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LA THÈSE A ÉTÉ MICROFILMÉE TELLE QUE NOUS L'AVONS REÇUE
TURBULENT TRANSPORT PHENOMENA
BETWEEN A LARGE BODY OF WATER
AND SURROUNDING ATMOSPHERE

by

Chia-Yu Shaw

A thesis submitted to the School of Graduate
Studies in partial fulfillment of the requirements
for the degree of Ph.D. in Mechanical Engineering

UNIVERSITY OF OTTAWA
OTTAWA, CANADA, 1975

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ABSTRACT

This dissertation presents the study of the problem of turbulent heat and mass transfer from a wavy water surface induced by wind. Two approaches are used for the calculation of the heat and mass transfer coefficients.

In the first approach, the mathematical model is developed on the basis of a hypothetically smooth surface. The predicted transfer coefficients are then modified to include surface with wind-induced waves using a correlation deduced from the energy or the diffusion equation. The heat and mass transfer Stanton Numbers are compared with available field data and satisfactory agreement is obtained.

In the second approach, both analytical and experimental studies are carried out on a wavy water surface which is generated in the air-water tunnel. The air-water tunnel has been designed and built specially for this study. It has a long and smooth entrance section so that a reasonably well developed mean air velocity profile can be produced prior to its encounter of the water surface. The airflow rate, temperature and relative humidity in the test section can also be controlled.

The experimental study involves measuring the fetch, wave amplitude, mean air velocity profile and dry-bulb as well as wet-bulb temperature profiles. A technique is developed to measure the wave amplitude. Also a multiple function measuring probe is built to measure the mean air velocity profile and the temperature profiles simultaneously.
The mathematical model for the analytical study is deduced from the energy and the diffusion equations. Similarity criteria for flow system having a free water surface are given. The influence of fetch on various flow parameters such as shear velocity and dynamic roughness are included in the model. For these purposes, the relationships between the fetch and various flow parameters have been deduced from the experimental data obtained in the air-water tunnel.

Comparison between theory and experiment is made and close agreements between the predicted and the measured temperature as well as specific humidity profiles are obtained. The predicted Stanton Number is also found in good agreement with available field data.
ACKNOWLEDGEMENT

The author wishes to express his sincere gratitude to Dr. Y. Lee who has initiated and supervised the present work. His valuable help, guidance and innumerable time have contributed much to this work and are deeply appreciated.

The author is indebted to the staff members of the department of Mechanical Engineering, for their interest and encouragement throughout the course of the study.

Thanks are also due to the technical staffs of the department of Mechanical Engineering for their generous assistance.
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Chapter</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>ABSTRACT</td>
<td></td>
<td>i</td>
</tr>
<tr>
<td>ACKNOWLEDGEMENT</td>
<td></td>
<td>iii</td>
</tr>
<tr>
<td>TABLE OF CONTENTS</td>
<td></td>
<td>iv</td>
</tr>
<tr>
<td>LIST OF TABLES</td>
<td></td>
<td>ix</td>
</tr>
<tr>
<td>LIST OF FIGURES</td>
<td></td>
<td>x</td>
</tr>
<tr>
<td>NOMENCLATURE</td>
<td></td>
<td>xi</td>
</tr>
<tr>
<td>CHAPTER 1.</td>
<td>Introduction</td>
<td>1</td>
</tr>
<tr>
<td>CHAPTER 2.</td>
<td>Literature Survey</td>
<td>4</td>
</tr>
<tr>
<td>2.1 Velocity Profiles</td>
<td></td>
<td>4</td>
</tr>
<tr>
<td>2.1.1 Smooth Surface</td>
<td></td>
<td>5</td>
</tr>
<tr>
<td>2.1.2 Rough Surface</td>
<td></td>
<td>6</td>
</tr>
<tr>
<td>2.2 Dynamic Roughness</td>
<td></td>
<td>8</td>
</tr>
<tr>
<td>2.3 Drag Coefficient</td>
<td></td>
<td>9</td>
</tr>
<tr>
<td>2.4 Thermal Stratification</td>
<td></td>
<td>12</td>
</tr>
<tr>
<td>2.5 Wind Induced-Waves</td>
<td></td>
<td>13</td>
</tr>
<tr>
<td>2.6 Heat and Mass Transfer</td>
<td></td>
<td>14</td>
</tr>
<tr>
<td>2.7 Correlation of Laboratory Data For Field Application</td>
<td></td>
<td>16</td>
</tr>
<tr>
<td>2.8 Summary</td>
<td></td>
<td>18</td>
</tr>
<tr>
<td>CHAPTER 3.</td>
<td>Analysis</td>
<td>19</td>
</tr>
<tr>
<td>3.1 Basic Equations</td>
<td></td>
<td>19</td>
</tr>
<tr>
<td>3.2 Physical Model and Assumptions</td>
<td></td>
<td>22</td>
</tr>
<tr>
<td>3.3 Boundary Layer Approximations</td>
<td></td>
<td>23</td>
</tr>
<tr>
<td>3.4 Equations of Heat Flux and Mass Flux</td>
<td></td>
<td>24</td>
</tr>
<tr>
<td>3.5 Methods of Solution</td>
<td></td>
<td>24</td>
</tr>
<tr>
<td>3.5.1 Method I - Smooth-Wavy Surface Analogy Approach</td>
<td></td>
<td>25</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.5.1.1 Solutions of Energy and Diffusion Equations</td>
<td>26</td>
</tr>
<tr>
<td>3.5.1.2 Numerical Procedure</td>
<td>29</td>
</tr>
<tr>
<td>3.5.1.3 Heat and Mass Transfer Coefficients for a Smooth Water Surface</td>
<td>31</td>
</tr>
<tr>
<td>3.5.1.4 Heat and Mass Transfer Coefficients for a Wavy Water Surface</td>
<td>32</td>
</tr>
<tr>
<td>3.5.1.5 Drag Coefficients under Atmospheric Conditions</td>
<td>32</td>
</tr>
<tr>
<td>3.5.1.6 Dimensionless Transfer Coefficients over a Wavy Water Surface</td>
<td>34</td>
</tr>
<tr>
<td>3.5.2 Method II - Semi-Empirical Approach</td>
<td>35</td>
</tr>
<tr>
<td>3.5.2.1 Assumptions</td>
<td>36</td>
</tr>
<tr>
<td>3.5.2.2 Functional Relationships Among U, L, z₀ and Cᵢ</td>
<td>37</td>
</tr>
<tr>
<td>3.5.2.3 Dimensionless Energy, Diffusion and Continuity Equations</td>
<td>39</td>
</tr>
<tr>
<td>3.5.2.4 Solutions of Energy and Diffusion Equations</td>
<td>43</td>
</tr>
<tr>
<td>3.5.2.5 Transfer Coefficients for a Wavy Water Surface</td>
<td>46</td>
</tr>
<tr>
<td>3.6 Wind Drift Current</td>
<td>47</td>
</tr>
<tr>
<td>CHAPTER 4. Experimental Study</td>
<td>48</td>
</tr>
<tr>
<td>4.1 Closed Circuit Air-Water Tunnel</td>
<td>48</td>
</tr>
<tr>
<td>4.2 Measuring Apparatus</td>
<td>52</td>
</tr>
<tr>
<td>4.2.1 Multiple Function Measuring Probe</td>
<td>52</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.2.2 Wave Gauge</td>
<td>53</td>
</tr>
<tr>
<td>4.2.3 Physical Arrangement of Measuring Apparatus</td>
<td>53</td>
</tr>
<tr>
<td>4.3 Calibration of Measuring Apparatus</td>
<td>54</td>
</tr>
<tr>
<td>4.3.1 Static Pressure Tube</td>
<td>54</td>
</tr>
<tr>
<td>4.3.2 Thermocouples</td>
<td>54</td>
</tr>
<tr>
<td>4.3.3 Differential Pressure Transducer</td>
<td>54</td>
</tr>
<tr>
<td>4.4 Experimental Procedure and Results</td>
<td>55</td>
</tr>
<tr>
<td>4.4.1 Steady-State Conditions</td>
<td>55</td>
</tr>
<tr>
<td>4.4.2 Height of Probes Above Mean Water Level</td>
<td>56</td>
</tr>
<tr>
<td>4.4.3 Air Velocity Profile</td>
<td>56</td>
</tr>
<tr>
<td>4.4.4 Wave Amplitude</td>
<td>57</td>
</tr>
<tr>
<td>4.4.5 Temperature and Specific Humidity Profiles</td>
<td>58</td>
</tr>
<tr>
<td>4.5 Evaluation of Shear Velocity, Dynamic Roughness, Drag Coefficient, Temperature and Specific Humidity</td>
<td>58</td>
</tr>
<tr>
<td>4.5.1 Shear Velocity, Dynamic Roughness and Drag Coefficient</td>
<td>58</td>
</tr>
<tr>
<td>4.5.2 Temperature and Specific Humidity</td>
<td>59</td>
</tr>
<tr>
<td>4.6 Discussion</td>
<td>59</td>
</tr>
<tr>
<td>4.6.1 Investigation of Two-Dimensional Flow</td>
<td>59</td>
</tr>
<tr>
<td>4.6.2 Uncertainty Analysis</td>
<td>60</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>CHAPTER 5. Discussion</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1 Method I - Smooth-Wavy Surface Analogy Approach</td>
<td>61</td>
</tr>
<tr>
<td>5.1.1 Drag Coefficient</td>
<td>61</td>
</tr>
<tr>
<td>5.1.2 Transfer Coefficients</td>
<td>63</td>
</tr>
<tr>
<td>5.2 Method II - Semi-Empirical Approach</td>
<td>64</td>
</tr>
<tr>
<td>5.2.1 Mean Velocity Profile</td>
<td>64</td>
</tr>
<tr>
<td>5.2.2 Drag Coefficient and Dynamic Roughness</td>
<td>65</td>
</tr>
<tr>
<td>5.2.3 Comparison of Flow Parameters</td>
<td>66</td>
</tr>
<tr>
<td>5.2.3.1 Comparison with Experimental Data of the Present Study</td>
<td>66</td>
</tr>
<tr>
<td>5.2.3.2 Comparison with Experimental Data of Other Authors</td>
<td>66</td>
</tr>
<tr>
<td>5.2.4 Temperature and Specific Humidity Profiles</td>
<td>68</td>
</tr>
<tr>
<td>5.2.4.1 Comparison with Experimental Results</td>
<td>69</td>
</tr>
<tr>
<td>5.2.5 Comparison of Analytical Stanton Numbers with Field Measurements</td>
<td>71</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>CHAPTER 6. Conclusions and Comments</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>6.1 Conclusions</td>
<td>73</td>
</tr>
<tr>
<td>6.2 Comments</td>
<td>74</td>
</tr>
</tbody>
</table>
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>APPENDIX</th>
<th>PAGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>A1.</td>
<td>Boundary Layer Approximations</td>
</tr>
<tr>
<td>A2.</td>
<td>First Derivative of $F$ with Respect to $Y^+$ at Water Surface</td>
</tr>
<tr>
<td>A2.1</td>
<td>Relationship Between Transfer Coefficients and Drag Coefficients</td>
</tr>
<tr>
<td>A2.3</td>
<td>Drag Coefficient Over a Wavy Water Surface</td>
</tr>
<tr>
<td>A3.</td>
<td>Functional Relationships Among $u_1$, $u_2$, and $L$</td>
</tr>
<tr>
<td>A3.1</td>
<td></td>
</tr>
<tr>
<td>A4.</td>
<td>Evaluation of $\frac{\partial^2 x}{\partial y^2}$, $\frac{\partial u_1}{\partial y^2}$, $\frac{\partial u_2}{\partial x^2}$, and $\frac{\partial u_1}{\partial x^2}$</td>
</tr>
<tr>
<td>A5.</td>
<td>Heat and Mass Transfer Stanton Numbers</td>
</tr>
<tr>
<td>A6.</td>
<td>Wind Drift Current</td>
</tr>
<tr>
<td>A7.</td>
<td>References</td>
</tr>
<tr>
<td>TABLE</td>
<td>PAGE</td>
</tr>
<tr>
<td>---------</td>
<td>------</td>
</tr>
<tr>
<td>2.1</td>
<td>101</td>
</tr>
<tr>
<td>3.1</td>
<td>103</td>
</tr>
<tr>
<td>4.1</td>
<td>104</td>
</tr>
<tr>
<td>4.2</td>
<td>106</td>
</tr>
</tbody>
</table>

**LIST OF TABLES**

TABLE 2.1 Drag Coefficient Over a Large Body of Water with Reference to Wind Speed at 10 Meters

TABLE 3.1 Law of the Wall

TABLE 4.1 Experimental Data

TABLE 4.2 Experimental Data
# LIST OF FIGURES

| Fig. 3.1 | Measurement of Fluctuating Components in a Rectangular Channel (After Reichardt (23)) | 107 |
| Fig. 3.2 | Flow Field Grid - Method I | 108 |
| Fig. 3.3 | Effect of $\Delta Y^+$ on $F$ | 109 |
| Fig. 3.4 | Dimensionless Temperature and Specific Humidity Profiles Based on Different Eddy Diffusivity Models | 110 |
| Fig. 3.5 | Drag Coefficient Over a Wavy Water Surface - Method I | 111 |
| Fig. 3.6 | Transfer Coefficient Over a Wavy Water Surface - Method I | 112 |
| Fig. 3.7 | Functional Relationship Among $a_{rms}/L$, Froude Number and $u^+$ | 113 |
| Fig. 3.8 | Functional Relationship Between $a_{rms}/L$ and $F_{r y}$ | 114 |
| Fig. 3.9 | Drag Coefficient Over a Wavy Water Surface - Method II | 115 |
| Fig. 3.10 | Dynamic Roughness - Method II | 116 |
| Fig. 3.11 | Flow Field Grid - Method II | 117 |
| Fig. 3.12 | Stanton Numbers - Method II | 118 |
| Fig. 4.1 | Wind-Induced Water Surface Velocity | 119 |
| Fig. 4.1a | Closed Circuit Air-Water Tunnel | 120 |
| Fig. 4.2 | Closed Circuit Air-Water Tunnel - Overall View from Upstream | 121 |
| Fig. 4.2a | Working Section | 122 |
| Fig. 4.2b | Working Section - Front Elevation | 123 |
| Fig. 4.3 | Schematics of Environmental Controls | 124 |
| Fig. 4.4 | Multiple Function Measuring Probe | 125 |
| Fig. 4.4a | Multiple Function Measuring Probe | 126 |
| Fig. 4.5 | Schematics of Measuring Apparatus | 127 |
| Fig. 4.6 | Calibration of Static Pressure Probe | 128 |
| Fig. 4.7 | Calibration of Thermocouple | 129 |
| Fig. 4.8 | Calibration of Pressure Transducer | 130 |
| Fig. 4.9 - Fig. 4.19 | Velocity Profiles | 131 |
| Fig. 4.20 - Fig. 4.21 | Temperature Profiles | 142 |
| Fig. 4.22 - Fig. 4.23 | Specific Humidity Profiles | 144 |
| Fig. 4.24 | Investigation of Two-Dimensional Flow - Velocity | 146 |
| Fig. 4.25 | Investigation of Two-Dimensional Flow - Temperature and Specific Humidity | 147 |
| Fig. 5.1 | Comparison of Drag Coefficients Predicted by Method I and Field Data | 148 |
| Fig. 5.2 | Comparison of Stanton Numbers Between Values Predicted by Method I and Field Data | 149 |
| Fig. 5.3 | Comparison of Drag Coefficients Predicted by Method II and Experimental Data | 150 |
| Fig. 5.4 | Comparison of Dynamic Roughness Predicted by Method II and Experimental Data | 151 |
| Fig. 5.5 | Comparison of Dimensionless Velocity Predicted by Method II and Experimental Data | 152 |
| Fig. 5.6 | Comparison of Drag Coefficients Predicted by Method II and Experimental Data | 153 |
| Fig. 5.7 | Charnock's Constant as a Function of Fetch | 154 |
| Fig. 5.8 | Dimensionless Temperature and Specific Humidity Profiles | 155 |
LIST OF FIGURES

Fig. 5.9  Temperature and Specific Humidity at $Y^{++} = 1$  156
Fig. 5.10 - Fig. 5.12  Comparison of $F$ Obtained Under laboratory conditions  157
Fig. 5.13  Comparison of Stanton Numbers and Field Data  160
Fig. 5.14  Comparison of Stanton Numbers and Field Data  161
Fig.A4.1  $x^{++}$ vs $u_{y} \over k \sqrt{g(y-y_{0})}$  162
Fig.A6.1  Wind Induced Water Surface Velocity  163
NOMENCLATURE

- rms root mean square amplitude
- $A_1$ to $A_{19}$ constants
- $b$ Charnock's constant $= z_0 g / u^2$
- $c_p$ specific heat of moist air at constant pressure
- $C_i$ drag coefficient $= 2 \left( \frac{u_i}{u} \right)^2 = \frac{2}{(u_i)^2}$
- $d_1$ to $d_8$ constants
- $D_s$ standard error or probable error in $s$
- $D_c$ diameter of pipe
- $e$ convective mass transfer rate
- $f$ friction factor for fully developed air flow in rough pipe
- $f_1$, $f_2$ functions
- $F$ dummy variables $= \frac{m - m_0}{m_0 - m_r} = \frac{t - t_0}{t_0 - t_r}$
- $\tilde{F}$ dummy variables $= \frac{m_0 - m}{m_0 - m_r} = \frac{t - t_0}{t_0 - t_r}$
- $\text{Fr}_L$ Froude number with fetch as the characteristic length $= \frac{u}{\sqrt{gL}}$
- $\text{Fr}_L y$ Froude Number with the combination of fetch and vertical distance as the characteristic length $= \frac{u}{\sqrt{gL 0.8 y 0.2}}$
- $\text{Fr}_y$ Froude Number with vertical distance as the characteristic length $= \frac{u}{\sqrt{gy}}$
NOMENCLATURE

\( g \) standard acceleration of gravity,

\( h_D \) mass transfer coefficient

\( h_H \) heat transfer coefficient

\( i \) enthalpy

\( J \) Joule's constant

\( H_{1/3} \) significant wave height, (average height or the highest 1/3 of waves)

\( k \) von Karman's constant; a value of 0.4 is used

\( k_s \) equivalent sand grain roughness

\( L \) fetch

\( L^+ \) dimensionless fetch = \( L(u_\tau) / \nu \)

\( L_e \) Lewis Number = \( h_H / (h_D c_p) \)

\( m \) specific humidity or humidity ratio

\( M \) dimensionless specific humidity = \( \frac{m - m_0}{m_0 - m_r} \)

\( MWL \) mean water level

\( n_1, \ldots, n \) exponents

\( N \) number of readings

\( Nu \) Nusselt Number = \( h_H L / \lambda \)

\( p \) pressure

\( Pr \) Prandtl Number = \( \frac{\mu c_p}{\lambda} = \frac{\nu}{\alpha} \)

\( p_v \) vapour pressure

\( q \) convective heat transfer rate
NOMENCLATURE

$r_c$ radius of pipe

$R$ depth of water

$Re_L$ Reynolds Number = $uL/v$

$Re_x$ Reynolds Number = $ux/v$

$R_{s-in}$ Gradient Richardson Number = $(\frac{\partial t}{\partial y}) / (\frac{du}{dy})$

$S_{c}$ mean value of air velocity, temperature or specific humidity

$S_{in}$ instantaneous measurement of air velocity, temperature or specific humidity

$S_h$ Schmidt Number = $\nu/\delta$

$Sh$ Sherwood Number = $h_D L/\alpha$

$\text{mass transfer Stanton Number} = \frac{e}{\rho \cdot (m_r - m_0) u_r}$

$\text{heat transfer Stanton Number} = \frac{q}{C_p \rho (t_r - t_0) u_r}$

$t$ temperature

$T$ dimensionless temperature = $\frac{t - t_0}{t_r - t_0}$

$u$ wind velocity in the flow direction

$u^*$ dimensionless velocity = $u/u_r$

$u_r$ shear velocity at water surface = $\sqrt{\frac{\tau_0}{\rho}}$

$u$ wind velocity at the reference height, 10 m.

$v$ wind velocity in the $y$ direction

$V$ water velocity in the flow direction

$V_r$ relative velocity of water with respect to moving water surface
NOMENCLATURE

$w$  wind velocity in the $z$ direction

$x$  distance in the flow direction

$x^+$  dimensionless distance $= x/z_o$

$x^+$  dimensionless distance $= x u^+ / \nu$

$y$  distance above water surface

$y^+$  dimensionless distance $= y/z_o$

$y^+$  dimensionless distance $= y u^+ / \nu$

$z$  distance along water surface perpendicular to $x$

$z_o$  dynamic roughness
NOMENCLATURE

Greek Letters

\( \alpha \)  
thermal diffusivity  
\( = \lambda / ( c \cdot c_p ) \)

\( \beta \)  
diffusion coefficient

\( \Gamma \)  
dimensionless transfer coefficient defined by Eqs (3.37) and (3.38)

\( \delta \)  
reference order of magnitude

\( \delta_1 \)  
displacement thickness as defined by Eq. (2.13)

\( \delta_2 \)  
momentum thickness as defined by Eq. (2.13)

\( \epsilon \)  
eddy diffusivity

\( \epsilon_e \)  
coefficient as defined by Eq. (3.43)

\( \epsilon_v \)  
1/Pr for heat transfer; 1/Sc for mass transfer

\( \theta \)  
time

\( \lambda \)  
thermal conductivity

\( \mu \)  
viscosity

\( \nu \)  
kinematic viscosity = \( \mu / \rho \)

\( \pi \)  
coefficient

\( \pi \)  
dimensionless ratio

\( \rho \)  
density

\( \sigma \)  
standard deviation of displacement or wave height

\( \sigma_s \)  
standard deviation or root-mean-square deviation

\( \tau \)  
shear stress

\( \phi_1, \phi_2, \phi_3 \)  
function

\( \phi \)  
dissipation function

\( \psi \)  
dimensionless constant with respect to \( y = \frac{(u_i y_i / v)^2}{u_i^2 / g y_i} \)

\( \Omega \)  
Monin Obukhov length = \( \frac{a_0 u^3 C_p \tau}{k g q} \)
NOMENCLATURE

Subscripts

\( o \)  
- at water surface

\( a \)  
- air

\( b \)  
- at solid boundary

\( cL \)  
- centerline of air-water tunnel

\( D \)  
- mass transfer

\( e \)  
- dummy variable

\( H \)  
- heat transfer

\( i \)  
- elevation above mean water level; grid index corresponding to direction \( x \)

\( j \)  
- grid index corresponding to direction \( y \)

\( l \)  
- water

\( M \)  
- momentum

\( r \)  
- relative, Appendix A6 only

\( r \)  
- at a reference height above the mean water level

\( s \)  
- smooth water surface

\( t \)  
- dummy variable

\( v \)  
- dummy variable

\( w \)  
- wavy water surface induced by wind

\( 10 \)  
- at the reference height of 10 m

\( I \)  
- Method I; smooth-wavy surface analogy approach, Sec. (3.5.1)

\( II \)  
- Method II; semi-empirical approach, Sec. (3.5.2)

Superscripts

\(-\)  
- time-averaged value

\(*\)  
- dimensionless quantity

\( . \)  
- fluctuating component
CHAPTER 1

INTRODUCTION

The mechanisms of turbulent heat, mass and momentum transfer from a water surface to atmosphere have been studied by many investigators \((1, 2, 3)^*\) since the early part of the nineteenth century. These studies involved the measurements of the vertical distribution of temperature, humidity and wind velocity over lakes, water reservoirs and oceans. \((1, 2, 3)^*\) They also involved the measurement of the daily evaporation rates from lakes and reservoirs \((2, 3)^*\). These data are of great importance for the understanding of the exchange of heat and water vapour between ocean and atmosphere. They also enable engineers to develop means to predict water losses from various water bodies.

In recent years, interest in air-water interaction has been heightened by the increasing demand for large thermal or nuclear power plants. \(4^*\) These power plants can usually be most economically operated by using once-through cooling systems to dissipate the waste heat. In such a system, the cooling water is taken from a nearby river, lake or reservoir, passed through the condenser and returned to the source. The heat carried by the cooling water will contribute to physical and biological changes in the receiving water body. These changes may be detrimental as it will raise the temperature of the water body.

It is known that the water temperature plays a triple role in affecting the rate of oxidation of pollutants, the capacity of water to hold oxygen in solution and the rate of reaeration of water. As water provides

\* Number in bracket indicates the reference in Appendix A7.
the environment for many species of organisms, the changes in
temperature, chemical content larger than those normally
experienced, can affect the normal growth and reproduction of the
organisms. Because of the potential impact on the quality of water
bodies, a great deal of research has been conducted to study the
problem of thermal discharge in waterways so that the degree and
extent of affected reaches can be determined. One of the
main goals is to predict changes in the water temperature due to
such thermal discharge. To date most of the studies use the energy
budget approach which requires the estimation of the rates of heat
and mass transfer at the air-water interface.

There are many formulae available for the evaluation of the
rates of heat and mass transfer from a water surface. Comparisons
between the estimated results based on some of these formulae and
the actual measurements indicate that the estimated evaporation
rates are subject to large errors, possibly much more than 200% in
some cases. In addition, the accuracy of the formula varies with the
surface and the ambient conditions under which it is applied. One
possible reason behind the poor estimates is that the development of
the formula in general, relies heavily on the experimental measure-
ments. Consequently, a formula which is suitable for one site may not
be valid for other locations where the surface and the ambient conditions
are different.

The purpose of the present study is to propose a method for
the estimation of the rates of heat and mass transfer from a water
surface. In contrast with the development of existing formulae, the
mathematical models for turbulent heat and mass transfer used in
the present study have been deduced from the general energy and the general diffusion equations. The results, therefore, are not limited to the given body of water on which the investigation was made. Two approaches are used in the present study.

In the first approach, the theory is developed on the basis of a large body of water. To facilitate reaching a solution, the mathematical model is simplified by restricting the analysis to a hypothetical smooth surface. The result is then extended to account for the wind-induced wave based on a functional relationship between the heat or mass transfer coefficient and the drag coefficient. The derivation of the functional relationship among the coefficients and the formula for estimating the drag coefficient over a wavy water surface are also presented.

In the second approach, the studies are carried out on a wavy water surface induced in an environmentally controlled wind tunnel. Both an analytical study and experimental investigation are made. The mathematical model differs from the previous one in that it deals with a wavy water surface directly. It also takes into account the variation of flow parameters with fetch such as the shear velocity and the dynamic roughness. Comparisons of theory and experiment are made to verify the validity of the mathematical model.

In addition, the analytical results are compared with the field measurements cited in various literature sources. Based on these comparisons, the applicability and the limitation of the theory are discussed.
CHAPTER 2

LITERATURE SURVEY

2.1 Velocity Profiles

In the theoretical study of turbulent heat and mass transfer from a large body of water, accurate determinations of the wind profile in the surface boundary layer are essential. The surface boundary layer, extending to not more than 100 m above the earth surface, is the lowest airflow region in the atmosphere. In this region the effect of the earth's rotation, Coriolis force, may be disregarded in comparison with effects which arise from the surface itself\(^7\). If the temperature gradient in the region concerned is also small, the flow structure of the wind is quite similar to that found in laboratory experiments on turbulent flow over a plane surface\(^7,8\). Accordingly, the shear stress \(\tau\) in the direction of mean wind \(x\), may be related to the time-averaged parameters of the velocity field by the equation

\[
\frac{\tau}{\rho} = v \frac{\partial u}{\partial y} + (- \bar{u} \bar{v}) \quad (2.1)
\]

The term \(\bar{u} \bar{v}\) is the time-averaged value of the product of the fluctuating velocity components in \(x\) and \(y\) directions, and may be expressed through the concept of the eddy diffusivity of momentum \(\varepsilon_M\), by the equation

\[
\bar{u} \bar{v} = - \varepsilon_M \frac{\partial u}{\partial y} \quad (2.2)
\]

Hence Eq. (2.1) can be rewritten as

\[
\frac{\tau}{\rho} = (v + \varepsilon_M) \frac{\partial u}{\partial y} \quad (2.3)
\]
\( \tau \) varies from a maximum at the surface to zero somewhere in the main stream. However, in the region not too far from the wall where most velocity change takes place \( \tau \) will not vary markedly from its wall value \( \tau_0 \) \(^{(9)}\). Then Eq. (2.3) may be approximately written as,

\[
\frac{\tau_0}{c} = (v + \varepsilon_M) \frac{\partial u}{\partial y} \tag{2.4}
\]

\[2.4.1\] Smooth Surface

To deduce a possible form for the turbulent velocity profile from Eq. (2.4), one has to know the relation between \( \varepsilon_M \) and the mean flow parameters. Many expressions for \( \varepsilon_M \) have been proposed \(^{(9)}\). The one proposed by Reichardt \(^{(10)}\) takes the form

\[
\frac{\varepsilon_M}{v} = [(1 + 0.4y^+)^{-1} + \frac{7.8}{11}] \left[ \exp \left( \frac{v^+}{11} \right) + 0.33y^+ \right] \exp (-0.33y^+) 
\]

\( \text{for } y^+ \geq 0 \)

\[
(2.5)
\]

The corresponding velocity profile is

\[
u^+ = 2.5 \ln (1 + 0.4y^+) + 7.8 \left[ 1 - \exp \left( \frac{v^+}{11} \right) - \frac{v^+}{11} \exp (-0.33y^+) \right] \text{ for } y^+ \geq 0
\]

\(2.6\)

These equations were originally developed for fully developed turbulent flow over a smooth surface. It has been shown that with proper modification on \( y^+ \), it also works for developing turbulent flow in pipes \(^{(8)}\) and in annuli \(^{(11, 12)}\). Furthermore, these equations have the virtue of providing continuity throughout the region of laminar sublayer where momentum is transferred virtually by molecular process into the region of turbulent core where turbulence dominates.
the mechanism of momentum transfer. This continuity characteristic is preferable because the limits of these regions distinguished by different momentum transfer processes are not well defined.

2.1.2 Rough Surface

For a water surface subjected to wind, the formation of waves can cause the surface to be roughened with moving protuberances. These protuberances are different in size as well as in shape and subjected to continuous and irregular changes. The heights of the waves are likely large enough to penetrate into the region of turbulent core. As a result, the flow may be turbulent starting from a reference distance below the wave crests\(^{(7)}\). Under this condition, Eq. (2.4) becomes

\[
\frac{\tau_0}{\rho} = \frac{u_1}{\bar{y}} \frac{2u}{\bar{y}}
\]

(2.7)

Applying either Prandtl's mixing length hypothesis or von Karman's similarity hypothesis\(^{(9)}\), the following expression for the velocity distribution of wind over a wavy water surface can be deduced from Eq. (2.7)

\[
\frac{u}{u_1} = \frac{1}{k} \ln \frac{z}{z_0}
\]

(2.8)

where \(z_0\) is known as the roughness length or the dynamic roughness\(^{(1,7,8)}\). The validity of Eq. (2.8) has been investigated by numerous researchers by comparing Eq. (2.8) with the measured wind profiles obtained both in field\(^{(1,3,13,14,15)}\) and in laboratory\(^{(16,17,18,19)}\). A large number of the measured wind profiles over the ocean obtained before 1962 was collected and reviewed by Roll\(^{(1)}\). He reported that the vertical profile of mean wind speed
over the sea surface is close to logarithmic if thermal stratification is absent. Similar conclusions were also reported by the researchers in the USSR\textsuperscript{(15)} based upon a large quantity of profiles measured in the lower atmosphere over large reservoirs. Furthermore, extensive measurements of wind profiles over the ocean were made by Ruggles\textsuperscript{(13)} in recent years. In his study, a total of 299 profiles were collected over a period of two years. Each profile is based on wind speeds measured simultaneously from 4 - 8 anemometer stations. An examination of these profiles indicate that only 13% of the 299 profiles do not follow the logarithmic relationship. Of that 13% as reported by Ruggles\textsuperscript{(13)} more than half of the cases were discarded on valid physical grounds. Therefore better than 90% of the 299 measured profiles were logarithmic. A similar study was also made by Marciano and Harbeck\textsuperscript{(3)}. Based upon the analysis of 23000 individual wind profiles measured over Lake Hetmer, they found that wind speed varied logarithmically with height between 2 and 8 meters, within the limits ascribable to instrumental error, regardless of stability variations.

The logarithmic velocity profile as defined by Eq. (2.8) has also been found to be valid over the wavy water surfaces generated in laboratories by Wu\textsuperscript{(16)}, Sibul\textsuperscript{(17)}, Shemdin\textsuperscript{(18)} and Chambers et al.\textsuperscript{(19)}. Wu's\textsuperscript{(16)} measurements were conducted over a water surface with wind-generated waves. His results clearly show that the mean wind velocity varies linearly with the logarithmic height. Sibul\textsuperscript{(17)} measured the wind profiles over wavy water with different depths. Again the logarithmic profiles were observed.

Shemdin\textsuperscript{(18)} has studied the wind velocity over water surfaces with wind-generated waves and with combination of wind and mechanically generated waves. His study indicates that the presence
of the wave will affect the velocity measurements in the following two ways. First, it causes the velocity measuring instrument to shift streamlines continuously if the instrument is stationary with respect to the ground. Second, it creates the wave-induced perturbation in the air velocity. The influence of waves is seen to become small at a distance approximately three wave amplitudes above the mean water surface. Chambers et al. have measured the mean wind profile under diabatic conditions with an air-water temperature difference ranging from -4°C to 16°C. The results show that the mean wind profile follows the logarithmic law much closer at higher wind speeds than at lower wind speeds. Based on their results, however, even at low wind speed, the deviation from the logarithmic wind profile is no more than 3% of the corresponding value specified by the logarithmic law.

2.2 Dynamic Roughness

The logarithmic velocity profile consists of two parameters: a shear velocity \( u_\tau \) and a dynamic roughness \( z_0 \). Although the numerical value of \( z_0 \) can be easily evaluated, the physical interpretation of \( z_0 \) is not yet clear. Based upon the mean wind profile measurements over the sea surface, Roll \(^{(1)}\) suggests that \( z_0 \) can be expressed in terms of shear velocity as

\[
z_0 = \frac{v}{(2.1 \ u_\tau)} \tag{2.9}
\]

This equation implies that \( z_0 \) decreases as \( u_\tau \) increases. In contradiction to Eq. (2.9), Charnock \(^{(20)}\) proposes that \( z_0 \) is directly proportional to the square of \( u_\tau \), thus

\[
z_0 = b \ \frac{u_\tau^2}{g} \tag{2.10}
\]

Here \( b \) is a constant factor. Charnock's \(^{(20)}\) expression for \( z_0 \)
appears to be more realistic than that of Roll. The increase in \( z_0 \) with \( u_\tau \) can be observed from the logarithmic velocity profile. As indicated by Eq. (2.8), for a constant \( u_\tau \), the shear velocity is directly proportional to the dynamic roughness.

Eq. (2.10) has been confirmed by Wu who suggests that the values of 0.0112 and 0.0156 may be assigned to the constant in Eq. (2.10) under laboratory and atmospheric conditions respectively. On the other hand, Kitaigorodskii et. al believe that such a constant does not exist. Since the conclusions of the investigators are based on the measured wind profiles obtained by many researchers, the disagreement can hardly be attributed to the imperfection of these measured wind profiles. Thus the functional relationship between \( z_0 \) and \( u_\tau \) has not yet been satisfactorily specified but needs further study. This problem will therefore be discussed in Chapter 5.

2.3 Drag Coefficient

The second parameter in the logarithmic velocity profile is the shear velocity \( u_\tau \). It is desirable to discuss this parameter in terms of the friction factor or the drag coefficient \( c_1 \) defined as

\[
    c_1 = \frac{\tau_0}{\rho u_1^2} = 2 \left( \frac{u_\tau}{u_1} \right)^2
\]

(2.11)

where the subscript \( i \) denotes a certain reference distance above the mean water level.

There are direct and indirect methods developed for the estimation of the drag coefficient at an air-water interface. These methods are known as the eddy correlation method, the profile
method, the surface tilt method and the integral method.

The eddy correlation method involves the measurement of \( \overline{u'v'} \). Here \( \overline{u'v'} \) is the time-averaged product of the fluctuating velocity components in \( x \) and \( y \) directions. This quantity was measured by Reichardt\(^{23}\) over a smooth surface in a wind tunnel. The smooth surface is the bottom of the rectangular test section 1 m wide and 0.24 m high. The results are reproduced in Fig. (2.1). For comparison, the linear distribution of \( \overline{\gamma_0} \) which was obtained from the measured pressure distribution is also shown in this figure. It is clearly seen from the diagram that the \( \overline{-u'v'} \) and the \( \overline{\gamma_0} \) curves nearly coincide over the major portion of the height of the test section. Therefore, if the distribution of \( \overline{-u'v'} \) can be measured over a wavy water surface, the shear stress distribution can also be obtained by fitting the data measured in the region away from the MWL with a straight line. Over a solid boundary the measurement of \( \overline{-u'v'} \) can be easily made using a hot wire anemometer with a hot-wire X-probe. Over a wavy water surface, however, the spray of water may markedly increase the difficulty in such a measurement.

The profile method is based on the logarithmic velocity profile which can be rewritten as

\[
\overline{u} = \frac{u_r}{k} \ln \overline{y} + f_1(\overline{z_0})
\]

(2.12)

where \( f_1(\overline{z_0}) = \frac{u_r}{k} \ln \overline{z_0} \).

The two-variables \( u_r \) and \( \overline{z_0} \) may be evaluated by fitting the measured velocities with the curve defined by Eq. (2.12).

There are fundamental difficulties inherent in this method.
One of them is the validity of Eq. (2.12) in the region close to the wall. The other difficulty arises from the estimation of two unknowns from a single equation i.e. Eq. (2.12). Regardless of these difficulties, however, this method is still the most popular method and is widely used.

The accuracy of this method has been checked against the eddy correlation method over oceans and lakes by Miyake et al.\textsuperscript{(24)} and Elder et al.\textsuperscript{(25)} Miyake et al. measured the quantity of $-u'v'$ as well as the mean wind velocity profiles over the ocean. They reported that the averaged drag coefficients based on the profile and the eddy correlation methods were 0.00226 and 0.00218 respectively. The difference in the results is less than 4% and is well within the experimental errors.

Elder et al.\textsuperscript{(25)} conducted wind profile and $-u'v'$ measurements over Lake Michigan. The results also indicated that the two methods were compatible.

Under steady state conditions, Keulegan\textsuperscript{(26)} shows that it is possible to express the turbulent shear stress in terms of the surface slope of a given body of water. This is known as the surface tilt method. The accuracy of the method primarily depends on the accuracy achieved in the measurement of the surface slope and the depth of water. Consequently, this method may offer severe difficulties when it is applied in laboratory especially when waves are present.

Drag coefficient can also be evaluated based on the momentum integral equation\textsuperscript{(27)}, the calculation of the drag coefficient depends mainly upon the velocity profile. The measurement of the velocity profile as discussed in Section 2.1.2, is likely to be inaccurate in the region up to approximately three wave amplitudes above the MWL.
This error, in turn, may be multiplied in the calculation of the drag coefficient from the above equations.

The drag coefficients calculated using this method were compared with that obtained by the profile method by Chambers et al. (19). They found that the integral method, in general, underestimated the drag coefficient by about 30% in comparison with the profile method.

A comprehensive compilation of the measured drag coefficients made up to the year of 1962 was tabulated by Roll (1). This table, together with some of the results measured in recent years, is reproduced in Table (2.1). (16, 17, 21, 24-26, 28-74) A cursory examination of the table reveals that not only is there an exceptionally wide scattering in the results but their conclusions with regard to the drag coefficient, wind speed and fetch are also different.

2.4 Thermal Stratification

For conditions of thermal stratification Monin and Obukhov (6) have suggested that the dimensionless velocity and temperature profiles should be universal functions of the dimensionless height $y/\Omega$. In particular,

$$\frac{\kappa v}{u^*} \frac{\partial u}{\partial y} = \phi \left( \frac{v}{\Omega} \right); \quad \frac{k C_p}{c_o u^*} \frac{\partial T}{\partial y} = Pr \theta \left( \frac{v}{\Omega} \right)$$

(2.13)

where $\Omega$ is a length incorporating the effect of buoyancy and is defined by

$$\Omega = - \frac{c_o u^*}{\kappa q} C_p t$$

Since the Richardson Number (8) has long been used to characterize the thermal stratification, it can be expressed in terms of $\Omega$ as

$$R_i = \frac{\frac{\rho}{\kappa}}{\left( \frac{\partial u}{\partial y} \right)^2} = Pr \left( \frac{\frac{v}{\Omega}}{\phi \left( \frac{v}{\Omega} \right)} \right)$$

(2.14)

As neutral stratification is approached, i.e. for $q \to 0$ when $y/\Omega \to 0$, the velocity profile should be the same as that defined by Eq. (2.8).
It follows that $\phi(0) = 1$. Therefore,

$$Ri = \frac{\gamma}{\Omega} \quad (2.15)$$

The above equation suggests that the effect of thermal stratification decreases as the water surface is approached. In other words, there is a region immediately above the water surface where the influence of thermal stratification can be neglected. The thickness of the region varies from several meters in the cases of very strong instability or stability, to very large values under neutral stability. Based on the velocity profile measurement conducted by Marciano et. al., this thickness should be about 8 meters.

2.5 Wind-Induced Waves

Wind blowing over a water surface generates waves. The wave height increases both with the wind velocity and with the fetch. Here the fetch represents the distance from the leading edge of the water to a particular point downstream. This distance should modify by a correction factor for a water body having irregular boundary which affects the normal growth of the wave. An empirical relationship among the significant wave height which is defined as the average height of the highest one-third of the waves, the wind speed and the fetch has been formulated by Wiegel. When these three parameters are expressed in terms of two Froude Numbers, Wiegel shows that the logarithms of the two Froude Numbers increase linearly with each other.

Observations on the wind-induced wave were made by Hidy and Plate in an air-water tunnel. The tunnel was 0.61 m wide by 0.76 m high with a plexiglass test section of about 12 m long. They reported that the water surface was smooth till the wind speed at about the center of the tunnel $u_{cL}$ reached 3 m/sec. When $u_{cL}$ exceeded 3 m/sec, ripple with a wavelength of 1 to 3 cm developed on the water surface. The higher the wind speed was, the closer to the leading edge of the reservoir was the wave developed.
2.6 Heat and Mass Transfer

The problem of heat and mass transfer from a water surface to the atmosphere has been much studied both in the laboratory \(^{(77, 78)}\) and in the field \(^{(2, 3, 79, 80, 81)}\). Emphasis in these studies, in general, has been on the development of the formulae for evaluating the rate of mass transfer rather than on the understanding of the mechanism of the transfer phenomena. As a result, a number of empirical formulae for estimating the rate of evaporation have been deduced. These formulae usually take the form:

\[
e = r_1(u) \cdot r_2(A) \cdot (P_{vo} - P_v)
\]

(2.16)

where \(e\) is the evaporation rate; \(r_1(u)\), \(r_2(A)\) are functions of wind speed and size of water body; \(P_{vo}\) is the saturation vapour pressure corresponding to water temperature and \(P_v\) is the vapour pressure of air at the same height where wind speed is measured.

These empirical formulae, in general, are effective in estimating the rate of evaporation from a body of water similar to the one on which data have been obtained. However, when they are applied to other conditions, serious errors can be expected.

In order to develop a formula which is not limited to a certain condition, many researchers \(^{(82, 83, 84, 85)}\) have attempted to approach the problem in a general way using the mass flux equation or the diffusion equation as their bases. These approaches are known as the aerodynamic approach and the diffusion method.

The aerodynamic approach involves the conversion of the mass flux equation
\[ e = -80 \left( \frac{\partial m}{\partial y} \right) \]  

(2.17)

into a practical formula for estimating the rate of evaporation.

One of the best known and most widely tested evaporation formula

is Thornthwaite and Holzman's formula.\(^{82}\)

\[ e = \frac{c k^2 (u_2 - u_1) (m_2 - m_1)}{\ln \left( \frac{y_2}{y_1} \right)} \]  

(2.18)

where \( u_1, u_2 \) and \( m_1, m_2 \) are the mean wind velocities and the

specific humidities respectively at \( y_1 \) and \( y_2 \); \( k \) is von Karman's

constant. This equation has been tested against the measured

evaporation rate by Marciano and Harbeck\(^{3}\). They have concluded

that Eq. (2.16) would give satisfactory results with proper instru-

mentation which is extremely difficulty to set up in field.

Formulae which are derived using the aerodynamic

approach do not account for the influence of the developing specific

humidity profile. If these formulae are used for the estimation of

the evaporation rate in the region near the leading edge of a given

body of water, considerable errors can be expected.

The effects of the developing specific humidity profile

on the rate of evaporation were considered by Sutton.\(^{84}\) Using

a power law wind profile and a steady-state two-dimensional model

diffusion as the basis of his calculation, Sutton\(^{84}\) deduced an

equation for calculating the rate of evaporation over a smooth

surface. Later, Calder\(^{85}\) used a similar method and developed

an evaporation formula for a rough surface. In comparison with the

actual measurements obtained from Lake Mead\(^{79}\), Calder's\(^{85}\)

equation is found to overestimate the rate of evaporation considerably.

An extensive analysis was made by Kitaigorodskii and
Volkov\(^{(86)}\) on the measured heat and humidity profiles over the water surfaces. These profiles were collected from various literature sources. In the analysis the heat and the mass transfer Stanton Numbers, \(St_H\) and \(St_D\), were evaluated from the profiles. These two dimensionless numbers are defined as follows:

\[
St_H = \frac{Nu}{Re Pr} = \frac{q_c}{C_p \rho (t_r - t_o) u_r}
\]

\[
St_D = \frac{Sh}{Re Sc} = \frac{e}{\rho (m_r - m_o) u_r}
\]

(2.19) 

(2.20)

An attempt was made to establish a relationship between the two Stanton Numbers and the mean fluid parameters other than the rates of heat and mass transfer. It was found that \(St_H\) or \(St_D\) would appear to increase linearly with the logarithm of \((z_0 u_r / \nu)\). This conclusion was also reported by Mangarella et al.\(^{(77,78)}\) who carried out similar analysis based on their measurements in an air-water tunnel.

Both Kitaigorodskii\(^{(86)}\) and Mangarella\(^{(77,78)}\) assume that the two dimensionless numbers, \(St_H\) and \(St_D\), are identical. This assumption implies that the Lewis number is unity. This conclusion appears to be valid based on the experimental evidence obtained both in laboratory\(^{(87)}\) and in field\(^{(86)}\).

2.7 Correlation of Laboratory Data for Field Application

Studies on the problems of heat and mass transfer over a water surface are often made under laboratory conditions. In order to generalize the result under atmospheric conditions, the model used in laboratory should be in geometrical, kinematic and
dynamical similarities with the prototype. The geometrical and 
the kinematic similarities are not too difficult to achieve, but the 
dynamical similarity, based on Froude Number and Reynolds 
Number may be impossible to obtain. Therefore one has to seek 
a certain empirical correlation to generalize the laboratory data 
for field application.

It has been shown by Wu (88) that for drag coefficients 
the Froude Number defined as

\[ \text{Fr} = \frac{u}{\sqrt{g y_r}} \]

can be used to correlate the laboratory and the field data. To use 
this correlation, he further suggests that the numerical values of 
\[ y_r \] for laboratory and field should be 10 cm and 10 m respectively.
The numerical values selected for \[ y_r \] do not appear to have any 
physical meaning. The validity of the correlation will be discussed 
in Chapter 5.

For heat and mass transfer an attempt (75, 76) was made 
to correlate the Stanton Numbers obtained under laboratory as well 
as atmospheric conditions to the dimensionless quantity \[ (z_0 u_\tau / v) \].
To check the validity of the correlation, the Stanton Numbers obtained 
in laboratory were compared with those obtained in oceans on the 
basis of equal \[ (z_0 u_\tau / v) \] by Mangarell et. al (77, 78). It was found 
that the agreement between the laboratory and the field results 
was quite good as long as \[ z_0 u_\tau / v \] was greater than 1. For \[ z_0 u_\tau / v \] less than unity, however, the field data might be as 
much as 10 times lower than the laboratory Stanton number.
2.8 Summary

Based upon the above review the following conclusions may be made:

1) For a smooth water surface, the vertical distribution of wind speed under fully developed flow conditions, can be satisfactorily expressed by Reichardt's (10) velocity profile defined by Eq. (2.6);

2) For a wavy water surface, under neutral stable condition, the vertical distribution of wind speed can be satisfactorily expressed by the logarithmic velocity profile defined by Eq. (2.8);

3) For a wavy water surface, the drag coefficients may be satisfactorily estimated by either the profile or the eddy correlation method.

4) Further studies are required to establish a relationship among drag coefficient, dynamic roughness and other fluid parameters such as mean wind speed, wave amplitude and fetch.

5) Further studies are required to develop formulae for the estimation of the rates of heat and mass transfer. The derivation of these formulae should be based on the theoretical considerations rather than the experimental measurements.

6) Further studies are required to establish a correlation for the generalization of the laboratory data for application under atmospheric conditions.

Thus, in this study, efforts will be made to seek solutions to the problems outlined in the last three items.
CHAPTER 3

ANALYSIS

3.1 Basic Equations

When wind blows over a water surface, heat and mass are transported to and from the surface by turbulent convection. The mechanisms of these transfer processes are governed by the general energy and the general diffusion equations. These equations can be derived from the corresponding equations for laminar flow using the procedure initiated by Reynolds. This procedure involves replacing all the individual parameters such as temperature by the sum of a time-averaged value and its fluctuating component and time averaging each term in the resultant equations. In cartesian coordinates with x and y in the directions of down wind and perpendicular to the surface these equations with no internal heat generation, no Dufour nor Soret effects and no chemical reactions can be written as follows:

Energy equation

\[ \rho \frac{Di}{D\theta} = \left[ \frac{\partial}{\partial x} (\lambda \frac{\partial \theta}{\partial x}) + \frac{\partial}{\partial y} (\lambda \frac{\partial \theta}{\partial y}) + \frac{\partial}{\partial z} (\lambda \frac{\partial \theta}{\partial z}) \right] \cdot \frac{\theta}{g J} \cdot \frac{1}{J} \frac{DP}{D\theta} - \delta C_p \left[ \frac{\partial u^{i+1}}{\partial x} + \frac{\partial v^{i+1}}{\partial y} + \frac{\partial w^{i+1}}{\partial z} \right] \] \hspace{1cm} (3.1)

Diffusion equation

\[ \frac{Dm}{D\theta} = \left[ \frac{\partial}{\partial x} (\beta \frac{\partial m}{\partial x}) + \frac{\partial}{\partial y} (\beta \frac{\partial m}{\partial y}) + \frac{\partial}{\partial z} (\beta \frac{\partial m}{\partial z}) \right] = \left[ \frac{\partial u^{i+1} m^{i+1}}{\partial x} + \frac{\partial v^{i+1} m^{i+1}}{\partial y} + \frac{\partial w^{i+1} m^{i+1}}{\partial z} \right] \] \hspace{1cm} (3.2)
which are the energy and diffusion equations written in terms of time-averaged values, but with the addition of the extra terms in the bracket on the right-hand side. These terms, involving the fluctuating components indicated by the "primes", represent the transfer of heat and mass by turbulent convection. In these two equations, the notation $D/D_\delta$ is the substantial derivative, $\zeta$ is the dissipation function and $P$ is the time-averaged pressure.

\[(90)\]

It can be shown that air can be treated as incompressible if the Mach Number is much less than 0.5. Under this condition the term $\frac{Di}{D_\delta}$ can be replaced by

\[
\frac{Di}{D_\delta} = C_p \frac{Dt}{D_\delta} + \frac{1}{c} \frac{DP}{D_\delta}
\]

\[(3.3)\]

It also can be shown that the dissipation function depends not only on the velocity but also on the Prandtl number. For air, where the Prandtl number is about 0.7, the velocity must approach to the speed of sound, Mach Number $= 1$, before $\zeta$ is significant. Since the Mach Number corresponding to the range of wind speeds under consideration is less than 0.05, $\zeta$ can be neglected. Furthermore, if steady-state condition are also assumed, Eq. (3.1) and (3.2) can now be written as:

Energy equation

\[
\sigma C_p \left[ u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} + w \frac{\partial T}{\partial z} \right] = \lambda \left[ \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right] - \sigma C_p \left[ \frac{\partial u T}{\partial x} + \frac{\partial v T}{\partial y} + \frac{\partial w T}{\partial z} \right]
\]

\[(3.4)\]

Diffusion equation

\[
u \frac{\partial m}{\partial x} + v \frac{\partial m}{\partial y} + w \frac{\partial m}{\partial z} = \sigma \left[ \frac{\partial^2 m}{\partial x^2} + \frac{\partial^2 m}{\partial y^2} + \frac{\partial^2 m}{\partial z^2} \right] - \sigma \left[ \frac{\partial u m}{\partial x} + \frac{\partial v m}{\partial y} + \frac{\partial w m}{\partial z} \right]
\]

\[(3.5)\]
The terms in the first bracket on the right hand side represent the heat fluxes or mass fluxes caused by conduction, whereas the second bracket represents those caused by turbulent fluctuations. The corresponding terms in these two brackets can be combined by introducing the concept of eddy diffusivities for heat or mass. These eddy diffusivities are defined as:

Eddy diffusivities for heat, \( \varepsilon_{\text{Hx}}, \varepsilon_{\text{Hy}}, \varepsilon_{\text{Hz}} \)

\[
\begin{align*}
\overline{u' t'} &= -\varepsilon_{\text{Hx}} \frac{\partial}{\partial x} \\
\overline{v' t'} &= -\varepsilon_{\text{Hy}} \frac{\partial}{\partial y} \\
\overline{w' t'} &= -\varepsilon_{\text{Hz}} \frac{\partial}{\partial z}
\end{align*}
\] (3.6)

Eddy diffusivities for mass, \( \varepsilon_{\text{Dx}}, \varepsilon_{\text{Dy}}, \varepsilon_{\text{Dz}} \)

\[
\begin{align*}
\overline{u' m'} &= -\varepsilon_{\text{Dx}} \frac{\partial}{\partial x} \\
\overline{v' m'} &= -\varepsilon_{\text{Dy}} \frac{\partial}{\partial y} \\
\overline{w' m'} &= -\varepsilon_{\text{Dz}} \frac{\partial}{\partial z}
\end{align*}
\] (3.7)

Substituting Eq. (3.6) into Eq. (3.4); Eq. (3.7) into Eq. (3.5), we have the following equations:

Energy equation

\[
\begin{align*}
\frac{\partial}{\partial x} \theta + v \frac{\partial}{\partial y} \theta + w \frac{\partial}{\partial z} \theta &= \frac{\partial}{\partial x} \left( \frac{\lambda}{\rho C_p} + \varepsilon_{\text{Hx}} \right) \frac{\partial}{\partial x} \theta + \frac{\partial}{\partial y} \left( \frac{\lambda}{\rho C_p} + \varepsilon_{\text{Hy}} \right) \frac{\partial}{\partial y} \theta + \\
&+ \frac{\partial}{\partial z} \left( \frac{\lambda}{\rho C_p} + \varepsilon_{\text{Hz}} \right) \frac{\partial}{\partial z} \theta
\end{align*}
\] (3.8)
Diffusion equation

\[
\frac{\partial m}{\partial x} + u \frac{\partial m}{\partial y} + w \frac{\partial m}{\partial z} = \frac{\partial}{\partial x} \left( \beta + \varepsilon_{Dx} \right) \frac{\partial m}{\partial x} + \frac{\partial}{\partial y} \left( \beta + \varepsilon_{Dy} \right) \frac{\partial m}{\partial y} + \frac{\partial}{\partial z} \left( \beta + \varepsilon_{Dz} \right) \frac{\partial m}{\partial z} 
\]

(3.9)

The velocity components are related by the equation of continuity

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0
\]

(3.10)

3.2 Physical Model and Assumptions

The physical model consists of a large water surface which interacts with the turbulent wind. In Cartesian coordinates, the water surface is defined by the x and z axes with the x axis along the direction of the wind. The y axis is the vertical distance normal to the water surface.

The governing equations for predicting the temperature and the specific humidity profiles have been derived, based on the following assumptions:

1. Incompressible, steady, constant-property turbulent boundary layer flow is assumed.
2. The hydrodynamic-boundary layer is leading the thermal and specific humidity boundary layer.
3. The governing equations can be simplified by means of the boundary layer concept for large Reynolds number.
3.3 Boundary Layer Approximations

Eqs. (3.8) and (3.9) are so involved that a simplification is necessary in order to obtain solutions. Such simplification can be made by retaining only the dominant terms which are identified by estimating the order of magnitude of various terms in the equations.

If such a simplification is based on the concept of the thin boundary layer, it is known as the boundary layer approximations. The thickness of boundary layer is defined as the vertical distance where the velocity transition from zero at the mean water level to a certain reference value \( u_0 \) takes place. This thickness increases with \( x \) till a reference distance \( x_r \) is reached where the thickness of boundary layer becomes constant. This reference distance \( x_r \) is assumed to be very large in comparison with the boundary layer thickness.

The solutions obtained from the equations which are simplified by using the boundary layer approximations, are asymptotic and apply to very large Reynolds Numbers\(^{(91)}\). Eqs. (3.8), (3.9) and (3.10) are simplified by using the boundary layer approximations in Appendix (A.1). The resultant equations which contain only the dominant terms are as follows:

Energy equation

\[
u \frac{\partial u}{\partial x} + v \frac{\partial u}{\partial y} = \frac{\partial}{\partial y} \left[ \left( \alpha + \xi \right) \frac{\partial u}{\partial y} \right]
\]  
(3.11)

where \( \alpha = x/\rho C_p \)

Diffusion equation

\[
u \frac{\partial m}{\partial x} + v \frac{\partial m}{\partial y} = \frac{\partial}{\partial y} \left[ \left( \beta + \xi \right) \frac{\partial m}{\partial y} \right]
\]  
(3.12)
Continuity equation

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \] (3.13)

3.4 Equations of Heat Flux and Mass Flux

Eqs. (3.11), (3.12) and (3.13) are the energy, the diffusion and the continuity equations of the boundary layer. These equations permit the detailed distribution of temperature and specific humidity within the boundary layers to be predicted analytically or numerically. Based on temperature and specific humidity distribution, the local heat or mass flux can be evaluated by the equations:

Heat Flux

\[ q = -\rho C_p \left( a + \varepsilon_H \right) \frac{dt}{dy} \] (3.14)

Mass Flux

\[ e = -\rho \left( b + \varepsilon_D \right) \frac{dm}{dy} \] (3.15)

3.5 Methods of Solution

Eqs. (3.11), (3.12) and (3.13) will be solved using two different approaches. The first approach which does not consider the effects of the wind-generated wave on the motion of air may be called the smooth-wavy surface analogy approach \(^9\). It calculates the transfer coefficients based on a hypothetical smooth surface model from the governing equations. The result will then extend to the wavy surface using a correlation function which depends upon the drag coefficients for both the smooth and the wavy surfaces. This approach has been used in the studies of the heat transfer characteristics of tubes and other passages with rough surfaces.
to avoid considering the extremely complicated flow conditions caused by the surface roughness.\(^{(93,94,95)}\) The investigations were concerned with sand-roughned walls\(^{(94)}\) as well as with walls having discrete roughness elements like grooves or ridges.\(^{(95)}\) A similar concept is also used for the pressure drop prediction in two-phase fluid flow.\(^{(96)}\)

The second approach is referred to as the semi-empirical approach. It solves both energy and diffusion equations directly for air over a wavy water surface. The actual flow conditions in terms of the dynamic roughness are introduced into the governing equations based on the results deduced from the measurements obtained in a laboratory air-water tunnel, or in fields.

3.5.1 Method I - Smooth - Wavy Surface Analogy Approach

Consider a stream of turbulent air moving over a hypothetically smooth water surface. Since the hydrodynamic boundary layer is likely to be fully developed before it reaches the edge of the water surface, Eqs. (3.11) and (3.12) become

\[
\frac{\partial u}{\partial x} = \frac{\partial}{\partial y} \left[ (\alpha + \varepsilon_H) \frac{\partial u}{\partial y} \right] \tag{3.16}
\]

\[
\frac{\partial m}{\partial x} = \frac{\partial}{\partial y} \left[ (\varepsilon + \varepsilon_D) \frac{\partial m}{\partial y} \right] \tag{3.17}
\]

where \(\varepsilon_H\) and \(\varepsilon_D\) are eddy diffusivities for heat and mass transfer, respectively. Now Eqs. (3.16) and (3.17) can be written in dimensionless form as

\[
\frac{u^+}{\alpha} \frac{\partial F}{\partial x^+} = \frac{\partial}{\partial y^+} \left( \varepsilon_t \right) \frac{\partial F}{\partial y^+} \tag{3.18}
\]
where \( u^+ = u/u_\tau \); \( y^+ = yu_\tau / v \); \( x^+ = xu_\tau / v \);

\[
T = (t_0 - t) / (t_0 - t_\tau) ; \quad M = (m_0 - m) / (m_0 - m_\tau)
\]

and also

(a) \( F = T \) and

\[
\varepsilon_t = \frac{1}{Pr} + \frac{E_H}{v}
\]

for the energy equation;

(b) \( F = M \) and

\[
\varepsilon_t = \frac{1}{Sc} + \frac{E_D}{v}
\]

for the mass diffusion equation.

Both \( Pr \) and \( Sc \) are assumed to be constant since \( Pr \) varies from 0.72 at 32 F to 0.71 at 300F and \( Le \) is close to unity.

3.5.1.1 Solution of Energy and Diffusion Equations

The appropriate boundary conditions to be considered here are:

\[
y^+ = 0 ; \quad u^+ = 0 ; \quad T = M = F = 0
\]

\[
y^+ \to \infty ; \quad u^+ \to u_\tau^+ ; \quad T = M = F = 1
\]

In order to solve Eq. (3.18) a rather standard explicit finite differences technique was used. The method used is that of DuFort-Frankel type which is known to be one of the most direct and stable methods.

The flow field was divided into a nodal network of small increments \( \Delta x^+ \) and \( \Delta y^+ \) as shown in Fig. (3.1). The corresponding DuFort-Frenkel type finite difference approximations for Eq. (3.18) are:
\[ F_{i+1,j} = \frac{1}{A_3} (A_1 F_{i,j+1} - A_4 F_{i-1,j} + A_2 F_{i,j-1}) \]  

where

\[ A_1 = \frac{(\varepsilon_{t,j+1} + \varepsilon_{t,j})}{2(\Delta y^+)^2} \]

\[ A_2 = \frac{(\varepsilon_{t,j} + \varepsilon_{t,j-1})}{2(\Delta y^+)^2} \]

\[ A_3 = \frac{u'^+}{2(\Delta x^+)} + \frac{A_1}{2} + \frac{A_2}{2} \]

\[ A_4 = \frac{u'^+}{2(\Delta x^+)} + \frac{A_1}{2} + \frac{A_2}{2} \]

The necessary starting data for Eq. (3.21) are provided by the ordinary explicit finite difference equation as follows:

\[ F_{i+1,j} = \frac{\Delta x^+}{u'^+} A_1 F_{i,j+1} + \left[ 1 - \frac{\Delta x^+}{u'^+} (A_1 + A_2) \right] F_{i,j} \]

\[ + \frac{\Delta x^+}{u'^+} A_2 F_{i,j-1} \]  

(3.22)

Before the numerical calculation can be performed, both the velocity profile and the eddy diffusivity for momentum, heat and mass have to be specified. For the eddy diffusivity for momentum and hence, the velocity profile, the choice is straightforward. The expressions derived by Reichardt (23) among many others as shown in Table (3.1) (23, 98, 104) can be used with confidence since they have been verified by experimental data for various flow passages (8, 11, 12). It should be pointed out that, a single equation
for $\frac{\varepsilon_M}{\nu}$ and the corresponding velocity profile have also been proposed by Spalding. These equations also have the virtue of providing continuity throughout the entire flow regime. Since both velocity profiles fit the available experimental velocity data equally well, Reichardt's expressions are preferable for the reason of algebraic simplicity. These expressions for the eddy diffusivity of momentum and its corresponding velocity profile are:

\[
\frac{\varepsilon_M}{\nu} = \left\{ (1 + 0.4y^+)^{-1} + \left( \frac{7.8}{11} \right) \left[ \exp \left( \frac{-y^+}{11} \right) + 0.33y^+ - 1 \right] \exp \left( -0.33y^+ \right) \right\}^{-1} 
\]

\[
u^+ = 2.5 \nu_{in} (1 + 0.4y^+) + 7.8 \left[ 1 - \exp \left( -\frac{y^+}{11} \right) - \left( \frac{y^+}{11} \right) \exp \left( -0.33y^+ \right) \right] \quad \text{for } y^+ \geq 0
\] (2.5)

(2.6)

The three eddy diffusivities are related to each other in terms of turbulent Prandtl number, $Pr_t$, turbulent Schmidt number, $Sc_t$, and the turbulent Lewis number $Le_t$, as follows:

\[
Pr_t = \frac{\varepsilon_M}{\varepsilon_H}
\]

\[
Sc_t = \frac{\varepsilon_M}{\varepsilon_D}
\]

\[
Le_t = \frac{\varepsilon_D}{\varepsilon_H}
\] (3.23)

Since both theoretical considerations and experimental evidence appear to indicate that $Le_t$ has a value of unity, approximately Eq. (3.23) can be rewritten as

\[
\frac{\varepsilon_H}{\varepsilon_M} \approx \frac{\varepsilon_D}{\varepsilon_M} \approx \frac{1}{Pr_t}
\] (3.24)
Experimental results have shown that $Pr_t$ is a function of the distance from the wall (106). However, for heat transfer in flow of air in tube or over a solid wall, constant values of 1 (87), 0.9 (107), and 0.8 (106) for $Pr_t$ have been used and satisfactory agreements with measured results have been reported. Since it appears that no satisfactory agreement between the results obtained by various investigators can be reached, in this study both the constant and the variable $Pr_t$ will be used. The corresponding eddy diffusivities of heat and mass are

$$\frac{\varepsilon_H}{\varepsilon_M} = \frac{C_D}{C_M} = 1$$

$$\frac{\varepsilon_H}{\nu} = \frac{C_D}{\nu} = \frac{C_M}{\nu} \left( 3.157 + 0.01802y^+ - 0.000056y^{+2} \right)$$

$$- 0.73462 \ln y^+ \quad \text{for } y^+ \leq 100$$

$$= \frac{C_M}{\nu} \quad \text{for } y^+ > 100$$

The correction factor in (ii) was obtained by fitting the experimental data of Blom (106) for $y^+$ up to 100 by the least squares approximation. In the region where $y^+$ is greater than 100 since Blom's data shows that $\varepsilon_H/\varepsilon_M$ is about 1 till $y^+$ reaches about 500, and then it scatters widely with $y^+$, $\varepsilon_H/\varepsilon_M$ is taken to be unity throughout this region.

3.5.1.2 Numerical Procedure

The appropriate finite difference approximations for the velocity profile and the eddy diffusivity can then be written as follows:

...
\[ u^+ = 2.5 \ln(1 + 0.4 y_j^+) + 7.8 \left[ 1 - \exp \left( - \frac{y_j^+}{11} \right) \right] \]

\[ - \left( \frac{y_j^+}{11} \right) \exp \left( - 0.33 y_j^+ \right) \]

\[ \tau_j^* = \text{Pr}^{-1} \left[ (1 + 0.4 y_j^+) \left( \frac{7.8}{11} \right) \right] \exp \left( - \frac{y_j^+}{11} \right) \]

\[ + (0.33 y_j^+ - 1) \exp \left( - 0.33 y_j^+ \right) \] \quad (3.26)

\[ \tau_j^* = \text{Pr}^{-1} [2.5 \ln(1 + 0.4 y_j^+) + 7.8 \left[ 1 - \exp \left( - \frac{y_j^+}{11} \right) \right] - (\frac{y_j^+}{11}) \exp \left( - 0.33 y_j^+ \right)] \] \quad (3.27)

Eqs. (3.21), (2.22), (3-25), (3.26) and (3.27) are the complete set of finite difference equations. To generate results Eq. (3.22) requires F's from only one \( x^+ \) location. Thus, the initial F distribution, \( F = 0 \) at water surface; \( F = 1 \) at \( y^+ > 0 \), is sufficient to start the computation. After four complete columns of F's were calculated, Eq. (3.21) was then used for further computation.

In the above equations \( \Delta y_1^+ \) is the vertical distance of the nodal point immediately above the water surface. The program was executed with various values of \( \Delta y_1^+ \) ranging from 1 to 250 and their corresponding F's at a particular nodal point were plotted against the \( \Delta y_1^+ \). Fig. (3.2) shows that as long as \( \Delta y_1^+ \) is less than 5, the solution is not sensitive to the numerical values of \( \Delta y_1^+ \). The proper value of \( \Delta y_1^+ \) was therefore selected as 4. \( \Delta x^+ \) was selected to be 5. The calculated F based on the two expressions for the eddy diffusivities for momentum are plotted in

* For mass transfer \( \text{Pr}^{-1} \) is replaced by \( \text{Sc}^{-1} \)
Fig. (3.3) for different $x^+$. It is seen that the agreement between the calculated $F$ based on the two $\xi_M$'s is excellent. Hence, for the sake of simplicity the assumption of equal eddy diffusivities will be adopted in the study.

3.5.1.3 Heat and Mass Transfer Coefficients for a Smooth Water Surface.

The transfer coefficients $h_H$ and $h_D$ for a smooth water surface are calculated by the flux equations, Eqs. (3.14) and (3.15).

In terms of $\frac{\partial F}{\partial y^+} |_{y^+=0}$, the transfer coefficients are:

$$\frac{h_H}{\lambda} = \frac{h_D}{\rho C_p} = \left(\frac{u^*}{\nu}\right) \frac{\partial F}{\partial y^+} |_{y^+=0}$$

(3.28)

The evaluation of $\frac{\partial F}{\partial y^+} |_{y^+=0}$ from the dimensionless temperature or specified humidity is given in Appendix (A.2.1). Substituting the results into Eq. (3.28), the transfer coefficients can be expressed, in terms of the dimensionless fetch, $L_s^+$ as:

for: $0 \leq L_s^+ = \frac{L(u^*_s)}{\nu} \leq 63200$

$$\frac{h_H}{\lambda} = \frac{h_D}{\rho C_p} = 0.202 \left(\frac{u^*}{\nu}\right) (L_s^+)^{-0.168}$$

(3.29)

for $L_s^+ > 63200$

$$\frac{h_H}{\lambda} = \frac{h_D}{\rho C_p} = 0.032 \left(\frac{u^*}{\nu}\right)$$

(3.30)
3.5.1.4 Heat and Mass Transfer Coefficients for a Wavy Water Surface

The transfer coefficients given by Eqs. (3.29) and (3.30) are valid for an imaginary water body having a hydrodynamically smooth surface at a given reference air velocity. For an actual water surface, they can be obtained from the following equation:

\[
\frac{(h_D)_w}{(h_D)_s} = \frac{(h_H)_w}{(h_H)_s} = \frac{(u_\tau)_w}{(u_\tau)_s} = \frac{(C_{10})_w}{(C_{10})_s} \]

The derivation of the above equation is given in Appendix (A.2.2).

In order to apply the above equation, one has to know the drag coefficients both for a smooth and a wavy water surface. The derivation of the above three parameters will be given as follows.

3.5.1.5 Drag Coefficients under Atmospheric Conditions

Over a smooth water surface the drag coefficient can be easily calculated from Eq. (2.6) using a trial and error method. Over a wavy water surface it also can be calculated from the wind velocity profile given by Eq. (2.8) provided that the relationship between the average height of surface roughness element \( \xi \) and the characteristic roughness length \( Z_0 \) is known. The value of \( Z_0 \) can be assumed to be proportional to \( \xi \) as \( Z_0 = \xi \xi \). The proportional constant \( \xi \) is about 30 calculated from the Moody diagram but is ranging approximately between 15 to 5350 according to Sibul's(17) extensive measurements on a wavy water surface produced in a laboratory wind wave tunnel. Based on the above results it is not possible to assign a value for \( \xi \) and hence the drag coefficient for a wavy water surface cannot be
calculated directly from Eq. (2.8).

On the other hand the functional relationship among wind speed, wave height and fetch introduced by Wiegel (75) may be approximately expressed by the equation:

\[
\frac{gH^{1/3}}{U^2} = 0.0026 \left( \frac{gL}{U^2} \right)^{1.536}
\]  (3.32)

or

\[
\frac{H^{1/3}}{L} = 0.0026 \left( \frac{U^2}{gL} \right)^{1.536}
\]

Here \( H^{1/3} \) is the significant wave height defined as the average of the highest 33-1/3 percent of the waves. \( L \) is the fetch. \( H^{1/3}/L \) may be defined as the relative roughness. From Eq. (3.32) the following conclusions may be drawn:

(i) For a constant wind speed the relative roughness decreases as the fetch increases.

(ii) For a constant fetch, the relative roughness and the wave height increase with wind speed.

(iii) For a constant wind speed the significant wave height increases with fetch.

Since the relationship between relative roughness and friction factor for fully developed flow in a rough pipe is well established, it is not unreasonable to expect that a similar interrelation also exists in case of fully developed wind flow along a

* by curve fitting
water surface; hence

\[
\frac{1}{\sqrt{C_{10} w}} = A_4 \ln \frac{L}{H^{1/3}} + A_5
\]  

(3.33)

Eqs. (3.32) and (3.33) indicate that the drag coefficient increases with relative roughness which, in turn, is closely related to significant wave height, wind speed and fetch. Hence, the results observed by many investigators, such as that the drag coefficient increases with wind speed \(^{(16)}\) and wave height \(^{(25)}\), and decreases as fetch increases \(^{(50)}\) can now all be explained from Eq. (3.33).

In Eq. (3.33) \(A_4\) and \(A_5\) are two constants which are best evaluated using experimental data obtained in field at various fetches and wind speeds. The wide scatter shown in the available data obtained in the field indicates that this is not the case.

The approximate values of the constants are therefore evaluated by using the empirical equation for friction factor for fully developed flow in rough pipe with the relative roughness replaced by \(L/H^{1/3}\). Details of the derivation are given in Appendix (A.2.3). The resultant equation for estimating the drag coefficient over a wavy water surface is

\[
\frac{1}{\sqrt{C_{10}^w}} = 1.86 \ln \frac{1}{Fr_L} + 11.2
\]  

(3.34)

This equation is plotted in Fig. (3.4).

3.5.1.6 Dimensionless Transfer Coefficients Over a Wavy Water Surface.

The transfer coefficients for a wavy water surface can be obtained by combining Eq. (3.29) or Eq. (3.30) with Eqs. (3.31) and (3.34). In terms of dimensionless numbers, these coefficients are
for \( L_s^+ = \frac{L(u_c)_s}{v} < 63200 \)

(1) Heat Transfer

\[
\Gamma_H = \frac{Nu}{Re_L} = \frac{0.152}{\frac{1.86 \ln \left(\frac{Fr}{Fr_L}\right) + 11.2}{Re_L}} \left(\frac{C_{10}}{10}\right)_s^{-0.084} Re_L^{-0.168}
\]

(3.35)

(2) Mass Transfer

\[
\Gamma_D = \frac{Sh}{Re_L} = \frac{0.152}{\frac{1.86 \ln \left(\frac{Fr}{Fr_L}\right) + 11.2}{Re_L}} \left(\frac{C_{10}}{10}\right)_s^{-0.084} Re_L^{-0.168}
\]

(3.36)

\( L_s^+ > 63200 \)

(1) Heat Transfer

\[
\Gamma_H = \frac{Nu}{Re_L} = \frac{0.022}{\frac{1.86 \ln \left(\frac{Fr}{Fr_L}\right) + 11.2}{Re_L}}
\]

(3.37)

(2) Mass Transfer

\[
\Gamma_D = \frac{Sh}{Re_L} = \frac{0.022}{\frac{1.86 \ln \left(\frac{Fr}{Fr_L}\right) + 11.2}{Re_L}}
\]

(3.38)

The results are plotted in Fig. (3.5).

3.5.2 Method II - Semi-empirical Approach

Wind blowing over a water surface generates waves which interact with the air flow. Under steady-state conditions, the mechanism of the turbulent transport process are governed by the equations defined in the previous sections. These equations are:
Velocity profile

\[ \frac{u}{u_\tau} = \frac{1}{k} \ln \left( \frac{v}{z_o} \right) \quad (2.8) \]

Continuity equation

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \quad (3.13) \]

Energy equation

\[ u \frac{\partial \theta}{\partial x} + \nu \frac{\partial \theta}{\partial y} = \frac{\partial}{\partial y} \left[ \left( \alpha + \varepsilon H \right) \frac{\partial \theta}{\partial y} \right] \quad (3.11) \]

Diffusion equation

\[ u \frac{\partial m}{\partial x} + \nu \frac{\partial m}{\partial y} = \frac{\partial}{\partial y} \left[ \left( \beta + \varepsilon D \right) \frac{\partial m}{\partial y} \right] \quad (3.12) \]

It will be convenient to transform Eqs. (3.11), (3.12) and (3.13) into nondimensional form before attempting to obtain the solution. To do so, it is desirable to use \( z_o \) and \( u_\tau \) as the reference length and reference velocity so that Eq. (2.8) can be applied directly. These two quantities, \( z_o \) and \( u_\tau \), are all functions of the surface roughness which, in turn, is dependent on the axial distance for the case of a water surface roughened by wind. Thus, it is necessary to know the functional relationships between \( z_o \), \( u_\tau \), and \( x \) before Eqs. (3.11) to (3.13) can be transformed into dimensionless forms.

3.5.2.1 Assumptions

The following assumptions are required to seek solutions to Eqs. (3.11) and (3.12):
1) The effect of the Froude Number on the fluid flow is much larger than that of the Reynolds Number.

2) The functional relationship among $a_{rms}$, $L$ and $u$ obtained under laboratory conditions is similar to that obtained in open sea.

3) Turbulent Prandtl Number, turbulent Schmidt Number and turbulent Lewis Number are all assumed to be unity.

The first assumption may be justified as the physical model under study consists of low viscosity fluids and a free water surface with waves. The second assumption is also approximately valid based on the study made by Wiegel (75). The validity of third assumption has been investigated and verified by many investigators (86-87).

3.5.2.2 Functional Relationships among $u$, $L$, $z_0$ and $C_i$

Because of the complex nature of turbulent flow, theoretical studies on the problem of turbulent transport mechanism over a rough surface still have to rely extensively on empirical information. In other words, the results of theoretical studies permit the presentation of experimental data in their most general form by establishing the basic forms of the relationships between the essential parameters, in addition to specifying the significant parameters themselves.

The functional relationships among $u$, $a_{rms}$, $L$ and $y$ are therefore, determined using Eqs. (3.32), (3.33) and (3.34) as the basis of the considerations. Once the functional relationships among various parameters are established, the constants can then be evaluated using the experimental data discussed in Chapter 4.
The details of the establishment of these functional relationships are given in Appendix (A.3).

The final forms of these functional relationships are:

\[
\frac{k u_i}{u_\tau} = -0.7 \psi 0.033 \ln \left[ \frac{\text{a rms/L}}{\left( \frac{u_i}{\sqrt{g L 0.8 y_i}} \right)^{3.5}} \right]
\]  \tag{3.39}

\[
\frac{\text{a rms}}{L} = \exp \left[ -14.2 + 0.08 \left( \frac{u_i}{\sqrt{g y_i}} \right) \psi + 0.12 \right]
\]  \tag{3.40}

or \[\psi = \frac{(u_i y_i / u_i')^2}{u_i / g y_i}\]

where \[\psi = \frac{(u_i y_i / u_i')^2}{u_i / g y_i}\]

In Eq. (3.39) \(k\) is von Karman's constant and is assumed to be 0.4. These two equations are plotted in Figs. (3.6) and (3.7) respectively. As shown in these figures, the data has an average deviation of 100% and 75% from Eqs. (3.39) and (3.40) respectively.

Eqs. (3.39) and (3.40) describe the functional relationship between the dimensionless velocity \(u^+\), and the Froude Number with respect to the fetch \(L\). Since \(u^+\) can also be expressed in terms of \(z_o\) by Eq. (2.8), the combination of the three equations gives,

\[
\frac{z_o}{y_i} = \left\{ \frac{946 \exp \left[ -14.2 + 0.08 \left( \frac{u_i}{\sqrt{g y_i}} \right) \psi + 0.12 \right]}{0.7 \psi 0.033} \right\} \left( \frac{\left( \frac{u_i}{\sqrt{g L 0.8 y_i}} \right)^{3.5}}{0.2} \right)
\]  \tag{3.41}
Similarly, replace \( u^+ \) by the drag coefficient \( C_i \), which is defined as
\[
C_i = 2 \left( \frac{\tau}{u_i} \right)^2 = 2 \left( \frac{u^+}{(u^+)^2} \right)
\]

The combination of Eqs. (3.39) and (3.40) gives
\[
\frac{1}{\sqrt{C_i}} = -1.23 + 0.053 \ln \left\{ \frac{946 \exp \left[ -14.2 + 0.08 \left( \frac{u_i}{\sqrt{g y_i}} \right)^{0.12} \right]}{u_i} \right\} \quad 3.5 \left( \frac{g L^{0.8} y_i}{0.2} \right)
\]

The calculated drag coefficient and dynamic roughness are shown in Figs (3.8) and (3.9).

3.5.2.3 Dimensionless Energy, Diffusion and Continuity Equations

Eqs. (3.11), (3.12) and (3.13) can be transformed into dimensionless forms using the following parameters
\[
\begin{align*}
\eta^* &= \frac{m - m_{z_0}}{m_{z_0} - m_{z}} = \frac{t_{z_0} - t}{t_{z_0} - t_{z}} \quad (3.43) \\
u^+ &= \frac{u}{\nu_{\tau}} \\
u^+ &= \frac{v}{\nu_{\tau}} \\
x^{++} &= \frac{x}{z_0} \\
y^{++} &= \frac{y}{z_0} \\
a + \varepsilon_{H} &= \frac{(\psi + \varepsilon_{M})/\varepsilon_{e}}{a + \varepsilon_{H}} \\
g + \varepsilon_{D} &= \frac{(\psi + \varepsilon_{M})/\varepsilon_{e}}{g + \varepsilon_{D}}
\end{align*}
\]
where

\[ \varepsilon_c = \frac{1 + \frac{\varepsilon_M}{\nu}}{Pr_{t}^{-1} + Pr_{t}^{-1} (\varepsilon_M/\nu)} \] for heat transfer

\[ = \frac{1 + \frac{\varepsilon_M}{\nu}}{S_{ct}^{-1} + S_{ct}^{-1} (\varepsilon_M/\nu)} \] for mass transfer

Introducing \( \tilde{r} \) and \( (\nu + \varepsilon_M) \varepsilon_e^{-1} \) into Eqs. (3.11 and 3.12)

we have,

\[ \frac{\partial \tilde{F}}{\partial x} + \nu \frac{\partial \tilde{F}}{\partial y} = \frac{\partial}{\partial y} \left[ (\nu + \varepsilon_M) \varepsilon_e^{-1} \frac{\partial \tilde{F}}{\partial y} \right] \tag{3.44} \]

In the above equation, the term \( (\nu + \varepsilon_M) \) can be evaluated as follows:

In the region not too far from the mean water level where most velocity change takes place, \( \tau \) will not vary markedly from its wall value \( \tau_0 \). Therefore, Eq. (2.3) may be approximately written as

\[ \frac{\tau_0}{\nu} = (\nu + \varepsilon_M) \frac{\partial \tilde{u}}{\partial y} \tag{2.4} \]

For the region away from the wall boundary the derivative \( \frac{\partial \tilde{u}}{\partial y} \) can be evaluated from the logarithmic velocity profile defined by Eq. (2.8) as

\[ \frac{\partial \tilde{u}}{\partial y} = \frac{\sqrt{\tau_0}}{k \zeta_0} \frac{1}{y^{++}} \tag{3.45} \]

Substituting Eq. (3.45) into Eq. (2.4) and noting that \( \frac{\tau_0}{\nu} = \frac{2}{\tau} \), we obtain

\[ \nu + \varepsilon_M = k \frac{u}{\tau} \zeta_0 y^{++} \tag{3.46} \]
Substituting Eq. (3.46) into Eq. (3.44) gives

\[ u \frac{\partial F}{\partial x} + v \frac{\partial F}{\partial y} = \frac{u}{y} \left( ku z o y^{++} + \frac{1}{\xi} \frac{\partial F}{\partial y} \right) \]  
\[ \tag{3.47} \]

The dimensionless terms \( u^+, v^, x^{++} \) and \( y^{++} \) will then replace the corresponding terms in Eq. (3.47) as follows:

1) \( u \frac{\partial F}{\partial x} = u^+ \frac{\partial F}{\partial x^{++}} \)  
\[ \tag{3.48} \]

The derivative \( \frac{\partial x^{++}}{\partial x} \) can be deduced from Eq. (3.41) as shown in Appendix (A.4.1) as

\[ \frac{\partial x^{++}}{\partial x} = -0.82 \]  
\[ \tag{A.4.2} \]

Substituting \( \frac{\partial x^{++}}{\partial x} \) into Eq. (3.48), gives

\[ u \frac{\partial F}{\partial x} = -0.82 \frac{u}{z_o} u^+ \frac{\partial F}{\partial x^{++}} \]  
\[ \tag{3.49} \]

2) \( v \frac{\partial F}{\partial y} = v^+ \frac{\partial F}{\partial y^{++}} \)  
\[ \tag{3.50} \]

3) \( \frac{\partial}{\partial y} (ku z o y^{++} + \frac{1}{\xi} \frac{\partial F}{\partial y}) = k \frac{u}{z_o} \frac{\partial}{\partial y^{++}} (y^{++} \frac{\partial F}{\partial y^{++}}) + \frac{k}{\xi} \frac{\partial}{\partial y^{++}} \frac{\partial F}{\partial y^{++}} \)  
\[ \tag{3.51} \]

Again, the derivative \( \frac{\partial u^{++}}{\partial y} \) can be deduced from the experimental data included in Chapter 4. The details are given in Appendix (A.4.1).
It is shown that,
\[
\frac{\partial u}{\partial y} = -0.19 \frac{u}{y},
\]
(A.4.4)

hence
\[
\frac{\partial}{\partial y} (k \mu z^{\infty} e - \frac{\partial F}{\partial y}) = \frac{k}{z^{\infty} e} \frac{\partial}{\partial y} (y^{\infty} \frac{\partial F}{\partial y}) - 0.19 \frac{\partial F}{\partial y}
\]
(3.52)

Substituting Eqs. (3.49), (3.50) and (3.52) into Eq. (3.47), and rearranging, we obtain,
\[
-0.82 u^{+} \frac{\partial F}{\partial x^{+}} + (v^{+} + 0.19k) \frac{\partial F}{\partial y^{+}} = \frac{k}{\varepsilon} \frac{\partial}{\partial y^{+}} (y^{+} \frac{\partial F}{\partial y^{+}})
\]
(3.53)

In order to transform the continuity equation into a dimensionless form using the parameters specified in Eq. (3.43), it is necessary to know the quantities \( \frac{\partial u}{\partial x} \) and \( \frac{\partial u}{\partial x} \). Again, these quantities can be deduced from the experimental data. The details of the derivation are given in Appendix (A.4.1). The final forms of these quantities are
\[
\frac{\partial u}{\partial x} = -0.42 \frac{u}{x}
\]
(A.4.5)
\[
\frac{\partial u}{\partial x} = \frac{2.23}{k x}
\]
(A.4.6)

With the above two quantities evaluated, the transform of the continuity equation can be proceeded as follows:
\[
\frac{\partial u}{\partial x} = \frac{\partial u}{\partial x^{++}} + \frac{\partial x^{++}}{\partial x}
\]

\[
= (u^+ \frac{\partial u}{\partial x^{++}} + u^+ \frac{\partial x}{\partial x^{++}} + \frac{\partial u}{\partial x^{++}}) \frac{z - x}{z^2} \frac{\partial x}{\partial x^{++}}
\]

\[
= \frac{u}{z_o} (0.34 \frac{u}{x^{++}} - \frac{1.82}{k x^{++}})
\]

\[
\frac{\partial v}{\partial y} = \frac{\partial u}{\partial y^{++}} + \frac{\partial y^{++}}{\partial y}
\]

\[
= \frac{1}{z_o} (v^+ \frac{\partial u}{\partial y^{++}} + \frac{\partial v}{\partial y^{++}})
\]

\[
= \frac{u}{z_o} (-0.19 \frac{v}{y^{++}} + \frac{\partial v}{\partial y^{++}})
\]

Substituting Eqs. (3.54) and (3.55) into Eq. (3.13) and rearranging we obtain

\[
\frac{\partial v}{\partial y^{++}} = \frac{1.82}{k x^{++}} - 0.34 \frac{u}{x^{++}} + 0.19 \frac{v}{y^{++}}
\]

3.5.2.4 Solution of Energy and Diffusion Equations

The equation to be solved is the energy or diffusion equation, Eq. (3.53). To obtain the solution, the logarithmic velocity profile specified by Eq. (2.8) and the equation of continuity, Eq. (3.56) have to be applied. Again, these equations are solved using the DuFort-Frenkel\(^7\) type finite difference method.
The corresponding finite difference equations based on the flow field grid shown in Fig. (3.10) are:

1) Energy or diffusion equation,

\[ F_{i+1,j} = \frac{1}{d_3} \left( d_4 F_{i,j+1} + d_5 F_{i,j-1} + d_6 F_{i-1,j} \right) \]  \hspace{1cm} (3.57)

where

\[ d_1 = \frac{k (y_{j+1}^{++} + y_{j}^{++})}{\varepsilon_e (y_{j+1}^{++} - y_{j-1}^{++}) (y_{j+1}^{++} - y_{j}^{++})} \]

\[ d_2 = \frac{k (y_{j}^{++} + y_{j-1}^{++})}{\varepsilon_e (y_{j+1}^{++} - y_{j-1}^{++}) (y_{j}^{++} - y_{j-1}^{++})} \]

\[ d_3 = \frac{-0.82 u_{i,j}^{++}}{2 \Delta x^{++}} + \frac{d_1}{2} + \frac{d_2}{2} \]

\[ d_4 = \frac{v_{i,j}^{++} + 0.19 k/\varepsilon_e}{y_{j+1}^{++} - y_{j-1}^{++}} + d_1 \]

\[ d_5 = \frac{v_{i,j}^{++} + 0.19 k/\varepsilon_e}{y_{j+1}^{++} - y_{j-1}^{++}} + d_2 \]

\[ d_6 = \frac{0.82 u_{i,j}^{++}}{2 \Delta x^{++}} - \frac{d_1}{2} - \frac{d_2}{2} \]

2) Continuity equation

\[ v_{i,j}^{+} = (y_{j}^{++} - y_{j-1}^{++}) \left[ \left( \frac{1}{y_{j}^{++} - y_{j-1}^{++}} + 0.19 \right) v_{i,j-1}^{+} + \frac{1.82}{k x^{++}} - 0.34 \frac{u_{i,j-1}^{+}}{x^{++}} \right] \]  \hspace{1cm} (3.58)
3) Velocity profile

\[ u^+_{i,j} = \frac{1}{k} \ln y^+_j \]  \hspace{1cm} (3.59)

The starting finite difference equation for the energy or diffusion equation is,

\[ F_{i+1,j} = \frac{1}{d_7} \left( d_4 F_{i,j+1} + d_8 F_{i,j} + d_5 F_{i,j-1} \right) \]  \hspace{1cm} (3.60)

where

\[ d_7 = \frac{0.82 u^+_i}{\Delta x^{++}} \]

\[ d_8 = d_7 - d_1 - d_2 \]

\[ d_1, d_2, d_4 \text{ and } d_5 \text{ are defined in Eq. (3.57.)} \]

The boundary and initial conditions are:

\[ y^{++} = 1; \quad u^+ = 0; \quad v^+ = 0; \quad F = 0 \]

\[ y^{++} \rightarrow \infty; \quad u^+ = u^+_r; \quad F \leq 1 \]

Eqs. (3.57), (3.58), (3.59), and (3.60) are the complete set of finite difference equations for the calculation of the temperature as well as specific humidity profiles over a wavy water surface. Computation was started with Eq. (3.60) and then continued with Eq. (3.57) after four columns of F's were calculated. IBM 360 computer was used for the computation.

The appropriate grid size was selected by estimating the suitable vertical distance of the nodal point immediately above the water surface, \( y^{++}_1 \). This was done by assuming various values...
for \( y_1^{++} \) and comparing the corresponding values of \( F \) associated with the different values of \( y_1^{++} \). It was found that if a value which is less than 10 was assigned for \( y_1^{++} \), a discontinuity in \( F \) exists at the location where the computation was taken over by Eq. (3.57). It was also found that there is no significant difference in values of \( F \) for various values of \( y_1^{++} \) if \( y_1^{++} \) is between 10 to 20. Thus, \( y_1^{++} \) was selected as 10. The value of \( \Delta x^{++} \) was selected to be -20.

### 3.5.2.5 Transfer Coefficients for a Wavy Water Surface

The transfer coefficients for a wavy water surface can be evaluated from the heat flux and the mass flux equations. The detailed derivation of the heat and the mass transfer coefficients is shown in Appendix (A.5). The final forms of these coefficients in terms of heat and mass transfer Stanton Numbers as defined by Eq. (A.5.7) is

\[
St_D = St_H = k \left\{ \ln y_r^{++} \ln \left[ \frac{k z_0 \frac{u'}{v}}{\nu} y_1^{++} + \varepsilon_v - 1 \right] \right\}^{-1}
\]

(3.61)

where

\[
\frac{u'}{v} = \sqrt{\frac{T_0}{c}}
\]

\[
\varepsilon_v = \frac{1}{Pr}
\text{ for heat transfer}
\]

\[
\varepsilon_v = \frac{1}{Sc}
\text{ for mass transfer}
\]

Since \( z_0 \) is a function of \( Fr_Ly \) and \( Fr_y \), the Stanton Number is also a function of the two quantities. The results are shown in Fig. (3.11).
3.6 Wind Drift Current

Thus far the water velocity at the air-water interface was not considered. This velocity which is induced by wind was found by Keulegan (26) to be about 3% of the free stream wind velocity at the fetch under consideration. Since this velocity is a function of the shear velocity it is also fetch dependent. Therefore direct measurement of the wind drift current is not feasible, especially for a wavy water surface. However, using Reichardt’s velocity profile, an approximate expression for the wind drift current can be proposed. The details of the derivation are given in Appendix (A6). The result is given as follows:

\[ V_o = -V_i^+ \frac{u}{rb} \quad (3.77) \]

where

\[ V_i^+ = 2.5 \mu (I + 0.4 R^+) + 7.8 \left[ 1 - \exp \left( -\frac{R^+}{11} \right) - \frac{R^+}{11} \exp \left( -0.33 R^+ \right) \right] \]

Finally, in calculating the drag as well as the transfer coefficients, the wind drift current can be accounted for by replacing the wind velocity, \( u \) by the velocity of wind relative to the water surface, \( u - V_o \). The values of \( V_o \) are shown in Fig. (3.14).
CHAPTER 4
EXPERIMENTAL STUDY

This chapter briefly describes the equipment and measuring technique used in this study. It also discusses various errors involved in estimating the mean values from the measurements.

4.1 Closed Circuit Air-Water Tunnel

Measurements were conducted in a closed circuit air-water tunnel which was built specially for the study. As shown in Figs. (4.1) and (4.1a) it is 1890 cm (744 in) long with a working section of 45.7 cm by 61 cm by 351 cm long (18 in by 24 in by 138 in long). It is composed of 5 parts, a fan section, an entrance section, a working section, a return duct and an environmental control system.

The fan section consists of an air filter chamber, a cooling coil, an electric heating coil and two centrifugal fans, enclosed in a common steel cabinet. The total capacity of the fans is 3.6 m³/sec. at a static pressure differential of 622 N/m² (6500 CFM at 2.5 inches of water). The fans are equipped with a common Torq-matic solid state variable speed drive. With this variable speed drive, the fan can be controlled at any speed up to 1800 rpm with an accuracy of ± 1%.

The entrance section is composed of a flow straightener and a 853 cm long (336 in) straight entrance duct. The long entrance duct is provided to ensure a constant flow condition and fluid parameter at the entrance of the working section. In steady of developing flow condition that of developed flow has been chosen for the present study. The flow straightener consists of a divergent diffuser, a rectangular duct with fine mesh screens, a convergent nozzle and a honeycomb straightener. The entrance duct is 45.7 cm wide by 30.5 cm high (18 in by 12 in.). It has a particle board casing with 26 GA stainless steel lining and is covered
by 2.54 cm (1 in.) thick fiberglass insulation. The stainless steel lining is used to provide a smooth surface and to prevent the formation of rust.

It is known that when a fluid enters into a pipe, the velocity distribution changes with distance, until a fully developed velocity profile is attained at a certain distance and remains constant downstream of it. The inlet distance required to attain the fully developed profile has been found to be 10 to 40 diameters (9, 91). Because of the high construction cost, the entrance length of the tunnel was chosen to be about 23 equivalent diameters of the cross sectional area of the duct.

The working section has a water reservoir as shown in Figs. (4.2) and (4.2a). The water level was kept, as close as possible, in alignment with the bottom of the entrance duct so that the air flow area is about 45.7 cm wide by 30.5 cm high (18 in. by 12 in.). A beach is installed at the front edge of the water reservoir. The angle of the beach should be small enough to ensure a smooth transition taking place between the adjoining air and water flow. On the other hand, it should not be too small so that the growth of the wave can be affected by the beach. Several beaches have been installed, the one which makes an angle of about 5 degrees with the water surface has been found to be satisfactory. The dimensions of the water reservoir are 45.7 cm wide by 30.5 cm deep by 351 cm long (18 in. by 12 in. by 138 in). A sloping beach together with a battle of folded wire mesh are installed at the end of the water reservoir to reduce reflection of waves. No attempt has been made to change the water depth since there are evidences to show that the velocity profile (17), the drag coefficient (76), and the wave height (17, 75) are not affected by the water depth.
The top plates consist of 5 pieces with their lengths varied from 30.5 cm to 91.4 cm (12 in to 36 in). All the measuring probes except the thermocouple for measuring the temperature at the mean water level are attached to one of the top pieces 45.7 cm long (18 in). The top pieces are interchangeable so that the measuring probe can be located at various positions along the working section by placing various combinations of the top pieces ahead of it. To minimize air leakage through the gap between two adjacent pieces, various lengths of polished stainless steel plates are provided so that one of these plates can be fitted underneath the top pieces. The measuring probes can also be positioned at any place laterally within 20.3 cm (8 in) away from the center by sliding the probes in a slot cut in the top piece. As shown in Fig. (4.2) the portion of the slot that is not occupied by the probes is filled with small pieces of well machined wood blocks to reduce air leakage.

The return duct has a particle board casing with GA 26 stainless steel lining and covered with fiberglass insulation. The dimensions of the return duct are 61 cm by 61 cm (24 in by 24 in). The transition piece that connects the working section and the return duct is waterproofed and is gradually tapered towards the edge of the water reservoir. Hence water spilled from the reservoir can flow back into it.

The environmental control system consists of a refrigeration plant, a cooling coil, an electric heating coil, a steam humidifier, a control system and a monitoring system.

The refrigeration plant has a capacity of 10.6 kw.
(3 tons). Its function is to provide chilled water to the cooling coil for cooling and dehumidifying the supply air. The chilled water supply temperature is accurately controlled using an evaporator pressure regulator with a temperature pilot whose set point is automatically adjusted according to the temperature of the chilled water supply. Moisture and heat can also be added to the supply air through the steam humidifier and the two stage electric heating coil, respectively. The steam for the humidifier is supplied by the heating plant of the University.

The operation of the heating and the cooling coils and steam humidifier are regulated by the three Honeywell Model 5500101 Serwotronik indicating controllers. The accuracy of these indicating controllers is ± 1% of the calibrated span. As shown in Fig. (4.3), the operation of the controllers for heating coil and steam humidifier is quite straightforward. It increases proportionally the supply of heat or moisture to the air if the dry bulb or dewpoint temperature of air at the entrance to the working section is lower than its set point. The reverse is the case if these temperatures are higher than their set points. An auxiliary controlling device has to be installed for the controller which operates the cooling coil. With the auxiliary controlling device, the controller can operate the cooling system based on the dewpoint temperature when the dry bulb temperature of air is lower than its set point. If, however, the dry bulb temperature is higher than its set point, the cooling system is operated based on the dry bulb temperature, even though the dewpoint controller calls for increase in humidity.
The monitoring system has a temperature sensor, a relative humidity sensor and a two-pen recorder. It is capable to record the temperature and the relative humidity of air at the exit of the working section continuously during the test.

4.2 Measuring Apparatus

4.2.1 Multiple Function Measuring Probe

A multiple function measuring probe as shown in Figs. (4.4) and (4.4a) about 2.3 cm wide by 0.32 cm high (0.9 in by 0.13 in) was designed for the measurement of the vertical distributions of velocity pressure, dry and wet bulb temperatures. This probe is composed of a static pressure tube, a total pressure tube, a dry and a wet bulb thermocouples and a water reservoir for the wet bulb thermocouple. As shown in Fig. (4.4) leads of the thermocouples and the connecting tubing of the pressure tubes are packed in a 45.7 cm (18 in) long hollow stainless steel stem of 1.27 cm (0.5 in) diameter. A needle is fastened to one end of the stem. The distance between the needle point and the centerline of the total pressure tube is 1.27 cm (0.5 in). Using the needle, the reference level of the still water level with respect to the centerline of the total pressure or other probes can be accurately estimated by obtaining the level where the pointer just touches its image. With the reference level determined, the vertical position of the probe can be read with an accuracy of 0.0025 cm per 15.24 cm (0.001 in per 6 in) from the Mitutoyo Model 509-204 dial height gauge whose arm of measuring slide is fastened to the other end of the stem. The height gauge is mounted on the exterior side of the top plate in such a way that when the measuring probe slides along laterally across the tunnel
in the slot of the top plate the height gauge moves along with it.

4.2.2 Wave Gauge.

The wave gauge was developed for the measurement of the wave amplitude. As shown in Fig. (4.5), this gauge is made of a copper tube of 0.64 cm (0.25 in.) in diameter. One end of the tube is bent to form an "u" so that when it is submerged in water its tip is normal to the mean water level. The other end of the tube is also mounted on the exterior side of the top plate similar to the multiple function measuring probe.

4.2.3 Physical Arrangement of Measuring Apparatus

A sketch of the physical arrangements of the measuring apparatus is shown in Fig. (4.5). The static and the total pressure tubes are connected to a Statham Model PM 97TC strain gauge differential pressure transducer. The output of the transducer is recorded on a Hewlett-Packard Model HP4100B two-channel strip-chart recorder. The thermocouples for measuring the dry bulb and the wet bulb temperatures of air as well as the temperature at the mean water level are recorded on a Hewlett-Packard Model 2010K high-speed data acquisition system which consists mainly of an integrator, a digital voltameter and a printer.

The wave gauge is connected to an Endevco Model 8505-0.2 strain gauge differential pressure transducer. The output of the transducer is connected to a DISA Model 55 D 35 RMS meter which, in turn, is connected to the two-channel
strip-chart recorder. In addition, a burette is connected to the wave amplitude circuit upstream of the pressure transducer for calibration before or during the test.

4.3 Calibration of Measuring Apparatus

4.3.1 Static Pressure Tube

The static pressure tube was calibrated against the wall taps. These wall taps are drilled in the wall of a 5.1 cm inside diameter (2 in.) plexiglas tube which forms the working section of a small wind tunnel. The multiple function measuring probe was placed in the center of the wind tunnel with the static pressure holes in alignment with the wall taps. The static pressure probe was then calibrated by comparing its results with those of the wall taps based on the same total pressure. These results, in terms of velocity pressure are shown in Fig. (4.6).

4.3.2 Thermocouples

The copper-constantan thermocouples used in the present study were calibrated indirectly using a thermocouple which is made from the same spool of wire. This thermocouple is calibrated against a Fisher Nro-15-155 precision thermometer in a temperature controlled oil bath. The resultant calibration curve is shown in Fig. (4.7).

4.3.3 Differential Pressure Transducers

The strain gauge differential pressure transducers were calibrated using a Betz micromanometer. For this purpose, both the transducer and the micromanometer were connected to a
single pressure source. The calibration was conducted by increasing the pressures in the source from 0.1 to 1 inch of water. To check the reproducibility of the results, the measurements were repeated by decreasing the pressures in the source from 1 to 0.1 inch of water. A typical calibration curve expressed in terms of the manometer readings versus the transducer outputs is shown in Fig. (4.8).

4.4 Experimental Procedure and Results

Prior to the experiment, the following tasks were performed:

1) The reservoir was filled with water up to its edge.
2) The reference distance between the centerline of the total pressure tube and the still water level was determined using the needle attached to the measuring probe. The height gauge was then zeroed accordingly.
3) The measuring probe was raised to the center of the tunnel.
4) The adjustments of zero and full scale were made on all the instruments used.
5) The fan was turned on and was adjusted to the predetermined speed.
6) The measurements were conducted after the steady-state condition was reached.

4.4.1 Steady-State Condition

For the tests in which the temperature and the specific humidity profiles were not taken, the steady state condition was
assumed to be reached when the mean centerline air velocity was invariable with time. However, for the experimental runs in which the temperature and the specific humidity profiles were taken, the approach of the steady-state condition was assumed when the mean temperature indicated by the monitoring system did not vary with time.

4.4.2 Height of Probes above Mean Water Level

Initially the height gauge of the traverse was zeroed at the still water level. To this the set-up of the water surface was added or subtracted and the zero of the height gauge was readjusted accordingly.

The set-up of water surface may be defined as the difference between the mean water level when the fan is in operation and the still water level. It was estimated by the following procedure: For lower wind speeds and fetches, the output of the differential pressure transducer for the wave gauge was connected directly to the recorder and the still water level was marked on the recording chart. The fan was then turned on and the instantaneous water levels were recorded. The set-up of the water surface was estimated from the still water level and the instantaneous water levels. For higher wind speeds and larger fetches, the wave gauge was connected to the burette as shown in Fig. (4.5). The still water surface was marked on the side of the burette. The fan was turned on and the mean water level under blowing wind was estimated from the instantaneous water levels in the burette.

4.4.3 Air Velocity Profile

As soon as the steady state condition was reached,
the vertical velocity traverse of air flow was taken using a set of total and static pressure tubes. These tubes were connected to a Statham Model PM 97 TC strain gauge differential pressure transducer. The output of the transducer was recorded on a strip-chart recorder.

The vertical traverse was measured starting from the position as close to the mean water level as possible. Because of the moving water surface, the velocity measurements of air flow at any traverse point were recorded for about one minute. Based upon these instantaneous readings, the mean velocity of air flow was determined. The range of the centerline velocities varied from 6.1 m/sec to 19.8 m/sec (20 ft/sec to 65 ft/sec).

In addition, the fetch which is the distance between the edge of the water surface and the tip of the probe was also measured for each experiment run.

4.4.4 Wave Amplitude

The wave amplitude was measured using a wave gauge together with a strain-gauge differential pressure transducer. The output of the transducer was time-averaged using a rms meter. Its mean value was then determined from the readings recorded on the strip-chart recorder.

To check the accuracy of the wave gauge, a burette was connected to the pressure transducer at the end of the test run, if the wave amplitude of that run was larger than 0.254 cm (0.1 inch). A column of water corresponding to the mean wave amplitude was introduced into the burette. The difference between the readings shown on the recorder before and after the connection of the burette
was considered as the experimental error. This difference was found to be less than 1% of the mean wave amplitude.

4.4.5 Temperature and Specific Humidity Profile's

Temperature and specific humidity traverses were taken simultaneously with the velocity measurements in some of the experimental runs. These measurements were made using a dry bulb and a wet bulb copper-constantan thermocouple. In addition a thermocouple was used to measure the temperature at the mean water level. The outputs of these thermocouples were time-averaged and recorded using a Hewlett-Packard Model 2010K high-speed data acquisition system.

The ranges of temperature and specific humidity at the center line of the tunnel were 21.1°C to 27.8°C (70°F to 82°F) and 26.3 g/g to 81.6 g/g, respectively.

4.5 Evaluation of Shear velocity, Dynamic Roughness, Drag Coefficient, Temperature and Specific Humidity

4.5.1 Shear velocity, Dynamic Roughness and Drag Coefficient

The measured velocity profiles are shown in Figs (4.9) to (4.19). The shear velocity and the dynamic roughness of each profile were evaluated by fitting the measurements to the equation

\[
\frac{u}{u_\tau} = \frac{1}{k} \frac{\nu}{z_0}
\]  \hspace{1cm} (2.8)

Based on the shear velocity, the drag coefficient was calculated from the definition

\[
C_i = 2 \left( \frac{u}{u_i} \right)^2
\]  \hspace{1cm} (2.11)
These results together with the corresponding fetch and wave amplitude are listed in Table (4.1).

4.5.2 Temperature and Specific Humidity

The measured temperature profiles are shown in Figs. (4.20) to (4.21). Based on the measured dry and wet bulb temperatures, the specific humidity was evaluated using a psychrometric chart. The resultant profiles are shown in Figs. (4.22) to (4.23). In addition, the temperature and the specific humidity at \( y = 0 \) and \( y = z_0 \) are also tabulated in Table (4.2).

4.6 Discussion

4.6.1 Investigation of Two-Dimensional Flow

The two dimensionality of the flow near the centerline of the air-water tunnel was investigated. For this purpose, the vertical velocity distributions taken at the centerline and at 3.81 cm (1.5 in) away from the centerline of the air-water tunnel are plotted in Fig. (4.24). As shown in the figure, deviations from the centerline velocity of less than 1% were obtained at 3.81 cm of both sides of the centerline. Similar investigations were also made for the temperature and the specific humidity distributions. As shown in Fig. (4.25), the measurements taken 3.81 cm away from the centerline are less than the centerline values by about 1% and 4% for the specific humidity and temperature, respectively. A part of the discrepancy between the temperature measurements may be attributed to the heat transfer through the working section which is not insulated.
4.6.2 Uncertainty Analysis

Errors in the study were minimized by suitable corrections and frequent checks on the calibration of various instruments.

Estimates of the random errors caused by the moving water surface, based on an assumed normal probability distribution, are calculated from the equation

\[ D_s = \frac{\sigma_s}{(N-1)^{1/2}} \]  \hspace{1cm} (4.1)

where

\[ \sigma_s = \left[ \frac{\sum (s_{in} - s_j)^2}{N} \right] \]  \hspace{1cm} (4.2)

The estimated errors based on Eqs. (4.1) and (4.2) are listed below:

1) Probable error in estimating the mean wind speed is less than 2% of the mean wind speed.

2) Probable error in estimating the mean wave amplitude is about 1% of the mean wave amplitude.

3) Probable error in estimating the mean temperature is less than 1% of the mean temperature.
CHAPTER 5

DISCUSSION

The analysis has been presented in two parts:

The first part which is referred to as Method I deals with the drag and the transfer coefficients over a large body of water roughened by wind. It has been seen that by generalizing the results available in the literature of momentum, heat and mass transfer under atmospheric conditions, the dimensionless Froude Number, \( \frac{u}{\sqrt{g L}} \), can be universally applied.

The second part which is referred to as Method II presents the study of turbulent momentum, heat and mass transfer over a wavy water surface generated in an air-water tunnel. The analysis is much more rigorous and the validity of the theory is more thoroughly tested than that in the first part, because a complete set of experimental data could be obtained from the air-water tunnel.

The analytical results predicted by the two methods will be compared with the experimental measurements in the following sections. Based on these comparisons, the applicability and the limitation of the theory will be discussed.

5. Method I - Smooth-wavy Surface Analogy Approach

5.1.1 Drag Coefficient

The functional relationship of transfer and drag coefficients between a smooth and a rough surface is given by Eq. (3.31). This result is identical to the empirical formula obtained by
Numer(93) for air flow in rough pipes. This equation indicates that the amount of heat or mass transfer is directly proportional to the square root of the drag coefficient over a wavy water surface.

Drag coefficients as shown in Fig. (3.4) can be correlated with wind speeds and fetches using the Froude number with the fetch as the characteristic length. It can be seen that increasing the wind speed or the fetch by a factor of 2 will cause a change in the drag coefficient by an amount of about 20%. Since most measurements obtained thus far, in the field, scatter beyond 20% of their mean values, the dependence of wind speed, fetch and hence Froude Number on the drag coefficient is difficult to detect, unless a systematic comparison of these data is made. For this purpose, some of the measured drag coefficients were plotted against the Froude Number, $u \sqrt{gL}$ in Fig. (5.1). The result clearly shows that the drag coefficient increases with the Froude Number.

For comparison, the calculated drag coefficient based on Eq. (3.34) is also shown in Fig. (5.1). The agreement between the calculated and the measured drag coefficients is surprisingly good in view of the crude assumptions that had to be made in the derivation of Eq. (3.34).

An attempt was made by Smith(73) to study the influence of the atmospheric stability on the drag coefficient. He could not establish any definite relationship to link the two parameters. As his measurements were conducted under stable, unstable and neutral stable conditions, the close agreement between his results and Eq. (3.34) may indirectly indicate that Eq. (3.34)
may also be valid under all conditions, especially at high wind speed.

5.1.2 Transfer Coefficients

The transfer coefficients predicted by Eqs. (3.35) to (3.38) were shown in Fig. (3.5). These coefficients are a function of both the Froude Number and the Reynolds Number for the dimensionless fetch, \( L^+ \) less than 63200 and are a function of the Froude Number alone for \( L^+ \) larger than or equal to 63200. The fetch corresponding to \( L^+ = 63200 \) varies according to wind speed. They can be translated into the values of 2.3 m and 13.8 m for the wind speeds of 15.7 m/sec and 2.24 m/sec. Therefore, for practical purposes, these coefficients can be considered as a function of the Froude Number only.

The calculated transfer coefficients expressed in terms of the heat transfer and the mass transfer Stanton Numbers are compared with the field measurements collected by Kitaigorodskii and Volkov (86) in Fig. (5.2). As shown, the Stanton Numbers predicted by Method I is in very good agreement with the field measurements at low roughness Reynolds Number defined by \( \frac{L}{u_m} \). At high roughness Reynolds Number, Method I appears to underestimate the rates of heat and mass transfer. On the basis of Eq. (3.34) and the logarithmic velocity profile it can be seen that at a constant velocity, in the region not too far from the edge of the water body, the roughness Reynolds Number decreases as the fetch increases. Thus, the poor agreement in high roughness Reynolds Number can be attributed to the omission of the variation of \( u_m \) with \( x \) in the mathematical model. On the other
hand, the close agreement obtained at low roughness Reynolds Number appears to indicate that the mathematical model used in Method I is quite adequate for the prediction of the rates of heat and mass transfer at larger fetches.

It is a well-known fact that transfer mechanism is very much dominated by the condition near the surface. In this region which extends up to 8 m above the mean water level, it has been found that the atmospheric stability appears to be of little practical significance so far as the determination of the daily evaporation is concerned \( \text{(3, 79)} \). Therefore if the velocity corresponding to a distance of 2 m above the mean water level is used in Eqs. (3.35) to (3.38), these equations may be expected to be valid under the conditions of neutral stable as well as nearly neutral stable.

5.2 Method II - Semi-empirical Approach

5.2.1 Mean Velocity Profile

The velocity of air was measured by obtaining the time-
averaged value of the instantaneous measurements using a strip-chart recorder. Since the probe was never closer than 5 \( \text{ms} \) to the mean water level, the error induced by the moving boundary which caused a continuous change in measuring height, should be negligible according to the study by Chambers et al. \( \text{(19)} \).

The measured mean velocity profiles are shown in
Figs. (4.9) to (4.19). It is seen that the experimental data can be represented quite well by the equation.

\[
\frac{u}{u_T} = \frac{1}{k} \ln \frac{y}{z_0}
\]  
(2.8)
In this study, about 10% of the measured profiles deviate slightly from the logarithmic profile. Of the 10%, except those stations that are near the mean water level and the centerline of the tunnel, deviations from the logarithmic velocity profile are less than 3%. Further examination on the 10% of the measured velocity profiles indicates that these measurements were conducted at large fetch under the condition of high centerline air velocity. Since the presence of water spray is usually observed at large fetch and high centerline air velocity, the poor agreement between the measured data and the logarithmic law near the mean water level may be caused by the partial blockage of the static pressure probe by water mist.

The air-water temperature difference does not appear to affect the mean velocity profile under the flow conditions of the centerline air velocities ranging from 3 to 85 m/sec and an air-water temperature difference up to 21°C. This conclusion was also reported by Chambers et al. (19) The values of Ri at 10 cm are less than 5 x 10^-4 which also indicates that the influence of thermal stratification is negligible.

5.2.2 Drag Coefficient and Dynamic Roughness

The functional relationships among $a_{\text{rms}} / L$, $Fr_y$, and $u^+$ as well as between $a_{\text{rms}} / L$ and $Fr_y$ are defined by Eqs. (3.39) and (3.40) respectively. These relationships were obtained by fitting the experimental data of the present study with the curves derived on the basis of principles of dimensional analysis. As shown in Figs. (3.6) and (3.7), the average deviations between the experimental data and the curves defined by Eqs. (3.39) and (3.40) are about 15% and 50% with reference to the two equations. These agreements can be considered to be very satisfactory in
view of the fact that the deviations of the drag coefficient which is inversely proportional to the square of \( u^+ \) are as large as 300% as reported in Table (2.1).

The drag coefficients predicted by Eqs. (3.39) and (3.40) are shown in Fig. (3.8). These graphs show that the drag coefficient is a function of both \( Fr_y \) and \( Fr_{Ly} \).

The dynamic roughness can also be calculated from Eqs. (3.39) and (3.40) utilizing the logarithmic velocity profile. The results are shown in Fig. (3.9). As shown in the figure, the dynamic roughness decreases as the \( Fr_y \) decreases and as \( Fr_{Ly} \) increases.

5.2.3 Comparison of Flow Parameters

5.2.3.1 Comparison with Experimental Data of the Present Study

The predicted and measured values of drag coefficient, dynamic roughness and \( u^+ \) are compared in Figs. (5.3), (5.4) and (5.5) respectively. As shown in these graphs, except for the dynamic roughness, close agreement between the analytical and the experimental results is obtained. The poor agreement between the calculated and measured dynamic roughness arises from the error involved in estimating the dimensionless velocity \( u^+ \). For the case of the drag coefficient the error increases by about a factor of two; for the case of the dynamic roughness, it increases exponentially.

5.2.3.2 Comparison with Experimental Data of Other Authors

The calculated drag coefficient is compared with the
experimental data obtained by Gottifred et al., Chambers et al. (19) and Hidy et al. (76) in Fig. (5.6). The work of Gottifred et al. (108) is very interesting in view of the method used for estimating the drag coefficient. The shear velocity was calculated by fitting the experimental data to the velocity profile

\[ u^+ = y^+ \quad 0 < y < y_1 \]  

(5.1)

Here \( y_1 \) is the thickness of the laminar sublayer which exists only if the wave is totally submerged within the layer. This method is superior than the profile method in that \( u_+ \) can be uniquely calculated. The reliability of the method depends upon the accuracy in the velocity measurements within the laminar sublayer which, in turn, is governed by the wave height. In general, if the air velocities are measured at small fetch under the condition of low wind, the accuracy of \( u_+ \) calculated based on these measurements should be highly dependable. Based on the above consideration, the result obtained by Gottifred (108) at low fetch (46 cm) and low air velocity \((u_* < 3.15 \text{ m/sec})\) was selected as the base for checking the validity of the analytical solution. As shown in Fig. (5.6) the agreement is extremely good.

The data obtained by Chambers et al. (19) and by Hidy et al. (76) are also shown in Fig. (5.6). As shown, the experimental data deduced using the profile method spread quite evenly on both sides of the 45° line. In general, except for the data deduced using the integral method, the deviation between the calculated and the measured drag coefficients is within 100% of the calculated value. Unlike the equipments used by the.
author or by Gottifred et al\(^{(108)}\) the air–water tunnels used by Chambers\(^{(19)}\) and Hidy\(^{(76)}\) do not have a long entrance section prior to the water tank. Since the drag coefficient is strongly affected by the air velocity profile over it as well as the local surface roughness, lack of an entrance section can change these two parameters considerably. The differences in the flow conditions under which the measurements were made are believed to result in, at least to a great extent, the wide gap existing between the analytical and the measured drag coefficients.

Charnock\(^{(20)}\) suggested an expression to link the dynamic roughness and the shear velocity as

\[
b = \frac{z_o^g}{u^2_\tau}
\]  

(5.2)

Based upon a large collection of wind profile measurements obtained in laboratory as well as in ocean, Wu\(^{(21)}\) suggested the values of 0.0112 and 0.0156 for \(b\) for the laboratory and ocean conditions respectively. To investigate the validity of this relationship, values of \(b\) deduced from the study are shown in Fig. (5.7). For comparison the values of \(b\) obtained by Chambers and Hidy are also shown in this figure. The result indicates that \(b\) is not a constant, but it is a function of fetch. Consequently, Eq. (5.2) can considerably overestimate \(z_o^g\) and hence \(C_i\) at small fetch if \(b\) is assumed to be constant as suggested by Wu\(^{(21)}\).

5.2.4 Temperature and Specific Humidity Profiles

The calculated temperature and specific humidity profiles are shown in Fig. (5.8). It is clearly shown in this figure that these profiles are functions of both \(x^{++}\) and \(y^{++}\).
The dependence of the profile on \( x^{++} \) becomes less pronounced as \( x^{++} \) increases.

These profiles were normalized using the temperature and the specific humidity at the distance \( z_o \) above the mean water level. The relationship between these values and those at the mean water level is shown in Fig. (5.9). An inspection of Fig. (5.9) shows that the differences in temperature as well as specific humidity between these two levels are higher at lower values of \( x^{++} \) than those at larger \( x^{++} \). This implies that both the temperature and the specific humidity at \( y = z_o \), i.e. \( y^{++} = 1 \), approach to their values at the mean water level, respectively at very large fetch where \( \frac{dx^{++}}{dx} \) approaches to constant.

5.2.4.1 Comparison with Experimental Results

The calculated and the measured profiles are compared in Figs (5.10) to (5.12). Very good agreement is achieved as shown in these figures. It is also shown in these graphs that, except for Runs Nos. 47, 62, 68, 73 and 92, the normalized temperature and specific humidity profiles coincide with each other. This result supports the validity of the assumption of unity for turbulent Lewis Number.

There is a fundamental difference in the normalization of the temperature and the specific humidity profiles between the present study and that of other investigators \( (77, 78) \). In the present study the temperature and specific humidity at \( y = z_o \) instead of the values at the water surface were used for the normalization of the profiles. As a result, for a wavy water surface
A significant discrepancy occurs between the normalized profiles based on the two different references. This dissimilarity in profiles, however, becomes less pronounced as the dimensionless fetch \( x^+ \) increases.

As shown in Fig. (5.9), the differences in temperature as well as specific humidity decrease as the dimensionless fetch \( x^+ \) increases. The choice of the reference may not have any effect on the presentation of the profiles, but it may lead to serious error in the evaluation of the transfer coefficients. These coefficients can be calculated either by evaluating the first derivative of the profile at the reference point or by estimating the enthalpy thickness for the case of heat transfer and its counterpart for the case of mass transfer. In either case, a complete description of the temperature or the specific humidity profiles, especially in the region where these profiles undergo a sharp change, is essential. Depending upon the surface roughness, for a wavy water surface as shown in Fig. (5.9), the differences in temperature and specific humidity between the levels \( y = z_0 \) and \( y = 0 \) are quite substantial. If the reference point is chosen to be the mean water level, because the temperature and specific humidity distributions in the region bounded by \( y = 0 \) and \( y = z_0 \) is not known, the transfer coefficients cannot be accurately determined. Besides, the velocity distribution of air in this region is also unknown. On the other hand if the reference point is chosen at the height \( z_0 \) above the mean water level where the air velocity is zero, all the profiles from this level upwards throughout the boundary layers are fully described. The calculation of the transfer coefficients based on this reference point can then be easily performed.
5.2.5 Comparison of Analytical Stanton Numbers with Field Measurements

The heat and mass transfer Stanton Numbers predicted by Method II were evaluated using different Prandtl Numbers and Schmidt Numbers. The Stanton Numbers based on $P_r = 0.72$ and $S_c = 0.6$ are shown in Fig. (5.13) whereas those based on $P_r = S_c = 1$ are shown in Fig. (5.14). As shown in these two figures, the Stanton Number is a function of both the roughness Reynolds and the Froude Number defined by $u_r \sqrt{g y}$. These figures also show that the Stanton Number based on $P_r = S_c = 1$ can be as much as 30% lower than that based on $P_r = 0.72$ and $S_c = 0.6$. Although both Prandtl Number and Schmidt Number vary with air temperature, their numerical values are approximately equal to 0.72 and 0.6 respectively for the temperature range under consideration. The Stanton Numbers based on $P_r = S_c = 1$ are shown so that they can be compared with the field data which are also evaluated based on $P_r = S_c = 1$.

The Stanton Numbers predicted by Method I and those obtained in field are also shown in Figs. (5.13) and (5.14). The field data were compiled by Kitaigorodskii and Volkov (86) from the profile measurements of other investigators (55, 56). Because the parameter $u_r \sqrt{g y}$ was not considered as a variable in the evaluation of the field data, a rigorous comparison between the theory and the field measurements cannot be made. However, it can be seen from Figs. (5.13) and (5.14) that the agreement between the analytical and the measured Stanton Numbers are quite satisfactory.
Figs. (3.13) and (3.14) show that there is a large scatter in the field data. Apart from the experimental errors, the scatter in the field data may be partially attributed to that \( Fr_y \) is not considered as one of the dominant parameters by those investigators. The influence of \( Fr_y \) on Stanton Number can be clearly observed from the analytical results shown in Figs. (3.13) and (3.14).

In addition, Figs. (3.13) and (3.14) show that the Stanton Number predicted by Method II is higher than that predicted by Method I. The difference is larger at high roughness Reynolds Number than that at low roughness Reynolds Number. As the approach used in Method II is direct and the validity of the model is thoroughly checked, it is reasonable to believe that the result predicted by Method II is more reliable than that predicted by Method I.
CHAPTER 6

CONCLUSIONS AND COMMENTS

6.1 Conclusions

A semi-analytical study has been made on the mechanisms of momentum, heat and mass transfer over a water surface subjected to a flow of turbulent air. The airflow over a wavy water surface has been found to a certain extent, to exhibit the properties of flow over a rough surface. Consequently, the drag coefficient has been found to follow a similar form as the empirical formula deduced for the friction factor in a rough pipe with the relative roughness replaced by the ratio of wave height to fetch. Good agreements between the analytical and measured drag coefficients have been achieved under conditions specified by Methods I and II.

The heat and mass transfer Stanton Numbers have been found to be a function of \( Fr_{Ly} \) and \( Fr_y \). The predicted Stanton Number has also been found in good agreement with the field results available in literature.

The main conclusions which may be drawn from the study are summarized as follows:

1) The airflow over a wavy water surface exhibits, to a certain extent, the properties of flow over a rough plate if the effect of fetch on the physical roughness is accounted for.

2) The mean wind velocity profile over a wavy water surface can be satisfactorily described by the logarithmic velocity profile.
3) The drag coefficient over a wavy water surface under atmospheric condition increases with \( \text{Fr}_L \).

4) The method of analysis described in Method II leads to promising result in generalizing the Stanton Number obtained under laboratory conditions to field conditions.

5) The agreement between the predicted Stanton Numbers and the available field data is satisfactory.

6) Additional field data are required to test the accuracy and the limitation of the mathematical models used in Methods I and II. These data should provide enough informations for the evaluation of \( \text{Fr}_L \), \( \text{Fr}_{Ly} \), \( \text{Fr}_y \) and roughness Reynolds Numbers.

7) The atmospheric stability appears to be of little practical significance for wind velocity profile, drag coefficient and Stanton Number. Further study is required.

6.2 Comments

The work presented in the preceeding chapters and the accompanying appendices can only be considered as an introduction to the investigation of the problem of transport phenomena over a large but finite body of water having a wavy surface induced by wind. We have, hopefully demonstrated that it is possible to approach the problem in a more general way than the widely adopted method based on empirical correlation, by using a simple two-dimensional model with an universal velocity profile as the basis of our calculations. We have also shown that this model can be refined on the basis of the experimental data, to account for the
variation of the flow parameters such as \( u_\tau \) and \( z_0 \) with \( x \).

The main variable that has been left out in the present analysis is the atmospheric stability. In laboratory, thermal stability does not appear to have any effect on the drag coefficient and the Stanton Numbers. This conclusion is supported by the close agreement between the dimensionless temperature and specific humidity profiles as well as the validity of the Reynolds similarity principle. In field, although the studies of Smith (1973) and Marciano et al (1983) lead to similar conclusions, the evidence is not as obvious as that in laboratory. Further studies may not improve the situation because of the difficulties involved in obtaining reliable experimental data in field.

Fortunately the mechanisms of turbulent momentum, heat and mass transfer are dominated by the flow conditions in the region not too far from the water surface. In this region the effect of atmospheric stability is greatly reduced according to Priestley (1979). If the formulae for drag coefficient and Stanton Numbers are deduced based on the flow parameters evaluated in this region, the influence of atmospheric stability may be disregarded.
Appendix A.1 Boundary Layer Approximations

Eqs. (3.8) and (3.9) are so involved that a simplification is necessary in order to obtain solutions. Such simplification can be made by retaining only the dominant terms which are identified by estimating the order of magnitude of various terms in the equations, as is often done in fluid flow studies. For this purpose, it is desirable to make these equations dimensionless by the use of the reference length, $x_r$, velocity, $u_r$, temperature, $t_r$, specific humidity, $m_r$, and kinematic viscosity $v$. $X_r$ is taken as the length in the direction of down wind $x$, where neither the temperature nor the specific humidity profile varies with $x$. $u_r$, $t_r$ and $m_r$ are the velocity, temperature and specific humidity at an arbitrary reference height, $y_r$. This reference height could be the boundary layer thickness under laboratory conditions.

In order to apply the boundary layer approximations, one has to assume that the ratio $y_r/x_r$ is very small with respect to unity ($y_r/x_r = \delta << 1$). With the reference parameters, the following dimensionless quantities are obtained:

\[
\begin{align*}
  u^* &= \frac{u}{u_r}; t^* &= \frac{t}{t_r}; \quad x^* = \frac{x}{x_r}; \quad v^* = \frac{v}{u_r}; \quad y^* = \frac{y}{x_r}; \quad w^* = \frac{w}{x_r}; \\
  z^* &= \frac{z}{x_r}; \quad c^* = \frac{\sqrt{\rho c p}}{v}; \quad \varepsilon^* = \frac{\varepsilon_{Hx}}{v}; \quad \varepsilon_{Hy}^* = \frac{\varepsilon_{Hy}}{v}; \quad \varepsilon_{Hz}^* = \frac{\varepsilon_{Hz}}{v}; \\
  m^* &= \frac{m}{m_r}; \quad \beta = \frac{\beta}{s c}; \quad \varepsilon_{Dx}^* = \frac{\varepsilon_{Dv}}{v}; \quad \varepsilon_{Dv}^* = \frac{\varepsilon_{Dz}}{v}; \quad Re^* = \frac{x_r u_r}{v}.
\end{align*}
\]

(A.1.1)
Since \( u, t \) and \( m \) approach to \( u^*, t^*, \) and \( m^* \) as \( y \) and \( x \) approach to \( y^* \) and \( x^* \) respectively, \( u^*, t^*, \) and \( m^* \) are each of order 1.

The boundary layer thicknesses for \( u, t \) and \( m \) are not necessarily the same, however they must be of the same order of magnitude, \( y^* \). It follows that \( y^* \) is of order \( \delta \) or less where \( \delta \) is the ratio of \( y^* \) to \( x^* \). Furthermore, as the water surface extends indefinitely in the directions \( x^* \) and \( z \), the two length parameters are, therefore of the same order of magnitude. For this reason \( z^* \) is also of order 1.

A flow field that has a very small reference height, \( y^* \) in comparison with \( x^* \) and a principal flow direction parallel to the \( x \) axis, requires that the velocity components \( v \) and \( w \) are of much smaller order of magnitude than the velocity component \( u \). Studies on flow over a plane surface \( (87,91) \) indicate that \( v^* \) is of order \( \delta \) or less and hence \( w^* \) may also have an order of \( \delta \).

Based upon the above considerations an order of magnitude evaluation of Eqs. (3.8), (3.9) and (3.10) can be made. The order of magnitude of each term is written beneath each of the equations which are written in dimensionless form using the dimensionless quantities as defined by Eq. (A.1.1).

Thus, the energy equation, Eq. (3.8) becomes

\[
\begin{align*}
    u^* \frac{\partial t^*}{\partial x^*} + v^* \frac{\partial t^*}{\partial y^*} + w^* \frac{\partial t^*}{\partial z^*} &= \frac{1}{\text{Re}^*} \left( \frac{1}{\text{Pr}^*} \frac{\partial^2 t^*}{\partial x^*} + \frac{1}{\text{Pr}^*} \frac{\partial^2 t^*}{\partial y^*} + \frac{1}{\text{Pr}^*} \frac{\partial^2 t^*}{\partial z^*} \right) \\
    \delta & \quad \delta & \quad \delta & \quad \delta & \quad \delta & \quad \delta \\
\end{align*}
\]
\[ \frac{\partial}{\partial x} \left( \frac{1}{Re_x} \right) + \frac{\partial}{\partial y} \left( \frac{1}{Sc_x + \varepsilon_{Dx}} \right) \frac{\partial m^*}{\partial z} + \frac{\partial}{\partial z} \left( \frac{1}{Sc_x + \varepsilon_{Dx}} \right) \frac{\partial m^*}{\partial y} \]

\[ + \frac{\partial}{\partial z} \left( \frac{1}{Sc + \varepsilon_{Dy}} \right) \frac{\partial m^*}{\partial y} \]  \hspace{1cm} (A.1.2)

Diffusion equation, Eq. (3.9) becomes

\[ u^* \frac{\partial m^*}{\partial x} + v^* \frac{\partial m^*}{\partial y} + w^* \frac{\partial m^*}{\partial z} = \frac{1}{Re_x} \left[ \frac{\partial}{\partial x} \left( \frac{1}{Sc_x + \varepsilon_{Dx}} \right) \frac{\partial m^*}{\partial x} + \frac{\partial}{\partial y} \left( \frac{1}{Sc_x + \varepsilon_{Dx}} \right) \frac{\partial m^*}{\partial y} \right. \]

\[ + \left. \frac{\partial}{\partial z} \left( \frac{1}{Sc + \varepsilon_{Dy}} \right) \frac{\partial m^*}{\partial y} \right] \]  \hspace{1cm} (A.1.3)

and equation of continuity, Eq. (3.10) becomes

\[ \frac{\partial u^*}{\partial x} + \frac{\partial v^*}{\partial y} + \frac{\partial w^*}{\partial z} = 0 \]  \hspace{1cm} (A.1.4)

Now it is possible to estimate the order of magnitude of each term in order to be able to drop small terms and thus to achieve the desired simplification of the equations.

In Eq. (A.1.2), the first two terms on the left hand side are each of order 1 and the third term is of order \( \delta \). Since as assumed, \( \delta \) is much smaller than 1, the third term w \( \frac{\partial m^*}{\partial y} \) can
be neglected with respect to either one of the first two terms. Furthermore, on the right-hand side of Eq. (A.1.2), the first and the third terms can be neglected with respect to the second term \( \frac{\partial}{\partial y^*} \left( \frac{1}{Pr} + \varepsilon_{Hy}^* \right) \frac{\partial t^*}{\partial y^*} \).

Likewise, in Eq. (A.1.3) the third term on the left-hand side can be neglected with respect to the first two terms. On the right-hand side, only the second term is retained.

Finally in Eq. (A.1.4), the third term \( \frac{\partial w^*}{\partial z^*} \) can be dropped from the equation.

Delete the terms based on the above discussions and revert to dimensional variables, Eqs. (A.1.2), (A.1.3) and (A.1.4) become:

Energy equation

\[ \frac{\partial}{\partial x} \left( \frac{\partial t}{\partial x} \right) + \frac{\partial}{\partial y} \left( \frac{\partial t}{\partial y} \right) = \frac{\partial}{\partial y} \left( \alpha + \varepsilon_{H} \right) \frac{\partial t}{\partial y} \]  

(A.1.5)

where \( \alpha = \lambda / \rho c_p \)

Diffusion equation

\[ \frac{\partial m}{\partial x} + \frac{\partial m}{\partial y} = \frac{\partial}{\partial y} \left[ (S + \varepsilon_D) \frac{\partial m}{\partial y} \right] \]  

(A.1.6)

Continuity equation:

\[ \frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \]  

(A.1.7)
Appendix A.2

A.2.1 First Derivative of \( F \) with respect to \( y^+ \) at Water Surface.

Because the method of numerical differentiation involves errors of great magnitude, the following method was used to evaluate \( \frac{dF}{dy^+} \bigg|_{y^+=0} \). This method involves a trial and error calculation together with the method of least squares for curve fitting. The technique is given as follows:

(a) Since the three eddy diffusivities were roughly to be identical, the dimensionless temperature or concentration profile was assumed to take the form:

\[
F = A_6 \ln (1 + A_7 y^+) + A_8 [1 - \exp (-\frac{y^+}{11}) - (\frac{y^+}{11}) \exp (-0.33 y^+)]
\]

where \( A_6, A_7 \) and \( A_8 \) are functions of \( x^+ \).

(b) Since \( A_7 \) has to be greater than zero, it was first assumed to be 0.05. With \( A_7 \) selected, \( A_6 \) and \( A_8 \) were calculated by fitting \( F \)'s and \( y^+ \)'s into the above equation using the method of least squares. Using the above results the standard error of estimate was also calculated.

(c) The calculation was continued with \( A_7 \) increased by an amount of 0.05 till the minimum standard error of estimate was obtained. The values of \( A_6, A_7 \) and \( A_8 \) associated with the minimum standard error of estimate were taken as the solution.
Once the dimensionless temperature or concentration profile was obtained, the value of \( \frac{\partial F}{\partial y} \bigg|_{y^+ = 0} \) was calculated from the equation:

\[
\frac{\partial F}{\partial y} \bigg|_{y^+ = 0} = A_6 A_7
\]

Again using the values obtained from Eq. (A.2.1) for various \( \chi^+ \)'s and the method of least squares, \( \frac{\partial F}{\partial y} \bigg|_{y^+ = 0} \) can be expressed by the following two equations:

For \( 0 \leq L_s^+ \leq 63200 \)

\[
\frac{\partial F}{\partial y} \bigg|_{y^+ = 0} = 0.202 (L_s^+) - 0.168 \tag{A.2.3}
\]

For \( L_s^+ = 63200 \)

\[
\frac{\partial F}{\partial y} \bigg|_{y^+ = 0} = 0.0315 \tag{A.2.4}
\]

The upper limit of Eq. (A.2.3) and the lower limit of Eq. (A.2.4) were selected so that the curves described by the two equations intersect at \( L_s^+ = 63200 \).
A.2.2 Relationship between Transfer coefficients 
and Drag coefficients

The mechanism of turbulent heat or mass and momentum transfer from a large body of water is governed by the equations:

\[
\frac{\partial m}{\partial x} = \frac{\partial}{\partial y} \left[ (\delta + \varepsilon_D) \frac{\partial m}{\partial y} \right] \quad \text{(3.16)}
\]

\[
h_D = [ - \alpha / (m_0 - m_r) ] \frac{\partial m}{\partial y} \bigg|_{y=0} \quad \text{(A.2.5)}
\]

\[
\frac{\tau_o}{\rho} = (\nu + \varepsilon_M) \frac{\partial u}{\partial y} \quad \text{(2.4)}
\]

From the boundary condition of constant mass flux, we obtain

\[
\frac{\partial m}{\partial x} = \frac{dm}{dx} = \frac{dm}{dx} = \frac{dm}{dx} = \frac{10}{dx} \quad \text{(A.2.6)}
\]

From Eq. (2.4), we obtain

\[
u = \frac{\tau_o}{\rho} \int_0^y \frac{1}{(\nu + \varepsilon_M)} \ dy \quad \text{(A.2.7)}
\]

Substituting Eqs. (A.2.6) and (A.2.7) into Eq. (3.16) and letting

\[
\varepsilon_D = \varepsilon_M \quad \text{yields}
\]

\[
\frac{dm}{dy} = \frac{1}{\delta + \varepsilon_M} \int_0^y \int_0^y \frac{dy}{\nu + \varepsilon_M} \ dy \quad \text{(A.2.8)}
\]

In order to obtain a relationship between \( \frac{\partial m}{\partial y} \bigg|_u \) and \( \frac{\partial m}{\partial y} \bigg|_s \)

* For the case of heat transfer Eq. (A.2.5) becomes:

\[
h_c = [- \lambda / (t_o - t_r)] \frac{\partial t}{\partial y} \bigg|_{y=0}
\]
we have to know the relationship between \( \frac{1}{\nu + \varepsilon_M} \) and \( \frac{1}{\nu + \varepsilon_M} \) \\
This can be shown from the velocity profiles for rough and smooth surfaces as follows:

The velocity profiles for rough and smooth surfaces are

1. **Rough surface**

   \[ u^+ = 2.5 \ln \left( \frac{\nu}{\nu_o} \right) \]  
   \[ (2.8) \]

   Taking the derivative, we have
   \[ \frac{du^+}{dy^+} = \frac{2.5}{y^+} \]  
   \[ (A.2.9) \]

2. **Smooth surface (Reynolds's universal velocity profile)**

   \[ u^+ = 2.5 \ln \left( 1 + 0.4y^+ \right) + 7.8 \left[ 1 - \exp \left( -\frac{y^+}{11} \right) \right] \]

   \[ - \left( \frac{y^+}{11} \right) \exp \left( -0.33y^+ \right) \]  
   for \( y^+ \geq 0 \)  
   \[ (2.6) \]

   In the turbulent layer where \( y^+ \) is much greater than zero, this profile can be rewritten as:

   \[ u^+ = 2.5 \ln \left( y^+ \right) + 2.5 \ln \left( 0.4 \right) + 7.8 \]  
   \[ (A.2.10) \]

   Taking the derivative, we have
   \[ \frac{du^+}{dy^+} = \frac{2.5}{y^+} \]  
   \[ (A.2.11) \]

   From Eq. (2.4) it can be shown that

   \[ \nu + \varepsilon_M = \frac{\nu}{\frac{du^+}{dy^+}} \]  
   \[ (A.2.12) \]
Substituting Eq. (A.2.9) and Eq. (A.2.11) into Eq. (A.2.12) respectively we have

\[
(v + \varepsilon_M^w)_{w} = \left(\frac{u^w}{u^s}\right) \frac{w}{s} \cdot (v + \varepsilon_M^s)_{s}
\]  

(A.2.13)

Substituting Eqs. (A.2.8) and (A.2.13) into Eq. (A.2.5) and noting that at the air water interface, i.e. \(y = 0\), \((v + \varepsilon_M^w)_{w} = v + \varepsilon_M^s = 0\), we have,

\[
\left(\frac{h_D^w}{h_D^s}\right)_w = \frac{\frac{dm_{10}}{dx}_{w}}{\frac{dm_{10}}{dx}_{s}} \cdot \left(\frac{u^w}{u^s}\right)_w = \left(\frac{u^w}{u^s}\right)_w
\]  

(A.2.14)

Since the atmosphere can be considered as a sink or source with a very large strength, it can be assumed that the mass flux generated by the water body has almost no effect on the specific humidity of the atmosphere at an elevation of approximately 10 meters. In other words, the ratio

\[
\frac{\frac{dm_{10}}{dx}_{w}}{\frac{dm_{10}}{dx}_{s}}
\]

is essentially equal to unity.

Therefore,

\[
\left(\frac{h_D^w}{h_D^s}\right)_w = \left(\frac{u^w}{u^s}\right)_w = \left(\frac{C_{10}^w}{C_{10}^s}\right) \frac{1}{2}
\]  

(A.2.15)
A.2.3 Drag Coefficient Over a Wavy Water Surface

The relationship between the drag coefficient and the significant wave height given by Eq. (3.33) is

\[ \frac{1}{\sqrt{\left( C_{10} \right)_w}} = A_4 \ln \frac{L}{H_{1/3}} + A_5 \]  

(3.33)

where \( A_4 \) and \( A_5 \) are two constants which are best evaluated using experimental data obtained in field at various fetches and wind speeds. Apart from economical considerations the large scatter revealed in the available measurements indicates that it is improbable that reliable data will be obtained. The two constants are therefore, estimated as follows.

The empirical formula for calculating the friction factor for fully developed flow in rough pipe \(^{(110)}\) is,

\[ \frac{1}{r} = 1.735 \ln \left( \frac{D_c}{K_s} \right) + 1.14 \]  

(A.2.16)

where \( D_c \) and \( K_s \) are the diameter of the pipe and the equivalent sand roughness. To apply the above equation for a rough flat plate, firstly, the radius of the pipe is replaced by \( \delta \) the thickness of the turbulent boundary layer. Replacing \( 2 \delta \) for \( D_c \) and \( C_{10\,w} \) for \( r \), Eq. (A.2.16) becomes,

\[ \frac{1}{\sqrt{\left( C_{10\,w} \right)_w}} = 1.735 \ln \left( \frac{2 \delta}{K_s} \right) + 1.14 \]  

(A.2.17)

The thickness of the turbulent boundary layer for a rough flat plate according to van der Hegge-Zijnen \(^{(111)}\) is

\[ \delta = 0.26 \frac{H_{1/3}}{\left( L/H_{1/3} \right)^{2/3}} \]  

(A.2.18)

Since the thickness of the turbulent boundary layer over a wavy water surface is unknown it may be estimated from Eq. (A.2.18) as an approximation.
From Eq. (3.32) for constant wind speed we obtain

$$H_{1/3} = L^{.464}$$  \hspace{1cm} (A.2.19)

Substituting Eqs. (A.2.18) and (A.2.19) into Eq. (A.2.17) yields,

$$\frac{1}{\sqrt{C_{10}}} = 1.735 \ln \left( A_{5} \frac{L^{.83}}{K_{s}} \right) + 1.14$$  \hspace{1cm} (A.2.20)

Since the equivalent sand roughness, $K_{s}$, is proportional to $H_{1/3}$, the significant wave height, Eq. (A.2.20) can be written as

$$\frac{1}{\sqrt{C_{10}}} = 1.735 \ln \left( A_{6} \frac{L^{.83}}{H_{1/3}} \right) + 1.14$$  \hspace{1cm} (A.2.21)

where $A_{6}$ is a constant. Since $L^{0.83}/H_{1/3}$ is a very large number in most cases, it is reasonable to assume that

$$A_{6} \frac{L^{0.83}}{H_{1/3}} \approx \frac{L}{H_{1/3}}$$

Therefore

$$\frac{1}{\sqrt{C_{10}}} = 1.735 \ln \left( \frac{L}{H_{1/3}} \right) + 1.14$$  \hspace{1cm} (A.2.22)

and in terms of Froude number

$$\frac{1}{\sqrt{C_{10}}} = 1.86 \ln \left( \frac{1}{F_{rL}} \right) + 11.2$$  \hspace{1cm} (A.2.23)
Appendix A.3

A.3.1 Functional Relationships Among \( u, a_{\text{rms}} \) and \( L \)

For air flow over a water surface roughened by wind, the mean velocity of air \( u \) at a distance \( y \) above the mean water level may be expressed by

\[
u = \phi_1(\zeta, L, \tau_\infty, a_{\text{rms}}, g)
\]

(A.3.1)

Let \( u, \zeta, L \) be the primary quantities, making a dimensional analysis, we have the following three dimensionless numbers:

\[
\begin{align*}
\text{for } \tau_\infty: \quad \pi_1 &= u \sqrt{\tau_\infty / \zeta} \\
\text{for } a_{\text{rms}}: \quad \pi_2 &= a_{\text{rms}} / L \\
\text{for } g: \quad \pi_3 &= u \sqrt{gL}
\end{align*}
\]

therefore,

\[
\frac{u}{\sqrt{\tau_\infty / \zeta}} = \phi_1 \left( \frac{a_{\text{rms}}}{L}, \frac{u}{\sqrt{gL}} \right)
\]

(A.3.2)

Since the power law velocity profile which is often used to approximate the logarithmic velocity profile, suggests that

\[
\frac{u}{y^n} = \frac{u_1}{y_1^n}
\]

the variation of \( u \) with \( y \) may be accounted for by dividing \( \pi_1 \) and \( \pi_2 \) by the quantity \( \psi^n \). Here the quantity \( \psi \) is
\[ \left( \frac{u_i y_i}{v} \right)^2 \frac{1}{\frac{u_i^2}{g y_i}} \] which is an expression for \( y_i \) in dimensionless form and \( n_1 \) is a constant. Introducing the modified \( \pi_1 \) and \( \pi_3 \) into Eq. (A.3.2) we have

\[ \frac{u_i}{\sqrt{\nu_0 \psi_i}} \frac{1}{n_1} = \phi_1 \left( \frac{a_{\text{rms}}}{L}, \frac{u_i}{\sqrt{gL \psi_i}} \frac{1}{n_1} \right) \]

Since studies\(^{(50,73)}\) appear to indicate that the quantity \( u_i \sqrt{\rho_0 / \rho} \) is not a strong function of fetch, the last term on the right-hand side of the equation may be approximated by

\[ \frac{u_i}{\sqrt{gL n_2 y_i (1-n_2)}} \]

Now the above equation may be rewritten as

\[ \frac{u_i}{\sqrt{\frac{\rho_0}{\rho} y_i}} \frac{1}{n_1} = \phi_1 \left[ \frac{a_{\text{rms}}}{L}, \frac{u_i}{\sqrt{gL n_2 (1-n_2)}}, \frac{1}{n_1} \right] \quad (A.3.3) \]

Furthermore, since Eqs. (3.33) and (3.34) suggest that \( u \sqrt{\rho_0 / \rho} \) varies with the logarithm of \( \frac{H}{1/3} \) or \( u / \sqrt{gL} \), the following equation may be established:

\[ \frac{u_i}{\sqrt{\frac{\rho_0}{\rho}}} \frac{1}{n_1} = A_9 \tan \left( \frac{A_{10} \frac{a_{\text{rms}}}{L}}{\sqrt{\frac{n_2}{gL y_i} (1-n_2)}} \right) \quad (A.3.4) \]

where \( A_9, A_{10}, n_1, n_2 \) and \( n_3 \) are constants which are to be determined based on the experimental data or the present study.
All the parameters except $a_{rms}$ on the right-hand side of Eq. (A.3.4) are easily obtainable. It is therefore, desirable to express the parameter $a_{rms}$ in terms of $u_i$ and $L$. Since the ratio $H_{1/3}/L$ as shown in Eq. (3.32) is a function of $u_i/\sqrt{gL}$, it may be assumed that similar relation also exists between $a_{rms}/L$ and $u_i/\sqrt{gL}$. A plot of $a_{rms}/L$ against $u_i/\sqrt{gL}$ based on the experimental data of the present study indicates that for a constant $L$, the logarithm of $a_{rms}/L$ varies linearly with $u_i/\sqrt{gL}$. Hence the following functional relationship between $a_{rms}/L$ and $u_i/\sqrt{gL}$ is established

$$\ln \frac{a_{rms}}{L} = \phi_2(L) \frac{u_i}{\sqrt{gL}} + \phi_3(L)$$  \hspace{1cm} (A.3.5)

Here $\phi_2(L)$ and $\phi_3(L)$ are all functions of fetch. Further examinations made on $\phi_2(L)$ and $\phi_3(L)$ indicate that $\phi_2(L)$ varies approximately with the square root of fetch and $\phi_3(L)$ is a very weak function of fetch. Accordingly Eq. (A.3.5) may be approximately written as

$$\ln \frac{a_{rms}}{L} = A_{11} u_i + A_{12}$$  \hspace{1cm} (A.3.6)

Expressing $u_i$ in terms of dimensionless quantities, we have,

$$\ln \frac{a_{rms}}{L} = A_{13} \frac{u_i}{\sqrt{g y_i}} + n_4 + A_{14}$$  \hspace{1cm} (A.3.7)

Here $A_{13}$, $A_{14}$ and $n_4$ are constants. These constants will be evaluated based on the experimental data of the present study.
A.3.2 Method of estimating the empirical constants in Eq. (A.3.4)

Eq. (A.3.4) can be rewritten as

\[
\frac{u_i}{\sqrt{\tau/c}} = -n_1 = A_9 \ln \frac{a_{\text{rms}/L}}{u_i} \left( \frac{n_3}{\sqrt{gL n_2 \frac{1-n_2}{n_3}}} \right) + A_{15} \quad (A.3.9)
\]

This equation can be simplified if \( u_i \) is measured at a single reference height above the water surface, say \( y_r \), and hence

\[
\frac{u_r}{u_i} = A_{16} \ln \left( \frac{a_{\text{rms}/L}}{u_r} \left( \frac{n_3}{\sqrt{gL n_2 \frac{1-n_2}{n_3}}} \right) \right) + A_{17} \quad (A.3.10)
\]

The constants \( A_{15} \) and \( A_{16} \) can be easily determined from the experimental data if \( n_2 \) and \( n_3 \) are known. The exponent \( n_2 \) should not be much smaller than 1 according Eq. (3.34). The exponent \( n_3 \), however, can only be assumed to be positive according to Eqs. (3.32) and (3.34). Both of the exponents will be determined by a trial and error technique as follows:

(a) Choose \( n_2 \) with values less than 1 and \( n_3 \) with values between .5 and 10.

(b) Use the method of least squares to obtain \( A_{17} \) and \( A_{18} \) from Eq. (A.3.10) and calculate the standard error of estimate for each combination of \( n_2 \) and \( n_3 \).

(c) Select the values of \( n_2 \) and \( n_3 \) and corresponding \( A_9 \) and \( A_{15} \) that give the lowest value of the standard error of estimate.
These values are taken as the solution to Eq. (A.3.10).

(d) Again, choose $n_1$ with values close to $(1-n_2)/2$ together with $n_2$ and $n_3$ obtained from (c) and use the same technique as outlined in (b) and (c) to obtain $A_9$ and $A_{15}$ as well as $n_1$ from Eq. (A.3.9).

The resultant equation is

\[ \frac{ku_i}{u_\tau^2} = -0.7 \psi \ln \left( \frac{946 \frac{a_{rms}}{L}}{\frac{u_i}{\sqrt{gL0.8\psi0.2}}} \right) \]  
(A.3.11)

where $\kappa$ is von Karman's constant and is equal to 0.4.

A.3.3 Method of estimating the empirical constants in Eq. (A.3.7).

Eq. (A.3.7) can be rewritten as

\[ \ln \frac{a_{rms}}{L} = A_{13} \frac{u_i}{\sqrt{g y_i}} y_i^{n_4} + A_{14} \]  
(A.3.12)

Since the experimental data indicate that if the value of $a_{rms}/L$ is less than about 0.008, it is a function of $u_i$ only, the combined exponent of $y_i$ must be approximately equal to $3n_1$ based on the power law velocity profile. Thus

\[ 3n_4 - .5 = -3 \times 0.033 \]

or \[ n_4 = .133 \]

To obtain the values of $n_4$, $A_{13}$ and $A_{14}$, the calculation proceeded by fitting the experimental data to Eq. (A.3.12) with a value of .133.
for $n_4$. This result was checked against those with different values of $n_4$. It was found that the least standard error of estimate was obtained with the value of $n_4$ equal to .12. The corresponding values of $A_{13}$ and $A_{14}$ are 0.08 and -14.2, respectively. Hence Eq. (A.3.12) becomes

$$\ln \frac{a_{rms}}{L} = 0.08 \frac{u_i}{\sqrt{g} y_i} \psi 0.12 - 14.2$$  \hspace{1cm} (A.3.13)

or

$$\frac{a_{rms}}{L} = \exp \left[ -14.2 + 0.08 \frac{u_i}{\sqrt{g} y_i} \psi 0.12 \right]$$  \hspace{1cm} (A.3.14)

for $\frac{a_{rms}}{L} \leq 0.008$

$$\frac{a_{rms}}{L} = 0.008$$  \hspace{1cm} for $\frac{a_{rms}}{L} > 0.008$
Appendix A.4

A.4.1 Evaluation of \( \frac{\partial x^{++}}{\partial x}, \frac{\partial u^{++}}{\partial y^{++}}, \frac{\partial u_r}{\partial x^{++}}, \text{ and } \frac{\partial u^+}{\partial x^{++}} \)

1) \( \frac{\partial x^{++}}{\partial x} \)

Replacing \( L \) by \( x \), Eq. (3.41) takes the form:

\[
z_o = y_r \left\{ \frac{946 \exp(-14.2 + 0.08 \frac{u_r}{\sqrt{g_y r}})}{0.12} \right\} \left\{ \frac{u_r}{3.5 \sqrt{g x 0.8 y_r 0.2}} \right\}
\]

(A.4.1)

Let

\[
\psi_1 = \frac{946 \exp(-14.2 + 0.08 \frac{u_r}{\sqrt{g_y r}})}{0.12}
\]

\[
\psi_1 = \frac{u_r}{3.5 \sqrt{g y_r 0.2}}
\]

then

\[
z_o = y_r (\psi_1 \times 1.4)
\]

\[
\frac{\partial z_o}{\partial x} = 0.7 \psi 0.033 y_r (\psi_1 \times 1.4)
\]

\[
= 0.98 \psi 0.033 \frac{z_o}{x}
\]

Let \( y = y_r = 6 \text{ in.} \) \( (2.36 \text{ cm}) \)

\[
\psi 0.033 = 1.86
\]

hence

\[
\frac{\partial z_o}{\partial x} = 1.82 \frac{z_o}{x}
\]

\[
\frac{\partial x^{++}}{\partial x} = \frac{z_o - x \frac{\partial z_o}{\partial x}}{z_o} = -0.82 \frac{z_o}{z_o}
\]

(A.4.2)
\begin{equation}
\frac{\partial u_\tau}{\partial y^{++}} = -0.19 \frac{z_o}{u_\tau}
\end{equation}

Dividing the above equation by \( y \) gives

\begin{equation}
\frac{\partial u_\tau}{\partial y^{++}} = -0.19 \frac{u_\tau}{y^{++}}
\end{equation}

The term \( \frac{\partial u_\tau}{\partial x^{++}} \) can also be evaluated from Eq. (A.4.4).

Taking the derivative of \( u_\tau \) with respect to \( x^{++} \) and rearranging, one obtains
\[ \frac{\partial u^+}{\partial x^{++}} = -0.42 \frac{u^+}{x^{++}} \]  \hspace{1cm} (A.4.5)

(4) \[ \frac{\partial u^+}{\partial x^{++}} \]

The logarithmic velocity profile as defined by Eq. (2.8) can be rewritten as

\[ u^+ = \frac{1}{k} \ln \frac{x}{z_o} + \frac{1}{k} \ln y - \frac{1}{k} \ln x \]

Taking the derivative of \( u^+ \) with respect to \( x^{++} \) gives

\[ \frac{\partial u^+}{\partial x^{++}} = \frac{1}{k} \frac{1}{x^{++}} - \frac{1}{k} \frac{1}{x} \frac{\partial x}{\partial x^{++}} \]

Since

\[ \frac{\partial x}{\partial x^{++}} = \frac{\partial x^{++}}{\partial x} \]

\[ = - \frac{z_o}{0.82} \]

Thus

\[ \frac{\partial u^+}{\partial x^{++}} = \frac{2.23}{kx^{++}} \]  \hspace{1cm} (A.4.6)
Appendix A.5  Heat and Mass Transfer Stanton Numbers

The total heat flux and the total mass flux due to the combined effects of molecular conduction and eddy transfer within the fluid at any point are given by the following equations:

\[ q = -\rho C_p v \left( \frac{1}{\rho_r} + \frac{\varepsilon_H}{\nu} \right) \frac{dT}{dy} \]  \hspace{1em} (A.5.1)

\[ e = -\rho v \left( \frac{1}{Sc} + \frac{\varepsilon_D}{\nu} \right) \frac{dm}{dy} \]  \hspace{1em} (A.5.2)

The above equation can be written in dimensionless form as

\[ G = (\varepsilon_V + \varepsilon_C) \frac{dT}{dy}^+ \]  \hspace{1em} (A.5.3)

where \( y^+ = \frac{y}{z_o} \)

and 1) for heat transfer

\[ \tilde{F} = \frac{t_{z_o} - t_T}{t_{z_o} - t_r} \]

\[ \varepsilon_V = \frac{1}{\rho_r} \]

\[ \varepsilon_C = \frac{\varepsilon_H}{\nu} \]

\[ G = \frac{q z_o}{C_p (t_{z_o} - t_r) \nu} \]
2) For mass transfer 

\[
\bar{F} = \left( \frac{m_z - m}{m_{z^0} - m_r} \right)
\]

\[
\bar{C}_v = \frac{1}{Sc}
\]

\[
\bar{C}_c = \frac{\varepsilon_D}{\nu}
\]

\[
G = \frac{\varepsilon_o z_o \sqrt{\frac{\tau_o}{\nu}} \, y_{++}}{\sigma (m_{z^0} - m_r) \nu}
\]

Substituting \( \frac{\varepsilon_M}{\nu} \) for \( \varepsilon_c \) and noting that

\[
\frac{\varepsilon_M}{\nu} = \frac{k z_o \sqrt{\tau_o / \nu}}{\nu} y_{++} + \varepsilon_v - 1
\]

Eq. (A.5.3) can be rewritten as

\[
G = \left( \frac{k z_o \sqrt{\tau_o / \nu}}{\nu} y_{++} + \varepsilon_v - 1 \right) \frac{\bar{F}}{dy_{++}}
\]

(A.5.4)

Integrating from \( y_{++} = 1 \) to \( y_{r=1} \) and noting that \( F = 0 \) at \( y_{++} = 1 \), we have

\[
\bar{F} = \frac{G \nu}{k z_o \sqrt{\tau_o / \rho}} \ln \left( \frac{k z_o \sqrt{\tau_o / \nu}}{\nu} y_{r=1} + \varepsilon_v - 1 \right)
\]

(A.5.5)

Dividing both sides of Eq. (A.5.5) by \( u_r \) and noting that

\[
u_r = \frac{\sqrt{\tau_o / \rho}}{k} \ln y_{r=1}
\]

(2.8)
we have

$$\frac{k_F}{\sqrt{\frac{\tau_0}{\gamma_r}} \ln \gamma_r^{++} \ln \left( \frac{k_z \sqrt{\frac{\tau_0}{\gamma_r}}}{\sqrt{\frac{\tau_0}{\gamma_0}}} \ln \left( \frac{k_z \sqrt{\frac{\tau_0}{\gamma_0}}}{\sqrt{\frac{\tau_0}{\gamma_0}}} \right) \right)} = \frac{G \gamma_r}{\sqrt{\frac{\tau_0}{\gamma_r}}} \ln \left( \frac{k_z \sqrt{\frac{\tau_0}{\gamma_0}}}{\sqrt{\frac{\tau_0}{\gamma_0}}} \right) + \varepsilon_v - 1$$

(A.5.6)

The heat transfer and mass transfer Stanton Numbers are defined by

$$St_H = \frac{q}{c_p u_r (t_{z_0} - t)}$$

and $$St_D = \frac{e}{c u_r (m_{z_0} - m)}$$

Substituting the heat transfer and the mass transfer Stanton Numbers into Eq. (A.5.6) yields

$$St_H = St_D = k \left( \frac{\ln \gamma_r^{++} \ln \left( \frac{k_z \sqrt{\frac{\tau_0}{\gamma_r}}}{\sqrt{\frac{\tau_0}{\gamma_0}}} \ln \left( \frac{k_z \sqrt{\frac{\tau_0}{\gamma_0}}}{\sqrt{\frac{\tau_0}{\gamma_0}}} \right) \right)}{-1} \right)$$

(A.5.7)

where $$\varepsilon_v = \frac{1}{Pr}$$ for heat transfer

$$\varepsilon_v = \frac{1}{Sc}$$ for mass transfer

and $$k = \text{von Karman's constant} \text{ (a value of 0.4 is used)}$$
Appendix A.6

Wind Drift Current

When a layer turbulent air moves over a water surface, a shear stress exists at the interface and the air drags water along with a velocity $v$. This velocity is related to the interfacial shear stress by the equation, (see Fig. A.6.1)

$$
\tau_L = \rho_L (\xi_M + v) \frac{dV}{d(R-r)} \quad \text{(A.6.1)}
$$

For an observer moving along with the air at the air-water interface, Eq. (A.6.1) becomes:

$$
\tau_L = \rho_L (\xi_M + v) \frac{dV}{d(R-r)} \quad \text{(A.6.2)}
$$

In the above equation, $\tau_L$ is the shear stress induced by wind and it decreases from its maximum value $\tau_L$ at the air-water interface to zero at the solid boundary. By analogy to the problem of turbulent flow in pipe, it is expected that the shear stress distribution is linear. Then, approximately, Eq. (A.6.2) can be written as:

$$
\tau_a \left( \frac{r}{R} \right) = \rho_L (\xi_M + v) \frac{dV}{d(R-r)} \quad \text{(A.6.3)}
$$

Eq. (A.6.3) is essentially the same equation which relates the shear stress and velocity profile of fully developed turbulent flow in pipe. It is, therefore, expected that all the universal velocity profiles for fully developed turbulent flow in pipe are equally applicable to Eq. (A.6.3). Because it is applicable throughout
the entire flow region (that is laminar sublayer, buffer layer and turbulent layer) the universal velocity profile originally derived by Reichardt is chosen as the solution to Eq. (A.6.3). This velocity profile is

\[
V_r^+ = 2.5 \ln \left[ 1 + 0.4 \frac{(R-r)^+}{11} \right] + 7.8 \left\{ 1 - \exp \left[ -\frac{(R-r)^+}{11} \right] \right. \\
- \left. \frac{(R-r)^+}{11} \exp \left[ -0.33 (R-r)^+ \right] \right\} (A.6.4)
\]

At the solid boundary where \( r \) is equal to zero, the dimensionless velocity is

\[
V_{rb}^+ = 2.5 \ln (1 + 0.4 R^+) + 7.8 \left[ 1 - \exp \left( -\frac{R^+}{11} \right) \right] - \left[\frac{R^+}{11}\right] \exp \left( -0.33 R^+ \right) \] \quad (A.6.5)

Finally, the absolute velocity at the interface (with reference to the solid boundary) is

\[
V_o = V_{rb}^+ u_r \bigg|_w \quad (A.6.6)
\]
Drag Coefficient Over a Large Body of Water
With Reference to Wind Speed at 10m, C_{10}

**TABLE 2.1**

<table>
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<th>Author</th>
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(Continued)
Drage Coefficient Over a Large Body of Water
With Reference to Wind Speed at 10m, $C_{10}$

**TABLE 2.1 continued**

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* P, E and S indicate Profile Method, Eddy Correlation Method and Surface Tilt Method respectively.
<table>
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<th>Range</th>
<th>Equation</th>
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<td>$0 \leq y^+ &lt; 11.5$</td>
<td>$u^+ = y^+$</td>
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<tr>
<td>$11.5 \leq y^+$</td>
<td>$u^+ = 2.5 \ln y^+ + 5.5$</td>
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<td>V&amp;n Karman (100)</td>
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<td>Reichardt (23)</td>
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<td>$0 \leq y^+$</td>
<td>$u^+ = 2.5 \ln (1 + 0.4 y^+) + 7.8 [1-\exp(-y^+/11)-(y^+/11)\exp(-0.33y^+)]$</td>
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<td>Deissler (101)</td>
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<td>$0 \leq y^+ &lt; 25$</td>
<td>$u^+ = \int_0^{y^+} \frac{dv^+}{1+(0.124)^2 u^+ y^+ [1-\exp(0.124)^2 u^+ y^+]^2}$</td>
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<td>van Driest (102)</td>
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<td>$0 \leq y^+$</td>
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<td>Rannie (103)</td>
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<td>$0 \leq y^+ &lt; 27.5$</td>
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<td>$y^+ = u^+ + 0.1108 [\exp(0.4u^+) - 1 - 0.4u^+ - (0.4u^+)^2/2! - (0.4u^+)^3/3! - (0.4u^+)^4/4!]$</td>
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### Table 4.1
**Experimental Data**

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Fig. 2.1 Measurement of Fluctuating Components in a Rectangular Channel (After Reichardt (23))
Fig. 3.1 Flow Field Grid — Method I
Fig. 3.2 Effect of $y^+$ on $F$
Fig. 3.3 Dimensionless Temperature and Specific Humidity Profiles Based on Different Eddy Diffusivity Models

\[ \frac{\varepsilon_H}{U} = \frac{\varepsilon_D}{U} = \frac{\varepsilon_M}{U} \]

\[ o, x : \frac{\varepsilon_H}{U} = \frac{\varepsilon_D}{U} = \frac{\varepsilon_M}{U} [\phi(Y^+)] \]

For \( Y^+ \leq 100 \)

\[ = \frac{\varepsilon_M}{U} \]

For \( Y^+ \geq 100 \)
Fig. 3.5 Transfer Coefficient Over A Wavy Water Surface - Method I
\[
\frac{\partial r_{\text{rms}}}{L} = \frac{F_{\text{LY}}}{3.5}
\]

\[
\psi = \frac{(U^2/\nu)^2}{u^2/gy}
\]

- \(\ast\) : \(\psi = 150 \times 10^5\)
- \(\circ\) : \(\psi = 18 \times 10^5\)

\[
\frac{kU^+}{0.033} \psi
\]

Fig. 3.6 Functional Relationship Among \(a_{\text{rms}}/L\), Froude Number and \(u^+\)

-15% - +15%
Fig. 3.7 Functional Relationship Between $a_{rms}/L$ and $Fr_y$
Fig. 3.10 Flow Field Grid - Method II
Fig. 4.1 Closed Circuit Air-Water Tunnel
Fig. 4.1a Closed Circuit Air-Water Tunnel
Overall View from Upstream
Fig. 4.3 Schematics of Environmental Controls
1) Wet-Blub Thermocouple
2) Water Reservoir
3) Support
4) Total Pressure Tube
5) Dry-Bulb Thermocouple
6) Static Pressure Tube
7) Traverse Tube
8) Make-Up Water Supply
9) Syringe
10) Cotton
11) Needle

Fig. 4.4 Multiple Function Measuring Probe
1) Static Pressure Tube, 2) Total Pressure Tube, 3 & 4) Dry & Wet Bulb Thermocouples

Fig. 4.4a Multiple Function Measuring Probe.
Fig. 4.5 Schematics of Measuring Apparatus

1) Total Pressure Tube
2) Static Pressure Tube
3) Dry-Bulb Thermocouple
4) Wet-Bulb Thermocouple
5) Wave Probe
6) Thermocouple
7) Statham Strain Gauge Differential Pressure Transducer
8) Endevco Strain Gauge Differential Pressure Transducer
9) Dlsa 55d35 RMS Voltmeter
10) Hewlett Packard HP4100B Two Channel Strip Chart Recorder
11) Burette for Wave Probe Calibration
12) Shut-Off Valve
13) Hewlett Packard 2010K High Speed Data Acquisition System
Fig. 4.6 Calibration of Static Pressure Probe
Fig. 4.8 Calibration of Pressure Transducer
Fig. 4.9 Velocity Profiles
Fig. 4.10 Velocity Profiles
Fig. 4.13 Velocity Profiles
Fig. 4.14 Velocity Profiles
Fig. 4.15 Velocity Profiles
Fig. 4.16 Velocity Profiles
Figure 4-17 Velocity Profiles

Y, IN
0.5
1
2
3
4
5
6

U
16
20
24
28
32
36
40
44
48
52
56

FPS

RUN 79

68, 70, 71, 72, 73, 74, 75, 76, 77, 78
Fig. 4.18 Velocity Profiles
Fig. 4.19 Velocity Profiles
Fig. 4-20 Temperature Profiles
Fig. 4-21 Temperature Profiles

- RUN 93
- **By Thermocouple Fixed at MWL**
- **By Multiple Measuring Probe**
- **By Interpolation, y = z**

Note: Shifted Horizontal Scale
Fig. 4.22 Specific Humidity Profiles
Fig. 4.23 Specific Humidity Profiles
Fig. 4.25 Investigation of Two-Dimensional Flow - Temperature and Specific Humidity
Fig. 5.1 Comparison of Drag Coefficients Predicted by Method I and Field Data
Fig. 5.2 Comparison of Stanton Numbers Between Values Predicted by Method I and Field Data
Fig. 5.3 Comparison of Drag Coefficients
Predicted by Method II and Experimental Data
Fig. 5.4 Comparison of Dynamic Roughness Predicted by Method II and Experimental Data
Fig. 5.5 Comparison of Dimensionless Velocity Predicted by Method II and Experimental Data
Fig. 5.6 Comparison of Drag Coefficients Predicted by Method II and Experimental Data
Fig. 5.7 Charnock's Constant as a Function of Fetch
Fig. 5.8 Dimensionless Temperature and Specific Humidity Profiles
Fig. 5.10. Comparison of F Obtained Under Laboratory Conditions
Fig. 5.11 Comparison of $F$ obtained under Laboratory Conditions
Fig. 5.12 Comparison of F Obtained Under Laboratory Conditions
Fig. 5.13 Comparison of Stanton Numbers and Field Data
Fig. 5.14 Comparison of Stanton Numbers and Field Data
Fig. A.6.1 Wind-Induced Water Surface Velocity
Appendix A.7 References


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