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On Weighted Quadrature Amplitude Modulation:

Power-Bandwidth Performance,

Synchronization and Detection

by

Andy D. Kucar

A thesis presented to the University of Ottawa in fulfillment of the thesis requirement for the degree of Doctor of Philosophy

Ottawa, Ontario

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ABSTRACT
This thesis investigates the performance characteristics of the newly introduced weighted quadrature amplitude modulation (WQAM) family and its corresponding new receivers.

A WQAM signal format consists of two quadrature components of the carriers that have generally unequal powers, and are modulated with independent data streams of different rates and different pulse shapes — composed of the rectangular and cosine waveforms, in general. Modulation schemes such as quadrature phase shift keying (QPSK), staggered QPSK (SQPSK), minimum shift keying (MSK), staggered quadrature overlapped raised cosine (SQORC), unbalanced QPSK, and M-ary QAM (MQAM) are all members of the WQAM family.

In order to reduce the adjacent channel interference, by using the gradient search technique, a few new pulse shapes giving a narrow mainlobe and minimal sidelobe levels are found.

The performance of 4-state WQAM schemes is evaluated in the adjacent channel interference linear and nonlinear channel environments. The results indicate that one of the new staggered WQAM schemes outperforms other known members of the WQAM family, i.e. (S)QPSK, MSK and (S)QORC in almost all practical situations.

The impact of amplitude and group delay distortions on MQAM schemes with more than 4 states is evaluated. Staggered QAM schemes perform better in the presence of linear group delay impairments, but nonstaggered schemes are less sensitive to linear amplitude gain impairments.

The correct detection of WQAM signals requires accurate carrier phase and symbol timing at the receiver. Therefore, a composite phase and timing estimation and data detection of WQAM signal sets is analyzed. By using a maximum likelihood approach, four new joint phase and timing estimators, for balanced and unbalanced WQAM schemes, are derived.

At high signal-to-noise ratios, phase estimator and detector schemes merge into the classical decision feedback carrier recovery loop (DFCRL). The performance of this loop, employing an integrate-and-sample device in each quadrature arm (active loop), is evaluated in the presence of both carrier uncertainty \( \phi \) and timing uncertainty \( \lambda \).

The performance of overlapped schemes is degraded if classical carrier recovery schemes are employed, e.g. by 1.09 dB if SQORC is used. The theoretical results are experimentally verified on our Intersymbol-Jitter-Free (IJF) 64 kb/s modem. New equalized DFCRL which employs a simple 3-tap transversal equalizer in each quadrature arm is proposed for the carrier recovery and the detection of overlapped schemes. Our new loop does not exhibit the performance degradation associated with classical schemes.

Using a maximum a posteriori probability approach and knowledge of the phase uncertainty \( \phi \), timing uncertainty \( \lambda \), and nonlinearity surface of the loop \( h(\phi, \lambda) \) — accumulated over corresponding observation intervals — improved loops, i.e. crosstalk cancellation DFCRL, timing intersymbol interference cancellation DFCRL, and FEEDLOOP, are introduced and analyzed. The theoretical results are verified by the Monte Carlo simulation. These loops
perform almost as well as a continuous wave (pilot tone) carrier recovery loop, do not exhibit quadrant ambiguities, and might be employed for phase and timing estimation and the detection of balanced and unbalanced WQAM schemes. The advantages of the new loops over the classical one become greater at lower probabilities of error and when higher state schemes are employed. For example at $P_e = 10^{-6}$ and in the presence of a normalized loop detuning of 0.7, the crosstalk cancellation loop (and FEEDLOOP) outperforms the classical one by 2.4 dB, assuming QPSK modulation is employed.

The impact of phase noise on the performance of MQAM, M-ary phase shift keying and M-ary quadrature partial response systems in a Gaussian noise environment is computed. For all cases the degradation due to phase noise is found to be less than 1 dB if a carrier-to-phase noise ratio in a double-sided Nyquist bandwidth $(C/N)_p$ is more than 10 dB higher than the carrier-to-thermal noise ratio $(C/N)_0$ required for the probability of error performance $P_e = 10^{-6}$. Performance graphs are presented which enable a fast first order approximation of the phase noise requirements of a system to be estimated. Our engineering rule-of-thumb approximations are in a close agreement with the results of measurements.
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<td>ACI</td>
<td>Adjacent Channel Interference</td>
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<td>AM/AM</td>
<td>Amplitude Modulation to Amplitude Modulation Transfer due to Nonlinearity</td>
</tr>
<tr>
<td>AM/PM</td>
<td>Amplitude Modulation to Phase Modulation Transfer due to Nonlinearity</td>
</tr>
<tr>
<td>BLOQAM</td>
<td>Blackman Quadrature Amplitude Modulation (QAM scheme employing the Blackman window as an elementary pulse shape)</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
</tr>
<tr>
<td>CCDFCRL</td>
<td>Crosstalk Cancellation Decision Feedback Carrier Recovery</td>
</tr>
<tr>
<td>CCI</td>
<td>Cochannel Interference</td>
</tr>
<tr>
<td>((C/N))</td>
<td>Carrier-to-noise ratio in a specified bandwidth</td>
</tr>
<tr>
<td>((C/N)_{6})</td>
<td>Carrier-to-thermal noise ratio in a specified bandwidth required for the probability of error performance (P_e = 10^{-6})</td>
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<tr>
<td>((C/N)_{p})</td>
<td>Carrier-to-phase noise ratio in a specified bandwidth</td>
</tr>
<tr>
<td>CR</td>
<td>Carrier Recovery</td>
</tr>
<tr>
<td>CW</td>
<td>Continuous Wave</td>
</tr>
<tr>
<td>DA</td>
<td>Data Aided (Approach)</td>
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<tr>
<td>DFCRL</td>
<td>Decision Feedback Carrier Recovery Loop</td>
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<tr>
<td>DTTTL</td>
<td>Data Transition Tracking Loop</td>
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<tr>
<td>FEC</td>
<td>Forward Error Correction (Codes)</td>
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<tr>
<td>FEEDLOOP</td>
<td>Feedback, Estimation of phase, Estimation of timing, Decision LOOP (an acronym)</td>
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<tr>
<td>GD</td>
<td>Group Delay</td>
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<tr>
<td>HL</td>
<td>Hard Limiter</td>
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<tr>
<td>HAMQAM</td>
<td>Hamming Quadrature Amplitude Modulation (QAM scheme employing the Hamming window as an elementary pulse shape)</td>
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<tr>
<td>HPA</td>
<td>High-Power Amplifier</td>
</tr>
<tr>
<td>IF</td>
<td>Intersymbol Jitter Free (Fehler's QPSK)</td>
</tr>
<tr>
<td>ISD</td>
<td>Integrate-Sample-and-Dump (Circuit)</td>
</tr>
<tr>
<td>ISI</td>
<td>Intersymbol Interference</td>
</tr>
<tr>
<td>LO</td>
<td>Local Oscillator</td>
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<tr>
<td>LPAM</td>
<td>Lary Pulse Amplitude Modulation</td>
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<td>MAPP</td>
<td>Maximum a Posteriori</td>
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<tr>
<td>ML</td>
<td>Maximum Likelihood</td>
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<td>MMSK</td>
<td>Modified Minimum Shift Keying</td>
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<td>M-ary Phase Shift Keying</td>
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<tr>
<td>MQAM</td>
<td>M-ary Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>MQPR</td>
<td>M-ary Quadrature Partial Response</td>
</tr>
<tr>
<td>NSK</td>
<td>Minimum Shift Keying</td>
</tr>
<tr>
<td>P/S</td>
<td>Parallel-to-Serial Converter</td>
</tr>
<tr>
<td>$P_e$</td>
<td>Probability of Error</td>
</tr>
<tr>
<td>$P(e</td>
<td>\phi)$</td>
</tr>
<tr>
<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
</tr>
<tr>
<td>QPSK</td>
<td>Quadrature Phase Shift Keying</td>
</tr>
<tr>
<td>QORC</td>
<td>Quadrature Overlapped Raised Cosine</td>
</tr>
<tr>
<td>RX</td>
<td>Receiver</td>
</tr>
<tr>
<td>SCPC</td>
<td>Single Channel per Carrier (Satellite System)</td>
</tr>
<tr>
<td>$\text{sign}(a)$</td>
<td>Signum of the Quantity $a$</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>S/N</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>$SNR_{IF}$</td>
<td>Signal-to-Noise Ratio within the Intermediate Frequency (IF) Bandwidth</td>
</tr>
<tr>
<td>SP</td>
<td>Signal Processor, Signal Processing</td>
</tr>
<tr>
<td>S/P</td>
<td>Serial-to-Parallel Converter</td>
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<td>SQAM</td>
<td>Staggered Quadrature Amplitude Modulation</td>
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<td>SQPSK</td>
<td>Staggered Quadrature Phase Shift Keying</td>
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<tr>
<td>SQORC</td>
<td>Staggered Quadrature Overlapped Raised Cosine</td>
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<tr>
<td>SSB</td>
<td>Single Sideband</td>
</tr>
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<td>S3MQAM</td>
<td>Staggered (Offset) Quadrature Amplitude Modulation scheme employing the class 3 minimal window as an elementary pulse shape</td>
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<tr>
<td>TDA</td>
<td>Transition Detector in the A Channel</td>
</tr>
<tr>
<td>TDB</td>
<td>Transition Detector in the B Channel</td>
</tr>
<tr>
<td>TR</td>
<td>Timing Recovery</td>
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<tr>
<td>TWTA</td>
<td>Travelling Wave Tube Amplifier</td>
</tr>
<tr>
<td>TX</td>
<td>Transmitter</td>
</tr>
<tr>
<td>UD</td>
<td>Uncertainty Diagram</td>
</tr>
<tr>
<td>$U\phi$</td>
<td>Phase Uncertainty (Diagram)</td>
</tr>
<tr>
<td>VCO</td>
<td>Voltage Controlled Oscillator</td>
</tr>
<tr>
<td>VCC</td>
<td>Voltage Controlled Clock</td>
</tr>
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<td>WQAM</td>
<td>Weighted Quadrature Amplitude Modulation</td>
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<td>3MQAM</td>
<td>Quadrature Amplitude Modulation scheme employing the class 3 Minimal window as an elementary pulse shape</td>
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<td>Quadrature Amplitude Modulation scheme employing the class 4 minimal window as an elementary pulse shape</td>
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- $\lambda$ .................................................... timing uncertainty
- $\phi$ .................................................... phase uncertainty
- $h(\phi, \lambda)$ ........................................ nonlinearity surface of the loop
- $p(\phi, \lambda)$ ........................................ probability density function of the process $\{\phi, \lambda\}$
- $\rho \equiv SNR_{L}$ .................................. Signal-to-Noise Ratio within the loop bandwidth
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INTRODUCTION
1.1 DEMAND FOR POWER-BANDWIDTH EFFICIENCY

The available radio frequency spectrum is a limited natural resource, which demands that efficient transmission of information be done. In a satellite system the maximum available power utilization is of primary concern, which requires a power efficient modulation schemes to be employed [B1]-[B11]. An efficient power utilization requires a spacecraft traveling wave tube amplifier (TWTA), or solid-state power amplifier (SSPA), and possibly an earth station high-power amplifier (HPA) to be operated in a nonlinear mode, which causes intermodulation, AM/AM and AM/PM transfer, and spectrum regeneration. To avoid a spectrum spillover into the neighboring channels — which will cause adjacent channel interference (ACI) — a careful design of the signal shaping filters in the modem, the output filter which follows the HPA, transponder input and output multiplex filters, and possibly linearization of the HPA and TWTA needs to be done. However, tight filtering might cause increased intersymbol interference (ISI). Multipath propagation within adjacent transponders on the spacecraft and reuse of frequency by other satellite or terrestrial links will cause a further degradation of the system performance. In this environment, nearly constant envelope modulation schemes with 4 or 8 states and a spectral efficiency of about 1 bit/s/Hz are used.

In cable, telephone line, and terrestrial radio systems (except a mobile radio) the bandwidth efficiency is of primary concern, which leads to the use of high-level modulation schemes [S4], [S6]. 256 state schemes with a spectral efficiency in the range of 6 bit/s/Hz are the field leaders. Such schemes are sensitive to amplitude and group delay imperfections associated with practical (analog) filters, selective fading channels, ACI and cochannel interference (CCI), and phase and timing uncertainties.

During the past years a tremendous amount of effort has been devoted to this subject. This includes novel modulation schemes, which perform well in a complex ACI and CCI environment [S1], [S3], [S5]; power efficient coding but at the expense of bandwidth efficiency [B19]; Ungerböck trellis coding in which an increased power efficiency is achieved without sacrifice of the bandwidth efficiency, but at the expense of the increased number of signal states and therefore increased sensitivity to carrier and timing uncertainties [P111], [S6]; combinations of modulation and coding, different linearization techniques to improve performance of a modulator-power amplifier combination [B10], [S3] — all these at the transmitter — and different carrier and timing recovery schemes with and without equalization at the receiver [B3], [S7]-[S8].

Many investigations into new modulation schemes (pulse shapes) have been based on trial-and-error methods. After a "new" pulse shape (modulation scheme) has been found, heuristically, the analysis — usually supported by computer simulation — followed. The procedure
was iterated until the "best" results were achieved. However, an optimal pulse shape for non-linearly amplified systems has not been described in the literature. Chapter 2 of this thesis deals with this problem in a different way. The idea and details are presented in the thesis outline and Chapter 2, respectively.

The bandwidth efficient modulation schemes usually operate in a linear channel environment, and the practical constraints differ significantly from those systems which operate in a power limited environment. Some of the typical problems, i.e. group delay and amplitude linearity impairments, estimation of the carrier phase and timing and detection, and degradation due to phase noise of the frequency sources are tackled in Chapter 3, Chapter 4 and Chapter 5, respectively.
1.2 DEMAND FOR ACCURATE CARRIER PHASE
AND TIMING ESTIMATION, AND DETECTION

The correct detection of WQAM signals requires accurate carrier phase and symbol timing. It is desired to estimate these parameters directly from measurements on the received data signal. Since WQAM schemes are of the suppressed carrier type, a nonlinear operation on the receiver data is necessary. When the number of states or the imbalance increases, estimation becomes more complex and a higher sensitivity to any phase uncertainty \( \phi \) and timing uncertainty \( \lambda \) might be expected. Some bandwidth and power efficient modulation schemes employ pulse weighting and overlapping to improve performance in nonlinear, ACI and CCI environments, which impose additional constraints on the receiver.

Numerous techniques for carrier and timing (clock) information recovery, based on a maximum likelihood (ML) or a maximum a posteriori (MAP) probability approach, have been analyzed. Among others, references [B3], [S7]–[S8] excellently summarize these efforts. Carrier recovery (CR) loops for an unbalanced QPSK scheme are analyzed in [P127]–[P130]. New CR loops for overlapped schemes are proposed in [P115]–[P116]. However, studies [P131]–[P136] show the superiority of a joint estimation of phase and timing, to which attention is devoted in Chapter 4 of this contribution. Kobayashi [P131] proposed a joint estimator and detector for QAM and SSB schemes which employs a decision-directed feedback. The proposed ML receiver is a recursive type which minimizes the probability of error of an entire data sequence and estimates phase and timing. Gaussian and generally unknown channels which invoke an adaptive receiver are analyzed. Falconer and Salz [P132] showed that the receiver of [P131] does not necessarily produce minimum symbol error probability. An ML receiver is proposed where phase and timing estimates are updated on a symbol-per-symbol basis, rather than by an average taken over an entire data sequence. Mengali [P133] analyzed two different recursive methods for carrier phase and timing acquisition. The coupling between phase and timing is discussed in view of its effects upon the stability and convergence rate of the synchronization algorithm. Franks [P134], and Meyers and Franks [P135] examined a joint carrier phase and symbol timing estimation in SSB and MQAM schemes. The importance of cyclostationarity for CR and timing recovery (TR) is highlighted and new joint CR+TR ML-based estimators are suggested. Poklemba [P136] presented a similar, although different, practical realization of a joint estimator-detector for the QPSK signal set. Kam [P83] pointed out some of the shortcomings of previous approaches and suggested a new ML receiver.

Most of the previous approaches dealt with optimization of phase and timing estimators and eventually the conditional probability of error for small uncertainties \( \phi \) and \( \lambda \). However, in practice, an average probability of error \( P_e = \int \phi \int \lambda P(\epsilon|x,y) p(x,y) \, dx \, dy \) is more important. Optimization of the system performance corresponds to the minimization of
the average probability of error $P_e$, which is equivalent to the minimization of the product $P(e|\phi, \lambda) p(\phi, \lambda)$. Herein, the product of this conditional probability of error $P(e|\phi, \lambda)$ times the probability density function $p(\phi, \lambda)$ of a joint estimator shall be minimized over the entire range of $\phi$ and $\lambda$. The probability density function $p(\phi, \lambda)$ of the decision feedback loop is difficult to obtain in a closed form. Therefore, we describe $p(\phi, \lambda)$ by the nonlinearity surface of the loop $h(\phi, \lambda)$, the importance and meaning of which is explained further, later. By knowing $P(e|\phi, \lambda)$ and $h(\phi, \lambda)$ we are able to minimize $P_e$ and therefore improve the overall system performance.
1.3 THESIS OUTLINE

The flowchart of the thesis is presented in Fig.1.1. In Chapter 2 the WQAM family is introduced. First, attention is focused on the pulse shaping and overlapping at the transmitter to keep the spectral density main lobe as narrow as possible while at the same time minimizing the sidelobe level within the prescribed bandwidth. Although coding might be combined with the WQAM schemes to improve their performance even further, coding analysis is outside the scope of this contribution. Our signal shape consists of weighted rectangular and cosine functions only, which allows an easy practical implementation of the modem even at the highest data rates. We use the gradient search technique to minimize sidelobe levels within an equivalent baseband $|fT_s| = 0$ to 5 (this corresponds to 1 to 2 adjacent channels in practice). The results of this minimization are the values of the weighting coefficients in the WQAM modulator, the meaning of which will be further explained, later. It might be expected that such a modulation scheme with low sidelobes will perform well in an ACI environment. Results which follow show that this hypothesis is correct. A comparison of 4 state WQAM schemes — new and known — is given, including signal shapes, power spectral densities, and performance in the linear and nonlinear ACI environments — typical for satellite systems.
Fig. 1.1. The flowchart of the thesis.
Chapter 1: Introduction

WQAM schemes with more than 4 states might combine different signal shapes at different signal levels to improve the performance in a particular environment. As an example of a possible application of high level WQAM schemes, in Chapter 3, a brief feasibility study of the transmission of North American T1 (DS1) 1544 kb/s or CCITT 2048 kb/s data stream over the analog 240 kHz wide CCITT supergroup is performed. The performance of the simplest, rectangular window, 256 and 1024 state, staggered and nonstaggered QAM schemes is analyzed in the presence of amplitude and group delay imperfections — typically associated with the supergroup filters.

In Chapter 2 and Chapter 3 a perfect knowledge of the carrier phase and symbol timing was assumed. However, this information is not available at the receiver in advance. Before any decision has to be made, the receiver must establish the correct phase and timing reference. This problem is considered in Chapter 4. First, by using a ML approach, four new joint phase and timing estimators are derived. Then, based on the knowledge of $\phi$, $\lambda$, and estimator-detector behavior accumulated over the observation interval(s), MAP detectors are derived. Since both phase and timing estimates depend on the probability of correct decision, this improved decision leads to an improved estimation, etc. As a result, a few new structures are suggested which cancel (or at least significantly attenuate) data-dependent pattern jitter, i.e. these loops perform nearly as well as a continuous wave (CW) loop.

The performance of any coherent digital modulation system is degraded by an excessive amount of phase noise. In this contribution the term phase noise is synonymous with non-thermal random interference such as phase noise of the frequency sources (local oscillators, up/down converters, voltage controlled oscillators), etc. In the past a considerable effort has been devoted by various authors to this subject [B3], [B13], [B15], [B16] [B20]. However, an exact mathematical solution for modulation schemes with more than 4 states has not been published. In Chapter 5 we adopt a practical approach, which yields useful performance curves in which the degradation due to phase noise can be readily seen.

Chapter 6 summarizes the thesis results. In the Chapter 7 a brief research proposal for the future work is outlined. This contribution concludes with an extensive list of references, and a printout of the computer programs.
4-state WQAM schemes
Chapter 2: 4-state WQAM schemes

A brief outline of the chapter follows. In Section 2.1 the WQAM system is introduced. A family of weighted cosine pulse shapes is defined. The general expressions for power spectral density are given. The new modulation schemes termed 3M-, 4M-, HM- and BLQAM are introduced. In Section 2.2 the performance of the WQAM schemes is evaluated in linear and hardlimited channels and in the presence of ACI by means of a computer simulation. A comparison of new and known modulation schemes is given in the table and graphs.
2.1. SYSTEM DESCRIPTION AND SPECTRUM OPTIMIZATION

A weighted quadrature amplitude modulation signal \( s(t) \) with data rates \( R_A = 1/T_A \) and \( R_B = 1/T_B \), where \( A \) and \( B \) represent the in-phase and quadrature channels respectively, Fig. 2.1.(a), is

\[
s(t) = \sqrt{P_A} \sum \alpha_i w_A(t - iT_A) \cos(2\pi f_c t) + \sqrt{P_B} \sum \beta_j w_B(t - jT_B + \delta) \sin(2\pi f_c t)
\]  

(2.1)

The quantities \( P_A \) and \( P_B \) are the signal powers, \( \alpha_i \) and \( \beta_j \) are independent binary \((\pm 1)\) random data sequences which could be obtained from an information sequence through a serial-to-parallel converter, \( w_A(t) \) and \( w_B(t) \) are the pulse shaping functions (weights, windows) in channels \( A \) and \( B \), defined in the interval \((0, T_A)\) and \((0, T_B)\), respectively, \( f_c \) is the carrier frequency, and \( \delta \) is an arbitrary delay in the \( B \) channel. The present concept might be applied to multilevel, multistate systems, where, in general, different pulse shapes might be employed on different baseband levels. As an example, \( M \)-ary QAM schemes employ \( \alpha_i, \beta_j \in \{-1, \ldots, -1, -1, \ldots, +1, \ldots, +1\} \) and rectangular signal shape on every baseband signal level. By allowing \( P_A \neq P_B \) and \( R_A \neq R_B \) an (intentionally) unbalanced system might be analyzed, or the effects of imbalance on the performance of balanced systems might be estimated. Generally \( P_A \neq P_B \) and \( R_A \neq R_B \); i.e., the QAM system is unbalanced. In this chapter, we limit our attention to the 4-state balanced system with \( P_A = P_B = P \), \( R_A = R_B = R = 1/2T = 1/T_S \) and \( w_A(t) = w_B(t) = w(t) \), particularly the nonstaggered schemes with \( \delta = 0 \) and the staggered schemes with \( \delta = T = T_S/2 \). The equivalent baseband power spectral density of the WQAM signal \( s(t) \) is \([B15,p.120] \)

\[
S_b(f) = \frac{2}{T_S} |W(f)|^2
\]  

(2.2)

where \( W(f) \) is the Fourier transform of the baseband signal pulse shape \( w(t) \).
Chapter 2: 4-state WQAM schemes

Figure 2.1. The weighted quadrature amplitude modulation system:
(a) Transmitter,
(b) Receiver.

SP = signal processor,
S/P = data signal.
We define the weighted cosine pulse shape as

\[ w_n(t) \stackrel{\text{def}}{=} \begin{cases} \sum_{i=1}^{n} g_i \cos \left( \frac{\pi K_i}{L_i} \right), & \text{for } |t| \leq L_i/2, \\ 0, & \text{for } |t| > L_i/2, \end{cases} \quad (2.3a) \]

where

\[ K_1 = 0, \quad \sum_{i=1}^{n} g_i = 1. \quad (2.3b) \]

The quantity \( g_i \) is the weighting factor (\( g_i = a_i \) or \( g_i = b_i \), refers to the A or B channel, respectively), \( K_i \) is the frequency parameter, \( L_i \) is the overlapping factor normalized to the symbol duration \( T_S \) and \( n \) is the class of the signal pulse shape. The Fourier transform of the weighted cosine signal is

\[
W_n(f) = \int_{-L_i/2}^{L_i/2} w_n(t) \exp(-j2\pi ft) \, dt \\
= \frac{1}{2} \sum_{i=1}^{n} a_i L_i [W_{io}(fL_i - K_i) + W_{io}(fL_i + K_i/2)] \quad (2.4)
\]

where

\[
W_{io} = \frac{\sin \pi x}{\pi x} = \text{sinc}(\pi x) \quad (2.5)
\]

The right choice of \( g_i, K_i \) and \( L_i \) parameters depends on the desirable goal. Here, our primary concern is the spectral density with narrow mainlobe and minimal sidelobe levels within \(|fT_s| = 0\) to \(5\). Since the signal with larger overlapping factor \( L_i \) possesses the narrower spectrum, \( L = L_i \) could be one of additional practical constraints. On the other hand, the signal with large \( L \) causes intersymbol interference at the sampling point. A further increase in \( L \) leads to a multilevel signal. The signal \( w_n(t) \) with a smooth transitions from maximum to zero and a good compromise between the mainlobe width and the sidelobe levels suggests the \( K_i \) to be an even integer. As an example, assume the overlapping factor \( L_i = L \), and \( n = 3 \). The Fourier transform \( W_3(f) \) of the class 3 signals, \( n = 3 \) in (2.4), is

\[
W_3(f) = \frac{\sin(\pi fL/2)}{(\pi fL/2)} \left[ \frac{\cos(\pi fL/2)}{(fL)^2 - 1} \right] \\
\quad \text{QPSK like} \quad \text{MSK like}
\]

\[
\times L^{-4a_1 - (5a_1 - 4a_2 + a_3)(fL)^2 + (a_1 - a_2 + a_3)(fL)^4 \over (fL)^2 - 1} \quad (2.6)
\]

The spectral density consists of a QPSK-like factor, a MSK-like factor, and a third factor which can be optimized to obtain a better spectral performance. Our results, reported in
Section 2.2 indicate that the signals obtained for the linear channel also have a good performance in non-linearly amplified (hardlimited) systems. The performance of modulation schemes is summarized in Table 2.1. The pulse shape which gives minimum sidelobes within the $|fT_s| = 0$ to 5 frequency band is termed as the minimum. The choice of a pulse duration $L$ will depend on an expected amount of ACI, frequency separation, and filter strategy. The subclass of windows termed as minimum, with $L = T_S$ is also known as the Blackman-Harris family [P63]. If $L > T_S$ the consecutive pulses will overlap, and the new elementary waveshape within the interval $(-T_S/2, T_S/2)$ will generally be asymmetric about the plane $t = 0$. The + and - pulses will also be asymmetric. However, the symmetry is maintained in average.

The QPSK, BPSK, and MQAM schemes analyzed in Chapter 3, are using rectangular signal shape (Dirichlet window). The MSK consists of Hann 1 signal in each of two quadrature channels, and the B channel is additionally staggered by $T_S/2$ to maintain a constant envelope of the signal $s(t)$, Fig. 2.1. However, all other modulation schemes employ overlapping techniques (overlapping factor $L > 1$) to obtain a narrower spectrum. The envelope of signal $s(t)$ is not constant in general. But spectrum sidelobes of schemes with the nonconstant envelope are far below the spectrum sidelobes of schemes with the constant envelope. They remain low even after hardlimiting, Fig. 2.3. The QORC scheme employs the Hann 2 signal window with the overlapping factor $L = 2T_S$. Further reduction of sidelobes level is achieved by employing the Hamming weighting and overlapping, i.e., the HMQAM.

In [P56] the MMSK signal family was introduced and the particular scheme with parameter $\Lambda = 0.8$ was found to be the best, based on a trial-and-error method. However, the BLQAM which is equivalent to the MMSK scheme with $\Lambda = 0.84$ seems to be the best.

The sidelobe level of the 3MQAM scheme is 12.7 dB lower than the BLQAM's and far below the QORC's. Additional sidelobe level reduction could be achieved at the expense of a wider mainlobe, i.e., by employing the 4MQAM.

The signal shapes $u_n(t)$ and the corresponding power spectral densities are given in Fig. 2.2 and Fig. 2.3, respectively.

The 3MQAM outperforms MSK in the whole frequency band, as well the QPSK, except around the first zero of the QPSK at $fT_S = 1$. From $fT_S = 1.17$ up, the sidelobes of 3MQAM are below the QORC. The 3MQAM represents a good compromise between the mainlobe width and the sidelobes level.
<table>
<thead>
<tr>
<th>Name of the window</th>
<th>Zero width</th>
<th>Sidelobe level, dB</th>
<th>Duration L/T</th>
<th>QAM name</th>
<th>a1 K1</th>
<th>a2 K2</th>
<th>a3 K3</th>
<th>a4 K4</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 Rectangular (Dirichlet)</td>
<td>1.00</td>
<td>-13.0</td>
<td>1.0</td>
<td>QPSK, QPSK</td>
<td>1.000000</td>
<td>0.000000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2 Cosine1 (Hann 1)</td>
<td>1.50</td>
<td>-23.0</td>
<td>1.0</td>
<td>MSK</td>
<td>-</td>
<td>1.000000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2 Cosine2 (Hann 2)</td>
<td>1.00</td>
<td>-31.4</td>
<td>2.0</td>
<td>QORC (IJJF)</td>
<td>0.500000</td>
<td>0.500000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>2 Hamming</td>
<td>1.17</td>
<td>-42.6</td>
<td>1.7</td>
<td>HMQAM</td>
<td>0.540000</td>
<td>0.460000</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>3 Blackman</td>
<td>1.50</td>
<td>-58.1</td>
<td>2.0</td>
<td>BLQAM</td>
<td>0.420000</td>
<td>0.500000</td>
<td>0.080000</td>
<td>-</td>
</tr>
<tr>
<td>3 Minimum Bl.-Harris3</td>
<td>1.50</td>
<td>-70.8</td>
<td>2.0</td>
<td>3MQAM</td>
<td>0.423333</td>
<td>0.497755</td>
<td>0.079222</td>
<td>-</td>
</tr>
<tr>
<td>4 Minimum Bl.-Harris4</td>
<td>2.00</td>
<td>-92.0</td>
<td>2.0</td>
<td>4MQAM</td>
<td>0.358755</td>
<td>0.488290</td>
<td>0.141288</td>
<td>0.01168</td>
</tr>
</tbody>
</table>

Table 2.1. The summary of WQAM system performance.
Figure 2.2. The signal shapes (windows) of the WQAM schemes: The QPSK (Dirichlet), MSK (cosine, Hann 1) — both the one symbol duration —, the HMQAM (Hamming) $L = 1.7 T_S$ symbol duration, the QORC, i.e. IJF (cosine squared, Hann 2), the 3MQAM (minimum class 3), and the 4MQAM (minimum class 4) modulation schemes — all two symbols duration.
Figure 2.3. The power spectral densities of WQAM schemes. Ordinata in dB.
2.2 PERFORMANCE IN ACI ENVIRONMENT

To evaluate the performance of WQAM schemes in ACI environment, the computer simulation procedure with the following assumptions is used. The WQAM receiver is illustrated in Fig.2.1(b). Here ideal carrier recovery (CR) and symbol timing recovery (STR) are assumed. The simulation model of a system is given in Fig.2.4. A 5th order phase equalized (constant group delay) Butterworth filter with single-sided bandwidth $B_1T_s = 0.55$ is assumed in every channel. The interferer's phases and symbol timings are randomized over the interval $(0, 2\pi)$ and $(0, T_s)$ respectively.

2.2.1 Linear channel

In the linear channel system model the hardlimiter in Fig.2.4 is by-passed. The performance of the WQAM systems vs. the normalized channel spacing frequency in the presence of two ACIs with the power level $I_1 = I_2 = 0$ dBr, equal to the power of the main channel, is summarized in Fig.2.5.(a). The performance of WQAM systems vs. the ACI interference level, i.e. the relative fading depth, is given in Fig.2.5.(b). The staggered S3MQAM using simple Butterworth filter is performing well even in the presence of strong ACI and outperforms QPSK, SQPSK, SQORC(SIJR) [B8],[P52], and MMSK [P56].
The simulation model of a QAM system in the Gaussian noise, Adjacent-Channel Interference (ACI) and/or Co-Channel Interference (CCI) environment. HL=Hard Limiter.

Figure 2.4. The simulation model of a QAM system.
a) The performance of the WQAM systems versus the normalized channel spacing frequency $fT_s$ in the presence of two equal power adjacent channel interferers.

The ordinates represent the degradation in dB, relative to the theoretical value $E_b/N_0 = 8.4$ dB at $P_e = 10^{-4}$. The (S)QPSK, MSK, (S)QORC, and (S)3MQAM—all, main channel and interferences are of the same type, correspondingly. The SQPSK & 3M labeled curve represents SQPSK main channel and two ACI interferences of the S3MQAM type.

b) The performance of the WQAM systems versus the adjacent channel interferers' level in dB (or flat fading depth in dB).

Figure 2.5. The linear channel performance of WQAM schemes.
2.2.2 Hardlimited channel

The power spectral density of the signal $s(t)$, Point 1 after the hardlimiter in Fig.2.4, is given in Fig.2.3. The sidelobes of S3MQAM scheme are about 10 dB below those of the SQORC and far below QPSK and MSK. In the present filtering strategy configuration, the constant envelope signals (QPSK, SQPSK and MSK) perform in exactly the same way as in the case of linear channel. The performance of the WQAM systems vs. the normalized channel space frequency and in the presence of two ACIs with the power level $I_1 = I_2 = 0$ dBr, equal to the level of the main channel, is summarized in Fig.2.6(a). The performance of the WQAM system vs. the ACI interference level, i.e. a relative flat fading depth, is illustrated in Fig.2.6(b).

The S3MQAM outperforms all competitors if a normalized space frequency of adjacent channels is larger than $1.35 fT_2$. As the fade depth increases the advantage of the S3MQAM becomes more obvious. As an example, at the fade depth of 12 dB, S3MQAM outperforms SQORC (IIF) modulation scheme by more than 1 dB, Fig.2.6b.

Single channel per carrier (SCPC) satellite systems above 10 GHz, and terrestrial and satellite systems for mobile, maritime, and aeronautical services, are some of the typical examples where these schemes might find an application. In practice, observed —main— channel might be faded due to the rain (above 10 GHz satellite SCPC systems) or due to the shadowing (mobile radio), while neighboring channels do not experience either fading or shadowing. Under this circumstances the main channel is experiencing an attenuation of the signal level, and in addition it must cope with an increased ACI. As it is shown, our new S3MQAM scheme outperforms its competitors under these practical conditions.
 normalized space frequency fTsymbel

a) The performance of the WQAM systems versus the normalized channel spacing frequency $fT_s$ in the presence of two equal power adjacent channel interferers.

The ordinates represent the degradation in dB, relative to the theoretical value $E_b/N_0 = 8.4$ dB at $P_e = 10^{-4}$. The (S)QPSK, MSK, (S)QORC, and (S)3MQAM—all, main channel and interferences are of the same type, correspondingly. The SQPSK & 3M labeled curve represents SQPSK main channel and two ACI interferences of the S3MQAM type.

Figure 2.6. The hardlimited channel performance of WQAM schemes.
2.3 CONCLUSION

In this chapter the weighted quadrature amplitude modulation (WQAM) was introduced. By using the gradient search technique, a family of pulse shapes with a narrow mainlobe and minimal sidelobe levels within an equivalent baseband \( |fT_s| = 0 \) to 5 (this corresponds to 1 to 2 adjacent channels in practice) was found. The performance of 4-state WQAM schemes was evaluated in the ACI linear and nonlinear channel environments, with the channel spacing and fading depth (signal attenuation) as parameters, by means of the computer simulation. One of the new staggered WQAM schemes, termed S3MQAM, outperforms other known members of the WQAM family, i.e. (S)QPSK, MSK and (S)QORC in almost all practical situations. By using the S3MQAM scheme more compact spacing —more efficient transmission, i.e. more b/s/Hz— than before might be possible.
256 and 1024QAM schemes
The WQAM family was introduced in the previous chapter. The performance of 4 state WQAM schemes was evaluated in the presence of ACI. In this chapter the performance of the 256 and the 1024QAM schemes, which employ rectangular window (pulse shape) at each symbol level, is evaluated in the presence of amplitude and group delay impairments. These are the typical impairments associated with the practical (analog) filters employed in the radio and cable systems.

An outline of the chapter follows. In Section 3.1 an overview of system considerations is given. The computer simulation model is presented in Section 3.2. The chapter concludes with the results of the computer simulation.
3.1 SYSTEM CONSIDERATIONS

To transmit 1544 (2048) kb/s over a 240 kHz wide CCITT supergroup, a digital modulation scheme with a spectral efficiency better than 1544(2048)/240=6.43 (8.53) b/s is required. To achieve this, (S)256QAM and (S)1024QAM schemes, with a theoretically maximum spectral efficiency of 8 b/s/Hz and 10 b/s/Hz respectively, are considered in this chapter. Forward error correcting (FEC) coding might be employed, which will bring the corresponding channel rate to the 1.6 Mb/s (2.1 Mb/s). Throughout this chapter the information rate of 1544 (2048) kb/s is associated with a channel rate of 1.6 (2.1) Mb/s. The 1.6 Mb/s might be transmitted by the 256QAM scheme, which will give the symbol rate of 0.2 MBaud and the spectral efficiency of 6.67 b/s/Hz. Should 1024QAM scheme be used, the symbol rate will be 0.16 MBaud, Table 3.1. In this case the less steep cosine filter with $\alpha = 0.5$ would be sufficient ($\alpha = 0.2$ if 256QAM is employed). The 2.1 Mb/s bit rate requires the 1024QAM to be employed, which gives the symbol rate of 0.21 MBaud, noise bandwidth of 210 kHz, channel spectral efficiency of 8.75 b/s/Hz and $\alpha = 0.14$ filters, Table 3.1.

<table>
<thead>
<tr>
<th>Mb/s</th>
<th>256QAM</th>
<th>1024QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MBaud</td>
<td>$\alpha$</td>
</tr>
<tr>
<td>1.6</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>2.1</td>
<td>—</td>
<td>—</td>
</tr>
</tbody>
</table>

Table 1. The 256 and the 1024QAM system considerations.
3.2 COMPUTER SIMULATION MODEL

In practice, at the transmitter the 1.544 (2.048) Mb/s data stream might be interleaved and FEC encoded to 1.6 (2.1) Mb/s and split into two streams by a serial-to-parallel converter S/P. Each stream might be differentially and Gray encoded into the 16 (32) level signals with the symbol rate of 200 (210) kBaud. These signals are fed into the $\alpha = 0.2$ ($\alpha = 0.14$) cosine filters and modulated by the in-phase and quadrature signals.

256QAM (1024QAM) signal is fed to the in-phase and quadrature receiver. The in-phase signal is used to extract the symbol timing clock. After demodulation, the baseband adaptive transversal equalizer might be applied to equalize the impairments caused by the supergroup-through-connect filters and other system impairments. After Gray and differential decoding, the in-phase and quadrature signals are parallel-to-serial converted into a 1.6 (2.1) Mb/s signal. This signal is de-interleaved and FEC decoded and becomes the 1.544 (2.048) Mb/s replica of the original signal.

To be able to simulate the influence of amplitude and group delay impairments on the 256QAM and 1024QAM systems we adopted a simplified computer model without coding and equalization and with the following properties: the pseudo-random data has period of $2^{31} - 1$, a half million samples is used for every run, the carrier frequency is set to the zero, an ideal carrier recovery is assumed, Gaussian noise is included analytically and the timing recovery has the memory which is 256 symbols long, i.e. practically perfect timing exists.†

† The computer model of the M-ary quadrature amplitude modulation MQAM, system is given in Fig. 3.1. The input data stream $d(t)$ with the data rate of 1.6 (2.1) Mb/s is split, by using the serial-to-parallel converter S/P, into the two independent data streams $A(t)$ and $B(t)$ respectively. The $B(t)$ could be staggered by a time delay $\delta = T_S/2$, where $T_S$ is the time period of $B(t)$. $A(t)$ and $B(t)$ are processed in the $2 \rightarrow L$ level converter, $z/\sin z$ and $\sqrt{\alpha}$ cosine filter to give the corresponding signals $a(t)$ and $b(t)$, respectively. $M = L \times L$ and $\alpha$ is the roll-off factor of the cosine filter. The corresponding parameters are given in Table 3.1. The channel is represented by the filter $H(f)$ and the white Gaussian noise. In the practice, the cascade of supergroup (SG)-through-connect filters might have a significant impact on the performance of transmission. At the receiver the in-phase and quadrature channels are fed into the $\sqrt{\alpha}$ cosine filters. Assuming wide $H(f)$, this filtering will satisfy both Nyquist and matched filter criteria, i.e. an optimum reception. If this is not the case, an adaptive equalizer should be employed. The carrier recovery and timing recovery circuits provide the correct phase and timing instants to make a correct decision. Finally, by $L \rightarrow 2$ mapping and the parallel-to-serial conversion P/S an estimation of the original data, $\hat{d}(t)$, is

† The carrier recovery and timing recovery schemes are analyzed in the Chapter 4.
available.
The results previously achieved by this computer model were in a close agreement with the measured results made on different hardware models with up to 256 states. Here, we present the performance of 256QAM and 1024QAM modems in the presence of amplitude or group delay imperfections similar to those associated with practical SG-through-connection filters.
Chapter 3: 256 and 1024 QAM schemes

Figure 3.1. The computer model of MQAM system.
Chapter 3: 256 and 1024QAM schemes

3.3. RESULTS

Three sets of results are presented. First, the performance of the 1.6 Mb/s modems using 256QAM schemes and a symbol rate of 0.2 MBaud is compared with the 1.6 Mb/s modems using 1024QAM schemes with a symbol rate of 0.16 MBaud.

- The degradation in dB is the difference between the carrier-to-noise ratio \((C/N)\) required to achieve the \(P_e = 10^{-6}\) performance when group delay or amplitude imperfections exist, and the \((C/N) = 33\) dB needed to achieve the same probability of error performance \(P_e\) with the 256QAM scheme in an ideal Gaussian channel environment.

The performance of 256QAM and 1024QAM systems in the presence of linear group delay is summarized in Fig.3.2. Staggered schemes seem to be less sensitive to the linear group delay imperfections than their nonstaggered counterparts. In the presence of strong group delay impairments the S1024QAM, with more relaxed filter \(\alpha = 0.5\), outperforms (S)256QAM schemes with \(\alpha = 0.2\). In the Hermitian symmetric channel (parabolic group delay as given in Fig.3.3 and parabolic amplitude gain as given in Fig.3.5) the staggered and nonstaggered schemes perform equally. However, in the presence of the linear amplitude gain impairments nonstaggered schemes outperform their staggered counterparts, Fig.3.4, i.e. 256QAM gives the best performance and it is followed by S256QAM, 1024QAM and S1024QAM. Usually amplitude distortions are less difficult to be equalized, which might prefer staggered schemes to be employed.

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1 The definition of group delay and amplitude gain corresponds to that given in [18, p.165].
2 A higher implementation margin for the 1024QAM schemes is assumed in figures which follow.
Figure 3.2. The performance degradation of the 256 and 1024QAM (1.544/1.6) Mb/s systems versus the linear group delay impairments. The degradation in dB is the difference between the C/N necessary to achieve the $P_e = 10^{-6}$ performance when impairments are present, and the C/N≈33 dB giving $P_e = 10^{-6}$ without impairments.
Figure 3.3. The performance degradation of the 256 and 1024QAM (1.544/1.6) Mb/s systems versus the parabolic group delay impairments. The degradation in dB is the difference between the C/N necessary to achieve the $P_e = 10^{-6}$ performance when impairments are present, and the C/N≈33 dB giving $P_e = 10^{-6}$ without impairments.
Figure 3.4. The performance degradation of the 256 and 1024QAM (1.544/1.6) Mb/s systems versus the linear amplitude gain impairments. The degradation in dB is the difference between the C/N necessary to achieve the $P_e = 10^{-6}$ performance when impairments are present, and the $C/N \approx 33$ dB giving $P_e = 10^{-6}$ without impairments.
Figure 3.5. The performance degradation of the 256 and 1024QAM (1.544/1.6) Mb/s systems versus the parabolic amplitude gain impairments. The degradation in dB is the difference between the C/N necessary to achieve the $P_e = 10^{-6}$ performance when impairments are present, and the C/N≈33 dB giving $P_e = 10^{-6}$ without impairments.
Second, the performance of the 2.1 Mb/s modems using the 1024QAM schemes is given.

- The degradation in dB is the difference between the \( (C/N) \) required to achieve the \( P_e = 10^{-6} \) performance when group delay or amplitude imperfections exist, and the \( (C/N) = 39 \) dB needed to achieve the same \( P_e \) performance with the 1024QAM system in an ideal Gaussian noise channel.

The staggered schemes seem to be less sensitive to the linear group delay while there is no difference in the performance in the presence of parabolic distortions, Fig. 3.6. However, the nonstaggered schemes perform better in the presence of linear amplitude gain distortion, Fig. 3.7.

Third, the computer generated eye diagram of the 256QAM system in the presence of 1.5 \( \mu s/\text{MHz} \) group delay is given in Fig. 3.8 (a). The central eyes are quite open, while the timing eyes are smeared, i.e. timing jitter exists. The eye diagram of the same system but without group delay distortion is given in Fig. 3.8 (b). In the S1024QAM system with the 0.5 \( \mu s/\text{MHz} \) linear group delay and \( \alpha = 0.14 \) cosine filter middle eyes are much narrower, intersymbol interference is present, while the small timing eyes are totally smeared, i.e. a strong timing jitter exists, Fig. 3.9. However, this will not be the case in an ideal \( \alpha = 1.0 \) Nyquist channel, i.e. both sampling and timing eyes are fully open, Fig. 3.10. Obviously, high level modulation schemes are very sensitive to any timing recovery uncertainty.
Figure 3.6. The performance degradation of 1024QAM (2.048/2.1) Mb/s systems versus the group delay impairments. The degradation in dB is the difference between the C/N necessary to achieve the $P_e = 10^{-6}$ performance when impairments are present, and the C/N≈39 dB giving $P_e = 10^{-6}$ without impairments.
Figure 3.7. The performance degradation of the 1024QAM (2.048/2.7) Mb/s systems versus the amplitude gain impairments. The degradation in dB is the difference between the C/N necessary to achieve the $P_e = 10^{-6}$ performance when impairments are present, and the C/N≈39 dB giving $P_e = 10^{-6}$ without impairments.
Figure 3.8. The 256QAM 0.2 MBd system with the $\alpha = 0.2$ filtering. The computer generated eye diagrams.
Figure 3.9. The S1024QAM 0.21 MBd system with the $\alpha = 0.14$ filtering. Linear group delay of 0.5 $\mu$s/MHz. The computer generated eye diagram.
Figure 3.10. The 1024QAM 0.21 MBd system with the $\alpha = 1.0$ filtering. No group delay impairments. The computer generated eye diagram.
3.4. CONCLUSION

In this chapter the performance analysis of 256 and 1024QAM schemes is performed in the presence of amplitude and group delay impairments. A brief feasibility study of the transmission of North American T1 (DS1) 1544 kb/s or CCITT 2048 kb/s data stream over the analog 240 kHz wide CCITT supergroup is given. Staggered QAM schemes perform better in the presence of linear group delay impairments, but nonstaggered schemes are less sensitive to linear amplitude gain impairments.

In this, and the previous chapter, it has been assumed that the receiver is provided with a perfect knowledge of the carrier phase and the timing instants. However, in practice, this information is not available in advance. Before any decision is to be made, the receiver must establish, by itself, a proper phase and timing. This topic is the concern of our next chapter.
ON COMPOSITE
PHASE AND TIMING ESTIMATION
AND DETECTION
OF WEIGHTED QAM SIGNALS
In Chapter 2 the WQAM family has been introduced. Its performance has been analyzed in the presence of different impairments such as: ACI, hardlimiter nonlinearity (Chapter 2), amplitude and group delay (Chapter 3) imperfections. An exact knowledge of the carrier phase and symbol timing(s) has been assumed. However, in practice, this is not the case and prior any decision has to be made, the proper carrier phase and symbol timing(s) need to be established. In this chapter, we search for new (improved) receivers for composite phase and timing estimation and detection of WQAM signals, and then, compare their performance with the known (conventional) ones.

An outline of the chapter follows. The WQAM signal has been defined in (2.1). However, for the purposes of this chapter a more precise definition is necessary. This, and the Bayes-based optimization strategy are given in the Section 4.1. Section 4.2 deals with ML-based classical and equalized DFCRL. The performance of these loops is presented in the form of an uncertainty diagram (UD) and a nonlinearity surface. Finally, MAP-based crosstalk cancellation DFCRL, timing intersymbol interference (ISI) cancellation DFCRL, and FEEDLOOP are presented in Section 4.3.
4.1. SYSTEM DESCRIPTION AND OPTIMIZATION STRATEGY

A weighted quadrature amplitude modulation signal $s(t, \alpha, \beta, \epsilon_1, \epsilon_2, \theta)$ with data rates $R_A = 1/T_A$ and $R_B = 1/T_B$ in the in-phase and quadrature channels respectively, can be represented by

$$s(t, \alpha, \beta, \epsilon_1, \epsilon_2, \theta) = \sqrt{P_A} \sum_i \alpha_i w_A[t - iT_A + \epsilon_1(t)] \cos[2\pi f_c t + \theta(t)]$$

$$+ \sqrt{P_B} \sum_j \beta_j w_B[t - (jT_B + \delta) + \epsilon_2(t)] \sin[2\pi f_c t + \theta(t)],$$

$i, j \in \{1\}$ (4.1)

Quantities $P_A$ and $P_B$ are the signal powers, $\alpha$ and $\beta$ are independent random data sequences, $w_A(t)$ and $w_B(t)$ are the pulse shaping functions (weights, windows) in channels $A$ and $B$, defined on the interval $(-T_A/2, +T_A/2)$ and $(-T_B/2, +T_B/2)$, respectively, $f_c$ is the carrier frequency, and $\delta$ is an arbitrary delay in the $B$ channel. $\epsilon_1(t)$ and $\epsilon_2(t)$ are random timing offsets assumed to be uniformly distributed on the interval $(-T_A/2, +T_A/2)$ and $(-T_B/2, +T_B/2)$, respectively— and independent of each other in general, and $\theta(t)$ is the random carrier phase assumed to be uniformly distributed on the interval $(-\pi, +\pi)$. Summations over $i, j$ extend over the set of all integers $\{1\}$. In the text which follows we use abbreviations $\epsilon_1 = \epsilon_1(t)$, $\epsilon_2 = \epsilon_2(t)$ and $\theta = \theta(t)$.

We would like to minimize the average probabilities of error $P_{eA}, P_{eB}$, where

$$P_{eA} = \int_{\phi} \int_{\lambda_1} P(e|x, y) p(x, y) \, dx \, dy$$

$$P_{eB} = \int_{\phi} \int_{\lambda_2} P(e|x, y) p(x, y) \, dx \, dy$$

Quantities $P(e|x = \phi, y = \lambda_1)$ and $P(e|x = \phi, y = \lambda_2)$ are the conditional error rates — where errors are committed by the decision devices within the loop and are conditioned on the phase uncertainty $\phi = \theta - \hat{\theta}$ and the timing uncertainties $\lambda_1 = \epsilon_1 - \hat{\epsilon}_1$ and $\lambda_2 = \epsilon_2 - \hat{\epsilon}_2$, respectively. $\hat{\theta}, \hat{\epsilon}_1$, and $\hat{\epsilon}_2$ are estimates of the phase and timings as provided by the corresponding loops. $p(x = \phi, y = \lambda_1)$ and $p(x = \phi, y = \lambda_2)$ are the corresponding probability density functions. Typical $P(e|\phi, \lambda = 0)$ and $p(\phi, \lambda = 0)$ functions are given in Fig.4.1.
Figure 4.1. The typical conditional probability of error $P(e|\phi)$, probability density function $p(\phi)$, and average probability of error $P_e$ curves of the MQAM scheme employing: a) classical carrier recovery loop (e.g. classical decision feedback carrier recovery loop DFCRL) with normalized loop detuning $\beta_N = 0$, b) classical DFCRL with $\beta_N = 0.2$, and c) crosstalk cancellation DFCRL.

The values of corresponding $P(e|\phi)$ curves are enlarged for illustrative purposes. In practice they are as low as $10^{-12}$ at $\phi \approx 0$, and approach to 1 as $\phi \to 45^\circ$. 
Chapter 4: On Composite Phase and Timing Estimation and Detection

The minimization of the $P_eA, P_eB$ is equivalent to the minimization of the corresponding products under the double integrals in (4.2a-b). Therefore, we would like to minimize these products over the entire range of $\{\phi, \lambda\}$. Both, yet unknown, functions depend on the known WQAM signal constellation and, yet unknown, circuit design which we would like to find.

To achieve this, a Bayes estimate is used with the following cost functions: $C_{ii} = 0$ for a correct decision and $C_{ij} = 1, i \neq j$ for an erroneous decision [B22]. Although a similar analysis might be found in many books related to communications, our approach has a few differences which lead to new solutions. These differences are emphasized in the text which follows, including analysis of a few new receivers. A few preliminaries need to be established. The received signal $r(t)$ consists of the signal $s(t)$ and the band-limited ($-B_{IF}/2 \to +B_{IF}/2$) white Gaussian noise with single sided spectral density $N_0$, i.e.

$$r(t) = s(t) + n(t) \quad (4.3)$$

where, for convenience, we have dropped the dependence of $s(t)$ and thus $r(t)$ on $\alpha, \beta, \epsilon_1, \epsilon_2, \theta$. We assume that the receiving $IF$ filter $H(f)$ is wide enough to cause no additional shaping of the signal pulse and narrow enough to allow a narrowband representation of the noise $n(t)$, i.e.

$$n(t) = n_c(t) \cos(2\pi f_c t + \theta) + n_q(t) \sin(2\pi f_c t + \theta) \quad (4.4)$$

Furthermore, we assume that $H(f)$ is symmetric around $f_c$, i.e., the in-phase noise component $n_c(t)$ and quadrature component $n_q(t)$ are statistically independent. In practice, a tight filtering might be imposed on both the transmitter and receiver side accompanied with nonlinearities, adjacent and cochannel interferences (ACI and CCI), selective fading, and other impairments. However, taking into account all of these factors at the same time leads to extremely complex expressions which remain to be solved. We limit our attention to a more tractable situation, where the WQAM signal is immersed in white Gaussian noise only. A responsibility of the receiver will be based on observation of $r(t)$ as given in (4.3), decide which of data pairs $(\alpha, \beta)$ are sent. However first, both phase coherence and timing synchronization need to be established. Composite phase and timing estimation and detection analysis is performed as follows. We use the sampled data approach [B21]. $r(t)$ is uniformly sampled on the interval $(0, T_N) N$ times. The spacing between samples is $\Delta t = t_k - t_{k-1} = 1/2 B_{IF}$, which creates independent samples. Now, (4.3) might be written in the sampled form

$$\bar{r} = \bar{s} + \bar{n} \quad (4.5)$$

Quantities $\bar{r}$ and $\bar{s}$ are the $N$-dimensional vectors of the received and transmitted signal, respectively, and $\bar{n}$ is the $N$-dimensional noise vector, the components of which are statistically independent zero-mean Gaussian random variables with variances $N_0 B_{IF}$. Without loss of generality, we assume that a decision on $r(t)$ occurs at $t_k = k$, i.e., at the $k$-th data
symbol. Carrier phase is observed on the interval \([k-1-T_e,k-1]\) where, with very little loss of generality, it might be assumed that \(T_e = V_{Ac}T_A\). \(V_{Ac}\) is an integer such that \(V_{Ac} > 1\), where the first subscript \(A\) refers to the \(A\) channel and the second subscript \(c\) refers to the carrier phase estimation. We arbitrarily assume that \(T_B = \xi T_A\) and \(\xi \geq 1\), i.e., symbol rate \(R_A\) in the \(A\) channel is equal to or higher than \(R_B\). During the interval \([k-1-T_e,k-1]\) there are \(V_{Ac}\) symbols \((\alpha_{k-1-T_e}, \alpha_{k-T_e}, \ldots, \alpha_{k-1})\) in the \(A\) channel if \(\epsilon_1 = 0\), \(V_{Ac} + 1\) symbols (the right most part of \(\alpha_{k-2-T_e}, \alpha_{k-1-T_e}, \ldots, \alpha_{k-2}\), the left most part of \(\alpha_{k-1}\)) if \(\epsilon_1 < 0\), and \(V_{Ac} + 1\) symbols (the right most part of \(\alpha_{k-1-T_e}, \alpha_{k-T_e}, \ldots, \alpha_{k-1}\), the left most part of \(\alpha_{k-1}\)) if \(\epsilon_1 > 0\). The data stream in the \(B\) channel is generally not aligned to the stream in the \(A\) channel. Depending on the data rate ratio \(\xi\), static offset \(\delta\), and amount of random offsets \(\epsilon_1\) and \(\epsilon_2\), different combinations exist. Generally, \(V_{Bc} = 1 = V_{Ac}/\xi + 1\) symbols in the \(B\) channel belong to the observation interval \([k-1-T_e,k-1]\). Timing epochs \(\epsilon_1\) and \(\epsilon_2\) are observed on the interval \([k-1-T_A,k-1]\) and \([k-1-T_B + \Delta \epsilon, k-1 + \Delta \epsilon]\) respectively, where \(\Delta \epsilon\) accounts for the relative offset between \(\epsilon_1\) and \(\epsilon_2\).

With these preliminaries, the Bayes estimate of \(\epsilon_1, \epsilon_2\) and \(\theta\) might be derived as follows: Choose \(\epsilon_1 = \hat{\epsilon}_1\), \(\epsilon_2 = \hat{\epsilon}_2\) and \(\theta = \hat{\theta}\) such that the conditional density function of the parameters \(\alpha, \beta, \epsilon_1, \epsilon_2, \theta\) given observation vector \(\vec{r}\), i.e., \(f(\epsilon_1, \epsilon_2, \theta | \vec{r})\), is maximum. Such an estimator is called nondata-aided (NDA), while a data-aided (DA) estimator employs data estimates \(\hat{\alpha}\) and \(\hat{\beta}\) instead. We use Bayes formula [B22]

\[
f(\epsilon_1, \epsilon_2, \theta | \vec{r}) = \frac{f(\vec{r} | \alpha, \beta, \epsilon_1, \epsilon_2, \theta) f(\epsilon_1, \epsilon_2, \theta)}{f(\vec{r})}
\]

where \(< \cdot >\) represents the statistical average on data \(\alpha, \beta\). Since \(f(\vec{r})\) is independent of \(\epsilon_1, \epsilon_2, \) and \(\theta\), the Bayes estimate of \(\epsilon_1, \epsilon_2, \) and \(\theta\) might be restated as follows: Choose \(\epsilon_1 = \hat{\epsilon}_1, \epsilon_2 = \hat{\epsilon}_2,\) and \(\theta = \hat{\theta}\) such that the product \(< f(\vec{r} | \alpha, \beta, \epsilon_1, \epsilon_2, \theta) >\) is maximum. Usually, \(f(\epsilon_1, \epsilon_2, \theta)\) is assumed to be uniformly distributed over the \((\epsilon_1, \epsilon_2, \theta)\) hyperplane, which leads to the following criterion: Choose \(\epsilon_1 = \hat{\epsilon}_1, \epsilon_2 = \hat{\epsilon}_2,\) and \(\theta = \hat{\theta}\) such that \(< f(\vec{r} | \alpha, \beta, \epsilon_1, \epsilon_2, \theta) >\) is maximum. We call this estimator maximum likelihood (ML), since the ML estimate corresponds mathematically to the limiting case of a maximum a posteriori (MAP) estimate in which the a priori knowledge approaches zero, i.e., \(\epsilon_1, \epsilon_2\) and \(\theta\) are uniformly distributed [B22, p.65]. Note that many authors treat \(\epsilon_1, \epsilon_2,\) and \(\theta\) as deterministic but unknown quantities in their ML approaches. \(f(\vec{r} | \alpha, \beta, \epsilon_1, \epsilon_2, \theta)\) is an a posteriori probability and very often such an estimator is (incorrectly) called MAP. Later, we use the knowledge of \(\epsilon_1, \epsilon_2,\) and \(\theta\) accumulated over corresponding observation intervals to develop improved estimator-detectors, which will be called MAP.
4.2. MAXIMUM LIKELIHOOD (ML) APPROACH

We maximize the likelihood function
\[
\Lambda(\bar{F}|\epsilon_1, \epsilon_2, \theta) = \ln[(f(\bar{F}|\alpha, \beta, \epsilon_1, \epsilon_2, \theta))],
\]
\[
(4.7)
\]
denote the corresponding \(\epsilon_1, \epsilon_2,\) and \(\theta\) with \(\bar{\epsilon}_1, \bar{\epsilon}_2,\) and \(\bar{\theta},\) and name the respective estimator as ML. The analysis proceeds as follows: Conditioned on \(\alpha, \beta, \epsilon_1, \epsilon_2,\) and \(\theta,\) the random vector \(\bar{F}\) is \(N\)-dimensional Gaussian with mean value which is equal to the vector \(\bar{\theta},\) i.e.
\[
f(\bar{F}|\alpha, \beta, \epsilon_1, \epsilon_2, \theta) = \frac{\exp[-(1/2) (\bar{F} - \bar{\theta})^T K_n^{-1}(N) (\bar{F} - \bar{\theta})]}{(2\pi)^{N/2} |K_n(N)|^{1/2}}
\]
\[
(4.8)
\]
Quantity \(K_n(N) = I(N)N_oB_{IF}\) is the noise covariance matrix, \(I(N)\) is the \(N\)-dimensional identity matrix, superscript \(T\) denotes transpose, and dependence of \(f(\bar{F}|\alpha, \beta, \epsilon_1, \epsilon_2, \theta)\) on \(\alpha, \beta, \epsilon_1, \epsilon_2,\) and \(\theta\) occurs in the vector \(\bar{\theta}.\) A more complex channel might be represented by a complex matrix \(K(N),\) while a corresponding equalizer is represented by its inverse \(K^{-1}(N).\) However, analysis of such a system is out of the scope of this contribution. Since the components of vector \(\bar{F}\) are mutually independent
\[
f(\bar{F}|\alpha, \beta, \epsilon_1, \epsilon_2, \theta) = (2\pi N_o B_{IF})^{-N/2} \exp\left(-\frac{1}{N_o} \sum_{k=1}^{N} |r_k|^2 \Delta t \right)
\]
\[
\times \exp\left(-\frac{1}{N_o} \sum_{k=1}^{N} |s_k|^2 \Delta t + \frac{2}{N_o} \sum_{k=1}^{N} r_k s_k \Delta t \right)
\]
\[
(4.9)
\]
where \(r_k = r(t_k)\) and \(s_k = s(t_k)\). \(r_k\) does not depend on \(\alpha, \beta, \epsilon_1, \epsilon_2,\) and \(\theta\) and might be absorbed within a constant. Very often, \(s(t)\) has been assumed to be a constant envelope process and a maximization of the terms in the last sum of (4.9) followed. This is valid for a rather limited number of systems such as nonfiltered QPSK, MSK, and MPSK. Any filtering of such a system introduces envelope fluctuations, which require an equalization to be employed at the receiver end. Here, we do not consider problems related to the filtering and equalization. However, an envelope fluctuation is inherited within the elementary structure of the multilevel and overlapped schemes and this must be taken into consideration. We explore this problem in Section 4.2.2. In this section a constant envelope process is assumed. Using (4.9), the likelihood function \(\Lambda(\bar{F}|\epsilon_1, \epsilon_2, \theta)\) might be written as
\[
\Lambda(\bar{F}|\epsilon_1, \epsilon_2, \theta) = \sum_{V} \ln \sum_{ij} e^{F} + C_1
\]
\[
(4.10)
\]
where summation over \( V \) denotes estimation on interval \([k - (V + 2), k - 1]\), summation over \( i_j \) denotes averaging over all possible data patterns \( \{\alpha_i, \beta_j\} \), and \( \Gamma \) will be described in (4.13), and \( C_1 \) is a constant which includes the a priori probabilities of the sequences \( \alpha \) and \( \beta \). Necessary but not sufficient conditions that must be satisfied by \( \epsilon_1, \epsilon_2, \) and \( \theta \) in order to be an ML estimate are

\[
\frac{\partial \Lambda(\hat{r}|\epsilon_1, \epsilon_2, \theta)}{\partial \theta} \bigg|_{\theta = \hat{\theta}, \epsilon_1 = \hat{\epsilon}_1, \epsilon_2 = \hat{\epsilon}_2} = 0 \quad (4.11a)
\]

\[
\frac{\partial \Lambda(\hat{r}|\epsilon_1, \epsilon_2, \theta)}{\partial \epsilon_1} \bigg|_{\theta = \hat{\theta}, \epsilon_1 = \hat{\epsilon}_1, \epsilon_2 = \hat{\epsilon}_2} = 0 \quad (4.11b)
\]

\[
\frac{\partial \Lambda(\hat{r}|\epsilon_1, \epsilon_2, \theta)}{\partial \epsilon_2} \bigg|_{\theta = \hat{\theta}, \epsilon_1 = \hat{\epsilon}_1, \epsilon_2 = \hat{\epsilon}_2} = 0 \quad (4.11c)
\]

In addition to (4.11a-c) the following conditions apply

\[
\begin{vmatrix}
\frac{\partial^2 \Lambda}{\partial \epsilon \partial \theta} & \frac{\partial^2 \Lambda}{\partial \epsilon^2} \\
\frac{\partial^2 \Lambda}{\partial \theta^2} & \frac{\partial^2 \Lambda}{\partial \epsilon \partial \theta}
\end{vmatrix} < 0. \quad (4.11d)
\]

\( \Lambda \) has a maximum if \( \frac{\partial^2 \Lambda}{\partial \epsilon^2} < 0 \) and \( \frac{\partial^2 \Lambda}{\partial \theta^2} < 0 \) at \((\epsilon_0, \theta_0)\)-this corresponds to a stable and correct (desirable) lock point of a recovery loop. \( \Lambda \) has a minimum if \( \frac{\partial^2 \Lambda}{\partial \epsilon^2} > 0 \) \( \frac{\partial^2 \Lambda}{\partial \theta^2} > 0 \) at \((\epsilon_0, \theta_0)\)-this corresponds to a false lock point of a recovery loop. Here we have tacitly assumed that the function \( \Lambda(\hat{r}|\epsilon_1, \epsilon_2, \theta) \) is differentiable over all \( \{\epsilon_1, \epsilon_2, \theta\} \), i.e. the first and second derivatives with respect to the \( \epsilon_1, \epsilon_2, \theta \) exist. However, this might not be true, in general, and any singularity must be taken into consideration. As an example: The derivative of a rectangular pulse must be precisely defined for \( t = +T_S/2 \) as \((a_k - a_{k+1})/2\); the derivative of \(+\) \((-\) \-) sequence of MSK (Hann pulse) has a discontinuity in the vicinity of \( t = T_S/2 \), i.e. the step from \(-1\) to \(+1\) \((+1\ to\ -1)\), and must be defined as zero for \( t = T_S/2 \) for both cases; etc. Then, (4.11a-c) become

\[
\frac{\partial \Lambda(\hat{r}|\epsilon_1, \epsilon_2, \theta)}{\partial \theta} = \sum_{\nu} \frac{\sum_{i,j} (\partial \Gamma / \partial \theta) e^\Gamma}{\sum_{i,j} e^\Gamma} \quad (4.12a)
\]

\[
\frac{\partial \Lambda(\hat{r}|\epsilon_1, \epsilon_2, \theta)}{\partial \epsilon_1} = \sum_{\nu} \frac{\sum_{i,j} (\partial \Gamma / \partial \epsilon_1) e^\Gamma}{\sum_{i,j} e^\Gamma} \quad (4.12b)
\]

\[
\frac{\partial \Lambda(\hat{r}|\epsilon_1, \epsilon_2, \theta)}{\partial \epsilon_2} = \sum_{\nu} \frac{\sum_{i,j} (\partial \Gamma / \partial \epsilon_2) e^\Gamma}{\sum_{i,j} e^\Gamma} \quad (4.12c)
\]
By using (4.1 and 4.8), a nondata-aided approach gives

\[
\Gamma = -\frac{1}{N_0}\int \left[ P_A x^2(V, \epsilon_1) + P_B y^2(V, \epsilon_2) \right] \cos 2(2\pi f_c t + \theta) \, dt \\
+ \frac{2}{N_0} \int \left[ \sqrt{P_A} r(t) x(V, \epsilon_1) \cos (2\pi f_c t + \theta) + \sqrt{P_B} r(t) y(V, \epsilon_2) \sin (2\pi f_c t + \theta) \right] \, dt
\]  
(4.13)

\[
\frac{\partial \Gamma}{\partial \theta} = \frac{1}{N_0} \int \left[ P_A x^2(V, \epsilon_1) - P_B y^2(V, \epsilon_2) \right] \sin 2(2\pi f_c t + \theta) \, dt \\
- \frac{2}{N_0} \int \left[ \sqrt{P_A} r(t) x(V, \epsilon_1) \sin (2\pi f_c t + \theta) - \sqrt{P_B} r(t) y(V, \epsilon_2) \cos (2\pi f_c t + \theta) \right] \, dt
\]  
(4.14)

\[
\frac{\partial \Gamma}{\partial \epsilon_1} = -\frac{P_A}{N_0} \int x(V, \epsilon_1) \frac{\partial x(V, \epsilon_1)}{\partial \epsilon_1} [1 + \cos 2(2\pi f_c t + \theta)] \, dt \\
+ \frac{2\sqrt{P_A}}{N_0} \int r(t) \frac{\partial x(V, \epsilon_1)}{\partial \epsilon_1} \cos (2\pi f_c t + \theta) \, dt + L_{AS} + L_{AC}
\]  
(4.15)

\[
\frac{\partial \Gamma}{\partial \epsilon_2} = -\frac{P_B}{N_0} \int y(V, \epsilon_2) \frac{\partial y(V, \epsilon_2)}{\partial \epsilon_2} [1 - \cos 2(2\pi f_c t + \theta)] \, dt \\
+ \frac{2\sqrt{P_B}}{N_0} \int r(t) \frac{\partial y(V, \epsilon_2)}{\partial \epsilon_2} \sin (2\pi f_c t + \theta) \, dt + L_{BS} + L_{BC}
\]  
(4.16)

where the summation over \( k \) is approximated with an integral

\[
\sum_{k=1}^{N} r(t_k) w(t - iT_s + \epsilon) \frac{\cos}{\sin} (2\pi f_c t + \theta) \, \Delta t = \\
\int_{-T_s/2+iT_s-cT_s/2}^{+T_s/2+iT_s-cT_s/2} r(t_k) w(t + \epsilon) \frac{\cos}{\sin} (2\pi f_c t + \theta) \, dt, \quad S = A, B
\]  
(4.17)

and for convenience the integral limits are omitted. \( L_{AS}, L_{AC}, L_{BS}, L_{BC} \) are terms due to Leibniz' rule given as

\[
L_{AS} = \frac{2\sqrt{P_A}}{N_0} \frac{T_A}{\pi} r(t) \cos (2\pi f_c t + \theta)
\]
\[ [x(iT_A - T_A/2 - \epsilon_1 T_A/\pi) - x(iT_A + T_A/2 - \epsilon_1 T_A/\pi)] p_A \]  \hspace{1cm} (4.18a)

\[ L_{AC} = \frac{P_A}{N_o} \left[ x^2(iT_A - T_A/2 - \epsilon_1 T_A/\pi) - x^2(iT_A + T_A/2 - \epsilon_1 T_A/\pi) \right] p_A \]  \hspace{1cm} (4.18b)

\[ L_{BS} = \frac{2\sqrt{P_B T_B}}{\pi} r(t) \sin(2\pi f_c t + \theta) \]

\[ [y(jT_B - T_B/2 - \epsilon_2 T_B/\pi) - y(jT_B + T_B/2 - \epsilon_2 T_B/\pi)] p_B \]  \hspace{1cm} (4.19a)

\[ L_{BC} = \frac{P_B}{N_o} \left[ y^2(jT_B - T_B/2 - \epsilon_2 T_B/\pi) - y^2(jT_B + T_B/2 - \epsilon_2 T_B/\pi) \right] p_B \]  \hspace{1cm} (4.19b)

where \( p_A \) and \( p_B \) are the probabilities of symbol transitions in the A and B channel, respectively. Note that for \( \epsilon_i > 0 \), two symbols, the \( k \)-th and \( (k - 1) \)-th, are involved in (4.18a–4.19b).

Equations (4.12a–c) are highly nonlinear, and explicit solutions for \( \epsilon_1, \epsilon_2, \) and \( \theta \) seem to be, at least, very complex if not impossible. Therefore, an ML estimator might not be practically realizable in general. Furthermore, these equations are not independent, which suggests that a joint carrier and timing solution might offer better results than separate solutions. In the following, we concentrate on some simplifications, which might lead to easier practical realizations. The integrals in (4.14–4.16) which contain double frequency terms equal zero if the carrier frequency \( f_c \) is an integer multiple of data rates \( R_A, R_B \). They are negligible if \( f_c \gg R_A, R_B \) and will be ignored in our analysis. Usually, the signal set in each quadrature channel is symmetric. Then, (4.12a–c) might be written as

\[ \frac{\partial \Lambda(\bar{r}|\epsilon_1, \epsilon_2, \theta)}{\partial \theta} = 0 = \sum \frac{N(\theta)}{D_o} \]  \hspace{1cm} (4.20a)

\[ \frac{\partial \Lambda(\bar{r}|\epsilon_1, \epsilon_2, \theta)}{\partial \epsilon_1} = 0 = \sum \frac{N(\epsilon_1)}{D_o} \]  \hspace{1cm} (4.20b)

\[ \frac{\partial \Lambda(\bar{r}|\epsilon_1, \epsilon_2, \theta)}{\partial \epsilon_2} = 0 = \sum \frac{N(\epsilon_2)}{D_o} \]  \hspace{1cm} (4.20c)

\[ D_o = \sum_{i,j} \epsilon_i \]  \hspace{1cm} (4.21a)

\[ \Rightarrow \sum_{i,j} W_i \cosh(C_x) W_j \cosh(S_y) \]  \hspace{1cm} (4.21b)

In (4.21a), \( i, j \in \{ I \} \), and the \( \Gamma \) term is given in (4.13). However, due to symmetry of the signal set, summations over \( i, j \) in (4.21b) and the equations which follow extend over
nonnegative integers only.

\[
N(\theta) = 4 \sum_{i,j} \left[ -S_x \sinh(C_x) \cosh(S_y) + C_y \cosh(C_x) \sinh(S_y) \right] \tag{4.22}
\]

\[
N(\epsilon_1) = 4 \sum_{i,j} \left[ -\frac{P_A}{N_0} \int x(V, \epsilon_1) \frac{\partial x(V, \epsilon_1)}{\partial \epsilon_1} \, dt + L_{AC} \right] W_i \cosh(C_x) W_j \cosh(S_y) + \left[ \frac{2\sqrt{PA}}{N_0} \int \frac{\partial x(V, \epsilon_1)}{\partial \epsilon_1} \cdot r(t) \cos(2\pi f_c t + \theta) \, dt + L_{AS} \right] W_i \sinh(C_x) W_j \cosh(S_y) \tag{4.23}
\]

\[
N(\epsilon_2) = 4 \sum_{i,j} \left[ -\frac{P_B}{N_0} \int y(V, \epsilon_2) \frac{\partial y(V, \epsilon_2)}{\partial \epsilon_2} \, dt + L_{BC} \right] W_i \cosh(C_x) W_j \cosh(S_y) + \left[ \frac{2\sqrt{PB}}{N_0} \int \frac{\partial y(V, \epsilon_2)}{\partial \epsilon_2} \cdot r(t) \cos(2\pi f_c t + \theta) \, dt + L_{BS} \right] W_i \cosh(C_x) W_j \sinh(S_y) \tag{4.24}
\]

where

\[
W_i = \exp\left[ -\frac{PA}{2N_0} \int x^2(V, \epsilon_1) \, dt \right] \tag{4.25a}
\]

\[
W_j = \exp\left[ -\frac{PB}{2N_0} \int y^2(V, \epsilon_2) \, dt \right] \tag{4.25b}
\]

are weighting factors proportional to the negative exponent of the signal-to-noise ratio (SNR) in the corresponding baseband levels, and

\[
C_x = \frac{2\sqrt{PA}}{N_0} \int r(t)x(V, \epsilon_1) \cos(2\pi f_c t + \theta) \, dt \tag{4.26a}
\]

\[
C_y = \frac{2\sqrt{PB}}{N_0} \int r(t)y(V, \epsilon_2) \cos(2\pi f_c t + \theta) \, dt \tag{4.26b}
\]

\[
S_x = \frac{2\sqrt{PA}}{N_0} \int r(t)x(V, \epsilon_1) \sin(2\pi f_c t + \theta) \, dt \tag{4.26c}
\]

\[
S_y = \frac{2\sqrt{PB}}{N_0} \int r(t)y(V, \epsilon_2) \sin(2\pi f_c t + \theta) \, dt \tag{4.26d}
\]

Leibnitz's terms and derivatives are now related to the nonnegative \(i, j\). Then, (4.20a-c) suggest a closed-loop structure for the joint carrier phase estimate \(\theta\) and timing estimates \(\epsilon_1, \epsilon_2\).
given in Fig. 4.2. In this and following figures, timing information paths to the integrals and delay elements necessary for a proper operation of the loops are omitted for the reason of clarity. Note that the timing loop consists of two essential parts: the first one which represents the first continuous part in (4.15–4.16) and the second one which represents Leibniz's transitional parts (4.18a–4.19b). The transitional part only is the essence of a digital data transition tracking loop (DTTL) [B16]. The probability of no transition of MQAM schemes equals $1/L$, where $L$ is the number of baseband levels, which suggests that this part might be essential for a TR device for multilevel schemes. The transitional part of rectangular schemes is independent of $\epsilon$, whereas it increases as $\epsilon$ increases for MSK, an example of a nonrectangular scheme.
Chapter 4: On Composite Phase and Timing Estimation and Detection

Figure 4.2. The joint phase and timing estimator for WQAM schemes, which corresponds to eqs. (4.11 a-c) in the text.
By approximating the continuous derivative by a finite difference, i.e.,

$$\frac{\partial \Lambda(\hat{f}|\epsilon_1, \epsilon_2, \theta)}{\partial \epsilon} = \Lambda(\epsilon + \Delta \epsilon/2) - \Lambda(\epsilon - \Delta \epsilon/2)$$

$$= \sum_V \frac{\Delta \epsilon}{\Delta \epsilon} \ln \frac{\sum_{i,j} \exp(+\Delta \epsilon) - \ln \sum_{i,j} \exp(-\Delta \epsilon)}{\Delta \epsilon}$$

$$\Rightarrow \sum_V \ln \frac{\sum_{i,j} \cosh(+\Delta \epsilon) - \ln \sum_{i,j} \cosh(-\Delta \epsilon)}{\Delta \epsilon}$$  \hspace{1cm} (4.27a)

where \(\{\pm \Delta \epsilon\}\) correspond to \(\Gamma(\epsilon \pm \Delta \epsilon/2)\) respectively, an early-late gate loop \([B3]\) is suggested.

The summations in (4.27a) assume that \(i, j \in \{I\}\) while the summations in (4.27b) assume nonnegative \(i, j\).

The structure given in Fig.4.2 seems to be complex, and the hyperbolic sine and hyperbolic cosine functions might not be practical to realize. The approximations \(\sinh(x) \approx x, \cosh(x) \approx 1 + x^2/2\) for small \(x\) (low SNR) and \(\cosh(x) \approx \text{sign}(x) \sinh(x)\) for large \(x\) (high SNR) might be employed to simplify the loop. However, we proceed in a different way. For 4-state 2x2 baseband levels schemes, (4.20a-c) reduce to

$$\frac{\partial \Lambda(\hat{f}|\epsilon_1, \epsilon_2, \theta)}{\partial \theta} = 0 = \sum_V \left[ -S_x \tanh(C_x) + C_y \tanh(S_y) \right]$$  \hspace{1cm} (4.28a)

$$\frac{\partial \Lambda(\hat{f}|\epsilon_1, \epsilon_2, \theta)}{\partial \epsilon_1} = 0 = \sum_V \left( -\frac{P_A}{N_o} \int x(V, \epsilon_1) \frac{\partial x(V, \epsilon_1)}{\partial \epsilon_1} \right) dt$$

$$+ \left[ \frac{2\sqrt{P_A}}{N_o} \int \frac{\partial x(V, \epsilon_1)}{\partial \epsilon_1} r(t) \cos(2\pi f_c t + \theta) dt + L_{AS} \right] \tanh(C_x)$$  \hspace{1cm} (4.28b)

$$\frac{\partial \Lambda(\hat{f}|\epsilon_1, \epsilon_2, \theta)}{\partial \epsilon_2} = 0 = \sum_V \left( -\frac{P_B}{N_o} \int y(V, \epsilon_2) \frac{\partial y(V, \epsilon_2)}{\partial \epsilon_2} \right) dt$$

$$+ \left[ \frac{2\sqrt{P_B}}{N_o} \int \frac{\partial y(V, \epsilon_2)}{\partial \epsilon_2} r(t) \sin(2\pi f_c t + \theta) dt + L_{BS} \right] \tanh(S_y)$$  \hspace{1cm} (4.28c)

(4.28a-c) suggest a device as given in Fig.4.3. Without the TR part (i.e. 4.28b and 4.28c) and for rectangular signal shapes, this becomes a CR loop as given in [P127].
Figure 4.3. The joint phase and timing estimator for 4 state 2x2 baseband level QAM schemes, which corresponds to eqns. (4.28 a-c) in the text.
We will not pursue unbalanced schemes further. However, note that QAM schemes with more than 4 states are balanced only on average i.e. the average power in the A channel is equal to the average power in the B channel, but they are not balanced on a per-symbol-basis. As an example, a 256QAM scheme exhibits the maximum imbalance of 15/1 when a baseband level of 1 in the A channel is accompanied with the level of 15 in the B channel and vice versa. Similarities between unbalanced schemes and MQAM schemes may be observed on the corresponding nonlinearity curves for unbalanced schemes [P127]–[P130] and MQAM schemes [P122]–[P125]. For 4 state balanced schemes with \( P_A = P_B, R_A = R_B = 1/T_S, C_x = C_y, S_x = S_y \) and rectangular signal shapes, (4.28a–c) reduce to

\[
\frac{\partial \Lambda(\vec{r}|\epsilon_1, \epsilon_2, \theta)}{\partial \theta} = 0 = \sum_V [-S_x \tanh(C_x) + C_x \tanh(S_x)]
\]

\[
\frac{\partial \Lambda(\vec{r}|\epsilon_1, \epsilon_2, \theta)}{\partial \epsilon} = 0 = \sum_V \left( -\sqrt{P_A} \int x(V, \epsilon) \frac{\partial x(V, \epsilon)}{\partial \epsilon} \right. \\
+ \tanh(C_x) \left( \frac{\partial x(V, \epsilon)}{\partial \epsilon} r(t) \cos(2\pi f_c t + \theta) dt \\
+ \tanh(S_x) \left( \frac{\partial y(V, \epsilon)}{\partial \epsilon} r(t) \sin(2\pi f_c t + \theta) dt \\
+ L_{AS} \tanh(C_x) + L_{BS} \tanh(S_x) \right) \right)
\]

which suggests the device given in Fig.4.4. The nonlinearity \( \tanh(x) \) might be approximated by \( x - x^3/3! + \ldots \) for low SNRs and by \( \text{sign}(x) \) for high SNRs. The high SNR loop approximation without the timing part is called a DFCRL since decisions on \( \alpha, \beta \), i.e. \( \hat{\alpha}, \hat{\beta} \) are used to (partly) wipe off the data. This type of loop is often called data-aided. However, a DA approach is not used here and this solution is, therefore, not necessary optimal in a DA sense. The DA approach calls for data estimates \( \hat{\alpha} \) and \( \hat{\beta} \) in (4.6–4.7). Both \( \hat{\alpha} \) and \( \hat{\beta} \) depend on \( \epsilon_1, \epsilon_2 \), in a yet unknown way and a strict DA approach seems to be rather difficult. Later, we discuss this dependence and a few improved circuits are suggested.
Figure 4.4. The joint phase and timing estimator for QPSK schemes, which corresponds to eqs. (4.29 a-c) in the text.
MSK is an example of schemes with a nonrectangular signal shape. By using (4.26a-d) and (4.28a-c) the phase and timing estimation equations become

\[
\frac{\partial \Lambda(r|e_1, e_2, \theta)}{\partial \theta} = 0 = \sum_V \left( - \int \cos\left(\frac{\pi t}{T} + \epsilon\right)r(t) \sin(2\pi f_c t + \theta) dt \right.
\]
\[\times \tanh\left[\frac{2\sqrt{P_A}}{N_0} \int \cos\left(\frac{\pi t}{T} + \epsilon\right)r(t) \cos(2\pi f_c t + \theta) dt\right] \]
\[+ \int \sin\left(\frac{\pi t}{T} + \epsilon\right)r(t) \cos(2\pi f_c t + \theta) dt \]
\[\times \tanh\left[\frac{2\sqrt{P_B}}{N_0} \int \sin\left(\frac{\pi t}{T} + \epsilon\right)r(t) \sin(2\pi f_c t + \theta) dt\right] \] (4.30a)

\[
\frac{\partial \Lambda(r|e_1, e_2, \theta)}{\partial e_i} = 0 = \sum_V \left( - \int \sin\left(\frac{\pi t}{T} + \epsilon\right)r(t) \cos(2\pi f_c t + \theta) dt + L_{AS} \right) \]
\[\times \tanh\left[\frac{2\sqrt{P_A}}{N_0} \int \cos\left(\frac{\pi t}{T} + \epsilon\right)r(t) \cos(2\pi f_c t + \theta) dt\right] \]
\[+ \int \cos\left(\frac{\pi t}{T} + \epsilon\right)r(t) \sin(2\pi f_c t + \theta) dt + L_{BS} \]
\[\times \tanh\left[\frac{2\sqrt{P_B}}{N_0} \int \sin\left(\frac{\pi t}{T} + \epsilon\right)r(t) \sin(2\pi f_c t + \theta) dt\right] \] (4.30b)

The corresponding joint estimator is given in Fig.4.5. Note the similarity of our estimator with that given in [P13S, Fig.3]. Our scheme uses additional transition detectors TDA and TDB to improve timing performance by accounting for the Leibniz terms. Since MSK is using \textit{cos}(x) and \textit{sin}(x) pulse shapes in \textit{A} and \textit{B} channels respectively, the derivatives suggested in (4.28a-c) are realized by employing the respective crossarm signals. In a data-aided approach, at high SNR, \textit{tanh}(x) = \textit{sign}(x) is used. The extension of this concept to other WQAM schemes is straightforward.
Figure 4.5. The joint phase and timing estimator for MSK schemes, which corresponds to eqs. (4.30 a-c) in the text.
Now, we proceed with the CR structures analysis. When schemes with more than 4 states are employed, a term within the second sum in (4.9) suggests that a quantizer

\[
Q_k(x) = \begin{cases} 
kd, & \text{for } (k-1)d \leq x < (k+1)d; \\
(L-1)d, & \text{for } (L-2)d \leq x < \infty; \\
-(L-1)d, & \text{for } -\infty < x \leq -(L-2)d.
\end{cases} 
\]

(4.31)

be used as a decision device. This loop, Fig. 4.6, which we call the classical DFCRL, serves as a reference to which all new schemes are compared. This type of loop with an integrate-sample-and-dump (ISD) circuit (box noted with an integral) within the loop arms is called active, while the other one, employing a lowpass filter instead, is called passive. We analyze the former one only and note that an active loop performs better than a passive one, at least for \(\phi, \lambda \neq 0\). Usually, an active loop with ideal sampling is assumed. Since this might not be a practical case, we analyze the loop performance in the presence of both CR uncertainty \(\phi\) and TR uncertainty \(\lambda\), which to the best of our knowledge has not been published in the open literature, yet.
Figure 4.6. The classical decision feedback carrier recovery loop.

nonstaggered: $a = (m-1)T$, $b = mT$

staggered: $a = (m-0.5)T$, $b = (m+0.5)T$
4.2.1. CLASSICAL DECISION FEEDBACK CR LOOP

The block diagram of the classical DFCRL is given in Fig.4. Combining both CR and TR uncertainties, the signal $z_A(t)$ at the decision device input is:

$$z_A(t) = a_m w_A [t - (m - 1)T - \varepsilon] \cos \phi + b_n w_B [t - (n - 1)T - \varepsilon] \sin \phi + \nu_A$$

(4.32)

where $\nu_A$ is the sample of Gaussian noise in the A channel. The conditional probability of error of an MQAM scheme is

$$U(\phi, \lambda) = P(e|\phi, \lambda) = P[e|\phi, \lambda, a_m = m = -(L-1)]$$

$$+ P[e|\phi, \lambda, a_m = m = +(L-1)] + P[e|\phi, \lambda, |a_m - m| < (L-1)]$$

(4.33)

$$P[e|\phi, \lambda, a_m = \pm (L-1), a_{m-1} = u, b_n = n, b_{n-1} = v] =$$

$$Q[\gamma_2 - (L-2) + [(L-1)R_A(|\lambda|) + uR_A(1 - |\lambda|)] \cos \phi + [nR_B(|\lambda|) + vR_B(1 - |\lambda|)] \sin \phi]$$

(4.34)

$$P[e|\phi, \lambda, |a_m - m| < (L-1), a_{m-1} = u, b_n = n, b_{n-1} = v] =$$

$$Q[\gamma_2 - (m-1) + [mR_A(|\lambda|) + uR_A(1 - |\lambda|)] \cos \phi + [nR_B(|\lambda|) + vR_B(1 - |\lambda|)] \sin \phi] +$$

$$Q[\gamma_2 + (m+1) - [mR_A(|\lambda|) + uR_A(1 - |\lambda|)] \cos \phi - [nR_B(|\lambda|) + vR_B(1 - |\lambda|)] \sin \phi]$$

(4.35)

The quantities $P[e|x, y, \ldots]$ are the conditional probabilities of error conditioned on $x, y, \ldots$ respectively. $Q[x]$ is the Gaussian probability function [B26,26.2.3], $\gamma$ is the average signal-to-noise ratio at the decision device input, and $R_A(x)$ and $R_B(x)$ are the crosscorrelation functions of reference signal and data in A and B channel respectively, in the corresponding TR loops. To distinguish between different conditional probabilities, the conditional probability of error $P(e|\phi, \lambda)$ is called the uncertainty diagram $U(\phi, \lambda)$, the meaning and importance of which will be explained further, later on. The conditional probability of error of staggered schemes is given by (4.33-4.35) if $R_B(|\lambda|)$ and $R_B(1 - |\lambda|)$ are replaced by $R_B(1/2 + |\lambda|)$ and $R_B(1/2 - |\lambda|)$, respectively.

The conditional probability of error performance $P(e|\phi)$ of MQAM schemes, conditioned on a CR uncertainty $\phi$ is summarized in Fig.4.7. For comparison purposes, a probability of

---

1 We assume that the receiving filter $H(f)$ is wide enough to cause no significant pulse reshaping and narrow enough to allow the narrow-band representation of additive white Gaussian noise of the channel. The loop bandwidth is much narrower than the data bandwidth. The loop is initially locked. As a consequence, the phase process $\phi$ in the loop varies much more slowly than the signal and the noise process correlation time is much less than the length of the symbol interval, i.e. the noise is practically white.
error of $10^{-10}$ is assumed. This corresponds to a typical non-faded value in terrestrial radio. To emphasize threshold crossings, the absolute value of arguments within the $Q(x)$ function, i.e. $Q(|x|)$, is used. The corresponding angles represent phase errors which cause threshold boundary crossings, i.e. erroneous decisions. We will attempt to relate these theoretically obtained curves to some practical cases as follows. As an example, an absolute CR phase uncertainty of 2 degrees causes a minimal degradation of the conditional probability of error, if BPSK is employed, see Fig.4.7. However, the same 2 degrees uncertainties causes the degradation from $10^{-10}$ to $10^{-3}$ if 256QAM scheme is concerned. The corresponding average probability of error $P_e$ depends further on the probability density function $p(\phi)$, which is dictated by the loop design. For this type of loop and an MQAM signal constellation, an exact expression for $p(\phi)$ is very difficult to obtain, and — in our best knowledge — is not known. As a consequence, $P_e$ is not known either. Therefore, for the practical purposes we will give some examples. If $p(\phi)$ is uniformly distributed, a receiver does not have the information about the carrier phase and coherent detection is not possible. If $p(\phi) = \delta(\phi)$, where $\delta$ is the delta function, the conditional probability of error $P(e|\phi)$ equals the average $P_e$ and the ordinate in Fig.4.7 reads as $P_e$. This occurs when the impairment dynamic is faster than the loop bandwidth, but slower than the data bandwidth, i.e. the phase error is virtually constant over a number of data symbols. A practical loop emphasizes the angles close to zero degrees. Therefore, $P_e$ results better than those given in Fig.4.7 might be expected. Fig.4.7 gives an estimate of maximum tolerable imperfections of the loop components, such as phase detector, amplifier and filter responses, d.c. wandering, etc. Obviously, as the number of levels (states) increases, these imperfections cause an increased degradation in the $P_e$ performance. In Section 4.3 we search for loops which are less prone to these effects. There, the $P_e$ performance of the classical DFCRL is given and compared with new improved loops.

The L-ary pulse amplitude modulation, LPAM ($M=L\times L$), conditional probability of error performance $P(e|\lambda)$ is summarized in Fig.4.8. To emphasize threshold crossings the absolute value of arguments within the $Q(x)$ function, i.e. $Q(|x|)$, is used. The corresponding fractions represent timing errors, normalized to the symbol duration, which cause erroneous decisions. A timing uncertainty of 1/30 of the symbol duration causes a performance degradation in probability of error from $10^{-10}$ to $10^{-9}$ in a two level baseband signal modulation scheme (BPSK, QPSK), but an erroneous decision in a 16 (256QAM) or higher level scheme. The meaning of this figure is similar to that of Fig.4.7.
Figure 4.7. The $P(e|\phi)$ performance of MQAM schemes. The performance of $10^{-10}$ is taken as the reference. To emphasize threshold crossings, the absolute value of arguments within the $Q(x)$ function, i.e. $Q(|x|)$, is used.
Figure 4.8. The $P(e|\lambda)$ performance of the LPAM schemes. The performance of $10^{-10}$ is taken as the reference. To emphasize threshold crossings, the absolute value of arguments within the $Q(x)$ function, i.e. $Q(|x|)$, is used.
Chapter 4: On Composite Phase and Timing Estimation and Detection

The combined effect of the phase and timing uncertainties on the system performance is given by (4.33–4.35). To get a better understanding of the nature of (4.33–4.35), we plotted them using two different forms: the uncertainty diagram (UD) $U(\phi, \lambda)$ and the equal probability of error or the isoper curves. To emphasize threshold crossings, $Q(\pi)$ is sometimes replaced by $Q(|\pi|)$. $U(\phi, \lambda)$ exhibits a periodicity of $2\pi$ radians, while $U(|\phi, \lambda|)$ of the BPSK and QAM schemes have periods of $\pi$ and $\pi/2$, respectively. The UD and isoper curves of the 256QAM scheme are given as an example in Fig.4.9a–b. These figures show a high sensitivity to any phase and timing uncertainty assuming that a classical DFCRL is used. The UD and isoper curves of the BPSK, 4PAM, QPSK, SQPSK and 16QAM schemes are given in [P117]. Similar curves for four level modulation schemes are generated by Harris and Kristiansen [P118] using a computer simulation technique. Further results are given in [P119]–[P121].
Figure 4.9. The 256QAM uncertainty diagram. The performance of 10^{-10} is taken as the reference.
More detailed analysis of the loop behavior including the probability density function, average probability of error, slipping rate, and acquisition properties is out of the scope of this presentation. However, most of these quantities, including \( p(\phi, \lambda) \) are functions of a nonlinearity surface which will be briefly assessed. The integro-differential equation of the loop is

\[
\dot{\phi}(t) = \varphi(t) - K_o F(p) e^{-p \Delta} P_{av} \left( [-g_1(\phi, \lambda) \sin \phi + g_2(\phi, \lambda) \cos \phi] + \nu \sqrt{N_e(\phi, \lambda) N_o} \right)
\]

(4.36)

where \( P_{av} = 2(M - 1)/3 \) is the average signal power, the dot represents the derivative with respect to the time variable \( t \), \( F(p) \) is the transfer function of the loop filter, \( K_o \) is the equivalent loop gain, \( \Delta \) is the delay within the loop, functions \( g_1, g_2 \) and \( N_e \) are

\[
g_1(\phi, \lambda) = \langle a \hat{a} + b \hat{b}|\phi, \lambda \rangle / P_{av}
\]

(4.37)

\[
g_2(\phi, \lambda) = \langle \hat{a} \hat{b} - \hat{b} a|\phi, \lambda \rangle / P_{av}
\]

(4.38)

\[
N_e(\phi, \lambda) = \langle \hat{a}^2 + \hat{b}^2|\phi, \lambda \rangle / P_{av}
\]

(4.39)

\( \nu \) is a Gaussian random variable, and \( \langle \cdot \rangle \) represents the time average. The function

\[
h(\phi, \lambda) = 2 \frac{-g_1(\phi, \lambda) \sin \phi + g_2(\phi, \lambda) \cos \phi}{N_e(\phi, \lambda) N_o}
\]

(4.40)

is termed the normalized nonlinearity surface or (two dimensional) restoring force of the CR loop, and becomes the nonlinearity S curve when \( \lambda \to 0 \). An integral of \(-h(\phi, \lambda)\) represents a potential function. The nonlinearity surface QPSK has period of \( \pi/2 \) radians, Fig. 4.10. However, the period of staggered QPSK is extended to \( \pi \) radians, Fig. 4.11. The nonlinearity surface of the SQPSK scheme has a favorable shape in a comparison with the shape of the QPSK scheme, because of an extended linear region near the origin (\( \phi = 0, \lambda = 0 \)) and the larger period. Therefore, SQPSK will perform better (will have lower \( P_e \)) than QPSK in the presence of phase uncertainties. Note that \( p(\phi, \lambda) \) depends on \( h(\phi, \lambda) \), i.e.

\[
p(\phi, \lambda) = C \frac{\exp[-\int \int h(\phi, \lambda) dx dy]}{N_e(\phi, \lambda) N_o} + \text{Ito terms}
\]

(4.41)

\( p(\phi, \lambda) \) is a nonelementary function due to a nonelementary integral in its exponent. By using (4.36-4.41), solutions \( U(\phi, \lambda) \) might be obtained, in principal, by a numerical integration. However, for more than 4 state schemes, an excessive amount of computer time is required. Nevertheless, for comparison purposes, the nonlinearity surface \( h(\phi, \lambda) \) and uncertainty diagram \( U(\phi, \lambda) \) are better figures of merit than the corresponding average probability of error \( P_e \). An ideal \( U(\phi, \lambda) \) is independent of \( \phi \) and \( \lambda \), while \( h(\phi) = \sin \phi \) is a good choice.
Figure 4.10. The QPSK nonlinearity surface.
Figure 4.11. The SQPSK nonlinearity surface.
4.2.2. EQUALIZED DFCRL

Although an equalization within the loop might be employed in any narrowband system to improve performance, here we analyze a wideband system employing an overlapped modulation scheme. Due to overlapping pulses extending beyond the \((-T/2, +T/2)\) interval, Fig. 2.2, ISI is present at the receiver. We analyze this ISI, which might be reduced by employing a simple equalizer in each of the quadrature arms of the DFCRL. Our analysis uses the SQORC (also known as Fehér's IJF, [BS]) modulation scheme, Table 2.1, Fig. 2.2, eqs. (2.1–2.3), as a model which might be straightforwardly applied to any other overlapped scheme. From (2.1–2.3), it is easy to show that SQORC does not have a constant envelope, i.e. the factor within the second sum in (4.9) is not a constant. Therefore, a ML approach should take this into consideration. By applying the previous, rather lengthy procedure, an ML-based receiver might be derived. Instead, we use a more effective approach based on the following logical reasoning: The autocorrelation function of an overlapped scheme spans more than a 2T second interval, Fig. 4.12, i.e. neighboring pulses cause ISI at the sampling point. We assume that an early-late gate loop, like a TR crosscorrelation device, is used to establish a timing reference, [B3], and sampling is made every T seconds. The crosscorrelation function of a presumably rectangular reference signal and input pulses (described by eqs. 2.3a-b), rather than the autocorrelation function given in Fig. 4.12, are used for our purposes. A T seconds long pulse might have four different forms, Fig. 4.13a, and an equal number of forms with opposite signum. The signal power after the ISD device (denoted with an integral in Fig. 4.6), i.e. at the decision device input, varies from symbol to symbol because of non-equal pulses. The probability of error performance is dictated by a pulse with the lowest energy, i.e., a degradation of 1.09 dB might be expected. This theoretical result is experimentally verified on our IJF modem, Fig. 4.13b. This translates into a conditional probability of error degradation from \(10^{-10}\) to \(10^{-8.59}\). See the UD in Fig. 4.14.
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Figure 4.12. The autocorrelation functions of WQAM schemes.
Figure 4.13. a) The SQORC signal shapes. b) The average probability of error performance measurements (•) of the IJF (SQORC) modem employing the classical carrier recovery loop.
Figure 4.14. The SQORC uncertainty diagram employing the classical DFCRL. The performance of $10^{-10}$ is taken as the reference.
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To smooth the signal fluctuations of overlapped schemes at the decision device input a simple 3-tap equalizer is placed between the ISD and the decision device inside the DFCRL in Fig.4.6, which gives a new loop called equalized DFCRL, Fig.4.15. The noise after the equalizer is Gaussian but not white with variance \( \sigma_n^2 = (c_0^2 + 2c_1^2)N_0 \), where \( c_0, c_1 \) are tap coefficients. As a consequence, the performance of SQORC is improved to \( 10^{-9.97} \) which is a minor, rather insignificant degradation (0.03 dB) from the \( 10^{-10} \) reference. The UD of SQORC using this transversal type of equalizer within the DFCRL is illustrated in Fig.4.16. Although the pulse shape of QORC schemes extends over parts of three \( T \) intervals— the right half of the \( (k-1) \)-th, all of the \( k \)-th, and the left half of the \( (k+1) \)-th interval, Fig.2.2, the energy in the \( (k-1) \)-th interval depends also on the pulse in the \( (k-2) \)-th interval, etc. Total ISI cancellation requires an infinitive number of taps. However, a simple 3-tap equalizer cancels ISI almost completely. Tap coefficients actually depend on the data streams in both quadrature channels, except for \( \phi = 0 \). By employing a more complex feedforward and decision feedback equalizer, we might be able to achieve a slightly better performance than given in Fig.4.16. However, a detailed analysis of the canceller optimization is out of the scope of this contribution. An ML-based device for SQORC (and other overlapped schemes) might also be realized in a parallel form. Three pairs of in-phase and quadrature arms at frequencies \( f_c \) and \( f_c + n/T_S \) will be followed by the ISD and decision devices plus a baseband combiner. No equalizers will be needed.

We have concluded the analysis of the ML approach and proceed with the MAP approach, in which a search for new improved loops is pursued.
Figure 4.15. The equalized decision feedback carrier recovery loop.
Figure 4.16. The SQORC uncertainty diagram employing equalized DFCRL. The performance of $10^{-10}$ is taken as the reference.
4.3 MAXIMUM A POSTERIORI (MAP) APPROACH

Previously, a uniform distribution of $f(\varepsilon_1, \varepsilon_2, \theta)$ in (4.6) was assumed and ML optimization followed. A structure with the $\tanh(x)$ nonlinearity was found, which was approximated with $\text{sign}(x)$ for high SNR and realized with a quantizer characteristic (4.31) for schemes with more than 2-baseband levels. The corresponding loop is called classical DFCRL, Fig.4.6. However, as shown by (4.31-4.39) and Figs.4.7-4.11, this loop seems to be optimal for $\phi = \lambda = 0$ only. In general, we might use the knowledge of $f(\varepsilon_1; \varepsilon_2; \theta)$ accumulated over previous $V_T$ and $V_c$ symbols to improve our decision on the $k$-th symbol. This value of $f(\varepsilon_1; \varepsilon_2; \theta)$ might be introduced in (4.5) and an MAP optimization procedure might follow. However, $f(\varepsilon_1, \varepsilon_2, \theta)$ and the related $p(\phi, \lambda)$ are nonelementary functions, which are rather difficult if not impossible to manipulate. Therefore, we use a more practical approach based on the knowledge of $\phi, \lambda$ and $h(\phi, \lambda)$ accumulated over previous $V_c$ and $V_T$ symbols respectively, as follows: According to (4.32-4.35), the phase uncertainty $\phi$ of the loop causes an attenuation of the in-phase signal proportional to $\cos \phi$ and a crosstalk proportional to $\sin \phi$. The timing uncertainty $\lambda$ causes an attenuation of the $k$-th pulse (on which the decision has to be made) proportional to $R(\lambda)$ and an ISI of the $(k - 1)$-th or $(k + 1)$-th neighboring pulse proportional to $R(1 - |\lambda|)$. A model of these degradations is shown in Fig.4.17a. Since we know the $V_c$ symbols long estimate of $\phi$, say $\hat{\phi}$, the $V_T$ symbols long estimate of $\lambda$, say $\hat{\lambda}$, and the $(V_c, V_T)$ symbols long estimate of $h(\phi, \lambda)$, say $\hat{h}(\phi, \lambda)$, we might be able to compensate for these degradations, Fig.4.17b. For the purpose of this contribution, we assume perfect estimates of $\phi, \lambda$ and $h(\phi, \lambda)$, i.e. $\phi = \hat{\phi}$, $\lambda = \hat{\lambda}$, and $h(\phi, \lambda) = \hat{h}(\phi, \lambda)$, which might be reasonable since high SNRs within the loop bandwidth are assumed. A compensation for the $\phi$-caused degradation is explained in Section 4.3.1, compensation for the $\lambda$-caused degradation is explained in Section 4.3.2, while a hybrid compensation scheme follows in Section 4.3.3.
Figure 4.17. a) The degradation model for the loop caused impairments due to the phase uncertainty \( \phi \) and the timing uncertainty \( \lambda \). b) The corresponding equalization model.
4.3.1 Crosstalk Cancellation DFCR Loop

The following conclusions might be drawn from (4.31-4.40): The quantizer represented by the characteristic in (4.31) is an optimum nonlinearity for $\phi = 0$ only. However, to minimize an average probability of error $P_e$ for any $\phi$, a device for which the decision is independent of $\phi$ must be found. In addition to the already applied ML criterion, different criteria (constraints) might apply, e.g., minimum mean square error, etc. We decide to minimize $P(e|\phi)$ over the entire range of $\phi$, which, as shown later, corresponds to the cancellation of pattern-jitter. Although an analysis might be quite complex regarding the difficulties of an exact calculation of the noise statistics at the VCO input, we ignore this noise, which is reasonable as long as the loop bandwidth is small in comparison with the data bandwidth. Then a new crosstalk cancellation (CC) DFCRL for M-ary quadrature amplitude modulation (MQAM) signal sets is given in Fig. 4.18. The loop employs two adders, two baseband amplifiers with gains equal to $\tan \phi$, and two adaptive quantizers (or a pair of amplifier-fix quantizer combination), which are governed by a phase error $\phi$ of the loop, to cancel (or at least attenuate) the crosstalk. As a result, the pattern dependent jitter is cancelled (at least attenuated) and the loop performs as a continuous wave (CW) CR loop. Here, we optimize the loop performance for $\lambda = 0$, while a more general case $\phi \neq 0, \lambda \neq 0$ is analyzed in Section 4.3.3. If $K_A(\phi) = K_B(\phi) = \tan \phi$, a signal at the A decision device input is

$$
\zeta_A = (a_m \cos \phi + b_n \sin \phi + \nu_A) - (-a_m \sin \phi + b_n \cos \phi + \nu_B) \tan \phi
= \frac{a_m}{\cos \phi} + (\nu_A - \nu_B \tan \phi)
$$

(4.42)

i.e. no crosstalk occurs and the signal-to-noise ratio remains constant, since both the desired signal and noise powers are increased in the same proportion. A decision is independent of CR uncertainties, i.e. the loop operates as a CW CR loop and the nonlinearity curves equal sin $\phi$, Fig. 4.19a (here an ideal timing is assumed). Note that the curve labeled $\lambda = 0.0$ corresponds also to any MQAM scheme (within a constant factor), assuming CCDFCRL is employed. The nonlinearity curves ($\lambda = 0.0$) of the MQAM schemes employing classical DFCRL are summarized in Fig. 4.19b. The advantage of our new crosstalk cancellation loop over the classical one is obvious.
Figure 4.18. The crosstalk cancellation DFCRL.
Figure 4.19a. The (S)QPSK nonlinearity curves employing the crosstalk cancellation decision feedback carrier recovery loop. The $\lambda = 0$ labeled curve corresponds to the MQAM schemes also.
Figure 4.19b. The MQAM nonlinearity curves employing classical decision feedback carrier recovery loop. The parameter $\Gamma = 4$ corresponds to the $P(e|\phi) \approx 10^{-8}$. 
In our analysis of the CCDFCRL we have assumed an optimal estimation of $\phi$, i.e. $\phi = \hat{\phi} = [h(\phi, \lambda = 0)]^{-1}$ where $[h(\phi, \lambda = 0)]^{-1}$ equals the inverse of $h(\phi, \lambda = 0)$. A detailed analysis of this estimator is out of the scope of this presentation. However, a brief heuristic explanation follows. Let's assume that the loop is initially locked and $\phi = 0$. As input symbols of data are arriving into the loop arms, the phase uncertainty $\phi(t)$ due to Gaussian noise fluctuates slowly (relative to the symbol rate) and might take any value between $(-\pi, +\pi)$. For small $\phi$, $h(\phi) \approx \phi$, i.e. $\phi$ is directly proportional to the voltage $h(\phi)$. The shaded part of the loop in Fig. 4.18 feeds back this (presumably correct) information on $\phi$ accumulated over the previous $V_c$ symbols and improves the decision at the $k$-th symbol, which is now virtually independent of $\phi$. This improved decision changes $h(\phi)$ from the complex form given in equation (4.40) and Fig. 4.19b to $h(\phi) = \sin \phi$, Fig. 4.19a. As a final result, the pattern jitter is cancelled (practically, it is attenuated by an amount proportional to the ratio of data and loop bandwidths $R_\phi = SNR_{IF}/SNR_L$) and, more importantly, the quadrant ambiguities previously present due to the combination of signal constellation and fixed threshold decision devices have vanished. Similar improvement might be expected in the tracking of unbalanced schemes while employing proper gains in the quadrature arms.

The average probability of error $P_e$ of the WQAM schemes employing CCDFCRL equals

$$P_e = P(e|\phi) \int_{-\pi}^{+\pi} p(\phi) d\phi = P(e|\phi = 0)$$  \hspace{1cm} (4.43)

i.e. no degradation (pattern jitter) exists due to the presence of data, assuming $P_e < 10^{-3}$. At high $P_e$ (above $10^{-3}$) the signal $h(\phi)$ which governs the loop is attenuated (by approximately 0.175 dB at $P_e \approx 10^{-1}$ when 16QAM is concerned) since the decision principal is employed within the loop. However, further analysis of these effects at high $P_e$ is out of the scope of this presentation.

In order to compare the average probability of error $P_e$ performances of the WQAM schemes employing the classical DFCRL and those schemes employing our new crosstalk cancellation DFCRL the knowledge of the probability density function $p(\phi)$ of the classical DFCRL is necessary. This is a rather difficult task, which requires a tedious mathematical procedure to be employed in an attempt to solve the stochastic partial differential equation of the Fokker-Planck-Kolmogorov type, or a time consuming computer simulation needs to be done. On the contrary, the probability density function $p(\phi)$ of the crosstalk cancellation DFCRL is practically equivalent to that of the CW loop and is given by [B16], Fig. 4.1,

$$p(\phi) \approx C \frac{\exp[\rho(\cos \phi + \beta N \phi)]}{2\pi I_0(\rho)}$$  \hspace{1cm} (4.44)

$\rho$ is the signal-to-noise ratio within the loop, $C$ is the normalization constant, $\beta_N = \beta/\rho$ is the normalized loop detuning, $\beta$ is the absolute detuning, and $I_0(\rho)$ is the modified Bessel
function of zero order and argument $p$. We assume that the classical DFCRL has the same probability density function $p(\phi)$ as the CW loop (and the crosstalk cancellation DFCRL) — except for the factor four and the periodicity of $\pi/2$ radians. The true probability density function of the classical DFCRL will have less desirable form. Therefore, our estimation might be biased in favor of the classical loop.

We were able to perform the Monte Carlo simulation of the performance of the QPSK schemes — employing both classical and crosstalk cancellation loops — at $P_e > 10^{-4}$ and that of the 16QAM at $P_e \approx 10^{-2}$. We also generated the theoretical performance curves of QPSK and 16QAM schemes at $p \leq 174$ (22.4 dB). The results are summarized in Figs. 20–21. In Fig. 20 the QPSK $\log_{10} P_e$ performance versus the carrier-to-noise ratio C/N in dB in the double-sided Nyquist bandwidth and the normalized loop detuning (loop stress) $\beta_N$ as a parameter is presented. The ratio $R_p = p/\text{SNR}_{IF} = 10$ is assumed. This is the lower end practical value, which allows us to generate these curves. Note that typical satellite systems operate at $\text{SNR}_{IF} > 5$ dB, i.e. $p > 15$ dB. The typical terrestrial (and cable) systems operate at even higher $\text{SNR}_{IF}$ ($p > 15$ dB). An estimation error of 0.46 dB is assumed when the crosstalk cancellation DFCRL is employed. An ideal ambiguity resolution is assumed when the classical DFCRL is employed — a bias in the favor of the classical loop. In practice, a degradation of 0.3–0.5 dB might be expected due to the ambiguity resolution. A random d.c. wandering within the loop, channel asymmetry (e.g. during a selective fading event), or any other dynamic impairment of such kind will cause a stress in the loop and consequently a degradation of the $P_e$ performance. The classical DFCRL suffers significant degradation, while our new loop is practically insensitive to these kind of impairments, Fig. 4.1, Fig. 4.20. At $P_e = 10^{-6}$ and $\beta_N = 0.7$ our new loop outperforms the classical one by 2.5 dB, assuming QPSK is employed. Results in Fig. 4.7, and Fig. 4.21 indicate that at lower $P_e$ and for higher state modulation schemes our new loop will outperform the classical one by even higher margin.

* E.g., let's brief the necessary performance of the $K_A(\phi)$ and $K_B(\phi)$ devices (amplifiers). Let's assume the symbol rate equals 10 Msymbols/s. Then, the necessary bandwidth of the $K_A(\phi)$ and $K_B(\phi)$ devices is in the range of the tens of MHz. Usually, the loop bandwidth is a one hundred or more narrower than the bandwidth of the data. Therefore, the phase changes $\phi$ and the necessary dynamic changes of the $K_A(\phi)$ and $K_B(\phi)$ amplifiers are in the range of one hundred of kHz, or lower. The necessary gain is proportional to the $\tan \phi$. If $|\phi| < 45^\circ$, $K_A(\phi)$ and $K_B(\phi)$ perform as attenuators. If the maximal available gains of $K_A(\phi)$, $K_B(\phi)$ equal 100, the $\tan \phi$ function might be modeled for any $\phi$ except in the $1^\circ$ wide strips around $\pm 90^\circ$. Therefore, the crosstalk cancellation DFCRL might be realized with the practically available components.
Figure 4.20. The QPSK $\log_{10} P_e$ performance versus the carrier-to-noise ratio $C/N$ in dB in the double-sided Nyquist bandwidth and the normalized loop detuning (loop stress) $\beta_N$ as a parameter, employing classical or crosstalk cancellation (CC) decision feedback carrier recovery loop (DFCRL). The ratio $R_p = SNR_L/SNR_{IF} = 10$ is assumed.
Figure 4.21. The 4 and 16QAM $\log_{10} P_e$ performance versus the normalized loop detuning (loop stress) $\beta_N$, employing classical or crosstalk cancellation (CC) decision feedback carrier recovery loop (DFCRL). The ratio $R_p = SNR_L/SNR_{IF} = 10$ is assumed.
4.3.2 Timing ISI Cancellation DFCR Loop

By observing equation (4.34), it is easy to see that a timing uncertainty $\lambda$ causes complex ISI. As an example, these negative effects might be partially overcome by employing a simple 3-tap equalizer, whose central tap gain is proportional to $+1/R(|\lambda|)$ and the gains of two neighboring taps are proportional to the product of $-R(1 - |\lambda|)$ and the corresponding step function ($c_{-1} = 0$ for $\lambda \geq 0$, while $c_{+1} = 0$ for $\lambda \leq 0$), Fig.4.17b. In a system employing narrowband filtering, an equalizer with more taps will be necessary. The resultant CR loop is called timing ISI cancellation DFCRL, Fig.4.22. The loops in Fig.4.15 and Fig.4.22 are the same except for the different tap gains of the corresponding equalizers. Here we have assumed perfect knowledge of $\lambda$ and have ignored crosstalk terms proportional to $\sin \phi$. The noise after the equalizer is Gaussian but not white with variance $\sigma_n^2 = [R^{-2}(|\lambda|) + R^2(1 - |\lambda|)]N_0$. However, SNR is proportional to

$$
\frac{R^2(|\lambda|)}{1 + R^2(|\lambda|)R^2(1 - |\lambda|)}
$$

(4.45)

Timing ISI pattern jitter is cancelled but SNR and the conditional probability of error deteriorate as $\lambda$ increases. However, this deterioration is not as severe as in the classical DFCRL. The relative improvement increases as the number of baseband levels increases. A decision feedback equalizer which cancels timing ISI on the previous symbol(s) and a transversal equalizer which acts on future symbol(s) might give better results. Further improvements might be expected with an optimization of the TR circuit.
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Figure 4.22. The timing intersymbol interference cancellation DFCRL.
4.3.3 FEEDLOOP

By using knowledge of both $\phi$ and $\lambda$ accumulated over $V_c$ and $V_T$ symbols respectively, a new feedback structure which uses an estimation of phase and an estimation of timing to improve detection is suggested in Fig.4.23. We call this loop FEEDLOOP, as an acronym for Feedback of data, Estimation of phase, Estimation of timing, and Detection LOOP. The uncertainty diagram of FEEDLOOP is independent of $\phi$ and dependance on $\lambda$ is significantly reduced, which becomes particularly important for multilevel schemes. SNR at the decision device input is proportional to

$$\frac{R^2(|\lambda|) \cos \phi}{\cos \phi + R^2(|\lambda|) R^2(1 - |\lambda|)} \quad (4.46)$$

for a FEEDLOOP employing a transversal equalizer, Fig.4.23. Based on the results of the Monte Carlo simulation, the UD of 256QAM scheme employing a FEEDLOOP is given in Fig.4.24. Note the deeps arround the singular points at $\phi = \pm 90$ degrees. The width of these deeps depend on the practical limitations of the components within the loop. The advantages of the FEEDLOOP, as seen in the results presented in Fig.4.24, and Fig.4.1c, over the classical DFCRL, Fig.4.10a-b, and Fig.4.1a, are obvious.

The SNR for the FEEDLOOP employing a decision feedback equalizer is rather difficult to calculate, in general. However, regarding $\lambda$ as a slow varying process relative to the symbol rate, i.e. $\lambda$ is constant during one symbol period,

$$\text{ISI} \propto a_{k \pm 1} R(1 - |\lambda|) P(e_k|\lambda) \quad (4.47a)$$
$$\text{signal} \propto a_k \{1 - [1 - R(|\lambda|)] P(e_k|\lambda)\} \quad (4.47b)$$
$$\text{noise} \propto \sigma^2 \quad (4.47c)$$

where $\propto$ means proportional to. Crosstalk is reduced by $P(e_k|\lambda)$, the drop in signal is reduced in the same proportion, and noise remains the same.

A few more sentences are devoted to the choice of UD and nonlinearity surface as figures of merit: An UD independent of phase uncertainty $\phi$ and timing uncertainty $\lambda$ represents the best possible performance, i.e. a CW loop. Any dependence on $\phi$ and $\lambda$ represents a degradation caused by the presence of data, which has been called in the literature "data introduced ISI," "data dependent pattern jitter," "squaring loss," "self noise," etc. The nonlinearity surface determines the probability of error function of the loop and other properties such as acquisition, skipping rate, etc. Therefore, both the UD and nonlinearity surface describe the performance of the particular signal format-carrier recovery loop combination.
Figure 4.23: The FEEDLOOP.
Figure 4.24. The 256QAM uncertainty diagram employing FEEDLOOP. The results of Monte Carlo simulation with 8124 symbols. \( P(e|\phi, \lambda) = -10 \log(PE) \). TR \((t/T_{\text{symbols}})\). CR (degrees).
4.4 CONCLUSION

In this chapter a composite phase and timing estimation and data detection of WQAM signal sets is analyzed. By using a maximum likelihood (ML) approach, four new joint phase and timing estimators are derived. The performance of the classical decision feedback carrier recovery loop (DFCRL) employing an integrate-and-sample device in each quadrature arm (active loop) is evaluated in the presence of both carrier uncertainty $\phi$ and timing uncertainty $\lambda$. Results are presented in the following new forms: the Uncertainty diagram $U(\phi, \lambda)$, the equal (iso) probability of error isoerror curves, and the nonlinearity surface $h(\phi, \lambda)$ which becomes loop's nonlinearity S-curve when $\lambda \to 0$. The performance of overlapped schemes is degraded if classical carrier recovery (CR) schemes are employed. The theoretical results are experimentally verified on our Intersymbol-Jitter-Free (IJF) 64 kb/s modem. New equalized DFCRL which employs a simple 3-tap transversal equalizer in each quadrature arm is proposed for CR and the detection of overlapped schemes. Our new loop does not exhibit performance degradation associated with classical CR loops. By using the MAP probability approach and knowledge of $\phi, \lambda$, and $h(\phi, \lambda)$ accumulated over corresponding observation intervals, improved loops, i.e. crosstalk cancellation DFCRL, timing intersymbol interference cancellation DFCRL, and FEEDLOOP, are introduced and analyzed. The theoretical results are verified by the Monte Carlo simulation. These loops perform almost as well as a continuous wave CR loop, do not exhibit quadrant ambiguities, and might be employed for phase and timing estimation and the detection of balanced and unbalanced WQAM schemes. The advantages of the new loops over the classical one become greater at lower probabilities of error and when higher state schemes are employed. E.g. at $P_e = 10^{-5}$ and in the presence of a normalized loop detuning of 0.7, the crosstalk cancellation loop (and FEEDLOOP) outperform the classical one by 2.4 dB, assuming QPSK modulation scheme is employed.

However, we are aware that our analysis is far to be complete. In practice, the total phase error might be composed of many (usually independent) components such as Doppler shift, angle modulation, instabilities of the transmitter and receiver frequency sources, additive noise (usually Gaussian), etc. Herein we dealt with the digital data modulated signals with suppressed carrier in the presence of white Gaussian noise. We analyzed the interaction of data and noise and introduced a method to wipe-off the data — in order to improve the performance. The analysis of effects of Doppler, ACI, discrete spurious, and other
impairments on the tracking and detection performance of the loop was not the part of this presentation. In the next chapter, we present a simplified performance analysis of the multistate modulation schemes influenced by the phase noise, i.e. instabilities of the frequency sources.
PERFORMANCE
OF M-ARY MODULATION SYSTEMS
IN THE PRESENCE OF PHASE NOISE
Chapter 5: Performance in the Presence of Phase Noise

In the Chapter 2 WQAM family is introduced. The performance of 4 state WQAM schemes was evaluated in the presence of different impairments typical for the satellite channels such as ACI and hardlimiter nonlinearity. In the Chapter 3 the performance of 256QAM and 1024QAM schemes is evaluated in the presence of the group delay and amplitude response impairments typical for the terrestrial radio and cable systems. An a priori knowledge of the carrier phase and symbol timing has been assumed. Chapter 4 deals with the estimation of the carrier phase and symbol timing of WQAM schemes, and a few new receivers are introduced and analyzed. In this chapter, the performance of WQAM and M-ary quadrature partial response (MQPR) systems is analyzed in the presence of phase noise. Herein we adopt a practical approach, which yields useful performance curves in which the degradation due to phase noise can be readily seen.

An outline of the chapter follows. In Section 5.1 the phase noise characterisation is given. In Section 5.2 the results for probability of error performance of MPSK, MQAM and MQPR modulation systems in the presence of thermal and phase noise are presented.
5.1 PHASE NOISE CHARACTERIZATION

The expressions for the performance of coherent digital modulation systems in a Gaussian channel are well known [B15]. However, in practice we rarely have a perfect knowledge of the local phase reference. The output signals from carrier and symbol timing recovery circuits are really random processes and produce errors in the estimation of phase and time, respectively. These random processes consist of two components: first contributed by thermal white Gaussian noise, and second contributed by all other random sources which is termed phase noise. The effects on the error probability performance due to this phase noise can be obtained in principle. However, this is a formidable task with only a few rather complex solutions existing [B3], [B12], [B16] for two- and four-state modulation systems only.

We compute the degradation of a system in the presence of phase noise in the following manner. Considering the sources of phase noise as independent random variables, the central-limit theorem says that, under certain general conditions, the resultant equivalent phase noise probability density function approaches a normal Gaussian curve as number of sources increase [B25, p.267]. Since the number of individual sources tends to be large in practice, and their magnitudes are of the comparable order, the assumptions of the central limit theorem should apply. This is the case of a well designed system. However, in a particular example, an individual source might dictate the overall system performance and Gaussianity might be destroyed, which might result in an increased degradation of the performance.

The phase noise is combined with assumed white Gaussian noise channel to produce a total carrier-to-noise ratio \((C/N)_T\) given by

\[
\frac{(C/N)_T}{(N/C)} = \left(\frac{(N/C)}{[(N/C) + (N/C)_p]}\right)^{-1} \tag{5.1a}
\]

\[
(N/C)_p = (N/C)_1 + (N/C)_2 + \ldots + (N/C)_n \tag{5.1b}
\]

where \((N/C)\) is the thermal noise-to-carrier ratio, \((N/C)_i\) \((i = 1, \ldots, n)\) is the noise-to-carrier ratio of the \(i\)-th random source and \((N/C)_p\) is the equivalent phase noise-to-carrier ratio. We use known expressions [B15] to calculate performance of coherent digital modulation systems in the presence of phase noise by replacing \((C/N)\) by \((C/N)_T\) and having \((C/N)_p\) as a variable parameter.

Because of a simplistic phase noise model, the accuracy of results might depend how close the assumed Gaussian probability density function fits the probability density function of the real phase noise. However, in practice we are dealing with the small amount of phase noise which causes degradations less than 1 dB at \(P_e = 10^{-6}\). In that case and for our purpose the Gaussian probability density function seems to be a good approximation for a real but unknown distribution of a phase noise. Our results of the measurement presented in Section 5.2 are in a close agreement with previous assumptions.
5.2 PERFORMANCE EVALUATION

The probability of error performance of Binary PSK (BPSK=2PSK), MPSK ($M > 2$), MQAM and MQPR modulation schemes are given by the following expressions [B15]

\[ P_B = \frac{1}{2} \text{erfc}(\sqrt{\gamma_b}), \quad \text{(BPSK)} \] (5.2)

\[ P_M = \text{erfc}(\sqrt{k \gamma_b \sin \frac{\pi}{M}}), \quad \text{(MPSK, } M > 2\text{)} \] (5.3)

\[ P_L = (1 - \frac{1}{L}) \text{erfc}(\sqrt{\frac{3}{M - 1} \frac{1}{2} \gamma_{av}}), \quad \text{(MQAM)} \] (5.4a)

\[ P_L = (1 - \frac{1}{M}) \text{erfc}(\sqrt{\frac{3}{M - 1} \frac{1}{4} \gamma_{av}^2}), \quad \text{(MQPR)} \] (5.5a)

\[ P_M = 2P_L(1 - \frac{1}{2}P_L) \quad \text{MQAM and MQPR (5.4b, 5.5b)} \]

where [B8,p.254]

\[ \gamma_b = \frac{E_b}{N_0} = \frac{C}{N} \frac{B}{f_b} \] (5.6)

\( \gamma_b \) stands for the approximately equal, \( P_B \) is the probability of error performance of BPSK, \( P_M \) is the corresponding symbol error rate for the MPSK ($M > 2$), MQAM and MQPR systems, while \( P_L \) corresponds to the probability of error of the baseband signal in each of the two quadrature components of QAM or QPR. \( \gamma_{av} \) is the average signal-to-noise ratio per k-bit symbol, where \( k = \log M \) and \( M \) is the number of levels. \( \gamma_b \) is the energy per bit-to-noise ratio, \( (C/N) \) is the carrier-to-thermal noise ratio; \( f_b \) is the bit rate bit/s and \( B \) is the double-sided noise bandwidth in Hz.

We assume \( B \) is equal to the double-sided Nyquist bandwidth. The expressions (5.2)-(5.5) give the probability of error performance curves illustrated in Fig.5.1. Inclusion of \( (C/N)_p \) as a variable parameter produces a series of curves, one of which (256QAM) is shown for illustration in Fig.5.2. The degradations \( (C/N)_T - (C/N)_p \) for \( P_e = 10^{-6}, \text{dBr, vs. the (C/N)_p are summarized in Figs.5.3—5.5. Fig.5.3 applies for the MPSK systems, the MQAM systems are shown in Fig.5.4 and the MQPR systems in Fig.5.5.} \)
Figure 5.1. The average probability of error performance of the M-ary schemes versus the carrier-to-thermal noise ratio. The white Gaussian channel only (no phase noise). The double-sided Nyquist bandwidth. 

- MQAM schemes,
- MPSK schemes,
- MQPR schemes.
Figure 5.2. The 256QAM average probability of error performance versus the carrier-to-thermal noise ratio with the carrier-to-phase noise ratio as a parameter. The double-sided Nyquist bandwidth.
Figure 5.3. The degradation of the MPSK systems in dB, with respect to the theoretical value necessary to achieve the $P_e = 10^{-6}$ performance, versus the carrier-to-phase noise ratio in the double-sided Nyquist bandwidth in dB. The pluses (+ + +) represent the results of measurements on 4 state SCPC satellite modems.
Figure 5.4. The degradation of the MQAM systems in dB, with respect to the theoretical value necessary to achieve the $P_e = 10^{-6}$ performance, versus the carrier-to-phase noise ratio in the double-sided Nyquist bandwidth in dB. The pluses (+ + +) represent the results of measurements on 4 state SCPC satellite modems.
Chapter 5: Performance in the Presence of Phase Noise

Figure 5.5. The degradation of the MQPR systems in dB, with respect to the theoretical value necessary to achieve the $P_e = 10^{-6}$ performance, versus the carrier-to-phase noise ratio in the double-sided Nyquist bandwidth in dB.
Chapter 5: Performance in the Presence of Phase Noise

In the BPSK system, Fig. 5.3, a \((C/N)_p > 20\) dB causes the degradation to be less than 0.5 dB. If \((C/N)_p > 30\) dB the degradation is negligible. This is an easily achievable goal, except in very low speed systems, where the spectral purity of signal source(s) could be critical [B28]. However, the 256QAM system — Fig. 5.2 and Fig. 5.4 — requires \((C/N)_p > 45\) dB to limit the phase noise caused degradation to the amount below 1 dB. Additionally, the high-ary modulation schemes are more fragile in a multiple fading and interference environment. Further, the interference equalizer, carrier and symbol timing recovery and data generation and decision circuits are more complex for the case of high-ary modulation schemes. Consequently, the performance required of each component in the high-ary system should be higher. In all practical cases we find that if we take \((C/N)_p\) to be 10 dB higher than the \((C/N)_6\), the degradation is less than 1 dB (Figs. 5.2—5.5). If \((C/N)_p > (C/N)_6 + 20\) dB the degradation is negligible.

Results of measurements, Figs. 5.3—5.4, which will be briefly assessed, are in a close agreement with our engineering rule-of-thumb approximation. In a low speed data system (e.g. single-channel-per-carrier SCPC satellite link) a phase noise of frequency sources might limit an overall performance of the system. To verify this, we performed sets of measurements on SCPC satellite links consisting of two different 70 MHz 32 symbols/s 4-state modems (staggered QPSK and IIF), University of Ottawa earth station with 70 MHz/14 GHz up-converter and 12 GHz/70 MHz down-converter and two different "space segments" (Telesat Canada ANIK 14/12 GHz satellite transponder and a laboratory transponder with a similar performance). Each link consisted of five local oscillators of a comparable quality, which phase noises were measured by an automated spectrum analyzer. Phase noise power of each oscillator was integrated over the error transfer function \([1-H(f)]\) of modem's carrier recovery loop (Costas loop type with a second order passive filter). The average of multiple measurements gave a noise power within the 16 kHz Nyquist bandwidth, which after multiplying by 2 and dividing by a total carrier power gave an \((N/C)_i\) of a particular phase noise contributor. Although a phase noise of frequency source itself is nonwhite (it might be represented by \(\sum cf^{-\mu}\), after it passes through an "whitening filter" (i.e. error transfer function of the loop, which might be approximated by the \(cf\) near the carrier where the noise contribution is the most important) it becomes near-white, but not necessary Gaussian. To get different values of \((N/C)_i\), the frequency stability of one of five local oscillators was intentionally degraded. Even with these unbalanced phase noise sources the measured results were in a close resemblance with previously assumed curves, Fig. 5.3—5.4.
5.3 CONCLUSION

In this chapter, the performance of MPSK, MQAM and MQPR modulation systems in the presence of phase noise and additive white Gaussian noise is presented in graphic forms. If a carrier-to-phase noise ratio is at least 10 dB higher than a carrier-to-thermal noise ratio required for the $P_c = 10^{-6}$, the degradation due to phase noise will be less than 1 dB. If $(C/N)_p > (C/N)_b + 20$ dB the degradation is negligible. The analysis is supported with results of measurements on 4-state SCPC satellite modems. Based on previous results a first order estimation of components requirements might be made. To minimize the degradation in the higher-ary modulation schemes the rigorous selection of components and use of advanced technologies is necessary.
THESIS SUMMARY
In the Introduction, the purpose of this study was outlined. It was the author's goal to find power-bandwidth efficient modulation schemes, which will perform superiorly in composite ACI, linear and nonlinear, channel environments. Synthesis and analysis of both, transmitter and receiver, were performed. The results, achievements, and our own contributions which are presented in the chapters 2—5 are summarized as follows.

In the Chapter 2, the weighted quadrature amplitude modulation (WQAM) was introduced. By using the gradient search technique, a family of pulse shapes with a narrow mainlobe and minimal sidelobe levels within an equivalent baseband \( |fT_s| = 0 \) to 5 (this corresponds to 1 to 2 adjacent channels in practice) was found. The performance of 4-state WQAM schemes was evaluated in the ACI linear and nonlinear channel environments, with the channel spacing and fading depth (signal attenuation) as parameters, by means of the computer simulation. One of the new staggered WQAM schemes, termed S3MQAM, outperforms other known members of the WQAM family, i.e., (S)QPSK, MSK and (S)QORC in almost all practical situations.

In the Chapter 3 the performance analysis of 256 and 1024QAM schemes is performed in the presence of amplitude and group delay impairments. A brief feasibility study of the transmission of North American T1 (DS1) 1544 kb/s or CCITT 2048 kb/s data stream over the analog 240 kHz wide CCITT supergroup is given. Staggered QAM schemes perform better in the presence of linear group delay impairments, but nonstaggered schemes are less sensitive to linear amplitude gain impairments.

In Chapter 4, a composite phase and timing estimation and data detection of WQAM signal sets is analyzed. By using a maximum likelihood (ML) approach, four new joint phase and timing estimators are derived. The performance of the classical decision feedback carrier recovery loop (DFCRL) employing an integrate-and-sample device in each quadrature arm (active loop) is evaluated in the presence of both carrier uncertainty \( \phi \) and timing uncertainty \( \lambda \). Results are presented in the following new forms: the Uncertainty diagram \( U(\phi, \lambda) \), the equal (iso) probability of error isoper curves, and the nonlinearity surface \( h(\phi, \lambda) \) which becomes loop's nonlinearity S-curve when \( \lambda \to 0 \). The performance of overlapped schemes is degraded if classical carrier recovery (CR) schemes are employed. The theoretical results are experimentally verified on our Intersymbol-Jitter-Free (IJF) 64 kb/s modem. New equalized DFCRL which employs a simple 3-tap transversal equalizer in each quadrature arm is proposed for CR and the detection of overlapped schemes. Our new loop does not exhibit performance degradation associated with classical CR loops. By using the MAP probability approach and knowledge of \( \phi, \lambda \), and \( h(\phi, \lambda) \) accumulated over corresponding observation intervals, improved loops, i.e., crosstalk cancellation DFCRL, timing intersymbol interference cancellation DFCRL, and FEEDLOOP, are introduced and analyzed. The theoretical results are verified by the Monte Carlo simulation. These loops perform almost as well as a continuous wave CR loop, do not exhibit quadrant ambiguities, and might be
employed for phase and timing estimation and the detection of balanced and unbalanced WQAM schemes. The advantages of the new loops over the classical one become greater at lower probabilities of error and when higher state schemes are employed. E.g., at $P_e = 10^{-6}$ and in the presence of a normalized loop detuning of 0.7, the crosstalk cancellation loop (and FEEDLOOP) outperform the classical one by 2.4 dB, assuming QPSK modulation scheme is employed.

In Chapter 5, the impact of phase noise on the performance of MQAM,M-ary phase shift keying and M-ary quadrature partial response systems in a Gaussian noise environment is computed. For all cases the degradation due to phase noise is found to be less than 1 dB if a carrier-to-phase noise ratio in a double-sided Nyquist bandwidth $(C/N)_p$ is at least 10 dB higher than the carrier-to-thermal noise ratio $(C/N)_t$ required for the probability of error performance $P_e = 10^{-6}$. Performance graphs are presented which enable a first order approximation of the phase noise requirements of a system to be estimated. Our engineering rule-of-thumb approximations are in close agreement with the results of measurements performed on the single-channel-per-carrier (SCPC) satellite links consisting of two different 70 MHz 32 ksymbols/s 4-state modems (SQPSK and IIF), University of Ottawa earth station with 70 MHz/14 GHz up-converter and 12 GHz/70 MHz down-converter and two different "space segments" (Telesat Canada ANIK 14/12 GHz satellite transponder and a laboratory transponder with a similar performance).

Thesis concludes with an extensive list of references, and a copy of the computer programs printout.

We highlighted some of the so far unresolved problems associated with data transmission over power and bandwidth limited nonlinear channels, we proposed new WQAM schemes at the transmitter and new CR loops at the receiver, and we have shown that our novel schemes outperform previously known devices.
This thesis outlined some of the so far unresolved problems related to the generation and reception of power-bandwidth efficient coherent digital modulation schemes. Several new modulation schemes were proposed and analyzed, including several new receivers for the corresponding WQAM schemes. In this chapter, a brief research proposal for future studies is given.

- The optimization of the signal shapes (i.e. the minimization of the spectral sidelobes) might be extended to include a cascade of nonlinearities, filters, and channel noise sources — typical in the satellite link. This might allow one to match the modulation scheme to the particular power amplifier nonlinearity, and as a result, improve the power and spectral efficiency of the system. Modulation might be combined with coding type — block, convolutional, or Ungerboeck trellis.

- The synthesis and analysis of the WQAM schemes might be extended to the multipath selective fading channels — typical for terrestrial radio systems, and to the Doppler effected channels — typical for mobile radio communications.

- The detailed study of different receivers might include stochastic Fokker-Planck-Kolmogorov partial differential equation analysis of the receiver behaviour in the presence of channel noise and intersymbol interferences, for acquisition performance during transients such as selective fading, discrete occasional interferences, etc.

- Possible application of the new decision feedback loops to coherent fiber optics and radionavigation, and further optimization of these devices might be investigated.
Chapter 8: References

A list of references consists of books (presented in the Section 8.1 and referred as Bxx), special issues (presented in the Section 8.2 and referred as Sxx) and other general publications (presented in the Section 8.3 and referred as Pxxx). Further references on synchronous communications systems, particularly phase locked loops, might be found in

8.1 Books

Chapter 8: References


Chapter 8 : References

8.2 Special Issues


8.3 Other Publications


Chapter 8: References


Chapter 8: References


Chapter 8: References


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Chapter 8: References


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Chapter 8: References


APPENDIX A

COMPUTER PROGRAMS PRINTOUT
Appendix A: Computer Programs Printout

This appendix contains a copy of the computer programs printout, which has been used in this thesis for the simulation purposes. Since joining Dr. Fehér's Digital Communications Group at the Department of Electrical Engineering, University of Ottawa, in September 1982, the author has been developing computer software for the Amdahl 470 (IBM 370 compatible) system in the CMS environment.

The functional software has been written in the FORTRAN 66 and FORTRAN 77 languages. PLOTCOMM plotter, Hewlett-Packard hp 7470 plotter, and Tektonix 4105 color graphic terminal have been used as the output devices. The author wrote the software in the corresponding languages. In addition, SAS language has been used for graphic outputs.

The software is organized in the functional subroutines, whose model particular black-boxes (modulator, filter, channel, nonlinearity, demodulator, synchronizer, equalizer, etc. — in practice). The main programs use these subroutines (black-boxes) and make appropriate connections, according to the author's wishes. Most of the programs have been written in an interactive form, which allows an user-friendly approach to the simulation. Program description is given in the program printout captions (comment lines).

It is assumed that a potential user of these programs is familiar with the mentioned computer languages and the CMS environment.
A. KUCAR, 1983 06 10

******************************************************************************
Subroutine calculates the Arcsin(x)/x function
Singular point Asinc(0)=1.0
******************************************************************************

SUBROUTINE ASINC(x)
IF (x.EQ.0.) GO TO 1
x = ARSIN(x)/x
RETURN
1: I = 1.
RETURN
END
**FILE: ATT**

**FORTRAN A**

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---

**KUCAR, 1984 01 08 12 08**

**Subroutine ATTENUATES the signal power**

---

**Subroutine ATT(EL, DATA, K)**

COMMON /R*/SAM, PL, BAUD, BW, AMPL(4, 6), TIMEL(4, 6), TIMPF(4, 30)

COMPLEX DATA(LD)

AT = 10.**(TIMBK(K, 11)/20.)

DO 1 I = 1, LD

1 DATA(I) = DATA(I) * AT

RETURN

END
A. KUCAR, 1983 10 10, 1985 02 25

-------------------------------------------------------------------------------------------------

Subroutine CRAWC Probability of error curves for a PE(Eb/No) as

LOW as TH

-------------------------------------------------------------------------------------------------

DIMENSION BERPHP (IPEN, PEI, IPR, ITO, TH)

SUBROUTINE BERPHP (PEI, PEI, IPR, ITO, TH)

M = ITO - IPR + 1

DO 1 K = 1, M

J = K - IPR - 1

IF (PEI(J) .LT. TH) RETURN

A2 = CLOG(PEI(J))

1 WRITE(2, 2) A2, A1, IPEN

2 FORMAT (2E15.6, I2)

RETURN

END
FILE: COMPAN

PORT P. A. ON P. E. P. OTIWA CMS RELEASE A

debugging and reports the binary of the data.

SUBROUTINE DATA(LB)

DATA(B:12-1)

COMPLEX(R(1),R(2))

REAL(DP:5,5)

DATA(SP:5,8)

DATA(TP:5,6)

DATA(JT:5,6)

END

END
FILE: CURPHP  FORTRAN  A  UNIV D*OF OTTAWA CMS RELEASE 4

A. KUCAR 1984 01 26 12 06

******************************************************************************

Subroutine PLOTS: 1) Pe=0.5*erfc(SQRRT(Wb/No))
******************************************************************************

Subroutine CURPHP(I,IFROM,ITO)

DO 1 J=IFROM,ITO

A1=FLOAT(J)
A0=10.**(A1)
A2=ERFC(SQRRT(A0))/2.
A3=ALOG10(A2)

1 WRITE(7,2)A1,A2,IPEN

2 FORMAT(2E15.5,1I2)

RETURN

END
**Subroutine DECODE:**

```
** SUBROUTINE DECODE(LARY, WOA DATA, MQAM DATA **

COMMON /IIF/ IOFF, LSAMPL, NSYMB, NO1, NO2, NUMT(4, 10), NT(4, 6096)
COMMON /R1/ SAM, PI, BAUD, BW, AMP(4, 4), TMP(4, 4), TMRP(4, 4)
COMPLEX DATA(LD)
DIMENSION PEF(50)

ART = FLOAT(LARY)
IF (LARY.EQ.1) ART = 2

HARY = (ART*ART - 1.0) / 3.0
IF (CN.EQ.0.0) EB = EB / ALOG(ART)*ALOG(2.0)
SNR = SQRT(2.0*P0*NOISE*EB)
PEF = (1.0 - SNR) / ART
NER = 0

1 PEI(M) = 0.0
IF (LARY.GT.1.0) GOTO 4

C

DO 2 K = 1, NSYMB
J1 = (K - 1) * LSAMPL + 1
J11 = (K - 1) * LSAMPL + 1
Y1B = REAL(DATA(J1))
Y1A = AIMAG(DATA(J1))
INDI = 0
2 IP(K) = (NI1, K, YB).LE.0.0) INDI = 1
INDO = 0
IP(INDK, K, YB).LE.0.0) INDO = 1
IP(INDK, YB).LE.0.0) INDO = 2
IB = ABS(YB) / SNR
YB = ABS(YB) / SNR

C Compute the probability of error for this symbol

FM = DEG - (FLOAT(M) / 20.0)
ARG = IB * INK
IF (ARG.GT.12.0) ARG = 12.0
BI = BPRC(ARG) / 2.0
IP (INDK, Y1B) = E1, -E1
ARG = YB * INK
IF (ARG.GT.12.0) ARG = 12.0
BO = BPRC(ARG) / 2.0
IP (INDQ, Y1B) = E1, -E2

2 PEI(M) = PEI(M) + (E1 + EQ) / 2
GOTO 9

C

4 HIGH = FLOAT(LARY - 1)
DO 7 K = 1, NSYMB
J1 = (K - 1) * LSAMPL + 1
J11 = (K - 1) * LSAMPL + 1
Y1B = REAL(DATA(J1))
Y1A = AIMAG(DATA(J1))
```

```
```
```fortran
DI1 = ABS(IB)
DI2 = ABS(YB)
INDI = 0
AI = FLOAT(NI(3:K))
IF((IB*AI).LE.0.) INDI = 1
AI = ABS(AI)
THRI1 = AI - 1.
THRI2 = AI + 1.
LAGI = 0.
IF (AI.EQ.HIGH) GOTO 30
IF (DI1.GE.THRI2.OR.DI1.LE.THRI1) INDI = 1
GOTO 40
30 IF (DI1.LE.THRI1) INDI = 1
LAGI = 1
40 INDO = 0
AI = FLOAT(NI(4:K))
IF((IB*AI).LE.0.) INDI = 1
AI = ABS(AI)
THRO1 = AI - 1.
THRO2 = AI + 1.
LAGI = 0.
IF (AI.EQ.HIGH) GOTO 50
IF (DI1.GE.THRO2.OR.DI1.LE.THRO1) INDI = 1
GOTO 60
50 IF (DI1.LE.THRO1) INDI = 1
LAGI = 1
60 IF(INDI.EQ.1.OR.INDO.EQ.1) NER = NER + 1
IF (NER.GT.3) GOTO 9
C ---------Compute the probability of error for this symbol---------
DI1 = ABS(DI1 - THRI2)/SNR
DI2 = ABS(DI2 - THRI1)/SNR
DI3 = ABS(DI3 - THRO2)/SNR
DI4 = ABS(DI4 - THRO1)/SNR
DO 7 M = 1,PRM,10
X(M) = 10.**(FLOAT(M)/20.)
ARG = DI1*X(M)
IF (ARG.GT.12.) ARG = 12.
ERFC = ERFC(ARG)/PEF
IF (LAGI.EQ.1) GOTO 5
ARG = DI2*X(M)
IF (ARG.GT.12.) ARG = 12.
ERFC = ERFC(ARG)/PEF
IF (LAGI.EQ.1) GOTO 6
ARG = DI3*X(M)
IF (ARG.GT.12.) ARG = 12.
ERFC = ERFC(ARG)/PEF
IF (LAGI.EQ.1) GOTO 7
ARG = DI4*X(M)
IF (ARG.GT.12.) ARG = 12.
ERFC = ERFC(ARG)/PEF
IF (LAGI.EQ.1) GOTO 8
7 PSI(M) = PSI(M) + (EI*EQ)/2.
8 PSEI = PSI(M) + (EI*EQ)/2.
9 WRITE(5,3) NER
WRITE(5,3) NER
3 FORMAT(1....ERRORS=','IS,' Try again')
```
Subroutine DECODE: LARY=1, WOAM DATA
LD is the total number of samples
DATA is the complex data matrix
MT (M) is the offset in samples in the I (Q) channel

SUBROUTINE DECODO(LD, DATA, NI, HQ, LARY)
COMMON /II, IOFF, LSAMPL, NSYM, H01, H02, WMLR(4, 20), W1(1, 4096)
COMMON /R1, SAM, PL, BAUD, BW, AMPL(4, 4), THET(4, 4), TIMBT(4, 30)
COMPLEX DATA(LD)
ARY=FLOAT(LARY)
IF (LARY.EQ.1) ARY=2.
IF (LARY.GT.2) GOTO 4

C. 2-level schemes
DO 2 I=1, LSAMPL
NI(1, I)=0
NI(2, I)=0
IMI=I+MI
IQ=I+MQ
DO 2 K=1, NSYM
JI=(K-1)*LSAMPL+IMI
JO=(K-1)*LSAMPL+IQ
JI=REAL(DATA(JI))
YB=AIMAG(DATA(JO))
INDI=0
IF (NI(3, K)*YB.LE.0.) INDI=1
INDO=0
IF (NI(3, K)*YB.LE.0.) INDO=1
NT(1, I)=NI(1, I)+INDI
NT(2, I)=NI(2, I)+INDO
C. IF error is detected WRITE sample number, generated, received samp.
C. IF (INDI.EQ.1 OR INDO.EQ.1) WRITE(2, 10) JI, NI(3, K), YB, JO, NT(4, K)
C
SYB
2 CONTINUE
RETURN
C. More than 2-level schemes
DO 7 I=1, LSAMPL
NI(1, I)=6
NI(2, I)=0
IMI=I+MI
IQ=I+MQ
DO 7 K=1, NSYM
JI=(K-1)*LSAMPL+IMI
JO=(K-1)*LSAMPL+IQ
JI=REAL(DATA(JI))
YB=AIMAG(DATA(JO))
DZ1=ABS(YB)
DY1=ABS(YB)
INDI=0
AY=FLOAT(NI(3, K))
IF (YB+AY).LE.0.) INDI=1
AY=ABS(AY)
THRI1 = AI - 1.
THRI2 = AI + 1.
IF (AI EQ HIGH) GOTO 30
IF (D1 GE THRI2 OR D1 LE THRI1) IND1 = 1
GOTO 40
INDQ = 0
AI = FLOAT(NI(4, K))
IF (YB AI LE 0.) INDQ = 1
AI = ABS(AI)
THRO1 = AI - 1.
THRO2 = AI + 1.
IF (AI EQ HIGH) GOTO 50
IF (D1 GE THRO2 OR D1 LE THRO1) IND2 = 1
GOTO 5
50 IF (D1 LE THRO1) INDQ = 1
5 NI(1, I) = NI(1, I) + INDI
NI(2, I) = NI(2, I) + INDQ
C...... IF error is selected WRITE sample number generated, received samp-
C... IF (IND1 EQ 1.0P INDQ EQ 1.0) WRITE(2, 10) 51, NI(3, K), YB, JQ, NI(4, K)
C
7 CONTINUE
10 FORMAT(2(I10, I6, F10.2))
RETURN
END
Subroutine GENERATES two channels of signal:

\[ \text{TAPE} = \text{WOAM signal} \]

\[ \text{LARY signal} \]

**Subroutine ENCODE**

COMMON /I1/IOPF, LSAAML, NSYMB, NO1, NO2, NUMR(4, 20), NR(4, 5096)
COMMON /R1/SAM, PT, BADD, AMPL(4, 4), TIMPL(4, 4), NMP(4, 30)
COMPLEX DATA(LD)
DOUBLE PRECISION PID, DSEED
REAL RI(512), RO(512)
PTD(3, 1415952653589793)
DSEED(4, 15129283, 30)
IF (LARY .GT. I1) GOTO 9

DO 1 I = 1, NSYMB
IF (3GUBFS(DSEED) .LE. 5) J = J
N1(I, I) = J
IF (3GUBFS(DSEED) .LT. 5) J = J
1 N1(I, J) = J
DO 2 K = 1, NSYMB
IF (TIMEK(1, J) .LE. 1) GOTO 3
KM1 = K - 1
KM2 = K - 2
IF (KM2 .EQ. 0) KM2 = NSYMB
IF (KM2 .EQ. -1) KM2 = NSYMB - 1
IF (KM2 .EQ. 1) KM2 = NSYMB
MI1 = N1(1, KM1)
MQ1 = N1(2, KM1)
MI2 = N1(1, KM2)
MQ2 = N1(2, KM2)
KM1 = K + 1
KM2 = K + 2
IF (NSYMB - K .EQ. 1) KM2 = 1
IF (NSYMB .EQ. K) KM2 = 2
IF (NSYMB .EQ. -K) KM1 = -1
LI1 = N1(1, KM1)
L01 = N1(2, KM1)
LI2 = N1(1, KM2)
L02 = N1(2, KM2)
3 IP = N1(K + 1)
CALL WINDOW(LD, MI1, L11, MI2, L12, IP, R1, NSYMB)
IF = N1(2, K + 1)
CALL WINDOW(LD, MQ1, LQ1, MQ2, LQ2, IP, R2, NSYMB)
IF = N1(K + 1) + 1
DO 5 S = 1, LSAAML
DATA(J1 + I) = CMPLX(RI(I), RQ(I))
GOTO 8
5 CONTINUE

M1 = LARY - 1
DO 2 I = 1, NSYMB
N1(I, I) = 2 * INT(LARY * GUBFS(DSEED)) - M1
2 N1(I, I) = 2 * INT(LARY * GUBFS(DSEED)) - M1
FILE: ENCODE FORTRAN A  UNIV D*OF OTTAWA CMS RELEASE 4

C

DO 6 K=1,NSYM
J=K-1
DO 6 II=1,JSAMPL
6 DATA(J1+I)=CMPLX(REAL(NI(1,K)),REAL(NI(2,K)))

C

8 IF (N.NE.1) GOTO 4
DO 7 II=1,JSYM
9 NI(3,II)=NI(1,II)
7 NI(4,II)=NI(2,II)
4 IF (IOPP.EQ.0) RETURN
L2=LD-1
DO 11 II=1,IOPP
A=AIMAG(DATA(I))
10 DO 12 II=1,L2
12 DATA(L)=CMPLX(REAL(DATA(L)),AIMAG(DATA(L+I)))
11 DATA(LD)=CMPLX(REAL(DATA(LD)),A)
RETURN
END
A. KUCAR, 1987-97 01

*************************************************************************************************

Subroutine generates two channels of signal: $\text{SN} = \text{FPY}^*$

*************************************************************************************************

SUBROUTINE ENCODO(LD, DATA, N, LARY)
COMMON /I1/IOFF, ISAMPL, NSYM, NO1, NO2, BUMB(4, 20), NI(1, 4096)
COMMON /R1/SAM, PI, BAUD, BW, AMPL(4, 4), TIMEL(4, 4), THRT(4, 30)
COMPLEX DATA(LD)
DOUBLE PRECISION PID, DSEED
DSEED=44592096.0 DO
M1=LARY DO
DO 2 I=1, NSYM
2 NI(I, 1)=2*INT(LARY*GGUBFS(DSEED))-M1
DO 6 K=1, NSYM
6 J1=(K-1)*ISAMPL
DO 1 I=1, ISAMPL
1 NI(4, I)=NI(2, I)
DO 7 I=1, NSYM
7 NI(3, I)=NI(1, I)
4 IF (IOPF.EQ.0) RETURN
LD=1 DO 12 L=1, LD
12 DATA(L)=CMPLX(REAL(DATA(L)), AIMAG(DATA(L)))
RETURN
END
FILE: FEEDLOOP FORTRAN * UNIV D/OP OTTAWA CPS RELEASE

KUCAR=1986 11 09, 1987 01 04

FEEDLOOP evaluates performance of FEEDLOOP by Monte Carlo method
JACK=0 No crosstalk compensation
JIM=1 Yes crosstalk compensation
JIO=0 No crosstalk compensation
JPZ=1 No coefficient definition within TRZQ subroutine
KPZ=1 Wb signal ISI due to timing uncertainty
KPR=2 Wb signal ISI compensation
BSTRP=999 Perfect estimation of the angle PT
BSTRM=999 Perfect estimation of the timing SAMP

COMMON /I1/IOPP, LSAMPL, NSYMB, NOA, NOB, NUMR(4, 20), NT(4, 4096)
COMMON /R1/SAM PT, BAUD, BW, AMPL1(4, 4), AMPL2(4, 4), TT(4, 4), TP(4, 4)
COMPLEX DATA(65536), T(65536), DATA(65536), X(65536), Y(65536)
DOUBLE PRECISION PID, DSEED
DIMENSION R(2), C(129)
PID=3.141592653589793D0
DSEED=475286.00
PI=3.141592653589793
PIR=180.0/PI
LSAMPL=32
NSYMB=2048
*IISAMPL*/
MIO=0
MO=0
IOPP=0
SAM=FLOAT(LSAMPL)
BAUD=100.
BW=SAM*BAUD
LD=LSAMPL*NSYMB
LS=LSAMPL/2
SAM1=1.0/SAM
ALG=ALG10(FLOAT(2*NSYMB))
JACK=1
JIM=1
JIO=1
BSTRP=1000.
BSTRM=1000.

C Window definition
NUMB(1:1)=1
TIME1(1:1)=1
TIME2(1:1)=0
AMPL(1:1)=1

C Number of levels
LARY=2
ART=FLOAT(LARY)

C Number of channels
NCH=1

C Calculate the average S/N for chosen symbol error rate SER
SER=1.0E-02
CALL MERFC1(SER,EB1,IER)
EB=EB1*SQT(2.)

Generate data
CALL ENCODO(LD,DATA,NCH,LARY)
C.... generate Gaussian noise, add it to the DATA and store it in DATA1
DO 1 I=1,LD
CALL GGNML(DSER,2,R)
1 DATA(I)=DATA(I)+TP(I)
C Introduce ISI
CALL TREQ(3,LSAMPL,LD,DATA,TP,C,LS?,JDF)
DO 3 I=1,LD
3 DATA1(I)=DATA(I)
C 5 degrees phase steps
DO 99 J=1,73
DEG=FLOAT(J-37)*5.
PI=DEG/PIR
CI=COS(PI)
SI=SIGN(PI)
C Calculate crosstalk
DO 1 DO 2 I=1,LD
2 DATA(I)=CMPLX(CX*REAL(DATA1(I))+SY*AIMAG(DATA1(I))
$+CI*AIMAG(DATA1(I))-SI*REAL(DATA1(I)))
C Crosstalk and timing ISI compensation
1 IF (JACK.EQ.0) GOTO 12
12 IF (PI.EQ.90. OR. PI.EQ.-90.) TPI=ATAN(PI)
IF (PI.EQ.90. OR. PI.EQ.-90.) TPI=1.5*PI
IF (ESTPI.GT.999.) GOTO 21
CALL GGNML(DSER,1,R)
TPI=TPI+R(I)/SQT(ESTPI)
11 DATA(I)=REAL(DATA1(I))-TP1*AIMAG(DATA1(I))
10 DATA(I)=CMPLX(REAL(DATA1(I))-TP1*AIMAG(DATA1(I))
11 DATA(I)=REAL(DATA1(I))-TP1*AIMAG(DATA1(I))
12 IF (JIM.EQ.1) CALL TREQ(3,LSAMPL,LD,DATA,TP,C,LS?,J)
IF (JACK.EQ.0) GOTO 14
DO 45 I=1,LD
45 DATA(I)=DATA1(I)*CX
14 CONTINUE
C Make decision once per sample and compare with sent sample
CALL DECD0(LD,DATA,NCH,LARY)
C Calculate signal at(t) before loop filter, then filter it
C DATA(I)=CMPLX(REAL(DATA(I)))*AD-AT1MAG(DATA1(I))*BN,0.
C CALL FILTER(LD,13,DATA,TF,PM,MP,MPM)
DO 99 K=1,LSAMPL
SAMB=FLOAT(K-1)/SAH-0.5
PEA=ALOG10(FLOAT(NI(K)+2.0))
WRITE(1,7) SAMB,DEG,PEA
99 CONTINUE
7 FORMAT(9E10.3)
10 FORMAT (2F5.0,5X,2F5.0,13)
END
**FILE: FILTER FORTRAN A**

**UNIV D/OF OTTAWA CMS RRLP#SP a**

```fortran
! KUCAR, 1984 01 04 1985 05 08

Subroutine performs the FILTERING of data

```

```fortran
SUBROUTINE FILTER(LD, L3, DATA, TP, TWK, N, NPTI)
COMMON /I1/IOPF, LSAMPLE, NSTATE, NO1, NO2, NUMB(4, 20), PT(8, 4, 0)
COMMON /R1/RSTATE, PI, BAUD, BW, AMPX(4, 4), TIMPL(4, 4), TWK(8, 4, 0)
COMPLEX DATA(LD), TP(LD)
INTEGER TWK(L3)
NP = NPTI(NPIL)
M1 = NP + 2
M2 = NP + 2 + 3
NOM = NUMB(N, M1)
CALL PT(I)(DATA, L3, IWK)
IPOP = IPOP(TIMER(N, 13)/BW1)
CALL TIMESH(IW, DATA, TP, IPOP)
SOTO (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, NO)
RETURN

Phased equalized BUTTERWORTH filter

1
```

```fortran
Mod(f) = -----------------------
\sqrt{1 + (f/FP)**2n}
```

```fortran
Arg(f) = 0
```

```fortran
FP = 3 dB point
```

```fortran
a = filter order
```

```fortran
2
```

```fortran
PN = TIMER(N, M2 + 1)/BW1
```

```fortran
NA1 = 2*NUMB(N, M1 + 1)
```

```fortran
TP(I) = CMPX(I, 1, 0, 0)
```

```fortran
DO 100 I = 2, NO1
```

```fortran
J = I
```

```fortran
A2 = 1/S3RT(J, 1, (FLOAT(J)/FP)**2NA1)
```

```fortran
100 TP(I) = CPMX(A2, 0)
```

```fortran
SOTO 99
```

```fortran
RETURN
```

Phased equalized SQUARE ROOT COSINE FILTER with arbitrary aloha

```fortran
1
```

```fortran
TP(I) = 1, ____________________________
```

```fortran
TP(I) = 0.5 / (1 - sin(PI/2/alpha*(f/FP - 1))),
```

```fortran
```

4

```fortran
IF (TIMKER(N, M2) .EQ. 0) THEN
```

```fortran
PN = TIMER(N, M2) = 0.0001
```

```fortran
FL = TIMER(N, M2) * FP
```

```fortran
```

**FILE: FILTER FORTRAN A**

**UNIV D/OF OTTAWA CMS RRLP#SP a**

```fortran
! KUCAR, 1984 01 04 1985 05 08

Subroutine performs the FILTERING of data

```

```fortran
SUBROUTINE FILTER(LD, L3, DATA, TP, TWK, N, NPTI)
COMMON /I1/IOPF, LSAMPLE, NSTATE, NO1, NO2, NUMB(4, 20), PT(8, 4, 0)
COMMON /R1/RSTATE, PI, BAUD, BW, AMPX(4, 4), TIMPL(4, 4), TWK(8, 4, 0)
COMPLEX DATA(LD), TP(LD)
INTEGER TWK(L3)
NP = NPTI(NPIL)
M1 = NP + 2
M2 = NP + 2 + 3
NOM = NUMB(N, M1)
CALL PT(I)(DATA, L3, IWK)
IPOP = IPOP(TIMER(N, 13)/BW1)
CALL TIMESH(IW, DATA, TP, IPOP)
SOTO (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, NO)
RETURN

Phased equalized BUTTERWORTH filter

1
```

```fortran
Mod(f) = -----------------------
\sqrt{1 + (f/FP)**2n}
```

```fortran
Arg(f) = 0
```

```fortran
FP = 3 dB point
```

```fortran
a = filter order
```

```fortran
2
```

```fortran
PN = TIMER(N, M2 + 1)/BW1
```

```fortran
NA1 = 2*NUMB(N, M1 + 1)
```

```fortran
TP(I) = CMPX(I, 1, 0, 0)
```

```fortran
DO 100 I = 2, NO1
```

```fortran
J = I
```

```fortran
A2 = 1/S3RT(J, 1, (FLOAT(J)/FP)**2NA1)
```

```fortran
100 TP(I) = CPMX(A2, 0)
```

```fortran
SOTO 99
```

```fortran
RETURN
```

Phased equalized SQUARE ROOT COSINE FILTER with arbitrary aloha

```fortran
1
```

```fortran
TP(I) = 1, ____________________________
```

```fortran
TP(I) = 0.5 / (1 - sin(PI/2/alpha*(f/FP - 1))),
```

```fortran```

4

```fortran
IF (TIMKER(N, M2) .EQ. 0) THEN
```

```fortran
PN = TIMER(N, M2) = 0.0001
```

```fortran
FL = TIMER(N, M2) * FP
```

```fortran
```
FILE: FILTER FORTRAN A UNIV D'OP OTTAWA CMS RELFSP A

41 TF(I) = CMPLX(10.,**((I-2-LD)*A1),0.)
300 199
400
C
C Parabolic Gain
C 11 A1 = BW1*BW1*TIMEK(N,M2)/20.
C DO 36 I = 1, NO1
C PI = FLOAT(I-1)
C AM1 = (A1*PI**PI)
36 TF(I) = CMPLX(10.,**AM1,0.)
GOTO 99
C
C RETURN
C 12 RETURN
13 RETURN
14 RETURN
15 RETURN
C
C
C
RUMMLER's SELECTIVE FADING model
C h(f) = (1 - b*exp(-12PI*(f-f0)/tau))
C a = flat fading depth (normalized to 1)
C B(dB) = 20*LOG(1+b)/(1-b) is a notch depth
C b=0 (flat fading),... 1 (infinite steep)
C c=nonminimum, minimum phase
C d=notch offset
C e=delay of second path
C
C 16 AR0 = TOWPI*TIMEK(N,15)*1.E-3
C TT = TIMEK(N,M2+1)
C B = 10.**(ABS(TT)/20.)
C B = (B-1.)/(B+1.)
C DO 57 I = 1, NO1
C AR1 = AR0*(FLOAT(I-1)/BW1-TIMEK(N,M2))
C AR2 = -SIGN(1,TT)*B*SIN(AR1)/(1.-COS(AR1))
C AR2 = ATAN(AR2)
C TF(I) = CMPLX(COS(AR2),-SIN(AR2))
C
C
C 99 DO 20 I = NO2, LD
C 20 TF(I) = CONJG(TF(LD+2-I))
300 199
C 98 DO 21 I = NO2, LD
C 21 TF(I) = TF(LD+2-I)
199 DO 111 I = 1, LD
111 DATA(I) = CONJG(DATA(I)*TF(I))
199 CALL FFT2C(DATA, L3, IWK)
222 DATA(I) = CONJG(DATA(I))/DIN
RETURN
END
FILE: INPL  FORTRAN A  UNIV D'OF OTTAWA CMS RELEASE A

A.KUCAR, 1983 11.02.1985  04:21

******************************************************************************
Input parameters for WQA, TRIANGULAR and KAISER-BEESPL signals
N is the channel number 1..4
AMPL(N,1-4)-amplitude parameter of the WQA signal
TIMEL(N,1-4)-symbol duration of the Kaiser-Bessel signal for channels 1-4
TIMEK(N,1-4)-frequency parameter normalized to Tsymbol
******************************************************************************

SUBROUTINE INPL(N,NWAY)
COMMON /IIP/IOFIP,L,SAMPL,N,LSTB,N07,N02,WTFP(4,2),X(4,400)
COMMON /R/STB,ST,BAUD,STB,AMPL(4,4),TIMEL(4,4),TIMEK(4,4)

2 FORMAT(I1)
   DO 4 K=1,N
       DO 4 I=1,F
       TIMEL(K,1)=C
       DO 55 J=1,N
       AM=1.
       TIMEK(J,1)=0.
       WRITE(8,3)J
   3 FORMAT('channel 'I1',':window TYPE:1=1 TO 4.0=Kaiser-Bessel')
   READ(7,2)NUM9(J,1)
   K=NUM9(J,1)
   IF (K.GT.0) GOTO 9
   WRITE(8,8)J
   4 FORMAT('channel 'I1',':PARAMETER 1.<alpha<90."
   READ(7,5)AMPL(J,1)
   WRITE(8,7)J
   5 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   TIMEK(J,1)=PI*AMPL(J,1)
   6 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   TIMEK(J,1)=PI*AMPL(J,1)
   7 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   TIMEK(J,1)=PI*AMPL(J,1)
   8 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   TIMEK(J,1)=PI*AMPL(J,1)
   9 DO 55 I=1,N
      IF (K.GE.I) GO TO 31
      WRITE(8,8)J
   10 FORMAT('channel 'I1',' L 1')
   READ(7,5)AMPL(J,1)
   11 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   12 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   13 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   14 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   15 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   16 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   17 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   18 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   19 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   20 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   21 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   22 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   23 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   24 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   25 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   26 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   27 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   28 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   29 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   30 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   31 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   32 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   33 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   34 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   35 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   36 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   37 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   38 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   39 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   40 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   41 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   42 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   43 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   44 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   45 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   46 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   47 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   48 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   49 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   50 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   51 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   52 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   53 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   54 FORMAT('channel 'I1',' L 1')
   READ(7,5)TIMEL(J,1)
   55 CONTINUE

5 FORMAT(3P13.7)
WRITE(6,*)
WRITE(8,*)
1 FORMAT(9L"Ampl",8X,"TIME",8X,"TIMEK")
DO 6 J=1,N
     K=NUMB(J,1)
! WRITE(6,5) (AMPL(J,J),J=I,I)!
WRITE(5,5) (AMPL(J,J),J=I,I)
WRITE(5,5) (TIME(J,J),J=I,I)
RETURN
END
**FILE: INP2**

**FORTRAN A**

**UNIV D/OP OTTAWA CMS RELFAS 4**

```
*KUCAR, 1983 12 13, 1985 05 02
******************************************************************************
Input data for the MAIN and ACT/CCI channels
N is the number of channels, max 4
NUMB(I,1) - modulation window type
   (N,B) - number of filters in each of max 4 systems, max 4
   (N,5) - filter type #1
   (N,6) - filter order #1
   (N,8-10) - filter #2
   (N,11-13) - filter #3
   (N,14-16) - filter #4
   (N,20) - Nonlinearity type
TIMEK(I,5) - 1st filter parameter,
   or linear: GDns/Hz,
   or parabolic: GDns/Hz
   or linear: GDdB/Hz
   or parabolic: GDdB/Hz
   or notch offset: Hz
   (N,6) - filter bandwidth Hz #1, or notch depth: dB
   (N,7-8) - filter #2
   (N,9-10) - filter #3
   (N,11-12) - filter #4
   (N,13) - frequency offset Hz
   (N,14) - channel level dB
   (N,15) - 2nd path delay ns
******************************************************************************

SUBROUTINE INP2(N)
COMMON /T1, T0P, LSAMPL, SYMB, NO1, NO2, NUMB(4,20), TI(T, 4096)
COMMON /R1, SAM, PI, BAAU, BW, AMPL(4,4), TI(M, 4), TIMEK(4,30)
TIMEK(1,13) = 0.
TIMEK(1,14) = 0.
IF (M.EQ.1) GOTO 22
DO 2 I = 2, N
   WRITE(8,3) I
   3 PFORM1(1,1) = OFFSET - MHz * I)
   READ(7,12) TIMEK(1,13)
   WRITE(8,5) I
   5 PFORM1(1,1) = LEVEL - dB * I
   READ(7,12) TIMEK(1,14)
   22 IF (NUMB(2,4).GT.0) GO TO 13
   DO 8 I = 1, N
   8 WRITE(8,21) I
   21 PFORM1(1,1) = NONLINEARITY = linear - 1.79 * I = hardlimiter /,
   #18I, 1 = twa (1real) /, 18I, 3 = HP, /, 18I, 4 = saas FFT /
   READ(7,14) NUMB(1,20)
   WRITE(8,7) I
   7 PFORM1(1,1) = NUMB. of filters - max 4*
   READ(7,4) NUMB(I,4)
   IF (KK.EQ.0) GO TO 8
   KK = NUMB(I,4)
   M = I + 2
   M = I + 2
   NUMB(I,1 +M) = 0
```

TIMEK(I,11) = 0.

WRITE(8,9) (J)
9 FORMAT('CH.',I1,'filter',I1,:01='Bessel',T45,'11=Parabolic gain',
      I1,'=Sine phase ripple',T45,'12=Cosine phase ripple',
      I1,'=Cosine amplitude ripple',T45,'13=Elliptical',
      I1,'=Sine amplitude ripple',T45,'14=Linear gain',
      I1,'=Linear gain',T45,'15=Parabolic GP',
      I1,'=Parabolic GP',)
READ(7,44) NUMB(I,M)
WRITE(8,27) (J)
57 FORMAT('CH.',I1,'filter',I1)
IP (NUMB(I,M),EQ.8) WRITE(8,108)
108 FORMAT(15X,'linear gain/MHz')
IP (NUMB(I,M),EQ.9) WRITE(8,109)
109 FORMAT(15X,'parabolic gain/MHz')
IP (NUMB(I,M),EQ.10) WRITE(8,110)
110 FORMAT(15X,'linear gain/MHz')
IP (NUMB(I,M),EQ.11) WRITE(8,111)
111 FORMAT(15X,'parabolic gain/MHz')
IP (NUMB(I,M),EQ.12) WRITE(8,112)
112 FORMAT(15X,'Ripplesin')
IP (NUMB(I,M),EQ.13) WRITE(8,113)
113 IF (NUMB(I,M),EQ.14) WRITE(8,114)
114 FORMAT(15X,'Number of periods within Nyquist bandwidth')
IP (NUMB(I,M),EQ.15) WRITE(8,115)
115 IF (NUMB(I,M),EQ.16) WRITE(8,116)
116 FORMAT(15X,'Notch offset/MHz')
IP (NUMB(I,M),EQ.17) WRITE(8,117)
117 IF (NUMB(I,M),EQ.18) WRITE(8,118)
118 READ(7,12) TIMEK(I,1)
IP (NUMB(I,M),GT.4) AND NUMB(I,M),LT.16) GOTO 8
119 READ(7,16) WRITE(8,119)
120 IF (NUMB(I,M),EQ.16) WRITE(8,20)
15 FORMAT(15X,'Nonminimum phase fading')
JP (NUMB(I,M),EQ.17) WRITE(8,21)
20 FORMAT(15X,'Word bandwidth/MHz')
READ(7,12) TIMEK(I,11)
117 IF (NUMB(I,M),EQ.17) WRITE(8,117)
119 IF (NUMB(I,M),EQ.18) WRITE(8,118)
120 READ(7,12) TIMEK(I,15)
8 CONTINUE
117 IF (NUMB(I,M),EQ.17) WRITE(8,117)
119 IF (NUMB(I,M),EQ.18) WRITE(8,118)
120 READ(7,12) TIMEK(I,15)
8 CONTINUE
13 DO 10 I = 1, N
WRITE(6,1)
5 WRITE(8,1)
1 FORMAT(6X,'ch.NUMBER ch.OFFSET-ch.WRITE(6,1)
6 FORMAT(6X,'ch.OFFSET-ch.WRITE(6,1)
11 FORMAT(6X,'ch.OFFSET-ch.WRITE(6,1)
FILE: INP2          PORTER A       UNIV D*OP OTTAWA CMS RELEASE 4

WRITE(6,14) I, TIMEK(I, 13), TIMEK(I, 14)
WRITE(6, 16)
WRITE(6, 16)
K = Numb(I, 1)
DO 10 J = 1, K
M = J*3+2
M1 = J*2+3
WRITE(6, 214) Numb(I, M), Numb(I, M+1), TIMEK(I, M+1), TIMEK(I, M)
10 CONTINUE
        FORMAT(I1)
        FORMAT(I2)
        FORMAT(P13.7)
        FORMAT(I15, 2F15.7)
        FORMAT(2F15, 2F15.7)
RETURN
END
***DECODES the 4QAM data using MONTE CARLO techniques***

Subroutine MCARL4(LD, DATA, MC4, ERROR, MT, MO)

COMMON /R1, R2, PI, BAUD, SAMP, TIMEI, TIMEO, MAT(4, 4)

COMMON /R1, R2, PI, BAUD, SAMP, TIMEI, TIMEO, MAT(4, 4)

COMPLEX DATA(LD)

DO 1 K=1, NSYMB

JI = (K-1) * SAMPLE + MT

JO = (K-1) * SAMPLE + MO

YB = NI(4, K) * AIMAG(DATA(JI))

IF(YB .LT. 0.) GOTO 2

WRITE (6, 4) K, MC4

WRITE (6, 4) K, MC4

2 IF(YB .LT. 0.) GOTO 6

WRITE (6, 5) K, MC4

IF(YB .LE. 0. OR. YB .LE. 0.) NPROP = NERROR + 1

CONTINUE

4 FORMAT ('1 channel error in symbol', 'R', 'I')

5 FORMAT ('Q channel error in symbol', 'R', 'I')

RETURN

END
A. KUCAR, 1985 10 27, 1986 04 25
Program MDPL FORTRAN
This program calculates the NONLINEARITY SURFACE and CONDITIONAL PROBABILITY OF ERROR of decision feedback carrier recovery loop for any M-ary QAM (staggered and non-staggered) scheme.

Pe/Pep is calculated rather than Pe/Pea*Peb=Pep*Peh

Pe(Pe) = carrier recovery error, SAE=timing recovery error.

The output of this program is used as input to the SAS programs DPL, G2, GPL, HI and GPHI

For M=4, the argument in "erfc(arg)" function is out of range for particular values of PI and AMP, which causes an error.

EXTERNAL PI, AMP,
AERR, TPEP, 0.

AT=0.5

CALL ERRSET(208.700,-1.1.1,208)

PI=3.141592653589793

PT=180/PI

WRITE(8,10)

READ(*,29)

IP=10

MIP=100*MIP

WRITE(8,1)

READ(*,2)

L1=2

WRITE(7,101)

READ(*,2)ISTAGG

ISTAGG=0

L2=L1+1

L1=L1-1

L2=L2-2

PL=FLOAT(L)

PN=PL*FL

PT=3.0/(PN-1.)

P2=1./PN/PM/2.

P3=P2*P1

PEP=PL

PPE=PPE*(SP*IP)*PEP

CALL MERFCI(PEP,EBNO,IPP)

EBN=EBNO

IF (L.GT.0) GOTO 77

PPE=FLOAT(IPP)

PEP=PEP

EBN=EBN+1

GPE=PEP/PEPA

4

PT=0.

AM=0.

RA1=1.-AMB

RA=AMB

RB=RA

RD=RA1

IP=0.
FILE: MDFL1
FORTRAN A UNIV D'OF OTTAWA GNS RELEASE 0

C NPE=INT(100.*PEA)
C WRITE(8,8)NPE,EBN
C IF (NPE-RE*E1P) GOTO 4
C WRITE(8,6)ERN,IXP
C DO 5 I=1,25
C WRITE(8,6)I
C SAMB=FLOAT(I-13)/24.
C *AMB
C R=AMB
C AMB=ABS(SAMB)
C RA=1.-AMB
C IF (ISTAGG.EQ.0) RB=RA
C IF (ISTAGG.EQ.0) RR=RA1
C IF (ISTAGG.EQ.0) RB=0.*5+AMB
C IF (ISTAGG.EQ.0) RB1=0.*5-AMB
C DO 5 J=46,46
C DEG1=FLOAT(J-46)*9.
C FI=DEG1/PI
C WRITE(1,7)SAM,DEG1,PEA,G1,G2,AN1,GPI
C 1 FORMAT (15H OF LEVELS I=04)
C FORMAT(T,2)
C FORMAT (NPE and EbNo=,I9,E16.3)
C FORMAT (I=3,14)
C 6 FORMAT (I=3,14)
C 8 FORMAT (EB=10.3)
C 10 FORMAT (BE=6.5 at Pe=1E-4,I2)
C 101 FORMAT (Staggered=01,Non-staggered=10)
C END

REAL FUNCTION F(PR,AMB)
COMMON L,L2,LM1,LM2,LM10,LM20,LM100,LM200
COMMON PEA,PER,EBN,RA1,RAI,RR,RR1,G1,G2,AN1,PR,PI,PI2,PI3
COMMON PLM2,PEA1,PRB1,PR11,PR12,PR13,PLM1,PI1,PI2,PI3
COMMON PEB1,PEB2,PI1,PI2,PI3
PRB=0.
PR12=0.
PR13=0.
PI1=0.
PI2=0.
PI3=0.
G1=0.
G2=0.
AN1=0.
L2=0.
L1=0.

FI=COS(PI)
XI=1IN(lF)

C DO 14 M0=1,L2,2
M=M0-L
FM1=FLOAT(M-1)
FI1=FM1+1
RAN1=M+RA1
IM=IARS(M)
PEB=0.
PRB=0.
N0=1,L2,2
N=M0-L
RAN1=N*RB1
PEA1=0.
FILE: MDPL
FORTRAN A' UNIV D'OP OTTAWA C'S RELFAS A

DO 11 N9=1,L2,2
M1=M9=L
RAN=M1*RA
YAI=(RAM1+RAM)*CI
DO 11 N9=1,L2,2
N1=N9-L
RAN=N1*RB
YB1=(RAN1+RAN)*SI
YAB1=YAI+YB1
IF (N0.EQ.1) PEA=ERFC((YAB1-PHI1)*PKN)
IF (1.EQ.LM1) PEA=ERFC((YAB1-PHI1)*PKN)
PEA=ERFC((YAB1-PHI1)*PKN) ERFC((YAB1-PHI1)*PKN)
PEA=PEA1+PEA
PEN=PEA+PEA1
11 PEIN=PEIN+N*PEA1
PEN=PEA+PEN
G1=G2+N*PEIN
C1=C2+N*PEN
DO 14 I9=1,L2,2
IF (I9.EQ.0) GOTO 14
14 T1=1/L
PPI=PLOT1(T1+1)
PNI=PPI-1.0
IA=IARS(I1)
PENI=0
PBJ=0
DO 13 N0=1,L2,2
N1=N0-L
RAN=N1*RB
PEI=0
DO 16 N9=1,L2,2
M1=M9-L
RAN=M1*RA
YAI=(RAM1+RAM)*CI
DO 16 N9=L2,2
N1=N9-L
RAN=N1*RB
YB1=(RAN1+RAN)*SI
YAB1=YAI+YB1
IF (I9.EQ.1) PEI=ERFC((YAB1-PHI1)*PKN)
IF (1.EQ.LM1) PEI=ERFC((YAB1-PHI1)*PKN)
IF ((IA+LT-LM1).AND.(II+GT-M)) %PEI=ERFC((YAB1-PHI1)*PKN)-ERFC((YAB1-PHI1)*PKN)
IF ((IA+LT-LM1).AND.(II+GT-M)) %PEI=ERFC((YAB1-PHI1)*PKN)-ERFC((YAB1-PHI1)*PKN)
16 PEI1=PEI+PEI
PEIN=PEIN+N*PEI1
PEI=PEI+PEI1
G2=G2+1.0
C2=C2+1.0
CONTINUE
13 G1=C1-2
AN1=C1-PEI2
FILE: MDPL1  FORTRAN  A  UNIV D'OF OTTAWA CMS RELEASE 4

DO 24 I0=1,L2,2  
M=M0=L  
R1AT(M1=1)  
PP1=PM1+2  
RM1=M*RB  
IM=IABS(R)  
PEIN=0  
DO 22 N0=1,L2,2  
N=N0=L  
RM1=M*RB  
PEA1=0  
DO 21 M9=1,L2,2  
M1=M9=L  
RM1=M1*RA  
YAT1=(RM1+R4)*CT  
DO 27 N9=1,L2,2  
N1=N9=L  
RM1=M1*RB  
YB1=(RM1+R4)*SX  
YAB1=YAT1-TR  
T1=(M0.EQ.1)  
PBA=ERFC((PP1-YAB1)*EB1)  
IP (N.EQ.LM1)  
PBA=ERFC((YAB1-PM1)*EB1)  
IP (I1.LT.LM1)  
PBA=ERFC((YAB1-PM1)*EB1)-ERFC((PP1-YAB1)*EB1)  
PBA=PBA1+PEA  
PEN=PEN+PEA  
DO 21 N9=1,L2,2  
N1=N9=L  
RM1=M1*RB  
YAT1=(RM1+R4)*CT  
DO 27 N9=1,L2,2  
N1=N9=L  
RM1=M1*RB  
YB1=(RM1+R4)*SX  
YAB1=YAT1-TR  
T1=(M0.EQ.1)  
PBA=ERFC((PP1-YAB1)*EB1)  
IP (N.EQ.LM1)  
PBA=ERFC((YAB1-PM1)*EB1)  
IP (I1.LT.LM1)  
PBA=ERFC((YAB1-PM1)*EB1)-ERFC((PP1-YAB1)*EB1)  
PBA=PBA1+PEA  
PEN=PEN+PEA  
DO 21 N9=1,L2,2  
N1=N9=L  
RM1=M1*RB  
YAT1=(RM1+R4)*CT  
DO 27 N9=1,L2,2  
N1=N9=L  
RM1=M1*RB  
YB1=(RM1+R4)*SX  
YAB1=YAT1-TR  
T1=(M0.EQ.1)  
PBA=ERFC((PP1-YAB1)*EB1)  
IP (N.EQ.LM1)  
PBA=ERFC((YAB1-PM1)*EB1)  
IP (I1.LT.LM1)  
PBA=ERFC((YAB1-PM1)*EB1)-ERFC((PP1-YAB1)*EB1)  
PBA=PBA1+PEA  
PEN=PEN+PEA.
PROGRAM TO EVALUATE THE PERFORMANCE OF MQAM MODEM IN
A NONLINEARLY AMPLIFIED MULTICOMPONENT ENVIRONMENT.

COMMON /1/IOFF,LSAMPL,NSYMB,NO1,NO2,NMPL(4,20),VT(4,096)
COMMON /B/SH,ST,BAUD,BW,AMPL(4,4),TIME(4,30)
COMPLEX DATA(32768),TP(32768),NATA(32768),TA(514)
DOUBLE PRECISION PID,DSEP
DIMENSION XP(65536),PSI(257),MK(9),TK(15),KK(256)
DIMENSION PEL(50),BER(50)
DIMENSION NAME(10)
CALL ERRSET(200,700,-1,1,1,208)

IJK=0
JKL=0
JLM=1

PI=3.141592653589793
P12=PI/2
P22=PI/22

DSEP=4569286.0
DSEP=4569286.0

TRM=30
T1=50
TR=1
LSAMPL=64
NSYMB=512
MI=LSAMPL/2
MO=MI
IOFF=0
SYN=FLOAT(NSYMB)
RUNS=FLOAT(NR)
CNEB=1
WRITE(8,101)

101 FORMAT('Baud rate:',MBD')
READ(7,222)RAOD
SA=FLOAT(LSAMPL)
BM=SA*RAOD
LB=LOW*NSYMB
LD=2*LD
L1=NSYMB
L2=16

LL3=9
L1=L1/2
L5=L1+1
NO1=LL/2+1
NO2=NO1+1
NUMB(2,4)=0

WRITE(8,3)
3 FORMAT('Number of channels')
READ(7,102)NCH
WRITE(8,8)
8 FORMAT('Number of levels M,L=G1 means shaping')
READ(7,12)LARY
FILE: MOAM  FORTRAN A  UNIV D D OF OTTAWA CMS RELPASA

2  CALL INP2(NCH)
WRITE(8,1)
4  FORMAT(AOPEN TYPE=1 TO 9*)
READ(7,102)IPEV
LPEV=IPEN+4
WRITE(6,6)IPEN,IOFF,NUMB(1,20),LARY
WRITE(8,8)IPEN,IOFF,NUMB(1,20),LARY
6  FORMAT(10,1X,=10,1X,=10,1X,OPFSET,NONL,LARY,GT$)
DO 5 I=IPROM,ITO
5  BER(I)=0
ANORM=SORT(1)
DO 99 J=1,MR
CALL ENCODE(LD,DATA,LARY)
CALL P1(LD,L2,L3,L5,DATA,TA,FB,PSI,NNK,K,3,IPEV,1)
IF (J.NE.1) GOTO 7
CALL PLTSAS(LD,DATA,60,MI,MO,JKL,IKJ,LARY,2)
7 CALL NCRWIN(LD,DATA,ANORM,1)
8  IF (IPEN.GE.2) GOTO 66
CALL P1(LD,L2,L3,L5,DATA,TA,FB,PSI,NNK,K,3,IPEV,1)
CALL FILTER(LD,L3,DATA,TF,IKJ,1,1)
9  IF (NUMB(NUM,GT,1) CALL FILTER(LD,L3,DATA,TF,IKJ,1,1)
IF (NUMB(NUM,GT,1) CALL FILTER(LD,L3,DATA,TF,IKJ,1,1)
10 CALL CHANN(LD,DATA,SNR,1)
11  IF (NCR.LT.2) GOTO 66
DO 10 K=2,NCH
CALL ENCODE(LD,DATA,NNK,LARY)
CALL NCRWIN(LD,DATA,ANORM,K)
PSH=I2*GGUBS(SEEK)
CALL PHASS(LD,DATA,PSH)
ITS=INT(SAM*GGUBS(SEEK)-5)
CALL TIMESH(LD,DATA,TP,ITS)
CALL ATT(LD,DATA,K)
12 CALL FILTER(LD,L3,DATA,TF,IKJ,K,N)
DO 10 T=1,79
10 DATA(I,J)=DATA(I,J)+DATA(J)(I)
66 CALL POWER(LD,DATA,PAE,PAE,EB,0)
CALL SYCHD(LD,DATA,TF,NOISE,MT,MI,EB,LARY)
CALL PLTSAS(LD,DATA,60,MT,MO,JKL,IKJ,LARY,2)
CALL PLTSAS(LD,DATA,250,MI,MO,JKL,IKJ,LARY,2)
CALL DDECODE(LD,DATA,NOISE,PEI,IPROM,ITO,EB,MT,NNK,LARY,CNER,SNR)
TF(NCR,GT,0) GOTO 123
DO 99 I=IPROM,ITO
PE1(I)=PEI(I)/SYM
99 BBP(I)=BBR(I)+PEI(I)
WRITE(6,1)
1 FORMAT(15X,=10X,=10X,Prob.of symbol error=100)
IP(CRBB.EQ.1.1) WRITE(8,1)
111 FORMAT(10X,=10X,Prob.of symbol error=100)
DO 99 I=IPROM,ITO
PSH*FLOAT(I)
M OA00560
M OA00570
M OA00580
M OA00590
M OA00600
M OA00610
M OA00620
M OA00630
M OA00640
M OA00650
M OA00660
M OA00670
M OA00680
M OA00690
M OA00700
M OA00710
M OA00720
M OA00730
M OA00740
M OA00750
M OA00760
M OA00770
M OA00780
M OA00790
M OA00800
M OA00810
M OA00820
M OA00830
M OA00840
M OA00850
M OA00860
M OA00870
M OA00880
M OA00890
M OA00900
M OA00910
M OA00920
M OA00930
M OA00940
M OA00950
M OA00960
M OA00970
M OA00980
M OA00990
M OA01000
M OA01010
M OA01020
M OA01030
M OA01040
M OA01050
M OA01060
M OA01070
M OA01080
M OA01090
M OA01100
FILE: MQAM
FORTRAN A
UNIV D/OF OTTAWA CMS RELEASE 0

PEI(I) = BPD(I) / SUMS
9 WRITE(8,172)ESH,PEI(I)
172 FORMAT(2E15.6,1x)
C
123 WRITE(8,173)
173 FORMAT(1x)

C #NEW PARAM>0, NEW BB SIGNAL<0...

READ(7,221)GO
IP (30) 100, 11, 2
11 CONTINUE
12 FORMAT(I2)
102 FORMAT(I4)
222 FORMAT(F15.7)
END
A. KUCAR 1984 01 04 1985 04 18

***************************************************************************
Subroutine COMPUTES the effective noise (PNOISE)

PNOISE = NO * Integral(|TF(f)|**2*df)
NO = 0.5
(White Gaussian Noise)
***************************************************************************

SUBROUTINE NOISE1(LD, TF, PNOISE)
COMMON /IT/IOPF, LSAMPL, NSYM, I1, I2, NUMB(4,20), MT(4, 0.06)
COMMON /R1/SAM, BI, BAUD, BW, AMPL(4, 3), TEM1(4, 8), TMPL(4, 30)
COMPLE: TF(LD)
SUM = 0.0
DO 1 L = 1, LD
1 SUM = SUM + (CABS(TF(L)))**2.
PNOISE = SUM * BW / FLOAT(LD) / 2.
RETURN
END
**FILE: NOWLIN  FORTRAN A  UNIV D'OP OTTAWA CMS RELEASE A**

```fortran
A.KUZAR 1984 07 04, 1985 04 18
******************************************************************************
Subroutine simulates an ideal HARDLIMITRP:Out=I1Top!!
******************************************************************************

SUBROUTINE NOWLIN(LD, DATA, ANORM)
COMMON /1/IOFF, LSAMPL, HSAMPL, NO1, NO2, NUMB(4,20), W1(4,4096)
COMPLEX DATA(LD)
IF (NUMB(N20).EQ.0) RETURN
DO 1 I = 1, LD
† DATA(I) = ANORM * DATA(I) / CABS(DATA(I))
1 RETURN
END
```
Subroutine COMPENSATES the PHASE SHIFT due to the carrier offset.

```fortran
SUBROUTINE PHASSH(LD, DATA, PSHIFT)
COMPLEX DATA(LD), EPS
EPS = CMPLX(COS(PSHIFT), -SIN(PSHIFT))
DO 1 I = 1, LD
  DATA(I) = DATA(I) * EPS
1 RETURN
END
```
**FILE: PLTSAS  FORTRAN A  UNIV D'OF OTTAWA CMS RELEASE A**

Subroutine DRAWS:IWHAT=1...EYE-PATTERN for 2 symbol duration

---

**---POLAR (state space) DIAGRAM**

---

SUBROUTINE PLTSAS(LD,DATA,M2,ML,M2,JJK,LASH,LARY,IWHAT)

COMMON /I1/IOFF,LSAMPL,NSAM,NO1,NO2,HOWN(4,20),HTM(4,4096)

COMMON /R1/SAM,PI,BAUD,BW,AMPL(4,4),ATMET(4,5),ATMP(4,30)

COMPLEX DATA(LD)

ARY=FLOAT(LARY)-1.

IF (ARY=-20.0.) ARY=1.

GO TO (1,2),IWHAT

---

**---EYE---**

---

1 JJ=2*LSAMPL+1

J1=JJ-1

DO 11 KK=1,M2

11 I=1,JJ

T1=I-1

II=K1*J1+T1+MT

A2=REAL(DATA(I1))/ARY

A1=(FLOAT(I1)-SAM)/SAM

WRITE(1,12)A1,A2,JJK

12 FORMAT(2F10.6,I8)

RETURN

---

**---POLAR---**

---

2 JJ=LSAMPL+1

IF (TJK.GT.0.) JJ=1

DO 13 KK=1,M2

K2=KK*LSAMPL

K1=K2+MI-1

T2=K2+MO-1

DO 13 I=1,JJ

A1=REAL(DATA(K1+I))/ARY

A2=ATM(5,DATA(K2+I))/ARY

WRITE(9,13)A1,A2,JJK

13 RETURN

END
Subroutine COMPUTES the MEAN and PEAK POWER and EB

SUBROUTINE POWER(LD, DATA, PAVE, PEAK, EB, N)

COMMON /I/OFF, LSAMPLE, SYMBOL, NO1, NO2, NUMB(4,20), HI(4,4096)
COMMON /RT/SAM, PL, BAUD, BW, ANPL(4,4), TIRRL(4,4), TIMP(4,30)

COMPLEX DATA(LD)

PEAK=0.
PAVE=0.
DO J=1, LD
A=ABS(DATA(I))
PAVE=PAVE+A
IF (PEAK .LT. A) PEAK=A
CONTINUE
PAVE=PAVE/FLOAT(LD)/2.
PEAK=10.*LOG10(PEAK/PAVE)
IF (N .EQ. 1) EB=PAVE/BAUD**2.
RETURN
END
SUBROUTINE P1(LD, LD2, L1, L2, L3, L4, L5, DATA, X, Y, PSX, PSY, WW, WK, X, Y, P, T)
COMMON /I1, IOFF, LSAOPL, ASYM, NO1, NO2, HUMRB(4, 4), HT(4, 4), O96
COMMON /R1, SAM, IT, BAOD, EB, AMPY(4, 4), AMPY(4, 4), TMT, TMT, N, 30
COMMON /DATA(LD), *A(LD), *B(LD)
DIMENSION EB(LD), *PSX(L2), *WW(L3), *WK(L4)
IF (JN.EQ.0) GOTO 98
L1 = L2 + 1
L2 = L2 - 2
P = 0
DO 1 I = 1, LD
I = I - 1
IF (I .EQ. 0) I = I + LD
EB(I) = REAL (DATA(I))
EB(I) = AIMAG (DATA(I))
1 P = EB(I) + EB(I + LD)
P = P / LD2
DO 9 I = 1, LD2
9 EB(I) = EB(I) - P
WRITE (6, 10)
10 FORMAT ("The mean of I+Q channel: \( \pm \) 17.5")
CALL P4PS (X, Y, LD2, L1, 0, PSX, PSY, XP, YP, WW, WK, X, Y, P, T)

169
FILE: P1   FORTRAN A   UNIV DI/OP OTTAWA CMS RELEASE 4

IF (Y LT 1.E-10) Y = 1.E-10
WRITE(6) XLOG10(Y)
WRITE(6) (PSX(I), XB(I), I = 1, L2)
FORMAT(2F16.7)
M = INT(PLOAT(L2-1)*10)/SAM + 1
SS = SAM/POAT(NSTMB)
DO 22 I = 1, M
Y = (I-1)*SS
22 WRITE(3, 23) Y, PSX(I), IP
FORMAT(2E15.6, I)
RETURN

TRIANG = 0.
IF (NUMBER(N) EQ 1) TRIANG = 1.
TIME1(N) = TIME1(N)/(1. + TRIANG)
NMB = IABS(NUMBER(N))
F = 0.
CALL PSD(N, NNN, TRIANG, P, SIGLOG, SZ)
J1 = 2001
IF (IP EQ 2) J1 = 1001
DO 12 J = J1, J1
P = POAT(J-1001)/200.
CALL PSD(N, NNN, TRIANG, P, SIGLOG, SZ)
WRITE(3, 23) P, SIGLOG, IP
RETURN

SUBROUTINE PSD(N, NNN, TRIANG, P, SIGLOG, SZ)
COMMON /11/ OPP, SP, AMP, NSMB, NO, N, N2, NMB, 4, 20, 4, 40, 4, 30
COMMON /1/ SAM, PI, BAU, BR, AMP, 4, 40, TIME1(4, 4), AMP1(4, 4)
IF (NNN EQ 0) GOTO 77
WD = TIMEK(N)/SINH(TIMEK(N, 1))
FL = P*TIM1(N)
IP = PI*SQR(ABS(AMP1(N, 1) = AMPl(N, 1) - FL * FL))
IP = AMPL(N, 1) = E*FL) GOTO 66
I = SINH(N) / I
GOTO 67
66 CALL SINC(I)
67 SLIN = W0 = I
SLIN = SLIN * SLIN
IP (P, EQ, 0) = S = SLIN
SLIN = SLIN / 2
GOTO 78
77 IP = PI*TIM1(N, 1) = F
CALL SINC(I)
GM = AMP1(N, 1) = TIM1(N, 1) = X
GNN = 0.
IF (TRIANG EQ 1) GM = GM * GM
IF (NNN EQ 1) GOTO 17
DO 16 K = 2, NNN
DO 16 L = 1, 2
IP = PI*TIM1(I, K) = F = (FLOAT(L-2) = TIMEK(N, K) / TIMEK(N, K)/2)
GOTO 17
CALL SINC(I)
16 CONTINUE
17 G = G + G / 2.
IF (F.EQ.0.) S2 = 2. * G * G
SLIN = 2. * G * G / S2
78 IF (SLIN .LT. 1E-10) SLIN = 1.0E-10
SLOG = 10. * ALOG10(SLIN)
RETURN
END
Subroutine PLOTS (JOE = 0) I = SUM(A*COS(PI*T*K/L)) signal
                    KBY = I0(N+500+500) 10000
                    T1 = T0(VL-F(L+T+1)) /

**************************************************************************
SUBROUTINE SIGPHP(LD, DATA, JOE, IPEB, N)
COMMON /L7/ IOPI, LAMDC, LAMDA, C0, M02, NUMRA(q, 20), HT(n, q, 1, 20)
COMMON /R1/ SAM, PI, BAUSD, BW, AMPL(4, 4), TIMPL(4, 4), TIPF(4, 30)
COMPLEX DATA(I)
REAL MBSIO
IF (JOE.EQ.1) GOTO 4
A0 = AMPL(N, 1)
DO 3 K = 1, N
KNH = NUMBA(K, 1)
KPEN = IPEB + K - 1
KN = KNH
IF (KN.EQ.0) KN = 1
A2 = A0
DO 3 J = 1, KN
DO 3 L = 1, 1081
I = L + 41
A1 = AMPL(K, 1) * SIN(1.4145 + PI * L / TIMPL(K, 1)) / TIMPL(K, 1)
A2 = MBSIO(1, A0, IER) / MBSIO(1, A0, IER)
GOTO 3
3 WRITE(10, 3) A2, KPEN
8 FORMAT(2F10.4, I2)
RETURN
K = LAMDC * 10 + 1
DO 5 I = 1, K
A1 = AMPL(I)
A2 = REAL(DATA(I))
A4 = 1.5 + ATAR2(A1, A2) * 6. / PI
A1 = FLOAT(I - 1) / SAM
A2 = 7.5 + REAL(DATA(I))
A3 = 4.0 + ABS(DATA(I))
WRITE(10, 5) A1, A2, A3, A4, KPEN
5 WRITE(10, 5) A1, A2, A3, A4, KPEN
8 FORMAT(2F10.4, I2)
RETURN
END
Subroutine CALCULATES \( \frac{\sin(x)}{x} \)

Singular point \( x=0 \) is included, i.e. \( \frac{\sin(0)}{0}=1 \).

Subroutine SINC(I)

IF (I.EQ.0.0) GO TO 1

I = \( \frac{\sin(I)}{I} \)

RETURN

1

I = 1

RETURN

END
Subroutine SYNCHRONIZES the M-ary QAM received data

SUBROUTINE SYNCH(IL, DA, TF, NOISE, MT, MQ, ET, LARY)
COMMON /J1, OFF, LSAMPL, NSYM, NO, 402, SNMB(4, 20), NT(4, 4096)
COMMON /R1, SAM, PI, BAUD, BW, AMPL(4, 4), IMPL(4, 4), FMAX(4, 30)
COMPLEX DATA(IL), TP(IL)
IF (OFF .EQ. 0) GOTO 111
DO 13 I = 1, OFF
A = AIMAG (DATA(IL))
DO 14 J = 1, LD
J1 = J + 1
14 DATA(J1) = COMPLX (REAL (DATA(J1)), AIMAG (DATA(J1-1)))
13 CONTINUE
111 HOLD = 0
NOF = 0
NS2 = 2 * NSYM
K = 1
3 NEW = 0
DO 4 J = 1, NSYM
J1 = J + 1
LSAMPL
IF (J1 .GT. LD) J1 = J - LD
IF (REAL (DATA(J1)) + REAL (DATA(J1+1))) / 2.
12 NEW = NEW + 1
ABC = NI(3, J) * XP
IF (ABC .GT. 0.) NEW = NEW + 1
ABC = NI(4, J) * YD
IF (ABC .GT. 0.) NEW = NEW + 1
4 CONTINUE
IF (NEW .NE. NEW) GOTO 5
5 K = K + 1
IF (HOLD .LT. NS2 .AND. K .LE. LD) GOTO 3
NOF = NOF + 1
CALL TIMESH(LD, DATA, TF, NOF)
BOI = FLOAT(NSYM) + 1.
BOI = BOI
MI = 1
MQ = 1
SNR = SQRT (NOISE * EB / 10.)
IF (LARY .GT. 1.) GOTO 2
DO 7 J = 1, LSAMPL
EQ = 0.
DO 6 K = 1, NSYM
J1 = (K-1) * LSAMPL + J
IF (REAL (DATA(J1)) + REAL (DATA(J1+1))) / 2.
IPI (SIGN(1, XR) .NE. NI(3, K)) EQ = EQ + 1.
IPI (SIGN(1, YB) .NE. NI(4, K)) EQ = EQ + 1.
XR = ABS (XR) / SNR
YB = ABS (YB) / SNR
7 CONTINUE
SYN000010
SYN000020
SYN000030
SYN000040
SYN000050
SYN000060
SYN000070
SYN000080
SYN000090
SYN000100
SYN000110
SYN000120
SYN000130
SYN000140
SYN000150
SYN000160
SYN000170
SYN000180
SYN000190
SYN000200
SYN000210
SYN000220
SYN000230
SYN000240
SYN000250
SYN000260
SYN000270
SYN000280
SYN000290
SYN000300
SYN000310
SYN000320
SYN000330
SYN000340
SYN000350
SYN000360
SYN000370
SYN000380
SYN000390
SYN000400
SYN000410
SYN000420
SYN000430
SYN000440
SYN000450
SYN000460
SYN000470
SYN000480
SYN000490
SYN000500
SYN000510
SYN000520
SYN000530
SYN000540
SYN000550
IF (IB .GE. 12.) IB = 12.
BE = BE + BRPC(IB)/2.
IB = ABS(IB)/SNR
IF (IB .GE. 12.) IB = 12.
BE = BE + BRPC(IB)/2.
CONTINUE
IF (E01 .LE. EI) GO TO 8
E01 = EI
M1 = J
8 IF (E0Q .LE. EQ) GO TO 7
EQ = EQ
M0 = J
7 CONTINUE
GOTO 1

C

2 HIGH = FLOAT(LARY - 1)
DO 17 J = 1, LSNAPL
BE = 0.
EQ = 0.
DO 16 K = 1, NSYMB
J1 = (K - 1) * LSNAPL + J
KP = REAL(DATA(J1)) + REAL(DATA(J1 + 1))/2.
KB = AIMAG(DATA(J1)) + AIMAG(DATA(J1 + 1))/2.
DX1 = ABS(IB)
DY1 = ABS(YB)
IND1 = 0
AI = FLOAT(NI(K, K))
IF (IB * AI .LE. 0.) INDI = 1
AI = ABS(AI)
THR1 = AI - 1.
THR2 = AI + 1.
LAG1 = 0.
IF (AI .EQ. HIGH) GOTO 30
IF (DX1 .GE. THR2 OR DX1 .LE. THR1) INDI = 1
GOTO 40
30 IF (DX1 .LE. THR1) INDI = 1
LAG1 = 1
40 INDO = 0.
AI2 = FLOAT(NI(K, K))
IF (IB * AI2 .LE. 0.) INDQ = 1
AI2 = ABS(AI2)
THQ1 = AI - 1.
THQ2 = AI + 1.
LAG2 = 0.
IP (AI2 .EQ. HIGH) GOTO 50
IF (DY1 .GE. THQ2 OR DY1 .LE. THQ1) INDQ = 1
GOTO 60
50 IF (DY1 .LE. THQ1) INDQ = 1
LAG2 = 1
60 DX2 = ABS(DX1 - THR1)/SNR
DY2 = ABS(DY1 - THR1)/SNR
IF (DX2 .GE. 12.) DX2 = 12.
BE = BE + BRPC(DX2)/2.
IF (LAG2 .EQ. 1) GOTO 10
10 DX2 = ABS(DX1 - THR2)/SNR
FILE: SYNC
FORTRAN A
UNIV D*/OF OTTAWA CMS RELEASE 4

IF (DX2.GT.12.) DX2=12.
10 IF (IND2.GE.1) EI=EI+1.
   IF (DY2.GT.12.) DT2=12.
   EQ=EQ+ERFC(DY2)/2.
   IF (LAQ.LT.EQ-1) GOTO 12
   DT2=ABS(DY1-THRO2)/SNR
   IF (DT2.GT.12.) DT2=12.
   EQ=EQ+ERFC(DT2)/2.
12 IF (IND2.GE.1) EQ=EQ+1.
16 CONTINUE
   IF (EQ.LE.EI) GO TO 18
   EO=EI
   NQ=J
18 IF (EOQ.LE.EQ) GO TO 17
   EOQ=EQ
17 CONTINUE
   EI=FLOAT(MI)/SAM
   EO=FLOAT(NQ)/SAM
   WRITE(6,9)EI,EQ
   WRITE(8,9)EI,EQ
9 FORMAT('Sampling points for I and Q channels, ?8.2)
RETURN
END
SUBROUTINE SHIFTS the signal in TIME DOMAIN

SUBROUTINE TIMESH(LD, DATA, TF, IFPOF)
COMMON DIY/IOFF, LSPACE, NSYM, RO1, RO2, NUM(4, 20), NT(4, 4096)
IF (IFPOF.EQ.0.0) RETURN
J = LD-IABS(IFPOF)
K = J + 1
IF (IFPOF.LT.0) GOTO 4
1 TP(I) = DATA(I)
DO 2 I = 1, J
2 DATA(I) = DATA(I)
DO 3 I = 1, J
3 DATA(I) = TP(I)
RETURN
4 L = 1 - IFPOF
DO 5 I = L, LD
5 TP(I) = DATA(I)
DO 6 I = 1, IFPOF
6 DATA(I) = DATA(I)
DO 7 I = 1, J
7 DATA(I) = TP(I)
RETURN
END
Subroutine TREQ models TRANSVERSAL EQUALIZER with an odd number of taps NTAP=2*N+1
NEOSAM is the number of samples of each of delay elements, etc.
if NEOSAM=LSAMPL a full tap equalizer is assumed,
if NEOSAM=LSAMPL/2 an 1/2 fractional equalizer is assumed, etc.
LD is the total number of samples
DATA is the complex data matrix
C is the tap coefficients matrix
TF is the dummy matrix
LS2=LSAMPL/2
JOE=0 tap coefficients defined previously
  =1 NRZ signal ISI due to timing uncertainty
  =2 NRZ signal ISI compensation

*******************************************************************

SUBROUTINE TREQ(NTAP,NEOSAM,LD,DATA,TF,C,LS2,JOE)
COMMON /11/ IOPF,LSAMPL,NSYMB,NO1,NO2,NO3,NO4(4,50),RT(4,4096)
COMMON /R1/ SAM,ST,BAUD,BW,AMPL(4,4),TIME(4,4),TWF(4,30)
COMPLEX DATA(LD),TF(LD),D1(1)
DIMENSION C(NTAP)
NTA=(NTAP-1)/2
NTAP=NTA+1
DO I=1,NSYMB
  IF (JOE.EQ.0) GOTO 4
  IF (J.GE.LE.LS2) C(1)=PL
  IF (J.GE.LE.LS2) C(3)=PL
  IF (JOE.EQ.1) GOTO 4
  C(2)=1./C(2)
  C(1)=C(1)
  C(3)=C(3)
  D1(1)=CPLX(0.,0.)
  DO 1 L=1,NTA
    LI=L*NEOSAM
    KM=K-L1
    KP=K+L1
    IF (KM.LT.1) KM=LD+KM
    IF (KP.GT.LD) KP=KP-LD
    D1(1)=D1(1)+C(NTAP0-L)*DATA(KM)+C(NTAP0+L)*DATA(KP)
  1 CONTINUE
  TF(K)=D1(1)+C(NTAP0)*DATA(KP)
  IF (JOE.EQ.1) GOTO 4
  SUBROUTINE RETURN
END

C

D1(1)=CPLX(0.,0.)
DO 1 L=1,NTA
  LI=L*NEOSAM
  KM=K-L1
  KP=K+L1
  IF (KM.LT.1) KM=LD+KM
  IF (KP.GT.LD) KP=KP-LD
  D1(1)=D1(1)+C(NTAP0-L)*DATA(KM)+C(NTAP0+L)*DATA(KP)
  1 CONTINUE
  TF(K)=D1(1)
  RETURN
END
Subroutine calculates WINDOW (WEIGHT) functions

Common variables:
- TIMEL: Array to store weights
- SAM, PM, BAUD, SW, ANML: Arrays
- N, M, L: Constants

Real variables:
- R(54), 1: Temporary variables

Do loop 1:
- i = 1 to LSAMPL
  - TL = TIMEL (i, 1)
  - IF (TL LE 1.) RETURN

Do loop 2:
- i = 1 to LSAMPL
  - J = 1 to KN
    - IP = TL(i, j)

Do loop 3:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 4:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 5:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND, (i, j, R(i), R(i + 1), IP)

Do loop 6:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 7:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 8:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 9:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 10:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 11:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 12:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 13:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 14:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 15:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 16:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 17:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 18:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 19:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 20:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 21:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 22:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 23:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 24:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 25:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 26:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 27:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 28:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 29:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 30:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 31:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 32:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 33:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 34:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 35:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 36:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 37:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 38:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 39:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 40:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 41:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 42:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 43:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 44:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 45:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 46:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 47:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 48:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 49:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)

Do loop 50:
- i = 1 to LSAMPL
  - J = 1 to KN
    - CALL WIND1 (i, j, R(i), R(i + 1), IP)
```fortran
DO 7 J=1,KN
   ITL=INT(TIMEL(N,J)*SAM)-L8-LSAMPL/2
   IF (ITL.EQ.0) ITL=1
   DO 7 I=1,ITL
      I=LSAMPL+I-1
      U=I*5+(i-1)/SAM
      CALL WIND1(J,U,Y,N,KHN,M2)
      R(I)=R(I)*Y
      CALL WIND1(J,U,Y,N,KHN,L2)
    7 R(I)=R(I)+Y
RETURN
END

********************************************************************

KAI塞尔-BESSEL and WEIGHTED COSINE windows--->Subr.WINDOW

********************************************************************

SUBROUTINE WIND1(J,U,Y,N,KHN,JIP)
COMMON /PL,SAMPI,BAOD,AMPL(4,4),TIMEL(4,4),TIMEK(4,30)
REAL HBSIO
IF (KHN.GT.0) GOTO 1
Y=AMPL(N,1)
A=AMPL(N,1)*SORT(1,-4.*T*T/TIMEL(N,1)/TIMEL(N,1))
Y=HBSIO(1,JIP,IER)/HBSIO(1,Y,IER)
Y=JIP*Y
RETURN
7 Y=JIP*AMPL(N,J)*COS(PI*TIMEK(N,J)/TIMEL(N,J)*T)
RETURN
END
```
FILE: WDPL  FORTRAN  A  UNIV D'OF OTTAWA CMS RELEASE 4

CALL DO 1
5 WRITE(17) SMB, DEG1, PEB, G1, G2, AN1, GPI, GPN
1 FORMAT(# of levels L=04)
2 FORMAT(T2)
3 FORMAT(#PE and ENo=1, I9, E16.3)
6 FORMAT(T1, T4)
8 FORMAT(EBNO=, P7.2, at Pe=1E-*, T2)
10 FORMAT(BBR exp=06)
101 FORMAT(Staggered=01, Non-staggered=00)
BND

C  *-----------------------------------------------------------------------

SUBROUTINE UD1
COMMON L,L2,LM1,LM2,LM1S0,L2L,L4L
COMMON PEL,AMB,PEA,PEB,EBN,RA,RA1,PR,RR1,GB1,G1,G2,AN1,PL,F1,R2,P3
COMMON PEL1,PRL1,PEI1,PEI2,PE1J1,RA1L,RBL,GBP,GPIN,AN1D,BD2,BD3,RA
COMMON PIP,PIP3,PE2,PE3
PEB=0.
PE2=0.
PE1=0.
G2=0.
G1=0.
G2=0.
PE=0.
PI=0.

C  *-----------------------------------------------------------------------

DO 14 M0=1,L2,2
M0=0-L
PM=PMAT(M-1)
PP1=PM1+2
IN=IABS(M)
PEN=0.
PIH=0.

DO 12 M0=1,L2,2
N0=0-L
PE1=0.

DO 11 M0=1,L2,2
M0=0-L

DO 11 M0=1,L2,2
M0=0-L

IF (13.EQ.-3) RAM1=M*RA1
IF (M1.NE.7) RAM1=M*BD1
IF ((M7.EQ.-1).AND.(M7.NE.8)) RAM1=**BD1*PIP3
DO 11 M0=1,L2,2
M0=N=L

DO 11 M0=1,L2,2
M0=N=L

DO 11 M0=1,L2,2
M0=N=L

IF (14.EQ.3) RAM1=M*RA1
IF (M8.NE.M) RAM1=M*RA2
IF (M.NE.M) RAM1=M*RA
IF (M.NE.M) RAM1=M*RA

DO 11 M0=1,L2,2
M0=N=L

DO 11 M0=1,L2,2
M0=N=L

C  *-----------------------------------------------------------------------

WDP000560
WDP000580
WDP000590
WDP000600
WDP000610
WDP000620
WDP000630
WDP000640
WDP000650
WDP000660
WDP000670
WDP000680
WDP000690
WDP000700
WDP000710
WDP000720
WDP000730
WDP000740
WDP000750
WDP000760
WDP000770
WDP000780
WDP000790
WDP000800
WDP000810
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WDP000870
WDP000880
WDP000890
WDP000900
WDP000910
WDP000920
WDP000930
WDP000940
WDP000950
WDP000960
WDP000970
WDP000980
WDP000990
WDP001000
WDP001010
WDP001020
WDP001030
WDP001040
WDP001050
WDP001060
WDP001070
WDP001080
WDP001090
WDP001100
N7 = N3 - L
J3 = IABS(N + N1 + N7)
IF (J3 .EQ. 3) RAN1 = N*RAB1
IF ((N7 .EQ. N1) .AND. (N7 .NE. N)) RAM1 = N*BD3*P1P3
IF ((N7 .EQ. N1) .AND. (N7 .NE. N)) RAM1 = N*RAB1/P1P1
DO 11 N6 = 1,3,2
N5 = N6 - L
J3 = IABS(N + N1 + N5)
IP (J4 .EQ. 3) RAN1 = N*RAB
IF ((N4 .EQ. N5) .AND. (N4 .NE. N)) RAN1 = N*BD4*P1P3
IP (N4 .EQ. N5) RAN1 = N*BD4
Y1 = (RAN1 + RAN)*SX
TAB1 = Y1 + YB1
IF (N0 .EQ. 1) PEA = ERFC((FP1 - YAB1)*ER1)
IF (N0 .EQ. 1) PEA = ERFC((FP1 - YAB1)*ER1)
IP + I1 =1,1,2,2
Y1 = 19 - L
IP1 = PLOT(I1 + 1)
PM1 = PPI1 + 2
IA = IABS(I1)
PPI1 = 0
PB1 = 0
DO 12 NO = 1, L2, 2
M6 = 0
L6 = 0
DO 13 M9 = 1, L2, 2
M1 = M9 - L
DO 14 I9 = 1, 3, 2
M7 = M9 - L
J3 = IABS(M + M1 + M7)
IP (J3 .EQ. 3) RAM1 = M*RA1
IP (M1 .EQ. M7) RAM1 = M*BD1
IP ((M7 .EQ. M1) .AND. (M7 .NE. M)) RAM1 = M*BD1*P1P3
DO 15 I6 = 1, 3, 2
M5 = M6 - L
J5 = IABS(M + M1 + M5)
IP (J4 .EQ. 3) RAM1 = M*RA1
IP (M4 .EQ. M5) RAM1 = M*BD2
IP ((M5 .EQ. M4) .AND. (M5 .EQ. M1)) RAM1 = M*RA1
IP ((M4 .EQ. M5) .AND. (M5 .EQ. M1)) RAM1 = M*RA1
TA1 = RAM1 + RAM2
CI = (RAM1 + RAM2)/CI
DO 16 I9 = 1, L2, 2
M1 = M9 - L
DO 16 M9 = 1, 3, 2
N7 = N3 - L
J3 = IABS(N + N1 + N7)
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IF (J3 .EQ. 3) RAN1 = M * RB1
IF (N7 .EQ. N1) AND * (N7 .NE. N)) RAN1 = M * RD3 * PIP3
IF (N7 .NE. N1) AND * (N7 .EQ. N)) RAN1 = M * RD3
DO 16 N6 = 1, 3, 2
N5 = N6 - L
J6 = IABS(N + M1 + N5)
IF (J4 .EQ. 3) RAN1 = M * RB
IF (N. EQ. N5) AND * (N. NE. N)) RAN1 = M * RD4 * PIP3
IF (N .NE. N5) RAN1 = M * RD4
YB1 = (RAN1 + RAN1) * SX
YB1 = YB1 + YB1
IF (I9 .EQ. 1) PEI = ERFC((YAB1 - FPI1) * ERN)
IF (I9 .EQ. 1) PEI = ERFC((FPI1 - YAB1) * ERN)
IF (I9 .EQ. 1) PEI = ERFC((FPI1 - YAB1) * ERN)
IF (I9 .EQ. 1) PEI = ERFC((FPI1 - YAB1) * ERN)
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IF (13,EQ.,3) RAN1=N*RNB1
IF ((N7,EQ.,N1)*AND.(N7,NE.,N)) RAN1=N*RND3*PTP3
IF ((N7,NE.,N1)*AND.(N7,NE.,N)) RAN1=N*RND3
DO 26 N6=1,3,2
R5=R6=L
J4=TAB5(N+N1+N5)
IF (J4,EQ.,3) RAN=N1*RBB
IF ((N,BQ.,N5)*AND.(N1,NE.,N)) RAN=N1*RD4*PTP3
IF (N,NE.,N5) RAN=N1*BD4
YB1=(RAN+RAN)*S6
TAB1=TAB1-YB1
IF (I9,EQ.,1) PE1=ERFC((YAB1-PFI1)*P9W)
IF (I1,EQ.,LM1) PEI=ERFC((PMT1-YAB1)*ERN)
IF ((IA,LT.,LM1) AND.(I1,GT.,N))
*PE1=ERFC((PMT1-YAB1)*EBN)-ERFC((PPT1-YAR1)*ERN)
IF ((IA,LT.,LM1) AND.(I1,LT.,N))
*PE1=ERFC((YAB1-PFI1)*EBN)-ERFC((YAR1-PMT1)*ERN)
26 PE1=PE1+PE1
PE11=PE11+PEI1
23 PE11=PE11+N*PEI1
22 G2=I1*PEI1
24 CONTINUE

G2=G2*P3/2.
G11=G1*P3
ANT1=-ANT1*F3
PE11=ALOG10(PEB*F2)
GFI=G1*SX+G2*CI
3FIN=2.*GFI/ANT1
RETURN
END
IF (LARY.EQ.1) CALL IMP1(NCH,1)
CALL IMP2(NCH)
WRITE(8,4)
FORMAT(100,12)IPEN
READ(7)IPEN
IPEN=IPEN+4
WRITE(6,6)IPEN,TOFF,NUMP(1,20),LARY
WRITE(8,6)IPEN,TOFF,NUMP(1,20),LARY
FORMAT(100,100,100)PEN,OFFSET,NORM,LARY,5TS)
DO 5 I=1,IPROM+1
5 BERR(I)=0.
ANORM=SORT(1.)
DO 99 J=1,ANR
CALL ENCODE(LD,DATA,LARY)
CALL PLTSAS(LD,DATA,60,HO,JKL,TYK,LARY,2)
CALL PLTSAS(LD,DATA,60,HO,JKL,TYK,LARY,1)
7 CALL NONLIN(LD,DATA,ANORM)
DO 99 J=1,ANR
CALL ENCODE(LD,DATA,1,LARY)
CALL PLTSAS(LD,DATA,60,HO,JKL,TYK,LARY,2)
CALL PLTSAS(LD,DATA,60,HO,JKL,TYK,LARY,1)
CALL PLTSAS(LD,DATA,60,HO,JKL,TYK,LARY,2)
CALL CHANN(LD,DATA,SHR)
IF (SHR.EQ.1) GOTO 66
DO 10 K=1,NCH
10 CALL ENCODE(LD,DATA,1,K,LARY)
DO 10 K=1,NCH
99 WRITE(6,1)I,PROM
1 IF (LARY.EQ.0) WRITE(8,1)ED/NO-----------prob.of symbol error------------
1 FORMA1.---C/N-----------prob.of symbol error------------
DO 9 IPROM+1
9 PSH=SPECTRUM(NCH,EB,MT,NO,LARY,CHB,EB)
WRITE(6,1)I,EB
1 FORMAT(100,100,100)PEN,OFFSET,NORM,LARY,5TS)
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9 WRITE (6,172) PSB, PBI(I)
172 FORMAT (2B75, 6, I2)

CALL BERRPH(IPEN, PEB, IPRM, ITO, 1.E-12)

123 WRITE (6,173)
173 FORMAT ('---------------------------
"NEW PARAM:0, NEW BB SIGNAL<0..."
"
READ(7,222) GO I
IF (30) 100, 11, 2
11 CONTINUE
12 FORMAT (I2)
102 FORMAT (I1)
222 FORMAT (P15.7)

END