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UMI®
Solutions to Controllability and Observability Problems in Distributed Testing

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A Thesis
Submitted to the School of Graduate Studies and Research of the University of Ottawa in Partial Fulfillment of the Requirements for the Degree of Masters in Computer Science.*

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Abstract

The objective of protocol testing is to confirm that a protocol implementation under test conforms to its specification. However, in protocol test architectures that utilize multiple remote testers in a distributed environment, this objective can be complicated by the fact that testers may encounter problems relating to controllability and observability during the application of a test sequence. Solutions in literature to these problems usually involve first generating a test sequence from the specification of an implementation under test, then inserting coordination messages or appending selected test subsequences that will allow the testers to solve the controllability and observability problems.

This thesis proposes a method that uses a set of transformation rules to construct a directed graph from a given specification. A transition tour of this directed graph based on a rural Chinese postman tour will result in a test sequence with no potential for controllability or observability problems, and where the use of coordination messages is either minimized or, if possible, avoided altogether.
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To Maya, Taro and Shoji

Thanks for being there.
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Chapter One

Introduction

1.1 Background

The objective of protocol testing is to confirm that a protocol Implementation Under Test (IUT) conforms to its specification. [II95] outlines a number of Abstract Test Methods, referred to in this thesis as Abstract Test Architectures, in which an IUT can be tested, and an abstract test case can be specified. Open Distributed Processing (ODP) [IS95] specifies a generic framework for building open distributed systems, and considerable effort has been spent with a view to defining test architectures for ODP implementations.

The majority of the literature published on the topic of testing refers to an IUT that has only one point of control and observation, otherwise known as a "port," which is accessed by one tester. However, the introduction of multiple ports and multiple testers within what is known as a distributed test architecture introduces the possibility of coordination problems among testers during the application of a test sequence, known as problems of controllability or observability. A controllability problem, also known as a synchronization problem, occurs if a tester cannot determine when to apply a particular input to an IUT. The literature deals with this problem by proposing various methods of constructing a synchronizable test sequence, i.e., a test sequence free of any synchronization problems. An observability problem occurs if a tester cannot determine whether a particular output from an IUT was generated in response to the related input.
Solutions to the observability problem vary depending on whether the distributed test architecture supports indirect communication, i.e., when the testers communicate with each other through their interaction with the IUT [II95], or direct communication, when the testers communicate with each other using external coordination messages over a multicast channel [IS95]. In this thesis we propose a method that will solve the controllability and observability problems within a distributed test architecture with multiple testers.

1.2 Motivation and Objectives of the Thesis

The problem that will be studied is, in its most general form, how to detect and eliminate potential controllability and observability problems that may be encountered while testing an implementation of a given Finite State Machine (FSM) that represents the specification of a communication protocol within the ISO/ODP distributed test architecture. Many researchers have studied the problems of controllability and observability, and a review of these studies is included later in this thesis. Among these studies, the observability problem has received relatively less attention than the controllability problem. Those researchers who have studied the observability problem have proposed solutions where first a synchronizable test sequence is generated by applying some test construction method to a given FSM. The synchronizable test sequence is then examined for instances of observability problems and corrective action is taken, depending on whether the test architecture being used supports direct communication between testers through the exchange of external coordination messages, or indirect communication between testers through their interaction with the IUT.
Our review of these solutions has led us to study a method that identifies controllability and observability problems before constructing a synchronizable test sequence, in an effort to minimize the amount of corrective action needed for the elimination of these problems. Thus, the objective of this thesis is to propose a method of generating a synchronizable test sequence whereby the length of the test sequence and the number of corrective actions needed to eliminate controllability and observability problems are minimized, and that can be applied in a distributed test architecture regardless of whether the test architecture supports direct or indirect communication between testers.

1.3 Contributions of the Thesis

We propose a method that generates a minimum-length test sequence with no possibility of potential controllability or observability problems when applied to a 2-port FSM in a distributed test architecture. This method can be used to generate test sequences that can be applied to distributed test architectures where testers are able to communicate amongst themselves directly, via external coordination messages and, with a minor augmentation, can be used to generate test sequences that can be applied in distributed test architectures where testers communicate amongst themselves indirectly, via their interactions with the IUT.

We prove that the proposed method ensures the absence of potential controllability and observability problems.

We show the proposed method to perform at least as well as other related methods proposed in literature, and show it to perform better than each one under certain conditions.
Finally, we extend the proposed method so that it can be applied to an $n$-port FSM in a distributed test architecture, $n > 2$.

1.4 Organization of the Thesis

Chapter 2 outlines the preliminaries needed to describe the proposed method, including the FSM model and the basics of protocol conformance testing. Chapter 3 reviews the existing methods in literature that have been proposed to solve controllability and observability problems. Chapter 4 describes our proposed method and compares it with the existing methods, and Chapter 5 presents our conclusions, with a summary of contributions and directions for future research.
Chapter Two

Preliminaries

2.1 FSM Model and its Graphical Representation

An \( n \)-port Finite State Machine (\( np \)-FSM) \( M = (S, \Sigma, \Gamma, \delta, \lambda, s_0) \) where

- \( S \) is a finite set of states of \( M \),
- \( s_0 \in S \) is the initial state of \( M \),
- \( \Sigma \) is an \( n \)-tuple \( (\Sigma_1, \Sigma_2, \ldots, \Sigma_n) \), where \( \Sigma_k \) is the input alphabet of port \( k \), and \( \Sigma_i \cap \Sigma_j = \emptyset \) for \( i \neq j, i, j = 1, 2, \ldots, n \). Let \( I = \Sigma_1 \cup \Sigma_2 \cup \ldots \cup \Sigma_n \cup \{-\} \), where \( - \) means null input,
- \( \Gamma \) is an \( n \)-tuple \( (\Gamma_1, \Gamma_2, \ldots, \Gamma_n) \), where \( \Gamma_k \) is the output alphabet of port \( k \), and \( \Gamma_i \cap \Gamma_j = \emptyset \) for \( i \neq j, i, j = 1, 2, \ldots, n \). Let \( O = \{<a_1, a_2, \ldots, a_n> \mid \exists a_i \in \Gamma_i \cup \{-\}, 1 \leq i \leq n\} \), where \( - \) means null output,
- \( \delta \) is the transition function that maps \( S \times I \) to \( S \), i.e., \( \delta : S \times I \rightarrow S \), and
- \( \lambda \) is the output function that maps \( S \times I \) to \( O \), i.e., \( \lambda : S \times I \rightarrow O \).

An FSM \( M \) is deterministic if, for each input \( x \in I \), there is at most one transition defined at each state of \( M \). An FSM \( M \) is said to be minimal if none of its states are equivalent (i.e., \( \forall s_i, s_j \in S, s_i \neq s_j, \exists \) an input sequence \( x \in I^* \) such that \( \lambda(s_i, x) \neq \lambda(s_j, x) \)). An FSM \( M \) is said to be completely specified if, for each input \( x \in I \), there is a transition defined at each state of \( M \).

An FSM \( M \) can be represented by a directed graph \( G = (V, E) \) where a set of vertices \( V \) represents the set \( S \) of states of \( M \), and a set of directed edges \( E \) represents all specified transitions of \( M \). A transition of an \( np \)-FSM \( M \) is a 3-tuple \( t_r = (s_r, s_i, x/y) \), where \( s_r, s_i \in S, x \in I, \) and \( y \in O \) such that \( \delta(s_r, x) = s_i, \lambda(s_r, x) = y, \) and \( x/y \) is known as an
input/output pair. Each edge \( e_k = (v_j, v_i; x/y) \in E \) represents a state transition from state \( v_j \) to state \( v_i \) with input \( x \) and output \( y \) where the input/output pair \( x/y \) is the label of \( e_k \). denoted by \( label(e_k) \), \( v_j \) is called the head of \( e_k \), denoted by \( head(e_k) \), and \( v_i \) is called the tail of \( e_k \), denoted by \( tail(e_k) \).

A path \( P = (v_1, v_2; x_1/y_1)(v_2, v_3; x_2/y_2)\ldots(v_{k-1}, v_k; x_{k-1}/y_{k-1}), k \geq 1 \) in \( G = (V, E) \) is a finite sequence of adjacent (but not necessarily distinct) edges in \( G \), where \( v_1 \) and \( v_k \) are head\((P)\) and tail\((P)\), and \( x_1/y_1, x_2/y_2, \ldots, x_{k-1}/y_{k-1} \) is label\((P)\). The cost or length of each edge of \( G \) is equal to the number of input/output pairs in its label. The cost of a path (or length of a path) \( P \) in \( G \) is the sum of the costs (or lengths) of edges included in \( P \).

A digraph \( G = (V, E) \) is strongly connected if, for any pair of vertices \( v_j \) and \( v_k \), there exists a path from \( v_j \) to \( v_k \). It is weakly connected if its underlying undirected graph is connected. A tour of \( G \) is a path in \( G \) that starts and ends at the same vertex of \( G \). An Euler tour of \( G \) is a tour that contains every edge of \( E \) exactly once. A postman tour (PT) of \( G \) is a tour that contains every edge in \( E \) at least once. A rural postman tour (RPT) of \( G \) over a set \( E_C \subseteq E \) is a tour traversing every edge in \( E_C \) at least once. A Chinese postman tour (CPT) is a minimum-cost tour that contains every edge in \( E \) at least once. A rural Chinese postman tour (RCPT) of \( G \) over a set \( E_C \subseteq E \) is a minimum-cost tour that traverses every edge in \( E_C \) at least once. The edge-induced subgraph of \( G[EC] \) is the subgraph of \( G \) whose vertex set is the set of ends of edges in \( E_C \) and whose edge set is \( E_C \). \( G[EC] \) is an edge-induced spanning subgraph of \( G \) if its vertex set is \( V \).

Given a vertex \( v \in V \), the in-degree of \( v \), \( d_i(v) \), is defined as \(|\{(u, v): (u, v) \in E\}|\) and the out-degree of \( v \), \( d_o(v) \), is defined as \(|\{(v, w): (v, w) \in E\}|\). A digraph \( G = (V, E) \) is
symmetric if \( d_i(v) = d_j(v), \forall v \in V \). Given a postman tour \( P \) of \( G \), let \( \chi(v_p, v_i; x/y) \geq 1 \) be the number of times edge \((v_p, v_i; x/y)\) is contained in \( P \). If edge \((v_p, v_i; x/y)\) is replicated \( \chi(v_p, v_i; x/y) \) times, a symmetric graph \( G^\wedge \), called a symmetric augmentation of \( G \), is obtained and \( P \) is an Euler tour of \( G^\wedge \). A min-cost symmetric augmentation of \( G \) is achieved by replicating edges of \( G \) such that \( G^\wedge \) is symmetric and the number of replicated edges are minimized.

2.2 Protocol Conformance Testing

The aim of protocol conformance testing is to determine, by means of testing, if a protocol implementation under test (IUT) conforms to its specification [IS94]. Typically, protocol conformance testing involves constructing a test sequence from the protocol specification, applying the test sequence to the IUT, and analyzing the result of the application of the test sequence to determine whether the IUT conforms to its specification. A test sequence is either a sequence of test cases (pairs of test input and expected output) or a collection of subsequences of test cases, in which case it is referred to as a test suite. The application of a test sequence is carried out in a test architecture that consists of testers and an IUT. ISO and ODP define a collection of abstract test architectures.

An Abstract Test Architecture (ATA), called an Abstract Test Method (ATM) in the context of the OSI model, is a description of how an IUT which is in a System Under Test (SUT) is to be tested, given at an appropriate level of abstraction to make the description independent of any particular realization. It must, however, include enough detail to enable abstract test cases to be specified for this test architecture [II95]. An
ATA in the context of a $2p$-FSM is defined in terms of the relationship between the IUT and the Upper and Lower Testers, through:

**Points of Control and Observation (PCO),** or points within a testing environment where the occurrence of test inputs and actual outputs is to be controlled and observed, as defined in an ATA [II95],

**Abstract Service Primitives (ASPs),** or implementation independent descriptions of an interaction between a service user and a service provider at a service boundary, as defined in an OSI service definition [II95], and

**Protocol Data Units (PDUs),** or information that is delivered as a unit among peer entities of a network and that may contain control information, address information, or data.

The rules for cooperation between Upper and Lower Testers are defined in the applicable Test Coordination Procedures (TCPs) for each architecture. These ATAs have analogous models in other architectures such as ODP [IS95], [FK95], [RA95]. Within an ATA, an abstract test suite (ATS) is defined as a test suite composed of abstract test cases, the complete and independent specifications of the actions of the testers required to achieve a specific test purpose.

The Upper Tester (UT) is defined as the representation of the means of providing, during test execution, control and observation of the upper service boundary of the IUT, as defined by the chosen ATA. The Lower Tester (LT) is defined as the representation of the means of providing, during test execution, indirect control and observation of the lower service boundary of the IUT via the underlying service provider. The ISO ATAs are defined as follows [II95]:

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- **Local** — an ATA in which both the Upper and Lower Testers are located within the Test System and there is a PCO at the upper service boundary of the IUT (Figure 1). The local test architecture requires the upper service boundary of the IUT to be a standardized hardware interface. In the ODP model, this ATA is called **centralized**, which refers to one tester.

![Figure 1. Local Test Architecture](image)

- **Coordinated** — an ATA in which the Upper Tester is within the SUT and for which a standardized Test Management Protocol is defined for the Test Coordination Procedures, enabling the control and observation to be specified solely in terms of the Lower Tester Activity. This includes the control and management of Test Management PDUs (Figure 2).
**Distributed** – an ATA in which the Upper Tester is within the SUT and there is a PCO at the upper service boundary of the IUT (Figure 3). The upper service boundary of the IUT may be either human user interface or standardized programming language interface. While requirements for TCP are specified, procedures are not.

**Remote** – an ATA in which the control and observation of test inputs and outputs is specified solely in terms of Lower Tester activity (Figure 4). Some requirements for Test
Control Procedures may be implied or informally expressed in the Abstract Test Suite, but no assumption is made regarding their feasibility or realization. There is no Upper Tester as such, but some Upper Tester functions may be performed by the SUT.

![Remote Test Architecture Diagram]

Figure 4. Remote Test Architecture

Among the ATAs listed above, an IUT has the highest observability and controllability in the local test architecture, and the lowest one in the remote test architecture. If a set of detectable faults \( \{S_r, S_d, S_c, S_i\} \) are associated with each test architecture (remote, distributed, coordinated, and local, respectively) the fault detectability in these architectures is shown to be the following [LB94]:

\[
S_r \subseteq S_d \subseteq S_c \subseteq S_i
\]

In this thesis, we will consider a sequence of test cases, i.e., a test sequence, rather than a collection of subsequences of test cases, i.e., a test suite, in order to compare our method with the existing methods.

A **global test sequence** \( \omega \) of an \( np \)-FSM \( M \) is the label of a path of the digraph \( G = (V, E) \) representing \( M \), which is of the form \( x_1/y_1, x_2/y_2, \ldots, x_m/y_m \), where \( x_i \in I \) and
\( y_i \in O, 1 \leq i \leq m \). Given a global test sequence \( \omega \), a **local test sequence** \( \omega_k \) for port \( k \) of an np-FSM \( M \) is of the form \( \alpha_1, \alpha_2, \ldots, \alpha_m \), where \( \alpha_i (1 \leq i \leq m) \) is either:

- \( x_i/y_i, x_i \in \Sigma_i \cup \{-\}, y_i = \langle a_1, a_2, \ldots, a_i \rangle \), where \( a_i \in \Gamma_i \cup \{-\} \) and \( a_j = - \), for \( j \neq k \),

  \( 1 \leq j, k \leq n \), or

- an external coordination message reception or transmission to ensure observability or controllability.

Note that \( a_i \in \Gamma_i \cup \{-\} \) together with \( x_i \in \Sigma_i \cup \{-\} \) allows \(-/-\) transitions within a local test sequence \( \omega_k \) for port \( k \) of an np-FSM. In an ATA involving multiple testers, each tester executes a local test sequence constructed from the global test sequence of a given FSM \( M \). In the local test sequence, tester \( k \) knows only that there are transitions involving itself or others, and doesn’t know the inputs or outputs of the other testers.

A **fault model** is defined as a set of models of non-conforming implementations of a given FSM \( M \) [IS94]. For a particular specification, two types of fault models may be recognized – fault models with:

- **transfer faults**, i.e., the final state of a tested transition of an implementation may be different from the final state specified by the specification, and

- **output faults**, i.e., the observed output of an implementation after a specified input may not be the one specified by the specification.

A **hybrid fault** is defined as a combination of the above.

**Controllability** refers to the ease of affecting the specified outputs. **Observability** refers to the ease of determining if specified inputs affect the outputs. **Detectability** refers to the ease of detecting a fault.
As mentioned earlier, some ATAs may cause controllability or observability problems, depending on the ability of the testers to coordinate the application of their respective local test sequences. It is said that a controllability (synchronization) problem exists when a tester is required to send an input to the IUT in the current transition, and because it is not involved in the previous transition, i.e., it didn’t send the input or receive the output in the previous transition, it does not know when to send the input. An observability problem exists when a tester is expecting to receive an output from the IUT and, because it did not send the input in the current transition nor did it receive an output in the previous transition, it does not know when to start or stop waiting for the reception of the output of the IUT.

Given an np-FSM \( M \) and a global test sequence \( \omega = x_1/y_1, x_2/y_2, \ldots, x_m/y_m \) of \( M \), where \( x_i \in I \) and \( y_i \in O, 1 \leq i \leq m \), a synchronization problem occurs when, in the labels \( x_j/y_j \) and \( x_{j+1}/y_{j+1} \) of any two consecutive transitions, there exists a tester \( k \) that sends \( x_{j+1} \) that is neither the one sending \( x_j \) nor one of those receiving an output belonging to \( y_j \), \( 1 \leq j \leq m-1 \).

Given an np-FSM \( M \) and a global test sequence \( \omega = x_1/y_1, x_2/y_2, \ldots, x_m/y_m \) of \( M \), where \( x_i \in I \) and \( y_i \in O, 1 \leq i \leq m \), any two consecutive transitions \( t_{ij} \) and \( t_{ik} \) whose labels are \( x_j/y_j \) and \( x_{j+1}/y_{j+1} \) form a synchronizable pair of transitions if \( t_{ik} \) can follow \( t_{ij} \) without causing a synchronization problem. For a transition \( t_{ij} = (v_i, v_j; x_j/y_j) \), each transition \( t_{ik} = (v_i, v_k; x_j/y_k) \) that forms a synchronizable pair of transitions with \( t_{ij} \) is called an eligible successor of \( t_{ij} \). Any (sub)sequence of transitions in which every pair of transitions is synchronizable is called a synchronizable transition (sub)sequence. A global test sequence is said to be synchronizable if it is the label of a synchronizable transition sequence.
Given an np-FSM $M$ and a global test sequence $\omega = x_1/y_1, x_2/y_2, \ldots, x_m/y_m$ of $M$, where $x_i \in I$ and $y_i \in O$, $1 \leq i \leq m$, a **1-shift output fault** in an implementation $N$ of $M$ exists when, in the labels $x/y_j$ and $x_{j+1}/y_{j+1}$ of any two consecutive transitions, there exists one $a_k$ in $y_j = <a_1, a_2, \ldots, a_n>$ of $M$ which occurs in $y_{j+1} = <a_1, a_2, \ldots, a_n>$ in $N$ (and not in $y_j$ in $N$) or there exists one $a_k$ in $y_{j+1} = <a_1, a_2, \ldots, a_n>$ of $M$ which occurs in $y_j = <a_1, a_2, \ldots, a_n>$ in $N$ (and not in $y_{j+1}$ in $N$), $1 \leq j \leq m-1$, $1 \leq k \leq m$. An instance of the observability problem manifests itself as an **undetectable 1-shift output fault** if there is a 1-shift output fault related to $a_k \in \Gamma_k$ in any two consecutive transitions whose labels are $x/y_j$ and $x_{j+1}/y_{j+1}$, such that tester $k$ satisfies the condition ($a_k$ is in $y_j$ XOR $a_k$ is in $y_{j+1}$) AND $x_{j+1} \notin \Sigma_k$. In this case, we say that tester $k$ is **involved** in the shift, and would not be able to detect it.
Chapter Three

Review of Test Coordination Methods

3.1 Synchronizable Test Sequence Generation

Earlier work on test coordination has focused mainly on defining methods for the generation of synchronizable global test sequences. In [SB84], Sarikaya and Bochman were the first team of researchers to formally address the synchronization problem and to propose a method for constructing synchronizable global test sequences from a given FSM. In general terms, each transition or test subsequence that is added to the global test sequence in the course of following a transition tour of the digraph of an FSM is examined in order to see if it is synchronizable with its predecessor. Non-synchronizable transitions or subsequences are discarded, and a different one from the present state is considered. This process is continued until all the transitions of the FSM are covered, or it is decided, through backtracking, that there is no synchronizable global test sequence for the given FSM. It is accepted in [SB84] that this method may result in a synchronizable global test sequence that is not of minimum length. Sarikaya and Bochman also discuss the concept of protocol specifications that are intrinsically nonsynchronizable. A protocol specification is intrinsically nonsynchronizable if it contains a transition that does not have an eligible successor. They consider the X.25 DTE protocol as an example of an intrinsically nonsynchronizable protocol, and propose a procedure that would enhance such a protocol for the purpose of testing. This is done
by modifying the state table of the protocol so that a synchronizable global test sequence can be found.

In [BU91], Boyd and Ural give an efficient algorithm to determine the existence of a synchronizable global test sequence for a given FSM. They also analyze the computational complexity involved in constructing a minimum length synchronizable global test sequence from a given $2p$-FSM, and conclude that it would be unlikely that a polynomial time algorithm will be found. Specifically, they define two problems that are based on the following definitions: a digraph $G = (V, E)$ is said to be order-specified if for each edge $e_{ij} \in E$, a subset of the edges leaving vertex $v_i$ is specified as eligible successors of $e_{ij}$, and a path in $G$ is said to be correctly ordered if for every consecutive pair of edges $e_{jk} = (v_j, v_k; x_j/x_k)$ and $e_{kl} = (v_k, v_{l}; x_k/x_{l})$, $e_{kl}$ is an eligible successor of $e_{jk}$.

Then, given an order-specified digraph $G = (V, E)$ and a specified vertex $v \in V$, the **Ordered Euler Tour Problem** (OETP) is defined as determining whether there exists a correctly ordered Euler tour of $G$ that starts at $v$. Given an order-specified digraph $G = (V, E)$, a specified vertex $v \in V$ and a cost function defined on $E$, the **Ordered Chinese Postman Problem** is defined as finding a minimum cost correctly ordered postman tour of $G$ that starts at $v$. Boyd and Ural prove that the OETP and the OCPP are both NP-complete problems. Their work provides justification for heuristic techniques for generating synchronizable global test sequences that are not necessarily of minimum length.

In [UW93], [GU95], and [CU95], the authors present methods for constructing synchronizable global test sequences. In general, these methods involve the modification of a digraph of an FSM such that a tour of the modified graph is a synchronizable global
test sequence. In [UW93], if a given 2p-FSM $M$ can be represented by an order specified
digraph $G = (V, E)$, then a synchronizable global test sequence may be generated from $G$
by constructing a digraph $G' = (V', E')$, identifying test subsequences, adding these test
subsequences to $G'$ to form $G^\wedge = (V^\wedge, E^\wedge)$ and finding an RPT in $G^\wedge$ over the set of edges
representing the test subsequences. [GU95] uses the concept of a correctly ordered
digraph and a given set of test subsequences to construct a synchronizable global test
sequence. This sequence can then be minimized by the elimination of redundant
subsequences and judicious choice of transfer sequences and test subsequences. [CU95]
also constructs a digraph $G' = (V', E')$ by adding the set $E_c$ of all test subsequences in a
given FSM $M$ to $G = (V, E)$, and finding an RCPT of $G'$ over $E_c$. Chen and Ural show
that if the edge induced spanning subgraph $G[E_c]$ of $G'$ is weakly connected, then
finding an RCPT of $G'$ over $E_c$ can be achieved in polynomial time. This is justified as
follows: in [AA88], Aho et al utilize the Rural Chinese Postman Problem to find a
rural postman tour of a digraph with minimum cost. Although computing such a tour is
known to be NP-complete, the authors present a class of graphs for which a minimum
cost global test sequence can be constructed in polynomial time. Specifically, they prove
that if an edge-induced subgraph $G[E']$ of a digraph $G = (V, E)$, $E' \subset E$, is a weakly
connected spanning subgraph of $G$, then any rural symmetric augmentation graph $G^\wedge$ of
$G$ is strongly connected and therefore has an Euler tour. When $G[E']$ is weakly
connected, an RCPT over $E'$ can therefore be found in polynomial time.

[BU91], [UW93], and [GU95] point out that not all FSMs will yield synchronizable
global test sequences using their methods, as they make the assumption that the testers
may only communicate with each other through the IUT. As well, it is recognized that some FSMs are intrinsically nonsynchronizable. [CU95] supports the use of an additional communication channel that allows the exchange of external coordination messages amongst the testers, and includes the cost of those messages in their minimization algorithm. The use of external coordination messages between testers hence can be gainfully used to construct a synchronizable global test sequence from any FSM, including the ones that are intrinsically nonsynchronizable.

In [TY98], Tai and Young introduce a definition of synchronizable that is slightly different than the one presented in this thesis and the other earlier work in the literature, a definition that depends on the current state and eligible transitions of an IUT as well as the transitions in the local test sequences. Specifically, a global test sequence for an FSM $M$ is said by Tai and Young to be synchronizable if any execution of $M$ and the testers according to the global test sequence is deterministic (i.e., the current state of $M$ has at most one eligible transition). The synchronization problem as it is defined in this thesis is referred to as the \textbf{pair-wise synchronization problem} by Tai and Young. [TY98] considers the testing strategy of port-based testing, which allows testers to communicate with each other only indirectly via their interactions with the IUT. Tai and Young also expand on the idea of port-based testing by introducing the concept of group-based testing, which divides the ports of an IUT into groups and allows the testers for ports in the same group to communicate with each other directly using external coordination messages.
3.2 Observability Problems in Global Test Sequences

Although some research has been done into the issue of observability problems in distributed testing, it has received much less attention than the problem of synchronization. The methods proposed by [CR99] and [LB94], described in detail in the next two sections, involve generating a global test sequence from the original digraph of an FSM, and inserting corrective actions into the global test sequence to ensure that it is free from controllability and observability problems. The most significant difference between the two methods is in their treatment of the interaction between testers. In [LB94], direct communication between the testers is not supported, and testers must communicate indirectly through the IUT. In [CR99], direct communication is allowed through the use of a multi-cast channel connecting the testers to each other, which allows for the exchange of external coordination messages specifically related either to observability or controllability.

In [YT98], Young and Tai describe three types of implementation faults that cause incorrect test observations. Given an FSM $M$, an implementation $N$ of $M$, and a global test sequence $\omega$, $N$ is said to have an input exchange fault with respect to $\omega$ if, for two transitions $t_i$ and $t_u$ in $\omega$, $N$ is modified from $M$ by exchanging the input symbols of $t_i$ and $t_u$. $N$ is said to have a forward output shifting fault with respect to $\omega$ if $N$ is modified from $M$ by removing an output symbol from $t_i$ and adding it to $t_u$, provided the input symbol of $t_u$ is associated with a different port than the port of the output being shifted. $N$ is said to have a backward output shifting fault with respect to $\omega$ if $N$ is modified from $M$ by removing an output symbol from $t_u$ and adding it to $t_i$, provided the input symbol of $t_u$ is associated with a different port than the port of the output being shifted. They
propose a strategy for solving the observability problem as follows: given a global test sequence consisting of transitions \( t_1, t_2, t_3, \ldots, t_m \) for a given FSM \( M \) and the implementation \( N \) of \( M \), first test \( N \) by using \( t_1 \), \( t_2 \), then \( t_1, t_2, t_3 \), and so on. The intent of this strategy is to validate the transitions in the global test sequence one at a time in the given order, and assumes a test architecture whereby the testers communicate amongst themselves indirectly via their interactions with the IUT.

In [HR00], Hieron proposes a method of augmenting the digraph \( G \) of an FSM \( M \) by adding edges that represent test subsequences, then finding a minimal length tour of the augmented digraph that contains each test subsequence. Hieron also discusses the concept of forward output-shifting faults and backward output-shifting faults, and proposes two types of observability-related coordination messages that can be used to detect these faults. Given two transitions \( t_j \) and \( t_k \), a **post-transition framing message** is sent to the tester sending the input to the IUT in \( t_k \) from each tester receiving an output from the IUT in \( t_j \), in order to preclude any possibility of a forward output shifting fault from \( t_j \) to \( t_k \). If a tester is receiving an output in \( t_j \) and sending an input in \( t_k \), no post-transition framing message for that tester is required. Given two transitions \( t_j \) and \( t_k \), a **pre-transition framing message** is sent from the tester sending the input to the IUT in \( t_k \) to each tester receiving an output from the IUT in \( t_j \), in order to preclude any possibility of a backward output-shifting fault from \( t_k \) to \( t_j \). Hieron's method assumes a test architecture whereby testers may communicate directly amongst themselves using external coordination messages.
3.3 General Distributed Test Architecture

![Diagram of General Distributed Test Architecture]

**Figure 5. General Distributed Test Architecture**

In the general distributed test architecture depicted in Figure 5, there are several testers located at different sites within the test system. However, testers communicate and synchronize with one another indirectly, via their interactions with the IUT. When \( n = 2 \), this model is reduced to the ISO Distributed Test Architecture.

Because no external coordination messages are possible in this architecture, [LB94] solves the problem of controllability by constructing a synchronizable global test sequence for an FSM as follows:

- for each transition \( t \) to be added to the global test sequence constructed so far, check to see if there is a synchronization problem with the previous transition of the sequence.
- if no, add \( t \) to the global test sequence and carry on.
- if yes, consider a different transition from the current state.
- if problem still exists, backtrack to the previous state and carry on.
Note that, given a transition, there may not exist any synchronizable sequence that can force the machine to traverse this transition. If this is the case, stop.

Consider the 2p-FSM $M_1$ in Figure 6. By applying the above steps, we can construct a synchronizable global test sequence:

$$\Pi = label(t_1, t_3, t_2, t_4),$$

i.e., $0/b, a>$, $1/c$, $1/b, ->$, $0/b, ->$, which solves the controllability problem.

![Figure 6. Digraph $G = (V, E)$ of 2p-FSM $M_1$](image)

The observability problem is defined by [LB94] as follows: for two adjacent transitions $t_x$ and $t_y$ in a given specification $M$, a faulty implementation $N$ can be obtained from $M$ by removing an output from one of these two transitions and adding the output to the other transition. [LB94] calls this class of output faults output shifting faults (referred to in this thesis as 1-shift output faults), and proposes an approach that adds specific transitions to the synchronizable global test sequence, with the intent of detecting all 1-shift output faults during the application of the sequence in the distributed test architecture. [LB94] proposes the following algorithm to accomplish this:

- generate a synchronizable global test sequence $\Pi$ by using one of the test generation methods for FSMs,
- find a set $\Omega$ of all consecutive transition pairs along the path caused by applying $\Pi$, where each pair may have a potential undetectable 1-shift output fault,
- if $\Omega$ is empty, stop. Otherwise, add a set of additional synchronizable test subsequences to $\Pi$ such that $\Pi$ can ensure the absence of potential undetectable 1-shift output faults in the transition pairs of $\Pi$.

Figure 7 shows a faulty implementation of the $2p$-FSM $M1$ in Figure 6, where a 1-shift output fault has caused the output $b$ intended for tester 1 in transition $t1$ to shift to transition $t3$. This fault is not detectable using the synchronizable global test sequence $\Pi = 0/b, a>, 1/<-, c>, 1/<b, ->, 0/<b, ->$. This is because the Upper Tester only knows that its second interaction with the IUT after sending the input “0” is the receipt of the output “b”, and has no way of knowing whether that output was generated by transition $t1$ (as is required by the specification) or transition $t3$ (as occurs in the faulty implementation). In either case, the Upper Tester will not detect the fault. As the Lower Tester is not able to observe the output “b”, it will also not detect the fault.

![Figure 7. Faulty Implementation of 2p-FSM M1](image)

To detect the fault in the example shown in Figure 7, and any others, the synchronizable global test sequence $\Pi$ is examined for instances of potential undetectable
1-shift output faults. In this example there are two such faults: the potential shift of output \( b \) from \( t1 \) to \( t3 \), and the potential shift of output \( b \) from \( t2 \) to \( t3 \). In each instance, additional synchronizable test subsequences are appended to \( \Pi \):

- adding \( 0/b,a \), \( 0/b,\rightarrow \) will detect a 1-shift output fault from \( t1 \) to \( t3 \). This is because the Upper Tester will not send the input “0” in transition \( 0/b,\rightarrow \) until it receives the output “\( b \)” in transition \( 0/b,a \). If that output has shifted, the Upper Tester will therefore detect the shift.

- adding \( 0/b,a \), \( 1/-,c \) will detect a 1-shift output fault from \( t2 \) to \( t3 \). This is because the Upper Tester is not expecting the second output “\( b \)” in transition \( 1/-,c \) from the IUT after it sends the input “0”. If that output has shifted, and given that \( 1/-,c \) is the last transition in the appended synchronizable global test sequence, the Upper Tester will therefore detect the shift.

The addition of the synchronizable test subsequences results in a new synchronizable global test sequence of:

\[
0/b,a,1/-,c,1/b,\rightarrow,0/b,\rightarrow,0/b,a,0/b,\rightarrow,0/b,a,1/-,c,\
\]

in which the potential undetectable 1-shift output faults in the original synchronizable global test sequence are rendered detectable. Note that appending additional test subsequences has resulted in three additional potential undetectable 1-shift output faults. Specifically, there is the potential for the output “\( a \)” to shift from \( 0/b,a \) to \( 0/b,\rightarrow \) undetected, as shown below:

\[
0/b,a,1/-,c,1/b,\rightarrow,0/b,\rightarrow,0/b,a,0/b,\rightarrow,0/b,a,1/-,c.\
\]

However, the appended transitions have been selected in such a manner that these additional potential undetectable 1-shift output faults are rendered detectable by the
transition pair 0/\langle b, a \rangle, 1/\langle -, c \rangle in the original synchronizable global test sequence. This situation will be examined in more detail in Section 4.4.

3.4 ODP Distributed Test Architecture

![Figure 8. ODP Distributed Test Architecture](image)

The ODP distributed test architecture, depicted in Figure 8, involves several testers located at different sites within the test system. Direct communication between testers is supported by a multicast channel.

![Figure 9. Digraph G = (V, E) of 2p-FSM M2](image)

\[ \Sigma_U = \{a\}, \Sigma_L = \{b\} \]
\[ \Gamma_U = \{a\}, \Gamma_L = \{b\} \]

\[ t_1 = a/\langle a, - \rangle \]
\[ t_2 = a/\langle a, - \rangle \]

\[ t_3 = b/\langle - , b \rangle \]

\[ t_4 = b/\langle - , a \rangle \]
In [CR99], the controllability (synchronization) problem is demonstrated in the 2p-FSM $M_2$ of Figure 9. In this example, the label $(a/\langle a, \rightarrow, a/\langle a, \rightarrow, \beta/\langle, b\rangle, \beta/\langle a, \rightarrow\rangle)$ of the transition tour $TT = t_1, t_2, t_3, t_4$ is a global test sequence of $M_2$. However, a synchronization problem exists between transitions $t_2$ and $t_3$. Specifically, in transition $t_3$, the Lower Tester is required to send an input $\beta$ to the IUT, but is not involved in transition $t_2$, and therefore does not know when to send its input. It is important to note that since $t_3$ causes a synchronization problem whether it follows $t_1$ or $t_2$, there is no synchronizable global test sequence for this FSM without external coordination between testers.

Recall from Section 2.2 that, given an $np$-FSM $M$ and a global test sequence $\omega = x_i/y_i$, $x_2/y_2$, \ldots, $x_m/y_m$ of $M$, where $x_i \in I$ and $y_i \in O$, $1 \leq i \leq m$, a synchronization problem occurs when in the labels $x_i/y_i$ and $x_{i+1}/y_{i+1}$ of any two consecutive transitions, there exists a tester $k$ that sends $x_{i+1}$ that is neither the one sending $x_i$ nor one of those receiving an output belonging to $y_j$, $1 \leq j \leq m-1$. In the algorithm proposed by [CR99] to eliminate controllability problems, tester $h$ can be the tester that receives an output from the IUT in $y_j$ or, in the case where there exists no output from the IUT in $y_j$, tester $h$ is the tester that sends an input to the IUT in $x_j$. For the purposes of this explanation we will assume tester $h$ is the tester sending an input to the IUT in $x_j$.

The solution proposed by [CR99] to the synchronization problem, one that is reflected at least in some degree in the solutions of [BM99] and [HR00], is to insert the external coordination message exchange relating to controllability "$<-C_h, +C_i>$" between $x_j/y_j$ and $x_{j+1}/y_{j+1}$ in the global test sequence, which will result in the insertion of:
- "-Cᵢ" in the local test sequence of tester h just after sending the input xᵢ to the IUT to indicate that it is to send an external coordination message to tester k, and

- "+Cᵦ" in the local test sequence of tester k just before sending the input xᵦ, to the IUT, to indicate that it is to receive an external coordination message from tester h telling tester k it is time to send its input to the IUT.

The synchronization problem that occurs between transitions α/<a, ->, b> and β/<-, b> can be solved by inserting an external coordination message exchange <-Cᵦ, +Cᵦ> after α/<a, ->> and before β/<-, b> in the global test sequence. Thus, the global test sequence ω of M₂, which is:

\[
\alpha/<a, ->, a/<a, ->, \beta/<-, b>, \beta/<a, ->, b>
\]

becomes

\[
\alpha/<a, ->, a/<a, ->, <-Cᵦ, +Cᵦ>, \beta/<-, b>, \beta/<a, ->>
\]

which can be used to generate the following two local test sequences for the Upper and Lower Tester respectively:

- \(ω_u = α/<a, ->, a/<a, ->, -cᵦ, ->, ->, ->/<a, ->>\)
- \(ω_κ = ->/<-, ->, ->/<-, ->, +cᵦ, β/<-, b>, β/<-, ->>\)

The problem of observability is demonstrated as follows. Given the digraph \(G = (V, E)\) of the 2p-FSM M₂ shown in Figure 9, consider a faulty implementation of M₂ shown in Figure 10. In this faulty implementation, note that the output message "a" intended for the Upper Tester in transition t₄ has now been shifted to transition t₃. This 1-shift output fault is not detectable by the application of the two local test sequences derived above, since the Upper Tester receives the proper output from the IUT but has no way of knowing that this output has been generated by the wrong transition.
In order to ensure that 1-shift output faults are detected, [CR99] proposes a solution to the observability problem that includes the use of the external coordination message "O".

Recall from Section 2.2 that, given an np-FSM \( M \) and a global test sequence \( \omega = x_1y_1, x_2y_2, \ldots, x_my_m \) of \( M \), where \( x_i \in I \) and \( y_i \in O \), \( 1 \leq i \leq m \), a 1-shift output fault in an implementation \( N \) of \( M \) exists when, in the labels \( x_jy_j \) and \( x_{j+1}y_{j+1} \) of any two consecutive transitions, there exists one \( a_k \) in \( y_j = <a_1, a_2, \ldots, a_n> \) of \( M \) which occurs in \( y_{j+1} = <a_1, a_2, \ldots, a_n> \) in \( N \) (and not in \( y_j \) in \( N \)), or there exists one \( a_k \) in \( y_{j+1} = <a_1, a_2, \ldots, a_n> \) of \( M \) which occurs in \( y_j = <a_1, a_2, \ldots, a_k> \) in \( N \) (and not in \( y_{j+1} \) in \( N \), \( 1 \leq j \leq m-1, 1 \leq k \leq m \). An instance of the observability problem manifests itself as an undetectable 1-shift output fault if there is a 1-shift output fault related to \( a_k \in \Gamma_k \) in any two consecutive transitions whose labels are \( x_jy_j \) and \( x_{j+1}y_{j+1} \), such that tester \( k \) satisfies the condition \( (a_k \text{ is in } y_j \text{ XOR } a_k \text{ is in } y_{j+1}) \) AND \( x_{j+1} \not\in \Sigma_k \). In this case, we say that tester \( k \) is involved in the shift, and would not be able to detect it.

To eliminate the observability problem (i.e., to render the 1-shift output fault detectable), [CR99] proposes the insertion of an external coordination message exchange
relating to observability "\(-O_k, +O_h\)" between \(x/y_j\) and \(x_{j+1}/y_{j+1}\) in the global test sequence which will result in the insertion of (In the following cases, tester \(k\) refers to the tester involved in the shift, and tester \(h\) refers to the tester sending the input to the IUT in \(x_{j+1}\)):

- Case 1: \((a_i \in y_{j+1})\) an external coordination message "\(+O_h\)" in the local test sequence for tester \(k\) just before receiving the output \(a_i\) from the IUT, sent by tester \(h\). Tester \(h\) will have a message "\(-O_k\)" inserted in its local test sequence just before sending the input \(x_{j+1}\) to the IUT. This message exchange has the effect of tester \(h\) telling tester \(k\): "prepare to receive an output from the IUT."

- Case 2: \((a_i \in y_j)\) an external coordination message "\(+O_h\)" in the local test sequence for tester \(k\) just after receiving the output \(a_i\) from the IUT, sent by tester \(h\). Tester \(h\) will have a message "\(-O_k\)" inserted in its local test sequence just before sending the input \(x_{j+1}\) to the IUT. This message exchange has the effect of tester \(h\) telling tester \(k\): "you should have received an output from the IUT by now."

The observability problem demonstrated in Figure 10 can be detected by inserting the external coordination message exchange relating to observability \(-O_{u_u}, +O_{l_u}\) between \(\beta/\langle -, b \rangle\) and \(\beta/\langle a, \rightarrow\) in the global test sequence to detect the potential shift of the output "\(a\)". Similarly, an external coordination message exchange relating to observability \(-O_{u_u}, +O_{l_u}\) is inserted between \(a/\langle a, \rightarrow\) and \(\beta/\langle -, b \rangle\) in the global test sequence to detect the potential shift of output "\(a\)". Thus, the synchronizable global test sequence \(\omega\) of \(M2\), which is:

\[ a/\langle a, \rightarrow\rangle, a/\langle a, \rightarrow\rangle, \langle-C_{L_u}, +C_{L_u}\rangle, \beta/\langle -, b \rangle, \beta/\langle a, \rightarrow\rangle, \]

with the insertion of the external coordination message exchange relating to observability, becomes:

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\( \alpha/\langle a, \to \rangle, \alpha/\langle a, \to \rangle, \langle -C_U, +C_U \rangle, \langle -O_U, +O_U \rangle, \beta/\langle -, b \rangle, \langle -O_U, +O_U \rangle, \beta/\langle a, \to \rangle \), which can then be used to generate the two local test sequences for the Upper and Lower testers, respectively:

- \( \omega_u = \alpha/\langle a, \to \rangle, \alpha/\langle a, \to \rangle, -C_L, +O_L, -/\langle \rangle, -/\langle \rangle, +O_L, -/\langle a, \to \rangle \)

- \( \omega_u = -/\langle \rangle, -/\langle \rangle, -/\langle \rangle, +C_U, -O_U, \beta/\langle -, b \rangle, -O_U, \beta/\langle \rangle, -/\langle \rangle \)
Chapter Four

The Proposed Method

4.1 Motivation

The methods described in [CR99], [LB94], and [YT98] examine a given global test sequence for instances of potential controllability and observability problems and take corrective measures in an attempt to eliminate them in the given sequence. As a result, the synchronizable global test sequence generated may or may not be minimal, either in terms of the total length of the test sequence or in the number of external coordination messages that are used.

We propose a method that takes into consideration both external coordination and input/output costs and consists of a set of transformation rules that construct a modified digraph from the specification of a given np-FSM \( M \), allowing for an RCPT of this graph to correspond to a synchronizable global test sequence of \( M \). This synchronizable global test sequence will ensure also that no 1-shift output fault remain undetected, and is minimal both in terms of the total number of external coordination message exchanges and the total length of the sequence.

Formally, the proposed method is a solution of the following problem:

Consider a minimal and deterministic np-FSM \( M = (S, \Sigma, \Gamma, \delta, \lambda, s_0) \) which is represented by a strongly connected digraph \( G = (V, E) \). As in the case of [LB94], [CR99], [YT98] and [HR00], let \( \Phi(M) = \{ M' = (S, \Sigma, \Gamma, \delta', \lambda, s_0) \} \). That is, \( \Phi(M) \) is the set of all those implementations of \( M \), each of which has no transfer fault and has the
same sets of states, inputs, outputs and the initial state as $M$. Suppose that the test
architecture to be used for testing implementations of $M$ in $\Phi(M)$ is the same as that of
[CR99], i.e., it consists of $n$ testers and supports direct communication between testers
through the use of a multicast channel and exchange of external coordination messages.
Suppose also that the cost of executing an external coordination message is higher than
that of a transition of $M$, and thus should be avoided when possible. Then, given $M$ and
$\Phi(M)$, construct a synchronizable global test sequence such that it:

- distinguishes $M$ from any faulty implementation of $M$ in $\Phi(M)$,
- will not cause any controllability and observability problems when applied in a
distributed test architecture, and
- is minimal, both in terms of the number of transitions of $M$ and the number of
external coordination message exchanges.

Below, we will present the proposed method as the solution of the above stated
problem for an $np$-FSM where $n = 2$ and the ports are labelled $U$ (Upper) and $L$ (Lower).
In Section 4.6, we will extend the solution for $n > 2$.

4.2 Description of the Proposed Method

In the first phase of the proposed method, the digraphs $G' = (V', E')$ and
$G'' = (V'', E'')$ are constructed from the digraph $G = (V, E)$ of the given $2p$-FSM $M$ for
the purpose of identifying the eligible successors of each transition of $M$ and inserting
external coordination message exchanges relating to controllability to eliminate potential
controllability problems.

Given $G = (V, E)$, perform the following steps:

- Step 1. For each vertex $v_i \in V$:  

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- create a pair of vertices \( i^v \) and \( i^e \) in \( V' \), and

- create a pair of dashed edges \((i^v, i^e; <-C_L, +C_U>)\) and \((i^v, i^e; <-C_U, +C_L>)\) in \( E' \),

which indicate the external coordination message exchanges relating to controllability.

- Step 2. For each edge \( e_{ji} = (v_j, v_i; x/y) \in E \), create the following edges in \( E' \):

  - \((j^v, k^v; x/y)\), if \( x \in \Sigma_u \), and \( \not\exists \) an \( a_L \) in \( y \),

  - \((j^v, k^v; x/y)\) and \((j^v, k^v; x/y)\), if \( x \in \Sigma_u \) and \( \exists \) an \( a_L \) in \( y \),

  - \((j^v, k^v; x/y)\), if \( x \in \Sigma_L \), and \( \not\exists \) an \( a_U \) in \( y \),

  - \((j^v, k^v; x/y)\) and \((j^v, k^v; x/y)\), if \( x \in \Sigma_L \) and \( \exists \) an \( a_U \) in \( y \).

- Step 3. For each vertex \( v \in V' \) where only dashed edges are arriving and leaving, remove from \( E' \) dashed edges arriving and leaving \( v \) and then remove \( v \) from \( V' \).

After this step is complete, the resulting digraph will be known as \( G' = (V', E') \).

- Step 4. Initially, let \( V'' = V' \) and \( E'' = E' \).

For each vertex \( v_i \) in \( V' \):

  - for each pair of edges \((v_i, v_j^v; x/y)\) and \((v_i, v_j^e; x/y)\):

    - create a null vertex \( v_i^* \) in \( V'' \),

    - create a solid bold edge \((v_i, v_i^*; x/-)\) in \( E'' \),

    - create two edges \((v_i^*, v_j^v; x/y)\) and \((v_i^*, v_j^e; x/y)\) in \( E'' \) and eliminate \((v_i, v_j^v; x/y)\)

      and \((v_i, v_j^e; x/y)\) from \( E'' \).

  - any remaining solid edges leaving \( v_i \) are made bold.

After this step is complete, the resulting digraph will be known as \( G'' = (V'', E'') \).
In the first step, each vertex \( j^V \) (or \( j^V \)) represents the starting state \( v_j \) of a transition with the input operation related to \( U(L) \), and each dashed edge \( (j^V, j^V; <-C_L, +C_L>) \) (or \( (j^V, j^V; <-C_H, +C_H>) \)) represents an external coordination message exchange relating to controllability. In the second step, each edge represents a transition of \( M \). Note that in any traversal of \( G' \), any two adjacent transitions of \( G \) will be covered without creating a synchronization problem. Note also that for any two states \( v_j \) and \( v_k \), the shortest paths from \( j^V \) (or \( j^V \)) to \( k^V \) (or \( k^V \)) can be used to generate the minimum-cost synchronizable transfer sequences which transfer \( M \) from state \( v_j \) to \( v_k \). After the fourth step, a rural Chinese postman tour of \( G'' \) over the set of bold edges can be generated as a minimum-cost synchronizable global test sequence of \( M \). If there is a requirement to minimize the total number of external coordination message exchanges, each dashed edge is given a sufficiently high cost so that a rural Chinese postman tour avoids those edges whenever possible.

Consider the digraph \( G = (V, E) \) of a 2p-FSM \( M3 \) shown in Figure 11. Figures 12 and 13 show the digraphs \( G' = (V', E') \) and \( G'' = (V'', E'') \). A rural Chinese postman tour over the bold edges of \( G'' \) given in Figure 13 yields a minimum-cost synchronizable global test sequence that does not use any external coordination message exchanges as the label of the path:

\[ t_1, t_3, t_4, t_5, t_6, t_4, t_5, t_1, t_2, \]

with a total of nine input/output pairs.
Figure 11. Digraph $G = (V, E)$ of 2p-FSM $M3$

Figure 12. Digraph $G' = (V', E')$ of 2p-FSM $M3$
In the second phase of the proposed method, the set $T_0$ of all transition pairs with a potential undetectable 1-shift output fault is created and the digraph $G'' = (V'', E'')$ is modified using the information in $T_0$ to form the digraph $G''' = (V''', E''')$. $T_0$ can be constructed from $G = (V, E)$ as follows:

for each vertex $v_j \in V$ do

for each edge $e_{ij}$ (say $t_{ij}$) = $(v_i, v_j; x_i/y_j)$ entering vertex $v_j$ do

for each edge $e_{pj}$ (say $t_{pj}$) = $(v_p, v_j; x_{j+1}/y_{j+1})$ leaving vertex $v_j$ do

if for some output $a_k \in F_b$, $a_k$ is in $y_j$ XOR $a_k$ is in $y_{j+1}$ AND $x_{j+1} \notin \Sigma_a$ then

add $(t_{pj}, t_{ip}, <-O_b, +O_k>)$ to $T_0$ where $h$ is the tester sending the input $x_{j+1}$ to the IUT in $t_{ip}$ and $k$ is the tester involved in the shift.
By applying this algorithm, we discover 6 pairs of transitions in which a potential undetectable 1-shift output fault may occur, i.e.:

\[ T_0 = \{ (t_1, t_2, <-O_u, +O_v>), (t_2, t_1, <-O_u, +O_v>), (t_2, t_6, <-O_u, +O_v>), (t_3, t_4, <-O_u, +O_v>), (t_4, t_5, <-O_u, +O_v>), (t_6, t_4, <-O_u, +O_v>) \} \]

The digraph \( G'' = (V'', E'') \) can now be modified to form the digraph \( G''' = (V''', E''') \), using the information in \( T_0 \):

- Step 1. For each \((t_{ij}, t_{ip}, <-O_u, +O_v>)\) triple in \( T_0 \), identify the vertex \( v_j \) in \( V'' \) (i.e., either \( j^u \) or \( j^f \)) such that \( v_j \) is tail\((t_{ij})\). Add a new vertex \( v_j^* \) in \( V''' \) (if one does not exist already).

- Step 2. A dashed edge from \( v_j^* \) to \( v_j \) is inserted in \( E''' \), with the label \( "t_{ij}, t_{ip}, <-O_u, +O_v>" \) for each \((t_{ij}, t_{ip}, <-O_u, +O_v>)\) triple in \( T_0 \). This dashed edge indicates an external coordination message exchange relating to observability. If there exists a triple \((t_{im}, t_{mq}, <-O_n, +O_v>)\) in \( T_0 \) where, in order to traverse the transition pair \((t_{im}, t_{mq})\), the same dashed edge from \( v_j^* \) to \( v_j \) must be traversed as is traversed for the transition pair \((t_{ip}, t_{ip})\), then an alternate label \( "t_{im}, t_{mq}, <-O_n, +O_v>" \) is added to this dashed edge.

- Step 3. For each solid edge \( t_{ij} \) in \( E'' \) whose label is not \(-/-\), if the edge \( t_{ij} \) leaves vertex \( v \) in \( V'' \), then it will leave the same vertex in \( V''' \). If the edge \( t_{ij} \) is also contained in some transition pair \((t_{ik}, t_{ik})\) in a triple in \( T_0 \) and arrives at a vertex \( v_j \) in \( V'' \), then the edge \( t_{ij} \) will go to \( v_j^* \) in \( V''' \), otherwise it will go to \( v_j \) in \( V''' \). The resulting digraph is \( G''' = (V''', E''') \)

- Step 4. After constructing an RCPT of \( G''' \) over the bold edges, post processing of the graph may be required in the case where alternate labels exist for dashed edges indicating coordination message exchanges relating to observability. The correct
alternative must be chosen in order to coincide with the transition pair selected in a particular subsequence of the tour. As well, we adopt the convention that any two consecutive transitions on a path in \( G \) that would be covered in the same order in a synchronizable traversal of \( G''' \), but with the inclusion of external coordination message exchanges relating to both observability and controllability, will have the controllability message exchange processed first. For example, in an RCPT over the bold edges of \( G''' = (V''', E''') \) of the 2p-FSM \( M3 \) shown in Figure 14, the test subsequence \( t2, t1 \) would require only the inclusion of the \(<-O_L, +O_U>\) coordination message exchange relating to observability, while the test subsequence \( t2, t6 \) would require only the inclusion of \(<-C_L, +C_U>, <-.O_U, +O_U>\).

Note that any two consecutive transitions on a path in \( G \) that would be covered in the same order in a synchronizable traversal of \( G''' \) do so without any possibility of a potential undetectable 1-shift output fault. Indeed, an RCPT of \( G''' \) over the set of bold edges will yield a minimum-cost synchronizable global test sequence with no possibility of potential undetectable 1-shift output faults. A proof for this statement is given in the next section. If there is a requirement to minimize the total number of external coordination message exchanges, each dashed edge is given a sufficiently high cost so that an RCPT avoids those edges whenever possible. Note, however, that the existence of duplicate edges (of the form \((v_i^*, v_j^{U}; x/y)\) and \((v_i^*, v_j^{L}; x/y)\) as described in Step 4 of the first phase of the proposed method) may result in a case where two consecutive transitions \( t_i \) and \( t_k \) on a path in \( G''' \) may be covered with either a coordination message exchange relating to observability or a coordination message exchange relating to controllability inserted between them. For example, in Figure 14, the transitions \( t4 \) and \( t5 \)
may be traversed in $G''$ of $2p$-FSM $M3$ as $t4$, $<-C_U, +C_L>$, $t5$ or $t4$, $<-O_L, +O_U>$, $t5$. In these cases we ensure that the path containing the observability message exchange is selected by assigning the cost of observability message exchange edges lower than the cost of controllability message exchange edges.

Figure 14 shows the digraph $G''$, where an RCPT over the bold edges yields a minimum-cost synchronizable global test sequence that has no potential undetectable 1-shift output faults as the label of:

$t1, t3, <-O_U, +O_L>, t4, <-O_L, +O_U>, t5, t6, <-O_U, +O_L>, t4, <-O_L, +O_U>, t5, t1,$

$<-O_L, +O_U>, t2,$

for an addition of five external coordination message exchanges relating to observability.

Figure 14. Digraph $G'' = (V'', E'')$ for $2p$-FSM $M3$
4.3 Showing Absence of Undetectable 1-shift Output Faults

We make the claim in the previous section that an RCPT of $G''' = (V''', E''')$ over the set of bold edges will be a minimum-cost synchronizable global test sequence with no possibility of potential undetectable 1-shift output faults. Here we show the validity of this claim through the examination of the graph construction method.

**Theorem 1:** Given a 2$p$-FSM that is minimal, deterministic and represented by a strongly connected digraph $G = (V, E)$, the proposed method constructs a global test sequence that is synchronizable and has no potential undetectable 1-shift output faults.

**Proof:** After the first phase of the proposed method, any two consecutive transitions on a path in $G = (V, E)$ that do not represent a pair of synchronizable transitions become separated in $G'' = (V'', E'')$ by a dashed edge (less null edges) representing an external coordination message exchange relating to controllability. Any two adjacent edges of $G$ that remain adjacent in $G''$ (less null edges) form a synchronizable pair of transitions. Thus, any two solid edges (less null edges) in the digraph $G''$ that represent two consecutive transitions on a path in $G$ form a synchronizable sequence of transitions, either through their own properties (i.e. they are adjacent in $G''$) or through the insertion of an external coordination message exchange relating to controllability. A global test sequence generated from $G''$ would therefore be synchronizable.

By a similar argument, after the second phase of the proposed method any two solid edges (less null edges) in $G''' = (V''', E''')$ that represent two consecutive transitions on a path in $G$ with a potential undetectable 1-shift output fault become separated by a dashed edge representing an external coordination message exchange relating to observability.
Any two adjacent solid edges (less null edges) in $G'''$ form a synchronizable pair of transitions without potential undetectable 1-shift output faults. Thus, any two solid edges (less null edges) in the digraph $G'''$ that represent two consecutive transitions on a path in $G$ form a synchronizable sequence of transitions without any potential undetectable 1-shift output faults, either through their own properties (i.e. they are adjacent edges in $G'''$) or through the insertion of external coordination message exchanges relating to observability and/or controllability.

A global test sequence generated from $G'''$ would therefore be synchronizable, and would have no potential undetectable 1-shift output faults.

Q.E.D.

As we have shown, the proposed method can eliminate the existence of any potential undetectable 1-shift output fault of an implementation of an FSM during testing. The proposed method can also be used to show the absence of any potential undetectable 1-shift output fault in any synchronizable global test sequence that can be derived from a given FSM. Consider the digraph $G = (V, E)$ of the 2p-FSM $M4$ shown in Figure 15. The associated digraph $G'' = (V'', E'')$ is given in Figure 16. When we attempt to form the set $T_0$ of all transition pairs with a potential undetectable 1-shift output fault, $T_0$ is shown to be empty. Therefore, there is no possibility of an undetectable 1-shift output fault in any synchronizable global test sequence generated from $G''$. In this case, $G''$ may be used to generate a synchronizable global test sequence with no possibility of potential undetectable 1-shift output faults, and using no external coordination message exchanges relating to observability.
Although computing an RCPT of a given graph is known to be NP-complete [AA88], [AA88] and [CU95] present a class of graphs in which an RCPT can be found in polynomial time. By applying their findings to our method, we can say that if an edge induced spanning subgraph $G[E_C]$ of $G''' = (V''', E''')$ is weakly connected, $E_C$ being the set of bold edges in $G'''$, then finding an RCPT of $G'''$ over $E_C$ can be done in polynomial time. We improve our likelihood of finding a weakly connected subgraph by assuming that since nodes connected by edges representing coordination message exchanges relating to controllability are always strongly connected, they may be considered as single nodes.

![Figure 15. Digraph $G = (V, E)$ of 2p-FSM M4](image)

$\Sigma_u = \{1, 2\} \Sigma_e = \{0\}$
$
\Gamma_u = \{a, c\} \Gamma_L = \{b\}$
4.4 Comparison with Other Methods

Using the FSM shown in Figure 9 (from [CR99]) as an example reveals no difference in the resulting synchronizable global test sequences generated by the proposed method and by the method of [CR99], as there is only one way to obtain a Chinese postman tour for that graph. However, there are cases where [CR99] may or may not obtain a synchronizable global test sequence free from external coordination message exchanges relating to controllability and observability, while our method guarantees that, if such a synchronizable global test sequence exists, then it will be found.

For example, consider the digraph $G = (V, E)$ of the 2p-FSM $M5$ shown in Figure 17. There are 24 possible Chinese postman tours that could be used to generate a synchronizable global test sequence for this FSM, shown in Table 1, one of which would be selected by [CR99] to first generate a transition tour and then to obtain a
synchronizable global test sequence. Note that although a global test sequence based on any of the 24 Chinese postman tours contains no controllability problems, it will contain observability problems. This is because any instance of \( t1 \) or \( t2 \) followed directly by \( t4 \) or \( t5 \) (or vice versa) will result in a potential undetectable 1-shift output fault. A method like [CR99] that obtains a global test sequence before checking for controllability or observability problems would therefore require the insertion of an external coordination message exchange relating to observability into any of the synchronizable global test sequences shown in Table 1. With our method, an RCPT over the bold edges of the digraph \( G''' = (V'', E'') \) obtained from \( G \) is guaranteed to result in a synchronizable global test sequence free from external coordination message exchanges relating to observability, at the cost of one extra transition. The digraphs \( G' = (V', E') \), \( G'' = (V'', E'') \) and \( G''' = (V''', E''') \) that were constructed from \( G = (V, E) \) of the 2p-FSM \( M5 \) using our method are shown in Figures 18, 19 and 20. \( T_0 \) for this example is:

\[
\{(t1, t4, < -O_u, +O_v >), (t1, t5, < -O_u, +O_v >), (t2, t4, < -O_u, +O_v >), (t2, t5, < -O_u, +O_v >), (t4, t1, < -O_u, +O_v >), (t4, t2, < -O_u, +O_v >), (t5, t1, < -O_u, +O_v >), (t5, t2, < -O_u, +O_v >)\}
\]

It can be seen, for example, that a synchronizable global test sequence which is the label of \( < t1, t3, t4, t6, t2, t3, t5 > \), obtained by following an RCPT of \( G''' \) over the bold edges, has no controllability or observability problems, nor does it require the insertion of any external coordination message exchange to ensure the absence of controllability or observability problems.
Figure 17. Digraph $G = (V, E)$ of 2p-FSM $M_5$

<table>
<thead>
<tr>
<th>$t_1$, $t_3$, $t_4$, $t_2$, $t_5$, $t_6$</th>
<th>$t_2$, $t_3$, $t_4$, $t_1$, $t_5$, $t_6$</th>
<th>$t_6$, $t_1$, $t_3$, $t_4$, $t_2$, $t_5$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_1$, $t_3$, $t_4$, $t_6$, $t_2$, $t_5$</td>
<td>$t_2$, $t_3$, $t_4$, $t_6$, $t_1$, $t_5$</td>
<td>$t_6$, $t_1$, $t_3$, $t_5$, $t_2$, $t_4$</td>
</tr>
<tr>
<td>$t_1$, $t_3$, $t_5$, $t_2$, $t_4$, $t_6$</td>
<td>$t_2$, $t_3$, $t_5$, $t_1$, $t_4$, $t_6$</td>
<td>$t_6$, $t_1$, $t_4$, $t_2$, $t_3$, $t_5$</td>
</tr>
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<td>$t_1$, $t_3$, $t_5$, $t_6$, $t_2$, $t_4$</td>
<td>$t_2$, $t_3$, $t_5$, $t_6$, $t_1$, $t_4$</td>
<td>$t_6$, $t_1$, $t_5$, $t_2$, $t_3$, $t_4$</td>
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<tr>
<td>$t_1$, $t_4$, $t_2$, $t_3$, $t_5$, $t_6$</td>
<td>$t_2$, $t_4$, $t_1$, $t_3$, $t_5$, $t_6$</td>
<td>$t_6$, $t_2$, $t_3$, $t_4$, $t_1$, $t_5$</td>
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<td>$t_1$, $t_4$, $t_6$, $t_2$, $t_3$, $t_5$</td>
<td>$t_2$, $t_4$, $t_6$, $t_1$, $t_3$, $t_5$</td>
<td>$t_6$, $t_2$, $t_3$, $t_5$, $t_1$, $t_4$</td>
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<td>$t_1$, $t_5$, $t_2$, $t_3$, $t_4$, $t_6$</td>
<td>$t_2$, $t_5$, $t_1$, $t_3$, $t_4$, $t_6$</td>
<td>$t_6$, $t_2$, $t_4$, $t_1$, $t_3$, $t_5$</td>
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<tr>
<td>$t_1$, $t_5$, $t_6$, $t_2$, $t_3$, $t_4$</td>
<td>$t_2$, $t_5$, $t_6$, $t_1$, $t_3$, $t_4$</td>
<td>$t_6$, $t_2$, $t_5$, $t_1$, $t_3$, $t_4$</td>
</tr>
</tbody>
</table>
Figure 18. Digraph $G' = (V', E')$ of 2p-FSM $M5$. 

Figure 19. Digraph $G'' = (V'', E'')$ of 2p-FSM $M5$. 

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[HR00] identifies two cases of potential undetectable 1-shift output faults that are used to determine where in the local test sequences to insert the external coordination message exchanges relating to observability. Case 1, which is referred to as a backward output-shifting fault in [HR00] and [YT98], requires an external coordination message to inform tester $k$ that it should expect to receive output $a_i$ from the IUT. Case 2, which is referred to as a forward output-shifting fault in [HR00] and [YT98], requires an external coordination message to inform tester $k$ that it should have received output $a_i$ already. The method proposed in [HR00] uses a different message for detecting forward output-shifting faults (the post transition framing message) than for backward output-shifting faults (the pre transition framing message). We will see that in our proposed method, it is not necessary to formally distinguish between the two cases in order to properly place the
external coordination message exchanges relating to observability, nor are different types of messages required for different types of output shifts.

Consider a synchronizable global test sequence which is the label of the path

\[ t_1, t_3, <-O_u, +O_l>, t_4, <-O_u, +O_l>, t_5, t_6, <-O_u, +O_l>, t_4, <-O_u, +O_u>, t_5, t_1, <-O_u, +O_u>, t_2, \]

derived from an RCPT over the bold edges of the digraph \( G''' = (V''', E''') \) of 2p-FSM M3, shown in Figure 14. For demonstration purposes, in each case of an external coordination message exchange relating to observability we annotate the message “bak” if it is of Case 1, and “fwd” if it is of Case 2. The resulting synchronizable global test sequence \( \omega \) now reads:

\[
\begin{align*}
\omega &= a<-, 1>, b<-, 1>, <-O_u, +O_l>_{\text{bak}}, b<0, ->, <-O_u, +O_u>_{\text{bak}}, a<-, 1>, b<-, 2>, \\
&\quad <-O_u, +O_u>_{\text{bak}}, b<0, ->, <-O_u, +O_u>_{\text{bak}}, a<-, 1>, a<-, 1>, <-O_u, +O_u>_{\text{fwd}}, a<0, ->.
\end{align*}
\]

The following two local test sequences can then be generated from \( \omega \):

- \( \omega_L = a<-, ->, /<-, ->, +O_l, /<0, ->, -O_l, a<-, ->, /<-, ->, +O_l, /<0, ->, -O_l, a<-, ->, a<-, ->, -O_l, a<-, -> \)

- \( \omega_L = /<-, 1>, b<-, 1>, -O_u, b<-, ->, +O_u, /<-, 1>, b<-, 2>, -O_u, b<-, ->, +O_u, /<-, 1>, +O_u, /<-, -> \)

Consider the local test sequence \( \omega_U \) and the first “+O_L” in that test sequence, indicating the expected receipt of an external coordination message relating to observability from the Lower Tester. Note that this “+O_L” message is preceded by the transition “-/<-, ->” and succeeded by the transition “-/<0, ->.” Since only the succeeding transition contains an output to the Upper Tester from the IUT, it is clear that this output “0” is the only one that could result in a potential undetectable 1-shift output fault. It is also clear that the output could only shift from the transition succeeding the “+O_L”
message to the transition preceding it, or a backward output-shifting fault in the terminology of [HR00]. We know then that the "+$O_e$" message in this case is intended to tell the Upper Tester to "prepare to receive an output from the IUT."

Similarly, the last "+$O_v$" message in the local test sequence $\omega_e$ is preceded by the transition "-/<!- 1>" and succeeded by the transition "-/<!-, ->." Since only the preceding transition contains an output to the Lower Tester from the IUT, that output could only result in a potential forward output-shifting fault in the terminology of [HR00], and the "+$O_v$" message could only mean "you should have received an output from the IUT by now."

In this way, 1-shift output faults in our method do not need to be formally designated as forward output-shifting faults or backwards output-shifting faults, nor do we require two types of coordination message exchanges, as is the case with [HR00]'s post-transition framing message and pre-transition framing message. A single message type, the external coordination message exchange relating to observability, is sufficient to detect all potential undetectable 1-shift output faults.

It is somewhat more challenging to compare the method described in [LB94] with our proposed method, as the two methods make use of different test architectures. [LB94] assumes the use of the general distributed test architecture, where testers communicate amongst themselves indirectly via their interactions with the IUT, while the proposed method assumes a multi-cast channel, as in [CR99], whereby testers may communicate directly through the exchange of external coordination messages. However, with a minor augmentation, the proposed method can be shown to be at least as effective as the approach in [LB94], even using the general distributed test architecture.
In the case of an FSM where one or more synchronizable global test sequences exists, both the method in [LB94] and our proposed method will be successful in finding one. This is because in [LB94], a transition may not be added to the existing global test subsequence unless it is an eligible successor of the final transition in the sequence constructed so far. The proposed method in this thesis entails a tour of a digraph $G''' = (V'''', E'''')$ whereby each transition leaving a particular state is the eligible successor of all transitions (or edges representing coordination message exchanges) entering it. Every pair of synchronizable transitions that are traversable in the digraph $G = (V, E)$ (assuming a synchronizable test sequence exists) will therefore also be traversable in $G'''$ (following the proposed method) without the necessity of resorting to external coordination message exchanges. Because of the heuristic nature of [LB94]'s method, it is difficult to assess which approach is more effective in constructing a minimum length synchronizable global test sequence. However, given an FSM where one or more synchronizable global test sequences do exist, while [LB94]'s method may or may not generate a minimum length synchronizable global test sequence, our method guarantees finding one with minimal length, without inserting any external coordination message exchanges relating to controllability. Consider the digraph $G = (V, E)$ of the $2p$-FSM $M6$ shown in Figure 21 and disregard observability problems for the moment. Our method will construct a minimum length synchronizable global test sequence that is the label of:

$t3, t5, t6, t11, t10, t9, t7, t1, t2, t6, t9, t8, t5, t6, t11, t12, t4$,

for a total of 17 input/output pairs. While the method of [LB94] may construct the same synchronizable global test sequence, there are many others that it may consider from this
digraph that have a larger cost in terms of input/output pairs. For example, another synchronizable global test sequence for this digraph is one that is the label of:

\[ t_3, t_5, t_6, t_{11}, t_{10}, t_9, t_7, t_1, t_2, t_6, t_{11}, t_{12}, t_5, t_6, t_8, t_5, t_6, t_{11}, t_{12}, t_4, \]

for a total of 21 input/output pairs.

\[ \Sigma_c = \{a\} \Sigma_e = \{b\} \]
\[ \Gamma_c = \{1\} \Gamma_e = \{2\} \]

\[ \begin{array}{c}
1 \\
2 \\
3 \\
4 \\
5 \\
6 \\
\end{array} \]

\[ \begin{array}{cccc}
t_{1} & = & a/\langle 2,2 \rangle & t_{6} = b/\langle 1,1 \rangle \\
t_{2} & = & a/\langle 1,2 \rangle & \\
t_{3} & = & b/\langle 1,1 \rangle & t_{5} = b/\langle 1,2 \rangle \\
t_{4} & = & a/\langle 1,1 \rangle & t_{7} = a/\langle 1,1 \rangle \\
t_{9} & = & b/\langle 1,1 \rangle & t_{8} = b/\langle 1,2 \rangle \\
t_{10} & = & b/\langle 1,2 \rangle & t_{11} = a/\langle 1,2 \rangle \\
t_{12} & = & a/\langle 1,2 \rangle & \\
\end{array} \]

Figure 21. Digraph \( G = (V, E) \) of 2p-FSM \( M_6 \)

In terms of observability problems, the proposed method can be augmented slightly to produce a synchronizable global test sequence with no potential undetectable 1-shift output faults and without using any external coordination message exchanges relating to observability, as the same test architecture used in [LB94] will be considered. The augmented method is as follows:

- Use the method proposed in this thesis to construct a digraph \( G''' = (V''', E''') \) from the digraph \( G = (V, E) \) of a given \( np \)-FSM.

- Find an RCPT of \( G''' \) over the set of bold edges that minimizes the number of external coordination message exchanges relating to observability and controllability.
- If an external coordination message exchange relating to controllability exists in the resulting global test sequence then stop, since the digraph $G$ has no synchronizable global test sequence that does not contain an external coordination message exchange relating to controllability. In this case, the method of [LB94] would also fail to find a synchronizable global test sequence.

- Otherwise, for each instance of an external coordination message exchange relating to observability in the generated synchronizable global test sequence, remove the external coordination message exchange and append the synchronizable global test sequence with a synchronizable test subsequence that will ensure the absence of potential undetectable 1-shift output faults between the pairs of transitions identified by the external coordination message exchanges relating to observability.

As an example, consider the digraph of the $2p$-FSM $M1$ shown in Figure 6. Using the proposed method, the digraphs $G' = (V', E')$, $G'' = (V'', E'')$ and $G''' = (V''', E''')$ are constructed as shown in Figures 22, 23, and 24. $T_0$ as derived from $G$ is:

$\{(t1, t3, <O_u, +O_i>), (t1, t4, <O_u, +O_i>), (t2, t3, <O_u, +O_i>), (t3, t2, <O_u, +O_i>), (t4, t1, <O_u, +O_i>)\}$.

An RCPT over the bold edges of $G'''$ yields a minimum-length synchronizable global test sequence that is the label of:

$t1, <O_u, +O_i>, t3, <O_u, +O_i>, t2, t4,$

which requires no external coordination message exchanges relating to controllability.

Removing the external coordination message exchanges relating to observability and appending the synchronizable global test sequence with the appropriate synchronizable test subsequences will result in a synchronizable global test sequence with the label of:
$t_1, t_3, t_2, t_4, t_1, t_4, t_1, t_3$.

This is the same synchronizable global test sequence with no potential undetectable 1-shift output faults that is found using the method of [LB94].

Figure 22. Digraph $G' = (V', E')$ of 2p-FSM M1

Figure 23. Digraph $G'' = (V'', E'')$ of 2p-FSM M1
Figure 24. Digraph $G'' = (V'', E'')$ of 2p-FSM $M1$

This augmented method as well as the method of [LB94] selects in an ad hoc manner the synchronizable test subsequences that will be appended to the synchronizable global test sequence in order to ensure the absence of potential undetectable 1-shift output faults. However, a set of rules may be applied to assist in the selection process. Synchronizable test subsequences may be selected according to the following steps:

- Step 1. **Identify $<-O_\nu, +O_\nu>$ to be replaced.** For each transition pair $(t_{ij}, t_{jk})$ from a triple $(t_{ij}, t_{jk}, <-O_\nu, +O_\nu>)$ in $T_0$ such that $label(t_{ij}), <-O_\nu, +O_\nu>$, $label(t_{jk})$ is a subsequence of a given synchronizable global test sequence for a given 2p-FSM $M$, form a new column in a table to be constructed.

- Step 2. **Identify candidates to replace $<-O_\nu, +O_\nu>$.** Under the column for a transition pair $(t_{ij}, t_{jk})$ in the table, form a new row for each possible pair of transitions $(t_{im}, t_{ij}), (t_{ij}, t_{jk}), (t_{im}, t_{jk})$ and $(t_{jk}, t_{ik})$ that represent pairs of adjacent transitions including $t_{ij}$ and $t_{jk}$ in $G = (V, E)$ of $M$. Remove all non-synchronizable pairs of transitions in that column. One of the remaining transition pairs will be selected according to Step 3 and the selected transition pair will be appended on to the synchronizable global test
sequence in order to detect the potential undetectable 1-shift output fault for that column.

- Step 3. Select one candidate to replace \(-O_a, +O_a\).

  - a) For each column and for each row in that column, consider the transition pair 
    \((t_{in}, t_{nn})\) added in Step 2, and record head\((t_{in})\) and tail\((t_{nn})\) from \(G' = (V', E')\). In the case where \(t_{nn}\) is a duplicate edge (i.e., of the form \((v_a^r, v_a^{u_r}, x/y)\) and \((v_a^r, v_a^{L_r}, x/y)\) within \(G'\), \(v^{uL}\) may be written for tail\((t_{nn})\).

  - b) For each column and for each row in that column, if the transition pair \((t_{in}, t_{nn})\) in that row is such that:

    i) tail\((t_{nn})\) is the same as head\((t_{ry})\) of a transition pair \((t_{ry}, t_{rc})\) in another column, 
    then that transition pair \((t_{in}, t_{nn})\) shall be referred to as right-attachable. If 
    tail\((t_{nn})\) is of the form \(v^{uL}\), then head\((t_{ry})\) from another column that is either \(v^u\) or \(v^l\) may 
    be considered,

    ii) head\((t_{in})\) is the same as tail\((t_{rc})\) of a transition pair \((t_{ry}, t_{rc})\) in another column, 
    then that transition pair \((t_{in}, t_{nn})\) shall be referred to as left-attachable. If tail\((t_{rc})\) 
    from another column is of the form \(v^{uL}\), then head\((t_{in})\) that is either \(v^u\) or \(v^l\) may 
    be considered,

Transition pairs that are neither right-attachable or left-attachable are removed.

- c) For each column and for each row in that column, if \(t_{nn}\) from any \((t_{in}, t_{nn})\) pair left 
  after b) does not involve an input to the IUT from the tester involved in the shift for 
that column that occurs after the occurrence of the output being shifted, then find an 
eligible successor \(t_{rc}\) of \(t_{nn}\) that does involve an input to the IUT from the tester 
involved in the shift and place it in brackets after \(t_{nn}\). This is to indicate that in order
to detect a potential undetectable 1-shift output fault, \((t_{im}, t_{mn})\) must be followed by a transition pair \((t_{mn}, t_{np})\) from another column. If such a pair does not exist, \((t_{im}, t_{mn})\) may only be used if it is included as the final transition pair in the synchronizable global test sequence and if its inclusion would detect the potential 1-shift output fault for that column. Add empty brackets after \(t_{mn}\) in this case, indicating \((t_{im}, t_{mn})\) may only be used as the final pair in the synchronizable global test sequence. Otherwise, it is removed.

- d) Select one transition pair \((t_{im}, t_{mn})\) from each column to append onto the synchronizable global test sequence such that \(t_{im}\) from the first pair chosen is an eligible successor to the last transition in the synchronizable global test sequence, and each subsequent transition pair is chosen so that its head is the same as the tail of its immediate predecessor.

For example, consider the potential undetectable 1-shift output fault between \(t1\) and \(t3\) in the minimum length synchronizable global test sequence for the 2p-FSM \(M1\), which is the label of \(t1, t3, t2, t4\). According to Step 1 described above, this transition pair is first inserted as the first column in Table 2. In Step 2, all possible pairs that represent adjacent transitions in \(G = (V, E)\) of \(M1\) that include either \(t1\) or \(t3\) (i.e., \((t3, t1), (t1, t4), (t4, t1), (t2, t3), (t3, t2)\)) are added as rows to the column. All transition pairs in this case are synchronizable except \((t3, t1)\), which is discarded. In Step 3a), the head and tail of each transition pair from \(G' = (V', E')\) of \(M1\) is recorded. For example, in the first transition pair \((t1, t4)\), \(t1\) leaves and \(t4\) enters node \(1'\). In Step 3b), transition pairs \((t4, t1)\) and \((t3, t2)\) are discarded, as they cannot be preceded or followed by any transition pair in the other column without the insertion of an additional transition. In Step 3c), transition
pair \((r1, r4)\) is examined to determine the result of the removal of the "b" in \(r1\) as the result of a 1-shift output fault from \(r1\) to \(r3\). It would be detected by the Upper Tester, since the Upper Tester would not be able to send the input "0" in \(r4\) without first receiving the "b" in \(r1\). An insertion of a "b" into \(r3\) in a \((r2, r3)\) transition pair would not be detectable unless \(r3\) was followed by a transition where the Upper Tester sends an input to the IUT. There is not enough information to determine whether it would be detectable if \(r3\) was the last transition in the appended synchronizable global test sequence. Since there exists no eligible successor for \(r3\) that involves an input to the IUT from the Upper Tester, the transition pair \((r2, r3)\) is discarded. In Step 3d), the test subsequence \(r1, r4, r1, r3\) can then be chosen to be appended to the synchronizable global test sequence for \(M1\).

As noted in Section 3.3, the transition pairs appended to the synchronizable global test sequence for \(M1\) result in potential undetectable 1-shift output faults that did not exist before they were appended. Specifically, there is now a potential undetectable 1-shift output fault in each of transition pairs \((r4, r1)\) and \((r1, r4)\). However, Table 3 shows that for each new potential undetectable 1-shift output fault that is caused by the appended part, there exists a transition pair in the original synchronizable global test sequence that detects that fault.
Table 2. Selecting transition pairs for potential undetectable 1-shift output faults – FSM M1

Note: $\Sigma_U = 0$, $\Sigma_L = 1$, $\Gamma_U = b$, $\Gamma_L = a, c$

<table>
<thead>
<tr>
<th>$t1 \rightarrow t3$ shift</th>
<th>$t3 \leftarrow t2$ shift</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Gamma_U &lt;, e&gt;, 0/\Gamma_U &lt;, b, a&gt;$</td>
<td>$\Gamma_U &lt;, e&gt;, 0/\Gamma_U &lt;, b, a&gt;$</td>
</tr>
<tr>
<td>$t3, t1$</td>
<td>$t3, t1$</td>
</tr>
<tr>
<td>$1^U$</td>
<td>$1^U$</td>
</tr>
<tr>
<td>$0/\Gamma_U &lt;, b, a&gt;, 0/\Gamma_U &lt;, b, -&gt;$</td>
<td>$0/\Gamma_U &lt;, b, a&gt;, 0/\Gamma_U &lt;, c&gt;, ()$</td>
</tr>
<tr>
<td>$t1, t4$</td>
<td>$t1, t3$</td>
</tr>
<tr>
<td>$2^U$</td>
<td>$1^L$</td>
</tr>
<tr>
<td>$0/\Gamma_U &lt;, b, a&gt;, 0/\Gamma_U &lt;, b, a&gt;$</td>
<td>$0/\Gamma_U &lt;, b, -&gt;, 1/\Gamma_U &lt;, b, -&gt;$</td>
</tr>
<tr>
<td>$t4, t1$</td>
<td>$t4, t2$</td>
</tr>
<tr>
<td>$1^L$</td>
<td>$1^L$</td>
</tr>
<tr>
<td>$1/\Gamma_U &lt;, b, -&gt;, 0/\Gamma_U &lt;, e&gt;, ()$</td>
<td>$0/\Gamma_U &lt;, b, -&gt;, 0/\Gamma_U &lt;, b, -&gt;$</td>
</tr>
<tr>
<td>$t2, t3$</td>
<td>$t2, t4$</td>
</tr>
<tr>
<td>$2^L$</td>
<td>$1^L$</td>
</tr>
<tr>
<td>$1/\Gamma_U &lt;, e&gt;, 0/\Gamma_U &lt;, b, -&gt;$</td>
<td>$1/\Gamma_U &lt;, e&gt;, 0/\Gamma_U &lt;, b, -&gt;$</td>
</tr>
<tr>
<td>$t3, t2$</td>
<td>$t2, t3$</td>
</tr>
</tbody>
</table>

Table 3. Transition pairs in synchronizable global test sequence for FSM M1 that detect potential undetectable 1-shift output faults in appended part

<table>
<thead>
<tr>
<th>Pairs in appended part causing potential fault</th>
<th>Pairs in original test sequence to detect that fault</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t4, t1 \ (0/\Gamma_U &lt;, b, -&gt;, 0/\Gamma_U &lt;, b, a&gt;)$</td>
<td>$t1, t3 \ (0/\Gamma_U &lt;, b, a&gt;, 1/\Gamma_U &lt;, c&gt;, )$</td>
</tr>
<tr>
<td>$t1, t4 \ (0/\Gamma_U &lt;, b, a&gt;, 0/\Gamma_U &lt;, b, -&gt;)$</td>
<td>$t1, t3 \ (0/\Gamma_U &lt;, b, a&gt;, 1/\Gamma_U &lt;, c&gt;, )$</td>
</tr>
</tbody>
</table>

4.5 $k$-shift Output Faults

Since our definition of global test sequence requires an input $x_i \in I$ for each transition, there can be only one instance of a potential undetectable 1-shift output fault in any two consecutive transitions of a synchronizable global test sequence for a $2p$-FSM. This is because, given two transition $t_i$ and $t_{i+1}$ of a synchronizable global test sequence, the definition of potential undetectable 1-shift output fault requires that the output being shifted be contained in either $t_i$ or $t_{i+1}$, but not both. Furthermore, any potential 1-shift output fault between $t_i$ and $t_{i+1}$ (or vice versa) will be detectable if the tester involved in the shift (i.e., $T_{shift}$) is the same tester that is sending the input to the IUT in $t_{i+1}$. That means a potential undetectable 1-shift output fault will only occur when the tester
sending the input to the IUT is not $T_{shift}$. The output being shifted may be contained in either $t_i$ or $t_{i+1}$, but not both. Hence, there can only be one instance of a potential undetectable 1-shift output fault in any two consecutive transitions of a synchronizable global test sequence for a 2p-FSM. Similarly for an np-FSM, the maximum number of potential undetectable 1-shift output faults in two consecutive transitions is $n-1$.

The potential undetectable $k$-shift output faults would be determined in much the same way as potential undetectable 1-shift output faults.

**Definition:** Given a 2p-FSM $M$ and a synchronizable global test sequence $\omega = x/y_1, x_2/y_2, \ldots, x_n/y_n$ of $M$, where $x_i \in I$ and $y_i \in O$, $1 \leq i \leq m$, the observability problem will manifest itself as an undetectable $k$-shift output fault in an implementation $N$ of $M$ when, in any two transitions (henceforth called terminal transitions) with labels $x_i/y_j$ and $x_{i+k}/y_{j+k}$, $k \geq 2$, the following conditions exist:

- The terminal transitions must satisfy the conditions for an undetectable 1-shift output fault,

- All intermediate transitions between the terminal transitions must be such that the $k$-shift output fault is not detected by the testers involved in the intermediate transitions.

For example, the synchronizable global test sequence of the 2p-FSM $M3$ described in Section 4.2, which is the label of the path:

$$a/<<, 1>, b/<<, 1>, b/<<0, ->, a/<<, 1>, b/<<, 2>, b/<<0, ->, a/<<, 1>, a/<<, 1>, a/<<0, ->,$$

contains a potential undetectable 2-shift output fault between the third transition in the sequence ($b/<<0, ->$) and the first ($a/<<, 1>$).

**Theorem 2:** Given a synchronizable global test sequence for a 2p-FSM where all potential 1-shift output faults are detectable when applied in the distributed test
architecture (with or without the insertion of external coordination message exchanges relating to observability), and where each output associated with a particular transition within the synchronizable global test sequence is unique, there will be no undetectable $k$-shift output faults, $k \geq 2$, in the synchronizable global test sequence.

**Proof:** We assume the existence of a synchronizable global test sequence for a $2p$-FSM in which all 1-shift output faults are detectable, or each potential undetectable 1-shift output fault has resulted in the insertion of an external coordination message exchange relating to observability. To prove the absence of undetectable $k$-shift output faults, we examine the direction of the shift and all combinations of the intermediate transitions given two terminal transitions that satisfy the conditions in the definition of an undetectable $k$-shift output fault. For the purposes of demonstration, a number of examples are given in Table 4. Note that in all cases the shift involves the Upper Tester; however, the same arguments in the proof can be applied if the testers were reversed.

1. Let $t_i$ and $t_{j+k}$ be two transitions that exhibit a potential undetectable $k$-shift output fault in accordance with the definition, and let $T_{\text{shift}}$ represent the tester involved in the shift, while $T_{\text{noshift}}$ represents the other tester.

2. The shift must be of the form "$j$ to $j+k$" or "$j+k$ to $j$." That is, the shift must be a forward shifting fault where an output $a_{\text{shift}}$ is removed from $y_j = \langle a_1, a_2, \ldots, a_n \rangle$ and inserted into $y_{j+k}$, or it must be a backward shifting fault where an output $a_{\text{shift}}$ is removed from $y_{j+k} = \langle a_1, a_2, \ldots, a_n \rangle$ and inserted into $y_j$.

3. If the shift is of the form "$j$ to $j+k$", then one of two cases can occur. Either there is at least one transition $t_i$ between $t_j$ and $t_{j+k}$ that contains an output to $T_{\text{shift}}$ from the IUT, or no transition between $t_j$ and $t_{j+k}$ contains an output to $T_{\text{shift}}$ from the IUT.
3.1 If there is at least one transition $t_i$ that contains an output to $T_{\text{shift}}$ (Case 1 of Table 4), $T_{\text{shift}}$ is expecting to receive the unique output associated with $t_j$ before it receives the unique output associated with $t_i$. In the case of a $k$-shift output fault from $t_j$ to $t_{j+k}$, $T_{\text{shift}}$ will receive the unique output associated with $t_j$ first, and will therefore detect the $k$-shift output fault.

3.2 If no transition between $t_j$ and $t_{j+k}$ contains an output to $T_{\text{shift}}$, then one of two cases must be true regarding $t_{j+1}$. If $t_{j+1}$ has an input to the IUT from $T_{\text{shift}}$ (Case 2a of Table 4), then $T_{\text{shift}}$ would not send that input until it had received the output associated with $t_j$, and the $k$-shift would therefore be detected. If $t_{j+1}$ has an input to the IUT from $T_{\text{no shift}}$, then a potential undetectable 1-shift output fault would exist between $t_j$ and $t_{j+1}$. This situation would have resulted in the insertion of an external coordination message exchange relating to observability into the synchronizable global test sequence between $t_j$ and $t_{j+1}$, and would enable $T_{\text{shift}}$ to detect the $k$-shift output fault.

4 If the shift is of the form "$j+k$ to $j$", then one of two cases can occur. Either there is at least one transition $t_i$ between $t_j$ and $t_{j+k}$ that contains an output to $T_{\text{shift}}$ from the IUT, or no transition between $t_j$ and $t_{j+k}$ contains an output to $T_{\text{shift}}$ from the IUT.

4.1 If there is at least one transition $t_i$ that contains an output to $T_{\text{shift}}$ (see Case 3 of Table 4), $T_{\text{shift}}$ is expecting to receive the unique output associated with $t_{j+k}$ after it receives the unique output associated with $t_i$. In the case of a $k$-shift output fault from $t_{j+k}$ to $t_j$, $T_{\text{shift}}$ will receive the unique output associated with $t_{j+k}$ first, and will therefore detect the $k$-shift output fault.
4.2 If no transition between \( t_i \) and \( t_{i+k} \) contains an output to \( T_{\text{shift}} \) (see Case 4 of Table 4), then a potential undetectable \( 1 \)-shift output fault would exist between \( t_{i+k-1} \) and \( t_{i+k} \). This situation would have resulted in the insertion of an external coordination message exchange relating to observability into the synchronizable global test sequence between \( t_{i+k-1} \) and \( t_{i+k} \), and would enable \( T_{\text{shift}} \) to detect the \( k \)-shift output fault.

Q.E.D.

**Theorem 3:** There exists an FSM \( M \) whereby a synchronizable global test sequence constructed from \( M \) may contain undetectable \( k \)-shift output faults, \( k \geq 2 \), if the outputs associated with each transition within that synchronizable global test sequence are not unique.

**Proof:** If the assumption regarding unique outputs is removed, \( k \)-shift output faults may be undetected when the synchronizable global test sequence is applied. Consider the following test subsequence (input and output languages are the same as in Table 4), which is the label \( a/\langle , 2\rangle , a/\langle 1, 2\rangle , b/\langle 1, -\rangle \) of a transition sequence \( t_1, t_2, t_3 \).

A shift of the output "1" from \( t_3 \) to \( t_1 \) may be undetected if the Upper Tester manages to send the input "a" for \( t_2 \) before receiving the shifted output. The Upper Tester could interpret this shifted output as the one generated by \( t_2 \), and the second output "1" in \( t_2 \) as the one generated by \( t_3 \), thus passing the faulty implementation. The shift would be detectable if the shifted output was received prior to sending the input "a" in \( t_2 \), but this detectability could not be assured without the use of a global clock.

Q.E.D.
Table 4. Examples of k-shift output faults

<table>
<thead>
<tr>
<th>2p-FSM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Σ_u = a, Σ_d = b</td>
</tr>
<tr>
<td>Γ_u = 1, 1, 1, 1, ... Γ_d = 2, 2, 2, 2, ...</td>
</tr>
</tbody>
</table>

T_shift in all cases is the Upper Tester.

Assume: k-shift output fault, t_i to t_{i+k}

ex: t_i = a/1, 2>, t_{i+k} = b/<-, 2, >, and after the k-shift output fault t_i = a/<-, 2>, t_{i+k} = b/1, 2, >.

Case 1:

a/1, 2>, b/1, 2>, a/<-, 2>, b/<-, 2>,

where t_i = b/1, 2>

Case 2:

a/a/1, 2>, a/<-, 2>, b/<-, 2>, b/<-, 2>,

b/a/1, 2>, <-O, +O>, b/<-, 2>, b/<-, 2>, b/<-, 2>,

Assume: k-shift output fault, t_{i+k} to t_i

ex: t_i = a/<-, 2>, t_{i+k} = b/1, 2, >, and after the k-shift output fault t_i = a/1, 2>, t_{i+k} = b/<-, 2, >

Case 3:

a/<-, 2>, a/<-, 2>, a/1, 2>, b/<1, 2>,

where t_i = a/1, 2>

Case 4:

a/<-, 2>, b/<-, 2>, b/<-, 2>, <-O, +O>, b/1, 2,>

4.6 Extending the Proposed Method to np-FSMs

The proposed method is generalized to accommodate np-FSMs, where n > 2.

Consider the digraph G = (V, E) of an np-FSM M, where n testers interacting with the IUT are labelled 1, 2, 3, ... , n. In the first phase, a digraph G' = (V', E') is constructed from G according to the following steps:

- Step 1. For each vertex v_i ∈ V:
- create a set of vertices \( i^l, i^r, \ldots, i^p \) in \( V \) and,

- \( \forall j, k, j \leq n, k \leq n, j \neq k \), create a set of dashed edges \( (i^l, i^r; \langle -C_i, +C_i \rangle) \) and \( (i^l, i^r; \langle -C_j, +C_j \rangle) \) in \( E' \) that indicate the external coordination message exchanges relating to observability.

- Step 2. For each edge \( e_v = (v_x, v_y; x/y) \in E \) create the following edge(s) in \( E' \):
  - \( (j^l, k^r; x/y) \), if \( x \in \Sigma_y \) and \( \exists a \neq - \) in \( y, r \neq q, 1 \leq r \leq n. \)
  - \( (j^l, k^r; x/y) \) and \( (j^l, k^r; x/y) \), if \( x \in \Sigma_y \) and \( \forall r, r \neq q, \exists a \neq - \) in \( y, 1 \leq r \leq n. \)

- Step 3. For each vertex \( v \in V \) where only dashed edges are arriving and leaving, remove from \( E' \) dashed edges arriving and leaving and then remove \( v \) from \( V' \). After this step is complete, the resulting digraph will be known as \( G' = (V, E'). \)

- Step 4. Initially, let \( V'' = V \) and \( E'' = E' \).

For each vertex \( v_i \in V' \), if there exists at least two edges \( (v_i, v_i^k; x/y) \) in \( E', 1 \leq k \leq n \), then

- create a null vertex \( v_i^* \) in \( V'' \),

- create a solid bold edge \( (v_i, v_i^*; x/y) \) in \( E'' \),

- create edges \( (v_i^*, v_i^k; x/y) \) in \( E'' \), \( \forall k \) where \( (v_i, v_i^k; x/y) \) exists in \( E' \) and

- eliminate edges \( (v_i, v_i^k; x/y) \) from \( E'' \).

Any remaining solid edges leaving \( v_i \) are made bold. The resulting digraph will be known as \( G'' = (V'', E'') \).

In the second phase of the proposed method, the set \( T_0 \) of triples \( (t_{ip}, t_{ip}, \langle -O_i, +O_i \rangle) \) where \( (t_{ip}, t_{ip}) \) is a transition pair with a potential undetectable 1-shift output fault, and \( \langle -O_i, +O_i \rangle \) represents an external coordination message exchange relating to
observability sent by tester $h$ and received by tester $k$. The digraph $G''$ is modified using the information in $T_0$ to form the digraph $G''' = (V''', E''')$ such that in any traversal of $G'''$ any two adjacent edges in $G$ that are covered by the traversal of $G''$ will not create a potential controllability or observability problem. $T_0$ is constructed by the following algorithm:

for each vertex $v_j \in V$, $1 \leq j \leq n$, do

for each edge $e_q$ (say $t_{ij}$) = ($v_u$, $v_j$; $x_j/y_j$) entering vertex $v_j$ do

for each edge $e_p$ (say $t_{jp}$) = ($v_p$, $v_j$; $x_{j+1}/y_{j+1}$) leaving vertex $v_j$ do

if for some output $a_k \in \Gamma$, $a_k$ is in $y_j$ XOR $a_k$ is in $y_{j+1}$, AND $x_{j+1} \notin \Sigma_h$ then

add ($t_{ij}$, $t_{jp}$, $-O_i + O_j$) to $T_0$ where $h$ is the tester sending the input $x_{j+1}$ to the IUT in $t_{ip}$ and $k$ is the tester involved in the shift.

Note that in this algorithm, for each tester involved in a possible shift in the transition pair $(t_{ip}, t_{jp})$, a new triple will be added to $T_0$.

The digraph $G'' = (V'', E'')$ can now be modified to form the digraph $G''' = (V''', E''')$, using the information in $T_0$:

- Step 1. For each $(t_{ip}, t_{jp}, -O_i, + O_j)$ triple in $T_0$, identify the vertex $v_j$ in $V''$ such that $v_j$ is $tail(t_{ip})$. Add a new vertex $v_j^*$ in $V'''$ (if one does not exist already).

- Step 2. A dashed edge from $v_j^*$ to $v_j$ is inserted in $E'''$, with the label “$t_{ip}$, $t_{jp}$, $-O_i, + O_j$” for each $(t_{ip}, t_{jp}, -O_i, + O_j)$ in $T_0$. This dashed edge indicates an external coordination message exchange relating to observability. Any additional triples $(t_{ip}, t_{jp}, -O_i, + O_j)$ in $T_0$ will have the label “$-O_i, + O_j$” appended onto the label of this edge. If there exists a triple $(t_{im}, t_{m+1}, -O_i, + O_j)$ in $T_0$ where, in order to traverse the transition pair $(t_{im}, t_{m+1})$, the same dashed edge from $v_j^*$ to $v_j$ must be traversed as is
traversed for the transition pair \((t_{ip}, t_{ip})\), then an alternate label \("t_{ip}, t_{ip}, \langle-O, +O\rangle\)" is added to this dashed edge. The alternate label may also be appended with additional labels of coordination message exchanges relating to observability as required.

- Step 3. For each solid edge \(t_{ij}\) in \(E''\) whose label is not \(-/\), if that edge \(t_{ij}\) leaves a vertex \(v\) in \(V''\) then it will leave the same vertex in \(V'''\). If the edge \(t_{ij}\) is also contained in some transition pair \((t_{ij}, t_{jk})\) in a triple in \(T_0\) and arrives at a vertex \(v_j\) in \(V''\), then the edge \(t_{ij}\) will go to \(v_j^*\) in \(V'''\), otherwise it will go to \(v_j\) in \(V'''\). The resulting digraph will be known as \(G''' = (V''' , E''' )\).

- Step 4. After constructing an RCPT of \(G'''\) over the bold edges, post processing of the graph may be required in the case where alternate labels exist for dashed edges indicating coordination message exchanges relating to observability. The correct alternative must be chosen in order to coincide with the transition pair selected in a particular subsequence of the tour. As well, we adopt the convention that any two consecutive transitions on a path in \(G\) that would be covered in the same order in a synchronizable traversal of \(G''''\), but with the inclusion of external coordination message exchanges relating to both observability and controllability, will have the controllability message exchange processed first. For example, in an RCPT over the bold edges of \(G'''' = (V'''' , E'''' )\) of the 3p-FSM \(M7\) shown in Figure 28, the test subsequence \((t_9, t_1)\) would require only the inclusion of the \(\langle-O_1, +O_1\rangle\) coordination message exchange relating to observability, while the test subsequence \((t_9, t_4)\) would require only the inclusion of \(\langle-C_3, +C_3\rangle, \langle-O_1, +O_1\rangle, \langle-O_2, +O_2\rangle\).

Consider the digraph \(G = (V, E)\) of the 3p-FSM \(M7\) shown in Figure 25, where:

- \(V\) is the set of vertices \(\{A, B, C\}\) representing the states of \(M7\).
- The three testers interacting with the IUT are labelled 1, 2, and 3,
- \( E \) is the set of directed edges representing the transitions of \( M7 \), and
- one of the vertices \( v_s \in V \) corresponds to the initial state \( s_0 \) of \( M7 \).

Figures 26, 27 and 28 show the digraphs \( G' = (V', E') \), \( G'' = (V'', E'') \) and \( G''' = (V''', E''') \), obtained after applying the above method to \( G \). \( T_0 \) for this example is:

\[
\{(t1, t2, \langle -O_1, +O_2 \rangle), (t1, t2, \langle -O_3, +O_2 \rangle), (t1, t6, \langle -O_2, +O_1 \rangle), (t1, t6, \langle -O_3, +O_1 \rangle),
\]
\[
(t2, t8, \langle -O_1, +O_2 \rangle), (t2, t8, \langle -O_3, +O_2 \rangle), (t2, t9, \langle -O_3, +O_1 \rangle), (t3, t1, \langle -O_3, +O_1 \rangle),
\]
\[
(t3, t1, \langle -O_3, +O_1 \rangle), (t3, t4, \langle -O_2, +O_3 \rangle), (t3, t5, \langle -O_3, +O_2 \rangle), (t4, t5, \langle -O_1, +O_2 \rangle),
\]
\[
(t4, t5, \langle -O_1, +O_2 \rangle), (t5, t2, \langle -O_1, +O_2 \rangle), (t5, t7, \langle -O_2, +O_3 \rangle), (t6, t7, \langle -O_1, +O_3 \rangle),
\]
\[
(t6, t7, \langle -O_2, +O_3 \rangle), (t7, t3, \langle -O_3, +O_2 \rangle), (t7, t3, \langle -O_3, +O_2 \rangle), (t7, t9, \langle -O_2, +O_1 \rangle),
\]
\[
(t8, t3, \langle -O_1, +O_2 \rangle), (t9, t1, \langle -O_2, +O_1 \rangle), (t9, t4, \langle -O_1, +O_3 \rangle), (t9, t4, \langle -O_2, +O_3 \rangle),
\]
\[
(t9, t5, \langle -O_1, +O_2 \rangle)\}.
\]

As is the case when the proposed method is applied to a 2p-FSM, the method when applied to an \( np \)-FSM may also result in a case where two consecutive transitions \( t_y \) and \( t_x \) on a path in \( G''' \) may be selected so that either a coordination message exchange relating to controllability or some combination of controllability and observability message exchanges is inserted between the two. For example, the test subsequence \( t_8, t_3 \) may be traversed in \( G''' \) of 3p-FSM \( M7 \) as \( t_8, \langle -C_3, +C_1 \rangle, t_3 \) or \( t_8, \langle -C_3, +C_2 \rangle, \langle -O_1, +O_3 \rangle, t_3 \). Similar to the case with the 2p-FSM, we ensure that the path containing the coordination message exchange(s) relating to observability is selected.

A rural Chinese postman tour over the bold edges of \( G''' \) as shown in Figure 28 can then be constructed. This tour will yield a minimum-cost synchronizable global test sequence that uses no external coordination message exchanges relating to controllability,
but seven external coordination message exchanges relating to observability as the label of:

\((t_1, <O_1, +O_2>, <O_1, +O_2>, t_2, t_3, <O_2, +O_2>, t_4, <O_1, +O_2>, <O_3, +O_2>, t_5, t_6, <O_1, +O_2>, <O_2, +O_3>, t_7, t_8, t_9)\).

Figure 25. Digraph \(G = (V, E)\) of 3p-FSM \(M7\)
Figure 26. Digraph $G' = (V', E')$ of 3p-FSM $M7$
Figure 27. Digraph $G'' = (V'', E'')$ of 3p-FSM $M7$
Figure 28. Digraph $G'' = (V'', E'')$ of 3-$p$-FSM $M7$
Chapter Five

Conclusions

5.1 Final Remarks

Two general approaches can be considered in order to solve the problems of controllability and observability that may occur during the application of a global test sequence in a distributed test architecture. In the first general approach, first a global test sequence is generated by applying some test construction method to the given FSM, and then the global test sequence is examined for instances of controllability and observability problems and corrective actions are taken. The corrective actions depend on whether the distributed test architecture being used supports direct communication between testers through the use of external coordination message exchanges, or indirect communication between testers through their interaction with the IUT. In the second general approach, which is introduced in this thesis, controllability and observability problems are considered before a global test sequence is constructed. This minimizes both the length of the global test sequence and the number of corrective actions required to eliminate controllability and observability problems.

We have studied the problem of controllability and observability in distributed testing, proposing a method whereby a minimum-length synchronizable global test sequence with no potential undetectable 1-shift output faults may be generated. If an FSM being implemented is such that a synchronizable global test sequence with no potential undetectable 1-shift output faults exists that does not require an external coordination message exchange, the proposed method will find it. If external
coordination message exchanges are required, then the proposed method can generate a synchronizable global test sequence such that the number of external coordination message exchanges is minimized. The proposed method is shown to be flexible such that it may be used whether the testers in the distributed test architecture are able to communicate amongst themselves directly or indirectly.

5.2 Summary of Contributions

Below we list the major contributions of the thesis:

We have proposed a method that generates a minimum-length synchronizable global test sequence with no possibility of potential observability problems when applied to a 2p-FSM in a distributed test architecture.

We have proven that, given a 2p-FSM that is minimal, deterministic and whose underlying graph is strongly connected, our proposed method constructs a synchronizable global test sequence with no potential undetectable 1-shift output faults.

We have shown the proposed method to perform at least as well as other methods proposed in the literature, and we have shown that it performs better than each one under certain conditions.

We have proven that, in a synchronizable global test sequence for a 2p-FSM, if no potential undetectable 1-shift output faults exist, then there can exist no potential undetectable k-shift output faults, k > 1, provided that each output in the sequence is unique.

We have extended the proposed method so that it can be applied to an np-FSM in a distributed architecture, n > 2.
5.3 Directions for Future Research

It would be interesting to see this work improved and/or extended in the following directions:

We have shown that, in a synchronizable global test sequence for a 2p-FSM, if no potential undetectable 1-shift output faults exist, then there can exist no potential undetectable k-shift output faults, $k > 1$, provided that each output in the sequence is unique. It would be interesting to determine under what conditions, if any, a similar claim could be made for np-FSMs, $n > 2$.

We have shown that our proposed method can be augmented slightly to produce a synchronizable global test sequence with no potential undetectable 1-shift output faults without using any external coordination message exchanges, using the same test architecture as in [LB94]. The example used in this thesis seemed to indicate that, even though additional potential undetectable 1-shift output faults are introduced by appending test subsequences to the original synchronizable global test sequence, they are rendered detectable by transition pairs that already exist in the original synchronizable global test sequence. It would be interesting to see a set of rules that yields test subsequences that, when appended to the original synchronizable global test sequence, result only in potential 1-shift output faults detectable by transition pairs within the original synchronizable global test sequence.

Our proposed method assumes a fault model that includes only output faults, and assumes transfer faults do not exist. It would be interesting to construct a method that would be effective if the fault model were to be expanded. Specifically, how would we ensure that we could detect every combination of transfer faults and output faults
(particularly 1-shift output faults) in an implementation \( N \) of the specification \( M \) of a given FSM?

We have given the conditions by which an RCPT of \( G'' = (V'', E'') \) over the set of bold edges of \( G''' \) may be done in polynomial time. Following a similar method, [CU95] allows for some dashed edges in their derived graph to be made bold in order to increase the likelihood of finding a weakly connected subgraph. It will be interesting to find some sufficient conditions for the existence of a polynomial time algorithm to derive an RCPT over the set of bold edges of \( G'' \).
REFERENCES


