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POWER EFFICIENT
MULTI-ARY QAM SYSTEMS

By
Jong-Soo Seo

A THESIS
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ABSTRACT

Techniques to improve the performance of spectrally efficient Multi-ary Quadrature Amplitude Modulation (QAM) systems operating in channels with various system distortions are studied.

To operate M-ary QAM at higher power efficiency, i.e., close to the saturation region of transmit high power amplifiers (HPAs), we propose a simple, robust baseband signal mapping technique. Our mapped 16-QAM compensates for system impairments caused by AM/AM and AM/PM non-linearities of HPAs, thus enables us to operate the HPAs closer to the saturation region while the CNR degradation is significantly reduced. In conjunction with a new signal constellation at the transmitter, we apply an Adaptive Decision Threshold (ADT) detector at the receiver to improve the system performance. As an illustrative result, the CNR degradation of our proposed system operating at 0.9 dB output back-off (OBO) of Siemens TWTA is only 1.9 dB at $P(e) = 10^{-6}$. This corresponds to 7 dB (or even more) CNR improvement compared to conventional 16-QAM systems. To maximize the overall system gain, we determine the optimum operating point of the HPAs by compromising the HPA power OBO and $P(e)$ performance degradation.

For the operation of M-ary QAM in the most power efficient mode (i.e., 0 dB OBO from the saturation) of HPAs, a new Non-Linearly Amplified (NLA) – M-ary Superposed-QAM (SQAM) is introduced. The signal waveshape of NLA- M-ary SQAM as well as the receiver LPF bandwidth is optimized to achieve the best $P(e)$ performance. The system sensitivity of NLA-16-SQAM modem is analyzed with respect to the output power level variation and the propagation time difference of the transmit HPAs. For comparison purposes, the performance of Multi-Amplitude MSK (MAMSK) and also a conventional 64-QAM are studied. Retaining a compact power spectrum and allowing a simple filtering strategy, NLA-16/64-SQAM operates within 0.75 dB CNR degradation at $P(e) = 10^{-6}$ in a saturated channel. To demonstrate the spectral robustness of NLA-16-SQAM, we further investigate the performance in a nonlinearly amplified multi-channel interference environment in the presence of additive white
Gaussian noise (AWGN), intersymbol interference (ISI), adjacent channel interference (ACI), and cochannel interference (CCI). Various channel conditions, such as channel spacings between the main and the adjacent channels, and flat (i.e., wide-band) fades on the desired main channel, are examined. Our results show that NLA-16-SQAM system outperforms MAMS, and thus permits a more efficient utilization of the available spectrum.

The impact of residual amplitude fluctuations of the received carrier after Automatic Gain Control (AGC) amplifiers, and the transmission system caused phase jitter on the performance of M-ary QAM systems are studied. We derive theoretical average symbol error probabilities of 16-, and 64-QAM, and also tight-bound symbol error probabilities of 256-, and 1024-QAM. To confirm the practical validity of our theoretical results and also the accuracy of the tight-bound symbol error probabilities, we perform computer simulations on 16-, 64-, and 256-QAM systems, and also experimental measurements on 256 kb/s rate 16-QAM modem. Our study shows that in the M-ary QAM the outermost signal states are most susceptible to the system impairments, thus result in the worst symbol error probability. Whereas, the innermost states are least susceptible, thus result in the best symbol error probability. An overall average symbol error probability, therefore, is mostly determined by the behaviour of the outermost signal states. For this reason, we introduce ADT detectors for the M-ary QAM receiver which could adjust the decision threshold levels to the midpoints of the demodulated eye openings, and demonstrate that this adaptive technique improves the system performance significantly. We also found that error floors (i.e., residual error rates) exist on the system performance for certain levels of phase jitter.

Finally, we analyze the impact of non-Gaussian impulsive noise (Middleton’s Class ‘A’ noise) combined with Gaussian thermal noise on the performance of M-ary QAM systems. $P(e)$ performance of the system is evaluated in terms of CNR, impulsive index ($\Lambda$) of the noise, and power ratio of the Gaussian noise-to- Impulsive noise ($\Gamma'$). Our results show that in the Multi-ary QAM, the non-Gaussian impulsive noise degrades the system performance significantly even at high CNR. It is also found that an upper bound on the error probability exists for $\Gamma' \leq 0.01$. 
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Chapter 1

INTRODUCTION

1.1 EFFICIENT OPERATION OF DIGITAL RADIO SYSTEMS AND ASSOCIATED PROBLEMS

Two major criteria considered in performance evaluations of digital radio systems may be the Power Efficiency and the Spectral Efficiency. Problems often encountered to meet these criteria may arise from the following aspects.

1. To achieve high spectral efficiencies in digital radio systems, high-level (i.e., multi-phase/amplitude) modulation techniques, such as M-ary QAM and M-ary QPRS, have been introduced. However, the sensitivity of these modems to the various amplitude and phase distortions of the transmission channels, increases significantly with increasing number of signal levels.

2. For the power efficient operation of the digital transmission systems, it is required to operate the transmit high power amplifiers (HPAs) close to the saturation region. However, in the multi-ary modulation systems, the AM/AM compression and AM/PM conversion nonlinearities of the transmit HPAs cause significant performance degradations.
3. Multi-carrier or multi-channel systems, such as FDMA (Frequency Division Multiple Access) and SCPC (Single Channel Per Carrier) systems, have been employed for the efficient use of the allocated spectrum. The frequency utilization efficiency of these systems may be mostly determined by the adjacent channel interference caused by the out-of-band energy of the adjacent channel signals. This problem is even more severe in nonlinearly amplified transmission channels, where the bandlimited power spectrum regrows near the unfiltered spectrum.

1.2 GENERAL REQUIREMENTS FOR THE HIGH POWER AND HIGH SPECTRAL EFFICIENCIES

1) Requirements for power efficient operation of the digital radio systems include:

- Deliver the maximum output power from the available transmit HPAs — Operation of the transmit HPAs at the minimum output back-off (OBO) or near the saturation region.

- Minimize the required $E_b/N_0$ or $C/N$ for satisfactory $P(e)$ performance — Minimize the intersymbol interference and nonlinear distortions caused by the bandlimiting filters and nonlinear power amplifiers, and also minimize various channel distortions and system impairments.

To meet these requirements, the modulated signals should have a constant or quasi-constant envelope and a compact power spectrum.

2) To achieve a high spectral efficiency, the power spectrum of a modulated signal should retain:

- A narrow main-lobe bandwidth,

- A fast spectral roll-off,
• A low out-of-band energy, and

• A low spectral regrowth after the nonlinear amplifications.

1.3 PREVIOUS APPROACHES AND UNSOLVED PROBLEMS

1.3.1 HPA Linearization Techniques

For the operation of HPAs at the minimum OBO while retaining a good $P(e)$ performance, several types of HPA linearization techniques have been proposed. One of these may be a fixed (baseband, IF or RF) predistortion linearizer, which has an inverse transfer characteristics of the HPA such that the cascade of these two devices could provide a near linear input/output (I/O) relationship [2,3]. The other may be an adaptive linearizer which consists of a predistorter at the transmitter and an adaptive linear equalizer at the receiver in order to minimize the combined effects of the intersymbol interference (ISI) and nonlinear distortions [4,6].

The implementation of a predistortion-type linearizer may not be feasible in rather simple digital circuits, as it requires various analog circuits including some nonlinear devices. The more powerful adaptive linearizer may be rather complicated to implement, and yet its behavior (e.g., $P(e)$ performance) on the practical HPAs has not been analyzed in depth.

1.3.2 Operation of M-ary Modems at the Saturation Mode of HPAs

For the operation of multi-ary modems at the highest power efficiency (i.e., at the saturation mode) of the HPAs, Miyachi [9] proposed a parallel-type modulation, Morais [10] showed $16\cdot NLA\cdot QAM$ leads to approximately 6 dB more power at the modulator output than the conventional 16-QAM, and Weber [11] improved the spectral properties by introducing a sinusoidal pulse shape.
These modems, however, require additional pre/post-modulation spectral shaping filters to achieve a high spectral efficiency, especially in the multi-carrier system operations. Premodulation LPFs prior to the saturated HPAs cause a significant amount of ISI, envelope fluctuations and spectral spreading, thus severely degrade the system performance. Conceptually, BPFs located after the saturated HPAs could reduce these degradations due to the nonlinear effects. However, in many applications, it is not practical to design spectral shaping BPFs following the HPA nonlinearity. Especially,

- Implementation of the post-HPA BPF for a low bit-rate-to-carrier frequency ratio (e.g., 64 kb/s to 0 GHz) is impractical.
- Difficulties are also due to the filter insertion loss, group-delay,
- Precise symmetry of the phase and amplitude characteristics of the BPF with respect to the carrier frequency, and
- Possible needs for the frequency agility such as in TDMA (Time Division Multiple Access) subscriber radios or satellite systems.

1.3.3 Spectrally Efficient M-ary QAM Systems

M-ary QAM may be one of the most popular candidates in the digital modulation techniques to achieve a high spectral efficiency. Being a hybrid modulation of the phase and the amplitude, the performance of the M-ary QAM would be very sensitive to the various phase and amplitude distortions introduced in the transmission channels. An in-depth analysis of these distortions, therefore, is mandatory for the efficient operation of the system.

1.3.3.1 Amplitude Fluctuations Caused by Flat (Wide-Band) Fade

Fading is known to be one of the most destructive impairments on the performance of digital radio systems. In general, two kinds of fade, such as a flat fade and a frequency selective fade, are considered in transmission
systems. A flat (or wide-band) fade may be compensated by AGC amplifiers, and a frequency selective fade may be compensated by adaptive fade equalizers [31,32].

To minimize the rapid (e.g., 100 dB/sec) carrier level fluctuations [33,34] of Multi-ary QAM systems, fast AGC circuits, which could maintain a constant or almost constant average carrier power at the demodulator input, are required. The fast response AGC circuits, however, require fairly wide-band detectors, which may not filter out an AWGN, interference and intermodulation noise presented at the input of the detectors. Consequently, the resultant noisy control signal would cause an incidental, or undesired amplitude modulation in the AGC circuit, and thus a residual amplitude fluctuation is present on the received QAM data streams. This problem is even worse in non-regenerative digital or hybrid (e.g., 1.544 Mb/s DIV) microwave systems in which up to 30 radio hops are used prior to the regeneration [35]. Alternatively, if the bandwidth of the AGC detector circuit is narrow, then such an incidental amplitude modulation due to the noisy control signal is negligible. However, in this case the fade caused fast amplitude variations are not tracked, and again an undesired amplitude modulation (fluctuation) is present in the received QAM signal.

1.3.3.2 Transmission System Caused Phase Jitter

In long-haul terrestrial microwave radio systems, phase jitter, which is also known as phase noise, is introduced by noise interference and/or instabilities in the transmitter local oscillators (LO), up/down converters, frequency multipliers, and receiver carrier recovery (CR) circuits. Most of the efforts have been devoted to analyze and reduce the effects of the phase jitter caused by the noise/instability in the CR circuits. The transmission system (e.g., transmitter LOs, up/down converters, and frequency multipliers) introduced phase jitter, however, would remain in the demodulated signal even if a pure sinusoidal carrier signal is provided by the receiver CR circuits, and distort the information carrying phase of the signal.
1.3.3.3 Non-Gaussian Impulsive Noise

Most of the analysis on the M-ary QAM systems has been performed under the assumption that the noise presented at the receiver front-end is pure Gaussian. However, noise generated by a variety of natural and man-made electromagnetic sources (e.g., atmospheric noise, electro-magnetic interference (EMI), ignition noise, etc.) exhibits impulsive characteristics [42,43]. Such a non-Gaussian impulsive noise is known to be one of the major source of errors in digital transmission systems [42-44].

1.4 OBJECTIVE OF THIS THESIS

1.4.1 Improve the Power and Spectral Efficiencies

1) Introduce a simple, robust Baseband Digital Signal Mapping Technique which may

- Allow the HPA to operate closer to the saturation region, thus improve the power efficiency, and

- Reduce or compensate the effect of the AM/AM compression and AM/PM conversion nonlinearities of the transmit HPAs, thus achieve a good $P(e)$ performance.

2) Determine the Optimum Operating Point of some practical HPAs in order to maximize the overall system gain by trading off

- HPA output power back-off, and

- $P(e)$ performance degradation.

3) Introduce a new Wave-shaped M-ary Offset-QAM technique which may

- Operate through fully saturated HPAs to achieve the highest power efficiency, while retaining a good $P(e)$ performance, and
• Retain a compact power spectrum, and a low spectral regrowth after nonlinear amplifications, thus

• Achieve a good $P(e)$ performance in the spectrally congested multi-carrier systems, thereby improve the frequency utilization efficiency, and also

• Not require postmodulation spectral shaping filters, and allow simple filtering strategies for the good performance.

Possible applications of these modulation techniques include:

• Satellite communications requiring a high spectral efficiency (e.g., more than 2b/s/Hz) as well as a high power efficiency, and

• Terrestrial microwave radio communications requiring a high power efficiency (i.e., operation near the saturation region of HPAs) as well as a high spectral efficiency.

1.4.2 In-depth Analysis of Phase and Amplitude Distortions

Perform an in-depth analysis of the phase and amplitude distortions introduced in the transmission channels of the multi-ary QAM systems including

• Theoretical study,

• Computer simulations, and

• Verification by Experimental measurements.

1) The theoretical study characterizes the behavior or the error mechanisms of the multi-ary modulation systems, thus enables us to investigate

• The best symbol error probability,

• The worst (or upper-bound) symbol error probability.
- An exact average symbol error probability,
- A tight-bound symbol error probability, and
- A residual error rate (or error floor).

The result of the best and the worst symbol error probabilities gives us an idea of a new detection technique for the improved performance.

With the tight-bound symbol error probability, we could eliminate the tedious and lengthy computations required for the exact average symbol error probabilities of the high-level (e.g., $M \geq 256$) QAM systems, and yet could evaluate fairly accurately the average symbol error probabilities.

The result of the residual error rate may help the system designers to define the system specifications to meet the system reliability objective.

2) The computer simulated and experimental measured eye diagrams and state-space diagrams of the distorted transmission systems provide us better physical insights of the system impairments, which lead us to the design of

- A new signal constellation or modulation technique, and
- A new receiver structure to improve the system performance.

1.4.3 Analysis of the Non-Gaussian Impulsive Noise

Until now, we have assumed that the noise presented at the receiver front-end is pure Gaussian. Since the noise generated by a variety of natural and man-made electromagnetic sources exhibits impulsive characteristics, it might be of a great importance to analyze the Impact of Non-Gaussian Impulsive Noise on the performance of M-ary QAM systems in order to

- Determine the nature of the system impairments,
- Predict the limit of the system performance, thus
• Design a new receiver structure to compensate the performance degradation.

1.5 THESIS ORGANIZATION

Including this introductory chapter, this thesis consists of seven chapters and two appendices, and is organized as follows.

Chapter 2 introduces a new simple and robust baseband digital signal mapping technique for 16-QAM which allows to operate the transmit HPAs closer to the saturation while retaining a good $P(e)$ performance. Three kinds of practical HPAs, such as Hughes TWTA, Siemens TWTA and Fujitsu GaAs FET are considered in the analysis. In conjunction with a new signal constellation at the transmitter, a new ADT detector is applied at the receiver to improve the system performance. In order to maximize the overall system gain, the optimum operating point of the HPAs is to be determined by compromising the HPA power OBO and the $P(e)$ performance degradation. The implementation of an equivalent baseband 16-QAM signal mapper including a nonlinear amplifier simulator, and measurement results are also presented to prove our concept of the signal mapping. Detailed circuit diagrams of this subsystem are given in Appendix A.

Chapter 3 introduces a new NLA - M-ary SQAM technique which can operate at the most power efficient - saturation mode of HPAs while retaining a compact power spectrum and a good $P(e)$ performance. The signal waveshape of NLA - M-ary SQAM as well as the receiver LPF bandwidth is optimized to achieve the best $P(e)$ performance. For comparison purposes, the performance of MAMS and conventional M-ary QAM are also analyzed. To demonstrate the spectral robustness of NLA-16-SQAM, we further investigate the performance in a nonlinearly amplified multi-channel interference environment, and compare this to the performance of MAMS.

Chapter 4 analyzes the impact of the residual amplitude fluctuations of the received carrier after AGC amplifiers on the performance of M-ary QAM systems.
In Section 4.1, we derive the theoretical average symbol error probabilities of 16- and 64-QAM systems in the presence of the residual amplitude fluctuations and AWGN. The best and the worst symbol error probabilities are also computed to demonstrate the sensitivities of the inner- and the outer-most signal states to the system impairments. The maximum tolerance of M-ary QAM system to the residual amplitude fluctuation is also studied. To confirm our theoretical results, we perform computer simulations on 16- and 64-QAM systems, and also experimental measurements on 256 kb/s data rate 16-QAM modem. Analyzing these results, we introduce an ADT detector at the receiver to improve the system performance.

In Section 4.2, we calculate the tight-bound symbol error probabilities of 64-, 256-, and 1024-QAM systems in the presence of the residual amplitude fluctuations and AWGN. To confirm their accuracies, the exact average symbol error probabilities of 64-QAM and also computer simulation results on 256-QAM are compared to the corresponding tight-bound symbol error probabilities.

Chapter 5 analyzes the impact of the transmission system (e.g., transmitter LOs, up/down converters, and frequency multipliers, etc.) introduced phase jitter on the performance of M-ary QAM systems.

In Section 5.1, we derive the theoretical average symbol error probabilities of 16- and 64-QAM systems in the presence of Gaussian distributed phase jitter and an AWGN. The best and the worst symbol error probabilities are also computed. Error floors (i.e., residual error rates) are found to exist. Experimental measurement results on 256 kb/s data rate 16-QAM modem, are reported and compared to our theoretical results.

In Section 5.2, we calculate the tight-bound average symbol error probabilities of 64-, 256-, and 1024-QAM systems in the presence of Gaussian distributed phase jitter and AWGN. To confirm the accuracy of our tight-bound symbol error probability, we compare the tight-bound symbol error probability of 64-QAM to its exact average symbol error probability.

Chapter 6 analyzes the impact of the non-Gaussian impulsive noise combined with the Gaussian thermal noise on the performance of M-ary QAM systems. $P(e)$ performance of the system is analyzed in terms of the CNR, impulsive index ($\Lambda$) of the noise, and power ratio of the Gaussian
noise-to- Impulsive noise ($\Gamma'$).

Finally, Chapter 7 summarizes the conclusions of this thesis and related further research proposals.

The computer programs used for the performance evaluations of the various M-ary modulation techniques are presented in the Appendix B.
Chapter 2

SIGNAL MAPPING OPTIMIZATION OF 16-QAM FOR NON-LINEAR MODE OPERATION

To achieve high spectral efficiencies in digital radio systems, multi-ary modulation techniques, such as M-ary QAM and QPRS have been introduced. One of the major problems, however, encountered in the power efficient operation of these systems may be the AM/AM compression and AM/PM conversion nonlinearity of the transmit high power amplifiers (HPAs).

In this chapter, we analyze the performance of 16-QAM systems in non-linearly amplified channels via computer simulations. The simulated state space diagrams of 16-QAM provide us with better physical insights of the HPA nonlinearity effects, which indicate that in nonlinear channels some new signal constellations may perform better than conventional equally-spaced rectangular signal constellations. For this reason, we propose a simple, robust baseband digital signal mapping technique which allows the HPA operation closer to the saturation and retain a good $P(e)$ performance, thereby enhance the power efficiency and overall system gain. To verify our concept, we also implement an equivalent baseband 16-QAM signal mapper.
2.1 SYSTEM CONFIGURATION

A system model used in the analysis is shown in Fig. 2.1. The microwave

![Diagram](image)

Figure 2.1: Analysis model of a 16-QAM modem operating in the non-linear mode of transmit HPAs.

HPAs considered in the analysis are Hughes TWTA, Siemens linearized TWTA (model #189), and Fujitsu linearized solid state GaAs FET amplifier. The AM/AM and AM/PM nonlinearity characteristics of these HPAs are shown in Fig. 2.2. Note that for Hughes TWTA, AM/AM compression and AM/PM conversion nonlinearities are significant, whereas for Siemens TWTA and Fujitsu GaAs FET, these nonlinearities are considerably compensated. The choice of transmit and receive filters is another critical factor controlling the $P(e)$ performance in nonlinear radio channels. In our system we use an aperture equalizer ($X/\sin X$) followed by $\alpha = 0.4$ square-root raised-cosine filter at the transmitter side, and another $\alpha = 0.4$ square-root raised-cosine filter at the receiver side. This filtering strategy is known to be one of the suboptimum filters in the nonlinear channel and has been adopted in numerous systems such as the INTELSAT TDMA/DSI system [1,2]. In the simulation, the carrier and symbol timing are assumed to be ideally recovered at the demodulator.
Figure 2.2: AM/AM and AM/PM nonlinearity characteristics of Hughes TWTA, Siemens linearized TWTA, and Fujitsu linearized GaAs FET amplifiers.
2.2 SIGNAL MAPPING
OPTIMIZATION OF 16-QAM
IN NON-LINEAR CHANNEL

For multi-ary digital modems, a signal state space diagram (or signal constellation) at the receiver gives us a good physical insight of the overall system impairments. An illustrative demodulated and low pass filtered ($\alpha = 0.4$) state space diagram of a conventional 16-QAM signal operating at 7 dB input backoff (IBO), or 2.2 dB OBO, of Hughes TWTA is shown in Fig.2.3(a). The corresponding eye diagram is shown in Fig.2.3(b). Note that due to the excessive ISI, the openings of outer two eyes are very small as compared to the opening of an inner eye. Such degradations are mainly caused by the amplitude compression and phase rotations of the modulated signals, where for the outermost signal states (i.e., $\{\pm3, \pm3\}$) these nonlinear distortions are most significant. Therefore, observing the nonlinear effects of HPAs on 16-QAM signals, we may optimize the transmit signal constellation, where for the conventional 16-QAM it is an equally-spaced rectangular constellation. Our objective of mapping a new signal constellation is to allow HPA operation closer to saturation thus improve the power efficiency, and to produce a more ideal rectangular signal constellation at the receiver thus achieve a good $P(e)$ performance. Such a signal mapping might be different depending on the nonlinear characteristics and also the operating point (e.g., OBO) of the HPAs. In general, the outer signal states will be more expanded in amplitude, and have a larger phase pre-rotation as compared to the inner signal states.

A block diagram of a 16-QAM transmitter including a signal mapping mechanism is shown in Fig.2.4. The mapping is accomplished within Digital Signal Processor (DSP), which generates the desired in-phase (I) and quadrature-phase (Q) amplitudes of 16 signals on QAM constellation. The output data of DSP are stored in Random Access Memory (RAM) then converted to analog voltages via Digital-to-Analog (D/A) converters to be quadrature modulated. The feedback path in Fig.2.4 gives adaptive features to the system which may update the nonlinear distortions, thus compensate for any drifts in HPA nonlinearities. To update DSP data, HPA output is sampled by a directional coupler and demodulated then converted to digi-
Figure 2.3: Demodulated and filtered ($\alpha = 0.4$) (a) State-space diagram, and (b) Eye diagram of 16-QAM operating at 7 dB IBO (2.2 dB OBO) of Hughes TWTA.
Figure 2.4: Functional block diagram for the generation of the suboptimally mapped 16-QAM signals in a nonlinear channel.
signal via Analog-to-Digital (A/D) converter. DSP compares Serial-to-
Parallel (S/P) converter output data with A/D feedback data, and keeps
on processing to match both data. The demodulator at the transmitter
side is hard-wired (or with some adjustments for phase shift or time delay)
to the modulator for the carrier and symbol timing recovery. For simpler
fixed type of signal mapping, the DSP/RAM including feedback path may
be replaced by a nonlinear look-up table. However, this approach might
not be suitable, particularly if the nonlinear HPA has drift problems.

The signals in the block diagram are processed as follows. Assuming
\{a_k\} and \{b_k\} are the mapped and D/A converted signal states of I-channel
and Q-channel respectively, the filtered signals after premodulation low pass
filter (LPF) having an impulse response \( h_T(t) \) are expressed as

In the I-channel:
\[
a(t) = \sum_k a_k p(t - kT_s) * h_T(t)
\]  
(2.1)

In the Q-channel:
\[
b(t) = \sum_k b_k p(t - kT_s) * h_T(t)
\]  
(2.2)

where,
\[
p(t) = \begin{cases} 
1, & 0 \leq t \leq T_s \\
0, & \text{otherwise} 
\end{cases}
\]  
(2.3)

and \( T_s \) is a data symbol duration, and \( * \) denotes a convolution operation.

The modulated 16-QAM signal transmitted through the nonlinear HPA
is
\[
s(t) = r(t) g[r(t)] \cos(\omega_c t + \theta(t) + f[r(t)])
\]  
(2.4)

where,
\[
r(t) = \sqrt{a^2(t) + b^2(t)}
\]
g[] : AM/AM conversion function of HPA
f[] : AM/PM conversion function of HPA
\( \omega_c \) : Angular frequency of carrier
\( \theta(t) = \tan^{-1}[b(t)/a(t)] \)
After demodulation and postdetection LPF having an impulse response $h_R(t)$, the I-ch and Q-ch signals are expressed as:

For the I-channel:

$$x(t) = r(t)g[r(t)]\cos\{\theta(t) + f[r(t)]\} * h_R(t)$$
$$= g[r(t)]\{a(t)\cos\{f[r(t)]\} - b(t)\sin\{f[r(t)]\}\} * h_R(t) \quad (2.5)$$

For the Q-channel:

$$y(t) = r(t)g[r(t)]\sin\{\theta(t) + f[r(t)]\} * h_R(t)$$
$$= g[r(t)]\{b(t)\cos\{f[r(t)]\} + a(t)\sin\{f[r(t)]\}\} * h_R(t) \quad (2.6)$$

In Eqs. (2.5) and (2.6), $b(t)\sin\{f[r(t)]\}$ and $a(t)\sin\{f[r(t)]\}$ terms represent the cross-talks introduced by the AM/PM conversion nonlinearity of HPA.

Denote the I-ch and Q-ch outputs of the threshold detector at the sampling instants $mT$, by $\{x_m\}$ and $\{y_m\}$, and respective outputs of A/D converter by $X$ and $Y$. DSP compares each digit of $X$ and $Y$ to those of S/P outputs $U$ and $V$ respectively, and processes to map $A$ and $B$ such that $U \oplus X$ and $V \oplus Y$ are minimized, where $\oplus$ represents modulo-2 summation. In our computer simulation, the mapping is done in iterative manners.

### 2.3 PERFORMANCE OF 16-QAM OPERATING IN NONLINEAR MODE OF HPAs

Theoretical analysis of complex nonlinear systems are not feasible in many situations. For this reason, we analyze the performance of 16-QAM systems in a nonlinearly amplified channel with the aid of computer simulation. The simulation model is the same as the one depicted in Fig.2.1. In the receiver, a new adaptive decision threshold (ADT) detector is applied to demonstrate an improved performance as compared to a conventional fixed decision threshold (FDT) detector. The conventional decision thresholds for 16-QAM signals having normalized amplitudes $\{\pm 1, \pm 3\}$ are set at $\{0, \pm 2\}$. Although these thresholds are optimum in AWGN linear channel, they may not be optimum in nonlinear channels where the outer signal
amplitudes are subject to more AM/AM compression nonlinearity of HPAs than the inner ones. Therefore, rather than a conventional FDT detector, an ADT detector may be preferable for a better $P(e)$ performance. The ADT detector sets the decision threshold levels at the midpoint of the demodulated eye openings. Detailed block diagram and operation of ADT detector is described in [8].

2.3.1 With Hughes TWTA

The $P(e)$ performance of 16-QAM operating at 11 dB, 9 dB, and 7 dB IBO of Hughes TWTA is shown in Fig.2.5. ADT detectors are applied in the receiver to ensure the optimum threshold levels ($THD_{opt}$), which are $\{0, \pm 1.6\}$, $\{0, \pm 1.5\}$, and $\{0, \pm 1.4\}$ for 11 dB, 9 dB, and 7 dB IBO, respectively. We note that with a conventional FDT detector, the system fails to detect signals operating at less than 11 dB IBO. From the eye diagram in Fig.2.3(b), an excessive ISI is noticed at the sampling instants due to the envelope fluctuations and cross-talks caused by AM/AM and AM/PM nonlinearities of HPA. The corresponding CNR degradations at $P(e) = 10^{-6}$ are 4.0 dB, 6.7 dB, and 14.1 dB respectively.

2.3.2 With Siemens Linearized #189 TWTA

Demodulated, and low pass filtered ($\alpha = 0.4$) state-space diagrams of conventional and suboptimally mapped 16-QAM signals, operating at 1 dB IBO of Siemens TWTA, are shown in Fig.2.6(a) and (b), respectively. From Fig.2.6(a), it is observed that the amplitudes of the outermost 4 signal states (i.e., $\{\pm 3, \pm 3\}$) are compressed and their phases are rotated clockwise. For the other signal states, the amplitude compression or phase rotation is less significant. Accounting for such nonlinear distortions, DSP maps a suboptimum signal constellation as shown in Fig.2.7, where the amplitude of the outermost 4 signal states are expanded by 25 % and their phases are pre-rotated counter-clockwise. The phases of the other 12 signal states are slightly rotated counter-clockwise, but their envelopes are not changed. Note that the minimum signal distance $S_2$ in Fig.2.6(a) is increased in Fig.2.6(b). Fig.2.8(a) and (b) show demodulated, filtered eye diagrams of the conventional and suboptimally mapped 16-QAM signals. Note that in
16-QAM

Note:
1. Tx. LPF; \( \frac{X}{\sin X} \sqrt{RC} \) (\( \alpha = 0.4 \))
2. Rx. LPF; \( \sqrt{RC} \) (\( \alpha = 0.4 \))
   * \( \sqrt{RC} \); Square-root Raised-cosine

Figure 2.5: \( P(e) \) performance of 16-QAM operating with Hughes TWTA. ADT (Adaptive Decision Threshold) detectors are applied in the receiver.
Figure 2.6: Demodulated and filtered state-space diagrams of (a) Conventional 16-QAM, and (b) Suboptimally mapped 16-QAM operating at 1 dB IBO (or 0.9 dB OBO) of Siemens linearized #189 TWTA.
16-QAM STATE-SPACE DIAGRAM

Figure 2.7: Signal constellation of 16-QAM suboptimally mapped for the operation of Siemens #189 TWTA at 1 dB IBO (0.9 dB OBO).
Figure 2.8: Demodulated and filtered eye diagrams of (a) Conventional 16-QAM, and (b) Suboptimally mapped 16-QAM operating at 1 dB IBO (0.9 dB OBO) of Siemens #189 TWTA.
Fig. 2.8(b) the ISI is reduced thus the eye opening is widened. \( P(e) \) performance of these two systems are shown in Fig. 2.9. The CNR degradation at \( P(e) = 10^{-6} \) for the conventional 16-QAM with ADT (\( THD_{opt} = \{0, \pm 1.7\} \)) detectors is 8.9 dB, whereas without ADT detectors the system fails. For the suboptimally mapped 16-QAM with ADT (\( THD_{opt} = \{0, \pm 1.9\} \)) detectors, the degradation is only 1.9 dB, that is, 7 dB (or more) improvement in CNR is achieved.

The \( P(e) \) performance of the conventional and suboptimally mapped 16-QAM systems operating at 3 dB IBO of Siemens #189 TWTA is shown in Fig. 2.10. It reveals that the CNR degradation at \( P(e) = 10^{-6} \) is 3.3 dB and 6.5 dB for the conventional 16-QAM with and without ADT (\( THD_{opt} = \{0, \pm 1.8\} \)) detectors, respectively. For the suboptimally mapped 16-QAM with ADT (\( THD_{opt} = \{0, \pm 2.0\} \)) detectors, the degradation is only 0.5 dB, that is, 6 dB improvement in CNR is achieved.

### 2.3.3 With Fujitsu Linearized GaAs FET Amplifier

Demodulated, filtered state-space diagrams of conventional and suboptimally mapped 16-QAM signals, operating at 1 dB IBO of Fujitsu GaAs FET, are shown in Fig. 2.11(a) and (b), respectively. It is noticed that the amplitudes of the outermost 4 signal states are compressed, whereas the phase rotation is almost negligible. The constellation of a suboptimally mapped 16-QAM signal is shown in Fig. 2.12, where the amplitudes of the outermost signal states are expanded by 40%. Fig. 2.13(a) and (b) show demodulated, filtered eye diagrams of the conventional and suboptimally mapped 16-QAM signals. \( P(e) \) performance of the corresponding systems are shown in Fig. 2.14. Note that the CNR degradation at \( P(e) = 10^{-6} \) is 5.4 dB and 2.9 dB for the conventional 16-QAM with ADT (\( THD_{opt} = \{0, \pm 1.6\} \)) detectors and the suboptimally mapped 16-QAM with ADT (\( THD_{opt} = \{0, \pm 1.9\} \)) detectors respectively. For the conventional 16-QAM without ADT detectors, an error floor occurs between \( 10^{-3} \) and \( 10^{-4} \).

The \( P(e) \) performance of the conventional and suboptimally mapped 16-QAM systems operating at 3 dB IBO of Fujitsu GaAs FET is shown in Fig. 2.15. Note that the CNR degradation at \( P(e) = 10^{-6} \) is 2.4 dB and
16-QAM

Note:
1. Tx. LPF; $\frac{X}{\sin X} \sqrt{RC}$ ( $\alpha=0.4$ )
2. Rx. LPF; $\sqrt{RC}$ ( $\alpha=0.4$ )
* $\sqrt{RC}$ ; Square-root Raised-cosine
** ADT ; Adaptive Decision Threshold

Figure 2.9: $P(e)$ performance of the conventional and suboptimally mapped 16-QAM operating at 1 dB IBO (0.9 dB OBO) of Siemens #189 TWTA.
16-QAM

Note:
1. Tx. LPF: \( \frac{x}{\sin x \sqrt{RC}} \) (\( \alpha = 0.4 \))
2. Rx. LPF: \( \sqrt{RC} \) (\( \alpha = 0.4 \))
* \( \sqrt{RC} \); Square-root Raised-cosine
** ADT; Adaptive Decision Threshold

Conventional 16-QAM
- Without ADT detector
- With ADT detector

Suboptimally mapped 16-QAM
- With ADT detector

6 dB improvement

Figure 2.10: \( P(e) \) performance of the conventional and suboptimally mapped 16-QAM operating at 3 dB IBO (2.84 dB OBO) of Siemens #189 TWTA.
Figure 2.11: Demodulated 
filtered state-space diagrams of (a) Conventional 16-QAM, and (b) Suboptimally mapped 16-QAM operating at 1 dB IBO (or 0.3 dB OBO) of Fujitsu linearized GaAs FET.
16-QAM STATE-SPACE DIAGRAM

Figure 2.12: Signal constellation of 16-QAM suboptimally mapped for the operation of Fujitsu GaAs FET at 1 dB IBO (0.3 dB OBO).
(a) Conventional 16-QAM

(b) Suboptimally mapped 16-QAM

Figure 2.13: Demodulated and filtered eye diagrams of (a) Conventional 16-QAM, and (b) Suboptimally mapped 16-QAM operating at 1 dB IBO (0.3 dB OBO) of Fujitsu GaAs FET.
Figure 2.14: $P(e)$ performance of the conventional and suboptimally mapped 16-QAM operating at 1 dB IBO (0.3 dB OBO) of Fujitsu GaAs FET.
16-QAM

Note:

1. Tx. LPF: \( \frac{X}{\sin X} \sqrt{RC} \) (\( \alpha=0.4 \))
2. Rx. LPF: \( \sqrt{RC} \) (\( \alpha=0.4 \))
   * \( \sqrt{RC} \); Square-root Raised-cosine
   ** ADT; Adaptive Decision Threshold

Conventional 16-QAM
   Without ADT detector
   With ADT detector

Suboptimally mapped 16-QAM
   With ADT detector

Ideal curve in Nyquist linear channel

2.3 dB improvement

Figure 2.15: \( P(e) \) performance of the conventional and suboptimally mapped 16-QAM operating at 3 dB IBO (1.5 dB OBO) of Fujitsu GaAs FET.
3.6 dB for the conventional 16-QAM with and without ADT ($THD_{opt} = \{0, \pm 1.9\}$) detectors, respectively. For the suboptimaly mapped 16-QAM with ADT ($THD_{opt} = \{0, \pm 2.0\}$) detectors, the degradation is 1.3 dB, that is, 2.3 dB improvement in CNR is achieved.

### 2.4 HARDWARE IMPLEMENTATION AND VERIFICATION

To confirm our concept of a signal mapping optimization in a nonlinearly amplified channel, we implemented an equivalent baseband 16-QAM signal mapper and a nonlinear amplifier simulator. A functional block diagram of this subsystem is depicted in Fig.2.16, and detailed circuit diagrams are included in Appendix A.

Input serial NRZ data (256 kb/s) is split into 4 parallel data (64 kBaud) streams via S/P converter. Programmable Read Only Memory (PROM) contains algorithms for a new signal mapping, and data to map an ideal constellation $\{U, V\}$ of 16-QAM. DSP reads in instructions and data of PROM, and also output data $\{X, Y\}$ of A/D converter.

DSP maps a new constellation based on the following algorithms.

$$A_{n+1} = A_n + C_1(U_n \oplus X_n) \tag{2.7}$$

$$B_{n+1} = B_n + C_2(V_n \oplus Y_n) \tag{2.8}$$

where $C_1$ and $C_2$ are scale factors.

Calculations of $U \oplus X$, $V \oplus Y$ and other arithmetic operations are performed in ALU (arithmetic logic unit) of DSP. Updated data $\{A, B\}$ of a new signal constellation is stored in RAM of DSP, where each point of the constellation occupies two memory locations (e.g., X- and Y- axis). The data $\{A, B\}$ in digital format is converted to analog signals (i.e., $\{\pm1, \pm3\}$) via D/A converter followed by current-to-voltage (C/V) converter.

A nonlinear device is inserted in the feedback loop of our test set-up to simulate AM/AM compression nonlinearity of HPA. An attenuator changes the input signal level, thus an operating point of the nonlinear device. Port address (PA) decoder decodes 3-bit address signal from DSP, and generates
Figure 2.16: Hardware implementation of an equivalent baseband 16-QAM signal mapper in a nonlinear channel.
chip select signals to activate latches. The latches are used as bidirectional
bus drivers.

Data is transferred over data bus to and from data RAM by two in-
dependent strobes of DSP, that is, data enable ($DEN$) and write enable
($WE$). Memory enable ($MEN$) allows instructions and data to be trans-
ferred between external ROM and on-chip RAM.

In the RAM of DSP, data $\{A, B\}$ is retained until an interrupt signal
is received to up-date the data. In our set-up, an up-date or conversion
speed is 31.25 $\mu$s (=1/32 kHz), that is, data $\{A, B\}$ can be up-dated for
every other symbols. This limitation is due to the conversion time of A/D
converter, which is 25 $\mu$s for ADC 80Z-12. DIP switches are provided to
enable or freeze the adaptation of the system.

Fig.2.17(a) shows an experimental measured state space diagram of an
equivalent baseband 16-QAM signal (i.e., analog version of $\{X, Y\}$) after a
nonlinear amplifier simulator. A nominal signal constellation of 16-QAM
(i.e., $\{U, V\}$) has been transmitted in this case. Note that amplitude com-
pressions on the outer signal states are much larger due to the AM/AM
nonlinearity of the simulator. Fig.2.17(b) shows the measured state space
diagram of an optimally mapped 16-QAM signal (i.e., analog version of
$\{A, B\}$) at the input of a nonlinear device. Note that the amplitudes of
outer states are expanded to compensate the AM/AM compression non-
linearity. Fig.2.17(c) shows the state space diagram of optimally mapped
16-QAM after a nonlinear amplifier simulator. Note that an ideal constel-
lation (i.e., $\{U, V\}$) of 16-QAM is recovered.

2.5 OPTIMIZATION OF HPA OPERATION

A useful parameter for assessing the overall system performance in a non-
linearly amplified channel is the Overall System Gain Reduction (OSGR),
which is the sum of HPA OBO and CNR degradations (e.g., at $P(e) = 10^{-6}$)
caused by nonlinear distortions of HPA. In this section, we try to find out
an optimum operating point of HPAs which could maximize the overall
system gain, or minimize the overall system gain reduction.

Tables 2.1–3 summarize the OSGR of conventional and suboptimally
Figure 2.17: Experimental measured state space diagram of an equivalent baseband 16-QAM signal. (a) Without mapper; after nonlinear amplifier simulator, (b) With mapper; prior to nonlinear amplifier simulator, (c) With mapper; after nonlinear amplifier simulator.
mapped 16-QAM systems, operating with three types of HPAs, with and without ADT detectors in the receiver. For Hughes TWTA, it is noticed that the operation at less than 11 dB IBO is possible only with the ADT detectors applied in the receiver. The OSGR, however, is still excessive due to the high nonlinearities of TWTA. For Siemens TWTA, the optimum operating point is at 1 dB IBO (or 0.9 dB OBO), where the OSGR at $P(e) = 10^{-6}$ is 2.8 dB. For Fujitsu GaAs FET, the optimum operating point is at 3 dB IBO (or 1.5 dB OBO), where the OSGR at $P(e) = 10^{-6}$ is 2.8 dB. These results imply that for the power efficient operations of HPAs, the suboptimally mapped 16-QAM in conjunction with ADT detectors is to be recommended.

<table>
<thead>
<tr>
<th>Transmitter</th>
<th>Conventional 16-QAM</th>
<th>Subopt. Mapped 16-QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiver</td>
<td>Without ADT*</td>
<td>With ADT</td>
</tr>
<tr>
<td>IBO (dB)</td>
<td>11.00</td>
<td>9.00</td>
</tr>
<tr>
<td>OBO (dB)</td>
<td>5.69</td>
<td>3.89</td>
</tr>
<tr>
<td>CNR Deg.† (dB)</td>
<td>➡</td>
<td>➡</td>
</tr>
<tr>
<td>OSGR* (dB)</td>
<td>➡</td>
<td>➡</td>
</tr>
</tbody>
</table>

Table 2.1: Overall system gain reduction of 16-QAM operating with Hughes TWTA. * Adaptive Decision Threshold detector. † CNR degradation at $P(e) = 10^{-6}$. * Overall System Gain Reduction defined as the sum of the HPA OBO and the CNR degradation at $P(e) = 10^{-6}$.

<table>
<thead>
<tr>
<th>Transmitter</th>
<th>Conventional 16-QAM</th>
<th>Subopt. Mapped 16-QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiver</td>
<td>Without ADT</td>
<td>With ADT</td>
</tr>
<tr>
<td>IBO (dB)</td>
<td>4.00</td>
<td>3.00</td>
</tr>
<tr>
<td>OBO (dB)</td>
<td>3.84</td>
<td>2.84</td>
</tr>
<tr>
<td>CNR Deg.(dB)</td>
<td>2.20</td>
<td>6.50</td>
</tr>
<tr>
<td>OSGR (dB)</td>
<td>6.04</td>
<td>9.34</td>
</tr>
</tbody>
</table>

Table 2.2: Overall system gain reduction of 16-QAM operating with Siemens Linearized #189 TWTA.
<table>
<thead>
<tr>
<th>Transmitter</th>
<th>Conventional 16-QAM</th>
<th>Subopt. Mapped 16-QAM</th>
</tr>
</thead>
<tbody>
<tr>
<td>Receiver</td>
<td>Without ADT</td>
<td>With ADT</td>
</tr>
<tr>
<td>IBO (dB)</td>
<td>5.00</td>
<td>3.00</td>
</tr>
<tr>
<td>OBO (dB)</td>
<td>3.50</td>
<td>1.50</td>
</tr>
<tr>
<td>CNR Deg.(dB)</td>
<td>1.10</td>
<td>3.60</td>
</tr>
<tr>
<td>OSGR(dB)</td>
<td>4.60</td>
<td>5.10</td>
</tr>
</tbody>
</table>

Table 2.3: Overall system gain reduction of 16-QAM operating with Fujitsu linearized GaAs FET.

2.6 CONCLUSION

A simple, robust baseband signal mapping technique has been proposed for the power efficient operation of M-ary (e.g., $M = 16$) QAM systems. Our suboptimally mapped 16-QAM compensates the system impairments caused by the AM/AM and AM/PM nonlinearities of the HPAs, thus enables us to operate the HPAs closer to the saturation region at the significantly reduced CNR degradation. Adaptive decision threshold (ADT) detectors have been also applied at the receiver to improve the system performance. An illustrative CNR degradation of our proposed system operating at 0.9 dB OBO of Siemens TWTA is 1.9 dB at $P(e) = 10^{-6}$. This corresponds to 7 dB (or more) CNR improvement as compared to the conventional 16-QAM system. Therefore, for the power efficient operation of the M-ary QAM systems, it is recommended to apply the baseband digital signal mapping technique in conjunction with ADT detectors.
Chapter 3

OPERATION OF BANDWIDTH COMPRESSIVE MULTI-ARY SQAM SYSTEMS AT THE SATURATION MODE OF HPAS

For the operation of $M$-ary QAM at the highest power efficiency, i.e., at the saturation mode of HPAs, several types of modulation techniques have been proposed [9–11]. However, to achieve a high spectral efficiency, these modems require additional pre/post-modulation spectral shaping filters. Such filters may either cause serious performance degradations in the non-linearly amplified channels, or not be practical to design in many applications.

It has been demonstrated that overlapping baseband pulses, where each pulse duration is wider than one symbol interval ($T_s$), has spectral advantages over non-overlapping pulses [12–15]. Smooth and correlated phase transitions are required to attain good spectral properties [16–19]. We introduced SQAM (Superposed QAM) technique which exhibits a good power
and spectral efficiency in linearly and nonlinearly amplified radio systems [20–22]. The SQAM signal is generated by superposing its double-interval (2T_s) pulses, and has a smooth and correlated phase transitions to produce a compact power spectrum.

In this chapter, exploiting the properties of our SQAM signals, we introduce new M-ary (e.g., M=16,64) Offset-QAM techniques which may operate through fully saturated HPAs while retaining a compact power spectrum and a good P(e) performance. To demonstrate the robustness of our system, we analyze the performance of non-linearly amplified (NLA) 16-SQAM system in a multichannel interference environment, and compare this to the performance of MAMSK system [11]. The MAMSK system has been developed at JPL with a similar objective as ours.

3.1 BANDWIDTH COMPRESSIVE
16-SQAM MODEMS

3.1.1 Generation of 16-SQAM Signals

16-SQAM modulator consists of the SQAM baseband signal processors and a conventional 16-Offset-QAM modulator structure. Fig.3.1 illustrates a block diagram of 16-SQAM modulator. Demodulator of 16-SQAM is the same as a conventional coherent 16-Offset-QAM demodulator.

16-SQAM signal in a linear channel is represented as:

\[ p(t) = \sum_k [a_k s(t - kT_s) \cos \omega_c t + b_k s(t - kT_s - T_s/2) \sin \omega_c t] \quad (3.1) \]

where,

\( \{a_k, b_k\} = \{\pm 1, \pm 3\}, \) independent and equiprobable.

\( T_s = \) Symbol duration (i.e., \( T_s = 4T_b \)).

\( \omega_c = \) Angular frequency of carrier.

\( s(t) \): Impulse response of SQAM baseband signal defined as [20]
s(t) : Impulse response of 16SQAM baseband signal processor

Figure 3.1: Block diagram of 16-SQAM modulator.
\[ s(t) = \frac{1}{2} \left( 1 + \cos \frac{\pi t}{T_s} \right) - \frac{1 - A}{2} \left( 1 - \cos \frac{2\pi t}{T_s} \right) \]  

where,

\[-T_s \leq t \leq T_s, \text{ and } A \text{ is an amplitude parameter of the SQAM signal to be selected to control the power spectrum and envelope fluctuations of the quadrature amplitude modulated signals.}\]

Examples of the impulse response \( s(t) \) are shown in Fig.3.2. Note that \( s(t) \) spans over a \( 2T_s \) duration, while the in-phase (I) or quadri-phase (Q) channel input signal \( \{a_k\} \) or \( \{b_k\} \) spans over a \( T_s \) duration. Therefore, the 16-SQAM output signal at the \( (k+1)T_s \) period is obtained by overlapping the double-interval pulses \( a_k s(t-kT_s) \) and \( a_{k+1} s(t-(k+1)T_s) \). Detailed generation of 2-level SQAM signals can be found in Refs.[20,22]. Equivalent baseband signal waveforms of 16-SQAM for \( A = 0.7, 0.8, 0.9, \) and \( 1.0 \) in a linear channel are illustrated in Fig.3.3.

### 3.1.2 Power Spectrum of 16-SQAM Signals

The normalized power spectral density (PSD) of the equivalent 16-SQAM baseband signals in a linear channel is deduced as [20]

\[
\frac{|S(f)|^2}{|S(0)|} = \frac{1}{A^2} \left( \frac{1}{1 - 4T_s^2 f^2} + \frac{A - 1}{1 - T_s^2 f^2} \right)^2 \left( \frac{\sin 2\pi f T_s}{2\pi f T_s} \right)^2
\]

where, \( T_s = 4T_b \).

A useful measure of the spectral compactness can be obtained from the fractional out-of-band power \( P_{O.B} \) defined as:

\[
P_{O.B} = 1 - \left[ \int_{-B}^{+B} S'(f) df / \int_{-\infty}^{+\infty} S'(f) df \right]
\]

where

\[
S'(f) = \frac{|S(f)|^2}{|S(0)|}
\]

and \( B \) is the channel bandwidth normalized to the data symbol rate (i.e., \( (f - f_c)/f_s \)).
Figure 3.2: Impulse response \( s(t) \) of 16-SQAM baseband signal processor. \( s(t) = (1 + \cos \pi t/T_s)/2 - (1 - A)(1 - \cos 2\pi t/T_s)/2 \), \((A:\) Amplitude parameter of 16-SQAM signal.)
Figure 3.3: 16-SQAM baseband signal waveforms. (a) $A = 1.0$, (b) $A = 0.9$, (c) $A = 0.8$, (d) $A = 0.7$. (Vert. : Amplitude, Hori. : Time ($T_s$).)
Fig. 3.4 shows the power spectrum of 16-SQAM for the different amplitude parameter \( A \) of the signal. Note that a decrease of \( A \) leads to a faster spectral roll-off at higher frequencies at the expense of a slightly wider main-lobe bandwidth. Depending on the particular system applications, desirable 16-SQAM signals may be selected based on the trade-off between the main-lobe occupancy and side-lobe roll-off. Fig. 3.5 compares the fractional out-of-band power of 16-SQAM to those of a conventional 16-QAM and a sinusoidal-shaped 16-state QAM - so called, MAMSK (*Multi-Amplitude MSK*) [11]. Note that 16-SQAM achieves the most compact power spectrum. In the case of \( A = 0.9 \), 99.9% of the signal power is contained inside the bandwidth of the data symbol rate \( f_s \).

### 3.1.3 Performance of 16-SQAM in AWGN Linear Channel

The \( P(e) \) performance of 16-SQAM modems in the Gaussian noise linear channel is analyzed for different values of the amplitude parameter \( A \) of 16-SQAM signals. Fig. 3.6 illustrates an analysis model of 16-SQAM system. Phase-equalized fourth-order Butterworth LPFs are used to bandlimit the receive noise power. The error probability of 16-SQAM is calculated by a quasi-analytical method of a computer simulation [23]. In the simulation, we assumed that the carrier and the symbol timing are ideally recovered at the demodulator. The demodulated signal is synchronized for the optimum sampling at the maximum eye openings of the data symbol. The threshold comparator enables us to read the signal distance \( d_n \) to the neighbouring decision threshold. The symbol error probability is calculated as:

\[
P_e = \frac{1}{2} \text{erfc} \left( \frac{d_n}{\sqrt{2}\sigma} \right) \tag{3.6}
\]

where, \( \sigma \) represents the rms voltage of the noise at the output of the receive LPF. In our simulation, the overall average symbol error probability \( P(e) \) of 16-SQAM is calculated by averaging the \( P_e \) for \( (2^{11} - 1) \) symbols. Fig. 3.7 shows the \( P(e) \) performance of 16-SQAM for the different amplitude parameter \( A \). As one of the optimum receiver LPF 3 dB bandwidth, we assume \( f_{3dB} = 1.07f_N \), where \( f_N \) is the Nyquist frequency. That is, the receive filter 3 dB bandwidth and the symbol duration product \( BT_s \) is
Figure 3.4: Power spectrum of 16-SQAM in a linear channel.
Figure 3.5: Fractional out-of-band power of 16-SQAM, MAMSK and 16-QAM signals in a linear channel.
Figure 3.6: Analysis model of 16-SQAM system in the Gaussian noise linear channel.
16-SQAM

Note: 4th order Butterworth LPFs ($BT_s=0.535$) used in the receiver.

![Graph showing probability of error vs. $EB/NO$ in dB for 16-SQAM with different values of $A$.]

Figure 3.7: $P(e)$ performance of 16-SQAM modem in the AWGN linear channel. ($A$: Amplitude parameter of 16-SQAM signal.)
assumed to be 0.535. The ideal curve represents a theoretical ideal performance of 16-QAM in the Nyquist linear channel defined as

\[ P(e) = \frac{3}{2} \text{erfc} \left( \frac{\sqrt{\text{CNR}}}{3\sqrt{2}} \right) \quad (3.7) \]

where \( \text{CNR} \) (or C/N) is an average carrier-to-noise power ratio of 16-QAM.

### 3.2 Non-Linearily Amplified (NLA)-16-SQAM MODEMS

In order to operate the 16-SQAM modems at the most power efficient saturation mode of HPAs, a new NLA-16-SQAM technique is introduced in this section [30]. Parallel-type modulation techniques [9–11] will be applied to generate NLA-16-SQAM signals.

#### 3.2.1 Power Efficient NLA-16-SQAM Signals and Modem

A block diagram of NLA-16-SQAM transmitter is shown in Fig.3.8. The input NRZ data enters 1-to-4 serial-to-parallel converter to be split into 4 parallel NRZ signals \( \{a_k\}, \{b_k\}, \{c_k\}, \) and \( \{d_k\} \). The quadri-phase input signals \( \{b_k\} \) and \( \{d_k\} \) are offset by half a symbol interval (i.e., \( T_s/2 \)) from the in-phase input signals \( \{a_k\} \) and \( \{c_k\} \). These four signals are then waveform-shaped and quadrature modulated through two parallel SQAM modulators.

The respective output signals of SQAM1 and SQAM2 are expressed as

\[ y_1(t) = \sum_k [a_k s(t - kT_s) \cos \omega_c t + b_k s(t - kT_s - \frac{T_s}{2}) \sin \omega_c t] \quad (3.8) \]

\[ y_2(t) = \sum_k [c_k s(t - kT_s) \cos \omega_c t + d_k s(t - kT_s - \frac{T_s}{2}) \sin \omega_c t] \quad (3.9) \]

Experimentally measured baseband waveshapes of the 2-level SQAM signals \( y_1(t) \) and \( y_2(t) \) are shown in Fig.3.9, where the illustrative amplitude
Figure 3.8: Block diagram of NLA-16-SQAM transmitter. 

d(p(t) = 2z_1(t) + z_2(t)).
Figure 3.9: Experimental measured baseband waveshapes of the 2-level SQAM signals $y_1(t)$ and $y_2(t)$. ($A = 0.8$, $f_0 = 128$ kb/s.)
parameter $A$ is 0.8 and the data bit rate $f_b$ is 128 kb/s. The hardlimiters remove the envelope fluctuations (e.g., 0.7 dB for $A = 0.7$, 1.3 dB for $A = 0.8$) of the SQAM signals $y_1(t)$ and $y_2(t)$, and produce constant envelope signals $z_1(t)$ and $z_2(t)$ which are expressed as:

$$z_1(t) = \frac{1}{\sqrt{a^2(t) + b^2(t)}}[a(t) \cos \omega_c t + b(t) \sin \omega_c t] \quad (3.10)$$

$$z_2(t) = \frac{1}{\sqrt{c^2(t) + d^2(t)}}[c(t) \cos \omega_c t + d(t) \sin \omega_c t] \quad (3.11)$$

where

$$a(t) = \sum_k a_k s(t - kT_s)$$

$$b(t) = \sum_k b_k s(t - kT_s - T_s/2)$$

$$c(t) = \sum_k c_k s(t - kT_s)$$

$$d(t) = \sum_k d_k s(t - kT_s - T_s/2)$$

Therefore, the hardlimited SQAM signals $z_1(t)$ and $z_2(t)$ suffer no degradation from the nonlinear amplifications of the HPA1 and HPA2. The output voltage level of the HPA1 is twice that of the HPA2, that is, the saturated output power of the HPA1 is 6 dB higher than that of the HPA2. The outputs of two HPAs are then vector-summed to generate the nonlinearily amplified (NLA) 16-SQAM signals.

The NLA-16-SQAM output signal $p(t)$ is expressed as:

$$p(t) = 2z_1(t) + z_2(t) \quad (3.12)$$

Equivalent baseband signal waveforms of NLA-16-SQAM for $A = 0.7, 0.8, 0.9$ and 1.0 in a hardlimited channel are illustrated in Fig.3.10. Fig.3.11(a) and (b) show the transmit state-space diagrams of NLA-16-SQAM signal for $A = 0.7$ and $A = 0.8$, respectively. The state space diagram of Weber's MAMS K is also shown in Fig.3.11(c).
Figure 3.10: Equivalent baseband NLA-16-SQAM signal waveforms. (a) $A = 1.0$, (b) $A = 0.9$, (c) $A = 0.8$, (d) $A = 0.7$. (Vert.: Amplitude, Hori.: Time ($T_s$).)
Figure 3.11: State-space diagrams of (a) NLA-16-SQAM \((A = 0.7)\), (b) NLA-16-SQAM \((A = 0.8)\), (c) MAMSK.
3.2.2 Power Spectrum of NLA-16-SQAM Signals

The normalized power spectral density (PSD) of the NLA-16-SQAM signal in a hardlimitted channel is calculated by using IMSL (International Mathematics and Statistics Library) routine of FFTPS (Fast Fourier Transform Estimates of Power Spectra), which uses the Parzen window for its power spectrum calculation. Fig.3.12 shows the out-of-band to total power ratios of NLA-16-SQAM, MAMSK and 16-QAM signals in a hardlimitted channel. Note that NLA-16-SQAM signals have steeper spectral roll-off and lower out-of-band energy. In the case of \( A = 0.8 \), 99.3% of the signal power is contained inside the bandwidth of the data symbol rate \( f_s \). Also note that the spectral regrowth of NLA-16-SQAM signal is not significant even after the nonlinear amplification. This is because in the SQAM signals, the phase transitions are smooth and correlated [20,21], and also the SQAM modem has an Offset mode of a quadrature modulation structure which reveals superior spectral properties to the Non-Offset mode structure in nonlinear channels. Fig.3.13 compares the fractional out-of-band power of a Non-Offset mode of NLA-16-SQAM \( (A = 0.8) \) to that of a conventional (i.e., Offset mode) NLA-16-SQAM \( (A = 0.8) \). Note that the spectral regrowth in the non-offset mode is very significant.

3.2.3 Performance of NLA-16-SQAM Modems

The performance of NLA-16-SQAM modems operating at the saturation mode of transmit HPAs are analyzed in this section. We analyze the effect of the baseband pulse shaping for different values of the amplitude parameter \( A \) of NLA-16-SQAM, and the receive filter 3 dB bandwidth and the symbol duration product \( BT \), on the \( P(e) \) performance. Our objective is to achieve a good \( P(e) \) performance by optimizing the waveshape of NLA-16-SQAM and the receive filter bandwidth.

3.2.3.1 Optimum Signal Waveshape of NLA-16-SQAM

The \( P(e) \) performance of NLA-16-SQAM modems for different values of \( A \) are shown in Fig.3.14. We found that NLA-16-SQAM for \( A = 0.7 \) shows the best performance (i.e., 0.75 dB degradation at \( P(e) = 10^{-6} \) from the
Figure 3.12: Out-of-band to total power ratios of NLA-16-SQAM, MAMSK and 16-QAM signals in a hardlimited channel.
Figure 3.13: Out-of-band to total power ratios of Offset and Non-Offset mode of NLA-16-SQAM ($A = 0.8$) in a hardlimited channel. Spectral regrowth in the Non-Offset mode is very significant.
Figure 3.14: $P(e)$ performance of NLA-16-SQAM modem operating at the saturation mode of the transmit HPAs. ($A$ is defined in Eq.(3.2).)
theoretical ideal performance of 16-QAM in the Nyquist linear channel), and more degradations with the increasing values of $A$. This is because for smaller values of $A$, the envelope fluctuation and thus the ISI due to the nonlinear amplification is less significant at the expense of a wider main-lobe bandwidth but lower sidelobes. The state-space diagrams in Fig.3.15 may clarify the effect of $A$ on the $P(e)$ performance. As one of suboptimum filters, 4th order Butterworth LPFs ($BT_s = 0.535$) are chosen for the NLA-16-SQAM receiver. The space diagram of a conventional 16-QAM, operating at 9 dB HPA IBO, is also shown in Fig.3.15(e). As a compromise, NLA-16-SQAM signals with $A = 0.8$ is to be considered for our further analysis.

For the comparison purpose, the $P(e)$ performance of MAMS using 4th order Butterworth LPFs in the receiver, and also the performance of a conventional 16-QAM operating at 9 dB HPA IBO are shown in Fig.3.14. The ideal curve represents the theoretical ideal performance of the 16-QAM modem in the Nyquist linear channel. Fig.3.16(a) shows the demodulated and filtered ($BT_s = 0.535$) eye diagram of NLA-16-SQAM ($A = 0.8$), while Fig.3.16(b) shows the eye diagram of a conventional 16-QAM operating at 9 dB HPA IBO. In Figs.3.14, 3.15, and 3.16, for the conventional 16-QAM, square-root raised-cosine ($\alpha = 0.4$) filters are used in the transmitter (with $X/sinX$ aperture equalizer) and in the receiver.

### 3.2.3.2 Optimum Bandwidth of the Receive Low Pass Filter

The effect of the receive LPF bandwidth on the performance of NLA-16-SQAM ($A = 0.8$) is analyzed. Our objective is to obtain the optimum filter bandwidth (or $BT_s$) by compromising the performance degradations caused by the thermal noise and the intersymbol interference. Fig.3.17 shows the $P(e)$ performance for different values of $BT_s$, where $B$ is the 3 dB bandwidth of the LPF, and $T_s$ is the data symbol duration. We noted that the filter bandwidth should not be too large because of the thermal noise, and should not be too small because of the ISI effect. We found that the near optimum $BT_s$ is 0.535 for NLA-16-SQAM ($A = 0.8$).
Figure 3.15: State-space diagrams (one quadrant, top right-hand side) of NLA-16-SQAM operating at the saturation mode of HPAs. The illustrative conventional 16-QAM is operating at 9 dB HPA IBO.
(a) NLA-16-SQAM \((A=0.8, B_{T_s}=0.53)\).

(b) Conventional 16-QAM.

Figure 3.16: Demodulated and low-pass filtered eye diagrams of (a) NLA-16-SQAM operating at the saturation mode of HPAs, and (b) Conventional 16-QAM operating at 9 dB HPA IBO.
Figure 3.17: $P(e)$ performance of NLA-16-SQAM ($A = 0.8$) modem for different values of the receiver LPF bandwidth and the symbol duration product ($BT_s$).
3.2.4 System Sensitivity of NLA-16-SQAM Modems

In the following sections, the system sensitivity of NLA-16-SQAM modems to the output power level variation, and to the propagation time difference (or equivalently, the static phase shift) of the transmit HPAs are analyzed in terms of $P(e)$ performance.

3.2.4.1 Sensitivity to Variations of HPA Output Power

NLA-16-SQAM transmitter requires two parallel HPAs which have a normal 6 dB power difference. A variation of the HPAs output power level from the normal level may result in anomalous signal levels or eye diagrams. As an illustrative example, Fig.3.18(a) shows the eye diagram of NLA-16-SQAM where the output voltage level of HPA1 is 90% of the normal level, that is, HPA1 suffers $-0.92$ dB power deviation from the normal level. In Fig.3.18(b), the output voltage level of HPA2 is 80% of the normal level, that is, HPA2 suffers $-1.94$ dB power deviation from the normal level.

We analyze the sensitivity of NLA-16-SQAM systems to the deviation of the HPA output power level. To filter the received NLA-16-SQAM ($A = 0.8$) signals, 4th order Butterworth LPFs ($BT_s = 0.535$) are used in the analysis. Fig.3.19(a) and (b) show the performance degradations, as compared to the performance of NLA-16-SQAM having normal output power levels of HPA1 and HPA2, at $P(e) = 10^{-4}$ vs. output power level deviations of the HPA1 and HPA2. In the figures, the solid curve represents the performance degradations when the decision threshold levels are fixed at the conventional levels (i.e., $\{0, \pm 2\}$). The dotted curve represents the degradations when the decision threshold levels are adjusted to the vertical center of the received eye openings. Note that the performance degradations are remarkably reduced after adjusting the threshold levels. Therefore, in the presence of an HPA output power variation, it is desirable to monitor the received eye diagrams and adjust the decision threshold levels to the midpoint of the eye openings. This can be done by using an Adaptive Decision Threshold (ADT) detector which adjusts the decision threshold levels in accordance with the HPA output power level. Detailed description of the ADT detector is given in Chapter 5. Automatic gain control (AGC)
(a) HPA1 suffers -0.92 dB power deviation from the normal level.

(b) HPA2 suffers -1.94 dB power deviation from the normal level.

Figure 3.18: Eye diagrams of NLA-16-SQAM (A = 0.8) modem having anomalous signal levels due to the variation of the HPA output power level. 4th order Butterworth LPFs (BT = 0.53) are used in the receiver.
on one of the HPAs may also reduce the power variation in such a way to
slave its output power to that of the other HPA.

3.2.4.2 Sensitivity to Propagation Time Difference of HPAs

In NLA-16-SQAM, the output signal envelopes of two hardlimiters are con-
stant, thus neither the AM/AM compression nor the AM/PM conversion
nonlinearity is introduced after passing the HPAs. However, if the signal
through the HPA1 acquire a different propagation time (or equivalently, a
static phase shift) from that through the HPA2, the resulting static an-
gular rotation of the signal states would distort the signal constellation
and degrade the $P(e)$ performance. We analyzed the effect of the static
phase shift (up to $\pm 8.0^\circ$) in the HPA1 and HPA2 on the performance of
the NLA-16-SQAM. Fig.3.20 shows the eye diagrams of the demodulated
NLA-16-SQAM signals distorted by the static phase shift of $\pm 4^\circ$ and $\pm 8^\circ$
in the HPA1 and in the HPA2, respectively. The corresponding $P(e)$ per-
formance of this system is shown in Figs.3.21 and 22, respectively. Note
that the system performance is more sensitive to the phase shift in the
HPA1 than in the HPA2. This is because the output signal level of the
HPA1 is twice that of the HPA2, thus the absolute amount of the angular
displacement is larger in the HPA1 than in the HPA2. Such a phase shift
or time delay could be compensated by using a variable phase shifter or a
delay line before or after the HPAs.

3.3 BANDWIDTH COMPRESSIVE
64-SQAM MODEMS

In this section, we study the performance of 64-SQAM and NLA-64-SQAM
systems which could improve the spectral efficiency by 1.5 times as com-
pared to 16-SQAM or NLA-16-SQAM systems.
Figure 3.19: Performance degradation of NLA-16-SQAM due to the HPA1 and HPA2 output power deviation.

(a) HPA1.

(b) HPA2.

Note:
- : With conventional fixed decision threshold detector
- : With adaptive decision threshold detector
Figure 3.20: Eye diagrams of demodulated and low-pass filtered NLA-16-SQAM ($A = 0.8$) signal distorted by the static phase shift in the HPA1 and HPA2. 4th order Butterworth LPFs ($BT_s = 0.53$) are used in the receiver.
Figure 3.21: \( P(e) \) performance of NLA-16-SQAM \( (A = 0.8) \) distorted by the static phase shift \( (\Delta \phi) \) in HPA1.
Figure 3.22: $P(e)$ performance of NLA-16-SQAM ($A = 0.8$) distorted by the static phase shift ($\Delta \phi$) in HPA2.
3.3.1 64-SQAM Signals

The block diagram of 64-SQAM modulator is similar to that of 16-SQAM as shown in Fig.3.1, where 2-to-4 level converters are to be replaced with 2-to-8 level converters. 64-SQAM signal in a linear channel is represented as Eq.(3.1), where \( a_k \), \( b_k \) = \{±1, ±3, ±5, ±7\} and \( T_s = 6T_b \).

The normalized PSD of the equivalent 64-SQAM baseband signals in a linear channel is expressed as Eq.(3.3), where \( T_s = 6T_b \). Therefore, the power spectrum and the fractional out-of-band power of 64-SQAM, in terms of the normalized frequency \((f - f_c)T_s\), would be the same as shown in Figs.3.4 and 3.5, respectively. However, the frequency scale normalized with respect to the data bit rate (i.e., \((f - f_c)T_b\) has to be changed to 1/6, 2/6, 3/6, and 4/6 instead of 1/4, 2/4, 3/4, and 4/4, respectively.

3.3.2 Performance of 64-SQAM in AWGN Linear Channel

The \( P(e) \) performance of 64-SQAM for different values of \( A \) in the Gaussian noise linear channel is shown in Fig.3.23. Phase-equalized 4th order Butterworth LPFs are used in the receiver. The optimum \( BT_s \) (i.e., LPF 3 dB bandwidth and the symbol duration product) is found to be 0.5, 0.53, 0.57, and 0.7 for \( A = 0.7 \), 0.8, 0.9, and 1.0, respectively.

The ideal curve represents a theoretical ideal performance of 64-QAM in the Nyquist linear channel defined as:

\[
P(e) = \frac{7}{4} \text{erfc} \left( \frac{\sqrt{\text{CNR}}}{7\sqrt{2}} \right)
\]  

(3.13)

where CNR is an average carrier-to-noise power ratio of 64-QAM.
Figure 3.23: $P(e)$ performance of 64-SQAM modem in the AWGN linear channel.

Note: 4th order Butterworth LPFs used in the receiver.

64-SQAM

- $A = 1.0$
- $A = 0.9$
- $A = 0.8$
- $A = 0.7$

Theoretical ideal curve of 64QAM
3.4 NON-LINEARLY AMPLIFIED (NLA)-64-SQAM MODEMS

3.4.1 NLA-64-SQAM Signals and Modem

A block diagram of NLA-64-SQAM modulator is illustrated in Fig.3.24. The input NRZ data is split into 6 parallel NRZ signals \( \{a_k\}, \{b_k\}, \{c_k\}, \{d_k\}, \{e_k\}, \text{ and } \{f_k\} \) via 1-to-6 S/P converter. The quadri-phase input signals \( \{b_k\}, \{d_k\}, \text{ and } \{f_k\} \) are offset by \( T_s/2 \) from the in-phase input signals \( \{a_k\}, \{c_k\}, \text{ and } \{e_k\} \), respectively. These six signals are then waveshaped and quadrature modulated through three parallel SQAM modulators. The output signals of SQAM1 and SQAM2 are expressed as Eqs.(3.8) and (3.9), respectively.

The output signal of SQAM3 is expressed as:

\[
y_3(t) = \sum_k [e_k s(t - kT_s) \cos \omega_c t + f_k s(t - kT_s - \frac{T_s}{2}) \sin \omega_c t]
\]  

(3.14)

The hardlimited constant envelope signal of \( y_3(t) \) is expressed as:

\[
z_3(t) = \frac{1}{\sqrt{e^2(t) + f^2(t)}} [e(t) \cos \omega_c t + f(t) \sin \omega_c t]
\]  

(3.15)

where

\[
e(t) = \sum_k e_k s(t - kT_s)
\]

\[
f(t) = \sum_k f_k s(t - kT_s - T_s/2)
\]

The output voltage level of HFA1 is twice that of HPA2, or four times that of HPA3; that is, the saturated output power of HFA1 is 6 dB higher than that of HPA2, or 12 dB higher than that of HPA3. Therefore, the NLA-64-SQAM output signal is expressed as:

\[
p(t) = 4z_1(t) + 2z_2(t) + z_3(t)
\]  

(3.16)

where \( z_1(t) \) and \( z_2(t) \) are defined as Eqs.(3.10) and (3.11).

An equivalent baseband signal waveform of NLA-64-SQAM for \( A = 0.7 \) in a hardlimited channel is illustrated in Fig.3.25.
Figure 3.24: Block diagram of NLA-64-SQAM transmitter.

\( p(t) = 4z_1(t) + 2z_2(t) + z_3(t) \)

** HPA1, HPA2 & HPA3 are operated in saturation mode.**
Figure 3.25: Equivalent baseband NLA-64-SQAM signal waveform ($\lambda = 0.7$).
The out-of-band to total power ratios of NLA-64-SQAM signals in a hardlimited channel, in terms of the normalized frequency \((f - f_c)T_s\), would be the same as shown in Fig. 3.12. However, the frequency scale normalized with respect to the data bit rate (i.e., \((f - f_c)T_s\)) has to be changed to 1/6, 2/6, 3/6, and 4/6 instead of 1/4, 2/4, 3/4, and 4/4, respectively.

3.4.2 Performance of NLA-64-SQAM Modems

The \(P(e)\) performance of NLA-64-SQAM modems operating at the saturation mode of HPAs is analyzed. We investigate the optimum waveshape of NLA-64-SQAM and the receiver filter (e.g., 4th order Butterworth LPF) 3 dB bandwidth which may result in a good \(P(e)\) performance. Fig. 3.26 shows that NLA-64-SQAM for \(A = 0.7\) results in the best performance, i.e., 0.75 dB CNR degradation at \(P(e) = 10^{-6}\) from the theoretical ideal performance of 64-QAM in the Nyquist linear channel. Fig. 3.27 shows \(P(e)\) performance of NLA-64-SQAM (\(A = 0.7\)) for different values of \(BT_s\) (i.e., receiver LPF 3 dB bandwidth and symbol duration product). An optimum \(BT_s\) is found to be 0.5, 0.53 and 0.57 for \(A = 0.7, 0.8\) and 0.9, respectively.

For comparison purpose, we also analyzed the performance of a conventional 64-QAM operating with Hughes TWTA and Siemens linearized TWTA. Fig. 3.28(a) shows the demodulated and low-pass filtered (\(BT_s = 0.5\)) state-space diagram of NLA-64-SQAM (\(A = 0.7\)) operating at the saturation mode of HPAs. The space diagrams of a conventional 64-QAM operating at 13 dB IBO (or 7.55 dB OBO) of Hughes TWTA and at 4 dB IBO (or 3.84 dB OBO) of Siemens linearized TWTA are shown in Fig. 3.28(b) and (c), respectively. Note that in Fig. 3.28(b) and (c) the AM/AM and AM/PM nonlinearity impairments are more significant on the outer signal states. The corresponding eye diagrams at the input of the receiver threshold detectors are shown in Fig. 3.29. For the conventional 64-QAM, we use aperture equalizers (X/sinX) followed by \(\alpha = 0.4\) square-root raised-cosine filters at the transmitter, and another \(\alpha = 0.4\) square-root raised-cosine filters at the receiver. This filtering strategy is known to be one of the suboptimum filters in a nonlinear channel. The \(P(e)\) performance of a conventional 64-QAM operating at 18 dB IBO (or 12.42 dB OBO) of Hughes TWTA, and at 5.5 dB IBO (or 5.33 dB OBO) of Siemens linearized TWTA are illustrated in Fig. 3.30. In comparison with the performance of NLA-64-
Figure 3.26: $P(e)$ performance of NLA-64-SQAM modem operating at the saturation mode of the transmit HPAs.
NLA-64-SQAM

Note: 4th order Butterworth LPFs are used in the receiver

$BT_s = 0.567$

$BT_s = 0.467$

$BT_s = 0.533$

$BT_s = 0.50$

Figure 3.27: $P(e)$ performance of NLA-64-SQAM ($A = 0.7$) for different values of the receiver filter $BT_s$. 

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Figure 3.28: State-space diagrams (one quadrant, top right-hand side) of (a) NLA-64-SQAM ($A = 0.7$) operating at the saturation mode of HPAs, (b) Conventional 64-QAM operating at 13 dB IBO (7.56 dB OBO) of Hughes TWTA, and (c) At 4 dB IBO (3.84 dB OBO) of Siemens linearized TWTA.
Figure 3.29: Demodulated and low-pass filtered eye diagrams of (a) NLA-64-SQAM ($A = 0.7$) operating at the saturation mode of HPAs, (b) Conventional 64-QAM operating at 13 dB IBO (7.56 dB OBO) of Hughes TWTA, and (c) At 4 dB IBO (3.84 dB OBO) of Siemens linearized TWTA.
SQAM (i.e., 0.75 dB CNR degradation at $P(e) = 10^{-6}$), a comparable performance of a conventional 64-QAM may be achieved by operating Hughes TWTA at 22 dB IBO (or 16.38 dB OBO), or Siemens TWTA at 6 dB IBO (or 5.33 dB OBO). These OBOs may correspond to the overall system gain of NLA-64-SQAM system over conventional 64-QAM systems.

3.5 PERFORMANCE OF NLA-16-SQAM IN MULTI-CHANNEL INTERFERENCE ENVIRONMENT

In a multichannel or multi-carrier system, such as FDMA (Frequency Division Multiple Access) and SCPC (Single Channel Per Carrier), the spectrum (or frequency utilization) efficiency of the system depends on the intersymbol interference (ISI), spectral spreading, and adjacent channel interference (ACI) effects caused by the nonlinear amplifiers and bandlimiting filters. Guard bands between the adjacent channels could minimize the ACI effects, but also reduce the frequency utilization efficiency. The required size of the guard band depends in part on the residual sidebands (or out-of-band power) of the transmitted signals. Consequently, signals having a compact power spectrum would be desirable to achieve a high spectral efficiency in the multicarrier system. For an efficient utilization of the allocated spectrum, frequency-reuse techniques by means of a dual polarization have been applied. Co-channel interference (CCI), however, is one of the major sources of the performance impairment in such systems.

In this section, we analyze the performance of NLA-16-SQAM systems in a nonlinerly amplified multichannel interference environment. The $P(e)$ performance is analyzed under various channel conditions, such as different channel spacings, flat fade depths on the main (desired) channel, and carrier-to-interference ratios. The signal waveshape of NLA-16-SQAM and the receive LPF bandwidth are to be optimized for a better performance. To demonstrate the spectral advantages of this system, the $P(e)$ performance is compared to that of MAMSK (Multi-Amplitude MSK) system [11].
Figure 3.30: $P(e)$ performance of NLA-64-SQAM operating at the saturation mode of HPAs, conventional 64-QAM operating at 18 dB IBO (12.42 dB OBO) of Hughes TWTA, and at 5.5 dB IBO (5.33 dB OBO) of Siemens linearized TWTA.
3.5.1 System Configuration

The performance of NLA-16-SQAM system in the presence of an AWGN and the multichannel interference (e.g., ACI and CCI) is evaluated with the aid of a computer simulation. In most practical applications, the interference caused by the immediate adjacent channels has the most significant impact on the performance degradation, whereas the impact of the more distant channels is not as harmful. Therefore, the simulation model of a multi-channel NLA-16-SQAM (or MAMSK) system may be represented as in Fig.3.31, where two (i.e., one upper and one lower) adjacent interfering channels are present. Fig.3.32 illustrates the corresponding frequency allocation, where two adjacent channels are assumed to be equally spaced. The main channel has a carrier frequency \( f_c \), and the adjacent channels have carrier frequencies \( f_{\pm 1} = f_c \pm \Delta F \), where \( \Delta F \) represents a channel spacing or a channel frequency separation.

Assumptions used in the computer simulations are as follows:

- As an illustrative data bit rate, \( f_s = 200 \text{ Mb/s} \) (or symbol rate \( f_s = 50 \text{ MBaud} \)) is used.

- Interfering channel signals have the same modulation format as the desired main channel signal.

- To avoid any coherence or synchronization between the main and the interfering channel signals, carrier phases and symbol timings of the interfering channel signals are randomized separately over \( \{0, 2\pi\} \) and \( \{-T_s/2, T_s/2\} \), respectively.

- The main and the interfering channel transmitters are operated in a fully saturated mode of HPAs.

In the computation of the average error probability of the multichannel system, each 16 values of the random carrier phases and symbol timings are taken for each interfering signal. Let \( P_e(\theta_u, \theta_l, \tau_u, \tau_l) \) be a marginal error probability assuming the carrier phases \( \theta_u, \theta_l \) and the symbol timings \( \tau_u, \tau_l \) of the interfering adjacent channels, where subscripts \( u \) and \( l \) represent the upper adjacent channel and the lower adjacent channel, respectively.
Figure 3.31: Simulation model of NLA-16-SQAM multi-channel system in the presence of two adjacent interfering channels.
Figure 3.32: Frequency allocation of NLA-16-SQAM multi-channel system.  
($f_c$: Main channel carrier frequency, $f_{\pm 1}$: Adjacent channel carrier frequency; $f_{\pm 1} = f_c \pm \Delta F$, $\Delta F$: Channel spacing.)
Then, the overall average error probability is obtained as;

\[ P(e) = \frac{1}{16} \sum_{k=1}^{16} P_e(\theta_{u_k}, \theta_{l_k}, \tau_{u_k}, \tau_{l_k}) \]  

(3.17)

### 3.5.2 Optimization of NLA-16-SQAM

**Signal Waveshape in ACI Environment**

The performance in the ACI environment is mostly governed by the channel spacings \( \Delta F \) and the out-of-band powers of the adjacent channel signals. In the NLA-16-SQAM, the signal waveshape could be optimized for the best \( P(e) \) performance by compromising the performance degradations caused by the envelope fluctuations (or ISI) and the spectral spreading due to the nonlinear amplification. Figs.3.33 and 34 show the \( P(e) \) performance of NLA-16-SQAM in the presence of AWGN and ACI for different values of the amplitude parameter \( A \). The main channel and two interfering adjacent channel signals are assumed to have the same powers (i.e., \textit{fade depth} \( FD = 0 \text{ dB} \)). In Figs.3.33 and 34, the illustrative channel spacings \( \Delta F \) are 1.8\( f_s \) and 1.6\( f_s \), respectively, where \( f_s \) is a data symbol rate. As one of suboptimum filters in the ACI environment, phase equalized 5th order Butterworth LPFs (\( f_{3dB} = 26 \text{ MHz} \)) are used in the receiver. Note that NLA-16-SQAM with \( A = 0.8 \) performs best, whereas in the AWGN single channel, \( A = 0.7 \) performed best. Also note that as the channel spacing becomes narrower, \( A = 0.9 \) performs better than \( A = 0.7 \). The out-of-band power of NLA-16-SQAM in Fig.3.12 may explain this tendency, that is, \( A = 0.7 \) signal contains less envelope fluctuations (i.e., less ISI) but higher out-of-band power than \( A = 0.9 \) signal. In the following evaluations of the system performance, NLA-16-SQAM signal of \( A = 0.8 \) will be considered.

### 3.5.3 Optimization of Receive Filter Bandwidth

Due to the thermal noise and/or ACI effects, a receive filter bandwidth cannot be too wide, and also it cannot be too narrow due to the ISI effects. An optimum filter bandwidth may exist which could minimize the performance degradations caused by the thermal noise, ISI and ACI effects.
NLA-16-SQAM

Note:

1. Data bit rate;
   $f_b = 200\text{Mb/s} \quad (f_s = 50\text{MBAud})$
2. Equal power channels
   (FD=0dB)
3. Rx filter;
   5th order Butterworth LPF
   ($f_{3dB} = 26\text{MHz}$)
4. Channel spacing;
   $\Delta F = 1.8f_s$

---

Figure 3.33: $P(e)$ performance of NLA-16-SQAM in the presence of AWGN and ACI. (Channel spacing; $\Delta F = 1.8f_s$). The amplitude parameter $A$ is defined in Eq.(3.2).
NLA-16-SQAM

Note:
1. Data bit rate;
   \( f_b = 200 \text{Mb/s} \) (\( f_s = 50 \text{MBaud} \))
2. Equal power channels
   (FD=0dB)
3. Rx filter;
   5th order Butterworth LPF
   (\( f_{3dB} = 26 \text{MHz} \))
4. Channel spacing;
   \( \Delta F = 1.6 f_s \)

Figure 3.34: \( P(e) \) performance of NLA-16-SQAM in the presence of AWGN
and ACI. (Channel spacing; \( \Delta F = 1.6 f_s \))
In this section, we optimize the bandwidth (or $BT_s$) of the receive Butterworth LPFs of NLA-16-SQAM ($A = 0.8$) system in the ACI environment, where $B$ (= $f_{3dB}$) is a 3 dB bandwidth of LPFs and $T_s$ is a data symbol duration. Figs.3.35 and 36 show the $P(e)$ performance for different values of the filter bandwidth, where we assumed that the main and the adjacent channels have equal power (i.e., $FD=0$ dB) and their channel spacings $\Delta F$ are $1.8f_s$ and $1.5f_s$, respectively. Note that an optimum filter bandwidth exists near $f_{3dB} = 26$ MHz (or $BT_s = 0.52$) and $f_{3dB} = 25$ MHz (or $BT_s = 0.5$), respectively. Also note that the optimum bandwidth increases with the increasing value of the channel spacing, and decreases with the increasing value of the fade depth $FD$ on the main channel signal.

3.5.4 Effect of Channel Spacing between Adjacent Channels

For this analysis, the adjacent interfering channel signals are assumed to have the same power as the main (desired) channel signal, and the receive filter bandwidth is optimized in any channel spacing. The $P(e)$ performance of NLA-16-SQAM in the ACI environment is shown in Fig.3.37 for different values of the channel spacing. In the figure, the theoretical ideal curve represents an ideal performance of 16-QAM in the Nyquist linear channel. For the comparison purpose, $P(e)$ performance of MAMSK is also shown in Fig.3.38. Note that NLA-16-SQAM, owing to its compact power spectrum, outperforms MAMSK, thus improves the spectrum (or frequency utilization) efficiency over MAMSK. Fig.3.39(a) and (b) show the eye diagrams of demodulated and filtered NLA-16-SQAM ($A = 0.8$) signal in the presence of ACI, where the illustrative channel spacings are $1.9f_s$ and $1.7f_s$, respectively. Corresponding eye diagrams of MAMSK are shown in Fig.3.39(c) and (d), respectively.

3.5.5 Effect of Flat Fade on the Main Channel Signal

In a terrestrial microwave radio link or in a satellite radio uplink, the transmission signal power is often suppressed due to the fading presented in the
NLA-16-SQAM

Note:
1. Data bit rate; \( f_b = 200 \text{Mb/s} \) (\( f_s = 50 \text{MBaud} \))
2. Equal power channels (FD=0dB)
3. Rx filter:
   5th order Butterworth LPF (\( f_{3dB} \))
4. Channel spacing:
   \( \Delta F = 1.8f_s \)

Figure 3.35: \( P(e) \) performance of NLA-16-SQAM (\( A = 0.8 \)) in the presence of AWGN and ACI for different values of the receive filter bandwidth (\( f_{3dB} \)).
( Channel spacing; \( \Delta F = 1.8f_s \) )
NLA-16-SQAM

Note:
1. Data bit rate: \( f_b = 200 \text{Mb/s} \) \( (f_s = 50 \text{MHz}) \)
2. Equal power channels \( \text{PD} = 0 \text{dB} \)
3. Rx filter:
   5th order Butterworth
   LPF \( (f_{3dB}) \)
4. Channel spacing:
   \( \Delta F = 1.5 f_s \)

\[ f_{3dB} = 25 \text{MHz} \quad (BT_s = 0.50) \]
\[ f_{3dB} = 26 \text{MHz} \quad (BT_s = 0.52) \]
\[ f_{3dB} = 24 \text{MHz} \quad (BT_s = 0.48) \]
\[ f_{3dB} = 27 \text{MHz} \quad (BT_s = 0.54) \]

Figure 3.36: \( P(e) \) performance of NLA-16-SQAM \( (A = 0.8) \) in the presence of AWGN and ACI for different values of the receive filter bandwidth \( (f_{3dB}) \).
( Channel spacing; \( \Delta F = 1.5 f_s \) )
NLA-16-SQAM

Note:

1. Data bit rate;
   \( f_b = 200 \text{Mb/s} \) (\( f_s = 50 \text{MBaud} \))

2. Equal power channels
   (FD = 0dB)

3. Rx filter;
   5th order Butterworth
   LPF

\( \Delta F = 2.0f_s \) (inner)
\( 1.9f_s \) (middle)
\( 1.8f_s \) (outer)

\( \Delta F = 1.7f_s \)
\( \Delta F = 1.6f_s \)
\( \Delta F = 1.5f_s \)

Figure 3.37: \( P(e) \) performance of NLA-16-SQAM (\( A = 0.8 \)) in the ACI environment for different values of the channel spacing (\( \Delta F \)).
MAMSK

Note:
1. Data bit rate; 
   $f_b=200$Mb/s ($f_s=50$MBaud)
2. Equal power channels 
   (FD=0dB)
3. Rx filter; 
   5th order Butterworth LPF

$\Delta F = 1.6f_s$
$\Delta F = 2.0f_s$
$\Delta F = 1.9f_s$
$\Delta F = 1.8f_s$
$\Delta F = 1.7f_s$

Figure 3.38: $P(e)$ performance of MAMSK in the ACI environment for different values of the channel spacing ($\Delta F$).
Figure 3.39: Demodulated and low-pass filtered (5th order Butterworth) eye diagrams of NLA-16-SQAM and MAMSK in the nonlinearly amplified ACI environment. ($\Delta F$: Channel spacing)
channel. As the worst channel condition, let us assume that only the desired main channel suffers a flat (i.e., wide-band) fade of a fade depth $FD$. We analyzed the system sensitivity of NLA-16-SQAM and MAMSK to the flat fade on the main channel signal in the ACI environment. Figs.3.40 and 41 show the $P(e)$ performance of NLA-16-SQAM ($A = 0.8$) for different values of $FD$, where the illustrative channel spacing $\Delta F$ is $1.9f_s$ and $1.7f_s$, respectively. The $P(e)$ performance of MAMSK having the channel spacing $\Delta F = 1.9f_s$ is shown in Fig.3.42, in which case the system cannot tolerate a fading depth of more than 6 dB. In Figs.3.40–42, C/N represents the available carrier-to-noise power ratio after the flat fade. We found that in the fading channel MAMSK system fails when the channel spacing is narrower than $1.7f_s$. This reveals that NLA-16-SQAM system would be much more tolerable to the flat fading than MAMSK system in the ACI environment.

### 3.5.6 Performance of NLA-16-SQAM in CCI Environment

For the efficient utilization of the allocated spectrum, a frequency-reuse technique has been applied. Co-channel interference (CCI), however, caused by the imperfect polarization of the system is one of the major sources of the performance impairment. We analyzed the impact of the CCI on the performance of NLA-16-SQAM and MAMSK systems. Fig.3.43 illustrates the simulation model of a cochannel (i.e., frequency reuse) system. The corresponding frequency allocation is shown in Fig.3.44.

Similarly as in the ACI case, the overall average error probability in the CCI environment is calculated as;

$$P(e) = \frac{1}{16} \sum_{k=1}^{16} P_e(\theta_k, \tau_k)$$  \hspace{1cm} (3.18)

where $\theta_k$ and $\tau_k$ are the carrier phase and symbol timing of the co-channel signal randomized over $\{0, 2\pi\}$ and $\{-T_s/2, T_s/2\}$, respectively.

The $P(e)$ performance of NLA-16-SQAM and MAMSK for different values of the main channel Carrier-to-cochannel Interference Ratio (CIR) is shown in Figs.3.45 and 46, respectively. The result shows that for a
NLA-16-SQAM

Note:
1. Data bit rate:
   \( f_b = 200 \text{Mb/s} \) (\( f_s = 50 \text{OMBaud} \))
2. Rx filter:
   5th order Butterworth LPF
3. Channel spacing:
   \( \Delta F = 1.9 f_s \)
4. FD = Fade depth of main channel signal

Figure 3.40: \( P(e) \) performance of NLA-16-SQAM (\( A = 0.8 \)) in the ACI environment for different values of the fade depth (FD). (Channel spacing; \( \Delta F = 1.9 f_s \))
Figure 3.41: \( P(e) \) performance of NLA-16-SQAM \( (A = 0.8) \) in the ACI environment for different values of the fade depth \( (FD) \). (Channel spacing; \( \Delta F = 1.7f_s \))
MAMSK

Note:
1. Data bit rate:
   \( f_b = 200 \text{Mb/s} \ (f_s = 50 \text{MBaud}) \)
2. Rx filter:
   5th order Butterworth LPF
3. Channel spacing:
   \( \Delta F = 1.9 f_s \)

Figure 3.42: \( P(e) \) performance of MAMSK in the ACI environment for different values of the fade depth (FD). (Channel spacing: \( \Delta F = 1.9 f_s \))
Figure 3.43: Simulation model of NLA-16-SQAM co-channel system.

Note: *
* Desired main channel
** Interfering cochannel
Figure 3.44: Frequency allocation of NLA-16-SQAM co-channel system.
reasonable system performance, it is required to maintain the CIR greater than 20 dB.

3.6 CONCLUSION

A new NLA-M-ary SQAM modem, operating at the most power efficient — saturation mode of the HPAs, has been introduced and analyzed. We optimized the waveshape of NLA-16/64-SQAM signals as well as the receiver LPF bandwidth to achieve the best P(e) performance. As compared to the conventional 16-QAM and MAMSK, NLA-16-SQAM reveals much lower out-of-band energy, and performs with only 0.75 dB CNR degradation at $P(e) = 10^{-6}$ using simple 4th order Butterworth LPFs at the receiver.

For the comparison purpose, the performance of a conventional 64-QAM, operating with Hughes TWTA and Siemens linearized TWTA, has been also studied. It has been found that the corresponding overall system gain of NLA-64-SQAM is 5.83 dB and 16.38 dB respectively.

To demonstrate the spectral advantages of the NLA-M-ary SQAM systems, we further investigated the performance of NLA-16-SQAM and MAMSK systems in a nonlinearly amplified multichannel interference (e.g., ACI and CCI) environment. Our results show that in a spectrally congested channel, especially in a fading channel, NLA-16-SQAM outperforms MAMSK, thus improves the efficiency of a frequency utilization.
Figure 3.45: $P(e)$ performance of NLA-16-SQAM ($A = 0.8$) in the CCI environment.
Figure 3.46: $P(\epsilon)$ performance of MAMSK in the CCI environment.
Chapter 4

IMPACT OF RESIDUAL AMPLITUDE FLUCTUATIONS ON THE PERFORMANCE OF MULTI-ARY QAM SYSTEMS

Spectrally efficient Multi-ary QAM systems, being a hybrid modulation of the amplitude and the phase, are very sensitive to various amplitude and phase distortions of the transmission channels. Fading is known to be one of the major sources of the amplitude distortions (or fluctuations) in the system. The highest rate of fade depth variations has been demonstrated to be about 100 dB/sec [33,34]. Such a fast fade rate may occur only in a small percentage of time. However, this small percentage of time might be a very critical parameter in the overall system reliability objective. A straight-forward extrapolation of this fast fade variation leads to the results in Table 4.1. This extrapolation is merely provided to highlight the possible cause of “fade chatter” or rapid change in the receive carrier level. At some subintervals, the fade might change slower or faster. Table 4.1 reveals that a 1 dB power level variation could occur in a time interval of 10 ms. If we assume that the signal strength recovers in an almost periodic pattern or part of a periodic pattern, then we could assume that the fundamental
frequency of this fade variation is 100 Hz in this case. Similarly, a fade chatter of 0.1 dB could occur at a rate of 1,000 Hz.

<table>
<thead>
<tr>
<th>Power Level Variation</th>
<th>Time Interval</th>
<th>Fundamental Frequency†</th>
</tr>
</thead>
<tbody>
<tr>
<td>100 dB</td>
<td>1 sec</td>
<td>1 Hz</td>
</tr>
<tr>
<td>10 dB</td>
<td>100 msec</td>
<td>10 Hz</td>
</tr>
<tr>
<td>1 dB</td>
<td>10 msec</td>
<td>100 Hz</td>
</tr>
<tr>
<td>0.1 dB</td>
<td>1 msec</td>
<td>1 kHz</td>
</tr>
<tr>
<td>0.01 dB</td>
<td>0.1 msec</td>
<td>10 kHz</td>
</tr>
</tbody>
</table>

Table 4.1: Possible variations of the received carrier power caused by a 100 dB/sec flat fade. † The minimum frequency of the power level variation when the variation is assumed to be sinusoidal.

A functional block diagram of AGC (Automatic Gain Control) subsystems compensating for a flat (i.e., wide-band) fade is shown in Fig.4.1. To track and minimize the rapid carrier level fluctuations caused by the fast fade, fast response AGC circuits are required. These circuits, however, require fairly wide-band detectors, which may not filter out various noise and interference present at the input of the detectors. Consequently, the resultant noisy control signal would cause an incidental, or undesired amplitude modulation in the AGC circuit, and thus a residual amplitude fluctuation is present on the received QAM data streams. Alternatively, if the bandwidth of the AGC detector is narrow, then such an incidental amplitude modulation due to the noisy control signal is negligible. However, in this case the fade caused fast amplitude fluctuations are not tracked, and again an undesired amplitude modulation (fluctuation) is present in the received QAM signal.

In this chapter, we analyze the impact of the residual amplitude fluctuations of the received carrier after AGC amplifiers on the performance of M-ary QAM systems. We derive theoretical average symbol error probabilities of 16- and 64-QAM, and also tight-bound symbol error probabilities of 256- and 1024-QAM systems in the presence of the residual amplitude fluctuations and AWGN. Experimental measurements as well as computer
Figure 4.1: Functional block diagram of AGC subsystems compensating a flat fading.
simulation results are reported to confirm our theoretical results.

4.1 ANALYSIS ON THE EFFECT OF RESIDUAL AMPLITUDE FLUCTUATIONS ON THE P(e) PERFORMANCE

An illustrative fade pattern and residual amplitude fluctuation after AGC amplifiers compensating for a flat fading of 100 dB/sec is shown in Fig.4.2. In this example, we assume that the received carrier level fluctuation caused by the flat fade is reduced by AGC amplifiers to about 0.6 dB, which is a residual amplitude fluctuation of the final AGC amplifiers output prior to the demodulator. As a first-order approximation, we assume that this residual fluctuation is sinusoidal. Fig.4.3 shows an illustrative simulated state-space diagram of 16-QAM perturbed by a residual amplitude fluctuation of 0.6 dB. In the state-space diagram, note that the effect of such a fluctuation is to agitate the signal envelopes along the axis connecting the origin \( \{0, 0\} \) and the transmitted ideal in-phase and quadri-phase signal states \( \{X, Y\} \), where \( \{X\}, \{Y\} = \{\pm 1, \pm 3\} \).

4.1.1 System Model

Referring to Figs.4.3 and 4.5, let us denote the equivalent baseband ideal signal points on the \( M \)-ary QAM constellation by the complex number

\[
X_k + jY_k = A_k e^{j\theta_k}
\]  

(4.1)

where \( k = 1, 2, 3, \ldots M \), and \( \{X_k\}, \{Y_k\} = \{\pm 1, \pm 3, \ldots, \pm(\sqrt{M} - 1)\} \), and \( A_k \) and \( \theta_k \) is the nominal envelope and phase of the \( M \)-ary QAM signal, respectively.

As an analytically tractable model of a residual amplitude fluctuation, let us characterize the fluctuation by a relatively slowly varying (i.e., as compared to the data bit rate \( f_b \)) sinusoidal waveform which has a normal-
Figure 4.2: Illustrative residual amplitude fluctuations after AGC amplifiers compensating for a flat fade of 100 dB/sec.
Figure 4.3: State-space diagram of 16-QAM signal perturbed by the residual amplitude fluctuation of 0.6 dB.
ized (i.e., in terms of a signal envelope \( A_k \)) amplitude \( u \) and a frequency \( f_u \).

In the presence of the residual amplitude fluctuation and an AWGN, the received signal is represented as

\[
r(t) = A_k[1 + u \sin(2\pi f_u t + \phi)]e^{j\theta_k} + n(t) \tag{4.2}
\]

where, \( \phi \) is a relative initial phase of the amplitude fluctuation uniformly distributed in \([0, 2\pi]\), and \( n(t) \) is an equivalent baseband thermal noise.

In Eq.(4.2), \( u = 0 \) implies that all the flat fading caused amplitude fluctuation is removed after AGC amplifier, while \( u = 1 \) implies that 100\% of the residual amplitude fluctuation is present at the input of the demodulator. This \( u \) can be also regarded as an AM index of the incidental amplitude modulation. Therefore, an analysis model of this system can be represented as Fig.4.4. The transmit and receive filters are assumed to be ideal Nyquist filters, that is, these filters do not introduce intersymbol interference at the optimum sampling instants.

### 4.1.2 Derivation of Symbol Error Probability of 16-QAM

An illustrative state-space diagram (one quadrant; top right-hand side) of 16-QAM signal perturbed by the residual amplitude fluctuation of \( uA_k \sin(2\pi f_u t + \phi) \) is shown in Fig.4.5, where \( d_{\min} \) represents the minimum signal distance between two neighbouring ideal case (i.e., no amplitude fluctuation) signal states. Due to a symmetry of QAM signal constellation, we do not need to investigate performance of the other quadrants. Since we are interested in the overall average symbol error probability, the value of \( f_u \) is immaterial as long as the relative initial phase of the fluctuation \( \phi \) is varied in \([0, 2\pi]\) or \([−\pi/2, \pi/2]\).

Decomposing the sinusoidal amplitude fluctuation into the in-phase (I) component fluctuation \( uA_k \cos \theta_k \sin \phi \) and the quadri-phase (Q) component fluctuation \( uA_k \sin \theta_k \sin \phi \), the probability density function (PDF) of the
Residual Amplitude Fluctuation
1 + u \sin(2\pi f_u t + \phi)

M-ary QAM Transmitter

Data bit rate: f_b

M-ary QAM Receiver

AWGN

Figure 4.4: Analysis model of M-ary QAM system in the presence of residual amplitude fluctuations and AWGN. (u : Normalized amplitude, \phi : Relative initial phase, f_u : Frequency of the fluctuation, f_u \ll f_b.)
Figure 4.5: One quadrant (top right-hand side) of a state-space diagram of 16-QAM signal perturbed by the residual amplitude fluctuation. (+: Locations of the ideal signal states.)
in-phase component fluctuation \((a_k \sin \phi)\) is given by:

\[
p_k(x) = \begin{cases} 
\frac{1}{\pi \sqrt{a_k^2 - x^2}} & |x| \leq a_k \\
0 & |x| > a_k 
\end{cases}
\]  
(4.3)

where \(a_k\) is a peak amplitude of the \(k\)-th in-phase component fluctuation defined as:

\[
a_k = u A_k \cos \theta_k, \quad k = 1, 2, 3, 4.
\]  
(4.4)

The PDF of a Gaussian thermal noise is given by:

\[
p(y) = \frac{1}{\sqrt{2\pi \sigma_G^2}} \exp\left(-\frac{y^2}{2\sigma_G^2}\right)
\]  
(4.5)

The total noise interference at the receiver input is the sum of the residual amplitude fluctuation and the Gaussian thermal noise. Since these two impairments are independent in nature, the PDF of their sum equals a convolution of their respective PDFs. Therefore, the PDF of an in-phase component of the total noise interference is obtained as:

\[
p_k(n) = \int_{-\infty}^{+\infty} p_k(z)p(n-z)dz
\]

\[
= \int_{-a_k}^{a_k} \frac{1}{\pi \sqrt{a_k^2 - z^2}} \frac{1}{\sqrt{2\pi \sigma_G^2}} \exp\left[-\frac{(n-z)^2}{2\sigma_G^2}\right]dz
\]  
(4.6)

By replacing \(z\) with \(a_k \sin \phi\), we obtain

\[
p_k(n) = \frac{1}{\pi \sqrt{2\pi \sigma_G^2}} \int_{-\pi/2}^{\pi/2} \exp\left[-\frac{(n-a_k \sin \phi)^2}{2\sigma_G^2}\right]d\phi
\]  
(4.7)

Similarly, the PDF of a quadrature-phase component of the combined noise interference is obtained as:

\[
p_k(q) = \frac{1}{\pi \sqrt{2\pi \sigma_G^2}} \int_{-\pi/2}^{\pi/2} \exp\left[-\frac{(q-b_k \sin \phi)^2}{2\sigma_G^2}\right]d\phi
\]  
(4.8)

where \(b_k\) is a peak amplitude of the \(k\)-th quadrature-phase component fluctuation defined as:

\[
b_k = u A_k \sin \theta_k, \quad k = 1, 2, 3, 4.
\]  
(4.9)
Error occurs when the combined noise interference exceeds $d_{\text{min}}/2$. Therefore, the symbol error probability of the innermost signal states (i.e., $A_3$ in Fig.4.5) in the presence of a residual amplitude fluctuation and AWGN is obtained as:

$$P_{e_k} = 2\int_{d_{\text{min}}/2}^{+\infty} p_k(n)dn + 2\int_{d_{\text{min}}/2}^{+\infty} p_k(g)dg$$
$$= \frac{2}{\pi\sqrt{2\pi}\sigma_G^2} \int_{d_{\text{min}}/2}^{+\infty} \int_{-\pi/2}^{\pi/2} \{\exp[-\frac{(n-a_k\sin\phi)^2}{2\sigma_G^2}]d\phi dn$$
$$+ \exp[-\frac{(g-b_k\sin\phi)^2}{2\sigma_G^2}]d\phi dq\}$$
$$= \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} [\text{erfc}\left(\frac{d_{\text{min}}/2-a_k\sin\phi}{\sqrt{2}\sigma_G}\right)$$
$$+ \text{erfc}\left(\frac{d_{\text{min}}/2-b_k \sin \phi}{\sqrt{2}\sigma_G}\right)]d\phi$$

(4.10)

where $k = 1, 2, 3, 4$.

The signal mean power in terms of a signal peak amplitude $A_k$ is given by:

$$C_{\text{avg}} = \frac{1}{4} \sum_{k=1}^{4} (\frac{A_k}{\sqrt{2}})^2$$

(4.11)

Since $A_2 = A_4 = \sqrt{5}A_1/3$, and $A_3 = A_1/3$, the signal mean power in terms of the outermost signal amplitude $A_1$ is

$$C_{\text{avg}} = \frac{5}{18}A_1^2$$

(4.12)

Therefore, the mean carrier-to-noise power ratio (CNR) of 16-QAM is given by

$$\text{CNR} = \frac{C_{\text{avg}}}{\sigma_G^2}$$
$$= \frac{5}{18} \left(\frac{A_1}{\sigma_G}\right)^2$$

(4.13)

where the noise bandwidth is a double-sided Nyquist bandwidth.
From Eqs. (4.4), (4.9), (4.13), and replacing the signal minimum distance with $d_{\text{min}} = \sqrt{2}A_1/3$, Eq. (4.10) is rewritten as

$$P_{e_k} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \left[ \text{erfc} \left( \sqrt{\frac{\text{CNR}}{10}} - 3\sqrt{\frac{\text{CNR}}{5}} \rho_k u \cos \theta_k \sin \phi \right) \right. \\
+ \left. \text{erfc} \left( \sqrt{\frac{\text{CNR}}{10}} - 3\sqrt{\frac{\text{CNR}}{5}} \rho_k u \sin \theta_k \sin \phi \right) \right] d\phi$$  

(4.14)

where $\rho_k = A_k/A_1$, $k = 1, 2, 3, 4$. (e.g., $\rho_1 = 1$, $\rho_2 = \rho_4 = \sqrt{5}/3$, and $\rho_3 = 1/3$.)

In 16-QAM, the number of the closest neighbouring signal states which may cause primary errors depends on the particular signal states transmitted. That is, referring to Fig. 4.5, $A_1$ is subject to a 2-directional error, $A_2$ and $A_4$ are subject to a 3-directional error, and $A_3$ is subject to a 4-directional error.

Therefore, an overall average symbol error probability of 16-QAM is given by

$$P(e) = \frac{1}{16} \left( 4 \times \frac{2}{4} P_{e_1} + 4 \times \frac{3}{4} P_{e_2} + 4 \times P_{e_3} + 4 \times \frac{3}{4} P_{e_4} \right)$$  

(4.15)

Since $P_{e_2} = P_{e_4}$:

$$P(e) = \frac{1}{5} (P_{e_1} + 3P_{e_2} + 2P_{e_3})$$  

(4.16)

Fig. 4.6 illustrates theoretically calculated average symbol error probabilities of 16-QAM in the presence of the residual amplitude fluctuation and AWGN, where the normalized amplitude $u$ of the fluctuation varies in the range [0.0, 0.36] or equivalently, [0.0, 2.67 dB]. Computer simulated $P(e)$ performance of 16-QAM system is also shown in Fig. 4.6, where $u$ varies in [0.0, 0.2] or [0.0, 1.58 dB]. Note that the theoretical and simulation results agree well. Fig. 4.7 shows the simulated eye diagram of a demodulated and low pass filtered 16-QAM signal distorted by the residual amplitude fluctuation having the normalized amplitude $u$ of 0.2 (1.58 dB). Note that the ISI in the outer eyes are more significant than the ISI in the inner eye. In the simulation, ideal square-root of $\alpha = 0.2$ raised-cosine filters are used in the transmitter and receiver, and $X/\sin X$ aperture equalizers are used in the transmitter. We also performed a number of simulations with different roll-off factors ($\alpha$) of the filters. As we expected from our theoretical analysis, $\alpha$ did not change these results.
Figure 4.6: Theoretical average symbol error probabilities of 16-QAM in the presence of the residual amplitude fluctuations and AWGN. *Computer simulation* results are shown in *dotted curves*. *(u : Normalized amplitude of the residual amplitude fluctuation.)*
Figure 4.7: Eye diagram of a demodulated and low-pass filtered ($\alpha = 0.2$) 16-QAM signal distorted by the residual amplitude fluctuation having the normalized amplitude $u = 0.2$ (1.58 dB). The ISI in the outer eyes are more significant than the ISI in the inner eye.
4.1.3 Derivation of Symbol Error Probability of 64-QAM

One quadrant (top right-hand side) of a state-space diagram of 64-QAM signal is shown in Fig.4.8. Following the same procedures as in 16-QAM case, the symbol error probability of 64-QAM in the presence of a sinusoidal amplitude fluctuation and AWGN is derived as Eq.(4.10), where $k = 1, 2, 3, \ldots 16.$

![Diagram](image)

Figure 4.8: One quadrant (top right-hand side) of a state-space diagram of 64-QAM signal. (Ideal case; No residual amplitude fluctuation.)
The signal mean power in terms of a signal peak amplitude \( A_k \) is given by

\[
C_{\text{avg}} = \frac{1}{16} \sum_{k=1}^{16} \left( \frac{A_k}{\sqrt{2}} \right)^2
\]

(4.17)

Or, in terms of the outermost signal amplitude \( A_1 \),

\[
C_{\text{avg}} = \frac{3}{14} A_1^2
\]

(4.18)

Therefore, the mean carrier-to-noise power ratio (CNR) of 64-QAM is given by

\[
\text{CNR} = \frac{3}{14} \left( \frac{A_1}{\sigma_G} \right)^2
\]

(4.19)

The minimum signal distance is expressed as

\[
d_{\text{min}} = \frac{\sqrt{2}}{7} A_1
\]

(4.20)

From Eq.(4.4),(4.9),(4.19) and (4.20), Eq.(4.10) is rewritten as

\[
P_{e_k} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \text{erfc}\left( \sqrt{\frac{\text{CNR}}{42}} - \sqrt{\frac{7\text{CNR}}{3}} \rho_k \cos \theta_k \sin \phi \right)

+ \text{erfc}\left( \sqrt{\frac{\text{CNR}}{42}} - \sqrt{\frac{7\text{CNR}}{3}} \rho_k \sin \theta_k \sin \phi \right) d\phi
\]

(4.21)

where \( \rho_k = A_k/A_1 \), \( k = 1, 2, 3, \ldots 16 \).

In Fig.4.8, the signal states \( A_{11}, A_{12}, A_{13}, A_{14}, A_{15} \), and \( A_{16} \) have the same behaviours as the symmetrical counter-parts \( A_2, A_6, A_3, A_3, A_7 \), and \( A_4 \), respectively. Therefore, to calculate the overall average symbol error probability, we need to know \( \rho_1, \rho_2, \ldots \rho_{10}, \theta_1, \theta_2, \ldots \theta_{10} \), and \( P_{e_1}, P_{e_2}, \ldots , P_{e_{10}} \). Table 4.2 tabulates the values of \( \rho_k \) and \( \theta_k \) for \( k = 1, 2, 3, \ldots 10 \).

Since, \( A_1 \) is subject to a 2-directional error, \( A_2, A_3, A_4, A_{11}, A_{14} \), and \( A_{16} \) are subject to a 3-directional error, and the others are subject to a 4-directional error, the overall average symbol error probability of 64-QAM is given by;

\[
P(e) = \frac{1}{16} \left[ \frac{2}{4} \times P_{e_1} + 2 \times \frac{3}{4} \times (P_{e_2} + P_{e_3} + P_{e_4}) \right]
\]

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<table>
<thead>
<tr>
<th>$k$</th>
<th>$\rho_k = A_k/A_1$</th>
<th>$\theta_k$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>$\tan^{-1}(1)$</td>
</tr>
<tr>
<td>2</td>
<td>$\sqrt{37}/7$</td>
<td>$\tan^{-1}(5/7)$</td>
</tr>
<tr>
<td>3</td>
<td>$\sqrt{29}/7$</td>
<td>$\tan^{-1}(3/7)$</td>
</tr>
<tr>
<td>4</td>
<td>5/7</td>
<td>$\tan^{-1}(1/7)$</td>
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<tr>
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<td>3/7</td>
<td>$\tan^{-1}(1)$</td>
</tr>
<tr>
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<td>$\sqrt{5}/7$</td>
<td>$\tan^{-1}(1/3)$</td>
</tr>
<tr>
<td>10</td>
<td>1/7</td>
<td>$\tan^{-1}(1)$</td>
</tr>
</tbody>
</table>

Table 4.2: Normalized signal envelope ($\rho_k$) and phase ($\theta_k$) of 64-QAM. The symbols $A_1 - A_{10}$ are depicted in Fig.4.8.

\[
\begin{align*}
&\frac{1}{4} \times (P_{c_5} + P_{c_6} + P_{c_{10}}) + 2 \times \frac{4}{4} \times (P_{c_6} + P_{c_7} + P_{c_8}) \\
&= \frac{1}{32} [P_{c_1} + 3(P_{c_2} + P_{c_3} + P_{c_4}) \\
&\quad + 2(P_{c_5} + P_{c_6} + P_{c_{10}}) + 4(P_{c_8} + P_{c_7} + P_{c_9})]
\end{align*}
\] (4.22)

Fig.4.9 illustrates theoretically calculated average symbol error probabilities of 64-QAM in the presence of the residual amplitude fluctuation and AWGN, where the normalized amplitude $u$ of the fluctuation varies in the range $[0.0, 0.16]$ or equivalently, $[0.0, 1.29 \text{ dB}]$. Computer simulated $P(e)$ performance of 64-QAM system is also shown, where $u$ varies in $[0.0, 0.1]$ or $[0.0, 0.83 \text{ dB}]$. Note the close agreement between the theoretical and the simulation results. A simulated state-space diagram of 64-QAM signal perturbed by the residual amplitude fluctuation of $u = 0.04$ (or 0.34 dB) is shown in Fig.4.10. Fig.4.11 illustrates the eye diagram of a demodulated and low pass filtered 64-QAM signal for $u = 0.08$ (or 0.67 dB). The modem filtering strategies of 64-QAM are the same as in 16-QAM modem.
Figure 4.9: Theoretical average symbol error probabilities of 64-QAM in the presence of the residual amplitude fluctuations and AWGN. Computer simulation results are shown in dotted curves. ($u$: Normalized amplitude of the residual amplitude fluctuation.)
Figure 4.10: State-space diagram of 64-QAM signal perturbed by the residual amplitude fluctuation of \( u = 0.04 \) (0.34 dB).
Figure 4.11: Eye diagram of a demodulated and low-pass filtered ($\alpha = 0.2$) 64-QAM signal distorted by the residual amplitude fluctuation having the normalized amplitude $u = 0.08$ (0.67 dB).
4.1.4 Experimental Verification of Theoretical and Simulation Results on 16-QAM Modem

To confirm our theoretical and computer simulation results on the performance of 16-QAM system, we developed an experimental test bed, and measured eye diagrams, state space diagrams and $P(e)$ performance of 16-QAM modem perturbed by the residual amplitude fluctuation, or equivalently, an incidental amplitude modulation. Fig.4.12 illustrates the experimental set-up for this measurement. Our 16-QAM modem transmits a data rate of 256 kbps (or 64 kBaud). The modulated signal (with a modulator carrier frequency $f_c = 1$ MHz) is up-converted to 70 MHz by a frequency synthesizer (PTS 160) and a frequency up-converter, then down-converted to 1 MHz by a frequency down-converter followed by a signal generator (Marconi 2019). The signal generator also provides an incidental amplitude modulation at the receiver input. Assuming that the carrier signal is ideally recovered, the carrier recovery circuit of this modem is hard-wired. Three different values of $AM$ index, which has been defined as $u$ in the analysis, are selected in the signal generator, that is, $u = 0.1, 0.2$ and $0.3$. Fig.4.13(a), (b) and (c) show the measured eye diagrams, and Fig.4.14(a), (b) and (c) show the corresponding state-space diagrams of demodulated and filtered 16-QAM signals for $u = 0.1$, $0.2$ and $0.3$, respectively. Note that the outer signal states or the outer eye diagrams contain more ISI than the inner ones, that is, the outer ones are more sensitive to the amplitude fluctuations than the inner ones. This is because for a given value of $AM$ index (or $u$), the absolute amounts of the amplitude variation on the outer signal states are larger than that on the inner states. The measured $P(e)$ performance of 16-QAM is compared to our theoretical results in Fig.4.15. The $P(e)$ performance difference between the measured and the theoretical results are due to the ISI caused by the imperfect channel filters of our experimental modem.
Figure 4.12: Experimental set-up for the measurement of the performance of 16-QAM system in the presence of the residual amplitude fluctuation (or equivalently, incidental amplitude modulation) and AWGN.
Figure 4.13: Experimental measured eye diagrams of a demodulated and low-pass filtered 16-QAM signal perturbed by the residual amplitude fluctuations of (a) $u = 0.1$, (b) $u = 0.2$, (c) $u = 0.3$. 
Figure 4.14: Experimental measured state-space diagrams of a demodulated and low-pass filtered 16-QAM signal perturbed by the residual amplitude fluctuations of (a) $u = 0.1$, (b) $u = 0.2$, (c) $u = 0.3$. 

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Figure 4.15: Experimental measured $P(e)$ performance (dotted curves) of 16-QAM in the presence of an AWGN and the residual amplitude fluctuations of $u = 0.1, 0.2, \text{ and } 0.3$. Theoretical results are shown in solid curves.
4.1.5 The Best and the Worst Symbol Error Probabilities and the Maximum System Tolerance of 16- and 64-QAM

Our simulated and experimentally measured eye diagrams and state-space diagrams reveal that in the M-ary QAM the outer signal states are more susceptible to the residual amplitude fluctuations than the inner signal states. In this section, we analyze particular symbol error probabilities including the best and the worst symbol error probabilities of 16- and 64-QAM, and also the maximum tolerance of the system to the residual amplitude fluctuation. Figs.4.16 and 17 show the particular symbol error probabilities and overall average symbol error probabilities of 16-QAM in the presence of an AWGN and the residual amplitude fluctuations of \( u = 0.24 \) (1.87 dB) and \( u = 0.32 \) (2.41 dB), respectively. These theoretical results are calculated from Eqs.(4.14) and (4.16). Figs.4.18 and 19 show typical symbol error probabilities and overall average symbol error probabilities of 64-QAM for \( u = 0.1 \) (0.83 dB) and \( u = 0.14 \) (1.14 dB), respectively. The symbol error probabilities \( P_{e_1}, P_{e_2}, P_{e_3} \), and \( P_{e_{10}} \) correspond to those of illustrative symbols \( A_1, A_5, A_8, \) and \( A_{10} \) in Fig.4.8, respectively, and are calculated from Eq.(4.21). The results in Figs.4.16 – 19 show that the outermost symbol (i.e., \( A_1 \)) performs worst, whereas the innermost symbol (i.e., \( A_5 \) for 16-QAM, \( A_{10} \) for 64-QAM) performs best, and the overall error performance is mostly determined by the worst symbol error probability. A loose upper-bound (i.e., the worst) symbol error probability, thus, can be obtained from the outermost symbol error probability \( P_{e_1} \). Also note that the performance degradations (or the system sensitivities to the channel impairments) caused by the outermost symbols are much more distinct than those by the innermost symbols. Therefore, it may be deduced from the results that the tolerance of M-ary QAM system to the residual amplitude fluctuation is inversely proportional to the outermost signal levels (e.g., \(|\pm 3|\) and \(|\pm 7|\) for 16- and 64-QAM respectively) of the QAM. For example, the maximum tolerance (i.e., the maximum allowable amplitude fluctuation before the received signal is detected erroneously) in terms of the normalized amplitude \( u \) are \( 1/3 \) (or 2.5 dB) for 16-QAM, and \( 1/7 \) (or 1.16 dB) for 64-QAM. For the higher level QAM, the maximum tolerance would be more tight, that is, \( 1/15 \) (or 0.56 dB) for 256-QAM, and \( 1/31 \) (or 0.28 dB).
16-QAM

\[ u = 0.24 \text{ (1.87 dB)} \]

**Note**

\( \Delta : P_{e_1} \) (Outermost symbol error prob.)

\( + : P_{e_2} = P_{e_4} \)

\( \times : P_{e_3} \) (Innermost symbol error prob.)

\( \circ : P(e) \) (Average symbol error prob.)

Figure 4.16: Symbol error probabilities and an overall average symbol error probability \( P(e) \) of 16-QAM in the presence of AWGN and the residual amplitude fluctuation of \( u = 0.24 \) (1.87 dB). The symbol error probability \( P_{e_i} \) is calculated from Eq.(4.14). ( \( P_{e_1} \) : Error probability of the outermost symbol. \( P_{e_3} \) : Error probability of the innermost symbol.)
Figure 4.17: Symbol error probabilities and an overall average symbol error probability $P(e)$ of 16-QAM in the presence of AWGN and the residual amplitude fluctuation of $u = 0.32$ (2.41 dB). ($P_{e1}$: Error probability of the outermost symbol. $P_{e4}$: Error probability of the innermost symbol.)
Figure 4.18: Typical symbol error probabilities and an overall average symbol error probability $P(e)$ of 64-QAM in the presence of AWGN and the residual amplitude fluctuation of $u = 0.1$ (0.83 dB). The symbol error probability $P_{e\alpha}$ is calculated from Eq.(4.21). ($P_{e1}$: Error probability of the outermost symbol. $P_{e10}$: Error probability of the innermost symbol.)
Figure 4.19: Typical symbol error probabilities and an overall average symbol error probability $P(e)$ of 64-QAM in the presence of AWGN and the residual amplitude fluctuation of $u = 0.14$ (1.14 dB). ($P_{e1}$: Error probability of the outermost symbol. $P_{e10}$: Error probability of the innermost symbol.)
for 1024-QAM. Fig.4.20 illustrates the $P(e)$ performance degradation (i.e., the required increase in CNR of the system to maintain $P(e) = 10^{-6}$) as a function of a normalized amplitude $u$ of the fluctuation in 16- and 64-QAM systems. An error floor occurs, that is, the system performance could not be improved by increasing CNR, when the residual amplitude fluctuation exceeds the specified maximum tolerance.

To demonstrate the different amount of the performance degradations or ISI imposed on the inner and the outer signal states, we measured the vertical ISI (or eye closure) at the sampling instants of the demodulated 16-QAM signals. For this measurement, in addition to the experimental setup illustrated in Fig.4.12, a function generator triggered by a data symbol clock rate ($f_s = 64$ kBaud) is used to generate a blank signal to highlight the ISI effects at the sampling instant. Fig.4.21(a), (b) and (c) show the experimental measured vertical ISI (or eye closure) of the demodulated 16-QAM signals perturbed by the residual amplitude fluctuations of $u = 0.1$, $u = 0.2$ and $u = 0.3$, respectively. Fig.4.21(d) shows $u = 0.33$ (or $1/3$) case, where the ISI in the outer eyes touches the decision thresholds (i.e., $THD_1$ and $THD_3$), thus an error floor occurs in the system.

### 4.1.6 Performance Improvement by Using Adaptive Decision Threshold Detectors

As we have noticed from the theoretical results of the best/worst symbol error probabilities of 16- and 64-QAM, and the experimental results on the eye closures of 16-QAM system, the outer signal states of the M-ary QAM are more susceptible to the residual amplitude fluctuations than the inner ones, and the overall system performance is mostly controlled by the outer signal states. This means that at the receiver the minimum signal distance of the outer signal state may become smaller than that of the inner one, whereas in the AWGN linear channel the minimum signal distance is equal for any signal states. For this reason, we propose an adaptive decision threshold (ADT) detector for the M-ary QAM receiver to improve the $P(e)$ performance as compared to a conventional fixed decision threshold (FDT) detector. In the ADT detector, the decision threshold level is adjusted to the midpoint of the demodulated eye openings. A block diagram of a 16-QAM receiver exploiting the ADT detector is shown in Fig.4.22. This
* u : Normalized amplitude of the residual fluctuation.

Figure 4.20: $P(e)$ performance degradation (i.e., the required increase in CNR of the system to maintain $P(e) = 10^{-6}$) as a function of a normalized amplitude $u$ of the residual amplitude fluctuation in 16- and 64-QAM systems.
Figure 4.21: Experimental measured vertical ISI (eye closure) of demodulated and low-pass filtered 16-QAM signals at the sampling instants perturbed by the residual amplitude fluctuations of (a) \( u = 0.1 \), (b) \( u = 0.2 \), (c) \( u = 0.3 \), (d) \( u = 0.333 \ (1/3) \). Fig.4.21(d) illustrates the case of the maximum tolerance of 16-QAM, where the ISI in the outer eyes touches the decision thresholds \( THD_1 \) and \( THD_3 \), thus causes errors.
Figure 4.22: Block diagram of 16-QAM receiver exploiting the Adaptive Decision Threshold (ADT) detector.
detector may be applied to any signalling levels of M-ary QAM or QPRS receiver. In order to explain the conceptual operation of the ADT detector, an illustrative eye diagram of 16-QAM is shown in Fig.4.23. (See also Figs.4.7, 4.13, and 4.21.) Demodulated and low pass filtered 16-QAM base-

![Eye Diagram](image)

**Figure 4.23:** Illustrative typical eye diagram of 16-QAM signal distorted by the residual amplitude fluctuation.

band signals are converted into digital data via A/D converter. Up/down counter in A/D converter reads the discrete values of $M_1, M_2, \ldots, M_6$, and these data are stored in RAM. Algorithms to compute optimum threshold levels, i.e., $THD_{opt_1} = (M_1 + M_2)/2$, $THD_{opt_2} = (M_3 + M_4)/2$, and $THD_{opt_3} = (M_5 + M_6)/2$, are stored in the ROM. These threshold levels are to be taken at the optimum sampling instants where the values of $(M_1 - M_2), (M_3 - M_4)$, and $(M_5 - M_6)$ reach the maximum. In DSP, a decision is made by comparing the outputs of A/D converter with the optimum threshold levels which are stored in the RAM. Data in the RAM will
be renewed upon receiving new data streams from A/D converter. Fig.4.24 illustrates the improved performance of 16-QAM, where the normalized amplitude of the fluctuation is assumed to be 0.16 (1.29 dB) and 0.24 (1.87 dB). Note that the corresponding improvement in CNR at $P(e) = 10^{-6}$ is 1.8 dB and 4.9 dB, respectively. Such improvements may relax the specifications for the design of AGC amplifiers where a good $P(e)$ performance is required in a flat fading channel.

4.2 TIGHT-BOUND SYMBOL ERROR PROBABILITIES OF HIGH-LEVEL QAM

Exact average symbol error probabilities of higher-level QAM systems (e.g., 256-, 1024-QAM, ...) may be calculated following the same procedures done in the previous analysis. However, the calculations might be very tedious and require a fairly long computing time. As we can notice from the typical symbol error probabilities of 64-QAM in Figs.4.18 and 19, those signal states on the axis connecting $A_1$ and $A_{10}$ (i.e., $A_1, A_5, A_8$, and $A_{10}$) show typical order ranges of error probability, and the others behave similarly to one of these 4 signal states. Observing this, without going into tedious and lengthy computations of an exact average symbol error probability, we can calculate the tight-bound average symbol error probabilities of the high-level QAM systems.

4.2.1 Verification of the Accuracy of the Tight-Bound Symbol Error Probability of 64-QAM

A tight-bound (T.B) average symbol error probability of 64-QAM is obtained as:

$$P(e)_{T.B} = \frac{1}{4} \left( \frac{1}{2} P_{c_1} + P_{c_5} + P_{c_8} + P_{c_{10}} \right)$$

(4.23)

where the symbol error probabilities $P_{c_1}, P_{c_5}, P_{c_8},$ and $P_{c_{10}}$ are obtained from Eq.(4.21).
Figure 4.24: Improved performance of 16-QAM by using Adaptive Decision Thresholded detectors in the presence of the residual amplitude fluctuation and AWGN.
Fig. 4.25 shows both the exact average symbol error probabilities (in solid curves) and the tight-bound average symbol error probabilities (in dotted curves) of 64-QAM in the presence of residual amplitude fluctuations and AWGN. Note that the tight-bound symbol error probabilities closely fit the exact symbol error probabilities.

4.2.2 Tight-Bound Symbol Error Probability Of 256-QAM

One quadrant (top right-hand side) of a state-space diagram of 256-QAM signal is shown in Fig. 4.26. The symbol error probability of 256-QAM in the presence of a sinusoidal amplitude fluctuation and AWGN is derived as Eq. (4.10), where \( k = 1, 2, 3, \ldots 64 \).

The signal mean power in terms of a signal peak amplitude \( A_k \) is given by:

\[
C_{\text{avg}} = \frac{1}{64} \sum_{k=1}^{64} \left( \frac{A_k}{\sqrt{2}} \right)^2
\]

(4.24)

Or, in terms of the outermost signal amplitude \( A_1 \),

\[
C_{\text{avg}} = \frac{17}{90} A_1^2
\]

(4.25)

Therefore, the mean carrier-to-noise power ratio (CNR) of 256-QAM is given by

\[
CNR = \frac{17}{90} \left( \frac{A_1}{\sigma_G} \right)^2
\]

(4.26)

The minimum signal distance is expressed as

\[
d_{\text{min}} = \frac{\sqrt{2}}{15} A_1
\]

(4.27)

From Eqs. (4.4), (4.9), (4.26), and (4.27), Eq. (4.10) is rewritten as

\[
P_{ek} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \left[ \text{erfc} \left( \sqrt{\frac{CNR}{170}} - 3 \sqrt{\frac{5CNR}{17}} \rho_k u \cos \theta_k \sin \phi \right) \\
+ \text{erfc} \left( \sqrt{\frac{CNR}{170}} - 3 \sqrt{\frac{5CNR}{17}} \rho_k u \sin \theta_k \sin \phi \right) \right] d\phi
\]

(4.28)
Figure 4.25: Exact average symbol error probabilities (solid curves) and tight-bound average symbol error probabilities (dotted curves) of 64-QAM in the presence of the residual amplitude fluctuations and AWGN. (u : Normalized amplitude of the residual amplitude fluctuation.)
Figure 4.26: One quadrant (top right-hand side) of a state-space diagram of 256-QAM signal.
where \( \rho_k = A_k / A_1 \), \( k = 1, 2, 3, \ldots 64 \).

To calculate a tight-bound average symbol error probability, we need to know \( A_1, A_2, \ldots A_8 \) (or \( \rho_1, \rho_2, \ldots \rho_8 \)), and \( \theta_1, \theta_2, \ldots \theta_8 \), which are readily given as:

\[
\rho_n = \frac{17 - 2n}{15}
\]

\[
\theta_n = \tan^{-1}(1)
\]

where \( n = 1, 2, \ldots 8 \).

The tight-bound average symbol error probability of 256-QAM is obtained as:

\[
P(e)_{TB} = \frac{1}{8} \left( \frac{1}{2} P_{e_1} + P_{e_2} + \cdots + P_{e_8} \right)
\]

Fig.4.27 shows the calculated tight-bound average symbol error probabilities of 256-QAM system in the presence of residual amplitude fluctuations and AWGN. An illustrative range of the amplitude fluctuations in terms of a normalized amplitude \( u \) is \([0.0, 0.08]\) or equivalently \([0.0, 0.67\) dB]. To confirm the accuracy of our results, we also calculate the exact average symbol error probabilities of 256-QAM with the aid of computer simulation. The number of symbols used in the simulation is 16,384 (= \( 2^{14} \)). The simulation model as well as Tx/Rx filter strategies of 256-QAM is the same as those of 16- and 64-QAM systems. The computer simulation results in Fig.4.27 show very close agreements with our theoretically calculated tight-bound symbol error probabilities. Fig.4.28 illustrates the simulated eye diagram of 256-QAM in the presence of the residual amplitude fluctuation of \( u = 0.04 \) (or 0.34 dB).

### 4.2.3 Tight-Bound Symbol Error Probability of 1024-QAM

One quadrant (top right-hand side) of a state-space diagram of 1024-QAM signal is shown in Fig.4.29. The symbol error probability of 1024-QAM in the presence of a sinusoidal amplitude fluctuation and AWGN is derived as Eq.(4.11'), where \( k = 1, 2, 3, \ldots 256 \).
Figure 4.27: Tight-bound average symbol error probabilities of 256-QAM in the presence of the residual amplitude fluctuations and AWGN. Calculated from Eqs.(4.28) and (4.31). Computer simulation results are shown in dotted curves. (u : Normalized amplitude of the residual amplitude fluctuation.)
Figure 4.28: Eye diagram of a demodulated and low-pass filtered ($\alpha = 0.2$) 256-QAM signal distorted by the residual amplitude fluctuation having the normalized amplitude $u = 0.04$ (0.34 dB).
Figure 4.29: One quadrant (top right-hand side) of a state-space diagram of 1024-QAM signal.
The signal mean power in terms of a signal peak amplitude \( A_k \) is given by:
\[
C_{\text{avg}} = \frac{1}{256} \sum_{k=1}^{256} \left( \frac{A_k}{\sqrt{2}} \right)^2
\]  
(4.32)

Or, in terms of the outermost signal amplitude \( A_1 \),
\[
C_{\text{avg}} = \frac{341}{1922} A_1^2
\]  
(4.33)

Therefore, the mean carrier-to-noise power ratio (CNR) of 1024-QAM is given by
\[
\text{CNR} = \frac{341}{1922} \left( \frac{A_1}{\sigma_G} \right)^2
\]  
(4.34)

The minimum signal distance is expressed as
\[
d_{\text{min}} = \frac{\sqrt{2}}{31} A_1
\]  
(4.35)

From Eqs. (4.4), (4.9), (4.34), and (4.35), Eq. (4.10) is rewritten as
\[
P_{e_k} = \frac{1}{\pi} \int_{-\pi/2}^{\pi/2} \left\{ \text{erfc} \left( \sqrt{\frac{\text{CNR}}{682}} - \sqrt{\frac{31\text{CNR}}{11}} \rho_k u \cos \theta_k \sin \phi \right) 
+ \text{erfc} \left( \sqrt{\frac{\text{CNR}}{682}} - \sqrt{\frac{31\text{CNR}}{11}} \rho_k u \sin \theta_k \sin \phi \right) \right\} d\phi
\]  
(4.36)

where \( \rho_k = A_k/A_1, \ k = 1, 2, 3, \ldots 256 \).

To calculate a tight-bound average symbol error probability, we need to know \( A_1, A_2, \ldots A_{16} \) (or \( \rho_1, \rho_2, \ldots \rho_{16} \)), and \( \theta_1, \theta_2, \ldots \theta_{16} \), which are readily given as:
\[
\rho_n = \frac{33 - 2n}{31}
\]  
(4.37)

\[
\theta_n = \tan^{-1}(1)
\]  
(4.38)

where \( n = 1, 2, 3, \ldots 16 \).

The tight-bound average symbol error probability of 1024-QAM is obtained as:
\[
P(e)_{T,B} = \frac{1}{16} \left( \frac{1}{2} P_{e_1} + P_{e_2} + \cdots + P_{e_{16}} \right)
\]  
(4.39)

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Fig. 4.30 shows the calculated tight-bound average symbol error probabilities of 1024-QAM system in the presence of residual amplitude fluctuations and AWGN. An illustrative range of the amplitude fluctuations in terms of a normalized amplitude $u$ is $[0.0, 0.035]$ or equivalently $[0.0, 0.30 \text{ dB}]$. Note that in 1024-QAM, the system performance is very sensitive to the residual amplitude fluctuation (or incidental amplitude modulation) of the AGC amplifiers. For example, with 0.09 dB residual amplitude fluctuations in the system, the CNR degradation at $P(e) = 10^{-6}$ is about 2 dB.

4.3 CONCLUSION

M-ary QAM system performance degradations caused by the residual amplitude fluctuations of the received carrier after AGC amplifiers have been studied. We derived theoretical average symbol error probabilities of 16- and 64-QAM, and also tight-bound symbol error probabilities of 256- and 1024-QAM in the presence of residual amplitude fluctuations and AWGN.

Our study shows that the inner signal states of M-ary QAM have larger tolerance to the system impairments than the outer signal states. For this reason, we introduced the adaptive decision threshold (ADT) detectors which could adjust the decision threshold levels to the midpoints of the demodulated eye openings, and demonstrated that these adaptive subsystems improve significantly the system performance. We also found that the tolerance of M-ary QAM systems to the residual amplitude fluctuation is inversely proportional to the outermost signal levels of the QAM. Our experimental results on 16-QAM agree well with the theoretical and computer simulation results, and our tight-bound symbol error probabilities closely fit the exact symbol error probabilities and also the computer simulation results.
Figure 4.30: Tight-bound average symbol error probabilities of 1024-QAM in the presence of the residual amplitude fluctuations and AWGN. Calculated from Eqs.(4.36) and (4.39). (u : Normalized amplitude of the residual amplitude fluctuation.)
Chapter 5

IMPACT OF TRANSMISSION SYSTEM CAUSED PHASE JITTER ON THE PERFORMANCE OF MULTI-ARY QAM SYSTEMS

In a Multi-ary QAM, the phase as well as the amplitude of the signal carries information. The performance of the M-ary (e.g., $M \geq 16$) QAM system, therefore, is very sensitive to the phase distortions of the transmission channels [37,38]. The transmission system (e.g., transmitter LOs, up/down converters, and frequency multipliers, etc.) caused phase jitter remains in the demodulated signal even if a pure sinusoidal carrier signal is provided by the receiver CR circuits. Fig.5.1 illustrates a measured state-space diagram of a 1.544 Mb/s rate 256-QAM system corrupted by the transmission system caused phase jitter, where a pure sinusoidal carrier signal is provided at the CR circuit of 256-QAM receiver [84]. The outer dots, which are more smeared than the inner ones due to the phase rotation, clearly indicate the phase noise nature of the impairment. This field measurement result was performed over a section of a long-haul commercial
$6$ GHz microwave system.

Figure 5.1: Measured state-space diagram of a $1.544$ Mb/s rate $256$-QAM system corrupted by the transmission system caused phase jitter. This field measurement was performed over a section of a long-haul commercial $6$ GHz microwave radio [84].

In this chapter, we analyze the impact of the transmission system introduced phase jitter on the performance of $M$-ary QAM systems. We derive theoretical average symbol error probabilities of $16$- and $64$-QAM, and also tight-bound average symbol error probabilities of $256$- and $1024$-QAM systems in the presence of the Gaussian distributed phase jitter and AWGN. Experimental measurement results are also reported to confirm our theoretical results.
5.1 ANALYSIS OF THE EFFECT OF
PHASE JITTER
ON THE P(ε) PERFORMANCE

The PDF of a practical system caused phase jitter is not known in an
exact form. As a first-order approximation, we assume that the PDF of
this phase jitter is Gaussian. This approximation based on the central
limit theorem is valid, especially if there are several sources which cause
the phase jitter such as is the case in long-haul microwave systems. In
particular, in a nonregenerative radio system, up to 30 hops and over 60
sources may contribute to the phase jitter [39]. Therefore, the PDF of a
phase jitter (noise) may be given by;

$$p(ε) = \frac{1}{\sqrt{2\pi σ_ε^2}} \exp\left(-\frac{ε^2}{2σ_ε^2}\right)$$ (5.1)

where $σ_ε$ is the rms deviation of the phase jitter $ε$.

Since the phase jitter tends to perturb the angular locations of the trans-
mitted signal, the margin against the Gaussian thermal noise is reduced.
An error occurs when the received signal states cross the decision boundary
from the transmitted signal states. For the signal states further away from
the origin, the angular displacements become larger, thus they are subject
to a larger degradation. Fig.5.2 shows an illustrative simulated state-space
diagram of 16-QAM signal perturbed by Gaussian distributed phase jitter.

5.1.1 Derivation of Symbol Error Probability of
16-QAM

Denote the in-phase and quadrature channel baseband signals of M-ary
QAM by $i(t)$ and $q(t)$. In the ideal system, the quadrature modulated,
transmitted signal is represented as;

$$s(t) = i(t) \cos ω_c t + q(t) \sin ω_c t$$ (5.2)
Figure 5.2: Simulated state-space diagram of 16-QAM perturbed by Gaussian distributed phase jitter. The number of the phase jitter is truncated to 16,384 samples in the simulation.
or,
\[ s(t) = a(t) \cos[\omega_c t + \theta(t)] \]  \hspace{1cm} (5.3)

where,
\[ a(t) = \sqrt{i^2(t) + q^2(t)} \]
\[ \theta(t) = \tan^{-1}\left(\frac{q(t)}{i(t)}\right) \]
\[ \omega_c = \text{Angular frequency of carrier.} \]

With the phase jitter \( \varepsilon(t) \) introduced at the transmission channel, the transmitted signal is represented as;
\[ s'(t) = a(t) \cos[\omega_c t + \theta(t) \pm \varepsilon(t)] \]  \hspace{1cm} (5.4)

Assuming a pure sinusoidal carrier signal (i.e., \( \cos \omega_c t, \sin \omega_c t \)) is recovered at the receiver, the demodulated and low pass filtered signal is given by;
\[ r(t) = a(t) \cos[\theta(t) \pm \varepsilon(t)] + n(t) \]  \hspace{1cm} (5.5)

where, \( n(t) \) represents an equivalent baseband thermal noise.

Therefore, an analysis model of this system can be represented as Fig.5.3. Fig.5.4 illustrates one quadrant (i.e., top right-hand side) of the state-space diagram of 16-QAM at the receiver, where the transmitted signal having nominal envelope \( A_k \) and phase \( \theta_k \) is perturbed by Gaussian distributed phase jitter \( \varepsilon \).

Decomposing the perturbed, received signal into the in-phase (I) and quadri-phase (Q) components, the signal level varies in the range of
[\[ A_k \cos(\theta_k + \varepsilon), A_k \cos(\theta_k - \varepsilon) \] in the I-channel, and [\[ A_k \sin(\theta_k - \varepsilon), A_k \sin(\theta_k + \varepsilon) \] in the Q-channel.

In the I-channel, error occurs when the thermal noise exceeds
\[ s_1 \equiv d_{\text{min}}/2 - A_k[\cos(\theta_k - \varepsilon) - \cos \theta_k] \]  \hspace{1cm} (5.6)
or,
\[ s_2 \equiv d_{\text{min}}/2 - A_k[\cos \theta_k - \cos(\theta_k + \varepsilon)]. \]  \hspace{1cm} (5.7)
Phase jitter $\varepsilon(t)$

$$e^{j\varepsilon(t)}$$

AWGN

M-ary QAM Transmitter

$s(t)$

$x$

$s'(t)$

$+$

$r(t)$

M-ary QAM Receiver

Figure 5.3: Analysis model of M-ary QAM system perturbed by the transmission system caused phase jitter $\varepsilon(t)$.

In the Q-channel, error occurs when the thermal noise exceeds

$$t_1 \equiv d_{\text{min}}/2 - A_k[\sin(\theta_k + \varepsilon) - \sin\theta_k]$$

(5.8)

or,

$$t_2 \equiv d_{\text{min}}/2 - A_k[\sin\theta_k - \sin(\theta_k - \varepsilon)]$$

(5.9)

Therefore, the impact of the phase jitter on the desired signal is to reduce a noise margin by the amount of $A_k[\varepsilon]$ terms of the above equations.

A conditional probability of error in the AWGN channel is given by;

$$P_{e_x|\varepsilon} = \int_{t_1}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma^2_G} \exp\left(-\frac{w^2}{2\sigma^2_G}\right)dw$$

$$+ \int_{t_2}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma^2_G} \exp\left(-\frac{x^2}{2\sigma^2_G}\right)dx$$

$$+ \int_{t_1}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma^2_G} \exp\left(-\frac{y^2}{2\sigma^2_G}\right)dy$$

$$+ \int_{t_2}^{+\infty} \frac{1}{\sqrt{2\pi}\sigma^2_G} \exp\left(-\frac{z^2}{2\sigma^2_G}\right)dz$$
Figure 5.4: One quadrant (i.e., top, right-hand side) of a state-space diagram of 16-QAM perturbed by Gaussian distributed phase jitter. (Nominal envelope $A_k$ and phase $\theta_k$ of the signal states are defined in Fig.4.5.)
= \frac{1}{2} [\text{erfc}\left(\frac{s_1}{\sqrt{2}\sigma_G}\right) + \text{erfc}\left(\frac{s_2}{\sqrt{2}\sigma_G}\right) + \text{erfc}\left(\frac{t_1}{\sqrt{2}\sigma_G}\right) + \text{erfc}\left(\frac{t_2}{\sqrt{2}\sigma_G}\right)] \tag{5.10}

where, erfc( ) is a complementary error function defined as

\text{erfc}(u) = \frac{2}{\sqrt{\pi}} \int_u^{+\infty} e^{-v^2} dv \tag{5.11}

Since the PDF of a phase jitter is given by \( p(\varepsilon) \), the symbol error probability is derived by averaging the conditional error probability of Eq.(5.10) with respect to the phase jitter \( \varepsilon \), that is,

\[ P_{e_k} = E\{P_{e_k|\varepsilon}\} | \varepsilon \]
= \int_{-\pi}^{\pi} p(\varepsilon) P_{e_k|\varepsilon} d\varepsilon
= \frac{1}{2\sqrt{2\pi\sigma_G^2}} \int_{-\pi}^{\pi} [\text{erfc}\left(\frac{s_1}{\sqrt{2}\sigma_G}\right) + \text{erfc}\left(\frac{s_2}{\sqrt{2}\sigma_G}\right) + \text{erfc}\left(\frac{t_1}{\sqrt{2}\sigma_G}\right) + \text{erfc}\left(\frac{t_2}{\sqrt{2}\sigma_G}\right)] \exp\left(-\frac{\varepsilon^2}{2\sigma_G^2}\right) d\varepsilon \tag{5.12}

where \( k = 1, 2, 3, 4 \).

In terms of the outermost signal envelope \( A_1 \), the mean carrier-to-noise power ratio (CNR) of 16-QAM is given by (See Section 1.2 of Chapter 4);

\[ \text{CNR} = \frac{5}{18} \left(\frac{A_1}{\sigma_G}\right)^2 \tag{5.13} \]

and the minimum signal distance is expressed as

\[ d_{\text{min}} = \frac{\sqrt{2}}{3} A_1 \tag{5.14} \]

Therefore, the terms inside the complementary error functions of Eq.(5.12) are rewritten as;

\[ \frac{\delta_1}{\sqrt{2}\sigma_G} = \sqrt{\text{CNR}} \frac{10}{3} - 3 \sqrt{\text{CNR}} \frac{5}{3} \rho_k [\cos(\theta_k - \varepsilon) - \cos \theta_k] \]
\[
\frac{s_2}{\sqrt{2} \sigma_G} = \sqrt{\frac{CNR}{10}} - 3\sqrt{\frac{CNR}{5}} \rho_k [\cos \theta_k - \cos(\theta_k + \varepsilon)]
\]
\[
\frac{t_1}{\sqrt{2} \sigma_G} = \sqrt{\frac{CNR}{10}} - 3\sqrt{\frac{CNR}{5}} \rho_k [\sin(\theta_k + \varepsilon) - \sin \theta_k]
\]
\[
\frac{t_2}{\sqrt{2} \sigma_G} = \sqrt{\frac{CNR}{10}} - 3\sqrt{\frac{CNR}{5}} \rho_k [\sin \theta_k - \sin(\theta_k - \varepsilon)]
\] (5.15)

where \( \rho_k = A_k/A_1, k = 1, 2, 3, 4 \) (e.g., \( \rho_1 = 1, \rho_2 = \rho_4 = \sqrt{5}/3, \rho_3 = 1/3 \)).

Theoretically, an ideal Gaussian noise process has infinitely high peaks. The probability that a noise sample exceeds \( 5\sigma \) (i.e., 5 times rms value of the noise) is known to be only \( 1.5 \times 10^{-12} \) [40]. Even though this probability is very low, it is still finite and would be a major contribution of an error-generating mechanism in digital transmission systems. For our purpose of error probability calculations, an accuracy in the order of \( 10^{-12} \) is good enough. Therefore, we do not consider noise samples beyond \( 5\sigma \), that is, the integration range of \([-\pi, \pi]\) in Eq.(5.12) is truncated to \([-5\sigma, 5\sigma]\), where \( \sigma \) is an rms phase jitter in radians. Computer simulations, either the quasi-analytical method or the Monte Carlo method, may not be suitable for the precise evaluation of the error performance in the presence of the Gaussian distributed phase jitter, especially for the high-level QAMs. It is because, in order to obtain a fairly accurate error probability, we need to observe \( 10^{12} \) (or at least \( 10^6 \)) samples of the phase jitter, which requires an extremely large size of a computer memory and a long computation time.

An overall average symbol error probability of 16-QAM is obtained as (See Section 1.2 of Chapter 4);
\[
P(e) = \frac{1}{8} (P_{e_1} + 3P_{e_2} + 2P_{e_3})
\] (5.16)

Fig.5.5 illustrates \( P(e) \) performance of 16-QAM in the presence of Gaussian distributed phase jitter and AWGN, where the phase jitter varies in the range \([0.0^\circ, 8.0^\circ\text{rms}]\).

An approximate conversion, widely used in industries, of an rms phase jitter \( \sigma_\varepsilon \) (in radian) into a mean Carrier-to-Phase Noise power ratio is given by;
\[
\frac{C}{N_p} = 10 \log_{10}(\frac{1}{\sigma_\varepsilon^2})
\] (5.17)
Figure 5.5: $P(\varepsilon)$ performance of 16-QAM in the presence of Gaussian distributed phase jitter ($\varepsilon$) and AWGN. Calculated from Eqs.(5.12),(5.15) and (5.16). The values of the phase jitter converted into the mean Carrier-to-Phase noise power ratio are tabulated in Table 5.1.
Table 5.1 tabulates the corresponding phase jitters in rms degree and in the carrier-to-phase noise power ratio (C/Np). Note that for a phase jitter larger than 3°rms, an error floor occurs and \( P(e) \) performance could not be improved by increasing CNR (i.e., even for the infinitely large CNR).

<table>
<thead>
<tr>
<th>RMS Phase Jitter ( \sigma_x ) (in degree)</th>
<th>Carrier-to-Phase Noise Power Ratio ( C/N_p ) (in dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1°</td>
<td>35.16 dB</td>
</tr>
<tr>
<td>2°</td>
<td>29.14 dB</td>
</tr>
<tr>
<td>3°</td>
<td>25.62 dB</td>
</tr>
<tr>
<td>4°</td>
<td>23.12 dB</td>
</tr>
<tr>
<td>5°</td>
<td>21.18 dB</td>
</tr>
<tr>
<td>6°</td>
<td>19.60 dB</td>
</tr>
<tr>
<td>7°</td>
<td>18.26 dB</td>
</tr>
<tr>
<td>8°</td>
<td>17.10 dB</td>
</tr>
</tbody>
</table>

Table 5.1: RMS phase jitter \( (\sigma_x) \) in degrees and the corresponding carrier-to-phase noise power ratios \( (C/N_p) \).

5.1.2 Derivation of Symbol Error Probability of 64-QAM

One quadrant (top right-hand side) of a state-space diagram of 64-QAM signal is shown in Fig. 4.8 of Chapter 4. Following the same procedures as in 16-QAM, a symbol error probability of 64-QAM in the presence of Gaussian distributed phase jitter and AWGN is derived as Eq.(5.12), where \( k = 1, 2, 3, \ldots 16 \).

In terms of the outermost signal envelope \( A_1 \), the mean carrier-to-noise power ratio (CNR) of 64-QAM is given by (See Section 1.3 of Chapter 4)

\[
CNR = \frac{3}{14} \left( \frac{A_1}{\sigma_G} \right)^2 \quad (5.18)
\]

and the minimum signal distance is expressed as

\[
d_{\text{min}} = \frac{\sqrt{2}}{7} A_1 \quad (5.19)
\]
Therefore, the terms inside the complementary error functions of Eq.(5.12) are rewritten as

\[
\frac{s_1}{\sqrt{2}\sigma_G} = \sqrt{\frac{CNR}{42}} - \sqrt{\frac{7CNR}{3}}\rho_k[\cos(\theta_k - \varepsilon) - \cos\theta_k]
\]

\[
\frac{s_2}{\sqrt{2}\sigma_G} = \sqrt{\frac{CNR}{42}} - \sqrt{\frac{7CNR}{3}}\rho_k[\cos\theta_k - \cos(\theta_k + \varepsilon)]
\]

\[
\frac{t_1}{\sqrt{2}\sigma_G} = \sqrt{\frac{CNR}{42}} - \sqrt{\frac{7CNR}{3}}\rho_k[\sin(\theta_k + \varepsilon) - \sin\theta_k]
\]

\[
\frac{t_2}{\sqrt{2}\sigma_G} = \sqrt{\frac{CNR}{42}} - \sqrt{\frac{7CNR}{3}}\rho_k[\sin\theta_k - \sin(\theta_k - \varepsilon)]
\]

(5.20)

where \(\rho_k = A_k/A_1, \ k = 1, 2, 3, \ldots, 16\), and the values of \(\rho_k\) and \(\theta_k\) are given in Table 4.2 of Chapter 4.

An overall average symbol error probability of 64-QAM is obtained as (See Section 1.3 of Chapter 4)

\[
P(\varepsilon) = \frac{1}{32}[P_{e_1} + 3(P_{e_2} + P_{e_3} + P_{e_4}) + 2(P_{e_5} + P_{e_8} + P_{e_{10}}) + 4(P_{e_6} + P_{e_7} + P_{e_9})]
\]

(5.21)

Fig.5.6 illustrates \(P(\varepsilon)\) performance of 64-QAM in the presence of Gaussian distributed phase jitter and AWGN, where the phase jitter varies in the range \([0.0^\circ, 4.0^\circ rms]\). The carrier-to-phase noise power ratio of 64-QAM is calculated from Eq.(5.17). Table 5.2 tabulates the corresponding phase jitters in rms degree and in carrier-to-phase noise power ratios. Note that for a phase jitter larger than \(1.5^\circ rms\), an error floor occurs and \(P(\varepsilon)\) performance could not be improved by increasing CNR.

5.1.3 Experimental Verification of Theoretical Result on 16-QAM Modem

To get a physical insight of the behaviour of the phase jitter and to compare with our theoretical results on the performance of 16-QAM system, we measured the eye diagrams, state-space diagrams and \(P(\varepsilon)\) performance
Figure 5.6: $P(e)$ performance of 64-QAM in the presence of Gaussian distributed phase jitter ($\varepsilon$) and AWGN. Calculated from Eqs.(5.12),(5.20) and (5.21). The values of the phase jitter converted into the mean Carrier-to-Phase noise power ratio are tabulated in Table 5.2.
<table>
<thead>
<tr>
<th>RMS Phase Jitter $\sigma_\varepsilon$ (in degree)</th>
<th>Carrier-to-Phase Noise Power Ratio $C/N_p$ (in dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.5°</td>
<td>41.18 dB</td>
</tr>
<tr>
<td>1.0°</td>
<td>35.16 dB</td>
</tr>
<tr>
<td>1.5°</td>
<td>31.64 dB</td>
</tr>
<tr>
<td>2.0°</td>
<td>29.14 dB</td>
</tr>
<tr>
<td>2.5°</td>
<td>27.20 dB</td>
</tr>
<tr>
<td>3.0°</td>
<td>25.62 dB</td>
</tr>
<tr>
<td>3.5°</td>
<td>24.28 dB</td>
</tr>
<tr>
<td>4.0°</td>
<td>23.12 dB</td>
</tr>
</tbody>
</table>

Table 5.2: RMS phase jitter ($\sigma_\varepsilon$) in degrees and the corresponding carrier-to-phase noise power ratios ($C/N_p$). Calculated from Eq.(5.17).

of 16-QAM modem perturbed by Gaussian distributed phase jitters. An experimental set-up for this measurement is shown in Fig.5.7. The transmission data rate of our 16-QAM modem is 256 kb/s (or 64 kBaud), and the carrier frequency is 1 MHz. As a substituting source of a transmission system caused phase jitter, a white Gaussian noise is added to the input of a PLL carrier recovery circuit in the 16-QAM receiver. Such a set-up may be valid for the first-order approximation of our laboratory measurement, especially for the high signal-to-noise ratio measurement, where the actual PDF of the phase jitter will closely resemble the Gaussian density [37]. The substitution of the phase jitter in Fig.5.7 is also valid, because from Fig.5.8 the overall system response of $s'(t)$ in Eq.(5.4) multiplied by $\cos \omega_c t$ in Model ‘A’ is equivalent to $s(t)$ in Eq.(5.3) multiplied by $\cos[\omega_c t \pm \varepsilon(t)]$ in Model ‘B’. This implies that the transmission system caused phase jitter may impose the same effects on the overall system as the phase jitter in the CR circuits does. The fundamental difference between these two models, however, is as follows. In Model ‘A’, the phase which carries the information of the transmitted signal is distorted in the transmission channels, as is a typical case in the long-haul microwave radio systems, and a pure sinusoidal carrier signal is assumed to be recovered at the receiver by means of well designed CR circuits or pilot tone insertion/extraction methods [41]. Whereas, in Model ‘B’ the transmission channel is assumed to be transparent, therefore the phase of the transmitted signal is not distorted, at the
Figure 5.7: Experimental set-up for the measurement of the performance of a 16-QAM system in the presence of the phase jitter and AWGN. \( N \): Thermal noise at the receiver front-end. \( N_p \): Phase noise at the CR circuit.
Figure 5.3: Overall system response of M-ary QAM in the presence of (a) Transmission system caused phase jitter, and (b) Phase jitter in the CR circuits.
receiver, however, the phase of the recovered carrier is perturbed by the noise.

Fig.5.9(a) and (b) show the experimental measured state-space diagrams of 16-QAM system having $C/N_p$ of 30 dB and 25 dB, or the equivalent rms phase jitter of $2^\circ$ and $3^\circ rms$, respectively. Note that the outer dots are more smeared than the inner dots due to the larger angular displacements of the phase jitter. The corresponding eye diagrams are shown in Fig.5.9(c) and (d). Fig.5.10(a) and (b) illustrate the respective measured power spectra of these phase noise. The measured $P(e)$ performance of 16-QAM is compared to our theoretical results in Fig.5.11. The $P(e)$ performance difference between the measured and theoretical results are mostly due to the ISI caused by the imperfect channel filters of our experimental modem.

5.1.4 The Best and the Worst Symbol Error Probabilities of 16- and 64-QAM

The signal state-space diagrams in Fig.5.9(a) and (b) reveal that in the Multi-ary QAM system the outer signal states may be subject to the larger CNR degradations as compared to the inner signal states due to the larger angular displacements of a phase jitter. In this section, we analyze particular symbol error probabilities including the best and the worst symbol error probabilities. Figs.5.12 and 13 show the particular symbol error probabilities and overall average symbol error probabilities of 16-QAM in the presence of AWGN and Gaussian distributed phase jitters of $3^\circ rms$ and $4^\circ rms$, respectively. Figs.5.14 and 15 show typical symbol error probabilities and average symbol error probabilities of 64-QAM for $1.5^\circ rms$ and $2.5^\circ rms$ phase jitters, respectively. The symbol error probabilities $P_{e_1}, P_{e_3}, P_{e_8}$, and $P_{e_{10}}$ correspond to those of illustrative symbols $A_1, A_3, A_8,$ and $A_{10}$ in Fig.4.8, respectively, and are calculated from Eqs.(5.12) and (5.20). Note that the outermost symbol (i.e., $A_1$) performs worst, whereas the innermost symbol (i.e., $A_3$ for 16-QAM, $A_{10}$ for 64-QAM) performs best, and the overall error performance is mostly determined by the worst symbol error probability. A loose upper-bound (i.e., the worst) symbol error probability, therefore, can be obtained from the outermost symbol error probability $P_{e_1}$. Also note that the performance degradations (or system
Figure 5.9: Experimental measured (a)&(b) State-space diagrams, and (c)&(d) The corresponding eye diagrams of 16-QAM perturbed by the phase noise of $C/N_p = 30$ dB (or $2^\circ\text{rms}$) in (a)&(c), and $C/N_p = 25$ dB (or $3^\circ\text{rms}$) in (b)&(d).
Figure 5.10: Experimental measured power spectra of the phase noise of (a) $C/N_P = 30$ dB, and (b) $C/N_P = 25$ dB. (Hori: 2 kHz/div, Vert: 10 dB/div, Resolution B.W: 0.1 kHz)
Figure 5.11: Experimental measured and theoretically calculated $P(e)$ performance of 16-QAM in the presence of AWGN and Gaussian distributed phase jitters of $\sigma_e = 2^\circ \text{rms (or } C/N_p = 30 \text{ dB)}$ and $c_e = 3^\circ \text{rms (or } C/N_p = 25 \text{ dB).}$
Figure 5.12: Symbol error probabilities and an overall average symbol error probability $P(e)$ of 16-QAM in the presence of Gaussian distributed phase jitter ($\sigma_e = 3^\circ rms$) and AWGN. ($P_{e_1}$: The worst (or upper-bound) symbol error probability caused by the outermost signal states. $P_{e_3}$: The best symbol error probability caused by the innermost signal states.)
Figure 5.13: Symbol error probabilities and an overall average symbol error probability $P(e)$ of 16-QAM in the presence of Gaussian distributed phase jitter ($\sigma_e = 4^\circ$rms) and AWGN.
Figure 5.14: Typical symbol error probabilities and an overall average symbol error probability $P(e)$ of 64-QAM in the presence of Gaussian distributed phase jitter ($\sigma_e = 1.5^{\circ}$rms) and AWGN. ($P_{e1}$: The worst (or upper-bound) symbol error probability caused by the outermost signal states. $P_{e_{10}}$: The best symbol error probability caused by the innermost signal states.) $P_e$ is the symbol error probability of $A_e$ depicted in Fig.4.8.
Figure 5.15: Typical symbol error probabilities and an overall average symbol error probability $P(e)$ of 64-QAM in the presence of Gaussian distributed phase jitter ($\sigma_e = 2.5^\circ \text{rms}$) and AWGN.
sensitivities to the channel impairments) caused by the outermost symbol are much more distinct than those by the innermost symbol.

5.2 TIGHT-BOUND SYMBOL ERROR PROBABILITIES OF HIGH-LEVEL QAM

Exact average symbol error probabilities of the higher-level QAM systems (e.g., 256-, 1024-QAM ...) may be calculated following the same procedures done in the previous analysis. However, the calculations might be very tedious and require a fairly long computing time. As we have noticed from the typical symbol error probabilities of 64-QAM in Figs.5.14 and 15, the signal states on the axis connecting $A_1$ and $A_{10}$ (i.e., $A_1$, $A_5$, $A_8$, and $A_{10}$) show typical order ranges of error probability, and the others behave similarly to one of these 4 signal states. Observing this, without going into tedious and lengthy computations of an exact average symbol error probability, we can calculate tight-bound average symbol error probabilities of the high-level QAM systems.

5.2.1 Verification of the Accuracy of the Tight-Bound Symbol Error Probability of 64-QAM

A tight-bound (T.B) average symbol error probability of 64-QAM is obtained as:

$$P(e)_{T.B} = \frac{1}{4} \left( \frac{1}{2} P_{e_1} + P_{e_5} + P_{e_8} + P_{e_{10}} \right)$$  \hspace{1cm} (5.22)

where the symbol error probabilities $P_{e_1}$, $P_{e_5}$, $P_{e_8}$, and $P_{e_{10}}$ are obtained from Eqs.(5.12) and (5.20).

Fig.5.16 shows both the exact average symbol error probabilities (in solid curves) and the tight-bound average symbol error probabilities (in dotted curves) of 64-QAM in the presence of Gaussian distributed phase jitter and AWGN. Note that the tight-bound symbol error probabilities closely fit the exact symbol error probabilities.
Figure 5.16: Exact average symbol error probabilities (in solid curves) and tight-bound average symbol error probabilities (in dotted curves) of 64-QAM in the presence of Gaussian distributed phase jitter and AWGN.
5.2.2 Tight-Bound Symbol Error Probability Of 256-QAM

One quadrant (top right-hand side) of a state-space diagram of 256-QAM signal is shown in Fig. 4.23. The symbol error probability of 256-QAM in the presence of Gaussian distributed phase jitter and AWGN is expressed as Eq. (5.12), where \( k = 1, 2, 3, \ldots 64 \).

In terms of the outermost signal envelope \( A_1 \), the mean carrier-to-noise power ratio (CNR) of 256-QAM is given by (See Section 2.2 of Chapter 4)

\[
\text{CNR} = \frac{17}{90} (\frac{A_1}{\sigma_G})^2 \tag{5.23}
\]

and the minimum signal distance is expressed as

\[
d_{\text{min}} = \frac{\sqrt{2}}{15} A_1 \tag{5.24}
\]

Therefore, the terms inside the complementary error functions of Eq. (5.12) are rewritten as

\[
\frac{s_1}{\sqrt{2}\sigma_G} = \sqrt{\frac{\text{CNR}}{170}} - 3\sqrt{\frac{5\text{CNR}}{17}} \rho_k [\cos(\theta_k - \varepsilon) - \cos \theta_k]
\]

\[
\frac{s_2}{\sqrt{2}\sigma_G} = \sqrt{\frac{\text{CNR}}{170}} - 3\sqrt{\frac{5\text{CNR}}{17}} \rho_k [\cos \theta_k - \cos(\theta_k + \varepsilon)]
\]

\[
\frac{t_1}{\sqrt{2}\sigma_G} = \sqrt{\frac{\text{CNR}}{170}} - 3\sqrt{\frac{5\text{CNR}}{17}} \rho_k [\sin(\theta_k + \varepsilon) - \sin \theta_k]
\]

\[
\frac{t_2}{\sqrt{2}\sigma_G} = \sqrt{\frac{\text{CNR}}{170}} - 3\sqrt{\frac{5\text{CNR}}{17}} \rho_k [\sin \theta_k - \sin(\theta_k - \varepsilon)] \tag{5.25}
\]

where \( \rho_k = A_k/A_1 \), \( k = 1, 2, 3, \ldots 64 \).

To calculate a tight-bound average symbol error probability, we need to know \( A_1, A_2, \ldots A_8 \) (or \( \rho_1, \rho_2, \ldots \rho_8 \)), and \( \theta_1, \theta_2, \ldots \theta_8 \), which are readily given as:

\[
\rho_n = \frac{17 - 2n}{15} \tag{5.26}
\]

\[
\theta_n = \tan^{-1}(1) \tag{5.27}
\]
where \( n = 1, 2, \ldots 8 \).

The tight-bound average symbol error probability is obtained as:

\[
P(e)_{T,B} = \frac{1}{8} \left( \frac{1}{2} P_{c_1} + P_{c_2} + \cdots + P_{c_8} \right)
\]  

(5.28)

Fig. 5.17 shows calculated tight-bound average symbol error probabilities of 256-QAM system in the presence of Gaussian distributed phase jitter and AWGN. An illustrative range of the phase jitter is \([0.0^\circ, 2.0^\circ \text{rms}]\). The corresponding carrier-to-phase noise power ratios of these rms phase jitter are given in Table 5.3. Note that an error floor occurs for the phase jitter larger than 0.75\(^\circ\) rms. In the Fig. 5.1 of a 1.544 Mb/s data rate 256-QAM system, the measured \( P(e) \) performance resulted in the residual error rate of \( 10^{-6} \).

<table>
<thead>
<tr>
<th>RMS Phase Jitter ( \sigma_z ) (in degree)</th>
<th>Carrier-to-Phase Noise Power Ratio ( C/N_p ) (in dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25(^\circ)</td>
<td>47.20 dB</td>
</tr>
<tr>
<td>0.50(^\circ)</td>
<td>41.18 dB</td>
</tr>
<tr>
<td>0.75(^\circ)</td>
<td>37.66 dB</td>
</tr>
<tr>
<td>1.00(^\circ)</td>
<td>35.16 dB</td>
</tr>
<tr>
<td>1.25(^\circ)</td>
<td>33.22 dB</td>
</tr>
<tr>
<td>1.50(^\circ)</td>
<td>31.64 dB</td>
</tr>
<tr>
<td>1.75(^\circ)</td>
<td>30.30 dB</td>
</tr>
<tr>
<td>2.00(^\circ)</td>
<td>29.14 dB</td>
</tr>
</tbody>
</table>

Table 5.3: RMS phase jitter \( \sigma_z \) in degrees and the corresponding carrier-to-phase noise power ratios \( C/N_p \). Calculated from Eq.(5.17).

### 5.2.3 Tight-Bound Symbol Error Probability of 1024-QAM

One quadrant (top right-hand side) of a state-space diagram of 1024-QAM signal is shown in Fig. 4.25. The symbol error probability of 1024-QAM in
Figure 5.17: Tight-bound average symbol error probabilities of 256-QAM in the presence of Gaussian distributed phase jitter and AWGN. The values of the phase jitter converted into the mean Carrier-to-Phase noise power ratio are tabulated in Table 5.3.
the presence of Gaussian distributed phase jitter and AWGN is derived as Eq.(5.12), where \( k = 1, 2, 3, \ldots 256 \).

In terms of the outermost signal envelope \( A_1 \), the mean carrier-to-noise power ratio (CNR) of 1024-QAM is given by (See Section 2.3 of Chapter 4)

\[
CNR = \frac{341}{1922} \frac{(A_1)^2}{\sigma_g^2}
\]

and the minimum signal distance is expressed as

\[
d_{\text{min}} = \frac{\sqrt{2}}{31} A_1
\]

Therefore, the terms inside the complementary error functions of Eq.(5.12) are rewritten as

\[
\begin{align*}
\frac{s_1}{\sqrt{2} \sigma_g} &= \sqrt{\frac{CNR}{682}} - \sqrt{\frac{31CNR}{11}} \rho_k [\cos(\theta_k - \varepsilon) - \cos \theta_k] \\
\frac{s_2}{\sqrt{2} \sigma_g} &= \sqrt{\frac{CNR}{682}} - \sqrt{\frac{31CNR}{11}} \rho_k [\cos \theta_k - \cos(\theta_k + \varepsilon)] \\
\frac{t_1}{\sqrt{2} \sigma_g} &= \sqrt{\frac{CNR}{682}} - \sqrt{\frac{31CNR}{11}} \rho_k [\sin(\theta_k + \varepsilon) - \sin \theta_k] \\
\frac{t_2}{\sqrt{2} \sigma_g} &= \sqrt{\frac{CNR}{682}} - \sqrt{\frac{31CNR}{11}} \rho_k [\sin \theta_k - \sin(\theta_k - \varepsilon)]
\end{align*}
\]

where \( \rho_k = A_k / A_1, k = 1, 2, 3, \ldots 256 \).

To calculate a tight-bound average symbol error probability, we need to know \( A_1, A_2, \ldots A_{16} \) (or \( \rho_1, \rho_2, \ldots \rho_{16} \)), and \( \theta_1, \theta_2, \ldots \theta_{16} \), which are readily given as;

\[
\begin{align*}
\rho_n &= \frac{33 - 2n}{31} \\
\theta_n &= \tan^{-1}(1)
\end{align*}
\]

where \( n = 1, 2, \ldots 16 \).

The tight-bound average symbol error probability is obtained as;

\[
P(e)_{T.B} = \frac{1}{16} \left( \frac{1}{2} P_{\epsilon_1} + P_{\epsilon_2} + \cdots + P_{\epsilon_{16}} \right)
\]
Fig. 5.18 shows calculated tight-bound average symbol error probabilities of 1024-QAM system in the presence of Gaussian distributed phase jitter and AWGN. An illustrative range of the phase jitter is $[0.0^\circ, 1.0^\circ_{\text{rms}}]$. The corresponding carrier-to-phase noise power ratios of these rms phase jitter are given in Table 5.4.

<table>
<thead>
<tr>
<th>RMS Phase Jitter $\sigma_e$ (in degree)</th>
<th>Carrier-to-Phase Noise Power Ratio $C/N_p$ (in dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.125\degree</td>
<td>53.22 dB</td>
</tr>
<tr>
<td>0.250\degree</td>
<td>47.20 dB</td>
</tr>
<tr>
<td>0.375\degree</td>
<td>43.68 dB</td>
</tr>
<tr>
<td>0.500\degree</td>
<td>41.18 dB</td>
</tr>
<tr>
<td>0.625\degree</td>
<td>39.25 dB</td>
</tr>
<tr>
<td>0.750\degree</td>
<td>37.66 dB</td>
</tr>
<tr>
<td>0.875\degree</td>
<td>36.32 dB</td>
</tr>
<tr>
<td>1.000\degree</td>
<td>35.16 dB</td>
</tr>
</tbody>
</table>

Table 5.4: RMS phase jitter ($\sigma_e$) in degree and the corresponding carrier-to-phase noise power ratios ($C/N_p$). Calculated from Eq. (5.17).

Note that an error floor occurs for the phase jitter larger than $0.375^\circ_{\text{rms}}$. The result shows that the tolerance of M-ary QAM system to the phase jitter would become very strict with the increasing number of the modulation levels. Table 5.5 summarizes the respective phase jitter of M-ary ($M = 16$, 64, 256, and 1024) QAM system which may result in a residual error rate of $P(e) \approx 10^{-8}$. In other words, to maintain the system reliability objective of 99.9999% the phase jitter of the system should be kept below the levels given in Table 5.5.

5.3 Conclusion

M-ary QAM system performance degradations caused by the transmission system caused phase jitter have been studied. We derived theoretical average symbol error probabilities of 16- and 64-QAM, and also tight-bound
Figure 5.18: Tight-bound average symbol error probabilities of 1024-QAM in the presence of Gaussian distributed phase jitter and AWGN. The values of the phase jitter converted into the mean Carrier-to-Phase noise power ratio are tabulated in Table 5.4.
symbol error probabilities of 256- and 1024-QAM in the presence of Gaussian distributed phase jitter and AWGN. To confirm our theoretical results, we also performed experimental measurements on 256 kb/s data rate 16-QAM modem.

Our study shows that the outermost signal states of M-ary QAM system result in the worst symbol error probability, whereas the innermost states result in the best symbol error probability. An overall average symbol error probability is mostly determined by the worst symbol error probability. We also found that error floors exist on the system performance for certain levels of phase jitter, which should be considered for the system reliability objective.

<table>
<thead>
<tr>
<th>M-ary QAM</th>
<th>RMS Phase Jitter $\sigma_e$ (in degree)</th>
<th>Carrier-to-Phase Noise Power Ratio : $C/N_p$</th>
</tr>
</thead>
<tbody>
<tr>
<td>16-QAM</td>
<td>$3.8^\circrms$</td>
<td>23.57 dB</td>
</tr>
<tr>
<td>64-QAM</td>
<td>$1.8^\circrms$</td>
<td>30.06 dB</td>
</tr>
<tr>
<td>256-QAM</td>
<td>$0.85^\circrms$</td>
<td>36.57 dB</td>
</tr>
<tr>
<td>1024-QAM</td>
<td>$0.425^\circrms$</td>
<td>42.60 dB</td>
</tr>
</tbody>
</table>

Table 5.5: RMS phase jitter and the corresponding carrier-to-phase noise power ratios of M-ary QAM causing a residual error rate of $P(e) \approx 10^{-6}$.
In Chapters 2–5, we have assumed that the noise present at the receiver front-end is purely Gaussian. Since the noise generated by a variety of natural and man-made electromagnetic sources exhibits impulsive characteristics, it might be of a great importance to analyze the impact of non-Gaussian impulsive noise on the performance of M-ary QAM systems. As an analytically tractable model of a Gaussian/non-Gaussian noise, Middleton’s Class ‘A’ model is known to fit closely a variety of non-Gaussian noise [45–47]. In this model, the frequency components of the impulsive noise are constrained within the bandwidth of the receiver BPF (i.e., Narrow-band non-Gaussian noise). Such a noise model might be a good approximation for the analysis of non-Gaussian noise effects on the high-capacity (e.g., 135 Mbit/sec or 200 Mbit/sec) digital microwave radio systems, where the data bit duration is fairly small.

In this chapter, we analyze the impact of the non-Gaussian impulsive
noise combined with the Gaussian thermal noise on the performance of M-ary (e.g., $M = 16, 64, 256$) QAM systems. $P(e)$ performance of the system is analyzed in terms of the CNR, impulsive index of the noise, and the power ratio of the Gaussian noise-to-Impulsive noise.

6.1 SYMBOL ERROR PROBABILITY OF M-ARY QAM IN THE PRESENCE OF NON-GAUSSIAN IMPULSIVE NOISE

In this section, we derive the symbol error probability of M-ary QAM in the Gaussian/non-Gaussian impulsive noise environments. Fig. 6.1 illustrates an analysis model of M-ary QAM system. M-ary QAM signal, during

![M-ary QAM System Diagram]

Figure 6.1: Analysis model of M-ary QAM system in the presence of the non-Gaussian Impulsive noise combined with the Gaussian thermal noise.

the $n$th period having a symbol interval $T_s$ and a carrier frequency $f_c$, is represented as:

$$s(t) = A_k \cos 2\pi f_c t + B_k \sin 2\pi f_c t$$

(6.1)
where, \( nT_s \leq t \leq (n + 1)T_s \), and the signal amplitudes \( \{A_k\}, \{B_k\} = \{\pm 1, \pm 3, \ldots \pm (\sqrt{M} - 1)\} \).

For the non-Gaussian noise, we use Middleton's Class 'A' model noise, which is a generalized model of the Gaussian noise combined with a non-Gaussian impulsive noise. At the output of the receive BPF, the noise is represented as:

\[
    n(t) = E(t) \cos[2\pi f_c t + \psi(t)]
    = n_I(t) \cos 2\pi f_c t - n_Q(t) \sin 2\pi f_c t \tag{6.2}
\]

where, \( E(t) \) is an instantaneous envelope of the noise, and \( \psi(t) \) is a relative initial phase of the noise which is uniformly distributed in \([0, 2\pi]\), i.e.,

\[
    p(\psi) = 1/2\pi \tag{6.3}
\]

\( n_I(t) \) and \( n_Q(t) \) represent the equivalent baseband in-phase (I) and quadrature (Q) noise components respectively, and are expressed as

\[
    n_I(t) = E(t) \cos \psi(t) \tag{6.4}
    \]

\[
    n_Q(t) = E(t) \sin \psi(t) \tag{6.5}
\]

The PDF of the instantaneous noise envelope \( E(t) \) is deduced as [45,48]:

\[
    p(E) = \frac{1}{N} e^{-\Lambda} \sum_{j=0}^{+\infty} \frac{\Lambda^j}{j!} \frac{E}{\sigma^2} \exp\left(-\frac{E^2}{2\sigma^2 N}\right) \tag{6.6}
\]

where

\( \Lambda \): Impulsive index, i.e., product of the received average number of impulses in a second and the duration of the impulse (\( \Lambda \leq 1 \)).

\( N \): Total noise power (\( N = \sigma^2_G + \sigma^2_I \)).

\( \sigma^2_I = (j/\Lambda + \Gamma')/(1 + \Gamma') \)

\( \Gamma' \): Mean power ratio of the Gaussian noise component (\( \sigma^2_G \)) to non-Gaussian impulsive noise component (\( \sigma^2_I \)).
For the calculation of the I-channel or Q-channel symbol error probability, let us derive the PDF of the I-channel (or Q-channel) noise component. The joint PDF of \( n_I(t) \) and \( n_Q(t) \) is derived from;

\[
p(n_I, n_Q) = \frac{p(E, \psi)}{|J(E, \psi)|} \tag{6.7}
\]

Since \( E \) and \( \psi \) are independent random variables, the joint PDF of \( E \) and \( \psi \) is given by

\[
p(E, \psi) = p(E)p(\psi) \tag{6.8}
\]

The Jacobian is calculated as

\[
J(E, \psi) = \begin{vmatrix}
\frac{\partial E}{\partial E} \cos \psi & \frac{\partial E}{\partial \psi} \\
\frac{\partial E}{\partial \sin \psi} & \frac{\partial \sin \psi}{\partial \psi}
\end{vmatrix}
\]

Therefore,

\[
J(E, \psi) = E \tag{6.9}
\]

Since,

\[
E^2(t) = n_I^2(t) + n_Q^2(t) \tag{6.10}
\]

\[
p(n_I, n_Q) = \frac{1}{2\pi N} e^{-\Lambda} \sum_{j=0}^{+\infty} \frac{\Lambda^j}{j!} \frac{1}{\sigma_j^2} \exp\left(-\frac{n_I^2 + n_Q^2}{2\sigma_j^2 N}\right) \tag{6.11}
\]

Since \( n_I \) and \( n_Q \) are independent random variables, the marginal PDF of \( n_I \) (or \( n_Q \)) is obtained as:

\[
p(n_I) = \int_{-\infty}^{+\infty} p(n_I, n_Q)dn_Q
\]

\[
= e^{-\Lambda} \sum_{j=0}^{+\infty} \frac{\Lambda^j}{j!} \frac{1}{\sqrt{2\pi N} \sigma_j^2} \exp\left(-\frac{n_I^2}{2N\sigma_j^2}\right) \tag{6.12}
\]

Note that PDF of the pure Gaussian noise is given by:

\[
p_G(n) = \frac{1}{\sqrt{2\pi \sigma_G^2}} \exp\left(-\frac{n^2}{2\sigma_G^2}\right) \tag{6.13}
\]

In the M-ary QAM, symbol errors occur when the noise exceeds \( d_{\text{min}}/2 \), where \( d_{\text{min}} \) represents the minimum distance between two neighbouring
Figure 6.2: State-space diagram of $M$-ary QAM signal. ($M = 64$ case, top right-hand side one quadrant is shown.) \( S \); Peak envelope of the $M$-ary QAM signal, \( d_{\text{min}} \); Minimum signal distance, \( d_{\text{min}} = \sqrt{2S} / (\sqrt{M} - 1) \).
signal states. The state-space diagram (one quadrant, top right-hand side) of an illustrative M-ary QAM (e.g., 64-QAM) signal is shown in Fig.6.2, where \( S \) represents the peak envelope of the signal. In terms of the peak signal envelope, the minimum signal distance is expressed as:

\[
d_{min} = \frac{\sqrt{2}S}{\sqrt{M} - 1} \tag{6.14}
\]

Therefore, the symbol error probability of \( M \)-ary QAM is obtained as:

\[
P_e = \int_{d_{min}/2}^{+\infty} p(n)dn
\]

\[
= e^{-\Lambda} \sum_{j=0}^{+\infty} \frac{\Lambda^j}{j!} \frac{1}{\sqrt{2\pi N\sigma_j^2}} \int_{d_{min}/2}^{+\infty} \exp(-\frac{n^2}{2N\sigma_j^2})dn
\]

\[
= e^{-\Lambda} \sum_{j=0}^{+\infty} \frac{\Lambda^j}{j!} \frac{1}{2\sqrt{2N\sigma_j}} \operatorname{erfc}\left(\frac{d_{min}}{2\sqrt{2N\sigma_j}}\right)
\]

\[
= \frac{e^{-\Lambda}}{2} \sum_{j=0}^{+\infty} \frac{\Lambda^j}{j!} \operatorname{erfc}\left(\frac{\sqrt{2S}/(\sqrt{M} - 1)}{2\sqrt{2N\sigma_j}}\right)
\]

\[
= \frac{e^{-\Lambda}}{2} \sum_{j=0}^{+\infty} \frac{\Lambda^j}{j!} \operatorname{erfc}\left(\frac{\sqrt{CNR}}{2(\sqrt{M} - 1)\sigma_j}\right) \tag{6.15}
\]

where,

\[
\operatorname{erfc}(x) = \frac{2}{\sqrt{\pi}} \int_{x}^{\infty} \exp(-t^2)dt \tag{6.16}
\]

\[
\text{CNR} = \frac{S^2/2N}{PF^{(M)}} \tag{6.17}
\]

where \( PF^{(M)} \) is a signal Peak Factor of the \( M \)-ary QAM defined as Signal peak power-to-Signal mean-power Ratio of the \( M \)-ary QAM. (e.g., in dB, \( PF^{(16)} = 2.55, PF^{(64)} = 3.67, \) and \( PF^{(256)} = 4.23. \)

In the \( M \)-ary QAM, the number of the closest neighbouring signal state depends on the transmitted signal state, i.e., the number of the signal state which is subject to 2-directional error, 3-directional error, or 4-directional error is \( 4, 4(\sqrt{M} - 2), \) or \( M - 4(\sqrt{M} - 1), \) respectively. Therefore, the
average symbol error probability of M-ary QAM is obtained as:

\[ P(e) = \frac{1}{M} \{ 4 \times 2 P_e + 4(\sqrt{M} - 2) \times 3 P_e + [M - 4(\sqrt{M} - 1)] \times 4 P_e \} = \frac{4(M - \sqrt{M})}{M} P_e \] (6.18)

Hence, the average symbol error probability of M-ary QAM, in the Gaussian/Impulsive noise channel, is given by:

\[ P(e) = \frac{2(M - \sqrt{M})}{M} e^{-\Lambda} \sum_{j=0}^{\infty} \frac{\Lambda^j}{j!} \text{erfc}\left[ \frac{\sqrt{\text{CNR}}}{\sqrt{2(\sqrt{M} - 1)\sigma}} \right] \] (6.19)

Note that the average symbol error probability of M-ary QAM in the Gaussian noise channel is given by:

\[ P(e)^G = \frac{2(M - \sqrt{M})}{M} \text{erfc}\left[ \frac{\sqrt{\text{CNR}}}{\sqrt{2(\sqrt{M} - 1)}} \right] \] (6.20)

### 6.2 ANALYSIS OF THE IMPACT OF THE IMPULSIVE NOISE

Computing Eq.(6.19) for different values of \( \Lambda \) and \( \Gamma' \), we analyze the error performance of M-ary (e.g., 16-, 64-, and 256-) QAM in various non-Gaussian impulsive noise environments.

In Fig.6.3, \( P(e) \) performance is evaluated by assuming the mean power ratio of the Gaussian-to-impulsive noise \( \Gamma' = 1 \) (i.e., 0 dB), and varying the impulsive index (\( \Lambda \)) of the noise in the range \( [10^{-3}, 10^0] \). As the impulsive index \( \Lambda \) becomes smaller, the noise impulsiveness becomes stronger thus causes larger performance degradation. Table 6.1 illustrates this tendency in terms of the instantaneous power ratio \( (I_p) \) of Impulsive-to-Gaussian noise defined as \( 10 \log(1/\Lambda/\Gamma') \). For example, with \( \Gamma' = 1 \) and \( \Lambda = 10^{-3} \), the instantaneous power ratio \( I_p \) is 30 dB, in which case the performance degradation is most significant. In Fig.6.4, impulsive index \( \Lambda \) is assumed to be 1, and the mean power ratio (\( \Gamma' \)) of the Gaussian-to-Impulsive noise varies in the range \( [10^{-2}, 10^1] \), that is, \([-20 \, \text{dB}, 10 \, \text{dB}] \). The result shows
Figure 6.3: $P(e)$ performance of $M$-ary ($M = 16, 64, 256$) QAM in the non-Gaussian impulsive noise environments ($\Gamma' = 1$).
<table>
<thead>
<tr>
<th>Impulsive Index ($\Lambda$)</th>
<th>$I_p = 10 \log(1/\Lambda/\Gamma')$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0 dB</td>
</tr>
<tr>
<td>0.1</td>
<td>10 dB</td>
</tr>
<tr>
<td>0.01</td>
<td>20 dB</td>
</tr>
<tr>
<td>0.001</td>
<td>30 dB</td>
</tr>
</tbody>
</table>

Table 6.1: Instantaneous power ratios of Impulsive-to-Gaussian noise ($I_p$).

that when $\Lambda$ is 1 and $\Gamma'$ becomes larger (e.g., $\Gamma' > 10$), the $P(e)$ performance approaches closely to the case of pure Gaussian noise. This is because when $\Lambda$ equals 1, an impulsive noise occurs continuously, thus the impulsiveness becomes weaker and weaker as $\Gamma'$ becomes larger.

In Fig.6.5, the impulsive index $\Lambda$ is assumed to be 0.1, and $\Gamma'$ varies in $[10^{-2}, 10^1]$. It is found that an upper bound (i.e., the worst case) of the error probability exists for $\Gamma' \leq 0.01$, that is, for the mean power ratio $I_m$ (i.e., $-10 \log \Gamma'$) of the Impulsive-to-Gaussian noise larger than 20 dB. This implies that an impulsive noise having the mean power ratio ($I_m$) larger than 20 dB may approximate an ideal mathematical impulse. In Figs.6.6 and 6.7, $\Lambda$ is assumed to be 0.01 and 0.001, and $\Gamma'$ varies in the range $[10^{-1}, 10^2]$ and $[10^0, 10^3]$, respectively. In both cases, it is found that an upper bound of the error probability exists for $\Gamma' \leq 0.01$ or $I_m \geq 20$ dB. Fig.6.8 summarizes an upper bound error probability of M-ary QAM for different values of impulsive index $\Lambda$.

6.3 CONCLUSION

We analyzed the impact of a non-Gaussian impulsive noise combined with the Gaussian thermal noise on the performance of 16-, 64-, and 256-QAM systems. Our result shows that at low CNR, the $P(e)$ performance is primarily dominated by the Gaussian noise component. At high CNR, however, the non-Gaussian impulsive noise degrades the performance significantly. Therefore, when the noise contains a strong impulsiveness (i.e., $\Lambda \ll, \Gamma' \ll$), the $P(e)$ performance could not be improved much by increas-
Figure 6.4: $P(e)$ performance of $M$-ary $(M = 16, 64, 256)$ QAM in the non-Gaussian impulsive noise environments ($\Lambda = 1$).
Figure 6.5: $P(e)$ performance of $M$-ary ($M = 16, 64, 256$) QAM in the non-Gaussian impulsive noise environments ($\Lambda = 0.1$).
Figure 6.6: $P(e)$ performance of $M$-ary ($M = 16, 64, 256$) QAM in the non-Gaussian impulsive noise environments ($\Lambda = 0.01$).
M-ARY QAM \((M = 16, 64, 256)\)

\[ \Lambda = 0.001 \]

**Note:**
- \( \Gamma' \): Mean power ratio of Gaussian-to-Impulsive noise
- \( \Lambda \): Impulsive index

Figure 6.7: \( P(e) \) performance of \( M \)-ary \((M = 16, 64, 256)\) QAM in the non-Gaussian impulsive noise environments \((\Lambda = 0.001)\).
Figure 6.8: The upper-bound (i.e., the worst case) error probabilities of M-ary ($M = 16, 64, 256$) QAM in the non-Gaussian impulsive noise environments.
ing the CNR. It has been also found that an upper bound of the error probability exists for $\Gamma' \leq 0.01$. 
Chapter 7

CONCLUSIONS AND SUGGESTIONS FOR FURTHER RESEARCH

7.1 CONCLUSIONS

Techniques to improve the performance of spectrally efficient Multi-ary QAM systems operating in channels with various transmission distortions have been studied.

For the operation of M-ary QAM close to the saturation region of the transmit high power amplifiers (HPAs), we proposed a simple, robust Base-band Digital Signal Mapping technique. Our mapped 16-QAM compensates for the system impairments caused by the AM/AM and AM/PM non-linearities of the HPAs, thus enables us to operate the HPAs closer to the saturation region at significantly reduced CNR degradation.

In conjunction with a new signal constellation at the transmitter, we applied an Adaptive Decision Threshold (ADT) detector at the receiver to improve the system performance. As an illustrative result, the CNR degradation of our proposed system operating at 0.9 dB OBO of Siemens TWTA is only 1.9 dB at $P(e) = 10^{-6}$. This corresponds to 7 dB (or even more) CNR improvement as compared to the conventional 16-QAM
system. To maximize the overall system gain, we have found the optimum operating point of the HPAs by trading off the HPA power OBO with the $P(e)$ performance degradation. The optimum operating point is 0.9 dB OBO for Siemens TWTA, and 1.5 dB OBO for Fujitsu GaAs FET. The corresponding overall system gain reduction at $P(e) = 10^{-6}$ is 2.8 dB in both cases. Therefore, for the power efficient operation of the M-ary QAM systems, it is recommended to apply the baseband digital signal mapping technique in conjunction with the ADT detectors.

For the operation of M-ary QAM at the most power efficient mode (i.e., at the full saturation) of HPAs, a new NLA (Non-Linearly Amplified) - M-ary SQAM (Superposed-QAM) has been introduced. The signal wave shape of NLA M-ary SQAM as well as the receiver LPF bandwidth has been optimized to achieve the best $P(e)$ performance. The system sensitivity of NLA-16-SQAM modem is analyzed with respect to the output power level variation, and the propagation time difference of the transmit HPAs.

Without need for the additional spectral shaping filters, NLA-16-SQAM reveals much more compact power spectrum than the conventional 16-QAM or MAMSK (Multi-Amplitude MSK). Using simple 4th order Butterworth LPFs ($BT = 0.535$) in the receiver, NLA-16/64-SQAM ($A = 0.7$) operates within 0.75 dB CNR degradation at $P(e) = 10^{-6}$ in a saturated channel. It has been found that the corresponding overall system gain of NLA-64-SQAM is 5.83 dB or 16.38 dB as compared to the conventional 64-QAM operating with Siemens linearized TWTA or Hughes TWTA, respectively.

To demonstrate the spectral robustness of NLA-16-SQAM, we have further investigated the performance in a nonlinearily amplified multi-channel interference environment in the presence of AWGN, ISI, ACI, and CCI. Various channel conditions, such as the channel spacings between the main and the adjacent channels, and the flat fade on the desired main channel, have been examined. Our results show that in a spectrally congested channel, especially in a fading channel, NLA-16-SQAM system outperforms MAMSK, thus improves the efficiency of a frequency utilization.

The impact of residual amplitude fluctuations of the received carrier after AGC amplifiers, and the transmission system caused phase jitter on the performance of M-ary QAM systems have been studied. We derived
theoretical average symbol error probabilities of 16- and 64-QAM, and also tight-bound symbol error probabilities of 256- and 1024-QAM. To confirm our theoretical results and also the accuracy of the tight-bound symbol error probabilities, we performed computer simulations on 16-, 64-, and 256-QAM systems, and also experimental measurements on 256 kb/s rate 16-QAM modem.

Our study shows that in the M-ary QAM the outermost signal states are most susceptible to the system impairments, thus result in the worst symbol error probability. Whereas, the innermost states are least susceptible, thus result in the best symbol error probability. An overall average symbol error probability, therefore, is mostly determined by the behaviour of the outermost signal states. For this reason, we introduced the ADT (adaptive decision threshold) detectors for the M-ary QAM receiver to improve the system performance. As an illustrative example, for the normalized amplitude $u = 0.16$ (1.29 dB) and $0.24$ (1.87 dB) of the residual amplitude fluctuations in the 16-QAM, the corresponding improvement in CNR at $P(e) = 10^{-6}$ is 1.8 dB and 4.9 dB, respectively. It has been found that the tolerance of M-ary QAM systems to the amplitude fluctuation is inversely proportional to the outermost signal levels of the QAM. That is, the maximum tolerance in terms of the normalized amplitude are 1/3 (2.5 dB), 1/7 (1.16 dB), 1/15 (0.56 dB), and 1/31 (0.28 dB) for 16-, 64-, 256-, and 1024-QAM, respectively. Our experimental results on 16-QAM agree well with the theoretical and computer simulation results, and our tight-bound symbol error probabilities closely fit the exact symbol error probabilities and also computer simulation results.

In the presence of the transmission system caused phase jitter, it has been found that error floors (i.e., residual error rates) exist on the system performance for certain levels of the phase jitter. Such error floors must be considered for the system reliability objective. As an illustrative example, the error floor would occur at around $P(e) = 10^{-6}$ due to the phase jitters of $3.6^\circ$, $1.7^\circ$, $0.85^\circ$, and $0.4^\circ$ rms, or equivalent carrier-to-phase noise power ratios of 24.0, 30.6, 36.6, and 43.1 dB for 16-, 64-, 256-, and 1024-QAM systems, respectively.

Finally, we analyzed the impact of the non-Gaussian impulsive noise (Middleton's Class 'A' noise) combined with the Gaussian thermal noise on
the performance of M-ary QAM systems. $P(e)$ performance of the system is evaluated in terms of CNR, impulsive index ($\Lambda$) of the noise, and power ratio of the Gaussian noise-to-Impulsive noise ($\Gamma'$). Our results show that at low CNR, the $P(e)$ performance is primarily dominated by the Gaussian noise component. At high CNR, however, the non-Gaussian impulsive noise degrades the performance significantly. Therefore, when the noise contains a strong impulsiveness (i.e., $\Lambda \ll, \Gamma' \ll$), the $P(e)$ performance could not be improved much by increasing the CNR. It has been also found that an upper bound on the error probability exists for $\Gamma' \leq 0.01$.

7.2 SUGGESTIONS FOR FURTHER RESEARCH

7.2.1 Bandwidth Compressive
Partial Response M-ary SQAM Systems

The SQAM signals are characterized by their compact output power spectra and ISI- and timing jitter-free transmission properties. Partial Response (PR) systems have been introduced to keep the desirable spectral properties of the signal, to make efficient use of the available bandwidth, and to cope better with the channel distortions [49–54].

In order to improve further the spectral properties of the SQAM systems, the partial response signaling technique may be combined with the baseband signal processing techniques of the SQAM to generate new Partial Response (PR) M-ary SQAM signals. The power spectra of PR-SQAM signals may result in a faster spectral roll-off and a lower out-of-band energy than those of SQAM signals or conventional QPRS (Quadrature PRS) signals.

With the increasing number of the signal levels in the M-ary QAM or M-ary QPRS, the timing jitter of the signal would become more serious, thus the system would be more sensitive to the sampling deviations in the symbol timing recovery (STR) [50,55]. Owing to the timing jitter-free characteristics of the SQAM signals, our proposed PR-SQAM systems may have less timing jitter (or wider horizontal eye openings), and slower rate
of eye closures with the offsets in the sampling time as compared to the conventional QPRS systems, thus may result in better $P(e)$ performance. Operations of PR-SQAM systems in the highly power efficient – saturation mode of HPAs may be also a valuable study [54].

7.2.2 Impact of Residual Amplitude Fluctuations and Transmission System Caused Phase Jitter on the M-ary QPRS Systems

In Chapters 4 and 5, we analyzed the impact of the residual amplitude fluctuations caused by a wide-band flat fading, and the transmission system caused phase jitter on the performance of the M-ary QAM systems.

Since these channel distortions may also cause serious performance degradations in the other multi-ary modulation systems, it might be worthwhile to extend the analysis to M-ary (e.g., $M = 9, 49, 225, \cdots$) QPRS systems, which is another excellent candidate for the digital transmission systems of a high spectral efficiency [56,57].

7.2.3 Performance of M-ary QAM Operating in the Hybrid Microwave Systems

In many transmission systems, the analog and the digital radio systems share the same radio frequency band. For example, the time division multiplexed (TDM) data and the frequency division multiplexed (FDM) voice signals are transmitted simultaneously over the hybrid microwave systems such as 

*Data-under-Voice* (DUV), *Data-in-Voice* (DIV), and *Data-above-Voice* (DAV) systems [58].

Therefore, it might be valuable to analyze the intersystem interference between the analog and the digital radio systems. Such an analysis may include the effect of the FDM voice signals on the performance of the M-ary QAM or the effect of the M-ary QAM on the performance of the FDM voice channels. The optimal operating conditions (e.g., frequency allocation, bandwidth, and transmit power) of the digital data transmission systems may be found throughout the analysis.

203
7.2.4 Combined Coding and Multi-ary Modulation Techniques Associated with Baseband Pulse Shaping

Coded modulation technique has become popular owing to its superior performance in the linear and nonlinear transmission channels as compared to the uncoded ones [59,60,61]. Therefore, it may be desirable to combine the coding (e.g., FEC, convolutional) techniques with the multi-ary modulation (e.g., M-ary QAM/QPRS) techniques in order to improve the performance in the distorted transmission channels.

By means of coding, the information is encoded into the expanded set of the channel signals such that it may keep the maximum free Euclidean distance between the transmitted data sequence. The coded bits are then mapped into the multi-phase/amplitude channel signals to be modulated. Baseband pulse shaping techniques (e.g., SQAM) may also be associated with the coded modulation technique to attain a desirable power spectrum.
Appendix A

DETAILED CIRCUIT DIAGRAMS OF 16-QAM SIGNAL MAPPER

This appendix documents detailed circuit diagrams and parts list required to implement a 16-QAM signal mapper and a nonlinear amplifier simulator whose functional block diagram is illustrated in Fig.2.16 of Chapter 2.

Detailed circuit diagrams of an equivalent baseband 16-QAM signal mapper and a nonlinear amplifier simulator are depicted in Figs.A.1–2 and A.3, respectively. Devices required to build these circuits are listed in Table A.1.
Figure A.1: Circuit diagram of equivalent baseband 16-QAM signal mapper.
Figure A.2: Circuit diagram of equivalent baseband 16-QAM signal mapper.
Figure A.3: Circuit diagram of nonlinear amplifier simulator.
<table>
<thead>
<tr>
<th>Device</th>
<th>Function</th>
</tr>
</thead>
<tbody>
<tr>
<td>TMS32010</td>
<td>Digital Signal Processor (DSP)</td>
</tr>
<tr>
<td>28S86</td>
<td>Programmable Read Only Memory (PROM)</td>
</tr>
<tr>
<td>74LS138</td>
<td>Port Address (PA) Decoder</td>
</tr>
<tr>
<td>74374</td>
<td>Octal D-type transparent Latches</td>
</tr>
<tr>
<td>ADC80-12</td>
<td>12-bit A/D Converter</td>
</tr>
<tr>
<td>AD567</td>
<td>12-bit D/A Converter</td>
</tr>
<tr>
<td>74393</td>
<td>Dual 4-bit Binary Counters</td>
</tr>
<tr>
<td>74LS74</td>
<td>Dual D-type Flip-Flops</td>
</tr>
<tr>
<td>74LS164</td>
<td>8-bit Parallel-out Serial Shift Registers</td>
</tr>
<tr>
<td>LF398</td>
<td>Sampler and Holder (S/H)</td>
</tr>
</tbody>
</table>

Table A.1: Required devices and functions to implement a 16-QAM signal mapper.
Appendix B

COMPUTER SIMULATION PROGRAMS

Theoretical analysis of the performance of communication systems are not feasible in many situations, especially when the channel contains various sources of nonlinearities and/or interferences. For the evaluation of such complex systems, computer simulations have been widely used to provide meaningful results.

We have evaluated the performance of M-ary QAM/SQAM systems in various channel conditions with the aid of computer simulation. Error probability of the system, power spectrum, eye diagram, waveshape and signal state space diagram can be obtained from the simulation. The programs are written in FORTRAN and run on the Amdahl 470/V8 computer.

Followings are the list of our computer programs used for the simulation.

1. 16NLA FORTRAN:
   Map optimum signal constellations of 16-QAM in a nonlinearly amplified channel.
2. 16SQ FORTRAN:
Evaluate the performance of NLA-16-SQAM and MAMSK in a hardlimited channel.

3. ACI FORTRAN:
Evaluate the performance of NLA-16-SQAM and MAMSK in a hardlimited multichannel interference environment.

4. 256QC FORTRAN:
Evaluate the performance of 16, 64, 256-QAM in the presence of residual amplitude fluctuations and AWGN.

5. 16AM FORTRAN:
Calculate the best, the worst and average symbol error probabilities of 16-QAM in the presence of residual amplitude fluctuations and AWGN.

6. 256A FORTRAN:
Calculate the best, the worst and tight-bound symbol error probabilities of 256-QAM in the presence of residual amplitude fluctuations and AWGN.

7. 16PM FORTRAN:
Calculate the best, the worst and average symbol error probabilities of 16-QAM in the presence of Gaussian distributed phase jitter and AWGN.

8. 256P FORTRAN:
Calculate the best, the worst and tight-bound symbol error probabilities of 256-QAM in the presence of Gaussian distributed phase jitter and AWGN.

9. IMPULS FORTRAN:
Calculate the symbol error probabilities of $\frac{1}{4}$, 16, 64, and 256-QAM in the presence of non-Gaussian impulsive noise.
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C   SIGNAL MAPPING OPTIMIZATION OF 16-QAM
C   FOR NON-LINEAR MODE OPERATION
C   TX. POWER AMPLIFIERS AVAILABLE ARE: HUGHES HPA/TWTA,
C   LINEARIZED SIEMENS-189 TWTA, FUJITSU GAAS FET.
C   ADT DETECTOR IS AVAILABLE AT RX.
C   ++++++++ FILE NAME : 16NLA FORTRAN A1 ++++++++ 
C+++++++++++++++++++++++++++++++++++++++++++++++++++++++++++++
C
COMPLEX DATA(8192),DATA1(8192),DATA2(8192),TF(8192),PS1,PS2
DIMENSION PEI(32),EBNO(32),NI(512),NQ(512)
DIMENSION XARRAY(32),YARRAY(32)
DIMENSION IWK(17),PO(257),TPO(257)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYM,BAUD
COMMON /BB/B
COMMON /CC/C
COMMON /DD/D
LDIM=8192
NSYM=512
LSAMPL=16
NSNR=32
BAUD=512.
C
FBW=256.
IORDER=4
ALPHA=0.4
IOFF=0
BAKOFF=15.0
NRUNS=2
C
++ DATA=( B*DATA1 + C*DATA2 ), ADT-THD-LEVEL=D,
C ++ FOR OPTIMAL MAPPER ; ENLARGE COEFF. FOR OUTERMOST 4 STATES=E,
C + ROTATION(DEG) FOR INNER 12 STATES=DEG1,
C + ROTATION FOR OUTERMOST 4 STATES=DEG2
C
B=1.0
C=2.0
D=1.0
E=1.00
DEG1=0.0
DEG2=0.0

C

DO 100 NR=1,NRUNS
OFF=FLOAT(IOFF)/FLOAT(LSAMPL)
WRITE(6,132) ALPHA,OFF,B,C
132 FORMAT(7H ALPHA=,F5.3,5X,5H OFF=,F5.3,5X,3H B=,F5.3,5X,3H C=,F5.3)
WRITE(6,133) D,DEG1,DEG2
133 FORMAT(7X,5H THD=,F10.5,5X,6H DEG1=,F9.4,5X,6H DEG2=,F9.4)
CALL LOAD16(DATA1,DATA2,NL,NQ)
PSFT1=DEG1/180.*3.14159
PSFT2=DEG2/180.*3.14159
CALL SUMM(DATA,DATA1,DATA2)
DO 22 I=1,LDIM
PS1=CMPLX(COS(PSFT1),SIN(PSFT1))
PS2=CMPLX(COS(PSFT2),SIN(PSFT2))
ENV=CABS(DATA(I))
MAX=SQR(18.)
MIN=SQR(2.)
IF(ENV.LT.MAX) GOTO 11
C ——— ROTATE OUTER 4 STATES ———
DATA(I)=DATA(I)*PS2
GOTO 12
C ——— ROTATE INNER 12 STATES ———
11 DATA(I)=DATA(I)*PS1
12 IF(ENV.LT.MAX) GOTO 22
C ——— ENLARGE OUTER 4 STATES AMP. ———
DATA(I)=E*DATA(I)
22 CONTINUE
C CALL FSPACE(DATA,NR,NRUNS,8)
CALL RCOSTX(TF)
CALL FILTER(DATA,TF)
C ——— SELECT TX. POWER AMP. ———
CALL HPA(DATA,BAKOFF)
C CALL TWT(DATA,BAKOFF)
C CALL TWTMOD(DATA,BAKOFF)
C CALL FETMOD(DATA,BAKOFF)
C ——— END OF TX. ———
CALL ENERGY(DATA,EB)
CALL RCOSRX(TF)
CALL HHGG(TF,PNOISE)

213
CALL FILTER(DATA,TF)
CALL SYNCRO(DATA,PNOISE,N1,NQ,MI,MQ,EB)

C ----- DRAW SUBRoutines ------
C
CALL PSpace(DATA, NR, NRUNS, MI)
CALL DECODE(DATA, PNOISE, N1, NQ, MI, MQ, EBN0, PE, ELEB)
DO 99 I=1, NSNR
   XARRAY(I)=EBNO(I)
99 YARRAY(I)=PEI(I)

C

C

CALL DRAWCN(XARRAY, YARRAY, NR, NRUNS)
C
CALL EYEQ(DATA)

C

C

D=D-0.15
C
E=E+0.05
C
DEG1=DEG1+1.0
C
DEG2=DEG2+1.0
C
BAKOFF=BAKOFF-2.0
C

100 CONTINUE
   STOP
   END
C

C********************************************************************
C
C          GENERATE EQUIVALENT BASEBAND 16-QAM SIGNALS.
C********************************************************************

SUBROUTINE LOAD16(DATA1, DATA2, NI, NQ)
COMPLEX DATA(1), DATA(1), DATA(1)
DIMENSION NY(11), NI(1), NQ(1), NX(2048)
DIMENSION NI(1), NQ(1), NQ(512), NQ(512), NQ(512), NQ(512)
COMMON /NUMB1/ FBW, ALPHA, LDIM, IOff, LSAMPL
COMMON /BB/B
COMMON /CC/C
DATA NY(1), NY(2), NY(3), NY(4), NY(5), NY(6), NY(7), NY(8), NY(9),
  NY(10), NY(11), -1, 1, 1, 1, -1, 1, -1, 1, -1/
DATA JLast, JtAp/11, 1/
KKK=2**JLast
KK=KKK-(1+JLast)
DO 6 I=1, JLast
   NX(I)=NY(I)
6   CONTINUE
C
C GENERATE $2^{**JLAST-1}$ LENGTH PRBS USING GIVEN DATA AND 
C GENERATOR POLYNOMIAL.

C

DO 1 J=1,KK
   I=J+JLAST
   NX(I)=NX(J)*NX(J+2)
   NX(I)=0-NX(I)
1 CONTINUE

C GENERATE 4 INTERLEAVED VERSIONS OF 
C PRS OF LENGTH $2^{**JLAST}$.
C

DO 2 I=1,512
   K3=4*I-3
   K4=4*I-2
   K5=4*I-1
   K6=4*I
   IF(K6.EQ.KKK) K6=1
   NI1(I)=NX(K3)
   NQ1(I)=NX(K4)
   NI2(I)=NX(K5)
   NQ2(I)=NX(K6)
   NI(I)=NI1(I)+NI2(I)**2
   NQ(I)=NQ1(I)+NQ2(I)**2
2 CONTINUE

C LOAD INTO SAMPLE ARRAY, 16 SAMPLES PER SYMBOL
C

J1=(I-1)*LSAMPL+1
   J2=I*LSAMPL
DO 3 J3=J1,J2
   DATA1(J3)=CMPLX(FLOAT(NI1(I)),FLOAT(NQ1(I)))
3 DATA2(J3)=CMPLX(FLOAT(NI2(I)),FLOAT(NQ2(I)))
2 CONTINUE
IF(IOFF.EQ.0) RETURN
LL=LDIM-1
DO 4 I=1,IOFF
   XX=AIMAG(DATA1(1))
   YY=AIMAG(DATA2(1))
DO 5 K=1,LL
   DATA1(K)=CMPLX(REAL(DATA1(K)),AIMAG(DATA1(K+1)))
5 DATA2(K)=CMPLX(REAL(DATA2(K)),AIMAG(DATA2(K+1)))
DATA1(LDIM)=CMPLX(REAL(DATA1(LDIM)),XX)
4 DATA2(LDIM)=CMPLX(REAL(DATA2(LDIM)),YY)
RETURN
END

C

*******************************************************************************

SUBROUTINE SUMM(DATA,DATA1,DATA2)
*******************************************************************************

COMPLEX DATA(1),DATA1(1),DATA2(1)
COMMON /NUMB1/ FBW,ALPHA,LDIM,JOFF,LSAMPL
COMMON /BB/B
COMMON /CC/C
DO 1 I=1,LDIM
DATA2(I)=DATA2(I)*C
DATA(I)=DATA1(I)*B+DATA2(I)
1 CONTINUE
RETURN
END

*******************************************************************************

C RAISED COSINE FILTER WITH ARBITRARY ALPHA AND X/SINX
*******************************************************************************

SUBROUTINE RCOSTX(TF)
COMPLEX TF(1)
COMMON /NUMB1/ FBW,ALPHA,LDIM,JOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
SBANDW=BAUD*LSAMPL
NO=LDIM/2
NO1=NO+1
IF (ALPHA.EQ.0) ALPHA=0.0001
FN=LDIM*(FBW/SBANDW)
F1=(1.-ALPHA)*FN
F2=(1.+ALPHA)*FN
IFN=IFIX(FN)
IF1=IFIX(F1)+1
IF2=IFIX(F2)+1
A1=3.1415926/(2.*FLOAT(IFN))
DO 8 I=2,IF1
TF(I)=CMPLX(1.0,0.0)
8 J=I-1
C ——— X/SIN(X) EQUALIZATION ———
A2=(FLOAT(J)*A1)/(SIN(FLOAT(J)*A1))
TF(I) = CMPLX(A2,0.0)
8 CONTINUE
   JK = IF1 + 1
   DO 9 J = JK, IF2
      I = J - 1
C ---- ROOT OF RAISED COSINE ----
   A3 = (FLOAT(I) * A1) / SIN(FLOAT(I) * A1)
   A = (3.1415926 / (2.0 * ALPHA)) * ((FLOAT(I) / FLOAT(IFN)) - 1.)
   TF(J) = CMPLX(SQRT(0.5 * (1.0 - SIN(A))), 0.0)
   TF(J) = TF(J) * CMPLX(A3, 0.0)
9 CONTINUE
   JH = IF2 + 1
   DO 10 I = JH, NO1
      TF(I) = CMPLX(0.0, 0.0)
10 CONTINUE
   NO2 = NO1 + 1
   DO 5 I = NO2, LDIM
      TF(I) = CONJG(TF(LDIM + 2 - I))
5 CONTINUE
   RETURN
END
C=============================================
C RAISED COSINE FILTER WITH ARBITRARY ALPHA
C=============================================

SUBROUTINE RCOSRX(TF)
COMPLEX TF(1)
COMMON /NUMB1/ FBW, ALPHA, LDIM, IOFF, LSAMPL
COMMON /NUMB2/ NSNR, NSYM, BAUD
SBANDW = BAUD * LSAMPL
NO = LDIM / 2
NO1 = NO + 1
IF (ALPHA.EQ.0) ALPHA = 0.0001
FN = LDIM * (FBW / SBANDW)
F1 = (1. - ALPHA) * FN
F2 = (1. + ALPHA) * FN
IFN = IFIX(FN)
IF1 = IFIX(F1) + 1
IF2 = IFIX(F2) + 1
C
C AMPLITUDE CHARACTERISTIC
C
217
A1=3.141592/(2.*FLOAT(INF))
DO 8 I=2,IF1
  TF(I)=CMPLX(1.0,0.0)
  J=I-1
  TF(J)=CMPLX(1.,0.0)
8 CONTINUE
  JK=IF1+1
  DO 9 J=JK,IF2
    I=J-1
7
C ROOT OF RAISED COSINE
C
A=(3.141592/(2.0*ALPHA))*((FLOAT(I)/FLOAT(INF))-1.)
TF(J)=CMPLX(SQRT(0.5*(1.0-SIN(A))),0.0)
9 CONTINUE
  JH=IF2+1
  DO 10 I=JH,NO1
    TF(I)=CMPLX(0.0,0.0)
10 CONTINUE
  NO2=NO1+1
  DO 5 I=NO2,LDIM
    TF(I)=CONJG(TF(LDIM+2-I))
5 CONTINUE
  RETURN
END
C******************************************************************************
C FILTERING PROCESS ON THE DATA SEQUENCE.
C******************************************************************************

SUBROUTINE FILTER(DATA,TF)
COMPLEX DATA(1),TF(1)
DIMENSION IWK(17)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
CALL FFT2C(DATA,13,IWK)
DO 1 I=1,LDIM
  1 DATA(I)=CONJG(DATA(I)*TF(I))
CALL FFT2C(DATA,13,IWK)
DO 2 I=1,LDIM
  2 DATA(I)=CONJG(DATA(I))/FLOAT(LDIM)
RETURN
END
C*****************************************************************************
C           COMPUTE THE EFFECTIVE NOISE (PNOISE)
C AT THE OUTPUT OF THE RECEIVE FILTER
C*****************************************************************************
SUBROUTINE HHGG(TF,PNOISE)
COMPLEX TF(1)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
SUM=0.0
SBANDW=BAUD*LSAMPL
DO 1 L=1,LDIM
HH=(CABS(TF(L)))**2
 1 SUM=SUM+HH
PNOISE=SUM*SBANDW/FLOAT(LDIM)/2.
RETURN
END

C*****************************************************************************
C           CALCULATE BIT ENERGY
C*****************************************************************************
SUBROUTINE ENERGY(DATA,EB)
COMPLEX DATA(1)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
WATTS=0.
DO 1 I=1,LDIM
WATTS=WATTS+((CABS(DATA(I)))**2.)
 1 CONTINUE
WATTS=WATTS/(FLOAT(LDIM))
EB=WATTS/(4.*BAUD)
RETURN
END

C*****************************************************************************
C           SYNCHRONIZE THE RECEIVED DATA.
C*****************************************************************************
SUBROUTINE SYNCRO(DATA,PNOISE,NI,NQ,MI,MQ,EB)
INTEGER Q?FLAG
COMPLEX DATA(1),AMP
DIMENSION NI(1),NQ(1)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
COMMON /INPUT/ NI,NQ

219
COMMON /DD/D
NERROR=0
IF (IOFF.EQ.0) GO TO 111
DO 6 K=1,IOFF
XX=AIMAG(DATA(LDIM))
LL=LDIM-1
DO 7 KK=1,LL
LDKK=LDIM+1-KK
7 DATA(LDKK)=CMPLX(REAL(DATA(LDKK)),AIMAG(DATA(LDKK-1)))
6 DATA(1)=CMPLX(REAL(DATA(1)),XX)
C ------- SYNCHRONIZE THE RECEIVED DATA -------
111 NOLD=0
NOF=0
K=1
300 CONTINUE
NEW=0
DO 200 J=1,NSYMB
J1=K+(J-1)*LSAMPL
IF(J1.GT.LDIM) J1=J1-LDIM
AXBAR=REAL(DATA(J1))
AYBAR=AIMAG(DATA(J1))
SS=AXBAR*NI(J)
IF (SS .GT. 0.) NEW=NEW+1
SS=AYBAR*NQ(J)
IF (SS .GT. 0.) NEW=NEW+1
200 CONTINUE
IF(NOLD.GE.NEW) GO TO 399
NOLD=NEW
NOF=K
399 K=K+1
LFU=2*NSYMB
NSFT=LSAMPL*5
IF (NOLD.LT.LFU.AND.K.LE.NSFT) GO TO 300
NOF=NOF-1
KT=K-2
WRITE(6,56) NOLD,NOF,KT
56 FORMAT(1X,'NO OF MAX. CORRECT SYMBOLS = ',I4,1X,/*NO OF SHIFT IN SAMPLES FOR MAX. CORRECT SYMBOLS= ',I4,1X,/*1X,TOTAL NO OF SHIFT FOR SEARCHING = ',I4)
C ------------------------------------------------------

IF (NOF.EQ.0) GO TO 230

220
LO= LDIM-1
DO 250 I=1, NOF
AMP= DATA(1)
DO 240 J=1, LO
DATA(J)= DATA(J+1)
240 CONTINUE
DATA(LDIM)= AMP
250 CONTINUE

C ----------- OPTIMIZE THE SAMPLING INSTANT -----------
230 MI=1
MQ=1
EOI= FLOAT(NSYMB) + 1.
EOQ= FLOAT(NSYMB) + 1.
VARIAN= PNOISE*EB/19.
SIGMA= SQRT(VARIAN)
DO 80 J=1, LSAMPL
EI=0.
EQ=0.
DO 70 K=1, NSYMB
C VERIFY THAT NO SAMPLED SYMBOL IS IN ERROR
J1=(K-1)*LSAMPL+J
AXBAR=(REAL(DATA(J1))+REAL(DATA(J1+1)))/2.
AYBAR=(AIMAG(DATA(J1))+AIMAG(DATA(J1+1)))/2.
INDEXI=0
SS=FLOAT(NI(K))*AXBAR
IF (SS .LT. 0.) INDEXI=1
AMPX=ABS(AXBAR)
AMPI=ABS(FLOAT(NI(K)))
THR1I= AMPI - 1.0
THR2I= AMPI + 1.0
C ----------- ADJUST THD LEVELS -----------
THR1I= THR1P*D
THR2I= THR2P*D
IFLAG=0
IF (AMPI.EQ. 3.0) GO TO 30
IF (( AMPX.GE.THR2I).OR.(AMPX.LE.THR1I)) INDEXI=1
GO TO 40
30 IF((AMPX.LE.THR1I) INDEXI=1
IFLAG=1
40 CONTINUE
INDEXQ=0
SS = FLOAT(NQ(K)) * AYBAR
IF (SS .LT. 0.) INDEXQ = 1
AMPY = ABS(AYBAR)
AMPQ = ABS(FLOAT(NQ(K)))
THR1Q = AMPQ - 1.0
THR2Q = AMPQ + 1.0

C ------------ ADJUST THD LEVELS ------------

THR1Q = THR1Q * D
THR2Q = THR2Q * D
Q?FLAG = 0
IF (AMPQ .EQ. 3.0) GO TO 50
IF ((AMPY .GE. THR2Q) .OR. (AMPY .LE. THR1Q)) INDEXQ = 1
GO TO 50

50 IF (AMPY .LE. THR1Q) INDEXQ = 1
Q?FLAG = 1
CONTINUE

IF (INDEXEQ .EQ. 1) EI = EI + 1.
IF (INDEXEQ .EQ. 1) EQ = EQ + 1.

C

C COMPUTE THE PROBABILITY OF ERROR FOR THIS SYMBOL AT EB/NO=12dB

C

D1 = ABS(AMPX - THR11)
ARG = D1 / (SIGMA * SQRT(2.))

C CHECK IF ARG IS LARGE IN WHICH CASE PE IS INSIGNIFICANT.

C

IF (ARG .GT. 12.) ARG = 12.
EI = EI + ERFC(ARG) / 2.
IF (Q?FLAG .EQ. 1) GO TO 65
D2 = ABS(THR2Q - AMPX)
ARG = D2 / (SIGMA * SQRT(2.))
IF (ARG .GT. 12.) ARG = 12.
EI = EI + ERFC(ARG) / 2.

65 D1 = ABS(AMPY - THR1Q)
ARG = D1 / (SIGMA * SQRT(2.))
IF (ARG .GT. 12.) ARG = 12.
EQ = EQ + ERFC(ARG) / 2.
IF (Q?FLAG .EQ. 1) GO TO 70
D2 = ABS(THR2Q - AMPY)
ARG = D2 / (SIGMA * SQRT(2.))
IF (ARG .GT. 12.) ARG = 12.
EQ = EQ + ERFC(ARG) / 2.
70 CONTINUE
   IF (EOL.LE.EL) GO TO 75
   EOL=EL
   MI=J
75 CONTINUE
   IF (EOQ.LE.EQ) GO TO 80
   EOQ=EQ
   MQ=J
80 MOFF=INT(MI-MQ)
   RETURN
END

C
C******************************************************************************
C
C DECODE THE RECEIVED DATA.
C******************************************************************************

SUBROUTINE DECODE(DATA,PNOISE,N1,NQ,MI,MQ,EBNO,PE,EB)
INTEGER Q7FLAG
COMPLEX DATA(1),AMP
DIMENSION EBNO(1),PE(1),N1(1),NQ(1)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD

C
C COMMON /INPUT/ NI,NQ
C COMMON /DD/D
NERROR=0
DO 1 I=1,NSNR
   PE(I)=0.
1   CONTINUE
DO 2 K=1,NSYMB
   J1=(K-1)*LSAMPL+MI
   J2=(K-1)*LSAMPL+MQ
   AXBAR=REAL(DATA(J1))+REAL(DATA(J1+1)))/2.
   AYBAR=AIMAG(DATA(J2))+AIMAG(DATA(J2+1)))/2.
   INDEXI=0
   SS=FLOAT(NI(K))*AXBAR
   IF (SS .LT. 0.) INDEXI=1
   AMPX=ABS(AXBAR)
   AMPI=ABS(FLOAT(NI(K)))
   THR1I=AMPI - 1.0
   THR2I=AMPI + 1.0
   THR1I=THR1I*D
   THR2I=THR2I*D
   Q7FLAG=0
IF (AMPLEQ .30) GO TO 30
IF (AMPX.GE.THR21).OR.(AMPX.LE.THR11)) INDEXI=1
GO TO 40
30 IF(AMPX.LE.THR11) INDEXI=1
Q7FLAG=1
40 CONTINUE
INDEXQ=0
SS=FLOAT(NQ(K))*AYBAR
IF(SS .LT. 0.) INDEXQ=1
AMPY=ABS(AYBAR)
AMPQ=ABS(FLOAT(NQ(K)))
THRIQ=AMPQ - 1.0
THR2Q=AMPQ + 1.0
THRIQ=THRIQ*D
THR2Q=THR2Q*D
Q7FLAG=0
IF (AMPQ.EQ. 3.0) GO TO 50
IF ((AMPY.GE.THR2Q).OR.(AMPY.LE.THR1Q)) INDEXQ=1
GO TO 60
50 IF(AMPX.LE.THR1Q) INDEXQ=1
Q7FLAG=1
60 CONTINUE
IF((INDEXLEQ.1).OR.(INDEXQ.EQ.1)) NERROR=NERROR+1

C __________________________
C COMPUTE THE PROBABILITY OF ERROR FOR THIS SYMBOL.
C THE VARIABLES ARE AS FOLLOWS: M IS EB/NO IN dB; PNOISE IS
C 1/2*INTEGRAL ( [H(F)**2] DF); AND VARIAN = NO*PNOISE.
C FOR THE END POINTS (Q7FLAG=1) THE PE=1/2*ERFC(D1/(SIGMA*SQRRT(2)))
C AND FOR THE INNER POINTS THE PE=1/2*ERFC(D1/(SIGMA*SQRRT(2))) +
C 1/2*ERFC(D1/(SIGMA*SQRRT(2))).
C __________________________
C
DO 4 M=1,NSNR
VARIAN=PNOISE*EB**(10.*(-0.1*FLOAT(M)))
SIGMA=SQRRT(VARIAN)
D1=ABS(AMPX-THR11)
ARG=D1/(SIGMA*SQRRT(2))
C
IF (INDEXLEQ.1) GO TO 153
IF (ARG.GT.12.) ARG=12.
PEI=ERFC(ARG)/2.
IF (Q7FLAG .EQ. 1) GO TO 100
D2=ABS(THR2I-AMPX)
ARG=D2/(SIGMA*SQR(T(2)))
IF (ARG.GT.12.) ARG=12.
PEI=PEI + ERFC(ARG)/2.
100 D1=ABS(AMPY-THR1Q)
ARG=D1/(SIGMA*SQR(T(2)))
IF (INDEXQ.EQ.1) GO TO 153
IF (ARG.GT.12.) ARG=12.
PEQ=ERFC(ARG)/2.
IF (Q7FLAG .EQ. 1) GO TO 110
D2=ABS(THR2Q-AMPY)
ARG=D2/(SIGMA*SQR(T(2)))
IF (ARG.GT.12.) ARG=12.
PEQ=PEQ + ERFC(ARG)/2.
110 IF(PEILT.1.E-15) PEI=0.
IF(PEQLT.1.E-15) PEQ=0.
4 PE(M)=PE(M)+PEI+PEQ
2 CONTINUE
DO 5 I=1,NSNR
PE(I)=PE(I)/FLOAT(NSYM)
5 EBN0(I)=FLOAT(I)
PRINT 150
150 FORMAT(5X,'EB/N0',10X,'PROB. OF ERROR',/)
WRITE (6,151) (EBN0(I),PE(I),I=1,NSNR)
151 FORMAT(5X,F5.1,10X,E13.3)
WRITE (6,152) NERROR
152 FORMAT(5X,'ERRORS=','I5',/)
GO TO 155
153 PRINT 154
154 FORMAT (10X,'SYMBOL WAS IN ERROR, RUN WAS TERMINATED',/)
155 CONTINUE
RETURN
END

C***************************************************************
C
C DRAW PROBABILITY OF ERROR CURVES FOR P(E)
C AS LOW AS 1.0E-8 AND C/N RATIO AS HIGH AS 35 DB.
C***************************************************************

SUBROUTINE DRAWCN(XARRAY,YARRAY,JCURV,ILAST)
DIMENSION XARRAY(34),YARRAY(34),X(29),Y(29)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYM,BAUD

225
M=NSNR
CF=4.
DO 1 K=1,NSNR
   KK=K+5
   IF(KK.GT.M) KK=M
   XARRAY(K)=XARRAY(KK)
   YARRAY(K)=YARRAY(KK)
   XARRAY(K)=XARRAY(K) + 10**(Aalog10(CF))
   IF(YARRAY(K).GE.1.0) YARRAY(K)=1.0
   IF(YARRAY(K).LE.1.0E-8) GOTO 3
GOTO 1
1 M=K-1
   GOTO 4
3 CONTINUE
4 CONTINUE
   I=M+1
   II=I+1
   XARRAY(I)=12.0
   XARRAY(II)=2.0
   YARRAY(I)=1.E-8
   YARRAY(II)=0.4
   IF(JCURV.GT.1) GOTO 2
C ------- ESTABLISH THE SURFACE AREA -------
   CALL PLOTS(30.0,27.5)
   CALL FACTOR(0.8)
C ------- ESTABLISH THE ORIGIN -------
   CALL PLOT(3.0,8.5,-3)
C ------- DRAW THE LOGARITHMIC Y-AXIS -------
   CALL LGAXS(0.0,0.0,20.0,PROBABILITY OF ERROR,20.0,
   +50.0,1.0E-8,4)
C ------- DRAW THE LINEAR X-AXIS -------
   CALL AXIS(0.0,0.0,9HC/N IN dB,-9,
   +15.0,0.0,12.0,2.0)
   GO TO 2
2 CONTINUE
   CALL LGLIN(XARRAY,YARRAY,M,1,2,JCURV,1)
   IF(JCURV.EQ.JLAST) CALL PLOT(0.0,0.0,999)
   RETURN
END
C*****************************************************************************
C SIMULATE THE EQUIVALENT BASEBAND

226
C NONLINEARITY OF THE INTELSAT-V HPA.
C***************************************************************************
SUBROUTINE HPA(DATA,BAKOFF)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
COMPLEX DATA(1)
REAL MP1
DATA ZPMAX,ZQMAX,VMAX,VO/1.068,0.928,1.414,1.1/
DATA A1,A3,A5,A7,A9,A11,A13,A15,A17/1.916288,-9.358132E-1,
71.942392E-1,-2.832262E-2,1.840305E-2,-4.2823407E-3,0,0,0,0,0,0/
DATA B1,B3,B5,B7,B9,B11,B13,B15,B17/-5.96599E-2,1.907669,
-1.846724,8.615552E-1,-1.999954E-1,1.815157E-2,0,0,0,0,0,0/
P(X)=(((A17*X**2+A15)*X**2+A13)*X**2+A11)*X**2+A9)*
?)X**2*A7)*X**2*A5)*X**2*A3)*X**2+A1
ZP(X)=P(X)*X
Q(X)=(((B17*X**2+B15)*X**2+B13)*X**2+B11)*X**2+B9)*
?)X**2+B7)*X**2+B5)*X**2+B3)*X**2+B1
ZQ(X)=Q(X)*X
Z(X)=SQRT(ZP(X)**2+ZQ(X)**2)
TWTIN=VO*10.**(-BAKOFF/20.)
PSHIFT=ATAN(ZQ(TWTIN*1.414)/ZP(TWTIN*1.414))
SSS=20.*alog10(ZP(TWTIN))
WRITE (6,2) BAKOFF,PSHIFT,SSS
2 FORMAT(5X,'HPA INPUT B/0:,F6.2,' DB',
?" OUTPUT PHASE ',F6.2,' OUTPUT B/0:,F6.2,' DB')
CALL POWER(DATA,MP1,PF)
FNORMI=TWTIN/SQRT(MP1)
DO 11 I=1,LDIM
DATA(I)=DATA(I)*FNORMI
11 CONTINUE
DO 10 I=1,LDIM
X=REAL(DATA(I))
Y=AIMAG(DATA(I))
R=SQRT(X**2+Y**2)
IF(R.GT.VMAX) GOTO 12
DATA(I)=CMPLX(P(R)*X-Q(R)*Y,P(R)*Y+Q(R)*X)
CO TO 10
12 DATA(I)=CMPLX(ZPMAX*X-ZQMAX*Y,ZQMAX*X+ZPMAX*Y)/R
CONTINUE
DO 16 I=1,LDIM
DATA(I)=DATA(I)/FNORMI/A1
227
CONTINUE
CALL PHASE(DATA,PSHIFT)
RETURN
END

C*****************************************************************************
C SIMULATE THE EQUIVALENT BASEBAND
C NONLINEARITY OF THE INTLSAT-V TWTA.
C*****************************************************************************

SUBROUTINE TWT(DATA,BAKOFF)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD

C
C REAL MP1
DATA ZPMAX,ZQMAX,VMAX,VO/.986,1.017,1.414,1./
 DATA A1,A3,A5,A7,A9,A11,A13,A15,A17/2.185509,-1.934083,
 ?9.807836E-1,-2.313386E-1,1.963255E-2,0.0,0.0,0.0,0.0/
 DATA B1,B3,B5,B7,B9,B11,B13,B15,B17/2.641891E-1,1.338875,
 ?9.973087E-1,2.675812E-1,-2.400961E-2,0.0,0.0,0.0,0.0/
 DATA P(X)=(((((A17*X**2+A15)*X**2+A13)*X**2+A11)*X**2+A9)*
 ?X**2+A7)*X**2+A5)*X**2+A3)*X**2+A1
 DATA ZP(X)=P(X)*X
 DATA Q(X)=(((((B17*X**2+B15)*X**2+B13)*X**2+B11)*X**2+B9)*
 ?X**2+B7)*X**2+B5)*X**2+B3)*X**2+B1
 DATA ZQ(X)=Q(X)*X
 DATA Z(X)=SQRZ(ZP(X)**2+ZQ(X)**2)
 DATA TWTIN=VO**10.**(-BAKOFF/20.)
 DATA PPP=ZQ(TWTIN**1.414)/ZP(TWTIN**1.414)
 DATA PSHIFT=ATAN(PPP)
 DATA SSS=20.**(ALOG10(ZP(TWTIN)))
 WRITE (6,2) BAKOFF,PSHIFT,SSS

2 FORMAT(5X,'TWT INPUT B/O:','F6.2,' DB',
 ?' OUTPUT PHASE ','F6.2,' OUTPUT B/O:','F6.2,' DB'/)
 CALL POWER(DATA,MP1,PF)
 FNORMI=TWTIN/SQRZ(MP1)
 DO 11 I=1,LDIM
 DATA(I)=DATA(I)*FNORMI
11 CONTINUE
 DO 10 I=1,LDIM
 X=REAL(DATA(I))
 Y=AIMAG(DATA(I))
 R=SQRZ(X**2+Y**2)

228
IF(R.GT.VMAX) GOTO 12
DATA(j)=CMPLX(P(R)*X-Q(R)*Y,P(R)*Y+Q(R)*X)
GO TO 10
12 DATA(j)=CMPLX(ZPMAX*X-ZQMAX*Y,ZQMAX*X+ZPMAX*Y)/R
10 CONTINUE
DO 16 I=1,LDIM
DATA(I)=DATA(I)/FNORMI/A1
16 CONTINUE
CALL PHASE(DATA,PSHIFT)
RETURN
END

C******************************************************************************
C SIMULATE THE EQUIVALENT BASEBAND
C NONLINEARITY OF GAAS FET (FUJITSU) AMPLIFIER.
C******************************************************************************

SUBROUTINE FETMOD(DATA,BAKOFF)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYM,BAUD
COMPLEX DATA(I)
REAL MP1,DEL
DATA A1,A3,A5,A7,A9,A11/1.06930256,0.81646901,-1.73904705,
?,1.49673802,-0.59510630,0.08753961/
DATA B1,B3,B5,B7,B9,B11/0.0069088,0.03509040,0.22796949,
?,0.30945796,0.14790106,-0.02370892/
DATA ZPMAX,ZQMAX,VMAX,VO/1.48063646,-0.03101959,1.414,1.0/
P(X)=(((A11*X**2+A9)*X**2+A7)*X**2+A5)*X**2+A3)*X**2+A1
ZP(X)=P(X)*X
Q(X)=(((B11*X**2+B9)*X**2+B7)*X**2+B5)*X**2+B3)*X**2+B1
ZQ(X)=Q(X)*X
Z(X)=SQRT(ZP(X)**2+ZQ(X)**2)
FETIN=VO*10**(-BAKOFF/20.)
PPP=ZQ(FETIN)**2/ZP(FETIN)**2
PHASE=ATAN(PPP)
SSS=20*(ALOG10(ZP(FETIN)))
WRITE (6,2) BAKOFF,PHASE,SSS
2 FORMAT(2X,'FET INPUT B/O:',F4.1,' DB',
?,OUTPUT PHASE ',F6.2,' OUTPUT B/O:',F6.1,' DB')
CALL POWER(DATA,MP1,PF1)
FNORMI=FETIN/SQRT(MP1)
DO :1 I=1,LDIM
DATA(I)=DATA(I)/FNORMI
229
11 CONTINUE
   DO 10 I=1,LDIM
   X=REAL(DATA(I))
   Y=AIMAG(DATA(I))
   R=SQRT(X**2+Y**2)
   IF(R.GT.VMAX) GO TO 12
   DATA(I)=CMPLX(P(R)*X-Q(R)*Y,P(R)*Y+Q(R)*X)
   GO TO 10
12 DATA(I)=CMPLX(ZPMAX*X-ZQMAX*Y,ZQMAX*X+ZPMAX*Y)/R
10 CONTINUE
   DO 16 I=1,LDIM
   DATA(I)=DATA(I)/FNORMI/A1
16 CONTINUE
   CALL PHASE(DATA,PSHIFT)
   RETURN
END

C**************************************************************
C SIMULATE THE EQUIVALENT BASEBAND
C NONLINEARITY OF THE LINEARIZED SIEMENS-189 TWTA.
C**************************************************************

SUBROUTINE TWTMOD(DATA,BAKOFF)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
COMPLEX DATA(1)
REAL MP1
DATA A1,A3,A5,A7,1.02503681,-0.01817265,0.02024002,-0.00901092/
DATA B1,B3,B5,B7/-0.01563649,0.04791214,-0.036956490,-0.0002606/
DATA ZPMAX,ZQMAX,VMAX,VO,1.5666125,-0.30451861,1.414,1./
P(X)=(((A11*X**2)+A9)*X**2+A7)*X**2+A3)*X**2+A1
ZP(X)=P(X)*X
Q(X)=(((B11*X**2)+B9)*X**2+B7)*X**2+B5)*X**2+B3)*X**2+B1
ZQ(X)=SQRT(ZP(X)**2+ZQ(X)**2)
TWTIN=VO**10**(-BAKOFF/20.)
PPP=ZQ(TWTIN*1.414)/ZP(TWTIN*1.414)
PSHIFT=ATAN(PPP)
SSS=20.*(ALOG10(ZP(TWTIN*1.414)/1.414))
WRITE (6,2) BAKOFF,PSHIFT,SSS
2 FORMAT(8X,'TWT INPUT B/0:',F6.2,' DB',
?" OUTPUT PHASE ','F6.2,' OUTPUT B/0:',F6.2,' DB')
CALL POWER(DATA,MP1,PF)
FNORMI=TWTIN/SQRT(MP1)
DO 11 I=1,LDIM
DATA(I)=DATA(I)*FNORMI
11 CONTINUE
DO 10 I=1,LDIM
X=REAL(DATA(I))
Y=AIMAG(DATA(I))
R=SQRT(X**2+Y**2)
IF(R.GT.VMAX) GOTO 12
DATA(I)=CMPLX(P(R)*X-Q(R)*Y,P(R)*Y+Q(R)*X)
GO TO 10
12 DATA(I)=CMPLX(ZPMAX*X-ZQMAX*Y,ZQMAX*X+ZPMAX*Y)/R
10 CONTINUE
DO 16 I=1,LDIM
DATA(I)=DATA(I)/FNORMI/A1
16 CONTINUE
CALL PHASE(DATA,PSHIFT)
RETURN
END

C******************************************************************************
C COMPUTE MEAN POWER AND PEAK FACTOR
C******************************************************************************

SUBROUTINE POWER(DATA,MP,PF)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
COMPLEX DATA(I)
REAL MP,PF,NP
MP=0.0
PF=0.0
DO 10 I=1,LDIM
MP=MP+((CABS(DATA(I)))**2.)
IF(PF.LT.CABS(DATA(I))) PF=CABS(DATA(I))
10 CONTINUE
WRITE(6,11) PF
11 FORMAT(5X,'PEAK VALUE:',F5.2)
MP=MP/FLOAT(LDIM)
MP=MP/2.
WRITE(6,16) MP
16 FORMAT(5X,'MP=',F7.3)
PF=PF**2./(2.*MP)
GO TO 15

231
15    PF=10.*ALOG10(PF)
       WRITE(6,20) PF
20   FORMAT(5X,'PEAK FACTOR: ',F5.2,' DB ')
       RETURN
END

C***********************************************************************
C COMPENSATE THE PHASE SHIFT CAUSED BY NONLINEAR AMP.
C***********************************************************************

SUBROUTINE PHASE(DATA,PSHIFT)
  COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
  COMMON /NUMB2/ NSNR,NSYMB,BAUD
  COMPLEX EPS,DATA(1)
  EPS=CMPLX(COS(PSHIFT),-SIN(PSHIFT))
  DO 10 I=1,LDIM
       DATA(I)=DATA(I)*EPS
10   CONTINUE
       RETURN
END

C***********************************************************************
C DRAW SIGNAL STATE SPACE DIAGRAM
C***********************************************************************

SUBROUTINE PSFACE (DATA,NR,NRUNS,MI)
  COMPLEX DATA(1)
  REAL XLEN,YLEN
  REAL X(514),Y(514)
  INTEGER NR,NRUNS
  COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
  COMMON /NUMB2/ NSNR,NSYMB,BAUD
C ESTABLISH THE SURFACE AREA.
  IF(NR.NE.1) GO TO 2
       XLEN=25.0*FLOAT(NRUNS)
       YLEN=27.5
       CALL PLOTS(XLEN,YLEN)
       CALL FACTOR(0.75)
C ESTABLISH THE ORIGIN.
       CALL PLOT(15.0,12.5,-3)
       GO TO 3
2   CALL PLOT(25.0,0.0,-3)
C WRITE TITLE -----
C 3 CALL SYMBOL(-3.0,11.0,0.49,13)SPACE DIAGRAM,0.0,13)
C DRAW AXES
CALL AXIS(-6.0,0.0,1H ,1,12.0,0.0,0.0,0.5)
CALL AXIS(0.0,-6.0,1H ,1,12.0,90.0,-3.0,0.5)

C PLOT THE DATA.
X(513)=0.0
X(514)=0.5
Y(513)=0.0
Y(514)=0.5
L=MI
DO 4 I=1,511
K=LSAMPL*I+L
IF (K.GT.LDIM) K=K-LDIM
X(I)=REAL(DATA(K))+REAL(DATA(K+1))/2
Y(I)=AIMAG(DATA(K))+AIMAG(DATA(K+1))/2
X(512)=X(1)
Y(512)=Y(1)
C CALL NEWPEN(NR)
CALL LINE (X,Y,512,1,1,11)
IF(NR.EQ.NRUNS) CALL PLOT(0.0,0.0,0.999)
WRITE(6,61)
61 FORMAT(4X,'—SPACE DIAGRAM IS REQUESTED —')
RETURN
END

C*******************************************************************************
C DRAW THE EYE DIAGRAM
C*******************************************************************************

SUBROUTINE EYEQ(DATA)
DIMENSION DATA3(256),DATA4(256)
COMPLEX DATA(1)
COMMON /NUMB1/ FBW,ALPHA,LM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
C ESTABLISH THE SURFACE AREA.
CALL PLOTS(35.0,27.5)
CALL FACTOR(0.7)
C ESTABLISH THE ORIGIN.
CALL PLOT(3.0,13.0,0,3)
C WRITE THE TITLE OF THE GRAPH.
CALL SYMBOL(10.0,12.0,0.49,17H16QAM EYE DIAGRAM,0.0,0.17)
C DRAW THE TIME AXIS.
CALL AXIS(0.0,-12.0,1H ,1,31.0,0,1,0,2.0)
C DRAW THE AMPLITUDE AXIS.
CALL AXIS(0.0,-12.0,1H ,1,24.0,90.0,-6.0,0.50)
C PLOT THE DATA.
JJ=2*LSAMPL
DATA3(JJ+1)=0.0
DATA3(JJ+2)=0.5
DATA4(JJ+1)=1.0
DATA4(JJ+2)=2.0
M2=NSYMB/2
DO 4 KK=1,M2
DO 2 I=1, JJ
II=(KK-1)*2*LSAMPL+I
DATA3(I)=REAL(DATA(I))
2 DATA4(I)=FLOAT(I)
CALL LINE(DATA4,DATA3, JJ,1,0,0)
4 CONTINUE
CALL PLOT(0.0,0.0,0,999)
RETURN
END

C***************************************************************************************
C PERFORMANCE OF NLA-16-SQAM (OR MAMSK) IN HARDLIMITED CHANNEL.
C FILE NAME : 16SQ FORTRAN A1
C ALSO ANALYZE THE EFFECTS OF GAIN (AMP.) VARIATION
C AND STATIC PHASE SHIFT OF TX. HPAS.
C***************************************************************************************
C
C COMPLEX DATA(2048),DATA1(2048),DATA2(2048),TF(2048)
DIMENSION PEI(32),EBNO(32),NI(128),NQ(128)
DIMENSION XARRAY(32),YARRAY(32)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
COMMON /INPUT/ NI,NQ
COMMON /AA/A
COMMON /BB/B
COMMON /CC/C
LDIM=2048
NSYMB=128
LSAMPL=16
BAUD=15
NSNR=28

C-----------------------
IOFF=8

234
ALPHA=0.4
OFF=FLOAT(IOFF)/FLOAT(LSAMPL)
IORDER=4
FBW=.8.
NRUNS=4
A=.8
B=1
C=1.0
DEG1=0.
DEG2=0.

DO 100 NR=1,NRUNS
WRITE(6,111) A,B,C
111 FORMAT(10H 16SQAM A=,F4.2,3X,11H AMP,VAR B=,F6.3,3X,
?11H THRESHLD C=,F6.3)
BTS=FBW/BAUD**2
WRITE(6,112) BTS,DEG1,DEG2
112 FORMAT(5H BTS=,F6.3,3X,6H DEG1=,F4.2,3X,6H DEG2=,F4.2)
CALL LOAD16(DATA1.DATA2,NI,NQ)

CALL MAMK(DATA1.DATA2,NI,NQ)

PSFT1=DEG1/180.*3.14159
PSFT2=DEG2/180.*3.14159
CALL HLIM(DATA1)

CALL PHAERR(DATA1,PSFT1)
CALL HLIM(DATA2)

CALL PHAERR(DATA2,PSFT2)

CALL SUMM(DATA,DATA1.DATA2)

CALL WAVEH2(DATA)

CALL SPACE(DATA,NI,NRUNS)

CALL BUT(TF,IORDER)
CALL ENERGY(DATA,EB)

CALL HHGG(TF,PNOISE)

235
CALL FILTER(DATA,TF)
CALL SYNCRO(DATA,PNOISE,M1,MQ,EB)

C

C CALL PSPACE(DATA,NR,NRUNS,M1)
CALL DECODE(DATA,PNOISE,M1,MQ,EBNO,PELEB)
DO 99 I=1,NSNR
XARRAY(I)=EBNO(I)
99 YARRAY(I)=PEI(I)

C ------- DRAW PE CURVE, EYE -------
CALL DRAWCN(ID,XARRAY,YARRAY,NR,NRUNS)
C
CALL EYEQ(DATA)
C

C A=A+0.1
C B=B+0.05
C C=C-0.025
C FBW=FBW-0.25
C IORDER=IORDER+1
C DEG1=DEG1+2.
C DEG2=DEG2+2.
C IOFF=IOFF-8
C

100 CONTINUE
STOP
END
C

C******************************************************************************
C
C GENERATE 4 PARALLEL (II,Q1,I2,Q2) SQAM SIGNALS.
C******************************************************************************

SUBROUTINE LOAD16(DATA1,DATA2,N1,NQ)
COMPLEX DATA1(1),DATA1(1),DATA2(1)
DIMENSION NY(7),N1(128),NQ(128),NI(1),NQ(1),NX(7)
DIMENSION RI1(16),RQ1(16),RI2(16),RQ2(16)
DIMENSION NV(7),NW(7),N2(128),NQ2(128)
COMMON /NUMB1/, FBW,ALPHA,LDIM,IOFF,LSAMPL
DATA NV(1),NV(2),NV(3),NV(4),NV(5),NV(6),NV(7)/,1,1,1,-1,1,-1,1/
DATA NX(1),NX(2),NX(3),NX(4),NX(5),NX(6),NX(7)/,-1,1,1,-1,1,1,1/
NW(1)=1
NW(2)=-1
NW(3)=1
NW(4)=-1
NW(5)=1

236
NW(6)=-1
NW(7)=1
NY(1)=-1
NY(2)=-1
NY(3)=1
NY(4)=1
NY(5)=1
NY(6)=1
NY(7)=-1
DATA JLAST/7/
I=0
J=6
KKK=2**JLAST
DO 1 K=1,KKK
C GENERATING ONE SYMBOL.
IF (I.GE.JLAST) I=0
IF (J.GE.JLAST) J=0
I=I+1
J=J+1
NV(I)=NV(J)*NV(I)
NW(I)=NW(J)*NW(I)
NX(I)=NX(J)*NX(I)
NY(I)=NY(J)*NY(I)
NI1(K)=NV(I)
NQ1(K)=NW(I)
NI2(K)=NX(I)
NQ2(K)=NY(I)
NI(K)=NI1(K)+NI2(K)**2
NQ(K)=NQ1(K)+NQ2(K)**2
C LOAD INTO THE ARRAY TO BE FOURIER TRANSFORMED.
DO 10 K=1,KKK
KM1=K-1
IF (KM1.EQ.0) KM1=KKK
MI1=NI1(KM1)
MQ1=NQ1(KM1)
MI2=NI2(KM1)
MQ2=NQ2(KM1)
CALL SIG(MI1,NI1(K),RI1)
CALL SIG(MQ1,NQ1(K),RQ1)
CALL SIG(MI2,NI2(K),RI2)
CALL SIG(MQ2,NQ2(K),RQ2)
J1=(K-1)*LSAMPL
DO 10 I=1,16
DATA1(J1+I)=CMPLX(R11(I),RQ1(I))
10 DATA2(J1+I)=CMPLX(R12(I),RQ2(I))
IF (IOFF.EQ.0) GO TO 20
DO 15 I=1,IOFF
A=AIMAG(DATA1(LDIM))
B=AIMAG(DATA2(LDIM))
DO 13 L=2,LDIM
K=LDIM+2-L,
DATA1(K)=CMPLX(REAL(DATA1(K)),AIMAG(DATA1(K-1)))
13 DATA2(K)=CMPLX(REAL(DATA2(K)),AIMAG(DATA2(K-1)))
DATA1(1)=CMPLX(REAL(DATA1(1)),A)
DATA2(1)=CMPLX(REAL(DATA2(1)),B)
15 CONTINUE
20 RETURN
END

C

C==================================================================
C SQAM BASEBAND SIGNAL PROCESSOR
C==================================================================

SUBROUTINE SIG(M,IP,R)
COMMON /AA/A
REAL R(16)
P=3.4159265/16.
IF(M.EQ.IP) GOTO 20
IF(IP.EQ.1) GOTO 10
DO 1 I=1,16
1 R(I)=COS((I+0.5)*P)
GOTO 40
10 DO 11 I=1,16
11 R(I)=-COS((I+0.5)*P)
GOTO 40
20 IF(IP.EQ.1) GOTO 30
DO 21 I=1,16
21 R(I)=-A-(1.0-A)*COS(2.*(I+0.5)*P)
GOTO 40
30 DO 31 I=1,16
31 R(I)=A+(1.0-A)*COS(2.*(I+0.5)*P)
40 CONTINUE
RETURN

238
END
C*******************************************************************************
C                 GENERATE 4 PARALLEL MSK SIGNALS.
C*******************************************************************************
SUBROUTINE MAMK(DATA1,DATA2,NI,NQ)
  COMPLEX DATA(1),DATA1(1),DATA2(1)
  DIMENSION N(1),NI1(128),NI2(128),NI(1),NQ(1),NX(7)
  DIMENSION RI1(16),RI2(16),RI(16),RIQ2(16)
  DIMENSION NV(7),NW(7),NI2(128),NQ2(128)
  COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
  DATA NV(1),NV(2),NV(3),NV(4),NV(5),NV(6),NV(7)/-1,-1,1,-1,1,1,1/
  DATA NX(1),NX(2),NX(3),NX(4),NX(5),NX(6),NX(7)/-1,-1,1,-1,1,1,1/
  NW(1)=1
  NW(2)=-1
  NW(3)=1
  NW(4)=-1
  NW(5)=1
  NW(6)=-1
  NW(7)=1
  NY(1)=-1
  NY(2)=-1
  NY(3)=1
  NY(4)=1
  NY(5)=1
  NY(6)=1
  NY(7)=1
  DATA JLAST/7/
  I=0
  J=6
  KKK=2**JLAST
  DO 1 K=1,KKK
C                 GENERATING ONE SYMBOL.
     IF (I.GE.JLAST) I=0
     IF (J.GE.JLAST) J=0
     I=I+1
     J=J+1
     NV(I)=NV(J)*NV(I)
     NW(I)=NW(J)*NW(I)
     NX(I)=NX(J)*NX(I)
     NY(I)=NY(J)*NY(I)
     NI2(K)=NV(I)
 1 CONTINUE
  RETURN
END
NQ1(K)=NW(I)
N12(K)=NX(I)
NQ2(K)=NY(I)
N1(K)=N11(K)+N12(K)*2

1   NQ(K)=NQ1(K)+NQ2(K)*2
C  LOAD INTO THE ARRAY TO BE FOURIER TRANSFORMED.
DO 10 K=1,KKK
   CALL MSK(N11(K),RI1)
   CALL MSK(NQ1(K),RQ1)
   CALL MSK(N12(K),RI2)
   CALL MSK(NQ2(K),RQ2)
   J1=(K-1)*LSAMPL
   DO 10 I=1,16
      DATA1(J1+I)=CMPLX(RI1(I),RQ1(I))
      DATA2(J1+I)=CMPLX(RI2(I),RQ2(I))
10    IF (IOFF.EQ.0) GO TO 20
   DO 15 I=1,IOFF
      A=AIMAG(DATA1(LDIM))
      B=AIMAG(DATA2(LDIM))
      DO 13 L=2,LDIM
         K=LDIM+2-L
         DATA1(K)=CMPLX(REAL(DATA1(K)),AIMAG(DATA1(K-1)))
         DATA2(K)=CMPLX(REAL(DATA2(K)),AIMAG(DATA2(K-1)))
      13   DATA1(1)=CMPLX(REAL(DATA1(1)),A)
      DATA2(1)=CMPLX(REAL(DATA2(1)),B)
   15   CONTINUE
20   RETURN
END

C**************************************************************************************
C  MSK BASEBAND SIGNAL PROCESSOR
C**************************************************************************************

SUBROUTINE MSK(NN,RR)
REAL RR(16)
P=3.14159265/16.
IF(NN.EQ.1) GOTO 55
DO 22 I=1,16
   RR(I)=-SIN((I+0.05)*P)
GOTO 40
22 CONTINUE
55 DO 33 I=1,16
33 RR(I)=SIN((I+0.05)*P)
40 CONTINUE
RETURN
END

C ******************************************************************************
C DRAW TRANSMITTED SIGNAL STATE-SPACE DIAGRAMS.
C ******************************************************************************

SUBROUTINE SPACE(DATA,JCURV,JLAST)
REAL X(2050),Y(2050)
COMPLEX DATA(1)
IF(JCURV.GT.1) GOTO 2

C ----- ESTABLISH SURFACE AREA -----
CALL PLOYS(30,0,27.5)
CALL FACTOR(0.5)

C ----- ESTABLISH ORIGIN -----
CALL PLOT(15.0,12.5,-3)

C ----- WRITE TITLE -----
CALL SYMBOL(-6.0,12.5,0.49,13HSPACE DIAGRAM,0.0,13)

C DRAW AXIS
CALL AXIS(-8.0,0.0,0.1H ,1,16,0,0.0,-4.0,0.50)
CALL AXIS(0.0,-8.0,1H ,1,16,0,0.0,-4.0,0.50)
X(2049)=0.0
X(2050)=0.50
Y(2049)=0.0
Y(2050)=0.50

C PLOT DIAGRAM
2 CONTINUE
DO 21 I=1,2048
X(I)=REAL(DATA(I))
Y(I)=AIMAG(DATA(I))
21 CONTINUE
CALL LINE(X,Y,2048,1,0,0)
IF(JCURV.EQ.JLAST) CALL PLOT(0.0,0.0,0.999)
WRITE(6,61)
61 FORMAT(5X,'SPACE DIAGRAM IS REQUESTED --')
RETURN
END

C******************************************************************************
C PHASE-EQUALIZED BUTTERWORTH LPF
C******************************************************************************

SUBROUTINE BUT(TF,JORDER)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYM,BAUD

241
COMPLEX TF(1)
NO1=LDIM/2 +1
NO2=NO1 +1
A1=1./NSYM
FENOR=FBW/BAUD
TF(1)=CMPLX(1.0,0.0)
DO 10 I=2,NO1
   J=I-1
   A2=1./SQRT(1.+(A1*FLOAT(J)/FBNOR)**(2*IORDER))
10   TF(I)=CMPLX(A2,0.)
   DO 20 I=NO2,LDIM
   TF(I)=TF(LDIM+2-I)
   RETURN
END
C*******************************************************************************
C THIS SUBROUTINE SIMULATES A HARD LIMITER
C*******************************************************************************
SUBROUTINE HLIM(DATA)
   COMPLEX DATA(1)
   COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
   DO 10 I=1,LDIM
   DATA(I)=DATA(I)/CABS(DATA(I))
   RETURN
END
C*******************************************************************************
C COMPUTE IN-BAND OR OUT-OF-BAND P.S.D.
C*******************************************************************************
SUBROUTINE SPECT(SIGNAL,PO,TPO)
   COMMON LDIM,IOFF
   COMMON/ PARA/ LSAMPL,NSYM,NO1,NO2,BAUD,SBANDW
   COMPLEX SIGNAL(1),CWK(256)
   DIMENSION X(32768),PO(129),IWK(8),WK(128),TPO(129)
   S=0.
   DO 10 I=1,LDIM
   L=I-IOFF
   IF(L.LE.0) L=L+LDIM
   X(I)=REAL(SIGNAL(I))
   X(I+16384)=AMAG(SIGNAL(L))
10   S=S+X(I)+X(I+16384)
   S=S/FLOAT(32768)
   DO 20 I=1,32768

242
20  X(I)=X(I)-S
     CALL FTPS(X,Y,32768,256,0,PO,PSY,XPS,IWK,WK,CWK,IER)
     PMAX=PO(1)
     TPO(129)=PO(129)
C
C      ------  CALCULATE IN-BAND P.S.D.  ------
C
C
C      DO  30 I=1,129
C30     TPO(I)=-10.*ALOG10(PO(I)/PMAX)
C      WRITE(6,1) (TPO(I),I=1,129)
C     FORMAT(2X,'POWER SPECTRUM'/10(2X,F6.1))
C      ------  CALCULATE OUT-OF-BAND P.S.D.  ------
C
C      DO  40 I=1,127
C40     TPO(129-I)=TPO(130-I)+2.*PO(129-I)
C     TPO(1)=TPO(2)+PO(1)
C     TP MAX=TPO(1)
C
C      DO  50 I=1,129
C50     TPO(I)=-10.*ALOG10(TPO(I)/TP MAX)
C      WRITE(6,2) (TPO(I),I=1,129)
C     FORMAT(2X,'OUT OF BAND TO TOTAL POWER RATIO'/10(2X,F6.1))
C
C      RETURN
C      END
C
C*******************************************************************************
C
C      DRAW THE NORMALIZED P.S.D.
C*******************************************************************************
C
SUBROUTINE DRAWS(TPO,JCURV,JLAST)
COMMON LDIM,IOFF
COMMON PARA/LSAMPL,NSYMB,NO1,NO2,BAUD,SBANDW
DIMENSION TPO(1),DATA2(131),DATA3(131)
COMPLEX DATA(1)
IF(JCURV.GT.1) GOTO 2
C
C     ESTABLISH THE SURFACE AREA.
C     CALL PLOTS(20.0,27.5)
C
C     ESTABLISH THE ORIGIN.
C     CALL PLOT(3.0,7.0,-3)
C
C     ------  CHOOSE TITLE OF THE GRAPH  ------
C     CALL SYMBOL(3.0,17.0,0.35,21H16SQAM POWER SPECTRUM,0.0,24)

243
CALL SYMBOL(3.0,17.0,0.35,17HOUT-OF-BAND POWER,0.0,24)
C
DRAW THE FREQ AXIS.
CALL AXIS(0.0,0.0,20HNORMALIZED FREQUENCY,-20,.0,.0,.0,0.5)
C
DRAW THE PSD AXIS.
C
--------- CHOOSE Y-AXIS TITLE ---------
C
CALL AXIS(0.0,0.0,18HNORMALIZED PSD DB,18,17.,+90.,-90.0,5.)
CALL AXIS(0.0,0.0,21HOUT-OF-BAND POWER DB,18,17.,+90.,-90.0,5.)
DATA2(66)=-80.
DATA2(67)=5.
DATA3(66)=0.0
DATA3(67)=0.5

2 CONTINUE
DO 4 KK=1,65
DATA2(KK)=-TP(TM(KK))
DATA3(KK)=(FLOAT(KK)-1.0)*16./256.

4 CONTINUE
CALL LINE(DATA3,DATA2,65,1,0,0)
IF(JCURV.EQ.JLAST) CALL PLOT(0.0,0.0,0.999)
RETURN
END

C
DRAW THE SIGNAL WAVE-SHAPE
C
SUBROUTINE Wavesh2(DATA)
DIMENSION DATA2(259),DATA3(259),MOD(2)
COMPLEX DATA(1)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYM,BAUD
C
ESTABLISH THE SURFACE AREA.
CALL PLOTS(36.0,27.5)
C
ESTABLISH THE ORIGIN.
CALL PLOT(2.3,19.5,-3)
C
DRAW THE TIME AXIS.
CALL AXIS(0.0,0.0,1H ,1.325,0.0,0.0,1.0)
C
DRAW THE AMPLITUDE AXIS.
CALL AXIS(0.0,-3.0,1H ,1.0,90.0,-3.0,1.0)
C
PLOT THE DATA.
DATA2(258)=0.0
DATA2(259)=1.0
DATA3(258)=0.0
DATA3(259)=16.
DO 4 KK=1,257
DATA2(KK)=REAL(DATA(KK))
4 DATA3(KK)=FLOAT(KK)
CALL LINE(DATA3(DATA2,257,1,0,0)
C ESTABLISH THE ORIGIN.
CALL PLOT(0.0,-13.0,-3)
C DRAW THE TIME AXIS.
CALL AXIS(0.0,0.0,1,0,-1,32.5,0.0,17,0.1,0)
C DRAW THE AMPLITUDE AXIS.
CALL AXIS(0.0,-3.0,1,0,0,0,-3.0,1,0)
C PLOT THE DATA.
DO 7 KK=1,256
DATA2(KK)=REAL(DATA(KK+256))
7 DATA3(KK)=FLOAT(KK)
DATA2(257)=REAL(DATA(1))
DATA3(257)=FLOAT(257)
CALL LINE(DATA3(DATA2,257,1,0,0)
CALL PLOT(0.0,0.0,0,999)
RETURN
END

C----------------------------------------------------------------------
C STATIC PHASE ERROR OF REFERENCE CARRIER
C----------------------------------------------------------------------
SUBROUTINE PHAERR(DATA,PSHIFT)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
COMPLEX EPS,DATA(1)
EPS=CMPLX(COS(PSHIFT),SIN(PSHIFT))
DO 10 I=1,LDIM
10 DATA(I)=DATA(I)*EPS
CONTINUE
RETURN
END

C----------------------------------------------------------------------
C PERFORMANCE OF NLA-16-SQAM AND MAMSK MODEMS IN A NONLINEARLY
C AMPLIFIED (HARDLIMITED) MULTICHANNEL INTERFERENCE ENVIRONMENT.
C ------- FILE NAME : ACI FORTRAN A1 -------
C----------------------------------------------------------------------
C
COMPLEX DATA(2048),DATA1(2048),DATA2(2048),TF(2048)

245
COMPLEX DATA3(2048), DATA4(2048), DATA5(2048), DATA6(2048)
COMPLEX DATA7(2048), DATA8(2048), BUFF(2048)
DIMENSION PE(32), EBNQ(32), NI(128), NQ(128), APEI(32)
DIMENSION XARRAY(32), YARRAY(32)
DIMENSION NIA(128), NQA(128), NIB(128), NQB(128)
REAL MP, MP1, MP2, MP0, MPF
COMMON /NUMB1/ FBW, ALPHA, LDIM, IOFF, LSAMPL, SBANDW
COMMON /NUMB2/ NSNR, NSYM, BAUD
COMMON /INPUT/ NI, NQ
COMMON /AA/A
COMMON /BB/B
COMMON /CC/C

C
C INITIALIZE PROGRAM.
C
OFF=FLOAT(IOFF)/FLOAT(LSAMPL)
SBANDW=FLOAT(LSAMPL)*BAUD
LDIM=2048
NSYM=128
LSAMPL=16
BAUD=100.
NSNR=28
IOFF=8
IORDER=5
NRUNS=16

C

DATA FOF, FOF1, FOF2/0., 200., -200./
DATA AT1, AT2/ -0., -0./
DATA NOA/1/
FBW=52.
A=0.8
B=1
C=1.0

C

DO 600 KA=1, NOA
PSHIF1=1.7
PSHIF2=2.9
ITSH1=672
ITSH2=1136
DO 19 I=1, NSNR
APEI(I)=0.0
19 CONTINUE
   DO 500 KM=1,NRUNS
C ----------- MAIN-CHANNEL -------------
   CALL LOAD16(DATA1,DATA2,N1,NQ)
C   CALL MAMK(DATA1,DATA2,N1,NQ)
   CALL HLIM(DATA1)
   CALL HLIM(DATA2)
   CALL SUMM(DATA,DATA1,DATA2)
   CALL BUT(TF,IORDER,FOF)
   CALL POWER(DATA,MP,PF)
   CALL ENERGY(DATA,EB)
   CALL FILTER(DATA,TF,FOF)
C ----------- ADJACENT-CHANNEL 1 --------
   CALL LOAD16(DATA3,DATA4,N1A,NQA)
C   CALL MAMK(DATA3,DATA4,N1A,NQA)
   CALL HLIM(DATA3)
   CALL HLIM(DATA4)
   CALL SUMM(DATA7,DATA3,DATA4)
   CALL PHASE(DATA7,PSHIFT1)
   CALL FSM1(DATA7,ITSH1)
   CALL ATT(DATA7,AT1)
   CALL POWER(DATA7,MP1,PF1)
   CALL FILTER(DATA7,TF,FOF1)
   DO 10 I=1,LDIM
10  DATA(I)=DATA(I)+DATA7(I)
C ----------- ADJACENT-CHANNEL 2 -----------
   CALL LOAD16(DATA5,DATA6,N1B,NQB)
C   CALL MAMK(DATA5,DATA6,N1B,NQB)
   CALL HLIM(DATA5)
   CALL HLIM(DATA6)
   CALL SUMM(DATA8,DATA5,DATA6)
   CALL PHASE(DATA8,PSHIFT2)
   CALL FSM1(DATA8,ITSH2)
   CALL ATT(DATA8,AT2)
   CALL POWER(DATA8,MP2,PF2)
C   CALL FILTER(DATA8,TF,FOF2)
   DO 11 I=1,LDIM
11  DATA(I)=DATA(I)+DATA8(I)
   CALL POWER(DATA,MPF,PFF)
   CALL HGGG(TF,PNOISE)
AC11=10.*ALOG10(MP/MP1)
AC12=10.*ALOG10(MP/MP2)
CALL SYNCRO(DATA,PNOISE,MI,MQ,EB)
CALL DECODE(DATA,PNOISE,MI,MQ,EBNO,PE,EB)
DO 5 I=1,NSNR
PE(I)=PE(I)/FLOAT(NSYMB)
EBNO(I)=FLOAT(I)
APEI(I)=APEI(I)+PE(I)
5 CONTINUE
C ---------------- RANDOMIZE TIME AND PHASE ----------------
ITSH1=ITSH1-1
ITSH2=ITSH2+1
PSHIF1=PSHIF1+.22
PSHIF2=PSHIF2+.47
500 CONTINUE
WRITE(6,111) A,B,C
111 FORMAT(10H 16SQAM A=,F4.2,3X,11H AMP,VAR B=,F6.3,3X,?
?11H THRESHLD C=,F6.3)
BTS=FBW/BAUD
WRITE(6,112) BTS
112 FORMAT(5H BTS=,,F6.3)
N=1
WRITE(6,3) N,FOF1,AT1,AC11
N=2
WRITE(6,3) N,FOF2,AT2,AC12
3 FORMAT(///2X,'CHANNEL ','I1,' CHARACTERISTICS'//2X,?
?'OFFSET FREQ.: ',F6.1,' MHZ',,' ATTENUATION: ',F4.1,' DB',
?'/ CARRIER TO INT. RATIO: ',F5.1,' DB')
DO 64 I=1,NSNR
APEI(I)=APEI(I)/FLOAT(NRUNS)
64 CONTINUE
WRITE(6,150)
150 FORMAT(5X,'EB/NO',10X,'PROB. OF. ERROR', //)
WRITE(6,172) (EBNO(I),APEI(I),I=1,NSNR)
172 FORMAT(5X,F5.1,10X,E13.6)
DO 99 I=1,NSNR
XARRAY(I)=EBNO(I)
99 YARRAY(I)=APEI(I)
C
C ---------------- CALL THE DRAWING ROUTINE ----------------
CALL DRAWCN(ID,XARRAY,YARRAY,KA,NOA)
C CALL EYEQ(DATA)
C
C  CHANGE SYSTEM PARAMETERS
C  A=A+0.1
C  AT1=AT1-3.
C  AT2=AT2-3.
C  FOF1=FOF1-10.
C  FOF2=FOF2+10.
C  C=C+0.05
C  FBW=FBW+2.0
C  IORDER=IORDER+1
C
600 CONTINUE
STOP
END
C
C**************************************************************************
C     THIS SUBROUTINE SHIFTS THE SIGNAL
C**************************************************************************

SUBROUTINE TIME(DATA,IT)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL,SBANDW
COMMON /NUMB2/ NSNR,NSYMB,BAUD
COMPLEX DATA(1),DAT
DO 10 I=1,IT
    DAT=DATA(I)
    DO 20 J=2,LDIM
  20    DATA(J-1)=DATA(J)
  10   DATA(LDIM)=DAT
RETURN
END
C**************************************************************************
C     ATTENUATE THE SIGNAL POWER.
C**************************************************************************

SUBROUTINE ATT(DATA,AT)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL,SBANDW
COMMON /NUMB2/ NSNR,NSYMB,BAUD
COMPLEX DATA(1)
FA=10.**(AT/20.)
DO 10 I=1,LDIM
  10   DATA(I)=DATA(I)*FA

249
RETURN
END
C****************************************************************************
C       COMPUTE MEAN POWER
C*****************************************************************************

SUBROUTINE POWER(DATA,MP,PF)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL,SBANDW
COMMON /NUMB2/ NSNR,NSYM,BAUD
COMPLEX DATA(1)
REAL MP
PF=0.
MP=0.0
DO 10 I=1,LDIM
MP=MP+((CABS(DATA(I)))**2.)
IF(PF.LT.CABS(DATA(I))) PF=CABS(DATA(I))
10 CONTINUE
MP=MP/FLOAT(LDIM)/2.
PF=(PF**2.)/MP
PF=10.*ALOG10(PF)
RETURN
END
C****************************************************************************
C       COMPENSATE THE PHASE SHIFT
C       DUE TO THE CARRIER OFFSET.
C*****************************************************************************

SUBROUTINE PHASE(DATA,PSHIFT)
COMMON LDIM
COMPLEX EPS,DATA(1)
EPS=CMPLX(COS(PSHIFT),-SIN(PSHIFT))
DO 10 I=1,LDIM
DATA(I)=DATA(I)*EPS
10 CONTINUE
RETURN
END
C****************************************************************************
C       SHIFT THE SIGNAL IN TIME DOMAIN.
C*****************************************************************************

SUBROUTINE FSM1(DATA,JOF)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL,SBANDW
COMMON /NUMB2/ NSNR,NSYM,BAUD
COMPLEX DATA(1),BUFF(2048)

250
J=LDIM-IABS(IFO)
K=J+1
IF (IFO.EQ.0) GOTO 9
IF (IFO.LT.0) GOTO 4
DO 1 I=1,J
1  BUFF(I)=DATA(I)
   DO 2 I=K,LDIM
2  DATA(I-J)=DATA(I)
   DO 3 I=1,J
3  DATA(I+IOF)=BUFF(I)
   GOTO 9
4  IOF=-IOF
   L=IOF+1
   DO 5 I=L,LDIM
5  BUFF(I-IOF)=DATA(I)
   DO 6 I=1,IOF
6  DATA(I+J)=DATA(I)
   DO 7 I=1,J
7  DATA(I)=BUFF(I)
9  RETURN
END

C******************************************************************************
SUBROUTINE BUT(TF,IORDER,FOF)
C******************************************************************************
COMMON /NUMBI/ FBW,ALPHA,LDIM,IOFF,LSAMPL,SBANDW
COMMON /NUMB2/ NSNR,NSYMB,BAUD
COMPLEX TF(1)
NO1=LDIM/2 +1
NO2=NO1 +1
A1=1./NSYMB
FBNOR=FBW/BAUD
TF(1)=CMPLX(1.0,0.0)
DO 10 I=2,NO1
   J=I-1
   A2=1./SQRT(1.+(A1*FLOAT(J)/FBNOR)**(2*IORDER))
10  TF(I)=CMPLX(A2,0.)
   DO 20 I=NO2,LDIM
20  TF(I)=TF(I+LDIM-2-I)
IFOF=IFIX(FOF*FLOAT(LDIM)/SBANDW)
CALL FSM1(TF,IFOF)
RETURN

251
END

C******************************************************************************
C PERFORMANCE OF 16, 64, 256-QAM IN THE PRESENCE OF
C RESIDUAL AMPLITUDE FLUCTUATION AND/OR PHASE JITTER
C ++++++ FILE NAME : 256QC FORTRAN ++++++
C CHANGE (EB) IN ENERGY TO 4,6,8 ALSO (CF) IN DRAWCN
C TO 4,6,8 FOR 16, 64, 256-QAM SIMULATION, RESPECTIVELY.
C******************************************************************************

C COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD,NBPSYM,CN1
COMMON /DD/D
DIMENSION PEI(38),EBNO(38),NI(2048),NQ(2048)
DIMENSION XARRAY(38),YARRAY(38)
COMPLEX DATA(65536),TF(65536)
REAL SS(65536),AMAX,AMIN

LDIM=65536
NSYMB=2048
LSAMPL=32
NSNR=36
BAUD=2048
FBW=1024
IOFF=0

C******************************************************************************
C FOR 16QAM  NBPSYM=4 (BITS/S/Hz)  LMAX=3
C 64QAM     6  7
C 256QAM    8 15
C******************************************************************************

NBPSYM=6
LMAX=7

OFF=FLOAT(IOFF)/FLOAT(LSAMPL)
CN1=FLOAT(NBPSYM)*2.+4.
KN=(NBPSYM-2)/2
IQAM=2**NBPSYM

C— AM : AM INDEX, FM : FREQ. OF AM FLUCTUATION ——
C— DEG : RMS DEG. OF PHASE JITTER, ————
C— D : THRESHOLD LEVEL OF ADT DETECTOR ————
ALPHA=0.4
NRUNS=2

252
DEG=0.
AM=0.0
FM=1.
D=1.0

C

DO 100 NR=1,NRUNS
NAP=LSAMPL
WRITE(6,23) DEG,ALPHA,OFF
23 FORMAT(5H DEG=,F10.6,5X,7H ALPHA=,F9.3,5X,5H OFF=,F6.3)
WRITE(6,24) AM,IQAM,D
24 FORMAT(4H AM=,F10.6,5X,13,-QAM=,F3.0,5X,3H D=,F10.6)

C --- CHOOSE M-ARY QAM ------
GO TO(11,12,13), KN
11 CALL LOAD16(DATA,NI,NQ,NAP)
GO TO 14
12 CALL LOAD64(DATA,NI,NQ,NAP)
GO TO 14
13 CALL LD256(DATA,NI,NQ,NAP)
GO TO 14

C
14 CALL RCOSTX(TF)
CALL FILTER(DATA,TF)
CALL AMF(DATA,AM,FM)
CALL ENERGY(DATA,EB)

C

C ------ END OF TRANSMITTER ------
C
C
CALL RCOSRX(TF)
CALL HHGG(TF,PNOISE)
CALL FILTER(DATA,TF)

C ------ PHASE JITTER ------
C
CALL GAUCPE(DATA,DEG)
C
CALL SINCPE(DATA,DEG,FM)
CALL SYNCHRO(DATA,PNOISE,NI,NQ,Mi,MQ,EB,NAP)

C
C
CALL PSpace(DATA,NR,NRUNS,MI,KN)
CALL DECODE(DATA,PNOISE,Mi,MQ,NI,NQ,EBNO,PE,EB,NAP)

C
DO 99 I=1,NSNR
XARRAY(I)=EBNO(I)
99 YARRAY(I)=PE(I)

253
C

C ----- CALL THE DRAWING ROUTINE -----

CALL DRAWCN(XARRAY,YARRAY,NR,NRUNS)
C CALL EYEQ(DATA,KN)

C

C         DEG=DEG+1
C         AM=AM+0.1
C         D=D-0.025

100 CONTINUE
  STOP
  END

C

C*******************************************************************************
C
C GENERATE A BASEBAND 256-QAM SIGNAL.
C*******************************************************************************

SUBROUTINE LD256(DATA,NI,NQ,NAP)
COMPLEX DATA(1)
DIMENSION NY(11),NI(1),NQ(1),NX(2048)
COMMON /NUMB1/ FBW,ALPHA,LDIM,OFF,LSAMPL
DATA NY(1),NY(2),NY(3),NY(4),NY(5),NY(6),NY(7),NY(8),NY(9),
?NY(10),NY(11),-1.1.1.1,-1.1.-1.1,-1.1.1/
C
NY(12)=1
DATA JLAST,JTAP/11,1/
KK=2**JLAST
KK=KK-(1+JLAST)
DO 6 I=1,JLAST
  NX(I)=NY(I)
6 CONTINUE

C

C GENERATE 2**JLAST -1 LENGTH SEQUENCE USING GIVEN DATA AND
C GENERATOR POLYNOMIAL.
C

DO 1 J=1,KK
  I=J+JLAST
  'NX(I)=NX(J)*NX(J+2)
  NX(I)=0-NX(I)
1 CONTINUE

C

C GENERATE 8 CYCLICALLY SHIFTED VERSIONS OF ORIGINAL
C SEQUENCE OF LENGTH 2**JLAST.
C

251
K1=1
K2=2
K3=3
K4=4
K5=5
K6=6
K7=7
K8=8
DO 2 I=1,KKK
  IF (K1.EQ.KKK) K1=1
  IF (K2.EQ.KKK) K2=1
  IF (K3.EQ.KKK) K3=1
  IF (K4.EQ.KKK) K4=1
  IF (K5.EQ.KKK) K5=1
  IF (K6.EQ.KKK) K6=1
  IF (K7.EQ.KKK) K7=1
  IF (K8.EQ.KKK) K8=1
  NI(I)=NX(K1)+NX(K3)*2+NX(K5)*4+NX(K7)*8
  NQ(I)=NX(K2)+NX(K4)*2+NX(K6)*4+NX(K8)*8
C
C  LOAD INTO SAMPLE ARRAY, 16 SAMPLES PER SYMBOL
  J1=(I-1)*LSAMPL+1
  J11=(I-1)*LSAMPL+(LSAMPL-NAP)
  J2=I*LSAMPL
  IF(J1.GT.J11) GO TO 666
  DO 3 J3=J1,J11
  3 DATA(J3)=CMPLX(0.0,0.0)
666 CONTINUE
  J12=J11+1
  DO 33 J33=J12,J2
  DATA(J33)=CMPLX(FLOAT(NI(I)),FLOAT(NQ(I)))
  C
  DATA(J33)=DATA(J33)*FLOAT(LSAMPL)/FLOAT(NAP)
33 CONTINUE
  K1=K1+1
  K2=K2+1
  K3=K3+1
  K4=K4+1
  K5=K5+1
  K6=K6+1
  K7=K7+1
  K8=K8+1
CONTINUE
IF(IOFF.EQ.0) RETURN
LL=LDM-1
DO 4 I=1,IOFF
XX=AIMAG(DATA(I))
DO 5 K=1,LL
5 DATA(K)=CMPLX(REAL(DATA(K)),AIMAG(DATA(K+1)))
4 DATA(LDM)=CMPLX(REAL(DATA(LDM)),XX)
RETURN
END

C******************************************************************************
C              GENERATE A BASEBAND 64-QAM SIGNAL.
C******************************************************************************
SUBROUTINE LOAD84(DATA,NI,NQ,NAP)
COMPLEX DATA(I)
DIMENSION NY(14),NI(1),NQ(1),NX(16384)
COMMON /NUMB1/ FBW,ALPHA,LDM,IOFF,LSAMPL
DATA NY(1),NY(2),NY(3),NY(4),NY(5),NY(6),NY(7),NY(8),NY(9),
?NY(10),NY(11),NY(12),NY(13),NY(14),NY(15),NY(16)/
NY(12)=1
NY(13)=-1
NY(14)=1
DATA JLAST,JTAAP/1,1,JLAST/
KKK=2**JLAST
KK=KKK-(J+JLAST)
DO 6 I=1,JLAST
NX(I)=NY(I)
6 CONTINUE
C
C              GENERATE 2**JLAST -1 LENGTH SEQUENCE USING GIVEN DATA AND
C              GENERATOR POLYNOMIAL.
C
DO 1 J=1,KK
I=J+JLAST
NX(I)=NX(J)*NX(J+2)
NX(I)=0-NX(I)
1 CONTINUE
C
C              GENERATE 6 INTERLEAVED VERSIONS OF ORICINAL
C              SEQUENCE OF LENGTH 2**JLAST.

256
C
   DO 2 I=1,12048
   K1=8*I-5
   K2=8*I-4
   K3=8*I-3
   K4=8*I-2
   K5=8*I-1
   K6=8*I
   IF (K6.EQ.KKK) K6=1
   NI(I)=(NX(K1)**4)+(NX(K3)**2)+NX(K5)
   NQ(I)=(NX(K2)**4)+(NX(K4)**2)+NX(K6)
C
C LOAD INTO SAMPLE ARRAY, 16 SAMPLES PER SYMBOL
   J1=(I-1)*LSAMPL+1
   J2=I*LSAMPL
   DO 3 J3=J1,J2
3    DATA(J3)=CMPLX(FLOAT(NI(I)),FLOAT(NQ(I)))
2    CONTINUE
   IF (IOFF.EQ.0) RETURN
   LL=LDIM-1
   DO 4 I=1,IOFF
   XX=AIMAG(DATA(1))
   DO 5 K=1,LL
5    DATA(K)=CMPLX(REAL(DATA(K)),AIMAG(DATA(K+1)))
   DATA(LDIM)=CMPLX(REAL(DATA(LDIM)),XX)
   RETURN
END
C*******************************************************************************
C GAUSSIAN DISTRIBUTED CARRIER PHASE JITTER
C*******************************************************************************
SUBROUTINE GAUCPE(DATA,DEG)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYM,BAUD
REAL R(65536)
COMPLEX DATA(1),EPS(65536)
DOUBLE PRECISION DSEED
DS=KDE=123457.0D0
CA1+ GGNML(DSEED,LDIM,R)
DO 10 I=1,LDIM
10   R(I)=R(I)**DEG/180.0**3.14159
    EPS(I)=CMPLX(COS(R(I)),SIN(R(I)))
    257
DATA(I)=DATA(I)*EPS(I)
10 CONTINUE
RETURN
END

C**********************************************************************
C SINUSOIDAL DISTRIBUTED CARRIER PHASE JITTER (DEG=P,P)
C**********************************************************************

SUBROUTINE SINCPE(DATA,DEG,FM)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
REAL R(65536)
COMPLEX DATA(1),EPS(65536)
PI=3.1415927
SAMP=1./(FLOAT(LSAMPL)*BAUD)
DO 10 I=1,LDIM
   R(I)=COS(2.*PI*FM*FLOAT(I-1)*SAMP)
   R(I)=R(I)*DEG/180.0*PI
   EPS(I)=CMPLX(COS(R(I)),SIN(R(I)))
DATA(I)=DATA(I)*EPS(I)
10 CONTINUE
RETURN
END

C**********************************************************************
C RESIDUAL AMP FLUCTUATION OR INCIDENTAL A.M.
C**********************************************************************

SUBROUTINE AMF(DATA,AM,FM)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
PI=3.1415927
SAMP=1./(FLOAT(LSAMPL)*BAUD)
DO 11 I=1,LDIM
   AMM=1.+AM*COS(2.*PI*FM*FLOAT(I-1)*SAMP)
DATA(I)=DATA(I)*AMM
11 CONTINUE
RETURN
END

C**********************************************************************
C TIKHONOV DISTRIBUTED CARRIER PHASE JITTER
C**********************************************************************

SUBROUTINE TIKCPE(DATA)
COMMON /NUMB1/ FBW,ALPHA,LDIM,IOFF,LSAMPL
COMMON /NUMB2/ NSNR,NSYMB,BAUD
COMMON /EE/E
REAL R(8192)
COMPLEX DATA(1),EPS(8192)
DOUBLE PRECISION DSEED
DSEED=123457.D0
CALL GGVMS(DSEED,E,LDIM,R)
DO 10 I=1,LDIM
EPS(I)=CMPLX(COS(R(I)),SIN(R(I)))
DATA(I)=DATA(I)*EPS(I)
10 CONTINUE
RETURN
END

C

THEORETICAL CALCULATION OF THE SYMBOL ERROR PROBABILITY OF
16-QAM IN THE PRESENCE OF RESIDUAL AMPLITUDE FLUCTUATIONS
AND AWGN. (THE WORST AND THE BEST SYMBOL ERROR PROBABILITIES
ARE GIVEN BY PE1 AND PE3, RESPECTIVELY.)

+++++++ FILE NAME : 16AM FORTRAN+++++++ 

INPUT PARAMETER : AM ( AM INDEX )

OUTPUT PARAMETER : SPE ( SYMBOL ERROR PROB.)

C

DIMENSION CN(40),SPE(40),XARRAY(42),YARRAY(42)
DIMENSION P1(40),P2(40),P3(40)
DOUBLE PRECISION CNR,PE1,PE2,PE3,PI,ROW1,ROW2,ROW3,AM
DOUBLE PRECISION SETA1,SETA2,SETA3,TPE
COMMON /NUMB2/ NSNR
PI=3.1415927D0

NSNR=40

C  -- ROW, SETA; ENVELOPE, PHASE OF SIGNAL --

ROW1=1.000
ROW2=DSQRT(5.D0)/3.D0
ROW3=1.00/3.D0
SETA1=DATAN(1.0D0)
SETA2=DATAN(ROW3)
SETA3=SETA1
AM=0.D0
NRUNS=4

DO 11 I=1,NSNR
   P1(I)=0.
   P2(I)=0.
   P3(I)=0.
11 SPE(I)=0.
DO 100 NR=1,NRUNS
WRITE(6,33) AM
33 FORMAT(10H AM INDEX=,D10.5)
DO 44 J=1,NSNR
   CNR=10.D0**(DFLOAT(J)/10.D0)
   CALL INTEG(CNR,AM,ROW1,SETA1,PE1)
   CALL INTEG(CNR,AM,ROW2,SETA2,PE2)
   CALL INTEG(CNR,AM,ROW3,SETA3,PE3)
C — OVERALL AVG. SYMBOL ERROR PROB. ——
   TPE=(PE1+3.D0*PE2+2.D0*PE3)/8.D0
   IF(TPE.LE.1.D-40) TPE=0.D0
   TPE=TPE-1.D0/4.D0*TPE**2.D0
   CN(J)=FLOAT(J)
   SPE(J)=SNGL(TPE)
C —— TYPICAL SYMBOL ERROR PROB. ——
   PE1=PE1*0.5D0
   PE2=PE2*0.75D0
   P1(J)=SNGL(PE1)
   P2(J)=SNGL(PE2)
   P3(J)=SNGL(PE3)

WRITE(6,22) J,SPE(J)
22 FORMAT(6H *CNR=,'5X,'5H *PE=,'E16.6)
C WRITE(6,23) P1(J),P2(J),P3(J)
C23 FORMAT(1X,5H PE1=,'E16.4,3X,5H PE2=,'E16.4,3X,5H PE3=,'E16.4)
44 CONTINUE
DO 99 I=1,NSNR
   XARRAY(I)=CN(I)
99 YARRAY(I)=SPE(I)
C ______________________________
C GOTO(1,2,3,4,NR
C1 YARRAY(I)=SPE(I)
C GOTO 99
C2 YARRAY(I)=P1(I)
C GOTO 99
C3 YARRAY(I)=P2(I)
C GOTO 99
C4 YARRAY(I)=P3(I)
C99 CONTINUE
   CALL DRAWCN(XARRAY,YARRAY,NR,NUNS)
   AM=AM+0.10D0
100 CONTINUE
STOP
END

C*****************************************************************************
C THIS SUBROUTINE PERFORMS INTEGRATION.
C INPUT PARAMETER : CNR, AM, ROW, SETA
C OUTPUT PARAMETER : PE (SYMBOL ERROR PROB.)
C*****************************************************************************

SUBROUTINE INTEG(CNR,AM,ROW,SETA,PE)
DOUBLE PRECISION CNR,PL,AM,ROW,SETA,PE,BL,P,AA,BB
DIMENSION AERF(501)
A(P)=DSQRT(CNR/10.D0)-3.D0*DSQRT(CNR/5.D0)*ROW*AM*DCOS(SETA)"
?DSIN(P)
B(P)=DSQRT(CNR/10.D0)-3.D0*DSQRT(CNR/5.D0)*ROW*AM*DSIN(SETA)"
?DSIN(P)
PI=3.1415927D0
DO 20 I=1,501
ERA=0.D0
ERB=0.D0
DELTA=DFLOAT(I-1)/500.D0
BL=PI/2.D0+PI*DELTA
AA=A(BL)
BB=B(BL)
IF(AA.GT.12.D0) AA=12.D0
IF(BB.GT.12.D0) BB=12.D0
ERA=DERFC(AA)
IF(ERA.LT.0.D0) ERA=0.D0
ERB=DERFC(BB)
IF(ERB.LT.0.D0) ERB=0.D0
AERF(I)=ERA+ERB
20 CONTINUE
PE=0.D0
DO 40 N=1,500
PE=(AERF(N)+AERF(N+1))/2.D0*PI/500.D0/PI+PE
IF(P,E.LE.1.D-40) PE=0.D0
40 CONTINUE
RETURN
END

C   THEORETICAL CALCULATION OF T.B SYMBOL ERROR PROBABILITY OF
C   256-QAM IN THE PRESENCE OF RESIDUAL AMPLITUDE FLUCTUATIONS
C   AND AWGN. (THE WORST AND THE BEST SYMBOL ERROR PROBABILITIES
C   ARE GIVEN BY PE1 AND PEs, RESPECTIVELY.)
C   TIGHT-BOUND SYMBOL ERROR PROB. IS OBTAINED BY AVERAGING
C   PE1,PE2,..., PEs.
C   ++++++++  FILE NAME : 256A FORTRAN ++++++++  
C   INPUT PARAMETER : AM ( AM INDEX )
C   OUTPUT PARAMETER : APE ( TIGHT-BOUND SYMBOL ERROR PROB.)
C***************************************************************************
C
DIMENSION CN(52),SPE(52),XARRAY(54),YARRAY(54),APE(52)
DOUBLE PRECISION CNR,PE1,PE2,PE3,PE,ROW1,ROW2,ROW3,AM
DOUBLE PRECISION SETA1,TFE,PE4,PE5,PE6,PE7,PE8
DOUBLE PRECISION ROW4,ROW5,ROW6,ROW7,ROW8,PPE
COMMON /NUMB2/ NSNR
PI=3.1415927D0

C
NSNR=52
ROW1=1.0D0
ROW2=13.0D0/15.D0
ROW3=11.0D0/15.D0
ROW4=9.0D0/15.D0
ROW5=7.0D0/15.D0
ROW6=5.0D0/15.D0
ROW7=3.0D0/15.D0
ROW8=1.0D0/15.D0
SETA1=DATAN(1.D0)

C
AM=0.D0
NRUNS=9
C
DO 11 J=1,NSNR

262
APE(I)=0.
DO 100 NR=1,NRUNS
WRITE(6,33) AM
33 FORMAT(16H AM INDEX=,D10.5)
DO 44 J=1,NSNR
CNR=10.D0**(DFLOAT(J)/10.D0)
CALL INTEG(CNR,AM,ROW1,SETA1,PE1)
CALL INTEG(CNR,AM,ROW2,SETA1,PE2)
CALL INTEG(CNR,AM,ROW3,SETA1,PE3)
CALL INTEG(CNR,AM,ROW4,SETA1,PE4)
CALL INTEG(CNR,AM,ROW5,SETA1,PE5)
CALL INTEG(CNR,AM,ROW6,SETA1,PE6)
CALL INTEG(CNR,AM,ROW7,SETA1,PE7)
CALL INTEG(CNR,AM,ROW8,SETA1,PE8)
CN(J)=FLOAT(J)
PPE=(0.5D0*PE1+PE2+PE3+PE4+PE5+PE6+PE7+PE8)/8.D0
IF(PPE.LE.1.D-35) PPE=0.D0
PPE=PPE-1.D0/4.D0*PPE**2.D0
APE(J)=SNGL(PPE)
WRITE(6,22) J,APE(J)
22 FORMAT(6H "CNR=",15,6H APE=,E16.6)
44 CONTINUE
DO 99 I=1,NSNR
XARRAY(I)=CN(I)
99 YARRAY(I)=APE(I)
CALL DRAWCN(XARRAY,YARRAY,NR,NRUNS)
AM=AM+0.01D0
100 CONTINUE
STOP
END

C*****************************************************************************

C THEORETICAL CALCULATION OF THE SYMBOL ERROR PROBABILITY OF
C 16-QAM IN THE PRESENCE OF GAUSSIAN DISTRIBUTED PHASE JITTER
C AND AWGN. (THE WORST AND THE BEST SYMBOL ERROR PROBABILITIES
C ARE GIVEN BY PE1 AND PE3, RESPECTIVELY.)
C
C +++++++  FILE NAME : 16PM FORTRAN +++++++
C INPUT PARAMETER : DEG ( PHASE JITTER IN RMS DEGREE )
C OUTPUT PARAMETER : SPE ( SYMBOL ERROR PROB. )
C CP ( CARRIER-TO-PHASE NOISE RATIO IN DB )
C*****************************************************************************
C
DIMENSION CN(40),SPE(40),XARRAY(42),YARRAY(42)
DIMENSION P1(40),P2(40),P3(40)
DOUBLE PRECISION CNR,PE1,PE2,PE3,PI,ROW1,ROW2,ROW3
DOUBLE PRECISION SETA1,SETA2,SETA3,TPE,DEG,SIGMA,SIG,CP
COMMON /NUMB2/ NSNR
PI=3.1415927D0
C
NRUNS=4
NSNR=40
ROW1=1.0D0
ROW2=DSQRT(5.0D0)/3.0D0
ROW3=1.0D0/3.0D0
SETA1=DATAN(1.0D0)
SETA2=DATAN(ROW3)
SETA3=SETA1
C
DEG=3.2D0
C
DO 11 I=1,NSNR
P1(I)=0.
P2(I)=0.
P3(I)=0.
11 SPE(I)=0.
DO 100 NR=1,NRUNS
WRITE(6,33) DEG
33 FORMAT(9H RMS DEG=,'D10.3)
SIGMA=DEG/180.0D0*PI
SIG=SIGMA**2.0D0
CP=-10.0D0*DLOG10(SIG)
WRITE(6,55) CP
55 FORMAT(5X, 'CARRIER-TO-PHASE NOISE RATIO=','D10.5,' DB')
DO 44 J=1,NSNR
CNR=10.0D0**(DFLOAT(J)/10.0D0)
CALL INTEG(CNR,SIGMA,ROW1,SETA1,PE1)
CALL INTEG(CNR,SIGMA,ROW2,SETA2,PE2)
CALL INTEG(CNR,SIGMA,ROW3,SETA3,PE3)
C
TPE=(PE1+3.0D0*PE2+2.0D0*PE3)/8.0D0
IF(TPE.LE.1.D-40) TPE=0.0D0
TPE=TPE-1.0D0/4.0D0*TPE**2.0D0
CN(J)=FLOAT(J)
SPE(J)=SNGL(TPE)

C

PE1=PE1*0.5D0
PE2=PE2*0.75D0
P1(J)=SNGL(PE1)
P2(J)=SNGL(PE2)
P3(J)=SNGL(PE3)

C

WRITE(6,22) J,SPE(J)
22 FORMAT(6H "CNR=",IS, DB',5X,5H "PE=",E16.6)

C

C WRITE(6,23) P1(J),P2(J),P3(J)
C23 FORMAT(1X,5H PE1=,E16.4,3X,5H PE2=,E16.4,3X,5H PE3=,E16.4)

C CONTINUE
44 DO 99 I=1,NSNR
59 XARRAY(I)=CN(I)
99 YARRAY(I)=SPE(I)

C

C GOTO(1,2,3,4),NR
C1 YARRAY(I)=SPE(I)
C GOTO 99
C2 YARRAY(I)=P1(I)
C GOTO 99
C3 YARRAY(I)=P2(I)
C GOTO 99
C4 YARRAY(I)=P3(I)
C99 CONTINUE
C

CALL DRAWCN(XARRAY,YARRAY,NR,NRUNS)

C

DEG=DEG+0.2D0
100 CONTINUE
STOP
END

C

C******************************************************************************
C THIS SUBROUTINE PERFORMS INTEGRATION.
C INPUT PARAMETER : CNR, SIGMA, ROW, SETA
C OUTPUT PARAMETER : PE (SYMBOL ERROR PROB.)
C******************************************************************************
SUBROUTINE INTEG(CNR,SIGMA,ROW,SETA,PE)
DOUBLE PRECISION CNR,PI,ROW,SETA,PE,AL,E,AA,BB,CC,DD,EE
DOUBLE PRECISION SIGMA,EX,CL
DIMENSION A Erf(501),X Erf(501)
A(E)=DSQRT(CNR/10.D0)-3.D0*DSQRT(CNR/5.D0)*ROW*(DCOS(SETA-E)
?-DCOS(SED))
B(E)=DSQRT(CNR/10.D0)-3.D0*DSQRT(CNR/5.D0)*ROW*(DCOS(SETA)
?-DCOS(SETA+E))
C(E)=DSQRT(CNR/10.D0)-3.D0*DSQRT(CNR/5.D0)*ROW*(DSIN(SETA+E)
?-DSIN(SETA))
D(E)=DSQRT(CNR/10.D0)-3.D0*DSQRT(CNR/5.D0)*ROW*(DSIN(SETA)
?-DSIN(SETA-E))
ES(E)=-(E/SIGMA)**2.D0)/1.D0
PI=3.1415927D0
DO 20 I=1,501
ER=0.D0
ERB=0.D0
ERC=0.D0
ERD=0.D0
DELTA=DFLOAT(I-1)/500.D0
AL=1.5D0*SIGMA+10.D0*SIGMA*DELTA
CL=DABS(AL)
AA=A(AL)
BB=B(AL)
CC=C(AL)
DD=D(AL)
IF(AA.GT.12.D0) AA=12.D0
IF(BB.GT.12.D0) BB=12.D0
IF(CC.GT.12.D0) CC=12.D0
IF-DD.DT.12.D0-DD=12.D0
ERA=DERF(AD)
ERB=ERF(BB)
ERC=ERF(CC)
ERD=ERF-DD)
IF(ERA.LT.0.D0) ERA=0.D0
IF(ERB.LT.0.D0) ERB=0.D0
IF(ERC.LT.0.D0) ERC=0.D0
IF(ERD.LT.0.D0) ERD=0.D0
A Erf(I)=ERA+ERB+ERC+ERD
EE=ES(ED)
IF(EE.LT.-45.D0) EE=-45.D0
EX=DEXP(EE)/DSQRT(2.D0*PI)/SIGMA/2.D0
XERF(I)=AERF(I)*EX

20 CONTINUE
PE=0.D0
DO 30 N=1,499,2
PE=(XERF(N)+4.D0*XERF(N+1)+XERF(N+2))/3.D0*10.D0*SIGMA/500.D0+PE
IF(PE.LE.1.D-40) PE=0.D0
30 CONTINUE
RETURN
END

C**************************************************************************
C THEORETICAL CALCULATION OF T.B SYMBOL ERROR PROBABILITY OF
C 256-QAM IN THE PRESENCE OF GAUSSIAN DISTRIBUTED PHASE JITTER
C AND AWGN. (THE WORST AND THE BEST SYMBOL ERROR PROBABILITIES
C ARE GIVEN BY PE1 AND PE8, RESPECTIVELY.)
C TIGHT-BOUND SYMBOL ERROR PROB. IS OBTAINED BY AVERAGING
C PE1, PE2, ..., PE8.
C
+++++++ FILE NAME : 256P FORTRAN +++++++
C INPUT PARAMETER : DEG (PHASE JITTER IN RMS DEGREE )
C OUTPUT PARAMETER : APE (TIGHT-BOUND SYMBOL ERROR PROB.)
C CP (CARRIER-TO-PHASE NOISE RATIO IN DB)
C**************************************************************************
C
DIMENSION CN(52),SPE(52),XARRAY(54),YARRAY(54),APE(52)
DOUBLE PRECISION CNR,PE1,PE2,PE3,PI,ROW1,ROW2,ROW3
DOUBLE PRECISION SETA1,TPE,PE4,PE5,PE6,PE7,PE8,SIG,CP
DOUBLE PRECISION ROW4,ROW5,ROW6,ROW7,ROW8,PPE,DEG,SIGMA
COMMON /NUMB2/ NSNR
PI=3.1415927D0
C

NRUNS=4
NSNR=52
ROW1=1.0D0
ROW2=13.0D0/15.D0
ROW3=11.0D0/15.D0
ROW4=9.0D0/15.D0
ROW5=7.0D0/15.D0
ROW6=5.0D0/15.D0
ROW7=3.0D0/15.D0
ROW8=1.0D0/15.D0

267
SETA1=DATAN(1.D0)

C

DEG=0.89D0

C

DO 11 I=1,NSNR

11 APE(I)=0.

DO 100 NR=1,NRUNS

WRITE(6,23) DEG

33 FORMAT(9H RMS DEG=',D10.5)

SIGMA=DEG/180.D0*PI

SIG=SIGMA**2D0

CP=-10D0*DLOG10(SIG)

WRITE(6,55) CP

55 FORMAT(5X, 'CARRIER-TO-PHASE NOISE RATIO=',D10.5,' DB')

DO 44 J=1,NSNR

CNR=10D0**((DFLOAT(J)/10.D0)

CALL INTEGR(CNR,SIGMA,ROW1,SETA1,PE1)

CALL INTEGR(CNR,SIGMA,ROW2,SETA1,PE2)

CALL INTEGR(CNR,SIGMA,ROW3,SETA1,PE3)

CALL INTEGR(CNR,SIGMA,ROW4,SETA1,PE4)

CALL INTEGR(CNR,SIGMA,ROW5,SETA1,PE5)

CALL INTEGR(CNR,SIGMA,ROW6,SETA1,PE6)

CALL INTEGR(CNR,SIGMA,ROW7,SETA1,PE7)

CALL INTEGR(CNR,SIGMA,ROW8,SETA1,PE8)

C

CN(J)=FLOAT(J)

PPE=(0.5D0*PE1+PE2+PE3+PE4+PE5+PE6+PE7+PE8)/8D0

IF(PPE.LE.1D-35) PPE=0.D0

PPE=PPE-1D0/4D0*PPE**2D0

APE(J)=SINGL(PPE)

WRITE(6,22) J,APE(J)

22 FORMAT(6H "CNR=',5S, 'DB', 5X, 6H APE=',E16.6)

44 CONTINUE

DO 99 I=1,NSNR

XARRAY(I)=CN(I)

99 YARRAY(I)=APE(I)

CALL DRAWCN(XARRAY,YARRAY,NR,NRUNS)

C

DEG=DEG+0.05D0

100 CONTINUE

STOP

268
END

C******************************************************************************
C COMPUTE THE PROBABILITY OF ERROR OF 4-, 16-, 64-, 256-QAM
C IN THE PRESENCE OF NON-GAUSSIAN IMPULSIVE NOISE.
C ++++++++ FILE NAME : IMPULS FORTRAN A1 ++++++++ 
C L= LEVELS OF M-ARY QAM
C A= OCCURRENCE OF IMPULSES
C GAMA= POWER RATIO OF GAUSSIAN-TO-IMPULSIVE NOISE
C******************************************************************************

DOUBLE PRECISION X,X1,Y,Y0,Y1,Z1,W,W1,W10,F1,YM,GM,FL,F,YM,GAMA,PI
DOUBLE PRECISION R1,R2,R3,D,Z,CIR,CNR,SGMMAD,HAP,HAP1,X2,X3,X4,X5
?X6,Y2,Y3,Y4,Y5,Y6,Y7,Y8,Y9,RATIO1,RATIO2,SM,SM1,S,A,PE1,PE
?,W2,F2,W2,F,T,DT,M1,XQ,F,T,DD,P,AA
DIMENSION PCNR(50),PEC(50),YARRAY(50),XARRAY(50)
COMMON /NUMB1/ JCNR,INC,CON
C — NQAM=1,2,3,4 FOR 4-,16-,64-,256-QAM, RESPECTIVELY —
NQAM=1
L=4**NQAM
A=0.01D0
GAMA=1.D0
C ————— NOB=NO. OF RUNS —————
NOB=3

S=1.D-9
PI=3.1415927D0
JCNR=36
ICN=4**(NQAM-1)+4
CN1=FLOAT(ICN)
DO 150 NOA=1,NOB
C A=1.D0
C AA=FLOAT(NOAA)
C A=A*10.D0***(1.D0-AA)
FL=FLOAT(L)
WRITE(6,11) L
11 FORMAT(23,'-QAM')
WRITE(6,12) A,GAMA
12 FORMAT(3H A=,D10.5,6X,6H GAMA=,D10.5)
C DO 99 J=1,JCNR

269
R2=FLOAT(J)
GOTO(90,91,92,93), NQAM
90  R2=R2+0.D0
     GOTO 94
91  R2=R2+2.55D0
     GOTO 94
92  R2=R2+3.67D0
     GOTO 94
93  R2=R2+4.23D0
94  CNR=10.D0**((R2/10.D0)
C
D=0.D0
SUM=0.D0
Z=1.D0
C
22  SIGMAD=(D/A+GAMA)/(1.D0+GAMA)
    XQ=DSQRT(2.D0*CNR)/2.D0/(DSQRT(FL)-1.D0)/DSQRT(SIGMAD)
    IF(XQ.GT.5.5D0) XQ=5.5D0
    CALL ERRFC(XQ,E)
    SUM1=A**D/Z*E
    SUM=SUM+SUM1
    D=D+1.D0
    Z=Z*D
    IF(E.EQ.0.D0) GOTO 22
    IF(SUM.EQ.0.D0) GOTO 22
    RATIO2=DABS(SUM1/SUM)
    IF(RATIO2.GT.S) GOTO 22
    PE1=1.D0/DEXP(A)/2.D0*SUM
    PE=4.D0/FL**(FL-DSQRT(FL))*PE1
    IF(P.EQ.0.D0) PE=0.D0
    PE=PE-1.D0/4.D0*PE**2.D0
    FEE(J)=SNGL(PE)
    PCNR(J)=FLOAT(J)
    XARRAY(J)=PCNR(J)
    YARRAY(J)=FEE(J)
90  CONTINUE
     WRITE(6,41) (PCNR(J),FEE(J),J=1,JCNR)
41  FORMAT(5X,SH CNR=F5.1,’ DB’,5X,4H PE=,E16.8)
    CALL DRAWCN(XARRAY,YARRAY,NOA,NOB)
    GAMA=GAMA*10.D0
150  CONTINUE
STOP
END

C

C********************************************************************
C
C CALCULATE COMPLEMENTARY ERROR FUNCTION
C********************************************************************

SUBROUTINE ERRFC(XQ,E)
DOUBLE PRECISION Y,T,DD,P,E,XQ
Y=1.42421356D0*XQ
T=1.D0/(1.D0+0.2316419D0*Y)
DD=0.3989423D0*DEXP(-1.D0*Y*Y/2.D0)
P=1.D0-DD*T*((((1.330274D0*T-1.821256D0)*T+1.781478D0)*T-70.356668D0)*T+0.3193815D0)
P=2.D0*P-1.D0
P=1.D0-P
E=P
RETURN
END
Bibliography


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