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FREE VIBRATION ANALYSIS
OF RECTANGULAR CANTILEVER PLATES
WITH SYMMETRICALLY DISTRIBUTED POINT SUPPORTS

by
H.T. Saliba

Thesis presented to the School of Graduate Studies
as partial fulfillment of the requirements
for the degree of M.A.Sc. in
Mechanical Engineering

UNIVERSITY OF OTTAWA
OTTAWA, CANADA, 1982
ABSTRACT

The objective of the present thesis is to provide an analytical "exact" solution to the free vibration problem of the cantilever plate with symmetrically distributed point supports using the method of superposition.

Starting with a brief review of the underlying theory in Chapter 1, Chapter 2 looks into the free vibration of the cantilever plate establishing the general procedure of the superposition method. Chapters 3 and 4 deal specifically with the application of this method to the free vibration problem of the cantilever plate with edge and lateral point supports respectively. Eigenvalues obtained are tabulated throughout this work. Mode shapes are illustrated in Appendix 2, while Appendix 3 shows a complete list of computer programs used to generate both eigenvalues and mode shapes. The following is a review of the historical progress of plate analysis.

HISTORICAL

According to Todhunter, and Pearson(1), the development of structural mechanics began with the investigation of static problems. However, the first analytical and experimental studies on plates were almost exclusively devoted to the problem of free
vibration(2). In 1766, Euler was the first to formulate a mathematical approach to the membrane theory of plates, and to use the analogy of two systems of stretched strings perpendicular to each other to solve the problem of rectangular and circular elastic membranes free vibrations(3). In 1789, by introducing the gridwork analogy, Euler's student, J. Bernoulli extended Euler's analogy to plates(4). However, Bernoulli found only resemblance between theory and experiments but no general agreement. That is to be expected since the torsional resistance of plates was not included in the differential equation of motion.

As to the various modes of free vibrations they were first discovered by the German physicist Chlandi(1). On a horizontal plate, he used evenly distributed powder, which, after induction of vibration, accumulated along the nodal lines.

Later, using the calculus of variations, Sophie Germain, a French mathematician, obtained a differential equation for the vibration of plates. Although, the Parisian Academy presented a prize award to Germain for her work in 1816, the work done by warping of the middle surface was not represented in her expression for the strain energy. However, one of Germain's judges, Lagrange, corrected the strain energy expression by adding the missing term, and he is believed to be the first to use the correct differential equation of the free vibration of plates.
Although, the correct differential equation of the free vibration of plates was found in Lagrange's notes in 1813, its derivation is attributed to Navier (1785-1836). A great engineer and bridge designer, Navier is known to be the real originator of the modern theory of elasticity. The solution of various plate problems is one of his numerous scientific activities. He derived the correct differential equation of rectangular plates with flexural resistance, and using the trigonometric series introduced by Fourier in the same decade, he provided an exact method for the solution of certain boundary value problems. This method yields mathematically exact solutions for the plate with Navier type boundary conditions, and that is the simply supported plate. Then in 1829, Poisson extended Navier's solution to the lateral vibration problem of thick circular plates.

Later, Kirchhoff came along (1824-1887), to go a step further in introducing the extended plate theory which takes into account the combined bending and stretching (5). He found that in analyzing large deflections of plates, the nonlinear terms could no longer be neglected. He also introduced the method of virtual displacement and the equation of frequency of plates.
Clebsch, in his translation to Kirchhoff's book(6), provided numerous valuable comments by Saint-Venant, among which, the extension of Kirchhoff's differential equation of thin plates, is considered to be the most important. In this extended equation the combined action of bending and stretching were correctly considered.

Russian scientists made a significant contribution to naval architecture by being the first to replace the ancient trade traditions by solid mathematical theories. However, due to the language barrier, Western scientists could not make use of these achievements until Timoshenko came along. It is through his work that Western scientists started to direct their attention toward the Russian research in the field of theory of elasticity. Among these Russian scientists are Krylov(1863-1945)(7), and his student Boobnov(8)(9)(10). As for Timoshenko's numerous important books, and contributions we list(11)(12)(13).

Among the numerous scientists that have worked on plate related problems, we name Foppl, who in his book on engineering mechanics(14) treated the nonlinear theory of plates, and Karman(15), who developed the final form of the differential equation of the large deflection theory.
For the period between the two world wars, we list only Bleich, Federhofer, Wagner, and Lévy. Then from the Soviet Union, Oniashvili and Gontkevitsh investigated the free and forced vibration of plates. Then came Johansen with the yield-line analysis (16) to introduce the first important deviation from the classic theory of elasticity in the solution of plate problems.

It is only recently that high speed electronic computers influenced the static and dynamic analysis of plates. Although, Hrennikoff had already developed an equivalent gridwork system for the static analysis of complex plate problems in 1941 (17), his technique could not be fully utilized due to the lack of high speed computers. Then with the introduction of high speed computers (1950's) Turner Clough, Martin, and Topp in 1956, introduced the finite element method, which permits the numerical solution of complex plate and shell problems in an economical way (18). However, the use of this method anticipates the availability of high speed computers with considerable storage capacity. One should also name here Argyris and Zienkiewicz for their numerous contributions in this field. Another numerical method for the static and dynamic analysis of plates of arbitrary shape subjected to arbitrary loads is based on improved finite difference techniques and was developed by Stussi and Collatz.
Although, considerable amounts of time and effort were put into the solution of the problem of free vibration of plates by numerous scientists and researchers for the past several centuries, up to the year 1969, it had been designers' experience that there is no single reference to which they can turn for a comprehensive list of available data and results on the subject of plate free vibration. It is thanks to Leissa of the Ohio State University, Columbus, Ohio, that such a reference came into existence. Writing of the manuscript began in the summer of 1965, and its publication came about in 1969(19). In his book Leissa provides a comprehensive list of available results for the frequencies and mode shapes of free vibration of plates. He also lists an exhaustive list of references on related research and publications. Although, Leissa in his book claims a "reasonable completeness of results published through the end of the year 1965," he acknowledges that some significant publications were not included in his book for the reason of unavailability.

With the publication of Leissa's book it became even clearer that most of the available solutions of plate vibration problems do not satisfy exactly the governing differential equation, the prescribed boundary conditions, or even both. Furthermore, none of the references that Leissa lists
in his book treats the subject of rectangular plate free vibration in a clear and orderly manner.

Most recently (1982) it is to D.J. Gorman's credit that such an orderly book came into existence (20). In this book, Gorman not only provides the graduate students and the design engineers with a reference "to which they can turn for a clear and orderly exposition of the general subject of rectangular plate free vibration" as he sees it, but he added a new dimension to an old concept, the superposition concept (21), and by so doing, he provided an exact analytic method for the solution of the plate free vibration problem. The method of superposition itself is not new, in fact it has been in use for solving static plate problems for decades. However, its introduction to the dynamic plate problems is credited to Gorman. In his book and numerous other publications, Gorman treated the problem of rectangular plate free vibration with numerous different types of boundary conditions and point supports. One case that Gorman did not approach in his book, or any of his publications is the case of the rectangular cantilever plate with edge or lateral point supports. Furthermore, to the author's knowledge, this particular problem has not been published or approached by any other researcher. Therefore, this thesis will concern itself with the study of this particular problem, providing a
solution using two different methods. First, the method of 
superposition will be used as it was introduced by Gorman, 
to provide an exact analytical solution to the problem of 
the free vibration of the rectangular cantilever plate with 
or without point supports. Then the method of improved finite 
element will be used to generate eigenvalues for comparison 
purposes.
ACKNOWLEDGEMENTS

The author wishes to acknowledge the excellent cooperation and help given by members of the Mechanical Engineering Department in particular and all of the University of Ottawa in general.

It is the author's wishes to pay particular thanks to Dr. D.J. Gorman, who was the advisor for this thesis and who provided countless thoughtful suggestions and ideas throughout this work that enabled the end product to be reached. The interest, patience, encouragement, and dedication offered by Dr. Gorman are highly valued.

Thanks are also due to the staff of the University Computing Centre for their understanding and cooperation.

Thank You All

The Author
### LIST OF SYMBOLS

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>a</td>
<td>Plate lateral dimension in the x direction</td>
</tr>
<tr>
<td>b</td>
<td>Plate lateral dimension in the y direction</td>
</tr>
<tr>
<td>D</td>
<td>Plate flexural rigidity $= \frac{Eh^3}{12(1-\nu^2)}$</td>
</tr>
<tr>
<td>E</td>
<td>Young's modulus of the plate material</td>
</tr>
<tr>
<td>f</td>
<td>Plate vibration frequency in Hertz</td>
</tr>
<tr>
<td>h</td>
<td>Plate thickness</td>
</tr>
<tr>
<td>t</td>
<td>Time in seconds</td>
</tr>
<tr>
<td>u</td>
<td>Distance between the concentrated force and the $\eta$ axis divided by side length $a$</td>
</tr>
<tr>
<td>$u^*$</td>
<td>$= 1 - u$</td>
</tr>
<tr>
<td>v</td>
<td>Distance between the concentrated force and the $\xi$ axis divided by the side length $b$</td>
</tr>
<tr>
<td>$v^*$</td>
<td>$= 1 - v$</td>
</tr>
<tr>
<td>$\xi$</td>
<td>Distance between the concentrated edge force and the plate central axis divided by the plate edge dimension</td>
</tr>
<tr>
<td>$W(\xi, \eta, t)$</td>
<td>Plate lateral displacement</td>
</tr>
<tr>
<td>$W(\xi, \eta)$</td>
<td>Amplitude of the plate lateral displacement</td>
</tr>
<tr>
<td>$x, y$</td>
<td>Plate spatial coordinates</td>
</tr>
<tr>
<td>$\lambda^2$</td>
<td>$= \omega a^2 \sqrt{D}$, plate eigenvalue</td>
</tr>
<tr>
<td>$\rho$</td>
<td>Mass of the plate per unit area</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Circular frequency of plate vibration</td>
</tr>
<tr>
<td>$\phi$</td>
<td>$= b/a$, plate aspect ratio</td>
</tr>
<tr>
<td>$\Phi^*$</td>
<td>$= a/b$, inverse of the plate aspect ratio</td>
</tr>
<tr>
<td>Symbol</td>
<td>Explanation</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Poisson's ratio for the plate material</td>
</tr>
<tr>
<td>$\nu^*$</td>
<td>$= 2 - \nu$</td>
</tr>
<tr>
<td>$\xi$</td>
<td>$= x/a$, dimensionless plate spatial coordinate</td>
</tr>
<tr>
<td>$\eta$</td>
<td>$= y/b$, dimensionless plate spatial coordinate</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Plate solution parameter, as defined in the thesis</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Plate solution parameter, as defined in the thesis</td>
</tr>
</tbody>
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Chapter 1

THE THEORY OF PLATES FREE VIBRATION

The objective of this thesis is to provide an analytical "exact" solution to the free vibration problem of the cantilever plate with edge and lateral point supports. However, it seems appropriate to start with a brief discussion on the underlying theory of the free vibration of plates in general and the method of superposition as introduced by Gorman (20) in particular. Although the author appreciates the importance of providing the reader with the general theory and the procedure that are followed throughout this thesis, and form the basis of the final results, it is left up to the reader to consult Gorman's book for any detailed derivations or explanation.

The Differential Equation

The correct differential equation governing the pure bending of plates subjected to lateral static loading is presented with detailed derivation by Timoshenko (11), and is written as

\[
\frac{\partial^4 W(x,y)}{\partial x^4} + 2\frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y)}{\partial y^4} = \frac{q(x,y)}{D}
\]
where \( q(x,y) \) is the applied static loading.

The governing differential equation for the free vibration of rectangular plates is obtained by replacing the lateral force \( q(x,y) \) of Equation (1.1) by the inertial force and by introducing the time variable parameter \( t(20) \).

\[
\frac{\partial^4 W(x,y,t)}{\partial x^4} + 2\frac{\partial^4 W(x,y,t)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y,t)}{\partial y^4} + \frac{\partial^2 W(x,y,t)}{\partial t^2} = 0 \quad \text{\ldots (1.2)}
\]

by replacing \( W(x,y,t) \) by its equivalent \( W(x,y)T(t) \), it can be shown that \( T(t) = \text{Asin}(\omega t + \alpha) \). And Equation (1.2) can be written as

\[
\frac{\partial^4 W(x,y)}{\partial x^4} + 2\frac{\partial^4 W(x,y)}{\partial x^2 \partial y^2} + \frac{\partial^4 W(x,y)}{\partial y^4} - \frac{\omega^2 D}{\rho} W(x,y) = 0 \quad \text{\ldots \ldots\ldots\ldots (1.3)}
\]

in its dimensionless form Equation (1.3) can be written as

\[
\frac{\partial^4 W(\xi,\eta)}{\partial \xi^4} + 2\frac{\partial^4 W(\xi,\eta)}{\partial \xi^2 \partial \eta^2} + \frac{\partial^4 W(\xi,\eta)}{\partial \eta^4} - \lambda^4 W(\xi,\eta) = 0 \quad \text{\ldots\ldots\ldots\ldots (1.4)}
\]

or

\[
\frac{\partial^4 W(\xi,\eta)}{\partial \eta^4} + 2\phi^2 \frac{\partial^4 W(\xi,\eta)}{\partial \eta^2 \partial \xi^2} + \phi^4 \frac{\partial^4 W(\xi,\eta)}{\partial \xi^4} - \phi^4 \lambda^4 W(\xi,\eta) = 0 \quad \text{\ldots (1.5)}
\]

**The Lévy Type Solution**

In 1820, Navier presented a paper to the French Academy of Sciences on the solution of bending of simply supported rectangular plates by double trigonometric series. This solution
is sometimes called the forced solution. At the turn of the century, in 1899, a solution by single Fourier series was introduced by Levy (22). This powerful method obtains the solution of Equation (1.5) in the form (20)

$$\lim_{\kappa \to \infty} W(\xi, \eta) = \sum_{m=1}^{\kappa} Y_m(\eta) \sin(m \pi \xi)$$  \hspace{1cm} (1.6)

by substituting this solution in Equation (1.5) and rearranging we have:

$$d^4 Y_m(\eta)/dn^4 - 2 \phi^2 (m \pi)^2 d^2 Y_m(\eta)/dn^2 + \phi^4 ((m \pi)^4 - \lambda^4) Y_m(\eta) = 0 \hspace{1cm} (1.7)$$

the solution of Equation (1.7) is well known and it depends on whether $\lambda^2 - (m \pi)^2$ is negative or positive.

If $\lambda^2 > (m \pi)^2$ then

$$Y_m(\eta) = A_m \cosh \beta_m \eta + B_m \sinh \beta_m \eta + C_m \sin \gamma_m \eta + D_m \cos \gamma_m \eta \hspace{1cm} (1.8)$$

If $\lambda^2 < (m \pi)^2$ then

$$Y_m(\eta) = A_m \cosh \beta_m \eta + B_m \sinh \beta_m \eta + C_m \sinh \gamma_m \eta + D_m \cosh \gamma_m \eta \hspace{1cm} (1.9)$$

where $\beta_m = \phi \sqrt{\lambda^2 + (m \pi)^2}$

and $\gamma_m = \phi \sqrt{\lambda^2 - (m \pi)^2}$ or $\phi \sqrt{(m \pi)^2 - \lambda^2}$ \hspace{1cm} (1.10)

whichever is real. And where $A_m$, $B_m$, $C_m$, and $D_m$ are constants.
to be determined by means of prescribed boundary conditions!

Limitations

Although, Lévy's method, which uses a single trigonometric series is more general than Navier's solution, it does not have an entirely general character since in its original form it can only be applied to rectangular plate vibration problems where at least two opposite edges have simple supports. However, it has been shown that by making use of the superposition method(20)(21), this Lévy-type solution is readily employed to solve not only rectangular plate vibration problems of all possible combinations of classical boundary conditions, but also to analyze numerous rectangular plates with nonclassical type boundary conditions. Next, a word about the classical boundary conditions and their mathematical formulation.

Classical Boundary Conditions

Whenever classical boundary conditions are mentioned, our attention is directed toward the three types of boundary conditions that have been studied so thoroughly in the classical literature, and they are, the clamped, the simply supported, and the

1. unless stated otherwise, the coordinate system used will always be that of Figure 1.1 throughout this work
free edge conditions. Although, we are dealing with the problem of free vibration, the formulation of these boundary conditions is identical to that found in Timoshenko's work\(^{(11)}\) on static plate analysis. This formulation will be presented here in both conventional and dimensionless coordinate systems. Reference is made to figures 1.1 and 1.2.

Clamped Edges:

Conventional coordinates

\[
W(x,y) = \frac{\partial W(x,y)}{\partial x} = 0 \tag{1.11}
\]

Dimensionless coordinates

\[
W(\xi,\eta) = \frac{W(\xi,\eta)}{\partial \xi} = 0 \tag{1.12}
\]

Figure 1.1. Conventional rectangular coordinates system
Simply Supported Edges:

Conventional coordinates

\[ W(x,y) = \frac{\partial^2 W(x,y)}{\partial x^2} = 0 \]

\[ \text{(1.13)} \]

Dimensionless coordinates

\[ W(\xi,\eta) = \frac{\partial^2 W(\xi,\eta)}{\partial \xi^2} = 0 \]

\[ \text{(1.14)} \]

Free Edges:

Conventional coordinates

\[ \frac{\partial^2 W(x,y)}{\partial x^2} + \nu \frac{\partial^2 W(x,y)}{\partial y^2} = 0 \mid x=a \]

\[ \text{(1.15)} \]

\[ \frac{\partial^3 W(x,y)}{\partial x^3} + \nu \frac{\partial^3 W(x,y)}{\partial x \partial y^2} = 0 \mid x=a \]

\[ \frac{\partial^2 W(x,y)}{\partial y^2} + \nu \frac{\partial^2 W(x,y)}{\partial x^2} = 0 \mid y=b \]

\[ \text{(1.16)} \]

Dimensionless coordinates

\[ \frac{\partial^2 W(\xi,\eta)}{\partial \xi^2} + \frac{\nu}{\phi^2} \frac{\partial^2 W(\xi,\eta)}{\partial \eta^2} = 0 \mid \xi=1 \]
\[
\frac{\partial^3 W(\xi, \eta)}{\partial \xi^3} + \frac{\nu}{\phi^2} \frac{\partial^3 W(\xi, \eta)}{\partial \xi \partial \eta^2} = 0 \mid \xi = 1
\]
\[
\frac{\partial^2 W(\xi, \eta)}{\partial \eta^2} + \frac{\nu \phi^2}{\phi} \frac{\partial^2 W(\xi, \eta)}{\partial \xi^2} = 0 \mid \eta = 1
\]
\[
\frac{\partial^3 W(\xi, \eta)}{\partial \eta^3} + \frac{\nu \phi^2}{\phi} \frac{\partial^3 W(\xi, \eta)}{\partial \eta \partial \xi^2} = 0 \mid \eta = 1.
\]

where the displacement and coordinate x are divided by plate dimension a, while the coordinate y is divided by dimension b. And where \( \phi = \frac{b}{a} \).

As we progress further ahead, it will become apparent that mathematical formulations for bending moments and vertical edge reactions are also required, as presented in the next paragraph.

![Figure 1.2. Rectangular plate dimensionless coordinate system.](image-url)
Edge Bending Moments and Vertical Reactions

The mathematical expressions for bending moments and vertical edge reactions distributed along the edges of a rectangular plate have been developed in the conventional coordinates by Timoshenko (11), and in the dimensionless coordinates by Gorman (20). These expressions are as follows:

Distributed Bending Moments:

Conventional coordinates

\[ M_x = -D \left( \frac{\partial^2 W(x,y)}{\partial x^2} + \nu \frac{\partial^2 W(x,y)}{\partial y^2} \right) \]

\[ M_y = -D \left( \frac{\partial^2 W(x,y)}{\partial y^2} + \nu \frac{\partial^2 W(x,y)}{\partial x^2} \right) \]  \hspace{1cm} (1.17)

Dimensionless coordinates

\[ \frac{M_a}{D} = - \left( \frac{\partial^2 W(\xi,\eta)}{\partial \xi^2} + \frac{\nu}{\phi^2} \frac{\partial^2 W(\xi,\eta)}{\partial \eta^2} \right) \]

\[ \frac{M_b}{D} = - \left( \frac{\partial^2 W(\xi,\eta)}{\partial \eta^2} + \nu \phi^2 \frac{\partial^2 W(\xi,\eta)}{\partial \xi^2} \right) \]  \hspace{1cm} (1.18)

Reference is made to Figure 13.
Vertical Edge Reactions:

Conventional coordinates

\[
V_x = -D \left( \frac{\partial^3 W(x,y)}{\partial x^3} + \nu^* \frac{\partial^3 W(x,y)}{\partial x \partial y^2} \right)
\]

\[
V_y = -D \left( \frac{\partial^3 W(x,y)}{\partial y^3} + \nu^* \frac{\partial^3 W(x,y)}{\partial y \partial x^2} \right)
\]  

(1.19)

Dimensioless coordinates

\[
\frac{V_x a^2}{D} = -\left( \frac{\partial^3 W(\xi,\eta)}{\partial \xi^3} + \nu^* \frac{\partial^3 W(\xi,\eta)}{\partial \xi \partial \eta^2} \right)
\]

\[
\frac{V_y b^2}{D} = -\left( \frac{\partial^3 W(\xi,\eta)}{\partial \eta^3} + \nu^* \frac{\partial^3 W(\xi,\eta)}{\partial \eta \partial \xi^2} \right)
\]  

(1.20)

where \(\nu^* = 2 - \nu\) and where reference is made to Figure 1.4

![Diagram of bending moments along the edges of a rectangular plate.](image)

Figure 1.3. Bending moments acting along the edges of a rectangular plate.
The vertical concentrated force that will act at each corner of a rectangular plate, as explained by Timoshenko(11) is represented by the following expressions.

**Corner Vertical Reaction:**

Conventional coordinates

\[ R = 2D(1-v) \frac{\partial^2 w(x,y)}{\partial x \partial y} \] ..........................(1.21)

Dimensionless coordinates.

\[ R = \frac{2D}{b^3(1-v)} \frac{\partial^2 w(\xi,\eta)}{\partial \xi \partial \eta} \] ..........................(1.22)

So far we have looked at all the basic equations that will be used throughout this work. Next, a word about the superposition method as it applies to rectangular plate free vibration problems(21).

![Diagram of a rectangular plate with vertical forces acting along the edges.](image)

**Figure 1.4:** Vertical edge reactions acting along the edges of a rectangular plate.
The Method of Superposition

It has been mentioned that Lévy-type solutions are easily found, and well established for the family of rectangular plates with at least two opposite edges simply supported (20). A second family of rectangular plates is that of plates for which no two opposite edges are simply supported. Most of these plates were analyzed by Gorman using the method of superposition. In this method, two or more appropriate plate problems whose Lévy-type solutions can be obtained are superimposed, and the constants, appearing in their boundary condition formulation, are adjusted in such a way so that their combination provides boundary conditions similar to those specified in the original problem. The plate problems that are superimposed are often called building blocks. Another example of the application of this powerful method is demonstrated in the forthcoming chapters.

We now conclude this chapter with few words on limitations and assumptions that one should keep in mind when using the above theory.

Discussion and Conclusion

We have looked so far at the rectangular plate free vibration underlying theory, and we have also stated all the basic equations for the exact analytical solution of any rectangular plate free vibration problem for which the following assumptions,
- Plate thickness is small compared to its lateral dimensions.

- For higher vibration modes, plate thickness is small compared to the distance between nodal lines.

- Lateral displacement ($W$) is small compared to the thickness of the plate.

- Negligible rotary inertia effects.

- No significant in plate forces.

Fortunately, most rectangular plate vibration problems satisfy the above assumptions well enough for most practical purposes. In the next chapter we will be dealing with the application of the superposition techniques to solve the problem of the rectangular cantilever plate.
Chapter 2

THE CANTILEVER PLATE

In chapter one we discussed the method of superposition and its applications to the rectangular plate free vibration problems in general. The application of this method will be demonstrated in the present chapter by solving the problem of a plate that has traditionally presented the analyst with serious difficulties when trying to satisfy its free edge conditions, namely the cantilever plate. Solution for this particular problem already exists (20). However, the present solution is being presented here for three important reasons.

1. To demonstrate first hand the steps involved in the application of the superposition techniques. Once that is done repetition will be avoided as much as possible.

2. By using different building blocks than those used by Gorman to solve this same problem, it is hoped to demonstrate the freedom in choosing the appropriate building blocks without altering the final results, even though the formulation of the problem might look somewhat different. Therefore, one would be inclined to select building blocks that are as simple, and as
easy to solve as possible.

3.- The last and most important reason for the choice of the cantilever plate problem is the usefulness of its solution in the analysis of the cantilever plate with symmetrically distributed point supports. This usefulness will become apparent to the reader in the forthcoming chapters.

The plate under consideration is shown in Figure 2.1. The length of the clamped edge is designated $2b$, and the other dimension is $a$. The $\xi$ axis is chosen to lie along the center line normal to the clamped edge of the plate. As discussed by Gorman, all free vibration modes of the cantilever plate will be either symmetric or antisymmetric with respect to the $\xi$ axis. Dealing with each of these two possible families of modes separately will help greatly simplify the analysis.

![Figure 2.1. The cantilever plate.](image-url)
Symmetric Modes

In order to determine its symmetric modes, only half of the cantilever plate need to be analyzed. This half plate with the three building blocks used in its solution are shown in Figure 2.2. Dotted lines along the edges of a building block refer to the slope normal to that edge. A small pair of circles attached to a building block's edge indicates that along that edge the plate has zero vertical edge reaction and that the slope of the plate taken normal to that edge is equal to zero. These boundary conditions are referred to as slip shear conditions (20). Extended edges indicate simple support.

The First Building Block:

Superimposing solutions of the three building blocks of Figure 2.2, a solution for the symmetric modes of the cantilever plate can be obtained. We begin by formulating the Lévy-type solution for the first of these building blocks.

Slip shear conditions are to be imposed along the edges η = 0, and ξ = 1, and simple support conditions on the edge ξ = 0. The remaining edge η = 1 is to have zero vertical edge reaction and a prescribed slope that varies cyclicly with time with the same frequency as the plate vibration. This last condition is an important deviation from Gorman's solution where bending moment is prescribed. This deviation eliminates rejection modes as explained later.
Figure 2.2: Building blocks used in the solution of the free vibration symmetric modes of the cantilever plate.
From Equation 1.6, 1.8, and 1.9 the Lévy-type solution of this block may be written as (20)

\[ W_1(\xi, \eta) = \sum_{m=1,3,5}^5 \left( A_m \cosh \beta_m \eta + D_m \cos \gamma_m \eta \right) \sin \frac{m \pi \xi}{2} \]

\[ + \sum_{m=k+2}^5 \left( A_m \cosh \beta_m \eta + D_m \cosh \gamma_m \eta \right) \sin \frac{m \pi \xi}{2} \]

...... (2.1)

where

\[ \beta_m = \phi \sqrt{\lambda^2 + (m \pi / 2)^2} \]

and

\[ \gamma_m = \phi \sqrt{\lambda^2 - (m \pi / 2)^2} \text{ or } \phi \sqrt{(m \pi / 2)^2 - \lambda^2} \]

whichever is real, and where the first summation applies to terms for which \( \lambda^2 > (m \pi / 2)^2 \). Also note that since we are interested in symmetric modes, antisymmetric terms are deleted from Eqn. 2.1 and therefore, boundary conditions at \( \eta = 0 \) are automatically satisfied. Furthermore, Equation 2.1 satisfies exactly the prescribed boundary conditions at the edges \( \xi = 0 \) and \( \xi = 1 \), as required of Levy solutions. The two boundary conditions along the edge \( \eta = 1 \) remain to be satisfied. We begin by enforcing zero vertical edge reaction. Equation 1.20 and the first summation of Equation 2.1 permit us to write for each value of \( m \)

\[ \frac{\partial^3}{\partial \eta^3} \left\{ \left( A_m \cosh \beta_m \eta + D_m \cos \gamma_m \eta \right) \sin \frac{m \pi \xi}{2} \right\} \]

...... (2.2)

\[ + \nu \phi^2 \frac{\partial^3}{\partial \eta^3} \left\{ \left( A_m \cosh \beta_m \eta + D_m \cos \gamma_m \eta \right) \sin \frac{m \pi \xi}{2} \right\}_{\eta = 1} = 0 \]
or

\[ A_m \left( \beta_m \left( \beta_m^2 - (m\pi/2)^2 \nu \phi^2 \right) \right) \sinh \beta_m + D_m \left( \gamma_m \left( \gamma_m^2 + (m\pi/2)^2 \nu \phi^2 \right) \right) \sin \gamma_m = 0 \quad \text{...............}(2.3) \]

solving for \( D_m \) in terms of \( A_m \) and substituting into Equation 2.1, the first summation of Equation 2.1 takes the following form

\[ \sum_{m=1,3,5} \kappa^* A_m (\cosh \beta_m n + \theta_m \cosh \gamma_m n) \sin \frac{m\pi \xi}{2} \quad \text{...............}(2.4) \]

A similar expression is obtained for terms in which we have \( \lambda^2 < (m\pi/2)^2 \) by using the second summation of Equation 2.1

\[ \sum_{m=k+2}^{\infty} A_m (\cosh \beta_m n + \theta_m \cosh \gamma_m n) \sin \frac{m\pi \xi}{2} \quad \text{...............}(2.5) \]

and \( W_1(\xi, n) = \text{Equation 2.4} + \text{Equation 2.5} \quad \text{...............}(2.6) \)

where

\[ \theta_{1m} = \frac{\beta_m \left( \nu \phi^2 (m\pi/2)^2 - \beta_m^2 \right) \sinh \beta_m}{\gamma_m \left( \nu \phi^2 (m\pi/2)^2 + \gamma_m^2 \right) \sin \gamma_m} \]

and

\[ \theta_{2m} = \frac{\beta_m \left( \nu \phi^2 (m\pi/2)^2 - \beta_m^2 \right) \sinh \beta_m}{\gamma_m \left( \nu \phi^2 (m\pi/2)^2 - \gamma_m^2 \right) \sin \gamma_m} \]

Next, in order to satisfy the last prescribed boundary condition along the edge \( \eta = 1 \) the slope along lines normal to this
edge is expanded in series form

\[
\frac{\partial W_1(\xi, n)}{\partial n} = \sum_{m=1, 3} E_m \sin \frac{m\xi}{2} \bigg|_{n=1} \tag{2.7}
\]

and in keeping with Equation 2.6 we have\(^1\).

\[
\frac{\partial W_1(\xi, n)}{\partial n} = k^* \sum_{m=1, 3, 5} A_m (\beta_m \sinh \beta_m n - \theta_{1m} \gamma_m \sin \gamma_m n) \sin \frac{m\xi}{2} + \sum_{m=k^*+2} A_m (\beta_m \sinh \beta_m n + \theta_{2m} \gamma_m \sinh \gamma_m n) \sin \frac{m\xi}{2} \tag{2.8}
\]

equating the right hand sides of Equations 2.7, and 2.8, and setting \(n=1\) we obtain

\[
E_m = A_m (\beta_m \sinh \beta_m n - \theta_{1m} \gamma_m \sin \gamma_m n) \tag{2.10}
\]
or if \(\lambda^2 < \left(\frac{m\pi}{2}\right)^2\)

\[
E_m = A_m (\beta_m \sinh \beta_m n + \theta_{2m} \gamma_m \sinh \gamma_m n) \tag{2.11}
\]
solving for \(A_m\) and substituting into Equation 2.6 we obtain

1. It was proven by Gorman(20) that such derivations are legitimate.
\[ W_1(\xi, \eta) = \sum_{m=1,3,5 \ldots}^{k} \frac{E_m}{2} (\cosh \theta_m n + \theta_1 m \cos \gamma_m n \sin \frac{m \pi \xi}{2} \right] \\
+ \sum_{m=k+2,2k+2}^{\infty} \frac{E_m}{2} (\cosh \theta_m n + \theta_2 m \cos \gamma_m n \sin \frac{m \pi \xi}{2} \right] \] \tag{2.12}

where

\[ \theta_{1m} = \theta_m \sinh \theta_m - \theta_1 m \gamma_m \sin \gamma_m \]

and

\[ \theta_{2m} = \theta_m \sinh \theta_m + \theta_2 m \gamma_m \sin \gamma_m \]

Equation 2.12 represent the solution of the first building block. We next, turn to the second building block.

The Second Building Block:

In finding a Lévy-type solution to the second building block of Figure 2.2, the reader will agree that in light of the boundary conditions the trigonometric part of this solution has to run in the direction of the \( \eta \) axis. In order to take greater advantage of equilibrium equations and forms of solutions already developed(20), a solution for the building block of Figure 2.3 will first be found. Then, a simple interchange of coordinates \( \xi \) and \( \eta \) will lead to the solution of the second block of interest.

The Lévy-type solution of the building block of Figure 2.3 may be written as(20)

\[ W(\xi, \eta) = \sum_{m=0,1,2}^{\infty} Y_m(\eta) \cos m \pi \xi \] \tag{2.13}
it can easily be verified that Equation 2.13 fully satisfies the boundary conditions along the edges $\xi=0$ and $\xi=1$. Then substituting Equation 2.13 into Equation 1.5 we obtain Equation 1.7, for which a solution is expressed by Equations 1.8, and 1.9.

Next, the simple support conditions along the edge $\eta=0$ is enforced by deleting the symmetric terms from Equations 1.8 and 1.9. The expression for $W(\xi, \eta)$ of Equation 2.13 becomes

$$W(\xi, \eta) = \sum_{m=0,1,2}^{k} (B_m \sinh \alpha_m \eta + C_m \sin \gamma_m \eta) \cos m\pi \xi$$

$$+ \sum_{m=k+1}^{\infty} (B_m \sinh \alpha_m \eta + C_m \sin \gamma_m \eta) \cos m\pi \xi$$

where the first summation pertains to terms for which $\lambda^2 > (m\pi)^2$

and where

$$\alpha_m = \phi \sqrt{\lambda^2 - (m\pi)^2}$$

and

$$\gamma_m = \phi \sqrt{(m\pi)^2 - \lambda^2}$$

whichever is real.

Figure 2.3. Intermediate block used for the solution of the second block of Figure 2.2
The next step is to enforce the prescribed boundary conditions along the edge \( n = 1 \) and in the process eliminate the two unknowns of Equation 2.14 \( B_m \) and \( C_m \). Note that these boundary conditions are the same as those pertaining along the same edge of the first building block of Figure 2.2. Therefore the steps followed here would be the same as those followed when the first building block was under study, with the exception of the prescribed slope that should be expanded in a cosine series form as

\[
\frac{\partial W}{\partial \eta} = \sum_{m=0,1} E_m \cos m \xi \tag{2.15}
\]

and in order to avoid repetition, only the final results will be shown.

\[
W(\xi, \eta) = \sum_{m=0,1} E_m (\sinh \eta \gamma_m + \theta_1 m \sin \gamma_m \eta) \cos m \xi
\]

\[
+ \sum_{m=k+1}^{\infty} E_m (\sinh \eta \gamma_m + \theta_2 m \sin \gamma_m \eta) \cos m \xi \tag{2.16}
\]

where

\[
\theta_1 m = \frac{\gamma_m (\psi^2 - \nu \phi^2 (m \pi)^2) \cosh \eta}{\gamma_m (\gamma^2 + \nu \phi^2 (m \pi)^2) \cos m}
\]

\[
\theta_2 m = \frac{\gamma_m (\psi^2 - \nu \phi^2 (m \pi)^2) \cosh \eta}{\gamma_m (\gamma^2 + \nu \phi^2 (m \pi)^2 - \gamma_m^2) \cosh \gamma_m}
\]

\[
\theta_{11 m} = \gamma_m \cos \gamma_m + \theta_{1 m} \gamma_m \cos \gamma_m
\]

\[
\theta_{22 m} = \gamma_m \cos \gamma_m + \theta_{2 m} \gamma_m \cos \gamma_m
\]
Now, the solution of the second building block of Figure 2.2 is extracted from the above solution, by interchanging the coordinates \( \xi \) and \( n \). However, because we are using side length \( a \) to allow a dimensionless eigenvalue \( \lambda^2 \), in addition to the interchange of coordinates, it is necessary to replace the aspect ratio by its inverse, and to multiply the eigenvalue \( \lambda^2 \) by the square of the aspect ratio. The solution of the second block is then given by Equation 2.17. Note that the subscript \( m \) is replaced by \( n \) to distinguish this solution from that of the first building block.

\[
W_2(\xi, n) = \sum_{n=0,1,2} \frac{E_n}{k^2} (\sinh \beta_n \xi + \theta_1 n \sin \gamma_n \xi) \cos n \pi
\]

\[
+ \sum_{n=k+1} \frac{E_n}{k^2} (\sinh \beta_n \xi + \theta_2 n \sin \gamma_n \xi) \cos 2n \pi \tag{2.17}
\]

where the first summation includes terms for which \( \lambda^2 \phi^2 > (n\pi)^2 \) only. And where

\[
\beta_n = \frac{1}{\phi} \sqrt{\lambda^2 \phi^2 - (n\pi)^2}
\]

\[
\gamma_n = \frac{1}{\phi} \sqrt{\lambda^2 \phi^2 - (n\pi)^2} \quad \text{or} \quad \frac{1}{\phi} \sqrt{(n\pi)^2 - \lambda^2 \phi^2}
\]

whichever is real. And

\[
\theta_n = \frac{\beta_n (\beta_n^2 - \nu \phi (n\pi)^2 \cosh \beta_n)}{\gamma_n (\nu \phi (n\pi)^2 + \gamma_n^2 \cos \gamma_n)}
\]
\( \theta_{2n} = \frac{\theta_n (\theta_n^2 - \gamma_n^2 (n\pi)^2) \cosh \gamma_n}{\gamma_n (\gamma_n^2 (n\pi)^2 - \gamma_n^2) \cosh \gamma_n} \)

\( \theta_{11n} = \theta_n \cosh \gamma_n + \theta_{1n} \gamma_n \cos \gamma_n \)

\( \theta_{22n} = \theta_n \cosh \gamma_n + \theta_{2n} \gamma_n \cosh \gamma_n \)

and the slope of lines normal to the edge \( \xi = 1 \) is expressed as

\[ \frac{\partial W_2(\xi, n)}{\partial \xi} = \sum_{n=0, 1} \theta_n \cos n\pi \]

Next, we look at a solution for the third building block.

The Third Building Block:

Following the same reasoning as before, the solution for this third and final building block of Figure 2.2 will be found by first finding a solution to the block of Figure 2.4 then transforming to that of Figure 2.5 and then, following the same steps we followed for the solution of the second building block, the expression for \( W_3(\xi, n) \) is found.

We now focus our attention on the building block of Figure 2.4, the Lévy-type solution may be written as

\[ W(\xi, n) = \sum_{m=0, 1, 2} Y_m(n) \cos m\pi \xi \] (2.18)
The reader should appreciate by now how the Lévy-type solution is selected to satisfy the boundary conditions along the edges \( \xi = 0 \) and \( \xi = 1 \).

Substituting Equation 2.18 back into the equilibrium equation we obtain Equation 1.7 that has as a solution Equations 1.8 and 1.9. Deleting the antisymmetric terms from this solution will enforce the slip shear conditions prescribed at the edge \( \eta = 0 \). And \( W(\xi, \eta) \) will assume the form

\[
W(\xi, \eta) = k^* \sum_{m=0,1,2} (A_m \cosh \beta_m \eta + D_m \cos \gamma_m \eta) \cos m \xi + \sum_{m=k^*+1} (A_m \cosh \beta_m \eta + D_m \cosh \gamma_m \eta) \cos m \xi \quad \text{(2.19)}
\]

enforcing zero displacement at the edge \( \eta = 1 \) we have

\[
W(\xi, \eta) = 0 \quad \text{at} \quad \eta = 1
\]

![Figure 2.4](image)

**Figure 2.4.** Intermediate building block used in arriving at a solution for the building block of Figure 2.5
or

$$A_m\cosh\beta_m + D_m\cos\gamma_m = 0 \quad \cdots \cdots (2.20)$$

or if \(\lambda^2 < (m\pi)^2\)

$$A_m\cosh\beta_m + D_m\cosh\gamma_m = 0 \quad \cdots \cdots (2.21)$$

then

$$D_m = -A_m\frac{\cosh\beta_m}{\cos\gamma_m} = -A_m\theta_1m$$

or

$$D_m = -A_m\frac{\cosh\beta_m}{\cosh\gamma_m} = -A_m\theta_2m$$

where

$$\theta_1m = \frac{\cosh\beta_m}{\cos\gamma_m}$$

and

$$\theta_2m = \frac{\cosh\beta_m}{\cosh\gamma_m}$$

therefore,

$$W(\xi,\eta) = \sum_{m=0,1,2} A_m(\cosh\beta_m\eta - \theta_1m\cos\gamma_m)\cos m\xi$$

$$+ \sum_{m=k+1}^\infty A_m(\cosh\beta_m\eta - \theta_2m\cosh\gamma_m)\cos m\xi \quad \cdots \cdots (2.22)$$

we now express the prescribed slope \(\partial W/\partial \eta\) at \(\eta=1\) in series form

$$\frac{\partial W(\xi,\eta)}{\partial \eta} = \sum_{m=0,1,2} F_m \cos m\xi \quad \text{at} \ \eta=1 \quad \cdots \cdots (2.23)$$

but from Equation 2.22 \(\partial W(\xi,\eta)/\partial \eta\) is
\[
\frac{\partial W(\xi, n)}{\partial n} = A_m (\beta_m \sinh \beta_m + \gamma_m \gamma_{1m} \sin \gamma_m) \cos m\pi \xi
\]
or if \(\lambda^2 < (m\pi)^2\)

\[
\frac{\partial W(\xi, n)}{\partial n} = A_m (\beta_m \sinh \beta_m - \gamma_m \gamma_{2m} \sinh \gamma_m) \cos m\pi \xi
\]

equating the right hand sides of Equations 2.24 and 2.23, and solving for \(A_m\) we obtain

\[
A_m = \frac{E_m}{\theta_{11m}} \quad \text{or} \quad A_m = \frac{E_m}{\theta_{22m}}
\]

where

\[
\theta_{11m} = \beta_m \sinh \beta_m + \gamma_m \theta_{1m} \sin \gamma_m
\]

\[
\theta_{22m} = \beta_m \sinh \beta_m - \gamma_m \theta_{2m} \sinh \gamma_m
\]

the expression for \(W(\xi, n)\) then becomes

\[
W(\xi, n) = \sum_{m=0, 1, 2}^{\infty} \sum_{k=0}^{m} \frac{E_m}{\theta_{11m}} (\cosh \beta_m n - \theta_{1m} \cos \gamma_m n) \cos m\pi \xi
\]

\[
+ \sum_{m=k+1}^{\infty} \frac{E_m}{\theta_{22m}} (\cosh \beta_m n - \theta_{2m} \cos \gamma_m n) \cos m\pi \xi
\]

\(\ldots (2.25)\)

where

\[
\beta_m = \phi \sqrt{\lambda^2 + (m\pi)^2}
\]

and

\[
\gamma_m = \phi \sqrt{\lambda^2 - (m\pi)^2} \quad \text{or} \quad \phi \sqrt{(m\pi)^2 - \lambda^2}
\]

whichever is real, and where \(\partial W(\xi, n)/\partial n\) at \(n=1\) is assumed to be

\[
\frac{\partial W(\xi, n)}{\partial n} = \sum_{m=0, 1, 2}^{\infty} E_m \cos m\pi \xi
\]
The solution for the building block of Figure 2.5 is now obtained from Equation 2.25 simply by replacing \( \eta \) by \( 1 - \eta \). Note that the slope will change sign and Equation 2.23 becomes

\[
\frac{\partial W(\xi, \eta)}{\partial \eta} = \sum_{m=0,1,2} E_m \cos m \xi 
\] at \( \eta = 0 \) ........ (2.26)

and the solution of the building block of Figure 2.5 is

\[
W(\xi, \eta) = \sum_{m=0,1,2} E_m \left( \cosh \beta_m (1 - \eta) - \theta_{1m} \cos \gamma_m (1 - \eta) \right) \cos m \xi 
\] + \sum_{m=k^*+1} E_m \left( \cosh \beta_m (1 - \eta) - \theta_{2m} \cosh \gamma_m (1 - \eta) \right) \cos m \xi 
\] ........ (2.27)

Furthermore, by interchanging the variables \( \xi \) and \( \eta \) of Equation 2.27 we arrive at a solution for the third building block of Figure 2.2. Following the same transformation rules established earlier, the aspect ratio is replaced by its inverse and \( \lambda^2 \) is multiplied by \( \phi^2 \). This final solution is

\[
W_3(\xi, \eta) = \sum_{p=0,1,2} E_p \left( \cosh \beta_p (1 - \xi) - \theta_{1p} \cos \gamma_p (1 - \xi) \right) \cos p \eta 
\] + \sum_{p=k^*+1} E_p \left( \cosh \beta_p (1 - \xi) - \theta_{2p} \cosh \gamma_p (1 - \xi) \right) \cos p \eta 
\] ........ (2.28)

where

\[
\beta_p = \frac{1}{\phi} \sqrt{\lambda^2 \phi^2 + (p \pi)^2} 
\]
\[ \gamma_p = \frac{1}{\phi} \sqrt{\lambda^2 \phi^2 - (p\pi)^2} \quad \text{or} \quad \frac{1}{\phi} \sqrt{(p\pi)^2 - \lambda^2 \phi^2} \]
whichever is real, and

\[ \theta_{1p} = \frac{\cosh \theta_p}{\cos \gamma_p} \]
\[ \theta_{2p} = \frac{\cosh \theta_p}{\cosh \gamma_p} \]
\[ \theta_{11p} = \theta_p \sinh \theta_p + \gamma_p \theta_{1p} \sin \gamma_p \]
\[ \theta_{22p} = \theta_p \sinh \theta_p - \gamma_p \theta_{2p} \sinh \gamma_p \]

and where the slope at \( \xi = 0 \) is expressed as

\[ \frac{\partial W(\xi, \eta)}{\partial \xi} = \xi - \frac{\eta}{p_0} \cos \pi \eta \quad \text{at} \quad \xi = 0 \]

\[ p = 0, 1, 2 \]

---

Figure 2.5. Intermediate building block used in arriving at a solution for the third building block of Figure 2.2.
Equations 2.12, 2.17, and 2.28 provide Lévy-type solutions for the first, second, and third building blocks of Figure 2.2 respectively. The next step is to superimpose these solutions constraining the Fourier coefficients that are present in them so that the net effect is to satisfy the boundary conditions of the plate of the left hand side of Figure 2.2. These conditions are, first, the slope of line's normal to the edge \( \xi = 0 \) must be set to zero; second the bending moments along the edges \( \xi = 1 \) and \( \eta = 1 \) must also be set to zero. Since these solutions, \( W_1(\xi, \eta) \), \( W_2(\xi, \eta) \), and \( W_3(\xi, \eta) \) satisfy exactly the plate vibration governing differential equation, their summation, \( W(\xi, \eta) \) will also satisfy this equation.

We begin by examining the contribution to bending moment along the edge \( \xi = 1 \) of each term of the solution \( W_1(\xi, \eta) \). From Equations 1.18, and 2.12 we obtain

\[
\frac{M_r}{D} = -\frac{\partial^2 W_1}{\partial \xi^2} + \frac{V}{4} \frac{\partial^2 W_1}{\partial \eta^2}
\]

at \( \xi = 1 \)

\[
= E_m \frac{\sin m \pi/2}{\theta_1 m} ((m \pi/2)^2 - \nu \beta_m^2/\phi^2) \cosh \beta_m \eta
\]

\[
+ \theta_1 m ((m \pi/2)^2 + \nu \gamma_m^2/\phi^2) \cos \gamma_m \eta)
\]

or if \( \lambda^2 < (m \pi/2)^2 \)

\[
= E_m \frac{\sin(m \pi/2)}{2m} ((m \pi/2)^2 - \nu \beta_m^2/\phi^2) \cosh \beta_m \eta
\]

\[
+ \theta_2 m ((m \pi/2)^2 - \nu \gamma_m^2/\phi^2) \cos \gamma_m \eta)
\]
The contribution to the slope along the edge $\xi=0$ of each term of the solution $W_1(\xi,\eta)$ is

$$\frac{\partial W_1}{\partial \xi} = \frac{E_m (m \pi / 2)}{\theta_{11m}} (\cosh \beta_m \eta + \theta_{1m} \cos \gamma_m \eta)$$

or if $\lambda^2 < (m \pi / 2)^2$ .................................................. (2.30)

$$= \frac{E_m (m \pi / 2)}{\theta_{22m}} (\cosh \beta_m \eta + \theta_{2m} \cosh \gamma_m \eta)$$

The contribution to bending moment along the edge $\eta=1$ of each term of the solution $W_1(\xi,\eta)$ is

$$\frac{M_0 \phi b}{D} = -\left( \frac{\partial^2 W_1}{\partial \eta^2} + \nu \frac{\partial \phi}{\partial \xi} \frac{\partial^2 W_1}{\partial \xi^2} \right)_{\text{at } \eta = 1}$$

$$= \frac{E_m}{\theta_{11m}} \left( (\nu \phi^2 (m \pi / 2)^2 - \theta_m^2) \cosh \beta_m + \theta_{1m} (\nu \phi^2 (m \pi / 2)^2 + \nu_m^2) \cos \gamma_m \right) \sin \frac{m \pi \xi}{2}.$$

or if $\lambda^2 < (m \pi / 2)^2$ .................................................. (2.31)

$$= \frac{E_m}{\theta_{22m}} \left( (\nu \phi^2 (m \pi / 2)^2 - \theta_m^2) \cosh \beta_m + \theta_{2m} (\nu \phi^2 (m \pi / 2)^2 - \nu_m^2) \cosh \gamma_m \right) \sin \frac{m \pi \xi}{2}.$$

Similar derivations are required for solutions $W_2(\xi,\eta)$ and $W_3(\xi,\eta)$. 
The contribution to bending moment at the edge $\xi=1$ of each term of the solution $W_2(\xi, \eta)$ is

$$\frac{M_a}{D} = -\left( \frac{\partial^2 W_2}{\partial \xi^2} + \frac{\nu}{\phi} \frac{\partial^2 W_2}{\partial \eta^2} \right)$$

at $\xi=1$

$$= \frac{E}{\theta_{11n}} \left( \frac{\nu}{\phi} (n\pi)^2 - \beta_1^2 \right) \sinh \beta_1 \eta$$

$$+ \theta_{1n} \left( \frac{\nu}{\phi} (n\pi)^2 - \gamma_1^2 \right) \sin \gamma_1 \eta \cos n\pi \eta$$

or if $\lambda^2 \phi^2 < (n\pi)^2$ .............................. (2.32)

$$= \frac{E}{\theta_{22n}} \left( \frac{\nu}{\phi} (n\pi)^2 - \beta_1^2 \right) \sinh \beta_1 \eta$$

$$+ \theta_{2n} \left( \frac{\nu}{\phi} (n\pi)^2 - \gamma_1^2 \right) \sinh \gamma_1 \eta \cos n\pi \eta$$

The contribution to the slope at the edge $\xi=0$ of each term of the solution $W_2(\xi, \eta)$ is

$$\frac{\partial W_2}{\partial \xi} = \frac{E}{\theta_{11n}} \left( \beta_n + \theta_{1n} \gamma \right) \cos n\pi \eta$$

or if $\lambda^2 \phi^2 < (n\pi)^2$ .............................. (2.33)

$$= \frac{E}{\theta_{22n}} \left( \beta_n + \theta_{2n} \gamma \right) \cos n\pi \eta$$

The contribution of each term of the solution $W_2(\xi, \eta)$ to bending moment along the edge $\eta=1$ is
\[ \frac{M_{\xi}}{B} = -\left( \frac{\partial^2 W_2}{\partial n^2} + \nu \phi^2 \frac{\partial^2 W_2}{\partial n^2} \right) \quad \text{at } n=1 \]

\[ = \frac{E_0 \cos \pi}{\theta_{11n}} \left( (\pi^2) - \nu \phi^2 \gamma_1 \right) \sinh \theta_{1n} \xi \]

\[ + \theta_{1n} ((\pi^2) + \nu \phi^2 \gamma_1^2) \sin \gamma_{1n} \xi \]

or if \( \lambda^2 \phi^2 < (\pi^2) \) \hspace{5cm} \hspace{5cm} \hspace{5cm} \hspace{5cm} (2.34) \]

\[ = \frac{E_0 \cos \pi}{\theta_{22n}} \left( (\pi^2) - \nu \phi^2 \gamma_1 \right) \sinh \theta_{2n} \xi \]

\[ + \theta_{2n} ((\pi^2) - \nu \phi^2 \gamma_1^2) \sin \gamma_{2n} \xi \]

The contribution of each term of the solution \( W_3(\xi, n) \) to bending moment along the edge \( \xi=1 \) is

\[ \frac{M_{\xi}}{B} = -\left( \frac{\partial^2 W_3}{\partial n^2} + \nu \phi^2 \frac{\partial^2 W_3}{\partial n^2} \right) \quad \text{at } \xi=1 \]

\[ = \frac{E_p}{\theta_{11p}} \left( \frac{\psi}{\phi^2} (p^2) (1-\theta_{1p}) - \beta_\theta^2 - \gamma_1^2 \theta_{1p} \right) \cosp \pi n \]

or if \( \lambda^2 \phi^2 < (p^2) \) \hspace{5cm} \hspace{5cm} \hspace{5cm} \hspace{5cm} (2.35)

\[ = \frac{E_p}{\theta_{22p}} \left( \frac{\psi}{\phi^2} (p^2) (1-\theta_{2p}) - \beta_\theta^2 + \gamma_1^2 \theta_{2p} \right) \cosp \pi n \]

The contribution to the slope along the edge \( \xi=0 \) of each term of the solution \( W_3(\xi, n) \) is

\[ \left. \frac{\partial W_3}{\partial \xi} \right|_{\xi=0} = -E_p \cosp \pi n \quad \text{.................} \hspace{5cm} (2.36) \]
The contribution to bending moment along the edge \( \xi = 1 \) of each term of the solution \( W_3(\xi, n) \) is

\[
\frac{M_{n2b}}{D} = -\left( \frac{\partial^2 W_3}{\partial n^2} + \nu \phi^2 \frac{\partial^2 W_3}{\partial \xi^2} \right)
\]

at \( n = 1 \)

\[
= E_\theta \cos \phi \pi \left( ((p\pi)^2 - \nu \phi^2 \beta^2 \gamma) \cosh \beta \gamma (1-\xi) \right)
\]

\[
- \theta_1 \pi \left( (p\pi)^2 + \nu \phi^2 \gamma^2 \cos \gamma (1-\xi) \right)
\]

or if \( \lambda^2 \phi^2 < (p\pi)^2 \)

\[
= E_\theta \cos \phi \pi \left( ((p\pi)^2 - \nu \phi^2 \beta^2 \gamma) \cosh \beta \gamma (1-\xi) \right)
\]

\[
- \theta_2 \pi \left( (p\pi)^2 - \nu \phi^2 \gamma^2 \cos \gamma (1-\xi) \right)
\]

Focusing our attention on the edge \( \xi = 1 \), where the net effect of all contributions to bending moment must be zero we write

Equation 2.29 + Equation 2.32 + Equation 2.35 = 0 \( \ldots \ldots \) (2.38)

It is seen that the second and third terms in Equation 2.38 which are Equations 2.32 and 2.35 respectively are in the form of Fourier series involving the trigonometric functions \( \cos(n\pi \eta) \) and \( \cos(p\pi \eta) \). However, since \( n \) and \( p \) vary in the same order, \( n = p = 0, 1, 2, \ldots \ldots \), both functions are one and the same. The coefficients of these series involve the quantities \( E_n \) and \( E_p \). It is easily seen that if one could expand the first term on the left hand side of Equation 2.38 in the same type of series,
one could impose the constraint that the sum of the coefficients before each of the Fourier trigonometric functions must equal zero. Expanding Equation 2.29 in a Fourier series of the type mentioned above we obtain

\[ f(\eta) = \sum_{n=0,1} A_n \cos n \pi \eta \]  
\[ \text{...............}(2.39) \]

where,

\[ \int_0^1 f(\eta) \cos n \pi \eta d\eta \quad \text{if n=0} \]

\[ A_n = \]  
\[ \text{...............}(2.40) \]

\[ 2 \int_0^1 f(\eta) \cos n \pi \eta d\eta \quad \text{if n\neq0} \]

Appendix 1 provides a list of the type of integrals needed for the determination of all such Fourier coefficients that will be required throughout this work. From Equation 2.40 it follows that these coefficients are obtained by multiplying Equation 2.29 by \(2\cos(n \pi \eta)\) and integrating between zero and one where \(n\) takes on values \(0,1,\ldots,k-1\), and dividing by 2 if \(n=0\). The results will be shown later in this chapter.

We next turn to the net contribution of the three building block's solutions to the slope at \(\xi=0\), and set it to zero. We obtain

\[ \text{Equation 2.30 + Equation 2.33 + Equation 2.36 = 0} \]  
\[ \text{...............}(2.41) \]
It is clearly seen that Equations 2.33, and 2.36 are in the form of a Fourier series involving the trigonometric function \( \cos n\pi \). Therefore, following the same reasoning as before, Equation 2.30 is expanded in a Fourier series of the same type. Its coefficients are easily obtained by multiplying the appropriate expressions in Equation 2.30 by \( 2\cos n\pi \), and integrating between zero and one, and dividing by 2 when \( n=0 \).

Finally, we turn to the net contribution of the three building block solutions of Figure 2.2 to the bending moment along the edge \( \eta = 1 \) and set it to zero. We write

Equation 2.31 + Equation 2.34 + Equation 2.37 = 0  \ldots \ldots (2.42)

The first term on the left hand side of Equation 2.42 is in the form of a Fourier series involving the trigonometric function \( \sin (m\pi \eta /2) \), where \( m = 1,3,5, \ldots, 2k-1 \). Following the same reasoning as before, it is clear that the second and third term of the left hand side of Equation 2.42 must be expanded in a Fourier series of the same type.

The coefficients for a function \( f(\eta) \), expanded in a series of the type \( \sin (m\pi \eta /2) \) are known to be given by

\[
A_m = 2 \int_0^1 f(\eta) \sin (m\pi \eta /2) d\eta
\]
It follows that the coefficients of Equation 2.34 expanded in a Fourier series of this type are obtained simply by multiplying the appropriate expressions of Equation 2.34 by $2\sin(m\pi\xi/2)$ and integrating between zero and one. The coefficients of the expanded Equation 2.37 are obtained in the same way.

In their proper expanded form, and if one choose to have $k$ term in each of the involved series, Equations 2.38, 2.41, and 2.42 provide $3k$ homogeneous algebraic equations relating $3k$ unknown coefficients $E_m's$, $E_n's$, and $E_p's$. By matrix techniques these equations are handled with relative ease. Figure 2.6 is a schematic representation of such matrix showing only the first three terms of the relevant series. By trial and error, a value of $\lambda^2$ that causes the coefficient matrix determinant to vanish is established. Since a vanishing determinant indicates that a nontrivial solution for $E_m's$, $E_n's$, and $E_p's$ exists, the associated $\lambda^2$ is in effect an eigenvalue.

Since we are more interested in the shape of the vibration function rather than its magnitude, one could arbitrarily set one of the unknown coefficients to unity, and by doing so transforms the $3k$ homogeneous equations into a set of nonhomogeneous equations with $3k-1$ unknowns. Using any $3k-1$ equations of the set, we can easily solve for these $3k-1$ unknowns using well known standard computer techniques.
In generating the matrix of Figure 2.6 it must be noted that the first quadrant, referred to as (1,1), represents the first term on the left hand side of Equation 2.42. The second quadrant (1,2) represents the second term. And the third (1,3) quadrant represents the third term on the left hand side of this equation. And in the same order Equation 2.38 is represented by Quadrants (2,1), (2,2), and (2,3). Also Equation 2.41 is represented by Quadrants (3,1), (3,2), and (3,3). These quadrants are

\[
\begin{array}{ccc|ccc|ccc}
 m&1&3&5&n&0&1&2&p&0&1&2 \\
 \hline
 - & 0 & 0 & - & (1,1) & - & (1,2) & - & (1,3) & - \\
 0 & - & 0 & - & - & - & - & - & - \\
 0 & 0 & - & - & - & - & - & - & - \\
 \hline
 - & (2,1) & - & 0 & 0 & - & 0 & 0 & 0 \\
 0 & - & 0 & - & (2,2) & 0 & 0 & 0 & 0 \\
 0 & 0 & - & 0 & 0 & 0 & - & 0 & 0 \\
 \hline
 - & (3,1) & - & 0 & 0 & -1 & 0 & 0 & 0 \\
 0 & - & 0 & 0 & (3,2) & 0 & 0 & 0 & -1 \\
 0 & 0 & - & 0 & 0 & 0 & 0 & 0 & -1 \\
 \hline
 E_m & E_n & E_p \\
 \end{array}
\]

Eqn. 2.42

Eqn. 2.38

Eqn. 2.41

\[\text{Figure 2.6. Matrix schematic representation for the symmetric mode problem of the cantilever plate.}\]
Quadrant (1, 1):

For $m = 1, 3, 5, \ldots, 2k-1$

\[
A_{(m+1)/2, (m+1)/2} = \{(v^2(m\pi/2)^2 - \beta^2_m)\cosh\beta_m \\
+ \theta_{1m}(v^2(m\pi/2)^2 + \gamma^2_m)\cos\gamma_m\}/\theta_{11m}
\]

or if $\lambda^2 < (m\pi/2)^2$

\[
= \{(v^2(m\pi/2)^2 - \beta^2_m)\cosh\beta_m \\
+ \theta_{2m}(v^2(m\pi/2)^2 - \gamma^2_m)\cosh\gamma_m\}/\theta_{22m}
\]

Quadrant (1, 2):

For $m = 1, 3, 5, \ldots, 2k-1$ ; $n = 0, 1, 2, \ldots, k-1$

\[
A_{(m+1)/2, k+n+1} = 2\cos(n\pi)\sin(m\pi/2)(x_1-x_2)/\theta_{11n}
\]

where

\[
x_1 = \{(n\pi)^2 - v^2\beta^2_n\}\beta_n\cosh\beta_n/\{\beta^2_n + (m\pi/2)^2\}
\]

\[
x_2 = \{(n\pi)^2 + v^2\gamma^2_n\}\gamma_n\theta_{1n}\cos\gamma_n/\{\gamma^2_n - (m\pi/2)^2\}
\]

or if $\lambda^2 \phi^2 < (n\pi)^2$

\[
= 2\cos(n\pi)\sin(m\pi/2)(x_1+x_2)/\theta_{22n}
\]

where

\[
x_1 = \{(n\pi)^2 - v^2\beta^2_n\}\beta_n\cosh\beta_n/\{\beta^2_n + (m\pi/2)^2\}
\]

\[
x_2 = \{(n\pi)^2 - v^2\gamma^2_n\}\gamma_n\theta_{2n}\cosh\gamma_n/\{\gamma^2_n + (m\pi/2)^2\}
\]
Quadrant (1,3):

For \( m = 1, 3, 5, \ldots, 2k-1 \); \( p = 0, 1, 2, \ldots, k-1 \)

\[
A_{(m+1)/2, 2k+p+1} = 2\cos(p\pi)(m\pi/2)(x_1^2 + x_2^2)/\theta_{11p}
\]

where

\[
x_1 = ((p\pi)^2 - \nu^2\beta_p^2)\cosh\beta_p / (\beta_p^2 + (m\pi/2)^2)
\]

\[
x_2 = ((p\pi)^2 + \nu^2\gamma_p^2)\theta_1 p\cos\gamma_p / (\gamma_p^2 - (m\pi/2)^2)
\]

or if \( \lambda^2 < (p\pi)^2 \)

\[
= 2\cos(p\pi)(m\pi/2)(x_1^2 - x_2^2)/\theta_{22p}
\]

where

\[
x_1 = ((p\pi)^2 - \nu^2\beta_p^2)\cosh\beta_p / (\beta_p^2 + (m\pi/2)^2)
\]

\[
x_2 = ((p\pi)^2 - \nu^2\gamma_p^2)\theta_2 p\cosh\gamma_p / (\gamma_p^2 + (m\pi/2)^2)
\]

Quadrant (2,1):

For \( n = 0, 1, 2, \ldots, k-1 \); \( m = 1, 3, 5, \ldots, 2k-1 \)

\[
A_{(k+n-1), (m+1)/2} = 2\sin(m\pi/2)\cos(n\pi)(x_1^2 + x_2^2)/\theta_{11m}
\]

where

\[
x_1 = ((m\pi/2)^2 - \nu^2\beta_m^2)\beta_m\sinh\beta_m / (\beta_m^2 + (n\pi)^2)
\]

\[
x_2 = ((m\pi/2)^2 + \nu^2\gamma_m^2)\gamma_m\theta_1 m\sin\gamma_m / (\gamma_m^2 - (n\pi)^2)
\]

or if \( \lambda^2 < (m\pi/2)^2 \)
\[ A_{k+n+1, (m+1)/2} = \frac{2\sin(m\pi/2)\cos(n\pi)(x_1 + x_2)}{\theta_{22m}} \]

where

\[
x_1 = \left\{ \left( \frac{m\pi}{2} \right)^2 - \nu \phi_1^2 \right\} \beta_n \sinh \beta_n \left/ \left( \beta_n^2 + (n\pi)^2 \right) \right. \]

\[
x_2 = \left\{ \left( \frac{m\pi}{2} \right)^2 - \nu \phi_1^2 \right\} \theta_{2m} \gamma_m \sinh \gamma_m \left/ \left( \gamma_m^2 + (n\pi)^2 \right) \right. \]

the above results must be divided by 2 if \( n = 0 \)

Quadrant (2,2):

For \( n = 0, 1, 2, \ldots, k-1 \)

\[ A_{k+n+1, k+n+1} = \frac{\left( \nu \phi_1^2 (n\pi)^2 - \beta_n^2 \right) \sinh \beta_n + \theta_{11n} \left( \nu \phi_1^2 (n\pi)^2 + \gamma_n^2 \right) \sinh \gamma_n}{\theta_{11n}} \]

or if \( \lambda^2 \phi_1^2 < (n\pi)^2 \)

\[ ={\left( \nu \phi_1^2 (n\pi)^2 - \beta_n^2 \right) \sinh \beta_n + \theta_{22n} \left( \nu \phi_1^2 (n\pi)^2 - \gamma_n^2 \right) \sinh \gamma_n} / \theta_{22n} \]

Quadrant (2,3):

For \( p = 0, 1, 2, \ldots, k-1 \)

\[ A_{k+p+1, 2k+p+1} = \frac{\left( \nu \phi_1^2 (p\pi)^2 (1-\theta_1 p) - \beta_p^2 - \gamma_p^2 \theta_2 p \right)}{\theta_{11p}} \]

or if \( \lambda^2 \phi_1^2 < (p\pi)^2 \)

\[ ={\nu \phi_1^2 (p\pi)^2 (1-\theta_2 p) - \beta_p^2 + \gamma_p^2 \theta_2 p} / \theta_{22p} \]

Quadrant (3,1):

For \( n = 0, 1, 2, \ldots, k-1 \) ; \( m = 1, 3, 5, \ldots, 2k-1 \)
\[ A_{2k+n+1,(m+1)/2} = \frac{2\cos(n\pi)(m\pi/2)(x_1+x_2)}{\theta_{11m}} \]

where

\[ x_1 = \frac{\beta_m \sin \beta_m}{(\beta_m^2 + (n\pi)^2)} \]
\[ x_2 = \frac{\gamma_m \theta_{1m} \sin \gamma_m}{(\gamma_m^2 - (n\pi)^2)} \]

or if \( \lambda^2 < (m\pi/2)^2 \)

\[ = \frac{2\cos(n\pi)(m\pi/2)(x_1+x_2)}{\theta_{22m}} \]

where

\[ x_1 = \frac{\beta_m \sin \beta_m}{(\beta_m^2 + (n\pi)^2)} \]
\[ x_2 = \frac{\gamma_m \theta_{2m} \sin \gamma_m}{(\gamma_m^2 + (n\pi)^2)} \]

above results must be divided by 2 if \( n=0 \)

Quadrant \((3,2)\):

For \( n = 0, 1, 2, \ldots, k-1 \)

\[ A_{2k+n+1,k+n+1} = \frac{\beta_n + \theta_{1n} \gamma_n}{\theta_{11n}} \]

or if \( \lambda^2 \phi^2 < (n\pi)^2 \)

\[ = \frac{\beta_n + \theta_{2n} \gamma_n}{\theta_{22n}} \]

Quadrant \((3,3)\):

For \( p = 0, 1, 2, \ldots, k-1 \)

\[ A_{2k+p+1,2k+p+1} = -1 \]
Note that all of the off-diagonal elements in Quadrants (1,1), (2,2), (2,3), (3,2), and (3,3) are zero.

Now, we are ready to feed the above information to a digital computer, and solve for eigenvalues, and mode shapes. Program 1 in Appendix 3 is specially designed for eigenvalue searches. While program 2 will solve for the unknown coefficients and shape data of this symmetric modes problem. In order to determine the number of terms $k$, that should be used in each of the Fourier series involved in the solution of this problem, a convergence test like the one shown in Figure 2.7, proved to be very effective, specially in the case of point supports.

![Graph](image)

**Figure 2.7.** Eigenvalue $\lambda^2$ versus the number of terms $k$ for the first symmetric mode of a square plate, $\phi$=1.
this conclude our discussion on the free vibration analysis of the symmetric modes of the rectangular cantilever plate. Eigenvalues for this family of modes will be tabulated and discussed later in this chapter. Next, we turn to the analysis of the antisymmetric modes.

Antisymmetric Modes:

The steps followed in this analysis are identical to those followed in the case of symmetric modes. We only need to analyze half of the full plate as shown in Figure 2.8, where the right hand side illustrates the three building blocks used in this solution with their boundary conditions.

The First Building Block:

We begin by finding a Levy-type solution for the first building block of Figure 2.8. In light of the boundary conditions along the edges $\xi=0$ and $\xi=1$ this solution may be written as

$$W_1(\xi,\eta) = \sum_{m=1,3,5} Y_m(\eta) \sin \frac{m\pi \xi}{2} \quad \text{...............(2.43)}$$

where $Y_m(\eta)$ is given by Equations 1.8 and 1.9. In order to enforce the simple support conditions along the edge $\eta=0$, symmetric terms must be deleted from the expression of $Y_m(\eta)$. Therefore, we may write
Figure 2.8. Building blocks used in the solution of the free vibration antisymmetric modes of the cantilever plate.
\[ \gamma_m(\eta) = B_m \sinh \beta_m \eta + C_m \sin \gamma_m \eta \]
or if \[ \lambda^2 < (m\pi/2)^2 \]
\[ = B_m \sinh \beta_m \eta + C_m \sin \gamma_m \eta \]
where
\[ \beta_m = \phi \sqrt{\lambda^2 - (m\pi/2)^2} \]
\[ \gamma_m = \phi \sqrt{\lambda^2 + (m\pi/2)^2} \]
or \[ \phi \sqrt{(m\pi/2)^2 - \lambda^2} \]
whichever is real.

It remains to satisfy the boundary condition at the edge \( \eta = 1 \).
Enforcing zero vertical edge reaction at \( \eta = 1 \) we write
\[ \frac{\partial^3 W_1(\xi, \eta)}{\partial \eta^3} + v \phi^2 \frac{\partial^3 W_1(\xi, \eta)}{\partial \eta^3 \xi^2} \bigg|_{\eta = 1} = 0 \quad \ldots \quad (2.45) \]
performing the necessary derivations and solving for \( B_m \) we have
\[ B_m = \theta_1 m C_m \]
or if \[ \lambda^2 < (m\pi/2)^2 \]
\[ = \theta_2 m C_m \]
where
\[ \theta_1 m = \frac{\gamma_m (\gamma_m^2 + (m\pi/2)^2 v \phi^2)^2 \cos \gamma_m}{\beta_m (\beta_m^2 - (m\pi/2)^2 v \phi^2)^2 \cosh \beta_m} \]
\[ \theta_2 m = \frac{\gamma_m (v \phi^2 (m\pi/2)^2 - \gamma_m^2) \cosh \gamma_m}{\beta_m (\beta_m^2 - v \phi^2 (m\pi/2)^2) \cosh \beta_m} \]
We now represent the slope at \( \eta = 1 \) by a series of the same type as \( W_1(\xi, \eta) \), or

\[
\frac{\partial W_1}{\partial \eta} = \sum_{m=1,3,5} E_m \sin \frac{m \pi \xi}{2} \tag{2.47}
\]

Substituting Equation 2.46 into 2.44, then Equation 2.44 into 2.43, and differentiating with respect to \( \eta \) at \( \eta = 1 \) and equating the results to the right hand side of Equation 2.47 we have

\[
E_m = C_m (\theta_{11m} \cosh \beta_m + \gamma_m \cos \gamma_m)
\]

or if \( \lambda^2 < (m\pi/2)^2 \)

\[
= C_m (\theta_{22m} \cosh \beta_m + \gamma_m \cosh \gamma_m)
\]

Solving for \( C_m \), and substituting into the expression of \( W_1(\xi, \eta) \) we write

\[
W_1(\xi, \eta) = \sum_{m=1,3,5} E_m (\theta_{11m} \sinh \beta_m \eta + \sin \gamma_m \eta) \sin \frac{m \pi \xi}{2} + \sum_{m=k+2} \hat{E}_m (\theta_{22m} \sinh \beta_m \eta + \sinh \gamma_m \eta) \sin \frac{m \pi \xi}{2} \tag{2.48}
\]

where

\[
\theta_{11m} = \theta_{11m} \cosh \beta_m + \gamma_m \cos \gamma_m
\]

\[
\theta_{22m} = \theta_{22m} \cosh \beta_m + \gamma_m \cosh \gamma_m
\]

With the solution of the first building block being completed, we next turn to the solution of the second block.
The Second Building Block:

A look at the second building block of Figure 2.8 reveals that the solution for this block can easily be extracted from the solution of the first block of the same figure by exchanging the variables \( \xi \) and \( \eta \). Also, in keeping with transformation rules discussed earlier \( \lambda^2 \) must be multiplied by \( \phi^2 \) and \( \phi \) must be changed into \( 1/\phi \). Therefore, we may write

\[
\bar{w}_2(\xi, \eta) = \sum_{n=1, 3, 5}^{\infty} \frac{E_n}{\delta_{11n}} (\theta_{1n} \sinh \beta_n \xi + \sin \gamma_n \xi) \sin \frac{\eta \pi n}{2} + \sum_{n=k^* + 2}^{\infty} \frac{E_n}{\delta_{22n}} (\theta_{2n} \sinh \beta_n \xi + \sinh \gamma_n \xi) \sin \frac{\eta \pi n}{2}
\]

\[\text{(2.49)}\]

where

\[
\theta_{1n} = \frac{\gamma_n (\gamma_n^2 + (n\pi/2)^2 v^2 \phi^2)}{\beta_n (\beta_n^2 - (n\pi/2)^2 v^2 \phi^2)} \cosh \beta_n
\]

\[
\theta_{2n} = \frac{\gamma_n (v^2 \phi^2 (n\pi/2)^2 - \gamma_n^2)}{\beta_n (\beta_n^2 - v^2 \phi^2 (n\pi/2)^2)} \cosh \gamma_n
\]

\[
\beta_n = \frac{1}{\phi} \sqrt{\lambda^2 \phi^2 - (n\pi/2)^2}
\]

\[
\gamma_n = \frac{1}{\phi} \sqrt{\lambda^2 \phi^2 - (n\pi/2)^2} \quad \text{or} \quad \frac{1}{\phi} \sqrt{(n\pi/2)^2 - \lambda^2 \phi^2}
\]

whichever is real, and

\[
\theta_{11n} = \beta_n \theta_{1n} \cosh \beta_n + \gamma_n \cos \gamma_n
\]

\[
\theta_{22n} = \theta_{2n} \beta_n \cosh \beta_n + \gamma_n \cosh \gamma_n
\]

\[
\frac{\partial \bar{w}_2}{\partial \xi} = \sum_{n=1, 3, 5}^{\infty} \frac{E_n}{\delta_{11n}} \sin \frac{\eta \pi n}{2}
\]

at \( \xi = 1 \).
The Third Building Block:

Here, consideration is given to the solution of the third and last building block of Figure 2.8. A careful examination of the building block under study reveals that the task of finding a solution is made easier by first solving the building block shown in Figure 2.9. The solution of the building block of figure 2.10 is then extracted from that of Figure 2.9 simply by changing the variable $n$ into $(1-n)$. Furthermore, this solution will in turn lead to the required solution of the last building block of Figure 2.8 as it is shown below.

**Figure 2.9.** Intermediate building block used in the solution of the building block of Figure 2.10.

**Figure 2.10.** Intermediate building block used in the solution of the third building block of Figure 2.8.
Focusing our attention on the building block of Figure 2.9, we write its Lévy-type solution as

\[ W(\xi, \eta) = \sum_{m=1,3,5} \frac{Y_m(\eta)}{m!} \sin \frac{m\pi \xi}{2} \]  

(2.50)

where

\[ Y_m(\eta) = A_m \cosh \gamma_m \eta + B_m \sinh \gamma_m \eta + C_m \sin \gamma_m \eta + D_m \cos \gamma_m \eta \]

or if \( \lambda^2 < (m\pi/2)^2 \)

\[ = A_m \cosh \gamma_m \eta + B_m \sinh \gamma_m \eta + C_m \sin \gamma_m \eta + D_m \cos \gamma_m \eta \]  

(2.51)

In light of the slip shear conditions along the edge \( \eta = 0 \), anti-symmetric terms must be deleted from Equation 2.51

\[ - Y_m(\eta) = A_m \cosh \gamma_m \eta + D_m \cos \gamma_m \eta \]

or if \( \lambda^2 < (m\pi/2)^2 \)

\[ = A_m \cosh \gamma_m \eta + D_m \cos \gamma_m \eta \]  

(2.52)

Enforcing zero displacement along the edge \( \eta = 1 \) we write

\[ A_m \cosh \gamma_m + D_m \cos \gamma_m = 0 \]

or

\[ A_m \cosh \gamma_m + D_m \cosh \gamma_m = 0 \]

and

\[ D_m = -A_m \theta_1 \]

or

\[ D_m = -A_m \theta_2 \]

where

\[ \theta_1 = \cosh \gamma_m / \cos \gamma_m \]

\[ \theta_2 = \cosh \gamma_m / \cosh \gamma_m \]
Therfore,

\[ W(\xi, \eta) = \sum_{m=1,3,5} A_m (\cosh m \eta - \theta_{1m} \cos \gamma_m \eta) \sin \frac{m \pi \xi}{2} \]

\[ + \sum_{m=k^*+2}^\infty A_m (\cosh m \eta - \theta_{2m} \cosh \gamma_m \eta) \sin \frac{m \pi \xi}{2} \]  

...(2.53)

Introducing,

\[ \frac{\partial W(\xi, \eta)}{\partial \eta} = \sum_{m=1,3,5} E_m \sin \frac{m \pi \xi}{2} \] \text{ at } \eta = 1

and equating to the first derivative of Equation 2.53 with respect to \( \eta \) at \( \eta = 1 \), we have

\[ E_m = A_m (\theta_{1m} \sinh \gamma_m \theta_m \sin \gamma_m) \]

or if \( \lambda^2 < (m\pi/2)^2 \)

\[ = A_m (\theta_{1m} \sinh \gamma_m \theta_m \sinh \gamma_m) \]

Solving for \( A_m \) we write

\[ A_m = \frac{E_r}{\theta_{1m}} \quad \text{or} \quad A_m = \frac{E_r}{\theta_{22m}} \]

where

\[ \theta_{1m} = \theta_m \sinh \gamma_m + \theta_{1m} \gamma_m \sin \gamma_m \]

\[ \theta_{22m} = \theta_m \sinh \gamma_m - \theta_{2m} \gamma_m \sinh \gamma_m \]

Substituting \( A_m \) by its value in Equation 2.53 we obtain
\[ W(\xi, \eta) = \sum_{m=1,3,5}^k E_m (\cosh \beta_m n - \theta_1 m \cos \gamma_m n) \sin \frac{m \pi \xi}{2} \]
\[ + \sum_{m=k+2} E_{22m} (\cosh \beta_m n - \theta_2 m \cos \gamma_m n) \sin \frac{m \pi \xi}{2} \]  \hspace{1cm} \text{(2.54)}

Now, the solution of the building block of Figure 2.10 is extracted from Equation 2.54, simply by changing the variable \( n \) into \( (1-n) \)
\[ W(\xi, \eta) = \sum_{m=1,3,5}^k E_m (\cosh \beta_m (1-n) - \theta_1 m \cos \gamma_m (1-n)) \sin \frac{m \pi \xi}{2} \]
\[ + \sum_{m=k+2} E_{22m} (\cosh \beta_m (1-n) - \theta_2 m \cos \gamma_m (1-n)) \sin \frac{m \pi \xi}{2} \]  \hspace{1cm} \text{(2.55a)}

Finally, the solution of the third building block of Figure 2.8 is obtained from Equation 2.55a by exchanging the variables \( \xi \) and \( \eta \), multiplying \( \lambda^2 \) by \( \phi^2 \), and replacing \( \phi \) by its inverse. Therefore,
\[ W_3(\xi, \eta) = \sum_{p=1,3,5}^k E_p (\cosh \beta_p (1-\xi) - \theta_1 p \cos \gamma_p (1-\xi)) \sin \frac{p \pi \eta}{2} \]
\[ + \sum_{p=k+2} E_{22p} (\cosh \beta_p (1-\xi) - \theta_2 p \cos \gamma_p (1-\xi)) \sin \frac{p \pi \eta}{2} \]  \hspace{1cm} \text{(2.55)}

where
\[ \theta_1 p = \cosh \beta_p / \cos \gamma_p \]
\[ \theta_2 p = \cosh \beta_p / \cosh \gamma_p \]
\[ \theta_{11p} = \beta_p \sinh \beta_p + \gamma_p \theta_1 p \sin \gamma_p \]
\[ \theta_{22p} = \beta_p \sinh \delta_p - \gamma_p \theta_{2p} \sinh \gamma_p \]

and

\[ \sum_{p=1,3,5} \frac{W_3(\xi, \eta) \partial \xi}{\xi} = -\sum_{p=1}^{\infty} E_p \sin \frac{p\pi \eta}{2} \]

\[ \beta_p = \frac{1}{\phi} \sqrt{\lambda^2 \delta_p^2 + (p\pi/2)^2} \]

\[ \gamma_p = \frac{1}{\phi} \sqrt{\lambda^2 \delta_p^2 - (p\pi/2)^2} \quad \text{or} \quad \frac{1}{\phi} \sqrt{(p\pi/2)^2 - \lambda^2 \delta_p^2} \]

whichever is real. Note that when changing a variable $\xi$ into $1-\xi$, the first derivative with respect to that variable changes sign as shown in Equation 2.56.

We now have solutions for all three building blocks of Figure 2.8. The next step is to superimpose these solutions, adjusting their unknown coefficients to satisfy the prescribed boundary conditions of the left hand side plate of Figure 2.8. We begin by examining the contribution of each of the building blocks to these boundary conditions.

The contribution of each term of the solution $W_1(\xi, \eta)$ to the bending moment along the edge $\xi=1$ is

\[ M = \frac{a}{D} \left( \frac{\partial^2 W_1}{\partial \xi^2} + \frac{\nu \partial^2 W_1}{\partial \eta^2} \right) \bigg|_{\xi=1} \]

\[ = \frac{E_m}{611m} \sin \frac{m\pi \eta}{2} \left[ ((m\pi/2)^2 - \nu \phi_1^2 \phi_2^2) \theta_{1m} \sin \delta_{1m} \eta \right. \]

\[ + \left. ((m\pi/2)^2 + \nu \phi_1^2 \gamma_2^2) \sin \gamma_{2m} \eta \right] \]
or if \( \lambda^2 < (m\pi/2)^2 \) ........................ (2.57)

\[
\frac{M_{\xi}}{D} = \frac{E_m}{\theta_{22m}} \sin \frac{m\pi}{2} \left( (m\pi/2)^2 - \phi_m^2 \right) \partial_1 \eta \sin \theta_m \eta \\
+ (m\pi/2)^2 - \phi_m^2 \eta \sin \gamma_m \eta 
\]

The contribution to the slope at \( \xi = 0 \) of each term of \( W_1(\xi, \eta) \) is

\[
\frac{\partial W_1(\xi, \eta)}{\partial \xi} = \frac{E_m}{\theta_{11m}} \left( (m\pi/2)^2 - \phi_m^2 \right) \left( \theta_1 \sin \theta_m \eta + \sin \gamma_m \eta \right)
\]

or if \( \lambda^2 < (m\pi/2)^2 \) ........................ (2.58)

\[
= \frac{E_m}{\theta_{22m}} \left( (m\pi/2)^2 - \phi_m^2 \right) \left( \theta_2 \sin \theta_m \eta + \sin \gamma_m \eta \right)
\]

The contribution to bending moment along the edge \( \eta = 1 \) of each term of \( W_1(\xi, \eta) \) is

\[
\frac{M_{\xi \phi}}{D} = -\left( \frac{\partial^2 W_1}{\partial \eta^2} + \phi_m^2 \frac{\partial^2 W_1}{\partial \xi^2} \right) \bigg|_{\eta = 1}
\]

\[
= \frac{E_m}{\theta_{11m}} \left( \phi_m^2 (m\pi/2)^2 - \phi_1^2 \right) \theta_1 \sin \theta_m \\
+ (\phi_m^2 (m\pi/2)^2 + \gamma_m^2) \sin \gamma_m \sin \frac{m\pi}{2}
\]

or if \( \lambda^2 < (m\pi/2)^2 \) ........................ (2.59)

\[
= \frac{E_m}{\theta_{22m}} \left( \phi_m^2 (m\pi/2)^2 - \phi_2^2 \right) \theta_2 \sin \theta_m \\
+ (\phi_m^2 (m\pi/2)^2 + \gamma_m^2) \sin \gamma_m \sin \frac{m\pi}{2}
\]
The contribution of each term of $W_2(\xi, \eta)$ to bending moment along the edge $\xi=1$ is

$$
\frac{M_\xi \phi}{D} = -\left(\frac{\partial^2 W_2}{\partial \xi^2} + \frac{\nu}{\phi^2} \frac{\partial^2 W_2}{\partial \eta^2}\right)_{\xi=1}
$$

$$
= \frac{E_n}{\theta_{11n}} \left( (\nu \phi^2 (n\pi/2)^2 - \beta_1^2) \theta_1 \sinh \beta_1 \right.
$$

$$
+ (\nu \phi^2 (n\pi/2)^2 + \gamma_1^2) \sin \gamma_1 \sin \frac{n\pi \eta}{2}.
$$

or if $\lambda^2 \phi^2 < (n\pi/2)^2$ ...........................................(2.60)

$$
= \frac{E_n}{\theta_{22n}} \left( (\nu \phi^2 (n\pi/2)^2 - \beta_2^2) \theta_2 \sinh \beta_2 \right.
$$

$$
+ (\nu \phi^2 (n\pi/2)^2 - \gamma_2^2) \sinh \gamma_2 \sin \frac{n\pi \eta}{2}.
$$

Contribution to the slope at $\xi=0$ of each term of $W_2$ is

$$
\frac{\partial W_2(\xi, \eta)}{\partial \xi} = \frac{E_n}{\theta_{11n}} (\theta_1 \beta_1 \gamma_1 \sin \frac{n\pi \eta}{2}).
$$

or if $\lambda^2 \phi^2 < (n\pi/2)^2$ ...........................................(2.61)

$$
= \frac{E_n}{\theta_{22n}} (\theta_2 \beta_2 \gamma_2 \sin \frac{n\pi \eta}{2}).
$$

Contribution to bending moment along the edge $\eta=1$ of each term of $W_2(\xi, \eta)$

$$
\frac{M_\eta \phi b}{D} = -\left(\frac{\partial^2 W_2}{\partial \eta^2} + \nu \phi^2 \frac{\partial^2 W_2}{\partial \xi^2}\right)_{\eta=1}
$$
\[
\frac{M_{b}}{D} = \frac{E_{p}}{\delta_{11n}} \sin \frac{n\pi}{2} \{((n\pi/2)^2 - \psi^2\delta_n^2)\delta_{1n}\sin \delta_n \xi \\
+ ((n\pi/2)^2 + \psi^2\gamma_n^2)\sin \gamma_n \xi \}
\]

or if \( \lambda^2 \psi^2 < (n\pi/2)^2 \).

\[
\frac{E_{p}}{\delta_{22n}} \sin \frac{n\pi}{2} \{((n\pi/2)^2 - \psi^2\delta_n^2)\delta_{2n}\sin \delta_n \xi \\
+ ((n\pi/2)^2 - \psi^2\gamma_n^2)\sin \gamma_n \xi \}
\]

The contribution of each term of \( W_3(\xi, n) \) to bending moment along the edge \( \xi = 1 \) is

\[
\frac{M_{b}}{D} = -\left( \frac{\partial^2 W_3}{\partial \xi^2} + \frac{\psi^2}{\delta_n^2} \frac{\partial^2 W_3}{\partial \xi \partial n} \right) |_{\xi = 1}
\]

\[
= \frac{E_{p}}{\delta_{11p}} \{(\psi^2(p\pi/2)^2(1-\beta_{1p}) - \beta_{1p} - \beta_{1p} \gamma_{1p}^2)\sin \frac{p\pi n}{2} \}
\]

or if \( \lambda^2 \psi^2 < (p\pi/2)^2 \)

\[
= \frac{E_{p}}{\delta_{22p}} \{(\psi^2(p\pi/2)^2(1-\beta_{2p}) + \beta_{2p} + \beta_{2p} \gamma_{2p}^2)\sin \frac{p\pi n}{2} \}
\]

Contribution to the slope at \( \xi = 0 \) of each term of \( W_3 \) is

\[
\frac{\partial W_3(\xi, n)}{\partial \xi} = -E_{p}\sin \frac{p\pi n}{2}
\]

The contribution to bending moment along the edge \( n = 1 \) of each term of \( W_3(\xi, n) \) is
\[
\frac{M_n \cdot \phi}{D} = - \left( \frac{\partial^2 W_3}{\partial n^2} + \nu \psi^2 \frac{\partial^2 W_3}{\partial \xi^2} \right) \bigg|_{n=1}
\]

\[
= \frac{E_p}{\theta_{11p}} \sin \frac{D \pi}{2} \{(p \pi/2)^2 - \nu \psi^2 \beta_p^2 \} \cosh \gamma_p (1-\xi)
\]

\[
- \{(p \pi/2)^2 + \nu \psi^2 \gamma_p^2 \} \theta_{1p} \cos \gamma_p (1-\xi) \}
\]

or if \( \lambda^2 \phi^2 < (p \pi/2)^2 \)

...... (2.65)

We now adjust and add relevant contributions to satisfy the boundary conditions of the plate under study. First, for zero slope at \( \xi = 0 \) we write

Equation 2.58 + Equation 2.61 + Equation 2.64 = 0 \( \ldots (2.66) \)

Second, for zero bending moment along the edge \( \xi = 1 \) we have

Equation 2.57 + Equation 2.60 + Equation 2.63 = 0 \( \ldots (2.67) \)

Finally, for zero bending moment along the edge \( n = 1 \) we write

Equation 2.59 + Equation 2.62 + Equation 2.65 = 0 \( \ldots (2.68) \)
The final step is to generate the coefficients matrix by expanding the necessary terms of Equations 2.66, 2.67, and 2.68 in appropriate Fourier series as it was done for the symmetric case. The generated matrix represented schematically by Figure 2.11 is as follows.

Quadrant (1,1):

For \( m=1,3,5, \ldots, 2k-1 \)

\[
A_{(m+1)/2,(m+1)/2} = \frac{1}{\theta_{11m}} \left( (\nu \phi^2 (m\pi/2)^2 - \beta_m^2) \theta_1 \sinh \beta_m \right)
+ (\nu \phi^2 (m\pi/2)^2 + \gamma_m^2) \sin \gamma_m
\]

or if \( \lambda^2 < (m\pi/2)^2 \)

\[
A_{(m+1)/2,(m+1)/2} = \frac{1}{\theta_{22m}} \left( (\nu \phi^2 (m\pi/2)^2 - \beta_m^2) \theta_2 \sinh \beta_m \right)
+ (\nu \phi^2 (m\pi/2)^2 - \gamma_m^2) \sinh \gamma_m
\]

Quadrant (1,2):

For \( m=1,3,5, \ldots, 2k-1 \); \( n=1,3,5, \ldots, 2k-1 \)

\[
A_{(m+1)/2,(n+1)/2+k} = 2 \sin (m\pi/2) \sin (n\pi/2) (x_1 - x_2) / \theta_{11n}
\]

where

\[
x_1 = \left( (n\pi/2)^2 - \nu \phi^2 \beta_n^2 \right) \theta_1 \beta_n \cosh \beta_n / \{ \beta_n^2 + (m\pi/2)^2 \}
\]

\[
x_2 = \left( (n\pi/2)^2 + \nu \phi^2 \gamma_n^2 \right) \gamma_n \cos \gamma_n / \{ \gamma_n^2 - (m\pi/2)^2 \}
\]
or if $\chi^2 \phi^2 < (n\pi/2)^2$

$$A_{(m+1)/2,(n+1)/2+k} = 2\sin(m\pi/2)\sin(n\pi/2)(x_1 + x_2)/\varepsilon_{22n}$$

where

$$x_1 = \{(n\pi/2)^2 - \nu\psi^2/\varepsilon_n^2\} \varepsilon_2 n \varepsilon_n \cosh \varepsilon_n / \{(\varepsilon_n^2 + (m\pi/2)^2)\}$$

$$x_2 = \{(n\pi/2)^2 - \nu\psi^2/\varepsilon_n^2\} \varepsilon_n \cosh \varepsilon_n / \{(\varepsilon_n^2 + (m\pi/2)^2)\}$$

Figure 2.11. Matrix schematic representation for the antisymmetric mode problem.
Quadrant (1,3):

For \( m = 1, 3, 5, \ldots, 2k-1 \); \( n = 1, 3, 5, \ldots, 2k-1 \)

\[
A(m+1)/2, (p+1)/2+2k = 2\sin(p\pi/2)(m\pi/2)(x_1+x_2)/\theta_{11p}
\]

where

\[
x_1 = ((p\pi/2)^2 - \psi^2 \beta_p^2) \cosh \beta_p / (\beta_p^2 + (m\pi/2)^2)
\]

\[
x_2 = ((p\pi/2)^2 + \psi^2 \gamma_p^2) \theta_{1p} \cos \gamma_p / (\gamma_p^2 + (m\pi/2)^2)
\]

or if \( \lambda^2 \phi^2 < (p\pi/2)^2 \)

\[
= 2\sin(p\pi/2)(m\pi/2)(x_1-x_2)/\theta_{22p}
\]

where

\[
x_1 = ((p\pi/2)^2 - \psi^2 \beta_p^2) \cosh \beta_p / (\beta_p^2 + (m\pi/2)^2)
\]

\[
x_2 = ((p\pi/2)^2 - \psi^2 \gamma_p^2) \theta_{2p} \cosh \gamma_p / (\gamma_p^2 + (m\pi/2)^2)
\]

Quadrant (2,1):

For \( m = 1, 3, 5, \ldots, 2k-1 \); \( n = 1, 3, 5, \ldots, 2k-1 \)

\[
A(n+1)/2+k, (m+1)/2 = 2\sin(m\pi/2)\sin(n\pi/2)(x_1-x_2)/\theta_{11m}
\]

where

\[
x_1 = ((m\pi/2)^2 - \psi^2 a_m^2) \theta_{1m} a_m \cosh a_m / (a_m^2 + (n\pi/2)^2)
\]

\[
x_2 = ((m\pi/2)^2 + \psi^2 a_m^2) \gamma_m \cos \gamma_m / (\gamma_m^2 + (n\pi/2)^2)
\]
or if \( \lambda^2 < (n\pi/2)^2 \)

\[
A_{(n+1)/2+k,(n+1)/2} = 2\sin(n\pi/2)\sin(n\pi/2)(x_1 + x_2)/\theta_{22m}
\]

where

\[
x_1 = \left\{ (m\pi/2)^2 - \nu_{1m}^2 \right\} \theta_{2m} \beta_m \cosh \beta_m / \left\{ \beta_m^2 + (n\pi/2)^2 \right\}
\]

\[
x_2 = \left\{ (m\pi/2)^2 - \nu_{1m}^2 \right\} \gamma_m \cosh \gamma_m / \left\{ \gamma_m^2 + (n\pi/2)^2 \right\}
\]

Quadrant (2,2):

For \( n=1,3,5,\ldots,2k-1 \)

\[
A_{(n+1)/2+k,(n+1)/2+k} = \left\{ (\nu_{1}^2 (n\pi/2)^2 - \beta_n^2) \theta_{1n} \sinh \beta_n \right\} \theta_{11n}
\]

\[
+ (\nu_{1}^2 (n\pi/2)^2 + \gamma_n^2) \sin \gamma_n \right\} \theta_{11n}
\]

or if \( \lambda^2 \phi^2 < (n\pi/2)^2 \)

\[
A_{(p+1)/2+k,(p+1)/2+k} = \left\{ (\nu_{1}^2 (p\pi/2)^2 - \beta_p^2) \theta_{1n} \sinh \beta_n \right\} \theta_{22n}
\]

\[
+ (\nu_{1}^2 (p\pi/2)^2 - \gamma_n^2) \sin \gamma_n \right\} \theta_{22n}
\]

Quadrant (2,3):

For \( p=1,3,5,\ldots,2k-1 \)

\[
A_{(p+1)/2+k,(p+1)/2+k} = \left\{ \nu_{1}^2 ((p\pi/2)^2 - (\theta_{1p})^2 - \beta_p^2 - \theta_{1p} \gamma_p^2) \right\} \theta_{11p}
\]
or if \( \lambda^2 \phi^2 < (p\pi/2)^2 \)

\[
A_{p+1}/2+k, (p+1)/2+2k = \left\{ \nu \phi_1^2 (p\pi/2)^2 (1- \theta_2 p) - \beta_p^2 + \theta_2 p \gamma_p^2 \right\} / \theta_2 2p
\]

Quadrant (3,1):

For \( m=1,3,5, \ldots, 2k-1 \) ; \( n=1,3,5, \ldots, 2k-1 \)

\[
A_{n+1}+2k, (m+1)/2 = 2(m\pi/2) \sin(n\pi/2)(x_1-x_2)/\theta_1 1m
\]

where

\[
x_1 = \theta_1 m \beta_m \cosh \beta_m / (\beta_m^2 + (m\pi/2)^2)
\]

\[
x_2 = \gamma_m \cos \gamma_m / (\gamma_m^2 - (m\pi/2)^2)
\]

or if \( \lambda^2 < (m\pi/2)^2 \)

\[
= 2(m\pi/2) \sin(n\pi/2)(x_1+x_2)/\theta_2 2m
\]

where

\[
x_1 = \theta_2 m \beta_m \cosh \beta_m / (\beta_m^2 + (m\pi/2)^2)
\]

\[
x_2 = \gamma_m \cosh \gamma_m / (\gamma_m^2 + (m\pi/2)^2)
\]

Quadrant (3,2):

For \( n=1,3,5, \ldots, 2k-1 \)

\[
A_{n+1}/2+2k, (n+1)/2+k = (\theta_1 n \beta_n + \gamma_n) / \theta_1 1n
\]
or if \( \lambda^2 \phi^2 < (n\pi/2)^2 \)

\[
\tilde{A}(n+1)/2+2k, (n+1)/2+k = (\beta_{2n}\beta_n + \gamma_n)/\beta_{22n}
\]

Quadrant (3,3):

For \( p=1,3,5, \ldots, 2k-1 \)

\[
\tilde{A}(p+1)/2+2k, (p+1)/2+2k = -1
\]

We now feed the above information to a digital computer through specially designed program 3 of Appendix 2 to generate appropriate eigenvalues, and through program 4 of the same appendix to generate mode shapes. Results obtained will be tabulated and discussed in the following paragraph.

**Eigenvalues for the Cantilever Plate**

An eigenvalue search was conducted for the cantilever plate problem using programs 1 and 3 of Appendix 2. Results are tabulated in Tables 2.1 and 2.2. Table 2.1 shows eigenvalues for the first five symmetric vibration modes. It must be recalled that analyses were made for half the plate where the aspect ratio is denoted \( \phi' \) and equal \( b/a \). The full plate has dimensions \( 2b \) by \( a \) as indicated in Figure 2.1, and its aspect ratio is denoted \( \phi \) where \( \phi=2\phi'=2b/a \).
Earlier in this chapter we have said that this particular problem, the problem of the free vibration of the rectangular cantilever plate, has been discussed and analyzed by Gorman in earlier publications using a set of building blocks different than the set used in the present analysis. We have also said that our intention is not to repeat the work that has already been done but to compare our results with those of Gorman's, and to try to show that one has the complete freedom in choosing any appropriate set of building blocks as long as their superposition can lead to the prescribed boundary conditions of the plate under study, and as long as no nonlinearity is introduced in any of the boundary conditions. Results found by Gorman are shown in Tables 2.3 and 2.4, where it is seen that the above are satisfied.

Tables 2.1 through 2.4 show a very good agreement between the two sets of results. In fact, both sets are identical as expected. The insignificant difference in some of the higher modes is attributed to computational errors and/or to a difference in the number of terms used in the series involved. However, some building blocks have the advantage of simplicity and ease in arriving at their individual solutions over the others. As one becomes more involved, his ability in choosing those blocks improves. Simplicity and ease of solution are not the only differences one should look for. Another rather
significant advantage is the fact that no rejection modes are possible when using the set of building blocks discussed in the present analysis. Rejection modes are modes of zero net displacement and yet nontrivial solutions for the unknown coefficients are indicated. For a full understanding of these modes, one is referred to Gorman’s work (20).

Mode shape data for the symmetric and antisymmetric vibration modes for the rectangular cantilever plate may easily be obtained using programs 3 and 4 of Appendix 2. Also in Appendix 3, a full set of mode shape graphics corresponding to the set of eigenvalues of Table’s 2.1 and 2.2 is found. All of these mode shapes enable the analyst to locate in advance those areas of the plate under consideration that are likely to undergo high displacement when vibrating in a specific mode, not to mention their usefulness in helping to establish, in advance, those areas where the highest bending stresses are likely to be encountered.

The task of determining the vibration circular frequencies for a given plate from known eigenvalues is easily accomplished as illustrated in the following example.

Example 2.1- Let us consider the plate of Figure 2.12 where the following are given; \( E=10.0 \times 10^6 \text{psi} \), \( \rho=9.75 \times 10^{-2} \text{lb. mass/in}^3 \)
and \( v = 0.333 \), \( h = 0.125 \), and \( a = b = 10 \text{ in.} \). Determine the second lowest symmetric and antisymmetric vibration mode frequencies.

Solution:

we have \( \rho = \frac{(9.75 \times 10^{-2} \times 0.125)}{386} = 3.16 \times 10^{-5} \frac{\text{lb}}{\text{in}^2 \cdot \text{sec}^2} \)

where the gravitational constant \( g \) is taken to be 386 in./sec²

we also have \( D = \frac{Eh^3}{12(1-v^2)} = \frac{10,000 \times 10^6 \times 0.125^3}{12(1-0.333^2)} = 1690 \text{ in.} \text{lb.} \)

from Tables 2.1 and 2.2 we have for the second lowest symmetric mode \( \lambda^2 = 21.09 \). Therefore, from the relation

\[ \omega = \frac{\lambda^2}{a^2} \sqrt{\frac{D}{\rho}} \]

we have \( \omega = \frac{21.09}{10 \times 10} \sqrt{\frac{1690}{3.16 \times 10^{-5}}} = 1542.3 \text{ rad/sec.} \)

and \( f = \omega / 2\pi = 245.5 \text{ Hz.} \)

for the second lowest antisymmetric mode \( \lambda^2 = 30.55 \), and

\[ \omega = \frac{(1542.3 \times 30.55)}{21.09} = 2234.1 \text{ rad/sec.} \]

and \( f = \omega / 2\pi = 355.6 \text{ Hz.} \)

\[ a = 10 \text{ in.} \]

\[ b = 10 \text{ in.} \]

Figure 2.12. Square cantilever plate for Example 2.1
\[ \lambda^2 = \omega^2 \sqrt{\frac{1}{D}} \quad (v = 0.333) \]

<table>
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<tr>
<th>(\phi = 2b/a)</th>
<th>1/3</th>
<th>1/2.5</th>
<th>1/2</th>
<th>1/1.5</th>
<th>1/1.25</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
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<tr>
<td>(\phi' = b/a)</td>
<td>1/6</td>
<td>1/5</td>
<td>1/4</td>
<td>1/3</td>
<td>1/2.5</td>
<td>1/2</td>
<td>5/8</td>
<td>3/4</td>
<td>1</td>
<td>1.25</td>
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<td>61.12</td>
<td>56.03</td>
<td>33.88</td>
<td>28.52</td>
<td>26.53</td>
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</tbody>
</table>

Table 2.1. First five eigenvalues for the symmetric vibration modes of the cantilever plate of Figure 2.1.

<table>
<thead>
<tr>
<th>(\phi = 2b/a)</th>
<th>1/3</th>
<th>1/2.5</th>
<th>1/2</th>
<th>1/1.5</th>
<th>1/1.25</th>
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<td>1/4</td>
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<td>1/2.5</td>
<td>1/2</td>
<td>5/8</td>
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<td>1</td>
<td>1.25</td>
<td>1.5</td>
</tr>
<tr>
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<td>65.07</td>
<td>56.14</td>
<td>47.33</td>
<td>38.71</td>
<td>34.53</td>
<td>30.55</td>
<td>27.58</td>
<td>25.59</td>
<td>18.84</td>
<td>13.63</td>
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<td>134.1</td>
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<td>65.17</td>
<td>52.43</td>
<td>36.44</td>
<td>32.10</td>
</tr>
</tbody>
</table>

Table 2.2. First five eigenvalues for the antisymmetric vibration modes of the cantilever plate of Figure 2.1.
\[ \lambda^2 = \omega a^2 \sqrt{\frac{D}{\rho}} \quad \quad (\nu = 0.333) \]

<table>
<thead>
<tr>
<th>(\phi = \frac{2b}{a'})</th>
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<th>1/2.5</th>
<th>1/2</th>
<th>1/1.5</th>
<th>1/1.25</th>
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<th>2</th>
<th>2.5</th>
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<tbody>
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<td>1/2.5</td>
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<td>60.94</td>
<td>56.03</td>
<td>33.88</td>
<td>28.53</td>
<td>26.53</td>
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</tbody>
</table>

Table 2.3. First five eigenvalues for the symmetric vibration modes of the cantilever plate as found by Gorman(20).

<table>
<thead>
<tr>
<th>(\phi = \frac{2b}{a})</th>
<th>1/3</th>
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<th>1/2</th>
<th>1/1.5</th>
<th>1/1.25</th>
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<th>2.5</th>
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<tr>
<td>(\phi' = \frac{b}{a})</td>
<td>1/6</td>
<td>1/5</td>
<td>1/4</td>
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<td>1.25</td>
<td>1.5</td>
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<tr>
<td>2</td>
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<td>65.17</td>
<td>52.43</td>
<td>36.44</td>
<td>32.10</td>
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</table>

Table 2.4. First five eigenvalues for the antisymmetric vibration modes of the cantilever plate as found by Gorman(20).
In this chapter we have discussed the method of superposition as it applies to the problem of free vibration of rectangular plates in general. We have also shown in great detail its application in the analysis of the free vibration of rectangular cantilever plates. Major steps followed in solving this problem are summarized below.

1- The introduction of the building blocks to be employed in the analysis, making sure that the governing differential equation is satisfied exactly by each of these building blocks, and that their superposition may lead to a solution that will satisfy the prescribed boundary conditions of the plate under study to any desired degree of accuracy by increasing the number of terms in the series involved. One must be careful not to introduce any nonlinearity.

2- Establish a Levy-type solution for each of the building blocks involved.

3- Establish the contribution of each of these solutions to the boundary conditions of the plate under study.

4- Expand the relevant expressions in one type of Fourier series. Adjust their coefficients to satisfy the prescribed boundary conditions they represent, and in so doing we develop a set of homogeneous algebraic equations
relating the \( n_k \) coefficients. Where \( n \) is the number of blocks involved, and \( k \) is the number of terms used in each of the series involved. \( k \) is dictated by the degree of accuracy required.

5- Handle these \( n_k \) equations by matrix techniques to establish the required number of eigenvalues.

6- Solve for the unknown coefficients by arbitrarily setting one of them to unity, and using well known computer techniques to solve for the remaining \( n_k-1 \), using any \( n_k-1 \) non homogenous equations formed from the previous \( n \) homogenous equations we had.

7- Using the computed values of these coefficients, generate the various mode shapes.

We have come now to the end of the second chapter of this thesis. In the following chapter we will look at the problem of the rectangular cantilever plate with symmetric point supports along the edges. Although, the above seven steps will be followed in analysing this family of plates, a new function, known as the Dirac Delta Function will have to be introduced to deal with the problem of the point supports.
Chapter 3

THE CANTILEVER PLATE
WITH EDGE POINT SUPPORTS

In Chapter one we discussed the general theory of plates, we also discussed the introduction of the method of superposition to dynamic plate problems. In Chapter two we went a step further to show the application of the concept of superposition in analyzing the free vibration problem of the cantilever plate. In the present Chapter an even further step will be taken by introducing point supports on the boundary of the cantilever plate of Chapter two. These point supports are assumed to be symmetric with respect to the $\xi$ axis as shown in Figure 3.1 and Figure 3.2.

Cantilever plates with point supports will be divided into three distinct families. The first family being that of Figure 3.1 where point supports are situated on edges parallel to the $\xi$ axis. The second family is that of Figure 3.2 where point supports are located on the edge parallel to the $\eta$ axis. And the third family is that of the cantilever plate with lateral point supports. The first two families will be
analyzed fully in the present chapter, while the third will be the subject of the next chapter.

Figure 3.1. The cantilever plate with point supports along edges parallel to the $\zeta$ axis. $x$ indicates a point support.

Figure 3.2. The cantilever plate with point supports along the edge parallel to the $\eta$ axis. $x$ indicates a point support.
Analysis of The First Family

Due to the symmetry of the point supports with respect to the $\xi$ axis of Figure 3.1, physical reasoning tells us that all free vibrations of this plate will either be symmetric or antisymmetric with respect to this axis. Each of these two families of vibration modes will be analyzed separately.

Symmetric Modes

Due to the symmetry that exist about the $\xi$ axis, only half the plate of Figure 3.1 need to be analyzed, as shown in Figures 3.3 and 3.5. It will be noted that building blocks of Figure 3.3 are exactly the same as those of Figure 2.2 with the exception of the fourth building block of Figure 3.3 which is added here to account for the effect of the point support. Consequently, solutions for the first three building blocks of Figure 3.3 are given by Equations 2.2, 2.17, and 2.28. Therefore, our attention is focused on the solution of the fourth building block.

The Fourth Building Block:

Concentrating on the last building block of Figure 3.3, we note that the solid dots indicate that slope taken
Figure 3.3. Building blocks for analyzing the symmetric modes of the cantilever plate with point supports symmetrically located on opposite edges.
normal to the edge is forbidden. We also note that the force exerted on the plate by the point support is represented schematically as a Dirac Function just below this fourth block. The Lévy-type solution of this building block may be written as

\[ W_l(\xi, \eta) = \sum_{l=1,3,5} \phi Y_l(\eta) \sin \frac{\pi \xi}{2} \quad \ldots (3.1) \]

It can easily be shown that each term of the above solution satisfies exactly the boundary conditions along the edges \( \xi = 0 \) and \( \xi = 1 \). Due to the symmetry which \( Y(\eta) \) must have with respect to the edge \( \eta = 0 \), we may write \( Y_l(\eta) \) as

\[ Y_l(\eta) = A_1 \cosh \beta_i \eta + D_1 \cos \gamma_1 \eta \]

or if \( \lambda^2 < (1/2)^2 \)

\[ = A_1 \cosh \beta_i \eta + D_1 \cosh \gamma_1 \eta \]

where

\[ \beta_i = \phi \sqrt{\lambda^2 + (1/2)^2} \]

and

\[ \gamma_1 = \phi \sqrt{\lambda^2 - (1/2)^2} \quad \text{or} \quad \phi \sqrt{(1/2)^2 - \lambda^2} \]

which ever is real.

The next step is to eliminate \( A_1 \) and \( D_1 \). Enforcing the condition of zero slope along the edge \( \eta = 1 \) we have

1. Note the difference between the EL (1) and the ODE (1).
\[ A_1 \beta_1 \sinh \beta_1 - D_1 \gamma_1 \sin \gamma_1 = 0 \]

or if \( \lambda^2 < (1\pi/2)^2 \)

\[ A_1 \beta_1 \sinh \beta_1 + D_1 \gamma_1 \sin \gamma_1 = 0 \]

(3.3)

Eliminating the unknown \( D_1 \) from Equation 3.2 we write:

\[ Y_1(\eta) = A_1 (\cosh \beta_1 \eta + \theta_1 \cos \gamma_1 \eta) \]

or if \( \lambda^2 < (1\pi/2)^2 \)

\[ = A_1 (\cosh \beta_1 \eta + \theta_2 \cosh \gamma_1 \eta) \]

(3.4)

where

\[ \theta_1 = \beta_1 \sinh \beta_1 / (\gamma_1 \sin \gamma_1) \]

\[ \theta_2 = \beta_1 \sinh \beta_1 / (\gamma_1 \sinh \gamma_1) \]

The other edge condition is the application of a time varying concentrated force of amplitude \( P \) at a distance \( \xi \) from the \( \eta \) axis. This load varies sinusoidally with time with the same frequency as the vibrating plate, and represents the force that the point support exerts on the plate. Expanding this concentrated edge reaction in a Dirac sine series we have (20)

\[ V(x) = 2P \sum_{m=1,3,5} \sin \frac{m\pi x}{2} \sin \frac{m\pi \xi}{2} \]

(3.5)
Introducing the dimensionless force amplitude \( P^* = Pb^3/Da \), Equation 3.5 may be written as

\[
\frac{V(\xi)b^3}{Da} = 2P^* \sum_{m=1,3,5} \sin \frac{m\pi \xi}{2} \sin \frac{m\pi}{2} \quad \ldots \ldots \ldots \ldots \ldots (3.6)
\]

From Equation 1.20 we have

\[
\frac{V(\xi)b^3}{Da} = \left. \left( \frac{\partial^3 W_\eta}{\partial \eta^3} + \nu_4 \frac{\partial^3 W_\eta}{\partial \eta^2 \partial \xi} \right) \right|_{\eta=1} \quad \ldots \ldots \ldots (3.7)
\]

where \( \phi \) of the left hand side of Equation 1.20 has been replaced by \( b/a \). Equating the right hand side of Equations 3.6 and 3.7, performing the necessary derivations, and solving, it is easily shown that

\[
Y_1(\eta) = P^* \phi_{111} (\cosh \theta_1 \eta + \theta_{11} \cos \gamma_1 \eta)
\]

or if \( \lambda^2 < (1\pi/2)^2 \)

\[
= P^* \phi_{221} (\cosh \theta_1 \eta + \theta_{21} \cos \gamma_1 \eta)
\]

where

\[
\phi_{111} = \frac{-2 \sin(1\pi/2)}{\beta_1(\beta_1^2-\nu_4^2(1\pi/2)^2) \sinh \theta_1 + \theta_{11} \gamma_1 (\gamma_1^2+\nu_4^2(1\pi/2)^2) \sin \gamma_1}
\]

and

\[
\phi_{221} = \frac{-2 \sin(1\pi/2)}{\beta_1(\beta_1^2-\nu_4^2(1\pi/2)^2) \sinh \theta_1 + \theta_{21} \gamma_1 (\gamma_1^2+\nu_4^2(1\pi/2)^2) \sin \gamma_1}
\]
Therefore,

\[ W_4(\xi, n) = \sum_{l=1,3,5} p_{\theta 11}(\cosh \theta_{11} n + \theta_{11} \cos \gamma_1 n) \sin \frac{1 \pi \xi}{2} \]

\[ + \sum_{l=k^*+2} p_{\theta 221}(\cosh \theta_{21} n + \theta_{21} \cosh \gamma_1 n) \sin \frac{1 \pi \xi}{2} \]  \hspace{1cm} (3.9)

We next turn to the contribution of each of the building blocks on the right hand side of Figure 3.3 to the prescribed boundary conditions of the plate on the left hand side of this figure. These boundary conditions are exactly the same as those of the plate on the left hand side of Figure 2.2 with the exception of the point support where displacement is forbidden. Hence Equations 2.29 through 2.37 provide contributions of the first three building blocks to bending moments along the edges \( \xi = 1 \) and \( n = 1 \) as well as their contribution to the slope at \( \xi = 0 \). We now evaluate their contribution to displacement at the point support.

The contribution of each term of \( W_4(\xi, n) \) to displacement at \( n = 1 \) and \( \xi = \xi \) is

\[ \sum_{m} F_m (\cosh \theta_m + \theta_m \cos \gamma_m) \sin \frac{1 \pi \xi}{2} \]

or if \( \lambda^2 < (m^2/2)^2 \) \hspace{1cm} (3.10)
\[
\frac{E_m}{\theta_{22m}} (\cosh \beta_m + \theta_{2m} \cosh \gamma_m) \sin \frac{m \pi \xi}{2} 
\]

The contribution of each term of \( W_2(\xi, \eta) \) to displacement at \( n=1 \) and \( \xi=\xi \) is

\[
\frac{E_n}{\theta_{11n}} (\sinh \beta_n + \theta_{1n} \sin \gamma_n \xi) \cos(n\pi) 
\]

or if \( \lambda^2 \eta^2 < (n\pi)^2 \) \( ...................(3.11) \)

\[
\frac{E_n}{\theta_{22n}} (\sinh \beta_n \xi + \theta_{2n} \sin \gamma_n \xi) \cos(n\pi) 
\]

The contribution of each term of \( W_3(\xi, \eta) \) to displacement at \( n=1 \) and \( \xi=\xi \) is

\[
\frac{E_p}{\theta_{11p}} (\cosh \beta_p (1-\xi) - \theta_{1p} \cos \gamma_p (1-\xi)) \cos(p\pi) 
\]

or if \( \lambda^2 \eta^2 < (p\pi)^2 \) \( ...................(3.12) \)

\[
\frac{E_p}{\theta_{22p}} (\cosh \beta_p (1-\xi) - \theta_{2p} \cosh \gamma_p (1-\xi)) \cos(p\pi) 
\]

We remain with the task of determining the contribution of the fourth building block to the prescribed boundary conditions. Starting with the contribution of each term of \( W_4(\xi, \eta) \) to bending moments along the edge \( \xi=1 \) we write
\[
\frac{M_{x, \phi}}{D} = -\left(\frac{\partial^2 W_\phi}{\partial \xi^2} + \frac{\nu \partial^2 W_\phi}{\partial \eta^2}\right)\bigg|_{\xi = 1}
\]

\[
= p*\theta_{11} \sin \frac{\pi}{2} \left((1/2)^2 - \nu\phi^2 \theta_1^2 \cosh \theta_1 \eta\right)
\]

\[\quad + \theta_{11} \left((1/2)^2 + \nu\phi^2 \gamma_1^2 \cos \gamma_1 \eta\right)\}
\]

or if \( \chi^2 < (1/2)^2 \) .............. (3.13)

\[
= p*\theta_{21} \sin \frac{\pi}{2} \left((1/2)^2 - \nu\phi^2 \theta_1^2 \cosh \theta_1 \eta\right)
\]

\[\quad + \theta_{21} \left((1/2)^2 - \nu\phi^2 \gamma_1^2 \cosh \gamma_1 \eta\right)\}
\]

The contribution to slope at \( \xi = 0 \) of each term of \( W_\phi (\xi, \eta) \) is

\[
\frac{W_\phi (\xi, \eta)}{\partial \xi} = p*\theta_{11} \left((1/2)^2 \cosh \theta_1 \eta + \theta_{11} \cos \gamma_1 \eta\right)
\]

or if \( \chi^2 < (1/2)^2 \) .............. (3.14)

\[
= p*\theta_{21} \left((1/2)^2 \cosh \theta_1 \eta + \theta_{21} \cosh \gamma_1 \eta\right)
\]

The contribution to bending moment along the edge \( \eta = 1 \) of each term of \( W_\phi (\xi, \eta) \) is

\[
\frac{M_{\eta, \phi}}{D} = -\left(\frac{\partial^2 W_\phi}{\partial \eta^2} + \frac{\nu \partial^2 W_\phi}{\partial \xi^2}\right)\bigg|_{\eta = 1}
\]

\[
= p*\theta_{11} \left((\nu\phi^2 (1/2)^2 - \theta_1^2 \cosh \theta_1 \eta\right)
\]

\[\quad + \theta_{11} \left((\nu\phi^2 (1/2)^2 + \gamma_1^2 \cos \gamma_1 \eta\right)\sin \frac{\pi}{2}
\]

\[
+ \theta_{11} \left((\nu\phi^2 (1/2)^2 + \gamma_1^2 \cos \gamma_1 \eta\right)\sin \frac{\pi}{2}
\]
or if \( \lambda^2 < (1\pi/2)^2 \)

\[
\frac{M_n \phi b}{D} = \sum P^* \theta_{221} \left( \upsilon \phi^2 \left(1\pi/2\right)^2 - \beta_1^2 \right) \cosh \beta_1 \\
+ \upsilon \phi \left( \upsilon \phi^2 \left(1\pi/2\right)^2 - \beta_1^2 \right) \cosh \gamma_1 \sin \frac{1\pi \xi}{2}
\]

The contribution to displacement at \( n=1 \) and \( \xi=\zeta \) of

\[
W_1(\xi, n) = \sum_{l=1, 3, 5} \frac{k^*}{\rho} P^* \theta_{121} \left( \cosh \beta_1 + \beta_1 \cos \gamma_1 \right) \sin \frac{1\pi \xi}{2}
\]

\[+ \sum_{l=k^*+2} P^* \theta_{221} \left( \cosh \beta_1 + \beta_1 \cosh \gamma_1 \right) \sin \frac{1\pi \xi}{2}
\]

Next, in order to satisfy the prescribed boundary conditions of the plate under study we write

1.- The net bending moment along the edge \( \xi=1 \) is zero.

Equation 2.29 + Equation 2.32 + Equation 2.35 + Equation 3.13 = 0

\[ \sum \]

2.- The net slope at \( \phi = 0 \) is zero. Or

Equation 2.30 + Equation 2.32 + Equation 2.36 + Equation 3.14 = 0

\[ \sum \]

3.- The net bending moment along the edge \( n=1 \) is zero.
Or

Equation 2.31 + Equation 2.34 + Equation 2.37 + Equation 3.15 = 0

............... (3.19)

4. - The net displacement at \( \eta = 1 \) and \( \xi = \zeta \) is also zero.

Or

Equation 3.10 + Equation 3.11 + Equation 3.12 + Equation 3.16 = 0

............... (3.20)

We now focus our attention on solving Equations 3.17 through 3.20 using the same matrix techniques we have used in chapter two. Note that the required matrix is generated simply by adding one column and one row to the matrix of Figure 2.6. The additional column represents the coefficients of \( P^* \) and the additional row represents the displacement at the point support. The resulting matrix is shown schematically by Figure 3.4. Elements of the last column and row are as follow.

Quadrant (1):

For \( l = 1, 3, 5, \ldots \ldots 2k-1 \)

\[
A_{(l+1)/2, 3k+1} = \theta_{11} \left( \psi^2 (l\pi/2)^2 - \beta_1^2 \cosh \beta_1 \right)
\]

\[
+ \theta_{11} \left( \psi^2 (l\pi/2)^2 + \gamma_1^2 \cos \gamma_1 \right)
\]
or if \( \lambda^2 < (l\pi/2)^2 \)

\[
A_{(l+1)/2,3k+1} = \theta_{21} \{(v\phi^2(l\pi/2)^2 - \beta_1^2)\cosh \beta_1 \\
+ \theta_{21}(v\phi^2(l\pi/2)^2 - \gamma_1^2)\cosh \gamma_1\}
\]

Quadrant (2):

For \( l=1,3,5, \ldots, 2k-1 \), \( n=0,1,2, \ldots, k-1 \)

\[
A_{k+n+1,3k+1} = k^* \sum_{l=1,3,5} \theta_{11\lambda} \sin(l\pi/2)(x_1 + x_{11}) \\
+ \sum_{l=k^*+2}^{2k-1} \theta_{22\lambda} \sin(l\pi/2)(x_2 + x_{22})
\]

where

\[
x_1 = \{(l\pi/2)^2 - v\phi_1^2 \beta_1^2\} \beta_1 \cos(n\pi) \sinh \beta_1 / (\beta_1^2 + (n\pi)^2).
\]

\[
x_{11} = \{(l\pi/2)^2 + v\phi_1^2 \gamma_1^2\} \gamma_1 \theta_{11\lambda} \sinh \gamma_1 \cos(n\pi) / (\gamma_1^2 - (n\pi)^2)^2
\]

\[
x_2 = \{(l\pi/2)^2 - v\phi_1^2 \beta_1^2\} \beta_1 \cos(n\pi) \sinh \beta_1 / (\beta_1^2 + (n\pi)^2)
\]

\[
x_{22} = \{(l\pi/2)^2 + v\phi_1^2 \gamma_1^2\} \theta_{21\lambda} \gamma_1 \cos(n\pi) \sinh \gamma_1 / (\gamma_1^2 + (n\pi)^2)
\]

the above must be divided by 2 if \( n=0 \).
Quadrant (3):

For \( l=1,3,5, \ldots, 2k-1 \); \( n=0,1,2, \ldots, , k-1 \)

\[
A_{2k+n+1,3k+1} = \sum_{l=1,3,5}^{k} 2e_{11} \cos(n\pi)(l\pi/2)(x_1+x_{11})
\]

\[
+ \sum_{l=k^*+2}^{2k-1} 2e_{22} \cos(n\pi)(l\pi/2)(x_2+x_{22})
\]

where

\[
x_1 = \beta_1 \! \! \! \! / (a_1^2 + (n\pi)^2)
\]

\[
x_{11} = \gamma_1 \! \! \! \! / (a_1^2 + (n\pi)^2)
\]

\[
x_2 = \beta_1 \! \! \! \! / (a_1^2 + (n\pi)^2)
\]

\[
x_{22} = \gamma_1 \! \! \! \! / (a_1^2 + (n\pi)^2)
\]

the above must be divided by 2 if \( n=0 \).

Quadrant (4):

For \( m=1,3,5, \ldots, 2k-1 \)

\[
A_{3k+1,(m+1)/2} = (\cosh a_m + \theta_{1m} \cos \gamma_m) \sin (m\pi \xi/2)/\theta_{11m}
\]

or if \( \lambda^2 < (m\pi/2)^2 \)

\[
= (\cosh a_m + \theta_{2m} \cosh \gamma_m) \sin (m\pi \xi/2)/\theta_{22m}
\]
Quadrant (5):

For \( n=0,1,2, \ldots, k-1 \)

\[
A_{3k+1,k+n+1} = (\sinh \theta n \zeta + \theta_{1n} \sin \gamma n \zeta) \cos(n\pi) / \theta_{11n}
\]

or if \( \lambda^2 \phi^2 < (n\pi)^2 \)

\[
= (\sinh \theta n \zeta + \theta_{2n} \sinh \gamma n \zeta) \cos(n\pi) / \theta_{22n}
\]

Quadrant (6)

For \( p=0,1,2, \ldots, k-1 \)

\[
A_{3k+1,2k+p+1} = (\cosh \theta p (1-\zeta) - \theta_{1p} \cos \gamma p (1-\zeta)) \cos(p\pi) / \theta_{11p}
\]

or if \( \lambda^2 \phi^2 < (p\pi)^2 \)

\[
= (\cosh \theta p (1-\zeta) - \theta_{2p} \cosh \gamma p (1-\zeta)) \cos(p\pi) / \theta_{22p}
\]

Quadrant (7):

For \( l=1,3,5, \ldots, 2k-1 \)

\[
A_{3k+1,3k+1} = \sum_{l=1}^{k} \theta_{1l1} (\cosh \theta l + \theta_{1l} \cos \gamma l) \sin \frac{l\pi \xi}{2}
\]

\[
+ \sum_{l=k+2}^{2k-1} \theta_{2l1} (\cosh \theta l + \theta_{2l} \cosh \gamma l) \sin \frac{l\pi \xi}{2}
\]
The above matrix is fed to a digital computer through programs 5 and 6 of Appendix 3 in order to generate eigenvalues that are listed in Table 3.1, and to generate mode shapes shown in Appendix 2. Program 5 may be used to generate eigenvalues for any combination of plate aspect ratio, point support position, and vibration mode. Also, program 6 can be used.

<table>
<thead>
<tr>
<th>m=1</th>
<th>3</th>
<th>5</th>
<th>n=0</th>
<th>1</th>
<th>2</th>
<th>p=0</th>
<th>1</th>
<th>2</th>
<th>l=1,3,5</th>
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<td>0</td>
<td>-</td>
<td>-</td>
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<td>0</td>
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<td>Eqn. 3.17</td>
</tr>
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<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Eqn. 3.17</td>
</tr>
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<td></td>
<td>0</td>
<td>0</td>
<td>1</td>
<td>0</td>
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<td>Eqn. 3.18</td>
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<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>Eqn. 3.18</td>
</tr>
<tr>
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<td></td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>-1</td>
<td>0</td>
<td>0</td>
<td>Eqn. 3.18</td>
</tr>
</tbody>
</table>

Figure 3.4. Matrix schematic representation for the symmetric mode problem. Point support at m=1 and ξ=ε.
to generate corresponding mode shape data.

We turn next to the antisymmetric vibration modes of the present plate problem.

**Antisymmetric Modes**

Building blocks used in this analysis are shown in Figure 3.5, and are exactly the same as those of Figure 2.8 with the exception of the fourth block which is added to account for the point support condition. Therefore, solutions for the first three building blocks of Figure 3.5 are given by Equations 2.48, 2.49, and 2.55. Hence, we only need to solve for the fourth building block and the necessary contributions to the prescribed boundary conditions. Steps followed here are exactly the same as those followed in the analysis of the symmetric case of Figure 3.3.

**Fourth Building Block:**

The Lévy-type solution can be written as

$$\bar{w}_n(\xi, \eta) = \sum_{l=1,3,5} Y_l(\eta) \sin \frac{l \pi \eta}{2}$$

It is relatively easy to show that each term of this solution
Figure 3.5. Building blocks used in the solution of the free vibration antisymmetric modes of the cantilever plate with point support along the edge n=1.
satisfies exactly the boundary conditions for \( \zeta = 0 \) and \( \zeta = 1 \).

In view of the antisymmetric nature of this block in the \( n \)

direction we may write

\[
Y_1(n) = B_1 \sinh \beta_1 n + C_1 \sinh \gamma_1 n
\]

or if \( \lambda^2 < (1/2)^2 \)

\[
\lambda = B_1 \sinh \beta_1 n + C_1 \sinh \gamma_1 n
\]

where

\[
\beta_1 = \phi \sqrt{\lambda^2 + (1/2)^2}
\]

\[
\gamma_1 = \phi \sqrt{\lambda^2 - (1/2)^2} \quad \text{or} \quad \phi \sqrt{(1/2)^2 - \lambda^2}
\]

which ever is real.

The zero slope condition for \( n=1 \) leads to

\[
B_1 \beta_1 \cos \beta_1 + C_1 \gamma_1 \cos \gamma_1 = 0
\]

or

\[
B_1 \beta_1 \cosh \beta_1 + C_1 \gamma_1 \cosh \gamma_1 = 0
\]

and

\[
Y_1(n) = B_1 (\sinh \beta_1 n + \theta_1 \sin \gamma_1 n)
\]

or

\[
Y_1(n) = B_1 (\sinh \beta_1 n + \theta_2 \sinh \gamma_1 n)
\]

where

\[
\theta_1 = -\beta_1 \cosh \beta_1 / \gamma_1 \cos \gamma_1
\]
and

\[ g_{21} = -B_1 \cosh \beta_1 / \gamma_1 \cosh \gamma_1 \]

Next, representing the vertical edge reaction along the edge \( n=1 \) by a Dirac function of the sin series type we have

\[
\frac{\nu b^3}{D_\alpha} = 2P^* \sum_{l=1,3,5} \sin \frac{l\pi \xi}{2} \sin \frac{l\pi \xi}{2} \quad \ldots \ldots \quad (3.24)
\]

where \( P^* = P b^3 / D_\alpha \)

Equating the right hand side of Equation 3.24 to the right hand side of Equation 1.18 and solving we have

\[ Y_1(n) = P^* g_{111} (\sinh \beta_1 n + \theta_1 \sin \gamma_1 n) \]

or if \( \lambda^2 < (l\pi/2)^2 \)

\[ Y_1(n) = P^* g_{221} (\sinh \beta_1 n + \theta_2 \sin \gamma_1 n) \quad \ldots \ldots \quad (3.25) \]

where

\[ \theta_{111} = \frac{-2 \sin(l\pi\xi/2)}{B_1 (\gamma_1^2 - \nu^2 \phi^2 (l\pi/2)^2) \cosh \beta_1 + \theta_{11} \gamma_1 \{ \gamma_1^2 + \nu \phi^2 (l\pi/2)^2 \} \cos \gamma_1} \]

\[ \theta_{221} = \frac{-2 \sin(l\pi\xi/2)}{B_1 (\gamma_1^2 - \nu^2 \phi^2 (l\pi/2)^2) \cosh \beta_1 + \theta_{21} \gamma_1 \{ \gamma_1^2 + \nu \phi^2 (l\pi/2)^2 \} \cosh \gamma_1} \]
The contribution of this fourth building block to the prescribed boundary conditions are as follow.

The contribution of each term of \( W_h(\xi, \eta) \) to bending moment at the edge \( \eta = 1 \) is

\[
\frac{M_{\eta 1}}{D} = P^* \theta_{111} \left\{ \left( (\pi/2)^2 \nu \phi^2 - \beta_1^2 \right) \sinh \beta_1 \\
+ \theta_{11} \left( (\pi/2)^2 \nu \phi^2 + \gamma_1^2 \right) \sin \gamma_1 \right\} \sin^2 \xi
\]

or if \( \lambda^2 < (\pi/2)^2 \)

\[
= P^* \theta_{221} \left\{ \left( (\pi/2)^2 \nu \phi^2 - \beta_1^2 \right) \sinh \beta_1 \\
+ \theta_{21} \left( (\pi/2)^2 \nu \phi^2 + \gamma_1^2 \right) \sinh \gamma_1 \right\} \sin^2 \xi
\]

The contribution of each term of \( W_h(\xi, \eta) \) to bending moment at \( \xi = 1 \) is

\[
\frac{M_{\xi 1}}{D} = P^* \theta_{111} \sin \left\{ \left( (\pi/2)^2 - \nu \phi^2 \beta_1^2 \right) \sinh \beta_1 \xi \\
+ \theta_{11} \left( (\pi/2)^2 + \nu \phi^2 \gamma_1^2 \right) \sin \gamma_1 \xi \right\}
\]

or if \( \lambda^2 < (\pi/2)^2 \)

\[
= P^* \theta_{221} \sin \left\{ \left( (\pi/2)^2 - \nu \phi^2 \beta_1^2 \right) \sinh \beta_1 \xi \\
+ \theta_{21} \left( (\pi/2)^2 - \nu \phi^2 \gamma_1^2 \right) \sinh \gamma_1 \xi \right\}
\]
The contribution of each term of \( W_4(\xi, \eta) \) to slope at the edge \( \xi=0 \) is

\[
\frac{\partial W_4}{\partial \xi} = \sum_{l=1,3,5} \phi_{11}(1\pi/2)(\sinh_1 \eta + \delta_1 \sin \gamma_1 \eta)
\]

or if \( \lambda^2 < (1\pi/2)^2 \),

\[
= \sum_{l=1,3,5} \phi_{22}(1\pi/2)(\sinh_2 \eta + \theta_2 \sinh \gamma_1 \eta)
\]

The contribution of \( W_4(\xi, \eta) \) to displacement at \( \eta=1 \) and \( \xi=\xi \) is

\[
W_4(\xi,1) = \sum_{l=1,3,5} \phi_{11}(1\pi/2)(\sinh_1 + \theta_1 \sin \gamma_1 \sin \frac{1\pi \xi}{2})
\]

\[
+ \sum_{l=k+2}^{2k-1} \phi_{22}(1\pi/2)(\sinh_2 + \theta_2 \sinh \gamma_1 \sin \frac{1\pi \xi}{2})
\]

The necessary matrix is generated by adding an extra column and an extra row to that of Figure 2.11. The additional column is as follows:

**Quadrant (1):**

For \( l=1,3,5, \ldots, 2k-1 \),

\[
A(1+1)/2,3k+1 = \phi_{11}((1\pi/2)^2 \nu_4^2 - \nu_4^2)\sinh_1
\]

\[
+ \phi_{11}((1\pi/2)^2 \nu_4^2 + \gamma_1^2)\sin \gamma_1
\]
or if \( \lambda^2 < (1\pi/2)^2 \)

\[
A^{(l+1)/2,3k+1} = \theta_{21}^2 (((1\pi/2)^2\varphi^2 - \gamma_1)\sinh \beta_1 \\
+ \theta_{21}^2 ((1\pi/2)^2\varphi^2 - \gamma_1^2)\sinh \gamma_1)
\]

Quadrant (2)

For \( l=1,3,5,\ldots,2k-1 \) ; \( n=1,3,5,\ldots,2k-1 \)

\[
A^{k+(n+1)/2,3k+1} = \sum_{l=1,3,5}^{k^*} \sin(1\pi/2)\sin(n\pi/2)(x_1-x_{11}) \\
+ \sum_{l=k^*+2}^{2k-1} \sin(1\pi/2)\sin(n\pi/2)(x_2-x_{22})
\]

where

\[
x_{11} = \frac{(1\pi/2)^2 - \nu_1^2 \beta_1}{\beta_1} \cosh \beta_1 / (\beta_1^2 + (n\pi/2)^2)
\]

\[
x_{11} = \frac{(1\pi/2)^2 + \nu_1^2 \beta_1}{\beta_1} \cos \gamma_1 / (\gamma_1^2 - (n\pi/2)^2)
\]

\[
x_{2} = \frac{(1\pi/2)^2 - \nu_1^2 \beta_1}{\beta_1} \cosh \beta_1 / (\beta_1^2 + (n\pi/2)^2)
\]

\[
x_{22} = \frac{(1\pi/2)^2 - \nu_1^2 \gamma_1^2}{\gamma_1^2} \cosh \gamma_1 / (\gamma_1^2 + (n\pi/2)^2)
\]

Quadrant (3):

For \( l=1,3,5,\ldots,2k-1 \) ; \( n=1,3,5,\ldots,2k-1 \)
\[
A_{2k+(n+1)/2,3k+1} = \sum_{l=1,3,5}^{k^*} 2e^{111(l\pi/2)\sin(n\pi/2)}(x_1-x_{11})
\]
\[
+ \sum_{l=k^*+2}^{2k-1} 2e^{221(l\pi/2)\sin(n\pi/2)}(x_2+x_{22})
\]

where

\[
x_{11} = \beta_1 \cosh \beta_1 /[\beta_1^2 + (\pi/2)^2]
\]
\[
x_{11} = \beta_1 \gamma_1 \cos \gamma_1 /[\gamma_1^2 - (\pi/2)^2]
\]
\[
x_2 = \beta_1 \cosh \beta_1 /[\beta_1^2 + (\pi/2)^2]
\]
\[
x_{22} = \beta_1 \cosh \beta_1 /[\gamma_1^2 + (\pi/2)^2]
\]

The last row in the coefficient matrix is

Quadrant (4):

For \(m=1,3,5,\ldots,2k-1\)

\[
A_{3k+1,(m+1)/2} = (\theta_1 m \sinh \theta_m + \sin \gamma_m) \sin(m\pi/2) / \gamma_1 m
\]

or if \(-\lambda^2 < (m\pi/2)^2\)

\[
= (\theta_2 m \sinh \theta_m + \sinh \gamma_m) \sin(m\pi/2) / \theta_2 m
\]
Quadrant (5):

For \( n=1, 3, 5, \ldots, 2k-1 \)

\[ A_{3k+1, k+(n+1)/2} = (\theta_1 n \sinh \beta_n \zeta + \sin y_n \zeta) \sin(n\pi/2)/\theta_{11n} \]

or if \( \lambda^2 \phi^2 < (n\pi/2)^2 \)

\[ = (\theta_2 n \sinh \beta_n \zeta + \sinh y_n \zeta) \sin(n\pi/2)/\theta_{22n} \]

Quadrant (6):

For \( p=1, 3, 5, \ldots, 2k-1 \)

\[ A_{3k+1, 2k+(p+1)/2} = \begin{cases} \cosh \beta_p (1-\zeta) - \theta_1 p \cos y_p (1-\zeta) \sin(p\pi/2)/\theta_{11p} \end{cases} \]

or if \( \lambda^2 \phi^2 < (p\pi/2)^2 \)

\[ = (\cosh \beta_p (1-\zeta) - \theta_2 p \cosh y_p (1-\zeta) \sin(p\pi/2)/\theta_{22p} \]

Quadrant (7):

For \( l=1, 3, 5, \ldots, 2k-1 \)

\[ A_{3k+1, 3k+1} = \sum_{l=1,3,5}^{k} \theta_{11l} (\sinh \beta_l + \theta_{11} \sin y_l) \sin \frac{l\pi x}{2} \]

\[ + \sum_{l=k+2}^{2k-1} \theta_{22l} (\sinh \beta_l + \theta_{21} \sinh y_l) \sin \frac{l\pi x}{2} \]
We now have all the necessary information to generate the coefficient matrix and start our eigenvalue search. Using program 7, eigenvalues listed in Table 3.2 were found. Also mode shapes were generated using program 8 of Appendix 3. Results are shown in Appendix 2. In the remainder of this chapter we will look at the free vibration problem of the rectangular cantilever plate with point supports symmetrically located on the edge $\xi = 1$ with respect to the $\xi$ axis. This type of plate is referred to here as the second family.

**Analysis of The Second Family**

Once again only half the plate need to be examined. Also vibration modes will either be symmetric or antisymmetric with respect to the $\xi$ axis. We choose to start with the symmetric family.

**Symmetric Modes**

Building blocks used in analyzing this family of vibration modes are shown in Figure 3.6, where it is seen that solutions $W_1$, $W_2$, and $W_3$ are readily available from Equations 2.12, 2.17, and 2.28 respectively. Therefore, we need only analyze the fourth building block in order to establish its solution $W_4$.
Figure 3.6: Building blocks used in the solution of the symmetric mode problem of the rectangular cantilever plate with point supports symmetrically located on the edge.  

\[ W_1(z, n) + W_2(z, n) + W_3(z, n) = 0 \]
The Fourth Building Block:

Concentrating on the fourth block of Figure 3.6, and comparing it closely with the fourth building block of Figure 3.5, it becomes clear that the solution of the present block can be extracted from the readily existing solution of that of Figure 3.5. To obtain the thought solution, the following changes must be made.

1. Change the sine function in the Levy-type solution to a cosine function and change the quantities \((m\pi/2)\), to \((1\pi)\), and divide the first term by two.

2. Interchange the variables \(\xi\) and \(\eta\), replace the aspect ratio \(\phi\) by its inverse \(1/\phi\), and multiply \(\lambda^2\) by \(\phi^2\).

Therefore, \(W_4(\xi, \eta)\) is

\[
W_4(\xi, \eta) = \sum_{l=0,1,2} \Sigma_{1} \Sigma_{11}(\sinh \beta_{1} \xi + \theta_{1} \sin \gamma_{1} \xi) \cos(1\pi \eta)
\]

\[
+ \sum_{l=1,k=2} (\sinh \beta_{1} \xi + \theta_{2} \sin \gamma_{1} \xi) \cos(1\pi \eta)
\]

\[
\text{where}
\]

\[
\beta_{1} = \frac{1}{2} \sqrt{\phi^2 \lambda^2 + (1\pi)^2}
\]

and
\[ \gamma_1 = \frac{1}{c} \sqrt{\lambda^2 \phi^2 - (l\pi)^2} \text{ or } \frac{1}{c} \sqrt{\lambda^2 \phi^2 - \chi^2 \phi^2} \]

whichever is real, and

\[ \theta_{11} = -\beta_1 \frac{\cosh\beta_1}{\gamma_1} \cos\gamma_1 \]
\[ \theta_{21} = -\beta_1 \frac{\cosh\beta_1}{\gamma_1} \cosh\gamma_1 \]

\[ \theta_{11} = \frac{-2\cos(l\pi\xi)}{\delta_1 (\beta_1^2 - \nu \phi^2 (l\pi)^2) \cosh\beta_1 + \theta_{11} \gamma_1 (\gamma_1^2 + \nu \phi^2 (l\pi)^2) \cos\gamma_1} \]
\[ \theta_{22} = \frac{-2\cos(l\pi\xi)}{\delta_1 (\beta_1^2 - \nu \phi^2 (l\pi)^2) \cosh\beta_1 + \theta_{21} \gamma_1 (\gamma_1^2 + \nu \phi^2 (l\pi)^2) \cosh\gamma_1} \]

and where the first summation pertaining to terms for which \( \lambda^2 \phi^2 > (l\pi)^2 \). And where \( \delta_1 = 2 \) if \( l = 0 \), and \( \delta_1 = 1 \) if \( l \neq 0 \).

Turning now to the contribution of \( W_0(\xi, \eta) \) to the prescribed boundary conditions we write

The contribution of each term of \( W_0(\xi, \eta) \) to bending moment at the edge \( \xi = 1 \) is

\[ \frac{M_{\xi\eta}}{D} = p \theta_{11} \left( (\nu \phi^2 (l\pi)^2 - \beta^2_1) \sinh\beta_1 \right. \]
\[ \left. + \theta_{11} (\nu \phi^2 (l\pi)^2 + \gamma_1^2) \sin\gamma_1 \cos(l\pi\eta) \right) \]
or if \( \lambda^2 \phi^2 < (1\pi)^2 \) \hspace{1cm} \text{...............(3.27)}

\[
\frac{M_{P \theta a}}{D} = P^* \theta_{221} ((v_i^2 (1\pi)^2 - \beta_1^2) \sinh \gamma_1 \\
+ \theta_{21} (\nu_i^2 (1\pi)^2 - \gamma_1^2) \sinh \gamma_1) \cos(1\pi \eta)
\]

The contribution of each term of \( W_4(\xi, \eta) \) to the slope at the edge \( \xi = 0 \) is

\[
\frac{\partial W_4}{\partial \xi} = P^* \theta_{111} (\beta_1 + \theta_1 \gamma_1) \cos(1\pi \eta)
\]

or if \( \lambda^2 \phi^2 < (1\pi)^2 \) \hspace{1cm} \text{...............(3.28)}

\[
= P^* \theta_{221} (\beta_1 + \theta_{21} \gamma_1) \cos(1\pi \eta)
\]

The contribution of each term of \( W_4(\xi, \eta) \) to bending moment at the edge \( \eta = 1 \) is

\[
\frac{M_{P \phi b}}{D} = P^* \theta_{111} \cos(1\pi) \{(1\pi)^2 - \nu \phi^2 \beta_1^2) \sinh \gamma_1 \xi \\
+ \theta_{11} ((1\pi)^2 + \nu \phi^2 \gamma_1^2) \sinh \gamma_1 \xi \}
\]

or if \( \lambda^2 \phi^2 < (1\pi)^2 \) \hspace{1cm} \text{...............(3.29)}

\[
= P^* \theta_{221} \cos(1\pi) \{(1\pi)^2 - \nu \phi^2 \beta_1^2) \sinh \gamma_1 \xi \\
+ \theta_{21} ((1\pi)^2 - \nu \phi^2 \gamma_1^2) \sinh \gamma_1 \xi \}
\]
The coefficient matrix is next generated simply by adding the following column and row to the matrix of Figure 2.6.

Quadrant (1):

For \( l = 0, 1, 2, \ldots, k-1 \); \( m = 1, 3, 5, \ldots, 2k-1 \)

\[
A_{(m+1)/2, 3k+1} = \sum_{l=0,1,2}^{k^*} 2\theta_{111} \cos(l\pi)\sin(m\pi/2)(x_1-x_{11})
+ \sum_{l=k^*+1}^{221} 2\theta_{211} \cos(l\pi)\sin(m\pi/2)(x_2+x_{22})
\]

where

\[
x_1 = \{(l\pi)^2 - \nu^2 \beta^2_1\} \beta_1 \cosh \beta_1 / \{\beta^2_1 + (m\pi/2)^2\}
\]

\[
x_{11} = \{(l\pi)^2 + \nu^2 \gamma^2_1\} \gamma_1 \theta_{111} \cos \gamma_1 / \{\gamma^2_1 - (m\pi/2)^2\}
\]

\[
x_2 = \{(l\pi)^2 - \nu^2 \beta^2_1\} \beta_1 \cosh \beta_1 / \{\beta^2_1 + (m\pi/2)^2\}
\]

\[
x_{22} = \{(l\pi)^2 - \nu^2 \gamma^2_1\} \gamma_1 \theta_{211} \cosh \gamma_1 / \{\gamma^2_1 + (m\pi/2)^2\}
\]

Quadrant (2):

For \( l = 0, 1, 2, \ldots, k-1 \)

\[
A_{k+1, 3k+1} = \theta_{111} \{(\nu^2 (l\pi)^2) - \beta^2_1\} \sin \beta_1
+ \theta_{111} \{(\nu^2 (l\pi)^2) - \gamma^2_1\} \sin \gamma_1
\]
or if $\lambda^2 \phi^2 < (l\pi)^2$

$$A_{k+1,3k+1} = \theta_{221}[(\nu \phi^2(l\pi)^2 - \beta_1^2) \sinh \beta_1 + \theta_{21} \nu \phi^2(l\pi)^2 - \gamma_1^2 \sinh \gamma_1]$$

Quadrant (3):

For $l=0,1,2,\ldots,k-1$

$$A_{2k+1,3k+1} = \theta_{111} (\beta_1 + \theta_{11} \gamma_1)$$

or if $\lambda^2 \phi^2 < (l\pi)^2$

$$= \theta_{221} (\beta_1 + \theta_{21} \gamma_1)$$

Quadrant (4):

For $m=1,3,5,\ldots,2k-1$

$$A_{3k+1,(m+1)/2} = (\cosh \beta_m \gamma + \theta_{1m} \cos \gamma \gamma_m \gamma) \sin (m\pi/2)/\theta_{11m}$$

or if $\lambda^2 < (m\pi/2)^2$

$$= (\cosh \beta_m \gamma + \theta_{2m} \cosh \gamma_m \gamma) \sin (m\pi/2)/\theta_{22m}$$

Quadrant (5):

For $n=0,1,2,\ldots,k-1$
\[ A_{3k+1, k+n+1} = (\sinh \theta_n + \theta_1 \sin \gamma_n) \cos(n \pi \xi) / \theta_1 \theta_n \]

or if \( \lambda^2 \phi^2 < (n \pi)^2 \)

\[ = (\sinh \theta_n + \theta_2 \sinh \gamma_n) \cos(n \pi \xi) / \theta_2 \theta_n \]

Quadrant (6):

For \( p=0, 1, 2, \ldots, k-1 \)

\[ A_{3k+1, 2k+p+1} = (1-\theta_1 p) \cos(p \pi \xi) / \theta_1 \theta p \]

or if \( \lambda^2 \phi^2 < (p \pi)^2 \)

\[ = (1-\theta_2 p) \cos(p \pi \xi) / \theta_2 \theta p \]

Quadrant (7):

For \( l=0, 1, 2, \ldots, k-1 \)

\[
A_{3k+1, 3k+1} = \sum_{l=0}^{k} \theta_{11l} (\sinh \beta_l + \theta_{11} \sin \gamma_l) \cos(l \pi \xi) \\
+ \sum_{l=k+1}^{2k} \theta_{22l} (\sinh \beta_l + \theta_{22} \sinh \gamma_l) \cos(l \pi \xi)
\]

Finally, program 9 of Appendix 2 is designed to generate eigenvalues for this family of vibration modes as listed
in Table 3.3. The corresponding mode shape data are generated using program 10 of Appendix 3. Generated shapes are also illustrated in appendix 2. We now turn to the analysis of the antisymsmetric modes.

Antisymmetric Modes

This family of vibration modes is analyzed using building blocks illustrated in Figure 3.7. Solutions for the first three of these blocks are available from those of Figure 2.8, and are given by Equations 2.48, 2.49, and 2.55 respectively. The solution of the fourth building block is easily obtained from the readily available solution of the fourth building block of Figure 3.5 by interchanging the variables \( \xi \) and \( \eta \). As it was discussed earlier, such interchange implies the replacement of the aspect ratio \( \phi \) by its inverse and the multiplication of \( \lambda^2 \) by \( \phi^2 \). Hence \( W_4(\xi, \eta) \) is

\[
W_4(\xi, \eta) = \sum_{k} \sum_{l=1, 3, 5} P_k^l \theta_{11l}(\sinh_1 \xi + \theta_{11} \sin_1 \xi) \sin \frac{1n \eta}{2}
\]

\[+ \sum_{k} \sum_{l=k+2} P_k^l \theta_{21l}(\sinh_1 \xi + \theta_{21} \sin_1 \xi) \sin \frac{1n \eta}{2}
\]  

(3.30)

where

\[
\theta_{ik} = \frac{1}{\phi} \sqrt{\frac{\lambda^2}{\phi^2 \lambda^2 + (1n/2)^2}}
\]
\[ \gamma_1 = \frac{1}{\phi^2} \sqrt{\lambda^2 \phi^2 - (1\pi/2)^2}, \quad \text{or} \quad \frac{1}{\phi^2} \sqrt{(1\pi/2)^2 - \lambda^2 \phi^2} \]

whichever is real, and

\[ \theta_{11} = -\frac{1}{\gamma_1} \cosh \beta_1 \cosh \gamma_1 \]

\[ \theta_{21} = -\frac{\beta_1 \cosh \beta_1}{\gamma_1} \cosh \gamma_1 \]

\[ \theta_{111} = \frac{-2\sin(1\pi/2)}{\beta_1 (\beta_1^2 - \nu \phi^2 (1\pi/2)^2) \cosh \beta_1 + \theta_{21} \gamma_1 (\gamma_1^2 + \nu \phi^2 (1\pi/2)^2) \cosh \gamma_1} \]

\[ \theta_{221} = \frac{-2\sin(1\pi/2)}{\beta_1 (\beta_1^2 - \nu \phi^2 (1\pi/2)^2) \cosh \beta_1 + \theta_{21} \gamma_1 (\gamma_1^2 + \nu \phi^2 (1\pi/2)^2) \cosh \gamma_1} \]

The contribution of \( W_\theta (\xi, \eta) \) to the prescribed boundary conditions are found to be

The contribution of each term of \( W_\theta (\xi, \eta) \) to bending moment at \( \eta = 1 \) is

\[ \frac{M_{n\phi \theta}}{D} = p_s \sin \frac{\lambda \phi}{2} \left[ \left( 1\pi/2 \right)^2 - \nu \phi^2 (1\pi/2)^2 \right] \sinh \beta_1 \xi \]

or if \( \chi^2 \phi^2 \ll (1\pi/2)^2 \).
Figure 3.7. Building blocks used in the solution of the antisymmetric modes of the rectangular cantilever plate with point supports symmetrically located on the edge $\xi=1$ with respect to the $\eta$ axis.
\[
\begin{align*}
\frac{M_{\gamma b}}{D} &= P*\theta_{221} \sin \frac{\pi}{2} \{(1\pi/2)^2 - \nu_2^2 \xi_1^2 \} \sinh \xi_1 \xi \\
&+ \theta_{21} \{(1\pi/2)^2 - \nu_2^2 \gamma_1^2 \} \sinh \gamma_1 \xi
\end{align*}
\]

The contribution of each term of \( W_\theta (\xi, \eta) \) to the slope at \( \xi = 0 \) is

\[
\frac{\partial W_\theta}{\partial \xi} = P*\theta \gamma_{11} \{(\xi_1^2 + \theta_{11} \gamma_1) \sin \frac{\pi \eta}{2}
\]

or if \( \gamma_2^2 < (1\pi/2)^2 \)

\[
= P*\theta \theta_{221} \{(\xi_1^2 + \theta_{21} \gamma_1) \sin \frac{\pi \eta}{2}
\]

The contribution of each term of \( W_\theta (\xi, \eta) \) to bending moment at \( \xi = 1 \) is

\[
\begin{align*}
\frac{M_{\gamma a}}{D} &= P*\theta \gamma_{11} \{(\nu_1^2 (1\pi/2)^2 - \xi_1^2) \sinh \xi_1 \\
&+ \theta_{11} \{(\nu_1^2 (1\pi/2)^2 + \gamma_1^2) \sinh \gamma_1 \} \sin \frac{\pi \eta}{2}
\end{align*}
\]

or if \( \gamma_2^2 < (1\pi/2)^2 \)

\[
= P*\theta \theta_{221} \{(\nu_1^2 (1\pi/2)^2 - \xi_1^2) \sinh \xi_1 \\
+ \theta_{21} \{(\nu_1^2 (1\pi/2)^2 + \gamma_1^2) \sinh \gamma_1 \} \sin \frac{\pi \eta}{2}
\]
The coefficient matrix is now generated by adding the following column and row to the matrix of Figure 2.11. Note that in these matrices the additional column is always multiplied by $P^*$. 

Quadrant (1):

For $l=1, 3, 5, \ldots, 2k-1$; $m=1, 3, 5, \ldots, 2k-1$

\[
A_{(m+1)/2, 3k+1}^{(m+1)/2, 3k+1} = \sum_{l=1,3,5}^{2k-1} 2 \theta_1 \gamma_1 x_1 \sin(l \pi/2) \sin(m \pi/2) (x_{11} - x_{11})
\]

\[
+ \sum_{l=k^*+2}^{2k-1} 2 \theta_2 \gamma_2 x_2 \sin(l \pi/2) \sin(m \pi/2) (x_{22} + x_{22})
\]

where

\[
x_1 = \frac{((l/2)^2 - \nu \phi^2 \epsilon^2) \gamma_1 \cosh \gamma_1}{\gamma_1^2 + (m \pi/2)^2}
\]

\[
x_{11} = \frac{((l/2)^2 + \nu \phi^2 \epsilon^2) \gamma_1 \gamma_1 \cos \gamma_1}{\gamma_1^2 - (m \pi/2)^2}
\]

\[
x_2 = \frac{((l/2)^2 - \nu \phi^2 \epsilon^2) \gamma_1 \cosh \gamma_1}{\gamma_1^2 + (m \pi/2)^2}
\]

\[
x_{22} = \frac{((l/2)^2 - \nu \phi^2 \epsilon^2) \gamma_1 \cosh \gamma_1}{\gamma_1^2 + (m \pi/2)^2}
\]

Quadrant (2):

For $l=1, 3, 5, \ldots, 2k-1$
\[ A_{k+1/2,3k+1} = \theta_{111} \left( (\nu \phi_1^2 l \pi/2)^2 - \beta_1^2 \right) \sinh \beta_1 \]
\[ + \theta_{11} (\nu \phi_1^2 l \pi/2 + \gamma_1^2) \sin \gamma_1 \]

or if \( \lambda^2 \phi^2 < (l \pi/2)^2 \)

\[ = \theta_{221} \left( (\nu \phi_1^2 l \pi/2)^2 - \beta_1^2 \right) \sinh \beta_1 \]
\[ + \theta_{21} (\nu \phi_1^2 l \pi/2 - \gamma_1^2) \sinh \gamma_1 \]

Quadrant (3):

For \( k = 1, 3, 5, \ldots, 2k-1 \)

\[ A_{2k+1/2,3k+1} = \theta_{111} (\beta_1 + \theta_{11} \gamma_1) \]

or if \( \lambda^2 \phi^2 < (l \pi/2)^2 \)

\[ = \theta_{221} (\beta_1 + \theta_{21} \gamma_1) \]

Quadrant (4):

For \( m = 1, 3, 5, \ldots, 2k-1 \)

\[ A_{3k+1, (m+1)/2} = (\theta_{1m} \sinh \beta_m + \sin \gamma_m) \sin (m \pi/2) / \theta_{11m} \]

or if \( \lambda^2 < (m \pi/2)^2 \)
\[ A_{3k+1, (m+1)/2} = (\theta_{2m} \sinh \beta_m \tau + \sinh \gamma_m \tau) \sin(m\pi/2)/\theta_{22m} \]

Quadrant (5):

For \( n=1, 3, 5, \ldots, 2k-1 \)

\[ A_{3k+1, k+(n+1)/2} = (\theta_{1n} \sinh \beta_n + \sin \gamma_n) \sin(n\pi/2)/\theta_{11n} \]

or if \( \lambda^2 < (n\pi/2)^2 \)

\[ = (\theta_{2n} \sinh \beta_n + \sin \gamma_n) \sin(n\pi/2)/\theta_{22n} \]

Quadrant (6):

For \( p=1, 3, 5, \ldots, 2k-1 \)

\[ A_{3k+1, 2k+(p+1)/2} = (1-\theta_{1p}) \sin(p\pi/2)/\theta_{11p} \]

or if \( \lambda^2 < (p\pi/2)^2 \)

\[ = (1-\theta_{2p}) \sin(p\pi/2)/\theta_{22p} \]

Quadrant (7):

For \( l=1, 3, 5, \ldots, 2k-1 \)
\[ A^{(3k+1),(3k+1)} = \sum_{l=1}^{5} \theta_{1ll}^*(\sinh \gamma_1 + \frac{\theta_{1l}}{2} \sin 2l\pi \gamma_1) \sin \frac{l\pi \gamma_1}{2} \]
\[ + \sum_{l=k+2}^{2k} \theta_{22l}^*(\sinh \gamma_1 + \frac{\theta_{2l}}{2} \sinh 2l\pi \gamma_1) \sin \frac{l\pi \gamma_1}{2} \]

Eigenvalues for this family of vibration modes are generated using program 11 of Appendix 3 and listed in Table 3.4. Corresponding mode shapes are also illustrated in Appendix 2 using data generated by specially designed program 12 of Appendix 3.

In Tables 3.1 through 3.4 eigenvalues are obtained with four digits accuracy. While this practice is followed throughout this work, greater accuracy is easily obtained by increasing the number \(k\) of terms used in the appropriate series involved. \(k\) may be decided upon following a convergence test similar to the one shown in Figure 2.7. Since point supports may assume arbitrarily their symmetric positions with respect to the \(x\) axis along the edges of the cantilever plate, it is virtually impossible to list all possible combinations of plate aspect ratios and point support positions along the edges. However, programs 5 through 12 are provided to generate eigenvalues and mode shape data for any possible combination one might encounter. As for Tables 3.1
\[ \lambda^2 = \omega a^2 \sqrt{\rho / D} \] 

| \( \phi = b/a \) | 1/3 | 1/2.5 | 1/2 | 1/1.5 | 1/1.25 | 1 | 1.25 | 1.5 | 2 | 2.5 | 3 |
| \( \phi' = b/a \) | 1/6 | 1/5.5 | 1/4 | 1/3 | 1/2.5 | 1/2 | 5/8 | 3/4 | 1 | 1.25 | 1.5 |

Mode 1
- 66.75 | 59.87 | 59.76 | 58.55 | 49.81 | 38.64 | 31.40 | 27.74 | 24.57 | 22.77 | 19.31 |

Table 3.1. First three eigenvalues for the symmetric vibration modes of the cantilever plate of Figure 3.1. \( \zeta = 0.5 \).

\[ \lambda^2 = \omega a^2 \sqrt{\rho / D} \] 

| \( \phi = b/a \) | 1/3 | 1/2.5 | 1/2 | 1/1.5 | 1/1.25 | 1 | 1.25 | 1.5 | 2 | 2.5 | 3 |
| \( \phi' = b/a \) | 1/6 | 1/5.5 | 1/4 | 1/3 | 1/2.5 | 1/2 | 5/8 | 3/4 | 1 | 1.25 | 1.5 |

Mode 1
- 41.53 | 35.31 | 29.05 | 22.70 | 19.43 | 16.03 | 13.14 | 11.08 | 8.384 | 6.808 | 5.857 |
- 107.5 | 96.81 | 86.33 | 75.22 | 65.62 | 50.76 | 38.30 | 30.59 | 21.96 | 17.17 | 13.98 |
- 142.4 | 124.8 | 106.2 | 84.62 | 75.04 | 68.80 | 53.63 | 43.31 | 33.16 | 28.40 | 24.99 |

Table 3.2. First three eigenvalues for the antisymmetric vibration modes of the cantilever plate of Figure 3.1. \( \zeta = 0.5 \).
\[ \lambda^2 = \omega a^2 \sqrt{\rho/D} \quad (\nu = 0.333) \quad \zeta = 0.5 \]

<table>
<thead>
<tr>
<th>( \phi = 2b/a )</th>
<th>1/3</th>
<th>1/2.5</th>
<th>1/2</th>
<th>1/1.5</th>
<th>1/1.25</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi' = b/a )</td>
<td>1/6</td>
<td>1/5</td>
<td>1/4</td>
<td>1/3</td>
<td>1/2.5</td>
<td>1/2</td>
<td>5/8</td>
<td>3/4</td>
<td>1</td>
<td>1.25</td>
<td>1.5</td>
</tr>
<tr>
<td></td>
<td>47.99</td>
<td>47.95</td>
<td>47.72</td>
<td>46.47</td>
<td>38.51</td>
<td>27.02</td>
<td>19.54</td>
<td>15.94</td>
<td>13.24</td>
<td>11.20</td>
<td>9.369</td>
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<tr>
<td></td>
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<td>99.61</td>
<td>91.30</td>
<td>56.09</td>
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<td>46.17</td>
<td>40.94</td>
<td>36.19</td>
<td>27.62</td>
<td>22.85</td>
<td>20.34</td>
</tr>
</tbody>
</table>

Table 3.3. First three eigenvalues for the symmetric vibration modes of the plate shown in Figure 3.2. \( \zeta = 0.5 \).

\[ \lambda^2 = \omega a^2 \sqrt{\rho/D} \quad (\nu = 0.333) \quad \zeta = 0.5 \]

<table>
<thead>
<tr>
<th>( \phi = 2b/a )</th>
<th>1/3</th>
<th>1/2.5</th>
<th>1/2</th>
<th>1/1.5</th>
<th>1/1.25</th>
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<th>1.25</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi' = b/a )</td>
<td>1/6</td>
<td>1/5</td>
<td>1/4</td>
<td>1/3</td>
<td>1/2.5</td>
<td>1/2</td>
<td>5/8</td>
<td>3/4</td>
<td>1</td>
<td>1.25</td>
<td>1.5</td>
</tr>
<tr>
<td>Mode 1</td>
<td>40.18</td>
<td>34.19</td>
<td>28.33</td>
<td>22.70</td>
<td>20.00</td>
<td>17.33</td>
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<td>86.75</td>
<td>69.42</td>
<td>53.60</td>
<td>44.17</td>
<td>34.23</td>
<td>29.25</td>
<td>26.71</td>
</tr>
</tbody>
</table>

Table 3.4. First three eigenvalues for the antisymmetric vibration modes of the plate shown in Figure 3.2. \( \zeta = 0.5 \).
through 3.4, we have chosen the point support position to be
at a distance of 1/2 from the axis perpendicular to the edge
on which it is located. Further discussions on eigenvalues
listed above and others will be forthcoming in later chapters.

We now come to the end of the present chapter where
we have analyzed rectangular cantilever plates with edge point
supports. The next chapter will concern itself with the study
of the free vibration problem of the rectangular cantilever
plate with lateral point supports. Also in the next chapter
a discussion and a comparison of some of the eigenvalues found
so far and others with existing eigenvalues will be carried
out whenever is possible.
Chapter 4

THE RECTANGULAR CANTILEVER PLATE
WITH LATERAL POINT SUPPORTS

Consider the cantilever plate of Figure 4.1. The free vibration problem of this family of plates was fully discussed in Chapter 2. Then in Chapter 3 we introduced the condition of point supports on the boundary of the cantilever plate as illustrated by Figures 3.1 and 3.2 where lateral movement of the plate is forbidden at these points. In this chapter internal point supports are being introduced, where lateral movement of the plate is forbidden where the coordinates $\xi$ and $\eta$ of the lower portion of the plate shown in Figure 4.2 assume the values $u$ and $v$. Here also point supports are considered to be symmetric with respect to the $\xi$ axis. Therefore, only the lower half of the plate of Figure 4.2 will be analyzed. If we assume that the plate is vibrating in one of its vibration modes with a circular frequency $\omega$, then lateral motion at the fixed point will have to be prevented by a time varying force that may be represented as $P\sin(\omega t)$. Solution for this type of problems was described by Nowaki(23), and then discussed by
Figure 4.1. Cantilever plate discussed in Chapter 2.

Figure 4.2. Cantilever plate with lateral point supports.
Gorman in his "Free Vibration Analysis of Rectangular Plates". Following our practice of dividing the free vibration modes of our plate into two families, we will first focus our attention on the symmetric mode family.

**Symmetric Modes**

Building blocks used in the solution of this family of free vibration modes consist of four blocks the first three of which are the same as those used in analyzing the same family of free vibration modes of the cantilever plate of Figure 3.1 and for which solutions are readily developed in Chapter 2. Figure 4.3 illustrates all four building blocks needed for this analysis. Focusing our attention on the last building block of Figure 4.3, we divide this block into two segments I and II. Each segment has its own coordinate system with the point support lying on the common line between the two segments as shown in Figure 4.4.

The Fourth Building Block:

This fourth building block was used by Gorman in his analysis of modes symmetric with respect to the $\xi$ axis and antisymmetric with respect to the $\eta$ axis for the rectangular plate with symmetrically distributed point supports(20).
Figure 4.3. Building blocks used in the solution of the symmetric free vibration modes of the cantilever plate with lateral point supports.
Although this solution is readily available, it will be reproduced here for convenient reference and to show briefly the various steps involved.

In view of the boundary conditions, Lévy-type solution may be written for each segment as follow

\[ W_{41}(\xi, \eta) = \sum_{m=1,3,5}^{k^*} \left( A_1 m \cosh \phi_m \eta + B_1 m \cos \gamma_m \eta \right) \sin \frac{m\pi \xi}{2} \]

\[ + \sum_{m=k^*+2}^{\infty} \left( A_2 m \cosh \phi_m \eta + B_2 m \cos \gamma_m \eta \right) \sin \frac{m\pi \xi}{2} \]  \hspace{2cm} (4.1)

and

\[ W_{42}(\xi, \eta) = \sum_{m=1,3,5}^{k^*} \left( C_1 m \cosh \phi_m \eta + D_1 m \cos \gamma_m \eta \right) \sin \frac{m\pi \xi}{2} \]

\[ + \sum_{m=k^*+2}^{\infty} \left( C_2 m \cosh \phi_m \eta + D_2 m \cos \gamma_m \eta \right) \sin \frac{m\pi \xi}{2} \]  \hspace{2cm} (4.2)

\[ \text{Figure 4.4: Fourth building block of Figure 4.3 divided into two segments by a line parallel to the } \xi \text{ axis and passing through the lateral point support.} \]
where

\[ \theta_m = \phi \sqrt{\lambda^2 + (m\pi/2)^2} \]

\[ \gamma_m = \phi \sqrt{\lambda^2 - (m\pi/2)^2} \quad \text{or} \quad \phi \sqrt{(m\pi/2)^2 - \lambda^2} \]

whichever is real. Then the concentrated force is represented as

\[ P(\xi) = \sum_{m=1,3,5} \sum_{u=1}^{\infty} P \sin \frac{mu\pi}{2} \sin \frac{m\pi \xi}{2} \]

(4.3)

The next step is to enforce the conditions of continuity and force equilibrium along the common edge. These conditions are:

1.- The plate displacement must be continuous across the boundary between the two segments.

2.- The slope of the plate taken in a direction normal to the boundary must also be continuous across the boundary.

3.- Bending moments must be continuous across the boundary between the two segments.

4.- There must be an equilibrium force balance between the applied force and the vertical edge reactions taken along the intersegment boundary.
By enforcing the above conditions we have

\[ A_1 \cos h \beta_m v + B_1 \cos \gamma_m v = C_1 \cos h \beta_m v^* - D_1 \cos \gamma_m v^* = 0 \]

\[ A_1 \sin h \beta_m v - B_1 \sin \gamma_m v + C_1 \beta_m \sin h \beta_m v^* - D_1 \gamma_m \sin \gamma_m v^* = 0 \]

\[ A_1 \beta_m \cos h \beta_m v - B_1 \gamma_m \cos \gamma_m v + C_1 \beta_m \cos h \beta_m v^* + D_1 \gamma_m \cos \gamma_m v^* = 0 \]

\[ A_1 \beta_m \sin h \beta_m v + B_1 \gamma_m \sin \gamma_m v + C_1 \beta_m \beta_m \sin h \beta_m v^* + D_1 \gamma_m \gamma_m \sin \gamma_m v^* = \pm \sin \frac{m \mu}{2} \]

and \( \lambda^2 < (m \pi/2)^2 \)

\[ A_2 \cos h \beta_m v + B_2 \cos \gamma_m v = -D_2 \cos h \gamma_m v^* = 0 \]

\[ A_2 \beta_m \sin h \beta_m v + B_2 \gamma_m \sin \gamma_m v + C_2 \beta_m \beta_m \sin h \beta_m v^* + D_2 \gamma_m \gamma_m \sin \gamma_m v^* = 0 \]

\[ A_2 \beta_m \cos h \beta_m v + B_2 \gamma_m \cos \gamma_m v - C_2 \beta_m \beta_m \cos h \beta_m v^* - D_2 \gamma_m \gamma_m \cos \gamma_m v^* = 0 \]

\[ A_2 \beta_m \sin h \beta_m v + B_2 \gamma_m \sin \gamma_m v - C_2 \beta_m \beta_m \sin h \beta_m v^* - D_2 \gamma_m \gamma_m \sin \gamma_m v^* = \pm \sin \frac{m \mu}{2} \]

where \( v^* = 1 - v \)

Solving the above for \( A_1, \ldots, D_2 \) we have

\[ A_1 = \frac{\pm \sin (m \mu/2) \cos h \beta_m v}{(\beta^2_m + \gamma^2_m) \beta_m \sin h \beta_m} \]

\[ B_1 = \frac{\pm \sin (m \mu/2) \cos \gamma_m v}{(\beta^2_m + \gamma^2_m) \gamma_m \sin \gamma_m} \]

\[ C_1 = \frac{\pm \sin (m \mu/2) \cos h \beta_m v}{(\beta^2_m + \gamma^2_m) \beta_m \sin h \beta_m} \]
\[ D_{1m} = P \sin(mu/2) \cos \gamma_m v / (\dot{\xi}_m^2 + \gamma_m^2) \gamma_m \sin \gamma_m \]

\[ A_{2m} = P \sin(mu/2) \cosh \beta_m v / (\dot{\xi}_m^2 - \gamma_m^2) \beta_m \sinh \beta_m \]

\[ B_{2m} = -P \sin(mu/2) \cos \gamma_m v / (\dot{\xi}_m^2 - \gamma_m^2) \gamma_m \sinh \gamma_m \]

\[ C_{2m} = P \sin(mu/2) \cosh \beta_m v / (\dot{\xi}_m^2 - \gamma_m^2) \beta_m \sinh \beta_m \]

\[ D_{2m} = -P \sin(mu/2) \cos \gamma_m v / (\dot{\xi}_m^2 - \gamma_m^2) \gamma_m \sinh \gamma_m \]

We now have a solution for the fourth building block of Figure 4.3 with an intersegment line running in the direction of the \( \xi \) axis. However, since we require the bending moment contribution of this building block along the edge \( \xi = 1 \), and the slope at the edge \( \xi = 0 \), although not necessary, it would be helpful to have a solution with intersegment line running parallel to these edges, as demonstrated by Figure 4.5, to eliminate any irregularity along these edges.

Solution of the block of Figure 4.5 may easily be obtained from that of the block of Figure 4.6 by interchanging the variables \( \xi \) and \( \eta \) and following transformation rules already discussed. Focussing our attention on Figure 4.5, in view of the boundary conditions along the edges \( \xi = 0 \) and \( \eta = 0 \) we may write
Figure 4.5. Same as Figure 4.4 with intersegment line running in the $n$ direction.

Figure 4.6. Intermediate block used in the solution of the block of Figure 4.4.
\[ W_{\ell 1}(\xi, \eta) = \sum_{m=0,1,2}^k (A_m \sin \beta_m \eta + B_m \sin \gamma_m \eta) \cos(m\pi \xi) \]
\[ + \sum_{m=k+1}^{k^*} (A_2m \sin \beta_m \eta + B_2m \sin \gamma_m \eta) \cos(m\pi \xi) \] 
\[ \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdots \cdrightarrow
and

\[ A_{2m} = P^* \cos(m \pi u) \cosh \zeta_m \sinh \theta_m \sin \frac{\pi m}{m^*} \delta_{m} \sinh \gamma_m \cos \gamma_m \]
\[ B_{2m} = -P^* \cos(m \pi u) \cosh \gamma_m \sin \frac{\pi m}{m^*} \delta_{m} \sinh \zeta_m \cos \gamma_m \]
\[ C_{2m} = P^* \cos(m \pi u) \sin \theta_m \sin \frac{\pi m}{m^*} \delta_{m} \sinh \gamma_m \cos \gamma_m \]
\[ D_{2m} = -P^* \cos(m \pi u) \sin \gamma_m \sin \frac{\pi m}{m^*} \delta_{m} \sinh \zeta_m \cos \gamma_m \]

Where the concentrated force has been expanded as a Dirac function using the cosine series

\[ P(\xi) = \sum_{m=0}^{\infty} P_m^* \frac{1}{m} \cos(m \pi u) \cos(m \pi \xi) \]  \hspace{1cm} (4.6)

where

\[ \delta_m = \begin{cases} 2 & \text{if } m = 0 \\ 1 & \text{if } m \neq 0 \end{cases} \]

and

\[ P^* = -2Pb^3/Da^2 \]

We now interchange the variables \( \xi \) and \( \eta \) in Equations 4.4 and 4.5 to obtain solutions for the two segments of Figure 4.5. Note that \( m \) is also replaced by \( n \).

\[ W_{11}(\xi, \eta) = \sum_{n=0,1,2}^{k^*} \left( A_n \sinh \theta_n \xi + B_n \sin \gamma_n \xi \right) \cos(n \pi \eta) \]
\[ + \sum_{n=k^*+1}^{\infty} \left( A_n \sinh \theta_n \xi + B_n \sin \gamma_n \xi \right) \cos(n \pi \eta) \]

and
\[ W_{2}(*,*) = \sum_{n=0,1,2}^{k^*} \left( C_{1n} \cosh \beta_n \xi + D_{1n} \cos \gamma_n \xi \right) \cos(n \pi n) \]

\[ + \sum_{n=k^*+1}^{\infty} \left( C_{2n} \cosh \beta_n \xi + D_{2n} \cosh \gamma_n \xi \right) \cos(n \pi n) \]

where:

\[ \delta_n = \frac{1}{\phi} \sqrt{\lambda^2 \phi^2 + (n \pi)^2} \]

\[ \gamma_n = \frac{1}{\phi} \sqrt{\lambda^2 \phi^2 - (n \pi)^2} \quad \text{or} \quad \frac{1}{\phi} \sqrt{(n \pi)^2 - \lambda^2 \phi^2} \]

We may also deduce the coefficients if we replace subscript \( m \) by \( n \), and interchange the quantities \( u \) and \( u^* \) with \( v \) and \( v^* \) respectively. Furthermore, we must divide the new coefficients by \( \phi^4 \) to leave unchanged our definition for \( P^* \). Displacement will still be dimensionless with respect to side length \( a \).

\[ A_{1n} = P^* \cos(n \pi v) \cosh \beta_n u^*/\delta_n \phi^4 (\beta_n^2 + \gamma_n^2) \beta_n \cosh \beta_n \]

\[ B_{1n} = -P^* \cos(n \pi v) \cos \gamma_n u^*/\delta_n \phi^4 (\beta_n^2 + \gamma_n^2) \gamma_n \cos \gamma_n \]

\[ C_{1n} = P^* \cos(n \pi v) \sinh \beta_n u^*/\delta_n \phi^4 (\beta_n^2 + \gamma_n^2) \beta_n \cosh \beta_n \]

\[ D_{1n} = -P^* \cos(n \pi v) \sin \gamma_n u^*/\delta_n \phi^4 (\beta_n^2 + \gamma_n^2) \gamma_n \cos \gamma_n \]

and:

\[ A_{2n} = P^* \cos(n \pi v) \cosh \beta_n u^*/\delta_n \phi^4 (\beta_n^2 - \gamma_n^2) \beta_n \cosh \beta_n \]

\[ B_{2n} = -P^* \cos(n \pi v) \cosh \gamma_n u^*/\delta_n \phi^4 (\beta_n^2 - \gamma_n^2) \gamma_n \cosh \gamma_n \]
With the solution of the final building block developed we next turn to derive expressions to its contribution to the prescribed boundary conditions. Contributions of the first three building blocks are readily available from Chapter 2, and they are given by Equations 2.29 to 2.37. Focussing our attention on the contribution of the last building block we write

The contribution to bending moment at the edge $\xi = 1$ of each term of Equation 4.8 is

$$\frac{M_{x}}{D_{1}} = \{(\nu \phi_{I}^{2}(n\pi)^{2} - \beta_n^2)C_{1n} + (\nu \phi_{I}^{2}(n\pi)^{2} + \gamma_n^2)D_{1n}\}\cos(n\pi \eta)$$

or if $\lambda^{2} \phi^{2} < (n\pi)^{2}$

$$= \{(\nu \phi_{I}^{2}(n\pi)^{2} - \beta_n^2)C_{2n} + (\nu \phi_{I}^{2}(n\pi)^{2} + \gamma_n^2)D_{2n}\}\cos(n\pi \eta)$$

The contribution to slope at $\xi = 0$ of each term of Equation 4.7 is

$$\beta_{I} = (A_{1n} \beta_n + B_{1n} \gamma_n)\cos(n\pi \eta)$$

or if $\lambda^{2} \phi^{2} < (n\pi)^{2}$

$$= (A_{2n} \beta_n + B_{2n} \gamma_n)\cos(n\pi \eta)$$
The contribution to bending moment at the edge \( n=1 \) of each term of Equation 4.2 is

\[
\frac{M_{n=b}}{D} = \left\{ (\nu \phi^2 (m \pi /2)^2 - \beta_m^2)C_{1m} + (\nu \phi^2 (m \pi /2)^2 - \gamma_m^2)D_{1m} \right\} \sin \frac{m \pi \xi}{2}
\]

or if \( \lambda^2 < (m \pi /2)^2 \)

\[
= \left\{ (\nu \phi^2 (m \pi /2)^2 - \beta_m^2)C_{2m} + (\nu \phi^2 (m \pi /2)^2 - \gamma_m^2)D_{2m} \right\} \sin \frac{m \pi \xi}{2}
\]

The contribution of Equation 4.1 to displacement at \( n=v \) and \( \xi=u \) is

\[
W_{u1}(u,v) = \sum_{m=1,3,5}^{k^*} (A_{1m} \cosh \beta_m v + B_{1m} \cos \gamma_m v) \sin \frac{m \pi u}{2} + \sum_{m=k^*+2}^{\infty} (A_{2m} \cosh \beta_m v + B_{2m} \cosh \gamma_m v) \sin \frac{m \pi u}{2}
\]

The coefficient matrix is generated by adding the following column and row to the matrix of Figure 2.6.

**Quadrant (1):**

For \( m=1,3,5, \ldots, 2k-1 \)

\[
A_{(m+1)/2,3k+1} = \left\{ (\nu \phi^2 (m \pi /2)^2 - \beta_m^2)C_{1m} + (\nu \phi^2 (m \pi /2)^2 + \gamma_m^2)D_{1m} \right\}
\]

or if \( \lambda^2 < (m \pi /2)^2 \)

\[
= \left\{ (\nu \phi^2 (m \pi /2)^2 - \beta_m^2)C_{2m} + (\nu \phi^2 (m \pi /2)^2 - \gamma_m^2)D_{2m} \right\}
\]
Quadrant (2):

For \( n=0,1,2, \ldots, k-1 \)

\[ A_{k+n+1,3k+1} = \{\nu^{2}\lambda(n\pi)^2 - \beta_n^2\} C_{1n} + \{\nu^{2}\lambda(n\pi)^2 + \gamma_n^2\} D_{1n} \]

or if \( \lambda^2 \phi^2 < (n\pi)^2 \)

\[ = \{\nu^{2}\lambda(n\pi)^2 - \beta_n^2\} C_{2n} + \{\nu^{2}\lambda(n\pi)^2 - \gamma_n^2\} D_{2n} \]

Quadrant (3):

For \( n=0,1,2, \ldots, k-1 \)

\[ A_{2k+n+1,3k+1} = A_1n B_1 + B_1n \gamma_n \]

or if \( \phi^2 \lambda^2 < (n\pi)^2 \)

\[ = A_2n B_2 + B_2n \gamma_n \]

Quadrant (4):

For \( m=1,3,5, \ldots, 2k-1 \)

\[ A_{(3k+1),(m+1)/2} = (\cos \theta_m v + \theta_{1m} \cos \gamma_m v) \sin (m\pi u/2) / \theta_{11m} \]

or if \( \lambda^2 < (m\pi/2)^2 \)
\[ A_{3k+1, (m+1)/2} = (\cosh \theta_m v + \theta_2 m \cosh \gamma_m v) \sin(m \pi u / 2) / \theta_{22m} \]

Quadrant (5):

For \( n = 0, 1, 2, \ldots, k - 1 \)

\[ A_{3k+1, k+n+1} = (\sinh \theta_n u + \theta_1 n \sin \gamma_n u) \cos(n \pi v) / \theta_{11n} \]

or if \( \lambda^2 \phi^2 < (n^2) \)

\[ = (\sinh \theta_n u + \theta_2 n \sin \gamma_n u) \cos(n \pi v) / \theta_{22n} \]

Quadrant (6):

For \( p = 0, 1, 2, \ldots, k - 1 \)

\[ A_{3k+1, 2k+p+1} = \cosh \theta_p (1-u) - \theta_1 p \cos \gamma_p (1-u) \cos(p \pi v) / \theta_{11p} \]

or if \( \lambda^2 \phi^2 < (p^2) \)

\[ = \cosh \theta_p (1-u) - \theta_2 p \cosh \gamma_p (1-u) \cos(p \pi v) / \theta_{22p} \]

Quadrant (7):

For \( m = 1, 3, 5, \ldots, 2k - 1 \)
\[ A_{3k+1,3k+1}^{k*} = \sum_{m=1,3,5} (A_{1m} \cosh \gamma_m v + B_{1m} \cos \gamma_m v) \sin \frac{m \pi u}{2} \]

\[ + \sum_{m=k^*+2}^{2k-1} (A_{2m} \cosh \gamma_m v + B_{2m} \cos \gamma_m v) \sin \frac{m \pi u}{2} \]

Next, using specially designed Program 13 of Appendix 3 we generate the coefficient matrix to establish those values of \( \lambda^2 \) which cause the determinant of the matrix to vanish. A vanishing determinant indicates that the associated value of \( \lambda^2 \) is an eigenvalue. Due to the fact that the coordinates of the point support of Figure 4.3 may assume any combination of real numbers between zero and one, it is physically impossible to cover eigenvalues for this infinitesimal number of possibilities. However, Appendix 3 provides Program 13 to handle any possibility one may encounter, and Program 14 will generate the corresponding mode shape data. Meanwhile, Table 4.1 lists the first three eigenvalues for eleven different plate aspect ratio where \( u \) and \( v \) assume the value \( 1/2 \). Corresponding mode shapes are also illustrated in Appendix 2.

We next turn to the antisymmetric free vibration modes of the cantilever plate with lateral point supports.
Antisymmetric Modes

We start by studying the four building blocks of Figure 4.7. It soon becomes apparent that solutions for the first three building blocks are readily available from the analysis of the antisymmetric free vibration modes of the cantilever plate of Chapter 2. Hence, we start with the analysis of the fourth building block by dividing it into two segments as shown in Figure 4.8.

The Fourth Building Block:

The block of Figure 4.8 was also used by Gorman in his analysis of the fully antisymmetric modes of the rectangular plate with symmetrically distributed point supports on the lateral surface. Hence, its solution is readily available and is repeated here for convenient reference. Using a second subscript to refer to the relevant segment of the building block we have

\[ W_{41}(\xi, \eta) = \sum_{m=1,3,5}^{k^*} (A_m \sin \beta_m \eta + B_m \sin \gamma_m \eta) \sin \frac{m \pi \xi}{2} \]  \hspace{1cm} \ldots (4.9)

\[ + \sum_{m=k^*+2}^{\infty} (A_m \sin \beta_m \eta + B_m \sin \gamma_m \eta) \sin \frac{m \pi \xi}{2} \]
Figure 4.7. Building blocks used in the solution of the antisymmetric free vibration modes of the cantilever plate with lateral point supports.
and

\[ W_{u2}(\xi, \eta) = \sum_{m=1,3,5}^{k^*} (C_{1m} \cosh \eta \, m \eta + D_{1m} \cos \gamma m \eta) \sin \frac{m \pi \xi}{2} \]

\[ + \sum_{m=k^*+2}^{\infty} (C_{2m} \cosh \eta \, m \eta + D_{2m} \cosh \gamma m \eta) \sin \frac{m \pi \xi}{2} \quad \ldots \ldots \quad (4.10) \]

where

\[ \phi_m = \phi \sqrt{\lambda^2 - (m\pi/2)^2} \]

\[ \gamma_m = \phi \sqrt{\lambda^2 - (m\pi/2)^2} \quad \text{or} \quad \phi \sqrt{(m\pi/2)^2 - \lambda^2} \]

whichever is real, following our established procedure, the first summation pertains to terms for which \( \lambda^2 > (m\pi/2)^2 \).

Figure 4.8. Fourth building block of Figure 4.7 with intersegment line parallel to the \( \xi \) axis.
Two of the original constants in Equations 4.9 and 4.10 have been eliminated by means of the boundary conditions prescribed at the edge \( n=0 \). If we represent the concentrated force by a sine Dirac function,

\[
P(\xi) = \sin\frac{m\pi u}{2}\sin\frac{m\pi \xi}{2}
\]

(4.11)

the continuity and force equilibrium equations become

\[
A_1 m \sinh \beta_m v + B_1 m \sin \gamma_m v - C_1 m \cosh \beta_m v^* - D_1 m \cos \gamma_m v^* = 0
\]

\[
A_1 m \cosh \beta_m v + B_1 m \gamma_m \cos \gamma_m v + C_1 m \beta_m \sinh \beta_m v^* - D_1 m \gamma_m \sin \gamma_m v^* = 0
\]

\[
A_1 m \beta_m^2 \sinh \beta_m v - B_1 m \gamma_m^2 \sin \gamma_m v - C_1 m \beta_m^2 \cosh \beta_m v^* + D_1 m \gamma_m^2 \cos \gamma_m v^* = 0
\]

\[
A_1 m \beta_m^3 \cosh \beta_m v - B_1 m \gamma_m^3 \cos \gamma_m v + C_1 m \beta_m^3 \sinh \beta_m v^* + D_1 m \gamma_m^3 \sin \gamma_m v^* = P \sin \frac{m\pi u}{2}
\]

and

\[
A_2 m \sinh \beta_m v + B_2 m \sin \gamma_m v - C_2 m \cosh \beta_m v^* - D_2 m \cos \gamma_m v^* = 0
\]

\[
A_2 m \beta_m \cosh \beta_m v + B_2 m \gamma_m \cos \gamma_m v + C_2 m \beta_m \sinh \beta_m v^* + D_2 m \gamma_m \sin \gamma_m v^* = 0
\]

\[
A_2 m \beta_m^2 \sinh \beta_m v + B_2 m \gamma_m^2 \sin \gamma_m v - C_2 m \beta_m^2 \cosh \beta_m v^* - D_2 m \gamma_m^2 \cos \gamma_m v^* = 0
\]

\[
A_2 m \beta_m^3 \cosh \beta_m v + B_2 m \gamma_m^3 \cos \gamma_m v + C_2 m \beta_m^3 \sinh \beta_m v^* + D_2 m \gamma_m^3 \sin \gamma_m v^* = P \sin \frac{m\pi u}{2}
\]
Solving the above equations we have

\[ A_{1m} = P \cdot \sin(m \pi u/2) \cosh(\beta_m v)/( \beta_m^2 + \gamma_m^2 ) \beta_m \cosh \beta_m \]

\[ B_{1m} = -P \cdot \cos \gamma_m v \sin(m \pi u/2)/( \beta_m^2 + \gamma_m^2 ) \gamma_m \cos \gamma_m \]

\[ C_{1m} = P \cdot \sin(m \pi u/2) \sinh \beta_m v/( \beta_m^2 + \gamma_m^2 ) \beta_m \cosh \beta_m \]

\[ D_{1m} = -P \cdot \sin \gamma_m v \sin(m \pi u/2)/( \beta_m^2 + \gamma_m^2 ) \gamma_m \cos \gamma_m \]

and

\[ A_{2m} = P \cdot \sin(m \pi u/2) \cosh \beta_m v/( \beta_m^2 - \gamma_m^2 ) \beta_m \cosh \beta_m \]

\[ B_{2m} = -P \cdot \sin(m \pi u/2) \cosh \gamma_m v/( \beta_m^2 - \gamma_m^2 ) \gamma_m \cosh \gamma_m \]

\[ C_{2m} = P \cdot \sin(m \pi u/2) \sinh \beta_m v/( \beta_m^2 - \gamma_m^2 ) \beta_m \cosh \beta_m \]

\[ D_{2m} = -P \cdot \sin(m \pi u/2) \sinh \gamma_m v/( \beta_m^2 - \gamma_m^2 ) \gamma_m \cosh \gamma_m \]

With the solution for the building block of Figure 4.8 available, it will be recalled from the solution technique for the symmetric modes that we also require a solution for this fourth building block with the segments divided by a line parallel to the \( \eta \) axis as shown in Figure 4.9. This solution may easily be inferred from the solution
of the block of Figure 4.8 which is given by Equations 4.9
and 4.10 by a simple interchange of the variables \( \xi \) and \( \eta \),
at the same time replacing subscript \( m \) by \( n \).

\[
W_{41}(\xi, \eta) = \sum_{n=1,3,5}^{K^*} \left( A_{1n} \sinh \beta_n \xi + B_{1n} \sin \gamma_n \xi \right) \sin \frac{n \pi \eta}{2} + \sum_{n=k^*+2}^{\infty} \left( A_{2n} \sinh \beta_n \xi + B_{2n} \sin \gamma_n \xi \right) \sin \frac{n \pi \eta}{2}
\]

and

\[
W_{42}(\xi, \eta) = \sum_{n=1,3,5}^{K^*} \left( C_{1n} \cosh \beta_n \xi + D_{1n} \cos \gamma_n \xi \right) \sin \frac{n \pi \eta}{2} + \sum_{n=k^*+2}^{\infty} \left( C_{2n} \cosh \beta_n \xi + D_{2n} \cosh \gamma_n \xi \right) \sin \frac{n \pi \eta}{2}
\]

Figure 4.9. Fourth building block of Figure 4.7
with intersegment line running parallel to the \( \eta \) axis.
where

\[ \beta_n = \frac{1}{\phi} \sqrt{\lambda^2 - (\pi/2)^2 (n\pi/2)^2} \]
\[ \gamma_n = \frac{1}{\phi} \sqrt{\lambda^2 + (n\pi/2)^2} \] or \[ \frac{1}{\phi} \sqrt{(n\pi/2)^2 - \lambda^2} \] whichever is real, and

\[ A_{1n} = P \sin(n\pi v/2) \cosh \beta_n u^* / (\beta_n^2 + \gamma_n^2) \beta_n \phi^4 \cosh \beta_n \]
\[ B_{1n} = -P \cos \gamma_n u \sin(n\pi v/2) / (\beta_n^2 + \gamma_n^2) \gamma_n \phi^4 \cos \gamma_n \]
\[ C_{1n} = P \sin(n\pi v/2) \sinh \gamma_n u / (\beta_n^2 + \gamma_n^2) \beta_n \phi^4 \cosh \gamma_n \]
\[ D_{1n} = -P \sin \gamma_n u \sin(n\pi v/2) / (\beta_n^2 + \gamma_n^2) \gamma_n \phi^4 \cos \gamma_n \]

and

\[ A_{2n} = P \sin(n\pi v/2) \cosh \beta_n u^* / (\beta_n^2 - \gamma_n^2) \beta_n \phi^4 \cosh \beta_n \]
\[ B_{2n} = -P \sin(n\pi v/2) \cosh \gamma_n u^* / (\beta_n^2 - \gamma_n^2) \gamma_n \phi^4 \cosh \gamma_n \]
\[ C_{2n} = P \sin(n\pi v/2) \sinh \beta_n u / (\beta_n^2 - \gamma_n^2) \beta_n \phi^4 \cosh \beta_n \]
\[ D_{2n} = -P \sin(n\pi v/2) \sinh \gamma_n u / (\beta_n^2 - \gamma_n^2) \gamma_n \phi^4 \cosh \gamma_n \]
Contributions of the fourth building block to the prescribed boundary conditions are:

The contribution of each term of Equation 4.12 to bending moment at $\xi = 1$ is

$$\frac{M_f \phi}{D} = \{C_{1n} (\nu \phi_1^2 (n\pi/2)^2 - \beta_n^2) + D_{1n} (\nu \phi_1^2 (n\pi/2)^2 + \gamma_n^2)\} \sin \frac{n\pi n}{2},$$

or if $\phi^2 \lambda^2 < (n\pi/2)^2$

$$= \{C_{2n} (\nu \phi_1^2 (n\pi/2)^2 - \beta_n^2) + D_{2n} (\nu \phi_1^2 (n\pi/2)^2 - \gamma_n^2)\} \sin \frac{n\pi n}{2}.$$

The contribution of each element of Equation 4.11 to slope at $\xi = 0$ is

$$W_{n1}(\xi, n) = (A_{1n} \beta_n + B_{1n} \gamma_n) \sin \frac{n\pi n}{2},$$

or if $\lambda^2 \phi^2 < (n\pi/2)^2$

$$= (A_{2n} \beta_n + B_{2n} \gamma_n) \sin \frac{n\pi n}{2}.$$

The contribution of each term of Equation 4.10 to bending moment along the edge $n=1$ is

$$\frac{M_m \phi}{D} = \{C_{1m} (\nu \phi_2^2 (m\pi/2)^2 - \beta_m^2) + D_{1m} (\nu \phi_2^2 (m\pi/2)^2 + \gamma_m^2)\} \sin \frac{m\pi \xi}{2}.$$
or if \( \lambda^2 < (m \pi/2)^2 \)

\[
\frac{M_{m} \phi_{b}}{D} = (C_{2m}(\nu \phi^2(m \pi/2)^2 - \beta_{m}^2) + D_{2m}(\nu \phi^2(m \pi/2)^2 - \gamma_{m}^2)) \sin \frac{m \pi \xi}{2}
\]

The contribution of Equation 4.11 to displacement at the point support \((u, v)\) is

\[
W_{u1}(u, v) = \sum_{n=1}^{k^*} (A_{1n} \sinh \xi_n u + B_{1n} \sin \gamma_n u) \sin \frac{nv}{2} + \sum_{n=k^*+2}^{\infty} (A_{2n} \sinh \xi_n u + B_{2n} \sin \gamma_n u) \sin \frac{nv}{2}
\]

The coefficient matrix may now be generated by adding the following column and row to the matrix of Figure 2.11.

Quadrant (1):

For \( m=1, 3, 5, \ldots, 2k-1 \)

\[
\hat{A}_{(m+1)/2, 3k+1} = C_{1m}(\nu \phi^2(m \pi/2)^2 - \beta_{m}^2) + D_{1m}(\nu \phi^2(m \pi/2)^2 + \gamma_{m}^2)
\]

or if \( \lambda^2 < (m \pi/2)^2 \)

\[
\hat{A}_{(m+1)/2, 3k+1} = C_{2m}(\nu \phi^2(m \pi/2)^2 - \beta_{m}^2) + D_{2m}(\nu \phi^2(m \pi/2)^2 - \gamma_{m}^2)
\]
Quadrant (2):

For $n=1, 3, 5, \ldots, 2k-1$

$$A_{k+(n+1)/2, 3k+1} = C_1 \left( \nu \phi (n\pi/2)^2 - \beta_n^2 \right) + D_1 \left( \nu \phi^2 (n\pi/2)^2 + \gamma_n^2 \right)$$

or if $\lambda^2 \phi^2 < (n\pi/2)^2$

$$= C_2 \left( \nu \phi (n\pi/2)^2 - \beta_n^2 \right) + D_2 \left( \nu \phi^2 (n\pi/2)^2 + \gamma_n^2 \right)$$

Quadrant (3):

For $n=1, 3, 5, \ldots, 2k-1$

$$A_{2k+(n+1)/2, 3k+1} = A_1 \beta_n + B_1 \gamma_n$$

or if $\lambda^2 \phi^2 < (n\pi/2)^2$

$$= A_2 \beta_n + B_2 \gamma_n$$

Quadrant (4):

For $m=1, 3, 5, \ldots, 2k-1$

$$A_{3k+1, (m+1)/2} = (\theta_m \sinh \beta_m n + \sin \gamma_m n) \sin (m\pi u/2)/\theta_1$$

or if $\lambda^2 < (m\pi/2)^2$
\[ A_{3k+1, (m+1)/2} = (\theta_{2m} \sinh \theta_{m} v + \sinh \theta_{m} v) \sin(m\pi u/2)/\theta_{22m} \]

Quadrant (5):

For \( n=1, 3, 5, \ldots, 2k-1 \)

\[ A_{3k+1, k+(n+1)/2} = (\theta_{1n} \sinh \theta_{n} u + \sinh \theta_{n} u) \sin(n\pi v/2)/\theta_{11n} \]

or if \( \lambda^2 \phi^2 < (n\pi/2)^2 \)

\[ = (\theta_{2n} \sinh \theta_{n} u + \sinh \theta_{n} u) \sin(n\pi v/2)/\theta_{22n} \]

Quadrant (6):

For \( p=1, 3, 5, \ldots, 2k-1 \)

\[ A_{3k+1, 2k+(p+1)/2} = (\cosh \theta_{p} (1-u) - \theta_{1p} \cos \theta_{p} (1-u)) \sin(p\pi v/2)/\theta_{11p} \]

or if \( \lambda^2 \phi^2 < (p\pi/2)^2 \)

\[ = (\cosh \theta_{p} (1-u) - \theta_{2p} \cosh \theta_{p} (1-u)) \sin(p\pi v/2)/\theta_{22p} \]

Quadrant (7)

For \( n=1, 3, 5, \ldots, 2k-1 \)
\[ A_{3k+1,3k+1} = \sum_{n=1,3,5}^{k^*} \left( A_{1n} \sinh \beta_n u + B_{1n} \sin \gamma_n u \right) \sin n \pi v \frac{\sin \frac{n \pi v}{2}}{2} \]

\[ + \sum_{n=k^*+2}^{2k-1} \left( A_{2n} \sinh \beta_n u + B_{2n} \sin \gamma_n u \right) \sin n \pi v \frac{\sin \frac{n \pi v}{2}}{2}. \]

Using the above information, Program 15 of Appendix 3 was designed to generate eigenvalues of any cantilever plate with lateral point supports symmetrically located with respect to the \( \xi \) axis as shown in Figure 4.2. These eigenvalues are obtained simply by feeding Program 15 to a digital computer with the plate aspect ratio and its point support coordinate \( u \) and \( v \). As an example, eigenvalues for the first three antisymmetric modes of the cantilever plate of different aspect ratios, and point support coordinate of \((1/2, 1/2)\) are listed in Table 4.2. Corresponding mode shape data are obtained using Program 16 of Appendix 3. Mode shapes corresponding to eigenvalues of Table 4.2 are illustrated in Appendix 2.

**Results and Discussion**

At the beginning of this work we mentioned that most of the published data and research work relating to the problem of plate vibrations up to the year 1965 were grouped
\[ \lambda^2 = \omega a^2 \sqrt{\rho / D} \quad \nu = 0.333 \quad u = v = 0.5 \]

<table>
<thead>
<tr>
<th>( \phi = 2b/a )</th>
<th>1/3</th>
<th>1/2.5</th>
<th>1/2</th>
<th>1/1.5</th>
<th>1/1.25</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi' = b/a )</td>
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<td>1/5</td>
<td>1/4</td>
<td>1/3</td>
<td>1/2.5</td>
<td>1/2</td>
<td>5/8</td>
<td>3/4</td>
<td>1</td>
<td>1.25</td>
<td>1.5</td>
</tr>
<tr>
<td>2</td>
<td>59.52</td>
<td>59.64</td>
<td>59.62</td>
<td>51.66</td>
<td>37.20</td>
<td>26.07</td>
<td>18.43</td>
<td>14.26</td>
<td>10.11</td>
<td>8.207</td>
<td>7.114</td>
</tr>
<tr>
<td>3</td>
<td>86.10</td>
<td>85.37</td>
<td>81.05</td>
<td>80.24</td>
<td>60.60</td>
<td>52.58</td>
<td>42.89</td>
<td>36.74</td>
<td>27.42</td>
<td>21.51</td>
<td>18.00</td>
</tr>
</tbody>
</table>

Table 4.1. First three symmetric mode eigenvalues for the cantilever plate with symmetrically located lateral point supports.

<table>
<thead>
<tr>
<th>( \phi = 2b/a )</th>
<th>1/3</th>
<th>1/2.5</th>
<th>1/2</th>
<th>1/1.5</th>
<th>1/1.25</th>
<th>1</th>
<th>1.25</th>
<th>1.5</th>
<th>2</th>
<th>2.5</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi' = b/a )</td>
<td>1/6</td>
<td>1/5</td>
<td>1/4</td>
<td>1/3</td>
<td>1/2.5</td>
<td>1/2</td>
<td>5/8</td>
<td>3/4</td>
<td>1</td>
<td>1.25</td>
<td>1.5</td>
</tr>
<tr>
<td>Mode 1</td>
<td>41.26</td>
<td>35.02</td>
<td>28.76</td>
<td>22.41</td>
<td>19.19</td>
<td>15.93</td>
<td>13.27</td>
<td>11.45</td>
<td>9.065</td>
<td>7.579</td>
<td>6.599</td>
</tr>
<tr>
<td>2</td>
<td>106.9</td>
<td>96.15</td>
<td>85.58</td>
<td>73.45</td>
<td>61.93</td>
<td>46.73</td>
<td>34.62</td>
<td>27.37</td>
<td>19.81</td>
<td>16.08</td>
<td>13.76</td>
</tr>
<tr>
<td>3</td>
<td>141.0</td>
<td>122.9</td>
<td>103.7</td>
<td>82.02</td>
<td>73.98</td>
<td>68.21</td>
<td>56.09</td>
<td>45.54</td>
<td>34.57</td>
<td>28.90</td>
<td>23.96</td>
</tr>
</tbody>
</table>

Table 4.2. Eigenvalues for the first three antisymmetric free vibration modes of the cantilever plate with lateral point supports.
together and presented in a single reference thanks to Leissa(19). Leissa's book lists a good number of references relating to the analysis of the rectangular cantilever plate. He also lists their published data most of which agrees favorably with the results of Gorman's work(20) relating to the cantilever plate problem. Gorman's results were listed in Tables 2.3 and 2.4 for convenient reference. In Chapter two we have compared these results with our own of Tables 3.1 and 3.2 and found a near total agreement. Leissa's book also lists a good number of references relating to the analysis of point supported plates, none of which dealt with the point supported cantilever plate. Although, a more recent literature survey shows that researchers and scientists are still devoting time and effort to the analysis of the point supported plates including Gorman(24) and G. Venkateswara Rao, I.S. Raju and C.L. Amba-Rao(25), none of them has dealt with the cantilever type plates. Therefore there is no apparent published data with which one may compare in order to test the validity of one's results. However, one can think of numerous options open to test these results, some of which will be explored in the following.

Focussing our attention on Figure 4.3, it is readily seen that by removing the third building block we would end
up with three building blocks that could be used to analyze modes symmetric with respect to the $\xi$ axis and antisymmetric with respect to the $\eta$ axis of the rectangular plate with symmetrically distributed point supports, readily analyzed by Gorman(20). With this in mind, the following test was conducted. The effects of the third building block of Figure 4.3 was eliminated by dropping rows and columns representing these effects from the appropriate coefficient matrix. Eigenvalues for plates similar to those analyzed by Gorman were generated and general agreement was found.

We next turn to Figure 4.7, where by dropping the third building block we end up with a set of building blocks that can be used to analyze the fully antisymmetric modes of the rectangular plate with symmetrically distributed point supports on the lateral surface, also analyzed by Gorman. Therefore, effects of the third building block of Figure 4.7 were removed by dropping out appropriate rows and columns from the coefficient matrix. Relevant eigenvalues were generated, and when compared with existing results, general agreement was also found.

In the above two tests we have checked and compared our derivations and programming of the present chapter. We
now turn to check for those of Chapter 3 as well. A close
look at Figures 4.3 and 4.7 shows that by decreasing the
value of the dimensionless quantity $u$, and as $u$ becomes
closer and closer to zero, at the limit we should end up
with the same eigenvalues as those of the cantilever plate,
and that proved to be the case. The same was found for the
plate of Figure 3.1 as $\tau$ became close to zero. Furthermore,
it is clearly seen that if the dimensionless quantity $u$ of
Figure 4.3 and 4.7 was to be increased so as to become very
close to unity, at the limit we expect the same eigenvalues
as those of Figure 3.2 That was also proven to be the case.
CONCLUSION

One of the most commonly used structures in the industrialized world is the rectangular plate. In many design problems, specifications merely ensuring that plates will withstand applied static loads prove to be inadequate. It is for this reason that analysis of the free vibration of plates has been and still is gaining momentum. The cantilever plate with symmetrically distributed point supports constitutes one of the classical plate free vibration problems. It has many industrial applications, as well as applications in civil engineering structures. Analysis of this type of plate has traditionally presented the analyst with serious difficulties because of the formidable problem of trying to satisfy the free edge conditions and the condition of zero displacement at the point supports.

In this paper, such difficulties were obviated, and a highly accurate analytical type solution was provided for the first time for this particular engineering problem. Through exploitation of the method of superposition, and techniques developed by Gorman(21), a solution for the above
problem was obtained by superimposing a number of solutions that were obtained by classical methods. Unlike numerical and other approximate solutions, the analytical solution satisfies exactly the differential equation throughout the plate. It also satisfies all boundary conditions to any desired degree of accuracy.

Highly accurate eigenvalues and mode shapes have been tabulated for both the symmetric and antisymmetric modes of a wide selection of plates with a value of Poisson's ratio equal 0.333. All of the results reported here were computed with a view to providing accuracy to four significant digits in the eigenvalues. Greater accuracy is easily obtained simply by increasing the number of terms used in the appropriate series involved. After conducting numerous convergence tests of the type described in Chapter 2, it was recognized that thirty term expansions would exceed the number of terms required for many plate configurations and modes. But in order to take care of the most demanding cases, it was decided to go with thirty terms throughout, in the case of point supported plates. However, seven term expansions proved to be adequate for most plate configurations and modes of the cantilever plate. It was decided to go with fifteen terms in this case to satisfy the most demanding cases.
A study of the literature reveals that exact or highly accurate solutions have been obtained for many rectangular thin-plate free vibration problems including the rectangular cantilever plate. However, to the author's knowledge, this proves not to be the case for the problem at hand. Consequently, no comparison of results is possible. But, eigenvalues generated were carefully examined using various techniques such as those discussed at the end of Chapter 4. Mode shape data were also generated and carefully analyzed using specially designed computer programs of Appendix 3.

The analytical solution presented herein may be considered as an improvement and an extension to the solution of the cantilever plate of Reference (20). In this paper, rejection modes introduced by the solution of the above reference were eliminated. Furthermore, point supports were introduced. A highly accurate analytical solution was provided using a technique already described in the literature. This mathematical technique is not limited in application to problems with symmetrically distributed point supports. Its application could be extended to any combination of point supports along the edges or the lateral surface by prudently choosing a sufficient number of building blocks to
satisfy the boundary conditions. The only requirement is that these boundary conditions and the differential equation do not introduce any non-linearities. In fact, a solution for the free vibration problem of the cantilever plate with four symmetrically distributed point supports, two along the edges parallel to the $\xi$ axis and two along the edge opposite to the clamped base, can easily be obtained by superimposing appropriate building blocks already discussed in this thesis. As indicated earlier, a search of the literature has failed to provide eigenvalue and mode shape information for this important industrial problem. A main goal of this paper is to fill this serious gap in the technical literature.
REFERENCES


(10). BOBNOV, I.G., Trudy po teorii plastin (Contributing Works to Plate Theory), Gos. Izd.-vo Techn.-Teoret. Lytry, Moscow 1953.


Appendix 1

In this Appendix a list of integrals is reproduced from Reference (20) for convenient reference. This list, although brief, it is sufficient for all the integrations required in this thesis.

\[ \int_0^1 \cosh \beta \xi \cos m \pi \xi \, d\xi = \frac{\beta \cos m \pi \sinh \beta}{\beta^2 + (m \pi)^2} \quad (A1.1) \]

\[ \int_0^1 \sinh \beta \xi \cos m \pi \xi \, d\xi = \frac{\beta}{\beta^2 + (m \pi)^2} (\cosh \beta \cos m \pi - 1) \quad (A1.2') \]

\[ \int_0^1 \cos \beta \xi \cos m \pi \xi \, d\xi = \frac{\beta \sin \beta \cos m \pi}{\beta^2 - (m \pi)^2} \quad (A1.3) \]

\[ \int_0^1 \sin \beta \xi \cos m \pi \xi \, d\xi = \frac{\beta}{\beta^2 - (m \pi)^2} (1 - \cos \beta \cos m \pi) \quad (A1.4) \]

\[ \int_0^1 \cosh \beta \xi \sin \frac{m \pi \xi}{2} \, d\xi = \frac{m \pi/2 + \beta \sinh \beta \sin m \pi/2}{\beta^2 + (m \pi/2)^2} \quad (A1.5) \]

\[ \int_0^1 \sinh \beta \xi \sin \frac{m \pi \xi}{2} \, d\xi = \frac{\beta \cosh \beta \sin m \pi/2}{\beta^2 + (m \pi/2)^2} \quad (A1.6) \]

\[ \int_0^1 \cos \beta \xi \sin \frac{m \pi \xi}{2} \, d\xi = \frac{m \pi/2 - \beta \sin(m \pi/2) \sin \beta}{(m \pi/2)^2 - \beta^2} \quad (A1.7) \]

\[ \int_0^1 \sin \beta \xi \sin \frac{m \pi \xi}{2} \, d\xi = \frac{-\beta \cos \beta \sin m \pi/2}{\beta^2 - (m \pi/2)^2} \quad (A1.8) \]

Equation (A1.6) is valid for \( m=1,3,5, \ldots \) only.

\[ \int_0^1 \cosh(\beta(1-\xi)) \sin \frac{m \pi \xi}{2} \, d\xi = \frac{(m \pi/2) \cosh \beta}{\beta^2 + (m \pi/2)^2} \quad (A1.9') \]
\begin{align}
\int_0^1 \sinh(\beta(1-\xi)) \sin \frac{m\pi \xi}{2} \, d\xi &= \frac{(m\pi/2) \sinh \beta - \beta \sin(m\pi/2)}{\beta^2 + (m\pi/2)^2} \tag{A1.10} \\
\int_0^1 \cos(\beta(1-\xi)) \sin \frac{m\pi \xi}{2} \, d\xi &= \frac{(m\pi/2) \cos \beta - \beta \cos(m\pi/2)}{\beta^2 - (m\pi/2)^2} \tag{A1.11} \\
\int_0^1 \sin(\beta(1-\xi)) \sin \frac{m\pi \xi}{2} \, d\xi &= \frac{\beta \sin(m\pi/2) - (m\pi/2) \sin \beta}{\beta^2 - (m\pi/2)^2} \tag{A1.12} \\
\end{align}

Equations (A1.9) and (A1.11) are valid for \( m = 1, 3, 5, \ldots \) only.

\begin{align}
\int_0^1 \sin \beta \xi \sin m\pi \xi \, d\xi &= \frac{-m\pi \cos m\pi \sinh \beta}{\beta^2 + (m\pi)^2} \tag{A1.13} \\
\int_0^1 \cosh \beta \xi \sin m\pi \xi \, d\xi &= \frac{m\pi(1-\cos m\pi \cosh \beta)}{\beta^2 + (m\pi)^2} \tag{A1.14} \\
\int_0^1 \sin \beta \xi \sin m\pi \xi \, d\xi &= \frac{m\pi \cos m\pi \sin \beta}{\beta^2 - (m\pi)^2} \tag{A1.15} \\
\int_0^1 \cos \beta \xi \sin m\pi \xi \, d\xi &= \frac{m\pi(1-\cos m\pi \cos \beta)}{(m\pi)^2 - \beta^2} \tag{A1.16} \\
\int_0^1 \sinh(\beta(1-\xi)) \sin m\pi \xi \, d\xi &= \frac{m\pi \sinh \beta}{(m\pi)^2 + \beta^2} \tag{A1.17} \\
\int_0^1 \cosh(\beta(1-\xi)) \sin m\pi \xi \, d\xi &= \frac{m\pi(\cosh \beta - \cos m\pi)}{\beta^2 + (m\pi)^2} \tag{A1.18} \\
\int_0^1 \sin(\beta(1-\xi)) \sin m\pi \xi \, d\xi &= \frac{m\pi \sin \beta}{(m\pi)^2 - \beta^2} \tag{A1.19} \\
\int_0^1 \cos(\beta(1-\xi)) \sin m\pi \xi \, d\xi &= \frac{m\pi(\cos \beta - \cos m\pi)}{(m\pi)^2 - \beta^2} \tag{A1.20} \\
\int_0^1 \cosh(\beta(1-\xi)) \cos m\pi \xi \, d\xi &= \frac{\beta \sinh \beta}{\beta^2 + (m\pi)^2} \tag{A1.21} \\
\int_0^1 \sinh(\beta(1-\xi)) \cos m\pi \xi \, d\xi &= \frac{\beta(\cosh \beta - \cos m\pi)}{\beta^2 + (m\pi)^2} \tag{A1.22} \\
\end{align}
\begin{align}
\int_0^1 \cos(\beta(1-\xi)) \cos \pi \xi d\xi &= \frac{\beta \sin \beta}{\beta^2 - (m \pi)^2} \quad (A1.23) \\
\int_0^1 \sin(\beta(1-\xi)) \cos \pi \xi d\xi &= \frac{\beta(\cos \pi - \cos \beta)}{\beta^2 - (m \pi)^2} \quad (A1.24)
\end{align}
FREE VIBRATION ANALYSIS

OF RECTANGULAR CANTILEVER PLATES

WITH SYMMETRICALLY DISTRIBUTED POINT SUPPORTS

VOLUME II
Appendix 2

MODAL SHAPES

This Appendix contains a comprehensive computer mode shape printouts illustrating both symmetric and antisymmetric mode shapes for the cantilever plates discussed in Chapters 2, 3, and 4. In these illustrations (PHI) indicates the full plate aspect ratio divided by two. U and V indicate the point support coordinates on the lower half of the cantilever plate as explained in the list of symbols. It also must be noted that in these illustrations only the lower half of the plate is shown.

The original computer printouts show three different colors, black, blue, and red. Where black indicates positive displacements, blue indicates zero displacements, and red indicates negative displacements. In future copies these displacements are indicated by (+), (0), and (-) respectively.
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER BEAM

THIRD ANTISYMMETRIC MODE WITH PHI = 180
WAVE PLATE GRAPHIC

********** THE RECTANGULAR CANTILEVER PLATE **********

******** FOURTH ANTISYMMETRIC MODE WITH PHI = 180 ********

[Diagram of wave plate with contour lines and axis labels]
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

FIFTH ANTISYMMETRIC MODE WIDTH 1.6
WAVE PLATE GRAPHIC

******* THE RECTANGULAR CANTILEVER PLATE *******

******* FIRST ANTSYMMETRIC MODE WITH PHI - 74 *******
WAVE PLATE GRAPHICS

THE RECTANGULAR CANTILEVER PLATE

SECOND ANISOTROPIC MODE WITH PHASE

Diagram showing the wave plate graphic with contour lines and phase indications.
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

THIRD ANTISYMMETRIC MODE WITH THE +, −, +
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

FIRST ANTISYMMETRIC MODE WITH PHI = 45°
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

SECOND ANTISYMMETRIC MODE WITH PHI = 90°
WAVE PLATE GRAPHIC

****** THE RECTANGULAR CANTILEVER PLATE ******

****** FOURTH ANTISYMMETRIC MODE WITH PHI = 45° ******
WAVE PLATE GRAPHIC

*********** THE RECTANGULAR CANTILEVER PLATE ***********

******** FIFTH ANTISYMMETRIC MODE WITH PHI - 30° **********
THE RECTANGULAR CANTILEVER PLATE
SECOND HARMONIC VIBRATION WITH OUT-OF-PLANE
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

FIRST ANTISYMMETRIC MODE WITH PHI
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

SECOND ANTISYMMETRIC MODE WITH \phi = 0
WAVE PLATE GRAPHIC

THE RECTANGULAR CANCELLER PLATE

THIRD ANTISYMMETRIC MODE WITH φ = 11
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

SECOND ANTISYMMETRIC MODE WITH \( \theta = \frac{\pi}{4} \)
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

FIRST SYMMETRIC MODE WITH PHI = 1/6
WAVE PLATE GRAPHIC

*********** THE RECTANGULAR CANTILEVER PLATE ***********

*********** THIRD SYMMETRIC MODE WITH PHI = 1/6 ***********
WAVE PLATE GRAPHIC

*********** THE RECTANGULAR CANTILEVER PLATE ***********

*********** FOURTH SYMMETRIC MODE WITH PHI = 1/6 ***********
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

FIFTH SYMMETRIC MODE WITH PHI = 1/6
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

FIRST SYMMETRIC MODE

∠ PHI = 1/4

\[ \theta \]

\[ \phi \]

X AXIS

Y AXIS

0 10 20 30 40 50
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

THIRD SYMMETRIC MODEL WITH PHI = 1/4

Y AXIS: X = 0
X AXIS: Y = 0.05
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

FIRST SYMMETRIC MODE WITH REL. - 1/2
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

SECOND SYMMETRIC MODE WITH PHASE 90°
WAVE PLATE GRAPHIC

******* THE RECTANGULAR CANTILEVER PLATE *******

******* FOURTH SYMMETRIC MODE WITH PHI = 0/2 *******
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

FIRST SYMMETRIC MODE WITH \phi_1 = 3.4

[Diagram of wave plate graphic with labeled axes and a linear function graph]
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

SECOND SYMMETRIC MODE WITH PHI = 3.6°
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

THIRD SYMMETRIC MODE WITH PHI = 3/4
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

FIRST SYMMETRIC MODE WITH PHI
WAVE PLATE GRAPHIC

- THE RECTANGULAR CANTILEVER PLATE -
- SECOND SYMMETRIC MODE WITH PUT -
THE Rectangular CANTILEVER PLATE

THIRD SYMMETRIC MODE WITH INCREASED CURVATURE
WAVE PLATE GRAPHICS

THE TRIANGULAR HOMOGENEOUS PLATE

FOURTH SYMMETRIC MODE WITH PATH
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

SECOND SYMMETRIC MODE WITH PHI = 3/2
WAVE PLATE GRAPHIC

THE RECTANGULAR CANTILEVER PLATE

THIRD SYMMETRIC MODE WITH AN L = 1

[Diagram showing wave patterns on a rectangular plate with labeled axes and contours indicating the third symmetric mode.]
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

FIRST SYMMETRIC MODE WITH PHI = 1/5, U = 1/2; V = 1/1
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGES, POINT SUPPORT ***

SECOND SYMMETRIC MODE WITH \( \Phi_1 = 1/5, \ J = 1/2, \ V = 1/1 \)

AXIS

AXIS
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

THIRD SYMMETRIC MODE WITH PHI = 1/5, U = 1/2, V = 1/1
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

FIRST SYMMETRIC MODE V Y = 0.14, U = 0.40, V =
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT-SUPPORT ***
SECOND SYMMETRIC MODE WITH PHI = \pi/4, \theta = \pi/2, V = 1/1
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LOCK POINT SUPPORT ***

THIRD SYMMETRIC MODE WITH PHI = 1/4, \( u = 1/2, v = 1/1 \)
WAVE PLATE GRAPHIC

---

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

THIRD SYMMETRIC MODE WITH PHI = 1/2ϕ, ϕ = 3ϕ/2, θ = ϕ/2, V = 1/1

---

AXIS X

---

SIXS AXI
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

FIRST SYMMETRIC MODE WITH $\phi = \frac{1}{2}$, $u = \frac{1}{2}$, $v = \frac{1}{4}$.
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

SECOND SYMMETRIC MODE WITH PHI = \( \frac{1}{2} \), U = \( \frac{1}{2} \), V = \( \frac{1}{3} \)
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

FIRST SYMMETRIC MODE WITH $\phi = 0$, $x = x_0$.
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE BOUNDARY SUPPORT...
SECOND SYMMETRIC MODE WITH THE POINTS (0.1, 0.1, 0.1, 0.1)
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE, POINT SUPPORT ***

THIRD SYMMETRIC MODE WITH Θ = 5.16, R = 1.02, η = 1.74
THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

FIRST SYMMETRIC MODE WITH PHI = 0.00
WAVE PLATE GRAPHICS

THE CARRIER PLATE WITH X-AXIS POINT SUPPORT
THIRD SYMMETRY WITH THE X-AXIS
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

SECOND SYMMETRIC MODE WITH PHI = 90°, 0°.
WAVE PLATE GRAPHIC

The WAVE PLATE with T-GOS point support:
A third, symmetric mode with parameters 0.45, 0.75.
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

FIRST ANTISYMMETRIC MODE WITH PHI = 1/6, J = 3, V = 1/1
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT
SECOND ANTISYMMETRIC MODE WITH PHI = 1/6, U = 3/2, V = 1/1
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***
THIRD ANTISYMMETRIC MODE WITH PHI = 1/6, U = 1/2, V = 1/1
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

FIRST ANTISYMMETRIC MODE WITH PHI = 1.74, U = .72, V = 1/1
WAVE PLATE GRAPHIC

SECOND ANTISYMMETRIC MODE WITH PHI = 1/4, 0, 1/2, V = 1/1

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***
FIRST ASYMMETRIC MODE WITH PHI = 1/2, U = 1/2, V = 1/1
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ---

THIRD ASYMMETRIC MODE WITH PHI = 1/2, u = 1/2, v = 1/2
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ---

FIRST ANTISYMMETRIC MODE WITH PHI = 3.4, Q = 1.2.
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

THIRD ANTISYMMETRIC MODE WITH PHI = \( \frac{\pi}{4} \), \( \phi = \frac{\pi}{2} \).
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT
FIRST ANTSYMMETRIC MODE WITH PHI = 90, Z = 0.2
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

SECOND ANTISYMMETRIC MODE WITH PHI = M/2, J = 1/2
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE BOUNDARY SUPPORT

THIRD ANTISYMMETRIC MODE WITH W/2 = 0.18, L = 0.18
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE SUPPORT

FIRST ANTISYMMETRIC MODE WITH PHI = 3/2, b = 1/2, c = -

Diagram showing a wave plate graphic with contour lines and axis labels.
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

SECOND ANTISYMMETRIC MODE WITH PH: 3/2, U: -1/2, V: 1/2

-X AXIS-
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

THIRD ANTISYMMETRIC MODE WITH PHI = 3/2, U = -1/2, V = -1/1.
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

FIRST SYMMETRIC MODE WITH PHI = 1/6, J = -1/4, V = -1/2
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***
SECOND SYMMETRIC MODE WITH PHI = 1/6, \( \phi = 1/4 \), \( V = 1/2 \)
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ---

THIRD SYMMETRIC MODE WITH PHI = 4/6, J = 1/6, V = 1/2
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ---

FIRST SYMMETRIC MODE WITH PHI = 175, S = 171. V = 74
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***
SECOND SYMMETRIC MODE WITH PHI = 1/3, \( \phi = \pi/2, \theta = \pi/2 \).
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

THIRD SYMMETRIC MODE WITH PHI = 1/3, \theta = \pi/1; \nu = \pi/2.
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***
FIRST SYMMETRIC MODE WITH PHI = 1/4, U = 1/1, V = 1/2.
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***
SECOND SYMMETRIC MODE WITH PHI = 1/4, U = 1/4, V = 1/2
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ---
FIRST SYMMETRIC MODE WITH PHI = 1/3, \( \phi = 0 \), \( \lambda = n/2 \)
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ---
SECOND SYMMETRIC MODE WITH $\phi = 1/3$, $\psi = 1/1$, $\nu = 1/2$
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ---

THIRD SYMMETRIC MODE WITH PHI = 1/3, x = ..., y = ...
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

FIRST SYMMETRIC MODE WITH PHI = 1/2, θ = 1/15, v = 1/2
WAVE PLATE GRAPHIC

- THE CANTILEVER PLATE WITH EDGE POINT SUPPORT -
SECOND SYMMETRIC MODE WITH PHI - 1/2.5, J - 1/1, V - 1/2
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

THIRD SYMMETRIC MODE WITH PHI = 1/2, J = 1/4, V = 1/2
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

FIRST SYMMETRIC MODE WITH PHI - 1/2, U - 1/1, V - 1/2
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

SECOND SYMMETRIC MODE WITH PHI - 1/2, U - 1/1, V - 1/2
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

THIRD SYMMETRIC MODE WITH $C_{11} = 1/2$, $U = 1/1$, $V = 1/2$.
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ---

FIRST SYMMETRIC MODE WITH PHI = 5/6, U = 1/2, V = 1/2.
WAVE PLATE GRAPHIC

- THE CANTILEVER PLATE WITH EDGE POINT SUPPORT -

SECOND SYMMETRIC MODE WITH PH = 5/8, U = 1, I = 1/2
WAVE PLATE GRAPHIC

- THE CANTILEVER PLATE WITH EDGE POINT SUPPORT -

THIRD SYMMETRIC MODE WITH PHI = 5/8, U = 1/1, V = 1/2
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

FIRST SYMMETRIC MODE WITH PH1 = 3 x 14, 0 = 0
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ---

SECOND SYMMETRIC MODE WITH \( \psi = 3.74, \phi = 0^\circ, \gamma = 1.72 \)
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

THIRD SYMMETRIC MODE WITH PHI = 3/4.
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH FREE EDGE SUPPORT

FIRST SYMMETRIC MODE WITH PHI = 0

-X AXIS-
WAVE PLATE GRAPHIC

THREE-LEVEL PLATE WITH EDGE PxY POINT SUPPORT.
SECOND SYMMETRIC MODEL WTH THE - X AXIS.-
THE NATURAL FREQUENCY OF A SLAB ON POINT SUPPORT

FIRST SYMMETRIC MODE WITH $\phi = 0.25$
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE PLATE SUPPORT ***

SECOND SYMMETRIC MODE WITH PHI = 1.20 deg - 11.7 deg
WAVE PLATE GRAPHIC

THIRD SYMMETRIC MODE WITH Q = 1.25 U = 1.0 V = 0.0
THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

FIRST SYMMETRIC MODE WITH PHI = 3/2
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

SECOND SYMMETRIC MODE WITH PHI = 3/4, 0, 5/4 ... 3/2
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

FIRST ANTISYMMETRIC MODE WITH PHI - FR. 0 - 1/1, 1 - 1/2
WAVE PLATE GRAPHIC
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH JIGGLED POINT SUPPORT ---

THIRD ANTISYMMETRIC MODE WITH PHI = 1/6, 0 = 71/3, V = 1/3

--- GRAPH ---
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

FIRST ANTISYMMETRIC MODE WITH \( \psi_1 = \frac{1}{\sqrt{3}} \), \( \psi_2 = \frac{1}{\sqrt{3}} \).
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***
SECOND ANTISYMMETRIC MODE WITH PHI = 1/2, 0 = 0.411, X = 0.12
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE-POINT SUPPORT ***

THIRD ANTISYMMETRIC MODE WITH \( c = 0.75, v = 0.71, \gamma = 0.72 \)
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH EDGE POINT SUPPORT

SECOND ANTISYMMETRIC MODE WITH θ=1.4, c=0.4, v=0.72
WAVE PLATE GRAPHIC

...THE CANTILEVER PLATE WITH EDGE POINT SUPPORT...

THIRD, ANTISYMMETRIC MODE WITH \( \pi \), \( \phi = \pi / 4 \), \( \gamma = \pi / 2 \)
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***
FIRST ASYMMETRIC MODE WITH \( \mu = 1/3, v = 0, w = 1/2 \)
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***

THIRD ANTSYMMETRIC MODE WITH PHI = 0.73   C = 0.71   W = 0.2
WAVE PLATE GRAPHIC

*** THE CANTILEVERED PLATE WITH EDGE POINT SUPPORT ***
SECOND ANTSYMMETRIC MODE WITH PHI = 0, j = 1, v = 1/2
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH EDGE POINT SUPPORT ***
FIRST ANTISYMMETRIC MODE WITH PHI = 1/2, U = -1, V = 1/2
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH "EDGE" POINT SUPPORT

SECOND ANTI-SYMMETRIC MODE WITH \( \nu = 3.6 \), \( \alpha = 1/11 \), \( \beta = 1/2 \)
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE ***

THIRD ANTISYMMETRIC NODE ***

X AXIS
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH SINGLE POINT SUPPORT

FIRST ANTISYMMETRIC MODE WITH D11 = 1.0, G = 1.0, V = 0.2

X AXIS

Y AXIS
WAVE PLATE GRAPHIC

... THE BAYLIFF PSTAR WITH TRUE PLANE SURFACE ...

SECOND HARMONIC MODE WITH PHASE JUMPS - 180°, V = 1.2
WAVE PLATE GRAPHIC

FIRST ASYMMETRIC MODE WITH PHI = 1.25°, C = 0.06
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH ROOF POINT SUPPORT

A SECOND AXISYMMETRIC MODE WITH PHI = 35 DEGREES
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***
SECOND SYMMETRIC MODE WITH PHI = 1/6, \( \theta = -1/2, \psi = 1/2 \)
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***
THIRD SYMMETRIC MODE WITH PHI - 1/6, U - 1/2, V - 1/2
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

FIRST SYMMETRIC MODE WITH \( \phi = \frac{1}{4}, \phi = \frac{1}{2}, x = \frac{1}{2} \).
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

SECOND SYMMETRIC MODE WITH PHI = 0.74, \( \lambda = 0.72, \nu = 0.72 \)
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT

THIRD SYMMETRIC MODE WITH PHI = 1/4, \( \Theta = 1/2, \gamma = 1/2 \)
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

SECOND SYMMETRIC MODE WITH \( \delta_1 = 1/2, \phi = 1/2, \gamma = 1/2 \).
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT

FIRST SYMMETRIC MODE WITH R = 1/4, \( \phi = \pi/2 \), \( l = 1/2 \)
WAVE PLATE GRAPHIC

---THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT---

SECOND SYMMETRIC MODE WITH PIN = 3.4, 0 = 0, V = 12
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT

THIRD SYMMETRIC MODE WITH DH = 6/4, a = 1/2, l = 1/2
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ---

FIRST SYMMETRIC MODE WITH Φ = ...., Ω = 1/2, V = 1/2
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT

SECOND SYMMETRIC MODE WITH Phi = 1.14, U = 1/32, V = 1/32
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT

THIRD SYMMETRIC MODE WITH PHASES: 0, 90, 180, 270
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT

FIRST SYMMETRIC MODE WITH PHI = 3/2, 0 = ...2, I = ...2
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT

SECOND SYMMETRIC MODE WITH PHI = 3/2, C = 1/2, V1 = 1/2.
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH CATHEDRAL POINT SUPPORT

THIRD SYMMETRIC MODE WITH \Theta = 3/2, \Phi = 4/2, \Lambda = 4/2
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

FIRST ANTI-SYMMETRIC MODE WITH Φ = 0, θ = π/2, Ψ = π/2
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***
SECOND ANTISYMMETRIC MODE WITH PHI = 1/6, \theta = \pi/2, \Omega = \Omega_2

[Diagram of wave plate with indicated modes and parameters]
WAVE PLATE GRAPHIC

THE HUMMER PLATE WITH LATERAL POINT SOURCE

FIRST ASYMMETRIC MODE WITH PHI = 12
WAVE PLATE GRAPHIC

The canonical case with layered medium support...
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT

THIRD ASYMMETRIC MODE WITH PHI = 1.4, v = 0.2, h = 1/2

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WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

FIRST ANISYMMETRIC MODE WITH FREQ. 2.0 2.4 2.8 3.2...
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***
SECOND ANTISYMMETRIC MODE WITH \( c = 1/2, u = 1/2, \psi = \pi/2 \)
WAVE PLATE GRAPHIC

*** THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ***

THIRD ANTI-SYMMETRIC MODE WITH PHI = 1/2, 0 = -1/2, 0 = 1/2
WAVE PLATE GRAPHIC

*** THE CANTILEVERED PLATE WITH LATERAL POINT SUPPORT ***

FIRST ANTISYMMETRIC MODE WITH PHI = 3/4, U = 1/2, V = 1/2.
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT

SECOND ANTISYMMETRIC MODE WITH

\[ \sin \theta = 0.67, \quad \theta = 30^\circ \]
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ---

THIRD ANTI-SYMMETRIC MODE WITH \( \phi_1 = 3/4, \theta = 12, \gamma = 1/2 \)
WAVE PLATE GRAPHIC

--- THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT ---

FIRST ANTISYMMETRIC MODE WITH \phi = \pi/2, \theta = \pi/2, \psi = \pi/2
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT

SECOND ANTI-SYMMETRIC MODE WITH 

\[ u(x, y) = \sin \frac{\pi x}{2} \sin \frac{\pi y}{2} \]
WAVE PLATE GRAPHIC

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THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT

THIRD ANTISYMMETRIC MODE WITH PHI = 90°, U = 1/2, V = 1/2
WAVE PLATE GRAPHIC

THE CANTILEVER PLATE WITH LATERAL POINT SUPPORT

FIRST ANTISYMMETRIC MODE

FIRST SYMMETRIC MODE
FREE VIBRATION ANALYSIS
OF RECTANGULAR CANTILEVER PLATES
WITH SYMMETRICALLY DISTRIBUTED POINT SUPPORTS

VOLUME III
Appendix 3

COMPUTER PROGRAMS
USED IN THE FREE VIBRATION ANALYSIS
OF THE RECTANGULAR CANTILEVER PLATE

This appendix presents a complete list of computer program listings to determine the eigenvalues and mode shapes for the symmetric and antisymmetric modes of the cantilever plate of Chapters 2, 3, and 4. While Programs one through 16 are based on the analytical solutions of Chapters 2, 3, and 4, Programs 17 and 18 use the finite element techniques. To generate desired eigenvalues or mode shapes, a full understanding of these programs is not required for enough explanatory comments have been entered at the beginning of each program, and no further description should be required here.
PROGRAM 1

THIS IS A SYMMETRIC MODE EIGENVALUE SEARCH PROGRAM
FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM.

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:

1. - K = THE NUMBER OF TERMS TO BE USED IN THE
    SERIES INVOLVED. FOR 5 DIGITS ACCURACY
    USE K = 10.

2. - PHIR = 2d/A, IS THE FULL PLATE ASPECT RATIO.

3. - ALMDS = AN INITIAL STARTING VALUE FOR THE EIGEN-
    VALUE SEARCH.

4. - DLIM = A FINISHING OR EIGENVALUE SEARCH ENDING
    LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO
    HALT EXECUTION.

5. - DEL = EIGENVALUE INCREMENT.

6. - PUL = POISSON'S RATIO.

IMPLICIT REAL*8(A-H,U-Z)
DIMENSION A(45,45)
K=??????????????????????????
Z=3600
PHIR=????????????????????
PHI1=PHIR/2
PRINT50,PHIR,K

C FJRMAT(*1,*PHIR=*,FD,4,1JX,*K='1.1S)
K1=2*K-1
C=1.
PI=4.*JATAN(C)
PHI2=PHI1*PHI1
PUL=????????????????????
PUL2=2-PUL
PHIN=1./PHI1
PHIN2=PHIN*PHIN
JFS=PUL*PH1S
JFS2=PUL*PHIN
I=1
ALMDS=????????????????????
DL1=????????????????????
DEL=????????????????????
CONTINUE
ALMDS=ALMDS+DEL
INITIALIZE THE MATRIX A

D1 2 N=1,K
D2 2 N=1,K
A(M,N)=0.0

2 CONTINUE
D3 3 N=1,K
A(N+2*K,N+2*K)=-1.
D4 10 M=1,K+2
D5 10 N=1,K
JN=N-1
EMP2=KPI/2.
EMP2=EMP2*EMP2
ENP=JN*P1
ENP=ENP*ENP
ENS=(1./PHIS)*(ENP*ENP+ALMDS*PHIS)
JN=DSRT(BNS)
X1=ALMDS*PHIS*ENP*ENP
IF (X1*LT.*0.0) GO TO 8
GNS=(1./PHIS)*X1
GNS=DSRT(GNS)
X=PO1S*PHNS*ENPS
T01=(BN*(BN-X)*DCOSH(BN))/(GNS*(GNS+X)*DCOS(GN))
T01P=DCOSH(BN)/DCOS(GN)
X=UF1S*ENPS
T01N=BN*DCOSH(BN)+T01*GN*DCUS(GN)
T01P=BN*DSINH(BN)+GN*T01P*DSIN(GN)
IF (XN*GT.*1.0) GO TO 4
A(N+K,N+K)=(UF1S*ENPS-BNS)*DSINH(BN)+T01*(UF1S*ENPS+GNS)*
DSIN(GN)/T01N
A(N+K,N+K)=(BN+TD1*GN)/TD1N
X=PO1S*PHNS*ENPS
T01=(BN*(BN-X)*DCOSH(BN))/(GNS*(GNS+X)*DCOS(GN))
T01P=DCOSH(BN)/DCOS(GN)
X2=(T01*(ENPS-UFS*BNS)/(BNS+EMP2S) )*GN*DCUS(GN)
XX=(DCUS(ENP)*DSIN(EMP2)/T01N)*(X1-X2)
A(M+1)/Z.N+K)**X2*0.
X2=(T01*(ENPS-UFS*BNS)/(BNS+EMP2S) )*DCOSH(BN)
XX2=T01P*(ENPS-UFS*GNS)/(GNS-EMP2S))*DCUS(GN)
XX2=T01P*(DSINH(EMP)*ENP2*(X1+X2)/TD1P)
A(M+1)/Z.N+K)**X2*0.
GO TO 10

5 X1=-X1
GNS=(1./PHIS)*X1
GNS=DSRT(GNS)
X=PO1S*PHNS*ENPS
IF (GNS*LT.*Z2) GO TO 8
T02=BN*(BN-X)/(GNS*(X-GNS))
X=UF1S*ENPS
T02F=BN-GN
T02F=T02*GN
IF (XN*GT.*1.0) GO TO 7
A(N+K,N+K)=(X-BNS+T02*(X-GNS)) /TD2N
A(N+2*K,N+K)=0.0
A(N+K,N+2*K)=0.0
GO TO 7

6 T02=(BN*(BN-X)*DCOSH(BN))/(GNS*(X-GNS)*DCOSH(GN))
T02P=DCOSH(BN)/DCOSH(GN)
X=UF1S*ENPS
T02F=BN*DSINH(BN)-GN*T02P*DSINH(GN)
TD22N = BN * DCOSH(BN) + T02 * GN * DCOSH(GN)
IF (M * GT +1.0) GO TO 8
A(N*K, N+K) = (X - BNS) * DSINH(BN) + TD2 * (X - GNS) * DSINH(GN) / TD22N
A(N + 2*K, N+K) = (BN + TD2 * GN) / TD22N
A(N + N + 2*K) = (UF1S * ENPS * (1 - TD2P) - BNS + GNS * TD2P) / TD22P
GO TO 8
7 X1 = ((ENPS - UFS * BNS) / (BNS + EMP2S)) * BN
X2 = ((ENPS - UFS * GNS) / (GNS + EMP2S)) * GN
* TD2
XX = (UCOS(ENP) * DSIN(EMP2) / TD22N) * (X1 + X2)
A((M + 1)/2, N + K) = XX * 2 = 0
X1 = (ENPS - UFS * BNS) / (BNS + EMP2S)
X2 = (ENPS - UFS * GNS) / (GNS + EMP2S)
XX = 2 * DCOS(ENP) * EMP2 * (X1 - X2) / TD22P
A((M + 1)/2, N + 2*K) = XX
GO TO 10
9 X1 = ((ENPS - UFS * BNS) / (BNS + EMP2S)) * BN * DCOSH(BN)
X2 = ((ENPS - UFS * GNS) / (GNS + EMP2S)) * GN * DCOSH(GN) * TD2
XX = (UCOS(ENP) * DSIN(EMP2) / TD22N) * (X1 + X2)
A((M + 1)/2, N + K) = XX * 2 = 0
X1 = ((ENPS - UFS * BNS) / (BNS + EMP2S)) * DCOSH(BN)
X2 = (TD2P * (ENPS - UFS * GNS) / (GNS + EMP2S)) * DCOSH(GN)
XX = 2 * DCOS(ENP) * EMP2 * (X1 - X2) / TD22P
A((M + 1)/2, N + 2*K) = XX
CONTINUE
GO 20 N=1, K
DJ 20 M=1, K+1
Q=1-N
ENPS = ENP * P1
ENPS = ENP * LP
EMP2 = M * P1 /2.
EMP2 = EMP2 * EMP2
BMS = PHIS* (ALMUS * EMP2S)
EM = UCSRT (BMS)
X1 = ALMUS * EMP2S
IF (X1 = LT 0.01) GO TO 13
Q=4 = PHIS* X1
GM = UCSRT (GMS)
ENPS = POIS * PHIS * EMP2S * DSINH (BM) / (GM * (GMS + POIS + PHIS * EMP2S))
1 * DSIN (GM)
T01 = TD1 + (1)
T01 = BM * DSINH (BM) - TD1 * GM * DSINH (GM)
XX = (BMS + ENPS) * TD1 * GM * DSINH (GM) / (GMS -
1 * ENPS) * 2 * EMP2 * DCOS (ENP) / TD11
IF (JN + GT0.0) GO TO 11
XX = XX / 2
A((M + 1)/2, N + 2*K) = XX
X1 = ((EMP2S - UFS * BMS) / (BMS + ENPS)) * BM * DCOS(ENP) * DSINH(BM)
X2 = ((EMP2S + UFS * GMS) / (GMS + ENPS)) * GM * DCOS(ENP) * DSINH(GM) * TD1
XX = (X2 + X1) * 2 * DSIN(EMP2) / TD11
IF (JN + GT0.0) GO TO 12
XX = XX / 2
A((M + 1)/2, N + 1/2) = (UFS * EMP2S - BMS) * DCOSH(BM) + TD1 *
1 * UFS * EMP2S + GM * DSOS (GM) / TD11
12 A(N*K, M+1)/2 = XX
GO TO 20
13 X1 = - X1
Q=4 = PHIS* X1
GM = UCSRT (GMS)
IF (BMS * LT 0.01) GO TO 45
T02 = (BM * (BMS - POIS * PHIS * EMP2S) * DSINH (BM)) * (1) / (GM * (GMS - POIS */
1PHIS*EMP2S*DSINH(GM))
TD2=BM*DSINH(BM)/TD2*GM*DSINH(GM)
X1=BM*DSINH(BM)/(BMS+ENPS)
X2=GM*TD2*DSINH(GM)/(GMS+ENPS)
XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD2
X1=(EMP2S+U**S+BMS)*BM*DSINH(BM)/(BMS+ENPS)
X2=(EMP2S+U**S+GMS)*TD2*GM*DSINH(GM)/(GMS+ENPS)
X1=2*DSINH(EMP2)*DCOS(ENP)*(X1+X2)/TD2
IF (JN*GM*0.0) GO TO 14
XX=XX/2
X1=XX/2
A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM)+TD2*(UFS*EMP2S-
GMS))*DCOSH(GM))/TD2
14 A(N+K+1,(M+1)/2)=XX
A(N+K+1,(M+1)/2)=X
GO TO 20
15 TD2=(BM*BM*POIS*PHIS*EMP2S))*/(-1)/GM*(GMS-POIS*PHIS*EMP2S))
TD2=BM*TD2*GM
X1=BM/(BM+ENPS)
X2=GM*TD2/(GMS+ENPS)
XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD2
X1=(EMP2S+U**S+BMS)*BM/(BM+ENPS)
X2=(EMP2S+U**S+GMS)*TD2*GM/(GMS+ENPS)
XX=2*DSINH(EMP2)*DCOS(ENP)*(X1+X2)/TD2
IF (JN*GM*0.0) GO TO 14
XX=XX/2
X1=XX/2
A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)+TD2*(UFS*EMP2S-GMS))/TD2
GO TO 14
20 CONTINUE
CALL DETERM (A,B,DET)
PRINT 110,ALMDS,DET
110 FORMAT(*,*,ALMDS=*,F10.4,10X,*DET=*,F20.5)
IF (ALMDS.LT.DLIM) GO TO 1
STOP
END
SUBROUTINE DETERM (A,N,DET)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(4S,4S)
SIGN=1.
LAST=N-1
C
START OVERALL LOOP FOR(N-1).PIVOTS
C
DO 200 I=1,LAST
C
FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
BIG=0.
DO 30 K=1,N
TERM=ABS(A(K,1))
IF (TERM.BIGG.50.0,50.30
BIG=TERM
L=K
30 CONTINUE
C
CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
C
IF (BIG).EQ.0.0,60.80
C
L-TH ROW HAS THE BIGGEST TERM --- IS I=L

80 IF (I-L)90,120,90

I IS NOT EQUAL TO L, SWITCH ROWS I AND L

90 SING=-SING

DO 100 J=1,N

TEMP=A(I,J)

A(I,J)=A(L,J)

100 A(L,J)=TEMP

NOW START PIVOTAL REDUCTION

120 PIVOT=A(I,1)

NEXTR=I+1

FOR EACH OF THE ROWS AFTER THE I-TH

DO 200 J=NEXTR,N

MULTIPLYING CONSTANT FOR THE J-TH ROW IS

CONST=A(J,1)/PIVOT

NOW REDUCE EACH TERM OF THE J-TH ROW

DO 200 K=1,N

A(J,K)=A(J,K)-CONST*A(I,K)

200 END OF PIVOTAL REDUCTION --- NOW COMPUTE DETERMINANT

DET=SIGN

DO 300 I=1,N

DET=DET*A(I,1)

300 GO TO 01

60 DET=0.

61 RETURN

END
PROGRAM 2

THIS IS A SYMMETRIC MODE SHAPE PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREQUENCY VIBRATION PROBLEM.

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:

1. K = AS DEFINED IN PROGRAM 1.
2. PHIR = 2B/A FULL PLATE ASPECT RATIO.
3. ALMDS = EIGENVALUE.
4. KS = NUMBER OF POINTS AT WHICH THE DISPLACEMENT IS REQUIRED.
5. POI = POISSON'S RATIO.

IMPLICIT REAL*8(A-H,O-Z)
COMMON Z2,PHI,P1,PHI5,POI,POIS,PHIN,PHINS,UF5,UFIS,ALMDS,K,K1,KS
DIMENSION A(45,45)
K=????????????????????????
KS=????????????????????
Z2=3600
PHIR=????????????????????
ALMDS=????????????????
PHI=PHIR/2
PRINT50,PHIR,K
50 FORMAT(1x,*PHIR=,1x,F8.4,10x,*K=*,IS)
K1=2*K-1
C=1.
P1=4.*CATAN(C)
PHI5=PHI*PHI
FUI=????????????????????
POIS=2-POI
PHIN=1./PHI
PHINS=PHIN*PHIN
UF5=POI*PHI5
UFIS=POI*PHINS
I=3*K
CONTINUE

1

INITIALIZE THE MATRIX A.

DO 2 M=1,1
DO 2 N=1,1
A(M,N)=0.0
2 CONTINUE
DO 3 K=1,K
A(N+2*K,N+2*K)=-1.
DO 10 M=1,K+2
DU 10 N=1,K
JN=N-1
EMP2=M*PI/2.*
EMP2=EMP2*EMP2
EMP=JN*PI
ENPS=EMP*ENP
BNS=(1./PHIS)*((ENP*ENP+AMDS*PHIS))
EN=DSORT(BNS)
X1=AMDS*PHIS-ENPS-ENP
IF (X1.LT.0.0) GO TO 5
GNS=(1./PHIS)*X1
GN=DSORT(GNS)
X=PHIS*PHINS*ENPS
T11=BN*(BNS-X)*DCOSH(BN)/(BN*(GNS*X)*DCOS(GN))
T11P=DCOSH(BN)/DCOS(GN)
A=UFIS*ENPS
T11N=BN*DSINH(BN)+TD1*GN*DCOS(GN)
T11P=BN*DSINH(BN)+GN+TD1P*DSINH(GN)
IF (M.T.E.1.0) GO TO 4
A(N+K,N+K)=(UFIS*ENPS-BNS)*DSINH(BN)+TD1*(UFIS*ENPS+GNS)*
1.0*DSINH(GN))/TD11N
A(N+2*K,N+K)=BN+TD1*N+1)/TD11N
4 X1=(ENPS-UFBS-BNS)/(BN+EMPS/BN))
X1=TD1*(BN+EMPS/BN)*GN*DCOS(GN)
XX=(DCOS(ENP)*DSINH(EMP))/(TD11N)*(X1-X2)
A((X1+1)/2,N+K)=XX/2.0
X1=(ENPS-UFBS-BNS)/(BN+EMPS/BN))
X2=(TD1P*ENPS+UFBS/GNS)/(GNS-EMPS/BN))
XX=(UDCOS(ENP)+EMPS/BN))/TD11N*X1+X2)
A(1+%1)/2,N+2*K)=XX
GO TO 10
5 X1=X1
GNS=(1./PHIS)*X1
GN=DSORT(GNS)
X=PHIS*PHINS*ENPS
IF (BNS.LT.2.0) GO TO 6
T22=BN*(BNS-X)/(GNS*(X-GNS))
X=UFIS*ENPS
T22F=BN-GN
T22N=BN+TD2*GN
IF (M.GT.1.0) GO TO 7
A(N+K,N+K)=(X-BNS+TD2*(X-GNS))/TD22N
A(N+K,N+K)=0.0
A(N+K,N+K)=0.0
GO TO 7
6 T22=(BN*(BNS-X)*DCOSH(BN))/(GNS*(X-GNS)*DCOSH(GN))
T22F=DCOSH(BN)/DCOS(GN)
X=UFIS*ENPS
T22F=BN*DSINH(BN)-GN+TD2P*DSINH(GN)
T22N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
IF (M.GT.1.0) GO TO 8
A(N+K,N+K)=(X-BNS+TD2*(X-GNS)*DSINH(GN))/TD22N
A(N+K,N+K)=0.0
A(N+K,N+K)=0.0
GO TO 7
7 X1=(ENPS-UFBS/BN)/(BN+EMPS/BN))
X2=((ENPS-UF5*GNS)/(GNS*EMP25))*GN
XX=(DCOS(ENP)*DSIN(EMP2)*/TD2N)*(X1*X2)
A((M+1)/2,N+K)=XX*2.0
X1=(ENPS-UF5*BN5)/(BN5*EMP25)
X2=(ENPS-UF5*GNS)/(GNS*EMP25)
XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD2P
A((M+1)/2,N+K)*=XX
G0 TO 10
8 X1=(ENPS-UF5*BN5)/(BN5*EMP25)*YN*DCOS(BN)
X2=(ENPS-UF5*GNS)/(GNS*EMP25)*GN*DCOS(GN)*TD2
XX=(DCOS(ENP)*DSIN(EMP2)/TD2N)*(X1*X2)
A((M+1)/2,N+K)*=XX*2.0
X1=(ENPS-UF5*BN5)/(BN5*EMP25)*DCOS(BN)
X2=(TD2P*(ENPS-UF5*GNS)/(GNS*EMP25)*DCOS(GN)
XX=2*DCOS(ENP)*EMP2*(X1-X2)/TD2P
A((M+1)/2,N+K)*=XX
10 CONTINUE
JU 20 N=1,K
JU 20 M=1,K,2
J=H-1
ENPS=ENP*ENP
EMP2=MP1/2
EMP2=EMP2*EMP2
BN5=PHIS*(ALMDS+EMP2)
BM=DSRT(BN5)
X1=ALMDS-EMP2
IF(X1<LT0.0) GU TO 13
GMS=PHIS*X1
GM=DSRT(GMS)
TD1=(BM*(BN5-POIS*PHIS*EMP25)*DSINH(BM))/(GM*(GMS-POIS*PHIS*EMP25)
1*DSIN (GM))
TU1=TD1*(1-1)
AX1=BM*DSINH(BM)-TD1*GM*DSINH(GM)
XX=
AX3=((BM*DSINH(BM))/GM*ENPS)+TD1*GM*DSINH(GM)/(GMS=
1*ENPS)*X2*EMP2*DCOS(ENP)/TD11
IF(JA@GT0.0) GU TO 11
XX=XX/2
11 A(N+K,M+1)/2=XX
X1=(EMP25-UF5*BN5)/(BN5*ENPS)*BM*DCOS(ENP)*DSINH(BM)
X2=(EMP25+UF5*GNS)/(GNS*ENPS)*GM*DCOS(ENP)*DSINH(GM)*TD1
XX=(X2*X1)/2*DSIN(EMP2)/TD11
IF(JA@GT0.0) GU TO 12
XX=XX/2
A((M+1)/2,(M+1)/2)=$(UF5*EMP25-BMS)*DCOSH(BM)+TD1*(
UF5*EMP25+GMS)*DCOSH(GM))/TD11
12 A(N+K,M+1)/2=XX
G0 TO 20
13 X1=-X1
GM=DSRT(GMS)
IF(BM+GT0.22) GU TO 15
TD2=(BM*(BM-POIS*PHIS*EMP25)*DSINH(BM))/(-1)/(GM*(GMS-POIS*
PHIS*EMP25)*DSINH(GM))
TD2=BM*DSINH(BM)+TC2*GM*DSINH(GM)
X1=BM*DSINH(BM)/(GM*ENPS)
X2=GM*TD2*DSINH(GM)/(GMS*ENPS)
XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD22
X1=(EMP25-UF5*BN5)*BM*DSINH(BM)/(BM*ENPS)
X2=(EMP25-UF5*GMS)*BM*DSINH(GM)/(GMS*ENPS)
XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD2
IF (JN*GT*0.0) GO TO 14
XX=XX/2
XX1=XX1/2
A1(N+1)/(2*(M+1)/2)=((UFS*EMP25-BMS)*DCOSH(BM)+TD2*(UFS*EMP25-
1*GMS)*DCOSH(GM))/TD2
14 A1(N+2*K+*(M+1)/2)=XX
A1(N+K+*(M+1)/2)=XX1
GO TO 20
15 TD2=(BM*(BMS-P01S*PHIS*EMP2S))*(-1)/(GMS*(GMS-P01S*PHIS*EMP2S))
TD2=BM+TD2*GM
XX=BM/(BMS+ENPS)
XX2=GM+TD2/(GMS+ENPS)
xx=2*DCOS(ENP)*EMP2*(X1+X2)/TD2
XX1=(EMP25-UFS*BMS)*BM/(BMS+ENPS)
XX2=(EMP25-UFS*GMS)*TD2*GM/(GMS+ENPS)
XX1=2*DSIN(EMP2)*DCOS(ENP)*(X1+X2)/TD2
IF (JN*GT*0.0) GO TO 14
XX=XX/2
XX1=XX1/2
A1(N+1)/(2*(M+1)/2)=((UFS*EMP25-GMS)+TD2*(UFS*EMP25-GMS))/TD2
GO TO 14
20 CONTINUE
CALL DETERM (A,A,DET)
PRINT25
25 FORMAT(*1*,*END*)
STOP
END
SUBROUTINE DETERM (A,A,DET)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(45,45),X(45),EM(45),EN(45),EP(45)
SING=1
M=N-1
LAST=M-1
START OVERALL LOOP FOR(N-1) PIVOTS
DJ 200 I=1,LAST
FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
BIG=0
DO 50 K=1,M
TERM=DAYSL(A(K,I))
IF (TERM-BIG)50,50,30
BIG=TERM
L=K
50 CONTINUE
CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
IF (EIG)80,60,80
L-TH ROW HAS THE BIGGEST TERM----IS I=L
60 IF (I-L)90,120,90
I IS NOT EQUAL TO L, SWITCH ROWS I AND L
90 DD 130 J=1,N
TEMP = A(I,J)
A(I,J) = A(L,J)
100 A(L,J) = TEMP

* START PIVOTAL REDUCTION

120 PIVOT = A(1,1)
NEXT = I + 1

FOR EACH OF THE ROWS AFTER THE I-TH
DO 200 J = NEXTR, M

MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C = A(I,J) / PIVOT

NOW REDUCE EACH TERM OF THE J-TH ROW
DO 200 K = 1, N
200 A(J,K) = A(J,K) - C * A(I,K)

END OF PIVOTAL REDUCTION -- PERFORM BACK SUBSTITUTION

N = N - 1
DO 500 I = 1, M

IREV IS THE BACKWARD INDEX, GOING FROM M BACK TO 1
IREV = M + 1 - I

GET Y(IREV) IN PREPARATION
Y = A(IREV, N)
IF (IREV = M) 400, 500, 400

NOT WORKING ON LAST ROW IF I IS 2 OR GREATER

400 DO 450 J = 2, I

# CHECK BACKWARD FOR X(N), X(N-1) -- SUBSTITUTING PREVIOUSLY
# FOUND VALUES
K = N + 1 - J
450 Y = Y - A(IREV, K) * X(K)

FINALLY, COMPUTE X(IREV)
500 X(IREV) = Y / A(IREV, IREV)

FIND AND PRINT EM, EN AND EP
L = N / 3
DO 550 I = 1, L
J = 2 * I - 1
K = I + 1
EM(J) = X(I)
EN(J) = X(I + L)
IF (J EQ L) GO TO 540
EP(J) = X(I + 2 * L)
550 CONTINUE
GO TO 550

540 EP(J) = 1
550 PRINT 00, K, K, J, EM(J), K, EN(J), K, EP(J)
600 FORMAT(*5i4, 12.5x, *5e12.5, 12.5, *1X, *5e12.5, 12.5) = *.D14.5
GO 601 I = 1, L
J = 1, K
EN(J) = EN(J)
EP(I) = EP(J)
601 CONTINUE
PRINT 650
650 FORMAT(*10H, "THE SYMMETRIC MUJE SHAPE DATA ARE: \\
CALL SHAPL (EM, LN, EP)
60 RETURN
END
SUBROUTINE SHAPE (EM, LN, EP)
IMPLICIT REAL*8 (A-H, U-Z)
COMMON Z2, PHI, P1, PHIS, POI, PSI, PHINS, UFS, UFIS, ALMDS, K, K1, KS
DIMENSION *1(21, 21), *2(21, 21), *3(21, 21), EM(45), EN(45),
1EP(45),
KS1 = KS + 1
E = E + 0, 0
DO 650 J = 1, KS1
PSI = J, 0
DO 640 J = 1, KS1
*11 = 0, 0
*22 = 0, 0
*33 = 0, 0
DO 620 N = 1, K
JN = N - 1
ENP = JN * P1
EN = ENP + ENP
BN = (1./PHIS) * (ENP * ENP + ALMDS * PHIS)
BN = UFS * (BN)
X1 = ALMDS * PHIS * ENP * ENP
IF (X1 < 10.) GO TO 606
GN = (1./PHIS) * X1
GN = UFS * (GN)
X = PSI * PHINS * ENP
TJ1 = (ENP * (BN - X) * DCUSH(BN)) / (GN * (GN + X) * DCUSH(GN))
TJ1 = 3 * UFS * (BN) / DCUSH(GN)
X = UFS * ENP
TJ1 = (BN - DCUSH(BN)) / (GN * (GN + X) * DCUSH(GN))
TJ1 = 3 * UFS * (BN) / DCUSH(GN)
IF (ETA < EQ. 0, 0) GO TO 601
IF (ENP < EQ. 0, 0) GO TO 601
X = DCUSH(ENP * ETA)
601 CONTINUE
702 IF (FSI < EQ. 1, 0) GO TO 604
X = EN(N) * (DSINH(BN * PSI) + TD1 * USIN(GN * PSI)) * X / TD11H
IF (FSI < EQ. 1, 0) GO TO 603
X = EP(N) * (DCUSH(BN) - TD1 * DCUSH(GN) * X / TD11P
GO TO 617
603 XX = 1, 0
604 IF (FSI < EQ. 1, 0) GO TO 604
XX = XX + 1
XX = XX + 1
XX = XX + 1
GO TO 617
606 X = 1, 0
XX = XX + 1
XX = XX + 1
XX = XX + 1
GO TO 617
GNS=(1./PHIS)*X1
GN=DSQRT(GNS)
X=POIS*PHINS*ENPS
IF (ENS=GT*.22) GO TO 612
T02=(BN*(BNS-X)*DCOSH(BN))/(GN*(X-GNS)*DCUSH(GN))
T02P=DCOSH(BN)/DCUSH(GN)
T02P=P*N*DENSION(BN)-GN*T02*DS1NH(GN)
T02B=BN*DCOSH(BN)+T02*GN*DCUSH(GN)
IF (ETA=EQ.0.0) GO TO 667
IF (ENP=EQ.0.0) GO TO 607
XX=DCOS(ENP*ETA)
GO TO 608
007 XX=1.0
008 IF (PSI=EQ.0.0) GO TO 610
X=x2=EN(N)*(DENSITY(BN)*PSI)+T02*DS1NH(GN)*SI)*X/T02B
IF (PSI=EQ.1.0) GO TO 609
X=x3=EP(N)*(DCOSH(BN*(1+PSI))-T02P*DCOSH(BN*(1-1-SI)))*X/T02P
GO TO 617
609 X=x3=EP(N)*(1-T02P)*X/T02P
GO TO 617
610 X=x2=0.0
X=x3=EP(N)*(DCOSH(BN)-T02P*DCUSH(BN))*X/T02P
GO TO 617
612 T02B=3N*(ENS-POIS*PHINS*ENPS)/(GN*(PSI*PHINS*ENPS-GNS))
IF (ETA=EQ.0.0) GO TO 613
IF (ENP=EQ.0.0) GO TO 613
XX=DCOS(ENP*ETA)
GO TO 614
613 XX=1.0
614 IF (PSI=EQ.0.0) GO TO 616
B=1.0
TEST=BN-BN*PSI
IF (TEST=GT.60.) B=0.0
X=x2=EN(N)*(DEXP((BN*PSI-BN)*B)+T02N*DEXP((GN*PSI-GN)*B))*X/(1*(BN+T02N*GN))
IF (PSI=EQ.1.0) GO TO 615
TEST=BN*PSI
IF (TEST=GT.60.) GO TO 615
X=x3=EP(N)*(DEXP(-(BN*PSI)-DEXP(-(GN*PSI)))*X/(BN-GN)
GO TO 617
615 X=x3=0.0
GO TO 617
616 X=x2=0.0
X=x3=0.0
617 #2(1,J)=x2+x2
#3(1,J)=x3+x3
#2=x2(1,J)
#3=x3(1,J)
620 CONTINUE
DU 630 M=1,K1,2
EMP2=EMP1/2.
EMP2=EMP*EMP2
BNS=PHIS*(ALMOD*EMP2)
EM=DSQRT(BMS)
X1=ALMOD*EMP2
IF (X1=LT.0.0) GO TO 623
GMS=PHIS*X1
GM=DSQRT(GMS)
T01=(BN*(GMS-POIS*PHIS*EMP2)*DS1NH(BN))/(GM*(GMS+POIS*PHIS*EMP2))
/ (1*DSIN(GB))
TD1=TD1*(-1)
TD1=BM*DSINH(BM)-TD1*GM*DSINH(GM)
IF (PS1.EQ.0.0) GO TO 622
IF (ETA.EQ.0.0) GO TO 621
X1*=EM(M)*DCOSH(BM*ETA)+TD1*DCOS(GM*ETA)*DSIN(EMP2*PS1)/TD11
GO TO 629
621 X1*=EM(M)*(1+TD1)*DSIN(EMP2*PS1)/TD11
GO TO 629
622 X1*=0.0
GO TO 629
623 X1=XM
GMS=PHIS*X1
GM=DSQRT(GMS)
IF (BMS.GT.2.0) GO TO 625
TD2=(dM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM))/(GMS*(GMS-POIS*PHIS*EMP2S))
TD2=TD2*(-1)
TD2=3*DSINH(BM)+TD2*GM*DSINH(GM)
IF (PS1.EQ.0.0) GO TO 622
IF (ETA.EQ.0.0) GO TO 624
X1=EM(M)*(DCOSH(BM*ETA)+TD2*DCOSH(GM*ETA))*DSINH(EMP2*PS1)/TD22
GO TO 629
624 X1=EM(M)*(1+TD2)*DSINH(EMP2*PS1)/TD22
GO TO 629
625 TD2=BM*(POIS*PHIS*EMP2S-BMS)/(GMS*(GMS-POIS*PHIS*EMP2S))
IF (TEST.GT.0.0) GO TO 627
IF (TEST.GT.0.0) GO TO 627
TEST=BMS*ETE
IF (TEST.GT.0.0) GO TO 627
X1=EM(M)*(DEXP(EM*ETA-BM)+TD2M*DEXP(BM*ETA-GM))*DSINH(EMP2*PS1)
1*(BM+TD2M*GM)
GO TO 629
627 X1=0.0
629 X1=X1+X1+X1
630 CONTINUE
630 CONTINUE
630 CONTINUE
640 CONTINUE
640 CONTINUE
RETURN
END
PROGRAM 3

THIS IS AN ANTISYMMETRIC MODE EIGENVALUE SEARCH PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM.

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:

1. - K = THE NUMBER OF TERMS TO BE USED IN THE SERIES INVOLVED. FOR 5 DIGITS ACCURACY, USE K = 10.
2. - PHIR = 2B/A, IS THE FULL PLATE ASPECT RATIO.
3. - ALMDS = AN INITIAL STARTING VALUE FOR THE EIGENVALUE SEARCH.
4. - DLIM = A FINISHING OR EIGENVALUE SEARCH ENDING LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO HALT EXECUTION.
5. - DEL = EIGENVALUE INCREMENT.
6. - POI = PIUSSEN'S RATIO.

IMPLICIT REAL*8(A-H,U-Z)
DIMENSION A(45,45)
K=??????????????????????????
K2=2*K
K1=2*K-1
22=2000
PHIR=??????????????????????
PHI=PHIR/2.
PHIR=PHIR*K
50 FORMAT(*1*,*PHIR=*,F8.4,10x,*K=*,15)
C=1.
PL=4.*DATAN(C)
PHIS=PHI*PHI
POI=??????????????????????
PUIS=2-POI
PHIN=1./PHI
PHINS=PHIN*PHIN
UFS=PLI*PHIS
UFS=POI*PHINS
I=3*K
ALMDS=????????????????????
DEL=??????????????????????
DLIM=??????????????????????
CONTINUE

INITIALIZE THE MATRIX TO 0.0

DO 2 M=1,1
DO 2 N=1,1
2 A(M,N)=0.0
DO 10 M=1,K1,2
DO 10 N=1,K1,2
EMP2=M*PI/2*
EMP25=EMP2*EMP2
EMP2=N*PI/2*
EMP2S=EMP2*EMP2
ENS=PHINS*(EMP2S+ALMD*PHIS)
BN=DSQRT(BNS)
XI=ALMD*PHIS-EMP2S
IF (XI.LT.0.0) GO TO 4
WNS=PHINS*XI
WN=DSQRT(GNS)
T01N=GNS*DCOS(GN)*((WNS+EMP2S*PHIS)*WNS)/(BN*DCOSH(BN)*(BNS-EMP2S*
1*FIS+PHINS))
T01H=GNS*DCOSH(BN)*GNS*DCOSH(GN)
T01P=DCOSH(BN)/DCOSH(GN)
T01P=BN*DSINH(BN)+EN*DSINF(GN)*T01P
IF (N.GT.1) GO TO 3
AT((N+1)/2+K,N+1)/2+K)=(T01N*BN*BN)/T01N
AT((N+1)/2+K,(N+1)/2+K)=((UFIS+EMP2S-BNS)*T01N*DSINH(BN)+UFIS
1*EMP2S+GNS)*DSINF(GN))/T01N
AT((N+1)/2+K*(N+1)/2+K)=(UFIS+EMP2S*(1-T01P)-BNS-T01P*GNS)/T01P
X1=((EMP2S-UF5S*GNS)/(BNS+EMP2S)))*T01N*BN*BN
X2=((EMP2S-UF5S*GNS)/(GNS-EMP2S)))*GNS*DCOSH(GN)
X1=2*DSINF(EMP2S)*DSINF(EMP2S)*T01P*(X1-X2)/T01N
AT((N+1)/2+K,N+1)/2+K)=XX
X1=((EMP2S-UF5S*GNS)/(GNS+EMP2S)))*DSINF(BN)
X2=((EMP2S-UF5S*GNS)/(GNS+EMP2S)))*DCOSH(BN)
XX=2*DSINF(EMP2S)*EMP2S*(X1+X2)/T01P
AT((N+1)/2+K,N+1)/2+K)=XX
GO TO 10

X1=X1
WNS=PHINS*X1
WN=DSQRT(GNS)
IF (BN.SLT.Z2) GO TO 6
T01H=GNS*(PHINS*EMP2S-GNS)/(BN*(BNS-PHINS*EMP2S))
T02N=T02N*BN*BN
T02P=1.
T022P=BN*BN*T02P
IF (N.GT.1) GO TO 5
AT((N+1)/2+K,N+1)/2+K)=.0.0
AT((N+1)/2+K,N+1)/2+K)=.0.0
AT((N+1)/2+K,N+1)/2+K)=((UFIS+EMP2S-BNS)*T02N+UFIS*EMP2S
1*GNS)/T022N
X1=((EMP2S-UF5S*GNS)/(BNS+EMP2S)))*T02N*BN
X2=((EMP2S-UF5S*GNS)/(GNS+EMP2S)))*GNS
XX=2*DSINF(EMP2S)*DSINF(EMP2S)*T01N+X1+X2)/T022N
AT((N+1)/2,XX)=XX
X1=((EMP2S-UF5S*GNS)/(BNS+EMP2S))
X2=((EMP2S-UF5S*GNS)/(GNS+EMP2S))
XX=2*DSINF(EMP2S)*EMP2S*(X1-X2)/T022P
AT((N+1)/2,XX)=XX
GO TO 10
0 TD2N=GN*(PO1S*PHIS*EMP2S-UNS)*DCOSH(GN)/(BN*(BNS-PUIS*PHINS*1*EMP2S)*DCOSH(BN))
TD22N=TD2N*BN*DCOSH(BN)*GN*DCOSH(GN)
TD22P=DCOSH(BN)/DCOSH(GN)
TD22P=BN*DSINH(BN)*GN*TD2P*DSINH(GN)
IF (M+GT.1) GO TO 7
A((N+1)/2+K2,*(N+1)/2+K)=((TD2N*BN*GN)/TD22N
A(N+1)/2+K,N+1)/2+K)=((UFS*EMP2S-BNS)*TD2N*DSINH(BN)+(UFS*
1*EMP25-GRS)*DSINH(GN))/TD22N
A((N+1)/2+K,N+1)/2+K)=((UFS*EMP2S*1-TD2P)-DNS+TD2P*GNS)/TD22P
7 X1=((EMP2S-UFS*UNS)/(UNS+EMP25))#TD2N*BN*DCOSH(BN)
X2=(1-EMP2S-UFS*GNS)/(GNS+EMP25)*GN*DCOSH(GN)
XX=2*DSIN(EMP2)*DSIN(ENP2)*((X1+X2)/TD22N
A(M+1)/2,(N+1),#K)=XX
X1=((EMP2S-UFS*BNS)/(BNS+EMP25))#DCOSH(BN)
X2=(1-EMP2S-UFS*GNS)/(GNS+EMP25)*TD2P*DCOSH(GN)
XX=2*DSIN(EMP2)*EMP2*(X1-X2)/TD22P
A((M+1)/2,(N+1)/2+K2)=XX
10 CONTINUE
DO 11 N=1,K
11 A(N*K2+K2)=-.1
DO 20 M=1,K
DO 20 M=1,K
20 ENP2=2*K/P1/2.
EMP2S=EMP2*EMP2
EMP2S=EMP2*EMP2
ENS=PHIS*(ALMDS+EMP2S)
BM=DSORT(BMS)
X1=ALMDS-EMP2S
IF (X1.LT.0.0) GO TO 13
GMS=PHIS*X1
GMS=DSORT(GMS)
TD11M=GM*(GMS+EMP2S*POIS*PHIS)*DCOSH(GM)/(BN*(BMS-EMP2S*
1*PO1S*PH1S)*DCOSH(BM))
TD11M=BM*TD11M*DCOSH(BM)+GM*DCOSH(GM)
IF (N+GT.1) GO TO 12
A((N+1)/2+K2,*(M+1)/2)=((UFS*EMP2S-BMS)*TD1M*DSINH(BM)+(UFS*
1*EMP2S+BMS)*DSINH(GM))/TD11M
12 X1=((EMP2S-UFS*BMS)/(BMS+EMP2S))#TD1M*BM*DCOSH(BM)
X2=((EMP2S-UFS*GMS)/(GMS+EMP2S))#GM*DCOSH(GM)
XX=2*DSIN(EMP2)*DSIN(ENP2)*((X1-X2)/TD11M
A((N+1)/2+K2,*(M+1)/2)=XX
X1=TD1M*BM*DCOSH(BM)/(LMS+EMP2S)
X2=M+DCOSH(GM)/(GMS-EMP2S)
XX=2*EMP2*DSIN(ENP2)*((X1-X2)/TD11M
A((N+1)/2+K2,*(M+1)/2)=XX
GJ TC 20
13 X1=X1
GMS=PHIS*X1
GMS=DSORT(GMS)
IF (BMS.LT.22) GO TO 13
TD2M=GM*(PO1S*PHIS*EMP2S-UNS)/(EM*(BMS-Pouis*PHIS*EMP2S))
TD22M=TD2M*BM+GM
IF (N+GT.1) GO TO 14
A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*TD2M+(UFS*EMP2S-GMS))/TD22M
14 X1=((EMP2S-UFS*BMS)/(BMS+EMP2S))#TD2M*BM
X2=((EMP2S-UFS*GMS)/(GMS+EMP2S))#GM
XX=2*DSIN(EMP2)*DSIN(ENP2)*((X1+X2)/TD22M
A((N+1)/2+K2,*(M+1)/2)=XX
\[
\begin{align*}
x_1 &= \text{TD2M} \times B M / (\text{BMS} + \text{ENP2S}) \\
x_2 &= \text{GM} / (\text{GMS} + \text{ENP2S}) \\
x_2 &= 2 \times \text{ENP2S} \times \text{DSIN ENP2S} \times (x_1 + x_2) / \text{TD2M} \\
A((N + 1)/2 + K_2(M + 1)/2) &= x_2 \\
\text{GO TO} 20 \\
15 & \text{TD2M} = \text{GM} \times (\text{POIS} \times \text{PHIS} \times \text{EMP2S} - \text{GMS}) \times \text{DCOSH}(\text{GM}) / (\text{BMS} \times (\text{BMS} \times \text{POIS} \times \text{PHIS} \\
& \times \text{EMP2S}) \times \text{DCOSH}(\text{BM})) \\
& \text{TD2M} = \text{TD2M} \times \text{BM} \times \text{DCOSH}(\text{BM}) + \text{GM} \times \text{DCOSH}(\text{GM}) \\
& \text{IF} (N > M - 1) \text{ GO TO} 16 \\
& A((N + 1)/2 + (M + 1)/2) = (\text{UFS} \times \text{EMP2S} - \text{BMS}) \times \text{TD2M} \times \text{DSINH}(\text{BM}) + (\text{UFS} \times \text{EMP2S} - \text{GMS}) \times \text{DSINH}(\text{GM}) / \text{TD2M} \\
16 & \text{x1} = (\text{EM2S} - \text{UFS} \times \text{BMS}) / (\text{BMS} + \text{ENP2S}) \times \text{TD2M} \times \text{BM} \times \text{DCOSH}(\text{BM}) \\
& \text{x2} = (\text{EM2S} - \text{UFS} \times \text{GMS}) / (\text{GMS} + \text{ENP2S}) \times \text{GM} \times \text{DCOSH}(\text{GM}) \\
& \text{XX} = 2 \times \text{DSIN(EM2S)} \times \text{DSIN(ENP2S)} \times (x_1 + x_2) / \text{TD2M} \\
& A((N + 1)/2 + K_2 + (M + 1)/2) = x_2 \\
& \text{X1} = \text{TD2M} \times \text{BM} \times \text{DCOSH}(\text{BM}) / (\text{BMS} + \text{ENP2S}) \\
& \text{X2} = \text{GM} \times \text{DCOSH}(\text{GM}) / (\text{GMS} + \text{ENP2S}) \\
& \text{XX} = 2 \times \text{EM2S} \times \text{DSIN(ENP2S)} \times (x_1 + x_2) / \text{TD2M} \\
& A((N + 1)/2 + K_2 + (M + 1)/2) = x_2 \\
20 & \text{CONTINUE} \\
& \text{CALL DETERM} (A, N, DET) \\
& \text{PRINT 100, ALMDS, DET} \\
110 & \text{FORMAT(*'**ALMDS=**F10.4, **DET=*D20.5)} \\
& \text{ALMDS} = \text{ALMDS} \times \text{DEL} \\
& \text{IF} (\text{ALMDS} \leq \text{DEL}) \text{ GO TO} 1 \\
& \text{STOP} \\
& \text{END} \\
& \text{SUBROUTINE DETERM} (A, N, DET) \\
& \text{IMPLICIT REAL*8(A-H, U-Z)} \\
& \text{DIMENSION A(45, 45)} \\
& \text{SIGN} = 1 \\
& \text{LAST} = N - 1 \\
& \text{START OVERALL LOOP FOR(N-1) PIVOTS} \\
& \text{DU 200 I=1, LAST} \\
& \text{FIND THE LARGEST REMAINING TERM IN 1-TH COLUMN FOR PIVOT} \\
& \text{BI1} = 0 \\
& \text{DO 50 K1=1, N} \\
& \text{TERM} = \text{ABS(A(IK1))} \\
& \text{IF} (\text{TERM} > \text{BIG}) \text{30} \\
& \text{30 BIG} = \text{TERM} \\
& \text{L} = K \\
& \text{50 CONTINUE} \\
& \text{CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND} \\
& \text{IF} (\text{BIG} \leq \text{80}) \text{80} \\
& \text{80 L-TH ROW HAS THE BIGGEST TERM-----IS I=L} \\
& \text{IF} (\text{I-L} \geq 0) \text{90} \\
& \text{90 I IS NOT EQUAL TO L, SWITCH ROWS I AND L} \\
& \text{Y} \text{V SIGN} = \text{SIGN} \\
& \text{DO 100 J=1, N} \\
& \text{TEMP} = A(I, J) \\
\end{align*}
\]
A(I,J)=A(L,J)
100 A(L,J)=TEMP

NOW START PIVOTAL REDUCTION

120 PIVOT=A(I,I)
    NEXTR=I+1
    FOR EACH OF THE ROWS AFTER THE I-TH
    DO 200 J=NEXTR,N
    MULTIPLYING CONSTANT FOR THE J-TH ROW IS
    CONST=A(J,I)/PIVOT
    NOW REDUCE EACH TERM OF THE J-TH ROW
    DO 200 K=1,N
    A(J,K)=A(J,K)-CONST*A(I,K)
200 END OF PIVOTAL REDUCTION

END OF PIVOTAL REDUCTION—NOW COMPUTE DETERMINANT

J=1 SIGN
DO 300 I=1,N
300 DET=DET*A(I,I)
GO TO 61
DET=0
RETURN
END
PROGRAM 4

THIS IS AN ANTI-SYMMETRIC MODE SHAPE PROGRAM FOR
THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM.

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:
1. K = AS DEFINED IN PROGRAM 3.
2. PHI K = 2D/A FULL PLATE ASPECT RATIO.
3. ALMDS = EIGENVALUE.
4. K = NUMBER OF POINTS AT WHICH THE DISPLACEMENT
   IS REQUIRED.
5. PUI = POISSON'S RATIO.

INPLICIT REAL*8(A-H,O-Z)
DIMENSION A(45,45)
COMMON Z2,PHI,P1,PHIS,PUI,POI,PHIN,PHINS,ALMDS,K,K1,K2
K=??????????????????????????
K1=2*K-1
Z2=2600
PHIK=??????????????????????
ALMDS=??????????????????????
PHI=PHIK/2
PHIN=P1,PHIK,P1,K,ALMDS
50 FORMAT(*1*,*PHIK=*:F5.4,1UX,*PHI=*:F5.4,1UX,*K=*:I4,
1UX,*ALMDS=*:F5.4)
PHIN=0
60 FORMAT(*-*,*THE VALUES OF EM,EN,AND EP ARE*:///)
C=1
PI=4.*CATAN(C)
PHIS=PHI*PHI
POI=??????????????????????
POI=2-POI
PHIN=1./PHI
PHINS=PHIN*PHIN
UFIS=POI*PHIS
UFIS=POI*PHINS
I=1*K

INITIALIZE THE MATRIX TO 0.0
0J 2 I=1.1
DO 2 N=1,1
A(N,N)=0.0
DJ 10 N=1,K*2
DJ 10 N=1,K*2
EMP2=EXP2=N=1/2

ENS=PHINS*(EMP2S+ALMDS*PHIS)
BNS=DSCRT(BNS)
X1=ALMDS*PHIS-EMP2S
IF (X1.LT.0.0) GO TO 4
GNS=PHINS*X1
GNS=DSCRT(GNS)
T11N=GN+DCOS(GN)*(GNS+EMP2S*PUIS*PHINS)/(BN+DCOSH(BN)*(BNS-EMP2S+
1*FUIS+PHINS))
T11N=BN*T11N+DCOSH(BN)+GN+DCOS(GN)
T11P=DCOSH(BN)/DCOS(GN)
T11P=BN*DSINH(BN)+GN*DSINH(GN)*T11P
IF (M.GT.1.) GO TO 3
A((N+1)/2+K,*N+1)/2+K)=(T11N*BN+GN)/T11N
A((N+1)/2+K,*N+1)/2+K=((FUIS*EMP2S-BNS)*T11N*DSINH(BN)+(FUIS
dnS)*DSINH(GN))/T11N
A((N+1)/2+K,*N+1)/2+K)=((FUIS*EMP2S-1-T11P)-BNS-T11P*GNS)/T11P
X1=((EMP2S-UF*BNS)/(BNS+EMP2S))*T11N*GN+DCOSH(BN)
X2=((EMP2S-UF*GNS)/(GNS+EMP2S))*GN+DCOSH(GN)
X2=2*DSIN(EMP2)*(DSIN(EMP2)*(X1-X2)/T11N
A((N+1)/2+K,*N+1)/2+K)=X
X1=((EMP2S-UF*GNS)/(GNS+EMP2S))*DCOSH(BN)
X2=((EMP2S-UF*GNS)/(GNS+EMP2S))*T11P*DCOSH(GN)
X2=*(DSIN(EMP2))T11P*EMP2*(X1+X2)/T11P
A((N+1)/2+K,*N+1)/2+K)=X
GO TO 10
X1=X1
GNS=PHINS*X1
GNS=DSCRT(GNS)
IF (ENS.LT.0) GO TO 6
T12N=GN*(PHINS*EMP2S-GNS)/(BN*(BNS-PUIS*PHINS*EMP2S))
T12Nh=T12N*BN+GN
T12P=1.*
T12F=BN-GN*T12P
IF (M.GT.1.) GO TO 5
A((N+1)/2+K,*N+1)/2+K)=0.0
A((N+1)/2+K,*N+1)/2+K)=0.0
A((N+1)/2+K,*N+1)/2+K)=(FUIS*EMP2S-BNS)*T12Nh+ENS*EMP2S
1-NS=T12Nh

X1=((EMP2S-UF*GNS)/(GNS+EMP2S))*T12Nh
X2=((EMP2S-UF*GNS)/(GNS+EMP2S))*GN
XX=2*DSIN(EMP2)*(DSIN(EMP2)*(X1-X2)/T12Nh
A((M+1)/2+K,*N+1)/2+K)=X
X1=(EMP2S-UF*GNS)/(GNS+EMP2S)
X2=(EMP2S-UF*GNS)/(GNS+EMP2S)
X2=2*DSIN(EMP2)*EMP2*(X1-X2)/T22P
A((M+1)/2+K,*N+1)/2+K)=X
GO TO 10
T12Nh=GN*(PHINS*EMP2S-GNS)*DCOSH(GN)/(BN*(BNS-PUIS*PHINS*EMP2S)*
1*EMP2S)*DCOSH(BN)
T12Nh=T12Nh*BN+DCOSH(BN)+GN*DCOSH(BN)
T12P=DCOSH(BN)/DCOSH(BN)
T12P=BN*DSINH(BN)-GN+T12P*DSINH(GN)
IF (M$GT; 1) GO TO 7
A((N$+1)/2+K2, (N$+1)/2+K2) = (TU2N$*BN$+GN$)/TD22N
A((N$+1)/2+K2, (N$+1)/2+K2) = ((UF$*ENP2S-BNS) * TD2N$*DSINH(BN$) + (UF$*1
ENP2S-GNS) * DSINH(GN$))/TD22N
A((N$+1)/2+K2, (N$+1)/2+K2) = (UF$*ENP2S*(1-TD2P) - BNS+TD2P*GNS)/TD22P
X1 = (ENP2S-UF$*BNS)/TD2N$*BN$*DCOSH(BN$)
X2 = ((ENP2S-UF$*GNS)/(GNS-EMP2S) * GN$*DCOSH(GN$)
X2 = 2*DSIN(EMP2P) * DSIN(EMP2P) * (X1+X2)/TD22N
A((M$+1)/2, (N$+1)/2+K2) = X2
X1 = (ENP2S-UF$*BNS)/(BN$-EMP2S) * DCOSH(BN$)
X2 = (ENP2S-UF$*GNS)/(GNS-EMP2S) * TD2P*DCOSH(GN$)
X2 = 2*DSIN(EMP2P) * EMP2P * (X1+X2)/TD22P
A((M$+1)/2, (N$+1)/2+K2) = X2
10 CONTINUE
DO 11 N=1,K
11 A(N$+K2, N$+K2) = -1
DJ 20 N=1,K,1,2
DJ 20 M=1,K,1,2
ENP2S=EMP2S*EMP2S
EMP2S=EMP2S*EMP2S
BMS=PHTS*(ALM$S+EMP2S)
EM=DSGRT(BMS)
X1 = ALM$S-EMP2S
IF (X1. LT. 0.0) GO TO 13
GM$ = PHIS*X1
GM$=DSGRT(GMS)$
TF1M=GM$*(GMS+EMP2S*POIS*PHIS)*DCUS(GM$)/(GM$*(GMS-EMP2S*
1POIS*PHIS)*DCUS(BM$)
TF1M=GM*TF1D1=DCUS(BM$)+GM*DCUS(GM$)
IF (N$*GT. 1) GO TO 2
A((M$+1)/2, (M$+1)/2) = (UF$*ENP2S-BMS) * TD1M*DSINH(BM$) + (UF$*
1*ENP2S-GMS) * DSINH(BM$)/TD11M
12 X1 = (ENP2S-UFIS*GMS)/(GMS-EMP2S) * TD1M*BM$*DCOSH(BM$)
X2 = ((ENP2S-UFIS*GMS)/(GMS-EMP2S) * GM$*DCUS(GM$)
X2 = 2*DSIN(EMP2P) * DSIN(EMP2P) * (X1+X2)/TD11M
A((N$+1)/2+K2, (M$+1)/2) = X2
X1 = TD1M*EM*DCOSH(BM$)/(GMS+EMP2S)
X2 = 2*EMP2D*DSIN(EMP2P) * (X1+X2)/TD11M
A((N$+1)/2+K2, (M$+1)/2) = X2
13 GM$=PHIS*X1
GM$=DSGRT(GMS)$
IF (BMS. LT. 22) GO TO 15
TD2M=GM$*(P01S*PHIS*EMP2S-GMS)/(GM$*(BMS-P01S*PHIS*EMP2S))
TD2M=TD2M+UMP*GM$
14 IF (N$*GT. 1) GO TO 14
A((M$+1)/2, (M$+1)/2) = (UF$*ENP2S-BMS) * TD2M+(UF$*EMP2S-GMS))/TD22M
X1 = (ENP2S-UFIS*GMS)/(GMS+EMP2S) * TD2M*BM$+TF1M+BMS
X2 = (ENP2S-UFIS*GMS)/(GMS+EMP2S) * GM$+GM$*DCUS(BM$)
X2 = 2*DSIN(EMP2P) * DSIN(EMP2P) * (X1+X2)/TD22M
A((N$+1)/2+K2, (M$+1)/2) = X2
X1 = TD2M*EM*/(BMS+EMP2S)
X2 = GM$/(GMS+EMP2S)
X2 = 2*EMP2D*DSIN(EMP2P) * (X1+X2)/TD22M
A((N$+1)/2+K2, (M$+1)/2) = X2
GO TO 20
15 \text{TD2M} = \text{GM}^2 \cdot (\text{POIS} \cdot \text{PHIS} \cdot \text{EMP2S} - \text{GMS}) \cdot \text{DCOSH} \cdot \text{GM} \cdot (\text{BM} \cdot (\text{BMS} - \text{POIS} \cdot \text{PHIS}) \cdot \text{EMP2S} - \text{GMS}) \cdot \text{DCOSH} \cdot \text{BM}) \\
\text{TD22M} = \text{TD2M} \cdot \text{BM} \cdot \text{DCOSH} \cdot \text{BM} \cdot \text{GM} \cdot \text{DCOSH} \cdot \text{GM} \\
\text{IF} \ (N \cdot \text{ST} \cdot A) \ \text{GO TO} \ 16 \\
\text{A}((N+1) / \text{ST})((N+1) / \text{ST}) = ((\text{UFS} \cdot \text{EMP2S} - \text{BMS}) \cdot \text{TD2M} \cdot \text{DSINH} \cdot \text{BM}) + ((\text{UFS} \cdot \text{EMP2S} - \text{GMS}) \cdot \text{DSINH} \cdot \text{GM}) / \text{TD22M} \\
16 \text{X1} = ((\text{EMP2S} - \text{UFS} \cdot \text{BMS}) / (\text{BMS} + \text{EMP2S})) \cdot \text{TD2M} \cdot \text{BM} \cdot \text{DCOSH} \cdot \text{BM} \\
\text{X2} = (\text{EMP2S} - \text{UFS} \cdot \text{GMS}) / (\text{GMS} + \text{EMP2S}) \cdot \text{GM} \cdot \text{DCOSH} \cdot \text{GM} \\
\text{XX} = 2 \cdot \text{DSIN} \cdot \text{EMP2S} \cdot \text{DSIN} \cdot \text{EMP2S} \cdot (\text{X1} + \text{X2}) / \text{TD22M} \\
\text{A}((N+1) / \text{ST})((N+1) / \text{ST}) = \text{XX} \\
20 \text{CONTINUE} \\
\text{CALL DETERM}(A, 1, \text{DET}) \\
\text{PRINT} 125 \\
25 \text{FORMAT}(1, *, \text{END}) \\
\text{STOP} \\
\text{END} \\
\text{SUBROUTINE DETERM}(A, N, \text{DET}) \\
\text{IMPLICIT REAL*8(A-N, 0-Z)} \\
\text{DIMENSION A(45, 45), X(45, 45), EM(45), EN(45), EP(45)} \\
\text{SIGN} = 1 \\
N = N - 1 \\
\text{LAST} = N - 1 \\
\text{START OVERALL LOOP FOR (N-1) PIVOTS} \\
\text{DO 200 I=1, \text{LAST}} \\
\text{FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT} \\
\text{BIG} = 0. \\
\text{DO 50 K=1, N} \\
\text{TERM} = \text{CABS}(A(K, I)) \\
\text{IF} \ (\text{TERM} > \text{BIG}) \text{50, 50, 30} \\
30 \text{BIW} = \text{TERM} \\
L = K \\
50 \text{CONTINUE} \\
\text{CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND} \\
\text{IF} \ (\text{BIG} < 10^{-60}) \\
\text{L-TH ROW HAS THE BIGGEST TERM --- IS I=L} \\
80 \text{IF} \ (I-\text{L}) < 90 \cdot 120 \cdot 90 \\
\text{1. IS NOT EQUAL TO L, SWITCH ROWS I AND L} \\
90 \text{J} = 100 \text{J} = 1 \cdot N \\
\text{TEMP} = A(I, J) \\
A(I, J) = A(L, J) \\
A(L, J) = \text{TEMP} \\
\text{NJ#. START PIVOTAL REDUCTION} \\
120 \text{PIVOT} = A(I, I) \\
\text{NEXR} = I + 1
FOR EACH OF THE ROWS AFTER THE I-TH
DO 200 J=NEXT,R,M
MULTIPLYING CONSTANT FOR THE J-TH ROW IS
CONST=A(J,1)*PIVOT
NEXT REDUCE EACH TERM OF THE J-TH ROW
DO 200 K=1,N
200 A(J,K)=A(J,K)-CONST*A(1,K)
END OF PIVOTAL REDUCTION-- PERFORM BACK SUBSTITUTION
M=N-1
DO 300 I=1,M
IREV IS THE BACKWARD INDEX, GOING FROM M BACK TO 1
IREV=M+1-I
GET Y(IREV) IN PREPARATION
Y=A(IREV,N)
IF (IREV=M) 400,500,400
NOT WORKING ON LAST ROW, I IS 2 OR GREATER
400 DO 450 J=2,I
#ORK BACKWARDS FOR X(N),X(N-1)------SUBSTITUTING PREVIOUSLY
FOUND VALUES
K=N+I-J
450 Y=Y-A(IREV,K)*X(K)
FINALLY, COMPUTE X(IREV)
500 X(IREV)=Y/A(IREV,IREV)
FIND AND PRINT EM,EN,AND EP
L=N/J
DO 550 I=1,L
J=2*I-1
EM(J)=X(I)
EN(J)=X(I+L)
IF(J EQ L) GO TO 540
EP(J)=X(I+2*L)
GO TO 550
540 EM(J)=1.
550 PRINT 600, J, EM(J), J, EN(J), J, EP(J)
600 FORMAT('J',12,10X, 'EM(',12X,')=',15.5,10X, 'EN(',12X,')=',
15.5,10X, 'EP(',12X,')=',15.5)
PRINT 650
650 FORMAT('THE ANTI-SYMMETRIC MODE SHAPE DATA ARE://////)
CALL SHAPE(EM,EN,EP)
60 RETURN
END
SUBROUTINE SHAPE(EM, EN, EP)
IMPLICIT REAL*8(A-H,O-Z)
COMMON Z2, PHI, P1, PHIS, POI, POIS, PHIN, PHINS, ALMDS, K, K1, KS
DIMENSION W(51, 51), W2(51, 51), W3(51, 51), W(51, 51), EM(45), EN
1(45), EP(45)
KS1 = KS + 1
ETA = 0.0
DO 650 I = 1, KS1
PSI = 0.0
DO 640 J = 1, KS1
11 = 0.0
22 = 0.0
33 = 0.0
DO 620 N = 1, K1, 2
ENP2 = K * P1 / 2.
ENP2S = ENP2 * ENP2
EN5 = PHINS * (ENP2S + ALMDS * PHIS)
BN = DSGRT(BNS)
X1 = ALMDS * PHIS * ENP2S
IF (X1, LT, 0.0) GO TO 605
SN = PHINS * X1
GN = DSGRT(GNS)
TD1N = GN * DCOSH(GN) * (GNS * ENP2S * POIS * PHINS) * (BN * DCOSH(BN) * (BN - ENP2S * POIS * PHINS))
TD1N = (BN * TD1N * DCOSH(BN) + GN * DCOSH(GN)) * (BN * DCOSH(BN) + GN * DCOSH(GN) * TD2P)
IF (ETA, EQ, 0.0) GO TO 602
IF (PSI, EQ, 1.0) GO TO 603
X1 = EN(H) * (TD1N * DSIN(H) * DSIN(H)) * DSIN(ENP2S * ETA) / TD1N
IF (PSI, EQ, 1.0) GO TO 601
X1 = EN(H) * (DCOSH((1 - PSI) * dN) - TD1P * DCOS((1 - PSI) * dN)) * DSIN(1 * ENP2 * ETA) / TD1P
GO TO 604
601 X3 = EP(N) * (1 - TD1P) * DSIN(ENP2 * ETA) / TD1N
GO TO 604
602 X2 = 0.0
X3 = 0.0
GO TO 604
603 X2 = 0.0
X3 = 0.0
GO TO 604
604 X2 = 0.0
X3 = 0.0
GO TO 604
605 X1 = X1
GNS = PHINS * X1
GN = DSGRT(GNS)
IF (ENS, GT, 22) GO TO 610
TD2N = GN * (POIS * PHINS * ENP2S - GNS) * DCOSH(GN) / (BN * (BN - POIS * PHINS * ENP2S) * DCOSH(BN))
TD2N = TD2N * BN * DCOSH(BN) + GN * DCOSH(GN)
TD2P = DCOSH(BN) / DCOSH(GN)
TD2P = BN * DSINH(BN) - GN * TD2P * DSINH(GN)
IF (ETA, EQ, 0.0) GO TO 607
IF (PSI, EQ, 1.0) GO TO 608
X2 = EN(H) * (TD2N * DSINH(BN) * PSI) + DSINH(GN * PSI) * DSIN(ENP2 * ETA) / TD2N
IF (PSI, EQ, 1.0) GO TO 606
X#3=EP(N)*(DCUSH((1-PSI)*BN)-TD2P*DCOSH(1-PSI)*GN))*DSIN(ENP2*ETA)/TD22P
GO TC 609

606 X#3=EP(N)*(1-TD2P)*DSIN(ENP2*ETA)/TD22P.
GO TC 609

607 X#2=0.0
X#3=0.0
GO TC 609

608 X#2=0.0
X#3=EP(N)*(DCOSH(BN)-TD2P*DCOSH(GN))*DSIN(ENP2*ETA)/TD22P

609 Z1*(I,J)=Z2+XW2
Z3(I,J)=Z3+XW3
Z2=Z1*(I,J)
Z3=Z3*(I,J)
GO TC 620

610 TD2N=BN*(PO1S*PHINS*ENP2S-GNS)/(BN*(BNS-PO1S*PHINS*ENP2S))
IF (ETA<EO.0.0) GO TO 612
IF (PSI<EO.0.0) GO TO 612
B=1.0
TEST=BN-BN*PSI
IF (TEST<GT.60.) B=0.0
X#2=BN*(1+D2N)*EXP((BN*PSI-BN)*B)+D2N*(GN*PSI-GN)*B)+
DSIN(ENP2*ETA)/(BN*TD2N+GN)
TEST=BN*PSI
IF (TEST<GT.60.) GO TO 611
X#3=EP(N)*(D2N*PSI-BN)-D2N*(-PSI*GN))*DSIN(ENP2*ETA)/(BN-GN)
GO TO 614

611 X#2=0.0
GO TO 614

612 X#2=0.0
X#3=0.0

614 Z1*(I,J)=Z2+XW2
Z3(I,J)=Z3+XW3
Z2=Z1*(I,J)
Z3=Z3*(I,J)

620 CONTINUE
GO 630 M=1.K1.2
ENP2=ENP2*EPS

622 M=PO1S*PHINS

624 M1=EM(N)*D2N*(BN*ETA)*DSIN(BN*ETA))
IF (BNS<GT.22) GO TO 627
15. TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCUSH(GM)/(BM*(BMS-POIS*PHIS*
EMP2S)*DCOSH(BM))
TD2M=TD2M*BM*DCSHE(BM)+GM*DCOSH(GM)
IF (ETA.EQ.0.0) GO TO 626
IF (PSI.EQ.0.0) GO TO 626
XW1=EM(M)*(TD2M*DSINH(BM*ETA)+DSINH(GM*ETA)) *DSIN(EMP2*PSI)/TD2M
GO TO 629
629 XW1=0.0
GO TO 629
627 TD2M=GM*(POIS*PHIS*EMP2S-GMS)/(BM*(BMS-POIS*PHIS*EMP2S))
IF (ETA.EQ.0.0) GO TO 628
IF (PSI.EQ.0.0) GO TO 628
TEST=BM-BM*ETA
IF (TEST.GT.60.) GO TO 628
XW1=EM(M)*(TD2M*DEXP(BM*ETA-BM)+DEXP(GM*ETA-GM)) *DSIN(EMP2*PSI)/
1(BM+TD2M+GM)
GO TO 629
628 XW1=0.0
629 W1(I,J)=W1+XW1
W1=W1(I,J)
CONTINUE
430 CONTINUE
PRINT 635, PSI, ETA, I, J, W1(I,J), W2(I,J), W3(I,J), I, J, W1(I,J)
635 FORMAT('PSI = ', 5F5.3, ' ETA = ', 5F5.3, ' W1 = ', 5F5.3, ', W2 = ', 5F5.3, ', W3 = ', 5F5.3, ', I = ', I3, ', J = ', I3, ', W1 = ', 5F5.3)
PS1=PSI+1.0/DFLOAT(KS)
CONTINUE
RETURN
END
PROGRAM 5

This is a symmetric mode eigenvalue search program for the rectangular cantilever plate free vibration problem, with point supports symmetrically located on the edges normal to the clamped base (one point support on each edge.)

Usage: The following variables must be provided:

1. K = the number of terms to be used in the series involved. For 5 digits accuracy, use k = 30.

2. CSI = 0 to 1, providing the dimensionless distance of the point support to the X or PSI axis. See Figure 3.1.

3. PHIR = 2π/A, is the full plate aspect ratio.

4. ALMDS = an initial starting value for the eigenvalue search.

5. DLIM = a finishing or eigenvalue search ending limit. It instructs the computer when to halt execution.

6. DEL = eigenvalue increment.

7. POI = Poisson's ratio.

IMPLICIT REAL*6(A-H,O-Z)
DIMENSION A(99,99)
K=????????????????????????????
Z2=3600
CSI=????????????????????
PHIR=????????????????????
PHI=PHIR/2
PRINT 50,PHIR,K,CSI
50 FORMAT('1',10X,'PHIR=',F8.4,10X,'K=',I4,10X,'CSI=',F8.4,1///)
K1=2*K-1
C=1
PI=4.*DATANIC
PHIS=PHI*PHI
POI=????????????????????
POI=2-POI
PHIN=1./PHI
PHINS=PHIN*PHIN
UFIS=POI*PHIS
UFIS=POI*PHINS
I=3*K
ALMDS=1
DLIM=1
DEL=0
1. CONTINUE
ALMDS=ALMDS+DEL

C. INITIALIZE THE MATRIX A

L=1+1
DO 2 M=1,L
DO 2 N=1,L
A(M,N)=0.0
2 CONTINUE
DO 3 N=1,K
DO 10 M=1,K+1
DO 10 N=1,K
JN=N-1
EMP2=M+P*I/2
EMP25=EMP2*EMP2
ENP=JN+1
ENPS=ENP*ENP
BNS=[1.+PHIS]*[ENP*ENP*ALMDS*PHIS]
BN=DSQRT(BNS)
XI=ALMDS*PHIS*ENP*ENP
IF (XI.LT.0.0) GO TO 5
GNS=[1.+PHIS]*X1
GN=DSQRT(GNS)
X=POIS*PHINS*ENPS
TD1=(BN+[BNS-X1]*DCOSH(BN))/([GN*GNS+X1]*DCOS(GN))
TD1P=DCOSH(BN)/DCOS(GN)
TUFS=ENPS
TD1IN=-BN*DCOSH(BN)+TD1*GN*DCOS(GN)
TD1IP=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
IF (M.GT.1.0) GO TO 4
A11+1,N+2*K]=[(DCOSH(BN+[1-CS1])]-TD1P*DCOS(GN)*([1-CS1]*DCOS(ENP))/
TD1IP
A11+1,N+2*K]=[(DSINH(BN+CS1)*TD1*DSIN(GN)*CS1)*DCOS(ENP)/TD1IN
A(N+2*K,N+2*K]=(X*[(1-TD1P)-BNS-GNS*TD1P]/TD11P
X1=([ENPS-UFS*BN]/[BNS*EMP2S]+BNS*DCOSH(BN)
X2=TD1P*[ENPS-UFS*GNS]/([GNS-EMP2S]*/DCOS(GN)
XX=TD1P*DSINH+EMP2S*DCOSH(BN)
XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD11P
A11+1,N+2,K]=XX
GO TO 10
5 X1=X1
GNS=[1.+PHIS]*X1
GN=DSQRT(GNS)
K=POIS*PHINS*ENPS
IF (BNS.LT.Z2) GO TO 6
TD2=BN*BN-BNS-X1)/(GNS*(X-GNS))
X=UFIS*ENPS
TD2P=BN-GN
TD22N = BN + TD2 * GN
IF (N GT 1.0) GO TO 7
B = 1.
TEST = BN * CSI
IF (TEST GT 60) B = 0.0
A(N+2 * K) = DCOS(ENP) * (DEXP((-BN * CSI + B) - DEXP((-GN * CSI + B)) + B)
1(BN = GN)
B = 1.0
TEST = BN - BN * CSI
IF (TEST GT 60) B = 0.0
A(L + N + K) = DCOS(ENP) * (DEXP((-BN + BN * CSI + B) * TD2 + DEXP((-GN + GN * CSI - B)) + 1)
B/B / (BN + TD2 + GN)
A(N + K, N + K) = (X - BNS * TD2 * (X - GNS)) / TD2N
2(A + 2 * K, N + K) = 0.0
A(N + K, N + 2 * K) = 0.0
GO TO 7
6 TD2 = [BN * BNS - X] * DCOSH(BN) / [GN * (X - GNS) * DCOSH(GN)]
TD2P = DCOSH(BN) / DCOSH(GN)
X = UFS * ENPS
TD2P = BN * DSINH(BN) + BN * TD2P * DSINH(GN)
TD22N = BN * DCOSH(BN) + TD2 * GN * DCOSH(GN)
IF (M GT 1.0) GO TO 8
7 XX = (X + 1.0 * N + K) * (X - BNS) * DSINH(BN) + TD2 * (X - GNS) * DSINH(GN) / TD2N
2(A + K, N + K) = DCOSH(BN + 1.0 - CSI) * TD2P * DSINH(GN) * (1.0 - CSI) * DCOS(ENP)
1/TO2P
A(N + 1.0 * N + K) = (X + 1.0 * N + K) * (X - BNS) * DSINH(BN) + TD2 * (X - GNS) * DSINH(GN) / TD2N
2(A + 2.0 * K, N + K) = (UFS * ENPS * (1.0 - TD2P)) - BNS + GNS * TD2P / TO2P
GO TO 8
8 XX = (X + 1.0 * ENPS - UFS * BNS) / (BNS + EMP25) * GN
XX = (X + 1.0 * ENPS - UFS * BNS) / (BNS + EMP25)
XX = (X + 1.0 * ENPS - UFS * GNS) / (GNS + EMP25)
XX = 2 * DCOS(ENP) * EMP2 * (X1 - X2) / TO2P
A(N + 1.0 * N + K) = XX
GO TO 10
10 CONTINUE
DO 20 M = 1, K1, 2
JNX = N - 1.
ENP = JN * PI
ENPS = ENP * ENP
EMP2 = M * PI / 2.
EMP2S = EMP2 * EMP2
BNS = PHS * (ALMDS + EMP2S)
BN = DSORT(BNS)
XX1 = ALMDS + EMP2
IF (XX1 LT 0.0) GO TO 13
GN = DSORT(GNS)
GO TO 13

TD1L = BM * DSINH(BM) / (GM * DSIN(GM))
TD1L = -2 * DSIN(EMP2 * CSI) / (BM * (BMS - POIS * PHIS * EMP2S) * DSINH(BM))
1 + TD1L * GM / (GM * DSINH(BM) / DSIN(GM))
TD1 = (BM * (BMS - POIS * PHIS * EMP2S) * DSINH(BM)) / (GM * (GM + POIS * PHIS * EMP2S) * DSINH(GM))
1 = DSIN(GM)
TD1 = TD1 * (-1)
TD1L = BM * DSINH(BM) / TD1 * GM / DSINH(GM)
XX = (BM * DSINH(BM)) / (BMS + ENPS) + TD1 * GM / DSINH(GM) / (GM - 1) * ENPS / 2 * EMP2 * DCOS(ENP) / TD11
X1 = BM * DCOS(ENP) * DSINH(BM) / (BMS + ENPS)
X2 = GM * TD1L * DCOS(ENP) * DSINH(GM) / (BMS - ENPS)
XX1 = 2 * TD1L * EMP2 * (X1 + X2)
IF (JN - GT, 0.0) GO TO 11
XX = XX2
XX1 = XX1 / 2
11
A(N + 2, K, 1 + 1 / 2) = XX
A(N + 2, K, I + 1) = A(N + 2, K, I + 1) + XX1
X1 = (EMP2S - UFIS * BMS) / (BMS + ENPS) * BM * DCOS(ENP) / DSINH(BM)
X2 = (EMP2S - UFIS * GM) / (BMS + ENPS) * GM * DCOS(ENP) / DSINH(GM) / TD1
XX = (X2 + X1) / 2 * DSINH(EMP2) / TD11
XX1 = 2 * TD1L * DCOS(EMP2) / (X1 + X2)
XX2 = TD1L * (DCOSH(BM) + TD1L * COSH(GM)) / DSINH(EMP2 * CSI)
XX3 = TD1L * (UF5 * EMP2S - BMS) / DCOSH(BM) + TD1L * (UF5 * EMP2S + GM) / 1 * DCOSH(GM)
XX4 = (DCOSH(BM) + TD1L * COSH(GM)) * DSINH(EMP2 * CSI) / TD11
IF (JN - GT, 0.0) GO TO 12
XX = XX2
XX1 = XX1 / 2
A((M + 1) / 2, (M + 1) / 2) = (UF5 * EMP2S - BMS) / DCOSH(BM) + TD1 * (1)
UF5 * EMP2S + GM / DCOSH(GM) / TD11
A((M + 1) / 2, I + 1) = XX3
A(I + 1, (M + 1) / 2) = XX4
A(I + 1, I + 1) = A(I + 1, I + 1) + XX2
A(N + K, (M + 1) / 2) = XX
A(N + K, I + 1) = A(N + K, I + 1) + XX1
GO TO 20
12
X1 = -X1
GM = PHIS * X1
GN = DSGRT(GM)
IF (BMS + GT = 22) GO TO 15
TD2 = (BM * (BMS - POIS * PHIS * EMP2S) * DSINH(BM)) / (-1) / (GM * (GM - POIS * 1PHIS * EMP2S) * DSINH(GM))
TD2 = BM * DSINH(BM) + TD2 * GM / DSINH(GM)
TD2L = -2 * DSINH(EMP2 * CSI) / (BM * (BMS - POIS * PHIS * EMP2S) * DSINH(BM) + 1 * TD2L * GM * (GM - POIS * GM * EMP2S) / DSINH(GM))
X1 = BM * DSINH(BM) + (BMS + ENPS)
X2 = GM * TD2 * DSINH(GM) / (GM + ENPS)
XX = 2 * DCOSH(ENP) * EMP2 * (X1 + X2) / TD2
X1 = (EMP2S - UFIS * BMS) / BMS + DSINH(BM) / (BMS + ENPS)
X2 = (EMP2S - UFIS * GM) / BMS + DSINH(GM) / (BMS + ENPS)
XX1 = 2 * DSINH(EMP2) * DCOSH(ENP) * (X1 + X2) / TD2
XX = BM + DSINH(BM) / (BMS + ENPS)
XX2 = TD2L * DSINH(GM) / (BMS + ENPS)
XX2 = 2 * TD2L * EMP2 * (X1 + X2) * DCOS(ENP)
X1 = (EMP2S - UFIS * BMS) / GM * DSINH(BM) / (BMS + ENPS)
X2 = (EMP2S - UFIS * GM) / GM * DSINH(GM) / (BMS + ENPS)
XX3 = 2 * TD2L * DSINH(EMP2) * (X1 + X2) * DCOS(ENP)
IF (JN - GT, 0.0) GO TO 14
XX4=TD22L*(UFS*EMP2S-BMS)*DCOSH(BM1)*TD2L*(UFS*EMP2S-GMS)
1*DCOSH(GM1)
XX5=TD22L*(DCOSH(BM1)*TD2L*DCOSH(GM1)*DSIN1EMP2*CSI)
XX6=DCOSH(BM1)*TD2*DCOS1HIGH1*DSIN1EMP2*CSI)/TD22
A(I+1,M+1/2)=XX6
A(I,M+1/2,I+1)=XX4
A(I+1,I+1)=A(I+1,I+1)+XX5
XX=XX/2
XX1=XX1/2
XX2=XX2/2
XX3=XX3/2
A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS)*DCOSH(BM1)+TD2*(UFS*EMP2S-
1GMS)*DCOSH(GM1)/TD22
14A(N+2*K,(M+1)/2)=XX
A(N+K,(M+1)/2)=XX
A(N+2*K,(I+1))=A(N+2*K,(I+1))+XX2
A(N+K,(I+1))=A(N+K,(I+1))+XX3
GO TO 20
15TD22=(BM1*(BMS-POIS*PHIS*EMP2S1)*(1-GM1*(GMS-POIS*PHIS*EMP2S))
TD22=BM1*(TD2*GM)
TD2L=-BM1/GM
TD22L=2*DSIN1EMP2*CSI)/(BM1*(BMS-POIS*PHIS*EMP2S)+TD2L*GM1*(GMS-
1POIS*PHIS*EMP2S))
XX1=BM1/(BMS+ENPS1)
XX2=GM1/TD21/(GMS+ENPS)
XX2=DCOSH1ENP)+EMP2*(XX1+XX2)/TD22
XX1=EMP2S-UFS1BMS1*BM1/(BMS+ENPS)
XX2=EMP2S-UFS1GMS1*TD2*GM1/(GMS+ENPS)
XX1=2*DSIN1EMP2)*DCOSH1ENP)*(XX1+XX2)/TD22
XX1=BM1/(BMS+ENPS)
XX2=GM1/TD21/(GMS+ENPS)
XX2=2*TD22L*EMP2*(XX1+XX2)*DCOSH(EMP)
XX1=EMP2S-UFS1BMS1*BM1/(BMS+ENPS)
XX2=EMP2S-UFS1GMS1*TD2*GM1/(GMS+ENPS)
XX3=2*TD22L*DSIN1EMP2)*(XX1+XX2)*DCOSH(EMP)
IF (IJN. GT. 0.0) GO TO 14
XX=XX2/2
XX1=XX1/2
XX2=XX2/2
XX3=XX3/2
XX4=TD22L*(UFS*EMP2S-BMS)+TD2L*(UFS*EMP2S-GMS)
XX5=TD22L*(1+TD2L)*DSIN1EMP2*CSI)
XX6=(1+TD2)*DSIN1EMP2*CSI)/TD22
A(I+1,(M+1/2)=XX6
A(I,(M+1/2,I+1)=XX4
A(I+1,I+1)=A(I+1,I+1)+XX5
A((M+1)/2,(M+1)/2)=((UFS*EMP2S-BMS1+TD2*(UFS*EMP2S-GMS)))/TD22
GO TO 14
20 CONTINUE
CALL DETERM (A, L, DET)
PRINT I10, ALMDS, DET
110 FORMAT (9, ALMDS=' ', F12.6, 10X, 'DET=' , F20.6)
IF (ALMDS. LT. DLIM) GO TO 1
STOP
END
SUBROUTINE DETERM (A, N, DET)
IMPLICIT REAL*8 (A-M, O-Z)
DIMENSION A(39,39)
SIGN=0
LAST=N-1
START OVERALL LOOP FOR (N-1) PIVOTS

DO 200 I=1, LAST

FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT

BIG=0.

DO 50 K=1, N
TERM=ABS(A(K, I))
IF (TERM-BIG) > 0, 50, 30

30 BIG=TERM

L=K

50 CONTINUE

CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND

IF (BIG) > 0, 60, 80

L-TH ROW HAS THE BIGGEST TERM----IS I=L

80 IF (I-L) > 0, 120, 90

I IS NOT EQUAL TO L, SWITCH ROWS I AND L

90 SIGN=-SIGN

DO 100 J=1, N

TEMP=A(I, J)

A(I, J)=A(L, J)

100 A(L, J)=TEMP

NOW START PIVOTAL REDUCTION

120 PIVOT=A(I, I)

NEXTR=I+1

FOR EACH OF THE ROWS AFTER THE I-TH

DO 200 J=NEXTR, N

MULTIPLYING CONSTANT FOR THE J-TH ROW IS

CONST=A(J, I)/PIVOT

NOW REDUCE EACH TERM OF THE J-TH ROW

DO 200 K=1, N

200 A(J, K)=A(J, K)-CONST*A(I, K)

END OF PIVOTAL REDUCTION----NOW COMPUTE DETERMINANT

DET=SIGN

DO 300 I=1, N

300 DET=DET*A(I, I)/10.

GO TO 61

60 DET=0.

61 RETURN

END
PROGRAM D

THIS IS A SYMMETRIC MODE SHAPE PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM. WITH POINT SUPPORTS SYMMETRICALY LOCATED ON THE EDGES NORMAL TO THE CLAMPED BASE. ONE POINT SUPPORT ON EACH EDGE.

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:

1. - K  = AS DEFINED IN PROGRAM 5.
2. - CSI  = AS DEFINED IN PROGRAM 5.
3. - PHIR  = 2B/A FULL PLATE ASPECT RATIO.
4. - ALMDS = EIGENVALUE.
5. - KS  = NUMBER OF POINTS AT WHICH THE DISPLACEMENT IS REQUIRED.
6. - POI  = POISSON'S RATIO.

IMPLICIT REAL*8(A-H,O-Z)
COMMON Z2*,PHI,P1,P1S,P01,POI,PHINS,UFIS,UFIS,ALMDS,CSI,K,K1

1K
DIMENSION A(99,99)
K=????????????????????????????
KS=?????????????????????????
Z2=3600
CSI=????????????????????????
PHIR=???????????????????????
ALMDS=??????????????????????
PHI=PHIR/2
PHINTS0,K,KS,CSI,PHIR,ALMDS
50 FORMAT(*11,10X,'K'=13,10X,'KS'=13,10X,'CSI'=1F7.4,10X,
'PHIR'=1F7.4,10X,'ALMDS'=1F8.4)///)
K1=2*K-1
C=1
PI=4.4ATAN(C)
PHIS=PH1*PHI
POI=???????????????????????
POI/=POI
PHIN=1./PHI
PHINS=PHIN*PHIN
UFIS=FC1*PHI'S
UFIS=POI*PHINS
I=3*K
1 CONTINUE
INITIALIZE THE MATRIX A

L=1+1
DO 2 M=1,L
GO TO 2
3 N=1+1
A(N+2*K, N+2*K) = -1.0
DO 10 M=1,K
DO 10 N=1,K
JN=N-1
EMP2=M*PI/2
EMP2=EMP2*EMP2
ENP=JN*PI
ENPS=ENP*ENP
BN=(1.0/PHIS)*(ENP*ENP+ALMDS*PHIS)
BN=DSINH(BN)
X1=ALMDS*PHIS*ENP*ENP
IF (X1.LT.0.0) GC TO 5
GNS=(1.0/PHIS)*X1
GNS=DSINH(GNS)
X=PHIS*ENPS
T11N=BN*DCOSH(BN)+TD1*GNS*DCOS(GNS)
TD1=BN*DSINH(BN)+GNS*TD1*DSINH(GNS)
IF (X.GT.1.0) GO TO 4
A11+1,N+2*K)=(DCOSH(BN*(1-CSI))-TD1*DCOS(GNS*(1-CSI)))*DCOS(ENP)/
10
A11+1,N+K)=(DSINH(BN+CSI)+TD1*DSINH(GNS+CSI))*DCOS(ENP)/TD11N
A(N+K,K+1)=(DSINH(BN+CSI)+TD1*DSINH(GNS+CSI))*DCOS(ENP)/TD11N
10 CONTINUE
X1=(ENPS=UFS+BN)/(BN+EMP2)*BN*DCOSH(BN)
X2=(TD1*ENPS*UFS+GNS)/(GNS-EMP2)*GNS*DCOS(GNS)
XX=(DCOS(EMP2)*DSIN(EMP2)/TD11N)*(X1-X2)
A11*(N+1)/2.N+K)=XX.X+2.*0
X1=(ENPS=UFS+BN)/(BN+EMP2)*DCOS(BN)
X1=(TD1*ENPS=UFS+GNS)/(GNS-EMP2)*DCOS(GNS)
XX=2*DCOS(ENP)*EMP2*(X1+X2)/TD11P
A11*(N+1)/2.N+2*K)=XX
GO TO 10
5 X1=X1
GNS=(1.0/PHIS)*X1
GNS=DSINH(GNS)
X=PHIS*ENPS
IF (BN.LT.22) GO TO 6
T22=BN*(BN-X)/(GNS*(X-GNS))
X=UFS*ENPS
T22=BN-GN
T22=BN+TD2*GN
IF (X.GT.1.0) GO TO 7
BN=1.0
TEST=BN+CSI
IF (TEST.GT.60) B=0.0
A(N+2*K)=DCOS(ENP)*(DEXP((-BN+CSI)*A)-DEXP((-BN+CSI)*B))/B
1(BN-GN)
B=1
TEST=BN-BN*CSI
IF (TEST<GT.60) B=0
A(N+N+K)=DCOS(EXP)*DEXP((-BN+BN*CSI)*B)+TD2*DEXP((-GN+GN*CSI)*B)
1+B/(BN+TD2+GN)
A(N+N+K)=B/(X-BS*TD2*(X-GNS))/TD2N
A(N+2+K,N+K)=0.0
A(N+N+2*K)=0.0
GO TO 7

6 TD2=BN*(BN-X)*DCOSH(BN)/*GN*(X-GNS)*DCOSH(GN)
TD2P=DCOSH(BN)/DCOSH(GN)
T=UFS*ENPS
TD2P=BN*DSIN(BN)-GN*TD2P*DSINH(GN)
TD2N=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
IF (N+N+1)<1.0) GO TO 8
A(I+1,1+N+2*K)=TD2P*DCOSH(1-CISI)*/DCOS(ENP)
1/TD2P
A(I+1,1+N+2*K)=TD2P*DCOSH(1-CISI)*/DCOS(ENP)
1/TD2P
A(I+1,1+N+2*K)=TD2P*DCOSH(1-CISI)*/DCOS(ENP)
1/TD2P

7 X1=(ENPS-UFS*BN)/(BNS+EMP25)*BN
X2=(ENPS-UFS*GNS)/(GNS+EMP25)*GN
X=DCOS(ENP)*DSINH(BN)/TD2N*(X1*X2)
A(I+1,1+2+K,1+2+K)=X
X=ENPS-UFS*BN/(BNS+EMP25)
X=ENPS-UFS*GNS/(GNS+EMP25)
X=DCOS(ENP)*EMP2*(X1*X2)/TD2P
A(I+1,1+2+K,1+2+K)=X

8 X1=(ENPS-UFS*BN)/(BNS+EMP25)*BN*DCOSH(BN)
X2=(ENPS-UFS*GNS)/(GNS+EMP25)*GN*DCOSH(GN)*TD2
X=DCOS(ENP)*DSINH(EMP2)*TD2N*(X1*X2)
A(I+1,1+2+K,1+2+K)=X
X=ENPS-UFS*BN/(BNS+EMP25)*DCOSH(BN)
X=TD2P*DCOSH(BN)*ENPS-UFS*GNS/(GNS+EMP25)*DCOSH(GN)
X=DCOS(ENP)*EMP2*(X1*X2)/TD2P
A(I+1,1+2+K,1+2+K)=X

CONTINUE
DJ 20 N=1,K
DJ 20 N=1,K
JN=N-1
ENPS=ENPS+1
EMPS=EMP2*EMPS/2
EMPS=EMPS*EMPS
BNS=PHIS*(ALMDS+EMP25)
EM=DSRT(BNS)
X1=ALMDS-EMP25
IF(X1<T-4.0) GO TO 13
GNS=PHIS*1
GNS=DSRT(GNS)
TD1L=BN*DSINH(BN)/(BNS+DSIN(BN))
TD1LL=2*DSINH(EMP2)*CSI/(BNS+GNS*PO1*PHIS*EMP25)*DSINH(BN)
TD1=TD1L+GNS*PO1*PHIS*EMP25+GNS*PO1*PHIS*EMP25
1*DSINH(BN)/(GNS*PO1*PHIS*EMP25)
1*DSINH(BN)/(GNS*PO1*PHIS*EMP25)
1*DSINH(BN)/(GNS*PO1*PHIS*EMP25)

TD1=TD1T1(-1)
A(I+1,J+1) = A(I+1,J+1) + XX5
XX = XX / 2
XX1 = XX1 / 2
XX2 = XX2 / 2
XX3 = XX3 / 2
A(N+1,M+1) = ((UFS*EMP2S-BMS)*DCUSH(BM) + TD2*(UFS*EMP2S-GMS)) / TD22

14 A(N+2*K,M+1) / 2 = XX
A(N+2*K,M+1) / 2 = XX
A(N+2*K,1+1) = A(N+2*K,1+1) + XX2
A(N+K,1+1) = A(N+K,1+1) + XX3

GO TO 20

15 TD2 = (BM*(BMS-POIS*PHIS*EMP2S)) / (GM*(GMS-POIS*PHIS*EMP2S))
TD22 = BM + TD2 * GM
TD2L = BM / GM
TD2 = 2 * DSIN(EMP2*CS1) / (BM*(BMS-POIS*PHIS*EMP2S) + TD2L*GM*(GMS-POIS*PHIS*EMP2S))
X1 = BM / (BMS+ENPS)
X2 = GM * TD2 / (GMS+ENPS)
XX = 2 * DCOS(ENP) * EMP2 / (X1 + X2) / TD22
X1 = (EMP2S-UFIS*BMS) * BM / (BMS+ENPS)
X2 = (EMP2-UFIS*GMS) * TD2 * GM / (GMS+ENPS)
XX1 = 2 * DSIN(EMP2) * DCOS(ENP) / (X1 + X2) / TD22
X1 = BM / (BMS+ENPS)
X2 = GM * TD2L / (GMS+ENPS)
XX2 = 2 * TD22L * EMP2 / (X1 + X2) * DCOS(ENP)
X1 = (EMP2-UFIS*BMS) * BM / (BMS+ENPS)
X2 = (EMP2-UFIS*GMS) * TD2L * GM / (GMS+ENPS)
XX3 = 2 * TD22L * DSIN(EMP2) * (X1 + X2) * DCOS(ENP)
IF (JN,GT,0,0) GO TO 14

XX = XX / 2
XX1 = XX1 / 2
XX2 = XX2 / 2
XX3 = XX3 / 2
XX4 = TD22L * ((UFS*EMP2S-BMS) + TD2L*(UFS*EMP2S-GMS))
XX5 = TD22L * (1+TD2L) * DSIN(EMP2*CS1)
XX6 = (1+TD2) * DSIN(EMP2*CS1) / TD22
A(I+1,N+1) / 2 = XX6
A(N+1,M+1) / 2 = XX4
A(I+1,J+1) = A(I+1,J+1) + XX5
A(N+1,M+1) / 2 = ((UFS*EMP2S-BMS) + TD2*(UFS*EMP2S-GMS)) / TD22
GO TO 14

20 CONTINUE
DQ 30 N = 1, L
TEMP = A(M+1)
A(M+1) = A(N+1,1+1)
A(N+1,1+1) = TEMP

30 CONTINUE
CALL DETERM (A,L,DET)
STOP
END

SUBROUTINE DETERM (A,N,DET)
IMPLICIT REAL*8 (A-M,0-2)
DIMENSION A(99,99), X(99), EM(75), N(75), EP(75)
SIGN = 1
N = 1
LAST = M-1

START OVERALL LOOP FOR (N-1) PIVOTS
DO 200 I=1, LAST
FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
   BIG=0.
   DO 50 K=1, N
      TERM=ABS(A(K, I))
      IF (TERM-BIG) 50, 50, 30
   30   BIG=TERM
       L=K
   50   CONTINUE
CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
   IF (BIG) 80, 60, 80
     L-TH ROW HAS THE BIGGEST TERM----IS I=L
     80   IF (I-L) 90, 120, 90
       I IS NOT EQUAL TO L, SWITCH ROWS I AND L
   90   DO 100 J=1, N
        TEMP=A(I, J)
        A(I, J)=A(L, J)
     100   A(L, J)=TEMP
NOW START PIVOTAL REDUCTION
   120   PIVOT=A(I, I)
       NEXTR=I+1
       FOR EACH OF THE ROWS AFTER THE I-TH
       DO 200 J=NEXTR, N
           MULTIPLYING CONSTANT FOR THE J-TH ROW IS
           CONST=A(J, I)/PIVOT
           NOW REDUCE EACH TERM OF THE J-TH ROW
           200   DO 200 K=1, N
                A(J, K)=A(J, K)-CONST*A(I, K)
           END OF PIVOTAL REDUCTION----PERFORM BACK SUBSTITUTION
       M=N-1
       DO 500 I=1, M
           IREV IS THE BACKWARD INDEX, GOING FROM M BACK TO 1
           IREV=M+1-I
           GET Y(IREV) IN PREPARATION
           Y=A(IREV, N)
           IF (IREV=M) 400, 500, 400
           NOT WORKING ON LAST ROW, I IS NOT GREATER
400 DO 420 J=2,1
      *WORK BACKWARD FOR X(N),X(N-1)------SUBSTITUTING PREVIOUSLY
      FOUND VALUES

      K=N+1-J

450 Y=Y-A(IREV,K)*X(K)

      FINALLY, COMPUTE X(IREV)

500 X(IREV)=Y/A(IREV,IREV)

      FIND AND PRINT EM, EN, EP, AND P*

      L=(N-1)/3
      DO 550 I=1,L
          J=2*I-1
          K=I-1
          EM(J)=X(J)
          EN(J)=X(J+L)
          IF(I.EQ.L) GO TO 540
          EP(J)=X(J+2*L)
          P=X(3*L)
      GOTO 550

540 EP(J)=1.0
550 PRINT 600, K,K,J,EM(J),K,EN(J),K,EP(J),P

600 FORMAT('K,K,J,EM,EN,EP,P')

601 DO 601 I=1,L
      J=2*I-1
      K=I-1
      EM(J)=EN(J)
      EP(J)=EP(J)
601 CONTINUE

650 PRINT 650

650 PRINT 650, 'THE SYMMETRIC MODE SHAPE DATA ARE://///)

CALL SHAPE (EM, EN, EP, P)

690 RETURN
      END

SUBROUTINE SHAPE (EM, EN, EP, P)
      IMPLICIT REAL*8(A-H,O-Z)
      COMMON 22, PHI, PI, PHI1, PSI1, PSI2, PHI2, PHII, PHIS, UFS, UFI1, ALMOS, CSI, K, K1,
      K2, KS
      DIMENSION M(21,21), W(21,21), W(21,21), W(21,21), W(21,21), W(21,21)
      1, M(75), EN(75), EP(75)
      KSI=KS+1
      ETA=0.0
      DO 650 I=1,KSI
          PSI=1.0
          DO 640 J=1,KSI
              W1=0.0
              W2=0.0
              W3=0.0
              W4=0.0
              DO 620 N=1,K
                  JN=N-1
                  ENP=JN+PI
                  EPS=ENP*ENP
                  DNS=(1./PHIS)*(ENP*ENP+ALMOS*PHIS)
EN=DSGRT(BNS)
X1=ALMDS*PHIS-ENP*ENP
IF (X1.LT.0.0) GO TO 606
GNS=(1.*PHIS)*X1
GN=DSGRT(GNS)
X=PGIS*PHINS*ENPS
TD2=(BN*(BNS-X)*DCUSH(BN))/(GN*(GNS+X)*DCOS(GN))
TD2P=DCUSH(BN)/DCOS(GN)
X=UFIS*ENPS
TD21N=BN*DCUSH(BN)+TD1*GN*DCOS(GN)
TD21P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
IF (ETA.EQ.0.0) GO TO 601
IF (ENP.EQ.0.0) GO TO 601
XX=DCOS(ENP*ETA)
GO TO 602

601
XX=1.0
602 IF (PSI.EQ.0.0) GO TO 604
XX2=EN(N)*(DSINH(BN+PSI)+TD1*DSIN(GN+PSI))*XX/TD21N
IF (PSI.EQ.1.0) GO TO 603
XX4=EP(N)*(DCOSH(BN+(1-PSI))-TD1P*DCOS(GN+(1-PSI)))*XX/TD21P
GO TO 617
603
XX3=EP(N)*(1-TD1P)*XX/TD21P
GO TO 617
604
XX2=0.0
XX3=0.0
GO TO 617
606 X1=-X1
GNS=(1.*PHIS)*X1
GN=DSGRT(GNS)
X=PGIS*PHINS*ENPS
IF (BNS.GT.22) GU TO 612
TD2=(BN*(BNS-X)*DCUSH(BN))/(GN*(X-GNS)*DCOS(GN))
TD2P=DCUSH(BN)/DCOS(GN)
TD22N=BN*DCUSH(BN)-GN*TD2P*DSINH(GN)
TD22N=BN*DCUSH(BN)+TD2*GN*DCUSH(GN)
IF (ETA.EQ.0.0) GO TO 607
IF (ENP.EQ.0.0) GO TO 607
XX=DCOS(ENP*ETA)
GO TO 608
607
XX=1.0
608 IF (PSI.EQ.0.0) GO TO 610
XX2=EN(N)*(DSINH(BN+PSI)+TD2*DSINH(GN+PSI))*XX/TD22N
IF (PSI.EQ.1.0) GO TO 609
XX4=EP(N)*(DCOSH(BN+(1-PSI))-TD2P*DCOSH(GN+(1-PSI)))*XX/TD22P
GO TO 617
609
XX3=EP(N)*(1-TD2P)*XX/TD22P
GU TO 617
610
XX2=0.0
XX3=0.0
GO TO 617
612 TD2=EN*(BNS-POIS*PHINS*ENPS)/(GN*(POIS*PHINS*ENPS-GNS))
IF(ETA.EQ.0.0) GO TO 613
IF (ENP.EQ.0.0) GO TO 613
XX=DCOS(ENP*ETA)
GO TO 614
613
XX=1.0
614 IF (PSI.EQ.0.0) GO TO 616
B=1.0
TEST=BN-BN*PSI
IF (TEST.GT.60.0) B=0.0.
624 \( x_{w1} = EM(x)(1+TD2)/(TD2) * DSIN(EMP2 * PSI)/TD22 \)
625 \( x_{w4} = P * TD22L * (1+TD2L) * DSIN(EMP2 * PSI) \)
626 GO TO 629
627 \( TD2M = BM * (POIS * PHIS * EMP2S - 3MS)/(BM * (GMS - POIS * PHIS * EMP2S)) \)
628 IF (PSI = EQ. 0.0) GO TO 627
629 IF (ETA EQ. 0.0) GO TO 627
630 \( B = 1.0 \)
631 TEST = BM - BM * ETA
632 IF (TEST GT. 60.0) B = 0.0
633 \( x_{w1} = EM(x)(1+DEXP((BM * ETA - BM) * B)) * TD2M * DEXP((GMS * ETA - GM) * B)) * DSIN(1 * EMP2 * PSI * B)/(BM * TD2M * GM) \)
634 \( TD2L = -BM * GM \)
635 \( TD2L = -2 * DSIN(EMP2 * CSI)/(BM * (BMS - POIS * PHIS * EMP2S) + TD2L * GM * (GMS - 1 * POIS * PHIS * EMP2S)) \)
636 \( x_{w4} = P * TD22L * (DEXP((BM * ETA - BM) * B)) * TD2L * DEXP((GMS * ETA - GM) * B)) * B * DSIN(1 * EMP2 * PSI) \)
637 GO TO 629
638 \( x_{w1} = 0.0 \)
639 \( x_{w4} = 0.0 \)
640 \( n(1,J) = x_{w1} + 1.0 \)
641 \( n(1,J) = x_{w4} + 1.0 \)
642 \( n(1,J) = x_{w4} + 1.0 \)
643 \( n(1,J) = x_{w4} + 1.0 \)
644 \( n(1,J) = x_{w4} + 1.0 \)
645 CONTINUE
646 \( x(1,J) = W1(1,J) + W2(1,J) + W3(1,J) + W4(1,J) \)
647 PRINT 635, PSI, ETA, I, J, x(1,J)
648 FORMATT(34, PSI = F5.3, ETA = F5.3, X = E12)
649 PRINTE(34, PSI, ETA, I, J, x(1,J)
650 CONTINUE
651 ETA = ETA + 1.0 / DFLOAT(KS)
652 CONTINUE
653 RETURN
654 END
PROGRAM 7

THIS IS AN ANTISYMMETRIC NUDE EIGENVALUE SEARCH PROGRAM FOR THE RECTANGULAR CANTILEVER-PLATE FREE VIBRATION PROBLEM WITH POINT SUPPORTS SYMMETRICALLY LOCATED ON THE EDGES NORMAL TO THE CLAMPED BASE-TONE SUPPORT ON EACH EDGE

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:


2. CSI = 0 TO 1, PROVIDING THE DISTANCE BETWEEN THE CONCENTRATED EDGE FORCE (POINT SUPPORT) AND THE PLATE CENTRAL AXIS DIVIDED BY THE PLATE EDGE DIMENSION.

3. PHIR = 2B/A, IS THE FULL PLATE ASPECT RATIO.

4. ALMOS = AN INITIAL STARTING VALUE FOR THE EIGENVALUE SEARCH.

5. DLIM = A FINISHING OR EIGENVALUE SEARCH ENDING LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO HALT EXECUTION.

6. DEL = EIGENVALUE INCREMENT.

7. POI = POISSON'S RATIO.

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(99,99)
K=?????????????????????????
CSI=??????????????????????
K2=2*K
K1=2*K-1
2=36*U
PHIR=????????????????????????
PHI=PHIR/2.
PRINT 50: PHIR,K,CSI
50 FORMAT(18*PHIR = .F7.4,10X,K =*.15,10X,CSI =*.F7.4,////)
C=1.
PI=4.*DATA(N,C)
PHIS=PHI*PHI
POI=????????????????????????
POIS=2-PI1
PHIK=1./PHI
PHINS=PH1N*PH1N  
VFS=PO1*PH1S  
UF1S=PO1*PH1NS  
I=1+K+1.  
ALMDS=?????????????????????  
DLIM=??????????????????????  
DEL=??????????????????????  
CONTINUE

1.  
INITIALIZE THE MATRIX TO 0.0  

DO 2 M=1,1  
GO TO 2

2.
A(M,N)=0.0  
DO 10 M=1,K+2  
GO TO 10

EMP2S=EMP2*EMP2  
ENP2=ENP2*EMP2  
ENP2S=ENP2S*ENP2  
BN=PHINS*(EMP2S+ALMDS*PH1S)  
EN=DSRT(BN)  
X1=ALMDS*PH1S-ENP2S  
IF (X1<LT.0.0)  
GO TO 4  

GNS=PHINS*X1  

GO TO 1


10.
X1=ALMDS*PH1S-ENP2S  
IF (X1>GT.0.0)  
GO TO 3

GNS=PHINS*X1  

GO TO 1


1.  

DO 2 M=1,1  
GO TO 2

2.  

A(M,N)=0.0  
DO 10 M=1,K+2  
GO TO 10

EMP2S=EMP2*EMP2  
ENP2=ENP2*EMP2  
ENP2S=ENP2S*ENP2  
BN=PHINS*(EMP2S+ALMDS*PH1S)  
EN=DSRT(BN)  
X1=ALMDS*PH1S-ENP2S  
IF (X1<LT.0.0)  
GO TO 4  

GNS=PHINS*X1  

GO TO 1


10.  
X1=ALMDS*PH1S-ENP2S  
IF (X1>GT.0.0)  
GO TO 3

GNS=PHINS*X1  

GO TO 1

3.

X1=((ENP2S-UF5*BN)*BN*EMP2S)/(BN+EMP2S)*BN*DSRT(BN)  
X2=((ENP2S-UF5*BN)/(BN+EMP2S))**DSRT(BN)  
XX=2*DSRT(EMP2)*DSRT(ENP2)*(X1-X2)/TD*IN  
A(M+1)/2_*N+1)/2+K)=XX  

GO TO 4

4.

X1=-(X1  

GO TO 4

5.

X1=-(X1  

GO TO 4

6.

X1=-(X1  

GO TO 4

7.

X1=-(X1  

GO TO 4

8.

X1=-(X1  

GO TO 4

9.

X1=-(X1  

GO TO 4
A(1+(N+1)/2*K)=B*(TC2N*DEXP((BN*CSI-BN)*B)+DEXP((GN*CSI-GN)*B))
1*(DSINV2P)/(TD2N*BN+GN)
TEST=BN*CSI
B=1.0
IF (TEST.GT.60.) B=0.0
A(1+(N+1)/2*K)=B*(DSINV2P/(BN-GN))+(DEXP(-BN*CSI)*B)
1-DEXP((BN*CSI)*B))
A((N+1)/2+K2,(N+1)/2+K)=0.0
A((N+1)/2+K,(N+1)/2+K2)=0.0
A((N+1)/2+K,(N+1)/2+K)=(UFI5*DSINV2P*BN)*TD2N+UFI5*DSINV2P
1-GNS)/TD22N
5 X1=(ENP2S-UFBS*BNS)/(BNS*EMP2S)*TD2N*BN
X2=(ENP2S-UFBS*GNS)/(GNS*EMP2S)*GN
X3=2*DSINV2P*DSINV2P†(X1+X2)/TD22N
A((N+1)/2+2,(N+1)/2+2)=XX
X1=(ENP2S-UFBS*BNS)/(BNS*EMP2S)
X2=(ENP2S-UFBS*GNS)/(GNS*EMP2S)
X3=2*DSINV2P*EMP2S*(X1-X2)/TD22P
A((N+1)/2,(N+1)/2+2)=XX
GJ TO 10
6 TD2N=BN*(PUI5*PHIS*DSINV2S-GNS)*DCUSH(GN)/(BN*(BNS-PUI5*PHIS*
1-ENP2S)*DCUSH(BN))
TD2N=TD2N*BN*DCUSH(BN)+GN*DCUSH(GN)
TD2P=DCUSH(BN)/DCUSH(GN)
TD22P=BN*DSINV2P(BN)-GN*TD2P*DSINV2P(GN)
IF (N.GT.1) GO TO 7
A((N+1)/2+K2)=DCOSH(BN*(1-CSI))
1-2DSINV2P(TD2N*BN*CSI)+DSINV2P(BN*CSI))/TD22N
A((N+1)/2+K)=10DSINV2P(BN*CSI)/TD22N
1-DSINV2P(TD2N*BN*CSI)+DSINV2P(BN*CSI))/TD22N
7 A((N+1)/2+K2,(N+1)/2+K2)=(UFI5*DSINV2P*BN)*(1-TD2P)-BNS+TD2P*GNS)/TD22P
X1=(ENP2S-UFBS*GNS)/(GNS*EMP2S)*GNS*DCUSH(BC)
X2=(ENP2S-UFBS*GNS)/(GNS*EMP2S)*GNS*DCUSH(BN)
X3=2*DSINV2P*DSINV2P†(X1+X2)/TD22N
A((N+1)/2,(N+1)/2+2)=XX
X1=(ENP2S-UFBS*BNS)/(BNS*EMP2S)/DCUSH(BN)
X2=(ENP2S-UFBS*GNS)/(GNS*EMP2S)*GNS*DCUSH(BC)
X3=2*DSINV2P*EMP2S*(X1-X2)/TD22P
A((N+1)/2,(N+1)/2+2)=XX
CONTINUE
10 GJ 11 N=1.
11 A(N+K2,N+K2)=-1.
DJ 20 N=1.K1.2
DJ 20 M=1.K1.2
ENP2=ENP2*ENP2
EMP2=EMP2*EMP2
DSM=PHIS*(ALMDS*EMP2S)
BM=DSUSH(BNS)
X1=ALMDS*EMP2S
IF (XI.GT.0.0) GO TO 13
GSM=PHIS*X1
GSM=DSUSH(GNS)
TD1M=GMDS*(GMS+EMP2S+PUI5*PHIS)*DCUSH(GM)/(GM+(GNS+EMP2S*
1UFI5*PHIS)*DCUSH(BM))
TD1L=BM*TD1M*DCUSH(BM)+GM*DCUSH(GM)
TJ1L=-BM*DCUSH(BM)/(GM*DCUSH(GM))
10
X1= X1
GMS=PHI5*X1.
DQ=DSQRT(GMS)
IF (BMS>LT*Z2) G0 TO 15
td22=gd22*(PO15*PHI5*EMP25-GMS)/(bm*(BMS-PO15*PHI5*EMP25))
td22=gd22*M+M
TOD2=di/M
TOD2=2*D5IN(EMP25CSI)/(BM*(BMS-PO15*PHI5*EMP25)+TOD2+GDS
1(GM=PHI5*PHI5*EMP255)
X2=EMP25-UF15*BMS)/M/(BMS*EMP255)
X2=2*EMP25-UF15*/(GMS*EMP255)*GMS*DQ(DS)
A4=(A4+42)/2
X1=T024*(EMP25*EF2(N2)+EMP25*EF2(N2))*X2+X242
A(N+1)/2+2*K,11=1(A(N+1)/2+2*K,11)+XX
A(1,1)=A(1,1)+XX
A(N+1)/2+2*K,11=1((A[N+1]/2+2*K,11)+XX
A(N+1)/2+2*K,12)=((GM*EMP25*(BMS-EMP25)*GM*/GMS=EMP255)
X2=2*EMP25*DSIN(EMP25)*(X1*X2)/TOD2M
A(N+1)/2+2*K,12=1(A(N+1)/2+2*K,12)+XX
A1=GM/GMS*EMP255
X2=2*EMP25*DSIN(EMP25)*(X1*X2)/TOD2M
A(N+1)/2+2*K,12=1(A(N+1)/2+2*K,12)+XX
G0 TO 20
13
T02M=GMA(PHIS*EMP25-GMS)*DCOSH(GM)/(BM*(BMS-PHIS*PHI5*EMP25)
10 EMP2S*DCOSH(BM)
TD22M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
TD2L=-BM*DCOSH(BM)/(GM*DCOSH(GM))
TD2L=2*DSIN(EMP2CS1)/(BM*(GM*DCOSH(BM))
10TD2L*GM*(GMS*P01S*PH1S*EMP2S)*DCOSH(GM)
X2=(EMP2S*UFIS*BMS)*3M*DCOSH(BM)/(GM*ENP2S)
XX=2*TD22L*DSIN(EMP2)*DSIN(EMP2)*(X2+X2)
A((N+1)/2+K1)=A((N+1)/2+K1)+XX
X2=BM*DCOSH(BM)/(BMS*ENP2S)
X2=TD2L*GM*DCOSH(GM)/(GMS*ENP2S)
XX=2*TD22L*EMP2*DSIN(EMP2)*(X2+X2)
A((N+1)/2+K2)=A((N+1)/2+K2)+XX
IF (NGT.1) GO TO 16
XX=TD22L(DSINH(BM)+TD2L*DSINH(GM))*DSIN(EMP2CS1)
A(1,1)=A(1,1)+XX
A((M+1)/2)=(TD2M*DSINH(BM)+DSINH(GM))*DSIN(EMP2CS1)/TD22M
A((M+1)/2)=TD2L*(EMP2S*UFIS-BMS)*DSINH(BM)+TD2L*
1EP2S*UFIS-GMS*DSINH(GM)/TD22M
10 X1=((EMP2S-UFIS*BMS)/(BMS*ENP2S))*TD2M*BM*DCOSH(BM)
X2=((EMP2S-UFIS*GMS)/(GMS*ENP2S))*GM*DCOSH(GM)
XX=2*DSIN(EMP2)*DSIN(EMP2)*(X1*X2)/TD22M
A((N+1)/2+K1,((M+1)/2)=XX
X1=TD2M*BM*DCOSH(BM)/(BMS*ENP2S)
X2=GM*DCOSH(GM)/(GMS*ENP2S)
XX=2*EMP2*DSIN(EMP2)*(X1*X2)/TD22M
A((N+1)/2+K2,((M+1)/2)=XX
20 CONTINUE
CALL DETERM (A1,DET)
PRINT 11G,ALMDS,DEI
110 FORMAT(' ',ALMDS=' F10.4',DET=' D20.5')
ALMDS=ALMDS+DEL
IF (ALMDS.LE.DLIM) GO TO 10
STOP
END
SUBROUTINE DETERM (A1,DET)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(99,99)
SIGN=1.
LAST=N-1
START UVEHALL LOOP FOR (N-1) PIVOTS
DO 200 I=1,LAST
FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
BIG=0.
DO 50 K=I,N
TERM=GBS(A(K,I))
IF (TERM-BIG)50,50,30
BIG=TERM
L=K
50 CONTINUE
CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
IF (BIG)80,60,80
L-TH ROW HAS THE BIGGEST TERM----IS I=L

90 IF (I=L)90,120,90
I IS NOT EQUAL TO L,SWITCH ROWS I AND L

90 SIGN=-SIGN
DO 100 J=1,N
   TEMP=A(I,J)
   A(I,J)=A(L,J)
   100 A(L,J)=TEMP

NOW START PIVOTAL REDUCTION

120 PIVOT=A(I,I)
   NEXTR=I+1
   FOR EACH OF THE ROWS AFTER THE I-TH
   DO 200 J=NEXTR,N
   MULTIPLYING CONSTANT FOR THE J-TH ROW IS
   CONST=A(J,I)/PIVOT
   NOW REDUCE EACH TERM OF THE J-TH ROW
   DO 200 K=1,N
   200 A(J,K)=A(J,K)-CONST*A(I,K)

END OF PIVOTAL REDUCTION----NOW COMPUTE DETERMINANT

DET=SIGN
DO 300 I=1,N
   300 DET=DET*A(I,I)/10.
GO TO 61.

60 DET=0.
61 RETURN
PROGRAM B

THIS IS AN ANTISYMMETRIC WAVE SHAPE PROGRAM FOR THE
RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM, WITH
POINT SUPPORTS SYMMETRICALY LOCATED ON THE EDGES NORMAL TO
THE CLAMPED BASE. (ONE POINT SUPPORT ON EACH EDGE)

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:

1. - K = AS DEFINED IN PROGRAM 7.
2. - CSI = AS DEFINED IN PROGRAM 7.
3. - PHIR = 2B/A FULL PLATE ASPECT RATIO.
4. - ALMDS = EIGENVALUE.
5. - KS = NUMBER OF POINTS AT WHICH THE DISPLACEMENT W IS REQUIRED.
6. - POI = POISSON'S RATIO.

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(69,99)
COMMON Z2,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,CSI,K,K1,
1K2,K3,K4,K5,K6,K7,K8,K9,K10,K11,K12,K13,K14,K15,K16,K17,K18,K19,K20,
2K21,K22,K23,K24,K25,K26,K27,K28,K29,K30,K31,K32,K33,K34,K35,K36,K37,
3K38,K39,K40,K41,K42,K43,K44,K45,K46,K47,K48,K49,K50,K51,K52,K53,
4K54,K55,K56,K57,K58,K59,K60,K61,K62,K63,K64,K65,K66,K67,K68,K69,
5K70,K71,K72,K73,K74,K75,K76,K77,K78,K79,K80,K81,K82,K83,K84,K85,
6K86,K87,K88,K89,K90,K91,K92,K93,K94,K95,K96,K97,K98,K99

50 FORMAT('11,3F7.4,10X,'K = ',15.10X,'CSI = ',F7.4,10X,'ALMDS = ',F7.4)
PRINT1
31 FORMAT(*10X,'THE VALUES OF EM,EN,EP,AND P* ARE:!!!:///)
C1=1,
PI=4.*DATAN(C)
PHIS=PHI*PHI
FOI=????????????????????
POIS=2-POI
PHIN=1./PHI
PHINS=PHIN*PHIN
UFS=POI*PHIS

PRINT
UFI=POI*PHINS
I=3*K+1
CONTINUE

INITIALIZE THE MATRIX TO 0.0

DO 2 N=1,N+1
DO 2 M=1,1
A(M,N)=0.0
2 CONTINUE

DO 10 M=1,K+1
DO 10 N=1,K+1
EM*2=M*PI/2.
EMPS=EMP*2
ENPS=EMP*2
ENPS=EMP*2
ENPS=EMP*2
BN=PHINS*(ENPS+KMD*PHINS)
CN=DSRT(GNS)
X1=ALWUS*PHIMS-ENPS
IF (X1<LT.0.0) GO TO 4
X1=PHINS*X1
CN=DSRT(GNS)
TD1=GN*DCUS(GN)*(GNS+ENPS*POI*PHINS)/(BN*DCUS(BN)*(BNS-ENPS*POI*PHINS))
TD11=BN*TD1*DCUS(BN)+GN*DCUS(GN)
TD11=DCUS(BN)/DCUS(GN)
TD11=TD1*DS1N(BN)+GN*DS1N(GN)*TD1
IF (M.GT.1) GO TO 3
A(1,N+1/2+K)=(TD1*DS1N(BN+C1)+DS1N(GN*C1))*DS1N(ENPS)/TD11
A(1,N+1/2+K)=(DCUS(BN+C1)-TD1*DS1N(GN))*/DS1N(ENPS)
TD11=TD11
A(N+1/2+K,N+1/2)+K=(DCUS(BN)*TD1*DS1N(BN)+(UFI)
1*ENPS*GNS)*DS1N(GN)/TD11
A(N+1/2+K,N+1/2+K)=(UFI*ENPS*GNS)/(BN*DCUS(BN))
X1=(ENPS+UFS*GNS)/(BN*DCUS(BN))
X2=(ENPS+UFS*GNS)/(BN*DCUS(BN))
X1=DS1N(ENPS)*DS1N(ENPS)*DS1N(ENPS)*DS1N(ENPS)
X2=DS1N(ENPS)*DS1N(ENPS)*DS1N(ENPS)*DS1N(ENPS)
X1=(EXPS+UFS*GNS)/(BN*DCUS(BN))
X2=(EXPS+UFS*GNS)/(BN*DCUS(BN))
GJ TO 10
4 CONTINUE
X1=X1
GJ=DSRT(GNS)
IF (GJ<LT.Z2) GO TO 6
TD2N=GN*(POI*PHINS*ENPS-GNS)/(BN*(BNS-POI*PHINS*ENPS))
TD2N=TD2N*BN+GN
TD2P=1.
TD2P=BN-GN*TD2P
IF (M.GT.1) GO TO 5
TEST=BN-BN*C1
B=1.
IF (TEST>=GT.60.0) D=0.
A(1,N+1/2+K)=B*(TD2N*EXP((BN-C1-BN)*B)+EXP((GN*C1-GN)*B))
I*(DS1N(ENPS)/(TD2N*BN+GN))
TEST=BN*C1
B=0.
IF (TEST<GT.60.0) D=0.
A((n+1)/2+k2) = B*(csn(enp2)/(bn-gn))*C(exp([-bn*csi]*b))
1. DEXP([-bn*csi]*b))
A((n+1)/2+k2, (n+1)/2+k2) = 0.0
A((n+1)/2+k2, (n+1)/2+k2) = ((ufis*enp2s-bns)*td2n+ufis*enp2s
1-gns)*td2n
5
x1=( (enp2s-usf*bn)/(bn+emp2s)) * td2n*bn
x2=( (enp2s-usf*gn)/(gn+emp2s)) * gn
x2=2*dsn(enp2)*dsn(enp2)+(x1+x2)/td2n
A((n+1)/2,n+1)/2+k2)-xx
A((n+1)/2,n+1)/2+k2) = xx
x2=2*dsn(enp2)*emp2*(x1-x2)/td2n
A((n+1)/2,n+1)/2+k2)=xx
6
TDO2N=GR*(POIS*PHINS*ENP2S-GN) * DCOSH(GN)/(BN*(BNS-POIS*PHINS*ENP2S)*DCOSH(BN))
TDO2N = TD2N*BN*DCOSH(BN) + GN*DCOSH(IGN)
TDO2P = DCOSH(BN) / DCOSH(IGN)
if (M+G+1) go to 7
A((n+1)/2+k2) = (td2n*dsn(enp2)+dsn(enp2)*COSH(1-COSI)) * TDO2N
A((n+1)/2+k2) = ((ufis*enp2s-bns)*TDO2N+DSN1NH(BN)+TFIS*ENP2S-GNS)*DSN1NH(BN)
7
x1=( (enp2s-usf*bn)/(bn+emp2s)) * (1-TDO2P-BNS*TD2P*GNS) / TD2P
x2=( (enp2s-usf*gn)/(gn+emp2s)) / GNSDCOSH(IGN)
xx=2*dsn(enp2)*dsn(enp2)+(x1+x2)/TD2N
A((n+1)/2,n+1)/2+k2)=xx
8
if (x1+LT+0.0) go to 13
9
GNS = PHIS * x1
G = DSCRT(GNS)
TDO1 = GM*(GNS*EMP2S-PHIS*PHIS)*DCUSI(GM)/(BM*(BNS-EMP2S*1PHIS+PHIS)*DCUSI(BM))
TDO1 = BM*TD1M*DCOSH(BM) + GM*DCUSI(GM)
TDO1 = 2*DSN1ENP2*CSI)/(BM*(BNS-POIS*PHIS*EMP2S)*DCOSH(BM)
1-TO1L = GM*(GNS*POIS*PHIS*EMP2S)*DCOSI(GM)
1-TD1L = GM*(GNS*POIS*PHIS*EMP2S)*DCOSI(GM)
xx = (XENP2S-USFIS*BN)/(BN+DCUSI(BN)+GM*EMP2S)
X11 = (EMP2S-USFIS*GNS)*TD1LI*GM*DCOSI(GM)/(GNS-EMP2S)
xx = 2*TD1LI*DSN1ENP2*DSN1ENP2*(X1-X11)
A((N+1)/2+K1) = A((N+1)/2+K1) + XX
AX1 = *DCGM(BM)/(BMS*EMP2S)
AX1 = TD1L*GM*DCSH(GM)/(BMS*EMP2S)
AX2 = 2*TD1L*EMP2*DSIN(EMP2)*(X1-X11)
A((N+1)/2+K2.1) = A((N+1)/2+K2.1) + XX
IF (K = GT 1) GO TO 12
XX = TD1L*(DSIN(BM)*TD1L*DSIN(GM)*DSIN(EMP2*CS1)
A1 = A(I, I) + XX
A((N+1)/2+K1) = (TDM1*CSIN(BM)*DSIN(GM)*DSIN(EMP2*CS1)/TD11M
A((N+1)/2+K2.1) = TD1L*(DSIN(EMP2*UFS*BM)*DSIN(BM)*TD1L*(DSIN(EMP2*UFS*BM)
A((N+1)/2+K2.1) = XX
AX1 = TCI1*BM*DSIN(EMP2)*(X1-X2)/TD11M
AX2 = G*M*DSIN(EMP2)*(X1-X2)/TD11M
GO TO 20
XG = XG + 1
IF (XG = LT 22) GO TO 19
T2M = U*(POLS*PH1*EMP2S-GM)/(BMS*POL1*PH1*EMP2S)
T2M = T2M + BM*GM
T2L = U/GM
TX2L = TX2L*2*DSIN(EMP2*CS1)/(EM*(BMS*POL1*PH1*EMP2S) + TD2L*GM
1 = (GMS-POLS*PH1*EMP2S)
XX = 2*(EMP2S*UFS*BM)/(BMS*EMP2S)
XX = 2*(EMP2S*UFS*BM)/(BMS*EMP2S)
XX = 2*(TD2L*DSIN(EMP2)*DSIN(EMP2)*(X2+X22)
A((N+1)/2+K1) = A((N+1)/2+K1) + XX
AX = 2*TH2L*EMP2*DSIN(EMP2)*(X2+X22)
A((N+1)/2+K2.1) = A((N+1)/2+K2.1) + XX
IF (N = GT 1) GO TO 14
XX = TD2L*(1+TD2L)*DSIN(EMP2*CS1),
A1 = A1 + 1 + XX
AX = 2*(TD2M*1*DSIN(EMP2*CS1)/TD2M
A((N+1)/2+K1) = (EMP2S*UFS*BM)/TD2M
A((N+1)/2+K2.1) = (EMP2S*UFS*BM)/TD2M
XX = 2*DSIN(EMP2)*DSIN(EMP2)*(X1+X2)/TD2M
A((N+1)/2+K1) = A((N+1)/2+K1) + XX
AX = TD2M*(1+TD2M)*BMS
AX = TD2L*EMP2*(BMS*EMP2S)
1 = TD2L*EMP2*(BMS*EMP2S)
XX = 2*EMP2*DSIN(EMP2)*(X1+X2)/TD2M
A((N+1)/2+K1) = A((N+1)/2+K1) + XX
GO TO 20
13 TD2M = GW1*PH1*EMP2S*(GS)*DCWSH(GM)/(BMS*POL1*PH1*EMP2S)
TD2M = TD2M*GM*DCWSH(GM)
TD2L = TD2L*GM*DCWSH(GM)
X2=(EMP2S-UF1S*BMS)*BM*DCOSH(BM)/(BMS+EMP2S)
X22=(EMP2S-UF1S*GMS)*TD2L*GM*DCOSH(GM)/(GMS+EMP2S)
X2=2*TD2L*DSIN(EMP2)*DSIN(EMP2)*(X2+X22)
A((N+1)/2+K, I)=A((N+1)/2+K, I)+XX
X2=BM*DCOSH(BM)/(BMS+EMP2S)
X22=TD2L*GM*DCOSH(GM)/(GMS+EMP2S)
X2=2*TD2L*EMP2*DSIN(EMP2)*(X2+X22)
IF (H GT 1) GO TO 10
XX=TD2L*(DSINH(BM)+TD2L*DSINH(GM)) * DSINH(EMP2*CSI)
A(I, I)=A(I, I)+XX
A(I, (M+1)/2)=(TD2L*DSINH(BM)+DSINH(GM))* DSINH(EMP2*CSI) / TD2M
A((M+1)/2, I)=TD2L*(EMP25*UFS-BMS)*DSINH(BM)+TD2M*(
I*EMP25*UFS-GMS)*DSINH(GM)
A((M+1)/2, (M+1)/2)=(UFS*EMP25-BMS)*TD2M*DSINH(BM)+(UFS*EMP25-
GMS)*DSINH(GM))/TD2M
IX=((EMP2S-UF1S*BMS)/(BMS+EMP2S))*TD2M*BM*DCOSH(BM)
X2=(EMP2S-UF1S*GMS)/(GMS+EMP2S)*GM*DCOSH(GM)
X2=2*DSINH(EMP2)*DSINH(EMP2)*(X1+X2)/TD2M
A((N+1)/2+K, (M+1)/2)=XX
X1=TD2M*EMP2*DCOSH(BM)/(BMS+EMP2S)
X2=GM*DCOSH(GM)/(GMS+EMP2S)
X2=2*EMP2*DSIN(EMP2)*(X1+X2)/TD2M
A((N+1)/2+K2, (M+1)/2)=XX
20 CONTINUE
DO 30 N=1,1
TEMP=A(M, M-1)
A(M, M-1)=A(M, M)
A(M, M)=TEMP
30 CONTINUE
CALL DETERM (A, I, DET)
PRINT 31
31 FORMAT(15*10X,'XXXXXXXXXXXXXXXXXXXXXXXXXXXX',10X,'END',10X,
'XXXXXXXXXXXXXXXXXXXXXXXXXX')
STOP
END
SUBROUTINE DETERM (A, N, DET)
IMPLICIT REAL*8(A-H,0-Z)
DIMENSION A(50,50),X(50),EM(75),EN(75),EP(75)
SIGN=1
M=N-1
LAST=M-1
START OVERALL LOOP FOR(N-1) PIOOTS
DO 200 I=1,LAST
FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT:
BIG=0
DO 50 K=1,M
TERM=DSBS(A(K, I))
IF (TERM-BIG) GO TO 50
50 BIG=TERM
L=K
END DO 50
CONTINUE
CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
IF (BIG) GO TO 100
100 STOP
L-TH ROW HAS THE BIGGEST TERM----IS I=L  
80 IF (I-L) .LE. 120.90  
' I IS NOT EQUAL TO L,SWITCH ROWS I AND L  
90 DO 100 J=1,N  
   TEMP=A(I,J)  
   A(I,J)=A(L,J)  
   A(L,J)=TEMP  
100  
NOW START PIVOTAL REDUCTION  
120 PIVOT=A(I,I)  
   NEXTR=I+1  
   FOR EACH OF THE ROWS AFTER THE I-TH  
   DO 200 J=NEXITR,M  
   MULTPLYING CONSTANT FOR THE J-TH ROW IS  
   CONST=A(J,I)/PIVOT  
   NOW REDUCE EACH TERM OF THE J-TH ROW  
   DO 200 K=1,N  
   A(J,K)=A(J,K)-CONST*A(I,K)  
   END OF PIVOTAL REDUCTION--PERFORM BACK SUBSTITION  
   N=N-1  
   DO 500 I=1,M  
   IREV IS THE BACKWARD INDEX.GOING FROM M BACK TO 1  
   IREV=M+I-1  
   GET Y(IREV) IN PREPARATION  
   Y=A(IREV,N)  
   IF (IREV=M) 400,500,400  
   NEXT CRKING CN-LAST ROW,i IS 2 OR GREATER  
400 DO 450 J=2,N  
   WORK BACKWARD FOR X(N),X(N-1)--SUBSTITUTING PREVIOUSLY FOUND VALUES  
   X=K=N+1-J  
   450 Y=Y-A(IREV,K)*X(K)  
   FINALLY,COMPUTE X(IREV)  
   500 X(IREV)=Y/A(IREV,IREV)  
   FIND AND PRINT EM,EN,EP,AND P*
L=(N-1)/3
DO 550 J=1,L
K=J
EM(J)=X(I)
EM(J)=X(I+1)
IF(I.EQ.0.L) GO TO 540
EP(J)=X(I+2*L)
P=X(3*L)
GO TO 550
540 EP(J)=E
550 PRINT600,J,K,EM(J),EM(J),EP(J),P
560 FORMAT(3F10.2,L20)
P01=13.5X,EN(*,12,*)=*D13.5X,EP(*,12,*)=*D13.5X
570 FORMAT(*13.0X,*30X,*THE ANTI-SYMMETRIC MODE SHAPE DATA ARE:*/,) CALL SHAPE (EM,EN,EP,P)
580 RETURN
END
SUBROUTINE SHAPE (EM,EN,EP,P)
IMPLICIT REAL*(A-H,O-Z)
COMMON ZK,PHI,P1,PHIS,P01,POIS,PHIN,PHINS,UF1,UF1S,ALMD,CS1,K,K1,
1 KS
DIMENSION W1(21,21),W2(21,21),W3(21,21),W4(21,21),W5(21,21)
1,EM(75),EN(75),EP(75)
KS=KS+1
ETA=0.0
DO 650 I=1,KS
PSI=0.0
DO 660 J=1,KS
650 W(I,J)=0.0
660 W=0.0
W=0.0
W=0.0
DO 670 N=1,K1,2
670 ENP=EPS/2.0
ENP=EPS/2.0
BN=DSQRT(BNS)
XI=ALMD*PHIS-ENP2
IF (XI*LT.0.0) GO TO 605
GNS=PHINS*X1
GNS=PHINS*X1
BN=TDSH(BN)
1,TDH=(1-PSI)*GN-TDH1*DSINH(BN*PSI)+DSIN(GN*PSI)*DSIN(ENP2*ETA)/TD1
1,ENP2*ETA)/TD11
1
601 X=EP(I)*(1-TD1P)*CSIN(ENP2*ETA)/TD11P
602 X=0.0
GO TO 604
603 X2=0.0
X3=EP(N)*(DCOSH(BN)-TD2P*DCOSH(GN)) *DSIN(ENP2-ETA)/TD11P
604 2(1,J)=W22+X*2
3(1,J)=W33+X*3
22=2(1,J)
33=3(1,J)
GO TO 620
605 X1=-X1
GNS=PHINS*X1
IF (BNS.GT.22) GO TO 610
6 TD2N=GNS*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS*ENP2S))*DCOSH(BN)
6 TD2h=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
6 TD2P=U*GSH(BN)/DCOSH(GN)
6 TD2=x*GSH(BN)-GS*TD2P*DSINH(GN)
6 IF (ETA.EQ.0.0) GO TO 607
6 IF (PS1.EQ.0.0) GO TO 608
6 X2=EN(N)*(TD2N*DSINH(BN*PSI)+DSINH(GN*PSI))*DSIN(ENP2-ETA)/TD2N
6 IF (PS1.EQ.0.0) GO TO 666
6 X3=EP(N)*(DCOSH((-1-PSI)*BN)-TD2P*DCOSH((-1-PSI)*BN))*DSIN(ENP2-ETA)/TD22P
606 GO TO 609
607 X2=0.0
X3=0.0
GO TO 609
608 X2=0.0
X3=EP(N)*(DCOSH(BN)-TD2P*DCOSH(GN)) *DSIN(ENP2-ETA)/TD22P
609 2(1,J)=W22+X*2
3(1,J)=W33+X*3
22=2(1,J)
33=3(1,J)
GO TO 620
610 TD2N=GNS*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
6 IF (ETA.EQ.0.0) GO TO 612
6 IF (PS1.EQ.0.0) GO TO 612
6 B=1.0
6 TEST=BN*BN*PSI
6 IF (TEST.GT.60.0) J=0.0
6 X2=EN(N)*(TD2N*DEXP(BN*PSI-BN)*BN)+DEXP((GN*PSI-GN)*BN)
6 *DSIN(ENP2-ETA)/(BN*TD2N+GN)
6 TEST=BN*PSI
6 B=1.0
6 IF (TEST.GT.60.0) B=0.0
6 X3=EP(N)*DEXP((-BN*PSI)*B)-DEXP((-GN*PSI)*B))*DSIN(ENP2-ETA)*B
6*(BN-GN)
6 GO TO 614
612 X2=0.0
X3=0.0
614 2(1,J)=W22+X*2
3(1,J)=W33+X*3
22=2(1,J)
33=3(1,J)
620 CONTINUE
620 DD 630 M=1.1*K1+2
6 EMP2=MEP2/2.
6 EMP2S=EMP2*EMP2
\text{BMS=PHIS*(ALMDS*EMP2S)}
\text{BM=DSORT(BMS)}
\text{XI=ALMDS-EMP2S}
\text{IF (XI.LT.0.0) GO TO 625}
\text{GMS=PHIS*X1}
\text{GMS=DSORT(GMS)}
\text{TD1M=GMS*EMP2S*PO1S*PHIS)*DCGS(GM)/(BM*(BMS-EMP2S*}
\text{PO1S*PHIS)*DCOSH(BM))}
\text{TJ11M=BM*TD1M*DCOSH(BM)+GM*DCUS(GM)}
\text{TQ11L=BM*DCUS(BM)/(GM*DCGS(GM))}
\text{TQ11L=-2*DSIN(EMP2*CS1)/(BM*(BMS-PO1S*PHIS*EMP2S)*DCOSH(BM)-TD1L*}
\text{GM*(GMS*PO1S*PHIS*EMP2S)*DCOS(GM))}
\text{IF (ETA.EQ.0.0) GO TO 632}
\text{IF (PSI.EQ.0.0) GO TO 622}
\text{Xw1=EM(M)*(TD1M*DSINH(BM*ETA)+DSINH(GM*ETA))*DSIN(EMP2*PSI)/TD11M}
\text{Xw4=P*TD11L*(DSINH(BM*ETA)+TD11L*DSIN(GM*ETA))*DSIN(EMP2*PSI)}
\text{GO TO 624}

622 Xw1=0.0
Xw4=0.0

624 \text{X1(I,J)=X11+Xw1}
\text{X4(I,J)=X44+Xw4}
\text{X11=X1(I,J)}
\text{X44=X4(I,J)}
\text{GO TO 630}

625 X1=-X1
\text{GMS=PHIS*X1}
\text{GMS=DSORT(GMS)}
\text{IF (BMS.GT.22) GO TO 627}
\text{15 TQ2N=GMS*(PO1S*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-PO1S*PHIS*}
\text{EMP2S)*DCUSH(BM))}
\text{TQ2M=TQ2M*BM*DCOSH(BM)+GM*DCOSH(GM)}
\text{TQ2L=BM*DCUSH(BM)/(GM*DCOSH(GM))}
\text{TQ2M=-2*DSIN(EMP2*CS1)/(BM*(BMS-PO1S*PHIS*EMP2S)*DCOSH(BM)+}
\text{GM*(GMS-PO1S*PHIS*EMP2S)*DCOSH(GM))}
\text{IF (ETA.EQ.0.0) GO TO 626}
\text{IF (PSI.EQ.0.0) GO TO 626}
\text{Xw1=EM(M)*(TD2M*DSINH(BM*ETA)+DSINH(GM*ETA))*DSIN(EMP2*PSI)/TD22M}
\text{Xw4=P*TD22L*(DSINH(BM*ETA)+TD22L*DSINH(GM*ETA))*DSIN(EMP2*PSI)}
\text{GO TO 626}

626 Xw1=0.0
Xw4=0.0

627 TQ2M=GMS*(PO1S*PHIS*EMP2S-GMS)/(BM*(BMS-PO1S*PHIS*EMP2S))
\text{TQ2L=BM*GM}
\text{TQ2L=-2*DSIN(EMP2*CS1)/(BM*(BMS-PO1S*PHIS*EMP2S)+TD2L*GM*(GMS-}
\text{1*PO1S*PHIS*EMP2S))}
\text{IF (ETA.EQ.0.0) GO TO 628}
\text{IF (PSI.EQ.0.0) GO TO 628}
\text{B=1}
\text{TEST=GM-BM*ETA}
\text{IF (TEST.GT.60.0) B=0}
\text{Xw1=EM(M)*(TD2M*DEXP((BM*ETA-BM)*B)+DEXP((GM*ETA-GM)*B))*B*DSIN(}
\text{EMP2*PSI))/TD2L*BM*GM)
\text{Xw4=BP*TD22L*(DEXP((BM*ETA-BM)*B)+TD22L*DEXP((GM*ETA-GM)*B))*DSIN(}
\text{EMP2*PSI)}
\text{GO TO 629}

628 Xw1=0.0
Xw4=0.0

629 X11(I,J)=X11+Xw1
X44(I,J)=X44+Xw4
W11 = w1(I,J)
W44 = w4(I,J)
630 CONTINUE
PRINT 635, PSI, ETA, I, J, W(1,J)
635 FORMAT(' ', 'PSI = ', 'F5.3,3X, ETA = ', 'F5.3,3X, W1( ', '12, ', '12, ') = ',
      'F12.5,3X, W2( ', '12, ', '12, ) = ', 'F12.5,3X, W3( ', '12, ', '12, ) = ', 'F12.5,3X, W4( ', '12, ', '12, ) = ', 'F12.5,3X')
PRINT 636, PSI, ETA, I, J
636 FORMAT(' ', '20X, 'PSI = ', 'F5.3,4X, ETA = ', 'F5.3,4X, ',
      '1D( ', '12, ', '12, ) = ', 'F12.5')
PSI = PSI + 1.0/DFLOAT(KS)
640 CONTINUE
ETA = ETA + 1.0/DFLOAT(KS)
650 CONTINUE
RETURN
END
PROGRAM 5

THIS IS A SYMMETRIC MODE EIGENVALUE SEARCH PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM WITH POINT SUPPORTS SYMMETRICALLY LOCATED ON THE EDGE PARALLEL TO THE CLAMPED EDGE.

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:


2. CSI = 0 TO 1, PROVIDING THE DIMENSIONLESS DIstance OF THE POINT SUPPORT TO THE X OR PSI AXIS. SEE FIGURE 3.2.

3. PHIR = EB/A, IS THE FULL PLATE ASPECT RATIO.

4. AMLS = AN INITIAL STARTING VALUE FOR THE EIGENVALUE SEARCH.

5. CLIM = A FINISHING OR EIGENVALUE SEARCH ENDING LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO END EXECUTION.

6. CEL = EIGENVALUE INCREMENT.

7. FOI = PI/SIND'S RATIO.

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(99,99)
K=21

CSI=???

PHIR=?????

PHI=PHIR/2
PRINT56,PHIR,K,CSI
STOP

FORMAT(1',10X,'PHIR = ',F8.4,10X,'K = ',I4,10X,'CSI = ',F8.4,10X)

C=1
FI=4*CAIANCE
PHIS=PHI

FI=???

FI=1.*PHI

PHINES=PHI+PHIN

VER=FI*PHIS

VFIS=FI*PHIN
1. INITIALIZE THE MATRIX A

L=I+1
DO 2 K=1,L
C0 2 K=1,L
A(K,K)=0.0
C0 3 K=1,L
A(N+2*K,N+2*K)=-1.0
C0 10 K=1,L+2
C0 10 K=1,L
JN=K-1
EMP2=K*FI/2.
REM1=REM2*EMP2
EMP=JN*FI
FNS=EMP*EMP
ENS=(1.0*PHIS)*(EMP*EMP+AL*DS*PHIS)
EN=DSRT(BN)
X1=AL*DS*PHIS*EMP*EMP
IF (X1.LT.0.0) GC TC 5
GNS=(1.0*PHIS)*X1
GCRT(GNS)
X=PC15#PHNS*ENFS
TD1=(CH*(BN-X)*DCOSH(BN))*(CH*(CN+S)*DCCS(GN))
TD1=XCCS(BN)/DCOS(GN)
TD1=-BN*DCCS(BN)/GN*CCCS(GN))
CL=0.0
IF (NC.GT.1.0) DL=2*C
T11=2*CCS(EMP*CSI)/((DL*(BN*ENS-X)*DCCS(BN))-TD1*GN*(GNS*X))
T00=DCCS(GN))
X=UFIS*ENFS
T11=BN*DSGH(BN)+TD1*GN*DCOS(GN)
T11P=BN*DSINH(BN)+GN*TC1P*DSIN(GN)
IF (N.GT.1.0) GC TO 4
A(L,N+K)=IDSIDH(BN)+TD1*CGS(DC)(GN))=DCCS(EMP*CSI)/TD1N
A(L,N+2*K)=IDSIDH(BN)+TD1*N+2*2*K)
A(N+2*K,L)=IDSIDH(BN)+TD1*ID1L*(X-BN)*DSINH(BN)+TD1L*2*X-GNS)*DSIN(GN))
A(N+2*K,L)=IDSIDH(BN)+TD1L*DSINH(BN)+TD1L*DSINH(BN)*DCOS(EMP*CSI)
A(L,L)=AL(L,L)*XX
4 X1=(ENFS+UF*BN)*BN*EMP2S)*BN*DCCS(BN)
X2=(T11*T(ENFS+UF*BN)*BN*DCCS(BN))=GNS*EMP2S)*GNS*DCCS(GN)
XX=(CCS(BN)*DCS(CN)+(EMP2)*TD11N)+(X1-X2)
((X+1)/2,K+K)=X*X2=0
X1=(ENFS+UF*BN)*BN*EMP2S)*BN*DCCS(BN)
X2=(T11P+(ENFS+UF*BN)*BN*DCCS(BN))=GNS*EMP2S)*GNS*DCCS(GN)
XX=2*CCS(EMP)*EMP2*(X1+X2)*TC11P
A((X+1)/2,N+2*K)=XX
X1=(ENFS+UF*BN)*(BN*EMP2S)*BN*DCOSH(BN)
X11=(ENFS+UF*BN)*(BN*EMP2S)*BN*DCOSH(BN)
\[ xx = 2 \times TD11L \times DCOS(ENP) \times DSIN(EMP2) \times (x1 - x11) \]
\[ A((k+1)/2, L) = A((k+1)/2, L) + xx \]

**GO TO 10**

**5**

\[ X = -1 \]
\[ GNS = \{ 1, /PHIS \} \times X \]
\[ G = DCOSCRT(GNS) \]
\[ X = FCISP*PHINS*ENFS \]
\[ IF (ENS*,LT*,22) \{ GO TO 6 \}
\[ TD2 = ENS*(BN*-X)/(GN*(X-GNS)) \]
\[ TD2L = BN*GN \]
\[ TD2F = 1.0 \]
\[ X = UFIS*ENPS \]
\[ TD2F = BN*GN \]
\[ TD2H = BN*TD2G*GN \]
\[ CL = 1.0 \]

**IF (X1 < CL) \{ GO TO 7 \}

\[ A(L, N+K) = \{ (1+TD2) \times DCOS(ENP*CSI) \}/TD2N \]
\[ A(L, N+K) = 0.0 \]
\[ A(N+2*K, L) = 0.0 \]
\[ A(N+K, L) = TD2L*\{ (X-ENS) \times TD2L*(X-GNS) \} \]
\[ A(L, L) = A(L, L) \times X \]
\[ A(N+K, N+K) = 0.0 \]
\[ A(N+2*K, N+2*K) = 0.0 \]

**GO TO 7**

**6**

\[ TD2 = (BN*\{ (X-GNS) \times DCOSH(BN) \})/(GN*\{ X-GNS \}) \]
\[ TD2L = BN*DCOSH(BN)/DCOSH(GN) \]
\[ TD2L = BN*DCOSH(BN)/GN*DCOSH(GN) \]
\[ X = UFIS*ENPS \]
\[ TD2F = BN*DSINH(EN) \times GN*TD2F*DSINH(GN) \]
\[ TD2H = BN*DCOSH(BN) \times TD2G*DCOSH(GN) \]

**GO TO 7**

**7**

\[ X = TD2L*\{ (X-GNS) \times DSINH(BN) \} \times TD2G*DCOSH(ENP*CSI) \]
\[ A(L, L) = A(L, L) \times X \]

**GO TO 8**
XX=24*TD2*DCOS(EMP1)*DSIN(EMP2)*(X2+X2)
A((N+1)/2+L)=A((N+1)/2+L)+XX
CO TC 10

8 \( x_1 = ((E_{NFS}-UFS*BN)) / (BNS+EMP2S) \) * EN*DCOSH(BN)
\( x_2 = ((E_{NFS}-UFS*GNS)) / (GNS+EMP2S) \) * EN*CCOSH(GN) * TD2
\( xx = (CCOS(EMP1)*DSIN(EMP2)/TD22) * (X1+X2) \)
A((N+1)/2+L)*N+K=XX*2+0
P= ((E_{NFS}-UFS*BN)) / (BNS+EMP2S) * CCOSH(BN)
P2= (CCOS(EMP1)*DSIN(EMP2)/TD22)*XX
A((N+1)/2+L)*N+K=XX
XX=((E_{NFS}-UFS*BN)) / (BNS+EMP2S) * EN*CCOSH(BN)
XX=2*TD2*DCOS(EMP1)*DSIN(EMP2)*(X2+X2)
A((N+1)/2+L)=A((N+1)/2+L)+XX

10 CONTINUE
CO 20 M=1*K
CO 20 M=1*K
JN=1-K
ENFS=ENFS*ENFS
EFS=EF*EF
EMP2=EFS+EMP2
EVS=EV*EV
E=Y=E*Y
DCC(GM)
1=11*TDC1(-1)
TDC1=EM*DSINH(BM)-TM*GMS*DSINH(GM)
XX= (E_{NFS}) * 2 * DSINH(EM)*TD1
A((N+1)/2+L)=XX
X1=(E_{EMP2S}-UFS*EMS)/((BNS+ENFS)*BM*CCOSH(EMP1)*DSINH(BM))
X2=(E_{ENFS}+UFS*GNS)/((GNS+EMP2S)*GM*CCOSH(EMP1)*DSINH(GM)) * TD1
XX=(X1+X2)*TD1
XX=(CCOSH(BM)+CSI)*TD1*CCOSH(GM)*TD1
TD1=TDC1*TD1
A((N+1)/2+L)=XX
X2=(E_{EMP2S}-UFS*EMS)*CCOSH(BM)*TD1
TDC1=EM*DSINH(BM)-TM*GMS*DSINH(GM)
XX= (E_{NFS}) * 2 * DSINH(EM)*TD1
A((N+1)/2+L)=XX
CO TC 20

13 \( x_1 =x_1 \)
GMS=FI*FI
GMS=CGSRT(GMS)
IF (EMS+GTS*2) GC TC 15
TDC2=EM*(BNS*PCIS+PHIS*EMP2S)*DSINH(BM)*(-1)/(GMS*PS-PCIS*
PHIS*EMP2S)*CSINH(GM))
TDC2=GM*DSINH(BM)*TD2*GMS*DSINH(GM)
X2=2*CCOS(EMP1)*EMP2*(X1+X2)/TD22
X1=(EMP2S-UFS*EMS)*BM*DSINH(BM)*BM*DSINH(GM)

\[ x_2 = (E + P_2 S - U F I S \times G S) \times T D 2 \times G M \times D S H \times (G M) / (G M S + E N P S) \]

\[ x_{11} = 2 \times D S H \times (E N P) \times (x_1 + x_2) / T D 2 \]

**IF (J \times GT \times 0) GO TC 14**

\[ x_6 = (C C S S H \times (B M \times C S I) + T D 2 \times C C S S H \times (G M + C S I)) \times D S H \times (E N P) / T D 2 \]

\[ x_{11} = (x_1 + x_2) / 2 = x_6 \]

**GO TC 20**

\[ T D 2 = (E M \times (G M S - P O I S \times P H I S \times E M P 2 S)) \times (-1) / (G M \times (G M S - P O I S \times P H I S \times E M P 2 S)) \]

**IF (J \times GE \times 0 \times 0) GO TC 14**

**END**
CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND

IF (E10) E0, 60, 80
  L-TH ROW HAS THE BIGGEST TERM----IS I=L
  80 IF (I-L)90, 120, 90
  I IS NOT EQUAL TO L, SWITCH ROWS I AND L
  50 SIGN=SIGN
  CC 100 J=1, N
  TEMP=A(I, J)
  A(I, J)=A(L, J)
  100 A(L, J)=TEMP

NOW START PIVOTAL REDUCTION

120 PIVCT=A(I, I)
  NEXTF=I+1

FOR EACH OF THE ROWS AFTER THE I-TH
CO 200 J=NEXTF+1

MULTIPLYING CONSTANT FOR THE J-TH ROW IS
CONST=A(J, I)/PIVCT

NOW REDUCE EACH TERM OF THE J-TH ROW
CO 200 K=I, N
200 A(J, K)=A(J, K)-CONST*A(I, K)

END OF PIVOTAL REDUCTION---NOW COMPUTE DETERMINANT

DET=SIGN
CO 300 I=1, N
300 DET=DET*A(I, I)/10.0
CO TC E1
60 DET=DET
61 END
END
PROGRAM 10

THIS IS A SYMMETRIC MODE SHAPE PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM WITH POINT SUPPORTS SYMMETRICALY LOCATED ON THE EDGE PARALLEL TO THE CLAMPED BASE.

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:
1. - K = AS DEFINED IN PROGRAM 9.
2. - CSI = AS DEFINED IN PROGRAM 9.
3. - PHIR = 28/A FULL PLATE ASPECT RATIO.
4. - ALMDS = EIGENVALUE.
5. - KS = NUMBER OF POINTS AT WHICH THE DISPLACEMENT IS REQUIRED.
6. - POI = POISSON'S RATIO.

IMPLICIT REAL*8(A-H,0-Z)
COMMON 22,PH1,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,CSI,K,K1,
1,K1
DIMENSION A(99,99)
K=????????????????????????
KS=????????????????????
Z2=3600
CSI=????????????????????
PHIR=????????????????????
ALMDS=????????????????????
PHI=PHIR/2
PRINT50;K,KS,CSI,PHIR,ALMDS
30 FORMAT(*1;10X,*K = *,I3,10X,*KS = *,I3,10X,*CSI = *,F7.4,10X,
1*PHIR = *,F7.4,10X,*ALMDS = *,F8.4,10X)
K1=2*K-1
C=1
PI=4.*DATAN(C)
PHIS=PHI*PHI
POI=????????????????????
POIS=2-POI
PHIN=1./PHI
PHINS=PHIN*PHIN
UFS=POI*PHIS
UFIS=POI*PHINS
I=I*K
CONTINUE
INITIALIZE THE MATRIX A

L=1+1
00 2 M=1*L
00 2 N=1*L
A(M,N)=0.0
2 CONTINUE
00 3 N=1*K
3 A(N+2*K,N+2*K)=-1*
00 10 M=1,K,1,2
00 10 N=1,K
JN=N-1
EMP2=M*PI/2.*
EMP2S=EMP2*EMP2
ENP=J*PI
ENPS=ENP*ENP
BN=1.*PHIS*(ENP*ENP+ALHDS*PHIS)
BN=DSRT(BNS)
X1=ALHDS*PHIS*ENP*ENP
IF (X1.1.LT.0.0) GO TO 5
BN=(1.*PHIS)*X1
GN=DSRT(GNS)
K=POIS*PHIS*ENPS
TO1=BN*(BNS-X)*DCUSH(GN)/(GN*(GNS+X)*DCOS(GN))
TO1P=DCUSH(BN)/DCOS(GN)
TO1L=3N*DCOS(BN)/GN*DCUS(GN)
DL=1.0
IF (X1.1.LT.0.0) DL=2.0
D11=2*DCOS(ENP*CSI)/DL*(BN*(BNS-X)*DCUSH(BN)-TD1L*GN*(GNS+X))
1*DCOS(GN)
X=UFIS*ENPS
TO11N=3N*DCUSH(BN)*TD1*GN*DCUS(GN)
TO11P=BN*DSINH(BN)+GN*TD1P*DSIN(GN)
IF (X1.1.LT.1.0) GO TO 4
A[L,N+K]=-(DSINH(BN)*TD1*DSIN(GN))*/DCOS(ENP*CSI)*TD11N
A[L,L]=TD1L*(X*TD1P+GNS+X)*DSINH(BN)+TD1L*(X+GNS+X)*DSIN(GN))
A[L,N+K]=TD1L*(BN+TD1L*GN)
X=TD1L*(DSINH(BN)+TD1L*DSIN(GN))*DCOS(ENP*CSI)
4 X1=((ENPS*UF*S+BNS)/(BNS+EMP2S))*BN*DCUSH(BN)
X2=(TD1*(ENPS*UF*S+GNS)/(GNS-EMP2S))*GN*DCUS(GN)
XX=(JDUS(EMP)*DSIN(EMP2)/TD11N)*(X1-X2)
A(N+1)/2*N=XX*2.0
XX=(ENPS*UF*S+BNS)/(BNS+EMP2S)*DCUSH(BN)
X2=(TD1*(ENPS*UF*S+GNS)/(GNS-EMP2S))*GN*DCUS(GN)
XX=2*DCUS(EMP)*EMP2*(A1+X2)/TD11P
A(N+1)/2*N=XX
X1=((ENPS*UF*S+BNS)/(BNS+EMP2S))*BN*DCUSH(BN)
X11=((ENPS*UF*S+GNS)/(GNS-EMP2S))*GN*TD1*DCUS(GN)
XX=2*TD1L*DCOS(ENP)*DSIN(EMP2)*(X1-X11)
A(N+1)/2*L=A(N+1)/2,L+XX
5 X1=-X1
GNS=(1.*PHIS)*X1
G=DCOSH(GNS)
x=POIS*PHINS*ENPS
IF (DNS.LT.22) GO TO 0
TD2=BN*(BNS-X)/(GN*(X-GNS))
TD2L=BN/GN
TD2P=1.0
X=UF1S*ENPS
TD2F=BN-GN
TD22=BN+TD2*GN
DL=1.0
IF (X.EQ.0.0) DL=2.0
TD2=-(2*DCOS(ENP*CS1))/(DL*(BN*(BNS-POIS*PHINS*ENPS)+TD2L*GN)
1*(GNS-POIS*PHINS*ENPS)))
IF (X.GT.1.0) GO TO 7
A(L,N+2,K)=0.0
A(N+2,K,L)=0.0
A(N+2,K,L)=TD22L*(X-BNS)*TD2L*(X-GNS))
X=TD22L*(1+TD2L)*DCOS(ENP*CS1)
A(L,L)=A(L,L)+XX
A(N+2,K,N+K)=(X-BNS)*TD2*(X-GNS))/TD22N
A(N+2,K,N+K)=0.0
A(N+2,K,N+K)=0.0
GO TO 7
6 TD22=BN*(BNS-X)*DCOSH(GN))
TD2P=DCOSH(BN)/DCOSH(GN)
TD2L=BN*DCOSH(BN)/GN*DCOSH(GN))
*UF1S*ENPS
TD2G=BN*DSINH(BN)-GN*TD2P*DSINH(GN)
TD22=BN*DCOSH(BN)+TD2*GN*DCOSH(GN)
DL=1.0
IF (X.EQ.1.0) DL=2.0
TD2=-(2*DCOS(ENP*CS1))/(DL*(BN*(BNS-POIS*PHINS*ENPS)*DCOSH(BN))
1*TD2L*GNS*(GNS-POIS*PHINS*ENPS)*DCOSH(BN))+
IF (X.GT.1.0) GO TO 8
A(L=N+2,K,N+K)=TD2*DSINH(GN))/DCOS(ENP*CS1)/TD22N
A(N+2,K,N+K)=(1-TD2P)*DCOS(ENP*CS1)/TD22N
A(N+2,K,N+K)=(X-BNS)*DSINH(BN)+TD2*(X-GNS)*DSINH(GN))/TD22N
A(N+2,K,N+K)=BN+TD2*GN)/TD22N
A(N+2,K,N+K)=UF1S*ENPS*(1-TD2P)-BN+GNS*TD2P)/TD22P
A(N+2,K,L)=TD22L*(X-BNS)*DSINH(BN)+TD2L*(X-GNS)*DSINH(GN)
A(N+2,K,L)=TD22L*(BN+TD2L*GN)
X=TD22L*(X-BNS)*DSINH(GN))*DCOS(ENP*CS1)
A(L,L)=A(L,L)+XX
GO TO 8
7 X1=((ENPS-UFS*BN)/(BNS+EMP2S))*BN
X2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN
XX=(DCOSH(ENP)*DSINH(EMP2[TD22N]))*(X1+X2)
A(N+1)/2,N+2*K)=XX
X1=(ENPS-UFS*BNS)/(BNS+EMP2S)
X2=(ENPS-UFS*GNS)/(GNS+EMP2S)
XX=2*DCOSH(ENP)*EMP2*(X1-X2)/TD2P
A(M+1)/2,N+2*K)=XX
X2=(ENPS-UFS*BN)/(BNS+EMP2S)
X2=(ENPS-UFS*GNS)*GN*TD2L/(GNS+EMP2S)
X2=2*DCOSH(ENP)*DSINH(EMP2)*(X2+X22)
A(M+1)/2,L)=A(M+1)/2,L)*XX
GO TO 10
8 X1=((ENPS-UFS*BNS)/(BNS+EMP2S))*BN*DCOSH(BN)
A2=((ENPS-UFS*GNS)/(GNS+EMP2S))*GN*DCOSH(GN)*TD2
Continued...

DO 20 N=1*K
DO 20 K=1,K1+2
J1=N+1
ENP=JN*PI
ENS=ENP*ENP
EMP2=EMP1+1/2
EMP2S=EMP2*EMP2
BMS=PHIS*(ALMDS*EMP2S)
BM=D*SORT(BMS)
X1=ALMDS-EMP2S
IF(X1.LT.0.0) GO TO 13
GMS=PHIS*X1
GM=D*SORT(GMS)
TD1=BM*(BMS-POLS*PHIS*EMP2S)*DSINH(BM)/(GM*(GMS+POLIS*PHIS*EMP2S))
1*DSINH(GM)
TD1=TD1*(-1)
TD11=BM*DSINH(BM)-TD1*GM*DSINH(GM)/((BMS+ENPS)+TD1*GM*DSINH(GM)/(GMS-ENPS))*2*EMP2*DCOS(ENP)/TD11
IF(JN.GT.0.0) GO TO 11
XX=XX/2
11 A(N+2*K, (M+1)/2)=XX
X1=(EMPS-UFIS+GMS)/(BMS+ENPS))**BM*DCOS(ENP)*DSINH(BM)
X2=(EMPS-UFIS+GMS)/(GMS-ENPS))**GM*DCOS(ENP)*DSINH(GM)/TD1
XX=(X1*X2)**2*DSINH2(BM)/TD11
A(N+2*K, (M+1)/2)=GM*DSINH2(BM)**2*DSINH2(GM)/TD11
IF(JN.GT.0.0) GO TO 12
XX=XX/2
A(N+2*K, (M+1)/2)=XX
GO TO 20
30 TO 20
X1=X1
GMS=PHIS*X1
GM=D*SORT(GMS)
IF (ENS.GT.22) GO TO 15
TD2=(BM*(BMS-POLS*PHIS*EMP2S)*DSINH(BM))/(GM*(GMS-POIS*414- PHIS*EMP2S)*DSINH(GM))
TD2=BM*DSINH2(BM)/BM+GMS*DSINH2(GM)
X1=GM*TD2*DSINH2(GM)/(GMS+ENPS)
X2=GM*TD2*DSINH2(BM)/(GMS+ENPS)
XX=2*DCOS(ENP)*EMP2*(X1*X2)/TD22
XX=EMPS-UFIS*BM*DSINH2(BM)/(GMS+ENPS)
XX=EMPS-UFIS*GMS*TD2*GM*DSINH2(GM)/(GMS+ENPS)
XX=2*DSINH(EMP2)**2*DCOS(ENP)**(X1*X2)/TD22
IF(JN.GT.0.0) GO TO 14
XX6=DCOSH(BM**CS1+COS(H)DOS(BM**CS1))**DSINH(EMP2)/TD22
A(1+1, (M+1)/2)=XX6
XX=XX/2
XX1=XX1/2
A(M+1)/2,(M+1)/2)=((UFS+EMP2S-BMS)*DCOSH(BM)+TD2*(UFS+EMP2S-
GMS)*DCCSH(GM))/TD2
14 A(N+2)*K_1,(M+1)/2)=XX
A(N+1),(M+1)/2)=XX1
GO TO 20
15 TD2=(BM*(BMS-POIS*PHIS*EMP2S)*(-1)/(GMS*(GMS-POIS*PHIS*EMP2S))
TD2=(BM*TD2*GM
X1=BM/(BMS+ENPS)
X2=GMS/ENPS
XX=2*DCOSH(ENP)*EMP2*(X1+X2)/TD2
X1=(EMP2S-UF1S*BMS)*BM/(BMS+ENPS)
X2=(EMP2S-UF1S*GMS)*TD2*GM/(GMS+ENPS)
XX1=2*DSIN(EMP2)*DCUS(ENP)*(X1+X2)/TD2
IF (JN GT 0.0) GO TO 14
XX=XX/2
XX1=XX1/2
B=1.0
TEST=BM-BM*CSI
IF (TEST LT 60.0) B=0.0
A(I+1),(M+1)/2)=XX6
A((M+1)/2,(M+1)/2)=((UFS+EMP2S-BMS)+TD2*(UFS+EMP2S-GMS))/TD2
GO TO 14
20 CONTINUE
DO 30 M=1,L
TEMP=A(M,1)
A(M,1)=A(M,1+1)
A(M,1+1)=TEMP
30 CONTINUE
CALL DETERM (A,L,DET)
STOP
END
SUBROUTINE DETERM (A,N,DLT)
IMPLICIT REAL*8(A-H,U-Z)
D14ENSILN A(59,99),X(99),EM(75),EN(75),EP(75)
SIGN=1
N=N-1
LAST=M-1
START OVERALL LOOP FOR(N-1) PIVOTS
DO 200 I=1,LAST

FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
BIG=0
DO 50 K=1,M
TERM=ABS(A(K,1))
IF (TERM BIG) 50,50,30
BIG=TERM
L=K
50 CONTINUE

CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
IF (BIG GT 80) 60,60,80
L-TH ROW HAS THE BIGGEST TERM----IS I=L 
80 IF ((I-L)90,120,90
   I IS NOT EQUAL TO L, SWITCH ROWS I AND L 
90 DO 100 J=1,N 
   TEMP=A(I,J) 
   A(I,J)=A(L,J) 
   A(L,J)=TEMP 
100 A(L,J)=TEMP 
   NO* START PIVOTAL REDUCTION 
120 PIVOT=A(I,I) 
   NEXTR=I+1 
   FOR EACH OF THE ROWS AFTER THE I-TH 
   DO 200 J=NEXTR,N 
   MULTIPLYING CONSTANT FOR THE J-TH ROW IS 
   CONST=A(J,I)/PIVOT 
   NOW REDUCE EACH TERM OF THE J-TH ROW 
   DO 200 K=1,N 
   A(J,K)=A(J,K)-CONST*A(I,K) 
200 END OF PIVOTAL REDUCTION---PERFORM BACK SUBSTITUTION 
   M=N-1 
   DO 500 I=1,M 
   IREV IS THE BACKWARD INDEX, GOING FROM M BACK TO 1 
   IREV=N+1-I 
   GET Y(IREV) IN PREPARATION 
   Y=A(IREV,N) 
   IF (IREV-M) 400,500,400 
   NOT WORKING ON LAST ROW, I IS 2 OR greater. 
400 DO 450 J=2,1 
   WORK BACKWARD FOR X(N), X(N-1)------SUBSTITUTING PREVIOUSLY 
   FOUND VALUES 
   K=N+1-J 
450 Y=Y-A(IREV,K)*X(K) 
   FINALLY, COMPUTE X(IREV) 
500 X(IREV)=Y/A(IREV,IREV) 
   FIND AND PRINT EM,EN,EP, AND P* 
   L=(N-1)/3
DO 550 I=1,L
  J=2*I-1
  K=I-1
  EM(J)=X(I)
  EN(J)=X(1+I)
  IF(J.EQ.L) GO TO 540
  EP(J)=X(I+2*L)
  P=X(3*L)
GO TO 550

540 EP(J)=1.
550 PRINT 600,J,K,K,EM(J),K,EN(J),K,EP(J),P
  FORMAT(*= 'M=12.3X','H=12.3X','P=12.5X','EN('=12.5X,'1P'=1D13.5)
  DO 601 I=1,L
    J=2*I-1
    EN(I)=EN(J)
  CALL SHAPE (EM,EN,EP,P)
  CONTINUE
  PRINT 650
  FORMAT(*= 'THE SYMMETRIC MODEL SHAPE DATA ARE: '/)
RETURN.

END
SUBROUTINE SHAPE (EM,EN,EP,P)
IMPLICIT REAL*8(A-H,O-Z)
COMMON 2*PHI,PI,PHIS,PO1,PO2,PHIN,PHINS,UF1,UF2,ALMDS,CS1,K,K1,
  KS
DIMENSION W1(21,21),W2(21,21),W3(21,21),W4(21,21),W(21,21)
EM(75),EN(75),EP(75)
KS1=KS+1
ETA=0.0
DO 650 I=1,KS1
  PSI=0.0
  DO 640 J=1,KS1
    XI=1.0
    YI=0.0
    ZI=0.0
    D2=0.0
    D4=0.0
    DO 620 N=1,K
      JN=N-1
      EN=JN*PI
      ENPS=EN*ENP
      ENS=(1.0/PHIS)*(ENP*ENP+ALMDS*PHIS)
      BN=DSGR(BN)
      XI=ALMDS*PHIS*ENP*ENP
      IF (XI.LE.0.0) GO TO 996
      GNS=1.0/PHIS*XI
      GN=DSGR(GNS)
      X=PHIS*PHINS*ENPS
      TO1L=BN*(BN-X)*DCUSH(BN)/GN*(GNS+X)*DCUS(GN)
      TO1P=DCUSH(BN)/DCUS(GN)
      TO1L=BN*DCOUSH(BN)/GN*DCUS(GN)
      XL=0.0
      IF (XL.EQ.1) DL=2.0
      TO1L=-2.0*DCUS(ENP*CS1)/(DL*(BN*(BN-X)*DCUSH(BN)-TO1L*GN*(GNS+X)*
      DCUS(GN)))
      X=UFS1*ENPS
      TD1IN=BN*DCOUSH(BN)+TD1*GN*DCUS(GN)
      TD1IP=BN*DS1IN(BN)+GN*TD1P*DS1N(GN)
IF (ETA = EQ.0.0) GO TO 601
IF (ENP = EQ.0.0) GO TO 601
XX = DCOS1(ENP*ETA)
GO TO 602

601 XX = 1.0

602 IF (PS1 = EQ.0.0) GO TO 604
XX2 = EN(N)*((DSINH(BN*PSI) + TD1*DSIN(1-PS1)) * XX / TD11)
XX4 = P * TD11L * (DSINH(BN*PSI) + TD1L * DSIN(1-PS1)) * XX
IF (PS1 = EQ.1.0) GO TO 603
XX3 = EP(N)*((DSOSHBN*1-PS1)) - TD1P * DCOSH(GN*(1-PS1)) * XX / TD11P
GO TO 617

603 XX4 = EP(N)*(1-TD1P)*XX / TD11P
GO TO 617

604 XX2 = 0.0
XX3 = 0.0
XX4 = 0.0
GO TO 617

605 XX = X1
GNS = (1+PH1S)*X1
GN = DSQRT(GNS)
X = PSI*PSI*ENPS
IF (BN*GT*Z2) GO TO 612
TD2L = -(BN*(BNS-X)*DCOSHBN)/(GN*(X-GNS)*DCOSH(GN))
TD2P = DCOSHBN/GNSSHBN
TD2L = -(BN*DCOSHBN)/(GN*DCOSH(GN))
TD2P = 0.0
GO TO 617

606 IF (X = EQ.1.0) DL = 2.0
TD2L = -2*DCOSH(ENPS1)/(DL*(BN*(BNS-X)*DCOSHBN)+TD2L*GN*(GNS-X))
TD2P = DCOSH(1-PS1)
IF (ETA = EQ.0.0) GO TO 607
IF (ENP = EQ.0.0) GO TO 607
XX = DCOS1(ENP*ETA)
GO TO 608

607 XX = 1.0

608 IF (PS1 = EQ.0.0) GO TO 610
XX2 = -EN(N)*((DSINHN(BN*PSI) + TD2*DSINHBN*1-PS1)) * XX / TD22
XX4 = P * TD22L * (DSINHN*BN*PSI) + TD2L * DSINHBN*1-PS1)) * XX
IF (PS1 = EQ.1.0) GO TO 609
XX3 = EP(N)*((DCOSHBN*1-PS1)) - TD2P*DCOSH(GN*(1-PS1)) * XX / TD22P
GO TO 617

609 XX3 = EP(N)*(1-TD2P)*XX / TD22P
GO TO 617

610 XX2 = 0.0
XX3 = 0.0
XX4 = 0.0
GO TO 617

612 TD2N = BN*PSI*PHIS*ENPS/(GN*(PSI*PHIS*ENPS-GNS))
IF (ETA = EQ.0.0) GO TO 613
IF (ENP = EQ.0.0) GO TO 613
XX = DCOS1(ENP*ETA)
GO TO 614

613 XX = 1.0

614 IF (PS1 = EQ.0.0) GO TO 616
G = 1.0
IF (TEST = EN-BN*PSI) 0.0
XX = E4((DEXP((BN*PSI-BN)*B) + TD2N*DEXP((GN*PSI-GN)*B))*CN(N)/
1(BN+TD2N+B))
C1=2*COS(EMP*CSI)
C2 = BN*(BNS-POIS*PHIS*EMP5)
C3 = GN*(GNS-POIS*PHIS*EMP5)
DL = 1.0
IF (N*EQ.1) DL = 2.0
TD2L = -BN/GN
TD2L = C1/(DL*C2+TD2L*C3))
X#4 = B*TD2L*XX *(DEXP((BN*PSI-BN)*B)+TD2L*DEXP((GN*PSI-GN)*B))
IF (PSI*EQ.1.0) GO TO 615
TD2P = 1.
TD2P = 8N-GN*TD2P
B = 1.
TEST = BN*PSI
IF (TEST*GT.60.0) B = 0.
X#3 = EXP(N)*B*XX *(DEXP((-BN*PSI)*B)-TD2P*DEXP((-GN*PSI)*B))/TD2P
GO TO 617
615 X#3 = 0.0
GO TO 617.
616 X#2 = 0.0
X#3 = 0.0
X#4 = 0.0
617 X#2 = 22+XW2
X#3 = 33+XW3
X#4 = 21.0
X#3 = 31.0
X#4 = 41.0
620 CONTINUE
DD 630 M = 1, K1, 2
EMP2 = EMP2*EMP2
BMS = PHIS*(ALMDS+EMP2)
EM = DSGRT(BMS)
X#1 = ALMDS-EMP2
IF (X#1*LT.0.0) GO TO 623
BMS = PHIS*X1
GM = DSGRT(GMS)
TJ1 = (BMS*(BMS-POIS*PHIS*EMP2)*DSINH(BM))/(GM*(GMS-POIS*PHIS*EMP2)
1 * DSINH(GM))
T1 = TD1-(-1)
TJ1 = EM*DSINH(BM)-TD1*GM*DSINH(GM)
IF (PSI*EQ.0.0) GO TO 622
IF (ETA*EQ.0.0) GO TO 621
X#1 = EM(M)*((DCUSH(4*ETA)+TD1*UCGS(GM*ETA)))*DSIN(EMP2*PSI)/TD11
X#1 = TD11
621 X#1 = EM(M)*(1+TD1)*DSIN(EMP2*PSI)/TD11
GO TO 629
622 X#1 = 0.0
GO TO 629
623 X#1 = X1
BMS = PHIS*X1
GM = DSGRT(GMS)
IF (BMS*GT.22) GO TO 625
T2 = (BMS*(BMS-POIS*PHIS*EMP2)*DSINH(BM))/(GM*(GMS-POIS*PHIS*EMP2)
1 * DSINH(GM))
T2 = TD2-(-1)
T2 = EM*(BMS-POIS*PHIS*EMP2)*DSINH(BM)/(GM*(BMS-POIS*PHIS*EMP2)
1 * DSINH(GM))
T2 = TD2+TD2*EM*DSINH(GM)
IF (PSI*EQ.0.0) GO TO 622
IF (ETA*EQ.0.0) GO TO 624
X#1 = EM(M)*((DCUSH(BM*ETA)+T2*DCUSH(GM*ETA)))*DSIN(EMP2*PSI)/TD22
GO TO 624
GO TO 629
624 XL1= EM(M)*(1+TD2)*DSIN(EMP2*PSI)/TD22
   GO TO 629
625 A=BM*(POIS*PHIS*EMP25-0.0)/(GM*(GMS-POIS*PHIS*EMP2))
   IF (PSI.EQ.0.0) GO TO 627
   IF (ETA.EQ.0.0) GO TO 627
   TEST=BM-VM*ETA
   IF (TEST.GT.60.0) GO TO 627
   XL1=EM(M)*(DEXP(BM*ETA-BM)+A*DEXP(GM*ETA-GM)))*DSIN(EMP2*PSI)/
   1(BM+A*GM)
   GO TO 629
627 XW1=0.0
629 X1(I,J)=XW1+W11
   W11=W1(I,J)
630 CONTINUE
   PRINT 635,PSI,ETA,w1(I,J),w2(I,J),w3(I,J),w4(I,J)
635 FORMAT(*-40,1X,PSI=*,F5.3,3X,ETA=*,F5.3,3X,W1(*,*,12,*,12,*)=*,
   1D12.5,3X,W2(*,12,*,12,*)=*,D12.5,3X,W3(*,12,*,12,*)=*,
   1D12.5,3X,W4(*,12,*,12,*)=*,D12.5,7)
   PRINT 635,PSI,ETA,w1(I,J)
636 FORMAT(*-40,20X,PSI=*,F5.3,4X,ETA=*,F5.3,4X,
   1D(*,12,*,12,*)=*,D12.5)
   PS1=PS1+1.0/DFLOAT(KS)
640 CONTINUE
   ETA=ETA+1.0/DFLOAT(KS)
650 CONTINUE
   RETURN
   END
PROGRAM 11

THIS IS AN ANTISYMMETRIC MODE EIGENVALUE SEARCH
PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION
PROBLEM WITH POINT SUPPORTS SYMMETRICALLY LOCATED ON THE
EDGE PARALLEL TO THE CLAMPED BASE.

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:

1. - K = THE NUMBER OF TERMS TO BE USED IN THE
       SERIES INVOLVED. FOR 5 DIGITS ACCURACY
       USE K = 30.

2. - CSI = 0 TO 1 PROVIDING THE DISTANCE BETWEEN
       THE CONCENTRATED EDGE FORCE(POINT SUP.)
       AND THE PLATE CENTRAL AXIS DIVIDED BY THE
       PLATE EDGE DIMENSION.

3. - PHIR = 2B/A, IS THE FULL PLATE ASPECT RATIO.

4. - ALMDS = EIGENVALUE.

5. - DLIM = EIGENVALUE SEARCH ENDING LIMIT. IT
       INSTRUCTS THE COMPUTER WHEN TO HALT
       EXECUTION.

6. - DEL = EIGENVALUE INCREMENT.

7. - PUI = POISSON'S RATIO.

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(59,99)
K=??????????????????????????????????
CSI=????????????????????????????????
K1=2*K
K1=2*K-1
Z2=3600.0
PHI=R=??????????????????????????
PHI=PHIR/2.
PRINT50,PHIR,K,CSI
50 FORMAT(1'***PHIR=',F7.4,10X,'K=',I5,10X,'CSI=',F7.4,10X)
C=12
P1=4.*DATAN(C)
PHIS=PHI*PHI
PUI=??????????????????????
POIS=2-PUI
PHIN=1./PHI
PHINS=PHIN*PHIN
UFIS=PO1*PHINS
I=J+K+1
ALMDS=????????????????????
DLM=????????????????????
DEL=????????????????????
1 CONTINUE

INITIALIZE THE MATRIX TO 0.0

DO 2 M=1,1
DO 2 N=1,1
2 A(M,N)=0.0
DO 10 M=1,K1+2
DO 10 N=1,K1+2
EMP2=N*PI/2.
EMP2=EMP2*EMP2
EMP2=A*PI/2.
EMP2=EMP2*EMP2
BNS=PHINS*(EMP2+ALMDS*PHIS)
EN=DSCRT(UNS)
X1=ALMDS*PHIS-EMP2
IF (X1+L*T.0.0) GO TO 4
GNS=PHINS*X1
GNS=DSCRT(GNS)
D11=GNS*DCOS(GN)*(GNS*EMP2*PHINS)/(BN*DCOS(BN)*(BNS-EMP2*)
1 FOIS*PHINS)
T011=BN*TD1N*DCOSH(BN)+GN*DCUS(GN)
T01P=DCOSH(BN)/DCUS(GN)
T11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
T01L=-BN*DCUSH(BN)/(GN*DCOS(GN))
T11L=GN*DSINH(EMP2*CSI)/(BN*(BNS-P01S*PHINS*EMP2)*DCUSH(BN)
1-TD1L*GN*DSINH(GN)*DSINH(GN)
GNS=PHINS*X1
GNS=DSCRT(GNS)
D11=GNS*DCOS(GN)*(GNS*EMP2*PHINS)/(BN*DCOS(BN)*(BNS-EMP2*)
1 FOIS*PHINS)
T011=BN*TD1N*DCOSH(BN)+GN*DCUS(GN)
T01P=DCOSH(BN)/DCUS(GN)
T11P=BN*DSINH(BN)+GN*DSIN(GN)*TD1P
T01L=-BN*DCUSH(BN)/(GN*DCOS(GN))
T11L=GN*DSINH(EMP2*CSI)/(BN*(BNS-P01S*PHINS*EMP2)*DCUSH(BN)
1-TD1L*GN*DSINH(GN)*DSINH(GN))
IF (M+GT.+1.) GO TO 3
A(I,N+1)/2+K)=TD11A*DSINH(BN)+DSIN(GN)*DSIN(EMP2*CSI)/TD11N
A(I,N+1)/2+K)=TD1P*DSIN(EMP2*CSI)/TD1P
A(N+1)/2+K)=TD11A*DSINH(BN)+DSIN(GN)*DSIN(EMP2*CSI)/TD11N
A(N+1)/2+K)=TD1P*DSIN(EMP2*CSI)/TD1P
X1=USIN(EPS2+GNS)*DSINH(BN)+TD1L*USIN(BN)+(UFIS*
1 EPS2+GNS)*DSIN(GN))
TD11N
A(N=1)/2+K)=TD11L*(USINH(BN)+TD1L*USIN(GN))*DSINH(EMP2*CSI)
1-1I
A(N=1)/2+K)=TD11L*USINH(BN)+TD1L*(USINH(BN)+TD1L*USIN(GN))
A(N=1)/2+K)=TD11L*USINH(BN)+TD1L*USINH(BN)+TD1L*(USINH*
1 EPS2+GNS)*DSINH(GN))
A((N+1)/2,(N+1)/2+K2)=XX 
X2=(((EMP2S-UF5*BMS)/(BMS+EMP2S))\*BMS*COSH(BM) 
X2=(((EMP2S-UF5S)/(GMS+EMP25))\*TD2L*GMS*COSH(GM) 
XX=2*TD2L*DSIN(EMP2)\*DSIN(EMP2)*(X+X2) 
A((N+1)/2,1)=A((N+1)/2,1)+XX 

CONTINUE 
DO 11 N=1,K 
11 A(N+K2,N+K2)=-1. 
DO 20 N=1,K1,2 
DO 20 M=1,K1,2 
EMP2S=EMP2S*EMP2 
EMP2S=EMP2S*EMP2 
BMS=PHIS*(ALMDS+EMP25) 
EM=DSORT(BMS) 
X=ALMDS-EMP2S 
IF (X.LT.0.0) GO TO 13 
GMS=PHIS*X1 
GM=DSORT(GMS) 
TJ1M=GM*(GMS+EMP2S*POIS+PHIS)*COSGM*(BMS*EMP2S* 
FUS+PHIS)*COSHGM 
TJ1M=GM+TD1M*COSH(BM)+GM*COSGM 
IF (N.GT.1) GO TO 12 
A((N+1)/2,(N+1)/2)=((UF5S*EMP2S-BMS)+TD1M*DSINGM)+GM*EMP2S 
A((N+1)/2,(N+1)/2)=((UF5S*EMP2S-BMS)+TD1M*DSINGM)+GM*EMP2S 
12 X1=(((EMP2S-UF5S*BMS)/(BMS+EMP2S))\*TD1M*GM*COSH(BM) 
X2=(((EMP2S-UF5S*BMS)/(BMS+EMP2S))\*GM*COSH(BM) 
XX=2*DSIN(EMP2)\*DSIN(EMP2)*(X1-X2)/TD1M 
A((N+1)/2,X2)=X 
X1=TD1M*GM+COSH(BM)/(BMS+EMP2S) 
X2=GM*COSGM/(BMS+EMP2S) 
X=2*EMP2*DSIN(EMP2)*(X1-X2)/TD1M 
A((N+1)/2+K2,(N+1)/2)=X 

GO TO 20 
13 X1=-X1 
GMS=PHIS*X1 
GM=DSORT(GMS) 
IF (N.GT.0) GO TO 15 
T2N=GM*(POIS+PHIS*EMP2S-GMS)/(UM*(BMS-POIS+PHIS*EMP2S)) 
T2N=GM*BM+GM 
IF (N.GT.1) GO TO 14 
TEST=BM*BM*CSI 
B=1.0 
IF (TEST.GT.60.) B=0.0 
A((N+1)/2,(N+1)/2)=B*DSIN(EMP2)*(TD2M+EXP((DM*CSI-BM)*B)+EXP((GM*CSI 
1.0)/B)) 
A((N+1)/2,(N+1)/2)=((UF5S*EMP2S-BMS)*TD2M+(UF5S*EMP2S-GMS))/TD2M 
14 X1=(((EMP2S-UF5S)/(BMS+EMP2S))\*TD2M*BM 
X2=((EMP2S-UF5S)/(BMS+EMP2S))\*GM 
XX=2*DSIN(EMP2)\*DSIN(EMP2)*(X1+X2)/TD2M 
A((N+1)/2,X2)=X 
X1=TD2M*BM/(BMS+EMP2S) 
X2=GM/(BMS+EMP2S) 
XX=2*EMP2*DSIN(EMP2)*(X1+X2)/TD2M 
A((N+1)/2+K2,(N+1)/2)=XX 
GO TO 20 
15 TD2M=GM*(POIS+PHIS*EMP2S-GMS)*COSH(GM)/(BMS*GM*POIS+PHIS* 
1.0*EMP2S)*COSH(DM) 


TD22M = TD2M * BM * DCOSH(BM) + GM * DCUSH(GM)

IF (A GT .1) GO TO 16

M(1, (M+1)/2) = (TD2M * DSINH(BM * CSI) + DSINH(GM * CSI)) * DSINH(EMP2) / TD22M
A((N+1)/2, (M+1)/2) = ((UFS * EMP2 - BMS) * TD2M * DSINH(BM) + (UFS * EMP2 -
16) * DSINH(GM)) / TD22M

16 X1 = ((EMP2 - UFS * BMS) / (BMS + ENP2S) * TD2M * BM * DCUSH(BM)
X2 = ((EMP2 - UFS * GMS) / (GMS + ENP2S) * GM * DCOSH(GM)
XX = 2 * DSIN(EMP2) * DSIN(ENP2) * (X1 + X2) / TD22M
A((N+1)/2, K, (M+1)/2) = XX
X1 = TD2M * BM * DCOSH(BM) / (BMS + ENP2S)
X2 = GM * DCOSH(GM) / (GMS + ENP2S)
XX = 2 * EMP2 * DSIN(ENP2) * (X1 + X2) / TD22M
A((N+1)/2, K, (M+1)/2) = XX

20 CONTINUE
CALL DETERM (A, I, DET)
PRINT 110, ALMDS, DET
110 FORMAT ('* ALMDS= ', F13.7, ' X= ', D20.5)
IF (ALMDS GT .0) GO TO 21
ALMDS = ALMDS + DEL
GO TO 1
21 CONTINUE
STOP
END

SUBROUTINE DETERM (A, N, DET)
IMPLICIT REAL*8(A-H, O-Z)
DIMENSION A(99, 99)
SIGN = 1.
LAST = N - 1

START OVERALL LOOP FOR(N-1) PIVOTS
DO 200 I = 1, LAST

FIND THE LARGEST REMAINING TERM ON I-TH COLUMN FOR PIVOT
BIG = 0.
DO 50 K = 1, N
TERM = DABS(A(I, K))
IF (TERM GT BIG) GO TO 50
BIG = TERM
L = K
50 CONTINUE

CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
IF (BIG LT .0) GO TO 90
L-TH ROW HAS THE BIGGEST TERM ---- IS I=L
DO 90 IF (I - L) 90, 120, 90

I IS NOT EQUAL TO L, SWITCH ROWS I AND L
90 SIGN = -SIGN
DO 100 J = 1, N
TEMP = A(I, J)
A(I, J) = A(L, J)
A(L, J) = TEMP
100 ALMDS = TEMP

NO* START PIVOTAL REDUCTION
C 120 PIVOT=A(1,1)  
   NEXTR=1+1
C FOR EACH OF THE ROWS AFTER THE I-TH
C DO 200 J=NEXTR,N
C MULTIPLYING CONSTANT FOR THE J-TH ROW IS
C CGN=1=PIVOT
C NOW REDUCE EACH TERM OF THE J-TH ROW
C DO 200 K=I,N
C 200 A(J,K)=A(J,K)-CGN*A(I,K)
C END OF PIVOTAL REDUCTION--NOW COMPUTE DETERMINANT
C DET=SIGA
C DO 300 I=1,N
C 300 DET=DET*A(I,I)/10
C GO TO 61
C 60 DET=0
C 61 RETURN
C END
PROGRAM 12

THIS IS AN ANISOTROPIC PLATE SHAPE PROGRAM FOR THE
RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM, WITH
POINT SUPPORTS SYMMETRICALLY LOCATED ON THE EDGE PARALLEL TO
THE CLAMPED BASE.

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:

1. - K = AS DEFINED IN PROGRAM 11.
2. - CSI = AS DEFINED IN PROGRAM 11.
3. - PHIR = 2B/A FULL PLATE ASPECT RATIO.
4. - ALMDS = EIGENVALUE.
5. - KS = NUMBER OF POINTS AT WHICH THE DISPLACE-
   MENT W IS REQUIRED.
6. - POI = POISSON'S RATIO.

IMPLICIT REAL*8 (A-H,O-Z)
DIMENSION A(99,99)
COMMON ZZ,PHI,PI,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,CSI,K,K1,
1KS
K=1:5
KS=1
CSI=1
K2=2*K
K1=2*K-1
ZZ=3600.
PHIR=PHI/2.
ALMDS=1.
PRINT50,PHIR,K,CSI,ALMDS
50 FORMAT('1''PHIR =',F7.4,10X,'K =',I5,10X,'CSI =',F7.4,10X,'ALMDS
1 =',F7.4,///)
PRINT51
51 FORMAT('1''30X,'THE VALUES OF EM,EN,EP,AND P ARE:','///)
C=1.
PI=4.*DAN4(C)
PHIS=PHI*PI
POI=0.333
POIS=2*POI
PHIN=1./PHI
PHINS=PHIN*PHIN
UFS=POI*PHIS
UFIS=POI*PHINS
I=3*K+1
1 CONTINUE

INITIALIZE THE MATRIX TO 0.0

DO 2 M=1,I
DO 2 N=1,I
2 A(M,N)=0.0
DO 10 K=1,K1,2
DO 10 N=1,K1,2
EMP2=M*PI/2.
EMP2S=EMP2
EMP2=N*PI/2.
EMP2S=EMP2
BNS=PHINS*(ENP2S+ALMDS*PHIS)
BV=DSQRT(BNS)
X1=ALMDS*PHIS-EMP2S
IF (X1.LT.0.0) GO TO 4
GNS=PHINS*X1
GN=DSQRT(GNS)
TD1N=GN*DCOS(GN)*(GNS+ENP2S*POIS*PHINS)/(BN*DCOSH(BN)*(BNS-ENP2S*1POIS*PHINS))
TD11N=BN*TD1N*DCOSH(BN)+GN*DCOSIGN)
TD1P=DCOSH(BN)/DCOSIGN)
TD1P=BN*DSINH(1BN)+GN*DSIN(1GN)*TD1P
TD1NL=BN*DCOSH(BN)/IGN*DCOS(IGN))
TD1IL=-2*DSIN(ENP2*CS1)/(BN+BNS-POIS*PHINS*ENP2S)*DCOSH(BN)
1-TD11N*GN*(GNS+POIS*PHINS*ENP2S)*DCOS(IGN))
IF (M.GT.1.) GO TO 3
A(N+1)/2+K=TD1N*DSINH(BN)+DSIN(IGN))
A(N+1)/2+K=TD1P*DSIN(ENP2*CS1)/TD1P
A(N+1)/2+K=TD11N*BN*BN+GN)/TD1N
A(N+1)/2+K=TD1IL*IGN*IGN+(UDIS*ENP2S*BN)*TD1N*DSINH(BN)+UDIS*
1-ENP2S+GNS)*DSINH(GN))/TD1NL
A(N+1)/2+K=UDIS*ENP2S*1-1TD1P*GNS)/TD1NL
XX=TD1IL*(DSINH(BN)+TD1L*DSIN(IGN))/DSIN(ENP2*CS1)
XX=AI,1=AI,1+XX
A(N+1)/2+K=TD1IL*(BN+TDI1L*GN)
A(N+1)/2+K=UDIS*ENP2S*BN)*DSINH(BN)*TD1IL*UDIS*
1-ENP2S+GNS)*DSINH(GN))
3 X1=(ENP2S-UF*BNS)/(BN*EMP2S)*TD1N*BN*DCOSH(1BN)
X2=(ENP2S-UF*GNS)/(GNS-EMP25)*GN*DCOSIGN)
XX=2*DSIN(EMP2*ENP2)*(X1-X2)/TD1N
A(M+1)/2,(N+1)/2+K)=XX
X1=(ENP2S-UF*BNS)/(BN*EMP2S)*BN*DCOSH(1BN)
X2=(ENP2S-UF*GNS)/(GNS-EMP2S)*TD1P*DCOSIGN)
XX=2*DSIN(EMP2*ENP2)*(X1-X2)/TD1P
A(M+1)/2,(N+1)/2+K)=XX
X1=(ENP2S-UF*BNS)/(BN*EMP2S)*BN*DCOSH(1BN)
X1=(ENP2S-UF*GNS)/(GNS-EMP2S)*TD1L*GN*DCOSIGN)
XX=2*TD1IL*DSIN(EMP2)*DSIN(EMP2)*(X1-X11)
A(N+1)/2,1=A(M+1)/2,1+XX
GO TO 10
4 XX=X1
GNS=PHINS*XX
GN=DSQRT(GNS)
IF (BNS.LT.22) GO TO 6
TD2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
TD2LN=TD2N*BN+GN

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TD2P = 1.
TD2P = GN * GN * TD2P

TD2L = BN * GN
TD2L = 2 * DSIN(ENP2 * CSI) / IBN * (BNS * POIS * PHINS * ENP2S) + TD2L * GN

IF (M * GT * 1) GO TO 5
A(I1 * (N + 1) / 2 + K1) = TD2N + 1 * DSIN(ENP2 * CSI) / TD22N
A(I1 * (N + 1) / 2 + K1) = 0.0
A(I1 * (N + 1) / 2 + K1) = 0.0
A(I1 * (N + 1) / 2 + K1) = TD2L * (UFIS * ENP2S * BNS + TD2L * (UFIS * ENP2S * GNS))
A(I1 * (N + 1) / 2 + K1) = DSIN(ENP2 * CSI)
A(I1 * (N + 1) / 2 + K1) = 0.0
A(I1 * (N + 1) / 2 + K1) = 0.0
A(I1 * (N + 1) / 2 + K1) = (UFIS * ENP2S * BNS) * TD2N + UFIS * ENP2S

1-GNS) / TD22N

5 X1 = 1 * ENP2S * UFS * BNS) / (BNS * ENP2S) * TD2N * BN
X2 = (ENP2S * UFS * GNS) / (GNS * ENP2S) * GN
XX = 2 * DSIN(ENP2) * DSIN(ENP2) * (X1 + X2) / TD22N
A(I1 * (N + 1) / 2 + K1) = X1
X1 = (ENP2S * UFS * BNS) / (BNS * ENP2S)
X2 = (ENP2S * UFS * GNS) / (GNS * ENP2S)
XX = 2 * DSIN(ENP2) * ENP2 * (X1 + X2) / TD22P
A(I1 * (N + 1) / 2 + K1) = X2
X2 = (ENP2S * UFS * BNS) / (BNS * ENP2S) * BN
XX2 = (ENP2S * UFS * GNS) / (GNS * ENP2S) * TD2L * GN
XX = 2 * TD2L * DSIN(ENP2) * DSIN(ENP2) * (X2 + X2)
A(I1 * (N + 1) / 2 + K1) = (A(I1 * (N + 1) / 2 + K1) + XX)

GO TO 10

6 TD2N = GN * (POIS * PHINS * ENP2S * GNS) * DCOSH(GN) / IBN * (BNS * POIS * PHINS * ENP2S)
DCOSH(BN)),
TD2N = BN * DCOSH(BN) / DCOSH(GN)
TD2P = DCOSH(BN) / DCOSH(GN)

TD2N = BN * DSINH(BN) - GN * TD2P * DSINH(GN)
TD2L = BN * DCOSH(BN) / (GNS * DCOSH(GN))
TD2L = -2 * DSIN(ENP2 * CSI) / IBN * (IBNS * POIS * PHINS * ENP2S) * DCOSH(BN) +
TD2L = GN * POIS * PHINS * ENP2S) * DCOSH(GN)

IF (M * GT * 1) GO TO 5
A(I1 * (N + 1) / 2 + K1) = TD2N * DSINH(BN) / DSINH(GN) * DSIN(ENP2 * CSI) / TD22N
A(I1 * (N + 1) / 2 + K1) = 0.0
A(I1 * (N + 1) / 2 + K1) = TD2P * DSINH(ENP2 * CSI) / TD22P
A(I1 * (N + 1) / 2 + K1) = 0.0
A(I1 * (N + 1) / 2 + K1) = TD2L * DSINH(GN) / DSINH(ENP2 * CSI)
A(I1 * (N + 1) / 2 + K1) = (UFIS * ENP2S * BNS) * TD2N * DSINH(BN) * (UFIS *
1-ENP2S * GNS) * DSINH(ENP2) / TD22N
A(I1 * (N + 1) / 2 + K1) = UFIS * ENP2S * (1 - TD2P * BNS * TD2P * GNS) / TD22P
A(I1 * (N + 1) / 2 + K1) = DSIN(ENP2) * DSIN(ENP2) * (X1 + X2) / TD22P
A(I1 * (N + 1) / 2 + K1) = X1
XX = 2 * DSIN(ENP2) * ENP2 * (X1 + X2) / TD22P
A(I1 * (N + 1) / 2 + K1) = X2
XX = 2 * TD2L * DSIN(ENP2) * DSIN(ENP2) * (X2 + X2)

IF (M * GT * 1) GO TO 5
A(I1 * (N + 1) / 2 + K1) = TD22L * (BN * TD2L * GN)
A(I1 * (N + 1) / 2 + K1) = TD2L * (UFIS * ENP2S * BNS) * DSINH(BN) + TD2L *
1-ENP2S * GNS) * DSINH(GN))
10 A(M+1)/2,I= =A(M+1)/2,I+XX
CONTINUE
DO 11 N=1,K
11 A(N+K2,N+K2)=-1.
DO 20 N=1,K1,2
20 M=1,K1,2
ENP2=NP1/2.
ENP2S=ENP2*ENP2
ENP2=NP1/2.
ENP2S=ENP2*ENP2
BMS=PHIS*(ALMD5+EMP2S)
BMS=DSQRAT(BMS)
X1=ALMD5-EMP2S
IF (X1.LT.0.0) GO TO 13
GMS=PHIS*X1
GMS=DSQRAT(GMS)
TM1=GM*(GMS+EMP2S*POIS*PHIS)*DCOS(GM)/(BMS*(BMS-EMP2S*POIS*PHIS)*DCOSH(BM))
TD11=GM*TD1M*DCOSH(BM)+GM*DCOS(GM)
IF (N,GT,1) GO TO 12
A(M+1)/2=TD1M*DSINH(BM)*CSI)*DSIN(GM)*TD11M/(TD11+TD1M)*DSINH(BM)+UFS*POIS*PHIS*DSIN(GM)/TD11M
12 X1=(EMP2S-UFIS*GM)/(BMS+EMP2S)*TD1M*BM+DCOSH(BM)
X2=(EMP2S+UFIS*GM)/(BMS-EMP2S)*GM*DCOS(GM)
X=X2*DSIN(EMP2)*DSIN(EMP2)/(X1-X2)/TD11M
A(M+1)/2+X1,(M+1)/2=XX
X1=TD1M*BM+DCOSH(BM)/(BMS+EMP2S)
X2=GM*DCOS(GM)/(BMS-EMP2S)
X=X2*EMP2*DSIN(EMP2)/(X1-X2)/TD11M
A(M+1)/2+X2,(M+1)/2=XX
GO TO 20
13 X1=XX
GMS=PHIS*X1
GMS=DSQRAT(GMS)
IF (BMS.LT.22) GO TO 15
TD2M=GM*(POIS*EMP2S-GMS)/(BMS*(BMS-POIS*PHIS*EMP2S))
TD2M=TD2M*BM+GMS
IF (N,GT,1) GO TO 14
TEST=BM*BM*CSI
B=1.0
IF (TEST.GT.60.0) B=0.0
A(M+1)/2=2*DSIN(EMP2)*(TD2M*DEXP((BM*CSI-BM)*B)*DEXP((GM*CSI-1)*(BM*CSI-BM)*B))
14 X1=(EMP2S-UFIS*GMS)/(BMS+EMP2S)*TD2M+UFS*EMP2S
X2=(EMP2-UFIS*GMS)/(BMS-EMP2S)*GM
X=X2*DSIN(EMP2)*DSIN(EMP2)/(X1*2)/TD2M
A(M+1)/2+X1,(M+1)/2=XX
X1=TD2M*BM*(BMS+EMP2S)
X2=GM*GMS*EMP2S
X=X2*EMP2*DSIN(EMP2)/(X1*2)/TD2M
A(M+1)/2+X2,(M+1)/2=XX
GO TO 20
15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BMS*(BMS-POIS*PHIS*EMP2S)*DCOSH(BM))
TD2M=TD2M*BM+DCOSH(BM)+GM*DCOSH(GM)
IF (N,GT,1) GO TO 16
A(M+1)/2=(TD2M+DSINH(BM)*CSI)*DSINH(GM)*TD2M
A(M+1)/2=(UFS+EMP2S-BMS)*TD2M+DSINH(BM)+UFS*EMP2S-
16 IGMS1=DSINH(GM))/TD22M
   X1=I*EMP2S-UFIS+BMS)/(BMS+ENP2S)*TD2*BM*DCOSH(BM)
   X2=I*EMP2S-UFIS+GMS)/(GMS+ENP2S)*GM*DCOSH(GM)
   XX=2*DSIN(EMP2)*DSIN(ENP2)*(X1+X2)/TD22M
   A((N+1)/2+K, (M+1)/2)=XX
   X1=TD2M*BM*DCOSH(BM)/(BMS+ENP2S)
   X2=GM*DCOSH(GM)/(GMS+ENP2S)
   XX=2*EMP2*DSIN(ENP2)*(X1+X2)/TD22M
   A((N+1)/2+K2, (M+1)/2)=XX

20 CONTINUE
   DO 30 M=1, I
   TEMP=A(M, I-1)
   A(M, I-1)=A(M, I)
   A(M, I)=TEMP
   30 CONTINUE
   CALL DETERM (A, I, DET)
   PRINT 31
   FORMAT (6H1°, 10X, 8X'XXXXXXXXXXXXX'°, 10X, 'END'°, 10X,
   1°'XXXXXXXXXXXXX'°)
   STOP

END

SUBROUTINE DETERM (A, N, DET)
IMPLICIT REAL*8(A, M, O, Z)
DIMENSION A(99, 99), X(99), EM(75), EN(75), EP(75)
SIGN=1.
M=N=1
LAST=M-1

C START OVERALL LOOP FOR (N-1) PIVOTS

DO 200 I=1, LAST

C FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT

BIG=0.
   DO 50 K=I, M
      TERM=ABS(A(K, I))
      IF (TERM>BIG) 50, 50, 30
   50 BIG=TERM
   L=K
   30 CONTINUE

C CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND

IF (BIG)<0.60, 60, 80

L-TH ROW HAS THE BIGGEST TERM----IS I=L

80 IF (I=L) 90, 120, 90.

C I IS NOT EQUAL TO L, SWITCH ROWS I AND L

90 DO 100 J=1, N
   TEMP=A(I, J)
   A(I, J)=A(L, J)
   A(L, J)=TEMP
100

C NOW START PIVOTAL REDUCTION

120 PIVOT=A(I, I)
NEXTX=I+1

FOR EACH OF THE ROWS AFTER THE I-TH
DO 200 J=NEXTX,M

MULTIPLYING CONSTANT FOR THE J-TH ROW IS
CONST=A(J,I)/PIVOT

NOW REDUCE EACH TERM OF THE J-TH ROW

DO 200 K=I,N

200 A(J,K)=A(J,K)-CONST*A(I,K)

END OF PIVOTAL REDUCTION—PERFORM BACK SUBSTITUTION

M=N-1
DO 500 I=1,M

IREV IS THE BACKWARD INDEX, GOING FROM M BACK TO 1
IREV=M+1-I

GET Y(IREV) IN PREPARATION
Y=A(IREV,N)
IF (IREV=M) 400, 500, 400

NOT WORKING ON LAST ROW, I IS 2 OR GREATER

400 DO 450 J=2,I

WORK BACKWARD FOR X(N), X(N-1)----SUBSTITUTING PREVIOUSLY FOUND VALUES

K=N+1-J
450 Y=Y-A(IREV,K)*X(K)

FINALLY, COMPUTE X(IREV)
500 X(IREV)=Y/A(IREV,IREV)

FIND AND PRINT EM, EN, EP, AND P

L=(N-1)/3
DD 550 I=1,L
J=2*I-1
K=J
EM(J)=X(J)
EN(J)=X(I+L)
IF(I.EQ.L) GO TO 540
EP(J)=X(I+2*L)
P=X(3*L)
GO TO 550

540 EP(J)=-1.
550 PRINTS00,J,K,EM(J),EN(J),I2,EP(J),P
600 FORMAT('T=S','I2,3X,'S'I2,3X, 'S'P='T'I2,5X,'S'EM('I2,4')='T'D13.5,5X,'S'EP('I2,4')='T'D13.5,5X,'S')
PRINT 601
601 FORMAT("10,3X,"THE ANTI-SYMMETRIC MODE SHAPE DATA ARE=",/)"
CALL SHAPE (EM, EN, EP, P)
RETURN.
END
SUBROUTINE SHAPE (EM, EN, EP, P)
IMPLICIT REAL*8(A-H, O-Z)
COMMON Z2, PHI, PI, PHIS, POIS, PHINS, UFS, UFIS, ALMDS, CSI, K, K1,
K2
DIMENSION X1(21, 21), X2(21, 21), X3(21, 21), X4(21, 21), X5(21, 21).
X1, EM(75), EN(75), EP(75).
K1 = KS + 1
ETA = 0.0
DO 650 I = 1, KS1
PSI = 0.0
DO 660 J = 1, KS1
X1(I, J) = 0.0
W22(I, J) = 0.0
W33(I, J) = 0.0
W44(I, J) = 0.0
DO 680 N = 1, K1, 2
ENP2 = N * PI / 2.
ENP25 = ENP2 * ENP2
BNS = PHINS + (ENP25 + ALMDS * PHIS)
BN = DSGRT(BNS)
X1 = ALMDS * PHIS - ENP25
IF (X1, LT, 0.0) GO TO 605
GNS = PHINS * X1
GNS = DSGRT(GNS)
TDIN = GNS * DCOS(GN) * (GNS + ENP25 * POIS + PHINS) / (BN * DCOSH(BN) * (BNS = ENP25 +
1 * POIS * PHINS))
TD11 = BN * TDIN * DCOSH(BN) * GNS * DCOS(GN)
TD1P = DCOSH(BN) / DCOS(GN)
TD11P = BN * DSINH(BN) * GNS * DSINH(GN) * TD1P
TD11L = BN * DCOSH(BN) / (GN * DCOS(GN))
TD11L = -2 * DSIN(ENP2 * CSI) / IBN + (BNS - POIS * PHINS * ENP25) * DCOSH(BN) -
1 * TD11L * GN * (GNS + POIS * PHINS * ENP25 * DCOSIGN)
IF (ETA, EQ, 0.0) GO TO 602
IF (PSI, EQ, 0.0) GO TO 603
X1(I, J) = ENP2 * TD11L * DSINH(BN) * PSII) * DSIN(ENP2 * ETA) / TD11N
X44(I, J) = X3(I, J) + X1(I, J) * TD11L * DSINH(BN) * PSII) * DSIN(ENP2 * ETA)
IF (PSI, EQ, 1.0) GO TO 601
X33(I, J) = EP(I, N) * (DCOSH((1 - PSI) * BN) - TD1P * DCOS((1 - PSI) * GN)) * DSIN
1 * ENP2 * ETA) / TD11P
GO TO 604
601 X33(I, J) = EP(I, N) * (1 - TD1P) * DSIN(ENP2 * ETA) / TD11P
GO TO 604
602 X33 = 0.0
X33 = 0.0
X33 = 0.0
X33 = 0.0
603 X33 = 0.0
X33 = 0.0
X33 = EP(I, N) * DCOSH(BN) - TD1P * DCOSIGN) * DSIN(ENP2 * ETA) / TD11P
604 X33 = 0.0
X33 = 0.0
X33 = 0.0
X33 = 0.0
W33 = W33(I, J)
W44 = W44(I, J)
GO TO 620
X1=X1
GNS=PHINSX1
GN=DSQRT(GNS)
IF (BNS GT Z2) GO TO 616
6 T2=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS*1ENP2S)*DCOSH(BN))
TD2N=TDN*BNS*DCOSH(BN)+GN*DCOSH(GN)
TD2P=DCOSH(BN)/DCOSH(GN)
T22P=BNS*DSINH(BN)-GN*TD2P*DSINH(GN)
TD2L=-BNS*DCOSH(BN)/(IGN*DCOSH(GN))
TD2L1=-2*DSIN(ENP2*CSI)/IBN*(BNS-POIS*PHINS*ENP2S)*DCOSH(BN)+1/TD2L*GNS*(GNS-POIS*PHINS*ENP2S)*DCOSH(GN)
IF (ETA EQ 0) GO TO 607
IF (PSI EQ 0) GO TO 608
XW2=EN(N)*TSI*DSINH(BN*PSI)+DSINH(BN*PSI)*DSIN(ENP2*ETA)/TD2N
XW4=TD2L*(DSINH(BN*PSI)+TD2L*DSINH(BN*PSI))*DSIN(ENP2*ETA)
IF (PSI EQ 1) GO TO 606
XW3=EP(N)*(DCOSH(1-PSI)*BNS-2*DSINH(1-PSI)*GN))
T22P=DSIN(ENP2*ETA)/TD2P
XW3=EP(N)*(1-T22P)*DSIN(ENP2*ETA)/TD2P
GO TO 609
606 XW3=0,0
607 XW3=0,0
608 XW2=0,0
609 XW2=0,0
610 T2N=GN*(POIS*PHINS*ENP2S-GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
T2L=BN/GN
C1=-2*DSIN(ENP2*CSI)
C2=BN*(BNS-POIS*PHINS*ENP2S)
C3=GNS*(GNS-POIS*PHINS*ENP2S)
T22L=C1/(C2+TD2L*C3)
IF (ETA EQ 0) GO TO 612
IF (PSI EQ 0) GO TO 612
B=1,0
IF (TEST=BN*BN*PSI)
IF (TEST GT 60) B=0,0
XW2=ES(N)*TDN*DEXP((BN*PSI-BN)*B)*DEXP((GN*PSI-GN)*B)*DSIN(1ENP2*ETA)/IBN*TD2N*GN)
XW4=BD*TD2L*(DEXP((BN*PSI-BN)*B)+TD2L*DEXP((GN*PSI-GN)*B))
IF (TEST=BN*PSI)
IF (TEST GT 60) B=0,0
XW3=2*EN(N)*BD*DEXP((1-GN*PSI)*B))
GO TO 614
612 XW2=0,0
XW3=0,0
800  PRINTS36, PSI, ETA, I, J, W(I, J)
     PSI=PSI+1.0/DFLOAT(KS)
640  CONTINUE
     ETA=ETA+1.0/DFLOAT(KS)
650  CONTINUE
     RETURN
     END
PROGRAM 13

THIS IS A SYMMETRIC MODE EIGENVALUE SEARCH PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM WITH SYMMETRICALLY DISTRIBUTED POINT SUPPORTS ON THE LATERAL SURFACE. (ONE POINT SUPPORT ON EACH SIDE OF THE PLATE CENTRAL AXIS.)

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:


2. U = 0 TO 1, PROVIDING THE DISTANCE BETWEEN THE CONCENTRATED FORCE (POINT SUPPORT) AND THE PLATE CLAMPED EDGE. AXIS DIVIDED BY SIDE LENGTH A.

3. V = 0 TO 1, PROVIDING THE DISTANCE BETWEEN THE CONCENTRATED FORCE (POINT SUPPORT) AND THE PLATE CENTRAL AXIS DIVIDED BY SIDE LENGTH B.

4. PHIR = 2B/A, IS THE FULL PLATE ASPECT RATIO.

5. ALMDS = AN INITIAL STARTING VALUE FOR THE EIGENVALUE SEARCH.

6. DLIM = A FINISHING OR EIGENVALUE SEARCH ENDING LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO HALT EXECUTION.

7. DEL = EIGENVALUE INCREMENT.

8. POI = POISSON'S RATIO.

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(99,99)
K=?????????????????????????
Z2=3600
U=??????????????????????
PHIR=???????????????????
PHI=PHIR/2.
PRINT50,PHIR,K,U,V
50 FORMAT(1X,6X,*PHIR=F8.4,*K=I4,*6X,*U=F8.4,*6X,*V=F8.4,/*6X,*12X)
K1=2*K-1
C=1.
PI=4.*DATA(N,1)
PHIS=PHI+PHI
POI=????????????????????
POIS=2-POI
PHI PHIN=1./PHI
PHINS=PHIN*PHIN
UF3=POI*PHIS
UFIS=POI*PHINS
I=3*K
DLX=???????????????????
ALMS=??????????????????
DEL=??????????????????
CONTINUE

INITIALIZE THE MATRIX A

L=1+1
DO 2 M=1,L
DO 2 N=1+L
A(M,N)=0.0
2 CONTINUE
DO 3 N=1,K
A(N+2*K=N+2*K)=1.0
DO 10 M=1+K,2
DO 10 N=1,K
JN=N-1
EMP2=EMP2/2.
EMP2=EMP2-EMP2
EMP=JN/PI
ENP=EMP*ENP
BN=1.*PHIS*PHIS+/ENP+ENP+ALMS*PHIS
BN=DSORT(BN)
X1=ALMS*PHIS-ENP*EMP
IF (X1=LE.0.0) GO TO 5
GNS=(1.+PHIS)*X1
GNS=DSORT(GNS)
KPOIS=PHINS*ENPS
TD1=1.+BNS-X)*DCOSH(BN)/IGN+GNS+X)*DCOS(GN)
TD1P=DCOSH(BN)/DCOSIGN
DL=1.0
IF (N.EQ.1.0) DL=2.0
A(N)=DCOS(EMP*V)+DCOSH(BN*(1-U))/1DL*PHIS*PHIS*(BNS+GNS)*BN*
1DCOSH(BN)
BN=DCOS(EMP*V)+DCOSH(GN*(1-U))/1DL*PHIS*PHIS*(BNS+GNS)*GN*
1DCOSIGN
C1N=DCOS(EMP*V)+DSINH(GN-U)/1DL*PHIS*PHIS*(BNS+GNS)*BN*DCOSH(BN)
D1N=DCOS(EMP*V)+DSINH(GN-U)/1DL*PHIS*PHIS*(BNS+GNS)*GN*DCOSIGN
X=UF1*ENPS
TD11N=BN*DCOSH(BN)*TD1*GN*DCOS(GN)
TD11P=BN*DSINH(BN)*GN*TD1P*DSINH(GN)
IF (N.GT.1.0) GO TO 4
A(N+K,N+K)=TD1*DSINH(GN-U)*DCOS(EMP*V)/TD11N
A(L+K+1,N+K+1)=DCOSH(BN*(1-U))-TD1*DCOS(GN*(1-U))*DCOS(EMP*V)/TD11P
A(N+K,N+K)=X*8**8-8**8+GNS*TD1P/8**8
A(N+K,N+K)=X-BNS-C1N+(X+GNS)*D1N
4 \( A(N+2*K, L) = A(N+BN+BN+GNN+GN) \)
\( X1 = (1/ENPS-UFS*BN) / (BN+EMS*BN) * BN*DCOSH(BN) \)
\( X2 = TD2*CNPS*UFS*GNN / (GNN-EMS*BN) * GNN*DCOSH(GNN) \)
\( XX = 1DCOS[ENPS] * DSIN[EPS2] / TD1[N] * (X1+X2) \)
\( A(M+1)/2, N+K) = XX * 2.0 \)
\( X1 = (1/ENPS-UFS*BN) / (BN+EMS*BN) * DCOSH(BN) \)
\( X2 = TD2[N*ENPS-UFS*GNN / (GNN-EMS*BN) * DCOSH(GNN) \)
\( XX = 2DCOS[ENPS] * EMS2 * (X1+X2) / TD1[N] \)
\( A(M+1)/2, N+2*K) = XX \)
GO TO 10

5 \( X1 = X1 \)
GNN = (1/PHIS) * X1
GNN = DSORTI(GNN)
X = POIS * PHINS * ENPS
IF (BN*S.LT.2Z) GO TO 6
TD2 = BN+BN-S-X / XGNS-NEGNS)
TD2P = 1.0
X = UFS*ENPS
TD2P = BN+GN
TD2N = BN+TD2*GN
DL = 1.0
IF (M.EQ.1.0) DL = 2.0
A(N+2K, L) = (DL*PHIS*PHIS*BN-NEGNS)*BN
B2N = -DCOSI[ENPS] * DL*PHIS*PHIS*BN-NEGNS)*BN
C2N = DCOSI[ENPS] * DL*PHIS*PHIS*BN-NEGNS)*BN
D2N = -DCOSI[ENPS] * DL*PHIS*PHIS*BN-NEGNS)*BN
IF (M.GT.1.0) GO TO 7
B = 1.0
TEST = BN-BO*U
IF (TEST.GT.60.0) B = 0.0
B = 1.0
TEST = BN-U
IF (TEST.GT.60.0) B = 0.0
GO TO 7

6 \( TD2 = (BN*BN-X)*DCOSH(BN) / (BN*BN-X)*DCOSH(BN) \)
TD2P = DCOSH(BN) / DCOSH(BN)
X = UFS*ENPS
TD2P = BN+DSINH(BN) - GN*TD2P*DSINH(BN)
TD2N = BN+DCOSH(BN) + TD2*GN*DCOSH(BN)
DL = 1.0
IF (N.EQ.1.0) DL = 2.0
B2N = -DCOSI[ENPS] * DL*PHIS*PHIS*BN-NEGNS)*BN
C2N = DCOSI[ENPS] * CQHBN / (DL*PHIS*PHIS*BN-NEGNS)*BN
D2N = -DCOSI[ENPS] * CQHBN / (DL*PHIS*PHIS*BN-NEGNS)*BN
IF (M.GT.1.0) GO TO 8
\[ A(L, N+K) = \{(\text{DCOSH}(BN*U) + TD2*\text{DSINH}(GN*U) + DCOS(ENP*V1) / TD22N) \}
\]
\[ A(L, N+2*K) = \{ \text{DCOSH}(BN*(1-U)) - TD2*\text{DCOSH}(GN*(1-U)) \} + DCOS(ENP*V1) / TD22P \]
\[ A(N+K, N+K) = \{(X-BNS) * \text{DSINH}(BN) + TD2*(1-GNS) * \text{DSINH}(GN) \} / TD22N \]
\[ A(N+2*K, N+1) = (BN + TD2*GN) / TD22N \]
\[ A(N+K, N+2*K) = \{ \text{UFS} * \text{ENPS} * (1-TD2*U) - BNS + GNS * TD2P \} / TD22P \]
\[ A(N+K, L) = (X-BNS) * C2N + (1-GNS) * D2N \]
\[ A(N+2*K, L) = A2N * BN + B2N * GN \]

**GO TO 6**

7
\[ X1 = \{(\text{ENPS} - \text{UFS} + BN) / (BNS + EMP25) \} * BN \]
\[ X2 = \{(\text{ENPS} - \text{UFS} + GNS) / (GNS + EMP25) \} * GN * TD2 \]
\[ XX = \{(\text{DCOS(ENP)} + \text{DSIN(EMP2)} / TD22N) * (1+X1 + X2) \} \]
\[ A1(M+1) / 2, N+K = X1 = X2 \]
\[ X1 = \{(\text{ENPS} - \text{UFS} + BNS) / (BNS + EMP25) \} * BN * \text{DCOSH}(BN) \]
\[ X2 = \{(\text{ENPS} - \text{UFS} + GNS) / (GNS + EMP25) \} * GN * \text{DCOSH}(GN) \]
\[ XX = \{(\text{DCOS(ENP)} + \text{DSIN(EMP2)} / TD22N) * (X1 + X2) \} \]
\[ A1(M+1) / 2, N+K = X1 = X2 \]

**CONTINUE**

8
\[ DO 20 N = 1, K \]
\[ DO 20 K = 1, K1, 2 \]
\[ JN = N - 1 \]
\[ ENP = JN + PI \]
\[ ENPS = ENP - ENP \]
\[ EMP2 = M1 + PI / 2. \]
\[ EMP2S = EMP2 * EMP2 \]
\[ BNS = \text{PHIS} * (\text{ALMD} + EMP2S) \]
\[ BM = \text{DSQRT}(BNS) \]
\[ X1 = \text{ALMD} * \text{EMP2S} \]
\[ IF (X1 LT 0.0) \text{GO TO 13} \]
\[ GMS = \text{PHIS} * X1 \]
\[ GM = \text{DSQRT}(GMS) \]
\[ A1N = \text{DSIN(EMP2-U)} * \text{DCOSH(BM*(1-V))} / \{(\text{BMS} + GMS) * BM * \text{DSINH}(BM)\} \]
\[ B1N = \text{DSIN(EMP2-U)} * \text{DCOS(GM*(1-V))} / \{(\text{BMS} + GMS) * GM * \text{DSINH}(GM)\} \]
\[ C1N = \text{DSIN(EMP2-U)} * \text{DCOSH(BM*V)} / \{(\text{BMS} + GMS) * BM * \text{DSINH}(BM)\} \]
\[ D1N = \text{DSIN(EMP2-U)} * \text{DCOS(GM*V)} / \{(\text{BMS} + GMS) * GM * \text{DSINH}(GM)\} \]
\[ TD1 = (BM * BMPS - DDIS * PHIS * EMP2S * DSINH(BM)) / \{(GM * GMPS * PHIS * EMP2S) \} \]
\[ P1 = \text{DSIN (GM)} \]
\[ TD1 = TD1 + TD1 \]
\[ TD11 = BM * DSINH(BM) - TD1 * GM * DSINH(GM) \]
\[ XX = (BM * BMPS * DSMH + GMS + ENPS) + TD1 * GM * DSMG(GM) / \{GMS - 1\} * ENPS \} * (2 * EMP2 * DCOS(ENP) / TD11 \]
\[ IF (JN, GT, 0.0) \text{GO TO 11} \]
\[ XX = XX / 2 \]
\[ A1(M+1) / 2, L = \{(UFS * EMP25 - BMS) * C1N + (UFS * EMP25 + GMS) * D1M \]
\[
\begin{align*}
XX_{2} &= (A_{1} \times \text{DCOSH}(BM \times V) + B_{1} \times \text{DCOS}(GM \times V)) \times \text{DSIN}(EMP2 \times U) \\
A_{[L+C]} &= A_{[L+C]} \times XX_{2} \\
A_{(M+1)/2, (M+1)/2} &= (UFS \times EMP2S-BMS) \times \text{DCOSH}(BM) + TD1 \times I \\
1UFS \times EMP2S + GMS \times \text{DCOS}(GM)1 / TD11 \times \text{DSQRT}(GMS) \\
A_{I} + I_{y} (M+1)/2 &= XX_{4} \\
A_{N+K_{y} (M+1)/2} &= XX \\
G0 TO 20 \\
X1 &= X1 \\
GMS &= \text{PHIS} \times X1 \\
G0 = \text{DSQRT}(GMS) \\
IF (BMS.GT.22) G0 TO 15 \\
TD2 &= BM \times (BMS-P0IS+\text{PHIS} \times EMP2S) \times \text{DSINH}(BM) \times (\text{BM-P0IS}+\text{PHIS} \times \text{EMP2S}) \times \text{DSINH}(BM) \\
A_{[L+C]} &= A_{[L+C]} \times TD2 \\
A_{2} &= \text{DSIN}(\text{EMP2} \times U) \times \text{DCOSH}(BM \times (1-V)) \times (BMS \times GMS) \times (BMS \times GMS) \times \text{DSINH}(BM) \\
A_{2} &= \text{DSIN}(\text{EMP2} \times U) \times \text{DCOSH}(BM \times (1-V)) \times (BMS \times GMS) \times \text{DSINH}(BM) \\
A_{2} &= \text{DSIN}(\text{EMP2} \times U) \times \text{DSINH}(BM) \times (BMS \times GMS) \times \text{DSINH}(BM) \\
X1 &= BM \times \text{DSINH}(BM) \times (BMS \times ENPS) \\
X2 &= GMS \times \text{TD2} \times \text{DSINH}(BM) \times (GMS \times ENPS) \\
X1 &= 2 \times \text{DCOS}(\text{ENP}) \times \text{EMP2} \times (X1 \times X2) / TD22 \\
X1 &= (\text{EMP2S-UFIS} \times BMS) \times BM \times \text{DSINH}(BM) \times (BMS \times ENPS) \\
X2 &= (\text{EMP2S-UFIS} \times BMS) \times \text{TD2} \times \text{BM} \times \text{DSINH}(BM) \times (GMS \times ENPS) \\
X1 &= 2 \times \text{DSIN}(\text{EMP2}) \times \text{DCOS}(\text{ENP}) \times (X1 \times X2) / TD22 \\
IF (\text{JN.GT.0} .0) G0 TO 15 \\
X6 &= (\text{DCOSH}(BM \times V) + \text{TD2} \times \text{DCOSH}(BM \times V)) \times \text{DSIN}(\text{EMP2} \times U) / TD22 \\
A_{I} + I_{y} (M+1)/2 &= XX_{6} \\
XX &= XX \times X \times X2 \\
X1 &= X1 \times X2 \\
A_{(M+1)/2, (M+1)/2} &= (UFS \times EMP2S-BMS) \times \text{DSINH}(BM) \times (BMS \times ENPS) \\
X1 &= 2 \times \text{DCOS}(\text{ENP}) \times \text{EMP2} \times (X1 \times X2) / TD22 \\
X1 &= \text{DSIN}(\text{EMP2S-UFIS} \times BMS) \times BM \times (BMS \times ENPS) \\
X1 &= (\text{EMP2S-UFIS} \times GMS) \times \text{TD2} \times \text{BM} \times \text{DSINH}(BM) \times (GMS \times ENPS) \\
X1 &= 2 \times \text{DSIN}(\text{EMP2}) \times \text{DCOS}(\text{ENP}) \times (X1 \times X2) / TD22 \\
IF (\text{JN.GT.0} .0) G0 TO 14 \\
XX &= XX / 2 \\
X1 &= XX / 2 \\
D &= BM \times (\text{POIS} \times \text{PHIS} \times \text{EMP2S-BMS}) / (GMS \times \text{POIS} \times \text{PHIS} \times \text{EMP2S}) \\
B &= 1.0 \\
TEST &= \text{BM} \times V \\
IF (\text{TEST} \times 60 \times 0) B &= 0.0 \\
XX &= B \times (\text{DEXP}((BM \times V) - B) + \text{DEXP}((BM \times V) - B)) \times \text{DSIN}(\text{EMP2} \times U) / \\
1 \times (BM \times 0.GM) \\
A_{I} + I_{y} (M+1)/2 &= XX_4 \\
A_{(M+1)/2, (M+1)/2} &= (UFS \times EMP2S-BMS) + TD2 \times (UFS \times EMP2S-GMS) / TD22 \\
A_{2} &= \text{DSIN}(\text{EMP2} \times U) \times (BM \times (BMS \times GMS)) \\
B &= \text{DSIN}(\text{EMP2} \times U) / (BM \times (BMS \times GMS)) \\
B &= \text{DSIN}(\text{EMP2} \times U) / (BM \times (BMS \times GMS)) \\
B &= \text{DSIN}(\text{EMP2} \times U) / (BM \times (BMS \times GMS)) \\
B &= \text{DSIN}(\text{EMP2} \times U) / (BM \times (BMS \times GMS))
B=1.0
TEST=BM-BM*V
IF (TEST .GT. 60.0) B=0.0
A(I+1)/2,L)=B*((UFS*EMP2S-BMS)*C2M*DEXP((B*M*V-BM)*B)+
1(UFS*EMP2S-GMS)*D2M*DEXP((GM*V-GM)*B))
TEST=BM-BM*V
IF (TEST .LT. 60.0) GO TO 16
XX=(A2M/2)*[(1+DEXP(-2*BM*V))+(B2M/2)*(1+DEXP(-2*GM*V))]
GO TO 17
16 AI=L,LI=AI(L,L)+XX
GO TO 14
20 CONTINUE
CALL DETERM(A,L,D)
PRINT 110,ALMDS,DET
110 FORMAT(7(E9.6,1X,10X,4D20.5))
ALMDS=ALMDS+DEL
IF (ALMDS .GT. 0.3D0) GO TO 21
GO TO 1
21 CONTINUE
STOP
END
SUBROUTINE DETERM(A,N,DET)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(99,99)
SIGN=1
LAST=N-1

START OVERALL LOOP FC(N-1). PIVOTS
DO 200 I=1,LAST

FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
BIG=0.
DO 50 K=I,N
TERM=DABS(A[I,K])
IF (TERM .GT. BIG) 50,50,30
30 BIG=TERM
L=K
50 CONTINUE

CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND.
IF (BIG .GT. 60.0,60,80
L-TH ROW HAS THE BIGGEST TERM----IS I=L
40 IF (I-L.900) 120,90
I IS NOT EQUAL TO L, SWITCH ROWS I AND L
90 SIGN=-SIGN
DO 100 J=1,N
TEMP=A[I,J]
100 A[L,J]=TEMP

NOW START PIVOTAL REDUCTION
120 PIVOT=A[I,I]
NEXTR = I + 1

FOR EACH OF THE ROWS AFTER THE I-TH
DO 200 J = NEXTR, N

MULTIPLYING CONSTANT FOR THE J-TH ROW IS
CONST = A(J, I) / PIVOT

NOW REDUCE EACH TERM OF THE J-TH ROW
DO 200 K = I, N
200 A(J, K) = A(J, K) - CONST * A(I, K)

END OF PIVOTAL REDUCTION --- NOW COMPUTE DETERMINANT

DET = SIGN
DO 300 I = 1, N
300 DET = DET * A(I, I) / 10
GO TO 61
60 DET = 0
61 RETURN
END
PROGRAM 14

THIS IS A SYMMETRIC MODE SHAPE PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM, WITH POINT SUPPORTS SYMMETRICALLY DISTRIBUTED ON THE LATERAL SURFACE OF THE PLATE. (ONE POINT SUPPORT ON EACH SIDE OF THE CENTRAL AXIS)

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:

1. - K = AS DEFINED IN PROGRAM 13.
2. - U = AS DEFINED IN PROGRAM 13.
4. - PHIR = 28/A FULL PLATE ASPECT RATIO.
5. - ALMOS = EIGENVALUE.
6. - KS = NUMBER OF POINTS AT WHICH THE DISPLACEMENT W IS REQUIRED.
7. - POI = POISSON'S RATIO.

IMPLICIT REAL*8(A-H,C-Z)
COMMON Z2,PHI,P1,PHIS,POI,POIS,PHIN,PHINS,UFS,UFIS,ALMDS,U,V,K,K1,
1KS
DIMENSION A(99,99)
K=11111111111111111111111
KS=11111111111111111111111
Z2=3600
V=11111111111111111111111
U=11111111111111111111111
PHIR=11111111111111111111111
ALMDS=11111111111111111111111
PHI=PHIR/2
PRINT 50,PHIR,K,CSI
50 FORMAT(1*",10X,"PHIR =",F8.4,10X,"K =",I4,10X,"CSI =",F8.4,10X)
K1=2*K-1
C=1.
P1=4.*DATAN(C)
PHIS=PHI*PHI
POI=PHIN*PHIN
POIS=2-P0I
PHIN=1./PHI
PHINS=PHIN*PHIN
UFS=POI*PHIS
UFIS=POI*PHINS
I=3*K

INITIALIZE THE MATRIX A

L=1+1
DO 2 M=1,L
DO 2 N=1,L
A(M,N)=0.0
2 CONTINUE
DO 3 N=1,K
A(N+2*K,N+2*K)=-1.
DO 10 M=1,K
DO 10 N=1,K
J=N-1
EMP2=M*PI/2.
EMP2=EMP2*EMP2
EMP=JN*PI
ENPS=ENP*ENP
BNS=[1.0/PHIS]*[ENP*ENP+ALMDS*PHIS]
GNS=DOSORT(BNS)
XI=ALMDS*PHIS-ENP*ENP
IF (XI.LT.0.0) GO TO 5
GNS=1.0/PHIS*X1
GNS=DOSORT(GNS)
X=PDIS*PHINS*ENPS
TD1=(BNS+(N-X)*DCOSH(BN))/(GNS+(GNS+X)*DCOS(GN))
TD1P=DCOSH(BN)/DCOS(GN)
DL=1.0
IF (N.EQ.1.0) DL=2.0
A(N)=DCOSH(ENP*V)*DCOSH(BN*(1-U))/(DL*PHIS*PHIS*(BNS+GNS)*BN*
1DCOSH(BN))
B(N)=DCOSH(ENP*V)*DCOSH(GN*(1-U))/(DL*PHIS*PHIS*(BNS+GNS)*GN*
1DCOSIGN))
C(N)=DCOSH(ENP*V)*DSINH(BN*U)/(DL*PHIS*PHIS*(BNS+GNS)*BN*DCOSH(BN))
D(N)=DCOSH(ENP*V)*DSINH(GN*U)/(DL*PHIS*PHIS*(BNS+GNS)*GN*DCOS(GN))
X=UFIS*ENPS
TD11N=BNS+DCOSH(BN)TD1*GNS+DCOSIGN
TD11P=BNS+DSINH(N*GNS+TD1P+DSINIGN)
IF (M.NE.1.0) GO TO 4
A(N+K,N+K)=(DSINH(BN*U)+TD1*DSINH(GN*U))*DCOS(ENP*V)/TD11N
A(L,N+2*K)=(DCOSH(BN*(1-U))-TD1P*DCOS(GN*(1-U))*DCOS(ENP*V)/TD11P
A(N+K,N+K)=(UFIS*ENPS-BNS)*DSINH(BN)+TD1*(UFIS*ENPS*GNS)*
1DSINIGN)/TD11N

A(N+K,N+K)=(BN+TD1*GNS)/TD11N
A(N+K,N+K)=A(N+K,N+K)*BN+81*N*GN
A(N+K,N+K)=A(N+K,N+K)*81*N*GN

X1=(1ENPS-UFPS*BN)/(BN+EMP2S)*BN*DCOSH(BN)
X2=(TD1*ENPS+UFPS*GNS)/(GNS-EMP2S)*GNS*DCOSIGN
XX=(DCOS(ENP)*DSIN(EMP2)/TD11A)*X1-X2
A(M+1,J)/2,N+2*K=XX

GO TO 10

X1=-X1
GNS=1.0/PHIS*X1
GNS=DOSORT(GNS)
X=PDIS*PHINS*ENPS
IF (BN* < 22) GO TO 6
T02 = B0*(BNS-X)/(IGN*(X-GNS))
T02P = 1.0
X = UF1S*ENPS
T02P = B0-GN
T02N = B0 + T02 * GN
DL = 1.0
IF (N*E0 = 1.0) DL = 2.0
A2 = DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS) * BN)
B2N = DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS) * GN)
C2N = DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS) * GN)
D2N = DCOS(ENP*V)/(DL*PHIS*PHIS*(BNS-GNS) * GN)
IF (K* GT 1.0) GO TO 7
B = 1.0
TEST = B0-BN-U
IF (TEST*GT*60.0) B = 0.0
A1L = N*+2*K*L = B*(DEXP(BN-U-BN) + T02*DEXP(GN-U-GN)) / (B0+1.0
B = 1.0
TEST = B0-BN-U
IF (TEST*GT*60.0) B = 0.0
A1N*+2*KN*+2*K1 = B*(A2*K*N+2*BN*+C2*K*+2*X-GNS) / T02N
A1N*+2*KN*+2*K1 = 0.0
A1N*+2*K1 = 0.0
GO TO 7
6
T02 = B0*(BNS-X)/(IGN*(X-GNS) * DCOSH(GN))
T02P = DCOSH(BN) / DCOSH(GN)
X = UF1S*ENPS
T02P = B0*DSINH(BN) - GN*T02*DSINH(GN)
T02N = B0*DCOSH(BN) + T02*GN*DCOSH(GN)
DL = 1.0
IF (N*E0 = 1.0) DL = 2.0
A2N = DCOS(ENP*V) * DCOSH(BN*(1-U))/((DL*PHIS*PHIS*(BNS-GNS) * BN*
10 * DCOSH(BN))
B2N = DCOS(ENP*V) * DCOSH(BN*(1-U))/((DL*PHIS*PHIS*(BNS-GNS) * GN*
10 * DCOSH(GN))
C2N = DCOS(ENP*V) * DSINH(BN * U)/(DL*PHIS*PHIS*(BNS-GNS) * BN*DCOSH(BN))
D2N = DCOS(ENP*V) * DSINH(GN*U)/(DL*PHIS*PHIS*(BNS-GNS) * GN*DCOSH(GN))
IF (K*GT*1.0) GO TO 8
A1L = N*+2*K*L = DSINH(BN * U) + T02*DSINH(GN*U) / T02N
A1L = N*+2*K*L = DCOSH(BN*(1-U)) - T02P*DCOSH(GN*(1-U)) / (T02P*DCOS(ENP*V) / T02N)
A1N*+2*KN*+2*K1 = (X-BNS)*DSINH(BN) + T02*(X-GNS)*DSINH(GN) / T02N
A1N*+2*KN*+2*K1 = 0.0
A1N*+2*K1 = 0.0
GO TO 8
7
X1 = UF1S*USFS/BNS*/(BNS*EMP25)/*BN
X2 = UF1S*USFS/GNS*/(GNS*EMP25)/*GN *TD2
XX = DCOS(ENP) * DSIN(EMP2)*TD2/2
X1 = X1 + 2*N*+2*K1 = X1 + 2.0
X1 = UF1S*USF*USFS/BNS*/(BNS*EMP25)
X2 = UF1S*USFS/GNS*/(GNS*EMP25)
\[XX = 2 \ast \text{DCOS}(\text{ENP}) \ast \text{EMP}2 \ast (X1 \times X2) / \text{TD2P} \]
\[A([N+2]/2, N+2) = XX \]
\[X1 = (\text{ENPS} \ast \text{UFS} \ast \text{BNS} \ast (\text{BNS} \ast \text{EMP2S})) \ast \text{BN} \ast \text{DCOSH} (\text{BN}) \]
\[X2 = (\text{ENPS} \ast \text{UFS} \ast \text{GNS} \ast (\text{GNS} \ast \text{EMP2S})) \ast \text{GN} \ast \text{DCOSH} (\text{GN}) \ast \text{TD2} \]
\[XX = (\text{DCOS} (\text{ENP}) \ast \text{DSIN} (\text{EMP}2) / \text{TD2P}) \ast (X1 \times X2) \]
\[A([M+1]/2, N+2) = XX \]
\[X1 = (\text{ENPS} \ast \text{UFS} \ast \text{BNS} \ast (\text{BNS} \ast \text{EMP2S})) \ast \text{DCOSH} (\text{BN}) \]
\[X2 = (\text{TD2P} \ast (\text{ENPS} \ast \text{UFS} \ast \text{GNS} \ast (\text{GNS} \ast \text{EMP2S})) \ast \text{DCOSH} (\text{GN}) \]
\[XX = 2 \ast \text{DCOS} (\text{ENP}) \ast \text{EMP}2 \ast (X1 \times X2) / \text{TD2P} \]
\[A([M+1]/2, N+2) = XX \]

CONTINUE

DO 20 N = 1, K
DO 20 K = 1, K, 2
JN = N-1
ENP = JN * PI
ENPS = ENP * ENP
EMP2 = M * PI / 2
EMP2S = EMP2 * EMP2
BNS = PHIS * (ALMDS + EMP2S)
BMS = PHIS * (ALMS + EMP2S)
X1 = ALMDS - EMP2S
IF (X1.LT.0.0) GO TO 13
GMS = PI * PI * X1
GMS * DSORT (GMS)
A1M = DSIN (EMP2 + U) * DCOSH (BM * (1 - V)) / (BM + GMS) * BM * DSINH (BM)
B1M = DSIN (EMP2 + U) * DCOSH (BM * V) / (BM + GMS) * BM * DSINH (BM)
C1M = DSIN (EMP2 + U) * DCOSH (BM * V) / (BM + GMS) * BM * DSINH (BM)
D1M = DSIN (EMP2 + U) * (GMS * GM * DSINH (GM))
TD1 = (BM + GMS + POIS * PHIS * EMP2S) * DSINH (BM) / (BM + GMS + POIS * PHIS * EMP2S)
1 EMPS = 2 * EMP2 * DCOS (ENP) / TD11
TD11 = BM * DSINH (BM) - TD1 * GM * DSINH (GM)
XX = (BM * DSINH (BM) / (BMS + ENPS) + TD1 * GM * DSINH (GM) / (GMS -
\[1 EMPS) = 2 * EMP2 * DCOS (ENP) / TD11 \]
IF (JN .GT. 0.0) GO TO 11
XX = XX/2
11 A1(M+2) = A1(M+1) / 2
X1 = (EMP2S - UFS * BNS) / (BMS + ENPS) * BM * DCOS (ENP) * DSINH (BM)
X2 = (EMP2S - UFS * GMS) / (GMS + ENPS) * GM * DCOS (ENP) * DSINH (GM) * TD1
XX = (X1 + X2) / 2 * DSINEP2) / TD11
X4 = (DCOSH (BM * V) * TD1 * GM * DSINH (GM)) / (BM + GMS) * GM * DSINH (GM)
IF (JN .GT. 0.0) GO TO 12
XX = XX/2
A2(M+1) = A1(M/2)
A2(M+1) = UFS * EMP2S * BMS * C1M * UFS * EMP2S * GMS * D1M
XX = (A1M * DCOSH (BM * V) + B1M * DCOS (GM * V)) / DSINEP2 * U
A3(M) = A1(L/2) * XX
A4(M+1) = UFS * EMP2S * BMS * DCOSH (BM) * TD1 * (UFS * EMP2S * BMS * DCOSH (BM) + TD1 * (UFS * EMP2S * BMS * DCOSH (BM) / TD11
A5(M+1) = (M+1) / 2
A6(M+1) = XX
12 A1(M+2) = A1(M+1) / 2
GO TO 20
13 X1 = X1
GMS = PHIS * X1
GM = DSORT (GMS)
IF (BMS .GT. 22) GO TO 15
TD2 = (BMS - POIS * PHIS * EMP2S) * DSINH (BM) * (-1) / (GM + (GMS - POIS * 1 PHIS * EMP2S) * DSINH (GM))
TD22 = BM * DSINH (BM) + TD2 * GM * DSINH (GM)
A2M = DSIN (EMP2 + U) * DCOSH (BM * (1 - V)) / (BMS - GMS) * BM * DSINH (BM)
A(M+1)=TEMP
CONTINUE
CALL DETERM(A,L,DET)
STOP
END
SUBROUTINE DETERM(A,N,DET)
IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(99,99),X(99),EM(75),EN(75),EP(75)
SIGN=1
M=N-1
LAST=M-1
START OVERALL LOOP FOR (N-1) PIVOTS
DO 200 I=1,LAST
FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
BIG=0.
DO 50 K=1,N
TERM=ABS(A(K,I))
IF (TERM>BIG) 50,50,30
50 BIG=TERM
L=K
50 CONTINUE
CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
IF (BIG) 60,60,80
L-TH ROW HAS THE BIGGEST TERM---- IS I=L
80 IF (I-L) 90,120,90
I IS NOT EQUAL TO L, SWITCH ROWS I AND L
90 DO 100 J=1,N
TEMP=A(I,J)
A(I,J)=A(L,J)
100 A(L,J)=TEMP
NOW START PIVOTAL REDUCTION
120 PIVOT=A(I,I)
NEXTR=I+1
FOR EACH OF THE ROWS AFTER THE I-TH
DO 200 J=NEXR,N
MULTIPLYING CONSTANT FOR THE J-TH ROW IS
CONST=A(J,I)/PIVOT
NOW REDUCE EACH TERM OF THE J-TH ROW
DO 200 K=1,N
200 A(J,K)=A(J,K)-CONST*A(I,K)
END OF PIVOTAL REDUCTION--PERFORM BACK SUBSTITUTION
M=N-1  
DO 500 I=1,M  
IREV IS THE BACKWARD INDEX, GOING FROM M BACK TO 1  
IREV=M+1-I  
GET Y(IREV) IN PREPARATION  
Y=A(IREV,N)  
IF (IREV=M) 400,000,400  
NOT WORKING ON LAST ROW, I IS 2 OR GREATER  
400 DO 450 J=2,I  

WORK BACKWARD FOR X(K), X(N-1) --- SUBSTITUTING PREVIOUSLY FOUND VALUES  
K=N+1-J  
450 Y=Y-A(IREV,K)*X(K)  
FINALLY, COMPUTE X(IREV)  
500 X(IREV)=Y/A(IREV,IREV)  

FIND, AND PRINT EM, EN, EP, AND P  
L=(N-1)/3  
DO 550 I=1,L  
J=2*I-1  
K=I-1  
EM(I)=X(I)  
EN(I)=X(I+L)  
IF(I.EQ.L) GO TO 540  
EP(I)=X(3*L)  
P=X(3*L)  
GO TO 550  
540 EP(I)=-L  
550 PRINTS00, J, K, K, J, EM(I), K, EN(I), K, EP(I), P  
600 FORMAT('I',I, 'M=',I2, '3X', 'N=',I2, '3X', 'P=',I2, '5X', 'EM(',I2, '),')  
   1D13.5,5X, 'EN(',I2, ')=',D13.5,5X, 'EP(',I2, ')=',D13.5,5X,  
1P*  1D13.5)  
DO 601 I=1,L  
J=2*I-1  
EN(I)=EM(I)  
EP(I)=EP(I)  
601 CONTINUE  
PRINT 650  
650 FORMAT('THE SYMMETRIC MODE SHAPE DATA ARE: / / / /')  
CALL SHAPE (EM, EN, EP, P)  
60 RETURN  
END  
SUBROUTINE SHAPE (EM, EN, EP, P)  
IMPLICIT REAL*8 (A-H,O-Z)  
COMMON Z2, PHI, PI, PHIS, POI, POIS, PHIN, PHINS, UFS, UFIS, ALMDS, U, V, K, K1, K8  
DIMENSION W(21,21), W2(21,21), W3(21,21), W4(21,21), W(21,21)  
1, EM(75), EN(75), EP(75)
KS1 = KS1 + 1
ETA = 0.0
DO 650 I = 1, KS1
PSI = 0.0
DO 640 J = 1, KS1
W11 = 0.0
W22 = 0.0
W33 = 0.0
W44 = 0.0
DO 620 N = 1, K
JN = N - 1
ENP = JN * PI
ENPS = ENP * ENP
BN = (1. / PHIS) * (ENP * ENP * ALMDS * PHIS)
BN = SQRT(BN)
X1 = ALMDS * PHIS * ENP * ENPS
IF (X1 LT 0.0) GO TO 606
GNS = (1. / PHIS) * X1
GN = SQRT(GNS)
X = POIS * PHIS * ENPS
TD1 = BN * (X + DCOSH(BN)) / (GN * (GNS + X) * DCOS(GN))
TD1P = DCOSH(BN) / DCOS(GN)
DL = 1.0
IF (GNS .EQ. 1) DL = 2.0
A1N = P * DCOS(ENP * VI) / DCOSH(BN * (1 - U)) / (DL * PHIS * PHIS * (BN + GNS) * GNS)
BN = DCOSH(BN)
D1N = - P * DCOS(ENP * VI) / DCOS(GN * (1 - U)) / (DL * PHIS * PHIS * (BN + GNS) * GNS)
ICOS = DCOS(GN)
C1N = P * DCOS(ENP * VI) / DSINH(BN * U) / (DL * PHIS * PHIS * (BN + GNS) * GNS)
ICOS = DCOS(BN)
X = U * P * ENPS
TD1N = BN * DCOSH(BN) + TD1N * GN * DCOS(GN)
TD1P = BN * DSINH(BN) + TD1P * DSINH(GN)
IF (ETA .EQ. 0.0) GO TO 601
IF (ENP .EQ. 0.0) GO TO 601
XX = DCOS(ENP * ETA)
GO TO 602
601 XX = 1.0
602 IF (PSI .EQ. 0.0) GO TO 604
X4 = ENP * (DSINH(BN * PSI) * TD1 * DSIN(GN * PSI)) + XX / TD1N
IF (PSI LT 0.0) GO TO 603
XX = ICOS * ICOS / DSINH(BN * (1 - PSI)) / TD1N / DCOS(GN) / DCOS(IN)
IF (PSI .EQ. 1.0) GO TO 603
X4 = ENP * ICOS * ICOS / DSINH(BN * PSI) * TD1N / DCOS(IN)
GO TO 617
603 XX = ENP * (1 - TD1P) * XX / TD1P
GO TO 617
604 XX = 0.0
XX = 0.0
XX = 0.0
GO TO 617
606 X1 = X1
GNS = (1. / PHIS) * X1
GN = SQRT(GNS)
X = POIS * PHIS * ENPS
IF (GNS .LT. 22) GO TO 642
TD1 = (BN + X - BN - X) * DCOSH(BN) / (GNS * (X - GNS) * DCOSH(IN))
TD2P = DCOSH(BN) / DCOSH(IGN)
TD2N = BN * DCOSH(BN) + TD2G * GN * DCOSH(IGN)
TD2P = BN * DSINH(BN) - GN * TD2P * DSINH(IGN)
DL = 1.0
IF (N_EQ. 1) DL = 2.0
A2N = P * DCOS(ENP*V) * DCOSH(BN*(1-U)) / (DL * PHIS * PHIS * (BNS-GNS) * BN * DCOSH(BN))
B2N = P * DCOS(ENP*V) * DCOSH(IGN*(1-U)) / (DL * PHIS * PHIS * (BNS-GNS) * GN * DCOSH(IGN))
C2N = P * DCOS(ENP*V) * DSINH(BN*U) / (DL * PHIS * PHIS * (BNS-GNS) * BN * DCOSH(BN))
D2N = P * DCOS(ENP*V) * DSINH(IGN*U) / (DL * PHIS * PHIS * (BNS-GNS) * GN * DCOSH(IGN))
IF (ETA_EQ. 0.0) GO TO 607
IF (ENP_EQ. 0.0) GO TO 607
XX = DCOS(ENP*ETA)
GO TO 608
607 XX = 1.0
608 IF (PSI_EQ. 0.0) GO TO 610
XX2 = EN(N) * DSINH(BN*PSI) + TD2 * DSINH(IGN*PSI) * XX / TD22N
IF (PSI_EQ. 0.0) GO TO 609
XX4 = IC2N * DCOSH(BN*(1-PSI)) + D2N * DCOSH(IGN*(1-PSI)) * XX
IF (PSI_EQ. 1.0) GO TO 609
610 XX3 = EP(IN) * DCOSH(BN*(1-PSI)) - TD2P * DCOSH(IGN*(1-PSI)) * XX / TD22P
XX5 = A2N * DSINH(BN*PSI) + B2N * DSINH(IGN*PSI) * XX
GO TO 617
609 XX3 = EP(IN) * (1 - TD2P) * XX / TD22P
GO TO 617
610 XX2 = 0.0
XX3 = 0.0
XX4 = 0.0
GO TO 617
612 A = BN * (BNS-POIS*PHIS*ENPSI/IGN*(POIS*PHIS*ENPSI*GNS))
IF (ETA_EQ. 0.0) GO TO 613
IF (ENP_EQ. 0.0) GO TO 613
XX = DCOS(ENP*ETA)
GO TO 614
613 XX = 1.0
614 IF (PSI_EQ. 0.0) GO TO 616
B = 1.0.
TEST = BN - BN*PSI
IF (TEST_GT. 60.0) B = 0.0
XX2 = B * (DEXP(BN*PSI-BN) + A * DEXP(IGN*PSI-IGN) * EN(N) * XX / (BN+A*IGN)
DL = 1.0
IF (N_EQ. 1) DL = 2.0
A2N = P * DCOS(ENP*V) / (DL * PHIS * PHIS * (BNS-GNS) * BN)
B2N = P * DCOS(ENP*V) / (DL * PHIS * PHIS * (BNS-GNS) * GN)
C2N = P * DCOS(ENP*V) / (DL * PHIS * PHIS * (BNS-GNS) * BN)
D2N = P * DCOS(ENP*V) / (DL * PHIS * PHIS * (BNS-GNS) * GN)
A1 = 0.0
A2 = 0.0
A3 = 0.0
B1 = 0.0
B2 = 0.0
B3 = 0.0.
TEST = U - 0.00001
IF (PSI GT TEST) GO TO 74
IF (U - BN*PSI GT. 60.0) GO TO 71
A1 = DEXP(BN*U+BNS*PSI)
B1=DEXP(-GN*U+GN*PSI)
71 IF (2*BN-BN*U-BN*PSI+GT.60.0) GO TO 72
    A2=DEXP(-2*BN+BN*U+BN*PSI)
    B2=DEXP(-2*GN+GN*U+GN*PSI)
72 IF (BN*U+BN*PSI+GT.60.0) GO TO 73
    A3=DEXP(-BN*U-BN*PSI)
    B3=DEXP(-GN*U-GN*PSI)
    GO TO 65.
74 IF (BN*PSI-BN*U+GT.60.0) GO TO 75
    A1=DEXP(BN*U-BN*PSI)
    B1=DEXP(GN*U-GN*PSI)
75 IF (BN*U+BN*PSI+GT.60.0) GO TO 76
    A2=DEXP(-BN*U-BN*PSI)
    B2=DEXP(-GN*U-GN*PSI)
76 IF (2*BN-BN*U-BN*PSI+GT.60.0) GO TO 77
    A3=DEXP(-2*BN+BN*U+BN*PSI)
    B3=DEXP(-2*GN+GN*U+GN*PSI)
65 IF (PSI.EQ.1.0) GO TO 615
66 TEST=BN*(1-PSI)
    IF (TEST.LT.40.) GO TO 619
    X#3=EP(N)+IDEXP(-BN*PSI)-DEXP(-GN*PSI)*XX/(BN-GN)
    GO TO 617.
615 X#3=0.0
    GO TO 617.
616 X#2=0.0
    X#3=0.0
    X#4=0.0
617 W#I(J)=W22+X#2.
71 X#3=0.0
    W#4=0.0
620 CONTINUE
    DO 630 M=1,K1,2
        EMP2=EMP1/2.
        EMP2S=EMP2+EMP2
        BM=PHIS*(ALMDS+EMP2S)
        BM=DOSRT(BM)
        X#1=ALMDS-EMP2S
        IF (X#1.LT.0.0) GO TO 623
        GMS=PHIS*X#1
        GMS=DOSRT1(GMS)
        TD1=BM*(GMS+POIS+PHIS*EMP2S)*DSINH(BM)/(GMS*(GMS+POIS+PHIS*EMP2S).
    1 *DSIN ((M))
    TD1=TD1*(-1)
    TD11=8*DSINH(BM)-TD1*GMS*DSIN(GM)
    IF (PSI.EQ.0.0) GO TO 622.
    GMS=GMS*(DCOSH(BM)*ETA)+TD1*DCOS(GM)*ETA)*DSIN(EMP2*PSI)/TD11
    GO TO 629.
621 X#1=EM(N)*(1+TD1)*DSIN(EMP2*PSI)/TD11
    GO TO 629.
622 X#1=0.0
    GO TO 629.
623 X#1=X#1
    GMS=PHIS*X#1
    GMS=DOSRT1(GMS)
IF (BMS.GT.2) GO TO 625
TD2=IBM*(BMS-POIS*PHIS*EMP2S)*DSINH(BM)/IGM*(GMS-POIS*PHIS*EMP2S)
1*DSINH(GM)
TD2=TD2*(-1)
TD22=BM*DSINH(BM)+TD2*GM*DSINH(GM)
IF (PSI.EQ.0.0) GO TO 622
IF (ETA.EQ.0.0) GO TO 624
XW1=EM(M)*(DCOSH(BM+ETA)+TD2*DCOSH(GM+ETA))*DSIN(EMP2*PSI)/TD22
GO TO 629
624 XW1=EM(M)*(1+TD2)*DSIN(EMP2*PSI)/TD22
GO TO 629
625 A=BM*(POIS*PHIS*EMP2S-BMS)/IGM*(GMS-POIS*PHIS*EMP2S)
IF (PSI.EQ.0.0) GO TO 627
IF (ETA.EQ.0.0) GO TO 627
TEST=8M-BMS*ETA
IF (TEST.GT.60.) GO TO 627
XW1=EM(M)*(DEXP(BM*ETA-8M)+A*DEXP(GM*ETA-8M))&CSIN(EMP2*PSI)/
1*(8M+8M)
GO TO 629
627 XW1=0.0
629 D1(I,J)=XW1+W1
W1=W1(I,J)
CONTINUE
630 CONTINUE
635 FORMAT('** PSI = ',F5.3,4X,' ETA = ',F5.3,4X,' W1(',I2,'),',I2,') = ',
1D12.5,4X,' W2(',I2,'),',I2,') = ',D12.5,4X,' W3(',I2,'),',I2,') = ',
1D12.5,4X,' W4(',I2,'),',I2,') = ',D12.5)
PRINT636,PSI,ETA,I,J,W1(I,J)
636 FORMAT('** PSI = ',F5.3,4X,' ETA = ',F5.3,4X,' W1(',I2,'),',I2,') = ',
1D12.5)
PSI=PSI+1.0/DFLOAT(KS)
640 CONTINUE
ETA=ETA+1.0/DFLOAT(KS)
650 CONTINUE
RETURN
END
PROGRAM 15

THIS IS AN ANTISYMMETRIC MODE EIGENVALUE SEARCH PROGRAM FOR THE RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM WITH POINT SUPPORTS SYMMETRICALY DISTRIBUTED ON THE LATERAL SURFACE OF THE PLATE (ONE SUPPORT ON EACH SIDE OF THE CENTRAL AXIS).

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED;

1. - K = THE NUMBER OF TERMS TO BE USED IN THE SERIES INVOLVED. FOR 5 DIGITS ACCURACY USE K = 30.
2. - U = 0 TO 1, PROVIDING THE DISTANCE BETWEEN THE CONCENTRATED FORCE (POINT SUPPORT) AND THE PLATE CLAMPED EDGE DIVIDED BY SIDE LENGTH A.
3. - V = DISTANCE BETWEEN THE CONCENTRATED FORCE AND THE PLATE CENTRAL AXIS DIVIDED BY SIDE LENGTH B.
4. - PHIR = 2B/A, IS THE FULL PLATE ASPECT RATIO.
5. - ALMDS = AN INITIAL STARTING VALUE FOR THE EIGENVALUE SEARCH.
6. - DLIM = A FINISHING OR EIGENVALUE SEARCH ENDING LIMIT. IT INSTRUCTS THE COMPUTER WHEN TO HALT EXECUTION.
7. - DEL = EIGENVALUE INCREMENT.
8. - POI = POISSON'S RATIO.

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(150,150)
U=????????????????????????
V=????????????????????????
K=????????????????????????
K2=2*K
K1=2*K-1
Z2=3600
PHIR=????????????????????
PHI1=PHIR/2.
PRINT50,PHER,K1,U,V
50 FORMAT('PHIR=',F8.4,10X,'K=',I5,10X,'U=',F8.4,10X,'V=',)
IF 8 = 4 , // // //
C = 1
PI = 4. * DAIAN(C1)
PHIS = PHI * PHI
POI = \ldots
POI = 2 * POI
PHIN = 1. / PHI
PHIN = PI * PHIN
UFS = POI * PHIS
UFIS = POI * PHINS
I = 3 * K
I = I + 1
ALMDS = \ldots
DEI = \ldots
DLIM = \ldots
CONTINUE

INITIALIZE THE MATRIX TO 0.0

DO 2 M = 1, N1
DO 2 N = 1, N1
2 A(M, N) = 0.0
DO 10 M = 1, K1, 2
DO 10 N = 1, K1, 2
EMP2 = M / PI / 2.
EMP2 = EMP2 * EMP2
EMP2 = N / PI / 2.
EMP2 = EMP2 * EMP2
BNS = PHINS * (EMP2 + ALMDS * PHIS)
BNS = PHINS * (EMP2 + ALMDS * PHIS)
X1 = ALMDS * PHIS - EMP2
IF (X1 .LT. 0.0) GO TO 4
GNS = PHINS * X1
GNS = PHINS * X1
TD11 = GN * DCON(GN) * (GNS + ENP2 + POI * PHINS) / (BN * DCON(BN) * (BN - ENP2 + POI * PHINS))
TD11 = GN * DCON(GN) * (GNS + ENP2 + POI * PHINS) / (BN * DCON(BN) * (BN - ENP2 + POI * PHINS))
TD1P = DCON(BN) / DCON(GN)
TD1P = DCON(BN) / DCON(GN)
A(IN) = BNS + TD1N + DCON(BN) + GN * DCON(GN) * TD1P
A(IN) = BNS + TD1N + DCON(BN) + GN * DCON(GN) * TD1P
1 B1N = - DCON(GN) * (1-U) * DCON(ENP2) / (BN + DCON(BN) * (BN - ENP2))
1 B1N = - DCON(GN) * (1-U) * DCON(ENP2) / (BN + DCON(BN) * (BN - ENP2))
C1 = DCON(ENP2) * (1) * DCON(HB(N-U)) / ((BN + GNS) * (BN - ENP2))
C1 = DCON(ENP2) * (1) * DCON(HB(N-U)) / ((BN + GNS) * (BN - ENP2))
D1N = - DCON(GN-U) * DCON(ENP2) / (BN + GNS) * (BN - ENP2)
D1N = - DCON(GN-U) * DCON(ENP2) / (BN + GNS) * (BN - ENP2)
IF (M.GT.1.) GO TO 3
A(IN+1) / 2 + K2, (N+1) / 2 * K1 = TD1N * BN + GN / TD1N
A(IN+1) / 2 + K2, (N+1) / 2 * K1 = TD1N * BN + GN / TD1N
2 ENP2 + GNS) * DCON(GN) / TD1N
2 ENP2 + GNS) * DCON(GN) / TD1N
A(IN+1) / 2 + K2, (N+1) / 2 * K1 = (UFIS * ENP2 - GNS) * TD1N + DCON(HB(N-U)) + UFIS
A(IN+1) / 2 + K2, (N+1) / 2 * K1 = (UFIS * ENP2 - GNS) * TD1N + DCON(HB(N-U)) + UFIS
1 / TD1P
1 / TD1P
3 X1 = (ENP2 - UFS * BNS) / (BN + ENP2)
3 X1 = (ENP2 - UFS * BNS) / (BN + ENP2)
X2 = (ENP2 + UFS * GNS) / (GNS - ENP2)
X2 = (ENP2 + UFS * GNS) / (GNS - ENP2)
A(M+1) / 2, (N+1) / 2 * K2 = X1
A(M+1) / 2, (N+1) / 2 * K2 = X1
X2 = (ENP2 - UFS * BNS) / (BN + ENP2)
X2 = (ENP2 - UFS * BNS) / (BN + ENP2)
X1 * (ENP2 + UFS * GNS) / (GNS - ENP2) * DCON(BN)
X1 * (ENP2 + UFS * GNS) / (GNS - ENP2) * DCON(BN)
\[
XX = 2 \cdot \text{DSIN}(\text{EMP2}) \cdot \text{EMP2} \cdot (X1 + X2) / TD2N
\]
\[
A((N+1)/2, (N+1)/2 + K) = XX
\]
GO TO 10

4

X1 = X1
GNS = PHINS \cdot X1
GN = DSGRT(GNS)
IF (BN \cdot LTZ2) GO TO 6
TD2N = GN \cdot (POIS \cdot PHINS \cdot EMP2 - GNS) / (BN \cdot (BNS - POIS \cdot PHINS \cdot EMP2))
TD2N = TD2N \cdot (BN + GN)
TD2P = 1.
TD2P = BN - GN \cdot TD2P
IF (M > GT1) GO TO 5
A((N+1)/2 + K2, (N+1)/2 + K) = 0.0
A((N+1)/2 + K1, (N+1)/2 + K2) = 0.0
A((N+1)/2 + K2, (N+1)/2 + K) = ((UFIS \cdot EMP2 - BNS) \cdot TD2N + UFIS \cdot EMP2)
1-GNS) / TD2N
TD2N = GN \cdot (POIS \cdot PHINS \cdot EMP2 - GNS) / (BN \cdot (BNS - POIS \cdot PHINS \cdot EMP2))
TEST = BN - BN + U
B = 1.0
IF (TEST \cdot GT1.60) B = 0.0
A1 = 11\((N+1)/2 + K2) = (((\text{EXP}((-BN-U) \cdot B) - \text{EXP}((-GN-U) \cdot B)) \cdot \text{DSIN}(\text{EMP2} \cdot V) / (\text{TD2N} \cdot (BN + GN))) \cdot B
TEST = BN + U
B = 1.0
IF (TEST \cdot GT1.60) B = 0.0
A11 = 1((N+1)/2 + K2) = (((\text{EXP}((-BN-U) \cdot B) - \text{EXP}((-GN-U) \cdot B)) \cdot \text{DSIN}(\text{EMP2} \cdot V) / (\text{TD2N} \cdot (BN + GN))) \cdot B
TEST = BN + U
B = 1.0
IF (TEST \cdot GT1.60) B = 0.0
A111 = 1((N+1)/2 + K2, 11) = (((\text{EXP}((-BN-U) \cdot B) + B \cdot (\text{EXP}((-GN-U) \cdot B)) \cdot B
X1 = ((E2P25 - UFS \cdot BNS) / (BNS + EMP2)) \cdot TD2N \cdot (BN + GN)
X2 = ((E2P25 - UFS \cdot BNS) / (BNS + EMP2)) \cdot GN
XX = 2 \cdot \text{DSIN}(\text{EMP2}) \cdot \text{EMP2} \cdot (X1 + X2) / TD2N
A((N+1)/2, (N+1)/2 + K) = XX
A111 = 1((N+1)/2 + K2, 11) = XX

5

X1 = ((E2P25 - UFS \cdot BNS) / (BNS + EMP2)) \cdot TD2N
A((N+1)/2 + K, (N+1)/2 + K2) = XX

6

TD2N = 6A \cdot (POIS \cdot PHINS \cdot EMP2 - GNS) \cdot DCSVH(BN) / (BN \cdot (BNS + POIS \cdot PHINS \cdot EMP2))
TD2N = TD2N \cdot (BN + DCSVH(BN) \cdot GN + DCSVH(GN)
TD2P = DCSVH(BN) / DCSVH(GN)
TD2P = (BN + DSINV(BN) - GN) \cdot TD2P \cdot DSINV(GN)
A2N = DSIN(EMP2 \cdot V) \cdot DCSVH(BN) \cdot (1 - U) / (((BNS - GNS) \cdot (BN + PHIS \cdot PHIS \cdot DCSVH(BN)) \cdot (1 - U))
1)
B2N = DSIN(EMP2 \cdot V) \cdot DCSVH(GN) \cdot (1 - U) / (((BNS - GNS) \cdot (BN + PHIS \cdot PHIS \cdot DCSVH(GN))

1)
B2N = DSIN(EMP2 \cdot V) \cdot DCSVH(BN) \cdot (1 - U) / (((BNS - GNS) \cdot (BN + PHIS \cdot PHIS \cdot DCSVH(BN)) \cdot (1 - U))
D2N = DSIN(EMP2 \cdot V) \cdot DSINV(BN) \cdot U / (((BNS - GNS) \cdot (BN + PHIS \cdot PHIS \cdot DCSVH(BN)) \cdot (1 - U))

IF (M GT 1) GO TO 7
A1(N+1)/2+K, IN+1/2+K) = (TD02N*BN*GN) / TD22N
A1(N+1)/2+K, IN+1/2+K) = (UFIS*EMP2S-BNS) * TD2N*DSINH(BN)*D2N*DSINH(BN)*D2N*DSINH(BN)*
1EMP2S-GNS)*DSINH(GN) / TD22N
A1(N+1)/2+K, IN+1/2+K) = (UFIS*EMP2S-12-TD02P-BNS*MDP*GNS) / TD22P
A1(N+1)/2+K, IN+1/2+K) = (UFIS*EMP2S-BNS) * D2N*UFIS*EMP2S-GNS)
A11111(N+1)/2+K2) = (TD2N*DSINH(BN*U) + DSINH(GN*U) + DSINH(Emp2S) / TD22N
A11111(N+1)/2+K2) = (DCOSH(BN*1-1-U) / TD2P*DCOSH(BN*1-1-U)) / TD22P

7
X1 = (EMP2S-UFIS*BNS)/(BNS+EMP2S) * TD2N*BN*DCOSH(BN)
X2 = (EMP2S-UFIS*GNS)/(GNS+EMP2S) * GN*DCOSH(GN)
XX = 2*DSIN(EMP2S)*DSIN(EMP2S)*X1*X2 / TD22N
A1(M+1)/2, IN+1/2+K) = XX
X1 = (EMP2S-UFIS*BNS)/(BNS+EMP2S) * DCOSH(BN)
X2 = (EMP2S-UFIS*GNS)/(GNS+EMP2S) * TD2P*DCOSH(GN)
XX = 2*DSIN(EMP2S)*EMP2S*(X1-X2) / TD22P
A1(M+1)/2, IN+1/2+K2) = XX
10 CONTINUE
DO 11 N1 = 1, K
A1(N+4, N1+K2) = -1.
DO 20 M1 = 1, K1, 2
ENP2S = M1*PI/2.
11 CONTINUE

IF (X1<LT.0.01) GO TO 13
GMS = PHIS*X1
GM = DSQRT(GMS)
TD1M = GM*GM + ENP2S*PHIS*PS*GMS/DCOSH(BM) / BMS*EMP2S
POIS = PHIS/DCOSH(BM)
C1M = DSIN(EMP2S*U)*DSINH(BM*V) / (BMS*GMS) * BM*DCOSH(BM)
D1M = DSINH(GM*V)/DSINH(U)/BMS*GMS/GMS*DCOSH(GM)
GMS = DSQRT(GMS)
A1(M+1/2, MP+1/2) = (UFIS*EMP2S-BMS) * TD1M*DSINH(BM) + (UFIS*EMP2S-BMS) / TD1M
A1(N+1/2, N+1/2) = (UFIS*EMP2S-BMS) * TD1M*DSINH(BM) + (UFIS*EMP2S-BMS) / TD1M
A1(M+1/2, N+1/2) = (UFIS*EMP2S-BMS) * TD1M*DSINH(BM) + (UFIS*EMP2S-BMS) / TD1M
A1(N+1/2, M+1/2) = (UFIS*EMP2S-BMS) * TD1M*DSINH(BM) + (UFIS*EMP2S-BMS) / TD1M
A1(M+1/2, N+1/2) = (UFIS*EMP2S-BMS) * TD1M*DSINH(BM) + (UFIS*EMP2S-BMS) / TD1M
A1(N+1/2, M+1/2) = (UFIS*EMP2S-BMS) * TD1M*DSINH(BM) + (UFIS*EMP2S-BMS) / TD1M

12
X1 = (EMP2S-UFIS*BMS)/(BMS*EMP2S) * TD1M*BM*DCOSH(BM)
XX = 2*DSIN(EMP2S)*DSINH(EMP2S)*X1*X2 / TD1M
A1(M+1/2, M+1/2) = XX
X1 = (EMP2S-UFIS*BMS)/(BMS*EMP2S) * TD1M*BM*DCOSH(BM)
XX = 2*DSIN(EMP2S)*DSINH(EMP2S)*X1*X2 / TD1M
A1(N+1/2, N+1/2) = XX
GO TO 20

13
X1S = X1
GMS = PHIS*X1
GM = DSQRT(GMS)
IF (BMS<LT.22) GO TO 15
TD2M = GM*GM * POIS*PHIS*EMP2S-GMS / (BMS*GMS*POIS*PHIS*EMP2S-GMS)
TD22M = TD2P*BM*GM
C2M = DSINH(EMP2S*U) / (BMS*GMS*POIS*PHIS*EMP2S-GMS)
D2M=DSIN(EMP2*U)/{(BMS-GMS)*GM)}
IF (N.GT.1) GO TO 14
B=1
TEST=BM-B*M*V
IF (TEST.GT.60) B=0.
A1111(1+1)/2=({TD2M*DEXP([-B*M+B*V]*B)+DEXP([-GM+GM*V]*B)}*1)
B=DSIN(EMP2*U)/{TD2M*BM+GM})} B
A1111(1+1)/2=({C2M+DEXP([-B*M+B*V]*B)+1*GM*EMP2S-BMS}+D2M*DEXP
1{(-GM+GM*V)*B}+1*GM*EMP2S-BMS}}/TD2M*B
A1111(1+1)/2={(BMS*EMP2S-GMS)*TD2M+(BMS*EMP2S-GMS))}/TD22M
B=1
TEST=BM-2*BM*V
IF (TEST.GT.60) B=0.
A11111=2*(C2M+DEXP([-B*M+B*V]*B)+1*GM*EMP2S-GMS)*BM
14 x11={C2M+DEXP([-B*M+B*V]*B)+1*GM*EMP2S-GMS)}*TD2M*BM
x2=(C2M+DEXP([-B*M+B*V]*B)+1*GM*EMP2S-GMS)}*GM
xx=x11+2*DSIN(EMP2)*DSIN(EMP2)*1X1+2)/TD22M
A1(1+1)/2+1=xx
x1=TD2M*BM/(BMS*EMP2S)
x2=GM+(BMS*EMP2S)
xx=x11+2*DEXP(EMP2)*DSIN(EMP2)*1X1+2)/TD22M
A1(1+1)/2+1=xx
GO TO 20
15 TD2M=GN(IPOIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BMS*(BMS-POIS*PHIS*1EMP2S)*DCOSH(BM))
TD2M=TD2M*B*BM*BM*BM+GM*GM*GM
C2M=DSIN(EMP2*U)*DSINH(BM*V)/(BMS-GMS)*BM*DCOSH(BM))
D2M=DSIN(EMP2*U)*DSINH(BM*V)/(BMS-GMS)*GM*DCOSH(GM)
IF (N.GT.1) GO TO 16
A1(1+1)/2=1{(UFS*EMP2S-BMS)*TD2M*DSINH(BM)+(UFS*EMP2S-1GMS)*DSINH(GM))}/TD22M
A1111,1111(1+1)/2=({TD2M*DSINTH(BM*V)*DSINTH(GM*V)}*DSIN(EMP2*U)/TD22M
A1111,1111(1+1)/2=({C2M+UFS*EMP2S-BMS}+D2M*(UFS*EMP2S-GMS)
A111111=1{(C2M+DEXP([-B*M+B*V]*B)+1*GM*EMP2S-GMS)}*GM*DSINH(BM*V)
X2=1{(C2M+DEXP([-B*M+B*V]*B)+1*GM*EMP2S-GMS)}*GM*DSINH(GM)
X2=x11+2*DSIN(EMP2)*DSIN(EMP2)*1X1+2)/TD22M
A1(1+1)/2+1=xx
X1=TD2M*BM*BM*BM*BM/(BMS*EMP2S)
x2=GM*DCOSH(BM)/(GWS*EMP2S)
xx=x11+2*DEXP(EMP2)*DSIN(EMP2)*1X1+2)/TD22M
A1(1+1)/2+1=xx
GO TO 20
CONTINUE
CALL DETERM (A111,DET)
PRINT 110,ALMDS,DET
110 FORMAT (A,11,9,E13.7,10X,9,9,E13.7,10X,9,9,D20.5)
ALMDS=ALMDS+DEL
IF (ALMDS.LT.30M) GO TO 1
STOP
END
SUBROUTINE DETERM (A,N,DET)
IMPLICIT REAL*8(A,N,0-Z)
DIMENSION A(150,150)
SIGN=1
LAST=N-1
START OVERALL LOOP FOR (N-1) PIVOTS
DO 200 I=1,LAST
FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT

BIG=0.
DO 50 K=1,N
TERM=ABS(A(I,K))
IF (TERM>BIG) GOTO 50
BIG=TERM
L=K
50 CONTINUE
CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND
IF (BIG) GOTO 90
L-TH ROW HAS THE BIGGEST TERM----IS I=L
80 IF (I-L) GOTO 90
IF I IS NOT EQUAL TO L, SWITCH ROWS I AND L
90 SIGN=-SIGN
DO 100 J=1,N
TEMP=A(I,J)
A(I,J)=A(L,J)
100 A(L,J)=TEMP
NOW START PIVOTAL REDUCTION
120 PIVOT=A(I,I)
NEXTR=I+1
FOR EACH OF THE ROWS AFTER THE I-TH
DO 200 J=NEXTR,N
MULTIPLYING CONSTANT FOR THE J-TH ROW IS
CONST=A(J,I)/PIVOT
NOW REDUCE EACH TERM OF THE J-TH ROW.
DO 250 K=1,N
200 A(J,K)=A(J,K)-CONST*A(I,K)
END OF PIVOTAL REDUCTION----NOW COMPUTE DETERMINANT
DET=SIGN
DO 300 I=1,N
300 DET=DET*A(I,I)/10.
GO TO 61
60 DET=0.
61 RETURN
END
PROGRAM 16

THIS IS AN ANTISYMMETRIC MODE SHAPE PROGRAM FOR THE
RECTANGULAR CANTILEVER PLATE FREE VIBRATION PROBLEM WITH
SYMMETRICALLY DISTRIBUTED POINT SUPPORTS ON THE PLATE LATERAL
SURFACE. (ONE SUPPORT ON EACH SIDE OF THE CENTRAL AXIS)

USAGE: THE FOLLOWING VARIABLES MUST BE PROVIDED:

1. K = AS DEFINED IN PROGRAM 15.
2. U = AS DEFINED IN PROGRAM 15.
3. V = AS DEFINED IN PROGRAM 15.
4. PHIR = 2B/A FULL PLATE ASPECT RATIO.
5. ALMDS = EIGENVALUE.
6. KS = NUMBER OF POINTS AT WHICH THE DISPLACE-
MENT W IS REQUIRED.
7. POI = POISSON'S RATIO.

IMPLICIT REAL*8(A-H,O-Z)
DIMENSION A(193,69)
COMMON Z2,PHI,P1,PHIS,PO1,POIS,PHIN,PHINS,ALMDS,U,V,K,K1,KS
U=????????????????????????
V=????????????????????????
K=????????????????????????
KS=????????????????????????
K2=2*K
K1=2*K-1
Z2=3600
PHIR=????????????????????
ALMDS=????????????????????
PHI=PHIR2
PRINT50,PHIR,PHI,K,ALMDS,U,V
FORMAT(19,PHIR=*,F8.4,10X,PHI=*,F8.4,10X,K=*,15.10X,ALMDS=19,F8.4,10X,U=*,F8.4,10X,V=*,F8.4,10X)
C=1
PI=4.*DATAN(1)
PHIS=PHI*PHI
PO1=????????????????????
POIS=2*PO1
PHIN=1./PHI
PHINS=PHIN*PHIN
UFS=POI*PHIS
UFIS=POIS*PHINS
I=3*K
I1=I+1
2 CONTINUE

INITIALIZE THE MATRIX TO 0.0

DO 2 M=1,I1
DO 2 N=1,I1
A(M,N)=0.0
DO 10 M=1,K1*2
DO 10 N=1,K1*2
EMP2=M*PI/2.
EMP2S=EMP2*EMP2
EMP2=2*N*PI/2.
EMP2S=EMP2*EMP2
BNS=PHINS*PENP2S+ALMDS*PHIS
BN=DSQRT(BNS)
X1=ALMDS*PHIS+EMP2S
IF (X1 LT 0.0) GO TO 4
GNS=PHINS*X1
GNS=DSQRT(GNS)
TDBN=GN*DCOS(I1)*GNS+EMP2S*POIS*PHINS)/(BNSDCOSH(BN)*(BNS-EMP2S*POIS*PHINS))
TDB=DCOSH(BN)/DCOS(I1)
TB1=DCOSH(BN)/DCOS(I1)
A1N=DSINH(BN)+GN*DSINH(BN)+GN*DCOS(I1)
1 X1=1*(ENP2S-ENS*BN)/(BNS-EMP2S)*TDBN*DSINH(BN)+(UFIS*EMP2S*GNS)*DSINH(I1)/TDB1
A1N+1/2*2+K1=1*(UFIS*EMP2S*1-TD1P)-BNS-TPD*GNS)/TB11
A1N+1/2*2+K1=C1N*(UFIS*EMP2S-BNS)*TDB1*GNS)/TB11
A1N+1/2*2+K1=A1N*BN+B1N*GN
A1N+1/2*2+K1=(TD1N*DSINH(BN*U1)+DSINH(BN*G1))/DSINH(1-U1)+DCOSH(BN)+1-U1)+DSINH(1-U1)
1/TC1P

2 X1=(ENP2S-ENS*BN)/(BNS-EMP2S)*TDBN*DSINH(BN)
X2=(ENP2S-ENS*BN)/(BNS-EMP2S)*GNS+EMP2S)*GNS+DCOSH(BN)
XX=2*DSINH(I1)*EMP2S*1*XX)/TDB1
A1(N+1/2,1+1/2+K1)=X1
1
X1=1*(ENP2S-ENS*BN)/(BNS-EMP2S)*TDBN*DSINH(BN)
X2=(ENP2S-ENS*BN)/(BNS-EMP2S)*GNS+EMP2S)*GNS+DCOSH(BN)
XX=2*DSINH(I1)*EMP2S*1*XX)/TDB1
A1(N+1/2,1+1/2+K1)=X1
1
GNS=PHINS*X1
GNS=DSQRT(GNS)
IF (GNS LT 0.2) GO TO 6
TDB2=GN*1*POIS*PHINS*EMP2S*GNS)/(BNS-POIS*PHINS*EMP2S)
TDB2=TD2P=1
TDB2P=BN*GN*TD2P
IF (M .GE. 1) GO TO 5
A((N+1)/2+K2, (N+1)/2+K) = 0
A((N+1)/2+K, (N+1)/2+K2) = 0
A((N+1)/2+K, (N+1)/2+K2) = 0
A((N+1)/2+K2, (N+1)/2+K) = (UFIS*ENP2S-BNS)TD2N+UFIS*ENP2S
1-GNS)TD2N
TD2N=GN*(POIS*PHINS*ENP2S=GNS)/(BN*(BNS-POIS*PHINS*ENP2S))
TEST=BN=BN*U
B=1.0
IF (TEST GT 60.), B = 0.
A((1, (N+1)/2+K) = (TD2N*DEXP(BN*U-BN*1*B)+DEXP(GN*U-GN*B))
1DSIN(ENP2*V)/TD2N*BN+GN1*B
TEST=BN=BN*U
B=1.0
IF (TEST GT 60.), B = 0.
A((1, (N+1)/2+K2) = (DEXP((BN*U*B)+DEXP((-GN*U*B)))*DSIN(ENP2*V)
)/(BN*GN))
C2N=DSIN(ENP2*V)/(BNS-GNS)*BN*PHIS*PHIS
D2N=DSIN(ENP2*V)/(BNS-GNS)*GN*PHIS*PHIS
TEST=BN=BN*U
B=1.0
IF (TEST GT 60.), B = 0.
A((N+1)/2+K, (N+1)/2+K2) = (DEXP((-BN*U*B)+DEXP((-GN*U*B))))*DSIN(ENP2*V)
)/(BN*GN)
A((1, (N+1)/2+K) = (C2N*(UFIS*ENP2S-BNS)+T02N*(UFIS
)*ENP2S-GNS)*-DEXP((BN*U-BN*1*B)+D2N*(UFIS
)*ENP2S-GNS)*DEXP((GN*U-GN*B)))*B
A2N=DSIN(ENP2*V)/(BNS-GNS)*BN*PHIS*PHIS
B2N=DSIN(ENP2*V)/(BNS-GNS)*GN*PHIS*PHIS
TEST=BN=BN*U
B=1.0
IF (TEST GT 60.), B = 0.
A((N+1)/2+K2, (N+1)/2+K) = (A2N*BN*DEXP((-BN*U*B))+B2N*GN*DEXP((-GN*U*B))))*B
x1 = (ENP2S-UF*S/BNS)/(BNS-ENP2S/BN)*TD2N*BN
X2 = (ENP2S-UF*S/BNS)/(BNS-ENP2S/BN)*GN
XX = 2*DSIN(ENP2)*DSIN(ENP2)*(X1+X2)/TD22N
A1((4+1)/2, (N+1)/2+K) = XX
X1 = (ENP2S-UF*S/GNS)/(BNS-ENP2S/BN)*GN
X2 = (ENP2S-UF*S/GNS)/(BNS-ENP2S/BN)*GN
XX = 2*DSIN(ENP2)*EMP2*(X1+X2)/TD22P
A1((4+1)/2, (N+1)/2+K2) = XX
GO TO 10
6 TD2N=GN*(POIS*PHINS*ENP2S-GNS)*DCOSH(GN)/(BN*(BNS-POIS*PHINS* 1ENP2S/DCOSH(BN))
TD2N=TD2N*BN*DCOSH(BN)+GN*DCOSH(GN)
TD2P=DCOSH(BN)/DCOSH(GN)
TD2P=BN*DSINH(BN)-GN*TD2P*DSINH(BN)
A2N=DSIN(ENP2*V)*DCOSH(BN+1-U))/(BNS-GNS)*BN*PHIS*PHIS*DCOSH(BN)
1
B2N=DSIN(ENP2*V)*DCOSH(GN*(1-U))/(BNS-GNS)*BN*PHIS*PHIS*DCOSH(IGN 1)
C2N=DSIN(ENP2*V)*DSINH(BN+1-U)/(BNS-GNS)*BN*PHIS*PHIS*DCOSH(BN)
D2N=DSIN(ENP2*V)*DSINH(BN+1-U)/(BNS-GNS)*BN*PHIS*PHIS*DCOSH(IGN)
1
IF (M, GT, 1) GO TO 7
A((N+1)/2+K2, (N+1)/2+K) = (TD2N*BN*GN)/TD22N
A((N+1)/2+K, (N+1)/2+K) = (UFIS*ENP2S-BNS)+TD2N*DSINH(BN)+UFIS
1ENP2S-GNS)*DSINH(BIN))/TD22N
A((N+1)/2+K2, (N+1)/2+K2) = (UFIS*ENP2S-BNS)+TD2P*DSINH(BIN)/TD22P
A((N+1)/2+K, (N+1)/2+K) = (UFIS*ENP2S-BNS)+D2N*UFIS*ENP2S-GNS)
A((N+1)/2+K2, (N+1)/2+K2) = A2N*BN+B2N*GN
A1((N+1)/2+K2) = (TD2N*DSINH(BN+1-U)))*DSIN(ENP2*V)/TD22N
A1((N+1)/2+K2) = (DCOSH(BN+1-U))*TD2P*DCOSH(IGN*(1-U))*DSIN(ENP2*V)
X1 = (ENP2S-UF*S/BNS)/(BNS-ENP2S/BN)*TD2N*BN*DCOSH(BN)
X2 = (ENP2S-UF*S/GNS)/(BNS-ENP2S/BN)*GN*DCOSH(BN)
\[
X = 2 \cdot \text{DSIN}(\text{EMP2} - \text{NPS}) \cdot \text{DSIN}(\text{EMP2} - (X_1 + X_2)) / TD22N
\]
\[
A(1 + M + 1) / 2, (M + 1) / 2, (N + 1) / 2 + K) = XX
\]
\[
X_1 = (\text{ENP2} - \text{NPS} + \text{BN}) / (\text{NPS} + \text{EMP2} - \text{BN}) \cdot \text{DCOSH}(\text{BN})
\]
\[
X_2 = (\text{ENP2} - \text{NPS} + \text{GN}) / (\text{NPS} + \text{EMP2} - \text{GN}) \cdot TD22P \cdot \text{DCOSH}(\text{GN})
\]
\[
X = 2 \cdot \text{DSIN}(\text{EMP2} - \text{EMP2} - (X_1 - X_2)) / TD22P
\]
\[
A(M + 1) / 2, (M + 1) / 2 + K) = XX
\]

10 CONTINUE

**DD**

11 N = 1, K

12 A(N + K2, N + K2) = -1

13 DD 10

14 M = 1, K, 2

15 ENP2 = ENP2 - EMP2

16 EMP2 = EMP2 - EMP2

17 BMS = PHIS * (ALMDS + EMP2)

18 B4 = DSQRT(BMS)

19 X1 = ALMDS - EMP2

20 IF (X1 * LT. 0.0) GO TO 13

21 GS = PHIS * X1

22 GM = DSQRT(GMS)

23 TD1M = GM * (GMS + EMP2 * PHIS * DCOSH(GM) / (BM * (BM - EMP2) * 1)

24 POIS = PHIS * DCOSH(BM)

25 TD1M = BM * TD1M * DCOSH(BM) + GM * DCOSH(GM)

26 C1M = DSIN(EMP2 * U) * DSINH(BM * V) / ((BMS + GMS) * BM * DCOSH(BM))

27 D1M = DSINH(EMP2 * U) + DSINH(BM * V) / ((BMS + GMS) * GM * DCOSH(GM))

28 IF (N * GT. 0) GO TO 12

29 A(M + 1) / 2, (M + 1) / 2 = (UFS * EMP2 - BMS) * TD1M * DSINH(BM) * IFS *

30 EMP2 * GMS) * DSINH(GM) / TD1M

31 A(I, I1) = TD1M * DSINH(BM * V) + DSINH(GM * U) / TD1M

32 A(M + 1) / 2, I1 = C1M * IFS * EMP2 - BMS) * D1M * (UFS * EMP2 * GMS)

33 A(I, I1) = A(I, I1) + C1M * DCOSH(BM * V) + D1M * DCOSH(GM * U) / DSINH(EMP2 * U)

34 X2 = (EMP2 - UFI * BMS) / (BMS * EMP2) * TD1M * BM * DCOSH(BM)

35 XX = 2 * DSINH(EMP2) * DSINH(EMP2) / (X1 - X2) / TD1M

36 A(M + 1) / 2, (M + 1) / 2 = XX

37 X1 = TD1M * BM * DCOSH(BM) / (BMS * EMP2)

38 X2 = GM * DCOSH(GM) / (GMS - EMP2)

39 XX = 2 * EMP2 * DSINH(EMP2) / (X1 - X2) / TD1M

40 A(M + 1) / 2, K, (M + 1) / 2 = XX

41 X1 = X1

42 GM = DSQRT(GMS)

43 IF (BMS * LT. 22) GO TO 15

44 TD2M = GM * POIS * PHIS * EMP2 - GMS) / (BM * (BMS - POIS * PHIS * EMP2))

45 C2M = DSINH(EMP2) / ((BMS - GMS) * BM)

46 D2M = -DSINH(EMP2 * U) / ((BMS - GMS) * GM)

47 IF (N * GT. 1) GO TO 14

48 B = 1

49 TEST = BM - BM * V

50 IF (TEST .GT. 60.) B = 0.

51 A(I, I1) / 2 = (TD2M * DEXP((-BM + BM * V) * B) + DEXP((-GM * GM * V) * B)) * 1

52 DSINH(EMP2 * U) / ((BMS - BM * GMS) * BM)

53 A(M + 1) / 2, I1 = C2M * DEXP((-BM + BM * V) * B) * (UFS * EMP2 - BMS) * D2M * DEXP

54 (-GM * GM * V) * B) * (UFS * EMP2 - GMS) * B

55 A(M + 1) / 2, (M + 1) / 2 = (UFS * EMP2 - BMS) * TD2M + (UFS * EMP2 - GMS) / TD22M

56 B = 1

57 TEST = BM - BM * V
IF (TES.GT.60) B=0.
A(I1,II)=A(I1,II)+5*(C2M*DEXP((2*BM*V-BM)*B)+D2M*DEXP((2*GM*V-
1GM)*B))B*DSIN(EMP2*U)
14 X1=((EMP25-UFIS*BMS)/(BMS+EMP2S5)+TD2M*BM
X2=((EMP25-UFIS*GMS)/(GMS+EMP2S5)+GM
XX=2*DSIN(EMP2)*DSIN(EMP25)*(X1+X2)/TD23M
A((N+1)/2+K,II)=X1
X1=TD2M*BM/(BMS+EMP25)
X2=GM/(GMS+EMP2S5)
XX=2*EMP2*DSIN(EMP25)*(X1+X2)/TD23M
A((N+1)/2+K,II)=XX
GO TO 20
15 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*DCOSH(GM)/(BM*(BMS-POIS*PHIS*
EMP2S)+DCOSH(BM))
X2M=TD2M*BM*DCOSH(BM)+GM*DCOSH(GM)
C2M=DSIN(EMP2*U)*DSINH(BM*V)/(BMS-GMS)*BM*DCOSH(BM)
X2M=DSIN(EMP2*U)*DSINH(GM*V)/(BMS-GMS)*GM*DCOSH(GM)
IF(N.GT.1) GO TO 16
A((N+1)/2,II)=((UF5*EMP2S-BMS)*TD2M*DSINH(BM)+UF5*EMP2S-
1GMS)*DSINH(GM)/TD23M
A((N+1)/2,II)=TD2M*DSINH(BM)+DSINH(GM)/(BM-(BMS-POIS*PHIS*
EMP2S)+DCOSH(BM))
A((N+1)/2,II)=TD2M*DSINH(BM)/(BMS-GMS)*GM*DCOSH(GM)
16 X1=((EMP2S-UFIS*BMS)/(BMS+EMP2S5)+TD2M*BM
X2=((EMP2S-UFIS*GMS)/(GMS+EMP2S5)+GM*DCOSH(GM)
XX=2*DSIN(EMP25)*DSINH(EMP25)*(X1+X2)/TD23M
A((N+1)/2+K,II)=XX
X1=TD2M*BM/DCOSH(BM)/(BMS+EMP2S5)
X2=GM*DCOSH(GM)/(GMS+EMP2S5)
XX=2*EMP2*DSINH(EMP25)*(X1+X2)/TD23M
A((N+1)/2+K,II)=XX
CONTINUE
DO 22 J=1,II.
TEMP=A((J,II)
END
22 CONTINUE
CALL DETERM (A, I1, DET)
PRINT 25
25 FORMAT(*10,39X,*END*,///)
STOP
END
SUBROUTINE DETERM (A,N,DET)
IMPLICIT REAL(*A-N,0-Z*)
DIMENSION A(99,99),X(99),EN(75),EP(75)
SIGN=1.
N=N+1
LAST=N-1
START OVERALL LOOP FOR(N-1) PIVOTS
DO 200 I=1,LAST
DO 99 K=N
TERM=ABS(A(K,I))
IF (TERM.BR.050.50,30)
99 TERM=TERM
DO 200 K=I,M.
TERM=ABS(A(K,I))
IF (TERM.BR.050.50,30)
200 CONTINUE
FIND THE LARGEST REMAINING TERM IN I-TH COLUMN FOR PIVOT
BIG=0.
DO 30 K=1,M.
TERM=ABS(A(K,I))
IF (TERM.BR.050.50,30)
30 BIG=TERM
L=K
50 CONTINUE

CHECK WHETHER A NON-ZERO TERM HAS BEEN FOUND

IF (BIG=0,60,80)
L-TH ROW HAS THE BIGGEST TERM----IS I=L
60 IF (I-L)90,120,90
I IS NOT EQUAL TO L, SWITCH ROWS I AND L
90 DO 100 J=1,N
      TEMP=A(I,J)
      A(I,J)=A(L,J)
    100 A(L,J)=TEMP

NOW START PIVOTAL REDUCTION

120 PIVOT=A(I,I)
      NEXTR=I+1

FOR EACH OF THE ROWS AFTER THE I-TH
DO 200 J=NEXTR,N
MULTIPLYING CONSTANT FOR THE J-TH ROW IS
      CONST=A(I,J)/PIVOT.
NOW REDUCE EACH TERM OF THE J-TH ROW
DO 200 K=1,N
      A(J,K)=A(J,K)-CONST*A(I,K)
      200 END OF PIVOTAL REDUCTION-- PERFORM BACK SUBSTITUTION

M=N-1
DO 500 I=1,M

IREV IS THE BACKWARD INDEX, GOING FROM M BACK TO 1
IREV=M+1-I
GET T(IREV) IN PREPARATION
T=A(IREV,N)
IF (IREV=M) 400,500,400
NOT WORKING ON LAST ROW, I IS 2 OR GREATER
400 DO 450 J=2,I
WORK BACKWARD FOR T(N), T(N-1)------SUBSTITUTING PREVIOUSLY
FOUND VALUES
      K=N+1-J
      450 T=T-A(IREV,K)*T(K)
      END OF PIVOTAL REDUCTION--
FINALLY, COMPUTE X[IREV]

500 X[IREV] = Y/A[IREV, IREV]

FIND AND PRINT EM, EN, EP, AND P:

L = N/3
DO 550 I = 1, L
J = 2*I - 1
EM(J) = X(I)
EN(J) = X(I+L)
P = X(M)
END
EP(I, EQ, L) GO TO 540
EP(J) = X(I+2*L)
END
GO TO 550
540 EP(J) = 1.
GO TO 1
550 PRINT 600, J, J, EM(J), J, EN(J), J, EP(J), P
END
600 FORMAT('J, J, EM(J), J, EN(J), J, EP(J), P')
10.0 FORMAT(1, L9, 'THE ANTI-SYMMETRIC MODE SHAPE DATA ARE:

1 CALL SHAPE(EM, EN, EP, P)
10 RETURN
END
SUBROUTINE SHAPE(EM, EN, EP, P)

EXPlicit Real*8(A-H, O-Z)
COMMON Z2, PHI, PI, PHIS, POIS, PHINS, ALMDS, U, V, K, K1, KS
DIMENSION X(M), W(M), X1(M), X2(M), X3(M), W1(M), W2(M), W3(M), W4(M), W5(M)
1 EM(75), EN(75), EP(75)
K5 = K5+1
ETA = 0.0
DO 630 I = 1, K5
PSI = 0.0
DO 640 J = 1, K5
W11 = 0.0
W22 = 0.0
W33 = 0.0
W44 = 0.0
DO 620 N = 1, K5
620 ENP2 = N*PI/2.
630 ENP2 = ENP2*ENP2
BNS = PHINS*(ENP2*ALMDS+PHIS)
BN = D50R(T(BNS)
Xi = ALMDS*PHIS-ENP2
IF (Xi > 1.0) GO TO 605
GNS = PHINS*Xi
GNS = D50R(GNS)
TD1N = GNS*DC0S(GN)*GNS*ENP2*POIS*PHINS)/(BN*DC0SHBN1*BNS-ENP2*POIS*PHINS)
TD1N = GNS*DC0S(HBN1)*GNS*DC0SIGN
TD1P = DC0SH(BN)/DC0SIGN
TD1F = BNS*DSINHBN1*GNS*DSINIGN1*TD1P
A1M = multiplying constants
10 DC0SH(BN1)
BNS = -1*DC0S(GN+1-U1)*DSIN(ENP2*V)/((BNS+GNS)*GNS*PHIS*PHIS)
10 DC0SIGN)
C1N = multiplying constants
D1N = multiplying constants
IF (ETA < 0.0) GO TO 402
IF (PSI.EQ.0.0) GO TO 603
X42=EN(N)*(TD1N*DSINH*(BN*PSI1)-DSIN(HEP2*ETA)*TD11N
IF (PSI.EQ.1.0) GO TO 601
X43=EPIN*(DCOSH*(1-PSI)-BN)-TD1P*DCOS((1-PSI)*GN)*DSIN
1(ENP2*ETA)/TD11P
W41=(A1N*DSINH*(BN*PSI1)+B1N*DSIN(HEP2*ETA)
W42=(C1N*DCOSH*(BN*1-PSI1)+D1N*DCOS(1-PSI)*GN)*DSIN(IENP2*ETA)
X44=W41
IF (PSI.GT.0) X34=W42
GO TO 604
601 X3=EPIN*(1-1TD1P)*DSIN(ENP2*ETA)/TD11P
X44=(C1N*D1N)*DSIN(ENP2*ETA)
GO TO 604
602 X22=0.0
X3=0.0
X4=0.0
GO TO 604
603 X3=EPIN*(DCOSH*(BN)-TD1P*DCOS(1-PSI)*DSIN(ENP2*ETA)/TD11P
X4=0.0
604 X21=172+X22
X31=X33+X3
X41=(I,J)=X44+X4
X22=X2(I,J)
X33=X3(I,J)
X44=X4(I,J)
GO TO 620
605 X1=-X1
G&N=PSIN*X1
G1=DSORT(IN1)
IF (BN.GT.22) GO TO 610
6 TD2N=GN*(POIS*PHIN*ENP2*-GNS)*DCOSH*(BN)/BN*(BN*POIS*PHIN*1ENP2*-DSOSH*(BN))
TD2N=TD2N*BN*DCOSH*(BN)+GN*DCOSH*(BN)
TD2P=DCOSH*(BN)/DCOSH*(GN)
TD2P=BN*DSINH*(BN)-GN*TD2P*DSINH*(GN)
A2N=P*DSIN(ENP2*V)*DCOSH*(BN*1-U1)/BN*GNS*BN*PHIS*PHIS*DCOSH*BN)
B2N=P*DSIN(ENP2*V)*DCOSH*(BN*1-U1)/BN*GNS*BN*PHIS*PHIS*DCOSH*BN)
G1N=P*DSIN(ENP2*V)*DSINH*(BN*U)/(BN*GNS*BN*PHIS*PHIS*DCOSH*BN)
D2N=P*DSIN(ENP2*V)*DSINH*(BN*U)/(BN*GNS*BN*PHIS*PHIS*DCOSH*BN)
IF (ETA.EQ.0.0) GO TO 607
IF (PSI.EQ.0.0) GO TO 608
X2=EN(N)*TD2N*DSINH*(BN*PSI1)-DSINH*(1-PSI)*DSIN(ENP2*ETA)/TD22N
IF (PSI.EQ.1.0) GO TO 606
X3=EPIN*(DCOSH*(1-PSI1)*BN)-TD2P*DCOSH*(1-PSI1)*GN)*DSIN(ENP2)
1ETAI/TD22P
W41=(A2N*DSINH*(BN*PSI1)+B2N*DSINH*(1-PSI1)*DSIN(ENP2*ETA)
W42=(C2N*DCOSH*(BN*1-PSI1)+D2N*DCOSH*(1-PSI1))*DSIN(ENP2*ETA)
X44=W41
IF (PSI.GT.0) X34=W42
GO TO 609
608 X3=EPIN*(1-1TD2P)*DSIN(ENP2*ETA)/TD22P
X44=(C2N*D2N)*DSIN(ENP2*ETA)
GO TO 609
607 X22=0.0
X3=0.0
X4=0.0
GO TO 609
608 XW2=0.0
XW3=EXP[(N) *(D.*DSh(BN)-TD2P*DCSh(GN))*DSIN(ENP2*ETA)/TD2P
XW4=0.0
609 W22(I,J)=W22+W2
W31(I,J)=W33+W3
W4(I,J)=W44+W4
W22=W2(I,J)
W33=W3(I,J)
W44=W4(I,J)
GO TO 620
610 TD2N=GN*(POIS*PHINS*ENP2-S-GNS)/(BN*(BNS-POIS*PHINS*ENP2))
IF (ETA.EQ.0.0) GO TO 612
IF (PSI.EQ.0.0) GO TO 612
B=1.0
TEST=BN-BN*PSI
IF (TEST.GT.60.0) B=0.0
A1=DEXP((BN*PSI-BN)*B)*B
A2=DEXP((GN*PSI-GN)*B)*B
XW2=EN[N]*{TD2N*AI+AI2)*DSIN(ENP2*ETA)/(TD2N*BN+GN)
B=1.0
TEST=BN*PSI
IF (TEST.GT.60.0) B=0.0
B1=DEXP(-(BN*PSI)*B)*B
B2=DEXP(-GN*PSI)*B)*B
XW2=EN[N]*{B1-B2)*DSIN(ENP2*ETA)/(BN-GN)
A2N=P*DSIN(ENP2*V2)*{(BNS-GNS)*BN*PHINS*PHINS
B2N=P*DSIN(ENP2*V2)*{(BNS-GNS)*BN*PHINS*PHINS
C2N=P*DSIN(ENP2*V2)*{(BNS-GNS)*BN*PHINS*PHINS
D2N=P*DSIN(ENP2*V2)*{(BNS-GNS)*BN*PHINS*PHINS
B=1.0
C=1.0
D=1.0
TEST1=BN*PSI-BN*U
TEST2=BN*U+BN*PSI
TEST3=BN*PSI-BN*U
IF (TEST1.GT.60.0) BB=0.0
IF (TEST2.GT.60.0) B=0.0
IF (TEST3.GT.60.0) D=0.0
IF (PSI.GT.0.0) GO TO 611
D1=DEXP((BN*PSI-1BN*U)*B)*B-DEXP((1-BN*U-BN*PSI)*C)*C+DEXP((BN*PSI
1*BN*U-2*BN)*D)*D)*D)*D)*D)
D2=DEXP((IGN*PSI-GN*U)*B)*B-DEXP((IGN*GNS)*C)*C+DEXP((IGN*PSI
1-GN*U-2*GN)*D)*D)*D)*D)
X4={A2N*D1*BN*V2)*DSIN(ENP2*ETA
GO TO 614
611 D1=DEXP((U*BN*PSI)*B)*B-DEXP((U-BN*PSI)*C)*C+DEXP((I-2
1*BN*BN*BN*PSI)*D)*D)*D)*D)
D2=DEXP((IGN*U-GN*PSI)*B)*B-DEXP((IGN*U-GN*PSI)*C)*C+DEXP((I-2
1*GN*GN*BN*PSI)*D)*D)*D)*D)
X4=C2N*D1*BN*V2)*DSIN(ENP2*ETA
GO TO 614
612 XW2=0.0
XW3=0.0
XW4=0.0
614 W22(I,J)=W22+W2
W31(I,J)=W33+W3
W44=W4(I,J)
W44=W4(I,J)
620 CONTINUE
5 DO 630 M=1,K/2
6 EMP2=M*PI/2
7 EMP2S=EMP2*EMP2
8 BMS=PHIS*(ALMDS+EMP2S)
9 BM=DSQRT(BMS)
10 X1=ALMDS-EMP2S
11 IF (X1.LT.0.0) GO TO 625
12 GMS=PHIS*X1
13 GM=DSQRT(GMS)
14 TD1M=GM*(GMS+EMP2S*POIS*PHIS)*DCOSIGNM/(BM*(BMS-EMP2S*POIS*PHIS)*DCOSIGNM)
15 TD11M=BM*TD1M*DCOSH(BM)+GM*DCOSIGNM
16 IF (ETA.EQ.0.0) GO TO 622
17 IF (PSI.EQ.0.0) GO TO 622
18 XW1=EM1*TD1M*DSINH(BM*ETA)+DSINH(GM*ETA)*DSIN(EMP2*PSI)/TD11M
GO TO 624
622 XW1=0.0
624 W1(W,J)=W11+W1
6 W1=W11(I,J)
GO TO 630
625 X1=X1
626 GM=DSQRT(GMS)
627 15 IF (BM.GT.22) GO TO 627
628 TD2M=GM*(POIS*PHIS*EMP2S-GMS)*(BM*POIS*PHIS*EMP2S)*DCOSH(BM)/1BM*(BMS-POIS*PHIS*EMP2S)*DCOSH(BM)
629 TD22M=TD2M*BM*DCOSH(BM)*GM*DCOSH(BM)
630 IF (ETA.EQ.0.0) GO TO 625
631 IF (PSI.EQ.0.0) GO TO 626
632 XW1=EM1*TD2M*DSINH(BM*ETA)+DSINH(GM*ETA)*DSIN(EMP2*PSI)/TD22M
GO TO 629
626 XW1=0.0
629 W11=1
630 CONTINUE
632 PRINT*11,J=$ (I,J)
633 FORMAT(17X,12,X'=I2,12,X'=I2,91=I,F7.0)
634 IF (PI.EQ.0) GO TO 632
637 10D12.4X,'(I2,91=I,F12.4)*',D12.4X,'(I2,91=I,F12.4)'=,D12.4X,'(I2,91=I,F12.4)
638 PRINT636,PSI,ETA,11(I,J),I,J

636 FORMAT('=',20X,'PSI =',F5.3,4X,'ETA =',F5.3,4X,'W(',I2,',',I2,') =')
632 CONTINUE
   PSI=PSI+1.0/DFLOAT(KS)
640 CONTINUE
   ETA=ETA+1.0/DFLOAT(KS)
650 CONTINUE
   RETURN
END