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On imperfect commitment in contracts*

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Abstract

In this paper I consider a repeated buyer-seller relationship wherein a seller has private information on his fixed cost parameter. Once a buyer pays for the good - but before its delivery - he may fear opportunistic behavior by the seller; the latter may prefer not to produce, in which case he pays a penalty and the trade is terminated. Depending on the magnitude of the penalty and the valuation of the future I identify three contractual regimes corresponding to the strength of legal system. The optimal stationary contract consists of two distinct parts. For the most efficient types of seller, the contract entails bunching with a fixed payment and a fixed output. For higher costs output is significantly reduced below the optimal static mechanism.

Key words: *Adverse selection, penalty for breach, discount rate.*

JEL Classification: D82, D86.

1 Introduction

Advance payment options create an incentive to breach a contract: the contracting party may decide not to provide the services agreed and instead either run, provide a poor quality service, or engage in lengthy and costly delays.[1] In construction a general contractor can take the prepayment and run without providing the service, a sub-contractor may fail to show up, a material supplier can delay the delivery and so on. In international trade, the cash-in-advance payment method creates enforcement problems especially in developing countries and transition economies where the legal system is weak.[2]

Breach of contract almost always involves financial penalties. These penalties may be enforced by court decision (as liquidated damages in construction) or they can be paid using bonds.[3] However, the penalty alone may not be sufficient to enforce the contract. Another enforcement device is the revocation of the business or professional license of the party responsible for breach. In some cases courts are too weak to impose severe sanctions. Then given easy, low cost access to information for third parties, a company, responsible for breach of contract may not find customers. Macchiavello and Morjaria (2014) consider the case of the Kenyan rose industry. They argue that transacting parties are sufficiently concerned with the future benefits to sustain the contracts. Levin (2003, p. 835) pointed out "Sovereign nations comply with trade agreements and repay foreign debt because they desire the continued goodwill of trading partners". However, termination of the bilateral relationship alone also may be insufficient for contract enforcement. In case of sovereign debt Bulow and Rogoff (1989) argue the threat of exclusion from future borrowing is unable to sustain positive lending by itself.

In this paper I consider a repeated buyer-seller relationship where the seller faces two sanctions in case of breach of the contract: he pays a penalty and trade is terminated. The main questions I address are when and how each of these enforcement devices matter. In the model the seller has private information on the fixed cost of provision, which is drawn from continuous distribution. Enforcement of contractual terms matters once a buyer pays for the good and before its delivery. The buyer may fear the seller's opportunistic behavior; that is the latter may prefer not to produce.

The nature of the optimal *stationary* contract depends on valuation of the future, summarized by the discount rate and the magnitude of the (financial) penalty for breach of contract. For large penalties, – or equivalently for a strong legal system – enforcement of the contract is assured, and infinite repetition of the optimal static mechanism (*o.s.m.* Baron and Myerson, 1982) is optimal. With medium penalties the transfer for the most efficient type – which is the most attractive for the deviating seller – is equal to the penalty. If the agent decides to deviate and breach the contract he can take the money which is just enough to cover the penalty. Thus, the principal

[1] Even though construction contracts often include a pre-agreed dispute resolution mechanism,

"When faced with a long and arduous dispute, there could be circumstances in which that dispute, or part of that dispute, could be determined by adjudication on an interim basis, which could see some cash in the door in the interim." (Civil Mining & Construction Pty Ltd v Isaac Regional Council, Take the money and run... by Paul Brennan, http://www.swaab.com.au/)

[2] Berglöf and Claessens (2006) have rich factual material related to enforcement in developing countries and transition economies.

[3] In order to perform any type of construction work in the US, a person or entity must be registered with the State. Registration requires a construction bond which can be seized in case of breach of contract which includes improper work in the conduct of the construction business. Unregistered companies without a bond may be subject to criminal and civil penalties (see http://www.lni.wa.gov/tradeslicensing/contractors/howreg/default.asp.).

solves the enforcement problem: the agent decides to continue the trade because he values it. For these two cases the optimal contract does not depend on the discount factor: the penalty plays the enforcement role.[4] For small penalties – or for weak legal systems – the highest transfer is greater than the penalty because it is inefficient to equalize the maximal transfer to the small penalty. This would distort the whole transfer schedule downward because of incentive compatibility and thus reduce the principal's payoff. In this case the future valuation of trades plays an important role. The seller may not deliver the good and he will have enough to cover the penalty. However, he prefers not to breach the contract because he values future trades. In this case enforcement of the contract requires both devices.

In non-trivial enforcement problem the optimal contract is made of two distinct parts. For the most efficient types of the seller the contract entails bunching with a fixed payment and a fixed output. For less efficient types the contract is separating, and is determined by the optimality condition which involves *adjusted* virtual costs. The threshold between bunching and separation reflects a trade-off between information rent provision and incentive to breach the contract. As in a standard second – best problem, more efficient types want to hide behind less efficient types creating information rents for efficient types. This leads to higher transfers (and higher output) for these types. However, the increase in transfers to efficient types creates incentives for inefficient types to breach the contract. Thus, to limit the incentive to deviate, the contract assumes pooling for the most efficient types.

Literature review. The paper is related to two actively developing areas of incentive theory: dynamic mechanism design and relational contracts. In dynamic adverse selection models the seller commits to the contract: he cannot breach it. This literature generally assumes that the agent's type is constant over time.[5] By assuming that the parties can commit to the contract but not to continuing the relationship I can employ the Revelation principle in its full strength. Thus, the model weakens the full commitment assumption in long-run contracting with fixed types where the equilibrium is the infinite repetition of the *o.s.m.* (Baron and Besanko 1984, Laffont and Tirole 1993).

In contrast the relational contracts literature mostly focuses on self-enforcing incentive contracts with independent types (Levin 2003, Wolitzky 2010, and Halac 2012). In his important contribution Levin (2003) considers an enforcement constraint which balances the incentives to benefit from the current contracting terms with the present value of the relationship. The optimal contract is stationary, and like my paper has partial pooling for efficient types and exhibits more distortion than the second-best solution.[6] Malcomson (2013) consider a variant of Levin's model with persistent types. In his model bunching arises because of the ratchet effect (see for example Laffont and Tirole 1993).

Martimort et al. (2013) present a model with two-sided enforcement. Type is drawn from a binary distribution and it is fixed. The optimal contract is non-stationary: transfers are increasing for the most efficient type in order to prevent opportunistic behavior by the seller in the initial rounds. Penalties for breach of contract for the buyer and seller are aggregated and only the aggregated penalty that matters. Thus if there is only an agent's enforcement problem – as in my

[4] Berglöf and Claessens (2006, p.123) argue that: "The limited empirical evidence suggests that private enforcement tools are often more effective than public tools."

[5] Baron and Besanko (1984), Laffont and Tirole (1996), and Battaglini (2005) extend this literature and consider types that are correlated over time. The general result with correlated types is that the optimal long-term contract is non-stationary and has a memory.

[6] Similar results can be found in Halac (2012).

paper – then the contract is enforced at no cost. The aggregated penalty is infinite since the buyer cannot breach the contract.

The paper is organized as follows. Section 2 presents the model and discusses the enforcement constraint. In Section 3 I present the principal's problem and formulate the main result. In Section 4 I present an algorithm to solve the model using the reduction to two simple programs. Section 5 introduces some possible extensions and discusses assumptions of the model. Optimality conditions and proofs are relegated to the Appendix.

2 Model

• **Preferences and Information.** I consider a long-term relationship between a buyer (the principal) and a seller (the agent). The buyer buys each period a (non-durable) good q from the seller and pays a transfer t.[7] The seller and the buyer have per-period utility functions given respectively by

$$V(q,t) = S(q) - t, \text{ and } U(q,t,\theta) = t - \theta q,$$

where θ is the seller's marginal cost – the agent's type. Assume that the agent's type is drawn once and for all. The agent privately learns his cost parameter θ which is drawn from the atomless distribution $F(\cdot)$ on the interval $\Theta = [\underline{\theta}, \bar{\theta}]$ with the corresponding positive density function $f(\cdot)$. Distribution $F(\cdot)$ is common knowledge. The gross surplus function $S(\cdot)$ is increasing and strictly concave ($S'(\cdot) > 0 > S''(\cdot)$) and satisfies the Inada conditions $S'(0) = +\infty, S(0) = 0$. I assume that the surplus is large enough that $\max_{q \geq 0} S(q) - \overline{\theta} q \geq 0$.

The time horizon is infinite, discrete, and the parties have a common discount factor δ, $0 \leq \delta \leq 1$.

Output is observable each period. The buyer offers a mechanism (a contract) to the seller which runs for all periods. The seller, after learning the parameter θ, accept or rejects this contract. Up to this point the framework is identical to Baron and Besanko (1984). The main differences with Baron and Besanko (1984) and Battaglini (2005) models are the focus on *stationary* mechanisms and imperfect commitment. At each period the agent – after receiving payment from the buyer – may decide not to deliver the good, in which case he pays for the breach of contract a (financial) penalty Π. This penalty is exogenously specified and is a part of the contractual environment.

• **Timing.** Without loss of generality I consider a direct mechanism $\{t(\hat{\theta}), q(\hat{\theta})\}_{\hat{\theta} \in \Theta}$ (see for example Baron and Besanko, 1984). The contracting game unfolds as follows (see Figure 1):

[7] The good q can be interpreted as the quantity, or quality in case of a single unit.

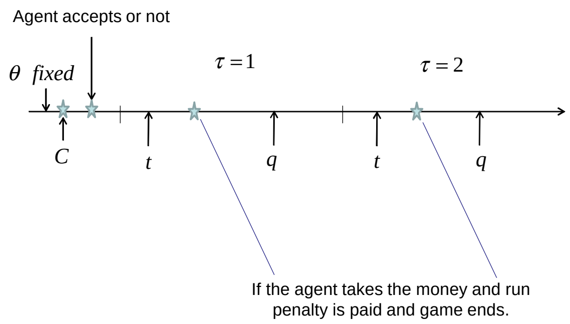


Figure 1: Timing of the model

1. The agent learns the value of θ which is his private information;
2. The principal offers a contract $\mathcal{C} = \{t(\hat{\theta}), q(\hat{\theta})\}_{\hat{\theta}\in\Theta}$ that runs for all periods of the relationship;
3. The agent accepts or rejects the contract, and if he accepts he reports his type $\hat{\theta}$, which may be different from the true type θ;
4. In each given period τ, the agent – after receiving the per-period payment $t(\hat{\theta})$ – decides to stay in the relationship or to walk away. In the latter case the agent pays Π for the breach of contract and the contractual relationship ends.[8]

By restricting attention to stationary mechanisms I avoid dependence of mechanisms on the history of play. I comment on stationarity assumption in Section 5.

• **Incentive Compatibility and Participation Constraints.** The equilibrium concept that I adopt is perfect Bayesian equilibrium, where truthful reporting is an equilibrium. Thus the set incentive feasible allocations is defined by the agent's incentive compatibility and participation constraints.

The agent's intertemporal payoff when accepting the contract and optimally choosing his reported type is $\frac{1}{1-\delta}(t(\theta) - \theta q(\theta))$. Hence the agent's *ex-post* participation constraint is satisfied when

$$t(\theta) - \theta q(\theta) \geq 0 \quad \forall\theta \in \Theta. \tag{1}$$

Incentive compatibility can be expressed by the following standard Lemma (whose proof is omitted).

Lemma 1 *1. $t(\theta)$ is almost everywhere differentiable and at points of differentiability satisfies*

$$\dot{t}(\theta) = \theta z(\theta), \tag{2}$$

[8] Here the penalty Π is included in the formal description of the contract. Importantly, this amount is enforceable by courts. Thus, even if a large penalty is written in the contract but cannot be enforced, I assume that the legal system is weak and the real amount of Π is considerably smaller than the one designated in the formal contract.

2. $q(\theta)$ is non-negative, almost everywhere differentiable and at points of differentiability satisfies

$$\dot{q}(\theta) = z(\theta) \leq 0. \tag{3}$$

We call the pair $(t(\theta), q(\theta))_{\theta\in\Theta}$ admissible they satisfy the conditions of the Lemma and both functions $t(\theta)$ and $q(\theta)$ are continuous.[9]

• **Enforcement Constraint.** Taking into account the fact that the agent may not deliver the output requested by the contract in any given period of his relationship with the buyer, the following *incentive enforcement* constraint must be satisfied

$$\frac{1}{1-\delta}\left[t(\theta) - \theta q(\theta)\right] \geq \max_{\hat{\theta}\in\Theta,\tau\geq 0}\left\{(1+\delta+...+\delta^{\tau-1})(t(\hat{\theta}) - \theta q(\hat{\theta})) + \delta^{\tau}\left(t(\hat{\theta}) - \Pi\right)\right\}. \tag{4}$$

This incentive constraint says that for all τ, the agent with type θ should prefer to take the corresponding contract rather than mimicking a type $\hat{\theta}$ for $\tau - 1$ periods, delivering the corresponding output for those periods, and then stopping delivery at date τ and being punished for the breach of contract from date τ onwards. This constraint can be significantly simplified as shown in the next lemma.

Lemma 2 *The contract $(t(\theta), q(\theta))_{\theta\in\Theta}$ that satisfies Lemma 1 is enfoceable if and only if*

$$t(\theta) - \theta q(\theta) \geq (1-\delta)(t(\underline{\theta}) - \Pi) \quad \forall \theta \in \Theta. \tag{5}$$

Proof: Consider the right-hand side of (4) and take $\tau = +\infty$. It immediately follows that $t(\theta) - \theta q(\theta) \geq t(\hat{\theta}) - \theta q(\hat{\theta}) \quad \forall (\hat{\theta}, \theta) \in \Theta^2$. This implies

$$t(\theta) - t(\hat{\theta}) \geq \theta(q(\theta) - q(\hat{\theta})) \geq 0 \text{ when } \theta \leq \hat{\theta}.$$

Therefore, $\max_{\hat{\theta}\in\Theta} t(\hat{\theta}) = t(\underline{\theta})$ and (5) immediately follows. ■

When an agent adopts a non-compliant behavior, his best strategy is to look like an efficient type, get the very large incentive payment targeted to that type, and then breach the contract. Although developed in a different context, this strategy is reminiscent of the well-known *"take the money and run"* strategy that is sometimes found in models with spot contracting.[10] As in those models, that deviation is less of a concern when the allocation to an efficient type is made less attractive.

Indeed, when the penalty for breach Π is not so large, the right-hand side of (5) is positive and the enforcement constraint significantly reduces the set of implementable. allocations. The incentive to breach is strongest for the most efficient agent. Reducing the payment $t(\underline{\theta})$ then facilitates enforcement for this agent. However, by incentive compatibility such distortion also requires the reduction of payments for all the least efficient types. This might have a detrimental effect on production.

When the penalty for breach Π is large enough, the right-hand side of (5) is negative and the enforcement constraint is necessarily implied by the participation constraint for the least efficient agent. This case is less interesting because it boils down to the usual description of the *o.s.m.*

[9] Continuity of the contract is assumed for simplicity. It can be proven that the optimal contract is continuous.

[10] See for instance, Laffont and Tirole (1993, Chapter 9). I thank David Martimort for this observation.

3 Optimal Contract

The principal's problem is

$$P: \quad \max_{\{(t,q)\text{ - admissible}\}} \int_{\underline{\theta}}^{\bar{\theta}} \left(S(q(\theta)) - t(\theta)\right) f(\theta) d\theta$$

subject to (1) and (5).

Note that the problem P is obtained from the standard second - best problem by adding the enforcement constraint (5). Recall the classical results of Baron and Besanko (1984) and Laffont and Tirole (1993) in a setting without an enforcement constraint (5) (or alternatively assume that the penalty is very large). The solution in this second-best contracting environment involves infinite repetition of the *o.s.m.* $q^{osm}(\theta)$ (Baron and Myerson 1982) and $t^{osm}(\theta) = \theta q^{osm}(\theta) + \int_{\theta}^{\bar{\theta}} q^{osm}(x) dx$, where $q^{osm}(\theta)$ is defined by

$$S'\left(q^{osm}(\theta)\right) = \theta + \frac{F(\theta)}{f(\theta)} \quad \forall \theta \in \Theta.$$

This is the second-best requirement: the buyer's marginal surplus must be equal to the seller's *virtual cost* $\theta + \frac{F(\theta)}{f(\theta)}$. This virtual cost exceeds true cost so as to induce some downward distortion of the output and reduce the costly information rent left to the agent. The next definition generalizes the *o.s.m.*

Definition 1 *For any non-negative r define the generalized o.s.m. output $q^{(r)}(\theta)$ as the solution of*

$$S'\left(q^{(r)}(\theta)\right) = \theta + \frac{r + F(\theta)}{f(\theta)} \quad \forall \theta \in \Theta.$$

Note that $q^{osm}(\theta) = q^{(0)}(\theta)$. If $r > 0$ then this output is smaller than that given by the *o.s.m.* The next assumption is reminiscent of the hazard rate assumption that assures that $q^{osm}(\theta)$ is a decreasing function.

Assumption 1 *Assume that $\frac{r+F(\theta)}{f(\theta)}$ is an increasing function for all non-negative values of r.*

Under this assumption the generalized *o.s.m.* is a decreasing function. Among examples of distributions satisfying this assumption are any weakly decreasing distributions (e.g., uniform and exponential distributions).[11]

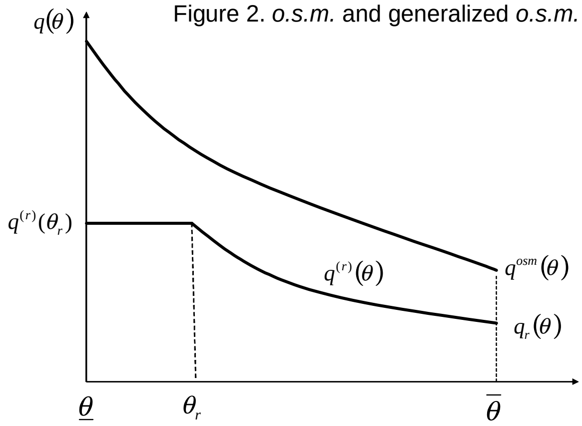
Now I define output which plays an important role in describing the optimal contract.

Definition 2 *For any non-negative r define the output $q_r(\theta)$ by*

$$q_r(\theta) = \begin{cases} q^{(r)}(\theta_r) & \text{if} \quad \theta < \theta_r, \\ q^{(r)}(\theta) & \text{if} \quad \theta_r \le \theta \le \bar{\theta}, \end{cases} \tag{6}$$

[11] This assumption can be revoked at the cost of more notation without altering any of main results. If $\frac{r+F(\theta)}{f(\theta)}$ increases on some intervals then the optimal contract may involve bunching for intermediate types as in Guesnerie and Laffont (1984).

Figure 2. *o.s.m.* and generalized *o.s.m.*



where $\theta_r \in \left[\underline{\theta}, \overline{\theta}\right]$ *is the (minimal) solution of*

$$r = \frac{F^2\left(\theta_r\right)}{\theta_r f\left(\theta_r\right) - F\left(\theta_r\right)}, \tag{7}$$

if it exists, otherwise $\theta_r = \overline{\theta}$.

This output involves bunching for types smaller than θ_r and for types greater than θ_r it is equal to the generalized *o.s.m.* (see Figure 2). For $r = 0$ the threshold θ_r is $\underline{\theta}$ and the corresponding output $q_r(\theta)$ is the *o.s.m.* In the case where no solution of (7) exists the output $q_r(\theta)$ consists of total bunching at $q^{(r)}\left(\overline{\theta}\right)$.

The corresponding transfers that implement the output $q_r(\theta)$ can be found by

$$t_r(\theta) = \theta q_r(\theta) + \int_{\theta}^{\bar{\theta}} q_r(\theta) d\theta + \left(t_r(\bar{\theta}) - \bar{\theta} q_r(\bar{\theta})\right). \tag{8}$$

The following Lemma establishes that output $q_r(\theta)$ exists for all non-negative r.

Lemma 3 *For any non-negative* r *either the solution* $\theta_r \leq \bar{\theta}$ *of (7) exists or there exists unique* $\widehat{r}$ *such that* $r(\overline{\theta}) = \widehat{r}$.

Proof. See Appendix ■

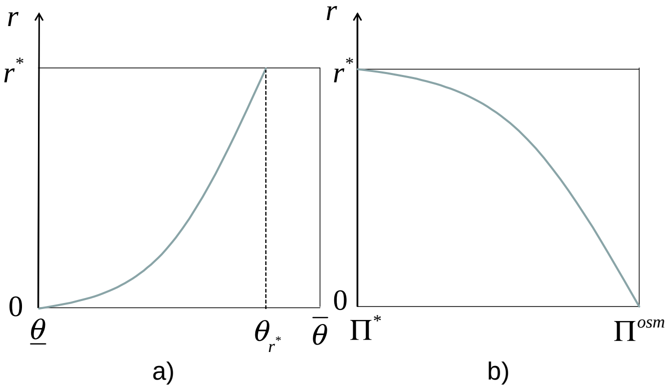
Figure 3 illustrates this Lemma. Denote $r^* = \frac{(1-\delta)}{\delta}$. Equation (7) defines the locus of thresholds θ_r and r that determine the output $q_r(\theta)$. In Figure 3 panel a) r, as a function of θ_r, increases and intersects the line r^* at $\theta_{r^*} < \overline{\theta}$. Thus r^* determines the output $q_{r^*}(\theta)$ with non-trivial bunching for $\theta \leq \theta_{r^*}$ and separating for $\theta > \theta_{r^*}$. If on the other hand $\theta_{r^*} = \overline{\theta}$ then $q_{r^*}(\theta) = q^{(r^*)}\left(\overline{\theta}\right)$ for all θ.

In Figure 3 panel b) each $r \in [0, r^*]$ corresponds to only one penalty $\Pi \in [\Pi^*, \Pi^{osm}]$ via

$$\Pi = \underline{\theta} q_r(\underline{\theta}) + \int_{\underline{\theta}}^{\bar{\theta}} q_r(x) dx. \tag{9}$$

This correspondence is decreasing. Note also that by (8) the right hand side of (9) is equal to $t_r(\underline{\theta})$ assuming the rent $\left(t_r(\bar{\theta}) - \bar{\theta}q_r(\bar{\theta})\right)$ is equal to zero.

Figure 3. r as a function of θ_r and Π



Using (9) define the penalties Π^{osm} and Π^{*} corresponding to $r = 0$ and $r = r^{*}$ respectively. The penalty Π^{osm} defines the threshold above which the enforcement constraint is implied by the agent's participation constraint and the corresponding output is the *o.s.m.* The penalty Π^{*} corresponds to parameter r^{*} and is smaller than Π^{osm}.

The optimal contract for the agent's enforcement problem can be described as follows.

Theorem 1 *1.* ***Strong enforcement.*** *For $\Pi \geq \Pi^{osm}$, the optimal contract is infinite repetition of the o.s.m.*

2. ***Weak enforcement.*** *For $\Pi \in [\Pi^{*}, \Pi^{osm}]$ the optimal contract is $(t_r(\theta), q_r(\theta))$, where the parameter r is determined by*

$$t_r(\theta_r) = \Pi(= t(\underline{\theta})). \tag{10}$$

In this case the most inefficient type has zero rent, $t_r(\bar{\theta}) - \bar{\theta}q_r(\bar{\theta}) = 0$.

3. ***Very weak enforcement.*** *For $\Pi \leq \Pi^{*}$ the optimal contract is $(t(\theta), q_{r^{*}}(\theta))$. The transfers $t(\theta)$ are defined by*

$$t(\theta) = t_{r^{*}}(\theta) + \frac{1-\delta}{\delta}\left(\Pi^{*} - \Pi\right).$$

If $\Pi < \Pi^{*}$ then the most inefficient type has a positive rent, $t_r(\bar{\theta}) - \bar{\theta}q_{r^{*}}(\bar{\theta}) = \frac{1-\delta}{\delta}\left(\Pi^{*} - \Pi\right) > 0$.

For a strong legal system – one that can enforce $\Pi \geq \Pi^{osm}$ – enforcement is not an issue: the constraint (5) is not binding. Clearly, valuation of the future does not play a role. The optimal contract is the *o.s.m.* - the one that will be optimal in the absence of enforcement, i.e., when the seller commits not to breach the contract.

When the legal system is weak, $\Pi \in [\Pi^{*}, \Pi^{osm}]$, then the crucial observation is that the highest transfer – the one which will be appropriated by opportunistic seller – is equal to the penalty.

Neither agent type wants to breach the contract because in this case they will lose the surplus from trade. Again, the discount factor does not play a role as soon as the payoff is non-negative.

In the most interesting case of the very weak legal system, the buyer for all $\Pi \leq \Pi^*$ uses the same optimal quantity $q_{r^*}(\theta)$. The transfer is greater than the small penalty. Thus any type of seller can breach the contract and enjoy positive gain equal to $t(\underline{\theta}) - \Pi$. To deter such deviation, the principal alters transfers so that the most inefficient type obtains a positive payoff equal to the difference $t(\underline{\theta}) - \Pi$ he obtains should he deviate.

When the penalty for breach Π diminishes below Π^{osm}, a regime with weak enforcement appears and presents very similar features to those studied in Levin (2003). In particular, to relax the enforcement constraint the payment $t(\underline{\theta})$ is reduced so that less information rent is distributed to the most efficient types. But such reduction in payment is made easier by also reducing the output for the most efficient types. This introduces countervailing incentives at the very top of the type distribution, with types less efficient than $\underline{\theta}$ – but not too much below – being now definitively attracted to the latter allocation. Those countervailing incentives induce some bunching at the very top of the distribution (see Figure 2).

It is interesting to observe that the bunching area is determined by two important conditions. First, condition (10) expresses the fact that the upper bound of the bunching interval – namely type θ^A – is just indifferent between enforcing the contract or not. Second, that indifference carries over to all types below the threshold and the optimality of doing so is captured by the condition (7).

Consider the weak enforcement case. The optimal output consists of two parts. For efficient types, $\theta \leq \theta_r$, it is a constant and transfers are $t_r(\theta) = \Pi$. For inefficient types, $\theta \geq \theta_r$, it consists of the generalized *o.s.m.* with adjusted for enforcement virtual costs $\theta + \frac{r+F(\theta)}{f(\theta)}$. Consider $\theta > \theta_r$. The extra term $\frac{F(\theta)}{f(\theta)}$ keeps information rents decreasing. If the information rent for θ increases then rents for all types more efficient than θ also increase. Hence transfers (and outputs) increase. This, on the other hand, increases the incentive for θ to take the highest transfer and not deliver the good designated for the most efficient type. This is captured in the extra term $\frac{r}{f(\theta)}$, where r is defined optimally when the benefits of bunching before θ_r are equal to separation benefits after θ_r.

Assume that for efficient types output strictly decreases with $q_r(\underline{\theta}) > q_r(\theta_r)$. Consider any positive variation $dq_r(\theta)$ on the interval $\theta \leq \theta_r$ such that $dq_r(\underline{\theta}) = dq_r(\theta_r) = 0$ and $q_r(\theta) + dq_r(\theta)$ is decreasing for $\theta \leq \theta_r$. Then the new contract defined by this output is closer to the *o.s.m.* and thus delivers more payoff. It is incentive compatible and moreover since $t_r(\underline{\theta})$ is unchanged, and only this value is relevant for the enforcement constraint, the enforcement constraint is satisfied. Thus it must be that $q_r(\underline{\theta}) = q_r(\theta_r)$. Note that intuition for bunching is similar to bunching for intermediate types when the hazard rate condition is not satisfied (see Guesnerie and Laffont, 1984).

4 Proof of Theorem: Reduction to Two Programs

First I discuss the constraints of the problem P. For convenience I reproduce constraints (1) and (5):

$$t(\theta) - \theta q(\theta) \geq 0 \quad \forall \theta \in \Theta \text{ and} \tag{11}$$

$$t(\theta) - \theta q(\theta) \geq (1-\delta)(t(\underline{\theta}) - \Pi) \quad \forall \theta \in \Theta. \tag{12}$$

The enforcement constraint (5) has the transfer for the most efficient type on the right-hand side. In case of breach of contract this transfer $t(\underline{\theta})$ is the net gain for the seller. The right-hand side of (12) is non-positive when this net gain is smaller than the penalty. In this case the enforcement constraint becomes innocuous and the problem is a second - best problem with the cap Π on transfers. Intuitively when the net gain from deviation is less than the penalty, the total gain for the seller, who breaches the contract is non-positive. Hence, the seller will not breach the contract provided that he obtain non-negative rents ex-post, i.e., when (11) is satisfied.

The more interesting case is when the net gain for the seller in case of breach of contract is greater than the penalty. In this case the participation constraint follows from the enforcement constraint. Indeed, the seller always has the option to breach the contract and be able to pay the penalty from the gain. To prevent contract's breach the seller has to obtain enough rent in case of continuation of contract. Following this discussion I introduce two systems of constraints

$$A : \begin{cases} t\left(\underline{\theta}\right) - \Pi \geq 0, \\ t(\theta) - \theta q(\theta) \geq (1-\delta)(t(\underline{\theta}) - \Pi) \quad \forall \theta \in \Theta, \end{cases}$$

and

$$B : \begin{cases} t\left(\underline{\theta}\right) - \Pi \leq 0, \\ t(\theta) - \theta q(\theta) \geq 0. \end{cases}$$

I split the problem P of the principal into two. The problems P^A and P^B are obtained from P by replacing (1) and (5) by systems A and B correspondingly.

An optimal solution of P is necessarily an optimal solution for P^A or for P^B. Conversely, the solution of problems P^A and P^B that yields the highest payoff to the principal is necessarily the optimal solution for P.

Program P^A

Consider first the problem P^A. To get rid of the transfer $t\left(\underline{\theta}\right)$ in the right-hand side of (12) I introduce the adjusted transfer schedule $y(\theta)$ as[12]

$$y(\theta) = t(\theta) - (1-\delta)\left(t\left(\underline{\theta}\right) - \Pi\right).$$

Then the system of inequalities A can be written as

$$\begin{cases} y\left(\underline{\theta}\right) - \Pi \geq 0, \\ y(\theta) - \theta q(\theta) \geq 0 \quad \forall \theta \in \Theta. \end{cases}$$

Note that the second constraint is necessarily binding:

$$y(\overline{\theta}) - \overline{\theta} q(\overline{\theta}) = 0. \tag{13}$$

Indeed, if $y(\overline{\theta}) - \overline{\theta} q(\overline{\theta}) > 0$, consider the contract $\left(q(\theta) + \varepsilon, y(\theta)\right)$, for ε such that $y(\overline{\theta}) - \overline{\theta}\left(q(\overline{\theta}) + \varepsilon\right) = 0$. This change does not affect the constraints of the problem and leads to a strictly higher payoff to the principal.

[12]Notice that $y\left(\underline{\theta}\right) = \delta t\left(\underline{\theta}\right) + (1-\delta)\Pi$ and $t\left(\underline{\theta}\right) = \frac{y(\underline{\theta}) - (1-\delta)\Pi}{\delta}$. Thus $t(\underline{\theta}) = \Pi$ if and only if $y(\underline{\theta}) = \Pi$.

The problem P^A can be re-written as

$$\max_{\{y(\cdot),q(\cdot)\}-admissible} \int_{\underline{\theta}}^{\overline{\theta}} \left(S(q(\theta)) - y(\theta)\right) f(\theta) d\theta - \frac{1-\delta}{\delta} \left[y\left(\underline{\theta}\right) - \Pi\right]$$

subject to (13) and

$$y(\underline{\theta}) - \Pi \geq 0. \tag{14}$$

The problem P^A depends on the penalty Π which enters in the objective function and in the constraint (14). To solve the problem P^A I consider it as an optimal control problem with boundary constraints (13)-(14) and a scrap value $-\frac{1-\delta}{\delta}\left[y\left(\underline{\theta}\right) - \Pi\right]$.

Lemma 4 *1. If $\Pi \geq \Pi^*$ then the optimal output is $q_r(\theta)$ where r is defined by the equation*

$$\theta_r q^{(r)}\left(\theta_r\right) + \int_{\theta_r}^{\overline{\theta}} q^{(r)}\left(\xi\right) d\xi = \Pi. \tag{15}$$

The constraint (14) is binding, $y(\underline{\theta}) = \Pi$.

2. If $\Pi < \Pi^*$ then the optimal output is $q_{r^*}(\theta)$.

The constraint (14) is not binding, $y(\underline{\theta}) > \Pi$.

Note that in case 1 the constraint $t\left(\underline{\theta}\right) - \Pi \geq 0$ binds and in case 2 this constraint is slack. Thus in case 1 enforcement is not an issue and the optimal contract does not depend on δ. The most interesting case, that of the weak enforcement, arises when $\Pi < \Pi^*$. In this case output is fixed at $q_{r^*}(\theta)$. The principal cannot sacrifice more efficiency in favor of enforcement. Since the penalty is small the enforcement now depends on the discount factor. This is reflected in $r^* = \frac{1-\delta}{\delta}$ - the parameter that determines the optimal contract.

Program P^B

Consider the problem P^B:

$$P^B : \max_{\{t(\cdot),q(\cdot)\}-admissible} \int_{\underline{\theta}}^{\hat{\theta}} \left(S(q(\theta)) - t(\theta)\right) f(\theta) d\theta$$

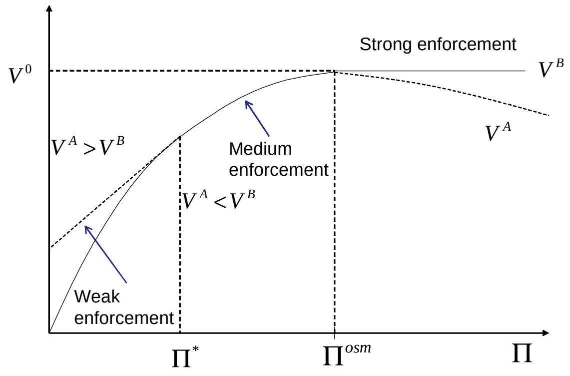
subject to (11) and

$$\Pi - t\left(\underline{\theta}\right) \geq 0. \tag{16}$$

The problem P^B is a second - best problem but with caps on transfers. This problem does not depend on the discount factor. There are two differences between programs P^B and P^A: there is no longer any scrap value in the objective function in P^B and the constraint (16) is the reverse of (14). The *o.s.m.* solution gives the upper bound on what the can be achieved in the presence of the enforcement constraint. Obviously for $\Pi \geq \Pi^{osm}$ the optimal contract is the *o.s.m.*

When $\Pi \leq \Pi^{osm}$ the constraint $\Pi - t\left(\underline{\theta}\right) \geq 0$ is binding. Hence, for $\Pi \in [\Pi^*, \Pi^{osm}]$ the optimal outputs and the transfer schedules are identical for both programs P^A and P^B. The following Lemma to summarize these findings.

Figure 4: Payoff of the principal as a function of penalty



Lemma 5 *For problem P^B:*

1. *If $\Pi \leq \Pi^{osm}$ then the optimal output is $q_r(\theta)$ where r is defined by (15). The constraint (16) is binding, $t(\underline{\theta}) = \Pi$;*

2. *If $\Pi > \Pi^{osm}$ the optimal output is the o.s.m. The constraint (16) is slack, $t(\underline{\theta}) < \Pi$.*

Comparison between P^A and P^B

Intuitively for small Π the value of the problem P^A is greater than the value of the problem P^B and the reverse is true for large Π. For $\Pi \geq \Pi^{osm}$ the problem P^B has the *o.s.m.* as the solution. On the other hand the constraint of the highest transfer in the problem P^A already distorts the transfer away from the *o.s.m.* Indeed, the gain to the seller in case of breach $t(\underline{\theta})$ must be greater or equal to the already large penalty which distorts the otherwise optimal $t^{osm}(\underline{\theta})$ upward. Therefore, for $\Pi > \Pi^{osm}$ the value of P^B is greater than that of P^A and the optimal contract is the *o.s.m.*

For $\Pi \in [\Pi^*, \Pi^{osm}]$ both programs P^A and P^B have the same necessary and sufficient conditions and, therefore, the same solutions. Moreover, since the constraint $y(\underline{\theta}) - \Pi$ is binding, the scrap value in the objective for the problem P^A is zero. Thus both programs have the same value, $V_B = V_A$ (see Figure 3).

For each $\Pi < \Pi^*$ consider the optimal contract $(t(\theta), q(\theta))$ for the problem P^B. For this contract the constraint (16) for the problem P^B is binding, $t(\underline{\theta}) = \Pi$. Hence, the contract $(t(\theta), q(\theta))$ is feasible for the problem P^A. Thus, $V_A \geq V_B$ and the solution to the problem P^A is optimal for P.

Proposition 1 1. *For $\Pi < \Pi^*$ the solution of the problem P^A given in Lemma 4 is optimal;*

2. *For $\Pi \in [\Pi^*, \Pi^{osm}]$ both programs deliver the same result;*

3. *For $\Pi > \Pi^{osm}$ the solution of the problem P^B given in Lemma 5 is optimal.*

5 Concluding Remarks

In this paper I characterize the optimal stationary contract when the agent can commit to a contract but cannot commit not to renege on it. I fix the discount rate and consider the comparative statics when the penalty for breach of contract changes. For large penalties the optimal contract is repetition of the *o.s.m.* and it does not depend on the discount factor. When the penalty is medium-sized, the optimal contract departs from the *o.s.m.* and involves bunching for efficient types, reflecting the trade-off between information rent provision and enforcement incentives. It is less efficient types who are attracted to the transfer of the most efficient type which must be the highest due to incentive compatibility. Still, the optimal contract does not depend on the discount rate. Since the highest transfer is equal to the penalty, the seller does not have an incentive to breach the contract as soon as he is provided with a non-negative payoff in the following trades. More interestingly, when the penalty is small, it is smaller than the highest transfer and the seller has an incentive to breach the contract. However, the contract is designed in such a way that the less efficient type receives a positive payoff exactly equal to the gross gain from deviation. Thus he will not breach the contract.

To establish these results I fix the discount rate and consider comparative statics as the penalty varies. Generally, the contract involves less bunching when the penalty increases. Similar results can be obtained with respect to changes in discount rate:

Corollary 1 *As the legal system becomes more efficient (*Π *increases) or the valuation of future trades* δ *increases,*

1. *the parameter* r *decreases and the optimal contract exhibits less bunching,*
2. *the principal and the agent benefit from this.*

The first point follows from the proof of the Theorem. Because the enforcement constraint relaxes when Π or δ increases, the optimal contract available for a smaller penalty is still available for a greater penalty. Thus, the principal benefits from the increase in penalty or the discount factor. Since output is greater for all θ, the agent's payoff $t(\theta) - \theta q(\theta) = \int_{\theta}^{\bar{\theta}} q(x)dx$ also increases. This result highlights complementarity between enforcement through the legal system and through the continuation of the relationship.

Finally, in the paper I focus on stationary contracts. In Baron and Besanko (1984) when enforcement is costless the optimal contract in the repeated relationship is an infinite replica of the *o.s.m.* Hence the stationarity restriction is without loss of generality. Nevertheless, stationary contracts remain of particular interest in relational contracting. Such contracts have been shown to be optimal for some environments (Levin, 2003). Note also that contracts in many real situations are stationary. Lafontaine and Shaw (1999) argue that franchising contracts are quite stationary. They found that contract terms such as royalty rates and franchise fees do not change over time.

6 Appendix

Proof of Lemma 3. Note first that equation (7) yields $r(\underline{\theta}) = 0$. Second, the denominator $\theta f(\theta) - F(\theta)$ is positive for $\theta = \underline{\theta}$. Consider an interval $[0, \varepsilon]$ such that this denominator is positive for all $\theta \in [0, \varepsilon]$. Differentiating (7) with respect to r leads to $r'(\theta) > 0$ for all $\theta \in [0, \varepsilon]$. Indeed,

$$r'(\theta_r)\left[\theta_r - \frac{F(\theta_r)}{f(\theta_r)}\right] = \left(\left(\frac{F(\theta_r)}{f(\theta_r)}\right)' F(\theta_r) + F(\theta_r) + r(\theta_r)\left(\left(\frac{F(\theta_r)}{f(\theta_r)}\right)' - 1\right)\right).$$

$$= \underbrace{\frac{F(\theta_r)}{f^2(\theta_r)}\left(2f^2(\theta_r) - (F(\theta_r) + r(\theta_r)) f'(\theta_r)\right)}_{(+)}.$$

Proceeding by adding small amounts to the intervals, either I reach θ' such that $\theta' f(\theta') - F(\theta') = 0$ or for all $\theta \in [\underline{\theta}, \overline{\theta}]$ the denominator is positive. In the first case for all $r \geq 0$ there exists unique $\theta_r \in [\underline{\theta}, \overline{\theta}]$ such that $r = \frac{F^2(\theta_r)}{\theta_r f(\theta_r) - F(\theta_r)}$. In the second case I can assume that $\theta_r = \overline{\theta}$ for all $r \geq \widehat{r} = r(\overline{\theta})$. ■

Proof of Lemma 4

Optimality conditions for the problem P^A : I impose the monotonicity condition explicitly in the problem. For that purpose I introduce a variable $z(\theta) = \dot{q}(\theta)$. Consider the problem P^A as an optimal control problem with state variables $y(\theta)$ and $q(\theta)$ and control variable $z(\theta)$. Co-state variables associated with (2) and (3) are $\lambda_1(\theta)$ and $\lambda_2(\theta)$ correspondingly.

The Hamiltonian

$$H(y, q, z, \lambda_1, \lambda_2, \theta) = (S(q) - y) f(\theta) + \lambda_1 \theta z + \lambda_2 z$$

is a concave in (y, q, z) function for all θ. Let $(y(\theta), q(\theta), z(\theta))$ be an admissible triplet with continuous, almost everywhere differentiable $y(\theta)$, $q(\theta)$ and piece-wise continuous $z(\theta)$. Then $(y(\theta), q(\theta), z(\theta))$ is a solution if and only if there exist continuous, piecewise differentiable co-state variables $(\lambda_1(\theta), \lambda_2(\theta))$ such that the following conditions (see Seierstad and Sydsaeter, p. 85 and 396) are satisfied:

$$z(\theta) \in \arg\max_z H(y(\theta), q(\theta), z, \lambda_1(\theta), \lambda_2(\theta), \theta) \quad \forall \theta, \tag{17}$$

$$\dot{y}(\theta) = \theta z(\theta), \tag{18}$$

$$\dot{q}(\theta) = z(\theta) \leq 0, \tag{19}$$

$$\dot{\lambda}_1(\theta) = f(\theta), \text{ a.e.}, \tag{20}$$

$$\dot{\lambda}_2(\theta) = -S'(q(\theta)) f(\theta), \text{ a.e.}, \tag{21}$$

$$\lambda_1(\underline{\theta}) = \frac{1-\delta}{\delta} - \beta(1-\delta), \lambda_2(\underline{\theta}) = 0, \text{ where } \beta \geq 0 \quad (= 0 \text{ if } y(\underline{\theta}) - \Pi > 0), \tag{22}$$

$$\lambda_1(\overline{\theta}) = \gamma, \lambda_2(\overline{\theta}) = -\gamma\overline{\theta}, \text{ where } \gamma \geq 0. \tag{23}$$

Denote by

$$r = \lambda_1(\underline{\theta}) = \frac{1-\delta}{\delta} - \beta(1-\delta) = \frac{(1-\delta)(1-\beta\delta)}{\delta}. \tag{24}$$

Conditions (18), (19) and the transversality conditions (22) imply

$$\lambda_1(\theta) = r + F(\theta), \tag{25}$$

and

$$\lambda_2(\theta) = -\int_{\underline{\theta}}^{\theta} S'(q(\xi))\, f(\xi)d\xi. \tag{26}$$

Define an auxiliary function $\psi(\theta) = \lambda_1(\theta)\theta + \lambda_2(\theta)$. Then from (25) and (26) I get

$$\psi(\theta) = (r + F(\theta))\,\theta - \int_{\underline{\theta}}^{\theta} S'(q(\xi))\, f(\xi)d\xi.$$

The optimality condition (17) yields

$$\psi(\theta)\, z(\theta) = 0 \text{ and } \psi(\theta) \geq 0 \text{ for all } \theta. \tag{27}$$

Derivation of r^* **:** Note that

$$\psi(\underline{\theta}) = r\underline{\theta} \geq 0. \tag{28}$$

Thus, $r \geq 0$. From (24) I obtain the range of possible values of r. The minimal value $r = 0$ corresponds to $\beta = \frac{1}{\delta} > 0$. The maximal value of r is $r^* = \frac{1-\delta}{\delta}$ which corresponds to $\beta = 0$.

The form of optimal $q(\theta)$**:** If $\psi(\theta) > 0$ on a non-degenerate interval, then $z(\theta) = 0$ on this interval and, therefore, by (18) and (19), both state variables $(q(\theta), y(\theta))$ are constant. If $\psi(\theta) = 0$ on the non-degenerate interval Θ' then $\dot{\psi}(\theta) = 0$. Thus I obtain

$$S'(q(\theta)) = \theta + \frac{F(\theta) + r}{f(\theta)} \text{ for all } \theta \in \Theta'.$$

This implies $q(\theta) = q^{(r)}(\theta)$ for all $\theta \in \Theta'$.

First I show that there cannot be more than two intervals of bunching. Suppose there are more than two separable bunching intervals. Then there must be at least two intervals of separation where output is equal to $q^{(r)}(\theta)$. This contradicts continuity of $q(\theta)$.

Denote intervals of bunching by $\Theta_1 = [\underline{\theta}, \theta_1)$ and $\Theta_3 = \left(\theta_2, \overline{\theta}\right]$.[13] On the interval $\Theta_2 = (\theta_1, \theta_2)$ I have $\psi(\theta) = 0$. Hence $q(\theta) = q^{(r)}(\theta), \forall\theta \in \Theta_2$. For all $\theta \in \Theta_1$, $q(\theta) = q^{(r)}(\theta_1) = q_1$ and for all $\theta \in \Theta_3$, $q(\theta) = q^{(r)}(\theta_2) = q_3$. I prove that Θ_3 is empty. Assume it is not empty. Then $q_3 > q^{(r)}(\theta), \forall\theta \in \Theta_3$, and, therefore, $S'(q_3) < S'(q^{(r)}(\theta))$. Thus $\dot{\psi}(\theta) = r + F(\theta) + f(\theta)\theta - S'(q_3)f(\theta) > r + F(\theta) + f(\theta)\theta - S'(q^{(r)}(\theta))f(\theta) = 0$, for all $\theta \in \Theta_3$. Because, $\psi(\theta_2) = 0$ and by (22) $\psi(\overline{\theta}) = 0$ – a contradiction. Thus, there may be only one interval of bunching $[\underline{\theta}, \theta_1]$.

Derivation of equation 7: The first equation to determine the values of θ_1 and r is $\psi(\theta_1) = 0$. This implies

$$0 = (r + F(\theta_1))\,\theta_1 - \int_{\underline{\theta}}^{\theta_1} S'(q(\xi))\, f(\xi)d\xi. \tag{29}$$

We also have $S'(q_1) = \theta_1 + \frac{r+F(\theta_1)}{f(\theta_1)}$. These two equations lead to

$$r\theta_1 = (r + F(\theta_1))\,\frac{F(\theta_1)}{f(\theta_1)}, \tag{30}$$

[13] Note that θ_1 can be equal to $\underline{\theta}$.

which is the same as (7).

Construction of the optimal contract: I established that the optimal output has a form depicted on Figure 2 with $\theta_1 = \theta_r$.

For all $\Pi \geq \Pi^{osm}$, $r = 0$. Hence the equation (15) has only a trivial solution $\theta_r = \underline{\theta}$. Then the optimal output is the *o.s.m.* In this case $\beta = \frac{1}{\sigma} > 0$. Hence, the constraint (14) is binding and $y(\underline{\theta}) = \Pi^{osm}$.

For all $\Pi \in [\Pi^*, \Pi^{osm}]$ the constraint (14) is binding. Hence, $y(\underline{\theta}) = \Pi$, and, therefore, $t(\underline{\theta}) = \Pi$. Define r via

$$\theta_r \, q(\theta_r) + \int_{\theta_r}^{\overline{\theta}} q^{(r)}(\theta) d\theta = \Pi.$$

Differentiating this expression w.r.t. r, I get $\Pi'(r) < 0$ and, therefore, r is uniquely defined for $\Pi \in [\Pi^*, \Pi^{osm}]$.

For $\Pi < \Pi^*$, (14) is not binding and the solution to the problem is the same as for Π^*.

■

Proof of Lemma 5. Modulo the change of variables, the optimality conditions (17)-(21) are the same as for the problem P^A. The transversality condition (22) is replaced by

$$\lambda_1(\underline{\theta}) = \beta'(1-\delta), \lambda_2(\underline{\theta}) = 0, \text{ where } \beta' \geq 0 (= 0 \text{ if } \Pi - t(\underline{\theta}) > 0).$$

Note that in this case the parameter $r = \beta'(1-\delta)$ is unbounded. If $r = 0$ then the optimal output is the *o.s.m.* and corresponding transfers are $t_0(\theta)$ defined by (8) with zero rent for $\overline{\theta}$. If $r > 0$ then the optimal output is $q_r(\theta)$ and transfers are also defined by (8) with zero rent for $\overline{\theta}$.

■

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