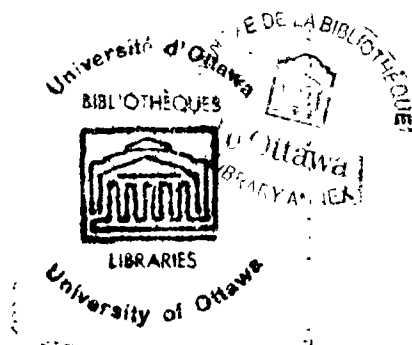


(THE INFLUENCE OF THE STUDY OF PLANE GEOMETRY
ON CRITICAL THINKING)
by Carmen Charles Massimiano

Thesis presented to the Faculty of Arts
of the University of Ottawa through the
Institute of Psychology as partial ful-
fillment of the requirements for the
degree of Doctorate of Philosophy.



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CURRICULUM STUDIORUM

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INTRODUCTION

In the influential report, Education for All American Youth, prepared by the Educational Policies Commission of the National Education Association, the following is stated:

Schools should be dedicated to the proposition that every youth in these United States—regardless of sex, economic status, geographic location, or race—should experience a broad and balanced education which will (a) equip him to enter an occupation suited to his abilities and offering reasonable opportunity for personal growth and social usefulness; (b) prepare him to assume the full responsibility of American citizenship; (c) give him a fair chance to exercise his right to the pursuit of happiness through the attainment and preservation of mental and physical health; stimulate intellectual achievement; and cultivate the ability to think rationally; and (d) help him to develop an appreciation of the ethical values which should undergird all life in a democratic society.¹

It is clearly implied in the report that the ability to think rationally on the part of every citizen is one of this nation's most effective defenses against the persuasive illogic of the enemies of democracy.

In studies that have surveyed purposes of teaching geometry, such reasons as "to teach the students to think

¹ National Education Association and American Association of School Administrators Educational Policies Commission, Education for All American Youth, Washington, The Commission, 1952, p. 32.

logically" and "to understand the meaning of deductive proof" were most frequently mentioned². What is logical reasoning? Is it the same as critical thinking or rational thinking? Do teachers of geometry mean by critical thinking what is meant by the teachers of social studies or the teachers of logic or the psychologist?

Brown tabulated the opinions of five hundred classroom teachers concerning the important objectives of teaching geometry. The objective receiving the highest rating was "to develop the habit of clear thinking and precise expression"³. However, the definition of "clear thinking" was not offered by Brown, nor by Shibli in 1930 who originated the questionnaire. In recent mathematics meetings and in recent articles, phrases concerning clear thinking have been used in as vague a way as the objective printed on the questionnaire of Shibli⁴ and Brown.

Robert E. Pingry in an important article noted the multitude of descriptive phrases of thinking appearing in

2 U.S. Department of Health, Education and Welfare, Mathematics in Public High Schools, Bulletin 1953, No. 5, Washington, Government Printing Office, 1953, p. 15.

3 Kenneth E. Brown, "Why Teach Geometry" in The Mathematics Teacher, Vol. 43, No. 3, issue of March 1950, p. 103-104.

4 Jabir Shibli, Recent Developments in the Teaching of Geometry, State College, Pa., J. Shibli, 1932, p. 213-215.

books and journals on teaching, psychology, philosophy and mathematics⁵.

Pingry enumerated the following phrases found in his reading:

clear thinking	elaborative thinking
straight thinking	scientific thinking
reflective thinking	postulational thinking
organic thinking	autonomous thinking
sublimen thinking	human thinking
factual thinking	logical thinking
	rational thinking

inventive, productive, creative, empirical, hypological, and on and on and on . . .⁶

It may be inferred from the above list that when mathematics teachers state one of the objectives of a course in mathematics, especially geometry, to be the development of the habit of clear thinking and precise expression, they may lack clearness and precision in their own thinking. J.W. Young avoided all of the usual adjectives to describe thinking, and said: ". . . it is clear that the chief end of mathematical study must be to make the pupil think"⁷.

Some consider a student a good critical thinker who is good at collecting and organizing data related to a

⁵ Robert E. Pingry, "Critical Thinking, What is It?" in The Mathematics Teacher, Vol. 44, No. 7, issue of November 1951, p. 466.

⁶ Pingry, Op. cit., p. 466.

⁷ J.W. Young, Fundamental Concepts of Algebra and Geometry, New York, Macmillan, 1936, p. 4.

problem; who can evaluate the appropriateness and the validity of sources and information derived from them; who can distinguish between fact and opinion; who can compare data making use of tables, graphs, and statistics; and who has developed the habit of withholding final judgment until sufficient facts are present. All of these skills and habits have been used by various people as a basis for measurement of a student's ability to think critically⁸.

With allowances for varying phraseology and varying definitions or lack of them, there thus seems to be a consensus that important values to be derived from the study of mathematics, particularly demonstrative geometry, are related to an acquaintance with the nature of proof and to a familiarity with critical thinking as a method of thought.

The ability to think critically, as concerned in this study, involves three elements:

- (1) An attitude of being disposed to consider in a thoughtful way the problems and subjects that come within the range of one's experiences,
- (2) Knowledge of the methods of logical inquiry and reasoning, and
- (3) Some skill in applying these methods.⁹

⁸ M.F. Roskopf, "The Present State of Evaluation of Critical Thinking in Algebra and Geometry", in The Mathematics Teacher, Vol. 43, No. 4, issue of April 1950, p. 142-148.

⁹ Edward M. Glaser, An Experiment in the Development of Critical Thinking, New York, Bureau of Publications, Columbia University, 1941, p. 5-6.

The chief problems of this study are: (1) Do high school sophomores who have studied plane demonstrative geometry improve more in critical thinking than those pupils who have not taken geometry? (2) What is the relationship between achievement in geometry and critical thinking ability? (3) What is the relationship between studying other subjects during the sophomore year and critical thinking ability?

The major assumptions made in this study are based upon experimental evidence reported in the literature or upon this writer's analysis of the data obtained from his testing program. Chapter I is devoted to the studies reported in the literature bearing on the processes of thinking and reasoning, and upon the abilities involved in critical thinking.

Chapter II deals with the history and teaching of geometry with special attention to the course of study in geometry as it is taught at Pittsfield High School.

Chapter III describes the instruments and tests measuring those aspects of the ability to think critically which were deemed important and subject to measurement by pencil-and-paper tests. This chapter also outlines the method used for establishing the control and experimental groups in the present research.

Chapter IV sets forth the results obtained from the administration of the tests and from the analysis of differences in the results between the control and the experimental groups.

The final chapter summarizes the data obtained and states the conclusions that may be inferred from the data.

CHAPTER I

REVIEW OF RESEARCH RELATED TO THE PROBLEM

Many studies have been reported in the literature bearing upon the different aspects of the processes of thinking and reasoning, and upon the development of the ability to think critically. The studies reviewed in this chapter are aspects of the problem described by these headings:

1. The Nature of the Thinking Process.
2. Learning and Reasoning.
3. Intelligence and Reasoning.
4. Reasoning Ability and Academic Grades.
5. Transfer of Training to Think Critically.
6. Efficacy of Subjects in Promoting Transfer.

1.- The Nature of the Thinking Process.

The nature and details of the thought process and the mental processes used in the solution of arithmetic problems are of long-standing psychological as well as philosophical interest.

Father Brennan, following St. Thomas Aquinas, describes the nature of the thinking process as follows:

Now, the basic law of method for all knowledge is advance from fact, or what we grasp by immediate experience, to the principle behind the fact. We start with what is least known, and work by degrees towards what is least known. After all, this is the only sensible way of learning. As St. Thomas counsels the student John: If you plunge at once into the sea, you are lost. But, if you enter first by a stream that empties into a river, then go by the river till you get to the sea, there is reason to believe that you can keep afloat. Or, to change the figure, we set off on our journey at the fact-finding level which is the stage of invention. Then we travel step by step, up the ladder of generalized knowledge. Our goal is synthesis; and our arrival there is heralded by the appearance of some ultimate law or principle that explains the facts with which we started, at the same time that it furnishes a clue to the meaning of other facts as yet unknown to us. Once the law or principle is mastered, we can make it the point of departure for further and more particular inferences.¹

Father Brennan continues:

. . . the first task of the thinker is the establishment of a goal or the creation of an anticipatory scheme that will serve as a frame of reference for the exploration of means. A careful search for appropriate ideas follows, with trial and error movements on the intellectual level, the elimination of useless material, and choice of what is best suited to our purpose. Finally there is the actual filling out of the original outline.²

¹ Robert Edward Brennan, General Psychology, A Study of Man Based on St. Thomas Aquinas, New York, Macmillan, 1952, p. 5.

² Ibid., p. 328.

In his book How We Think, Dewey stated that the origin of thinking is a felt difficulty, state of doubt, or perplexity.

Given a difficulty, the next step is suggestion of some way out - the formation of some tentative plan or project, the entertaining of some theory that will account for the peculiarities in question, the consideration of some solution for the problem. The data at hand cannot supply the solution; they can only suggest it. What, then, are the sources of the suggestion? Clearly, past experiences and a fund of relevant knowledge at one's command. If the person had some acquaintance with similar situations, if he had dealt with material of the same sort before, suggestions more or less apt and helpful will arise. But unless there has been some analogous experience, confusion remains. Even when a child (or a grown-up) has a problem, it is wholly futile to urge him to think when he has no prior experience that involve some of the same conditions.³

Dewey differentiated between ordinary thinking and reflective thinking:

There may . . . be a state of perplexity and also previous experience out of which suggestions emerge, and yet thinking need not be reflective. For the person may not be sufficiently critical about the ideas that occur to him. He may jump at a conclusion without weighing the grounds on which it rests; he may forego or unduly shorten the act of hunting, inquiring; he may take the first "answer" or solution that comes to him because of mental sloth, torpor, impatience to get something settled.⁴

One of the most complete analyses of the total process was provided by Dewey when he analyzed what he called reflective thinking or problem solving in five phases:

³ John Dewey, How We think, New York, Heath, 1933, p. 15.

⁴ Ibid., p. 16.

1. Becoming aware of the problem. A perplexing situation has arisen. The individual is aware that a problem is present.
2. Defining the problem. The rather vague, perplexing situation is now made more sharp and specific. The goal to be achieved is now known.
3. Suggesting a hypothesis for the solution of problem. Conjectures, suppositions, guesses are made. "The inference involves a leap, a jump, the propriety of which cannot be absolutely warranted in advance".
4. Reasoning out the implications of the suggested hypotheses. Tracing out the consequences of the various hypotheses.
5. Experimental corroboration. A test of the hypothesis is made against experience."

Symonds⁶, however, pointed out that attempts to apply Dewey's five formal phases to the solution of various types of problems did not result in altogether satisfactory returns since not all thinking is carried out in the same kind of problem solution, nor does it always require the complete act of thinking.

In order to discover the mental processes which required different kinds of thought processes, Symonds met with a group of interested and willing graduate students in education for one evening a week during the greater part of a school year. At each meeting he tried to stimulate by appropriate questions one type of mental process such as selective recall, formulation of a definition, and seventeen

5 Ibid., p. 106-115.

6 Percival M. Symonds, Education and the Psychology of Thinking, New York, McGraw-Hill, 1936, p. 7-8.

other thought processes, each of which Symonds regards as a single aspect of thought. The group was advised that each member would be asked in turn what mental processes he went through in order to reach his solution. Symonds states:

Out of this more or less introspective form of analysis, several interesting observations were made. In the first place, we are all very dependent on previous experience in the solution of any problem Another factor in problem solving which this analysis seems to discover is the importance of the concept. The words in the problem as verbally stated and the meaning which they naturally convey are the starting point for the solution. Each word brings up its own trend of associations and the processes of analysis and selection immediately start to operate. With meager concepts, one is limited to the actual concrete association which has been previously made in experience. With more adequate concepts, which have been freed from their concomitants, thinking is more free and versatile.⁷

When the analysis "of each of these separate mental processes was completed" Symonds found that:

there seemed to be an interpretation of mental processes in thinking at all levels, which did not permit one to say that this process was more elementary than that Certain processes, like analysis, selection, generalization, and organization seem to occur frequently, suggesting that most processes of reasoning depend on these fundamental operations.⁸

Symonds believed that thinking is a complex act capable of being divisible into components, and that each of these components is a matter of skill and learning in itself, and

7 Ibid., p. 12.

8 Ibid., p. 12.

lastly he suggested that the operations of thought conveniently fall into the following classification and the complete act of thought is an organization of very separate abilities:

- I. Fundamental Psychological Processes.
- II. Concepts.
- III. Organization of Concepts.
- IV. Judgment:
 - a. Judgments of facts.
 - b. Judgments of value.
 - c. Evaluation.
- V. Organization of Judgments.
- VI. Syllogistic Reasoning.
- VII. Argument - the Organization of Syllogism.⁹

Commins¹⁰ held that:

the occasion for thought is commonly a new situation dissimilar in some respect to the old. Or it may be that the new situation is not very different, but the available organization has not been retained in a detailed, workable form. Thus, if we forget the actual steps for extracting a square root, we may yet be thinking of the principle involved to reconstruct them.¹¹

Commins further stated that:

earlier schemes of organization (of experiences) become most readily available through abstraction and generalization, the two distinctive features of thought . . . Under the guidance of dynamic integrative tendencies, new properties and relationships appear in objects in the perceptual field . . . The development of thought is a matter of fitting unconnected facts into generalized schemes of understanding this comprehension.

⁹ Ibid., p. 183.

¹⁰ W.D. Commins, Principles of Educational Psychology, New York, Ronald Press, 1937, p. 469.

¹¹ Ibid., p. 469.

It is only when the difficulty is rather clearly defined and when this is seen as fitting into a particular background that the individual undertakes directive thinking . . . The results of previous trends of development are borne in general "attitudes", "mental sets", "determining tendencies" which manifest themselves as general forms of readiness to think in certain ways or along certain lines.¹²

The Harvard Committee suggested:

Abstractions in themselves are meaningless unless connected with experience . . . The teacher can do a great deal; . . . he can relate theoretical content to the students' life . . . and he can deliberately simulate in the class-room situations from life . . . he can be persistent in directing the attention of the student from the symbols to the things they symbolize.¹³

In an analysis of the trend of human thinking as demonstrated by language usage, Judd¹⁴ arrived at the conclusion that active thinking tends to go in the direction of the synthesis of idea. The mind does not dwell on separate isolated items of experience but combines all these items into integrated systems.

Strasheim¹⁵ told a series of ten stories to children between the ages of five and ten. All of these tales revolved around a problem that could be solved by the same

12 Ibid., p. 470, 501-502.

13 Harvard Committee Report, General Education in a Free Society, Cambridge, Harvard University Press, 1945, p. 70-71.

14 C.H. Judd, Education as the Cultivation of the Higher Mental Processes, New York, Macmillan, 1936, p. 30.

15 J.J. Strasheim, A New Method of Mental Testing, Baltimore, Warwick and York, 1926, 158 p.

general method, that of "flipping" a coin or by arriving at a decision by chance.

The first story laid down the rule. The stories that followed required the child to apply the relations he had discovered in the previous situation to entirely changed elements found in the new problem. In the last three stories, the child had to find the elements for himself, either by visualizing the situation, or by remembering various features, and different qualities associated with it, or by selecting from such "reproducts". For example, in the tenth problem two swimmers wanted to find out by chance which one should go back to the boat and which one should swim to shore. The story ended with the statement that since they were in the water they had no money, so they had to find another way.

Strasheim concluded that in the early stages of education, the relationships are understood as being more or less specific to the "fundaments" between which they have been educed, "with the result that they are suitable for future occasions only when the same or similar fundaments occur". In the later stages of education, relations are abstracted from their contact with the fundaments, and, as such may be widely applied to the most varied fundaments.

Burt devised tests to measure reasoning ability in school children between the ages of seven and fourteen.

In this study Burt concludes:

All the elementary mental mechanisms essential to formal reasoning are present . . . by the mental age of seven if not somewhat before. Development consists primarily in an increase in the extent and variety of the subject matter to which those mechanism can be applied, and in an increase in the precision and elaboration with which those mechanisms can operate. . . A child's reasoning ability thus appears to be a function of the degree of organic complexity of which his attention is capable.¹⁶

Haslitt¹⁷ concluded that children observe the same general patterns of reasoning as adults, but that their ability to render explicit all the implications of the concepts they arrive at comes slowly with age.

Piaget¹⁸ observed that childish thought is egocentric, and stated that logical reasoning develops in connection with the "socialization" of language and thought. As the child grows older he learns to think and speak partly, at least, for the benefit of others. He is often called upon to explain and justify his statements, beliefs and actions. This is apt to center his attention upon the logical background of his thinking and toward the widely accepted generalizations which make his particular statements or actions appear reasonable in the eyes of others.

¹⁶ C. Burt, "The Development of Reasoning in School Children", in the Journal of Experimental Pedagogy, Vol. 5, No. 3, issue of December 1919, p. 127.

¹⁷ V. Haslitt, "Children's Thinking", in the British Journal of Psychology, Vol. 20, Part 4, issue of April 1930, p. 354-361.

¹⁸ J. Piaget et al., Judgment and Reasoning in the Child, New York, Harcourt, Brace, 1926, VIII-260 p.

Dunker¹⁹, student of Kohler and Max Wertheimer, introduced the concept of the "Search Model" as useful in understanding the psychological process of solving a problem. The search model evolves as the individual clarifies the problem, bridging the gap between what is given and what is required, and serves for a period of time to direct or channel the individual's deliberation.

Dunker pointed out the relationship of the given and the solution: "A solution always arises out of the demands made by what is required on what is given". The student, then, should be able to organize his efforts and secure a solution to the problem²⁰.

Downing²¹ devised a test to measure the use of some of the elements and safeguards of scientific thinking. This test was given to over one thousand pupils in each of the grades eight to twelve and to College freshmen in the science classes. On the basis of the means of the test scores for all these grades, Downing concluded that ability to think scientifically consists of a complex number of component abilities and that these abilities develop not only at varying rates but at different rates for the different communities.

19 Karl Dunker, "A Problem Solving", in the Psychological Monographs, Vol. 58, No. 5, 1945, p. 8.

20 Ibid., p. 35.

21 E.R. Downing, "The Elements and Safeguards of Scientific Thinking", in The Scientific Monthly, Vol. 26, issue of March 1928, p. 231-243.

Strauss used the test prepared by Downing and found the percentage of pupils, in grades eight through twelve, who made errors or very slight errors in their responses to each of the fifteen parts of the test. Strauss reported:

The ability to recognize a problem, the ability to observe (i.e. two of the first steps in the thinking process) seem to be the best mastered of the elements studied. On the other hand, the ability to reason, to analyze, and to see essential relationships seem to be the least developed of these processes.²²

Henderson and Pingry²³ believed that mathematicians are particularly cognizant of the role played by concepts and generalizations in the deliberative process in problem-solving. These abstractions make it possible to restructure or reorganize past experience and bring it to bear on the problem at hand. They observed:

The production and retention of thought material is dependent on the individual's "span of apprehension", sometimes called "immediate memory span". The individual must be able to remember what the "given" is and what he is expected to find as he continues to work on the problem. He must also be able to select from his past learning whatever is relevant to the problem.²⁴

22 S. Strauss, "Some Results for the Test of Scientific Thinking", in Science Education, Vol. 16, No. 2, issue of December 1931, p. 90-91.

23 Kenneth B. Henderson and Robert E. Pingry, "Problem Solving in Mathematics", in The Learning of Mathematics, Its Theory and Practice, Twenty-First Yearbook, Washington, National Council of Teachers of Mathematics, 1953, p. 241.

24 Ibid., p. 241.

Heidbreder²⁵ set out on the task of discovering the relationship in connection with the placing of geometrical figures in various combinations. She stressed the behavior of the participant, that is, an attitude of activity toward the problem, shown by calling up various hypothesis as some of the possible solutions, as the one activity indispensable to the solution of the problem. She also concluded that the experiment showed the importance of successful experience and that concrete successes, occurring in the course of an unsolved problem, have a certain advantage over failure as stimuli to thought.

Gengerelli²⁶, studying the effect of established mental patterns upon the learning of new patterns, concluded that after we have learned to conceive of or react to a thing in one way, it is relatively more difficult to conceive of or react to it in a different way than if the first way had not been learned at all.

A summary of the studies of past researches concerning the thinking process indicates that the elementary mental processes necessary to formal reasoning are usually present but undeveloped at the mental age of seven. In early stages

25 E. Heidbreder, "Problem Solving in Children and Adults", in the Journal of Genetic Psychology, Vol. 35, No. 4, issue of December 1928, p. 522-545.

26 J.A. Gengerelli, "Mutual Interference in the Evolution of Concepts", in the American Journal of Psychology, Vol. 38, No. 4, issue of October 1927, p. 639-646.

of education the child apprehends relations between things as being more or less specific to the particular things between which the relations have been inferred; the apprehended relations are consequently suitable for future occasions only when the same or similar elements occur. In later stages of education, relations are abstracted into guiding principles or generalizations and thus become more widely applicable; unconnected facts come to be fitted into generalized schemes of understanding. This then is comprehension.

The development of logical reasoning is aided through the process of "socialization" as the child finds it important to justify his statements, actions and beliefs to other persons.

Thinking of a problem-solving kind originates in a felt state of doubt, difficulty, or perplexity, and the way this difficulty is felt or defined limits the types of responses that will occur to the thinker. To "get out of the rut" requires a reformulation of the issue.

If a problem is stated verbally, the meaning which the words convey is the starting point for the solution. Each word brings up its own trend of associations and the process of analysis and selection immediately starts. The interpretation of language by the individual thus influences in a very definite and important way the individual's thinking.

2.- Learning and Reasoning.

Maier²⁷ believed that his experiments substantiated the belief that reasoning and learning are separate abilities - reasoning requiring the reorganization of isolated experiences in terms of a goal, and learning requiring repetition of the experiences so as to establish the relationship. Billings²⁸ found correlations ranging from .35 to .59 between information and problem-solving ability in various fields. Commins, writing from a Gestalt approach in his chapter on "Learning and Thinking", stated:

An essential feature of learning is the organization of facts into an apprehended whole of experience or behavior. . . . When the child is learning to cut with scissors, for example, it is the actual steps in the operation itself which are the important things. . . . There are many ends of training which can be served only by preparing the child in advance to meet situations in a rather specific manner. There are many other occasions in life, however, for which the child cannot be specifically prepared beforehand. For such situations as these he can be made ready only in a much more general manner, through the development of comprehensive insights, tendencies, and methods of approach. To make these directly applicable to the problem at hand usually calls for "thinking". . . . As a special form of learning . . . thinking has its own characteristic features. . . . Thought, instead of being directly concerned with the details of actions or events, as are skill and memory, deals immediately with the

27 N.R.F. Maier, "Reasoning in Humans, Part I: On Direction", in the Journal of Comparative Psychology, Vol. 10, No. 2, issue of April 1930, p. 115-143.

28 M.L. Billings, "Problem Solving in Different Fields of Endeavor", in The American Journal of Psychology, Vol. 46, No. 2, issue of April 1934, p. 267.

scheme or plan or organization which holds these details together. If, in place of actually doing the cutting with scissors, we direct our attention to the principle of movement according to which the cutting takes place we are then said to be thinking . . . Whether thinking plays an important role in learning will depend upon our approach to the field of fact approaching organization.²⁹

The evidence suggests that the ability to learn and the ability to think or reason are related but not identical. Learning may take place through the mere repetition of given experiences, while reasoning appears to require the reorganization of isolated experiences in terms of a goal.

Reasoning requires a greater degree of intellectual development than the ability to learn. Critical or reflective thinking involves a still higher order of intellectual development, in which the ability to reason is included.

3.- Intelligence and Reasoning.

To the layman the terms intelligence and reasoning are very closely related, if not the same. To the psychologist as well it seems that there is a great deal of similarity between reasoning, on pencil and paper laboratory tests at any rate, and what is measured by tests of intelligence. What is the character and degree of this relationship?

29 W.D. Commins, op. cit., p. 468-469.

Billings³⁰ offered a provocative opinion on the status of problem-solving as an ability. He set out to discover whether a person is as good a reasoner in one academic field as another, and gave his subjects problems from such areas as arithmetic, geometry, physics, mechanics, and sociology. Materials necessary for solution were taught first. Correlations between these various fields ran from .53 to .78; between problem-solving in these fields and the Army Alphatest the correlation ranged between .42 and .59. He found that the correlations satisfied the criterion of the tetrad equation, and concluded that problem-solving is a general factor, if not "intelligence" itself.

Sells³¹ found significant correlation between syllogistic reasoning and intelligence, found a correlation of .50 between abstract and concrete syllogism scores when intelligence has partialled out. Sells interpreted his results as indicating two separate factors of ability, possibly a reasoning factor and an intelligence factor.

30 Billings, op. cit., p. 270.

31 S.B. Sells, "The Atmosphere Effect: An Experimental Study of Reasoning", in the Archives of Psychology, No. 200, issue of May 1936, p. 25.

Smoke³² reported a correlation of .52 between speed of concept-formation and intelligence. He noted further the use of "hypothesis" in formulating the concept, and the fact that his more intelligent students were speedier in abstracting. Roslow³³ reported multiple correlations ranging from .32 to .81 between a battery of rational learning problems and various intelligence tests. A multiple-factor analysis of these results of Roslow indicated the presence of a common factor in the problems and intelligence tests.

Powers³⁴ correlated scores on a test of common superstition with the score on the University of Washington Intelligence Examinations, and obtained a coefficient of .15, concluding "that intelligence does not appear to be a safeguard against credulity or false belief".

32 K.L. Smoke, "An Objective Study of Concept Formation", in the Psychological Monograph, Vol. 67, No. 4, Whole No. 191, 1932, p. 40.

33 S. Roslow, "A Statistical Analysis of Rational Learning Problems", in the Journal of Genetic Psychology, Vol. 48, No. 2, issue of June 1936, p. 441-467.

34 F.F. Powers, "The Influence of Intelligence and Personality Traits Upon False Beliefs", in the Journal of Social Psychology, Vol. 2, No. 4, issue of November 1931, p. 492.

Stressing the difference between intelligence and ability to think, Commins stated:

. . . Although intelligence is sometimes defined as the ability to think, the two functions are not really the same. A person's thinking is a reflection of his intelligence plus other factors. While we may measure general mental maturity with very little stress upon knowledge, we cannot estimate the quality of a person's thinking apart from the truthfulness and value of the knowledge he attains in this manner. To think is to think about something, and to think well is to attain true knowledge about this something through the reorganization of experience. Thought is a step closer to knowledge than is intelligence. The teacher may be a better thinker than his student, the old man better than the boy, the urbanite better than the farmer, the high school student better than the savage youth, in spite of very little difference in I.Q.³⁵

Since there are so many explanations of what reasoning and what intelligence signify, and since there are many ways of measuring each or both, it is extremely difficult to describe their relationship adequately.

Some researchers believe that problem-solving ability is a general factor if not "intelligence" itself. Most investigators find that while reasoning ability or problem-solving ability is an important part of what we measure by tests of "general intelligence", reasoning ability is not the

35 Commins, Op. cit., p. 499.

whole of what we consider to be manifestations of intelligence. As Commins stated it:

An individual's ability to think out a problem at any level of education or scientific achievement will depend upon his general mental maturity, his background of experience, certain character or volitional traits, and his facility in the use of language.³⁵

4.- Reasoning Ability and Academic Grades.

Do students who score high on tests of reasoning ability tend to get significantly higher school grades than students who score relatively low in reasoning ability, and do those who receive high grades tend to be superior in their reasoning ability?

Washburn³⁶ selected 113 students at Vassar College on the basis of their ability in mathematics. One group made of fifty-four students with marked ability and another group of fifty-nine students with marked lack of ability were given a test in logical reasoning, not mathematical in character. There were twenty arguments to be answered "sound" or "false". Results were compared with mathematical records and academic records. Conclusions based on comparison of mean scores were as follows:

³⁵ Commins, Op. cit., p. 499.

³⁶ M.F. Washburn, "Mathematical Ability, Reasoning and Academic Standing", in The American Journal of Psychology, Vol. 50, Nos. 1-4, issue of November 1937, p. 484-488.

1) Intellectual ability as represented by high academic standing, and reasoning ability as represented by results of the test are related.

2) High academic standing does not necessarily guarantee good reasoning ability, but good reasoning ability is an excellent guarantee of high academic standing.

3) Poor reasoning ability does not exclude high academic standing but low academic standing excludes good reasoning ability.

4) Good reasoning ability guarantees high academic standing more than it guarantees high academic standing plus good mathematical ability.

5) Where academic standing is high, reasoning ability is only a little higher in the group that is good in mathematics than in the group that is good in other subjects only. The ability to reason with mathematical, quantitative relations does not add much to the ability to reason with non-quantitative relations.

6) Mathematical ability may exist without good reasoning ability and with low academic standing.

7) Poor mathematical ability excludes good reasoning, more than poor reasoning excludes good mathematical ability.³⁷

Glaser concluded in his study that school marks were found to be related to the scores on the Watson-Glaser Critical Thinking Tests. "Among the factors measured in this study, intelligence, reading ability, and school marks are most closely related to the scores made on the tests of critical thinking"³⁸.

Washburn's study provides evidence that academic standing and reasoning ability are related, that high academic standing does not ensure good reasoning ability as frequently

³⁷ Ibid., p. 488.

³⁸ E.M. Glaser, An Experiment in the Development of Critical Thinking, Contributions to Education, No. 843, New York, Teachers College, Columbia University, p. 150.

as good reasoning ability ensures high academic standing, and that poor reasoning ability does not exclude high academic standing but low academic standing definitely tends to exclude good reasoning ability. Glaser indicates in his extensive study that school marks were found to be related to the scores on the Watson-Glaser Critical Thinking Tests.

5.- Transfer of Training to Think Critically.

The term "transfer of training" was defined by Thorndike as "the influence which an improvement or change in one mental function has upon other mental functions"³⁹. Orata explained transfer as "that process of using or applying previously acquired information, habit, skill, attitude, or ideals in dealing with a relatively new or novel situation"⁴⁰.

A distinction should be drawn between the terms "transfer of training" and the "doctrine of formal discipline" (often referred to as the mental discipline theory). Formal discipline has been variously defined as the sharpening of the wits, the ability to remember and apply, a facility with words, or an ability to rationalize. According to the doctrine of formal discipline the transfer from one field to

³⁹ E.L. Thorndike, Educational Psychology, Briefer Course, New York, Teachers College, Columbia University, 1919, p. 259.

⁴⁰ P.T. Orata, "Transfer of Training and Reconstruction of Experience" The Mathematics Teacher, Vol. 30, No. 3, issue of March, 1937, p. 99.

another unrelated field takes place not through the application of any specific content or methods learned, but through the increased mental ability and power developed as a result of such training; a mind so sharpened and so stored is ready for any calling or any test.

Much of the research relevant to thinking may be summarized under the heading of "formal discipline", which generally postulated the existence of faculties such as will, memory, and judgment that could be trained by exercise. The material learned was considered less important than the mental work involved in the learning of the material.

This point of view was expressed in the report of the Committee of Ten of the National Education Association in 1894. The report claimed that the improvement in reasoning acquired in mathematics, for example, would so develop the faculty of logical thinking that there would be transfer of the ability to reason logically to history, sciences or languages⁴¹.

The doctrine of formal discipline is not generally accepted today. As a matter of fact even some members of the Committee of Ten disagreed with the majority report and presented their own views on mental training.

⁴¹ National Education Association, Report of the Committee of Ten on Secondary School Studies, New York, American Book, 1894, xii-249 p.

Redden and Ryan observed:

The attack, some years ago, on both transfer of training and the theory of formal discipline was directed against what is commonly called "the old faculty psychology". Some writers have falsely ascribed faculty psychology to scholasticism. Aristotle and the scholastics did use the term "faculties" in reference to the ways in which the soul manifests its activities, but they never taught that these "faculties" were separate and independent, or that they were airtight compartments of the mind, as it were, each carrying out its own particular role in its own peculiar costume. Both Aristotle and the scholastics taught that soul and body together form a substantial union; that the soul is a spiritual substance and "informs" the body; that the "mind" acts as a unit. Now, a spiritual substance, because it is spiritual, is not capable of extension. In other words, it cannot be divided into parts or separate faculties, such as reason, memory, imagination, intellect, and will. The soul must be a unity and have unity of action.⁴²

The doctrine of formal discipline was challenged by the publication in 1901 of the investigations of Thorndike and Woodworth, who formulated the theory of identical elements. They believed that the spread of any improvement due to practice takes place only where identical elements are used in the influencing and influenced function⁴³.

Thorndike and his followers developed and expanded his theory as evidence was accumulated. The early

⁴² J.D. Redden and F.A. Ryan, A Catholic Philosophy of Education, Milwaukee, Bruce, 1948, p. 172-173, (Italics by author).

⁴³ E.L. Thorndike and R.S. Woodworth, "Influence and Improvement in One Mental Function Upon the Efficiency of Other Functions", Psychological Review, Vol. 8, No. 3, issue of May 1901, p. 247-261.

interpretations of identical elements as specific components of a learning situation were extended to include words or complex components.

What the theory of identical elements demands, wrote Woodworth, is that transfer should be of concrete performances; whether simple or complex makes no difference to the theory. Confusion will be avoided by using the word "constituent" or "component" in place of "element" and by speaking of the theory of identical components.⁴⁴

Rosskopf declared that

according to an eclectic point of view, the interpretation of "identical elements" can be as broad as the interpretation of "structure" by gestaltists.⁴⁵

However, Thorndike's influence was felt to be in support of learning by direct practice when he stated:

. . . every bond formed should be formed with due consideration of every other bond that has been or will be formed; every ability should be practiced in the most effective possible relations with other abilities.⁴⁶

The phrase "every bond formed" in the foregoing quotation might be interpreted to mean that an operation in arithmetic, or mathematics, is to be learned through direct practice of the operation; an operation becomes "fixed" in

⁴⁴ R.S. Woodworth, Experimental Psychology, New York, Holt, 1938, p. 177.

⁴⁵ M.F. Rosskopf, "Transfer of Training", The Learning of Mathematics, Its Theory and Practice, Twenty-first year-book of the National Council of Teachers of Mathematics, Washington, 1953, p. 208.

⁴⁶ E.L. Thorndike, The Psychology of Arithmetic, New York, Macmillan, 1922, p. 140.

the mind of a student through repetitive doing of exercises involving the operation⁴⁷. This view has been described by the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics as a fundamental weakness of Thorndike's position, since no transfer would be expected to take place "from processes involving rote learning, mechanical habits and drill"⁴⁸.

Judd⁴⁹ was one of the first psychologists in America to advance a doctrine different from that of identical elements. Judd experimented with subjects who were taught the principles of generalization involved in a task and compared their performance with that of subjects who did not know the principle. His objective was to test the effect of generalization on transfer. Although his test group was small and there was no adequate control group, yet his conclusion that knowledge of a principle facilitates transfer had a great effect.

For many years Judd advocated the importance of training in generalizing as a way of promoting transfer.

⁴⁷ Rosskopf, Op. Cit., p. 209.

⁴⁸ National Council of Teachers of Mathematics, The Place of Mathematics in Secondary Education, Fifteenth Yearbook, New York, Bureau of Publications, Columbia University, 1940, p. 218-219.

⁴⁹ C.H. Judd, "The Relation of Special Training to General Intelligence", Educational Review, Vol. 36, No. 1, issue of June 1908, p. 28-42.

He emphasized that much depends on how the subject is handled, and warned that:

The most promising subject in the curriculum can be turned into a formal and intellectually stagnant drill if it is presented by a teacher who has no desire to teach pupils how to generalize experience. On the other hand, a teacher who has the ability to train pupils to look beyond particular facts and to see their relations and their broader meanings can stimulate thinking with any material of instruction that comes to his hands.⁵⁰

In 1936 Judd restated his faith in his early doctrine with emphasis on "reconstruction of experience", perhaps under the influence of Gestalt:

It is of importance for an understanding of the nature of the higher mental processes that there be a clear realization of the fact that it requires time and laborious reorganization of experience for the individual to gain full comprehension of the meaning of the words which make up the number system.⁵¹

Judd continued:

At the higher levels of arithmetical thought and manipulation as well as at the lower levels, it is not enough that the mind acquire mere rules or successions of isolated ideas. There must be an organization of experience of a form that is . . . described by the term "conceptual".

50 C.H. Judd, Psychology of Secondary Education, Boston, Ginn, 1927, p. 422.

51 C.H. Judd, Education as Cultivation of the Higher Mental Processes, New York, Macmillan, 1936, p. 72.

Judd concluded that:

The view that all mental activities can be explained in terms of elements which are of the simplest and most primitive type overlooks altogether the principle that . . . organization . . . accounts for life.

If psychology is to rescue education from the new formalism, which consists in devotion to mere acquisitions of detached and unorganized facts - if mathematics, the natural sciences, and all other school subjects are to be taught by some method other than mere drill - there will have to be clear recognition of the difference between the lower and higher forms of mental activity. The higher forms of experience will have to be reached by the processes of education.⁵²

Lasar developed a method of teaching geometry in such a manner as to encourage generalization and showed how geometry might be utilized as "a medium for making the pupils conscious of the existence of logical patterns of valid and invalid reasoning in mathematics as well as in the thinking of every day life"⁵³.

Katona attempted to distinguish between meaningful and senseless learning by measuring the effect of understanding and repetition, as methods of learning, to solve life problems upon the retention and ability to apply what was learned to new school and life situations. He repeated these experiments many times, and found that while senseless learning did not transfer, meaningful learning did. In this

⁵² Ibid., p. 154-165.

⁵³ Nathan Lasar, "The Importance of Certain Concepts and Laws of Logic for the Study and Teaching of Geometry", The Mathematics Teacher, Vol. 31, No. 3, issue of March 1938, p. 100.

latter case, "100 percent transfer is the rule, not the exception"⁵⁴. In fact transfer effect amounting to more than 100 percent, which, according to commonly accepted interpretation of transfer experiments, is impossible, if not a contradiction in terms, is not at all unusual. Katona found that thirty six of the students in the understanding groups had better scores in the "new" tasks than in the "practiced" tasks; and that the occurrence of "infinite transfer effects" was shown to be at least theoretically possible⁵⁵.

McConnell⁵⁶ held that meaningful generalization is not so much concerned with procedure as with fundamental mathematical relations, emphasizing that the newer trends in the psychology of learning stressed the primacy of organization. It is the structure or organization or relationship that a pupil sees in a mathematics problem that allows him to arrive at a solution.

⁵⁴ George Katona, Organizing and Memorizing, New York, Columbia University Press, 1940, p. 130.

⁵⁵ Ibid., p. 278.

⁵⁶ T.R. McConnell, "Recent Trends in Learning Theory", Arithmetic in General Education, Sixteenth Yearbook of National Council of Teachers of Mathematics, New York, Bureau of Publications, Columbia University, 1941, p. 268-269.

6.- Efficacy of Subjects in Promoting Transfer.

There now remains for consideration the possibility that certain school subjects and activities are superior to others as a means of encouraging the growth and transfer of critical thinking. In his famous study of 1924, seeking to discover the disciplinary values of the high school studies, Thorndike reported: "The expectation of any large differences in general improvement of the mind from one study rather than another seems doomed to disappointment"⁵⁷. These investigations revealed that one might as well study sewing, cooking, physical training or bookkeeping as an equal amount of mathematics, Latin, Civics, or physics so far as any development of thinking habits was concerned. Thorndike's comment on his results was: "The differences are so small and the unreliabilities so large that the influence of the subject studied seems unimportant"⁵⁸. However, near the end of the report one reads, "A repetition of this experiment with 16,000 or 18,000 more cases is needed before final conclusions should be stated"⁵⁹.

⁵⁷ E.L. Thorndike, "Mental Discipline in High School Studies", Journal of Educational Psychology, Vol. 15, No. 127, issue of February 1924, p. 98.

⁵⁸ Ibid., p. 95.

⁵⁹ Ibid., p. 97.

Parker set up a controlled experiment in an effort to answer the question, "Can pupils of geometry be taught to prove theorems more economically and effectively when trained to use consciously a technique of logical thinking; and furthermore, does such training more than the usual method, increase the pupils' ability to analyze and see relationship in other, non-geometrical, situations?" An experimental and a control group were taught by the same teacher. The pupils in the experimental group studied the thought processes used in proving geometric theorems and gained some understanding of the nature of logical thought, while in the control group the theorems themselves were recognized as of major importance and little attention was given to the thought processes involved in proving them. The results of the training were measured by requiring pupils to prove geometric theorems before and after their preparation. Parker concluded:

These data would seem to offer conclusive evidence, insofar as one experiment can be considered to do so, that when pupils are taught to use consciously a technique of logical thinking, they try more varied methods of attack, reject erroneous suggestions more readily, and without becoming discouraged maintain an attitude of suspended judgment until the method has been shown to be correct. The data on the reasoning tests . . . indicate that . . . training in logical thinking . . . tends to carry over these methods of attack and this attitude . . . to other problem situations not concerned with geometry.⁶⁰

⁶⁰ E. Parker, "Teaching Pupils the Conscious Use of a Technique of Thinking", The Mathematics Teacher, Vol. 17, No. 4, issue of April 1924, p. 200-201.

Perry constructed a special reasoning test by selecting items from several standardized reasoning and intelligence tests, and equated two control groups and one experimental group on the basis of scores received on her special composite reasoning test. Each group was composed of three divisions of reasoning ability -- average, superior, and very superior. In each control group instruction in geometry was definitely guided by a textbook. In one of these groups, the book propositions were emphasized, while in the other the emphasis was placed on the proving of original exercises. No attention was given in either group to any particular method of thinking and in each case, the class was conducted by the question and answer method. The experimental geometry group was first taught the technique of reasoning such as analysis of facts into their elements, and purposive thinking when formulating and testing possible solutions. Emphasis was placed on the "if-then" type of thinking as well as the analytic method. The techniques learned were then applied to geometry problems. The results showed that the experimental technique markedly aided the average pupils, but was detrimental to the very superior group. Perry concluded that the experimental method in reasoning helped decrease difficulties in the solution of exercises in geometry and also developed worthwhile habits

leading to successful solutions. Perry did not include non-mathematical subject matter in the course but in the experimental group the ability to solve problems of non-mathematical character was markedly improved, following the period of training in the solution of exercises in geometry. This improved ability was more noticeable in the results of those tests which were similar to the type of reasoning emphasized in demonstrative geometry in form and content⁶¹.

Hall⁶² purposely selected students for their lack of ability to do the work in a regular geometry class, and approached the subject with them by showing the need of proof in the students' every day lives. The experiences of the pupils were used for illustration in trying to bring about an understanding of the methods of drawing valid conclusions and the concepts of proof as used in geometry. Hall concluded that geometry can be used as an effective and convenient vehicle for teaching the nature of valid inference if the whole structure of geometric applications and life applications are interwoven. Hall held that the fact of transfer of training from one activity to related activities seems established. He emphasized further that the equally

⁶¹ W.M. Perry, A Study in the Psychology of Learning in Geometry, Contributions to Education No. 179, New York, Teachers College, Columbia University, 1925, p. 34.

⁶² L.E. Hall, "Applying Geometric Methods of Thinking to Life Situations", The Mathematics Teacher, Vol. 31, No. 8, issue of December 1938, p. 379-384.

important consideration -- that in order to obtain transfer we must teach for it -- has not received the attention due it by teachers of geometry.

Fawcett⁶³, in a study that had a pronounced impact on the teaching of geometry, described classroom procedures which might be used to cultivate critical and reflective thought and evaluated the effect of such procedures on the thinking of pupils. He investigated three related problems: (1) the problem of leading the pupil to understand the nature of deductive proof through the study of geometric situations, (2) the problem of generalizing this experience so that effective transfer will result, and (3) the problem of evaluating the resulting change in the behavior of the student. His class of twenty-five students met four times a week for a period of two school years. The first four weeks were spent in discussing undefined terms, definitions, and assumptions. Classroom practice centered on the method by which theorems are established. The method of proof was applied to the analysis of arguments in connection with the fields to which transfer was desired. Editorials, advertisements, political speeches, and court evidence were analyzed

63 H.P. Fawcett, The Nature of Proof, Thirteenth Yearbook of the National Council of Teachers of Mathematics, New York, Bureau of Publications, Columbia University, 1938, VIII-146 p.

for basic assumptions and for the evidence offered in support of the given conclusions. Fawcett reported the following results:

1. Mathematical method illustrated by a small number of theorems yields a control of the subject matter of geometry at least equal to that obtained from the usual formal course.

2. By following the procedures outlined, it is possible to improve the reflective thinking of secondary school pupils.

3. This improvement in the pupil's ability for reflective thinking is general in character and transfers to a variety of situations.

4. The usual formal course in demonstrative geometry does not improve the reflective thinking of the pupils.⁶⁴

After Thorndike and Woodworth's theory of identical elements, the most important formulation of the process of transfer of training was Judd's generalization theory. Katona stressed the importance of discovery, of explanation, of reconstruction and reorganization of experience as well as the importance of non-verbalized knowledge of a principle. McConnell, Fawcett, Hall, Lazar, Parker and Perry, in the field of mathematics, all supported the view that the learning of content alone of any subject is not likely to give general training to the mind, nor to develop a generalized ability to think critically.

⁶⁴ Ibid., p. 119.

Father Castiello, S.J.⁶⁵, an authority in the field of transfer, expressed the following truths:

The amount of transfer is in direct proportion to the degree of intelligence possessed and the method of teaching. A powerful intellect is capable of seeing many relations, unifying his knowledge more and, therefore, making much more transfer. That has been shown in an experiment by Overman. What Overman probably did not know is that he was restating St. Thomas' thesis on the synthetical power of "genius". The very intimate connection between method of teaching and amount of transfer has also been proved by numerous experiments. In fact, it has been shown that the amount of transfer can be multiplied by five or six if the method is changed. That is what one would expect. Narrow views and sheer mechanical drill coop up the mind. Breadth of intelligence and a capacity for linking up and unifying all knowledge must effect much more transfer from one field to another.⁶⁶

Orata⁶⁷ summarized the results of research as follows:

First, transfer is a fact, as revealed by nearly eighty percent of the studies; second, transfer is not automatic process that can be taken for granted, but it is to be worked for . . .; and third, the amount of transfer is conditioned by many factors, among which are: age; mental ability; (possibly) time interval between learning and transfer; degree of stability attained by the learned pattern; "knowledge of directions, favorable attitude toward the learning

⁶⁵ J. Castiello, A Humane Psychology of Education, New York, Sheet and Ward, 1936, XXXI-254 p.

⁶⁶ Ibid., p. 172

⁶⁷ P.T. Orata, "Recent Research Studies on Transfer of Training with Implications for Curriculum, Guidance, and Personnel Work", Journal of Educational Research, Vol. 35, No. 2, issue of October 1941, p. 81-101.

situation, and efficient use of past experience"; accuracy of learning; "conscious acceptance by the learner of methods, procedures, principles, sentiments and ideals"; meaningfulness of the learning situation; the personality of the subject --greater transfer in extroverts than introverts; method of study; suitable organization of subject matter presentation; and provision for continuous reconstruction of experience.⁶⁸

It is evident that the entire doctrine of mental transfer is in a state of flux, changing with the various research findings. Many teachers of geometry continue to be bogged down by the quicksands of shifting theories of mental training. In fact methods of instruction used by some teachers today are based on formal discipline. The appeals of lay people, especially, "for a return to the good old days of really 'hard' instruction imply a theory of learning based on formal discipline"⁶⁹.

Statement of the Problem.- In the years -- in many cases decades -- since the research summarized in this chapter was published, the teaching of geometry has undergone substantial development, and procedures for measuring critical thinking ability have been improved. It may therefore be timely to inquire in what ways and to what degree the learning of geometry as it is now organized in Pittsfield High School,

⁶⁸ Ibid., p. 81-82.

⁶⁹ Rosskopf, Op. cit., p. 206-207.

contributes to an improvement in the ability to think critically in non-geometric contexts.

The principal questions to be answered by this research are: (1) Do high school sophomores who have taken plane demonstrative geometry improve more in critical thinking ability than those pupils who have not taken geometry? (2) What is the relationship between achievement in geometry and critical thinking ability? (3) What is the relationship between studying other subjects during the sophomore year and critical thinking ability?

CHAPTER II

HISTORY AND TEACHING OF GEOMETRY

At this point it will not be amiss to sketch the history of geometry and to recall the methods that have been proposed to teach it.

1.- The History of Geometry.

The word "geometry" is derived from the Greek words for "earth measure". In both Egypt and Babylonia geometry was developed as the art of measuring land in connection with agriculture and irrigation. Here, then, we have the origin of the word geometry -- the "earth measurement" born of the necessity for the frequent measurements brought about by the overflow of the banks of the Nile¹.

The very fact, however, that the geometry of the Egyptians was comprised almost entirely of constructions and measurements, goes a long way to explain certain of its defects. The Egyptians failed to construct a rigorously logical system of geometry resting upon a few axioms and postulates; they were unable to bring special cases under a more general

¹ D.E. Smith, History of Mathematics, New York, Ginn, 1925, Vol. II, p. 270.

view and thereby arrive at broad and fundamental theories². Their rules of practical geometry were based merely on experiment, and their geometry was wholly empirical and intuitive without the idea of logical demonstration.

The active commercial intercourse that sprang up between Greece and Egypt about the seventh century B.C. resulted, naturally, in the interchanging of ideas as well as of goods. The Greeks, always striving for the acquisition of knowledge, sought out instruction from the Egyptian priests. Thales, Pythagoras, Plato, and Eudoxus, all visited Egypt and from the time these Greek philosophers started to study Egyptian geometry, this branch of mathematics assumed a radically different aspect. The Greeks sought continually to discover the reasons for things. They enjoyed the "contemplation of ideal relations, and they loved science as science"³.

To Thales of Miletus, an energetic and enterprising merchant who went to Egypt to make money, falls the distinction of having introduced the study of geometry into Greece. On his return to Asia Minor, Thales founded the Ionian School of philosophy and mathematics in Miletus, then a city of great

² F. Cajori, A History of Mathematics, New York, Macmillan, 1931, p. 11.

³ Cajori, Op. cit., p. 15.

wealth and influence. Here mathematics was studied not for its own sake, but as a general preparation for philosophy⁴.

Thales, in 600 B.C. was the first person who logically demonstrated a theorem in geometry. In all he proved only five theorems⁵, but he is given credit for being the first to organize geometry as a science and to prove his conclusions.

The greatest and most famous disciple of Thales was Pythagoras (580-500 B.C.). No mean traveler himself, Pythagoras was unquestionably influenced by his visits to Egypt, Babylonia, and probably Persia and India. In later life he established a school at Crotona, a Greek colony in South-eastern Italy. His school actually constituted a secret brotherhood devoted to philosophy and mathematics, the members of which were united for life. They were under oath never to divulge the discoveries and doctrines of their school. It was therefore very difficult to determine to whom each particular discovery was to be ascribed as the Pythagoreans were in the habit of referring every discovery back to Pythagoras⁶.

4 Cajori, Op. cit., p. 16.

5 Alva W. Stamper, History of Teaching of Geometry, New York, Teachers College, Columbia, 1909, p. 11.

6 Cajori, Op. cit., p. 18.

Pythagoras and his followers left behind no mathematical treatises, but although sources are scanty, it has been established that mathematics was the principal study. Pythagoras raised mathematics to the rank of a science, stressing arithmetic as strongly as geometry. His geometry reflected this closeness to arithmetic. Much like the geometry of Egypt, the geometry of the Pythagoreans is concerned a great deal with areas. The famous theorem that the square on the hypotenuse is equal to the sum of the squares on the other two sides of a right triangle is ascribed to Pythagoras because it is supposed that Pythagoras was the first to prove it; he probably learned the truth of the theorem from the Egyptians in the special case when the sides are 3, 4, 5, respectively, and applied this truth to any right triangle⁷.

It is believed that the political activities of the Pythagorean School led a fearful Greek government to disband the fraternity. Conflicting stories indicate that Pythagoras and many of his followers were either killed or driven into exile⁸.

The history of geometry now passes to the year 400 B.C. A short distance outside the city of Athens was a pleasant

⁷ Cajori, Op. cit., p. 18.

⁸ Thomas Smith, Euclid, His Life and System, New York, Scribners, 1902, p. 25.

garden, surrounded by a wall and containing groves and fountains. This estate was believed to have belonged to Akademos and so it was called Akademia after him. Here Plato taught his philosophy⁹.

Plato was very "modern" in his approach to the teaching of mathematics: mathematical applications were to be connected with boxing and wrestling matches to keep the pupil wide awake.

Although Plato was supposed to have placed the inscription "Let no man destitute of geometry enter my doors" over the entrance to his school, he did very little either to extend or consolidate geometrical science, and his school did not send out any great mathematicians¹⁰. He laid major stress on the principles of reasoning and methods of demonstration as they applied to the proof of geometric facts. In the Academy geometry was studied as a means rather than an end, a philosophy rather than a science, a discipline that was conducive always to the right study of all philosophy. It was due to Plato's influence that mathematicians who followed began the subject with a carefully worded list of definitions, postulates, and axioms. It was Plato who limited geometricians to the use of the unmarked ruler and

⁹ Alfred Hooper, Makers of Mathematics, New York, Random House, 1948, p. 44.

¹⁰ Thomas Smith, Op. cit., p. 35-38.

compass, and his influence kept curves other than the circles out of plane geometry. Unconcerned with the uses of mathematical knowledge, Plato saw in geometry one of the means for the best possible mental training of future leaders of men¹¹.

Geometry was so used by one of the greatest leaders of all time; while still an unknown lawyer, Abraham Lincoln at the age of forty sought to train his mind by studying the first six books of Euclid, written some three hundred years before the Christian era¹². Euclid was not the world's greatest mathematician, but he may be credited with writing the world's best selling mathematical textbook of all times.

Very little is known of the author whose book was to be studied for more than two thousand years. Until about fifty years ago, school geometries had the name Euclid as their only title. There is reason to believe that Euclid studied at Athens before he became a teacher of mathematics at Alexandria in Egypt. His was a keen and orderly mind, able to collect and arrange the mathematical knowledge of his time (little better than a maze of unrelated facts) into a clear and logical structure¹³. No doubt there were many

11 Hooper, Op. cit., p. 44.

12 E.T. Bell, Men of Mathematics, New York, Simon and Shuster, 1937, p. xx.

13 Hooper, Op. cit., p. 47.

propositions that were original to Euclid; but the feature which made his Elements famous, and which accounts for the fact that it is the oldest scientific textbook still in use, is found in its simple but logical sequence of theorems and problems¹⁴. He arranged his Elements in books, each written on a separate parchment scroll:

- I Congruence,
- II Identities,
- III Circles,
- IV Inscribed and Circumscribed Regular Polygons,
- V Proportion,
- VI Similarity,
- VII-IX Arithmetic,
- X Incommensurables,
- XI-XIII Solid Geometry.¹⁵

Euclid's influence on our own present geometry is evident from this list; it seems uncanny that for twenty centuries his first six books were the student's usual introduction to geometry.

An examination of his Elements discloses the following features: (1) lack of practical applications; (2) omission of original exercises, every proposition being fully proved; (3) exclusion of hypothetical constructions (hence Euclid began his geometry with constructions); (4) a general plan for the proof of all propositions; (5) restriction of constructions to those possible by use of the compass and straight edge only.

¹⁴ D.E. Smith, History of Mathematics, New York, Ginn, 1925, Vol. I, p. 103-106.

¹⁵ D.E. Smith, Op. cit., p. 106.

Since the time of Euclid much that has been discovered in geometry has not been incorporated into high school courses. About 200 B.C. Appollonius studied at the school in Alexandria where Euclid had taught, and he became one of its most illustrious alumni. He did his greatest work in conic sections; history records the fact that Appollonius contributed more to this subject than Euclid had done to elementary geometry. Appollonius showed how the parabola, hyperbola, and ellipse (all named by him) could be produced from sections of a cone.

His methods were so effective that it was not until 1637 that they were changed by the great French mathematician Descartes in the book La Geometrie that completely revolutionized the treatment of conic sections, and for that matter, all Greek geometry¹⁶. Descartes expressed geometric relations algebraically in a coordinate system still used today. This discovery tended to widen the scope of geometry by associating many parts of it with algebraic symbols, although the influence on modern high school geometry is still largely unfelt.

Another great geometrician belonging to the Alexandrian School was Archimedes, although he is usually remembered for his mechanical inventions and his work in connection

¹⁶ Hooper, Op. cit., p. 53.

with mechanics and hydrostatics. He used the experimental method to discover geometric truths which he later proved logically. This great mathematician of antiquity perished at the hand of a Roman soldier as he helped to draw up the strategy in the defense of Syracuse¹⁷.

In modern times the content of geometry has been enlarged by the contributions of many capable mathematicians, mostly from France, Germany, and England¹⁸.

The most outstanding French work on geometry was Legendre's Elements de Geometrie, which appeared in 1794 and became widely studied both in Europe and the United States. Legendre departed from Euclid in some important respects. He referred to arithmetic and algebra for the treatment of proportion and assumed the correspondence between line segment and number. Euclid has insisted that all constructions as well as theorems be proved before they could be used; Legendre, however, permitted "hypothetical constructions". Although Legendre simplified the treatment, he was rigorous in his logic. He grouped the propositions by topic and separated the theorems from the problems, while Euclid had showed little interest in such matters but aimed primarily to arrange the propositions in the most logical chain of

17 Hooper, Op. cit., p. 63.

18 Stamper, Op. cit., p. 55-103.

reasoning. Legendre went beyond Euclid in his more complete treatment of solid geometry. Perhaps the principal contribution of Legendre was his demonstration that geometry can be placed on a sound logical basis different from that of Euclid in sequence and method¹⁹.

Geometry has had a long and stimulating history. Unquestionably its outstanding feature is the manner in which the ideas and teachings of Plato and Euclid have dominated plane demonstrative geometry for over 2000 years. Teachers and students of geometry have been followers of Euclid, usually indifferent to other geometric materials equally valuable and to other geometers equally challenging. Some of the higher branches of mathematics today make use of geometries that are based on first principles quite different from those chosen by Euclid. The geometries of Lobachevsky (1793-1856) and Riemann (1826-1866) are the outstanding examples of non-Euclidean geometries, equally as logical as Euclid's²⁰.

19 Stamper, Ibid., p. 86.

20 Hooper, Op. cit., p. 47.

2.- Teaching of Geometry.

The Greeks made the first definite contribution to the formal educational curriculum of the western peoples. Their liberal arts or studies were music, rhetoric, dialectic, arithmetic, geometry, and astronomy. Arithmetic and geometry were "mentioned as the more advanced studies"²¹. It may be observed that even then there was the beginning of the division between what would later be termed the trivium and the quadrivium. There is no record of changes in the Greek curriculum of the liberal studies until the first century of the Christian era²².

The organizing genius of the Romans was a great preserving force for the conservation of the Greek contribution to civilization, especially in the development of the Roman system of secondary education. Although at first the education of the Roman youth was carried out by Greek teachers in the Greek language, strong efforts were exerted to establish a national literature and a national education.

M.T. Varro (116-27 B.C.) attempted to construct a system of education based on Greek concepts, but with Roman rather than Greek literature as a foundation. To implement

²¹ Paul Abelson, The Seven Liberal Arts, A Study in Medieval Culture, New York, Teachers College, Columbia University, 1906, p. 3.

²² Ibid., p. 3.

this objective, Varro wrote treatises on every subject field taught in Greece, but omitted drawing and included astronomy. His last work, entitled Disciplinarum Libri Novem, is known to have included works on grammar, rhetoric, dialectic, geometry, arithmetic, astrology, music, medicine, and architecture. Regarding the treatises of Varro, Abelson has stated:

The books were undoubtedly written for the secondary schools, which were at this time already clearly differentiated from the higher rhetorical schools. The practical secondary curriculum of the day was grammar in its narrow sense, and also literature, arithmetic, geometry and music -- a repetition of the Greek curriculum of the third century before Christ²³.

Varro's efforts to establish a modified Greek curriculum in Rome were unsuccessful, perhaps because of the fact that rhetoric and dialectic soon came to be regarded as advanced studies. The study of mathematics was reduced to a minimum in the Roman course of study with major emphasis placed not on the contemplative aspects but on the practical applications.

Toward the close of the fourth century the fixed character of the curriculum represented by the practices of pagan schools underwent marked changes. The Christian Church, with its foundations based on the principles of universal

23 Ibid., p. 4-5.

peace, forged an influence that transcended the bonds of national and ethnic groups. The best of the Greek and Roman culture persisted or was absorbed bodily by the Church. The seven liberal arts, as expounded and outlined by Martius Capella about the middle of the fifth century, served as a pattern for the secondary school curriculum throughout the Middle Ages.

Although Capella's curriculum may not have been generally followed in toto, the underlying principles, based on those of Varro, were significant. The trivium (grammar, rhetoric, and dialectic) was the "core of general education; the quadrivium (geometry, arithmetic, music, and astronomy) was to comprise the "content" subjects. The manner of grouping the subjects into two categories perhaps provided the basis for the struggle in later centuries between the proponents of the humanities and those of the sciences. The teachers of Rome and Greece, however, never intended the grouping to become an antithesis because they believed both the trivium and quadrivium necessary to their concept of a complete education of their youth²⁴.

Under the direction of the Church, mediaeval education wielded a great influence in the preservation of the culture of the ancient world, especially since the mediaeval teacher's

²⁴ Ibid., p. 7-10.

dependence upon the book as his source of information and wisdom led to the widespread use of prepared "texts", including a formidable list of books used in the study of the seven liberal arts. The early texts in geometry, the source of knowledge of geometry throughout the Middle Ages, were written almost entirely by hand²⁵. The interest in geometry during this period tended to be cursory since more emphasis was placed on the humanities.

At the close of the sixteenth century, interest in geometry was revived largely because of the influence of Kepler. More famous as an astronomer, Kepler did not produce a systematic exposition of geometry²⁶; he dealt with general geometric principles and illustrated them by a few cases.

The history of teaching of geometry during the past two hundred years clearly reveals two tendencies of importance to this study. First, there was a tendency to place geometry earlier in the curriculum. In 1726 the curriculum of Harvard and other colleges included geometry as a part of the liberal arts training for seniors only. During the latter part of the eighteenth century Euclid was taught to under-class men at Harvard, Yale, and Pennsylvania. This downward trend

25 H.R. Douglas, Editor, The High School Curriculum, New York, Ronald Press, 1947, p. 7-10.

26 W.W. Rouse Ball, A Short Account of the History of Mathematics, London, Macmillan, 1927, p. 256-257.

continued until today we find demonstrative geometry taught to sophomores and even freshmen in our high schools.

The second tendency was the outgrowth of the first. Euclid had to be modified to make it attainable by the less mature learner. This was to prove a long and extremely difficult task, as might be expected in the case of a book with such prestige and wealth of tradition. Rigid adherence to the original Euclid as a sacred duty continued to dominate the curriculum for a long time, leading to an unproductive formalism²⁷.

This inflexible state of affairs contributed to widespread attacks on high school geometry. Defended as a course in reasoning, geometry often degenerated into sterile memory work, with the result that many educators advocated the elimination of geometry from the programs of most high school pupils. David Eugene Smith warned in 1928.

It would be disastrous to sacrifice the courses in algebra and geometry that we now have, since we have nothing worthy of the student's vigorous mental attack that can take their place.²⁸

27 L.E. Mensenkamp, "Some Desirable Characteristics in a Plane Geometry Text" in The Teaching of Geometry, Fifth Yearbook, National Council of Teachers of Mathematics, New York, Bureau of Publications, Teachers College, Columbia University, 1938, p. 200.

28 David Eugene Smith, Mathematics in the Training for Citizenship, Third Yearbook, National Council of Teachers of Mathematics, New York, Bureau of Publications, Teachers College, Columbia University, 1928, p. 11-23.

Nevertheless the percentage of pupils enrolled in geometry in high schools declined from 30.9 per cent in 1910 to 11.6 per cent in 1953²⁹. Part of this decline was due to the increased holding power of secondary education and the provision of a variety of curricula in which geometry may not be studied. However, according to Brown, there was a decline between 1934 and 1953 not only in percentages but in actual numbers of pupils studying geometry in high school.

For a generation friends of geometry have championed long-overdue changes in the content and teaching of geometry: that propositions receive less attention and that emphasis be transferred to original exercises and the development of analysis, even if a reduction of the sum total of the elements taught in geometry had to be accepted³⁰. The National Committee on the Reorganization of Mathematics in Secondary Education devoted a major portion of its findings to geometry. In its report the Committee recommended a list of fundamental theorems with two bases of selection: (1) the extent to which the propositions and corollaries were used in subsequent

29 K.E. Brown, "Status of Mathematics Education in Public Secondary Schools", Mathematics in Secondary Schools Today, Bulletin of National Association of Secondary School Principals, Vol. 38, No. 203, issue of May 1954, p. 56.

30 Jack D. Wilson, "Trends in Geometry", in The Mathematics Teacher, Vol. 46, No. 2, issue of February 1953, p. 67.

proofs of important propositions and exercises; (2) the value of the propositions in completing important pieces of theory³¹.

Since colonial times the textbook has been the chief tool in schools, and mathematics has been no exception to this rule. In recent decades publishers of mathematics textbooks have been alert to new ideas and have generally been willing, often with the danger of financial loss, to incorporate new teaching trends into textbooks. Frequently such innovations have become generally accepted and have contributed significantly to educational progress. The great responsibility which rests on the authors and publishers of textbooks is emphasized by the tendency of many adults of today to accept without question what they read on the printed page, -- a reflection of the dominant role that textbooks may have played in their own education.

It is axiomatic that a course in geometry must depend, in addition to good teaching, on suitable instructional materials, of which there is none more important than the textbook.

Authors of textbooks written since the report of the National Committee on the Reorganization of Mathematics

³¹ National Committee on Mathematics Requirements, The Reorganization of Mathematics in Secondary Education, 1927, p. 55-60.

in Secondary Education have increasingly recognized that Euclid did not write for high school students but for mature minds whose intellectual interests and abilities conditioned them for the challenge of severe postulational thinking.

Authors of effective geometry textbooks are likely to be mathematicians who combine a thorough grounding in classical and modern mathematics with successful teaching experience in present day high schools and an understanding of how adolescents learn.

The difference between the Euclidean treatment and a modern approach is clarified by the following description of the geometry textbook in use in Pittsfield High School:

(1) The older division into the usual books is disregarded and the material is organized into twelve divisions, each with a high degree of unity³². The newer basis of organization was stated in the second report of the Commission on Post-War Plan as follows: "The work of each year should be organized into a few large units built around key concepts and fundamental principles"³³.

(2) The psychology of learning is recognized and applied consistently. Carefully planned and graded material

³² Theodore Herberg and Joseph B. Orleans, A New Geometry for Secondary Schools, Second Edition, Boston, Heath, 1948, p. 111.

³³ National Council of Teachers of Mathematics, "The Second Report of the Commission of Post-War Plans", in The Mathematics Teacher, Vol. 38, No. 5, issue of May 1945, p. 209.

is provided for the growth, formulation, and application of concepts. This is particularly true of the introduction to the study of congruence, parallels, locus, similarity, and areas. As a basis for developing clear concepts, provisions are included for the visual recognition of relationships in figures before "word exercises" are presented³⁴. Ogden and Richards called attention to the danger of assuming that technical terms have independent and uniform meaning for learners:

Words, as everyone now knows, "mean" nothing by themselves, although the belief that they did . . . was once equally universal. It is only when a thinker makes use of them that they stand for anything, or in one sense, have "meaning". They are instruments.³⁵

(3) The textbook employs and students are encouraged to acquire mathematically adequate language -- clear, concise, correct, and cogent. Mathematical language, complementing ordinary language, is a tool for thinking, of great value in dealing with novel quantitative situations. The effective use of mathematical language is stimulated by correlation with every day life, industry, science, and aviation³⁶.

³⁴ Herberg and Orleans, Op. cit., p. iii. (Italics by authors).

³⁵ C.K. Ogden, I.A. Richards, The Meaning of Meaning, New York, Harcourt Brace, 1927, p. 9-10.

³⁶ Herberg and Orleans, Op. cit., p. iii.

Brune stressed the semantic implications of mathematical instruction:

Teachers seek to transfer this consciousness of semantic principles to situations outside the classroom. Citizens in a democracy need to be wary of words. They encounter at every hand words designed for them to accept with a minimum of careful evaluation. Instead of encouraging pupils to memorize empty symbols, teachers in America should help pupils to think critically and weigh their own and others' words.³⁷

The geometry textbook encourages the pupil to read the printed page for guidance, instead of depending entirely on the recall of oral classroom explanations.

Ability to comprehend mathematical discourse is considered to be a valid and significant goal. Henderson and Pingry emphasized this aspect of mathematical instruction when they wrote: "The mathematics teacher is a reading teacher"³⁸.

(4) The number of assumptions has been increased. By treating as assumptions the conditions for congruence, parallelism, and similarity, a thoroughly logical course has been built that is much simpler than the usual course. The

³⁷ I.H. Brune, "Language in Mathematics", in The Learning of Mathematics, Its Theory and Practice, Twenty-first Yearbook, Washington, National Council of Teachers of Mathematics, 1953, p. 189.

³⁸ K.B. Henderson and R.E. Pingry, "Problem-Solving in Mathematics" in The Learning of Mathematics, Its Theory and Practice, National Council of Teachers of Mathematics, Washington, 1953, p. 250.

place of the assumption in geometry is described as follows:

The goal of reducing the number of assumptions to a minimum is admittedly one of advanced rather than of elementary mathematics. For slow and average pupils much can be gained by postponing to the end of the course proofs of theorems which are well known to be stumbling blocks. The proofs of all assumed theorems have been placed at the end of the book where they can readily be used if desired.³⁹

Birkoff and Beatley, in discussing postulational thinking stated:

Should we not rather . . . impress on them the need of certain assumptions, and show that this need is not peculiar to geometry, but is inherent in all logical systems? Could anything but good come from our indicating the possibility of some latitude in the choice of assumptions for geometry, and showing that each such choice leads to a slightly different approach to geometry, each valid with respect to its own groups of assumptions and to no other?⁴⁰

The postponement of the proofs of assumed theorems agrees with the teaching procedure that prevails in many classrooms, where a proof may be omitted, because it is not required or postponed until later in the year because of its difficulty. Although teachers may be justified in this practice, they recognize the undesirability of having pupils meet numerous text-presented proofs which they are told to skip.

³⁹ Herberg and Orleans, Op. cit., p. iv.

⁴⁰ G.D. Birkhoff, Ralph Beatley, "A New Approach to Elementary Geometry", in The Teaching of Geometry, Fifth Yearbook, National Council Teachers of Mathematics, New York, Teachers College, Columbia University, 1930, p. 87.

(5) A limited number of theorems are proved deductively, but other theorems and a large number of corollaries and formulas are treated by a modified inductive method, in which formal reproduction of text-presented proofs is minimized. Although the deductive method is given the attention its importance deserves, the pupil is encouraged and guided in discovering geometric relationships analytically as well as inductively.

Nygard urged the departure from the traditional emphasis on text-presented theorems as follows:

In each topic only a few key theorems should be completely proved in the text book. Much of each topic should be presented so as to challenge the students to make their own investigations and to report the results of their own research.⁴¹

Skolnik recommended a balance between the deductive and inductive approaches:

For the study of geometry to encompass a general way of thinking, induction must of course be an integral part of the work. But it must not be an escape from deductive proof, nor must it be a deprecation of the inductive method.⁴²

The theorems, corollaries, and formulas, inductively developed, constitute a rather complete collection of tools for the development of original proofs and the solution of computational exercises.

⁴¹ P.H. Nygard, "A Functional Revision of Plane Geometry", in The Mathematics Teacher, Vol. 34, No. 6, issue of October 1941, p. 272.

⁴² David Skolnik, "Let's Forget Mental Transfer", in The Mathematics Teacher, Vol. 45, No. 3, issue of March 1952, p. 164.

(6) Integration of mathematics is sought in numerous ways within the fundamental framework of demonstrative geometry: arithmetic and algebra are utilized in the development of relations involving angles, proportions, areas, etc., and in the applications of these relations; numerical trigonometry is introduced naturally and applied frequently; concepts and skills are applied to three-dimensional situations⁴³.

(7) Relating geometric thinking to non-geometric situations is frequently employed as a means of motivating the logical emphasis of the course in demonstrative geometry. Since pupils learn subjects best in connection with their use in meaningful situations, the study of non-geometric exercises of logical reasoning that have relevance to geometric thinking facilitates the learning of geometric content, as in the treatment of the meaning of the definition, indirect proofs, truth of converses, fallacious reasoning, etc. A critical and appreciative attitude toward reasoning in every-day life can be made to contribute to greater insight into geometric thinking, and conversely.

The New York State Mathematics Committee stated the importance of conscious attention to the nature of logical

⁴³ Herberg and Orleans, Op. cit., p. v.

thinking as follows:

The concepts of the nature of proof as well as the logical structure exhibited in a chain of propositions are essential objectives in the teaching of formal geometry.⁴⁴

William David Reeve reiterated in March 1954 his long-standing convictions on the dominant role that must be assigned to the nature of proof in geometry:

The purpose of the geometry course is to show the student the meaning of a demonstration, to enable him to have the satisfaction of knowing something, to let him see the significance of mathematical precision and feel the joy that comes from discovering absolute truth. In other words, the work in geometry is the only course in deductive logic which a student can get in a secondary school. It is better than pure logic because it has in it figures that illustrate the things that are said and done. If geometry is not taught to give a training in logic or in the power to prove one's statements, or if it does not do both, then it is not worth teaching.⁴⁵

The course of geometry, at Pittsfield High School (see Appendix A), is organized around the following objectives, based on the exhaustive study of the Joint Commission of the Mathematical Association of America and the National Council of the Teachers of Mathematics⁴⁶, which reported that

⁴⁴ Mathematics Syllabus Committee, Tenth Year Mathematics, Albany, University of the State of New York, 1943, p.5.

⁴⁵ William David Reeve, "General Mathematics in the Secondary School-II", in the Mathematics Teacher, Vol. 47, No. 3, issue of March 1954, p. 174. (Italics mine).

⁴⁶ National Council of Teachers of Mathematics, The Place of Mathematics in Secondary Education, Fifteenth Year-book, New York, Bureau of Publications, Columbia University, 1940, p. 92-93.

the effective course in geometry had to:

1. Increase the pupils' knowledge of geometrical forms and his understanding of geometrical concepts.

2. Acquaint the pupil with the basic geometric propositions and their applications.

3. Encourage further geometric investigation and independent thinking by means of well selected and up-to-date exercises.

4. Make clear the need for well-defined terms, precision of statement, and accuracy of thinking for the purpose of establishing conclusions, both geometric and non-geometric.

5. Develop the power of deductive reasoning as a method of thinking, and to apply this ability in other fields.

6. Arouse in the pupils an appreciation of the significance of geometry in its applications to science, engineering, and the arts.

7. Correlate the skills of arithmetic and algebra with those of geometry to give a deeper understanding of mathematical relationship.

8. Encourage three-dimensional thinking as an outgrowth of two-dimensional thinking, and thus stimulate the development of spatial imagination.

CHAPTER III

EXPERIMENTAL PROCEDURE

The present experimenter made a survey of the tests which had been developed to measure critical thinking ability, valid for use with high school students, and of sufficient length and discriminatory power to provide high enough reliability for group testing. The Watson-Glaser Critical Thinking Appraisal (1951-52 Editions) (Appendix 2), was determined to be nearest to satisfying the standards mentioned above and it was this test that was used in the experiment. The form used was the third revision of the 1941 edition of the Watson-Glaser Tests of Critical Thinking, devised particularly for use with high school and college groups. It should be noted that the 1941 edition was revised in 1949 and reissued as an experimental edition. Many improvements of the items as well as to the subtests resulted from repeated tryouts on various groups through the use of data derived from item analysis, intercorrelation studies and analysis studies bearing upon the concept and measurement of critical thinking. The 1951 edition¹ constituted an entirely new battery, and included only those tests and items which were found to be most reliable, valid and useful!

¹ Goodwin Watson and Edward M. Glaser, Watson-Glaser Critical Thinking Appraisal Manual of Directions, Yonkers, World Book, 1952, p. 9.

1.- Description of Test.

The Critical Thinking Appraisal, considered as a whole, is intended to measure the following constituents of the ability to think, as analyzed by the authors of the test:

- (a) An attitude of wanting to have supporting evidence for opinions or conclusions before assuming them to be true.
- (b) Knowledge of the methods of logical inquiry which help determine the weight of different kinds of evidence and which help one to reach warranted conclusions.
- (c) Skill in employing the above attitude and knowledge.

A critical thinker effectively examines beliefs or proposals in the light of supporting evidence, of the relevant facts in the case, instead of jumping prematurely to a conclusion.

Critical thinking requires that one be able to comprehend and use language for accurate and discriminating communication of thought, recognize the existence (or non-existence) of logical relationships between propositions, interpret data and draw warranted conclusions or generalizations, appraise the adequacy and weight of alleged evidence, weigh it and judge between different degrees of probability of certain conclusions, recognize unstated assumptions, and evaluate arguments.

Critical thinking also involves skill in searching out the "what", "when", "where", and "who", which helps in defining an unanswered question or problem; it involves skill in sizing up a given situation or problem, examining possible explanations and alternative courses of action which might be taken to meet the problem, and then arriving at sound conclusions. These often entail choosing the most effective courses of action for achieving the desired objectives.²

Future references to critical thinking ability in this study will be in terms of this definition.

The Critical Thinking Appraisal, available in two carefully equated forms, AM and BM, is composed of five subtests (see Appendix 2). In Test 1, concerned with the ability of the student to reason by "inference", each exercise begins with a statement of facts which the student is to regard as true, and is followed by five inferences which some persons might possibly make from the stated facts. The student is to examine each of these inferences and determine whether it is true, probably true, or probably false, or conclude that no decision can be made concerning it because of insufficient data. The first forms of Test 1

2 Ibid., p. 8.

were described as follows by Glaser:

The inference test is designed to measure ability, to judge the probable truth or falsity and the relevancy of inferences drawn from given statements of fact . . . By this test, one may obtain a measure (within the limitations of paper and pencil responses) of (1) the student's general accuracy in judging the degree of truth or falsity of inferences or conclusions drawn from given facts; (2) the tendency to err in the direction of (a) uncritical acceptance of conclusions which are not supported by the evidence, or (b) failure to recognize a conclusion as probable or improbable when the evidence does not warrant such an inference; and (3) hypercritical rejection of conclusions which are supported by the evidence.³

Test 2 is entitled "Recognition of Assumptions".

A number of statements are presented, each followed by three assumptions. The student is to decide for each assumption whether it is necessarily implied by the preceding statement. An "assumption" is defined as "something supposed or taken for granted"⁴. This test is designed to measure the student's ability to recognize the assumptions which are involved in a statement.

Test 3, labeled "Deduction", is made up of individual exercises, each containing two statements which the student is instructed to accept as being true without exception.

³ Edward M. Glaser, An Experiment in the Development of Critical Thinking, New York, Teachers College, Columbia, 1941, p. 89-90. (Although this book was published in connection with the 1942 edition of the test, Dr. Glaser informed the present experimenter that the material of the book which is referred to in this thesis is also applicable to the 1951 edition.)

⁴ Goodwin Watson and Edward M. Glaser, Watson-Glaser Critical Thinking Appraisal, Form AM, Yonkers, World Book, 1951-52, p. 4.

Several conclusions are proposed following each set of two statements and the student is to decide which of these conclusions necessarily follow from the statements given and which do not. The deduction subtest is intended to measure one's ability to recognize the objective relation or implication between premises and conclusions; to determine whether what seems an implication or necessary inference between one proposition and another is indeed such⁵.

Test 4, involving "Interpretation", consists of exercises in each of which a short paragraph is presented which the student is to accept as being true. Again, several conclusions are proposed after each paragraph and the student is to decide which conclusions can properly be drawn from the information given and which cannot be. The conclusion is to be considered valid if it is well enough supported by the facts presented so that it is true beyond a "reasonable doubt". This test is "designed to sample ability to weigh evidence and to distinguish between unwarranted generalizations and probable inferences which, though not conclusive or necessary, are warranted beyond a reasonable doubt"⁶.

⁵ Goodwin Watson and Edward M. Glaser, Watson-Glaser Critical Thinking Appraisal, Manual of Directions, Yonkers, World Book, 1952, p. 1.

⁶ Ibid., p. 1.

It will be noticed that Test 3 and 4 consist of items about "neutral" materials such as music, scientific experiments, etc. Other items in these two subtests are similar in format but they pertain to problems which are commonly emotionally loaded, e.g., references to the different races, political parties, and theories.

Test 5 is the "Evaluation of Arguments" test, in which a series of questions is presented, each followed by three or four arguments which the student is to regard as true. Some of the arguments support the affirmative side of the question and others support the negative side. The student is to decide whether each argument is a strong argument and important to the question at issue, or weak and unimportant or irrelevant⁷.

The test is to be given to college or high school groups in its entirety in one continuous testing period. The maximum working time usually required for this test is 44 minutes, with allotments as follows: inference test, 13 minutes; recognition of assumptions test, 6 minutes; deduction test, 10 minutes; interpretation test, 9 minutes; and test on evaluation of arguments, 6 minutes⁸.

7 Ibid., p. 1.

8 Ibid., p. 3.

However, the total time for the battery and times for the subtests were suggested only as guides. Approximately 95 per cent of the students taking the test at Pittsfield High School completed the tests in less than 40 minutes.

2.- Evaluation of the Watson-Glaser.

The Watson-Glaser Critical Thinking Appraisal has been used in the present research as a measure of skill in the ability to think critically. Obviously, no conclusions which are based on results of this test could be considered any more valid or more reliable than the test itself. Therefore, before any results are appraised or conclusions drawn, it would be proper to evaluate the test as to its validity and reliability. Specifically, there are two questions which should be asked: (1) Does the test actually measure critical thinking ability? (2) Does the test measure the ability consistently?

Validity.- According to Wert, "Validity is defined as the degree to which a test measures what it is supposed to measure"⁹. Relating this definition to the test used in this experiment, one should ask how accurately the test measures critical thinking ability, keeping in mind how this ability has been defined by the authors of this test.

⁹ James Wert, Educational Statistics, New York, McGraw-Hill, 1938, p. 205.

It is commonly accepted that mental traits or abilities, unlike such quantities as height and weight, are intangible in character and can be measured only indirectly in terms of their manifestations in the overt behavior of individuals. As Lindquist has pointed out, the selection of a sample of test items for the measurement of any mental trait depends in large part upon the somewhat arbitrary subjective opinion of the person who makes the selection. The actual magnitude of the errors in this subjective selection could be determined directly only if a corresponding infallible or "perfect" measure of the same trait were available. If this comparison were possible, the coefficient of correlation between the infallible and the fallible measures would be considered as the true coefficient of validity of the latter. However, as Lindquist stated:

The true validity of an educational or psychological test must always remain a hypothetical concept, since there is never available an infallible "criterion" measure against which the fallible obtained measures may be evaluated.¹⁰

That which is selected as the criterion to serve as the yardstick of validity for the test in question may itself be fallible, and hence the correlation coefficient obtained

¹⁰ E.F. Lindquist, A First Course in Statistics: Their Use and Interpretation in Education and Psychology, Revised Edition, Boston, Houghton Mifflin, 1942, p. 206-215.

would not be a true coefficient of validity, but would only be indicative of the amount of agreement between one fallible measure and another which is perhaps somewhat less fallible.

The subjective opinion of the person who selects certain test items to measure a certain mental ability can be compared, however, with the subjective opinions of other persons who are considered to be authorities in the field which is concerned with the development and training of the ability which is purportedly to be measured. The greater the degree of agreement among these persons as to the desirability of the test items, the greater the degree of "validity" the test exercises may be said to have.

This was the procedure followed in establishing the validity of the test used in this experiment. The authors of the test first selected various exercises which they believed would measure the aspects of thinking which have been referred to and defined above. These items were then submitted to a board of thirty-five "experts" selected on the basis of their extensive training in logic, language meaning and scientific method. The board agreed unanimously that proficiency in individual aspects of thinking ability which were involved in the test, would when considered collectively be indicative of critical thinking ability. For any exercise to be included in the final form of the 1951-52 edition of

the test, the authorities had to agree unanimously that, in their opinions, it did measure that aspect of thinking ability which it had been said to measure, keeping in mind the way in which this aspect had been previously defined. At least ninety per cent of the board of experts also had to agree on the key answer to each of the items before the item could be included in the final copy of the test. In practice, if there was any disagreement as to the "correct" answer to any of the exercises or if any member of the group was of the opinion that one answer could be considered as correct as another answer, the exercise in question was not included in the final copy of the 1951-52 editions¹¹.

This procedure is believed to contribute to the highest possible degree of test validity. While there are statistical methods which might be employed to ascertain the degree of validity of a test such as the Critical Thinking Appraisal, Lindquist concluded:

In most instances, particularly in educational achievement testing, final judgment as to the validity of the test must be based primarily on a subjective appraisal of the detailed content of the test in relation to an authoritative description and competent logical analysis of the trait or objectives to be measured.¹²

11 Watson-Glaser, Manual of Instructions, p. 11.

12 Lindquist, Op. cit., p. 226-227.

Reliability.- When reference is made to the "reliability" of a test, the quality in question is the extent of "consistency in measurement"¹³. Applying this concept to the Watson-Glaser Critical Thinking Appraisal, one would try to determine if it consistently measures the same ability to the same degree; that is, would the test yield the same measurement of an individual's critical thinking ability if it were possible for the individual to repeat the test under the same physical conditions and before any change whatsoever could occur in the individual's make-up? While this situation must always remain a hypothetical concept and can never be actually realized, various methods of computation have been devised so that the situation is approximated as nearly as possible.

The split-half reliability coefficient for the total score of the 1949 Experimental Edition of the Watson-Glaser Critical Thinking Appraisal, based on 758 Brooklyn College Pre-Sophomores tested February 1950, was 0.83¹⁴. An equivalent-form reliability of 0.81 was obtained when Forms AM and BM of the 1949 edition were administered to 48 Juniors of the Oak Ridge (Tenn.) High School and an equivalent-form reliability coefficient of 0.79 was obtained when Forms BM

¹³ Ibid., p. 216.

¹⁴ Watson-Glaser, Manual, p. 10.

and AM of the 1949 edition were administered to 30 Sophomores of the same High School. Since the latter two reliability coefficients were inter-form reliabilities they are understandably lower than the split-half reliabilities¹⁵. The standard error of measurement resulting from the testing program at Oak Ridge High School was computed as approximately 5. It should be noted that these reliability coefficients and standard error reported above were obtained from the 118-item standardization edition of the tests from which 19 of the least reliable items were deleted in constructing the final published edition. The authors stated their opinion that such changes would not tend to reduce the reliability of similar data if obtained from the 1951 editions; the data from the 1949 edition may be considered as closely approximating the data that would be obtained from the administration of both Forms AM and BM and using the average of each individuals' scores on the two forms¹⁶. The equivalent-form coefficient of reliability obtained at Oak Ridge High School from the average score on both forms was 0.90. Table I presents data arising out of the administration of Form AM of the Critical Thinking Appraisal to 260 girls and 271 boys at Pittsfield High School in October 1953, as well as published reliability coefficients reported by the publisher.

15 Ibid., p. 9.

16 Ibid., p. 10.

TABLE I.-
Comparative Reliabilities on the Watson-Glaser.

	Oak Ridge H.S., Tenn.		Pittsfield H.S., Mass.	
	Juniors	Sophomores	Boys (Soph.)	Girls (Soph.)
Form of Test	AM	AM, BM	AM	AM
No. of Items	118	118	99	99
Year of Admin.	1950	1950	1951	1951
No. of Testees	48	30	271	260
Mean	60.9	58.3	51.2	50.6
Sigma	12.0	Not Given	9.6	8.4
Coef. Nature	Inter-Form	Inter-Form	Kuder-Rich.	Kuder-Rich.
Coef. Value	0.81	0.79	0.70	0.70

The data arising out of the present research furnish additional evidence for the determination of estimates of reliability of the Critical Thinking Appraisal. The present author has utilized this information in the computation of reliability coefficients, using the Kuder-Richardson Formula¹⁷:

$$r_{tt} = \frac{n}{n-1} \cdot \frac{\sigma_t^2 - n\bar{p}\bar{q}}{\sigma_t^2}$$

Where r_{tt} = reliability coefficient of the total test,

n = number of items in the test,

σ_t = standard deviation of scores on the total test,

M_t = mean of test scores,

$$\bar{p} = \frac{M_t}{n}$$

$$\bar{q} = 1 - \bar{p}$$

It will be noted that the standard deviation in Table I reported by the publishers of Critical Thinking Appraisal is substantially higher than those secured at Pittsfield High School by the present author.

Lindquist has stated that:

. . . Particular consideration should be given to the range of talent upon the coefficient of reliability. When computed for a group that is widely variable in the trait measured, the coefficient of reliability of a given test may be considerably higher than when computed for a group that is relatively homogeneous in the same trait.¹⁸

¹⁷ G.F. Kuder and M.W. Richardson, "The Theory of the Estimation of Test Reliability", Psychometrika, Vol. 2, No. 3, issue of September 1937, p. 152-159.

¹⁸ Lindquist, Op. cit., p. 217.

For a power test such as the Critical Thinking Appraisal (where the speed factor is negligible, the reliability estimates obtained by the Kuder-Richardson Formula 21 are never overestimates.

Underestimates may arise from sources such as the following:

1. Heterogeneity of item content.- Since the assumption of a general factor embodied in test items is not perfectly realized in this test, underestimation may be expected from heterogeneity of item content.

2. Variation in item difficulty.- Since the assumption that all items of the test are equal in difficulty is only approximately realized, underestimation may be expected from variation in item difficulty.

It therefore seems reasonable to conclude that the reliability of the Critical Thinking Appraisal for the groups involved in the present research is not lower than .70 and is probably equal to about .80. This level of reliability may be considered adequate for group analysis.

Watson and Glaser reported that the scores on the Critical Thinking Appraisal tended to correlate to the extent of approximately 0.45 with measures of intelligence and concluded that "the Critical Thinking Appraisal differs

considerably from abstract tests designed to measure basic intellectual abilities"¹⁹.

In the present investigation a measure of verbal mental ability was included as an element in the experimental design. The instrument used for this purpose was the Terman-McNemar Test of Mental Ability, Form C (Appendix 3)²⁰. This test is a completely revised edition of the Terman Group Test of Mental Ability, which was first published in 1920. Like the Watson-Glaser Critical Thinking Appraisal the Terman-McNemar is now published in new Forms C and D which were paired for difficulty in terms of the average per cent passing each item. A final check to determine the equality and the reliability coefficients of Forms C and D was carried out by means of a controlled rotated group experiment in grades 7, 9, and 11 in the schools of Concord, New Hampshire. The data obtained from this experiment involving 1,500 students showed the two forms comparable throughout the range of scores²¹.

The split-half reliability coefficient for the Terman-McNemar Test of Mental Ability, in the Concord, New Hampshire

¹⁹ Watson-Glaser, Manual, p. 9.

²⁰ Lewis M. Terman and Quinn McNemar, Terman-McNemar Test of Mental Ability, Form C. Yonkers, World Book, 1941.

²¹ Lewis M. Terman and Quinn McNemar, Terman-McNemar Test of Mental Ability, Manual of Instructions, Yonkers, World Book, 1942, p. 2-4.

controlled experiment, was found to be 0.96. The equivalent-form reliability in this administration, comprising 239 cases in grades 7 and 9, was determined as 0.95²².

The Probable Error of Measurement for the Terman-McNemar Test is approximately 2.2 standard score points for the entire age range covered by the test²³.

The norm for the test were established through a cooperative national testing program sponsored by the publisher in which 190,000 tests were administered in 200 communities in 37 states including 307 parochial schools in the diocese of Philadelphia, Pennsylvania²⁴.

The test scores on the administration of the Terman-McNemar Test of Mental Ability and the two forms of the Watson-Glaser Critical Thinking Appraisal at Pittsfield High School as well as pupil-personnel records comprise the basic data of the present investigation. Pittsfield High has been recently described as follows:

Pittsfield High School, with curriculum offerings in college preparatory, commercial, general, household arts, retail sales, technical, and vocational fields, can sustain valid claims to being a comprehensive high school, offering varied educational opportunities to boys and girls with diverse patterns of aptitudes and interests.²⁵

22 Terman-McNemar, Manual of Instructions, p. 3.

23 Ibid., p. 3.

24 Ibid., p. 4.

25 Edward J. Russell, Pittsfield High School Faces the Future, Pittsfield, Public School Department, 1954, p. 31.

Geometry is a required course for all pupils selecting college preparatory, technical (open to boys only), and drafting. Although college preparatory and technical are both designed to prepare for college, their differences are important. The college preparatory curriculum is intended for boys and girls who are planning to enter liberal arts colleges or specialized college programs with a literary concentration. The technical curriculum is designed for boys who are looking forward to an engineering education, to college majors in science, or to employment in technical occupations presupposing a substantial knowledge of mathematics and science.

The commercial curriculum has attracted an average of half or more of the tenth-grade girls and perhaps one tenth of the boys. This curriculum is designed to prepare pupils for general clerical positions, stenographers, private secretaries, general business and for civil service examinations. The courses of study are arranged for three levels of achievement, adapted to the ability of the student.

The general curriculum aims to provide a broad training and makes it possible to adapt the pupil's program to his individual need and preference. It does not prepare for college, technical schools, or apprentice schools.

The drafting course prepares for employment as a draftsman, for admission to high grade apprentice courses.

The household arts curriculum requires the pupils taking it to devote one-half of their time to household arts and one-half to academic subjects. The course gives training in the selection, purchase and preparation of foods, in the selection, purchase and construction of clothing, and in general home management including nursing, home decorating and child care.

The vocational courses for boys consist of offerings in printing, sheet metal, cabinet-making, machine shop, welding, automobile mechanics, auto body. In all these courses a full week of shop work alternates with a full week of classroom work throughout the year.

3.- Control and Experimental Groups.

On October 5, 1953, all 531 Sophomore (tenth-grade) pupils at Pittsfield High School were given the Watson-Glaser Critical Thinking Appraisal, Form AM. The test was administered according to the instructions contained in the Manual of directions which accompanies the test. (See Appendix 4). During the following week, under the direction of the Director of Testing, the Sophomores were given Form C of the Terman-McNemar Test of Mental Ability.

In order to compare the "geometry" groups and the "non-geometry" groups, pupils from the two groups were paired

on the basis of (1) intelligence as indicated by the verbal mental ability score on the Terman-McNemar Test; (2) score on the Watson-Glaser Test administered on October 5, 1953; (3) sex; (4) age. On this basis three matched sets of paired pupils as described in Table II were constituted (Appendix 5).

Group E₁, consisting entirely of boys, was drawn from the vocational curriculum and consisted of students in the automobile mechanics, sheet metal, cabinet-making and welding courses. These boys took English, shop science, mathematics, shop drawing and United States history.

Group C₁, consisting entirely of boys, was drawn from the drafting course. They took English, United States history, drafting, and geometry.

Group E₂, consisting entirely of boys, was drawn from the drafting and technical courses. Eight boys took English, geometry, drafting and United States history. Fourteen took English, geometry, physics and drafting.

Group C₂, consisting entirely of boys, was drawn from the technical and college preparatory courses. Ten boys took English, geometry, Latin and biology. Twelve boys took English, geometry, French, physics and drafting.

TABLE II.-
Matching of the Groups.

	E ₁	E ₂	E ₃	C ₁	C ₂	C ₃
No.	20	22	45	20	22	45
Age (Years)						
Mean	15.3	15.2	15.04	15.2	15.2	15.15
Range	14.3- 16.3	14.3- 16.7	14.- 16.2	14.3- 16.7	14.6- 16.2	14.3- 16.2
Sigma	.4	.5	.4	.5	.4	.4
<u>Terman-McNemar</u>						
Mean	109.0	113.0	113.7	109.0	113.6	114.2
Range	96- 126	96- 132	93- 128	96- 126	99- 135	93- 128
Sigma	8.9	9.9	7.1	8.9	9.0	7.7
<u>Watson-Glaser AM</u>						
Mean	51.2	52.1	53.6	51.2	53.1	54.0
Range	40- 66	40- 63	36- 64	43- 61	42- 76	46- 73
Sigma	6.6	6.2	6.5	5.5	6.5	6.7

Group E₃, consisting entirely of girls, was drawn from the commercial course. They took English, mathematics, United States history, and typing.

Group C₃, consisting entirely of girls, was drawn from the college preparatory course. They took English, geometry, biology, French or Latin.

The teachers and students were informed that the test scores would be included in the pupil-personnel folders and that the scores would be made available to the. No further elaboration of the reasons for the testing was made. All the teachers, both control and experimental, were rated by the principal as being competent in their professional field and in their teaching ability.

A rating scale on all teachers in Pittsfield High School is prepared by the principal for submission to the Superintendent of Schools every year (See Appendix 6).

CHAPTER IV

PRESENTATION AND EVALUATION OF THE RESULTS

The nature and character of the evaluation procedure to be employed in a given situation depend upon the goals and objectives with respect to which achievement is to be measured. It may therefore be advisable at this point to restate briefly the objectives of this study.

The chief questions of this research, asked in Chapter I, are: (1) Do high school sophomores who have taken plane demonstrative geometry improve more in critical thinking ability than those pupils who have not taken geometry? (2) What is the relationship between achievement in geometry and critical thinking ability? (3) What is the relationship between achievement in all other major subjects during the sophomore year and critical thinking ability?

The test scores of the subjects were the basic data, the analysis and interpretation of which were expected to provide the answers for the questions listed above.

(1) Do high school sophomores who have taken plane geometry for seven months improve more in critical thinking ability than those pupils who have not taken geometry?

The answer to this major question was first sought by noting the increase in the mean of the scores made in the two administrations of the Watson-Glaser Critical Thinking

Appraisal to students in each of the experimental and control groups, and ascertaining whether such differences were significant.

The distribution of t may be used to test the significance of the differences between two means. The formula for t which applies here is:

$$t = \frac{\overline{WG}_2 - \overline{WG}_1}{\sigma M_d}$$

in which $\overline{WG}_2$ = mean of the second test,

$\overline{WG}_1$ = mean of the first test,

and σM_d = standard error of the difference between the means.

The value of t for the available number of degrees of freedom ($n-1$) corresponds to the value of the probability that such results could be obtained by chance. If the probability that the difference between the means could be due to chance is small (for example 1%), then the probability that a real difference exists is great (99%) and it may be stated that these means are significantly different (at the 1% level). In most phases of scientific work, a 5% level of significance is required before it can be asserted that a real (not a chance or random) difference is present. A probability greater than 5% means that the chances of a real difference are less, while a probability of less than .01% is highly significant.

Table III presents the initial and final mean scores on the Watson-Glaser Critical Thinking Appraisal for each experimental and control group. Table IV gives the significance of the gains shown in Table III.

All the groups listed in Table III made gains which varied from 2.60 to 7.95. However the gain made by the E_1 group (significance level is 20%) cannot be said to be statistically significant; the other groups made gains significant at the 0.1% level. It will be recalled that the E_1 group was composed of boys in the vocational curriculum (not taking geometry), and that the E_3 group was composed of girls in the commercial curriculum (not taking geometry). The members of the other groups all took geometry. Thus, all the geometry groups made significant gains; one of the two non-geometry groups did not.

Are the mean gains in critical thinking (as measured by the Watson-Glaser Critical Thinking Appraisal) significantly different between each of the paired groups? This was one of the principal questions involved in this study. The formula used for this aspect of the analysis is:

$$t = \frac{\overline{WG, C_1} - \overline{WG, E_1}}{\sigma M_d}$$

TABLE III.-

Mean Scores and Gains in Mean Score for the Two Administrations of the Watson-Gleser Critical Thinking Appraisal for Each of the Six Groups.

Group	N	$\overline{WG}_1$	$\overline{WG}_2$	$\Delta \overline{WG}'s$
C ₁	20	51.25	59.05	7.80
E ₁	20	51.45	54.05	2.60
C ₂	22	53.13	60.77	7.64
E ₂	22	52.14	60.09	7.95
C ₃	45	53.96	61.89	7.93
E ₃	45	53.62	59.09	5.47

TABLE IV.-

Significance of the Mean Gains in the Watson-Gleser Test Scores for Each Group, Using the t-Test.

Group	N	$\Delta \overline{WG}'s$	d.f.	t	Significance Level
C ₁	20	7.80	19	3.77	0.1%
E ₁	20	2.60	19	1.36	20%
C ₂	22	7.64	21	3.98	0.1%
E ₂	22	7.95	21	3.88	0.1%
C ₃	45	7.93	44	8.72	0.1%
E ₃	45	5.47	44	5.21	0.1%

in which $\overline{WG, C_1}$ - mean gain in WG score for Group C_1

$\overline{WG, E_1}$ - mean gain in WG score for Group E_1

and σM_d - standard error of the difference
between these means

$(n_1 + n_2 - 2)$ degrees of freedom are available for this test of significance.

Table V presents the paired-group differences between the mean gains in Total Scores (i.e., for each set of paired groups the mean gain of the C group less the mean gain of the E group) and also the significance of these differences using the t-test.

Although Table V reveals a difference of 5.20 points between the mean gains of Groups E_1 and C_1 , a difference of -0.31 points between the mean gains of Groups C_2 and E_2 (both of which took geometry), and a difference of 2.46 points between the mean gains of Groups C_3 and E_3 , it also indicates that none of these differences may be claimed to be statistically significant at the 5% level. However, 5.20 reaches the 7% level and 2.5 the 8% level, so that both non-geometry groups did tend to increase less in the Watson-Glaser Critical Thinking Appraisal than the corresponding geometry groups, with probabilities of better than 1 in 10 that the differences are not due to chance. To corroborate this conclusion, the difference of 0.31 between the two matched geometry groups gave a probability of 90% that the

TABLE V.-

Significance of Paired-Group Differences Between Mean Gains
in Watson-Glaser Scores, Using the t-Test.

Group	N	Mean Gain	$\overline{WG}_E - \overline{WG}_C$	d.f.	t	Signif. Level
C ₁	20	7.80				
E ₁	20	2.60	5.20	38	1.85	7%
C ₂	22	7.64				
E ₂	22	7.95	-0.31	42	0.11	90%
C ₃	45	7.93				
E ₃	45	5.47	2.46	88	1.77	8%

differences were only due to chance. It should be noted that Group C₂ took both geometry and French or Latin while Group E₂ took geometry without taking a foreign language; since there was no significant difference between these two groups there is no evidence of any influence that a foreign language might exert on the difference in improvement in critical thinking ability of these groups.

What is the relationship between achievement in geometry and critical thinking ability? To measure this relationship, the correlations between geometry grades and WG₁, WG₂, and WG were obtained by using the following formula:

$$r_{xy} = \frac{\sum xy}{\sqrt{\sum x^2 \sum y^2}}$$

The data used for the computation of the correlations are presented in Appendix 8. The coefficients of correlations are found in Table VI.

Only in Group C₂, consisting of boys who took geometry and a foreign language, are the correlations sufficiently large to indicate a positive relationship between the second Watson-Glaser test scores and geometry grades and between the change in the Watson-Glaser test scores and the geometry grades. However, the low negative correlation between the initial Watson-Glaser test scores and the geometry grades

TABLE VI.-

Correlation Between Geometry Grades and Initial, Final, and Change in Watson-Glaser Critical Thinking Appraisal Scores.

Group	N	Correlation Coefficients Between Geometry and		
		WG ₁	WG ₂	Δ WG
G ₁	20	.003	-.207	-.162
G ₂	22	-.035	.469	.505
E ₂	22	.079	-.099	-.128
E ₃	45	.157	.222	.058

eliminates the possibility of being able to predict success in geometry by administering the Watson-Glaser Critical Thinking Appraisal prior to taking the course. Also in Group C₂, the second Watson-Glaser test scores and the change in Watson-Glaser test scores have about the same correlations with grades in the other major subjects as with the geometry grades (English .546 and .497; Biology or Physics .358 and .425; French or Latin .474 and .562). Data on correlations between grades in each of the major subjects and the initial, final and change in Watson-Glaser Critical Thinking Appraisal scores are found in Appendix 8.

Significance data on the partial regression coefficients were obtained by means of the t-test, using the following formula:

$$t_{b_1} = \frac{b_1}{s_{b_1}}$$

where b_1 = the regression coefficient (for each grade)

s_{b_1} = standard deviation of the regression coefficient with (n-5) degrees of freedom.

The data on the t-tests, processed on the electronic computer, were obtained from line 18 of the multiple regression analysis sheets found in Appendix 7; they are consistent with the findings discussed in the preceding section dealing with the correlations between geometry grades and

the initial, final and change in the Watson-Glaser Critical Thinking Appraisal scores. The t-tests of significance of the regression coefficients are presented in Table VII. All the t-tests for all the regression coefficients are combined in Appendix 9.

Correlation with English as found in Table VIII may to a large degree be due to correlation with the ability to read adequately or to understand language discriminatingly. Correlation with U.S. History may be due to the inclusion of much social study-type material found in the Watson-Glaser Critical Thinking Appraisal. A study of the t-tests in Appendix 9 substantiated these findings as they pertain to the study of English and the influence on critical thinking ability.

The data do not seem to support the hypothesis that success in geometry (as measured by grades) is closely associated with ability in critical thinking, (as measured by the Watson-Glaser Critical Thinking Appraisal scores); they show very little correlation between geometry grades and these critical thinking scores.

TABLE VII.-

Significance of the Partial Regression Coefficients Between Grades in Geometry and Initial, Final, and Change in Watson-Glaser Critical Thinking Appraisal Scores.

Group	N	t-Test for		
		WG ₁	WG ₂	Δ WG
C ₁	20	-.429	-1.539	.174
C ₂	22	-1.188	.982	2.234
E ₂	22	-.257	.525	.553
E ₃	45	1.518	1.688	.144

TABLE VIII.-

Correlation Between English Grades and the Initial, Final,
and Change in Score of the Watson-Glaser Critical Thinking
Appraisal.

Group	N	r between English and		
		WG ₁	WG ₂	Δ WG
C ₁	20	-.065	.289	.262
E ₁	20	.204	.267	.116
C ₂	22	.125	.576	.497
E ₂	22	.145	.318	.147
C ₃	45	.412	.339	-.102
E ₃	45	-.052	.088	.104

To measure the relationship between critical thinking scores and grades in all the major subjects during the sophomore year, multiple correlation coefficients were computed on an electronic calculator and are presented in Table IX.

These multiple correlation coefficients indicate how closely a combination of subject grades is related to the Watson-Glaser scores or changes in score, when the subject grades are combined at optimal weights. They represent the maximum correlation with Watson-Glaser scores that can be obtained by any combination of the subject grades. Thus, no correlation coefficient between grades in any one subject and the Watson-Glaser scores can be greater than the multiple correlation coefficient between grades in all subjects and those Watson-Glaser scores.

Table IX indicates that the grades correlate more closely with the second Watson-Glaser test scores (range of r equals .42 to .64) than with the first Watson-Glaser test scores (range equals .19 to .55) or with the change in Watson-Glaser scores (range equals .20 to .71), and that grades and Watson-Glaser scores correlate most closely in Group C₂. However, only 6 of the 18 multiple correlation coefficients exceed .5 (the highest being .713), while 5 are less than .3.

TABLE IX.-

Multiple Correlation Coefficients Between Grades in All Major Subjects During the Sophomore Year and the Initial, Final, and Change in the Watson-Glaser Critical Thinking Appraisal Scores.

Group	N	Multiple Correlation Coefficients Between All Grades and		
		WG ₁	WG ₂	Δ WG
C ₁	20	.233	.504	.502
E ₁	20	.475	.421	.268
C ₂	22	.444	.638	.713
E ₂	22	.185	.580	.453
C ₃	45	.548	.477	.200
E ₃	45	.203	.437	.357

The variance in Watson-Glaser is explained by R to the extent indicated by the following per cents: $R=.3$, less than 9% of the variance; $R=.5$, 25%; $R=.580$, 34%; $R=.638$, 41%; $R=.713$, 51%.

The multiple regression equations contains regression coefficients indicating the best relative weights for combining the subject grades for each group. Thus, the multiple regression equations may be used to correlate critical thinking scores with the best-weighted sum of the subject grades (assuming a linear relationship exists between all of the correlated variables).

Table X presents the multiple regression equation calculated for each group. The multiple regression equations show the relationship between critical thinking ability and grades in all the major subjects taken during the sophomore year. The regression equations do not contribute directly toward substantiating or validating the hypotheses studied in this investigation, but may be of value in other studies of the relation of high school work on the development of critical thinking ability.

TABLE X.- Multiple Regression Equations for Estimating the Initial, Final or Change in Score of the Watson-Glaser from Grades in all Major Subjects in Sophomore Year.

Group	Multiple Regression Equations						$\sigma_{est_{WG}}$
C ₁	46.657 WG1	-0.092 G	-0.075 E	+0.085 H	+0.132 D		6.129
	70.438 WG2	-0.037 G	+0.309 E	-0.297 H	-0.099 D		6.933
	23.781 Δ WG	+0.055 G	+0.384 E	-0.382 H	-0.231 D		8.989
E ₁	13.718 WG1	+0.299 M	-0.044 E	-0.046 H	+0.308 S		6.781
	33.439 WG2	-0.145 M	+0.042 E	+0.304 H	+0.058 S		7.637
	5.062 Δ WG	-0.302 M	+0.091 E	-0.338 H	-0.189 S		11.477
C ₂	43.150 WG1	-0.215 G	+0.328 E	+0.270 BP	-0.280 L		6.781
	1.214 WG2	+0.210 G	+0.382 E	+0.023 BP	+0.183 L		7.872
	-41.936 Δ WG	-0.424 G	+0.054 E	-0.247 BP	+0.463 L		7.010
E ₂	42.531 WG1	-0.067 G	+0.050 E	+0.101 BP	+0.048 D		6.935
	49.021 WG2	+0.132 G	+0.373 E	-0.629 BP	+0.260 D		6.660
	6.490 Δ WG	+0.200 G	+0.323 E	-0.731 BP	+0.212 D		9.539
C ₃	24.603 WG1	+0.186 G	-0.657 E	-0.182 B	-0.305 L		5.899
	32.714 WG2	+0.204 G	+0.454 E	-0.017 B	-0.279 L		5.838
	8.111 Δ WG	+0.019 G	-0.203 E	+0.164 B	+0.026 L		6.264
E ₃	57.464 WG1	+0.030 M	-0.071 E	+0.119 H	-0.135 T		6.809
	40.714 WG2	+0.160 M	-0.123 E	+0.322 H	-0.143 T		6.829
	-16.750 WG	+0.130 M	-0.052 E	+0.203 H	-0.008 T		6.878

WG1 = Initial Watson-Glaser Test Score
 WG2 = Final Watson-Glaser Test Score
 Δ WG = Final-Initial Watson-Glaser Test Score
 G = Geometry Grade
 E = English Grade
 S = Science Grade
 B = Biology Grade

L = Latin or French Grade
 M = Mathematics Grade
 H = U.S. History Grade
 D = Drafting Grade
 P = Physics Grade
 T = Typing Grade

SUMMARY AND CONCLUSIONS

The major problem which was involved in this study was the evaluation of the training received in seven months' study of plane demonstrative geometry with respect to its effect on the critical ability of the students. The comparison of the total test scores on two administrations of the Watson-Glaser Critical Thinking Appraisal made by all sophomores furnished the data from which the answers to the problem were derived.

Although the sample and rationale used in this study present certain limitations, certain conclusions may be tentatively drawn from the results that were obtained. These results and conclusions may be relevant to other high school sophomores only insofar as it could be said that those pupils had been subjected to the same conditions which prevailed while this experiment was being conducted.

It will be recalled in Chapter II that the course of geometry, as taught at Pittsfield High School, emphasized that relating geometric thinking to non-geometric situations is frequently employed as a means of motivating the logical emphasis of the course of geometry. Since pupils learn subjects best in connection with their use in meaningful situations, the study of non-geometric exercises of logical reasoning that have relevance to geometric thinking facilitates

the learning of geometric content. A critical and appreciative attitude toward reasoning in every-day life can be made to contribute to greater insight into geometric thinking.

During the seven-month period, both the students who took geometry and those who did not take geometry, with the exception of Group E₁, gained in critical thinking ability as measured by this test to the point of high statistical significance. Group E₁ was composed of boys in the vocational curriculum (not taking geometry).

The differences disclosed between paired groups in the mean gains in critical thinking were not significant at the 5% level. However, both non-geometry groups did tend to increase in the Watson-Glaser Critical Thinking Appraisal scores less than the corresponding groups that did take geometry.

It should be observed that Group C₂ (which took both geometry and French or Latin) and Group E₂ (which took geometry without taking a foreign language), did not present a significant difference, so that there is no evidence of any influence that a foreign language might exert on the variation.

Correlation between English grades and the Watson-Glaser test scores may to a large degree be due to correlation with the ability to read adequately. The literature on the

ability to think critically indicates clearly that a variety of skills are involved. Although certain parts of the Watson-Glaser Critical Thinking Appraisal seem to test some of the abilities involved in critical thinking, language ability and general knowledge may be more influential in the selection of some answers by pupils than by the ability to do critical thinking. Much work must admittedly still be done on the selection of test items, even assuming an agreement on the meaning of critical thinking itself.

The data do not seem to support the hypothesis that grades in geometry are closely associated with ability in critical thinking, (as measured by the Watson-Glaser Critical Thinking Appraisal scores).

This conclusion does not rule out the possibility that significant correlations might be found between success in geometry and critical thinking ability if success in geometry were measured by objective tests, since differences among teachers are factors in variations in geometry class grades.

Perhaps teachers of geometry have made excessive claims for geometry as an aid to critical thinking. For the reasoning in non-geometric or life situations is often unlike the type of reasoning required in geometry, and in life situations the best solutions of problems are not

always those derivable by strict deductive reasoning. Decisions may be influenced by social customs, personal interests, and other non-deductive factors. To prepare pupils to solve these problems requires training in some ways of thinking and reasoning not permitted in the study of geometry.

It is possible that the influence of studying geometry on the development of critical thinking ability (revealed in this study at an inadequate significance level) might be established at a higher level if the research in this study were extended with additional numbers of students. This would seem to be a fruitful field of research, since quantitative data are needed to interpret current hypotheses as to the power of geometry, as it is customarily taught, to develop the ability to think critically in non-geometric contexts.

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A fine study in modern English of the traditional school.

Bell, E.T., Men of Mathematics, New York, Simon and Schuster, 1937, xx-592 p.

Rather than provide an historical development of mathematics, Professor Bell provides biographies of famous men of mathematics in such an order that an historic perspective is provided. This view of history through biographies is written very well.

Betz, William, "The Transfer of Training with Particular Reference to Geometry", The Teaching of Geometry, Fifth Yearbook, National Council of Teachers of Mathematics, New York, Bureau of Publications, Teachers College, Columbia, 1930, p. 149-198.

This is a remarkably fine summary and critical evaluation of the relevant literature on transfer of training with particular reference to Geometry.

Billing, Marion L., "Problem Solving in Different Fields of Endeavor", in The American Journal of Psychology, Vol. 46, No. 2, issue of April 1934, p. 259-272.

An analysis of the data shows that students who were able to solve the problems had the required information, but not all the pupils who had the information were able to solve the problems.

Brown, Kenneth E., Mathematics Education Research Studies, Circular No. 377, Department of Health, Education and Welfare, Washington, Superintendent of Documents, 1952, 25 p.

Contains summaries of 57 studies in the field of mathematics completed in 1952.

-----, Mathematics in Public High Schools, Bulletin 1953, No. 5, U.S. Department of Health, Education and Welfare, Washington, Government Printing Office, 1953, viii-47 p.

This study reports data for the school year 1951-52 and the first semester of 1952-53. This information is based on replies received from 857 randomly selected public high schools.

----- and Johnson, Philip G., Education for the Talented in Mathematics and Science, Bulletin 1952, No. 13, Washington, Superintendent of Documents, 1952, 34 p.

Discusses what is being done by leading educators to provide increased facilities and methods for the maximum development of rapid learners in these fields.

Brune, T.H., Editor, Guidance Pamphlet in Mathematics for High School Students (Revision), Washington, National Council of Teachers of Mathematics, 1953, Paper, vi-40 p.

This pamphlet answers the questions: "Who should study mathematics?" "Why?" "What is mathematics?" "What kind of mathematics is needed for personal use, by trained workers' in various fields, for college preparation, by professional workers in different areas?" It represents an honest effort to furnish authoritative answers to these very important questions about mathematical study.

Burt, Cyril, "The Development of Reasoning in School Children", Journal of Experimental Pedagogy, Vol. V, Nos. 2 and 3, issues of June and December 1919, p. 68-77, 191-227.

Burt developed this series of tests to determine the way in which the reasoning capacities of children develop from year to year; by means of reasoning capacities, to estimate the level of general intelligence among older and brighter children. The correlations with intelligence were high. The development of reasoning appears to consist essentially in an increase in the number, variety, originality, and compactness of the relations which the mind can perceive and integrate into a coherent whole.

Butler, Charles H., and Wren, F. Lynwood, The Teaching of Secondary Mathematics, New York, McGraw-Hill, 1941, xii-513 p.

The book discusses in detail the place and function of mathematics; the improvement and evaluation of instruction in secondary mathematics and the teaching of the special subject matter of secondary mathematics. The exercises and the bibliographies at the end of each chapter are well chosen and extremely useful.

Cajori, Dorian, History of Mathematics, New York, Macmillan, 1931, 516 p.

This volume is a scholarly and carefully documented work for those who wish to attain perspective in the history of mathematics.

Commins, W.D., Principles of Educational Psychology, New York, Ronald Press, 1937, xiv-596 p.

An excellent book with worthwhile material on the subject of transfer of training.

Cook, Inez M., "Developing Reflective Thinking Through Geometry", in The Mathematics Teacher, Vol. 36, No. 2, issue of February 1943, p. 79-82.

A group of teachers of mathematics at Lincoln, Nebraska High School felt that in teaching Geometry improvement might be effected by reorganizing their classroom procedure so that the transfer of training was made a conscious and continuous procedure -- from geometric to non-geometric problems and vice versa.

Christofferson, H.C., Geometry Professionalized for Teachers, New York, Teachers College, Columbia University, 1933, vi-204 p.

Contains a wealth of material written to review and extend geometric concepts on the college level and to serve as a guide to the teaching of geometry.

Dewey, John, How We Think, New York, Heath, 1933, x-301 p.

One of Dewey's most influential books on education that deals with the nature of thought. The book was written in the conviction that education would find a unifying factor if it adapted as its central purpose the cultivation of a scientific attitude of mind in meeting the problems of individual and social life.

Dunker, Karl, "On Problem Solving", Psychological Monographs, Vol. 58, No. 5, 1945, v-113 p.

A highly scientific study in the realm of thought with a philosophical development which has a high "gestaltist" content.

Eby, Frederick, and Charles Flinn Arrowood, The Development of Modern Education. In Theory Organization and Practice, New York, Prentice-Hall, 1947, xxiv-922 p.

The best general account on this subject. Full of information for one interested in the history of geometry. Useful bibliographical references.

Fawcett, Harold P., The Nature of Proof, Thirteenth Yearbook, The National Council of Teachers of Mathematics, New York, Bureau of Publications, Columbia University, 1938, viii-146 p.

It is the purpose of this research study, by a distinguished teacher of Mathematics, to describe classroom activities and methods by which geometric proof may be used as a means "for cultivating critical and reflective thought and to evaluate the effect of such experiences on the thinking of the pupils".

Fehr, Howard F., Secondary Mathematics. A Functional Approach for Teachers, Boston, Heath, 1951, xi-431 p.

This book suggests ways in which the functional approach can be used to make various topics in junior and senior high schools mathematics more meaningful to students.

Flesch, Rudolf, The Art of Clear Thinking, New York, Harper, 1951, 212 p.

This book is a practical self-help book for anyone who wishes to improve his thinking and increase his flow of ideas according to the author. It offers a simple practical technique for picking out fallacies, analyzes propaganda, and applies the methods of scientific research to ordinary problems. However, the author tells you from the very start that it "would be impudent to tell intelligent, grown-up people how to think". This book is very readable. The bibliography is valuable.

Gengerelli, J.A., "Mutual Interference in the Evolution of Concepts", The American Journal of Psychology, Vol. 38, No. 4, issue of October 1927, p. 639-646.

Gengerelli wanted to determine if it was really true that if we have learned to conceive a thing in a certain way, it will be relatively more difficult to conceive that same thing in a second and different way.

Glaser, Edward M., An Experiment in the Development of Critical Thinking, Columbia Contributions of Education, No. 843, New York, Bureau of Publications, Columbia University, 1941, ix-212 p.

This book is a report on the experimental study of the ability to think critically. Dr. Glaser is co-author of the Watson-Glaser Critical Thinking Appraisal used as the basis of this study. The comprehensive review and bibliography of previous research in the psychology, training, and measurement of critical thinking ability made this volume a useful reference book.

Hall, Elizabeth Loetzer, "Applying Geometric Methods of Thinking to Life Situation", The Mathematics Teacher, Vol. 31, No. 8, issue of December 1938, p. 379-384.

Hall purposely selected students for their lack of ability to do work in a regular geometry class, and approached the subject with them by showing the need of proof in the students' every day lives and experiences. The experiences of the pupils were used for illustration in trying to bring about an understanding of the methods of drawing valid conclusions and the concepts of proof as used in Geometry. Hall concludes that Geometry can be used as an effective and convenient vehicle for teaching the nature of valid inference if the whole structure of geometric applications and life applications are interwoven. He states that the fact of transfer (of training) from one activity to related activities seems established. But the equally important consideration that in order to obtain this transfer we must teach for it, has not yet received due emphasis.

Harvard Committee Report, General Education in a Free Society, Cambridge, Harvard University Press, 1945, xix-267 p.

Although the matter of college preparation still tends to dominate much of high school curricular planning, there is increasing concern that the high school become much more than a pre-college institution. This report indicates the fact that the colleges themselves are growing more aware of this vital problem. The report is at once a landmark in the development of more effective educational policy and a sample of the best in pedagogical prose.

Hazlitt, Victoria, "Children's Thinking" in the British Journal of Psychology, Vol. 20, Part 4, issue of April 1930, p. 354-361.

Hazlitt tried to collect evidence bearing on the validity of Piaget's most fundamental fact about childish thinking being egocentric, and how it differs from that of adults. Hazlitt concludes that "Piaget's picture of a striking difference between adult and childish thinking is due to an over-valuation of verbal expressions as a measure of thinking and an exaggerated view of the logicity of adult thought."

Heidbreder, E.F., "Problem Solving in Children and Adults", in the Journal of Genetic Psychology, Vol. 35, No. 4, issue of December 1928, p. 522-545.

The results of this experiment may be considered as converging lines of evidence indicating that the activity involved in solving problems included several modes of reaction which showed a gradual development with increasing maturity.

Hogben, Lancelot, Mathematics for the Millions, New York, Norton, 1946, xiv-7690 p.

This is one of the few books on mathematics which has been a best seller. Much historical information is provided on many aspects of Geometry.

Hooper, Alfred, Makers of Mathematics, New York, Random House, 1948, ix-402 p.

The reader is introduced not only to the fascination of the story of mathematics and mathematicians but also to actual mathematical processes.

Judd, Charles Hubbard, Education as Cultivation of the Higher Mental Processes, New York, Macmillan, 1936, vii-206 p.

This very important yet short book is devoted to reports on a number of studies which explain the nature of the more advanced forms of thinking and on the effectiveness or lack of it of the educational methods used in the effort to encourage and nurture these forms of thinking.

-----, "The Relation of Special Training to General Education", Educational Review, Vol. 36, No. 1, issue of June 1908, p. 28-42.

Judd determined by experiment that the most important educational principle is not one of special mental functions, but the principle that good teaching aims at generalizations.

Katona, George, Organizing and Memorizing, New York, Columbia University Press, 1940, xvi-318 p.

The theory of transfer as reconstruction of experience gained much support from this outstanding work on transfer. Katona experimented with large numbers of human subjects in school and lifelike situations and attempted to distinguish between senseless and meaningful learning.

Kinney, Lucien Blair, and C. Richard Purdy, Teaching Mathematics in the Secondary School, New York, Rinehart, 1952, xvi-381 p.

This book is excellent for the teacher or prospective teacher of mathematics. This book, written by mathematicians, analyses all the modern principles of educational psychology as they relate to the teaching of mathematics. The need for a sound mathematics program for future leaders is emphasized, and it has an especially good discussion of mathematics in general education.

Laser, Nathan, "The Importance of Certain Concepts and Laws of Logic for the Study and Teaching of Geometry", The Mathematics Teacher, Vol. 31, Nos. 3, 4, 5, issues of March, April, May, 1938, p. 99-113, 156-174, 216-240.

Laser developed a method of teaching geometry in such a manner as to encourage generalization and showed how geometry might be utilized as a medium for making the pupils conscious of the existence of logical patterns of valid and invalid reasoning in mathematics as well as in the thinking of every day life.

Lieber, Lilian R., Mits, Wits and Logic, New York, Norton, 1947, 240 p.

All of the Lieber's books have an unusual style, a format making the prose appear to be blank verse and deal whimsically but soundly with the nature and topics of modern mathematics, emphasizing axiomatics. The section on logic, written by this mathematician is valuable to the teacher of Geometry.

Maier, Norman R., "Reasoning in Humans on Direction", in the Journal of Comparative Psychology, Vol. 10, No. 2, issue of April 1930, p. 115-143.

When the solution of a problem is broken into three parts and given to a subject as three separate experiences, then such experiences are not sufficient to bring about the solution of the problem. Thus a selected presentation of the experience is not enough. The parts or experiences must be combined in a certain manner and a "direction" or way the problem is attacked, seems to be a factor which determines the nature of the combination. "Trial and error" may be present in the attempts at the solution, but is inadequate to explain the sudden appearance of the correct or successful solution, when such solution requires productive rather than reproductive thinking.

National Committee on Mathematics Requirements, The Reorganization of Mathematics in Secondary Education, Boston, Houghton Mifflin, 1927, x-180 p.

This report resulted from a comprehensive study of the whole problem concerned with the improvement of Mathematical education. It was of particular importance because over 100 organizations took part in the work of investigating every aspect of mathematical instruction.

National Council of Teachers of Mathematics, The Learning of Mathematics, Its Theory and Practice, Twenty-First Yearbook, Washington, The Council, 1953, 355 p.

Twelve authors contributed various aspects of the process of learning, motivation, problem-solving, and language are discussed. Excellent bibliographies are provided at the ends of chapters.

-----, The Place of Mathematics in Secondary Education, Fifteenth Yearbook, New York, Teachers College, Columbia University, 1940, xvi-253 p.

The most important contribution to mathematical education since the 1923 Report of the National Committee on Mathematical Requirements. Excellent chapters on aims and objectives.

-----, The Teaching of Geometry, The Fifth Yearbook, New York, Teachers College, Columbia University, 1930, x-206 p.

This yearbook was intended to supplement and assist the National Committee in studying the feasibility of a combined one-year course in plain and solid geometry. Every approach to the teaching of geometry is explored.

-----, The Teaching of Mathematics in Secondary Schools, Eighth Yearbook, New York, Teachers College, Columbia University, 1933, 264 p.

The purpose of the Eighth Yearbook was to present some of the important phases of the teaching of mathematics in the secondary schools.

Ogden, C.K., and I.A. Richards, The Meaning of Meaning, New York, Harcourt Brace, 1930, xxii-363 p.

A study of the influence of language upon thought and the science of symbolism.

Orata, Redro T., "Recent Research Studies on Transfer of Training with Implications for Curriculum, Guidance and Personnel Work", Journal of Educational Research, Vol. 35, No. 2, issue of October 1941, p. 81-101.

A leading student of transfer of training summarizes the research in transfer of training. Orata emphasizes the point that eighty percent of the studies reveal that transfer is a fact.

Orata, Redro T., "Transfer of Training and Reconstruction of Experience", in The Mathematics Teacher, Vol. 30, No. 3, issue of March 1937, p. 99-109.

Orata states that maximum transfer can take place only if three active processes are provided; namely: experience, continuity, and reconstruction.

Parker, Elsie Johnson, "Teaching Pupils the Conscious Use of a Technique of Thinking", Mathematics Teacher, Vol. 17, No. 4, issue of April 1924, p. 191-201.

Parker set up a controlled experiment in an effort to answer the question, "Can pupils of Geometry be taught to prove theorems more economically and effectively when trained to use consciously a technique of logical thinking; and furthermore, does such training, more than the usual method, increase the pupils' ability to analyze and see relationships in other, non-geometrical, situations?"

Perry, Winona A., A Study in the Psychology of Learning Geometry, Contributions to Education, No. 179, New York, Teachers College, Columbia University, 1925, 60 p.

The purpose of this investigation is a study of the psychology of thinking as it appears in the solution of exercises in Geometry based on the comparative effect of two techniques, upon students of three levels of potential ability in reasoning.

Pingry, Robert E., "Critical Thinking -- What Is It?" in The Mathematics Teacher, Vol. 44, No. 7, issue of November 1951, p. 466-470.

Pingry pointed out that critical thinking has a great number of aspects and means many different things to different people. The phrase "critical thinking" must be defined or supplemented by specific outcomes of learning in terms of actual behavior characteristics and skills desired.

Powers, Francis F., "The Influence of Intelligence and Personality Traits Upon False Beliefs", in Journal of Social Psychology, Vol. 2, No. 4, issue of November 1931, p. 490-493.

This experiment described the results of correlating measures of intelligence, introversion, and superstition.

Roslow, Sydney, "A Statistical Analysis of Rational Learning Problems", in the Journal of Genetic Psychology, Vol. 48, No. 2, issue of June 1936, p. 441-467.

The statistical analysis of the results obtained with the rational learning problems indicate that intelligence is an important factor in the ability to solve these problems. The hypothesis indicated that there may be two methods of solution, a verbal and a visual, and that the visual method was more efficient.

Roszkopf, Myron F., "Transfer of Training", The Learning of Mathematics, Its Theory and Practice, Twenty-first Yearbook, National Council of the Teachers of Mathematics, Washington, 1953, p. 205-227.

This is an excellent chapter that brings teachers of mathematics up to date on transfer of training with special reference to mathematics.

Sells, Saul B., "The Atmosphere Effect: An Experiment Study of Reasoning", in the Archives of Psychology, No. 200, issue of May 1936, 72 p.

The relationships of abstract and concrete syllogisms to each other and to intelligence are such that no serious sacrifice is made by limiting the research to abstract items only. The correlation between abstract and concrete is high, and both have a high correlation with intelligence.

Skolnik, David, "Let's Forget Mental Transfer", in The Mathematics Teacher, Vol. 45, No. 3, issue of March 1952, p. 161-167.

Skolnik is one of many teachers who are making serious attempts to realize the broader purposes of Geometry and who has adopted a course of study with that general aim.

Smith, Thomas, Euclid, His Life and System, New York, Scribner, 1902, vi-227 p.

This volume is one of a series of the World's Epoch-Makers. Although this volume was undertaken to popularize Euclid, it does trace, very effectively, the history of the all ancients who contributed to geometry. This book is exhaustive from the scientific standpoint, but is sufficiently easy and popular in style to be of considerable interest and attention to the non-scientific reader.

Smoke, Kenneth L., "An Objective Study of Concept Formation", in Psychological Monograph, Vol. 67, No. 4, whole No. 191, 1932, 46 p.

Smoke indicates that the learning that took place in his experiments was true to the "learning that occurs in the work-a-day world".

Stamper, Alva W., History of Teaching of Geometry, New York, Teachers College, Columbia University, 1909, x-163 p.

This thesis is an excellent concise history of the teaching of geometry. It contains a very complete and useful bibliography.

Thorndike, E.L., "Mental Discipline in High School Studies", Journal of Educational Psychology, Vol. 14, Nos. 1, 2, issues of January, February, 1924, p. 1-22, 83-98.

Thorndike undertook to determine to what extent a year's training in each of the many high school subjects would increase ability in a series of tests of "selective and relational thinking" or reasoning. About 13,400 pupils in grades ten, eleven and twelve, were tested. The implications of these investigations is that no one subject has outstanding merit as an easy and sure means of developing ability to think in general.

----- and R.S. Woodworth, "The Influence of Improvement in One Mental Function Upon the Efficiency of Other Functions", Psychology Review, Vol. 8, No. 3, issue of May 1901, p. 247-261.

This study is one of the earliest experiments on transfer in the field of perception ever carried out.

Washburn, M.F., "Mathematical Ability, Reasoning and Academic Standing", in The American Journal of Psychology, Vol. 50, Nos. 1-4, issue of November 1937, p. 484-488.

The purpose of this study was to determine the relationship of mathematical ability to reasoning and academic standing.

Welkowitz, Samuel, "Tenth Year Geometry for All American Youth", in The Mathematics Teacher, Vol. 39, No. 3, issue of March 1946, p. 99-112.

Welkowitz proposed the enrichment of geometry through the juxtaposition of the manual and the mental, the pure and the applied, the concrete and the abstract, the inductive and the deductive, the synthetic and analytical.

Wilson, Jack D., "Trends in Geometry", in The Mathematics Teacher, Vol. 46, No. 2, issue of February 1953, p. 67-70.

This article is an excellent résumé of the reasons advanced for teaching geometry during the period 1920-1954.

Woodworth, Robert S., "Transfer of Training", Experimental Psychology, New York, Holt, 1938, p. 176-208.

This is an especially excellent chapter on transfer organized to show the development of specific aspects or problems on this subject.

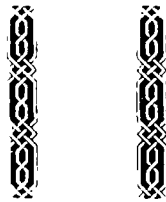
APPENDIX 1

CURRICULUM STUDY USED AS BASIS OF THIS RESEARCH

PLANE GEOMETRY

Curriculum Study

SEPTEMBER, 1953



MATHEMATICS DEPARTMENT

Pittsfield High School

PITTSFIELD PUBLIC SCHOOL DEPARTMENT

Pittsfield, Massachusetts

PLANE GEOMETRY

Book-propositions are studied for understanding only; pupils are therefore not responsible for the reproduction of the proof of a text theorem, unless the proof is considered to be an original exercise.

GENERAL AIMS

1. To increase the pupil's knowledge of geometrical forms and his understanding of geometrical concepts.
2. To acquaint the pupils with the basic geometric propositions and their applications.
3. To encourage further geometric investigation and independent thinking by means of selected exercises.
4. To make clear the need for well-defined terms, precision of statement, and accuracy of thinking for the purpose of establishing conclusions.
5. To develop the power of deductive reasoning as a method of thinking.
6. To arouse in the student an appreciation of the significance of geometry in its application to science, engineering, and the arts.
7. To correlate the skills of arithmetic and algebra with those of geometry to give a deeper understanding of mathematical relationships.
8. To encourage three-dimensional thinking as a outgrowth of two-dimensional thinking and thus stimulate the development of spatial imagination.

Unit I

Introduction: brief review of intuitive geometry; the meaning of proof; simple informal proofs; using axioms and preliminary theorems.

Desirable Outcomes: growth in

1. Confidence in studying geometry.
2. Understanding that geometry was developed through the experiences of people to meet the needs of man in a growing civilization.
3. Interest in and enthusiasm for learning more about geometry.
4. A basic vocabulary of geometric terms.
5. Habits of concise and exact speech.
6. Understanding the role of assumptions, clearly defined and agreed upon, as basics of making valid conclusions.
7. Readiness for theorems and exercises on triangle congruence.

Unit II

Congruent triangles; formal proof of the congruence of triangles by using the *asa*, *sas*, *sss* congruence theorems (no overlapping figures).

Desirable Outcomes: growth in

1. Comprehension of ability to define geometric terms introduced in this unit.
2. Ability to classify triangles according to sides and angles.
3. Understanding the nature of superposition as a method of proving triangles congruent.
4. Appreciation of the pleasure of discovering and establishing truths through the process of reasoning.

Unit III

Constructions; bisectors, perpendiculars, principal lines in a triangle; proof of constructions.

Desirable Outcomes: growth in

1. Comprehension of and the ability to define geometric terms introduced in this unit.
2. Construction of neat and accurate figures.
3. Familiarity with fundamental concepts by the construction of simple figures.
4. A sense of triangle rigidity.
5. Acquisition of a working vocabulary of fundamental angle situations.

Unit IV

Parallel lines; numerical relationships; congruent triangles; use of axioms.

Desirable Outcomes: growth in

1. The use of axioms and postulates as reasons for steps in the solution of exercises.
2. Correlating the skills of arithmetic and algebra with geometric facts through the use of numerical exercises.
3. Appreciating the usefulness of parallel lines.

Unit V

Angle-sum; numerical relationships; advanced congruent triangles (including overlapping figures).

Desirable Outcomes: growth in

1. Appreciation of the usefulness of perpendicular lines in the world around us.
2. Understanding the relation between a statement and its converse.
3. Insight into simple three-dimensional thinking.
4. Recognition of direct and indirect proofs.
5. Ability to handle the advanced congruent triangles.

Unit VI

Parallelograms; congruent triangles; special parallelograms; line joining midpoints; series of parallels.

Desirable Outcomes: growth in

1. Using the skills of arithmetic and algebra for the practical application of the geometric theory learned in this unit.
2. Comprehension of the meaning of the basic principles of the unit and applying them to the solution of original exercises.
3. Discovering new truths as a result of independent investigation.
4. Appreciation of the use of polygons in nature, art, and industry.

Unit VII

Circles; arcs; chords; tangents; congruent triangles.

Desirable Outcomes: growth in

1. Understanding the relationship between central angles, arcs, chords, and tangents in equal circles.
2. Recognizing the value of tangent principles in mechanics and industry.
3. Appreciation of the use of circles in decoration, architecture, and industry.
4. Expanding two-dimensional thinking into three dimensional thinking.

Unit VIII

Angle-measurement; numerical relationship; equal arcs and angles.

Desirable Outcomes: growth in

1. Understanding the meaning of "measurement" and the limitations of its use.
2. Acquiring the concept of "degree" as a unit of measurement of (a) an angle, (b) an arc.
3. Understanding the numerical relations between the arcs of a circle and the angles, which determine these arcs.
4. Recognizing the basic principle of continuity underlying the theorems of measurement of angles appreciating the methods of generalized thinking.
5. Skillful application of the principles of measurement of angles.

Unit IX

Proportions; similar triangles; proving triangles similar; writing a proportion; simple numerical applications.

Desirable Outcomes: growth in

1. Understanding the meaning of ratio and the fundamental properties of equal ratios.
2. Comprehension of algebraic equations through work with proportions.
3. Realization of the meaning and importance of a common unit of measure.
4. A basic understanding of proportional line segments and their use in "scale" drawings.

Unit X

Applications of similar triangles; complex numerical relations; chords, secants, tangents.

Desirable Outcomes: growth in

1. Appreciating the importance of similarity in the many fields in which geometry plays a part.
2. Understanding the value of similarity of indirect measurement.
3. Understanding mathematical principles and concepts through the use of arithmetic and algebra in geometric situations.

Unit XI

Application of similar triangles; right triangles; Pythagorean relation; 30 degree and 45 degree right triangles.

Desirable Outcomes: growth in

1. Attaining a broader insight into the practical uses of the principles of similarity.
2. Ability to use mathematical tables.
3. The concept of indirect measurement and mastering its practical applications.

Unit XII

Areas; fundamental formulas; numerical relations.

Desirable Outcomes: growth in

1. An understanding of the concept of measurement as related to surfaces.
2. Ability to differentiate between congruence, equality, and similarity of figures.
3. Skillful application of the area formulas to practical situations.

Unit XIII

Comparison of areas; equivalent figures; transformation; areas of similar triangles and polygons; algebraic analysis.

Desirable Outcomes: growth in

1. The skills and concepts of arithmetic and algebra used in the computation and comparison of areas.
2. Appreciation of the possibilities of sequential thinking as illustrated by the unfolding of the area formulas from the basic propositions.

Unit XIV

Regular polygons; properties; constructions; area; circumference and area of circles; sectors; segments.

Desirable Outcomes: growth in

1. Attaining a basic understanding of the properties of regular polygons as a background for the measurement of the circle.
2. Appreciation of the importance of the regular polygon in art and nature.
3. Skillful construction of figures which satisfy definite specifications.

Unit XV

Constructions; fundamental triangle constructions; fundamental loci and combinations original loci.

Desirable Outcomes: growth in

1. Ability to discover a locus and to prove that it satisfies the given condition.
2. Using analytical thinking in the solution of construction problems.
3. Further insight into three-dimensional thinking.
4. Ability to prove that the construction actually accomplishes what is claimed for it.
5. Analyzing a problem, understanding its requirements, and discovering the most effective method of making the construction from the given data.

Unit XVI

General Review

Desirable Outcomes:

1. Retention of the facts learned.
2. Opportunity for the use of skills and processes that have been developed.
3. Ability to see the relationship of the parts to each other and to the whole will be clearly established and understood.

APPENDIX 2

WATSON-GLASER CRITICAL THINKING APPRAISAL,
FORM AN AND FORM BM

WATSON-GLASER

CRITICAL THINKING

APPRAISAL

Form AM

by GOODWIN WATSON

*Professor of Education, Teachers College,
Columbia University*

and EDWARD MAYNARD GLASER

*Consulting Psychologist, Rohrer, Hibler, and Replogle,
Los Angeles*

DIRECTIONS:

This booklet contains several different types of tests designed to find out how well you are able to reason analytically and logically.

Do not turn this page until instructed to do so. Do not make any marks on this test booklet. All answers are to be marked on the separate Answer Sheet provided. If you wish to change an answer, be sure to erase your old answer completely.

TEST 1. Inference

DIRECTIONS. An *inference* is a conclusion which a person draws from certain observed or supposed facts. Thus, from the electric light visible behind the window shades and from the sound of piano music in a house, a person might *infer* that someone is at home. But this inference may or may not be correct. Possibly the people in the house went out leaving the lights on, and the piano music could be coming from a radio or phonograph they left playing.

In this test each exercise begins with a *statement of facts* which you are to regard as true. After each statement of facts you will find several possible inferences — that is, inferences which some persons might make from the stated facts. Examine each inference separately, and make a decision as to its degree of truth or falsity.

On the Answer Sheet you will find for each inference spaces marked with the letters T, PT, ID, PF, and F. For each inference make a mark on the Answer Sheet under the appropriate letter as follows:

T — if you think the inference is *definitely true*; that it properly follows from the statement of facts given.

PT — if, in the light of the facts given, you think the inference is *probably true*; that there is better than an even chance that it is true.

ID — if you decide that there are *insufficient data*; that you cannot tell from the facts given whether the inference is likely to be true or false.

PF — if, in the light of the facts given, you think the inference is *probably false*; that there is better than an even chance that it is false.

F — if you think the inference is *definitely false*; that it cannot possibly be drawn from the facts given or that in some manner it contradicts the facts.

Sometimes, in deciding whether an inference is probably true or probably false, you will have to use certain commonly accepted knowledge or information which practically every person knows. This will be illustrated in the example which follows.

Here is the example; the correct answers are indicated in the block at the right.

EXAMPLE. A thousand eighth-grade students recently attended a voluntary week-end conference in a Midwestern city. At this conference questions of race relations and means of achieving lasting world peace were discussed, since these were the problems the students felt to be most vital today.

1. As a group, the students who attended this conference had a keener interest in humanitarian or broad social problems than most eighth-grade students have.....
2. The majority of these students were between the ages of 17 and 18.....
3. The students came from all sections of the country.....
4. The students came to discuss trade-union problems.....
5. Some eighth-grade students felt that discussion of race relations and means of achieving world peace might be worthwhile.....

TEST 1

	T	PT	ID	PF	F
1		■			
2				■	
3			■		
4					■
5	■				

In the above example, inference 1 is *probably true* (PT) because (as is common knowledge) most eighth-grade students are not likely to evidence such serious concern with broad social problems.

Inference 2 is *probably false* (PF) because (common knowledge) there are relatively few eighth-grade students in the United States between 17 and 18 years of age.

There is no evidence for inference 3. Thus there are *insufficient data* (ID) for making a judgment in the matter.

Inference 4 is *definitely false* (F) because it is given in the statement of facts that race relations and means for achieving world peace were the problems discussed.

Inference 5 necessarily follows from the given facts; it therefore is *true* (T).

In the exercises which follow, more than one of the inferences from a given statement of facts may be true (T), or false (F), or probably true (PT), or probably false (PF), or have insufficient data (ID) to warrant any conclusion. That is, you are to consider each inference by itself.

Make a heavy black mark in the space under the letter that you think best describes each inference. If you change an answer, erase thoroughly. Make no extra marks on the answer sheet.

An English teacher arranged for the students in one of her classes to see the movie *Great Expectations*, while the students in other classes studied the book itself, without seeing the picture. Tests to measure appreciation and understanding of the story were administered immediately upon completion of each type of instruction. On all tests the class which was taught with the aid of the movie did better. The class which saw the movie became so interested that before the semester was over most of those students read the book, entirely on their own initiative.

1. The tests to measure appreciation and understanding of the story were administered both to the students who saw the picture and to those who only studied the book.....
2. The children who were taught with the aid of the motion picture were required to read the book before the end of the semester.
3. Pupils who see movies instead of reading books lose interest in reading.....
4. Most of the children in the class which saw the picture would have preferred to study the book *Great Expectations* in the usual way without the aid of the movie.....
5. The teacher who conducted the experiment will hereafter try to use motion pictures when they are available, as an aid in teaching literary appreciation.
6. Pupils can learn more about any given subject from motion pictures than they can from books.....

The first newspaper in America, edited by Ben Harris, appeared in Boston September 25, 1690, and was banned the same day by Governor Simon Bradstreet. The editor's long fight to continue his little paper and print what he wished marks an important episode in the continuing struggle to maintain a free press.

7. The editor of the first American newspaper died within a few days after his paper was banned.....
8. Governor Bradstreet felt he had the legal authority to ban Ben Harris's paper.....
9. The editor of this paper wrote articles against taxes of the kind which later brought about the "Boston Tea Party.".....
10. Ben Harris was a man of persistence in holding to some of his interests and convictions.....

Some time ago a crowd gathered in Middletown, Mississippi, to hear the new president of the local Chamber of Commerce speak. He said, "I am not asking, but demanding, that labor unions accept their full share of responsibility for civic betterment and community interests. I am not asking, but demanding, that they join the Chamber of Commerce." The listening representatives of the Central Labor Unions applauded enthusiastically. Three months later all the labor unions in Middletown were represented in the Chamber of Commerce, where they served enthusiastically on committees, spoke their minds, and participated actively in the civic betterment projects.

11. Both the labor union representatives and the other members of the Chamber of Commerce came to recognize one another's problems and viewpoints better through their Chamber of Commerce contacts.
12. Labor unions' participation in the Middletown Chamber of Commerce has largely eliminated worker-management disputes in that town....
13. The active participation of the labor unions caused friction at the meetings of the Chamber of Commerce.
14. The union representatives soon regretted having accepted the invitation to participate in the Chamber of Commerce.....
15. Many of the Chamber of Commerce members came to feel that their president had been unwise in asking the union representatives to join the Chamber.....
16. The representatives of the Central Labor Unions joined the Chamber of Commerce against the desires of the great majority of their membership.....

Studies have shown that there is relatively much more tuberculosis among Negroes in the United States than among whites. There is no difference, however, in rate of tuberculosis between Negroes and whites who have the same level of income. The average income of whites in the United States is considerably higher than the average income of Negroes.

17. Tuberculosis can be cured.
18. Raising the economic level of Negroes would reduce tuberculosis.....
19. Tuberculosis is less prevalent among Negroes with relatively high incomes than among Negroes with relatively low incomes.
20. Whether a white person is rich or poor makes no difference in the likelihood of his getting tuberculosis.

TEST 2. *Recognition of Assumptions*

DIRECTIONS. An *assumption* is something supposed or taken for granted. When someone states, "I'll graduate in June," he takes for granted or assumes that he will be alive in June, that he will remain in school until that time, that he will pass his courses, and similar things.

Below are a number of statements. Each statement is followed by several proposed assumptions. *You are to decide for each assumption whether it necessarily is taken for granted in the statement.*

If you think the given assumption is taken for granted in the statement, make a heavy mark between the dotted

lines under "ASSUMPTION MADE" in the proper place on the Answer Sheet. If you think the assumption is not necessarily taken for granted in the statement, make a heavy line under "ASSUMPTION NOT MADE" on the Answer Sheet.

Below is an example: the block at the right shows how these items should be marked on the Answer Sheet.

If you do not see why the answers marked are right, ask the examiner to explain. In some cases more than one of the given assumptions is necessarily made; in other cases none of the given assumptions is made.

EXAMPLE. STATEMENT: "We need to save time in getting there, so we'd better go by plane."

PROPOSED ASSUMPTIONS:

1. Going by plane will take less time than going by some other means of transportation. (It is assumed in the statement that greater speed of a plane over other means of transportation will enable the group to get to their destination in less time.)
2. It is possible to make plane connections to our destination. (This is necessarily assumed in the statement, since, in order to save time by plane, it must be possible to go by plane.) . .
3. Travel by plane is more convenient than travel by train. (This assumption is not made in the statement — the statement has to do with saving time, and says nothing about convenience or about any other specific mode of travel.)

TEST 2

	ASSUMPTION MADE	NOT MADE
1	☒	☐
2	☒	☐
3	☐	☒

STATEMENT: "Let us immediately build superior armed force and thus keep peace and prosperity."

PROPOSED ASSUMPTIONS:

21. If we have superior armed force, that will insure the maintenance of peace and prosperity.....
22. Unless we increase our armaments immediately we shall have war.
23. We now have peace and prosperity.....

STATEMENT: "A wise man will save at least twelve dollars each week out of his earnings."

PROPOSED ASSUMPTIONS:

24. No fools have sense enough to save twelve dollars a week.....
25. A person needs to be wise in order to save twelve dollars a week.....

STATEMENT: "Even if all the wealth in the country suddenly were to be distributed equally, some people soon would again become rich and others poor."

PROPOSED ASSUMPTIONS:

26. The real causes of wealth and poverty would not be much affected by such Socialism.
27. Our present economic system is better than such

STATEMENT: "Mary isn't going to invite John to her party."

PROPOSED ASSUMPTIONS:

28. Mary hasn't yet had her party.....
29. Mary now doesn't like John.....
30. The party will be at Mary's house.....

STATEMENT: "Live in the city of Zenith — lowest taxes."

PROPOSED ASSUMPTIONS:

31. Efficient management of a city implies lower taxes.
32. An important consideration in deciding where to live is avoidance of high taxes.....
33. The people of Zenith are content with their present city government.....

STATEMENT: "Our school is fortunate in having all American pupils, so we have no race problems."

PROPOSED ASSUMPTIONS:

34. American pupils do not present any race problems.
35. If we practiced democracy, there would be no race problem.....
36. A school is unfortunate if its pupils are of varied nationalities.....

TEST 3. *Deduction*

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DIRECTIONS. Each exercise below consists of two statements (premises) followed by several proposed conclusions. For the purposes of this test, consider the two statements in each exercise as true without exception. Read the first conclusion beneath the statements, and if you think it *necessarily follows from the statements given*, answer by making a heavy black mark between the pair of dotted lines under "CONCLUSION FOLLOWS" in the corresponding blank on the Answer Sheet. If you think it is *not a necessary conclusion* from the given statements,

then put a heavy black mark under "CONCLUSION DOES NOT FOLLOW," even though you may believe it to be true from your general knowledge.

Likewise read and judge each of the other conclusions. Try not to let your prejudices influence your judgment — just stick to the given statements and judge each conclusion as to whether it necessarily follows from them. Mark all your answers on the Answer Sheet.

Here is an example; the block at the right shows how your answers should be marked on the Answer Sheet.

EXAMPLE. Some holidays are rainy. All rainy days are boring. Therefore —

1. No clear days are boring. (The conclusion does not follow, as you cannot tell from these statements whether or not clear days are boring and some may be)
2. Some holidays are boring. (The conclusion necessarily follows from the statements, since, according to them, the rainy holidays must be boring)
3. Some holidays are not boring. (The conclusion does not follow from the statements even though you may know that some holidays are very pleasant)

TEST 3	
CONCLUSION	
FOLLOWS DOES NOT FOLLOW	
1	☐
2	☒
3	☐

All musicians are temperamental. Some musicians are not proud. Therefore —

37. All temperamental people are musicians.
38. No proud people are temperamental.
39. Some proud people are musicians.

No jockey is a heavyweight boxer. All heavyweight boxers are large men. Therefore —

40. No jockey is a small man.
41. No heavyweight boxer is a small man.
42. Jockeys are small men

Some cannibals are sincere idealists. All cannibals are fanatics. Therefore —

43. Some sincere idealists are fanatics.
44. Some fanatics are sincere idealists.
45. No fanatics are sincere idealists.
46. All fanatics are cannibals.

All mice that are injected with substance "A" develop disease "X." Mouse #24 was not injected with substance "A." Therefore —

47. Mouse #24 did develop disease "X."
48. Not all mice with numbers between 20 and 30 were injected with substance "A."
49. Mouse #24 did not develop disease "X."

No Republican is a Democrat. All Democrats favor prosperity. Therefore —

50. Republicans favor prosperity.
51. No Republican opposes prosperity.
52. No Democrat opposes prosperity.
53. No Republican favors prosperity.

All Jews feel friendly toward the State of Israel. David feels friendly toward the State of Israel. Therefore —

54. David is not friendly toward the Arabs.
55. David is Jewish.
56. Some non-Jews also feel friendly toward the State of Israel.

If an adult has the ability to give love to others, he must have received love as a child. Some adults did not receive love when they were children. Therefore —

57. Some adults do not have the ability to give love to others.
58. If an adult received love as a child, he has the ability to give love to others.

If a person is superstitious, he believes fortunetellers. Some people do not believe fortunetellers. Therefore —

59. No superstitious person doubts fortunetellers.
60. If a person is not superstitious, he will not believe fortunetellers.
61. If a person believes fortunetellers, he is superstitious.

TEST 4. *Interpretation*

DIRECTIONS. Each exercise below consists of a short paragraph followed by several proposed conclusions.

For the purpose of this test assume that everything in the short paragraph is true. The problem is to judge whether or not each of the proposed conclusions logically follows beyond a reasonable doubt from the information given in the paragraph.

If you think that the proposed conclusion follows *beyond a reasonable doubt* (even though it may not follow

absolutely and necessarily), then make a heavy black mark between the appropriate dotted lines under the "CONCLUSION FOLLOWS" column on the Answer Sheet. If you think that the conclusion does *not* follow beyond a reasonable doubt from the facts given, then make a mark under "CONCLUSION DOES NOT FOLLOW."

In some cases more than one of the proposed conclusions may follow; in other cases none of the conclusions may follow.

A report of the U. S. Census states that during 1940 there were approximately 1,656,000 marriages and 264,000 divorces granted in the United States.

62. Getting a divorce is a quick and easy matter in the United States.....
63. If the above ratio still holds true, then about six times as many people get married each year as get divorced.....
64. The divorce rate in the United States is much too high.....

Usually I fall asleep promptly, but about twice a month I drink coffee in the evening; and whenever I do, I lie awake and toss for hours after I go to bed.

67. My problem is mostly mental; I am over-aware of the coffee when I drink it at night, anticipating that it will keep me awake, and therefore it does... ..
68. I don't fall asleep promptly after drinking coffee at night because the caffeine in coffee stimulates my nervous system for several hours after drinking it..

At the end of the semester the pupils in Mr. Black's class averaged 10 points higher than the pupils in Miss Walter's class on the same geometry test. Mr. Black and Miss Walter used a somewhat different method of teaching geometry.

69. Mr. Black probably is a better teacher than Miss Walter.....
70. The pupils in Mr. Black's class were brighter as a group than the pupils in Miss Walter's class, and therefore they learned more easily.. ..
71. The method used by Mr. Black in teaching geometry was superior to the method used by Miss Walter... ..

When Great Britain began to offer free public medical service, the government was surprised because far more people than they had expected came for eyeglasses and dental work.

72. People who previously had neglected their eyes and teeth now chose to have such treatment.. ..
73. People who didn't really need these services sought them because they were free.... ..
74. People in Great Britain previously had been careless about the state of their eyes and teeth.. ..
75. The British public was pleased with the government health program.....

Victims of radiation sickness (for example, after an atomic explosion) are likely to die of anemia because the blood-building properties of the bone marrow are damaged. In everyday medical practice, X-ray dosages have to be worked out with utmost care to keep the patient from falling prey to radiation sickness. Experimenting on rabbits, Dr. Leon Jacobson found that when the spleen and appendix were protected with lead, the animals survived what would otherwise have been a fatal overdose of X rays. The undamaged spleen and appendix make enough blood to enable the damaged tissue to recover.

65. If from the blood-forming organs a substance could be isolated which would speed an individual's recovery from radiation sickness, that substance probably would also enable X-ray patients to take heavier doses.....
66. Dr. Jacobson's experiments on rabbits should be tried on a sufficiently large scale with people to see whether the same results would hold true.....

TEST 4. *Interpretation (Continued)*

125

The *Los Angeles Times* made a survey of the number of men and women drivers involved in automobile accidents in the Los Angeles area during a given period of time. They found that men drivers were involved in 1210 accidents while women drivers were involved in only 920 accidents.

76. If the survey figures constitute a representative sample, men drivers are involved in accidents more frequently than women drivers in the Los Angeles area.
77. More men than women drive cars in the Los Angeles area.
78. Women are safer drivers than men in the Los Angeles area.

Intelligence tests show that Negro children in Northern cities surpass Negro children in Southern cities but do not score as high as white children in Northern cities.

79. White children as a group score higher because they are born with higher native intelligence than Negro children
80. The Negro families who moved to the North are on the average more intelligent than those who remained in the South . .

81. Northern Negroes receive better schooling than Southern Negroes, which in turn influences performance on the tests

The history of the last two thousand years shows that wars have become steadily more frequent and more destructive, the twentieth century being the bloodiest on record

82. Mankind has not advanced as much in the art of keeping peace as it has in the science of waging war.
83. Wars are caused by basic traits of selfishness, greed, and pugnacity, which are rooted in human nature. .
84. Increased industrialization, competitiveness, and improved weapons bring on increasingly frequent wars
85. There will be increasingly frequent future wars, and they will become steadily more destructive than past wars.

Go on to the next test.

TEST 5. *Evaluation of Arguments*

DIRECTIONS. In making decisions about important questions it is desirable to be able to distinguish between arguments that are *strong* and those which are *weak* in so far as the question at issue is concerned.

Strong arguments must be both important and directly related to the question.

Weak arguments may not be directly related to the question, even though they may be of great general importance; or they may be of minor importance; or they may be related to trivial aspects of the question.

Below is a series of questions. Each question is followed by three or four arguments. *For the purpose of this test you are to regard each argument as true.* The problem then is to decide whether it is a **STRONG** argument or a **WEAK** argument.

You are to answer by making a heavy mark on the Answer Sheet under "STRONG" if you think the argument is strong, or by making a heavy mark under "WEAK" on the Answer Sheet if you think the argument is weak. When evaluating an argument, judge it on its own merit; try not to let counter-arguments or your own attitude toward the question influence your judgment. Judge each argument separately. In some questions all the arguments may be **STRONG**, in others all may be **WEAK**.

Here is an example. The block at the right shows how these arguments should be marked on the Answer Sheet. Study them carefully until you know just what is expected of you. Note that the argument is evaluated as to how well it supports the side of the question indicated.

EXAMPLE. Should all young men go to college?

1. Yes; college provides an opportunity for them to learn school songs and cheers (This would be a silly reason for spending years of one's life in college)
2. No; most young men profit more from work experience than from college classes. (According to the directions, we must accept this argument as true, hence it is a strong and important one against all young men going to college)
3. No; excessive studying permanently warps an individual's personality. (This argument, although of great general importance when accepted as true, is not directly related to the question, because attendance at college does not necessarily require excessive studying)

TEST 5	
ARGUMENT	
STRONG	WEAK
1	☒
2	☒
3 ..	☒

Remember that for the purpose of this test each argument is to be regarded as true.

Can rich and poor people who happen to oppose each other at law obtain approximately equal justice from the courts?

86. No; a rich person can hire better lawyers and technical experts, pay for the time of more witnesses, and continue the fight in higher courts.....
87. No; rich people win the majority of their lawsuits against poor people.....
-

Should married women be eligible for employment as public school teachers if they are otherwise qualified?

88. No; there are more single women in our country than there are school-teaching jobs.....
89. Yes; women tend to become better teachers after marriage.....
90. No; a mother's first responsibility is to her own children.....
-

Should infants be fed by regular schedule rather than whenever they seem to be hungry?

91. No; babies know best when they are hungry and ready to eat.....
92. Yes; children must sooner or later learn that they can't always have their own way.....
93. Yes; a regular schedule is easier for the parents....

Should the government take over all the main industries in the country, employ all who want to work, and offer the products at cost prices?

94. No; so much concentration of economic and bureaucratic power in government would undermine our personal and political freedom.
95. No; elimination of competition and the profit motive would result in much less initiative for production of useful new goods and services.
96. Yes; the government already operates post offices, highways, parks, military forces, public health services, and other public services.....
-

Should groups in this country who are opposed to some of our government's policies be allowed unrestricted freedom of press and speech?

97. Yes; a democratic state thrives on free and unrestricted discussion, including criticism.....
98. No; if given full freedom, opposition groups would disunite the American people, weaken our position, and ultimately lead to loss of our democracy.....
99. No; the countries opposed to our form of government do not permit the free expression of our point of view in their territory.....

Go back and check your work.

WATSON-GLASER

CRITICAL THINKING

APPRAISAL

Form BM

by GOODWIN WATSON

*Professor of Education, Teachers College,
Columbia University*

and EDWARD MAYNARD GLASER

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TEST 1. *Inference*

DIRECTIONS. An *inference* is a conclusion which a person draws from certain observed or supposed facts. Thus, from the electric light visible behind the window shades and from the sound of piano music in a house, a person might *infer* that someone is at home. But this inference may or may not be correct. Possibly the people in the house went out leaving the lights on, and the piano music could be coming from a radio or phonograph they left playing.

In this test each exercise begins with a *statement of facts* which you are to regard as true. After each statement of facts you will find several possible inferences — that is, inferences which some persons might make from the stated facts. Examine each inference separately, and make a decision as to its degree of truth or falsity.

On the Answer Sheet you will find for each inference spaces marked with the letters T, PT, ID, PF, and F. For each inference make a mark on the Answer Sheet under the appropriate letter as follows:

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PT — if, in the light of the facts given, you think the inference is *probably true*; that there is better than an even chance that it is true.

ID — if you decide that there are *insufficient data*; that you cannot tell from the facts given whether the inference is likely to be true or false.

PF — if, in the light of the facts given, you think the inference is *probably false*; that there is better than an even chance that it is false

F — if you think the inference is *definitely false*; that it cannot possibly be drawn from the facts given or that in some manner it contradicts the facts.

Sometimes, in deciding whether an inference is probably true or probably false, you will have to use certain commonly accepted knowledge or information which practically every person knows. This will be illustrated in the example which follows.

Here is the example; the correct answers are indicated in the block at the right

EXAMPLE. A thousand eighth grade students recently attended a voluntary week-end conference in a Midwestern city. At this conference questions of race relations and means of achieving lasting world peace were discussed, since these were the problems the students felt to be most vital today

1. As a group, the students who attended this conference had a keener interest in humanitarian or broad social problems than most eighth grade students have
2. The majority of these students were between the ages of 17 and 18
3. The students came from all sections of the country . .
4. The students came to discuss trade-union problems
5. Some eighth-grade students felt that discussion of race relations and means of achieving world peace might be worthwhile

TEST 1

	T	PT	ID	PF	F
1		■			
2				■	
3			■		
4					■
5	■				

In the above example, inference 1 is *probably true* (PT) because (as is common knowledge) most eighth-grade students are not likely to evidence such serious concern with broad social problems.

Inference 2 is *probably false* (PF) because (common knowledge) there are relatively few eighth-grade students in the United States between 17 and 18 years of age

There is no evidence for inference 3. Thus there are *insufficient data* (ID) for making a judgment in the matter.

Inference 4 is *definitely false* (F) because it is given in the statement of facts that race relations and means for achieving world peace were the problems discussed

Inference 5 necessarily follows from the given facts, it therefore is *true* (T)

In the exercises which follow, more than one of the inferences from a given statement of facts may be true (T), or false (F), or probably true (PT), or probably false (PF), or have insufficient data (ID) to warrant any conclusion. That is, you are to consider each inference by itself

Make a heavy black mark in the space under the letter that you think best describes each inference. If you change an answer, erase thoroughly. Make no extra marks on the answer sheet.

A study in the Elmtown High School showed that the students from high-income homes received better grades on the average than did pupils from low-income homes. Differences in school grades given by teachers were considerably greater than differences on standard tests of intelligence and achievement. Students from high-income homes took part in many more of the extracurricular school activities which cost money than did the students from low-income homes. Student officers were usually chosen from the high-income group.

1. Elmtown sent children of rich and poor to the same high school.....
2. Many students from low-income homes felt they couldn't afford to participate in extracurricular school activities which cost money.....
3. The students from low-income homes actually studied harder than did the students from wealthy homes.
4. There were no differences between students from high-income homes and students from low-income homes on standard tests of intelligence and achievement.....
5. The majority of the students from low-income homes, recognizing the general superiority of the students from high-income homes, wanted them to lead the student groups.....

The town of Westfield, beginning twenty years ago, has gradually bought up farms abandoned by owners who failed to pay taxes, and by this time has set out some 3600 acres of community forest. The oak trees have grown rapidly. The town forests yielded \$9000 net profit on lumber last year and \$8500 the year before. Local authorities believe that the net profit on the lumber will eventually be \$40,000 a year.

6. It costs the town more to cut and sell the lumber than it realizes from the sales.....
7. The operating expenses were less than \$9000 last year.....
8. The owners who abandoned their farms and failed to pay their taxes were either incompetent farmers or lazy ones.....
9. The town of Westfield is continuing to buy up tax-delinquent farms to be set aside as community forests.....
10. The Westfield community forests will yield an annual net profit of \$40,000 within two or three years.....

When a Negro doctor bought a home in a suburban residential district where only whites of various nationalities had lived before, a mob stoned the house and broke many windows. The mayor, at the request of his Committee on Race Relations, sent extra police to keep order in this district.

11. The immediate neighbors of the Negro doctor liked him and did not join the mob.....
12. The Negro doctor was a better citizen than the white people who lived around him.....
13. The mob was composed of citizens who upheld democratic principles.....
14. The Negro family sold the house and moved elsewhere.....
15. The white families in the district all quickly sold their homes and moved away.....

Mr. Brown, who lives near the town of Salem, was brought before the Salem municipal court for the fourth time in the past month on a charge of keeping his dance hall open after midnight. He again pleaded guilty and was fined the maximum, \$100, as in each earlier instance.

16. Mr. Brown thought that it would pay him financially to keep his place open after 12 o'clock, even though he had to risk paying frequent fines.....
17. Mr. Brown's dance hall is within the legal jurisdiction of the town of Salem.....
18. The regular patrons of Mr. Brown's dance hall are habitual lawbreakers.....
19. The maximum fine of \$100 was fully effective in keeping all dance halls in Salem closed after midnight.
20. The midnight closing law was enacted because one of Mr. Brown's neighbors complained about the noise.....

Go on to the next page.

TEST 2. *Recognition of Assumptions*

DIRECTIONS. An *assumption* is something supposed or taken for granted. When someone states, "I'll graduate in June," he takes for granted or assumes that he will be alive in June, that he will remain in school until that time, that he will pass his courses, and similar things.

Below are a number of statements. Each statement is followed by several proposed assumptions. *You are to decide for each assumption whether it necessarily is taken for granted in the statement.*

If you think the given assumption is taken for granted in the statement, make a heavy mark between the dotted

lines under "ASSUMPTION MADE" in the proper place on the Answer Sheet. If you think the assumption is not necessarily taken for granted in the statement, make a heavy line under "ASSUMPTION NOT MADE" on the Answer Sheet.

Below is an example: the block at the right shows how these items should be marked on the Answer Sheet.

If you do not see why the answers marked are right, ask the examiner to explain. In some cases more than one of the given assumptions is necessarily made; in other cases none of the given assumptions is made.

EXAMPLE. STATEMENTS: "We need to save time in getting there, so we'd better go by plane."

PROPOSED ASSUMPTIONS:

1. Going by plane will take less time than going by some other means of transportation. (It is assumed in the statement that greater speed of a plane over other means of transportation will enable the group to get to their destination in less time.)
2. It is possible to make plane connections to our destination. (This is necessarily assumed in the statement, since, in order to save time by plane, it must be possible to go by plane.)
3. Travel by plane is more convenient than travel by train. (This assumption is not made in the statement — the statement has to do with saving time, and says nothing about convenience or about any other specific mode of travel.)

TEST 2

	ASSUMPTION MADE	NOT MADE
1	☒	☐
2	☒	☐
3	☐	☒

STATEMENT: "The coach isn't going to let Ed play in any more games this season."

PROPOSED ASSUMPTIONS:

21. The coach can prevent Ed from playing.
22. Ed must have done something terribly wrong.
23. The games for this season are not yet over.
24. There will be a chance for Ed to play next season.

STATEMENT: "The less government interferes with business, the better for everyone."

PROPOSED ASSUMPTIONS:

25. Government is inefficient.
26. Most businessmen are superior in character to most government officials.
27. Any good cause is injured if it gets mixed up in politics.
28. If free from government controls, business would do what is good for us.

STATEMENT: "I want to be sure I don't get typhoid fever while I'm in South America, so I shall go to my physician and get typhoid injections before I sail."

PROPOSED ASSUMPTIONS:

29. If I don't take the injections, I shall become ill with the fever.
30. Only a physician can give typhoid injections.
31. I am going to South America very soon.

STATEMENT: "Only a fool would try to keep the pleasant taste of wine by having his mouth always full of it."

PROPOSED ASSUMPTIONS:

32. Some fools don't like wine.
33. A wise man never has his mouth full of wine.
34. Some fools get drunk on wine.

STATEMENT: "If war is inevitable, we'd better launch a preventive war now while we have the advantage."

PROPOSED ASSUMPTIONS:

35. If we fight now, we are more likely to win than we would be if forced to fight later.
36. War is inevitable.

Go on to the next page.

TEST 3. *Deduction*

128

DIRECTIONS. Each exercise below consists of two statements (premises) followed by several proposed conclusions. For the purposes of this test, consider the two statements in each exercise as true without exception. Read the first conclusion beneath the statements, and if you think it *necessarily follows from the statements given*, answer by making a heavy black mark between the pair of dotted lines under "CONCLUSION FOLLOWS" in the corresponding blank on the Answer Sheet. If you think it is *not a necessary conclusion* from the given statements,

then put a heavy black mark under "CONCLUSION DOES NOT FOLLOW," even though you may believe it to be true from your general knowledge.

Likewise read and judge each of the other conclusions. Try not to let your prejudices influence your judgment — just stick to the given statements and judge each conclusion as to whether it necessarily follows from them. Mark all your answers on the Answer Sheet.

Here is an example; the block at the right shows how your answers should be marked on the Answer Sheet.

EXAMPLE. Some holidays are rainy. All rainy days are boring. Therefore —

1. No clear days are boring. (The conclusion does not follow, as you cannot tell from these statements whether or not clear days are boring and some may be.)
2. Some holidays are boring. (The conclusion necessarily follows from the statements, since, according to them, the rainy holidays must be boring.)
3. Some holidays are not boring. (The conclusion does not follow from the statements even though you may know that some holidays are very pleasant.)

TEST 3	
CONCLUSION FOLLOWS	CONCLUSION DOES NOT FOLLOW
1 : : :	■
2 ■	: : :
3 : : :	■

All good typists must be able to spell correctly. Anne can spell correctly. Therefore —

37. Anne is not a good typist.....
38. Anne's typing ability cannot be inferred from the information given.
39. Anne is a good typist.....

All glass is transparent. Lucite is not glass. Therefore —

40. Lucite is transparent.....
41. Some forms of glass are not transparent.....
42. Lucite is not transparent.....

All radicals are foreign-born. No patriotic citizen is a radical. Therefore —

43. No radical is a patriotic citizen.....
44. Liberals and Democrats work together for the same ends.....
45. No patriotic citizen is foreign-born.....
46. Some foreign-born people are patriotic citizens....

Some Russians would like to control the world. All Russians seek a better life for themselves. Therefore —

47. Some people who would like to control the world seek a better life for themselves.....
48. All people who seek a better life for themselves would like to control the world.....
49. Many Russians find life under their dictatorship miserable, and therefore seek a change.....

Some students keep late hours. All students hate to get up in the morning. Therefore —

50. All students who hate to get up in the morning were up late the night before.....
51. No student likes to get up in the morning.....
52. All students who keep late hours hate to get up in the morning.....
53. Keeping late hours causes students to hate to get up in the morning.

If Negroes are segregated, there is racial discrimination. If democratic principles are not violated, Negroes are not segregated. Therefore —

54. Racial discrimination is limited to Negroes.....
55. If there is racial discrimination, Negroes are segregated.....
56. If democratic principles are violated, there is racial discrimination.....

All children who are spanked develop repressions. John never was spanked. Therefore —

57. John has never developed repressions.....
58. All people who have repressions probably were spanked while children.....

If a person thinks straight, he will define the problem to be solved. Some people do not define the problem to be solved. Therefore —

59. If a person does not think straight, he will not define the problem to be solved.
60. If a person defines the problem to be solved, he thinks straight.
61. Some people do not know how to define the problem to be solved.

TEST 4. *Interpretation*

DIRECTIONS. Each exercise below consists of a short paragraph followed by several proposed conclusions.

For the purpose of this test assume that everything in the short paragraph is true. The problem is to judge whether or not each of the proposed conclusions logically follows beyond a reasonable doubt from the information given in the paragraph.

If you think that the proposed conclusion follows *beyond a reasonable doubt* (even though it may not follow

absolutely and necessarily), then make a heavy black mark between the appropriate dotted lines under the "CONCLUSION FOLLOWS" column on the Answer Sheet. If you think that the conclusion does *not* follow beyond a reasonable doubt from the facts given, then make a mark under "CONCLUSION DOES NOT FOLLOW."

In some cases more than one of the proposed conclusions may follow; in other cases none of the conclusions may follow.

A salesman for Brown's Liniment claimed that his product would promptly soothe sore muscles in the body because it would penetrate very quickly to the affected parts. In order to demonstrate the penetrating qualities of Brown's Liniment, the salesman poured ten drops on a thick piece of sole leather, and the liniment quickly went through this substance.

62. The liniment which the salesman poured on the sole leather was effective in penetrating the kind of sole leather used in the demonstration.
63. The salesman deliberately misrepresented his product.
64. The salesman's demonstration was good evidence for his claim that the liniment would promptly soothe sore muscles in the body.
65. There is no relationship between the liniment's ability to penetrate this particular piece of sole leather and its ability to penetrate into the human body.

In a certain city where school attendance laws are rigidly enforced, it was found that only 15 per cent of the male school population had a perfect attendance record during any single school semester. Among those who sold newspapers, however, 25 per cent had a perfect attendance record during the same period.

66. If truants were given jobs selling newspapers, their school attendance would improve.
67. Newsboys in another city would have a similarly superior attendance record.
68. Those who carry partial responsibility for their own support tend to take their schooling more seriously..

Among the people listed in *Who's Who in America* those with college degrees are about 15 times more numerous than those with only a grade-school education.

69. If the given figures hold constant, college graduates as a group have a 15 to 1 better chance of being listed in *Who's Who* than have people with only a grade-school education.
70. The editors of *Who's Who* give undue weight to the types of achievement made by college-trained people.
71. People listed in *Who's Who* tend to come from wealthy families who can afford to give their children many advantages, including a college education. ...

Jane's posture used to be poor; she dressed in bad taste, had very few friends, was ill at ease in company, and in general was quite unhappy and maladjusted. Then someone recommended that she visit Dr. Baldwin, a reputed expert on helping people to improve their personality. Jane took this recommendation, and after three months of treatment by Dr. Baldwin she carried herself well, dressed attractively, was more at ease and more popular, and in general felt much happier.

72. Jane's improvement may be only temporary. . . .
73. Jane's marked improvement must have been brought about by important factors other than Dr. Baldwin's treatment.
74. Jane's improvement was caused solely by the treatment given her by Dr. Baldwin.

Several studies have shown that Southern Negroes make lower scores on intelligence tests than Northern Negroes, but that the average measured intelligence of Southern Negro children who move North at an early age increases each year until their average measured intelligence approaches that of the Northern Negroes.

75. The children of Southern Negroes who move North tend to equal the children of Northern Negroes in the kinds of ability measured by the intelligence tests used.
76. The increase is due to the relative superiority of the schools Negroes attend in the North.

TEST 4. *Interpretation (Continued)*

A national weekly magazine published some articles criticizing the action of the Catholic Church in matters of health and censorship, and was promptly banned from the high school libraries by the school board of an Eastern city.

77. The majority of the people on that school board were afraid of the power of the Catholic Church
78. A majority of the people in that city must have been Catholics
79. The magazine should not have published those articles

Statistics for a certain city in the United States, which has a well-organized Boy Scout program, indicate that during the past ten years no Scouts have been convicted of juvenile delinquency

80. The Scout program prevents delinquency
81. Scouting is a good way to meet the most important needs of boys today

A sleeping wife was awakened by the sensation of a sharp blow across her mouth. Later that night her husband returned from a fishing trip, his front teeth knocked out by a sharp blow from the oar of his boat. Apparently the husband's accident and the wife's dream came at about the same time.

82. Under the circumstances described there was no way for any ordinary form of direct communication between the husband and wife at the time of the husband's accident
83. If the facts reported in this case are true, they can be accounted for only by the operation of mental telepathy
84. The dream was a chance coincidence which was not really influenced by the accident.
85. There are many occurrences in life which we do not understand.

Go on to the next test.

TEST 5. *Evaluation of Arguments*

DIRECTIONS In making decisions about important questions it is desirable to be able to distinguish between arguments that are *strong* and those which are *weak* in so far as the question at issue is concerned

Strong arguments must be both important and directly related to the question.

Weak arguments may not be directly related to the question, even though they may be of great general importance, or they may be of minor importance, or they may be related to trivial aspects of the question.

Below is a series of questions. Each question is followed by three or four arguments. *For the purpose of this test you are to regard each argument as true.* The problem then is to decide whether it is a **STRONG** argument or a **WEAK** argument

You are to answer by making a heavy mark on the Answer Sheet under "STRONG" if you think the argument is strong, or by making a heavy mark under "WEAK" on the Answer Sheet if you think the argument is weak. When evaluating an argument, judge it on its own merit; try not to let counter-arguments or your own attitude toward the question influence your judgment. Judge each argument separately. In some questions all the arguments may be **STRONG**, in others all may be **WEAK**.

Here is an example. The block at the right shows how these arguments should be marked on the Answer Sheet. Study them carefully until you know just what is expected of you. Note that the argument is evaluated as to how well it supports the side of the question indicated.

EXAMPLE. Should all young men go to college?

1. Yes, college provides an opportunity for them to learn school songs and cheers. (This would be a silly reason for spending years of one's life in college)
2. No, most young men profit more from work experience than from college classes. (According to the directions, we must accept this argument as true, hence it is a strong and important one against all young men going to college) . . .
3. No; excessive studying permanently warps an individual's personality. (This argument, although of great general importance when accepted as true, is not directly related to the question, because attendance at college does not necessarily require excessive studying)

TEST 5
ARGUMENT
STRONG WEAK

1 ☐ ☒

2 ☒ ☐

3 ☐ ☒

Remember that for the purpose of this test each argument is to be regarded as true.

Should the government provide "baby bonuses" to help support each dependent child in a family so that the family standard of living is not lowered by having children?

86. Yes; every family then could afford proper child care, which would greatly improve the general health of the nation.....
87. No; such bonuses would require much higher taxes, thus actually lowering the general standard of living.....
88. No; people who do not wish to have children then would be taxed to help provide "baby bonuses" for other people.....

Should the government continue to pay farmers the cost of soil-conservation practices on their own land?

89. Yes; food is a necessity to the entire nation, and farmers are much more likely to undertake the necessary practices which will assure abundant future crops if they get paid for the extra work involved.....
90. No; soil conservation is simply good farming practice which will increase the owner's likelihood of making a good living from his land; to tax others to pay him for helping himself is needless and unjust.
91. No; to pay a special group such as farmers out of public funds for helping themselves financially is opposed to the general welfare of the people as a whole.....

Is it possible for man to develop a death ray that will kill all living beings on whom it is focused?

92. No; no one has ever seen such a death ray outside the comics.
93. Yes; powerful concentrations of X rays and other types of radiant energy do kill animals and human beings.....
94. No; some very competent scientists doubt that such a death ray is possible.....
95. No; for if man ever does develop such a ray, he also will work on counter measures to offset it.. . .

Would a labor party be a good thing for the people of the United States?

96. Yes; differences between Republicans and Democrats today are not so great as differences between liberals and conservatives within those parties.. . .
97. No; labor unions have sometimes called strikes which hurt the public at large.....

Should pupils be excused from public schools to receive religious instruction during school hours in their own churches?

98. No; having children go back and forth between school and church is inefficient and would violate our basic and time-tested concept of the desirability of separation between church and state.
99. Yes; if children do not receive religious instruction during school hours, it may be difficult to get them to church at any other time.....

Go back and check your work.

APPENDIX 3

TERMIN McLEMAR TEST OF MENTAL ABILITY.

FORM C

C

Stanford University, California

Name.....

Date of birth.....
Month Day Year

Age.....Grade.....Boy.....Girl.....

School.....

City.....State.....

Teacher.....Date.....

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TEST 7. BEST ANSWER

Read each statement and mark the answer space which has the same number as the answer which you think is BEST.

SAMPLE. We should not put a burning match in the wastebasket because

1 Matches cost money. 2 We might need a match later.

3 It might go out. 4 It might start a fire.

1	2	3	4

1. The saying, "Idle brains are the devil's workhouse," means

1 The devil is lazy. 2 People who are idle get into trouble.

3 Many hands make light work. 4 The devil works with his brains. 1

1	2	3	4

2. The saying, "It's an ill wind that blows nobody good," means that

5 Winds bring rain. 6 That which brings misfortune to some may help others.

7 Trade winds help commerce. 8 It's easy to catch cold in a storm. 2

5	6	7	8

3. Farmers rotate crops because

1 Variety is the spice of life. 2 It confuses the plant pests.

3 It helps maintain soil fertility. 4 It gives the farmer a balanced diet. 3

1	2	3	4

4. The saying, "Little strokes fell great oaks," means

5 Continued effort brings results. 6 Oak trees are weak.

7 Little strokes are best. 8 Anyone can fell an oak. 4

5	6	7	8

5. The saying, "A miss is as good as a mile," means

1 A girl can walk just about a mile. 2 Errors are unpardonable.

3 The evil men do lives after them. 4 A failure is a failure, no matter how small. 5

1	2	3	4

6. The saying, "It never rains but it pours," means

5 Salt stays dry when it rains. 6 Every cloud has a silver lining.

7 Troubles seldom come singly. 8 Storms are more frequent than showers. 6

5	6	7	8

7. The cause of echoes is

1 Static electricity in the air. 2 The reflection of sound waves.

3 The absence of anything in the air. 4 Not known. 7

1	2	3	4

8. The saying, "Look before you leap," means

5 Consider first, act afterward. 6 Trust your eyes, but not your feet.

7 Anything is right which looks right. 8 Never take chances. 8

5	6	7	8

9. The saying, "Never ride a free horse to death," means

1 Never accept free rides. 2 Never abuse privileges granted as favors.

3 One should prize anything that is free. 4 A horse is to ride, not to kill. 9

1	2	3	4

10. The saying, "If the shoe fits, wear it," means

5 Be sure to buy shoes that fit. 6 Give the devil his due.

7 Don't take unnecessary steps. 8 Recognize your own faults and virtues. 10

5	6	7	8

11. Copper is used for electric wiring because

1 It is decorative. 2 It is easily bent.

3 It retains heat. 4 It is a good conductor. 11

1	2	3	4

12. The saying, "Don't cross your bridges till you come to them," means

5 Where there's a will there's a way. 6 Everything comes to him who waits.

7 Don't anticipate possible troubles. 8 Bridges are dangerous. 12

5	6	7	8

Score

TEST 1. INFORMATION

131

Mark the answer space which has the same number as the word that makes the sentence TRUE.

SAMPLE. Our first President was

1 Adams 2 Washington 3 Lincoln 4 Jefferson 5 Monroe....

1	2	3	4	5

1. Polo is a kind of
1 disease 2 work 3 bear 4 game 5 language.....1
 2. Herring is a kind of
6 wig 7 flower 8 pattern 9 jewel 10 fish.....2
 3. The lyre was an early instrument used in
1 music 2 writing 3 mining 4 farming 5 sculpturing.....3
 4. Linen is made from
6 hair 7 jute 8 flax 9 rayon 10 latex.....4
 5. Burlap is a kind of
1 lumber 2 stone 3 hood 4 fabric 5 comedy.....5
 6. Quicksilver is another name for
6 chromium 7 tin 8 mercury 9 aluminum 10 lead.....6
 7. The number of pounds in a ton is
1 (1000) 2 (2000) 3 (3000) 4 (4000) 5 (5280).....7
 8. Chinchilla is a kind of
6 fur 7 seasoning 8 chemical 9 malady 10 furniture.....8
 9. The fathom is a measure of
1 weight 2 curvature 3 hardness 4 depth 5 strength.....9
 10. Larceny is a term used in
6 forestry 7 medicine 8 theology 9 pedagogy 10 law.....10
- 1 →
11. Napoleon's final defeat was at
1 Waterloo 2 Paris 3 Verdun 4 Elba 5 Leipzig.....11
 12. The dynamo produces
6 dynamite 7 powder 8 electricity 9 gas 10 steam.....12
 13. Pasteur was a famous
1 traveler 2 boxer 3 artist 4 bacteriologist 5 physicist.....13
 14. The Pharaohs were kings of
6 Babylon 7 Jerusalem 8 Syria 9 Greece 10 Egypt.....14
 15. Sonata is a term used in
1 drawing 2 drama 3 music 4 poetry 5 phonetics.....15
 16. The Colosseum was an
6 amphitheater 7 aqueduct 8 aquarium 9 archway 10 army.....16
 17. The larynx is in the
1 abdomen 2 throat 3 head 4 ear 5 pelvis.....17
 18. Among birds that migrate widely are
6 eagles 7 cardinals 8 owls 9 robins 10 quail.....18
 19. Emeralds are usually
1 red 2 yellow 3 green 4 purple 5 blue.....19
 20. Sirloin is a cut of
6 mutton 7 beef 8 veal 9 lamb 10 pork.....20
 21. The head of a museum is called a
1 musician 2 curator 3 mortician 4 pastor 5 collector.....21
 22. A six-sided figure is called a
6 pentagon 7 hexagon 8 sextet 9 helix 10 scholium.....22
 23. The bat is most closely related to the
1 butterfly 2 swallow 3 owl 4 mouse 5 moth.....23
 24. A character in "David Copperfield" is
6 Tiny Tim 7 Uriah Heep 8 Scrooge 9 Goliath 10 Darnell.....24
 25. Quinine comes from
1 leaves 2 roots 3 medicine 4 minerals 5 bark.....25

1	2	3	4	5
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TEST 6. OPPOSITES

Mark the answer space which has the same number as the word which is OPPOSITE, or most nearly opposite, in meaning to the beginning word of each line.

SAMPLE. north — 1 hot 2 east 3 west 4 down 5 south.....

- | | | | | | | |
|-----------------|---------------|-----------------|----------------|--------------|---------------|---------|
| 1. exit — | 1 emit | 2 transcend | 3 entrance | 4 origin | 5 arrival | 1 |
| 2. amateur — | 6 novitiate | 7 musical | 8 professional | 9 inventor | 10 experience | 2 |
| 3. genuine — | 1 stolen | 2 counterfeit | 3 sincere | 4 original | 5 unworthy | 3 |
| 4. abundance — | 6 liberality | 7 frugality | 8 luxury | 9 hunger | 10 scarcity | 4 |
| 5. alert — | 1 illiterate | 2 pert | 3 sluggish | 4 disabled | 5 easy | 5 |
| 6. waste — | 6 refuse | 7 conserve | 8 devastate | 9 dole | 10 generate | 6 |
| 7. humiliated — | 1 honored | 2 refreshed | 3 satisfied | 4 lively | 5 arrogant | 7 |
| 8. gravity — | 6 fragility | 7 specificity | 8 purity | 9 constancy | 10 levity | 8 |
| 9. limitation — | 1 explanation | 2 encouragement | 3 ability | 4 freedom | 5 speed | 9 |
| 10. monotony — | 6 difficulty | 7 diversion | 8 harmony | 9 repetition | 10 variety | 10 |

6→

- | | | | | | | |
|------------------|---------------|-------------|--------------|--------------|--------------|---------|
| 11. obtuse — | 1 accessible | 2 abstruse | 3 acute | 4 corpulent | 5 agile | 11 |
| 12. expel — | 6 remain | 7 propel | 8 exile | 9 retain | 10 contract | 12 |
| 13. asset — | 1 bankruptcy | 2 descent | 3 misery | 4 mortgage | 5 liability | 13 |
| 14. acid — | 6 alkaline | 7 neutral | 8 pepsin | 9 briny | 10 chemical | 14 |
| 15. eccentric — | 1 particular | 2 stupid | 3 egocentric | 4 ordinary | 5 virtuous | 15 |
| 16. disperse — | 6 approve | 7 remove | 8 gather | 9 spare | 10 whisper | 16 |
| 17. wax — | 1 pale | 2 waive | 3 shine | 4 age | 5 wane | 17 |
| 18. blithe — | 6 helpless | 7 cheerless | 8 stingy | 9 lazy | 10 slow | 18 |
| 19. active — | 1 past | 2 careless | 3 passive | 4 pensive | 5 dull | 19 |
| 20. depress — | 6 press | 7 elate | 8 oppress | 9 exhort | 10 climb | 20 |
| 21. concede — | 1 deny | 2 recede | 3 finesse | 4 usurp | 5 resign | 21 |
| 22. recline — | 6 succumb | 7 stretch | 8 erect | 9 stand | 10 decline | 22 |
| 23. invincible — | 1 susceptible | 2 weak | 3 stubborn | 4 visible | 5 broken | 23 |
| 24. rash — | 6 prudent | 7 worthy | 8 smooth | 9 irrational | 10 stringent | 24 |
| 25. defile — | 1 confess | 2 file | 3 excel | 4 purify | 5 beautify | 25 |

TEST 2. SYNONYMS

132

Mark the answer space which has the same number as the word which has the SAME or most nearly the same meaning as the beginning word of each line.

SAMPLE. correct — 1 neat 2 fair 3 right 4 poor 5 good

	1	2	3	4	5
1. comic —	1 clumsy	2 laughable	3 universal	4 tricky	5 peculiar 1
2. hazardous —	6 adventurous	7 hard	8 annoying	9 speedy	10 dangerous 2
3. comfort —	1 entreat	2 console	3 pity	4 anger	5 discourage 3
4. equip —	6 decorate	7 establish	8 surround	9 load	10 furnish 4
5. introductory —	1 social	2 advisory	3 transitory	4 preliminary	5 internal 5
6. writhe —	6 squeal	7 twist	8 split	9 hurt	10 crawl 6
7. trivial —	1 amusing	2 rare	3 unimportant	4 detailed	5 uneven 7
8. evade —	6 vacate	7 invade	8 shun	9 hide	10 desist 8
9. denounce —	1 beguile	2 detest	3 inform	4 discover	5 condemn 9
10. opposed —	6 censored	7 adverse	8 involved	9 oppressed	10 morose 10
2→					
11. counsel —	1 quarrel	2 yield	3 advise	4 assemble	5 represent 11
12. abhor —	6 detest	7 fear	8 avoid	9 defy	10 disgust 12
13. depict —	1 vanish	2 slump	3 portray	4 edit	5 fabricate 13
14. expel —	6 repel	7 eject	8 injure	9 expend	10 depart 14
15. encumbrance —	1 contrivance	2 memory	3 burden	4 saddle	5 prison 15
16. docile —	6 facile	7 cozy	8 agile	9 submissive	10 intractable 16
17. quell —	1 inquire	2 dispel	3 instill	4 quest	5 subdue 17
18. abandon —	6 flee	7 escape	8 refuse	9 forsake	10 refrain 18
19. jubilant —	1 exultant	2 judicious	3 playful	4 pleasant	5 jeweled 19
20. hoax —	6 danger	7 deception	8 persuasion	9 revelation	10 mischief 20
21. curdle —	1 coagulate	2 spoil	3 snuggle	4 condense	5 churn 21
22. reverence —	6 revenge	7 relevance	8 obedience	9 veneration	10 mercy 22
23. incite —	1 instigate	2 revolt	3 announce	4 disrupt	5 consume 23
24. boldness —	6 hypocrisy	7 ferocity	8 audacity	9 frankness	10 confidence 24
25. appease —	1 praise	2 pique	3 entice	4 gladden	5 placate 25

Score

• Ear is to hear as eye is to

1 cry 2 glasses 3 spy 4 wink 5 see.....

Hat is to head as shoe is to

6 arm 7 leg 8 foot 9 fit 10 glove

DO THEM ALL LIKE THE SAMPLES.

- | | | | | | | |
|---|--------------|-------------|------------|-----------|--------------------|----|
| 1. Zoo is to animals as aquarium is to | 1 birds | 2 fish | 3 bees | 4 statues | 5 butterflies..... | 1 |
| 2. Linoleum is to floor as sheet is to | 6 cotton | 7 piano | 8 blanket | 9 linen | 10 bed..... | 2 |
| 3. Food is to hunger as water is to | 1 desert | 2 thirst | 3 quench | 4 drink | 5 milk..... | 3 |
| 4. Add is to subtract as multiply is to | 6 arithmetic | 7 increase | 8 fraction | 9 add | 10 divide..... | 4 |
| 5. Stone is to marble as wood is to | 1 brick | 2 soft | 3 cut | 4 oak | 5 pile..... | 5 |
| 6. Abide is to depart as stay is to | 6 play | 7 leave | 8 away | 9 over | 10 home..... | 6 |
| 7. Author is to book as artist is to | 1 painter | 2 brush | 3 picture | 4 easel | 5 paint..... | 7 |
| 8. You is to yours as me is to | 6 his | 7 ours | 8 mine | 9 theirs | 10 my..... | 8 |
| 9. Singing is to opera as dancing is to | 1 masquerade | 2 orchestra | 3 movie | 4 drama | 5 ballet..... | 9 |
| 10. Shell is to nut as skin is to | 6 hull | 7 animal | 8 white | 9 soft | 10 cover..... | 10 |

5➡

- | | | | | | | | | |
|-----|--|-------------|-----------------|-------------|------------|---------------|-------|----|
| 11. | Cub is to bear as gosling is to | 1 fox | 2 grouse | 3 goose | 4 rabbit | 5 duck | | 11 |
| 12. | Liberty is to freedom as bondage is to | 6 slavery | 7 free | 8 suffer | 9 serf | 10 revolution | | 12 |
| 13. | Imitate is to copy as invent is to | 1 inventory | 2 copyright | 3 originate | 4 machine | 5 patent | | 13 |
| 14. | 1 is to 3 as 9 is to | 6 (18) | 7 (27) | 8 (36) | 9 (45) | 10 (81) | | 14 |
| 15. | Complex is to simple as hard is to | 1 tough | 2 work | 3 easy | 4 smooth | 5 brittle | | 15 |
| 16. | Tree is to forest as person is to | 6 women | 7 couple | 8 human | 9 crowd | 10 men | | 16 |
| 17. | City is to mayor as army is to | 1 soldier | 2 navy | 3 private | 4 admiral | 5 general | | 17 |
| 18. | Wolf is to sheep as cat is to | 6 milk | 7 fur | 8 kitten | 9 mouse | 10 dog | | 18 |
| 19. | $\frac{4}{3}$ is to $\frac{1}{3}$ as 28 is to | 1 (7) | 2 (14) | 3 (33) | 4 (34) | 5 (43) | | 19 |
| 20. | Hog is to bristles as snake is to | 6 fangs | 7 scales | 8 venom | 9 coil | 10 rattle | | 20 |
| 21. | Seldom is to never as little is to | 1 none | 2 neither | 3 small | 4 often | 5 large | | 21 |
| 22. | Day is to 365 as week is to | 6 (7) | 7 (31) | 8 (48) | 9 (52) | 10 (60) | | 22 |
| 23. | Corrupt is to depraved as sacred is to | 1 hallowed | 2 Sunday | 3 depressed | 4 Bible | 5 prayer | | 23 |
| 24. | Square is to cube as circle is to | 6 round | 7 circumference | 8 sphere | 9 dice | 10 line | | 24 |
| 25. | Thermometer is to temperature as metronome is to | 1 intensity | 2 weight | 3 distance | 4 pressure | 5 time | | 25 |

TEST 3. LOGICAL SELECTION

133

Mark the answer space which has the same number as the word which tells what the thing ALWAYS has or ALWAYS involves.

SAMPLE. A cat always has

1 kittens 2 spots 3 milk 4 mouse 5 hair.....

1 2 3 4 5

- | | | | | | | | | | | |
|----------------------------------|-----------------|---------------|----------------|-----------------|------------------------|----|---|---|---|----|
| 1. An orchestra always has | 1 violinists | 2 piano | 3 musicians | 4 saxophone | 5 singers.....1 | 1 | 2 | 3 | 4 | 5 |
| 2. A museum always has | 6 visitors | 7 minerals | 8 collections | 9 guides | 10 paintings....2 | 6 | 7 | 8 | 9 | 10 |
| 3. School always involves | 1 children | 2 students | 3 arithmetic | 4 geography | 5 sports....3 | 1 | 2 | 3 | 4 | 5 |
| 4. A box always has | 6 contents | 7 wood | 8 lid | 9 hinge | 10 depth.....4 | 6 | 7 | 8 | 9 | 10 |
| 5. Contentment always involves | 1 devotion | 2 position | 3 satisfaction | 4 elation | 5 recognition 5 | 1 | 2 | 3 | 4 | 5 |
| 6. A newspaper always has | 6 pictures | 7 editor | 8 puzzles | 9 fiction | 10 cartoons.....6 | 6 | 7 | 8 | 9 | 10 |
| 7. A wheel always has | 1 circumference | 2 spokes | 3 tire | 4 wood | 5 metal.....7 | 1 | 2 | 3 | 4 | 5 |
| 8. A policeman always has | 6 club | 7 cap | 8 beat | 9 uniform | 10 authority.....8 | 6 | 7 | 8 | 9 | 10 |
| 9. A nation always has | 1 states | 2 colonies | 3 seaports | 4 laws | 5 navy.....9 | 1 | 2 | 3 | 4 | 5 |
| 10. Night always has | 6 stillness | 7 moon | 8 clouds | 9 ghosts | 10 hours.....10 | 6 | 7 | 8 | 9 | 10 |
| | | | | | | 3→ | | | | |
| 11. A ship always has | 1 engine | 2 guns | 3 hull | 4 passengers | 5 freight.....11 | 1 | 2 | 3 | 4 | 5 |
| 12. A message always involves | 6 telepathy | 7 messenger | 8 speech | 9 communication | 10 writing 12 | 6 | 7 | 8 | 9 | 10 |
| 13. Discipline always involves | 1 revenge | 2 anger | 3 morale | 4 whipping | 5 training.....13 | 1 | 2 | 3 | 4 | 5 |
| 14. A bottle always has | 6 hollowness | 7 label | 8 cork | 9 glass | 10 transparency.....14 | 6 | 7 | 8 | 9 | 10 |
| 15. Anxiety always involves | 1 awe | 2 grief | 3 insomnia | 4 uneasiness | 5 discouragement..15 | 1 | 2 | 3 | 4 | 5 |
| 16. Compromise always involves | 6 respect | 7 friendship | 8 adjustment | 9 law | 10 violation....16 | 6 | 7 | 8 | 9 | 10 |
| 17. An heir always has | 1 money | 2 lawyer | 3 heirlooms | 4 property | 5 predecessor...17 | 1 | 2 | 3 | 4 | 5 |
| 18. An invention always involves | 6 usefulness | 7 originality | 8 patent | 9 value | 10 imitation....18 | 6 | 7 | 8 | 9 | 10 |
| 19. A dance always has | 1 music | 2 partners | 3 rhythm | 4 audience | 5 costume.....19 | 1 | 2 | 3 | 4 | 5 |
| 20. A debt always involves | 6 interest | 7 creditor | 8 mortgage | 9 payment | 10 worry.....20 | 6 | 7 | 8 | 9 | 10 |
| 21. Rebuke always involves | 1 criticism | 2 help | 3 resignation | 4 postponement | 5 despair..21 | 1 | 2 | 3 | 4 | 5 |
| 22. Admiration always involves | 6 affirmation | 7 generosity | 8 flattery | 9 esteem | 10 love.....22 | 6 | 7 | 8 | 9 | 10 |
| 23. Annihilation always involves | 1 surprise | 2 destruction | 3 pain | 4 punishment | 5 vengeance 23 | 1 | 2 | 3 | 4 | 5 |
| 24. Abhorrence always involves | 6 aversion | 7 rage | 8 fear | 9 irreverence | 10 nausea.....24 | 6 | 7 | 8 | 9 | 10 |
| 25. Ostentation always involves | 1 simplicity | 2 modesty | 3 wealth | 4 display | 5 perfection....25 | 1 | 2 | 3 | 4 | 5 |

Score.....

TEST 4. CLASSIFICATION

In each line below, four of the words belong together. Pick out the ONE WORD which does not belong with the others, and mark the answer space bearing its number.

SAMPLES. 1 dog 2 cat 3 horse 4 chicken 5 cow
6 hop 7 run 8 stand 9 skip 10 walk

1	2	3	4	5
:	:	:	■	:
6	7	8	9	10
:	:	■	:	:

1. 1 Catholic 2 Methodist 3 Presbyterian 4 Republican 5 Baptist 1
2. 6 damp 7 wet 8 moist 9 soggy 10 soft 2
3. 1 telegraph 2 train 3 automobile 4 bicycle 5 boat 3
4. 6 often 7 seldom 8 safely 9 always 10 rarely 4
5. 1 oats 2 rye 3 wheat 4 clover 5 barley 5
6. 6 cello 7 harp 8 drum 9 violin 10 guitar 6
7. 1 Scottie 2 Holstein 3 Collie 4 Shepherd 5 Spitz 7
8. 6 digestion 7 smell 8 sight 9 hearing 10 taste 8
9. 1 pepper 2 cinnamon 3 nutmeg 4 pickle 5 mustard 9
10. 6 chapel 7 temple 8 tabernacle 9 cathedral 10 casino 10

1	2	3	4	5
:	:	:	:	:
6	7	8	9	10
:	:	:	:	:
1	2	3	4	5
:	:	:	:	:
6	7	8	9	10
:	:	:	:	:
1	2	3	4	5
:	:	:	:	:
6	7	8	9	10
:	:	:	:	:
1	2	3	4	5
:	:	:	:	:
6	7	8	9	10
:	:	:	:	:

4→

11. 1 reason 2 pity 3 joy 4 anger 5 fear 11
12. 6 arithmetic 7 geometry 8 history 9 trigonometry 10 algebra 12
13. 1 mosquito 2 ladybug 3 gnat 4 snail 5 beetle 13
14. 6 grosbeak 7 swallow 8 oriole 9 lark 10 gazelle 14
15. 1 nail 2 brad 3 awl 4 staple 5 tack 15
16. 6 large 7 tall 8 high 9 short 10 low 16
17. 1 priest 2 organist 3 minister 4 rabbi 5 bishop 17
18. 6 devotion 7 adoration 8 reverence 9 avarice 10 admiration 18
19. 1 pine 2 fir 3 maple 4 cedar 5 spruce 19
20. 6 Christ 7 Caesar 8 Moses 9 Mohammed 10 Confucius 20
21. 1 hither 2 recent 3 whence 4 near by 5 down 21
22. 6 lead 7 brass 8 iron 9 tin 10 copper 22
23. 1 verdict 2 testimony 3 subpoena 4 court 5 evidence 23
24. 6 inherit 7 lend 8 beg 9 borrow 10 earn 24
25. 1 moreover 2 besides 3 also 4 furthermore 5 however 25

1	2	3	4	5
:	:	:	:	:
6	7	8	9	10
:	:	:	:	:
1	2	3	4	5
:	:	:	:	:
6	7	8	9	10
:	:	:	:	:
1	2	3	4	5
:	:	:	:	:
6	7	8	9	10
:	:	:	:	:
1	2	3	4	5
:	:	:	:	:
6	7	8	9	10
:	:	:	:	:
1	2	3	4	5
:	:	:	:	:
6	7	8	9	10
:	:	:	:	:
1	2	3	4	5
:	:	:	:	:
6	7	8	9	10
:	:	:	:	:

Score

APPENDIX 4

DIRECTIONS FOR ADMINISTERING THE WATSON-GLASER CRITICAL THINKING APPRAISAL

Directions for Administering Critical Thinking Appraisal

134

1. Distribute Answer Sheets to pupils and tell them to fill out names at top and all information called for at bottom of Answer Sheet.
2. Distribute Test Booklets, warning pupils not to write in them at any time. Answers must be recorded on the separate Answer Sheet.
3. Read to pupils: "This booklet contains tests designed to measure five aspects of your ability to think logically and analytically. Each test is preceded by its own specific instructions. If you cannot readily determine what the directions mean or why the samples are marked as they are, tell me and I will explain them. Work as rapidly as you wish and let me know when you have finished."
4. Start the test and record the time here :
Time for test, 45 minutes : 45
5. Time to say: "Stop work. Pencils down." :
6. Collect Answer Sheets.
7. Collect Test Booklets.
8. Deliver Answer Sheets and Test Booklets to Office, along with this time table.

Note: If a pupil finishes before time is called, his materials may be collected and he may be allowed to work QUIETLY on something else.

Teacher _____ Room _____ Date _____

Please notify the Office by telephone when your room has finished.

APPENDIX 5

FACTORS UPON WHICH STUDENTS WERE WATCHED

TABLE XI.- Group E₁, Matching Factors.

Pupil	TMc	WG ₁	WG ₂	Δ WG	Age
1	97	51	49	-2	15.3
2	105	60	48	-12	15.0
3	113	52	51	-1	15.2
4	128	53	59	6	14.8
5	94	55	35	-20	15.9
6	99	50	51	1	15.2
7	98	43	56	13	16.3
8	112	40	45	5	15.2
9	109	41	63	22	15.7
10	113	66	57	-9	15.6
11	102	55	49	-6	15.3
12	122	56	69	13	14.9
13	115	55	53	-2	14.8
14	115	50	57	7	14.7
15	99	53	53	0	16.1
16	108	56	60	4	15.2
17	117	58	51	-7	15.3
18	119	50	61	11	15.4
19	109	43	52	9	14.3
20	105	42	62	20	15.5

TABLE XII.- Group C₁, Matching Factors.

Pupil	Thc	WG ₁	WG ₂	Δ WG	Age
1	96	48	68	20	15.8
2	105	58	66	8	16.7
3	113	57	57	0	15.7
4	126	54	60	6	15.1
5	96	54	57	3	14.5
6	99	48	56	8	15.1
7	98	47	52	5	15.3
8	111	43	47	4	14.8
9	111	44	62	18	15.2
10	111	61	63	2	14.3
11	101	51	57	6	16.2
12	124	52	57	5	14.7
13	115	54	42	-12	14.8
14	117	54	63	9	14.9
15	99	55	56	1	15.0
16	107	56	65	9	15.3
17	115	56	58	2	14.9
18	122	47	67	20	15.0
19	109	40	72	32	15.2
20	105	46	56	10	15.4

TABLE XIII.- Group E₂, Matching Factors.

Pupil	TMc	WG ₁	WG ₂	Δ WG	Age
1	96	48	68	20	15.8
2	105	58	66	8	16.7
3	130	63	68	5	15.1
4	126	54	60	6	15.1
5	99	48	56	8	15.1
6	99	55	56	1	15.0
7	111	43	47	4	14.8
8	111	44	62	18	15.2
9	111	61	63	2	14.3
10	101	51	57	6	16.2
11	124	46	56	18	15.2
12	124	52	57	5	14.7
13	115	54	42	-12	14.8
14	117	54	63	9	14.8
15	107	56	65	9	15.3
16	115	56	58	2	14.9
17	113	56	56	0	15.1
18	122	47	67	20	15.0
19	132	52	71	19	14.9
20	113	63	56	-7	15.1
21	109	40	72	32	15.2
22	105	46	56	10	15.4

TABLE XIV.- Group C₂, Matching Factors.

Pupil	TMc	WG ₁	WG ₂	Δ WG	Age
1	99	48	44	-4	15.1
2	108	54	51	-3	15.1
3	128	68	73	5	15.1
4	125	59	71	12	15.1
5	102	46	42	-4	15.0
6	101	57	60	3	16.0
7	111	42	66	24	14.9
8	113	47	58	11	15.0
9	111	58	69	11	15.2
10	122	48	62	14	14.6
11	104	55	51	-4	15.5
12	122	55	62	7	15.4
13	116	56	51	-5	15.2
14	117	53	70	17	14.8
15	109	56	68	12	16.2
16	116	54	63	9	15.2
17	111	57	53	-4	15.9
18	122	49	64	15	14.8
19	135	54	76	22	14.7
20	113	65	66	1	15.8
21	110	41	56	15	14.7
22	105	47	61	14	15.2

TABLE XV.- Group E₃, Matching Factors.

Pupil	TMc	WG ₁	WG ₂	Δ WG	Age
1	109	58	58	0	15.1
2	105	53	55	2	15.3
3	128	51	73	22	14.6
4	111	50	58	8	15.0
5	119	56	58	2	15.1
6	122	64	70	6	14.5
7	116	56	70	14	14.8
8	103	48	52	4	15.5
9	111	60	59	-1	15.7
10	110	59	58	-1	14.8
11	115	52	57	5	15.1
12	108	50	50	0	14.8
13	113	56	61	5	15.2
14	120	63	61	-2	14.9
15	118	49	58	9	15.4
16	108	42	50	8	14.4
17	118	48	57	9	14.6
18	103	57	48	-9	15.2
19	117	55	67	12	15.2
20	113	52	63	11	14.8
21	120	57	74	17	14.8
22	120	57	62	5	14.9
23	128	54	63	9	14.8
24	108	48	58	10	15.4
25	111	45	53	8	15.0
26	108	55	58	3	15.7
27	108	56	64	8	16.2
28	117	56	62	6	15.2
29	126	54	59	5	14.5
30	117	59	62	3	14.0
31	117	53	62	9	15.1
32	127	64	68	4	15.2
33	115	64	67	3	14.8
34	119	60	59	-1	14.7
35	108	60	52	-8	15.1
36	113	59	51	-8	14.7
37	93	48	39	-9	15.2
38	105	45	51	6	15.2
39	111	53	50	-3	15.3
40	115	55	65	10	16.0
41	112	63	70	7	14.8
42	123	51	58	7	15.0
43	108	37	57	20	15.1
44	111	45	58	13	15.0
45	110	36	50	14	15.3

TABLE XVI.- Group C₃, Matching Factors.

Pupil	TMc	WG ₁	WG ₂	Δ WG	Δ go
1	107	57	52	-5	15.3
2	105	55	64	9	15.2
3	127	48	55	7	14.8
4	111	50	59	9	14.8
5	119	57	64	7	15.2
6	122	60	65	5	15.2
7	119	55	47	-8	14.5
8	102	45	56	11	16.2
9	112	61	68	7	15.8
10	111	61	66	5	15.3
11	115	54	56	2	15.8
12	108	53	53	0	15.5
13	113	53	62	9	15.9
14	119	60	71	11	15.5
15	118	51	73	22	15.3
16	109	45	62	17	15.3
17	117	54	59	5	14.9
18	104	56	66	10	14.8
19	117	55	68	13	15.2
20	116	52	67	15	15.4
21	123	57	59	2	15.1
22	128	67	67	12	15.1
23	127	59	66	7	14.3
24	106	51	71	20	16.1
25	111	47	64	17	15.5
26	109	55	65	10	14.8
27	109	61	65	4	15.7
28	118	53	63	10	14.9
29	128	54	64	10	14.8
30	113	54	62	8	14.9
31	116	57	67	10	14.8
32	126	69	66	-3	15.0
33	118	64	69	5	14.6
34	120	62	63	1	14.8
35	108	61	59	-2	15.2
36	115	56	63	7	15.1
37	93	50	60	10	15.2
38	105	43	51	8	14.9
39	114	53	56	3	15.1
40	116	52	63	11	15.0
41	111	60	69	9	15.4
42	126	56	65	9	15.4
43	107	40	51	11	14.5
44	114	46	58	12	14.8
45	108	31	46	15	14.7

APPENDIX 6

TEACHER RATING FORM, PUBLIC SCHOOL DEPARTMENT,
PITTSFIELD, MASSACHUSETTS

Pittsfield, Massachusetts

TEACHER RATING FORM

Teacher

Date

School

Rated by

Rating is indicated by a check in appropriate column.	Very Superior	Superior	Satisfactory	Unsatisfactory
1. Vitality, endurance, health.				
2. Personality, character.				
3. Understanding of, interest in pupils.				
4. Cooperation with associates.				
5. Cooperation with parents and community.				
6. Professional spirit, growth.				
7. General culture, preparation.				
8. Class management, control.				
9. Attendance, punctuality, clerical reports.				
10. Teaching technique.				
11. Pupil achievement in subject matter.				
12. Pupil growth in character and personality.				
..				

To the Principal: The rating "Very Superior" should be reserved exclusively for a description of very unusual merit.

The rating "Unsatisfactory" should result in suggestions for improvement in a personal conference with the teacher.

APPENDIX 7

MULTIPLE REGRESSION ANALYSIS OF SCORES ON THE
WATSON-GLASER CRITICAL THINKING APPRAISAL
FOR ALL GROUPS TESTED

4 - Variable Continuous Dept for Regression

143

[illegible]

Multiple Regression Analysis of Scores of First Watson-Gleser Test for Boys in groups E-1

W-G1	English	Science	Mathematics	History
2000000051	1500000051	6500000051	9000000051	7200000051
5100000051	7000000051	7500000051	9000000051	9000000051
6000000051	8000000051	8400000051	8000000051	8300000051
5200000051	8000000051	7200000051	7500000051	8200000051
5500000051	7500000051	6800000051	6500000051	6300000051
5000000051	7000000051	7700000051	7500000051	8000000051
4300000051	8000000051	7700000051	7500000051	8000000051
4000000051	7500000051	6000000051	6000000051	6500000051
4100000051	6500000051	7500000051	8100000051	9200000051
6600000051	8000000051	8500000051	7800000051	9200000051
5500000051	8000000051	6000000051	7000000051	7200000051
5600000051	8000000051	7600000051	8100000051	9000000051
5500000051	8500000051	6000000051	7500000051	8000000051
5000000051	4000000051	6700000051	5900000051	5000000051
5300000051	6000000051	7600000051	7800000051	9500000051
5600000051	8500000051	7000000051	7200000051	7500000051
5800000051	8000000051	7200000051	7000000051	8000000051
5000000051	8500000051	6500000051	7500000051	8700000051
4300000051	7500000051	8000000051	7500000051	8000000051
5300000051	7500000051	8000000051	7500000051	8000000051
1000000050	1500000053	1449000053	1496000053	1588000053
2000000050	1146000053	1059970055	1131140055	1284220055
3000000050	7745500054	7491100054	7737700054	8212100054
4000000050	1095650055	1126950055	1201950055	1187810055
5000000050	2100000053	8900000052	4950000052	1095000053
6000000050		1016950053	3958000052	7144000052
7000000050			1213200053	1185600053
8000000050				2334800053
9000000050	8909500052	2800000052	3599500052	4078000052
1000000051	8409525746	5893534846	1158376246	2728911346
1100000051		1668405747	1490302346	1584191046
1200000051			1655860647	8495646346
1300000051				1036163447
1400000051	9999996049	9999998049	9999998049	1000000150
1500000051	1371810551	4361014048	3084665849	2985932149
1600000051	5145000051	7500000051	7245000051	7480000051
1700000051	1966318049	2769691249	2759253149	2182677249
1800000051	2217857049	1113721950	1082152150	2123729549

1029000053
5383300054
1157650055
1199680055

1840000052

$$W_{G1} = A + b_1(\text{English}) + b_2(\text{Science}) + b_3(\text{Math}) + b_4(\text{History})$$

x_1 x_2 x_3 x_4

W-62	English	Science	Mathematics	History		
2000000051	1500000051	6500000051	9000000051	7200000051		
4900000051	7000000051	7500000051	9000000051	9000000051		
4800000051	8000000051	8400000051	8000000051	8300000051		
5100000051	8000000051	7200000051	7500000051	8200000051		
5900000051	7500000051	6800000051	6500000051	6300000051		
3500000051	7000000051	7700000051	7500000051	8000000051		
5100000051	8000000051	7700000051	7500000051	8000000051		
5600000051	7500000051	6000000051	6000000051	6500000051		
4500000051	6500000051	7500000051	8100000051	9200000051		
6300000051	8000000051	8500000051	7800000051	9200000051		
5700000051	8000000051	8000000051	7000000051	7200000051		
4900000051	8000000051	7600000051	8100000051	9000000051		
6900000051	8500000051	6000000051	7500000051	8000000051		
5300000051	4000000051	6700000051	5900000051	5000000051		
5700000051	6000000051	7600000051	7800000051	9500000051		
5300000051	8500000051	7000000051	7200000051	7500000051		
6000000051	8000000051	7200000051	7200000051	8000000051		
5100000051	8000000051	6500000051	7000000051	8000000051		
6100000051	8500000051	6500000051	7500000051	8700000051		
5200000051	7500000051	8000000051	7500000051	8000000051		
6200000051	7500000051					
1000000050	1500000053	1449000053	1496000053	1588000053	1081000053	
2000000050	1146000055	1059970055	1131140055	1284220055	5949100054	
3000000050	8147500054	7857400054	8108700054	8645000054		
4000000050	1095650055	1126950055	1201950055	1087810055	1157650055	1199680055
5000000050	2100000053	8900000052	4950000052	1095000053		
6000000050		1016950053	3958000052	7144000052		
7000000050			1213200053	1185600053		
8000000050				2334800053		
9000000050	106295005	4000000052	2555500052	2282000052	6246000052	
1000000051	8409525746	5893534846	1158376246	2728911546		
1100000051		1668405747	1490302346	1584191046		
1200000051			1655860647	8495646346		
1300000051				1036163447		
1400000051	9999996049	9999998049	9999998049	1000000150		
1500000051	3343870151	4175810048	5760242048	1445203149	3036765649	
1600000051	5405000051	7500000051	7245000051	7480000051	7040000051	
1700000051	2214430849	3119173749	3107418049	2458089649	7636612850	
1800000051	1885726149	1848644149	4650815049	1235417050	4207080049	

$$W_{G2} = A + b_1(x_1) + b_2(x_2) + b_3(x_3) + b_4(x_4)$$

ΔW-0	English	Science	Mathematics	History		
2000000051	1500000051	6500000051	9000000051	7200000051		
2000000050	7000000051	7500000051	9000000051	9000000051		
1200000051	8000000051	8400000051	8000000051	8300000051		
1000000050	8000000051	7200000051	7500000051	8200000051		
6000000050	7500000051	6000000051	6600000051	6300000051		
8000000051	7000000051	7700000051	7500000051	8000000051		
1000000050	8000000051	7500000051	7500000051	8000000051		
1300000051	7500000051	6000000051	6000000051	6500000051		
3000000050	6500000051	7500000051	8100000051	8200000051		
2200000051	8000000051	8500000051	7800000051	9200000051		
9000000050	8000000051	8000000051	7000000051	7200000051		
6000000050	8000000051	7600000051	8100000051	9000000051		
1300000051	8500000051	6000000051	7500000051	8000000051		
2000000050	4000000051	6700000051	5900000051	5000000051		
7000000050	6000000051	7400000051	7200000051	7500000051		
50	8500000051	7000000051	7200000051	8000000051		
4000000050	8000000051	7200000051	7200000051	8000000051		
7000000050	8000000051	6500000051	7000000051	8000000051		
1100000051	8500000051	6500000051	7500000051	8700000051		
2000000050	7500000051	8000000051	7500000051	8000000051		
2000000051	7500000051	8000000051	7500000051	8000000051		
1000000050	1500000053	1449000053	1496000053	1588000053	5000000051	
2000000050	1146000053	1059970053	1131140053	1284220053	2254000053	
3000000050	3995000053	3634000053	3745000053	4367000053		
4000000050	1095650053	1124950053	1201950053	1087810053	1157650053	1199680053
5000000050	2100000053	8900000052	4950000052	1095000053		
6000000050		1016950053	3950000052	7144000052		
7000000050			1213200053	1185600053		
8000000050				2334800053		
9000000050	2129000053	2450000052	1150000051	5000000050	3970000052	
1000000051	8409525746	5893534846	1158376246	2728411346		
1100000051		1668405747	1490302346	1584191046		
1200000051			1655860647	8495646346		
1300000051				1036163447		
1400000051	9999996049	9999998049	9999998049	1000000150		
1500000051	5062484050	9149723048	1288424649	3023314949	3384249049	
1600000051	2500000050	7500000051	7245000051	7480000051	7940000051	
1700000051	3327956449	4687648749	4669982449	3694138849	1147668051	
1800000051	2749351849	4020511449	46473932149	9161239449	4681520049	

$$\Delta WG = A + b_1(\text{English}) + b_2(\text{Science}) + b_3(\text{Mathematics}) + b_4(\text{History})$$

W-G1	English	Geometry	History	Drafting		
2000000051	1500000051					
4800000051	7200000051	7600000051	8000000051	6200000051		
5800000051	5500000051	6500000051	6000000051	6000000051		
5700000051	6000000051	7500000051	8400000051	7800000051		
5400000051	7800000051	7500000051	7000000051	8000000051		
5400000051	8500000051	6500000051	7600000051	7600000051		
4800000051	7000000051	7500000051	7200000051	8000000051		
4700000051	6200000051	7200000051	8000000051	8200000051		
4300000051	6800000051	8500000051	7700000051	7200000051		
4400000051	7000000051	5500000051	6000000051	7100000051		
6100000051	6800000051	8400000051	7500000051	9000000051		
5100000051	6000000051	8000000051	7000000051	8000000051		
5200000051	8500000051	8800000051	9500000051	9500000051		
5400000051	5000000051	7600000051	8200000051	8000000051		
5400000051	6500000051	7300000051	7700000051	8000000051		
5500000051	8000000051	8500000051	8000000051	8200000051		
5600000051	8200000051	7000000051	8000000051	8200000051		
5600000051	6500000051	7000000051	7000000051	7000000051		
4700000051	7800000051	8500000051	7200000051	7500000051		
4800000051	6800000051	7400000051	7400000051	8000000051		
4600000051	7000000051	8000000051	7200000051	8000000051		
1000000050	1391000053	1508000053	1506000052	1561000052	1025000053	
2000000050	9851700054	1149660055	11492005	1231170055	5312700054	
3000000050	7122200054	7728800054	7728600054	8014300054		
4000000050	1051550055	1051340055	109003005	1142440055	1183110055	118262005
5000000050	1772950053	2736000052	3917000052	4374500052		
6000000050		1262800053	6916000052	6116000052		
7000000050			1190200053	7187000052		
8000000050				1280950052		
9000000050	5957500052	6675000051	3000000050	1035000052	1417500051	
1000000051	6260488246	3316380043	1171004146	1469112		
1100000051		1224740947	5422295046	2804221		
1200000051			1532731447	5612670		
1300000051				1279427		
1400000051	1000000150	1000000050	100000050	9999999040		
1500000051	4665650251	7474140748	9217423845	8526785045	132236	4
1600000051	5125000051	6955000051	7540000051	7530000051	7505000051	
1700000051	1533406549	2144776849	2399366849	2192133349	612872445	
1800000051	4874207049	4297614549	3553764749	6032304249	2527780049	

Multiple Regression Analysis of Scores of Second Watson-Gleser Test
for Boys in Group C-1

W-02	English	Geometry	History	Drafting	
29000000051	15000000051				
68000000051	72000000051	76000000051	80000000051	6200000000	
66000000051	55000000051	65000000051	60000000051	6000000000	
57000000051	60000000051	75000000051	84000000051	7800000000	
60000000051	72000000051	75000000051	70000000051	8000000000	
57000000051	85000000051	45000000051	76000000051	7600000000	
56000000051	70000000051	75000000051	72000000051	8000000000	
52000000051	62000000051	72000000051	80000000051	8200000000	
47000000051	68000000051	85000000051	77000000051	7200000000	
62000000051	70000000051	55000000051	60000000051	7100000000	
63000000051	68000000051	84000000051	75000000051	9000000000	
57000000051	60000000051	80000000051	70000000051	8000000005	
57000000051	85000000051	88000000051	95000000051	9500000000	
42000000051	50000000051	76000000051	82000000051	8000000000	
63000000051	65000000051	73000000051	77000000051	8500000000	
56000000051	80000000051	85000000051	80000000051	8200000005	
65000000051	82000000051	70000000051	80000000051	8200000005	
58000000051	65000000051	70000000051	70000000051	7100000000	
67000000051	78000000051	85000000051	72000000051	7500000000	
72000000051	68000000051	74000000051	74000000051	8000000000	
56000000051	70000000051	80000000051	72000000051	8000000000	
10000000050	13910000053	15080000053	15060000053	1561000000	11810000053
20000000050	98517000054	11496600055	11459200055	12311700055	70705000054
30000000050	82517000054	88819000054	88600000054	91949000054	
40000000050	10515500055	10513400055	10900300055	11424400055	11831100055 11826200055
50000000050	17729500053	27360000052	39170000052	43545000052	
60000000050		12628000053	69160000052	61100000052	
70000000050			11902000053	71470000052	
80000000050				13809500053	
90000000050	96695000052	37845000052	22840000051	32930000052	22805000052
10000000051	82604800246	33163800043	1171004146	14696100046	
11000000051		1224740947	5422295046	2404100046	
12000000051			1532731447	56120000046	
13000000051				1279427847	
14000000051	1000000150	10000000050	10000000050	94490000050	
15000000051	7043835151	3090796249	3734980048	29720290049	351733048
16000000051	59050000051	69550000051	75400000051	75300000051	78000000051
17000000051	1734704949	2426333149	2714344549	2479406449	93335405
18000000051	1781741050	1539355049	1094934450	3972622949	500000049

Multiple Regression Analysis of Change in Scores of Two Watson-Glaser Tests for Boys in Groups C-1

Age-1	Swelling	Geometry	History	Drawing
30000000051	15000000051	76000000051	80000000051	62000000051
20000000051	12000000051	65000000051	60000000051	60000000051
80000000051	50000000051	65000000051	84000000051	74000000051
60000000051	60000000051	15000000051	70000000051	40000000051
50000000051	78000000051	65000000051	12000000051	76000000051
80000000051	70000000051	75000000051	12000000051	80000000051
50000000051	62000000051	82000000051	80000000051	82000000051
40000000051	68000000051	82000000051	77000000051	72000000051
18000000051	70000000051	82000000051	60000000051	71000000051
20000000051	68000000051	84000000051	75000000051	70000000051
50000000051	60000000051	40000000051	70000000051	40000000051
50000000051	60000000051	58000000051	95000000051	95000000051
12000000051	50000000051	76000000051	82000000051	80000000051
90000000051	20000000051	75000000051	77000000051	85000000051
10000000051	20000000051	75000000051	80000000051	82000000051
20000000051	82000000051	70000000051	80000000051	82000000051
20000000051	65000000051	00000000051	70000000051	71000000051
20000000051	50000000051	85000000051	72000000051	75000000051
72000000051	50000000051	74000000051	72000000051	80000000051
15000000051	50000000051	70000000051	72000000051	80000000051
10000000051	34100000051	15040000051	15060000051	15610000051
20000000051	78517000051	11492600051	11409200051	12311700051
30000000051	11293000051	11531000051	11314000051	11800000051
40000000051	10515500051	10517400051	10900300051	11424400051
50000000051	17225500051	27360000051	39170000051	43545000051
60000000051	260488240	1224740947	5422295046	2404221646
70000000051	16212000051	44520000051	25140000051	41280000051
80000000051	260488240	1224740947	5422295046	2404221646
90000000051	17225500051	27360000051	39170000051	43545000051
10000000051	260488240	1224740947	5422295046	2404221646
11000000051	16212000051	44520000051	25140000051	41280000051
12000000051	260488240	1224740947	5422295046	2404221646
13000000051	17225500051	27360000051	39170000051	43545000051
14000000051	260488240	1224740947	5422295046	2404221646
15000000051	16212000051	44520000051	25140000051	41280000051
16000000051	260488240	1224740947	5422295046	2404221646
17000000051	17225500051	27360000051	39170000051	43545000051
18000000051	260488240	1224740947	5422295046	2404221646
19000000051	16212000051	44520000051	25140000051	41280000051
20000000051	260488240	1224740947	5422295046	2404221646
21000000051	17225500051	27360000051	39170000051	43545000051
22000000051	260488240	1224740947	5422295046	2404221646
23000000051	16212000051	44520000051	25140000051	41280000051
24000000051	260488240	1224740947	5422295046	2404221646
25000000051	17225500051	27360000051	39170000051	43545000051
26000000051	260488240	1224740947	5422295046	2404221646
27000000051	16212000051	44520000051	25140000051	41280000051
28000000051	260488240	1224740947	5422295046	2404221646
29000000051	17225500051	27360000051	39170000051	43545000051
30000000051	260488240	1224740947	5422295046	2404221646

1183110055 1102620055

W-01	English	Geometry	Physics & Chemistry	Drafting	
32000000051	17000000051				
48000000051	72000000051	76000000051	42000000051	80000000051	
58000000051	55000000051	65000000051	60000000051	60000000051	
53000000051	81000000051	82000000051	80000000051	80000000051	
54000000051	78000000051	75000000051	70000000051	80000000051	
48000000051	70000000051	78000000051	72000000051	80000000051	
55000000051	80000000051	85000000051	82000000051	82000000051	
43000000051	68000000051	85000000051	77000000051	72000000051	
44000000051	75000000051	55000000051	60000000051	71000000051	
51000000051	68000000051	40000000051	75000000051	90000000051	
51000000051	60000000051	80000000051	70000000051	80000000051	
46000000051	61000000051	90000000051	80000000051	77000000051	
52000000051	88000000051	88000000051	95000000051	95000000051	
54000000051	60000000051	76000000051	82000000051	80000000051	
54000000051	65000000051	73000000051	77000000051	85000000051	
56000000051	82000000051	70000000051	70000000051	82000000051	
56000000051	60000000051	70000000051	60000000051	71000000051	
56000000051	70000000051	83000000051	80000000051	80000000051	
47000000051	78000000051	85000000051	70000000051	75000000051	
52000000051	93000000051	97000000051	90000000051	90000000051	
63000000051	83000000051	84000000051	80000000051	80000000051	
40000000051	68000000051	74000000051	74000000051	80000000051	
46000000051	68000000051	80000000051	72000000051	80000000051	
10000000050	15820000050	17350000053	16380000053	17500000053	11470000053
20000000050	11585000050	13836900055	12374400055	14031800055	60647000054
30000000050	82673000054	90553000054	85586000054	91385000054	
40000000050	12551800050	11876300053	12670700055	13054200055	13880500055 13131800055
50000000050	20898200053	75573000052	97591000052	86609000052	
60000000050		17405900050	13633600053	79364000052	
70000000050			17874500053	10225000053	
80000000050				11134500053	
90000000050	84600000052	19007300052	96400000051	10663600050	14636400052
1000000005	7250436946	0508203044	0518279246	0226593446	
11000000051		1427803647	1060180347	0497040145	
12000000051			2009160247	0663964046	
13000000051				2139301447	
14000000051	10009000050	9999980049	10000000050	9990900049	
15000000051	4253060951	4974900048	6731054148	1073098749	4769206048
16000000051	5213636451	7190909151	7880363651	7445454551	7954545551
17000000051	1868224049	2620071949	3108358149	3207378549	934711250
18000000051	2662908249	2568740049	3259272849	1486948349	1851249049

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U-S R	English	Geometry	Physion and History	Drafting
22000000051	17000000051	16000000051	62000000051	80000000051
66000000051	17200000051	65000000051	60000000051	60000000051
66000000051	50000000051	62300000051	30000000051	60000000051
66000000051	80000000051	73000000051	70000000051	60000000051
66000000051	73000000051	73000000051	70000000051	60000000051
56000000051	80000000051	73000000051	70000000051	60000000051
56000000051	80000000051	73000000051	70000000051	60000000051
47000000051	68000000051	85000000051	77000000051	72000000051
63000000051	73000000051	55000000051	90000000051	71000000051
57000000051	60000000051	40000000051	70000000051	80000000051
57000000051	61000000051	80000000051	80000000051	77000000051
42000000051	80000000051	80000000051	90000000051	95000000051
63000000051	60000000051	76000000051	82000000051	80000000051
63000000051	62000000051	73000000051	77000000051	85000000051
56000000051	82000000051	70000000051	70000000051	82000000051
56000000051	60000000051	70000000051	60000000051	71000000051
67000000051	73000000051	13000000051	80000000051	80000000051
56000000051	80000000051	85000000051	70000000051	75000000051
56000000051	80000000051	85000000051	70000000051	80000000051
71000000051	80000000051	84000000051	80000000051	80000000051
72000000051	80000000051	97000000051	90000000051	90000000051
56000000051	80000000051	74000000051	74000000051	80000000051
10000000050	15820000053	17350000053	16380000053	17500000053
20000000050	11585000055	13550000055	12374400055	14031800055
30000000050	95554000054	10411800055	98114000054	10523300055
40000000050	12351800055	11876300055	12670700055	13054200055
50000000050	20898200053	75573000052	97591000052	86609000052
60000000050		17005900053	13634600053	79364000052
70000000050			17874500053	10225500053
80000000050				11134500053
90000000050	1135818053	49018200052	13973000052	31490900052
10000000051	7258436046	3509233044	1518279246	4226593446
11000000051		1427383847	106712247	3497240446
12000000051			2009182247	966364046
13000000051				213301447
14000000051	10000000050	9499998043	10000000050	9999999949
15000000051	4902093251	372858549	1322426349	629441849-2601493849
16000000051	60000000051	7190909151	7886363651	7443434551
17000000051	1794214149	2516565549	983420049	3000317749
18000000051	2078118550	5254885349	2104527550	844537349
				5797320049

Multiple Regression Analysis of Change in Scores
of Two Watson-Gleser Tests for
Boys in Group E-2

Δ W-G	English	Geometry	Physics & History	Drafting
22000000051	17000000051	76000000051	62000000051	80000000051
20000000051	72000000051	65000000051	60000000051	60000000051
80000000050	55000000051	80000000051	80000000051	80000000051
50000000050	80000000051	75000000051	70000000051	80000000051
80000000051	78000000051	78000000051	72000000051	80000000051
80000000050	70000000051	85000000051	82000000051	82000000051
10000000050	80000000051	85000000051	77000000051	72000000051
40000000050	88000000051	55000000051	60000000051	71000000051
16000000051	75000000051	84000000051	75000000051	90000000051
20000000050	68000000051	80000000051	70000000051	90000000051
90000000051	60000000051	90000000051	80000000051	77000000051
10000000051	61000000051	88000000051	95000000051	95000000051
50000000050	88000000051	76000000051	82000000051	80000000051
10000000051	60000000051	73000000051	77000000051	85000000051
90000000050	65000000051	70000000051	70000000051	82000000051
90000000050	82000000051	70000000051	60000000051	71000000051
20000000050	60000000051	70000000051	80000000051	80000000051
50	70000000051	83000000051	80000000051	80000000051
20000000051	78000000051	85000000051	70000000051	75000000051
19000000051	93000000051	97000000051	90000000051	90000000051
70000000050	83000000051	84000000051	80000000051	80000000051
32000000051	68000000051	74000000051	74000000051	80000000051
10000000051	68000000051	80000000051	72000000051	80000000051
10000000050	15820000053	17350000053	16380000053	17500000053
20000000050	11585000055	13856900055	12374400055	14031800055
30000000050	12881000054	13565000054	12528000054	13848000054
40000000050	12551800055	11876300055	12670700055	13054200055
50000000050	20895200053	75573000052	97591000052	86609000052
60000000050		17405900053	13633600053	79364000052
70000000050			17874500053	10225500053
80000000050				11134500053
90000000050	1946954553	29690900052	23613600052	-50154500052
10000000051	7258436946	3508233044	-1518279246	-4226593446
11000000051		1427883847	1067182347	-3497240145
12000000051			2009182247	9663964046
13000000051				2139301447
14000000051	10000000050	9999998049	10000000050	9999999049
15000000051	6490270050	3231102549	1995574849	7307539549
16000000051	7954545550	7190909151	7866363651	7445454551
17000000051	2569924149	3604576749	4275849349	4412061549
18000000051	1257275550	5536225149	1709026450	-4815324549
				4532250049

1313180055

W-G 1	English	Geometry	French and Latin	Biology and Physics		
22000000051	17000000051					
48000000051	71000000051	68000000051	53000000051	60000000051		
54000000051	71000000051	70000000051	65000000051	70000000051		
68000000051	80000000051	83000000051	63000000051	82000000051		
59000000051	95000000051	92000000051	81000000051	90000000051		
46000000051	70000000051	68000000051	65000000051	80000000051		
57000000051	75000000051	75000000051	60000000051	60000000051		
42000000051	81000000051	78000000051	78000000051	60000000051		
47000000051	65000000051	77000000051	73000000051	80000000051		
78000000051	76000000051	70000000051	78000000051	84000000051		
48000000051	80000000051	85000000051	71000000051	82000000051		
55000000051	78000000051	73000000051	60000000051	86000000051		
55000000051	85000000051	70000000051	70000000051	86000000051		
56000000051	60000000051	75000000051	62000000051	73000000051		
55000000051	78000000051	88000000051	55000000051	70000000051		
56000000051	70000000051	60000000051	62000000051	80000000051		
54000000051	70000000051	85000000051	61000000051	80000000051		
57000000051	78000000051	60000000051	71000000051	62000000051		
49000000051	78000000051	78000000051	78000000051	75000000051		
54000000051	92000000051	95000000051	97000000051	95000000051		
65000000051	76000000051	60000000051	78000000051	77000000051		
41000000051	79000000051	80000000051	80000000051	86000000051		
47000000051	78000000051	60000000051	73000000051	60000000051		
10000000050	16860000053	16600000053	15340000053	16780000053	11690000053	
20000000050	13052000053	12749600055	10920800055	13034400055	63059000054	
30000000050	89727000054	81550000054	81357000054	89482000054		
40000000050	12807800055	11857200055	12925400055	11634900055	12778400055	11800900055
50000000050	13110900053	86164000052	10118200053	65818000052		
60000000050		22414500053	60173000052	11712700053		
70000000050			22463600053	10066400053		
80000000050				23583600053		
90000000050	94259100052	13909100052	51364000051	15418200052	31918200052	
10000000051	1457870347	4352413846	562193346	4923595445		
11000000051		7345196846	1339507446	3005028446		
12000000051			7691054846	2379263546		
13000000051				6610822446		
14000000051	1000000150	9999999049	10000000050	10000000050		
15000000051	4315034551	3275193849	2148339249	2795928049	2699728349	
16000000051	5313636451	7663636451	7545454551	6972727351	7627272751	
17000000051	2547278049	1808066649	1850142449	1715319949	6671652250	
18000000051	1285762250	1188196950	1511196150	1573892050	4440610049	

W-G 2	English	Geometry	French and Latin	Biology and Physics	
22000000051	17000000051				
44000000051	71000000051	68000000051	53000000051	60000000051	
51000000051	71000000051	70000000051	65000000051	70000000051	
73000000051	80000000051	83000000051	63000000051	82000000051	
71000000051	95000000051	92000000051	81000000051	90000000051	
42000000051	70000000051	68000000051	65000000051	80000000051	
60000000051	75000000051	75000000051	60000000051	60000000051	
66000000051	81000000051	78000000051	78000000051	60000000051	
58000000051	65000000051	77000000051	73000000051	80000000051	
69000000051	76000000051	70000000051	78000000051	84000000051	
62000000051	80000000051	85000000051	71000000051	82000000051	
51000000051	78000000051	73000000051	60000000051	86000000051	
62000000051	85000000051	80000000051	70000000051	86000000051	
51000000051	60000000051	75000000051	62000000051	73000000051	
70000000051	78000000051	88000000051	55000000051	70000000051	
68000000051	70000000051	60000000051	62000000051	80000000051	
63000000051	70000000051	85000000051	61000000051	80000000051	
53000000051	78000000051	60000000051	71000000051	62000000051	
64000000051	78000000051	78000000051	78000000051	75000000051	
76000000051	92000000051	95000000051	97000000051	95000000051	
76000000051	76000000051	60000000051	78000000051	77000000051	
56000000051	79000000051	80000000051	80000000051	86000000051	
51000000051	78000000051	60000000051	73000000051	60000000051	
10000000050	16860000053	16600000053	15340000053	16780000053	13370000053
20000000050	13052000053	1274960055	1092080055	1303440055	8302900054
30000000050	1033440053	1018180055	9417200054	1027110055	
40000000050	1280780055	1185720055	1292540055	1163490055	1277840055 1180090055
50000000050	1311090053	6616400052	1011820053	6581800052	
60000000050		2241450053	6017300052	1171270053	
70000000050			2246360053	1006640053	
80000000050				2358360053	
90000000050	1775864053	8811800052	9352700052	9466360052	7343600052
10000000051	1457870347	4352413846	5621393346	4923595445	
11000000051		7345196846	1339507446	3005028446	
12000000051			7691054846	2379263546	
13000000051				6610822846	
14000000051	1000000150	9999999049	1000000050	1000000050	
15000000051	1214319950	3815935949	2095735449	1832735349	2257718048
16000000051	6077272751	7663636451	7545454551	6972727351	7627272751
17000000051	3005619349	2133398849	2183045449	202396349	7872107650
18000000051	1269600550	9823458249	8395314649	1115493349	6377710049

Multiple Regression Analysis of Scores
in Second Nation-Grader Test for
Boys in Group C-P

Α-70	English	Geometry	French and Latin	Biology and Physics
22000000051	17000000051	68000000051	53000000051	60000000051
40000000050	71000000051	70000000051	65000000051	70000000051
30000000050	71000000051	83000000051	63000000051	42000000051
50000000050	80000000051	83000000051	81000000051	90000000051
12000000051	95000000051	92000000051	65000000051	80000000051
40000000050	70000000051	88000000051	60000000051	40000000051
20000000050	72000000051	75000000051	73000000051	60000000051
11000000051	81000000051	78000000051	73000000051	80000000051
11000000051	76000000051	77000000051	78000000051	82000000051
11000000051	80000000051	70000000051	71000000051	84000000051
40000000050	78000000051	45000000051	60000000051	46000000051
70000000050	85000000051	40000000051	70000000051	46000000051
50000000050	60000000051	45000000051	62000000051	77000000051
11000000051	60000000051	80000000051	55000000051	70000000051
12000000051	70000000051	60000000051	62000000051	80000000051
90000000050	70000000051	85000000051	61000000051	80000000051
40000000050	75000000051	60000000051	71000000051	62000000051
12000000051	75000000051	80000000051	78000000051	75000000051
20000000050	92000000051	95000000051	97000000051	95000000051
40000000050	60000000051	60000000051	78000000051	77000000051
12000000051	75000000051	60000000051	80000000051	80000000051
11000000051	80000000051	60000000051	73000000051	60000000051
00000000050	16000000051	16000000051	15000000051	17800000051
00000000050	305200051	12749600055	10920800051	13034400051
00000000050	1261700051	12663000054	12815000051	13229000051
00000000050	1280700051	11857200055	12924400055	11634900055
00000000050	13110900051	6164000052	10118400053	65418000051
00000000050	13110900051	62414600053	60173000052	11714700053
00000000050	13110900051	22463600053	22463600053	10066400053
00000000050	13110900051	23583600053	23583600053	23583600053
00000000050	13110900051	11008180053	11008180053	11518200051
10000000051	1457870547	4352413846	5621393446	4923595445
11000000051	1457870547	345196846	1339507446	3005028445
12000000051	1457870547	7691054846	7691054846	2379463546
13000000051	1457870547	661082246	661082246	661082246
14000000051	10000000150	9999999049	10000000050	10000000050
15000000051	4193618551	5407059248	424068449	4628645049
16000000051	7636363650	7663636451	7545454551	6972727351
17000000051	2676447149	1899751349	1949960649	1802301549
18000000051	2020454449	44012850	2381034450	1372453350
19000000051	2020454449	44012850	2381034450	1372453350
20000000051	2020454449	44012850	2381034450	1372453350
21000000051	2020454449	44012850	2381034450	1372453350
22000000051	2020454449	44012850	2381034450	1372453350
23000000051	2020454449	44012850	2381034450	1372453350
24000000051	2020454449	44012850	2381034450	1372453350
25000000051	2020454449	44012850	2381034450	1372453350
26000000051	2020454449	44012850	2381034450	1372453350
27000000051	2020454449	44012850	2381034450	1372453350
28000000051	2020454449	44012850	2381034450	1372453350
29000000051	2020454449	44012850	2381034450	1372453350
30000000051	2020454449	44012850	2381034450	1372453350
31000000051	2020454449	44012850	2381034450	1372453350
32000000051	2020454449	44012850	2381034450	1372453350
33000000051	2020454449	44012850	2381034450	1372453350
34000000051	2020454449	44012850	2381034450	1372453350
35000000051	2020454449	44012850	2381034450	1372453350
36000000051	2020454449	44012850	2381034450	1372453350
37000000051	2020454449	44012850	2381034450	1372453350
38000000051	2020454449	44012850	2381034450	1372453350
39000000051	2020454449	44012850	2381034450	1372453350
40000000051	2020454449	44012850	2381034450	1372453350
41000000051	2020454449	44012850	2381034450	1372453350
42000000051	2020454449	44012850	2381034450	1372453350
43000000051	2020454449	44012850	2381034450	1372453350
44000000051	2020454449	44012850	2381034450	1372453350
45000000051	2020454449	44012850	2381034450	1372453350
46000000051	2020454449	44012850	2381034450	1372453350
47000000051	2020454449	44012850	2381034450	1372453350
48000000051	2020454449	44012850	2381034450	1372453350
49000000051	2020454449	44012850	2381034450	1372453350
50000000051	2020454449	44012850	2381034450	1372453350
51000000051	2020454449	44012850	2381034450	1372453350
52000000051	2020454449	44012850	2381034450	1372453350
53000000051	2020454449	44012850	2381034450	1372453350
54000000051	2020454449	44012850	2381034450	1372453350
55000000051	2020454449	44012850	2381034450	1372453350
56000000051	2020454449	44012850	2381034450	1372453350
57000000051	2020454449	44012850	2381034450	1372453350
58000000051	2020454449	44012850	2381034450	1372453350
59000000051	2020454449	44012850	2381034450	1372453350
60000000051	2020454449	44012850	2381034450	1372453350
61000000051	2020454449	44012850	2381034450	1372453350
62000000051	2020454449	44012850	2381034450	1372453350
63000000051	2020454449	44012850	2381034450	1372453350
64000000051	2020454449	44012850	2381034450	1372453350
65000000051	2020454449	44012850	2381034450	1372453350
66000000051	2020454449	44012850	2381034450	1372453350
67000000051	2020454449	44012850	2381034450	1372453350
68000000051	2020454449	44012850	2381034450	1372453350
69000000051	2020454449	44012850	2381034450	1372453350
70000000051	2020454449	44012850	2381034450	1372453350
71000000051	2020454449	44012850	2381034450	1372453350
72000000051	2020454449	44012850	2381034450	1372453350
73000000051	2020454449	44012850	2381034450	1372453350
74000000051	2020454449	44012850	2381034450	1372453350
75000000051	2020454449	44012850	2381034450	1372453350
76000000051	2020454449	44012850	2381034450	1372453350
77000000051	2020454449	44012850	2381034450	1372453350
78000000051	2020454449	44012850	2381034450	1372453350
79000000051	2020454449	44012850	2381034450	1372453350
80000000051	2020454449	44012850	2381034450	1372453350
81000000051	2020454449	44012850	2381034450	1372453350
82000000051	2020454449	44012850	2381034450	1372453350
83000000051	2020454449	44012850	2381034450	1372453350
84000000051	2020454449	44012850	2381034450	1372453350
85000000051	2020454449	44012850	2381034450	1372453350
86000000051	2020454449	44012850	2381034450	1372453350
87000000051	2020454449	44012850	2381034450	1372453350
88000000051	2020454449	44012850	2381034450	1372453350
89000000051	2020454449	44012850	2381034450	1372453350
90000000051	2020454449	44012850	2381034450	1372453350
91000000051	2020454449	44012850	2381034450	1372453350
92000000051	2020454449	44012850	2381034450	1372453350
93000000051	2020454449	44012850	2381034450	1372453350
94000000051	2020454449	44012850	2381034450	1372453350
95000000051	2020454449	44012850	2381034450	1372453350
96000000051	2020454449	44012850	2381034450	1372453350
97000000051	2020454449	44012850	2381034450	1372453350
98000000051	2020454449	44012850	2381034450	1372453350
99000000051	2020454449	44012850	2381034450	1372453350
10000000051	2020454449	44012850	2381034450	1372453350
10100000051	2020454449	44012850	2381034450	1372453350
10200000051	2020454449	44012850	2381034450	1372453350
10300000051	2020454449	44012850	2381034450	1372453350
10400000051	2020454449	44012850	2381034450	1372453350
10500000051	2020454449	44012850	2381034450	1372453350
10600000051	2020454449	44012850	2381034450	1372453350
10700000051	2020454449	44012850	2381034450	1372453350
10800000051	2020454449	44012850	2381034450	1372453350
10900000051	2020454449	44012850	2381034450	1372453350
11000000051	2020454449	44012850	2381034450	1372453350
11100000051	2020454449	44012850	2381034450	1372453350
11200000051	2020454449	44012850	2381034450	1372453350
11300000051	2020454449	44012850	2381034450	1372453350
11400000051	2020454449	44012850	2381034450	1372453350
11500000051	2020454449	44012850	2381034450	1372453350
11600000051	2020454449	44012850	2381034450	1372453350
11700000051	2020454449	44012850	2381034450	1372453350
11800000051	2020454449	44012850	2381034450	1372453350
11900000051	2020454449	44012850	2381034450	1372453350
12000000051	2020454449	44012850	2381034450	1372453350
12100000051	2020454449	44012850	2381034450	1372453350
12200000051	2020454449	44012850	2381034450	1372453350
12300000051	2020454449	44012850	2381034450	1372453350
12400000051	2020454449	44012850	2381034450	1372453350
12500000051	2020454449	44012850	2381034450	1372453350
12600000051	2020454449	44012850	2381034450	1372453350
12700000051	2020454449	44012850	2381034450	1372453350
12800000051	2020454449	44012850	2381034450	1372453350
12900000051	2020454449	44012850	2381034450	1372453350
13000000051	2020454449	44012850	2381034450	1372453350
13100000051	2020454449	44012850	2381034450	1372453350
13200000051	2020454449	44012850	2381034450	1372453350
13300000051	2020454449	44012850	2381034450	1372453350
13400000051	2020454449	44012850	23810344	

W-C 1	English	Mathematics	U. S. History	Typing	
45000000051	40000000051	80000000051	90000000051	75000000051	
58000000051	75000000051	80000000051	90000000051	75000000051	
53000000051	76000000051	75000000051	80000000051	75000000051	
51000000051	90000000051	86000000051	94000000051	91000000051	
50000000051	80000000051	75000000051	60000000051	45000000051	
56000000051	88000000051	90000000051	95000000051	95000000051	
64000000051	85000000051	83000000051	90000000051	80000000051	
56000000051	80000000051	98000000051	90000000051	90000000051	
48000000051	60000000051	60000000051	75000000051	75000000051	
60000000051	83000000051	75000000051	78000000051	55000000051	
59000000051	85000000051	00000000051	95000000051	80000000051	
52000000051	83000000051	90000000051	82000000051	85000000051	
50000000051	72000000051	70000000051	70000000051	70000000051	
56000000051	60000000051	80000000051	94000000051	73000000051	
63000000051	83000000051	90000000051	80000000051	75000000051	
49000000051	94000000051	81000000051	92000000051	70000000051	
42000000051	80000000051	75000000051	74000000051	70000000051	
48000000051	75000000051	82000000051	75000000051	70000000051	
57000000051	75000000051	70000000051	74000000051	72000000051	
55000000051	80000000051	81000000051	90000000051	70000000051	
52000000051	80000000051	72000000051	75000000051	78000000051	
57000000051	85000000051	79000000051	90000000051	85000000051	
57000000051	85000000051	70000000051	92000000051	77000000051	
54000000051	75000000051	70000000051	75000000051	75000000051	
48000000051	87000000051	83000000051	95000000051	90000000051	
45000000051	85000000051	83000000051	85000000051	75000000051	
55000000051	90000000051	78000000051	80000000051	82000000051	
56000000051	60000000051	70000000051	70000000051	63000000051	
56000000051	71000000051	75000000051	72000000051	70000000051	
54000000051	83000000051	85000000051	85000000051	80000000051	
59000000051	68000000051	75000000051	80000000051	70000000051	
53000000051	84000000051	87000000051	87000000051	80000000051	
64000000051	81000000051	73000000051	78000000051	40000000051	
64000000051	80000000051	84000000051	80000000051	74000000051	
60000000051	80000000051	60000000051	87000000051	70000000051	
60000000051	81000000051	85000000051	92000000051	80000000051	
59000000051	80000000051	72000000051	78000000051	90000000051	
48000000051	70000000051	70000000051	44000000051	72000000051	
45000000051	80000000051	95000000051	94000000051	90000000051	
53000000051	90000000051	72000000051	92000000051	75000000051	
55000000051	75000000051	75000000051	93000000051	60000000051	
63000000051	75000000051	77000000051	78000000051	84000000051	
51000000051	80000000051	70000000051	80000000051	75000000051	
37000000051	85000000051	83000000051	90000000051	90000000051	
45000000051	75000000051	65000000051	90000000051	87000000051	
36000000051	83000000051	81000000051	85000000051	85000000051	
10000000050	3572000050	35100000053	37250000053	34590000053	24130000053
20000000050	2861200050	2767980055	3130330050	2705750055	1313200055
30000000050	1914200055	1881980055	1999140050	1450970055	
40000000050	2798240050	2972480055	2757140055	2921000055	2137300055 2887270055
50000000050	2584580050	12080000053	1565780050	1146270053	
60000000050		30180000053	16100000050	15710000053	
70000000050			4685780053	2398070053	
80000000050				4693200055	
90000000050	1934580050	11658000052	16000000051	1712200050	38227000052
10000000051	5338926046	1380492446	1190117240	2336143540	
11000000051		4697497046	7047819045	8750557945	

T-G 1	English	Mathematics	U. S. History	Typing
1200000051			3391791646	1206434147 -
1300000051				3097574347
1400000051	9999999049	9999999049	1000000050	1000000150
1500000051	5746420251	7147922248	2996126748	1192138249 134951449
1600000051	5362272251	7937777851	7800001051	8277777811 7557666751
1700000051	1573246642	1475736049	1253986549	1198182849 6809205450
1800000051	4543421449	2030259349	9506786649	1126122150-2011645049

Multiple Regression Analysis of Scores
to First High-Ability Test
for Girls in Group E-8

Multiple Regression Analysis of Scores
in Second Watson-Gleser Test
for Girls in Group B-3

W-02	ENGLISH	MATHEMATICS	U.S. HISTORY	TYPING	
45000000051	40000000051				
58000000051	75000000051	80000000051	90000000051	75000000051	
55000000051	76000000051	75000000051	80000000051	75000000051	
73000000051	90000000051	86000000051	94000000051	91000000051	
58000000051	80000000051	75000000051	80000000051	45000000051	
58000000051	88000000051	90000000051	95000000051	95000000051	
70000000051	85000000051	83000000051	90000000051	80000000051	
70000000051	80000000051	98000000051	90000000051	90000000051	
52000000051	60000000051	60000000051	75000000051	75000000051	
59000000051	83000000051	75000000051	78000000051	55000000051	
58000000051	85000000051	70000000051	95000000051	80000000051	
57000000051	85000000051	90000000051	82000000051	85000000051	
50000000051	72000000051	70000000051	70000000051	70000000051	
65000000051	60000000051	80000000051	94000000051	73000000051	
61000000051	85000000051	90000000051	80000000051	75000000051	
58000000051	94000000051	81000000051	92000000051	82000000051	
50000000051	80000000051	75000000051	74000000051	70000000051	
57000000051	75000000051	82000000051	75000000051	70000000051	
48000000051	75000000051	70000000051	74000000051	72000000051	
67000000051	80000000051	81000000051	90000000051	72000000051	
63000000051	80000000051	72000000051	75000000051	75000000051	
74000000051	85000000051	79000000051	90000000051	85000000051	
62000000051	85000000051	70000000051	82000000051	77000000051	
63000000051	75000000051	70000000051	75000000051	75000000051	
58000000051	87000000051	83000000051	95000000051	90000000051	
53000000051	85000000051	83000000051	85000000051	75000000051	
58000000051	90000000051	78000000051	80000000051	62000000051	
64000000051	60000000051	70000000051	70000000051	53000000051	
62000000051	71000000051	75000000051	72000000051	77000000051	
59000000051	83000000051	85000000051	85000000051	80000000051	
62000000051	68000000051	75000000051	80000000051	70000000051	
62000000051	84000000051	87000000051	87000000051	80000000051	
68000000051	81000000051	73000000051	78000000051	80000000051	
67000000051	80000000051	84000000051	80000000051	74000000051	
59000000051	80000000051	60000000051	87000000051	70000000051	
52000000051	81000000051	85000000051	92000000051	80000000051	
51000000051	80000000051	72000000051	78000000051	95000000051	
39000000051	70000000051	70000000051	44000000051	72000000051	
51000000051	80000000051	90000000051	94000000051	80000000051	
50000000051	90000000051	72000000051	92000000051	75000000051	
65000000051	75000000051	75000000051	93000000051	60000000051	
70000000051	75000000051	77000000051	78000000051	84000000051	
58000000051	80000000051	70000000051	80000000051	50000000051	
57000000051	85000000051	83000000051	90000000051	90000000051	
58000000051	75000000051	65000000051	90000000051	70000000051	
50000000051	83000000051	81000000051	85000000051	85000000051	
10000000050	35720000053	35300000053	5725000005	34900000053	26590000053
20000000050	28612200055	27679000055	31303300055	27057000055	15942300055
30000000050	21128100055	20803000055	22173700055	20450000055	
40000000050	27987400055	29724800055	27571400055	29215000055	27137300055 28872700055
50000000050	25845900053	12080000053	15657800053	14627000053	
60000000050		30180000053	16100000053	15710000053	
70000000050			46857000053	23935000053	
80000000050				45932000053	
90000000050	23056400053	21549000052	62800000052	12308400053	21155000052
10000000051	5338926046	1380492446	1190117246	2336143545	
11000000051		6697497046	7047819045	8750557045	

Multiple Regression Analysis of Scores
in Second split n-Glaser Test
for Girls in Group W-3

W-3	ENGLISH	MATHS. I.C.	U.S. HISTORY	TYPE
1200000051			3391793646	1206434440-
1300000051				3097574346
1400000051	9999999049	9999999049	1000000000	1000000150
1500000051	4071384851	1230783849-	1599036344	3220000050
1600000051	5900000001	7937777851	7800000051	8277777751
1700000051	1577820049	1400034349	1257638943	1201003349
1800000051	7800489649-	1081013050	2560799050	1190017650-

1430250449

768666675

6829038150

4349070049

Avg	English	Mathematics	U. S. History	Typing
4530000051	4030000051	4000000051	7000000051	7500000051
4000000051	5000000051	8000000051	9000000051	7500000051
51	7500000051	8000000051	9000000051	7500000051
2000000051	7600000051	7500000051	9000000051	7500000051
2200000051	9000000051	8000000051	9400000051	9100000051
8000000051	6000000051	7500000051	6000000051	4500000051
2000000051	8900000051	9000000051	9000000051	9000000051
6000000051	8300000051	8300000051	9000000051	8000000051
1400000051	4000000051	9800000051	9000000051	9000000051
1000000051	7000000051	7500000051	7400000051	5500000051
1000000051	8500000051	4000000051	9500000051	8000000051
5000000051	8300000051	8000000051	8200000051	5500000051
51	7200000051	7000000051	7000000051	7000000051
9000000051	6000000051	8000000051	7400000051	7300000051
2000000051	8000000051	9000000051	8000000051	7500000051
0000000051	9000000051	8100000051	9200000051	8200000051
8000000051	4000000051	7500000051	7400000051	7000000051
9000000051	7500000051	8000000051	7500000051	7000000051
9000000051	7500000051	7000000051	7400000051	7200000051
1200000051	8000000051	8100000051	9000000051	7200000051
1100000051	8000000051	7200000051	7500000051	7800000051
1700000051	8300000051	7900000051	9200000051	8500000051
5000000051	6500000051	7000000051	9200000051	7000000051
9000000051	7500000051	7000000051	7500000051	7500000051
1000000051	5700000051	8000000051	9500000051	9000000051
3000000051	8500000051	8000000051	8500000051	7500000051
3000000051	9000000051	7800000051	8000000051	6200000051
8000000051	6000000051	7000000051	7000000051	6300000051
6000000051	7100000051	7500000051	7200000051	7000000051
5000000051	8000000051	8500000051	8500000051	8000000051
3000000051	6600000051	7500000051	8000000051	7000000051
9000000051	8400000051	8700000051	8700000051	8000000051
4000000051	8100000051	7300000051	7800000051	8000000051
3000000051	8000000051	8400000051	8000000051	7400000051
1000000051	8000000051	6000000051	8700000051	7000000051
8000000051	9100000051	8500000051	9200000051	8600000051
8000000051	4000000051	7200000051	7500000051	9600000051
9000000051	7000000051	7000000051	4400000051	7200000051
6000000051	8000000051	8500000051	4400000051	9000000051
3000000051	9000000051	7200000051	9200000051	7500000051
1000000051	7500000051	7500000051	7500000051	6000000051
7000000051	7500000051	7700000051	7500000051	8400000051
7000000051	8000000051	7000000051	4000000051	7500000051
2000000051	8500000051	8000000051	9000000051	9000000051
1300000051	7000000051	6500000051	9000000051	8700000051
1400000051	8300000051	8100000051	8500000051	4500000051
1300000051	3572000053	3510000053	3720000053	3459000053
3000000051	2861220053	7679800053	3100000053	2705750053
3000000051	1985900054	1983400054	2142000054	1950000054
4000000051	2798240053	2972480053	2757140053	2921600053
5000000051	2584580053	1208000053	1563780053	1143070053
6000000051		3018000053	1610000053	1571000053
7000000051			4685740053	2348670053
8000000051				4695200053
9000000051	2169200053	3320670052	6440000052	1059667053
1000000051	5338926046	1380494446	1190117246	2336143545
1100000051		4697497046	7047819045	8750557945

Multiple Key-pressing Analysis of Chinese in Grades
 of Two Nation-Class sets
 for Jiple in Grade L-1

Multiple Regression Analysis of Change in Scores
of Twelfth-Grade Tests
for Girls in 1964-65

A 2-3	English	Mathematics	U. S. History	Writing
1200000051				1206934642
1300000051			3391793046	3097574342
1400000051	9993999009	9999999049	1000000050	1000000150
1500000051	1675028051	5160041048	1300329943	2028415749
1600000051	5446666750	793777851	7800000051	8277777851
1700000051	1583163649	1498666449	1260673449	1210507249
1800000051	3247016949	8723144949	1401372350	6468145348
				571953043

U-61	ENGLISH	GEOMETRY	BIOLOGY	LATIN AND FRENOCH	
4500000051	4000000051	7000000051	8000000051	8000000051	
5700000051	8300000051	7800000051	8500000051	8000000051	
5500000051	8500000051	7800000051	8500000051	8000000051	
4800000051	8500000051	8000000051	9100000051	8500000051	
5000000051	7800000051	4500000051	7000000051	8000000051	
5700000051	9000000051	7200000051	8000000051	8000000051	
6000000051	8500000051	7500000051	8000000051	7500000051	
5500000051	8700000051	8000000051	8700000051	9000000051	
4500000051	7500000051	8700000051	7500000051	7500000051	
6100000051	8500000051	6500000051	7600000051	7500000051	
6100000051	8600000051	7000000051	8000000051	8400000051	
5400000051	7000000051	8800000051	8500000051	8500000051	
5300000051	7400000051	6800000051	6500000051	7000000051	
5300000051	8200000051	8000000051	8500000051	8400000051	
6000000051	8500000051	9000000051	8600000051	8300000051	
5100000051	7500000051	8000000051	8000000051	8000000051	
4500000051	8500000051	7000000051	9000000051	8300000051	
5400000051	7500000051	6000000051	7500000051	3900000051	
5800000051	8500000051	8500000051	8000000051	8000000051	
5500000051	8000000051	8500000051	8000000051	8000000051	
5200000051	8400000051	7000000051	8200000051	8700000051	
5700000051	7800000051	7800000051	8000000051	8000000051	
5500000051	8800000051	7800000051	8000000051	8000000051	
5800000051	8500000051	8000000051	7500000051	8000000051	
5100000051	9300000051	9000000051	9000000051	8600000051	
4700000051	7000000051	8000000051	7800000051	7200000051	
5500000051	8200000051	8000000051	7000000051	8000000051	
6100000051	7800000051	7500000051	7500000051	7600000051	
5300000051	7100000051	7800000051	8000000051	7500000051	
5400000051	9600000051	9700000051	9500000051	9500000051	
5400000051	8400000051	8000000051	8000000051	8300000051	
5700000051	8100000051	8100000051	8000000051	7800000051	
6900000051	9100000051	8800000051	7500000051	8500000051	
6400000051	8000000051	6800000051	7500000051	8000000051	
6200000051	8700000051	7800000051	7500000051	8000000051	
6100000051	8500000051	7200000051	8800000051	8600000051	
5600000051	8200000051	6800000051	7000000051	8700000051	
5000000051	9000000051	6800000051	7500000051	8400000051	
4500000051	8300000051	7200000051	8000000051	9000000051	
5300000051	7500000051	9000000051	8000000051	9000000051	
5200000051	8800000051	7000000051	7500000051	7800000051	
6000000051	7500000051	7000000051	9000000051	8800000051	
5600000051	7000000051	7500000051	8000000051	9000000051	
4000000051	7200000051	7600000051	7000000051	8200000051	
4600000051	8300000051	8000000051	7500000051	8500000051	
5100000051	7500000051	6000000051	7500000051	8600000051	
1000000050	3713000053	3455000053	3578000053	3651000053	2428000053
2000000050	3280730055	2683450055	2862740055	2992030055	1329940055
3000000050	2010980055	1868060055	1931770055	1968350055	
4000000050	2856520055	2760770055	3022880055	2756600055	2815910055 2912210055
5000000050	1709240053	5761100052	8422400052	1039930053	
6000000050		1077780053	9491100052	1275330053	
7000000050			1783240053	9259300052	
8000000050				2985200053	
9000000050	1989910053	7610200052	3895600052	1240700052	1567300052
1000000051	8584602946	2228750845	3064513546	2135242846	

Multiple Regression Analysis of Scores
in First Watson-Gleser Test
for Girls in Group C-3

N-01	ENGLISH	GEOMETRY	BIOLOGY	LATIN AND FRENCH
1100000051		4301084046	1668832346	1397519946-
1200000051			8366301346	8144907045-
1300000051				4943375146
1400000051	1000000050	1000000050	1000000150	1000000050
1500000051	2460325051	0574481349	1857223344	1817022649-3045168749-
1600000051	5395555651	8251111151	7677777451	7951111151 8113333351
1700000051	1728340949	1223360749	1706224449	1311542249 5899118850
1800000051	3803926250	1518132250	1064037650	2321822950-5481360049

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[illegible]

Multiple Regression Analysis of Change in Scores
of Two Watson-Glaser Tests
for Girls in Group C-3

Δ W-G	English	Geometry	Biology	Latin and French
1000000051	8584002940	2228956845	3064513540	-213524240-
1100000051	4301084646	1668832346	1668832346	-1397519946-
1200000051		8366301340	8366301340	8144907045-
1300000051			4943375140	4943375140
1400000051	1000000050	1000000050	1000000150	1000000050
1500000051	8111365050	2032696549	1872864548	1044074449
1600000051	7933333350	8251111151	7677777851	7951111151
1700000051	1835228149	1299018049	1811743949	1392053049
1800000051	1107598850	-1441754149	9074540849	1844234440
				1997435049

APPENDIX 8

**SIMPLE CORRELATIONS BETWEEN FINAL GRADES IN
SCHOOL SUBJECTS AND INITIAL, FINAL AND CHANGE IN SCORES
OF THE WATSON-GLASER CRITICAL THINKING APPRAISAL**

TABLE XVII.- Simple Correlations Between Final Grades in School Subjects and Initial, Final and Change in Scores of the Watson-Glaser Critical Thinking Appraisal.

Groups	Variables	N	WG ₁	WG ₂	Δ WG
E ₁ Boys	Mathematics	20	.329	.201	.003
	Science	20	.378	.246	.078
	English	20	.204	.267	.116
	U.S. History	20	.290	.396	.178
	Multiple R		.475	.421	.268
C ₁ Boys	Geometry	20	.003	-.207	-.162
	Drafting	20	.163	-.205	-.257
	English	20	-.065	.289	.262
	U.S. History	20	.122	-.307	-.311
	Multiple R		.233	.504	.502
E ₃ Girls	Mathematics	45	-.006	.237	.251
	U.S. History	45	.057	.037	.033
	English	45	-.052	.088	.140
	Typing	45	-.127	.064	.119
	Multiple R		.203	.437	.357
C ₃ Girls	Geometry	45	.157	.222	.057
	Biology	45	.066	.162	.094
	Latin or French	45	-.064	-.502	.019
	English	45	.412	.339	-.102
	Multiple R		.548	.477	.200
E ₂ Boys	Geometry	22	.079	-.099	-.128
	Phys. or History	22	.151	-.220	-.268
	English	22	.145	.318	.147
	Drafting	22	.151	.066	-.049
	Multiple R		.185	.580	.453
C ₂ Boys	Geometry	22	-.035	.469	.505
	Biol. or Physics	22	.214	.358	.425
	English	22	.125	.576	.497
	French or Latin	22	-.106	.474	.562
	Multiple R		.444	.638	.713

APPENDIX 9

**t-TEST FOR ALL PARTIAL REGRESSION COEFFICIENTS
FOR ALL GROUPS TESTED**

TABLE XVIII.- t-Test for All Partial Regression Coefficients
for All Groups Tested.

Groups	Variables	N	WG ₁	WG ₂	ΔWG
E ₁	English	20	-.222	.188	.274
	Science	20	1.113	.184	-.403
	Mathematics	20	1.082	-.465	-.647
	U.S. History	20	.212	1.234	.916
C ₁	English	20	-.487	1.781	1.706
	Geometry	20	-.429	-1.539	.174
	U.S. History	20	.355	-1.095	-1.087
	Drafting	20	.603	-.397	-.718
E ₂	English	22	.266	2.078	1.257
	Geometry	22	-.257	.525	.553
	Phys. or History	22	.326	-2.108	-1.709
	Drafting	22	.149	.844	.481
C ₂	English	22	1.286	1.269	.202
	Geometry	22	-1.188	.982	2.234
	French or Latin	22	-1.511	.839	2.381
	Biol. or Physics	22	1.574	.111	-1.372
E ₃	English	45	-.454	-.780	-.324
	Mathematics	45	.203	1.081	.872
	U.S. History	45	.951	2.560	1.601
	Typing	45	-1.126	-1.190	-.067
C ₃	English	45	3.804	2.655	-1.107
	Geometry	45	1.518	1.688	.144
	Biology	45	-1.065	-1.024	.907
	Latin or French	45	-2.321	-2.148	.184

APPENDIX 10

AN ABSTRACT OF
THE INFLUENCE OF THE STUDY OF PLANE GEOMETRY
ON CRITICAL THINKING

APPENDIX 10

ABSTRACT OF

The Influence of the Study of Plane Geometry on Critical Thinking

In studies that have surveyed the purposes of teaching geometry, such reasons as "to teach the students to think logically" and "to understand the meaning of deductive proof" were most frequently mentioned. In one important survey that tabulated the opinions of five hundred classroom teachers concerning the important objectives of teaching geometry, the one receiving the highest rating was "to develop the habit of clear thinking and precise expression".

With allowances for varying phraseology and varying definitions or lack of them, the literature indicated that the important values to be derived from the study of demonstrative geometry are related to an acquaintance with the nature of proof and to a familiarity with critical thinking as a method of thought.

The ability to think critically, as concerned in this study, involves three elements: (1) an attitude of being disposed to consider in a thoughtful way the problems and subjects that come within the range of one's experiences; (2) knowledge of the methods of logical inquiry and reasoning; and (3) some skill in applying these methods.

The chief problems of this study are: (1) Do high school sophomores who have studied plane demonstrative geometry improve more in critical thinking than those pupils who have not taken geometry? (2) What is the relationship between achievement in geometry and critical thinking ability? What is the relationship between achievement in all other major subjects during the sophomore year and critical thinking ability?

Special attention is given to the course of study in geometry as it is taught at Pittsfield High School. This course is organized around the objectives based on the exhaustive study of the Joint Commission of the Mathematical Association of America and the National Council of Teachers of Mathematics.

Relating geometric thinking to non-geometric situations is frequently employed as a means of motivating the logical emphasis of the course in demonstrative geometry. Since pupils learn subjects best in connection with their use in meaningful situations, the study of non-geometric exercises of logical reasoning that have relevance to geometric thinking facilitates the learning of geometric content, as in the treatment of the meaning of the definition, indirect proofs, fallacious reasoning, etc. At Pittsfield High School,

a critical and appreciative attitude toward reasoning in every-day life is made to contribute to greater insight into geometric thinking.

On October 5, 1953, all 531 sophomore pupils at Pittsfield High School were given the Watson-Glaser Critical Thinking Appraisal, Form AM. During the following week, the sophomores were given Form C of the Terman-McNemar Test of Mental Ability.

In order to compare the "geometry" groups and the "non-geometry" groups, students from the two groups were paired on the basis of (1) intelligence as indicated by the verbal mental ability score on the Terman-McNemar Test; (2) score on the Watson-Glaser Test administered on October 5, 1953; (3) sex; (4) age. On this basis three matched sets of paired pupils were constituted. On May 6, 1954, all Sophomores were given Form BM of the Watson-Glaser Critical Thinking Appraisal.

The test scores on the Terman-McNemar Test of Mental Ability and the two forms of the Watson-Glaser Critical Thinking Appraisal as well as pupil-personnel records comprise the basic data of this investigation.

The answer to the major question, as to whether high school sophomores who have taken plane geometry for seven months improve more in critical thinking than those pupils

who have not taken geometry, was first sought by noting the increase in the mean of the scores made in the two administrations of the Watson-Glaser Critical Thinking Appraisal, and ascertaining whether such differences were significant.

The distribution of t was used to test the significance of the differences between the two means. All groups tested made gains varying from 2.60 to 7.95. The gain made by the boys not taking geometry is significant at the low 20% level; however gains made by the other groups are highly significant at 0.1%. Included in the groups that did make significant gains was one made up of girls in the commercial curriculum who did not take geometry.

The paired groups revealed a difference of 5.20 points between the mean gains of the boys who took geometry and those who did not take geometry; a difference of -0.31 points between the mean gains of the boys who took geometry and its paired group who took geometry and a foreign language; and a difference of 2.46 points between the mean gains of the girls who took geometry and those who did not. None of these differences is significant at the 5% level. However, 5.20 reaches the 7% level and 2.46 the 8% level, so that both non-geometry groups did tend to increase less in the Watson-Glaser Critical Thinking Appraisal than the corresponding geometry groups, with probabilities of better than 1 in 10

that the differences were not due to chance. Consistently with the hypothesis, the difference of -0.31 between the two matched geometry groups (one taking geometry but no foreign language, the other taking geometry and French or Latin) had a probability of 90% that the differences were only due to chance.

Only in group C_2 are the correlations sufficiently large to indicate a positive relationship between geometry grades and the second Watson-Glaser test scores or the difference in Watson-Glaser test scores. In this group the second Watson-Glaser test scores and the difference in the Watson-Glaser test scores have about the same correlations with grades in other major subjects as with geometry grades (about .4 or .5). The ability to read adequately or to understand language discriminatingly may be a factor in the correlation with English grades. The inclusion of social study-type material in the Watson-Glaser Critical Thinking Appraisal may be a factor with the correlation with U.S. History grades.

The data do not seem to support the theory that success in geometry (as measured by grades) is closely associated with critical thinking (as measured by the Watson-Glaser test scores).

Multiple correlation coefficients between grades in all major subjects during the sophomore year and the initial, final and difference in Watson-Glaser Critical Thinking Appraisal scores indicate that subject grades correlate more closely with the second Watson-Glaser test scores than with the first Watson-Glaser test scores or the change in Watson-Glaser test scores, and that subject grades and Watson-Glaser scores correlate most closely in the group of boys who took geometry and a foreign language.

Significance data for the regression coefficients, obtained by processing on an electronic computer, were consistent with the findings dealing with the correlations between geometry grades and the initial, final and difference in Watson-Glaser Critical Thinking Appraisal test scores.

It possible that the influence of studying geometry on the development of critical thinking ability (revealed in this study at an inadequate significance level) might be established at a higher level if the research in this study were extended with additional numbers of students. This would seem to be a fruitful field of research, since quantitative data are needed to interpret current hypotheses as to the power of geometry, as it is customarily taught, to develop the ability to think critically in non-geometric contexts.