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Peirong Jia

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Abstract

Three-dimensional (3-D) surface-geometry measurement using structured light is commonly used for digital modeling of an object or environment when the geometry is unknown. Digital fringe projection with phase-shifting has become an increasingly common measurement method as it is a full-field method to improve acquisition speed over point-by-point and line-by-line methods, and it encodes all lines in the pattern to reduce the ambiguity problem that occurs with multiple-line projection methods. However, fringe-projection phase-shifting techniques are still limited in the speed of measurement, partly due to the use of projected light patterns that require complex computation, or that require several steps of repeated measurements with several images in order to reconstruct the 3-D object surface.

This doctoral dissertation addresses the development of new methods for real-time full-field range sensing by fringe-projection phase-shifting techniques, to use more efficient algorithms. It also addresses the design, calibration, and measurement-accuracy testing of a new full-field real-time range-sensing system based on the developed methods. A novel 3-D shape measurement technique, called triangular-pattern phase-shifting profilometry, is developed in this research. The proposed method utilizes triangular gray-level-coded fringe patterns and newly developed intensity-ratio generation algorithms, essential to reconstruct a 3-D object surface. The minimum number of measurements required to reconstruct an unknown 3-D object is two, which is less than the minimum number of measurements required for the traditional sinusoidal-pattern and trapezoidal-pattern phase-shifting methods. Compared with the sinusoidal-pattern and trapezoidal-pattern phase-shifting methods, the new method involves less processing because of the simple intensity-ratio computation used and because fewer images or measurement steps are required to reconstruct the 3-D object. Extension of the method by increasing the number of phase-shifting steps was found to increase measurement accuracy. An optimal value of pitch of the projected triangular fringe pattern was found for each extended method. Two error compensation methods, repeated phase-offset and intensity-ratio error compensation, are proposed to reduce the measurement error, mainly caused by fringe projection non-linearity and image defocus. Error reductions of 24.0% and 28.5% were achieved, respectively. An off-line 3-D shape measurement system and a real-time 3-D shape measurement system have been developed with the proposed triangular-pattern phase-shifting method.

Dedicated to my son: Eric

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1 Introduction

In recent decades, there has been an increasing interest among computer-vision researchers in three-dimensional (3-D) surface-shape measurement. The aim of three-dimensional surface-geometry measurement is to build a model of an object or environment by acquiring 3-D geometric point data. This is achieved either by scanning across the object surface or by full-field approaches where the 3-D surface data is obtained with the sensor at a single viewpoint. The latter approaches are of much interest in computer vision as they have great potential for real-time 3-D shape measurement.

1.1 Applications of 3-D Shape Measurement

Three-dimensional shape measurement techniques are widely used in diverse areas such as computer vision, robotics, product inspection, reverse engineering, archaeology, and biomedical applications.

- Computer vision: 3-D object detection, recognition, tracking and scene understanding are the major domains in computer vision. Constructing the 3-D model of the object or scene is often a necessary step for the process.
- Robotics: The location of obstacles in the environment is represented by a geometric model. 3-D scanners have been used in the mobile-robot navigation system to build geometric models of unknown areas.
- Product inspection: Automated part inspection in manufacturing industries involves measurement and analysis of various 3-D features on an actual part and comparison with a computer-aided design model to determine if a feature is within the tolerances and specifications or not. The differences can serve as a guide for modifying the manufacturing process until the part is acceptable.
- Reverse engineering: Reproduction of existing mechanical parts for which there is no computer model is often necessary in modern engineering. The reverse engineering process involves measuring an object and then reconstructing it as a 3-D model. Parts

made by hand-made moulds can be digitized by measuring them with 3-D measuring techniques and re-manufacturing them either by machining or by Rapid Prototyping (RP) technology.

- Archaeology: Some human archaeological treasures, such as Emperor Qin's Terra-cotta in China, that are not suitable to exposure to the elements, can be digitized to create a 3-D model. Three-dimensional models of an archaeological site permit visitors to explore the archaeological site virtually.
- Medicine: Computerized Tomography (CT) scanners are commonly utilized in medicine to visualize internal organs, perform diagnosis, and plan surgery. Shape digitization in medicine is wide ranging from performing cranial reconstruction to manufacture of sockets for artificial limbs.

1.2 Traditional versus Modern Techniques in 3-D Shape Measurement

Traditionally, 3-D shape measurement has been carried out using a coordinate measurement machine (CMM), which usually uses a touch trigger probe to acquire dimensional data by moving a sensing probe along the object surfaces. The drawbacks of this technique are: (1) the measurement speed is slow due to the mechanical movement; (2) the mechanical contact on the measured surface during measurement could cause wear of the probe or damage to the measured surface (Shiou 2003); objects with fragile surfaces are not suitable for CMM measurement; (3) the accuracy could be affected by wear of the probe; and (4) measurement density is low. Range scanning techniques are also employed in 3-D shape measurement. However, they are usually based on point-by-point (Parthasarathy 1982 and Rioux 1984) or line-by-line (Popplestone 1975 and Porter 1982) scanning, which is slow and hard to use to carry out real-time full-field 3-D shape measurement.

Recently, full-field optical 3-D shape measurement techniques have been developed and gained great interest in the field of computer vision (Hung 2000, Quan 2001, Huang 2003a and Miyasaka 2002). Surface-geometry information is acquired over a region of a surface rather than just a point or line. Compared with other techniques, it has the beneficial factor of fast measurement speed due to the fact that it does not require scanning to cover the

whole object surface. Moiré interferometry (Takasahi 1970) and fringe projection methods (Creath 1988, Halioua 1989, and Greivenkamp 1992) are good representatives of this technique. The methods allow relatively simple image-processing algorithms to extract the 3-D coordinate information, high-speed image grabbing, reliable quantitative surface measurement, non-contact and noninvasive characteristics, as well as potential for real-time 3-D shape measurement. White light is generally used in full-field systems. This is advantageous in not causing speckle problem and being eye safe.

Various optical methods for 3-D shape measurement have been proposed and developed, with some reaching commercialization (Chen 2000a, Huntley 2000 and Blais 2003). However, there is still a need for sufficiently fast and accurate range-sensing systems that can provide real-time 3-D shape measurement. Applications include model-based localization, 3-D object recognition, real-time pose tracking, mobile-robot navigation, simultaneous localization and mapping, and virtual reality environment construction. A full review of optical 3-D shape measurement techniques including 3-D data acquisition, 3-D shape reconstruction and display is given in Chapter 2.

1.3 Research Objectives

The main objective of this research is to develop new methods for real-time full-field range sensing and an implementation mechanism for three-dimensional (3-D) surface-geometry measurement to use more efficient algorithms than previously used. The research challenge is to improve on previous full-field phase-shifting methods that use programmable projectors to enable real-time measurement, by:

- determining the most appropriate projection pattern that is suitable for real-time 3-D shape measurement,
- developing calibration and 3-D reconstruction algorithms that minimize computation in the 3-D reconstruction in order to minimize processing time,
- developing methods of synchronizing the pattern projection and camera-image acquisition, and maximizing the projection and acquisition rates, to maximize the speed of measurement,

- developing methods to select the best system parameters to obtain the best trade-off in resolution and accuracy with high-speed 3-D surface geometry acquisition,
- developing real-time graphic rendering of acquired 3-D surface geometry, and
- demonstrating the application of the system in practical surface-geometry applications.

Toward meeting the scientific research objectives, the proposed research includes the design, construction, calibration, and measurement accuracy testing of the new methods for real-time range sensing to perform 3-D surface-geometry measurement, as well as the development of processing algorithms to perform calibration, 3-D object reconstruction, rendering and display. The new full-field range-sensing techniques are developed with the aim of maximizing 3-D surface geometry measurement speed toward performing real-time 3-D shape measurement, while achieving the best measurement resolution and accuracy.

The ideal outcome would be a measurement system that is capable of acquiring and rendering full-field surface geometry at a fast speed or in real-time. The measurement system could be applied to fast or real-time 3-D surface-geometry modeling with specific applications such as model-based localization, 3-D object recognition, real-time pose tracking, simultaneous localization and mapping, mobile-robot navigation, 3-D mapping of environment surfaces for virtual-reality environments, real-time inspection, range-image-guided robot-arm guidance, and real-time measurement of deforming surfaces such as in body-tissue 3-D surface-modeling during surgery. The measurement system could also be applied to stationary objects, such as for reverse engineering and environment modeling. It is intended that the methods of this research be eventually applicable to a working volume small enough to measure a coin or large enough to measure the shape of an automobile body. However, the applicability to such working volumes was not investigated.

1.4 Overview of Approaches to Real-Time Full-Field Range Sensing and Proposed Approach

In this section, an overview of approaches to real-time full-field range sensing is given to provide the motivation and understanding of the proposed approach of the current research. The methods summarized in this section are described in more detail in Chapters 2 and 3.

Several real-time 3-D shape measurement techniques and systems (Raskar 1998, Morimoto 1999, Huang 1999b, Fujigaki 2000, Hall-Holt 2001, Rusinkiewicz 2002, Guan 2003, Zhang 2004, and Huang 2005) have been developed, with some success and some limitations. Almost all the existing real-time 3-D shape measurement systems measure the 3-D object by projecting a structured-light pattern, capturing the distorted pattern images from another direction, and processing them using certain algorithms to retrieve the 3-D information of the object. Single-coded-pattern methods (Harding 1991, Geng 1996, Huang 1999b, and Guan 2003) usually use color patterns to retrieve the 3-D information of the object. These techniques usually can achieve high acquisition speeds, but low measurement accuracy due to the impact of the color of the object surface or the color coupling and intensity imbalance. Multiple-coded-pattern methods (Zhang 2004, Rusinkiewicz 2002, and Huang 2003a) project and switch a series of coded patterns rapidly onto the object, and the 3-D coordinates of points on the object surface can be calculated by processing the images captured from another direction. These techniques usually can achieve high measurement accuracy, but low measurement speed because of the complicated computation and more images required to retrieve the 3-D information. For most systems, much and often most of the time is spent on phase calculations and image processing. Different patterns have different performances, and an efficient pattern can reduce the burden of calculation. Also, the best selected design for an entire model-acquisition pipeline is vital to a real-time system. Therefore, for real-time 3-D shape measurement, it is not sufficient just to speed up projection and acquisition by means of efficient hardware. It is necessary to design efficient patterns for fast manipulation and processing to increase the efficiency of the entire system.

The traditional intensity-ratio method (Carrihill 1985 and Miyasaka 2002) has the advantage of fast processing speed, but with lower measurement accuracy. Also, the problem of ambiguity arises for measuring objects with discontinuous surface shape if the intensity-ratio ramp is repeated (Chazan 1995) to increase the robustness to measurement noise. The sinusoidal-pattern phase-shifting method works well in terms of measurement accuracy. However, the traditional sinusoidal-pattern phase-shifting algorithms involve the calculation of an arctangent function to obtain the phase, which results in more complex and time consuming computation and reduces the measurement speed. Considering this issue, an approach called trapezoidal-pattern three-step phase-shifting method (Zhang 2004, Huang

2005) was proposed for increasing the processing speed. The latter method motivated the current approach. In order to exert the advantages of the traditional intensity-ratio and sinusoidal-pattern phase-shifting methods, in this research, the two methods were combined in a novel 3-D shape measurement approach with new patterns. This newly developed method is called *triangular-pattern phase-shifting profilometry*.

In the triangular-pattern phase-shifting method, a triangular grey-level coded pattern is generated by computer software and used for projection. The measurement technique is developed for triangular-pattern phase-shifting using two, three, four steps, etc. to reconstruct the 3-D object. The phase shifting algorithms to retrieve the intensity-ratio, which is essential to obtain the object height or depth, are developed for each method.

The minimum number of measurements (sample images) of the triangular-pattern phase-shifting method that are required to reconstruct the unknown 3-D object is two, which is less than the minimum number of measurement requirements of the traditional sinusoidal-pattern phase-shifting and trapezoidal-pattern three-step phase-shifting methods (Zhang 2004 and Huang 2005). The use of only two linear triangular patterns has an advantage of less processing compared to current methods that process at least three images, or methods that process more complex functions than intensity ratio. This would be an advantage for high speed or real-time 3-D object-shape measurement. Furthermore, the proposed approach has more options for different applications. For applications that require higher processing speed, few phase-shifting steps could be used. On the other hand, more phase-shifting steps would be suitable when high accuracy is required.

1.5 Outline of the Dissertation

The remainder of this dissertation is organized as follows. Chapter 2 reviews optical methods for 3-D shape measurement, with a focus on digital fringe-projection techniques. The detailed review of optical 3-D shape measurement techniques suggests that the full-field optical methods based on digital fringe projection and phase-shifting techniques have the beneficial factors of fast measurement speed and therefore a high potential for real-time 3-D shape measurement, relatively simple image-processing algorithms to extract the 3-D coordinate information, and non-contact characteristics.

Chapter 3 describes the experimental setup, the software structure and functions of the off-line 3-D shape measurement system developed in this research, and basic algorithms of fringe projection and phase-shifting techniques.

In Chapter 4, the novel 3-D shape measurement approach, namely, *triangular-pattern phase-shifting profilometry*, is developed based on algorithms that compute intensity-ratio and intensity-ratio-to-height conversion. Analytical and experimental analyses of the measurement accuracy of the proposed method are detailed. Object measurement experiments, carried out by applying the proposed triangular-pattern phase-shifting method to verify its practical performance, are also described.

Chapter 5 addresses the system calibration with linear and non-linear mapping functions developed in Chapter 3. Computer simulation and physical experiments in calibration and measurement, carried out to compare the accuracy of linear and non-linear calibrations, are described. Experimental measurements of a 3-D object, carried out with sinusoidal-pattern phase-shifting method using the calibration results, are also detailed.

Measurement accuracy analysis and error compensation for the triangular-pattern phase-shifting method are discussed in Chapter 6. Both projector gamma non-linearity and image defocus errors have been analyzed and compensation methods are proposed.

Chapter 7 describes the principle and implementation of real-time 3-D shape measurement developed in this research.

Finally, Chapter 8 summarizes the conclusions and contributions of this research, and gives recommendations for future work.

2 Review of Optical 3-D Shape Measurement Techniques and Previous Work

This chapter reviews optical methods, introduces techniques developed and provides a more detailed introduction to digital fringe-projection techniques which were the techniques ultimately adopted as the focus of this research. Figure 2-1 introduces a taxonomy that will be followed for the review. This taxonomy is by no means comprehensive; rather, it is intended to introduce the reader to the variety of methods available.

Optical 3-D shape-measurement techniques can be divided into *active* methods, where a controlled projecting beam is used as the light source, and *passive* methods, where there is no controlled source of light. Active approaches make contact with the object or project some kind of energy onto it, whereas passive methods do not interact with the object. A comprehensive comparison of active and passive methods is presented in El-Hakim (1995).

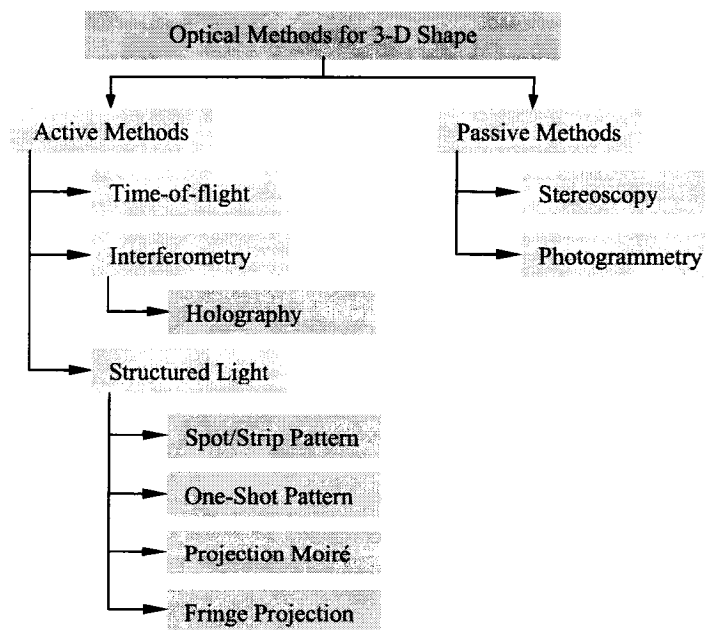


Figure 2-1 Classification of optical 3-D shape measurement techniques.

2.1 Passive Methods

Without projecting a controlled light onto the object, passive range-sensing methods compute 3-D geometry of the object by only using images acquired in natural light. This is the challenge for passive methods. Passive range-sensing methods obtain 3-D information of a measured object from two or more digitized intensity images. Passive methods attempt to find matching image features between a pair of general images about which nothing is known *a priori*. Passive systems provide high accuracy on well-defined features, such as targets, corners and edges; however, freeform and unmarked surfaces are hard to measure. A review of passive methods is given in (Seitz 1999 and Mada 2003). Passive methods can be subdivided into *stereoscopy* and *photogrammetry* methods.

2.1.1 Stereoscopy

If the same object is seen from two different positions, its 3-D information can be determined using stereo vision. It is the most important way in which humans capture range information. In computer vision, a stereo camera system, commonly two CCD cameras, are used to view a scene, and extract 3-D information by matching features of the images taken from the two different viewpoints. A true stereo effect is achieved when two overlapping images of a common area captured from two different points, are rendered and viewed simultaneously. The primary computational problem of stereo vision is to find the correspondence of various points in the two images obtained by the two cameras. Once correspondences are determined, the 3-D information of those points is easily computed from the pairs of points in both images. The shape of the measured object can be calculated when the geometry parameters of the system are known. The geometry parameters include the distance between cameras, distance between cameras and object, angle between optical axis of both cameras, and optical properties of the camera lens.

Stereo vision methods rely on finding a common feature that is visible in both images, such as edges and corners. Various matching-based methods, such as area-based matching (Cunningham 1978), feature-based matching (Ayache 1987), and profile matching (Gray 1998) have been proposed. Depending heavily on the presence of scene texture, passive

stereo vision techniques have a slow process to determine the correspondences between left and right views by means of image matching. Stereo vision is thus computationally intensive and for unmarked freeform surfaces, is hard to achieve in real-time.

2.1.2 Photogrammetry

Photogrammetry is another passive technique to measure 3-D objects from two or more photogrammes or photographs or more recently, digital images, taken from different points of view. Photogrammetry techniques provide higher accuracy measurement for objects with high contrast ratios and well-defined edges. It is mainly used for feature type object measurement because untargeted or featureless surfaces may not be measured properly. Moreover, the ability to extract all the desired features is affected by ambient light. Because of its high accuracy, photogrammetry can be combined with the fringe-projection techniques to measure complex objects (Reich 2000), or combined with laser-scanner systems to perform real-time 3-D pose estimation (Blais 2001).

2.2 Active Methods

In contrast to passive methods, by projecting specially prepared light and observing the projected images by conventional devices, many active range-scanning methods can rapidly acquire dense, highly accurate range data and have relative computational simplicity. Active range-scanning methods are the most popular because they perform well in the absence of scene texture and accurate range data can be obtained. The drawbacks for active methods are: (1) the ambient energy sources may impact the measurement accuracy; (2) the accuracy is sensitive to surface properties, such as transparency, reflectance, and color variations; (3) most techniques are limited to measurement of small objects; for large objects and scenes, the range-sensor must be moved to different viewpoints and the range data merged by registration or alignment of the different 3-D images. A review of active methods is given in Besl (1989), Beraldin (2003), and Mada (2003). Active methods can be subdivided into *time-of-flight*, *interferometry* and *structured light* methods.

2.2.1 Time-Of-Flight (TOF)

An optical time-of-flight rangefinder typically consists of a transmitter that emits a collimated light beam, a scanning mechanism, and a receiver that detects the portion of reflected light on the target. The technique is based on the direct measurement of the time of flight of light from a laser or other light source during its round trip from the range sensor to the surface of interest (Massa 1998). By sending out a short pulse of light, the time-of-flight systems estimate distance by calculating the time difference between the emitted and received pulses. Long-range sensors are usually based on time-of-flight techniques. The accurate measurement of depth is limited by the requirement of high precision in time measurements. Moreover, the measurement speed is limited by the scanning of the light beam to obtain a full frame of an image (Moring 1989 and Massa 1998).

2.2.2 Interferometry

Interferometry is an old but extensively used technique in optical metrology. Lasers are used to generate a bright beam to make the time and spatial projection coherent. The light from the source is modified by an optic or lens and split into two beams: a reference and an object beam. The reference beam reflects from a reference mirror and goes to a detector plane. The object beam reflects from an object surface and goes to the detector plane too. Both beams interfere with each other. An interferogram contains full information about the object surface, and the shape of the object can therefore be easily reconstructed. Higher resolution and accuracy could be achieved with solid-state detector arrays and high-resolution graphic board microprocessors. However, this technique is sensitive to environmental disturbances, such as vibrations. Moreover, it is only suitable to the application of surface measurement of an object with a smooth surface.

Holographic interferometry is an extension of interferometric measurement techniques where at least one of the waves that interfere is reconstructed by a hologram. The basic principle of holographic interferometry involves generation of a contour fringe pattern by the interference of two reconstructed images of a double-exposure hologram. It is often used to measure the strains in complex structures and to detect object deformations in real-time.

2.2.3 Structured Light

The structured light method is one of the most widely used techniques to retrieve the 3-D shape of an unknown object. It can be easily understood as a technique where the second stereo camera in a passive stereo-vision system is replaced by a light source, which projects a known light pattern such as a stripe, grid, dot matrix, or more complex shape onto an object. With the camera positioned at an angle to the light source, the light pattern that forms on the object surface appears distorted in the camera image. The 3-D coordinates of points of the distorted pattern on the object surface can then be simply calculated by triangulation, a principle based essentially on the geometry of the camera-light projector system. Repeating light patterns such as multiple lines, pattern distortion by the object surface, missing data, occlusion or hiding of points, or spurious reflections can create problems in determining the correspondence between the projection and observed image for geometry-based methods. The ambiguities in finding a correspondence between the image and the structured light can be removed by projecting coded patterns in multiple frames, where the patterns carry information of the coordinates of the projected points, without considering geometrical constraints. The principle of triangulation and use of coded patterns will be explained below in this section. Three comprehensive surveys on coded structured light techniques as well as a new classification of patterns for structured light sensors are presented in Batlle (1998), Pagès (2003), and Salvi (2004). Structured light methods include spot/stripe pattern, one-shot pattern, moiré and fringe projection techniques, which are explained below.

2.2.3.1 Spot/stripe Pattern Technique

One of the most common and simplest triangulation methods involves projecting a spot beam or light stripe, usually a laser sheet, onto the object to be scanned. A sensor, which is frequently a CCD camera, senses the reflected light from the object from an angle to the light source (Figure 2-2). The angle and the pixel position of the observed light pattern are related and the depth information can be computed by analyzing the image of the distorted laser light pattern along the detected profile. In order to get full depth or range information of the entire surface, the laser sheet has to be moved across the object surface by a positioning device, and images are acquired for multiple positions of the projected light.

This technique is common because of its simplicity. Projecting a single laser point (stripe) to scan progressively over the surface of the target object removes the correspondence problem between the projection and observed image, which occurs in the presence of discontinuities. However, the method requires complicated scanning mechanisms or positioning devices (Blais 1988). Moreover, the processing speed can be low because the entire scene has to be scanned point-by-point (Parthasarathy 1982 and Rioux 1984) or line-by-line (Popplestone 1975 and Porter 1982) to build the entire 3-D object.

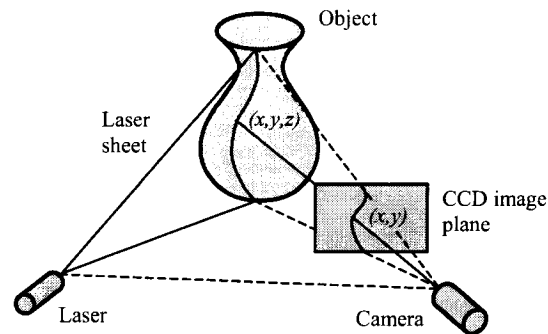


Figure 2-2 Principle of laser triangulation system.

Projection of multiple stripes simultaneously onto the object or scene is an alternative technique used in order to achieve fast 3-D data acquisition. An entire image is obtained at once rather than by multiple images of a single stripe or point by scanning, and data-acquisition speed may be increased without sacrificing data integrity. However, the method suffers from the correspondence problem of matching the correct lines in the acquired 2-D camera image with the lines on the object surface (Boyer 1987, Chen 1997, and Rusinkiewicz 2002). In order to identify the stripes or the elements of an array of projected rays, various methods based on coded structured light (color or binary) have been proposed (Salvi 2004). The coded structured light approach is an absolute measurement method that encodes all lines in the pattern from left to right, requiring only a small number of images to obtain a full depth-image.

Boyer (1987), Chen (1997), and Zhang (2002a) applied color-structured light to mitigate the matching problem and accelerate the procedure of range data acquisition. Both Boyer

and Zhang used a single camera while Chen's method is based on stereovision. Boyer assigned an index number to each stripe to calculate the range of the illuminated image points on the stripes. However, the indexing algorithm may be misleading in some occlusive areas because the color of the stripes in the observation may be different from that of the stripes in the projection due to different reflection properties of the object surface. By applying two cameras, Chen (1997) removed the lighting-to-image correspondence problem that occurs with the single-camera method, but a relatively simple image-to-image correspondence problem arose. Zhang (2002a) employed equal-width colored stripe patterns to reconstruct a 3-D surface from one or more images. Dynamic and multi-pass dynamic programming techniques were applied to find the optimal match between the projected color transition sequence and the detected edge sequence for solving correspondence problems and dropouts. Mislabeling and occlusions problems were solved by assigning a label to every stripe in the image according to its probability of matching.

Hall-Holt (2001) demonstrated a method of coding the stripe over time to deal with the correspondence problem. With time-coded structured-light, the identity of the stripe that illuminated the observed pixel of the frame could be resolved. Horn (1999) proposed a method that encodes adjacent stripes with a codeword. A unique temporal code was projected for each of the light planes. An alphabet of codes based on multiple gray levels was employed. The noisy transmission is received and subsequently decoded at each point in the image plane. The aim of the work was to find the smallest set of patterns that meet the accuracy requirements of a certain application producing the best performance under certain noise conditions.

Another coded structured-light method called *intensity-ratio depth sensing* was proposed by Carrihill (1985) and Miyasaka (2002). In this method, two patterns, a linear gray-level pattern and a constant flat pattern, are used to measure the 3-D object. The linear gray-level pattern consists of many vertical thin slit-like areas. Each area in the pattern is one-valued function against horizontal position of the target space. Every slit is identified by its own intensity. By projecting these two patterns onto the object and capturing images of the patterns deformed by the object, an intensity-ratio is calculated for every pixel between two consecutive frames. The 3-D coordinates of the object are determined by triangulation. This method has the advantage of fast processing speed because of the simple calculation, but the

accuracy is poor and the ambiguity problem arises for measuring objects with discontinuous surface shape if the intensity-ratio ramp is repeated (Chazan 1995) to decrease the measurement noise. Poor accuracy was achieved in Carrihill's method because the sensor used was highly sensitive to noise and nonlinearities. To improve measurement accuracy, Miyasaka utilized a three-CCD video camera and a Digital Light Processing (DLP) video projector in the system. Only the linear part of the gamma curve was employed with distortion of the lens system taken into account through a least-squares method. Repeated linear-coded triangular patterns have been used (Chazan 1995, Fang 1997a, and Fang 1997b) instead of a ramp pattern toward reducing the sensitivity to image noise; however, the decoding of images requires at least three image samples of the measured object, and the problem of ambiguity arises for measurement of objects with discontinuous surface shape.

According to Pagès (2003), the encoding techniques described above can be classified as space encoding (Posdemer 1982, Boyer 1987, Chen 1997, and Zhang 2002a), temporal encoding (Horn 1999 and Hall-Holt 2001), and direct encoding (Carrihill 1985, Chazan 1995, and Miyasaka 2002). Additional details about the coded structured-light methods are available in Batlle (1998), Pagès (2003), and Salvi (2004).

2.2.3.2 One-Shot Pattern Technique

One-shot systems project a simple pattern of squares onto a scene and view it from a different angle by a camera to recover the range data from only a single image. One system design question is whether to project a smooth or a piecewise-constant pattern. Other approaches use a limited set of masks that are projected sequentially to build the position codes. Most of such patterns are binary (black and white) for reasons of robustness.

Several one-shot projection patterns have been proposed to be able to recover the range data from a single image (Vuylsteke 1990, Maruyama 1993, Proesmans 1996, and Winkelbach 2002). Vuylsteke (1990) introduced a single fixed binary encoded pattern, where each line neighborhood has its own signature, to identify the grid line in the image with locally distributed indexing information. A constraint of this nonsequential encoding method is the limitation of the surface texture interference. A single pattern with multiple slits was presented by Maruyama (1993). Random gaps in the line pattern were used to identify each line. It is suitable for smooth surfaces measurement but is sensitive to highly

textured surfaces and discontinuities. The method of Proesmans (1996) was also based on projection of a single grid pattern of squares. Different with Vuylsteke's method, the pattern does not carry a code; the identification of the lines was done by the assumption of pseudo-orthographic projection. Winkelbach (2002) applied a single camera shot of a simple binary-stripe pattern for rapid object surface reconstruction. This technique is based on the idea that the directions and widths of the projected stripes in the observed image depend on the local orientation of the object's surface. The surface normals, which can be calculated by analyzing the stripe directions and widths in the camera images, are used for 3-D feature determination and surface reconstruction. It is noted that this method did not use triangulation but was simple. However, erroneous stripe angles and widths tend to exist in areas with high tilt angles.

More recently, a systematic method that combines multiple patterns into a single composite pattern was proposed by Guan (2003). It was completed by modulating each individual pattern along an orthogonal direction. This non-ambiguous one-shot structured light pattern is suitable for real-time 3-D modeling. However, the abrupt albedo and depth variation caused problems in the edge regions and the spatial resolution needed improvement.

2.2.3.3 Moiré Technique

Moiré is referred to as the “beat pattern” produced between two gratings, a master grating and a reference grating, of approximately equal spacing. Figure 2-3¹ shows a moiré pattern between two straight-line gratings of the same pitch. If the gratings are not identical straight-line gratings, the moiré pattern (bright and dark fringes) will not be straight equally-spaced fringes.

The fundamental principle of the moiré method is that a grating pattern is projected onto an object and the projected fringes are distorted according to the shape of the object. The object surface together with the projected fringes is imaged through a grating structure called a reference grating as shown in Figure 2-4. The image interferes with the reference grating to form moiré fringe patterns. The moiré fringe patterns contain information about the shape of the object. When the geometry of the measurement system is known, analysis

¹ Figure 2-3 obtained from: <http://www.faraday.gla.ac.uk/moire.htm>

of the patterns then gives accurate descriptions of changes in depth and hence shapes of the object. Shadow moiré (Takasahi 1970, Meadows 1970, and Chaing 1975) is the simplest method of moiré technique for measuring 3-D shape using a single grating placed in front of the object. The grating in front of the object produces a shadow on the object that is viewed from a different direction through the grating. One advantage of this method is that few calculations are required to convert image data into profile information of the measured object. A detailed description of the moiré technique is given by Takasahi (1970).

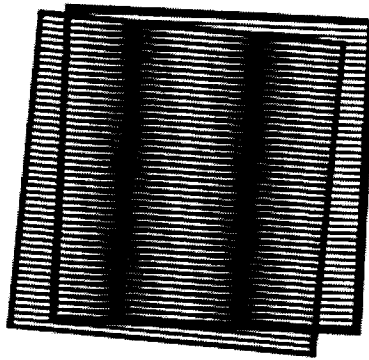


Figure 2-3 Moiré between two straight-line gratings of the same pitch.

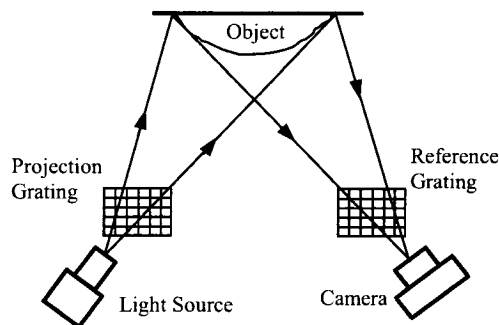


Figure 2-4 Optical setup for moiré projection system.

The use of moiré fringes to measure 3-D geometry information is well established. Early work in this field employed the manual fringe method to measure the 3-D shape from

photographs produced by the shadow moiré method (Takasahi, 1970, Meadows 1970, and Chaing 1975). Shadow moiré can provide accurate 3-D surface information but it requires a grating screen larger than the size of the object. Moiré techniques are frequently employed because of the high accuracy of measurements and fast measurement speed, as they do not require scanning to cover the whole object surface. The sensitivity of the moiré method is dependent upon the pitch or frequency of the reference grating and the configuration of the experimental setup. The use of a fringe shifting technique can raise the sensitivity of the method.

2.2.3.4 Fringe-Projection and Phase Shifting

The fringe projection technique (Takasahi 1970, Creath 1988, and Halioua 1989), an alternative approach to the moiré method, uses a fringe or grating pattern that is projected onto an object surface and then viewed from another direction. The projected fringes or grating is distorted according to the topography of the object. Instead of using the moiré phenomenon, however, the 3-D surface is measured directly from the fringe projection by triangulation. The image intensity distribution of the deformed fringe pattern or grating is imaged into the plane of the CCD array, then sampled and processed to retrieve the phase distribution through phase extraction techniques (explained in more detail in Chapter 3). The phase may be measured from some arbitrary zero reference and expressed as an angle. This phase distribution contains the height information of the 3-D object surface. The coordinates of the 3-D object is determined by triangulation. Detailed description of the fringe-projection technique is given by Creath (1988), Halioua (1989) and Greivenkamp (1992), and details in the context of the proposed and presented research are given in later sections.

To increase the measurement resolution, phase measuring interferometry techniques (Creath 1988, Halioua 1989, and Greivenkamp 1992) have been implemented in moiré and fringe projection methods to extract phase information, among which phase-shifting methods (He 1998 and Choi 1998) are the most widely used. The principle of this technique is that periodically varying grey-level intensity fringe patterns are projected onto an object surface and then viewed from another direction (Figure 2-5). In general, the minimum number of measurements of the interferogram that are required to reconstruct the unknown phase distribution is three (Creath1988). A sinusoidal fringe pattern is usually used and three

or four phase steps are applied with an appropriate phase increment to generate the fringe patterns. Traditional phase-shifting systems use hardware such as a piezoelectric transducer to produce continuous as well as discrete phase shifts (Creath1988). In these cases, the accuracy of the extracted phase is limited by the accuracy of the mechanical shifting process. The accuracy depends on the number of images. More phase steps usually can generate higher accuracy in 3-D shape reconstruction. The trade-off, however, is the longer time needed both in image acquisition and processing, which is fairly limited for real-time analysis.

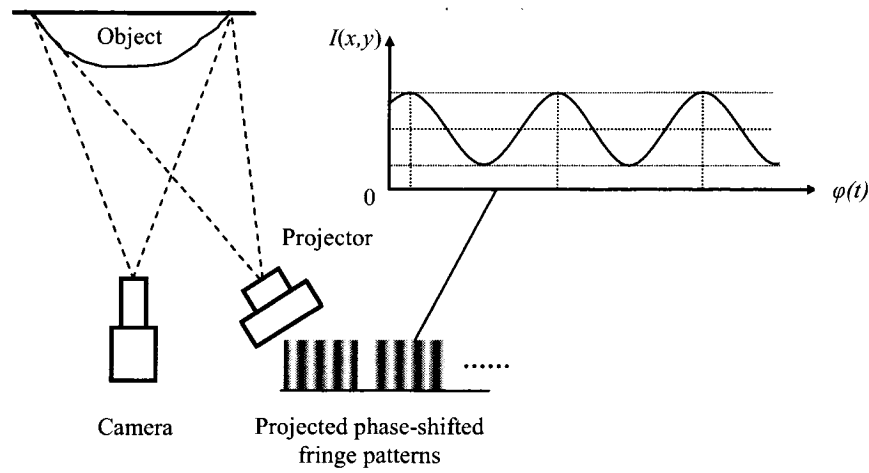


Figure 2-5 Method of fringe projection with phase-shifting.

Another phase measurement technique is using Fourier transform analysis (Takeda 1982, Kreis 1987, Freischlad 1990, Malcolm 1991, Gorecki 1992, Gu 1995, and Su 2001). In this method, only one deformed fringe pattern image is required to retrieve the phase distribution. In order to separate the pure phase information in the frequency domain, the Fourier transform usually requires carrier fringes; this brings difficulty in practice because the frequency of the carrier fringe must be controlled accurately. Another critical limitation of the Fourier transform technique is the inability to handle discontinuities. Moreover, the complicated mathematical calculation of Fourier transforms is computationally intensive (Almazan-Cu  llar 2003) and makes the technique less suitable for high-speed 3-D shape measurement.

The phase distribution (explained in more detail in Chapter 3) obtained by applying the phase-shifting algorithm is wrapped into the range 0 to 2π , due to the feature of the arctangent function. A phase unwrapping process (Macy 1983, Judge 1994, Huntley 1998 and Ghiglia 1998) is necessary to convert the modulo 2π phase data into its natural phase distribution, which is a continuous representation of the phase map. This measured phase map contains the height information of the 3-D object surface (Halioua 1989). Therefore, a phase-to-height conversion algorithm (Zhou 1994, Hung 2000, Chen 2000b, Li 2001, Zhang 2002, Hu 2003, Liu 2003, Sitnik 2005, and Guo 2005) is usually necessary to retrieve the 3-D coordinates of the object surface.

More recently, digital-fringe projection techniques for 3-D surface reconstruction have been developed using high-resolution programmable projectors. Computer-generated digital fringe patterns are projected onto an object and the projected patterns are electronically shifted with high accuracy. By using a computer to control the pattern and phase, these techniques have the advantage of being able to precisely manipulate and easily generate any grating pattern. The following section gives more detailed information about digital fringe-projection techniques.

2.3 Digital Fringe-Projection Techniques

Compared to the traditional fringe-projection and laser interferometric fringe-projection techniques, the digital fringe-projection technique has many advantages: (1) any high quality fringe pattern can be precisely and quickly generated by software; (2) the fringe pitch can be easily modified to adapt to the object surface, and thus optimize the range measurement of the object; (3) the phase can be shifted precisely and quickly by software according to the specific algorithm, without the need for a physical mechanical phase shifter; (4) the use of a high and constant brightness and high contrast-ratio projector improves the accuracy of the 3-D shape measurement; and (5) with proper synchronization between the projection and image acquisition, real-time 3-D reconstruction could be achieved.

However, digital fringe projection technique also has some disadvantages, such as the gamma non-linearity of the video projector, intensity noise, and the image defocus error of both the projector and camera.

The most popular methods using computer-generated fringe projection with phase-shifting can be roughly divided into grayscale phase-shifting (Hung 1993, Huang 1999a, Fujigaki 2000, Hung 2000, Quan 2001, Quan 2003, Hu 2003, Zhang 2004, and Huang 2005), color-encoded phase-shifting (Cao 2002, Huang 1999b, and Pan 2004) and fringe projection based on Digital Micromirror Device (DMD) methods (Huang 2003a, Hung 1993, Zhang 2002b, and Zhang 2004). The digital fringe projection techniques are described in more detail below.

2.3.1 Measurement System Setup

Figure 2-6 shows a simple schematic diagram of a 3-D shape measurement system setup based on digital fringe-projection and phase-shifting techniques. A video projector is used to project the fringe pattern, which is generated digitally by computer, onto the object surface. In the figure, $I(x, y)$ is the grey level intensity which varies periodically, $\varphi(x, y)$ is the phase, and $\delta(t)$ is the time-varying phase shift, and x and y refer to the horizontal and vertical positions, respectively, of points corresponding to each image pixel. A CCD camera is used to capture the image contour of fringe patterns from another direction. The distorted fringe pattern captured by the CCD camera is then sent to the computer via a frame grabber for processing. The reconstructed 3-D shape is displayed on the monitor.

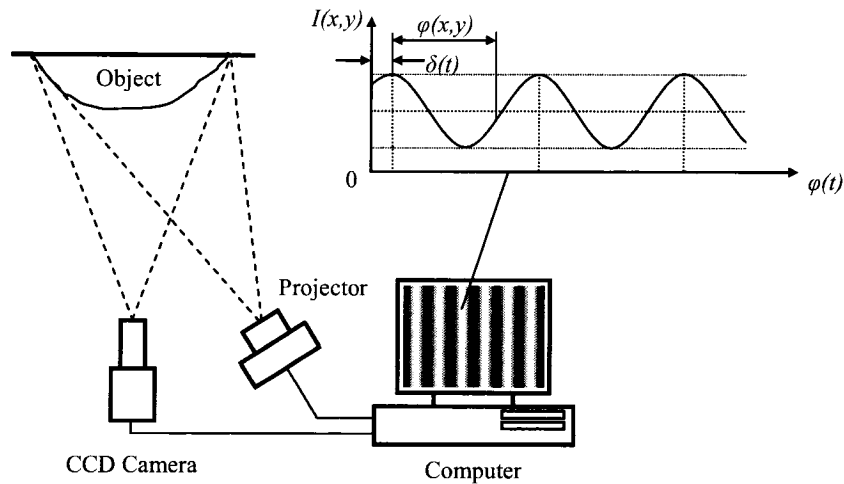


Figure 2-6 3-D shape measurement system setup based on digital fringe projection technique.

2.3.2 Grayscale Phase-Shifting Method

The grayscale phase-shifting method projects a series of spatially phase-shifted sinusoidal grayscale fringe patterns onto an object and then a camera from another direction captures the images of the perturbed fringe pattern sequentially for processing. As the phase map acquired directly from the computation of the captured images is limited from 0 to 2π , the natural phase distribution of the pixels that carries the 3-D surface information is generated by applying phase-unwrapping techniques (Macy 1983, Goldstein 1988, Judge 1994, Huntley 1998, and Ghiglia 1998). The 3-D shape information for each pixel is extracted by use of a phase-to-height conversion algorithm (Zhou 1994, Hung 2000, Fujigaki 2000, Chen 2000b, Li 2001, Zhang 2002, Hu 2003, Liu 2003, Sitnik 2005, and Guo 2005).

Hung (1993) proposed a method using a sinusoidally encoded gray level pattern to reconstruct the 3-D shape of an object. This method is based on the assumption that a planar surface should have a constant first derivative in the propagating phase. The depth of a point corresponding to a pixel can be obtained after the calculation of the phase variation of the observed pattern and the frequency of the signal. Hung (2000) presented a grayscale phase-shifting technique for full-field surface measurement. A car model, $100 \times 400 \times 50 \text{ mm}^3$ in size, was measured with a resolution of 512×512 pixels, yielding a 0.5 mm depth resolution. The total time of measurement, including the projection and acquisition of all fringe patterns and phase determination, was less than a minute using of a Pentium 133 MHz computer. In addition to projecting the phase-shifted patterns, a centerline pattern was used (Hu 2003) in a grayscale method to determine the absolute phase map for measuring objects of a master gauge and a sheet metal panel with a triangular shape. The standard deviation of measurement error for the calibration plate was 0.23 mm. The measurement error of the height of the triangular shape metal panel was less than 0.5 mm. The whole measurement process, including fringe-pattern projection and acquisition, phase map and object-surface coordinates determination, and display of the measured shape on the computer screen, took 3 seconds to finish. Because of the existence of errors in the measurement system, such as imaging errors due to the projector and camera, and calibration errors, Huang (2003b)

presented an error compensation method to improve the measurement accuracy based on the method of Hu (2003). The error map of the system was constructed and the error compensation effect was evaluated through distance measurement using a target, a coordinate measuring machine (CMM), and statistical analysis. The average error was reduced from -0.058 to -0.009 mm and the standard deviation of the error from 0.274 to 0.103 mm, which is an improvement of more than 60%. The limitation of this method, however, is that the establishment of the error map is a time-consuming process. Simplification of the process to improve the speed would be useful.

Instead of projecting a sinusoidal fringe pattern, Morimoto (1999) and Fujigaki (2000) proposed an integrated phase-shifting method which projects a rectangular grayscale pattern multiplied by two weighting functions, respectively. Morimoto built a signal processing board for real-time phase analysis. The system, which records four frames of the deformed grating on the object during one cycle of the phase shift, can obtain the phase distribution every $1/30$ second. In Fujigaki's method, thirty-two phase-shifted grayscale images with rectangular distribution are projected and captured to determine the phase difference that corresponds to the height distribution of the object. In the case of worst focus, the average phase error is 3% and the maximum phase error is 5% when the object is stable. For a moving object, the error increases linearly with the phase-shifting aberration ratio.

For phase-shifting techniques, projecting and grabbing images sequentially consume time especially when more phase-shift procedures are included. Usually a minimum of three phase-shifted fringe patterns is necessary for the sinusoidal-pattern phase-shifting method. To further increase the measurement speed, a two-step phase shifting technique (Quan 2003 and Almazan-Cuéllar 2003) was presented. Unlike the conventional phase-shifting algorithm, which requires a minimum of three phase-shifted fringe patterns, Quan (2003) and Almazan-Cuéllar (2003) presented a two-step sinusoidal-pattern phase-shifting grayscale method for calculation of the phase values. As the phase unwrapping is carried out by use of an arccosine function, which is also a time-consuming computation, the use of this new algorithm simplifies the optical system and speeds up the measurement with limitation. The drawback of this method is that the measurement accuracy is lower because the accuracy is dependent on the number of images (Morimoto 1999 and Huang 1999a).

2.3.3 Color-Encoded Phase-Shifting Method

The color-encoded method uses a DLP/LCD (Liquid Crystal Display) video projector to project a color-encoded pattern onto the object. Only a single image which integrates three phase-shifted images (RGB components) is captured by a color CCD camera. The image is then separated into its RGB components which creates three phase-shifted gray-scale images. These images are then used to reconstruct the 3-D object. The problems for this technique include the overlapping between the color channels that make the separation of RGB components difficult, and the intensity imbalance between the separated images of the red, green and blue fringe patterns. A method to effectively separate the captured image into its RGB components to create three phase-shifted images of the object and compensate the imbalance is still necessary for this technique to be useful.

Huang (1999b) employed a color-encoded phase shifting technique to measure a plaster sculptured object with the size of a human head. The resolution was 505×454 pixels without considering the fact that only 25% of the pixels provided true surface information in the red and blue channels. If this is considered, the real resolution is as low as 126×113 pixels. Although the measurement accuracy of the color-encoded phase shifting technique is low compared with grayscale phase-shifting methods, the measurement speed is restricted only by the frame rate of the camera (Huang 1999b and Pan 2004), as this technique requires only one image to reconstruct the 3-D shape of an object. By employing a high brightness LCD projector and a three-CCD video camera, the resolution and accuracy of the color-encoded phase-shifting technique were improved by Pan (2004). The size of the measured object had the dimensions of approximately $250 \times 180 \times 40$ mm³. Although the errors caused by color coupling and intensity imbalance were decreased using a look-up-table (LUT), reduction of errors and increase in measurement accuracy can still be further improved.

2.3.4 Fringe Projection Based on the Digital Micromirror Device (DMD)

The method of fringe projection based on DMD technique projects a color-encoded fringe pattern onto the object using a DLP video projector. In the DMD-based video projector, the RGB color channels are sequentially projected. With the removal of the color filter of the DLP video projector and synchronization between the pattern projection and image acquisition, three grayscale phase-shifted images can be obtained with high speed

(Huang 2003a, Zhang 2002b, and Zhang 2004). The 3-D shape of the object is reconstructed using phase wrapping and unwrapping algorithms and a phase-to-coordinate conversion algorithm. Further research of this technique could be focused on simplifying the system, increasing the speed for image grabbing, and achieving a true real-time 3-D shape measurement system in which not only the images are obtained in real time, but also the reconstruction of the 3-D shape is completed in real time.

Huang (2003a) presented a digital fringe projection method which takes full advantages of the single-chip DLP technology for rapid switching of three coded fringe patterns. Three grayscale phase-shifted images were obtained in about 50 ms with a grayscale CCD camera having a maximum sampling rate of 85 frames/sec. A reasonable accuracy and measurement resolution of 1 mm were achieved with a measurement area of 250×180 mm. If the sampling speed of a camera is fast enough, for example, a camera with “block readout mode” feature can reach 300 frames/second by blocking a portion of the image, and three phase-shifted fringe images can be obtained in less than 10 ms. Thus a potential measurement speed of 100 Hz is possible for this technique. As the synchronization is achieved by employing a self-designed external timing signal generation circuit for the DMD based video projector, this makes the system more complicated. Based on the method of Huang (2003a), an improved approach called trapezoidal-pattern phase-shifting method (Zhang 2004 and Huang 2005) was proposed for further increasing the processing speed. This method is based on the consideration that the traditional sinusoidal-pattern phase-shifting algorithms involve the calculation of an arctangent function to obtain the phase and this computation results in a slow measurement speed. By projecting three phase-shifted trapezoidal patterns, and calculating the intensity-ratio at each pixel instead of the phase, less processing time is required. For an image size of 532×500 pixels, the processing time needed for this method was 4.6 ms on a system with two Pentium 4 2.8 GHz computers compared to the sinusoidal-pattern phase-shifting method, which needed 20.8 ms to process. The total processing time for 3-D shape reconstruction was about 24.2 ms per frame, which is fast enough to make real-time 3-D shape reconstruction possible. The drawback of this method, however, is that the texture mapping became more difficult.

2.3.5 Summary of Existing Digital Fringe-Projection Techniques

Considering the above methods, full-field optical methods based on digital fringe-projection techniques have the capability of high-speed image grabbing, and potential for real-time 3-D shape measurement. Furthermore, the phase shifting method has proven to be a powerful tool for fringe analysis to complete 3-D object reconstruction with high resolution if it is used appropriately, but it requires multiple images for the analysis (Fang, 1997b). Compared with other techniques, which usually have correspondence problems, fringe-projection based on phase-shifting techniques has the advantages that it only deals with the pixels in the phase map, which can then be converted to 3-D coordinates of the object. It is not only suitable for measuring relatively large-scale objects by patching sub-regions together, such as a sandy pool with the size of $9 \times 5 \text{ m}^2$ (Li 2001), but it is also suitable for measuring small-scale objects or even the fibrous structure of a surface if the system is equipped with microscopes. For example, the numbers on a coin of size of $2 \times 1.5 \text{ mm}^2$, the electronic components on an electronic printed circuit board of size 1 mm (Quan 2001 and Quan 2003), and a diffusive surface with a height variation around $30 \text{ }\mu\text{m}$ (Zhang 2002b) have been measured.

The grayscale phase-shifting method usually can obtain high accuracy and resolution, but with low measurement speed. The speed can be increased if the synchronization between the pattern projection and image acquisition is achieved by software or hardware. Using the color-encoded fringe projection technique could increase the speed, but the cost of the system is also increased due to the use of the high-brightness projector and expensive camera, such as the three-CCD color video camera. Compared with the grayscale phase-shifting method, the limitation of the color-encoded phase-shifting method is that the accuracy of the measurement is low because of the problems of the color coupling and intensity imbalance. Moreover, the original colors of the object surface could affect the measurement results. Therefore, it may occur that only the object with neutral color surfaces could allow meaningful results using the color-encoded fringe projection technique. Compared with the color-encoded phase-shifting method, the speed of the fringe projection based on DMD technique is slower, but it does not have the problems associated with color because of the use of grayscale phase-shifted images. Details of measurement speed,

resolution and accuracy of existing digital fringe projection techniques are summarized in Table 2-1.

Table 2-1 Summary of computer-generated fringe projection methods.

	Author	Setup	Description	Object	Speed	Resolution	Accuracy reported by authors
Grayscale Phase-shifting Method	Morimoto 1999	-LCD projector -CCD camera	Four phase-shifted rectangular grayscale images are projected and captured to determine the phase difference that corresponds to the height distribution of object. A signal processing board for real-time phase analysis was developed.	Human with an electric light bulb.	The system can display the phase distribution i.e. the equal-height distribution every 1/30 second.		Accuracy is high.
	Fujigaki 2000	-LCD projector -CCD camera	Thirty-two phase-shifted rectangular grayscale images are projected and captured to determine the phase difference that corresponds to the height distribution of object.	-A rubber ball and a metal spoon. -Human hand grasping a rubber ball.	The processing time is about 2-3 seconds.		The error is about 3% for a phase of 2π in the case of the worst focus when the object is stable. The error increases linearly with the phase-shifting aberration ratio when the object is moving.

Grayscale Phase-shifting Method	Hu 2003	-DLP video projector (InFocus LitePro 620) -CCD grayscale camera (Kodak Megaplug 1.6i) -Processor board (Matrox Genesis)	Three phase-shifted sinusoidal grayscale images and a centerline pattern are projected and captured to determine the absolute phase map of the object, which is used to obtain the 3-D information of the object surface. No synchronization.	-A master gauge. -A sheet metal panel with a triangular shape.	Each measurement takes 3 s to finish.		The standard deviation of measurement error for the calibration plate was 0.23 mm. Measurement error of the height of the triangular shape of sheet metal panel was less than 0.5 mm.
	Huang 2003b	-LCD projector -CCD grayscale camera	Similar to Hu 2003, but an error compensation method is presented to compensate the imaging errors of the projector and camera, and the calibration errors.	A Boeing airplane model			For the specific target, the average error was reduced from -0.058 to -0.009 mm and the standard deviation of the error from 0.274 to 0.103 mm - an improvement of more than 60%.
	Huang 1999a	-DLP video projector -Grayscale CCD camera	Three phase-shifted sinusoidal grayscale images are projected and captured. No synchronization.	- Plaster cast of a cat. - Plaster cast of a human face	Reasonably fast speed	High resolution	

Grayscale Phase-shifting Method	Hung 2000	-Projector -CCD camera	Four phase-shifted sinusoidal grayscale images are projected and captured. No synchronization.	Car model 100×400×50 mm ³	Very high. <1min with Pentium 133MHz computer	Spatial resolution: 512×512 pixels. Depth resolution: 0.5mm Slope resolution: 0.001 for reflected fringe technique.	
	Quan 2001	-LCD projector -CCD camera -Long-working-distance microscope	Four phase-shifted sinusoidal grayscale images are projected and captured. No synchronization.	-Number 99 on a coin. 2×1.5 mm ² - A resistor with 1mm length.		500 nm 50 nm	
	Quan 2003	-LCD projector -CCD camera -Long-working-distance microscope	Two phase-shifted sinusoidal grayscale images are projected and captured. No synchronization.	The number 20 on a coin.			
Color-encoded Phase-shifting	Cao 2002	-DLP video projector -Color CCD camera	Only one image is projected and captured. The color image is split into its RGB components, which form the three phase-shifted images.		Very quick.		Measurement accuracy has been improved.

Color-encoded Phase-shifting Method	Huang 1999b	-DLP video projector (InFocus LitePro 620) -Color CCD camera (Kodak DC210)	Similar to Cao 2002	Plastic head Sculpture	Contouring speed is limited only by the frame rate of the camera.	505×454 pixels (without considering that only 25% of pixels were providing true surface information) Real resolution: 126×113 pixels	Reasonable measurement accuracy.
	Pan 2004	-LCD projector (Epson PowerLite8000i) -3-CCD color video camera (Sony DXC-3000) -Frame grabber (Matrox Corona)	Similar to Huang 1999b, but a Look-Up-Table (LUT) method is developed to compensate for the color coupling and imbalance errors.	A sea horse made of white plaster. 250×180×40 mm ³	Measurement speed is limited only by the frame rate of the camera.	Much better compared to Huang 1999b	Significantly improves the measurement accuracy of the system.
Fringe projection based on DMD technique method	Huang 2003a	-DLP video projector -Grayscale CCD camera (Kodak ES310)	A color-encoded fringe pattern that consists of three phase shift fringes is sent to the DLP video projector. With the removal of the color filter of the DLP video projector and the synchronization between the projector and camera, three grayscale phase-shifted images are obtained with high speed.	-Plastic head -Adult chest	Capture all three phase shifted images in 50 ms with the camera having a maximum sampling rate of 85 frames/sec.	A measurement resolution of 1 mm was achieved with a measurement area of 250×180 mm.	Reasonable measurement accuracy.

Fringe projection based on DMD technique method	Zhang 2002b	-DLP video projector -Grayscale CCD camera (Kodak ES310) -Frame grabber (Matrox Meteor-II) - Stereomicroscope (Nikon SMZ800)	Similar to Huang 2003a, but a microscope was used in the system to measure the fibrous structure of a surface.	Measurement area ranges from $7.6 \times 5.7 \text{ mm}^2$ to $1.2 \times 0.9 \text{ mm}^2$ depending on the zoom ratio used.		Experimental results demonstrated micrometer level resolution when a $63\times$ zoom ratio was used. Approximately $2\text{-}3\mu\text{m}$	Peak-to-peak height variations are 2.2 and $3.0\mu\text{m}$ for the upper and lower surfaces. The measured step height has a mean of $22.3\mu\text{m}$ with a peak-to-peak variation of $2.6\mu\text{m}$.
	Zhang 2004 Huang 2005	-DLP projector (Kodak DP900) -Grayscale CCD camera (Dalsa CA-D6-0512W) -Color camera (Uniq Vision UC-930)	Similar to Huang 2003a, but trapezoidal phase-shifting method was used to perform real-time measurement.	Human faces	The total processing time for 3-D shape reconstruction was around 24.2 ms per frame (532×500 pixels) with a Pentium 4, 2.8 GHz PC.	Both the sinusoidal phase-shifting and trapezoidal phase-shifting methods provide a pixel-level resolution. The depth resolution of the trapezoidal phase-shifting method is six times better than former intensity-ratio methods.	The noise level was RMS 0.05 mm in an area of $260 \times 244\text{ mm}$ for the sinusoidal phase-shifting method and 0.055 mm for the trapezoidal phase-shifting method. The system is able to measure moving objects with good accuracy.

2.4 Summary

A variety of 3-D shape measurement techniques based on the adoption of optical technology have been reviewed. The basic description for each technique was given, and variety methods to perform the technique were reviewed and summarized. This chapter has also presented a classification and comparison of digital fringe projection techniques. A detailed review of optical 3-D shape measurement techniques suggests that the full-field

optical methods based on digital fringe projection and phase-shifting techniques have the advantages of high-speed image grabbing, high accuracy and resolution, and ease to generate fringe patterns and perform phase shifting precisely and quickly. It therefore has the potential for real-time 3-D shape measurement. The above review suggests that improvements of full-field digital fringe projection and phase-shifting techniques through modifications to the image manipulation software would be a significant contribution toward real-time full-field surface measurement. The potential of modifying the projected patterns to achieve the improved image processing was the motivation for the direction taken in this research. In adopting this approach, it is understood that improvements based on modified pattern projection and associated image processing could be used in less expensive and less complicated systems than those involving more complex hardware, discussed above. Furthermore, new pattern projection and associated image processing methods could be combined with the more complex hardware systems described above, to improve those systems even further.

3 Phase-Shifting Measurement System and Principles

In this chapter, first, the principle of the proposed 3-D shape measurement system, the features of the hardware in the system, and the system modules will be introduced in Section 3.1. Then the design of the experimental setup, including selection of an appropriate lens for the camera and initial system parameter decisions, will be addressed in Section 3.2. This is followed by discussion of the major issues related to the measurement principle of phase-shifting algorithms (Section 3.3), how to obtain the phase distribution from the captured images using the sinusoidal-pattern phase-shifting method (Section 3.4), and how to calculate the 3-D coordinates of the object from the phase map (Section 3.5).

In this research, the mapping relationship between the phase and the height of the object surface for phase-measuring profilometry was formulated to linear and non-linear equations through simplified geometrical derivation. A comparison will be made between the two approaches in Chapter 5 using a least-squares method.

3.1 Off-Line 3-D Shape Measurement System

As discussed in Section 2.3, digital fringe projection and phase-shifting techniques have the advantages of high-speed, full-field image acquisition, reliable quantitative surface-geometry measurement, as well as non-contact and non-invasive characteristics, and potential for real-time 3-D shape measurement. Therefore, the full-field optical method based on digital fringe projection and phase-shifting techniques has been fully adopted in this research.

The 3-D shape measurement system consists of an off-line 3-D shape measurement system and a real-time 3-D shape measurement system. The real-time 3-D shape measurement system will be described in detail in Chapter 7. Here, the off-line 3-D shape measurement system will be introduced.

3.1.1 Principle of 3-D Shape Measurement

Figure 3-1 shows a schematic diagram of an experimental set-up for the proposed fringe-projection phase-shifting measurement system. A projector is used to project a series of fringe patterns, generated digitally by computer, onto the object surface. The projected fringe patterns are distorted according to the topography of the object. A CCD camera is used to capture the image of the fringe patterns from another direction. The video signal of the fringe pattern from the CCD camera is then sent to the computer via a frame grabber for processing. The intensity-ratio/phase distribution (explained in more detail later in this chapter) is retrieved through intensity-ratio/phase extraction techniques and then the coordinates of the 3-D object is determined by triangulation. The reconstructed 3-D object can then be displayed on the monitor. Figure 3-2 is a photograph of the experimental setup.

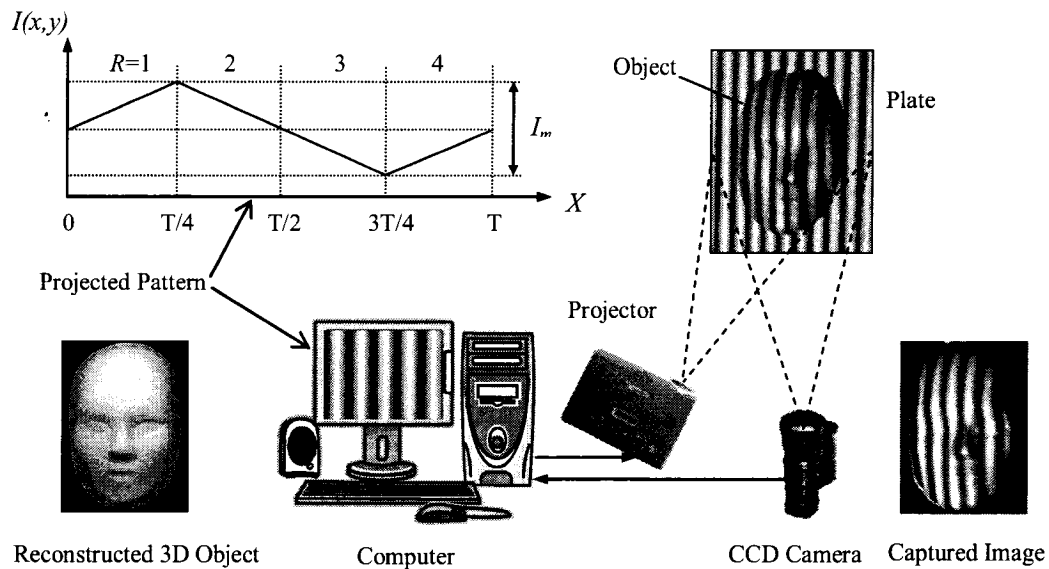


Figure 3-1 Schematic diagram of the 3-D shape measurement system based on fringe projection.

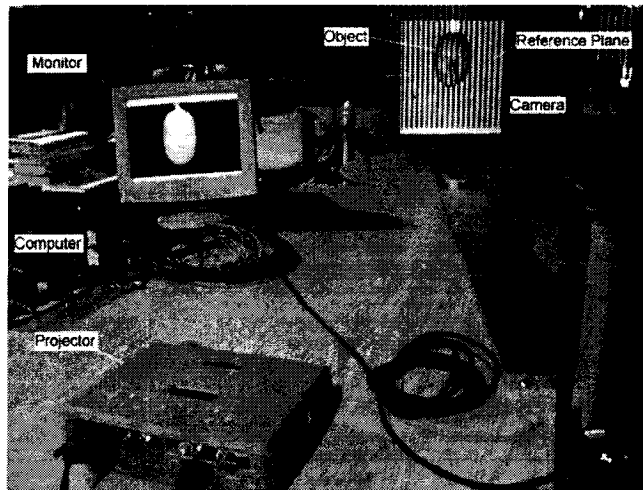


Figure 3-2 Photograph of the experimental full-field shape-measurement system setup.

3.1.2 Equipment

The following equipment for the experiment setup has been used: a Pentium 4 computer with 3.04 GHz processor and 1.0 GB memory, an InFocus LP600 DLP video projector, a Sony XCHR50 progressive-scan black and white CCD camera, and a Matrox Odyssey XA vision processor board.

The digital projection system (In Focus LP600) has a brightness of 2000 ANSI lumens, a resolution of XGA 1024×768, and a contrast ratio of 1000:1. It provides control over brightness, contrast ratio, manual zoom lens, and automatic vertical keystone. For image capture, a Sony XCHR50 Progressive Scan Black and White CCD Camera was used. It delivers detailed images with the equivalent VGA resolution of 648×494 pixels. The CCD has square pixels, eliminating the need for aspect ratio conversion. A Matrox Odyssey XA vision processor board, which integrates a quad-input analog frame grabber, was used to digitize the image signal to the computer.

3.1.3 Structure of the Off-Line 3-D Shape Measurement System

Figure 3-3 shows the structure of the off-line 3-D shape measurement system. The system consists of twelve modules including Pattern Generation, Phase Shifting, Image Capture, Intensity-ratio/Phase Wrapping, Intensity-ratio/Phase Unwrapping, 3-D Coordinate

Calculation, 3-D Object Rendering/Display, System Calibration, Accuracy Analysis, Error Compensation, Image Processing, and File I/O.

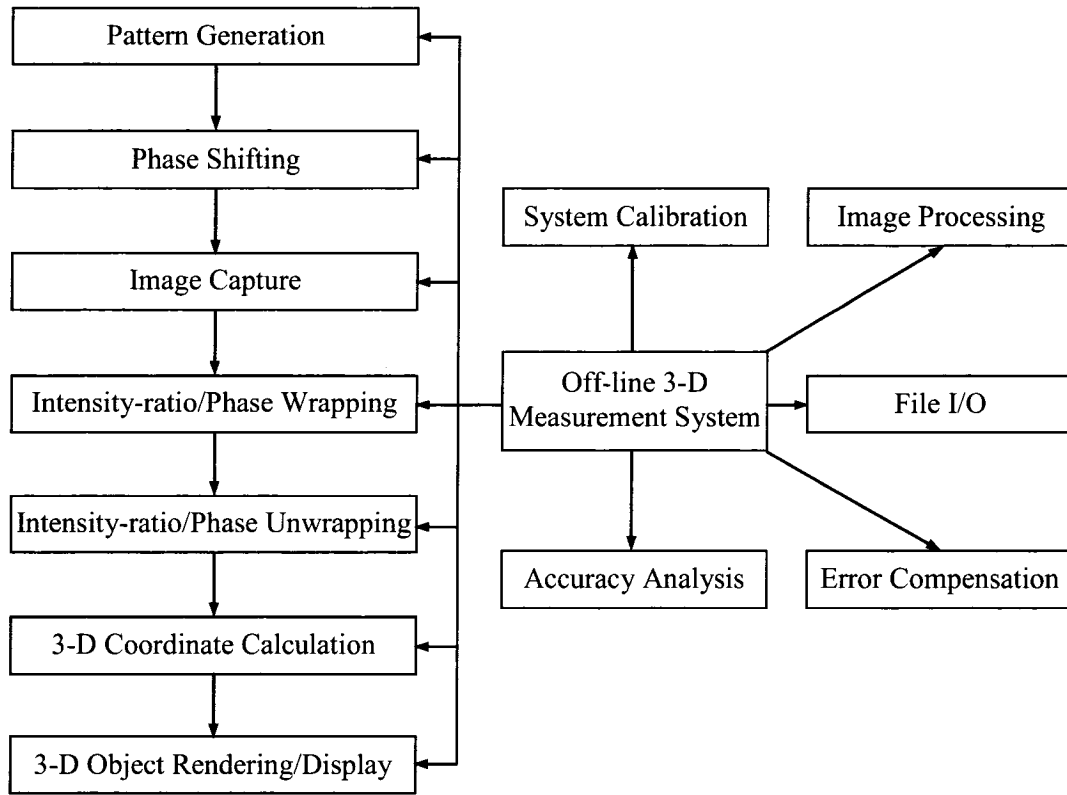


Figure 3-3 Structure of the off-line 3-D shape measurement system.

The Pattern Generation module can generate triangular and sinusoidal fringe patterns with given parameters, such as the value of the pitch of the pattern, and maximum and minimum input intensity values of the pattern. The Phase Shifting module is responsible for shifting the pattern with the given step. The Image Capture module controls the CCD camera to capture the image of the projected pattern and send it to the computer for processing. The Intensity-ratio/Phase Wrapping module processes the captured images using intensity-ratio/phase measuring algorithms, and obtains a wrapped intensity-ratio/phase map. Intensity-ratio/Phase Unwrapping module remove the discontinuities of the wrapped intensity-ratio/phase map to obtain a natural intensity-ratio/phase distribution. The 3-D

Coordinate Calculation module is used to determine the coordinates of the measured object surface by applying intensity-ratio/phase-to-height conversion algorithms. The 3-D Object Rendering/Display module is used to generate graphical display of the measured results by using OpenGL features, such as lighting effects, smooth shading, and material properties with functions of solid, wireframe, and shaded display. The System Calibration module is used to calibrate the system to obtain system-related calibration parameters. Both linear and non-linear calibrations have been implemented in this module. The Accuracy Analysis module is carried out to perform measurement accuracy analysis for both sinusoidal-pattern and triangular-pattern phase-shifting methods. The Error Compensation module reduces measurement error caused by projector gamma non-linearity and image defocus. This is done in the two-step triangular-pattern phase-shifting method through repeated two-step triangular-pattern phase-shifting method or in any phase-shifting method through intensity-ratio error compensation. The Image Processing module is used to smooth the reconstructed object surface by applying either median or averaging filtering or both with a selected mask size and number of runs, or by applying Bezier surface fitting. File I/O handles data format conversion and data opening and saving.

Each module in the system was implemented with Visual C++ 6.0, Matrox Imaging Library, and OpenGL software. Matlab software was used to analyze the results of the calibration, simulation, and measurement.

3.2 Design of Experimental Setup

Based on the previous work done by some researchers, the work volume of the proposed measurement system is aimed for 220 mm in width, 280 mm in length, and 50 mm in depth. The rectangular section size accommodates a typical human-head. The calibration setup is based on the proposed work volume and available equipment. Firstly, a suitable lens for the XC-HR 50 CCD camera is chosen.

3.2.1 Lens Selection

Based on the decided work volume of the proposed measurement system, a proper lens should be chosen for the CCD camera.

Some of the XC-HR 50 CCD camera specifications are listed below:

- Image pickup device: 1/3 type interline transfer CCD
- Number of effective pixels: 659×494 (H×V)
- Optical black: 33 pixels per horizontal scan line
- Cell size: 7.4×7.4 μm (H×V)
- Chip size: 5.84×4.94 mm (H×V)
- Flange back: 17.526 mm
- Effective lines: 648×494 (H×V)

Figure 3-4 shows the diagram for lens focal-length calculation. From the diagram, the lens focal length for both horizontal and vertical direction can be calculated by the following two equations:

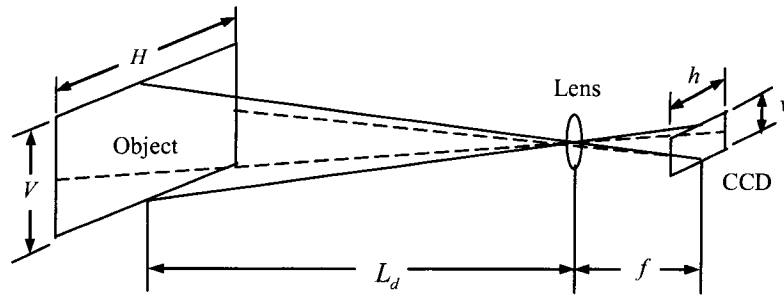


Figure 3-4 Lens focal length calculation.

To cover the width of the object (horizontal direction), the focal length of the lens should be:

$$f_h = \frac{L_d}{H} h \quad (3-1)$$

To cover the height of the object (vertical direction), the focal length of the lens should be:

$$f_v = \frac{L_d}{V} v \quad (3-2)$$

To cover both the width and height of the object, the focal length of the lens should be chosen by:

$$f = \min(f_h, f_v) \quad (3-3)$$

where

f_h — Lens focal length determined by horizontal direction

f_v — Lens focal length determined by vertical direction

f — Lens focal length

V — Vertical size of object

H — Horizontal size of object

v — Vertical size of image

h — Horizontal size of image

L_d — Distance from lens to object

For the XC-HR 50 camera:

$$h = 648 \times 0.0074 = 4.795 \text{ mm}$$

$$v = 494 \times 0.0074 = 3.656 \text{ mm}$$

Because the XC-HR 50 camera is a 1/3 type interline transfer CCD camera, from the optical manual the standard values are:

$$h = 4.8 \text{ mm}$$

$$v = 3.6 \text{ mm}$$

According to the work volume, the field of view of the camera is chosen as: $H \times V = 350 \times 300 \text{ mm}^2$. The throw distance of the camera (or object-to-camera standoff distance) is chosen as: $L_d = 1560 \text{ mm}$. This is based on the minimum throw distance of the InFocus

LP600 DLP video projector, which is 1524 mm. By substituting the values of the parameters L_d, H, V, h , and v into Equations (3-1) and (3-2), the focal length for the horizontal and vertical directions should be:

$$\text{Horizontal: } f_h = \frac{L_d}{H} h = \frac{1560}{350} \times 4.8 = 21.394 \text{ mm}$$

$$\text{Vertical: } f_v = \frac{L_d}{V} v = \frac{1560}{300} \times 3.6 = 18.72 \text{ mm}$$

Therefore, the lens focal-length should be chosen as:

$$f = \min(f_h, f_v) = \min(21.394, 18.72) = 18.72 \text{ mm}$$

However, from the available lens, the most suitable lens for this research has a focal length of 25 mm. Therefore this lens has been chosen for the experimental setup and the throw distance of the camera is recalculated based on not changing the field view of the camera as follows:

$$\text{Horizontal: } L_d = \frac{H}{h} f = \frac{350}{4.8} \times 25 = 1823 \text{ mm}$$

$$\text{Vertical: } L_d = \frac{V}{v} f = \frac{300}{3.6} \times 25 = 2083 \text{ mm}$$

Therefore, the throw distance of the camera should be chosen as: $L_d = 2083 \text{ mm}$.

3.2.2 Initial System-Parameter Selection

Figure 3-5 shows the layout of projector and camera and the calibration positions in the proposed measurement system. In the coordinate system $O-XYZ$ (Y is in the direction perpendicular to the XZ plane), the projector is offset from the camera in the X direction. The projector and camera are both aimed at the center of the object space shown by point E . The meanings of the parameters are as follows:

- α — Full angle of view of projector
 β — Full angle of view of camera
 θ — Angle between axes of projector and camera
 d — Offset of projector in the XOZ
 W_1 — Minimum width covered
 W_2 — Maximum width covered
 L_d — Distance between camera and reference plane

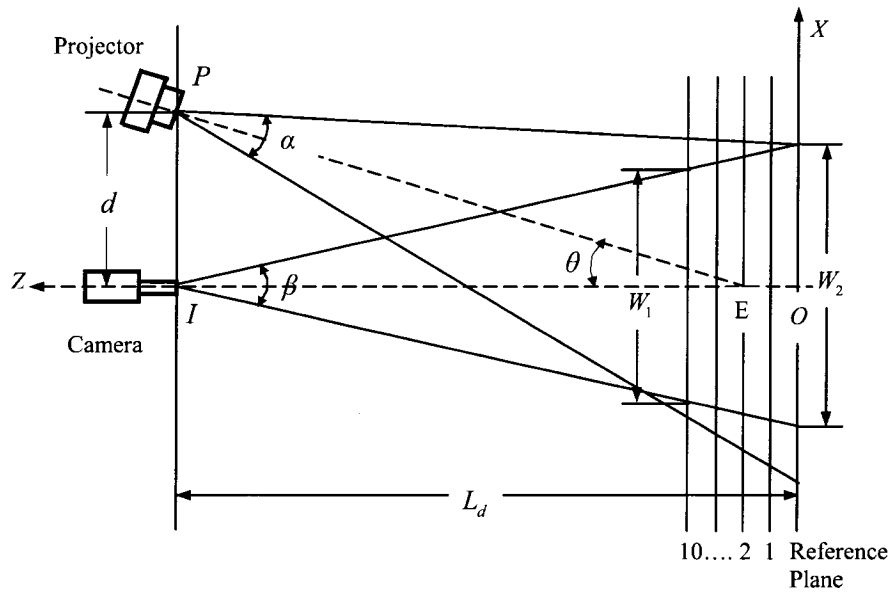


Figure 3-5 Layout of projector and camera.

The system parameters include the angle between the axes of projector and camera, the distance between the camera lens and the CCD sensor, the distance between the camera and reference plane and the fringe frequency. Directly determining these system parameters (Hu 2003) is time consuming and difficult to do accurately. In this research, an initial design to determine their approximate values was performed, in order to design the calibration setup. A reverse operation is then carried out to calibrate the system by either measuring the calibration object or the reference plane in sequence at different positions.

Because the sensitivity of the system depends on the angle θ between the axes of the projector and the camera, an initial angle is chosen as $\theta = 15^\circ$, and the angle is then determined during the experimental process. The throw distance of the camera is chosen as $L_d = 1900$ mm. (This is slightly shorter than the calculation value of 2083 mm, because the actual object mask used has a height of 210 mm. This distance can be adjusted to a suitable value according to the size of the object being measured.) Based on the determined work volume (5×10 mm = 50 mm depth) and the design requirements, iterative calculations were carried out in Microsoft® Excel spreadsheets (Figure 3-6) to find the most practical and optimized layout parameters for the system. Figure 3-7 shows the graphical results from the iteration. The following layout parameters are obtained from the iteration and are chosen to be the initial parameters for testing. They can be modified as necessary.

$$\alpha = 17.97^\circ, \beta = 18^\circ, \theta = 15^\circ, L_d = 1900 \text{ mm}$$

$$d = 501.06 \text{ mm}, W_1 = 582.85 \text{ mm}, W_2 = 601.86 \text{ mm}$$

Input				
Camera	Location	Z	X	Unit
		0	0	mm
	Distance L_d		1900	mm
	Depth of Object		60	mm
Projector	Full angle of view		18	deg
		Z	X	Unit
	Location	0		mm
	Center Point Of View	1870	0	mm
Angle between axes of projector & camera			15	deg
Output				
		Z	X	Unit
Location of Projector		0	501.06	mm
Min Width Covered CD			582.85	mm
Max Width Covered AB			601.86	mm
Distance O2A			1910.51	mm
Distance O2E			1935.97	mm
Distance O2D			2003.41	mm
Distance AE			302.42	mm
Distance ED			292.97	mm
Left angle of projector Alfa 1			8.99	deg
Right angle of camera Alfa 2			8.30	deg
Full Angle of View of Projector Alfa			17.97	deg
Angle OAF			96.01	deg
Angle OFA			66.01	deg
Distance AF			645.28	mm

Figure 3-6 Calculation of layout parameters

Based on the design of the calibration setup, the experimental jig was designed and manufactured. A modification was made on the system to account for the projection direction.

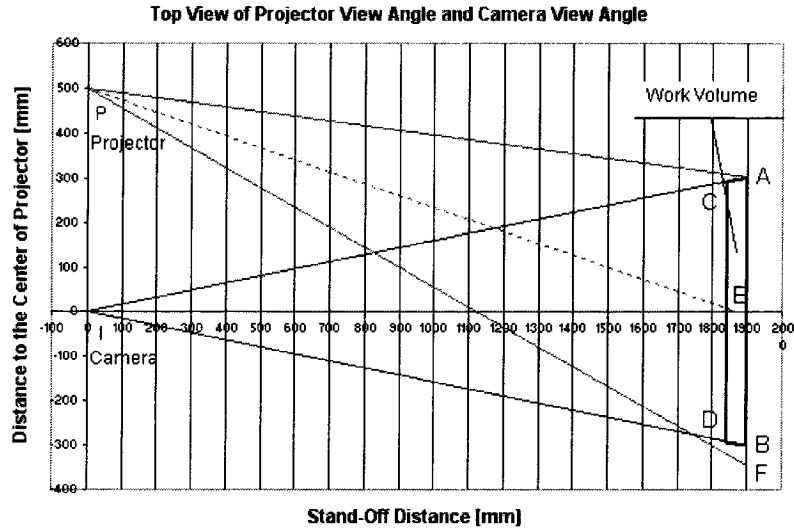


Figure 3-7 Top view of projector-view and camera-view angles.

3.3 Phase-Shifting Algorithms

The fundamental concept of fringe-projection techniques, described in detail in Creath (1988), Halioua (1989), and Greivenkamp (1992), is reviewed briefly here in order to define the important parameters and to give some original formulas.

When a sinusoidal fringe pattern, shown in Figure 3-8, is projected onto a 3-D diffusing object, the intensity distribution of the image of the distorted pattern, as seen by the imaging system, can be expressed as follows (Halioua 1989):

$$I(x, y) = a(x, y) + b(x, y) \cos[\varphi(x, y) + \delta(t)] \quad (3-4)$$

This equation has the following three unknowns: $a(x, y)$ represents the background intensity, also called the dc background, $b(x, y)$ is the amplitude of modulation, and $\varphi(x, y)$ is the phase distribution. $I(x, y)$ is the intensity detected by the CCD and $\delta(t)$ is the time-varying phase shift, which is known since this is the shift in the known patterns that are projected. The value of phase $\varphi(x, y)$ is related to the shape of an object. The methods for obtaining the phase distribution will be discussed first, and then the phase-to-height conversion algorithm will be explained.

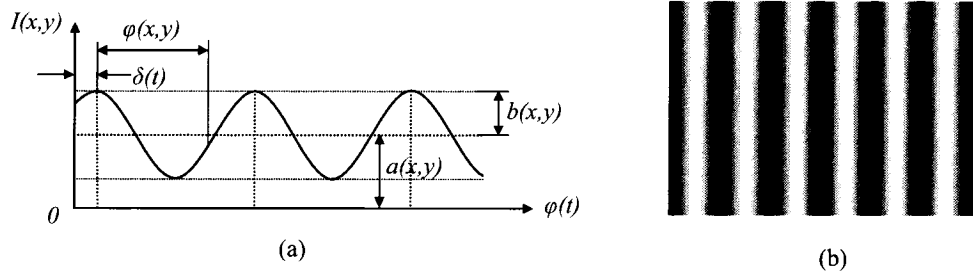


Figure 3-8 Sinusoidal fringe pattern. (a) The sinusoidal variation of intensity, (b) Computer generated sinusoidal fringe pattern.

Many algorithms can be used to calculate the phase distribution from the captured fringe images, and the phase-shifting algorithm (Creath 1988) is the widely used suitable solution. In order to solve the three unknowns in Equation (3-4), the minimum number of measurements of the interferogram intensity that are required to reconstruct the unknown phase distribution is three. Three equations are set up by shifting the phase of the fringe pattern three times, obtaining three intensity values, and then the phase distribution $\varphi(x, y)$ can be retrieved. Here a general case of the N -phase algorithm is considered, in which the N measurements are equally spaced over one modulation period. The intensity distribution can be expressed as follows:

$$I_i(x, y) = a(x, y) + b(x, y) \cos[\varphi(x, y) + \delta_i] \quad (3-5)$$

for $i = 1, 2, 3, \dots, N, N \geq 3$

where δ_i is the introduced phase shift. For the total of N recorded intensity measurements the phase can be calculated using a least-squares technique – Gaussian least-squares approach. For the convenience of using least-squares analysis, the intensity distribution can be rewritten in the following equation:

$$\begin{aligned} I_i(x, y) &= a(x, y) + b(x, y) \cos[\varphi(x, y) + \delta_i] \\ &= C_1 + C_2 \cos \delta_i + C_3 \sin \delta_i \end{aligned} \quad i = 1, 2, 3, \dots, N \quad (3-6)$$

where

$$\begin{aligned} C_1 &= a(x, y) \\ C_2 &= b(x, y) \cos \varphi(x, y) \\ C_3 &= -b(x, y) \sin \varphi(x, y) \end{aligned} \quad (3-7)$$

To determine the values of the three variables C_1, C_2 and C_3 , the variance ε is minimized, as defined by the following expression:

$$\varepsilon = \frac{1}{N} \sum_{i=1}^N [I_i(x, y) - (C_1 + C_2 \cos \delta_i + C_3 \sin \delta_i)]^2 \quad (3-8)$$

By taking the partial derivatives of the variance ε with respect to the three unknowns C_1, C_2 and C_3 , three equations are obtained by letting the derivatives equal to zero, and they can be written in matrix form as:

$$\begin{pmatrix} N & \sum \cos \delta_i & \sum \sin \delta_i \\ \sum \cos \delta_i & \sum \cos^2 \delta_i & \sum \sin \delta_i \cos \delta_i \\ \sum \sin \delta_i & \sum \cos \delta_i \sin \delta_i & \sum \sin^2 \delta_i \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ C_3 \end{pmatrix} = \begin{pmatrix} \sum I_i \\ \sum I_i \cos \delta_i \\ \sum I_i \sin \delta_i \end{pmatrix} \quad (3-9)$$

By solving the above matrix, the following parameters are obtained:

$$C_1 = \frac{1}{N} \sum_{i=1}^N I_i(x, y) \quad (3-10)$$

$$C_2 = \frac{2}{N} \sum_{i=1}^N I_i(x, y) \cos \delta_i \quad (3-11)$$

$$C_3 = \frac{2}{N} \sum_{i=1}^N I_i(x, y) \sin \delta_i \quad (3-12)$$

From Equation (3-7) we have:

$$\tan \varphi(x, y) = -\frac{C_3}{C_2} \quad (3-13)$$

By substituting Equations (3-11) and (3-12) into the above equation, the phase distribution $\varphi(x, y)$ can be obtained as the following expression:

$$\varphi(x, y) = -\tan^{-1} \frac{\sum_{i=1}^N [I_i(x, y) \sin \delta_i]}{\sum_{i=1}^N [I_i(x, y) \cos \delta_i]}, \quad i = 1, 2, 3, \dots, N \quad (3-14)$$

where the phase shift $\delta_i = \frac{2\pi i}{N}$.

As mentioned earlier, at least three intensity measurements must be carried out to determine all three unknowns. Many phase-shifting algorithms have been developed, all of them with different properties. Table 3-1 summarizes the equations of some of these algorithms (Creath 1988, Halioua 1989, and Greivenkamp 1992). They can be derived from the interference Equations (3-9) to (3-14) using three, four and five phase shifting steps.

In this part of the research, three and four steps sinusoidal-pattern phase-shifting will be used for testing. For three steps, three angles of 0° , 120° , and 240° are chosen as phase steps; for four steps, the angles of 0° , 90° , 180° , and 270° are chosen as phase steps.

All of these algorithms described above share common characteristics: they require that a series of measurements be recorded as the reference phase is varied; the phase

distribution $\varphi(x, y)$ is calculated by the arctangent function. Normally the arctangent is defined only over a range of $-\pi/2$ to $\pi/2$; however, in phase-shifting interferometry the signs of sine and cosine are known. This means that the quadrant the phase is in is known, so when the arctangent is performed, the correction can be made to extend the calculated phase to the range from 0 to 2π (or $-\pi$ to π), which will yield a wrapped phase distribution ranging from 0 to 2π (or $-\pi$ to π), as illustrated in Table 3-2. Therefore, a saw-tooth-like phase-wrapped image is generated (Figure 3-9 (a), Figure 3-10 (a), and Figure 3-11 (a)). A process that is referred to as phase unwrapping is used to remove the 2π discontinuities in order to provide the necessary physical information.

Table 3-1 Equations of phase-shifting algorithms developed from phase-shifting interferometry.

Phase Step	Phase Angle	Equation for $\varphi(x, y)$
3	$60^\circ(0^\circ, 60^\circ, 120^\circ)$	$\varphi(x, y) = \tan^{-1} \frac{2I_1 - 3I_2 + I_3}{\sqrt{3}(I_2 - I_3)}$
3	$90^\circ(-90^\circ, 0^\circ, 90^\circ)$	$\varphi(x, y) = \tan^{-1} \frac{I_1 - I_3}{2I_2 - I_1 - I_3}$
3	$90^\circ(0^\circ, 90^\circ, 180^\circ)$	$\varphi(x, y) = \tan^{-1} \frac{I_1 - 2I_2 + I_3}{I_1 - I_3}$
3	$120^\circ(-120^\circ, 0^\circ, 120^\circ)$	$\varphi(x, y) = \tan^{-1} \frac{\sqrt{3}(I_1 - I_3)}{2I_2 - I_1 - I_3}$
3	$120^\circ(0^\circ, 120^\circ, 240^\circ)$	$\varphi(x, y) = \tan^{-1} \frac{\sqrt{3}(I_3 - I_2)}{2I_1 - I_2 - I_3}$
4	$60^\circ(0^\circ, 60^\circ, 120^\circ, 180^\circ)$	$\varphi(x, y) = \tan^{-1} \frac{5(I_1 - I_2 - I_3 + I_4)}{\sqrt{3}(2I_1 + I_2 - I_3 - 2I_4)}$
4	$90^\circ(0^\circ, 90^\circ, 180^\circ, 270^\circ)$	$\varphi(x, y) = \tan^{-1} \frac{I_4 - I_2}{I_1 - I_3}$
5	$90^\circ(0^\circ, 90^\circ, 180^\circ, 270^\circ, 360^\circ)$	$\varphi(x, y) = \tan^{-1} \frac{7(I_4 - I_2)}{4I_1 - I_2 - 6I_3 - I_4 + 4I_5}$

Table 3-2 Determination of the phase modulo 2π .

Numerator ($\sin\varphi$)	Denominator ($\cos\varphi$)	Adjusted phase	Range of phase values
$= 0$	> 0	0	0
> 0	> 0	φ	$0 - \pi/2$
> 0	$= 0$	$\pi/2$	$\pi/2$
> 0	< 0	$\pi - \varphi$	$\pi/2 - \pi$
$= 0$	< 0	π	π
< 0	< 0	$\pi + \varphi$	$\pi - 3\pi/2$
< 0	$= 0$	$3\pi/2$	$3\pi/2$
< 0	> 0	$2\pi - \varphi$	$3\pi/2 - 2\pi$

3.4 Phase-Unwrapping Techniques

The phase unwrapping process converts the modulo 2π phase data into its natural range, yielding is a continuous representation of the phase map, usually called the unwrapped phase map. Various phase unwrapping techniques have been developed (Macy 1983, Goldstein 1988, Judge 1994, Huntley 1998, and Ghiglia 1998) and new phase unwrapping algorithms are being proposed. Here the basic fundamentals of phase unwrapping techniques and the algorithm adopted to perform the phase unwrapping process are described.

Phase unwrapping is simple in concept. According to the sampling theorem–Nyquist criterion (there are at least two pixels per fringe period), the phase value difference (This should not be confused with the phase difference term, discussed in Section 3.5 and Chapter 5) between any two consecutive phase samples (pixels) should not change by more than $\pm\pi$. This criterion is used to reconstruct the phase map by successive comparisons of neighboring pixels. If the phase value difference calculated for two adjacent pixels exceeds

π or is smaller than $-\pi$, then 2π or multiples of 2π must be subtracted from or added to the calculated value of the second pixel until the phase difference is back to the range between $-\pi$ and π . Therefore, $\pm\pi$ is selected as the threshold, and the entire phase map is calculated by working outward from the starting location.

During the phase unwrapping process, traditionally the phase values at different pixel locations are sequentially unwrapped line by line following certain paths in the wrapped phase map. The simplest choices of those paths are straight rows and columns. In the case of one-dimensional signals, assuming that the phase value before phase unwrapping is $\varphi_w(x_i, y_i)$, and the phase value after phase unwrapping is $\varphi_u(x_i, y_i)$, define the phase value difference between any two neighboring pixels as:

$$\Delta\varphi_w(x_i, y) = \varphi_w(x_i, y) - \varphi_w(x_{i-1}, y) \quad (3-15)$$

The most basic strategy is as follows. The phase 2π must be added to or subtracted from the phase of all wrapped pixels according to the following conditions:

$$\begin{aligned} \text{if } \Delta\varphi_w(x_i, y) > \pi, \quad \text{then } \varphi_u(x_i, y) &= \varphi_w(x_i, y) - 2\pi \\ \text{if } \Delta\varphi_w(x_i, y) < -\pi, \quad \text{then } \varphi_u(x_i, y) &= \varphi_w(x_i, y) + 2\pi \end{aligned} \quad (3-16)$$

The above procedure will yield an unwrapped phase $\varphi_u(x, y)$, as shown in Figure 3-9. It is noted that multiples of 2π might be subtracted from or added to the calculated value of the second pixel based on the previous unwrapping process.

Phase unwrapping is a challenging step in phase-measurement techniques. Theoretically, phase unwrapping is simply carried out by comparing the wrapped phase $\varphi_w(x_i, y_i)$ at neighboring pixels and adding or subtracting multiples of 2π . However, in practice $\varphi_w(x_i, y_i)$ will be contaminated with measurement noise, such as local shadows, irregular surface brightness, surface anomalies, fringe discontinuities, and insufficient sampling frequencies, which make the unwrapping operation more complicated and may make the result inaccurate. All of these defects in a wrapped phase map may cause spurious phase jumps in the unwrapped phase map. Some unwrapping algorithms might not be able to

distinguish these phase jumps and may be fooled into adding or subtracting a value of 2π or multiples of 2π to the next pixel in the unwrapped phase map. This error may propagate over a large area and spoil many pixels. Figure 3-10 illustrates this phenomenon that results from a simple one-dimensional unwrapping process. Figure 3-10 (a) is a wrapped phase map. The 2π dynamic range is represented in gray levels. Black represents the phase value of 0, and white the value of 2π . All other gray levels represent intermediate and linearly mapped phase values. The effect of the phase unwrapping operation is shown in Figure 3-10 (b). Due to the limitation of this one-dimensional unwrapping algorithm and measurement noise, the unwrapping error propagates over a large area. Figure 3-10 (c) is the phase difference map, from which unwrapping error propagation is clearly illustrated. More detailed explanation about phase difference will be given in Section 3.5.

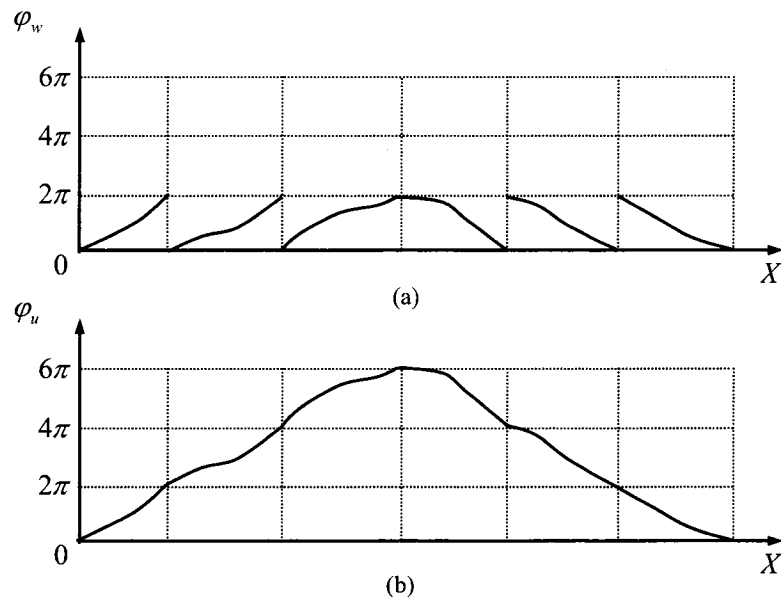


Figure 3-9 Phase-shifting unwrapping algorithm. (a) One-dimensional phase wrapping, (b) One-dimensional phase unwrapping.

An example of a two-dimensional phase unwrapping process is given in Figure 3-11. From the figure it can be seen that the wrapped phase map in (a) was properly unwrapped, as shown in (b).

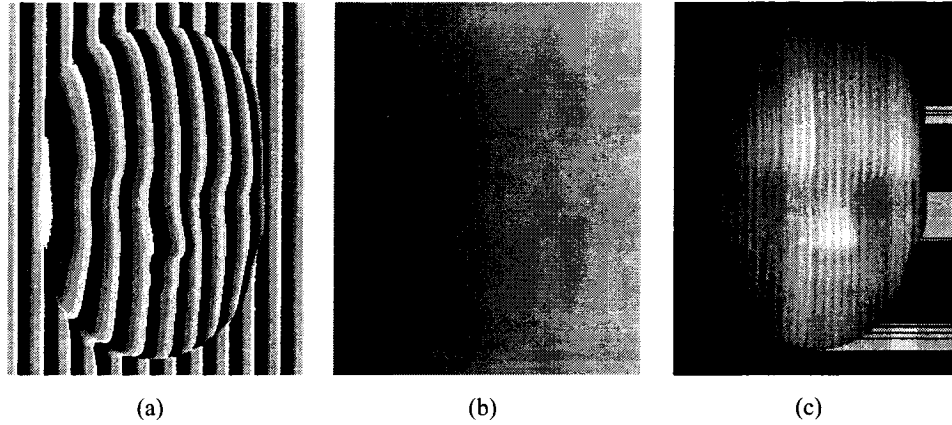


Figure 3-10 One-dimensional phase-unwrapping process result. (a) Wrapped phase map, (b) Unwrapped phase map with unwrapping error propagation, and (c) Phase difference map.

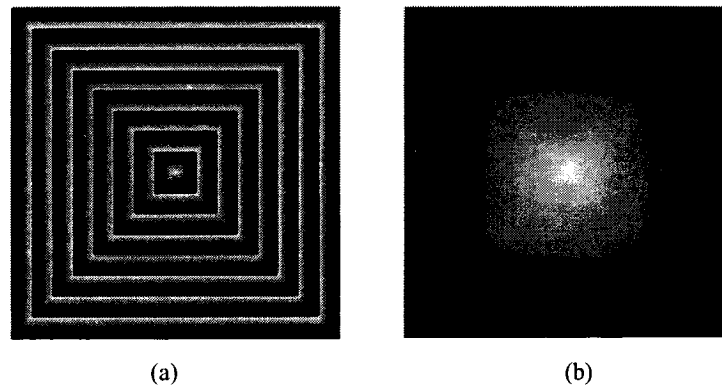


Figure 3-11 Two dimensional phase-unwrapping process result. (a) Wrapped phase map and (b) Unwrapped phase map (Marklund 1998).

In general, different unwrapping paths result in different unwrapped phases at a specific location. These inconsistencies are caused by abnormal points where spurious phase jumps occur. These points are called *residues* (Goldstein 1988), and their locations can be determined using the procedure shown in Figure 3-12. This is called path-dependence of the unwrapping process. As the unwrapped phase represents a physical quantity of the measured object and therefore should be single-valued, path-dependence is definitely undesired in the phase unwrapping process. Therefore, the ability to differentiate spurious jumps from true jumps for a phase unwrapping algorithm is very important. Moreover, the phase unwrapping

process should follow adaptively changing paths to circumvent those problematic phase data. For most unwrapping problems, the central task is to find the best unwrapping paths.

The existences of residues in the phase maps make the phase unwrapping path-dependent. If there are no residues, the phase can be unwrapped along any path, and the result will be independent of the path. To remove the impact of the residues, the residues are connected by a line or branch cut, which balances the number of positive and negative residues. This results in making the unwrapped phase the same for all allowed paths.

In this work, Goldstein's approach (Goldstein 1988), which is effective and extremely fast, is applied for phase unwrapping. The basic idea of Goldstein's method is to identify all residues in the wrapped phase map, generating the branch cuts put on between the residues to balance the residues, and then integrating around the branch cuts.

To detect the residual point's location, the procedure shown in Figure 3-12 is used. This figure shows a small part of a phase map consisting of the phase $\varphi(i, j)$ and its surrounding phase values. The following variables that present the wrapped phase differences around the closed path (Goldstein 1988) are defined.

$$\left\{ \begin{array}{l} \Delta_1 = \frac{\varphi(i, j+1) - \varphi(i, j)}{2\pi} \\ \Delta_2 = \frac{\varphi(i+1, j+1) - \varphi(i, j+1)}{2\pi} \\ \Delta_3 = \frac{\varphi(i+1, j) - \varphi(i+1, j+1)}{2\pi} \\ \Delta_4 = \frac{\varphi(i, j) - \varphi(i+1, j)}{2\pi} \end{array} \right. \quad (3-17)$$

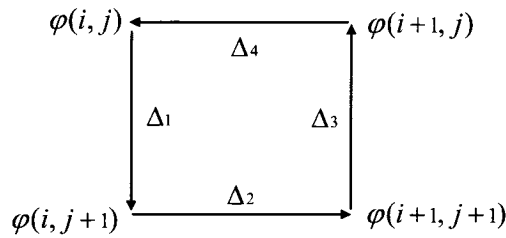


Figure 3-12 Small portion of a typical array of wrapped phase values.

The residue at point (i, j) can be calculated by summing the phase differences around the closed path as indicated in Figure 3-12 from the following equation (Goldstein 1988):

$$r(i, j) = \sum_{k=1}^4 \Delta_k \quad (3-18)$$

The points with a zero residue are normal points, which means no inconsistency or residue is present. Otherwise, they are abnormal points where path-independent unwrapping is not possible. Points with positive or negative value of residue are labeled positive or negative charge or polarity. The necessary and sufficient condition for path-independent phase unwrapping is that the phase-unwrapping paths never run across branch cuts connecting a pair of residual points with opposite polarities. The path-independent phase unwrapping is thus reduced to a problem of finding a good network of branch cuts.

In practice, the phase image is scanned pixel by pixel until a residue is detected. This residue is then marked and scanning continues to search for the next residue. A branch cut is placed between two residues no matter if they have the same polarity or not. If the residues have opposite polarities, they are designated “balanced” by the branch cut. Otherwise, the search for the residues continues. Whenever a residue is encountered during the search of the residue, it is always connected by a branch cut to the just found residue even if this residue has already been joined to some other residue by a branch cut. For those residues that have not yet been connected to other residues, their polarities are added to the sum of the polarities of the other residues. Otherwise, their polarities are not added. When the cumulative “charge” reaches zero, these residues are designated as balanced. After all residues are identified and balanced by the branch cuts, the phase can be unwrapped along any path that does not cross the branch cuts.

3.5 Phase-Measuring Profilometry

After performing the phase unwrapping process, a continuous phase distribution can be obtained. The measured phase map contains the height information of the 3-D object surface

(Halioua 1989). Therefore, a phase-to-height conversion algorithm is usually necessary to reconstruct the 3-D coordinates of the object surface. This algorithm is usually related to the system setup and to the relationship between the phase distribution and height of the object surface. The system parameters include the angle between the axes of the projector and camera, distance between the camera lens and the CCD sensor, distance between camera and reference plane and fringe frequency. Direct determination of these system parameters (Hu 2003) would be tedious and difficult to measure accurately. Alternatively, system calibration techniques have been developed to obtain the mapping relationship between the phase distribution and the 3-D object-surface coordinates, without explicitly determining the system-geometry parameters (Hu 2003). Instead, calibration parameters, which implicitly account for the system geometry, are determined. Based on geometric analysis of the measurement system, several phase-to-height mapping techniques (Zhou 1994, Chen 2000b, Hung 2000, Li 2001, Zhang 2002, Liu 2003, Guo 2005, and Sitnik 2005) have been developed.

One of the simplest methods to convert the phase to the height of the 3-D object surface is phase-measuring profilometry, which establishes the relationship between the object surface profile and phase map. Figure 3-13 shows the relationship between the phase of the projected fringe pattern and the height of the object. Point P is the center of the exit pupil of the projector, and point E is the center of the entrance pupil of the camera. The position at $z = 0$ in the coordinate system is defined as the reference plane. Points P and E are assumed to be in the same plane with a distance H to the reference plane. The distance between points P and E is d . The projector projects a fringe pattern which has a pitch p on the reference plane. The phase at point C is φ_C and at point A is φ_A .

In Figure 3-13, because $\triangle DPE$ and $\triangle DAC$ are similar, the following expression can be obtained:

$$\frac{d}{AC} = \frac{H-h}{h} \quad (3-19)$$

where h is the distance of point D on the object surface with respect to the reference plane. AC is the distance between points A and C .

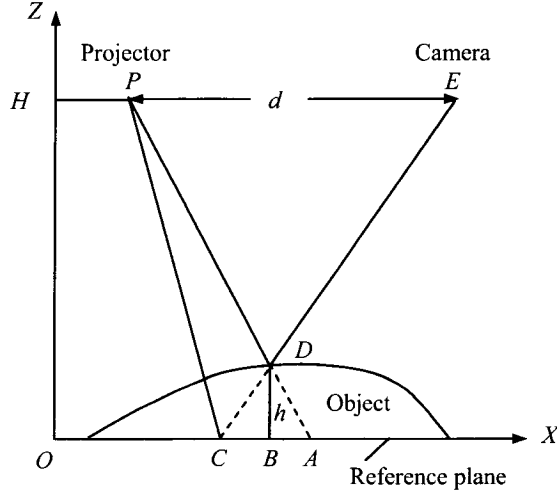


Figure 3-13 Relationship between the phase of the projected fringe pattern and height of the object.

For the reference plane, the following expression holds:

$$\frac{\overline{AC}}{p} = \frac{\varphi_A - \varphi_C}{2\pi} \quad (3-20)$$

where p is the fringe pitch on the reference plane.

Define $\Delta\varphi = \varphi_A - \varphi_C$ as the phase difference between points A and C . Then by combining Equations (3-19) and (3-20), the following expression is obtained:

$$h = \frac{H}{1 + \frac{2\pi d}{p\Delta\varphi}} \quad (3-21)$$

Equation (3-21) states that the distribution of the height of the object surface relative to the reference plane is a function of the distribution of the phase difference. During measurement, the reference plane is measured first and the phase distribution is obtained by phase wrapping and unwrapping computations. The measurement result is then used as a

reference for the object measurement, and the height of the object surface is measured relative to the reference plane. When measuring the object, on the CCD array, point D on the object surface will be imaged onto the same pixel as point C on the reference plane. As point D on the object surface has the same phase value as point A on the reference plane, $\varphi_D = \varphi_A$, by subtracting the reference phase map from the object phase map, $\Delta\varphi = \varphi_D - \varphi_C$, the phase difference $\Delta\varphi = \varphi_A - \varphi_C$ at this specific pixel can be easily obtained. A similar calculation can be done for the whole phase map.

To calculate the height of the object surface by using Equation (3-21), in addition to the phase difference $\Delta\varphi$ discussed above, other system related parameters are also needed to be determined. However, these parameters are not easy to measure, especially for pitch p on the reference plane. Therefore, system calibration, described in Chapter 5, is necessary to determine the system related parameters. Once the phase difference and system related parameters are obtained, the complete height distribution of the object surface relative to the reference plane can be obtained.

Equation (3-21) describes a non-linear relationship between the distributions of the height of the object surface and the phase difference. To carry out system calibration, a modified form of Equation (3-21) using linear and non-linear relationships between the distributions of the object height and phase difference is developed below, and the effect of these changes in formulation on measurement accuracy is analyzed.

3.5.1 Linear Mapping

In Equation (3-21), only when H is much larger than h (or when d is much larger than $\overline{AC}$), which is often true in general, Equation (3-21) can be simplified as:

$$h \approx \frac{pH}{2\pi d} \Delta\varphi \quad (3-22)$$

Thus, an approximate linear relationship between the phase difference map and the surface height of the object is derived. However, Equation (3-21) was obtained only by considering that the points P, E, C, A, D are located in the same X - Z plane (Figure 3-13). Actually, the object extends in the Y direction. This means that the parameter H is not a

constant parameter; it is a function of the X and Y coordinates. Therefore, considering the X - Y dimensions, the phase-to-height mapping function, Equation (3-22), for calculating the surface height of the object relative to the reference plane can be written as:

$$h(x, y) = K(x, y)\Delta\varphi(x, y) \quad (3-23)$$

where $K(x, y) = \frac{p(x, y)H(x, y)}{2\pi d}$. $K(x, y)$ is a coefficient of the optical setup and a function of (x, y) . The phase difference $\Delta\varphi(x, y)$ can be calculated by:

$$\Delta\varphi(x, y) = \varphi(x, y) - \varphi_r(x, y) \quad (3-24)$$

where $\varphi(x, y)$ is the distorted fringe phase distribution of the object surface, $\varphi_r(x, y)$ is the reference fringe phase distribution taken from a planar reference plane, and $\Delta\varphi(x, y)$ is the phase difference between $\varphi(x, y)$ and $\varphi_r(x, y)$. Both $\varphi(x, y)$ and $\varphi_r(x, y)$ can be obtained for any image point from Equation (3-14) by applying a phase-shifting technique (Creath 1988 and Greivenkamp 1992). Coefficient $K(x, y)$ can be determined from a system calibration, discussed in Chapter 5. For surface-geometry measurement of an unknown object, once $\Delta\varphi(x, y)$ is obtained by applying Equation (3-24), h , the height of the object, can be determined for any image point using Equation (3-23). If $K(x, y)$ and $h(x, y)$ are known for any point (x, y) , rearrangement of Equation (3-23) allows the phase difference to be calculated at that point as follows:

$$\Delta\varphi(x, y) = \frac{h(x, y)}{K(x, y)} \quad (3-25)$$

This will be applied in the experiments discussed in Chapter 5.

3.5.2 Non-Linear Mapping

To obtain the non-linear relationship between the phase difference map and the surface height of the object, Equation (3-21) can be rearranged as follows:

$$\Delta\varphi = \frac{\frac{2\pi d}{pH} h}{1 - \frac{1}{H} h} \quad (3-26)$$

Considering the x - y dimensions, Equation (3-26) can be expressed simply as:

$$\Delta\varphi(x, y) = \frac{m(x, y)h(x, y)}{1 - n(x, y)h(x, y)} \quad (3-27)$$

where $m(x, y) = \frac{2\pi d}{p(x, y)H(x, y)}$, $n(x, y) = \frac{1}{H(x, y)}$; $m(x, y)$ and $n(x, y)$ are the system related parameters relating to the optical setup, and x, y are the pixel coordinates. Equation (3-27) can be rewritten as the following phase-to-height mapping function:

$$h(x, y) = \frac{\Delta\varphi(x, y)}{m(x, y) + n(x, y)\Delta\varphi(x, y)} \quad (3-28)$$

This is the non-linear relationship between the phase-difference map and the surface height of the object. Some researchers obtained a similar form of equations by complicated coordinate transformation and geometrical derivation (Zhou 1994, Liu 2002, and Guo 2005). However, here it has been obtained by a rather simple derivation.

3.6 Summary

Based on the proposed 3-D shape measurement approaches, the appropriate hardware for the system was determined. The structure and functions of the off-line 3-D shape measurement system were developed. The design of the calibration setup for determining

unknown system parameters was implemented. Based on the determined work volume and the design requirements, iterative calculations were carried out in Microsoft® Excel spreadsheets to determine the most practical and optimized layout parameters for the system. The phase-measuring algorithms, including phase-wrapping, phase-unwrapping, and phase-to-height conversion, have been analyzed and implemented in the system. The relationship between the phase difference and the height of an object surface was formulated as linear and non-linear calibrations of a fringe-projection phase-measuring system for 3-D surface-geometry measurement. A simplified geometrical derivation was used for the non-linear method.

4 Triangular-Pattern Phase-Shifting Profilometry

Based on digital fringe-projection, intensity-ratio, and phase-shifting techniques, a novel 3-D shape measurement approach, *triangular-pattern phase-shifting profilometry*, is presented in this research. This chapter addresses this technique in detail.

4.1 Fringe-Projection Technique Difficulties

In 3-D shape measurement using fringe-projection techniques, the projection of different patterns results in different performances. For most fringe projection techniques, much (often most) time is spent on phase and phase-to-height conversion calculations. For real-time 3-D shape measurement, it is not sufficient just to speed up the pattern projection and image acquisition by means of efficient hardware. The system must employ a suitable pattern for fast and efficient manipulation.

The traditional sinusoidal-pattern phase-shifting method works well in terms of measurement accuracy. However, the calculation of the arctangent function is time consuming and slows down the measurement speed. The common intensity-ratio method (Carrihill 1985, Chazan 1995, and Miyasaka 2002) has the advantage of fast processing speed, but the problem of ambiguity arises for measuring objects with discontinuous surface shape if the intensity-ratio ramp is repeated (Chazan 1995) to increase robustness to measurement noise. In order to apply the advantages of the traditional sinusoidal-pattern phase-shifting method and the common intensity-ratio method, the two methods are combined and a novel triangular-pattern phase-shifting approach for 3-D shape measurement is proposed in this research.

The remainder of this chapter is organized as follows. Section 4.2 summarizes the proposed triangular-pattern phase-shifting method first and then describes the algorithms for two-, three-, four-, five-, and six-step phase-shifting to retrieve the intensity-ratio distributions. Section 4.3 describes multiple triangular-pattern phase-shifting and intensity-ratio unwrapping. Section 4.4 addresses intensity-ratio-to-height conversion algorithms.

Section 4.5 presents accuracy-analysis experiments of the proposed method. Section 4.6 presents the object-shape measurement experiments and compares each method, and a summary is given in Section 4.7.

4.2 *Triangular-Pattern Phase-Shifting Technique*

4.2.1 Overview of the Triangular-Pattern Phase-Shifting Method

The triangular-pattern phase-shifting method is summarized here and then detailed below. In the proposed novel triangular-pattern phase-shifting method, a triangular gray-level coded pattern (rather than sinusoidal pattern) is generated by software and used for the projection. The triangular pattern is phase-shifted by two, three, four steps, etc. to reconstruct the 3-D object. The phase-shifting algorithms to generate the intensity ratio, essential for surface reconstruction, were developed for two-, three-, four-, five-, and six-step measurement methods. The algorithms can also be generalized to any N -step method for higher accuracy. A triangular shape intensity-ratio distribution (rather than phase distribution) is obtained by calculation of the captured triangular-pattern images. Removal of the triangular shape of the intensity-ratio over the full pattern pitch generates an intensity-ratio ramp. Measurement resolution and accuracy can be increased by using multiple triangular fringes. In this case, the intensity-ratio is wrapped into the range 0 to 4, 0 to 6, 0 to 8, etc, for the different phase-shifting methods, respectively. The unwrapped intensity-ratio distribution is obtained by removing the discontinuity of the wrapped image with an unwrapping method (Goldstein 1988) modified from that used in the sinusoidal-pattern phase-shifting method. An intensity-ratio-to-height conversion algorithm, based on a traditional phase-to-height conversion algorithm in the sinusoidal-pattern phase-shifting method, is then used to reconstruct the 3-D surface geometry of the object. Compared with the traditional sinusoidal-pattern phase-shifting-based method with the same numbers of phase shifts, the processing speed is expected to be faster with similar resolution. Compared with the traditional intensity-ratio-based method (Carrihill 1985 and Miyasake 2002), it has a better depth resolution and less ambiguity problem when the triangular pattern is repeated (Chazan 1995) to reduce the sensitivity to measurement noise.

In the following sections, the novel aspects of the algorithms for the different number of phase-shifting steps of the triangular-pattern phase-shifting method will be presented in detail.

4.2.2 Two-Step Triangular-Pattern Phase-Shifting Algorithm

Two triangular patterns phase-shifted by half of the pitch are needed to reconstruct the 3-D object in the two-step triangular-pattern phase-shifting method. The intensity distribution of the two-step phase-shifted triangular fringe patterns are shown in Figure 4-1 (a) and (b). The intensity equations for the two shifted triangular patterns are formulated as follows:

$$I_1(x, y) = \begin{cases} \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{I_m(x, y)}{2} & x \in [0, \frac{T}{4}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{3I_m(x, y)}{2} & x \in [\frac{T}{4}, \frac{3T}{4}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{3I_m(x, y)}{2} & x \in [\frac{3T}{4}, T) \end{cases} \quad (4-1)$$

$$I_2(x, y) = \begin{cases} -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{I_m(x, y)}{2} & x \in [0, \frac{T}{4}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{I_m(x, y)}{2} & x \in [\frac{T}{4}, \frac{3T}{4}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{5I_m(x, y)}{2} & x \in [\frac{3T}{4}, T) \end{cases} \quad (4-2)$$

$$I_m(x, y) = I_{\max}(x, y) - I_{\min}(x, y) \quad (4-3)$$

where $I_1(x, y)$ and $I_2(x, y)$ are the intensities for the two shifted triangular patterns respectively, T is the pitch of the patterns, $I_m(x, y)$ is the intensity modulation, and $I_{\max}(x, y)$ and $I_{\min}(x, y)$ are the maximum and minimum intensities of the two triangular patterns, respectively. The intensity difference between the two patterns $I_1(x, y) - I_2(x, y)$

can be computed and then normalized by the intensity modulation $I_m(x, y)$ as intensity-ratio $r_o(x, y)$:

$$r_o(x, y) = \frac{|I_1(x, y) - I_2(x, y)|}{I_m(x, y)} \quad (4-4)$$

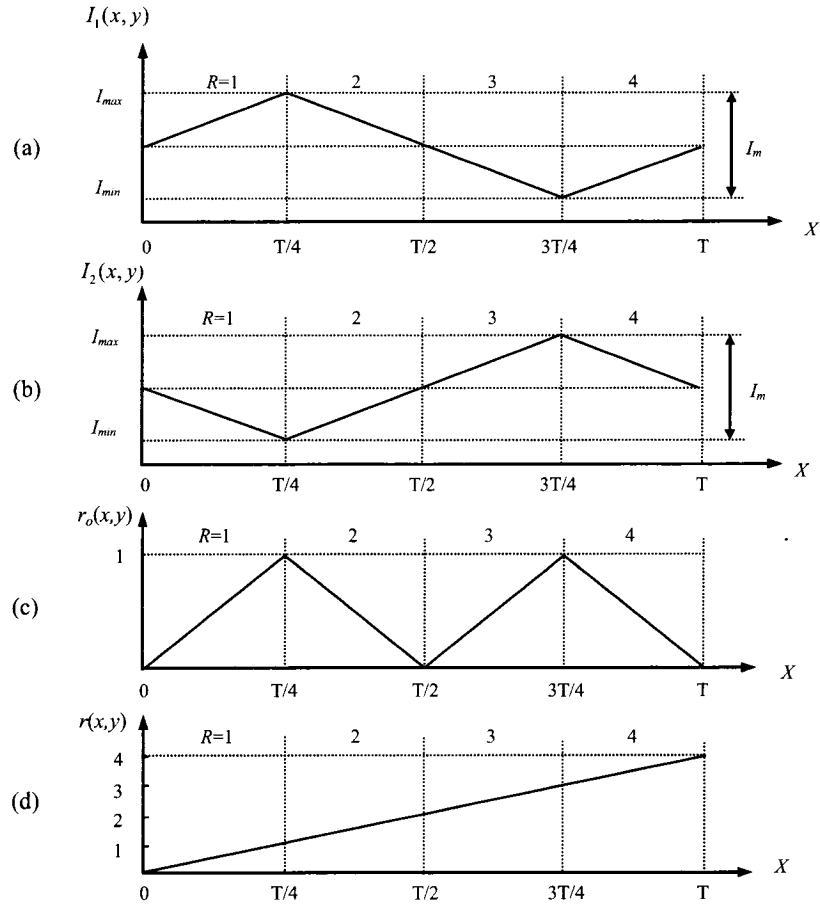


Figure 4-1 Two-step triangular-pattern phase-shifting algorithm. (a) and (b) Two phase-shifted triangular fringe patterns, (c) Intensity-ratio in a repeated triangular shape, and (d) Intensity-ratio ramp after removal of the repeated triangular shape.

After the operation by Equation (4-4), the triangular pattern is divided into four regions (Figure 4-1). Each region has different intensity-ratio patterns. The shape of the intensity-ratio $r_o(x, y)$ over the full pitch has a periodic triangular shape with values ranging from 0 to 1, as shown in Figure 4-1 (c). This periodic triangular shape can be converted to a ramp by applying the following equation:

$$r(x, y) = 2 \times \text{round}\left(\frac{R-1}{2}\right) + (-1)^{R+1} r_o(x, y) \quad R = 1, 2, 3, 4 \quad (4-5)$$

where R is the region number. The converted intensity-ratio ramp map $r(x, y)$, shown in Figure 4-1 (d), has intensity values ranging from 0 to 4.

4.2.3 Three-Step Triangular-Pattern Phase-Shifting Algorithm

Three triangular patterns phase-shifted by one-third of the pitch from each other are needed to reconstruct the 3-D object in the three-step triangular-pattern phase-shifting method. Figure 4-2 shows the cross sections of the three-step phase-shifted triangular fringe patterns. Figure 4-3 shows the three-step phase-shifted triangular fringe patterns separately as they correspond to the intensity equations. The intensity equations for the three shifted triangular patterns are formulated as follows:

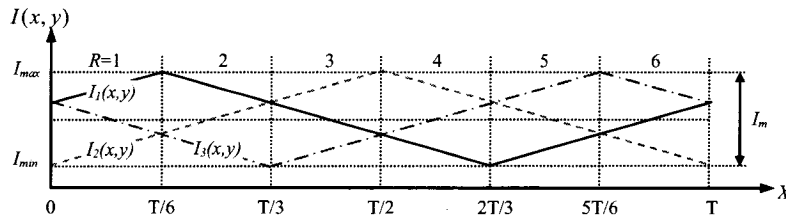


Figure 4-2 Cross sections of the three-step phase-shifted triangular fringe patterns.

$$I_1(x, y) = \begin{cases} \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{2I_m(x, y)}{3} & x \in [0, \frac{T}{6}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{4I_m(x, y)}{3} & x \in [\frac{T}{6}, \frac{2T}{3}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{4I_m(x, y)}{3} & x \in [\frac{2T}{3}, T) \end{cases} \quad (4-6)$$

$$I_2(x, y) = \begin{cases} \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) & x \in [0, \frac{T}{2}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + 2I_m(x, y) & x \in [\frac{T}{2}, T) \end{cases} \quad (4-7)$$

$$I_3(x, y) = \begin{cases} -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{2I_m(x, y)}{3} & x \in [0, \frac{T}{3}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{2I_m(x, y)}{3} & x \in [\frac{T}{3}, \frac{5T}{6}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{8I_m(x, y)}{3} & x \in [\frac{5T}{6}, T) \end{cases} \quad (4-8)$$

$$I_m(x, y) = I_{\max}(x, y) - I_{\min}(x, y) \quad (4-9)$$

where $I_1(x, y)$, $I_2(x, y)$ and $I_3(x, y)$ are the intensities for the three shifted triangular patterns respectively, T is the pitch of the patterns, $I_m(x, y)$ is the intensity modulation, and $I_{\max}(x, y)$ and $I_{\min}(x, y)$ are the maximum and minimum intensities of the three triangular patterns, respectively. The intensity-ratio $r_o(x, y)$ can be normalized by:

$$r_o(x, y) = \frac{I_{\text{high}}(x, y) - I_{\text{med}}(x, y) + I_{\text{low}}(x, y) + I_m(x, y)}{I_m(x, y)} \quad (4-10)$$

where $I_{\text{high}}(x, y)$, $I_{\text{med}}(x, y)$ and $I_{\text{low}}(x, y)$ are the highest, median and lowest intensities of the three shifted triangular patterns at the same position in the range of the pitch, respectively. After the operation by Equation (4-10), the triangular pattern is divided into six

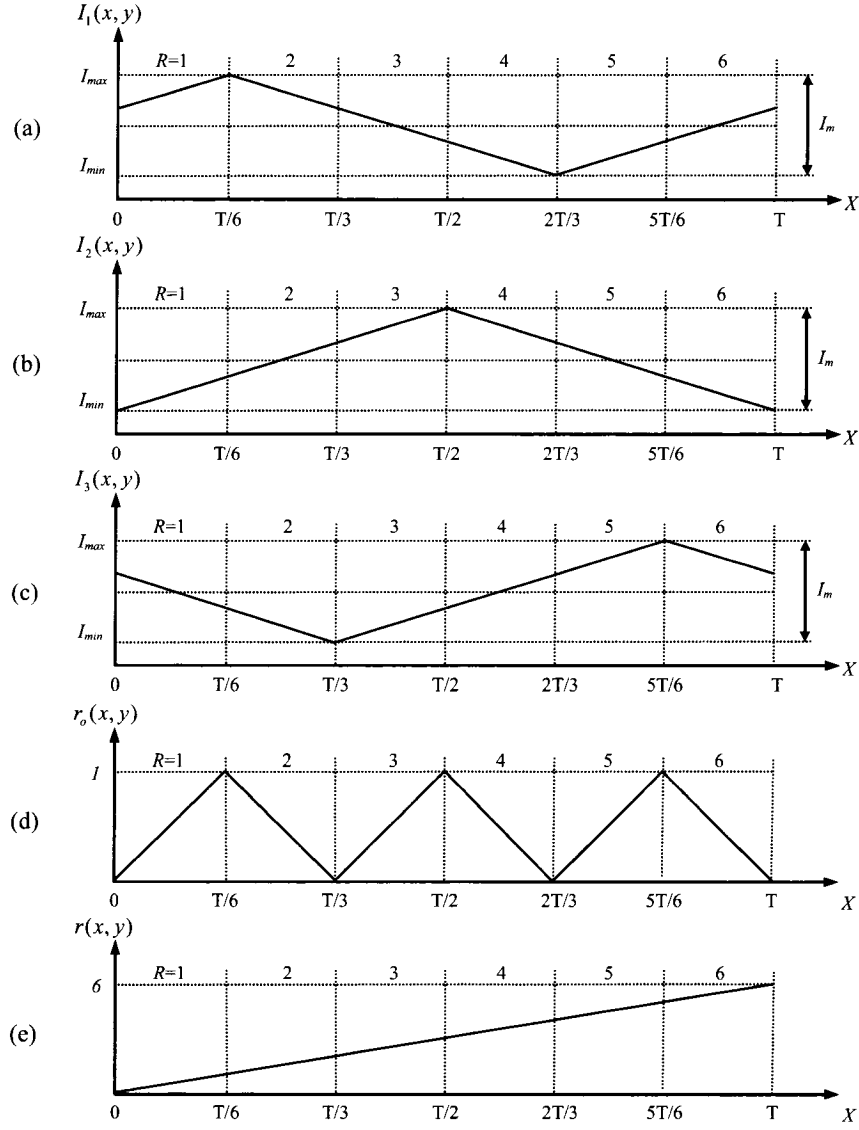


Figure 4-3 Three-step triangular-pattern phase-shifting algorithm. (a), (b) and (c) Three phase-shifted triangular fringe patterns, (d) Intensity-ratio in a periodic triangular shape, and (e) Intensity-ratio ramp after the removal of the periodic triangular shape.

regions, each with a different intensity-ratio pattern. The intensity ratio distribution $r_0(x, y)$ over the full pitch, shown in Figure 4-3 (d), is similar to that for the two-step triangular-pattern phase-shifting method, shown in Figure 4-1 (c) with values ranging from 0 to 1, but

with three repeated triangles and six regions over the full pitch. The repeated triangles can be converted to a ramp by applying Equation (4-5) with the region number R ranging from 1 to 6, as shown in Figure 4-3 (e).

4.2.4 Four-Step Triangular-Pattern Phase-Shifting Algorithm

Four triangular patterns phase-shifted by one-fourth of the pitch from each other are needed to reconstruct the 3-D object in the four-step triangular-pattern phase-shifting method. Figure 4-4 shows the cross sections of the four-step phase-shifted triangular fringe patterns. Figure 4-5 shows the four-step phase-shifted triangular fringe patterns separately as they correspond to the intensity equations. The intensity equations for the four shifted triangular patterns are formulated as follows:

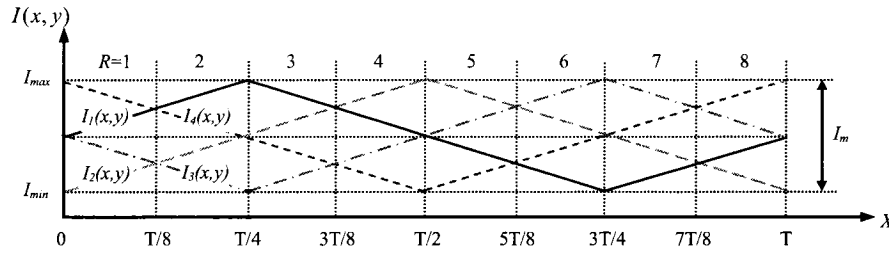


Figure 4-4 Cross sections of the four-step phase-shifted triangular fringe patterns.

$$I_1(x, y) = \begin{cases} \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{I_m(x, y)}{2} & x \in [0, \frac{T}{4}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{3I_m(x, y)}{2} & x \in [\frac{T}{4}, \frac{3T}{4}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{3I_m(x, y)}{2} & x \in [\frac{3T}{4}, T) \end{cases} \quad (4-11)$$

$$I_2(x, y) = \begin{cases} \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) & x \in [0, \frac{T}{2}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + 2I_m(x, y) & x \in [\frac{T}{2}, T) \end{cases} \quad (4-12)$$

$$I_3(x, y) = \begin{cases} -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{I_m(x, y)}{2} & x \in [0, \frac{T}{4}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{I_m(x, y)}{2} & x \in [\frac{T}{4}, \frac{3T}{4}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{5I_m(x, y)}{2} & x \in [\frac{3T}{4}, T) \end{cases} \quad (4-13)$$

$$I_4(x, y) = \begin{cases} -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + I_m(x, y) & x \in [0, \frac{T}{2}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - I_m(x, y) & x \in [\frac{T}{2}, T) \end{cases} \quad (4-14)$$

$$I_m(x, y) = I_{\max}(x, y) - I_{\min}(x, y) \quad (4-15)$$

where $I_1(x, y)$, $I_2(x, y)$, $I_3(x, y)$ and $I_4(x, y)$ are the intensities for the four shifted triangular patterns respectively, T is the pitch of the patterns, $I_m(x, y)$ is the intensity modulation, and $I_{\max}(x, y)$ and $I_{\min}(x, y)$ are the maximum and minimum intensities of the four triangular patterns, respectively. The intensity-ratio $r_o(x, y)$ can be normalized by:

$$r_o(x, y) = \frac{\|I_1(x, y) - I_3(x, y)\| - \|I_2(x, y) - I_4(x, y)\|}{I_m(x, y)} \quad (4-16)$$

After the operation by Equation (4-16), the triangular pattern is divided into eight regions, each with a different intensity ratio. The shape of intensity-ratio $r_o(x, y)$ distribution over the full pitch is shown in Figure 4-5 (e), and again has a periodic triangular shape with values ranging from 0 to 1. However, the distribution is different from the other triangular-pattern phase-shifting methods. This periodic triangular shape can be converted to a ramp by applying the following equation:

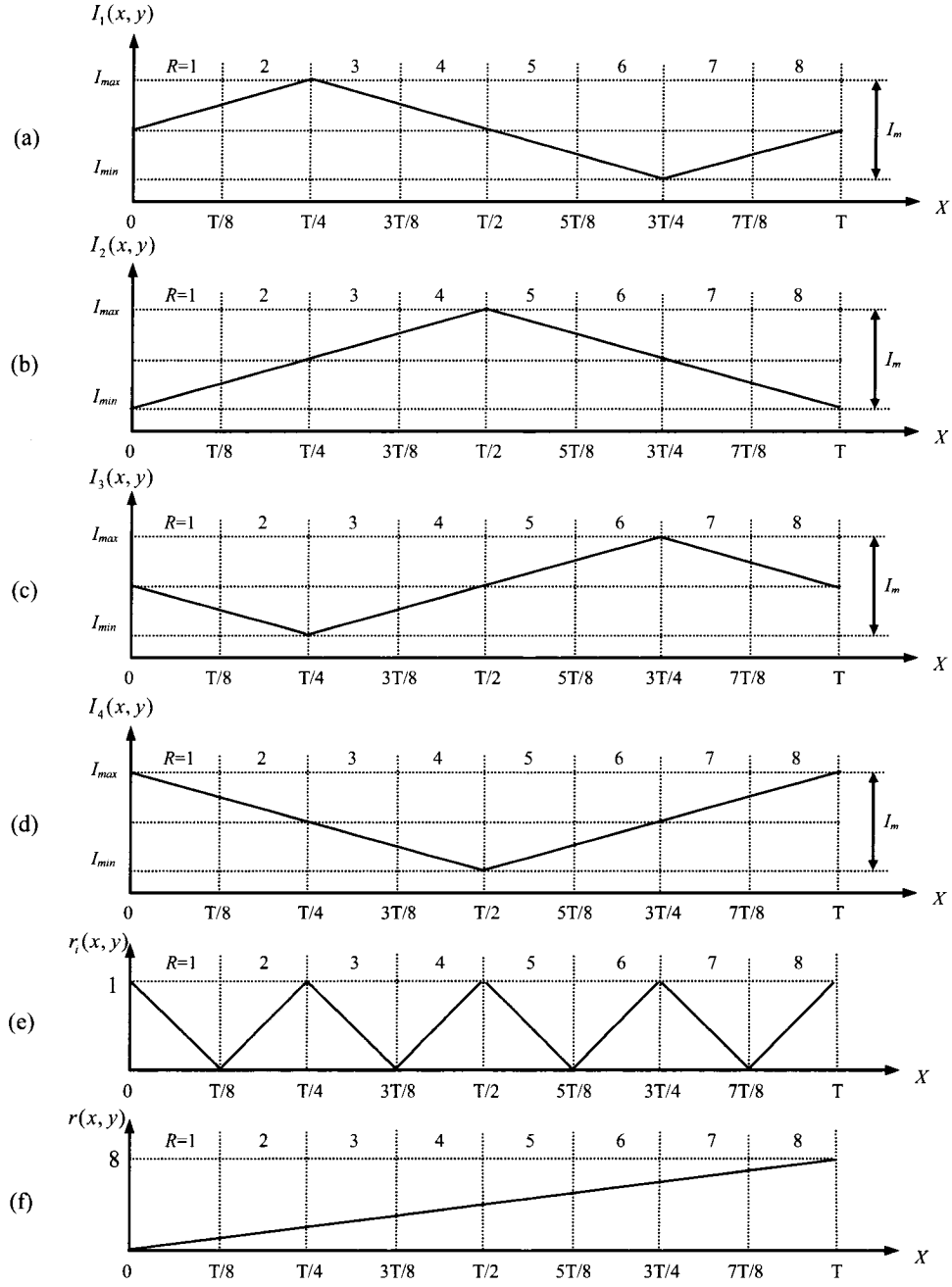


Figure 4-5 Four-step triangular-pattern phase-shifting algorithm. (a) - (d) Four phase-shifted triangular fringe patterns, (e) Intensity-ratio in a periodic triangular shape, and (f) Intensity-ratio ramp after the removal of the periodic triangular shape.

$$r(x, y) = 2 \times \text{round}\left(\frac{R}{2}\right) - 1 - (-1)^{R+1} r_o(x, y) \quad R = 1, 2, 3, \dots, 8 \quad (4-17)$$

where R is the region number. The converted intensity-ratio ramp map $r(x, y)$, shown in Figure 4-5 (f), has intensity values ranging from 0 to 8.

4.2.5 Five-Step Triangular-Pattern Phase-Shifting Algorithm

Five triangular patterns phase-shifted by one-fifth of the pitch from each other are needed to reconstruct the 3-D object in the five-step triangular-pattern phase-shifting method. Figure 4-6 shows the cross sections of the five-step phase-shifted triangular fringe patterns. Figure 4-7 shows the five-step phase-shifted triangular fringe patterns separately as they correspond to the intensity equations. The intensity equations for the five shifted triangular patterns are formulated as follows:

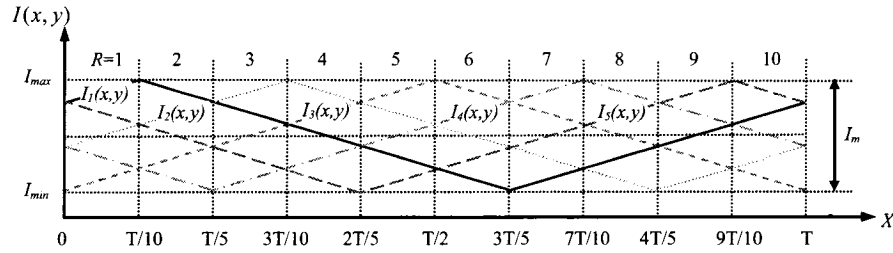


Figure 4-6 Cross sections of the five-step phase-shifted triangular fringe patterns.

$$I_1(x, y) = \begin{cases} \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{4I_m(x, y)}{5} & x \in [0, \frac{T}{10}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{6I_m(x, y)}{5} & x \in [\frac{T}{10}, \frac{3T}{5}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{6I_m(x, y)}{5} & x \in [\frac{3T}{5}, T) \end{cases} \quad (4-18)$$

$$I_2(x, y) = \begin{cases} \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{2I_m(x, y)}{5} & x \in [0, \frac{3T}{10}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{8I_m(x, y)}{5} & x \in [\frac{3T}{10}, \frac{4T}{5}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{8I_m(x, y)}{5} & x \in [\frac{4T}{5}, T) \end{cases} \quad (4-19)$$

$$I_3(x, y) = \begin{cases} \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) & x \in [0, \frac{T}{2}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + 2I_m(x, y) & x \in [\frac{T}{2}, T) \end{cases} \quad (4-20)$$

$$I_4(x, y) = \begin{cases} -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{I_m(x, y)}{5} & x \in [0, \frac{T}{5}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{I_m(x, y)}{5} & x \in [\frac{T}{5}, \frac{7T}{10}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{12I_m(x, y)}{5} & x \in [\frac{7T}{10}, T) \end{cases} \quad (4-21)$$

$$I_5(x, y) = \begin{cases} -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{4I_m(x, y)}{5} & x \in [0, \frac{2T}{5}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{4I_m(x, y)}{5} & x \in [\frac{2T}{5}, \frac{9T}{10}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{14I_m(x, y)}{5} & x \in [\frac{9T}{10}, T) \end{cases} \quad (4-22)$$

$$I_m(x, y) = I_{\max}(x, y) - I_{\min}(x, y) \quad (4-23)$$

All parameters in the above equations have the same meanings as in the other triangular-pattern phase-shifting methods described above. The intensity-ratio $r_o(x, y)$ can be normalized by:

$$r_o(x, y) = \frac{I_{high}(x, y) - I_{med1}(x, y) + I_{med2}(x, y) - I_{med3}(x, y) + I_{low}(x, y) - I_{\min}(x, y)}{I_m(x, y)} \quad (4-24)$$

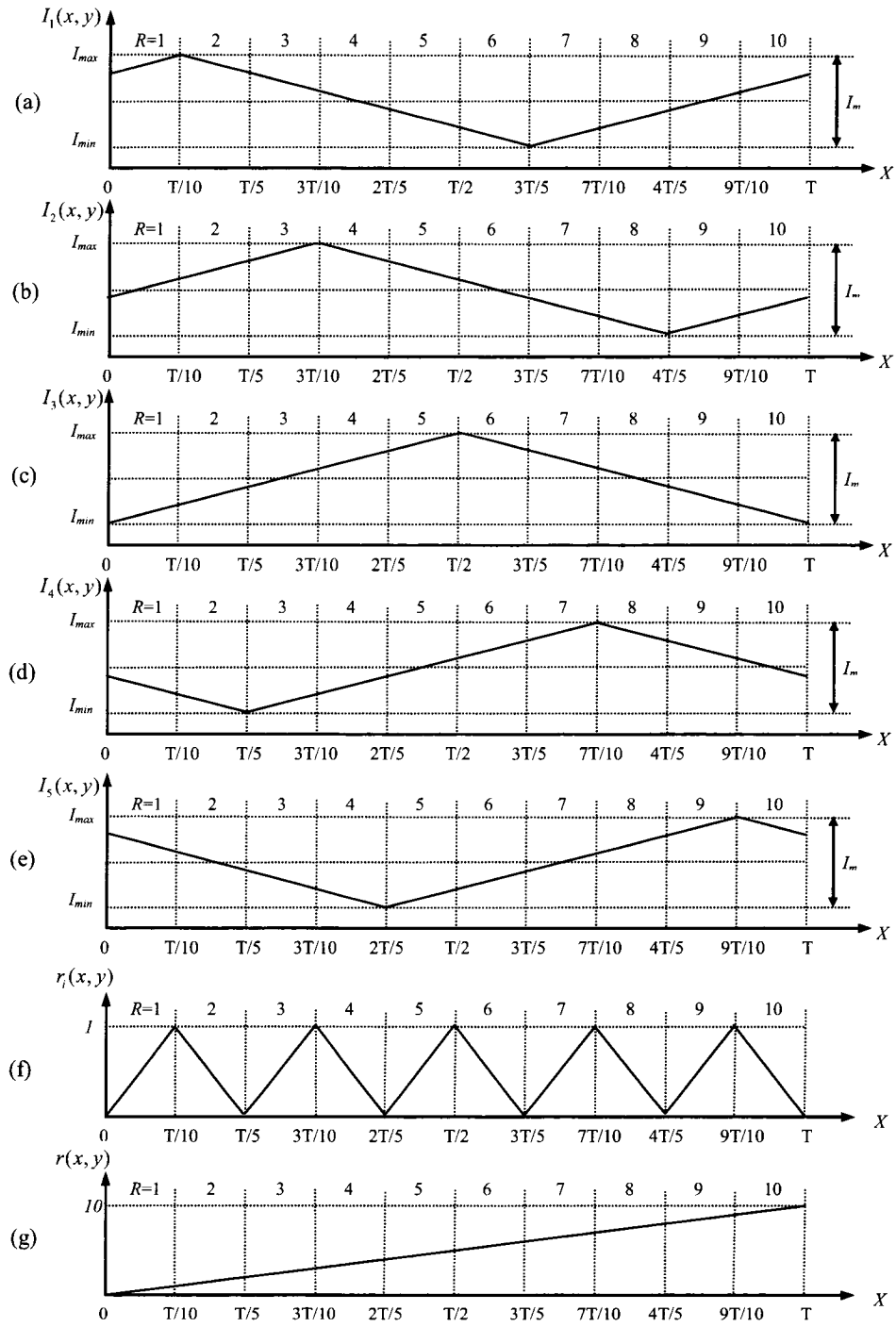


Figure 4-7 Five-step triangular-pattern phase-shifting algorithm. (a) - (e) Five phase-shifted triangular fringe patterns, (f) intensity-ratio in a repeated triangular shape, and (g) intensity-ratio ramp after removal of the repeated triangular shape.

where $I_{high}(x, y)$, $I_{med1}(x, y)$, $I_{med2}(x, y)$, $I_{med3}(x, y)$ and $I_{low}(x, y)$ are the highest, second highest, third highest, fourth highest and lowest intensities of the five shifted triangular patterns at the same position in the range of pitch, respectively. After the operation by Equation (4-24), the triangular pattern is divided into ten regions, each with a different intensity-ratio pattern. The intensity ratio distribution $r_0(x, y)$ over the full pitch is similar to that for the two-step triangular-pattern phase-shifting method, shown in Figure 4-1 (c) with values ranging from 0 to 1, but with five repeated triangles and ten regions over the full pitch. The repeated triangles can be converted to a ramp by applying Equation (4-5) with the region number R ranging from 1 to 10.

4.2.6 Six-Step Triangular-Pattern Phase-Shifting Algorithm

Six triangular patterns phase-shifted by one-sixth of the pitch from each other are needed to reconstruct the 3-D object in the six-step triangular-pattern phase-shifting method. Figure 4-8 shows the cross sections of the six-step phase-shifted triangular fringe patterns. Figure 4-9 shows six-step phase-shifted triangular fringe patterns separately as they correspond to the intensity equations. The intensity equations for the six shifted triangular patterns are formulated as follows:

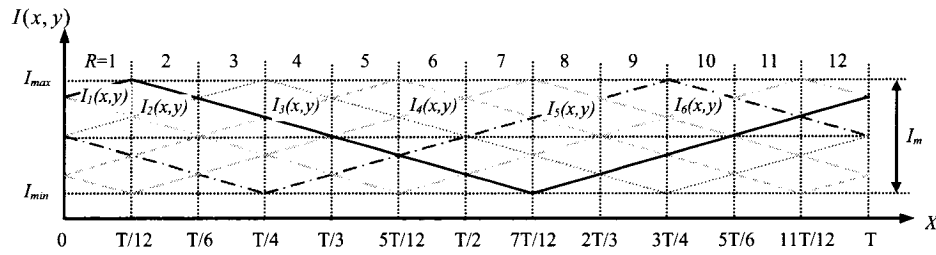


Figure 4-8 Cross sections of the six-step phase-shifted triangular fringe patterns.

$$I_1(x, y) = \begin{cases} \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{5I_m(x, y)}{6} & x \in [0, \frac{T}{12}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{7I_m(x, y)}{6} & x \in [\frac{T}{12}, \frac{7T}{12}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{7I_m(x, y)}{6} & x \in [\frac{7T}{12}, T) \end{cases} \quad (4-25)$$

$$I_2(x, y) = \begin{cases} \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{I_m(x, y)}{2} & x \in [0, \frac{T}{4}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{3I_m(x, y)}{2} & x \in [\frac{T}{4}, \frac{3T}{4}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{3I_m(x, y)}{2} & x \in [\frac{3T}{4}, T) \end{cases} \quad (4-26)$$

$$I_3(x, y) = \begin{cases} \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{I_m(x, y)}{6} & x \in [0, \frac{5T}{12}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{11I_m(x, y)}{6} & x \in [\frac{5T}{12}, \frac{11T}{12}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{11I_m(x, y)}{6} & x \in [\frac{11T}{12}, T) \end{cases} \quad (4-27)$$

$$I_4(x, y) = \begin{cases} -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{I_m(x, y)}{6} & x \in [0, \frac{T}{12}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{I_m(x, y)}{6} & x \in [\frac{T}{12}, \frac{7T}{12}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{13I_m(x, y)}{6} & x \in [\frac{7T}{12}, T) \end{cases} \quad (4-28)$$

$$I_5(x, y) = \begin{cases} -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{I_m(x, y)}{2} & x \in [0, \frac{T}{4}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{I_m(x, y)}{2} & x \in [\frac{T}{4}, \frac{3T}{4}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{5I_m(x, y)}{2} & x \in [\frac{3T}{4}, T) \end{cases} \quad (4-29)$$

$$I_6(x, y) = \begin{cases} -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{5I_m(x, y)}{6} & x \in [0, \frac{5T}{12}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{5I_m(x, y)}{6} & x \in [\frac{5T}{12}, \frac{11T}{12}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{17I_m(x, y)}{6} & x \in [\frac{11T}{12}, T) \end{cases} \quad (4-30)$$

$$I_m(x, y) = I_{\max}(x, y) - I_{\min}(x, y) \quad (4-31)$$

All parameters in the above equations have the same meanings as in the other triangular-pattern phase-shifting methods described above. The intensity-ratio $r_o(x, y)$ can be normalized by:

$$r_o(x, y) = \frac{I_{high}(x, y) - I_{med1}(x, y) + I_{med2}(x, y) - I_{med3}(x, y) + I_{med4}(x, y) - I_{low}(x, y)}{I_m(x, y)} \quad (4-32)$$

where $I_{high}(x, y)$, $I_{med1}(x, y)$, $I_{med2}(x, y)$, $I_{med3}(x, y)$, $I_{med4}(x, y)$ and $I_{low}(x, y)$ are the highest, second highest, third highest, fourth highest, fifth highest, and lowest intensities of the six shifted triangular patterns at the same position in the range of the pitch, respectively. After the operation by Equation (4-32), the triangular pattern is divided into twelve regions, each with a different intensity-ratio pattern. The intensity ratio distribution $r_o(x, y)$ over the full pitch is similar to that for the two-step triangular-pattern phase-shifting method, shown in Figure 4-1 (c), with values ranging from 0 to 1, but with six repeated triangles and twelve regions over the full pitch. The repeated triangles can be converted to a ramp by applying Equation (4-5) with the region number R ranging from 1 to 12.

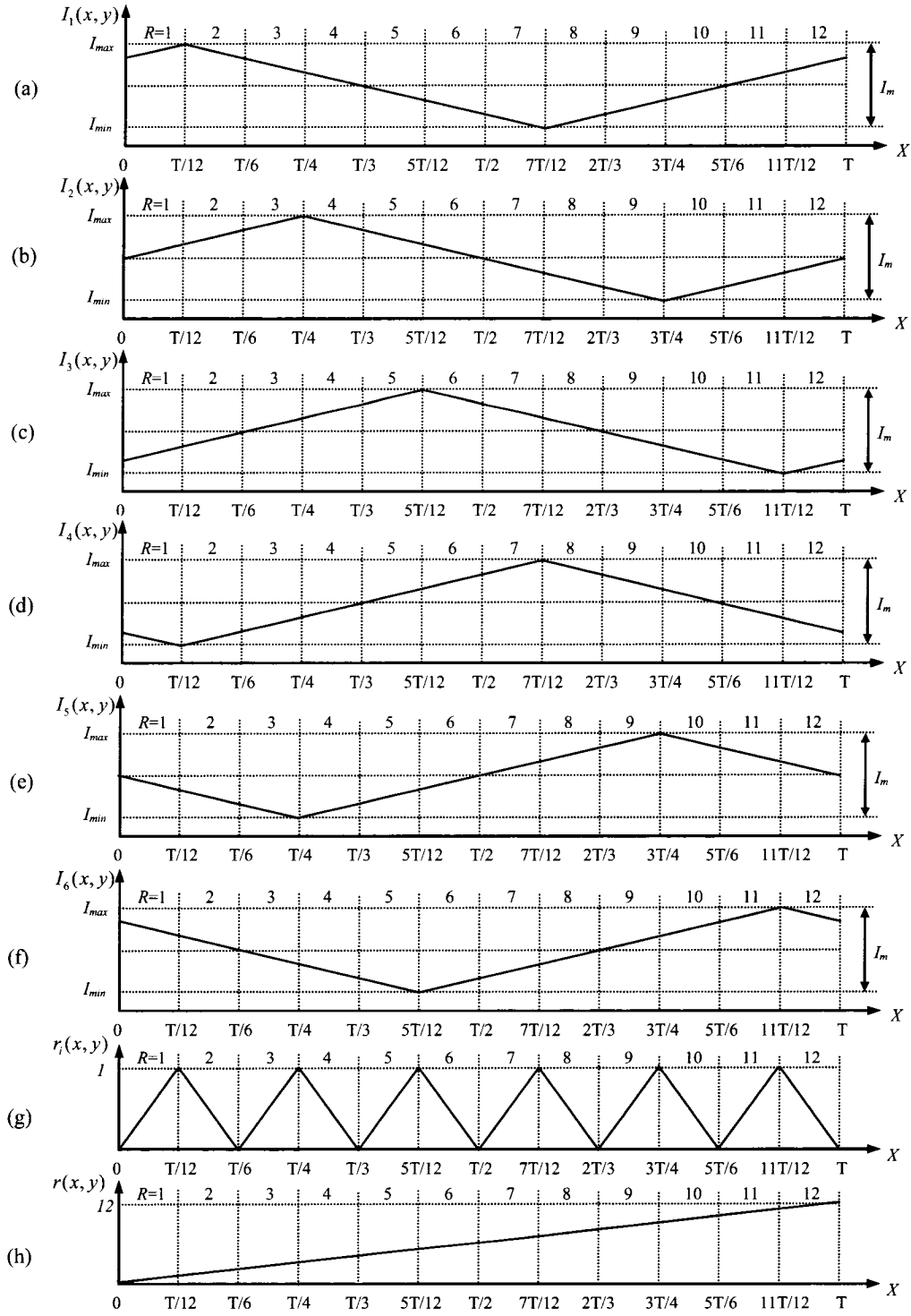


Figure 4-9 Six-step triangular-pattern phase-shifting algorithm. (a) - (f) Six phase-shifted triangular patterns, (g) Intensity-ratio in a triangular shape, and (h) Intensity-ratio ramp after the removal of the triangular shape.

4.3 Multiple Triangular-Pattern Phase-Shifting and Intensity-Ratio Unwrapping

In fringe projection phase-shifting methods, repeated fringe patterns are used in order to reduce sensitivity to image noise and increase resolution (Creath 1988, Halioua 1989, Greivenkamp 1992, He 1998, and Choi 1998). In the multiple triangular-pattern phase-shifting methods, the repeated triangular fringe patterns result in the intensity ratio being wrapped into the range 0 to 4 for two-step phase-shifting; 0 to 6, for three-step phase-shifting; and 0 to 8, for four-step phase-shifting, etc, which is different from the sinusoidal-pattern phase-shifting methods, where the phase is wrapped into the range of 0 to 2π (Greivenkamp 1992) for any number of phase-shifting steps. Both the wrapped intensity-ratio of the triangular-pattern phase-shifting method and the wrapped phase of the sinusoidal-pattern phase-shifting method have a sawtooth-like shape. Removal of the discontinuity of the wrapped intensity ratio requires a process similar to the phase unwrapping (Goldstein 1998) in the traditional sinusoidal-pattern phase-shifting method, discussed in Section 3.4. The depth (range) information of the object is contained in this unwrapped intensity-ratio map.

As just described, instead of wrapping into a fixed modulo of 2π in the sinusoidal-pattern phase-shifting method for any number of phase-shifting steps, the modulo values of the wrapped intensity-ratio are different for different number of phase-shifting steps in the triangular-pattern phase-shifting method. Some points need to be emphasized in order to perform intensity-ratio unwrapping correctly.

Define the modulo of the wrapped intensity-ratio for triangular-pattern phase-shifting method as r_m for any method involving different number of phase-shifting steps, according to the sampling theorem – Nyquist criterion (there are at least two pixels per fringe period). The intensity-ratio value difference (This should not be confused with the intensity-ratio difference term, discussed in Section 4.4 and Chapter 5) between any two consecutive intensity-ratio samples (pixels) should not change by more than $\pm \frac{r_m}{2}$. This criterion is used to reconstruct the intensity-ratio map by successive comparisons of neighboring pixels. If

the intensity-ratio value difference calculated for two adjacent pixels exceeds $\frac{r_m}{2}$ or is smaller than $-\frac{r_m}{2}$, then r_m or multiples of r_m must be subtracted from or added to the calculated value of the second pixel until the intensity-ratio value difference is back to the range between $-\frac{r_m}{2}$ and $\frac{r_m}{2}$. Therefore, $\pm\frac{r_m}{2}$ is selected as the threshold, and the entire intensity-ratio map is calculated by working outward from the starting location.

In the case of one-dimensional signals, assuming that the intensity-ratio value before intensity-ratio unwrapping is $r_w(x_i, y_i)$, and the intensity-ratio value after intensity-ratio unwrapping is $r_u(x_i, y_i)$, define the intensity-ratio value difference between any two neighboring pixels as:

$$\Delta r_w(x_i, y) = r_w(x_i, y) - r_w(x_{i-1}, y) \quad (4-33)$$

The most basic strategy is as follows. The modulo of the wrapped intensity-ratio r_m must be added to or subtracted from the intensity-ratio of all wrapped pixels according to the following conditions:

$$\begin{aligned} \text{if } \Delta r_w(x_i, y) > \frac{r_m}{2}, \quad \text{then } r_u(x_i, y) &= r_w(x_i, y) - r_m \\ \text{if } \Delta r_w(x_i, y) < -\frac{r_m}{2}, \quad \text{then } r_u(x_i, y) &= r_w(x_i, y) + r_m \end{aligned} \quad (4-34)$$

where $r_m = 4, 6, 8, 10, 12$ for two-, three-, four-, five-, and six-step triangular-pattern phase-shifting method, respectively.

The above procedure will yield an unwrapped intensity-ratio $r_u(x, y)$. Figure 4-10 shows this technique for the triangular-pattern phase-shifting method. It is noted that multiples of r_m might be subtracted from or added to the calculated value of the second pixel based on the previous unwrapping process.

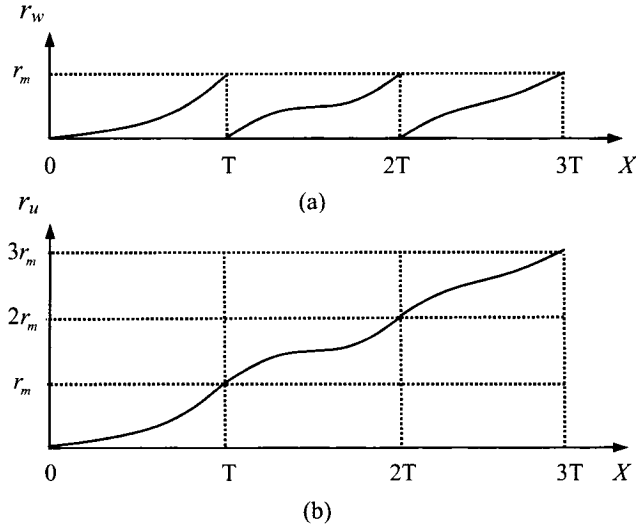


Figure 4-10 Intensity-ratio unwrapping in the triangular-pattern phase-shifting method. (a) One-dimensional wrapped intensity-ratio distribution, (b) One-dimensional unwrapped intensity-ratio distribution.

In the same way as the traditional phase unwrapping method described in Section 3.4, the relationship between the wrapped intensity-ratio and the unwrapped intensity-ratio for a two-dimensional intensity-ratio unwrapping may be stated as:

$$r_u(x_i, y_i) = r_w(x_i, y_i) \pm r_m n(x_i, y_i), \quad 1 \leq i \leq M, \quad 1 \leq j \leq N \quad (4-35)$$

where M and N are the numbers of pixels of the intensity-ratio map in two dimensions; n is the set of integers to be determined using measured wrapped intensity-ratio map $r_w(x_i, y_i)$; and the selection of the sign ($\pm$) depends upon the intensity-ratio change direction.

After the intensity-ratio unwrapping operation, a continuous intensity-ratio distribution can be obtained. The depth (range) information of the object is contained in this unwrapped intensity-ratio map.

4.4 Intensity-Ratio-to-Height Conversion Algorithm

To retrieve the 3-D surface coordinates of the object from the unwrapped intensity-ratio map, an intensity-ratio-to-height conversion algorithm is necessary for the triangular-pattern phase-shifting method. This algorithm is based on the traditional phase-to-height conversion algorithm (Fujigaki 2000 and Zhang 2002) commonly used in the sinusoidal-pattern phase-shifting method. The only difference in this algorithm is that in the triangular-pattern phase-shifting method, intensity-ratio values replace the phase values used in the sinusoidal-pattern phase-shifting method. Once the depth (range) information (z coordinate) is obtained, the x , y coordinates of the object can be determined by the lateral coordinate calibration (Liu 1999, Li 2001, Liu 2003, and Su 2004).

The intensity-ratio-to-height conversion equations are developed based on Figure 3-13, described in Section 3.5. However, here, the projected pattern is the triangular fringe pattern rather than the sinusoidal fringe pattern. Figure 3-13 therefore shows the relationship between the intensity-ratio of the projected triangular fringe pattern and the height (depth) of the object. Here, the projector projects a triangular fringe pattern whose pitch is p on the reference plane. The intensity-ratio at point C is r_C , and the intensity-ratio at point A is r_A .

In Figure 3-13, because $\triangle DPE$ and $\triangle DAC$ are similar, Equation (3-19) can be used to derive the equations for intensity-ratio-to-height conversion. On the reference plane, the following expression holds:

$$\frac{\overline{AC}}{p} = \frac{r_A - r_C}{T} \quad (4-36)$$

where p is the fringe pitch on the reference plane, T is the fringe pitch of the pattern generated by computer.

Define $\Delta r = r_A - r_C$ as intensity-ratio difference between points A and C . Then by combining Equations (3-19) and (4-36), the height of the object surface relative to the reference plane can be calculated by:

$$h = \frac{H}{1 + \frac{Td}{p\Delta r}} \quad (4-37)$$

Equation (4-37) states that the distribution of the height of the object surface relative to the reference plane is a function of the distribution of the intensity-ratio difference. During measurement, the reference plane is measured first. This measurement result will be used as the reference for the object measurement. The height of the object surface is then measured relative to the reference plane. When measuring the object, on the CCD array, point D on the object surface will be imaged onto the same pixel as point C on the reference plane. Because point D on the object surface has the same value of intensity ratio as point A on the reference plane, $r_D = r_A$, by subtracting the reference intensity-ratio map from the object intensity-ratio map, the intensity-ratio difference $\Delta r = r_A - r_C$ at this specific pixel can be easily obtained. This can be done for the whole intensity-ratio map. Once the intensity-ratio difference and system related parameters, determined by system calibration, describe in Chapter 5, are obtained, the complete height distribution of the object surface relative to the reference plane can be obtained.

Equation (4-37) describes a non-linear relationship between the distribution of the height of the object surface and the distribution of the intensity-ratio difference. To perform system calibration, described in Chapter 5, a modified form of Equation (4-37) using linear and non-linear relationships between the distribution of the object height and distribution of the intensity-ratio difference was developed. The formulation is exactly the same as that of the sinusoidal-pattern phase-shifting method, which is described in Section 3.5. Here the results are listed below.

Linear Mapping Function:

$$h(x, y) = K(x, y)\Delta r(x, y) \quad (4-38)$$

where $K(x, y) = \frac{p(x, y)H(x, y)}{Td}$. The coefficient $K(x, y)$ depends on the optical setup, and is a function of pixel coordinates (x, y) .

Non-linear Mapping Function:

$$h(x, y) = \frac{\Delta r(x, y)}{m(x, y) + n(x, y)\Delta r(x, y)} \quad (4-39)$$

where $m(x, y) = \frac{Td}{p(x, y)H(x, y)}$, $n(x, y) = \frac{1}{H(x, y)}$, $m(x, y)$ and $n(x, y)$ are system parameters relating to the optical setup, and (x, y) are the pixel coordinates.

The intensity-ratio difference $\Delta r(x, y)$ can be calculated by:

$$\Delta r(x, y) = r(x, y) - r_r(x, y) \quad (4-40)$$

where $r(x, y)$ is the distorted fringe intensity-ratio distribution of the object surface, $r_r(x, y)$ is the reference fringe intensity-ratio distribution taken from a planar reference plane, and $\Delta r(x, y)$ is the intensity-ratio difference between $r(x, y)$ and $r_r(x, y)$. Both $r(x, y)$ and $r_r(x, y)$ can be obtained from the calculation of Equation (4-5), for two-, three-, five-, and six-step triangular-pattern phase-shifting methods, and Equation (4-17), for the four-step triangular-pattern phase-shifting method.

The values of the parameters $K(x, y)$, for linear mapping Equation (4-38), and $m(x, y)$ and $n(x, y)$, for non-linear mapping Equation (4-39), can be determined by system calibration, which will be described in Chapter 5.

4.5 Accuracy-Analysis Experiments

For the digital fringe projection technique, in addition to the gamma non-linearity of the video projector, which decreases the accuracy and resolution of the measurement (Guo

2004), errors may be due to grey-level quantization, intensity noise, and the defocus of the projector and camera. For the triangular-pattern phase-shifting method, the number of phase steps and the value of the pitch of the triangular pattern also impact the accuracy of the measurement. The projector gamma non-linearity error will be addressed in Chapter 6. In the following, the impact of image defocus, pitch, and number of phase-steps on measurement accuracy will be investigated. It should be noted that for all simulations and physical experiments throughout this thesis, all RMS errors are computed based on known or ground truth reference values. As higher measurement accuracy was achieved with non-linear calibration than with linear calibration (discussed in Chapter 5), all triangular-pattern phase-shifting method experiments used non-linear calibration.

4.5.1 Image Defocus Error and Simulation Experiment

In a projector-camera based measurement system it may be expected that the projector and camera be adjusted to focus on the measured object for the best measurement accuracy. However, one characteristic of the DLP projector used in the measurement system is that the projected pattern is divided into numerous active square pixels by horizontal and vertical black lines. In order to remove the black lines, the focus of projector must be manually adjusted to slightly blur the image. This defocus of the projected light unfortunately blurs the projected triangular grey-scale pattern and results in a quasi-triangular pattern captured by the camera. This contributes to measurement error in the triangular-pattern phase-shifting method. Here, the defocus error for two-step triangular pattern method is analyzed. It also applies to the other triangular-pattern phase-shifting methods with different number of steps. The intensity equations in the case of defocus of the images for the two shifted triangular patterns can be modeled as follows:

$$I_1^D(x, y) = \begin{cases} \frac{2I_1(x, y)}{3} \left\{ 1 + \frac{1}{2} \left[\cos\left(\frac{2\pi x}{T}\right) \right]^S \right\} & \text{when } \cos\left(\frac{2\pi x}{T}\right) > 0 \\ I_1(x, y) & \text{when } \cos\left(\frac{2\pi x}{T}\right) = 0 \\ \frac{2I_1(x, y)}{3} \left\{ 1 + \frac{1}{2} \left[-\cos\left(\frac{2\pi x}{T}\right) \right]^S \right\} & \text{when } \cos\left(\frac{2\pi x}{T}\right) < 0 \end{cases} \quad (4-41)$$

$$I_2^D(x, y) = \begin{cases} \frac{2I_2(x, y)}{3} \left\{ 1 + \frac{1}{2} \left[\cos\left(\frac{2\pi x}{T}\right) \right]^S \right\} & \text{when } \cos\left(\frac{2\pi x}{T}\right) > 0 \\ I_2(x, y) & \text{when } \cos\left(\frac{2\pi x}{T}\right) = 0 \\ \frac{2I_2(x, y)}{3} \left\{ 1 + \frac{1}{2} \left[-\cos\left(\frac{2\pi x}{T}\right) \right]^S \right\} & \text{when } \cos\left(\frac{2\pi x}{T}\right) < 0 \end{cases} \quad (4-42)$$

where $I_1(x, y)$ and $I_2(x, y)$ are the intensities for the two shifted triangular patterns calculated by Equations (4-1) and (4-2), respectively; $I_1^D(x, y)$ and $I_2^D(x, y)$ are the intensities in the case of defocus of the images for the two shifted triangular patterns, respectively; x is the horizontal position of the pixel; and S is the defocus coefficient. The pattern can be changed continuously from a triangular pattern to sinusoidal pattern by changing the value of the defocus coefficient S , where $S = 0.0$ corresponds to the triangular pattern and $S = 1.0$ corresponds to the sinusoidal pattern. Figure 4-11 shows how the triangular pattern is blurred with different values of defocus coefficient S .

The triangular intensity-ratio $r(x, y)$, in Figure 4-1 (c), calculated by Equation (4-4), on the other hand, always has a maximum value of 1 whether the fringe image is defocused or not. Through simulation it was found that the intensity equations in the case of image defocus for the two-shift triangular pattern can be expressed more precisely as follows:

$$I_1^D(x, y) = \begin{cases} I_1(x, y) \left\{ 1 + 0.6S \left[\cos\left(\frac{2\pi x}{T}\right) \right]^S \right\} & \text{when } \cos\left(\frac{2\pi x}{T}\right) > 0 \\ I_1(x, y) & \text{when } \cos\left(\frac{2\pi x}{T}\right) = 0 \\ I_1(x, y) \left\{ 1 + 0.6S \left[-\cos\left(\frac{2\pi x}{T}\right) \right]^S \right\} & \text{when } \cos\left(\frac{2\pi x}{T}\right) < 0 \end{cases} \quad (4-43)$$

$$I_2^D(x, y) = \begin{cases} I_2(x, y) \left\{ 1 + 0.6S \left[\cos\left(\frac{2\pi x}{T}\right) \right]^S \right\} & \text{when } \cos\left(\frac{2\pi x}{T}\right) > 0 \\ I_2(x, y) & \text{when } \cos\left(\frac{2\pi x}{T}\right) = 0 \\ I_2(x, y) \left\{ 1 + 0.6S \left[-\cos\left(\frac{2\pi x}{T}\right) \right]^S \right\} & \text{when } \cos\left(\frac{2\pi x}{T}\right) < 0 \end{cases} \quad (4-44)$$

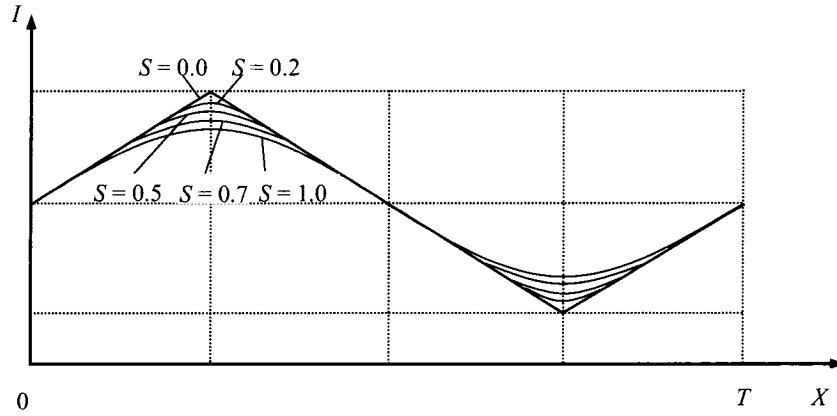


Figure 4-11 Effect of the defocused triangular pattern for different values of defocus coefficient S .

In this case, when the fringe images are defocused, the defocused intensity-ratio $r^D(x, y)$ can be calculated by:

$$r^D(x, y) = \frac{|I_1^D(x, y) - I_2^D(x, y)|}{I_m^D(x, y)} \quad (4-45)$$

where $I_m^D(x, y) = \max \{I_1^D(x, y), I_2^D(x, y)\} - \min \{I_1^D(x, y), I_2^D(x, y)\}$. Substituting Equations (4-43) and (4-44) into Equation (4-45), the following equation is obtained:

$$r^D(x, y) = \begin{cases} \frac{|I_1(x, y) - I_2(x, y)|}{I_m^D(x, y)} \left\{ 1 + 0.6S \left[\cos\left(\frac{2\pi x}{T}\right) \right]^S \right\} & \text{when } \cos\left(\frac{2\pi x}{T}\right) > 0 \\ \frac{|I_1(x, y) - I_2(x, y)|}{I_m^D(x, y)} & \text{when } \cos\left(\frac{2\pi x}{T}\right) = 0 \\ \frac{|I_1(x, y) - I_2(x, y)|}{I_m^D(x, y)} \left\{ 1 + 0.6S \left[-\cos\left(\frac{2\pi x}{T}\right) \right]^S \right\} & \text{when } \cos\left(\frac{2\pi x}{T}\right) < 0 \end{cases} \quad (4-46)$$

By changing the value of the defocus coefficient S from 0 to 1, the intensity-ratio error corresponding to the different level of defocus can be calculated as:

$$\Delta r(x, y) = r^D(x, y) - r(x, y) \quad (4-47)$$

The defocused intensity-ratio corresponding to the defocus coefficient S over one full pitch period is shown in Figure 4-12.

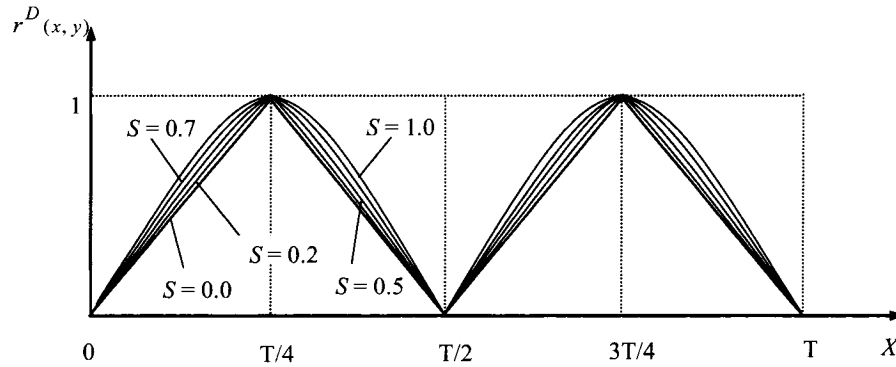


Figure 4-12 Effect of the defocused intensity-ratio corresponding to S .

The average intensity-ratio error Δr_{ave} and maximum intensity-ratio error Δr_{max} can then be defined as:

$$\Delta r_{ave} = \frac{1}{T} \sum_{x=0}^T \Delta r(x, y) \quad (4-48)$$

$$\Delta r_{max} = \max \{ \Delta r(x, y) \} \quad (4-49)$$

These errors depend on the value of the defocus coefficient S . They are calculated for various values of S from 0 to 1 to determine the influence of the defocus. Figure 4-13 shows the relationship between the intensity-ratio error and the defocus coefficient S . The result shows that the maximum and average errors due to the defocus increase with defocus coefficient S . However, the rate of growth decreases with the higher defocus coefficient S because the triangular pattern approaches a sinusoidal pattern and makes the errors stabilize.

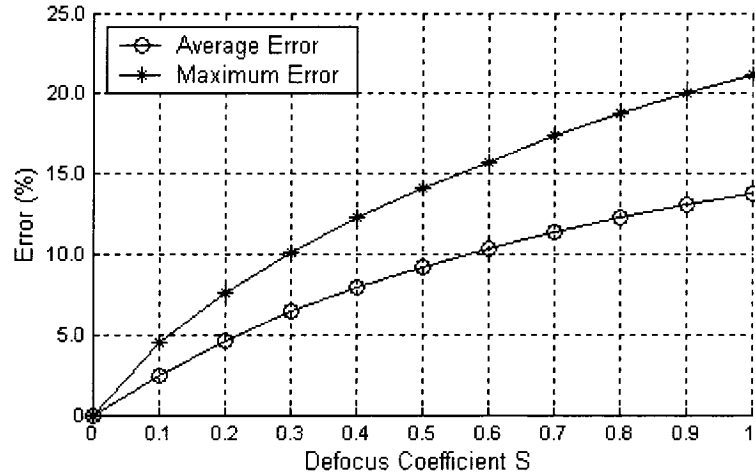


Figure 4-13 Maximum and average intensity-ratio errors corresponding to defocus coefficient S .

4.5.2 Impact of Pitch on Measurement Accuracy

To determine the influence of the pitch of the projected triangular pattern on measurement accuracy, measurement experiments were performed as follows: first, images of a flat reference plate at position $z = 0$ (zero depth) were acquired for each of the phase shifts and the intensity-ratio distribution was obtained by intensity-ratio wrapping and intensity-ratio unwrapping (Section 4.3). Thereafter, the plate was moved 25 mm toward the

camera, and measured again. At this position, images were acquired and the intensity-ratio distribution was computed as for the reference plane. The intensity-ratio differences between the measured plane and the reference plane at position $z = 0$ were determined and the distance between the two positions was obtained by applying intensity ratio-to-height conversion as discussed in Section 4.4. The root-mean-square (RMS) errors (RMSE) of the depth calculation between the two positions were calculated using all pixel points (648×494) of the measured plate. All experiments were performed using the same physical experimental system setup, with a camera-to-object standoff distance of 1.9 m.

To determine the impact of the pitch of the projected triangular pattern on measurement accuracy, measurement of the flat plate at a depth of 25 mm was performed using a pitch of 12, 13, 14... 24 pixels, respectively, for both the two- and three-step phase-shifting methods.

The RMS errors in depth measurement for the different values of pitch of the projected triangular fringe patterns are shown in Table 4-1 for the two- and three-step methods. For the two-step method, it is seen firstly, that lower RMSE occur for even values of pitch (shown in bold) than for the odd values. For the even values of pitch, the error decreases with lower value of pitch. However, a limiting value seems to occur at a pitch of 16, below which the error increases. For the three-step method, there is also a trend of decreasing error with lower value of pitch, again with a limiting value, this time of 15, below which the error increases. For the three-step method it can also be observed that lower errors occur for a pitch of 12, 15, 18, 21, and 24 (shown in bold) when the pitch can be evenly divided by three, which is the number of phase steps or phase shifts. This is consistent with the occurrence of lower errors for the two-step method for the pitch values 12, 14, 16, 18, 20, 22, and 24, when the pitch can be evenly divided by two, the number of phase steps or phase shifts. In general, the digital fringe projection generates a phase shifting error if the pitch of the pattern cannot be evenly divided by the number of phase steps, as the number of pixels in the image is integral. When selecting the pitch, this is therefore an important factor. An optimal value of pitch for the triangular-pattern phase-shifting methods also seems to exist. The higher errors at lower pitch values (1.868 mm for two-step and 0.743 for three-step, both for pitch of 12 pixels) have been found to occur as a result of intensity-ratio unwrapping failure.

Experiments for triangular-pattern phase-shifting method with other different phase-shifting steps were also carried out, and the results demonstrated that the optimal value of pitch of the triangular pattern with different phase-shifting steps may be different. The optimal pitches for the different phase-shifting methods are summarized in Table 4-2.

Table 4-1 Impact of pitch on measurement error for two- and three-step triangular-pattern phase-shifting methods. The measured distance is 25 mm.

Pitch (pixels)		12	13	14	15	16	17	18	19	20	21	22	23	24
RMSE (mm)	2 step	1.868	4.213	0.699	1.063	0.656	1.015	0.775	1.024	0.889	1.133	0.949	1.236	1.057
	3 step	0.743	1.638	1.483	0.616	1.019	0.977	0.749	0.980	1.273	0.822	1.339	1.038	0.943

Table 4-2 Optimal value of pitch for triangular-pattern phase-shifting method with different number of phase steps.

Phase Steps	2	3	4	5	6
Optimal Value of Pitch (pixels)	16	15	16	15	18

4.5.3 Impact of Phase-Step on Measurement Accuracy

To evaluate the impact of the number of phase steps in the triangular-pattern phase-shifting methods on the measurement accuracy, measurement of the flat plate at a depth of 25 mm was performed with different number of phase-shifting steps, from two to six, discussed in Section 4.2, and different values of pitch of the triangular pattern. Since each method has an optimal value of pitch, to evaluate the influence of the number of phase steps on measurement accuracy effectively, experiments had to be done for various values of pitch for each method. The experimental setup and measurement procedure is exactly the same as that described in Section 4.5.2. The measurements were performed using two-, three-, four-, five-, and six-step phase shifting in each of the following four groups: Group A with a pitch of 15 pixels, Group B: 16 pixels, Group C: 18 pixels, and Group D: 20 pixels. The RMS

errors for the four groups of measurements are shown in Table 4-3. Figure 4-14 shows the corresponding cross sections of flat plate depth measurements for the plate at 25 mm depth, Group A: Figure 4-14 (a) – (e), Group B: (f) – (j), Group C: (k) – (o), and Group D: (p) – (t).

From Table 4-3 and Figure 4-14, it can be seen that the measurement accuracy generally increases (RMS error decreases) with an increase in the number of phase-shifting steps (from left to right in the table and figure) if the appropriate values of pitch, divisible by the number of phase shifting steps, are used for the particular number of phase shifts, as discussed in Section 4.5.2. These values are shown in bold in Table 4-3. For example, considering Groups C and D with a pitch of 18 and 20 pixels respectively, the RMS errors decrease from two-step to six-step phase shifting from 0.775, 0.749, 0.652, 0.329, to 0.287 mm, respectively (shown in bold). Considering all four values of pitch (Groups A to D), the highest measurement accuracies, seen for the lower values of pitch, increase further as the number of phase-shifting steps increases. This is seen by the shaded values in Table 4-3, where the RMS errors are reduced from 0.656 mm for two-step phase shifting (16 pixel pitch), to 0.616 mm for three-step (15 pixel pitch), 0.423 for four-step (16 pixel pitch), 0.312 for five-step (15 pixel pitch), and 0.287 mm for six-step phase shifting (18 pixel pitch).

Table 4-3 RMS errors in depth measurement of a flat plate using the triangular-pattern phase-shifting method with different number of phase-shifting steps and different values of pitch of the projected triangular pattern. The measured distance is 25 mm. The values in bold are appropriately selected pitch values, evenly divisible by the number of phase-shifting steps. The optimal pitch values are indicated by shading.

Phase-Shifting Steps		2	3	4	5	6
RMS error (mm)	Group A: 15 pixel pitch	1.063	0.616	1.084	0.312	0.929
	Group B: 16 pixel pitch	0.656	1.019	0.423	1.169	1.492
	Group C: 18 pixel pitch	0.775	0.749	1.027	1.157	0.287
	Group D: 20 pixel pitch	0.889	1.273	0.652	0.329	0.991

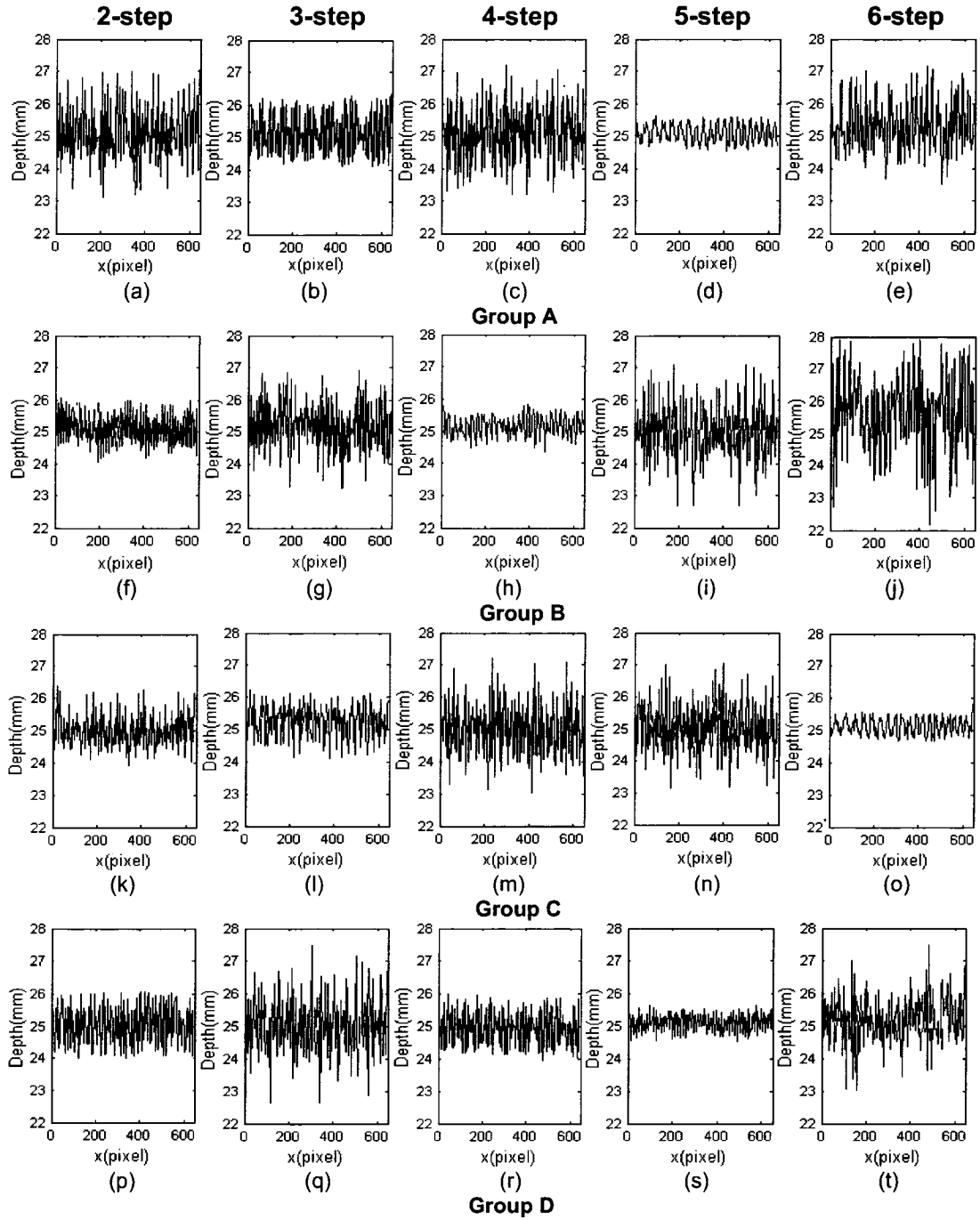


Figure 4-14 Depth measurements across the flat plate positioned 25 mm from the reference position using the triangular-pattern phase-shifting method with different number of phase-shifting steps and different values of pitch in the projected triangular patterns. For each group, measurement results are for two-, three-, four-, five-, and six-step triangular-pattern phase-shifting method, respectively as follows: (a) - (e) (Group A) for a pitch of 15 pixels, (f) - (j) (Group B) pitch of 16 pixels, (k) - (o) (Group C) pitch of 18 pixels, and (p) - (t) (Group D) pitch of 20 pixels.

It should be noted that appropriate values of pitch for the four-step phase shifting are 16 and 20, for the five-step are 15 and 20, and for the six-step phase shifting is 18, as these values can be divided evenly by the number of phase shifting steps. Furthermore, the shaded RMS values in Table 4-3 correspond to the optimal values of pitch, which is shown in Table 4-2, for the different phase-shifting methods.

In Figure 4-14, the lower errors can be seen where the pitch is evenly divisible by the number of phase steps. For example, for Group C (18 pixel pitch) 18 can be divided evenly by 2, 3 and 6 and the errors shown in Figure 4-14 (k), (l), and (o) for two-, three-, and six-step phase shifting methods, respectively, are therefore lower than the others in the same group. For Group D which used a pitch of 20 pixels, the two-, four- and five-step phase-shifting methods had lower errors (Figure 4-14 (p), (r), and (s), respectively), as 20 can be evenly divided by 2, 4, and 5.

4.5.4 Measurement Accuracy along the Depth

Measurement of the flat plate, using the optimal value of the pitch, determined in Section 4.5.2, for each phase-shifting method as discussed above, was repeatedly carried out over the full 50 mm calibrated range of depth in 2 mm increments. RMS errors of the depth were calculated based on every image pixel and are shown in Figure 4-15.

The RMS error varies with a pattern of increasing error with depth and then decreasing error with depth, in a cyclical pattern. The RMS error and the amplitude of the oscillation decrease with more phase-shifting steps; however, the number of cycles of the RMS error increases with more phase-shifting steps. For two-, three-, and four-step methods, the distributions of the RMS error curves overlap at some range of depth, but in general, more phase-shifting steps lead to more accurate measurements. In comparison to the four-step sinusoidal-pattern phase-shifting method, the six-step triangular-pattern phase-shifting method is most comparable in terms of measurement accuracy, where RMS errors relative to ground truth ranged from approximately 0.3 to 0.4 mm over the mid-range of depth. (Non-linear calibration was used for both methods and results for the sinusoidal method determined in real measurements in this research are given in Section 5.3.6 (Figure 5-12)). Repeated testing was carried out and the results indicate that the methods with fewer phase-shifting steps are more sensitive to noise, resulting in a change of shape of the RMS error

distributions. On the other hand, the methods with more phase-shifting steps are more robust, and have more repeatable measurement accuracy in repeated testing.

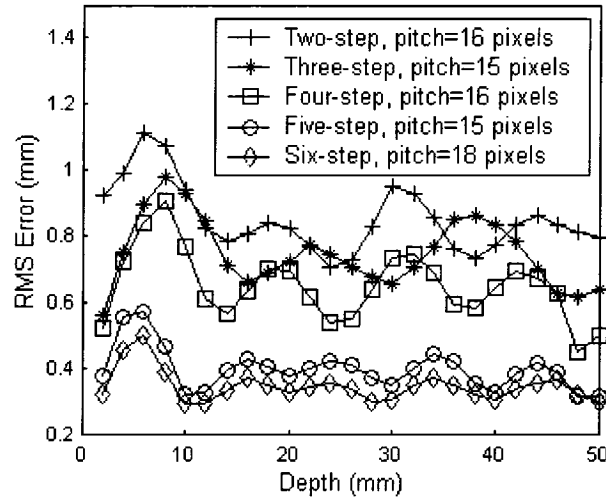


Figure 4-15 Measurement errors over the full range of depth using optimal values of pitch of the triangular-pattern phase-shifting methods for different number of phase-shifting steps, determined by measurement of a flat plate. RMS errors are calculated based on all pixels of the 648 x 494 resolution image.

4.5.5 Discussion

The above experiment results suggest that a design trade-off arises in the selection of the value of pitch. A small value of pitch implies higher measurement accuracy, but it is likely to lead to intensity-ratio unwrapping failures due to defocus errors. The angle between the optical axis of the camera and projector also has been found to affect the influence of pitch on measurement accuracy. In the experiments, the triangular patterns with optimal values of pitch have the highest measurement accuracy when the angle between the optical axis of the camera and projector is approximately 15° . When the pitch of the triangular pattern is less than the optimal values of pitch, the intensity-ratio unwrapping fails in some areas and therefore generates more error.

This conclusion is obtained under a set level of focus of the projector and camera. In testing the effect of increasing pitch of the triangular pattern, no adjustment of the focus of the projector and camera took place, and it was thus set to be the same for all tests.

However, the defocus level of the projected triangular pattern decreases with the increase of the pitch of triangular pattern under a fixed focus of the projector. The RMS values of the depth calculation error should ideally be obtained under the same defocus level of the projected triangular pattern. However, it is difficult to obtain triangular patterns with different values of pitch with the same defocus level just by manually adjusting the focus of the projector and camera during the testing. This issue should be investigated further.

A similar phenomenon happens in the sinusoidal-pattern phase-shifting method. To generate a true sinusoidal pattern with a specified value of pitch, the focus of the projector has to be adjusted to blur the pattern, as discussed earlier. When the value of the pitch changes, the focus of the projector has to be adjusted again to obtain a true sinusoidal pattern, otherwise, more error will be generated.

4.5.6 Conclusions

The measurement accuracy of the proposed triangular-pattern phase-shifting method was investigated analytically and experimentally. The defocus simulation result shows that the intensity-ratio error is approximately 22% in the case of the worst defocused lighting. The impact of the number of phase-shifting steps and pitch of the projected triangular fringe pattern on measurement accuracy were determined. More phase steps generate a higher measurement accuracy; however, the digital fringe projection generates phase-shifting errors if the pitch of the pattern cannot be evenly divided by the number of phase steps. Lower values of pitch lead to higher measurement accuracy; however, there is an optimal pitch value, below which, intensity-ratio unwrapping failure occurs. Higher measurement accuracy can thus be obtained using a greater number of phase-shifting steps and a lower value of pitch, as long as the pitch is appropriately selected to be divisible by the number of phase-shifting steps, and not below the optimal value, where failure intensity-ratio unwrapping would occur.

4.6 3-D Object Measurement Experiments

In addition to measurement accuracy experiments, further measurement experiments were carried out for the triangular-pattern phase-shifting methods to verify the general

performance of the proposed methods in measuring the surface of an object. A schematic diagram of the experimental system is shown in Figure 3-1, with triangular patterns as input. The object used to test the system is a plastic human-head mask with a white surface. The size of the mask is $210 \times 140 \times 70 \text{ mm}^3$. The 3-D shape measurement system was developed with Visual C++ 6.0 and OpenGL software. Matlab software was used to analyze the results of the calibration and simulation. The triangular and sinusoidal fringe patterns were generated by a newly written computer program and projected onto the object by the DLP video projector. The pattern was shifted with two steps, three steps, four steps, etc, and the camera captured the object images via the Matrox Odyssey XA vision processor board frame grabber. The computer performs computations for intensity-ratio/phase wrapping, intensity-ratio/phase unwrapping, intensity-ratio/phase difference, and intensity-ratio/phase-to-height calculation using the calibration data obtained during the calibration, which will be discussed in Chapter 5. The reconstructed 3-D object is displayed with OpenGL, with functions for displaying the reconstructed object in shaded, solid or wireframe modes. Averaging and median filters are applied to reduce noise and smooth the reconstructed 3-D object surface. The reconstructed object can be zoomed in or out and rotated.

Before the measurement experiment, system calibration (discussed in Chapter 5) was performed to obtain the system related calibration parameters, which are necessary for the measurement.

4.6.1 Triangular-Pattern Phase-Shifting Method Testing

To compare the measurement results of the triangular-pattern phase-shifting method with different number of shifting steps discussed in Section 4.2, the optimal values of the pitch of the triangular patterns determined in Section 4.5.2 were used to measure a 3-D object surface.

Figure 4-16 shows the two-step triangular pattern segments; (a) is the pattern generated by software using Equations (4-1) and (4-2), and (b) shows the captured fringe-pattern segments that were observed on a flat reference plane. A triangular-shape intensity-ratio distribution is obtained by the calculation of the two captured triangular patterns. Removing the repeated triangular shape of the intensity-ratio through each full pitch generates a wrapped intensity-ratio distribution ranging from 0 to 4, as shown in Figure 4-17 (a). The

unwrapped intensity-ratio distribution, which is shown in Figure 4-17 (b), is obtained by removing the discontinuity of the wrapped image with a modified unwrapping method (Goldstein 1988) used in the sinusoidal-pattern phase-shifting method. An intensity-ratio-to-height conversion algorithm, described in Section 4.4 is used to retrieve the 3-D coordinates of the object.

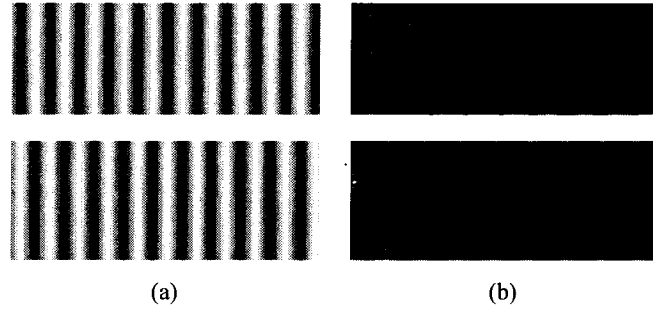


Figure 4-16 Triangular fringe pattern segments with two steps (0° and 180°). (a) Generated triangular patterns by software, (b) Captured triangular patterns by camera.

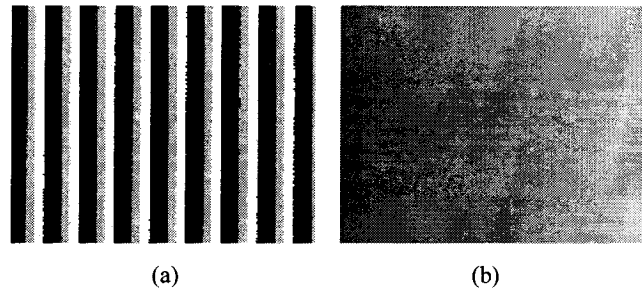


Figure 4-17 Wrapped and unwrapped intensity-ratio map of two-step triangular fringe pattern on the flat reference plane. (a) Wrapped intensity-ratio map, (b) Unwrapped intensity-ratio map.

Figure 4-18 shows the measurement procedure and results of the human-head mask by applying the two-step triangular-pattern phase-shifting method. Figure 4-18 (a) shows one of the two captured triangular patterns on the mask. Figure 4-18 (b) and (c) are the wrapped and unwrapped intensity-ratio distributions of the mask. By subtracting the unwrapped reference intensity-ratio map of Figure 4-17 (b) from the unwrapped object intensity-ratio

map of Figure 4-18 (c), the intensity-ratio difference distribution was obtained, as shown in Figure 4-18 (d). By applying the calibration result (Figure 5-2) to the intensity-ratio difference distribution map, the object height distribution was obtained by using either the linear mapping function, Equation (4-38), or the non-linear mapping function, Equation (4-39), and the 3-D model was constructed, as shown in Figure 4-18 (e) – (j).

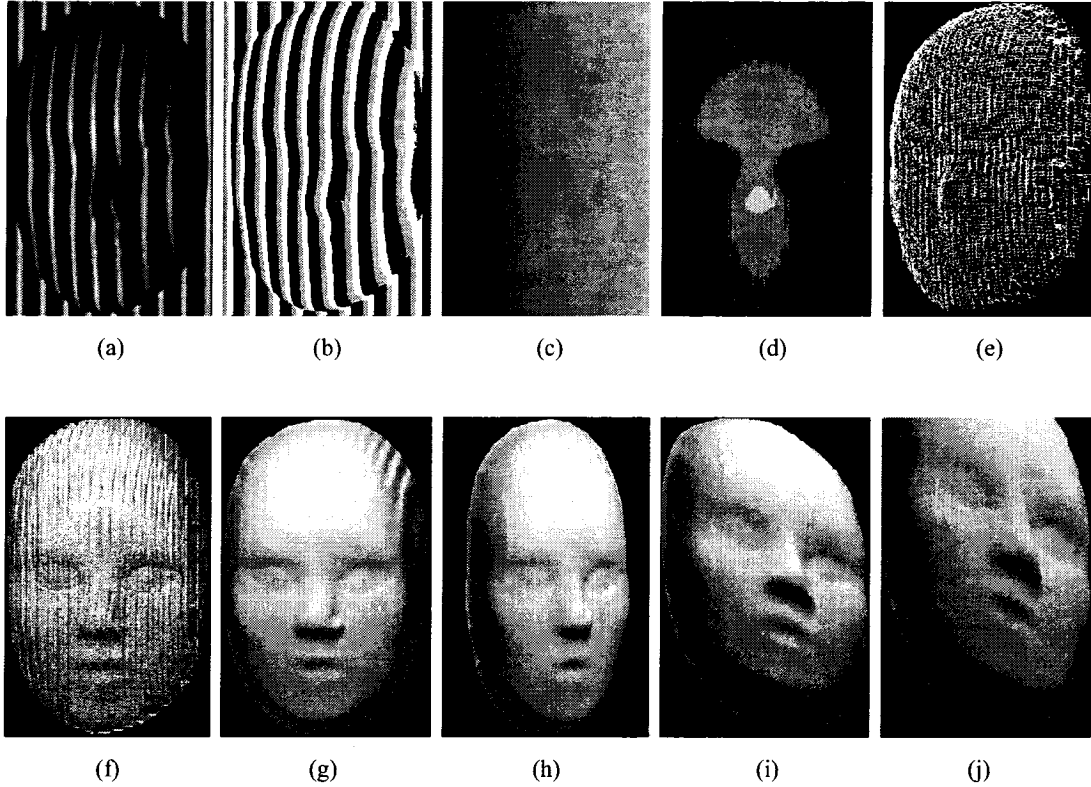


Figure 4-18 3-D shape measurement of a plastic human-head mask with the two-step triangular-pattern phase-shifting method. (a) One of the two captured triangular fringe pattern images, (b) Wrapped intensity-ratio map, (c) Unwrapped intensity-ratio map, (d) Intensity-ratio difference map after scaling, (e) Reconstructed 3-D wire-frame model, (f) Reconstructed 3-D shaded model with noise, and (g) – (j) Reconstructed 3-D shaded model after filtering. The triangular pattern was generated with a pitch of 16 pixels.

In the same way as described above for the two-step triangular-pattern phase-shifting method, the measurement of the same human-head mask was carried out by using three-step, four-step, five-step, and six-step triangular-pattern phase-shifting methods. For each phase-shifting method, the optimal values of pitch for each method were used. The measurement

results are shown in Figure 4-19, Figure 4-20, Figure 4-21, and Figure 4-22. In these figures, (a) shows one of the three, four, five, and six captured three-shift, four-shift, five-shift, and six-shift triangular patterns on the mask; (b) and (c) are the wrapped and unwrapped intensity-ratio distributions of the mask; (d) is the intensity-ratio difference distribution; (e) is 3-D wire-frame model; (f) shows 3-D shaded model with noise; and (g) – (j) are 3-D shaded models from different directions after filtering.

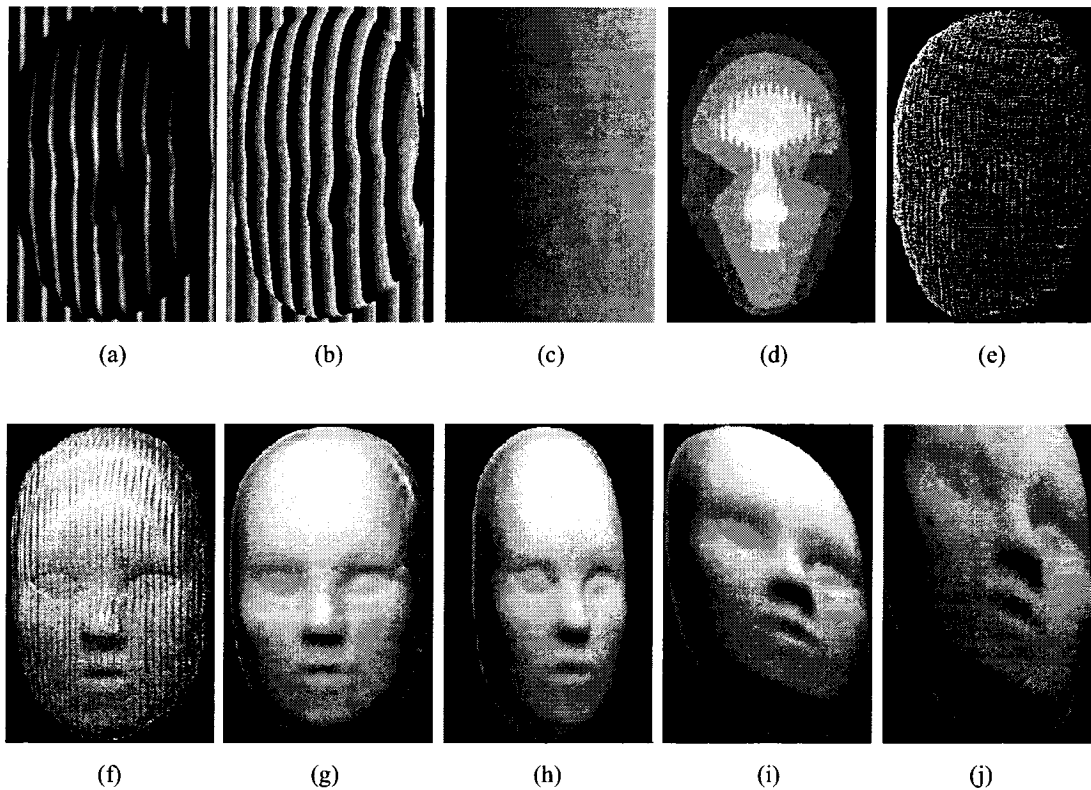


Figure 4-19 3-D shape measurement of a plastic human-head mask with the three-step triangular-pattern phase-shifting method. (a) One of the three captured triangular fringe pattern images, (b) Wrapped intensity-ratio map, (c) Unwrapped intensity-ratio map, (d) Intensity-ratio difference map after scaling, (e) Reconstructed 3-D wire-frame model, (f) Reconstructed 3-D shaded model with noise, and (g) – (j) Reconstructed 3-D shaded model after filtering. The triangular pattern was generated with a pitch of 15 pixels.

To compare the measurement results clearly, the 3-D shaded models from each measurement are shown together in Figure 4-23: (a) – (e) are the reconstructed 3-D shaded models with the two-, three-, four-, five-, and six-step triangular-pattern phase-shifting methods, respectively.

It can be seen from Figure 4-23 that the reconstructed surface of the human-head mask was smoother by the triangular-pattern phase-shifting method with the higher number of phase-shifting steps. This is consistent with the higher accuracy found for the higher number of phase-shifting steps.

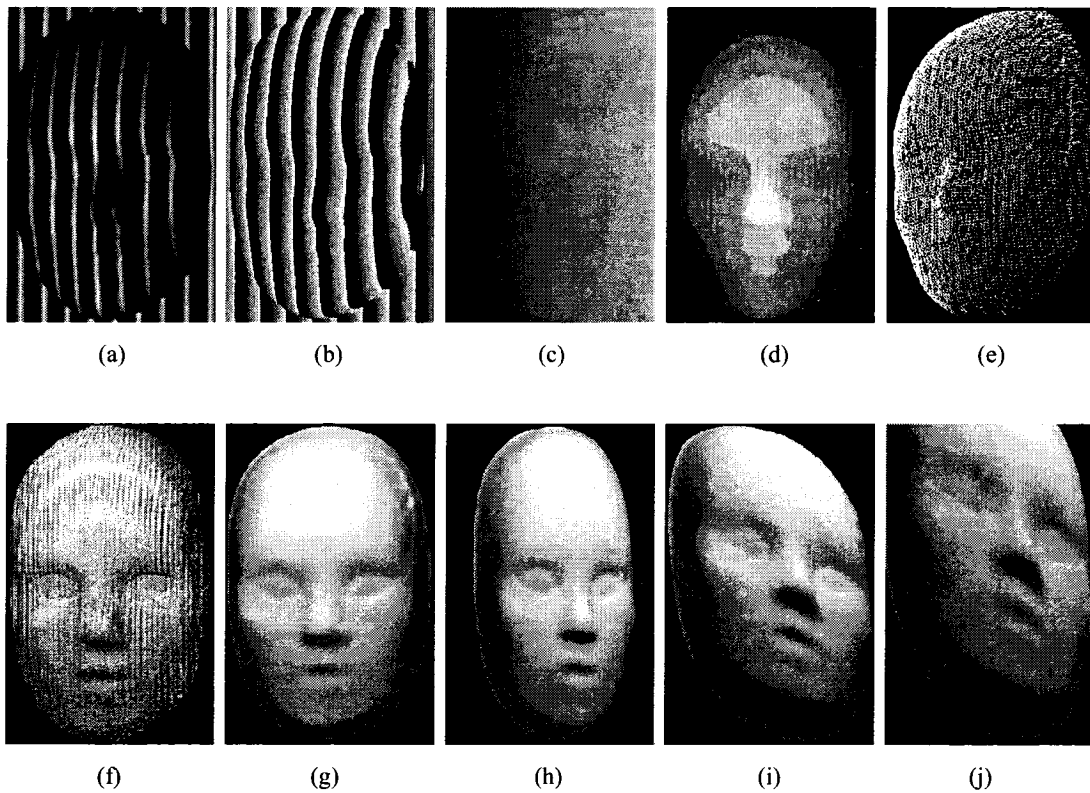


Figure 4-20 3-D shape measurement of a plastic human-head mask with the four-step triangular-pattern phase-shifting method. (a) One of the four captured triangular fringe pattern images, (b) Wrapped intensity-ratio map, (c) Unwrapped intensity-ratio map, (d) Intensity-ratio difference map after scaling, (e) Reconstructed 3-D wire-frame model, (f) Reconstructed 3-D shaded model with noise, and (g) – (j) Reconstructed 3-D shaded model after filtering. The triangular pattern was generated with a pitch of 16 pixels.

As the six-step triangular-pattern phase-shifting method had the highest accuracy among tested methods (0.287 mm RMS error) it was used to measure a wooden mask which had more features than the first mask. The 3-D shaded models are shown in Figure 4-24. (a) shows one of the six captured six-shift triangular patterns on the mask; (b) and (c) are the wrapped and unwrapped intensity-ratio distributions of the mask; (d) is the intensity-ratio difference distribution; (e) shows 3-D shaded model with noise; and (f) – (j) 3-D shaded models with different orientations, which demonstrate that the complex features can be well reconstructed by the triangular fringe-pattern phase-shifting method, using a higher number of phase-shifting steps when higher accuracy is needed.

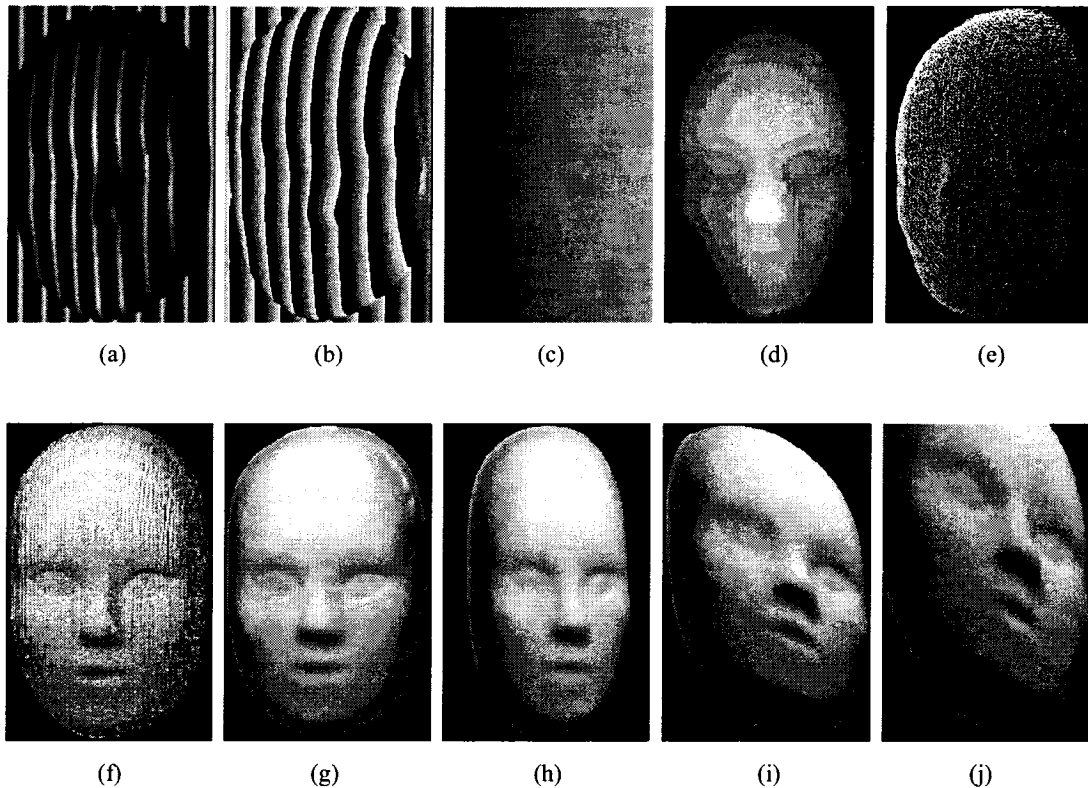


Figure 4-21 3-D shape measurement of a plastic human-head mask with the five-step triangular-pattern phase-shifting method. (a) One of the five captured triangular fringe pattern images, (b) Wrapped intensity-ratio map, (c) Unwrapped intensity-ratio map, (d) Intensity-ratio difference map after scaling, (e) Reconstructed 3-D wire-frame model, (f) Reconstructed 3-D shaded model with noise, and (g) – (j) Reconstructed 3-D shaded model after filtering. The triangular pattern was generated with a pitch of 15 pixels.

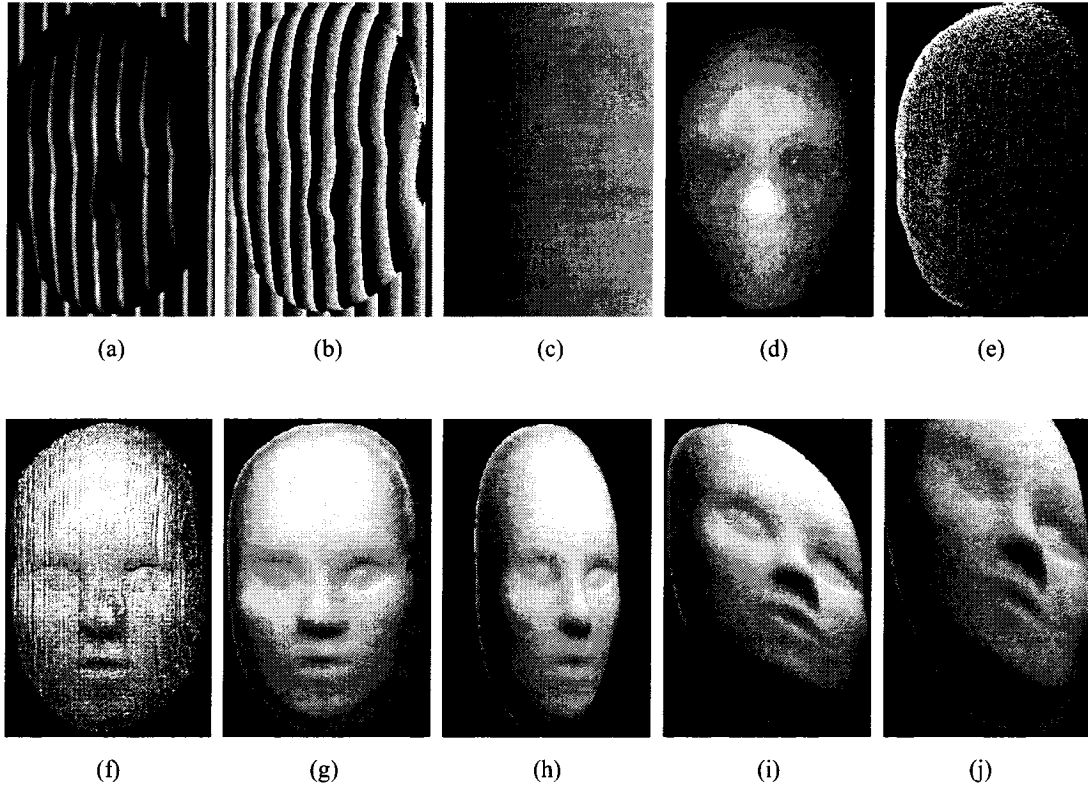


Figure 4-22 3-D shape measurement of a plastic human-head mask with the six-step triangular-pattern phase-shifting method. (a) One of the six captured triangular fringe pattern images, (b) Wrapped intensity-ratio map, (c) Unwrapped intensity-ratio map, (d) Intensity-ratio difference map after scaling, (e) Reconstructed 3-D wire-frame model, (f) Reconstructed 3-D shaded model with noise, and (g) – (j) Reconstructed 3-D shaded model after filtering. The triangular pattern was generated with a pitch of 18 pixels.

4.6.2 Discussion

The triangular-pattern phase-shifting method with different number of phase-shifting steps has been tested with different values of pitch of the input triangular pattern. The experimental results indicate that the triangular-pattern phase-shifting method with more phase-shifting steps can provide a higher accuracy of the 3-D object reconstruction. However, this depends on the pitch of the triangular pattern being evenly divided by the number of the phase-shifting steps. A smaller value of pitch of the triangular pattern also increases the measurement accuracy, as long as intensity-ratio unwrapping can be successfully completed. The drawback of the triangular-pattern phase-shifting method lies

in its sensitivity to image defocus. An error analysis and method of error compensation for the triangular-pattern phase-shifting method were therefore carried out, and are presented in Chapter 6.

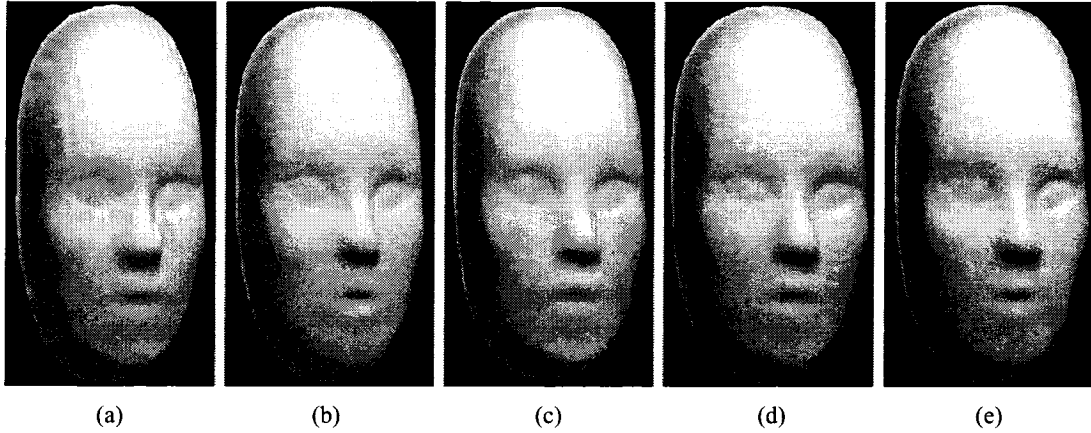


Figure 4-23 3-D shape measurement of a plastic human-head mask with the triangular-pattern phase-shifting method. (a) – (e) Reconstructed 3-D shaded models with different phase-shifting methods with optimal value of pitch of the triangular pattern; in (a) and (c), the triangular pattern was generated with pitch of 16 pixels, (b) and (d) with a pitch of 15 pixels, and (e) with 18 pixels.

4.7 Summary

A novel triangular-pattern phase-shifting approach was developed. The presented approach combines the advantages of both the traditional sinusoidal-pattern phase-shifting method and the intensity-ratio method, and it possesses the features of simple coding and decoding. Experiments determined that more phase steps generate higher measurement accuracy; however, the digital fringe projection generates phase-shifting errors if the pitch of the pattern cannot be evenly divided by the number of phase steps. Lower values of pitch lead to higher measurement accuracy; however, there is an optimal pitch value, below which, intensity-ratio unwrapping failure occurs. Higher measurement accuracy can thus be obtained using a greater number of phase-shifting steps and a lower value of pitch, as long as the pitch is appropriately selected to be divisible by the number of phase-shifting steps, and not below the optimal value, where failure intensity-ratio unwrapping would occur. The minimum number of sample images required to reconstruct the 3-D object is two, which is

less than the image-number requirement of current methods. Compared with the traditional sinusoidal-pattern phase-shifting method, which involves a time-consuming calculation of an arctangent function to obtain the phase, the processing speed is expected to be faster because fewer images are required to reconstruct the 3-D object and the intensity-ratio calculation is simpler. Compared with the traditional intensity-ratio-based methods (Carrihill 1985 and Miyasake 2002), it also has a better depth resolution and less ambiguity problem when the intensity-ratio is repeated (Chazan 1995) to reduce the measurement noise. Applications to various inspection tasks, mobile-robot navigation and 3-D surface modeling are expected.

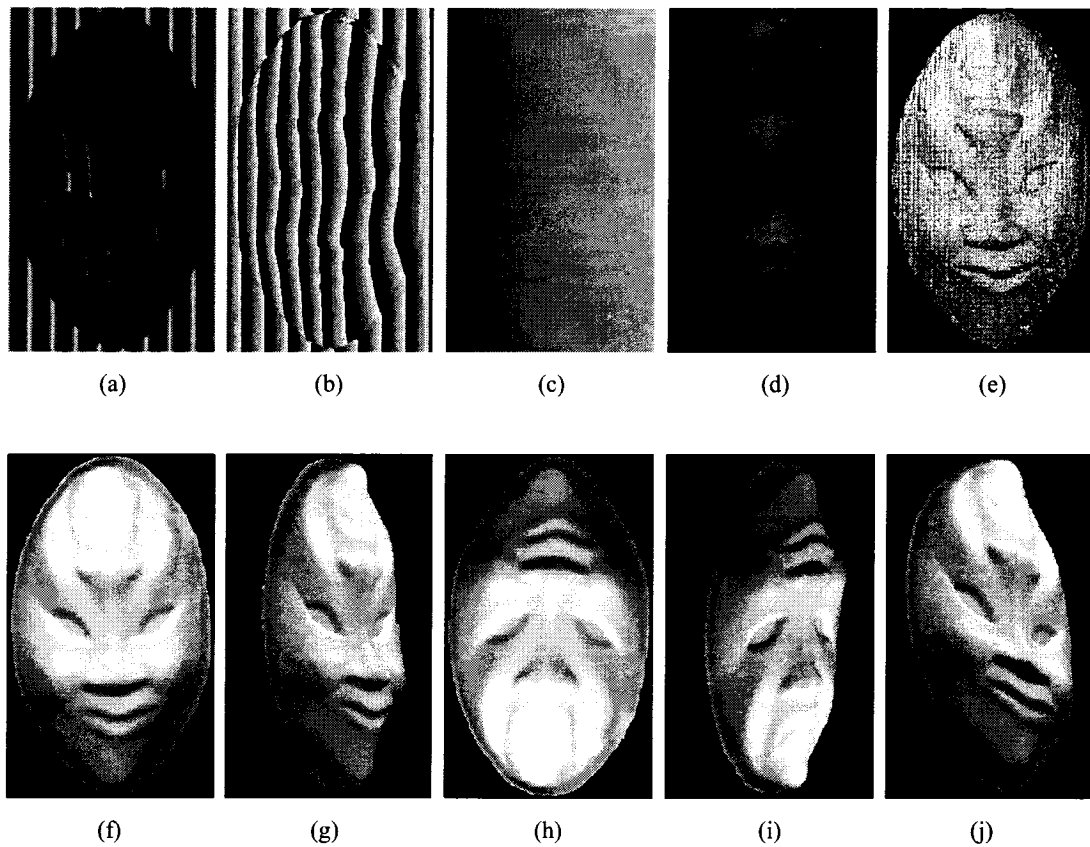


Figure 4-24 3-D shape measurement of a wooden mask with the six-step triangular-pattern phase-shifting method. (a) One of the six captured six-shift triangular fringe pattern images, (b) Wrapped intensity-ratio map, (c) Unwrapped intensity-ratio map, (d) Intensity-ratio difference map after scaling, (e) Reconstructed 3-D wire-frame model, (f) Reconstructed 3-D shaded model with noise, and (g) – (j) Reconstructed 3-D shaded model after filtering. The triangular pattern was generated with a pitch of 18 pixels.

5 System Calibration and 3-D Shape Measurement Experiments

The calibration procedure for the triangular-pattern phase-shifting method is the same as for the sinusoidal-pattern phase-shifting method, except that the triangular-pattern phase-shifting method deals with intensity-ratio, while the sinusoidal-pattern phase-shifting method deals with phase. Without loss of generality, the following description is only for the sinusoidal-pattern phase-shifting method.

5.1 Introduction

Both linear and non-linear mapping functions have been used to express the relationship between the phase distribution and height of the object in this research; however, often obtaining the non-linear mapping function has involved complex derivations. Also, there has not been any comparative study of linear and non-linear methods of phase-measuring calibration and measurement techniques. In this chapter, the mapping relationship between the phase and the height of the object surface is formulated by both linear and non-linear equations. The non-linear mapping function is obtained through a simple geometrical derivation, and a comparison is made between the two calibration approaches in terms of measurement accuracy. A least-squares method is applied in both linear and non-linear calibrations to obtain the system related calibration parameters. Both numerical simulation and measurement experiments are carried out to analyze the accuracy of the two calibrations. In Section 5.2, the principle of the system calibration is introduced, including the detailed descriptions about the linear and non-linear calibrations to determine the system related calibration parameters through a least-squares method, and the procedure to calculate the intensity-ratio/phase difference during the calibration. Section 5.3 presents calibration and measurement experiments, both in simulation and real, for error analysis of both linear and non-linear calibration methods, and discusses the accuracy of the linear and non-linear calibration methods. Section 5.4 presents the 3-D shape measurement experiment of the

sinusoidal-pattern phase-shifting method by applying the calibration results. A summary is given in Section 5.5.

5.2 Principle of System Calibration

Equations (3-21) and (4-37) can be used to calculate the height of the object surface relative to the reference plane only if all the system parameters and the fringe pitch on the reference plane are known. Although parameters H (in the $X-Z$ plane) and d in Equation (3-22) could be measured, it is difficult to precisely determine the parameter p , the fringe pitch in the reference plane, which is actually not constant across the image due to the divergence of rays from the projector (Figure 3-5). Instead, a calibration is performed to determine the unknown coefficients that relate the height of the object surface to the intensity-ratio/phase difference, without having to explicitly know the system parameters related to the system configuration. Accurate alignment of the camera, projector, and reference plate are therefore not necessary. As the relationship between the intensity-ratio/phase difference map and the surface height of the object is formulated as linear and non-linear equations discussed in Section 3.5 and Section 4.4, calibration by both of these methods was performed in this research to obtain the system-related calibration parameters. This involves determining $K(x, y)$ in Equations (3-23) and (4-38) for the linear calibration, and $m(x, y)$ and $n(x, y)$ in Equations (3-28) and (4-39), for the non-linear calibration, as explained in the remainder of Chapter 5. Figure 5-1 illustrates the system calibration setup.

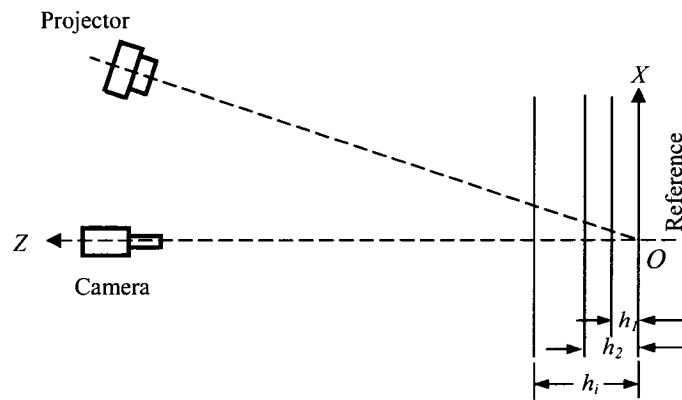


Figure 5-1 Schematic representation of the system calibration setup.

5.2.1 Linear Calibration

To determine the coefficient $K(x, y)$ in Equation (3-23) for the sinusoidal-pattern phase-shifting method, in the coordinate system of Figure 5-1, either the calibration object or the reference plane must be translated through a known distance along the Z direction relative to the reference plane at O . Here the reference plane is translated to different known calibration positions of depth h_i , where $i = 1, 2, 3, \dots, N$ is the calibration position and N is the number of positions. By applying Equation (3-23), the phase-to-height relationship for each pixel (x, y) can be determined at each calibration position as follows:

$$h_i(x, y) = K(x, y)\Delta\varphi_i(x, y) \quad i = 1, 2, \dots, N \quad (5-1)$$

where the phase difference $\Delta\varphi_i(x, y)$ is obtained by:

$$\Delta\varphi_i(x, y) = \varphi_i(x, y) - \varphi_r(x, y) \quad i = 1, 2, \dots, N \quad (5-2)$$

where $\varphi_i(x, y)$ is the calibration phase distribution due to the translation h_i of the calibration plane relative to the reference plane, $\varphi_r(x, y)$ is the phase distribution of the reference plane, and both $\varphi_i(x, y)$ and $\varphi_r(x, y)$ can be obtained from the captured intensity images acquired at each calibration position and the reference plane, respectively, by applying a specific phase-shifting technique described previously in Section 3.3. Because of the linear relationship, the coefficient $K(x, y)$ can be obtained by only one calibration position ($N = 1$). However, to calibrate over the entire working volume expected for object measurement and to increase the measurement accuracy, in practice, the system is calibrated by shifting the calibration plate to several different positions. Applying the least-squares algorithm to linear Equation (3-23), the following equation is used to obtain the coefficient $K(x, y)$ and thus complete the system calibration:

$$K(x, y) = \frac{\sum_{i=1}^N \Delta\varphi_i(x, y)h_i(x, y)}{\sum_{i=1}^N \Delta\varphi_i^2(x, y)} \quad (5-3)$$

With $K(x, y)$ determined, the height or depth of any object surface can be determined from Equation (3-23) by first acquiring the phase difference distribution using Equation (3-24).

The use of more calibration positions tends to increase the accuracy of the calibrated system.

For the triangular-pattern phase-shifting method, instead of phase difference, $\Delta\varphi(x, y)$, the intensity-ratio difference, $\Delta r(x, y)$, calculated by Equation (4-40), should be substituted into Equation (5-3) to calculate the coefficient $K(x, y)$, that is:

$$K(x, y) = \frac{\sum_{i=1}^N \Delta r_i(x, y)h_i(x, y)}{\sum_{i=1}^N \Delta r_i^2(x, y)} \quad (5-4)$$

5.2.2 Non-Linear Calibration

The non-linear calibration is similar to the linear calibration except that two parameters $m(x, y)$ and $n(x, y)$ of Equation (3-28) for sinusoidal-pattern phase-shifting method are determined instead of the single coefficient $K(x, y)$. The phase difference can be obtained using the same method as in the linear calibration. The minimum number of calibration positions for the non-linear calibration is two ($N = 2$). Again to calibrate over the entire object-space working volume and to increase the measurement accuracy, more calibration positions ($N > 2$) are used to perform the non-linear calibration. A least-squares method is applied to determine parameters $m(x, y)$ and $n(x, y)$ in Equation (3-28), which can be rearranged as:

$$\Delta\varphi(x, y) = m(x, y)h(x, y) + n(x, y)h(x, y)\Delta\varphi(x, y) \quad (5-5)$$

By choosing $h(x, y)$ and $h(x, y)\varphi(x, y)$ in Equation (5-5) as the basis functions, and applying the least-squares algorithm, the sum of squares is:

$$q = \sum_{i=1}^N [\Delta\varphi_i(x, y) - m(x, y)h_i(x, y) - n(x, y)h_i(x, y)\Delta\varphi_i(x, y)]^2 \quad (5-6)$$

and q depends on $m(x, y)$ and $n(x, y)$. A necessary condition for q to be minimum is:

$$\begin{aligned} \frac{\partial q}{\partial m(x, y)} &= -2 \sum_{i=1}^N [\Delta\varphi_i(x, y) - m(x, y)h_i(x, y) - n(x, y)h_i(x, y)\Delta\varphi_i(x, y)]h_i(x, y) = 0 \\ \frac{\partial q}{\partial n(x, y)} &= -2 \sum_{i=1}^N [\Delta\varphi_i(x, y) - m(x, y)h_i(x, y) - n(x, y)h_i(x, y)\Delta\varphi_i(x, y)]h_i(x, y)\Delta\varphi_i(x, y) = 0 \end{aligned}$$

which can be arranged as:

$$\left. \begin{aligned} m(x, y) \sum_{i=1}^N h_i^2(x, y) + n(x, y) \sum_{i=1}^N h_i^2(x, y)\Delta\varphi_i(x, y) &= \sum_{i=1}^N h_i(x, y)\Delta\varphi_i(x, y) \\ m(x, y) \sum_{i=1}^N h_i^2(x, y)\Delta\varphi_i(x, y) + n(x, y) \sum_{i=1}^N h_i^2(x, y)\Delta\varphi_i^2(x, y) &= \sum_{i=1}^N h_i(x, y)\Delta\varphi_i^2(x, y) \end{aligned} \right\} \quad (5-7)$$

Equation (5-7) can be written in matrix form as:

$$\begin{pmatrix} a_1(x, y) & a_2(x, y) \\ a_2(x, y) & a_3(x, y) \end{pmatrix} \begin{pmatrix} m(x, y) \\ n(x, y) \end{pmatrix} = \begin{pmatrix} b_1(x, y) \\ b_2(x, y) \end{pmatrix} \quad (5-8)$$

where

$$a_1(x, y) = \sum_{i=1}^N h_i^2(x, y)$$

$$a_2(x, y) = \sum_{i=1}^N h_i^2(x, y) \Delta \phi_i(x, y)$$

$$a_3(x, y) = \sum_{i=1}^N h_i^2(x, y) \Delta \phi_i^2(x, y)$$

$$b_1(x, y) = \sum_{i=1}^N h_i(x, y) \Delta \phi_i(x, y)$$

$$b_2(x, y) = \sum_{i=1}^N h_i(x, y) \Delta \phi_i^2(x, y)$$

The parameters $m(x, y)$ and $n(x, y)$ in Equation (5-8) can be solved as:

$$\left. \begin{aligned} m(x, y) &= \frac{a_3(x, y)b_1(x, y) - a_2(x, y)b_2(x, y)}{a_1(x, y)a_3(x, y) - a_2^2(x, y)} \\ n(x, y) &= \frac{a_1(x, y)b_2(x, y) - a_2(x, y)b_1(x, y)}{a_1(x, y)a_3(x, y) - a_2^2(x, y)} \end{aligned} \right\} \quad (5-9)$$

A procedure similar to that described in Section 5.2.1 is applied here to obtain the parameters $m(x, y)$ and $n(x, y)$. After completing the calibration and getting the phase difference distribution, the 3-D coordinates of the object surface can be calculated from Equation (3-28).

For the triangular-pattern phase-shifting method, instead of phase difference, $\Delta \phi(x, y)$, the intensity-ratio difference, $\Delta r(x, y)$, calculated by Equation (4-40), should be substituted into Equation (5-9) to calculate the coefficient $m(x, y)$ and $n(x, y)$. Thus the coefficients $a_1(x, y)$, $a_2(x, y)$, $a_3(x, y)$, $b_1(x, y)$, and $b_2(x, y)$ in Equation (5-9) should be calculated by the following equations:

$$a_1(x, y) = \sum_{i=1}^N h_i^2(x, y), \quad a_2(x, y) = \sum_{i=1}^N h_i^2(x, y) \Delta r_i(x, y), \quad a_3(x, y) = \sum_{i=1}^N h_i^2(x, y) \Delta r_i^2(x, y)$$

$$b_1(x, y) = \sum_{i=1}^N h_i(x, y) \Delta r_i(x, y), \quad b_2(x, y) = \sum_{i=1}^N h_i(x, y) \Delta r_i^2(x, y)$$

5.2.3 Intensity-Ratio Difference and Phase Difference Calculation

The intensity-ratio difference and phase difference calculations are major steps of the calibration procedure for the triangular-pattern and sinusoidal-pattern phase-shifting methods. There is a very important issue that needs to be addressed for the intensity-ratio difference and phase difference calculations. Otherwise, it may result in a wrong result for these calculations in practice.

The value of the starting point of the unwrapped intensity-ratio and unwrapped phase map depends on the position of the captured fringe patterns, which are related to the optical setup. However, the value of the starting point of the unwrapped intensity-ratio for triangular-pattern phase-shifting methods may be in the range from 0 to 4, for the two-step triangular-pattern phase-shifting method, 0 to 6, for the three-step triangular-pattern phase-shifting method, 0 to 8, for the four-step triangular-pattern phase-shifting method, etc. For the sinusoidal-pattern phase-shifting methods, the value of the starting point of the unwrapped phase map ranges from 0 to 2π for any phase step. During the calibration, under the coordinate system of the Figure 5-1, the sign of the intensity-ratio difference and phase difference calculated by Equations (4-40) and (5-2) depends on the direction of movement of the calibration plane. When the calibration plane is translated to different positions toward the positive Z direction, the value of the intensity-ratio difference for the triangular-pattern phase-shifting methods, and phase difference for sinusoidal-pattern phase-shifting methods, have a positive sign; however, when the reference plane is translated to different positions toward the negative Z direction, the value of the intensity-ratio difference and phase difference have a negative sign. For triangular-pattern phase-shifting methods, the value of the starting point of the unwrapped intensity-ratio map can be any value between 0 and 4, 6, 8, etc, for the different number of phase steps; for sinusoidal-pattern phase-shifting methods, the value of the starting point of the unwrapped phase map can be any value between 0 and 2π for any phase step. Therefore, several operations (given below) are necessary to calculate the intensity-ratio and phase difference (Equations (4-40) and (5-2)) in order to obtain a positive intensity-ratio difference for the triangular-pattern phase-shifting methods, and a positive phase difference for the sinusoidal-pattern phase-shifting methods, when the reference plane moves toward the positive Z direction. Similar operations are necessary to obtain a negative intensity-ratio difference for the triangular-pattern phase-

shifting methods, and a negative phase difference for the sinusoidal-pattern phase-shifting methods, when the reference plane moves toward the negative Z direction. This is an important issue to perform the calibration correctly.

For triangular-pattern phase-shifting methods, the following operations are necessary.

When the reference plane is translated to different positions toward the positive Z direction:

$$\text{if } \Delta r_i(x, y) < 0, \text{ then } \Delta r_i(x, y) = \Delta r_i(x, y) + nr_m \quad n = 1, 2, 3, \dots \quad (5-10)$$

When the reference plane is translated to different positions toward the negative Z direction:

$$\text{if } \Delta r_i(x, y) > 0, \text{ then } \Delta r_i(x, y) = \Delta r_i(x, y) - nr_m \quad n = 1, 2, 3, \dots \quad (5-11)$$

where n is an integer to be determined during the calibration process using the measured intensity-ratio difference map $\Delta r_i(x, y)$. The value of r_m is 4 for the two-step triangular-pattern phase-shifting method, 6 for the three-step triangular-pattern phase-shifting method, 8 for the four-step triangular-pattern phase-shifting method, etc.

For sinusoidal-pattern phase-shifting methods, the following operations are necessary.

When the reference plane is translated to different positions toward the positive Z direction:

$$\text{if } \Delta \phi_i(x, y) < 0, \text{ then } \Delta \phi_i(x, y) = \Delta \phi_i(x, y) + 2n\pi \quad n = 1, 2, 3, \dots \quad (5-12)$$

When the reference plane is translated to different positions toward the negative Z direction:

$$\text{if } \Delta \phi_i(x, y) > 0, \text{ then } \Delta \phi_i(x, y) = \Delta \phi_i(x, y) - 2n\pi \quad n = 1, 2, 3, \dots \quad (5-13)$$

where n is an integer to be determined during the calibration process using the measured phase difference map $\Delta\varphi_i(x, y)$.

5.3 Experiments in Calibration and Measurement

Experiments were carried out in both computer simulation and real measurement to compare the accuracy of the linear and non-linear calibration methods. This is described in this subsection.

5.3.1 Experimental Setup and Overview

A schematic diagram of the experimental system is shown in Figure 3-1. According to Section 3.2, Design of Experimental Setup, the distance between the reference plane and the camera lens was approximately 1.9 m and the angle between the axes of the projector and the camera was approximately 15°. The values of these parameters do not have to be precise however, as the system calibration parameters are determined using the proposed calibration methods. The equipment used in these experiments is described in Section 3.1.2.

For the real system calibration experiment, both triangular and sinusoidal grayscale pattern phase-shifting methods were used to calibrate the system for different measurement methods used for 3-D shape measurement, and this will be discussed later in Section 5.4. Simulation experiments for the comparison of the linear and non-linear calibrations were performed using only the four-step sinusoidal grayscale pattern phase-shifting method to verify the performance of the linear and non-linear calibrations and to compare their accuracies. The triangular and sinusoidal fringe patterns were generated by a computer program and projected onto a flat calibration plate of size 400 mm × 500 mm. Acquisition, computation and display were performed using Visual C++ 6.0 and OpenGL.

For the comparison of linear and non-linear calibration and measurement methods, in order to eliminate any differences in the system setup and environmental effects, such as lighting, mechanical positioning of the plate, and noise, all experiments and simulations below for the linear and non-linear methods used the exact same projection pattern with the same value of pitch, and the same captured input images. Thus the exact same phase

distributions $\varphi(x,y)$ and phase-difference distributions $\Delta\varphi(x,y)$ were input to both the linear and non-linear calibration and measurement algorithms to eliminate all differences due to environmental and physical experimental conditions. As well, all synthetic noise added in simulations for the linear method was identical to the noise added in the non-linear method.

To determine and compare the accuracies of the linear and non-linear calibrations, discussed in Sections 5.2.1 and 5.2.2, and to evaluate the effects of noise on the calibration and measurement procedures, several calibration and measurement simulations and experiments were performed. A similar approach was used by Guo (2005) for some experiments. Experiments include: (a) real system calibration (Section 5.3.2); (b) calibration and measurement simulation where noise is added at the calibration stage (Section 5.3.3); (c) calibration and measurement simulation where noise is added at both the calibration and measurement stages (Section 5.3.4); calibration and measurement simulation where no noise is added at both the calibration and measurement stages (Section 5.3.5); and (d) real measurement (Section 5.3.6). The discussion of the experimental results is presented in Section 5.3.7, and the conclusions are given in Section 5.3.8.

5.3.2 Real System Calibration

A real system calibration was first performed using the linear and non-linear calibration methods, discussed in Sections 5.2.1 and 5.2.2, respectively, to obtain the distribution of coefficients $K(x,y)$ for the linear method, and $m(x,y)$ and $n(x,y)$ for the non-linear method. Both triangular-pattern phase-shifting and sinusoidal-pattern phase-shifting methods were used, respectively, to determine the system related calibration parameters during the system calibration. First, images of the flat reference plate at position $z = 0$ were acquired for each of the phase shifts and the intensity-ratio/phase distribution was obtained by intensity-ratio/phase wrapping and unwrapping calculations. Thereafter, the plate was moved toward the camera in sequence to 10 different positions at 5 mm increments between each position. At each position, images were acquired and the intensity-ratio/phase distribution computed as for the reference plane. The intensity-ratio/phase differences between the measured plate and the reference plane at position $z = 0$ were determined using Equations (4-40) and (5-2). The distributions of coefficients $K(x,y)$ for the linear

calibration, and $m(x, y)$ and $n(x, y)$ for non-linear calibration were obtained using Equations (5-4) and (5-9), for triangular-pattern phase-shifting method, and Equations (5-3) and (5-9), for sinusoidal-pattern phase-shifting method (It should be noted that for non-linear calibration calculation, the parameters in Equation (5-9) have different calculation equations for triangular-pattern phase-shifting and sinusoidal-pattern phase-shifting methods). Figure 5-2, Figure 5-3 and Figure 5-4 illustrate the results of linear and non-linear calibrations for two-, three-, and six-step triangular-pattern phase-shifting methods; Figure 5-5 and Figure 5-6 show the results of linear and non-linear calibrations for three- and four-step sinusoidal-pattern phase-shifting methods.

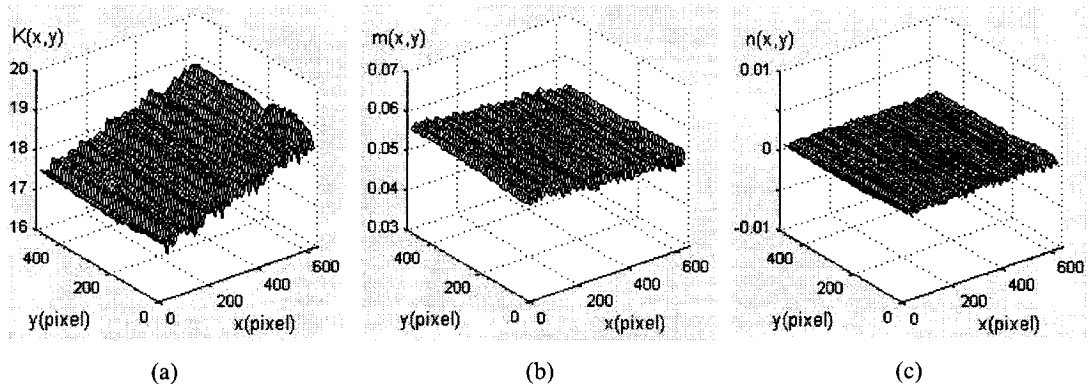


Figure 5-2 Results of the real linear and non-linear calibrations of two-step triangular-pattern phase-shifting method. (a) Distribution of $K(x, y)$ of the linear calibration, (b) and (c) Distribution of $m(x, y)$ and $n(x, y)$ of the non-linear calibration.

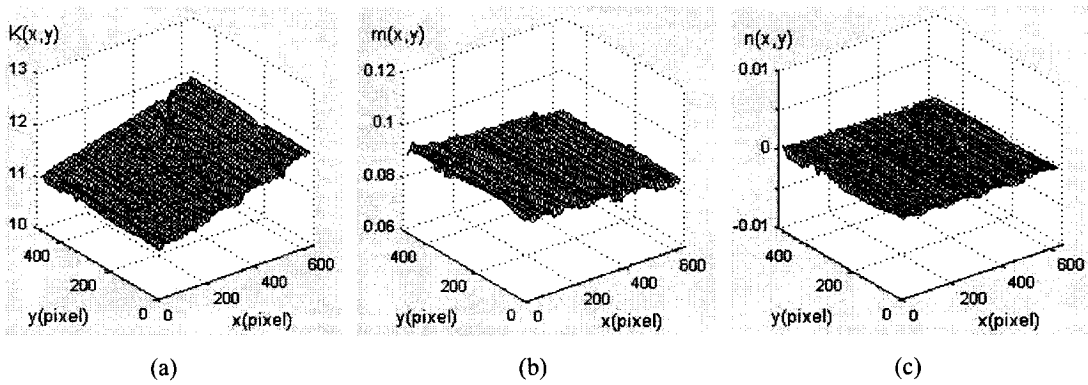


Figure 5-3 Results of the linear and non-linear calibrations of three-step triangular-pattern phase-shifting method. (a) Distribution of $K(x, y)$ of the linear calibration, (b) and (c) Distribution of $m(x, y)$ and $n(x, y)$ of the non-linear calibration.

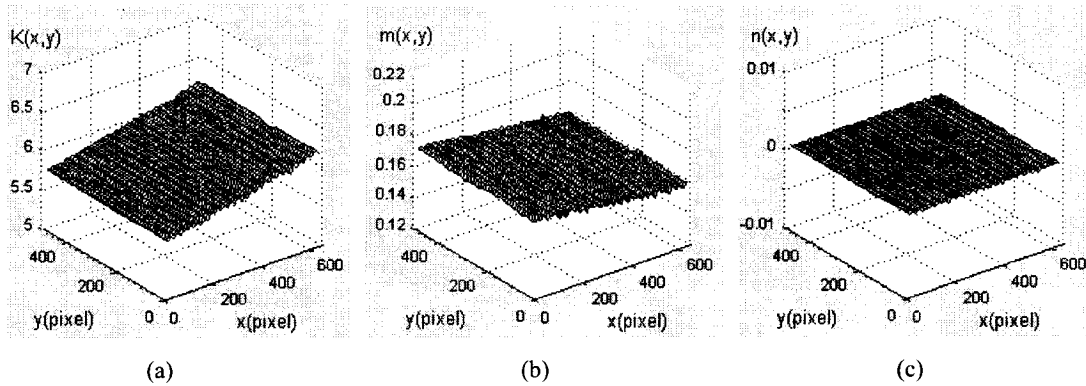


Figure 5-4 Results of the real linear and non-linear calibrations of six-step triangular-pattern phase-shifting method. (a) Distribution of $K(x, y)$ of the linear calibration, (b) and (c) Distribution of $m(x, y)$ and $n(x, y)$ of the non-linear calibration.

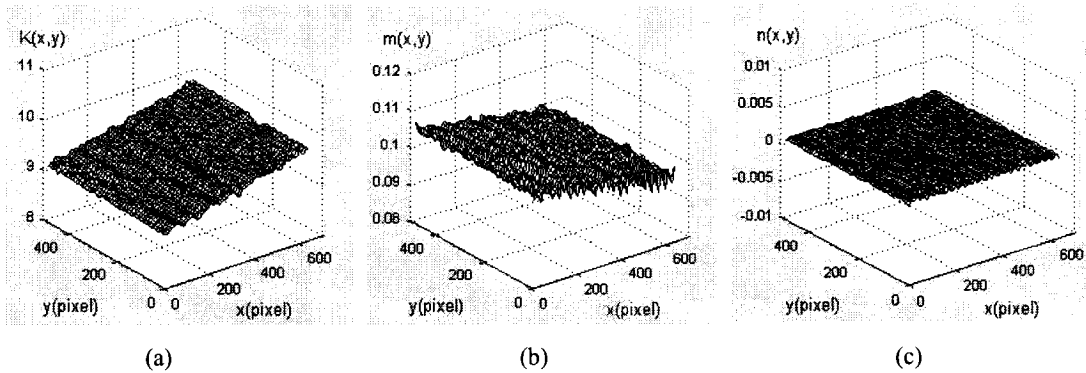


Figure 5-5 Results of the real linear and non-linear calibrations of three-step sinusoidal-pattern phase-shifting method. (a) Distribution of $K(x, y)$ of the linear calibration, (b) and (c) Distribution of $m(x, y)$ and $n(x, y)$ of the non-linear calibration.

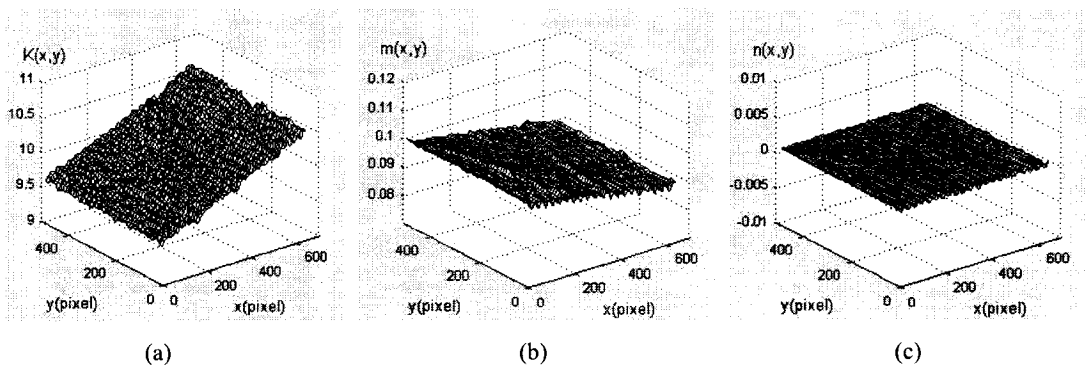


Figure 5-6 Results of the real linear and non-linear calibrations of four-step sinusoidal-pattern phase-shifting method. (a) Distribution of $K(x, y)$ of the linear calibration, (b) and (c) Distribution of $m(x, y)$ and $n(x, y)$ of the non-linear calibration.

5.3.3 Calibration and Measurement Simulation with Noise in Calibration

A first calibration and measurement simulation (Guo 2005) was carried out to determine the accuracy of both linear and non-linear calibrations under the influence of noise in the calibration. The simulation involved re-calibration using the coefficients $K(x, y)$ for the linear calibration, and $m(x, y)$ and $n(x, y)$ for the non-linear calibration, obtained by the real calibration (Section 5.3.2) but with noise synthetically introduced to the system in the re-calibration, followed by a measurement simulation.

To perform calibration and measurement simulation, a specific point (a single pixel location) with known values of coefficients $K = 9.648320$ for the linear calibration, and $m = 0.098381$ and $n = 0.000881$ for the non-linear calibration was selected from the respective distributions (Figure 5-6). The calibration stage of the simulation was accomplished by sampling the mapping function, Equation (3-23), for the linear method, and Equation (3-28) for the non-linear method, at 11 positions with an equal spacing of 5 mm between adjacent positions. Thus, the range of depth that was simulated was 50 mm. During the simulation, an independent additive noise was purposely added to the phase difference at the simulation point as follows:

$$\Delta\phi'(x, y) = \Delta\phi(x, y) + \eta(x, y) \quad (5-14)$$

where $\Delta\phi(x, y)$ is calculated by Equations (3-25) and (3-27) with known coefficients $K(x, y)$, $m(x, y)$, $n(x, y)$ and $h(x, y)$; and $\eta(x, y)$ is the noise with zero-mean Gaussian distribution, generated by the Box-Muller transformation (Box 1958). The standard deviation σ used in the simulation was chosen to be 0.01π . $\Delta\phi'(x, y)$ is the phase difference with added noise, and is the new synthetic data used to re-calibrate the system to get new values of coefficients $K'(x, y)$ for the linear method, and $m'(x, y)$ and $n'(x, y)$ for the non-linear method.

To evaluate the effect of noise on the calibrations, a measurement using these new values of $K'(x, y)$ for the linear method, and $m'(x, y)$ and $n'(x, y)$ for the non-linear method was simulated; however, noise was not added again in the measurement procedure. The simulated measurement was performed on a flat plate at 51 positions with equal spacing of

1 mm between adjacent positions (range of depth of 50 mm). The depth at each position with respect to the reference position was calculated using the phase-to-height mapping Equations (3-23) and (3-28) (linear and non-linear, respectively). At each position, one thousand runs with different noise values were performed, computing the depth and the root-mean-squares (RMS) errors over all runs, using the linear and non-linear calibrations.

The detailed simulation procedure is as follows:

- Select the true values of $K(x, y)$ for linear method, $m(x, y)$ and $n(x, y)$ for non-linear method from the real calibration results.
- For calibration positions, $h(x, y) = \{5, 10, \dots, 50\}$ with an increment of 5 mm.
- Using $K(x, y)$, $m(x, y)$, $n(x, y)$ and $h(x, y)$ compute the theoretical phase difference $\Delta\phi(x, y)$ using Equation (3-25) for the linear method, and Equation (3-27) for the non-linear method.
- Simulate phase difference $\Delta\phi'(x, y) = \Delta\phi(x, y) + \eta(x, y)$ by adding a random error.
- Using $h(x, y)$ and simulated $\Delta\phi'(x, y)$, compute the calibration results of the new $K'(x, y)$, $m'(x, y)$ and $n'(x, y)$ using Equation (5-3) for the linear method, and Equation (5-9) for the non-linear method.

Thus, the calibration procedure is finished. By making use of the above calibration results, a measurement can be performed. Because the purpose of the calibration simulation is to examine the effects of random error on calibration, the random error is not considered in the following measurement procedure.

- For theoretical measurement depths, $h(x, y) = 0-50$ mm (with 1 mm increment).
- Using true values of $K(x, y)$ for the linear method, $m(x, y)$ and $n(x, y)$ for the non-linear method, compute $\Delta\phi(x, y)$ at these measurement depths with Equation (3-25) for the linear method, and Equation (3-27) for the non-linear method.
- Using $\Delta\phi(x, y)$ determined in the last step and the calibration results of the new $K'(x, y)$, $m'(x, y)$ and $n'(x, y)$, compute the measurement results of these depths $h_c(x, y)$ using Equation (3-23) for the linear method, and Equation (3-28) for the non-linear method.

- Compute the measurement errors by subtracting the theoretical depths from the measured depths: $h_c(x, y) - h(x, y)$.

Thus, the measurement procedure is finished. By repeating this procedure 1000 times, the root-mean-square error of the depth calculation can be obtained.

- Compute RMS errors of the depth calculation.

Figure 5-7 shows the structured flow chat of the calibration simulation. The results for the linear and non-linear methods, discussed in Section 5.3.7, are compared in Figure 5-8 (a).

Select true values of K , m and n from real calibration results	
Number of calibration positions	$N_c = 10$
Gap between each calibration position	$G_c = 5$ mm
Number of measurement positions	$N_m = 50$
Gap between each measurement position	$G_m = 1$ mm
For calibration positions $i = 0$ to $N_c - 1$	
$h(x, y) = (i + 1) \cdot G_c$	
Compute theoretical phase difference $\Delta\phi(x, y)$	
Linear method:	$\Delta\phi(x, y)[i] = \frac{h(x, y)}{K(x, y)}$
Non-linear method:	$\Delta\phi(x, y)[i] = \frac{m(x, y)h(x, y)}{1 - n(x, y)h(x, y)}$
$i = i + 1$	
For measurement positions $t = 0$ to $N_m - 1$	
$h_error[t] = 0$	
$t = t + 1$	
For $s = 0$ to $N - 1$ (number of runs)	
$c_1 = 0, c_2 = 0, a_1 = 0, a_2 = 0, a_3 = 0, b_1 = 0, b_2 = 0$	
For calibration positions $i = 0$ to $N_c - 1$	
Simulate phase difference by adding a random error $\Delta\phi'(x, y)[i] = \Delta\phi(x, y)[i] + \eta(x, y)$	
$h(x, y) = (i + 1) \cdot G_c$	

	Compute the parameters for Equations (5-3) and (5-8) Linear method: $c_1 += \Delta\varphi'(x, y)[i]h(x, y)$ $c_2 += \Delta\varphi'(x, y)[i]\Delta\varphi'(x, y)[i]$ Non-linear method: $a_1 += h^2(x, y)$ $a_2 += h^2(x, y)\Delta\varphi(x, y)[i]$ $a_3 += h^2(x, y)\Delta\varphi^2(x, y)[i]$ $b_1 += h(x, y)\Delta\varphi(x, y)[i]$ $b_2 += h(x, y)\Delta\varphi^2(x, y)[i]$
	$i = i + 1$
	Compute the calibration results of new K_{new}, m_{new} and n_{new} Linear method: $K_{new}(x, y) = \frac{c_1}{c_2}$ Non-linear method: $m_{new}(x, y) = \frac{a_3(x, y)b_1(x, y) - a_2(x, y)b_2(x, y)}{a_1(x, y)a_3(x, y) - a_2^2(x, y)}$ $n_{new}(x, y) = \frac{a_1(x, y)b_2(x, y) - a_2(x, y)b_1(x, y)}{a_1(x, y)a_3(x, y) - a_2^2(x, y)}$
	For theoretical measurement depths $j = 0$ to $N_m - 1$ $h(x, y) = (j + 1) \cdot G_m$ Using true values of K, m and n compute $\Delta\varphi(x, y)$ at these measurement depths Linear method: $\Delta\varphi(x, y)[j] = \frac{h(x, y)}{K(x, y)}$ Non-linear method: $\Delta\varphi(x, y)[j] = \frac{m(x, y)h(x, y)}{1 - n(x, y)h(x, y)}$
	Using $\Delta\varphi(x, y)$ determined in the last step and the calibration results of the new K_{new}, m_{new} and n_{new} compute the measurement results of these depths $h_c(x, y)$ (calculated) Linear method: $h_c(x, y)[j] = K_{new}(x, y)\Delta\varphi(x, y)[j]$ Non-linear method: $h_c(x, y)[j] = \frac{\Delta\varphi(x, y)[j]}{m_{new}(x, y) + n_{new}(x, y)\Delta\varphi(x, y)[j]}$
	Compute the measurement errors:

	$h_error(x, y)[j] += [h_c(x, y)[j] - h(x, y)[j]]^2$
	$j = j + 1$
	$s = s + 1$
For $t = 0$ to $N_m - 1$	
Compute the root-mean-square values of depth calculation $RMS[t] = \sqrt{\frac{h_error[t]}{N}}$	
Save the value of the calculation $RMS[t]$.	
	$t = t + 1$

Figure 5-7 Structured flow chart for the calibration simulation experiment.

5.3.4 Calibration and Measurement Simulation with Noise in both Calibration and Measurement

A second calibration and measurement simulation was carried out using a procedure very similar to the simulation discussed above. That is, the calibration simulation was carried out first, and then the measurement was performed. However, to determine the system accuracy with noise in both the calibration and measurement, independent additive noise with zero-mean Gaussian distribution was also purposely added to the phase differences at the simulation point when performing the simulated measurement. The detailed simulation procedure is as follows:

- Select the true values of $K(x, y)$ for the linear method, $m(x, y)$ and $n(x, y)$ for the non-linear method from the real calibration results.
- For calibration positions, $h(x, y) = \{5, 10, \dots, 50\}$ with an increment of 5 mm.
- Using $K(x, y)$, $m(x, y)$, $n(x, y)$ and $h(x, y)$ compute theoretical phase difference $\Delta\phi(x, y)$ with Equation (3-25) for the linear method, and Equation (3-27) for the non-linear method.
- Simulate phase difference $\Delta'\phi(x, y) = \Delta\phi(x, y) + \eta(x, y)$ by adding a random error.

- Using $h(x, y)$ and simulated $\Delta'\phi(x, y)$, compute the calibration results of the new $K'(x, y)$, $m'(x, y)$ and $n'(x, y)$ with Equation (5-3) for the linear method, and Equation (5-9) for the non-linear method.
- For theoretical measurement depths, $h(x, y) = 0-50$ mm (with 1 mm increment).
- Using true values of $K(x, y)$ for the linear method, $m(x, y)$ and $n(x, y)$ for the non-linear method compute $\Delta\phi(x, y)$ at these measurement depths with Equation (3-25) for the linear method, and Equation (3-27) for the non-linear method.
- Simulate phase difference $\Delta\phi'(x, y) = \Delta\phi(x, y) + \eta(x, y)$
- Using $\Delta\phi'(x, y)$ determined in the last step and the calibration results of the new $K'(x, y)$, $m'(x, y)$ and $n'(x, y)$, compute the measurement results of these depths $h_c(x, y)$ with Equation (3-23) for the linear method, and Equation (3-28) for the non-linear method.
- Compute the measurement errors by subtracting the theoretical depths from the measured depths: $h_c(x, y) - h(x, y)$.
- Compute RMS errors of the depth calculation.

The results of the measurement simulation with noise added at both the calibration and measurement stages, discussed in Section 5.3.7, are shown in Figure 5-8 (b).

Different points with different values of parameters $K(x, y)$, $m(x, y)$ and $n(x, y)$ have also been selected from their respective distribution (Figure 5-6) for simulation. Table 5-1 lists the values of parameters $K(x, y)$, $m(x, y)$ and $n(x, y)$ for these selected points.

Figure 5-9 shows the RMS errors of the depth calculation for linear and non-linear calibration methods for these different points.

To further compare the accuracy of the linear and non-linear calibration methods, instead of choosing only one specific point and performing calculations ten thousand times with different values of noise, the entire regions of the three coefficient distributions (K , m , n , Figure 5-6) of size 648×494 pixels were used for the simulation experiment. In this experiment, each point had a different value of noise added and thus each point may have a different value of coefficient $K(x, y)$, for the linear method, and $m(x, y)$ and $n(x, y)$, for

the non-linear method. The simulation procedure was the same as with the one-point method described above; however, the RMS values of the depth calculation errors at specific depths were calculated from all pixel points instead of one point. The simulated results of the calibration and measurement simulation, discussed in Section 5.3.7, are shown in Figure 5-10.

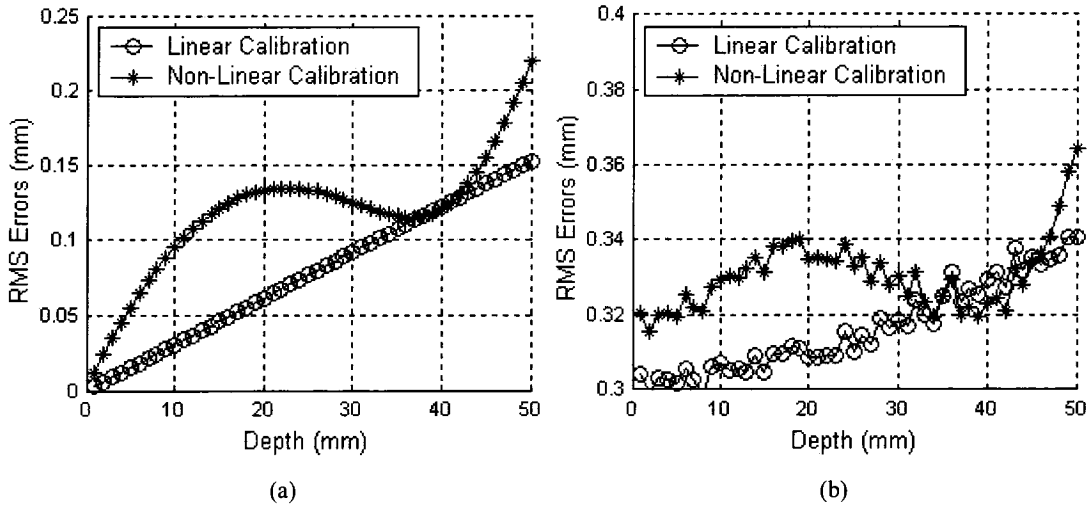
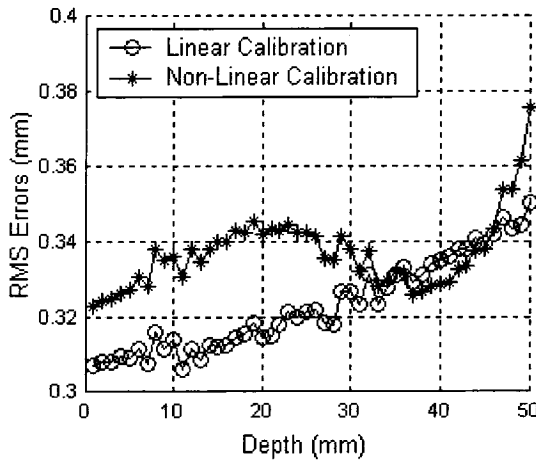


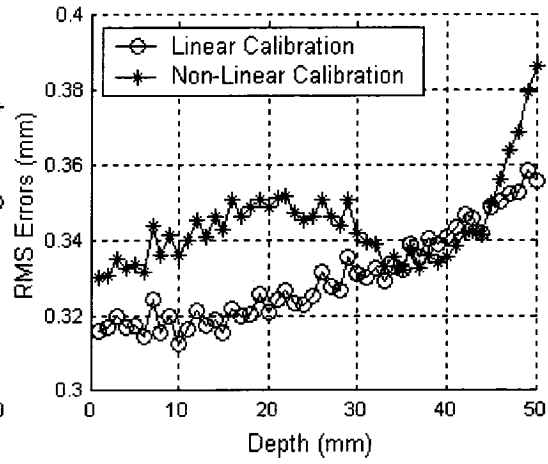
Figure 5-8 RMS errors of the depth calculation computed in ten thousand runs with different noise values for a specific point, at each of 51 measurement positions for the linear and non-linear calibration methods. (a) Measurement with noise added at the calibration stage only, (b) Measurement with noise added at both calibration and measurement stages.

Table 5-1 Values of parameter K , m and n of selected points for the measurement simulation experiment.

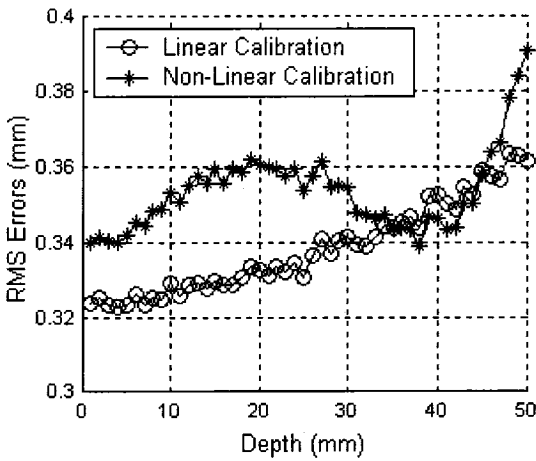
Calibration Methods	Linear Calibration	Non-linear Calibration	
Coefficient	$K(x, y)$	$m(x, y)$	$n(x, y)$
Point 1	9.837657	0.096614	0.000855
Point 2	10.077396	0.095042	0.000723
Point 3	10.314181	0.092270	0.000825
Point 4	10.487199	0.090869	0.000718



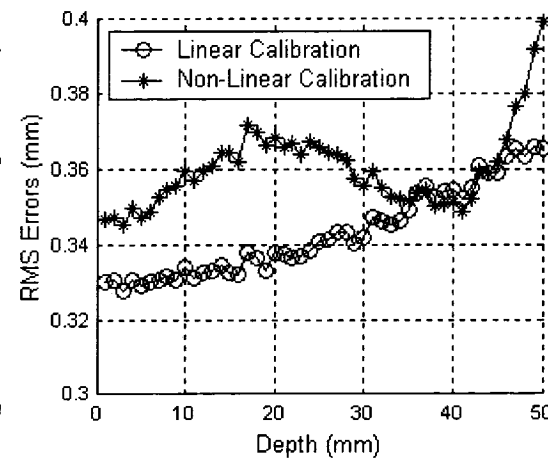
(a) $K = 9.837657$, $m = 0.096614$, $n = 0.000855$



(b) $K = 10.077396$, $m = 0.095042$, $n = 0.000723$



(c) $K = 10.314181$, $m = 0.092270$, $n = 0.000825$



(d) $K = 10.487199$, $m = 0.090869$, $n = 0.000718$

Figure 5-9 RMS errors of the depth calculation for linear and non-linear calibration methods where noise was added at both the calibration and measurement stages for different points. (a) $K = 9.837657$, $m = 0.096614$, $n = 0.000855$; (b) $K = 10.077396$, $m = 0.095042$, $n = 0.000723$; (c) $K = 10.314181$, $m = 0.092270$, $n = 0.000825$; (d) $K = 10.487199$, $m = 0.090869$, $n = 0.000718$.

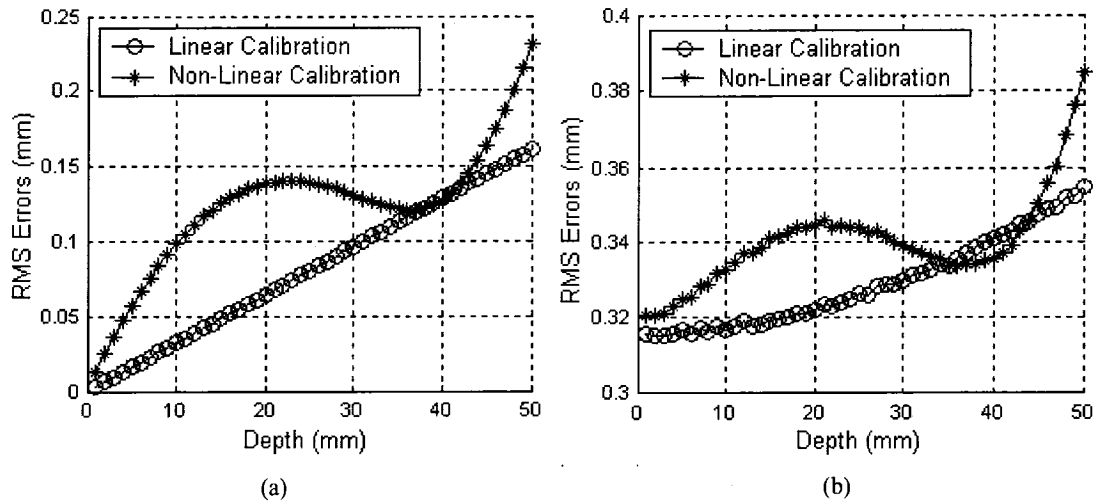


Figure 5-10 RMS errors of the depth calculation for linear and non-linear calibration methods calculated over all pixels (the entire distribution Figure 5-6) at each measurement position. (a) Measurement with noise added at calibration stage only, (b) Measurement with noise added at both calibration and measurement stages.

5.3.5 Calibration and Measurement Simulation with No Noise in both Calibration and Measurement

To confirm the validity of the calibration and measurement simulation discussed in Section 5.3.3 and Section 5.3.4, calibration and measurement simulations with no noise added in both calibration and measurement were carried out with both the one-point method and the entire-distribution method. The calibration and measurement simulation for both methods was carried out using the same procedures discussed in Section 5.3.3 and Section 5.3.4. The simulated results of the calibration and measurement simulation, discussed in Section 5.3.7, are shown in Figure 5-11.

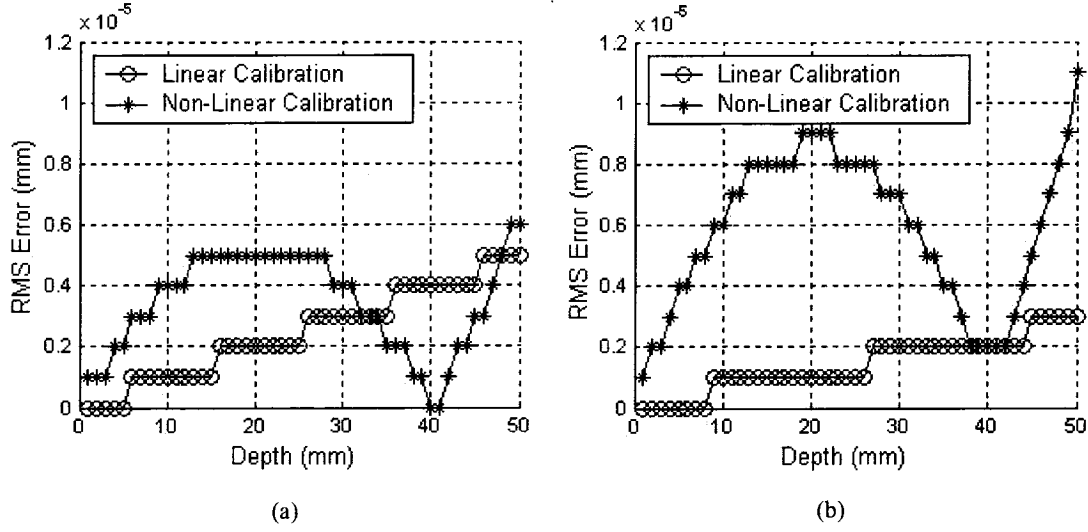


Figure 5-11 RMS errors of the depth calculation with no noise added in both calibration and measurement stages at each of 51 measurement positions for linear and non-linear calibration methods. (a) Calculated at a specific point at each measurement position, (b) Calculated over all pixels (the entire distribution Figure 5-6) at each measurement position. Note: the scale of RMS Error is multiplied by 10^{-5} .

5.3.6 Real Measurement

A real measurement experiment was carried out with the same system setup as the simulations, using the real calibration (Section 5.3.2) results for the linear and non-linear methods (Figure 5-6). First, images of a white flat plate at reference position $z = 0$ were acquired for each of the four phase shifts and the phase distribution was obtained by phase wrapping and unwrapping calculations. Thereafter, the plate was moved toward the camera to 25 different positions at 2 mm increments. At each position, images were acquired and the phase distribution was obtained in the same way as for the reference plate. The phase differences between the measured plate at each position and the reference plane at position $z = 0$ were determined using Equation (5-2). The measured distance at each position relative to the reference position was obtained by applying Equation (3-23), for the linear method, and Equation (3-28), for the non-linear method. The RMS values of the depth calculation errors at each position were calculated using all image pixel points of the measured plate and the result, discussed in Section 5.3.7, is shown in Figure 5-12.

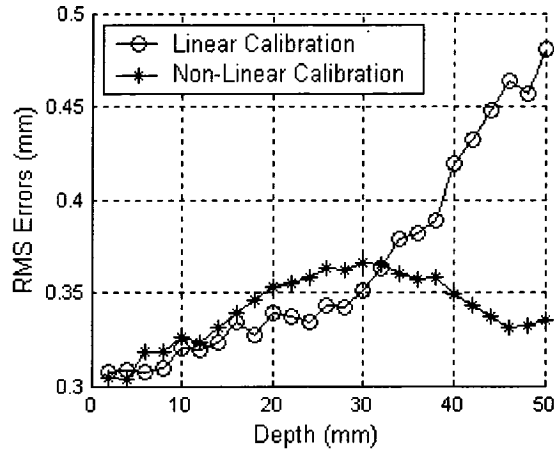


Figure 5-12 RMS errors of the depth calculation obtained by real measurement of a white flat plate at 25 different positions with 2 mm increments relative to the reference plane using linear and non-linear calibration results.

5.3.7 Results and Discussion

The distribution of coefficients $K(x,y)$ for the linear calibration, and $m(x,y)$ and $n(x,y)$ for the non-linear calibration for the four-step sinusoidal-pattern phase-shifting method are shown in Figure 5-6. The distribution of $K(x,y)$ was nearly planar and sloped while coefficient $m(x,y)$ was also nearly planar with a smaller gradient, and $n(x,y)$ was nearly planar and horizontal.

For the calibration and measurement simulations, the RMS errors of the depth calculated at different plate positions, for ten thousand runs at a specific point, consistently showed that the accuracy of the non-linear method was not superior to the linear method (Figure 5-8 and Figure 5-9). The accuracy of the non-linear method increased slightly in the middle range of depth (18 – 40 mm depth) and then decreased sharply at the higher depths (40 – 50 mm depth).

Based on the simulations, the linear calibration resulted in a higher measurement accuracy than the non-linear calibration in the low and middle range of depth (0 mm to 30 mm) and higher depth (45 mm to 50 mm), when noise was included in the calibration only (Figure 5-8 (a)) and in the calibration and measurement (Figure 5-8 (b) and Figure 5-9 (a) –

(d)). The same trend was seen when the entire image was analyzed instead of isolated points, for noise introduced at the calibration stage only (Figure 5-10 (a)) and noise introduced in both the calibration and measurement (Figure 5-10 (b)). This difference between the two calibration methods was approximately 0.02 mm in RMS error (RMSE) for most differences, and up to only 0.035 mm where the greatest differences occurred. While the value of this difference was small, it was approximately 10 % of the error. The RMSE generally increased from measurement at small depth from the reference plane (0 mm) to the full range of depth (50 mm), by approximately 0.32 mm to 0.40 mm RMSE for the non-linear calibration, and 0.32 mm to 0.36 mm RMSE for the linear calibration.

The error distribution for the simulation that introduced noise to the calibration only (Figure 5-8 (a)) is linearly distributed along the range of measurement depth, 0 – 0.15 mm RMSE for 0 – 50 mm depth for the linear calibration method. For the non-linear calibration method, the RMSE ranged from 0 to 0.23 mm over the full range of depth, and the measurement accuracy remained nearly constant in the middle range of the measurement depth (15 – 40 mm depth). The shapes of the curves for both linear and non-linear calibration methods were similar when noise was introduced in the measurement process (Figure 5-8 (b), Figure 5-9 (a) – (d), and Figure 5-10 (b)) compared to when noise was introduced in the calibration only (Figure 5-8 (a) and Figure 5-10 (a)), although the curves for the former show higher errors due to the additional noise. The accuracy of the linear calibration was higher than the non-linear calibration. A possible explanation is that the light source may have been far enough from the measured object, that the divergence of the projected light, which would make the relationship between the height of the object surface and the phase difference non-linear, was small enough for the linear relationship to be approximated, without contributing a high error to the measurement system.

The shape of the RMS-error versus depth curves was very similar for the different points (Figure 5-9 (a) – (d)) for both linear and non-linear calibrations. The influence of parameters $K(x, y)$, $m(x, y)$ and $n(x, y)$ on measurement accuracy can be seen in Figure 5-9. For the linear calibration method, when the value of $K(x, y)$ increased (from 9.837657 to 10.487199, Figure 5-9 (a) to Figure 5-9 (d), respectively), the accuracy decreased (RMSE increased approximately 0.02 mm). For the non-linear method, when $m(x, y)$ decreased

(from 0.096614 to 0.090869, Figure 5-9 (a) to Figure 5-9 (d), respectively), the accuracy decreased (RMSE errors increased about 0.02 mm).

The RMS errors of the depth based on simulation over the entire image (Figure 5-10) were nearly the same as with the one-point method (Figure 5-8 (b) and Figure 5-9 (a) – (d)) except that the curves were smoother with the former method. The one-point simulation would be preferred as it only took approximately 1 s to complete, compared to approximately 20 min for the simulation over the whole area.

The effect of adding noise in the measurement process can be seen by comparing the simulations with noise in the measurement and calibration processes (Figure 5-8 (b) and Figure 5-10 (b)) to simulations with noise in the calibration process only (no measurement noise) (Figure 5-8 (a) and Figure 5-10 (a)), respectively. The noise led to an increase in RMSE of approximately 0.15 mm to 0.3 mm for different depths.

The RMS error distributions of the depth calculation with no noise added in both calibration and measurement (Figure 5-11) have similar relative distribution shape with noise added in both the calibration and measurement processes (Figure 5-8 – Figure 5-10). However, the scale of the RMS error is very small because of no noise. Both the one-point method and entire-distribution methods verified the calibration and measurement simulation results discussed in Section 5.3.3 and Section 5.3.4.

The results of the real measurement experiment (Figure 5-12) are very similar to the results of the measurement simulations with synthetic noise (Figure 5-10 (b)) for both the linear and non-linear calibrations, over the range 0 – 30 mm depth. For the range 30 – 50 mm depth, the RMSE (based on the entire image) was higher in the real measurement than in the simulation, increasing up to 0.48 mm RMSE at 50 mm depth using the linear calibration. However, for the non-linear calibration, the real-measurement RMSE over the same range of depth (Figure 5-12) decreased to 0.33 mm RMSE at 50 mm depth, and was lower than the RMSE for the simulations (Figure 5-10 (b)).

At the lower range of 0 – 32 mm depth, the real-measurement RMSE for the linear method was similar, only slightly higher and lower (0 – 0.015 mm) over most of this range, to that for the non-linear calibration method (Figure 5-12). However, at the higher range of 32 – 50 mm depth, the real-measurement RMSE was considerably lower for the non-linear calibration method compared to that for the linear calibration method (Figure 5-12), up to

0.145 mm RMSE difference at 50 mm depth. A possible explanation is that, as the object approaches the projector and camera for the higher range of depth, the earlier assumption of linearity based on small divergence of light from the projector at the longer object-to-projector distances, becomes less valid (i.e. when the object-to-projector distance decreases at the higher range of depth). Note that the object approaches the projector and camera as the depth increases. Also, the non-linear method may be less sensitive to noise.

5.3.8 Conclusions

Simulation results indicate that the accuracy of the linear calibration is generally higher than non-linear calibration method. The RMS error distributions of real measurement experiment results of both calibration methods are close to the simulation results at the lower range of depth. However, at the higher range of depth, the non-linear calibration method had considerably higher accuracy. As the object approaches the camera and projector for the higher range of depth, the assumption of linearity based on small divergence of light from the projector becomes less valid. Repeated testing indicates that the linear method is more sensitive to the noise, which sometimes results in lower measurement accuracy in the real measurement; the non-linear method is more robust to noise, and usually can achieve more repeatable measurement accuracy in repeated testing.

5.4 3-D Object Measurement Experiments

In the measurement experiments, testing was performed for the sinusoidal-pattern phase-shifting method. The sinusoidal fringe pattern with three- and four-step phase shifts were performed and test results were achieved. The sinusoidal pattern can be generated using the following equation:

$$I_i(x, y) = \frac{255}{2} \left[1 + \cos\left(\frac{2\pi x}{P} + \delta_i\right) \right] \quad (5-15)$$

where $I_i(x, y)$ is the intensity value of the i th phase shift at pixel (x, y) ; P is the pitch of the sinusoidal pattern in pixels; δ_i is the i th phase shift angle; and x is the coordinate along the

horizontal direction. As the intensity along the vertical direction is the same at any horizontal position, there is no parameter y in Equation (5-15).

5.4.1 Three-Step Sinusoidal-Pattern Phase-Shifting Method Testing

The three-step sinusoidal pattern was generated by substituting 0° , 120° , and 240° into Equation (5-15), to obtain the following equations:

$$I_1(x, y) = \frac{255}{2} \left[1 + \cos\left(\frac{2\pi x}{P} + 0\right) \right] \quad (5-16)$$

$$I_2(x, y) = \frac{255}{2} \left[1 + \cos\left(\frac{2\pi x}{P} + 120\right) \right] \quad (5-17)$$

$$I_3(x, y) = \frac{255}{2} \left[1 + \cos\left(\frac{2\pi x}{P} + 240\right) \right] \quad (5-18)$$

Figure 5-13 shows the three-step sinusoidal pattern segments, (a) is the pattern generated by software using the above equations, and (b) shows the captured fringe pattern segments that were observed on a flat reference plane. The phase distribution of this three-step sinusoidal pattern with phase steps of 0° , 120° , and 240° can be obtained by applying the appropriate equation listed in Table 3-1. The obtained phase map is a wrapped phase distribution ranging from 0 to 2π (or $-\pi$ to π), as shown in Figure 5-14 (a). After applying the phase unwrapping algorithm, the 2π discontinuity was removed and a continuous phase map was generated, as shown in Figure 5-14 (b).

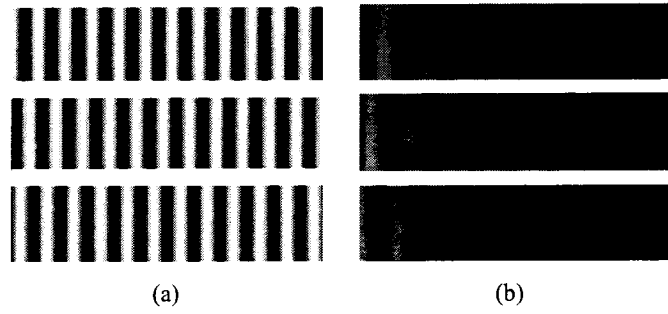


Figure 5-13 Sinusoidal fringe-pattern segments with three steps (0° , 120° , and 240°). (a) Generated patterns by software, (b) Captured patterns by camera.

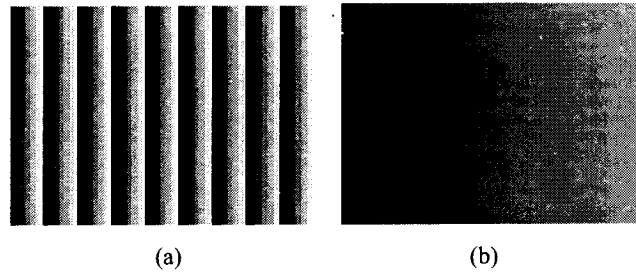


Figure 5-14 Wrapped and unwrapped phase map of three-step sinusoidal fringe pattern on the flat reference plane. (a) Wrapped phase map, (b) Unwrapped phase map.

Figure 5-15 shows the measurement results of the human-head mask by applying the three-step sinusoidal-pattern phase-shifting method. Figure 5-15 (a) shows one of the three-step fringe patterns on the mask. Figure 5-15 (b) and (c) are the wrapped and unwrapped phase distributions of the mask. By subtracting the unwrapped reference phase map of Figure 5-14 (b) from the unwrapped object phase map of Figure 5-15 (c), the phase difference distribution was obtained, as shown in Figure 5-15 (d). By applying the calibration result in Figure 5-5 to the phase difference distribution map, the object height distribution was obtained by using either the linear mapping function, Equation (3-23), or the non-linear mapping function, Equation (3-28), and the 3-D model was constructed, as shown in Figure 5-15 (e) the 3-D wire-frame model, (f) the 3-D shaded model with noise, and (g) – (j) the 3-D shaded model after filtering.

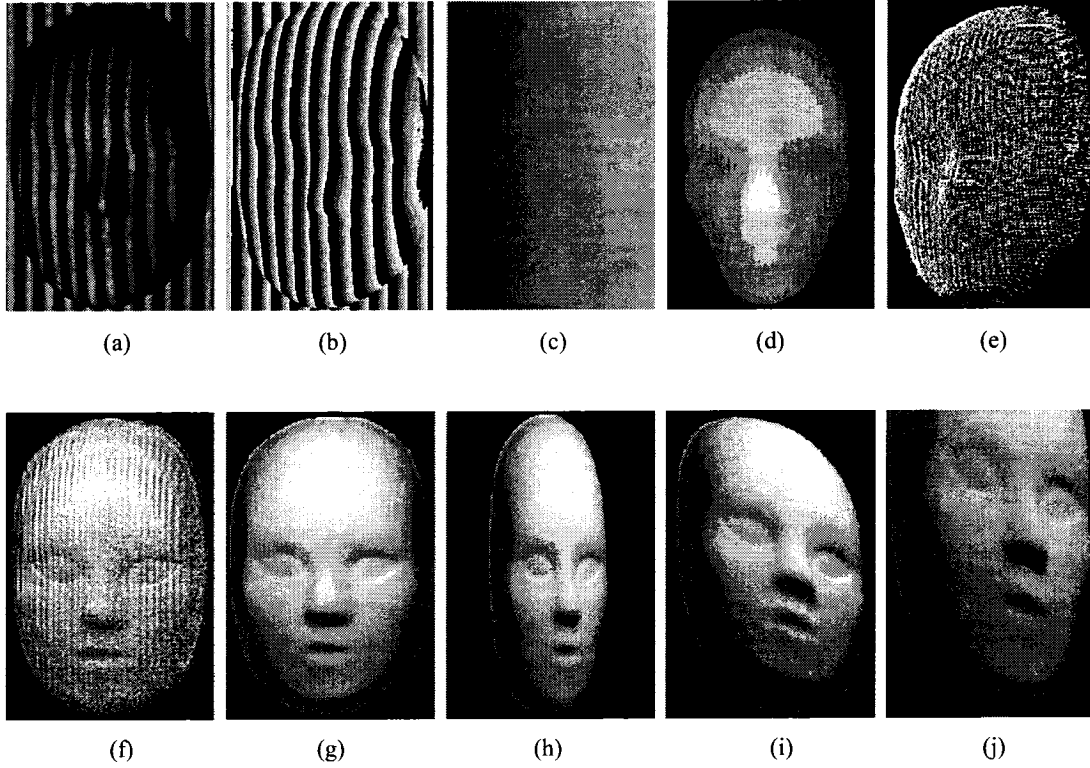


Figure 5-15 3-D shape measurement of a human-head mask obtained using the three-step sinusoidal-pattern phase-shifting method. (a) One of the captured shifted (0° , 120° , and 240°) fringe images, (b) Wrapped phase map, (c) Unwrapped phase map, (d) Phase difference map after scaling, (e) Reconstructed 3-D wire-frame model, (f) Reconstructed 3-D shaded model with noise, (g) and (j) Reconstructed 3-D shaded model after filtering. The sinusoidal pattern was generated with a pitch of 15 pixels.

5.4.2 Four-Step Sinusoidal-Pattern Phase-Shifting Method Testing

The four-step sinusoidal pattern was generated by substituting 0° , 90° , 180° , and 270° into Equation (5-15) to obtain the following equations:

$$I_1(x, y) = \frac{255}{2} \left[1 + \cos\left(\frac{2\pi x}{P} + 0\right) \right] \quad (5-19)$$

$$I_2(x, y) = \frac{255}{2} \left[1 + \cos\left(\frac{2\pi x}{P} + 90\right) \right] \quad (5-20)$$

$$I_3(x, y) = \frac{255}{2} \left[1 + \cos\left(\frac{2\pi x}{P} + 180\right) \right] \quad (5-21)$$

$$I_4(x, y) = \frac{255}{2} \left[1 + \cos\left(\frac{2\pi x}{P} + 270^\circ\right) \right] \quad (5-22)$$

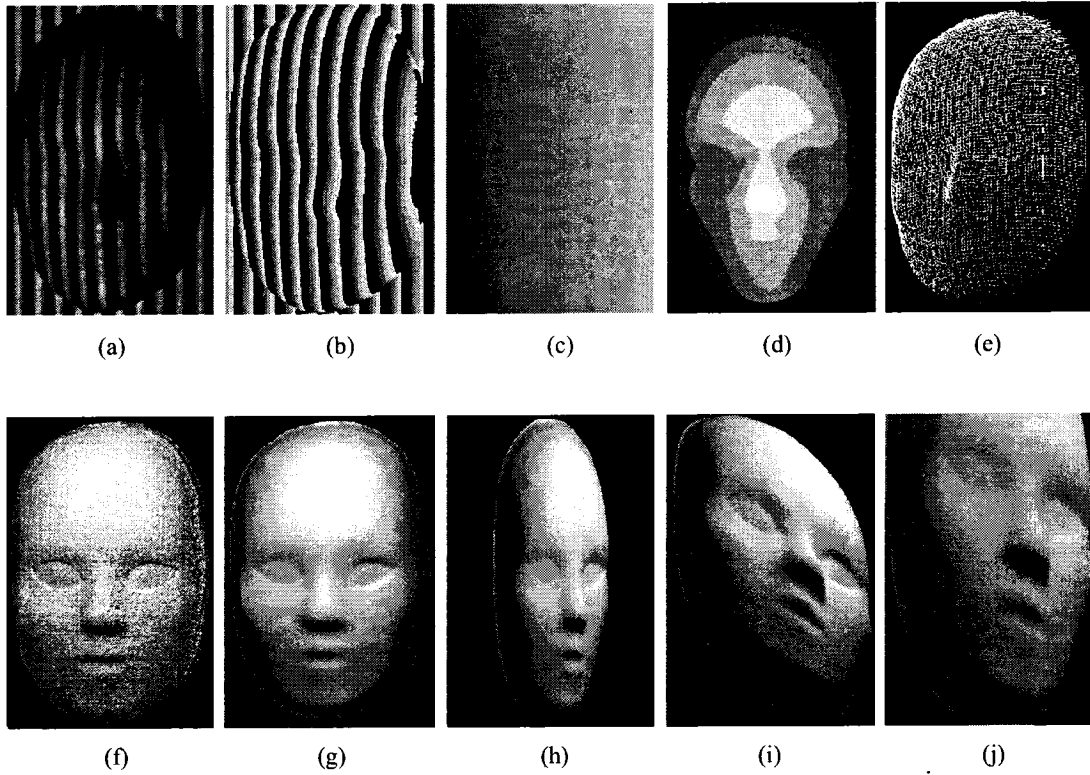


Figure 5-16 3-D shape measurement of a human-head mask obtained using the four-step sinusoidal-pattern phase-shifting method. (a) One of captured shifted (0° , 90° , 180° and 270°) fringe images, (b) Wrapped phase map, (c) Unwrapped phase map, (d) Phase difference map after scaling, (e) Reconstructed 3-D wire-frame model, (f) Reconstructed 3-D shaded model with noise, and (g) - (j) Reconstructed 3-D shaded model after filtering. The sinusoidal pattern was generated with a pitch of 16 pixels.

The measurement procedure of the four-step sinusoidal-pattern phase-shifting method is the same as with the three-step sinusoidal-pattern phase-shifting method described above. The captured phase-shifted images were processed by applying the appropriate equation listed in Table 3-1, and a wrapped phase map ranging from 0 to 2π was obtained. After applying the phase unwrapping algorithm, the 2π discontinuity was removed and a continuous phase map was generated. By applying the calibration results in Figure 5-6, the object height distribution was obtained using either the linear mapping function, Equation (3-23), or the non-linear mapping function, Equation (3-28), and the 3-D model was

constructed. The measurement procedure and results are shown in Figure 5-16: (a) one of the captured shifted (0° , 90° , 180° and 270°) fringe patterns, (b) the wrapped phase map, (c) the unwrapped phase distribution, (d) the phase difference map after scaling, (e) the 3-D wire-frame model, (f) the 3-D shaded model with noise, and (g) - (j) the 3-D shaded model after filtering.

5.4.3 Discussion

A design trade-off arises in the same way as described in the triangular-pattern phase-shifting method in the selection of the value of pitch. A small value of pitch implies increased accuracy, but it is likely to lead to phase unwrapping failures. This situation also depends on the angle between the optical axes of the camera and projector. In the experiment, for the three-step sinusoidal-pattern phase-shifting method, the pitch was 15 pixels, and for the four-step sinusoidal-pattern phase-shifting method, the pitch was 16 pixels. Perfect phase unwrapping can be achieved when the angle between the optical axes of the camera and projector is approximately 15° . In any situation, the four-step sinusoidal-pattern phase-shifting method can achieve better accuracy than the three-step method when the same value of pitch of the sinusoidal pattern was chosen and the pitch of the pattern can be evenly divided by the number of phase steps. The general conclusion, that more pattern shifting can generate more accurate depth at each pixel location for the measurement, was verified in the system.

5.5 Summary

A comparison is made in terms of measurement accuracy between the linear and non-linear calibration approaches. A least-squares method is applied in both linear and non-linear calibrations to obtain the system related calibration parameters. The accuracy for both methods was evaluated analytically and experimentally and the RMS error distributions of the depth calculation for both methods were obtained. Simulation results indicate that the accuracy of the linear calibration is generally higher than the non-linear calibration method. Based on real system measurements, at the lower range of depth, the accuracy of the linear calibration was similar to the non-linear calibration method. However, at the higher range of

depth, the non-linear calibration method had considerably higher accuracy. As the object approaches the camera and projector for the higher range of depth, the assumption of linearity based on small divergence of light from the projector becomes less valid. The linear method is more sensitive to noise, and sometime results in lower measurement accuracy in the real measurement; the non-linear method is more robust to the noise, and leads to higher measurement accuracy repeatability in repeated testing.

6 Error Analysis and Compensation

To improve the measurement accuracy for the triangular-pattern phase-shifting method, error sources and their impacts were investigated and the improved approaches are presented. For the digital fringe-projection technique, error sources may include the gamma non-linearity (Guo 2004 and Zhang 2005) of the video projector, grey-level quantization, intensity noise, and the image defocus of both the projector and camera. The triangular pattern is characterized by sharp corners, unlike the sinusoidal pattern; therefore, defocused lighting and gamma non-linearity become the important issues in the triangular-pattern phase-shifting method. To reduce the error caused by defocus of the projected lighting, the projector and camera should be adjusted to focus on the measured object. However, the triangular patterns projected by the DLP video projector used in this research were divided by many small squares with black lines, which introduced more noise to the system. The projector has to be adjusted to project a blurred image to remove these black lines on the triangular grey-scale patterns, thus the defocused lighting becomes worse. The impact of defocused lighting on the measurement accuracy of the triangular-pattern phase-shifting method has been analyzed by computer simulation in Chapter 4. This chapter will focus on the measurement-error compensation.

In the following, the error generated by projector gamma non-linearity and image defocus will be analyzed and compensated. Section 6.1 investigates the gamma non-linearity error with experiments. Section 6.2 presents an effective error compensation method for two-step triangular-pattern phase-shifting method and error compensation experiments. Section 6.3 presents an intensity-ratio error compensation method for triangular-pattern phase-shifting method with any number of phase-shifting steps and error compensation experiments. A summary is given in Section 6.4.

6.1 Experimental Gamma Non-Linearity Error Analysis

Because of the gamma non-linearity and defocus of the projector and camera, the relationship between the intensity values of the pattern input to the projector and the intensity values captured by the camera is non-linear. This non-linear relationship decreases the accuracy and resolution of the measurement. In this research, the intensity response of the measurement system is first analyzed and without any modifications to the physical measurement system, the input intensity values of the triangular pattern are then adjusted to yield the highest measurement accuracy. Compensation is then performed using a modified two-step triangular-pattern phase-shifting method and intensity-ratio error compensation method to reduce the residual errors after input pattern adjustment.

6.1.1 Gamma Non-Linearity

The intensity response of the measurement system was first determined by projecting a linear grayscale pattern with minimum intensity of 0 and maximum intensity of 255 onto a white flat plate. An image of the reflected pattern was captured by the CCD camera. The relationship between the intensity values input to the DLP projector and the intensity values captured by the CCD camera is shown in Figure 6-1. Figure 6-1 indicates that the intensity values of the captured image are only sensitive to input intensity values higher than a sensitivity threshold of about 40, and have very low sensitivity to input intensities up to 90. The captured image intensities increase non-linearly through the mid-range intensities to about 190; have a nearly linear distribution for the higher input intensity values beyond 190 to about 240; and then increase non-linearly to about 255. Figure 6-1 reveals the impact to the intensity mapping curve of both gamma non-linearity and image defocus of the projector and camera. It is noted that the non-linear response beyond about 240 was mainly generated by the defocus of the projector and camera.

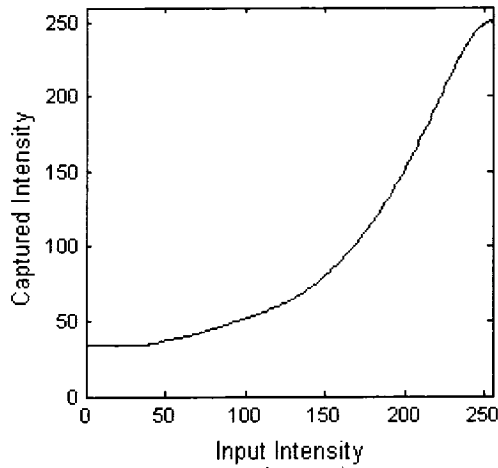


Figure 6-1 Gamma non-linearity of the video projector and camera.

6.1.2 Sensitivity Threshold and Non-Linearity Error

To analyze how the sensitivity threshold and non-linearity impact the measurement accuracy, a series of triangular patterns with different minimum intensity values of 0, 20, 40, 50, 60, 80 and maximum intensity value of 255 were generated by software and projected onto a white flat plate and captured by camera, as shown in Figure 6-2. All projected patterns had the same maximum intensity value as input intensity values higher than 190 had an almost linearly relationship with the output values. Due to the gamma non-linearity of the projector, the captured grayscale patterns were distorted from the input triangular greyscale patterns, especially for the lower input intensity values, as seen by the round valleys in the periodic patterns (Figure 6-2). The distortion of the captured greyscale patterns decreases when the minimum input intensity value of the triangular pattern is increased. When the minimum input intensity is 60 and 80, the captured intensity patterns become more triangular, as seen in Figure 6-2 (e) – (f) where the curves are less rounded, and nearly characterized by sharp valleys. However, choosing a higher minimum input intensity value, to use the linearly mapped region of the input-output intensity curve (Figure 6-1), limits the measurement accuracy because of the reduced dynamic range of input intensity values.

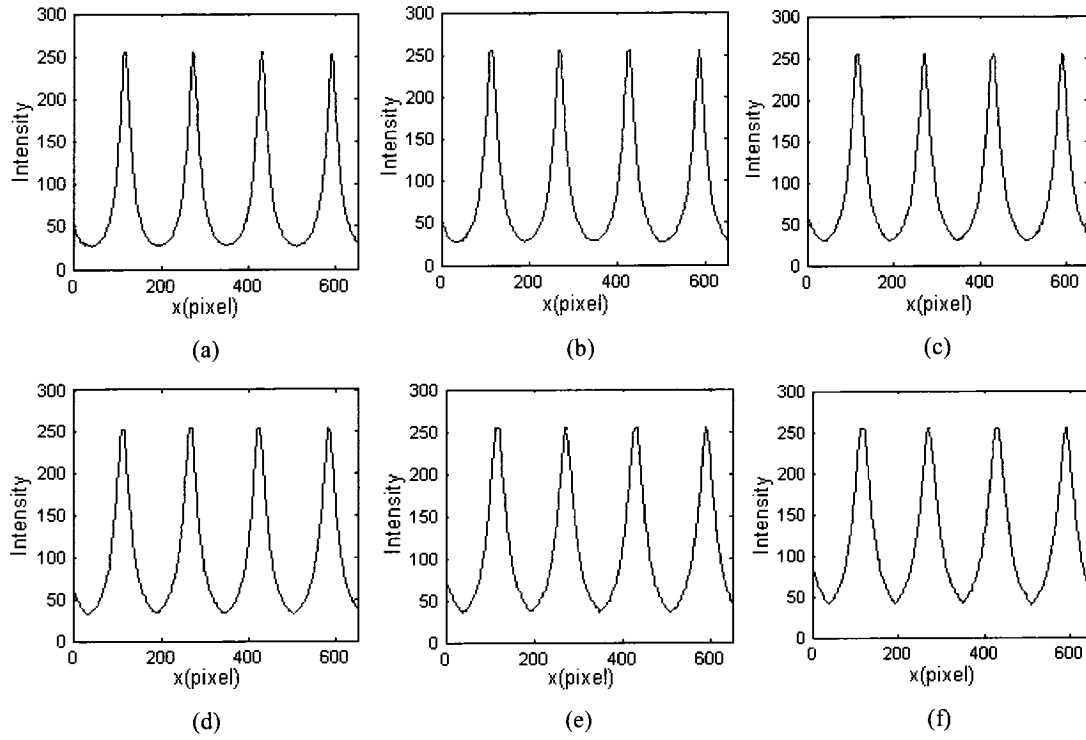


Figure 6-2 Intensity of captured triangular patterns with different intensity input. (a) - (f) all have a maximum input intensity value of 255, but with different minimum input intensity values: (a) 0, (b) 20, (c) 40, (d) 50, (e) 60, and (f) 80.

To determine the minimum intensity value that should be used to achieve the highest measurement accuracy, a plate positioned a distance of 25 mm from the reference position was measured with minimum intensity values of 0, 20, 40, 50, 60, and 80 and a maximum intensity of 255 in the projected patterns, using the two-step triangular-pattern phase-shifting method. First, images of the flat plate at reference plane position $z = 0$ were acquired for each of the phase shifts and the intensity-ratio distribution was obtained by intensity-ratio wrapping and intensity-ratio unwrapping. Thereafter, the plate was moved 25 mm toward the camera, and measured again. At this position, images were acquired and the intensity-ratio distribution computed as for the reference plane. The intensity-ratio differences between the measured plane and the reference plane at position $z = 0$ were determined, and the distance between the two positions was obtained by applying intensity-ratio-to-height conversion. The RMS error of the depth between the measured and reference plane positions were calculated based on all pixel points (648×494 pixels) in the image, and

the measurement results are summarized in Table 6-1. The triangular pattern with a minimum input intensity value of 40 had the highest measurement accuracy with the lowest RMS error of 0.667 mm. The trade-off required between choosing a higher minimum input intensity value to use the more linear region of the input-to-output intensity mapping, and a lower minimum input intensity to achieve a greater range of input intensity values is apparent. At minimum input intensity values lower than 40, the RMS error increased to 0.695 and 0.885 mm, for minimum intensity values of 20 and 0, respectively. At minimum input intensity values higher than 40, the RMS error increased to 0.730, 0.837 and 1.188 mm, for minimum intensity values of 50, 60 and 80, respectively.

To verify this result visually, a specific cross section of the distance measurement of the flat plate was selected out for displaying. Figure 6-3 shows the cross section of the measured flat plate at the position 25 mm toward the reference position with different minimum input intensity values of the projected triangular patterns: (a) – (f) all have maximum intensity value of 255, but with different minimum intensity values of (a) 0, (b) 20, (c) 40, (d) 50, (e) 60, and (f) 80. In this figure, the minimum input intensity value of 40, which is shown in (c), has minimum measurement error, which corresponds to the result in Table 6-1. Therefore, the value of 40 should be chosen as the minimum input intensity value of the triangular pattern in the present system to get the highest measurement accuracy.

Table 6-1 RMS error in flat-plate measurement using the two-step triangular-pattern phase-shifting method with different minimum input intensity values of the projected triangular pattern. The measured distance is 25 mm.

Min Input Intensity Value	0	20	40	50	60	80
RMS Error (Depth)(mm)	0.885	0.695	0.667	0.730	0.837	1.188

From Figure 6-1, it can be seen that the camera-captured image intensity first begins to increase at an input intensity value of approximately 40, which corresponds to the input intensity that yielded the highest measurement accuracy. Based on the above experimental analysis, it seems that the first intensity value for which an increase in camera-captured

intensity occurs may be the ideal choice for the minimum input intensity. The response curve between the input-pattern intensity and the intensity captured by the camera depends on the projector and camera. This response curve should be determined for the specific measurement system in order to determine the minimum intensity value used in the computer-generated input pattern. It should be noted that even with the appropriately selected minimum input intensity value of the triangular pattern, the non-linearity still exists in the measurement system.

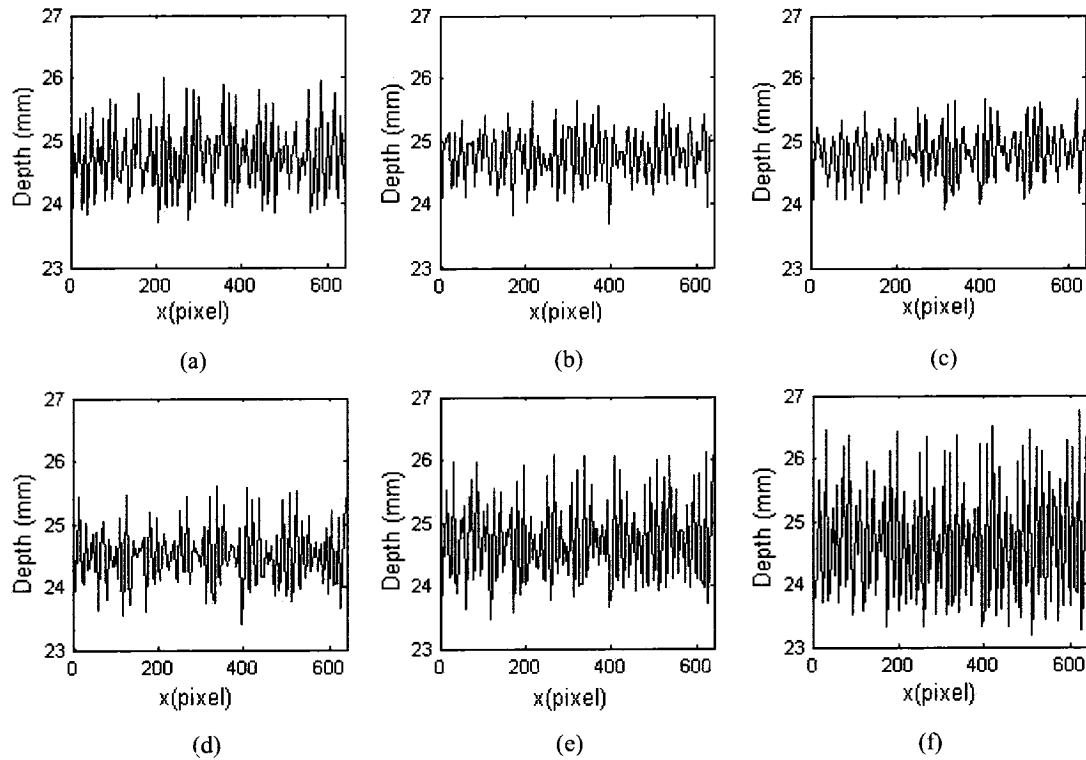


Figure 6-3 Depth measurement of the measured flat plate at 25 mm from the reference plane position with different input intensity values of the projected triangular patterns. (a) - (f) all have a maximum intensity value of 255, but with different minimum intensity values: (a) 0, (b) 20, (c) 40, (d) 50, (e) 60, and (f) 80.

In a projector-camera based measurement system it may be expected that the projector and camera be adjusted to focus on the measured object for the best measurement accuracy. As discussed earlier, the active square pixels seen as black lines must be removed by

manually adjusting the projector focus to slightly blur the image. This defocus of the projected light unfortunately blurs the projected triangular pattern and results in a quasi-triangular pattern captured by the camera. This contributes to measurement error in the triangular-pattern phase-shifting method.

6.1.3 Conclusions

Experimental error analysis of the two-step triangular-pattern phase-shifting method indicates that the minimum intensity of the computer-generated pattern input to the projector has an important affect on the measurement accuracy because of the gamma non-linearity. A trade-off is required in choosing a higher minimum input intensity value to use the more linear region of input-to-output intensity mapping, and a lower minimum input intensity to achieve a greater range of input intensity values. The first intensity value for which an increase in camera-captured intensity occurs may be the ideal choice for the minimum input intensity.

6.2 Error Compensation for the Two-Step Triangular-Pattern Phase-Shifting Method

Gamma non-linearity error compensation methods based on statistical analysis of the fringe images (Guo 2004), retrieving the inverse function according to the response curve (Pan 2004), calibrating the gamma curve of the projector for phase-error compensation (Zhang 2005), and repeated three-step phase shifting with initial phase offset of 60° (Huang 2002) have been proposed for sinusoidal-pattern phase-shifting methods. The latter method motivated the current approach.

To analyze the error contributed by projector gamma non-linearity and image defocus, a flat plate was measured by the two-step triangular-pattern phase-shifting method using a pitch of 30 pixels, and the wrapped and unwrapped intensity-ratio distributions were obtained. To clearly view errors in the resulting wrapped and unwrapped intensity-ratio distributions, a specific row of the distributions was selected for display. Figure 6-4 (a) – (b) show the wrapped and unwrapped intensity ratio, respectively, for the 250th row of the distributions over a length of 100 pixels from columns 201 to 300. The errors can be seen in the figures, where the sloped lines should be straight for both the wrapped and unwrapped

intensity ratios if neither defocus nor non-linearity in the input-to-output mapping occurs and no other noise exists. The more prominent errors indicated by arrows in Figure 6-4 (a) and (b) appear to be periodic, occurring at a frequency of 2 times per pitch of the triangular pattern. These errors are likely caused by image defocus while other errors are mainly caused by gamma non-linearity.

A further revelation of the periodic error is apparent in Figure 6-5 which shows a partial cross section of the surface height h (depth) of a human-head mask measured using the two-step triangular-pattern phase-shifting method. The measured profile of the human-head mask has a periodic wave pattern across the surface, likely due mainly to the projector gamma non-linearity and image defocus.

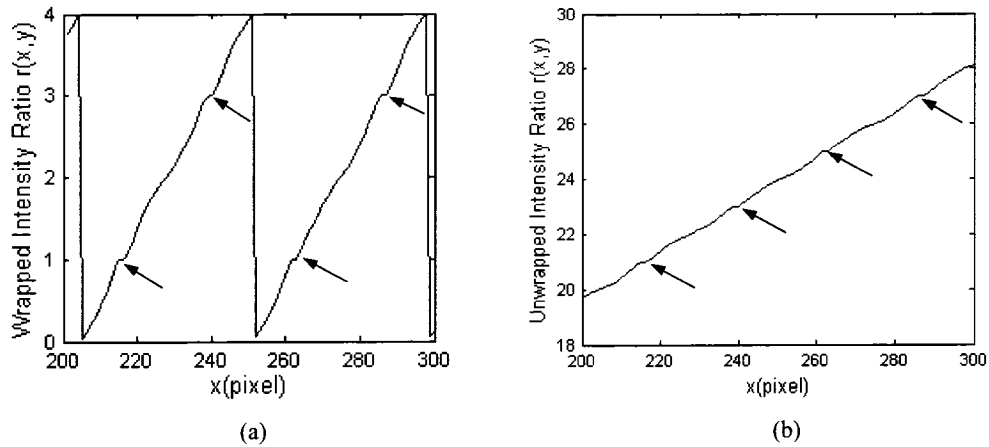


Figure 6-4 Wrapped and unwrapped intensity ratio obtained in measurement of a flat plate.
(a) Wrapped intensity ratio, (b) Unwrapped intensity ratio.

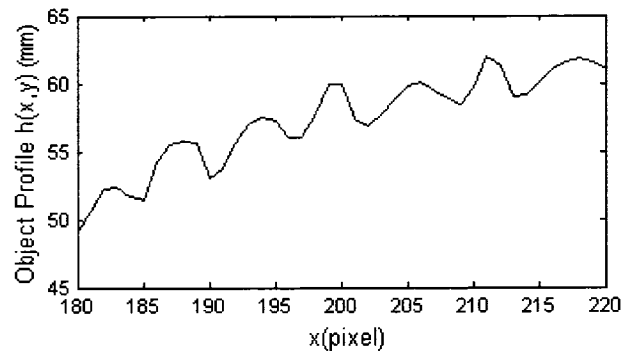


Figure 6-5 Cross section of the surface height h (depth) of a human-head mask measured using the two-step triangular-pattern phase-shifting method.

6.2.1 Repeated Two-Step Triangular-Pattern Phase-Shifting Method

To compensate for the residual periodic wave error, a modified two-step triangular-pattern phase-shifting method, called repeated two-step triangular-pattern phase-shifting method, was developed in this research. As the frequency of the error over a pitch of the triangular pattern is 2, measurement by two-step triangular-pattern phase-shifting is carried out twice, once with the two triangular patterns shown in Figure 4-1, and the second time with two triangular patterns that have an initial phase offset of one-eighth of the pitch, as shown in Figure 6-6. The second set of measurements is used to compensate for errors occurring in the first set of measurement, by averaging the two 3-D object height distributions obtained from the two sets of measurements, in order to generate the final 3-D object height distribution. The intensity equations of the two-step phase-shifted triangular patterns with the initial phase offset of one-eighth of the pitch, used in the second set of measurements, are formulated as follows:

$$I_3(x, y) = \begin{cases} \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{3I_m(x, y)}{4} & x \in [0, \frac{T}{8}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{5I_m(x, y)}{4} & x \in [\frac{T}{8}, \frac{5T}{8}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{5I_m(x, y)}{4} & x \in [\frac{5T}{8}, T) \end{cases} \quad (6-1)$$

$$I_4(x, y) = \begin{cases} -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{I_m(x, y)}{4} & x \in [0, \frac{T}{8}) \\ \frac{2I_m(x, y)}{T}x + I_{\min}(x, y) - \frac{I_m(x, y)}{4} & x \in [\frac{T}{8}, \frac{5T}{8}) \\ -\frac{2I_m(x, y)}{T}x + I_{\min}(x, y) + \frac{9I_m(x, y)}{4} & x \in [\frac{5T}{8}, T) \end{cases} \quad (6-2)$$

where $I_3(x, y)$ and $I_4(x, y)$ are the intensities for the two shifted triangular patterns respectively.

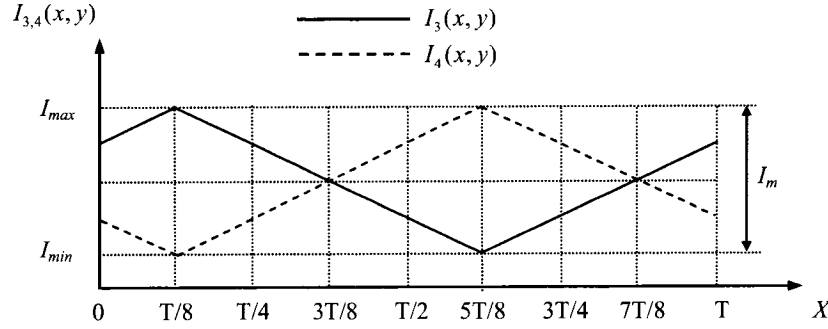


Figure 6-6 Two-step phase-shifted triangular patterns with the initial phase offset of one-eighth of the pitch, used in the second set of measurements of the modified two-step triangular-pattern phase-shifting method.

6.2.2 Error-Compensation Experiments

To determine the performance of the modified two-step triangular-pattern phase-shifting method in compensating errors, four tests were carried out. In the first test, two sets of measurements were carried out, one using the basic two-step triangular-pattern phase-shifting method, and the second using two-step phase-shifting with an initial phase offset of one-eighth of the pitch. The results are shown in Figure 6-7 for measurement of a human-head mask, where the peak and valley of the error waves generated by the two measurement processes are reversed. By averaging the two measurements, the wave error is greatly countervailed, and the measurement error is significantly reduced.

A second test was carried out to compare the measurement accuracy between the two-step triangular-pattern phase-shifting method and the modified method using error compensation, under different minimum intensity values of the triangular pattern. A flat plate located a distance of 25 mm from a reference plate position was measured by both techniques with different minimum intensity values and the RMS errors of the depth calculation were obtained. The optical setup and measurement procedures were the same as in Section 4.6, except that the repeated two-step triangular-pattern phase-shifting with phase offset was also used, as described in Section 6.2.1. The RMS error of the depth calculation, shown in Table 6-2, was lowest for the minimum input projected-pattern intensity of 40 for the modified method with error compensation, as it was for the unmodified method. The

measurement accuracy using the modified two-step triangular-pattern phase-shifting method increased over that using the unmodified method. The reduction in error was 24.0% for an input intensity value of 40, and an average of 22.5% for the different minimum intensity input values.

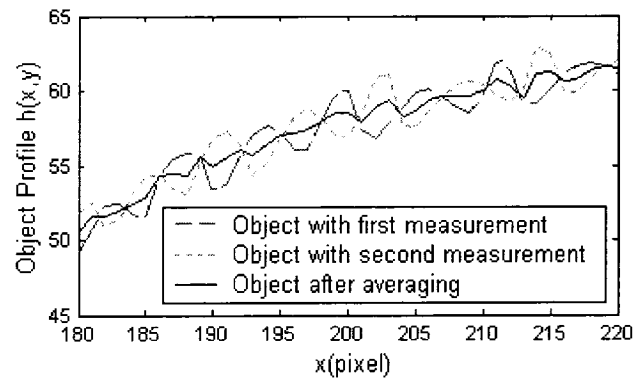


Figure 6-7 Cross section of the measured human-head mask with the modified two-step triangular-pattern phase-shifting method.

Table 6-2 RMS error of flat plate measurement using two-step and modified two-step triangular-pattern phase-shifting methods for different minimum input intensity values of the projected triangular pattern. The measured distance is 25 mm.

Minimum input intensity value		0	20	40	50	60	80
RMS depth error (mm)	Two-step triangular-pattern phase-shifting	0.885	0.695	0.667	0.730	0.837	1.188
	Modified two-step triangular-pattern phase-shifting	0.669	0.566	0.508	0.575	0.648	0.898
Error reduction (%)		24.4	18.6	23.9	21.3	22.6	24.4

In the third test, measurement of a flat plate was repeatedly carried out with both the two-step triangular-pattern phase-shifting method and the modified measurement method with error compensation, over the full 50 mm calibrated range of depth to cover the entire

calibrated volume. Measurements were performed in 2 mm increments through the depth, and a triangular pattern with a pitch of 16 pixels was used. RMS errors in depth were calculated based on errors at every image pixel and are shown in Figure 6-8. For the unmodified two-step triangular-pattern phase-shifting method, the RMS error varies with a pattern of increasing error and then decreasing error with depth, in a somewhat cyclical pattern. For the modified measurement method, the RMS error increases gradually with depth until 35 mm depth and then decreases sharply with depth. The measurement accuracy using the modified two-step triangular-pattern phase-shifting method with error compensation was consistently higher than that using the unmodified two-step triangular-pattern phase-shifting method, especially in the range of less than 20 mm and larger than 40 mm. It seems that where the unmodified method was most accurate, in this case at the valleys at 23-27 and 37-38 mm depth, the ability to compensate for errors by repeated measurement with phase-offset was limited. The greatest measurement accuracy of the modified method, shown by the low RMS error of 0.42 mm at 2, 8 and 50 mm depth, occurred where the unmodified measurement method had large errors. This pattern of error compensation may be due to the optimal focus of the projector and camera being made for the mid-range of depth, 25 mm, and error compensation being more possible at depths with less than optimal focus.

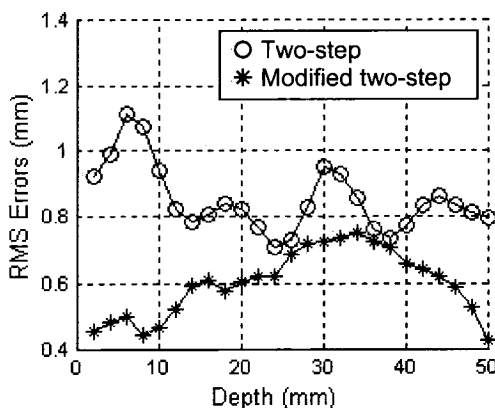


Figure 6-8 RMS errors of the two-step triangular-pattern phase-shifting method and the modified measurement method determined by measurement of a flat plate using a pitch of 16 pixels. RMS errors are calculated based on all pixels of the 648 x 494 resolution image.

In a fourth test, a human-head mask was measured with both the unmodified and modified two-step triangular-pattern phase-shifting methods. Figure 6-9 (a) – (e) and (f) – (j) show the measurement results using the unmodified method and modified methods, respectively. The two measurements were carried out under the same optical setup using a pitch of 16 pixels to generate the triangular pattern. The effectiveness of the error compensation is seen in Figure 6-9, where the reconstructed surface of the human-head mask was smoother with the modified two-step triangular-pattern phase-shifting than with the unmodified measurement. This is consistent with the higher measurement accuracy found when the error was compensated for.

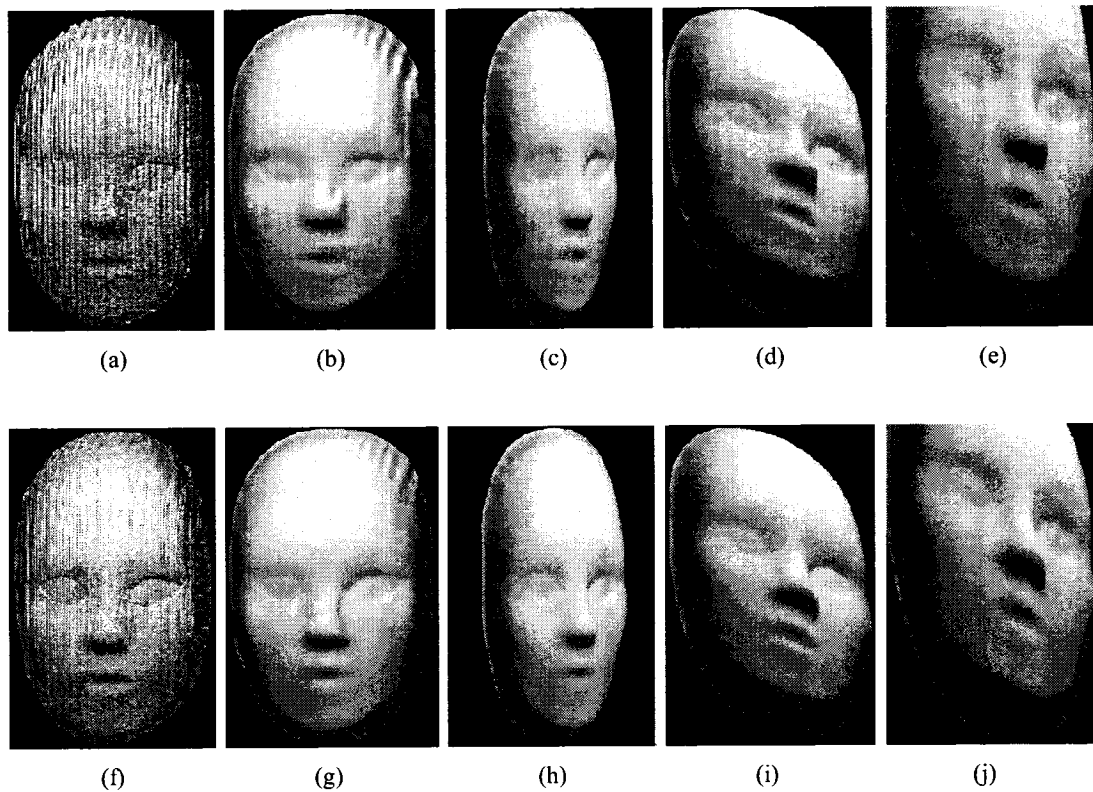


Figure 6-9 3-D shape measurement of a human-head mask by the (a)-(e) unmodified and (f)-(j) modified two-step triangular-pattern phase-shifting methods. (a) and (f) are reconstructed 3-D models with noise and all other figures have filtering applied. The triangular pattern was generated with a pitch of 16 pixels.

6.2.3 Conclusions

For the two-step triangular-pattern phase-shifting method, a periodic wave error pattern occurs in the wrapped and unwrapped intensity ratios and the object-surface height due to projector gamma non-linearity and image defocus. Error reduction was achieved by implementing a modified two-step triangular-pattern phase-shifting method, which performs the two-step triangular-pattern phase-shifting twice, the second time with an initial phase offset of one-eighth of the pitch. This compensates for the periodic errors when the two obtained object height distributions are averaged to generate the final 3-D object height distribution. The measurement accuracy for the modified two-step triangular-pattern phase-shifting method was higher than for the unmodified method over the full range of depth and for all minimum input intensity values tested.

6.3 *Intensity-Ratio Error Compensation*

The measurement accuracy of traditional phase-measuring profilometry (PMP) techniques (Creath 1988, Halioua 1989, and Greivenkamp 1992) depends largely on the measured phase map. Likewise, the measurement accuracy of the triangular-pattern phase-shifting profilometry relies on the accuracy of the measured intensity-ratio. The error compensation method described in Section 6.2 only applies to the two-step triangular-pattern phase-shifting method. In this section, a new intensity-ratio error compensation method is presented to decrease the measurement error due to projector gamma non-linearity and image defocus in triangular-pattern phase-shifting measurement with any number of phase-shifting steps. The method, motivated by the phase-error compensation approach (Zhang 2005), involves estimating the intensity-ratio error in a simulation of the triangular-pattern phase-shifting measurement process with both real and ideal captured triangular-pattern images obtained from real and ideal gamma non-linearity functions. A look-up-table (LUT) relating the measured intensity-ratio to the corresponding intensity-ratio error is constructed and used for intensity-ratio error compensation, and thus shape-measurement error compensation. The experimental results demonstrated that this intensity-ratio error compensation method significantly reduces the measurement error in the triangular-pattern phase-shifting method.

6.3.1 Principle of Intensity-Ratio Error Compensation

Without loss of generality, the principle of intensity-ratio error compensation is introduced for the two-step triangular-pattern phase-shifting method; however the intensity-ratio error compensation method developed below is applicable to all triangular-pattern phase-shifting methods with different numbers of phase-shifting steps.

The gamma non-linearity curve obtained in Section 6.1 is used to carry out the intensity-ratio error compensation (Figure 6-10).

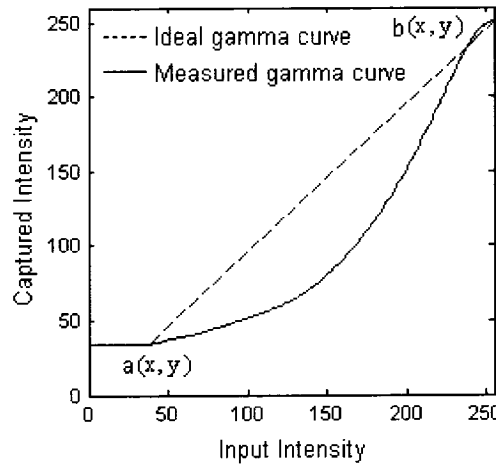


Figure 6-10 Non-linear mapping of projector input intensity to camera-captured image intensity.

To determine the intensity-ratio error caused by the non-linearity between the pattern input to the projector and the image captured by the camera, a simulation to retrieve the intensity-ratio error is carried out. First, a polynomial curve is fit to the measured gamma curve data (Figure 6-10), and an ideal gamma curve is drawn as a straight line between points $a(x,y)$, corresponding to the intensity sensitivity threshold, discussed in Section 6.1.2, and $b(x,y)$, the curve endpoint. Two phase-shifted triangular patterns are then generated by Equations (4-1) and (4-2), using a minimum input intensity value of 40 and maximum input intensity value of 255, as shown in Figure 6-11 (a). Using these two phase-shifted triangular patterns as input patterns to the projector, a simulation is performed to generate simulated captured triangular-pattern images corresponding to the measured and ideal gamma curves, shown in Figure 6-11 (b). The measured intensity error due to the

projector gamma non-linearity and image defocus is apparent. The intensity-ratio $r_0(x, y)$ by using Equation (4-4) computed for both measured and ideal triangular pattern images is shown in Figure 6-12, and the intensity-ratio ramp computed using Equation (4-5) for both measured and ideal intensity-ratios is shown in Figure 6-13. In this figure, the ideal intensity-ratio ramp is a sloped straight line, while the measured intensity-ratio ramp is characterized by a periodic error curve. The difference between them is the measured intensity-ratio error, shown in Figure 6-14. Errors indicated by arrows are mainly due to image defocus while other errors are mainly due to gamma non-linearity. The two maximum intensity-ratios errors occur where the intensity ratio $r(x, y)$ has values of 1 and 3. As can be seen from Figure 4-1 (a), (b) and (c), these maxima correspond to the peaks of the projected triangular-fringe pattern, at $T/4$ and $3T/4$, respectively. This indicates that the major intensity-ratio error is caused by the projected pattern defocus, where the sharp peaks are blurred into curves.

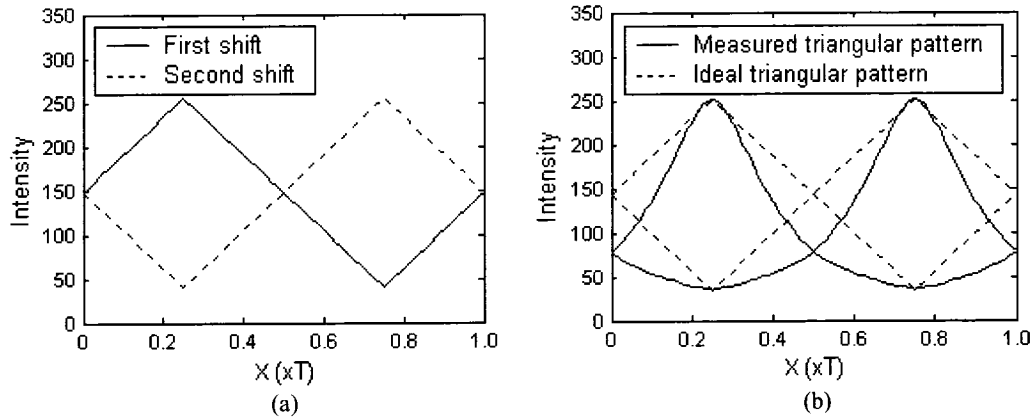


Figure 6-11 Triangular pattern simulation with measured and ideal gamma curve functions.
 (a) Input triangular pattern, and (b) Simulated captured triangular pattern image.

The intensity-ratio error, shown in Figure 6-14, is a single-valued function of the intensity ratio. By constructing a look-up table (LUT), which maps the intensity-ratio error as a function of measured intensity-ratio, intensity-ratio error compensation can be implemented. A LUT would have to be constructed separately for each triangular-pattern phase-shifting method, for different number of phase-shifting steps. However, for a given

method, the LUT would only need to be constructed once, as long as the relative positions and orientations of the projector and camera are not changed.

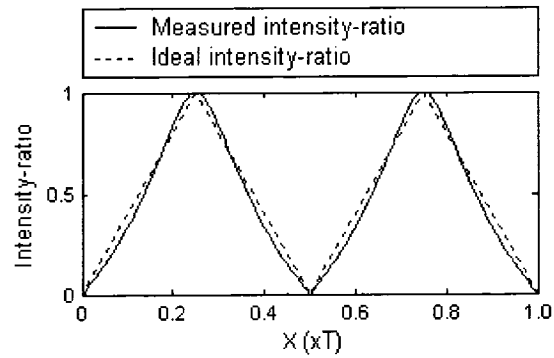


Figure 6-12 Measured and ideal intensity-ratio with periodic triangular shape.

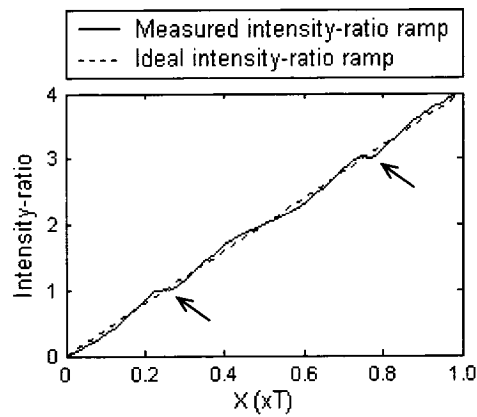


Figure 6-13 Measured and ideal intensity-ratio ramp after conversion from the triangular shape.

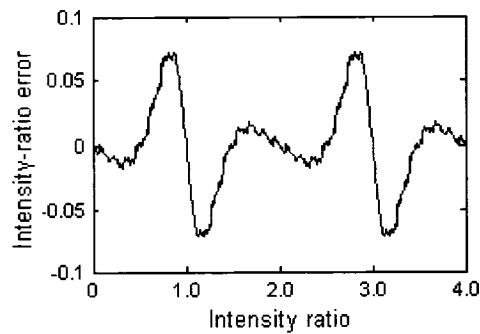


Figure 6-14 Intensity-ratio error as a function of intensity ratio.

6.3.2 Error-Compensation Experiments

To determine the performance of the intensity-ratio error compensation method, four tests were carried out. In the first test, a flat white plate was measured and the wrapped intensity-ratio distribution was obtained before and after applying intensity-ratio error compensation. The wrapped intensity-ratio before correction is characterized by a wave pattern, shown in Figure 6-15 (a), due to the projector gamma non-linearity and image defocus. By applying the intensity-ratio compensation, the wave error is greatly reduced, as shown in Figure 6-15 (b).

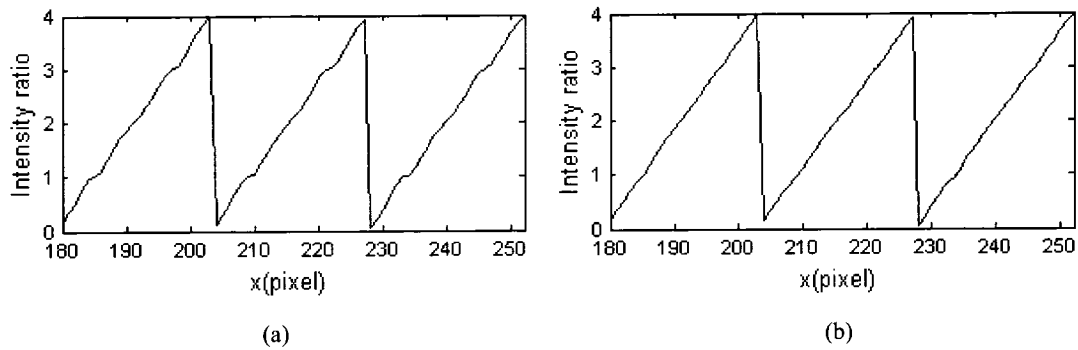


Figure 6-15 Intensity ratio compensation. (a) Wrapped intensity-ratio before correction, (b) Wrapped intensity-ratio after correction.

A second test, carried out to compare the measurement accuracy before and after error compensation, involved repeated measurement of a flat plate with the two-step triangular-pattern phase-shifting method over the full 50 mm calibrated range of depth to cover the entire calibrated volume. Measurements were performed in 2 mm increments through the depth, and a triangular pattern with a pitch of 16 pixels was used. RMS errors in depth, calculated based on every image pixel, are shown in Figure 6-16. For the two-step triangular-pattern phase-shifting method before error compensation, the RMS error varies with a pattern of increasing error and then decreasing error with depth, in a somewhat periodic pattern. After applying error compensation the RMS error decreases significantly across the entire range of depth, although a residual cyclical error pattern with lower amplitude than for the uncorrected method remains.

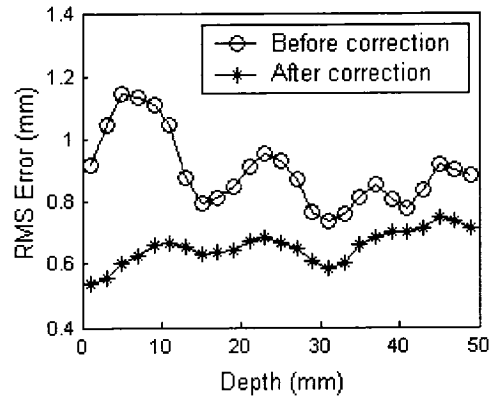


Figure 6-16 RMS errors of the two-step triangular-pattern phase-shifting method before and after applying intensity-ratio error compensation by measurement of a flat plate using a pitch of 16 pixels. RMS errors are calculated based on all pixels of the 648 x 494 resolution image.

In the third test, a measurement of a flat plate at the position of 20 mm from the reference plane was carried out. A cross section of a flat plate measurement, before and after error compensation, is shown in Figure 6-17. The measurement error is significantly reduced after error compensation by 28.5%.

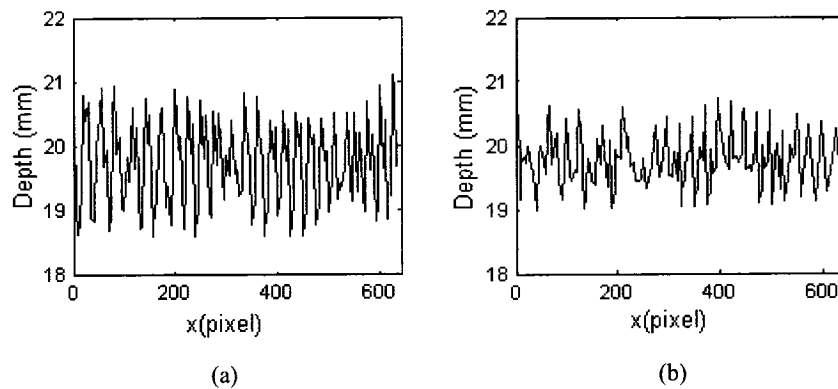


Figure 6-17 Depth measurement of the measured flat plate at 20 mm from the reference position. (a) Measured profile of the flat plate before correction, (b) Measured profile of the flat plate after correction.

In a third test, a human-head mask was measured with the two-step triangular-pattern phase-shifting method. Figure 6-18 (a) – (c) and (d) – (f) show the measurement results before and after correction, respectively. The measurement was carried out using a pitch of 16 pixels to generate the triangular pattern. The effectiveness of the error compensation is seen in Figure 6-18, where the reconstructed surface of the human-head mask was smoother after error compensation was applied.

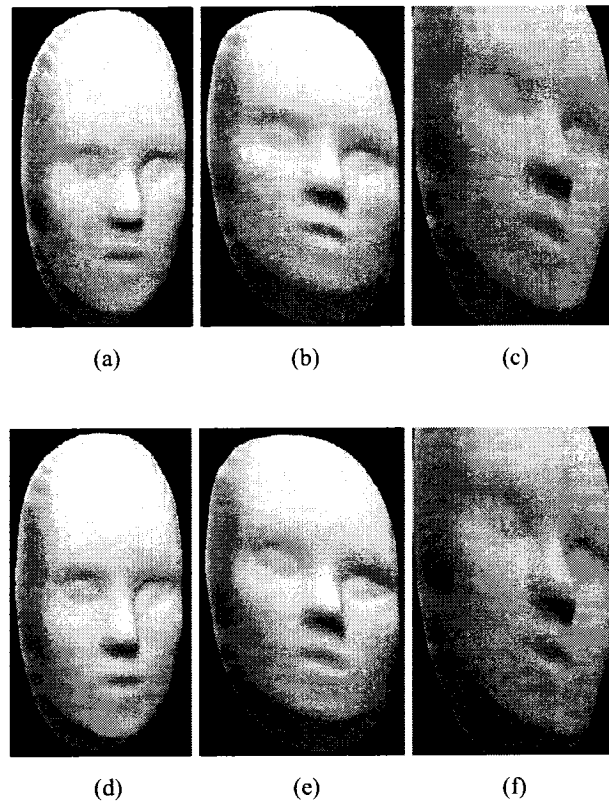


Figure 6-18 3-D shape measurement of a human-head mask. (a)-(c) before correction was applied, and (d)-(f) after error compensation. The triangular pattern was generated with a pitch of 16 pixels.

6.3.3 Conclusions

An effective intensity-ratio error compensation method suitable for phase-shifting with any number of steps was developed to reduce measurement error mainly caused by projector gamma non-linearity and image defocus in a triangular-pattern phase-shifting 3-D

measurement system. The intensity-ratio measurement error can be obtained by simulating the measurement process of a triangular-pattern phase-shifting method with real and ideal captured triangular-pattern images based on real and ideal gamma non-linearity functions. A LUT of intensity-ratio measurement error corresponding to the measured intensity-ratio can be used for intensity-ratio error compensation and therefore shape-measurement error compensation during the measurement. The experimental results demonstrated that the intensity-ratio error compensation method significantly reduces the measurement error in the triangular-pattern phase-shifting method.

6.4 Summary

The impact of projector gamma non-linearity and image defocus on the measurement accuracy has been investigated. The triangular pattern with minimum input intensity value of 40 can generate the highest measurement accuracy. An improved two-step triangular-pattern phase-shifting approach, repeated two-step triangular-pattern phase-shifting method, for 3-D object measurement was developed. The measurement accuracy using the modified two-step triangular-pattern phase-shifting method increased over that using the unmodified method. The reduction in error was 24.0% for an input intensity value of 40, and an average of 22.5% for the different minimum intensity input values. The repeated two-step triangular-pattern phase-shifting method also has less sensitivity to image noise.

An intensity-ratio error compensation method has also been presented in this chapter to decrease the measurement error due to the gamma non-linearity and defocus of the projector and camera for the triangular-pattern phase-shifting method with any number of phase-shifting steps. In the proposed method, the intensity-ratio measurement error is obtained by simulating the measurement using the triangular-pattern phase-shifting method with real and ideal captured triangular-pattern images. A LUT that stores the intensity-ratio measurement error corresponding to the measured intensity-ratio is constructed and used for intensity-ratio error compensation during the measurement. The experimental results demonstrated that this intensity-ratio error compensation method significantly reduces the measurement error in the triangular-pattern phase-shifting method.

7 Real-Time 3-D Shape Measurement

Real-time 3-D shape measurement developed in this research will be addressed in this chapter. The algorithms implemented in the real-time system are based on the off-line 3-D shape measurement system developed in this research with the additional implementation of synchronization between the pattern projection and image acquisition, and optimization of the whole processing procedure. Section 7.1 gives a basic introduction to the real-time 3-D shape measurement method developed in this research. Section 7.2 describes the basic requirements and implementation mechanism for a real-time 3-D shape measurement method. Section 7.3 presents the real-time 3-D shape measurement method developed in this research. Section 7.4 presents experiments in real-time 3-D shape measurement using the triangular-pattern phase-shifting method with different phase-shifting steps and preferred parameters. Section 7.5 provides a summary of this chapter.

7.1 Introduction

By synchronizing the pattern projection and the image observation, a real-time 3-D shape measurement system could be achieved. The synchronization could be accomplished by hardware or software. Hardware-based synchronization could achieve higher measurement speed, but a synchronization circuit board is necessary and a more expensive system would be needed. Software-based synchronization has the advantage of comparing the performance of the different patterns easily without any hardware investment. Through software-based synchronization 3-D shape measurement experiments, the efficiency of the patterns and the best algorithms can be tested and determined. Hardware-based synchronization can be implemented if further development to further increase measurement speed is necessary. The focus of this research is to develop a new full-field range sensing method and implementation mechanism that is suitable for real-time 3-D surface geometry measurement. Therefore, software-based synchronization was used in this research, and the speed is limited compared to the more expensive hardware-based approaches. As mentioned,

hardware synchronization could be combined with the software-synchronization approaches developed in this research. In this way, the hardware-based approaches could be improved in terms of measurement speed. To achieve real-time 3-D shape measurement by software, in addition to the synchronization between the pattern projection and image observation, the 3-D data acquisition, intensity-ratio/phase wrapping and unwrapping calculations, 3-D coordinates calculation and 3-D object rendering should be carried out simultaneously to increase the measurement speed. Parallel processing software, which fully employs multi-thread programming techniques to achieve real-time 3-D shape measurement, was developed in this research. All the algorithms developed in the off-line 3-D shape measurement system, including pattern generation, phase-shifting, intensity-ratio and phase wrapping calculations, intensity-ratio and phase unwrapping calculation, and intensity-ratio and phase to height conversion computations, are implemented in the real-time 3-D shape measurement system.

7.2 Principle of Real-Time 3-D Shape Measurement

The key issue of developing a real-time 3-D shape measurement system is implementing the system with high measurement speed. Several factors impact the measurement speed, such as whether the system is equipped with fast processing algorithms, efficient hardware and software, and if there is efficient communication between the hardware and software. All these factors will be discussed in the following subsections.

7.2.1 Basic Requirements for a Real-Time 3-D Shape Measurement System

To carry out real-time 3-D shape measurement, the following requirements should be satisfied in general.

- **Fast processing algorithms:** The algorithms play an important role in a real-time 3-D shape measurement system. Fast processing algorithms usually increase the measurement speed significantly. Here in the fringe-projection measurement methods, the processing algorithms depend on the properties of the projected patterns. Different patterns can be processed with different speeds.

- Efficient implementation mechanism: Each major step in a real-time 3-D shape measurement system requires processing time. The entire processing cycle can be separated into several groups. Different grouping schemes will impact the whole processing speed. An efficient real-time implementation mechanism to integrate the hardware and software together to carry out fast processing is important.
- Efficient compiler software: The measurement system implemented with different compiler software applied to the same processing algorithms would have different processing speeds. A good compiler can optimize the source codes and increase the execution speed.
- High speed hardware: This includes high-speed CCD camera, frame grabber, and computer. The continuous fringe patterns can be projected and captured in real-time by a projector and high-speed CCD cameras and sent to the computer through a frame grabber, or alternatively, firewire cameras could be used.

In this research, an InFocus LP600 DLP video projector and a high-speed progressive-scan Sony XC-HR 50 CCD camera were used to perform real-time image acquisition. The triangular-pattern phase-shifting method, which has the features of simple coding and decoding computation and fewer images required to reconstruct the 3-D object, were implemented in the real-time 3-D shape measurement system to provide fast processing speed. An optimized implementation mechanism, discussed in Section 7.2.2, was used to carry out real-time 3-D shape measurement. The 3-D object rendering was carried out by an optimized OpenGL rendering algorithm, which significantly reduces the time required to display the reconstructed 3-D object on the monitor. The whole system was implemented with an effective cooperation between OpenGL and the Win32 compiler system with multi-thread programming. The above configuration satisfies the basic requirements for a real-time 3-D shape measurement system.

7.2.2 Real-Time Implementation Mechanism

To achieve real-time 3-D shape measurement, it is necessary to design an efficient real-time implementation mechanism to integrate the hardware and software together to carry out fast processing. The entire measurement procedure includes several major processes. Each

major process should be optimized and communicate with each other effectively to increase the entire processing speed.

The pipeline of the real-time 3-D shape measurement system developed in this research is shown in Figure 7-1. From this figure it can be seen that the measurement process includes several major steps. However, the time used in each step is different. To optimize the entire processing, the entire procedure should be separated into several groups. Each group which consists of one or more major steps can be implemented with a single thread. In this research, the entire procedure was separated into two, three, four and five groups and the initial testing indicated that four groups can achieve the highest measurement speed. These four groups were implemented by four threads and they cooperated with each other effectively. With the above configuration and implementation, the real-time 3-D shape measurement system can perform 3-D shape measurement with the maximum measurement speed.

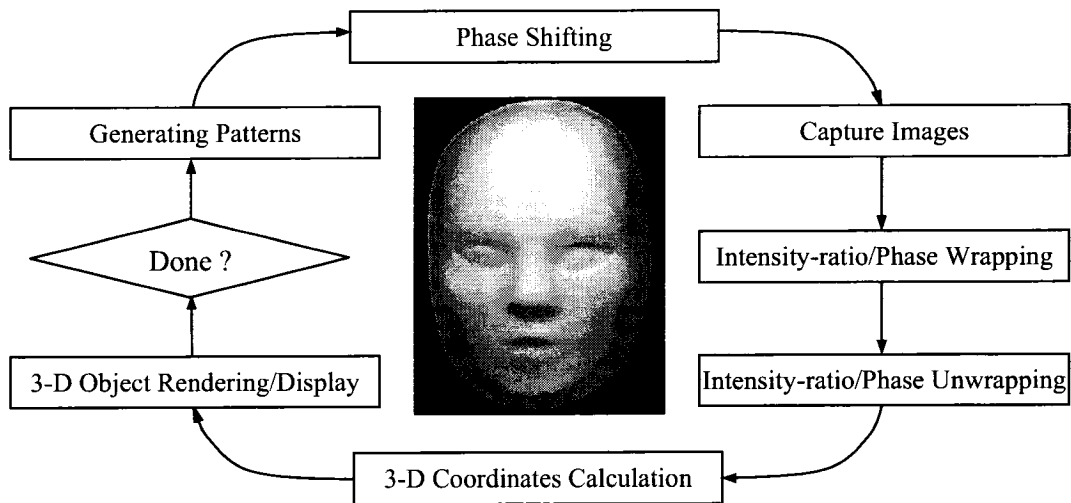


Figure 7-1 Real-time fringe-projection technique implementation pipeline.

7.3 Real-Time 3-D Shape Measurement System

Based on the considerations discussed above, a real-time 3-D shape measurement system based on software synchronization between the pattern projection and image acquisition was developed. The real-time 3-D shape measurement system is described in detail in the following subsections.

7.3.1 System Setup

Figure 7-2 shows the schematic diagram of the real-time 3-D shape measurement system developed in this research. The hardware used in the real-time 3-D shape measurement system is exactly the same as the equipment used in the off-line 3-D shape measurement system. The geometric positions of projector, camera, and reference plate remain unchanged in the real-time 3-D shape measurement system. However, unlike the off-line 3-D shape measurement system where operations are manually carried out, the hardware components cooperate with each other automatically in the real-time 3-D shape measurement system. The cooperation was implemented by multi-threading programming which will be described in detail in the following section.

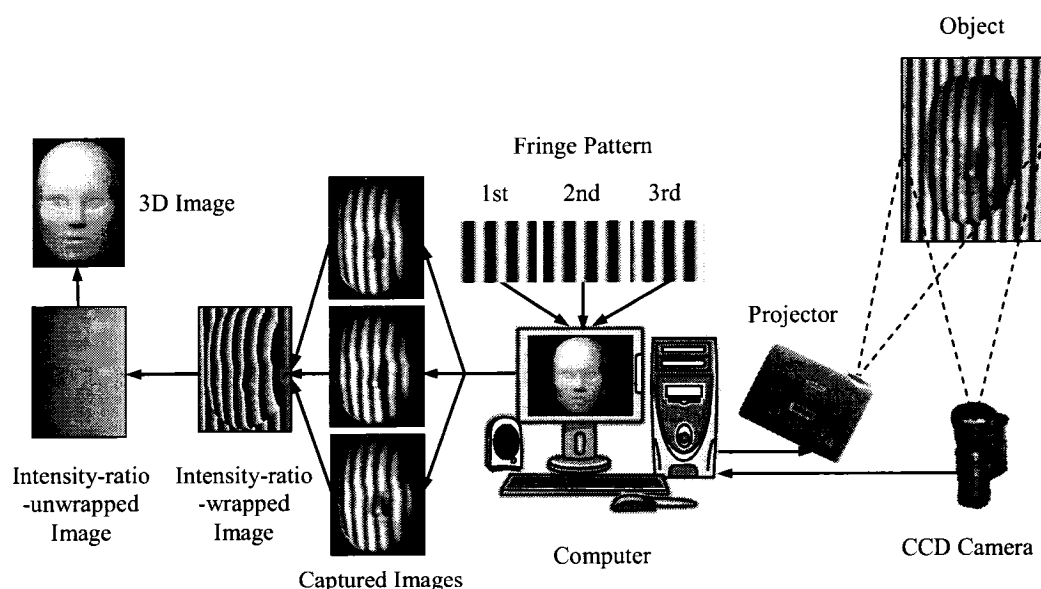


Figure 7-2 Schematic diagram of real-time 3-D shape measurement system.

7.3.2 Implementation

Through testing, the entire 3-D shape measurement process, shown in Figure 7-1, was separated into four major groups to achieve the fastest measurement speed. These four groups are called: acquisition, wrapping, unwrapping, and reconstruction. The acquisition group consists of generating patterns, phase shifting, and capturing images; the wrapping group consists of intensity-ratio wrapping; the unwrapping group consists of intensity-ratio unwrapping; and the reconstruction group consists of 3-D coordinate calculation, and 3-D object rendering and display. These four groups were implemented with the following four threads.

- Acquisition thread: this thread performs the pattern projection by the projector, phase shifting, and fringe pattern capture by camera continuously.
- Wrapping thread: this thread is responsible for the calculation of the intensity-ratio wrapping by applying phase-shifting algorithms to the images captured by the acquisition thread.
- Unwrapping thread: this thread is responsible for the computation of the intensity-ratio unwrapping by using the intensity-ratio unwrapping algorithm applied to the wrapped intensity-ratio map obtained by the wrapping thread.
- Reconstruction thread: this thread first performs 3-D coordinate computation by applying the intensity-ratio-to-height conversion algorithm to the unwrapped intensity-ratio distribution, acquired from the unwrapping thread, and then performs 3-D object rendering and display with OpenGL.

Figure 7-3 illustrates the implementation of the real-time 3-D shape measurement system with four threads and their communication. When the system starts, the initial parameters, such as the triangular or sinusoidal patterns, phase steps, pitch of the pattern, should be chosen and given. Then the patterns for each step will be generated and saved in memory for projection. The threads are synchronized by the synchronization event. The detailed execution procedure is described as follows:

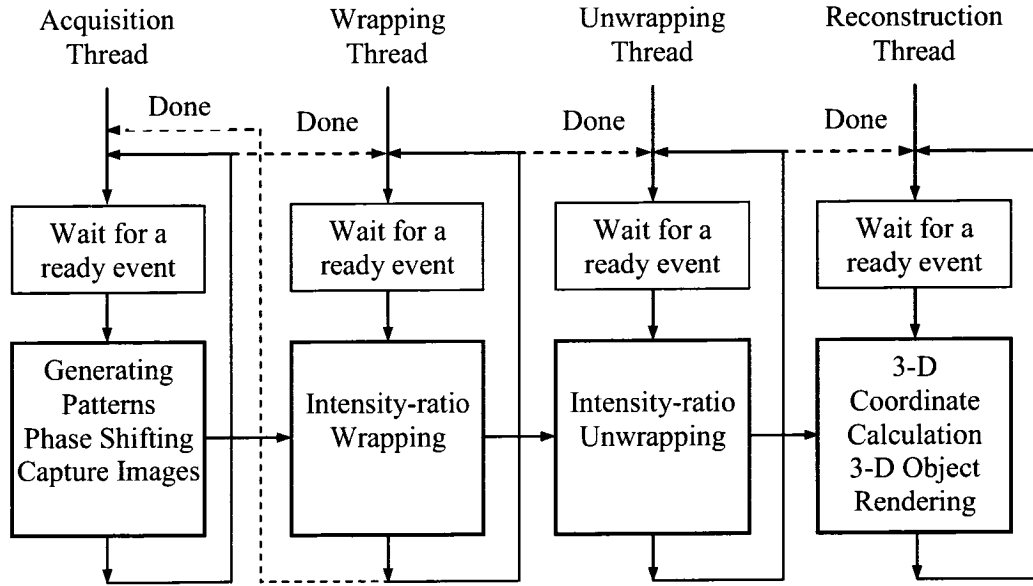


Figure 7-3 Real-time 3-D shape measurement implementation.

Four threads start at the same time, and then wait for a Ready Event to continue execution. The acquisition thread is given a Ready Event first. This thread projects the pattern with phase-shifting, and the camera captures the image at the same time. When the images for all steps have been captured, a ready event will be generated and sent to the wrapping thread. The acquisition thread will wait until a ready event coming from the wrapping thread is obtained, and then it will continue to perform data acquisition. When the wrapping thread gets a ready event from the acquisition thread, it will perform the intensity-ratio wrapping calculation by processing the captured images obtained by the acquisition thread. When finished, it will send a ready event to both the acquisition thread and the unwrapping thread, and then wait for the next ready event from the acquisition thread for the next iteration. When the unwrapping thread gets a ready event from the wrapping thread, it will apply the intensity-ratio unwrapping computation to the wrapped intensity-ratio maps generated by the wrapping thread. When finished, it will send a ready event to the reconstruction thread, and then wait for next ready event from the wrapping thread for the next iteration. When the reconstruction thread receives a ready event from unwrapping thread, it will calculate the 3-D coordinates of the measured object surface by applying the

intensity-ratio-to-height conversion algorithms, and then it will render the 3-D object and display the reconstructed 3-D object on the monitor using OpenGL. The process from acquisition to reconstruction will repeat until a stop event is received by any thread. Before quitting the system, any resources applied for the system need to be released.

Double buffering technique was used in the 3-D object rendering with OpenGL to speed up the process. Also, Win32 programming was also used instead of MFC, which involves too many Windows messages to deal with, to accelerate the processing speed.

7.4 Real-Time 3-D Shape Measurement Experiment

Before a measurement of a dynamic object is carried out using the real-time 3-D shape measurement system, the system needs to be calibrated using the methods described in Chapter 5.

7.4.1 Experiment with a Dynamic Object

In the real-time 3-D shape measurement experiment, a human facial expression measurement was carried out with triangular-pattern phase-shifting method with different phase-shifting steps. To get the best measurement results for each method with different phase-shifting steps, the pitch of the pattern for each method was chosen appropriately according to the conclusion obtained in Section 4.5. Figure 7-4 shows a human facial expression measurement procedure with the two-step triangular-pattern phase-shifting method. In this figure, (a) is a photo of the subject; (b) is one of the captured triangular-pattern fringe images of the two-shifted triangular pattern; and (c) – (e) are renderings of the reconstructed 3-D object. A sequence of dynamic human facial expression measurement by the two-step triangular-pattern phase-shifting method was recorded, and is shown in Figure 7-5. The measurement speed is approximately 5.6 fps. It is noted that during measurement, the subject was intentionally changing the expression and the facial expression was reconstructed by the system. The experiment results demonstrated that the real-time 3-D shape measurement system can measure a slowly moving object in real-time.

Triangular-pattern phase-shifting methods with different phase-shifting steps were also tested in the system with human facial expression measurements, the results are shown in

Figure 7-6, Figure 7-7, Figure 7-8, and Figure 7-9, respectively. The measurement speeds are 5.2, 4.8, 4.5, and 4.2 fps for the three-, four-, five-, and six-step methods, respectively. The more steps used in the triangular-pattern phase-shifting method, the slower the measurement speed was; however, the measurement accuracy increases with more phase-shifting steps. This can be seen by the smoother surfaces obtained when more steps are used.

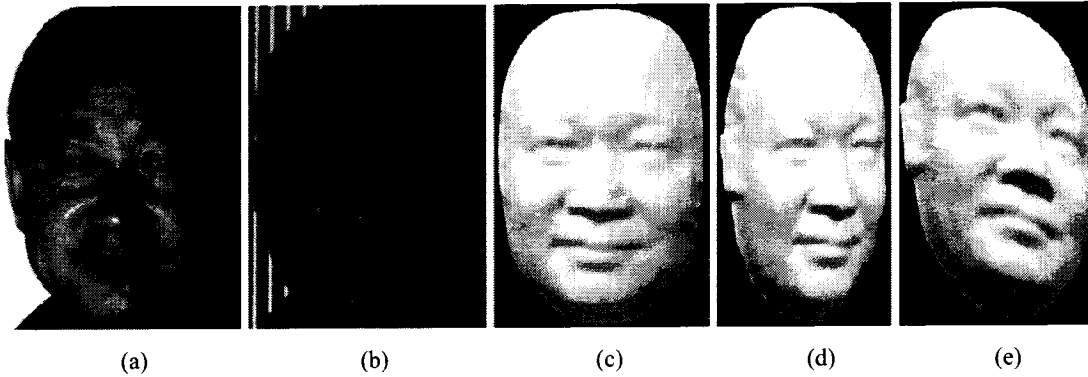


Figure 7-4 3-D shape measurement of a human face with the two-step triangular-pattern phase-shifting method. (a) 2-D photo, (b) One of the two captured two-shift triangular fringe pattern images, (c) – (e) Reconstructed 3-D shaded model. The triangular patterns were generated with a pitch of 16 pixels.

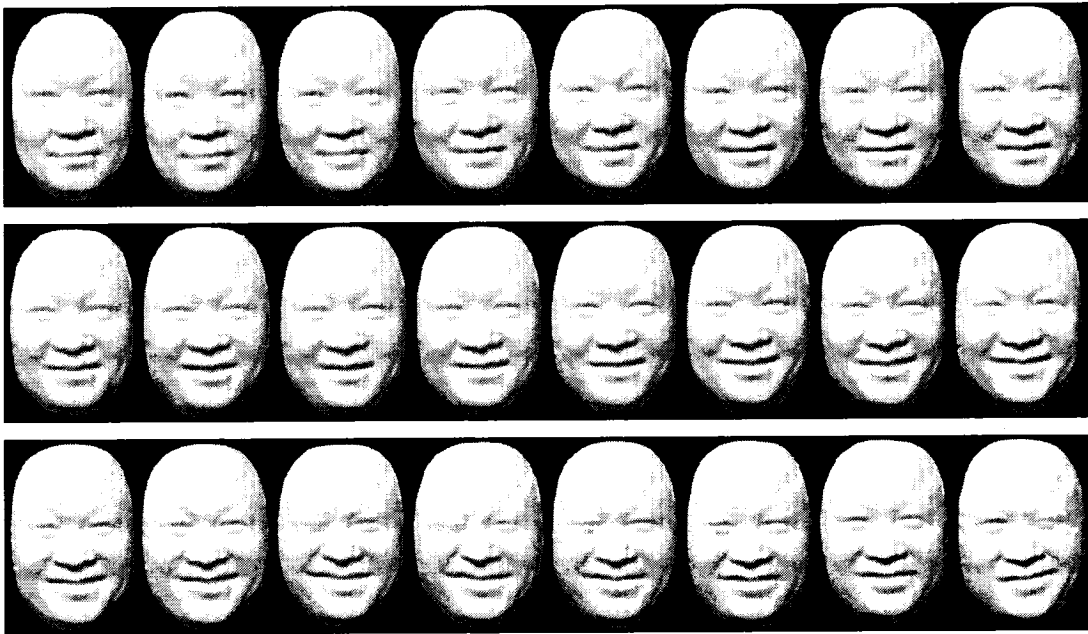


Figure 7-5 Sequence of measurement results of human facial expressions with the two-step triangular-pattern phase-shifting method. The triangular patterns were generated with a pitch of 16 pixels.

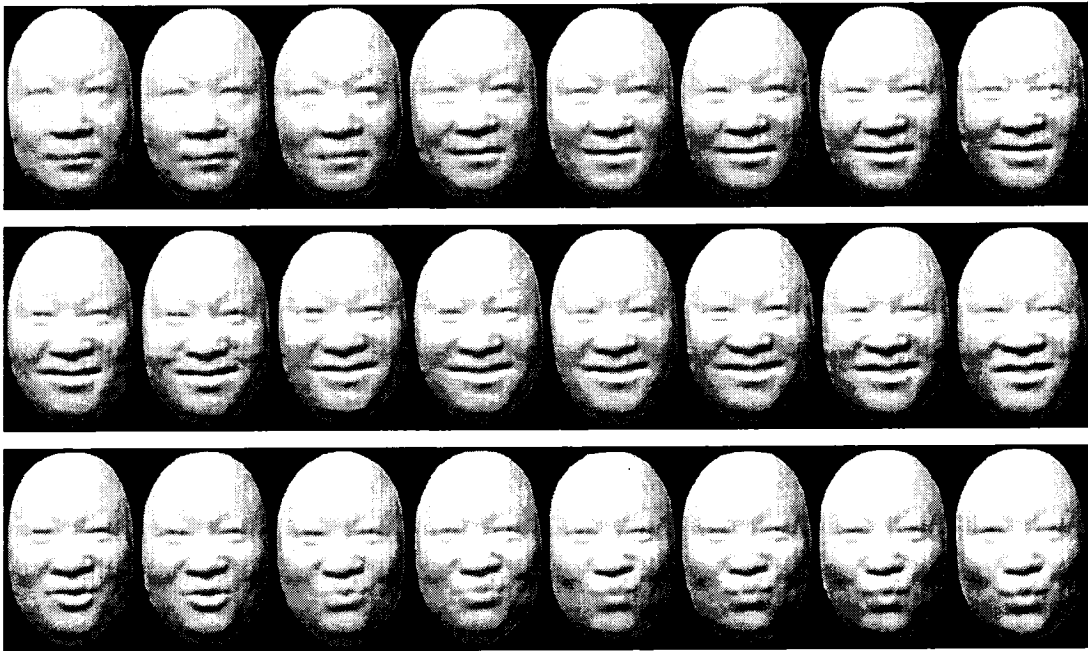


Figure 7-6 Sequence of measurement results of human facial expressions with the three-step triangular-pattern phase-shifting method. The triangular patterns were generated with a pitch of 18 pixels.



Figure 7-7 Sequence of measurement results of human facial expressions with the four-step triangular-pattern phase-shifting method. The triangular patterns were generated with a pitch of 16 pixels.



Figure 7-8 Sequence of measurement results of human facial expressions with the five-step triangular-pattern phase-shifting method. The triangular patterns were generated with a pitch of 20 pixels.



Figure 7-9 Sequence of measurement result of human facial expressions with the six-step triangular-pattern phase-shifting method. The triangular pattern were generated with a pitch of 18 pixels.

7.4.2 Discussion

The real-time 3-D shape measurement system developed in this research focused on the implementation mechanism to accomplish real-time image acquisition, fast image manipulation, and real-time 3-D object rendering. Software-based synchronization between the pattern projection and image acquisition was implemented in the system to verify the efficiency of the proposed real-time 3-D shape measurement pipeline. Although the measurement speed is limited, the experimental results of a slow moving subject indicated that the proposed pipeline and the four-thread implementation method are sufficient. If a dual CPU system is used or hardware-based synchronization is implemented, the measurement speed will be increased.

During the experiment, it was noted that the flash rate of the projector was not quick enough, and sometimes the projected triangular patterns were distorted. The distorted patterns due to the low flash rate of the projector reduce the measurement accuracy. This is the major error source, which is different from the off-line 3-D shape measurement system. This issue should be further investigated.

7.5 Summary

A real-time 3-D shape measurement system based on software synchronization between the pattern projection and image acquisition was developed. Multi-thread programming, OpenGL tools for 3-D object rendering, and double buffering techniques were adopted in the real-time 3-D shape measurement system. The entire processing of the 3-D shape measurement system was separated into four major groups and implemented with four parallel processing threads. All phase-shifting methods have been implemented in the real-time measurement system. Experiments indicated that more phase-shifting steps require more processing time. The measurement speed was approximately 0.35 fps slower with each additional step or image. The system generates a data cloud of 648×494 points per frame. Due to the implementation of software-based synchronization, the measurement speed is limited. The highest measurement speed is approximately 5.6 fps for two-step method. Experiment results demonstrated that the system is capable of measuring a slow moving object. However, as the proposed triangular-pattern phase-shifting method (two-step method)

has the advantages of fewer sample images required to reconstruct the 3-D object and simpler coding and decoding computations to retrieve the intensity-ratio, this method is expected to permit faster processing than current methods used in real-time 3-D object measurement systems under the same hardware implementation environment.

8 Conclusions, Contributions, and Recommendations for Future Work

This chapter addresses the conclusions of the research, major contributions which have been made, and recommendations for method improvement and further research.

8.1 Conclusions

This doctoral dissertation has addressed the design, calibration, and measurement-accuracy testing for a new real-time full-field range-sensing method to perform 3-D surface-geometry measurement using digital fringe-projection and phase-shifting techniques. This dissertation has focused on the development of 3-D information retrieving algorithms that are suitable for real-time 3-D shape measurement and an implementation mechanism of a real-time 3-D shape measurement pipeline. Software synchronization was used to synchronize the pattern projection and image acquisition to carry out real-time 3-D shape measurement to verify the efficiency of the algorithms and implementation pipeline developed in this research. A comparison of the present research with previous work follows and conclusions and limitations of the proposed approach are summarized thereafter.

8.1.1 Comparison of Present Research with Previous Work

The most significant real-time 3-D shape measurement systems developed so far have been those developed by Rusinkiewicz (2002), Huang (2003a), and Zhang (2004). In Rusinkiewicz's method, a structured light pattern consisting of many vertical stripes with different widths was used for projection, and a variant of the iterative closest point (ICP) algorithm was used to perform alignment. At least four frames of projected patterns were captured and used to decode the range information using a complex algorithm. The method allows the users to rotate an object by hand and fill holes in the model in real time. The projector was synchronized to an NTSC camera by hardware. A dual-CPU system with Intel Pentium III Xeon processors running at 1.0 GHz was used to carry out image grabbing,

stripe-boundary detection, matching, and identification from the accumulated illumination history, all of which were the first few stages of the range scanning pipeline; and 3-D coordinate calculation, 3-D object rendering, and display, which were the second set of operations of the pipeline. The system operates at a speed of 60 Hz in the first piece of the pipeline and 10 Hz in the second piece of the pipeline with lower quality results. High quality measurement results could be achieved with offline registration and surface reconstruction with noise of about 0.1 mm that depends on the quality of camera and digitizer used. However, it was not clear if the noise reported was the total error of the measurement relative to ground truth. Furthermore, the standoff distance was approximately 540 mm, approximately 28.4% of the 1.9 m standoff used in the current research. In general, a shorter standoff distance may lead to higher accuracy.

Huang (2003a) presented a high-speed 3-D shape measurement system that uses a color sinusoidal fringe pattern to measure the 3-D object. When the color sinusoidal fringe pattern is sent to a modified DMD-based video projector with the removal of the color filter, three gray-scale sinusoidal fringe patterns with a 120-deg phase shift are obtained. The 3-D information of the measured object is obtained by applying phase wrapping, unwrapping, and phase-to-height conversion calculations. The data are processed offline with the measurement resolution of 1 mm on a measurement area of 250 mm $\times$ 180 mm, but a potential sampling rate up to 100 Hz could be achieved if the speed of the camera is fast enough. Because the computation of phase involves an arctangent function which is time consuming to compute, Zhang (2004) presented an improved method, called trapezoidal-pattern phase-shifting method, to further increasing the processing speed. In this system, two Pentium 4 computers with 2.8 GHz processing speed were used, one for pattern projection and the other for image processing. The synchronization between projection and acquisition was implemented with a self-designed circuit board. By projecting three phase-shifted trapezoidal patterns with a modified DLP projector, the 3-D object is reconstructed by intensity-ratio wrapping, unwrapping, and intensity ratio-to-height conversion calculations instead of phase-based calculation at each pixel, and less processing time is required. For an image size of 532 $\times$ 500 pixels, the processing time needed for this method was 4.6 ms compared to the sinusoidal-pattern phase-shifting method, which needed 20.8 ms to process. The total processing time for 3-D shape reconstruction was about 24.2 ms per frame, which

is fast enough to make real-time 3-D shape reconstruction possible. In Huang, no measurement error was reported. In Zhang, the noise level of this method for a measured area of $260 \text{ mm} \times 244 \text{ mm}$ was reported to be RMS 0.055 mm. However, again, the error relative to ground truth was not reported. Furthermore, it is not clear what the standoff distance between the measured object and camera was.

The speed of a real-time 3-D shape measurement system depends on the processing algorithms, the speed permitted by the hardware, and the efficiency of communication between hardware and software, and the processing algorithms play a critical role. The real-time system developed in the present research has a measurement speed of 5.6 fps for measuring an object with the size of $380 \text{ mm} \times 280 \text{ mm}$ for the two-step method with an image size of 648×494 pixels, which seems slower than the systems described above. However, it is noted that this system is based on a single-CPU computer, software synchronization only, and without any modification of hardware. This is major difference from the other systems described above. The most important feature of the proposed triangular-pattern phase-shifting method developed in this research is that the minimum number of measurements (sample images) that are required to reconstruct the unknown 3-D object is two, which is less than the number of measurement requirements of the systems described above (Rusinkiewicz 2002, Huang 2003a, and Zhang 2004). It should be appreciated that each measurement, includes the image acquisition and computation on every pixel of an image, so the reduction of one image or phase-shifting step is a highly significant reduction of demand for processing. Furthermore, the coding and decoding algorithms to retrieve the 3-D information of the measured object are simpler than any methods described above. These two points suggest that with the same experimental setup including hardware synchronization, the two-step triangular pattern phase-shifting method would have the highest measurement speed. Synchronization of the pattern projection and image acquisition using hardware, to enhance the efficiency of the implementation mechanism developed in this research, is anticipated for future work.

Even under the same number of phase-shifting steps, the triangular-pattern phase-shifting methods are expected to be faster than the sinusoidal-pattern phase-shifting methods with similar resolution. This is due to the sinusoidal-pattern phase-shifting methods involving a time consuming computation of non-linear functions to calculate phase (for

phase-shifting with more than two steps using the arctangent function, for phase-shifting with two steps using the arccosine function (Quan 2003 and Almazan-Cuéllar 2003)), while only linear computation is involved in the triangular-pattern phase-shifting methods. It is also noted that the trapezoidal-pattern phase-shifting method (Zhang 2004) can only be applied with three phase-shifting steps.

8.1.2 Conclusions from Tests and Analyses

A novel triangular-pattern phase-shifting approach, which is suitable for different applications with different phase-shifting steps, was developed. The presented approach combines the advantages of both the traditional sinusoidal-pattern phase-shifting method and intensity-ratio method, and a triangular grey-level coded pattern with different phase shiftings was used for 3-D object surface measurement. The phase-shifting algorithms to generate the intensity ratio, essential for surface reconstruction, were developed for two-, three-, four-, five-, and six-step measurement methods. The minimum number of measurements (sample images) required to reconstruct an unknown 3-D object is two, which is less than the minimum number of measurements required for the traditional sinusoidal-pattern and trapezoidal-pattern phase-shifting methods. Compared with the traditional sinusoidal-pattern and trapezoidal-pattern phase-shifting methods (Zhang 2004 and Huang 2005) with the same number of phase-shifting steps, the new method has lower measurement accuracy, but is expected to have faster processing speed with similar resolution because of the simple intensity-ratio computation used and because fewer images are required to reconstruct the 3-D object. Compared with the traditional intensity-ratio based methods (Carrihill 1985, Chazan 1995, and Miyasaka 2002), it has a better depth resolution and less ambiguity problem when the triangular pattern is repeated (Chazan 1995) to reduce sensitivity to measurement noise. An efficient real-time 3-D shape measurement pipeline was also developed. With the combination of this pipeline, the newly developed triangular pattern, and the associate algorithms to retrieve the 3-D coordinates of the object, a system with the highest measurement speed could be achieved, if combined with the more advanced hardware found in other systems. Applications of the proposed method in various fields, such as inspection in manufacturing and surface modeling are expected.

8.1.3 Limitations of the Proposed Approach

The proposed triangular-pattern phase-shifting method for 3-D shape measurement described in this dissertation is not without limitations. Through testing with different lighting, and different colors of object surface, the following limitations of the proposed method exist:

- (1) The measurement accuracy is sensitive to the reflectance of the measured object surface. Black, grey, or colored surfaces may contribute to reduced measurement accuracy.
- (2) Regions of the measured object with surfaces parallel to the direction of the projected light may cause intensity-ratio distortion and unwrapping failure that will generate more measurement error.
- (3) The gamma non-linearity and image defocus of the projector and camera with the proposed triangular-pattern phase-shifting method has more impact on the measurement accuracy than the sinusoidal and trapezoidal-pattern phase-shifting methods.

In regard to the limitations described above, the reflectance of the measured object surface restricts the application of this technique to some areas. For the measurement error generated due to intensity-ratio distortion in regions parallel to the lighting direction and due to defocus of projector and camera, the error compensation technique is necessary to reduce the measurement error. This research has included repeated-offset and intensity-ratio compensation methods to compensate for the measurement error mainly caused by defocus and non-linearity of the projector and camera.

8.2 Contributions

This doctoral dissertation has made the following major contributions:

- (1) Developed a novel triangular-pattern phase-shifting approach for 3-D shape measurement.**

A novel triangular-pattern phase-shifting approach (Jia 2005a, Jia 2005c, and Jia 2006a) for 3-D shape measurement was developed in this research. The algorithms for determining

the intensity-ratio, essential for determining the 3-D coordinates of the measured object, were developed for different number of phase-shifting steps. The intensity-ratio-to-height conversion algorithm was also developed based on the phase-to-height conversion algorithm used in the traditional sinusoidal-pattern phase-shifting method. The proposed new method is suitable for different 3-D shape measurement applications with different phase-shifting steps. The presented approach combines the advantages of the traditional sinusoidal-pattern phase-shifting method and the traditional intensity-ratio method to reconstruct a 3-D object. The minimum number of measurements (sample images) required to reconstruct an unknown 3-D object is two, which is less than the minimum number of measurements required for the traditional sinusoidal-pattern phase-shifting method and trapezoidal-pattern phase-shifting method. Compared with the traditional sinusoidal-pattern phase-shifting and trapezoidal-pattern phase-shifting methods, the proposed method has faster processing speed because of the simple computation of the intensity-ratio and fewer images required to reconstruct the 3-D object. Compared with the traditional sinusoidal-pattern phase-shifting-based method with the same number of phase shifts, the processing speed would be faster with similar resolution. The proposed method also has a better depth resolution compared to traditional intensity-ratio based methods (Carrihill 1985 and Miyasake 2002) and less degree of ambiguity problems when the triangular pattern is repeated (Chazan 1995) to increase the measurement accuracy. The features of the proposed triangular-pattern phase-shifting approach are: (1) simple coding and decoding algorithms; (2) fewer images required to reconstruct the 3-D object. These features make the real-time 3-D shape measurement possible. Applications to various inspection tasks, mobile-robot navigation and 3-D surface digitizing are expected.

(2) Analyzed the measurement accuracy of the proposed triangular-pattern phase-shifting method analytically and experimentally.

One of the major error sources of the digital fringe projection is defocused lighting. The defocus simulation (Jia 2005a, and Jia 2005c) was carried out to analyze the effect of defocused image on the intensity-ratio, which contains the height information of the object, in the proposed triangular-pattern phase-shifting method. The simulation result shows that the intensity-ratio error is about 22% in the case of the worst defocused lighting. The

measurement accuracy of the triangular-pattern phase-shifting method with different phase-shifting steps and different values of pitch of the triangular pattern was analyzed by experiment (Jia 2006a and Jia 2005c). The experiment results indicate that a small value of pitch can increase the measurement accuracy, but if too small, may lead to failure of the intensity-ratio unwrapping calculations. More phase steps generate higher accuracy in the 3-D shape reconstruction; however, the digital fringe projection generates phase shifting error if the pitch of the pattern cannot be evenly divided by the number of phase steps.

(3) Simplified non-linear mapping equations derivation process and a comparison was made between linear and non-linear calibrations numerically and experimentally.

The non-linear mapping functions were obtained by a simplified geometrical derivation. A comparison of linear and non-linear system calibrations to determine the relationship between the phase difference and height of the object surface was performed (Jia 2005b and Jia 2006d). The accuracy for both methods was simulated numerically and the RMS error distributions of the depth calculation for both methods were obtained. The simulation experimental results indicate that the accuracy of the linear calibration is generally higher than the non-linear calibration method. The measurement accuracy using the non-linear calibration results is higher than when using the linear calibration result in a certain range of measurement depth between 35 mm and 45 mm. In general, the accuracy of measurement using linear calibration data is higher than using non-linear calibration data in most of the range of measurement depth.

A real measurement experiment was carried out to verify the simulation results under the same parameters configuration. The RMS errors distributions of the real measurement experiment of both calibration methods are close to the simulation results over the lower range of depth (0–32 mm). However, at the higher range of depth (32–50 mm), the real-measurement RMS errors were considerably lower for the non-linear calibration method compared to that for the linear calibration method. Repeated testing indicates that the linear method is more sensitive to the noise, which sometimes results in lower measurement accuracy in the real measurement; the non-linear method is more robust to noise, and usually can achieve a more repeatable measurement accuracy in repeated testing.

(4) Developed a repeated two-step triangular-pattern phase-shifting method to reduce the measurement error.

The major error sources for the triangular-pattern phase-shifting method are the gamma non-linearity and image defocus of the projector and camera. To improve the measurement accuracy of the two-step triangular-pattern phase-shifting method, an improved method, called repeated two-step triangular-pattern phase-shifting approach (Jia 2006b), to decrease the measurement error due to the gamma non-linearity and image defocus of the projector and camera was developed. In the proposed method, the object is measured twice by the two-step triangular-pattern phase-shifting method. For the second measurement, a phase offset of one-eighth of the pitch is introduced relative to the first measurement. For this second measurement, the peaks and valleys of the error wave are reversed from that of the first measurement. Averaging of the two measurement results for the complete image, therefore countervail the wave error of each other for the complete 3-D surface. The measurement accuracy using repeated two-step triangular-pattern phase-shifting method was increased over that using the two-step triangular-pattern phase-shifting method by about 24.0%.

(5) Developed an intensity-ratio error compensation method for triangular-pattern phase-shifting method.

An intensity-ratio error compensation method to decrease the measurement error due to the projector gamma non-linearity and image defocus for the triangular-pattern phase-shifting method with any number of phase-shifting steps was presented in this research. In the proposed method, the intensity-ratio measurement error is obtained by simulating the measurement with the triangular-pattern phase-shifting method with real and ideal captured triangular-pattern images. A LUT that stores the intensity-ratio measurement error corresponding to the measured intensity-ratio is constructed and used for intensity-ratio error compensation during the measurement. The experimental results demonstrated that this intensity-ratio error compensation method significantly reduces the measurement error in the triangular-pattern phase-shifting method. The measurement accuracy after error compensation increased over that before error compensation by 28.5%.

(6) Developed an advanced off-line 3-D shape measurement system.

An off-line 3-D shape measurement system has been efficiently developed in this research. The major functions of this system include system calibration with linear and non-linear methods; 3-D object measurement and reconstruction, rendering and display, measurement accuracy analysis in numerical simulation and real measurement, error compensation, image processing, etc. The system was developed with Microsoft Visual C++ 6.0, Matrox Image Library 8.0, OpenGL 2.0, and Matlab 7.0. Both the triangular-pattern phase-shifting method and sinusoidal-pattern phase-shifting method have been implemented in the system.

(7) Presented a novel real-time 3-D shape measurement pipeline and implementation mechanism.

A novel real-time 3-D shape measurement pipeline and implementation mechanism has been proposed and evaluated with a real-time 3-D shape measurement system in this dissertation. The real-time 3-D shape measurement system was implemented from a prototype of generating patterns, phase shifting, capturing images, intensity-ratio/phase wrapping, intensity-ratio/phase unwrapping, 3-D coordinate calculation, and 3-D object rendering pipeline, which is shown in Figure 7-1. The experimental results suggested that the entire 3-D shape measurement process shown in Figure 7-1, when separated into four major steps, can optimize the measurement procedure to obtain the maximum measurement speed. These four steps were implemented with four threads with parallel processing software and they cooperate with each other effectively. With this proposed 3-D shape measurement pipeline and implementation mechanism, the system can provide maximum processing speed.

(8) Developed a real-time 3-D shape measurement system.

A real-time 3-D shape measurement system using the triangular-pattern phase-shifting method has been successfully developed with the proposed real-time 3-D shape measurement pipeline and implementation mechanism in this research. The system was developed with Win32 programming with the integration of multi-threading, Matrox Imaging Library, and OpenGL. The system can simultaneously capture pattern images,

process the acquired images to retrieve the 3-D information, and reconstruct and display the 3-D object. Software-based synchronization was used to implement the system to verify the real-time implementation pipeline proposed in this research. The system generates a data cloud of 648×494 points per frame. The system is capable of measuring slow moving objects.

8.3 Future Work

Some recommendations for future improvements of the current research are summarized as follows:

(1) Develop a real-time 3-D shape measurement system by hardware synchronization with the proposed real-time implementation pipeline and mechanism.

Software-based synchronization has been implemented in the developed real-time 3-D shape measurement system to verify the effectiveness of the proposed real-time implementation pipeline, triangular-pattern phase-shifting algorithms, and implementation mechanism. The experimental results indicate that the maximum measurement speed can be achieved by the system equipped with the proposed real-time implementation pipeline and mechanism. However, to further increase the measurement speed and to confirm the efficiency of the proposed real-time implementation pipeline and mechanism, it is necessary to develop a hardware synchronization circuit board to synchronize the pattern projection and image acquisition, and modify the proposed real-time implementation pipeline and mechanism as necessary. In the hardware synchronization based system, two computers may be used; one for pattern generation and projection, and the other for image manipulation and 3-D object reconstruction and rendering.

(2) Increase the pattern projection and image manipulation speed for the real-time 3-D shape measurement system.

In the current experimental setup for real-time 3-D shape measurement, the pattern was generated beforehand and saved in memory for projection through a DLP projector. Also, the captured images are processed in the computer memory after the sample data is

transferred to the computer via the frame grabber. The measurement speed was partly limited by the response of the projector and the data transferring speed from the frame grabber to the PC through the PCI interface. A specially designed projection system and an on-board processing scheme are necessary to further increase the measurement speed for the current real-time 3-D shape measurement system. Upgrading the image acquisition system, including the camera and the camera-computer interface could lead to increase the image acquisition speed. This could be implemented with the software-based synchronization or even for a real-time 3-D shape measurement system with hardware synchronization developed in the future.

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