Empirical Tests of the Capital Asset Pricing Model

by

Yi Jin

(2849993)

Major Paper presented to the

Department of Economics of the University of Ottawa

In partial fulfillment of the requirements of the M.A. Degree

Supervisor: Professor Kathleen Day

ECO7997

Ottawa, Ontario

December, 2003

Empirical Tests of the Capital Asset Pricing Model

Abstract

In this paper I use recent Canadian stock market data to test the SLMB Capital Asset Pricing Model. I apply the same methodology as Fama and Macbeth (1973), but cannot obtain conclusions similar to theirs. Fama and Macbeth find that there exists a strong linear relationship between risk and expected returns, and that the risk coefficient is the only factor explaining expected returns. Instead I find that the risk beta has a weak relationship with expected returns. Other factors might explain returns.

Empirical Tests of the Capital Asset Pricing Model

I. Introduction

The Capital Assets Pricing Model is also called the SLMB model, because this asset pricing theory was developed by Sharpe (1964), Lintner (1965), Mossin (1966) and Black (1972). It was the first complete model of asset pricing which explained the relationship between the average returns of stocks and risk. The whole model is based on some of the main ideas of market portfolio theory offered by Markowitz (1959), who believed that portfolio investment was mean-variance efficient (the mean was the expected return of the portfolio investment, and the variance was the risk). This mean-variance efficiency implies that the expected returns on securities are a positive linear function of their beta coefficients (the beta coefficient is the slope of the regression function when the security's return is regressed on the market return).

After this model was established about 30 years ago, many economists tested it empirically, and obtained consistent results, i.e., the theory matched the empirical evidence. The representative article is Fama and Macbeth (1973). They used real world data to test the theoretical model, and their results regarding the expected returns of stocks were consistent with the model.

But things changed in the early 1980s. Many economists found that the risk coefficients were not the only factor explaining the expected returns of stocks. Many other variables – for example firm size, the leverage effect, and a firm's book value of common equity – also play a strong role in explaining the average returns of securities (Fama and French 1992). There are several empirical results that are inconsistent with the SLMB model. The most well-established is the firm size effect of Banz (1981). In his article he argued that firm size, as measured by market equity ME (a stock's price times the shares outstanding), can explain average returns, and found that some small sized firms (low ME) had too high returns relative to their beta coefficients and some large sized firms (high ME) had too low returns relative to their beta coefficients.

Bhandari (1988) suggested another important factor that can explain expected average returns. She pointed out that there was a positive relationship between the leverage effect – the effect of high debt/equity ratios - and average returns.

The CAPM has now been in existence for nearly forty years and there remain many debates in asset pricing theory. Some modifications to the CAPM and even distinctly new models have been developed. But the CAPM still has its appeals. Jagannathan and McGrattan (1995) have pointed that the attraction of the CAPM is its powerfully simple logic and good predictions about how to measure risk and about the relationship between expected return and risk. Perhaps because of its simplicity, the empirical record of the model is very poor. This empirical record provides some contrary examples when the CAPM is used in applications. These problems may reflect true failings, but they may also be due to shortcomings of the empirical tests, most notably the problem of poor proxies for the market portfolio (which plays a central role in the model's predictions).

In this paper I do not put the emphasis on the debate about CAPM theory. Instead, I try to examine the model with which Fama and Macbeth (1973) had obtained results consistent with the CAPM. This paper is famous because it successfully demonstrated that the predictions of the "two-parameter" capital asset pricing model were correct. What I want to do in this major paper is to use more recent Canadian data and the same model to test the predictions of the so called "two-parameter model." I use the same methodology as Fama and Macbeth (1973) to see whether we can get results similar to theirs.

II. Theoretical Framework

In this section I discuss the development of the CAPM. Discussion of the theory of stock prices began with the work of Markowitz (1952, 1959). The Markowitz model is a single-period mean-variance (MV) model, in which an individual investor forms a portfolio at the beginning of the period. The objective is to maximize the portfolio's expected return, at a given level of risk. The assumption of a single time period, plus the assumptions about the investor's attitude toward risk (rational individual investor), allow risk to be measured by the variance (or standard deviation) of the portfolio's return. Investors believe that the expected returns of securities and portfolios are closely related to their risk coefficients, called betas. This particular relationship between asset returns and risk betas is also called the SML—the Security Market Line (Sharpe 1985). Sharpe deduced this relation as "a key implication of the original CAPM" (Sharpe 1985,161). In Sharpe (1985) he states that the relationship between risk and expected returns of all securities and portfolios should form an upward-sloping straight line, when the expected returns are measured on the vertical axis and the risk beta coefficients are measured on the horizontal axis.

The single-period MV model may be the best-known model of optimal portfolio theory. From a theoretical point of view, it represents a perfect combination of logic and simplicity. Beta is an indicator that reveals the relationship between a single asset and the market. The variance of the market portfolio also reveals the part of risk that cannot be diversified away. From an empirical point of view, the model appears to be readily testable: betas are easily estimated from standard regressions, as will be shown later.

As Davis (2001) notes, the change in the expected return and standard deviation of a portfolio as securities are added to the portfolio will depend on the covariance between the added securities and the other securities in the portfolio. That is to say, if I want to add new security into the portfolio I must first calculate the covariance between the added security and the other securities. The covariance determines the proportion of the new security in the portfolio.

The Capital Asset Pricing Model was independently developed by Sharpe (1964), Lintner (1965) and Mossin (1966). All three based their work on Markowitz' framework. The capital asset pricing model of Sharpe, Lintner, Mossin and Black marks the birth of asset pricing theory (finally Sharpe got a Nobel Prize for it in 1990). Before their work, there were no asset pricing models with clear, testable predictions about risk and return. According to Jagannathan and McGrattan (1995), after forty years, the CAPM is still widely used in applications, used by financial managers and institutional investors in estimating the cost of equity capital for firms and in evaluating the performance of managed portfolios.

The important contribution of Markowitz's theory to the SLMB model involves the formation of portfolios. When Markowitz's theory of portfolio formation is applied, the following assumptions must hold: the stock market is efficient; there exists a risk free asset that has a certain return; returns are normally distributed; the investor has a one-period horizon; investors are risk-averse and utility-maximizing; information is homogenous and complete; there are no taxes or transaction costs; and unlimited short selling is possible. All these assumptions have been theoretically derived and analysed by Sharpe (1964) and Lintner (1965). In their articles they provide a more detailed explanation of the above assumptions. Here I offer only an overview of their explanations of the assumptions. To ensure market efficiency, Sharpe and Lintner add another two key assumptions to the Markowitz model to identify a portfolio that must be efficient if the market is to clear. The first key assumption is that they assume a common pure rate of interest, with all investors able to borrow or lend funds on equal terms. The second key assumption is that they assume homogeneity of investor expectations. Investors are assumed to agree on the prospects of various investments—the expected values, standard deviations and correlation coefficients. Only in an efficient market can portfolio theory diversify the investment risks.

Normally distributed returns are required for the mean-variance efficiency of market portfolio theory. To explain the assumption of a one-period horizon, I can use an example (Fama and Macbeth 1973): e.g., given market clearing prices at *t-1*, investors agree on the joint distribution of asset returns from *t-1* to *t*. This is the so-called "one-period horizon" assumption. No taxes or transaction costs means that there is borrowing and lending at a risk-free rate, r_f, which is the same for all investors. "Risk-averse utility-maximizing investors" means that individual investors are assumed to be rational: their behaviour is risk-averse when facing risk and uncertainty, and they seek to maximize personal utility. They demand that higher risk be associated with a higher expected return.

"Homogeneous information" means that when the capital market is perfect and efficient, and information is available without cost, all investors can derive the same correct understanding of the distribution of the future price of any assets or portfolios.

Homogenous information is something like the key assumption of market efficiency: homogeneous expectations. Under these assumptions of homogenous information and no short selling, I can write the market equilibrium weighting factor as $x_{im} \equiv$ total market value of all units of asset i / total market value of all assets (Fama and Macbeth 1973, 611). As Fama and Macbeth say in their article, "the homogeneous-expectations assumption implies a correspondence between ex ante assessments of return distributions and distributions of ex post returns" (Fama and Macbeth 1973, 611).

The CAPM model is heavily dependent on these assumptions. When I try to analyse something implied by this model, I must first assume that all of these prerequisites are satisfied. I know that only if the above assumptions are satisfied can I continue to apply the framework of the CAPM asset pricing theory.

The derivation of the equations of the CAPM proceeds as follows. Investors choose portfolios along the security market line, which shows the combinations of the risk free asset and the risky portfolio m. In order to ensure markets are in equilibrium (quantity supplied = quantity demanded), the portfolio m must be the market portfolio of all risky assets. So, all investors combine the market portfolio and the risk free asset, and the only risk that investors have to bear is the risk associated with the market portfolio.

All this leads to the CAPM equation:

$$E(R_j) = R_f + \beta_j \lfloor E(R_m) - R_f \rfloor,$$

where $E(R_j)$ and $E(R_m)$ are the expected returns to asset j and the market portfolio, R_f is the risk-free rate of return, and β_j is the beta coefficient for asset j (Fama and MacBeth 1973, 610). β_j measures the tendency of asset j to co-vary with the market portfolio. It represents the part of the asset's risk that cannot be diversified away through portfolios, and this is the risk for which investors are compensated.

The CAPM equation says that the expected return of any risky asset is a linear function of its co-variance with the market portfolio. Then if I believe that the CAPM theory does a good job of predicting asset prices, I should observe this positive linear relationship between the average portfolio returns and the portfolio betas. Furthermore, only beta is able to explain the cross-sectional differences in average returns. That is to say "beta should be the only thing that matters in the CAPM framework" (Davis 2001, 2).

In this study I plan to use the same model used by Fama and Macbeth (1973): the two-parameter CAPM model. The model consists of more than ten equations. Their derivation of the model is summarized in the remainder of this section.

If returns are normally distributed, then the risk of portfolio p can be measured by the standard deviation, $\sigma(\tilde{R}_p)$, of its return, $\tilde{R}_p$, and the risk of an asset for an investor who holds p is the contribution of the asset to $\sigma(\tilde{R}_p)$. First, let $\tilde{R}_p$ be the returns of portfolio p. By definition,

$$\tilde{R}_p = \sum_{i=1}^{N} x_{ip} \tilde{R}_i ,$$

where $\tilde{R}_i$ is the return on asset i, x_{ip} is the proportion of portfolio funds invested in asset i, and N is the number of assets. Since

$$\operatorname{var}(\tilde{R}_p) = \sum_{i=1}^{N} \sum_{j=1}^{N} x_{ip} x_{jp} \operatorname{cov}\left(\tilde{R}_i, \tilde{R}_j\right),$$

and

$$\operatorname{cov}\left(\tilde{R}_i, \tilde{R}_p\right) = \sum_{j} x_{jp} \operatorname{cov}\left(\tilde{R}_i, \tilde{R}_j\right),$$

the standard deviation of portfolio p can be written as

$$\sigma(\widetilde{R}_p) = \sum_{i=1}^{N} x_{ip} \left[\frac{\sum_{j=1}^{N} x_{ip}\sigma_{ij}}{\sigma(\widetilde{R}_p)} \right] = \sum_{i=1}^{N} x_{ip} \frac{\operatorname{cov}(\widetilde{R}_i, \widetilde{R}_p)}{\sigma(\widetilde{R}_p)} \quad . \qquad (1)$$

The contribution of asset i to $\sigma(\widetilde{R}_p)$, or the risk associated with asset i in the portfolio p, can be viewed as being proportional to

$$\sum_{j=1}^{N} x_{ip}\sigma_{ij} / \sigma(\widetilde{R}_p) = \operatorname{cov}(\widetilde{R}_i, \widetilde{R}_p) / \sigma(\widetilde{R}_p) \quad . \qquad (2)$$

For any individual investor the relationship between the risk of a certain asset and its expected return depends on the efficiency of the investor's optimal portfolio. Using optimal portfolio theory, I need to find portfolio m, defined by the weights x_{im}. According to optimal portfolio theory, the objective of the investor is to choose the weights x_{im} so as maximize the expected portfolio returns,

$$E(\widetilde{R}_m) = \sum_{i=1}^{N} x_{im} E(\widetilde{R}_i) \qquad (3)$$

subject to the constraints that the risk associated with the portfolio is equal to the market risk

$$\sigma(\widetilde{R}_p) = \sigma(\widetilde{R}_m) \qquad (4)$$

and

$$\sum_{i=1}^{N} x_{im} = 1 \quad . \qquad (5)$$

Choosing the portfolio in this manner will ensure that it is efficient. The Lagrangian method can be used to derive the following equation:

$$E(\tilde{R}_i) - E(\tilde{R}_m) = S_m \left[\frac{\sum_{j=1}^{N} x_{jm}\sigma_{ij}}{\sigma(\tilde{R}_m)} - \sigma(\tilde{R}_m) \right], \qquad (6)$$

where S_m is the rate of change of $E(\tilde{R}_p)$ with respect to $\sigma(\tilde{R}_p)$, the expected return. This equation, which is the basis of the two-parameter CAPM, implies that the difference between the expected return on asset *i* and the expected return on portfolio *m* is proportional to the difference between the risk of the asset and the risk of portfolio *m*, with the factor of proportionality being S_m. This conclusion is very important because it explains the relationship between risk and return.

Note that I can re-write equation (6) for convenience as:

$$E(\tilde{R}_i) = \left[E(\tilde{R}_m) - S_m\sigma(\tilde{R}_m)\right] + S_m\sigma(\tilde{R}_m)\beta_i, \qquad (7)$$

where $$\beta_i \equiv \frac{\operatorname{cov}(\tilde{R}_i, \tilde{R}_m)}{\sigma^2(\tilde{R}_m)} = \frac{\sum_{j=1}^{N} x_{jm}\sigma_{ij}}{\sigma^2(\tilde{R}_m)} = \frac{\operatorname{cov}(\tilde{R}_i, \tilde{R}_m)/\sigma(\tilde{R}_m)}{\sigma(\tilde{R}_m)} \quad . \qquad (8)$$

The parameter β_i is just the risk of asset *i* in the portfolio *m*, which was defined in equation (2), divided by $\sigma(\tilde{R}_m)$, the total risk of *m*. If I define

$$E(\tilde{R}_0) \equiv \left[E(\tilde{R}_m) - S_m\sigma(\tilde{R}_m)\right], \qquad (9)$$

then $$S_m = \frac{E(\tilde{R}_m) - E(\tilde{R}_0)}{\sigma(\tilde{R}_m)}. \qquad (10)$$

This means $E(\tilde{R}_0)$ is the expected return on a security that has a β of zero; i.e., a security for which the sample $\operatorname{cov}(\tilde{R}_i, \tilde{R}_m) = 0$. This security, although not risk-free, does not contribute to the risk of portfolio *m*. Using equation (10), equation (7) can be re-written as

$$E(\widetilde{R}_i) = E(\widetilde{R}_0) + \left[E(\widetilde{R}_m) - E(\widetilde{R}_0)\right]\beta_i \quad . \qquad (11)$$

Equation (11) has three implications that I will test later in this paper. They are described by Fama and Macbeth (1973) as follows: First, the relationship between the expected return on a security and its risk in any efficient portfolio *m* is linear. Second, β_i is a complete measure of the risk of security *i* in the efficient portfolio *m*; more importantly, no other factor influences the returns in equation (11). Third, in a market, a rational individual investor (a risk-averse investor) believes that higher risk should be associated with a higher expected return, i.e., $E(R_m) - E(R_0) > 0$. This implies that in an efficient market, returns on securities must be greater than returns on risk-free assets.

To test these implications I must rewrite equation (11) in a form which can be estimated using real data:.

$$\widetilde{R}_{it} = \alpha_i + \beta_i \widetilde{R}_{mt} + \widetilde{\varepsilon}_{it} \quad . \qquad (12)$$

Equation (12) is a standard regression equation and is known as the market model. In this equation α_i is the intercept coefficient of the Security Market Line; in general a security is under-priced if α_i is positive and over-priced if α_i is negative. β_i represents the individual firm's risk coefficient, and $\widetilde{\varepsilon}_{it}$ is a random error. With sufficient time series data I can use the method of ordinary least squares to obtain estimates of β_i and the standard errors of the regression equation for each individual firm.

But equation (12) is not the final estimating equation that I will use to analyse the model's predictive and explanatory power. Instead I use the following equation to test three hypotheses:

$$R_{pt} = \hat{\gamma}_{0t} + \hat{\gamma}_{it}\hat{\beta}_{p,t-1} + \hat{\gamma}_{2t}\hat{\beta}_{p,t-1}^{\ 2} + \hat{\gamma}_{3t}\overline{S}_{p,t-1}(\hat{\varepsilon}_i) + \hat{\eta}_{pt}. \qquad (13)$$

I can use the individual firms' monthly returns to calculate the returns of portfolios, R_{pt}, and use each firm's β to compute the β of each portfolio, and then average the standard errors of the regression equations (12) to compute $\overline{S}_{p,t-1}(\hat{\varepsilon}_i)$, a measure of non-beta risk. This is the same model that Fama and Macbeth (1973) used. What remains is to search for the appropriate data and use the same methodology to estimate equation (13).

The above is the theoretical framework of the capital asset pricing model that Fama and Macbeth used in 1973. Each of the following seven statements (provided by Grauer (2001, 7)) is good measure of whether the CAPM is true or false:

> First, the market portfolio is MV efficient. Second, there is a linear relation between the expected returns and market betas of securities, i.e., securities plot on the SML. Third, market betas are the only measures of risk needed to explain the cross-section of expected returns. Fourth, in the risk-free asset version of the model, the market portfolio is the tangency portfolio, i.e., the point of tangency between a ray emanating from the risk-free interest rate and the minimum-variance frontier of risky assets. Fifth, in the risk-free asset version of the model, the separation property implies that all investors will hold some combination of the risk-free asset and the market portfolio. Sixth, there is at least one positively weighted efficient portfolio. Seventh, the Invariance Law of Prices implies that if the CAPM determines prices and means, variances, and co-variances are constant over time, then there is an exact linear relation between the values of any three assets.

Fama and Macbeth (1973) evaluate the second and third statements in their empirical test of the CAPM theory. In this paper I also want to examine whether or not these two statements hold.

III. Literature Review

Most theories of asset pricing have been in existence for about forty years.[1] Jensen (1972) provides quite a good overview of the CAPM theory and early tests of the model. As he indicated in his article, equation (12) could provide a sufficient description of the relationship between risk and security returns. But Douglas (1969) found that the model may not provide a complete description of the relationship: he found the estimated slope (beta) of the security market line (equation (12)) was too flat and the intercept term (alpha) was too large. Miller and Scholes (1972) repeated the Douglas study as a replication exercise and obtained the same result. Miller and Scholes (1972) provided some further discussion of the problems caused by some possible econometric difficulties. They concluded that there existed measurement errors in the estimated betas. Measurement errors could make the significance of the coefficients in equation (12) misleading, and cause the estimated slope to be flatter than the actual slope.

Later, Black, Jensen, and Scholes (1972), Blume and Friend (1973), and Fama and MacBeth (1973) produced some extensive tests of the model. They focus on the cross-sectional regressions across portfolios of the relationship between the expected returns and betas (equation (13) estimated separately for each *t*). In the cross-section regression approach (proposed by Jensen) securities are grouped into portfolios to remove the measurement errors in the betas from the regressions. Their findings indicate that the relationship between the average return and beta is almost linear (i.e., equation (13) is linear), but the estimated slope of the SML is still too flat and the intercept is too high (a linear relationship does not exist in equation (12)). A flatter SML means that the

[1] This review of literature relies heavily on Davis (2001), and Grauer (2001).

empirical data support a zero-beta CAPM theory, not the original model.[2] Thus the evidence provides a strong basis for the rejection of the original Sharpe-Lintner model (equation (12)) and for the acceptance of Black's (1972) zero-beta CAPM theory.

Among these studies of the early 1970s I think that the first successful extensive tests of the MV CAPM were those of Fama and Macbeth (1973). After the publication of this study, economists accepted the argument that the CAPM was the only explanation of asset returns and that the risk coefficient beta was the only factor that influenced asset returns for quite a long period.

But as time went by, many economists in the 1970s proposed modifications to the SLMB model. For example, Jensen (1972,15-33) discussed the conditions under which one might relax the assumptions of CAPM theory as the extensions of the Mean-Variance CAPM theory. First, he thinks the assumption that all investors are single-period expected utility of terminal wealth maximizers is wrong. Second, he discusses the possibility of the existence of riskless borrowing and lending opportunities. Third, he considers the influence of the existence of non-marketable assets. Fourth, he discusses the effect of differential taxes on dividends and capital gains. Fifth, he discusses a model in continuous time. Sixth, he mentions the existence of heterogeneous expectations. Other examples include Merton's (1973) intertemporal capital asset pricing model (ICAPM), and Rubinstein's (1974) single-period linear risk tolerance (LRT) model. Ross' (1976, 1977) Arbitrage Pricing Theory offered a totally different theory of asset pricing, known as the APT model. Other authors, such as Rubinstein (1976), Lucas (1978), and Breeden (1979) proposed an intertemporal consumption-based model (CCAPM). All of these alternatives were tested against the SLMB CAPM theory.

---

[2] Zero-beta CAPM theory implies a very flat SML curve, flatter than in the original CAPM.

Roll's (1977) critique changed things in academic circles. Roll argued that if a test does not use the correct market portfolio, the SLMB model might be wrongly rejected; i.e., empirical tests of the CAPM might be mis-specified, and the choice of a proxy for the market portfolio can influence the estimated results. Roll suggests that economists should be more careful about the choice of proxy for the market portfolio when doing empirical testing. Roll cast doubt on the proxy chosen by Fama and Macbeth (1973), raising the concern that their perfect test results might not be valid.

Then Banz (1981) showed that the risk beta was not the only factor that determined the price of assets. He suggested that the size of companies also was very important, and at least exerted the same level of influence on asset prices as the traditional factor – risk coefficients. Banz's study examined the empirical relationship between returns and the total market value of NYSE common stocks. He found that smaller firms obtained higher returns to risk than larger firms – i.e., the average returns on small (low market equity) firms' stocks were too high compared with their estimated betas, and the average returns on larger firms' stocks were too low compared with their estimated betas. Thus market equity provided another important explanation of the cross-section variation in average returns. Furthermore, he also found that this kind of size effect had existed for at least forty years. This was very strong evidence that the CAPM was misspecified.

Following Banz's research, more and more articles showed that there existed many empirical contradictions of the SLBM model. DeBondt and Thaler (1985) show that stocks which tended to be either winners or losers (but never in the middle) in the recent past would exhibit a reverting behavior in future years. Reverting behavior means that past winners (losers) will become future losers (winners). They identified "losers" as stocks that had had poor returns over the past three to five years; "winners" were those stocks that had had high returns over the same period. Their main finding was that losers had much higher average returns than winners over the next three to five years. In contrast, Jegadeesh and Titman (1993) find that forming portfolios based on short-term past returns leads to a continuation of short-term returns, the so-called **Long-Term Return Reversals phenomenon.** Chopra, Lakonishok and Ritter (1992) indicated that beta cannot account for this difference (caused by winners' and losers' expected returns) in average returns. There existed a certain tendency for returns to be reversed over long time horizons (i.e., losers become winners). This was another contradiction of the CAPM. Losers would have to have much higher betas than winners in order to justify the return difference. Past losers tend to be future losers and past winners are future winners. Since the CAPM theory cannot explain these patterns of average returns, they are called CAPM anomalies.

Stattman (1980) and Rosenberg and Lanstein (1985) found that a special ratio was closely related to the average returns on U.S stocks. They defined this particular ratio as the ratio of a firm's book value of common equity (BE) to its market value (ME), simplified as the BtM Ratio. Later, Chan, Hamao and Lakonishok (1991) showed that the special ratio BE/ME (book-to-market equity) could also explain the cross-section of average returns on Japanese stocks.

Another early study that contradicted the predictions of the CAPM was Basu (1977). Using a sample period that extended from April 1957 to March 1971, Basu showed that stocks with high earnings/price ratios (or low P/E ratios) earned significantly higher returns than stocks with low earnings/price ratios. His results indicated that differences in betas could not explain the returns' differences. In a subsequent study, Basu (1983) showed again that this "E/P effect" was not just observed among small cap firms (firms with small development capitals). A later study by Jaffe, Keim and Westerfield (1989) confirmed this discovery and also showed that the E/P effect did not just appear in the month of January (the January effect). The E/P effect was a direct contradiction of the CAPM's implication that beta should be the only variable that matters. These test results indicated that E/P ratios could be another factor explaining average returns.

Bhandari (1988) suggested another important element which could explain the average returns of common stocks. She argued that there existed a positive relationship between the leverage effect and cross-section average returns.[3] She found that firms with high leverage (high debt/equity ratios) would have higher average returns than firms with low leverage for the 1948–1979 period. High leverage increased the riskiness of a firm's equity, but this increased risk should be reflected in a higher beta coefficient. In the SLMB model the risk associated with leverage management is included within the effect of market beta, so that the leverage effect should not have any additional influence on average returns. In Fama and Macbeth (1973), all financial companies are deleted to purge the data of the leverage effect. But in Bhandari's research leverage still had independent explanatory power.

With so many contradictions of the SLMB CAPM, Fama and French (1992) did an empirical test of all the relevant factors that influence average returns. In their cross-sectional model, they included the market beta coefficient, the size effect, book-to-market equity (BE/ME), the leverage effect, and the earnings-price ratio (E/P) to

[3] The leverage effect involves the relationship between stock returns and implied and realized volatility. This means that volatility will increase when the stock price falls. For example, changes in firm's equity depends on the degree of leverage in firm's capital structure. An increase in leverage will produce an increase in stock volatility.

evaluate their joint explanatory power. They used data from returns files for common stocks traded on the NYSE, AMEX and NASDAQ markets, for the period 1962 to 1989, from the CRSP.[4] Their results shocked the academic circle. Fama and French (1992) report that there existed only a weak positive relation between average returns and beta over the 1941-1990 period, and virtually no relation over the shorter 1963-1990 period. Even when other variables were excluded from the regression equation, leaving only beta, the relationship between market betas and average returns was flat. Furthermore unlike the simple regression of average returns on beta, the bivariate relations between average return and size, leverage, E/P, and book-to-market equity were very strong. Their results indicated that even when other variables were also included, there existed a strong negative relationship between firm size and average returns. There also existed a clearly positive relation between book-to-market equity and average returns. Among all the relevant factors the role of book-to-market equity was very important. On the basis of these results, they came to the conclusion that the CAPM was useless for its original purpose.

Fama and French's (1992) paper again stimulated further debate regarding the CAPM model. According to Grauer (2001, 24),

> Some believe that the CAPM is dead. Others await a meaningful test. Some believe a three-factor model that is consistent with rational behavior has replaced the CAPM. Others believe that taxes and liquidity are missing factors in a rationally based CAPM. Still others believe the three-factor model is consistent with irrational behavior.

[4] CRSP is "the Center for Research in Security Prices" at the Graduate School of Business and Management of the University of Chicago. It provides comprehensive security price data via two primary stock files, the NYSE/AMEX file and the Nasdaq file.

In the 1990s many arguments about whether the market betas were the only measures of risk that could explain expected returns appeared. Tests of this proposition were fascinating. The tests set off a multifaceted debate concerned with the efficacy of the testing methodology, and the interpretation of the results, etc. Moreover, the tests continue to blur the distinction between tests of market efficiency and tests of asset-pricing models.

Grauer (2001) summarizes all the debates on the CAPM theory by pointing out that after the 1990s four main schools of thought developed. One school argues that the CAPM may be spuriously rejected due to a misunderstanding of econometric methods. Lo and MacKinlay (1990), Black (1993), and MacKinlay (1995) argued that the CAPM anomalies might be the result of "data snooping" as Fama (1991) had noted.[5] Here Kothari, Shanken and Sloan (1995) argued that there was "survivorship bias" in the returns used to test the model, especially "in terms of the book-to-market returns".[6] Roll and Ross (1994), Kandel and Stambaugh (1995), and Grauer (1999) suggested that the cross-sectional tests were easily influenced by the use of poor proxies for the market portfolio. Roll and Ross showed that a proxy can be nearly MV efficient and yield a zero slope in a regression of means on betas (calculated relative to the proxy) using population parameters.

A second school of thought, advocated by Fama and French (1993, 1995, 1996,1997, 1998) believed that asset pricing was rational and that a three-factor ICAPM or APT did not reduce the reasonable basis of the CAPM. They suggested that the three-factor model

---

[5] Lo and MacKinlay (1990) argued that data snooping happened when a result was statistically significant but involved spurious relationships between empirical data. Financial analysis is particularly vulnerable to the problem of data snooping because we use large data sets, and we sometimes search for statistical relationships without considering the economic justification. Data snooping is unfriendly when seeking statistical relationships in financial economics.

[6] Lo and MacKinlay (1990) showed that survivorship bias meant the tendency for excluding failed companies from performance due to the fact that they no longer exist. Survivorship bias causes the results of some studies to be flawed, because only companies which have been successful survived until the end of the period are included. Portfolio performance may be misleading due to survivorship bias, for example if securities in the portfolio family merged or de-listed.

did not contradict the CAPM, i.e. they did not see a difference between them. Fama and French (1998) argued that the high returns earned by value stocks (those with low prices relative to earnings, dividends, book assets or other measures of fundamental value) arose because these stocks were definitely riskier than the growing stocks. They also argued that the empirical successes of their three-factor model suggested that it was an equilibrium asset-pricing model (I can call it a three-factor version of the ICAPM or the APT). In 1993, Fama and French extended the analysis of Fama and French (1992) to a three-factor model of asset pricing. Grauer (2001, 27) argues that:

> The expected return on a security in excess of the riskfree rate is explained by the sensitivity of its return to three factors. The first is the excess return on the market portfolio. The second is the difference between the return on a portfolio of small stocks and the return on a portfolio of large stocks (SMB, small minus big). The third is the difference between the return on a portfolio of high-book-to-market stocks and the return on a portfolio of low-book-to-market stocks (HML, high minus low).

Again Fama and French (1996) showed that the CAPM anomalies largely disappeared in a three-factor model except for the influence of short-term returns. The article offered some important contributions. They replicated the univariate time-series tests which were used by Black, Jensen and Scholes (1972) and multivariate time-series tests which had been used by Gibbons, Ross and Shanken (1989) with grouped data. The results were strong no matter which econometric method was used. Even more, the paper (Fama and French 1996) provided complete interpretations of the results. Later Fama and French (1998) provided some out-of-sample evidence (based on international data) for testing the risk premium earned by the net value of security (high book-to-market value minus low book-to-market). This evidence provided robust support for their arguments.

A third school argued that a rational CAPM should include the factors of taxes and liquidity. In fact Brennan (1970) had already pointed out that these should have some kind of influence according to the CAPM, caused by differential taxes on dividends and capital gains. Amihud and Mendelson (1986), Brennan and Subrahmanyam (1996), Brennan, Chordia, and Subrahmanyam (1998), Amihud (2000), and Pastor and Stambaugh (2001) examined higher expected returns on illiquid stocks.

A fourth school offered arguments which differed from those of the above three main schools. The first of these arguments in the fourth school was from Black (1993), who suggested that the Fama and French (1992) results were like an "artifact" of data mining. Former researchers had spent huge amount of time looking for relationships between stock returns and other variables, but they found only a few other variables related to stock returns. Very fortunately these variables could show a statistical relation to returns. Then Fama and French (1992) chose these related variables as their explanatory variables and found that these variables had significant coefficients. Thus the observed explanatory power of these variables could be due to a data mining exercise by these previous studies. Based on this, Black (1993) contended that some of the statistical tests in Fama and French (1992) were not properly specified. Since the relations between returns and size, BtM (book-to-market value), etc. were an artifact of data mining, he suggested that analysis using another data source and time period would help to remove the effect of data-mining. MacKinlay (1995) also mentioned the effect of data mining on the estimation results.

This theory quickly faced debate from Davis (1994). He had built a database of book values for large U.S. industrial firms from 1940 to 1963. The reason for building such a special database was that "for this period the Compustat coverage was either poor or nonexistent" (Davis 2001, 8). This database was constructed to get rid of survivorship bias, and it covered a period that preceded the period studied by Fama and French. If Fama and French's results were coming from data mining, this independent time period should produce different results. The spurious relation would not exist in a different period. Further more, the beta coefficients in this study were estimated with annual returns followed Kothari, Shanken and Sloan's (1995) suggestion. But the results of Davis (1994) confirmed Fama and French's (1992) results. In his study the explanatory power of BtM was observed during the period from 1940 to 1963, although in his data the magnitude of firms' returns was a little bit smaller. This difference may have been caused by the fact that the database for the Compustat period contained only large firms (large firms last longer and have long historic records). His results also indicated that the relation between beta and average returns was flat.

Other authors like Lakonishok, Schleifer and Vishny (1994) offered further detailed evidence that the Fama and French (1992) results were not simply due to survivorship bias. Carrying out tests for the 1968-1991 period, they found that if firms in CRSP and Compustat were properly matched, few of the missing firms from Compustat would have a significant effect on the Fama-French results. At the same time, they constructed a database including large firms for this particular period, and firm data in this dataset would have no survivorship bias. With this special dataset, they still found a robust BtM effect.

Barber and Lyon (1997) proposed a new, better way to examine the data mining problem. Instead of using the Compustat dataset, they formed a sample including financial firms for the 1973–1994 period. In this study they found a robust BtM effect among these firms. Since financial firms were excluded from the Fama-French sample (to remove the leverage effect from average returns), the results of Barber and Lyon provided additional evidence of the explanatory power of BtM.

Further independent evidence came from Fama and French (1998). They still found a robust BtM effect in several developed countries for the 1975-1995 period.[7] They also found there existed a risk premium in several emerging markets.[8] This international evidence provides strong support for Fama and French's (1992) results.

This completes our overview of the development of CAPM theory. There might be some previous study of CAPM using Canadian data, but I am not able to find them. Many exciting debates have arisen recently. I can see that there have been many contradictions of the model and also quite a number of interesting innovations in econometric methods for testing the model. But Grauer (2001) once argued that the fundamental doubt about the empirical evidence was due to the logic of these asset pricing theories rather than their statistical application. This seems like a very appropriate conclusion to these theoretical debates.

IV. Data

Fama and Macbeth (1972) used monthly data for all common stocks traded on the New York Stock Exchange during the period January 1926 through June 1968. These data came from the Centre for Research in Security Prices of the University of Chicago (CRSP).[9] Fama and Macbeth's data were monthly returns including dividends and capital gains, with the appropriate adjustments for capital changes (i.e., stock splits and

[7] There are thirteen developed countries: U.S., Japan, U.K., France, Germany, Italy, the Netherlands, Belgium, Switzerland, Sweden, Australia, Hong Kong, Singapore.
[8] There are sixteen emerging countries: Argentina, Brazil, Chile, Colombia, Greece, India, Jordan, Korea, Malaysia, Mexico, Nigeria, Pakistan, Philippines, Taiwan, Venezuela, Zimbabwe.
[9] CRSP website : http://gsbwww.uchicago.edu/research/crsp/

stock dividends), computed by the CRSP.

Fama and Macbeth divided the time span of their data into nine sub-periods. The first sub-period ranged from 1926 to 1938, altogether thirteen years; the second sub-period ranged from 1927 to 1942, a total of sixteen years; the next six sub-periods – 1931 to 1946, 1935-1950, 1939-1954, 1943-1958, 1947-1962, and 1951-1966 – also lasted sixteen years; while the last sub-period is from 1955 to 1968, fourteen years. Thus with the exception of the first and last sub-periods, all sub-periods are sixteen years in length. Using one of the sixteen-year periods as example, Fama and Macbeth use the first seven years to form portfolios, the next five years to estimate each individual firm's beta values and calculate the portfolio betas, and the remaining four years to test the model. I can call the first seven years the portfolio formation period, the next five years the estimation period, and the final four years the testing period. They also defined some criteria for deciding whether or not to include a firm in the sample. To be included, a firm's data must be available for a particular subset of the sixteen years. For example, in our case, according to Fama and Macbeth's rule any firm that is listed before Feburary1990 and continuously listed to January1999 will be included in the monthly return dataset.

In Canada there exists a research institution similar to CRSP in the U.S. named Toronto Stock Exchange Database Centre.[10] Like CRSP, the Toronto Stock Exchange Database Centre computes and makes available to researchers data on the returns of firms whose stocks are traded on the Toronto Stock Exchange. From this database I obtained monthly return data for common equities as well as the TSE 300 index. The latter is used as the market returns proxy.

I retrieved monthly return data for the sixteen-year period from January 1987 to

[10] http://dc1.chass.utoronto.ca/cfmrc

December 2002. These sixteen years are divided into three sub-periods: I will use January 1987 to December 1993 (seven years) to form portfolios, January 1994 to December 1998 to estimate beta coefficients, and January 1999 to December 2002 to test the model. Following Fama and Macbeth (1973), all the firms listed before January 1990 that are not de-listed before February 1999 are included in the dataset. The fact that not all firms have monthly return data beginning in January 1987 is not a problem – such firms should be included in the monthly return data set. But firms whose monthly return data begins in February 1990 should not be included in the database. For example, an individual firm whose monthly return data became available in any month before February 1990 and was continuously listed up to and including January 1999 will be included in the monthly return data set. Thus in the portfolio forming period (January 1987 to December 1993), the last four years are the most important. Thus a firm will be included in the dataset only if returns are available for at least the last four years of the portfolio formation period.

First I need to find a list of all firms listed on the Toronto Stock Exchange in January 1990. Such a list can be found in the *Toronto Stock Exchange Review*, which is the print publication of the Toronto Stock Exchange. In its monthly report I can easily find the names of all listed companies. Initially all the companies that appear in the January 1990 *Review* are included in the sample, with the exception of all financial companies. They are excluded to avoid the leverage effect. Then data availability is checked for each firm on the list. Any firm for which data did not become available until February 1990, or which was de-listed before February 1999 is excluded from the dataset. That is to say that firms that were de-listed even in December 1989 are not included. The fact that returns data are not available for the entire sixteen years for some firms is not a problem as long as the minimum requirements are met.

The number of the firms that were listed in January 1990 is 1752. From this total I need to exclude financial companies, such as banks, investment companies, financial groups, saving and mortgage companies, insurance companies, financial service companies, trust companies, and some holding companies. The number of financial firms excluded is 264, reducing the sample size to 1488. Of these 1488 firms, 234 companies satisfied the minimum requirements with respect to the availability of their returns data.

I need to be careful about the choice of proxy for market returns. In this major paper I use Canadian stock data from the Toronto Stock Exchange so it is best to use the TSE 300 Total Return Index as our proxy for market returns. In the database of Toronto Stock Exchange Database Centre I can download directly the monthly return data of individual firms, but for the TSE 300 Total Return Index; I need to calculate the returns, because from the database I only obtain the level of the TSE 300 Return Index. Using the formula

$$\text{Returns}=(P_t - P_{t-1}) / P_{t-1}$$

we can calculate monthly return of the TSE 300 Index, where P is the level of the TSE300 Index.

V. Methodology

Fama and Macbeth (1973) use a cross-section regression method involving the construction of portfolios of stocks to test the prediction of the CAPM. This method was first established by Blume (1970). I will use the same method here, but for only one four-year test period, rather than the nine periods that they examined. The Fama-MacBeth method is very complicated, so it is explained step by step in this section of the paper.

Due to some potential problems like errors in variables, Fama and Macbeth suggested using Blume's method, because forming portfolios can reduce estimation error (Blume 1970). Blume proved that for any portfolio *p*, defined by the weights x_{ip}, i=1,2,…,N;

$$\hat{\beta}_p \equiv \frac{\hat{\text{cov}}(\tilde{R}_p, \tilde{R}_m)}{\hat{\sigma}^2(\tilde{R}_m)} = \sum_{i=1}^{N} x_{ip} \frac{\hat{\text{cov}}(\tilde{R}_i, \tilde{R}_m)}{\hat{\sigma}^2(\tilde{R}_m)} = \sum_{i=1}^{N} x_{ip} \hat{\beta}_i . \qquad (14)$$

I too construct portfolios to test the predictive relationship of the two-parameter Capital Asset Pricing Model. In previous research, Blume found that the linear relationship would be stronger when using portfolios in the estimation. So it is easier to observe the linear relationship between risk and returns when using portfolios rather than the individual stocks.

Step 1:

Since equation (12) is a simple linear regression model, the OLS estimate of β_i is

$\hat{\beta}_i \equiv \frac{\hat{\text{cov}}(\tilde{R}_i, \tilde{R}_m)}{\hat{\sigma}^2(\tilde{R}_m)}$, where $\hat{\text{cov}}(\tilde{R}_i, \tilde{R}_m)$ and $\hat{\sigma}^2(\tilde{R}_m)$ are the estimates of $\text{cov}(\tilde{R}_i, \tilde{R}_m)$ and $\sigma^2(\tilde{R}_m)$ obtained from monthly returns. The proxy which is chosen for $\tilde{R}_{mt}$ is the return on the TSE300 Index.

The first step is to estimate equation (12) for each individual firm using ordinary least squares (OLS) estimation and the available monthly data for the period of January 1987 to December 1993. From these equations I obtain the estimated beta coefficient of the regression equation. Since there are 234 companies in the dataset, equation (12) is estimated a total of 234 times. To estimate equation (12) I need not only the monthly return data for each individual firm but also the monthly return data for the TSE300 Index.

Step 2:

Next I rank the beta coefficients estimated in Step 1 from the smallest to the largest, so that the last one is the largest of the estimated betas. Then the 234 securities are divided into twenty portfolios based on the ascending order of the estimated beta values. The first eighteen portfolios each consist of twelve securities; the last two portfolios (No. 19 and No. 20) each consist of nine firms. The first portfolio includes the securities with the lowest beta values, while the last portfolio includes the securities with the highest beta values.

Step 3:

After forming portfolios, I re-estimate the individual betas for each security in each portfolio using the next five years of monthly data (from January 1994 to December 1998), again using equation (12). The standard error of each regression is also saved. I call this period (from January 1994 to December 1998) the estimation period.

Step 4:

In this step I construct the data set for the test period of January 1999 to December 2002. I use the individual stock $\hat{\beta}_i$ to compute the $\hat{\beta}_{pt}$. This step is the most important and complicated step in Fama and Macbeth's method.

Although Blume (1970) demonstrated that $\hat{\beta}_{pt}$ could be written as a weighted average of the $\hat{\beta}_i$, as in equation (14), Fama and Macbeth (1973) compute $\hat{\beta}_{pt}$ as the simple average of $\hat{\beta}_i$ for the securities included in portfolio p at time t. When a security is de-listed after period t, it is excluded from the calculation of β_{pt} for all subsequent periods. More specifically, to generate the values of $\hat{\beta}_{pt}$ for each month during the period January 1999 to December 1999, I take the simple average, over all the securities in portfolio p that are still listed in period t, of the $\hat{\beta}_i$ computed in Step 4. Although computing the average itself is simple, the process is complicated by the fact that for each period t , one must verify whether security i is still listed; if it is not, it must be excluded from the calculation.

Before the $\hat{\beta}_{pt}$ are generated for the next twelve-month period, January 2000 to December 2000, the $\hat{\beta}_i$ are re-estimated, this time using data for the period January 1994 to December 1999. In order to compute the $\hat{\beta}_{pt}$ for January 2001 to December 2001, the $\hat{\beta}_i$ need to be re-estimated again using data for the period January 1994 to December 2000. Finally, the $\hat{\beta}_{pt}$ for the period January 2002 to December 2002 are based on $\hat{\beta}_i$ estimated using data for the period January 1994 to December 2001. Note that I do not in fact have to re-estimate equation (12) for all 234 firms each time the $\hat{\beta}_i$ s are re-estimated, because I can exclude the de-listed firms.

As a result of this procedure, the $\hat{\beta}_{pt}$ vary considerably from month to month. There are two sources of monthly variation: first, when firms are de-listed, they are excluded from the portfolio; and second, because the $\hat{\beta}_i$ are re-estimated every twelve months. Re-estimation of the $\hat{\beta}_i$ allows for the possibility that each individual firm's β_i may change over time.

The process of constructing the variable $\overline{S}_{pt}(\hat{\varepsilon}_i)$ of equation (13) is very similar to the process for $\hat{\beta}_{pt}$. $\overline{S}_{pt}(\hat{\varepsilon}_i)$ is computed as the simple average of the $S(\hat{\varepsilon}_i)$ for the firms included in portfolio p at time t, where $S(\hat{\varepsilon}_i)$ is the standard error of regression equation (12) for firm i. When a security is de-listed after period t it should be excluded from the calculation for all subsequent periods. I take the simple average using data for the period January 1994 to December 1998 to generate the values of $\overline{S}_{pt}(\hat{\varepsilon}_i)$ for each month in the period January 1999 to December 1999. I include all the securities in portfolio p that are still listed in period t. This process is repeated again in calculating $\overline{S}_{pt}(\hat{\varepsilon}_i)$ for the periods January 2000 to December 2000, January 2001 to December 2001 and January 2002 to December 2002. The estimates of $S(\hat{\varepsilon}_i)$ are updated each year as in the calculation of $\hat{\beta}_{pt}$. Finally I get the $\overline{S}_{pt}(\hat{\varepsilon}_i)$ for every month from January 1999 to December 2002.

Last I will explain how I compute R_{pt}. The method is the same as for computing $\overline{S}_{pt}(\hat{\varepsilon}_i)$ and $\hat{\beta}_{pt}$. R_{pt} is the simple average of R_{it} over all securities still listed in period t. The calculation is a little bit easier than computing $\overline{S}_{pt}(\hat{\varepsilon}_i)$ and $\hat{\beta}_{pt}$, because the R_{it} for listed firms come directly from the Toronto Stock Exchange Database.

Step 5:

Step 5 is to estimate equation (13):

$$R_{pt} = \hat{\gamma}_{0t} + \hat{\gamma}_{it}\hat{\beta}_{p,t-1} + \hat{\gamma}_{2t}\hat{\beta}_{p,t-1}^{\ 2} + \hat{\gamma}_{3t}\overline{S}_{p,t-1}(\hat{\varepsilon}_i) + \hat{\eta}_{pt},$$

using OLS and data for the three variables $R_{pt}, \hat{\beta}_{p,t-1}, \bar{S}_{p,t-1}(\hat{\varepsilon}_i)$ generated in Step 4. In total there are 940 (20*47) observations.[11] I define $\hat{\beta}^2{}_{p,t-1}$ as the simple square of $\hat{\beta}_{p,t-1}$ for easy calculation.[12] The equation is estimated separately for each month from February 1999 to December 2002. For each month the sample size is 20 (equal to the number of portfolios). After the equation has been estimated, I compute $\bar{\hat{\gamma}}_0, \bar{\hat{\gamma}}_1, \bar{\hat{\gamma}}_2, \bar{\hat{\gamma}}_3$ and $s(\bar{\hat{\gamma}}_0), s(\bar{\hat{\gamma}}_1), s(\bar{\hat{\gamma}}_2), s(\bar{\hat{\gamma}}_3)$, using the formulae

$$\bar{\hat{\gamma}}_i = \frac{1}{47}\sum_{i=1}^{47}\hat{\gamma}_i, \qquad i=0,1,2,3.$$

$$s(\bar{\hat{\gamma}}_i) = \frac{1}{47}\sum_{i=1}^{47}s(\hat{\gamma}_i), \quad \text{i=0,1,2,3,}$$

where 47 is the number of months. The average of R^2 and the sample standard error of R^2, $s(R^2)$ are also computed.

Step 6:

Now I can do some hypothesis tests to test some assumptions and implications of this SLMB two-parameter CAPM model. The test statistics for testing hypotheses are computed using the formula

$$t(\bar{\hat{\gamma}}_i) = \frac{\bar{\hat{\gamma}}_i}{s(\hat{\gamma}_i)/\sqrt{n}}, \qquad \text{n} = 47.$$

---

[11] Because the lagged values of $\bar{S}_{pt}(\hat{\varepsilon}_i)$ and $\hat{\beta}_{pt}$ appear in equation (13), it can only be estimated using monthly data from February 1999 to December 2002, not January 1999 to December 2002.

[12] Fama and Macbeth (1973) used the formula $\hat{\beta}^2{}_{p,t-1} = \frac{1}{N_p}\sum_{i=1}^{N_p}\hat{\beta}_i^2$, where N_p is the number of firms in portfolio p. The purpose of this variable is to test the linearity of the regression equation.

Using this formula I can obtain the t-statistics of the four coefficients — $t(\bar{\hat{\gamma}}_0), t(\bar{\hat{\gamma}}_1), t(\bar{\hat{\gamma}}_2), t(\bar{\hat{\gamma}}_3)$.

First I need to test some hypotheses relevant to the economic meaning of the model itself. The SLMB two-parameter CAPM model has three important implications:

Condition 1: $E(\tilde{\gamma}_{2t})=0$, a test of the linearity of the regression equation (13). It means that the relationship between the expected return on a security and its risk in any efficient portfolio M is linear.

Condition 2: $E(\tilde{\gamma}_{3t})=0$, a test of the hypothesis that there are no systematic effects of non-β risk. In other words, all unsystematic risks can be diversified through portfolios, and the remaining risk can be explained by the β coefficients of individual stocks.

Conditions 3: $E(\tilde{\gamma}_{1t}) > 0$, a test of the market efficiency hypothesis. The returns on a portfolio must be greater than the risk-free asset's returns; there is a payoff to risk-taking.

Note that $E(\tilde{\gamma}_{2t})$ is the expected population mean of $\tilde{\gamma}_{2t}$. I cannot observe the population mean, but I can estimate it. The sample mean of $\hat{\gamma}_{2t}$, i.e. $\bar{\hat{\gamma}}_{2t}$ is a good estimator of $E(\tilde{\gamma}_{2t})$. $\bar{\hat{\gamma}}_{2t}$ is easily computed, as explained in Step 5. I estimate $E(\tilde{\gamma}_{3t})$ and $E(\tilde{\gamma}_{1t})$ in the same manner. Thus I can use the t-statistics $t(\bar{\hat{\gamma}}_0)$, $t(\bar{\hat{\gamma}}_1)$, $t(\bar{\hat{\gamma}}_2)$, and $t(\bar{\hat{\gamma}}_3)$ to test these three hypotheses.

VI. Explanation of results

In this section of the paper I will explain the results of all the tests. I will discuss them in detail from step to step, and try to explain the results of each step from Step 1 to Step 6.

In Step 1, I obtain estimates of beta for all companies in our dataset. They are the basis for Step 2. In Step 2, the portfolios are constructed as shown in Table 1. The 234 companies are divided into twenty portfolios based on the data from January 1987 to December 1993. The firms in p1 have the lowest beta value in the period from 1987 to 1993, while the firms in p20 include the highest beta values among the 234 companies. The lowest beta value is -1.950947 in p1; the highest beta value is 3.325133 in p20. These values are similar to those of Fama and Macbeth (1973) in magnitude.

Table 2 provides some descriptive statistics for each portfolio, including the mean of the beta values for each portfolio, computed as the simple average of all the beta values in each portfolio. In addition, I list the minimum and the maximum beta value for each portfolio. From Table 1, I can see that there is only one portfolio (p1) that has some negative beta values. The mean beta value for p1 is also negative. Among the 234 firms only nine firms have negative beta values; the rest of the firms all have positive beta values.

Table 3 presents the β_i and $S(\varepsilon_i)$ values computed using data for the time period January 1994 to December 1998 in Step 3. These data are the basis of our future calculations. If I compare the results in Table 1 with the results in Table 3, I find some very interesting things. In Table 1 I only have one portfolio including securities with negative beta values, and they are the lowest beta values; but in Table 3 nine portfolios include securities with negative beta values. In Table 3, altogether I have ten firms with negative betas. Table 3 shows that the estimated beta for each security has changed, as a result of the change in sample period: some have changed from positive to negative (e.g., r71), some have changed from negative to positive (e.g., r203), and some have changed in magnitude but not sign, (e.g., r7, r168). This is good evidence of the volatility of the stock market – the performance of portfolios varies hugely.

In Step 4, I compute the monthly portfolio beta coefficients. The results for all four periods are listed in Table 4 for portfolios 1 to 8 (listing the results for all the portfolios would take too much space, so I chose only some as representatives). These are the data that are used to estimate equation (13). Although I have a few negative betas belonging to different portfolios, the portfolio betas are non-negative. Except three specific months (January 2000, January 2001, January 2002) the beta values of these eight portfolios are quite stable and do not vary much within each calendar year. Before January 2000 (from February 1999 to December 1999) only one portfolio displays some change in its portfolio beta: portfolio 1 changed its β_{pt} from 0.872 to 0.952 in June 1999. The other seven portfolios do not change their beta values until January 2000. I can see from Table 4 that in January 2000, the beta values of all eight portfolios change: from 0.952 to 0.91 for p1; from 0.879 to 0.63 for p2; from 0.72 to 0.75 for p3; from 0.70 to 0.60 for p4; from 0.63 to 0.61 for p5; from 0.729 to 0.65 for p6; from 0.68 to 0.60 for p7; from 0.84 to 0.80 for p8. The same situation happens in January 2001 and January 2002. In January 2000, beta increases only for portfolio 3. In January 2001 only one portfolio (p4) increases its portfolio beta while all others decrease; in January 2002 all portfolios have decreased portfolio betas! Among these changes there are three especially big variations: p3 changes from 1.13 to 0.63 in January 2001; p4, from 0.84 to 0.53 in January 2002; and p6, from 0.78 to 0.45 in January 2002. All of these big changes occur in January because that is when the β_i values change (they have been re-estimated). Almost all the firms must remain listed throughout the period February 1999 – December 2002; otherwise there would be more month-to-month variation within each calendar year.

In Step 5, I estimate equation (13). The results are presented in two tables. One is Table 5, which presents summary statistics for equation (13) for the four-year testing period as a whole. The other is Table 6, which shows the estimated results for each month in this four-year period. In Table 6 I can see that $\hat{\gamma}_0$, $\hat{\gamma}_3, \hat{\gamma}_1$ and $\hat{\gamma}_2$ and $S(\hat{\gamma}_0), S(\hat{\gamma}_3)$ are similar in value to Fama and Macbeth's (1973) results. In Table 5, the t-statistics of the four coefficients $\hat{\gamma}_0$, $\hat{\gamma}_3, \hat{\gamma}_1$ and $\hat{\gamma}_2$ have no significance either. Whether I set the significance level to 1%, 5% or 10%, I cannot reject the null hypothesis that the means of the coefficients are zero.

However in Table 6 I can see that in some particular time periods $\hat{\gamma}_1$, $\hat{\gamma}_2$ and $\hat{\gamma}_3$ are statistically significant. I can set the significance level equals to 10% or 5% or 1%; each yields a different result. For example, if I set the significance level to 10%, in Table 6 I can see that in June and July of 1999, November of 2001, May and June of 2002, and August of 2002, $\hat{\gamma}_1$ is significant. In the case of $\hat{\gamma}_2$, it has significance in June and July of 1999, January of 2000, November of 2001, June of 2002, and August of 2002. In January 2000, April 2000, June 2000, February 2001, and September 2001 $\hat{\gamma}_3$ is statistically significant.

Thus I can see that considering the 47 months as a whole, the estimation results for each coefficient are not significant. But when I separate the sample into 47 sub-samples, the estimation results for a particular month might have individual significance.

The last step is to carry out the hypothesis tests. I list all the relevant properties of the test statistics in Table 5. The statistic for testing condition 1 is $t(\bar{\hat{\gamma}}_2)$= -1.176944. The degrees of freedom for this test are equal to 46; since the 5% critical value is 2.014, I cannot reject the null hypothesis that $E(\tilde{\gamma}_{2t})$= 0. This means that the relationship between expected returns and the risk beta coefficient β_{pt} in the time period from February 1999 to December 2002 (the test period) is not nonlinear.[13]

For the hypothesis test for condition 2, the value of the test statistic is $t(\bar{\hat{\gamma}}_3)$= 0.3707813, and the critical value is the same as for the previous test. Thus once again I cannot reject the null hypothesis that there exists no systematic effects of non-β risk, at the 5% level of significance. This result implies that the data are consistent with this prediction of market portfolio theory. β can measure all the systematic risks in the Canadian stock market and any non-systematic risks can be diversified through portfolios. Since β represents all the systematic risks, there is nothing else that is also relevant to the systematic risks.

Looking more closely at the results in Table 5, as the previous tests showed, on average $\hat{\gamma}_{2t}$ and $\hat{\gamma}_{3t}$ have no significance in this regression equation. According to the hypothesis tests they have no power to explain expected returns. Furthermore, there exists only a weak statistical linear relationship between expected returns and the risk beta coefficients. The t statistic for $\bar{\hat{\gamma}}_1$ is also less than the critical value, implying that on

[13] Here I cannot say "the relationship is linear" because for it to be linear, $\hat{\gamma}_1$ must also be nonzero. But our results suggest that it is not.

average β_{pt} has no explanatory power either (the values are in Table 5). This result is similar to that of Fama and French (1992). They draw the conclusion that the linear positive relationship between the risk coefficient and expected returns is very weak.

For the period of 1935-1940 Fama and Macbeth (1973) obtained $\bar{\hat{\gamma}}_1 = 0.0109$, while I obtain $\bar{\hat{\gamma}}_1 = 0.02121$. Both the sign and the magnitude are the same; the result implies a positive relationship between portfolio beta and risk returns.

I can also compare the standard error of $\overline{S}_{p,t-1}(\hat{\varepsilon}_i)$. For this variable again the estimates are similar to those of Fama and Macbeth (1973). The values of $\overline{S}_{p,t-1}(\hat{\varepsilon}_i)$ for these two different stock markets are similar. The average R^2 of equation (13) is 0.20803 and its standard error is 0.15942. Fama and Macbeth's R^2 in one certain period (1935-1945) is 0.29 while the standard error is 0.30. There is no important difference between these numbers.

VII. Conclusion

In a few words, our test of the SLMB two-parameter CAPM model using Canadian stock market data has not been too successful. Although I have verified three of the implied assumptions of the model (conditions 1, 2, and 3), and found consistency with Fama and Macbeth (1973) in the estimated portfolio betas, four coefficients in equation (13) ($\hat{\gamma}_0$, $\hat{\gamma}_3$, $\hat{\gamma}_1$ and $\hat{\gamma}_2$), R^2 and the standard error of the regression equation $\overline{S}_{p,t-1}(\hat{\varepsilon}_i)$, the more important prediction test comes to the opposite conclusion. β and returns have no strong relationship. The coefficient of the portfolio betas has no significance, and therefore they have no explanatory power. Our results contradict those of Fama and Macbeth (1973), but are similar to those of Banz (1981) and Fama and French (1992). I get the same conclusion: the linear relationship between β and expected returns is weak.

In recent years many scholars have contributed to the debate about CAPM theory. Perhaps some new theory can explain our results. Some economists argue that Fama and Macbeth's results are very specific to their sample period. Things have changed since the late 1960s; stock market behaviour is different. Thus if I use more recent data to test the CAPM I may not get a similar conclusion.

I have tested some of the assumptions of the two-parameter model and verified that these assumptions hold. But recently economists have proposed a lot of modifications to the model such as changing two factors into three factors, arbitrage pricing theory, many other factors that might contribute to returns, and so on. Not only have they changed the explanatory factors of the model, but also they have totally changed the model itself. Many asset pricing theories have been developed, some of which were discussed in the literature review.

Using recent Canadian stock market data I do not find that there is a strong relationship between returns and risk, as defined by the CAPM framework. Our portfolio betas are similar, but the coefficient of β_{pt} has no significance. Thus I can conclude that β_{pt} might not be the only factor explaining expected returns. Other variables might explain the expected returns of portfolios. The only thing I can say is that with the same model and the same methodology and with Canadian stock market data I cannot draw the exact same conclusions as Fama and Macbeth (1973) did.

References

Amihud and Mendelson (1986) "Asset pricing and the bid-ask spread," *Journal of Financial Economics,* 17, 223-249.

Amihud, Yakov (2000), "Illiquidity and stock returns: Cross-section and time-series effects," *Manuscript of New York University*

Banz, Rolf W. (1981), "The relationship between return and market value of common stocks," *Journal of Financial Economics,* 9, 3-18.

Barber, Brad M., and John .Lyon (1997), "Firm Size, Book-to-Market Ratio, and Security Returns: A Holdout Sample of Financial Firms," *Journal of Finance* 52 : 875-883.

Basu, Sanjoy (1977), "Investment performance of common stocks in relation to their price-earnings ratios: A test of the efficient market hypothesis," *Journal of Finance,* 32, 663-682.

Basu, Sanjoy (1983), "The relationship between earnings yield, market value, and return for NYSE common stocks: Further evidence", *Journal of Financial Economics,* 12, 51-74.

Bhandari, Laxmi Chand (1988), "Debt/Equity ratio and expected common stock returns: Empirical evidence", Journal of Finance 43, 507-528

Black, Fischer (1972), "Capital market equilibrium with restricted borrowing", *Journal of Business*, 45, 444-455.

Black, Fischer (1993), "Estimating expected return", *Financial Analysts Journal*, 49, 36-38.

Black, Fischer, Michael C. Jensen and Myron Scholes (1972), "The capital asset pricing model: Some empirical evidence", in Michael C. Jensen, ed.: *Studies in the Theory of Capital Markets* (Praeger Publishers, New York), 79-124

Blume, Marshall E, (April 1970), "Portfolio Theory: A Step toward Its Practical Application", *Journal of Business*.43: 152-173

Blume, Marshall E. and Irwin Friend (1973), "A new look at the capital asset pricing model", *Journal of Finance,* 28, 19-33.

Breeden, Douglas T. (1979), "An intertemporal asset pricing model with stochastic consumption and investment opportunities", *Journal of Financial Economics,* 7, 265-296.

Brennan, Michael J. (1970), "Taxes, market valuation and corporate financial policy", *National Tax Journal*, 417-427.

Brennan, Michael J. and Avanindhar Subrahmanyam (1996), "Market microstructure and asset pricing: On the compensation for illiquidity in stock returns", *Journal of Financial Economics,* 41, 441-464.

Brennan, Michael J., Tarun Chordia, and Avanindhar Subrahmanyam (1998), "Alternative factor specifications, security characteristics and the cross-section of expected stock returns", *Journal of Financial Economics,* 49, 345-373.

Chan, Hamao and Lakonishok (1991), "Fundamentals and Stock Return in Japan, " *Journal of Finance*, 46, 1739-1764

Chopra, Lakonishok and Ritter (1992), "Measuring abnormal performance: Do stocks overreact?" *Journal of Financial Economics* 31, 235-268

Daniel, and Sheridan Titman (1997), "Evidence on the characteristics of cross sectional variation in stock returns", *Journal of Financ*e, 52, 1-33.

Davis, James L (1994), "The Cross-Section of Realized Stock Returns: The Pre-COMPUSTAT Evidence," *Journal of Finance* 49 (1994):1579-1593.

Davis, James L. "Explaining Stock Returns: A Literature Survey", www.ifa.com/Library/Support/Articles/ Scholarly/ExplainingStockReturns.pdf

Douglas, George W. (1969), "Risk in the equity markets: An empirical appraisal of market efficiency", *Yale Economic Essay*s, 9, 3-45.

DeBondt and Thaler (1985), " Does the stock market overreact?", *Journal of Finance*, 40, 793-805.

Elmar, Mertens (2002), "The CAPM and Regression Tests", class note for the course of Portfolio Theory and Capital Markets at the University of Basel. www.unibas.ch/wwz/finance/team/ mertens/LectureNoteCAPM.pdf

Fama, Eugene F, (1970), "Efficient Capital Markets: A Review of Theory and Empirical Work", *Journal of Finance* 25: 383-417

Fama, Eugene F, (1991), "Efficient capital markets: II", *Journal of Finance,* 46, 1575-1617.

Fama, Eugene F, (1996), "Multifactor portfolio efficiency and multifactor asset pricing", *Journal of Financial and Quantitative Analysi*s, 31, 441-465.

Fama, Eugene F, and Kenneth R. French (1992), "The cross-section of expected stock returns", *Journal of Finance,* 47, 427-466.

Fama, Eugene F, and Kenneth R. French (1993), "Common risk factors in the returns on bonds and stocks", *Journal of Financial Economics,* 33, 3-56.

Fama, Eugene F, and Kenneth R. French (1995), "Size and book-to-market factors in earnings and returns", *Journal of Finance,* 50, 131-155.

Fama, Eugene F, and Kenneth R. French (1996), "Multifactor explanations of asset pricing anomalies", *Journal of Finance* 51, 55-84.

Fama, Eugene F, and Kenneth R. French (1997), "Industry costs of equity", *Journal of Financial Economics,* 43, 153-193.

Fama, Eugene F, and Kenneth R. French (1998), "Value versus growth: The international evidence", *Journal of Finance* 53, 1975-1999.

Fama, Eugene F, and James D. MacBeth (1973), "Risk, return and equilibrium: Empirical tests", *Journal of Political Economy,* 81, 607-636.

Grauer, Robert R (1999), "On the cross-sectional relation between expected returns, betas, and size", *Journal of Finance,* 54, 773-789.

Grauer, Robert R. (2001) "Asset Pricing Theory and Tests", http://www.bus.sfu.ca/homes/rob_g/IntroductionNovember302001Single.pdf.

Gibbons, Michael R, Ross and Shanken (1989), "A Test of the Efficiency of a Given Portfolio", *Econometric*a, 57, 1121-52

Jaffe, Keim and Westerfield (1989), "Earnings Yields, Market Values, and Stock Returns", Journal *of Finance*, 44, 135-148.

Jegadeesh and Titman (1993), "Returns to buying winners and selling losers: Implications for stock market efficiency", *Journal of Finance*, 48, 65-91

Jensen, Michael C. (1972), "Capital markets: Theory and evidence", *Bell Journal of Economics and Management Science,* 3, 357-398.

Jensen, Michael C (1972), *studies in the Theory of Capital Markets*, Praeger Publishers, New York, 1972.

Kan, Raymond, and Chu Zhang (1999a), "Two Pass Tests of Asset Pricing Models With Useless Factors", *Journal of Financ*e, 54, 203-35.

Kan, Raymond, and Chu Zhang (1999b), "A Critique of the Stochastic Discount Factor Methodology", *Journal of Financ*e, 54, 1221-48.

Kandel, Shmuel, and Robert F. Stambaugh (1995), "Portfolio inefficiency and the cross-section of mean returns", *Journal of Finance,* 50, 157-184.

Kothari, S.P., J. Shanken and R. Sloan (1995), "Another Look at the Cross-section of Expected Stock Returns", *Journal of Financ*e, 50, 185-224.

Lakonishok, Josef, and Andrei Schleifer and Robert W. Vishny (1994), "Contrarian Investment, Extrapolation, and Risk", *Journal of Financ*e, 49 (5), 1541-78.

Lintner, John (1965), "The valuation of risk assets and the selection of risky investments in stock portfolios and capital budgets", *Review of Economics and Statistics,* 47, 13-47.

Lo, Andrew W., and MacKinlay A. Craig (1990), "Data-snooping Biases in Tests of Financial Asset Pricing Models", *Review of Financial Studie*s, 3, 431-67

Lucas (1978), "Asset prices in an exchange economy", *Econometrica,* 76, 1429-1445.

MacKinlay, A. Craig (1987), "On multivariate tests of the CAPM", *Journal of Financial Economic*s, 18, 341-371.

MacKinlay, A. Craig (1995), "Multifactor models do not explain deviations from the CAPM", *Journal of Financial Economics,* 38, 3-28.

Markowitz, Harry (1952), "Portfolio Selection", *Journal of Finance*, 7 (1), 77-91.

Markowitz, Harry (1959), *Portfolio selection: Efficient diversification of investments* New York: Wiley.

Merton, Robert C. (1973), 'An intertemporal capital asset pricing model', *Econometrica,* 41, 867-887.

Miller, Merton and Myron Scholes (1972), "Rates of return in relation to risk: A re-examination of some recent findings", in Michael C. Jensen, ed.: *Studies in the theory of capital markets* (Praeger Publishers, New York), 47-78.

Mossin (1966), "Equilibrium in a capital asset market", Econometrica 35, 768-783.

Jagannathan and McGrattan (1995), "The CAPM Debate", Federal Reserve Bank of Minneapolis Quarterly Review, Vol. 19(4), Fall 1995, pp. 2-17.

Pastor and Stambaugh (2001), "Liquidity risk and expected stock returns", manuscript University of Pennsylvania.

Roll, Richard (1977), "A critique of the asset pricing theory's tests; part I: On past and potential testability of the theory", *Journal of Financial Economics,* 4, 129-176.

Roll, Richard and Stephen A. Ross (1994), "On the cross-sectional relation between expected returns and betas", *Journal of Finance,* 49, 101-121.

Rosenberg and Lanstein (1985), "Pervasive evidence of market inefficiency", *Journal of Portfolio Management*, 11, 9-17

Ross, Stephen A. (1976), "The arbitrage theory of capital asset pricing", *Journal of Economic Theory,* 13, 341-360.

Ross, Stephen A. (1977), "Return, risk and arbitrage", in Irwin Friend and James Bicksler, eds.: *Risk and Return in Finance,* Vol. 1, (Ballinger Publishing, Cambridge, MA).

Rubinstein (1974), "An aggregation theorem of securities markets", *Journal of financial Economics,* 1, 225-244.

Rubinstein (1976), "The valuation of uncertain income streams and the pricing of options", *Bell Journal of Economics and Management Science*, 7, 407-425.

Sharpe, William F. (1964), "Capital asset prices: A theory of market equilibrium under conditions of risk", *Journal of Finance,* 19, 425-442.

Sharpe (1985), *Investments*, Third Edition, New Jersey: Prentice-Hall.

Stattman (1980), "Book Values and Expected Stock Returns ", unpublished MBA Honors paper, University of Chicago, Chicago, Illinois

Stambaugh, Robert F. (1982), "On the exclusion of assets from tests of the two-parameter model: A sensitivity analysis", *Journal of Financial Economics*, 10, 237-286.

Table 1 Formation of Twenty Portfolios (data from January 1987 to December 1993)

name		beta1987-1993	name		beta1987-1993
p1			p2		
asbestone	r214	-1.950947	SYNEX	r28	0.133924
corby.b	r203	-0.706874	FORTIS	r137	0.19982
cott	r204	-0.706874	brick	r235	0.268373
DEL	r157	-0.563969	QUEBECT	r63	0.28718
TIMMINCC	r21	-0.487708	TRANSLT	r19	0.306025
NEW P	r86	-0.470344	LYTTON	r99	0.326628
conso.mec	r200	-0.363872	bce	r224	0.342677
VITRAN	r7	-0.103651	corby	r202	0.360757
RIO.A	r53	-0.098729	chche.a	r189	0.364435
cansouthpe	r168	0.053533	TOROMO	r20	0.367712
becker milk	r225	0.065333	IMPERIAL	r114	0.394995
THIRD.CD	r23	0.069558	algoma	r211	0.398539
p3			p4		
candianu.b	r177	0.418898	bc.gas	r223	0.557732
PANTLAS	r75	0.429839	TRANSC	r18	5.59E-01
candianu.a	r176	0.429871	HAMMONI	r124	0.56267
DOVER	r149	0.437147	SCHNEID	r43	0.575477
coniaga	r198	0.438922	DENISON	r158	0.585283
budd	r236	0.49836	MAPLEL	r95	0.595035
MDS.A	r98	0.498684	alberta en	r208	0.607365
PNGAS	r77	0.501229	HYZEL	r116	0.612175
MRC	r91	0.509703	PHOENIX	r71	0.613977
ccl.a	r184	0.514016	JEAN	r107	0.61637
MDS.B	r97	0.536349	SAMUEL	r44	0.628559
GESCO	r133	0.546981	WINPARK	r2	0.631852
p5			p6		
DEVTEK	r156	0.645137	MOLSON.I	r89	0.722157
HARRIS.A	r123	0.649294	IRWINTOY	r110	0.735548
REVENUE	r55	0.656866	cinra	r193	0.736267
VENTRA	r11	0.66259	LOBLAW	r101	0.739353
amercian I	r212	0.665181	RANGERC	r58	0.742926
GOLDF	r128	0.675307	GENDIS.A	r134	0.752916
NUMAC	r79	0.684471	LEON	r103	0.769757
ROTH	r49	0.689112	bracknell	r231	0.792295
black hawk	r227	0.691338	candatire.a	r174	0.800162
SHAW	r40	0.693497	andres wi	r213	0.804678
IMPERI.O	r113	0.700506	colony	r196	0.818109
TRIPLE	r17	0.712418	SPAR	r33	0.82291
p7			p8		
SHELL	r39	0.824613	canbread	r167	0.914019
WESTON	r3	0.837333	canadex	r169	0.915245

Table 1 (continued)

chum.a	r191	0.841851	abit	r205	0.917594
IRWIN.TV	r109	0.843091	QUEBEC.E	r62	0.922724
ELEC.X	r146	0.862068	bema	r226	0.923152
WIC	r6	0.864927	TEMBEC	r24	0.923946
NOVAGOL	r81	0.867421	cambir	r165	0.940928
candatire.b	r175	0.875016	ST.TELE	r47	0.945968
QUEBECR	r64	0.876835	SCINTLOF	r42	0.946422
MOLSON./	r90	0.905563	GULF	r127	0.947766
REITMAN	r57	0.906861	TELE.A	r27	0.951095
TELE.B	r26	0.912324	IMASO	r115	0.955525
p9			p10		
candianse	r173	0.962339	SOUTHAM	r34	1.025669
MACCHIP	r93	0.978938	LINAMAR	r102	1.02664
LOEWEN	r100	0.994531	WAJAX	r5	1.034192
GULFSTRI	r126	0.995485	MOORE	r88	1.038221
HAWERKS	r122	1.004005	DONOHUE	r150	1.050265
ECHO	r147	1.004646	POWER	r66	1.051119
DUPONT	r148	1.004646	cascad	r182	1.062775
EXALL	r143	1.006872	atco.a	r217	1.065877
astral.a	r215	1.010796	XCAL	r1	1.08E+00
HUDSON	r117	1.017919	DOMTAR	r151	1.076755
PARAMO	r74	1.020735	FPI	r142	1.080543
FOUR S	r136	1.02E+00	SICO	r38	1.081102
p11			p12		
LAFARGE	r105	1.082649	SUMM	r29	1.140974
IPSCO	r111	1.086191	breakwater	r234	1.14259
EMPIRE	r144	1.093507	atco.b	r218	1.150058
DOMAN.A	r153	1.096386	cara.a	r179	1.150481
ccl.b	r185	1.099231	SEARS	r41	1.15735
cara.b	r180	1.100026	METALOR	r92	1.1598
DOMAN.B	r152	1.116294	DOFASCO	r154	1.162175
WESTF	r4	1.117075	THOMSON	r22	1.163697
cabre	r161	1.118475	LAIDLAW	r104	1.16633
canapacifi	r172	1.124308	RAM	r59	1.168912
FINNING	r139	1.13743	PETROM	r72	1.17304
GLN	r131	1.139969	HALEY	r125	1.173553
p13			p14		
RIO	r54	1.178238	agra	r207	1.247908
bombar.a	r228	1.180635	PLACE	r69	1.256031
chum.b	r192	1.187466	NORTHGA	r83	1.261735
UNICAN.A	r13	1.189912	ULSTER	r14	1.272582
ELEC.Y	r145	1.190997	cae	r162	1.274569
FAR WEST	r140	1.192814	VIDE	r8	1.295344
chateau	r188	1.192984	VICEROYH	r10	1.30417
FIRST MAI	r138	1.195986	bombar.b	r229	1.305973

Table 1 (continued)

cdeara	r186	1.200143	cambi	r164	1.315666
alcan	r209	1.201938	IVACO.A	r108	1.322502
HOLLING	r118	1.208878	TYLER	r15	1.325616
caled	r163	1.234165	PLACEDO	r68	1.326274
p15			p16		
GOLDEN F	r130	1.329458	daim	r160	1.437423
algo	r210	1.348512	TUDDR	r16	1.450524
bovar	r230	1.348997	ST.A	r46	1.455478
HIGHRIDG	r120	1.359012	circuit	r194	1.460339
SONORA	r35	1.362435	TECSYN	r25	1.466376
ST.L	r45	1.37128	SUDBURY	r30	1.466458
battle	r221	1.379486	QUENNST	r60	1.470276
brascan	r233	1.39416	canada air	r170	1.480978
GEAC	r135	1.397277	OXFORD	r78	1.486333
PANASLI	r76	1.402861	campbel	r166	1.49876
consumer.|	r201	1.414016	HIGHRIVE	r121	1.505132
DEXLEIGH	r155	1.426719	cheni	r190	1.52424
p17			p18		
channelres	r187	1.54678	consolida.e	r199	1.669295
carma	r181	1.548011	GOLDENS	r129	1.671993
HIGHWO	r119	1.54824	ROMANC	r50	1.686616
STELCO	r31	1.576242	MARK	r94	1.692399
cogn	r195	1.581195	MAGNA.A	r96	1.699324
astral.b	r216	1.593576	canfor	r178	1.711834
INTER.F	r112	1.597456	NEXUS	r85	1.721215
agnico	r206	1.601159	NUINSO	r80	1.727617
comin	r197	1.625589	GLAMIS	r132	1.738014
QUEENST	r61	1.627748	NORAND	r84	1.738951
UKH	r12	1.631288	ROGERS	r52	1.746346
DJE	r159	1.642321	VICE.R	r9	1.778082
p19			p20		
SILORP	r37	1.781685	PEBEN	r73	2.071927
PLATINOV	r67	1.788105	aur	r219	2.116423
STROUD	r32	1.788374	PUREGO	r65	2.135321
JONPOL	r106	1.78944	FAHNEST(	r141	2.160592
SLOCAN	r36	1.797314	NSR	r87	2.202384
canaturalre	r171	1.809472	NORTHW/	r82	2.422001
brampton	r232	1.842166	PIONEER	r70	2.797302
ROGERS.I	r51	1.866962	aurizon	r220	2.89111
REPAP	r56	1.891092	cassiar	r183	3.325133

The first column contains the abbreviated names of firms listed on the Toronto Stock Exchange. The second column, r**, is a symbol for the abbreviated firms names. Because the firms' names are too long, each firm has name, and each firm's name has a symbol.

Table 2 The Range of the Betas Estimated in Step 2

January 1987 to December 1993

Portfolio	No. of firms	Mean beta	Min beta	Max beta
1	12	-0.438712	-1.950947	0.069558
2	12	0.31259	0.133924	0.398539
3	12	4.69E-01	0.418898	0.418898
4	12	0.59545	0.557732	0.631852
5	12	0.67714	0.645137	0.712418
6	12	0.76976	0.722157	0.82291
7	12	0.86816	0.824613	0.912324
8	12	0.9337	0.914019	0.955525
9	12	1.00208	0.962339	1.02E+00
10	12	1.05569	1.025669	1.081102
11	12	1.1093	1.082649	1.139969
12	12	1.15908	1.140974	1.173553
13	12	1.19618	1.178238	1.234165
14	12	1.29236	1.247908	1.326274
15	12	1.37785	1.329458	1.426719
16	12	1.003063	1.437423	1.52424
17	12	1.5933	1.54678	1.642321
18	12	1.71514	1.669295	1.778082
19	9	1.81718	1.781685	1.891092
20	9	1.362606	2.071927	3.325133

Table 3 Estimated variables for twenty portfoios (data from January 1994 to December 1998)

name		beta94-98	se 94-98	name		beta94-98	se 94-98
p1				p2			
asbestone	r214	-1.81E+00	0.375327	SYNEX	r28	5.12E-01	0.110796
corby.b	r203	3.68E-01	0.044009	FORTIS	r137	1.74E+00	0.119234
cott	r204	7.63E-01	0.129507	brick	r235	4.23E-01	0.121186
DEL	r157	7.27E-01	0.233606	QUEBECT	r63	8.79E-01	0.045689
TIMMINCC	r21	8.52E-01	0.098487	TRANSLT	r19	5.36E-01	0.038212
NEW P	r86	3.42E-01	0.120204	LYTTON	r99	1.46E+00	0.246823
conso.mec	r200	6.25E+00	1.537413	bce	r224	9.89E-01	0.050392
VITRAN	r7	-3.05E-01	0.099084	corby	r202	2.05E-01	0.055761
RIO.A	r53	9.83E-01	0.104987	chche.a	r189	1.37E+00	0.126682
cansouthpe	r168	1.02E+00	0.135688	TOROMO	r20	8.45E-01	0.081123
becker mill	r225	4.78E-01	0.110742	IMPERIAL	r114	9.91E-01	7.57E-02
THIRD.CD	r23	8.06E-01	0.09949	algoma	r211	5.95E-01	0.087587
p3				p4			
candianu.b	r177	6.16E-01	0.033559	bc.gas	r223	4.79E-01	0.033672
PANTLAS	r75	1.12E+00	0.125441	TRANSC	r18	5.44E-01	0.049038
candianu.a	r176	5.88E-01	0.035109	HAMMONI	r124	5.17E-01	0.109119
DOVER	r149	4.49E-01	0.162095	SCHNEID	r43	1.64E-01	0.130355
coniaga	r198	1.36E+00	0.196073	DENISON	r158	9.87E-01	0.185103
budd	r236	8.35E-01	0.079581	MAPLEL	r95	1.19E+00	0.074688
MDS.A	r98	6.01E-01	0.067822	alberta en	r208	6.51E-01	0.059102
PNGAS	r77	5.74E-01	0.04625	HYZEL	r116	1.49E+00	0.234928
MRC	r91	4.01E-01	0.060566	PHOENIX	r71	-1.02E-01	0.084505
ccl.a	r184	5.58E-01	0.044583	JEAN	r107	7.97E-01	0.079262
MDS.B	r97	6.56E-01	0.061926	SAMUEL	r44	1.28E+00	0.088458
GESCO	r133	9.02E-01	0.028161	WINPARK	r2	4.58E-01	0.053766
p5				p6			
DEVTEK	r156	4.92E-01	0.096933	MOLSON.I	r89	7.32E-01	0.048501
HARRIS.A	r123	7.27E-01	0.129924	IRWINTOY	r110	0.341239	0.103129
REVENUE	r55	5.97E-01	0.069029	cinra	r193	8.16E-01	0.098952
VENTRA	r11	1.10E+00	0.096079	LOBLAW	r101	5.49E-01	0.06085
amercian l	r212	-1.28E-01	0.257961	RANGERC	r58	4.44E-01	0.083631
GOLDF	r128	4.01E-01	0.097463	GENDIS.A	r134	6.95E-01	0.127026
NUMAC	r79	3.74E-01	0.076412	LEON	r103	3.44E-01	0.059062
ROTH	r49	1.08E-01	0.053016	bracknell	r231	7.70E-01	0.093806
black hawk	r227	1.31E+00	0.129849	candatire.a	r174	8.68E-01	0.068565
SHAW	r40	7.40E-01	0.078131	andres wi	r213	3.13E-01	0.057991
IMPERI.O	r113	1.25E+00	0.089158	colony	r196	2.59E+00	0.43656
TRIPLE	r17	5.99E-01	0.09008	SPAR	r33	2.93E-01	0.140226
p7				p8			
SHELL	r39	4.79E-01	0.046927	canbread	r167	7.30E-01	0.064693
WESTON	r3	7.43E-01	0.058362	canadex	r169	-4.92E-01	0.20498

Table 2 (continued)

chum.a	r191	4.12E-01	0.096056	abit	r205	1.31E+00	0.06686
IRWIN.TV	r109	1.01E+00	0.106006	QUEBEC.E	r62	5.64E-01	0.057927
ELEC.X	r146	9.62E-01	0.051428	bema	r226	1.33E+00	0.210463
WIC	r6	6.21E-01	0.066558	TEMBEC	r24	8.94E-01	0.080864
NOVAGOL	r81	9.05E-01	0.345678	cambir	r165	8.59E-01	0.071407
candatire.b	r175	3.91E-01	0.064435	ST.TELE	r47	8.93E-01	0.114775
QUEBECR	r64	4.99E-01	0.056046	SCINTLOF	r42	1.27E+00	0.248367
MOLSON./	r90	5.90E-01	0.048693	GULF	r127	7.58E-01	0.108073
REITMAN	r57	2.88E-01	0.05665	TELE.A	r27	1.44E+00	0.091504
TELE.B	r26	1.31E+00	0.091433	IMASO	r115	5.54E-01	0.048081
p9				**p10**			
candianse	r173	3.21E-01	0.111973	SOUTHAM	r34	5.46E-01	0.067938
MACCHIP	r93	5.02E-01	0.13986	LINAMAR	r102	1.30E+00	0.075487
LOEWEN	r100	6.61E-01	0.125288	WAJAX	r5	7.50E-01	0.073828
GULFSTRI	r126	1.31E+00	0.19365	MOORE	r88	8.44E-01	0.05233
HAWERKS	r122	1.44E-01	0.133128	DONOHUE	r150	2.03E+00	0.074689
ECHO	r147	1.24E+00	0.157337	POWER	r66	9.98E-01	0.0475
DUPONT	r148	1.00E+00	0.11529	cascad	r182	9.63E-01	0.06414
EXALL	r143	5.80E-01	0.067073	atco.a	r217	6.23E-01	0.042262
astral.a	r215	2.35E-01	0.100729	XCAL	r1	2.05E+00	0.234609
HUDSON	r117	1.40E+01	3.859724	DOMTAR	r151	1.36E+00	0.050927
PARAMO	r74	1.12E+00	0.100824	FPI	r142	9.47E-01	0.110231
FOUR S	r136	8.82E-01	0.085775	SICO	r38	-1.15E+00	0.068552
p11				**p12**			
LAFARGE	r105	1.34E+00	0.092421	SUMM	r29	7.76E-01	0.132486
IPSCO	r111	1.38E+00	0.195591	breakwater	r234	1.81E+00	0.184997
EMPIRE	r144	3.57E-01	0.187156	atco.b	r218	-2.26E-01	0.038932
DOMAN.A	r153	9.89E-01	0.139957	cara.a	r179	5.35E-01	0.0717
ccl.b	r185	5.08E-01	0.051216	SEARS	r41	1.18E+00	0.086349
cara.b	r180	5.02E-01	0.06952	METALOR	r92	4.48E-01	0.157447
DOMAN.B	r152	7.49E-01	0.112105	DOFASCO	r154	8.43E-01	0.073626
WESTF	r4	8.63E-01	0.068978	THOMSON	r22	8.58E-01	0.041202
cabre	r161	4.65E-01	0.082278	LAIDLAW	r104	6.65E-01	0.056664
canapacifi	r172	9.66E-01	0.051219	RAM	r59	4.21E-01	0.237225
FINNING	r139	6.25E-01	0.05648	PETROM	r72	8.73E-01	0.112281
GLN	r131	1.24E+00	0.103181	HALEY	r125	7.82E-01	0.080019
p13				**p14**			
RIO	r54	6.74E-01	0.060497	agra	r207	8.18E-01	0.084479
bombar.a	r228	7.94E-01	0.058813	PLACE	r69	3.16E-01	0.096112
chum.b	r192	-4.32E+00	0.079368	NORTHGA	r83	3.29E-01	0.094268
UNICAN.A	r13	2.89E-01	0.081108	ULSTER	r14	7.09E-01	0.079801
ELEC.Y	r145	9.83E-02	0.016411	cae	r162	7.12E-01	0.066593
FAR WES	r140	5.83E-01	0.200475	VIDE	r8	6.71E-01	0.077678
chateau	r188	5.93E-01	0.094475	VICEROYH	r10	9.72E-01	0.172107
FIRST MAI	r138	1.20E+00	0.071301	bombar.b	r229	8.64E-01	0.058421

Table 2 (continued)

cdeara	r186	2.09E-01	0.140755	cambi	r164	1.33E+00	0.117662
alcan	r209	1.14E+00	0.055637	IVACO.A	r108	1.44E+00	0.147705
HOLLING	r118	8.03E-01	0.077736	TYLER	r15	1.58E+00	0.278982
caled	r163	3.74E-01	0.311241	PLACEDO	r68	1.41E+00	0.123738
p15				p16			
GOLDEN F	r130	1.32E+00	0.273265	daim	r160	9.04E-01	0.082014
algo	r210	6.20E-01	0.310224	TUDDR	r16	6.17E-01	0.351104
bovar	r230	9.19E-01	0.128315	ST.A	r46	9.15E-01	0.151603
HIGHRIDG	r120	8.78E-01	0.084542	circuit	r194	3.30E-02	0.263306
SONORA	r35	1.36E+00	0.253016	TECSYN	r25	1.20E+00	0.147881
ST.L	r45	8.09E-01	0.086394	SUDBURY	r30	1.07E-01	0.233792
battle	r221	1.25E+00	0.153738	QUENNST	r60	1.34E+00	0.182314
brascan	r233	9.03E-01	0.048209	canada air	r170	1.72E+00	0.162449
GEAC	r135	4.84E-01	0.030205	OXFORD	r78	8.64E-01	0.166092
PANASLI	r76	1.03E+00	0.294372	campbel	r166	0.482743	0.149827
consumer.|	r201	7.67E-01	0.17447	HIGHRIVE	r121	1.04E+00	0.224701
DEXLEIGH	r155	8.38E-01	0.088275	cheni	r190	7.39E-01	0.173532
p17				p18			
channelres	r187	2.19E+00	0.279912	consolida.e	r199	8.17E-01	0.29376
carma	r181	7.49E-01	0.094669	GOLDENS	r129	2.18E+00	0.234652
HIGHWO	r119	6.04E-01	0.101682	ROMANC	r50	8.58E-01	0.112352
STELCO	r31	7.98E-01	0.174574	MARK	r94	4.64E-01	0.105422
cogn	r195	1.32E+00	0.142069	MAGNA.A	r96	7.96E-01	0.065114
astral.b	r216	-4.34E-01	0.128618	canfor	r178	9.96E-01	0.074124
INTER.F	r112	3.99E-01	0.051122	NEXUS	r85	2.59E-02	0.262487
agnico	r206	1.56E+00	0.140242	NUINSO	r80	1.20E+00	0.312396
comin	r197	1.09E+00	0.067504	GLAMIS	r132	1.27E+00	0.17919
QUEENST	r61	1.29E+00	0.324351	NORAND	r84	1.08E+00	0.05242
UKH	r12	1.99E+00	0.205616	ROGERS	r52	1.07E+00	0.108678
DJE	r159	1.22E+00	0.153909	VICE.R	r9	6.26E-01	0.141147
p19				p20			
SILORP	r37	6.97E-01	0.082736	PEBEN	r73	1.15E+00	0.128508
PLATINOV	r67	3.80E-01	0.195021	aur	r219	1.23E+00	0.114174
STROUD	r32	1.23E+00	0.075155	PUREGO	r65	1.86E+00	0.263368
JONPOL	r106	1.09E+00	0.224574	FAHNEST(	r141	6.79E-01	0.069546
SLOCAN	r36	9.93E-01	0.116815	NSR	r87	-2.87E-01	0.390053
canaturalre	r171	1.14E+00	0.08843	NORTHW/	r82	1.43E+00	0.362127
brampton	r232	1.07E+00	0.187616	PIONEER	r70	5.22E-01	0.215965
ROGERS.I	r51	1.43E+00	0.119379	aurizon	r220	1.08E+00	0.143449
REPAP	r56	1.29E+00	0.205191	cassiar	r183	1.16E-01	0.281216

Table 4 Data for Variables Used in Equation (13)

For Some Selected Portfolios: P1-P8

	1	2	3	4	5	6	7	8
	Portfolios for Testing Period Feb.1999							
Variables								
Returns of p	0.01682	-0.048672	-0.026351	0.005389	-0.071501	0.005714	-0.053749	-0.032864
beta p	8.72E-01	8.79E-01	7.22E-01	7.05E-01	6.30E-01	7.29E-01	6.84E-01	8.42E-01
se bar	0.257379	0.096596	0.078431	0.0985	0.105336	0.114858	0.090689	0.114
	Portfolios for Testing Period March1999							
Returns of p	0.075718	0.011724	0.002973	0.053322	0.278045	0.026809	0.190094	0.055835
beta p	8.72E-01	8.79E-01	7.22E-01	7.05E-01	6.30E-01	7.29E-01	6.84E-01	8.42E-01
se bar	0.257379	0.096596	0.078431	0.0985	0.105336	0.114858	0.090689	0.114
	Portfolios for Testing Period April 1999							
Returns of p	0.076355	0.01246	0.047405	0.039481	0.081513	0.088477	0.106863	0.051431
beta p	8.72E-01	8.79E-01	7.22E-01	7.05E-01	6.30E-01	7.29E-01	6.84E-01	8.42E-01
se bar	0.257379	0.096596	0.078431	0.0985	0.105336	0.114858	0.090689	0.114
	Portfolios for Testing Period May 1999							
Returns of p	-0.023693	0.052754	0.012537	-0.010907	-0.028251	0.002724	-0.031412	-0.019785
beta p	8.72E-01	8.79E-01	7.22E-01	7.05E-01	6.30E-01	7.29E-01	6.84E-01	8.42E-01
se bar	0.257379	0.096596	0.078431	0.0985	0.105336	0.114858	0.090689	0.114
	Portfolios for Testing Period June 1999							
Returns of p	0.000515	-0.028075	0.018163	0.011275	0.030846	0.028493	0.036405	0.028983
beta p	9.52E-01	8.79E-01	7.22E-01	7.05E-01	6.30E-01	7.29E-01	6.84E-01	8.42E-01
se bar	0.280777	0.096596	0.078431	0.0985	0.105336	0.114858	0.090689	0.114
	Portfolios for Testing Period July 1999							
Returns of p	0.077064	0.015566	-0.001959	-0.008616	-0.027495	-0.026744	0.008305	-0.005758
beta p	9.52E-01	8.79E-01	7.22E-01	7.05E-01	6.30E-01	7.29E-01	6.84E-01	8.42E-01
se bar	0.280777	0.096596	0.078431	0.0985	0.105336	0.114858	0.090689	0.114
	Portfolios for Testing Period August 1999							
Returns of p	-0.053339	-0.041763	0.000193	-0.012551	0.003271	0.030756	0.033375	0.014369
beta p	9.52E-01	8.79E-01	7.22E-01	7.05E-01	6.30E-01	7.29E-01	6.84E-01	8.42E-01
se bar	0.280777	0.096596	0.078431	0.0985	0.105336	0.114858	0.090689	0.114
	Portfolios for Testing Period Sept. 1999							
Returns of p	-0.03516	-0.021177	-0.012159	0.053413	-0.016951	0.034581	0.034333	-0.004929
beta p	9.52E-01	8.79E-01	7.22E-01	7.05E-01	6.30E-01	7.29E-01	6.84E-01	8.42E-01
se bar	0.280777	0.096596	0.078431	0.0985	0.105336	0.114858	0.090689	0.114
	Portfolios for Testing Period Oct. 1999							
Returns of p	0.00882	-0.015524	0.039773	-0.015281	-0.018729	-0.018235	-0.056968	-0.032144
beta p	9.52E-01	8.79E-01	7.22E-01	7.05E-01	6.30E-01	7.29E-01	6.84E-01	8.42E-01
se bar	0.280777	0.096596	0.078431	0.0985	0.105336	0.114858	0.090689	0.114

Table 4 (continued)

Portfolios for Testing Period Nov. 1999								
Returns of p	0.020587	0.05458	-0.003633	-0.02358	-0.045178	-0.051726	-0.025166	-0.019484
beta p	9.52E-01	8.79E-01	7.22E-01	7.05E-01	6.30E-01	7.29E-01	6.84E-01	8.42E-01
se bar	0.280777	0.096596	0.078431	0.0985	0.105336	0.114858	0.090689	0.114

Portfolios for Testing Period Dec. 1999								
Returns of p	0.008323	0.047043	0.008287	0.004232	0.031119	0.027212	0.034609	0.022522
beta p	9.52E-01	8.79E-01	7.22E-01	7.05E-01	6.30E-01	7.29E-01	6.84E-01	8.42E-01
se bar	0.280777	0.096596	0.078431	0.0985	0.105336	0.114858	0.090689	0.114

Portfolios for Testing Period Jan.2000								
Returns of p	0.093233	0.048484	-0.014904	0.01414	0.008705	-0.012271	0.01063	0.038744
beta p	9.10E-01	6.36E-01	7.52E-01	6.02E-01	6.12E-01	6.50E-01	6.09E-01	8.09E-01
se bar	0.26258	0.113459	0.082093	0.098314	0.114386	0.124063	0.093042	0.12485

Portfolios for Testing Period Feb.2000								
Returns of p	0.081534	0.206506	0.072535	0.05699	-0.038283	0.146535	-0.047947	0.126311
beta p	1.00E+00	6.36E-01	7.52E-01	6.02E-01	6.12E-01	6.50E-01	6.09E-01	8.82E-01
se bar	0.288838	0.113459	0.082093	0.098314	0.114386	0.124063	0.093042	0.1362

Portfolios for Testing Period March 2000								
Returns of p	-0.089449	-0.077064	-0.047748	-0.004363	0.078623	0.005484	-0.02217	0.051566
beta p	1.00E+00	6.36E-01	7.52E-01	6.02E-01	6.12E-01	6.50E-01	6.09E-01	9.70E-01
se bar	0.288838	0.113459	0.082093	0.098314	0.114386	0.124063	0.093042	0.14982

Portfolios for Testing Period April 2000								
Returns of p	-0.010625	-0.020264	0.014917	-0.025867	0.036135	0.018224	0.061878	0.027055
beta p	1.00E+00	6.36E-01	8.20E-01	6.02E-01	6.12E-01	6.50E-01	6.09E-01	9.70E-01
se bar	0.288838	0.113459	0.089556	0.098314	0.114386	0.124063	0.093042	0.14982

Portfolios for Testing Period May 2000								
Returns of p	0.031583	0.05067	-0.05792	-0.004361	0.09479	0.096073	-0.038682	-0.006456
beta p	1.00E+00	6.36E-01	8.20E-01	6.02E-01	6.12E-01	6.50E-01	6.64E-01	9.70E-01
se bar	0.288838	0.113459	0.089556	0.098314	0.114386	0.124063	0.101501	0.14982

Portfolios for Testing Period June 2000								
Returns of p	0.045485	0.063047	0.017181	-0.043803	0.022921	0.004503	0.009124	-0.022645
beta p	1.00E+00	6.36E-01	9.03E-01	6.02E-01	6.67E-01	6.50E-01	6.64E-01	9.70E-01
se bar	0.288838	0.113459	0.098511	0.098314	0.124784	0.124063	0.101501	0.14982

Portfolios for Testing Period July 2000								
Returns of p	-0.023001	-0.039453	-0.01332	0.006945	-0.017945	-0.003676	-0.022091	-0.019334
beta p	1.00E+00	6.94E-01	9.03E-01	6.02E-01	6.67E-01	6.50E-01	6.64E-01	9.70E-01
se bar	0.288838	0.123774	0.098511	0.098314	0.124784	0.124063	0.101501	0.14982

Portfolios for Testing Period August 2000								
Returns of p	0.045239	0.003594	0.048328	0.432548	0.053268	0.031128	0.024167	0.18424
beta p	1.00E+00	6.94E-01	1.00E+00	6.02E-01	6.67E-01	6.50E-01	6.64E-01	9.70E-01
se bar	0.288838	0.123774	0.109457	0.098314	0.124784	0.124063	0.101501	0.14982

Table 4 (continued)

Portfolios for Testing Period Sept 2000

Returns of p	-0.043539	-0.026687	0.05169	-0.029559	0.005333	-0.026636	0.00475	0.030025
beta p	1.00E+00	6.94E-01	1.13E+00	6.02E-01	6.67E-01	7.09E-01	6.64E-01	1.08E+00
se bar	0.288838	0.123774	0.123139	0.098314	0.124784	0.135342	0.101501	0.166467

Portfolios for Testing Period Oct. 2000

Returns of p	-0.013879	-0.074851	0.017367	-0.020585	0.020116	0.008151	-0.076512	-0.031181
beta p	1.00E+00	6.94E-01	1.13E+00	6.02E-01	7.34E-01	7.09E-01	6.64E-01	1.08E+00
se bar	0.288838	0.123774	0.123139	0.098314	1.37E-01	0.135342	0.101501	0.166467

Portfolios for Testing Period Nov. 2000

Returns of p	-0.056214	-0.047509	-0.024182	-0.08447	0.015967	-0.030203	-0.057969	-0.012299
beta p	1.00E+00	6.94E-01	1.13E+00	6.02E-01	7.34E-01	7.09E-01	6.64E-01	1.08E+00
se bar	0.288838	0.123774	0.123139	0.098314	1.37E-01	0.135342	0.101501	0.166467

Portfolios for Testing Period Dec. 2000

Returns of p	-0.020957	0.021479	0.041201	-0.004436	0.092583	0.019209	0.105554	0.077878
beta p	1.00E+00	6.94E-01	1.13E+00	6.02E-01	7.34E-01	7.09E-01	6.64E-01	1.08E+00
se bar	0.288838	0.123774	0.123139	0.098314	1.37E-01	0.135342	0.101501	0.166467

Portfolios for Testing Period Jan. 2001

Returns of p	0.12334	0.062052	-0.005272	0.35351	-0.002087	0.016836	0.043216	-0.057156
beta p	8.91E-01	5.81E-01	6.34E-01	6.97E-01	6.01E-01	6.45E-01	5.38E-01	8.12E-01
se bar	0.285087	0.12869	0.082002	0.117145	1.21E-01	1.30E-01	1.08E-01	1.24E-01

Portfolios for Testing Period Feb. 2001

Returns of p	-0.073754	-0.060386	0.019456	-0.005668	0.037054	0.041328	0.028493	-0.003695
beta p	8.91E-01	5.81E-01	6.34E-01	6.97E-01	6.01E-01	6.45E-01	5.38E-01	8.12E-01
se bar	0.285087	0.12869	0.082002	0.117145	1.21E-01	1.30E-01	1.08E-01	1.24E-01

Portfolios for Testing Period March. 2001

Returns of p	-0.057448	0.062715	0.032603	-0.018721	5.32E-05	0.065717	0.020051	0.016956
beta p	8.91E-01	5.81E-01	6.34E-01	7.60E-01	6.68E-01	6.45E-01	5.38E-01	8.12E-01
se bar	0.285087	0.12869	0.082002	0.127794	1.34E-01	1.30E-01	1.08E-01	1.24E-01

Portfolios for Testing Period April. 2001

Returns of p	0.056109	-0.033332	0.066378	0.003259	0.092142	0.051286	0.07197	0.082116
beta p	8.91E-01	5.81E-01	6.34E-01	7.60E-01	6.68E-01	7.09E-01	5.38E-01	8.12E-01
se bar	0.285087	0.12869	0.082002	0.127794	1.34E-01	1.43E-01	1.08E-01	1.24E-01

Portfolios for Testing Period May 2001

Returns of p	0.092166	0.103845	0.014775	0.028637	0.025199	0.052968	0.078929	0.013863
beta p	8.91E-01	5.81E-01	6.34E-01	7.60E-01	6.68E-01	7.09E-01	5.38E-01	8.12E-01
se bar	0.285087	0.12869	0.082002	0.127794	1.34E-01	1.43E-01	1.08E-01	1.24E-01

Portfolios for Testing Period June 2001

Returns of p	-0.049812	0.009531	-0.004884	-0.111402	-0.012926	0.02413	-0.035495	-0.058732
beta p	8.91E-01	5.81E-01	6.34E-01	7.60E-01	6.68E-01	7.88E-01	5.92E-01	8.12E-01
se bar	0.285087	0.12869	0.082002	0.127794	1.34E-01	1.59E-01	1.19E-01	1.24E-01

Table 4 (continued)

Portfolios for Testing Period July 2001

Returns of p	-0.013894	-0.022585	0.04538	0.020301	-0.007103	-0.00132	0.047881	-0.025096
beta p	8.91E-01	5.81E-01	6.34E-01	7.60E-01	6.68E-01	7.88E-01	5.92E-01	8.12E-01
se bar	0.285087	0.12869	0.082002	0.127794	1.34E-01	1.59E-01	1.19E-01	1.24E-01

Portfolios for Testing Period August 2001

Returns of p	-0.067452	-0.034829	-0.004377	0.049677	-0.001666	-0.007708	0.055435	-0.003343
beta p	8.91E-01	5.81E-01	6.34E-01	7.60E-01	6.68E-01	7.88E-01	5.92E-01	9.13E-01
se bar	0.285087	0.12869	0.082002	0.127794	1.34E-01	1.59E-01	1.19E-01	1.39E-01

Portfolios for Testing Period Sept. 2001

Returns of p	-0.118468	-0.011965	-0.039108	0.041393	-0.068247	-0.070515	-0.058987	-0.036509
beta p	8.91E-01	5.81E-01	6.34E-01	7.60E-01	6.68E-01	7.88E-01	5.92E-01	9.13E-01
se bar	0.285087	0.12869	0.082002	0.127794	1.34E-01	1.59E-01	1.19E-01	1.39E-01

Portfolios for Testing Period Oct. 2001

Returns of p	-0.020525	-0.042581	-0.007941	0.001687	0.051486	-0.000339	-0.015701	0.024318
beta p	8.91E-01	5.81E-01	6.34E-01	7.60E-01	6.68E-01	7.88E-01	5.92E-01	9.13E-01
se bar	0.285087	0.12869	0.082002	0.127794	1.34E-01	1.59E-01	1.19E-01	1.39E-01

Portfolios for Testing Period Nov. 2001

Returns of p	0.069233	0.055	0.012044	0.022779	0.034185	0.063812	0.105236	0.00195
beta p	8.91E-01	5.81E-01	6.34E-01	7.60E-01	7.51E-01	7.88E-01	5.92E-01	9.13E-01
se bar	0.285087	0.12869	0.082002	0.127794	1.51E-01	1.59E-01	1.19E-01	1.39E-01

Portfolios for Testing Period Dec. 2001

Returns of p	0.08593	0.141406	0.058175	0.041855	0.030564	0.046603	0.040114	0.035636
beta p	8.91E-01	5.81E-01	6.34E-01	8.36E-01	7.51E-01	7.88E-01	5.92E-01	9.13E-01
se bar	0.793407	0.337582	0.402091	0.699701	0.564555	0.620843	0.350528	0.834294

Portfolios for Testing Period Jan. 2002

Returns of p	0.059195	0.109316	-0.011718	-0.035245	0.033527	0.077601	0.053047	0.017243
beta p	8.01E-01	5.22E-01	5.41E-01	5.28E-01	4.90E-01	4.55E-01	3.77E-01	7.97E-01
se bar	0.257192	0.131059	0.089099	0.101606	1.21E-01	9.42E-02	1.04E-01	1.29E-01

Portfolios for Testing Period Feb.2002

Returns of p	0.028005	0.04293	0.038509	-0.002543	0.099087	0.055864	0.021029	0.075938
beta p	8.01E-01	5.22E-01	5.41E-01	5.28E-01	4.90E-01	5.12E-01	3.77E-01	7.97E-01
se bar	0.257192	0.131059	0.089099	0.101606	1.21E-01	1.06E-01	1.04E-01	1.29E-01

Portfolios for Testing Period March 2002

Returns of p	0.026869	0.132698	0.03187	0.042568	0.064846	0.021159	0.07434	0.071395
beta p	8.01E-01	5.74E-01	5.41E-01	5.28E-01	4.90E-01	5.12E-01	3.77E-01	7.97E-01
se bar	0.257192	0.144165	0.089099	0.101606	1.21E-01	1.06E-01	1.04E-01	1.29E-01

Portfolios for Testing Period April 2002

Returns of p	-0.019553	0.024095	0.03192	-0.051778	0.022397	0.002853	0.045185	0.026176
beta p	8.01E-01	5.74E-01	5.41E-01	5.28E-01	4.90E-01	5.12E-01	3.77E-01	7.97E-01
se bar	0.257192	0.144165	0.089099	0.101606	1.21E-01	1.06E-01	1.04E-01	1.29E-01

Table 4 (continued)

Portfolios for Testing Period May 2001

Returns of p	0.034137	0.167234	0.034305	-0.004085	0.00736	0.053118	0.006765	0.049367
beta p	8.01E-01	5.74E-01	5.41E-01	5.86E-01	4.90E-01	5.12E-01	3.77E-01	7.97E-01
se bar	0.257192	0.144165	0.089099	0.112896	1.21E-01	1.06E-01	1.04E-01	1.29E-01

Portfolios for Testing Period June 2001

Returns of p	-0.05588	-0.057911	-0.05469	-0.075099	-0.004334	-0.016301	-0.037429	-0.003593
beta p	8.01E-01	5.74E-01	5.41E-01	5.86E-01	4.90E-01	5.12E-01	3.77E-01	7.97E-01
se bar	0.257192	0.144165	0.089099	0.112896	1.21E-01	1.06E-01	1.04E-01	1.29E-01

Portfolios for Testing Period July 2001

Returns of p	-0.069216	-0.082411	-0.041182	-0.075227	-0.060756	-0.090549	-0.076441	-0.063676
beta p	8.01E-01	5.74E-01	5.41E-01	5.86E-01	4.90E-01	5.12E-01	3.77E-01	7.97E-01
se bar	0.257192	0.144165	0.089099	0.112896	1.21E-01	1.06E-01	1.04E-01	1.29E-01

Portfolios for Testing Period Augustl 2001

Returns of p	0.013122	0.02253	0.006096	-0.032554	-0.004788	-0.016219	0.023102	-0.00401
beta p	8.01E-01	5.74E-01	5.41E-01	5.86E-01	4.90E-01	5.12E-01	3.77E-01	7.97E-01
se bar	0.257192	0.144165	0.089099	0.112896	1.21E-01	1.06E-01	1.04E-01	1.29E-01

Portfolios for Testing Period Sept. 2002

Returns of p	-0.080937	-0.065377	-0.028831	-0.063569	-0.005636	-0.089935	-0.056961	-0.025921
beta p	8.01E-01	5.74E-01	5.41E-01	5.86E-01	4.90E-01	5.12E-01	3.77E-01	7.97E-01
se bar	0.257192	0.144165	0.089099	0.112896	1.21E-01	1.06E-01	1.04E-01	1.29E-01

Portfolios for Testing Period Oct. 2002

Returns of p	0.020142	0.02824	-0.021729	0.020263	0.031063	0.042923	-0.010823	0.006744
beta p	8.01E-01	5.74E-01	5.41E-01	5.86E-01	4.90E-01	5.12E-01	3.77E-01	7.97E-01
se bar	0.257192	0.144165	0.089099	0.112896	1.21E-01	1.06E-01	1.04E-01	1.29E-01

Portfolios for Testing Period Nov. 2002

Returns of p	0.006765	0.000297	-0.019795	0.056528	-0.014348	0.066704	0.050259	-0.018107
beta p	8.01E-01	5.74E-01	5.41E-01	5.86E-01	4.90E-01	5.12E-01	3.77E-01	7.97E-01
se bar	0.257192	0.144165	0.089099	0.112896	1.21E-01	1.06E-01	1.04E-01	1.29E-01

Portfolios for Testing Period Dec. 2002

Returns of p	0.024567	0.020639	0.041871	-0.002182	0.004834	-0.016762	-0.039133	0.046042
beta p	8.01E-01	5.74E-01	5.41E-01	5.86E-01	4.90E-01	5.85E-01	3.77E-01	7.97E-01
se bar	0.257192	0.144165	0.089099	0.112896	1.21E-01	1.21E-01	1.04E-01	1.29E-01

Table 5. Summary of Results for Equation (13)

$$R_{pt} = \hat{\gamma}_{0t} + \hat{\gamma}_{it}\hat{\beta}_{p,t-1} + \hat{\gamma}_{2t}\hat{\beta}_{p,t-1}{}^{2} + \hat{\gamma}_{3t}\overline{S}_{p,t-1}(\hat{\varepsilon}_{i}) + \hat{\eta}_{pt}$$

Coefficient	Mean Estimate	Standard Error	t-statistic
γ_{0t}	0.73867E-02	0.97325E-01	0.5203254
γ_{1t}	0.21211E-01	0.21033	0.6913704
γ_{2t}	-0.17513E-01	0.10201	-1.176944
γ_{3t}	0.20698E-01	0.38270	0.3707813
R^2	0.18748	0.15913	

TABLE 6. Monthly Estimation Results For Equation (13), February 1999 to December 2002

$$R_{pt} = \hat{\gamma}_{0t} + \hat{\gamma}_{it}\hat{\beta}_{p,t-1} + \hat{\gamma}_{2t}\hat{\beta}_{p,t-1}{}^{2} + \hat{\gamma}_{3t}\overline{S}_{p,t-1}(\hat{\varepsilon}_i) + \hat{\eta}_{pt}$$

Period	Statistic												
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$S(\hat{\gamma}_0)$	$S(\hat{\gamma}_1)$	$S(\hat{\gamma}_2)$	$S(\hat{\gamma}_3)$	$t(\hat{\gamma}_0)$	$t(\hat{\gamma}_1)$	$t(\hat{\gamma}_2)$	$t(\hat{\gamma}_3)$	r^2
1999-02	-0.11604	0.17872	-0.10272	0.22113	0.09588	0.1714	0.09384	0.3366	-1.210	1.043	-1.095	0.6569	0.071220
1999-03	0.20690	-0.2566	0.09899	-0.21672	0.1237	0.2211	0.1216	0.4380	1.672	-1.161	0.8138	-0.4948	0.15541
1999-04	0.15041	-0.08605	0.0266	-0.20331	0.0824	0.1473	0.08103	0.2918	1.825	-0.5843	0.3283	-0.6968	0.18135
1999-05	0.00515	-0.05442	0.015516	0.069	0.06321	0.113	0.06215	0.2238	0.08144	-0.4817	0.2497	0.3085	0.02802
1999-06	0.13742	-0.22278	0.097834	-0.11013	0.04393	0.0793	0.04322	0.1540	3.128@	-2.808#	2 .264#	-0.7150	0.36161
1999-07	0.05709	-0.17355	0.11291	0.1554	0.04569	0.08233	0.04469	0.0226	1.249	-2.108#	2.527#	0.1452	0.54210
1999-08	0.03975	-0.00129	-0.0398	-0.11531	0.04032	0.07265	0.03944	0.1371	0.9858	-0.0177	-1.011	-0.8409	0.67875
1999-09	-0.078503	0.18643	-0.065092	0.05839	0.1147	0.2061	0.1117	0.3744	-0.6842	0.9047	-0.5828	0.1560	0.098749

Period	Statistic (continued)												
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$S(\hat{\gamma}_0)$	$S(\hat{\gamma}_1)$	$S(\hat{\gamma}_2)$	$S(\hat{\gamma}_3)$	$t(\hat{\gamma}_0)$	$t(\hat{\gamma}_1)$	$t(\hat{\gamma}_2)$	$t(\hat{\gamma}_3)$	r^2
1999-10	-0.10570	0.14771	-0.10732	0.32899	0.06592	0.1184	0.06417	0.2151	-1.603	1.248	-1.672	1.529	0.17592
1999-11	-0.00968	-0.0379	0.037041	-0.13813	0.07396	0.1328	0.07200	0.2413	-0.1308	-0.2854	0.5145	-0.5724	0.03100
1999-12	0.06532	-0.0316	0.052130	-0.26657	0.1829	0.3273	0.1775	0.5959	0.3571	-0.09648	0.2937	-0.4474	0.017999
2000-01	-006484	0.11671	-0.14338	0.52292	0.04931	0.09872	0.06259	0.1784	-1.315	1.182	-2.291#	2.931@	0.40709
2000-02	0.01307	0.09513	-0.12521	0.73350	0.1362	0.2748	0.1751	0.4908	0.09599	0.3462	-0.7152	1.494	0.12526
2000-03	0.05275	-0.0896	0.047806	-0.11032	0.06458	0.1304	0.08319	0.2325	0.8167	-0.6875	0.5747	-0.4745	0.052178
2000-04	0.06271	-0.0269	0.044000	-0.52695	0.06151	0.1238	0.07906	0.2203	1.020	-0.2176	0.5565	-2.392#	0.34416
2000-05	-0.03624	0.03284	-0.044155	0.28630	0.07507	0.1504	0.09598	0.2667	-0.4827	0.2184	-0.4600	1.073	0.070755
2000-06	-0.04342	0.05168	-0.05163	0.27287	0.04242	0.08469	0.05375	0.1479	-1.023	0.6103	-0.9606	1.845*	0.18195
2000-07	-0.02728	0.05985	-0.053585	-0.05557	0.02756	0.05495	0.03437	0.0933	-0.9893	1.089	-1.559	-0.5955	0.44147
2000-08	0.12318	-0.1277	0.021093	0.22487	0.1450	0.2900	0.1799	0.4825	0.8493	-0.4403	0.1173	0.4660	0.049191
2000-09	0.01766	-0.0121	-0.007004	-0.008935	0.0656	0.1318	0.08003	0.2087	0.2692	-0.09153	-0.0875	-0.0428	0.028477
2000-10	-0.13195	0.15974	-0.10542	0.23741	0.08762	0.1757	0.1067	0.2778	-1.506	0.9089	-0.9881	0.8547	0.07353
2000-11	-0.03359	0.01493	-0.0250	-0.06379	0.04383	0.08829	0.05337	0.1381	-0.7666	0.1691	-0.4685	-0.4621	0.16025

Period	Statistic (continued)												
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$S(\hat{\gamma}_0)$	$S(\hat{\gamma}_1)$	$S(\hat{\gamma}_2)$	$S(\hat{\gamma}_3)$	$t(\hat{\gamma}_0)$	$t(\hat{\gamma}_1)$	$t(\hat{\gamma}_2)$	$t(\hat{\gamma}_3)$	r^2
2000-12	0.03208	0.02026	-0.02346	-0.05026	0.05762	0.1152	0.0696	0.1765	0.5567	0.1760	-0.3371	-0.2848	0.06528
2001-01	-0.10327	0.39645	-0.15711	0.39645	0.2413	0.5354	0.3250	0.5354	-0.4280	0.7405	-0.4834	0.7405	0.06876
2001-02	0.08945	-0.1504	0.19283	-0.44337	0.1122	0.2485	0.1485	0.2574	0.7971	-0.6053	1.299	-1.722*	0.31468
2001-03	0.22594	-0.5594	-0.07238	1.8811	0.4982	1.097	0.6555	1.125	0.4535	-0.5099	-0.1104	1.672	0.24327
2001-04	0.08258	-0.0634	0.069573	-0.14945	0.1118	0.2324	0.1361	0.2751	0.7386	-0.2728	0.5112	-0.5432	0.053637
2001-05	-0.02163	0.10413	-0.06779	0.30659	0.1194	0.2339	0.1312	0.3066	-0.0181	0.4452	-0.5168	1.000	0.10813
2001-06	-0.05813	0.02849	-0.00989	0.13206	0.08664	0.1680	0.0936	0.2137	-0.6709	0.1696	-0.1058	0.6179	0.11789
2001-07	0.12820	-0.2862	0.14310	-0.0909	0.1016	0.1971	0.1098	0.2507	1.261	-1.452	1.303	-0.3626	0.11984
2001-08	-0.02015	0.08554	0.01248	-0.37303	0.09172	0.1510	0.07634	0.3167	-0.2197	0.5666	0.1636	-1.178	0.12544
2001-09	-0.04895	0.11774	0.02852	-0.58504	0.08788	0.1573	0.08262	0.2266	-0.5570	0.7484	0.3452	-2.581#	0.40219
2001-10	-0.0099	0.04633	-0.01032	-0.074693	0.09286	0.1546	0.07679	0.2759	-0.1073	0.2996	-0.1344	-0.2708	0.014719
2001-11	0.18282	-0.2632	0.11303	-0.02985	0.06651	0.1096	0.05434	0.1944	2.749@	-2.402#	2.080*	-0.1535	0.30082
2001-12	0.11993	-0.0871	-0.21467	0.047869	0.06809	0.1044	0.2290	0.04726	1.761*	-0.8342	-0.9374	1.013	0.085342
2002-01	-0.00238	0.16197	-0.06734	-0.01980	0.08413	0.2096	0.1125	0.04623	-0.0283	0.7729	-0.5985	-0.4284	0.06078

Period	Statistic (continued)												
	$\hat{\gamma}_0$	$\hat{\gamma}_1$	$\hat{\gamma}_2$	$\hat{\gamma}_3$	$S(\hat{\gamma}_0)$	$S(\hat{\gamma}_1)$	$S(\hat{\gamma}_2)$	$S(\hat{\gamma}_3)$	$t(\hat{\gamma}_0)$	$t(\hat{\gamma}_1)$	$t(\hat{\gamma}_2)$	$t(\hat{\gamma}_3)$	r^2
2002-02	-0.00720	0.14885	-0.00653	-0.12890	0.08030	0.1727	0.1185	0.2736	-0.08966	0.8621	-0.05513	-0.4711	0.33657
2002-03	-0.02297	0.14676	0.00131	-0.06477	0.1211	0.2627	0.1727	0.3907	-0.1897	0.5587	0.00759	-0.1658	0.26543
2002-04	-0.028255	0.12626	-0.1034	0.16809	0.08304	0.1801	0.1184	0.2679	-0.3402	0.7009	-0.8728	0.6274	0.05389
2002-05	-0.20454	0.69778	-0.2257	-0.38425	0.1601	0.3452	0.2270	0.5129	-1.278	2.021#	-0.9942	-0.7492	0.39594
2002-06	0.065762	-0.2741	0.15445	-0.04835	0.05856	0.1263	0.08305	0.1876	1.123	-2.171#	1.860*	-0.2577	0.24170
2002-07	-0.021921	-0.1152	0.03636	0.01535	0.05850	0.1271	0.08230	0.1864	-0.3747	-0.9062	0.4418	0.08235	0.16138
2002-08	-0.22831	0.55509	-0.33687	0.32045	0.07608	0.1647	0.1062	0.2400	-3.001@	3.371@	-3.172@	1.335	0.41794
2002-09	-0.10950	0.09120	-0.02626	0.03091	0.08523	0.1839	0.1192	0.2685	-1.285	0.4958	-0.2203	0.1151	0.10121
2002-10	-0.00759	0.02611	-0.0257	0.14313	0.07397	0.1596	0.1034	0.2330	-0.1027	0.1636	-0.2484	0.6142	0.044619
2002-11	0.053374	-0.10121	0.05169	-0.01804	0.05632	0.1215	0.07876	0.1774	0.9477	-0.8327	0.6563	-0.1017	0.047901
2002-12	-0.041833	0.22100	0.03739	-0.49221	0.1209	0.2592	0.1682	0.3733	-0.3460	0.8525	0.2224	-1.319	0.38788

* means t-statistic is significant at 10% level of significance; # means t-statistic is significant at 5% level of significance; @ means t-statistic is significant at 1% level of significance.