HIGH TEMPERATURE TRANSPORT MEASUREMENTS

ON n-TYPE GaSb AND GaAs

by

Joseph Basinski

Ph.D. Thesis 1972

LIST OF CONTENTS

ABSTRACT i
ACKNOWLEDGEMENTS iii
LIST OF TABLES iv
LIST OF FIGURES vii

CHAPTER I: INTRODUCTION 1
CHAPTER II: THEORY 14
CHAPTER III: EXPERIMENTAL TECHNIQUE AND EQUIPMENT 41
CHAPTER IV: CONDUCTION BANDS IN GaSb 67
CHAPTER V: CONDUCTION BANDS IN GaAs 137

APPENDIX A 178
REFERENCES 180
ABSTRACT

The considerable variation reported in the literature for the $\Gamma_1 - L_1$ minima separation and its temperature dependence in GaSb, suggested the need for further critical study to resolve these apparent discrepancies.

In this work measurements of magnetoresistance and Hall coefficient as a function of magnetic field and temperature were made on samples of Tellurium doped GaSb in the temperature range 25°C to 200°C. Also the Hall coefficient at constant magnetic field and the zero field conductivity were measured up to a temperature of 360°C on these n-type samples and up to 550°C on an undoped p-type sample. The results were analysed on the basis of a model including three conduction bands with minima at $\Gamma_1$, $L_1$ and $X_1$, and two valence bands. The non-parabolicity of the $\Gamma_1$ conduction band was taken into account. A zero temperature $\Gamma_1 - L_1$ band separation of 0.099 eV and a temperature dependence of $-3.4 \times 10^{-5}$ eV/°K was obtained. Also, the temperature variation of electron mobilities in the $\Gamma_1$ and $L_1$ minima was obtained from the results. This temperature variation of electron mobility $\mu_1(T)$ in the $\Gamma_1$ band was explained using a model involving four scattering mechanisms; viz. polar optical, screened Coulomb, interband and spacecharge. It is shown that the anomalously low electron mobility in the $\Gamma_1$ band of GaSb reported in the literature, can be understood when proper account is taken of the screened Coulomb scattering by both ionized impurities and the heavy electrons in the $L_1$ minima. These calculations also give an estimate of the value of the interband coupling constant, $D_{12}$, of $4.1 \times 10^8$ eV/cm for scattering between $\Gamma_1$ and $L_1$ bands.

It is well established that the lowest conduction band in GaAs has a minimum at the centre of the Brillouin zone. However, the positions in k-space of the next lowest set of minima have been in doubt. Experimental results strongly
favour these subsidiary minima to be in the <100> directions. On the other hand, calculations based on various theoretical models predict the sets of minima to lie in either or both <100> and <111> directions.

An attempt is made here to resolve this problem by making Hall coefficient and conductivity measurements on a number of heavily Tellurium doped n-type GaAs samples in the temperature range 20° to 600° C. The results were analysed using three possible conduction band models. From these analyses it was concluded that the most probable model involves the \( \Gamma_1 \), \( \Delta_1 \) and \( L_1 \) minima with absolute zero temperature band separations of 0.381 eV and 0.488 eV for the \( \Gamma_1 - \Delta_1 \) and \( \Gamma_1 - L_1 \) respectively.
ACKNOWLEDGEMENTS

First of all I wish to express my sincere thanks to my supervisor, Professor J. C. Woolley for his support, guidance and encouragement during the course of this work.

The help of all the professors in this department is deeply appreciated, in particular that of Professor Y. P. Varshni and Dr. K. S. Song for many discussions on scattering and band structure. I have benefited also from many stimulating and helpful discussions with my colleagues, especially Dr. C. C. Y. Kwan, Mr. R. Glinski, Mr. S. Rosenbaum and Mr. D. Demars.

I also wish to thank Miss L. Sawyer and Mrs. D. Perron for typing this manuscript.

The capable and prompt attention of all the workshop staff is gratefully acknowledged, as well as the able assistance of R. Hart.

I would also like to express my thanks to my wife Joan, daughters Teresa and Jane for their patience during the course of this work.

Finally, I acknowledge the financial assistance of the National Research Council of Canada through the P.I.E.R. Fellowship.
LIST OF TABLES

I-1 The values of $E_2$ and $\alpha_2$ reported in the literature. 12

II-1 Definitions of effective masses $m^*_e$, $m^*_c$ and $m^*_d$. 40

IV-1 The slopes (t, s) and intercepts (u, a) obtained from $\rho_o/\Delta \rho$ and $-R_o/\Delta R$ vs $B^{-2}$ plots and the zero field Hall coefficient $R_o$ and $\sigma_o$ for sample N2 of n-GaSb. 106

IV-2 Comparison of calculated values of $R_1$, $R_2$, $\sigma_1$ and $\sigma_2$ using the three combinations of experimental parameters (i) $R_o$, $\sigma_o$, s, t; (ii) $R_o$, $\sigma_o$, s, a; (iii) $R_o$, $\sigma_o$, t, u. 107

IV-2a Calculated values of $R_1$, $R_2$, $\sigma_1$ and $\sigma_2$ using magnetoresistance results together with zero field values of Hall coefficient ($R_o$) and conductivity ($\sigma_o$), i.e. combination (iii) $R_o$, $\sigma_o$, t and u. 108

IV-3 Calculated values of $n_1$, $n_2$, $\mu_1$ and $\mu_2$ using the values of Table IV-2a for $R_1$, $R_2$, $\sigma_1$ and $\sigma_2$ and assuming that $r_1 = 1.00$ and $r_2 = 1.18$, both constant with temperature. The value of $n_t$ is obtained by adding $n_1$ and $n_2$. 109

IV-4 Band parameters used in fitting the GaSb data on sample N2. The list of references on the right give the source from which the parameters were obtained. 110
IV-5 Results of fitting the temperature variation of $n_1$ given in Table IV-3 and using the band parameters given in Table IV-4, together with $E_{20} = 0.099 \, \text{eV}$, $\alpha_2 = -3.4 \times 10^{-5} \, \text{eV/K}$ and $n_t = 1.45 \times 10^{24} \, \text{m}^{-3}$.

IV-6 The results of analysis of measurements at 78$^0$K.

IV-7 The values of carrier concentrations $n_1$, $n_2$, $n_3$ and mobility $\mu_1$ and $\mu_2$ determined from fitting $R(T)$ at 0.87 Wb/m$^2$ and $\sigma_0(T)$ with $\mu_1$ and $\mu_2$ adjustable. Remaining parameters were those of Table IV-4 in addition to $E_{20} = 0.099 \, \text{eV}$, $\alpha_2 = -3.4 \times 10^{-5} \, \text{eV/K}$ and $n_t = 1.45 \times 10^{24} \, \text{m}^{-3}$.

IV-8 Parameters of the $<100>$ minima in three semiconductors Si, GaP and GaAs and estimates of the temperature dependence $\alpha_3$ ($\equiv \alpha_3'$) using these values together with $\alpha_0 = -5.0 \times 10^{-4} \, \text{eV/K}$ and $\theta_o = 112^0$K for the fundamental gap temperature variation of GaSb (c. f. Table IV-4).

IV-9 The conduction band parameters of the $\Gamma_1$ band used in calculations of electron mobility, $\mu_1$, in this band.

IV-10 Carrier concentrations and electron mobilities $\mu_1$ and $\mu_2$ determined from fitting experimental data on sample P1.

V-1 Room temperature values of $n_1$, $\sigma_1$ and $\mu_1$ for the five samples of n-GaAs.

V-2 List of band parameters for GaAs together with the references from which these were obtained.
Results of least square fitting of the optical data for the $E_1$ and $E_2$ peaks as a function of temperature from wavelength modulation reflectance data of Walter et al. (70W1)

The mobility, its temperature dependence and scattering parameter used in fitting the GaAs data.

Results of fitting to the three conduction band model, $\Gamma_1$, $L_1$ and $A_1$.

Results of fitting to the three conduction band model, $\Gamma_1$, $L_1$ and $X_1$.

Results of fitting to the two conduction band model, $\Gamma_1$ and $A_1$.

Approximate calculations of mobility $\mu_1(T)$ of the electrons in the $\Gamma_1$ band, for sample M0807/8 of GaAs. The ionized impurity scattering mobility $\mu_1$ was calculated using $N_I = 1.24 \times 10^{24} \text{ m}^{-3}$ and Ehrenreich-Moore correction factor of 1.45. The lattice mobility $\mu_2$ was taken from Ikoma (70I1) and acoustic mobility was calculated assuming the deformation potential of 7 eV.

Summary of results for the three models of conduction band structure of GaAs.

Comparison of analysis of Harland's (65H1) data of magneto-resistance as a function of magnetic field and Hall coefficient at 4.2 K.
LIST OF FIGURES

III-1 Temperature-controlled furnace. 55
III-2 Temperature-controlled furnace-sample area. 56
     Detail of construction.
III-3 Temperature control for controlled furnace. 57
III-4 Sample holder probe for controlled temperature furnace. 58
III-5 Schematic of high temperature drift furnace (20-800° C). 59
III-6 Temperature profile in the drift furnace at two rheostat settings. 60
III-7 Sample holder probe for the high temperature drift furnace. 61
III-8 Schematic of the measuring circuit. 62
III-9 Schematic of the Czochralski crystal growth method. 63
III-10 Phase diagram of (GaIn)Sb after Woolley and Smith (58W1). 64
III-11 Calibration for the 15" electro-magnet. 65
III-12 Parallelopiped sample showing evaporated gold strips. 66

IV-1 Hall coefficient R vs temperature T° C for two sets of Hall probes (2-6) and (3-5), GaSb sample N1. 117
IV-2 Average conductivity σ₀ vs temperature T° C for GaSb sample N1. 118
IV-3 Measured Hall coefficient R vs B at 25° C for GaSb sample N2. 119
IV-4 -R₀/ΔR vs B⁻² at two temperatures for GaSb sample N2. 120
IV-5  \( \rho_0/\Delta \rho \) vs \( B^{-2} \) at two temperatures for GaSb sample N2.  

IV-6  \(-R\) vs \( B^2 \) at low magnetic field for the determination of \( R_0 \).  

IV-7  The fitted curve (solid line) of \( n_1 \) vs \( T^0C \) and experimental points (X) for \( E_{20} = 0.099 \) eV, \( \alpha_2 = -3.4 \times 10^{-5} \) eV/°K.  

IV-8  Hall coefficient \(-R\) at 0.87 Wb/m and conductivity \( \sigma_0 \) vs temperature for GaSb sample N2.  

IV-9  Hall coefficient \((B = 0.8 \) Wb/m\(^2\)) and conductivity for p-type sample P1. The fitted curve is shown as solid line  

IV-10 Plots of \( \ln(\mu(T)/\mu(300)) \) vs \( \ln(300/T^0K) \) below 120°C for the estimate of the temperature dependence of the mobilities \( \mu_1 \) and \( \mu_2 \) for GaSb sample N2. The slopes \( n_1 \) and \( n_2 \) are 0.67 and 0.38 respectively.  

IV-11 Fitted mobility \( \mu_2(T) \) assuming acoustic and ionized impurity scattering and reciprocal addition of mobilities. The dots are the values of mobility \( \mu_2 \) obtained by the method of Section IV-331.  

IV-12 Plots of \( \mu_1 \) vs T showing the effect of varying the coupling constant \( D_{12} \). Top curve, calculated \( \mu_1(T) \) using \( \tau_{p0}, \tau_I, \tau_{ib} \) and \( D_{12} = 1 \times 10^8 \) eV/cm. Dashed curve shows the effect of adjusting \( D_{12} \) to fit the high temperature results of \( \mu_1(T) \); (bottom curve) obtained by fitting \( R(T) \) and \( \sigma(T) \) with \( \mu_1 \) and \( \mu_2 \) as adjustable parameters.  

IV-13 Semi-log plots of \( \mu_1(T) \) (circles) as obtained in Section IV-332. The solid line was derived by fitting these points using four relaxation times, i.e. \( \tau_{p0}, \tau_I, \tau_{ib} \) and \( \tau_{sc} \).
The four relaxation times as a function of reduced energy
\[ \varepsilon = \frac{E}{h\omega} \] for \( N_i = 2.2 \times 10^{24} \text{ m}^{-3} \), \( D_{12} = 3.2 \times 10^8 \text{ eV/cm} \) and \( (Q_s N_s) = 4.8 \times 10^4 \text{ cm}^{-1} \).

The fitted mobility \( \mu_1(T) \) calculated using \( \tau_{po} \), \( \tau_{ib} \) and \( \tau_i \) with \( D_{12} = 3.0 \times 10^8 \text{ eV/cm} \) and \( N_i = 4.2 \times 10^{24} \text{ m}^{-3} \).

The points were obtained by the method of Section IV-332.

Mobility \( \mu_1 \) vs \( T^0 \text{C.} \) —— for \( \tau^{-1} = \frac{1}{\tau_{po}} + \frac{1}{\tau_{ib}} \), —— calculated for sample N2, —— calculated for sample P1 and (X) are points determined from fitting experimental data from sample P1.

The scattering parameter \( r_1 \) as a function of temperature, at three magnetic fields.

The scattering parameter \( r_1 \) as a function of temperature, at \( B = 0.87 \text{ Wb/m}^2 \).

The magnetoresistance coefficient as a function of temperature at \( B = 3.0 \text{ Wb/m}^2 \).

Schematic of the band structure of GaSb. Includes the previously established parameters together with those determined in this thesis.

A semi-log plot of the experimental Hall coefficient vs temperature for four samples of n-GaAs.

A semi-log plot of the experimental conductivity vs temperature for five samples of n-GaAs.

A semi-log plot of carrier concentration \( n_1, n_2 \) and \( n_3 \) in the three conduction minima \( \Gamma_1, L_1 \) and \( \Delta_1 \), with \( \mu_2(300) = 0.065 \text{ m}^2/\text{V.sec} \) and \( E_{30} = 0.380 \text{ eV} \).
The Hall coefficient vs temperature for sample M0572/8 of n-GaAs. The solid line is the fitted curve for the three conduction band model $\Gamma_1, L_1, \Delta_1$ (or $X_1$). The crosses are experimental points.

The Hall coefficient vs temperature for sample NE22/1 of n-GaAs. The solid line is the fitted curve for the three conduction band model $\Gamma_1, L_1, \Delta_1$ (or $X_1$). The crosses are experimental points.

The Hall coefficient vs temperature for sample M0807/8 of n-GaAs. The solid line is the fitted curve for the three conduction band model $\Gamma_1, L_1, \Delta_1$ (or $X_1$). The crosses are experimental points.

The Hall coefficient vs temperature for sample M0370/55 of n-GaAs. The solid line is the fitted curve for the three conduction band model $\Gamma_1, L_1, \Delta_1$ (or $X_1$). The crosses are experimental points.

The Hall coefficient vs temperature for sample M0241/44 of n-GaAs. The solid line is the fitted curve for the three conduction band model $\Gamma_1, L_1, \Delta_1$ (or $X_1$). The crosses are experimental points.

The Hall coefficient vs temperature for samples M0807/8 and NE22/1 of n-GaAs. The solid line is the fitted curve for the two conduction band model $\Gamma_1$ and $\Delta_1$. The crosses and plus' are experimental points.
Electron mobility in the \( \Gamma_1 \) minimum vs temperature obtained by fitting \( R(T) \) with the three conduction band model \( \Gamma_1, L_1, \Delta_1 \) (or \( X_1 \)).

Electron mobility in the \( \Gamma_1 \) minimum vs temperature for sample M0807/8, obtained by fitting \( R(T) \) with the three conduction band model \( \Gamma_1, L_1, \Delta_1 \) (or \( X_1 \)).

Electron mobility in the \( \Gamma_1 \) minimum vs temperature for sample M0807/8. (1) Ionized impurity mobility \( \mu^*_I \) \( N_I = 1.24 \times 10^{24} \text{ m}^{-3} \). (2) Lattice mobility \( \mu_\lambda \) using Ikoma's results. (3) Combined \( \mu_I \) and \( \mu_\lambda \) mobilities. (4) Combined \( \mu_I, \mu_\lambda \) and \( \mu_a \).

Variation of the Hall scattering parameter, \( r_1 \), with magnetic field, \( B \), for sample NE22/1.

Variation of \( \rho_C/\Delta \rho \) with \( B^{-2} \) for sample M018/60, at 400\(^\circ\)C.

Schematic of the band structure of GaAs. Includes the previously established parameters as well as those estimated in this thesis.
CHAPTER I  INTRODUCTION

This thesis is mainly concerned with the conduction band structure of GaSb and GaAs, and electron mobility in their conduction bands. The introductory discussion given below is divided into two separate parts dealing with the problems of GaSb and GaAs respectively.

GaSb

The compound semiconductor GaSb has been investigated experimentally and theoretically since about 1955 and its band structure is now quite well understood. However, uncertainties still exist in the values of several band parameters.

It was established by Zwerdling et al. (59Z1) that the lowest conduction band minimum in GaSb is at the centre of the Brillouin zone \( \Gamma_1 \) * 0.813 eV above the valence band at absolute zero. This value, the principal energy gap in GaSb, was later confirmed by Adachi (69A2) who reanalysed Zwerdling et al.'s results including the non-parabolic effects.

Sagar (60S1) showed, from transport measurements under pressure, that a set of four equivalent minima are at the zone boundaries in the \( \langle 111 \rangle \) directions \((L_1)\)^*, at approximately 0.08 eV above the \( \Gamma_1 \) minimum. Ehrenreich (61E1) estimated that the next lowest set of minima are at the \( \langle 100 \rangle \) zone boundaries \( (X_1) \)^* and lie about 0.3 - 0.4 eV above the principal \( \Gamma_1 \) conduction minimum. Later, Kosicki and Jayaraman (68K1) obtained a value of 0.315 ± 0.015 eV for the \( \Gamma_1 - X_1 \) separation from the pressure dependence of the resistivity at room temperature.

* The notation for symmetry of the band extrema used here is a standard group theoretical notation first used by Bauckaert, Smoluchowski and Wigner (36B1). When the symmetry of a band extremum is in doubt, then standard crystallographic notation is used describing the directions in k-space.
Although theories of two band behaviour have been developed, in practice the application of these is difficult; either because the bands are of complex form (e.g. heavy hole valence band) or because experimental conditions which are required to ensure that both bands are populated (e.g. application of high pressures or temperatures, or heavy doping of samples) do not comply with assumptions on which the theories are based. However, the small energy separation of the $\Gamma_1$ and $L_1$ minima in GaSb ensures that samples can be chosen so that carriers are present in both bands at normal pressure and in a temperature range in which the theories are applicable. Consequently band parameters for the $\Gamma_1$ and $L_1$ bands can be determined from transport measurements on GaSb.

Table I-1 shows some reported values for the $\Gamma_1$ and $L_1$ minima separation, $E_2$, and its temperature dependence, $\alpha_2$. It can be seen from this Table that considerable disagreement exists between these values. This wide variation can be traced, in most cases, to assumptions made in the analyses of experimental results, in particular the values chosen for the electron effective mass in the two bands and, in the case of theoretical calculations, to the model chosen.

Part of the work described here is concerned with obtaining reliable values of $E_2$ and $\alpha_2$, as well as explaining the observed low mobility in the $\Gamma_1$ band. This was achieved by making accurate experimental measurements of the Hall coefficient and conductivity as a function of both magnetic field and temperature on appropriate samples of GaSb at temperatures where, (a) the experimental data are sensitive to $E_2$ and $\alpha_2$ and (b) analysis can be carried out with a minimum number of assumptions. To this end, experiments were carried out at and above room temperature, in contrast to previously reported measurements which were invariably made at lower temperatures. The reason for this is that above about 20°C n-type
samples can be chosen in which the total number of electrons, distributed over conduction bands, will remain constant with temperature. Under these conditions the analysis of experimental data is simplified and results are more reliable.

In the following sections we shall review the pertinent literature on GaSb and critically discuss the band parameters given.

The reported values of effective mass of electrons in the $\Gamma_1$ band vary between $0.040m$, determined by Yep and Becker (66Y1) from de Haas-Shubnikov measurements at liquid helium temperature, and $0.049m$ obtained by Piller (63P1) from Faraday rotation measurements at liquid nitrogen temperature. Since Kane (56K1) has shown that the $\Gamma_1$ band in III-V compounds cannot be considered parabolic, it is insufficient to quote effective mass values other than the bottom of the band effective mass at absolute zero, $m_{\infty}^*$, without quoting the Fermi energy and temperature. Only the mass given by Yep and Becker has been corrected for non-parabolicity of the band, but Piller (63P1) obtained a value of $0.053m$ at absolute zero using an approximate Kane band in his analysis of Faraday rotation data. This latter value can not be regarded as reliable because of the assumptions involved in its derivation. The optical results of Aubin et al. (69A3) on the alloy system (GaIn)Sb when extrapolated to the GaSb end of the ternary section, give the bottom of the band effective mass as between $0.042$ and $0.043m$ at room temperature.

Several values of the effective mass in the $L_1$ minima can be found in the literature. Becker et al. (61B1) report a value of $0.027m$ for the density of states effective mass, $m_{2d}^*$, which they derived from measurements of Hall effect, conductivity and magnetoresistance at low temperatures. Piller (63P1) on the other hand derived a value of $0.228m$ from his Faraday rotation data in agreement with $0.226$ by Van Tongerloo and Woolley (69T1) based on similar measurements. Theoretical
calculations of Cohen and Bergstresser (66C1), using pseudo-potential method lead to a value of 0.276m, but the band separations they calculate do not agree with experimentally obtained values. More recently Zhang and Callaway (68Z1, 69Z1) also using pseudo-potential method, but including non local terms, arrived at a value of 0.238m for this effective mass, but in contrast to Cohen and Bergstresser's calculation, they force the band separations to agree with experimentally determined values by adjusting their pseudo-potentials. The ellipticity parameter \( K_2 \), (the ratio of longitudinal to transverse effective mass, \( K_2 = \frac{m^*_{2L}}{m^*_{2T}} \)), was obtained experimentally by Piller (64P1), who reports a value of 8.4, from analysis of Faraday rotation measurements which he interprets using a two band model. A value of \( K_2 = 10 \) was also reported by Averous et al. (70A1) which was deduced from transport measurements on GaSb, analysis of which was also based on a two band model. The two theoretical calculations mentioned above give values of \( K_2 \) of 9.54 (66C1) and 13.5 (68Z1).

The effective mass of electrons in the \( X_1 \) band has not been obtained experimentally. In their analysis of resistivity as a function of pressure, Kosicki et al. (68K1) use 1.2m, established for the total density of states mass, \( m^*_{3d} \), in the \( \Delta_1 \) minima from Si, which they assume to be the appropriate value for the same band in GaSb.

Pitt (69P2), in analysing his experimental data of Hall coefficient and resistivity at high pressures, on the other hand used a value of 0.9m, which he obtained from similar measurements on GaAs for the minima in \(<100> \) direction. The pseudo-potential calculations, mentioned above, of Cohen and Bergstresser (66C1) and of Zhang and Callaway (68Z1) give values for the longitudinal, \( m^*_{L} \), and the transverse, \( m^*_{T} \), effective masses. If a multiplicity of three is assumed, i.e.
minima at the zone boundary with symmetry, $X_1$, this leads to total density of state effective mass values of 1.0m and 0.87m respectively. Also, these values of $m^*_L$ and $m^*_t$ give ellipticity parameters, $K_3$, of 5.6 and 6.9.

The low electron mobility in the $\Gamma_1$ conduction band, the value of which has usually been obtained as $\sim 0.4m^2/V\.sec.$ at room temperature (e.g. Sagar 60S1; Ehrenreich 61E1; Harland and Woolley 66H1) has long been thought to be anomalously low. Baxter et al. (67B1) suggested that this low mobility can be explained if the scattering is predominantly ionized impurity and if the high degree of compensation in n-type samples is taken into account and the heavy electrons populating the $L_1$ minima are considered to add to this scattering. That is, in addition to ionized impurities, the heavy electrons which have a mobility of $\sim 1/6$ of those in the $\Gamma_1$ band, will act as Coulomb scattering centres. Baxter et al. took as an example Sagar's (60S1) sample 1B and calculated the ionized impurity mobility based on this model to be $0.65m^2/V\.sec.$ They combined this with the total lattice mobility of $1.4m^2/V\.sec.$ estimated by Ehrenreich (61E1) to obtain a combined value of $0.44m^2/V\.sec.$ at room temperature, in good agreement with experiment.

Electron mobilities in the $L_1$ band also have been obtained from pressure experiments at room temperature by Pitt (69P2) and by Rosenbaum (72R1), values of between 0.04 and 0.05$m^2/V\.sec.$ being derived. These agree well with values of $0.04$ to $0.065m^2/V\.sec.$ deduced by Sagar (60S1) from the analysis of the two band Hall effect.

Pitt (69P1) has also obtained a value of $\sim 0.004m^2/V\.sec.$ for the electron mobility in the $X_1$ band at room temperature. This seems somewhat low in comparison with the mobility observed in Si for the $\Delta_1$ minima, but compares well with the mobility measured in the GaP principal conduction band which has the same symmetry
(68M1), and with the mobility of electrons in the \( \Delta_1 \) band of GaAs when doping is taken into account.

From the foregoing brief review of the conduction band parameters in GaSb, it seems clear that the largest disagreements exist in the value of the \( \Gamma_1 - L_1 \) separation and its temperature dependence. Also the model which was used by Baxter et al. (67B1) seems to oversimplify the problem of scattering in the \( \Gamma_1 \) band. We shall attempt to clarify these problems in Chapter IV of this thesis.

It was suggested by Bate (62B1) that group VI impurity levels in III-V compounds are associated with the minima of subsidiary conduction bands. Harland and Woolley (66H1) found that the total electron concentration in their Te-doped GaSb increased with increasing temperature. To account for this they postulated an impurity level situated between the \( \Gamma_1 \) and \( L_1 \) minima. Recently Pitt (69P2) obtained an ionization energy \( \sim 0.02 \) eV for Te donors in GaSb below the \( L_1 \) minima. For this reason the experimental measurements on n-type GaSb which are analysed in Chapter IV were performed at and above room temperature to ensure that all the available donors are ionized and hence the total electron concentration remains constant.

**GaAs**

Gallium Arsenide is probably one of the most studied III-V compounds because of its potential for device applications as well as purely academic interest. However, a serious gap exists in the understanding of the conduction band structure of this material. It is well established that, as in GaSb, the lowest conduction band in GaAs has its minimum at the centre of the Brillouin zone with \( \Gamma_1 \) symmetry. The problem arises as to the position in \( k \)-space of the next lowest set of minima.
Essentially, the experimental results suggest that the first subsidiary minima are \( \approx 0.4 \) eV above the \( \Gamma_1 \) minimum in the \( <100> \) direction and have \( A_1 \) or \( X_1 \) symmetry, but these results are not conclusive. Theoretical calculations using different methods of analysis, on the other hand, do not agree on whether the first subsidiary minima are in the \( <100> \) or \( <111> \) direction, nor do they agree on the relative energy separations of these sets of minima.

Since the first subsidiary minima in GaAs lie at about 0.4 eV above the \( \Gamma_1 \) minimum, in order to study these subsidiary minima by transport measurements, it is necessary to use heavily doped samples and elevated temperatures. Therefore measurements of Hall coefficient, at a constant magnetic field, and of conductivity were made from 20 to 600°C on samples with room temperature electron concentrations of from \( 5 \times 10^{23} \) to \( 5 \times 10^{24} \) m\(^{-3} \). Measurements of magnetoresistance as a function of magnetic field were also made at 450°C, but these could not be analysed on the same model as those as GaSb. The results of Hall coefficient and conductivity were analysed on three models involving possible arrangements of subsidiary conduction bands. It was possible to choose the most probable model from these analyses on the basis of overall consistency in explaining experimental data.

Data concerning the subsidiary valleys in the conduction band structure of GaAs are reviewed in the following paragraphs. A more extensive review of the conduction band structure of GaAs can be found in the article by Gray (70G1). Photoemission studies of James et al. (68J1, 68J2) and Spicer and Eden (68S1) show transitions to subsidiary minima at about 0.35, 0.45 and 0.95 eV above the \( \Gamma_1 \) minimum and were interpreted as being transitions to the minima at the \( X_1, L_1 \) and \( X_3 \) points in the Brillouin zone. The identification of the lowest of these minima is consistent with the results of an analysis based on a two band model by Ehrenreich (60E1) and
using data on the high temperature Hall coefficient. This analysis shows that there
are six equivalent minima at 0.36 eV above the $\Gamma_1$ minimum with a combined density of
states mass of 1.2m. By analysing data on the resistivity as a function of pressure,
Ehrenreich (60E1) was able to deduce a variation with pressure of the position of
these minima relative to the valence band. Since this pressure coefficient and the
above effective mass were reasonably close to the corresponding values observed for
the $\Delta_1$ minima in silicon, Ehrenreich concluded that the subsidiary minima in GaAs
were probably also in the $<100>$ direction. Experiments on Ga(AsP) alloys are in
agreement with Ehrenreich's conclusion. These show that the positions of the $<100>$
minima in the phosphide rich alloys, when extrapolated back to zero phosphorous content
lead to a $\Gamma_1 - \Delta_1$ separation of about 0.4 eV (60E1). The conclusion is also consistent
with work of Harris et al. (70H1) on the variation of the threshold field for Gunn
oscillations with uniaxial compression along various directions. Measurements by
Balslev (68B1) of the indirect absorption from the $\Gamma_1$ minimum to a subsidiary band,
as a function of uniaxial compression along various directions, also reveal minima in
the $<100>$ directions at a somewhat higher energy of 0.43 eV at liquid nitrogen
temperature.

Although the above evidence strongly suggests that the lowest subsidiary
minima are in the $<100>$ directions, it is worth noting that it cannot at this stage be
regarded as conclusive. The first reason for this is that Ehrenreich's analysis (60E1)
does not lead directly to the density of states in the subsidiary minima, but to the
product of the density of states and the factor $e^{-\alpha/k}$, where $\alpha$ is the linear tem-
perature coefficient of the $\Gamma_1 - \Delta_1$ band separation and $k$ is Boltzmann's constant.
With $\alpha$ assumed to be zero, Ehrenreich (60E1) could determine only that the density of
states mass lies between 0.9 and 1.7m. More recent experiments on GaAs under pressure
have given the value of 0.38 eV at room temperature for the energy separation of
the minima (67H1) and 0.85m for the density of states mass (70P1, 70P2), but it was necessary in both these cases to assume either the density of states (67H1) or energy separation (70P1, 70P2) to deduce these values. Also, even if the <100> minima are the lowest at high pressure in GaAs, as seems almost certain, it does not necessarily follow that the <100> minima are the lowest at zero pressure. The <100> and <111> minima could cross before the <100> minima could start to influence the transport properties. This arrangement of the minima would not necessarily conflict with the data on the band gap of Ga(AsP) alloys, particularly since any non-linear variation of the band gap with composition was not taken into account in the extrapolation of the data (68L2).

As was already pointed out, band structure calculations (66C1, 69J1, 66P1, 70C1, 68H1) do not consistently predict the relative position of the <100> and <111> minima, the latter being calculated to be from as much as 0.5 eV below to 0.4 eV above the <100> minima. Pseudo-potential calculations of Cohen and Bergstresser (66C1) predict the <111> minima to be 0.1 eV below the <100> minima, whereas Jones and Lettington (69J1), using the same method but a different form of pseudo-potential, have recently calculated the <111> minima to be 0.4 eV above the <100> minima. The k.p calculations of Pollak et al. (66P1) place the <111> and <100> minima at about the same energy. The <111> minima are found to be below the <100> minima in the OPW calculations of Collins et al. (70C1), the separation being 0.44 eV, although the <100> minima occur in this case 0.8 of the way to the X point, i.e. have Δ1 symmetry. On the other hand, Herman (68H1) has shown that it is possible to adjust the Fourier coefficients of the potential used in the OPW calculation to obtain close agreement with the observed transition energies between the various minima throughout the band structure. He found that if X1 and L1 minima were taken to lie 0.35 and 0.43 eV respectively above the Γ1 minimum then a good overall fit to the experimental data
was obtained.

The bottom of the band effective mass in the \( \Gamma_1 \) band of GaAs has been measured by various methods (65M1; 68S2; 69C1; 68V1), the values obtained vary between 0.064m and 0.067m, i.e. close agreement exists for this parameter. On the other hand, no experimentally determined values have been reported for the effective mass in the \(<111>\) minima for GaAs. However, the k.p perturbation calculations of Pollak et al. (66P1) predict values of 0.97m and 0.116m for the longitudinal and transverse mass respectively, from which a density of states mass of 0.235m and an ellipticity parameter of 8.4 are calculated.

Electron mobility in the \( \Gamma_1 \) band has been investigated by Ehrenreich (60E1), Fortini et al. (70F1), Moore and Ehrenreich (66M1), Moore (67M1) and Ikoma (70I1). Ehrenreich has shown that in pure samples, the electron mobility is limited by polar optical scattering with a resulting room temperature value of 0.93m/V sec. Polar optical mobility was also calculated for GaAs by Fortini et al. (70F1) and compared to Hall mobility measurements on a pure sample \( n \sim 10^{20} \text{m}^{-3} \) at 350°C over a wide temperature range. They found good agreement up to room temperature, but at higher temperatures, calculations lead to a higher mobility than those experimentally observed. Fortini et al. suggest that the disagreement may be accounted for by omitting in their calculations the effects of multiphonon scattering and neglecting the temperature dependence of the dielectric constant. Ehrenreich (60E1) has also made calculations of combined polar optical and ionized impurity scattering using variational techniques, he showed that at room temperature the calculated mobility was in good agreement with the measured mobility as a function of doping, except in the high doping range. Later Moore and Ehrenreich (66M1) showed that in order to have better agreement at high doping concentrations, i.e. \( N_I > 7.5 \times 10^{23} \text{m}^{-3} \), quantum mechanical corrections had to be made to the Brooks-Herring ionized impurity mobility relation.
Making these corrections, they found excellent agreement with experimental results at liquid nitrogen temperature, and slightly less acceptable agreement at 300°K. Ikoma (70I1), on the other hand, made measurements of Hall mobility up to 600°C on samples doped with less than $1 \times 10^{23} \text{m}^{-3}$ tellurium donors, and obtained reasonably satisfactory agreement above 200°C between his experimental results and calculated values. The calculations included two phonon resonance, polar optical and interband scattering between $\Gamma_1$ and $\Delta_1$ minima. However, some heavier doped samples had a measured mobility of roughly half the calculated value, indicating that scattering at elevated temperatures was underestimated by Ikoma's model.

The mobility at room temperature in the $\Delta_1$ (or $X_1$) minima of GaAs has been deduced from the Hall coefficient and resistivity measurements under pressure. Hutson et al. (67H1) deduced $0.011 \text{m}^2/\text{V} \cdot \text{sec}$. whereas Pitt and Lees (70P2) derived $0.035 \text{m}^2/\text{V} \cdot \text{sec}$. for the electron mobility in the $<100>$ minima. This discrepancy may be explained by the use of different effective mass values for these minima. Thus, Hutson et al. used the density of states mass of 1.2m obtained by Ehrenreich (60E1), i.e. six equivalent minima; whereas Pitt and Lees derived a value of 0.85m, which was based on a 3 equivalent minima model. Conwell and Vassel (68C) calculated the mobility to be $0.0145 \text{m}^2/\text{V} \cdot \text{sec}$. by taking into account acoustic, polar optical and equivalent intervalley scattering by optical phonons. This value agrees well with the value of $0.015 \text{m}^2/\text{V} \cdot \text{sec}$. obtained earlier by Lees et al. (67L1) from hydrostatic pressure measurements assuming 6 equivalent valleys. Calculations of James et al. (69J2) which include equivalent intervalley and deformation potential scattering, give a mobility in this band of $0.012 \text{m}^2/\text{V} \cdot \text{sec}$. 
TABLE I-1

<table>
<thead>
<tr>
<th>$E_2$ eV</th>
<th>$\alpha_2$ eV/°k x $10^4$</th>
<th>Ref.</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.074</td>
<td>-3.</td>
<td>(60S1)</td>
</tr>
<tr>
<td></td>
<td>$\sim$1.</td>
<td>(61C1)</td>
</tr>
<tr>
<td></td>
<td>1.1</td>
<td>(63P1)</td>
</tr>
<tr>
<td>0.084 *</td>
<td>0.8</td>
<td>(66H1)</td>
</tr>
<tr>
<td>0.084</td>
<td>0.59</td>
<td>(68L1)</td>
</tr>
<tr>
<td>0.083</td>
<td>0.55</td>
<td>(69B1)</td>
</tr>
<tr>
<td>0.08</td>
<td>2.1</td>
<td>(70A1)</td>
</tr>
<tr>
<td>0.077</td>
<td>-2.</td>
<td>(70R1)</td>
</tr>
<tr>
<td>0.04</td>
<td>-1.8</td>
<td>(70S1)</td>
</tr>
<tr>
<td>0.058</td>
<td>1.4</td>
<td>(71S1)</td>
</tr>
</tbody>
</table>

* Liang (68L1) uses the value of $E_2$ obtained by Harland and Woolley (66H1)

Table I-1: The values of $E_2$ and $\alpha_2$ reported in the literature.
CHAPTER II

THEORY

II-1 Effective mass

II-2 The electron band structure, the Kane band

II-3 Electron scattering in III-V compounds
   II-31 Lattice scattering
      II-311 Acoustic Phonon
      II-312 Optical Phonon
   II-32 Scattering by lattice defects
      II-321 Ionized impurity
      II-322 Spacecharge
   II-33 Other scattering mechanisms

II-4 Carrier mobility

II-5 The Hall effect
   II-51 "Two band" Hall effect
   II-52 Multiband Hall effect

II-6 Magnetoresistance
   II-61 "One band"
   II-62 "Two band"
II-7 Limits of validity of the two band picture in the presence of non-isotropic band

II-8 Combined analysis of two band Hall effect and Magnetoresistance

II-9 Charge neutrality and Fermi energy
CHAPTER II  THEORY

Theory necessary to the understanding of the work presented in this thesis will be reviewed here. When proofs of certain expressions may be found in the literature these will not be repeated but references will be given; any underlying assumptions used will be critically discussed. The author's own contribution when applicable will be pointed out.

II-1  The Effective Mass

The concept of effective mass has been discussed in various standard texts (e.g. Kittel, 53K1). However, since there are different definitions of effective mass depending on the nature of the external force being applied, a summary of those which will be used in later chapters in dealing with the analysis of experimental results is presented in Table II-1.

II-2  Form of the Energy Bands, the Kane Model

Theoretical calculations of band structure are generally a formidable problem. The necessity of determining band structure in solids has led to a number of different approximations which make the problem more tractable. By using the "one electron" approximation, various theories can be developed to give the energy band scheme of solids. One of these is the semi-empirical \( k.p \) perturbation theory developed by Kane (56K1) which describes the band shapes at and in the vicinity of critical points. The present work is in part concerned with the band parameters of the \( \Gamma_1 \) conduction band of GaSb and GaAs. In both these compounds the \( \Gamma_1 \) conduction band is limited to a small range in \( k \) space, so that of the available methods of calculations, the \( k.p \) representation is most suitable. The results of the \( k.p \) perturbation theory developed by Kane will thus be used.
We shall start with the $E$ vs $k$ relation obtained from the Kane model and use this relation to derive expressions which will later be used in the analysis of experimental data.

First order perturbation theory led Kane to an $8 \times 8$ secular determinant, the solution of which is

$$E'[E'(E' - E_o)(E' + \Delta) - k^2 P^2(E' + 2/3\Delta)] = 0 \quad \text{II-1}$$

where $E' = E - \hbar^2 k^2 / 2m$, energy being measured from the top of the valence band.

- $E_o$ is the principal gap
- $\Delta$ is the spin orbit splitting of the valence band
- $P$ is a matrix element
- $m$ is the electron rest mass

It should be pointed out at this point that solution II-1 is obtained when only three valence bands and one conduction band are taken into account, neglecting the interaction of higher conduction bands. This model has been used extensively and has proved to be satisfactory for the III-V compounds (e.g. InSb, 71Z1) so that the approximation seems to be justified in this case.

It has been shown by Aubin (69Al) that a general solution to equation II-1 for the conduction band is of the form

$$E' = \frac{E_o}{2} \left[1 + \left(1 + \frac{\beta k^2 P^2}{E_o^2}\right)^{1/2}\right] \quad \text{II-2}$$

where

$$\beta = \frac{4 + 8n/3}{1 + n}$$

and

$$n = \Delta/E_o$$
Hence, replacing $E'$ in II-2 gives

$$E = \frac{\hbar^2 k^2}{2m} + \frac{E_o}{2} \left[ (1 + \frac{\beta k^2 p^2}{E_o^2})^{1/2} - 1 \right]$$  \hspace{1cm} \text{II-3}$$

Using the definition of cyclotron and band curvature effective mass given in Table II-1, we can now derive the variation of the effective mass with $k$, i.e.

$$\frac{1}{m^*} = \frac{1}{m} + (E_o^2 + \beta k^2 p^2)^{-1/2} \frac{\beta p^2}{2\hbar^2}$$  \hspace{1cm} \text{II-4}$$

$$\frac{1}{m_e} = \frac{1}{m^*} - \frac{\beta^2 p^4 k^2}{2\hbar^2 E_o^3} \left( 1 + \frac{\beta p^2 k^2}{E_o^2} \right)^{-3/2}$$  \hspace{1cm} \text{II-5}$$

Thus the bottom of the band effective mass $m_e^*$ is obtained when $k = 0$, i.e.

$$\frac{1}{m_e^*} = \frac{1}{m} + \frac{\beta p^2}{2\hbar^2 E_o}$$  \hspace{1cm} \text{II-6}$$

which now gives the well known relation for the square of the matrix element $p^2$,

$$p^2 = \frac{3\hbar^2 / 2m}{E_o} \frac{E_o (E_o + \Delta) (m/m_e^* - 1)}{(3E_o + 2\Delta)}$$  \hspace{1cm} \text{II-7}$$

Eliminating $p^2$ and $\beta$ from II-3 we obtain the $E(k)$ relation in terms of the basic measurable band parameters

$$E = \frac{\hbar^2 k^2}{2m} + \frac{E_o}{2} \left[ \{1 + \frac{4\hbar^2 k^2}{2mE_o} (m/m_e^* - 1)\}^{1/2} - 1 \right]$$  \hspace{1cm} \text{II-8}$$

The same relation was obtained by Cardona (61C1) who assumed that $E' \ll E_o + 2/3\Delta$. It should be noted that for large energy gap $E_o$ and small effective mass the above relation reduces to the standard parabolic band, in which case the effective mass
is independent of energy

$$E = \frac{\hbar^2 k^2}{2m}$$  \hspace{1cm} \text{II-9}

We can invert relation II-8 to obtain $k(E)$ i.e.

$$k^2 = \left( \frac{2E}{\hbar^2 m_o} \right) \left[ 1 + \frac{2E}{E_o} \right] \left( 1 + \frac{4E}{E_o} \left( 1 - \frac{m^*}{m} \right) \right)^{1/2} \hspace{1cm} \text{II-10}$$

Another form of the $k(E)$ relation can be derived starting with equation II-1 and assuming that $E > \hbar^2 k^2 / 2m$ which simplifies the relation to

$$E(E + E_o)(E + E_o + \Delta) - k^2 P^2 (E + E_o + 2/3\Lambda) = 0 \hspace{1cm} \text{II-11}$$

from which one obtains

$$k^2 = \frac{E(E + E_o)(E + E_o + \Delta)}{(E + E_o + 2/3\Lambda)P^2} \hspace{1cm} \text{II-12}$$

It was shown by Aubin (69Al) that for InSb both relations II-10 and II-12 are good approximations to the Kane model and agreed very well with the $k(E)$ expression derived by Kolodziejczak (66K2) who included the effect of higher and lower bands by second order perturbation theory. We expect therefore that in the case of GaSb and GaAs where the interaction of valence and conduction bands is weaker, the above expressions would be a still better approximation to the Kane model than for InSb.

Indeed, it can be seen from Kolodziejczak's calculations (66K2) that for both GaAs and GaSb the inclusion of the upper and lower bands does not significantly affect the Kane model, at least in the Fermi energy range of about $+0.1\text{eV}$ to $-0.1\text{eV}$ encountered in the experiments described here. Seiler et al. (69S1) have considered the effect of the $L_1$ minima on the isotropy of the $\Gamma_1$ minimum in GaSb and found that the distortion is very small, hence to a good approximation we can consider
that the $\Gamma_1$ band in GaSb is isotropic.

Ehrenreich (57El) has pointed out that the Kane model is strictly valid only at absolute zero. Hence $E_o$ appearing in the Kane expression is the band gap at absolute zero. Further, at finite temperatures the value of $E_o$ is affected both by lattice distortion and interaction with the phonon field. However, Ehrenreich (57El) suggests that the band curvature, which determines the effective mass, depends only on the lattice dilation, and therefore that $E_o$ in the Kane model should be replaced by $E_o^*$, the "effective mass band gap" where $E_o^*$ is the value the band gap would have if only lattice dilation effects were considered. The above effect was considered also by Ravich (66R1) who derived the changes with temperature in the effective mass, due to electron phonon interaction, as a function of the temperature variation of the band gap for InSb and InAs.

The effective mass band gap $E_o^*$ may be expressed in the form (68T1)

$$E_o^* = E_{oo} - \left( \frac{E_{oo} - E_o}{a} \right)$$

where $E_{oo}$ is the intrinsic band gap at absolute zero and "a" is a parameter which determines the relative effect of lattice dilation and interaction of phonon field on the band gap.

II-3 Electron Scattering in III-V Compounds

It can be shown that under steady state conditions Boltzmann's transport equation can be written in the form

$$\kappa^{-1} \frac{df}{dE} (F \cdot \text{grad}_k E) = \left. \frac{df}{dE} \right|_{\text{coll}}$$
where \( F \) is the applied field, \( f \) is the distribution function under the applied field.

The collision term on the right of the equation is responsible for restoring \( f \) to its equilibrium value. If the deviation from equilibrium is small it is possible to write the collision term in the form,

\[
\frac{df}{dt} |_{\text{coll}} = \frac{f - f_o}{\tau}
\]

where \( f_o \) is the equilibrium distribution function and \( \tau \) is a relaxation time, then II-14 becomes

\[
\mathbf{h}^{-1} \frac{df}{dE} (F, \text{grad}_k E) = \frac{f - f_o}{\tau}
\]

The validity of the relaxation time approximation is discussed in many standard texts (e.g. Blatt, 68B2) and will be assumed here.

The magnitude, energy and temperature dependence of \( \tau \) will depend on the scattering mechanism, which will now be discussed. It should be pointed out that the following discussion does not attempt to exhaust all the possible scattering processes, nor does it treat them very rigorously since full derivations can be found in the references quoted.

II-31 Lattice Phonon Scattering

II-311 Acoustic Phonon, "deformation potential" scattering

For acoustic mode vibrations, the wavelength is large compared with the interatomic distance and the vibrational energy is relatively small. It was pointed out by Bardeen and Shockley (50B1) that the long wavelength acoustic vibrations which produce a small perturbation on the periodic potential of a crystal, can be
used to calculate the motion of an electron neglecting the periodic potential. They introduced the name "deformation potential" for this type of effective potential which can be produced by long wavelength acoustical waves. The relaxation time, due to this scattering is given by

$$\tau_{\text{al}} = \frac{\rho U_{\parallel}^2 \hbar^4}{8\pi^3 (2m^*)^{3/2} kT E_1^2 E_1^{1/2}} \quad \text{II-17}$$

where \(\rho\) is the density, \(U_{\parallel}\) is the velocity of the longitudinal waves in the crystal and \(E_1\) is the deformation potential energy defined by

$$\Delta E_c = E_1 \frac{\Delta V}{V_0} \quad \text{II-18}$$

Here \(\Delta E_c\) is the change in energy of the bottom of the conduction band due to a small change \(\Delta V\) in the original volume \(V_0\).

For ellipsoidal constant energy surfaces it was shown by Herring and Vogt (56H1) that \(E_1\) is related to \(\varepsilon_d\) and \(\varepsilon_u\) deformation potential constants which depend on the symmetry of the minima considered.

II-312 Optical Phonon Scattering

(a) It has been shown by Ehrenreich (60E1) that polar optical scattering is dominant in conduction bands with \(\Gamma_1\) symmetry in III-V compounds. Ehrenreich (60E1) derived, (using variational techniques) an expression for the mobility due to combined polar optical and ionized impurity scattering. The present measurements were taken at \(T > 300^\circ K\), hence other scattering mechanisms such as interband scattering have to be included so that Ehrenreich's expression becomes inapplicable. Therefore we shall make use of the expression for the relaxation time due to polar scattering derived by Conwell and Vassell (68C1). In the case of a parabolic band
the relaxation time $\tau_{po}$ is given by:

$$\frac{1}{\tau_{po}} = \frac{eE_o (2N_{q\ell} + 1)}{[2m^* E]^{1/2}}$$

and for a non-parabolic band with spherical symmetry this becomes

$$\frac{1}{\tau_{po}} = eE_o (2N_{q\ell} + 1) (2m_0^* \gamma)^{1/2} \frac{d\nu}{dE}$$

where

$$eE_o = \frac{e^2 m_0^* (\hbar \omega_{q\ell})}{4\pi \hbar^2} \left( \frac{1}{\kappa_\infty} - \frac{1}{\kappa_o} \right)$$

$$N_{q\ell} = \left[ \exp \left( \frac{\hbar \omega_{q\ell}}{kT} \right) - 1 \right]^{-1}$$

and

$$\gamma = \frac{\hbar^2 k}{2m_0^*}$$

where $\omega_{q\ell}$ is the optical phonon frequency, $\kappa_\infty$ and $\kappa_o$ are the high frequency and static dielectric constants respectively.

It was pointed out by Conwell and Vassell (68C1) that the expression for $\tau_{po}$ was derived assuming only wave functions with S symmetry. For higher energies the admixture of p functions causes the expression to be less accurate. However, at these higher energies other scattering mechanisms, such as interband, become more important in both GaAs and GaSb and hence this error may be neglected.

(b) In semiconductors with band structure such that two bands have minima close in energy, the scattering of carriers between the two bands may be important in determining the mobility of carriers in either one or both bands. This scattering is usually termed "interband" scattering.
In polar semiconductors such as III-V compounds and their alloys, this scattering mechanism is possible with an absorption or emission of an optical phonon, since a large change in wave vector \( k \) is necessarily required when the two minima are at different symmetry points in the Brillouin zone. An expression for \( \tau_{ib} \) was also derived by Conwell and Vassell (68C1) and is

\[
\frac{1}{\tau_{ib}} = \frac{D_{12}^2 (M_d^*)^{3/2}}{\sqrt{2} \pi \hbar^3 \rho \omega_{12}} \times \frac{1}{e^{x_{12}} - 1}
\]

\[
x_{12} = \left( E + \hbar \omega_{12} - E_{12} \right)^{1/2} + e^{x_{12}} \left( E - \hbar \omega_{12} - E_{12} \right)^{1/2}
\]

where \( D_{12} \) is the coupling constant, \( \omega_{12} \) the angular frequency for transition between the two bands, \( x_{12} = \hbar \omega_{12} / kT \) (\( k = \) Boltzmann constant). \( E_{12} \) is the energy separation of the two minima and \( M_d^* = \nu^2 \hbar m_d^* \). The integer \( \nu \) represents the number of equivalent minima.

II-32 Scattering by Lattice Defects

II-321 Scattering by ionized impurities

The problem of scattering by ionized impurities was originally treated by Conwell and Weisskopf (50C1) who considered Rutherford scattering by each ion independently. The divergence arising from the increasing contribution of small angle scattering was removed by arbitrarily cutting off the scattering cross sections at a distance \( r^{-1} = N_I^{1/3} / 2 \), where \( N_I \) is the impurity density. In later work Brooks and Herring (51Bl) and Dingle (55D1), used a screened Coulomb potential, which takes care of the divergence difficulty, this arises from the fact that charge carriers distribute themselves around the impurity and cancel its field at large
distances. This leads to a relaxation time $\tau_I$ given by an expression of the following type (59S1):

$$
\tau_I = \frac{16\pi (2m^*)^{1/2} \kappa^2_0 E^{3/2}}{e^4 n_I f(z)}
$$

where $f(z)$ is a slowly varying function of $E$ and is given by $f(z) = \ln(1+z) - z/(1+z)$, here $z \equiv 2(k a)^2$ and "a" is the screening distance which is a function of the carrier density, temperature and energy of the carrier.

The above screening relation was derived by quantum-mechanical treatment and is based on the Born approximation, which implies that the range of validity of the treatment is given by $z \gg 1$. When this is violated as in the case of high impurity concentration or at very low temperatures the accuracy of the calculation of mobility may be suspect.

Using the results obtained by Dingle (55D1), Beer (63B1) has given a relation for $z$ of the form:

$$
z = \frac{8 \kappa_0 m^* k T E}{e^2 h^2 n} \frac{F_{1/2}(\eta)}{F_{-1/2}(\eta)}
$$

where $n$ is the carrier concentration in the conduction band and $F$'s are Fermi integrals defined by:

$$
F_r(\eta) = \int_0^{\infty} \frac{x^r dx}{1 + e^{x-\eta}}
$$

where $\eta$ is the reduced Fermi energy $E_F/kT$.

More recently Moore and Ehrenreich (66M1) and later Moore (67M1) have extended the above theory by including quantum mechanical corrections to the semi-classical Brooks-Herring treatment. They derive the following expression for the mobility:

$$
\mu_I = \mu^*_I / \{1 + \sigma_B + \sigma_M + \sigma_D\}
$$
where $\mu^*$ is the Brooks-Herring mobility and the correction terms $\sigma_B$, $\sigma_M$ and $\sigma_D$ are defined by Ehrenreich and Moore (66El) and contain some formidable integrals which Moore (67M1) solved numerically for GaAs.

The above relations are valid only for parabolic bands. Taking the non-parabolicity into account, it was shown by Kolodziejczak (66K1) that the relaxation time for ionized impurity scattering may be written in the form

$$
\tau_I = \frac{8\pi \kappa_B^2 (2\gamma)^{3/2} m^*_o 1/2}{e^4 N_I f(z)} \frac{dE/d\gamma}{(2m^*_o)^{1/2}} (kT)^{1/2} L_1^{1/2} (\beta, \eta) \tag{II-29}
$$

where $z$ is now given by

$$
z = \frac{16\pi^2 \kappa_B \hbar}{e^2 (2m^*_o)^{1/2}} \frac{\gamma}{(2m^*_o)^{1/2}} \frac{L_1^{1/2} (\beta, \eta)}{(kT)^{1/2}} \tag{II-30}
$$

$L_1^{1/2} (\beta, \eta)$ is an integral defined as

$$
L_1 (\beta, \eta) = \int_0^\infty \left( \frac{df}{dx} \right) (x + \beta x^2)^n (1 + 2\beta x)^h dx \tag{II-31}
$$

and

$$
\beta = \frac{kT}{E_0} \quad , \quad \eta = \frac{E_F}{kT} \tag{II-31}
$$

However, if we have multiple charged scattering centres and carriers present in more than one band, Kolodziejczak relations II-30 and II-31 have to be modified to take this into account. Baxter et al. (67B1) have shown that this can be done by replacing $N_I$ in equation II-29 by $\sum_i Q_i N_i$, where $Q_i$ and $N_i$ are the charge and density of the $i^{th}$ scattering centre respectively. They have also shown that in order to take into account the difference of screening by the carriers in the different bands $z$ must be replaced by

$$
z = \frac{8m^*_o E \alpha^2 d\gamma/dE}{h^2} \tag{II-32}
$$
where $a$ is the screening distance given by Dingle (55D1)

$$\frac{1}{a^2} = \frac{e^2}{\kappa_0 kT} \frac{\delta n_i}{\delta n}$$  \hspace{1cm} \text{II-33}$$

and $n_i$ is the density of carriers in band $i$.

**Spacecharge Scattering**

It was suggested by Weisberg (62W1) that the anomalously low mobility of electrons observed in some III-V compounds is due to scattering by spacecharge regions in the material. These spacecharge regions are formed by randomly distributed acceptor impurities compensating the donors, which would tend to produce intrinsic or even p-type regions. To provide the appropriate potential difference with respect to the uncompensated material, a region of spacecharge will be formed.

By assuming that these regions are impenetrable spheres to the quasi free electrons, Conwell and Vassell (68Cl) obtain an expression for the relaxation time due to spacecharge scattering for parabolic bands,

$$\tau_{sc} = (N_s Q_s)^{-1} \left[ m^*/2E \right]^{1/2}$$  \hspace{1cm} \text{II-34}$$

where $N_s$ is the density of the impenetrable spheres and $Q_s$ their scattering cross section.

For non-parabolic bands this relaxation time is

$$\tau_{sc} = (N_s Q_s)^{-1} (m^*/2\gamma)^{1/2} \frac{d\gamma}{dE}$$  \hspace{1cm} \text{II-35}$$

It is of interest to note that unlike impurity scattering this process becomes more important as the energy of electrons increases.
II-33 **Other Scattering Mechanisms**

There are various other scattering mechanisms which play a role in carrier scattering in semiconductors, among these are piezo-electric and neutral impurity. Both of these can be shown to be unimportant in GaAs and GaSb at the temperatures at which experimental data for this thesis were obtained. However, electron-electron and electron-hole interactions can become important at elevated temperatures and in the intrinsic region. These interactions will be discussed further in a later chapter dealing with the experimental results on GaSb.

II-4 **Carrier Mobility**

When an electric field $E_x$ is applied to a semiconductor in direction $x$, the average drift velocity of a carrier will be given by

$$v_x = \mu E_x$$

where the proportionality constant $\mu$ is called the carrier mobility; it is positive for holes and negative for electrons. This mobility is given by

$$\mu = \frac{e <\tau>}{m^*_e}$$

for parabolic bands, where $<\tau>$ is an average over all energies of the electron distribution.

If the band is non-parabolic (e.g. Kane Band), the effective mass is a function of energy and therefore must be included in the averaging process, and the mobility may be written as

$$\mu = e \frac{<\tau>}{m^*_e}$$
However, it has been pointed out by Sosnowski (64S1) that the second term in equation II-5 cancels out in the Boltzmann equation. Consequently, it is the cyclotron mass that enters into the solution and all expressions entering the transport. Therefore $m_c^*$ appearing in II-38 should be replaced by $m_c^*$. Zawadzki (62Z1) has shown that for spherical constant energy surfaces the average $<\tau/m_c^*>$ is given by

$$<\frac{\tau}{m_c^*}> = \frac{\int_{0}^{\infty} (-\frac{\partial f}{\partial E}) \frac{\tau}{m_c^*} k^3 dE}{\int_{0}^{\infty} (-\frac{\partial f}{\partial E}) k^3 dE} \tag{II-39}$$

where $f = f_o$ the equilibrium Fermi-Dirac distribution factor for small $E_x$.

For parabolic bands this becomes

$$<\tau> = \frac{\int_{0}^{\infty} (-\frac{\partial f_o}{\partial E}) \tau E^{3/2} dE}{\int_{0}^{\infty} (-\frac{\partial f_o}{\partial E}) E^{3/2} dE} \tag{II-40}$$

and under the condition of non-degenerate statistics $-\frac{\partial f_o}{\partial E}$ may be replaced by $f_o$, and in this limit $f_o (= \exp[-E/kT])$ is the Boltzmann distribution function.

In cases when more than one scattering mechanism affects the carriers, the resultant relaxation time may be written in the form

$$\tau^{-1} = \sum_i \tau_i^{-1} \tag{II-41}$$

where "i" runs over all the scattering modes. This relation assumes that the scattering mechanisms are independent of one another, which in effect is an approximation, since in the case of lattice and ionized impurity scattering, for example, an electron may be scattered by a lattice phonon while in the field of an
impurity. However, this approximation is usually made in semiconductor work
and will be adopted here.

II-5 The Hall Effect

II-51 "Two band" Hall coefficient

In most III-V compounds the conduction band structure is such that when
two non-equivalent minima are partially occupied by carriers one of these is the \( \Gamma \)
minimum, which is isotropic but non-parabolic, and the others are the minima in
the \(<111>\) or \(<100>\) directions. These latter minima have ellipsoidal energy surfaces
and can be taken as parabolic. In addition, the electron effective mass of the
ellipsoidal minima are much greater than that in the minimum at \( \Gamma \) (e.g. Madelung
64ML, p. 352).

The theory developed below is therefore applicable to the band structures
of III-V compounds and, although it may be suitable for other materials it is
important to ensure that the assumptions made in deriving these expressions are
still valid.

For the case of two non-interacting bands it was shown by Chambers (52Cl)
that the Hall coefficient is given by

\[
R = \frac{R_1 \sigma_1^2 (1 + \sigma_2^2 R_2^2 B^2) + R_2 \sigma_2^2 (1 + \sigma_1^2 R_1^2 B^2)}{(\sigma_1 + \sigma_2)^2 + \sigma_1^2 \sigma_2^2 B^2 (R_1 + R_2)^2}
\]

II-42

This expression as it stands cannot be used directly with experimental
results to give useful information about the parameters of the two bands. However,
with some manipulation a more convenient form is developed here, from which the band
parameters $\sigma_1, \sigma_2, R_1$ and $R_2$ can be calculated from experimental results. This development is shown below. At $B = 0$ the expression II-42 reduces to:

$$R_0 = \frac{R_1\sigma_1^2 + R_2\sigma_2^2}{\sigma_o^2}$$  \hspace{1cm} \text{II-43}

where $\sigma_o = \sigma_1 + \sigma_2$, the zero field conductivity. Let the change in Hall coefficient $R - R_0 = \Delta R$, then:

$$\frac{\Delta R}{R_0} = \frac{\sigma_1\sigma_2^2(R_1 + R_2)[(\sigma_1 + \sigma_2)^2R_1R_2 - (R_1 + R_2)(R_1\sigma_1^2 + R_2\sigma_2^2)] B^2}{(R_1\sigma_1^2 + R_2\sigma_2^2)[(\sigma_1 + \sigma_2)^2 + \sigma_1^2\sigma_2^2 (R_1 + R_2)^2 B^2]} \hspace{1cm} \text{II-44}

with further manipulation we obtain:

$$-\frac{R_0}{\Delta R} = \frac{(R_1\sigma_1^2 + R_2\sigma_2^2) (\sigma_1 + \sigma_2)^2}{\sigma_1\sigma_2^2 (R_1 + R_2)(R_1\sigma_1 + R_2\sigma_2)^2} B^{-2}$$

$$+ \frac{(R_1\sigma_1^2 + R_2\sigma_2^2)(R_1 + R_2)}{(R_1\sigma_1 - R_2\sigma_2)^2}$$

$$= sB^{-2} + a \hspace{1cm} \text{II-45, II-46}

where

$$s = \frac{(R_1\sigma_1^2 + R_2\sigma_2^2) (\sigma_1 + \sigma_2)^2}{\sigma_1\sigma_2^2 (R_1 + R_2)(R_1\sigma_1 - R_2\sigma_2)^2} \hspace{1cm} \text{II-47}

$$a = \frac{(R_1\sigma_1^2 + R_2\sigma_2^2)(R_1 + R_2)}{(R_1\sigma_1 - R_2\sigma_2)^2} \hspace{1cm} \text{II-47}
Hence a plot of \(- \frac{R}{\Delta R}\) vs. \(B^{-2}\) will give a straight line, with the slope \(s\) and intercept "a" each depending on the four band parameters \(R_1, \sigma_1, R_2,\) and \(\sigma_2\). To solve for these factors two more relations between these and experimentally measurable quantities, are needed; these are given by the equations for \(R_o\) and \(\sigma_o\). The solution of the four equations then gives:

\[
\sigma_1 = \frac{\sigma_o}{2} \left[ 1 + \sqrt{\frac{c}{c + 4}} \right] \quad \text{II-49}
\]

\[
\sigma_2 = \frac{\sigma_o}{2} \left[ 1 - \sqrt{\frac{c}{c + 4}} \right] \quad \text{II-50}
\]

where

\[
c = - \frac{\sqrt{s a}}{R_o \sigma_o} \left[ R_o \sigma_o + \sqrt{\frac{s a}{s}} \right]^2 \quad \text{II-51}
\]

and

\[
R_1 = \frac{1}{2\sigma_1 - \sigma_o} \left[ R_o \sigma_o + \frac{\sigma_2}{\sigma_1} \sqrt{\frac{s a}{s}} \right] \quad \text{II-52}
\]

\(R_2\) can then be obtained from the equation II-43 for \(R_o\).

The choice of sign in the above expressions containing square roots depends on the relative magnitude of \(\sigma_1\) and \(\sigma_2\). The signs given correspond to \(\sigma_1 > \sigma_2\).

The above treatment shows that as long as the expression for two band Hall coefficient holds; i.e. \(B\) is small enough so that no quantum effects are present (Blatt 68B2, p. 216), measurement of \(R\) as a function of magnetic field \(B\) in principle can give information on the carrier concentration and mobility of carriers in the two bands. The main difficulty in obtaining this information, provided accurate experimental results of \(R\) and \(B\) can be obtained, is in the calculation of the carrier density \(n_i\) (i = 1, 2) from the respective Hall coefficients. This difficulty is inherent in the expression due to the usual lack of understanding of scattering
mechanisms in a band, and hence in the knowledge of the scattering factor \( r = \frac{\langle \tau_i \rangle}{\langle \tau_1 \rangle^2} \). Further discussion of this point is postponed till later chapters dealing with analysis of experimental results.

II-52 Multiband Hall coefficient

It is an easy step to extend the two band expression for \( R \) to any number of bands, say \( q \), i.e.:

\[
R = \frac{\sum_{i=1}^{q} D_i / B}{(\Sigma_{A_i}^2 + (\Sigma_{D_i}^2)^2)} , \quad i = 1, \ldots, q
\]

where

\[
A_i = \frac{\sigma_i}{(1 + \sigma_i^2 R_i^2 B^2)}
\]

and

\[
D_i = \frac{R_i \sigma_i^2 B}{(1 + \sigma_i^2 R_i^2 B^2)}
\]

We can write this expression in terms of carrier concentrations \( n_i \) and mobilities, \( \mu_i \), i.e.:

\[
R = \frac{\sum_{i=1}^{q} (r_i F_i e n_i \mu_i^2)/(1 + \zeta_i^2)}{\left[ \sum_{i=1}^{q} (e n_i \mu_i)/(1 + \zeta_i^2) \right]^2 + \left[ \sum_{i=1}^{q} (e n_i \mu_i \zeta_i)/(1 + \zeta_i^2) \right]^2}
\]

where \( \zeta_i = r_i F_i u_i B = \nu_{H_i} B \), and \( \nu_H \) is the Hall mobility defined by \( \nu_H = \frac{R \sigma}{\mu} \).

The conductivity \( \sigma \) is then given by

\[
\sigma = e \sum_{i=1}^{q} n_i \mu_i
\]

When the density of holes becomes appreciable so that they contribute to the transport, their effect is easily taken into account by additional terms in equations II-56 and II-57.
II-6 Magnetoresistance

When a magnetic field is applied to a conductor, as in the case of Hall effect, the resistance of the conductor will invariably increase. When the field is applied, the trajectories of electrons between collisions will acquire a curvature. This means that the component of motion in the direction of applied electric field will be reduced by the presence of the magnetic field, effectively increasing the resistance. This phenomenon will be briefly discussed for the case of two models.

II-61 One band magnetoresistance

It can easily be shown that for a simple band with isotropic constant energy surfaces and constant relaxation time, the magnetoresistance will be zero. There are two cases when the magnetoresistance for carriers in one band is non-zero; a) when the constant energy surfaces are non-spherical and b) when the relaxation time is a function of energy.

For a) in the case of low magnetic fields and ellipsoids with major axes along <100> and <111> directions the appropriate expressions can be found in Smith (59S1, p.124). Smith also gives a definition of the "magnetoresistance coefficient" ($\xi$) as

$$\xi + 1 = \frac{<\tau^3>} {<\tau^2>^2}$$  \hspace{1cm} \text{II-58}

Thus in case b) when the relaxation time is a function of energy and the band is not parabolic, the above expression becomes

$$\xi + 1 = \frac{<\tau^3>_m}{<\tau^2>_m^2}$$  \hspace{1cm} \frac{c}{m_c}$$  \hspace{1cm} \text{II-59}$$
II-62 Two band magnetoresistance

It is well known that for \( K = 1 \) and \( \tau = \text{constant}, \) magnetoresistance vanishes (e.g. Smith 59S1, p. 124). This is no longer true when we have two types of carriers. It was shown by Chambers (52C1) that in this case resistivity as a function of magnetic field is given by:

\[
\rho = \frac{(\sigma_1 + \sigma_2) + \sigma_1 \sigma_2 B^2 (\sigma_1 R_1^2 + \sigma_2 R_2^2)}{(\sigma_1 + \sigma_2)^2 + \sigma_1^2 \sigma_2^2 B^2 (R_1 + R_2)^2} \quad \text{II-60}
\]

Then \( \Delta \rho = \rho - \rho_o \)

\[
= \frac{\sigma_1 \sigma_2 B^2 (\sigma_1 R_1 - \sigma_2 R_2)^2}{(\sigma_1 + \sigma_2) \left[ (\sigma_1 + \sigma_2)^2 + \sigma_1^2 \sigma_2^2 B^2 (R_1 + R_2)^2 \right]} \quad \text{II-61}
\]

Kwan and Woolley (68K2) showed that as in the case of Hall coefficient \( R \), we can express \( \frac{\rho_o}{\Delta \rho} \) in the form of a straight line

\[
\frac{\rho_o}{\Delta \rho} = t B^{-2} + u \quad \text{II-62}
\]

where

\[
t = \frac{\sigma_0 - \sigma_1}{\sigma_1 \left( R_o \sigma_0 - R_1 \sigma_1 \right)^2} \quad \text{II-63}
\]

\[
u = \frac{\sigma_1 \left( R_o \sigma_0 + R_1 \sigma_1 \right) - 2 R_1 \sigma_1^2}{\left( \sigma_0 - \sigma_1 \right) \left( R_o \sigma_0 - R_1 \sigma_1 \right)^2} \quad \text{II-64}
\]

Using equations II-43, II-63 and II-64, we can again express \( \sigma_1, R_1, \sigma_2 \) and \( R_2 \) in terms of measurable quantities \( \sigma_0, R_0 \) and \( t, u \), i.e. \( \sigma_1 \) and \( \sigma_2 \) are again given by equations II-49 and II-50 where \( c \) now has the value

\[
c = t \left[ R_o \sigma_0 + \sqrt{\frac{u}{t}} \right]^2 \quad \text{II-65}
\]

and \( R_1 \) is then given by:

\[
R_1 = \frac{1}{\sigma_1} \left[ R_o \sigma_0 - \frac{\sqrt{\sigma_2}}{t \sigma_1} \right] \quad \text{II-66}
\]

As before \( R_2 \) can then be obtained from equation II-43.
II-7 Limits of Validity of Two Band Picture in the Presence of Non Isotropic Band

Two important assumptions have been made in the above analysis, viz. (a) that the bands can be treated as isotropic and independent and (b) that the variation of $R_1$, $R_2$, $\sigma_1$ and $\sigma_2$ with $B$ can be neglected, i.e. that single band effects are negligible. These assumptions will now be considered.

The materials to which this analysis has mainly been applied are the III-V compounds and their alloys, so that the two bands considered are the $\Gamma_1$ conduction band minimum and either the $<111>$ or the $<100>$ minima. It is generally accepted that the energy surfaces at $\Gamma$ can be taken as spherical, satisfying assumption (a). The energy surfaces at the $<111>$ and $<100>$ minima are ellipsoids of revolution, and are very similar to those of germanium and silicon respectively. The work of Gold and Roth (57G1) shows that in these later cases, the approximation to an isotropic form is satisfactory only if $\mu B << 1$. Applying this condition in the case of III-V materials, it is necessary to limit the magnetic field $B$ used in experimental work to a value such that $\mu_2 B << 1$ where $\mu_2$ is the mobility of electrons in the $<111>$ or $<100>$ minima. The values of $\mu_2$ obtained for different materials (71K1) show that in most cases a maximum value of $B$ of 3 Wb/m$^2$ will allow the above analysis to be used.

The question of whether the bands may be considered independent when inter-band scattering can occur has been discussed by Ehrenreich and Overhauser (56E1) and by Beer (63B1), who concluded that the above type of analysis is satisfactory.

With regard to assumption (b), that the single band contributions to the magnetic effects can be neglected, this depends upon the detail of the bands concerned and the temperature in any particular case. However, it has been pointed out by Blatt (68B2), that in the majority of cases such an assumption is valid. As a particular example, here we will consider the case of GaSb. The variation of $R_1$, $R_2$, 

\(\sigma_1\) and \(\sigma_2\) with B can be calculated from standard one band equations (59S1). For the \(\langle 111\rangle\) minima, assuming typical conditions, it has been shown (65HL) that the variation in \(\sigma_2\) due to both ellipsoidal energy surfaces and energy dependent relaxation time is less than 2.5\%. In the case of the Hall parameter \(R_2\), provided the condition \(\mu_2B \ll 1\) discussed above is satisfied, \(R_2\) is effectively independent of B. In the case of the \(\Gamma_1\) minimum, any single band effects will be due to non-parabolicity of the band and to energy dependent relaxation times. Assuming a Kane model for this band and determining room temperature relaxation time values from a suitable combination of polar optical, interband and ionized impurity scattering (Section IV-332), it is found that \(\frac{\Delta R_1}{R_1}\) will be less than 2.0\% and \(\frac{\Delta \sigma_1}{\sigma_1}\) less than 3.2\%.

The values are sufficiently small to justify the use of the above method for the analysis of the Hall effect and magnetoresistance data obtained for GaSb at room temperature. This is confirmed by the experimental data in Chapter IV. Thus in figures (IV-5, IV-6) the graphs of \(\frac{\rho_0}{\Delta \rho}\) and \(-\frac{R_0}{\Delta R}\) versus \(B^{-2}\) are straight lines as predicted by equations II-45 and II-62, and also the values of \(R_1\), \(R_2\), \(\sigma_1\) and \(\sigma_2\) obtained by the three different methods of analysis are in good agreement. Neither of these results would obtain if the analysis were invalid. Thus in the present work at higher temperature (see Section IV-31), it was found that at temperatures above 120\(^\circ\)C the variation of \(\frac{\rho_0}{\Delta \rho}\) with \(B^{-2}\) was no longer linear, and it can be shown that as the temperature is raised, the variation of \(R_1\), \(\sigma_1\) and \(\sigma_2\) with B increases, so that at temperatures above about 120\(^\circ\)C, in this case, the single band contributions can not be considered negligible. Thus, it is seen that the applicability of the above analysis must be considered for each individual case. However, to a considerable extent, the analysis provides in itself a check on whether the initial assumptions are valid. This is further illustrated by the results on a GaAs sample doped with
Te (Chapter V) on which magnetoresistance measurements were made at 400°C. The plot of $\frac{p_0}{\Delta p}$ versus $B^{-2}$ is not linear and has a negative intercept at $B^{-2} = 0$.

II-8 Combined Analysis of Two Band Hall Effect and Magnetoresistance

In practice the accuracy with which the band parameters $\sigma_1$, $R_1$, $\sigma_2$ and $R_2$ can be determined from the analysis of either two band Hall effect or magnetoresistance, depends on the accuracy of the intercept "a" or "u", and is quite sensitive to these factors. Since the values of the intercepts are usually small, of the order of unity, and depend on extrapolation of the straight line to infinite fields, "u" and "a" are less reliable than the slopes $s$ and $t$. Since only four equations are needed to determine the four band parameters, by combining the results of two band Hall effect and magnetoresistance, more accurate results can be expected using $\sigma_0$, $R_o$, $s$ and $t$. In this case $\sigma_1$ is again given by equation II-49, but now

$$c = t[R_o \sigma_0 (1 - \left(\frac{t}{s}\right)^{\frac{1}{2}})]^2$$  \hspace{1cm} \text{(II-67)}$$

and

$$R_1 = \frac{R_o \sigma_0 [s \sigma_1 - t(\sigma_0 - \sigma_1)]}{s(2\sigma_1 - \sigma_0)\sigma_1}$$  \hspace{1cm} \text{(II-68)}$$

II-9 Charge Neutrality and Fermi Energy

If we consider a semiconductor in the state of equilibrium, then the net charge within the material must be zero. This is often called the "charge neutrality" condition and may be stated in the form

$$N_d - N_a = \sum_i n_i - \sum_j p_j$$  \hspace{1cm} \text{(II-69)}$$
where \( N_d \) and \( N_a \) are respectively the concentration of ionized donors and acceptors and \( i \) and \( j \) run over bands containing electrons and holes respectively.

For a simple donor level, the net ionized donor density \( N_d \) is given by

\[
N_d = \frac{N_D}{1 + g_T \exp\left(\frac{-E_D + E_F}{kT}\right)}
\]

II-70

where \( g_T \) is the "degeneracy" of the donor level, and \( E_D \) and \( E_F \) are the ionization energy of the donors and Fermi energy respectively. Under suitable conditions of temperature and ionization energy \( N_d = N_D \). A similar expression can be written for the acceptors in a p-type semiconductor.

Considering an n-type semiconductor with three conduction bands (1 - 3) with extrema "close" in energy so that under specific conditions all three bands will contain electrons, as is the case for GaSb and GaAs at high temperatures. We shall also consider that the lowest band is at the centre of the Brillouin zone with \( \Gamma \) symmetry, and can be described by the Kane model, the remaining two bands will be assumed to be parabolic. The carrier concentrations will then be given by

\[
n_1 = 4\pi(2m_{10}^* kT/h^2)^{3/2} \left[F_{1/2}(\eta) + (5/2 - 5\xi)\beta F_{3/2}(\eta) + (7/2 - 21/2\xi)\beta^2 F_{5/2}(\eta) - (1/4 + 7/2\xi)\beta^3 F_{3/2}(\eta)\right]
\]

(68T1)

II-71

where

\[
\eta = \frac{E_F}{kT}, \quad \xi = \frac{m_{10}^*}{m}, \quad \beta = \frac{kT}{E_o^*}
\]

(57E1)

II-72

\[
m_{10}^*(T) = m_{10}^*(0) \left(\frac{E_o^*}{E_{oo}}\right)^\xi
\]

(68T1)

II-73

\[
E_o^* = E_{oo} - (E_{oo} - E_o)/2
\]

(67V1)

II-74

\[
E_o = E_{oo} + \alpha_o T^2/(\theta_o + T)
\]
$F_{\frac{1}{2}}$ etc are standard Fermi integrals. The Fermi energy $E_F$ is measured from the bottom of band 1, $m_{10}^*$ is the effective mass at the bottom of this band, $E_{\infty}$ is the absolute zero value of the band gap and $\alpha$, and $\theta$ are constants.

\[
n_2 = n_2 c \left( \frac{m_{2d}^*}{m} \right)^{3/2} F_{\frac{1}{2}}(n - \delta_2) \quad \text{II-75}
\]

\[
n_3 = n_3 c \left( \frac{m_{3d}^*}{m} \right)^{3/2} F_{\frac{1}{2}}(n - \delta_3) \quad \text{II-76}
\]

\[
p_h = c \left( \frac{m_{ph}^*}{m} \right)^{3/2} F_{\frac{1}{2}}(n - \Delta) \quad \text{II-77}
\]

\[
p_L = c \left( \frac{m_{pl}^*}{m} \right)^{3/2} F_{\frac{1}{2}}(n - \Delta) \quad \text{II-78}
\]

where

- $c = 4\pi (2mk/h^2)^{3/2}$

- $\delta_2 = E_2/kT$, $\delta_3 = E_3/kT$, $\Delta = E_\infty/kT$

$m_{2d}^*$, $m_{3d}^*$, $m_{ph}^*$ and $m_{pl}^*$ are density of states effective masses in bands 2, 3 and valence bands respectively, $n_2$ and $n_3$ the band multiplicity and $E_2$, $E_3$ are the energy separations of 1 to 2 and 1 to 3 conduction band minima respectively.

The temperature variation of $E_2$ and $E_3$ can be assumed linear, i.e.

\[
E_2 = E_{20} + \alpha_2 T \quad \text{II-79}
\]

\[
E_3 = E_{30} + \alpha_3 T \quad \text{II-80}
\]

at temperatures when the more complicated form, such as that given by Varshni (67V1), become linear with temperature. This point will be further discussed in the following chapters dealing with experimental results.
<table>
<thead>
<tr>
<th>General</th>
<th>Non-parabolic, but spherically symmetric bands</th>
<th>Parabolic bands with ellipsoidal energy surfaces</th>
</tr>
</thead>
<tbody>
<tr>
<td>Band curvature effective mass</td>
<td>( \frac{1}{m_{ij}} = \frac{1}{\hbar} \frac{\partial^2 E}{\partial k_i \partial k_j} )</td>
<td>( \frac{1}{m_e^*} = \frac{1}{\hbar} \frac{\partial^2 E}{\partial k^2} )</td>
</tr>
<tr>
<td>Cyclotron effective mass</td>
<td>( m_c^* = \frac{\hbar^2 k}{2\pi} \frac{\partial A}{\partial E} )</td>
<td>( m_c^* = \frac{\hbar^2 k}{2\pi} \frac{\partial A}{\partial E} )</td>
</tr>
<tr>
<td>Density of states effective mass</td>
<td>( n = \frac{1}{\pi} \int_0^\infty f_o k^2 \frac{dk}{dE} \ dE )</td>
<td>( n = \frac{1}{\pi} \int_0^\infty f_o k^2 \frac{dk}{dE} \ dE )</td>
</tr>
</tbody>
</table>

**Table II-1:** Definitions of effective masses, \( m_e^* \), \( m_c^* \) and \( m_d^* \).
# CHAPTER III

**EXPERIMENTAL TECHNIQUE AND EQUIPMENT**

## III-1 High Temperature furnaces and Sample holders  

### III-11 Temperature controlled furnace, $20^\circ - 450^\circ C$  

- **III-111 Sample holder**  
- **III-112 Measurement technique and apparatus**

### II-12 Temperature drift furnace, $20^\circ C - 800^\circ C$

- **III-121 Sample holder**  
- **III-122 Measurement technique and apparatus**

## III-2 Material and Sample Preparation  

### III-21 Growth of GaSb and (GaIn)Sb alloys.

### III-22 Sample preparation and ohmic contacts for high temperature measurements.
III-1 High Temperature Furnaces and Sample Holders

Two different furnaces were used in this work, this was necessary since essentially two kinds of experiments were carried out. The measurement of magnetoresistance and Hall coefficient as a function of magnetic field and temperature required long term temperature stability, whereas the measurement of Hall coefficient and conductivity as a function of temperature only, could be carried out rapidly with slow temperature drift. Essential features of the two types of furnace are described below.

III-11 Temperature controlled furnace, (20° - 450° C)

This furnace was originally designed by Dr. G. Dionne (71D1) for the purpose of making Hall effect and magnetoresistance measurements between 77 and 800°K. The principal features of this design are: a) small outside diameter, in order that it would fit between the pole pieces of the Harvey-Wells 15" electromagnet with a gap of 1¼"; b) low power consumption and good temperature stability. Schematic drawings of the furnace are shown in Figs. III-1 and III-2. Fig. III-1 shows the overall design and Fig. III-2 shows the region of the heater and sample holder. The manganin heater wire (48Ω/ft.) is wound around a thin copper capsule sealed at the bottom end. The wire is kept in place and insulated from the capsule with Sauereison cement. The capsule fits snugly into a quartz tube which is also sealed at the bottom end and forms the inner wall of a double vacuum jacket which reduces heat losses to the outside. Maximum temperature of the outer stainless steel wall does not exceed 70° C. The two vacuum jackets can be evacuated separately, the inner space can be filled with liquid nitrogen to facilitate measurements below room temperature. The sample holder fits inside the
copper capsule and the whole region can be evacuated, flushed and then filled with helium gas to prevent undue oxidation of the sample and aid in establishing temperature equilibrium. Because the copper capsule is a good conductor and is well insulated, no temperature gradients within the capsule could be observed when a thermocouple probe was used. The design of the sample holder (described below) is therefore critical. It was designed in such a way so as not to introduce excessive thermal leaks which would introduce undesirable gradients at the sample.

There are two (Pt. vs Pt. + 13% Rh.) thermocouples, (not shown in the schematics) one is attached to the copper capsule and responds rapidly to any temperature change of the capsule. The second is part of the sample holder, the junction is placed within 1 mm of the sample. This thermocouple was checked at steam point against N.B.S. 1962 tables. Temperature may be controlled by either thermocouple, although the use of the thermocouple on the sample holder gives a sluggish response due to thermal lag. The advantage of using this thermocouple for control is that it is also used for temperature measurement and hence only one potentiometer is required.

Temperature control is achieved by an electro-optical system shown in Fig. III-3. The signal from the thermocouple is fed into a Rubicon potentiometer set for a given temperature. The galvanometer spot is in off balance position, hence no light falls on the photocell and maximum current is demanded, since the plate current is proportional to photocell resistance. As the temperature rises and the light spot approaches the photocell the current falls reaching equilibrium when the potentiometer is at balance. Temperature control of better than \( \pm 0.01^\circ\mathrm{C} \) has been obtained using this system at a maximum temperature of \( 450^\circ\mathrm{C} \). Also long term stability of \( \approx 0.005^\circ\mathrm{C}/\text{hr.} \) was achieved. The drift problem arises mainly from
two sources, drift of the voltage supply to the potentiometer and secondly the
drift of temperature of the reference junction of the thermocouple. These were
minimized by placing the potentiometer supply battery in a styrofoam container
and keeping it connected continuously. The reference ice bath was stirred
periodically to maintain its temperature constant.

III-111 The Sample Holder

The schematic of the sample holder, designed by the author, is shown in
Fig. III-4. The sample holder itself was machined out of Pyrophylite and then
fired at 1500°C in an inert atmosphere. The sample is held by a stainless steel
leaf spring (not shown) which is insulated from the sample by a thin piece of
mica. The contacts to the sample are made with .005" platinum wire attached to
stainless steel (#304) screws. The copper lead wires are then attached to the
other end of the screws at the back of the holder and together with thermocouple
wires, are cemented in place in grooves cut for the purpose. The narrow neck of
the holder then fits inside a quartz tube which is attached to the upper flange
(Fig. III-1). The lead wires are passed through ceramic thermocouple feed-
throughs which are placed inside the quartz tube and then pass through covar feed-
throughs in the upper flange. In this way the lead and thermocouple wires are
continuous from the sample holder to the outside of the furnace reducing possible
thermal noise due to junctions.

The use of quartz tube in this construction is not completely satisfactory
since it is not mechanically strong. Definite improvement could be made using a
ceramic tube of low thermal conductivity.
III-112 Measuring Apparatus and Technique

The leads from the sample were soldered to a Leeds and Northrup eleven position selector switch which was connected to an Astodata nanovolt amplifier model 120, the output of which was in turn connected to a 5 figure Dana digital voltmeter model 5330/360. The resolution of the digital voltmeter is 1μV, but in conjunction with the nanovolt amplifier which was used at gains of $10^3$ and $5 \times 10^3$ gives resolution of 5 nV, limited by the input noise of the instrument.

The current to the sample was supplied by a 12V battery with suitable rheostats in the circuit. Currents of 1 and 10 mA were used depending on sample resistance and magnitude of the signal.

The electro-magnet employed was a Magnion-Harvey-Wells 15 inch model, with a pole gap of $\frac{1}{4}''$ and maximum field of 3.2 Webers/m$^2$. The accuracy of the field was five parts in $10^5$, and stability of one part in $10^6$. Uniformity over a 1.0 cm specimen placed in the centre of the pole pieces was one part in $10^5$ at maximum field. The calibration and procedure is described by Aubin (69Al) and is given in Fig. III-11.

The advantage of using the Astodata nanovolt amplifier as a preamplifier is that the input signal can be backed-off to zero on this instrument. When measuring magnetoresistance and Hall effect as a function of magnetic field, this capability of the preamplifier is very necessary and convenient, otherwise external back-off circuitry has to be used. It is due to the use of this amplifier that very accurate measurements were made on n-GaSb.

When the temperature had reached equilibrium, the zero field resistivity voltage was measured, by selecting the appropriate lead wires on the selection switch. This voltage was then backed-off to zero and the magnetic field turned on,
the magnetoresistance voltage was then obtained directly. The procedure was carried out with reversed magnetic field and sample current. The Hall voltage was measured in similar fashion except that in this case only the misalignment voltage was backed-off. With proper care in positioning contacts on the sides of a parallelepiped shaped specimen, the misalignment voltage was small. Time taken to complete the measurements at a given temperature was of the order of 2 hours.

III-12 Temperature Drift Furnace (20 - 800°C)

The design of this furnace was originally made by Coderre (69C2) and is shown in Fig. III-5. The heating element (Nichrome wire 1.26 ohms/ft.) was re-wound in order to improve the temperature profile, giving an essentially flat region at the position of the sample. The profile is shown in Fig. III-6 at two power settings, also the position of the sample in the furnace is shown.

The power to the furnace was supplied by a 2KVA variac. The temperature was continuously changed at a constant low rate by a motor connected by a pulley to the variac. The rate was adjusted by changing the pulley ratio between the motor and the variac. The average rate used was ~3°C/min. The time required to take readings was ~20 sec., hence giving temperature accuracy of ±0.5°C. The temperature was continuously monitored by a Pt. vs Pt. + 13% Rh. thermocouple junction placed within 1 mm. of the sample. The reference junction was maintained at ice point. The thermocouple emf was backed-off by a Tinsley four figure potentiometer and displayed with the aid of a Pye Scalamp light spot galvanometer.

Measurements were taken every 10°C from room to the highest temperature of 600°C. This temperature limit is set due to the high vapour pressure of As and
the subsequent loss of this element from the surface of GaAs samples at temperatures greater than 600°C, leaving a thin film of highly conducting Ga metal on the sample surface.

III-121 Sample Holder

The sample probe is shown in Fig. III-7 and is again described by Coderre (69C2). One modification was made here: the feed-throughs were changed to hollow type to allow electrical lead copper wires to pass through, maintaining continuity. This served to minimize the possibility of additional thermal emf appearing as voltage noise if all the pins were not at the same temperature.

III-122 Apparatus and Measuring Technique

The measuring circuit was essentially the same as that used by Coderre (69C2) and is shown in Fig. III-8. In order to monitor sample current accurately a 10Ω standard resistance was introduced in series with the sample, also a better constant current power supply (Electronic Measurements Model C612) was introduced in the sample circuit, the ripple was 0.01% or less and drift of less than 0.005% over a day after warm up.

The Hall and conductivity voltages were measured using a Hewlett-Packard model 419A null voltmeter as a preamplifier. The input impedance of this meter was >1MΩ, thus high enough for measurement of low resistivity samples. The output was nominally 1 volt on all scales, and was fed into a Hewlett-Packard 4 figure digital voltmeter.

It was found that the preamplifier gain would always be slightly lower than specified (1% - 2%), hence after each run it was calibrated against a Tinsley
potentiometer and voltage readings were corrected accordingly.

A magnetic field of .800 W/m² was used, supplied by an electro-magnet with 4" pole pieces separated by 1.4" gap. The magnetic field was measured using a Rawson-Lush rotating coil gaussmeter.

III-2 Material and Sample Preparation

This thesis is concerned with the determination of some conduction band parameters in GaSb and GaAs. Some of the crystal GaAs used for this work was obtained from Monsanto Chemicals Co., samples prepared from this material will be denoted by the prefix MO. The remaining samples of GaAs were cut from single crystals prepared by Dr. B. Lombos at Northern Electric R & D Laboratories, these will be denoted by the prefix NE.

A number of samples of n-GaSb-Te as well as one p-type sample were measured, these were cut from single crystal ingots prepared by the author and a slice obtained from Dr. Strauss of Lincoln Laboratories. In addition the author attempted to grow good quality alloys of Te doped $\text{Ga}_x\text{In}_{1-x}\text{Sb}$ in the composition range $.9 < x < 1$, for the purpose of studying interband scattering, $\Gamma_{1c}$ and $L_{1c}$ band separation and electron effective mass in the $\Gamma_{1c}$ and $L_{1c}$ bands through magnetoresistance, Hall effect and de Haas Shubnikov measurements at low temperatures. The method of preparation of GaSb and Ga$_x$In$_{1-x}$Sb is described below.

III-21 Preparation of Te doped GaSb and Ga$_x$In$_{1-x}$Sb

GaSb single crystals can easily be grown from a stoichiometric melt using the Czochralski (19C1) method.
It was shown by Trumbore et al. (62T1) that single crystals of 
Ga\textsubscript{x}In\textsubscript{1-x}Sb also can be grown from the melt using the same method. They report 
obtaining single crystals in the composition range $0.4 < x < 1$. More recently 
Robert (69R1) reported in his thesis that only single crystals in the range 
$0.95 < x < 1$. could be prepared by this method in their laboratory.

(i) The Czochralski method

The melt is contained in a crucible and a seed crystal which dips into 
it from above is rotated and slowly withdrawn, so that the rate of upward motion 
is equal to the rate of crystal growth, which is determined by the rate of loss 
of latent heat through the seed. If the two rates do not balance, the crystal 
either increases or decreases in diameter as growth proceeds. The purpose of the 
rotation is to maintain circular symmetry of the crystal, the shape of which would 
be unstable if the crystal were stationary. An added advantage of the rotation is 
that it causes the liquid near the interface to flow outwards thus improving mixing 
of the melt.

The basic vertical Czochralski crystal-pulling furnace is shown 
schematically in Fig. III-9. The charge is placed in a crucible which in turn fits 
into a graphite susceptor supported by an insulating pedestal. A seed holder 
containing a seed at the bottom, is attached to precision motors which withdraw 
and rotate the seed. In order to be able to grow under controlled atmosphere all is 
surrounded by a quartz tube which is sealed at both ends by steel flanges with 
O-rings, and gas inlet and outlet ports. The R-F coil is placed on the outside of 
the quartz tube surrounding the carbon susceptor.

The Czochralski apparatus used was on loan to the author from the 
Communications Research Centre at Shirley Bay, by the kind permission of 
Drs. C. D. Cox and J. Barrie.
(ii) **Alloy preparation**

In order to prepare heavily Te doped single crystals of GaSb and Ga\(_{x}\)In\(_{1-x}\)Sb in the composition range 0.9 < x < 1 it was necessary to divide the work in three stages: a) preparation of single crystal GaSb; b) Te doping of GaSb; c) preparation of single crystal Ga\(_{x}\)In\(_{1-x}\)Sb.

(a) Polycrystalline GaSb was initially synthesized by sealing off individual elements, in suitable proportions, in an evacuated quartz ampoule and placing it in a furnace at 900\(^\circ\)C for thirty minutes and then quenching in air. The ingot obtained in this way was coarsely polycrystalline and was used as a charge in the quartz crucibles in the Czochralski puller. Randomly oriented seed crystals were cut from parts of an ingot obtained from the Monsanto Chemical Co. However, it would have been preferable to use seeds with <111> orientation since this is the preferred growth direction in diamond and zincblende structures, but this was not possible with the available seed material.

It was evident from the start that the use of hydrogen atmosphere was not sufficient to keep the melt surface clean and the melt stoichiometric. Despite its relatively low vapour pressure at the melting point of GaSb, Sb is still lost from the surface. This has two undesirable effects, firstly that the melt becomes Ga rich and secondly that Sb is deposited on the quartz tube making observation of the crystal growth difficult. In order to overcome this, the well known "Liquid Encapsulation" technique was introduced using boric oxide glass. This has the advantage of overcoming the above two problems but also a disadvantage in that the glass is extremely viscous and limits seed rotation to about 10 revolutions per minute. In this way three single crystals of GaSb were pulled, varying in weight from 5 to 11 gms, using a pull rate of \(\sim\)5 mm/hr. and a rotation rate of about
7 - 10 revolutions per minute.

Two single crystals of undoped GaSb were grown about 1.5 cm in diameter and 3 cm. long. Undoped GaSb always grows p-type, it is thought (64M1) that lattice defects are responsible for the p-type nature of this material. The p-type GaSb grown was of good quality with uniform carrier distribution. A sample cut from one of the ingots was used by the author for high temperature measurements of Hall coefficient and conductivity. Results of those measurements will be discussed in Chapter IV. This material has also been used by other members of this group for various measurements, e.g. thermoreflectance, electroreflectance and photoconductivity.

(b) Tellurium doping of GaSb. Several crystals of GaSb doped with Te were pulled, none were perfect single crystals, all had at least two or three twin boundaries running from top to bottom or diagonally across the crystal. The twinning was not a consequence of Te doping, but rather of difficulties in obtaining clean melt surfaces. These crystals had room temperature Hall coefficients ranging from $1 \times 10^{-5}$ to $7 \times 10^{-5} \text{ m}^3/\text{O}$.\text{C.}

In calculating the necessary weight of Te to be added to the melt we initially used the segregation coefficient $k_{\text{Te}}$ value of 0.4 given by Madelung (64M1). The results of Hall effect measurements showed that this value was probably high, a value of 0.1 or 0.2 was closer from our results. The Hall effect results also showed that doping uniformity in crystals doped to $>10^{18}$ Te atms/cc was good, but in lighter doped crystals severe gradients could be observed. However, the gradients are not restrictive when measurements with Van der Pauuw geometries can be made. This material was therefore used by S. Rosenbaum (72S1) in investigation of the effect of hydrostatic pressure on transport properties of n-GaSb. (Further
discussion of gradient problems is given in Chapter IV which deals with the transport results on n-GaSb).

The pseudo-binary phase diagram of (GaIn)Sb was determined by Woolley et al. (58W1) and is shown in Fig. III-10. It can be seen from this figure that the segregation coefficient is ~0.2, viz., approximately 50 - 50% melt of (GaIn)Sb corresponds to solidus composition of Ga.9In.1Sb. This is the maximum InSb content necessary for the proposed measurements on this system. Hence as a crystal is being pulled from a melt, the melt becomes richer in InSb, causing composition gradients along the growth of the crystal. To overcome this, large melts (100 - 150 gms) were used to pull crystals of up to 10 gms wt.

Several single phase crystals were obtained using melts with InSb content from 20 to 50% using growth rates of between 2 and 3 mm/hr. These crystals were single over the initial 1 to 2 cms. but later, heavy twinning occurred, the reason for this is not understood. The composition of these crystals was analysed using the X-ray powder diffraction method. This requires a knowledge of the lattice parameter variation with composition, which was assumed to be linear to a first approximation. Since this method is accurate to only 1.5% this assumption is not restricting. The composition of three of the crystals grown was determined to be between 2 and 8% of InSb in (GaIn)Sb by the above method.

Although the first one to two cm. were single crystal, difficulties were encountered in increasing the diameter beyond about 0.8 cm. without heavy twinning occurring, this again could in part be ascribed to scum on the melt surface beneath the boric oxide glass and in part to constitutional supercooling.

At this point it was felt that to obtain good quality crystals of the alloy doped with Te by this method would be extremely hard and impractical, since
the growth rates may need to be cut down further to avoid constitutional super-
cooling extending preparation time to unreasonable limits. Further growth of the
alloys was therefore temporarily abandoned. The prepared crystals were not of
good enough quality for the purpose of the proposed measurements because of the
non uniform impurity distribution. However, this same material can be used for
measurements which do not require the use of relatively large samples and hence
do not impose such stringent requirements. For example, optical and those
electrical measurements for which the Van der Pauw geometries are possible, can be
made on this material.

III-22 Sample Preparation and Ohmic Contacts for High Temperature Measurements

All samples measured were parallelepiped shaped with dimensions of the
order of 1.5 x 0.2 x 0.1 cm. These were cut from slices using either a carborundum
disc or wire saw. They were then boiled in Trichloroethylene, vapour degreased and
lapped with #3200 alumina lapping powder.

Ohmic contacts to GaAs samples were obtained by evaporating 0.015" strips
of gold through a mask on each side of the specimen, as shown in Fig. III-12. Care
was taken to align the strips on each side of the specimen to minimize the Hall voltage
offset. The two ends of the specimen were completely covered with gold for current
contacts. The gold was then alloyed with GaAs at 530°C in He atmosphere for 2 min.
The sample was allowed to cool in the alloying furnace to prevent oxidation. The
specimen was placed in the sample holder and 0.005" Pt. wires were then spark welded
to the gold areas. This method produced good ohmic contacts which remained ohmic up
to the highest temperature reached. Problems were encountered with the mechanical
strength of these contacts, when transferring the sample probe into the furnace,
contact was sometimes lost. Also, at elevated temperatures, contacts were lost probably due to expansion of the Pt. wires. Apart from these small problems the method was basically sound.

It was found unnecessary to go through the alloying procedure with contacts to GaSb. Spark welded Pt. wires directly to the specimen produced ohmic contacts of low resistance and good mechanical strength.
Fig. III-1  Temperature-controlled furnace.
Fig. III-2 Temperature-controlled furnace-sample area. Detail of construction.
Fig. III-3  Temperature control for controlled furnace.
Fig. III-4  Sample holder probe for controlled temperature furnace.
Fig. III-5 Schematic of high temperature drift furnace (20-800°C).
Fig. III-6  Temperature profile in the drift furnace at two rheostat settings.
Fig. III-7  Sample holder probe for the high temperature drift furnace.
Fig. III-8  Schematic of the measuring circuit.
Fig. III-9  Schematic of the Czochralski crystal growth method.
Fig. III-10 Phase diagram of (GaIn)Sb after Woolley and Smith (58Wl)
Fig. III-12  Parallelopiped sample showing evaporated gold strips.
CHAPTER IV

CONDUCTION BANDS IN GaSb

IV-1 Material and Experimental conditions

IV-2 Experimental Results

IV-21 Measurements of magnetoresistance and Hall coefficient as a function of magnetic field and temperature on sample N2

IV-22 Measurements of low field Hall coefficient and conductivity as a function of temperature.

IV-3 Method of Analysis

IV-31 Determination of $\Gamma_1 - L_1$ minima separation and its temperature dependence from magnetoresistance results.

IV-32 Determination of carrier mobility in the $\Gamma_1$ and $L_1$ bands from Hall coefficient and conductivity.

IV-31 Sample N2

IV-32 Sample P1

IV-33 Analysis of electron mobility in $\Gamma_1$ and $L_1$ bands.

IV-331 Analysis of the mobility $\mu_2(T)$ in the $L_1$ band for $T < 360^\circ$C.

IV-332 Analysis of the mobility $\mu_1(T)$ in the $\Gamma_1$ band
IV-333 Calculation of the scattering parameter $r_1$ in the $\Gamma_1$ band.

IV-334 Calculation of magnetoresistance coefficient.
CHAPTER IV  CONDUCTION BANDS IN GaSb

IV-1 Material and Experimental Conditions

The conduction band structure of GaSb satisfies the two primary conditions required for investigation of the band parameters by the two band model and by measurements of Hall coefficient and conductivity as a function of both magnetic field and temperature. These two conditions are (a) appreciable number of carriers are present in each band and (b) mobilities of the carriers in the two bands are not the same. For optimum conditions, the carrier densities in the two bands should be of the same order of magnitude with a mobility ratio of about 10:1.

Although the above mentioned conditions are satisfied, the range of Tellurium doped n-GaSb on which useful measurements can be made is further restricted. The additional restrictions are imposed by (a) the material and (b) the analysis of experimental data. However, the amount of information on the band structure of GaSb which can be obtained from measurements of Hall coefficient and conductivity as a function of magnetic field and temperature, even within the limits of the above restrictions, make such an investigation of considerable value.

It is well known that grain boundaries and impurity gradients can strongly affect galvanomagnetic measurements and their interpretation (e.g. Beer 63B1, p. 308 and references therein). It is therefore desirable to have single crystal samples with uniform impurity distribution, when galvanomagnetic measurements are used to investigate semiconducting materials.
Single crystal Te doped n-GaSb can be prepared with total electron concentrations of between about $10^{23}$ m$^{-3}$ and $5 \times 10^{24}$ m$^{-3}$. The lower limit is imposed by compensating acceptors which are always present in concentrations in the order of $10^{23}$ m$^{-3}$, whereas the upper limit is set by Te impurities which strain the lattice so that above this concentration single crystal growth is impossible. Because of the effect of compensation by the acceptors, material doped to $<10^{24}$ m$^{-3}$ tends to have impurity gradients. Although it is not impossible to obtain large samples (required for magneto-resistance measurements) with uniform impurity distribution, nevertheless it is extremely difficult. These considerations therefore, limit the useful material to a doping range from about $10^{24}$ m$^{-3}$ to $5 \times 10^{24}$ m$^{-3}$. Additional conditions which restrict the useful range of material are further imposed by the analysis of the data. These further restrictions will now be discussed.

In order to calculate the respective carrier concentrations $n_1$ and $n_2$ from $R_1$ and $R_2$, the Hall scattering parameters $r_1$ and $r_2$ must be known. Since, as will be seen later, the dominant scattering in the $\Gamma_1$ band of GaSb near room temperature is by screened Coulomb scattering centres, then, for electron concentrations of $<10^{24}$ m$^{-3}$, $r_1$ will have a value of between 1.0 and 1.9 depending on carrier degeneracy. Therefore without prior knowledge of the effective relaxation time for electrons in the $\Gamma_1$ band, $r_1$ can not be calculated and hence the value of $n_1$ can not be determined accurately from $R_1$. For this reason it is difficult, if not impossible, to make accurate analyses on n-GaSb with electron density $<10^{24}$ m$^{-3}$.

The above mentioned problems of homogeneity and the difficulty in analyses of galvanomagnetic data, because of uncertainty in $r_1$, do not apply for material in which the electron concentration is $> 10^{24}$ m$^{-3}$. However, other factors impose an upper limit of about $2 \times 10^{24}$ m$^{-3}$. Firstly, the value of $r_2$ becomes uncertain, at
higher electron concentrations, because of increased impurity scattering. Secondly, the total carrier concentration \( n_t = n_1 + n_2 \) no longer remains constant with temperature, which makes the analyses of experimental data more difficult and unreliable. The temperature variation of \( n_t \) is due to the presence of the Te impurity states which are associated with the \( L_1 \) minima. When the doping increases and the Fermi level approaches the impurity level, some electrons will be trapped by the impurities. Since the ionization energy of the Te impurities associated with the \( L_1 \) minima is not well known, this brings in an additional uncertainty into the analyses. In addition, it is not known at what doping concentrations the impurity level becomes a band and overlaps with the \( L_1 \) minima. This would have a twofold effect on the \( L_1 \) band, i.e. it would distort the band near the minima and would reduce the \( \Gamma_1 - L_1 \) band separation (69T1). Both effects would further unnecessarily complicate the analyses. The foregoing considerations therefore limit the doping range for useful experiment and analyses to a range of electron concentrations of roughly 1 to \( 2 \times 10^{24} \) m\(^{-3}\).

It should be made clear that whereas the upper limit is set by the physical properties of heavily doped n-GaSb, the lower limit is principally set by the experimental difficulty of growing homogeneous material. Since, the problem arising from uncertainty in \( r_1 \) can be overcome once the electron scattering mechanisms in the \( \Gamma_1 \) band are known, then \( r_1 \) can be determined by a "closed loop iteration" procedure.

IV-2 Experimental Results

In view of the discussion of the previous section, the choice of samples for measurements of Hall coefficient, \( R \), and conductivity, \( \sigma \), as a function of magnetic field, \( B \), was based on two criteria; (a) homogeneity and (b) range of Hall coefficient
satisfying the electron concentration range for accurate analyses. Therefore, preliminary measurements of conductivity and constant field Hall coefficient were made on five n-type samples of GaSb in the temperature range $20^\circ < T^\circ C < 550^\circ$. In addition, for reasons discussed later, a p-type sample of GaSb also was measured in the same temperature range. The n-type samples had room temperature Hall coefficients of between 9 and $70 \times 10^{-6}$ $m^3/C$. Out of these five samples one was chosen for measurements of $R$ and $\sigma$ as a function of magnetic field and temperature. The remaining four samples were rejected for reasons discussed below.

Typical results of Hall coefficient and conductivity as a function of temperature, for one of the rejected samples, N1, are shown in fig. IV-1 and IV-2. It can be seen from fig. IV-1 that considerable impurity gradient exists in this sample. The two sets of Hall probes (2-6, 3-5; Fig. III-12) were spaced 5.00 mm. apart and gave Hall coefficients differing by about 20% at $B = 0.87$ Wb/m$^2$. Fig. IV-1 shows also that the maxima in the separate Hall coefficient curves, corresponding to the two sets of Hall probes, occur at different temperatures. The conductivity, $\sigma_0$, over the same temperature range shows an inflection, Fig. IV-2. Since $\sigma_0$ is a product of the carrier concentration and mobility between the two sets of Hall probes, the inflection in $\sigma_0$ is therefore due to some average effect of carrier concentration and mobility variation along the sample. On the other hand, the value of $R$ is usually calculated by taking the average of the two Hall coefficients corresponding to the two sets of probes. Therefore the values of $R$ and $\sigma_0$ are not compatible, since they correspond to different averages.

Three other n-type samples had similar impurity gradients and together with sample N1 were rejected on this basis alone. However, the general shape of the Hall coefficient curves of all of these samples is significant, and will be referred to later.
The remaining n-type sample, N2, with room temperature Hall coefficient of about \(1 \times 10^{-5} \text{ m}^3/\text{C}\), had a gradient of less than 1% between the two sets of Hall probes spaced 5.4 mm. apart. This amount of inhomogeneity was considered acceptable for both magnetoresistance measurements and subsequent analyses based on the two band model discussed in Chapter II. Impurity gradients were further checked in this sample by making contacts at two other positions along the sample and measuring both conductivity and Hall coefficient. The values of \(\sigma_o\) and \(R\) calculated for the three sets of positions agreed to better than 1%.

Similar measurements were performed on a p-type sample, P1, which was also found to be homogeneous.

Sample N2 was subsequently used by Demars (72D1) for measurements of magneto-Seebeck coefficient in fields from 0.0 to 3.2 Wb/m\(^2\) at room temperature. A small Van der Pauw type specimen was then cut from sample N2 and was used by Rosenbaum (72R1) for measurements of resistivity and constant field Hall coefficient as a function of pressure up to 12 Kbars at room temperature. Since Van der Pauw samples can be made small (e.g. 2 x 2 mm. in area), in contrast to the much larger samples required for magnetoresistance measurements (e.g. 1 x 15 mm.), Rosenbaum was also able to find large enough areas of uniform impurity distribution for Van der Pauw samples from material which showed severe gradients in the large parallelepiped shaped specimens. He used a specimen with room temperature and zero pressure Hall coefficient of about \(28 \times 10^{-6} \text{ m}^3/\text{C}\) for measurements up to 12 Kbars to complement the measurements made on the sample cut from N2. The results of analyses of the data obtained by Demars and Rosenbaum will be discussed in a later section. However, it may be indicated here that the good agreement obtained between the present data on both n and p samples, the magneto-Seebeck data of Demars and both sets of pressure data by Rosenbaum justify the assumption that the present data is representative of GaSb.
IV-21 Measurements of Magnetoresistance and Hall coefficient as a function of magnetic field and temperature on sample N2.

The measurements were made on the apparatus described in Chapter III, at liquid Nitrogen temperature (78°C) and from room temperature (usually between 20° and 25°C) to 360°C in intervals of 20°. Typical results of $R$ vs. $B$ are shown in Fig. IV-3. Similar shaped curves were obtained for $\sigma$ vs. $B$, those will not be shown for the sake of brevity. These results are replotted in Figs. IV-4 and IV-5 in the form $-R_o/\Delta R$ and $\rho_o/\Delta \rho$ against $B^{-2}$ for several temperatures. The value of $R_o$ is obtained from a plot of $-R$ vs. $B^2$ at low fields ($B < .9 \text{ Wb/m}^2$) extrapolated to $B = 0$, these plots being shown in Fig. IV-6. To justify this method of obtaining $R_o$, it is necessary to expand equation II-42 in ascending powers of $B^2$ as was done by Harland and Woolley (66H1), who showed that the coefficient of $B^4$ is very small and hence at low fields II-42 approximates to a straight line of the form

$$R = R_o + \alpha B^2$$

IV-1

It can be seen from Fig. IV-6 that, within the limits of experimental error, the points do in fact give a straight line, and hence $R_o$ can be found.

Equations II-46 and II-62 predict that plots of $-R_o/\Delta R$ and $\rho_o/\Delta \rho$ vs. $B^{-2}$ should be linear, if the conditions under which these equations were derived are satisfied. Figs. IV-4 and IV-5 do indeed show that straight lines are obtained experimentally, at least below 140° and even at 200°C the deviations from a straight line are found to be small, although at this temperature we would expect the contribution to magnetoresistance from individual bands to become significant.

The results of least squares fitting these data to a straight line are tabulated in Table IV-1, together with the values of $R_o$ and $\sigma_o$. 
IV-22 Measurements of low field Hall coefficient and conductivity as a function of temperature

The results of measurements of low field Hall coefficient (B < 0.9 Wb/m²) and conductivity on samples N2 and P1 are shown in Fig. IV-8 and IV-9 respectively. It is of interest to note that the inflection in R(T) observed for the purer sample N1, Fig. IV-1, is also present in this heavier doped sample.

IV-3 Method of Analysis

IV-31 Determination of Γ - L minima separation and its temperature dependence from magnetoresistance results

The separation of the two minima Γ₁ and L₁ (i.e. E₂) and its temperature dependence (dE₂/dT) can in principle be obtained with the use of the charge neutrality condition, equation II-69, if the carrier distribution between the bands and the band parameters are known.

The carrier distribution can be obtained from the magnetoresistance and Hall coefficient measurements described in section IV-21. The plots of -R₀ / ΔR and ρ₀ / Δρ vs. B⁻² have been shown to be linear, giving slopes s, t and intercepts a, u respectively; these are tabulated in Table IV-1 together with R₀ and σ₀. Since we have only four unknowns R₁, R₂, σ₁ and σ₂, to determine these we need to use only four of the experimentally determined quantities. The question arises as to which four should be picked. Obviously, the values of R₀ and σ₀ will be accurate since σ₀ is obtained directly from experiment and R does not vary strongly at low B, hence extrapolation to B = 0 should give an accurate value of R₀. Also the slopes s and t will be accurate as long as straight lines are obtained experimentally. On the other
hand, the values of $a$ and $u$ are obtained by extrapolation to $B = \infty$ and hence will be the least accurate values. Therefore, best results will be obtained if we use the combination (i) $R_o$, $\sigma_o$, $s$ and $t$, the other combinations (ii) $R_o$, $\sigma_o$, $s$, $a$ and (iii) $R_o$, $\sigma_o$, $t$, $u$ will give less accurate results. We have carried out these calculations with the three sets of values at three different temperatures, using equations II-43; II-49 to II-52; II-65 to II-68, and tabulated the results in Table IV-2. From this Table, it can be seen that the calculated values of $R_1$, $R_2$, $\sigma_1$ and $\sigma_2$ agree well for all three cases, in particular for (i) and (iii). This could be expected, since the value of $a$ is probably the least accurate one, because the relative change of $R$ with $B$ is smaller than that of $\sigma$, and hence $a$ will be less accurate than $u$. We can also conclude that the two band picture is valid in this temperature range. Because of the good agreement between the results using case (i) and (iii), magnetoresistance results above are, in fact, sufficient to determine the quantities $R_1$, $R_2$, $\sigma_1$ and $\sigma_2$, which are tabulated in Table IV-2a. The accuracy of the data is reflected to the largest extent in the value of $R_2$. From equation II-43, the value of $R_2$ is determined from the difference of two terms $R_o(\sigma_o/\sigma_2)^2$ and $R_1(\sigma_1/\sigma_2)^2$ which are comparable in magnitude and therefore $R_2$ is the least accurate value of the four parameters calculated.

Using the values of $R_1$ and $R_2$ as a function of temperature, carrier distributions $n_1$ and $n_2$ can be calculated when the scattering parameters $r_1$ and the ellipticity $K_1$ are known. No values have been published for either $r_1$ or $r_2$ since carrier scattering in neither band is understood. We shall assume that $r_1 = 1.0$ and does not vary with temperature within the experimental range (it will be shown in section IV-333 that this assumption is valid). We shall also assume that the scattering in the $L_1$ band is dominated by longitudinal acoustic waves and
therefore \( r_2 = 1.18 \) and is temperature invariant. The value of \( K_2 \) was deduced by Piller (64P1) to be 8.6 and this value will be used throughout. The carrier concentrations \( n_1 \) and \( n_2 \) were then calculated from \( R_1 \) and \( R_2 \), the mobilities \( \mu_1 \) and \( \mu_2 \) were obtained from \( \sigma_1 \) and \( \sigma_2 \), and the results are tabulated in Table IV-3, together with the total carrier density \( n_1 + n_2 = n_t \).

The energy level of Te donors was reported by Pitt (69P2) to be about 0.02 eV below the bottom of the \( L_1 \) minima, and we therefore expect that for sample N2 the total carrier density should remain constant above room temperature. This, of course, is true only for temperatures below that of intrinsic conduction. Table IV-3 shows that \( n_t = 1.45 \times 10^{24} \text{ m}^{-3} \) and does indeed remain constant within the accuracy inherent in the calculation of \( R_2 \) and hence of \( n_2 \). The charge neutrality can now be expressed in the form:

\[
N_D - N_A = n_1 + n_2 = n_t = 1.45 \times 10^{24} \text{ m}^{-3}
\]

and thus determines the position of the Fermi level at a given temperature. The condition IV-2 together with equations II-71 and II-75 for \( n_1 \) and \( n_2 \) were then used to determine \( E_2 \) and \( \alpha_2 \) by the following procedure. The carrier densities \( n_1 \) and \( n_2 \) were calculated from equations II-71 and II-75 for a given set of values of \( E_2 \) and \( \alpha_2 \) together with the parameters given in Table IV-4; the Fermi energy being adjusted so that condition IV-2 was satisfied. The carrier concentrations thus calculated were then compared to the values of \( n_1 \) and \( n_2 \) obtained from analyses of magneto-resistance data (Table IV-3) over the temperature range \( 20 < T < 120^\circ \text{C} \). \( E_2 \) and \( \alpha_2 \) were adjusted until satisfactory agreement was obtained between the two sets of results for \( n_1 \) and \( n_2 \). Since the relative change with temperature of \( n_1 \) is much greater than that of \( n_2 \), the fit was judged acceptable when \( n_1(T) \) was fitted.

The above procedure was then repeated with the \( X_1 \) band being included in
the charge neutrality condition, i.e.

$$N_D - N_A = n_1 + n_2 + n_3 = n_t$$

IV-3

to determine at which temperature an appreciable number of electrons will be excited to the third set of minima. The additional parameters for the $X_1$ band which were required for the calculations are given in Table IV-4. The density of states effective mass of 1.2 m that was used, was assumed to be the same as that for the $<100>$ band in GaAs (60E1). The band separation, $E_{30}$, of 0.315 eV was determined at room temperature by Kosicki et al. (68K1) and here its temperature dependence was assumed to be zero. It will be shown later that this assumption is quite reasonable and not very restricting, since only at the highest experimental temperatures are there any appreciable number of electrons in the $X_1$ band. The results show that at $100^\circ$C, $n_3$ is only 1% of $n_1$; at $140^\circ$C it is 3% and at $200^\circ$C it is 9%. Since the analysis of the magnetoresistance was based on the two band model, the fitting of $n_1$ is valid up to $\sim 120^\circ$C only since, as will be shown in section IV-3, the contribution to magnetoresistance by individual bands could become appreciable above $\sim 120^\circ$C. Both these causes of uncertainty combine to make the results (Table IV-3) above $120^\circ$C unreliable for use in the determination of $E_2$ and $\alpha_2$. Hence, only the values obtained in the temperature range $25^\circ$ to $120^\circ$C were used. The results are shown in Table IV-5 and Fig. IV-7, where we have plotted the fitted carrier density in the $\Gamma_1$ band (solid line) and the values of $n_1$ from Table IV-3 as a function of temperature. The solid line represents the best fit obtained with $E_{20} = 0.099 \pm .002$ eV and $\alpha_2 = -3.4 \pm .01 \times 10^{-5}$ eV/°K. Because a linear form of temperature dependence of $E_2$ has been used (equation II-79), it is to be expected that the value obtained for $E_{20}$ is slightly higher than the actual band gap at 0°K. Nevertheless, the present value agrees well with that of 0.095 ± .005 eV found by Yep and Becker (67Y1).
at 4.2⁰K from de Haas-van Alphen measurements, although it is higher than earlier reported values which range from 0.04 - 0.09 eV. The value of α₂ appears quite reasonable, although it is nowhere near the values reported by some authors, which are of the order of 10⁻⁴ eV, e.g. -2 x 10⁻⁴ eV/⁰K reported by Robert and Barjon (70R1). However, it is close to a value of -5 x 10⁻⁵ eV/⁰K which was estimated from existing experimental data by assuming linear temperature dependences for the two direct energy gaps involved; viz. the A₃ᵥ → A₁c peak (given by the E₁ reflectance peak) and the principal Σ₅ᵥ → Σ₁c gap (E₀). Bahan (70B1) reported a value of 5.0 ± 0.05 x 10⁻⁴ eV/⁰K for the temperature coefficient of the E₁ peak and a value of -4.5 x 10⁻⁴ eV/⁰K was obtained by Woolley and Evans (61W1) for the coefficient of the principal gap, by fitting the optical absorption data. Further discussion and comparison with published values of E₂₀ and α₂ is postponed to section IV-42.

Measurements were also made of magnetoconductivity, low field Hall coefficient (B < .9 Wb/m²) and conductivity at liquid Nitrogen temperature (78⁰K), the results being given in Table IV-6. It is clear from these results that even if we use a maximum possible value of r₂ = 1.93, i.e. if the carriers in the L₁ band are assumed to be non-degenerate and the scattering wholly due to ionized impurities, the maximum possible total carrier concentration then becomes nₜ = 9.923 x 10²³ m⁻³, which is considerably less than nₜ = 1.45 x 10²⁴ m⁻³ obtained at room temperature and above. We conclude therefore that the model proposed by Bate (62B1) and used by Pitt (70P2) is correct, that group VI impurities are associated with subsidiary minima and their energies lie above the bottom of the principal minimum. Hence as the temperature drops and the Fermi level approaches the impurity level, "freeze out" of mobile carriers to the impurity level occurs, and therefore the total carrier
concentration drops. For this reason, as was mentioned in the Introduction, the measurements carried out for this thesis were made at $T > 300^\circ K$ on samples which were not too heavily doped. In this way, the exact energy position of the impurity level and their density is not required for the analysis since the impurity states will remain ionized at all the temperatures of the experiment.

**IV-32** Determination of carrier mobility in the $\Gamma$ and $L$ bands from Hall coefficient and conductivity

**IV-321 Sample N2**

The case of electrons being distributed between two conduction bands, one with high and one with low carrier mobility, is analogous to the case encountered in many semiconductors at low temperature when carriers are distributed between a conduction band and an impurity band, (e.g. Basinski and R. Olivier 67B2). The electron mobility in the impurity band is always considerably lower than in the conduction band. In this case, as the temperature is lowered, the electrons freeze-out into the impurity band, the Hall coefficient starts rising and the conductivity drops. As the temperature is lowered further, provided the mobility ratio remains constant, $R$ will pass through a maximum and will then level off at some still lower temperature. To a first approximation, the position of the maximum is at a temperature where the two conductivities in the conduction and impurity band are equal (67B2).

The analogy with the case of $n$-GaSb, having a high mobility $\Gamma_1$ band and low mobility $L_1$ minima, is clear, except that in this case the role of temperature is reversed, since we require an increase in temperature for transfer of electrons to the low mobility minima. We would therefore expect for sample N2 that as the temperature is raised, the Hall coefficient would pass through a maximum and then
start to fall. This is not observed, however, (see Fig. IV-8), for the curve
starts to level off but then rises again. (N.B. the inflection in R(T) around
150°C is similar to that seen in the Hall coefficient vs. temperature data of
sample N1, Fig. IV-1). There are two possible reasons for this, either (a) electrons
are lost to the \( \chi_1 \) minima where they will have very low mobility, \( \sim 0.0045 \text{ m/V. sec.} \)
at room temperature according to Pitt (69P2), or (b) the mobility ratio \( \mu_1/\mu_2 \)
increases with increasing temperature due to some change in the effective scattering
mechanism.

Initially, we tried to fit the results of R(T) vs. T given in Fig. IV-8
assuming case (a) only. To do this, we used the parameters for the \( \Gamma_1 \) and \( L_1 \) minima
determined from the magnetoresistance data and assumed that the temperature dependences
of mobilities \( \mu_1 \) and \( \mu_2 \) obtained from magnetoresistance results below 120°C remain
the same up to 360°C, and take the following form:

\[
\mu = \mu_300 \left( \frac{300}{T} \right)^q
\]  

Equation IV-4

Values for \( q_1 \) and \( q_2 \) were determined by least squares fit of the data for \( \mu_1(T) \)
and \( \mu_2(T) \) respectively, (Table IV-3) and the results are shown in Fig. IV-10.

An attempt was then made to fit the Hall coefficient temperature variation,
R(T), using equation II-56 by varying \( \alpha_3 \) and \( m_3^* \). The contribution of intrinsic
carriers was included in the charge neutrality condition in order to take care of any
increase in the total electron concentration. The mobility \( \mu_3 \) was calculated from:

\[
\mu_3 = (\sigma_0 - \sigma_1 - \sigma_2)/en_3
\]  

Equation IV-5

and \( E_{30} \) was given by \( E_{30} = 0.315 - 300 \alpha_3 \), the value of \( E_3 = 0.315 \) at room tem-
perature having been reported by Kosicki et al. (68K1) and by Pitt (70P1). The
heavy hole valence band was taken into consideration but not the light holes, the
effective mass of the heavy holes being taken as 0.33 m (68W1) and a hole mobility of 0.14 m²/V. sec. (64M1) at 300^oK was assumed with a temperature dependence of T^{-1.5}. Actually, as can be seen from Table IV-7, even at 360^oC the density of holes is only 4 x 10^{21} m^{-3}, and hence the fit is very insensitive to the value of the hole mobility or its temperature dependence. In any case, no satisfactory fit could be obtained by varying α₃ and m_3* within reasonable limits. We concluded therefore that although the third set of minima may indeed contribute to the increase in R above 150^oC, this contribution is small. In view of the above result, an attempt was then made to fit the R(T) vs. T data assuming case (b) only. It was now assumed that all the parameters of the bands were known except µ₁(T) and µ₂(T), and these were to be obtained by simultaneously fitting R(T) and σ_o(T) data to equations II-56 and II-57. The parameters for the two lowest conduction band minima, Γ₁ and L₁, and the heavy hole valence band were taken as before. In addition the following parameters were assumed for the <100> minima:

- m_{3d}^* = 0.36 m
- α₃ = 0.0
- E₃ = 0.31 eV
- µ₃ = .01 \left(\frac{300}{T}\right)^{1.5}

The choice of the parameters for the <100> minima was dictated by the known values of these parameters in Si, GaP and GaAs for minima with the same symmetry. It seems rather unlikely that the <100> band in GaSb would be very different from the band of the same symmetry in these three semiconductors. The values of the parameters of the <100> band in Si, GaP and GaAs are given in Table IV-8. From this Table, we can see that for both III-V compounds the effective mass and mobility in
the <100> minima are roughly the same. In the estimate of the value of \( \alpha_3 \) to be used, two results are relevant. It was observed that the Varshni (67V1) expression II-74 holds for direct and indirect gap materials with valence band as reference, and at high temperatures, this expression becomes linear with T. Therefore to estimate \( \alpha_3 \), we calculated the difference between the temperature coefficients of the <100> band in GaAs and GaP, and of the \( \Gamma_1 \) band in GaSb, from values at two temperatures, 500 and 700°K. The results are included in Table IV-8 and are denoted by \( \alpha_3 \). These values indicate that \( \alpha_3 \) should be positive and of the order of 
\[ +5 \times 10^{-5} \text{ eV/°K} \] 
It will be shown in Chapter V that in the case of GaAs, \( \alpha_3 \approx 1 \times 10^{-5} \text{ eV/°K} \).

Since the values of the parameters, \( m_3^* \), \( \alpha_3 \), and \( \mu_3 \), are uncertain, we have varied them to determine what effect, if any, a wrong choice would have on the values of \( \mu_1(T) \) and \( \mu_2(T) \) obtained from the fit. The following ranges of variations were employed:

\[ 0.34 \text{ m} < m_{3d}^* < 0.38 \text{ m} \]
\[ -5 \times 10^{-5} \text{ eV/°K} < \alpha_3 < +5 \times 10^{-5} \text{ eV/°K} \]
\[ 0.005 \text{ m}^2/\text{V. sec.} < \mu_3(300) < 0.020 \text{ m}^2/\text{V. sec.} \]

The values of \( \mu_1(T) \) and \( \mu_2(T) \) determined from the fitting, using the above range of parameters, agreed within 1% and 8% respectively. The final results are shown in Fig. IV-8 as a solid line and are presented in numerical form in Table IV-7.

IV-322 Sample P1

The results of measurements of Hall coefficient and conductivity on the p-type samples are shown in Fig. IV-9. The curves are typical of a p-type sample in which the Hall coefficient changes sign from positive to negative due to an
increase in the concentration of the highly mobile electrons as they are excited across the band gap with increasing temperature. These data were analysed using the conduction band parameters obtained from the n-type N2 sample, together with known values of the parameters of the valence bands. The method of analysing is described below.

The data were divided into two parts, a low temperature section, below 250°C where purely hole conduction is present, and a high temperature section above 250°C where mixed conduction occurs.

Considering first the low temperature portion of the Hall coefficient curve, the initial drop in the value of R is due to ionization of acceptor impurities with activation energy $E_a$ and density $N_a$. The curve tends to level off at ~200°C, when the impurity level is practically completely ionized. We shall treat both $E_a$ and $N_a$ as adjustable parameters which can be obtained from the low temperature data. Before this can be done, the values of mobilities in the light and heavy hole valence bands, as well as the Hall scattering parameters, must be known.

It has been pointed out by Blatt (68B1, p. 169), that in Ge and Si, the mobilities in the two valence bands would be expected to be approximately in the ratio of the two effective masses, since the relaxation times of the two bands of holes are approximately equal. This comes about because the scattering probability depends essentially on the density of final states, which in either case is the density of states of the heavy hole band. This follows if it is assumed that the scattering of the light holes is dominated by interband scattering into the heavy hole band and that of the heavy holes by intraband scattering, both of which are directly dependent on the density of states in the heavy hole band. It has been shown by Dresselhaus et al. (55D1) that for Ge the two relaxation times are
\[ \tau_p = 7 \times 10^{-11} \text{ sec and } \tau_{ph} > 5 \times 10^{-11} \text{ sec. For convenience, the same assumption will be made about the mobility ratio for the case of the valence bands in GaSb, i.e. the ratio of a light to heavy hole mobilities is then given by:} \]

\[ \frac{\mu_l}{\mu_h} = \frac{m_p^*}{m_h^*} = 0.33 / 0.045 = 7.34 \]

the values of effective masses of the holes have been determined by Walton and Mishra (68WL) from cyclotron measurements. It will also be assumed that both the Hall scattering parameters are equal to unity and remain constant with temperature.

With the above assumptions the values of \( N_a \) and \( E_a \) as well as \( \mu_h \) (and hence \( \mu_l \)) can be deduced using the following relations,

\[ P_h + P_l = \frac{N_a}{1 + g_a \exp \left[ (E_F - E_0 + E_a)/kT \right]} \quad \text{(IV-6)} \]

where \( g_a \) is the spin degeneracy factor, taken to be 4 for the acceptors, as was also assumed by Compas (69C3),

\[ \sigma = e\mu_h (P_h + bP_l) \quad \text{(IV-7)} \]

\[ R = (P_h + b^2 P_l)/e(P_h + bP_l)^2 \quad \text{(IV-8)} \]

where \( b \) is the mobility ratio of light to heavy holes.

For a given value of \( N_a \) and \( E_a \), the hole densities \( p_h \) and \( p_l \) can be calculated from the charge balance equation IV-6 together with \( p_h/p_l = (m_h^*/m_l^*)^{3/2} \).

By substituting these densities into equation IV-8 and making use of the experimental value of \( \sigma \), the Hall coefficient is then calculated and compared with the experimental value. \( N_a \) and \( E_a \) are adjusted so as to give a best fit to the Hall curve below 200°C. The values thus determined were \( N_a = 4.44 \times 10^{23} \text{ m}^{-3} \) and \( E_a = 0.042 \text{ eV} \). The hole mobilities were then determined from the experimental value
of the conductivity using equation IV-7. These display an exponential dependence on temperature of the form

$$\mu_h = \mu_{ho} e^{-T/\phi}$$  \hspace{1cm} \text{IV-9}$$

The values of $$\mu_{ho}$$ and $$\phi$$ were then obtained by fitting the results to equation IV-9, resulting in $$\mu_{ho} = 0.0764 \text{ m}^2/\text{V. sec.}$$ and $$\phi = 296^\circ\text{K}.$$ With the values of $$N_a$$, $$E_a$$ and the magnitude and temperature dependence of the mobility of light and heavy holes determined from the low temperature data, the complete curves of $$R$$ and $$\sigma$$ were then simultaneously fitted, using $$\mu_1$$ and $$\mu_2$$ as adjustable parameters. The conduction band parameters used were those obtained from the n-type sample N2. The fitted curves for $$R$$ and $$\sigma$$ are shown in Fig. IV-9, and the values of $$\mu_1$$ and $$\mu_2$$ required for the fit are tabulated in Table IV-10.

Below $$440^\circ\text{C}$$ the fit is insensitive to the values of $$\mu_1$$ and $$\mu_2$$. Thus, considerable scatter in the values of mobilities was obtained in this range and hence these values are not given in Table IV-10, nor are they plotted in Fig. IV-16.

In the transition region of $$R$$, from positive to negative sign, (250 < $$T < 440^\circ\text{C}$$), the fitting is most sensitive to the relative density of holes and electrons and therefore to the temperature dependence of the principal gap, $$\alpha_0$$. The value of $$\alpha_0$$ had to be increased from $$-5.0 \times 10^{-4} \text{ eV/K}$$ to $$-5.4 \times 10^{-4} \text{ eV/K}$$ in order to obtain a good fit to the experimental results of $$R$$ and $$\sigma$$. The value of $$\theta (=112^\circ\text{K})$$ however was left unchanged. The resulting good agreement with the values of Hall coefficient and conductivity measured as a function of temperature, on the P1 sample, give confidence in the values of conduction band parameters determined from experimental data on the n-type sample N2.
Analysis of electron mobility in the $\Gamma$ and $L$ bands

Analysis of the mobility $\mu_2(T)$ in the $L_1$ band for $T < 360^\circ C$

Scattering of electrons in the $L_1$ band of GaSb is not well understood. Although it is possible to make an educated guess at what scattering mechanisms may be important in the temperature range below $360^\circ C$, this type of estimate becomes practically worthless at still higher temperatures because of the interaction of the $L_1$ band with higher lying minimum in the $<100>$ direction. Therefore an attempt will be made here to analyse by an approximate method only the $\mu_2(T)$ data obtained from the n-type sample N2 below $360^\circ C$. It was assumed that the scattering in the $<111>$ minima is essentially due to ionized impurities and acoustic phonons only, since these mechanisms are important for heavy carriers. In that case, one can write the total mobility in the approximate form:

$$\frac{1}{\mu_2(T)} = \frac{1}{\mu_a(T)} + \frac{1}{\mu_I(T)}$$  \hspace{1cm} \text{IV-10}$$

where $\mu_a(T)$ and $\mu_I(T)$ are the mobilities due to scattering by acoustic phonons and ionized impurities respectively. Since the Fermi energy at the lowest temperature is more than 2 kT below the bottom of the $L_1$ minima (c.f. Table IV-7), classical statistics can be used in the integration of equation II-40 to give $\mu_a$ and $\mu_I$ (equation II-17, II-25) and hence obtain $\mu_a(T)$ and $\mu_I(T)$. The values of $\mu_a$ and $\mu_I$ so obtained will have the following dependence on temperature:

$$\mu_a = aT^{-3/2}$$  \hspace{1cm} \text{IV-11}$$

where

$$a = \frac{e (8\pi)^{1/2} \hbar \frac{4}{3} c_{L\perp}^2}{3 \bar{E}_1^{3/2} m_{2d}^{3/2} m_{2e}^{3/2} k^{3/2}}$$  \hspace{1cm} \text{IV-12}$$

$$c_{L\perp} = \rho U^2$$
and \( \mu_I = cT^{3/2} \)  

where \( c = \frac{64\pi^{1/2}}{\kappa O^2} \frac{(2k)^{3/2}}{m_{2d}^*} \frac{1/2}{e^3 m_{2e}^* N_1 f(z)} \) \( \)  

In carrying out the integration for \( \mu_I \), \( f(z) \) was taken outside the integral, since it is a slowly varying function of energy (59S1). \( \mu_2(T) \) can now be written in the form:

\[
\frac{1}{\mu_2(T)} = \frac{1}{aT^{-3/2}} + \frac{1}{cT^{3/2}}
\]  

This expression was fitted to the experimental values of \( \mu_2(T) \), Fig. IV-11, a non-linear least squares fitting program (IBM library programme BMDX-85) being used, and the temperature invariant parameters \( a = 4.4 \times 10^2 \text{ m}^2/\text{V. sec. K}^{3/2} \) and 
\( c = 2.3 \times 10^{-5} \text{ m}^2/\text{V. sec. K}^{3/2} \) were obtained. In doing this, it was assumed that \( f(z) \) is independent of temperature, where in fact it is a slowly varying function of temperature. This will cause some error in the value of \( c \). From these values of \( a \) and \( c \) the deformation potential \( E_1 \) and the ionized impurity density \( N_1 \) were estimated. To calculate \( E_1 \) from \( a \), a value for \( c_{\ell\ell} \) was required and this was calculated using the Herring and Vogt (56H1) expression:

\[
c_{\ell\ell} = c_{12} + 2c_{44} + \frac{3}{5} (c_{11} - c_{12} - 2c_{44})
\]  

where \( c_{ii} \) are elastic constants, which are given for GaSb by Madelung (64M1). 
Thus, a value of approximately 9 eV was deduced for \( E_1 \), and this compares well with the value of about 10 eV reported by Sherwood (70S4).

In estimating the total ionized impurity concentration \( N_1 \) from \( c \), we used \( f(z) = .11 \) obtained from \( E = 3kT \) at \( T = 300^0\text{K} \), together with \( n_t = 1.45 \times 10^{24} \text{ m}^{-3} \), resulting in a value of \( 2 \times 10^{24} \text{ m}^{-3} \) for \( N_1 \).
Another way of estimating $N_i^*$ was suggested by Baxter et al. (67B1). They found that in order to explain their measurements of mobility on n-GaSb at $78^\circ K$, they required a greater density of acceptors than is usually found in undoped p-GaSb, i.e. $1 - 2 \times 10^{23} \text{ m}^{-3}$ (67B1). To explain their results, they postulated that the acceptor centres are doubly charged and therefore the effective total ionized impurity concentration $N_i^*$ is given by:

$$N_i^* = n_t + 6N_a^*$$

In that case if the lower limit of $1.0 \times 10^{23} \text{ m}^{-3}$ is taken for the density of acceptors $N_a^*$, a value of $N_i^* = 2.1 \times 10^{24} \text{ m}^{-3}$ is obtained. On the other hand, taking the upper limit of $2 \times 10^{23} \text{ m}^{-3}$ for $N_a$ gives $N_i^* = 2.6 \times 10^{24} \text{ m}^{-3}$. The value above from a least squares fit of $n_2(T)$ is closer to the lower limit, therefore somewhat arbitrarily we chose to use $N_i = 2.2 \times 10^{24} \text{ m}^{-3}$ in the analysis, given below, of the mobility $\mu_1(T)$ of electrons in the principal conduction band.

IV-332 Analysis of the mobility $\mu_1(T)$ in the $\Gamma_1$ band

(a) $\mu_1$ mobility deduced from measurements on sample N2 ($T < 360^\circ C$)

As was pointed out earlier, $\mu_1(T)$ is insensitive to the parameters chosen for the <100> band and therefore was determined to much better accuracy than $\mu_2(T)$. Hence the approximate analysis which we employed for $\mu_2(T)$ is no longer sufficient, and a more accurate approach should be used in analysing the temperature dependence of $\mu_1$.

Conwell and Vassell (68C1) used three scattering mechanisms to analyse their mobility results for the $\Gamma_1$ band in GaAs, and all three are appropriate for the $\Gamma_1$ band in GaSb. In addition, since we are dealing with a moderately heavily doped sample, ionized impurity scattering must also be included. This is supported by the findings of Baxter et al. (67B1) who found that at room temperature the scattering
of electrons in the $\Gamma_1$ band of GaSb by ionized impurities was important.

Initially we tried to simulate the behaviour of $\mu_1(T)$ with the following three scattering modes:

(i) polar optical,

(ii) interband, $\Gamma_1$ to $L_1$,

(iii) ionized impurity.

In this case the net relaxation time $\tau$ is given by:

$$\frac{1}{\tau} = \frac{1}{\tau_{\text{po}}} + \frac{1}{\tau_{\text{ib}}} + \frac{1}{\tau_{\text{I}}}$$  \hspace{1cm} \text{IV-18}

where $\tau_{\text{po}}$, $\tau_{\text{ib}}$, $\tau_{\text{I}}$ are given by equations II-20, II-24 and II-29 respectively. For the purpose of calculations, $\tau$'s will be rewritten in a form more adaptable for computer solution.

Using the definition of $\gamma$ (equation II-23) and equation II-10 for $\gamma_k$, it can be shown that the relaxation time due to polar scattering is given by:

$$\frac{1}{\tau_{\text{po}}} = 5.716 \times 10^{-27} \rho (m^*_o)^{1/2} \left(2N_{q\lambda} + 1\right)/\varepsilon^{1/2}$$  \hspace{1cm} \text{IV-19}

where

$$\rho = \frac{m}{m^*_o} - \left(\frac{m^*}{m^*_o} - 1\right) \left\{ 1 + \frac{4e\varepsilon^*_o}{\varepsilon^*_m} \left(1 - \frac{m^*_o}{m}\right) \right\}^{-1/2}$$  \hspace{1cm} \text{IV-20}

$$\varepsilon = E/h\omega_{\lambda} \text{ and } \varepsilon^*_o = E^*_o/h\omega_{\lambda}, \text{ i.e. all the energies are normalized to } h\omega_{\lambda}. \text{ In order to simplify writing these expressions, we have written } m^*_o \text{ for } m^*_{10}(T), \text{ i.e. the bottom of the band } (\Gamma_1) \text{ effective mass as a function of temperature which is given by equation II-72 in terms of the mass at absolute zero.}\n
\text{‡ For the purpose of calculations we have chosen throughout this section to normalise all the energies to } h\omega_{\lambda}.\n
Similarly the expression for the interband relaxation time $\tau_{ib}$ (cf. equation II-24) can be written as:

$$\frac{1}{\tau_{ib}} = \frac{6.775 \times 10^{40} (M_d^*)^{3/2} D_{12}^{2}}{e^{x_{12}} - 1} [(c + 0.8 - \varepsilon_2)^{1/2} + e^{x_{12} (c-0.8 - \varepsilon_2)^{1/2}}] \quad \text{IV-21}$$

where $M_d^* \, m_2^*$ is in M.K.S. units and $D_{12}$ in eV/cm. It was also assumed that $\omega_{12} = 0.8 \omega_c$ (72V1).

Kolodziejczak's equation II-29 for the ionized impurity scattering relaxation time $\tau_I$ was derived on the basis of a single non-parabolic band. Therefore the Fermi energy $E_F$, appearing in the integral $L(\beta, \eta)$ within the expression for the screening radius, describes the distribution of electrons in a single band. Hence it does not correspond to the Fermi energy obtained here in section IV-321, which is affected by the carriers in the $L_1$ band. In addition, we have two sets of electrons, light ($n_1$) and heavy ($n_2$), and the $n_2$ electrons will shield the ionized impurities more effectively than the $n_1$ electrons (67B1). Therefore, the screening correction given by Kolodziejczak cannot be applied in this case. It was also shown by Moore and Ehrenreich (66M1, 67M1) that at high doping levels, quantum corrections are important, and both relations II-25 and II-29 are inexact, even when screening is excluded. It was therefore initially thought unnecessary to make corrections to the screening radius due to two band effects without making the more important but very tedious quantum corrections. With these reservations in mind, we used the Brooks-Herring relation for the ionized impurity scattering relaxation time $\tau_I$ with only one change, i.e. the inclusion of the non-parabolicity effect through the effective mass, i.e.

$$\tau_I = 277.5 \left(\frac{m^*}{m_c}\right)^{1/2} \varepsilon^{3/2} / f(z) \quad \text{IV-22}$$

where

$$z = 1.61 \times 10^{-29} \, m^*_c$$

and $N_1 = 2.2 \times 10^{24} \, m^{-3}$ was used.
To calculate mobility $\mu = e \left< \frac{\tau}{m^*} \right>$, equation II-39 was used with $\tau$ given by IV-18 and $m^*$ by II-4. The values of various parameters appearing in the equations for the relaxation times $\tau_{po}$, $\tau_{ib}$ and $\tau_I$, which depend on band structure of GaSb, are given in Table IV-9. The numerical integrations required in evaluating the infinite integrals in II-39 were done on an IBM 360 model 65 computer using Simpson's method. It was found that the integrals converged rapidly above $\varepsilon \sim 10$. At $\varepsilon = 35$ the magnitude of the functions in the integrals had values about $10^{-10}$ smaller than the maximum value, hence integration was cut off at this value of $\varepsilon$.

The results of the calculations are shown in Fig. IV-12, the upper curve for $D_{12} = 1 \times 10^8$ eV/cm. It can be seen from Fig. IV-12, that with these three relaxation times, the calculated mobility is roughly twice as large as that obtained from the fitting of $R(T)$ and $\sigma_0(T)$. Since we do not know the proper magnitude of $D_{12}$, we can use this as an adjustable parameter and the result for $D_{12} = 4.6 \times 10^8$ eV/cm, is shown as a dashed line, from which it can be seen that by adjusting $D_{12}$ it is possible to fit the high temperature portion of the $\mu_1(T)$, but not the low temperature part. This suggests that (a) additional scattering mechanisms are present which are more important at lower temperatures than the interband scattering, i.e. spacecharge scattering, or (b) that the density of scattering centres is higher than that estimated, and that the mobility is governed mainly by $\tau_I$.

Taking the first possibility, and including spacecharge scattering in the analysis, the total relaxation time becomes

$$\frac{1}{\tau} = \frac{1}{\tau_{po}} + \frac{1}{\tau_{ib}} + \frac{1}{\tau_I} + \frac{1}{\tau_{sc}} \quad \text{IV-23}$$

where

$$\frac{1}{\tau_{sc}} = 9.666 \times 10^{-11} \left( \frac{N_5 Q_5}{p} \right) \frac{e^{1/2}}{m^*_0^{1/2}} \quad \text{IV-24}$$

The magnitude of $(N_5 Q_5)$ is unknown, hence we end up with two adjustable parameters
$D_{12}$ and $(N_{Q_{s}})$. Fortunately, $D_{12}$ and $(N_{Q_{s}})$ do not have the same effect on $\tau$ over the whole temperature range of measurement. While $(N_{Q_{s}})$ affects the low temperature portion of the $\mu_1(T)$ curve, $D_{12}$ is more important at higher temperatures. Therefore both these parameters can be estimated by fitting $\mu_1(T)$. Best results were obtained for $D_{12} = 3.2 \times 10^8$ eV/cm and $(N_{Q_{s}}) = 4.8 \times 10^4$ cm$^{-1}$ and are shown as solid lines through the points in Fig. IV-13. These values can be compared with those obtained by Conwell and Vassell (68C1) for GaAs. These authors found the value of the coupling constant $D_{12}$ to be $\sim 10^8$ eV/cm between the $\Gamma_1$ and $<100>$ minima, and $(N_{Q_{s}}) = 3.2 \times 10^4$ cm$^{-1}$. To the best of our knowledge, this is the first estimate of the magnitude of the coupling constant between the $\Gamma_1$ and $L_1$ valleys of GaSb.

In order to see the relative importance of the scattering mechanisms, we have plotted in Fig. IV-14 the calculated relaxation times at room temperature as a function of reduced energy $\varepsilon (= \frac{E}{\hbar \omega_L})$ with the above mentioned values for $D_{12}$ and $(N_{Q_{s}})$. It is seen from this graph that polar scattering is the least important mechanism in determining the mobility in the $\Gamma_1$ band in this sample. At low energies ionized impurity and spacecharge effects are important, whereas, at higher energies, as expected, the interband scattering takes over.

If we now take the other possibility, and consider $N_I$ as an adjustable parameter together with $D_{12}$, the relaxation time is:

$$\frac{1}{\tau} = \frac{1}{\tau_{po}} + \frac{1}{\tau_{ib}} + \frac{1}{\tau_I}$$  \hspace{1cm} \text{IV-25}

where

$$\tau_I = 6.11 \times 10^{26} \ (m_c^*)^{1/2} \ \varepsilon^{3/2} / N_1 f(z)$$  \hspace{1cm} \text{IV-26}

The resulting fit to $\mu_1(T)$ is shown in Fig. IV-15 for $N_I = 4.2 \times 10^{24}$ m$^{-1}$ and $D_{12} = 3.0 \times 10^8$ eV/cm. It is noticeably worse than the previous fit when spacecharge
scattering was included. However, because of the approximate nature of the ionized impurity relaxation time $\tau_I$ which is employed, and the importance of $\tau_I$ in determining the mobility when $\tau_{sc}$ is neglected, it is not possible to make a positive choice between the two models. With $N_I = 4.2 \times 10^{24} \text{ m}^{-3}$, the mobility at room temperature due to ionized impurity scattering alone is calculated to be $0.54 \text{ m}^2/\text{V sec}$. The interband scattering is then responsible for lowering the mobility to the observed value of $0.38 \text{ m}^2/\text{V sec}$. This value of $N_I$ seems to be somewhat high when compared with the previously estimated values $\sim 2 \times 10^{24} \text{ m}^{-3}$ from analysis of $\mu_2(T)$. If, however, the slow $n_2$ electrons are considered to contribute to the ionized impurity scattering, as has been suggested by Baxter et al. (67B1), this high value of $N_I$ no longer seems so unreasonable.

We have therefore repeated these calculations following the method of Baxter et al. which considers the heavy electrons in the same way as ionized impurities. In that case the effective ionized impurity concentration $N_I^*$ now becomes:

$$N_I^*(T) = n_t + (6N_a^- + n_2(T))$$  \hspace{1cm} \text{IV-27}$$

In addition, the effect of the difference in screening between the light and heavy electrons is allowed for through $z$ (in equation II-32). In this case the two adjustable parameters are $N_a^-$ and $D_{12}$. Good agreement between the calculated and experimental results was obtained for $N_a^-$ of $1.1 \times 10^{23} \text{ m}^{-3}$ and $D_{12}$ of $4.1 \times 10^8 \text{ eV/cm}$. The value of $1.1 \times 10^{23} \text{ m}^{-3}$ for $N_a^-$ is in excellent agreement with the value of $0.95 \times 10^{23} \text{ m}^{-3}$ obtained by Baxter et al. from results of measurements on n-GaSb at $78^\circ \text{K}$.  

(b) $\Gamma_1$ mobility deduced from measurements on sample P1 ($T > 360^\circ$C)

The temperature variation of electron mobility in the $\Gamma_1$ band of GaSb deduced from measurements on the p-type sample are shown in Fig. IV-16, together with calculated results based on the relaxation time approximation. The scattering mechanisms considered in the calculation were:

(i) polar optical,
(ii) interband, $\Gamma_1$ to $L_1$ and $\Gamma_1$ to $X_1$,
(iii) screened Coulomb.

The relaxation time expression for the polar optical and interband ($\Gamma_1$ to $L_1$) scattering have already been given in the previous section, equations IV-19 and IV-21. For scattering between $\Gamma_1$ and $X_1$ bands a density of states mass of 1.2 m was assumed for the $X_1$ band together with a temperature independent minima separation of 0.315eV. In the screened Coulomb scattering relaxation time, it was assumed that both the heavy electrons in the $L_1$ and $X_1$ conduction bands and the heavy holes contribute to the same extent as the ionized impurities. Hence, the effective density of scattering centres is given by:

$$N^*_1 = N_a + n_2(T) + n_3(T) + p_h(T)$$  \text{II-28}

This method of accounting for scattering of the light electrons by heavy holes was proposed by Ehrenreich (57El) and used in calculations of electron mobility in the $\Gamma_1$ band of InSb.

With the above assumptions about the total density of effective Coulomb scattering centres, the relaxation time $\tau_1$ was calculated using equation II-29 and II-32, together with the Fermi energy determined from fitting $R(T)$ and $\sigma(T)$ data on the p-type sample. The Fermi energy is required in the calculation of the screening length, from equation IV-33.
It can be seen from Fig. IV-16 that the calculated mobility curve lies about 10% above the values of mobility deduced from the experiment. This is not unexpected since we have neglected in these calculations any effect of electron-electron interaction which would tend to reduce the calculated mobility, as was pointed out by Ehrenreich (59E1) with regard to the electron mobility in the \( \Gamma_1 \) band of InSb at high temperatures. With these considerations in mind, the agreement between the calculated and experimentally deduced temperature dependence of the mobility of electrons in the \( \Gamma_1 \) band of GaSb can be considered satisfactory.

**IV-333 Calculation of the scattering parameter \( r \) in the \( \Gamma \) band**

Having obtained the total relaxation time \( \tau(e, T) \), taking into account the two scattering models discussed in the last section, it was then possible to calculate the scattering parameter \( r \) as a function of temperature \( T \) and magnetic field \( B \). It is easily shown that \( r_1 \) is given by:

\[
r_1(B) = \frac{\langle \frac{\tau^2}{m_c^* (1 + \omega^2 \tau^2)} \rangle}{\langle \frac{\tau}{m_c^* (1 + \omega^2 \tau^2)} \rangle^2 + \langle \frac{\omega}{m_c^* (1 + \omega^2 \tau^2)} \rangle^2} \tag{IV-29}
\]

where

\[
\omega = \frac{eB}{m_c^*}
\]

at low fields ( \( \omega^2 \langle \tau^2 \rangle \ll 1 \) ) this reduces to

\[
r_1 = \frac{\langle \frac{\tau^2}{m_c^*} \rangle}{\langle \frac{\tau}{m_c} \rangle^2} \tag{IV-30}
\]
With the aid of equation II-39 which defines the average $<\rho>$, $r_1$ was calculated for $B = 0.2, 0.87, 2.0$ and $3.0$ Wb/m$^2$ as a function of temperature $(25^\circ < T < 360^\circ)$. The results are shown in Figs. IV-17 and IV-18. From these, it can be seen that $r_1$ is nearly unity over the full range of temperature, the total variation being less than 6% between $25^\circ$C and $360^\circ$C, and <1% over the range $(25^\circ$C - $120^\circ$C) where magnetoresistance measurements were analysed. It can also be seen that over the range of magnetic fields used in magnetoresistance measurements, i.e. 1.4 to 3.2 Wb/m$^2$, the scattering parameter $r_1$ is only weakly dependent on $B$. Hence, the assumption that $r_1 = 1.00$, which was used in the analysis of magnetoresistance results, is justified.

IV-334 Calculation of the magnetoresistance coefficient

The analysis of the two band magnetoresistance and Hall coefficient was carried out on the assumption that the magnetoresistance due to individual bands was negligible in comparison with the two band effect. In order to check whether this assumption is valid for the $\Gamma_1$ band, in which the magnetoresistance arises because of the energy dependence of the relaxation time, we have calculated the coefficient, $\xi$, (Smith 59S1, p. 124, eqn. 167), using $r$'s obtained for the two models in section IV-332. The results are shown in Fig. IV-19. From these, we estimated $\Delta \rho_1/\rho_1$ for three temperatures and $B = 3.0$ Wb/m$^2$, using the values of $R_1$ and $\sigma_1$ from Table IV-2a.

The following results are obtained:

<table>
<thead>
<tr>
<th>$T^\circ$C</th>
<th>$\Delta \rho_1/\rho_1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>3.2%</td>
</tr>
<tr>
<td>120</td>
<td>3.5%</td>
</tr>
<tr>
<td>200</td>
<td>4.8%</td>
</tr>
</tbody>
</table>

It is seen that the contribution of the $\Gamma_1$ band magnetoresistance to the total magnetoresistance is
resistance increases with temperature, as expected, but even at 200°C is not very large.

IV-4  **Summary, Discussion and Conclusion**

IV-41  **Summary**

Magnetoresistance measurements as a function of field B and temperature T (25 < T < 120°C) together with low field (B < 0.9 Wb/m²) Hall coefficient were used to determine \( n_1, n_2, \mu_1 \) and \( \mu_2 \). The variation of \( n_1 \) and \( n_2 \) with temperature was then employed to show that \( E_{20} = 0.099 \pm 0.02 \) eV and \( \alpha_2 = -3.4 \pm 0.1 \times 10^{-5} \) eV/°K.

The Kane model was used for the \( \Gamma_1 \) band and the \( L_1 \) band was assumed to be parabolic. The low field Hall coefficient in the temperature range 25 - 360°C was then analysed taking into account the third set of conduction band minima with \( X_1 \) symmetry and the heavy hole valence band, \( \Gamma_{15} \).

It was found that including these two extra bands (\( X_1 \) and \( \Gamma_{15} \)) in the analysis of \( R(T) \) and \( \sigma_0(T) \) was insufficient to explain the shape of these curves, whatever parameters were chosen for the \( X_1 \) band. The effect of the valence band was found to be negligible. The shape of the \( R \) vs. \( T \) curve was then explained on the basis of a change in the dominant scattering mechanism in both bands. The parameters for the \( <100\) minima were estimated by comparison with the \( <100\) minima in GaAs and GaP. Using these parameters, together with previously determined parameters for the \( L_1 \) and \( \Gamma_1 \) minima, \( R(T) \) and \( \sigma_0(T) \) were fitted simultaneously giving \( \mu_1(T) \) and \( \mu_2(T) \). The value of \( \mu_2(T) \) was affected considerably more than \( \mu_1(T) \) by the choice of parameters for the \( X_1 \) band. The mobility \( \mu_2(T) \) was then fitted using an approximate method which assumed that the two operative scattering mechanisms were ionized impurity and deformation potential, and use being made of Matthiessen's rule for the
addition of mobilities. The fitting gave estimates of the total ionized impurity
$N_I \sim 2 \times 10^{24} \text{ m}^{-3}$ and the deformation potential of $\approx 9 \text{ eV}$.

The mobility of electrons in the $\Gamma_1$ band was then fitted assuming two
models. The first model included four scattering mechanisms with relaxation times
$\tau_{po}, \tau_{ib}, \tau_I$ and $\tau_{sc}$. The assumption of a value of ionized impurity density $N_I$
of $2.2 \times 10^{24} \text{ m}^{-3}$, as estimated above, left two undetermined parameters $D_{12}$ and
$(N_S Q_S)$ in the expression for the effective relaxation time $\tau$. These were used as
adjustable parameters in fitting $\mu_1(T)$ and a good fit was obtained with
$D_{12} = 3.2 \times 10^8 \text{ eV/cm}$ and $(N_S Q_S) = 4.8 \times 10^4 \text{ cm}^{-1}$. In the second model, spacecharge
scattering was not included, and so the mobility was determined by only $\tau_{po}, \tau_{ib}$
and $\tau_I$. In this case, $D_{12}$ and $N_I$ were used as adjustable parameters, with the best
fit being obtained with $D_{12} = 3.0 \times 10^8 \text{ eV/cm}$ and $N_I = 4.2 \times 10^{24} \text{ m}^{-3}$. This value
of $N_I$ is double that determined from the fitting of the $\mu_2(T)$ data. However, in
obtaining the latter value, no account was taken of (a) scattering of the $\Gamma_1$
electrons by the heavy electrons in the $L_1$ minima, (b) the possibility that the
acceptors are doubly charged and (c) the difference in the effective screening by
the heavy and light electrons. Taking account of these effects in the calculation
of $\tau_I$, the $\mu_1(T)$ data were fitted using $N_a^\infty$ and $D_{12}$ as adjustable parameters, with
best fit being obtained for $N_a^\infty = 1.1 \times 10^{23} \text{ m}^{-3}$ and $D_{12} = 4.1 \times 10^8 \text{ eV/}^0\text{K}$.

Having obtained the relaxation time $\tau(E, T)$ on the basis of the two models
the scattering parameter $\tau_1$ was then calculated for the $\Gamma_1$ band as a function of
temperature at four fields, $B = 0.2, 0.87, 2$ and $3 \text{ Wb/m}^2$. It was found that $\tau_1$ was
close to unity and varied only slowly with both $B$ and $T$.

Since the analysis of magnetoresistance measurements against field was
based on the assumption that the contributions of the individual bands to the total
magnetoresistance were small in comparison with the two band effect, we also cal-
culated the magnetoresistance coefficient $\xi$ for the $\Gamma_1$ band. The magnitude of $\Delta \sigma_{1}/\sigma_1$
was found to be 3.2% at $B = 3 \text{ Wb/m}^2$ at $25^\circ\text{C}$, increasing to 4.8% at $200^\circ\text{C}$.

The analysis of the data on the p-type sample was made in two stages. First the "low" temperature data were analysed to determine the mobility variation of the light and heavy holes as a function of temperature. Then the complete curves of $R$ and $\sigma_0$ were analysed using these temperature dependent mobilities for the holes, together with conduction band parameters obtained from analyses of data on sample N2, and using $\mu_1$ and $\mu_2$ as adjustable parameters. These analyses yielded the temperature dependence of mobilities $\mu_1$ and $\mu_2$ at temperatures above $400^\circ\text{C}$. The electron mobility $\mu_1$ in the $\Gamma_1$ band was then calculated from a relaxation time approximation for the same range in temperature, and values obtained were in reasonable agreement with the above experimental values.

IV-42 Discussion

The analysis of the magnetoresistance measurements as a function of magnetic field and temperature were based on the assumption that the observed magnetoresistance is purely a two band effect. The validity of this assumption has already been discussed in section II-7, where we have used the results for the magnetoresistance coefficient of the $\Gamma_1$ band given in the previous section. From these considerations alone, we would be justified in using the analysis even above $120^\circ\text{C}$. However, because of the proximity of the $<100>$ band, and the resulting loss of electrons to this band, the validity of the two band model is questionable at temperatures above this value. Therefore, only the results obtained at temperatures below $120^\circ\text{C}$ were used in determining the energy gap of the $\Gamma_1$ and $L_1$ band and its temperature dependence. To do this, we had to make some further assumptions. Firstly, to obtain $n_1$ and $n_2$ from $R_1$ and $R_2$ we assumed that the scattering parameters were equal to 1.0
and 1.18 respectively and remained constant with temperature. It was shown that putting \( r_1 = \text{constant} = 1.0 \) was quite reasonable, its variation (Fig. IV-17) with temperature and magnetic field was a maximum of 8% over the full temperature range. On the other hand, the value of \( r_2 = 1.18 \) was used because of a lack of understanding of the scattering processes in the \( L_1 \) minima, and although the magnitude may be in error, the fact that the resulting values of \( n_1 + n_2 = n_t \) remained constant over the temperature range \( 25 < T < 120^\circ C \) indicates that the assumption that \( r_2 \) remains constant with \( T \) is indeed a valid one.

We also had to assume a linear form for the temperature dependence of \( E_2 \). We should have, in principle, used the Varshni (67V1) relation for the temperature dependences of the direct \( (\Gamma_{15v} - \Gamma_{1c}) \) and the indirect \( (\Gamma_{15v} - L_{1c}) \) gaps, and taken the difference to obtain \( E_2 \). But, although we know the values of \( \alpha_o \) and \( \theta_o \) for the direct, no values are available for the indirect gap. This would leave us with one additional unknown parameter. Since as has already been pointed out in section II-8, the Varshni relation becomes linear at elevated temperatures, it was felt that rather than have one more adjustable parameter, making the fitting more difficult and unreliable, the use of a linear form of \( E_2(T) \) would be preferable. As a consequence, the value of \( E_{20} = 0.099 \pm 0.002 \) eV which was obtained, is somewhat higher than the actual band gap at 0\(^\circ\)K. The value agrees very well however with the value of \( E_{20} = 0.095 \pm 0.005 \) eV obtained by Yep and Becker (67Y1) at 4.2\(^\circ\)K and also with results obtained by re-analysing Harland's (65H1) data of magnetoresistance on GaSb at 4.2\(^\circ\)K, which gave a value of \( E_{20} = 0.093 \pm 0.003 \) eV (see Appendix A). It is also in agreement with the value of 0.087 eV at room temperature which was obtained by Rosenbaum (72R1) from pressure measurements on two samples. One of these was cut from our sample N2 and the second also from material doped with Te, but with lower total electron density.
It should be pointed out that the values of $E_{20}$ and $\alpha_2$ obtained from this analysis are dependent on the effective masses in the two bands. We have used a value of $m_{10}^*(0) = 0.040m$ for the bottom of the band effective mass in the $\Gamma_1$ band. This value was determined by Yep and Becker (66Y1) at 4.2$^0$K and was later used by Baxter et al. (67B1) in the analysis of mobility at 78$^0$K on n-GaSb. The choice of this value over other published values, e.g. $0.052 \pm 0.002m$ (61B1) and $0.047 \pm 0.003m$ (59Z1), was made because in the determination of $m_{10}^*(0) = 0.040m$, Yep and Becker (66Y1) took into account the non-parabolicity of the band, and this was not done in the other cases.

The value of the density of states effective mass in the $L_1$ band, $m_{2d}^*$, is still less certain. We have used Van Tongerloo's (69T1) value of 0.226m obtained from Faraday rotation measurements at room temperature. It agrees with the result of Piller (63P1, 64P1) who found $m_{2d}^* = 0.228m$ and with the Germanium value of 0.22m (59S1). However, these values are intricately dependent on other assumptions and parameters used in the analysis by which they were obtained, hence can at best be considered to be estimates of the value of $m_{2d}^*$. The value of $E_{20}$ is quite insensitive to $m_{2d}^*$, but $\alpha_2$ is sensitive, and this is the reason, in part, for such large variations in the reported values of $\alpha_2$.

From consideration of the published values of $m_{2d}^*$, we estimate that the value of 0.226m is probably the lower limit for the density of states effective mass of the electrons in the $L_1$ minima. In that case, our value of $-3.4 \times 10^{-5}$ eV/$^0$K$^{-1}$ is also a lower limit. Any increase in $m_{2d}^*$ will make this value more positive. If we double $m_{2d}^*$ (i.e. $m_{2d}^* = .452m$), we obtain a value of $\alpha_2 = +5.4 \times 10^{-5}$ eV/$^0$K$^{-1}$.

Both scattering models gave an essentially good fit to the $\mu_1(T)$ values determined from measurements on the n-type sample N2, and thus on this basis alone, we are unable to reject either model. In fact, we believe that although we obtained
good agreement between the experimental and calculated results when spacecharge scattering was neglected and scattering by heavy electrons included, the correct model can not completely exclude $\tau_{sc}$. We are led to this conclusion for two reasons. Firstly, the scattering cross section of the heavy electrons must be smaller than that of ionized impurities because the former are not stationary. Secondly, the results of measurements by Demars (72D1) of longitudinal and transverse Nernst-Ettingshausen (N-E) coefficients as a function of magnetic field on the same sample at $300^{0}\text{K}$ indicate that some spacecharge scattering must be present. Demars used the relaxation times calculated by the present author for $300^{0}\text{K}$ in the calculation of both longitudinal and transverse N-E coefficients, and found that although good agreement between the calculations and experiment existed for the variation of the longitudinal N-E coefficient as a function of $B$, less satisfactory agreement was obtained for the transverse N-E coefficient when spacecharge scattering was completely neglected. From our results it is not possible to determine to what extent spacecharge must be included since that would add an extra adjustable parameter ($N_{s}Q_{s}$) which has a similar effect on $\mu_{1}$ at low temperatures as does $N_{a}$. These calculations have shown that the mobility of $\approx 0.4\text{m}^{2}/\text{V} \cdot \text{sec}$ for electrons in the $\Gamma_{1}$ minimum at $300^{0}\text{K}$ is not at all unreasonable when account is taken of Coulomb scattering by both ionized impurities and the heavy $L_{1}$ electrons. A further improvement in calculating the mobility in this band can be made by variational calculations, but this type of calculation is beyond the scope of this thesis. Such calculations could take into account quantum mechanical corrections such as those described by Moore (67M1), and any effect of electron-electron interactions of the $\Gamma_{1}$ band electrons which may become important at higher temperatures.

A rough comparison with the electron mobility in the $\Gamma_{1}$ band of GaAs further
shows that a value of \(\mu_1 \approx 0.4m^2/V.s\) is not unreasonable for the mobility in the \(\Gamma_1\) band of GaSb. Using the value of \(\mu_1(GaAs) = 0.22m^2/V.s\) obtained from Moore's (67M1) data for a sample with \(N_1^* = 4 \times 10^{24} \text{m}^{-3}\) and \(\mu_1(GaAs) = 0.93m^2/V.s\) for a pure sample (60E1) at room temperature, then the mobility due to ionized impurities scattering alone is estimated to be \(\mu_1(GaAs) = 0.288m^2/V.s\). If all other effects are assumed to be equal, the ionized impurity mobility is inversely proportional to \((m_e^*)^{1/2}\), and hence:

\[
\frac{\mu_1(GaAs)}{\mu_1(GaSb)} = \left(\frac{0.04}{0.07}\right)^{1/2}
\]

Thus \(\mu_1(GaSb) = 1.3 \times \mu_1(GaAs) = 0.375m^2/V.s\). which is close to the value of 0.376\(m^2/V.s\) obtained for sample N2 and is in the range of values quoted in the literature by other authors.

The analyses of the experimental results of \(R\) and \(\sigma\) on the P1 sample also lead to an estimate of the temperature dependence of electron mobility \(\mu_1\) in the \(\Gamma_1\) conduction band at \(T > 400^\circ\text{C}\). Calculations of the \(\Gamma_1\) electron mobility from a theoretical model, based on relaxation time approximations and including scattering by optical phonons, Coulomb centres and interband scattering into \(L_1\) and \(X_1\) band, agree reasonably well in magnitude and temperature with the results determined from fitting \(R\) and \(\sigma\). The calculated values lie about 10% above the determined values. This discrepancy can be explained by the neglect of electron-electron interactions in the \(\Gamma_1\) band which would tend to reduce the calculated mobility still further. However, within the scope of the relaxation time approximation the electron-electron interactions could not be taken into account. The agreement of 10% that was obtained can therefore be regarded as satisfactory.
IV-43 Conclusion

The two band model was used in the analysis of the magnetoresistance data on sample N2. It was then possible to determine the $\Gamma_1 - L_1$ band separation ($E_{20} = 0.099$ eV) and its temperature dependence ($\alpha_2 = -3.4 \times 10^{-5}$ eV/$^0$K). The value of 0.099 eV for $E_{20}$ agrees very well with the value $0.095 \pm 0.005$ eV determined by Yep and Becker (66Y1) at liquid helium temperature and with Rosenbaum's (72R1) room temperature value of 0.087 eV.

It was found that screened Coulomb scattering plays an important role in determining the electron mobility in the $\Gamma_1$ band. A value for the coupling constant $D_{12}$ of $4.1 \times 10^8$ eV/cm was also estimated as well as the deformation potential of $\omega 9$ eV for the $L_1$ band. This estimate of the deformation potential agrees very well with the value of $\omega 10$ eV reported recently by Sherwood (70S4).

The analyses of the data on the p-type sample led to the following conclusions:

(a) scattering in the valence band is dominated by polar optical interactions,

(b) the temperature coefficient of the principal gap, $\alpha_o$, is $-5.4 \times 10^{-4}$ eV/$^0$K.

In addition, electron mobility in the $\Gamma_1$ band of GaSb was estimated at $T > 400^0$C by fitting the experimental results of $R(T)$ and $\sigma(T)$. These results were then compared with calculated values of mobility based on the relaxation time approximation and an acceptable agreement was obtained.

The overall agreement obtained between our results on GaSb and those of Demars and Rosenbaum supports the claim that the parameters which were determined are in fact representative of GaSb.
<table>
<thead>
<tr>
<th>$\text{T}^\circ\text{C}$</th>
<th>$t$ $\text{Wb}^2/\text{m}^4$</th>
<th>$u$</th>
<th>$s$ $\text{Wb}^2/\text{m}^4$</th>
<th>$a$</th>
<th>$-R_o \times 10^6$ $\text{Coul/m}^3$</th>
<th>$\sigma_o \times 10^{-4}$ $\Omega^{-1}\text{m}^{-1}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>47.09</td>
<td>0.806</td>
<td>101.5</td>
<td>2.58</td>
<td>9.249</td>
<td>3.103</td>
</tr>
<tr>
<td>40</td>
<td>48.35</td>
<td>0.929</td>
<td>97.55</td>
<td>2.50</td>
<td>9.370</td>
<td>2.881</td>
</tr>
<tr>
<td>60</td>
<td>49.28</td>
<td>1.042</td>
<td></td>
<td></td>
<td>9.521</td>
<td>2.633</td>
</tr>
<tr>
<td>80</td>
<td>51.75</td>
<td>1.154</td>
<td></td>
<td></td>
<td>9.660</td>
<td>2.388</td>
</tr>
<tr>
<td>100</td>
<td>53.87</td>
<td>1.253</td>
<td>69.17</td>
<td>2.80</td>
<td>9.764</td>
<td>2.191</td>
</tr>
<tr>
<td>120</td>
<td>57.18</td>
<td>1.427</td>
<td></td>
<td></td>
<td>9.836</td>
<td>2.016</td>
</tr>
<tr>
<td>140</td>
<td>60.33</td>
<td>1.531</td>
<td></td>
<td></td>
<td>9.899</td>
<td>1.854</td>
</tr>
<tr>
<td>160</td>
<td>64.79</td>
<td>1.846</td>
<td></td>
<td></td>
<td>10.018</td>
<td>1.653</td>
</tr>
<tr>
<td>200</td>
<td>70.77</td>
<td>2.066</td>
<td></td>
<td></td>
<td>10.121</td>
<td>1.466</td>
</tr>
</tbody>
</table>

Table IV-1: The slopes ($t$, $s$) and intercepts ($u$, $a$) obtained from $\sigma_o / \Delta \sigma$ and $-R_o / \Delta R$ vs. $B^{-2}$ plots and the zero field Hall coefficient $R_o$ and $\sigma_o$ for sample N2 of n-GaSb.
<table>
<thead>
<tr>
<th>$T^\circ C$</th>
<th>(i)</th>
<th>(ii)</th>
<th>(iii)</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>$\sigma_1 \times 10^{-4}$</td>
<td>2.275</td>
<td>2.221</td>
</tr>
<tr>
<td></td>
<td>$\sigma_2 \times 10^{-4}$</td>
<td>0.827</td>
<td>0.882</td>
</tr>
<tr>
<td></td>
<td>$-R_1 \times 10^5$</td>
<td>1.648</td>
<td>1.670</td>
</tr>
<tr>
<td></td>
<td>$-R_2 \times 10^6$</td>
<td>5.48</td>
<td>6.73</td>
</tr>
<tr>
<td>40</td>
<td>$\sigma_1 \times 10^{-4}$</td>
<td>2.057</td>
<td>1.995</td>
</tr>
<tr>
<td></td>
<td>$\sigma_2 \times 10^{-4}$</td>
<td>0.824</td>
<td>0.886</td>
</tr>
<tr>
<td></td>
<td>$-R_1 \times 10^5$</td>
<td>1.755</td>
<td>1.791</td>
</tr>
<tr>
<td></td>
<td>$-R_2 \times 10^6$</td>
<td>5.20</td>
<td>8.23</td>
</tr>
<tr>
<td>100</td>
<td>$\sigma_1 \times 10^{-4}$</td>
<td>1.283</td>
<td>1.152</td>
</tr>
<tr>
<td></td>
<td>$\sigma_2 \times 10^{-4}$</td>
<td>0.908</td>
<td>1.039</td>
</tr>
<tr>
<td></td>
<td>$-R_1 \times 10^5$</td>
<td>2.561</td>
<td>2.877</td>
</tr>
<tr>
<td></td>
<td>$-R_2 \times 10^6$</td>
<td>5.72</td>
<td>8.06</td>
</tr>
</tbody>
</table>

Table IV-2: Comparison of calculated values of $R_1$, $R_2$, $\sigma_1$ and $\sigma_2$ using the three combinations of experimental parameters (i) $R_o$, $\sigma_o$, s and t, (ii) $R_o$, $\sigma_o$, s and a, (iii) $R_o$, $\sigma_o$, t and u.


<table>
<thead>
<tr>
<th>T°C</th>
<th>$-R_0 \text{m}^3/\text{Coul} \times 10^6$</th>
<th>$\sigma_0 (\Omega \cdot \text{m})^{-1} \times 10^{-4}$</th>
<th>$\sigma_1 (\Omega \cdot \text{m})^{-1} \times 10^{-4}$</th>
<th>$\sigma_2 (\Omega \cdot \text{m})^{-1} \times 10^{-4}$</th>
<th>$-R_1 \text{m}^3/\text{Coul} \times 10^5$</th>
<th>$-R_2 \text{m}^3/\text{Coul} \times 10^6$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>9.249</td>
<td>3.103</td>
<td>2.260</td>
<td>.843</td>
<td>1.664</td>
<td>5.33</td>
</tr>
<tr>
<td>40</td>
<td>9.370</td>
<td>2.881</td>
<td>2.039</td>
<td>.842</td>
<td>1.777</td>
<td>5.48</td>
</tr>
<tr>
<td>60</td>
<td>9.521</td>
<td>2.633</td>
<td>1.773</td>
<td>.860</td>
<td>1.974</td>
<td>5.37</td>
</tr>
<tr>
<td>80</td>
<td>9.660</td>
<td>2.388</td>
<td>1.530</td>
<td>.858</td>
<td>2.189</td>
<td>5.26</td>
</tr>
<tr>
<td>100</td>
<td>9.764</td>
<td>2.191</td>
<td>1.336</td>
<td>.855</td>
<td>2.416</td>
<td>5.13</td>
</tr>
<tr>
<td>120</td>
<td>9.836</td>
<td>2.016</td>
<td>1.160</td>
<td>.856</td>
<td>2.689</td>
<td>5.19</td>
</tr>
<tr>
<td>140</td>
<td>9.899</td>
<td>1.854</td>
<td>1.013</td>
<td>.841</td>
<td>2.967</td>
<td>5.01</td>
</tr>
<tr>
<td>160</td>
<td>9.956</td>
<td>1.714</td>
<td>.872</td>
<td>.842</td>
<td>3.376</td>
<td>5.03</td>
</tr>
<tr>
<td>170</td>
<td>10.018</td>
<td>1.652</td>
<td>.815</td>
<td>.837</td>
<td>3.574</td>
<td>5.13</td>
</tr>
<tr>
<td>200</td>
<td>10.121</td>
<td>1.466</td>
<td>.662</td>
<td>.804</td>
<td>4.224</td>
<td>4.98</td>
</tr>
</tbody>
</table>

Table IV-2a: Calculated values of $R_1$, $R_2$, $\sigma_1$ and $\sigma_2$ using magnetoresistance results together with zero field values of Hall coefficient. $(R_0)$ and conductivity $(\sigma_0)$, i.e. combination (iii) $R_0$, $\sigma_0$, t and u.
<table>
<thead>
<tr>
<th>$T^\circ C$</th>
<th>$n_1 \text{ m}^{-3} \times 10^{-23}$</th>
<th>$n_2 \text{ m}^{-3} \times 10^{-24}$</th>
<th>$n_t \text{ m}^{-3} \times 10^{-24}$</th>
<th>$\mu_1 \text{ m}^2/\text{V.sec}$</th>
<th>$\mu_2 \text{ m}^2/\text{V.sec}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>3.757</td>
<td>1.07</td>
<td>1.45</td>
<td>.3760</td>
<td>.0491</td>
</tr>
<tr>
<td>40</td>
<td>3.517</td>
<td>1.11</td>
<td>1.46</td>
<td>.3623</td>
<td>.0475</td>
</tr>
<tr>
<td>60</td>
<td>3.167</td>
<td>1.13</td>
<td>1.45</td>
<td>.3500</td>
<td>.0474</td>
</tr>
<tr>
<td>80</td>
<td>2.855</td>
<td>1.16</td>
<td>1.44</td>
<td>.3348</td>
<td>.0464</td>
</tr>
<tr>
<td>100</td>
<td>2.587</td>
<td>1.19</td>
<td>1.45</td>
<td>.3229</td>
<td>.0448</td>
</tr>
<tr>
<td>120</td>
<td>2.106</td>
<td>1.22</td>
<td>1.43</td>
<td>.3007</td>
<td>.0432</td>
</tr>
<tr>
<td>160</td>
<td>1.852</td>
<td>1.22</td>
<td>1.40</td>
<td>.2944</td>
<td>.0432</td>
</tr>
<tr>
<td>170</td>
<td>1.749</td>
<td>1.19</td>
<td>1.36</td>
<td>.2914</td>
<td>.0441</td>
</tr>
<tr>
<td>200</td>
<td>1.480</td>
<td>1.22</td>
<td>1.37</td>
<td>.2797</td>
<td>.0411</td>
</tr>
</tbody>
</table>

Table IV-3: Calculated values of $n_1$, $n_2$, $\mu_1$ and $\mu_2$ using the values of Table IV-2a for $R_1$, $R_2$, $\sigma_1$ and $\sigma_2$ and assuming that $r_1 = 1.00$ and $r_2 = 1.18$, both constant with temperature. The value of $n_t$ is obtained by adding $n_1$ and $n_2$. 
### TABLE IV-4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{10}^*$</td>
<td>0.0402 m</td>
<td>(66Y1)</td>
</tr>
<tr>
<td>$m_{2d}^*$</td>
<td>0.226 m</td>
<td>(69T1)</td>
</tr>
<tr>
<td>$v_2$</td>
<td>4</td>
<td>(69P1)</td>
</tr>
<tr>
<td>$m_{3d}^*$</td>
<td>0.36 m</td>
<td>(60E1)</td>
</tr>
<tr>
<td>$v_3$</td>
<td>6</td>
<td>(60E1)</td>
</tr>
<tr>
<td>$m_h^*$</td>
<td>0.33 m</td>
<td>(68W1)</td>
</tr>
<tr>
<td>$E_{oo}$</td>
<td>0.813 eV</td>
<td>(64M1)</td>
</tr>
<tr>
<td>$a_o^*$</td>
<td>$-5.0 \times 10^{-4}$ eV/°K</td>
<td></td>
</tr>
<tr>
<td>$\theta_o^*$</td>
<td>112 °K</td>
<td></td>
</tr>
<tr>
<td>$E_2$</td>
<td>Variable</td>
<td></td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>Variable</td>
<td></td>
</tr>
<tr>
<td>$E_3$</td>
<td>0.315 eV</td>
<td>(68K1)</td>
</tr>
<tr>
<td>$\alpha_3$</td>
<td>0.0</td>
<td></td>
</tr>
<tr>
<td>$K_2$</td>
<td>8.6</td>
<td>(64P1)</td>
</tr>
<tr>
<td>$K_3$</td>
<td>3.6</td>
<td>(60E1)</td>
</tr>
</tbody>
</table>

**Table IV-4:** Band parameters used in fitting the GaSb data on sample N2. The list of references on the right give the source from which the parameters were obtained.

* Values obtained by fitting the absorption data as a function of temperature of Woolley and Evans (61W1) for GaSb.
Table IV-5: Results of fitting the temperature variation of $n_1$ given in Table IV-3 and using the band parameters given in Table IV-4, together with $E_{20} = 0.099$ eV, $\alpha_2 = -3.4 \times 10^{-5}$ eV/°K, and $n_t = 1.45 \times 10^{24}$ m$^{-3}$.
TABLE IV-6

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>T</strong></td>
<td>78°C</td>
</tr>
<tr>
<td><strong>R_0</strong></td>
<td>$6.483 \times 10^{-6}$ m$^{-3}$/Coul.</td>
</tr>
<tr>
<td><strong>σ_0</strong></td>
<td>$8.525 \times 10^4$ (Ω.m.)$^{-1}$</td>
</tr>
<tr>
<td><strong>s</strong></td>
<td>119.3</td>
</tr>
<tr>
<td><strong>a</strong></td>
<td>0.744</td>
</tr>
</tbody>
</table>

| **σ_1**  | $8.239 \times 10^4$ (Ω.m.)$^{-1}$ |
| **σ_2**  | $0.286 \times 10^4$ (Ω.m.)$^{-1}$ |
| **R_1**  | $-6.683 \times 10^{-6}$ m$^{-3}$/Coul. |
| **R_2**  | $-2.13 \times 10^{-4}$ m$^{-3}$/Coul. |

| **n_1**  | $9.356 \times 10^{23}$ m$^{-3}$ for $r_1 = 1.00$ |
| **n_2**  | $0.570 \times 10^{23}$ m$^{-3}$ for $r_2 = 1.93$ |
| **n_t**  | $9.926 \times 10^{23}$ m$^{-3}$ |

Table IV-6: The results from analysis of measurements at 78°C.
<table>
<thead>
<tr>
<th>T°C</th>
<th>( n_1 \times 10^{-3} )</th>
<th>( n_2 \times 10^{-4} )</th>
<th>( n_3 \times 10^{-2} )</th>
<th>( \mu_1 m^2/V.\text{sec} )</th>
<th>( \mu_2 m^2/V.\text{sec} )</th>
<th>( E_F \text{ eV} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>3.806</td>
<td>1.066</td>
<td>0.03035</td>
<td>0.3738</td>
<td>0.04822</td>
<td>0.0315</td>
</tr>
<tr>
<td>40</td>
<td>3.518</td>
<td>1.096</td>
<td>0.04656</td>
<td>0.3621</td>
<td>0.04784</td>
<td>0.0268</td>
</tr>
<tr>
<td>60</td>
<td>3.170</td>
<td>1.131</td>
<td>0.07733</td>
<td>0.3496</td>
<td>0.04737</td>
<td>0.0203</td>
</tr>
<tr>
<td>80</td>
<td>2.863</td>
<td>1.162</td>
<td>0.1211</td>
<td>0.3343</td>
<td>0.04595</td>
<td>0.0136</td>
</tr>
<tr>
<td>100</td>
<td>2.591</td>
<td>1.187</td>
<td>0.1805</td>
<td>0.3224</td>
<td>0.04488</td>
<td>0.0067</td>
</tr>
<tr>
<td>120</td>
<td>2.558</td>
<td>1.213</td>
<td>0.2585</td>
<td>0.3104</td>
<td>0.04342</td>
<td>-0.0003</td>
</tr>
<tr>
<td>140</td>
<td>2.147</td>
<td>1.230</td>
<td>0.3559</td>
<td>0.2984</td>
<td>0.04196</td>
<td>-0.0076</td>
</tr>
<tr>
<td>160</td>
<td>1.965</td>
<td>1.246</td>
<td>0.4761</td>
<td>0.2876</td>
<td>0.04045</td>
<td>-0.0150</td>
</tr>
<tr>
<td>170</td>
<td>1.886</td>
<td>1.255</td>
<td>0.546</td>
<td>0.2831</td>
<td>0.03960</td>
<td>-0.0187</td>
</tr>
<tr>
<td>200</td>
<td>1.670</td>
<td>1.274</td>
<td>0.7906</td>
<td>0.2667</td>
<td>0.03679</td>
<td>-0.0302</td>
</tr>
<tr>
<td>220</td>
<td>1.544</td>
<td>1.284</td>
<td>0.9846</td>
<td>0.2570</td>
<td>0.03507</td>
<td>-0.0381</td>
</tr>
<tr>
<td>240</td>
<td>1.349</td>
<td>1.294</td>
<td>1.208</td>
<td>0.2476</td>
<td>0.03328</td>
<td>-0.0460</td>
</tr>
<tr>
<td>260</td>
<td>1.339</td>
<td>1.300</td>
<td>1.454</td>
<td>0.2385</td>
<td>0.03162</td>
<td>-0.0542</td>
</tr>
<tr>
<td>280</td>
<td>1.254</td>
<td>1.306</td>
<td>1.730</td>
<td>0.2297</td>
<td>0.02985</td>
<td>-0.0624</td>
</tr>
<tr>
<td>300</td>
<td>1.179</td>
<td>1.313</td>
<td>2.033</td>
<td>0.2212</td>
<td>0.02815</td>
<td>-0.0707</td>
</tr>
<tr>
<td>320</td>
<td>1.110</td>
<td>1.316</td>
<td>2.359</td>
<td>0.2131</td>
<td>0.0266</td>
<td>-0.0792</td>
</tr>
<tr>
<td>340</td>
<td>1.048</td>
<td>1.319</td>
<td>2.709</td>
<td>0.2056</td>
<td>0.02513</td>
<td>-0.0878</td>
</tr>
<tr>
<td>360</td>
<td>0.9934</td>
<td>1.323</td>
<td>3.089</td>
<td>0.1982</td>
<td>0.02363</td>
<td>-0.0964</td>
</tr>
</tbody>
</table>

Table IV-7: The values of carrier concentrations \( n_1, n_2, n_3 \) and mobility \( \mu_1 \) and \( \mu_2 \), obtained from fitting \( R(T) \) at 0.87 Wb/m\(^2\) and \( \sigma_0(T) \) with \( \mu_1 \) and \( \mu_2 \) adjustable. Remaining parameters were those of Table IV-4 in addition to \( E_{20} = 0.099 \text{ eV}, \ \alpha_2 = -3.4 \times 10^{-5} \text{ eV/°K}, \) and \( n_t = 1.45 \times 10^{24} \text{ m}^{-3} \).
<table>
<thead>
<tr>
<th></th>
<th>Si</th>
<th>GaP</th>
<th>GaAs</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m^*_3$</td>
<td>0.33 m$^a$</td>
<td>0.34 m$^b$</td>
<td>0.36 m$^c$</td>
</tr>
<tr>
<td>$\mu_{300}$ (m$^2$/Vsec.)</td>
<td>0.13 m$^a$</td>
<td>0.01 m$^d$</td>
<td>0.0155 m$^c$</td>
</tr>
<tr>
<td>$\alpha_3$ eV/°K $\times 10^4$</td>
<td>-7.0 m$^e$</td>
<td>-6.2 m$^f$</td>
<td>-5.4 m$^g$</td>
</tr>
<tr>
<td>$\theta_3$ °K</td>
<td>1110 m$^e$</td>
<td>460 m$^f$</td>
<td>208 m$^g$</td>
</tr>
<tr>
<td>$\alpha_3'$ (500) $\times 10^5$</td>
<td>+9</td>
<td>+3</td>
<td></td>
</tr>
<tr>
<td>$\alpha_3'$ (700) $\times 10^5$</td>
<td>+6</td>
<td>+1</td>
<td></td>
</tr>
</tbody>
</table>

(a) Smith (59S1), (b) Madelung (64M1), (c) Conwell and Vassell (68Cl),
(d) Montgomery (68M1), (e) Varshni (67V1), (f) Panish and Casey (69P1),
(g) This thesis-Chapter V.

Table IV-8: Parameters of the <100> minima in three semiconductors Si, GaP and GaAs and estimates of the temperature dependence $\alpha_3 (\equiv \alpha_3')$ using these values together with $\alpha_o = -5.0 \times 10^{-4} \text{ eV/°K}$, $\theta_o = 112°\text{K}$ for the fundamental gap temperature variation (c.f. Table IV-4).
Table IV-9: The conduction band parameters of the $\Gamma_1$ band used in calculations of electron mobility $\mu_1$ in this band.
<table>
<thead>
<tr>
<th>T°C</th>
<th>$n_1 \times 10^{22}$ m$^{-3}$</th>
<th>$n_2 \times 10^{23}$ m$^{-3}$</th>
<th>$n_3 \times 10^{22}$ m$^{-3}$</th>
<th>$p_h \times 10^{24}$ m$^{-3}$</th>
<th>$p_x \times 10^{22}$ m$^{-3}$</th>
<th>$\mu_1$ m$^2$/V.sec.</th>
<th>$\mu_2$ m$^2$/V.sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>23.5</td>
<td>.2668</td>
<td></td>
<td>.3224</td>
<td></td>
<td>.3342</td>
<td></td>
<td>1.344</td>
</tr>
<tr>
<td>40</td>
<td>.2790</td>
<td>.3028</td>
<td>.3224</td>
<td>.6279</td>
<td>.8496</td>
<td>1.405</td>
<td></td>
</tr>
<tr>
<td>80</td>
<td></td>
<td>.3506</td>
<td>.3608</td>
<td></td>
<td></td>
<td>1.525</td>
<td></td>
</tr>
<tr>
<td>120</td>
<td>.3506</td>
<td>.3608</td>
<td>.3608</td>
<td>.4502</td>
<td></td>
<td>1.623</td>
<td></td>
</tr>
<tr>
<td>160</td>
<td>.3506</td>
<td>.3608</td>
<td>.3608</td>
<td>.4502</td>
<td></td>
<td>1.683</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>.3506</td>
<td>.3608</td>
<td>.3608</td>
<td>.4502</td>
<td></td>
<td>1.683</td>
<td></td>
</tr>
<tr>
<td>240</td>
<td>.3506</td>
<td>.3608</td>
<td>.3608</td>
<td>.4502</td>
<td></td>
<td>1.683</td>
<td></td>
</tr>
<tr>
<td>280</td>
<td>.0132</td>
<td>.0127</td>
<td>.0028</td>
<td>.0398</td>
<td>.2169</td>
<td>.1817</td>
<td></td>
</tr>
<tr>
<td>320</td>
<td>.0528</td>
<td>.0585</td>
<td>.0174</td>
<td>.3808</td>
<td>.2169</td>
<td>.1817</td>
<td></td>
</tr>
<tr>
<td>360</td>
<td>.1742</td>
<td>.2191</td>
<td>.0853</td>
<td>.414</td>
<td>.2169</td>
<td>.1817</td>
<td></td>
</tr>
<tr>
<td>400</td>
<td>.4721</td>
<td>.6632</td>
<td>.3278</td>
<td>.4502</td>
<td>.2169</td>
<td>.1817</td>
<td></td>
</tr>
<tr>
<td>420</td>
<td>.7198</td>
<td>1.065</td>
<td>.5864</td>
<td>.4924</td>
<td>.2169</td>
<td>.1817</td>
<td></td>
</tr>
<tr>
<td>440</td>
<td>1.046</td>
<td>1.625</td>
<td>.9921</td>
<td>.5499</td>
<td>.2169</td>
<td>.1817</td>
<td></td>
</tr>
<tr>
<td>460</td>
<td>1.451</td>
<td>2.364</td>
<td>1.590</td>
<td>.6279</td>
<td>.2169</td>
<td>.1817</td>
<td></td>
</tr>
<tr>
<td>480</td>
<td>1.945</td>
<td>3.311</td>
<td>2.441</td>
<td>.7262</td>
<td>.2169</td>
<td>.1817</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>2.522</td>
<td>4.473</td>
<td>3.605</td>
<td>.8496</td>
<td>.2169</td>
<td>.1817</td>
<td></td>
</tr>
<tr>
<td>520</td>
<td>3.188</td>
<td>5.893</td>
<td>5.156</td>
<td>1.000</td>
<td>.2169</td>
<td>.1817</td>
<td></td>
</tr>
<tr>
<td>540</td>
<td>3.945</td>
<td>7.580</td>
<td>7.181</td>
<td>1.181</td>
<td>.2169</td>
<td>.1817</td>
<td></td>
</tr>
</tbody>
</table>

Table IV-10  Results of analyses of the Hall coefficient and conductivity data on sample P1.
Fig. IV-1 Hall coefficient $R$ vs. Temperature $T^\circ C$ for two sets of Hall probes (2-6) and (3-5), GaSb sample M1.

$-R \times 10^{-6} \text{ m}^3/\text{C}$
Fig. IV-2 Average conductivity $\sigma_0$ vs. temperature $T^\circ C$ for GaSb sample N1.
Fig. IV-3  Measured Hall coefficient R vs. B at 25°C for GaSb sample N2.
Fig. IV-4  Plots of $-R/\Delta R$ vs $B^{-2}$ at two temperatures for GaSb sample N2.
Fig. IV-5  Plots of $\rho_0/\Delta \rho$ vs $B^{-2}$ at two temperatures for GaSb sample N2
Fig. IV-6  \(-R\) vs. \(B^2\) at low magnetic field for the determination of \(R_0\).
Fig. IV-7  Plot of the fitted curve (solid line) of $n_1$ vs $T^\circ C$ and experimental points (X) for $E_{20} = 0.099$ eV, $\alpha_2 = -3.4 \times 10^{-5}$ eV/°K.
Fig. IV-8 Hall coefficient $-R$ at $0.87 \text{ Wb/m}^2$ and conductivity $\sigma_0$ vs temperature in °C. GaSb sample N2
Fig. IV-9  Hall coefficient ($B = 0.8 \text{ Wb/m}^2$) and conductivity for p-type sample Pl. The fitted curve is shown as solid line.
Fig. IV-10 Plots of $\ln[\mu(T)/\mu(300)]$ vs. $\ln(300/T^\circ K)$ below $120^\circ C$ for the estimate of the temperature dependence of the mobilities $\mu_1$ and $\mu_2$ for GaSb sample N2. The slopes $n_1$ and $n_2$ are 0.67 and 0.38 respectively.
Fig. IV-11  Fitted mobility $\mu_2(T)$ assuming acoustic and ionized impurity scattering and reciprocal addition of mobilities. The dots are the values of mobility $\mu_2$ obtained by the method of Section IV-331.
Fig. IV-12  Plots of $\mu_1$ vs. $T$ showing the effect of varying the coupling constant $D_{12}$.

Top curve, calculated $\mu_1(T)$ using $\tau_{po}, \tau_1, \tau_{ib}$ and $D_{12} = 1 \times 10^8$ eV/cm.

Dashed curve shows the effect of adjusting $D_{12}$ to fit the high temperature results of $\mu_1(T)$; (bottom curve) obtained by fitting $R(T)$ and $\sigma(T)$ with $\mu_1$ and $\mu_2$ as adjustable parameters, sample N2.
Fig. IV-13  Semi-log plot of $\mu_1(T)$ (circles) as obtained in Section IV-332. The solid line was derived by fitting these points using four relaxation times, i.e. $\tau_{po}$, $\tau_I$, $\tau_{ib}$ and $\tau_{sc}$. 
The four relaxation times as a function of reduced energy $\varepsilon = E/\hbar\omega_\lambda$ for

- $N_I = 2.2 \times 10^{24}$ m$^{-3}$,
- $D_{12} = 3.2 \times 10^8$ eV/cm.
- $(Q_s N_s) = 4.8 \times 10^4$ cm$^{-1}$.
Fig. IV-15 The fitted mobility $\mu_1(T)$ calculated using $\tau_{po}$, $\tau_{ib}$ and $\tau_I$ with

$$D_{12} = 3.0 \times 10^8 \text{ eV/cm and } N_I = 4.2 \times 10^{24} \text{ m}^{-3}.$$
Fig. IV-16  Mobility $\mu_1$ vs. $T^\circ C$ ---- for $\tau^{-1} = \frac{\tau^{-1}_p + \tau^{-1}_i}{\tau_p + \tau_i}$,  . . . .  Calculated for sample N2,  --- calculated for sample P1 and (X) are points determined from fitting experimental data from sample P1.
Fig. IV-17 The scattering parameter $r_1$ as a function of $T^\circ C$, at three magnetic fields.
Fig. IV.18 The scattering parameter $r_1$ as a function of temperature, at $B = 0.87$ Wb/m².
Fig. IV-19
The magnetoresistance coefficient as a function of temperature at $B = 3.0 \text{ W/m}^2$.
Fig. IV-20  Schematic of the band structure of GaSb. Includes the previously established parameters together with those determined in this thesis.
CHAPTER V

CONDUCTION BAND IN GaAs.

V-1 Preliminary Considerations

V-2 Experimental Results

V-3 Analysis of Results
   V-31 The three models of the conduction band structure of GaAs
   V-32 Analysis based on three conduction band model, \( \Gamma_1, \Delta_1 \)
      (or \( X_1 \)) and \( L_1 \).
   V-33 Two band model, \( \Gamma_1 \) and \( \Delta_1 \) (or \( X_1 \))
   V-34 Two band model, \( \Gamma_1 \) and \( L_1 \).

V-4 Mobility in the \( \Gamma_1 \) band

V-5 Magnetoresistance at 400° C

V-6 Summary and Discussion

V-7 Conclusion
CHAPTER V  CONDUCTION BAND IN GaAs

V-1 Preliminary Considerations

In contrast to GaSb where the as-grown crystals are always p-type, bulk GaAs always grows in the n-type form when no intentional dopant is added. The n-type nature of such GaAs crystals is thought to be due to Si impurities, which enter into solid solution from surrounding growth apparatus (e.g. Cochran and Foster 62C1), and not due to non-stoichiometry as is the case for p-type GaSb. It is by now well known that Si behaves as an amphoteric impurity in GaAs (e.g. Kressel and H. von Philipsborn 69K1). Further evidence of the amphoteric behaviour of Si in GaAs has been provided by Hicks and Green (70H2 and references therein). They found that in the growth of layers by the liquid phase epitaxial method, at temperatures between about 700 and 850°C, p-type layers were obtained, because at these temperatures Si shows a preference for the As sites. At higher growth temperatures n-type material is obtained, in which case Si prefers the Ga sites. It was also shown by Hicks and Green that the silicon-doped GaAs layers exhibit a marked degree of compensation. For this reason, Si doped material was considered not suitable for the proposed measurements.

Other well known donor type impurities are Te, Se and S. The latter two are found to give more than one donor level (Pitt and Lees 70P2, Hudson et al. 67H1) and therefore would tend to unduly complicate any analyses of experimental data. Since this work is not concerned with the determination of activation energies of impurities in GaAs, such complications should be avoided. Te doped GaAs was therefore used for
the present work. The doping range used was $5 \times 10^{23} - 5 \times 10^{24} \text{ m}^{-3}$. The lower limit is that below which the transfer of carriers into subsidiary bands would not be sufficient at reasonable temperatures. The upper limit is that above which complications due to overlap of the impurity band with subsidiary minima would arise.

Because the separation of the two lowest conduction bands in GaAs is about four times that found for GaSb, essentially no electrons will be in the subsidiary band at room temperature and the highest doping attainable in single crystals. In order therefore to transfer electrons into this band, higher temperatures have to be used. Measurements of constant field Hall coefficient and conductivity show that at around $200^\circ\text{C}$ electron transfer into the subsidiary bands just becomes noticeable. Therefore to get an appreciable effect, temperatures $> 200^\circ\text{C}$ must be used.

It has been pointed out in the last chapter which dealt with GaSb, that the conditions required for useful analyses of magnetoresistance and Hall coefficient as a function of magnetic field data are such that the relative density of electrons in the two bands are comparable and that the ratio of mobilities is about 10:1. The mobility ratio condition is satisfied for GaAs, but in order to satisfy the electron density condition high temperatures ($> 400^\circ\text{C}$) have to be used. In this case, the conditions under which the equations II-46 and II-62 were derived would no longer be valid, because it is expected that the magnetoresistance of the individual bands would now become comparable with the two band effect. This is borne out by the results of measurements of resistance as a function of magnetic field at $400^\circ\text{C}$, on a heavily doped sample. Therefore only measurements of constant field Hall coefficient and conductivity were made on GaAs.
V-2 Experimental Results

Measurements of conductivity and Hall coefficient at a field of 0.80 Wb/m$^2$ were made on six samples of Te doped GaAs in the temperature range from 20°C to 600°C. The samples were doped with Te to concentrations between $4 \times 10^{23}$ m$^{-3}$ and $9 \times 10^{24}$ m$^{-3}$. The results are shown in Figs. V-1 and V-2 where Hall coefficient and conductivity are plotted against temperature. Room temperature values of $n_1$, $\nu_1$ and $\sigma_1$ for the six samples are given in Table V-1.

Two of these samples were also measured up to ~800°C. The results were not reproducible on cycling the temperature between 20° and 800°C. However, as long as the maximum temperature did not exceed ~600°C, reproducible results were obtained. This lack of reproducibility above 600°C is ascribed to dissociation of the sample surface due to the loss of As. Various encapsulation techniques were tried to maintain an As overpressure which would prevent As evaporation. However, none were successful. The method of quartz encapsulation used by Roberts (65R1) was considered, but due to obvious practical difficulties was discarded. We therefore limited the measurements to below ~600°C. Even with this temperature limitation, the change in Hall coefficient was large enough (Fig. V-1) to allow a meaningful analysis.

Measurements of conductivity as a function of magnetic field (from 0 to 3.2 Wb/m$^2$) were made at 400°C on the heaviest doped sample ($n \sim 10^{25}$ m$^{-3}$). This high temperature was chosen in order that enough carriers would be excited into the first subsidiary band to give a measurable two band effect. The results are shown in Fig. V-14, in the form of $\rho_0/\Delta\rho$ vs. $B^{-2}$ plot.
V-3 Analysis of Results

V-31 The three models of the conduction band structure of GaAs

The uncertainty in the conduction band structure of GaAs has already been fully discussed in Chapter I. It suffices to say in summary here that we have three possible models for the conduction band structure which should be reflected in the experimental results; these are:

(i) Three band model, with band extrema at Γ₁, Δ₁ (or X₁) and L₁

The first subsidiary band minima are either in the <100> direction, (i.e. have Δ symmetry), or are at the edge of the Brillouin zone and hence have X₁ symmetry. The next set of minima have L₁ symmetry and are close in energy to the first, i.e. any analysis based on this model must include three sets of minima Γ, Δ(or X₁) and L₁. This model is supported by the theoretical calculations of Cohen and Bergstresser (66C1), Pollak et al. (66P1) and Herman (68H1), and experimentally by James et al. (68J1).

(ii) Two band model with extrema at Γ₁ and Δ₁ (or X₁)

The first subsidiary minima are Δ₁ (or X₁) but the L₁ minima are well removed in energy to have no effect on electron distribution. This model was assumed by Ehrenreich (60E1) and is supported by the theoretical calculations of Jones and Lettington (69J1).

(iii) Two band model with extrema at Γ₁ and L₁

The first subsidiary minima have L₁ symmetry, the next highest minima with Δ₁(or X₁) symmetry are well removed and will not affect electron distribution. This model is based on theoretical calculations of band structure by Collins et al. (70C1).
Analysis based on three conduction band model, $\Gamma_1$, $\Delta_1$ (or $X_1$) and $L_1$

The experimental results of Hall coefficient and conductivity as a function of temperature were analysed using the three band model. The charge neutrality condition then takes the following form:

$$N_D - N_A = n_1 + n_2 + n_3 - p = n_t$$  \hspace{1cm} V-1

where again subscripts 1, 2, 3 correspond to the $\Gamma_1$, $\Delta_1$ (or $X_1$) and $L_1$ minima respectively. Since all the samples were doped with Te and it has been shown (70P2) that these impurities are associated with a subsidiary band and have ionization energies $\sim 0.02$ eV below the bottom of this band, they will be ionized at all temperatures. Therefore the carrier concentration measured at room temperature is equal to $N_D(\text{Te}) - N_A$, hence the neutrality condition can now be written as:

$$n_t = n_1 + n_2 + n_3 - p$$  \hspace{1cm} V-2

where $n_t = n(300^0\text{K})$. Thus electron distribution in the bands can be calculated provided the necessary band parameters and band shapes are known. Here again, as in GaSb, we shall take account of the non-parabolicity of the $\Gamma_1$ conduction band using the Kane model; the other bands will be considered parabolic. The carrier concentrations $n_1$, $n_2$, $n_3$ and $p$ are then calculated using equations II-71 through II-77 with appropriate parameters which are given in Table V-2. Since there are many values reported in the literature for some of the band parameters, the choice of the parameters listed in Table V-2 will now be discussed.

The value of $m^*_1 = 0.0648m$ was determined by Chamberlain and Stradling (69C1) through cyclotron resonance measurements on very pure expitaxial n-GaAs. It is therefore preferred over other values, e.g. 0.067m obtained using interband magneto-optical measurements (68V1) or 0.064m determined from Faraday rotation measurements at room
temperature (65M1). The effective mass of electrons in the subsidiary minima are not well established to date, (see Chapter I).

The density of states effective mass in the L₁ band of GaAs has not been experimentally determined, we have therefore used values of \( m_{2z}^* \) and \( m_{2t}^* \) calculated by Pollak et al. (66P1) by the \( k.p \) method. These values lead to a density of states effective mass which agrees quite well with the experimental density of states mass for the L₁ band in GaSb and Ge, therefore should be reasonably representative of the density of states mass in the L₁ band of GaAs. The density of states effective mass of 1.2m was determined by Ehrenreich (60E1) for a band with \( \Delta_1 \) symmetry in GaAs. The most recent determination of the heavy hole effective mass was reported by Walton (68W1) and was determined from cyclotron measurements. Panish and Casey (69P1) derived values of \( \sigma_0 \) and \( \theta_0 \) by fitting absorption data over a wide temperature range. To do this a value of the principal gap at absolute zero had to be used. They used a value of 1.522 eV determined by Sturje (62S1) which is very close to the latest and most accurate value of 1.5189 eV reported by Bimberg and Scheirer (72B1). Because of the small difference between these two values of \( E_{\infty} \), it was not considered necessary to refit the data of Panish and Casey using the more accurate values for \( E_{\infty} \). In any case our analyses are not very sensitive to \( E_{\infty} \).

The remaining parameters \( \sigma_2, \theta_2, \sigma_3 \) and \( \theta_3 \), for the temperature coefficients of the energy gaps in the \( <111> \) and \( <100> \) directions, were derived using Walter et al. (70W1) data of the shift with temperature of the \( E_1 \) and \( E_2 \) reflectance peaks. These data were fitted to the Varshni equation II-74 by the least squares method and the results are given in Table V-3. According to Kane (66K1) and Higginbotham et al. (68H2) optical transitions do not occur at band extrema but over a range in \( k \)-space, nevertheless the temperature dependence of the peaks should be reasonably represen-
tative of the relative conduction band motion with respect to the valence band.

Using these values for the \( \alpha_2 \), \( \theta_2 \), \( \alpha_3 \) and \( \theta_3 \) parameters together with those of \( \alpha_0 \) and \( \theta_0 \) given by Panish and Casey, the energy separation \( E_2 \) and \( E_3 \) can be calculated from:

\[
E_2 = E_{20} + \left( \frac{\alpha_2}{\theta_2 + T} - \frac{\alpha_0}{\theta_0 + T} \right) T^2 \tag{V-3}
\]

\[
E_3 = E_{30} + \left( \frac{\alpha_3}{\theta_3 + T} - \frac{\alpha_0}{\theta_0 + T} \right) T^2 \tag{V-4}
\]

knowing \( E_{20} \) and \( E_{30} \). However, neither of these values are known.

Using the value for \( E_{23} = E_2 - E_3 = 0.09 \) eV at 300°K determined by James et al. (68J2), together with equations V-3 and V-4, it is then possible to determine the value of \( E_{20} \) as a function of \( E_{30} \). This then, is the only unknown parameter required to calculate the carrier distribution in the bands when use is made of the neutrality condition. Before it is possible to fit the experimental results of the Hall coefficient and conductivity as a function of temperature, it is further necessary to know the mobilities in the bands and their temperature dependences, as well as the respective scattering parameters. Since the valence and the subsidiary bands are only populated at elevated temperatures where ionized impurity scattering is relatively unimportant, the values of room temperature mobility that are required are those measured on pure samples. The magnitude of the mobility in the \( L_1 \) band of GaAs at room temperature is not known, it will therefore be used as the second adjustable parameter. For the electron mobility in the \( \Delta_1 \) band we used the value of 0.015m²/V.sec. This value was determined by Lees et al. (67L1) and is very close to 0.0145m²/V.sec calculated by Conwell and Vassell (68C1) by combining deformation potential, intraband and interband scattering. The magnitude of hole mobility at
300°K in lightly doped p-type GaAs never exceeds 0.045 m²/V·sec., this value was then employed in our calculation. Since the conductivity will be determined principally by the carriers in the $\Gamma_1$ conduction band, use was made of the experimentally measured conductivity $\sigma_0$ to calculate $\nu_1(T)$ which is given by:

$$\nu_1 = \left( e\sigma_0 - n_2\mu_2 - n_3\mu_3 - p\mu_p \right) / n_1$$  \hspace{1cm} \text{V-5}

It was assumed that since the carriers in the two subsidiary conduction bands and the valence band have large effective masses, the mobility at elevated temperatures in these bands will be limited by acoustic phonon scattering. Hence, for these three bands, mobility will have $T^{-1.5}$ dependence and the scattering parameters are then all equal to 1.18. The temperature variation of the scattering parameter $r_1(T)$ in the $\Gamma_1$ band has been calculated by Ikoma (70II) and measured by Stillman et al. (70S3) for lightly doped GaAs. Both the calculated and experimental results lead to essentially the same variation of $r_1(T)$ of between 1.03 and 1.10 over the temperature range of interest here. To check these values we made Hall coefficient measurements as a function of magnetic field on sample NE22/1 at about -50°C. The results are shown in Fig. V-10; it can be seen that these agree very well with the results of Stillman et al., when account is taken of the experimental temperature. This variation of $r_1(T)$ is small in comparison with $n_1(T)$ and hence to a good approximation it can be assumed that $r_1$ is constant with temperature and has a value of 1.08. All the mobility parameters discussed above are given in Table V-4.

The two unknowns $\mu_2(300\degree\text{K})$ and $E_{30}$ were then used as adjustable parameters in fitting the experimental results of the Hall coefficient temperature variation. The method of fitting is the same as that described in the previous chapter on GaSb. The results of the fitting for the five samples are shown in Table V-5. The smallest
increments used were $\delta E_{30} = 0.001$ eV and $\delta \mu_{20}(300^\circ K) = 0.005 m^2/V\cdot sec$. It is of interest to note that the fitting was sensitive only to the value of $E_{30}$ but not to $\mu_{2}(300^\circ K)$, this was not unexpected since $n_{3}(T)$ would always be higher than $n_{2}(T)$ (Fig. V-3) and with $\mu_{2}(T) > \mu_{3}(T)$ at all temperatures, the $L_{1}$ band would have only a small effect on $R(T)$.

The recent room temperature results of Pitt and Lees (70P2) on Hall effect and conductivity as a function of pressure indicate that the <100> minima lies on the zone boundary and hence have $X_{1}$ symmetry. From these measurements they determined the density of states mass $M_{3d}^* = 0.85 \pm 0.10 m$, the zero pressure mobility $\mu_{3}(300) = 0.033 \pm 0.005 m^2/V\cdot sec.$ and the band separation $E_{3} = 0.38 \pm 0.01$ eV.

Using Pitt and Lees' values of $M_{3d}^*$ and $\mu_{3}(300)$, our Hall effect and conductivity data were refitted, with $E_{30}$ as an adjustable parameter. All the other parameters were the same as used for the previous analysis. However, to obtain an acceptable fit it was found necessary to adjust the temperature dependence of the band separation, $\alpha_{3}$, in addition to $E_{30}$, keeping $\theta_{3}$ constant. To do this and still maintain only two variables (i.e. $E_{30}$ and $\alpha_{3}$) we set $\mu_{2}(300) = 0.065 m^2/V\cdot sec.$ This was considered to be justified since it was found in the previous analysis that the fitting was insensitive to the value of $\mu_{2}$.

The fitted curves are shown in Figs. V-4 through V-8 and the parameters $E_{3}$ and $\alpha_{3}$ resulting from this fitting procedure are tabulated in Table V-6. The curves apply to both the <100> band symmetries, $A_{1}$ and $X_{1}$, since the agreement between the calculated and experimental curves was good in both cases. For the case of minima with $A_{1}$ symmetry, we obtained $E_{30} = 0.38 \pm .004$ eV where we used the value of $\alpha_{3} = -5.45 \times 10^{-4}$ eV/K derived from optical experiments. Since this value of $\alpha_{3}$ could not be used for the minima with $X_{1}$ symmetry, in this case we obtained a value
of $\alpha_3 = -6.0 \times 10^{-4}$ eV/°K and $E_{30} = 0.374$ eV. Comparing the two temperature dependences with that of the principal gap, i.e. $\alpha_0 = -5.8 \times 10^{-4}$ eV, it can be seen that in the first case the separation increases with temperature, while in the second case it decreases. The band separation at absolute zero is also smaller in the second case. This difference is due to the lower density of states mass and higher mobility in the <100> band for the case of minima with $X_1$ symmetry.

V-33 The two band model, $\Gamma_1$ and $\Delta_1$ (or $X_1$)

The two band model proposed by Ehrenreich (60E1) will now be considered. In this case there is only one set of subsidiary minima in the <100> direction.

Using the parameters for the two bands ($\Gamma_1, \Delta_1$) from Table V-2 it was again found necessary to adjust $\alpha_3$ as well as $E_{30}$ to obtain a satisfactory fit to the experimental results of all five samples. We also had to assume a linear temperature dependence for the band separation in order to maintain a maximum of two adjustable parameters. Bearsing this in mind we expected that $E_{30}$ obtained from this fit would be slightly larger than the true value. Good agreement was again obtained between the experimental and the calculated Hall coefficient for all five samples; the fitted curves for two of them are shown in Fig. V-9 and the values of the adjustable parameters, corresponding to the best fit, are given in Table V-7. It can be seen that the most consistent values of $E_{30}$ and $\alpha_3$ are 0.386 eV and $-3.6 \times 10^{-5}$ eV respectively.

It was pointed out in the last section (V-32) that the effect of reducing the density of states effective mass in the subsidiary minima and increasing electron mobility in these minima was to lower the band separation and increase the rate at which the two band minima approach each other with increase in temperature. It was
therefore considered not necessary to repeat the analysis using Pitt and Lees' values of $M_{3d}$ and $\nu_3(300)$ for minima with $X_1$ symmetry, since the results are predictable.

V-34  The two band model, $\Gamma_1$ and $L_1$

The recent calculations of Collins et al. (70C1) give the following values for the band separations: $E_{20} = 0.38$ eV and $E_{30} = 0.82$ eV, and the density of states mass of 1.78m for the lowest subsidiary band, $L_1$. Since $E_{30}$ is large, no electrons can be excited into the $X_1$ minima either thermally or by means of applying a high electric field (Gunn effect). Hence all available electrons would be distributed between the two lowest bands with minima at $\Gamma_1$ and $L_1$. In this case the Gunn effect can only involve the $L_1$ and $\Gamma_1$ bands and one would expect that the threshold voltage for oscillations would therefore increase with uniaxial stress in the <111> direction, because the $L_1$ minima tend to move away from the $\Gamma_1$ minimum under applied stress (68C1). However, this is not found experimentally by Harris et al. (70H1), on the contrary, the threshold remains constant up to about 10 Kbars and then decreases by $\sim 10\%$ when the stress is increased to 20 Kbars. On the other hand Harris et al. show that with <100> stress applied the threshold voltage decreases appreciably and the oscillations are quenched at just below 10 Kbars. In addition, it can be pointed out that the density of states effective mass of 1.78m seems rather high for a band with $L_1$ symmetry when compared to the density of states effective masses in the principal conduction band of Ge and the first subsidiary band of GaSb, both of which have the same symmetry and density of states masses of 0.55m and 0.57m respectively. The foregoing arguments rule out this band model as being representative of the conduction band structure of GaAs.
The temperature variation of the electron mobility of the $\Gamma_1$ band determined from the fitting procedure for the three conduction band model ($\Gamma_1$, $\Delta_1$, $L_1$) is shown in Fig. V-10. This temperature dependence can be explained on the basis of a simple model. As an example, we will take the results for sample M0807/8, Fig. V-11. The compensation in these samples should be negligible, in which case the total ionized impurity concentration will be given by the room temperature carrier concentration, i.e. $N_I = N_D = n_I(300)$. The ionized impurity mobility $\mu_I$ can then be approximately calculated using the Brooks-Herring expression obtained by averaging $\tau_I$ (equation II-25) over the carrier distribution and then applying the quantum correction factors calculated by Moore (67M1). These are given in Moore's paper in Figs. 2 and 4 as a function of both carrier concentration and temperature. From these figures it can be seen that the total correction factor $(\sigma_B + \sigma_M + \sigma_D)$ is about 1.45 and remains fairly constant over the temperature range $20^\circ$C - $600^\circ$C. The mobility $\mu_I$ (equation II-28) is then given by $\mu_I = \mu_I^* / 1.45$, and is given for six temperatures in Table V-8, column 2.

The lattice scattering mobility $\mu_L$ was calculated by Ikoma (70I1) and included polar optical, two phonon resonant scattering and interband scattering. His results are given in column 3 of Table V-8 and apply for temperatures greater than 500 K. In addition we calculated the mobility due to acoustic scattering, $\mu_a$; this is given in the 4th column of Table V-8. The combined mobilities, using Matthiessen's addition rule, are shown in columns 5 and 6 of Table V-8 for several temperatures. The values given in column 5 include $\mu_I$ and $\mu_L$ whereas those of column 6 also include $\mu_a$. The last column gives the mobility results derived for sample M0807/8 by fitting the three conduction band model to the experimental results of Hall coefficient and conductivity. These mobilities are plotted in Fig. V-12 against temperature. As can
be seen from Fig. V-12 the magnitude and temperature dependence of mobility based on this rather approximate calculation agree fairly well with the results obtained from curve fitting. It is evident from Fig. V-12 that ionized impurity scattering plays an important role in determining the mobility of electrons in the $\Gamma_1$ band. It is therefore important to have a proper expression for the relaxation time due to ionized impurity scattering which must take into consideration both the quantum mechanical correction factors discussed by Moore, and the screening due to both light and heavy electrons. It is understandably a difficult problem to carry out these calculations and perhaps for this reason has not been carried out to date. We have used the Moore correction factor of 1.45, but ignored any effect of screening due to heavy electrons in calculating $\mu_1^*$. It was necessary to include the correction factor to lower $\mu_1$ so that reasonable agreement between the calculated values of mobility and those obtained from the fitting procedure could be obtained. Even then, at highest temperatures the calculated $\mu_1$ is greater than the fitted value. This discrepancy could be explained by the fact that our approximate calculations ignored interband scattering between $\Gamma_1$ and $L_1$ bands, since Ikoma used only a two band model ($\Gamma_1$ and $\Delta_1$) in his calculations. It can also be seen from Fig. V-10 that for the heaviest doped sample, MO241/44, the mobility at high temperatures is considerably lower than for the three more lightly doped samples. This suggests that interband scattering may become stronger at these temperatures and would support the existence of the $L_1$ band close in energy to the $\Delta_1$ (or $X_1$) minima. The computer programs written for the calculations of the electron mobility in the $\Gamma_1$ band of GaSb could easily be adopted for similar calculations in GaAs. However, such calculations of the mobility in the $\Gamma_1$ band of GaAs were considered not worth while until the band structure of GaAs has been settled and more parameters have been experimentally determined for the $L_1$ band.
V-5 Magnetoresistance measurements at 400°C

The heaviest doped sample available was chosen for these measurements in order to obtain maximum possible electron transfer into the subsidiary band at a reasonable temperature. The sample used (MO18/60) had a room temperature electron concentration of $7.93 \times 10^{24}$ m$^{-3}$. It was not employed in the determination of band parameters since, as was pointed out earlier, with such high doping the possibility arises of impurity band overlap with the subsidiary band and thus lowers the energy separation. However, this would only increase the number of carriers excited into the subsidiary band and therefore give better conditions for observation of the two band effect. The results of measurements at 400°C are shown in Fig. V-14 where $\rho_o/\Delta\rho$ is plotted against $B^{-2}$. Two points are readily apparent from this Figure:

a) systematic deviation from a straight line behaviour is observed and

b) the intercept at $B^{-2} = 0$ is negative.

Neither the non-linearity of the plot nor the negative intercept can be explained by experimental error and are not predicted by the two band model with the assumption employed. It can be seen that the intercept $u$ given by equation II-64 can not be negative.

Calculation of $\Delta\rho/\rho_o$ at 400°C for sample MO241/44 at a field of 3.0 Wb/m$^2$ gives 0.21% for the two band magnetoresistance. This is less than half the value of the observed magnetoresistance at 3.0 Wb/m$^2$, i.e. $\Delta\rho/\rho_o = 0.60\%$. Although the carrier density in sample MO18/60 is higher than in MO241/44, hence more carriers will be transfered to the subsidiary band, at the same temperature, which would increase $\Delta\rho/\rho_o$ above 0.21%, this increase will not be large enough to give $\Delta\rho/\rho_o$ of 0.60%. A rough estimate of the magnetoresistance due to the two band effect for sample MO18/60 gives a value of 0.25% at 400°C. The observed magnetoresistance of
0.6% must therefore, to a greater part, be due to a contribution from individual bands. Hence, these types of measurements can not be used to determine band parameters in GaAs as was done for GaSb.

V-6 Summary and Discussion

The results of the analyses for the two models of the conduction bands are summarized in Table V-9. It can be seen that in all cases, \( E_{30} \) is \( \approx 0.38 \) eV, the lowest value of 0.374 eV is obtained for the three band model \( (\Gamma_1, \chi_1 \text{ and } L_1) \). This is reasonable since electron mobility given by Pitt and Lees (70P1) for the subsidiary band with \( \chi_1 \) symmetry is double the value calculated for the \( \Delta_1 \) band. Hence, to account for the experimentally observed rise in \( R(T) \) more electrons have to be present in the subsidiary band in the case of \( \chi_1 \) symmetry.

It is quite evident from the results of the analyses that a clear cut choice can not be made as to the correct conduction band structure of GaAs, based purely on analyses of measurements of the Hall coefficient and conductivity as a function of temperature. However, as can be seen from Tables V-5 through V-7, the most consistent results for the five samples are obtained with the three band model with minima having \( \Gamma_1, \Delta_1 \text{ and } L_1 \) symmetries. The results based on this model are shown schematically in Fig. V-15. It should however be pointed out that since some uncertainty exists in the values of several band parameters used in the analyses, the larger scatter in \( E_{30} \) and \( \alpha_3 \) obtained in the other cases does not completely rule them out.
V-7 Conclusion

It has been shown that the Hall effect and conductivity measurements on heavily doped GaAs can not, by themselves, conclusively distinguish between the three conduction band models for GaAs. It is felt though, that the investigation reported here has shed more light on the problem at hand and has allowed a more unified understanding of this part of the field. However, there is a need for more work to help in solving some of the points still not fully understood. The position of the $L_1$ minima is probably the most important of these. This problem can be solved by transport measurements on GaAs$_x$Sb$_{1-x}$ alloys. The $<100>$ bands in the compounds GaAs and GaSb are at about 0.4 and 0.3 eV above the $\Gamma_1$ band, these therefore do not move to any extent throughout the alloy system. On the other hand the $L_1$ band minima in GaSb are about 0.1 eV above the $\Gamma_1$ minimum and about 0.5 eV in GaAs (as suggested by the present work). If this is actually the case, then in alloys with $x$ between 0.3 and 0.5, the $L_1$ minima would lie below the $<100>$ minima, in which case electrons would primarily transfer into these sets of minima when high temperatures, pressures or electric fields are applied. Unfortunately, the problem of growth of these alloys has not been solved to date, therefore, until such time as these alloys are available and measurements of the Hall coefficient and conductivity as a function of temperature are analysed the results of the present work, although not conclusive, give best agreement with available data. Hence, the assignment of the hierarchy of the bands is $\Gamma_1$, $\Delta_1$ and $L_1$. 
TABLE V-1

<table>
<thead>
<tr>
<th>Sample</th>
<th>$T^\circ \text{C}$</th>
<th>$n_1 \text{m}^{-3} \times 10^{-23}$</th>
<th>$\sigma_1 \Omega^{-1} \text{m}^{-1} \times 10^{-4}$</th>
<th>$\mu_1 \text{m}^2/\text{V sec.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MO572/8</td>
<td>20.1</td>
<td>4.25</td>
<td>2.42</td>
<td>.360</td>
</tr>
<tr>
<td>NE22/1</td>
<td>21.5</td>
<td>6.33</td>
<td>3.23</td>
<td>.323</td>
</tr>
<tr>
<td>MO807/8</td>
<td>20.0</td>
<td>12.4</td>
<td>5.86</td>
<td>.295</td>
</tr>
<tr>
<td>MO370/55</td>
<td>20.4</td>
<td>34.3</td>
<td>9.33</td>
<td>.170</td>
</tr>
<tr>
<td>MO241/44</td>
<td>23.0</td>
<td>44.3</td>
<td>15.1</td>
<td>.214</td>
</tr>
<tr>
<td>MO18/60</td>
<td>23.2</td>
<td>79.3</td>
<td>21.70</td>
<td>.166</td>
</tr>
</tbody>
</table>

Table V-1 Room temperature values of $n_1$, $\sigma_1$, and $\mu_1$ for the five samples of n-GaAs.
**TABLE V-2**

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_{10}^*$</td>
<td>0.0648m</td>
<td>(69C1)</td>
</tr>
<tr>
<td>$m_{2t}^*$</td>
<td>0.970m</td>
<td>(66P1)</td>
</tr>
<tr>
<td>$m_{2t}^*$</td>
<td>0.116m, $K_2 = 8.4$</td>
<td>(66P1)</td>
</tr>
<tr>
<td>$m_{3t}^*$</td>
<td>0.88m</td>
<td></td>
</tr>
<tr>
<td>$m_{3t}$</td>
<td>0.23m, $K_3 = 3.8$</td>
<td>(66P1)</td>
</tr>
<tr>
<td>$M_{2d}^*$</td>
<td>0.59m, 4 minima</td>
<td>(66P1)</td>
</tr>
<tr>
<td>$M_{3d}^*$</td>
<td>1.2m, 6 minima</td>
<td>(60E1)</td>
</tr>
<tr>
<td>$m_h^*$</td>
<td>0.50m</td>
<td>(68W1)</td>
</tr>
<tr>
<td>$E_{23}$</td>
<td>0.09 eV at 300°K</td>
<td>(68J2)</td>
</tr>
<tr>
<td>$E_3$</td>
<td>Variable</td>
<td></td>
</tr>
<tr>
<td>$E_{oo}$</td>
<td>1.522 eV</td>
<td>(69P1)</td>
</tr>
<tr>
<td>$\alpha_0$</td>
<td>$-5.8 \times 10^{-4}$ eV °K$^{-1}$</td>
<td>(69P1)</td>
</tr>
<tr>
<td>$\theta_0$</td>
<td>300°K</td>
<td>(69K1)</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>$-6.52 \times 10^{-4}$ eV °K$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>$\theta_2$</td>
<td>214°K</td>
<td></td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>$-5.45 \times 10^{-4}$ eV °K$^{-1}$</td>
<td></td>
</tr>
<tr>
<td>$\theta_3$</td>
<td>208°K</td>
<td></td>
</tr>
</tbody>
</table>

† This value was calculated using Ehrenreich's value of $M_{3d}^* = 1.2m$ and Pollak's value of $m_{3t}^* = 0.23m$.

††† These values were obtained by least squares fitting of optical data reported by Walter et al. (70W1).

Table V-2: List of band parameters for GaAs together with the references from which these were obtained.
<table>
<thead>
<tr>
<th>$E_1$ peak</th>
<th>$T^\circ K$</th>
<th>$E_1$(expt)eV</th>
<th>$E_1$(fit)eV</th>
<th>$\delta$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.015</td>
<td>3.013</td>
<td>-.001</td>
<td></td>
</tr>
<tr>
<td>25</td>
<td>3.014</td>
<td>3.009</td>
<td>-.001</td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>3.010</td>
<td>3.002</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>3.002</td>
<td>2.994</td>
<td>-.002</td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>2.996</td>
<td>2.985</td>
<td>-.001</td>
<td></td>
</tr>
<tr>
<td>125</td>
<td>2.986</td>
<td>2.975</td>
<td>-.001</td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>2.976</td>
<td>2.964</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>175</td>
<td>2.964</td>
<td>2.952</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>2.952</td>
<td>2.940</td>
<td>.001</td>
<td></td>
</tr>
<tr>
<td>225</td>
<td>2.939</td>
<td>2.927</td>
<td>.001</td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>2.926</td>
<td>2.914</td>
<td>.000</td>
<td></td>
</tr>
<tr>
<td>275</td>
<td>2.914</td>
<td>2.901</td>
<td>-.001</td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>2.902</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\alpha_2 = -6.52 \times 10^{-4}$, $\theta_2 = 214$

<table>
<thead>
<tr>
<th>$E_2$ peak</th>
<th>$0$</th>
<th>5.114</th>
<th>5.113</th>
<th>5.113</th>
<th>.000</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>5.113</td>
<td>5.109</td>
<td>.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>50</td>
<td>5.110</td>
<td>5.103</td>
<td>-.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>75</td>
<td>5.105</td>
<td>5.096</td>
<td>-.002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>100</td>
<td>5.098</td>
<td>5.088</td>
<td>-.002</td>
<td></td>
<td></td>
</tr>
<tr>
<td>125</td>
<td>5.090</td>
<td>5.080</td>
<td>.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>150</td>
<td>5.080</td>
<td>5.070</td>
<td>.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>175</td>
<td>5.070</td>
<td>5.061</td>
<td>+.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>200</td>
<td>5.060</td>
<td>5.050</td>
<td>.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>225</td>
<td>5.050</td>
<td>5.040</td>
<td>+.001</td>
<td></td>
<td></td>
</tr>
<tr>
<td>250</td>
<td>5.039</td>
<td>5.029</td>
<td>.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>275</td>
<td>5.029</td>
<td>5.018</td>
<td>.000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>300</td>
<td>5.018</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

$\alpha_2 = -5.45 \times 10^{-4}$, $\theta_3 = 208$

Table V-3: Results of least squares fitting of the optical data for the $E_1$ and $E_2$ peaks as a function of temperature from wavelength modulation reflectance data of Walter et al. (70W1).
### Table V-4

<table>
<thead>
<tr>
<th>Band</th>
<th>Mobility at $300,^\circ K\ m^2/V\cdot Sec.$</th>
<th>Temperature dependence of $\mu$</th>
<th>Scattering parameter</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_{1c} (000)$</td>
<td></td>
<td></td>
<td>1.08</td>
</tr>
<tr>
<td>$L_{1c} &lt;111&gt;$</td>
<td>Variable</td>
<td>$T^{-1.5}$</td>
<td>1.18</td>
</tr>
<tr>
<td>$\Delta_{1c} &lt;100&gt;$</td>
<td>0.015</td>
<td>$T^{-1.5}$</td>
<td>1.18</td>
</tr>
<tr>
<td>$\Gamma_{15V} &lt;000&gt;$</td>
<td>0.045</td>
<td>$T^{-1.5}$</td>
<td>1.18</td>
</tr>
</tbody>
</table>

Table V-4: The mobility, its temperature dependence and scattering parameter used in fitting the GaAs data.
<table>
<thead>
<tr>
<th>Sample</th>
<th>$E_{30}$ eV</th>
<th>$E_{20}$ eV</th>
<th>$\mu_2(300^\circ\text{K})$ $\text{m}^2/\text{V.sec.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>M0241/44</td>
<td>0.380</td>
<td>0.487</td>
<td>.065</td>
</tr>
<tr>
<td>M0370/55</td>
<td>0.381</td>
<td>0.488</td>
<td>.070</td>
</tr>
<tr>
<td>M0807/8</td>
<td>0.380</td>
<td>0.487</td>
<td>.065</td>
</tr>
<tr>
<td>NE22/1</td>
<td>0.384</td>
<td>0.491</td>
<td>.065</td>
</tr>
<tr>
<td>M0572/8</td>
<td>0.381</td>
<td>0.488</td>
<td>.060</td>
</tr>
</tbody>
</table>

Table V-5: Results of fitting to the three conduction band model, $\Gamma_1$, $L_1$ and $\Delta_1$. 
<table>
<thead>
<tr>
<th>Sample</th>
<th>$E_{30}$ eV</th>
<th>$\alpha_3 \text{ eV} \cdot \text{K}^{-1} \times 10^4$</th>
<th>$E_{20}$ eV</th>
</tr>
</thead>
<tbody>
<tr>
<td>MO241/44</td>
<td>0.373</td>
<td>-6.1</td>
<td>0.472</td>
</tr>
<tr>
<td>MO320/55</td>
<td>0.372</td>
<td>-6.1</td>
<td>0.471</td>
</tr>
<tr>
<td>M0807/8</td>
<td>0.374</td>
<td>-6.0</td>
<td>0.473</td>
</tr>
<tr>
<td>NE22/1</td>
<td>0.376</td>
<td>-5.9</td>
<td>0.475</td>
</tr>
<tr>
<td>M0572/8</td>
<td>0.376</td>
<td>-6.0</td>
<td>0.475</td>
</tr>
</tbody>
</table>

Table V-6: Results of fitting to the three conduction band model, $\Gamma_1$, $L_1$ and $X_1$. 
Table V-7: Results of fitting to the two conduction band model $\Gamma_1$ and $\Delta_1$.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$E_{30}$ eV</th>
<th>$\alpha_3$ eV$^oK^{-1}\times10^5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>MO241/44</td>
<td>0.385</td>
<td>-3.5</td>
</tr>
<tr>
<td>MO370/55</td>
<td>0.388</td>
<td>-3.8</td>
</tr>
<tr>
<td>MO807/8</td>
<td>0.386</td>
<td>-3.4</td>
</tr>
<tr>
<td>NE22/1</td>
<td>0.386</td>
<td>-3.6</td>
</tr>
<tr>
<td>MO572/8</td>
<td>0.384</td>
<td>-3.2</td>
</tr>
<tr>
<td>$T^\circ K$</td>
<td>$\mu_1^* m^2/V \cdot sec.$</td>
<td>$\mu_\Lambda m^2/V \cdot sec.$</td>
</tr>
<tr>
<td>-----------</td>
<td>-----------------</td>
<td>----------------</td>
</tr>
<tr>
<td>293</td>
<td>.462</td>
<td>.930</td>
</tr>
<tr>
<td>503</td>
<td>.582</td>
<td>.362</td>
</tr>
<tr>
<td>603</td>
<td>.645</td>
<td>.294</td>
</tr>
<tr>
<td>693</td>
<td>.693</td>
<td>.255</td>
</tr>
<tr>
<td>813</td>
<td>.744</td>
<td>.210</td>
</tr>
</tbody>
</table>

Table V-8: Approximate calculations of mobility $\mu_1(T)$ of the electrons in the $\Gamma_1$ band, for sample M0807/8 of GaAs. The ionized impurity scattering mobility $\mu_1^*$ was calculated using $N_1 = 1.24 \times 10^{24} \text{m}^{-3}$ and Ehrenreich-Moore correction factor of 1.45. The lattice mobility $\mu_\Lambda$ was taken from Ikoma (7011) and acoustic mobility was calculated assuming the deformation potential of 7 eV.
<table>
<thead>
<tr>
<th>Model</th>
<th>Conduction bands</th>
<th>( E_{20} ) eV</th>
<th>( \alpha_2 ) eV ( ^0 K )^{\text{-1}} x 10^-4</th>
<th>( \theta_2 ) (^0 K)</th>
<th>( E_{30} ) eV</th>
<th>( \alpha_3 ) eV ( ^0 K )^{\text{-1}} x 10^-4</th>
<th>( \theta_3 ) (^0 K)</th>
<th>( \mu_2(300) ) m²/V·sec.</th>
<th>( \mu_3(300) ) m²/V·sec.</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( \Gamma_1, \Lambda_1 ) and ( L_1 )</td>
<td>0.488</td>
<td>-6.52</td>
<td>214</td>
<td>.381</td>
<td>-5.45</td>
<td>208</td>
<td>.065</td>
<td>.015</td>
</tr>
<tr>
<td></td>
<td>( \Gamma_1, X_1 ) and ( L_1 )</td>
<td>0.473</td>
<td>-6.52</td>
<td>214</td>
<td>.374</td>
<td>-6.00</td>
<td>208</td>
<td>.065</td>
<td>.035</td>
</tr>
<tr>
<td>2</td>
<td>( \Gamma_1 ) and ( \Lambda_1 )</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>.386</td>
<td>- .35</td>
<td>-</td>
<td>-</td>
<td>.015</td>
</tr>
</tbody>
</table>

Table V-9: Summary of results for the two models of the conduction band structure of GaAs.
**Fig. V-1** A semi-log plot of the experimental Hall coefficient vs temperature for four samples of n-GaAs.
Fig. V-2 A semi-log plot of the experimental conductivity vs temperature for five samples of n-GaAs.
Fig. V-3  A semi-log plot of carrier concentration $n_1$, $n_2$ and $n_3$ in the three conduction minima $\Gamma_1$, $L_1$ and $\Delta_1$, with $\mu_2(300) = 0.065 \text{ m}^2/\text{V.s}$ and $E_{30} = 0.380 \text{ eV}$.
Fig. V-4 The Hall coefficient vs temperature for sample MO572/8 of n-GaAs. The solid line is the fitted curve for the three conduction band model \( \Gamma_1, L_1, \Delta_1 \) (or \( X_1 \)). The crosses are experimental points.
Fig. V-5 The Hall coefficient vs temperature for sample NE22/1 of n-GaAs. The solid line is the fitted curve for the three conduction band model $\Gamma_1$, $L_1$, $\Delta_1$ (or $X_1$). The crosses are experimental points.
Fig. V-6  The Hall coefficient vs temperature for sample MO807/8 of n-GaAs. The solid line is the fitted curve for the three conduction band model $\Gamma_1$, $L_1$, $\Delta_1$ (or $X_1$). The crosses are experimental points.
Fig. V-7  The Hall coefficient vs temperature for sample MO370/55 of n-GaAs. The solid line is the fitted curve for the three conduction band model \( \Gamma_1, L_1, \Delta_1 \) (or \( X_1 \)). The crosses are experimental points.
Fig. V-8  The Hall coefficient vs temperature for sample MO241/44 of n-GaAs. The solid line is the fitted curve for the three conduction band model $\Gamma_1, I_1, \Lambda_1$ (or $X_1$). The crosses are experimental points.
Fig. V-9 The Hall coefficient vs temperature for samples MO807/8 and NE22/1 of n-GaAs. The solid line is the fitted curve for the two conduction model $\Gamma_1$ and $\Delta_1$. 
Fig. V-10 Electron mobility in the $\Gamma_1$ minimum vs temperature obtained by fitting $R(T)$ with the three conduction band model $\Gamma_1$, $L_1$, $\Delta_1$ (or $X_1$).
Fig. V-11 Electron mobility in the $\Gamma_1$ minimum vs temperature for sample MO807/8, obtained by fitting $R(T)$ with the three conduction band model $\Gamma_1$, $L_1$, $\Delta_1$ (or $X_1$).
Fig. V-12  Electron mobility in the $\Gamma_1$ minimum vs temperature for sample MO807/8. (1) Ionized impurity mobility $\mu_I$ (B-H) $N_I = 1.24 \times 10^{24}$ m$^{-3}$. (2) Lattice mobility $\mu_L$ using Ikoma's results. (3) Combined $\mu_I$ and $\mu_L$ mobilities. (4) Combined $\mu_I$, $\mu_L$ and $\mu_a$. 
Fig. V-13: Variation of the Hall scattering parameter, $r_1$, with magnetic field, $B$, for sample NE22/l at -50°C.
Fig. V-14: Variation of $\rho_o/\Delta \rho$ with $B^{-2}$ for sample M018/60, at $400^\circ$C.
Fig. V-15 Schematic of the band structure of GaAs. Includes the previously established parameters as well as those estimated in this thesis.
APPENDIX A

Harland (65H1) made magnetoresistance and Hall coefficient measurements as a function of magnetic field on several n-type GaSb samples at 4.2°K. He analysed these to obtain $R_1$, $R_2$, $\sigma_1$ and $\sigma_2$, from which values he calculated $n_1$, $n_2$, $\mu_1$ and $\mu_2$. His analysis involved expanding the two band Hall coefficient and magnetoresistance in terms of ascending powers of $B^2$ and relating the coefficients to the band parameters $R_1$, $R_2$, $\sigma_1$ and $\sigma_2$. We have re-analysed his data using our method which does not involve the approximation made by expansion in terms of $B^2$, the results are shown in Table A-1, together with Harland's results. It can be seen that the two methods give very similar results for $R_1$, $\sigma_1$ and $\sigma_2$ but the value of $R_2$ obtained by Harland is smaller than that determined by the present method. This would tend to over-estimate the total carrier density. The results based on the present analysis were fitted to determine the value of $E_2$ at 4.2°K, a value of $E_2 = 0.093$ eV was obtained. This essentially is the value at absolute zero, i.e. $E_{20}$. 
Table A-1: Comparison of analysis of Harland's (65H) data of magnetoresistance as a function of magnetic field and Hall coefficient at 4.2°K.

<table>
<thead>
<tr>
<th>Sample</th>
<th>$R_1 \times 10^6$</th>
<th>$-R_2 \times 10^6$</th>
<th>$\sigma_1 \Omega^{-1} \times 10^{-5}$</th>
<th>$\sigma_2 \Omega^{-1} \times 10^{-4}$</th>
<th>$\sigma_2 \Omega^{-1} \times 10^{-5}$</th>
<th>$\sigma_1 \Omega^{-1} \times 10^{-5}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>6</td>
<td>4.817</td>
<td>8.59</td>
<td>.944</td>
<td>2.005</td>
<td>2.007</td>
<td>.955</td>
</tr>
<tr>
<td>7</td>
<td>3.898</td>
<td>5.158</td>
<td>5.158</td>
<td>2.083</td>
<td>2.083</td>
<td>5.158</td>
</tr>
</tbody>
</table>

Table A-1: Comparison of analysis of Harland's (65H) data of magnetoresistance as a function of magnetic field and Hall coefficient at 4.2°K.
REFERENCES


            (London), A130, 499.


1953  (53B1)  L. BRILLOUIN, "Wave Propagation in Periodic Structures",
            (53K1)  C. KITTEL, "Introduction to Solid State Physics", J. Wiley

1955  (55B1)  H. BROOKS, in Advances in Electronics and Electron Physics

1955  (55D1)  R. G. DINGLE, Phil. Mag., 46, 831.


      (S6H1)  C. HERRING and E. VOGT, Phys. Rev., 101, 944.
      (S6K1)  E. O. KANE, Phys. Chem. Sol., 1, 83.

      (S7G1)  LOUIS GOLD and LAURA M. ROTH, Phys. Rev., 107, 358.
      (S7K1)  E. O. KANE, Phys. Chem. Sol., 1, 249.

      (S8S1)  R. J. SLADEK, Phys. Chem. Sol. 5, 155.

1959  (S9E1)  H. EHRENREICH, Phys. Chem. Sol., 9, 129.
      (S9S1)  R. A. SMITH, "Semiconductors", Cambridge University Press.
      (S9Z1)  S. ZWERDLING, B. LAX, K. J. BUTTON and L. M. ROTH,
              Phys. Chem. Sol. 9, 320.

      (S0S1)  A. SAGAR. Phys. Rev., 117, 93.

              32, 2094.
      (S1C1)  M. CARDONA, Phys. Rev., 121, 752.
      (S1E1)  H. EHRENREICH, J. Appl. Phys. Suppl. 32, 2155.

1962

1963
(63S1) M. SHYAM, J. W. ALLEN and G. L. PEARSON, Trans. I.E.E.E., ED-13, 63

1964
(64M1) D. MADELUNG, "Physics of II-V Compounds", John Wiley & Sons, Inc.
(64S1) L. SOSNOWSKI, Proc. of 7th Intr. Conf. on Phys. of Semiconductors, Paris, 341.

1965
(65R1) F. E. ROBERTS, Phys. Letters, 17, 21.
1966


1967


(67V1) Y. P. VARSHNI, Physica, 34, 149.

1968
(68C1) E. M. CONWELL and M. O. VASSELL, Phys. Rev., 166, 797.
(68J1) L. W. JAMES, Ph.D. Thesis, Stanford University.
(68L2) J. LEES, Solid State Comm., 6, 11.
(68S2) R. A. STRADLING and R. A. WOOD., J. Phys., C1, 1711.
(68Z1) H. I. ZHANG and J. CALLAWAY, Solid State Comm., 6, 515.
1969  
(69P2) G. D. PIT, High Temperature-High Pressure, 1, 111.
(69R1) JEAN LOUIS ROBERT, Thesis, University of Montpellier.
(69S1) D. G. SEILER and W. M. BECKER, Phys. Rev., 183, 784.

1970  

(70F1) A. FORTINI, D. DIGNET and J. LUGAND, J. Appl. Phys., 41 3121.


(70S2) N. STELJAN, Rev. Romanian Physics, 15, 153.


(70S4) N. T. SHERWOOD, Ph.D. Thesis, Purdue University.


1971


1972


(72V1) Y. P. VARSHNI, Private communication.