Base Station Placement
in Integrated Aerial and Terrestrial
Wireless Cellular Networks

by

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Abstract

Base station (BS) location is an important problem in designing cellular networks since its solution has a profound impact on the overall network performance. An optimal BS location depends on the traffic distribution, propagation pathloss and many system parameters, which renders its analytical study difficult so that numerical algorithms are widely used instead. In this thesis, the problem is studied analytically from a fundamental perspective. By formulating the problem as a convex optimization problem, we can characterize the globally-optimum location and obtain a number of closed-form solutions and insights.

Afterwards, we consider drones as aerial BSs that can enhance network coverage or capacity by moving supply towards demand when required. Utilizing drone-BSs (DBSs) is a promising approach to boost the agility and flexibility of future wireless networks. The specific attributes of drones such as mobility could be especially useful for future applications with extreme demands. DBSs can play a remarkable role as a communication network facilitator when the temporal and spatial variations in user densities and rates are expected to result in difficult-to-predict traffic patterns. However, deploying DBSs in a network presents several challenges. One important issue is finding the efficient 3D placement of DBSs to satisfy the dynamic requirements of the system. Another challenge is the limited wireless backhaul capacity of DBSs and consequently, higher latency incurred. This issue can be alleviated by providing content caching in DBSs to decrease backhaul congestion and latency. In this thesis, we find locations of DBSs for various network objectives such as minimizing the number of DBSs or the transmit power, maximizing the number of covered users or total rate of the users in a wireless network, while considering the restrictions of the network such as finite backhaul capacity, disparate quality of service requirement of users, and limited access bandwidth into account. To this end, we propose various mathematical frameworks and efficient algorithms to design, optimize, and deploy drone-based communication systems. We also obtain user-BS associations and bandwidth allocations in different scenarios which is an involved problem due to mobility of DBSs. Extensive simulation results demonstrate the effectiveness of the proposed algorithms and provide useful insights that can be used to develop design guidelines.
To

my beloved parents, Nahid and Mohammad Rasoul,

my lovely husband, Ehsan,

and my sweet little daughter, Elsa.
Acknowledgements

First and foremost, I would like to express my sincere appreciation to my supervisors, Professor Abbas Yongacoglu and Professor Halim Yanikomeroglu, for their unrelenting support, invaluable guidance, and constructive suggestions throughout my studies. It was a great privilege and honor to work and study under their guidance. I also wish to show my gratitude to Dr. Maher Arar who made this journey possible in the first place. My grateful thanks are also extended to all the members of our wonderful research group for their cooperation, support, and encouragement during the course of this research.

A heartfelt gratitude and love goes to my parents, Nahid and MohammadRasoul, for providing me with abundant support and continuous encouragement throughout my years of study. I am also very grateful to my sister, Reyhane, who always stood by me with smile and cheered me up.

Last, but not least, I wish to express my infinite appreciation and love to my husband, Ehsan. Without his endless patience and unfailing support, completing this thesis would not have been possible.
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<th>Meaning</th>
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<tr>
<td>3D</td>
<td>Three Dimensional</td>
</tr>
<tr>
<td>1G</td>
<td>First Generation of Wireless Telecommunication Technologies</td>
</tr>
<tr>
<td>2G</td>
<td>Second Generation of Wireless Telecommunication Technologies</td>
</tr>
<tr>
<td>3G</td>
<td>Third Generation of Wireless Telecommunication Technologies</td>
</tr>
<tr>
<td>4G</td>
<td>Fourth Generation of Wireless Telecommunication Technologies</td>
</tr>
<tr>
<td>5G</td>
<td>Fifth Generation of Wireless Telecommunication Technologies</td>
</tr>
<tr>
<td>3GPP</td>
<td>The 3rd Generation Partnership Project</td>
</tr>
<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
</tr>
<tr>
<td>B&amp;B</td>
<td>Branch and Bound</td>
</tr>
<tr>
<td>BILP</td>
<td>Binary Integer Linear Programming</td>
</tr>
<tr>
<td>BS</td>
<td>Base Station</td>
</tr>
<tr>
<td>CapEx</td>
<td>Capital Expense</td>
</tr>
<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
</tr>
<tr>
<td>CoV</td>
<td>Coefficient of Variation</td>
</tr>
<tr>
<td>D2D</td>
<td>Device to Device</td>
</tr>
<tr>
<td>DBS</td>
<td>Drone Base Station</td>
</tr>
<tr>
<td>eMBB</td>
<td>Enhanced Mobile Broadband</td>
</tr>
<tr>
<td>FDMA</td>
<td>Frequency-Division Multiple Access</td>
</tr>
<tr>
<td>FSO</td>
<td>Free Space Optics</td>
</tr>
<tr>
<td>FSPL</td>
<td>Free Space Path Loss</td>
</tr>
<tr>
<td>Gbps</td>
<td>Gigabit per Second</td>
</tr>
<tr>
<td>GPRS</td>
<td>General Packet Radio Service</td>
</tr>
<tr>
<td>GSM</td>
<td>Global System for Mobile Communication</td>
</tr>
<tr>
<td>HetNet</td>
<td>Heterogeneous Network</td>
</tr>
<tr>
<td>iid</td>
<td>Independent and Identically Distributed</td>
</tr>
<tr>
<td>Acronym</td>
<td>Description</td>
</tr>
<tr>
<td>---------</td>
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</tr>
<tr>
<td>InP</td>
<td>Infrastructure Provider</td>
</tr>
<tr>
<td>IoT</td>
<td>Internet of Things</td>
</tr>
<tr>
<td>Kbps</td>
<td>Kilobits per Second</td>
</tr>
<tr>
<td>KKT</td>
<td>Karush-Kuhn-Tucker</td>
</tr>
<tr>
<td>LDPC</td>
<td>Low-Density Parity-Check</td>
</tr>
<tr>
<td>LoS</td>
<td>Line of Sight</td>
</tr>
<tr>
<td>Mbps</td>
<td>Megabits per Second</td>
</tr>
<tr>
<td>MBS</td>
<td>Macro Base Station</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multiple-Input Multiple-Output</td>
</tr>
<tr>
<td>MMS</td>
<td>Multimedia Messaging Service</td>
</tr>
<tr>
<td>mMTC</td>
<td>Massive Machine-Type Communication</td>
</tr>
<tr>
<td>NLoS</td>
<td>Non Line of Sight</td>
</tr>
<tr>
<td>NFV</td>
<td>Network Function Virtualization</td>
</tr>
<tr>
<td>OpEx</td>
<td>Operational Expense</td>
</tr>
<tr>
<td>PSD</td>
<td>Positive Semidefinite</td>
</tr>
<tr>
<td>PSO</td>
<td>Particle Swarm Optimization</td>
</tr>
<tr>
<td>QCQP</td>
<td>Quadratic Constraints Quadratic Programming</td>
</tr>
<tr>
<td>QoE</td>
<td>Quality of Experience</td>
</tr>
<tr>
<td>QoS</td>
<td>Quality of Service</td>
</tr>
<tr>
<td>RF</td>
<td>Radio Frequency</td>
</tr>
<tr>
<td>SDN</td>
<td>Software-Defined Network</td>
</tr>
<tr>
<td>SDP</td>
<td>Semi-Definite Programming</td>
</tr>
<tr>
<td>SDR</td>
<td>Semi-Definite Relaxation</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Interference-plus-Noise Ratio</td>
</tr>
<tr>
<td>SIR</td>
<td>Signal-to-Interference Ratio</td>
</tr>
<tr>
<td>SMS</td>
<td>Short Messaging Service</td>
</tr>
<tr>
<td>SNR</td>
<td>Signal-to-Noise Ratio</td>
</tr>
<tr>
<td>SON</td>
<td>Self Organization Network</td>
</tr>
<tr>
<td>SP</td>
<td>Service Provider</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
</tr>
<tr>
<td>uLLRC</td>
<td>Ultra Reliable Low Latency Communication</td>
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<table>
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<th>Notation</th>
<th>Meaning</th>
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<tr>
<td>$h$</td>
<td>Altitude of a DBS</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Elevation angle between a DBS and a user</td>
</tr>
<tr>
<td>$d$</td>
<td>Distance between a BS and a user</td>
</tr>
<tr>
<td>$v$</td>
<td>Horizontal distance from a DBS to a user</td>
</tr>
<tr>
<td>$PL$</td>
<td>Pathloss</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Excessive pathloss due to LoS or NLoS</td>
</tr>
<tr>
<td>$f_c$</td>
<td>Carrier frequency</td>
</tr>
<tr>
<td>$c$</td>
<td>Speed of light</td>
</tr>
<tr>
<td>$P(\text{LoS})$</td>
<td>Probability of LoS</td>
</tr>
<tr>
<td>$P(\text{NLoS})$</td>
<td>Probability of NLoS</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Pathloss exponent</td>
</tr>
<tr>
<td>$N$ or $I$</td>
<td>Number of Users</td>
</tr>
<tr>
<td>$R$ or $r$</td>
<td>Target rate of a user</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>SNR/SINR of a user receiving service from a BS</td>
</tr>
<tr>
<td>$P$</td>
<td>Transmit power of a BS</td>
</tr>
<tr>
<td>$\sigma_0^2$</td>
<td>Noise power</td>
</tr>
<tr>
<td>$N_{U_{BS}}$</td>
<td>The maximum number of users that a DBS can serve</td>
</tr>
<tr>
<td>$C_{BS}$</td>
<td>Capacity of a DBS</td>
</tr>
<tr>
<td>$B$</td>
<td>Total bandwidth of a BS</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Spectral efficiency</td>
</tr>
<tr>
<td>$N_{BS}$</td>
<td>Estimated number of DBSs</td>
</tr>
<tr>
<td>$\gamma_{th}$</td>
<td>Minimum SINR level required for a user</td>
</tr>
<tr>
<td>$m$</td>
<td>How much the area of a DBS is inside a subarea</td>
</tr>
<tr>
<td>$a$</td>
<td>Mutual area between a DBS and a subarea</td>
</tr>
<tr>
<td>$A$</td>
<td>Total area of a DBS</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$D$</td>
<td>User density function of a subarea</td>
</tr>
<tr>
<td>$S$</td>
<td>Total area of a subarea</td>
</tr>
<tr>
<td>$\epsilon$</td>
<td>An indicator function that shows if a DBS should be used or is redundant</td>
</tr>
<tr>
<td>$\rho$</td>
<td>An indicator function that shows association between a user and a BS</td>
</tr>
<tr>
<td>$C$</td>
<td>Wireless backhaul capacity of a DBS</td>
</tr>
<tr>
<td>$b$</td>
<td>Required bandwidth by a user</td>
</tr>
<tr>
<td>$\mathcal{I}$</td>
<td>Set of users</td>
</tr>
<tr>
<td>$\mathcal{J}$</td>
<td>Set of BSs</td>
</tr>
<tr>
<td>$i$</td>
<td>Index of users</td>
</tr>
<tr>
<td>$j$</td>
<td>Index of BSs</td>
</tr>
<tr>
<td>$J$</td>
<td>Total number of BSs</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Normalized bandwidth resource</td>
</tr>
<tr>
<td>$\tau$</td>
<td>An indicator that shows whether a user is delay-tolerant or delay-sensitive</td>
</tr>
<tr>
<td>$l$</td>
<td>3D location of a DBS</td>
</tr>
<tr>
<td>$G$</td>
<td>Antenna gain</td>
</tr>
<tr>
<td>$G_0$</td>
<td>Maximum gain of a directional antenna</td>
</tr>
<tr>
<td>$\theta_B$</td>
<td>Directional antenna’s half-power beamwidth</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Percentage of bandwidth allocation between access and backhaul</td>
</tr>
<tr>
<td>$g$</td>
<td>Channel gain</td>
</tr>
<tr>
<td>$N_0$</td>
<td>Noise power spectral density</td>
</tr>
<tr>
<td>$K$</td>
<td>Total files in the network</td>
</tr>
<tr>
<td>$E$</td>
<td>Caching matrix</td>
</tr>
<tr>
<td>$U$</td>
<td>Request matrix</td>
</tr>
<tr>
<td>$F$</td>
<td>Cache association matrix</td>
</tr>
<tr>
<td>$W$</td>
<td>Total available bandwidth for backhauling</td>
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Chapter 1

Introduction

1.1 Motivation

Wireless networks have evolved over the past 30 years to meet the growing demand of wireless users, from 1G to 5G and beyond. The classification of generations is generally due to a fundamental change in the nature of the service, advancements in transmission technology, providing higher peak bit rates, new frequency bands, wider channel frequency bandwidth, and higher capacity for many simultaneous data transfers. A new generation of cellular technology has appeared approximately every ten years since the first generation (1G) systems introduced in 1981. 1G systems used analog signals with a maximum speed of 2.4 kilobits per second (kbps). The second generation of wireless technologies (2G), was launched in Finland in 1991. 2G systems such as the global system for mobile communications (GSM) used digital signals and the maximum data speed they could support was 64 kbps. In this generation, data services for mobile communications (short message service (SMS) and multimedia messaging service (MMS)) were introduced. Afterwards, 2.5G systems that implemented a packet-switched domain in addition to the circuit-switched domain were introduced. In that generation, the speed increased up to 144 kbps and email messages could be sent and received. The third generation (3G) of wireless technologies was launched in 2001. In 3G systems, data transmission increased to 2 megabits per second (Mbps). Using that technology, the wireless mobile network was transformed from a pure
telephony system to a network that could transmit rich multimedia contents. The fourth generation of wireless telecommunication technologies (4G) started from the late 2000s. 4G networks can support data rates of up to 1 gigabit per second (Gbps) for nomadic and low mobility communication and 100 Mbps for high mobility communication [1–4].

Wireless traffic volume is predicted to increase about 1000 times in the next decade. To support this amount of traffic, the fifth generation of wireless telecommunication technologies (5G) are also expected to achieve a 1000-fold increase in capacity. With such explosive growth of traffic demands and due to high costs and scarcity of radio spectrum resources, cell planning is of utmost importance for service providers. It includes determining optimum locations and the number of base stations (BSs) to meet the traffic and quality of service (QoS) demands at the minimum cost [5].

Such a huge increment in the capacity that is expected in 5G networks and beyond will also allow higher device densities per area unit and a large portion of the population are able to consume high-quality streaming media many hours per day in their mobile devices using wireless cellular technology. 5G research and development also aims at improved support of machine to machine communication, also known as the Internet of things (IoT). Therefore, the other challenging requirements of 5G systems are very low latency, high traffic density, high energy efficiency, and high reliability and availability [6–8].

The brute-force way to provide ubiquitous high-rate coverage is very well known: deploy a very dense network of BSs. However, this solution is not feasible in terms of capital expenses (CapEx) and operational expenses (OpEx), due to the fact that a high percentage of these BSs will be lightly loaded or even will not have any load at a given time and space. Moreover, the temporal and spatial variations in user densities and user application rates are expected to result in difficult-to-predict traffic patterns. The utilization of drone base stations (DBSs) is a promising solution in such scenarios.

DBSs can assist a ground network of BSs to provide high data rate service whenever there is an excessive need in space and time, especially in situations when this demand is difficult to be predicted. Due to the fast deployment of DBSs, they can also address temporary coverage issues in remote areas, or when terrestrial wireless infrastructure is
damaged due to a natural disaster such as earthquakes or floods. Although the OpEx of a DBS may be more than that of a ground small-cell BS, if engineered properly, DBSs may result in substantial savings in the overall network deployment costs.

Figure 1.1 is an illustrative diagram representing some use cases of DBSs in beyond 5G networks. As depicted in this figure, DBSs can assist ground BSs to temporary inject capacity and alleviate congestion in certain spots such as stadiums. They can also provide coverage in remote areas or when the ground infrastructure is broken due to inclement weather conditions, vandalism, transmission problems or during the aftermath of natural disasters. It is worth noting that DBSs are not suggested to replace the terrestrial ones, rather they are proposed as add-ons to the system when needed. By moving forward, it is expected to have a more agile and flexible access network. One important approach in this regard which has recently attracted a lot of attention in literature, and currently is a major discussion item in 3GPP working groups for 5G networks and beyond, is utilizing aerial networks [9].

Drones are part of the broader class of aerial vehicles called unmanned aerial vehicles (UAVs). Currently, UAVs are remotely controlled by human operators at ground control
stations. This approach is not scalable, as the complexity dramatically increases by the number of UAVs and the possibility of human error rises [10]. Therefore, there is a need for exploiting autonomous UAVs. Autonomous DBSs, which are able to accomplish missions relying on on-board computers without real-time human control, will not be part of 5G networks, but it is envisioned to utilize them in wireless networks beyond 5G systems.

The coordination between UAVs based on current methods can be done either in a centralized or distributed fashion. In a centralized coordination approach, a single authority is used to coordinate the activities in all devices. This approach may achieve high levels of consistency. However, there are some fragility concerns and in addition, all the nodes should communicate with the coordinator which might be distant, resulting in high communication overhead. On the other hand, in a distributed coordination manner, each device solely decides about its own actions [11], so this approach is more resilient to failure, but also remarkably more difficult to be developed. This approach may also not work well in applications requiring real-time coordination. Consequently, both traditional centralized and distributed coordination models may not be suitable for the UAV coordination in wireless cellular networks; therefore, a novel approach is required [10].

According to U.S. Federal aviation regulations, the maximum allowable altitude of drones that can freely fly without any permit is 400 feet. In general, it is easy and fast to deploy and integrate drones with terrestrial networks and they also have low implementation costs. Such specifications make them suitable for unexpected or limited-duration missions. Moreover, having a low altitude combines both coverage superiority and limited cell radius [12]. In addition, they probably have better communication channels thanks to the presence of line of sight (LoS) links. Due to their mobility, new opportunities for performance improvement are offered through the dynamic adjustment of drone state to best fit the communication environment. Moreover, using adaptive communication jointly with mobility adjustment can further improve the performance of the network [13]. However, the utilization of highly mobile and energy-constrained drones in wireless communication systems also introduces many new challenges.

Exploiting DBSs in wireless cellular networks needs close collaboration between regu-
tors, network and technology providers. From the regulator’s side, safety is a foremost issue that should be addressed. Performance standards, registration and licensing programs are other important concerns that need to be investigated. In the network provider’s side, optimizing existing networks in terms of providing network coverage, determining performance metrics and managing coexistence with a terrestrial network of BSs are the main issues. Interference management caused by other DBSs and a high number of neighbour ground BSs is necessary. Handover optimization is another major issue in DBSs deployment. From the technology provider’s side, developing on-board intelligence and providing safe and autonomous operations are important. Moreover enabling reliable communication is very crucial especially when critical data needs to be handled. Using redundancy and replicating data and partitioning it into grades of required reliability might be useful. By using drones in civilian services and applications, large volumes of streaming data will be expected; therefore, it is needed to deal with such data by labelling, processing and analysis [14].

1.2 Objective and Scope

In this thesis, we first study optimal BS location at a fundamental level and then consider drones as flying BSs that can provide temporary coverage or inject capacity in a wireless network.

An optimal BS location depends on the traffic (user) distribution, propagation pathloss and many system parameters, which renders its analytical study difficult so that numerical algorithms are widely used instead. In this thesis, the problem is studied analytically. First, it is formulated as a convex optimization problem to minimize the total BS transmit power subject to QoS constraints, which also account for fairness among users. Due to its convex nature, Karush-Kuhn-Tucker (KKT) conditions are used to characterize a globally-optimum location as a convex combination of user locations, where convex weights depend on user parameters, pathloss exponent and overall geometry of the problem. Based on this characterization, a number of closed-form solutions are obtained.
Afterwards, we consider DBSs and address some of the technical challenges related to utilizing them in wireless networks. One of the most fundamental issues in drone-based wireless communications is three-dimensional (3D) deployment of DBSs. Another major difference between a ground BS and a DBS is that the latter one has a major limitation in the backhaul link. In order to address these issues, in this thesis, we jointly find the 3D placement of DBSs, the user-BS associations, and their corresponding bandwidth allocations considering the network objectives and constraints of the system.

As the system model slightly changes from chapter to chapter and for better understanding and flow of the thesis, we include a system model section at the beginning of each chapter; however, some of the assumptions are repetitive in different chapters.

1.3 Thesis Contributions

In this thesis, we develop a fundamental framework to address the BS location problem in wireless cellular networks and then consider drones as a special form of aerial BSs and investigate the optimization of drone-enabled wireless networks. In particular, we propose frameworks for efficient deployment of DBSs and optimizing the performance of the system in terms of minimizing the number of DBSs or maximizing the capacity or energy efficiency while taking into account the distinctive features of the DBSs such as their flexible altitude and limited backhaul capacity. The major contributions of this thesis are summarized as follows:

- We analytically study the optimum BS location and the impact of pathloss exponent and traffic distribution on the optimum location. To this end, we model the problem as a convex optimization problem and obtain a number of closed-form solutions and insights they facilitate.

- We consider drones as flying BSs that can provide temporary coverage or inject capacity to the network and find the efficient 3D deployment of DBSs for various network design parameters while taking user requirements and system limitations into account.
• One of the main differences in 5G networks and beyond is the capacity to provide seamless service for different applications with diverse demands. Accordingly, we assume that delay-sensitive users co-exist with regular delay-tolerant users and take into account a heterogeneous network (HetNet) including both ground and aerial BSs in the system. The ground BS can provide service for the users and is also considered as the backhaul source for DBSs.

• The major difference between a ground BS and a DBS is that the latter one has a major limitation in the backhaul link. A ground BS usually has a fixed wired/wireless backhaul connection and can relatively offer very high data rates to a core network. A DBS on the other hand should have a wireless backhaul; therefore, the peak data rate it can support is limited and it may dramatically decrease due to inclement weather conditions. We consider a limited variable backhaul capacity in DBSs and propose content caching to alleviate network congestion and decrease latency.

1.4 Publications

The following is a list of publications generated during the course of the Ph.D. Program.


### 1.4.1 Papers under preparation


### 1.5 Organization of the Thesis

The rest of this thesis is organized as follows.

Chapter 2 provides a literature review of the problem considered herein.

In Chapter 3, the problem of optimum BS location is studied at the fundamental level. The problem is modelled as a convex optimization problem to minimize the total BS
transmit power, subject to the QoS constraints, and a number of closed-form solutions and useful insights are obtained.

Chapter 4, finds the minimum number of DBSs and their 3D placement through a heuristic algorithm to cover a set of high data rate users distributed with different densities in an area.

Chapter 5, introduces wireless backhaul as one of the limiting factors in utilizing DBSs and proposes a backhaul-limited optimal DBS placement algorithm for various network design parameters and heterogeneous rate requirements in a clustered user distribution. It also investigates the robustness of DBS placement and studies how much the users’ movements may affect the proposed optimal solution.

In Chapter 6, a number of DBSs are connected to a ground BS for backhauling, and 3D placement of DBSs, user-BS associations and their corresponding bandwidth allocations are jointly optimized in order to maximize the total rate of the users in a proportional fairness approach.

In Chapter 7, a HetNet including both ground and backhaul-limited DBSs is considered and caching is proposed to alleviate backhaul congestion in DBSs and decrease latency. Then a framework is developed to jointly optimize the 3D placement of DBSs, the association of users with BSs, and their corresponding bandwidth allocations while minimizing the total transmit power of the DBSs.

Finally, a summary of this work, along with proposed future research directions are presented in Chapter 8.
Chapter 2

Background and Related Works

To put our work into perspective, in this chapter we provide a brief overview of the existing literature about cell-planning and BS placement and also utilizing drones as aerial BSs in wireless networks.

2.1 BS Placement

The problem of BS location in wireless cellular networks has been extensively studied in the existing literature [15–24]. A large number of optimization algorithms have been proposed to attack this problem numerically, taking into account a number of practically-important parameters and limitations. Many of the proposed algorithms use a pre-selected finite list of candidate sites where the BS could potentially be located and look for ones that optimize some objective function amongst that list [15, 16]. The considered problems are formulated as mixed integer programming or combinatorial optimization and the methods to solve them include simulated annealing [17], Tabu search [15, 18, 19], simplex method, and branch and bound (B&B) algorithm [20], etc. While these approaches can be useful in practice, their common feature is that the considered problems are NP-hard (i.e. the numerical complexity grows very fast with the problem size), and convergence of algorithms to a global optimum cannot be guaranteed, due to the lack of convexity of the underlying optimization problems. Furthermore, the sub-optimality gap is also unknown. Due to the
nature of numerical algorithms, they offer limited insight into the problem, for which no analytical solution is known either.

A different approach is adopted in [21, 22], where an optimal BS location is searched over the whole area of interest (without assuming a finite number of candidate locations). Additionally, after finding sub-optimal BS locations, the number of BSs is minimized by removing redundant BSs so that the quality of network service is not affected. Numerical algorithms are proposed for this two-stage optimization process. A pattern search algorithm is used in [21] to minimize the total power consumption of the network by properly locating BSs subject to the signal to interference ratio (SIR) constraints. It is based on the mesh-adaptive direct search extended to include non-linear inequalities via the augmented Lagrangian. While the algorithm converges to a KKT point, this is not sufficient for global optimality since the underlying optimization problem is not convex (so that a KKT point can be a local rather than global minimum, an inflection point, or even a maximum rather than minimum). A combinatorial optimization problem is formulated in [22] to find BS locations that satisfy area coverage and cell capacity constraints. Two heuristic numerical algorithms, namely particle swarm optimization (PSO) and gray-wolf optimization, are used and afterward the redundant BSs are eliminated to achieve the minimum required number of BSs. Although these algorithms can be useful in practice, their convergence to a globally-optimal solution is not guaranteed and their numerical complexity can be very large for a large problem size.

Yet another approach is adopted in [23, 24], where the weighted sum pathloss (to all users) is minimized by properly locating a BS. Various numerical algorithms for local optimization are used, such as Hooke-Jeeves’, quasi-Newton, conjugate gradient search, steepest descent, simplex or Rosenbrock methods, simulated annealing, and genetic algorithm. The entire area of interest is partitioned into a finite grid, which is sequentially refined while looking for an optimal location. None of these methods guarantee a globally-optimal solution due to their intrinsic limitations or due to the non-convexity of underlying optimization problem. In addition, the cost function (the weighted sum pathloss) is introduced in an ad-hoc manner, without any link to system-level performance indicators (e.g.
total transmit power or energy efficiency subject to QoS constraints).

Recently, there is a growing interest in utilizing UAVs as flying BSs to increase capacity or cover some areas temporarily. There are a number of papers that investigate UAVs placement in the wireless network that will be discussed in Section 2.2.3.

2.2 Drones in Wireless Networks

2.2.1 Different use cases of drones

Although military and governmental organizations have traditionally exploited drones to conduct mission-critical inspection and monitoring operations, they are now getting attention for civilian applications [25]. There are mainly 3 different categories of use cases of drones enabled by wireless communication that could be further explored.

- **Drone as a User:** Drones may be considered as airborne users, using wireless technology for control, command and flying. Numerous new applications in the civilian and commercial domains have emerged including weather monitoring, forest fire detection, traffic control, cargo transport, emergency services, search and rescue mission, surveillance and videography, delivery, agricultural visual inspection, automated planting, infrastructure inspection like bridges and cell towers, inspection of hard to reach assets like oil, gas, wind turbines and so on [14, 26–28]. Many of these applications require a highly reliable connection with low latency, ultra high availability, and seamless mobility. Therefore, a network needs to distinguish a drone-user from a ground-user to optimize services for the users. Moreover, the drones have likely a LoS channel with ground BSs; hence, the effect of such users on the ground network especially in uplink mode should be carefully studied to ensure their compatibility with the existing infrastructure, because the high interference they may cause can severely affect the quality of the network [29].

- **Drone as a relay:** Drones can assist wireless networks as relays. Relaying is a well known technique in terrestrial communication systems to improve throughput
and/or reliability as well as extending the communication range. Due to the practical
constraints such as limited mobility and wired backhauls, most terrestrial relays are
deployed in fixed locations. Using drones as mobile relays is a great idea particularly
for delay-tolerant applications. A drone-relay can fly continuously between the source
and destination to reduce the link distances during both information reception and
relaying phases [13, 29]. The collected data can be relayed to a ground station by
equipping the drone with a cellular radio. If real time data is required, a group of
drones can communicate with each other in multi-hop fashion to decrease transferring
data time from source to destination. One of the major issues in aerial relays is the
dynamic change in the communication links due to continuous mobility. Moreover,
differences in height of the sender and the receiver antennas makes the aerial relaying
distinct from the traditional mobile networks. It is therefore necessary to investigate
and derive wireless communication properties in aerial wireless relay networks to
design robust protocols [30].

- **Drone as a BS:** Drones can also be equipped with BSs. Such aerial BSs can assist a
ground network of BSs to decrease congestion in the network or increase coverage area
whenever and wherever it is required by moving supply towards demand. Compared
to terrestrial BSs, the DBSs benefit from potential mobility and adjustable altitude to
enhance the likelihood of establishing an LoS connection with the users. Throughout
this thesis we consider drones as aerial BSs.

### 2.2.2 Air to ground channel model

A successful design of an aerial communication system requires a complete understanding
of multipath propagation between aerial platforms and ground users. In terrestrial chan-
nels, radio frequency (RF) signal's amplitude reduces as a function of the traveled distance
and the pathloss is usually modelled by a log-distance relation and a pathloss exponent.
It is observed that air to ground (ATG) radio signal propagation greatly differs from the
terrestrial case. The radio signal transmitted from an aerial platform propagates through
free space until reaching the urban environment where it incurs shadowing, blocking, scat-
tering, and other effects caused by buildings and structures. It is found that the ATG pathloss depends on the height of the airborne transceiver and the elevation angle between the aerial platform and the ground user, given by $\theta$, as demonstrated in Figure 2.1 [31].

There are a limited number of studies related to characterizing the ATG propagation model. The authors in [32] and [33] present an overview of the existing models. In [34], a statistical propagation model between an airborne transmitter and a ground receiver in a dense urban environment is developed. Pathloss and shadowing are considered as a function of the elevation angle and represented from the simulated propagation data extracted from a 3D outdoor ray-tracing model. The pathloss is modelled based on three classes of links, LoS, obstructed LoS and non-LoS (NLoS) and the model is derived to operate at frequencies between 200 MHz and 5 GHz. This study is based on a single model city; therefore, it cannot be generalized for different kinds of urban environments. Another approach is adopted in [35], where a theoretical model is proposed to determine the likelihood of an LoS link for ATG channels based on local building geometry and knife-edge diffraction theory.

One of the most widely used pathloss models for ATG communications is presented in [36] and we adopt it throughout this thesis. In this paper, the authors focus on the lower part of the stratosphere, i.e. heights between 200 and 3000 meters and assume quasi-stationary aerial platforms, so Doppler effect which happens due to the high velocity is ignored in their analysis. Moreover, transmitters and receivers are supposed to be isotropic. They conduct Ray tracing simulation for three different frequencies (700, 2000, and 5800
MHz) which cover a wide range of applications. Based on the simulation results, three propagation groups are observed, corresponds to receivers with LoS or near LoS connections, receivers with NLoS connection that still receive the signal via strong reflection and diffraction, and receivers suffering deep fading due to successive reflections and diffractions. The effect resulting from the last group is disregarded as it includes only a small percentage of the total samples set.

The total power reduction of a signal transmitted from the aerial platform to the ground receiver can be written in decibel form as

\[ PL \ (dB) = FSPL + \psi_i, \]  \tag{2.1}  

where \( FSPL \) is the free space pathloss according to Friis equation and it is equal to

\[ FSPL = 20 \log\left(\frac{4\pi f_c d}{c}\right), \]  \tag{2.2}  

where variable \( f_c \) is the carrier frequency, \( c \) stands for the speed of light, \( d \) stands for the distance between an aerial platform and a user and it is equal to \( \sqrt{h^2 + v^2} \), where \( h \) and \( v \) are the altitude of the aerial platform and its horizontal distance from the ground user, respectively. Also, variable \( \psi_i, i = \{\text{LoS, NLoS}\} \) shows the excessive pathloss due to LoS or NLoS channel between the transmitter and receiver and can be modelled by a Gaussian distribution as \( \psi_i = N(\mu_i, \sigma_i^2) \), where \( \mu_i \) is the mean excessive pathloss shown by a constant value depending only on the environment and \( \sigma_i = k_i \exp(-l_i \cdot \theta) \), where \( k_i \) and \( l_i \) are frequency and environment dependent parameters, so the variance, \( \sigma_i \), depends on the elevation angle and type of the environment.

The probability of having an LoS connection between an aerial transmitter and a ground user depends on the elevation angle and urban statistical parameters and it can be formulated as [12]

\[ P(\text{LoS}) = \frac{1}{1 + \kappa \exp(-\omega(\theta - \kappa))}, \]  \tag{2.3}  

where \( \kappa \) and \( \omega \) are constant values depending on the environment and \( \theta \) is the elevation angle in degree and it is equal to \( \frac{180}{\pi} \arctan(\frac{2}{v}) \). Table 2.1 provides ATG channel model
Table 2.1. Urban environment parameters for $f_c = 2$ GHz

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Sub Urban</th>
<th>Urban</th>
<th>Dense Urban</th>
<th>Highrise Urban</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mu_{\text{LoS}}$</td>
<td>0.1</td>
<td>1</td>
<td>1.6</td>
<td>2.3</td>
</tr>
<tr>
<td>$\mu_{\text{NLoS}}$</td>
<td>21</td>
<td>20</td>
<td>23</td>
<td>34</td>
</tr>
<tr>
<td>$(k_{\text{LoS}}, l_{\text{LoS}})$</td>
<td>(11.25, 0.06)</td>
<td>(10.39, 0.05)</td>
<td>(8.96, 0.04)</td>
<td>(7.37, 0.03)</td>
</tr>
<tr>
<td>$(k_{\text{NLoS}}, l_{\text{NLoS}})$</td>
<td>(32.17, 0.03)</td>
<td>(29.6, 0.03)</td>
<td>(35.97, 0.04)</td>
<td>(37.08, 0.03)</td>
</tr>
<tr>
<td>$(\kappa, \omega)$</td>
<td>(4.88, 0.43)</td>
<td>(9.61, 0.16)</td>
<td>(12.08, 0.11)</td>
<td>(27.23, 0.08)</td>
</tr>
</tbody>
</table>

Figure 2.2. Probability of LoS versus elevation angle in different types of environments.

Parameters for $f_c = 2$ GHz in different types of environments.

Fig. 2.2 shows that by increasing the elevation angle between an aerial platform and a ground user, the probability of establishing an LoS connection is increased in different types of environments. When the location of a ground user is fixed, increasing the elevation angle can be obtained by increasing the altitude of an aerial platform.

Without considering the random behaviors of the radio channel, the average pathloss can be presented in a probabilistic manner as [12]

$$\text{PL}(\text{dB}) = 20 \log\left(\frac{4\pi f_c d}{c}\right) + P(\text{LoS}) \cdot \mu_{\text{LoS}} + P(\text{NLoS}) \cdot \mu_{\text{NLoS}},$$  \hspace{1cm} (2.4)
where \( P(\text{NLoS}) = 1 - P(\text{LoS}) \). The ATG pathloss versus altitude in different types of environments with \( f_c = 2 \) GHz for \( v = 200 \) meters is shown in Fig. 2.3. As seen in this figure, by increasing the altitude of a DBS, the pathloss first decreases and then increases. That is due to the fact that in low altitudes the probability of NLoS connection is much higher than that of LoS, because of reflections by buildings and other objects, and the additional loss of an NLoS connection is higher than an LoS connection; but when the altitude increases the LoS probability increases as well and in turn pathloss decreases. On the other hand, pathloss also depends on the distance between a transmitter and a receiver, so after a specific height this factor dominates and by increasing the altitude, the pathloss increases, too. These results show that, the DBSs can be considered as a new tier of access nodes in wireless communication systems. Instead of changing the transmit power, which makes different coverage areas, the desired coverage area can be attainable by changing the height of a DBS.

2.2.3 Drone BSs

There has been a growing interest in utilizing drones as aerial BSs over the past few years [37–39]. One of the fundamental issues in deploying DBSs is finding their 3D placements.
Several papers have investigated the problem of the 3D placement of DBSs in wireless networks. In [12], an analytical approach to find an optimal altitude of an aerial platform in order to maximize the coverage is presented. The authors find the altitude of a UAV as a function of maximum allowed pathloss and statistical parameters of the environment. It is a 1D placement problem similar to [40] in which another study about the optimal coverage area of DBSs with and without interference is presented. First the optimal altitude which leads to maximum ground coverage and minimum required transmit power for a single DBS and two DBSs is derived and then the impact of the distance between drones on the coverage area is studied and the optimal distance that maximizes the coverage area is derived.

Some papers have focused on finding 3D placement of a single DBS in a wireless network [41–44]. In [41], the authors suggest to use a DBS to maximize the revenue of the network. They formulate a 3D placement problem to maximize the covered users in an area and find the DBS location through numerical methods. They extend their work in [42] by considering the mobility of users as an additional degree of freedom. Then jointly optimize the 3D location of a DBS and the incentives that are offered to each user in order to move to a place with a better coverage. In [43], an algorithm to find the location of a DBS that maximizes the number of covered users is proposed and then extended to different QoS requirements in [44]. In [43], the the problem is decoupled into vertical and horizontal dimensions. For the vertical dimension, the optimum angle that maximizes the coverage radius is found, followed by the optimum height; in the horizontal dimension, the deployment is modelled as a circle placement problem. In the case of different QoS requirements in [44], the problem is formulated as a mixed integer problem, and the approximated optimum height is found numerically via exhaustive search. Afterwards, a multiple circle placement problem is considered and solved numerically by the branch and cut method.

To model more realistic scenarios, other papers have considered wireless networks with more than one DBS or with the integration of terrestrial BSs. In [45], the authors minimize the number of DBSs that cover a set of users by proposing a successive BS placement, where the DBSs are placed sequentially starting on the area perimeter of the uncovered
users along a spiral path towards the center, until all users are covered. They consider fixed altitude for all the DBSs and LoS channels between DBSs and users. They also assume no interference between DBSs. In [46], multiple DBSs with directional antennas provide coverage for ground users. Firstly, the downlink coverage probability as a function of the altitude and the antenna gain is found and then using circle packing theory, the locations of the DBSs are determined to maximize the total coverage area. Furthermore, the minimum number of DBSs needed to guarantee a target coverage probability for a given area is determined. In doing so, it is assumed that all the DBSs have similar altitudes and coverage is the only constraint for receiving service, while capacity can be an important issue especially in dense environments. A user might be under the coverage of a BS without getting service from it due to lack of bandwidth resource and congestion in the network. In [47], optimal deployment of multiple DBSs considering downlink scenario is investigated. The goal is to minimize the total required transmit power of DBSs while satisfying the users’ rate requirements. The authors find the optimal locations of DBSs as well as the cell boundaries in a sequential manner. To do so, the problem is divided into two sub-problems that are solved iteratively. In the first sub-problem, given the cell boundaries corresponding to each DBS, the optimal locations of them are derived using the facility location framework. In the second sub-problem, the locations of DBSs are assumed to be fixed, and the optimal cell boundaries are obtained using optimal transport theory. They suppose that DBSs transmit over different frequency bands and hence they do not interfere with each other. In [48], two heuristic approaches, namely genetic algorithm and PSO, are used to maximize the user satisfaction with provided data rates and to find the user-BS associations and optimal positions of DBSs. In [49], an integration of DBSs and drone users is proposed. DBSs are deployed on the basis of the notion of truncated octahedron shapes, and cell associations for minimizing the latency is performed using optimal transport theory. In [50], the trajectory and deployment of DBSs to minimize the power consumption of the users is investigated. In [51], the total transmit powers of the users are minimized while satisfying some QoS constraints in the uplink, and by converting the problems into subproblems, the resource allocations, user associations, and placements of DBSs are found. In [52], the placement of DBSs to minimize the UAV recall frequency.
of the network is determined for single time-slot and multiple time-slot durations. In the dynamic case with multiple time-slot duration, a sequential Markov-greedy decision strategy is applied to find a near-optimal solution.

It is important to ensure not only the communication links between the UAVs and users are reliable, but also the links between neighboring UAVs are stable in order to prevent communication link disruptions; therefore, in [25], the distance between the centers of two neighboring UAVs, referred to as the inter UAV distance, is considered as an essential parameter in maintaining stable communication between the UAVs. If such distance is small, the capabilities of UAVs for coverage are under-utilized. On the other hand, if it is significantly high, the communication throughput degrades dramatically as the communication delay increases substantially. To deal with this issue, a dynamic trajectory control algorithm for the UAVs in order to improve the performance of the network is proposed.

An important constraint in operational duration of drones is their limited onboard energy. Although energy storage technologies have greatly improved over the past few years, limited available energy can still severely affect drone endurance. This issue can be addressed through two approaches. First, energy replenishment should be done by exploiting inter-drone cooperation. For instance, at any time only one drone can leave the serving area to refill the energy and during that time the neighboring drones should do its job by increasing their transmit power and/or readjusting their positions. Energy replenishment can also be matched to the dynamic load pattern that is supported by drones. For example, it can be scheduled during low data traffic at nights. Secondly, energy-efficient operation through smart energy management is required, which means the missions should be accomplished with minimum energy expenditure. The main energy usage of drone-based communication systems is drone propulsion and wireless communication; therefore, in order to have energy-efficient operation, on one hand the movement of drones should be carefully controlled by taking into account the energy consumption associated with every maneuver, on the other hand data transmission should be done by minimum energy consumption by communication circuits, signal transmission, and etc. This can be done by maximizing energy efficiency in bits per Joule (i.e., the number of successfully transmit-
ted data bits per unit of energy consumption) [13]. In [53], the authors suppose some of the users act as cluster heads and aggregate and relate terrestrial data to an aerial platform. They believe that this mechanism can introduce significant energy saving when the information is compressible. Afterwards, they present a cluster-head selection algorithm based on Matérn hard-core point process and obtain the analytical formulation to optimize the spacing between cluster heads that minimizes the overall energy consumption. They suppose that the terrestrial users are deployed in a homogeneous manner, while in heterogeneous user distribution, clustering can lead to better results. Also they do not consider users with different applications and required rates which can dramatically affect the clustering. Moreover, interference is ignored for simplicity. They find that clustering is inefficient when there is no compression, while this observation is due to their assumption that the radius of the deployment area is much smaller than the altitude of the platform, so the elevation angle is approximated to hold the same for all the users. Thus, all the users have the same fixed pathloss towards the aerial platform. In addition, they do not change the placement of the aerial platform and suppose it is in the center of the area with altitude of 12 km. In [54], it is assumed that some users directly communicate by the DBSs while others are engaged through D2D communication. The coverage probability of both users that communicate directly to DBSs and D2D users is derived and it is shown that a maximum system sum-rate can be achieved if the altitude of the drone is appropriately adjusted based on the D2D users density. Also using the disc covering problem they find minimum stop points to completely cover a region with a minimum required transmit power.

Another major aspect of DBSs is their wireless backhaul. Unlike ground BSs, which are usually connected to the core network by high capacity fibre links, DBSs should have a wireless backhaul of limited capacity. Moreover, this capacity may also change dramatically due to inclement weather conditions, environmental parameters, and the distance of a DBS from a backhaul source. Therefore, it is crucial to consider a wireless backhaul with limited capacity in designing the network with DBSs. However, only a few works have considered this issue so far [55, 56]. In [55], a DBS and a macro base station (MBS) for backhauling are considered, and the 3D location of the DBS and the bandwidth allocation for maximizing
the profit data rate are found by applying a two-stage search algorithm, including Golden section search and the steepest descent algorithm. To overcome the limited backhaul capacity of DBSs, in [56], non-orthogonal multiple access (NOMA) is employed on backhaul transmission, and the authors try to jointly optimize the resource allocation at the MBS and DBSs along with the decoding order of the NOMA process and the positions of the DBSs to maximize the total achievable rates of the users. They approximate the original non-convex problem by a series of convex ones utilizing the difference of convex program characterized by the Lipschitz continuity and by sequentially solving each approximate problem until convergence.

The diversity of use cases in 5G is classified in three broad categories: enhanced mobile broadband (eMBB), massive machine-type communications (mMTC), and ultra-reliable and low latency communications (uRLLC) [57]. All of these have different challenging demands such as seamless user experience, very low latency or very high data rates. Edge-caching is a propitious technique to address very low latency requirements of future wireless networks by bringing content closer to end-users via distributed storage in the network. Edge-caching also helps alleviate backhaul congestion especially during peak traffic times in networks with limited backhaul capacity [58, 59]. Having wireless backhaul in DSBs and using terrestrial BSs as hubs for backhauling can be an issue for some delay-sensitive applications. Moreover, DBSs also have limited battery life. Therefore, it is crucial to equip DBSs with local caches to help with both the backhaul congestion and the latency issue. Enabling DBSs with caching will also reduce energy consumption and prolong battery life. To increase the probability of users requesting files that are available in the local cache of a DBS, content caching is often designed using popularity distributions, e.g., the Zipf distribution [60]. However, files may be cached at a DBS whose channel to the user requesting the file is poor. In this case, data transmission may not be reliable or high transmit power will be needed. Therefore, it may be required to send the requested files via backhaul to other DBSs who have better channels to that user or directly via the terrestrial BSs in the network. The caching placement phase is usually implemented during an offline interval when the DBS is being charged at a DBS park. When the DBS is on duty, the delivery phase happens after users send their requests. There are a
few papers which consider caching in DBSs. In [61], an algorithm based on the machine
learning framework of conceptor based echo state networks is proposed to find user-DBS
associations, the optimal location of DBSs, and the contents to be cached at each DBS
while increasing the QoE of users and minimizing the transmit power of DBSs. In the
model proposed in [61], the network leverages some human-centric information such as
users’ visited locations, requested contents, gender, job, and device types to predict the
mobility pattern of the users and their requested contents. In [62], DBSs capture videos
on live games and send them to small BSs that serve the virtual reality users. To meet
the delay requirements, BSs are able to cache some popular contents from the data. A
distributed deep learning algorithm based on echo liquid state machine is proposed to solve
the optimization problem. In [63], throughput among IoT devices is maximized by finding
the location of a DBS and the placement of content caching. To solve the optimization
problem, it is divided into two parts. First, for fixed content, the location problem is broken
down into horizontal and vertical dimensions, and an enumeration method is employed to
obtain the 2D position. Then, for a fixed location, a concave problem for probabilistic
caching placement is formulated. In [64], a cache-enabled network of DBSs and small BSs
for secure video streaming is studied. Interference is managed by interference alignment
and the idle small BSs are utilized to disrupt the potential eavesdropping by generating
the jamming signals.
Chapter 3

On Optimal Location of BSs

3.1 Introduction

Cell deployment has a major importance in designing cellular wireless networks for service providers mainly due to high costs and scarcity of radio resources. It includes determining optimum locations and the number of BSs to meet the traffic and QoS demands at the minimum cost [5]. Communication technologies are currently responsible for around 5% of the total generated carbon footprint and this amount is expected to increase significantly with deployment of 5G systems as the data traffic and the number of connected devices will increase significantly. This calls for energy-efficient approach to wireless communications, which already became an active area of research [65]- [66]. In this chapter, we address the energy efficiency issue by optimizing a BS location to minimize its total transmit power (to all users) and finding an optimal power allocation among users subject to rate constraints for each user, which represent QoS requirements.

While the algorithms mentioned in Section 2.1 are useful from the practical perspective, they have a number of limitations at the fundamental level. In all those algorithms, convergence to a global optimum cannot be guaranteed either due to non-convexity of underlying optimization problems or inherent limitations (approximations) of the algorithms. Furthermore, a gap to a globally-optimal solution is not known or bounded either. Due to the numerical nature of the algorithms, very limited or no insights are available. No
closed-form solutions to the considered problems are known either.

In this chapter, we address the problem at the fundamental level. The optimal BS location problem is modeled as a convex optimization problem to minimize the total BS transmit power, subject to the QoS constraints. Due to the convex nature of this formulation, the respective KKT conditions are sufficient for global optimality, so that a number of closed-form solutions can be obtained and numerical algorithms can be built with a guaranteed convergence to a global optimum (using e.g. the barrier method) [67]. The emphasis of this chapter is on the analysis and fundamental insights rather than on numerical solutions.

The system model is introduced in Section 3.2, where a BS serves a given number of users with known locations and rate requirements. This may represent actual users in a cellular system as well as expected user distributions (e.g. in business or apartment buildings, shopping centers and other social attractors); expected traffic demands in different locations can also be represented in this way via virtual users. While we consider a single BS scenario, the obtained results can be used as a building block to solve higher-level problems with multiple BSs (an approach used extensively in the current literature). The considered model and approach are general enough to include any rate that is a monotonically-increasing function of the signal to noise ratio (SNR) and hence can include fading, in addition to the average pathloss, as well as nonuniform user (traffic) distributions. Different propagation conditions to different users are accommodated as well. An optimal BS location subject to QoS constraints is formulated as a convex optimization problem, which is a key to the further development.

Based on this formulation, Section 3.3 characterizes an optimal BS location in the general 3D case as a convex combination of user locations, where the convex weights depend on user bandwidth and rate demands, some system and propagation parameters, and overall geometry of the problem. This characterization is subsequently used to obtain a number of explicit closed-form solutions for an optimal BS location. In the case of users with identical parameters (rate and bandwidth) and free space propagation, the optimal BS location is shown to be the mean of user locations. This also applies to other
propagation environments, provided that the set of users is symmetric (as defined in Section 3.3.3). Furthermore, the optimal BS location is also independent of pathloss exponent $\nu$ in this case while $\nu$ has a profound impact on it for asymmetric user sets. In the case of large pathloss exponent, the optimal BS location is determined by the most distant user locations. In general, users with higher rate demands are shown to contribute more to an optimal BS location (via their convex weights). Clustered environments are also considered, where all users are grouped into several clusters, and an optimal BS location is shown to be the mean of cluster centers provided that the inter-cluster distances are significantly larger than the cluster sizes. An unusual property is observed whereby an optimal BS location is not necessarily unique. While the optimal BS location is always unique when the pathloss exponent $\nu > 1$, this is not necessarily the case with $\nu = 1$, as shown for collinear users in Section 3.3.5. However, it is always unique for the elevated BS case.

While the study above considers the general 3D user locations, Section 3.4 considers an elevated BS scenario, where all users are located on a 2D (ground) plane and the BS is elevated above it. This may represent typical BS locations in cellular systems as well as UAV-based BSs. The optimal BS location is characterized as a convex combination of user locations, where the weights depend on user parameters and pathloss exponent, subsequently elevated above the ground plane.

Section 3.5 extends the original problem to include extra constraints on a BS location (where the BS must be located within a certain available area, e.g. a rooftop or a hill). The characterization of Section 3.3 as well as many other results are shown to hold for this extended problem as well.

Based on the characterization of optimal BS location in Section 3.3, an iterative algorithm is proposed in Section 3.6 and its convergence is proved in some special cases. Numerical examples are given in Section 3.7. They illustrate and validate the analytical results as well as the proposed algorithm. In particular, they suggest that the algorithm always converges provided that the pathloss exponent is not too large.

It is interesting to note that, in the special case of $\nu = 1$ and identical user parameters,
the problem considered here reduces to the celebrated "Fermat-Weber" problem [68, 69], which is to find a point that minimizes the sum of its distances to a set of given points. An algorithm was proposed by Weiszfeld in 1937 [70] to solve that problem and a significant amount of work investigating its convergence and improving it followed [68–73]. To quote [68], "The Weber problem ... has a long and convoluted history. Many players, from many fields of study, stepped on its stage, and some of them stumbled. The problem seems disarmingly simple, but is so rich in possibilities and traps that it has generated an enormous literature dating back to the seventeenth century, and continues to do so."

3.2 System Model and Problem Formulation

Let us consider a BS serving $N$ users located at $x_k$, $k = 1, \ldots, N$, via some form of orthogonal multiple-access technique (e.g. frequency-division multiple access (FDMA)). User rates $R_k$ can be expressed as follows:

$$R_k = \Delta f_k \log(1 + \gamma_k / \Gamma_k), \quad (3.1)$$

where $\Delta f_k$ and $\gamma_k = \frac{P_{rk}}{\sigma_0^2}$ are the bandwidth and the SNR of user $k$, the channel is frequency-flat with AWGN noise of power $\sigma_0^2$ and $P_{rk}$ is the signal power received by user $k$; $\Gamma_k \geq 1$ is the SNR gap to the capacity of user $k$ [76, 77]. When efficient (capacity-approaching) codes are used for each user, $\Gamma_k \rightarrow 1$. In practice, the assumption of rates being close to the capacity is justified due to the existence of codes which operate very close to channel capacity, e.g. turbo, polar or low-density parity-check (LDPC) codes [78]. In fact, this model can be further extended to include any rate which is a monotonically-increasing function of the SNR, see below. The received power $P_{rk}$ is related to the transmit power $P_k$ allocated by the BS to user $k$ via the pathloss model, see e.g. [79],

$$P_{rk} = \alpha_k P_k / d_k^{\nu_k}, \quad (3.2)$$
where $d_k = |c - x_k|$ is the distance between the BS located at $c$ and user $k$ located at $x_k$, $|x|$ is the Euclidean norm (length) of vector $x$, $\nu_k$ is the pathloss exponent, and $\alpha_k$ is a constant related to the propagation environment, which is independent of distance, but may depend on frequency. For example, in the case of free space propagation environment, e.g. when LoS path is dominant, $\nu_k = 2$ and $\alpha_k = (\lambda_k/(4\pi))^2$, where $\lambda_k$ is the wavelength of user $k$, while for the 2-ray ground reflection model $\nu_k = 4$ and $\alpha_k = h_t^2 h_{rk}^2$, where $h_t$, $h_{rk}$ are the transmit (BS) and user $k$ antenna heights [79], all in the far-field.

We assume that the BS knows pathloss to each user (or, equivalently, its SNR). To satisfy QoS requirements, each user rate must not be less than its target rate $R_{0k}$: $R_k \geq R_{0k}$. To achieve this objective in an energy-efficient way, the operator selects BS location $c$ in an optimal way to minimize its total transmit power $P_T = \sum_k P_k$ subject to the QoS constraints as follows:

$$\textbf{P1:} \min_{\{P_k\},c} \sum_k P_k$$
subject to $R_k \geq R_{0k}, \forall k = 1, \ldots, N$, \hspace{1cm} (3.3a)

where the optimization variables are BS location $c$ as well as per-user powers $\{P_k\}$, so that the BS performs optimal per-user power allocation as well. Noting that the constraints $R_k \geq R_{0k}$ are equivalent to $\gamma_k \geq \gamma_{0k} = \left(2^{R_{0k}/\Delta f_k} - 1\right)\Gamma_k$, the problem $\textbf{P1}$ can be re-formulated as follows:

$$\textbf{P2:} \min_{\{P_k\},c} \sum_k P_k$$
subject to $P_k \geq \beta_k |c - x_k|^\nu_k, \forall k = 1, \ldots, N$, \hspace{1cm} (3.4b)

where $\beta_k = \gamma_{0k} \sigma_{0k}^2 / \alpha_k$. Note that $\sigma_{0k}^2$ may also include interference power as a part of it. We further note that problem $\textbf{P1}$ and hence $\textbf{P2}$ can also accommodate any rate model that is a monotonically-increasing function of the SNR $R_k(\gamma_k)$, not only that in (3.1), so that the condition $R_k \geq R_{0k}$ is equivalent to $\gamma_k \geq \gamma_{0k}$ with properly-selected $\gamma_{0k} = R^{-1}_k(R_{0k})$. This generalized model can also include fading, where $R_k$ and $\gamma_k$ are interpreted as the
average (ergodic) rate and SNR, respectively.

### 3.3 Optimal BS Location and Power Allocation

Following the model of the previous section, an optimal BS location and power allocation to minimize the total transmit power subject to the QoS constraints can be characterized as follows.

**Theorem 1.** An optimal BS location \(c^*\) for \(P2\) in (3.4) can be expressed as a convex combination of user locations \(\{x_k\}\),

\[
e^* = \sum_k \theta_k x_k, \quad \theta_k = \frac{\beta_k \nu_k |c^* - x_k|^{\nu_k - 2}}{\sum_k \beta_k \nu_k |c^* - x_k|^{\nu_k - 2}},
\]

(3.5)

if either (i) \(\nu_k \geq 2\) or/and (ii) \(c^* \neq x_k\) and \(\nu_k \geq 1\). Transmission with the least per-user power is optimal: \(P_k^* = \beta_k |c^* - x_k|^{\nu_k}\).

A proof is given in Appendix. Next, we explore some properties of an optimal BS location.

**Proposition 1.** When \(\nu_k > 1\) for some \(k\), an optimal BS location is unique. This is not necessarily the case if \(\nu_k = 1\) for all \(k\).

A proof is given in Appendix. To proceed further, we need the following definition [67].

**Definition 1.** Let \(\{y_k\}\) be a set of points. Its convex hull \(\text{conv}\{y_k\}\) is the set of all convex combinations of the points in \(\{y_k\}\),

\[
\text{conv}\{y_k\} = \left\{ \sum_k q_k y_k : q_k \geq 0, \sum_k q_k = 1 \right\}.
\]

(3.6)

Figure 3.1 illustrates this definition. Note that \(\text{conv}\{y_k\}\) is always a convex set, regardless of \(\{y_k\}\).
Corollary 1. The optimal BS location $c^*$ in (3.5) is in the convex hull of all user locations,

$$c^* \in \text{conv}\{x_k\}.$$  \hspace{1cm} (3.7)

Proof. Notice from (3.5) that $0 \leq \theta_k \leq 1$, $\sum_k \theta_k = 1$, and then apply Definition 1. \hfill \Box

While (3.5) characterizes an optimal location of the BS, no closed-form solution of this relationship is known in the general case (note that (3.5) is not a closed-form solution itself since $\theta_k$ depends on $c^*$). The above Corollary gives a property of such solution. Furthermore, it implies that the search of $c^*$ can always be confined to $\text{conv}\{x_k\}$, without loss of optimality. For example, if all users are located on a line or in a building, the optimal BS is also on this line or in this building.

We obtain below a number of explicit closed-form solutions for $c^*$ in some special cases.

### 3.3.1 Free space propagation

The first important special case is that of free space propagation, where $\nu_k = 2$. In practice, $\nu_k$ is close to 2 when propagation is close to free space, i.e. most of the 1st Fresnel zone is free of obstructions [79]. This is also the case in a multipath channel when multipath components are much weaker than LoS; therefore, LoS dominates and the propagation becomes almost the same as in free space. $\nu_k$ is close to 2 in many indoor environments when...
LoS is present [79] and \( \nu_k = 2 \) appears often in the 3GPP LTE propagation models [80]. Using Theorem 1, \( c^* \) can be expressed as follows in the general case.

**Corollary 2.** If \( \nu_k = 2 \) for all \( k \), the optimal BS location \( c^* \) is a weighted mean of the user locations,

\[
c^* = \sum_k \theta_k x_k, \quad \theta_k = \frac{\beta_k}{\sum_i \beta_i}.
\]  

(3.8)

*Proof.* Use (3.5) with \( \nu_k = 2 \). \( \square \)

Note that (3.8) is an explicit closed-form solution, since \( \theta_k \) are now independent of \( c^* \). It follows that users with larger \( \beta_k \), i.e. those requiring higher rates, contribute more to \( c^* \) so that as \( \beta_k \) increases, \( c^* \) moves closer to \( x_k \). In the limiting case of \( \beta_1 > 0, \beta_i = 0, i \neq 1 \), the optimal location \( c^* = x_1 \).

Further simplification is possible when all users require the same rate and have the same system settings, so that \( \beta_k = \beta, \forall k \).

**Corollary 3.** If \( \nu_k = 2 \) and \( \beta_k = \beta, \forall k \), the optimal BS location is the arithmetic mean of user locations,

\[
c^* = \bar{x} = \frac{1}{N} \sum_k x_k.
\]  

(3.9)

The total BS transmit power \( P_T = \sum_k P_k \) is proportional to the user location variance \( \sigma^2 \),

\[
P_T = N \beta \sigma^2,
\]  

(3.10)

where \( \sigma^2 = N^{-1} \sum_k |\bar{x} - x_k|^2 \).

*Proof.* Use (3.8) with \( \beta_k = \beta \). (3.10) follows from (3.9). \( \square \)

Note that (3.10) also represents the total BS power when all \( N \) users are located at the same distance \( \sigma \) from the BS, i.e. on a circle (or sphere) of radius \( \sigma \) centered on the BS, where \( P = \beta \sigma^2 \) represents per-user BS power. Hence, the original, possibly highly irregular, user setting can be equivalently substituted by a highly-symmetric (circular) user locations, keeping the same total BS power as well as the same total and per-user rates (however, the per-user powers are not necessarily the same). This is illustrated in Fig. 3.2.
3.3.2 Large pathloss exponent

To obtain further insights, we consider the limiting case of large pathloss exponent \( \nu_k \to \infty \), which serves as an approximation to large but finite \( \nu_k \) (as will be seen from numerical experiments in Section 3.7). To simplify the discussion, we further assume that all users have identical parameters so that \( \beta_k = \beta, \forall k \).

Proposition 2. If \( \nu_k \to \infty \), the optimal BS location is the mean of most distant user locations.

Proof. Without loss of generality, arrange users according to their distances to the BS in a descending order, i.e. \( d_1 = d_2 = \ldots = d_p > d_{p+1} \geq \ldots \geq d_N \), where \( p \) is the number of most distant users. Then, \( \theta_i \) can be expressed as

\[
\lim_{\nu_k \to \infty} \theta_i = \lim_{\nu_k \to \infty} \frac{d_i^{\nu_k-2}}{\sum_k d_k^{\nu_k-2}} = \lim_{\nu_k \to \infty} \left( \sum_{k=1}^p \left( \frac{d_k}{d_i} \right)^{\nu_k-2} + \sum_{k=p+1}^N \left( \frac{d_k}{d_i} \right)^{\nu_k-2} \right)^{-1} = \begin{cases} p^{-1}, & 1 \leq i \leq p, \\ 0, & i > p, \end{cases}
\]

(3.11)

and therefore,

\[
c^* = \frac{1}{p} \sum_{k=1}^p x_k.
\]

(3.12)
Hence, for large pathloss exponent, it is the most distant users who determine the optimal BS location, while nearby users contribute little. Finding most distant users in a set can be expressed geometrically as follows. First, generate a sphere large enough to enclose all the users. Then, shrink it until no further shrinkage is possible while keeping all the users inside, as illustrated in Fig. 3.3, thus obtaining the smallest enclosing sphere. The most distant users are those on the sphere surface. This can also be expressed as a convex optimization problem below, where optimization variables are the sphere center $c$ and its radius $\alpha$.

\[
\begin{align*}
\min_{\alpha, c} & \quad \alpha \\
\text{subject to} & \quad |c - x_k| \leq \alpha, \forall k = 1, \ldots, N.
\end{align*}
\]

### 3.3.3 Symmetric sets of users

To obtain closed-form solutions for $c^*$ beyond those above, we consider now scenarios where user location sets posses some symmetry properties. This should also approximate (due to the continuity of the problem in user locations) scenarios where users are nearly-symmetric.
We will need the following definitions of symmetric sets.

**Definition 2.** Let $\Omega_l = \{x_k : k \in I_l\}$ be a set of $|I_l|$ points (users), where $I_l$ is an index set and $|I_l|$ is its cardinality. The set $\Omega_l$ is called elementary symmetric if the distance between its center $a_l = |I_l|^{-1} \sum_{k \in I_l} x_k$ and any of its points is the same, i.e. $|a_l - x_k| = d_l$, $\forall k \in I_l$.

**Definition 3.** Set $\Omega$ is symmetric if it is a union of disjoint elementary symmetric sets with the same centers, i.e. $\Omega = \bigcup_l \Omega_l$ and $a_l = a$, $\forall l$.

While an elementary symmetric set is also symmetric, the converse is not true in general, i.e. a symmetric set does not need to be elementary symmetric, as Fig. 3.4 illustrates, so the former is more general than the latter. Equipped with these notions of symmetry, we are now able to obtain the optimal BS location in a closed form.

**Proposition 3.** Let the set $\Omega$ of user locations be symmetric, i.e. $\Omega = \bigcup_l \Omega_l$, where $\Omega_l$ are disjoint and elementary-symmetric, $\nu_k = \nu_l$ for any $k \in I_l$, and $\beta_k = \beta$, $\forall k$. Then, for any pathloss exponents $\nu_k > 1$ for all $k$, the optimal BS location is its center $a$, i.e. the mean of the users’ locations,

$$c^* = a = \bar{x} = \frac{1}{N} \sum_k x_k.$$  

(3.14)
Proof. Since (3.5) is necessary for optimality of \( c^* \) and since optimal location is unique when \( \nu_k > 1 \), it is also sufficient, i.e. any \( c^* \) that satisfies (3.5) is optimal. We demonstrate below that \( c^* = a \), where \( a \) is the center of \( \Omega \), does satisfy (3.5). Since all user locations form a union of elementary symmetric sets \( \Omega_l \) of the same center \( a \), it follows that

\[
a = \frac{1}{N} \sum_k x_k = \frac{1}{|I_l|} \sum_{i \in I_l} x_i.
\]

(3.15)

Note that the distance \( |a - x_i| \) between any user in \( \Omega_l \) and its center \( a \) is the same, i.e. \( d_l = |a - x_i| \) for any \( i \in I_l \), since \( \Omega_l \) is elementary symmetric. Using \( c^* = a \) in 2nd part of (3.5),

\[
\theta_i = \frac{|a - x_i|^{\nu_i - 2}}{\sum_k |a - x_k|^{\nu_k - 2}} = \frac{d_l^{\nu_i - 2}}{\sum_k |a - x_k|^{\nu_k - 2}} = p_l, \ \forall i \in I_l,
\]

(3.16)

i.e. all weights \( \theta_i \) are the same for all users in the same symmetric set \( \Omega_l \). Now using these \( \theta_i \) in 1st part of (3.5), we obtain

\[
c^* = \sum_k \theta_k x_k = \sum_{l_i} \sum_{i \in l_i} \theta_i x_i = \sum_{l_i} p_l \sum_{i \in l_i} x_i = \sum_{l_i} p_l |I_l| a = a,
\]

(3.17)

as required, where the last 2 equalities are due to \( \sum_{i \in I_l} x_i = |I_l| a \) and \( \sum_k \theta_k = \sum_{l_i} p_l |I_l| = 1 \).

It should be emphasized that this result holds for any \( \nu_k > 1 \), not just for \( \nu_k = 2 \), as in Corollary 3, so this result is more general in terms of \( \nu_k \) but more restrictive in terms of user locations as symmetry is required here, unlike Corollary 3. Note also that, unlike the general case, the optimal BS location is independent of pathloss exponent \( \nu_k \) as long as the user set is symmetric. This Proposition also implies that when new users are added to existing ones, the optimal BS location is not affected as long as new users do not disturb symmetry. It can be further shown that the BS location in (3.14) also minimizes the amount of co-channel interference to the users of other cells provided they satisfy certain symmetry requirement.
3.3.4 Clustering of users

Let us consider a scenario when users are clustered around some points (social attractors, e.g. business or shopping centers, apartment buildings, etc.), as illustrated in Fig. 3.5. When the cluster sizes (radii) are much smaller than the distance between them, the optimal BS location can be approximated as follows. When only 2 clusters are present, their centers can be set, without loss of generality (via appropriate choice of the reference frame), to be $c_1$ and $-c_1$, and, choosing 1st basis vector along the same directions, the (scalar) coordinates are $c_1$ and $-c_1$.

**Proposition 4.** Let $\nu_k = \nu > 1$, $\beta_k = \beta$ for all $k$, and all the users be clustered in two sets, $C_1$ and $C_2$, and their cluster centers be $-c_1$ and $c_1$; let the distance between the clusters $D = 2|c_1|$ be much larger that the cluster radii $r_1$ and $r_2$: $D \gg r_1, r_2$ (see Fig. 3.5). Then, the optimal BS location can be approximated as follows: it is on the line connecting the cluster centers and its coordinate $c^*$ is

$$c^* \approx c_1 \left( \frac{m_2/m_1}{\nu - 1} \right)^{\frac{1}{\nu - 1}} - 1, \quad \left( \frac{m_2/m_1}{\nu - 1} + 1 \right), \quad (3.18)$$

where $m_1$ and $m_2$ are the number of users in each cluster.

**Proof.** When all users have the same parameters, $\beta_k = \beta$, $\forall k$, the optimization problem $P2$ in (3.4) is equivalent to

$$\min_c \sum_k d_k^\nu, \quad (3.19)$$

Since the cluster sizes are much smaller than the distance between them, each cluster can be approximated by a point (located at its center) where all users of this cluster are located. Applying Corollary 1 under this approximation, the BS is located on the line segment connecting $c_1$ and $-c_1$, which is characterized by its coordinate $c$, so that $d_k \approx |c + c_1|$ for all users in $C_1$, and $d_k \approx |c - c_1|$ for all users in $C_2$. Under this approximation, the problem in (3.19) is simplified to

$$\min_c m_1|c + c_1|^\nu + m_2|c - c_1|^\nu. \quad (3.20)$$
Setting the derivative of the objective in (3.20) to zero, one obtains

\[ m_1 \nu (c + c_1)^{\nu-1} - m_2 \nu (c_1 - c)^{\nu-1} = 0, \]  

(3.21)

from which (3.18) is obtained after some manipulations.

Using this Proposition, we make the following observations.

1. \( c^* \) depends on \( m_2/m_1, \ c_1 \) and \( \nu \), but not on cluster sizes, provided that they are much smaller than the inter-cluster distance.

2. \( c^* \) is a monotonically-increasing function of \( m_2 \): if \( m_2 > m_1 \), then \( c^* > 0 \), i.e. the BS is closer to the bigger cluster centered at \( c_1 > 0 \), and it is getting closer to it as the number of its users grows.

3. If \( m_1 = m_2 \), then \( c^* \approx 0 \), i.e. the BS is in the middle of the clusters when they have the same number of users, which is an intuitively-appealing conclusion.

4. If \( \nu = 2 \), then \( c^* \approx (m_2 - m_1)/(m_2 + m_1) \).

5. Finally, if \( m_2 \gg m_1 \), then \( c^* \approx c_1 \), i.e. the BS approaches the bigger cluster center.

### 3.3.5 Collinear users

In this section, we consider the case where all users are located on a line. This is motivated by practical settings on highways, in tunnels, street canyons or corridors. Following Corollary 1, an optimal BS location is also on the line, while its specific location depends
on users’ locations and pathloss exponent. We consider below the case of $\nu_k = 1$ for all $k$ and demonstrate some unusual properties such as non-uniqueness of optimal BS location. Note that $\nu < 2$ represents an environment more favorable for propagation than free space and it is possible in channels with guided wave structure, such as tunnels, corridors, street canyons [79].

**Proposition 5.** Let all users have the same system parameters, $\nu_k = 1$, $\beta_k = \beta$, $\forall k$, and be located on a line as represented by their scalar coordinates $x_k$, $k = 1 \ldots N$; without loss of generality, set $x_1 \leq x_2 \leq \ldots \leq x_N$. If $\nu_k = 1$, an optimal BS location is a median of users’ locations,

$$
c^* = \begin{cases} 
  x_{(N+1)/2}, & N \text{ is odd}, \\
  \text{any } a \in [x_{N/2}, x_{N/2+1}], & N \text{ is even}.
\end{cases} \tag{3.22}
$$

**Proof.** Since all users as well as the BS are located on a line and transmission with the least per-user power is optimal, the problem $\mathbf{P2}$ is equivalent to

$$
\min_c \sum_k |x_k - c|, \tag{3.23}
$$

which is a convex problem. Since there are no constraints, the KKT conditions reduce to the stationarity condition (zero derivative at optimal point). When the number of users is even, consider any point $a$ between two middle points, i.e. $x_{N/2} \leq a \leq x_{N/2+1}$, as illustrated in Fig. 3.6. Below, we demonstrate that this point is optimal. Indeed,

$$
f(a) = \sum_{k=1}^N |x_k - a| = \sum_{k=1}^{N/2} (a - x_k) + \sum_{k=N/2+1}^N (x_k - a) = \sum_{k=1}^{N/2} x_k - \sum_{k=1}^{N/2} x_k, \tag{3.24}
$$

so that $df(a)/da = 0$ for any $a \in [x_{N/2}, x_{N/2+1}]$ and hence $c^* = a$. When the number of users is odd, consider any $a \in [x_{(N-1)/2}, x_{(N+1)/2+1}]$, so that

$$
f(a) = |x_{(N+1)/2} - a| - \sum_{k=1}^{(N-1)/2} x_k + \sum_{k=(N+1)/2+1}^N x_k, \tag{3.25}
$$

which is clearly minimized by $a = x_{(N+1)/2}$. It is straightforward to see that any $a$ not in
Figure 3.6. If $\nu_k = 1$ and the number of users is even, an optimal BS location is not unique: it can be anywhere between two middle-point users.

The interval $[x_{(N-1)/2}, x_{(N+1)/2+1}]$ cannot be optimal since it gives larger $\sum_k |x_k - a|$. Hence, $c^* = x_{(N+1)/2}$.

An illustration of Proposition 5 is given in Fig. 3.6 when the number of users is even. Note that an optimal BS location is not unique in this case, which is ultimately due to the fact that $|x|$ is not strictly convex. However, if $\nu > 1$, then it is always unique, according to Proposition 1, since $|x|^\nu$ is strictly convex in this case. To see the impact of $\nu$, let us consider 3 special cases as shown in Fig. 3.7:

1. For $\nu = 1$, an optimal BS location is a median point, which is not unique (can be anywhere between users 3 and 4).

2. For free space propagation, $\nu = 2$, the optimal BS location is the (unique) mean of the users’ locations, according to Corollary 3.

3. For asymptotically-large $\nu$, the optimal BS location is the mean of the most distant users’ locations, according to Proposition 2, so that most distant users contribute most to optimal BS location in this case.

Thus, $\nu$ has a profound impact on optimal BS location for asymmetric user sets. This is in stark contrast with symmetric user sets (Proposition 3), where the optimal BS location is independent of $\nu$.

3.4 Elevated Base Station

In practice, BS is often located at some elevation above ground to provide clear LoS to most users hence improving coverage. This also includes scenarios with an airborne...
Figure 3.7. Optimum BS locations for different pathloss exponents. (a) For $\nu = 1$, it is a median point, which is not unique; (b) for $\nu = 2$ - the mean of the user locations; (c) for $\nu \to \infty$ - the mean of the most distant users. As $\nu$ increases, the impact of the distant user on the right increases too.

Figure 3.8. An elevated BS scenario, where all users are located on the ground plane while the BS is elevated to a given height $h$.

communication node (e.g. a drone). To model this scenario, we consider the setting of Fig. 3.8, where users are located on a (ground) plane with 2D vector $x_k$ representing user $k$, while the BS is above the ground at a given height $h$ and $c$ is its 2D location (projection) on the ground plane. The distance between the BS and user $k$ is therefore $\sqrt{|c - x_k|^2 + h^2} = |c - x_k|_h$. Thus, the problem $P2$ becomes

$$\min_{\{P_k\}, c} \sum_{k=1}^{N} P_k \quad (3.26a)$$

subject to $P_k \geq \beta_k |c - x_k|_h^{\nu_k}, \forall k = 1, ..., N. \quad (3.26b)$

The following Theorem characterizes its solutions.

**Theorem 2.** Consider the elevated BS location problem in (3.26) when $\nu_k \geq 1$. Its solution
\( c^* \) can be expressed as a convex combination of user locations \( \{x_k\} \),

\[
c^* = \sum_k \theta_k x_k, \quad \theta_k = \frac{\beta_k v_k |c^* - x_k|_{h}^{v_k - 2}}{\sum_i \beta_i v_i |c^* - x_i|_{h}^{v_i - 2}}.
\]

(3.27)

Proof. Follows from 2nd part of the proof of Theorem 1, see (A.6)-(A.11).

Note that while Theorem 1 needs special consideration for singular cases, Theorem 2 is not restricted in this way, since \( |x|_h \) is differentiable for any \( x \) when \( h \neq 0 \). The characterization of \( c^* \) in Theorem 2 is similar, in its functional form, to that in Theorem 1, with the substitution \( |\cdot| \to |\cdot|_h\). Hence, a number of properties/solutions pointed above also hold for the elevated BS problem in terms of its 2D projected location \( c^* \). In particular, Corollaries 1-3, Propositions 2, 3, do hold for the elevated BS as well. Proposition 1 is strengthened as follows.

**Proposition 6.** The optimal elevated BS location is unique for any \( v_k \geq 1 \) if \( h \neq 0 \).

Proof. Follows the steps of that of Proposition 1 by observing that \( |x|_{h}^{v} \) is strictly convex for any \( v \geq 1 \) if \( h \neq 0 \).

It is tempting to conclude that the optimal elevated BS location can be found by first solving the problem with \( h = 0 \) (no elevation) and then using its solution \( c^* \) and "elevating" it by \( h \) above the ground plane, but this is incorrect in general as shown by examples in Section 3.7. However, it is indeed the case if \( v_k = 2 \) for all \( k \), as follows from (3.27).

### 3.5 Additional Location Constraints

When locating a BS in practice, quite often there are some additional constraints due to existing infrastructure, such as a limited roof-top area available for a BS location. In such a case, the problem \( P2 \) can be modified to include extra constraint on BS location as
follows:

$$\textbf{P3}: \min_{\{P_k, c\}} \sum_k P_k \quad (3.28a)$$

subject to \( P_k \geq \beta_k |c - x_k|^\nu_k, \ \forall k = 1, ..., N, \)

\(|c - a_l| \leq r_l, \ \forall l = 1, ..., L, \quad (3.28c)\)

where the additional constraints \(|c - a_l| \leq r_l\) account for physical limitations or preferences, as discussed above, for given \(a_l, r_l\).

An optimal BS location under these extra constraints can be characterized as follows.

**Theorem 3.** When (i) \(\nu_k \geq 2\), or/and (ii) \(\nu_k \geq 1\) and \(c^* \neq x_k\), the optimal BS location for the problem \(P3\) can be expressed as a convex combination of user and constraint locations,

\[ c^* = \sum_{k=1}^{N+L} \theta_k x_k, \quad (3.29) \]

where \(x_{N+l} = a_l, \ l = 1...L, \)

\[ \theta_k = \frac{\nu_k \beta_k |c^* - x_k|^{\nu_k-2}}{\sum_{k=1}^N \beta_k \nu_k |c^* - x_k|^{\nu_k-2} + 2 \sum_{l=1}^L \mu_l}, \ k = 1...N, \quad (3.30) \]

\[ \theta_{N+l} = \frac{2 \mu_l}{\sum_{k=1}^N \beta_k \nu_k |c^* - x_k|^{\nu_k-2} + 2 \sum_{l=1}^L \mu_l}, \ l = 1...L, \quad (3.31) \]

and dual variables \(\mu_l \geq 0\) are found from

\[ \mu_l(|c^* - a_l| - r_l) = 0, \quad (3.32) \]

subject to \(|c^* - a_l| \leq r_l\). Signaling with the least per-user power is optimal: \(P_k^* = \beta_k |c^* - x_k|^\nu_k\).

**Proof.** The proof is similar to that of Theorem 1. The Lagrangian is

\[ L = \sum_k P_k + \sum_k \lambda_k (\beta_k |c - x_k|^\nu_k - P_k) + \sum_l \mu_l (|c - a_l|^2 - r_l^2), \quad (3.33) \]

where \(\mu_l \geq 0\) are Lagrange multipliers responsible for the additional location constraints.
The stationarity condition is

\[ \frac{\partial L}{\partial c} = \sum_k \lambda_k \beta_k \nu_k (c - x_k) |c - x_k|^{\nu_k - 2} + 2 \sum_l \mu_l (c - a_l) = 0, \quad (3.34) \]

from which, after some manipulation, (3.29)-(3.31) follow. (3.32) are the complementary slackness conditions associated with $|c - a_l| \leq r_l$.

Note that if $\mu_l > 0$ (active $l$-th location constraint), then $|c^* - a_l| = r_l$, i.e. an optimal BS location is on the circle of radius $r_l$ centered at $a_l$. Otherwise, the constraint is inactive and can be discarded. When all extra location constraints are inactive, $\mu_l = 0$ for all $l$ and Theorem 3 reduces to Theorem 1.

Some of the properties above can be also extended to include additional location constraints. In particular, Proposition 1 applies verbatim and Corollary 1 is extended to $c^* \in \text{conv}\{x_k, a_l\}$, i.e an optimal BS location is in the convex hull of $\{x_k, a_l\}$. Since $0 \leq \theta_k \leq 1$ and $\sum_k \theta_k = 1$, it follows from (3.29) that the optimal BS location is a weighted mean of user locations and additional constraint centers.

If $\nu_k = 2$ and $\beta_k = \beta$, i.e. free space propagation and identical user parameters,

\[ \theta_k = \frac{\nu \beta}{\nu \beta N + 2 \sum_l \mu_l}, \quad k = 1...N, \quad (3.35) \]
\[ \theta_{N+l} = \frac{2 \mu_l}{\nu \beta N + 2 \sum_l \mu_l}, \quad l = 1...L, \quad (3.36) \]

so that the optimal BS location is the weighted mean of the mean user location and extra constraint centers.

### 3.6 An Iterative Algorithm for the General Case

While a number of closed-form solutions of the location problem $P_2$ have been presented above, no such solution is known in the general case. However, the characterization of a solution in Theorem 1 can be exploited to build an iterative algorithm for the general case as follows. First, select an initial BS location $c^1$ (not necessarily optimal) and use
Algorithm 1 (optimum BS location)

1: Initialization: $i = 1$, $c^1 = \sum_k \beta_k x_k / \sum_k \beta_k$, $\epsilon > 0$.
2: while $|c^{i+1} - c^i| > \epsilon$ do
3: \hspace{1em} $c^{i+1} = f(c^i)$.
4: \hspace{1em} $i \leftarrow i + 1$.
5: end while

It in (3.5) to compute the weights $\theta_k$. Second, use these weights to update the location according to the 1st equation in (3.5). The process can be repeated until some convergence condition is satisfied. To present the algorithm, let

$$f(c) = \frac{\sum_k \beta_k v_k |c - x_k|^{\nu_k - 2} x_k}{\sum_k \beta_k v_k |c - x_k|^{\nu_k - 2}}. \quad (3.37)$$

Then, the algorithm can be described as in Algorithm 1.

The convergence condition in line 2 of the algorithm can be substituted by some other suitable condition, for example, in term of the total BS power: $|P_T(c^{i+1}) - P_T(c^i)| < \epsilon$, where $P_T(c) = \sum_k \beta_k |c - x_k|^{\nu_k}$ is the total BS power at location $c$. Additional improvements of this basic algorithm are possible. For example, one can enforce a certain minimum number of iterations to ensure that the algorithm does not terminate prematurely. Moreover, one can select at each iteration the best overall location so far (it is sufficient to take the best of 2 most recent locations), in which case the sequence of total BS powers generated by the algorithm will be monotonically-decreasing.

When $\beta_k = \beta$ and $\nu_k = 1$, $\forall k$, this algorithm coincides with Weiszfeld’s algorithm [70] to solve the celebrated 350-years-old Fermat-Weber problem [69]. This problem as well as the algorithm have a long and convoluted history (including a number of false claims of convergence) [68]. Its convergence has been fully settled only recently [75].

It is beyond the scope of this chapter to study the convergence of Algorithm 1 in details. However, we do point out cases where such convergence is achieved in a single iteration. Numerical experiments below support the empirical conclusion that Algorithm 1 converges if $\nu_k \leq 3$.

**Proposition 7.** Algorithm 1 converges in a single iteration, i.e. $c^* = c^2 = c^1$, if any of
the following holds:

1. \( \nu_k = 2, \ \forall k \).

2. The conditions of Proposition 3 hold.

3. There are 2 identical users, i.e. \( \beta_1 = \beta_2, \nu_1 = \nu_2 \).

4. \( c^1 = c^* \), i.e. initial location is optimal.

**Proof.** To prove the last case, observe from (3.5) that using \( c^1 = c^* \) in (3.37) results in \( c^2 = c^1 = c^* \) and hence the algorithm terminates in 1 iteration. Cases 1 - 3 follow from Case 4 since, in these cases, \( c^1 = c^* \). \( \square \)

Note that condition 4 implies that an optimal location \( c^* \) is a convergence point of the algorithm, i.e. if the algorithm reaches an optimal point, it will stop there.

This algorithm can also be used to solve the elevated BS location problem in (3.26), with the substitution \(| \cdot | \rightarrow | \cdot |_h \) in (3.37). It is interesting to note that, in this case and when \( \nu_k = 1, \beta_k = \beta, \ \forall k \), the elevated BS problem coincides with that considered in [74], where \( h \) was introduced as a smoothing variable to avoid singularities and thus ensure the convergence of the barrier method. However, no physical justification for it was provided, beyond a computational convenience. In our setting, \( h \) appears naturally and has a solid meaning of the BS height above the ground plane.

### 3.7 Numerical Examples

In this section, we validate and illustrate the analytical results above, examine their accuracy as well as the convergence of Algorithm 1. In all examples, users are located within a square of side \( 2R_{\text{max}} \); all coordinates are normalized (except for clustered scenarios) by \( R_{\text{max}} \) to make the results independent of physical size, but dependent on geometry of user locations. In all examples, \( \beta_k = 1, \nu_k = \nu \) for all \( k \), and the total BS power \( P_T \) is normalized.
Table 3.1. The error norm $|c^* - c^*_{CVX}|$ averaged over $10^3$ randomly-generated user sets.

| $|c^* - c^*_{CVX}|$ | 5 users | 10 users | 50 users | 100 users |
|---------------------|---------|----------|----------|-----------|
| $\nu = 1$           | 7.64E-06 | 7.95E-06 | 4.96E-06 | 5.42E-06 |
| $\nu = 2$           | 1.36E-06 | 1.02E-06 | 1.14E-06 | 1.71E-07 |
| $\nu = 3$           | 2.27E-06 | 1.48E-06 | 6.24E-07 | 2.33E-07 |
| $\nu = 4$           | 5.78E-06 | 3.09E-06 | 1.62E-07 | 5.18E-08 |

The expression for an optimal BS location in Theorem 1 was validated by comparing $c^*$ in (3.5) with $c^*_{CVX}$ obtained via the convex optimization toolbox CVX [81] to solve the problem $P2$ in (3.4) numerically. In doing so, the optimal location $c^*_{CVX}$ obtained numerically via CVX was used in 2nd equality in (3.5) to evaluate $\theta_k$, which were subsequently used in 1st equality to evaluate $c^*$. $10^3$ user sets were randomly generated, where a given number of users were located with a unit square for each set, and the error norm $|c^* - c^*_{CVX}|$ averaged over all user sets was evaluated. No significant difference between $c^*$ and $c^*_{CVX}$ was observed for different pathloss exponents and different numbers of users in each set. As Table 3.1 shows, the average error does not exceed $10^{-5}$ in all considered scenarios. The optimal BS location $c^*$ was also compared with $c^*_{CVX}$ for $\nu = 2$ (see Corollary 3) using $10^3$ randomly-generated user sets as above. No significant difference was found either: in all tested cases, the average error did not exceed $10^{-5}$.

Next, we assess the accuracy of the approximation in Proposition 2 (see (3.12)) when applied to finite $\nu$, for asymmetric user locations in Fig. 3.9a. Fig. 3.9b shows the normalized BS power $P_{Tn}$ found via (3.12) and via CVX. Note a reasonably good agreement between the two methods, even when $\nu$ is not so large. When $\nu$ increases, the accuracy improves significantly. In many other tested cases, the convergence was much better. For example, Fig. 3.10a demonstrates a more symmetric user setting and optimal BS locations.
Figure 3.9. (a) User locations (asymmetric scenario) and optimal BS locations via the approximation (3.12) (for $\nu \to \infty$) and numerically via CVX for $\nu = 4$. The normalized BS powers are 3.96 and 3.34 dB, respectively, so that the approximation incurs only a small loss of 0.62 dB. (b) Normalized BS power versus pathloss exponent $\nu$ for the user setting in (a).

via (3.12) and CVX, while Fig. 3.10b shows the normalized BS power versus $\nu$. Note that the agreement here is much better than that in Fig. 3.9b for the whole considered range of $\nu$. To understand this, observe from Proposition 3 that the optimal BS location
Figure 3.10. (a) User locations and optimal BS locations via the approximation (3.12) and numerically via CVX, all for $\nu = 4$. (b) Normalized BS power versus pathloss exponent $\nu$ for the user setting in (a). Note good agreement between the two for all considered values of $\nu$.

is independent of $\nu$ if the user set is symmetric and, hence, nearly-independent if the set is nearly-symmetric (as in Fig. 3.10a), so that the actual BS power will be almost same for both BS locations (computed for a given $\nu$ and from Proposition 3).

To validate the clustering approximation (3.18), Fig. 3.11a shows two clusters of users
and optimal BS locations found via (3.18) and numerically by CVX, for $\nu = 2.5$ and $\nu = 4$. Note that the approximate and numerical solutions agree well with each other, even though the larger cluster size is not so small compared to the inter-cluster distance. The accuracy of approximation slightly decreases for larger $\nu$. For the same clusters, Fig. 3.11b shows the impact of inter-cluster distance. The accuracy of the approximation in (3.18) increases with the distance; while it is uniformly good for $\nu = 2.5$, it is slightly worse for $\nu = 4$ when
Figure 3.12. The projection of the optimum BS location on the ground plane for different heights, $v = 4$. Note that the height affects the optimum ground location as well, but this effect is not significant if $h \geq 5$.

the distance is not large enough. Notice that the evaluation of optimal BS location based on the approximation in (3.18) is much simpler than that for the whole setting (for which no closed-form solution is known), hence demonstrating its usefulness. While in general the accuracy of the approximation depends on user locations, in addition to pathloss exponent, good accuracy was observed in most tested cases.

Next, we consider an elevated BS location when all users are located on the ground. Fig. 3.12 shows the optimum BS location (projected on the ground) for $v = 4$ and various BS heights; $h = 0$ corresponds to no elevation. Observe that the projected optimal BS location is not the same as that with $h = 0$. Hence, finding an optimal BS location on the ground (no elevation) and then elevating it to height $h$ is not optimal in general. Once certain height is reached, its further increase does not have significant impact on optimal BS location.

To validate Algorithm 1 and its convergence, Fig. 3.13b shows the normalized BS power $P_{Tn}$ under BS location found by Algorithm 1 versus iteration number for user locations in Fig. 3.13a. The results via CVX are also shown for comparison. Note the fast convergence for all considered $v \leq 3$, while it takes only 1 iteration for $v = 2$, as expected from Proposition 7.
Figure 3.13. (a) A typical user setting. (b) Convergence of Algorithm 1 in terms of $P_{T_n}$ for the user setting in (a); the solutions via CVX are also shown for comparison. Note fast convergence for all considered $\nu \leq 3$, while it takes only 1 iteration for $\nu = 2$.

3.8 Conclusions

In this chapter, the problem of determining an optimal BS location for a given set of users was formulated as a convex optimization problem to minimize the total BS power subject to QoS (rate) constraints. Its globally-optimal solution was expressed as a convex combination of user locations. Based on this, a number of closed-form solutions were obtained, which revealed the impact of system and user parameters, propagation pathloss, and overall
system geometry. The symmetry in the user set was shown to make the optimal BS location independent of pathloss exponent, which is not the case in asymmetric sets. These results provide insights unavailable from numerical algorithms, and allow one to develop design guidelines for more involved systems. Additionally, based on the characterization of optimal BS location, an iterative algorithm was proposed, and its convergence was shown for some special cases. Overall, the chapter aims at building an analytical foundation for the BS location problem, that can facilitate system design and network planning, and can be further extended to include more complicated scenarios.
Chapter 4

On the Number and 3D Placement of Drone BSs

4.1 Introduction

One of the most important issues in drone-based wireless communication systems as mentioned earlier, is 3D deployment of DBSs. In terrestrial networks, 2D placement of BSs are usually found based on the long term average traffic. Once the BSs are installed, they are fixed and do not move, but DBSs can move and change their positions. Moreover, in addition to the horizontal movement, their altitude can also change; therefore, whenever it is required to use DBSs in a network, the number of required BSs as well as their 3D placement should be found in a quick way. By having an extra dimension which is height, the problem becomes even more complicated than 2D location problems. In this chapter, we formulate a problem to find 3D placement of DBSs to satisfy both coverage and capacity issues. The main contribution of this part is to provide an efficient algorithm to find the minimum number of DBSs and their 3D locations to serve an arbitrarily located set of users with high rate requirements.
4.2 System Model

We consider an urban area with a specific number of users. Our goal is to find the minimum number of DBSs and their 3D placements to give service to the users in that area. We limit our analysis to downlink, so DBSs are transmitting data. An important feature of a DBS is its ability to move; therefore, we do not need to cover a region while there is no user there. As users move, DBSs might follow them if needed. Here, we find the placement of DBSs for one snapshot of the users positions. Our assumptions are as follows:

- There is no ground BS in the area.
- There are a number of DBSs in the system.
- Co-channel interference is considered.
- The system is in downlink mode.
- There is no limitation in the backhaul capacity of DBSs.

4.2.1 Pathloss model

Random behaviors of the channel should not affect the placement of DBSs; hence in this chapter, we adopt the mean pathloss given in (2.4). Also the probability of having LoS connection is formulated as (2.3).

4.3 Optimization Problem

We adopt the framework of our optimization problem from [22]. To start the optimization problem, we need an initial estimation of the number of DBSs that can serve all the users. The number of DBSs should be estimated based on both coverage and capacity requirements. For coverage constraint, according to [12] only one DBS can cover a very large area by setting its altitude in the optimum height, so in our problem mainly the
data traffic requirement of the users enforces to use more DBSs. Therefore, for the initial estimation of the number of DBSs we only consider the capacity constraint.

First, we need to find the maximum number of users that a DBS can serve as below:

\[ N_{UBS} = \left\lfloor \frac{C_{BS}}{r} \right\rfloor, \] (4.1)

where \( \lfloor \cdot \rfloor \) denotes the floor function, \( r \) is the target download rate of the users, and \( C_{BS} \) is the capacity of a DBS and is equal to \( B \times \eta \), where \( B \) is the total bandwidth of a DBS and \( \eta \) is the average spectral efficiency of the system. The number of DBSs is estimated as

\[ N_{BS} = \left\lceil \frac{I}{N_{UBS}} \right\rceil, \] (4.2)

where \( \lceil \cdot \rceil \) denotes the ceiling function and \( I \) is the total number of users available in the region.

### 4.3.1 System constraints

Our goal, as mentioned earlier, is to cover the users and serve them based on their QoS requirements.

#### 4.3.1.1 Coverage constraints:

For the coverage constraints, we wish at least \( \zeta \) percent of all the users are covered by DBSs. This constraint can be formulated as

\[ P\{\gamma_{ij^*} > \gamma_{th}\} \geq \zeta, i = 1, \ldots, I, \] (4.3)

where \( j^* = \arg\max_{j} \gamma_{ij} \), and \( \gamma_{ij} \) is the SINR of user \( i \) receiving service from DBS \( j \), and \( \gamma_{th} \) is the minimum SINR level required for each user.
4.3.1.2 Capacity constraints:

For the capacity constraints, a parameter $m_{j,k}$ ($j = 1, ..., N_{BS}$, and $k = 1, ..., N_{subarea}$) is defined as [22]

$$m_{j,k} = \frac{a_{j,k}}{A_j}, \quad (4.4)$$

where $a_{j,k}$ is the mutual area between DBS $j$ and subarea $k$, and $A_j$ is the total area of DBS $j$; so $m_{j,k}$ is a number between zero and one and shows how much of the area of DBS $j$ is inside subarea $k$. A subarea is part of the region in which the user density is the same. Fig. 4.1 simply illustrates the concept of $m_{j,k}$. In this figure, $m_{1,1} = 1$ as the whole area of DBS 1 is in subarea 1; it leads to $m_{1,2} = 0$. Accordingly for DBS 2, $m_{2,1} \approx \frac{1}{3}$ and $m_{2,2} \approx \frac{2}{3}$ as around $\frac{1}{3}$ and $\frac{2}{3}$ of DBS 2 is inside subarea 1 and 2, respectively.

When a subarea is dense in terms of number of users, more DBSs are required to serve all the users. Therefore, to satisfy the capacity constraint the below inequality should hold:

$$\sum_{j=1}^{N_{BS}} N_{U_{BS}} m_{j,k} \geq D_k S_k, k = 1, ..., N_{subarea}, \quad (4.5)$$

where $D_k$ and $S_k$ are the user density function and the total area of subarea $k$, respectively.

4.3.1.3 Equivalent spectral efficiency constraint

We assume that the users are admitted to the network on a first-come, first-served basis. The bandwidth requirement of each user is defined as $b_i = \frac{r_i}{\eta_i}$, where $r_i$ and $\eta_i$ are downlink
rate requirement and spectral efficiency of user $i$, respectively. The number of users that a BS can support is limited due to limited available bandwidth and thus we have

$$\sum_{i=1}^{N} b_i \leq B, \text{ and } \sum_{i=1}^{N+1} b_i > B, \quad (4.6)$$

where $B$ is the available bandwidth in the BS and $N$ is the maximum number of users a BS can support. Since $\{b_1, b_2, ..., b_{N+1}\}$ is a sequence of positive independent and identically distributed (iid) random variables, the maximum number of users a BS can support is a renewal process [82] and can be defined as

$$\tilde{N}(w) = \max\{N \in \mathbb{N} : \sum_{i=1}^{N} b_i \leq w\}. \quad (4.7)$$

If $B$ is defined as the stopping time of this process, then $\tilde{N}(B) = N$ is the number of users that are admitted by a BS.

According to the Central Limit Theorem for renewal processes, for large $B$, the distribution of $\tilde{N}(B)$ is approximated to be Gaussian with mean $\frac{B}{\mathbb{E}\{b_i\}}$, where the average bandwidth that user $i$ requires is

$$\mathbb{E}\{b_i\} = \mathbb{E}\{\frac{r_i}{\eta_i}\} \overset{(a)}{=} \mathbb{E}\{r_i\} \cdot \mathbb{E}\{\frac{1}{\eta_i}\}, \quad (4.8)$$

where $(a)$ follows from the fact that $r_i$ and $\eta_i$ are independent random variables. Here, we suppose that all the users have the same rate requirement, so $r_i = r$. Then,

$$\mathbb{E}\{N\} \approx \frac{B}{r} \cdot \eta_{eq}, \quad (4.9)$$

where $\eta_{eq}$ is the equivalent spectral efficiency and is calculated as [83]

$$\eta_{eq} = \frac{1}{\mathbb{E}\{\frac{1}{\eta_i}\}}. \quad (4.10)$$

Therefore, to ensure that the equivalent spectral efficiency of the system is at least $\eta$ (the
value we used in initial estimation part), the following constraint should hold.

\[ \frac{1}{\mathbb{E}\{\frac{1}{\eta_i}\}} \geq \eta. \] (4.11)

### 4.3.2 Problem formulation

Our goal is to minimize the number of required DBSs so that the capacity and coverage constraints are satisfied. In doing so, the optimization problem is expressed as follows:

\[
\begin{align*}
\min_{x_j, y_j, h_j, (\epsilon_j)} & \quad \sum_{j=1}^{N_{BS}} \epsilon_j \quad (4.12a) \\
\text{subject to} & \quad \sum_{j=1}^{N_{BS}} N_{U_{BS}} m_{j,k} \geq D_k S_k, k = 1, \ldots, N_{\text{subarea}}, \quad (4.12b) \\
& \quad \sum_{i=1}^{L} \rho_i \geq \zeta L, \quad (4.12c) \\
& \quad \frac{1}{\mathbb{E}\{\frac{1}{\eta_i}\}} \geq \eta, \quad (4.12d)
\end{align*}
\]

where \(x_j, y_j, \) and \(h_j\) are 3D positions of the DBS \(j\). We also define the following indicator functions:

\[ \epsilon_j = \begin{cases} 
1, & \text{if DBS } j \text{ is used,} \\
0, & \text{if DBS } j \text{ is redundant},
\end{cases} \] (4.13)

and

\[ \rho_i = \begin{cases} 
1, & \text{if user } i \text{ is under the coverage of a DBS,} \\
0, & \text{otherwise.}
\end{cases} \] (4.14)

### 4.4 Proposed Algorithm

Finding BS locations while ensuring that nearly all the users are served and their traffic requirements are achieved is a very complicated problem in general. Adding a new dimension to the problem, which is the altitude of the aerial BSs, makes the problem even more complex. Meta heuristic algorithms such as genetic algorithm [84], simulated annealing [85],
PSO [86], tabu search [87], and ant colony optimization [88] are often used in such complex problems. Here we use PSO algorithm to find 3D placements of DBSs. The algorithm is an appropriately modified version of the one developed in [22] to fit our problem.

PSO is an optimization technique proposed by J. Kennedy and R. Eberhart in 1995 [86]. It is inspired by social behaviour of bird flocking or fish schooling. The algorithm starts with a population of random solutions and iteratively tries to improve the candidate solutions with regards to a given measure of quality. The best experience of each candidate as well as the best global experience of all the candidates in all iterations are recorded and the next movement of the candidates is influenced by these items.

In order to find 3D placements of DBSs, using PSO algorithm, we first consider the capacity constraint and find the locations of DBSs that minimize the below utility function.

$$U_1 = \sum_{k=1}^{N_{subarea}} \sum_{j=1}^{N_{BS}} \left\{ N_{BS} m_{j,k} - D_k S_k \right\}. \quad (4.15)$$

Then we use these 3D points as an initial solution and improve them so that the total number of uncovered users is minimized, taking into account that the capacity constraint should still hold. The utility function that satisfies coverage constraint, while keeping the capacity constraint active, is given below.

$$U_2 = \begin{cases} - \sum_{i=1}^{I} \rho_i, & \text{if } (4.5) \text{ holds,} \\ 0, & \text{otherwise.} \end{cases} \quad (4.16)$$

Finally, to satisfy (4.12d), we use the following utility function.

$$U_3 = \begin{cases} -I + \frac{1}{E\{\frac{1}{m}\}}, & \text{if } (4.5) \text{ holds,} \\ 0, & \text{otherwise.} \end{cases} \quad (4.17)$$

The PSO algorithm starts by generating $L$ particles of length $3 \times N_{BS}$ to form an initial population $W^l, l = 1, ..., L$. Each particle contains random positions of all the DBSs within
the region. The particle that provides the best utility in all the iterations is recorded as $W^{(global)}$. Also for each particle the best result is kept as $W^{(l,local)}$. In each iteration, $W^{(global)}$ and $W^{(l,local)}$ are updated and the velocity and movement of the particles are calculated based on them. The velocity term $V^{(l)}_w$, $w = 1, ..., 3N_{BS}$, at iteration $t + 1$ is computed as follows:

$$V^{(l)}_w(t + 1) = \phi V^{(l)}_w(t) + c_1\phi_1(W^{(l,local)}_w(t) - W^{(l)}_w(t)) + c_2\phi_2(W^{(global)}_w(t) - W^{(l)}_w(t)),$$  (4.18)

where $\phi$ is the inertia weight that controls speed of convergence. $c_1$ and $c_2$ are personal and global learning coefficients, and $\phi_1$ and $\phi_2$ are two random positive numbers. Afterward, the positions of the elements in a particle are updated as

$$W^{(l)}_w(t + 1) = W^{(l)}_w(t) + V^{(l)}_w(t + 1).$$  (4.19)

Details of the proposed algorithm is provided in Algorithm 2. This algorithm finds a feasible solution for DBS placements in the region so that at least $\zeta$ percent of all the users are served based on their data traffic requirements. After finding the 3D placement of the DBSs, we try to minimize their number by removing the ones whose elimination do not affect the quality of the network. It can iteratively be checked by removing one DBS in each iteration and then checking the constraints. If they hold, that DBS can be removed without violating the constraints. If more than one DBS can be removed based on this approach, at first step the one which results in fewer users disconnected from the system is selected as the redundant DBS. After removing this DBS, the algorithm is repeated until finally no redundant DBS remains.

4.5 Performance Evaluation

We consider an urban area with simulation parameters provided in Table 4.1. Matlab software is used as a simulation platform. The total area is 100 km$^2$; 1000 users are distributed in this area in a number of different ways.
Algorithm 2 PSO algorithm for 3D placement of DBSs

1: Generate an initial population including \( L \) random particles \( W(l)(0), l = 1, ..., L \). Each particle has size \( 3 \times N_{BS} \). Set \( t = 1, U = U_1, U^{(global)} = \min\{U(l)(0), l = 1, ..., L\} \) and \( U^{(local)} = U(l)(0) \).

2: while \( U^{(global)} > -I \) do
3: for \( l = 1, ..., L \) do
4: Compute \( V(l)(t), W(l)(t), U^{(l)}(t) \).
5: if \( U^{(l)}(t) < U^{(local)} \) then
6: \( W(l^{(local)}) = W(l)(t), U^{(l^{(local)})} = U^{(l)}(t) \).
7: if \( U^{(l^{(local)})} < U^{(global)} \) then
8: \( W^{(global)} = W(l^{(local)}), U^{(global)} = U^{(l^{(local)})} \).
9: end if
10: end if
11: end for
12: if \( U^{(global)} \leq 0 \) then
13: \( U = U_2 \).
14: end if
15: if \( U^{(global)} \leq -\zeta I \) then
16: \( U = U_3 \).
17: end if
18: \( t = t + 1 \).
19: end while

Table 4.1. Simulation parameters for Chapter 4

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_c )</td>
<td>2 GHz</td>
<td>( r )</td>
<td>1 Mbps</td>
</tr>
<tr>
<td>( B )</td>
<td>20 MHz</td>
<td>( P_t )</td>
<td>5 watts</td>
</tr>
<tr>
<td>( \eta )</td>
<td>1.7 bps/Hz</td>
<td>( \gamma_{th} )</td>
<td>-7 dB</td>
</tr>
<tr>
<td>( \zeta )</td>
<td>0.95</td>
<td>( h_{max} )</td>
<td>600 meters</td>
</tr>
</tbody>
</table>

In Scenario I, the total area is divided in two equal regions with 20 and 80 percent of the users uniformly distributed in the left and right regions, respectively. The initial estimation of the number of DBSs is 30; finally only 1 DBS is identified as redundant. The user distribution in this scenario and the 2D projection of DBSs locations, using PSO algorithm, are shown in Fig. 4.2a. As it is seen in this figure, the right region which has higher user density needs more DBSs to serve all the users and to avoid congestion. The Voronoi tessellation of the DBSs are also shown to give a better insight about the DBS placement. It should be noted that these lines do not show the actual frontiers for BS-user connection and the association policy is based on the best SINR. The 3D placement of the
Figure 4.2. Scenario I, (a) User distribution and 2D projection of DBSs locations, (b) 3D locations of DBSs. Users are uniformly distributed with different densities in the left and right regions. DBSs in the left and right regions are shown by green and red squares, respectively. Users are illustrated by blue dots.

DBSs in Scenario I is depicted in Fig. 4.2b. As seen in this figure, in the right region which has higher user density, the average altitude of the DBSs is less than that of in the left region. This is due to the fact that when a region is dense, all the resources of a DBS are utilized by the nearby users, as such a DBS usually can not serve farther away users.
Hence it is better that such a DBS decreases its altitude to make less interference to farther users which are served by another DBS. In the left region where the user density is lower, the DBSs increase their heights to decrease pathloss and cover more users in the region.
Figure 4.5. Scenario II, (a) User distribution and 2D projection of DBSs locations, (b) 3D locations of DBSs. 40 percent of users have a normal distribution in the central region and 60 percent of them are uniformly distributed in the remaining region. DBSs in the central region and in the remaining region are shown by red and green squares, respectively. Users are illustrated by blue dots.

The CDF curve for SINR distribution of the users and the convergence speed of the algorithm are shown in Fig. 4.3 and 4.4, respectively. It should be noted that the utility function in Fig. 4.4 is equal to $U_1$ if $U_1$ is greater than zero; the utility function for negative values of $U_1$ and also if $U_2$ is greater than $-\zeta N_U$ is equal to $U_2$; otherwise, it equals $U_3$.  

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In Scenario II, 40 percent of the users are normally distributed with standard deviation 1000 meters in the central region. The other 60 percent of the users are uniformly distributed in the remaining region. In this scenario, the final number of required DBSs to serve all the users is 29. The user distribution and the 2D projection of DBSs locations is depicted in Fig. 4.5. As seen in this figure, in the central region which has higher user density, the DBSs are closer to each other compared to the other subarea. Also like previous scenario, in the region with higher user density, the DBSs are placed in lower altitudes to make less interference for the users served by other DBSs.

Fig. 4.6 and 4.7 show the CDF curve for SINR distribution and the convergence speed of the algorithm in Scenario II, respectively.

4.6 Conclusions

In this chapter, we provided a new DBSs deployment plan, while minimizing the number of them, in order to serve the users based on their traffic requirements. Generally this is an optimization problem which is too complex to solve; therefore, a heuristic algorithm based on PSO was proposed. The number of DBSs and their 3D placements were estimated, while
Figure 4.7. Convergence speed of the PSO algorithm in Scenario II (refer to Figure 4.5).

satisfying coverage and capacity constraints of the system. Afterwards, the DBSs whose removal did not affect the QoS of the users, were removed. Simulation results considering regions with different user densities, confirmed the acceptable performance of the proposed method. It was observed that the number of DBSs in an area is proportional to the user density in that area. It was also noted that DBSs can change their altitudes in order to tackle coverage and capacity issues. A DBS decreases its altitude in a dense area to reduce interference to the users that are not served by it and increases its altitude to cover a larger area in a low density region.
Chapter 5

Backhaul-Aware Robust 3D Placement of Drone BSs

5.1 Introduction

A major difference between a ground BS and a DBS is that the latter one has a major limitation in the backhaul link. A ground BS usually has a fixed wired/wireless backhaul connection and can relatively offer very high data rates to a core network. A DBS on the other hand, should have a wireless backhaul; therefore, the peak data rate a DBS can support is limited and it may dramatically decrease due to inclement weather conditions especially if the link is based on the FSO or mmWave technology. Therefore, it is important to consider the limitations and requirements of the wireless backhaul link as one of the constraints in designing and deploying DBSs in future 5G+ networks. In this chapter we propose a backhaul limited optimal DBS placement algorithm for various network design parameters, such as the number of the served users or the sum-rate of the served users for heterogeneous rate requirements in a clustered user distribution. We also investigate the robustness of DBS placement and study how much the users movements may affect the proposed optimal solution.
5.2 System Model

We consider an urban area with a number of users. The goal is to find the optimum location of a DBS that maximizes our objective function. Our assumptions are as follows:

- There is no ground BS in the area.
- There is one DBS in the system; therefore, no co-channel interference is available.
- The system is in downlink mode.
- The DBS has a fixed limited backhaul capacity.

5.2.1 Pathloss model

The average pathloss model in (2.4) is adopted here.

5.2.2 Spatial users distribution

To obtain heterogeneity in spatial user distribution, we utilize a Matérn cluster process [89,90]. It is a doubly Poisson cluster process, where parent points which are the center of clusters are created by a homogeneous Poisson process. The daughter points, that represent users in our model, are uniformly scattered in circles with radius $R$ around parent points by using another homogeneous spatial Poisson process. Thus, the density function, $f(z)$, of a given user in location $z$ is

\[
f(z) = \begin{cases} 
\frac{1}{\pi R^2}, & \text{if } ||z|| \leq R, \\
0, & \text{otherwise}. 
\end{cases} \tag{5.1}
\]

5.3 Backhaul-Aware DBS Placement

We assume that an area is already covered by a number of ground BSs, but due to an extensive temporal increase in the number of users or their required rates, some of them
can not be served by the terrestrial network due to the lack of resources such as bandwidth. We propose to integrate a DBS with the existing cellular network infrastructure to offer coverage to such users. User selection may change based on the chosen approach, whether it is network-centric or user-centric. The users are assumed to operate different applications with a variety of rate requirements. The total bandwidth of the DBS and the wireless backhaul peak rate are the limiting factors in our formulation.

5.3.1 System constraints

5.3.1.1 Backhaul capacity constraint

For the backhaul constraint, we assume that the peak aggregate rate that the wireless backhaul link of a DBS can support is $C$ Mbps; so,

$$\sum_{i=1}^{I} r_i \cdot \rho_i \leq C,$$

(5.2)

where $I$ stands for the total number of users that are not served by the terrestrial network, $r_i$ denotes the data rate required by user $i$, and $\rho_i$ is the user indicator function defined as

$$\rho_i = \begin{cases} 1, & \text{if user } i \text{ is served by the DBS,} \\ 0, & \text{otherwise.} \end{cases}$$

(5.3)

5.3.1.2 Bandwidth allocation constraint

Another limiting factor is the total available bandwidth of the DBS. It can be formulated as

$$\sum_{i=1}^{I} b_i \cdot \rho_i \leq B,$$

(5.4)

where $B$ stands for the total bandwidth of the DBS, and $b_i$ denotes the bandwidth required by user $i$ which is equal to $\frac{r_i}{\eta_i}$, where $\eta_i = \log_2(1 + \gamma_i)$ represents the spectral efficiency and $\gamma_i$ stands for the SNR of user $i$. 

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5.3.1.3 Coverage constraints

We assume that a user is in the coverage of the DBS if its QoS requirement is satisfied. It can be formulated as

\[ \text{PL}_i \cdot \rho_i \leq \text{PL}_{\text{max}}, \forall i, \]  

(5.5)

where \( \text{PL}_i \) stands for the pathloss when the signal is received by user \( i \) and \( \text{PL}_{\text{max}} \) is the maximum pathloss that a user can tolerate before outage based on its QoS requirement. It should be noted that when there is no interference in the system, (5.5) is exactly equivalent to \( \gamma_i \geq \gamma_{\text{min}} \), where \( \gamma_i \) is the received SNR of user \( i \) and \( \gamma_{\text{min}} \) is the minimum received SNR before outage.

5.3.2 Problem formulation

Our optimization problem is formulated as follows:

\[
\begin{align*}
\max_{x,y,h,\{\rho_i\}} & \quad \sum_{i=1}^{I} \alpha_i \cdot \rho_i \\
\text{subject to} & \quad \sum_{i=1}^{I} r_i \cdot \rho_i \leq C, \\
& \quad \sum_{i=1}^{I} b_i \cdot \rho_i \leq B, \\
& \quad \text{PL}_i \cdot I_i \leq \text{PL}_{\text{max}}, \quad \forall i, \\
& \quad x_{\text{min}} \leq x \leq x_{\text{max}}, \\
& \quad y_{\text{min}} \leq y \leq y_{\text{max}}, \\
& \quad h_{\text{min}} \leq h \leq h_{\text{max}}, \\
& \quad \rho_i \in \{0,1\}, \quad \forall i,
\end{align*}
\]

where \( x, y, \) and \( h \) are 3D coordinates of the DBS placement. Variables \( x_{\text{min}}, x_{\text{max}}, y_{\text{min}}, \) and \( y_{\text{max}} \) represent the limits of the area coordinates and \( h_{\text{min}} \) and \( h_{\text{max}} \) are the minimum and maximum allowed altitudes of the DBS, respectively. The maximum height of a DBS
depends on its type, size, weight, power of the battery, and other features. It may also be limited by regulatory laws. Several organizations such as US Federal Aviation Authority, transport Canada, and Canadian Aviation Regulation Advisory Council are working to coordinate such laws [91]. Variable $\alpha_i$ is a coefficient related to user $i$ and it is determined based on the system, whether it is network-centric or user-centric. It also depends on the metric that is used to identify the priority of a user. These concepts will be explained in more details later in Section 5.3.4.1.

### 5.3.3 Proposed algorithm

We find the best 3D placement of a DBS that maximizes the number of users served with higher priority through an exhaustive search. In each candidate coordinates of the DBS, the problem can be transformed to a binary integer linear programming (BILP) as given below, which can then be solved through the B&B method.

\[
\max_{(\rho_i)} \sum_{i=1}^{I} \alpha_i \cdot \rho_i \quad (5.7)
\]

subject to \((5.6b), (5.6c), (5.6d), (5.6h)\).

B&B algorithm can find the global optimum in discrete and combinatorial optimization problems. It systematically enumerates candidate solutions by means of state space search. The set of candidate solutions form a rooted tree with the full set at the root. The algorithm explores branches of this tree, which are subsets of the solution set. Before enumerating a branch, it is compared with the upper and lower estimated bounds on the optimal solution. The branch is discarded if it cannot find a better solution than the best one found so far by the algorithm.

We consider an urban region with a total area of 16 km$^2$. For the user distribution, we suppose that the parents are created by a Poisson process with an average density of $10^{-7}$ per m$^2$ and daughters follow another Poisson distribution with an average density of $6 \times 10^{-3}$ per m$^2$. The cluster radius is taken as 700 meters. The step size to search 3D
location of the DBS is 100 meters. Other simulation parameters are provided in Table 5.1. Matlab software is used to carry out the simulations.

5.3.4 Performance evaluation

5.3.4.1 Network-centric versus user-centric

The network may select the users based on the network-centric or the user-centric approach. In the network-centric approach, the network tries to serve as many users as possible, regardless of their rate requirements. As a result, the majority of the served users are the ones who need less data rates. In this approach, \( \alpha_i \) in (5.7) is equal to 1 for all the users. In the user-centric approach, values of \( \alpha_i \) vary with the users and they are determined based on the priority of users. A large number of existing and future applications may require differentiation among the users and applications; therefore, offering service only to the users with low rates would not be fair. There are different metrics such as the sum-rate, price differentiation, signal strength, and content demand to identify users priorities. These metrics are explained below:

- **sum-rate**: One method of selecting users is to maximize the total sum-rate. In this way, by setting \( \alpha_i \) equal to \( r_i \), the users who require higher data rates would be given higher priority to access the network. In this chapter, we use such metric in the user-centric approach.

- **price differentiation**: Users may be categorized based on how much they are willing to pay for their subscribed services, for instance, as platinum, gold, and silver users.
The platinum users who pay higher, want to be connected to the network under almost every conditions, even if their channel is poor or they need high amount of resources. By assigning a large value to $\alpha_i$ of such users, the service provider guarantees that they are served.

- **signal strength:** The selection of the users can be based on their received signal strength, so the operator first serves the ones who have favorable channel conditions.

- **content demand:** In content-aware systems, the users who need to access the network urgently based on their required content, are given higher priority.

The user distribution and the 3D placement of a DBS in a network-centric and a user-centric approach are shown in Fig. 5.1a and 5.1b, respectively. It is observed that in both approaches the DBS moves to the highest possible altitude ($h_{\text{max}}$) to cover a larger area. As seen in this figure, in the network-centric approach more users are served compared to the user-centric approach. There is a license fee related to spectrum usage that a service provider has to pay which is based on how much bandwidth per person is utilized over a geographical area [92]. Therefore, the network-centric approach may be a more favorable option for a service provider as it pays less for the spectrum usage.

In Fig. 5.2 the CDF of required rates of the served users for both approaches is depicted. As seen in this figure, the CDF curve related to the network-centric approach is above the user-centric one, meaning that in the former one, there is a higher probability of serving users with lower rates. Therefore, more users in total are served in the network-centric approach as it has been seen earlier in Fig. 5.1.

5.3.4.2 Backhaul limitation

The backhaul link in a wireless system may be dedicated or in-band.

- **dedicated backhaul:** Dedicated backhaul may be FSO or mmWave link between access and core networks. Such links can provide very high backhaul capacity, but they are very sensitive to weather conditions; in foggy or rainy weather, the peak rate may dramatically decrease [93].
Figure 5.1. User distribution and 3D DBS placement in (a) network-centric and (b) user-centric approaches. The DBS and its projection on XY-plane are shown in asterisk and red circles, respectively.

- **in-band backhaul:** Currently in LTE, Wi-Fi, WiMAX, and HSPA networks, the main technology used for the wireless backhaul links is based on RF microwave [94]. Microwave backhaul can be deployed very quickly at a relatively low cost. By using RF for backhaul, the same spectrum is used in both the access and backhaul links, so it causes interference and the capacity of the backhaul connection is affected.
Fig. 5.3 compares the number of served users versus different wireless backhaul peak rates of a DBS in the network-centric and user-centric approaches. This range of wireless backhaul rates represents the various rates of different types of wireless links. As seen in this figure, low backhaul rates can severely limit the number of served users. By increasing the backhaul capacity, the number of served users is increased differently in two scenarios. It will stop increasing when the backhaul capacity is around 150 Mbps as there is no more spectrum resource in the DBS to serve more users. The speed of increase in the number of served users is almost fixed in the user-centric approach (see fixed slope of the yellow dashed line in Fig. 5.3), while it is decreasing in the network-centric approach (see decreasing slope of the blue dashed lines in Fig. 5.3). The fixed slope in the user-centric approach is due to the fact that in this scenario, high rate users are served first and when wireless backhaul capacity increases, low rate users receive service, so the amount of increase in the number of served users remain fixed. In the network-centric approach, the slope is not fixed, because low rate users are served first in this scenario; therefore, only a few high rate users get service by increasing the backhaul capacity and the amount of increment is reduced in each step of increasing the backhaul capacity.
Figure 5.3. Number of served users versus different wireless backhaul rates.

### 5.3.4.3 Robustness

Mobile DBSs change the radio channel persistently, so highly complicated interference management and resource allocation schemes are required. Moreover, constant movements of a DBS consume a lot of battery and decrease flight time. Hence, if a DBS flies to a predetermined good position and is not required to change its place constantly due to users movements, this will result in savings in energy and reduction in complexity. Fig. 5.4a shows the impact of users movements on the performance of the network if the DBS stays in its position. As seen, by increasing the movement distance, number of the served users will decrease, but this reduction is not significant and as Fig. 5.4b demonstrates, a very low percentage of users would be dropped out of the network if they move. For instance, if the users are moving within 100 meters, less that 2% of them in the network-centric approach and less than 1% in the user-centric approach would be disconnected. Therefore, the solution is robust. If a DBS flies to a suitable place, it can stay there for a while unless the network reaches a particular pre-determined user-dropped out threshold.
5.4 Conclusions

In this chapter, the optimal 3D placement of a DBS over an urban area with users having different rate requirements was investigated. The wireless backhaul peak rate and the bandwidth of a DBS were considered as the limiting factors in both network-centric and user-centric approaches. The network-centric approach maximized the total number of
served users regardless of their required rates, while the user-centric approach maximized their sum-rate. Our investigation results also showed that only a small percentage of the total served users would be in outage when the users move. This highlights the robustness of the proposed algorithm against the modest movement of users (within few meters).
Chapter 6

User Association and Bandwidth Allocation for Terrestrial and Aerial BSs with Backhaul Considerations

6.1 Introduction

In future wireless networks, three general use cases are being considered: eMBB, mMTC, and uRLLC [7]. All of these use cases have challenging demands and they are very different from each other. For instance, while mMTC applications tolerate low data rates and large delays, uRLLC applications can become very demanding to provide reliability and low-latency requirements. In such cases, providing isolated routes and caching to reduce latency, and allocating more wireless resources to provide reliability may be necessary [95]. In this chapter, we assume uRLLC users with delay-sensitive applications co-exist with regular eMBB users and jointly optimize resource allocations, user associations considering user types, and 3D placements of DBSs. Moreover, on the contrary to many other studies, the existence of a MBS is also taken into account. We assume that DBSs are associated with the MBS for backhauling; therefore, the backhaul capacity of a DBS is limited and depends on the available bandwidth and the distance between it and the MBS.
6.2 System Model

We consider a downlink wireless HetNet including two tiers of BSs, an MBS and a number of DBSs. DBSs are utilized to serve as small-cells assisting the wireless network in cases where the existing infrastructure is insufficient to address the demand. They are assumed to utilize wireless connection for both access and backhaul links. On the one hand, wireless links provide a mobility advantage to DBSs such that they can be positioned with respect to the users, which can increase spectral efficiency and decrease average pathloss. On the other hand, wireless links can be less reliable compared to wired connections, and energy expenditure increases too. Therefore, a careful system design is key.

We denote by $I$ the set of users, and $J$ the set of BSs. We use $i \in I = \{1, 2, ..., I\}$ and $j \in J = \{0, 1, ..., J\}$ to index users and BSs, respectively. Index 0 in $J$ denotes the only MBS considered in this system. We assume that high capacity fiber links carry information from the MBS to the core network; therefore, there is no congestion in the backhaul link of the MBS. We also assume that in-band wireless backhaul is employed for DBSs and the MBS is utilized as a hub to connect DBSs to the network. To avoid self-interference, orthogonal frequency channels in the backhaul and access side of the DBSs is employed; therefore, part of the bandwidth is shared between the access side of the MBS and DBSs and the remainder is dedicated for the backhaul of DBSs. The FSPL according to (2.2) is considered for backhaul links.

We assume there are wireless point-to-point $X_n$ links between BSs, which do not interfere with access and backhaul links. Considering non-ideal $X_n$ connections and the energy cost of wireless links, real-time coordination for interference management among BSs may not be efficient. Hence, to decrease inter-cell interference, reverse time division duplex is employed, which uses reversed uplink/downlink time slot configurations for MBS and DBSs [96]. When the MBS is in downlink mode, the DBSs are in the uplink mode. As a result, the only interference the MBS users receive is from the DBS users, which is negligible as the transmit power of user equipments is lower than that of an MBS. We also consider that the air-to-ground access links from DBSs to users and the air-to-air backhaul links among DBSs are operated over orthogonal frequency bands. Our assumptions are
summarized as follows:

- There is a MBS in the area.
- There are a fixed number of DBSs in the system.
- Backhaul and access spectrum of DBSs are orthogonal; therefore, there is no self interference.
- MBS and DBSs employ reverse time division duplex; therefore, there is no inter-cell interference between them.
- There is co-channel interference between DBSs as they share the same spectrum.
- The system is in downlink mode.
- The MBS serves as a backhaul source for DBSs and therefore, they have a variable limited backhaul capacity.

6.2.1 Channel models

In this chapter, we adopt the pathloss given in (2.1) and consider that the probability of having a LoS connection between a DBS and a user is formulated as (2.3). Moreover, we assume that DBSs are equipped with directional antennas which their gain can be approximated by [46]

$$G = \begin{cases} G_0, & -\frac{\theta_B}{2} \leq \phi \leq \frac{\theta_B}{2}, \\ g(\phi), & \text{otherwise}, \end{cases}$$

(6.1)

where $|\phi| = 90 - \theta$, $\theta_B$ denotes the DBS directional antenna’s half-power beamwidth as shown in Fig. 6.1 and $G_0 \approx \frac{30,000}{\theta_B}$ is the maximum gain of the directional antenna [97]. We assume $g(\phi)$ is negligible.

We adopt the MBS channel model from 3GPP TR 36.942 [98]. The average path loss in dB can be expressed as $128.1 + 37.6 \log_{10}(d')$, where $d'$ is the distance between the transmitter and receiver in kilometers. Also, the lognormal shadowing with standard
deviation 10 dB is assumed. Moreover, an omni-directional antenna is considered for the MBS in our model.

6.3 Optimization Problem

The mobility of DBSs and different types of users require that the following key issues are considered to provide wireless services efficiently:

- Finding the locations of DBSs,
- Determining the user-BS associations with consideration to user type,
- Bandwidth allocation for access and backhaul links.

6.3.1 System constraints

6.3.1.1 BS association constraints

In our framework, a user cannot be associated with more than one BS; therefore,

$$
\sum_{j \in J} \rho_{ij} = 1, \forall i \in I,
$$

(6.2)

where $\rho_{ij} \in \{0,1\}$ is the binary association indicator variable for user $i$ and BS $j$, and 1 indicates association.
6.3.1.2 Bandwidth allocation constraints

If the total bandwidth in the network is unity, we can denote the part assigned to backhaul of DBSs with $\alpha$, and the part assigned to the access of both the MBS and DBSs with $1 - \alpha$. The total amount of resources allocated by each BS to all the users cannot exceed its available bandwidth; therefore,

$$\sum_{i \in I} \rho_{ij} \cdot \beta_{ij} \leq 1 - \alpha, \forall j \in J,$$

(6.3)

where $\beta_{ij} \in [0, 1]$ is resource amount that is assigned to user $i$ from BS $j$.

6.3.1.3 Backhaul capacity constraints

The total data rate a DBS can support should not exceed its backhaul capacity; so,

$$\sum_{i \in J} r_{ij} \leq C_j, \forall j \in J\setminus 0,$$

(6.4)

where $r_{ij}$ is the total data rate of user $i$ receiving from BS $j$ and $C_j$ is the backhaul capacity of DBS $j$. Assuming Shannon capacity is achieved, $C_j$ can be written as

$$C_j = \alpha \cdot \eta_{j0},$$

(6.5)

where $\eta_{j0} = \log_2(1 + \gamma_{j0})$ and $\gamma_{j0}$ is the received SNR at the DBS $j$ from the MBS for the backhaul connection. It is equal to $\gamma_{j0} = \frac{P_0 g_{j0}}{P_N}$, $j \in J \setminus 0$, where $P_0$ denotes the transmit power of the MBS, $g_{j0}$ stands for the channel gain between the MBS and DBS $j$, and $P_N$ denotes the noise power. Similarly,

$$r_{ij} = \rho_{ij} \cdot \beta_{ij} \cdot \eta_{ij},$$

(6.6)
where $\eta_{ij}$ is the spectral efficiency of user $i$ associated with BS $j$, and is equal to $\log_2(1+\gamma_{ij})$. Moreover,

$$\gamma_{ij} = \begin{cases} 
\frac{P_{j}g_{ij}}{\sum_{l\in J\setminus j} P_{l}g_{il} + P_{N}}, & j \in J\setminus 0, \quad \text{(6.7a)} \\
\frac{P_{j}g_{ij}}{P_{N}}, & j = 0, \quad \text{(6.7b)}
\end{cases}$$

where $P_{j}$ denotes the transmit power of BS $j$ and $g_{ij}$ stands for the channel gain between user $i$ and BS $j$. Note that, (6.7a) is the received SINR of user $i$ if it is associated with DBS $j$, while (6.7b) is the received SNR of user $i$ if it is associated with the MBS.

### 6.3.1.4 User type constraints

As wireless backhaul links for DBSs may increase the latency, we assume that uRLLC users utilize a delay-sensitive application, and can only be associated with the MBS, which can be formulated as

$$\sum_{j\in J\setminus 0} \rho_{ij} \leq 1 - \tau_i, \forall i \in I, \quad \text{(6.8)}$$

where $\tau_i \in \{0, 1\}$; $\tau_i = 1$ indicates that the user $i$ is delay-sensitive and $\tau_i = 0$ indicates the opposite.

### 6.3.1.5 DBS association constraints

The probability of LoS connection is usually high in DBSs and as all of them share the same bandwidth, it might cause high interference. To mitigate such interference, we assume that DBSs are equipped with directional antennas and that only the users in the footprint of a DBS antenna coverage can be served by it. It can be formulated as

$$\rho_{ij} \cdot (\theta_{ij} - \theta^{*}) \geq 0, \forall i \in I, \forall j \in J\setminus 0, \quad \text{(6.9)}$$

where $\theta_{ij}$ is the elevation angle between user $i$ and DBS $j$, and $\theta^{*} = 90 - \frac{\eta_{ij}}{2}$.
6.3.1.6 Interference prevention constraints

Note that the coverage radius, \( u_j \), of DBS \( j \) is related to its altitude as \( \tan(\theta^*) = \frac{h_j}{u_j} \), where \( h_j \) is the altitude of DBS \( j \). Accordingly, the minimum distance required between two DBSs to prevent interference can be written as follows (Fig. 6.2):

\[
\frac{h_j + h_j'}{\tan(\theta^*)}, \forall j, j' \in J \setminus 0.
\] (6.10)

6.3.2 Problem formulation

In order to prevent over-loading in some DBSs (e.g., if the users are clustered) we consider proportional fairness. Therefore, a logarithmic utility function is assumed, where \( U(r_i) = \log r_i \). Hence, the problem formulation also considers fairness and can be cast as the following mixed-integer optimization problem:

\[
\max_{\{u_j \geq 0\}, \{\rho_{ij}\}, \{\beta_{ij}\}, \{\alpha\}} \sum_{i \in J} U(r_i) \quad \text{(6.11a)}
\]

subject to

\[
\sum_{j \in J} \rho_{ij} = 1, \forall i \in I, \quad \text{(6.11b)}
\]

\[
\sum_{i \in I} \rho_{ij} \cdot \beta_{ij} \leq 1 - \alpha, \forall j \in J, \quad \text{(6.11c)}
\]
\[
\sum_{i \in I} \rho_{ij} \cdot \beta_{ij} \cdot \eta_{ij} \leq \alpha \cdot \eta_{j0}, \forall j \in \mathcal{J} \setminus \{0\}, \quad (6.11d)
\]
\[
\sum_{j \in \mathcal{J} \setminus \{0\}} \rho_{ij} \leq 1 - \tau_i, \forall i \in \mathcal{I}, \quad (6.11e)
\]
\[
\rho_{ij} \cdot (\theta_{ij} - \theta^*) \geq 0, \forall i \in \mathcal{I}, \forall j \in \mathcal{J} \setminus \{0\}, \quad (6.11f)
\]
\[
D_{ij'} \geq \frac{h_j + h_{j'}}{\tan(\theta^*)}, \forall j, j' \in \mathcal{J} \setminus \{0\}, j \neq j'. \quad (6.11g)
\]
\[
\rho_{ij} \in \{0, 1\}, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \quad (6.11h)
\]
\[
\alpha \in [0, 1], \quad (6.11i)
\]
\[
\beta_{ij} \in [0, 1 - \alpha], \forall i \in \mathcal{I}, \forall j \in \mathcal{J}, \quad (6.11j)
\]

where \(l_{j \in \mathcal{J} \setminus \{0\}}\) is the 3D location of DBS \(j\) and \(r_i = \sum_{j \in \mathcal{J}} r_{ij}\) is the total rate of user \(i\).

Equal resource allocation is the optimal allocation for the logarithmic utility \([99]\); therefore, \(\beta_{ij} = \frac{1 - \alpha}{\sum_{k \in \mathcal{J}} \rho_{kj}}\) and the problem is transformed to

\[
\max_{\{l_{j \in \mathcal{J} \setminus \{0\}}\}, \{\rho_{ij}\}, \{\alpha\}} \sum_{i \in \mathcal{I}} \sum_{j \in \mathcal{J}} \rho_{ij} \log \frac{\eta_{ij} \cdot (1 - \alpha)}{\sum_{k \in \mathcal{J}} \rho_{kj}} \quad (6.12a)
\]

subject to

\[
\sum_{i \in \mathcal{I}} \rho_{ij} \cdot \frac{1 - \alpha}{\sum_{k \in \mathcal{J}} \rho_{kj}} \cdot \eta_{ij} \leq \alpha \cdot \eta_{j0}, \forall j \in \mathcal{J} \setminus \{0\}, \quad (6.12b)
\]

\((6.11b), (6.11e) - (6.11i)\).

Even after the above simplification, the optimization problem has a non-convex objective function with non-linear constraints and a combination of binary and continuous variables. In other words, it is a non-convex mixed-integer, NP-hard optimization problem.

### 6.4 Proposed Algorithm

To alleviate the difficulties mentioned in the preceding section, we first relax the binary cell association indicator, \(\rho_{ij}\); therefore,

\[
\rho_{ij} \in [0, 1], \forall i \in \mathcal{I}, \forall j \in \mathcal{J}. \quad (6.13)
\]
It upper bounds the performance and corresponds to the case where users can be associated with multiple BSs. Then, for fixed DBS locations, the optimization problem becomes a separable problem in $\rho_{ij}$ and $\alpha$ and can be solved through a primal decomposition algorithm [96]. This procedure includes three main processes:

1. The user-BS association problem can be written as a convex subproblem for a fixed $\alpha$, as

$$\max_{\{\rho_{ij}\}} \sum_{i \in I} \sum_{j \in J} \rho_{ij} \log \frac{\eta_{ij} \cdot (1 - \alpha)}{\sum_{k \in J} \rho_{kj}}$$  \hspace{1cm} (6.14a)

subject to  \hspace{1cm} (6.11b), (6.11e), (6.11f), (6.12b), (6.13).

This can be solved with convex optimization tools efficiently.

2. After finding $\rho_{ij}$ and rounding it, the following master problem, which is also convex, is solved.

$$\max \alpha \log (1 - \alpha)$$  \hspace{1cm} (6.15a)

subject to  \hspace{1cm} (6.11i), (6.12b).

Each iteration of the master problem requires solving the subproblem and updating $\rho_{ij}$ variables.

3. After finding variables $\rho_{ij}$ and $\alpha$, the location of DBSs is updated through the PSO method by maximizing utility function (6.16) where the required constraints are added as penalty functions to the objective function.

$$\sum_{i \in I} \sum_{j \in J} \rho_{ij} \log \frac{\eta_{ij} \cdot (1 - \alpha)}{\sum_{k \in J} \rho_{kj}} - \sum_{j, j' \in J \setminus \{0, \bar{j}, \bar{j}'} \left( \tan(\theta^*) \cdot D_{jj'} - h_j - h_{j'} \right) - \sum_{j \in J \setminus 0} \left( \alpha \cdot \eta_{j0} - \sum_{i \in I} \rho_{ij} \cdot \frac{(1 - \alpha) \cdot \eta_{ij}}{\sum_{k \in J} \rho_{kj}} - \sum_{i \in I} \rho_{ij} \cdot (\theta_{ij} - \theta^*) \right).$$  \hspace{1cm} (6.16)

Processes 1-3 are repeated until convergence is reached. The proposed algorithm is sum-
Algorithm 3  Finds 3D locations of DBSs, user-BS association and bandwidth allocation for access and backhaul of DBSs.

1: Inputs: Users locations, number of DBSs.
2: Initialization: Cluster the users based on the number of DBSs using k-means clustering. Assume the initial location of DBSs is the center of the clusters. Set $t = 1$, $m(t) = U(t) - U(t-1)$, $n(t') = U(t') - U(t' - 1)$; define $\alpha(1) = a$, $m(1) = n(1) = M$, where $M$ is a big number. $\nu$ and $\epsilon$ are small positive numbers.
3: while $n(t') \geq \nu$ do
4:     while $m(t) \geq \epsilon$ do
5:         Find $\rho_{ij}(t), U(t)$. Round $\rho_{ij}(t)$.
6:         $t = t + 1$.
7:     Find $\alpha(t), U(t)$.
8: end while
9: Find $U(t')$.
10: $t' = t' + 1$.
11: Update the 3D locations of DBSs using PSO algorithm. Find $U(t')$.
12: end while

6.5 Performance Evaluation

We consider an urban region with total area 250000 m$^2$, which is served by one MBS in the center of the area, and 3 DBSs at locations and altitudes to be determined. We assume that users have a Matérn distribution which is a doubly Poisson cluster process [89] as defined in Section 5.2.2. The heterogeneity of the users distribution is measured by the coefficient of variation (CoV) of the Voronoi area of the users [100, 101]. CoV is a scalar metric that measures the regularity of the user locations. It is defined as $\frac{1}{0.529} \frac{\sigma_V}{\mu_V}$, where $\sigma_V$ and $\mu_V$ are the standard deviation and the mean of the Voronoi tessellation areas of the users, respectively. CoV=1 corresponds to the Poisson point process, while CoV>1 represents clustered distribution of the users. To better visualize different CoV values, Fig. 6.3 shows sample user distributions with their corresponding CoVs. As we can see, higher CoV means that users are more clustered.

The probability of being a delay-sensitive or delay-tolerant user is 0.2 and 0.8, respectively. All results are averaged over 100 Monte Carlo simulations. The urban environment parameters are taken from Table 2.1 and the simulation parameters are provided in Ta-
Figure 6.3. Sample user distributions with different CoV values, (a) CoV=1, (b) CoV=2, and (c) CoV=4.

Table 6.1. Simulation parameters for Chapter 6

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_c$</td>
<td>2 GHz</td>
<td>$N_0$</td>
<td>-174 dBm/Hz</td>
</tr>
<tr>
<td>$P_0$</td>
<td>46 dBm</td>
<td>$P_j$, $\forall j \in \mathcal{J}\setminus0$</td>
<td>36 dBm</td>
</tr>
<tr>
<td>$h_{\text{max}}$</td>
<td>500 m</td>
<td>System Bandwidth</td>
<td>10 MHz</td>
</tr>
</tbody>
</table>

A random realization of the user distribution along with the BSs are shown in Fig. 6.4. The users that associate with different BSs are specified by different colours. It is observed that if a DBS has to serve farther users, it has to increase its altitude. Note that, a higher altitude creates a trade-off by yielding a larger probability of LoS links, as well as a higher pathloss, as can be seen in (2.2) and (2.3).

Fig. 6.5 illustrates the empirical CDFs of the users’ rates for two different CoVs along with exemplary distributions corresponding to each CoV. It is observed that in a more clustered distribution, the probability that each user receives a higher rate increases. This confirms that the proposed algorithm can increase the performance of the cellular network in terms of users’ satisfactions in more clustered distributions.

The number of users associated with both the MBS and the DBSs are depicted in Fig. 6.6. By increasing the CoV, more users could be associated with the DBSs which results in better load balancing in the system. On the one hand, clustered users can be covered with DBSs at lower altitudes, which can increase SNR (and rate), similar to the case with green users in Fig. 6.4. On the other hand, increasing the number of users served by DBSs,
Figure 6.4 A typical user distribution with CoV=3.3 along with the MBS and 3D placement of DBSs. The MBS is shown in a black square. DBSs and their projection on an XY-plane are shown using asterisk and red circles, respectively. Delay-sensitive and delay-tolerant users are shown by diamonds and circles, respectively. Also, different colours of users demonstrate association with different BSs.

Figure 6.5. CDF of users’ rates for two different CoVs.

decreases the wireless resources allocated to each user. Fig. 6.5 and Fig. 6.6 together show that this trade-off is in favor of rate, when CoV increases.

Finally, Fig. 6.7 illustrates the total rate of users associated with the DBSs for different
Figure 6.6. Number of users associated with the MBS and the DBSs for different CoV values.

Figure 6.7. Total rate of the users for different number of DBSs.

half-power beamwidths, $\theta_B$’s, and number of utilized DBSs. Note that, increasing $\theta_B$ increases the maximum possible coverage area. However, it also increases $D$ in (6.11g), which means that to prevent overlapping, DBSs have to keep a larger distance between each other. Hence, in Fig. 6.7, the total capacity of users decreases, although the coverage radius increases with increasing $\theta_B$. Moreover, the effect of $\theta_B$ becomes more severe as
the number of utilized DBSs increases. These results show that it is necessary to develop efficient interference cancellation methods for dense deployments of DBSs, since preventing overlaps between DBSs causes significant performance loss.

6.6 Conclusions

In this chapter, delay-sensitive users were associated with the MBS, while delay-tolerant users could be associated with either one of the BSs. As all the DBSs shared the same bandwidth, using directional antennas was proposed to relieve the effect of the interference. User-BS association and access and backhaul bandwidth allocation were found through a decomposition method and the locations of DBSs were updated using a PSO algorithm. Also, further insights on the effects of CoV and half-power beamwidth were obtained by simulations. The results showed that utilizing DBSs in case of clustered users could increase total rate of the users associated with DBSs, despite depleting the resources. In order to prevent interference, overlaps of coverage areas of different DBSs are not allowed in many studies. However, the half-power beamwidth should be chosen carefully for these scenarios as the results showed that increasing the beamwidth could decrease total rate by preventing DBSs to be deployed in beneficial locations.
Chapter 7

Wireless Networks with Cache-Enabled and Backhaul-Limited Aerial BSs

7.1 Introduction

Backhaul capacity constraint is an important limitation that should be considered in designing networks with DBSs. To alleviate this limitation, content-caching in DBSs is proposed. Caching the popular content can also decrease the latency of the network which is essential for delay-sensitive applications. To this end, in this chapter we provide a novel framework that accounts for limited backhaul capacity in DBSs while serving users with various QoS and rate requirements. In so doing, we propose an algorithm that determines backhaul-aware 3D placement of DBSs while minimizing their total transmit power. The proposed algorithm also finds user-BS associations and their corresponding bandwidth allocations in the network.

7.2 System Model

We consider a downlink wireless HetNet including two tiers of BSs, an MBS, and several DBSs as depicted in Fig. 7.1. DBSs utilize wireless connections for both access and backhaul
Figure 7.1. Graphical illustration for the integration of cache-enabled DBSs in a cellular network. Only users that are in LoS coverage of a DBS can be associated with it. User (a) is delay-sensitive and its requested content is not available in the DBS; therefore, it has to be served by the MBS. User (b) is delay-tolerant; therefore, it can connect to the DBS, even though its requested content is not available in that DBS. User (c) requests the data that is cached in the DBS; so whether it is delay-tolerant or delay-sensitive does not matter, and it can be served by the DBS. The DBS park is for energy charging and content caching.

links. We assume that high capacity fibre links carry information from the MBS to the core network; therefore, there is no congestion in the backhaul link of the MBS. We also consider MBS as a hub to connect DBSs to the network. Wireless links are capacity-limited and may suffer congestion during peak network times, and they can also increase the latency compared to a wired backhaul; therefore, we assume that DBSs can cache some data. Caching popular content is a very effective method to alleviate backhaul congestion and decrease delays in delivering service to end-users by bringing the data closer to the users. It will also decrease the total transmit power in the network as there is no need to fetch data every time a user sends a request to the core network. Hit probability is a principal metric that shows the probability of whether a user’s requested content is stored in a BS or not [102]. A lower probability means that more backhaul capacity has to be consumed. Different placement strategies yield different hit probabilities. The mechanism is commonly designed on the basis of the popularity of the content. It is observed that
content popularity follows a generalized Zipf law which states that the request rate \( q(n) \) for the \( n \)-th most popular content is proportional to \( \frac{1}{n^\alpha} \) for some \( \alpha \) \cite{103}. Typically, \( \alpha \) is between 0.64 and 0.83.

Similar to Chapter 6, we consider two groups of users in the system, namely delay-tolerant and delay-sensitive users. Such users can be defined either on the basis of the application they use or the fee they pay for their subscribed services. Delay-sensitive users are prone to high latency, while delay-tolerant users can tolerate some delay and receive service at a later time. To overcome the latency issue, a delay-sensitive user should either associate with an MBS that has a wired backhaul to the core network or connect to a DBS that has the requested data in its local cache to avoid the need for a 2 hub connection from the DBS to the core network and consequently lessen the delay.

We consider centralized decision-making in our network whereby a central entity is aware of the necessary information and the network parameters and makes the user association decision along with bandwidth allocation, power level transmission, and DBS locations. At the end of section 7.4.1, a partially distributed decision-making framework is also discussed.

We denote by \( I \) the set of users and by \( J \) the set of BSs. We use \( i \in I = \{1, 2, ..., I\} \) and \( j \in J = \{0, 1, ..., J\} \) to index users and BSs, respectively. Index 0 in \( J \) denotes the only MBS in the system. To avoid inter-cell interference and self-interference, we assume orthogonal frequency channels both in access and backhaul sides of the network. Our assumptions are summarized as follows:

- There is a MBS in the area.
- There are a fixed number of DBSs in the system.
- Backhaul and access spectrum of DBSs are orthogonal; therefore, there is no self interference.
- We assume orthogonal frequency channels in access sides of MBS and DBSs; therefore, there is no co-channel interference.
• The system is in downlink mode.

• The MBS serves as a backhaul source for DBSs and therefore, they have a variable limited backhaul capacity.

• All DBSs have local caches with limited size.

7.3 Power Minimization Problem

In this section, we propose an optimization framework that determines the user-BS associations and bandwidth allocations in addition to the 3D locations of the DBSs to minimize the total transmit power of the DBSs in downlink transmission. First, we introduce the constraints along with the design objectives considered in the system and then present the transmit power minimization problem formulation.

7.3.1 System constraints

7.3.1.1 BS association constraints

In our framework, each user should be served by only one BS. This yields the following constraint:

\[ \sum_{j \in J} \rho_{ij} = 1, \forall i \in I, \]  

(7.1)

where \( \rho_{ij} \in \{0, 1\} \) is a binary association indicator variable for user \( i \) and BS \( j \), and 1 indicates association.

7.3.1.2 Bandwidth allocation constraints

The total amount of resources allocated by each BS to all the users cannot exceed the available bandwidth of that BS. Therefore,

\[ \sum_{i \in I} \rho_{ij} \cdot \beta_{ij} \leq 1, \forall j \in J, \]  

(7.2)
where $\beta_{ij} \in [0, 1]$ is the normalized bandwidth resource of BS $j$ that is assigned to user $i$.

### 7.3.1.3 QoS constraints

The wide range of services requested by the users makes their QoS demands fairly disparate. We denote the QoS demanded by user $i$ by $r_i$ and measure it in Mbps. To ensure that QoS demands of the users are met, each user’s rate must not be less than its target rate. Therefore,

$$\sum_{j \in J} \rho_{ij} \cdot B_j \cdot \beta_{ij} \cdot \eta_{ij} \geq r_i, \forall i \in I,$$

(7.3)

where $B_j$ is the total bandwidth of BS $j$, variable $\eta_{ij}$ is the received spectral efficiency of user $i$ when connected to BS $j$, and $r_i$ is the minimum required rate of user $i$. Assuming Shannon capacity is achieved, $\eta_{ij} = \log_2(1 + \gamma_{ij})$, where $\gamma_{ij}$ is the SNR received by user $i$ from $j$th BS.

### 7.3.1.4 User type constraints

We assume that the total data in the network is $K$ files. We also consider a finite cache capacity in each DBS, which means each DBS can store part of the whole data in its local cache and refresh the contents periodically. Let us define the cache matrix $E^{J \times K} = [e_{jk}] = \{0, 1\}$ for DBSs, where $e_{jk} = 1$ denotes that DBS $j$ caches $k$th file and $e_{jk} = 0$ indicates the opposite. The user request matrix is also defined by $U^{I \times K} = [u_{ik}] = \{0, 1\}$, where $u_{ik} = 1$ means that user $i$ requests file $k$ and $u_{ik} = 0$ means the opposite. We assume that the central entity is aware of both matrices $E$ and $U$ and therefore can control the caching strategy by obtaining the cache association matrix $F^{I \times J} = [f_{ij}] = \{0, 1\}$, where $f_{ij} = 1$ means that the content requested by user $i$ is cached in DBS $j$; otherwise, $f_{ij} = 0$. Let us explain the caching strategy in more detail with an example: Assume that there are 10 contents available in the network and each DBS can cache 20 percent of the total contents. Based on certain placement strategies, DBS 1 decides to keep contents 2 and 4 in its local cache; therefore, $e_{12} = 1$ and $e_{14} = 1$. On the user side, if user 3 requests for content 4, $u_{34}$ becomes 1. As the central entity is aware of the whole matrix $E$ and $U$, it can obtain
the entries of matrix $F$. In the aforementioned example, $f_{31} = 1$ as the requested content of user 3 is available in DBS 1.

Let us consider the case of delay-sensitive users only associating with the MBS or DBSs that have their requested data in their local cache, hence

$$\sum_{j \in J} f_{ij} \cdot \rho_{ij} \geq \tau_i, \ \forall i \in I,$$

(7.4)

where $\tau_i \in \{0, 1\}$. $\tau_i = 1$ indicates that the user $i$ is delay-sensitive and $\tau_i = 0$ indicates the opposite. We consider $f_{ij} = 1$ for $j = 0$ as there is a wired connection from the MBS to the core network.

7.3.1.5 LoS constraints for association

In this chapter, we adopt the pathloss given in (2.1) and assume only the mean of excessive pathloss due to LoS or NLoS channel. We also consider that the probability of having a LoS connection between a DBS and a user is formulated as (2.3). Adjustable altitudes in DBSs can increase the likelihood of establishing an LoS connection to ground users. A weaker channel implies higher transmit power and resource usage; therefore, to decrease the transmit power, we assume that user $i$ can be associated with DBS $j$ only if it has an LoS channel with a high probability with that DBS. Therefore,

$$P(\text{LoS})_{ij} \geq \zeta \cdot \rho_{ij}, \ \forall i \in I, \forall j \in J\backslash 0,$$

(7.5)

where $\zeta$ is a number close enough to one.

7.3.1.6 Backhaul capacity constraints

The total data rate a DBS can support should not exceed its backhaul capacity. Note that by storing the content in the local cache, we can alleviate the backhaul consumption accordingly,

$$\sum_{i \in I} \rho_{ij} \cdot B_j \cdot \beta_{ij} \cdot \eta_{ij} \cdot (1 - f_{ij}) \leq C_j, \ \forall j \in J\backslash 0,$$

(7.6)
where $C_j$ is the backhaul capacity of DBS $j$.

### 7.3.2 Problem formulation

With the proliferation of data traffic and the number of connected devices, the total carbon footprint that communication technologies generate are expected to increase significantly. Therefore, due to both economic and environmental concerns, it is not only data rate and throughput that are important factors in network deployment these days, but also how much energy is spent delivering that rate. Moreover, by considering the limited energy in DBSs the importance is doubled. To address the issue of energy efficiency, in this chapter we optimize the locations of DBSs to minimize the total transmit power considering the QoS requirements of the users and limited backhaul capacity of the DBSs. Therefore, the optimization problem is expressed as follows:

\[
P_1: \min_{\{p_{ij}\}, \{l_j\}, \{\rho_{ij}\}, \{\beta_{ij}\}} \sum_{j \in J} \sum_{i \in I} p_{ij} \cdot \rho_{ij} \tag{7.7a}
\]

subject to

\[
\sum_{j \in J} \rho_{ij} = 1, \forall i \in I, \tag{7.7b}
\]

\[
\sum_{i \in I} \rho_{ij} \cdot \beta_{ij} \leq 1, \forall j \in J, \tag{7.7c}
\]

\[
\sum_{j \in J} \rho_{ij} \cdot B_j \cdot \beta_{ij} \cdot \eta_{ij} \geq r_i, \forall i \in I, \tag{7.7d}
\]

\[
\sum_{j \in J} f_{ij} \cdot \rho_{ij} \geq \tau_i, \forall i \in I, \tag{7.7e}
\]

\[
P(LoS)_{ij} \geq \zeta \cdot \rho_{ij}, \forall i \in I, \forall j \in J \setminus 0, \tag{7.7f}
\]

\[
\sum_{i \in I} \rho_{ij} \cdot B_j \cdot \beta_{ij} \cdot \eta_{ij} \cdot (1 - f_{ij}) \leq C_j, \forall j \in J \setminus 0, \tag{7.7g}
\]

\[
\rho_{ij} \in \{0, 1\}, \beta_{ij} \in [0, 1], \forall i \in I, \forall j \in J, \tag{7.7h}
\]

where $p_{ij}$ is the transmit power from BS $j$ to user $i$ and $l_j = (x_j, y_j, h_j)$ is the 3D location of DBS $j$. Here we are interested in keeping the MBS as free as possible in case there is an emergency need; therefore, we consider $p_{ij} = M$ for $j = 0$, where $M$ is a big number.
In this way, we force the users to be associated with the MBS only if one of the following situations hold: They are not in the LoS coverage of any DBSs; they are delay-sensitive and their requested content is not available in any of the DBSs with LoS coverage; or the DBSs do not have enough backhaul capacity. Otherwise, they are associated with one of the DBSs.

This optimization problem has a non-convex objective function and nonlinear constraints with a combination of binary and continuous variables. In other words, it is a non-convex, NP-hard optimization problem.

7.4 Proposed Algorithm

To alleviate the difficulties mentioned in the preceding section, we break the problem down into two subproblems and solve them iteratively until they converge into a local optimum. First, for fixed \( \{\rho_{ij}\} \) and \( \{\beta_{ij}\} \), we find \( \{l_j\} \), and then for new 3D locations of DBSs, we update \( \{\rho_{ij}\} \) and \( \{\beta_{ij}\} \). Finally, \( P_{ij} \) is calculated using the updated information. The detail is explained in the following.

7.4.1 3D locations of DBSs

In this step, we assume that \( \{\rho_{ij}\} \) and \( \{\beta_{ij}\} \) are known and hence we find \( \{l_j\} \).

According to (7.7b), each user is associated with only one BS; therefore, (7.7d) can be simplified to

\[
\rho_{ij} \cdot B_j \cdot \beta_{ij} \cdot \eta_{ij} \geq r_i \cdot \rho_{ij}, \quad \forall i \in I, \forall j \in J. \tag{7.8}
\]

By considering only DBSs and assuming that Shannon capacity is achieved, after some manipulations, we have

\[
P_{ij} \cdot \rho_{ij} \geq A_{ij} \cdot d_{ij}^2 \cdot \rho_{ij}, \tag{7.9}
\]

where \( A_{ij} = \vartheta \cdot N_0 \cdot B_j \beta_{ij} \cdot (2\pi f_c^j / \nu_j - 1) \) and \( \vartheta = (4\pi f_c^j /\nu_j)^2 \cdot 10^\psi / 10 \). Variable \( d_{ij} \) is the distance between DBS \( j \) and user \( i \) and is equal to \( ((x_j - x_i)^2 + (y_j - y_i)^2 + h_j^2)^{1/2} \), and \( N_0 \) denotes
the power spectral density of AWGN. Therefore, in P1 instead of minimizing the transmit power, one can minimize the weighted distance between the DBSs and their associated users.

Inequality (7.7f) can be rewritten as

$$\theta_{ij} \geq b \cdot \rho_{ij}, \quad (7.10)$$

where $b = \frac{-\pi}{180} \ln \frac{1-\xi}{\kappa \xi} + \frac{\pi \kappa}{180}$. Therefore,

$$((x_j - x_i)^2 + (y_j - y_i)^2 + V h_j^2) \cdot \rho_{ij} \leq 0, \quad (7.11)$$

where $V = 1 - \frac{1}{\sin^2 b}$.

Minimizing the total transmit power implies that each user receives service with the minimum required rate; therefore, (7.7g) can be changed to

$$\sum_{i \in I} r_i \cdot \rho_{ij} \cdot (1 - f_{ij}) \leq C_j, \forall j \in J \setminus 0. \quad (7.12)$$

By considering FSPL for backhaul links and assuming equal spectrum allocation for backhauls, $C_j = \frac{W}{n} \cdot \log_2(1 + \frac{P_0}{\frac{4\pi f_c}{c^2} d_{j0}^2 N_0 W})$, where $W$ is the total available bandwidth for backhaul connection from an MBS to DBSs and $n$ is the number of DBSs, $P_0$ is the MBS transmit power for backhauling, and $d_{j0}$ is the distance from the MBS to DBS $j$. After some manipulations, we have

$$L_j \cdot [(x_j - x_0)^2 + (y_j - y_0)^2 + h_j^2] \leq D,$$

where $L_j = 2 \sum_{i \in J} r_i \cdot (1 - f_{ij}) - 1$ and $D = \frac{P_0}{\frac{4\pi f_c}{c^2} d_{j0}^2 N_0 W}$.

Finally, P1 is simplified to

**P2:** min $\sum_{j \in J, i \in I} \sum A_{ij} [(x_j - x_i)^2 + (y_j - y_i)^2 + h_j^2] \cdot \rho_{ij}$ subject to $[(x_j - x_i)^2 + (y_j - y_i)^2 + V h_j^2] \cdot \rho_{ij} \leq 0, \forall i \in I, \forall j \in J \setminus 0, \quad (7.14a)$
\[ L_j[(x_j - x_0)^2 + (y_j - y_0)^2 + h_j^2] - D \leq 0, \quad \forall j \in \mathcal{J} \setminus 0. \]  

(7.14c)

The optimization problem \( P2 \) is in the form of separable QCQP problems \cite{104} whose general form is given as

\[
\begin{align*}
\min_{\xi_j} & \quad \sum_{j \in \mathcal{J} \setminus 0} \frac{1}{2} \xi_j^T G_{0j} \xi_j + q_{0j}^T \xi_j + n_{0j} \\
\text{subject to} & \quad \frac{1}{2} \xi_j^T G_{tj} \xi_j + q_{tj}^T \xi_j + n_{tj} \leq 0, \quad \forall t = \{1, ..., I + 1\}, \forall j \in \mathcal{J} \setminus 0.
\end{align*}
\]

(7.15a, 7.15b)

Here we have

\[ \xi_j = [x_j \ y_j \ h_j]^T, \]

(7.16)

and

\[ G_{0j} = \begin{bmatrix} 2 \sum_i A_{ij} \rho_{ij} & 0 & 0 \\ 0 & 2 \sum_i A_{ij} \rho_{ij} & 0 \\ 0 & 0 & 2 \sum_i A_{ij} \rho_{ij} \end{bmatrix}. \]

(7.17)

Also

\[ q_{0j} = \begin{bmatrix} \sum_i -2x_i A_{ij} \rho_{ij} \\ \sum_i -2y_i A_{ij} \rho_{ij} \\ 0 \end{bmatrix}, \]

(7.18)

and \( n_{0j} = \sum_i (x_i^2 + y_i^2) A_{ij} \rho_{ij} \).

For the constraints we have

\[ G_{tj} = \begin{cases} 
\begin{bmatrix} 2 \rho_{ij} & 0 & 0 \\
0 & 2 \rho_{ij} & 0 \\
0 & 0 & 2 \nu \rho_{ij} \end{bmatrix}, & \text{if } t = i, \\
\begin{bmatrix} 2L_j & 0 & 0 \\
0 & 2L_j & 0 \\
0 & 0 & 2L_j \end{bmatrix}, & \text{if } t = I + 1.
\end{cases} \]

(7.19)
Also

\[ q_{tj} = \begin{cases} 
-2x_i\rho_{ij} & \text{if } t = i, \\
-2y_i\rho_{ij} & \text{if } t = I + 1, 
\end{cases} \tag{7.20} \]

and

\[ n_{ij} = \begin{cases} 
(x_i^2 + y_i^2)\rho_{ij} & \text{if } t = i, \\
L_jx_0^2 + L_jy_0^2 - D & \text{if } t = I + 1. 
\end{cases} \tag{7.21} \]

Here we note that \( G_{tj} \) is not a positive semidefinite (PSD) matrix; therefore, the QCQP problem is not convex. To solve this issue, we apply the Suggest-and-Improve framework to get approximate solutions to the non-convex QCQP problem \([105]\).

1. **Suggest:** In this step a candidate point is found. To find a candidate point, we transform the problem to a semi-definite programming problem using the semi-definite relaxation (SDR) technique. SDR is a computationally efficient approximation approach to QCQP.

Using \( \xi_j^T G_0 j \xi_j = \text{Tr}(G_0 j \xi_j \xi_j^T) \) and by introducing the new variable \( X_j = \xi_j \xi_j^T \) we can rewrite each QCQP problem for every DBS as

\[
\begin{align*}
\min_{\{X_j\},\{\xi_j\}} & \quad \frac{1}{2} \text{Tr}(G_0 j X_j) + q_{0j}^T \xi_j + n_{0j} \\
\text{subject to} & \quad \frac{1}{2} \text{Tr}(G_{tj} X_j) + q_{tj}^T \xi_j + n_{tj} \leq 0, \quad \forall t = \{1, \ldots, I + 1\}, \\
& \quad X_j = \xi_j \xi_j^T. \tag{7.22c}
\end{align*}
\]

Using this transformation, we have embedded the original problem with three variables into a much larger space with 12 dimensions by obtaining the additional property that the objective and constraints are affine in \( X_j \) and \( \xi_j \) except the constraint \( (7.22c) \) which is not convex. The problem can be relaxed into a convex problem by replacing this non-convex equality constraint with a PSD constraint \( X_j - \xi_j \xi_j^T \geq 0 \). Then by solving the following convex problem in each DBS, a lower bound on the
optimum value of the problem $P_2$ is found.

$$
\begin{align}
\min_{(X_j), (\xi_j)} & \quad \frac{1}{2} \text{Tr}(G_{0j}X_j) + q_{0j}^T \xi_j + n_{0j} \\
\text{subject to} & \quad \frac{1}{2} \text{Tr}(G_{tj}X_j) + q_{tj}^T \xi_j + n_{tj} \leq 0, \quad \forall t = \{1, ..., I + 1\}, \\
& \quad \begin{bmatrix} X_j & \xi_j \\ \xi_j^T & 1 \end{bmatrix} \succeq 0,
\end{align}
$$

(7.23a, 7.23b, 7.23c)

where the last constraint is formulated as a Schur complement [67]. This problem is convex and can be conveniently solved by available software packages such as convex optimization toolbox CVX in MATLAB.

Afterwards, one can apply the randomization procedure explained in Algorithm 4 to improve the solution found via SDR. Gaussian randomization procedure minimizes the expected objective function subject to holding the constraints in expectation; therefore, there is no guarantee that the sampling points from the normal distribution give feasible points of the $P_2$ at all. However, these points are a good choice for the $Suggest$ method and can serve as a starting point for the $Improve$ method.

2. Improve: In this step, we run a local method to find a solution point that is not worse than the candidate point. As the candidate points might be infeasible, the goal in this step is to minimize the constraint violation and obtain a smaller value in the objective function. Here we apply a coordinate descent method to improve the candidate point $\delta_{l^*j}$. Coordinate descent method is an algorithm that solves optimization problems by successively performing minimizations along coordinate directions to find the local minimum of a function, and it includes two phases.

The goal in the first phase is to reduce the maximum constraint violation and achieve a feasible point if possible. Let us consider $\delta_{l^*j}$ as the candidate point. We repeatedly cycle over each coordinate of $\delta_{l^*j}$ and update it to the value that minimizes the maximum constraint violation. If the violations on all constraints along all the coordinates become zero or smaller, a feasible point $\gamma_j$ is found.

In the second phase, we look for other feasible points with better objective values. To
Algorithm 4 Gaussian randomization procedure for the QCQP problem

1: **Inputs:** SDR solution $X^*_j$ and $\xi^*_j$, the number of randomizations $L$. 
2: for $l = 1, ..., L$ do 
3: generate $\delta_{lj} \sim N(\xi^*_j, X^*_j - \xi^*_j \xi^*_j^T)$. 
4: end for 
5: determine $l^* = \arg\min_{l=1,\ldots,L} \frac{1}{2} \delta_{lj}^T G_{0j} \delta_{lj} + q_{0j}^T \delta_{lj} + n_{0j}$. 
6: **output:** $\delta_{l^*j}$, which is the approximate solution for the QCQP problem.

As shown above, the problem of finding the 3D placements of the DBSs was transformed to separable QCQPs; therefore, for decision making, a partially distributed solution is possible where the 3D location of each DBS is computed locally at that particular DBS, but the overall decisions for associations, bandwidth, and power allocations are made at a central controller. The distributed solution is indeed preferred due to not relying on a single controller in the network. Because if the centralized system suffers from a failure, the operation of the entire network is disrupted. A centralized solution may also endure signalling overhead, outdated information, and scalability problems [106].

7.4.2 User-BS associations and bandwidth allocations

In the second step, we assume that the locations of DBSs are known and hence the problem is transformed to

$$\textbf{P3:} \quad \min_{\{\rho_{ij}, \beta_{ij}\}} \quad \sum_{i \in \mathcal{I}} \left( \sum_{j \in \mathcal{J}} A_{ij} \cdot d_{ij}^2 \cdot \rho_{ij} + \sum_{j=0} M \cdot \rho_{ij} \right)$$

subject to

$$\sum_{j \in \mathcal{J}} \rho_{ij} = 1, \forall i \in \mathcal{I}, \quad (7.24a)$$

$$\sum_{i \in \mathcal{I}} \rho_{ij} \cdot \beta_{ij} \leq 1, \forall j \in \mathcal{J}, \quad (7.24b)$$

$$\sum_{i \in \mathcal{I}} f_{ij} \cdot \rho_{ij} \geq \tau_i, \forall i \in \mathcal{I}, \quad (7.24c)$$
\[ \rho_{ij} \leq P(\text{LoS})_{ij}/\zeta, \; \forall i \in \mathcal{I}, \forall j \in \mathcal{J}\setminus 0, \quad (7.24e) \]

\[ \sum_{i \in \mathcal{I}} r_i \cdot \rho_{ij} \cdot (1 - f_{ij}) \leq C_j, \; \forall j \in \mathcal{J}\setminus 0, \quad (7.24f) \]

\[ \rho_{ij} \in \{0, 1\}, \beta_{ij} \in [0, 1], \forall i \in \mathcal{I}, \forall j \in \mathcal{J}. \quad (7.24g) \]

This problem is non-convex due to the non-convexity of the objective function, binary variables \( \{\rho_{ij}\} \), and the product relationship between \( \{\rho_{ij}\} \) and \( \{\beta_{ij}\} \). To circumvent this difficulty, we consider equal resource allocation between all the users that are associated with a BS and assume all the users in LoS coverage of a DBS are associated with that DBS; therefore, \( \mathbf{P3} \) is transformed to

\[
\begin{align*}
\min_{\{\rho_{ij}\}} & \quad \sum_{i \in \mathcal{I}} \left( \sum_{j \in \mathcal{J}\setminus 0} A_{ij} \cdot d_{ij}^2 \cdot \rho_{ij} + \sum_{j=0} M \cdot \rho_{ij} \right) \\
\text{subject to} & \quad \sum_{j \in \mathcal{J}} \rho_{ij} = 1, \; \forall i \in \mathcal{I}, \\
& \quad \sum_{j \in \mathcal{J}} f_{ij} \cdot \rho_{ij} \geq \tau_i, \; \forall i \in \mathcal{I}, \\
& \quad \rho_{ij} \leq P(\text{LoS})_{ij}/\zeta, \; \forall i \in \mathcal{I}, \forall j \in \mathcal{J}\setminus 0, \\
& \quad \sum_{i \in \mathcal{I}} r_i \cdot \rho_{ij} \cdot (1 - f_{ij}) \leq C_j, \; \forall j \in \mathcal{J}\setminus 0, \\
& \quad \rho_{ij} \in \{0, 1\}, \forall i \in \mathcal{I}, \forall j \in \mathcal{J}. \quad (7.25f) 
\end{align*}
\]

Since this problem is a binary linear programming problem, it can be solved by optimization tools such as MOSEK [107]. To refine \( \beta_{ij} \) values, we solve the problem below in each DBS \( j \) for known \( \rho_{ij} \)’s.

\[
\begin{align*}
\min_{\{\beta_{ij}\}} & \quad \sum_{i \in \mathcal{I}} N_0 \cdot (2^{\frac{N}{\beta_{ij}}} - 1) \cdot d_{ij}^2 \cdot \rho_{ij} \\
\text{subject to} & \quad \sum_{i \in \mathcal{I}} \rho_{ij} \cdot \beta_{ij} \leq 1, \\
& \quad \beta_{ij} \in [0, 1], \forall i \in \mathcal{I}. \quad (7.26c) 
\end{align*}
\]
Algorithm 5 Finds 3D locations of DBSs, user-BS associations, and bandwidth allocations

1: Inputs: User locations, number of DBSs.
2: Initialization: Choose initial locations of the DBSs randomly. Set $t = 1$, $P(t) = \sum_{i \in I} (\sum_{j \in J} A_{ij} \cdot d_{ij}^2 \cdot \rho_{ij} + \sum_{j=0} M \cdot \rho_{ij})$, and $m(t) = P(t) - P(t-1)$. Define $m(1) = N$, where $N$ is a big number. $\epsilon$ is a small positive number.
3: while $m(t) \geq \epsilon$ do
4: Find $\rho_{ij}(t)$ and $\beta_{ij}(t)$ in $P_3$ and update $P(t)$.
5: $t = t + 1$.
6: Find $l_j$ in $P_2$ and update the DBS locations. Find $P(t)$ and $m(t)$, accordingly.
7: end while

<table>
<thead>
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<th>Parameter</th>
<th>Value</th>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_0$</td>
<td>40 dBm</td>
<td>$N_0$</td>
<td>-130 dBm/Hz</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>0.8</td>
<td>$f_c$</td>
<td>2 GHz</td>
</tr>
</tbody>
</table>

Optimization problem (7.26) is convex as the objective function and all the constraints are convex. The convexity of (7.26a) is proved by showing that its second derivative is greater than zero for all positive values of $\beta_{ij}$'s; therefore, (7.26) can be solved by existing optimization tools. Algorithm 5 explains the iterative procedure that solves the two subproblems and updates the variables until convergence to a local optimum.

7.5 Performance Evaluation

In this section, we investigate the performance of the proposed algorithm and provide numerical examples to illustrate the merits of these approaches for different network instances. We consider an urban square region with a side length of 1000 meters and a two-tier HetNet scenario. One MBS is located in the center of the area which can serve both the users and the DBSs (as a backhaul hub). There are also several DBSs in the network whose locations should be determined. The dedicated bandwidth for each BS in access side is 10 MHz and total bandwidth for backhaul of DBSs is 12 MHz. We adopt the MBS channel from 3GPP propagation model [98]. Denoting the distance between a user and the MBS by $d'$ in kilometres, the average path loss for the link between the MBS and user can be expressed as $128.1 + 37.6 \log_{10}(d')$. We assume that the total size of the available data in the system is 10 files and each DBS can store a fixed number of contents
in its local cache based on the popularity. For simplicity, we assume that all files have the same size.

The users are placed according to a uniform or Matérn distribution in different scenarios where the latter is a doubly Poisson cluster process as explained in Section 5.2.2. We also consider that the user types, i.e., delay-tolerant or delay-sensitive, are known to the BSs and the core network. The users have different QoS demands drawn from a set of \( \{0.5, 1, 1.5 \text{ Mbps}\} \). Other simulation parameters are provided in Table 7.1. All results are averaged over 100 Monte Carlo simulations. In the scenario considered herein, a centralized design in which a central node collects all the network parameters is required to associate users
Figure 7.3. Total and average transmit power of DBSs and average transmit power per user for 2 different CoVs. When the users are more clustered, the required transmit power is decreased, although the number of users associated with the DBSs is increased. There are 3 DBSs in the network and each one can cache 20% of the total contents. Also 10% of the users are delay-sensitive.

A typical user distribution with CoV=2 along with an MBS and 2 or 3 DBSs are shown
The number of iterations required for convergence to a local optimum. There are 60 users in the network with CoV=1 and 10% of them are delay-sensitive. Also the number of DBSs is 3, and each DBS can cache 20% of the total contents.

In Fig. 7.2. As we can see, the control center decides about the location of DBSs based on different parameters such as the traffic distribution, the QoS requirements of the users, and the number of DBSs, to name a few. When the number of DBSs is 2 as in Fig. 7.2a, to increase the chance of establishing an LoS connection with more users, the DBSs move to higher altitudes and consequently they have to increase their transmit power. On the other hand, when the number of DBSs is 3 as in Fig. 7.2b, the DBSs can move to lower altitudes when the users are highly clustered in a small area to decrease the required transmit power. As we can see, 2 DBSs can serve the same number of users for this typical user distribution simply by changing their 3D locations and in the expense of higher transmit power. Therefore, based on the traffic distribution and user requirements, sometimes using fewer number of DBSs is preferred as they can do the same job with lower complexity in the system.

In Fig. 7.3a, the total transmit power of 3 DBSs, the average transmit power in each DBS, and per user for 2 different CoVs are shown. It is observed that by increasing the CoV, although more users are served by DBSs, as seen in Fig. 7.3b, the required transmit power is decreased. This is due to the fact that by increasing the CoV, the average distance
Figure 7.5. Total transmit power of DBSs and the average transmit power in each DBS and per DBS user is decreased by increasing the number of DBSs, while the number of associated users with DBSs is increased. There are 60 users in the network with CoV=1 and 10% of them are delay-sensitive. Also each DBS can cache 20% of the total contents.

between users and DBs is reduced, which also decreases the required transmit power. The slight increase in the average required power per user is due to the fact that by increasing the number of users, it is more probable that farther users establish an LoS link with a DBS and therefore the average weighted distance between the users and their associated DBSs and consequently the required transmit power per user is increased.

Fig. 7.4 shows the number of required iterations to converge to a local optimum across different snapshots. There are 60 users in the network with uniform distribution and 10% of the users are delay-sensitive. The number of DBSs is 3. As shown, the algorithm converges quickly. In the majority of snapshots the local optimum is found in less than 3 iterations, and the maximum number of iterations in all the snapshots is less than 8.

Fig. 7.5 depicts the total transmit power and the average transmit power per DBS and user for different numbers of DBSs in the network. It is observed that by increasing the number of DBSs, the average transmit power is decreased. This is because the transmit power is proportional with the weighted distance between a DBS and its associated user, and by increasing the number of DBSs, the average distance will decrease. It is also observed that by increasing the number of DBSs in the network, the total number of users
Figure 7.6. (a) Average backhaul capacity usage in each DBS, (b) average number of users associated with DBSs, with and without content caching in DBSs. The cache-enabled DBSs can cache 10%, 20%, or 40% of the total contents. By increasing the cache size, the backhaul capacity usage is decreased, while the number of users associated with DBSs is increased. The number of DBSs is 3, and users are placed according to a uniform distribution with 10% of them being delay-sensitive.

associated with DBSs are increased as the probability of establishing an LoS connection between a DBS and a typical user is increased, although it is important to note that such increment is not linear as by increasing the number of DBSs, the dedicated backhaul
bandwidth in each DBS is decreased and therefore, due to backhaul capacity limitation, the number of users associated with DBSs will finally saturates.

Fig. 7.6 depicts the effect of caching in DBSs on backhaul capacity usage and number of users associated with DBSs. As shown in Fig. 7.6a, the backhaul is less consumed when the DBSs are cache-enabled and by increasing the size of local cache in DBSs, although more users are served by DBSs as seen in Fig. 7.6b, the backhaul capacity usage is reduced. It is observed that by increasing the network traffic, the number of users associated with DBSs is first increased and then saturates due to the limitation in backhaul capacity of DBSs. This saturation happens faster when the caching size is smaller.

Fig. 7.7 illustrates the number of users associated with DBSs for different percentage of delay-sensitive users. By increasing the number of delay-sensitive users, more users have to associate with the MBS, because delay-sensitive users can associate with DBSs only if their requested content is available in the local cache of a DBS that is in their LoS coverage. Therefore, by increasing the caching size as shown, this issue can be alleviated.

Fig. 7.8 shows the average transmit power in each DBS versus the available access bandwidth in BSs for different number of users in the network. As shown, by increasing
the available bandwidth, the transmit power is decreased with a higher rate at first and then the decrement gradually reduces. This is due to the fact that by increasing the bandwidth, the noise power is also increased. It is also observed that by increasing the traffic in the network, the required power and/or bandwidth will increase. Therefore, because available resources in the network are scarce, careful design is essential to meet the requirements of the system, while minimizing the resource consumption in the network.

### 7.6 Conclusions

In this chapter, we developed a novel framework to jointly optimize the 3D placement of DBSs, the association of users with BSs, and bandwidth allocations, while minimizing the total transmit power of the DBSs. To decrease both the latency and congestion issue in the backhaul, we proposed content caching in the DBSs. Based on different applications in future wireless systems, we defined two groups of users, delay-sensitive and delay-tolerant, and assumed that delay-sensitive users should either associate with the MBS or the DBSs which have their requested content in their local caches, while delay-tolerant users can con-
nect to all the DBSs with LoS coverage or the MBS if there is enough resource. Due to the intractability of the problem, we divided the joint optimization problem into subproblems and iteratively updated them. First, user-BS associations and bandwidth allocations were found using existing optimization tools, and then 3D placements of DBSs were updated using the SDR approach and coordinate descent method. Simulation results showed that the proposed algorithm yields significant performance gains and indicated that caching can extensively decrease backhaul usage and help congestion issue. It was also shown that in networks with highly clustered users, there is a higher chance of establishing LoS connections between DBSs and users and hence more users can be associated with DBSs.
Chapter 8

Summary and Future Work

8.1 Summary

Due to the rapid proliferation of data traffic and scarcity of the wireless resource, providing an optimum location for BSs to meet the traffic demands is very essential. On the other hand, wireless users expect unlimited capacity everywhere and all the time, at an affordable price, but the temporal and spatial variations in user density and user application rates are expected to result in difficult-to-predict traffic patterns. Therefore, using drones as flying BSs is a promising approach in such scenarios that will bring supply towards demand whenever and wherever it is required. By properly engineering DBSs there can be substantial savings in the overall network deployment. The network operators can design and locate ground BSs for average traffic demand and utilize DBSs when there is a sudden excessive need in part of the network. Due to the fast deployment of such BSs, they can also address temporary coverage issues in remote areas or when terrestrial infrastructure is damaged due to a natural disaster.

In this thesis, after explaining about the importance of cell planning and presenting the key features of drones as flying BSs in Chapter 1, we comprehensively studied the existing literature about cell planning in terrestrial BSs and integration of DBSs in wireless networks in Chapter 2.
In Chapter 3, we formulated the problem of determining an optimal BS location for a given set of users as a convex optimization problem to minimize the total BS power subject to QoS constraints. We then expressed a globally-optimal solution as a convex combination of user locations. Based on this, we obtained a number of closed-form solutions, which revealed the impact of system and user parameters, propagation pathloss, as well as the overall system geometry. We showed that the symmetry in the user set makes the optimal BS location independent of pathloss exponent, which is not true for asymmetric sets. The results provide insights unavailable from numerical algorithms and allow one to develop design guidelines for more complicated systems. In addition, we proposed an iterative algorithm based on the characterization of optimal BS location and proved its convergence for some special cases.

In Chapter 4, we introduced DBSs as the capacity injectors to the network to alleviate the high traffic congestion and serve users with high rate requirements. We found the minimum number of required DBSs and provided a novel DBSs deployment plan to serve a set of users based on their traffic requirements. Generally, this optimization problem is too complicated; therefore, a heuristic algorithm based on PSO was proposed. The simulation results for areas with different user densities showed the acceptable performance of the proposed method. The key outcome from this chapter was that the number of DBSs and their altitudes depend on the user densities in an area as such they can change their altitudes in order to tackle coverage or capacity issues. They decreased their altitudes in dense areas to reduce interference for the users that were not served by them and increased their altitudes to cover a larger area in low-density regions.

In Chapter 5, we considered limited backhaul capacity in DBSs as one of the fundamental constraints in deploying such BSs and found the optimal 3D placement of a DBS over an urban area with users having different rate requirements. The wireless backhaul peak rate and the bandwidth of a DBS were the limiting factors in both the network-centric and user-centric approaches. In the network-centric approach, the total number of served users regardless of their required rates was maximized and in the user-centric approach, the sum-rate of served users was maximized. Our investigations also showed the robustness
of the proposed algorithm against the modest movement of users.

In Chapter 6, based on different applications in future wireless systems, we introduced two groups of users: delay-sensitive and delay-tolerant. Delay-sensitive users were associated with the MBS, while delay-tolerant users could be associated with either one of the BSs. We assumed all the DBSs share the same bandwidth and hence proposed using directional antennas to mitigate the effect of interference. We then found user-BS association and wireless backhaul bandwidth allocation through a decomposition method and updated locations of DBSs using a PSO algorithm. We also obtained further insights into the effects of CoV and half-power beamwidth by simulations. The results showed that in the case of clustered users, utilizing DBSs can increase the total rate of the users associated with DBSs. In order to mitigate interference, we prevented overlapping of coverage areas for different DBSs. Although this method is useful, we found that the half-power beamwidth should be chosen carefully in such scenarios, as increasing the beamwidth could decrease the total rate by preventing DBSs to be deployed in beneficial locations.

In Chapter 7, we developed a novel framework to jointly optimize the 3D placement of DBSs, the association of users with BSs, and corresponding bandwidth allocations, while minimizing the total transmit power of the DBSs. To decrease both the latency and congestion issue in the backhaul, we proposed content caching in DBSs. We considered two groups of users in the network: delay-sensitive and delay-tolerant and assumed that delay-sensitive users should either associate with the MBS or the DBSs which cached their requested contents in their local caches. On the other hand, delay-tolerant users could connect to DBSs with LoS coverage or the MBS. Due to the high complexity of the problem, we divided it into subproblems and iteratively updated them. First, user-BS associations and bandwidth allocations were found, and then 3D placements of DBSs were updated using the SDR approach and coordinate descent method. Simulation results showed that the proposed algorithm provides significant performance gains and indicated that caching can substantially decrease backhaul usage and help the congestion issue. It was also shown that in networks with highly clustered users, the chance of establishing LoS connections between DBSs and users is higher and hence more users can be associated
8.2 Future Work

Despite a considerable number of studies on both cell-planning and drone-based communications, there are still some issues that should be addressed. In this section, we discuss a number of interesting possibilities for the extension of the current work.

8.2.1 Multiple BSs and stochastic distribution of users

In Chapter 3, we analytically found the BS location in a cellular network. An interesting extension to this work could be considering multiple BSs in the network and trying to obtain closed-form solutions and insights for such network. We had also assumed that the exact locations of users are known, while we can find the optimum BS location when a marginal distribution of users is known.

8.2.2 Channel modelling

Currently, there is a limited number of studies related to ATG channel modelling and the majority of the existing literature only assume LoS in such channels. Therefore, it is required to consider more realistic channel models that can adapt to different operational environments. Moreover, due to communications between drones, a true air to air channel model that captures the Doppler effect is also necessary.

8.2.3 Interference management

As the air to ground channel is different from the terrestrial channel model and the probability of having LoS connection in aerial BSs is higher than ground ones, the co-channel interference among DBSs and between DBSs and ground BSs is a critical issue that should
be managed in order to boost HetNets capacity. The user-BS association is a key challenge in this regard. It is clear that the user-BS association directly affects the interference and so a good association can enhance capacity. Therefore, novel interference mitigation techniques jointly with user association should be applied to the system to increase the network utility.

8.2.4 Mobile users and mobile DBSs

In this thesis, we considered a snapshot of the users and found the 3D placement of DBSs for that snapshot. A promising extension to this work would be considering the movement of users and find the trajectory of DBSs in such dynamic networks using machine learning techniques.

8.2.5 Network slicing

In traditional cellular networks, the network operator is the main service provider and owns the whole network including all the infrastructure, core network, radio access network, the transmission network, and the spectrum resource, while in future and through network slicing, different parts of the network may be virtualized and operated by different parties. This virtualization technique can enable one network to provide multiple services with extremely different requirements while maintaining isolation. This is a promising approach in drone-based communication to especially decrease the overall deployment cost of the network. Moreover, by having a global view of the network in a centralized network entity, better resource configuration and management is possible. Two important enabling technologies in this regard are explained below:

- **Network function virtualization (NFV):** By abstracting and sharing infrastructure and radio spectrum resources, the CapEx and OpEx of deploying a cellular network can be dramatically reduced. Moreover, migrating to newer products or technologies becomes much easier by virtualizing the wireless network and isolating part of it. By decoupling the infrastructure and resource from the services, different
services can share the same physical resources and maximize the utility; therefore, by virtualizing the network two different roles can be established: infrastructure provider (InP) and service provider (SP). InPs own the infrastructure and radio resources, while SPs lease and operate such resources to offer end-to-end service to mobile users [108, 109]. By virtualizing physical resources, the network can react faster to rapid changes compared to traditional systems. Also remotely and dynamically provisioning and configuration of the network is possible.

- **Software-defined network (SDN):** SDN is an approach in computer networks that decouples the control plane from the data plane. This decoupling lets the control plane to be programmable by softwares that might be running on a centralized network entity, while the infrastructure is completely separated from software; therefore, in principle, it separates network intelligence from network functions. The SDN controller can reside in the cloud. It is one of the promising technologies that can be applied in network virtualization [108]. Resource configuration and management can be efficiently done using SDN instead of traditional manual configurations. The logic control function and high-level strategies can be managed and configured dynamically through a central controller and a standard protocol such as Openflow can provide an appropriate method for the control plane to communicate with the data plane.

### 8.2.6 Self organization networks (SON)

Since aerial BSs will operate in dynamic conditions and in order for them to be highly responsive to users’ mobility, they need to be established with features for self organization networks including self-configuration, self-optimization, and self-healing. Finding the optimal placement for DBSs based on different parameters such as guaranteed QoS, limited wireless backhaul capacity, and limited available spectrum resource in areas with different user rates and densities can actually show how DBSs could be integrated/placed in a cellular network which is the concept of self-configuration which was covered in this thesis. Resource allocation, interference mitigation, energy consumption minimization could ad-
dress the self-optimization functions of a drone-based SON. An ultimate vision could be how efficiently different DBSs will form any desired performance by intelligent signalling and delegate control. Finally, the self-healing feature in DBSs can be explored to avoid any failure in the network by adjusting parameters/algorithms/placements and meet some minimum performance criteria.
APPENDIX
Appendix A

Proofs

A.1 Proof of Theorem 1 in Chapter 3

Since the problem $P2$ is convex and the strong duality holds (since Slater condition is satisfied), its KKT conditions are sufficient for optimality [67]. Its Lagrangian is

$$L(P_k, c) = \sum_k P_k + \sum_k \lambda_k (\beta_k |c - x_k|^{\nu_k} - P_k),$$

(A.1)

where $\lambda_k \geq 0$ are Lagrange multipliers responsible for the power constraints. First, we consider the non-singular case, when $c^{*} \neq x_k \ \forall k$, and deal with the singular case later on. In the non-singular case, the KKT conditions take the following form:

\[
\begin{align*}
\frac{\partial L}{\partial c} &= \sum_k \lambda_k \beta_k \nu_k (c - x_k)|c - x_k|^{\nu_k - 2} = 0, \\
\frac{\partial L}{\partial P_k} &= 1 - \lambda_k = 0, \\
\lambda_k (\beta_k |c - x_k|^{\nu_k} - P_k) &= 0, \\
P_k &\geq \beta_k |c - x_k|^{\nu_k}, \lambda_k \geq 0,
\end{align*}
\]

(A.2) (A.3) (A.4)

where (A.2) are the stationary conditions, (A.3) are the complementary slackness conditions, and (A.4) are primal and dual feasibility conditions. 1st condition in (A.2) was obtained from

$$\frac{\partial |x|^\nu}{\partial x} = \nu |x|^{\nu - 2},$$

(A.5)
if \( x \neq 0 \), which always holds in the non-singular case. The 2nd condition in (A.2) implies \( \lambda_k = 1 \) so that, from (A.3), \( P_k = \beta_k|c - x_k|^\nu_k \), i.e. transmitting with the least required power for each user is optimal. Combining this with 1st condition in (A.2) results, after some manipulations, in (3.5).

The singular case, when \( c^* = x_k \) for some \( k \), is more involved as, in this case, (A.5) and hence 1st condition in (A.2) do not hold (since \( x = 0 \) and \(|x|\) is not differentiable at \( x = 0 \)). To deal with this case, we consider a regularized version of Problem 2 of the following form:

\[
\begin{align*}
\min \{P_k, c\} & \quad \sum_k P_k \\
\text{subject to} & \quad P_k \geq \beta_k|c - x_k|^\nu_k, \, \forall \, k = 1, \ldots, N, \quad (A.6a) \\
\end{align*}
\]

where \(|x|^h = (|x|^2 + h^2)^{1/2}\), for some \( h \neq 0 \). Since \(|x|^h\) is differentiable for any \( x \) (including \( x = 0 \)) when \( h \neq 0 \), the singularity is always avoided and one can use the same KKT-based approach as above. The respective KKT conditions are:

\[
\begin{align*}
\frac{\partial L}{\partial c} &= \sum_k \lambda_k \beta_k \nu_k (c - x_k)|c - x_k|^\nu_k - 2 = 0, \quad \frac{\partial L}{\partial P_k} = 1 - \lambda_k = 0, \quad (A.7) \\
\lambda_k (\beta_k|c - x_k|^\nu_k - P_k) &= 0, \quad (A.8) \\
P_k \geq \beta_k|c - x_k|^\nu_k, \, \lambda_k \geq 0, \quad (A.9)
\end{align*}
\]

where we have used

\[
\frac{\partial |x|^\nu}{\partial x} = \nu x |x|^{\nu - 2}, \quad (A.10)
\]

that is valid for any \( x \), including \( x = 0 \), hence avoiding the singularity problem. Solving the KKT conditions in the same way as above, one obtains the optimal location \( c^*(h) \) as follows:

\[
c^*(h) = \sum_k \theta_k(h)x_k, \, \theta_k(h) = \frac{\beta_k \nu_k |c^*(h) - x_k|^\nu_k - 2}{\sum_k \beta_k \nu_k |c^*(h) - x_k|^\nu_k - 2}. \quad (A.11)
\]

To proceed further, let \( P_T = \sum_k \beta_k|c^* - x_k|^\nu_k \) and \( P_h = \sum_k \beta_k|c^*(h) - x_k|^\nu_k \) be the optimal total transmit powers of the BS for the original and regularized problems, and \( P_T(c) = \sum_k \beta_k|c - x_k|^\nu_k \) be the total BS transmit power for the original problem when the BS is
located at \(c\) (not necessarily optimal). Their relationship can be characterized as follows.

**Lemma 1.** The powers \(P_T\), \(P_T(c)\) and \(P_h\) are related as follows:

\[
P_T \leq P_T(c^*(h)) \leq P_h,
\]

where \(P_T(c^*(h))\) is the total BS power of the original problem when it is located at \(c^*(h)\), i.e. the optimal location of the regularized problem. Furthermore,

\[
\lim_{h \to 0} P_h = P_T = \lim_{h \to 0} P_T(c^*(h)),
\]

and, when the limit exists, \(\lim_{h \to 0} c^*(h) = c^*\).

**Proof.** Since

\[
|c - x_k|^\nu_k \leq |c - x_k_h^*|^\nu_k,
\]

for any \(c\), \(k\), and \(h\), it follows that

\[
P_h = \sum_k \beta_k |c^*(h) - x_k|^\nu_k \geq P_T(c^*(h)) \geq \min_c P_T(c) = P_T,
\]

as required. Since \(|c - x_k_h^*|^\nu_k\) is continuous and

\[
\lim_{h \to 0} |c - x_k_h^*|^\nu_k = |c - x_k|^\nu_k,
\]

it follows that \(\lim_{h \to 0} P_h = P_T\), and hence, from (A.15),

\[
\lim_{h \to 0} P_T(c^*(h)) = P_T,
\]

and, when the limit exists,

\[
\lim_{h \to 0} c^*(h) = c^*.
\]
since $P_T(c)$ is a continuous function. \qed

Now notice that the KKT conditions (A.7)-(A.9) of the regularized problem converge to those of the original problem in (A.2)-(A.4) if $\nu_k \geq 2$, since $|x|_h \rightarrow |x|$ as $h \rightarrow 0$ and the regularized KKT conditions are continuous in $h$ when $\nu_k \geq 2$ (even if $c = x_k$). Hence, $c^*(h) \rightarrow c^*$ as $h \rightarrow 0$, i.e. the regularized problem solution converges to that of the original one and thus (3.5) holds in full generality when $\nu_k \geq 2$ (even in the singular case $c^* = x_k$).

A.2 Proof of Proposition 1 in Chapter 3

We need the following technical lemma.

**Lemma 2.** The function $f(x) = |x|^\nu$ is strictly convex for any $\nu > 1$.

**Proof.** For any $x, y$, $0 \leq \theta \leq 1$, the following holds:

$$f(\theta x + (1 - \theta)y) = |\theta x + (1 - \theta)y|^\nu$$
$$\leq (\theta |x| + (1 - \theta)|y|)^\nu$$
$$\leq \theta |x|^\nu + (1 - \theta)|y|^\nu$$
$$= \theta f(x) + (1 - \theta)f(y), \quad (A.19)$$

where 1st inequality is due to the triangle inequality,

$$|\theta x + (1 - \theta)y| \leq \theta |x| + (1 - \theta)|y|, \quad (A.20)$$

and the fact that $x^\nu$ is strictly increasing, while 2nd inequality is due to the convexity of $x^\nu$ for $\nu > 1$. To establish strict convexity, let $x \neq y$ and $0 < \theta < 1$, and observe that 1st inequality in (A.19) is strict if $x \neq \alpha y$ for any $\alpha > 0$, due to the strict inequality in (A.20) in this case, and hence

$$f(\theta x + (1 - \theta)y) < \theta f(x) + (1 - \theta)f(y), \quad (A.21)$$
as required. On the other hand, if \( x = \alpha y \) for some \( \alpha > 0, \alpha \neq 1 \), then \( |x| \neq |y| \) and 2nd inequality in (A.19) is strict, due to the strict convexity of \( x^\nu \) for \( \nu > 1 \), hence implying (A.21).

\[ \blacksquare \]

Now, the uniqueness of the solution follows from the fact that \( \mathbf{P2} \) is equivalent to

\[
\min_c \sum_k \beta_k |c - x_k|^\nu_k, \tag{A.22}
\]

since transmitting with the least per-user power is optimal, and the objective here is strictly convex if \( \nu_k > 1 \) for some \( k \), from Lemma 2, and thus the solution is unique [67]. Non-uniqueness for \( \nu_k = 1 \) can be shown via examples, see Proposition 5 and Fig. 3.6.
References


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