss of productivity, UI payments, welfare payments etc), the fourth term represents the cost of administering immigration and the fifth term represents the cost of creating new jobs. For numerical simulation, the values chosen are $\lambda_1 = 0.5 \times 10^{15}$, $\lambda_2 = 0.5 \times 10^{16}$,$\lambda_3 = 0.5 \times 10^{13}$,$\lambda_4 = 0.5 \times 10^{12}$,$\lambda_5 = 0.5 \times 10^{12}$. These parameters can be arbitrarily chosen by the planner. Using the state equation and the cost functional, we have the Hamiltonian given by,

$$H(t, x, \psi, u) \quad = \quad (f(t, x(t), u(t)), \psi(t)) + \ell(t, x(t), u(t)) \tag{8.4}$$

where the adjoint system is given by

$$\dot{\psi} = -H_x \tag{8.5}$$

which after expansion gives:

$$\begin{cases} \dot{\psi}_1 = \tau_{12}\psi_1 + d_1\psi_1 + e_1\psi_1 - \tau_{12}\psi_2 - \tau_{12}\psi_4 + \beta \\ \dot{\psi}_2 = -b\psi_1 - p_1u_1\psi_1 - u_1(\psi_2 + \psi_4) + A(\psi_2 + \psi_4) - \tau_{23}\psi_3 - p_2u_1\psi_3 - 2\lambda_4u_1^2x_2 + \beta \\ \dot{\psi}_3 = d_3\psi_3 + e_3\psi_3 + \beta \\ \dot{\psi}_4 = -2\lambda_3x_4 + u_2\psi_4 - 2\lambda_5u_2^2x_4. \end{cases} \tag{8.6}$$

For notational convenience, we use A to denote the sum of terms as shown

$$A = \tau_{23} + d_2 + e_2 \tag{8.7}$$

and β as defined in equation (7.9). The terminal costate is given by

$$\psi_1(T) = \psi_2(T) = \psi_3(T) = \psi_4(T) = 0, \tag{8.8}$$

and the gradient vector is given by

$$g(u) = \begin{cases} x_2(p_1\psi_1 + \psi_2 + p_2\psi_3 + \psi_4) + 2\lambda_4u_1x_2^2 \\ -x_4\psi_4 + 2\lambda_5u_2x_4^2. \end{cases} \tag{8.9}$$

For numerical results, using the state equation (8.1) and the co-state equation (8.6) along with the gradient (8.9) as defined above, we use the algorithm as indicated in section 5.3.

8.2 Unemployment Rate

According to Statistics Canada definition, the unemployment rate is given by the ratio of the number of unemployed people in labour force to the number of people in the labour force. Here, we only consider labour force in the age group G2.

For composition of labour force, a certain percentage of the population in age group G2 is eliminated. These are people in the age group G2 who are not actively seeking for jobs (such as students, handicaps, terminally sick, etc.). According to Statistics Canada definition, the labour force is given by some percentage of this population. That is,

$$\text{labour force } = p \times x_2.$$

The factor p is a function of time as shown in Figure 8.1. The unemployment rate is then expressed by:

$$\begin{aligned} \text{Unemployment rate} &= \frac{\text{labour force } - (x_2 - x_4)}{\text{labour force}} \times 100\% \\ &= \frac{px_2 - (x_2 - x_4)}{px_2} \times 100\%. \end{aligned} \tag{8.10}$$

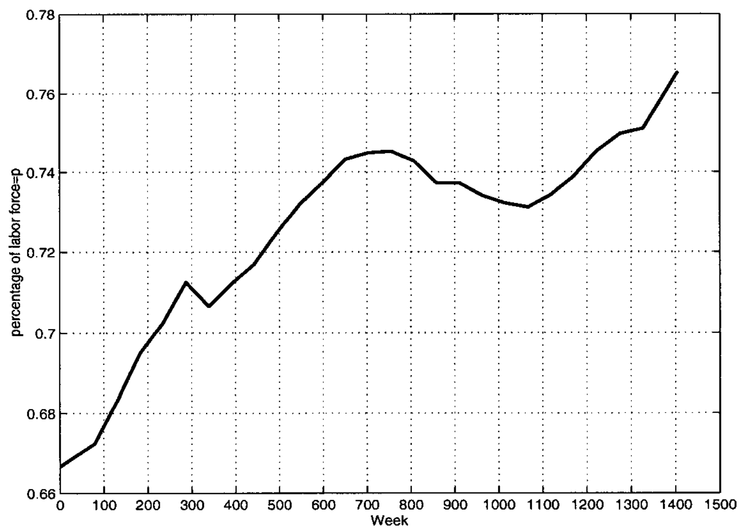

Figure 8.1: Percentage of labour Force in G2 (Statistics Canada)

8.3 Data Used for Simulation

For simulation results, the data used are as given below:

(A) Initial Condition of the State Equation

The initial condition for the state equation is given by:

$$\begin{cases} x_{01} = 5959921 \\ x_{02} = 15467175 \\ x_{03} = 2022697 \\ x_{04} = 6304875. \end{cases} \tag{8.11}$$

(B) Control Constraint

Control variable	Case A	Case B
u_1	$0 \leq u_1 \leq 5 \times 10^{-4}$	$0 \leq u_1 \leq 5 \times 10^{-4}$
u_2	$0 \leq u_2 \leq 1.63 \times 10^{-3}$	$0 \leq u_2 \leq 1.81 \times 10^{-3}$

Table 8.1: Control Constraints

For better illustration and comparison of results, we use two different sets of control constraints(A) and (B). The ranges of control variables u_1, u_2 are described in Table 8.1. It is clear that the upper limit of the job creation rate in case (B) is larger than that of case (A).

(C) Percentage of labour force (Fig.15)

Percentage of labour force, which is obtained from Statistics Canada, is shown in Figure 8.1.

8.4 Discussion of Simulation Results

Using the system equation (8.1) with immigration rate u_1 and job creation rate u_2 as control variables, corresponding to the target level $T_2 \equiv [x_m, x_M] = [2.9 \times 10^7, 3 \times 10^7]$, the expression (8.3) as the objective functional and the optimization procedure as described in section 8.1, we obtain the optimum immigration and job creation policies. The results are displayed in Figures 8.2 - 8.9.

The gradient $g(u_1)$ for Cases A-B is displayed in Figure 8.2. Figure 8.3 shows the gradient $g(u_2)$ for Cases A-B. The optimum immigration rate (or intake rate) of population of age group G2 is shown in Figure 8.4. The curves Case A and B represent the simulation results corresponding to the Cases A and B respectively. The optimum immigration policies corresponding to cases A and B are very close. Hence, the total population of the two cases (shown in Figure 8.6) coincides. But the optimum job creation rates indicated in Figure 8.5 are significantly different.

Again it is clear that for constant (non variable) target, the optimal control policy is nearly bang-bang. As a result of this control policy, the population rapidly increases initially and then slows down preventing large deviation from the upper boundary of the target set. Again, we expect control policies to be smooth if variable target $T(t)$ is chosen as in subsection 7.2.2.

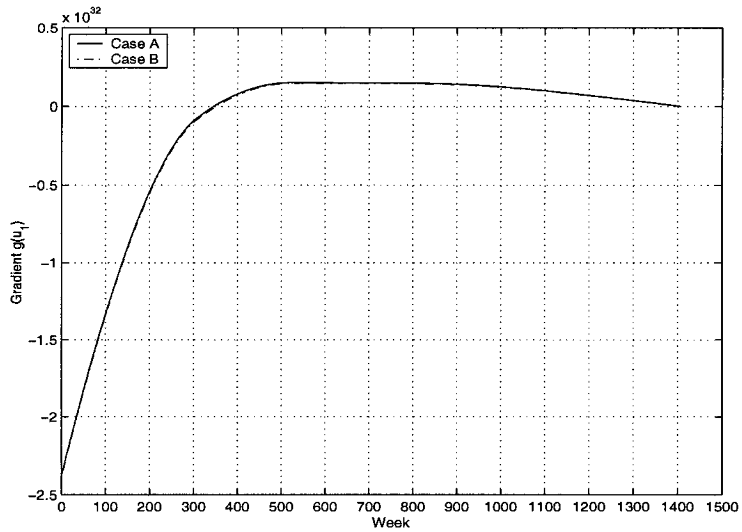

Figure 8.2: Gradient $g(u_1)$

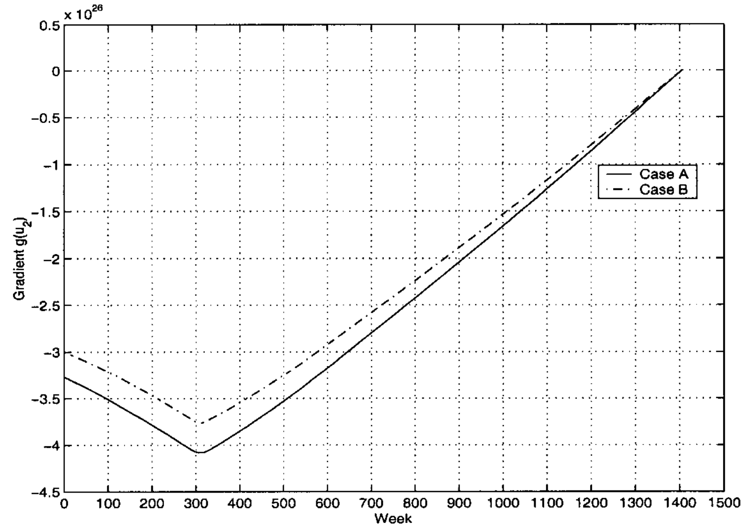

Figure 8.3: Gradient $g(u_2)$

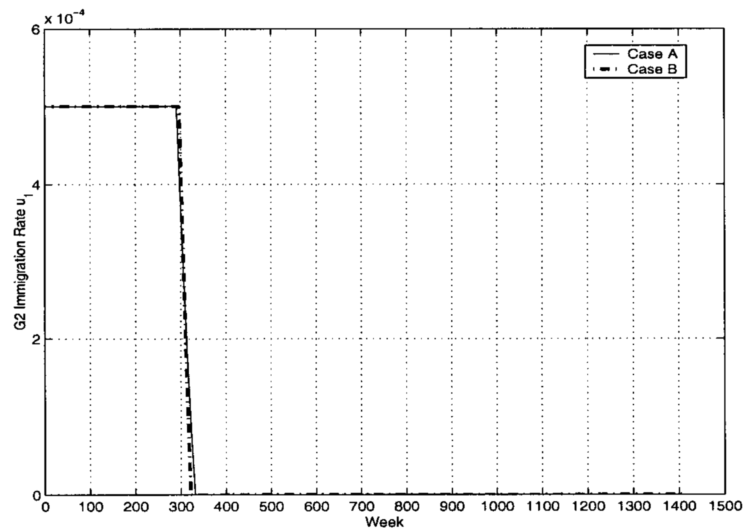

Figure 8.4: Optimum Immigration Rate of G2

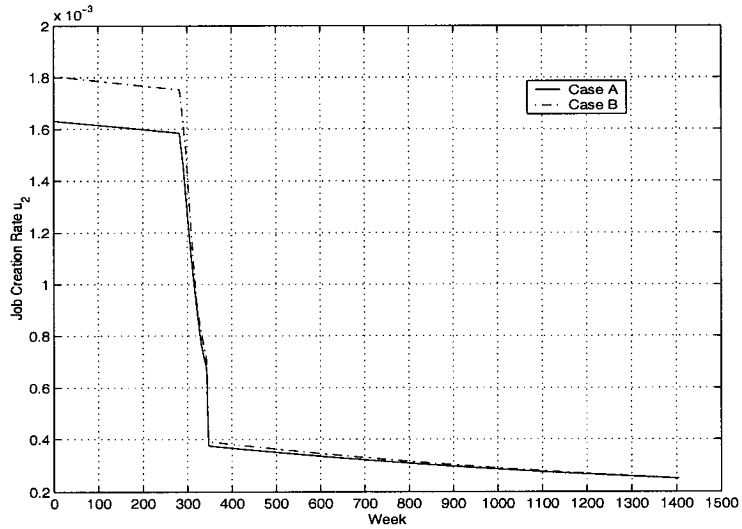

Figure 8.5: Optimum Job Creation Rate (for G2)

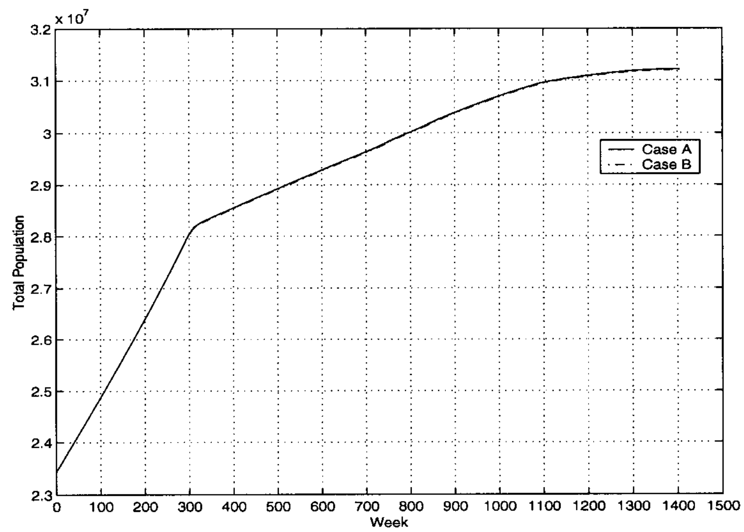

Figure 8.6: Total Population with Optimized Immigration and Job Creation Rates

The unemployment rate is calculated by equation (8.10), which is also represented as follows:

$$\begin{aligned} \text{Unemployment rate} &= \frac{px_2 - (x_2 - x_4)}{px_2} \times 100\% \\ &= (\frac{p-1}{p} + \frac{x_4}{px_2}) \times 100\%. \end{aligned} \tag{8.12}$$

Since the percentage of labour force p (Figure 8.1) is the same for cases A and B, the value of $\frac{p-1}{p}$ in case A is equal to that in case B. The working age group populations x_2 for cases A and B (Figure 8.7) are very close, but the job creation rate in case B is higher compared to that in case A (Figure 8.5), thus the unemployed population of age group G2 x_4 in case A is higher than that in case B (shown in Figure 8.8). Based on the above results, the value of $\frac{x_4}{px_2}$ in case A is higher than that in case B. From the above analysis, the unemployment rate $(\frac{p-1}{p} + \frac{x_4}{px_2})$ in case B is lower than that in case A, which is consistent with the simulation result shown in Figure 8.9.

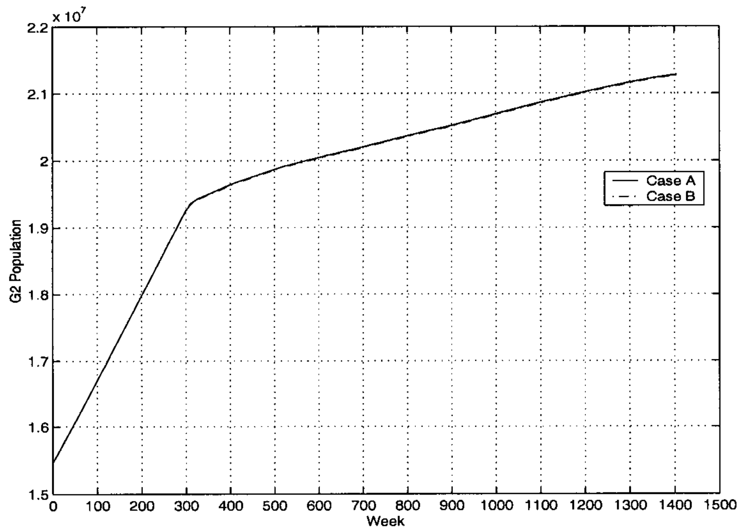

Figure 8.7: Population of G2

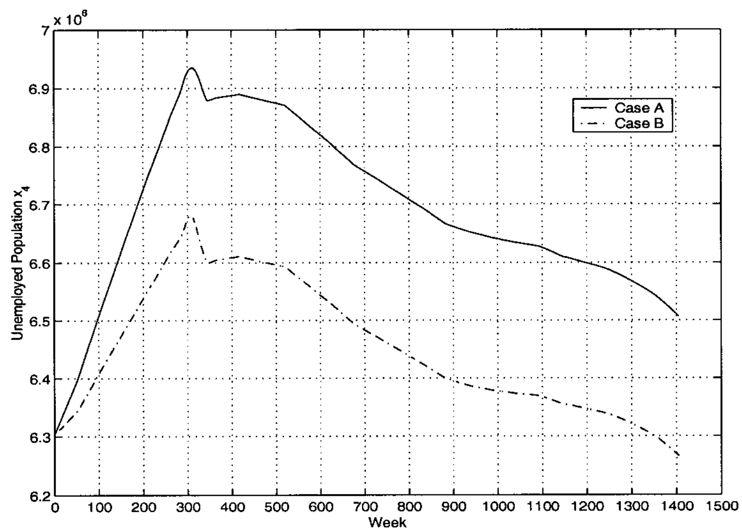

Figure 8.8: Unemployed Population (x_4)

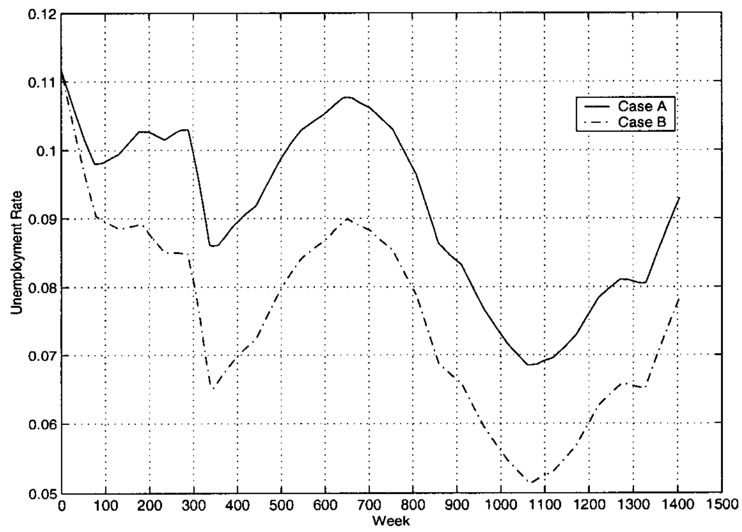

Figure 8.9: Unemployment Rate

Chapter 9

Conclusion and Future Work

9.1 Conclusion

In this thesis we have demonstrated that by using modern Systems and Optimal Control theory, that is possible to formulate optimum immigration and job creation strategies while maintaining population level close to certain pre-specified targets.

We have constructed in chapter 3, a simplified dynamic population model using the models proposed in [1]. Using the basic data (birth, death and transition rates etc.) from Statistics Canada [30], we found the simulation results (population) based on our model in close agreement with the actual population. This was presented in chapter 4.

Optimization methodologies, which are the basic methods employed in chapter 6, 7 and 8, were discussed in chapter 5. In case of non availability of the basic parameters, one must use available population statistics to determine the unknown parameters. This is known as identification, and we have demonstrated in chapter 6 that optimal control theory can be used to determine these parameters.

Based on the model constructed in chapter 3, we have formulated in chapter 7 a control problem with the objective of reaching a specified population target (constant as well as variable) by use of immigration rate as the control variable. Optimal control theory is used to determine the optimum immigration policy as illustrated by numerical results.

In chapter 8, the population model is augmented by including a fourth equation describing the dynamics of unemployment rate, including job creation rate as another control variable. Following our methodology optimum immigration and job creation policies were determined. Results are illustrated by numerical simulation and they are found to be very encouraging.

Using actual field data along with the desired objective functional reflecting important social concerns, and following the methodology presented here one can develop optimum immigration and job creation policies from the simulation results.

9.2 Future Work

The proposed dynamic demographic models in this thesis can be employed to identify unknown parameters and optimize immigration and job creation rates. Based on our work already done, future investigations will be on the following points: (1) Developing demographic models for sex, education and marital status; (2)Using the present dynamic models to predict the population in the future; (3) Adding labour forces into age group 3; (4)Using neural networks to construct more accurate nonlinear demographic models.

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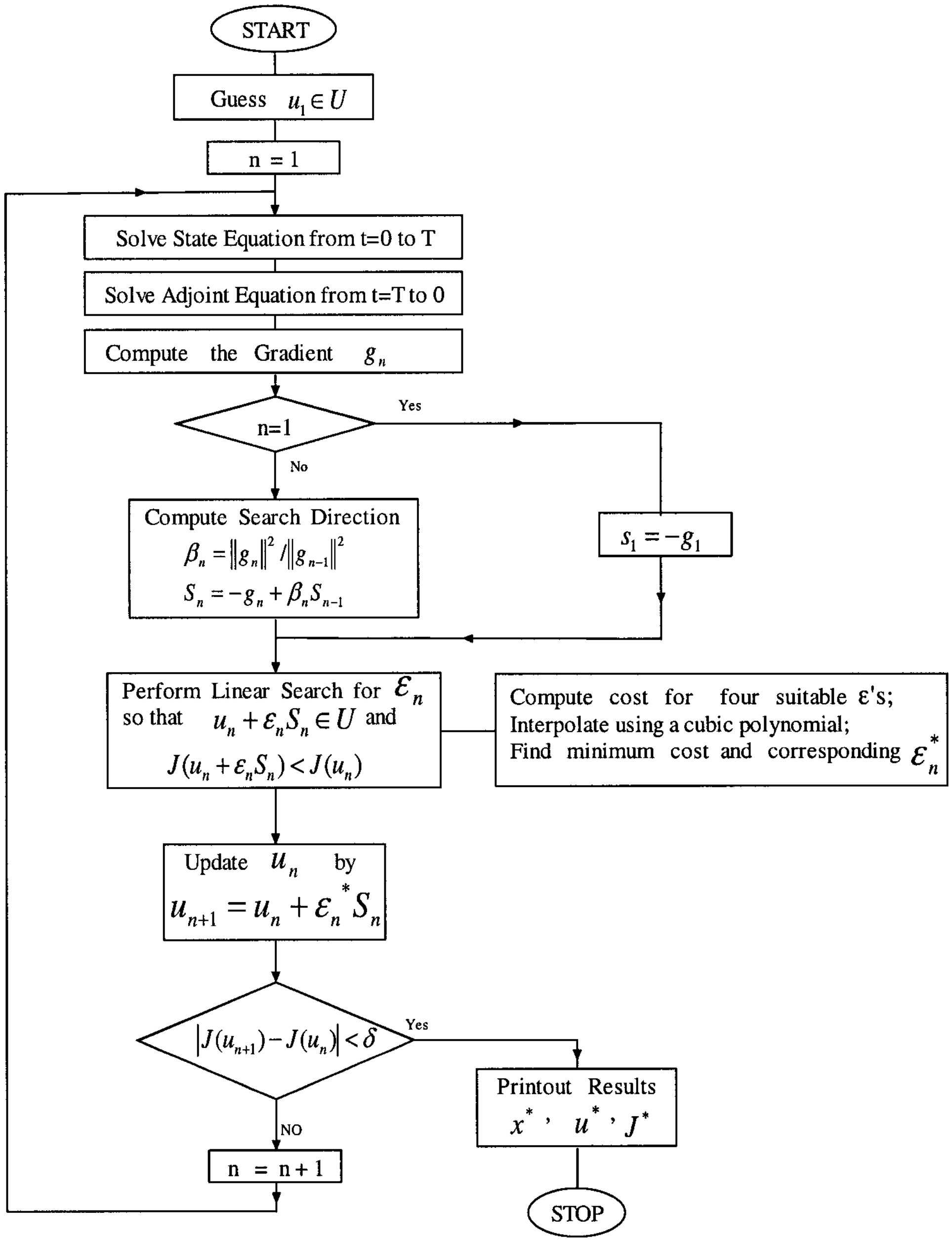
Appendix A The Gradient Algorithm Flow Chart



Appendix B Fortran Code

Parameter Identification Code

Optimum Immigration Policy Code

Optimum Immigration and Job Creation Rates Code

Because of the length limitation of thesis, only a portion of the code, optimum Immigration Policy Code has been included to the thesis. If requested, the full code can be provided.

```
                         Optimum Immigration Policy

C------------------------------------------------------------------------T  00520
C     MAIN PROGRAM                                                       T  00530
C------------------------------------------------------------------------T  00540
      IMPLICIT REAL *8(A-H,O-Z)
      REAL AGE1(31),AGE2(31),AGE3(31),IMM1(31),IMM2(31),IMM3(31)
     *              ,EM1(31),EM2(31),EM3(31),BIR(31)
     *              ,DEATH1(31),DEATH2(31),DEATH3(31)
     *              ,AGE14(31),AGE64(31)

      DIMENSION YY(3,1613),U(1,1613),GRD(1,1613),UNEW(1,1613),GR(1)
      DIMENSION Y(3),DERY(3),AUX(4,4),YX(4),X(3),Z(3),ZZ(3,1613)
      DIMENSION SRH(1,1613),PRMT(3),UU(1),GGG(1),XX(3,1613)
      DIMENSION R12(1613),d(3,1613),R(3,1613),dd(3),RR(3)
      DIMENSION B(31),DE(3,31),S(3,31),TR(2,31),EMMI(3,31)
      DIMENSION TRA(2),TTRA(3,1613),E(3,1613),EE(3),TOTALYY(1613)
      DIMENSION NMIN(1613),KMAX(1613),TRTRA(2),NUMBER(1613)
      COMMON/PARAM/MAXP,MINP,AA,BB,P1,P2
      PRMT(1)=0.0
      PRMT(2)=1612
      PRMT(3)=1
      NDIM=3
      NPTS=1613
      NCTL=1
      MAX=10

      STOP=1.0D-07

      MAXP=30000000
      MINP=29000000

      AA=0.5D19
      BB=0.5D20

      DO 15 J=1,NPTS
15    U(1,J)=0
      M=10

C     INITILIZATION OF CONTROL VECTOR                                    T  00860
C     -------------------------------------------------------------------
C     THIS PART IS TO LOAD DATA FROM FILE
C     -------------------------------------------------------------------
      OPEN(UNIT=16, FILE='data1.txt',STATUS='OLD')
C     REWIND 16
      I=1

10    READ(16,*,END=99)AGE1(I),AGE2(I),AGE3(I),IMM1(I),IMM2(I),IMM3(I)
C     WRITE(*,*)AGE1(I),AGE2(I),AGE3(I),IMM1(I),IMM2(I),IMM3(I)
      I=I+1
      GO TO 10
99    CONTINUE
      CLOSE(UNIT=16,STATUS='KEEP')

      OPEN(UNIT=16, FILE='data2.txt',STATUS='OLD')
      REWIND 16
      J=1
11    READ(16,*,END=100)EM1(J),EM2(J),EM3(J),BIR(J)
C     WRITE(*,*)EM1(J),EM2(J),EM3(J),BIR(J)
      J=J+1
      GO TO 11
100   CONTINUE
      CLOSE(UNIT=16,STATUS='KEEP')

      OPEN(UNIT=16, FILE='data3.txt',STATUS='OLD')
      REWIND 16
      K=1
12    READ(16,*,END=101)DEATH1(K),DEATH2(K),DEATH3(K),AGE14(K),AGE64(K)
C     WRITE(*,*)DEATH1(K),DEATH2(K),DEATH3(K),AGE14(K),AGE64(K)
      K=K+1
```

```
      GO TO 12
101   CONTINUE
      CLOSE(UNIT=16,STATUS='KEEP')

C     LOAD THE IDENTIFIED VARIABLE COEFFICIENTS FROM THE FILE


      OPEN(UNIT=16, FILE='initial.txt',STATUS='OLD')

      REWIND 16
      K=1
16    READ(16,*,END=102)TTRA(3,K),TTRA(1,K),TTRA(2,K),R12(K),d(1,K)
     *                    ,d(2,K),d(3,K)

      K=K+1
      GO TO 16
102   CONTINUE
      CLOSE(UNIT=16,STATUS='KEEP')
C     ------------------------------------------------------------------
C     THIS PART IS TO CALCULATE THE COEFFICIENT VALUE ACCORDING TO DATA
C     ------------------------------------------------------------------
C     %(here suppose the -1 year old is 1/16 of the total first age group)
C     PRINT*,'***********THE NET IMMIGRATION RATE'
      DO 13 I=1,31

C     %immigration rate in the three age groups
      S(1,I)=(IMM1(I)*15/16)/(AGE1(I)*52);
      S(2,I)=IMM2(I)/(AGE2(I)*52);
      S(3,I)=IMM3(I)/(AGE3(I)*52);

C     %emigration rate in the three age groups
      EMMI(1,I)=(EM1(I)*15/16)/(AGE1(I)*52);
      EMMI(2,I)=EM2(I)/(AGE2(I)*52);
      EMMI(3,I)=EM3(I)/(AGE3(I)*52);

C     BIRTH RATE
      B(I)=BIR(I)/(AGE2(I)*52);

C     death rate in the three age groups
      DE(1,I)=(DEATH1(I)*15/16)/(AGE1(I)*52);
      DE(2,I)=DEATH2(I)/(AGE2(I)*52);
      DE(3,I)=DEATH3(I)/(AGE3(I)*52);

C     TRANSITION RATE
      TR(1,I)=AGE14(I)/(AGE1(I)*52)
      TR(2,I)=AGE64(I)/(AGE2(I)*52)


13    CONTINUE

C     THE VALUES OF WEEKLY


      R(1,1)=S(1,1)
      R(2,1)=S(2,1)
      R(3,1)=S(3,1)
      ZZ(1,1)=AGE1(1)
      ZZ(2,1)=AGE2(1)
      ZZ(3,1)=AGE3(1)

      E(1,1)=EMMI(1,1)
      E(2,1)=EMMI(2,1)
      E(3,1)=EMMI(3,1)

      COUNT=0
      K=1
      DO 14 M=2,NPTS
      IF(COUNT .LT. 52)THEN

            R(1,M)=S(1,K)
            R(2,M)=S(2,K)
```

```
            R(3,M)=S(3,K)
            ZZ(1,M)=AGE1(K)
            ZZ(2,M)=AGE2(K)
            ZZ(3,M)=AGE3(K)

            E(1,M)=EMMI(1,K)
            E(2,M)=EMMI(2,K)
            E(3,M)=EMMI(3,K)
            COUNT = COUNT+1
C           WRITE(*,*)M,E(1,M),E(2,M),E(3,M),COUNT


      ELSE
            COUNT=0
            K=K+1
            R(1,M)=S(1,K)
            R(2,M)=S(2,K)
            R(3,M)=S(3,K)
            ZZ(1,M)=AGE1(K)
            ZZ(2,M)=AGE2(K)
            ZZ(3,M)=AGE3(K)

            E(1,M)=EMMI(1,K)
            E(2,M)=EMMI(2,K)
            E(3,M)=EMMI(3,K)
            COUNT = COUNT+1
C           WRITE(*,*)M,E(1,M),E(2,M),E(3,M),COUNT

      ENDIF
14    CONTINUE

      WRITE(16,888)
888   FORMAT(10X,'TRUE POPULATION '//)
C888   FORMAT(10X,'TRANSITION RATES '//)
      T=1
      DO 220 L=1,NPTS
      DO 333 I=1,NDIM
C     TRA(I)=TTRA(I,L)
333   Z(I)=ZZ(I,L)

      WRITE(16,19) T,(Z(I),I=1,NDIM)
C     WRITE(16,19) T,(TRA(I),I=1,NDIM)
19    FORMAT(F9.3,8E18.9)
C19    FORMAT(F9.2,8E18.9)
      TT=M
      T=T+PRMT(3)
220   CONTINUE
      PRINT*,'CALL INTFOR IN MAIN'
      PRINT*,'R12'
      WRITE(*,*)R12
      PRINT*,'d'
      WRITE(*,*)d
      PRINT*,'TTRA'
      WRITE(*,*)TTRA
      CALL INTFOR(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,U,UU,YY
     *           ,E,EE,R12,d,dd,TTRA,TRTRA)

      CALL  COST(NDIM,NCTL,NPTS,PRMT,YY,CST,TOTALYY,KMAX,NMIN)

      WRITE(16,555)
555   FORMAT(//10X,'TRAJECTORY WITH ASSUMED CONTROLS'//)
      CALL OUTP(NDIM,NPTS,NCTL,PRMT,Y,YY,U,UU,M,GRD,GGG)

      CALL CONJ(NDIM,NCTL,NPTS,MAX,PRMT,Y,X,DERY,AUX,YX,U,UNEW
     *  ,UU,YY,SRH,GRD,GR,CST,ALPA,STOP,Z,ZZ,XX,E,EE,TOTALYY,KMAX,NMIN
     *  ,R12,d,dd,TTRA,TRTRA)
      CALL GRADV(NDIM,NPTS,NCTL,PRMT,XX,YY,GRD)
C     WRITE(16,222)
222   FORMAT(10X,'FINAL RESULTS'//)

      WRITE(16,20) CST
```

```
20    FORMAT(10X,'COST=',E15.5//)
      WRITE(16,111)
111   FORMAT(10X,'OPTIMAL TRAJECTORY'//)
      WRITE(16,444)
444   FORMAT(10X,'TRAJECTORY WITH OPTIMAL CONTROLS '//)
      CALL OUTP(NDIM,NPTS,NCTL,PRMT,Y,YY,U,UU,M,GRD,GGG)

      STOP
      END

C------------------------------------------------------------------T 01110
      SUBROUTINE FCT1(NDIM,NCTL,T,Y,DERY,U,E,R12,d,TTRA)
C------------------------------------------------------------------T 01130
C                                                                  T 01140
C     SYSTEM DYNAMICS ( STATE EQUATIONS )                          T 01150
C                                                                  T 01160
      IMPLICIT REAL *8(A-H,O-Z)                                    T 01170
      DIMENSION Y(NDIM),DERY(NDIM),U(NCTL),E(NDIM),TTRA(3),d(NDIM)
      COMMON/PARAM/MAXP,MINP,AA,BB,P1,P2

      DERY(1)=R12*Y(2)-TTRA(1)*Y(1)-d(1)*Y(1)-E(1)*Y(1)+P1*U(1)*Y(2)
      DERY(2)=TTRA(1)*Y(1)-TTRA(2)*Y(2)-d(2)*Y(2)-E(2)*Y(2)+U(1)*Y(2)
      DERY(3)=TTRA(2)*Y(2)-d(3)*Y(3)-E(3)*Y(3)+P2*U(1)*Y(2)

      RETURN                                                       T 01240
      END                                                          T 01250
C


C------------------------------------------------------------------T 01930
      SUBROUTINE INTCON(NDIM,Y)                                    T 01940
C------------------------------------------------------------------T 01950
C                                                                  T 01960
C     INITIAL CONDITIONS FOR THE STATE EQUATIONS                   T 01970
C                                                                  T 01980
      IMPLICIT REAL *8(A-H,O-Z)                                    T 01990
      DIMENSION Y(NDIM)                                            T 02000

      Y(1)=6338787
      Y(2)=14073277
      Y(3)=1807496

      RETURN                                                       T 02060
      END                                                          T 02070
C                                                                  T 02080

C------------------------------------------------------------------T 03470
      SUBROUTINE INTFOR(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,U,UU,YY
     *     ,E,EE,R12,d,dd,TTRA,TRTRA)
C------------------------------------------------------------------T 03490
C                                                                  T 03500
C     DIFFERENTIAL EQUATION SOLVER USING RUNGE-KUTTA METHOD        T 03510
C                          FORWARD SOLUTION OF STATE EQUATIONS     T 03520
C                                                                  T 03530
      IMPLICIT REAL *8(A-H,O-Z)                                    T 03540
      DIMENSION YY(NDIM,NPTS),U(NCTL,NPTS),UU(NCTL)                T 03550
      DIMENSION Y(NDIM),DERY(NDIM),AUX(4,NDIM),YX(NDIM)            T 03560
      DIMENSION PRMT(3),E(NDIM,NPTS),EE(NDIM),R12(NPTS)
      DIMENSION d(NDIM,NPTS),dd(NDIM),TTRA(3,NPTS),TRTRA(2)

      CALL INTCON(NDIM,Y)                                          T 03580
      X=PRMT(1)                                                    T 03590
      H=PRMT(3)                                                    T 03600
      K=1                                                          T 03610
      DO 1 I=1,NDIM                                                T 03620
1     YY(I,K)=Y(I)
      DO 11 K=2,NPTS                                               T 03640
      XX=X                                                         T 03650
      DO 21 LL=1,NCTL
21    UU(LL)=U(LL,K)

```

```
      DO 22 J=1,NDIM
      dd(J)=d(J,K)
      EE(J)=E(J,K)
C     WRITE(*,*)K,NN,RR(NN)
22    CONTINUE
      DO 23 I=1,2
23    TRTRA(I)=TTRA(I,K)
      RR12=R12(K)

      CALL FCT1(NDIM,NCTL,XX,Y,DERY,UU,EE,RR12,dd,TRTRA)
      DO 2 I=1,NDIM                                                   T  03690
      AUX(1,I)=H*DERY(I)                                              T  03700
2     YX(I)=Y(I)+AUX(1,I)*0.5                                         T  03710
      XX=X+0.5*H                                                      T  03720
      CALL FCT1(NDIM,NCTL,XX,YX,DERY,UU,EE,RR12,dd,TRTRA)
      DO 3 I=1,NDIM                                                   T  03740
      AUX(2,I)=H*DERY(I)                                              T  03750
3     YX(I)=Y(I)+AUX(2,I)*0.5                                         T  03760
      XX=X+0.5*H                                                      T  03770
      CALL FCT1(NDIM,NCTL,XX,YX,DERY,UU,EE,RR12,dd,TRTRA)
      DO 4 I=1,NDIM                                                   T  03790
      AUX(3,I)=H*DERY(I)                                              T  03800
4     YX(I)=Y(I)+AUX(3,I)                                             T  03810
      XX=X+H                                                          T  03820
      CALL FCT1(NDIM,NCTL,XX,YX,DERY,UU,EE,RR12,dd,TRTRA)
      DO 5 I=1,NDIM                                                   T  03840
5     AUX(4,I)=H*DERY(I)                                              T  03850
      DO 8 I=1,NDIM                                                   T  03860
      DY=(AUX(1,I)+2.0*AUX(2,I)+2.0*AUX(3,I)+AUX(4,I))/6.0            T  03870
8     Y(I)=Y(I)+DY                                                    T  03880
      X=X+H                                                           T  03890
      DO 10 I=1,NDIM                                                  T  03900
10    YY(I,K)=Y(I)

11    CONTINUE

      RETURN                                                          T  03930
      END
C---------------------------------------------------------------------T  01270
      SUBROUTINE FCT2(NDIM,NCTL,T,X,Y,U,DERX,E,R12,d,TTRA)
C---------------------------------------------------------------------T  01290
C                                                                     T  01300
C     ADJOINT SYSTEM (ADJOINT EQUATIONS)                              T  01310
C                                                                     T  01320
      IMPLICIT REAL *8(A-H,O-Z)                                       T  01330
      DIMENSION X(NDIM),DERX(NDIM),Y(NDIM),U(NCTL)
      DIMENSION E(NDIM),TTRA(3),d(NDIM)
      COMMON/PARAM/MAXP,MINP,AA,BB,P1,P2

      IF((Y(1)+Y(2)+Y(3)).LT. MAXP .AND.(Y(1)+Y(2)+Y(3)) .GT. MINP) THEN
            IMAX=0
            IMIN=0
      ELSE IF((Y(1)+Y(2)+Y(3)) .LT. MINP) THEN
            IMAX=0
            IMIN=1
      ELSE IF((Y(1)+Y(2)+Y(3)) .GT. MAXP) THEN
            IMAX=1
            IMIN=0
      ELSE IF((Y(1)+Y(2)+Y(3)) .EQ. MAXP .OR. (Y(1)+Y(2)+Y(3)) .EQ. MINP)THEN
            IMAX=0
            IMIN=0
      ENDIF


      DERX(1)=TTRA(1)*X(1)-TTRA(1)*X(2)+d(1)*X(1)+E(1)*X(1)
     * +2*AA*(MAXP-Y(1)-Y(2)-Y(3))*IMAX
     * +2*BB*(MINP-Y(1)-Y(2)-Y(3))*IMIN


      DERX(2)=TTRA(2)*X(2)-TTRA(2)*X(3)-R12*X(1)+d(2)*X(2)-U(1)*X(2)
     * +E(2)*X(2)-P1*U(1)*X(1)-P2*U(1)*X(3)
```

```
     * +2*AA*(MAXP-Y(1)-Y(2)-Y(3))*IMAX
     * +2*BB*(MINP-Y(1)-Y(2)-Y(3))*IMIN

      DERX(3)=d(3)*X(3)+E(3)*X(3)
     * +2*AA*(MAXP-Y(1)-Y(2)-Y(3))*IMAX
     * +2*BB*(MINP-Y(1)-Y(2)-Y(3))*IMIN
      RETURN                                                            T  01400
      END                                                               T  01410

C---------------------------------------------------------------------- T  02090
      SUBROUTINE TERCON(NDIM,NPTS,X)
C-----------------------------------------------------------------------T  02110
C                                                                       T  02120
C     TERMINAL CONDITIONS FOR THE ADJOINT EQUATIONS                     T  02130
C                                                                       T  02140
      IMPLICIT REAL *8(A-H,O-Z)                                         T  02150
      DIMENSION X(NDIM)

      X(1)=0
      X(2)=0
      X(3)=0

      RETURN                                                            T  02300
      END                                                               T  02310
C                                                                       T  02320
C-----------------------------------------------------------------------T  03960
      SUBROUTINE INTBAK(NDIM,NPTS,NCTL,PRMT,Y,DERX,AUX,YX,X,U,YY,UU
     *          ,XX,E,EE,R12,d,dd,TTRA,TRTRA)
C-----------------------------------------------------------------------T  03990
C                                                                       T  04000
C     RUNGA-KUTTA SUBROUTINE FOR BACKWARD SOLUTION OF ADJOINT EQUATIONS T  04010
C                                                                       T  04020
      IMPLICIT REAL *8(A-H,O-Z)                                         T  04030
      DIMENSION YY(NDIM,NPTS),U(NCTL,NPTS),UU(NCTL)
      DIMENSION Y(NDIM),DERX(NDIM),AUX(4,NDIM),YX(NDIM),X(NDIM)         T  04050
      DIMENSION PRMT(3),Z(NDIM),ZZ(NDIM,NPTS),XX(NDIM,NPTS)
      DIMENSION E(NDIM,NPTS),EE(NDIM),R12(NPTS)
      DIMENSION d(NDIM,NPTS),dd(NDIM),TTRA(3,NPTS),TRTRA(2)

      CALL TERCON(NDIM,NPTS,X)

      T=PRMT(2)                                                         T  04080
      H=-PRMT(3)                                                        T  04090
      K=NPTS

      DO 11 J=1,NPTS
      DO 9 I=1,NDIM
      dd(I)=d(I,K)
      EE(I)=E(I,K)
      Z(I)=ZZ(I,K)
9     Y(I)=YY(I,K)

      DO 81 I=1,NCTL
81    UU(I)=U(I,K)
      DO 16 I=1,NDIM

      XX(I,K)=X(I)
16    CONTINUE

      DO 23 I=1,2
23    TRTRA(I)=TTRA(I,K)
      RR12=R12(K)

      TT=T                                                              T  04220
      CALL FCT2(NDIM,NCTL,TT,X,Y,UU,DERX,EE,RR12,dd,TRTRA)
      DO 2 I=1,NDIM                                                     T  04240
      AUX(1,I)=H*DERX(I)

2     YX(I)=X(I)+AUX(1,I)*0.5                                           T  04260
      TT=TT+0.5*H                                                       T  04270
      CALL FCT2(NDIM,NCTL,TT,YX,Y,UU,DERX,EE,RR12,dd,TRTRA)
```

```
      DO 3 I=1,NDIM                                                     T 04290
      AUX(2,I)=H*DERX(I)
3     YX(I)=X(I)+AUX(2,I)*0.5                                           T 04310
      TT=T+0.5*H                                                        T 04320
      CALL FCT2(NDIM,NCTL,TT,YX,Y,UU,DERX,EE,RR12,dd,TRTRA)
      DO 4 I=1,NDIM                                                     T 04340
      AUX(3,I)=H*DERX(I)
4     YX(I)=X(I)+AUX(3,I)                                               T 04360
      TT=T+H                                                            T 04370
      CALL FCT2(NDIM,NCTL,TT,YX,Y,UU,DERX,EE,RR12,dd,TRTRA)
      DO 5 I=1,NDIM                                                     T 04390
5     AUX(4,I)=H*DERX(I)

      DO 8 I=1,NDIM                                                     T 04410
      DX=(AUX(1,I)+2.0*AUX(2,I)+2.0*AUX(3,I)+AUX(4,I))/6.0              T 04420
8     X(I)=X(I)+DX

      T=T+H
      K=K-1
11    CONTINUE                                                          T 04460

      RETURN                                                            T 04470
      END                                                               T 04480

C-----------------------------------------------------------------------T 02330
      SUBROUTINE OUTP(NDIM,NPTS,NCTL,PRMT,Y,YY,U,UU,M,GRD,GGG)
C-----------------------------------------------------------------------T 02350
C                                                                       T 02360
C     OUTPUT SUBROUTINE FOR PRINTOUT OF SYSTEM TRAJECTORY               T 02370
C                                                                       T 02380
      IMPLICIT REAL *8(A-H,O-Z)                                         T 02390
      DIMENSION Y(NDIM),YY(NDIM,NPTS),U(NCTL,NPTS),PRMT(3),UU(NCTL)
      DIMENSION GRD(NCTL,NPTS),GGG(NCTL)
      T=1
      DO 222 L=1,NPTS
      DO 111 I=1,NDIM

111   Y(I)=YY(I,L)
      TOTALP=YY(1,L)+YY(2,L)+YY(3,L)
      DO 112 I=1,NCTL                                                   T 02450
112   UU(I)=U(I,L)                                                      T 02460
      WRITE(16,12) T,TOTALP,(Y(I),I=1,NDIM)

      TT=M                                                              T 02490
      T=T+PRMT(3)
12    FORMAT(F9.1,8E18.9)
222   CONTINUE

      WRITE(16,555)
555   FORMAT(////10X,'BIRTH RATE, DEATH RATES AND TRANSITION RATES '//)
      T=1
      DO 221 L=1,NPTS
      DO 113 I=1,NCTL
      GGG(I)=GRD(I,L)
113   UU(I)=U(I,L)                                                      T 02460
      WRITE(16,13) T,(UU(I),I=1,NCTL),(GGG(I),I=1,NCTL)

      TT=M                                                              T 02490
      T=T+PRMT(3)

13    FORMAT(F9.1,8E18.9)
221   CONTINUE
      RETURN
      END


C-----------------------------------------------------------------------T 01430
      SUBROUTINE COST(NDIM,NCTL,NPTS,PRMT,YY,CST,TOTALYY,KMAX,NMIN)
C-----------------------------------------------------------------------T 01450
C                                                                       T 01460
C     COMPUTE THE COST USING THE TRAPEZOIDAL RULE FOR INTEGRATION       T 01470
```

```
C                                                                         T  01480
      IMPLICIT REAL *8(A-H,O-Z)                                           T  01490
      DIMENSION YY(NDIM,NPTS),PRMT(3),ZZ(NDIM,NPTS)
      DIMENSION NMIN(NPTS),KMAX(NPTS),TOTALYY(NPTS)

      COMMON/PARAM/MAXP,MINP,AA,BB,P1,P2

      NN=NPTS-1
      TOTALYY(1)=YY(1,1)+YY(2,1)+YY(3,1)
      TOTALYY(NPTS)=YY(1,NPTS)+YY(2,NPTS)+YY(3,NPTS)


      IF(TOTALYY(1).LT. MAXP .AND.TOTALYY(1) .GT. MINP) THEN
            KMAX(1)=0
            NMIN(1)=0
      ELSE IF(TOTALYY(1) .LT. MINP) THEN
            KMAX(1)=0
            NMIN(1)=1
      ELSE IF(TOTALYY(1) .GT. MAXP) THEN
            KMAX(1)=1
            NMIN(1)=0
      ELSE IF(TOTALYY(1) .EQ. MAXP .OR. TOTALYY(1) .EQ. MINP)THEN
            KMAX(1)=0
            NMIN(1)=0
      ENDIF

      IF(TOTALYY(NPTS).LT. MAXP .AND.TOTALYY(NPTS) .GT. MINP) THEN
            KMAX(NPTS)=0
            NMIN(NPTS)=0
      ELSE IF(TOTALYY(NPTS) .LT. MINP) THEN
            KMAX(NPTS)=0
            NMIN(NPTS)=1
      ELSE IF(TOTALYY(NPTS) .GT. MAXP) THEN
            KMAX(NPTS)=1
            NMIN(NPTS)=0
      ELSE IF(TOTALYY(NPTS) .EQ. MAXP .OR. TOTALYY(NPTS) .EQ. MINP)THEN
            KMAX(NPTS)=0
            NMIN(NPTS)=0
      ENDIF

      SUM=0.5*AA*((TOTALYY(1)-MAXP)*(TOTALYY(1)-MAXP)*KMAX(1)
     *     +(TOTALYY(NPTS)-MAXP)*(TOTALYY(NPTS)-MAXP)*KMAX(NPTS))
     *    +0.5*BB*((TOTALYY(1)-MINP)*(TOTALYY(1)-MINP)*NMIN(1)
     *     +(TOTALYY(NPTS)-MINP)*(TOTALYY(NPTS)-MINP)*NMIN(NPTS))

      DO 2 J=2,NN
      TOTALYY(J)=YY(1,J)+YY(2,J)+YY(3,J)
      IF(TOTALYY(J).LT. MAXP .AND.TOTALYY(J) .GT. MINP) THEN
            KMAX(J)=0
            NMIN(J)=0
      ELSE IF(TOTALYY(J) .LT. MINP) THEN
            KMAX(J)=0
            NMIN(J)=1
      ELSE IF(TOTALYY(J) .GT. MAXP) THEN
            KMAX(J)=1
            NMIN(J)=0
      ELSE IF(TOTALYY(J) .EQ. MAXP .OR. TOTALYY(J) .EQ. MINP)THEN
            KMAX(J)=0
            NMIN(J)=0
      ENDIF
      SUM=SUM+AA*(TOTALYY(J)-MAXP)*(TOTALYY(J)-MAXP)*KMAX(J)
     *     +BB*(TOTALYY(J)-MINP)*(TOTALYY(J)-MINP)*NMIN(J)

 2    CONTINUE
      SUM=SUM*PRMT(3)
      CST=SUM

      RETURN
      END
C-------------------------------------------------------------------------T  03280
      SUBROUTINE FIRST(NCTL,NPTS,GRDO,SRH,VALUO)                          T  03290
```

```
C-----------------------------------------------------------------------T  03300
C                                                                      T  03310
C     INITIALIZES THE SEARCH PROCESS                                   T  03320
C                                                                      T  03330
      IMPLICIT REAL *8(A-H,O-Z)                                        T  03340
      DIMENSION GRDO(NCTL,NPTS),SRH(NCTL,NPTS)                         T  03350
      VALUO=0.0                                                        T  03360
      DO 1 I=1,NCTL                                                    T  03370
      DO 1 J=1,NPTS                                                    T  03380
      VALUO=VALUO+GRDO(I,J)*GRDO(I,J)                                  T  03390
1     CONTINUE                                                         T  03400
      DO 2 I=1,NCTL                                                    T  03410
      DO 2 J=1,NPTS                                                    T  03420
2     SRH(I,J)=-GRDO(I,J)                                              T  03430
      RETURN                                                           T  03440
      END                                                              T  03450
C                                                                      T  03460
C-----------------------------------------------------------------------T  03070
      SUBROUTINE SEARCH(NCTL,NPTS,GRDN,SRH,VALUO)                      T  03080
C-----------------------------------------------------------------------T  03090
C                                                                      T  03100
C     COMPUTES SEARCH DIRECTION                                        T  03110
C                                                                      T  03120
      IMPLICIT REAL *8 (A-H,O-Z)                                       T  03130
      DIMENSION GRDN(NCTL,NPTS),SRH(NCTL,NPTS)                         T  03140
      VALUN=0.0                                                        T  03150
      DO 2 I=1,NCTL                                                    T  03160
      DO 2 J=1,NPTS                                                    T  03170
2     VALUN=VALUN+GRDN(I,J)*GRDN(I,J)                                  T  03180
      BTA=VALUN/VALUO                                                  T  03190
C     BTA=0.0                                                          T  03200
      DO 3 I=1,NCTL                                                    T  03210
      DO 3 J=1,NPTS                                                    T  03220
3     SRH(I,J)=-GRDN(I,J)+BTA*SRH(I,J)                                 T  03230

      VALUO=VALUN                                                      T  03240
      RETURN                                                           T  03250
      END
C-----------------------------------------------------------------------T  01110
      SUBROUTINE GRADV(NDIM,NPTS,NCTL,PRMT,XX,YY,GRD)
C-----------------------------------------------------------------------T  01130
C                                                                      T  01140
C     SYSTEM DYNAMICS ( STATE EQUATIONS )                              T  01150
C                                                                      T  01160
      IMPLICIT REAL *8(A-H,O-Z)                                        T  01170
      DIMENSION YY(NDIM,NPTS),XX(NDIM,NPTS),PRMT(3),GRD(NCTL,NPTS)
      DIMENSION GR(NCTL)
      DO 1 I=1,NPTS
      GRD(1,I)=P1*YY(2,I)*XX(1,I)+YY(2,I)*XX(2,I)+P2*YY(2,I)*XX(3,I)
1     CONTINUE


      RETURN
      END

C-----------------------------------------------------------------------T  02560
      SUBROUTINE CONJ(NDIM,NCTL,NPTS,MAX,PRMT,Y,X,DERY,AUX,YX,U,UNEW   T  02570
     * ,UU,YY,SRH,GRD,GR,CST,ALPA,STOP,Z,ZZ,XX,E,EE,TOTALYY,KMAX,NMIN
     * ,R12,d,dd,TTRA,TRTRA)
C-----------------------------------------------------------------------T  02590
C                                                                      T  02600
C     CONJUGATE GRADIENT ALGORITHM                                     T  02610
C                                                                      T  02620
      IMPLICIT REAL *8(A-H,O-Z)                                        T  02630
      DIMENSION YY(NDIM,NPTS),U(NCTL,NPTS),UNEW(NCTL,NPTS),PRMT(3)     T  02640
      DIMENSION Y(NDIM),DERY(NDIM),AUX(4,NDIM),YX(NDIM),SRH(NCTL,NPTS) T  02650
      DIMENSION X(NDIM),GRD(NCTL,NPTS),UU(NCTL),GR(NCTL)
      DIMENSION Z(NDIM), ZZ(NDIM,NPTS), XX(NDIM,NPTS)
      DIMENSION E(NDIM,NPTS), EE(NDIM),R12(NPTS)
      DIMENSION d(NDIM,NPTS),dd(NDIM),TTRA(3,NPTS),TRTRA(2)
      DIMENSION NMIN(NPTS),KMAX(NPTS),TOTALYY(NPTS)
```

```
      ITER = 0
      CSTOD=CST
      CALL INTBAK(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,X,U,YY,UU
     *              ,XX,E,EE,R12,d,dd,TTRA,TRTRA)

      CALL GRADV(NDIM,NPTS,NCTL,PRMT,XX,YY,GRD)
      CALL FIRST(NCTL,NPTS,GRD,SRH,VALU)

      WRITE(16,2) ITER,CSTOD,ALPA
2     FORMAT(//5X,'ITER=',I3,3X,'COST=',E20.12,5X,'ALPA=',E20.7/)        T  02730
210   ITER=ITER+1                                                        T  02740
      KOUNT=0                                                            T  02750
      ALPAO=ALPA                                                         T  02760
      IF(ITER .GT. MAX) GO TO 999

      PRINT*,'4, HERE***************'
67    CALL ALPHA(NDIM,NCTL,NPTS,PRMT,Y,DERY,AUX,YX,U,UNEW,YY,
     * UU,SRH,CSTOD,ALPA,ZZ,E,EE,TOTALYY,KMAX,NMIN,R12,d,dd,TTRA,TRTRA)

      PRINT*,'CALL INTFOR IN CONJ 1 '

      CALL INTFOR(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,U,UU,YY
     *  ,E,EE,R12,d,dd,TTRA,TRTRA)
      CALL COST(NDIM,NCTL,NPTS,PRMT,YY,CST,TOTALYY,KMAX,NMIN)
      PRINT*,'7, HERE***************'
      WRITE(16,2) ITER,CST,ALPA

      IF(ITER .GT. MAX) GO TO 999

      CSTDIF=CSTOD-CST

      IF(CSTDIF .LT. STOP) GO TO 999

      CSVALUE=CSTOD - CST
      IF(CSTOD - CST) 65,65,66
65    ALPA=ALPAO*0.01                                                    T  02900
      DO 76 I=1,NCTL                                                     T  02910
      DO 76 J=1,NPTS                                                     T  02920
76    U(I,J)=U(I,J)-ALPA*SRH(I,J)                                        T  02930
      KOUNT=KOUNT+1                                                      T  02940
      IF(KOUNT - 3) 67,67,68                                             T  02950
68    WRITE(16,69)                                                       T  02960
69    FORMAT(//10X,'COST DOES NOT DECREASE ANY MORE'//)                  T  02970
      GO TO 999                                                          T  02980
66    CSTOD=CST

      CALL INTBAK(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,X,U,YY,UU
     *                   ,XX,E,EE,R12,d,dd,TTRA,TRTRA)
      CALL GRADV(NDIM,NPTS,NCTL,PRMT,XX,YY,GRD)

      CALL SEARCH(NCTL,NPTS,GRD,SRH,VALU)                                T  03020
      GO TO 210                                                          T  03030

999   CONTINUE
      RETURN
      END

C-----------------------------------------------------------------------T  04500
      SUBROUTINE ALPHA(NDIM,NCTL,NPTS,PRMT,Y,DERY,AUX,YX,U,UNEW,YNEW,
     * UU,SRH,CSTOD,STEP,ZZ,E,EE,TOTALYY,KMAX,NMIN,R12,d,dd,TTRA,TRTRA)
C-----------------------------------------------------------------------T  04530
C                                                                        T  04540
C     UPDATES THE PARAMETER VECTOR                                       T  04550
C                                                                        T  04560
C               LAGRANGE EXTRAPOLATION HAS BEEN USED                     T  04570

      IMPLICIT REAL *8(A-H,O-Z)                                          T  04590
      DIMENSION X(4),C(4),PRMT(3),UNEW(NCTL,NPTS),UU(NCTL)               T  04600
      DIMENSION YNEW(NDIM,NPTS),SRH(NCTL,NPTS),U(NCTL,NPTS)              T  04610
      DIMENSION Y(NDIM),DERY(NDIM),AUX(4,NDIM),YX(NDIM),ZZ(NDIM,NPTS)
```

```
      DIMENSION E(NDIM,NPTS),EE(NDIM),R12(NPTS)
      DIMENSION d(NDIM,NPTS),dd(NDIM),TTRA(3,NPTS),TRTRA(2)
      DIMENSION NMIN(NPTS),KMAX(NPTS),TOTALYY(NPTS)

      PRINT*,'IN ALPHA CALL CONTROL 1--SRH'
      WRITE(*,*)SRH

      PRINT*,'IN ALPHA CALL CONTROL 1---XX'
      WRITE(*,*)XX
      CALL CONTRL(NCTL,NPTS,STEP,U,UNEW,SRH)

      CALL INTFOR(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,UNEW,UU,YNEW
     *  ,E,EE,R12,d,dd,TTRA,TRTRA)

      CALL COST(NDIM,NCTL,NPTS,PRMT,YNEW,CST,TOTALYY,KMAX,NMIN)

      C(1)=CST                                                          T 04660
      X(1)=STEP                                                         T 04670
      STEP=2.0*X(1)
      CALL CONTRL(NCTL,NPTS,STEP,U,UNEW,SRH)                            T 04690
      CALL INTFOR(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,UNEW,UU,YNEW
     *  ,E,EE,R12,d,dd,TTRA,TRTRA)
      PRINT*,'3, ALPHA-----'
      CALL COST(NDIM,NCTL,NPTS,PRMT,YNEW,CST,TOTALYY,KMAX,NMIN)

      C(2)=CST                                                          T 04720
      X(2)=STEP
      CVALUE=C(1)-C(2)
      PRINT*,'CALL INTFOR IN ALPHA 2'
      IF(C(1)-C(2)) 3,3,4                                               T 04740
3     STEP=0.5*X(1)
      PRINT*,'IN ALPHA CALL CONTROL 3'
      WRITE(*,*)SRH
21    CALL CONTRL(NCTL,NPTS,STEP,U,UNEW,SRH)                            T 04760
      CALL INTFOR(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,UNEW,UU,YNEW
     *  ,E,EE,R12,d,dd,TTRA,TRTRA)
      PRINT*,'5, ALPHA-----'
      CALL COST(NDIM,NCTL,NPTS,PRMT,YNEW,CST,TOTALYY,KMAX,NMIN)

      C(3)=C(2)                                                         T 04790
      X(3)=X(2)                                                         T 04800
      C(2)=C(1)                                                         T 04810
      X(2)=X(1)                                                         T 04820
      C(1)=CST                                                          T 04830
      X(1)=STEP
      PRINT*,'5.1, ALPHA-----'
      IF(C(1)-C(2)) 9,9,10
      PRINT*,'5.2, CALL INTFOR IN ALPHA 3-----'
9     STEP=0.5*X(1)
      PRINT*,'IN ALPHA CALL CONTROL 4'
      WRITE(*,*)SRH
      CALL CONTRL(NCTL,NPTS,STEP,U,UNEW,SRH)                            T 04870
      CALL INTFOR(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,UNEW,UU,YNEW
     *  ,E,EE,R12,d,dd,TTRA,TRTRA)

      CALL COST(NDIM,NCTL,NPTS,PRMT,YNEW,CST,TOTALYY,KMAX,NMIN)

      C(4)=C(3)                                                         T 04900
      X(4)=X(3)                                                         T 04910
      C(3)=C(2)                                                         T 04920
      X(3)=X(2)                                                         T 04930
      C(2)=C(1)                                                         T 04940
      X(2)=X(1)                                                         T 04950
      C(1)=CST                                                          T 04960
      X(1)=STEP

      CC=C(1)-C(2)
      IF(C(1)-C(2)) 9,9,100
      PRINT*,'6.3, CALL INTFOR IN ALPHA 4-----'
4     STEP=2.0*X(2)
      WRITE(*,*)SRH
```

```
22    CALL CONTRL(NCTL,NPTS,STEP,U,UNEW,SRH)                                T  05000
      CALL INTFOR(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,UNEW,UU,YNEW
     *   ,E,EE,R12,d,dd,TTRA,TRTRA)
      CALL COST(NDIM,NCTL,NPTS,PRMT,YNEW,CST,TOTALYY,KMAX,NMIN)
      C(3)=CST                                                              T  05030
      X(3)=STEP
      A=C(2)-C(3)
      PRINT*,'VALUE C(2)-C(3) IS-------'
      IF(C(2)-C(3)) 200,19,19                                               T  05050
19    STEP=2.0*X(3)
      PRINT*,'IN ALPHA CALL CONTROL 6'
      WRITE(*,*)SRH
      CALL CONTRL(NCTL,NPTS,STEP,U,UNEW,SRH)                                T  05070
      CALL INTFOR(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,UNEW,UU,YNEW
     *   ,E,EE,R12,d,dd,TTRA,TRTRA)
      CALL COST(NDIM,NCTL,NPTS,PRMT,YNEW,CST,TOTALYY,KMAX,NMIN)
      C(4)=CST                                                              T  05100
      X(4)=STEP
      CVALUE3=C(3)-C(4)
      IF(C(3)-C(4)) 100,301,301                                             T  05120
301   C(1)=C(2)                                                             T  05130
      X(1)=X(2)                                                             T  05140
      C(2)=C(3)                                                             T  05150
      X(2)=X(3)                                                             T  05160
      C(3)=C(4)                                                             T  05170
      X(3)=X(4)                                                             T  05180
      GO TO 19                                                              T  05190
10    STEP=(X(1)+X(2))*0.5
      PRINT*,'IN ALPHA CALL CONTROL 7'
      WRITE(*,*)SRH
      CALL CONTRL(NCTL,NPTS,STEP,U,UNEW,SRH)                                T  05210
      CALL INTFOR(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,UNEW,UU,YNEW
     *   ,E,EE,R12,d,dd,TTRA,TRTRA)
      CALL COST(NDIM,NCTL,NPTS,PRMT,YNEW,CST,TOTALYY,KMAX,NMIN)
      C(4)=C(3)                                                             T  05240
      X(4)=X(3)                                                             T  05250
      C(3)=C(2)                                                             T  05260
      X(3)=X(2)                                                             T  05270
      C(2)=CST                                                              T  05280
      X(2)=STEP                                                             T  05290
      GO TO 100                                                             T  05300
200   STEP=(X(2)+X(3))*0.5
      PRINT*,'IN ALPHA CALL CONTROL 8'
      WRITE(*,*)SRH
      CALL CONTRL(NCTL,NPTS,STEP,U,UNEW,SRH)                                T  05320
      CALL INTFOR(NDIM,NPTS,NCTL,PRMT,Y,DERY,AUX,YX,UNEW,UU,YNEW
     *   ,E,EE,R12,d,dd,TTRA,TRTRA)
      CALL COST(NDIM,NCTL,NPTS,PRMT,YNEW,CST,TOTALYY,KMAX,NMIN)
      C(4)=C(3)                                                             T  05350
      X(4)=X(3)                                                             T  05360
      C(3)=CST                                                              T  05370
      X(3)=STEP                                                             T  05380
      GO TO 100                                                             T  05390
100   CALL POLY(C,X,STEP)                                                   T  05400
      WRITE(16,345)
345   FORMAT(10X,'LAGRANGE EXTRAPOLATION'/)
      WRITE(16,5) (X(I),I=1,4)
      WRITE(16,6) (C(I),I=1,4)
5     FORMAT(/10X,'ALPHA',4D14.5/)                                          T  05460
6     FORMAT(10X,'COST ',4D14.5)
      CALL CONTRL(NCTL,NPTS,STEP,U,U,SRH)                                   T  05480
      RETURN                                                                T  05490
```

```
      END                                                               T 05500
C                                                                       T 05510

C-----------------------------------------------------------------------T 05520
      SUBROUTINE POLY(C,X,ALPMIN)                                       T 05530
C-----------------------------------------------------------------------T 05540
C                                                                       T 05550
C     SUBROUTINE FOR CURVE FITTING BY LAGRANGE EXTRAPOLATION            T 05560
C                                                                       T 05570
      IMPLICIT REAL *8 (A-H,O-Z)                                        T 05580
      DIMENSION C(4),X(4)                                               T 05590
      F1=C(1)/((X(1)-X(2))*(X(1)-X(3))*(X(1)-X(4)))                     T 05600
      F2=C(2)/((X(2)-X(1))*(X(2)-X(3))*(X(2)-X(4)))                     T 05610
      F3=C(3)/((X(3)-X(2))*(X(3)-X(1))*(X(3)-X(4)))                     T 05620
      F4=C(4)/((X(4)-X(2))*(X(4)-X(3))*(X(4)-X(1)))                     T 05630
      A=F1+F2+F3+F4                                                     T 05640
      B=F1*(X(2)+X(3)+X(4))+F2*(X(1)+X(3)+X(4))+F3*(X(1)+X(2)+X(4))     T 05650
     *  +F4*(X(1)+X(2)+X(3))                                            T 05660
      CC=F1*(X(2)*X(3)+X(3)*X(4)+X(2)*X(4))+F2*(X(1)*X(3)+X(1)*X(4)+    T 05670
     *   X(3)*X(4))+F3*(X(1)*X(2)+X(1)*X(4)+X(2)*X(4))+F4*(X(1)*X(2)    T 05680
     *   +X(1)*X(3)+X(2)*X(3))                                          T 05690
      ALPMIN=(B+DSQRT(B*B-3.0*A*CC))/(3.0*A)                            T 05700
      RETURN                                                            T 05710
      END                                                               T 05720
C                                                                       T 05730
C-----------------------------------------------------------------------T 05740
      SUBROUTINE CONTRL(NCTL,NPTS,STEP,U,UNEW,SRH)                      T 05750
C-----------------------------------------------------------------------T 05760
C                                                                       T 05770
C     UPDATING THE CONTROL                                              T 05780
C                                                                       T 05790
      IMPLICIT REAL *8(A-H,O-Z)                                         T 05800
      DIMENSION U(NCTL,NPTS),UNEW(NCTL,NPTS),SRH(NCTL,NPTS)

      DO 2 J=1,NPTS
      DO 2 I=1,NCTL

      UNEW(I,J)=U(I,J)+STEP*SRH(I,J)

      WRITE(*,*)I,J,U(I,J),UNEW(I,J),STEP, SRH(I,J)
      IF(UNEW(I,J) .LT. 0) THEN
            UNEW(I,J)=0
      ELSEIF(UNEW(I,J) .GT. 5D-4) THEN
            UNEW(I,J)=5D-4

      ENDIF

2     CONTINUE

      RETURN
      END
```