

Shadow and Light: Casting a Shadow with Light and the Design of Geometric Phase Quantum Logic Gates in Waveguide Systems

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Abstract

Shadows are a common and familiar optical effect. We can all easily cast, observe, and characterize them. In our daily experience, they occur with the presence of an illumination source and an opaque object that casts the shadow. However, shadows cast by an opaque object are not the only possibility. In the first project of this thesis, we demonstrate what we call the “laser shadow” effect, where beams of light serve as both illumination source and object to create a new type of shadow. We then characterize the temperature and power dependence of the effect. Finally, we demonstrate how this laser shadow is indeed a shadow by comparing it to a genuine shadow.

Creating a quantum computer is a goal at the intersection of both quantum mechanics and computer science. It is of interest because quantum computers have been shown to have an advantage over classical computers in solving certain types of problems. In the second project of this thesis, we design a possible building block for such a quantum computer. The design relies on the “geometric phase”, which theory suggests can provide enhanced robustness against errors. We then test our design in simulations of waveguide systems using two material platforms in different coupling regimes. Finally, we propose a practical experiment to take the design out of the computer and test the waveguide system using a physical apparatus.

Acknowledgements

First, I would like to thank my supervisor, Prof. Jeff Lundeen, for his insight, generosity, and patience. From my time in his group, both as a co-op student and during my master's, he's shown enthusiasm for the scientific method, always making sure we are asking the right question, and always ready to turn uncertainty into a teachable moment. His focus on proper science communication has had a lasting effect on me, one which I will carry in my research career.

I would like to thank the members of the Lundeen Lab, both past and present, for the camaraderie we have had throughout the years. Though the pandemic may have made working with you all the more difficult, I hope we would have the chance to work together in normal times.

I would like to thank both postdocs of the Lundeen Lab, Dr. Lambert Giner and Dr. Raphael Abrahão, for their help and insights throughout my time in the group. The discussions we had proved invaluable in the course of my projects, whether it be troubleshooting problems or general advice.

I would like to thank the Canada Excellence Research Chair (CERC) group for the enlightening discussions that were offered, both formally at group meetings and informally during group lunches. They highlight the importance of keeping your head outside your own bubble, introducing me to topics I would not know about otherwise. They also provided for moments of reprieve from the work.

I would like to thank Dr. Orad Reshef for sharing his wide breadth of simulation knowledge, as it was indispensable for the second project in this thesis.

I would like to thank the Physics Graduate Student Association (PGSA) for continuing to organize social events during the course of my master's, which will have taken place completely during the pandemic. I was always looking forward to these events and they made isolation much more bearable.

I would like to thank my parents, Paul and Thérèse, for their love and support throughout my university studies, which simply grew during the pandemic. This thesis would not have been possible without them. *Merci beaucoup.*

Lastly, I acknowledge financial support from the Natural Sciences and Engineering Research Council (NSERC) during my master's.

Author Contributions

The work presented in this thesis contains the contributions of multiple individuals over several years. All the figures presented in this thesis were created by myself except for figures 2.1, 2.4, 4.7, 4.8a, 4.9, and 10.1.

The Laser Shadow project described in Part I began as a collaboration between Dr. Akbar Safari, a postdoctoral fellow in Prof. Robert Boyd’s group and Jordan Pagé, an undergraduate co-op student in Prof. Jeff Lundeen’s group before the onset of the pandemic. Dr. Raphael Abrahão, the postdoctoral fellow in the Lundeen Lab, and Jordan built the experimental apparatus. Jordan obtained initial data and continued troubleshooting the experiment, with the help of both postdocs. Dr. Abrahão and Jordan obtained the data where the shadow is split in two using a beam displacer. Dr. Abrahão recorded the video from which I extracted the images in fig. 4.10. Jordan obtained pictures of the shadow of a human hair. I obtained images of the laser shadow effect, both direct images and data images. Jordan created the initial `python` and `LabVIEW` codes used in the analysis of the images, which I then modified as necessary. I showed that temperature effects do occur in the system. I measured the temperature of the ruby as a function of input power. I created the shutter mechanism to control the heating of the ruby, edited the `LabVIEW` code to accommodate this change, and to increase the dynamic range of the pictures taken. I took the transmission measurements for both lasers and analyzed them. I measured the stability of the green laser (expected vs. measured power). I measured the beam waist of the green laser in both transverse dimensions. The mathematical description of the effect was derived by Dr. Safari, which I then used to model our experiment and compare with experimental data. Prof. Boyd suggested the possibility of an “antishadow” effect. I showed that this effect is much harder to see than the normal laser shadow. The project was supervised throughout by Prof. Lundeen.

The Geometric Phase Waveguide Logic Gate project of Part II, in its original form, was designed by Matthew Runyon, a former master’s student in Prof. Lundeen’s group back in 2017. The initial experimental setup was built by an undergraduate honours student, Michel Poisson. With Michel’s assistance, I took measurements which showed an inconsistency between theory and experiment. I performed all the simulations and re-derived the theory. During this initial period, the postdoctoral fellow of the group at the time,

Dr. Lambert Giner was of great help, proposing solutions and assisting with simulation issues. These initial contributions are primarily described in Chapter 8. I discovered a problem that converted the project from Si based logic gates to fused silica based. I reworked the simulations for the new material and geometry. I then proposed, and Prof. Lundeen agreed, that we move the project from the virtual, to real space. We reached out to the Extreme Ultrafast Photonics group at the University of Ottawa, headed by Prof. Ravi Bhardwaj, to help with the fabrication of our devices. The lead on the fabrication side was Athulya Thulaseedharan, a doctoral student. I reworked the experimental testing setup to reflect several changes in experimental parameters, such as the size of the desired gate, the change in wavelength, and the change from grating couplers to butt-coupling. During this second part, Dr. Abrahão helped with the design process of the modified apparatus. Another postdoctoral fellow from the Boyd group, Dr. Orad Reshef, was of great help in troubleshooting any simulation related question, at both stages of the project.

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Chapter 1

Outline

This thesis contains two distinct and unrelated projects.

In Part [I](#), we discuss the Laser Shadow project. Chapter [2](#) explains the inspiration behind the project as well as what we mean by a “laser shadow”. This Chapter also theoretically models the shadow. Chapter [3](#) details the experimental apparatus and additions that are needed to properly characterize the effect. Chapter [4](#) details the various results obtained, demonstrating the laser shadow. We conclude this Part in Chapter [5](#).

In Part [II](#), we discuss the Geometric Phase Waveguide Logic Gate project. The motivation, goals, and central concept behind the project are introduced in Chapter [6](#). The derivation of the designs of the desired logic gates are detailed in Chapter [7](#). Initial simulations of the logic gates and the problems that come with the simulations are discussed in Chapter [8](#). Subsequent simulations of the logic gates in a different material system and the continuation of simulation related issues are discussed in Chapter [9](#). The details of transitioning from simulations to a physical experiment are detailed in Chapter [10](#), where an outlook is given and the project is concluded.

Final remarks for the thesis as a whole are presented in Chapter [11](#).

Several appendices accompany this thesis. In Appendix [A](#), additional images relating to Part [I](#) are illustrated. In Appendix [B](#), supporting derivations for Part [II](#) are detailed. Finally, Appendix [C](#) prints important scripts which were used during the course of Part [II](#).

Part I

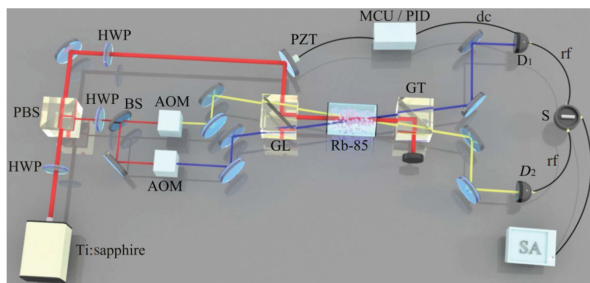
The Laser Shadow Project

Chapter 2

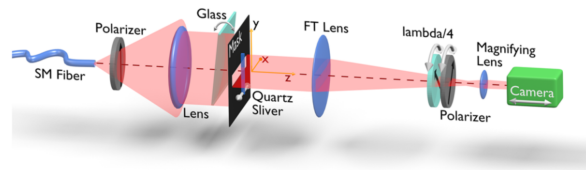
The Concept

2.1 Inspiration

The inspiration to pursue the creation of a laser shadow came from the literature. Specifically, experimental papers include a schematic of the apparatus. This is to help illustrate what and how the authors achieved their results. As computing power has progressed, the quality of these schematics has also improved. The use of commercial design software is prevalent to accomplish this goal. However, this software does not always take into account the real physics of the objects they are representing. This is particularly noticeable in three-dimensional schematics.



(a) Example 1. Reprinted figure with permission from Shengshuai Liu, Yanbo Lou, and Jietai Jing, Physical Review Letters, **123**, 113602, 2019. Copyright 2019 by the American Physical Society [1].



(b) Example 2. Reprinted figure with permission from Charles Bamber, Jeff S. Lundeen, Physical Review Letters, **112**, 070405, 2014. Copyright 2014 by the American Physical Society [2].

Figure 2.1: In both apparatuses presented in this figure, the experiments that each represents are not relevant to the thesis; it is the figure itself which is important. Within each figure, the object which represents the laser beam is casting a shadow within the software's design space. This neglects the fact that these objects are supposed to represent light and not cast a shadow.

Unlike electrons which can interact in free-space through their charge, light itself does not interact with

itself in free-space, just like waves passing through each other in the ocean. This is also noticeable for light through lasers, as we do not see the beam in free-space unless there are particles that can scatter the light. However, in schematics such as the ones presented in fig. 2.1, the laser beam *is* visible. This is because the software treats the object that the laser represents as an opaque object, neglecting the reality of the beam of light. The shadows in these figures have an illumination source and an object which are beams of light. Having a shadow created in this configuration should not be possible.

The key word above is “should” as this project aims to create such a shadow, where both object and illumination source are light.

2.2 What is a Shadow?

What is a shadow? We are all familiar with shadows. You simply need to have a light source nearby and you will cast a shadow. Large-scale shadows are also possible, such as the shade created by a cloud on a sunny day or a solar eclipse, where the shadow is on a planetary scale. In these three simple examples, the illumination source is the sun, and the opaque object which is blocking the light is either your body, a cloud, or the Moon. This light can be absorbed or reflected by the object. In this project, we aim to have light be both the source and the object. This is the laser shadow.

While we are focusing on what a shadow *is*, we must also be clear as to what a shadow *is not*. The overall effect of a shadow is to create a region of space where light is not present, relative to its surroundings. One can observe such a region of space by looking at the bottom of a pool whose surface is disturbed by waves. There will be regions which are darker than its surroundings. This is not a shadow: the light is bent out of the way rather than being absorbed by the water. A visual explanation of this difference is illustrated in fig. 2.2.

This observation allows us to set out one criteria for the laser shadow: we must have the presence of absorption, just like regular shadows. In the context of light, this requires an interaction medium. To guide us in the selection of the right medium we set out additional criteria for the laser shadow. We want to use visible light to achieve this effect. This is to reflect the fact that regular shadows are also visible to the unaided eye. This will restrict the wavelengths of light we will have to consider. The next criteria will be that the effect must be large scale. In our case, “large” will mean the effect can be seen without any additional optical aid, i.e., with our unaided eyes. We do not want to limit ourselves to a microscopic effect, as this does not relate to our experiences with shadows. We want this effect to have the same properties as a normal shadow. Our last criteria will be the speed of the effect. Consider a genuine shadow being cast by an object, such as a cloud. When the cloud moves, so does the shadow. There is no delay between the

objects movement and the shadows movement. Therefore, we want a fast material response to replicate this property.

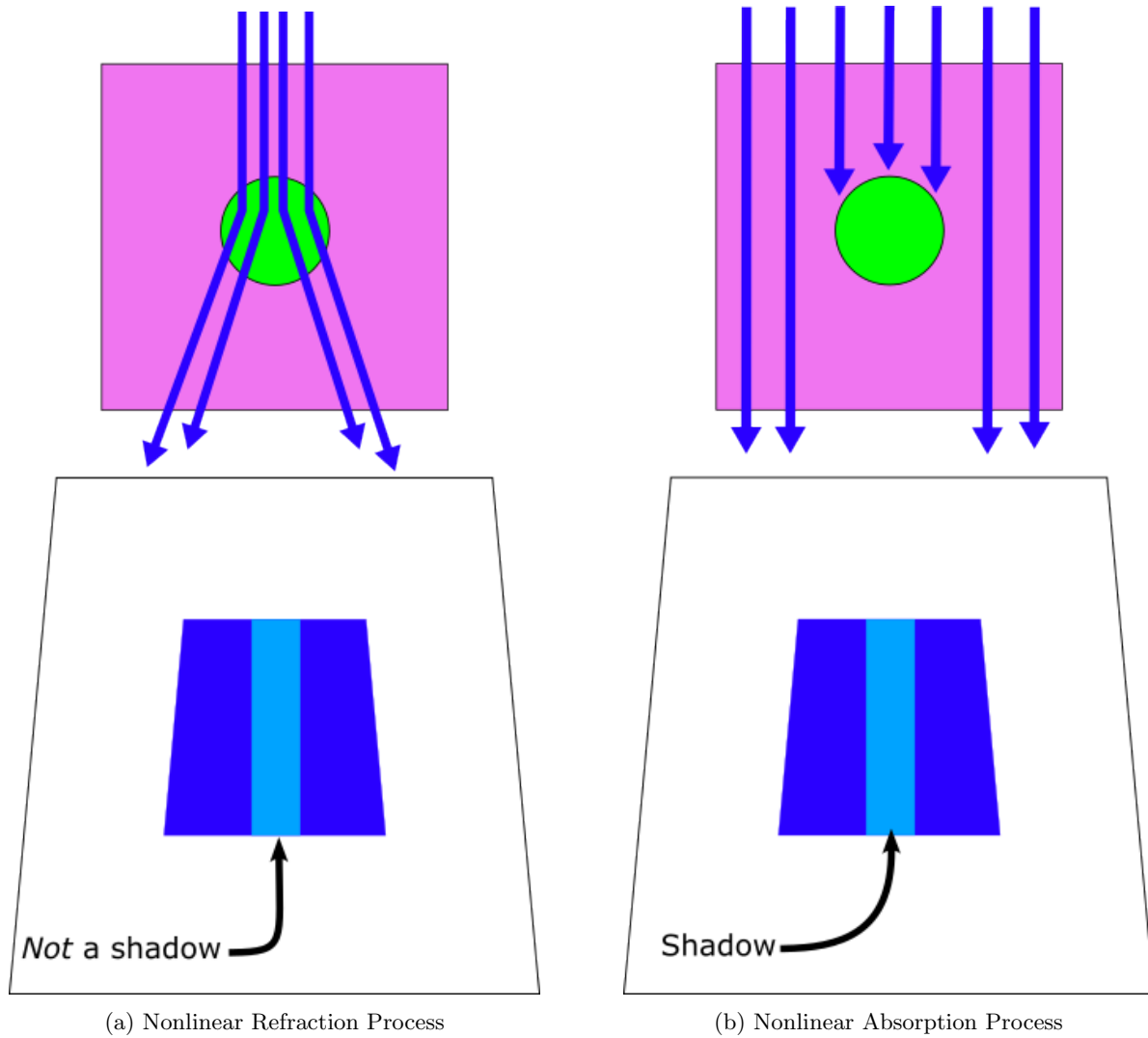
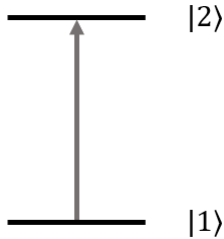


Figure 2.2: Comparing nonlinear processes in the context of laser shadows. In fig. 2.2a, the illumination beam (blue arrows) is bent out of the way to create a region of reduced brightness on the paper. This is not a shadow, as there is no obstruction due to the presence of an object. In fig. 2.2b, the illumination beam is partially blocked due to the presence of an object. This is a shadow.

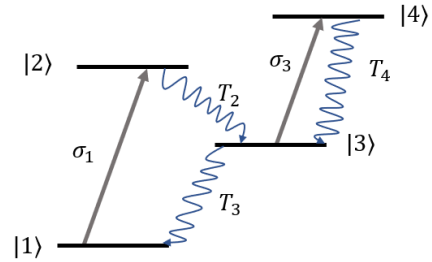
The first criteria we will tackle will be the absorption criteria. The typical case of absorption is that of “saturation of absorption”, whose energy level diagram is shown in fig. 2.3a. This is a two-level process, where one laser would be used to change the population from level $|1\rangle$ to level $|2\rangle$. In this process, as the incident intensity increases, the capacity of the material to absorb this intensity slowly decreases, up to a threshold. After the intensity passes the threshold, which is known as the “saturation intensity”, the absorption decreases significantly. However, this process is inadequate for our purposes. When our second

laser beam would encounter this system, the system would not be able to absorb any more light, hence creating a local *increase* of light. We have named this region of increased light the “antishadow”. What we need instead is a material which exhibits “inverse saturation of absorption”. The energy level diagram of this process is shown in fig. 2.3b.

We extend the two-level system to a four-level system. The first beam excites atoms from $|1\rangle$ to $|2\rangle$. This level then decays to an intermediate level $|3\rangle$. When illuminated by the second laser beam, the population is now moved from $|3\rangle$ to $|4\rangle$, being absorbed by the system. Put another way, in the presence of the first laser beam, the material will have increased absorption of the second beam. Hence, this is the nonlinear process which will give us the effect we want. We must now determine the right material which exhibits this nonlinear absorption process.



(a) Saturation of Absorption Process



(b) Inverse Saturation of Absorption Process

Figure 2.3: Energy diagrams of two different nonlinear absorption processes. In the two-level saturation of absorption process, the absorption will decrease as a function of intensity. In the four-level inverse saturation of absorption process, the absorption of the second transition will increase as a function of intensity of the first transition. This second process will allow us to observe the desired effect.

The important parameters in the inverse saturation of absorption process are the absorption cross-section σ for each transition, and the lifetimes T of each energy level. The cross-sections are a measure of the likelihood of the transition occurring and the lifetimes are the average amount of time that an atom will remain excited at that energy level. These parameters are schematically represented in fig. 2.3b.

2.3 The Plan

To summarize: we want to use light as both the illumination source and the object to create a shadow. This shadow will be created using a nonlinear absorption process. We want the effect to be visible with the unaided eye. As the material we will need does not exhibit the usual saturation of absorption, we are slightly restricted in our material choice. However, it was previously noted [3, 4] that ruby, whose chemical formula is $\text{Al}_2\text{O}_3:\text{Cr}$, exhibits the inverse saturation of absorption effect we are looking for. Furthermore,

this effect has been shown to be quite large [5]. Since ruby has a special place in optics due to it being the gain material in the first laser [6], it has been extensively studied. This knowledge can be used to help in the design process.

With the material selected, we must now select the wavelengths of both laser beams. We first consider the wavelength of the first transition. As indicated in fig. 2.3b, the main consideration is that we want σ_3 to be larger than σ_1 . We want this condition to allow atoms to be excited to $|4\rangle$. We also want the first laser to be absorbed by the ground state of ruby, 4A_2 . These conditions are satisfied by $\lambda_1 = 532$ nm [7]. This is convenient as we have easy access to a powerful laser at 532 nm (see details in sec. 3.1). Therefore, this green laser will be the object laser.

For the second laser beam, we are constrained in choosing a visible wavelength that can be absorbed by the ruby's 2E state [8] (which is the $|3\rangle$ state), and has a larger absorption cross-section [7, 8] for the second transition than the first transition. These conditions are satisfied with $\lambda_2 = 450$ nm [7]. Therefore, the blue laser will be the illumination laser. We now have, in theory, the general physical process which we want to implement. That is, we want to observe inverse saturation of absorption in ruby, using 532 nm and 450 nm lasers. Since these are the wavelengths of green and blue respectively, each laser will be referred to using either their colour or their wavelength interchangeably.

In addition to the above discussion regarding the energy levels of the ruby which will be involved in this experiment, we also want to add additional information regarding the decay routes of these levels. First, the $^4T_2 \rightarrow ^2E$ (i.e., $|2\rangle \rightarrow |3\rangle$) and $^4T_1 \rightarrow ^2E$ (i.e., $|4\rangle \rightarrow |3\rangle$) decay routes are fast and radiationless [5]. Second, the decay from $^2E \rightarrow ^4A_2$ (i.e., $|3\rangle \rightarrow |1\rangle$) is the optical decay route for a ruby laser, emitting at 694 nm [6]. This emission will be observed when we take images of the ruby as it is being excited (e.g., fig. 4.9a), and will be filtered out during measurements (see details in sec. 3.1).

Before moving to the experiment, however, we want to be able to quantify the shadow to see how large the effect is. To do so, we will model the shadow based on the population levels of the Cr^{3+} ions of the ruby.

2.3.1 Modelling of the Shadow

The modelling of the effect will be done with rate equations [9, 10] of the populations in the energy levels in ruby [8]. In this derivation, we follow Safari [5]. We begin with the number density of Cr^{3+} ions in our ruby sample, N , which is given by:

$$N = N_1 + N_3, \tag{2.1}$$

where N_1 is the population in level $|1\rangle$ (the ground state) and N_3 is the population in level $|3\rangle$ (the metastable state). We note that the lifetimes of levels $|2\rangle$ and $|4\rangle$ are short enough to be neglected. Thus, the change in population in the ground state is the sum of the population being excited out of the ground state (to level $|2\rangle$), and the population which is decaying back to the ground state (from $|3\rangle$ to $|1\rangle$), or given mathematically:

$$\frac{dN_1}{dt} = -\frac{I}{\hbar\omega}\sigma_1 N_1 + \frac{N_3}{T_3}, \quad (2.2)$$

where σ_1 is the absorption cross-section of the transition excited by the green laser, T_3 is the lifetime of the $|3\rangle$ energy state, $\hbar\omega$ is the energy of one green photon, and I is the intensity of the green laser. Furthermore, we also define the saturation intensity as:

$$I_S = \frac{\hbar\omega}{T_3\sigma_1}. \quad (2.3)$$

The saturation intensity is the intensity at which the material will no longer be able to absorb more energy i.e., it saturates. We now solve equation 2.2 at steady state ($\frac{dN_1}{dt} = 0$). Using the definition of the saturation intensity as well as eq. 2.1 to eliminate N_3 , we get:

$$N_1 = \frac{N}{1 + I/I_S}. \quad (2.4)$$

After obtaining this expression, we focus on the control parameters for the experiment. The main control parameter we will have over the experiment will be the power of the green laser. We therefore want to obtain an expression for the absorption of the blue beam in the ruby as a function of the power of the green beam. To obtain this expression, we first consider the definition of the absorption coefficient α . This coefficient, not to be confused with the absorption cross-section σ , is a measure of how much intensity a beam of light will lose to absorption as it progresses through a length of material. This coefficient α is given by [11, p. 167]:

$$\alpha = \sigma N, \quad (2.5)$$

where σ is the absorption cross-section and N is the total number density. Expanding this expression with eqs. 2.1 and 2.4 as well as the relevant absorption cross-sections we get:

$$\begin{aligned} \alpha(I) &= \sigma_1 N_1 + \sigma_3 N_3 \\ &= \frac{\sigma_1 N}{1 + I/I_S} + \sigma_3 N \left(1 - \frac{1}{1 + I/I_S} \right). \end{aligned} \quad (2.6)$$

Note that the above equation is a function of the absorption of the blue laser in terms of the intensity of the green laser. An obvious question to ask is: what about the intensity of the blue laser? Shouldn't we have $\alpha(I^{\text{green}}, I^{\text{blue}})$? The answer is no, for the reason that the blue laser will be of very low intensity, particularly when compared with the green laser. Therefore we can use the approximation $I \sim I^{\text{green}}$ to neglect the intensity of the blue beam.

Finally, we add additional labels on the previous equation to be explicit about which parameters are determined by the blue laser, and which by the green laser:

$$\alpha^{\text{blue}}(I) = \frac{\sigma_1^{\text{blue}} N}{1 + I \frac{\sigma_1^{\text{green}} T_3}{\hbar \omega}} + \sigma_3^{\text{blue}} N \left(1 - \frac{1}{1 + I \frac{\sigma_1^{\text{green}} T_3}{\hbar \omega}} \right). \quad (2.7)$$

Equation 2.7 allows us to model the absorption for the inverse saturation of absorption process. Let us compare this equation to the model of saturation of absorption [11]:

$$\alpha(I) = \frac{\alpha_0}{1 + I/I_S}, \quad (2.8)$$

where α_0 is the unsaturated absorption coefficient and I_S is the saturation intensity. Figure 2.3 compares the processes schematically but eqs. 2.7 and 2.8 allows us to compare the process more qualitatively.

In eq. 2.8, that is for the saturation of absorption process, when I increases, $\alpha(I)$ *decreases* in value. However in eq. 2.7, that is for the inverse saturation of absorption process, when I increases, as all other parameters are fixed values, the first term in the expression will decrease but the second term will increase. Thus, if $\sigma_3 > \sigma_1$, $\alpha(I)$ will *increase*. This is precisely what we want to happen with the absorption in our system.

The next step in the modelling requires us to consider how we will measure this result. We do not directly measure the absorption. What we do measure is the intensity distribution of the blue beam. Because of the increased absorption in the region where there is green light, this region will be darker than its surroundings. Therefore, we will be able to see the shadow and extract data from this intensity distribution. To capture this intensity distribution, a camera will be used. To maximize the size of the effect, we consider the shape of the green laser beam.

Laser beams are usually modelled as Gaussian distributions, such as:

$$I(x, y) = \frac{2P}{\pi w_{0x} w_{0y}} e^{-\frac{2x^2}{w_{0x}^2} - \frac{2y^2}{w_{0y}^2}}, \quad (2.9)$$

where P is the total power, x, y are the Cartesian coordinates of the distribution, and w_{0x} and w_{0y} are the beam waists in the x and y dimensions respectively. Figure 2.4 helps us visualize each dimension. The green

laser propagates in z and has a spatial distribution in x and y (hence the beam waists in x and y). These beam waists will be shaped to maximize the size of the effect. Since there are two laser beams to consider, we cannot use the convention that z is the direction of propagation for both beams. It will only apply to the green beam. From this figure, we can conclude that the camera will image in the (y, z) plane. The shadow will be observable as a cross-section in the y dimension on this image.

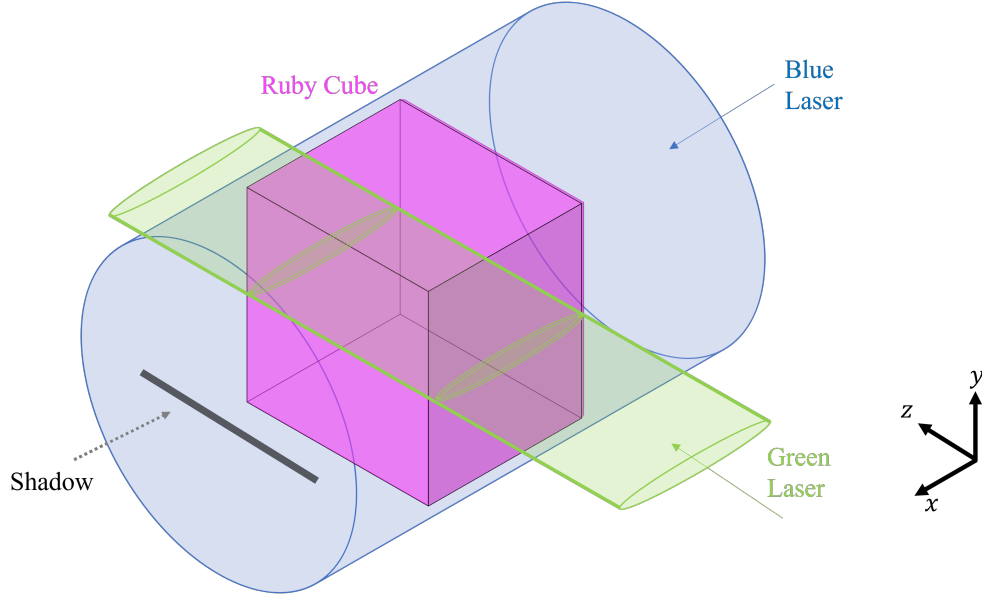


Figure 2.4: Schematic of the spatial dimensions at the ruby for the Laser Shadow. The ruby will is moved in the y dimension to obtain averages and statistics. The shadow appears as a region of reduced brightness in the (y, z) plane.

As the green laser propagates in z through the ruby, some of its intensity will be absorbed. Thus, we need to consider the possibility that the effect of the shadow changes with z . To observe this effect, one needs to slice the image at different points along z . This would allow for a “shadow vs. z ” measurement. However, for mathematical simplicity, the image is integrated along z , averaging out this effect. The result of this averaging is our shadow data, which takes the form:

$$I(y) = I_0 e^{-\int_0^L \alpha^{\text{blue}}(y, z) dz}, \quad (2.10)$$

where I_0 is the incident intensity of the blue laser, α^{blue} is the expression in eq. 2.7, and L is the length of the ruby in the z dimension. This result is a one-dimensional cross-section of the intensity of the blue laser as a function of one spatial dimension, y . It is in the y dimension where the shadow will be visible. As with

a real shadow, we will have a region of mostly constant intensity, with a region of reduced intensity at the location of the shadow.

2.4 Experimental Context

At the heart of this experiment lies the large inverse saturation of absorption of ruby, noted in [5]. Since ruby is a material that has been extensively studied [5, 6, 7, 8, 12] throughout the years, it is surprising that we can still find new properties and new uses for this material.

As well as exploring new material properties, this project also redefines what a shadow can be. A laser shadow is a genuine shadow which is induced by light and not by a solid object. Since shadows are also simple optical effects, it is intriguing to find something new from something well known. The study of shadows is not limited to a laser shadow as “normal” shadows themselves continue to be studied [13].

Chapter 3

Design of Experiment

3.1 Introduction

We now have a goal: to create and measure the shadow cast by a laser beam within a ruby crystal using a green beam as the object and a blue beam as the illumination source. This Chapter details the constraints and the requirements needed to experimentally achieve this goal.

We begin with the two laser sources. We will first detail the blue laser. It is a *ThorLabs CPS450*. This laser emits at 450 ± 10 nm at a constant power, nominally 4.5 mW. The light is emitted as a collimated elliptical beam ($3.2 \text{ mm} \times 1.0 \text{ mm}$). A $\varnothing 1''$ achromatic doublet, $f = 45$ mm, with an anti-reflective coating for 400-700 nm, lens focuses the light onto a $75 \mu\text{m}$ pinhole, which is used as a spatial filter to make the light more symmetrical, as the pinhole acts like a point source of light. The beam was then expanded using a $\varnothing 2''$ achromatic doublet, $f = 1000$ mm, with an anti-reflective coating for 400-700 nm lens. This was done for two reasons: first, the beam will be larger than the ruby, allowing the entire ruby face to be imaged. Second, this flattens the spatial mode to approximately a plane wave. We want the same spatial distribution to hit the ruby to have the same amount of blue light be absorbed across the face of the ruby.

Next, we detail the green laser. It is a *Spectra-Physics Millennia eV 25 System*. It emits at 532 nm at a maximum power of 25 W. It emits in the TEM_{00} spatial mode with a beam diameter of $2.3 \text{ mm} \pm 10\%$ ¹. A half-waveplate is used to control the beam's polarization. We want to optimize this beam shape for our experiment. Without any shaping, the beam would keep its initial shape and be symmetric on the input (x, y) plane. As the blue laser propagates through the ruby in the x dimension, it would only encounter the green laser, and thus the region of increased absorption, for a limited distance. We want to maximize the interaction length between the two lasers within the ruby. This means stretching the beam in the x

¹Beam diameter defined at $1/e^2$ points.

dimension to match the size of the ruby. Only changing this dimension, however, would have a net increase in the surface area of the green laser on the ruby, which decreases its intensity. As this is a nonlinear effect, we want to maximize the interaction length as well as the intensity. To compensate, the beam is also compressed in the y dimension.

Each of these transformations is accomplished with a 4f arrangement of a pair of cylindrical lenses, which were all mounted in individual rotation mounts (*ThorLabs RSP1X15* rotation mounts for $\varnothing 1''$ optics, 360° continuous). All four of these lenses are $\varnothing 1''$ plano-convex round cylindrical lenses with an anti-reflective coating for 350-700 nm. The squeezing in y is accomplished with cylindrical lenses of focal lengths 50 mm and 500 mm (in that order). The stretching in x is accomplished with cylindrical lenses of focal lengths 100 mm and 400 mm (in that order). This beam then terminates safely at a beam trap after the ruby crystal.

The third major component to this experiment is the ruby itself. The ruby was obtained from Wuzhou Yingma Jewelry Co., Ltd. [14] and is in the form of a cube. This facilitates the injection of both laser beams, which can both enter and exit the crystal at normal incidence. The ruby measures 12.0 ± 0.5 mm in each dimension. The last two surfaces of the cube will be used to hold it in place. The doping concentration of Cr^{3+} in the ruby was experimentally determined and will be discussed in sec. 4.4.1. Several rubies of different dopings were obtained from this manufacturer. However, only one doping was characterized as it led to most visible effect.

The last major component is related to data collection. As previously mentioned, the raw data we acquired are images. Thus, we need an imaging system. The camera used is an *IDS UI388xCP-M*. It is a CMOS, monochrome camera. The maximum resolution available (which was the resolution that was used) is 3088×2076 pixels (6.41 MPix). The pixels are $2.4 \mu\text{m}$ in size. The camera has a 12-bit bit depth. However, images were recorded as 8-bit. An example raw picture from the camera can be found in an appendix (see fig. A.3a). The maximum frame rate at the maximum resolution is 58.0 fps. A *ThorLabs NE40A* (absorptive ND filter with optical density of 4.0) is attached to the camera to ensure it does not saturate.

The camera is placed on a manual translation stage (*NewFocus Model 9064-X Linear Stage*). This allows us to scan the image over the ruby's length in the x dimension (to find the optimal imaging plane) without having to constantly move the camera and change the alignment. This imaging plane is the input face of the ruby in x . The imaging lenses ($\varnothing 1''$ achromatic doublet, $f = 300$ mm, with an anti-reflective coating for 400-700 nm and $\varnothing 1''$ achromatic doublet, $f = 50$ mm, with an anti-reflective coating for 400-700 nm) were selected to shrink the size of the beam so that it can fit on the camera's sensor in its entirety. Because we only want to see in the blue region of the spectrum (since that is where the absorption and thus the shadow is), a blue bandpass filter (*ThorLabs FB450-40*) is added in the imaging system as well. This filter is centred at 450 ± 8 nm, has a full-width half max (FWHM) of 40 ± 8 nm, and had a minimum transmission of 45%

at the centre wavelength.

3.2 Experimental Apparatus

The apparatus was contained within a fully enclosed box to contain any reflections or scattering. This is for safety reasons because of the large powers that are required in the nonlinear process. The light can only exit the box in one of two paths. The first path, named the “imaging” path, is to the camera, which sits right outside the box. The second path, called the “direct” path, requires some propagation length outside of the box. It terminates at a piece of white paper upon which the image is projected. A lens of $f = 125$ mm is used to focus the light to this piece of paper. The first path is to take quantitative measurements with the camera. The second is to be able to see the shadow with the unaided eye, which is one of our criteria. To change between the two paths requires either the inclusion of a removable mirror (for the direct path) or the lack of said removable mirror (for the imaging path). The removable mirror is mounted on a *ThorLabs KB1X1 Kinematic Base* with a *ThorLabs LMR1 Mount* to allow for repeatable placement at the same position and orientation. Whichever path is in use requires the blue bandpass filter. In the path of the green laser, the first two cylindrical lenses apply the stretching transformation in x and the second pair applies the squeezing transformation in y .

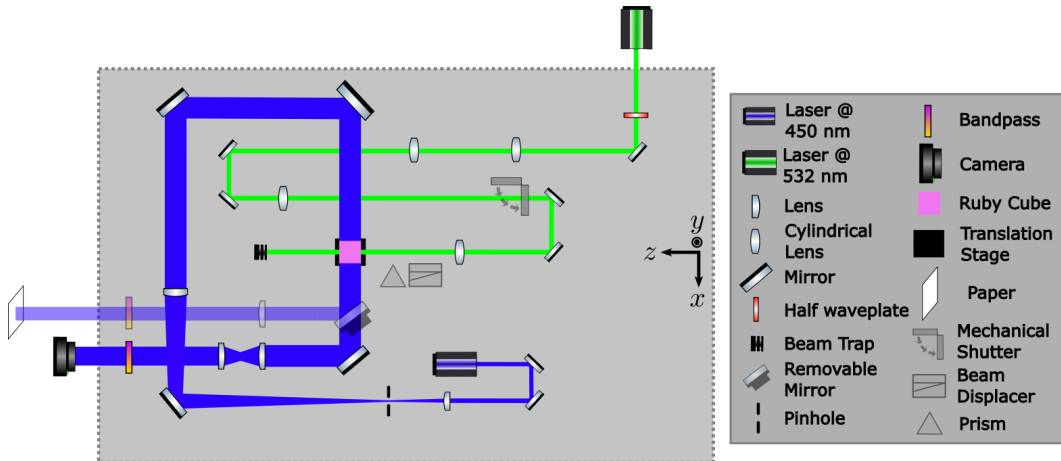


Figure 3.1: Essential overview of the experimental apparatus² for the observation and characterization of the Laser Shadow. The ruby is translated in and out of the page to get an average and statistics for measurements. We have two imaging paths for qualitative (“direct”) and quantitative (“imaging”) measurements. The green (object) laser’s spatial profile is modified to optimize the effect before hitting the ruby. The blue (illumination) laser’s spatial profile is modified to approximate a plane wave when it hits the ruby.

A schematic of the apparatus is presented in fig. 3.1. Only essential elements have been included. One element which has been neglected in the figure is the temperature monitor (*Omega CN740 Temperature*

²Figure created in part with [15].

Controller), which is connected to the ruby crystal by a K-type thermocouple and *ThorLabs TCDT1* thermal tape. The temperature monitor remained attached to the ruby throughout other measurements to allow us to keep monitoring its temperature. The other elements are discussed in sec. 3.4. The translation stage (*Newport CONEX-LTA-HL Motorized Actuator Integrated with CONEX-CC Controller* with a maximum travel range of 25 mm) moves the ruby in the y dimension. This allows us to obtain an average. We take one image at every step of the translation stage through its designated 2 mm movement, where the step size is 0.1 mm. The average is obtained by combining these images into one. This is repeated for each power. The averaging also allows us to smooth out imperfections which are on the ruby's surface.

The placement of the temperature sensor itself on the surface of the ruby is also considered. If the sensor is too close to the green laser, it will measure a change in temperature which is attributable to the *presence* of the laser and not the change in temperature *caused* by the green laser. Therefore, we want to measure the temperature from a point on the ruby which is the furthest away from the green laser's input. This will allow the heat caused by the green laser to dissipate through the ruby and it will be that temperature which is measured.

In the y dimension this point is above the laser. In the z dimension, this point is on the opposite side of the ruby, which is the output face of the green laser. The final position is visible in fig. 3.2b. Figure 3.2 shows both the input and output face of the ruby in the z dimension. This set of pictures show the placement of the sensor with respect to the green laser at both the input and output planes.

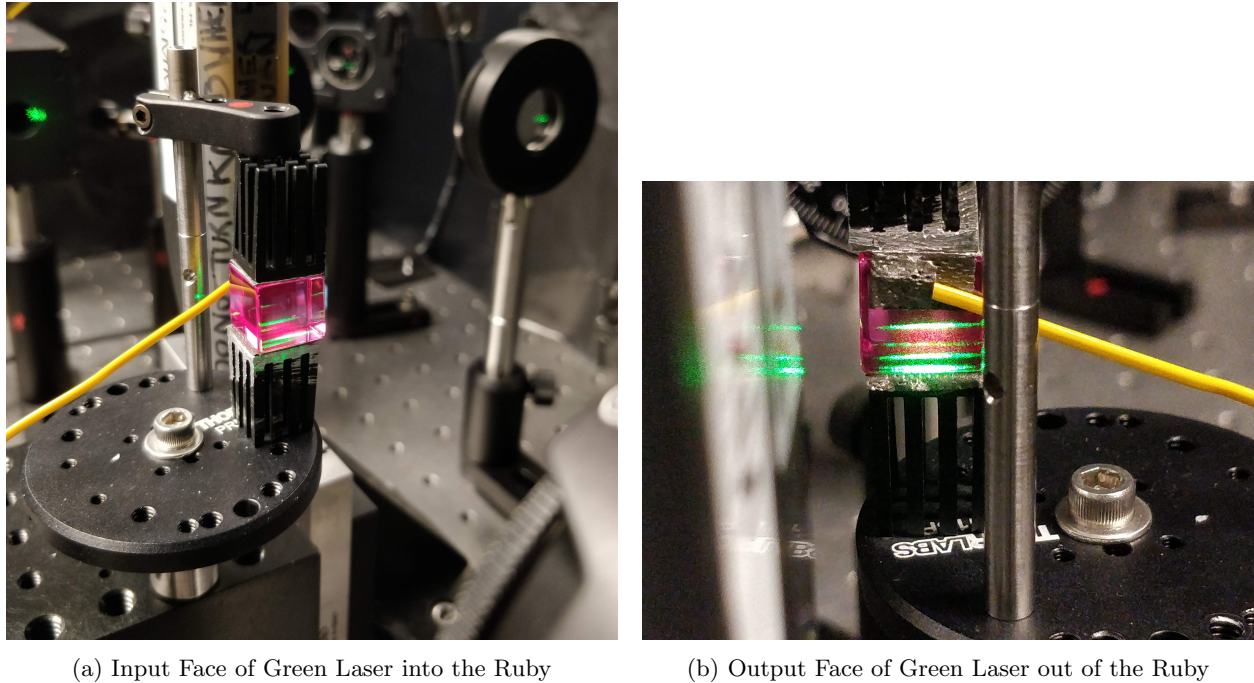


Figure 3.2: The ruby crystal is shown as its mounted in the apparatus with the green (object) laser visible. In both figures, the temperature sensor is visible. In fig. 3.2b, the placement of the temperature is shown to not be blocking the exit of the green laser out of the ruby.

3.3 Heating Problem

The ruby will heat up when hit by the powerful green laser as it is absorbing some of the green laser. Recall that the green laser inverts the population from the ground state $|1\rangle$ to the excited state $|2\rangle$. The excited state then de-excites to the meta-stable state $|3\rangle$ through the creation of phonons. These additional phonons increase the temperature of the crystal structure. This is a problem because the absorption cross-sections σ (as well as the index of refraction and absorption coefficient) depends on temperature [8, 12]. The derivation of sec. 2.3.1 was made assuming constant room temperature [5]. Therefore, increasing the temperature of the ruby by a significant amount will move the measurements away from the model.

The change in temperature that the ruby experiences without temperature control are noted in table 3.1. The temperatures in this table were recorded at thermal equilibrium. This thermal equilibrium was reached after the temperature monitor did not register a change in temperature for at least five uninterrupted minutes. These temperatures were measured with the ruby attached to two heatsinks which are attached on the ruby's top and bottom surfaces (see fig. 3.2 for heatsink placement). These temperatures are significantly different than room temperature. Hence, not controlling the amount of heat that reaches the ruby will have adverse effects. The unwanted effects to our figure of merit in particular, the contrast, are discussed in sec.

4.3.

Therefore, we must take active control of the amount of time that the ruby will be exposed to the green laser, as this will ensure it is heated by the smallest allowable amount.

Power (W)	Temperature (°C)
5	63
8	82
10	96
13	112
15	124
18	137
20	149

Table 3.1: Power vs. Temperature of Ruby Cube Without Temperature Control

This control over the temperature is obtained with the addition of a mechanical shutter, which is discussed in the following section.

3.4 Modifications Required for Characterizations

3.4.1 Controlling Temperature with a Mechanical Shutter

Controlling the temperature was done with the addition of a mechanical shutter in the path of the green beam. Due to geometric constraints, it was placed before the second last mirror in the green laser's path, instead of being closer to the ruby, which would have been ideal. We can achieve the desired control over the temperature of the ruby by limiting the amount of time it is exposed to the intense green laser, hence the shutter. Preventing the ruby from being heated in the first place is preferable to actively cooling it. This control method would apply an external temperature gradient which would have an unknown effect on the crystal structure.

The mechanical shutter is created by a combination of a motor (*TowerPro Micro Servo Digital 9g SG90*) which is connected to an Arduino UNO and controlled by the computer. Anodized aluminum foil is used with a foam-like material is used as the shutter material. This combination is directly attached to the motor. The foil is the material which the laser will hit. The foam-like material is used to give the foil some rigidity. A LabVIEW script is used to collect the data. The translation stage moves by 0.1 mm over 2 mm, giving us a total of 21 steps (and therefore pictures) per dataset. The camera was set at its maximum resolution. This allowed for a maximum frame rate of 58.0 fps. However, the camera was set at ~ 6 fps. To maximize the dynamic range of the images, the exposure was manually set at 0.125 s. We note that there was no gain set.

The script proceeds as follows. It initializes the connection with the shutter, translation stage, and camera. It sets the exposure on the camera and waits 500 ms for that change to take effect. The translation stage is then moved to the initial position. As there could be a distance to cover between the stage's current position and the initial position, the script waits 7 s before proceeding. It then proceeds to the main loop of the script. The stage is moved to the initial position. For the first iteration, this is already true. The script waits for the stage to move to that location with a delay of 2 s. This is because the script moves to the next instruction without checking the current instruction completes. The shutter is then opened. The script waits 500 ms to allow the shutter motor to actuate before taking the picture. The shutter is then closed, and the script waits another 500 ms to allow the shutter motor to return to its initial closed position. These specific timings of 500 ms were used as tests showed they allowed for the ruby to be exposed to the laser for the smallest amount of time. Once the shutter is closed, the script waits 8 s before finishing this iteration of the loop. This is to allow the ruby to cool somewhat before proceeding. Testing showed that there was no distinguishable difference in the ruby temperature between waiting ~ 10 s and 1 minute at this stage. The script then loops and moves the translation stage by one step to the next position. The loop continues until the stage has reached its final position and recorded those measurements. The script then completes by closing the connections to the instruments. Even by limiting the amount of time the ruby is exposed to the green laser, it will still have warmed slightly during this process (as shown in table 3.2). Therefore, we wait until the ruby is back at room temperature (~ 23 °C) before proceeding to the next dataset. An example cooldown process is presented in fig. 3.3.

In this figure, the first rise in temperature is the data taken with the blue and the green lasers on. The short decrease that follows is due to the time it took to change the settings in the script and to turn the blue laser on or off, before taking a background dataset with only the green laser on. This set was to confirm whether or not scattering from the green laser was detectable at the camera. Since high powers of the green laser were used, such scattering may be detectable at the camera if there is not enough attenuation. If this is detectable, then it must be subtracted from the initial set, as it would contribute to the intensity detected and would not be from the illumination source i.e., the blue laser. This data is the second rise in temperature. Once this concluded, the ruby was allowed to cool back to room temperature, which takes up the rest of the figure.

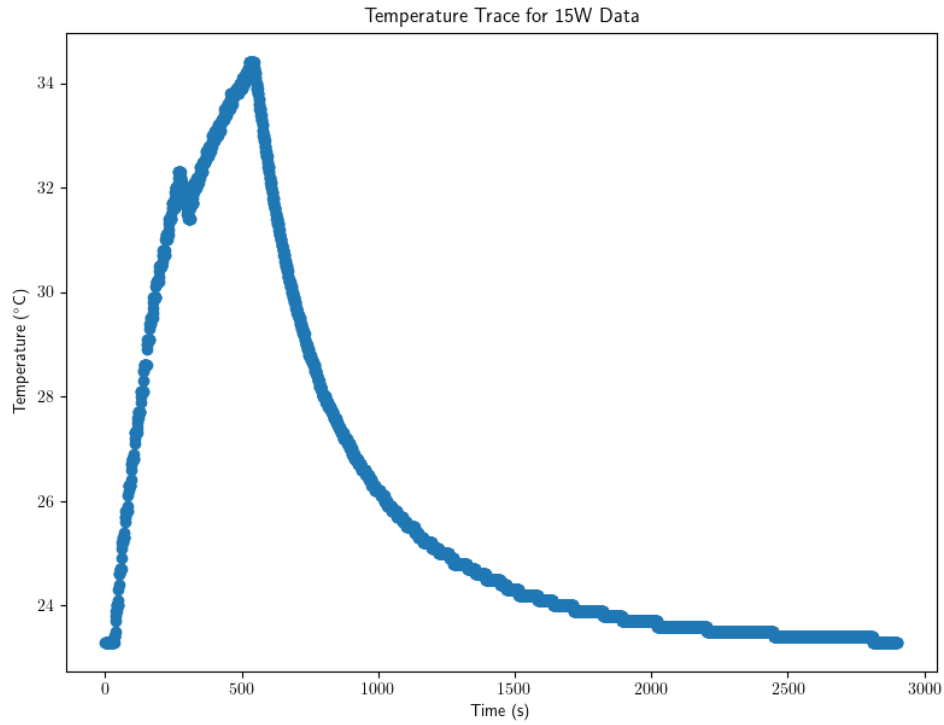


Figure 3.3: Example Temperature vs. Time Data While Taking Measurements With the Shutter

From the results in table 3.2, we can see the addition of the shutter successfully limited the temperature increase of the ruby. At the highest power used when collecting data, the ruby only reached around half of the temperature of the lowest power without the shutter. Furthermore, the difference between the initial and the highest temperature which was reached is on the order of 10°C . This is a small increase in temperature, which is desired.

Power (W)	Temperature ($^{\circ}\text{C}$) Without Shutter	Temperature ($^{\circ}\text{C}$) With Shutter
5	63	26
8	82	28
10	96	29
13	112	31
15	124	32
18	137	34

Table 3.2: Power vs. Temperature of Ruby Cube With Temperature Control

3.4.2 Experimental Determination of Ruby Doping

To determine the doping of the ruby, we use transmission measurements. To get these transmission values, we measure the power of each laser beam both before and after the ruby cube. For the blue beam, an aperture was inserted in the beam path. This was done to limit the light that reaches the sensor to only the light that hits the ruby, as the beam is larger. A moveable lens ($f = 75$ mm) was then used to focus this light onto the power meter’s sensor (*ThorLabs PM100D* power meter and *ThorLabs S120C* sensor). The power meter was connected to a computer which automated the data collection via a `python` script. This script records the power over a given amount of time (data was taken over 10 minutes). Two flags were used to denote the laser which was used for this dataset and whether the dataset was before or after the ruby. Therefore, a total of four datasets needed to be taken to determine the transmission for both lasers. The script was artificially delayed to allow for proper sampling of the power as a function of time. Without this delay (~ 0.8 s), the script would run at the clock speed of the computer and the sensor would not have the time to register a new power. Once the data is saved, it is plotted with a linear fit. We expect the power of each laser to be constant (within fluctuations) with time. The fit allows us confirm this on the spot.

For the green beam, the same lens and power meter combination were used. The green laser has two operational modes: the “alignment” mode and the “normal” mode. The alignment mode, as the name suggests, is only used for alignment and thus is very low power. However, this power fluctuates significantly. Therefore, normal mode was used for these measurements, with the laser set at 0.5 W. To compare low power with low power, a reflective neutral density filter (*ThorLabs ND30A* of optical density 3.0) was added to the green laser’s path. This allows both sets of measurements to be in the 100s of μW .

The transmission through the ruby will follow Beer’s Law [10]:

$$T_i = e^{-\sigma_1^i N L}, \quad (3.1)$$

where T_i is the transmission through the ruby of wavelength i , σ_1^i is the absorption cross-section at wavelength i , N is the number density of Cr^{3+} ions in the ruby, and L is the length of the ruby in the orientation that each wavelength is travelling. As the transmission measurements are performed in the low power regime, N is the number density in $|1\rangle$. Since the ruby is a cube, the length L has the same value for both wavelengths and therefore does not need an additional label to denote the length dimension. In this equation, the absorption cross-sections and the doping are the unknowns. Using transmission data, we will fit a doping value to obtain the cross-sections of both transitions simultaneously. The values for the doping and cross-sections are presented in sec. 4.4.1.

3.4.3 Beam Waist Measurements

Lastly, as mentioned in sec. 3.1, the green beam's shape is modified to a particular profile. As the beam waist of the green beam is a parameter in the modelling equations (see eq. 2.9), we must experimentally determine the beam waists of the green beam. This is done using the standard “razor blade” method. In this method, a razor blade is attached to a translation stage, orientated perpendicular to the dimension that will be measured. As it is translated across the beam, it will slowly start to block the beam. The position of the blade and the power are recorded at every step. This data will be used to fit an equation which will yield the beam waist.

Therefore an apparatus holding the blade had to be inserted in the green beam's path. It had to be adapted to accommodate both dimensions. This apparatus was placed ~ 10 mm (for the waist in x) and ~ 20 mm (for the waist in y) in front of the ruby before the green beam hits it. These positions were selected due to ensure that we measure the beam's shape as it hits the ruby, which are the expected beam waist parameters in the model as well as geometric constraints. The values for both beam waists are obtained in sec. 4.4.1.

Chapter 4

Observing and Characterizing a Laser Shadow

4.1 Direct Imaging of a Laser Shadow

We begin with results from the direct path which terminates at a piece of paper. All pictures presented in this section are unedited pictures direct from the camera. The camera used was an *Olympus OM-D EM-5 Mark II* with a *M.Zuiko* 17 mm f/1.8 prime lens. The camera settings, such as the autofocus, ISO, and exposure time, were changed between each kind of picture taken. These changes were done to optimize the results, such that the laser shadow was visible in each lighting condition. To help in setting the focus, a piece of paper with the university logo is placed beside the laser shadow image. The camera itself was placed to the side of the direct path on the optical table, at a distance of ~ 20 cm. A timer was used to limit vibrations that occur when manipulating the camera. Figures [4.1](#), [4.2](#), and [4.3](#) show the results with the green laser set at 15 W.

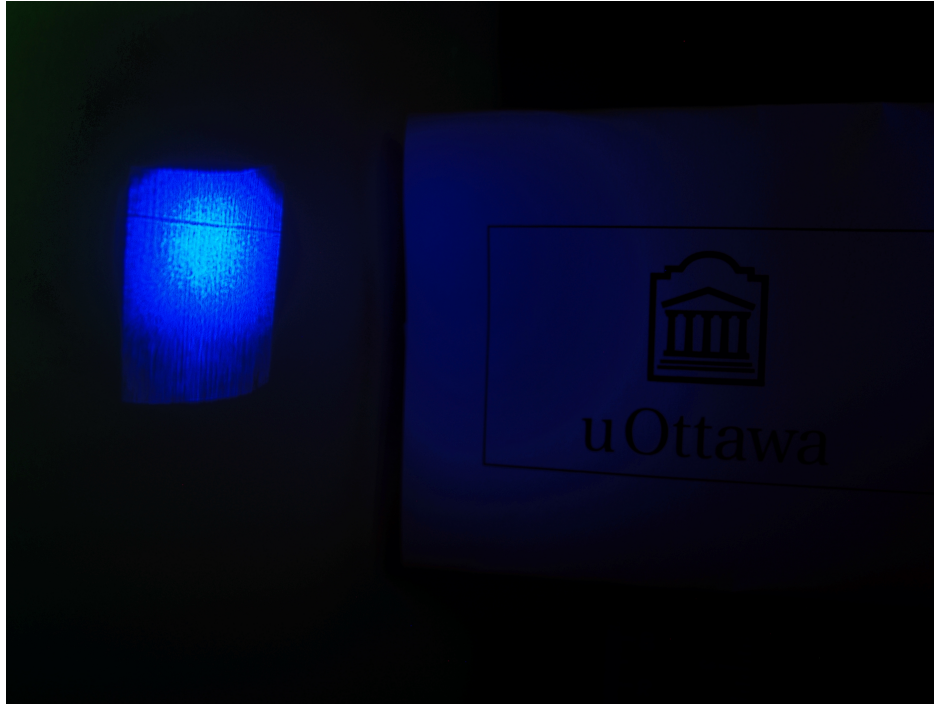


Figure 4.1: The Laser Shadow as imaged in the direct path. It is the line of reduced brightness on the left side of the figure, within the blue rectangle. The university logo can be observed on the right side of the image, as further evidence that this picture was taken from the direct path.

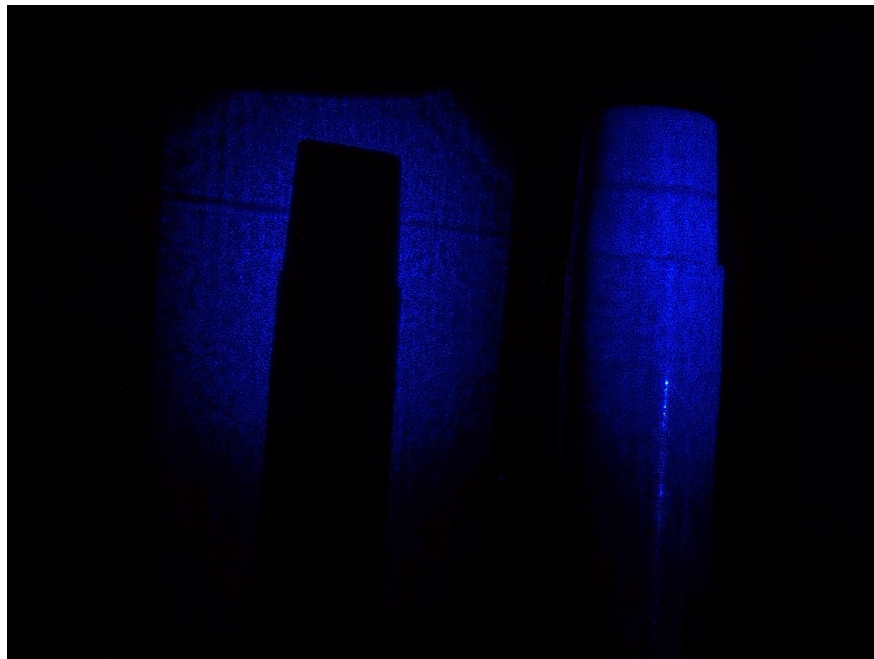


Figure 4.2: To show that the Laser Shadow is a genuine shadow, we show that it has the same properties as “regular” shadows. One such property illustrated in this figure is that the shadow takes the shape of an object, when the object is put in the illumination path. The opaque object is placed ~ 10 cm in front of the background. We can observe the Laser Shadow on both the opaque object and background.

Figure 4.2, shows an obscured opaque object which blocks the shadow, allowing it to take its shape. To be precise, the opaque object in question is a dry erase marker. It can be easily identified in fig. 4.3. The same laser settings were used between these two pictures. The difference lies in the camera settings as well as the addition of a white light source (cell phone flashlight) in fig. 4.3, which illuminates the region of interest from above.



Figure 4.3: This figure had the same experimental setup as fig. 4.2, with the exception of the additional white light source illuminating from above. This allows the observation of the opaque object and Laser Shadow in context. Due to the depth of field of the image, the Laser Shadow is visible in the foreground on the opaque object. The background is out of focus, which makes the observation of the Laser Shadow difficult from this figure.

How do these figures satisfy our shadow criteria? Recall the four criteria set out in sec. 2.2 we must satisfy to obtain a genuine shadow:

- Effect must be absorptive
- Effect must be large-scale
- Effect must use visible light
- Effect must be “fast”

Two criteria are easily satisfied by the above figures: the use of visible light (the blue) and the large scale shadow (the image is several cm large and can be seen with the unaided eye). Additional images taken with the camera further away from the paper, and where one can see the green light leak out of the box

are available in an appendix (see sec. A.1). The absorption of the green light can be confirmed in different ways. First, turning on and off the green light makes the shadow appear and disappear, so we know it is due to the existence of the green light within the ruby. Second, the shadow is *darker* than its surroundings. Since there is less blue light in the region of the shadow, it must have been absorbed by the ruby. There are no absorbing optical elements between the ruby and the camera which would explain this observation. Therefore, the absorption criterion is satisfied. Lastly, the speed criterion is satisfied because the lifetime of the metastable state of the ruby is short enough. As the green beam is moved, the shadow follows it after the excited state is saturated, which takes approximately a lifetime. After a lifetime, the absorption of the blue laser changes, and we get the desired increased absorption. Additional experimental evidence for satisfying the “speed” criteria is discussed in sec. 4.2.2.

Therefore, we can conclude that we have indeed observed a shadow cast by a laser beam. Furthermore, in fig. 4.2, we observe the shadow falling on an object and taking its shape. This is another way it behaves just like a shadow does. This image was initially at the same size as fig. 4.1, but it was cropped since the rest of the image is dark. As the depth of field in fig. 4.2 must be larger than fig. 4.1, the exposure must compensate, giving a much darker image overall.

With the successful demonstration of the shadow, we will now quantify it. To do so, we introduce our figure of merit, the *contrast function* $C(I)$. The contrast is defined as:

$$C(I) = \frac{\max\{I(y)\} - \min\{I(y)\}}{\max\{I(y)\}}. \quad (4.1)$$

where $I(y)$ is defined in eq. 2.10. The larger the contrast C , the “darker” the shadow. Since we expect the absorption of the blue light to increase with the power of the green laser, we also expect to see a relation between the contrast and the input power.

4.2 Analyzing the Laser Shadow

The quantified shadows we have obtained will allow us to validate the model as written in eq. 2.7. To obtain these results, we created an average set of pictures from the camera (see fig. A.3b for an example average) per power level of the green laser. In each of these pictures, the ruby was at a different position in y , but the green beam, and thus the shadow, remains at the same position in y . To obtain the average, we proceeded as follows. The raw images were taken at maximum resolution. We then cropped all images to the only relevant part, the illuminated ruby. From these cropped images, the average was created by treating them as an array and pixel-wise averaging over the entire set of 21 images. The average was necessary to smooth

out the imperfections on the surface of the ruby and to obtain uncertainties for the contrasts. From this result, we applied eq. 2.10, that is we integrated along the z dimension. The integration is approximated by summing up each row of the averaged image. The result of the integration of the image along z is our main result. However, since the shadow only means something when it is compared to the background. A shadow is a region that is darker than its *surroundings*. Therefore the set of background images of only the blue laser is put through this same analysis process and included in the results. One example result is highlighted in fig. 4.4.

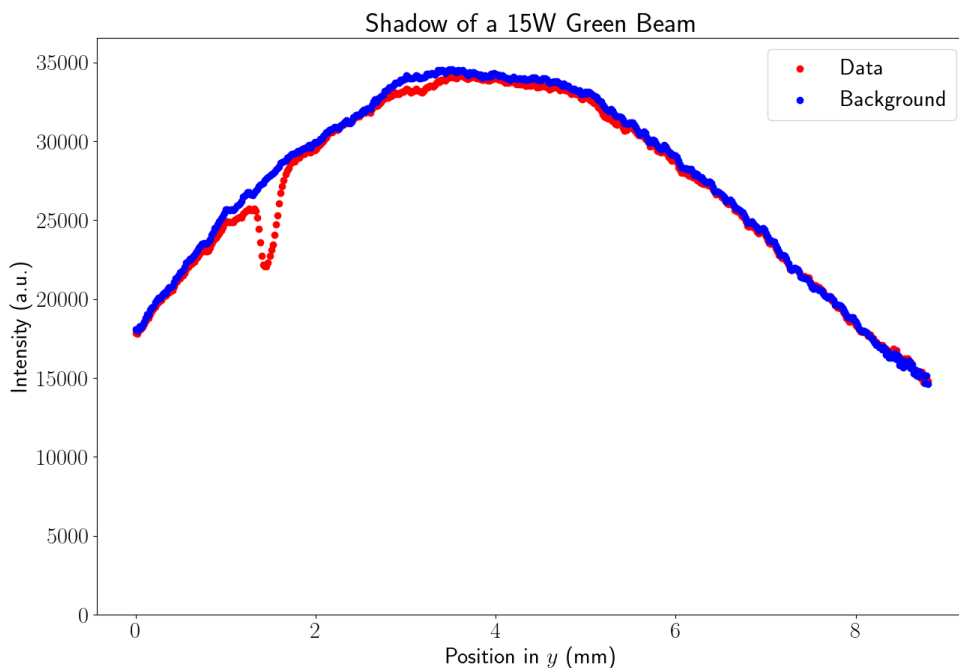


Figure 4.4: The Laser Shadow at 15 W

This particular result gives us a contrast of 19.9%. We can repeat the process for different powers. These raw results are individually presented in fig. 4.5. To properly observe the contrast increasing as a function of power, the data for each power is normalized to the background and all shadows are presented together in fig. 4.6. Using the contrast values and their respective powers, a relationship can be observed (discussed in sec. 4.3). The specific contrasts obtained for each power are listed in table 4.1.

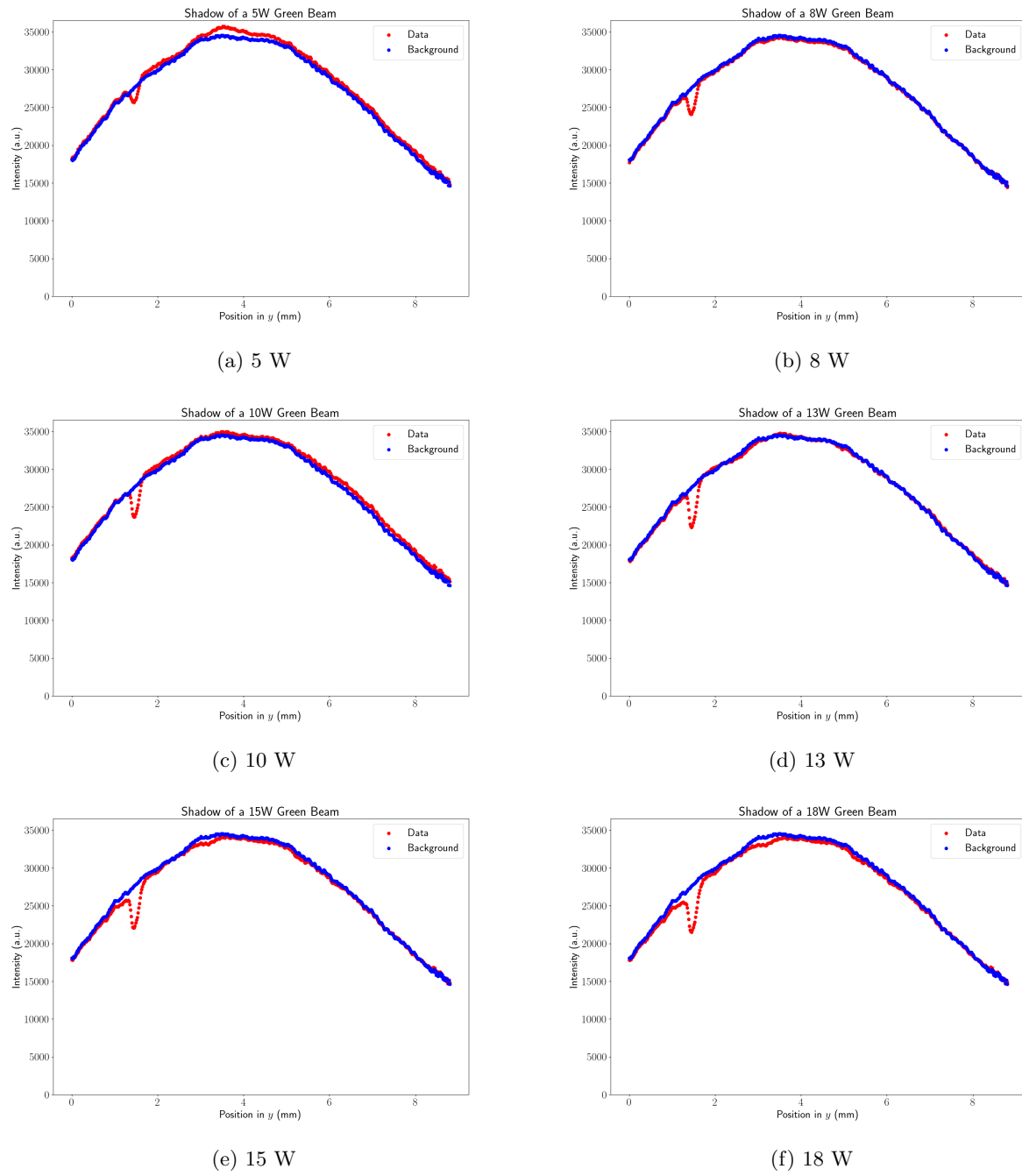


Figure 4.5: Individual Laser Shadows at Different Powers

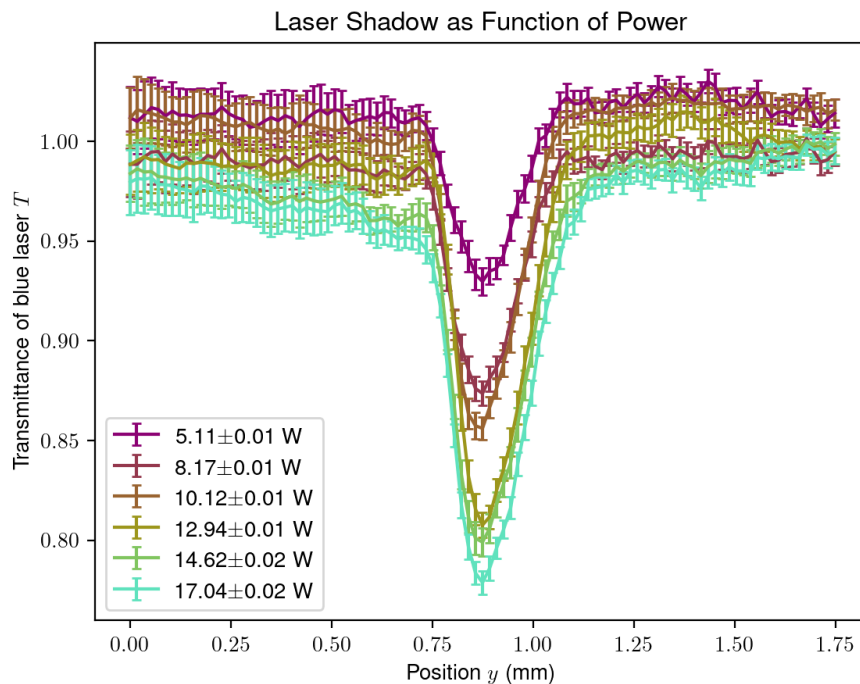


Figure 4.6: The Laser Shadow is plotted as a function of input power. The position space that is plotted is reduced and normalized when compared to figs. 4.5. We observe increasing contrast with increasing power.

Power (W)	Experiment (%)
5	7.03
8	12.65
10	14.39
13	19.24
15	19.9
18	22.13

Table 4.1: Experimental Contrast of the Laser Shadow as a Function of Power

4.2.1 Comparison with a Genuine Shadow

To further illustrate how the laser shadow is a genuine shadow, we can compare it with something that is known to be a shadow. Specifically, we use a strand of human hair from one of the contributors of this project as the object. This strand of hair was mounted and installed in front of the ruby, in the path of the blue laser (this is the x dimension). This is the only difference in the apparatus when compared with the laser shadow. With this additional element, both the laser shadow and the shadow from the strand of hair can be observed simultaneously (fig. 4.7).

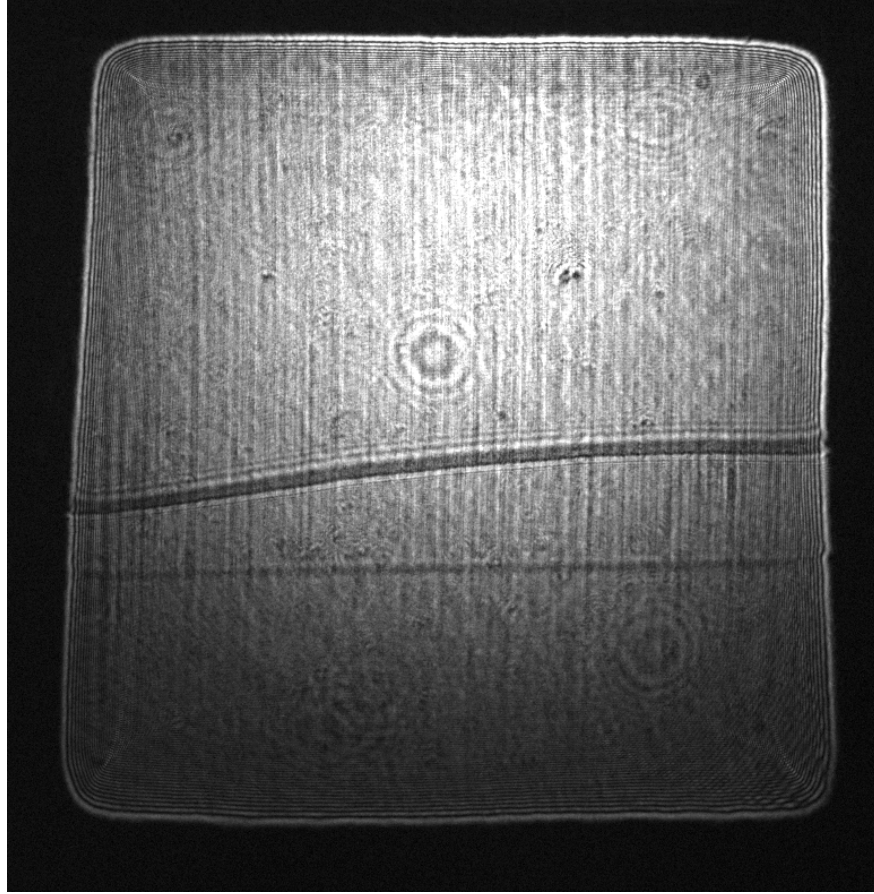
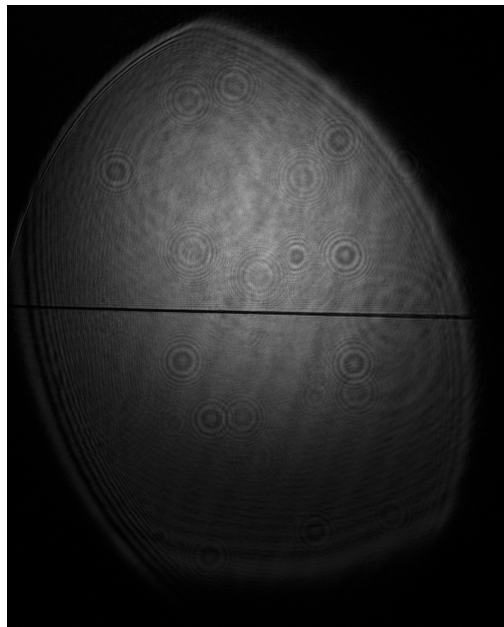
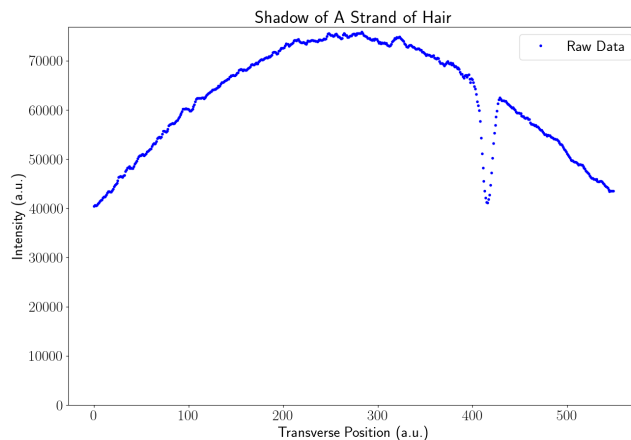


Figure 4.7: The Laser Shadow is compared with a genuine shadow from a strand of hair. The Laser Shadow is the shadow on the bottom of the image, as it is a straight line. The shadow of the strand of hair is the top shadow. The difference in shape is the factor which identifies the object casting the shadow.

It is possible to distinguish between the shadow of the hair and the shadow of the laser in fig. 4.7 due to the shape of the hair: the top shadow which bends, is the shadow of the strand of hair. The bottom shadow, which is a straight line in fig. 4.7, is the laser shadow. However, that is the only distinguishing feature, as both behave the same way i.e., both are genuine shadows. We can obtain the same plot of the integrated intensity along z as fig. 4.4 for the hair. This requires a picture of the shadow of the hair alone (fig. 4.8a). The result is in fig. 4.8b.



(a) Raw Data for Shadow of a Strand of Hair



(b) Integrated Shadow of a Strand of Hair

Figure 4.8: We can replicate the analysis from the Laser Shadow against the shadow of the strand of hair. Figure 4.8a for the hair is equivalent to fig. A.3b for the Laser Shadow and fig. 4.8b for the hair is equivalent to fig. 4.4 for the Laser Shadow.

The hair has a contrast of 35.9%. We can see that both the shadow of the strand of hair and the laser shadow behave in a similar fashion. Thus, this is another piece of evidence which supports the claim that the laser shadow is a genuine shadow.

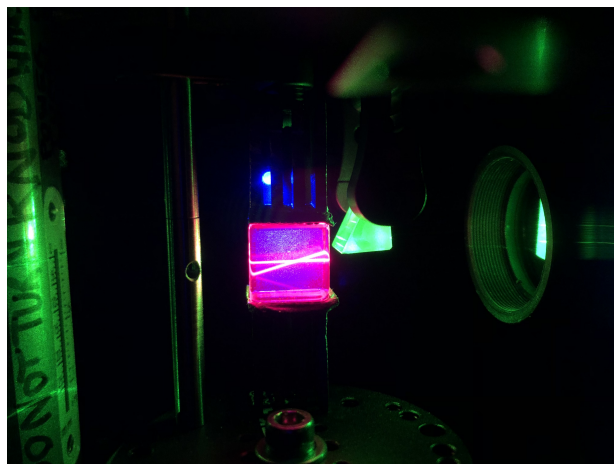
4.2.2 Splitting the Shadow in Two

There is another property of shadows that we can demonstrate with the laser shadow, that of two shadows overlapping. If there are multiple objects in the path of the illumination source, each object will cast a shadow. However, because of the projection, these shadows will overlap in the image.

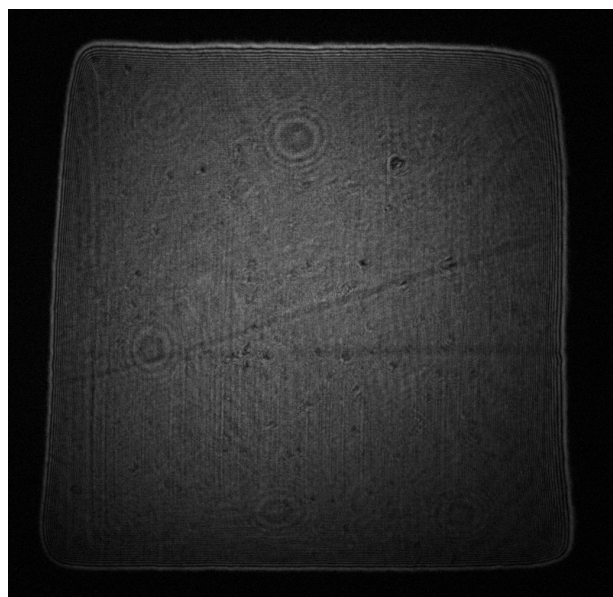
To observe this effect with the laser shadow, we need two intense green beams which will act as the two objects. This is accomplished by adding a beam displacer (*ThorLabs BD40 Calcite Beam Displacer* 4.0 mm separation) to the path of the green laser before the ruby. The beam displacer works by exploiting the birefringence of the material, which is calcite in this particular case. The polarization which is parallel to the optical axis of the crystal travels unperturbed and the polarization orthogonal to the optical axis is displaced. The displacement was created in the y dimension and creates two parallel beams travelling in the z dimension. Depending on the polarization which enters the calcite, the splitting between the intensity of the two beams can be controlled. Therefore, the half-waveplate at the beginning of the path of the green

laser is rotated such that each beam has roughly the same intensity, as judged by the naked eye. This means the splitting is roughly 50:50 and each beam should show the same “level” of the laser shadow effect.

As both beams are initially travelling in parallel, one must be diverted such that both beams overlap within the ruby. This is accomplished by adding a right-angle prism in front of the ruby. The prism is mounted in such a way that only the top beam hits the prism, which is refracted into the ruby. This mounting arrangement can be observed in fig. 4.9a. This picture was taken in the green laser’s alignment mode. Figure 4.9b shows the result of this overlap from the imaging path. For this image, the laser was set at 25 W. Since the set laser intensity was split into two beams of ~ 12 W, the effect is less intense as it would be for a single beam of 25 W.



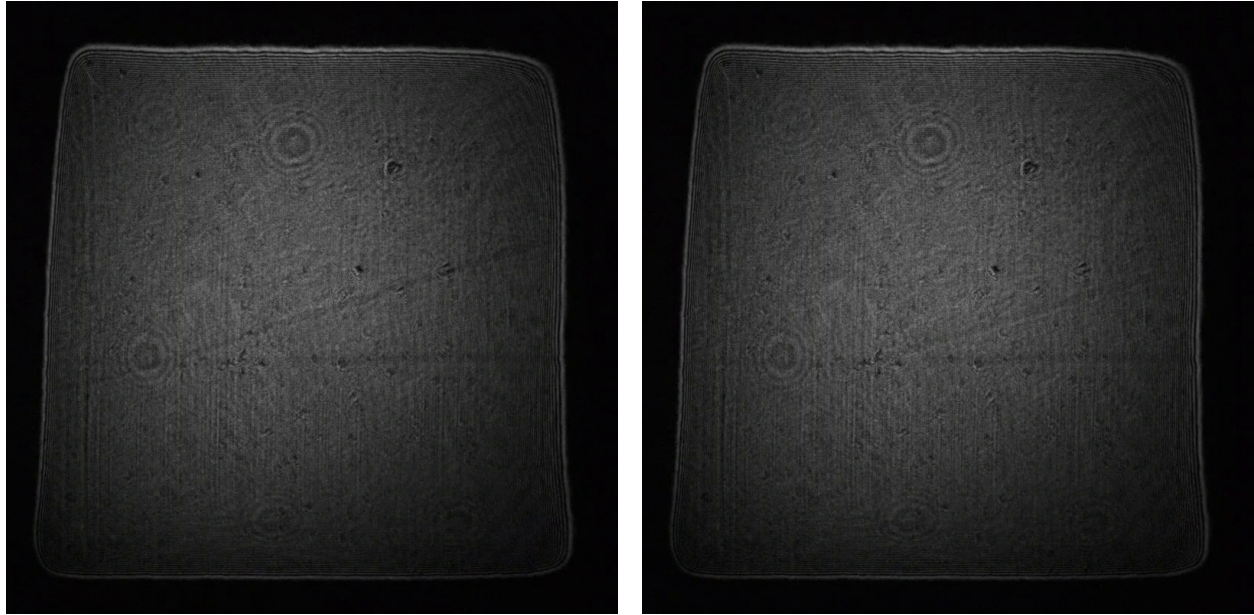
(a) Direct Image of Two Laser Shadows



(b) Data Image of Two Laser Shadows

Figure 4.9: When two objects cast shadows, these shadows overlap in the image. We replicate this property of shadows with two laser beams acting as the objects casting shadows. In fig. 4.9a, we can observe the changes in the experimental apparatus which allows us to obtain two object lasers (the beam displacer and the prism). In fig. 4.9b, we observe the result of this overlapping shadow from the imaging path.

Additionally, the two-beam configuration allows us to have more evidence to support the satisfaction of the “speed” criteria. The right-angle prism shown in fig. 4.9a can be moved independently by hand. This movement is then translated to the diverted beam. To observe this movement, we recorded a video from the camera in the imaging path to see this movement in time. For this recording, the laser was set at 25 W. Figure 4.10 shows two screenshots of this video extracted 0.5 s apart (the frame rate is 30 fps). There is clear movement in the vertical position of the diverted beam between the two pictures. Therefore, with this frame rate and laser intensity, the time-response of the material is shown to be fast enough for the effect to match our expectations of the behaviour of a shadow.



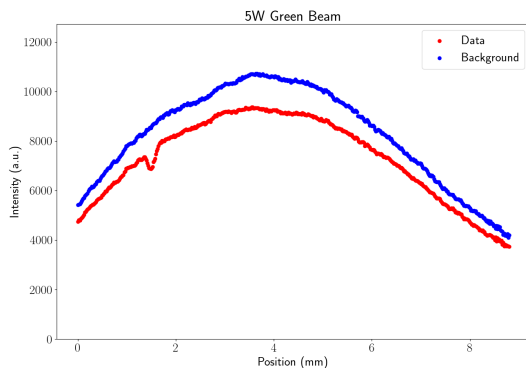
(a) Image Extracted at 22 s

(b) Image Extracted at 22.5 s

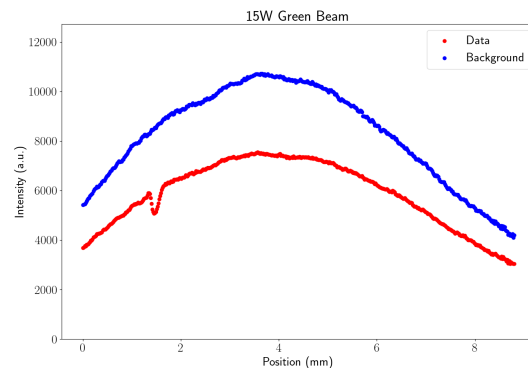
Figure 4.10: The two images in this figure were taken 0.5 s apart, when the prism which refracts one object beam into the ruby is in motion. We can see the effect of this movement on the angled object, which is at a different position between the two images. Therefore, this is additional evidence which supports the “speed” criteria for the Laser Shadow.

4.3 Impact of the Temperature of the Ruby on the Shadow

As noted in sec. 3.3, the ruby heats up quite considerably. We were able to observe the effect of this heating directly in results like fig. 4.4. For example, without temperature control of the ruby, one can get erroneous figures like the following:



(a) Low Power Bad Contrast



(b) High Power Bad Contrast

Figure 4.11: The gap that increases with power is due to temperature effects. Controlling the temperature makes the gap disappear.

There is a gap that appears between the background and data curves, unlike fig. 4.4. The contrasts we observed from these results also began to decrease with increasing power, which was unexpected. Once the temperature was controlled, we saw both the disappearance of the gap, as shown in fig. 4.4 as well as the measured contrasts increasing with power (fig. 4.12a).

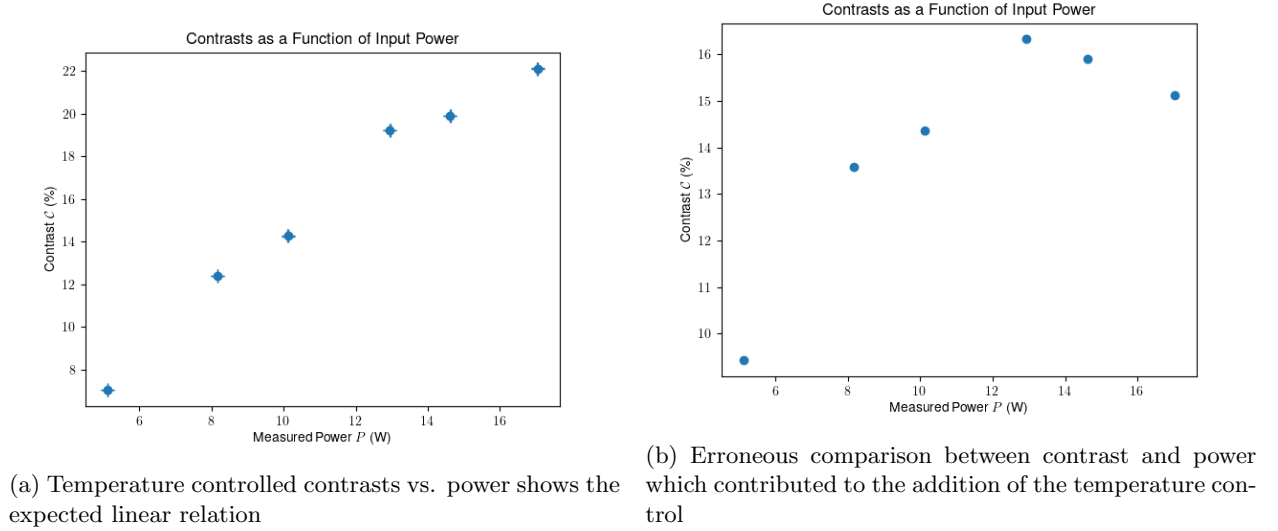


Figure 4.12: Contrast vs. Input Power, With and Without Temperature Control

We can see a linear relation in fig. 4.12a between the contrast and the power. As expected from eq. 2.7, the absorption (and therefore the contrast) increases with increasing input power.

4.4 Theoretical Simulations of a Laser Shadow

4.4.1 Experimentally Derived Parameters

We now want to compare the experimental results detailed in this Chapter with the theoretical model of eqs. 2.7 and 2.10. It is important to note that this model is not *purely* theoretical, as some parameters used in the model have been obtained experimentally. This is the reason for the error band that can be seen in fig. 4.13 below. The experimentally determined parameters are: the beam waists of the green laser in both dimensions, w_{0x} and w_{0y} , the absorption cross-sections σ_1^{blue} and σ_1^{green} , and the doping N .

In the literature [7, 8, 12], the absorption cross-sections of both transitions have been measured. However, when using eq. 3.1 to solve for the doping using the cross-sections from the literature as known values, we do not get the correct transmission values. Therefore, the ruby we have used has different physical parameters to those used in the literature. Thus we will not be using values from the literature for the

absorption cross-sections. To determine them experimentally, we therefore proceeded as follows: since we have an underdetermined system (i.e., two equations and three unknowns), the doping was manually set to a reasonable value. We then used eq. 3.1 to simultaneously fit for the absorption cross-sections that matched our measured transmission values for both wavelengths. We will now discuss the transmission measurements which will allow this computation.

We begin this discussion with a reminder that we must take into account Fresnel reflections at both interfaces between the ruby and the surrounding air. Recall the Fresnel amplitude transmission coefficients [16]:

$$t_{\perp} = \frac{2n_i \cos(\theta_i)}{n_i \cos(\theta_i) + n_t \cos(\theta_t)} \quad (4.2a)$$

$$t_{\parallel} = \frac{2n_i \cos(\theta_i)}{n_i \cos(\theta_t) + n_t \cos(\theta_i)}, \quad (4.2b)$$

where i denotes the “incident”, t denotes “transmitted” and $\perp$ and $\parallel$ denote the perpendicular and parallel polarizations, respectively. Since we are dealing with normal incidence at both interfaces, both equations reduce to the same form:

$$t = \frac{2n_i}{n_i + n_t}. \quad (4.3)$$

From the transmission amplitude, we can get the transmission coefficient T [16]:

$$T = \frac{n_t}{n_i} |t|^2. \quad (4.4)$$

This transmission coefficient is then used to correct the raw measurements. Applying eqs. 4.3 and 4.4 for $n_{\text{ruby}} = 1.77$ [14] and $n_{\text{air}} = 1.0003$ [17] with both indices taking both roles of “incident” and “transmitted”, we get a factor of 92.28% per interface. Therefore, our Fresnel corrected transmission values are $T_{\text{blue}} = 26.01 \pm 0.02\%$ and $T_{\text{green}} = 26.8 \pm 0.3\%$. These values are then used with a set doping of $N = 2.5 \times 10^{25} \text{ m}^{-3}$ to obtain $\sigma_1^{\text{blue}} = 4.489 \times 10^{-24} \text{ m}^2$ and $\sigma_1^{\text{green}} = 4.389 \times 10^{-24} \text{ m}^2$.

As for the beam waists, the procedure is the same for both dimensions. As noted in sec. 3.4.3, we use the “razor blade” method. We measure the power output that is not blocked and the position x of the razor blade. This information is then fitted to the following equation:

$$P(x) = \frac{p_0}{2} \operatorname{erfc} \left(\frac{\sqrt{2}(x_0 - x)}{w} \right), \quad (4.5)$$

where p_0 is the power, x_0 is the centre position, w is the beam waist, and $\operatorname{erfc} x = 1 - \operatorname{erf} x$ is the complementary error function. All three of these values are fit parameters, with the important result being the beam waist. We get $w_{0x} = 3.17 \pm 0.01$ mm and $w_{0y} = 0.168 \pm 0.003$ mm as experimental values. These values are consistent with the beam shaping that we had planned in sec. 3.1.

4.4.2 Comparison Between Theory and Experiment

To summarize, we list all model parameter values and relevant errors in the following table:

Parameter	Value	Error ($\pm$)	Source
T_3	1 ms	N/A	Free Parameter
σ_1^{blue}	$4.5 \times 10^{-24} \text{ m}^2$	$0.5 \times 10^{-24} \text{ m}^2$	Fit
σ_1^{green}	$4.4 \times 10^{-24} \text{ m}^2$	$0.5 \times 10^{-24} \text{ m}^2$	Fit
σ_3^{blue}	$17.2 \times 10^{-24} \text{ m}^2$	10%	Fit
N	$2.5 \times 10^{25} \text{ m}^{-3}$	10%	Fit
$\hbar\omega$	$\frac{\hbar 2\pi c}{532 \times 10^{-9}} \text{ 1/s}$	N/A	Constants
w_{0x}	3.17 mm	0.01 mm	Fit
w_{0y}	0.168 mm	0.003 mm	Fit

Table 4.2: Summary of Parameter Values for the Laser Shadow Model

Initially, T_3 was set by literature values for the lifetime of the meta-stable ^2E state [8]. However, this value proved to be inadequate, as the model could not simultaneously fit both the low power and the high-power regimes. An explanation for this is the underestimation of the saturation intensity (defined in eq. 2.3). If the saturation intensity is too low, then the population in the meta-stable state will increase with power before hitting a plateau. Increasing the power after this plateau will no longer increase the population in the meta-stable state. To increase the saturation intensity, the lifetime T_3 is reduced. This fit is further justified by recalling that the ruby is warmer than room temperature during the experiment. We can also expect the lifetime to be shorter due to these heating effects.

To get the fit for both N and σ_1 , the doping is set manually to a reasonable value (per the literature). Then, according to eq. 3.1, the last parameter to fit for is σ_1 . This fit uses the Fresnel corrected transmission in the computation. Both σ_1 's were fitted simultaneously with the same doping value, as both face the same ruby.

The remaining parameter was σ_3^{blue} . It was used as the final fit parameter for the model. Its value was determined by fitting the model to data at both low and high powers, making sure that the fit is within an acceptable error range for both regimes. The error bands on the theoretical curves are determined as follows.

Recall that we solve for σ_1 in eq. 3.1. The remaining three parameters (transmission, doping, and cube length) all have errors. Therefore, error propagation is used to obtain the uncertainty for both σ_1 's. We assume a $\pm 10\%$ error on the doping (as is done in the literature [7]). The uncertainty on the length of the ruby is ± 0.5 mm. Experimental error on the transmission of the blue laser is $\pm 0.02\%$ and $\pm 0.3\%$ for the green laser. With this, we have a total of eight parameters in the model with uncertainties. The error bands on the model are obtained by adding the parameter errors in quadrature. The error bars on the experimental data was obtained by taking the standard error of the data set and propagating the error of the background. They were then normalized to the same scale as the data.

An example in both the low and high power regimes are shown in fig. 4.13.

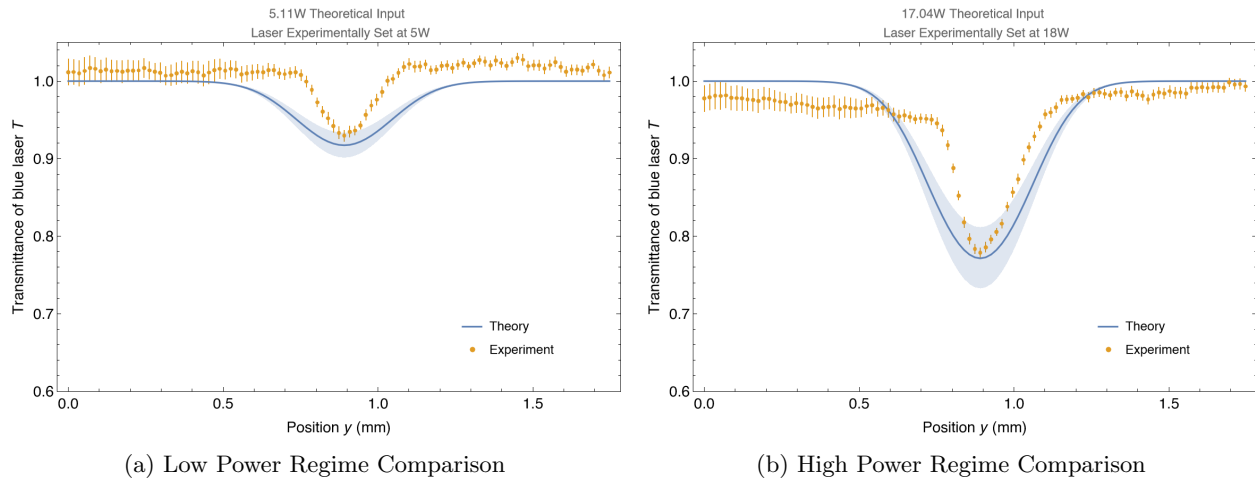


Figure 4.13: Comparison Between Laser Shadow Model and Experimental Data

From figs. 4.13, we can see that there is very good agreement between the theoretical model and the experimental data, as the contrast values obtained are within the error of the model. All contrast values between theory and experiment are given in 4.3.

Power (W)	Theory (%)	Experiment (%)
5	8.18	7.03
8	12.35	12.65
10	14.94	14.39
13	18.29	19.24
15	20.15	19.9
18	22.67	22.13

Table 4.3: Comparison of the Contrast of the Laser Shadow Between Theory and Experiment

After accounting for experimental errors as indicated in the error bars and the error bands if figs. 4.13, we can conclude that the model of eq. 2.7 is validated.

Chapter 5

Discussion and Conclusion

In this project, we set out to investigate and demonstrate the creation of a shadow where light takes the role of both the illumination source as well as the object casting the shadow. We have shown that this “laser shadow” is indeed possible. This redefines our understanding of what is a shadow, as something other than matter is used to cast a shadow. This demonstration is therefore the next step in the long history of the study and use of shadows.

We have shown that the laser shadow is indeed a genuine shadow, by comparing it to the shadow of a solid object, as well as demonstrating other properties we can expect from shadows. We have demonstrated that temperature effects have a significant impact on the results, hindering the appearance of the laser shadow. We have also demonstrated the laser shadow in motion and having two laser shadows overlap.

This experiment relied on the optical properties of ruby, particularly its large inverse saturation of absorption. However, other materials that present a similarly large inverse saturation of absorption would also show the laser shadow effect. One such material class are photochromic glasses [18]. These glasses darken when exposed to high-frequency light. Illuminating them with a different wavelength of light would show a shadow. However, these glasses have slow material responses, on the order of minutes rather than milliseconds. The effect of this slower response would be observable in camera by moving the intense beam and not seeing the image move with it. This does not match our experience of the time-response of a shadow and contradicts our “speed” criteria.

This experiment is a demonstration of a new application of a known nonlinear effect. New ideas bring about new applications. One such application that can exploit this effect is an optical switch. The presence or lack thereof of the shadow could be used as the trigger for a switch. Several shadows could also be created on the same material. The detection of such shadows could lead towards the implementation of a logical

operation purely in an optical system.

To conclude, we note that these results will be reported in a forthcoming publication [19].

Part II

The Geometric Phase Waveguide Logic Gate Project

Chapter 6

Motivation

6.1 Quantum Computing

Computers are useful tools. They drive our modern world. Their usefulness, however, depends on the task they are trying to perform. Certain tasks are easy to compute, such as multiplying numbers. Other tasks are inherently difficult, such as factoring large numbers. They can be so difficult that they are classically intractable. Classifying “how hard” problems are to solve is the domain of *Complexity Theory*.

To begin the discussion with complexity classes, we highlight two of the most important ones: **P** and **NP**. **P** refers to the easy tasks previously mentioned, specifically when the solution to a problem is easily found *and* easily checked. **NP** refers to tasks where the *solutions* are easily checked, but obtaining the solution is difficult. The factoring problem previously mentioned falls into this class [20]. One of the most important unsolved problems in theoretical computer science is the **P** vs. **NP** problem, which asks whether both classes completely overlap, or whether some problems exist which are inherently hard. The key question is to decide whether problems that can be quickly verified can also be quickly solved. The next complexity class to note is **BPP**, short for “Bounded-Error Probabilistic Polynomial”. If a problem can be solved with randomized algorithms in polynomial time with a bounded error of probability, it falls within this complexity class [20].

“Quickly” here refers to the time complexity of the problem: when the length of the input increases, how does the length of time required for the algorithm to complete increase? Two notable time complexity classes are: $\mathcal{O}(n^k)$ and $\mathcal{O}(2^{n^k})$, which are polynomial and exponential time, respectively (k is some positive integer). Problems which fall into polynomial time are called “tractable”, and those that require exponential time are called “intractable” [20, p. 139].

A natural question to ask at this point is how does quantum mechanics fit into complexity theory? The

answer lies in the quantum analogue of **BPP**, which is **BQP**. **BQP** stands for “Bounded-Error Quantum Polynomial”. It is currently unknown where **BQP** lies with respect to other complexity classes [20], making it a focal point for further study of computational problems. In 1994, Peter Shor demonstrated that the integer factorization problem belongs in **BQP**, making it efficiently *solvable* on a quantum computer. This result is of particular importance considering modern encryption protocols (such as RSA [21]) rely on the fact that factorization³ belongs to **NP**. If someone were able to create a quantum computer of sufficient size, they would be able to break these protocols. This is one motivation behind “quantum key distribution”. However that concept is outside the scope of this thesis. What *is* in scope is the possibility of creating a physical quantum computer, not just a theoretical one.

Before examining the various implementations of quantum computing, we highlight how quantum mechanics is able to achieve an advantage over classical algorithms by briefly presenting Grover’s Search algorithm. Suppose you want to know whether an element is in a given list of size n . A naive search protocol starts at the beginning of the list and compares that element with the element you are searching for. If they match, you are done. If they don’t, you continue down the list, one step at a time. Such an algorithm scales as $\mathcal{O}(n)$ as it checks every single element. Grover’s algorithm [20] requires a superposition over all the elements of the search space. We are in effect searching the entire space at once due to the superposition. Hence, this algorithm scales as $\mathcal{O}(\sqrt{n})$, which is faster than $\mathcal{O}(n)$ for the same sized list. This speedup is the quantum advantage. It is achievable due to uniquely quantum properties. Such properties include superposition and entanglement.

With quantum algorithms, we now need a quantum computer to be able to run them. Finding the optimal platform for quantum computing is an ongoing search, much like classical computing evolving through mechanical devices such as punch cards and electronic devices such as vacuum tubes before settling on silicon (Si) based hardware [22]. Various implementations have been suggested, such as superconducting artificial atoms [23], ion traps [24], topological quantum computing [25], and photonics [26]. In the ideal world of theory, these schemes could be realized without the introduction of errors. However, in a practical device, errors *will* be introduced, and therefore must be considered. It remains to be demonstrated which platform could be implemented at the largest scale with the smallest allowable amount of errors. Therefore, a key issue to address is the possibility of errors in the computation and how to mitigate these errors.

One method to suppress such errors are quantum error correcting schemes [27]. These schemes encode information in such a way to be resilient to errors [20, p. 425]. Another method, which is of interest to us, is to implement a quantum computing scheme with built-in “robustness”. These schemes allow for a

³Factorization is one particular problem which can be shown to have an advantage over classical algorithms. Others include search (Grover’s Algorithm) and simulations of quantum systems [20].

certain kind of error by their very nature, thus attracting a lot of interest in their development. The idea which underlies these robust schemes was initially developed by Pancharatnam [28] in 1956 in the context of polarization of light, long before the idea of quantum computing was postulated in the 1980s [20]. This concept, now known as the “geometric phase”, started to take hold in the literature with Sir Michael Berry’s seminal paper on the topic in 1984 [29]. In this work, Berry formulated the adiabatic geometric phase. Soon after, this was generalized to the non-adiabatic regime [30]. Furthermore, the geometric phase as introduced by Berry was also extended to be non-Abelian⁴ by Wilczek and Zee in 1984 [31]. The combination of a non-adiabatic, non-Abelian geometric phase was proposed by Anandan in 1988 [32]⁵. The geometric phase will be derived mathematically in the following section (see sec. 6.2.1).

These concepts are theoretical descriptions of different kinds of geometric phases. The next step is the formulation of quantum computing schemes which rely on a geometric phase. These computing schemes extend the ideas of classical computing to be compatible with quantum mechanics. Classical computers are able to process information by passing bits of information through *logic gates*. Therefore this extension requires the formulation of *quantum* logic gates. Two such kinds of schemes which design quantum logic gates around the geometric phase are *holonomic quantum computation* (HQC) [34] and *geometric quantum computation* (GQC) [35]. The term “holonomy” is a sophisticated term from the field of differential geometry, whose specifics are outside the scope of this thesis. For our purposes, it will mean a “non-Abelian geometric phase” [34]. Like the original Berry phase, HQC was developed in the adiabatic regime [34, 36].

However, what does it mean for a *phase* to be (non-)adiabatic? This terminology comes from the application of the Adiabatic Theorem. This theorem states the following: suppose that a time-dependent Hamiltonian, changes *gradually* from $\mathcal{H}(0)$ to $\mathcal{H}(\tau)$. If a particle was initially in the n th eigenstate of $\mathcal{H}(0)$, under the Schrödinger Equation it will evolve to the n th eigenstate of $\mathcal{H}(\tau)$ [37]. Thus, if the system starts out in the n th eigenstate of $\mathcal{H}(0)$, it will evolve as the n th eigenstate of the *instantaneous* Hamiltonian $\mathcal{H}(t)$, with time $0 < t < \tau$. As we will see in the following section, this evolution carries two different phases: the dynamical phase and the geometric phase. As this evolution must be done gradually⁶, we can conclude that the adiabatic regime is associated with long evolution time. Conversely, the non-adiabatic regime is associated with a short evolution time. Since faster evolution time is more advantageous when it comes to computations, so is the non-adiabatic regime.

It would therefore be advantageous to extend the HQC scheme to the non-adiabatic regime, allowing for shorter evolution time. This was accomplished in 2012 under the name “non-adiabatic holonomic quantum

⁴non-Abelian is a term from group theory denoting non-commutativity.

⁵A recent review [33] covers the differences between the kinds of geometric phases. An important distinction to note is that a Berry phase is a geometric phase, but a geometric phase is not in general, a Berry phase.

⁶Gradually, or adiabatically, means that the time scaling of the change in the external conditions is much longer than the time scaling of the internal process [37].

computation” (NHQC) [36], using the phases described by Anandan [32]. NHQC has previously been experimentally demonstrated in an artificial atom [23], among other implementations [38].

Lastly, the ideal quantum computing platform will be one that performs *universal* quantum computing. Certain tasks require access to non-commuting resources [36]. This is not possible to accomplish with Abelian phases (gates). Thus, non-Abelian phases (gates) are required. To demonstrate this non-commuting behaviour, at least two such gates must be created. Therefore, the goal of this project is to create two non-Abelian gates in a photonic circuit using NHQC. In the following section, we will more formally review the concept of the geometric phase as a whole and how it will be used in this experiment.

6.2 The Geometric Phase

6.2.1 Mathematical Derivation

Let $|\psi\rangle$ be a general quantum state and let $|\psi_n\rangle$ be its basis states. Evolving this state in time requires the application of the Schrödinger Equation. The result is:

$$|\psi\rangle = \sum_n e^{-iE_n t/\hbar} |\psi_n\rangle. \quad (6.1)$$

From this equation, we can conclude that it is a phase that drives evolution in quantum mechanics. It is (usually) driven by energy differences between the different energy eigenstates. The usual time-evolution factor $e^{-iE_n t/\hbar}$ as seen above is called the *dynamical* phase. However, as Berry pointed out in 1984 [29], it is not the only kind of phase one can get as a state evolves.

To derive the expression of the dynamical phase and this other phase, we follow the steps of Ekert [35]. Assume a cyclic state in time i.e., it takes the same value at time 0 and at a time $\tau > 0$. This can be written as:

$$|\psi(\tau)\rangle = e^{i\Phi} |\psi(0)\rangle, \quad (6.2)$$

allowing for a phase Φ that is obtained during this time evolution. We also define a basis for this state such that $|\psi(t)\rangle = e^{if(t)} |\tilde{\psi}(t)\rangle$ and $|\tilde{\psi}(t)\rangle = |\tilde{\psi}(0)\rangle$ [30]. This is to simplify the examination of the phase, as this is the component of the state we wish to investigate. With this basis, we can write $\Phi = f(\tau) - f(0)$.

With these preliminaries set, we examine what happens to the phase Φ as the cyclic state is evolved in time. This is accomplished with the Schrödinger equation, where $\mathcal{H}(t)$ is the Hamiltonian:

$$\begin{aligned}
i\hbar \frac{\partial |\psi(t)\rangle}{\partial t} &= \mathcal{H}(t) |\psi(t)\rangle \\
i\hbar \frac{\partial}{\partial t} \left(e^{if(t)} |\tilde{\psi}(t)\rangle \right) &= \mathcal{H}(t) e^{if(t)} |\tilde{\psi}(t)\rangle \\
i\hbar \left[if'(t) e^{if(t)} |\tilde{\psi}(t)\rangle + e^{if(t)} \frac{\partial}{\partial t} \left(|\tilde{\psi}(t)\rangle \right) \right] &= \\
-\hbar f'(t) e^{if(t)} |\tilde{\psi}(t)\rangle + i\hbar e^{if(t)} \frac{\partial}{\partial t} \left(|\tilde{\psi}(t)\rangle \right) &=
\end{aligned} \tag{6.3}$$

We left multiply eq. 6.3 by $\langle \tilde{\psi}(t) | = \langle \tilde{\psi}(t) | e^{-if(t)}$:

$$-\hbar f'(t) + i\hbar \left\langle \tilde{\psi}(t) \left| \frac{d}{dt} \right| \tilde{\psi}(t) \right\rangle = \langle \psi(t) | \mathcal{H}(t) | \psi(t) \rangle. \tag{6.4}$$

This equation is then rearranged and integrated. Since we have defined $\Phi = f(\tau) - f(0)$, we can write:

$$\Phi = -\frac{1}{\hbar} \int_0^\tau \langle \psi(t) | \mathcal{H}(t) | \psi(t) \rangle dt + i \int_0^\tau \left\langle \tilde{\psi}(t) \left| \frac{d}{dt} \right| \tilde{\psi}(t) \right\rangle dt. \tag{6.5}$$

The phase Φ now has two terms: the first term, is the usual dynamical phase. However the second term is the *geometric* phase. For mathematical convenience, the second term can be rearranged as a contour integral [35]:

$$\gamma = i \oint_{\mathcal{C}} \left\langle \tilde{\psi} \left| d \right| \tilde{\psi} \right\rangle, \tag{6.6}$$

where $\mathcal{C}$ is the path taken by the state in parameter space and d is the differential operator. This phase is geometric in nature because it only depends on the path taken in parameter space. It does not depend on the Hamiltonian (which drives the evolution), nor does it depend on the basis states $\{|\tilde{\psi}\rangle\}$ [35].

We have just derived an expression for a geometric phase. This expression is more general than the phase introduced by Berry and was used to highlight the two different phases that arise. To obtain *Berry's* phase, one must further apply the Adiabatic Theorem. Recall that the Adiabatic Theorem states that “a system that starts out in the n th eigenstate of the initial Hamiltonian ($\hat{H}(0)$) will evolve as the n th eigenstate of the instantaneous Hamiltonian ($\hat{H}(t)$)” [37, p. 429]. Put another way, if the Hamiltonian changes slowly enough, the system will remain in the same instantaneous eigenstate.

We now proceed as Berry did to find an expression for Berry's phase [29]. Suppose a Hamiltonian $\mathcal{H}$ that is dependent on a set of parameters $\mathbf{R}(t)$ which can change with time i.e., $\mathcal{H}(\mathbf{R}(t))$. As with the previous derivation, assume cyclic evolution. This means $\mathbf{R}(\tau) = \mathbf{R}(0)$ for a time $\tau > 0$.

Let $|\psi(t)\rangle$ be a state of this system and let it evolve according to the Schrödinger equation:

$$i\hbar \frac{\partial}{\partial t}(|\psi(t)\rangle) = \hat{\mathcal{H}}(\mathbf{R}(t)) |\psi(t)\rangle. \quad (6.7)$$

At any time t , the eigenstates of $\hat{\mathcal{H}}(\mathbf{R}(t))$ are the states $|n(\mathbf{R})\rangle$ which satisfy the eigenvalue equation

$$\hat{\mathcal{H}}(\mathbf{R}(t)) |n(\mathbf{R}(t))\rangle = E_n(\mathbf{R}(t)) |n(\mathbf{R}(t))\rangle, \quad (6.8)$$

where E_n are the energies. Because of the adiabatic evolution requirement, the state $|n(\mathbf{R}(0))\rangle$ will evolve due to $\hat{\mathcal{H}}$ to be $|n(\mathbf{R}(t))\rangle$ at time t . Therefore the state $|\psi(t)\rangle$ can be written as [29]:

$$|\psi(t)\rangle = \exp\left(\frac{-i}{\hbar} \int_0^t E_n(\mathbf{R}(t')) dt'\right) \exp(i\gamma_n(t)) |n(\mathbf{R}(t))\rangle, \quad (6.9)$$

where the first exponential is the usual dynamical phase once more and γ_n is the geometric phase. After a full trip around the contour, the total phase change is [29]:

$$|\psi(\tau)\rangle = \exp\left(\frac{-i}{\hbar} \int_0^\tau E_n(\mathbf{R}(t)) dt\right) \exp(i\gamma_n(\mathcal{C})) |\psi(0)\rangle, \quad (6.10)$$

where,

$$\gamma_n(\mathcal{C}) = i \oint_{\mathcal{C}} \langle n(\mathbf{R}) | \nabla_{\mathbf{R}} n(\mathbf{R}) \rangle \cdot d\mathbf{R}. \quad (6.11)$$

is the geometric phase change. This expression is a contour integral⁷. It can be interpreted as the parameters $\mathbf{R}(t)$ evolving around a closed path in parameter space. Due to Stokes's Theorem, this contour integral can be recast as a surface integral [29]:

$$\gamma_n(\mathcal{C}) = - \iint_{\mathcal{C}} d\mathbf{S} \cdot \mathbf{V}_n(\mathbf{R}) \quad (6.12a)$$

$$\mathbf{V}_n(\mathbf{R}) \equiv \text{Im} \sum_{m \neq n} \frac{\langle n(\mathbf{R}) | \nabla_{\mathbf{R}} \hat{\mathcal{H}}(\mathbf{R}) | m(\mathbf{R}) \rangle \times \langle m(\mathbf{R}) | \nabla_{\mathbf{R}} \hat{\mathcal{H}}(\mathbf{R}) | n(\mathbf{R}) \rangle}{(E_m(\mathbf{R}) - E_n(\mathbf{R}))^2}, \quad (6.12b)$$

where $\mathbf{R} = \mathbf{R}(t)$ for simpler notation and $d\mathbf{S}$ is an infinitesimal surface element in this space. This result can be interpreted as the geometric phase being dependent on an area enclosed by the path $\mathcal{C}$ taken in parameter space. This expression, while general, can be simplified. In the case of a two-state degeneracy, such as a qubit, the expression simplifies to [29]:

⁷ $\nabla_{\mathbf{R}}$ is called the covariant derivative with respect to $\mathbf{R}$ in the language of differential geometry.

$$\gamma_{\pm}(\mathcal{C}) = \mp \frac{1}{2} \Omega(\mathcal{C}), \quad (6.13)$$

where $\pm$ denotes each state of the two-state degeneracy and $\Omega(\mathcal{C})$ is the solid angle subtended by the contour $\mathcal{C}$. This result is the main reasoning behind the use of a Berry phase for quantum computing. If the path taken in parameter space has some perturbations, as long as the solid angle is the same, so too will the desired transformation. This property is known as “robustness”.

6.2.2 Illustration of the Concept

In the previous section, we have derived the mathematical expressions behind the geometric phase. It has been shown to be in general proportional to a path in parameter space (eq. 6.6), which is recast as a proportionality to an area (eq. 6.12), which itself is simplified in the particular case of a two-state degeneracy (eq. 6.13) to a dependency on the solid angle. As an aid to these equations, a visualization of this phase can help remove doubts as to its geometric nature.

Suppose you are located at the North Pole of a curved surface, such as fig. 6.1a. In the context of a qubit, this sphere can be thought of as the Bloch Sphere. You walk towards the equator, then side step the same distance, and backpedal the same distance, taking the same path as the arrow in the figure. You have returned to your starting location, but are now facing a different direction. This change in orientation is precisely the geometric phase. This example also shows why geometric phase is also called “a global rotation without a local rotation” [39].

The figure also illustrates two properties which are important to obtain a geometric phase: cyclic evolution and parallel transport⁸. This first condition means that you have started and ended your journey at the same location. In the example in the figure, this was the North Pole. Mathematically, this is the cyclical nature of the state (eq. 6.2). The second condition means that your orientation did not locally change at any point during your journey. This can be seen in the figure as the orientation of the arrow does not change along the path, it is the path itself which changes. Mathematically, the second condition means that the length of the arrow as it is being parallel-transported does not change and that the phase between one point on the path and a point that is a line element ds away is the same [30].

The first figure highlights what the geometric phase *is*. The second, fig. 6.1b, demonstrates the robustness of the geometric phase. In that figure, there are perturbations along the path. However, these imperfections have kept the area enclosed by the path constant. Therefore, the desired transformation is also constant, hence the robustness.

⁸Like “holonomy”, parallel transport is a mathematically sophisticated term from differential geometry which has a specific definition. This precise definition is outside the scope of the thesis.

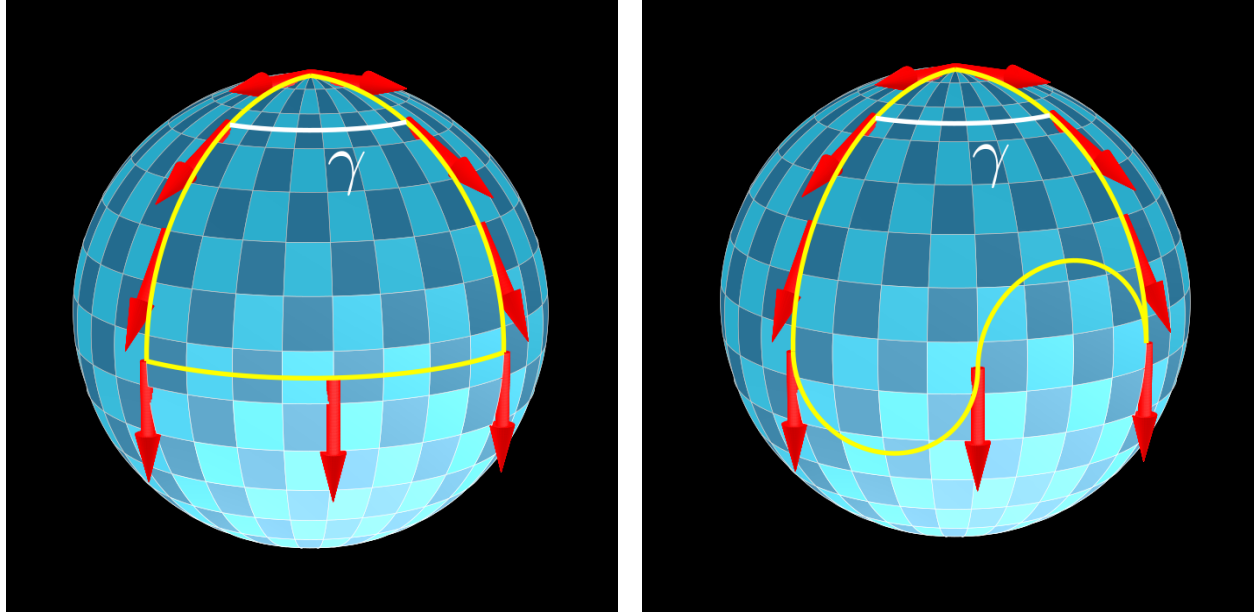
(a) Illustration of the Geometric Phase ⁹(b) Illustration of Robustness ⁹

Figure 6.1: Figure 6.1a illustrates the Geometric Phase as the change in orientation that one observes when travelling in a cyclical path on a curved surface, while satisfying parallel transport. Figure 6.1b illustrates the theorized robustness of the Geometric Phase, due to its dependence on the path taken.

To conclude this Chapter, we note that the geometric phase is not limited to the quantum regime. It is merely a consequence of moving along a curved surface, which is why it is difficult to escape differential geometry to explain this effect. We provide two classical examples of a geometric phase.

The first classical example of the geometric phase is Foucault's Pendulum [41]. Foucault's Pendulum is an important historical demonstration, as it was the first direct evidence of the Earth's rotation. Foucault's Pendulum is comprised a long pendulum located somewhere on the surface of the Earth. Taking care when launching the pendulum to not add additional forces, one needs only to wait to observe the geometric phase. After some time, the plane of oscillation of the pendulum will rotate. The speed of rotation depends on the latitude of the pendulum, according to [42]:

$$\omega = 360^\circ \sin(\phi)/\text{day}, \quad (6.14)$$

where ω is the rate of precession and ϕ is the latitude of the pendulum. The angular amount by which the plane will have rotated is the geometric phase, which is proportional to the solid angle subtended by the path (hence the dependency on the latitude).

The second classical extension of Berry's phase was introduced by John Hannay in 1985 [43]. It is obtained from a classical Hamiltonian which takes a closed path in parameter space and is moving adiabatically slowly.

⁹Made with [40].

It most easily appears in the formulation of classical mechanics known as the *Hamilton-Jacobi Equation*. This equation can be formulated in terms of canonical variables known as the *action* I and the *angle* θ [44]. Hannay then develops the “angle variable holonomy”. For an adiabatic path in parameter space, the rate of change of the action is identically 0, but there is no physical reason, according to Hannay, that the same applies to the angle. This change in the angle variable is then shown to be non-zero in general. Thus, in this formulation of classical mechanics, the angle change $\Delta\theta$ is the analogue to the geometric phase. This is why the angle change is known as the “Hannay Angle”.

Chapter 7

Design of Geometric Phase Logic Gates

In the literature, a non-Abelian, non-adiabatic geometric phase logic gate has previously been implemented in an atomic system [23] in the Λ configuration. Our goal in this project is to translate this scheme to a photonic system. Analogies which allow the translation between the two kinds of systems are reviewed in [45].

7.1 Analogy From Atoms to Waveguides

Atomic and photonic structures are quite different in terms of necessary equipment and control. In certain atomic systems, operating temperatures must be reduced to near absolute zero. This is done to reach the superconducting regime. These systems are called “artificial atoms” because they have quantized properties like natural atoms [46]. In the case of [23], their experiment relied on a superconducting transmon [47], which operates by using a Josephson junction qubit. This type of qubit has quantized charge [48]. It is not simple to control this structure at these low temperatures while the remaining measurements are performed at room temperature.

In contrast, in photonic structures, the control apparatus is much simpler. One example is the boson sampling structure demonstrated in [49]. At first approximation, these platforms seem to be quite different. However, the physics and the mathematics in which it is written are analogous to one another. Therefore, it is important to highlight the common aspects between the two implementation platforms. Important parameters for this analogy are highlighted in table 7.1.

Parameter	Atomic	Waveguide
Propagation	Time (t)	Space (z)
Coupling	Optical Pulses	Tunnelling
Three states	Three Energy Levels	Three Waveguides

Table 7.1: Comparing Parameters Between Atomic and Waveguide Theory

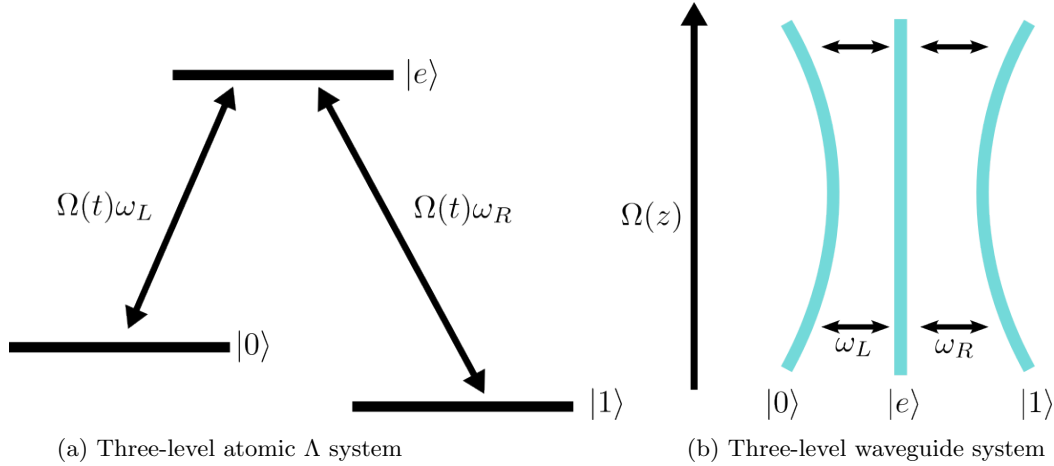


Figure 7.1: Comparing equivalent parameters between atomic and waveguide systems. For the atomic system in fig. 7.1a, we have two pulses of amplitudes ω_L and ω_R which have the same time-dependent envelope. The equivalent waveguide system of fig. 7.1b has two different coupling amplitudes given by the separation between the waveguides, and the same space-dependent envelope, as both outer waveguides are curved at the same rate.

We can draw several conclusions from this comparison. First, the three-level atomic system under a Λ configuration (fig. 7.1a) can be replicated as a three-waveguide system where the two outer waveguides do not couple (fig. 7.1b), mimicking the forbidden transition of the atomic ground states. Next is the interchange between time and spatial dimensions. In both cases, we have the coupling parameter which evolves as a function of propagation. Thus, a time-varying optical pulse is replicated by distance-varying waveguides [50]. This also means that other parameters which depend on the propagation will have analogous quantities between the systems. Lastly, we can draw analogies between the Hamiltonians which will drive each implementation. The Hamiltonian simply dictates how the states evolve, it does not specify the representation of the states themselves. Hence, the atomic Hamiltonian itself can be reused in the photonic system, as we can describe the states in the same manner.

7.2 Implementation of the Geometric Phase in a Waveguide System

Section 6.2 covered the concept of the geometric phase. In this section, we cover the mathematical concepts required to implement such a phase in the following derivations. To implement a non-Abelian geometric phase, two conditions must be satisfied: the parallel transport condition and the cyclic evolution condition.

The parallel transport condition is defined by [23]:

$$\langle \Psi_i | \mathcal{H} | \Psi_j \rangle = 0, \quad (7.1)$$

where $\mathcal{H}$ is the Hamiltonian of the system, and $\Psi_i, i \neq j$ are two different basis states for the system. This condition is satisfied when the ratio of the coupling amplitudes (ω_L/ω_R in fig. 7.1) is fixed to a time(space)-independent constant [23].

The cyclic evolution condition means that the time(space)-dependent parameter must start and end with the same value. As waveguides couple due to evanescent tail coupling (to be discussed in further detail in 7.4.1), we can change the coupling by varying the distance between waveguides. The cyclic evolution condition is given by [36]:

$$\int_0^\tau \tilde{\Omega}(t) dt = \pi, \quad (7.2)$$

where $\tilde{\Omega}(t)$ is the time(space)-dependent parameter of the system. Combined, these two conditions allow for a universal one-qubit logic gate [36].

A comment on eq. 7.2: why π and not 2π if this is the *cyclic* evolution condition? In the language of the atomic system, $\tilde{\Omega}(t)$ is a π pulse. The gate is accomplished by applying two π pulses of identical temporal overlap.

7.3 Deriving the Logical Parameters

We now wish to derive the necessary parameters for our waveguide system, based on the previously mentioned theory. We begin with deriving the unitary transformation that the waveguide logic gates will implement. These unitary parameters are implementation agnostic, as they only set the transformation.

7.3.1 Derivation of the Unitary Transformation

Since the implementation is based on an atomic system, our starting point in this derivation will be the atomic Hamiltonian [36]. Under the Rotating Wave Approximation (RWA), the system is described as:

$$\mathcal{H}(t) = \hbar \tilde{\Omega}(t) (\omega_L |e\rangle\langle 0| + \omega_R |e\rangle\langle 1| + \text{H.c.}), \quad (7.3)$$

where $\tilde{\Omega}(t)$ is the identical time-dependent coupling envelope, $|0\rangle$ is the logical 0 state, $|1\rangle$ is the logical 1 state, $|e\rangle$ is the excited state, ω_L and ω_R are the coupling amplitudes between the left (right) waveguides and the centre, and H.c. stands for the Hermitian conjugate. Combined, the $\{|0\rangle, |1\rangle\}$ states are called the *computational* basis. The coupling amplitudes must satisfy two conditions:

$$|\omega_L|^2 + |\omega_R|^2 = 1 \quad (7.4a)$$

$$\frac{\omega_L}{\omega_R} = C. \quad (7.4b)$$

The first represents a normalization condition, and the second condition is the required parametrization previously mentioned. The time(space)-independent constant C will be defined in eq. 7.23, but we note here that it parameterizes the ratio according to two parameters $\{\theta, \phi\}$ i.e., $C = C(\theta, \phi)$. Next, we define what are called the bright $|b\rangle$ and dark $|d\rangle$ states:

$$|b\rangle = \omega_L^* |0\rangle + \omega_R^* |1\rangle \quad (7.5a)$$

$$|d\rangle = -\omega_R |0\rangle + \omega_L |1\rangle. \quad (7.5b)$$

These two states define the *bright-dark* basis. These definitions may seem arbitrary but they will be both mathematically and physically useful. Both the computational and the bright-dark bases are defined in terms of two states and generate subspaces. Both can be extended to the full space by adding the $|e\rangle$ state, which is orthogonal in both bases. As there are several bases used in this derivation, we write them all in the following table to be explicit:

Computational	Bright \ Dark	Eigenbasis
$ 0\rangle$	$ b\rangle$	$ b\rangle + e\rangle$
$ e\rangle$	$ e\rangle$	$ b\rangle - e\rangle$
$ 1\rangle$	$ d\rangle$	$ d\rangle$

Table 7.2: Comparison of States in Different Bases

Using the state definitions of the bright and dark states, we can simplify eq. 7.3:

$$\mathcal{H}(t) = \hbar \tilde{\Omega}(t) (|e\rangle\langle b| + |b\rangle\langle e|) = \hbar \tilde{\Omega}(t) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (7.6)$$

From this simplified Hamiltonian, we see that the $|d\rangle$ state has decoupled from the system. This is important, as it shows that the complete vector space of the system can be decomposed into subspaces. In particular, the $|b\rangle$ and $|d\rangle$ states belong to different subspaces. Hence these subspaces will take the names of the *bright* subspace and the *dark* subspace. Furthermore, the existence of this dark subspace is the source of non-commuting behaviour [23, 51]. As for the non-adiabatic evolution, [51] showed that non-adiabatic evolution requires evolution within the bright subspace. Conversely, this means that adiabatic evolution requires evolution within the dark subspace. Equation 7.6 demonstrates that this design evolves within the bright subspace.

From this simplified Hamiltonian, we can define the unitary transformation it generates. This is accomplished with the definition of the time evolution operator, $U(t)$:

$$U(t) = e^{-\frac{i}{\hbar} \int \mathcal{H}(t) dt}. \quad (7.7)$$

Using eq. 7.6 in this definition:

$$U(t) = e^{-i \int \tilde{\Omega}(t) dt} \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (7.8)$$

To compute this matrix exponential, we proceed in the general case where $\int \tilde{\Omega}(t) dt = \beta$, where $\beta \in [0, 2\pi[$. This general case will then be reduced to the particular case of $\beta = \pi$ for the geometric phase, as indicated in eq. 7.2. We stress that the resulting matrix is in the $\{|b\rangle, |e\rangle\}$ subspace:

$$U(t, \beta) = \begin{pmatrix} \cos(\beta) & -i \sin(\beta) \\ -i \sin(\beta) & \cos(\beta) \end{pmatrix}. \quad (7.9)$$

Since $|d\rangle$ has decoupled from the dynamics of the system, it is not affected by U i.e., $U|d\rangle = |d\rangle$. Therefore, we can extend eq. 7.9 back to the full $\{|b\rangle, |e\rangle, |d\rangle\}$ space:

$$U(t, \beta) = \begin{pmatrix} \cos(\beta) & -i \sin(\beta) & 0 \\ -i \sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{pmatrix}. \quad (7.10)$$

From this result, we change basis back to the computational basis (details in sec. B.3). The general unitary transformation, as given by an arbitrary β , and parametrized by θ and ϕ , is:

$$U(\theta, \phi, \beta) = \begin{pmatrix} \cos^2\left(\frac{\theta}{2}\right) + \cos(\beta) \sin^2\left(\frac{\theta}{2}\right) & -ie^{i\phi} \sin(\beta) \sin\left(\frac{\theta}{2}\right) & -e^{i\phi} \sin^2\left(\frac{\beta}{2}\right) \sin(\theta) \\ -ie^{-i\phi} \sin(\beta) \sin\left(\frac{\theta}{2}\right) & \cos(\beta) & -i \cos\left(\frac{\theta}{2}\right) \sin(\beta) \\ -e^{-i\phi} \sin^2\left(\frac{\beta}{2}\right) \sin(\theta) & -i \cos\left(\frac{\theta}{2}\right) \sin(\beta) & \cos(\beta) \cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right) \end{pmatrix}. \quad (7.11)$$

From the general transformation, we now apply the geometric phase assumption of eq. 7.2, that is $\beta = \pi$. Simplifying, gives us the unitary specific to our implementation, written in the full $\{|0\rangle, |e\rangle, |1\rangle\}$ space:

$$U(\theta, \phi) = \begin{pmatrix} \cos(\theta) & 0 & -e^{i\phi} \sin(\theta) \\ 0 & -1 & 0 \\ -e^{-i\phi} \sin(\theta) & 0 & -\cos(\theta) \end{pmatrix}. \quad (7.12)$$

Finally, we can reduce this general unitary back to the computational subspace $\{|0\rangle, |1\rangle\}$:

$$U(\theta, \phi) = \begin{pmatrix} \cos(\theta) & -e^{i\phi} \sin(\theta) \\ -e^{-i\phi} \sin(\theta) & -\cos(\theta) \end{pmatrix}. \quad (7.13)$$

Therefore, eq. 7.13 is the unitary transformation which our system will implement.

7.3.2 Determining the Parameters for the Unitary Transformation

From eq. 7.13, we can obtain the specific values for θ and ϕ for the logic gates we want to implement. The logical NOT or Pauli-X transformation is given by [20]:

$$\text{NOT} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}. \quad (7.14)$$

Similarly, the Hadamard transformation is given by [20]:

$$H = \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \quad (7.15)$$

We now solve for θ and ϕ . Allowing for a global phase κ , we specifically solve:

$$\kappa * \{\text{NOT or } H\} = \begin{pmatrix} \cos(\theta) & -e^{i\phi} \sin(\theta) \\ -e^{-i\phi} \sin(\theta) & -\cos(\theta) \end{pmatrix}. \quad (7.16)$$

Solving for the NOT gate, we get:

$$U(\theta = \frac{\pi}{2}, \phi = 0) = -1 * \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad (7.17)$$

and solving for the H gate, we get:

$$U(\theta = \frac{3\pi}{4}, \phi = 0) = -1 * \frac{1}{\sqrt{2}} \begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}. \quad (7.18)$$

In both cases, there is a global phase difference of $\kappa = -1$ between the unitary and the ideal gate. As this is a *global* phase on a single qubit, it cannot be measured. Therefore this difference will not be an issue. Since θ and ϕ are periodic parameters, one can wonder if there are additional solutions. There are indeed four total solutions for both logic gates. However, only two of these solutions give physical solutions when used in eq. 7.32. Both remaining physical solutions are equivalent. Therefore, without loss of generality, we chose the solutions with $\phi = 0$.

7.4 Derivation of Design Equations

We next derive what we call the “design equations”. These are the specific parameters needed for the implementation in a waveguide system.

7.4.1 Review of Coupling Between Waveguides

We begin the derivation by reviewing the coupling between waveguides. First, one waveguide can be considered as a potential well. It has a fundamental mode. Thus, we can consider two waveguides as a double potential well [52]. In this view, a particle trapped in the well oscillates between the two supermodes of the two waveguide system, with transition time $t = \hbar\pi/(E_{as} - E_s)$, where E_{as} denotes the energy of the anti-symmetric (as) supermode and E_s denotes the energy of the symmetric (s) supermode. The supermodes are obtained from orthogonal *superpositions* of the two fundamental modes. The geometry of the two waveguide system is illustrated in fig. 7.2. Note that this figure is *not* to scale.

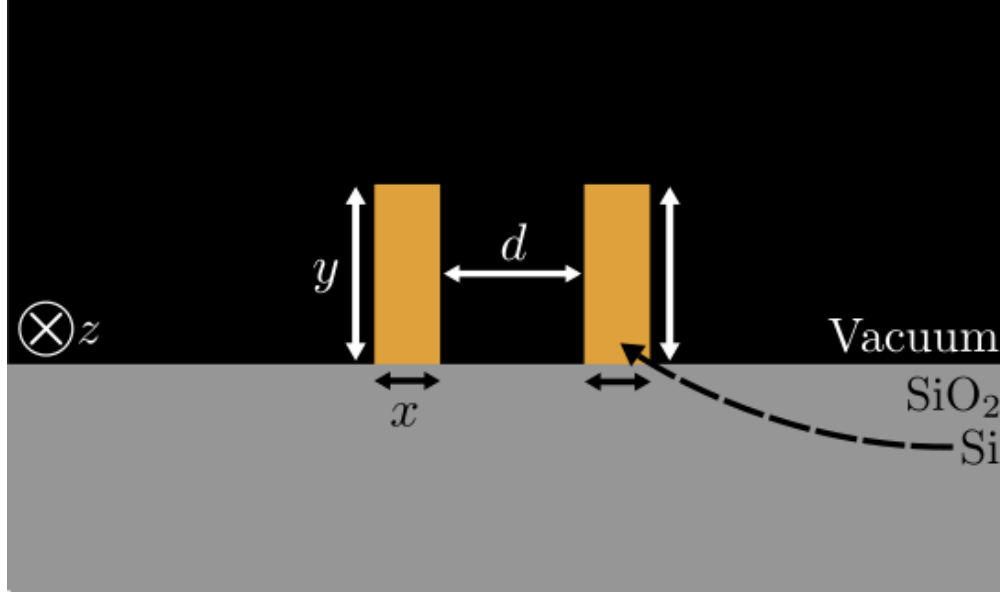


Figure 7.2: Schematic of Silicon Waveguide Structure and Dimensions

As highlighted in sec. 7.1, the time dimension t becomes the propagation dimension z in waveguides. Therefore we will be restricting the discussion to explicit z dependence in the remainder of the discussion.

Let us compare the wavefunction of the double potential well [52] with the generic form of a propagating mode [53]:

$$\Psi(x, t) = M e^{-iE_s t/\hbar} \left(\psi_s(x) + \psi_{as}(x) e^{-i\Delta E t/\hbar} \right) \quad (7.19a)$$

$$\mathcal{E}(x, y, z, t) = \mathcal{E}_m(x, y) \exp(i(\omega t - \beta_m z)), \quad (7.19b)$$

where $\Psi(x, t)$ is the system wavefunction, $\psi_i(x)$ are the supermode wavefunctions ($i = \{as, s\}$), $\Delta E = E_{as} - E_s$ the difference in energy between the two supermodes, M is a normalization constant, $\mathcal{E}_m(x, y)$ is the wavefunction of the m th mode, and β_m is the propagation constant of the m th mode. Since the energy difference ΔE between the supermodes will drive the oscillation in the potential well, so will the difference in the β_m will cause oscillations in the waveguides, per the time-space analogy previously discussed.

In the following derivations, it is convenient to define a new quantity, called the *effective index* n_{eff} [53], rather than directly using the propagation constant:

$$n_{\text{eff}} = \frac{\beta_m}{\omega/c}. \quad (7.20)$$

Each mode of propagation thus gets its own effective index and it will be the *difference* in effective index that will drive the evolution of the system, similar to the *difference* in energies. This analogy allows us to

write the coupling between waveguides as [52] :

$$\tilde{\Omega}(z) = \frac{\pi \Delta n(z)}{\lambda}, \quad (7.21)$$

where $\Delta n(z)$ is the difference between the effective indices of the two waveguide system¹⁰. This function of the effective index is dependent on the material as well as the size of the waveguide cross-section. It can be found by simulation (sec. 8.3) or by experiment (sec. 10.4). At a distance d_0 between the waveguides, it has the form:

$$\Delta n(z = z_0) = N e^{-\alpha d_0}, \quad (7.22)$$

where N is a numerical parameter from the fit, and α is the coupling coefficient. This function can also be obtained from the double potential well solution [52]. As our waveguides will be curved, we also fix this function at a particular point along the propagation direction z_0 . Equations 7.21 and 7.22 will be important in the derivations that follow.

7.4.2 Fixing Coupling Amplitudes

We now define the arbitrary z -independent constant C . This constant (see sec. 7.3.1) is the parametrization of ω_L/ω_R which defines the coupling amplitude. This ratio also needs to be constant (in the propagation dimension) to satisfy the parallel transport condition. We define C as:

$$C = \tan\left(\frac{\theta}{2}\right) e^{i\phi}. \quad (7.23)$$

From eq. 7.23, we have the coupling amplitude for each transition. We know that these amplitudes are modulated by an identical coupling envelope. We can combine these to obtain specific formulas for each coupling pair:

$$\Omega^L(z) = \tilde{\Omega}(z) \sin\left(\frac{\theta}{2}\right) e^{i\phi} \quad (7.24a)$$

$$\Omega^R(z) = \tilde{\Omega}(z) \cos\left(\frac{\theta}{2}\right), \quad (7.24b)$$

where $\Omega^L(z)$ is the coupling between the left and centre waveguide as a function of propagation distance, $\Omega^R(z)$ is the same for the centre and right waveguides, and $\tilde{\Omega}(z)$ is the identical coupling envelope. The waveguides are currently straight and therefore are a constant distance apart throughout the length. The

¹⁰Two straight waveguides are used to determine the material dependant coupling parameters for the three-waveguide system.

next step is to give them a shape to modulate the coupling.

7.4.3 Minimum Separation Between Waveguide Pairs, Δd

We assume a parabolic bending profile for the two outer waveguides. In its most general form, we have:

$$d^L(z) = d_0^L + a_L(z - z_0^L)^2 \quad (7.25a)$$

$$d^R(z) = d_0^R + a_R(z - z_0^R)^2, \quad (7.25b)$$

where the a_i are the bending coefficients, the z_0^i are the location of the minimum of the parabola along the direction of propagation, and d_0^i are the minimum distance between the centre waveguide and the i waveguide, where $i \in \{L, R\}$ and $L = \text{left}$ and $R = \text{right}$. In this general form, each side can have its own values for each of these parameters. However we can make some simplifications.

It can be shown (see appendix B.1) that $a_L = a_R \stackrel{\text{def}}{=} a$. This means that we will not have any difference in the “pulse length”, which is present in other schemes such as coherent tunnelling adiabatic passage (CTAP) [50]. Since there is no difference in the pulse length, we can also state $z_0^L = z_0^R \stackrel{\text{def}}{=} z_0$. This means the minimum of the parabola occurs at the same location along the propagation direction for both curved waveguides. Having different locations for the minimum would be precisely how one would create the different pulse lengths in the waveguide system.

As eq. 7.22 is only defined for the minimum distance z_0 , we can extend it to the entire propagation length of the device using eqs. 7.25. Applying this extension, we get the difference in the eigenmodes of the system over the entire propagation length of the system, or:

$$\Delta n(z) = N e^{-\alpha d_0^\eta} e^{-\alpha a(z-z_0)^2}, \quad (7.26)$$

where η is either L or R , as it is the only parameter which distinguishes between the couplings. With this general expression for $\Delta n(z)$, we can now insert it in eq. 7.21 to have a full expression for the coupling:

$$\tilde{\Omega}(z) = \frac{\pi}{\lambda} N e^{-\alpha a(z-z_0)^2} e^{-\alpha d_0^\eta}. \quad (7.27)$$

We now write this equation explicitly in terms of both couplings:

$$\Omega^L(z) = \underbrace{\frac{\pi}{\lambda} N e^{-\alpha a(z-z_0)^2}}_{\text{coupling}} e^{-\alpha d_0^L} \quad (7.28a)$$

$$\Omega^R(z) = \underbrace{\frac{\pi}{\lambda} N e^{-\alpha a(z-z_0)^2}} e^{-\alpha d_0^R}. \quad (7.28b)$$

This expression allows us to have a functional form to the identical coupling envelope mentioned in 7.1, given by the underbraced factor. Therefore, we define this identical envelope as:

$$\Omega(z) = \frac{\pi}{\lambda} N e^{-\alpha a(z-z_0)^2}, \quad (7.29)$$

which allows the simplification of eqs. 7.28 to:

$$\Omega^L(z) = \Omega(z) e^{-\alpha d_0^L} \quad (7.30a)$$

$$\Omega^R(z) = \Omega(z) e^{-\alpha d_0^R}. \quad (7.30b)$$

With the expressions in eqs. 7.30, we can now have an expression for the ratio of the couplings. This allows us to use the constant defined in eq. 7.23:

$$\frac{\Omega^L(z)}{\Omega^R(z)} = \frac{\Omega(z) e^{-\alpha d_0^L}}{\Omega(z) e^{-\alpha d_0^R}} = \frac{e^{-\alpha d_0^L}}{e^{-\alpha d_0^R}} \stackrel{\text{def}}{=} \tan\left(\frac{\theta}{2}\right) e^{i\phi}, \quad (7.31)$$

Expanding on the final equality in the above expressions, allows us to obtain our first design equation:

$$\begin{aligned} d_0^L &= d_0^R - \frac{\ln(\tan(\frac{\theta}{2}) e^{i\phi})}{\alpha} \\ \Delta d &= -\frac{\ln(\tan(\frac{\theta}{2}) e^{i\phi})}{\alpha}. \end{aligned} \quad (7.32)$$

This equation tells us that is is the *difference* between the minimum distances which will set the transformation for the logic gates. Individually, neither minimum separation is enough to apply the desired transformation. To get the difference, one minimum separation is set by the equation. The other is arbitrarily set based on eq. 7.22. This equation informs us on the strength of the coupling for our desired material system, thus it is a guide on how to choose the arbitrary distance. For the remainder of the derivations, it will be important to decide which parameter is set, and which is adjusted for the desired transformation, as both could play either role. We will therefore set d_0^R and have d_0^L adjust accordingly.

7.4.4 Bending Coefficient, a

The next design equation will be the definition of the bending coefficient, a . The starting point for this derivation will be the cyclic evolution condition (eq. 7.2) where time-dependence has been replaced with

space-dependence, and ℓ is the length of the device:

$$\int_0^\ell \tilde{\Omega}(z) dz = \pi. \quad (7.33)$$

Replacing $\tilde{\Omega}(z)$ with $\Omega^L(z)$ as given in eq. 7.30a and simplifying:

$$\int_0^\ell \Omega(z) e^{-\alpha d_0^L} dz = \pi \quad (7.34)$$

$$e^{-\alpha d_0^L} \int_0^\ell e^{-\alpha a(z-z_0)^2} dz = \pi \quad (7.35)$$

$$\frac{\pi}{\lambda} N e^{-\alpha d_0^L} \int_0^\ell e^{-\alpha a(z-z_0)^2} dz = \pi. \quad (7.36)$$

At this stage, it would be ideal to substitute a value for ℓ . This is not possible as we do not have an expression for the length. However, it is possible to eliminate it. Since the outer waveguides are parabolas, the minimum distance z_0 occurs at the exact middle of the device, or $z_0 = \ell/2$. Thus, with a substitution with $u = z - z_0$, we can write:

$$\frac{\pi}{\lambda} N e^{-\alpha d_0^L} \int_{-\ell/2}^{\ell/2} e^{-\alpha a u^2} du = \pi. \quad (7.37)$$

This is a Gaussian integral in u . Under the assumption of a long device, which means taking the limit of $\ell \rightarrow \infty$, yields the standard result of $\sqrt{\frac{\pi}{\alpha a}}$. This has eliminated ℓ and we can continue to evaluate eq. 7.37:

$$\frac{\pi}{\lambda} N e^{-\alpha d_0^L} \sqrt{\frac{\pi}{\alpha a}} = \pi. \quad (7.38)$$

The above expression can be solved for a giving us our second design equation:

$$a(\alpha, N, d_0^L, \lambda) = \frac{\pi}{\alpha} \left(\frac{N}{\lambda} \right)^2 e^{-2\alpha d_0^L}, \quad (7.39)$$

where the dependencies have been noted explicitly.

7.4.5 Device Length, ℓ

The last design equation is that of the length of the device, ℓ . The starting point of this derivation will be an intermediate step of the previous derivation, specifically eq. 7.37. We acknowledge that in reality, we cannot have a device of infinite length. A simplification must therefore be made to get a device of finite length.

This simplification is setting the integral to be a fraction of π . If we need an infinite device to obtain a *full* π , then it is logical that a finite device will give us a fraction of π . We choose this fraction to be 0.99π . The justification for this choice comes from the predictive equations derived in sec. B.4, which are based on the unitary derived in sec. 7.3.1. These predictive equations confirm that this is a satisfactory simplification.

As the integral is now no longer over $]-\infty, \infty[$, the standard result we used previously no longer applies. Instead, we use the Gaussian integral *inversion*, which uses the error function $\text{erf}(x)$. The error function is defined by:

$$\text{erf}(x) = \frac{2}{\sqrt{\pi}} \int_0^x e^{-t^2} dt. \quad (7.40)$$

This expression allows us to transform eq. 7.37 into (see B.2):

$$\frac{\pi}{\lambda} N e^{-\alpha d_0^L} \sqrt{\frac{\pi}{\alpha a}} \text{erf}\left(\frac{1}{2} \sqrt{\alpha a} \ell\right) = 0.99\pi, \quad (7.41)$$

which itself can be simplified and then solved for ℓ , giving us our final design equation:

$$\ell(\alpha, a) = \frac{2}{\sqrt{\alpha a}} \text{erf}^{-1}(0.99), \quad (7.42)$$

where $\text{erf}^{-1}(x)$ is the inverse error function.

Since a is now known (see eq. 7.39), we can use its definition to obtain an equivalent definition for ℓ which may be more useful as it shows the underlying dependencies:

$$\ell(\alpha, a, N, d_0^L, \lambda) = \frac{2 \text{erf}^{-1}(0.99)}{\sqrt{\pi}} \frac{\lambda}{N e^{-\alpha d_0^L}}. \quad (7.43)$$

7.5 Summary of All Parameters for the Design of Waveguide Logic Gates

To summarize, we have the logical parameters, which sets the type of gate we wish to design. In particular, we are interested in creating a NOT gate, and a Hadamard gate. The logical parameters for each gate are solved for in eqs. 7.17 and 7.18, respectively. Since we are designing waveguide systems, we also need to determine the coupling coefficient for the material system of choice. The material parameters (α and N) are defined in eq. 7.22. The wavelength λ must also be selected for the desired material. With the logical and material parameters set, we combine these results with the theory behind the geometric phase and are able to derive three design equations. These allow us to define design parameters for the gate structures. These

design equations are equation 7.32 (Δd), equation 7.39 (a), and equation 7.43 (ℓ). In particular, for the Δd , one minimum distance d_0^R is arbitrarily defined. This in turn sets the scale of the entire device.

The logical parameters are summarized in table 7.3 and the design parameters are summarized in fig. 7.3.

Gate	θ	ϕ
NOT	$\pi/2$	0
Hadamard	$3\pi/4$	0

Table 7.3: Unitary Transformation Parameters for Waveguide Logic Gates

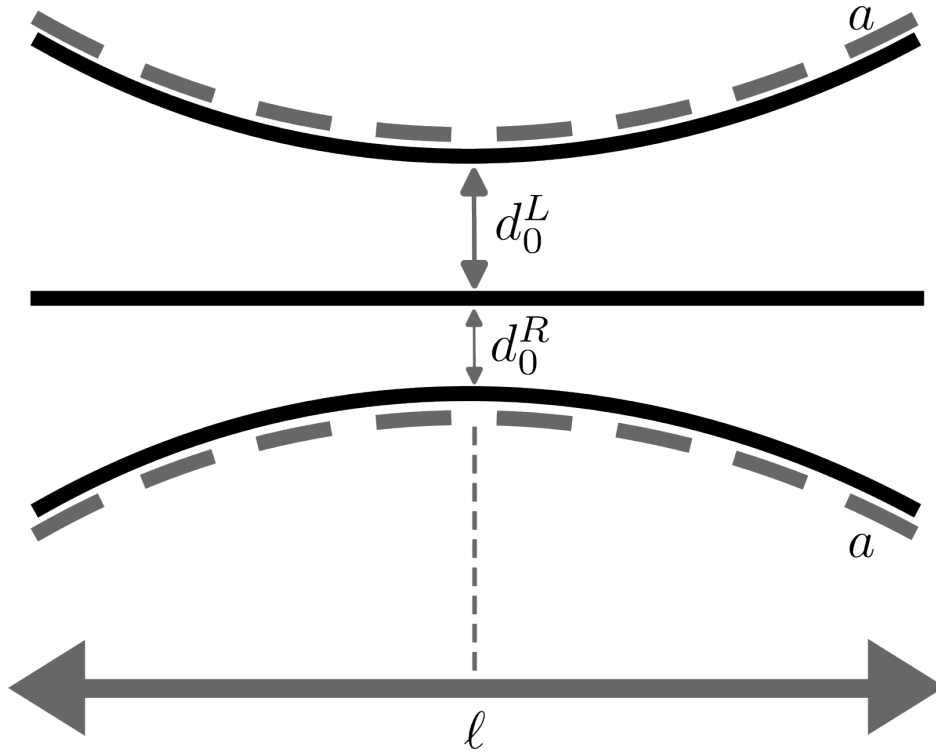


Figure 7.3: Logic Gate Schematic with All Design Parameters Labelled

In fig. 7.3 the dashed lines represents the curvature of both outer waveguides. This is to distinguish the curvature of the waveguides from the waveguides themselves. Other features to note from this schematic is the symmetry across the mid-point, where the minimum distances d_0^L and d_0^R are defined as well as the *lack* of symmetry across the central waveguide. The length of the device ℓ is the operational length. That is, this is the distance along which we must propagate to obtain the desired transformation. It is possible to have additional structures on both sides of the logic gates to help with coupling. It is not necessary to couple light directly into this structure. In the design process, we are not limited to the logic gates listed in table 7.3. This design process could apply to other one qubit logic gates.

Chapter 8

EME Simulations of Silicon Based Geometric Phase Logic Gates

8.1 Introduction

The original design for this project called for logic gates made in silicon. This would allow for easier integration with existing computing infrastructure. However, making a chip, testing it, analyzing errors, and starting the process over is expensive in both time and money. It would be more productive and cost-effective to only make a single chip with designs in which you are confident will work. This is the reasoning behind simulation software, as they allow for design iterations without having to fabricate a device nor a testing apparatus for said device. Simulations can be used to design a new device (e.g., [54]) or to compare a new simulation approach against the “standard” (e.g., [55]).

Simulation software are only tools, however, as they are only *representations* of reality, not reality itself. One particular way to illustrate this is in the discretization of space within the software. As computers do not have infinite resources, they are not able to compute the equations which describe the system of interest with infinite precision. One must therefore apply a mesh. Using an appropriate mesh for the structure one is simulating is one extremely important consideration when designing a simulation. This mesh also informs the choice of simulation algorithm. The effectiveness of different kinds of algorithms are studied. One such study is in [56]. Thus, another consideration is what solver to use, as not all solvers are created equal. These challenges will be explored in the following chapters.

The following two Chapters will describe the simulations undertaken during this project. Several differences will be highlighted between the Chapters, such as the waveguiding material and the simulation process

in general. This is because the simulations in the current Chapter were performed in 2018, whereas the simulations in the following Chapter were performed mostly in 2021. This time difference allowed for improvements to the simulations, such as the use of scripts, and the “model” feature in Lumerical. Important scripts are listed in an appendix (see appendix C).

8.1.1 Utility of Simulations

As indicated in the introduction, simulation software are merely tools one can use to test their devices without making them. This is advantageous, as it saves on time and monetary resources. However, due to the inherent discretization, they are an approximation of reality. In some circumstances (e.g., [54]), this approximation is of sufficient order to be relied upon such that the device can proceed to fabrication. However, as in the case of this project, simulations can lead to unphysical results. Reasons for these unphysical results could be computational in nature (e.g., finite difference algorithm blowing up at a boundary), or because the device itself will not work. It is important to be able to distinguish between these two classes of failures. Consistency is the key to differentiate between the two.

All of the simulations presented in this thesis were performed with commercial software (Lumerical MODE: Waveguide Simulator) from Lumerical Inc. [57]. Specifically, two different solvers from this software were used: the EigenMode Expansion (EME) solver [58] and the Finite Difference EigenMode (FDE) solver [59]. Both solvers will be detailed in the following sections.

8.2 EigenMode Expansion Solver

The main solver for the simulations of the logic gates is the “EigenMode Expansion” (EME) Solver. This is not the usual Finite-Difference Time Domain (FDTD) solver which is common in the literature [60, 61, 62], as it is a “brute force” method. The main reason for this choice is due to the length of the device, which increases the computational complexity of the problem. To understand this difference, we will quickly review both solvers and then compare them.

The EME [58] method is a two-step simulation process. The first step is to decompose (in the frequency-domain) the electromagnetic fields in terms of a basis set of local eigenmodes. This is accomplished by dividing the simulation space (in the propagation dimension) into several cells, and then solving for the modes at each interface between adjacent cells. As waveguides tend to have a limited number of guiding modes, the main approximation of this method is the finite number of modes that are used in this decomposition. Then, the modes need to be connected from one to the next. To join each cell, the scattering matrix (s-matrix) method is used. This allows the creation of a single s-matrix for the entire device. It is here where the

strength of the EME method for long or periodic devices comes into play, as the modes only need to be computed once. To add the next period of the device (for a periodic device for example), only the relevant s-matrix needs to be reused. The second step is to combine the modes of each cell to get the s-matrix of the entire device. If a single cell's propagation distance is modified at this stage, only that cell needs to be recomputed, the rest of the structure's s-matrices can be reused. As the structure changes its the propagation direction, it is important to have a good number of cells to ensure that there is a strong overlap between adjacent sets of modes.

The FDTD [63] method solves Maxwell's equation in the time-domain. The fields are discretized in a specific manner such that different field components lie on different parts of the unit cell. This is known as the "Yee cell". All derivatives are replaced by finite differences [64]. Then, by switching "on" a source, Maxwell's equations are solved in an iterative manner as an initial-value problem. This continues until the field has decayed below a given threshold value. Further details about the FDTD method can be found in [64]. However, it can be seen from this short description that the solution scales as the volume of the simulation space. Therefore, longer devices will be at a disadvantage.

A general comparison between important computational and physical criteria are presented for both methods in table 8.1. In this table, n is the refractive index of the material.

Parameter \ Scheme	EME	FDTD
Memory	Efficient for long or periodic devices	Not as good as EME for long devices
Speed	Efficient for long structures when cross-section changes slowly	Not as good as EME for long devices
Δn	Can model high n	Slows with high n
Geometries	Can adapt to different geometries	Fine with a large grid

Table 8.1: Comparison of Relevant Parameters Between Simulation Algorithms [62]

All of the elements presented table 8.1 are relevant to our conditions. First, memory and speed are restricted by the computer resources at our disposal. We were limited to a single computer with the following specifications:

Parameter	Value
CPU	Intel Core i5-6400 @ 2.70 GHz
RAM	64 GB
GPU	Intel HD Graphics 530
Storage Capacity	2 TB (not all usable)
OS	Windows 10 Enterprise
Age	~ 6 Years

Table 8.2: Simulation Computer Specifications

Therefore, the computer constraints limited the number of simulations that could be performed in a reasonable amount of time with a reasonable mesh size. Even when allowing for increased storage capacity (as this is the easiest part to upgrade), we were limited in what we could store for long periods of time. Next, the Δn , or the refractive index contrast of the structure. In Si-based structures, the refractive index contrast is on the order of ~ 2.5 , which is large. This large contrast implies the modes within those waveguides are more confined. The last parameter to review is the geometry of the simulation space. In the EME solver setup, the user can define different sections with a different number of cells, where the higher cell number would be used for a part of the structure which varies significantly. The smaller number of cells would cover the region where there is little change in the structure. Thus, optimization is possible here. For the FDTD solver, mesh refinement options are available, such as increasing the mesh density in regions of large index. Complex curved geometries, however, would require some approximations when using a rectangular mesh.

8.3 Finite Difference EigenMode Solver

The FDE solver is used to solve for the material parameters, α and N . To explain this process, we will briefly cover how the solver works [59]. The FDE solver calculates the mode field profiles, effective index, and loss of the structure that is used. It does this by taking a 2D slice of the structure, creating a mesh within this space, and then solving Maxwell's equations. This computation is performed as an eigenvalue problem. For our purposes, the important result is the effective index n_{eff} of each guiding mode. For a one waveguide structure, this will be the fundamental mode. For a two waveguide structure, this will be the symmetric and antisymmetric supermodes. The additional lossy modes are ignored.

Thus, several two waveguide structures are created, each with a different separation d . The effective indices of all these structures are obtained. We now have a set of points, $\{d, \Delta n_{\text{eff}}(d)\}$, which is then fitted to eq. 7.22. The fit parameters will be the material parameters for this specific material. Thus, there is only a need to repeat this process if the material changes. An example fit for fused silica waveguides is presented in fig. 8.1.

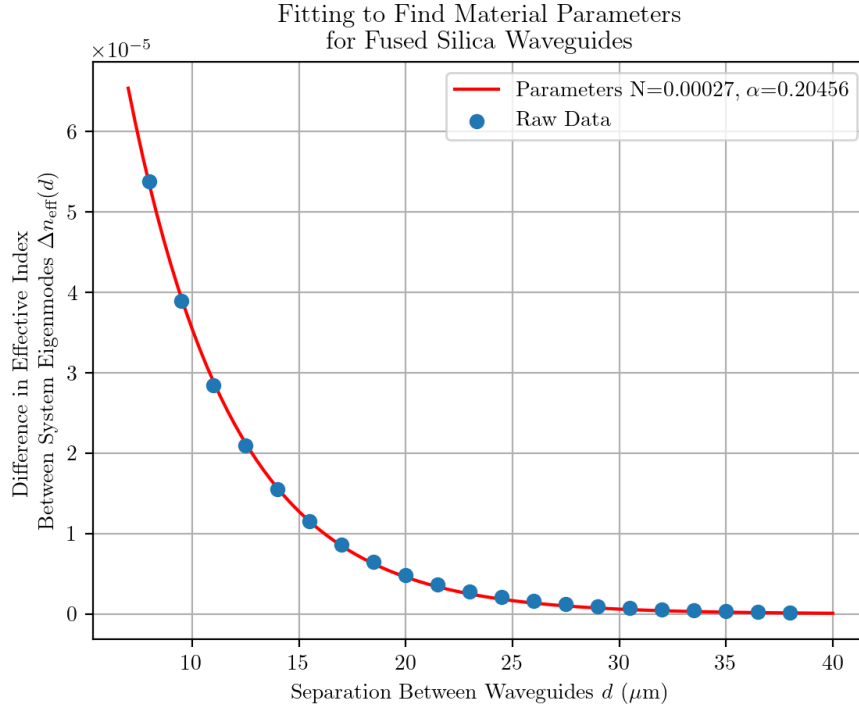


Figure 8.1: Several pairs of straight waveguides of varying separation d are simulated. The difference between the n_{eff} of the symmetric and antisymmetric supermodes is obtained for each pair. These values are then used in eq. 7.22 to fit and obtain the material parameters α and N . An example fit is presented in the figure for the fused silica material system.

Relevant example FDE scripts can be found in an appendix (see sec. C.2).

8.4 Silicon Based Simulations and Results

The silicon based waveguide structures that were simulated have a geometry as illustrated in fig. 7.2. Specifically, they are 500 nm wide (in y) and 220 nm tall (in z). The minimum separation d_0 changed depending on the gate. Overall, the design calls for strip waveguides made in the configuration known as silicon on insulator (SOI) type (the insulator is SiO_2). The rest of the simulation space is in vacuum (i.e., there is vacuum between each strip waveguide). The wavelength is set at 1550 nm, which is in the telecom C-band. This band is important for fibre based communications. Thus, having a quantum device which is compatible with this frequency band is of particular interest.

From the EME simulation, there are two results of interest. These are the *field profile* and the *s-matrix* of the entire device (s standing in for “scattering”). The field profile monitor is placed at the middle height of the waveguides i.e., at $z = 110$ nm. We use this result as the qualitative check on the performance of each device. The scattering matrix is used for quantitative analysis. We can use this matrix to obtain the

percentage of the light that goes from the input port to each of the three output ports. Hence, this is what allows us to compare to theory.

The results that follow are merely a sample of the silicon-based simulations that were performed. The specific design parameters that were used for these results are presented in table 8.3. The corresponding field profile results are presented in fig. 8.2 and the corresponding intensity splitting from the s-matrix are listed in table 8.4. From the s-matrix results, we can conclude that this particular set of gates does not follow theory. Furthermore, as these are just a sample of the simulations performed, we note that these differences were consistently observed with these structures. Various gates of different types were tested, and there was never an acceptable agreement between simulation performance and theory.

An important note regarding table 8.3. As these were some of the initial simulations that were performed, they do not follow the design equations as derived in sec. 7.4. This is particularly clear when noticing that ℓ and a are consistent between the two gates, whereas theory shows that these quantities should differ. We want to emphasize that this is not the source of the errors mentioned above, as the errors remained once these two parameters were updated.

The failure of the designs to adequately perform the desired transformations is a systematic error. Therefore, the solution demands a systematic answer. We do not simply want to obtain working gates (which could be accomplished by simulating many different gate designs until we find a set that works) but be able to explain *why* the initial set of gates does not work. To begin this investigation, the modes of the three-waveguide system are analyzed in more detail.

Parameter	Value
d_0^R	184 nm
d_0^L	184 nm
ℓ	300 μm
a	20.7 m^{-1}

(a) Parameters for NOT Gate

Parameter	Value
d_0^R	150 nm
d_0^L	237 nm
ℓ	300 μm
a	20.7 m^{-1}

(b) Parameters for Hadamard Gate

Table 8.3: Gate Parameters for Silicon Based Gates

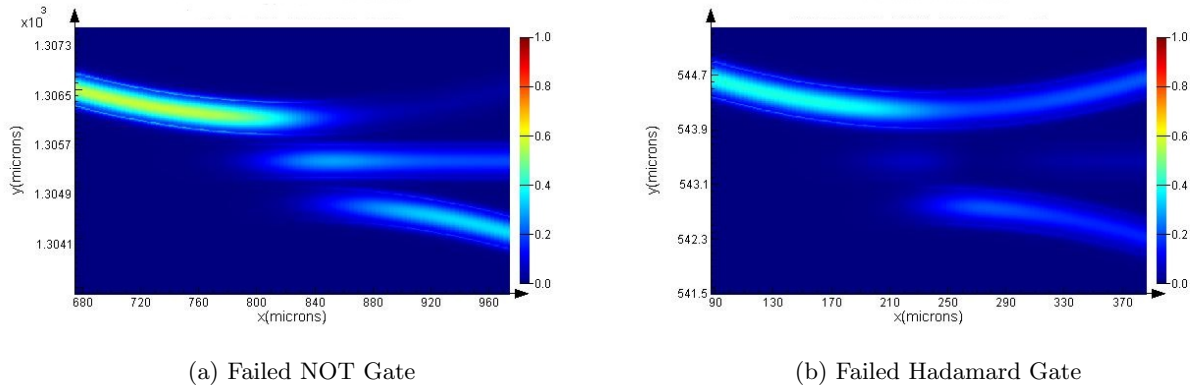


Figure 8.2: Examples of Failed Simulations for Both Gate Designs in Silicon

NOT	$ 0\rangle$	$ e\rangle$	$ 1\rangle$
Ideal	0	0	1
Simulation	0.026	0.60	0.37

(a) NOT Gate

Hadamard	$ 0\rangle$	$ e\rangle$	$ 1\rangle$
Ideal	0.5	0	0.5
Simulation	0.51	0.099	0.38

(b) Hadamard Gate

Table 8.4: Ideal vs. Simulation Intensity Splitting for Both Gates

8.5 Discussion and Proposed Hypothesis

The investigation as to the nature of the failure of the silicon-based designs is focused on a theoretical explanation. As the EME method relies on the *eigenmodes* of the system, we begin by inspecting the eigenmodes of the three-waveguide system and comparing them as given in theory and as given from the commercial software. The modes as obtained from the software are obtained with the FDE solver specifically.

As mentioned in sec. 7.4.1 when discussing the coupling between waveguides, the supermodes of the two-level system are formed by the superposition of the two fundamental modes of the two individual waveguides. This can be extended to the three-waveguide system as a superposition of the *three* fundamental modes. Therefore, we can compare the eigenmodes out of the software against a manual superposition of the three fundamentals. The fundamental mode is obtained using the FDE solver on a single waveguide. The spacing between waveguides is known and can be replicated.

From this manual superposition, we can conclude that the eigenmodes of the three-waveguide system are indeed a superposition of the three fundamental modes. This can be observed in fig. 8.3, where one particular eigenmode is plotted at short and long minimum separation.

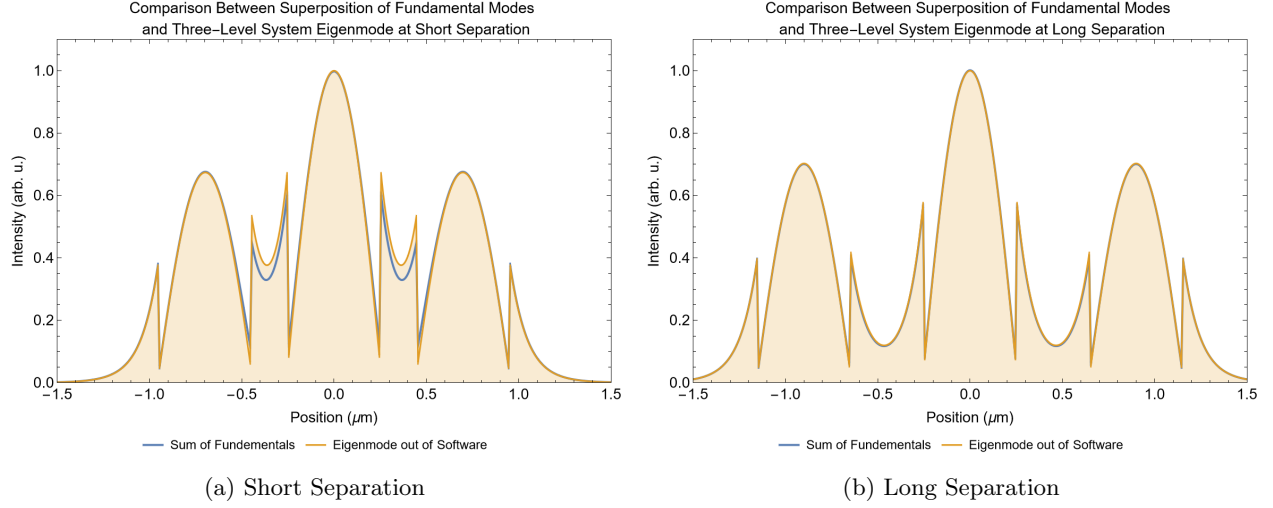


Figure 8.3: Supermode vs. Eigenmode at Short and Long Waveguide Separation

However, as demonstrated in fig. 8.3a, we are able to differentiate between the superposition of fundamentals and the eigenmode. This discrepancy vanishes at longer separation (fig. 8.3b). To compare these two cases of the same eigenmode, we analyze the eigenvalues directly.

For each eigenmode, there is an associated eigenvalue. In the atomic system, this value can be obtained with the Hamiltonian. In the waveguide system, these are the effective indices. Due to the previously discussed analogy between the atomic system and waveguides, the logic behind the atomic system can be applied to the waveguide system. However, we can also derive the same conclusion from coupled mode theory. We shall present both and conclude the same result.

We begin the comparison with the atomic system. Recall the atomic Hamiltonian of eq. 7.3. We rewrite this Hamiltonian in matrix form below:

$$\mathcal{H}(t) = \hbar\tilde{\Omega}(t) \begin{pmatrix} 0 & \omega_0^* & 0 \\ \omega_0 & 0 & \omega_1 \\ 0 & \omega_1^* & 0 \end{pmatrix}, \quad (8.1)$$

where ω_0 is the coupling amplitude between $|0\rangle$ and $|e\rangle$ and ω_1 is the coupling amplitude between $|1\rangle$ and $|e\rangle$. From this matrix, we obtain the eigenvectors and eigenvalues as listed in table 8.5.

Eigenmode Label	Eigenvector	Eigenvalue
0	$\{-\omega_1, 0, \omega_0\}$	0
−	$\{\omega_0^*, -\sqrt{ \omega_0 ^2 + \omega_1 ^2}, \omega_1^*\}$	$-\hbar\tilde{\Omega}(t)$
+	$\{\omega_0^*, +\sqrt{ \omega_0 ^2 + \omega_1 ^2}, \omega_1^*\}$	$\hbar\tilde{\Omega}(t)$

Table 8.5: Atomic Theory Eigenvectors and Eigenvalues

Several things of note in table 8.5. First, the dark state $|d\rangle$ is the state that is labelled 0, with eigenvalue of 0. It is so-called “dark” due to the lack of population in the middle waveguide. Furthermore, the other two eigenvalues are *symmetric* around this central value. Therefore, per the analogy between energies and effective indices, this symmetry should persist with effective indices of the waveguide eigenmodes. However, we have found that this is not the case.

Let n_{eff}^0 , n_{eff}^- , and n_{eff}^+ be the effective indices of the three eigenmodes of the waveguide system, where the labels $\{0, -, +\}$ reflect the desired symmetry of the eigenvalues i.e., mode 0 is the central value, mode $-$ is lower than the central value, and mode $+$ is higher than the central value. Let $\delta^{+0} = n_{\text{eff}}^+ - n_{\text{eff}}^0$ and $\delta^{0-} = n_{\text{eff}}^0 - n_{\text{eff}}^-$. Finally, let $\Delta_n = \delta^{+0} - \delta^{0-}$. We can expand on these definitions, to write the splitting between eigenvalue pairs as:

$$\Delta_n = n_{\text{eff}}^+ + n_{\text{eff}}^- - 2n_{\text{eff}}^0. \quad (8.2)$$

From the eigenvalues of the atomic Hamiltonian, $\Delta_n = 0$. For the simulated waveguide system, $\Delta_n \neq 0$, which shall be shown below. Going from shorter minimum distances to longer distances, the splitting decreases. That is, for $\mathfrak{C} \gg d$ we have: $d \rightarrow \mathfrak{C} \implies \Delta_n \rightarrow 0$.

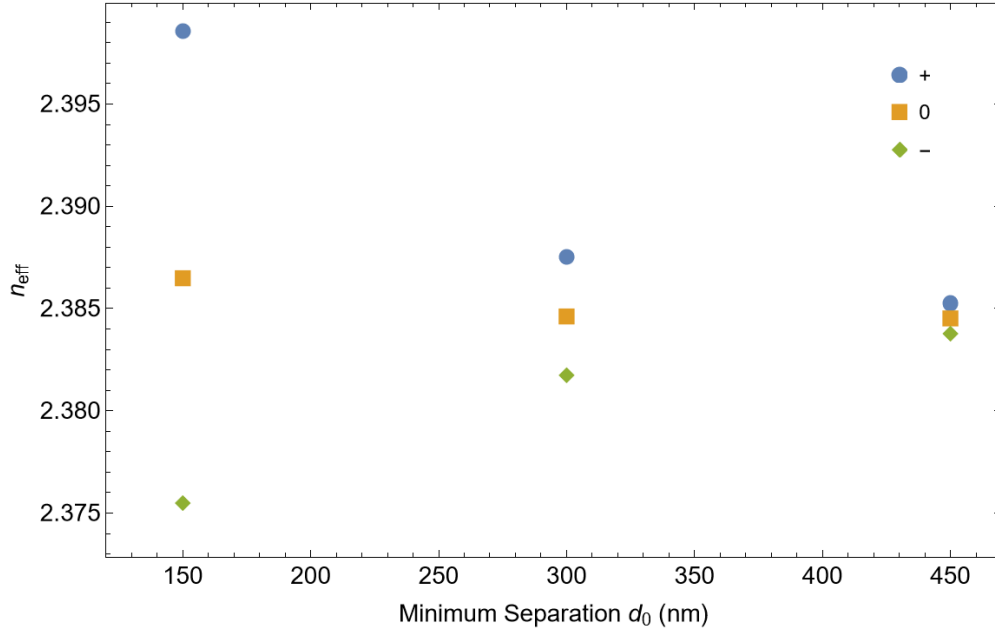


Figure 8.4: We compare the n_{eff} obtained from the simulation for the three eigenmodes of the waveguide system. According to theory, the extreme values should be symmetric with respect to the middle value. We can see this is not the case. Furthermore, this discrepancy gets smaller as the waveguide separation gets larger.

d_0 (nm)	δ^{+0}	δ^{0-}	Δ_n
150	0.0120897	0.0109944	0.00109529
300	0.00291351	0.00287411	0.0000393964
450	0.000745252	0.000744128	1.12429×10^{-6}

Table 8.6: n_{eff} Splitting Numerical Values

The legend of fig. 8.4 follows the previous notation, where mode 0 is the central value of the effective index, mode + is above and mode – is below the central value. To expand on the figure, these results are also listed in table 8.6. The splitting Δ_n does get smaller with increased distance. However, there should not be any differences in this quantity over the distances.

We now conclude this same result using coupled mode theory [53]. This is to demonstrate that the issue does not arise because of the analogy between atomic and waveguide systems, and that it appears directly when considering waveguides. Assuming three identical parallel waveguides that are equally spaced and assuming each individual waveguide supports only one propagating mode, we can write the following propagation constants. Recall that the n_{eff} is merely a rescaling of the propagation constant β (eq. 7.20) and so they are equivalent formulations:

$$\beta_1^2 = \beta_0 + K + \sqrt{2}J \quad (8.3a)$$

$$\beta_2^2 = \beta_0 + K \quad (8.3b)$$

$$\beta_3^2 = \beta_0 + K - \sqrt{2}J, \quad (8.3c)$$

where β_0 is the propagation constant of the confined mode of a single waveguide, β_i^2 are the propagation constants for the $i = \{1, 2, 3\}$ eigenmodes of the three waveguide system, K is the dielectric perturbation of one waveguide due to the other, and J is the exchange coupling between waveguide pairs. These quantities are given by [53]:

$$K = \left(\frac{\omega}{c}\right)^2 \iint \mathcal{E}_a^* \Delta m_b^2 \mathcal{E}_a \, dx \, dy \quad (8.4a)$$

$$J = \left(\frac{\omega}{c}\right)^2 \iint \mathcal{E}_a^* \Delta m_a^2 \mathcal{E}_b \, dx \, dy, \quad (8.4b)$$

where the $\mathcal{E}_j(x, y)$ is the transverse spatial component of the fundamental mode for the j th waveguide, Δm_j^2 is the function which defines the index distribution of the j th waveguide core, and a and b represent one waveguide of a coupling pair of waveguides. Thus, from eqs. 8.3, we can conclude the same result as the simulation data: that the splitting of the eigenvalues should be symmetrical, and that we do not have this

symmetry.

There is an assumption in this derivation that provides a way forward. Specifically, eqs. 8.3 are obtained under the assumption that the waveguide pairs are “not too close to each other” [53, p. 617]. This means that the fundamental mode of one waveguide is unperturbed by the presence of the neighbouring waveguide. Mathematically, this means there is a small overlap between the modes, i.e., $I \ll 1$ where [53, p. 620]:

$$I = \iint \mathcal{E}_b^* \mathcal{E}_a \, dx \, dy, \quad (8.5)$$

where I is known as the overlap integral, $\mathcal{E}_j(x, y)$ is the transverse spatial component of the fundamental mode for the j th waveguide and a and b represent one waveguide of a coupling pair of waveguides.

If the material contrast is small or if the fundamental mode is small at the location of interest, the assumption is valid. However, if the material contrast is high (such as in silicon i.e., ~ 2.5), or if the waveguides are too close together, the assumption breaks down. Hence, we cannot write the supermodes in terms of the unperturbed fundamental modes.

Thus, to further investigate the simulations of the geometric phase logic gate designs, we want to investigate low index contrast materials such that this assumption is valid.

Chapter 9

EigenMode Discrepancy Investigation

9.1 Investigation of Hypothesis

The simulations in silicon have been shown to not follow theory. One possible explanation for this discrepancy has been proposed. Specifically, we proposed that the strong coupling that arises due to the large material index contrast in the silicon-based systems violates the assumption that the supermodes of the three-waveguide system can be written as a superposition of the unperturbed fundamental modes. There are two possible ways to investigate a solution to this hypothesis.

As indicated in sec. 7.5, there is one parameter (d_0^R) which is arbitrary. Thus, this parameter sets the overall scale of the device. Hence, one way to investigate the hypothesis is by increasing this arbitrary value and monitoring the effect of this change on the overall gate performance. The physical implication is that these new devices will be larger overall, which will mean smaller coupling between each waveguide pair. However, this is not ideal.

We need the solver to be able to recognize there is *some* level of coupling between the waveguides. If we reduce the coupling too much, the solver may not be able to resolve the difference numerically. It will therefore interpret no coupling whatsoever. Furthermore, smaller coupling also leads to a longer device. This increases both computational requirements for simulations but also increases eventual fabrication costs.

The second possibility is to change the material system of the design. The index contrast in silicon-based systems is large. Hence, to get some coupling, the waveguides must be a certain distance apart. However, if this distance is too small, as we have shown, one waveguide may feel the perturbation of the other waveguide, which violates the unperturbed superposition assumption. Thus, the aim of the new material system is to reduce the index contrast. This will allow the waveguides to be further apart and still register some amount

of coupling. Hence, the individual waveguides should not be perturbed by the neighbouring waveguide. This will be the investigation method discussed in this Chapter.

9.1.1 Low Index Contrast Material Selection

The main criteria in the selection of the new material is to have a small material index contrast. Therefore, the need for compatibility with existing technologies is suppressed. This will also affect the wavelength selection for the new devices. We will not restrict ourselves to materials that guide in a telecom band.

There are various kinds of low index wave guiding materials available such as various kinds of glasses (which we will discuss) and polymers [65]. When considering what material systems to move to, we should not only consider the low index contrast exclusively but also the fabrication process. With the silicon-based waveguides, the fabrication of the final devices would not be possible in-house. This requires sending the design to a different factory, which will take time and money. Furthermore, the silicon-based structures are not easily iterable due to this long process. It would therefore be advantageous to use a material with both a low index contrast, as well as being “easy” to fabricate. In this context, “easy” means that we have the capacity to create the waveguides in-house.

There is a particular class of waveguide structures which satisfies these two requirements: femtosecond (fs) written waveguides. The precise details of this fabrication technique will be discussed in sec. 10.2. This technique is very general and can apply to various material systems. Specifically, we have selected fused silica glass as the new material system. This particular material was selected due to the relative simplicity of the fabrication process as well as having a very low material index contrast. This will facilitate the comparison between the large index contrast of the silicon-based system.

With the new material system selected, we can proceed to the design process of the new logic gates.

9.2 Simulations of Fused Silica Based Geometric Phase Logic Gates

9.2.1 Theoretical Parameters for Fused Silica Gates

The design of the fused silica glass waveguide structure uses the same design equations and process as derived in sec. 7.4. However, an additional parameter must also be taken into account: the V number [66]. This dimensionless parameter determines the number of modes a step-index fibre can support. If $V \lesssim 2.405$, then the waveguide is single-mode [66]. It is given by:

$$V = \frac{2\pi}{\lambda} r \sqrt{n_{\text{core}}^2 - n_{\text{clad}}^2}, \quad (9.1)$$

where λ is the wavelength the waveguide will support, r is the radius of the core, and n_i are the refractive index of the core and the clad, respectively. This was not considered in the silicon design as the waveguide structure was designed to the standard dimensions for single-mode operation. With the new material system, we are manually determining the size of the waveguides and thus must consider this parameter.

To obtain the index contrast, we take the cladding index to be that of the SiO_2 material in the simulation software. This material has an index of $n_{\text{clad}} = 1.457$. The fabrication technique (detailed in sec. 10.2), allows for a variable index difference [67]. We can therefore choose the index contrast for the waveguides, which can set the coupling regime that we want. We choose an index increase of 0.001, which gives us $n_{\text{core}} = 1.458$. This is in stark contrast to the index contrast of the silicon waveguides (0.001 vs. ~ 2.5), allowing us to properly test the aforementioned hypothesis. Another approximation is made in the geometry of the design. The fabrication technique creates elliptical waveguides, which can be tuned to be circular. In the simulations, however, we will approximate the circular waveguides with rectangular waveguides.

These assumptions are made such that we can compute eq. 9.1. The last parameters are defined the wavelength, which is set to $\lambda = 633 \text{ nm}$, and the core radius, which is set at $r = 2 \mu\text{m}$. From these parameters, we obtain $V = 1.07$, which is well below the cutoff for single mode operation. With the material considerations complete, we can repeat the simulation process described in sec. 8.3 to obtain the material parameters α and N . This fit process also informs the selection of the arbitrary parameter ($d_0^R = 15 \mu\text{m}$). With these values, we can use the design equations to determine the design parameters for the design of the logic gates. These are listed in tables 9.1. As we want to design the same two logic gates (NOT and Hadamard), the values for the logical parameters of table 7.3 remain unchanged.

Parameter	Value
d_0^R	$15 \mu\text{m}$
d_0^L	$15 \mu\text{m}$
ℓ	$102166 \mu\text{m}$
a	0.0062149 m^{-1}

(a) Parameters for NOT Gate

Parameter	Value
d_0^R	$15 \mu\text{m}$
d_0^L	$10.691 \mu\text{m}$
ℓ	$42318.5 \mu\text{m}$
a	0.0362233 m^{-1}

(b) Parameters for Hadamard Gate

Table 9.1: Gate Parameters for Fused Silica Based Gates

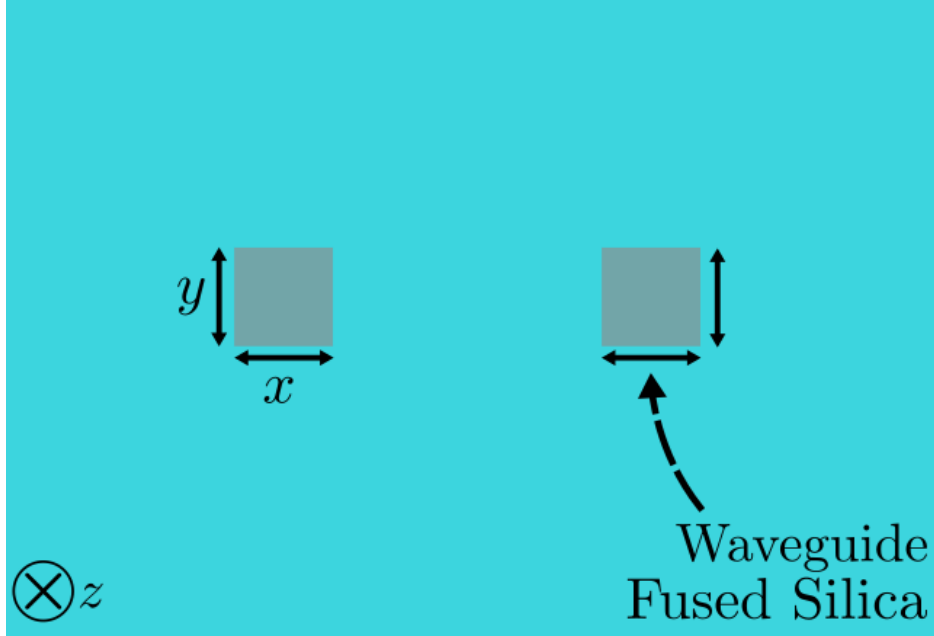


Figure 9.1: Schematic of Fused Silica Waveguide Structure and Dimensions

9.2.2 Fused Silica Simulation Results

In the simulations of these waveguides, there was an additional consideration for the overall structure that was simulated. As originally designed, the simulation injected the input mode directly into the *curved* waveguide. As there was some uncertainty as to whether injecting the mode with some angle with respect to the propagation direction would cause some additional errors, extra structures were added at the beginning and end of the operational length of the device. These “s-bend” structures allow for the injection and detection of the modes in each waveguide at normal incidence. Furthermore, since these waveguides have a low index contrast, they have weak confinement, which means they spread out. Thus, these structures also help prevent any overlap between the modes before the operational length of the device. They allow for the waveguides to be separate from each other by several times the minimum distance. We can therefore be confident that the modes do not couple during this section of the overall structure, only during the operational length of the device, ℓ . Similarly to sec. 8.4, the following simulation results are a representative sample of the total number of simulations performed with this material system.

First, we present two simulations of a NOT gate.

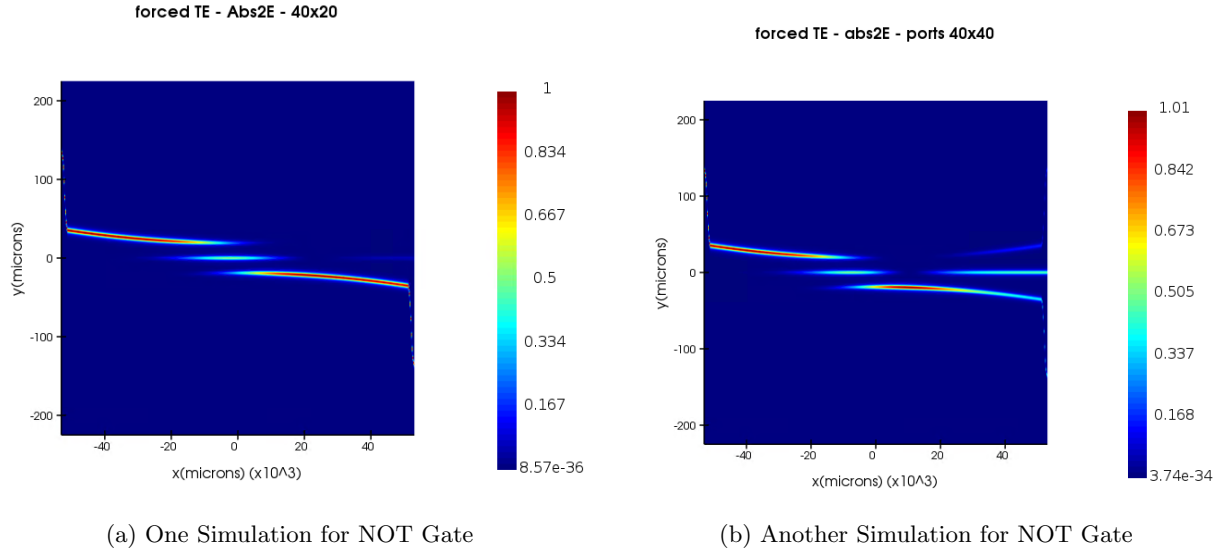


Figure 9.2: Two Simulations of the Same NOT Gate in Fused Silica

NOT	$ 0\rangle$	$ e\rangle$	$ 1\rangle$
Ideal	0	0	100
Fig. 9.2a	0.1	5.5	94.2
Fig. 9.2b	8.5	39.4	51.2

Table 9.2: Numerical Results for Failed Fused Silica NOT Gates

From fig. 9.2 and table 9.2, we can see that the two simulations do not give the same results. The gate listed in fig. 9.2a is the most promising of the two, as the splitting of the input light is closer to the ideal values. However, this splitting is still significantly different from the results obtained from the predictive equations (see table B.1). Furthermore, the lack of consistency between the simulations is concerning, as these are the same physical device being simulated. We will consider this point further shortly.

Next, we present two simulations of a Hadamard gate.

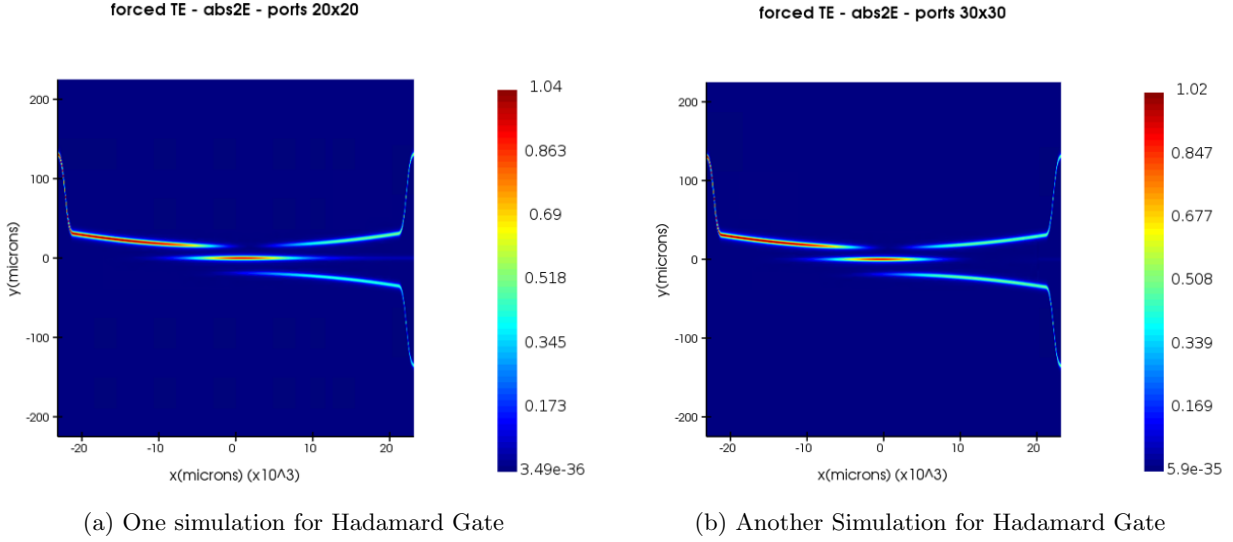


Figure 9.3: Two Simulations of the Same Hadamard Gate in Fused Silica

Hadamard	$ 0\rangle$	$ e\rangle$	$ 1\rangle$
Ideal	50	0	50
Fig. 9.3a	49.8	4.7	41.9
Fig. 9.3b	47.9	1.7	49.4

Table 9.3: Numerical Results for Failed Fused Silica Hadamard Gates

In fig. 9.3, it is difficult to notice a difference between the two simulations. Therefore, we compare them using the splitting listed in table 9.3. Here, we can see that the performance of the two simulations are much closer together than the NOT gate. Furthermore, the gate presented in fig. 9.3b is the closest overall to the ideal transformation. However, like the NOT gate, even this result is significantly different than what is predicted by the predictive equations (see table B.1).

From these results for both gates, we can conclude that the simulations continue to not perform as expected. As these results were obtained numerically through software, we must consider the possibility that numerical errors have a significant enough contribution to negatively impact the results. We investigate this further in the next section.

9.3 Numerical Artifacts as a Significant Source of Error

To further the discussion regarding numerical errors, we present a third set of logic gates, one for the NOT transformation, and one for the Hadamard transformation. The field profiles are presented in 9.4, and their respective error splitting are listed in table 9.4.

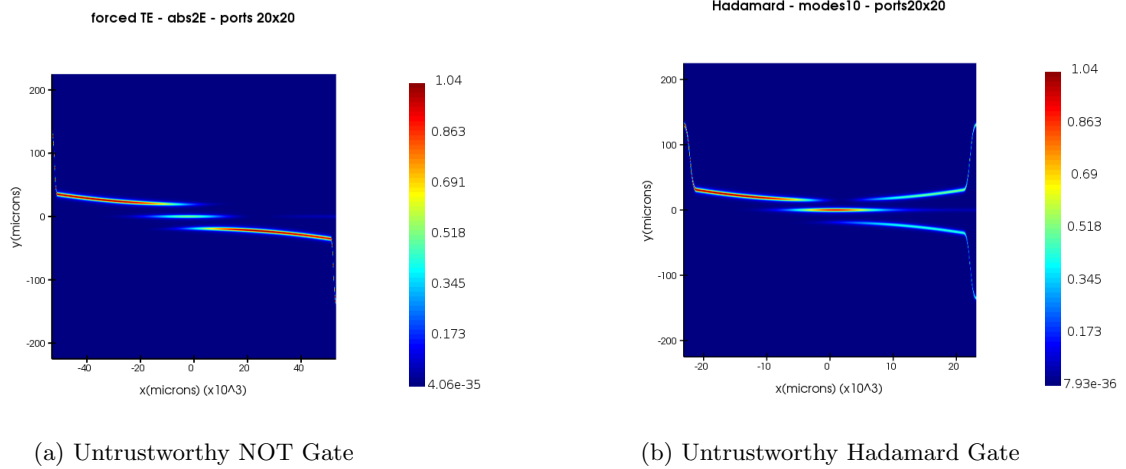


Figure 9.4: Untrustworthy Simulations for Both Gates

NOT	$ 0\rangle$	$ e\rangle$	$ 1\rangle$
Ideal	0	0	100
Fig. 9.4a	0.13	5.4	90.5

(a) Results for Third NOT Gate

Hadamard	$ 0\rangle$	$ e\rangle$	$ 1\rangle$
Ideal	50	0	50
Fig. 9.4b	49.8	0.05	41.4

(b) Results for Third Hadamard Gate

Table 9.4: Numerical Results for Both Untrustworthy Gates

First, from table 9.4, we can conclude that neither of these transformations perform as expected (i.e., do not perform according to ideal theory or from the predictive equations). Second, by comparing all the numerical results of three transformations for each gate (that is comparing tables 9.2 and 9.4a for the NOT transformation and tables 9.3 and 9.4b for the Hadamard transformation) we can also conclude that none of the simulations perform similarly to each other as well.

These conclusions are concerning because all the waveguide structures presented in this Chapter, per transformation, are *identical*. That is, all NOT gates presented in this Chapter are identical and all Hadamard gates are also identical. The difference lies exclusively in the *simulation solver* settings used. The main parameter that changes between these specific simulations is the size of the port objects (which are located in the (y, z) plane in the figures). If the simulation results accurately represented reality, changing these settings would *not* change the physics.

Even with ideal solver settings, one should confirm that the observed results are indeed physical. One such “verification” method is by making the mesh size coarser. This could be the mesh in either the propagation dimension or the transverse dimensions. Since reality does not have numerical artifacts, changing the mesh size in this fashion allows the user to be confident in their results. However, the mesh size of the ports is not necessarily the only source of the observed discrepancy. The difficulty lies in finding the specific parameter(s) which cause this discrepancy, as there is a large parameter space of solver settings to explore.

We have explored the role of the modes that the ports record, the mesh size in the propagation dimension, and the aforementioned port mesh size to name a few possible settings. The simulation results of these investigations continued to demonstrate inconsistency between the results.

There are two possible solutions to solve these issues at this stage: a systematic sweep of the large solver parameter space can be undertaken to confirm which parameter(s) is (are) the culprit(s). In our particular case, this is not ideal due to the length of time each simulation requires. It is computationally intractable to perform proper sweeps of these devices. Or one can remember that the simulation software is merely a *tool*. It is not a guaranteed substitute for an experiment. Even with this project, the simulations were intended to be used to *confirm* the designs, before being fabricated, not being the end results themselves. Therefore, the second solution is to forgo simulations altogether and test these devices directly in an experiment.

Chapter 10

Fabrication and Testing of Waveguide Systems

10.1 Introduction

Simulations are useful tools. They allow the creation and investigation of different structures without having to create them, possibly saving time and money. This is particularly notable in the case the material platform is not suitable for fast and frequent iterations. However, like all tools, they have their limits in the applicability. In the previous two Chapters, we have discovered such a limit. The commercial software was shown to be unable to simulate the kinds of devices we are asking of it. We have tested devices with both high and low index contrast, which translates to two different regimes of applications. Neither regime was shown to be adequate in the design of the geometric phase logic gates we are designing. To overcome this limit, we create and test the devices directly in an experiment in the lab, not in the virtual environment of simulation software.

For this experiment, there are two main parts to consider: the creation of the devices, and the creation of the apparatus which will test them. The former is detailed in sec. [10.2](#), and the latter in sec. [10.3](#).

10.2 Fabrication of Fused Silica Waveguide Systems

The fabrication technique of our waveguide systems is known under different names, such as femtosecond laser writing (fs LW) [\[68\]](#) or femtosecond laser micromachining (FLM) [\[69\]](#). This technique was first reported in 1997 [\[70\]](#) after being proposed the year prior [\[71\]](#). This technique has seen tremendous development since

being introduced [69, 72]. It can be used to create complicated structures and three-dimensional structures [51, 67, 73], and was used as the platform for early demonstrations of boson sampling [49, 69]. It is also a convenient method to use to create waveguides, as it does not require the lithographic equipment that is required for silicon-based structures. The specific pieces of equipment required for the fabrication of the waveguides are present in the Extreme Ultrafast Photonics group at the University of Ottawa. They will be fabricating our waveguide designs based on this technique. Schematically, the technique is succinctly summarized in fig. 10.1.

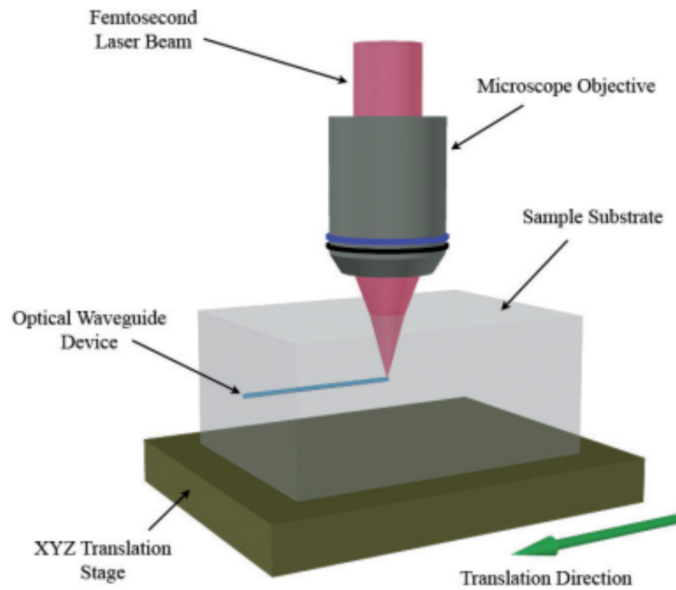


Figure 10.1: fs laser written waveguide technique. Reused with permission from M. Ams *et. al.*, Laser & Photonics Reviews, Wiley. ©2009 by WILEY-VCH Verlag GmbH & Co. KGaA, Weinheim [74].

The main idea of this writing technique is to focus a fs pulse in the material to induce a permanent change in the refractive index at the focal point. By simultaneously moving the sample, this point can be extended into a line. In the particular case of fused silica, the induced change in refractive index is an index increase [67]. Structurally, the increase in index is caused by a reordering of the silica bonds, which were broken by the fs pulse. This process is known as “densification”. This creates a line of higher index surrounded by a lower index. Therefore, this line is now the waveguide¹¹.

There are other types of usages of this fabrication technique, which depends on the material. The method we just described is called type I. In type II, two fs damage lines are used to create the waveguide, as these materials suffer an index decrease under a fs pulse [69]. This occurs in dielectric crystals such as LiNbO₃ [75]. The waveguide will now be *between* the damage lines instead of *being* the damage line.

¹¹Due to total internal reflection.

Important parameters to consider in this technique are the pulse duration, pulse energy, and writing speed. Different materials will call for different sets of parameters [69, 73, 75, 76, 77]. The same material can also support different regimes of parameters, as demonstrated in fused silica by [73] and [78]. As the sample is translated, we want the energy that is deposited at each location to be approximately the same. This allows for the same change in index to occur, which in turn allows for a uniform waveguide. A slower translation can allow for a higher change in index compared to a faster translation speed when keeping the pulse energy and duration constant. Ref. [67] studied the effect of the translation speed on the change in index in fused silica, showing almost an order of magnitude of possible index changes for the speeds that were investigated.

The current translation speeds are listed only in one dimension. However, since we know that the final design of our waveguide system has curved waveguides, we will require simultaneous translation in two dimensions to keep the induced index change constant. This simultaneous movement is a major issue in the fabrication of the waveguide logic gates [45]. Other challenges include possible birefringence in the waveguides due to their shape. A circular waveguide structure would therefore be preferred, as it does not have a preference between circular polarization states. In the case of fused silica waveguides, a cylindrical lens is used to focus the beam in the material to obtain a symmetric waveguide.

10.3 Creation of Experimental Waveguide Testing Apparatus

To characterize the waveguides, there are three main components that need to be considered: the input source, the fibre alignment, and the detection. We have designed the waveguides to guide at $\lambda = 633$ nm. Therefore, the laser source will be a HeNe laser (model *ThorLabs HNLS008L*). We must consider that backreflections are possible due to other optical equipment in the beam path, to be discussed shortly. We do not want to have such reflections re-entering the laser cavity. Therefore, a *ThorLabs IO-3D-633-VLP* free-space isolator is used. Next, we consider the polarization and its control mechanism. We want to know the polarization that we are sending into the waveguide system. To achieve this control, a linear polarizer (*ThorLabs LPVISE100-A 400–700 nm*) and a combination of two quarter-waveplates (QWP) and a half-waveplate (HWP) in QWP-HWP-QWP order are used to set the desired polarization. The laser is then coupled to a single-mode fibre, where it is led to the alignment part of the apparatus.

To protect against dust when the apparatus is not in use, the entire alignment portion is contained within a custom-made plexiglass box. This box has a removable lid which is only removed when needed. The input and output fibres are each mounted on *ThorLabs HFV002* fibre holders which are themselves mounted on *ThorLabs NanoMax 603D* nanopositionner. Between the input and output nanopositionner sits the sample,

mounted such that it and the two fibre holders are at the same height. To help with the alignment of the fibre tip and the sample, a microscope, which is mounted on a boom stand, can be moved into position above the sample. A camera mounted to see through the microscope objective. This camera is then connected to a computer, where we can monitor the alignment process with higher resolution. The mounted camera is a *Mightex BCE-B013-U* camera. It is a monochrome 8-bit CMOS camera with pixel size of $5.2\mu\text{m}$. At the maximum resolution of 1280×1024 (1.3 MPix) pixels, it has a frame rate of 25 fps. Once the laser goes through the sample, it is led to the detection portion. This comprises another single-mode fibre, which is connected to a *ThorLabs PM100D* detector with a *ThorLabs S120C* sensor.

The entire experimental apparatus is schematically represented in fig. 10.2. To obtain data for all three output ports, we will align with each of them individually. Once aligned, the input port will be fixed in place, such that there is no difference in the input for all three outputs.

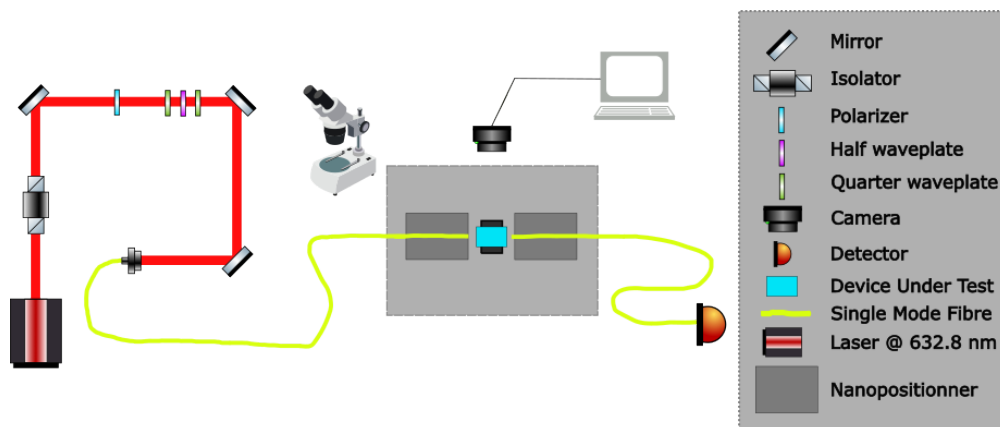


Figure 10.2: Schematic of the testing setup for the waveguide systems¹². The microscope and the overhead camera are used to help with the alignment of the fibres at the input and output ends of the sample. As the HeNe laser propagates through the sample, we will be able to observe fluorescence through the camera. The waveplates are used to compensate the polarization change induced from the fibre, such that we know what polarization is injected into the sample.

10.4 Characterization Procedure

Once the testing apparatus is created and aligned, we can proceed with the experiment. This is accomplished in several steps.

First, we must obtain the material parameters (α and N) for the specific fused silica sample we are using. This is done in the same manner as the simulations. Several pairs of straight waveguides are created at different separations d . We inject the laser light into each waveguide pair, and record the coupling between

¹²Figure created in part with [15].

the waveguides through the above camera. We will be able to observe the intensity oscillating between the two waveguides. This is due to the comparison we have previously discussed in sec. 7.4.1. If we consider the two waveguides a double potential well system, we can conclude that the light will oscillate with a transition time of $\frac{\hbar\pi}{\Delta E}$, where ΔE is the difference in energy between the symmetric and antisymmetric modes of the two well system [52]. This energy difference is analogous to the propagation constant changing in the waveguide system [45], which is a quantity related to the effective index. Thus, we will be able to extract the necessary $\Delta n(d)$ information from this picture. This procedure is repeated for several separations d .

With this data over several waveguide pairs, we fit to eq. 7.22 to obtain the material parameters. Once the fit has been determined, we use it to inform the arbitrary choice of d_0^R , as this fit is an indication of coupling strength. The initial straight waveguides were written with an aspheric lens with a numerical aperture of 0.25 and the laser parameters listed in table 10.1. Recall that the design of the waveguides in sec. 9.2.1 were of a $4\mu\text{m}$ square. This was an approximation done for the simulations. The waveguides that this method will write will be roughly circular in shape, with a target diameter of $\sim 9\mu\text{m}$.

Pulse Energy	Pulse Duration	Wavelength	Writing Speed	Repetition Rate
250 nJ	180 fs	1030 nm	1 mm/s	100 kHz

Table 10.1: Laser Parameters for Straight fs Written Fused Silica Waveguides

Once the material parameters have been obtained and the choice of d_0^R made, we can proceed with the application of the design equations and the creation of the logic gate structures. Performance will be determined by the fidelity of the experimental gates and the idealized gates. Several gate sizes can also be tested, with the maximum size limit being the separation between the fibre holders in the experiment. Robustness can be tested with the creation of waveguide structures where some parameters do not have the ideal value from the design. Recall that the geometric phase relies on the loop in parameter space be area-invariant. The parameter which travels this loop in the waveguide system is the coupling. Therefore, special focus will be on modifying the coupling away from its ideal value. This could be done artificially, by directly changing the value that the design is based on. Or this could be achieved by changing a parameter upon which the coupling depends on. Furthermore, the non-Abelian nature of the logic gates must also be tested. To do so, we can create a single waveguide structure that combines two logic gates which have been determined to function appropriately. The non-commuting nature will be confirmed by obtaining a different result depending on the order in which the beam will travel through the structure.

10.5 Project Summary and Outlook

During the course of this project, we have faced several setbacks. These include theoretical investigations of the designs, dealing with numerical issues in commercial software, and pandemic-related supply chain delays for parts. As of writing, there are some remaining issues with this project.

The supply chain issues have prevented our collaborators who fabricate the waveguide structures from obtaining the right optics to obtain the desired circular shape for the initial straight waveguides. The current compromise is the use of several damage lines to act as a single waveguide. However, this is not ideal due to the increase loss that this design brings. Additionally, the experiment as currently designed uses a continuous wave HeNe laser source. However, for quantum communication, it would be ideal to use a single photon source, as they can be put into superpositions and entangled with relative ease [20]. As our designs create devices which are long, ~ 10 cm, loss considerations will become more important, particularly at the single-photon level.

Furthermore, during the course of this project, we were made aware of a group at the University of Rostock in Germany who are working on a similar project [79, 80]. As of writing, they have shown fs written non-adiabatic non-Abelian logic gates, including the two gates (NOT and Hadamard) we have attempted to design. Their demonstrations use a single-photon source to demonstrate the effectiveness of the fabricated gates. The non-commuting nature was shown with a continuous wave laser. However, the claimed robustness of the geometric phase remains to be experimentally demonstrated.

Chapter 11

Final Remarks

In this thesis, we have discussed two distinct and unrelated projects.

In Part [I](#), we have experimentally demonstrated how a laser beam can cast a shadow, redefining what a shadow can be. We have shown that the temperature effects that occur are not negligible and can have a significant impact on the results. We have compared this laser shadow with the shadow of an opaque object, highlighting the fact that *both* are genuine shadows. To accomplish this effect, we have used a known nonlinear effect in a new way. This opens the possibility for new applications using the laser shadow effect.

In Part [II](#), we have examined the geometric phase, and how this concept can be used in the implementation of quantum computing. We have outlined a full design process for a waveguide system based on the analogous theory for an atomic system. We used this design process to generate waveguide structures in two material systems. These designs were then tested using simulation software as an initial step in our prototyping process. This highlighted issues when the simulations were shown to consistently not follow theory. A physical experiment was designed and is being prepared to confirm the validity of the underlying theory and the ability of the software to accurately simulate a geometric phase-based logic gate. This will prepare the way for future experiments.

Appendix A

Additional Pictures

In this appendix, additional pictures relating to Part I are presented.

A.1 Images with Green Laser Visible



Figure A.1: Image of the Laser Shadow and Background To Give Context



Figure A.2: Example Picture of the Laser Shadow with Green Light Visible

A.2 Images from Data Processing



(a) Example Raw Single Image (15 W)



(b) Example Superimposed Image (15 W)

Figure A.3: Example Image Processing Steps

A.3 Image with Additional White Light

The following image is similar to fig. 4.2, just without the marker in the path of the shadow.



Figure A.4: Laser Shadow on White Paper Illuminated by White Light

Appendix B

Additional Derivations

This appendix elaborates on derivations that do not fit directly in the main text but support certain claims in Part II.

B.1 Identical Bending Coefficients

At first glance, it may not be obvious why the bending coefficients a_i of eq. 7.25 must be equal. One possible confirmation is visual in nature, since this is a consequence of the requiring the minimum distance d_0 be at the same location in z for a parabolic bending profile. Therefore to be precise, we will show this mathematically in the following section.

We begin with the fact that the ratio of the couplings must be independent of “time” or z :

$$\frac{\Omega^L(z)}{\Omega^R(z)} = C, \tag{B.1}$$

where C does not depend on z . Therefore, this ratio is equal at all z . In particular, the ratio at $z = 0$ and $z = \ell$ must be equal:

$$\frac{\Omega^L(z=0)}{\Omega^R(z=0)} = \frac{\Omega^L(z=\ell)}{\Omega^R(z=\ell)}. \tag{B.2}$$

In eqs. 7.28, the result of this derivation is already used. However, we can use these equations without having to apply this result. We therefore get an expression where both a_i and d_0^i are different per side. We will use this definition in what follows:

$$\begin{aligned}
 \frac{\frac{\pi}{\lambda} N e^{-\alpha a_L (0-z_0)^2} e^{-\alpha d_0^L}}{\frac{\pi}{\lambda} N e^{-\alpha a_R (0-z_0)^2} e^{-\alpha d_0^R}} &= \frac{\frac{\pi}{\lambda} N e^{-\alpha a_L (\ell-z_0)^2} e^{-\alpha d_0^L}}{\frac{\pi}{\lambda} N e^{-\alpha a_R (\ell-z_0)^2} e^{-\alpha d_0^R}} \\
 \frac{e^{-\alpha a_L z_0^2}}{e^{-\alpha a_R z_0^2}} &= \frac{e^{-\alpha a_L (\ell-z_0)^2}}{e^{-\alpha a_R (\ell-z_0)^2}} \\
 e^{-\alpha a_L z_0^2 + \alpha a_R z_0^2} &= e^{-\alpha a_L (\ell-z_0)^2 + \alpha a_R (\ell-z_0)^2} \\
 -a_L z_0^2 + a_L (\ell-z_0)^2 + a_R z_0^2 - a_R (\ell-z_0)^2 &= 0 \\
 a_L ((\ell-z_0)^2 - z_0^2) + a_R (z_0^2 - (\ell-z_0)^2) &= 0 \\
 a_L (\ell^2 - 2\ell z_0) &= a_R (\ell^2 - 2\ell z_0) \\
 \frac{a_L (\ell^2 - 2\ell z_0)}{a_R (\ell^2 - 2\ell z_0)} &\stackrel{?}{=} 1.
 \end{aligned} \tag{B.3}$$

In general, the above equation can be simplified to obtain the desired result. However, one must take care with this division, hence the “?”, because our assumption of $z_0 = \frac{\ell}{2}$ further in the derivation in the main text causes a problem:

$$\ell^2 - 2\ell \left(\frac{\ell}{2} \right) = \ell^2 - \ell^2 = 0, \tag{B.4}$$

which means eq. B.3 reduces to:

$$\frac{a_L 0}{a_R 0} = \frac{0}{0} = \text{undefined}. \tag{B.5}$$

Thankfully, L'Hôpital's Rule can solve this kind of indeterminate form. Thus we compute,

$$\begin{aligned}
 \frac{a_L \lim_{z_0 \rightarrow \frac{\ell}{2}} \frac{d}{dz_0} (\ell^2 - 2\ell z_0)}{a_R \lim_{z_0 \rightarrow \frac{\ell}{2}} \frac{d}{dz_0} (\ell^2 - 2\ell z_0)} &= \\
 \frac{a_L \lim_{z_0 \rightarrow \frac{\ell}{2}} -2\ell}{a_R \lim_{z_0 \rightarrow \frac{\ell}{2}} -2\ell} &= \\
 \frac{a_L}{a_R} * 1 &= 1.
 \end{aligned}$$

Thus, we can be confident to simplify the division in eq. B.3 to obtain the desired result of $a_L = a_R$.

B.2 Simplification Using the Error Function

In this section, we elaborate on the steps to go from eq. 7.37 to eq. 7.41. We concentrate on the integral defined in eq. 7.37:

$$\int_{-\frac{\ell}{2}}^{\frac{\ell}{2}} e^{-\alpha a u^2} du. \quad (\text{B.6})$$

With the proper substitution of $t = \sqrt{\alpha a} u$, this becomes:

$$\int_{-\frac{\sqrt{\alpha a \ell}}{2}}^{\frac{\sqrt{\alpha a \ell}}{2}} e^{-t^2} \frac{dt}{\sqrt{\alpha a}} \quad (\text{B.7})$$

$$\frac{1}{\sqrt{\alpha a}} \frac{\sqrt{\pi}}{\sqrt{\pi}} \int_{-\frac{\sqrt{\alpha a \ell}}{2}}^{\frac{\sqrt{\alpha a \ell}}{2}} e^{-t^2} dt \quad (\text{B.8})$$

$$\frac{\sqrt{\pi}}{\sqrt{\alpha a}} \frac{2}{\sqrt{\pi}} \int_0^{\frac{\sqrt{\alpha a \ell}}{2}} e^{-t^2} dt. \quad (\text{B.9})$$

Which can be simplified using the definition of the error function:

$$\sqrt{\frac{\pi}{\alpha a}} \operatorname{erf}\left(\frac{1}{2}\sqrt{\alpha a \ell}\right), \quad (\text{B.10})$$

where this last expression is precisely the relevant factor resulting from the integration in eq. 7.41, without the prefactors.

B.3 Changing Basis

In the derivations of sec. 7.3.1, there were several changes of basis involved. This section will go over them in more detail. Preliminaries are the definitions of the bright and dark states (eqs. 7.5) and the normalization condition (eq. 7.4a). The unitary is initially given in terms of the $\{|b\rangle, |e\rangle, |d\rangle\}$ basis (eq. 7.10). We want to return to the computational basis of $\{|0\rangle, |e\rangle, |1\rangle\}$. From the definitions of the $|b\rangle$ and $|d\rangle$ states, we can define the matrix P :

$$P = \begin{pmatrix} \omega_0^* & 0 & \omega_1^* \\ 0 & 1 & 0 \\ -\omega_1 & 0 & \omega_0 \end{pmatrix}, \quad (\text{B.11})$$

such that

$$\begin{pmatrix} |b\rangle \\ |e\rangle \\ |d\rangle \end{pmatrix} = P \cdot \begin{pmatrix} |0\rangle \\ |e\rangle \\ |1\rangle \end{pmatrix}. \quad (\text{B.12})$$

Therefore, we can change the basis using:

$$U^{0e1} = P^{-1} \cdot U^{\text{bed}} \cdot P, \quad (\text{B.13})$$

where the superscript denotes the basis the unitary matrix is written in, and P^{-1} is the inverse of P . We can compute this matrix multiplication:

$$\begin{aligned} U^{0e1}(\theta, \phi, \beta) &= \begin{pmatrix} \omega_0 & *0 & -\omega_1^* \\ 0 & 1 & 0 \\ \omega_1 & 0 & \omega_0^* \end{pmatrix} \cdot \begin{pmatrix} \cos(\beta) & -i\sin(\beta) & 0 \\ -i\sin(\beta) & \cos(\beta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} \omega_0^* & 0 & \omega_1^* \\ 0 & 1 & 0 \\ -\omega_1 & 0 & \omega_0 \end{pmatrix} \\ &= \begin{pmatrix} |\omega_1|^2 + |\omega_0|^2 \cos(\beta) & -i\omega_0 \sin(\beta) & -\omega_0 \omega_1^* + \omega_0 \omega_1^* \cos(\beta) \\ -i\omega_0^* \sin(\beta) & \cos(\beta) & -i\omega_1^* \sin(\beta) \\ -\omega_1 \omega_0^* + \omega_1 \omega_0^* \cos(\beta) & -i\omega_1 \sin(\beta) & |\omega_0|^2 + |\omega_1|^2 \cos(\beta) \end{pmatrix}, \end{aligned} \quad (\text{B.14})$$

whereby inserting the definitions of ω_0 and ω_1 (eqs. 7.23 and 7.4b) we get:

$$U(\theta, \phi, \beta) = \begin{pmatrix} \cos^2\left(\frac{\theta}{2}\right) + \cos(\beta) \sin^2\left(\frac{\theta}{2}\right) & -ie^{i\phi} \sin(\beta) \sin\left(\frac{\theta}{2}\right) & -e^{i\phi} \sin^2\left(\frac{\beta}{2}\right) \sin(\theta) \\ -ie^{-i\phi} \sin(\beta) \sin\left(\frac{\theta}{2}\right) & \cos(\beta) & -i \cos\left(\frac{\theta}{2}\right) \sin(\beta) \\ -e^{-i\phi} \sin^2\left(\frac{\beta}{2}\right) \sin(\theta) & -i \cos\left(\frac{\theta}{2}\right) \sin(\beta) & \cos(\beta) \cos^2\left(\frac{\theta}{2}\right) + \sin^2\left(\frac{\theta}{2}\right) \end{pmatrix}. \quad (\text{B.15})$$

This result is the general unitary in terms of an uncertain β . In the ideal case, $\beta = \pi$ (eq. 7.2). Thus, inserting $\beta = \pi$ and simplifying gives us:

$$U(\theta, \phi) = \begin{pmatrix} \cos(\theta) & 0 & -e^{i\phi} \sin(\theta) \\ 0 & -1 & 0 \\ -e^{-i\phi} \sin(\theta) & 0 & -\cos(\theta) \end{pmatrix}, \quad (\text{B.16})$$

which is precisely the general unitary of eq. 7.12, as required.

B.4 Predictive Equations for Waveguide Systems

The starting point for the predictive equations is eq. 7.11. All of the simulation inputs are in $|0\rangle$, which in the computational basis is:

$$|0\rangle = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}. \quad (\text{B.17})$$

Therefore, for an input $|0\rangle$, the result will be:

$$\begin{pmatrix} \cos^2(\frac{\theta}{2}) + \cos(\beta) \sin^2(\frac{\theta}{2}) & -ie^{i\phi} \sin(\beta) \sin(\frac{\theta}{2}) & -e^{i\phi} \sin^2(\frac{\beta}{2}) \sin(\theta) \\ -ie^{-i\phi} \sin(\beta) \sin(\frac{\theta}{2}) & \cos(\beta) & -i \cos(\frac{\theta}{2}) \sin(\beta) \\ -e^{-i\phi} \sin^2(\frac{\beta}{2}) \sin(\theta) & -i \cos(\frac{\theta}{2}) \sin(\beta) & \cos(\beta) \cos^2(\frac{\theta}{2}) + \sin^2(\frac{\theta}{2}) \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos^2(\frac{\theta}{2}) + \cos(\beta) \sin^2(\frac{\theta}{2}) \\ -ie^{-i\phi} \sin(\beta) \sin(\frac{\theta}{2}) \\ -e^{-i\phi} \sin^2(\frac{\beta}{2}) \sin(\theta) \end{pmatrix}. \quad (\text{B.18})$$

To get the predicted intensities at each output port, we must take the absolute square of the previous result. Hence,

$$I_{|0\rangle} = \left| \cos(\beta) \sin^2\left(\frac{\theta}{2}\right) + \cos^2\left(\frac{\theta}{2}\right) \right|^2 \quad (\text{B.19a})$$

$$I_{|e\rangle} = \sin^2(\beta) \sin^2\left(\frac{\theta}{2}\right) \quad (\text{B.19b})$$

$$I_{|1\rangle} = \sin^4\left(\frac{\beta}{2}\right) \sin^2(\theta). \quad (\text{B.19c})$$

are the predictive equations. These equations are already normalized. One can use the process in eq. B.18 with $|e\rangle$ and $|1\rangle$ to get predictions for different inputs.

As mentioned in sec. 7.4.5, we can use these equations to show that the simplification of 0.99π is satisfactory. To do so, we use eqs. B.19, with $\beta = 0.99\pi$, and the values of θ and ϕ as given in table 7.3. We get the following results:

NOT	$ 0\rangle$	$ e\rangle$	$ 1\rangle$
$\beta = \pi$	0	0	1
$\beta = 0.99\pi$	$\approx 10^{-7}$	0.00049	0.999507

(a) NOT Gate with Ideal and Approximate β

Hadamard	$ 0\rangle$	$ e\rangle$	$ 1\rangle$
$\beta = \pi$	0.5	0	0.5
$\beta = 0.99\pi$	0.499405	0.00084	0.499753

(b) Hadamard Gate with Ideal and Approximate β

Table B.1: Application of Predictive Equations with $\beta = 0.99\pi$

The difference between the ideal and approximate parameters are small enough to justify the finite length simplification in sec. [7.4.5](#).

Appendix C

Lumerical Scripts

This appendix is a repository of Lumerical [57] scripts created during the thesis for Part II. Too many were created during the course of the thesis to list them all here. Relevant scripts which would allow replication of the results of Chapters 8 and 9 are therefore presented.

C.1 Initial Scripts

Listing C.1: Creation of Straight Waveguides in Si

```
1 #use this script to generate a 1,2,3
   waveguide straight waveguide
   structure
2 #comment out the top and/or bottom
   waveguides to get 1 or 2 waveguides
3
4 clear;
5 switchtolayout;
6 deleteall;
7
8 #variable declaration
9 #waveguide seperations from the middle
10 dleft=500e-9;
11 dright=500e-9;
12
13 #other
14 xoffset=-25e-6;
15 xspanbase=350e-6;
```

```
16 xspanwg=300e-6;
17 yspanwg=500e-9;
18
19 #adds base
20 addrect;
21 set("name","base");
22 set("x",xoffset);
23 set("x span",xspanbase);
24 set("y",0);
25 set("y span",25e-6);
26 set("z",-50e-6);
27 set("z span",100e-6);
28 set("material","SiO2 (Glass) - Palik");
29
30 #sets three straight parallel waveguides
31 #center
32 hcenter=220e-9;
33 addrect;
34 set("name","center guide");
```

```

35 set("x",xoffset);
36 set("x span",xspanwg);
37 set("y",0);
38 set("y span",yspanwg);
39 set("z",hcenter/2);
40 set("z span",hcenter);
41 set("material","Si (Silicon) - Palik");
42
43 #left
44 hleft=220e-9;
45 addrect;
46 set("name","left guide");
47 set("x",xoffset);
48 set("x span",xspanwg);
49 set("y",-dleft-yspanwg);
50 set("y span",yspanwg);

```

```

51 set("z",hleft/2);
52 set("z span",hleft);
53 set("material","Si (Silicon) - Palik");
54
55 #right
56 hright=220e-9;
57 addrect;
58 set("name","right guide");
59 set("x",xoffset);
60 set("x span",xspanwg);
61 set("y",dright+yspanwg);
62 set("y span",yspanwg);
63 set("z",hright/2);
64 set("z span",hright);
65 set("material","Si (Silicon) - Palik");

```

Listing C.2: Creation of Curved Waveguides in Si

```

1  #use this script to generate a curved 3
    waveguide structure
2  #based off of "create_straight_wgs.lsf"
3  #there is a similar baseline of variables
    for the eme scripts
4
5  clear;
6  switchtolayout;
7  deleteall;
8
9  #variable declaration
10 #waveguide seperations from the middle
11 dleft=150e-9;
12 dright=150e-9;
13
14
15 dlen=300e-6;#design length
16 xspanbase=dlen+50e-6;#to have extra room
17 xspanwg=dlen+4e-6;#extra room for the
    ports to pass through
18 xspandisc=-xspanwg/2:(xspanwg/50):xspanwg
    /2;# discretized x

```

```

19 a=20.7;#curvature
20
21 #other
22 center=0e-6;
23 yspanwg=500e-9;
24
25 #adds base
26 addrect;
27 set("name","base");
28 set("x",center-xspanbase/2);
29 set("x span",xspanbase);
30 set("y",0);
31 set("y span",25e-6);
32 set("z",-50e-6);
33 set("z span",100e-6);
34 set("material","SiO2 (Glass) - Palik");
35
36 #adds center waveguide, straight
37 hcenter=220e-9;
38 addrect;
39 set("name","center guide");
40 set("x",center-xspanbase/2);
41 set("x span",xspanwg);
42 set("y",0);

```

```

43 set("y span",yspanwg);
44 set("z",hcenter/2);
45 set("z span",hcenter);
46 set("material","Si (Silicon) - Palik");
47
48 #adds right, curved waveguide
49 hright=220e-9;
50 addpoly;
51 set("name","right guide");
52 set("x", center-xspanbase/2);
53 set("y",center);
54 set("z",hright/2);
55 set("z span", hright);
56 set("material","Si (Silicon) - Palik");
57
58 xR1=xspandisc;
59 yR1=dright+a*((xspandisc-center))^2+
    yspanwg/2;
60
61 xR2=flip(xR1,1);
62 yR2=flip(yR1,1)+yspanwg;
63
64 right_wg_vts=[xR1,yR1;xR2,yR2];

```

```

65 set("vertices",right_wg_vts);
66
67
68 #adds left, curved waveguide
69 hleft=220e-9;
70 addpoly;
71 set("name","left guide");
72 set("x", center-xspanbase/2);
73 set("y",center);
74 set("z",hleft/2);
75 set("z span", hleft);
76 set("material","Si (Silicon) - Palik");
77
78 xL1=xspandisc;
79 yL1=-dleft-a*((xspandisc-center))^2-
    yspanwg/2;
80
81 xL2=flip(xL1,1);
82 yL2=flip(yL1,1)-yspanwg;
83
84 left_wg_vts=[xL1,yL1;xL2,yL2];
85 set("vertices",left_wg_vts);

```

Listing C.3: Creation of EME Solver Structure

```

1 #structure must be created before this
  script is run
2
3 #variable setup
4 heme=0.6e-6;
5 meshsize=10e-9;
6
7 zport=25e-9;
8 yspanport=600e-9;
9 zspanport=550e-9;
10
11 #adds eme solver
12 addeme;
13 set("x min",-xspanbase/2);
14 set("y",0);

```

```

15 set("y span",10e-6);
16 set("z",heme/2-50e-9);
17 set("z span",heme);
18
19 set("number of cell groups",1);
20 set("group spans",[xspanwg]);
21 set("cells",300);
22 set("subcell method",[1]);
23
24 set("modes",3);
25 setnamed("EME","number of modes for all
    cell groups",3);
26
27 setnamed("EME","define y mesh by",1);
28 set("dy",meshsize);
29 setnamed("EME","define z mesh by",1);
30 set("dz",meshsize);

```

```

31
32 #edits port 1 (input)
33 select("EME::Ports::port_1");
34 set("use full simulation span",0);
35 set("y",dright+yspanwg);
36 set("y span",yspanport);
37 set("z",zport);
38 set("z span",zspanport);
39 set("mode selection","fundamental mode");
40 set("port location",1);
41
42 #edits port 2
43
44 select("EME::Ports::port_2");
45 set("use full simulation span",0);
46 set("y",dright+yspanwg);
47 set("y span",yspanport);
48 set("z",zport);
49 set("z span",zspanport);
50 set("mode selection","fundamental mode");
51 set("port location",2);
52
53 #adds and edits port 3
54 addmeport;
55 select("EME::Ports::port_3");
56 set("use full simulation span",0);
57 set("y",0);
58 set("y span",yspanport);
59 set("z",zport);
60 set("z span",zspanport);
61 set("mode selection","fundamental mode");
62 set("port location",2);
63
64 #adds and edits port 4
65 addmeport;
66 select("EME::Ports::port_4");
67 set("use full simulation span",0);
68 set("y",-dleft-yspanwg);
69 set("y span",yspanport);
70 set("z",zport);
71 set("z span",zspanport);
72 set("mode selection","fundamental mode");
73 set("port location",2);
74
75 #adds monitors.
76 addmeprofile;
77 set("monitor type",3);
78 set("x min",-xspanbase/2);
79 set("x max",-xspanwg/2+xspanwg+xoffset);
80 set("y",0);
81 set("y span",10e-6);
82 set("z",hcenter/2);
83
84 addmeindex;
85 set("monitor type",3);
86 set("x min",-xspanbase/2);
87 set("x max",-xspanwg/2+xspanwg+xoffset);
88 set("y",0);
89 set("y span",10e-6);
90 set("z",hcenter/2);
91
92
93 #commented or not depending if the script
    is being tested or used
94 #run;
95 #emepropagate;
96 #visualize(getresult("monitor","field
    profile"));

```

Listing C.4: Creation of FDE Solver Structure

```

1 #structure must be created before this
    script is run
2
3 #variable declaration
4 hfde=0.6e-6;
5 center=0;
6 meshsize=5e-9;
7
8 #sets FDE Solver
9 addfde;

```



```

10 set("solver type",1);
11 set("x",center+xoffset);
12 set("y",0);
13 set("y span",15e-6);
14 set("z",hcenter/2);
15 set("z span",1.0e-6);
16 set("number of trial modes",5);
17 set("wavelength",633e-9);
18
19 set("define y mesh by",1);
20 set("dy",meshsize);
21 set("define z mesh by",1);
22 set("dz",meshsize);
23
24 mesh;
25 run;

26 findmodes;
27
28 #comments decide if 1,2,3 modes are
    visualized
29 m1E=getresult("mode1","E");
30 m1E.setname("mode1");
31
32 ##m2E=getresult("mode2","E");
33 ##m2E.setname("mode2");
34
35 ##m3E=getresult("mode3","E");
36 ##m3E.setname("mode3");
37
38 visualize(m1E);
39 ##add2visualizer(m2E,1);
40 ##add2visualizer(m3E,1);

```

C.2 Sweep Scripts

Listing C.5: Creation of FDE Sweep

```

1 #script to get a proper sweep
2 addsweep;
3 setsweep("sweep","name","d-sweep-te");
4 setsweep("d-sweep-te","type","Ranges");
5 setsweep("d-sweep-te","number of points",
    21);
6
7 #add parameter which is swept
8 dpara= struct;
9 dpara.Name="distance";
10 dpara.Parameter="::model::d";
11 dpara.Type="Length";
12 dpara.Start=8e-6;
13 dpara.Stop=38e-6;
14 dpara.Units="microns";
15
16 addsweepparameter("d-sweep-te",dpara);
17
18 #add results for mode 1 (neff and TE pol
    frac)

19 res_model1_neff = struct;
20 res_model1_neff.Name="mode1 neff";
21 res_model1_neff.Result="::model::FDE::data
    ::mode1::neff";
22
23 addsweepresult("d-sweep-te",
    res_model1_neff);
24
25 res_model1_te = struct;
26 res_model1_te.Name="mode1 TE";
27 res_model1_te.Result="::model::FDE::data::
    mode1::TE polarization fraction";
28
29 addsweepresult("d-sweep-te",res_model1_te)
    ;
30
31
32 #add results for mode 2 (neff and TE pol
    frac)

```

```

33 res_mode2_neff = struct;
34 res_mode2_neff.Name="mode2 neff";
35 res_mode2_neff.Result="::model::FDE::data
    ::mode2::neff";
36
37 addsweepresult("d-sweep-te",
    res_mode2_neff);
38
39 res_mode2_te = struct;
40 res_mode2_te.Name="mode2 TE";
41 res_mode2_te.Result="::model::FDE::data::
    mode2::TE polarization fraction";
42
43 addsweepresult("d-sweep-te",res_mode2_te)
    ;
44
45 #add results for mode 3 (neff and TE pol
    frac)
46 res_mode3_neff = struct;
47 res_mode3_neff.Name="mode3 neff";
48 res_mode3_neff.Result="::model::FDE::data
    ::mode3::neff";
49
50 addsweepresult("d-sweep-te",
    res_mode3_neff);
51
52 res_mode3_te = struct;
53 res_mode3_te.Name="mode3 TE";
54 res_mode3_te.Result="::model::FDE::data::
    mode3::TE polarization fraction";
55
56 addsweepresult("d-sweep-te",res_mode3_te)
    ;
57
58
59 #add results for mode 4 (neff and TE pol
    frac)
60 res_mode4_neff = struct;
61 res_mode4_neff.Name="mode4 neff";
62 res_mode4_neff.Result="::model::FDE::data
    ::mode4::neff";
63
64 addsweepresult("d-sweep-te",
    res_mode4_neff);
65
66 res_mode4_te = struct;
67 res_mode4_te.Name="mode4 TE";
68 res_mode4_te.Result="::model::FDE::data::
    mode4::TE polarization fraction";
69
70 addsweepresult("d-sweep-te",res_mode4_te)
    ;

```

Listing C.6: Running First FDE Simulation Before Running a Sweep

```

1 mesh;
2 run;
3 findmodes;

```

Listing C.7: Exporting Data from an FDE Sweep To Analyze

```

1 #gets data from d_sweep_te and exports it
    for analysis
2 #will take the difference in python to
    have record of the two values
3 #outside of lumerical

```

```

4
5
6 #will need to have the proper d setup
    here
7 neff1=getsweepresult("d-sweep-te","mode1
    neff");
8 te_polfrac1=getsweepresult("d-sweep-te","

```

```

    model TE");
9
10 neff2=getsweepresult("d-sweep-te","mode2
    neff");
11 te_polfrac2=getsweepresult("d-sweep-te","
    mode2 TE");
12
13 neff3=getsweepresult("d-sweep-te","mode3
    neff");
14 te_polfrac3=getsweepresult("d-sweep-te","
    mode3 TE");
15
16 neff4=getsweepresult("d-sweep-te","mode4
    neff");
17 te_polfrac4=getsweepresult("d-sweep-te","
    mode4 TE");
18
19 fname="results\sweeps\glass-d-sweep-neff.
    txt";
20
21
22 distance_list = neff1.distance;
23 format long;
24
25 write(fname,"neff values","overwrite");
26 write(fname,"minimum distance, mode 1,
    mode 2, mode 3, mode4");

```

```

27
28 for (i=1:length(distance_list) ){
29     str = num2str(distance_list(i))+", "+
        num2str(neff1.neff(i))+", "+num2str(
        neff2.neff(i))+", "+num2str(neff3.
        neff(i))+", "+num2str(neff4.neff(i));
30     write(fname,str);
31 }
32
33 write(fname,"");
34 write(fname,"TE polarization");
35 write(fname,"minimum distance, mode 1,
    mode 2, mode 3, mode4");
36
37 for (i=1:length(distance_list) ){
38     str2 = num2str(distance_list(i))+", "
        +num2str(te_polfrac1.%TE polarization
        fraction%(i))+", "+num2str(
        te_polfrac2.%TE polarization fraction
        %(i))+", "+num2str(te_polfrac3.%TE
        polarization fraction%(i))+", "+
        num2str(te_polfrac4.%TE polarization
        fraction%(i));
39     write(fname,str2);
40
41 }

```

C.3 Parameters and Script for Model Structure

Name	Type	Value
dLeft	Length	15 μm
dRight	Length	15 μm
wg_length	Length	102 166 μm
curvature	Number	0.0062149
z_span	Length	60 μm
y_span	Length	300 μm
wg_side	Length	4 μm
mesh_y	Length	0.25 μm
mesh_z	Length	0.25 μm
background_material	Material	SiO2 (Glass) - Palik
wavelength	Length	0.633 μm
wg_index	Number	1.458
disc_mesh	Number	5000
eme_cells_wg	Number	50
eme_cells_sb	Number	50
port_yspan	Length	40 μm
port_zspan	Length	20 μm
offset	Length	20 μm
num_modes	Number	10
port_modes	String	fundamental TE Mode
sb_x_span	Length	2000 μm
sb_y_span	Length	100 μm
t_mesh	Number	2000
monitor_resolution	Number	500

Table C.1: User properties in Lumerical Model. Certain values were changed depending on the desired simulation. Used in conjunction with C.8.

Listing C.8: Creation of Fused Silica and Solver Structures

```

1 deleteall;
2
3 #discretizing the propagation direction
  for the parabolic waveguides
4 #disc_mesh = discretization mesh
5 xspandisc = -wg_length/2:(wg_length/
  disc_mesh):wg_length/2;
6
7 #computing the vertices of the right
  waveguide
8 xR1=xspandisc;
9 yR1=-dRight-curvature*((xR1))^2-wg_side
  /2;

```

```

10
11 xR2=flip(xR1,1);
12 yR2=flip(yR1,1)-wg_side;
13 VR = [xR1,yR1;xR2,yR2];
14 VR = [VR;
15       VR(1,1),VR(1,2)];
16
17 #computing the vertices of the left
  waveguide
18 xL1=xspandisc;
19 yL1=dLeft+curvature*((xL1))^2+wg_side/2;
20
21 xL2=flip(xL1,1);
22 yL2=flip(yL1,1)+wg_side;
23 VL = [xL1,yL1;xL2,yL2];

```

```

24 VL = [VL;
25     VL(1,1),VL(1,2)];
26
27 #computing the vertices of the middle
    waveguide
28 VC = [-wg_length/2,wg_side/2;
29     wg_length/2,wg_side/2;
30     wg_length/2,-wg_side/2;
31     -wg_length/2,-wg_side/2;
32     -wg_length/2,wg_side/2];
33
34
35 #####
36 #additions for s-bend extension
37 #sb is a standin for s-bend
38 t=0:1/t_mesh:1;
39
40 function get_bx(pol,t){
41     bx=(1-t)^3*pol(1,1)+3*(1-t)^2*t*pol
        (2,1)+3*(1-t)*t^2*pol(3,1)+t^3*pol
        (4,1);
42     return bx;
43 }
44 function get_by(pol,t){
45     by=(1-t)^3*pol(1,2)+3*(1-t)^2*t*pol
        (2,2)+3*(1-t)*t^2*pol(3,2)+t^3*pol
        (4,2);
46     return by;
47 }
48
49 pole_TR = [wg_length/2,VL(disc_mesh+1,2)+
        wg_side/2;
50     wg_length/2+sb_x_span/2,VL(
        disc_mesh+1,2)+wg_side/2;
51     wg_length/2+sb_x_span/2,VL(
        disc_mesh+1,2)+wg_side/2+sb_y_span;
52     wg_length/2+sb_x_span,VL(
        disc_mesh+1,2)+wg_side/2+sb_y_span];
53
54 #adds the top right s-bend
55 addpoly;

56 set("name","TR-sb");
57 set("index",wg_index);
58 set("z span",wg_side);
59 set("z",wg_side/2);
60 set("override mesh order from material
        database",1);
61 set("mesh order",1);
62
63 TR_bx = get_bx(pole_TR,t);
64 TR_by = get_by(pole_TR,t);
65
66 TR_fbx = flip(TR_bx,1);
67 TR_fby = flip(TR_by,1)+wg_side/2;
68
69 TR_b = [TR_bx,TR_by-wg_side/2;
70     TR_fbx,TR_fby];
71 TR_b = [TR_b;
72     TR_b(1,1),TR_b(1,2)];
73
74 set("vertices",TR_b);
75
76
77 pole_TL = [-wg_length/2,VL(1,2)+wg_side
        /2;
78     -wg_length/2-sb_x_span/2,VL
        (1,2)+wg_side/2;
79     -wg_length/2-sb_x_span/2,VL
        (1,2)+wg_side/2+sb_y_span;
80     -wg_length/2-sb_x_span,VL
        (1,2)+wg_side/2+sb_y_span];
81
82 #adds the top left s-bend
83 addpoly;
84 set("name","TL-sb");
85 set("index",wg_index);
86 set("z span",wg_side);
87 set("z",wg_side/2);
88 set("override mesh order from material
        database",1);
89 set("mesh order",1);
90 TL_bx = get_bx(pole_TL,t);

```

```

91 TL_by = get_by(pole_TL,t);
92
93 TL_fbx = flip(TL_bx,1);
94 TL_fby = flip(TL_by,1)+wg_side/2;
95
96 TL_b = [TL_bx,TL_by-wg_side/2;
97         TL_fbx,TL_fby];
98 TL_b = [TL_b;
99         TL_b(1,1),TL_b(1,2)];
100
101 set("vertices",TL_b);
102
103 pole_BL = [-wg_length/2,VR(1,2)-wg_side
104            /2;
105            -wg_length/2-sb_x_span/2,VR
106            (1,2)-wg_side/2;
107            -wg_length/2-sb_x_span/2,VR
108            (1,2)-wg_side/2-sb_y_span;
109            -wg_length/2-sb_x_span,VR
110            (1,2)-wg_side/2-sb_y_span];
111
112 #adds the bottom left s-bend
113 addpoly;
114 set("name","BL-sb");
115 set("index",wg_index);
116 set("z",wg_side/2);
117 set("z span",wg_side);
118 set("override mesh order from material
119      database",1);
120 set("mesh order",1);
121
122 BL_bx = get_bx(pole_BL,t);
123 BL_by = get_by(pole_BL,t);
124
125 BL_fbx = flip(BL_bx,1);
126 BL_fby = flip(BL_by,1)+wg_side/2;
127
128 BL_b = [BL_bx,BL_by-wg_side/2;
129         BL_fbx,BL_fby];
130 BL_b = [BL_b;
131         BL_b(1,1),BL_b(1,2)];
132
133 BL_b = [BL_bx,BL_by-wg_side/2;
134         BL_fbx,BL_fby];
135 BL_b = [BL_b;
136         BL_b(1,1),BL_b(1,2)];
137
138 set("vertices",BL_b);
139
140 pole_BR = [wg_length/2,-wg_side/2+VR(
141            disc_mesh+1,2);
142            wg_length/2+sb_x_span/2,-
143            wg_side/2+VR(disc_mesh+1,2);
144            wg_length/2+sb_x_span/2,-
145            wg_side/2+VR(disc_mesh+1,2)-sb_y_span
146            ];
147
148 #adds the bottom right s-bend
149 addpoly;
150 set("name","BR-sb");
151 set("index",wg_index);
152 set("z",wg_side/2);
153 set("z span",wg_side);
154 set("override mesh order from material
155      database",1);
156 set("mesh order",1);
157
158 BR_bx = get_bx(pole_BR,t);
159 BR_by = get_by(pole_BR,t);
160
161 BR_fbx = flip(BR_bx,1);
162 BR_fby = flip(BR_by,1)+wg_side/2;
163
164 BR_b = [BR_bx,BR_by-wg_side/2;
165         BR_fbx,BR_fby];
166 BR_b = [BR_b;
167         BR_b(1,1),BR_b(1,2)];
168
169 set("vertices",BR_b);
170
171 #adds the middle left straight part in
172 the s-bend region
173 addpoly;
174 set("name","ML-sb");

```

```

160 set("index",wg_index);
161 set("z",wg_side/2);
162 set("z span",wg_side);
163 set("override mesh order from material
    database",1);
164 set("mesh order",1);
165 ML_vtx = [-wg_length/2,wg_side/2;
166           -wg_length/2,-wg_side/2;
167           -wg_length/2-sb_x_span,-
    wg_side/2;
168           -wg_length/2-sb_x_span,
    wg_side/2;
169           -wg_length/2,wg_side/2];
170
171 set("vertices",ML_vtx);
172
173 #adds the middle right straight part in
    the s-bend region
174 addpoly;
175 set("name","MR-sb");
176 set("index",wg_index);
177 set("z",wg_side/2);
178 set("z span",wg_side);
179 set("override mesh order from material
    database",1);
180 set("mesh order",1);
181 MR_vtx = [wg_length/2,wg_side/2;
182           wg_length/2,-wg_side/2;
183           wg_length/2+sb_x_span,-
    wg_side/2;
184           wg_length/2+sb_x_span,
    wg_side/2;
185           wg_length/2,wg_side/2];
186
187 set("vertices",MR_vtx);
188
189
190 #####
191
192 VB_full = [-wg_length/2-sb_x_span,y_span
    /2+sb_y_span;

```

```

193           wg_length/2+sb_x_span,y_span
    /2+sb_y_span;
194           wg_length/2+sb_x_span,-y_span
    /2-sb_y_span;
195           -wg_length/2-sb_x_span,-
    y_span/2-sb_y_span;
196           -wg_length/2-sb_x_span,y_span
    /2+sb_y_span];
197
198 addpoly;
199 set("name","full background");
200 set("material",background_material);
201 set("z",0);
202 set("z span",z_span);
203 set("vertices",VB_full);
204
205 addpoly;
206 set("name","center guide");
207 set("index",wg_index);
208 set("z",wg_side/2);
209 set("z span",wg_side);
210 set("override mesh order from material
    database",1);
211 set("mesh order",1);
212 set("vertices",VC);
213
214 addpoly;
215 set("name","right guide");
216 set("x",0);
217 set("y",0);
218 set("z",wg_side/2);
219 set("z span",wg_side);
220 set("index",wg_index);
221 set("override mesh order from material
    database",1);
222 set("mesh order",1);
223 set("vertices",VR);
224
225 addpoly;
226 set("name","left guide");
227 set("x",0);

```

```

228 set("y",0);
229 set("z",wg_side/2);
230 set("z span",wg_side);
231 set("index",wg_index);
232 set("override mesh order from material
    database",1);
233 set("mesh order",1);
234 set("vertices",VL);
235
236 ##### end of structure
    creation #####
237 ##### creation of solver
    structure #####
238
239
240 #solver_yspan = 2*y_span-50e-6;#y_span is
    for the hole, not the whole
    structure
241 solver_yspan=2*y_span-150e-6;
242
243 #general solver setup
244 addeme;
245 set("x min",-wg_length/2+offset-sb_x_span
    );
246 set("y span", solver_yspan);
247 set("z", wg_side/2);
248 set("z span", port_zspan);
249 set("wavelength",wavelength);
250
251 set("number of cell groups",3);
252
253 set("group spans",[sb_x_span;wg_length-2*
    offset;sb_x_span]);
254 set("cells",[eme_cells_sb;eme_cells_wg;
    eme_cells_sb]);
255 set("subcell method",[1;1;1]);
256 set("MAX STORED MODES",num_modes);
257 setnamed("EME","number of modes for all
    cell groups", num_modes);
258 setnamed("EME","display cells",1);
259

```

```

260 setnamed("EME","define y mesh by",1);
261 set("dy",mesh_y);
262 setnamed("EME","define z mesh by",1);
263 set("dz",mesh_z);
264
265 #port1 setup (input)
266 select("EME::Ports::port_1");
267 set("use full simulation span",0);
268 set("y",VL(1,2)+wg_side/2+sb_y_span);
269 set("y span", port_yspan);
270
271 set("z span", port_zspan);
272 set("mode selection",port_modes);
273 #set("mode selection","fundamental mode")
    ;
274
275 #port 2
276 select("EME::Ports::port_2");
277 set("use full simulation span",0);
278 set("y",VL(disc_mesh+1,2)+wg_side/2+
    sb_y_span);
279 set("y span", port_yspan);
280 #set("z",);
281 set("z span", port_zspan);
282 set("mode selection",port_modes);
283 #set("mode selection", "fundamental mode
    ");
284
285 #port 3
286 addemepoint;
287 select("EME::Ports::port_3");
288 set("use full simulation span",0);
289
290 set("y span", port_yspan);
291
292 set("z span", port_zspan);
293 set("mode selection",port_modes);
294 #set("mode selection", "fundamental mode
    ");
295 set("port location",2);
296

```



```
297 #port 4
298 addmeport;
299 select("EME::Ports::port_4");
300 set("use full simulation span",0);
301 set("y",-wg_side/2+VR(disc_mesh+1,2)-
      sb_y_span);
302 set("y span", port_yspan);
303
304 set("z span", port_zspan);
305 set("mode selection",port_modes);
306 #set("mode selection", "fundamental mode
      ");
307 set("port location",2);
308
309 #adds monitors
310 addmeprofile;
311 set("monitor type", 3);
```

```
312 set("x",0);
313 set("x span",wg_length-2*offset+2*
      sb_x_span);
314 set("y span",solver_yspan);
315 set("z",wg_side/2);
316 set("name","profile_mon");
317 set("x resolution",monitor_resolution);
318
319 addmeindex;
320 set("monitor type", 3);
321 set("x",0);
322 set("x span",wg_length-2*offset+2*
      sb_x_span);
323 set("y span",solver_yspan);
324 set("z",wg_side/2);
325 set("name","index_mon");
326 set("x resolution",monitor_resolution);
```

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