COMBINED AXIAL LOAD - BENDING MOMENT

BEHAVIOUR OF REINFORCED CONCRETE SHORT COLUMNS

by

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ABSTRACT

Extensive and sometimes comprehensive research on the ultimate strength of reinforced concrete columns has been carried out almost since the first use of this structural material. This thesis reviews briefly some of the more significant experimental and analytical work in this field.

An experimental investigation has been carried out on 6" by 6" square section concrete columns, with symmetrical reinforcement. Loads were applied at eccentricities e/t equal to 0.00±0.01; 0.10±0.01; 0.30±0.01; 0.50±0.01; 0.75±0.01 and ∞. A total of 100 column specimens were tested, divided into 4 groups, each having 25 specimens. The percentage of reinforcement in each group was: 1.24; 2.27; 5.14 and 7.14 respectively. The variations of concrete strength considered in the project were 4,000 and 6,000 psi.

The scope of this thesis was purposely divided into three groups depending on the predominant type of stress to which the columns were subjected, namely: (1) pure flexure; (2) axial compression; (3) combined axial compression and flexure.

From the comparison of the results, it was found that the experimentally determined mean strength of the column specimens was in good agreement with a mean value of 1.04 and a standard deviation of 0.09 in comparison with the loads calculated from the formulae in the American Concrete Institute Building Code Requirement ACI 318-63 and a mean value of 0.99 and a standard deviation of 0.02 for the strain gradient method.
A computer program using the strain gradient method is presented. The input requires only the geometry of the member, stress-strain characteristics of the concrete, the longitudinal reinforcement and the magnitude and location of the applied loading. Final results of the calculated maximum axial loads and maximum external moments are obtained and printed out.

This thesis analysed and discussed the test results, reviewed the ultimate strength theories, thus gave an overall picture of the development and scope of the research on ultimate strength of reinforced concrete short columns.
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TABLE OF CONTENTS

PAGE NO.

ABSTRACT ......................................................... i

ACKNOWLEDGEMENTS ........................................... iii

TABLE OF CONTENTS ........................................... iv

LIST OF TABLES AND FIGURES ................................. viii

NOTATION ........................................................ xiii

CHAPTER I  INTRODUCTION .................................... 1

CHAPTER II  REVIEW OF LITERATURE .......................... 4

CHAPTER III  EXPERIMENTAL INVESTIGATION ................. 11

3.1 Object and Scope of Investigation ...................... 11

3.2 Material ..................................................... 11

3.2.1 Concrete Mixtures ..................................... 11

3.2.2 Reinforcing Steel ...................................... 13

3.3 Types of Specimens ....................................... 14

3.4 Fabrication and Curing ................................... 17

3.5 Testing Methods and Procedure .......................... 19

3.5.1 Reinforced Concrete Columns ......................... 19

3.5.1.a Columns Under Combined Axial and Bending ....... 19
### 3.5.1.b Columns Under Pure Flexure ...

### 3.5.2 Concrete Cylinders

<table>
<thead>
<tr>
<th>CHAPTER IV</th>
<th>THEORETICAL INVESTIGATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.1 Ultimate Strength Analysis of Reinforced Concrete Columns Under the Requirement of ACI Code</td>
<td>28</td>
</tr>
<tr>
<td>4.1.1 Reinforced Concrete Columns Under Balanced Failure</td>
<td>30</td>
</tr>
<tr>
<td>4.1.2 Reinforced Concrete Columns Under Compression Failure</td>
<td>33</td>
</tr>
<tr>
<td>4.1.3 Reinforced Concrete Columns Under Tension Failure</td>
<td>37</td>
</tr>
<tr>
<td>4.1.4 Reinforced Concrete Beams With Equal Tension and Compression Steel</td>
<td>40</td>
</tr>
<tr>
<td>4.2 Strain Gradient Method</td>
<td>43</td>
</tr>
<tr>
<td>4.2.1 Reinforced Concrete Columns Under Balanced Failure</td>
<td>47</td>
</tr>
<tr>
<td>4.2.2 Reinforced Concrete Columns Under Compression Failure</td>
<td>50</td>
</tr>
<tr>
<td>4.2.3 Reinforced Concrete Columns Under Tension Failure</td>
<td>51</td>
</tr>
</tbody>
</table>
CHAPTER V  DISCUSSION AND SUMMARY  
OF TEST RESULTS  

5.1 General Behavior and Modes of Failure  
of Specimens  

5.2 Analysis of Experimental Results  

5.3 Analysis of Theoretical Results  

5.4 Comparison of Test Results With  
the ACI Design Formulae  

5.4.1 Effects of Concrete Quality,  
Strength of Reinforcement  
and Eccentricity  

5.4.2 Interaction Diagrams  

5.4.3 Variation of $P_{exp.}/P_{cal.}$  

5.5 Comparison of Test Results With  
the Strain Gradient Method  

5.5.1 Effects of Concrete Quality,  
Strength of Reinforcement  
and Eccentricity  

5.5.2 Interaction Diagrams  

5.5.3 Variation of $P_{exp.}/P_{cal.}$  

5.6 Comparisons with Previous Investigations  

5.7 Studies of Deflection
CHAPTER VI  CONCLUSIONS AND RECOMMENDATIONS .......... 96

6.1 Conclusions ........................................... 96

6.2 Recommendations ................................. 98

REFERENCES .............................................. 99

APPENDIXES :-

APPENDIX A  Steel Properties ......................... 106

APPENDIX B  Individual Graphs of Specimens ........ 111

APPENDIX C  Examples .................................. 197

APPENDIX D  Computer Programs ....................... 205
# LIST OF TABLES

<table>
<thead>
<tr>
<th>TABLE</th>
<th>DESCRIPTION</th>
<th>PAGE NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2.1.a</td>
<td>MIX PROPORTION BY WEIGHT WITH COMPRESSION STRENGTH OF 4000 psi.</td>
<td>12</td>
</tr>
<tr>
<td>3.2.1.b</td>
<td>MIX PROPORTION BY WEIGHT WITH COMPRESSION STRENGTH OF 6000 psi.</td>
<td>12</td>
</tr>
<tr>
<td>3.2.2.a</td>
<td>AVERAGE TENSILE PROPERTIES OF REINFORCING STEEL</td>
<td>13</td>
</tr>
<tr>
<td>3.3.a</td>
<td>OUTLINE OF TESTS</td>
<td>15</td>
</tr>
<tr>
<td>5.1</td>
<td>TEST RESULTS OF GROUP 1, 2, 3 &amp; 4</td>
<td>53</td>
</tr>
<tr>
<td>5.3</td>
<td>CALCULATED VALUES Pult. OF ACI DESIGNED</td>
<td>62</td>
</tr>
<tr>
<td>5.4</td>
<td>COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS BY ACI FORMULAE</td>
<td>64</td>
</tr>
<tr>
<td>5.5</td>
<td>COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOAD</td>
<td>75</td>
</tr>
<tr>
<td>5.6.2</td>
<td>COMPARISON OF TEST AND COMPUTED ULTIMATE LOADS UNDER ECCENTRICAL LOADS — BY CHOW</td>
<td>86</td>
</tr>
<tr>
<td>5.6.3</td>
<td>COMPARISON OF TEST AND COMPUTED ULTIMATE LOADS UNDER ECCENTRICAL LOADS — BY HOGNESTAD</td>
<td>87</td>
</tr>
<tr>
<td>5.6.4</td>
<td>COMPARISON OF TEST AND COMPUTED ULTIMATE LOADS UNDER ECCENTRIC LOADS — BY ACI COLUMN INVESTIGATION</td>
<td>90</td>
</tr>
<tr>
<td>TABLE</td>
<td>PAGE NO.</td>
<td></td>
</tr>
<tr>
<td>------</td>
<td>---------</td>
<td></td>
</tr>
<tr>
<td>5.6.5 COMPARISON OF TEST AND COMPUTED ULTIMATE LOADS UNDER ECCENTRIC LOADS — BY BACH AND GRAF</td>
<td>91</td>
<td></td>
</tr>
<tr>
<td>5.7.1 COMPARISON BETWEEN OBSERVED AND CALCULATED CENTRE DEFLECTIONS</td>
<td>95</td>
<td></td>
</tr>
</tbody>
</table>
# List of Figures

<table>
<thead>
<tr>
<th>Figure</th>
<th>Description</th>
<th>Page No.</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.2.1</td>
<td>Grading curves of coarse aggregate &amp; sand</td>
<td>12</td>
</tr>
<tr>
<td>3.3.a</td>
<td>Detail of column specimens</td>
<td>16</td>
</tr>
<tr>
<td>3.4.a</td>
<td>Typical drawing for steel form</td>
<td>18</td>
</tr>
<tr>
<td>3.5.1</td>
<td>General view of a tested specimen under eccentric loading</td>
<td>20</td>
</tr>
<tr>
<td>3.5.2.a</td>
<td>Arrangement of testing</td>
<td>21</td>
</tr>
<tr>
<td>3.5.2.b</td>
<td>Arrangement of testing</td>
<td>22</td>
</tr>
<tr>
<td>3.5.3.a</td>
<td>General view of test specimen under pure bending</td>
<td>25</td>
</tr>
<tr>
<td>3.5.3.b</td>
<td>Sketch of test set up for beam column</td>
<td>26</td>
</tr>
<tr>
<td>3.5.4.a</td>
<td>6&quot; by 12&quot; concrete cylinder under test</td>
<td>25</td>
</tr>
<tr>
<td>4.1.a</td>
<td>Assumed stress-strain relationships for concrete in compression</td>
<td>29</td>
</tr>
<tr>
<td>4.1.1.a</td>
<td>Balanced condition — rectangular section</td>
<td>31</td>
</tr>
<tr>
<td>4.1.2.a</td>
<td>Interaction diagram, compression plus bending, ultimate-strength $P_0$ &amp; $M_{uo}$</td>
<td>34</td>
</tr>
<tr>
<td>4.1.2.b</td>
<td>Ultimate capacity controlled by compression</td>
<td>36</td>
</tr>
<tr>
<td></td>
<td>— rectangular section</td>
<td></td>
</tr>
<tr>
<td>4.1.3.a</td>
<td>Ultimate capacity controlled by tension</td>
<td>38</td>
</tr>
<tr>
<td></td>
<td>— rectangular section</td>
<td></td>
</tr>
<tr>
<td>4.1.4.a</td>
<td>Ultimate strength, eccentric compression</td>
<td>41</td>
</tr>
<tr>
<td></td>
<td>— rectangular section</td>
<td></td>
</tr>
<tr>
<td>4.1.4.b</td>
<td>Collapse mechanism of a beam column</td>
<td>43</td>
</tr>
<tr>
<td>FIGURE</td>
<td>PAGE NO.</td>
<td></td>
</tr>
<tr>
<td>--------</td>
<td>----------</td>
<td></td>
</tr>
<tr>
<td>4.2</td>
<td>METHOD OF ANALYSIS ................................ 45</td>
<td></td>
</tr>
<tr>
<td>4.2.1.1</td>
<td>INTERACTION DIAGRAM ................................ 48</td>
<td></td>
</tr>
<tr>
<td>4.2.1.2</td>
<td>STRESS-STRAIN RELATIONSHIP .......................... 49</td>
<td></td>
</tr>
<tr>
<td>4.2.1.2.a</td>
<td>POINT b .......................................... 49</td>
<td></td>
</tr>
<tr>
<td>4.2.1.2.b</td>
<td>LINE b-c .......................................... 49</td>
<td></td>
</tr>
<tr>
<td>4.2.1.2.c</td>
<td>LINE c-d .......................................... 49</td>
<td></td>
</tr>
<tr>
<td>4.2.1.2.d</td>
<td>LINE a-b .......................................... 49</td>
<td></td>
</tr>
<tr>
<td>5.1.1</td>
<td>TYPICAL FAILURE PHENOMENA OF GROUP 1 ............ 56</td>
<td></td>
</tr>
<tr>
<td>5.1.2</td>
<td>TYPICAL FAILURE PHENOMENA OF GROUP 2 ............ 57</td>
<td></td>
</tr>
<tr>
<td>5.1.3</td>
<td>TYPICAL FAILURE PHENOMENA OF GROUP 3 ............ 58</td>
<td></td>
</tr>
<tr>
<td>5.1.4</td>
<td>TYPICAL FAILURE PHENOMENA OF GROUP 4 ............ 59</td>
<td></td>
</tr>
<tr>
<td>5.4.1</td>
<td>$\frac{p_{\text{exp.}}}{p_{\text{cal.}}}$ VERSUS ULTIMATE CONCRETE STRESS .......... 67</td>
<td></td>
</tr>
<tr>
<td>5.4.2</td>
<td>$\frac{p_{\text{exp.}}}{p_{\text{cal.}}}$ VERSUS RATIO OF REINFORCEMENT .......... 67</td>
<td></td>
</tr>
<tr>
<td>5.4.2.1</td>
<td>INTERACTION DIAGRAM FOR GROUP 1 .................. 68</td>
<td></td>
</tr>
<tr>
<td>5.4.2.2</td>
<td>INTERACTION DIAGRAM FOR GROUP 2 .................. 69</td>
<td></td>
</tr>
<tr>
<td>5.4.2.3</td>
<td>INTERACTION DIAGRAM FOR GROUP 3 .................. 70</td>
<td></td>
</tr>
<tr>
<td>5.4.2.4</td>
<td>INTERACTION DIAGRAM FOR GROUP 4 .................. 71</td>
<td></td>
</tr>
<tr>
<td>5.4.3</td>
<td>$\frac{p_{\text{exp.}}}{p_{\text{cal.}}}$ VERSUS ECCENTRICITY RATIO OF LOAD .......... 67</td>
<td></td>
</tr>
<tr>
<td>5.4.3.1</td>
<td>HISTOGRAM AND FILTED NORMAL CURVE ............... 73</td>
<td></td>
</tr>
</tbody>
</table>
5.5.1 \( \frac{P_{\text{exp}}}{P_{\text{cal}}} \) VERSUS ULTIMATE CONCRETE STRESS ....... 78

5.5.2 \( \frac{P_{\text{exp}}}{P_{\text{cal}}} \) VERSUS RATIO OF REINFORCEMENT ............ 78

5.5.2.1 INTERACTION DIAGRAM FOR GROUP 1 ................. 79

5.5.2.2 INTERACTION DIAGRAM FOR GROUP 2 ................. 80

5.5.2.3 INTERACTION DIAGRAM FOR GROUP 3 ................. 81

5.5.2.4 INTERACTION DIAGRAM FOR GROUP 4 ................. 82

5.5.3 \( \frac{P_{\text{exp}}}{P_{\text{cal}}} \) VERSUS ECCENTRICITY RATIO OF LOAD ...... 78

5.5.3.1 HISTOGRAM AND FILTED NORMAL CURVE ............... 84

5.7.1 DEFLECTION SHAPE OF A COLUMN ...................... 92

5.7.2 COLUMN UNDER DEFLECTION ......................... 93

5.7.3 STRAIN RELATIONSHIP ............................... 93

A.1 STRESS-STRAIN CURVE OF #3 LONGITUDINAL
REINFORCEMENT (PLAIN BAR) ......................... 107

A.2 STRESS-STRAIN CURVE OF #4 LONGITUDINAL
REINFORCEMENT (DEFORM BAR) ......................... 108

A.3 STRESS-STRAIN CURVE OF #6 LONGITUDINAL
REINFORCEMENT (DEFORM BAR) ......................... 109

A.4 STRESS-STRAIN CURVE OF #7 LONGITUDINAL
REINFORCEMENT (DEFORM BAR) ......................... 110

B.1 to B.85 LOAD-DEFLECTION CURVE OF SPECIMEN ............ 112
NOTATION

\( A_s \) = area of tension steel, sq.in.

\( A_s' \) = area of compression steel, sq.in.

\( a \) = depth of equivalent rectangular stress block = \( K_c \), in.

\( a_b \) = depth of equivalent rectangular stress block = \( K_c c' \), in.

\( b \) = width of rectangular member, in.

\( c \) = distance from extreme compression fiber to neutral axis at ultimate strength, in.

\( C_b \) = distance from extreme compression fibre to neutral axis for balanced condition = \( d(87,000)/(87,000+f_y) \)

\( d \) = distance from extreme compression fiber to centroid of tension reinforcement

\( d' \) = distance from extreme compression fibre to centroid of compression reinforcement

\( d'' \) = distance from plastic centroid to centroid of tension reinforcement

\( E_s \) = modulus of elasticity of reinforcement

\( E_c \) = modulus of elasticity of concrete

\( e' \) = eccentricity of load with respect to centroid of tension reinforcement

\( e \) = eccentricity of load with respect to plastic centroid of section

\( e_b \) = eccentricity of load \( P_b \) measured from plastic centroid of section

\( f_c \) = concrete strength
\[ f_c' = \text{compression strength of 6" by 12" concrete cylinders, psi.} \]

\[ f_c'' = \text{flexural compressive strength of concrete member} \]

\[ f_y = \text{yield strength of reinforcement, psi.} \]

\[ f_{si} = \text{compressive steel stress, psi.} \]

\[ f_{sz} = \text{tensile steel stress, psi.} \]

\[ K_1 = \text{the ratio of average stress to maximum stress} \]

\[ K_d = \text{distance from extreme compression fibre to neutral axis at ultimate strength} \]

\[ L = \text{span length, ft.} \]

\[ M'_u = \text{ultimate moment, kip-in.} \]

\[ M''_u = \text{ultimate moment of column under pure bending, kip-in.} \]

\[ m = \frac{f_y}{0.85f_c'} \]

\[ P'_u = \text{ultimate load, kips.} \]

\[ P''_u = \text{ultimate lateral load of column under pure bending, kips.} \]

\[ p = \frac{A_s}{bd} \]

\[ p' = \frac{A_{s'}}{bd} \]

\[ P_b = \text{axial load capacity at simultaneous crushing of concrete and yielding of tension steel, kips.} \]

\[ (\text{balanced conditions}) \]

\[ P_o = \text{ultimate load of concentrically loaded column, kips.} \]

\[ q = \frac{A_s}{bd} \cdot \frac{f_y}{f_c'} \]
\[ t = \text{over-all depth of a rectangular section in the direction of bending} \]

\[ V = \text{coefficient of variation} \]

\[ \delta = \text{deflection} \]

\[ \varepsilon = \text{strain} \]

\[ \varepsilon_{s1}, \varepsilon_{s2} = \text{steel strains} \]

\[ \varepsilon_c = \text{compressive strain in concrete} \]

\[ \varepsilon_o = \text{compressive strain in concrete corresponding to maximum stress (Fig. 4.1.a)} \]

\[ \varepsilon_s = \text{steel strain} \]

\[ \varepsilon_x = \text{strain in the X-direction} \]

\[ \varepsilon_u = \text{useful limit of compressive strain in concrete (Fig. 4.1.a)} \]

\[ \rho = \text{radius of curvature} \]

\[ \phi = \text{angle change per unit length, curvature} \]

\[ \sigma = \text{standard deviation} \]
CHAPTER I

INTRODUCTION

The basic function of columns in most normal frameworks is to carry the vertical load down to some lower level. In ordinary construction, there are many cases in which members are subjected to a combination of bending moments and axial loads. Generally, in reinforced concrete members, the direct axial load in such a combination is a compressive force. Lateral earth pressures which act upon foundation walls, columns connected to beams eccentrically, together with the effect of the rigidity of the connections, will cause bending moments to be applied to the columns. Wind loads acting upon a building also force the columns to bend sideways. In other cases, internal columns in the framework of a symmetrical structure, are subjected to out of balance bending moments, which also cause the column to bend. All of these, are ordinary causes of combined compressive force and flexure in the members that are affected by the above conditions.

Generally, problems involving compression and bending come into one of the two classes: the first includes those members under compression failure, with the crushing of concrete before yielding of the tension steel; the second covers those under tension failure, with the tension steel yield first.
In the past forty years, the strength of reinforced concrete columns, their behaviour under various conditions of loading, and proper procedures for their design have been of intense interest among Structural and Civil Engineers.

In 1638, Galilei's work regarding flexure of beams, was exclusively devoted to ultimate strength. Hooke's Law was formulated 40 years later, and Navier developed the fundamental theorems of the theory of elasticity in 1821. By 1930, the phenomena of failure and ultimate strength of columns in axial compression were fairly well covered, and theories for ultimate strength had also been developed. Tests and theories for eccentrically loaded columns were developed sometime later.

All present knowledge regarding ultimate strength of reinforced concrete columns is based on the results of tests. Consequently, ultimate strength methods give an indication of being both accurate and useful, while the straight-line method does not produce either consistent or satisfactory results.

The investigation described in this thesis was carried out to determine the effects of concrete quality, percentage and yield strength of reinforcement, and eccentricity on the actual failure load of the reinforced concrete tied columns and to the predicted ultimate strength, while the length and the nominal dimensions of the cross-section remained unchanged. The validity of the ACI Design Formulae and the strain gradient
method were also checked.

The experimental results were analysed and compared with the theoretical results obtained from the equations of the American Concrete Institute Building Code, and the strain gradient method.
CHAPTER II

REVIEW OF LITERATURE

The subject of 'Mechanics of Materials' has been a science of empirical character from the earliest developments due to the nature of the problem involved. This is particularly the case with concrete and reinforced concrete. The first attempts to establish a mathematical design procedure for reinforced concrete members were characterised by loading to failure a number of specimens and trying to establish an equation which correlated with the test results. In current terminology, these early theories are referred to as inelastic or ultimate theories.

In 1638, the work done by Galilei, attempted to describe the behaviour of beams in flexure following ultimate load tests on masonry cantilever beams.

In 1821, Navier's theory of bending was developed, based on the earlier's work of Bernoulli and Hooke's Law.

In 1894, the Straight-Line theory, was presented by Coignet and tédesco (3)*, to explain the behavior of concrete beams and columns. The theory was mathematically simple and was accurate enough for design purpose at that time. This method assumed a linear distribution of strain and that the stresses of both the concrete and steel

* Numerals in parentheses refer to corresponding items in the List of References.
were directly proportional to strain (the concrete was assumed to have no tension capability). No account was taken of the plastic behavior of the material nor was there any specific determination of the maximum load that could be carried.

From 1920 to 1930, another important development was taking place in the philosophy of column design. Before that time, bending stresses were generally neglected in the design of concrete building columns, and such stresses together with other effects, were assumed to be allowed for in an overall factor of safety. The Slope-Deflection and the Moment-Distribution Methods developed to analyze monolithic structures, enabled the bending moments as well as the axial loads on columns in frames to be determined easily. Consequently, current design codes required columns to be designed for both the axial load and bending moment obtained from a rational analysis.

Members subjected to axial compression have been represented in tests by axially-loaded columns. Although this type of load may never encountered in actual construction, it represents a limiting case for the columns under combined axial load and bending.
About 1900, Considère (4) proposed the following equation for the allowable load of concentrically loaded reinforced concrete columns, using appropriate safety factors to determine the allowable unit stresses for concrete and steel. Strain compatibility was not considered.

\[ P = A_c f_c + A_{st} f_s \]

where \( f_c \) is allowable stress in extreme fiber of concrete, psi.

\( f_s \) is allowable tensile stress of steel, psi.

\( A_c \) and \( A_{st} \) are areas of concrete and steel respectively.

The Straight-Line Theory (Standard Theory) was established later, by using the transformed area formula, and the strain compatibility was taken into account.

\[ P = A_c f_c \left( 1 + (n - 1) \frac{p_{st}}{E_c} \right) \]

where \( n \) is the modular ratio \( (E_s/E_c) \)

\( p_{st} \) is the ratio of total longitudinal reinforcement area to the gross area of concrete.

In 1921, McMillan (5) studied column test data, and found that building columns under load may develop steel stresses, due to plastic action, considerably higher than those predicted by the Straight-Line theory. This important study led to a number of ACI column research investigations,
carried out by Slater & Lyse (66), Slater (7), and Richart (8), and others in the 1930's. From their work, rational equations for the ultimate load of concentrically loaded reinforced concrete columns were developed. The ultimate load for tied columns is simply the sum of the strengths of the longitudinal reinforcement and the concrete. The ultimate strength of tied columns were shown to be as follows:

\[ P = 0.85A f'_c + A f'_{st} \]

where \( f'_c \) is the ultimate concrete cylinder strength

\( f'_{st} \) is the yield point stress of longitudinal reinforcement.

In 1938, Richart and Olsen (11), carried out an investigation of 82 eccentrically-loaded columns, and presented an expression relating the ultimate load on an eccentrically-loaded column, to that of an axially-loaded column. In which,

\[ \frac{P_{ecc.}}{P_{axial}} = \frac{1}{1 + C \frac{e c}{k^2}} \]

where \( e \) is the eccentricity of the load with respect to the centroid of the column cross-section,

\( c \) is the distance from the centroid to the extreme fiber in compression,

\( C \) is an empirical constant less than unity,

\( k \) is the radius of gyration of the column section.
This empirical equation, based on experimental tests, was capable of predicting the ultimate strengths for failures in compression, but was not capable of predicting either the ultimate strengths for failures by yielding of the reinforcement nor if a member would fail in tension or by crushing of the concrete.

In 1937 and 1943, Whitney (12) (37) made an important contribution on eccentrically loaded columns. He presented ultimate theories for the compression and tension failures of members subjected to axial compression and bending. Whitney assumed that the non-linear concrete compressive stress distribution could be approximated to a rectangular stress block with a uniform stress intensity of 0.85f'c over some depth 'a' such that equilibrium is satisfied. The following equation for compression failures in combined bending and axial load was developed.

\[ P = \frac{2A_s f_y}{2e + 1} + \frac{bft_f c}{3te + 1.178} \]

where - \( A_s \) ' is the area of compression reinforcement,
- \( d' \) is the distance from extreme compression fibre to centroid of compression reinforcement,
- \( d \) is the distance from centroid of tension reinforcement to compression face of member,
- $b$ is the width of a rectangular member,
- $t$ is the total depth of section.

For compression failures, this equation reduces to the ultimate load for a centrically-loaded column, with $e = 0$.

In 1944, Jensen (13) made an attempt to use the inelastic properties of the stress-strain curve for concrete. He assumed a trapezoidal stress distribution and derived the properties of this trapezoid as a function of cylinder strength by analysis of the observed ultimate strength of reinforced concrete beams.

Followed by a number of extensive studies of Jensen's theory, the ultimate strength theories for eccentrically loaded columns were developed.

Many additional tests have been reported between the years 1900 to 1950, from both European and North American investigators. Among these, are included those of Von Thullie (14), 1896; Bach and Graf (15), 1913; Baumann (16), 1934; Bittner (17), 1935; Brandzaeg (18), 1935; Thomas (19), 1938; The Russian Specifications (20), 1938; Anderson (21), 1941; and Richart (22), 1947.

In 1951, Hognestad (25) published a more refined ultimate strength theory, based on more realistic, and more complex assumptions regarding the stress-strain relation of concrete, derived empirically from his tests. His investigation not only predicted the types of failure and the
ultimate strength of the columns, but also the behavior of the column throughout the entire loading range. In his paper, Hognestad also introduced the dimensionless form of interaction diagram; with the application of this diagram, one can determine the behaviour in which the strength varied between the limits of axial compression to pure flexure of the column, simply and accurately.

Both the ACI (American Concrete Institute) Building Code (ACI - 318 - 71) (23) and the NBC (National Building Code of Canada, 1965) (24) permit the ultimate strength method to be used in the design of reinforced concrete columns, using a $\phi$ factor to reduce the predicted actual strength capacity of the column, to obtain the design ultimate strength. These $\phi$ factors were intended to take into account the importance of the member and the accuracy of the predicted equation and allow for any errors in construction techniques. The ACI code specifies $\phi = 0.7$ for tied columns and $\phi = 0.75$ for spirally hooped columns; while the NBC code specifies $\phi = 0.75$ for both.
CHAPTER III

EXPERIMENTAL INVESTIGATION

3.1 Object and Scope of Investigation

The object of this investigation was to experimentally study the basic behavior of reinforced concrete short columns subject to combined bending and axial load.

The major variables studied in this program were the percentage of reinforcement, steel yield stress, concrete strength and eccentricity of the applied loading, while the length and nominal dimensions of the columns remained unchanged.

Details of the nominal dimension of the reinforced concrete columns is given in Figure (3.2.a).

3.2 Material

3.2.1 Concrete Mixtures

The concrete used for all specimens was supplied by Dominion Building Materials Ltd. The concrete was transit mixed during the transportation from the batch plant to the laboratory of the University of Ottawa. Two design compression strengths, of 4000 psi and 6000 psi at 28 days, were used in the project as being typical of concrete strengths presently used in construction. The mix proportions of the concrete are shown in Table (3.2.1.a) and Table (3.2.1.b) below. The grading curves of coarse and fine aggregates are shown in Fig. (3.2.1).
TABLE (3.2.1.a) Mix Proportion by Weight with Compression Strength of 4000 psi. (Slump = 3")

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>WEIGHT PER CUBIC YARD OF CONCRETE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Portland Cement</td>
<td>500 lbs.</td>
</tr>
<tr>
<td>Sand</td>
<td>1,530 lbs.</td>
</tr>
<tr>
<td>3/4&quot; Coarse Aggregate</td>
<td>1,910 lbs.</td>
</tr>
<tr>
<td>Water</td>
<td>294 lbs.</td>
</tr>
</tbody>
</table>

W/C = 294/500 = 0.59

TABLE (3.2.1.b) Mix Proportion by Weight with Compression Strength of 6000 psi. (Slump = 3")

<table>
<thead>
<tr>
<th>MATERIAL</th>
<th>WEIGHT PER CUBIC YARD OF CONCRETE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Standard Portland Cement</td>
<td>700 lbs.</td>
</tr>
<tr>
<td>Sand</td>
<td>1,140 lbs.</td>
</tr>
<tr>
<td>3/4&quot; Coarse Aggregate</td>
<td>1,960 lbs.</td>
</tr>
<tr>
<td>Water</td>
<td>286 lbs.</td>
</tr>
</tbody>
</table>

W/C = 286/700 = 0.41

FIGURE (3.2.1) GRADING CURVES OF COARSE AGGREGATE & SAND
3.2.2 Reinforcing Steel

Three sizes of deformed bars of Nos. 7, 6, and 4, and one size of plain bar of No. 3, were used in the tests. The ties for all the specimens were manufactured by hand in the laboratory from #2, 1/4 in. plain bars. Average properties of the various lots of reinforcement as determined from tension tests in the laboratory are given in Table (3.2.2.a), and used for the calculations in both the ACI Design Formulae and the Strain Gradient Method.

All longitudinal reinforcement was specified to be Grade 60 steel.

TABLE (3.2.2.a)

AVERAGE TENSILE PROPERTIES OF REINFORCING STEEL

<table>
<thead>
<tr>
<th>BAR SIZE (in.)</th>
<th>NO. OF TESTS</th>
<th>AVERAGE YIELD POINT (ksi.)</th>
<th>AVERAGE ULTIMATE STRENGTH (ksi.)</th>
<th>USED IN SERIES NO.</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MEAN</td>
<td>C.of V.</td>
<td></td>
</tr>
<tr>
<td>3/8</td>
<td>25</td>
<td>54.40</td>
<td>1.6%</td>
<td>76.00</td>
</tr>
<tr>
<td>4/8</td>
<td>25</td>
<td>65.50</td>
<td>2.9%</td>
<td>92.80</td>
</tr>
<tr>
<td>6/8</td>
<td>25</td>
<td>52.60</td>
<td>1.4%</td>
<td>110.00</td>
</tr>
<tr>
<td>7/8</td>
<td>25</td>
<td>67.40</td>
<td>0.5%</td>
<td>112.10</td>
</tr>
</tbody>
</table>

* The yield point is defined by a residual strain of 0.2% instead of 0.5% of the total strain under load; the yielding stress was found to be very close in both cases.

Typical stress-strain curves for various size of reinforcement are shown in Appendix A, from Fig. (A.1) page 107 to Fig. (A.4) page 110.
3.3 Types of Specimens

The 100 test specimens was divided into four groups, all of them being tied columns. Since the investigation was confined to combined axial and bending problems, the specimens were kept fairly short, \( \frac{L}{r} = 30 \) so that the results would not be affected by the occurrence of buckling failures. An outline of the test program is given in Table (3.3.a), which indicates that the major variables were: size and yield stress of reinforcement, concrete strength and the eccentricity of applied loading.

Columns were completely identified by a numeral, a capital letter and followed by another numeral. The first numeral -- 1 through 4 -- indicates the size of the reinforcement; the capital letter -- A, B, C, D, DE or E -- indicates the eccentricity of load; the final numeral -- 1 to 5 -- indicates one of five companion specimens.

For example:

1B - 1 is a column in group one, with 7/8 in. longitudinal reinforcement, loaded with an eccentricity of \( e/t = 0.1 \), and the first of this group of specimens.

The four groups of specimens with the different sizes of longitudinal reinforcement are shown in Fig. (3.2.a). Concrete cylinders, 6 by 12 in. were made as control specimens.
<table>
<thead>
<tr>
<th>GROUP NO.</th>
<th>ECCENTRICITY e/t</th>
<th>SIZE OF REINFORCEMENT (in.)</th>
<th>CONCRETE STRENGTH (psi.)</th>
<th>$\frac{f_y}{0.85f_c}$</th>
<th>TOTAL NO. OF COLUMNS</th>
<th>TESTED*</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>A (0.0)</td>
<td>7/8</td>
<td>4000</td>
<td>1.26</td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>B (0.1)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>C (0.3)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>D (0.5)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>E ($\infty$)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>H</td>
</tr>
<tr>
<td>2</td>
<td>B (0.1)</td>
<td>6/8</td>
<td>6000</td>
<td>0.404</td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>C (0.3)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>D (0.5)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>DE (0.75)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>E ($\infty$)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>H</td>
</tr>
<tr>
<td>3</td>
<td>A (0.0)</td>
<td>4/8</td>
<td>4000</td>
<td>0.479</td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>B (0.1)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>C (0.3)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>D (0.5)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>E ($\infty$)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>H</td>
</tr>
<tr>
<td>4</td>
<td>B (0.1)</td>
<td>3/8</td>
<td>6000</td>
<td>0.097</td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>C (0.3)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>D (0.5)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>H</td>
</tr>
<tr>
<td></td>
<td>DE (0.75)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>V</td>
</tr>
<tr>
<td></td>
<td>E ($\infty$)</td>
<td></td>
<td></td>
<td></td>
<td>5</td>
<td>H</td>
</tr>
</tbody>
</table>

* H = Horizontally tested specimens  
V = Vertically tested specimens
SECTION A—A

FIG. (3.2.a) DETAIL OF COLUMN SPECIMENS
3.4 Fabrication and Curing

The concrete was mixed and supplied by the Dominion Building Material Limited, and the specimens were cast in the structures laboratory of the University of Ottawa throughout the tests. The total time from mixing of concrete to the final placing and compacting operations, was kept within one hour and thirty minutes.

The reinforcement was assembled into a cage before it was placed in the forms for casting. The longitudinal bars were placed inside the ties and welded to them. Spacing blocks were used to provide accurate spacing and protecting coverings of the longitudinal reinforcement. Wooden blocks were inserted in the center portion of the steel forms before casting. Steel forms, as shown in Fig. (3.4.a), were used for each casting for the entire project.
SECTION A—A

FIG.(3.4.a) TYPICAL DRAWING FOR STEEL FORM
3.5 Testing Methods and Procedure

All five test specimens of a group of reinforced concrete columns and the twenty associated 6 by 12 in. concrete cylinders, were cast at the same time, and were tested within a 24 hour period, at the age of 28 days. The columns were subjected to short term loading, the time between the first and last increments of the applied load was approximately 30 minutes.

3.5.1 Reinforced Concrete Columns

3.5.1.a Columns under Combined Axial and Eccentric Loadings

A total of 75 column specimens were assigned to be tested under axial and eccentric loadings, with eccentricities e/t equal to 0.00±0.01; 0.10±0.01; 0.30±0.01; 0.50±0.01 and 0.75±0.01 etc.; of which 25 were tested horizontally in a special testing frame, with a 100 kips capacity. The other 50 specimens were tested vertically in a 600,000-lb. capacity Baldwin Southwark Tate-Emery hydraulic testing machine. A view of both the test set ups during a column test are shown in Fig. (3.5.1.a) and (b), and a sketch of the testing equipment for each is given in Fig. (3.5.2.a) and (b), respectively. The test method is indicated by an horizontal and vertical tested specimen, respectively in Figs. (3.5.2.a) and (b).

The columns were cast in a horizontal position in order to avoid a differential in concrete quality along the column length, but, on the other hand, this would cause a strength differential through the cross-section of the column.
FIG. (3.5.1) GENERAL VIEW OF A TESTED SPECIMEN UNDER ECCENTRIC LOADING
FIG. (3.5.1) GENERAL VIEW OF A TESTED SPECIMEN UNDER ECCENTRIC LOADING
FIG. (3.5.2.b) ARRANGEMENT OF TESTING
Five or ten test specimens were cast at one casting, and the concrete was compacted in the forms by means of a small electrically driven pencil vibrator.

The day after the columns were cast, the forms were removed, and the columns and the control cylinders (20 for each casting), were cured under wet sacks in the laboratory for 28 days before testing.

The method and procedure used with both testing apparatuses were the same. In order to avoid the crushing of the concrete at both ends of the specimen, two 1" thick steel bearing plates were used, one at each end face of the specimen.

For the specimens tested horizontally in the testing frame, a steel cap was used for both the ram and the load cell. To obtain single axis hinged ends at both ends of the specimen, two 1" diameter steel rollers were put in between the end bearing plates and the steel caps, both of which were provided with semi-circular grooves to keep the steel rollers in the desired positions.

For the specimens tested vertically in the 600,000-lb. Baldwin Southwark Tate-Emery hydraulic machine, two 1\(\frac{1}{2}\)" steel bearing plates, screwed to the movable head and the bed of the machine respectively, were provided with semi-circular grooves, to keep the 1" roller in place. Two - 12" x 6" x 1/2" steel plates were clamped to both capitals of the columns by 4 - 1/2" thread steel bars (which can be
seen in Fig. (3.5.2.b). These steel bars were placed on each capital of the column and prestressed before the test, by tightening the nuts, to reduce the possibility of diagonal tension failure in the capitals, because it was found impracticable to reinforce the capitals sufficiently with only embedded bar reinforcement.

The lateral deflection of the column was measured with three 0.001 in. dial indicators; one located at the midspan of the specimen, and the other two at 6" to the left and right of the midspan of the specimen respectively, in the plane of eccentricity.

All columns were carefully centered and levelled or plumbed in the testing machine. Initial readings of all gages were taken immediately before application of any loading. Loading proceeded to failure in 15 to 20 increments for every specimen.

3.5.1.b Columns under Pure Flexure

Twenty five specimens were tested under flexure, ie, with the eccentricity e/t equal to infinity. The method of testing was as for a simply-supported beam; loaded at two points located symmetrically about midspan. A view of the testing frame during the test of a specimen is shown in Fig. (3.5.3.a) and a sketch of the testing equipment is given in Fig. (3.5.3.b).
FIG.(3.5.3.a) GENERAL VIEW OF TESTED SPECIMEN UNDER PURE BENDING

FIG.(3.5.4.a) 6" BY 12" CONCRETE CYLINDER UNDER TEST
FIG. (3.5.3.a)  GENERAL VIEW OF TESTED SPECIMEN UNDER PURE BENDING

FIG. (3.5.4.a)  6" BY 12" CONCRETE CYLINDER UNDER TEST
The steel loading beam, located under the ram, was supported symmetrically by two load transfer blocks, both with 1" thick steel bearing plates resting directly on the upper surface of the specimen. Two 1" diameter steel rollers were put in between the steel blocks and the bearing plates. The load was applied by the ram, and transferred to the test specimen through the steel beam. The concrete beam-column was supported by two load cells each with a capacity of 50 kips, 6" from each end of the column. The load was measured from the reactions of the load cells.

The lateral deflections of the specimen, and the increments of the applied loading, were measured and applied the same way as for a column tested under combined axial and bending.

3.5.2 Concrete Cylinders

The compression strength of the concrete cylinders were tested on the same day as the specimen and thus gave the actual concrete cylinder compression strength at that age. A total of 20 test cylinders were cast at the same time as the corresponding columns, hence no differential time effects would be considered between the testing specimens and the cylinders.

Both end faces of the concrete cylinder were capped with standard sulfur capping compound and tested to failure in a 300,000-lb. capacity Forney hydraulic concrete testing machine. A view of a 6" by 12" concrete cylinder under test is shown in Fig. (3.5.4.a).
CHAPTER IV
THEORETICAL INVESTIGATION

4.1 Ultimate Strength Analysis of Reinforced Concrete

Columns as for ACI 318 - 56 or NBC 1965 or
CSA - A23.3 - 1970

The ultimate strength analysis of reinforced concrete
member presented herein is based on the following assumptions:

(1) A linear distribution of strain over the cross
section is assumed at all load levels.

(2) The stress-strain relationship assumed for concrete
in compression used in the strain-gradient method
is that derived by Hognestad as shown in Fig. (4.1.a).

\[ f_c = f_c'' \left[ \frac{2\varepsilon}{\varepsilon_0} - \left( \frac{\varepsilon}{\varepsilon_0} \right)^2 \right] \]

\[ \Delta f_c'' = 0.15 f_c'' \]

\[ \varepsilon_0 = \frac{2f_c''}{E_c} \]

\[ \varepsilon_u = 0.003 \]

FIG. (4.1.a) ASSUMED STRESS-STRAIN RELATIONSHIPS
FOR CONCRETE IN COMPRESSION
The initial modulus of elasticity, \( E_c \) is assumed in accord with Inge Lyse's Equation.

\[
E_c = 1,800,000 + 460f'_c \\
\]

\((4.1.1)\)

(3) It is assumed in the present analysis that no tension stresses can be taken by the concrete.

(4) It is assumed that no general slip occurs between concrete and reinforcing steel, although local slip must be present at tension cracks in the concrete.

(5) In the ACI Design Formulae, the maximum strain at the extreme compression fiber at ultimate strength shall be assumed equal to 0.003.

(6) At ultimate strength the concrete stress is not proportional to strain. The variation of compressive concrete stress can be assumed to be uniform at an intensity of \( 0.85 f'_c \) over an equivalent compression zone bounded by the edge of the cross section and a straight line located parallel to the neutral axis at a distance \( a = K_i c \) from the fiber of maximum compressive strain. The distance "c" from the fiber of maximum strain to the neutral axis is measured in a direction perpendicular to that axis. The fraction "K_i" shall be taken as 0.85 for \( f'_c \leq 4000 \text{ psi} \), and decreased by 0.05 for every 1000 psi. above 4000.
In the above assumptions, Nos. 1, 3 and 4 are common for both the ACI and strain gradient method.

All the theoretical calculations presented use the final eccentricity of the columns taken from the experimental results.

4.1.1 Reinforced Concrete Columns under Balanced Failure

The balanced condition is the loading condition which produces at ultimate strength, simultaneously, a strain of 0.003 in the extreme fiber of concrete and yield strain in the tension steel, and assume that the compression steel reaches its yield point stress, from the strain relationship is shown in Fig. (4.1.1.a):-
FIG. (4.1.1.a) BALANCED CONDITION—RECTANGULAR SECTION
\[ c_b = \frac{\epsilon_u}{f_y / E_s + 0.003} \]
\[ c_b = \frac{0.003}{f_y / (29 \times 10^6) + 0.003} \cdot d \]
\[ = \frac{87,000d}{f_y + 87,000} \] .......................... (4.1.1.a)

\[ C_c = 0.85f_c 'k_c b c_b \]
\[ = 0.85f_c 'a_b b \] .......................... (4.1.1.b)

\[ C_s = A_s ' (f_y - 0.85f_c ') \] .......................... (4.1.1.c)

\[ T = A_s f_y \] .......................... (4.1.1.d)

Force equilibrium requires.

\[ P_b = C_c + C_s - T \]
\[ = 0.85f_c 'k_c b c_b + A_s ' (f_y - 0.85f_c ') - A_s f_y \] .......................... (4.1.1.e)

Taking moments about the plastic centroid,

\[ P_e b_c = C_c (d - a_b/2 - d'') + C_s (d - d' - d'') + Td'' \]

\[ P_e b_c = M_b = 0.85f_c 'k_c b \cdot b \cdot (d - 0.5k_c b - d'') + A_s ' (f_y - 0.85f_c ') \cdot (d - d' - d'') + (A_s f_y) d'' \] .......................... (4.1.1.f)
4.1.2 Reinforced Concrete Columns under Compression Failure

When the ultimate eccentric load $P'_u$ exceeds the balanced value $P_b$ or when the eccentricity $e$ is less than the balanced values $e_b$, the capacity of the section is controlled by compression failure of the concrete. In this case, the tensile force $T$ will be based on a stress less than the yield strength and may actually be compressive.

Using the principles of statics, considering the actual strain variation as the unknown, the ultimate eccentric load can be found. However, two approximate procedures are provided in the ACI Code to simplify the calculations.

In one approach, $P'_u$ is assumed to decrease linearly from $P_o$ to $P_b$ as the moment increases from zero to $M_b = P_b \cdot e_b$, as shown in an interaction diagram of Fig. (4.1.2.a).

$$\frac{P_o - P'_u}{P_o - P_b} = \frac{P'_u}{P_b} \cdot \frac{e}{e_b} \quad \text{...............(4.1.2.a)}$$

Solving Equation (4.1.2.a) for $P'_u$ gives,

$$P'_u = \frac{P_o}{1 + \left(\frac{P_o}{P_b} - 1\right) \frac{e}{e_b}} \quad \text{...............(4.1.2.b)}$$

or

$$P'_u = P_o - \left(\frac{P_o}{P_b}ight) \frac{M'_u}{M_b} \quad \text{...............(4.1.2.c)}$$
FIG. (4.1.2a) INTERACTION DIAGRAM, COMPRESSION PLUS BENDING, ULTIMATE STRENGTH $P_o$ AND $M_{uo}$
A second approximate procedure may be applied for symmetrical reinforcement in terms of the column dimensions. Considering the moment equilibrium with respect to the tension steel, in Fig. (4.1.2.b) gives,

\[
P'_u \left( e + \frac{d - d'}{2} \right) = C_c \left( d - \frac{a}{2} \right) + C_s \left( d - d' \right)
\]

\[\text{..........................(4.1.2.a)}\]

where

\[
C_c = 0.85 f'c b a \quad \text{.........(4.1.2.b)}
\]

When compression controls, compression steel yields at ultimate strength, thus

\[
C_s = A_s' f_y \quad \text{..........................(4.1.2.c)}
\]

Substituting Equations (4.1.2.b) and (4.1.2.c) in Equation (4.1.2.a) gives,

\[
P'_u = \frac{f'c b t}{3te \frac{d^2}{d^2}} + \frac{A_s' f_y}{3 \left( \frac{d-d'}{2} \right) t} + \frac{A_s' f_y}{\left( \frac{e}{d-d'} + \frac{1}{2} \right)}
\]

\[\text{..........................(4.1.2.d)}\]

With the boundary condition of \(P'_u = P_o\) and \(e = 0\), then

\[
P_o = 0.85 f'c b t + 2 A_s' f_y \quad \text{..........(4.1.2.e)}
\]
FIG. (4.1.2.b) ULTIMATE CAPACITY CONTROLLED
BY COMPRESSION—RECTANGULAR SECTION
Substituting the above boundary condition in Equation (4.1.2.d), and neglecting the displacement of concrete by compression steel, it becomes,

\[ P_u' = \frac{b t f'_c}{\left( \frac{3\epsilon e}{d^2} + 1.18 \right)} + \frac{A_s' f_y}{\left( \frac{e}{d - d'} + 0.5 \right)} \]

.................................(4.1.2.f)

It is to be observed that the direct use of statics gives probably the more accurate answer; and the result will be the same if the reduction for concrete displaced by compression steel is being applied. The first approach of straight line interaction gives the most conservative value.

4.1.3 Reinforced Concrete Columns under Tension Failure

When the ultimate capacity \( P_u' \) is less than the balanced value \( P_b \) or the eccentricity \( e \) is greater than the balanced value \( e_b \), the capacity of the section is controlled by tension in the steel. A tension failure is initiated by yielding of the tension steel, this produces a movement of the neutral axis towards the compression zone. In this situation the ultimate strain in the tension steel will be greater than the yield strain. Referring to Fig. (4.1.3.a) and assuming that the strain in the compression steel is greater than the yield strain, one can obtain,
FIG. (4.1.3.a) ULTIMATE CAPACITY CONTROLLED BY TENSION — RECTANGULAR SECTION
\[ T = A_s f_y \]
\[ C_s = A_s' (f_y - 0.85f_c') \]
\[ C_c = 0.85f_c' K, cb \]

From force equilibrium,
\[ P'_u = 0.85f_c' K, cb + A_s' (f_y - 0.85f_c') - A_s f_y \quad \ldots \ldots (4.1.3.a) \]

Let
\[ p = \frac{A_s}{bd} \]
\[ p' = \frac{A_s'}{bd} \quad \ldots \ldots \ldots \ldots (4.1.3.b) \]
\[ m = \frac{f_y}{0.85f_c'} \]

From moment equilibrium with respect to the tension steel,
\[ P'_u = 0.85f_c' (K, c) b \left( d - \frac{K_c}{2} \right) + A_s' (f_y - 0.85f_c') (d - d') \]
\[ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots (4.1.3.c) \]
Substituting Equations (4.1.3.a) and (4.1.3.b) in Equation (4.1.3.c), and rearrange,

\[ p'_{u} = 0.85f_{c}'bd \left\{ p'(m - 1) - pm + (1 - e'/d) \right. \]

\[ + \sqrt{1 - e'/d}^2 + 2 \left\{ (e'/d)(pm - p'm + p') + p'(m - 1)(1 - d'/d) \right\} \]

\[ ...........................................(4.1.3.d) \]

In our case, \( A_s = A_{s'} \) and \( p = p' \), Equation (4.1.3.d) reduced to

\[ p'_{u} = 0.85f_{c}'bd \left\{ - p + 1 - e'/d + \sqrt{(1 - e'/d)^2 + 2p [(m - 1)(1 - d'/d) + e'/d]} \right\} \]

\[ ...........................................(4.1.3.e) \]

4.1.4 Reinforced Concrete Beams with Equal Tension and Compression Steel

When eccentricity 'e' becomes very large so that bending is predominant, the effect of the direct compression caused by \( P_u \) is small. The member acts as a beam column and can be analyzed as a flexural member when the tensile steel limits the strength. When \( e \) becomes infinite, it is approximately the case of pure moment resistance, with the moment capacity \( M_u \) in simple bending, and \( P_u = 0 \).

From force equilibrium as shown in Fig. (4.1.4.a) requires that
$\varepsilon_c = (0.001 - 0.003)$

\[ A_s f_y = A_s f_s + 0.85 f_c ' b a \] \hspace{0.5cm} (4.1.4.a)

where

\[ a = K_i \cdot c \]

From strain diagram

\[ \varepsilon_s = \frac{\varepsilon_c \times (c - d')}{c} \] \hspace{0.5cm} (4.1.4.b)

where $\varepsilon_c$ varied from 0.001 to 0.003

From stress-strain relationship of reinforcement, and a assigned value of $\varepsilon_c$ (varied from 0.001 to 0.003), $f_s$ can be determined.
Using Equations (4.1.4.a) and (b), and considering force equilibrium, c can be obtained by trial and error.

From moment equilibrium about the centroid of the tension steel, the moment is found to be:

\[ M_u \tau = A' \sigma \tau_s (d - d') + 0.85 f'_c x b x K_c c (d - K_c c/2) \]

\[ \ldots \ldots \ldots \ldots (4.1.4.c) \]

The maximum load capacity of a structural member is attained when a mechanism forms. A two-point loaded beam column collapses by forming a plastic hinge in the centre of the span, as shown in the Fig. (4.1.4.b).

Using either the equilibrium method or the mechanism method, the ultimate load \( P_u \tau \) can be found,

\[ P_u \tau = \frac{6M_u \tau}{L} \]

\[ \ldots \ldots \ldots \ldots (4.1.4.h) \]
4.2 Strain Gradient Method

For cross sections consisting of linear materials, it is generally possible, and usually most convenient to write a single continuous expression relating axial load, moment and curvature. Unfortunately, the response of reinforced concrete to external effects is very difficult to evaluate, due to the nonlinear behavior of the concrete. Instead, it is more convenient to define the relationship between $P$, $M$ and $\phi$ by a number of discrete points—The strain-gradient method.
The determination of the relation between \( P \), \( M \) and \( \phi \) evolves directly from the application of ordinary principles of mechanics of materials. If a linear distribution of strain across the section is assumed and the stress-strain relationships known or assumed for both concrete and steel, the elemental force on each element of material can be determined. By algebraically summing these elemental forces, the external moment and force necessary are found.

The method assumes a linear strain distribution over the depth of the section is as shown in Fig. (4.2). A generalized stress-strain characteristic for concrete, was derived by Hognestad as given in Sec. 4.1. The characteristics for the steel were taken directly from the experimental results. From the stress-strain relationships for the concrete and the reinforcement, the distribution of stresses over the cross-section can be determined as shown in Fig. (4.2.c) and (d).

From these stresses, the external force and moment necessary to keep the member in equilibrium is found. For instance, the external moment is the algebraic sum of the external applied moment plus the axial load deformation moment. The moment curvature relationship will be obtained by varying the strain gradient and keeping the axial load equal to the desired value by moving the position of the neutral axis.

It is obvious that, the moment capacity which is available to carry external applied moment is equal to the member moment capacity less the moment capacity required by the axial load-lateral deformation moment.
Strain-Gradient Method:

(a) CROSS-SECTION

(b) STRAIN DISTRIBUTION

(c) STRESS DISTRIBUTION IN THE CONCRETE

(d) FORCE IN THE REINFORCEMENT

FIG. (4.2.) METHOD OF ANALYSIS
The method is iterative, it is convenient to list the steps as follows:

(1) Specify an axial load.

(2) Select a strain gradient.

(3) Select the position of the neutral axis.

(4) Determine the curvature by computing the angle between the line representing the strain distribution and the assumed position of the neutral axis.

(5) Sum algebraically the stress times the area over which they act to obtain the calculated axial load under equilibrium. If the calculated axial load and the chosen load agree with 1% — go to next step. If not, repeat step (3).

(6) Sum the bending moment caused by the normal stress to obtain the resulting bending moment.

(7) Increase strain gradient and obtain the maximum external moment until the maximum strain in the concrete reaches 0.003.

(8) Write out the calculated results of maximum axial load and maximum external moment for that curvature.

(9) Assign a new value of axial load until all experimental results have been considered.
If a sufficient number of discrete points relating
P, M and Φ are determined, the load-moment-curvature rela-
tionship for the section will be described.

4.2.1 Reinforced Concrete Columns under Balanced Failure

As shown in Fig. (4.2.1.1), the balance point 'b' is
a common point to both the failure and yield interaction
diagram, simultaneously, a strain of 0.003 in the extreme
fibre of concrete and the yield strain in the tension steel.
From the stress-strain diagram in Fig. (4.2.1.2.a) gives,

\[
T = A_s f_{s1}
\]

\[
C_c = 0.85 f_c 'ab'
\]

See notation

\[
C_s = A_s f_{s2}
\]

where

\[
a = K_1 \cdot c
\]

\[
P_u' = C_c - T + C_s
\]

(4.2.1.1)

\[
M_u' = C_c (t/2 - a/2) + T (t/2 - d') + C_s (t/2 - d')
\]

(4.2.1.2)

Solve Equations (4.2.1.1) and (4.2.1.2) by trial and
error method.
FIG. (4.2.1.1) INTERACTION DIAGRAM
FIG. (4.2.1.2.a) POINT b
(FIG. 4.2.1.1.)

FIG. (4.2.1.2.b) LINE b—c
FIG. (4.2.1.2.c) LINE c—d

FIG. (4.2.1.2.d) LINE a—b

FIG. (4.2.1.2) STRESS—STRAIN RELATIONSHIP
4.2.2 Reinforced Concrete Columns under Compression Failure

For moderate eccentricities, when compression governs, the behavior of the column is represented by the line b-c in the interaction diagram in Fig. (4.2.1.1).

From the stress-strain diagram in Fig. (4.2.1.2.b) the equilibrium equation applies

\[ P' = C_c - T + C_s \] \hspace{2cm} (4.2.1.3)

\[ M' = C_c (t/2 - a/2) + C_s (t/2 - d') + T (t/2 - d') \] \hspace{2cm} (4.2.1.4)

For zero or small eccentricities, as the interval line d-c, in the interaction diagram, and considering equilibrium as in Fig. (4.2.1.2.c), gives

\[ P' = C_c + C_s + T \] \hspace{2cm} (4.2.1.5)

\[ M' = C_c (t/2 - c/2) - T (t/2 - d') + C_s (t/2 - d') \] \hspace{2cm} (4.2.1.6)
4.2.3 Reinforced Concrete Beam-columns under Tension Failure

For large eccentricities, when tension governs, as represented by the curve line a-b in the interaction diagram, and from the Fig. (4.2.1.2, d), we have,

\[ P_u' = C_c + C_s - T \] \hspace{1cm} (4.2.1.7)

\[ M_u' = C_c \left( \frac{t}{2} - \frac{a}{2} \right) + T \left( \frac{t}{2} - d' \right) + C_s \left( \frac{t}{2} - d' \right) \] \hspace{1cm} (4.2.1.8)

When eccentricity become infinity, as the point a in the interaction diagram, we have,

\[ T = C_c + C_s \] \hspace{1cm} (4.2.1.9)

\[ M_u = C_c \left( d - \frac{a}{2} \right) + C_s \left( d - d' \right) \] \hspace{1cm} (4.2.1.10)

and

\[ \frac{T}{P_u} = \frac{6M_u}{L} \]

where \( L = \) span length (53\text{\textquotedblmi})
CHAPTER V
DISCUSSION AND SUMMARY OF TEST RESULTS

5.1 General Behaviour and Modes of Failure of Specimens

All specimens with the eccentricity equal to 0.00±0.01; 0.10±0.01; 0.30±0.01; 0.50±0.01 and 0.75±0.01, failed in two modes: compression failure and tension failure. Only a few columns had the combination of the variables that the mode of failure was close enough to the balanced conditions.

All those specimens, which were designed as beam-columns, with the eccentricity equal to infinity failed by tension failure.

The observed modes of failure of the individual test columns are listed in Table (5.1), page 53, and the various typical failure phenomena are shown in Figs. (5.1.1) to (5.1.4), on page 56 to page 59. The load-deflection curves for each member are presented in Appendix B, from Fig. (B.1) page 112 to Fig. (B.85) page 196, the maximum load in each curve indicates the failure load for the member.
FIG. (5.1) TEST RESULT OF GROUP I, II, III, & IV

<table>
<thead>
<tr>
<th>SPECIMEN NO.</th>
<th>$f'_c$ (psi.)</th>
<th>REINFORCEMENT TYPE</th>
<th>$f_y$ (psi.)</th>
<th>$e/t$</th>
<th>$P_{exp.}$ (Kips)</th>
<th>MODE OF FAILURE</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A-1</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1A-2</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1A-3</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1A-4</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1A-5</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.0</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1B-1</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1B-2</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1B-3</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1B-4</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1B-5</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1C-1</td>
<td>3,500 c.of v. (3.5%)</td>
<td>Deformed</td>
<td>67,400</td>
<td>0.3</td>
<td>112.00</td>
<td>C</td>
</tr>
<tr>
<td>1C-2</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.3</td>
<td>110.00</td>
<td>C</td>
</tr>
<tr>
<td>1C-3</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.3</td>
<td>110.50</td>
<td>C</td>
</tr>
<tr>
<td>1C-4</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.3</td>
<td>112.12</td>
<td>C</td>
</tr>
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<td>1C-5</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.3</td>
<td>115.00</td>
<td>C</td>
</tr>
<tr>
<td>1D-1</td>
<td>3,500 c.of v. (3.5%)</td>
<td>Deformed</td>
<td>67,400</td>
<td>0.5</td>
<td>74.50</td>
<td>C</td>
</tr>
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<td></td>
<td>Deformed</td>
<td></td>
<td>0.5</td>
<td>72.50</td>
<td>C</td>
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<tr>
<td>1D-3</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.5</td>
<td>68.00</td>
<td>C</td>
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<td>1D-4</td>
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<td>Deformed</td>
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<td>0.5</td>
<td>69.00</td>
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</tr>
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<td>Deformed</td>
<td></td>
<td>0.5</td>
<td>70.00</td>
<td>C</td>
</tr>
<tr>
<td>1E-1</td>
<td>4,300 c.of v. (7.5%)</td>
<td>Deformed</td>
<td>67,400</td>
<td>$\infty$</td>
<td>25.00</td>
<td>T</td>
</tr>
<tr>
<td>1E-2</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>$\infty$</td>
<td>24.94</td>
<td>T</td>
</tr>
<tr>
<td>1E-3</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>$\infty$</td>
<td>25.00</td>
<td>T</td>
</tr>
<tr>
<td>1E-4</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>$\infty$</td>
<td>22.00</td>
<td>T</td>
</tr>
<tr>
<td>1E-5</td>
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<td>Deformed</td>
<td></td>
<td>$\infty$</td>
<td>24.98</td>
<td>T</td>
</tr>
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<td></td>
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</tr>
<tr>
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<td></td>
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<td></td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2B-3</td>
<td></td>
<td></td>
<td></td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2B-4</td>
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<td></td>
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</tr>
<tr>
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<td></td>
<td>0.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2C-1</td>
<td>4,600 c.of v. (5.0%)</td>
<td>Deformed</td>
<td>52,600</td>
<td>0.3</td>
<td>100.00</td>
<td>C</td>
</tr>
<tr>
<td>2C-2</td>
<td></td>
<td>Deformed</td>
<td></td>
<td>0.3</td>
<td>104.00</td>
<td>C</td>
</tr>
<tr>
<td>2C-3</td>
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<td>0.3</td>
<td>112.50</td>
<td>C</td>
</tr>
<tr>
<td>2C-4</td>
<td></td>
<td>Deformed</td>
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<td>105.50</td>
<td>C</td>
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<tr>
<td>2C-5</td>
<td></td>
<td>Deformed</td>
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<td>106.50</td>
<td>C</td>
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Continued
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<tr>
<th>SPECIMEN NO.</th>
<th>$\frac{f'_c}{f_y}$ (psi.)</th>
<th>REINFORCEMENT TYPE</th>
<th>$\frac{f'_c}{f_y}$ (psi.)</th>
<th>e/t</th>
<th>$P_{exp.}$ (Kips)</th>
<th>MODE OF FAILURE</th>
</tr>
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<tr>
<td>2D-1</td>
<td>4,000 (5.1%)</td>
<td>Deformed</td>
<td>52,600</td>
<td>0.5</td>
<td>59.85</td>
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</tr>
<tr>
<td>2D-2</td>
<td>4,000 (5.1%)</td>
<td>Deformed</td>
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<td>0.5</td>
<td>61.77</td>
<td>C</td>
</tr>
<tr>
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<td>Deformed</td>
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<td>0.5</td>
<td>57.43</td>
<td>C</td>
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<tr>
<td>2D-4</td>
<td>4,000 (5.1%)</td>
<td>Deformed</td>
<td>52,600</td>
<td>0.5</td>
<td>61.90</td>
<td>C</td>
</tr>
<tr>
<td>2D-5</td>
<td>4,000 (5.1%)</td>
<td>Deformed</td>
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<td>0.5</td>
<td>61.10</td>
<td>C</td>
</tr>
<tr>
<td>2DE-1</td>
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<td>Deformed</td>
<td>52,600</td>
<td>0.75</td>
<td>45.10</td>
<td>T</td>
</tr>
<tr>
<td>2DE-2</td>
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<td>Deformed</td>
<td>52,600</td>
<td>0.75</td>
<td>47.70</td>
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<td>Deformed</td>
<td>52,600</td>
<td>0.75</td>
<td>45.45</td>
<td>T</td>
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<td>4,600 (5.0%)</td>
<td>Deformed</td>
<td>52,600</td>
<td>0.75</td>
<td>45.50</td>
<td>T</td>
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<td>4,600 (5.0%)</td>
<td>Deformed</td>
<td>52,600</td>
<td>0.75</td>
<td>43.50</td>
<td>T</td>
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<tr>
<td>2E-1</td>
<td>4,000 (5.1%)</td>
<td>Deformed</td>
<td>52,600</td>
<td>∞</td>
<td>18.00</td>
<td>T</td>
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<tr>
<td>2E-2</td>
<td>4,000 (5.1%)</td>
<td>Deformed</td>
<td>52,600</td>
<td>∞</td>
<td>19.60</td>
<td>T</td>
</tr>
<tr>
<td>2E-3</td>
<td>4,000 (5.1%)</td>
<td>Deformed</td>
<td>52,600</td>
<td>∞</td>
<td>19.96</td>
<td>T</td>
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<td>Deformed</td>
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<td>∞</td>
<td>19.30</td>
<td>T</td>
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<tr>
<td>2E-5</td>
<td>4,000 (5.1%)</td>
<td>Deformed</td>
<td>52,600</td>
<td>∞</td>
<td>19.70</td>
<td>T</td>
</tr>
<tr>
<td>3A-1</td>
<td>4,000 (4.6%)</td>
<td>Deformed</td>
<td>65,500</td>
<td>0.0</td>
<td>165.00</td>
<td>C</td>
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<tr>
<td>3A-2</td>
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<td>Deformed</td>
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<td>0.0</td>
<td>156.50</td>
<td>C</td>
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<tr>
<td>3A-3</td>
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<td>Deformed</td>
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<td>0.0</td>
<td>164.60</td>
<td>C</td>
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<td>0.0</td>
<td>162.00</td>
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<tr>
<td>3A-5</td>
<td>4,000 (4.6%)</td>
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<td>0.0</td>
<td>171.00</td>
<td>C</td>
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<tr>
<td>3B-1</td>
<td>3,700 (5.3%)</td>
<td>Deformed</td>
<td>65,500</td>
<td>0.1</td>
<td>103.10</td>
<td>C</td>
</tr>
<tr>
<td>3B-2</td>
<td>3,700 (5.3%)</td>
<td>Deformed</td>
<td>65,500</td>
<td>0.1</td>
<td>103.90</td>
<td>C</td>
</tr>
<tr>
<td>3B-3</td>
<td>3,700 (5.3%)</td>
<td>Deformed</td>
<td>65,500</td>
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* C, T and C & T indicate compression, tension and near balance failure, respectively.
FIG. (5.1.1) TYPICAL FAILURE PHENOMENA OF GROUP 1
FIG. (5.1.1) TYPICAL FAILURE PHENOMENA OF GROUP 1

COMPRESSION FAILURE

TENSION FAILURE
FIG. (5.1.2) TYPICAL FAILURE PHENOMENA OF GROUP 2
FIG. (5.1.2) TYPICAL FAILURE PHENOMENA OF GROUP 2
Compression Failure

Tension Failure

Fig. (5.1.3) Typical Failure Phenomena of Group 3
FIG.(5.1.3) TYPICAL FAILURE PHENOMENA OF GROUP 3
FIG. (5.1.4) TYPICAL FAILURE PHENOMENA OF GROUP 4
Fig. (5.1.4) Typical failure phenomena of Group 4
5.2 Analysis of Experimental Results

Among the 100 specimens tested, 15 failed in shear through the capitals of the specimens. The reason was insufficient embedded bar reinforcement to reinforce the capitals against shear failure. To solve this problem, four auxiliary tension rods were used to clamp plates around each capital through the rest of the tests.

The results of the remaining 85 tests are believed to be significant, and yet are subjected to various errors as indicated below.

(1) Small initial eccentricities due to errors in centering and adjusting the specimens in the testing machines, and the nonhomogenous properties of concrete along the transverse direction.

(2) The position of the reinforcing bars in the cross-section of the member may vary ±1/8 in. from the nominal values. This error is not important for compression failure members, but for tension failure members, it changes the internal moment arm and thus affects the ultimate loads.

(3) The rate of applying load increments, and the total time that used for testing each column specimen can not be controlled to be identical.

Hence, it is possible but not too significant that the ultimate loads, may have been influenced by the uncontrolled time factors.

5.3 Analysis of Theoretical Results

Both the results of the American Concrete Institute design equations and the strain gradient method were obtained by
computer programs. Table (5.3) on page 62 shows the results obtained from the ACI Design Formulae, and from the strain gradient method.

5.4 Comparison of Test Results with the ACI (American Concrete Institute) Design Formulae

The test results show general agreement with the design formulae for the full range of conditions from axially-loaded columns to beam-columns under pure bending. The actual strengths for all specimens under all cases are slightly larger than the calculated values as shown in Table (5.4) on page 64. The arithmetic mean of the ratios of measured to computed ultimate load is 1.04, the overall standard deviation is 0.09, and the overall coefficient of variation is 8.94 per cent.

Due to the known variations of concrete strength, steel yield stress and dimension, the measured results will not necessarily be equal to the theoretical results but still may be significant.

From the measured variabilities of the concrete cylinder stress (8%) and the steel yield stress (3%) and assuming a dimensional variability of 3% it can be shown that only deviations of measured to calculated loads greater than 6% are significant.

Hence within the accuracy of the experimental results the ACI design formulae accurately predicts the failure load of columns under combined axial load and moment.
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**NOTE:** Results obtained from ACI (American Concrete Institute) Design Formulæ.

* Results obtained from Strain Gradient Method.
### TABLE (5.4) COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS

**BY ACI FORMULAE (KIPS)**

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Mean value of $P_{exp}/P_{cal} = 1.04$

Standard deviation $= 0.09$

Coefficient of variation $= 8.94\%$
5.4.1 Effects of Concrete Quality; Strength of Reinforcement and Eccentricity

Figs. (5.4.1) to (5.4.3) show the curves of ratio of measured* to computed ultimate load against concrete strength, reinforcement ratio and eccentricity ratio, respectively. It may be observed that the ratio of measured to computed ultimate load varied about ±10 percent from unity.

5.4.2 Interaction Diagram

Interaction diagrams are presented in Figs. (5.4.2.1) to (5.4.2.4), from page 68, to page 71; comparing the maximum measured load capacity of the specimens with the theoretical ultimate strengths. The material properties of the cross-section with a variation of concrete strength from 3400 to 4600 psi.; percentage of reinforcement varied from 1.27 (plain bar) to 7.14 percent; and steel yield strength from 43,000 to 67,400 psi.

For concentrically-loaded columns, small initial eccentricities due to errors in centering the specimens, as well as the inhomogeneous properties of the concrete along the lateral direction, will generally cause lateral deflections, and thus reduce the column strength. For eccentrically-loaded columns, the errors in entering may increase or reduce the column strength, depending on the direction of the error. The ultimate loads of concentrically-loaded columns are far more sensitive due to such centering errors, than those of eccentrically-loaded columns.

* each point represent the average ratio $\frac{P_{\exp}}{P_{\text{ACI}}}$ of each group.
\[ \frac{P_{\text{exp.}}}{P_{\text{cal.}}} \]

**FIG. (5.4.1) \( P_{\text{exp.}}/P_{\text{cal.}} \) VERSUS ULTIMATE CONCRETE STRESS**

\[ f_c' \quad (\text{CONC. STRESS, PSI.}) \]

**FIG. (5.4.2) \( P_{\text{exp.}}/P_{\text{cal.}} \) VERSUS RATIO OF REINFORCEMENT**

\[ p \quad (\text{RATIO OF REINF.}) \]

**FIG. (5.4.3) \( P_{\text{exp.}}/P_{\text{cal.}} \) VERSUS ECCENTRICITY RATIO OF LOAD (ACI DESIGN FORMULAE)**

- ○ - 3/8"φ, PLANE BAR
- ● - 4/8"φ, 6/8"φ AND
- 7/8"φ DEFORMED BAR
FIG. (5.4.2.1) INTERACTION DIAGRAM FOR GROUP I
(ACI DESIGN FORMULAE)
FIG.(5.4.2.2) INTERACTION DIAGRAM FOR GROUP 2
(ACI DESIGN FORMULAE)
FIG. (5.4.2.3) INTERACTION DIAGRAM FOR GROUP 3
(ACI DESIGN FORMULAB)
FIG. (5.4.2.4) INTERACTION DIAGRAM FOR GROUP 4
(ACI DESIGN FORMULAB)
From Figs. (5.4.2.1) to (5.4.2.4), it can be seen that the accuracy of the ACI Design formulae will generally be within ±10 percent.

5.4.3 Variation of $P_{\text{exp}}/P_{\text{cal}}$.

A histogram and the fitted normal curve of the ratios of the experimental results to the theoretical results using the ACI formulae are shown in Fig. (5.4.3.1) of page. The total population of 85 ratios was subjected to a statistical analysis.

The arithmetic mean of the total population is 1.04, the standard deviation of the ratios between the measured and calculated ultimate loads is 0.09 and the coefficient of variation is 8.94 percent. The probability of failure, calculated from the theoretical distribution of the ultimate load capacity, when using a $\phi$ factor of 0.70, under the ACI Building Code was found to be 0.03% and was 0.19% when using a $\phi$ factor of 0.75 under the NBC Building Code.

The test results appear to be distributed at random and no systematic trends of variation were found. The column design formulae, using the $\phi$ reduction factor were so conservative that there is a very low probability of the maximum column capacity would be less than the ultimate design strength.
FIG. (5.4.3.1) HISTOGRAM AND FITTED NORMAL CURVE

MEAN $\mu = 1.04$
6 = 0.09
$\nu = 8.94\%$
5.5 Comparison of Test Results with the Strain Gradient Method

The test results showed close agreement with the results obtained from the Strain-Gradient Method for the full range of conditions from axially-loaded columns to beam columns under pure flexure. The actual strengths for all specimens under all cases are slightly less than the calculated values as shown in Table (5.5) on page 75. The arithmetic mean of the ratios of measured to computed ultimate load is 0.99, the overall standard deviation is 0.02, and the overall coefficient of variation is 1.94 percent.
### TABLE (5.5) COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS BY STRAIN GRADIENT METHOD (KIPs)

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### TABLE (5.5) COMPARISON OF ACTUAL AND COMPUTED ULTIMATE LOADS

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Mean value of $\frac{P_{\text{exp}}}{P_{\text{cal}}} = 0.99$

Standard deviation $= 0.02$

Coefficient of variation $= 1.94\%$
5.5.1 Effects of Concrete Quality; Strength of Reinforcement and Eccentricity

Figs. (5.5.1) to (5.5.3) show the curves of ratio of measured* to computed ultimate load against concrete strength, reinforcement ratio and eccentricity ratio, respectively. It can be seen from these curves that the ratio of measured to computed ultimate load varied within the limit of ±3 percent from unity.

5.5.2 Interaction Diagram

From Figs. (5.5.2.1) to (5.5.2.4), a series of interaction curves, compare the maximum load capacity of the columns to the theoretical ultimate strengths. The material properties of the cross-section are: concrete strength from 3400 to 4600 psi.; percentage of reinforcement varied from 1.27 (plain bar) to 7.14 percent; and steel yield strength from 43,000 to 67,400 psi.

It can be seen from Figs. (5.5.2.1) to (5.5.2.4) that the experimental values lie well in between ±10 percent of the theoretical curves.

* each point represent the average ratio $P_{exp}/P_{ACI}$ of each group.
FIG. (5.5.1) \( P_{\text{exp.}} / P_{\text{cal.}} \) VERSUS ULTIMATE CONCRETE STRESS

FIG. (5.5.2) \( P_{\text{exp.}} / P_{\text{cal.}} \) VERSUS RATIO OF REINFORCEMENT

FIG. (5.5.3) \( P_{\text{exp.}} / P_{\text{cal.}} \) VERSUS ECCENTRICITY RATIO OF LOAD (STRAIN GRADIENT METHOD)
FIG. (5.5.2.1) INTERACTION DIAGRAM FOR GROUP I  
(STRAIN GRADIENT METHOD)
FIG. (5.5.2.2) INTERACTION DIAGRAM FOR GROUP 2 (STRAIN GRADIENT METHOD)
FIG. (5.5.2.3) INTERACTION DIAGRAM FOR GROUP 3
(STRAIN GRADIENT METHOD)
FIG. (5.5.2.4)  INTERACTION DIAGRAM
FOR GROUP 4
(STRAIN GRADIENT METHOD)
5.5.3 Variation of $P_{exp}/P_{cal}$

In Fig. (5.5.3.1) of page 84, shows the histogram and fitted normal curve of the ratios of the experimental results to the theoretical results given by the Strain Gradient Method. The total population of 85 ratios was studied and subjected to a statistical analysis.

The arithmetic mean of the total population is 0.99, the standard deviation of the ratios between the measured and calculated ultimate loads is 0.02, and the coefficient of variation is 1.94 percent.

Within the scope of the tests reported herein, the ultimate loads computed from the Strain Gradient Method slightly overestimate the column capacities, within one percent on the average.
FIG.(5.5.3.1) HISTOGRAM AND FITTED NORMAL CURVE

\[ \frac{P_{\text{exp.}}}{P_{\text{st.}}} \]

MEAN $x = 0.99$

$\sigma = 0.02$

$\nu = 1.94\%$
5.6 Comparisons with Previous Investigations

The ratios of the computed values from the ACI Design Formulas and the test results are compared with the ratios presented by Chow (26), Hognestad (27), The ACI Column Investigation (27), and Bach and Graf (15).

In Table (5.4), on page 64, shows the ratios of the measured to the calculated results obtained in the project, with the mean value of 1.04, the standard deviation of 0.09, and the coefficient of variation of 8.94 percent.

In Table (5.6.2), on page 86, shows the ratios of the measured to the calculated results presented by Chow, with the mean value of 1.15, the standard deviation of 0.0931 and coefficient of variation of 8.3 percent.

Table (5.6.3) on page 87, shows the ratios of the measured to the calculated results presented by Hognestad, with a mean value of 0.97, standard deviation of 0.059, and coefficient of variation of 6.08 percent.

Table (5.6.4) on page 90, gives the ratios of the measured to the calculated results presented by The ACI Column Investigation, with a mean value of 1.00, standard deviation of 0.074, and the coefficient of variation of 7.40 percent.

Table (5.6.5), on page 91, gives the ratios of the measured to the calculated results by Bach and Graf, with a mean value of 1.035, standard deviation of 0.0703, and coefficient of variation of 6.79 percent.
Similar results were found with the specimens reported by Hognestad, The ACI Column Investigation and Bach and Graf, and are in close agreement with results presented in Table (5.4).

The ratios of the computed values from the Strain Gradient Method and the test results are also compared with the ratios presented by Chow. The ratios of the measured to the calculated results presented by Chow, with the mean value of 0.97, the standard deviation of 0.076, and coefficient of variation of 7.9%, were compared to the results obtained in Table (5.5), with the mean value of 0.99, standard deviation of 0.02 and coefficient of variation of 1.94%.

Since the agreement between Chow's results and the ACI design formulae is much worse than that between Hognestad, The ACI Column Investigation, the Bach and Graf and the ACI design formulae, it is concluded that Chow's results must contain some experimental errors.
### TABLE (5.6.2) COMPARISON OF TEST AND COMPUTED ULTIMATE LOADS

**FOR UNDER ECCENTRICAL LOADS**

**BY CHOW**

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<th>$f'_c$ (psi.)</th>
<th>$f_y$ (ksi.)</th>
<th>% OF REINF.</th>
<th>$P_{exp}$ (Kips)</th>
<th>$P_{ult}$ (Kips)</th>
<th>$\frac{P_{exp}}{P_{cal}}$</th>
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**Mean value of $\frac{P_{exp}}{P_{cal}} = 1.15$**

**Standard deviation** = 0.0931

**Coefficient of variation** = 8.30%
### Table (5.6.3) Comparison of Test and Computed Ultimate Loads Under Eccentric Loads — by Hognestad

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<td>306.00</td>
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<td>343.00</td>
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<td>4300.00</td>
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<td>4010.00</td>
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Continued
### TABLE (5.6.3) Continued

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<th>SPECIMEN</th>
<th>$f'_C$ (psi.)</th>
<th>$f_y$ (ksi.)</th>
<th>% OF REINF.</th>
<th>$P_{exp}$ (Kips)</th>
<th>$P_{cal}$ (Kips)</th>
<th>$P_{exp}$ / $P_{cal}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>B 10 in.</td>
<td>2200.00</td>
<td>230.00</td>
<td>218.00</td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5350.00</td>
<td>220.00</td>
<td>227.00</td>
<td>0.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>4850.00</td>
<td>210.00</td>
<td>216.00</td>
<td>0.97</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3580.00</td>
<td>180.00</td>
<td>188.00</td>
<td>0.96</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>4290.00</td>
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<td>206.00</td>
<td>1.00</td>
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<td></td>
</tr>
<tr>
<td></td>
<td>2300.00</td>
<td>151.00</td>
<td>153.00</td>
<td>0.99</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>2070.00</td>
<td>137.00</td>
<td>148.00</td>
<td>0.93</td>
<td></td>
<td></td>
</tr>
<tr>
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<td>5350.00</td>
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<td></td>
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<tr>
<td></td>
<td>5100.00</td>
<td>153.00</td>
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<td>138.80</td>
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<td></td>
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<tr>
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<td>115.50</td>
<td>108.00</td>
<td>1.07</td>
<td></td>
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</tr>
<tr>
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<td>111.00</td>
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<td></td>
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<tr>
<td></td>
<td>5100.00</td>
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<td>1.06</td>
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<td>T 10 in.</td>
<td>4850.00</td>
<td>79.00</td>
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<tr>
<td></td>
<td>3800.00</td>
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<td></td>
<td>4630.00</td>
<td>84.00</td>
<td>82.00</td>
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<tr>
<td></td>
<td>1950.00</td>
<td>72.50</td>
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<td>0.97</td>
<td></td>
<td></td>
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<td></td>
<td>2070.00</td>
<td>74.50</td>
<td>73.50</td>
<td>1.01</td>
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<td></td>
</tr>
</tbody>
</table>

Mean value of $P_{exp} / P_{cal} = 0.97$

Standard deviation = 0.059

Coefficient of variation = 6.08%
TABLE (5.6.4) COMPARISON OF TEST AND COMPUTED ULTIMATE LOADS
UNDER ECCENTRIC LOADS—by ACI COL. INVESTIGATION

<table>
<thead>
<tr>
<th>SPECIMEN</th>
<th>$f_c'$ (psi.)</th>
<th>$f_y$ (psi.)</th>
<th>% OF REINF.</th>
<th>$P_{exp}$ (Kips)</th>
<th>$P_{cal}$ (Kips)</th>
<th>$\frac{P_{exp}}{P_{cal}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>2860.00</td>
<td>50,000</td>
<td>4</td>
<td>219.00</td>
<td>231.00</td>
<td>0.95</td>
</tr>
<tr>
<td>b</td>
<td>3090.00</td>
<td>50,000</td>
<td>4</td>
<td>255.00</td>
<td>242.00</td>
<td>1.05</td>
</tr>
<tr>
<td>a</td>
<td>2650.00</td>
<td>50,000</td>
<td>4</td>
<td>253.00</td>
<td>222.00</td>
<td>1.14</td>
</tr>
<tr>
<td>b</td>
<td>2850.00</td>
<td>50,000</td>
<td>4</td>
<td>238.00</td>
<td>231.00</td>
<td>1.03</td>
</tr>
<tr>
<td>a</td>
<td>4700.00</td>
<td>44,700</td>
<td>1.5</td>
<td>225.00</td>
<td>246.00</td>
<td>0.92</td>
</tr>
<tr>
<td>b</td>
<td>4150.00</td>
<td>44,700</td>
<td>1.5</td>
<td>227.00</td>
<td>222.00</td>
<td>1.02</td>
</tr>
<tr>
<td>a</td>
<td>4670.00</td>
<td>50,000</td>
<td>4</td>
<td>285.00</td>
<td>310.00</td>
<td>0.92</td>
</tr>
<tr>
<td>b</td>
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<td>4</td>
<td>320.00</td>
<td>313.00</td>
<td>1.02</td>
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<tr>
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<td>293.00</td>
<td>291.00</td>
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<td>50,000</td>
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<td>309.00</td>
<td>306.00</td>
<td>1.01</td>
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<td>6</td>
<td>317.00</td>
<td>315.00</td>
<td>1.01</td>
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<tr>
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<td>4985.00</td>
<td>42,200</td>
<td>6</td>
<td>291.00</td>
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<td>4</td>
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<td>410.00</td>
<td>0.94</td>
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<tr>
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<td>6245.00</td>
<td>50,000</td>
<td>4</td>
<td>410.00</td>
<td>379.00</td>
<td>1.08</td>
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<tr>
<td>b</td>
<td>6530.00</td>
<td>50,000</td>
<td>4</td>
<td>420.00</td>
<td>391.00</td>
<td>1.08</td>
</tr>
</tbody>
</table>

Mean value of $\frac{P_{exp}}{P_{cal}} = 1.00$

Standard deviation $= 0.074$

Coefficient of variation $= 7.40\%$
### TABLE (5.6.5) COMPARISON OF TEST AND COMPUTED ULTIMATE LOADS UNDER ECCENTRIC LOADS ---- BY BACHAND GRAF

<table>
<thead>
<tr>
<th>SPECIMEN</th>
<th>$f'_c$ (psi.)</th>
<th>$f_y$ (psi.)</th>
<th>% OF REINF.</th>
<th>$P_{exp}$ (Kips)</th>
<th>$P_{cal}$ (Kips)</th>
<th>$\frac{P_{exp}}{P_{cal}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>2830.00</td>
<td>53,700</td>
<td>1</td>
<td>735.00</td>
<td>729.00</td>
<td>1.01</td>
</tr>
<tr>
<td></td>
<td>2830.00</td>
<td>53,700</td>
<td>1</td>
<td>447.00</td>
<td>423.00</td>
<td>1.05</td>
</tr>
<tr>
<td></td>
<td>2830.00</td>
<td>53,700</td>
<td>1</td>
<td>273.00</td>
<td>257.00</td>
<td>1.06</td>
</tr>
<tr>
<td></td>
<td>2830.00</td>
<td>53,700</td>
<td>1</td>
<td>153.40</td>
<td>148.00</td>
<td>1.03</td>
</tr>
<tr>
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<td>2830.00</td>
<td>53,700</td>
<td>1</td>
<td>71.30</td>
<td>68.50</td>
<td>1.04</td>
</tr>
</tbody>
</table>

B = 16 in.  
T = 16 in.

|          | 2830.00       | 53,700       | 1.9         | 892.00           | 835.00          | 1.07             |
|          | 2830.00       | 53,700       | 1.9         | 496.00           | 486.00          | 1.02             |
|          | 2830.00       | 53,700       | 1.9         | 347.00           | 343.00          | 1.01             |
|          | 2830.00       | 53,700       | 1.9         | 231.50           | 219.00          | 1.05             |
|          | 2830.00       | 53,700       | 1.9         | 118.00           | 116.40          | 1.01             |

Mean value of $\frac{P_{exp}}{P_{cal}} = 1.035$

Standard deviation = 0.0703

Coefficient of variation = 6.79%
5.7 Studies of Deflection

The deflections of all columns tested were observed by means of three dial indicators; at centre span and 6" to the left and right of the centre of the column along the longitudinal direction. These dial indicators were attached to the compression face of the columns as shown in Figs. (3.5.2.a) and (b). The load-deflection curves for all columns presented in Figs. (B.1) to (B.85), on page 112 to page 196.

The deflections at mid-height with respect to the hinged ends were used to calculate the actual eccentricities at failure. All the ACI Design Equations for ultimate loads of eccentrically-loaded members reported in this thesis were referred to the actual eccentricities at failure.

The deflections at mid-height of the columns were also considered theoretically. They can be obtained easily by introducing the simple assumption that the deflected shape was a cosine wave, as shown in Fig. (5.7.1).

\[ Y = 8 \cos \frac{\pi x}{L} \quad \ldots (1) \]

\[ \frac{d^2 y}{dx^2} = -8 \left( \frac{\pi}{L} \right)^2 \cos \frac{\pi x}{L} \]

but

\[ \frac{d^2 y}{dx^2} = -\frac{1}{\rho} \]

\[ \therefore \frac{1}{\rho} = 8 \left( \frac{\pi}{L} \right)^2 \cos \frac{\pi x}{L} \quad \ldots \ldots (2) \]

\[ \text{FIG. (5.7.1) \qquad DEFLECTED SHAPE OF A COLUMN} \]
Apply boundary condition, when \( X = 0, \ Y = \delta \)

Thus

\[
\frac{1}{f} = \delta \left( \frac{\pi}{L} \right)^2 \quad \ldots \ldots (3)
\]

From Fig. (5.7.2), consider column under deflection.

\[
\frac{1}{f} = \frac{\varepsilon_x}{Y} = \frac{\varepsilon_x}{Kd} = \phi \quad \ldots \ldots (4)
\]

Substitute Eq. (4) in Eq. (3), we have,

\[
\delta \left( \frac{\pi}{L} \right)^2 = \phi
\]

\[
\therefore \ \delta = \phi \cdot \left( \frac{\pi}{L} \right)^2 \quad \ldots \ldots (5)
\]

In our case, as shown in Fig. (5.7.3) gives,

\[
(\varepsilon_c + \varepsilon_{sl}) = d \cdot \phi
\]

\[
\therefore \ \phi = \frac{\varepsilon_c + \varepsilon_{sl}}{d} \quad \ldots \ldots (6)
\]

Substitute Eq. (6) in Eq. (5), we obtain,

\[
\delta = \left( \frac{\pi}{L} \right)^2 \left( \frac{\varepsilon_c + \varepsilon_{sl}}{d} \right)
\]

\[
= K \cdot (\varepsilon_c + \varepsilon_{sl})
\]

where \( K = \frac{\pi^2}{L^2} \cdot d \), and \( \varepsilon_c \) and \( \varepsilon_{sl} \) can be obtained for each column, from the Strain Gradient Method.
The test results of the deflections at mid-height were compared with the theoretical results as shown in Table (5.7.1), and were found to be in a satisfactory agreement. The errors are probably due to the centering of the specimen; the beam-column effect under various eccentricities; the assumptions of the maximum concrete strain equal to 0.003 and the assumption that the deflected shape of the specimens is a cosine curve.
### TABLE (5.7.1) COMPARISON BETWEEN OBSERVED AND CALCULATE CENTRE DEFORMATIONS

<table>
<thead>
<tr>
<th>SPECIMEN *</th>
<th>INITIAL ECCENTRICITY e/t</th>
<th>DEFLECTION AT FAILURE</th>
<th>( \delta \text{ exp.} )</th>
<th>( \delta \text{ cal.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>MEAN</td>
<td>C.of V.</td>
<td>MEAN</td>
</tr>
<tr>
<td>4E (1 - 5)</td>
<td>0.75</td>
<td>0.481</td>
<td>11.1%</td>
<td>0.437</td>
</tr>
<tr>
<td>4DE (1 - 5)</td>
<td>0.75</td>
<td>0.261</td>
<td>7.7%</td>
<td>0.320</td>
</tr>
<tr>
<td>4D (1 - 5)</td>
<td>0.5</td>
<td>0.601</td>
<td>12.6%</td>
<td>0.642</td>
</tr>
<tr>
<td>4C (1 - 5)</td>
<td>0.3</td>
<td>0.537</td>
<td>9.6%</td>
<td>0.420</td>
</tr>
<tr>
<td>4B (1 - 5)</td>
<td>0.1</td>
<td>0.431</td>
<td>14.4%</td>
<td>0.448</td>
</tr>
<tr>
<td>2E (1 - 5)</td>
<td>0.75</td>
<td>0.363</td>
<td>16.5%</td>
<td>0.324</td>
</tr>
<tr>
<td>2DE (1 - 5)</td>
<td>0.75</td>
<td>0.463</td>
<td>12.9%</td>
<td>0.418</td>
</tr>
<tr>
<td>2D (1 - 5)</td>
<td>0.5</td>
<td>0.614</td>
<td>9.1%</td>
<td>0.550</td>
</tr>
<tr>
<td>2C (1 - 5)</td>
<td>0.3</td>
<td>0.380</td>
<td>5.5%</td>
<td>0.305</td>
</tr>
<tr>
<td>3E (1 - 5)</td>
<td>0.5</td>
<td>0.359</td>
<td>1.4%</td>
<td>0.384</td>
</tr>
<tr>
<td>3D (1 - 5)</td>
<td>0.5</td>
<td>1.050</td>
<td>6.3%</td>
<td>0.964</td>
</tr>
<tr>
<td>3C (1 - 5)</td>
<td>0.3</td>
<td>0.725</td>
<td>15.7%</td>
<td>0.734</td>
</tr>
<tr>
<td>3B (1 - 5)</td>
<td>0.1</td>
<td>0.413</td>
<td>2.0%</td>
<td>0.470</td>
</tr>
<tr>
<td>3A (1 - 5)</td>
<td>0.0</td>
<td>0.033</td>
<td>6.4%</td>
<td>0.034</td>
</tr>
<tr>
<td>1E (1 - 5)</td>
<td>0.5</td>
<td>0.460</td>
<td>4.8%</td>
<td>0.390</td>
</tr>
<tr>
<td>1D (1 - 5)</td>
<td>0.5</td>
<td>0.449</td>
<td>11.4%</td>
<td>0.415</td>
</tr>
<tr>
<td>1C (1 - 5)</td>
<td>0.3</td>
<td>0.312</td>
<td>10.3%</td>
<td>0.330</td>
</tr>
</tbody>
</table>

* Average Values of 5 Specimens in a Same Group.*
CHAPTER VI

CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

From this investigation, some main conclusions may be summarised as follows:-

1) The short column design formulae given in the ACI 318-56, NBC 1965 and CSA.A23.3-1970 Building Codes predicts the measured failure loads of columns within ±10%, provided that the actual eccentricity of the load at failure is used in the calculations. The average ratio of measured load / predicted load is 1.04 with a standard deviation of 0.09.

2) The ACI - CSA Design Formulae together with the recommended "ψ" factor of 0.70 implies a probability of the ultimate section capacity of a laboratory specimen being less than the design ultimate load of 0.03%.

3) The probability of the actual load capacity being less than the design ultimate load is 0.19%, using a "ψ" factor of 0.75 as recommended by the Canadian Standard (CSA.A23-1970), together with the ACI - NBC - CSA prediction formulae.

4) The strain gradient method using the measured steel stress-strain curve and Hognestad stress-strain curve for the concrete predicts the experimental values within ±3%. The average measured load / predicted load ratio is 0.99 with a standard deviation of 0.02.
5) Calculations of the lateral deflection from the strain gradient method assuming that the deflected shape of an eccentrically loaded long column deflects laterally in the form of a cosine agree reasonably well with the measured lateral deflections of the laboratorial specimens.

6) The accuracy of the results obtained from the ACI Design Formulae, increases as the percentage of the longitudinal reinforcement increases.

7) The ratio of the measured to computed ultimate load using the ACI Design Formulae, becomes close to unity as the ratio e/t approaching infinity.

8) The ratio of the measured to computed ultimate load using the strain gradient method, approaches close to unity as the ratio e/t increases.

9) The moment with respect to the balanced condition is increased with an increase of longitudinal reinforcing steel.

10) The ductility of the column section is decreased with the increase of the longitudinal reinforcing steel.

11) The experimental results using deformed bars rather than plain bars, give more consistent agreement with the predicted results from both the strain gradient method and ACI - NBC - CSA methods.
6.2 Recommendations

1) Longitudinal stirrups are recommended in the capitals to transmit the eccentric load to the prismatic section of the column.

2) Test the Strain Gradient Method experimentally, by installing strain gages in the specimen to check the correspondence of strain / strain gradient and load.

3) To develop a general computer program to represent the behavior of long column under uni-axial and bi-axial bending, by the Strain Gradient Method, for the analysis of the reinforced concrete columns.
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APPENDIX A

STEEL PROPERTIES

FIGURE (A.1) to FIGURE (A.4)

STRESS-STRAIN CURVES OF LONGITUDINAL REINFORCEMENTS
Fig. (A.1) Stress—Strain Curve of #3 longitudinal reinforcement (plain bar)
FIG. (A.2) STRESS—STRAIN CURVE OF #4 LONGITUDINAL REINFORCEMENT (DEFORM BAR)
FIG. (A.3) STRESS—STRAIN CURVE OF #6 LONGITUDINAL REINFORCEMENT (DEFORM BAR)
FIG. (A.4) STRESS—STRAIN CURVE OF #7 LONGITUDINAL REINFORCEMENT (DEFORM BAR)
APPENDIX B

INDIVIDUAL GRAPHS OF SPECIMENS

FIGURE (B.1) to FIGURE (B.85)

LOAD-DEFLECTIONS CURVES

OF SPECIMENS
FIG. (B. 1) LOAD—DEFLECTION CURVE OF SPECIMEN 4E-1
FIG. (B. 2) LOAD—DEFLECTION CURVE OF SPECIMEN 4E-2
FIG. (B. 3) LOAD—DEFLECTION CURVE OF SPECIMEN 4E-3
FIG. (B. 4) LOAD—DEFLECTION CURVE OF SPECIMEN 4E−4
FIG. (B. 5) LOAD—DEFLECTION CURVE OF SPECIMEN 4E-5
FIG. (B. 6) LOAD—DEFLECTION CURVE OF SPECIMEN 4D−1
FIG. (B. 7) LOAD—DEFLECTION CURVE OF SPECIMENT 4D—2
FIG. (B. 8) LOAD—DEFLECTION CURVE OF SPECIMEN 4D–3
FIG. (B. 9) LOAD—DEFLECTION CURVE OF SPECIMEN 4D-4

LOAD (KIPS) vs. DEFLECTION (INCH)
FIG.(B.10) LOAD—DEFLECTION CURVE OF SPECIMEN 4D—5
FIG. (B. 11) LOAD—DEFLECTION CURVE OF SPECIMEN 4C-1
FIG. (B. 12) LOAD—DEFLECTION CURVE OF SPECIMEN 4C–2
FIG. (B. 13) LOAD—DEFLECTION CURVE OF SPECIMEN 4C–3
FIG. (B.14) LOAD—DEFLECTION CURVE OF SPECIMEN 4C-4
FIG. (B. 15) LOAD—DEFLECTION CURVE OF SPECIMEN 4C-5
FIG. (B. 16) LOAD—DEFLECTION CURVE OF SPECIMEN 4DE-1
FIG. (B.17) LOAD–DEFLECTION CURVE OF SPECIMEN 4DE–2
**FIG. (B. 18) LOAD—DEFLECTION CURVE OF SPECIMEN 4DE-3**
FIG. (B. 19) LOAD—DEFLECTION CURVE OF SPECIMEN 4DE − 4
FIG. (B. 20) LOAD—DEFLECTION CURVE OF SPECIMEN 4DE—5
FIG. (B.21) LOAD—DEFLECTION CURVE OF SPECIMEN 4B-1
FIG. (B. 22) LOAD—DEFLECTION CURVE OF SPECIMEN 4B-2
FIG. (B. 23) LOAD—DEFLECTION CURVE OF SPECIMEN 4B–3
FIG. (B.24) LOAD—DEFLECTION CURVE OF SPECIMEN 4B—4
FIG. (B. 25) LOAD—DEFLECTION CURVE OF SPECIMEN 4B—5
FIG. (B.26) LOAD—DEFLECTION CURVE OF SPECIMEN 3E−1
FIG. (B.27) LOAD—DEFLECTION CURVE OF SPECIMEN 3E−2
FIG. (B.28) LOAD—DEFLECTION CURVE OF SPECIMEN 3E-3
FIG. (B.29) LOAD—DEFLECTION CURVE OF SPECIMEN 3E-4
FIG. (B.30) LOAD—DEFLECTION CURVE OF SPECIMEN 3E−5
FIG. (B. 31) LOAD—DEFLECTION CURVE OF SPECIMEN 3D-1
FIG. (B. 32) LOAD—DEFLECTION CURVE OF SPECIMEN 3D-2
FIG. (B. 33) LOAD—DEFLECTION CURVE OF SPECIMEN 3D—3
FIG. (B. 34) LOAD—DEFLECTION CURVE OF SPECIMEN 3D-4
FIG. (B. 35) LOAD—DEFLECTION CURVE OF SPECIMEN 3D-5
FIG. (B.36) LOAD—DEFLECTION CURVE OF SPECIMEN 3C-1
FIG. (B.37) LOAD—DEFLECTION CURVE OF SPECIMEN 3C–2
FIG. (B. 38) LOAD–DEFLECTION CURVE OF SPECIMEN 3C–3
FIG. (B.39) LOAD–DEFLECTION CURVE OF SPECIMEN 3C–4
FIG. (B.40) LOAD—DEFLECTION CURVE OF SPECIMEN 3C-5
FIG. (B. 41) LOAD—DEFLECTION CURVE OF SPECIMEN 3B–1
FIG. (B. 42) LOAD—DEFLECTION CURVE OF SPECIMEN 3B—2
FIG. (B.43) LOAD—DEFLECTION CURVE OF SPECIMEN 3B-3
FIG. (B.44) LOAD—DEFLECTION CURVE OF SPECIMEN 3B-4
FIG. (B. 45) LOAD—DEFLECTION CURVE OF SPECIMEN 3B-5
FIG. (B.46) LOAD—DEFLECTION CURVE OF SPECIMEN 3A-1
FIG.(B.47) LOAD–DEFLECTION CURVE OF SPECIMEN 3A–2
FIG. (B.48) LOAD—DEFLECTION CURVE OF SPECIMEN 3A—3
FIG. (B. 49) LOAD—DEFLECTION CURVE OF SPECIMEN 3A–4
FIG. (B. 50) LOAD—DEFLECTION CURVE OF SPECIMEN 3A-5
FIG. (B. 51) LOAD—DEFLECTION CURVE OF SPECIMEN 2E-1
FIG. (B. 52) LOAD—DEFLECTION CURVE OF SPECIMEN 2E−2
FIG. (B. 53) LOAD—DEFLECTION CURVE OF SPECIMEN 2E−3
FIG. (B. 54) LOAD—DEFLECTION CURVE OF SPECIMEN 2E-4
FIG. (B. 55) LOAD—DEFLECTION CURVE OF SPECIMEN 2E-5
FIG. (B. 56) LOAD—DEFLECTION CURVE OF SPECIMEN 2DE-1
FIG. (B. 57) LOAD—DEFLECTION CURVE OF SPECIMEN 2DE—2
FIG. (B. 58) LOAD—DEFLECTION CURVE OF SPECIMEN 2DE-3
FIG. (B. 59) LOAD—DEFLECTION CURVE OF SPECIMEN 2DE-4
FIG. (B.60) LOAD—DEFLECTION CURVE OF SPECIMEN 2DE-5
FIG. (B. 61) LOAD—DEFLECTION CURVE OF SPECIMEN 2D—1
FIG. (B. 62) LOAD—DEFLECTION CURVE OF SPECIMEN 2D—2
FIG. (B. 63) LOAD—DEFLECTION CURVE OF SPECIMEN 2D-3
FIG. (B.64) LOAD—DEFLECTION CURVE OF SPECIMEN 2D—4
FIG. (B. 65) LOAD—DEFLECTION CURVE OF SPECIMEN 2D-5
FIG. (B. 66) LOAD—DEFLECTION CURVE OF SPECIMEN 2C-1
FIG. (B. 67) LOAD—DEFLECTION CURVE OF SPECIMEN 2C—2
FIG. (B.68) LOAD—DEFLECTION CURVE OF SPECIMEN 2C–3
FIG. (B.69) LOAD—DEFLECTION CURVE OF SPECIMEN 2C—4
FIG. (B.70) LOAD—DEFLECTION CURVE OF SPECIMEN 2C-5
FIG. (B. 71) LOAD—DEFLECTION CURVE OF SPECIMEN 1E-1
FIG. (B.72) LOAD–DEFLECTION CURVE OF SPECIMEN 1E–2
FIG. (B. 73) LOAD—DEFLECTION CURVE OF SPECIMEN 1E−3
FIG. (B. 74) LOAD—DEFLECTION CURVE OF SPECIMEN 1E−4
FIG. (B.75) LOAD—DEFLECTION CURVE OF SPECIMEN 1E−5
FIG. (B.76) LOAD—DEFLECTION CURVE OF SPECIMEN ID—1
FIG. (B.77) LOAD—DEFLECTION CURVE OF SPECIMEN 1D-2
FIG. (B.78) LOAD—DEFLECTION CURVE OF SPECIMEN ID—3
Fig. (B.79) Load—Deflection Curve of Specimen ID-4
FIG. (B.80) LOAD—DEFLECTION CURVE OF SPECIMEN ID—1
FIG. (B.81) LOAD-DEFLECTION CURVE OF SPECIMEN IC-1
FIG. (B. 82) LOAD—DEFLECTION CURVE OF SPECIMEN IC–2
FIG. (B.83) LOAD—DEFLECTION CURVE OF SPECIMENT IC-3
FIG. (B.84) LOAD—DEFLECTION CURVE OF SPECIMEN 1C—4
FIG. (B.85) LOAD—DEFLECTION CURVE OF SPECIMEN IC-5
APPENDIX C

EXAMPLES

EXAMPLE A

COLUMN DUE TO ECCENTRIC LOADING
BY
ACI DESIGN FORMULAE

EXAMPLE B

COLUMN DUE TO PURE BENDING
BY
STRAIN-GRADIENT METHOD
(A) Reinforced Concrete Columns due to Eccentric Loading

Example: Column 4D-4

Given: \( \frac{e}{t} = 0.3 \)

\( f_c' = 48,708 \text{ psi.} \)
\( f_y = 45,386 \text{ psi.} \)
\( d' = 4.57'' \)
\( d'' = 1.43'' \)
\( d'' = 1.57'' \)
\( A_s = A_s' = 0.11 \text{ sq.in.} \)
\( e = 3.38'' \)

Refer to Figs. (4.1.1.a) and (4.1.3.a)

\[
C_b = \frac{87d}{87 + f_y} = \frac{87 \times 4.57}{87 + 45.386}
\]

\[
= \frac{87 \times 4.57}{132.386} = 3.0''
\]

\( a = K_1C_b = 0.85 \times 3 = 2.55'' \)

\[
C_c = 0.85f_c'ab
\]

\[
= 0.85 \times 4.871 \times 2.55 \times 6
\]

\( = 63.4 \text{ kips.} \)
\[ C_s = A_s (f_y - 0.85f'_c) \]
\[ = 0.22 \times (45.39 - 0.85 \times 4.871) \]
\[ = 9.08 \text{ kips.} \]

\[ T = A_s f_y = 0.22 \times 45.386 = 9.99 \text{ kips.} \]

\[ P_b = C_s + C_c - T = 9.08 + 63.4 - 9.99 \]
\[ = 62.49 \text{ kips.} \]

Substitute in Eq. (4.1.3.c)

\[ P_{b'e_b} = C_c (d - \frac{a}{2}) + C_s (d - d') \]
\[ = 63.4 \left( f_y \times 0.57 - \frac{2.55}{2} \right) + 9.08 \left( 4.57 - 1.43 \right) \]
\[ = 173.5 \text{ k-in.} \]

\[ e_b = \frac{173.5}{62.49} = 2.78'' < e = 3.38'' \]

Therefore tension controls.

\[ p = \frac{A_s}{bd} = \frac{0.22}{6 \times 4.95} = 0.0074 \]

\[ m = \frac{f_y}{0.85f'_c} = \frac{45.386}{0.85 \times 4.871} = 1.097 \]
Substitute into Eq. (4.1.3.e)

\[ P_u = 0.85 f'_{cd} \left\{ \left( -p + 1 - e'/d \right) + \sqrt{(1 - \frac{e'}{d})^2 + 2p \left( (m - 1)(1 - \frac{d'}{d}) + \frac{e''}{d} \right)} \right\} \]

\[ = 0.85 \times 4.87 \times 6 \times 4.57 \]

\[ \left\{ -0.0074 + 1 - 4.95/4.57 \right\} + \sqrt{(1 - 4.95/4.57)^2 + 2 \times 0.0074 \left( 9.97 \left( 1 - \frac{1.43}{4.57} \right) + \frac{4.95}{4.57} \right)} \]

\[ = 28.8 \text{ kips.} \]

For NBC design load, \( \phi = 0.75 \)

\[ P \text{ design} = 0.75 \times 28.8 \]

\[ = 21.6 \text{ kips.} \]

For ACI design load, \( \phi = 0.7 \)

\[ P \text{ design} = 0.7 \times 28.8 \]

\[ = 20.2 \text{ kips.} \]
(B) Reinforced Concrete Columns due to Pure Bending

Example: Column 3E-5

From Strain Diagram as Shown below.

\[
\epsilon_y = \epsilon_u \left( \frac{1}{K} - 1 \right)
\]

\[
\epsilon_s = \left( \frac{Kd - d'}{Kd} \right) \epsilon_u
\]

If compression steel below N.A. then

\[
\epsilon_s = \left( \frac{d' - Kd}{Kd} \right) \epsilon_u
\]

Given: \( f'_c = 3.461 \text{ ksi.} \)

\( f_y = 65.525 \text{ ksi.} \)

\( d = 4.69'' \)

\( d' = 1.31'' \)

\( A_s = A_s' = 0.2 \text{ sq.in.} \)

\( e/t = \infty \)
1st TRIAL:

Assume:

\[
\begin{align*}
\epsilon_u &= 0.003 \\
K &= 0.4
\end{align*}
\]

\[K_d = 0.4 \times (4.69) = 1.876''\]

\[
\epsilon_y = \left(\frac{4.69}{1.876} - 1\right) \times 0.003
\]

\[= 0.003 \times (1.5)\]

\[= 0.0045\]

From Stress-Strain diagram of steel

\[f_y = 65,525 \text{ psi.}\]

\[T = 2 \times 0.2 \times 65.525\]

\[= 26.20 \text{ kips.}\]

\[\epsilon_s = 0.003 \times \left(\frac{1.876 - 1.31}{1.876}\right)\]

\[= 0.003 \times (0.218)\]

\[= 0.000646\]

\[C_s = 20 \times 0.2 \times 2 = 8.0 \text{ k}\]
From Stress-Strain diagram of concrete

\[ f'_c = 2.492 \text{ ksi.} \]

\[ C_c = 0.85f'_c a \times B \]

\[ = 0.85 \times 0.85 \times 2.492 \times 1.876 \times 6 \]

\[ = 28.05 \text{ k} \]

\[ C_c + C_s = 28.05 + 8.0 = 34.05 \text{ k} > T = 26.20 \]

(NO GOOD)

2nd TRIAL:

Assume:

\[ \left\{ \begin{array}{c}
\varepsilon_u = 0.0026 \\
K = 0.34 \\
\end{array} \right. \]

\[ \therefore K_d = 1.60 \]

From Stress-Strain diagram of concrete, obtains,

\[ f'_c = 2.720 \text{ psi.} \]

\[ \varepsilon_y = 0.0026 \left( \frac{1}{0.34} - 1 \right) = 0.0026 \times 1.94 \]

\[ = 0.00505 \]

\[ T = 2 \times 0.2 \times 65.525 = 26.20 \text{ k}. \]
\[ C_c = 0.72 \times 2.72 \times 1.60 \times 6 = 18.80 \text{ k.} \]

\[ \varepsilon_s = 0.0026 \left( \frac{1.60 - 1.31}{1.31} \right) = 0.0026 (0.221) = 0.000575 \]

From Stress-Strain diagram of steel, gives,

\[ f_s = 17,000 \text{ psi.} \]

\[ C_s = 3.4 \times 2 = 6.8 \text{ k.} \]

\[ C_c + C_s = 18.80 + 6.8 = 25.60 \div T = 26.20 \text{ k.} \]

For:

\[ P_u = \frac{6M_u}{L} \]

Take moment about the centroid of Rectangular concrete block,

\[ M_u = M_1 - M_2 \]

\[ M_1 = 26.20 (d - a/2) = 26.20 (4.69 - \frac{1.36}{2}) = 105 \text{ k'} \]

\[ M_2 = 6.8 (1.31 - 0.68) = 4.28 \text{ k'} \]

\[ \therefore M_u = -4.28 + 105.0 = 100.72 \text{ k'} \]

Therefore,

\[ P_u = \frac{6M/L}{53} = \frac{6 \times 100.72}{53} = 11.41 \text{ k'} \approx 11.37 \text{ k'} \]

(0.K.)
APPENDIX D

COMPUTER PROGRAMS

PROGRAM A

PROGRAM FOR ACI DESIGN

FORMULAE

PROGRAM B

PROGRAM FOR STRAIN-GRADIENT

METHOD
A). PROGRAM FOR ACI DESIGN FORMULAE

The arguments used in the program are defined as follows:

\[ TM = m \]
\[ PUA = \text{Ultimate load of column under concentric loading} \]
\[ PUB = \text{Ultimate load of column under pure bending} \]
\[ PUBB = \text{Ultimate load of column under balanced condition} \]
\[ PUC = \text{Ultimate load of column under eccentric loading (compression failure)} \]
\[ PUT = \text{Ultimate load of column under eccentric loading (tension failure)} \]
\[ D = d \]
\[ DP = d' \]
\[ E = \text{Initial eccentricity} \]
\[ EI = \delta \text{ (deflection)} \]
\[ FY = f_y \]
\[ FC = f'_c \]
\[ AS = A_s \]
\[ EC = e = E + \delta \]
EB = \( e_b \)

AB = \( a_b \)

EP = \( e' \)

MUB = Balanced moment, kip-in.

\[ P = p = \frac{A_s}{bd} \]
GENERAL SOLUTIONS FOR SHORT REINFORCED CONCRETE COLUMNS
SUBJECTED TO AXIAL AND BENDING—BY ACI DESIGN FORMULAE

COMMON AND DIMENSION STATEMENT

COMMON:
DIMENSION TM(100), PUC(100), PUBB(100), PUT(100),
& D(100), DP(100), E(100), EI(100), FY(20), FC(20), AS(20),
& EC(100), EB(100), AB(100), EP(100), CK(10), P(100), PP(20), NUB(100),
& E(100), CC(100), CS(100), EB1(100), ABB(100), AK(100), AKM(100),
& AP(100), AM(100), BP(100), CP(100), CM(100), DDP(100), BM(1000),
& EDMIN(100), EE(100), EM(100), C(100), DK(1000), PUB(100),
& EFORCE(100), AX(100), AXC(100), BXB(100), CX(100), CXM(100),
& EA(20), A1(20), A2(20), A3(20), A4(20), SC(10), STA(1000), FD(1000),
& EDX(100), DXM(100), EXP(100), EXM(100), XBM(100), FS(1000)

INPUT DATA:

GCG 2

*M=100

GCG 3

N=20

GCG 4

READ (1, 100) (E(I), I=1, M)

GCG 5

READ (1, 100) (D(I), I=1, M)

GCG 6

READ (1, 100) (DP(I), I=1, M)

GCG 7

READ (1, 100) (EI(I), I=1, M)

GCG 8

READ (1, 100) (FC(J), J=1, N)

GCG 9

READ (1, 100) (FY(J), J=1, N)

GCG 10

READ (1, 100) (AS(J), J=1, N)

GCG 11

READ (1, 100) (FORCE(I), I=1, M)

GCG 12

READ (1, 209) (STC(K), K=1, 5)

GCG 13

READ (1, 209) (CK(KK), KK=1, 10)

GCG 14

READ (1, 200) (AO(J), J=1, N)

GCG 15

READ (1, 200) (AI(J), J=1, N)

GCG 16

READ (1, 200) (A2(J), J=1, N)

GCG 17

READ (1, 200) (A3(J), J=1, N)
CC19 100 FFORMAT (8F10.5)
CC20 209 FFORMAT (5F10.5)
CC21 200 FFORMAT (5E13.6)
CC22 905 WRITE (3,909) (E(I),I=1,N)
CC23 908 FFORMAT (/6X,5X,10F12.5/)
CC24 WRITE (3,908) (D(I),I=1,N)
CC25 908 FFORMAT (/6X,5X,10F12.5/)
CC26 WRITE (3,908) (DP(I),I=1,N)
CC27 WRITE (3,908) (EI(I),I=1,N)
CC28 WRITE (3,908) (FC(J),J=1,N)
CC29 WRITE (3,908) (FY(J),J=1,N)
CC30 WRITE (3,908) (AS(J),J=1,N)
CC31 WRITE (3,908) (FORCE(I),I=1,N)
CC32 B=6.0
CC33 T=6.0
CC34 I=1
CC35 DO 110 J=1,N
CC36 CC 900 II=1,5
CC37 WRITE (3,111) I
CC38 11 FFORMAT (13)
CC39 R=EI(I)/T
CC40 WRITE (3,505)
CC41 505 FFORMAT (1H1,15X,'E/D'/)
CC42 PP=0.
CC43 PP=AS(J)/(AT)
CC44 WRITE (3,506) R
CC45 506 FFORMAT (12X,F6.2)
CC46 P=0.
CC47 P=AS(J)/(E*D(I))
CC48 WRITE (3,710)
CC49 710 FFORMAT (17,5X,'THE VALUE OF P'/)
CC50 WRITE (3,720) P(I)
CC51 720 FFORMAT (12X,F17.6)
CC52 AB(I)=0.
CC53 AB(I)=(87.0)/(87.0+FY(J))*0.85*D(I)
CC54 EC(I)=AB(I)
CC55 EC(I)=EC(I)+EI(I)
CC56 WRITE (3,410)
CC57 410 FFORMAT (17,5X,'THE VALUES OF EC'/)
CC58 WRITE (3,500) EC(I)
CC59 TH(J)=0.
CC60 TH(J)=FY(J)/(0.85*FC(J))
CC61  AK(J)=C.
CC62  AK(J)= ((P(I)*FY(J))/FC(J))
CC63  AKM(J)=0.
CC64  AKM(J)= P(I)*TM(J)
CC65  WRITE (3,810)
CC66  810 FORMAT (///,5X,*THE VALUES FO P*FY/FC* )
CC67  WRITE (3,560) AKM(I)
CC68  WRITE (3,820)
CC69  820 FORMAT (///,5X,*THE VALUE OF P*M )
CC70  WRITE (3,560) AKM(I)
CC71  IF (E(I)*LE.-.0) GO TO 120
CC72  IF (E(I)*GE. 1000.) GO TO 220
CC73  EBI(I)= C.
CC74  TTI(I)= 0.
CC75  CCI(I)= C.
CC76  CCI(I)= 0.
CC77  PUBB(I)=0.
CC78  TTI(I)=ASU(I)*FY(J)
CC79  CCI(I)=ASU(I)* (FY(J)-0.85*FC(J))
CC80  PUBB(I)=CC(I)*CS(I)*TT(I)
CC81  EBI(I)=
& (TTI(I)* (T/2-OP(I))
&+CC(I)* (T/2-AB(I)/2)
&+CS(I)* (T/2-OP(I)))/PUBB(I)
CC82  WRITE (3,430)
CC83  430 FORMAT (///,5X,*THE VALUES OF EBI)
CC84  WRITE (3,500) EBI(I)
CC85  C
C
C*****************************************************************************
C
C DEFINE FAILURE CCNIDON
C
C*****************************************************************************
C
CC66  IF (EC(I)-EBI(I)) 301,302,303
CC87  120 PUA(J)= 0.
CC88  APU(I)= 0.
CC89  AM(J)= 0.
CC90  C
C
C*****************************************************************************
C
C REPEAT TO CONCENTRIC LOADING
C
C*****************************************************************************
C
CC90  PUA(J)= 0.85*FC(J)*(B*T-2*AS(J)) + 2*AS(J)*FY(J)
CC52 WRITE (3,510)
CC53 510 FORMAT (/,5X,'THE VALUES OF PUA')
CC54 WRITE (3,500) PUA(J)
CC55 50C FORMAT (/,5X,F17.6)
CC56 WRITE (3,560)
CC57 560 FORMAT (/,3X,'THE VALUE OF P/PU',2X,'THE VALUE OF M/PU')
GO098 WRITE (3,601) AP(J),AM(J)
GO099 601 FORMAT (/,10X,2F10.2)
C100 AXP(I)=0.
C101 AXM(I)=0.
C102 AXP(I)=FGRCE(I)/(FC(J)*B*T)
C103 WRITE (3,599)
C104 599 FORMAT (/,3X,'THE VALUE OF P*EX*,2X,'THE VALUE OF MEX/P*'U')
C105 WRITE (3,601) AXP(I), AXM(I)
C106 GO TO 400
C107 220 BF(I)=0.
C
************
C
************
C
CC72 COMPUTE ULTIMATE MOMENT & LOAD DUE TO PURE BENDING
C
************
C
C108 221 K=1.5
C109 DC 221 K=1.5
C110 KK=1
C111 C(KK)=CK(KK)*D(I)
C112 281 C(KK)=CK(KK)*D(I)
C113 EK(KK)=0.85%C(KK)
C114 STA(KK)=ABS(STC(KK)*(C(KK)-DP(I))/C(KK))
C115 FS(KK)=A0(J)+A1(J)*STA(KK)+A2(J)*STA(KK)**2
C116 As(J)*STA(KK)**3+As4(J)*STA(KK)**4
C117 FC(KK)=A(S(J)*FS(KK)+0.85*FC(J)*B*DK(KK)-AS(J)*FY(J)
C118 IF (ABSF0(KK)) GT (0.01*AS(J)*FY(J)) GO TO 227
C119 IF (ABSF0(KK)) GT (0.01*AS(J)*FY(J)) GO TO 227
C120 PUB(KK)=AS(J)*FS(KK)*I(DI)-DI[P(I)]
C121 PUB(KK)=AS(J)*FS(KK)*I(DI)-DI[P(I)]
C122 PUB(KK)=As(J)*FS(KK)*I(DI)-DI[P(I)]
C123 PUB(KK)=As(J)*FS(KK)*I(DI)-DI[P(I)]
C124 520 FORMAT (/,5X,'THE VALUES OF PUB')
C125 WRITE (3,500) PUB(KK)
C126 WRITE (3,560) PUB(KK)
C127 WRITE (3,601) BF(I), BM(KK)
C128 B(KK)=K(KK)*2.5
C129 IF (K(KK)LE.10) GO TO 281
C130 CONTINUE
C127  BXF(I)=0.
C128  BXM(I)=0.
C129  XBM(I)=0.
C130  XBM(I)=FORCE(I)*53./6.
  BXP(I)=XBM(I)/(FC(J)*B*F*T)
C132  WRITE (3,501) BXP(I),BXM(I)
C133  GC TO 400
C134  301 PLC(I)=0.
C135  CP(I)=0.
C136  CM(I)=C.
C137
C138

******************************************************************************

C  COMPUTE THE ULTIMATE LOAD DUE TO ECCENTRIC LOADING(COMP.FAILED)

******************************************************************************

C139  PUC(I)=AS(J)*FY(J)/((EC(I)/(D(I)-DP(I)))+0.5)
  &*B*T*FC(J)/((3*B*EC(I)/(D(I)**2)))+1.*18.
  CP(I)=
    CM(I)=CP(I)*EC(I)/T
C140  & PUC(I)/(FC(J)*B*T)
C141  WRITE (3,530)
C142  FCRMAT (J,5X,THE VALUES OF PUC)
C143  WRITE (3,500) PUC(I)
C144  WRITE (3,560)
C145  WRITE (3,560) CP(I),CM(I)
C146  CXP(I)=0.
C147  CXM(I)=0.
C148  CXP(I)=FCRCE(I)/(FC(J)*B*T)
C149  CXM(I)=CXP(I)*EC(I)/T
C150  WRITE (3,599)
C151  WRITE (3,501) CXP(I),CXM(I)
C152  GC TO 400
C153  DCP(I)=0.
C154  LCM(I)=0.
C155
C  COMPUTE THE ULTIMATE LOAD DUE TO BALANCED CONDITION

******************************************************************************

C156  DDP(I)=
    & PUBB(I)/(FC(J)*B*T)
C157  LCM(I)=CDP(I)*EC(I)/T
0128  540 FORMAT (/";5X;"THE VALUES OF PUBB")
0129  WRITE (3,500) PUBB(I)
C160  WRITE (3,560)
0161  WRITE (3,601) DDP(I),DDM(I)
0162  EXP(I)=0.
C163  DXM(I)=0.
0164  DXP(I)=FORCE(I)/(FC(J)*B*T)
C165  CXM(I)=DXP(I)*EC(I)/T
C166  WRITE (3,599)
0167  WRITE (3,601) DXP(I),DXM(I)
C168  GC TO 400
C169  303 PUT(I)=0.
C170  EP(I)=0.
C171  EEP(I)=0.
C172  EM(I)=0.
C173  EP(I)=EC(I)+3.-DP(I)
C174  ******************************************
C175  COMPUTE THE ULTIMATE LOAD DUE TO ECCENTRIC LOADING
C176  (TENSION FAILURE)
C177  ******************************************
.C174
C175  PUT(I)=C.85*FC(J)*B*D(I)*(-P(I)+1.0-EP(I)/D(I))
0176  &+SQRT((1.0+EP(I)/D(I))**2+
C177  62.0*P(I)*((TM(J)-1.0)*(1.0-EP(I)/D(I))**2+EP(I)/D(I))))
.178  EEP(I)=PUT(I)/(FC(J)*B*T)
C179  EM(I)=EEP(I)*EC(I)/T
C180  WRITE (3,500) PUT(I)
C181  WRITE (3,601) EEP(I),EM(I)
C182  EXP(I)=0.
C183  EXM(I)=0.
0184  EXP(I)=FORCE(I)/(FC(J)*B*T)
0185  EXP(I)=EXP(I)*EC(I)/T
C186  WRITE (3,599)
0187  WRITE (3,601) EXP(I),EXM(I)
C188  GC TO 400
C189  400 I=I+1
0190  900 CONTINUE
C191  110 CONTINUE
B). PROGRAM FOR STRAIN-GRADIENT METHOD

The arguments used in the program are defined as follows:-

STC = Strain of Concrete
AK = Factor
AS = Area of tension steel, sq.in.
EC = Eccentricity of load with respect to plastic centroid of section
DP = Distance from extreme compression fibre to centroid of tension reinforcement
D = Distance from extreme compression fibre to centroid of tension reinforcement
DK = Distance from extreme compression fibre to neutral axis at ultimate strength
STA = Strain in tension reinforcement
STB = Strain in compression reinforcement
FY = Yield strength of tension reinforcement, psi.
FC = Compressive strength of concrete cylinders, psi.
FS = Tensile steel stress, psi.
AB = Depth of equivalent rectangular stress block = $K_1 c$
AO — A4
B1 — B4  Coefficients of equation
C1 — C4

BM0 = Internal resisting moment, kip-in.

BM1 = Internal resisting moment in terms of the strength of steel, kip-in.

BM2 = Internal resisting moment in terms of the strength of compression steel, kip-in.

PU = P_u, Ultimate load, kips.
CALCULATIONS OF ULTIMATE AXIAL LOAD AND ULTIMATE MOMENT
FROM THE STRAIN GRADIENT METHOD FOR REINFORCING CONCRETE
COLUMNS SUBJECTED TO AXIAL AND BENDING

0001

DIMENSION FORCE(50), STC(6), AK(45), AS(50), EC(50), DP(50),
& D(50), DK(3000), STA(3000), STB(3000), FCAL(3000),
& FCA(3000), FSB(3000), FSC(3000), MCAL(3000),
& MMAX(50), PEPS(50), MEPS(50),
& FC1(3000), FC2(3000),
& FY(3000), FC(3000), FS(3000), AB(3000), FC3(3000), FC4(3000),
& AO(50), AI(50), A2(50), FC5(3000), FC6(3000), A3(50), FC10(3000),
& C0(50), CI(50), C2(50), C3(50), R(50), FC7(3000), FC8(3000), FC9(3000)
& A4(50),
   C4(50), F0(300), BM0(300), BM1(300), BM2(300), PJ(300)

0002

REAL MCAL, MMAX, MEPS

0003

READ(1,100) (FORCE(I), I=1,35)

0004

FORMAT(8F10.5)

0005

READ(1,100) (D(I), I=1,35)

0006

READ (1,100) (EC(I), I=1,35)

0007

READ (1,100) (AS(I), I=1,35)

0008

READ (1,100) (DP(I), I=1,35)

0009

READ (1,100) (AK(J), J=1,42)

0010

READ (1,100) (STC(K), K=1,6)

0011

READ (1,100) (RI(I), I=1,35)

0012

READ (1,299) (AO(I), I=1,35)

0013

FORMAT (5E13.6)

0014

READ (1,299) (AI(I), I=1,35)

0015

READ (1,299) (A2(I), I=1,35)

0016

READ (1,299) (A3(I), I=1,35)

0017

READ (1,299) (A4(I), I=1,35)

0018

READ (1,200) (C0(I), I=1,35)

0019

READ (1,200) (CI(I), I=1,35)

0020

READ (1,200) (C2(I), I=1,35)

0021

READ (1,200) (C3(I), I=1,35)

0022

READ (1,200) (C4(I), I=1,35)

0023

200 FORMAT (6E13.6)

0024

WRITE (3,208)

0025

208 FORMAT(///,20X,'FORCE')

0026

WRITE (3,202) (FORCE(I), I=1,35)

0027

202 FORMAT(///,2X,8F16.6)

0028

WRITE(3,201)

0029

201 FORMAT (///,2X,'THEVALES OF D(I)'\})
0030 WRITE (3,202) (O(I),I=1,35)
0031 WRITE (3,204)
0032 204 FORMAT (///,2X,*THE VALVES OF EC*)
0033 WRITE (3,202) (EC(I),I=1,35)
0034 WRITE (3,205)
0035 205 FORMAT (///,2X,*AS*)
0036 WRITE (3,202) (AS(I),I=1,35)
0037 WRITE (3,601)
0038 601 FORMAT (///,2X,*DP*)
0039 WRITE (3,202) (DP(I),I=1,35)
0040 WRITE (3,206)
0041 206 FORMAT (///,2X,*AK*)
0042 WRITE (3,202) (AK(I),I=1,42)
0043 WRITE (3,207)
0044 207 FORMAT (///,20X,*STC*')
0045 WRITE (3,202) (STC(K),K=1,6)
0046 WRITE (3,302)
0047 302 FORMAT (///,20X,*AO*')
0048 WRITE (3,509) (AO(I),I=1,35)
0049 WRITE (3,303)
0050 303 FORMAT (///,*A1*')
0051 WRITE (3,509) (A1(I),I=1,35)
0052 WRITE (3,304)
0053 304 FORMAT (///,*A2*')
0054 WRITE (3,509) (A2(I),I=1,35)
0055 WRITE (3,305)
0056 305 FORMAT (///,*A3*')
0057 WRITE (3,509) (A3(I),I=1,35)
0058 WRITE (3,306)
0059 306 FORMAT (///,*A4*')
0060 WRITE (3,509) (A4(I),I=1,35)
0061 WRITE (3,406)
0062 406 FORMAT (///,*C0*')
0063 WRITE (3,509) (C0(I),I=1,35)
0064 WRITE (3,407)
0065 407 FORMAT (///,*C1*')
0066 WRITE (3,509) (C1(I),I=1,35)
0067 WRITE (3,408)
0068 408 FORMAT (///,*C2*')
0069 WRITE (3,509) (C2(I),I=1,35)
0070 WRITE (3,409)
0071 409 FORMAT (///,*C3*')
0072 WRITE (3,509) (C3(I),I=1,35)

217
C

0074  501 FORMAT ('//,'C4*')
0075  WRITE(3,509) (C4(I),I=1,35)
0076  509 FORMAT ('//,2X,8E13.6/')

C

0077  B=6.
0078  T=6.

C

0079  DO 1000 I=26,35
0080    PEPS(I)=0.
0081    PEPS(I)=PEPS(I) +0.01*FORCE(I)
0082    MMAX(I)=0.
0083    MMAX(I)=MMAX(I)+FORCE(I)*EC(I)
0084    MEPS(I)=0.
0085    MEPS(I)=MEPS(I) +0.01*MMAX(I)

C

0086  WRITE (3,199)
0087    19 FORMAT ('//,5X,'E/D*)
0088  WRITE (3,299) R(I)
0089  29 FORMAT ('//,F8.4')
0090  GO 999 K=1,J=1
0091  61 IF (STC(K).LT.0.0001) GO TO 999
0092  IF (STC(K).GT.0.003) GO TO 999
0093  26 IF (J.GT.42) GO TO 999
0094  DK(J)=AK(J)*O(I)
0095  STA(J)=0.
0096  STB(J)=0.
0097  FCA(J)=0.
0098  FSB(J)=0.
0100  FSC(J)=0.
0101  FCA(J)=0.
0102  MCAL(J)=0.
0103  FO(J)=0.
0104  BM0(J)=0.
0105  BM1(J)=0.
0106  BM2(J)=0.
0107  PU(J)=0.
0108  AB(J)=0.
0109  FY(J)=0.
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
</tr>
</thead>
<tbody>
<tr>
<td>0110</td>
<td>FS(T)=0.</td>
</tr>
<tr>
<td>0111</td>
<td>FC(J)=0.</td>
</tr>
<tr>
<td>0112</td>
<td>AB(J)=0.85*DK(J)</td>
</tr>
<tr>
<td>0113</td>
<td>IF (DK(J)-D(I)) 50,150,250</td>
</tr>
<tr>
<td>0114</td>
<td>50 IF (DK(J)-DP(I)) 10,20,30</td>
</tr>
<tr>
<td>0115</td>
<td>10 STA(J)=STA(J)+STC(K)*I(I,/AK(J)-1)</td>
</tr>
<tr>
<td>0116</td>
<td>STB(J)=STB(J)+STC(K)*(DP(I)-DK(J))/DK(J)</td>
</tr>
<tr>
<td>0117</td>
<td>IF (STA(J)+GT+0.1 OR. STB(J)+GT+0.1) GO TO 101</td>
</tr>
<tr>
<td>0118</td>
<td>FY(J)=FY(J)+A0(I)+A1(I)*STA(J)+A2(I)*STC(J)**2</td>
</tr>
<tr>
<td>0119</td>
<td>&amp;+A3(I)*STA(J)**3+4(A(I)*STA(J)**4</td>
</tr>
<tr>
<td>0120</td>
<td>FC1(J)=CO(I)+C1(I)<em>19/20</em>STC(K)+C2(I)<em>((19/20</em>STC(K))**2</td>
</tr>
<tr>
<td>0121</td>
<td>&amp;+C3(I)<em>((19/20</em>STC(K))**3+C4(I)<em>((19/20</em>STC(K))**4</td>
</tr>
<tr>
<td>0122</td>
<td>FC2(J)=CO(I)+C1(I)+17/20<em>STC(K)+C2(I)</em>((17/20*STC(K))**2</td>
</tr>
<tr>
<td>0123</td>
<td>&amp;+C3(I)<em>((17/20</em>STC(K))**3+C4(I)<em>((17/20</em>STC(K))**4</td>
</tr>
<tr>
<td>0124</td>
<td>FC3(J)=CO(I)+C1(I)+15/20<em>STC(K)+C2(I)</em>((15/20*STC(K))**2</td>
</tr>
<tr>
<td>0125</td>
<td>&amp;+C3(I)<em>((15/20</em>STC(K))**3+C4(I)<em>((15/20</em>STC(K))**4</td>
</tr>
<tr>
<td>0126</td>
<td>FC4(J)=CO(I)+C1(I)+13/20<em>STC(K)+C2(I)</em>((13/20*STC(K))**2</td>
</tr>
<tr>
<td>0127</td>
<td>&amp;+C3(I)<em>((13/20</em>STC(K))**3+C4(I)<em>((13/20</em>STC(K))**4</td>
</tr>
<tr>
<td>0128</td>
<td>FC5(J)=CO(I)+C1(I)+11/20<em>STC(K)+C2(I)</em>((11/20*STC(K))**2</td>
</tr>
<tr>
<td>0129</td>
<td>&amp;+C3(I)<em>((11/20</em>STC(K))**3+C4(I)<em>((11/20</em>STC(K))**4</td>
</tr>
<tr>
<td>0130</td>
<td>FC6(J)=CO(I)+C1(I)+9/20<em>STC(K)+C2(I)</em>((9/20*STC(K))**2</td>
</tr>
<tr>
<td>0131</td>
<td>&amp;+C3(I)<em>((9/20</em>STC(K))**3+C4(I)<em>((9/20</em>STC(K))**4</td>
</tr>
<tr>
<td>0132</td>
<td>FC7(J)=CO(I)+C1(I)+7/20<em>STC(K)+C2(I)</em>((7/20*STC(K))**2</td>
</tr>
<tr>
<td>0133</td>
<td>&amp;+C3(I)<em>((7/20</em>STC(K))**3+C4(I)<em>((7/20</em>STC(K))**4</td>
</tr>
<tr>
<td>0134</td>
<td>FC8(J)=CO(I)+C1(I)+5/20<em>STC(K)+C2(I)</em>((5/20*STC(K))**2</td>
</tr>
<tr>
<td>0135</td>
<td>&amp;+C3(I)<em>((5/20</em>STC(K))**3+C4(I)<em>((5/20</em>STC(K))**4</td>
</tr>
<tr>
<td>0136</td>
<td>FC9(J)=CO(I)+C1(I)+3/20<em>STC(K)+C2(I)</em>((3/20*STC(K))**2</td>
</tr>
<tr>
<td>0137</td>
<td>&amp;+C3(I)<em>((3/20</em>STC(K))**3+C4(I)<em>((3/20</em>STC(K))**4</td>
</tr>
<tr>
<td>0138</td>
<td>FC10(J)=CO(I)+C1(I)+1/20<em>STC(K)+C2(I)</em>((1/20*STC(K))**2</td>
</tr>
<tr>
<td>0139</td>
<td>&amp;+C3(I)<em>((1/20</em>STC(K))**3+C4(I)<em>((1/20</em>STC(K))**4</td>
</tr>
<tr>
<td>0140</td>
<td>FC(J)=1.0*FC1(J)+FC2(J)+FC3(J)+FC4(J)+FC5(J)+FC6(J)+FC7(J)</td>
</tr>
<tr>
<td>0141</td>
<td>&amp;+FC8(J)+FC9(J)+FC10(J)</td>
</tr>
<tr>
<td>0142</td>
<td>FCA(J)=FC(J)<em>B</em>DK(J)</td>
</tr>
<tr>
<td>0143</td>
<td>FSB(J)=FSB(J)+AS(I)*FY(J)</td>
</tr>
<tr>
<td>0144</td>
<td>FSC(J)=FSC(J)+AS(I)*FS(J)</td>
</tr>
<tr>
<td>0145</td>
<td>WRITE (3,7)</td>
</tr>
<tr>
<td>0146</td>
<td>7 FORMAT (/,,10X,'FSC',10X,'FSC',10X,'FCA')</td>
</tr>
<tr>
<td>0147</td>
<td>WRITE (3,9) FSB(J),FSC(J),FCA(J)</td>
</tr>
</tbody>
</table>
0166  FC5(JJ)= C0(I)+C1(I)*11/20*STC(K)+C2(I)*3/20*STC(K)***2
      &+C3(I)*((11/20*STC(K))**4+C4(I)*3/20*STC(K))**4
0167  FC6(JJ)= C0(I)+C1(I)*9/20*STC(K)+C2(I)*9/20*STC(K)***2
      &+C3(I)*((9/20*STC(K))**4+C4(I)*9/20*STC(K))**4
0168  FC7(JJ)= C0(I)+C1(I)*7/20*STC(K)+C2(I)*7/20*STC(K)***2
      &+C3(I)*((7/20*STC(K))**4+C4(I)*7/20*STC(K))**4
0169  FC8(JJ)= C0(I)+C1(I)*5/20*STC(K)+C2(I)*5/20*STC(K))**2
      &+C3(I)*((5/20*STC(K))**4+C4(I)*5/20*STC(K))**4
0170  FC9(JJ)= C0(I)+C1(I)*3/20*STC(K)+C2(I)*3/20*STC(K)***2
      &+C3(I)*((3/20*STC(K))**4+C4(I)*3/20*STC(K))**4
0171  FC10(JJ)=C0(I)+C1(I)*1/20*STC(K)+C2(I)*1/20*STC(K))**2
        &+C3(I)*((1/20*STC(K))**4+C4(I)*1/20*STC(K))**4
0172  FC(J)=1./10.*(FC1(J)+FC2(J)+FC3(J)+FC4(J)+FC5(J)+FC6(J)+FC7(J)
        &+FC8(J)+FC9(J)+FC10(J))

C
0173  FCA(J)=FC(J)*B*DK(J)
0174  FSB(J)=FSB(J)+AS(J)*FY(J)
0175  FSC(J)=0.
0176  IF(EC(I).LT.1000.) GO TO 330
0177  F0(J)=F0(J)+FSB(J)+FCA(J)+FSC(J)
0178  IF(ABS(F0(J)).LE.(0.01*FCA(J))) GO TO 130
0179  J=J+1
0180  GO TO 26

C
0181  130 BM1(J)=BM1(J)+FSB(J)*(D(I)-AB(J)/2)
0182  BM2(J)=BM2(J)+FSC(J)*(DP(I)-AB(J)/2)
0183  BM0(J)=BM0(J)+BM1(J)+BM2(J)
0184  PU(J)=PU(J)+S*BM0(J)/53.
0185  IF(ABS(PU(J)-FORCE(I)).LE.PEPS(I)) GO TO 210
0186  J=J+1
0187  GO TO 26

C
0188  330 FCAL(J)=FCAL(J)+FCA(J)-FSC(J)-FSB(J)
0189  MCAL(J)=MCAL(J)+FSB(J)*(T/2-OP(I))
      &-FSC(J)*(T/2-OP(I))
0190  &+FCA(J)*(T/2-AB(J)/2)
0190  GO TO 300

C
0191  3G STA(J)=STA(J)+STC(K)*D(I)/DK(J)-STC(K)
0192  STB(J)=STB(J)+((DK(J)-DP(I))*STC(K))/DK(J)

21
IF (STA(J) GT 0.1 OR STA(J) GT 0.1) GO TO 101

0154
C  C
FY(J) = FY(J) + A0(I) + A1(I) * STA(J) + A2(I) * STA(J)**2
& A3(I) * STA(J)**3 + A4(I) * STA(J)**4
FS(J) = FS(J) + A0(I) + A1(I) * STB(J) + A2(I) * STB(J)**2
& A3(I) * STB(J)**3 + A4(I) * STB(J)**4

0196
FC1(J) = C0(I) + C1(I) * 19/20 * STC(K) + C2(I) * (19/20 * STC(K))**2
& C3(I) * (19/20 * STC(K))**3 + C4(I) * (19/20 * STC(K))**4

0197
FC2(J) = C0(I) + C1(I) * 17/20 * STC(K) + C2(I) * (17/20 * STC(K))**2
& C3(I) * (17/20 * STC(K))**3 + C4(I) * (17/20 * STC(K))**4

0198
FC3(J) = C0(I) + C1(I) * 15/20 * STC(K) + C2(I) * (15/20 * STC(K))**2
& C3(I) * (15/20 * STC(K))**3 + C4(I) * (15/20 * STC(K))**4

0199
FC4(J) = C0(I) + C1(I) * 13/20 * STC(K) + C2(I) * (13/20 * STC(K))**2
& C3(I) * (13/20 * STC(K))**3 + C4(I) * (13/20 * STC(K))**4

0200
FC5(J) = C0(I) + C1(I) * 11/20 * STC(K) + C2(I) * (11/20 * STC(K))**2
& C3(I) * (11/20 * STC(K))**3 + C4(I) * (11/20 * STC(K))**4

0201
FC6(J) = C0(I) + C1(I) * 9/20 * STC(K) + C2(I) * (9/20 * STC(K))**2
& C3(I) * (9/20 * STC(K))**3 + C4(I) * (9/20 * STC(K))**4

0202
FC7(J) = C0(I) + C1(I) * 7/20 * STC(K) + C2(I) * (7/20 * STC(K))**2
& C3(I) * (7/20 * STC(K))**3 + C4(I) * (7/20 * STC(K))**4

0203
FC8(J) = C0(I) + C1(I) * 5/20 * STC(K) + C2(I) * (5/20 * STC(K))**2
& C3(I) * (5/20 * STC(K))**3 + C4(I) * (5/20 * STC(K))**4

0204
FC9(J) = C0(I) + C1(I) * 3/20 * STC(K) + C2(I) * (3/20 * STC(K))**2
& C3(I) * (3/20 * STC(K))**3 + C4(I) * (3/20 * STC(K))**4

0205
FC10(J) = C0(I) + C1(I) * 1/20 * STC(K) + C2(I) * (1/20 * STC(K))**2
& C3(I) * (1/20 * STC(K))**3 + C4(I) * (1/20 * STC(K))**4

0206
FC(J) = 1.10 * (FC1(J) + FC2(J) + FC3(J) + FC4(J) + FC5(J) + FC6(J) + FC7(J)
& FC8(J) + FC9(J) + FC10(J))

C  C
FCA(J) = FC(J) * B*OK(J)
0208
FSB(J) = FSB(J) + FY(J) * AS(I)
0209
FSC(J) = FSC(J) + FS(J) * AS(I)

C  C
0210
IF (EC(I) LT 1000.0) GO TO 630
0211
FO(J) = FO(J) + FSB(J) + FCA(J) - FSC(J)
0212
IF (ABSFO(J)) LE. (0.01 * FSB(J)) GO TO 430
0213
J = J + 1
0214
GO TO 26

C  C
0215
BMI(J) = BMI(J) + (FSB(J) - FSC(J)) * (D(I) - AB(I) / 2)
0216
BM2(J) = BM2(J) + FSC(J) * (D(I) - DP(I))
0217
BMO(J) = BMO(J) + BMI(J) + BM2(J)
PU(J) = PU(J) + 6 * RMO(J) / 51, 

IF (ABS(PU(J) - FORCE(I))) - PEPS(I)) GO TO 210

J = J + 1
GO TO 26

C

630 FCAL(J) = FCAL(J) + FCA(J) - FSB(J) + FSC(J)

MCAL(J) = MCAL(J) + FCA(J) * (T/2 - AB(J) / 2)

& * FSB(J) * (T/2 - DP(I))

GO TO 300

C

150 STA(J) = STA(J)

STB(J) = STB(J) + (STC(K) / DK(J)) * (DK(J) - DP(I))

IF (STA(J) .GT. 0.1) OR (STB(J) .GT. 0.1) GO TO 101

C

FY(J) = FY(J) + A0(I) + A1(I) * STA(J) + A2(I) * STA(J)**2

& + A3(I) * STA(J)**3 + A4(I) * STA(J)**4

FS(J) = FS(J) + A0(I) + A1(I) * STB(J) + A2(I) * STB(J)**2

& + A3(I) * STB(J)**3 + A4(I) * STB(J)**4

FC1(J) = C0(I) + C1(I) * 19/20 * STC(K) + C2(I) * (19/20 * STC(K))**2

& + C3(I) * (19/20 * STC(K))**3 + C4(I) * (19/20 * STC(K))**4

FC2(J) = C0(I) + C1(I) * 17/20 * STC(K) + C2(I) * (17/20 * STC(K))**2

& + C3(I) * (17/20 * STC(K))**3 + C4(I) * (17/20 * STC(K))**4

FC3(J) = C0(I) + C1(I) * 15/20 * STC(K) + C2(I) * (15/20 * STC(K))**2

& + C3(I) * (15/20 * STC(K))**3 + C4(I) * (15/20 * STC(K))**4

FC4(J) = C0(I) + C1(I) * 13/20 * STC(K) + C2(I) * (13/20 * STC(K))**2

& + C3(I) * (13/20 * STC(K))**3 + C4(I) * (13/20 * STC(K))**4

FC5(J) = C0(I) + C1(I) * 11/20 * STC(K) + C2(I) * (11/20 * STC(K))**2

& + C3(I) * (11/20 * STC(K))**3 + C4(I) * (11/20 * STC(K))**4

FC6(J) = C0(I) + C1(I) * 9/20 * STC(K) + C2(I) * (9/20 * STC(K))**2

& + C3(I) * (9/20 * STC(K))**3 + C4(I) * (9/20 * STC(K))**4

FC7(J) = C0(I) + C1(I) * 7/20 * STC(K) + C2(I) * (7/20 * STC(K))**2

& + C3(I) * (7/20 * STC(K))**3 + C4(I) * (7/20 * STC(K))**4

FC8(J) = C0(I) + C1(I) * 5/20 * STC(K) + C2(I) * (5/20 * STC(K))**2

& + C3(I) * (5/20 * STC(K))**3 + C4(I) * (5/20 * STC(K))**4

FC9(J) = C0(I) + C1(I) * 3/20 * STC(K) + C2(I) * (3/20 * STC(K))**2

& + C3(I) * (3/20 * STC(K))**3 + C4(I) * (3/20 * STC(K))**4

FC10(J) = C0(I) + C1(I) * 1/20 * STC(K) + C2(I) * (1/20 * STC(K))**2

& + C3(I) * (1/20 * STC(K))**3 + C4(I) * (1/20 * STC(K))**4

FC(J) = 1.10 + *(FC1(J) + FC2(J) + FC3(J) + FC4(J) + FC5(J) + FC6(J) + FC7(J)

& + FC8(J) + FC9(J) + FC10(J)
<table>
<thead>
<tr>
<th>Line</th>
<th>Code</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>0263</td>
<td>IF (T, LE, AB(J)) AB(J) = T</td>
<td></td>
</tr>
<tr>
<td>0264</td>
<td>FSB(J) = FSB(J) + FY(J) * AS(I)</td>
<td></td>
</tr>
<tr>
<td>0265</td>
<td>FSC(J) = FSC(J) + FS(J) * AS(I)</td>
<td></td>
</tr>
<tr>
<td>0266</td>
<td>FCA(J) = FCA(J) + B * OK(J)</td>
<td></td>
</tr>
<tr>
<td>0267</td>
<td>FCAL(J) = FCAL(J) + FCA(J) + FSC(J) + FSB(J)</td>
<td></td>
</tr>
<tr>
<td>0268</td>
<td>MCAL(J) = MCAL(J) + FCA(J) * (T / 2 - AB(J) / 2)</td>
<td></td>
</tr>
<tr>
<td>0269</td>
<td>GO TO 300</td>
<td></td>
</tr>
<tr>
<td>0270</td>
<td>300 IF (ABS(FCAL(J) - FORCE(I)).LE.PEPS(I)) GO TO 60</td>
<td></td>
</tr>
<tr>
<td>0271</td>
<td>J = J + 1</td>
<td></td>
</tr>
<tr>
<td>0272</td>
<td>GO TO 26</td>
<td></td>
</tr>
<tr>
<td>0273</td>
<td>WRITE (3, 59)</td>
<td></td>
</tr>
<tr>
<td>0277</td>
<td>WRITE (3, 69)</td>
<td></td>
</tr>
<tr>
<td>0280</td>
<td>WRITE (3, 79)</td>
<td></td>
</tr>
<tr>
<td>0282</td>
<td>WRITE (3, 600) DK(J), D(I), STA(J), STB(J), FCA(J), 6FSB(J), FSC(J)</td>
<td></td>
</tr>
<tr>
<td>0283</td>
<td>600 FORMAT ('//', 5X, 7F17.6)</td>
<td></td>
</tr>
<tr>
<td>0284 999 CONTINUE</td>
<td></td>
<td></td>
</tr>
<tr>
<td>0285 1000 CONTINUE</td>
<td></td>
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</tr>
<tr>
<td>0286</td>
<td>RETURN</td>
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</tr>
<tr>
<td>0287</td>
<td>END</td>
<td></td>
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</tbody>
</table>