A NOVEL MICROSTRIP TO CYLINDRICAL CAVITY TRANSITION
AND ITS APPLICATION IN THE DESIGN OF
HIGHLY STABLE 12 GHz GaAs FET OSCILLATORS

by

Guy R. Painchaud

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Department of Electrical Engineering
Faculty of Science and Engineering
University of Ottawa
Ottawa, Ontario
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ABSTRACT

The poor stability of present MIC fundamental oscillators is due to the lack of a suitably stable MIC resonator. In this report, this problem is overcome through the use of a high Q cavity coupled to the microstrip circuit by means of a novel transition. A simple analysis is given for the transition, based on Wheeler's equivalent energy concept for small hole coupling and an approximate parallel plate waveguide model for the microstrip. The analysis appears adequate for most design purposes. The transition was then incorporated in the designs of two different 12 GHz GaAs FET oscillators. The stability achieved by these oscillators is comparable to that of a crystal oscillator-multiplier unit. The oscillators are intended to find application as LO's in small satellite ground terminals.
ACKNOWLEDGEMENTS

The major part of the work described in this report was performed at the Communications Research Centre, Ottawa. I wish to thank Dr. R.W. Breithaupt, Director, Space Electronics for making the facilities available to me.

I also wish to express sincere thanks to Dr. D.S. James for his invaluable help and guidance throughout the entire period of this work. Some of the results described in Chapter Three were previously carried out by Dr. D.S. James and Professor W.J.R. Hoefer. These contributions are gratefully acknowledged. Professor W. Steenaart provided constant encouragement through numerous discussions. Mr. E. Minkus provided valuable technical assistance at various stages of the work.

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CHAPTER 1

PREFACE

1-1 Introduction: This report describes exploratory development work on ultra-stable fundamental microwave oscillators for use as LOs (local oscillator and transmit frequency sources in satellite communications systems. With the advent of high power communications satellites and consequently smaller ground terminals [1-2], there arises a need for a simple oscillator of high stability.

In order to meet the rigid requirements demanded, present sources are comprised of elaborate crystal oscillator-multiplier chains. Since multiplication factors in the order of 100 are common, the output power of such sources is of the order of a few milliwatts at X-band. For applications requiring moderate power, a cavity oscillator using either an Impatt or Gunn diode stabilized to a crystal reference is used. Stabilization can be accomplished in two ways. The first is to incorporate a loosely coupled varactor diode in the cavity oscillator and use a phase-lock loop to lock this oscillator to the output of the crystal oscillator-multiplier chain [3-4]. An alternate approach is to injection lock the cavity oscillator to the output of the multiplier unit [5-6]. Both of these techniques are complex and relatively costly.

The approach studied in this report is to construct a fundamental oscillator using a device that is inherently low noise. A high $Q_0$ resonator is then used to minimize the FM noise. The active device used is a GaAs FET; since it is a two port device, the oscillator is constructed in MIC form.

Current MIC designs either use planar resonators ($Q_0 < 500$) [7] or
"open" dielectric resonators \((Q_o<5000)\) [8]. Both these types of resonators are inadequate for use in a highly stable oscillator because of their relatively low values of \(Q_o\) and poor temperature stability. What is required is a cavity resonating in a high \(Q_o\) mode, such as the \(TE_{012}\) mode of a cylindrical cavity \((Q_o \approx 30,000 \text{ at X-band})\). What would then be required is a simple transition between microstrip and the cavity without resorting to bulky connectors. Such a transition was constructed. Although several authors [9-13] have described empirically designed microstrip and stripline to waveguide transitions none of these papers with the exception of [13] mention coupling to a resonant cavity.

This report consists of two parts. The first consists of chapters 1 thru 4 and describes the analysis and evaluation of a novel transition between microstrip and a cylindrical cavity [14-15]. A review of the properties of resonant cavities is contained in chapter 2. The second part of the report comprising chapters 5 thru 7 describes exploratory hardware. Chapter 5 concerns the implementation of the transition described in the first part of the report in the design of stabilizing cavities for MIC oscillators. Factors influencing the stability of the cavity are examined in detail. Chapters 6 and 7 describe two alternate approaches to the design of a highly stable GaAs FET oscillator using the cavities described in chapter 5. The tradeoff between FM noise performance and output power is examined. At present, output power is low due to the limitations of present GaAs FETs, however, as medium power FETs [16-18] become commercially available, output powers in excess of 100 mW at X-band should be feasible from such oscillators.
CHAPTER 2

CHARACTERISTICS OF CIRCULAR CYLINDRICAL RESONANT CAVITIES

2-1 INTRODUCTION:

At microwave frequencies resonant cavities are widely used as highly selective microwave circuit components. Unlike lumped element L, C, R resonant circuits, they possess more than one resonant frequency and have Q factors which are much greater than those obtainable from lumped L, C, R components. The cavities to be described later on will have Q's of the order 30,000 - 64,000.

In the most general terms, a cavity is any region of space that is completely enclosed by a conducting boundary. Solutions of Maxwell's equations are then found for the enclosed region.

For a source free region homogeneously filled with an isotropic medium the E and H fields satisfy Maxwell's equations in the following form [22-25]

\[ \nabla \times \vec{E} = -j\omega \vec{H} \]  
\[ \nabla \times \vec{H} = j\omega \vec{E} \]  

Furthermore, the E and H fields also satisfy the wave equation:

\[ \nabla^2 \left\{ \frac{\vec{E}}{\vec{H}} \right\} + \kappa^2 \left\{ \frac{\vec{E}}{\vec{H}} \right\} = 0 \]  

The following shapes of cavities have been extensively studied [19-25]

1) rectangular parallelepipeds
2) circular cylinders
3) elliptical cylinders
4) circular coaxial structures
5) hollow spheres

In the remainder of this chapter attention will be given to the circular cylinder as a practical cavity for the following reasons:
1) Of all the shapes mentioned above, it is the easiest to machine.

2) It will be mentioned that certain modes yield extremely high $Q_o$'s for cylindrical cavities.

3) The modes mentioned in (2) require no electrical contact between the side walls and cavity end plates (a fortunate fact as such a contact is difficult to realize); which all other modes do. These other modes can then be suppressed by not providing this contact.

2-2 MODE SOLUTIONS FOR THE CYLINDRICAL CAVITY:

The field expressions for the cylindrical cavity shown in figure (2-1) will be stated.

Fig. 2-1  Circular Cylindrical Cavity

The field expressions are obtained by solving (2-1) and (2-2) in cylindrical coordinates and applying the appropriate boundary conditions; these are given below for the TE and TM modes:

**TE Modes**

$E_r, E_\phi = 0$ at $z=0$ and $z=L$  \[2-3a\]

$\frac{\partial H_z}{\partial r} = 0$ at $\rho=R$  \[2-3b\]

**TM Modes**

$E_r, E_\phi = 0$ at $z=0$ and $z=L$  \[2-4a\]

$E_z = 0$ at $\rho=R$  \[2-4b\]
The method of solution is given in any text on electro-

magnetics, for example. The magnitude of the expressions are given below [19],

the time factor has been omitted and the E and H fields are in time quadrature.

\[ \begin{align*}
  \mathbf{E}_\rho &= \sqrt{\frac{\mu}{\varepsilon}} \frac{j_\ell(k_1) \sin(\phi) \sin(k_3 z)}{k_1} & 2-5a \\
  \mathbf{E}_\phi &= \sqrt{\frac{\mu}{\varepsilon}} \frac{j'_\ell(k_1) \cos(\phi) \sin(k_3 z)}{k_1} & 2-5b \\
  \mathbf{E}_z &= 0 & 2-5c \\
  \mathbf{H}_\rho &= \frac{k_3}{k} \frac{j_\ell(k_1) \cos(\phi) \cos(k_3 z)}{k_1} & 2-5d \\
  \mathbf{H}_\phi &= \frac{k_3}{k} \frac{j'_\ell(k_1) \sin(\phi) \cos(k_3 z)}{k_1} & 2-5e \\
  \mathbf{H}_z &= \frac{k_1}{k} \frac{j_\ell(k_1) \cos(\phi) \sin(k_3 z)}{k_1} & 2-5f 
\end{align*} \]

\[ \begin{align*}
  \mathbf{E}_\rho &= -\sqrt{\frac{\mu}{\varepsilon}} \frac{k_3}{k} \frac{j'_\ell(k_1) \cos(\phi) \sin(k_3 z)}{k_1} & 2-6a \\
  \mathbf{E}_\phi &= \sqrt{\frac{\mu}{\varepsilon}} \frac{k_3}{k} \frac{j_\ell(k_1) \sin(\phi) \sin(k_3 z)}{k_1} & 2-6b \\
  \mathbf{E}_z &= \sqrt{\frac{\mu}{\varepsilon}} \frac{k_1}{k} \frac{j_\ell(k_1) \cos(\phi) \cos(k_3 z)}{k_1} & 2-6c \\
  \mathbf{H}_\rho &= -\frac{j_\ell(k_1)}{k_1} \frac{\sin(\phi) \cos(k_3 z)}{k_1} & 2-6d \\
  \mathbf{H}_\phi &= -j'_\ell(k_1) \frac{\cos(\phi) \cos(k_3 z)}{k_1} & 2-6e \\
  \mathbf{H}_z &= 0 & 2-6f 
\end{align*} \]
In the above, the constants $k_1$ and $k_3$ satisfy the following:

$$k_1^2 + k_2^2 = k^2$$  \hspace{1cm} 2 - 7

2-3 THE MODE CHART:

A convenient graphical relation between $D$, $L$ and the resonant frequency $f$ of a particular mode can be obtained from (2-7). Substituting the expressions for $k$, $k_1$ and $k_3$ into (2-7) gives:

$$\omega^2 \mu e = \left(\frac{2r_{km}}{D}\right)^2 + \left(\frac{n \pi}{L}\right)^2$$  \hspace{1cm} 2 - 8

This last equation can be rearranged into:

$$(fD)^2 = \left(\frac{cr_{km}}{\pi}\right)^2 + \left(\frac{c}{2}\right)^2 \pi^2 \left(\frac{D}{L}\right)^2$$  \hspace{1cm} 2 - 9a

or

$$(fD)^2 = \Lambda + Bn^2 \left(\frac{D}{L}\right)^2$$  \hspace{1cm} 2 - 9b

The values for $A$ are given in table (2-1) for the first 30 modes given that $f$, $D$, and $L$ have the following units:

- $f$ – the resonant frequency of any mode in MHz
- $D$ – the (inner) diameter of the cavity in inches
- $L$ – the (inner) length of the cavity in inches
<table>
<thead>
<tr>
<th>Mode</th>
<th>$A \times 10^8$</th>
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<tbody>
<tr>
<td>01N</td>
<td>0.8156</td>
</tr>
<tr>
<td>02N</td>
<td>4.2975</td>
</tr>
<tr>
<td>03N</td>
<td>10.5617</td>
</tr>
<tr>
<td>11N</td>
<td>2.0707</td>
</tr>
<tr>
<td>12N</td>
<td>6.9415</td>
</tr>
<tr>
<td>13N</td>
<td>14.5970</td>
</tr>
<tr>
<td>TM</td>
<td></td>
</tr>
<tr>
<td>21N</td>
<td>3.7197</td>
</tr>
<tr>
<td>22N</td>
<td>9.9923</td>
</tr>
<tr>
<td>31N</td>
<td>5.7410</td>
</tr>
<tr>
<td>32N</td>
<td>13.4374</td>
</tr>
<tr>
<td>41N</td>
<td>8.1212</td>
</tr>
<tr>
<td>51N</td>
<td>10.8511</td>
</tr>
<tr>
<td>61N</td>
<td>13.9238</td>
</tr>
<tr>
<td></td>
<td>0.2070</td>
</tr>
<tr>
<td>2.0707</td>
<td>6.9415</td>
</tr>
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<td>10.5617</td>
<td>2.0707</td>
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<td>13.4374</td>
<td>8.1212</td>
</tr>
<tr>
<td>10.8511</td>
<td>13.9238</td>
</tr>
</tbody>
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Table 2-1. Values of $A$ for the first 30 modes.

For air at 25°C and 60% relative humidity, $B$ has the following value:

$$B = 0.34799 \times 10^8$$

Plotting $(fD)^2$ versus $(D/E)^2$ as related by (2-9b) gives the mode chart shown in figure (2-2).
Fig. 2-2. Mode chart for circular cylinder resonant cavity [19]

This chart not only gives values of $D$ and $L$ for the resonant frequency of a particular mode, but it also shows the proximity of the neighbouring modes. This last point is important when using the higher modes.

2-4 Q-FACTOR CONSIDERATIONS:

The $Q$-factor for any mode can be calculated from the following [22-25]:

$$ Q = \frac{2}{\delta} \frac{\int_{\text{cavity}} \bar{H} \cdot \bar{H}^* \, dv}{\int_{\text{cavity}} \bar{H} \cdot \bar{H}^* \, ds} $$

2 - 11
The values of $\bar{H}$ are taken from (2-5) or (2-6).

A more convenient factor for evaluating the performance of the various modes is the mode shape factor, $M.S.$ as given by:

$$M.S. = \frac{Q_0^*}{\lambda}$$

The $M.S.$ factor is useful because it is only a function of the ratio $\frac{D}{L}$ for a particular mode. Graphs of the $M.S.$ factor versus $\frac{D}{L}$ are found in the literature and are reproduced in figures (2-3) through (2-5) for some of the lower modes.

Fig. 2-3. Mode shape factor for circular cylinder resonator - TE modes with $k=0$ [19].
Fig. 2-4. Mode shape factor for circular cylinder resonator - TE modes with $l=0$.\textsuperscript{[9]}

Fig. 2-5. Mode shape factor for circular cylinder resonator - TM modes \textsuperscript{[9]}. 
It is interesting to note that in general, the $TE_{01n}$ modes have the highest $Q$ for a given volume [20]. It is very important to keep the volume of the cavity as small as possible since for a cavity of volume $V$, the number $N$ of modes having a resonant frequency lower than a given frequency $f$ is approximately:

$$N = \frac{8\pi Vf^3}{3e^3} \quad 2-13$$

On the basis of what was just said, the $TE_{01n}$ modes provide the best possible $Q$-factors. However, one particular problem must be taken into account. From table (2-1) or figure (2-2), it can be seen that every $TE_{0mn}$ mode has a companion $TM_{1mn}$ mode resonating at the same frequency. The $TM_{1mn}$ modes have a lower $M.S.$ factor than the $TE_{0mn}$ modes and will reduce the overall $Q$. In the presence of small geometrical irregularities, these two modes resonate at slightly different frequencies; the cavity response will then be split which is highly undesirable. However, the $TM_{1mn}$ modes can be effectively suppressed by perturbing the current density in the cavity walls at the appropriate position, as will be described in the next section.

2-5 MODE SUPPRESSION TECHNIQUES:

Expressions for the current density in the cavity walls will be given and then used to show how certain types of modes can be suppressed by perturbing the current density at the appropriate location.

The current density $J$ induced in the inner walls of the cavity is given by [21].

$$\overline{J} = \overline{n} \times \overline{H_T} \quad 2-14$$

Expressions for $J$ are given in table (2-2).
<table>
<thead>
<tr>
<th>TE_mn Modes</th>
<th>Side walls</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J_\phi = \frac{k_3}{k} J__2(k_1 \rho) \cos(\ell \phi) )</td>
<td>( J_\phi = \frac{k_1}{k} J__2(k_1 R) \cos(\ell \phi) \sin(k_3 z) )</td>
</tr>
<tr>
<td>( J_\rho = -\frac{k_3}{k} \frac{J__2(k_1 \rho)}{k_1 \rho} \sin(\ell \phi) )</td>
<td>( J_z = -\frac{k_3}{k} J__2(k_1 R) \sin(\ell \phi) \cos(k_3 z) )</td>
</tr>
<tr>
<td>TM_mn Modes</td>
<td></td>
</tr>
<tr>
<td>( J_\phi = \frac{J__2(k_1 \rho)}{k_1 \rho} \sin(\ell \phi) )</td>
<td>( J_\phi = 0 )</td>
</tr>
<tr>
<td>( J_\rho = -J__2(k_1 \rho) \cos(\ell \phi) )</td>
<td>( J_z = -J__2(k_1 R) \cos(\ell \phi) \cos(k_3 z) )</td>
</tr>
</tbody>
</table>

Table 2-2. Expressions for the current density induced in the inner walls of the cavity.

Consider the current density at the positions where the side walls join the end plates. On consulting table (2-2), it can be seen that only the TE\_0mn modes have no current paths between the end plates and side walls at these points. For the TE\_mn (\( \ell \neq 0 \)) and TM\_mn modes, the current density is zero only at certain points where \( \cos(\phi) \) or \( \sin(\phi) \) is null. By presenting an impedance to the current paths between the end plates and side walls, all modes except the TE\_0mn modes can be suppressed to some extent, the amount of suppression depending on the magnitude of the current path for a particular mode. This impedance can be realized by either cutting concentric slots in the end plates at the point where they join the side walls, or by separating the end plates from the side walls by an annulus made of a lossy material.

The mode suppression method described not only suppresses the companion TM\_mn modes, but all others not of the TE\_0mn type. This fact greatly improves the stop band of the cavity when used under transmission. This type of mode suppression will be used in the design of the cavities to be described in chapters 4 and 5.
EFFECTS OF ENVIRONMENTAL CONDITIONS ON THE RESONANT FREQUENCY:

As shown in section (2-3), the resonant frequency of the cavity is determined by the cavity dimensions D and L and the dielectric constant \( \varepsilon \) of the material filling the cavity. Frequency shift caused by slight changes in D, L and \( \varepsilon \) can be written as:

\[
\Delta \omega = \frac{\partial \omega}{\partial D} \Delta D + \frac{\partial \omega}{\partial L} \Delta L + \frac{\partial \omega}{\partial \varepsilon} \Delta \varepsilon
\]

Expressions for \( \frac{\partial \omega}{\partial D} \), \( \frac{\partial \omega}{\partial L} \), \( \frac{\partial \omega}{\partial \varepsilon} \) are obtained from (2-8):

\[
\frac{\partial \omega}{\partial D} = -\frac{4\pi^2 \xi_m}{\omega \varepsilon D^3} \quad \text{(2-16a)}
\]

\[
\frac{\partial \omega}{\partial L} = \frac{n^2 \pi^2}{\omega \varepsilon L^3} \quad \text{(2-16b)}
\]

\[
\frac{\partial \omega}{\partial \varepsilon} = \frac{-1}{2 \omega \varepsilon^2} \left[ \frac{4\pi^2 \xi_m^2}{D^2} + \frac{n^2 \pi^2}{L^2} \right] \quad \text{(2-16c)}
\]

If the cavity is made of a homogeneous material, the change in D and L can be expressed in terms of \( \alpha \), the coefficient of thermal expansion.

\[
\Delta D = \alpha D \Delta T \quad \text{(2-17a)}
\]

\[
\Delta L = \alpha L \Delta T \quad \text{(2-17b)}
\]

Combining (2-15) thru (2-17) yields the following expression for the normalized shift in resonant frequency:

\[
\frac{\Delta f}{f} = -\alpha \Delta T - \frac{1}{2} \frac{\Delta \varepsilon}{\varepsilon} \quad \text{(2-18)}
\]

If the cavity is air-filled, the following empirical formula [26-27] can be used for the dielectric constant:

\[
\varepsilon = 1 + 2.1 \times 10^{-4} \left( \frac{P_a}{T} \right) + 1.8 \times 10^{-4} \left( 1 + \frac{5.58 \times 10^3}{T} \right) \frac{P_w}{T} \quad \text{(2-19)}
\]
The variables $T$, $P_a$, and $P_w$ are defined below:

- $T$ - the temperature in °K.
- $P_a$ - the air pressure inside the cavity in mmHg.
- $P_w$ - the water vapor pressure inside the cavity in mmHg.

Expression 2-18 can then be rewritten as:

$$
\frac{\Delta f}{f} = -\alpha \Delta T - \frac{1}{2} \frac{1}{c} \left( \frac{3c_1}{2T} \Delta T + \frac{3c_2}{2P_a} \Delta P_a + \frac{3c_3}{2P_w} \Delta P_w \right)
$$

2-20

In the above, the cavity is assumed to be tuned at $T_0$, $P_{ao}$ and $P_{wo}$. The terms $\Delta T$, $\Delta P_a$ and $\Delta P_w$ represent the changes in temperature, air pressure and water vapour pressure from the reference values of $T_0$, $P_{ao}$ and $P_{wo}$. The quantities \( \frac{\Delta c_1}{\Delta T} \), \( \frac{\Delta c_2}{\Delta P_a} \) and \( \frac{\Delta c_3}{\Delta P_w} \) are obtained by differentiating (2-19):

$$
\frac{\Delta c_1}{\Delta T} = - \left\{ 2.1 \times 10^{-4} \frac{P_a}{T^2} + 1.8 \times 10^{-4} \left( 1 + \frac{1.1 \times 10^4}{T} \right) \frac{P_w}{T^2} \right\}
$$

2-21a

$$
\frac{\Delta c_2}{\Delta P_a} = 2.1 \times 10^{-4} \frac{1}{T}
$$

2-21b

$$
\frac{\Delta c_3}{\Delta P_w} = 1.8 \times 10^{-4} \left( 1 + \frac{5.58 \times 10^3}{T} \right) \frac{1}{T}
$$

2-21c

The above terms are evaluated at $T_0$, $P_{ao}$ and $P_{wo}$ and substituted into (2-20).

The frequency stability as given by (2-20) is a function of temperature, air pressure and water vapour pressure. An improvement in stability can be had by hermetically sealing the cavity with dry air or nitrogen. Under these conditions, the gas inside the cavity satisfies Boyle's Law.
\[ \frac{P_a}{T} = \frac{P_{ao}}{T_0} \quad 2-22a \]

\[ \frac{P_w}{T} = \frac{P_{wo}}{T_0} \quad 2-22b \]

The terms \( \frac{P_a}{T} \) and \( \frac{P_w}{T} \) in (2-19) are now constants under the above conditions and the frequency stability can be written as:

\[ \frac{\Delta f}{f} = -\alpha \Delta T - \frac{1}{2} \frac{\theta}{\varepsilon} \Delta T \quad 2-23 \]

The value of \( \frac{\partial \varepsilon}{\partial T} \) is obtained by differentiating (2-19) and treating \( \frac{P_a}{T} \) and \( \frac{P_w}{T} \) as constants.

\[ \frac{\partial \varepsilon}{\partial T} = -\frac{1}{2} \frac{P_{wo}}{T T_0} \quad 2-24 \]

It can be seen that the last term of (2-23) can be made null if the gas filling the cavity contains no water vapour (\( P_{wo} = 0 \)). No improvement in stability is obtained by sealing the cavity under vacuum. Graphs of frequency stability will be presented in chapter 5.
Due to machining tolerances, some form of tuning method is required to adjust the cavity resonant frequency to the precise value required. For cases where the cavity must be tuned over a broad band, one of the end plates can be moved relative to the other by means of a suitable mechanism. The change in resonant frequency can then be found from (2-16b) and is given below.

\[
\frac{\partial f}{\partial L} = -\frac{2n^2}{\pi L^3}
\]

If tuning over a broad band is not required, a simple post can be introduced into the cavity to fine tune the resonant frequency. The change in resonant frequency can be obtained using Slater's perturbation formula [22-25], which is given below.

\[
\frac{\Delta \omega}{\omega} = \frac{\int_{\text{post}} (\mu \epsilon^2 - c \epsilon^2) dV - \int_{\text{cavity}} (\mu \epsilon^2 - c \epsilon^2) dV}{4U \int_{\text{cavity}} (\mu \epsilon^2 + c \epsilon^2) dV}
\]

A small metallic post, of length \(L\) and radius \(r\), is located at the position shown in figure (2-6).

---

**Fig. 2-6** Fine tuning of a TE_{011} cavity by means of a small metallic post.
From (2-5), it is noted that the $H$ field tangential to the end plate of the cavity is null at the location of the post. Because of this, a good electrical contact is not required between the post and the end plate. Using (2-5), the numerator of (2-26) is evaluated as follows:

$$\int (\mu H^2 - \epsilon E^2) \, dV = \mu \pi r^2 \left[ \frac{1}{2} \frac{L}{4n^2} \sin \frac{2n\pi}{L} \right]$$  \hspace{1cm} 2-27$$

post

The energy stored in the cavity, $U$, is found by integrating the square of the $H$ field over the volume of the cavity at the instant when the $E$ field is zero. Using the value of $H$ given by (2-5) and performing the integration outlined above gives the following expression for $U$.

$$U = \int \mu H^2 dV = \frac{\mu L R^2}{2} \left[ J_0(k_1 R)^2 \right]$$  \hspace{1cm} 2-28$$
cavity

The final expression for the change in resonant frequency is obtained using (2-26) thru (2-28), along with the fact that $J_0(k_1 R) = 0.403$ for the $TE_{01n}$ modes.

$$\frac{\Delta f}{f} = 3.08r^2 \left[ \frac{L}{2} \frac{4n^2}{L} \sin \frac{2n\pi}{L} \right]$$  \hspace{1cm} 2-29$$

Since the post is located at the maximum of the $H$ field, increasing the size of the post increases the resonant frequency of the cavity.

In a practical case that requires the cavity to be tuned over a broad band it is likely that a combination of the two tuning techniques described would be used as the change in resonant frequency caused by moving an end plate slightly is still quite large. For a cavity designed to resonate in the $TE_{012}$ mode at 12GHz, (2-25) predicts that a .001" change in $L$ will shift the resonant frequency approximately 4MHz.
CHAPTER 3

ANALYSIS OF A NOVEL TRANSITION BETWEEN MICROSTRIP AND A CYLINDRICAL CAVITY

3.1 INTRODUCTION

The microstrip-to-cylindrical cavity transition to be analyzed is shown in figure (3-1). The ground plane of a microstrip substrate forms one of the end plates of the cavity. The cavity is designed to resonate in a $T_{E_{01n}}$ mode at the desired operating frequency; the $T_{E_{01n}}$ modes are used for the reasons mentioned in chapter 2. Coupling is by means of an aperture located in the ground plane at the point of maximum radial $H$-field in the cavity. The microstrip line terminates in an open circuit $3 \lambda / 4$ beyond the aperture; this length of line maximizes the magnetic coupling through the aperture. This configuration is quite practical as the cavity can be machined into the substrate holder.

![Diagram](image)

**FIG. 3-1** COUPLING ARRANGEMENT BETWEEN MICROSTRIP AND A $T_{E_{01n}}$ CAVITY. COUPLING IS BY MEANS OF AN APERTURE OF DIAMETER $d$ LOCATED IN THE SUBSTRATE GROUND-PLANE.
3.2 METHOD OF ANALYSIS

A rigorous analysis of the transition shown in figure (3-1) is quite formidable as the microstrip is an inhomogeneous structure and no closed-form solution is available. Since in most applications cavities are undercritically coupled to conserve a high $Q$ factor, Wheeler's equivalent energy concept [28] for calculating the coupling through small apertures is employed. This yields a simple analytic expression for the external $Q$ of the cavity, $Q_{\text{ext}}$, which has been found useful for design purposes.

Wheeler [28] has shown that the coupling between a waveguide and a resonant cavity can be evaluated by considering two symmetrical coupling problems. One problem is the coupling between two waveguides due to the normalized reactance $x$ of a small aperture in a common wall. In this case, the waveguides consist of two microstrip lines, each terminated at one end by an open circuited stub an odd number of quarter wavelengths from the aperture. The other problem is the evaluation of the coupling factor $k$ that exists between two identical cavities sharing an aperture in a common wall. The loading power factor $p = \frac{1}{Q_{\text{ext}}}$ between the cavity and the guide is then found to be the product of the normalized reactance $x$ and the coupling factor $k$, with error $O(p)$. In the following sections, $k$ and $x$ are calculated by employing the equivalent energy concept and a simplified parallel-plate waveguide model for the microstrip.
3.3 APERTURE COUPLING BETWEEN TWO IDENTICAL CAVITIES:

As shown by Wheeler the coupling factor $k$ between two identical resonant cavities is as given by (3-1a) and (3-1b) for electric and magnetic couplings respectively.

$$k_e = \frac{1}{4} \frac{V_{ec}}{V_e} \quad 3 - 1a$$

$$k_m = \frac{3}{4} \frac{V_{mc}}{V_m} \quad 3 - 1b$$

In (3-1), $V_{ec}$ and $V_{mc}$ are the effective volumes of the coupling aperture as defined by Wheeler for electric and magnetic coupling. Electric coupling occurs when there is a component of the cavity $E$ field perpendicular to the aperture, while magnetic coupling requires that a component of the cavity $H$ field be tangential to the coupling aperture. Values of $V_{ec}$ and $V_{mc}$ are given in table (3-1) for elliptical apertures located in thin conducting walls.

<table>
<thead>
<tr>
<th>Field Orientation</th>
<th>Circle $r_1 = r_1$</th>
<th>General $k = \sqrt{1 - (r_2/r_1)^2}$</th>
<th>Narrow $r_1 \ll r_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H$ - tangential to major axis ($2r_1$)</td>
<td>(\frac{4\pi r_1^2}{3}) (\frac{\pi}{\pi}) Sphere</td>
<td>(\frac{4\pi r_1^2}{3}) (\frac{k^2}{P(k) - E(k)})</td>
<td>(\frac{4\pi r_1^2}{3}) (\frac{1}{ln \frac{4r_1}{cr_2}})</td>
</tr>
<tr>
<td>$H$ - tangential to minor axis ($2r_2$)</td>
<td>(\frac{4\pi r_2^2}{3}) (\frac{\pi}{\pi}) Sphere</td>
<td>(\frac{4\pi r_2^2}{3}) (\frac{k^2}{P(k) - (1 - k^2)P(k)})</td>
<td>(\frac{4\pi r_2^2}{3})</td>
</tr>
<tr>
<td>$E$ - perpendicular</td>
<td>(\frac{4\pi r_1^2}{3}) (\frac{2}{\pi})</td>
<td>(\frac{4\pi r_1^2}{3}) (\frac{1}{E(k)})</td>
<td>(\frac{4\pi r_1^2}{3})</td>
</tr>
</tbody>
</table>

Table 3-1. Formulas for the effective volume of thin elliptical apertures.
The volume $V_e$ or $V_m$ in (3-1) is the effective volume of the cavity for electric or magnetic coupling. This is the volume which, when uniformly filled with the field that exists at the aperture, would store the same amount of energy as the actual cavity. The effective volume satisfies (3-2a) or (3-2b) for electric or magnetic coupling respectively.

$$\frac{1}{2} \varepsilon E_c^2 V = \frac{1}{2} \varepsilon \int_{\text{cavity}} E^2 \, dV \quad 3-2a$$

$$\frac{1}{2} \mu H_c^2 V = \frac{1}{2} \mu \int_{\text{cavity}} H^2 \, dV \quad 3-2b$$

In (3-2), the subscript $c$ denotes that $H_c$ is the field that occurs on the wall at the location of the aperture, but without the aperture present. As implies (3-2), $E_c$ and $H_c$ are assumed constant over the area of the aperture since the latter is assumed to be small compared to the dimensions of the cavity.

For the TE$_{01N}$ modes, there is no component of the E-field perpendicular to the cavity walls, because of this, only magnetic coupling can be used. The location of the aperture ($S$ in figure (3-1)) is chosen to coincide with the position of maximum radial H-field in the cavity. This gives the strongest coupling for a given aperture. From (2-5), the value of $H_\rho$ tangential to the cavity end plates is:

$$H_\rho = \frac{k_3}{k_0} J_1(k_1 \rho) \quad 3-3$$

The maximum value of $H_\rho$ is found by differentiating the expression for $H_\rho$ and equating it to zero.
\[
\frac{3}{9} \left( \frac{-k^3}{k_o} J_1(k_1 \rho) \right) \bigg|_{\rho=S} = 0
\]

Solving (3-4) for \( \rho=S \) gives:

\[
S = 0.481R
\]

The value of the \( H \) field at the coupling aperture is then given by:

\[
H_c = \frac{-k_3}{k_o} J_1(k_1 S)
\]

The value of the effective volume is, using (2-5), (3-2b) and (3-6):

\[
V_m = \int_{\text{cavity}} H_c^2 dV = \frac{\mu_0 R^2}{2} \left[ \frac{-k_3}{k_o} J_1(k_1 S) \right]^2
\]

For the circular aperture of diameter \( d \) shown in figure (3-1), the value of \( V_{mc} \) is obtained from table (3-1):

\[
V_{mc} = \frac{2}{3} d^3
\]

Finally, the value of \( k_m \) is obtained using (3-1b), (3-7) and (3-8):

\[
k_m = \frac{1}{4} \frac{V_{mc}}{V_m} = \frac{1}{4} \cdot \frac{2}{3} d^3 \left[ \frac{-k_3}{k_o} J_1(k_1 S) \right]^2
\]

\[
= \frac{\mu_0 R^2}{2} \left[ J_0(k_1 R) \right]^2
\]
A PARALLEL PLATE WAVEGUIDE MODEL FOR MICROSTRIP:

This section describes a simplified microstrip model consisting of a parallel plate waveguide with magnetic sidewalls. The model will then be used in the following section to calculate the normalized reactance of an aperture coupling two microstrips together.

The equivalence between microstrip and a parallel plate waveguide was used for discontinuity calculations by Leighton and Milnes [29], based on earlier work by Oliner [30] for stripline. This equivalence will be made on a T. E. M. basis although the microstrip does not support pure T. E. M. propagation. The microstrip and the parallel plate guide are shown in figure (3-2).

(a) MICROSTRIP LINE  (b) EQUIVALENT PARALLEL PLATE WAVEGUIDE WITH MAGNETIC WALLS.

FIG. 3-2 EQUIVALENCE BETWEEN MICROSTRIP AND A PARALLEL PLATE WAVEGUIDE WITH MAGNETIC SIDEWALLS (T.E.M.-MODE)
The boundary conditions for the parallel-plate guide are:

top and bottom plates: \( E \times \bar{n} = 0 \) (electric wall) \( 3 - 10a \)
side walls: \( H \times \bar{n} = 0 \) (magnetic wall) \( 3 - 10b \)

The following field expressions satisfy the wave equation for T.E.M. propagation and the above boundary conditions for the parallel plate guide:

\[
\begin{align*}
H_x &= H_0 e^{-j\beta z} & E_y &= E_0 e^{-j\beta z} \\
H_y &= H_z = E_x = E_z = 0
\end{align*}
\]

\( 3 - 11a \)

\[
\beta = \omega \sqrt{\mu \varepsilon} \quad \frac{E_y}{H_x} = \eta
\]

\( 3 - 11b \)

\( (\text{propagation in the +z direction assumed}) \)

The characteristic impedance of the parallel plate guide is:

\[
Z_0 = \frac{h}{\omega \eta}
\]

\( 3 - 12 \)

The values of \( Z_0 \) and \( \varepsilon_{\text{eff}} \) for the microstrip are given by Wheeler [31] (electro-static analysis) or by Gertsinger [32] (who accounts for dispersion).

The parallel plate guide is assumed to be uniformly filled with dielectric of relative value \( \varepsilon_{\text{eff}} \left( \mu = \mu_0 \right) \). The equivalent width of the guide is then found from (3-12).

\[
W' = \frac{h}{2} \frac{\eta_0}{\omega \sqrt{\varepsilon_{\text{eff}}}}
\]

\( 3 - 13 \)

Finally, the wavelength in the parallel plate guide is:

\[
\lambda_p = \frac{\lambda_0}{\sqrt{\varepsilon_{\text{eff}}}}
\]

\( 3 - 14 \)
3-5 APERTURE COUPLING BETWEEN TWO IDENTICAL MICROSTRIPS:

An expression for the normalized reactance of a small aperture coupling two microstrips together will now be derived. The actual coupling configuration is shown in figure (3-3a); the aperture (again of diameter d) is located in the common groundplane. Both microstrips terminate in open circuits $3\lambda g/4$ past the aperture. This length of line transforms the open circuit into a short circuit (maximum H-field) at the location of the aperture, thus yielding maximum magnetic coupling.

![Diagram of aperture coupling between two identical microstrips]

FIG. 3-3 THE COUPLING CONFIGURATION BETWEEN TWO PARALLEL PLATE WAVEGUIDES SHOWN IN (b) IS USED TO MODEL THE COUPLING BETWEEN THE TWO MICROSTRIPS SHOWN IN (a).
For the purpose of analysis, the coupling configuration of figure (3-3a) is modelled by that shown in figure (3-3b), which illustrates the coupling between two parallel plate waveguides that share an aperture in their common broad wall. As in figure (3-3a) both guides terminate in open circuits $3\lambda g/4$ beyond the aperture.

The fields about the location of the aperture in the guides of figure (3-3b) are given below for the TEM mode.

\[
H_x = 2H_0 \cos \frac{2\pi}{\lambda} g z \quad 3-15a
\]

\[
E_y = 2E_0 \sin \frac{2\pi}{\lambda} g z \quad 3-15b
\]

From [25] the transmission coefficient $T$ through an aperture as shown in figure (3-3b) is given below, with the variables as defined in [25].

\[
T = \frac{2\pi j}{S_0} \left( \frac{H_1}{H_0} \alpha \frac{H_2}{H_0} + \frac{H_2}{H_0} \right) \quad 3-16
\]

The $S_0$, a normalizing factor is given by:

\[
S_0 = \frac{1}{\eta} \int_{guide} \left( \n \cdot \nabla \right) ds \quad 3-17
\]

The transmission coefficient can also be expressed in terms of the normalized reactance of the aperture as follows [25]:

\[
T = \frac{2\pi j x}{2\pi j x + 1} \quad 3-18
\]

The value of $S_0$ is calculated using (3-11) and (3-17).

\[
S_0 = \frac{1}{\eta} \int_0^h \int_0^W |E_t| |H_x| dy dx = W'hH_0^2 \quad 3-19
\]
For the remainder of this section, it will be assumed that the aperture is small w.r.t. \( \lambda \). From figure (3-3b) and (3-15) it can then be seen that the \( E \) field is zero at the position of the aperture, while the \( H \) field can be assumed constant over the aperture with magnitude \( 2H_0 \). The normalized reactance of a small aperture (\( \times 1 \)) is obtained from (3-18) in terms of the transmission coefficient.

\[
x = \frac{T}{2j} \quad \text{for } x < 1
\]

3-20

In this particular case, the last two terms of (3-16) are zero. Since both guides in figure (3-3b) are identical, the term \( H_0 \xi^2 \) in (3-16) reduces to \( 2H_0^2 \). The value of the polarizability \( \xi \) used in (3-16) is one fourth of the effective volume of the aperture as defined by Wheeler [28].

For the circular aperture used, the value of \( V_{mc} \) is given by (3-8). Using (3-8), (3-16), (3-19) and (3-20) along with the fact that \( H_0 \xi^2 = 2H_0^2 \), the following expression is obtained for the normalized reactance of the coupling aperture.

\[
x = \frac{md^3}{3wh\lambda g}
\]

3-21

3-6 APERTURE COUPLING BETWEEN MICROSTRIP AND A TE_{01M} CAVITY:

An expression for the external \( Q \), \( Q_{\text{ext}} \), of the cavity due to the loading of the microstrip will now be given. As shown by Wheeler [28], the loading power factor \( p \) can be calculated by referring to figure (3-4)
The quantities $r$, $C$, $L$ in figure (3-4) are used to model the characteristics of the uncoupled cavity about a particular $TE_{01n}$ mode. The resonant frequency $f_o$ and unloaded $Q$, $Q_o$, of the cavity are given below.

$$ f_o = \frac{1}{2\pi \sqrt{LC}} $$
$$ Q_o = \frac{\omega L}{r} = \frac{X_L}{r} \tag{3-22} $$

The reactance of the coupling aperture can either be expressed as $kX_L$ (in terms of the coupling factor $k$ that exist between two aperture coupled cavities) or as $xZ_o$ (in terms of the normalized reactance $x$ that exist between two aperture coupled microstrips). Since both are equal:

$$ kX_L = xZ_o \tag{3-24} $$

The real and imaginary parts of the impedance $R+jX$ that is transformed into the cavity by the aperture are:

$$ R = Z_o x^2 \quad \quad X = Z_o x \tag{3-25} $$

Using the definition of the loading power factor and equations (3-24) - (3-25) yields the following expression.

$$ p = \frac{1}{Q_{ext}} = \frac{R}{X_L} = kx \tag{3-26} $$
The loaded $Q_L$ of the coupled cavity satisfies the following relation.

\[ \frac{1}{Q_L} = \frac{r}{Q_L} + \frac{R}{Q_L} = \frac{1}{Q_0} + \frac{1}{Q_{\text{ext}}} \]  

3-27

The ratio of the impedance coupled into the cavity by the aperture to that of the cavity at resonance is defined as the coupling coefficient $\beta$ and is given by:

\[ \beta = \frac{R}{r} \]  

3-28

From (3-23), (3-26) and (3-28), the coupling coefficient can be expressed as:

\[ \beta = \frac{Q_0}{Q_{\text{ext}}} \]  

3-29

Finally, using (3-9), (3-13), (3-14), (3-21) and (3-26), the following expression for $Q_{\text{ext}}$ is obtained.

\[ Q_{\text{ext}} = 5.423 \times 10^{-6} \, f \frac{L^3D^2h^2}{n^2Z_0} \]  

3-30

In a practical design problem, one is most often required to calculate the size of aperture required to give a certain $\beta$. Once the parameters of the microstrip and cavity have been established, the aperture diameter can then be found using (3-29) and (3-30). For high impedance lines, (3-29) - (3-30) can yield values of $d$ much greater than $W$, the width of the microstrip. If this occurs, a lower value of $Z_0$ (wider line) should be used for the coupling region. Although the $Q_0$ of the cavity can be found from figure (2-3), it should actually be measured as it depends critically on the surface finish of the cavity.
It should be noted that the imaginary part of the impedance \( R + jX \) that is transformed into the cavity will slightly perturb the resonant frequency from its value given by (3-22). The extent of this frequency pulling is mentioned in chapter 4 and can be neglected.

Something should be said at this point concerning the approximations made in deriving the expression for the external \( Q \). These approximations are:

1. The parallel plate waveguide model for the microstrip is based on a T.E.M. equivalence. However, it is assumed that this model is sufficient to characterize the fields in the microstrip at the position of the aperture which is located in the centre of the ground plane with respect to the microstrip (see figure (3-1)).

2. The error in the loading power factor \( \frac{1}{Q_{\text{ext}}} \) as given by (3-26) is of the order (\( kx \)).

3. The expression used for \( V_{\text{mc}} \) assumes that the aperture is located in a thin conducting wall; this is a realistic approximation as the ground plane metalization is thin (less than 30\( \mu \)m for the substrates to be described in chapters 4 and 5).

4. The value of \( Q'_{\text{ext}} \) as given by (3-30) assumes that the \( H \) field is constant over the aperture with magnitude \( 2H_0 \). As the frequency or the \( e_r \) of the substrate increases, this approximation deteriorates. A correction for this effect is given in the next section.

The expression for \( Q_{\text{ext}} \) derived in this section is experimentally verified in chapter 4.

3-7 EXTENSION OF THE ANALYSIS TO COVER LARGE \( \lambda/\text{APERTURE RATIO} \) AND OTHER APERTURE SHAPES.

As was mentioned in the last section, the assumption that the \( H \) field is constant over the aperture is not always valid. For certain values of substrate \( e_r \) and resonant frequency \( f_0 \), the \( H \) field can vary appreciably over the aperture in the direction of propagation. Under these conditions, it is shown in chapter 4 that the coupling coefficient \( \beta \) as
obtained from (3-29) and (3-30) yields values that are larger than those obtained experimentally. To account for this discrepancy, a first order correction is applied to the expression for $Q_{\text{ext}}$ by using the average value of the $H$ field over the aperture in the calculation of $q$. This average value $H_{\text{avg}}$ is obtained by integrating the $H$ field over the area of the aperture and dividing the result by the area of the aperture; it is convenient to normalize this result w.r.t. the maximum $H$-field. For the circular aperture shown in figure (3-1), the average $H$ field is calculated using (3-15) [33]

$$\frac{H_{\text{avg}}}{H_{\text{max}}} = \frac{\int \cos \left( \frac{\pi d}{\lambda g} \right) \, dA}{\text{aperture area}} = \frac{2 \lambda g}{\pi d} J_1 \left( \frac{\pi d}{\lambda g} \right)$$

(3-31)

From (3-16), (3-20) and (3-26) it can be seen that an expression for $Q_{\text{ext}}$ which accounts for the variation in the $H$ field over the aperture is obtained by multiplying the previous expression for $Q_{\text{ext}}$ (3-30) by a correction factor $q$. This correction factor $q$ is obtained using (3-14) and (3-31).

$$q = \left( \frac{2J_1(u)}{u} \right)^2, \quad u = \frac{\pi d}{\lambda g} = \frac{df_o/e_{\text{eff}}}{9.55 \times 10^{-2}}$$

(3-32)

A similar correction factor could be applied for the transverse variation in the $H$ field. However, the field variation in this direction is not as severe as that in the direction of propagation and it seems sufficient to include a correction for the first case only. From ground-plane current density distributions obtained by finite difference computations [34], it is estimated that the $H$ field variation in the $x$-direction is less than 10%, even for the largest anticipated aperture.

The analysis can be readily modified for use with other shapes of apertures by using the appropriate value of $V_m$. From a practical standpoint, it may be advantageous to use a rectangular aperture instead of a circular one.
This might be so in the case of MIC fabrication where a mask containing a rectangular aperture could be easier to cut than one with a circular aperture.

An expression for $Q_{\text{ext}}$ will now be given for coupling with the rectangular aperture shown in figure (3-5).

![Diagram of rectangular aperture in ground-plane](image)

**FIG. 3-5** SECTON OF AN MIC SUBSTRATE TO BE USED WITH THE CAVITY SHOWN IN FIG.(3-1). COUPLING IS THRU THE RECTANGULAR APERTURE IN THE GROUND-PLANE.

Although no theoretical expression is available for the magnetic polarizability of a rectangular aperture as shown in figure (3-5), experimental data obtained from electrolytic tank measurements is available [35]. This data is presented in figure (3-6) for the case where the H field is parallel to the width of the aperture (a dimension in figure (3-5)).
FIG. 3-6 MAGNETIC POLARIZABILITY OF A RECTANGULAR APERTURE WITH THE H-FIELD PARALLEL TO THE MAJOR DIMENSION. [35]
Using (3-1b), (3-7), (3-13), (3-14), (3-16), (3-19), (3-20) and (3-26) along with the value of \( M \) obtained from figure (3-6) gives the following expression for \( Q_{\text{ext}} \):

\[
Q_{\text{ext}} = 1.506 \times 10^{-7} \frac{f_0 L^3 b^2 \mu^2}{d^2 \varepsilon_0 \varepsilon_{\text{eff}} M^2}
\]

3-33

It can be seen using (3-29), (3-30) and (3-33) that compared to a circular aperture of diameter \( d \), a rectangular one of length \( d/2 \) and width \( d \) yields a coupling coefficient only 11% less for small apertures.

For cases where the b dimension becomes an appreciable fraction of \( \lambda_g \), the average value of the H field should be used in the calculation of \( \mu \). Performing the calculation outlined by (3-31) yields the following form for the correction factor \( q \) for the case of a rectangular aperture.

\[
q = \left( \frac{\sin(w)}{w} \right)^2, \quad \mu = \frac{\pi b}{\lambda_g} \frac{f_0 \varepsilon_{\text{eff}}}{9.55 \times 10^6}
\]

3-34
CHAPTER 4

EXPERIMENTAL VERIFICATION OF A NOVEL TRANSITION
BETWEEN MICROSTRIP AND A CYLINDRICAL CAVITY

INTRODUCTION:

The transition analyzed in chapter 3 was experimentally verified and the results will now be given. The measurements were done at S-band using oversized substrates in order to reduce the effects of geometrical irregularities which will have a more pronounced effect at higher frequencies.

DESIGN OF THE S-BAND CAVITY:

The cavity was chosen to resonate in the $TE_{012}$ mode at a frequency of 3.333 GHz. The $TE_{012}$ mode has a higher $Q$ than the $TE_{011}$ mode and the larger volume required for the $TE_{012}$ mode is not a problem.

A high $Q$ mode permits critical coupling to be achieved with a smaller aperture so that the analysis presented in chapter 3 may better approximate the coupling problem for values of $\beta$ up to critical coupling. A ratio of $\frac{D}{L} = 1.02$ yields the highest $Q_o$ for the $TE_{012}$ mode and fairly good separation between the neighboring modes. For the resonant frequency chosen, the values of $D$ and $L$ were found using (2-9b) and are: $D=5.628''$, $L=5.517''$.

The cavity was machined of brass and the inside was then silver plated. An end plate was also made with a small coupling loop that terminates into an O.S.M. type connector. The loop was oriented to couple to the $H$ field of the $TE_{012}$ mode. For silver ($\delta = 1.109 \times 10^{-4}$ cm at S-band), the $Q_o$ is approximately 64,000, as found from figure (2-3). A mode suppressor in the form of an annular gasket was machined of synthane and served to suppress modes not of the $TE_{0mn}$ type. A photograph of the cavity and mode suppressor is shown in Fig. (4-1).
Fig. 4-1. Photograph of the S-band Cavity and Mode Suppressor. The coupling loop can be seen mounted to the end plate.

The resonant frequencies of the modes closest to the TE\(_{012}\) mode were calculated using (2-9b) and are given in table (4-1).

<table>
<thead>
<tr>
<th>Mode</th>
<th>TE(_{212})</th>
<th>TE(_{311})</th>
<th>TE(_{012})</th>
<th>TM(_{112})</th>
<th>TM(_{210})</th>
<th>TE(_{113})</th>
<th>TE(_{312})</th>
</tr>
</thead>
<tbody>
<tr>
<td>f(_0)/GHz</td>
<td>2.954</td>
<td>3.001</td>
<td>3.333</td>
<td>3.333</td>
<td>3.427</td>
<td>3.435</td>
<td>3.526</td>
</tr>
</tbody>
</table>

Table 4-1. Resonant frequencies of the neighbouring modes.

The return loss of the cavity was measured on a H.P. swept frequency analyser. The results are given in figure (4-2a) and (4-2b); the two plots correspond to the response of the cavity with and without the mode suppressor. The plots agree with the results given in table (4-1); the dips in figure (4-2a) are all shifted up in frequency due to the absence of the mode suppressor. The values given in table (4-1) take the thickness of the mode suppressor into account. The measured resonant frequency of the TE\(_{012}\) mode is 3.356 GHz (with mode suppressor).
Fig. 4-2. Return loss of the cavity as a function of frequency (a) without the mode suppressor and (b) with the mode suppressor present.
DESIGN OF THE MICROSTRIP CIRCUITS:

The microstrip circuits were etched onto two (oversized) substrates with $h = 200$ mils. One was made of rexolite having $\varepsilon_r = 2.6$, the other being of styecast and having $\varepsilon_r = 10$.

At first though it seemed sensible to make the line impedance 50 ohms on both substrates, however for the $\varepsilon_r = 10$ substrate, this leads to an aperture diameter that is greater than the width of the microstrip at critical coupling (see section (3-6)). To overcome this difficulty, a lower value of $Z_0$ was used for the line on the styecast substrate. A value of $Z_0 = 22.8$ is found to yield values for the aperture diameter that are still smaller than the line width even beyond critical coupling.

The dimensions of the microstrip were calculated with a computer program based on Gertsinger's model [32]. As was mentioned in section (3-5), an open-circuited stub of electrical length $3/4 \lambda$ was used to present a short circuit condition at the aperture. The physical length of the stub includes a correction for the end effect [36].

The actual layout of the microstrip is shown in figure (4-3), with the corresponding parameters given in table (4-2). A photograph of the styecast substrate is shown in figure (4-4).

<table>
<thead>
<tr>
<th>Substrate</th>
<th>S</th>
<th>L</th>
<th>W</th>
<th>h</th>
<th>$Z_0$</th>
<th>$\varepsilon_{\text{eff}}$ (at $f_0$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rexolite</td>
<td>1.354&quot;</td>
<td>1.657&quot;</td>
<td>.541&quot;</td>
<td>.200&quot;</td>
<td>50.0</td>
<td>2.27</td>
</tr>
<tr>
<td>$\varepsilon_r = 2.6$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Styecast</td>
<td>1.354&quot;</td>
<td>.871&quot;</td>
<td>.626&quot;</td>
<td>.200&quot;</td>
<td>22.8</td>
<td>9.45</td>
</tr>
<tr>
<td>$\varepsilon_r = 10$</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4-2. Microstrip parameters for the rexolite and styecast substrates.
FIG. 4-3. MICROSTRIP CIRCUIT ON OVERSIZED (0.200") SUBSTRATE.

Fig. 4-4. Photograph of the microstrip circuit on the styecast substrate.
In order to simplify the Q measurement, a two port transmission cavity configuration was used. The aperture in the microstrip ground plane formed the input port and the coupling loop on the end plate served as the output (detector) port. Since a high Q cavity was used, a comb technique [37] was employed to accurately measure the 3 dB bandwidth of the coupled cavity.

The equivalent circuit used to interpret the measurements and the physical structure it represents are shown in Fig. (4-5). This equivalent circuit represents the coupled cavity about the resonant frequency of the TE_{012} mode. The symbols used are defined as:

\[ Y_{01} \] The characteristic admittance of the measurement setup

\[ n_1; jB_1 \] The transformer and susceptance represent the tuner

\[ jB_2 \] The discontinuity susceptance of the coax to microstrip transition

\[ Y_{02} \] The characteristic admittance of the microstrip line (lossy)

\[ n_2; jB_3 \] These parameters represent the coupling aperture. The susceptance represents the energy storage in the evanescent modes that match the boundary conditions at the aperture

\[ L, C, \xi \] This parallel tuned circuit represents the resonance of the TE_{012} mode

\[ n_3; jB_4 \] These represent the coupling loop.
The tuner was adjusted to match the admittance seen at the microstrip transition to that of the system. The shunt susceptances $jB_3$ and $jB_4$ shift the resonant frequency slightly. As the resultant frequency shift is small (less than 600 KHz) and does not effect the measurements, these susceptances are neglected.

The input and output coupling coefficients as defined in [25] are:

$$
\beta_1 = \frac{Y_{02}}{n_2^2 G} \quad \beta_2 = \frac{Y_{01}}{n_3^2 G}
$$

4-1

In order to evaluate the coupling between the microstrip and the cavity, the latter is considered as a one-port structure at reference plane $c-c'$ with the detector port $b-b'$ transformed into the cavity. The unloaded $Q$ and coupling coefficient for the cavity as a one port at $c-c'$ are:

$$
Q' = \frac{\omega C}{G(\beta_2 + 1)} = \frac{Q_0}{\beta_2 + 1}
$$

4-2

$$
\beta = \frac{Y_{02}}{n_2^2 G(\beta_2 + 1)} = \frac{\beta_1}{\beta_2 + 1}
$$

4-3
Fig. 4-5. (a) Schematic of the coupled cavity. The aperture is located at reference plane c-c'. (b) An equivalent circuit for the coupled cavity between reference planes a-a' and b-b'. (c) A simplified equivalent circuit at reference plane c-c' for the cavity at resonance with the detector port, b-b', transformed into the cavity.
The insertion loss measured between a-a' and b-b' in figure (4-5b) is equivalent to that measured between c-c' and the transformed detector impedance $Y_{02} \times \frac{\beta_2}{\beta_1}$ in figure (4-5c) and can be expressed as:

$$I.L = \frac{4\beta}{(\beta+1)^2} \times \frac{\beta_2}{(\beta_2+1)}$$  \hspace{1cm} 4-4

The minimum insertion loss occurs when port c-c' is critically coupled to the microstrip ($\beta = 1$). The ratio $r$ between the insertion loss and the minimum insertion loss is:

$$r = \frac{4\beta}{(\beta+1)^2}$$  \hspace{1cm} 4-5

The value of $\beta$ obtained from the relative insertion loss is then:

$$\beta = \left(\frac{2}{r} - 1\right) \pm \sqrt{\left(\frac{2}{r} - 1\right)^2 - 1}$$  \hspace{1cm} 4-6

Note that $\beta$ is only a function of the relative insertion loss, because of this, any losses in the microstrip do not influence the measurements. The value of $\beta_2$ should not be too small or else the minimum insertion loss will be high and the system noise could become a problem when measuring small values of $\beta$. For measurement convenience, the coupling loop was adjusted so that $\beta_2 = 1$.

From a measurement of the loaded $Q$, $Q_L$, $\beta$ is independently obtained as:

$$\beta = \frac{Q'}{Q_L} - 1$$  \hspace{1cm} 4-7

The setup used to perform the measurements is shown in block form in figure (4-6). A photograph of the actual layout is shown in figure (4-7).
Fig. 4.6. Block diagram of the instrumentation used to measure the insertion loss and loaded Q of the coupled cavity.
Fig. 4-7. Photograph of the layout used for the measurements.
EXPERIMENTAL RESULTS:

The substrates were in turn mounted on the cavity and the
diameter of the aperture was increased in steps.

The total insertion loss and bandwidth (of the loaded Q) were
measured each time and are plotted in figure (4-8) for the rexolite
substrate and in figure (4-9) for the styecast substrate as a function
of the normalized aperture diameter. From the data presented in
figures (4-8) and (4-9), the following quantities were calculated
and are tabulated in table (4-3) and (4-4) for the rexolite and
styecast substrates respectively.

The total insertion loss

\[ r \] - The ratio between the insertion loss of the cavity and
the minimum insertion loss of the cavity

\[ \beta_{m1} \] - The coupling coefficient obtained from the measured
insertion loss and given by (4-6).

\[ Q_{L_m} \] - The loaded Q of the cavity obtained from the bandwidth
measurement. \( Q_{L_m} = \frac{f_o}{\Delta f} \)

\[ Q_{o_m}^' \] - The unloaded Q of the cavity considered as a one-port structure.

\[ Q_{o_m}^' = Q_{L_m} (\beta_{m1} + 1) \]

\[ \beta_{m2} \] - The coupling coefficient obtained from the bandwidth measure-
ment and given by (4-7).

\[ Q_{ext_c} \] - The external Q as calculated from (3-30)

\[ \beta_{c1} \] - The coupling coefficient obtained from \( Q_{ext_c} \) and given by
(3-29)-(3-30).

\[ q \] - The correction factor given by (3-32)

\[ \beta_{c2} \] - The coupling coefficient obtained from \( Q_o \) and \( Q_{ext_c} \) as
modified by \( q \) and given by (3-29), (3-30), (3-32).
Fig. 4-8. Measured insertion loss and bandwidth for coupling with the microstrip on the rexolite substrate. The aperture diameter is normalized to the width of the microstrip.
Fig. 4-9. Measured insertion loss and bandwidth for coupling with the microstrip on the styecast substrate. The aperture diameter is normalized to the width of the microstrip.
<table>
<thead>
<tr>
<th>Aperture Diameter mm</th>
<th>Insertion Loss dB</th>
<th>$\Delta f$ kHz</th>
<th>$r$</th>
<th>$\beta_m^1$</th>
<th>$Q_{L_m}$</th>
<th>$Q_{Q_m}'$</th>
<th>$\beta_m^2$</th>
</tr>
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<tbody>
<tr>
<td>3.1</td>
<td>31.3</td>
<td>150</td>
<td>$1.738 \times 10^{-3}$</td>
<td>$4.35 \times 10^{-4}$</td>
<td>22,373</td>
<td>22,383</td>
<td></td>
</tr>
<tr>
<td>3.3</td>
<td>27.8</td>
<td>155</td>
<td>$3.890 \times 10^{-3}$</td>
<td>$9.73 \times 10^{-4}$</td>
<td>21,652</td>
<td>21,673</td>
<td></td>
</tr>
<tr>
<td>4.6</td>
<td>20.6</td>
<td>155</td>
<td>$2.042 \times 10^{-2}$</td>
<td>$5.16 \times 10^{-3}$</td>
<td>21,652</td>
<td>21,764</td>
<td></td>
</tr>
<tr>
<td>5.9</td>
<td>14.5</td>
<td>158</td>
<td>$8.318 \times 10^{-2}$</td>
<td>$2.17 \times 10^{-2}$</td>
<td>21,241</td>
<td>21,702</td>
<td></td>
</tr>
<tr>
<td>7.1</td>
<td>10.1</td>
<td>168</td>
<td>$2.291 \times 10^{-1}$</td>
<td>$6.50 \times 10^{-2}$</td>
<td>19,976</td>
<td>21,274</td>
<td></td>
</tr>
<tr>
<td>8.6</td>
<td>5.8</td>
<td>176</td>
<td>$6.166 \times 10^{-1}$</td>
<td>$2.35 \times 10^{-1}$</td>
<td>19,068</td>
<td>23,552</td>
<td>$2.31 \times 10^{-1}$</td>
</tr>
<tr>
<td>10.2</td>
<td>4.1</td>
<td>225</td>
<td>$9.120 \times 10^{-1}$</td>
<td>$5.40 \times 10^{-1}$</td>
<td>14,916</td>
<td>23,007</td>
<td>$5.74 \times 10^{-1}$</td>
</tr>
<tr>
<td>11.2</td>
<td>3.7</td>
<td>281</td>
<td>1.000</td>
<td>1.00</td>
<td>11,943</td>
<td>23,886</td>
<td>$9.66 \times 10^{-1}$</td>
</tr>
<tr>
<td>12.2</td>
<td>4.1</td>
<td>360</td>
<td>$9.120 \times 10^{-1}$</td>
<td>1.84</td>
<td>9,322</td>
<td>26,474</td>
<td>1.52</td>
</tr>
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<td>13.4</td>
<td>5.4</td>
<td>440</td>
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<td>3.64</td>
<td>7,627</td>
<td>35,389</td>
<td>2.08</td>
</tr>
<tr>
<td>14.3</td>
<td>5.5</td>
<td>594</td>
<td>$6.607 \times 10^{-1}$</td>
<td>3.79</td>
<td>5,650</td>
<td>27,064</td>
<td>3.16</td>
</tr>
<tr>
<td>16.6</td>
<td>7.0</td>
<td>956</td>
<td>$4.677 \times 10^{-1}$</td>
<td>6.40</td>
<td>3,510</td>
<td>25,960</td>
<td>5.69</td>
</tr>
</tbody>
</table>

Table 4-3. Calculated and measured parameters for coupling w.
<table>
<thead>
<tr>
<th>$Q_{L_m}$</th>
<th>$Q_{0_m}$</th>
<th>$\beta_{m2}$</th>
<th>$Q_{ext_c}$</th>
<th>$\beta_{c1}$</th>
<th>$q$</th>
<th>$\beta_{c2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>22,373</td>
<td>22,383</td>
<td>$6.24\times10^7$</td>
<td>$3.76\times10^{-4}$</td>
<td>.992</td>
<td>$3.73\times10^{-4}$</td>
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</tr>
<tr>
<td>21,652</td>
<td>21,673</td>
<td>$4.29\times10^7$</td>
<td>$5.48\times10^{-4}$</td>
<td>.993</td>
<td>$5.44\times10^{-4}$</td>
<td></td>
</tr>
<tr>
<td>21,652</td>
<td>21,764</td>
<td>$5.85\times10^6$</td>
<td>$4.02\times10^{-3}$</td>
<td>.985</td>
<td>$3.96\times10^{-3}$</td>
<td></td>
</tr>
<tr>
<td>21,241</td>
<td>21,702</td>
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<td>$1.75\times10^{-2}$</td>
<td></td>
</tr>
<tr>
<td>19,976</td>
<td>21,274</td>
<td>$4.32\times10^5$</td>
<td>$5.43\times10^{-2}$</td>
<td>.965</td>
<td>$5.24\times10^{-2}$</td>
<td></td>
</tr>
<tr>
<td>19,068</td>
<td>23,552</td>
<td>$2.31\times10^{-1}$</td>
<td>$1.37\times10^5$</td>
<td>.948</td>
<td>$1.63\times10^{-1}$</td>
<td></td>
</tr>
<tr>
<td>14,916</td>
<td>23,007</td>
<td>$5.74\times10^{-1}$</td>
<td>$1.72\times10^{-1}$</td>
<td>.948</td>
<td>$4.44\times10^{-1}$</td>
<td></td>
</tr>
<tr>
<td>11,943</td>
<td>23,886</td>
<td>$9.66\times10^{-1}$</td>
<td>$2.81\times10^4$</td>
<td>.915</td>
<td>$7.65\times10^{-1}$</td>
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<td>1.52</td>
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<tr>
<td>7,627</td>
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<td>27,064</td>
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<td>3.63</td>
<td>.865</td>
<td>3.14</td>
</tr>
<tr>
<td>3,510</td>
<td>25,960</td>
<td>5.69</td>
<td>$2.65\times10^3$</td>
<td>8.87</td>
<td>.822</td>
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</table>

Measured parameters for coupling with the rexolute substrate.
<table>
<thead>
<tr>
<th>Aperture Diameter (mm)</th>
<th>Insertion Loss (dB)</th>
<th>( \beta_m )</th>
<th>( Q_m' )</th>
<th>( \beta_m' )</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.2</td>
<td>37.0</td>
<td>220</td>
<td>5.370 \times 10^{-4}</td>
<td>1.27 \times 10^{-4}</td>
</tr>
<tr>
<td>2.8</td>
<td>31.5</td>
<td>220</td>
<td>1.905 \times 10^{-3}</td>
<td>4.78 \times 10^{-4}</td>
</tr>
<tr>
<td>3.4</td>
<td>28.2</td>
<td>230</td>
<td>4.074 \times 10^{-3}</td>
<td>1.02 \times 10^{-3}</td>
</tr>
<tr>
<td>3.8</td>
<td>24.5</td>
<td>245</td>
<td>9.550 \times 10^{-3}</td>
<td>2.40 \times 10^{-3}</td>
</tr>
<tr>
<td>4.8</td>
<td>19.4</td>
<td>300</td>
<td>3.090 \times 10^{-2}</td>
<td>7.85 \times 10^{-3}</td>
</tr>
<tr>
<td>5.4</td>
<td>16.0</td>
<td>240</td>
<td>6.761 \times 10^{-2}</td>
<td>1.75 \times 10^{-2}</td>
</tr>
<tr>
<td>6.0</td>
<td>13.0</td>
<td>230</td>
<td>1.349 \times 10^{-1}</td>
<td>3.62 \times 10^{-2}</td>
</tr>
<tr>
<td>6.6</td>
<td>10.5</td>
<td>236</td>
<td>2.399 \times 10^{-1}</td>
<td>6.85 \times 10^{-2}</td>
</tr>
<tr>
<td>7.4</td>
<td>8.0</td>
<td>240</td>
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<td>1.38 \times 10^{-1}</td>
</tr>
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<td>270</td>
<td>6.607 \times 10^{-1}</td>
<td>2.64 \times 10^{-1}</td>
</tr>
<tr>
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<td>5.0</td>
<td>300</td>
<td>8.511 \times 10^{-1}</td>
<td>4.43 \times 10^{-1}</td>
</tr>
<tr>
<td>10.2</td>
<td>4.4</td>
<td>340</td>
<td>9.772 \times 10^{-1}</td>
<td>7.38 \times 10^{-1}</td>
</tr>
<tr>
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<td>4.4</td>
<td>410</td>
<td>9.772 \times 10^{-1}</td>
<td>1.36</td>
</tr>
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<td>4.6</td>
<td>490</td>
<td>9.333 \times 10^{-1}</td>
<td>1.70</td>
</tr>
<tr>
<td>13.4</td>
<td>4.9</td>
<td>590</td>
<td>8.710 \times 10^{-1}</td>
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<tr>
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<td>5.4</td>
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<td>7.762 \times 10^{-1}</td>
<td>2.80</td>
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<td>900</td>
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<td>3.94</td>
</tr>
<tr>
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<td>7.0</td>
<td>1,110</td>
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<td>5.26</td>
</tr>
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</table>

Table 4-4. Calculated and measured parameters for coupling with
<table>
<thead>
<tr>
<th>$Q_{om}$</th>
<th>$Q'_{om}$</th>
<th>$\theta_{m2}$</th>
<th>$Q_{ext_c}$</th>
<th>$\beta_{c1}$</th>
<th>$q$</th>
<th>$\beta_{c2}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>15.254</td>
<td>15.256</td>
<td>2.64x10^8</td>
<td>6.82x10^{-5}</td>
<td>.987</td>
<td></td>
<td>6.73x10^{-5}</td>
</tr>
<tr>
<td>15.254</td>
<td>15.261</td>
<td>6.20x10^7</td>
<td>2.90x10^{-4}</td>
<td>.979</td>
<td></td>
<td>2.84x10^{-4}</td>
</tr>
<tr>
<td>14.606</td>
<td></td>
<td>1.93x10^7</td>
<td>9.29x10^{-4}</td>
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<td>1.74x10^{-3}</td>
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<td></td>
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<td>2.81x10^{-2}</td>
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<td>11.187</td>
<td>16.143</td>
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<td>9.871</td>
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<td>19.317</td>
<td>2.65x10^4</td>
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<td>6.849</td>
<td>18.492</td>
<td>8.19x10^{-1}</td>
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<td>.686</td>
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<td>8.57x10^{-1}</td>
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<td>17.747</td>
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<td>4.597</td>
<td>17.469</td>
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<td>3.729</td>
<td>18.421</td>
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<td>2.87</td>
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<td>18.924</td>
<td>3.82</td>
<td>3.35x10^3</td>
<td>.476</td>
<td></td>
<td>4.13</td>
</tr>
<tr>
<td></td>
<td></td>
<td>4.94</td>
<td>2.07x10^3</td>
<td>.428</td>
<td></td>
<td>5.38</td>
</tr>
</tbody>
</table>

...and measured parameters for coupling with the stycastrate substrate.
As can be seen from tables (4-3) and (4-4), the value of $Q_{o1}$ changed slightly between each measurement. This change is partially due to measurement errors in both $I_L$ and $\Delta f$ and also to the fact that the parallelism between the end plate and the microstrip ground plane depends on how the bolts securing both of these pieces are torqued. The value of $Q_{o1}$ used in calculating $\beta_{m1}$, $\beta_{c1}$, and $\beta_{c2}$ was obtained from the value of $\Delta f$ at critical coupling as taken from figures (4-8) and (4-9) and differed by a few percent from the average value as calculated from the tabulated values. The values of $Q_{o1}$ that are used in calculations are 23,482 for the rexolite substrate and 17,959 for the stycast substrate. The difference between these two values is probably due in part to the fact that the coupling loop became loose after the first set of measurements and had to be replaced (the size of the new loop being slightly different); also there may have been a slight variation in the conductivity of the ground planes between the two substrates. These two values of $Q_{o1}$ give values of $Q_o$ (using 4-2) that are 74% and 56% respectively of the theoretical $Q_o$, assuming $\beta_2=1$.

It can be seen from figures (4-8) and (4-9) that for small values of $\beta$ (aperture diameter), the insertion loss measurement is the more sensitive to a change in $\beta$, while for values of $\beta$ about critical coupling the bandwidth measurement is more sensitive. Using both measurements enables one to measure the coupling coefficient quite accurately over several decades.

The values of $\beta_{m1}$ and $\beta_{m2}$ (measured) are plotted in figures (4-10) and (4-11) for the rexolite and stycast substrates respectively along with $\beta_{c1}$ and $\beta_{c2}$ (theory) all as a function of the normalized aperture diameter.
Fig. 4-10. Calculated and measured values of $\beta$ for coupling with the microstrip on the rexolite substrate. The aperture diameter is normalized to the width of the microstrip.
Fig. 4-11. Calculated and measured values of $\beta$ for coupling with the microstrip on the styecast substrate. The aperture diameter is normalized to the width of the microstrip.
As can be seen from figures (4-10) and (4-11), the calculated and measured values of $\beta$ are in good agreement for values of $\beta$ up to and greater than critical coupling. The field variation in the $z$ direction over the aperture becomes significant for aperture diameters corresponding to values of $\beta$ about critical coupling. At critical coupling, the aperture diameter is 19% of $\lambda_g$ for the rexolite substrate and 38% of $\lambda_g$ for the styacast substrate. This fact explains why the measured values of $\beta$ for the styacast substrate seem to depart more from the values of $\beta_{c_1}$ (theory, without correction factor) than do those for the rexolite substrate for values of $\beta$ greater than approximately 2. In this region the values of $\beta_{c_2}$ (theory, with correction factor) are in good agreement with the measured values.

As far as experimental errors are concerned, the most sensitive parameter is the aperture diameter as can be seen from (3-30). Error bars on figures (4-10) and (4-11) show the change in $\beta_{c_1}$ due to an estimated worst case measurement error of .010" in the aperture diameter. The aperture diameter was increased by using a knife blade held in a drawing compass and the aperture circumference usually had an edge ripple of a few mils.

The analysis of the microstrip to cavity transition presented in chapter 3 was experimentally verified in this chapter and found to be useful for values of $\beta$ up to and greater than critical coupling using a high $Q_0$ cavity mode. The transition will be used in the second part of this report in the design of stabilizing cavities for experimental 12 GHz GaAs FET oscillators.
CHAPTER 5

IMPLEMENTATION OF THE MICROSTRIP TRANSITION IN
THE DESIGN OF STABILIZING CAVITIES FOR MIC OSCILLATORS

5-1 Introduction: The microstrip to cylindrical cavity transition described in the first part of this report was evaluated at 12 GHz. The transition was then implemented in the design of stabilizing cavities for MIC oscillators. The cavities were used to stabilize the GaAs FET oscillators to be described in chapters six and seven.

5-2 Implementation of the Microstrip to Cavity Transition at 12 GHz:

In order to evaluate the microstrip transition at 12 GHz, a cavity was designed to resonate in the TE_{012} mode in the vicinity of 12 GHz. The dimensions of the cavity were obtained from (2-9b) and are:

\[ D = 1.667'' \text{, } L = 1.416'' \]

These dimensions yield a mode shape factor of .78 as per figure (2-3); this corresponds to a value of \( Q_0 = 33,200 \) assuming silver plating. The cavity was machined of brass and the inside silver plated. A slot was milled into the top of the cavity to accept a 2" x 2" x .025" alumina substrate. The measured frequency of the TE_{012} mode was 11.950 GHz.

Four different coupling circuits were designed for use with the cavity, each one consisting of two coupling sections. For the reason mentioned in section (3-6), wider lines are used over the coupling apertures. In this case, tapered sections are used to match the lower \( Z_o \) sections to the 50\( \Omega \) sections (.022" wide lines). The tapered sections vary linearly in width, this gives an impedance variation along the taper that is approximately exponential [38]. The tapers are 2\( \lambda \)g long; the return loss from such a taper is greater than 22 dB assuming an impedance transformation of 2:1 [23]. The layout of the circuits is shown in figure (5-1).
The circuits were etched onto 2" x 2" x .025" alumina substrates using standard photolithography techniques. The substrates were first sputter-coated with TiW-Au metallization [39]. A mode suppressor consisting of a discontinuous circular slot as shown in figure (5-1) was etched into the ground plane of some of the circuits. The slot is interrupted under the microstrip lines to maintain continuity of the ground plane. The parameters used for the various circuits are given in table (5-1). Since $Q_{ext}$ is a critical function of the aperture diameter, the diameters of the coupling apertures on each circuit were measured using an optical comparator and are given in table (5-1). The microstrip dimensions were calculated using the methods mentioned in chapter 4.
<table>
<thead>
<tr>
<th>CIRCUIT</th>
<th>$S$ (mils)</th>
<th>$d_1$ (mils)</th>
<th>$d_2$ (mils)</th>
<th>$W_1$ (mils)</th>
<th>$W_2$ (mils)</th>
<th>$\ell_1$ (mils)</th>
<th>$\ell_2$ (mils)</th>
<th>$Z_1$ (\Omega)</th>
<th>$Z_2$ (\Omega)</th>
<th>MODE SUPPRESSOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIC-026</td>
<td>.401</td>
<td>34.4</td>
<td>35.0</td>
<td>43</td>
<td>43</td>
<td>255</td>
<td>255</td>
<td>35</td>
<td>35</td>
<td>No</td>
</tr>
<tr>
<td>MIC-033</td>
<td>.401</td>
<td>67.3</td>
<td>67.8</td>
<td>74</td>
<td>74</td>
<td>243</td>
<td>243</td>
<td>25</td>
<td>25</td>
<td>No</td>
</tr>
<tr>
<td>MIC-034</td>
<td>.401</td>
<td>68.6</td>
<td>68.8</td>
<td>74</td>
<td>74</td>
<td>243</td>
<td>243</td>
<td>25</td>
<td>.25</td>
<td>Yes</td>
</tr>
<tr>
<td>MIC-035</td>
<td>.401</td>
<td>62.8</td>
<td>88.3</td>
<td>74</td>
<td>99</td>
<td>243</td>
<td>237</td>
<td>25</td>
<td>20</td>
<td>Yes</td>
</tr>
</tbody>
</table>

TABLE 5-1  Values of the parameters used for the four microstrip coupling circuits.
A photograph of the cavity and circuit MIC-034 with the microstrip launchers attached is shown in figure (5-2). Figure (5-3) is a photograph of the ground plane of circuit MIC-034 showing the coupling apertures and mode suppressor.

Fig. 5-2 A photograph of the cavity and circuit MIC-033 with the microstrip launchers attached.

Fig. 5-3 A view of the ground plane of circuit MIC-034 showing the coupling apertures and mode suppressor.
The equivalent circuit shown in figure (5-4) is used to characterize the coupled cavity at resonance. The conductance $G$ represents the cavity losses and reference planes 1-1' and 2-2' are located at the input and output coupling apertures. $Z_1(Y_1)$ and $Z_2(Y_2)$ are the impedances (admittances) of the input and output microstrips over the coupling apertures. The tapered sections are represented by the ideal transformers with turn ratios $1:n_3$ and $1:n_4$; $Z_0 = 50\Omega$ is the impedance of the measurement system. The microstrip launchers are located at reference planes 3-3' and 4-4'.

![Equivalent Circuit Diagram]

Fig. 5-4 An equivalent circuit for the coupled cavity used to define the input and output coupling coefficients.

The input and output coupling coefficients $\beta_1$ and $\beta_2$ are defined below [25]. It is assumed that the tapered sections provide a perfect match between the $50\Omega$ lines and the lower impedance lines, that is $Y_0 = n_3^{-2} Y_1$ and $Y_0 = n_4^{-2} Y_2$.

$$\beta_1 = \frac{Y_1}{n_1^{-2} G} = \frac{Y_0}{n_1^{-2} n_3^{-2} G} \quad 5-1$$

$$\beta_2 = \frac{Y_2}{n_2^{-2} G} = \frac{Y_0}{n_2^{-2} n_4^{-2} G} \quad 5-2$$

The circuits were in turn mounted on the cavity and the insertion loss and loaded $Q$ were measured. In the case of circuit MIC-035, the return loss on the output port was also measured. This was done because unlike the other circuits, MIC-035 has unequal input and output coupling ports and
an additional measurement is required to uniquely determine \( \beta_1 \) and \( \beta_2 \). The insertion loss and return loss on the output port are given below in terms of \( \beta_1 \) and \( \beta_2 \).

\[
\begin{align*}
\text{IL} &= \frac{4\beta_1 \beta_2}{(1 + \beta_1 + \beta_2)^2} \quad 5-3 \\
\text{RL} &= \left| \frac{\beta_2 - \beta_1 - 1}{\beta_2 + \beta_1 + 1} \right| \quad 5-4
\end{align*}
\]

Values of \( \beta_1 \) and \( \beta_2 \) were calculated using the measurement data. The results are compared with the calculated values as given by (3-29), (3-30) and (3-32) in Table (5-2). The measured values of \( \beta_1 \) and \( \beta_2 \) given in the table for circuits MIC-026 thru MIC-034 assume that both apertures have equal diameters whereas the calculated values are based on the diameters given in Table (5-1). It can be seen on comparing tables (5-1) and (5-2) that the coupling coefficients are extremely sensitive to the aperture diameter and that there is good agreement between the measured and calculated values. The value of \( Q_0 \), approximately 29,000, is 87% of the theoretical value.

Finally, to show the effect of the mode suppressor, swept frequency measurements were made using circuits MIC-033 and MIC-034. These two circuits only differ in that the latter has a mode suppressor. The results are shown in figure (5-5). It is seen that the mode suppressor has a dramatic effect on those modes which are not of the \( \text{TE}_{11m} \) type. The modes closest to the \( \text{TE}_{012} \) mode that are not suppressed are the \( \text{TE}_{011} \) and the \( \text{TE}_{013} \); these occur at 9.587 GHz and 15.193 GHz respectively. These last two modes might present a problem in some circuit applications.
<table>
<thead>
<tr>
<th>CIRCUIT</th>
<th>INSERTION LOSS dB</th>
<th>OUTPUT PORT RETURN LOSS dB</th>
<th>QL MEASURED</th>
<th>β(_1) CALCULATED WITHOUT CORRECTION FACTOR</th>
<th>β(_1) CALCULATED WITH CORRECTION FACTOR</th>
<th>Qo MEASURED</th>
<th>β(_2) CALCULATED WITHOUT CORRECTION FACTOR</th>
<th>β(_2) CALCULATED WITH CORRECTION FACTOR</th>
</tr>
</thead>
<tbody>
<tr>
<td>MIC-026</td>
<td>37.1</td>
<td>---</td>
<td>2.85x10⁴</td>
<td>7.1x10⁻³</td>
<td>6.4x10⁻³</td>
<td>----</td>
<td>7.1x10⁻³</td>
<td>7.1x10⁻³</td>
</tr>
<tr>
<td>MIC-033</td>
<td>8.7</td>
<td>---</td>
<td>1.84x10⁴</td>
<td>.29</td>
<td>.28</td>
<td>.25</td>
<td>.29</td>
<td>.30</td>
</tr>
<tr>
<td>MIC-034</td>
<td>8.3</td>
<td>---</td>
<td>1.80x10⁴</td>
<td>.31</td>
<td>.32</td>
<td>.29</td>
<td>.31</td>
<td>.32</td>
</tr>
<tr>
<td>MIC-035</td>
<td>8.0</td>
<td>16.7</td>
<td>1.24x10⁴</td>
<td>.19</td>
<td>.19</td>
<td>.18</td>
<td>1.14</td>
<td>1.20</td>
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</tbody>
</table>

TABLE 5-2 Summary of the results obtained from the four coupling circuits. The measured values of the coupling coefficients are in good agreement with the calculated values.
Fig. 5-5 Measured insertion loss of the cavity using
(a) circuit MIC-033 and
(b) circuit MIC-034.
5-3 **Fine Tuning the Cavity:** A small metallic post was used to fine tune the resonant frequency of the $\text{TE}_{012}$ mode. A micrometer head was mounted on the cavity and the micrometer shank served as the tuning post. The micrometer head was located so that the shank would occupy a position inside the cavity as shown in figure (2-6). A photograph of the cavity with the micrometer head in place is shown in figure (5-6).

![Image](image-url)

**Fig. 5-6** A photograph of the cavity showing the position of the micrometer head used for fine tuning.

The radius of the shank used is 100 mils; the shank can be inserted into the cavity to a depth of 500 mils. The change in resonant frequency due to the shank was calculated using (2-29) and is plotted in figure (5-7) as a function of its length, $L$. Measured values of the change in resonant frequency are also indicated on the figure.

It was observed that the tuning post has a degrading effect on the $Q_0$ of the cavity. To reduce this effect, the edges of the post were rounded (50 mil radius) and it was silver plated. Even so, inserting the post into the cavity to a depth of 400 mils caused a 25% reduction in $Q_0$. The tuning post should be made of a material having a low coefficient of thermal expansion in order to minimize changes in the
Fig. 5-7  Measured and calculated shift in the resonant frequency of the cavity as a function of $l$.

resonant frequency. For example, if an invar ($\alpha = 1$ ppm/°C) post is used having $r = 0.100''$ and $l = 0.400''$, the change in resonant frequency due to thermal expansion of the post would be less than 1.0 kHz for a variation in temperature from -25°C to 75°C.

5-4 Design Considerations for Highly Stable Cavities: A stabilizing cavity is used to improve both the short term stability (FM noise) and long term stability (frequency drift) of an oscillator. Improvement of the short term stability is realized by using a high $Q_0$ cavity mode, such as the $TE_{01n}$ modes. The long term stability is enhanced by minimizing changes in the resonant frequency of the cavity due to environmental changes. These changes are due to two effects: (a) thermal expansion of the cavity material, (b) changes in the dielectric
constant of the gas (air) filling the cavity as functions of temperature, water vapour pressure and atmospheric pressure. The first effect is minimized by constructing the cavity of a material that has a low coefficient of thermal expansion [40-41] whereas the latter is diminished by hermetically sealing the cavity.

Cavities constructed of brass or aluminum ($\alpha = 20$ ppm/°C), invar ($\alpha = 1$ ppm/°C) and titanium silicate ($\alpha = 0.06$ ppm/°C) were compared with and without hermetic sealing. The titanium silicate used is type 7971, made by Corning Glass Works, [42] and has an extremely low coefficient of thermal expansion. A data sheet for this material is given in the appendix. The total normalized shift in resonant frequency for the unsealed case, $\Delta f/f$, was calculated for the three cavities using (2-18) thru (2-21). The temperature was allowed to vary from $-25^\circ$C to $75^\circ$C. The range of relative humidity considered was from 0% to 100%, however, the water vapour pressure inside the cavity was assumed to never be in excess of the saturated water vapour pressure at 50°C. This is because although the air temperature outside the cavity would never exceed 50°C, that inside the cavity could be greater than 50°C if the cavity is in sunlight. Values of atmospheric pressure of 760 mmHg and 500 mmHg were considered; these correspond to sea level and an altitude of approximately 11,500 feet. The above should encompass all environmental changes likely to be encountered by a cavity stabilized oscillator. The results are plotted in figure (5-8) with the normalized frequency shift given in parametric form.
Fig. 5-8  Plots of the normalized shift in the resonant frequency of cavities constructed of (a) brass or aluminum (b) invar and (c) titanium silicate.

It would appear from figure (5-8) that there is no improvement in stability by using a titanium silicate cavity instead of an invar one. To improve the stability, the cavity must be hermetically sealed. The normalized shift in resonant frequency was then re-calculated using (2-23) - (2-24). The water vapour pressure inside the cavities is taken to correspond to 2% relative humidity at 25°C, the temperature at which the cavities are assumed to be sealed. The results are given in table (5-3) along with those for the unsealed case. It can be seen from table (5-3) that the change in resonant frequency of the invar and titanium silicate cavities is primarily due to the change in the dielectric constant of the air in the cavity. It is evident that the titanium silicate cavity must be sealed in order to realize the full potential of the material.
<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>UNSEALED CAVITY</th>
<th></th>
<th></th>
<th>SEALED CAVITY</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Δf/f DUE TO</td>
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<td>Δf/f TOTAL</td>
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<td>Δf/f TOTAL</td>
<td>Δf/f DUE TO</td>
</tr>
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<td></td>
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<td>DIELECTRIC TOTAL</td>
<td>f = 12GHz</td>
<td>THERMAL</td>
<td>DIELECTRIC TOTAL</td>
<td>f = 12GHz</td>
<td>THERMAL</td>
</tr>
<tr>
<td></td>
<td>EXPANSION</td>
<td>CONSTANT</td>
<td>kHz/GHz</td>
<td>EXPANSION</td>
<td>CONSTANT</td>
<td>kHz/GHz</td>
<td>EXPANSION</td>
</tr>
<tr>
<td></td>
<td>kHz/GHz</td>
<td>kHz/GHz</td>
<td>kHz/Hz</td>
<td>kHz/Hz</td>
<td>kHz/Hz</td>
<td>kHz/Hz</td>
<td>kHz/Hz</td>
</tr>
<tr>
<td>Brass, Aluminum</td>
<td>-1000</td>
<td>-396.4</td>
<td>-1396.4</td>
<td>-1000</td>
<td>+.332</td>
<td>-999.7</td>
<td>-11.996</td>
</tr>
<tr>
<td>α = 20ppm/°C</td>
<td>+1000</td>
<td>+104.6</td>
<td>+1104.6</td>
<td>+1000</td>
<td>-.654</td>
<td>+999.3</td>
<td>+11.992</td>
</tr>
<tr>
<td>Invar</td>
<td>-50</td>
<td>-396.4</td>
<td>-346.4</td>
<td>-50</td>
<td>+.332</td>
<td>-49.67</td>
<td>-0.596</td>
</tr>
<tr>
<td>α = 1.0ppm/°C</td>
<td>+50</td>
<td>+104.6</td>
<td>+134.6</td>
<td>+50</td>
<td>-.654</td>
<td>+49.35</td>
<td>+0.92</td>
</tr>
<tr>
<td>Titanium Silicate</td>
<td>-3</td>
<td>-396.4</td>
<td>-399.4</td>
<td>-3</td>
<td>+.332</td>
<td>-2.67</td>
<td>-0.032</td>
</tr>
<tr>
<td>α = 0.6ppm/°C</td>
<td>+3</td>
<td>+104.6</td>
<td>+107.6</td>
<td>+3</td>
<td>-.654</td>
<td>+2.35</td>
<td>+0.028</td>
</tr>
</tbody>
</table>

Unsealed Cavity: 
-25 ≤ T ≤ 75°C  
500 ≤ P_a ≤ 760 mmHg  
0 ≤ RH ≤ 100%

Sealed Cavity: 
-25 ≤ T ≤ 75°C sealed with 2% relative humidity at 25°C

TABLE 5-3 Summary of the maximum shifts in the resonant frequency of cavities constructed of (a) brass or aluminum (b) invar and (c) titanium silicate. Worst case conditions are assumed for both the unsealed and sealed cases.
In order to evaluate the performance of a titanium silicate cavity, such a cavity was machined of the material on an optical lathe to the dimensions given in section (5-2). The cavity consists of a hollow cylindrical piece 100 mils thick and a separate end piece 300 mils thick. Both pieces weigh less than 3 ounces. The inner surface of each piece was evaporated with $T_4$W metallization to a thickness of a few hundred angstroms. The thickness was then increased to several skin-depths by electroplating the pieces with silver.

The end piece was then epoxied to one end of the cylindrical piece and a substrate similar to MIO-034 was epoxied to the other end. This was done in a bell jar filled with dry nitrogen. A jig was constructed to hold the titanium silicate cavity and the microstrip launchers. Figure (5-9) is a photograph of the titanium silicate cavity along with the substrate.

Fig. 5-9 A photograph of the titanium silicate cavity parts.
From table (5-3), it is seen that the normalized shift in resonant frequency due to the cavity material and that due to the air inside the cavity are of opposite signs for the sealed case. To examine this effect further, the total frequency shift $\Delta f$ was calculated using (2-23) - (2-24) for water vapour pressures corresponding to 0%, 2% and 5% relative humidity at $25^\circ C$. To improve accuracy, the term $\partial \varepsilon / \partial T$ was evaluated at each value of $T$. The results are plotted in figure (5-10) as a function of temperature. It is seen that increasing the water vapour pressure decreases $\Delta f$, however, the dew point of the water vapour in the cavity is increased. For a water vapour pressure corresponding to 5% relative humidity, condensation inside the cavity would occur around $-15^\circ C$. Thus it is not that desirable to increase the water vapour pressure in order to minimize $\Delta f$.

![Graph showing the shift in resonant frequency for different humidity levels and temperature]

**Fig. 5-10** Plots of the shift in resonant frequency of the sealed titanium silicate cavity for three values of initial relative humidity.
The titanium silicate cavity was evaluated in an environmental chamber. For a -25°C to 75°C variation in temperature, the resonant frequency changed approximately 1.5 MHz. This value is at least ten times that predicted by figure (5-10). It was found that this discrepancy was due to flexing of the substrate caused by the pressure differential. For spacecraft application, this is of no consequence as the cavity would be vented. In other applications, some form of support would have to be used to reinforce the substrate.

The one possible disadvantage of using a titanium silicate cavity is that of the cost of machining the material. However, for spacecraft applications, this disadvantage is offset by the light weight and excellent stability of such a cavity. For ground station applications, it might be more economical to use an invar cavity. In this case, the cavity could be placed in an oven if increased stability was required.
CHAPTER 6

A 12 GHz FEEDBACK OSCILLATOR UTILIZING A TE\textsubscript{012} CAVITY AND A FET AMPLIFIER MODULE

6-1 INTRODUCTION

This chapter describes the analysis and evaluation of an experimental oscillator consisting of an amplifier and a high $Q_o$ cavity arranged in the form of a feedback loop. This approach is similar to that commonly used for phase-shift oscillators at lower frequencies. A similar configuration described in the literature was used to measure the $\varepsilon_r$ and $\mu_r$ of materials inserted into the cavity. The results presented in chapter 5 are used in the design of the transmission cavity and its microstrip coupling circuit. A FET amplifier module provides the necessary loop gain.

6-2 OSCILLATOR CONFIGURATION

The configuration of the feedback oscillator is shown in figure (6-1).

![Diagram of 12 GHz cavity feedback FET oscillator]

Fig. 6-1 Configuration of the 12 GHz cavity feedback FET oscillator.
It consists of a loop which contains a PET amplifier module, a 3 port circulator and a transmission cavity. The amplifier is inevitably operated in a non-linear mode.

The circuit functions as follows. The output power of the amplifier is directed to the input port of the cavity via the circulator. The input port of the cavity is undercritically coupled (β₁ < 1) and as a consequence, some of the amplifier's output power is reflected. This reflected power then appears at the third port of the circulator and is the useful output of the oscillator. The remainder of the amplifier's output power flows into the input port of the cavity. A fraction of this power is dissipated by the cavity losses. The rest flows out the output port of the cavity and into the input of the amplifier to sustain the loop at unity gain. The output port of the cavity is critically coupled to provide an input match to the amplifier module.

A directional coupler could have been used instead of a circulator to obtain output power from the oscillator. However, there are disadvantages when using a coupler, these being:

1. Some power is dissipated in the terminated port of the coupler.
2. It is difficult to construct a coupled line coupler in MIC form that has fairly tight coupling as the sensitivity due to the gap width becomes severe. Other coupler types (e.g. Lange) are also difficult to fabricate.
3. To optimize the oscillator for maximum output power, the coupling has to be adjustable.
For the configuration shown in figure (6-1), it will be shown in the next section that the oscillator can be adjusted for maximum output power by simply varying $\beta_1$, the input coupling coefficient to the cavity.

6-3 ANALYSIS OF THE FEEDBACK OSCILLATOR

For the purpose of analysis, the oscillator configuration of figure (6-1) is modelled by the equivalent circuit shown in figure (6-2).

![Equivalent Circuit Diagram](image)

Fig. 6-2 Equivalent circuit of the oscillator configuration shown in figure (6-1).

The equivalence between figures (6-1) and (6-2) is as given below. The source $2V_1$ in series with the impedance $Z_0$ at reference plane 1-1' along with the impedance $Z_0$ at 5-5' represents the FET amplifier module. The amplifier is assumed to be unilateral with ideal input and output match. The parameter $\phi$ represents any phase shift in the amplifier and connecting lines that is in excess of a multiple of 360°. The impedance $Y_c$, a parallel tuned circuit consisting of $L$, $G$, $C$ and the ideal transformers represent the coupled cavity about the resonant frequency of the TE012 mode. The coupled cavity is characterized by its input and output coupling coefficients $\beta_1$ and $\beta_2$, its resonant frequency $f_0$ ($\omega_0$) and unloaded $Q$, $Q_0$, as defined below:
\[ Y_c = G \left( 1 + j 2 Q_c \delta \right) \]

\[ \delta = \frac{\omega^2 \omega_o}{\omega} \]

\[ \omega_o = \frac{1}{\sqrt{L C}} \]

\[ Q_o = \frac{\omega C}{G} \]

\[ \beta_1 = \frac{Y_o}{n_1^2 G} \]

\[ \beta_2 = \frac{Y_o}{n_2^2 G} \]

Since the cavity sees an admittance \( Y_o \) at reference plane 2-2', the admittance at resonance (\( \delta = 0 \)) seen at reference plane 4-4 is:

\[ \frac{Y_4}{Y_o} = \frac{n_2^2}{n_1^2} + n_2^2 \frac{G}{Y_o} = \frac{\beta_1 + 1}{\beta_2} \]

In order to present a perfect match to the input of the amplifier, the above normalized impedance at 4-4' must be unity. This condition fixes the value of \( \beta_2 \):

\[ \beta_2 = \beta_1 + 1 \]

The normalized input admittance of the cavity at 2-2' is:
\[
\frac{V_2}{V_o} = \frac{n_1^2}{n_2^2} + \frac{n_1^2 G}{V_o} (1+jQ_o \delta) = \frac{1+\beta_2 JQ_o \delta}{\beta_1}
\]

From (6-7), the reflection coefficient at 2-2' is:

\[
\rho_2 = \frac{1 - \frac{V_2}{V_o}}{1 + \frac{V_2}{V_o}} = \frac{1 + jQ_o \delta}{1+\beta_1 + jQ_o \delta}
\]

The power reflected at 2-2', \(P_r\), is given below normalized to \(P_o\), the output power of the amplifier. The reflected power \(P_r\) is the useful output power of the oscillator.

\[
\frac{P_r}{P_o} = |\rho_2|^2 = \frac{1+Q_0^2 \delta^2}{(\beta_1+1)^2 + Q_0^2 \delta^2}
\]

The power absorbed by the cavity losses \(P_a\) normalized to \(P_o\) is:

\[
\frac{P_a}{P_o} = \left(1-|\rho_2|^2\right) \frac{n_1^2 G}{n_2^2 + n_1^2 G} \left(1-|\rho_2|^2\right) \frac{1}{\beta_1} \left(\frac{\beta_1}{1+\beta_2}\right)
\]

Using (6-7) and (6-9), the above expression can be rewritten as:

\[
\frac{P_a}{P_o} = \frac{\beta_1}{(\beta_1+1)^2 + Q_0^2 \delta^2}
\]

Since the amplifier sees a perfect output match, the voltage at reference plane 5-5' is given by:
\[ V_5 = V_1 \left( \beta_1 + 1 \right) \frac{n}{n_2} e^{-j(2\pi n + \phi)} \]  

6-13

The above expression can be rewritten as below with the aid of (6-4), (6-5), (6-7) and (6-9).

\[ V_5 = V_1 \left( \beta_1 + 1 \right) \frac{1}{\left( \left[ \left( \beta_1 + 1 \right)^2 + Q_0^2 \delta^2 \right] \beta_1 \right)^{1/2}} \exp \left\{ -j \left[ \tan^{-1} \left( \frac{Q_0 \delta}{\beta_1 + 1} \right) + 2\pi n + \phi \right] \right\} \]  

6-14

For oscillation to occur there must be unity gain and zero phase shift around the loop. This implies that \( \frac{V_5}{V_1} = 1 \). If the electrical length \( \phi \) of the amplifier and connecting lines is not exactly a multiple of 360° at \( \omega_0 \), the frequency of oscillation will differ from \( \omega_0 \) slightly and is obtained using (6-1b) and (6-14).

\[ \omega_{osc} = \frac{\omega_0}{1 + \frac{\beta_1 + 1}{Q_0} \tan \left( 2\pi n + \phi \right)} \]  

6-15

normalized to

Finally, the power \( P \) transmitted through the cavity \( \text{tr} \) \( P \) can be obtained from (6-14)

\[ \frac{P_{t}}{P_0} = \left| \frac{V_5}{V_1} \right|^2 = \frac{\beta_1 (\beta_1 + 1)}{(\beta_1 + 1)^2 + Q_0^2 \delta^2} \]  

6-16

Note that \( \frac{P}{P_0} + \frac{P_{t}}{P_0} + \frac{P_{i}}{P_0} = 1 \), as is required for conservation of power.

In order to calculate the oscillator output power, an empirical expression that accounts for gain compression is required for the transfer function of the amplifier. The following expression was chosen for the reasons given in section (6-4).
\[ P_o = gP_1 \left(1 - \exp \left(-\gamma/P_1 \right) \right) \]  

In the above, \( P_o \) is the amplifier output power, \( P_1 \) the input power, \( g \) is the small signal power gain and \( \gamma \) is a factor that is a function of the compression point of the amplifier.

For unity loop gain, the amplifier gain must be equal to the loop transmission factor. Using (6-16) and (6-17), this is expressed as:

\[ \frac{(\beta_1 + 1)^2 + Q_o^2\delta^2}{\beta_1 (\beta_1 + 1)} = g \left[ 1 - \exp \left(-\gamma \frac{(\beta_1 + 1)^2 + Q_o^2\delta^2}{P_o \beta_1 (\beta_1 + 1)} \right) \right] \]  

Solving for \( P_o \) in (6-18) gives the following:

\[ P_o = \frac{-\gamma \left[ (\beta_1 + 1)^2 + Q_o^2\delta^2 \right]}{\beta_1 (\beta_1 + 1) \ln \left[ 1 - \frac{(\beta_1 + 1)^2 + Q_o^2\delta^2}{g\beta_1 (\beta_1 + 1)} \right]} \]  

The oscillator output power, \( P_{osc} \), is equal to \( P_o \) and is obtained using (6-10), (6-14) and (6-19).

\[ P_{osc} = \frac{-\gamma \left[ 1 + (\beta_1 + 1)^2 \sec^2 (2\pi + \phi) \right]}{\beta_1 (\beta_1 + 1) \ln \left[ 1 - \frac{(\beta_1 + 1) \sec^2 (2\pi + \phi)}{g\beta_1} \right]} \]  

For any value of \( \beta_1 \), \( g \) and \( \gamma \), the oscillator power is a relative maximum when \( \phi = 0 \). Under these conditions, the output power is as given below, normalized to \( \gamma \).
\[ P_{\text{osc}} = \frac{-1}{\gamma \beta_1(\beta_1+1) \ln \left(1 - \frac{\beta_1 + 1}{g \beta_1^2}\right)} \]

Once the value of \( g \) for the amplifier has been measured, it can be seen by plotting (6-21) as a function of \( \beta_1 \) that there is a certain value of \( \beta_1 \) for which the oscillator power is maximum. There is also a minimum value of \( \beta_1 \) below which there is no oscillation; this occurs when the product of the loop transmission factor \( P_T/P_0 \) and the small signal power gain \( g \) is equal to unity. An expression for \( \beta_{1\min} \) is obtained by noting that the above condition occurs when the argument of the logarithmic term in (6-21) is zero.

\[ \beta_{1\min} = \frac{1}{g - 1} \]

Curves of \( P_{\text{osc}}/\gamma \) are plotted in figure (6-3) as a function of \( \beta_1 \) for different values of \( g \); \( \phi \) is taken as equal to zero. The dashed curve drawn through the maximum of each curve gives the optimum \( \beta_1 \) for maximum output power as a function of the parameter \( g \).

Although no detailed analysis of oscillator AM and FM noise has been done at present, it can be seen from (6-15) that FM noise is only a function of \( \phi \) while from (6-20) it is seen that AM noise is a function of both \( g \) and \( \phi \).

Finally the DC to RF efficiency of the oscillator is obtained from (6-10),

\[ \eta_{\text{osc}} = \frac{P_0}{P_{\text{DC}}} \frac{1}{(\beta_1 + 1)^2} \]

where \( P_{\text{DC}} \) is the DC input power to the amplifier.
Fig. 6-3 Normalized oscillator output power $P_{\text{osc}}/\gamma$ as a function of the coupling coefficient $\beta_1$ with amplifier gain $g$ as a parameter (solid curves). The dashed curve indicates values of $\beta_1$ that yield maximum output power as a function of $g$. 

The oscillator configuration analyzed in this section was assembled and evaluated. The results are given in section (6-5). Section (6-4) concerns the characterization of the FET amplifier module used in the oscillator.

6-4 CHARACTERIZATION OF THE FET AMPLIFIER MODULE

The FET amplifier module used is a 3 stage, common source amplifier using NEC-V244 GaAs FET's. The design and construction of the amplifier has been described in detail elsewhere [ ]. A photograph of the amplifier is shown in figure (6-4).

![Photograph of the 3-stage FET amplifier module showing the bias networks and coupling capacitors. The device packages are inserted into the 3 holes drilled through the substrate. [46-47]](image)

The input and output match, and small signal power gain of the amplifier were measured and are presented in figures (6-5) and (6-6). Bias has been adjusted to maximize $g$ around 12 GHz.
Fig. 6-5 Input and output match of the FET amplifier shown in figure (6-4).
Fig. 6-6 Small signal power gain $g$ of the FET amplifier shown in figure (6-4).
The transfer characteristic of the amplifier was then measured by plotting the output power $P_0$ as a function of the input power $P_1$ as shown in figure (6-7). As was mentioned in section (6-3), an empirical expression is required for $P_0$ as a function of $P_1$. Polynomials up to the fourth order were first tried, but proved to give a poor fit. An exponential expression as given by (6-17) gives a much better fit. The terms $g$ and $\gamma$ obtained using the data presented in figures (6-6) and (6-7). The term $\gamma$ can be calculated using either the 1 dB or 3 dB compression point.

$$\gamma_{1\text{dB}} = \frac{1.99\ P_{01}}{8} \quad 6-24a$$

$$\gamma_{3\text{dB}} = \frac{1.39\ P_{03}}{8} \quad 6-24b$$

In the above, $P_{01}$ is the output power at the 1 dB compression point and $P_{03}$ that at the 3 dB compression point. Values of $P_0$ as given by (6-17) are plotted in figure (6-7) for $\gamma$ as given by (6-24a) and (6-24b). The values of $P_0$ calculated using (6-17) are in good agreement with measured values up to the 3 dB compression point. It should be noted that the exponential expression used (6-17) is not normally found in the literature. Most other authors \[ \text{[43,45]} \] seem to apply Van der Pol's expression to the voltage gain of an amplifier when accounting for non-linearity. As stated earlier, applying a polynomial to the power gain of the FET amplifier gave a poor fit; the exponential expression used proved adequate for the range of gain compression obtained from FET amplifiers ($P_1 < 4$ dBm).
Fig. 6-7 Transfer characteristic of the FET amplifier shown in figure (6-4) showing the 1 dB and 3 dB compression points. Values of $\gamma_{1\text{dB}}$ and $\gamma_{3\text{dB}}$ are calculated using (6-24).

BIAS:

$V_{ds} = -0.25\text{V}$, $V_{gs} = +5.3\text{V}$
$V_{gs} = -0.57\text{V}$, $I_0 = 48\text{mA}$
$V_{os} = -1.27\text{V}$

- MEASURED
- CALCULATED USING $\gamma_{1\text{dB}} = 0.40\text{mw}$
- CALCULATED USING $\gamma_{3\text{dB}} = 0.42\text{mw}$

$Q = 14\text{dB}$
$f = 12.0\text{GHz}$
6.5 **Experimental Evaluation of a 12 GHz Cavity Feedback Oscillator**

The performance of the feedback oscillator described in sections (6-2) and (6-3) was evaluated experimentally. The details of the test configuration are shown in figure (6-8).

![Diagram of a 12 GHz Cavity Feedback Oscillator](image)

**Fig. 6-8** Layout used for the evaluation of the 12 GHz feedback oscillator.

A phase shifter was included to study the effects of any additional phase shift in the loop. The loop losses in the setup were measured and found to be approximately 2 dB. For the purpose of obtaining $\beta_1$, the losses are subtracted from the small signal power gain of the amplifier (14 dB) giving values of $g = 15$ (11.8 dB) and $\gamma = .4$ mW. From figure (6-3) and (6-7), $\beta_1 = .18$ and $\beta_2 = 1.18$ are obtained. Circuit MIC-035 was designed to give graphs these values (see chapter 5) and was used for the measurements. Photos of the oscillator setup are shown in figure (6-9).
Fig. 6-9 Photographs of the layout used for the evaluation of the oscillator. The cavity can be seen in (a) to the left of the waveguide phase shifter; the FET amplifier is directly behind the phase shifter. A dolly in the upper right hand corner of (b) contains the equipment used for the FM noise measurement. The metal box with the panel meters houses the power supply for the FET amplifier.
Measurements of oscillator power and frequency were made as a function of phase shift $\phi$ and are given in figure (6-10) along with calculated values as given by (6-15) and (6-20). Bias was adjusted to the values indicated on the figures of section (6-4).

![Graph showing measured and calculated values of oscillator output power, $P_{osc}$, and shift in oscillator frequency, $\Delta f$, as a function of $\phi$.]

**Fig. 6-10** Measured and calculated values of oscillator output power, $P_{osc}$, and shift in oscillator frequency, $\Delta f$, as a function of $\phi$. 

- $f_0 = 11.9496$ GHz
- $f_{osc} = f_0 + \Delta f$
- $f_0 = 29,000$
- $\beta_1 = 0.18$
- $\beta_2 = 1.18$
- $g = 15$
- $\gamma = 0.4$ mw
The calculated and measured values of $\Delta f$ and $P_{osc}$ given in figure (6-10) are in good agreement. It is seen that the value of $P_{osc}$ is quite insensitive to $\phi$ over a broad range; a change of $60^\circ$ in $\phi$ causes $P_{osc}$ to change by only 400 kHz. Both of these facts indicate that any random change in $\phi$ due to the amplifier will have a minimal effect on the oscillator AM noise, while FM noise should be quite low.

The power at the output of the amplifier, $P_o$, was calculated to be 6.8 dBm using (6-10). From figure (6-7) it is seen that this value of $P_o$ is in the region where (6-17) is in good agreement with the measured transfer characteristics of the amplifier.

Photographs of the oscillator's output spectrum are shown in figure (6-11).
Fig. 6-11 Photographs of the output spectrum of the feedback oscillator showing the absence of spurious signals.

\[ P_{osc} = 5.4 \text{ dBm in both (a) and (b)} \]

The FM noise of the oscillator was measured using a Microwave Systems frequency stability analyzer and a Hewlett Packard model 3390A wave analyzer. For comparison, the FM noise of an oscillator multiplier unit was also measured; this particular unit comprised of a 108 MHz crystal oscillator followed by a \( \times 10^8 \) multiplier. The output power from the multiplier was approximately 10 mW. The results are given in figure (6-12). It is seen that the FM noise of the feedback oscillator is comparable to that of the oscillator-multiplier unit.
Fig. 6-12 Comparison of the FM noise of the feedback oscillator to that of the oscillator-multiplier source described in the text.
A feedback oscillator has been demonstrated having short term stability (FM noise) comparable to that of an oscillator-multiplier unit. At present output power is low simply due to the limitations of the FET amplifier. Use of a Gunn or IMPATT amplifier would overcome this problem. The long term stability (frequency drift) can be kept at a minimum by using a titanium silicate or inves cavity. (see chapter 5).

The effect of temperature on the small signal gain of the amplifier should be kept in mind as it will effect $P_{osc}$ if the oscillator is operated over a broad temperature range. This problem could be remedied by using a heater or adjusting the amplifier gate bias through a temperature compensation circuit. It is planned to integrate the oscillator unto one substrate and perform detailed temperature tests; temperature testing the oscillator in its present form gave rise to erroneous results due to the many connectors involved.
CHAPTER 7

A 12 GHz GaAs FET Oscillator in MIC Form

7-1 Introduction: This chapter describes the design and evaluation of an X-band GaAs FET oscillator in MIC form. It is intended to find applications as an L.O. in MIC communication subsystems. The basic oscillator was stabilized using a TE_012 transmission cavity. The performance of the oscillator was evaluated with and without the stabilizing cavity. Although several authors have recently described similar GaAs FET oscillators [48-51], this is apparently the first time that a cavity stabilized version is reported [52-53].

7-2 Characterization of the GaAs FETs: The devices used were the Nippon Electric (NEC) V-244 and to a lesser extent, the Plessey GAT-3. Data sheets for these devices are found in the appendix. The V-244 have values of I_DSS that are typically in the range 40-110 ma; for the GAT-3, 20-60 ma is typical. Both devices were procured in chip form and bonded into a special package developed at CRC [46-47]. The devices are mounted in common source configuration; single gate and drain bonds are made to the tabs on each side of the package, with double bonds for the source-to-pedestal connection. A drawing of the package is shown in Fig. (7-1).
Fig. 7-1. The package developed at CRC for use with GaAs FET chips. [46-47]

A jig was constructed to measure the S-parameters of the devices. It consists of a 50Ω section of microstrip mounted in a suitable carrier with OSM connectors at each end. A 0.085" hole is drilled through the substrate to accept the FET package. Rexolite fingers extending from the top plate of the jig keep the package tabs in contact with the microstrip. A photograph of a jig similar to that used is shown in Fig. (7-2).*

Fig. 7-2. A jig similar to the one used for 50Ω S-parameter measurements.

* The circuit of Fig. 2 has a matching transformer network on the drain side, not used for 50Ω S-parameters.
The packaged devices were inserted into the jig and S-parameter measurements were made using a Hewlett Packard network analyzer. The results obtained from one of the V-244 devices are plotted on the polar chart of Fig. (7-3) for 11 GHz < f < 13 GHz; bias used is indicated on the figure. These results are typical of all the devices measured.

**Fig. 7-3.** A polar plot of the S-parameters as obtained from one of the V-244 devices in a 500 jig. The frequency range covered is 11-13 GHz.

The gain compression points of the devices were obtained by measuring output power as a function of input power. Typical results as obtained from one of the devices are presented in Fig. (7-4) for the values of bias indicated.
Fig. 7-4, Transfer characteristics of one of the V-244 devices in a 500 jig for two different values of gate bias. The frequency is 12 GHz.

7-3 Analysis of the Oscillator Circuit: In order to achieve oscillation, the admittance looking into one port of the device must have a negative real part when the other port is terminated by a certain admittance. This is satisfied when the stability factor k is less than unity. The k factor is given below in terms of the device S-parameter [54-55].

\[
k = \frac{1 + |S_{11}S_{22} - S_{12}S_{21}|^2 - |S_{11}|^2 - |S_{22}|^2}{2|S_{12}||S_{21}|}
\]

7-1

For the S-parameter values given in Fig. (7-3), the k factor is calculated to be greater than unity about 12 GHz and external feedback must be applied to the device. Feedback can be applied in two ways.
(1) An impedance can be inserted into the source. This approach has been described in the literature [48-51]. For the V-244 device, a value of $X_L = 7.9 \, \Omega$ is required at 12 GHz.

(2) An impedance can be inserted between the gate and drain. Since packaged devices were used, the second approach was chosen as the most convenient. A transmission line connected between the gate and drain of the device as shown in Fig. (7-5a) provides the necessary feedback.

Fig. 7-5. (a) A schematic of the oscillator circuit. The gate and drain microstrip lines have $Z_0 = 50$. (b) Equivalent circuit using Z-parameters for (a) between reference planes 3-3' and 4-4'.

The lengths $L_1$ and $L_2$ must be adjusted so that $\text{Re}[Y_{osc}] < 0$. The S-parameters of the device at reference planes 3-3', 4-4' are given below in terms of those measured at 1-1', 2-2'.

---
\[ S_{3-4} = \Phi \cdot S_{1-2} \cdot \Phi \]

where \( \Phi = \begin{bmatrix} e^{-j\phi_1} & 0 \\ 0 & e^{-j\phi_2} \end{bmatrix} \)

Performing the above multiplication gives the following form for \( S_{3-4} \):

\[
S_{3-4} = \begin{bmatrix} S_{11} e^{-j2\phi_1} & S_{12} e^{-j(\phi_1+\phi_2)} \\ S_{21} e^{-j(\phi_1+\phi_2)} & S_{22} e^{-j2\phi_2} \end{bmatrix}
\]

(7-3)

The circuit between 3-3' and 4-4' can be described in terms of \( Z \) parameters as shown in Fig. (7-5b). Furthermore, the FET is assumed to be unilaterial; it can be seen from Fig. (7-3) that this is a valid approximation. That is \( S_{12} = Z_{12} = 0 \); the remaining \( Z \) parameters are given as follows.

\[
Z_{11} = Z_0 \frac{1 + |S_{11}| e^{j(\psi_{11}-2\phi_1)}}{1 - |S_{11}| e^{j(\psi_{11}-2\phi_1)}}
\]

(7-4a)

\[
Z_{22} = Z_0 \frac{1 + |S_{22}| e^{j(\psi_{22}-2\phi_2)}}{1 - |S_{22}| e^{j(\psi_{22}-2\phi_2)}}
\]

(7-4b)

\[
Z_{21} = \frac{2|S_{21}| e^{j(\psi_{21}-\phi_1-\phi_2)}}{(1-|S_{22}| e^{j(\psi_{22}-2\phi_2)}) (1-|S_{11}| e^{j(\psi_{11}-2\phi_1)})}
\]

(7-4c)

The admittance \( Y_{osc} \) seen at the output port of the oscillator is calculated by referring to Fig. (7-5).

\[
Y_{osc} = \frac{1}{Z_{11}} + \frac{1(\cdot 21)}{Z_{22}}
\]

(7-5)

Using (7-4), the above expression for \( Y_{osc} \) can be rewritten in terms of \( \phi_1, \phi_2 \) and the measured device \( S \)-parameters.

\[
\begin{align*}
\frac{(1-|S_{11}| e^{j(\psi_{11}-2\phi_1)})}{(1+|S_{11}| e^{j(\psi_{11}-2\phi_1)})} + \frac{(1-|S_{22}| e^{j(\psi_{22}-2\phi_2)})}{(1+|S_{22}| e^{j(\psi_{22}-2\phi_2))}} & = \frac{2|S_{21}| e^{j(\psi_{21}-\phi_1-\phi_2)}}{(1-|S_{11}| e^{j(\psi_{11}-2\phi_1)}) (1-|S_{22}| e^{j(\psi_{22}-2\phi_2)})}
\end{align*}
\]

(7-6)
In order to design the oscillator, the following conditions must be satisfied.

\[
\text{Re} \left[ Y_{\text{osc}} \left( f_{\text{osc}}, V_{\text{osc}} \right) \right] + \text{Re} \left[ Y_L \left( f_{\text{osc}} \right) \right] = 0 \quad 7-7a
\]

\[
\text{Im} \left[ Y_{\text{osc}} \left( f_{\text{osc}}, V_{\text{osc}} \right) \right] + \text{Im} \left[ Y_L \left( f_{\text{osc}} \right) \right] = 0 \quad 7-7b
\]

In the above, \( f_{\text{osc}} \) is the frequency of oscillation and \( V_{\text{osc}} \) is the corresponding RF voltage. Under large signal conditions, \( \text{Re}[Y_{\text{osc}}] \) is assumed to be a monotonically decreasing function of the RF voltage; as \( V \) increases, \( |\text{Re}[Y_{\text{osc}}]| \) decreases until it is equal to \( \text{Re}[Y_L] \). Since the output power is given as \( V^2_{\text{osc}} \text{Re}[Y_L] \), there is some value of \( \text{Re}[Y_L] \) for which the oscillator power \( P_{\text{osc}} \) will be a maximum. This value of \( \text{Re}[Y_L] \) is less than the small signal value of \( |\text{Re}[Y_{\text{osc}}]| \).

In order to obtain the value of \( \text{Re}[Y_L] \) that gives the optimum \( P_{\text{osc}} \), an expression for \( Y_{\text{osc}} \) is needed that accounts for large signal effects. To a first approximation, it might be assumed that only the magnitude of \( S_{21} \) decreases as the device is operated under large signal conditions. However, because of the form of (7-6), a change in \( |S_{21}| \) can actually change both the real and imaginary parts of \( Y_{\text{osc}} \). Because of this, it is difficult to attempt an exact design.

An approximate design is obtained by using the small signal S-parameter data of Fig. (7-3) and finding values of \( \lambda_1 \) and \( \lambda_2 \) that yield a maximum negative value of \( \text{Re}[Y_{\text{osc}}] \) in the vicinity of \( f_{\text{osc}} \). The values of \( f_{\text{osc}} \) and \( P_{\text{osc}} \) are then adjusted by varying the bias slightly and placing small dielectric chips on the microstrip lines to vary \( \phi_1 \) and \( \phi_2 \) slightly. Maximizing \( \text{Re}[Y_{\text{osc}}] \) enables the oscillator to deliver the optimum power to the load. Due to the complexity of (7-6), the design was done using a computer program. Values of \( \lambda_1 = 398 \) mils and \( \lambda_2 = 53 \) mils were obtained assuming an alumina \( (\epsilon_r = 10) \) substrate. The resulting values of \( \text{Re}[Y_{\text{osc}}] \) were negative over an interval greater than 11 GHz–13 GHz; the values of \( Y_{\text{osc}} \) are plotted in Fig. (7-10). It is
seen that \( \text{Re}[Y_{\text{osc}}] \) has a maximum negative value at practically the same frequency where \( Y_{\text{osc}} \) = 0; both are satisfied at \( f_{\text{osc}} = 11.3 \text{ GHz} \).

Because of this the load admittance need only be real.

As stated earlier, it is difficult to calculate the value of \( \text{Re}[Y_L] \) that will give the optimum \( f_{\text{osc}} \). It was decided to use a value of 20 m\( \mu \)ho. This means that the oscillator sees a 50\( \Omega \) load and no matching circuit is needed on the output port. From Fig. (7-10), the ratio of \( \text{Re}[Y_L] \) to the small signal value of \( |\text{Re}[Y_{\text{osc}}]| \) is calculated to be .714; a ratio of .3 was used in [49].

Several oscillator circuits were built, each one having slightly different values of \( l_1 \) and \( l_2 \). The topology of one of the circuits (FETAD62-B) is shown in Fig. (7-6).

![Diagram of FETAD62-B oscillator circuit]

**Fig. 7-6. Topology of one of the oscillator circuits (FETAD62-B).**
The two pads labelled "D" and "G" are for drain and gate bias.

The corresponding values of \( l_1 \) and \( l_2 \) are given in Table (7-1) along with those of \( l_1 \) and \( l_2 \). It is seen that the values are centered about \( l_1 = 398 \text{ mils} \) and \( l_2 = 53 \text{ mils} \), the values for which \( Y_{\text{osc}} \) is plotted in Fig. (7-10).
<table>
<thead>
<tr>
<th>Circuit</th>
<th>$l_a$-mils</th>
<th>$l_b$-mils</th>
<th>$l_1$-mils</th>
<th>$l_2$-mils</th>
</tr>
</thead>
<tbody>
<tr>
<td>FETA D 62-A</td>
<td>167</td>
<td>54</td>
<td>400</td>
<td>53</td>
</tr>
<tr>
<td>FETA D 62-B</td>
<td>167</td>
<td>50</td>
<td>392</td>
<td>53</td>
</tr>
<tr>
<td>FETA D 63-A</td>
<td>167</td>
<td>58</td>
<td>408</td>
<td>53</td>
</tr>
<tr>
<td>FETA D 63-B</td>
<td>167</td>
<td>62</td>
<td>416</td>
<td>53</td>
</tr>
</tbody>
</table>

Table 7-1. Values of $l_a, l_b, l_1, l_2$ used for the oscillator circuits.

The gate and drain bias circuits consist of $\lambda/4$ sections of high impedance line each terminated by a section of radial line. The latter provides a broad-band short circuit [46-47]. A 22 mil gap in $l_1$ is for a D.C. blocking capacitor to isolate gate and drain bias. The phase shift introduced by the capacitor is approximately equal to the electrical length of the line it replaces around 12 GHz. Mitered are used on the bends to reduce the VSWR, as described in [56]. To account for the tee junction, the lengths $l_1$ and $l_2$ are measured from the mid point of the junction; no compensation was made for any junction discontinuity. The cross and the two semi-circular lines shown in Fig. (7-6) serve as alignment marks for drilling the hole that accommodates the device package. The performance of the circuits is described in section (7-5); section (7-4) discusses the use of a high $Q_o$ cavity to stabilize the oscillator.

7-4 Cavity Stabilization of the FET Oscillator: The oscillator described in the preceding section was stabilized using a high $Q_o$ cavity. The cavity can be connected in either the transmission [57], reaction [58] or reflection [59] configurations. For this particular application, the transmission configuration shown in Fig. (7-7) was used. The transmission line is $\lambda/2$ long at $f_1$. 
Fig. 7-7. (a) Transmission cavity configuration used to stabilize the FET oscillator circuits. (b) Equivalent circuit for (a), used for analysis.

The values of $Y_0$, $\delta$, $\omega_0$, $Q_0$, $\beta_1$ and $\beta_2$ are as given by (6-1) - (6-5). The normalized admittance seen at the input port of the cavity (oscillator port) is:

$$
Y_1 = \frac{1}{Y_0} \left( \frac{\beta_2}{\beta_1} - \frac{20 \delta}{\omega_0 \beta_1} \right)
$$

(7-8)

In order to achieve the same loading as used in the unstabilized case, the oscillator must see a value of $Y_L = 20$ mho at resonance. Also, Mullen [60] showed that the loading of an oscillator that optimizes output power also gives the optimum AM noise performance. This implies that $Y_1/Y_0 = 1$; for this to hold, the value of $\beta_2$ (load port) must satisfy the following:

$$
\beta_2 = \beta_1 - 1
$$

(7-9)

A measure of the stabilization introduced by the cavity is given by the stabilization factor $S$ as defined below [57, 58].
\[
\frac{\text{d}w}{\text{d}B} \bigg|_{\text{w/o cavity}} = \frac{\omega_o}{2Q_{\text{ext}}} \quad \text{(7-10)}
\]

\[
\frac{\text{d}w}{\text{d}B} \bigg|_{\text{w/cavity}} = \frac{\omega_o}{2\left[\frac{\omega_o}{\beta_1} + Q_{\text{ext}}\right]} \quad \text{(7-10)}
\]

In the above, the terms \(\text{d}w/\text{d}B\) are evaluated at reference plane 3-3' with \(w = \omega_o\). It can be seen from the definition of \(S\) that adding the stabilization cavity reduces the FM noise by \(1/S\). The external Q of the unstabilized oscillator, \(Q_{\text{ext}}\), is obtained from the pulling figure \(L\) [58] which is a more commonly used parameter.

\[
L = 0.417 \frac{f_o}{Q_{\text{ext}}} \quad \text{(7-11)}
\]

The pulling figure is defined as the total shift in the frequency of the unstabilized oscillator as a 1.5 VSWR load is phased through more than 180°. Using (7-11), the expression for \(S\) can be rewritten as below.

\[
S = 1 + \frac{2.4 \cdot Q_o \cdot L \cdot f_o}{\beta_1}
\]

Referring to figure (7-7) and (7-9), the following expression is obtained for the insertion loss introduced by the cavity. \(P_L\) is the power delivered to the load at 2-2' in Fig. (7-7).

\[
\frac{P_L}{P_{\text{osc}}} = \frac{\beta_1 - 1}{\beta_1} \quad \text{(7-13)}
\]

In a practical application, the values of \(Q_o\), \(f_o\), and \(L\) are known. The choice of \(\beta_1\) involves a trade-off between stability and insertion loss. If the required value of \(S\) occurs for a value of \(\beta_1\) that gives an insertion loss that is too great, a larger value of \(Q_o\) must be used. Values of \(S\) and IL as given by (7-12) and (7-13) are plotted in Fig. (7-8) as functions of \(\beta_1\) with \(Q_o = 29,000\) and \(f_o = 11.950\) GHz. The value of \(L\) (21 MHz) is obtained from section (7-5).
In order to see how the conditions given by (7-7) are satisfied; the locus of $Y_L$ was calculated and plotted as a function of frequency. The expression for $Y_L$ is obtained referring to Fig. (7-7) and is given below. The transmission line is assumed to be $\lambda_g/2$ long at $f = f_1$.

$$Y_L = G_L + jB_L = Y_0 \left[ \frac{y_1 + j\tan(\pi f/f_1)}{1 + y_1 \tan(\pi f/f_1)} \right]$$

7-14

The value of $y_1$ is obtained using (6-1) - (6-5).

$$y_1 = \frac{\beta_2 + 1}{\beta_1} + \frac{2Q_0}{\beta_1 (1-f_0/E)}$$

7-15

Using (7-14) and (7-15) along with the value of $\beta_2$ as given by (7-9) gives the following expressions for $G_L$ and $B_L$. 

$Q_0 = 29,000$

$f_0 = 11.950$ GHz

$L = 21$ MHz
\[ G_L = Y_0 \frac{1 - \tan^2 (\pi f / f_1)}{1 - \frac{2Q_0 (1-f_o/f_1) \tan (\pi f / f_1)}{\beta_1}} \]

\[ B_L = Y_0 \frac{\frac{2Q_0 (1-f_o/f_1)}{\beta_1} \left[ 1 - \frac{2Q_0 (1-f_o/f_1) \tan (\pi f / f_1)}{\beta_1} \right] - \tan^2 (\pi f / f_1)}{1 - \frac{2Q_0 (1-f_o/f_1) \tan (\pi f / f_1)}{\beta_1}} \]

The values of \( G_L \) and \( B_L \) are plotted on the Smith chart of Fig. (7-9) for the case where \( \beta_1 = 1.18 \), \( Q_0 = 29,000 \) and \( f_o = f_1 = 11.950 \) GHz.

Fig. 7-9. A plot of \( Y_L(f) \) as seen at reference plane 3-3' of figure (7-7). The portion of the locust enclosed by the dashed line is re-plotted in figure (7-10).
These values are obtained using the TE$_{012}$ cavity and circuit MIC-035 described in section (5-2). The values of $G_L$ and $B_L$ enclosed by the dashed rectangle are re-plotted in Fig. (7-10) along with values of the negative of the oscillator admittance, $-Y_{osc}$.

**Fig. 7-19.** Plots of $-Y_{osc}$ as given by (7-6) and the portion of $Y_L$ enclosed by the dashed line in figure (7-9).
The locus $-Y_{osc}(V_1)$ represents the small signal value of $Y_{osc}$ as calculated from (7-6) and the S-parameter data of Fig. (7-3). The estimated locus $-Y_{osc}(V_2)$ where $V_2 < V_1$ satisfies the conditions required for oscillation as given by (7-7); this occurs at point (A) in Fig. (7-10). Values of $f_{osc} = 11.9497$ GHz and $V_{osc} = V_2$ are obtained from Fig. (7-10).

It is noted that the value of $f_{osc} = 11.9497$ GHz for the stabilized oscillator is different from that of $f_{osc} = 11.3$ GHz for the unstabilized oscillator; (viz.) $f_{osc}$ (unstabilized) = 11.3 GHz whereas $f_o = f_1 = 11.950$ GHz.

In a practical situation, one would wish to vary the frequency of the stabilized oscillator by tuning the cavity. It is quite difficult to predict the frequency of the stabilized oscillator and the corresponding output power under these conditions as the values of $f_{osc}$ (unstabilized), $f_1$ and $f_o$ are likely to be all slightly different. The simplest case occurs when $f_{osc}$ (unstabilized) = $f_o = f_1$ as the frequency of the stabilized oscillator is then the same as that for the unstabilized case. It should be mentioned that if the circuit is not properly adjusted, the stabilized oscillator can operate at more than one frequency. For example, if the value of $V = V_3$ was greater than $V_2$, the locus of $Y_{osc}$ ($V_3$) would be to the left of $Y_{osc}$ ($V_2$). It can be seen from Fig. (7-10) that (7-7) could then also be satisfied at points (B) or (C) depending on the value of $V_3$. However, since the value of $C_L$ is of the order $1 \times 10^{-6}$ mmho at these points, the corresponding value of $P_{osc}$ is much smaller than that at point (A).

This problem could be avoided by using a tightly coupled $\lambda/2$ resonator in the feedback path between the drain and gate of the device, as shown in Fig. (7-11). The effect of this would be to decrease the range over which Re$[Y_{osc}] < 0$. 
Fig. 7-11. An alternate topology incorporating a $\lambda/2$ resonator to reduce the frequency range over which $\text{Re}\{Y_{osc}\} < 0$.

For example, if the resonator is 164 mils long and a coupling gap of 1 mil is used, the coupled resonator has an insertion loss of 8 dB at 11.95 GHz. The insertion loss at 14 GHz is greater than 3 dB; at 8 GHz, it is greater than 11 dB. The range over which $\text{Re}\{Y_{osc}\} < 0$ using the above mentioned resonator is now no greater than approximately 1 GHz.

7-5 Experimental Evaluation of the FET Oscillator Circuits: The circuits described in section (7-3) were fabricated on 1"x1"x.025" alumina substrates ($\varepsilon_r = 10$). Each substrate contained two such circuits along with a section of line. A photograph of one of the substrates is shown in Fig. (7-12) mounted on top of a carrier plate. A device is located in the upper circuit under the bar that supports the two rexolite fingers. The blocking capacitor used to isolate gate and drain bias can be seen in the lower circuit along with the bias circuits and the hole drilled through the carrier plate for the device package.
Fig. 7-12. An alumina substrate containing two oscillator circuits mounted on a jig. The bar over the upper circuit supports the two rexolite fingers.

The performance of the four circuits was then evaluated using several devices in each circuit. It was found that a few of the devices with low values of $I_{DSS}$ didn't oscillate when inserted into the circuits. These devices could be made to oscillate by placing a small dielectric chip on the circuit and moving it about. For those devices having high $I_{DSS}$ values, the frequency of oscillation could be varied over several hundred MHz by placing a small dielectric chip on the feedback portion of the circuit. It was also observed that the devices didn't yield maximum RF power all in the same circuit. Initially, a modulated spectrum was obtained with some of the devices. This was found to be due to a low frequency (~10 MHz) bias circuit oscillation modulating the oscillator output. This oscillation was somewhat a function of gate bias and disappeared for certain values of $V_{GS}$. It was totally eliminated by inserting a 3.3 kΩ resistor in series with the gate bias port, physically close to the bias circuit.
The oscillator output power $P_{osc}$ and efficiency $\eta$ obtained from one of the circuits are plotted in Fig. (7-13) both as a function of gate bias $V_{GS}$ with constant drain bias and drain bias $V_{DS}$ with constant gate bias. The device used had values of $I_{DSS} = 64$ mA and $V_P = -3.56$V. The values of $P_{osc}$ and $\eta$ plotted in Fig. (7-13) can be considered as median values when compared to those obtained from different devices in different circuits.

![Graph showing $P_{osc}$ vs $V_{GS}$ and $P_{osc}$ vs $V_{DS}$](image-url)

*Fig. 7-13. Plots of oscillator output power and efficiency as functions of drain and gate bias.*
It was observed that the maximum value of $P_{osc}$ attainable from a device is proportional to $I_{DSS}$. The maximum value of $P_{osc}$ measured was 27 mW at an efficiency of 11% for a device having $I_{DSS} = 98\text{mA}$ and $V_P = 4.45\text{V}$. Bias used was $V_{DS} = 5.0\text{V}$ and $V_{GS} = 2.0\text{V}$. The results given in Fig. (7-13) are in good agreement with those reported for the FET oscillator described in [5] for comparable values of $V_{DS}$ and $V_{GS}$.

However, the authors of [48-49] continued to increase $V_{DS}$ to 9V for which they obtained values of $P_{osc} = 40\text{mW}$ and $\eta = 12\%$ from a device having $I_{DSS} = 76\\text{mA}$ and $V_P = 4.3\text{V}$. The values of $P_{osc}$ and $\eta$ shown in Fig. (7-13) weren't measured for values of $V_{DS}$ greater than 5.0V as this is the rated maximum value of $V_{DS}$ for the NEC-V-244 devices.

The same oscillator circuit (with a different device; $I_{DSS} = 60\text{mA}$ and $V_P = 3.33\text{V}$) was injection-locked to a stable source. The lock-in bandwidth $B_L$ was then measured as a function of the locking gain $P_{osc}/P_i$ where $P_i$ is the injected power. The results are given in Fig. (7-14). Bias was adjusted to give the values of $P_{osc}$ and $f_{osc}$ indicated on the figure.

The external $Q$ of the oscillator, $Q_{ext}$ was calculated using the data of Fig. (7-14) and Adler's equation [5, 6], given below.

$$B_L = \frac{2 f_{osc}}{Q_{ext}} \sqrt{\frac{P_i}{P_{osc}}}$$

Taking $B_L = 10\text{MHz}$, $f_{osc} = 11.5\text{GHz}$, $P_{osc} = 2\text{mW}$ and $P_i = 0.02\text{mW}$, a value of $Q_{ext} = 230$ is obtained. The slope of the curve of Fig. (7-14) is constant at 20 dB/decade as predicted by (7-17). The pulling figure of the oscillator was calculated using (7-11). A value of $L = 21\text{MHz}$ was obtained.
Fig. 7-14. A plot of the injection locking bandwidth as a function of the locking gain.

The oscillator was then stabilized using the TE$_{012}$ cavity and circuit MIC-035 described in chapter 5. For this combination, Fig. (7-8) predicts an insertion loss of 8.2 dB and a stabilization factor of 105. It should be noted that these may not be the optimum values for a particular application. A photograph of the FET oscillator with the stabilizing cavity attached is shown in Fig. (7-15).
Fig. 7-15. A photograph of the oscillator circuit shown in figure (7-12) attached to the stabilizing cavity. The oscillator frequency was then measured for both the unstabilized and stabilized cases as a function of gate and drain bias. The frequency of the unstabilized oscillator was first adjusted to approximately 11.95 GHz using a dielectric chip. The results are plotted in Fig. (7-16).
Fig. 7-16. Plots of the oscillator frequency for the stabilized and unstabilized cases as functions of (a) drain bias and (b) gate bias.

It is seen that adding the cavity caused a dramatic reduction in the pushing figures of the oscillator. For the unstabilized case, the pushing figures are calculated to be $\frac{\partial f_{osc}}{\partial V_{GS}} = \frac{38 \text{ MHz/V}}{}$ and $\frac{\partial f_{osc}}{\partial V_{DS}} = 9.2 \text{ MHz/V}$. With the stabilization cavity connected, these values dropped to $\frac{\partial f_{osc}}{\partial V_{GS}} = 0.8 \text{ MHz/V}$ and $\frac{\partial f_{osc}}{\partial V_{DS}} = 0.6 \text{ MHz/V}$. However, when the drain bias was decreased beyond +4.0V, the frequency jumped from 11.939 GHz to 8.342 GHz and the value of $P_L$ dropped to below .01 mW. The reason why this can occur was mentioned in section (7-4). To bring the frequency back up to 11.939 GHz, the drain voltage had to be increased to 5.0V. The measured value of $f_{osc} = 11.939 \text{ GHz}$.
is in good agreement with the value of $f_{osc} = 11.950$ GHz predicted by Fig. (7-10). It can be seen from Fig. (7-15) that the cavity was connected to the oscillator using an OSM adapter; because of this, the value of $f_1$ was probably not equal to 11.950 GHz, the value assumed in Fig. (7-10). Also, $f_{osc}$ (unstabilized) was adjusted to approximately 11.950 GHz, a value of 11.3 GHz is used in Fig. (7-10). The change in $f_1$ is the more significant of the two.

Photographs of the oscillator's output spectra are shown in Fig. (7-17) for both the unstabilized and stabilized cases showing the dramatic decrease in oscillator noise. Bias used was $V_{DS} = 5.0$V and $V_{GS} = -1.7$V. The output power for the unstabilized case was 13 mW; this dropped to approximately 1.3 mW with the cavity connected. The insertion loss of 10 dB is in good agreement with the value of 8.2 dB predicted by Fig. (7-8), which doesn't account for the losses in the microstrip lines and connector.
The FM noise of the unstabilized oscillator was measured using the setup described in chapter 5. Bias used was $V_{GS} = -1.7V$ and $V_{DS} = 5.0V$ corresponding to values of $f_{osc} = 11.943$ and $P_{osc} = 13$ mW. The results are given in Fig. (7-18).

The FM noise performance of the FET oscillator described in [49] is also indicated on Fig. (7-18) for comparison. No values were given for $f_m$ less than 10 kHz. This oscillator had a value of $Q_{ext} = 300$ and bias was adjusted to give values of $f_{osc} = 10.14$ GHz and $P_{osc} = 23$ mW as reported in [49]. Since the value of $Q_{ext}$ for the present oscillator is lower than that for the oscillator described in [49] and operates at a slightly higher frequency, the latter should have a lower value of $\Delta f$ at any given value of $f_m$. However, it can be seen from Fig. (7-18) that this is not so. This discrepancy is probably due to the different measurement systems used and in any case is relatively small.
Fig. 7-18. Plots of the oscillator r.m.s. frequency deviation for the unstabilized case (measured) and stabilized case (calculated, $S=105$). For comparison, the results for the FET oscillator described in [49] are also shown.
No change in $\Delta f$ at any given value of $f_m$ was noticed when the drain bias was varied from 5.0V to 3.5V provided the value of $f_{osc}$ was held constant by adjusting the gate bias. The value of $\Delta f$ obtained from the oscillator with the cavity connected was calculated assuming a value of $S=105$ and is also plotted on Fig. (7-18). Measuring this value proved to be a problem due to the low output power of the stabilized oscillator. The calculated value of $\Delta f$ is comparable to that of the feedback oscillator described in chapter 6.

Finally, the behaviour of the unstabilized oscillator was evaluated as a function of temperature $T$ using an environmental chamber. Bias was fixed at $V_{GS}=-1.0V$ and $V_{DS}=5.0V$. The value of $f_{osc}$ decreased from 11.869 GHz to 11.833 GHz in a linear fashion as $T$ was increased from 5°C to 55°C. The value of $P_{osc}$ decreased from 15 mW to 10.5 mW over the same temperature change. The device used for this test had $I_{dd}=58mA$ and $V_p=-3.22V$.

The value of $f_{osc}$ was then adjusted to 11.853 GHz at room temperature by adjusting the gate bias. The temperature was then varied from 5°C to 55°C and the frequency was adjusted to 11.853 GHz at each value of $T$ by adjusting $V_{GS}$. The value of $P_{osc}$ was also measured. The results are given in Fig. (7-19); drain bias was held constant at 5.0V.

Fig. (7-19) shows that the value of $f_{osc}$ can be made independent of $T$ by adjusting the gate bias. The relationship between $V_{GS}$ and $T$ is almost linear and a simple thermistor circuit can be used to compensate the oscillator. Even if a stabilizing cavity is used, the oscillator should be temperature compensated to avoid the possibility of mode jumping.

A 12 GHz GaAs FET oscillator in MIC form has been described. A transmission cavity has then been used to stabilize the basic oscillator. At present, power is low due to the limitation of the devices. However, the efficiency is at least twice that of present Impatt and Gunn oscillators.
Fig. 7-19. A plot of the gate bias required to keep the oscillator frequency constant as a function of temperature. The corresponding values of oscillator power are also shown.

With a stabilizing cavity, the oscillator can be used as an L.O. for mixers in MIC communication subsystems. For example, it can be seen from Fig. (7-8) that an unstabilized oscillator having a value of $P_{osc} = 27$ mW (14.3 dBm) connected to a cavity having parameters $f_0 = 11.950$ GHz, $Q_0 = 29,000$, $\beta_1 = 1.66$ will give a value of $P_L = 8.3$ dBm. From Fig. (7-18), the value of $\Delta f$ in a 100 Hz bandwidth at 1 KHz from the carrier is calculated to be .2 Hz/$\sqrt{Hz}$. These values should be sufficient for a number of L.O. applications.
CHAPTER 8

CONCLUSIONS

8-1 Summary

A novel microstrip to cavity transition has been analyzed. Values of the coupling coefficient calculated using the expression derived for the external Q of the cavity were found to be in good agreement with measured values. The transition was then used in the design of two different cavity stabilized FET oscillators. In summarizing, the work described in this report can be considered in two parts, (a) the microstrip to cavity transition and (b) the FET oscillators.

Although the transition was used in this report to stabilize FET oscillators, it could also be used to stabilize a microstrip IMPATT or Gunn diode oscillator. Consider, for example, the design of a low cost converter for the reception of signals from a broadcast satellite [1-2]. The front end of such a converter might consist of a rat race mixer, a Gunn L.O. and an I.F. amplifier. To minimize cost, the unit could be constructed in MIC form. The Gunn L.O. would be stabilized using the microstrip to cavity transition with the cavity machined into the substrate holder. L.O. tuning could easily be accomplished by inserting a post into the cavity as described in sections (2-7) and (5-3). It would probably be more economical to use a sealed invar cavity instead of a titanium silicate cavity in this case. A heater could be used to further improve the frequency drift if required. If, for a 12 GHz cavity, the TE_{011} mode is used instead of the TE_{012} mode, the length of the cavity would be .708" instead of 1.416" with the Q_{o} of the TE_{011} mode being 85% of that for the TE_{012} mode. It might be desirable to use the TE_{011} to reduce the depth of the cavity. A converter constructed using the above technique would make use of the desirable
properties of both microstrip (ease of production through photolithography and ease of connecting active and passive components) and waveguide cavities (high $Q_0$ and temperature stability, using appropriate materials).

Two different FET oscillator configurations were analyzed and experimentally evaluated. The F.M. noise spectra of both were comparable to that of a crystal oscillator multiplier source. However, no measurements of A.M. noise were done and the F.M. noise spectra were not compared with those of IMPATT or Gunn diode oscillators. Output power was low due to the limitations of the FET's used. As medium power FET's become commercially available, output powers in excess of 100 mW at X-band should be feasible. It is noted that in the oscillator configuration described in Chapter 6, the output power is obtained at the output of the FET amplifier. In the oscillator described in Chapter 7, the output power is obtained at the output port of the stabilizing cavity. It is yet not clear which of the two configurations is the optimum in terms of noise properties. For the case of Chapter 7, the stabilizing cavity could also be connected in the reaction [58] or reflection [59] configuration. The trade-off between output power, noise spectrum, and tuning range for these three configurations has not been examined. Also, the trade-off between cost and both short term and long term stability for (a) a crystal oscillator-multiplier source (b) a stabilized IMPATT oscillator (c) a stabilized Gunn oscillator and (d) a stabilized FET oscillator has not been examined for various L.O. applications. It might be possible to design an electronically tunable FET oscillator (without a stabilizing cavity) having a tuning bandwidth larger than that of either an electronically tunable IMPATT or Gunn oscillator of comparable power. This is because the frequency of oscillation of both the IMPATT and Gunn oscillators is a function of the transit time of the carriers in the device. The frequency of oscillation of an FET oscillator
is determined, primarily, by the feedback network and not by the device.

8-2 Recommendations For Future Work

The following recommendations for future work can be made based on the context of the previous section.

1. A comparison of the oscillator configuration of Chapter 6 with that of Chapter 7 using (a) a transmission cavity (b) a reflection cavity and (c) a reaction cavity in terms of output power, noise spectrum and tuning range.

2. A comparison of (a) a crystal oscillator-multiplier source (b) a cavity stabilized IMPATT oscillator (c) a cavity stabilized Gunn oscillator and (d) a cavity stabilized FET oscillator in terms of cost and both long term and short term stability for various source applications.

3. A study of the maximum output power obtainable from an FET oscillator using medium power FET's.

4. A study of the design of an electronically tuned FET oscillator to see how broad a tuning range is possible while still maintaining reasonable output power.
APPENDIX A

DATA SHEET FOR CORNING TITANIUM SILICATE.
JLE™ Titanium Silicate, Code 7971—Fused Silica, Code 7940

Thermal Properties

<table>
<thead>
<tr>
<th></th>
<th>7971</th>
<th>7940</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average expansion coefficient per°C</td>
<td>0.00 (10.03) x 10⁻⁶</td>
<td>0.50 x 10⁻⁶</td>
</tr>
<tr>
<td>5° to 35°C</td>
<td>0.03 (10.03) x 10⁻⁶</td>
<td>0.49 x 10⁻⁶</td>
</tr>
<tr>
<td>0° to 200°C</td>
<td>0.03 (10.03) x 10⁻⁶</td>
<td>0.39 x 10⁻⁶</td>
</tr>
<tr>
<td>-100° to +200°C</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Service temperature

- Normal: 800°C, 900°C
- Extreme: 1000°C, 1100°C
- Softening point: 1490°C, 1585°C
- Annealing point: 1600°C, 1075°C
- Strain point: 890°C, 990°C

Specific heat, 25°C, cal/gm°C: 0.183, 0.177

Thermal conductivity, 25°C, cal cm/cm² sec °C: 0.00313, 0.00329

Thermal diffusivity, 25°C, cm²/sec: 0.0079, 0.0084

Mean Specific Heat

Thermal Conductivity

Thermal Diffusivity
APPENDIX B

DATA SHEETS FOR THE PLESSEY GAT-3 AND NIPPON V-244 DEVICES
GENERAL DESCRIPTION
The GAT 3 is intended for use in microwave and oscillator applications up to a frequency of 12 GHz. The performance of the device depends upon the method of mounting. A P102 packaged device can be used effectively up to 8 GHz whereas a device mounted in the P103 microstrip package can be used up to 12 GHz. Optimum performance, however, is obtained by mounting the transistor chip directly into an alumina microstrip circuit. Devices are available as chips or mounted in either the P102 or the P103 microstrip package.

ELECTRICAL CHARACTERISTICS (AT 25°C)

<table>
<thead>
<tr>
<th>Characteristic</th>
<th>Min.</th>
<th>Typ.</th>
<th>Max.</th>
<th>Units</th>
<th>Test Conditions</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gate-source voltage</td>
<td>-10</td>
<td>-</td>
<td>-</td>
<td>Volts</td>
<td>lg = 10μA Vds = 0V</td>
</tr>
<tr>
<td>Drain current</td>
<td>-</td>
<td>-</td>
<td>50</td>
<td>mA</td>
<td>Vds = 5V Vgs = 0V</td>
</tr>
<tr>
<td>Pinch-off voltage</td>
<td>-</td>
<td>-5</td>
<td>-10</td>
<td>Volts</td>
<td>Vds = 5V Ids = 10μA</td>
</tr>
<tr>
<td>Maximum available power gain (M.A.G.)</td>
<td>-</td>
<td>8</td>
<td>-</td>
<td>dB</td>
<td>8 GHz</td>
</tr>
<tr>
<td>Chip</td>
<td>-</td>
<td>6</td>
<td>4</td>
<td>dB</td>
<td>Vds = 5V Vgs = 0V</td>
</tr>
<tr>
<td>In P102 package</td>
<td>-</td>
<td>8</td>
<td>-</td>
<td>dB</td>
<td>12 GHz</td>
</tr>
<tr>
<td>In P103 package</td>
<td>-</td>
<td>5</td>
<td>-</td>
<td>dB</td>
<td></td>
</tr>
<tr>
<td>Noise figure</td>
<td>-</td>
<td>5.5</td>
<td>-</td>
<td>dB</td>
<td>8 GHz</td>
</tr>
<tr>
<td>GAT 3 Chip</td>
<td>-</td>
<td>6.0</td>
<td>-</td>
<td>dB</td>
<td></td>
</tr>
<tr>
<td>In P102 package</td>
<td>-</td>
<td>5.5</td>
<td>-</td>
<td>dB</td>
<td>12 GHz</td>
</tr>
<tr>
<td>In P103 package</td>
<td>-</td>
<td>10.0</td>
<td>-</td>
<td>dB</td>
<td>Vds = 5V Ids = 15 mA</td>
</tr>
<tr>
<td>Chip</td>
<td>-</td>
<td>2.7</td>
<td>3.5</td>
<td>dB</td>
<td>12 GHz</td>
</tr>
<tr>
<td>In P102 package</td>
<td>-</td>
<td>4.0</td>
<td>4.5</td>
<td>dB</td>
<td></td>
</tr>
<tr>
<td>In P103 package</td>
<td>-</td>
<td>2.7</td>
<td>3.5</td>
<td>dB</td>
<td>12 GHz</td>
</tr>
<tr>
<td>GAT 3/010 Chip</td>
<td>-</td>
<td>5.0</td>
<td>6.0</td>
<td>dB</td>
<td></td>
</tr>
<tr>
<td>In P102 package</td>
<td>-</td>
<td>5.0</td>
<td>6.0</td>
<td>dB</td>
<td></td>
</tr>
<tr>
<td>In P103 package</td>
<td>-</td>
<td>5.0</td>
<td>6.0</td>
<td>dB</td>
<td></td>
</tr>
</tbody>
</table>

TYPICAL GAIN AND STABILITY PARAMETERS FOR A DEVICE OPERATING IN THE P102 PACKAGE IN THE COMMON SOURCE CONFIGURATION. Vds = 5V Vgs = 0V

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Maximum Stable Gain (M.S.G.) (dB)</th>
<th>Stability Factor (K)</th>
<th>Maximum Available Gain (M.A.G.) (dB)</th>
<th>Maximum Unilateral Gain (GUmax) (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>13.0</td>
<td>0.85</td>
<td>-</td>
<td>12.0</td>
</tr>
<tr>
<td>5.0</td>
<td>12.9</td>
<td>0.83</td>
<td>-</td>
<td>11.5</td>
</tr>
<tr>
<td>6.0</td>
<td>12.5</td>
<td>0.86</td>
<td>9.3</td>
<td>10.1</td>
</tr>
<tr>
<td>7.0</td>
<td>12.6</td>
<td>1.3</td>
<td>7.4</td>
<td>8.3</td>
</tr>
<tr>
<td>8.0</td>
<td>13.5</td>
<td>2.2</td>
<td></td>
<td>6.9</td>
</tr>
</tbody>
</table>
Typical small-signal S-parameters — GAT 3 series
For a Device in the P102 Package in the Common Source Configuration.

\[ V_{ds} = 5V \quad V_{gs} = 0V \quad Z_o = 50\Omega \]

<table>
<thead>
<tr>
<th>Frequency (GHz)</th>
<th>Magnitude</th>
<th>Angle (degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.0</td>
<td>0.050</td>
<td>64.6</td>
</tr>
<tr>
<td>5.0</td>
<td>0.055</td>
<td>67.1</td>
</tr>
<tr>
<td>6.0</td>
<td>0.067</td>
<td>66.5</td>
</tr>
<tr>
<td>7.0</td>
<td>0.067</td>
<td>63.3</td>
</tr>
<tr>
<td>8.0</td>
<td>0.056</td>
<td>79.9</td>
</tr>
</tbody>
</table>

Blas conditions
\[ V_{ds} = 5V \]
\[ I_{ds} = 10mA \]
Tuned for minimum noise
GALLIUM ARSENIIDE FIELD EFFECT TRANSISTOR

The NE244 is a Gallium Arsenide FET designed for Low Noise Amplifier and Oscillator applications up to X-band. The NE24406 is packaged in a stripline package with low parasitics. The designation for the chip is NE24406.

NE244 Features:
- Very High $f_{MAX}$: 55GHz (Packaged Device)
- High $f_{MAX}$: 11dB at 8GHz
- Low Noise Figure: 3dB at 8GHz
- New Stripline Package

ABSOLUTE MAXIMUM RATINGS ($T_a=25^\circ C$)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>SYMBOL</th>
<th>RATINGS</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drain to Source Voltage</td>
<td>$V_{DS}$</td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td>Gate to Source Voltage</td>
<td>$V_{GS}$</td>
<td>-10</td>
<td>V</td>
</tr>
<tr>
<td>Drain Current</td>
<td>$I_D$</td>
<td>100</td>
<td>mA</td>
</tr>
<tr>
<td>Total Power Dissipation</td>
<td>$P_{T^*}$</td>
<td>500</td>
<td>mW</td>
</tr>
<tr>
<td>Channel Temperature</td>
<td>$T_{ch}$</td>
<td>175</td>
<td>°C</td>
</tr>
<tr>
<td>Storage Temperature</td>
<td>$T_{stg}$</td>
<td>-65 to +175</td>
<td>°C</td>
</tr>
</tbody>
</table>

*Derate at 300°C/W from 25°C (channel to ambient)
Derate at 200°C/W from 25°C (channel to case)

ELECTRICAL CHARACTERISTICS ($T_a=25^\circ C$) (NE24406)

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>SYMBOLS</th>
<th>TEST CONDITIONS</th>
<th>MIN.</th>
<th>TYP.</th>
<th>MAX.</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drain Current</td>
<td>$I_{DSS}$</td>
<td>$V_{DS}=3V, V_{GS}=0$</td>
<td>30</td>
<td>60</td>
<td>100</td>
<td>mA</td>
</tr>
<tr>
<td>Pinchoff Voltage</td>
<td>$V_{GS}$ (off)</td>
<td>$V_{DS}=3V, I_D=100mA$</td>
<td>-2</td>
<td>-4</td>
<td>-10</td>
<td>V</td>
</tr>
<tr>
<td>Transconduction</td>
<td>$R_m$</td>
<td>$V_{DS}=3V, I_D=30mA$</td>
<td>15</td>
<td>20</td>
<td>55</td>
<td>mmho</td>
</tr>
<tr>
<td>Maximum Frequency of Oscillation</td>
<td>$f_{MAX}$</td>
<td>$V_{DS}=3V, I_D=30mA$</td>
<td>40</td>
<td>55</td>
<td>GHz</td>
<td></td>
</tr>
<tr>
<td>Maximum Available Power Gain</td>
<td>$N_{AG}$</td>
<td>$V_{DS}=3V, I_D=30mA$</td>
<td>15</td>
<td>14</td>
<td>11</td>
<td>dB</td>
</tr>
<tr>
<td>Noise Figure</td>
<td>$N_F$</td>
<td>$V_{DS}=3V, I_D=10mA$</td>
<td>1.8</td>
<td>2.5</td>
<td>3.5</td>
<td>dB</td>
</tr>
<tr>
<td>Output Power (1dB Compression)</td>
<td>$P_{O}$</td>
<td>$V_{DS}=3V, I_D=30mA$</td>
<td>10</td>
<td>2.5</td>
<td>10</td>
<td>dBm</td>
</tr>
</tbody>
</table>

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SALES AGENTS FOR Nippon Electric Company, Limited U.S.A. — CANADA — EUROPE
COMMON SOURCE S' PARAMETERS

ID=10mA, V_DDS=3V
Frequency in GHz

S11 and S22

S21

S12
### ELECTRICAL CHARACTERISTICS ($T_a=25^\circ$)

<table>
<thead>
<tr>
<th>PARAMETER</th>
<th>SYMBOLS</th>
<th>TEST CONDITIONS</th>
<th>MIN.</th>
<th>TYP.</th>
<th>MAX.</th>
<th>UNIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drain Current</td>
<td>$I_{DSS}$</td>
<td>$V_{DS}=3V, V_{GS}=0$</td>
<td>30</td>
<td>60</td>
<td>100</td>
<td>mA</td>
</tr>
<tr>
<td>Pinch-off Voltage</td>
<td>$V_{GS\ (off)}$</td>
<td>$V_{DS}=3V, I_D=100\mu A$</td>
<td>-2</td>
<td>-4</td>
<td>-10</td>
<td>V</td>
</tr>
<tr>
<td>Transconductance</td>
<td>$g_m$</td>
<td>$V_{DS}=3V, I_D=30mA$</td>
<td>15</td>
<td>20</td>
<td></td>
<td>mho</td>
</tr>
</tbody>
</table>

#### NE24400, CHIP DIMENSIONS, MICROMETERS

1. Source
2. Drain
3. Gate

**Chip size 500µ square**

---

**RECOMMENDED BONDING PROCEDURE:**

1. Clean chip thoroughly with trichloroethylene (CHCl₃:CC1₂), acetone (CH₂COCH₂) and iso-propyl-alcohol ((CH₃)₂CH-C₂H₅OH).
2. Dry thoroughly.
3. Use 20 micron pure gold wire.
4. Thermal compression bonding technique with unheated wedge bonding tip.
5. Chip temperature of 300°C ± 30°C.
6. Probe bonding pressure of 22 ± 2 grams.
REFERENCES


