PHOTONIC INTEGRATED CIRCUITS UTILIZING NANO-ELECTROMECHANICAL SYSTEMS ON SILICON-ON-INSULATOR PLATFORM FOR SOFTWARE DEFINED NETWORKING IN ELASTIC OPTICAL NETWORKS:
NEW INSIGHTS INTO PHASED ARRAY SYSTEMS, TUNABLE WDM, AND CASCADED FIR AND IIR ARCHITECTURES

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Doctorate in Philosophy
degree in Electrical and Computer Engineering

Ottawa-Carleton Institute for Electrical and Computer Engineering
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AUTHOR'S DEDICATION

THIS PAGE IS DEDICATED BY THE AUTHOR TO THANK ALLAH, AND TO THANK PROPHET MOHAMMED AND HIS PURE PROGENY (AHL AL-BAYT), MAY THE BLESSINGS AND PEACE OF ALLAH BE UPON THEM

ALL PRAISE IS DUE TO ALLAH THE LORD OF THE WORLDS


PROPHET MOHAMMED, MAY THE BLESSINGS AND PEACE OF ALLAH BE UPON HIM AND HIS PURE PROGENY (AHL AL-BAYT), SAID:

"I AM THE CITY OF KNOWLEDGE, AND ALI IS ITS GATE; SO WHOMEVER DESIRES KNOWLEDGE LET HIM ENTER THE GATE"

ALI A. HUSSEIN

TORONTO, ONTARIO, CANADA, OCTOBER 2015, DHU AL-HIJJA 1436 A.H.
Ali A. Hussein received the award displayed in the upper image in the 2013 IEEE Ottawa Section Annual General Meeting held at the Ottawa Convention Centre, Ottawa, Canada, in 2013. This image is shown here as a proof for awarding the refereed conference proceeding paper [P3] in the list of thesis publications.

Note: The signatures are removed intentionally from the image.
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I want to express my deep sense of gratitude to my supervisor Professor Trevor J. Hall for his valuable support and guidance throughout the development of this thesis. I am gratefully indebted to him for providing the elements needed to realize this achievement. Working with Professor Hall is pleasure and honor.

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ABSTRACT

Optical communications systems operate at the limits of their margins to respond to increasing capacity demands. Some of the signal processing functions required must soon operate at speeds beyond electronic implementation. Optical signal processors are fundamentally analog which requires precise control of the operating state. Programmable optical components are consequently essential. The thesis explores and elucidates the properties of meshes of generalized Mach-Zehnder interferometers (GMZIs) amenable to silicon (Si) photonics integration that are based on multimode interference couplers with programmability achieved via voltage controlled phase-shift elements within the interferometer arms to perform a variety of finite impulse response (FIR) and infinite impulse response (IIR) signal processing functions.

The thesis presents a novel class of integrated photonic phased array systems with a single-stage, multistage, and feedback architectures. The designed photonic integrated systems utilize nanoelectromechanical-system (NEMS) operated phase shifters of cascaded free suspended slot waveguides that are compact and require a small amount of power to operate. The structure of the integrated photonic phased array switch (IPPAS) elements is organized such that it brings the NEMS-operated phase shifters to the exterior sides of the construction; facilitating electrical connection. The transition slot couplers used to interconnect the phase shifters to the rest of the silicon structure are designed to enable biasing one of the silicon beams of each phase shifter from an electrode located at the side of the phase shifter. The other silicon beam of each phase shifter is biased through the rest of the silicon structure of the fabric, which is taken as a ground. Phased array processors of 2×2 and 4×4 multiple-input-multiple-output (MIMO) ports are conveniently designed within reasonable footprints native to the current fabrication technologies. The response of the single-stage 4×4 broadband IPPAS element is determined, and its phase synthesis states required for single-throw, double-throw and broadcast routing operations are predicted. The transmission responses of the single-stage wavelength division multiplexing (WDM) processors of 2×2 and 4×4 MIMO ports are simulated. The wavelength steering capability of the transmission interferograms by applying progressive phase shifts through the array of NEMS-operated phase shift elements of the single-stage 4×4 WDM (de)multiplexer is demonstrated.

The advantages of cascading broadband and WDM phased array sections are articulated through several study cases. Five different cascaded phased array architectures are trialed for the
construction of non-blocking 4×4 IPPAS broadband switches that are essential elements in the construction of universal photonic processors. A cascaded 2×2 WDM (de)multiplexer that can set the bandwidth of the (de)multiplexed cyclic channels into a binary number of programmable values is demonstrated. The envelope and wavelength modulations of the transmission responses utilizing a cascaded forward structure of three 2×2 sections that can be utilized for the (de)multiplexing of different bandwidth channels are demonstrated providing individual wavelength steering capability of the narrowband and wideband channels and the individual wavelength steering capability of the slow envelope and wavelength modulating functions. Innovative universal 2×2 and 4×4 cascaded phased array processors of advanced high-order architectures that can function as both non-blocking broadband routers and tunable WDM (de)multiplexers with spectrum steering and bandwidth control of the (de)multiplexed demands are introduced.

The multimode interference (MMI) coupler is utilized for the construction of several IIR feedback photonic processors. Tunable photonic feedback processors have the advantage of using less number of MMI couplers compared to their counterparts of FIR forward-path processors saving on the footprint and loss merits. A passive feedback 2×2 (de)multiplexer made of a 4×4 MMI coupler and two loopback paths is proposed. The inclusion of an imbalance in the lengths of the loopback paths of the same symmetrical feedback (de)multiplexer is demonstrated to achieve wavelength modulation of the (de)multiplexed transmission responses that are useful for the (de)multiplexing of different bandwidth channels. Several newly introduced IIR feedback architectures are demonstrated to function similarly as their counterparts of FIR forward-path processors as binary bandwidth variable (de)multiplexers, envelope and wavelength modulation (de)multiplexers, and universal feedback processors.

The investigation provided in this thesis is also supported with dynamic zero-pole evolution analysis in the complex plane of analysis of the studied FIR and IIR photonic processors to enhance understanding the principle of operation. This research expands the prospective for constructing innovative silicon-on-insulator (SOI) based optical processors for applications in modern optical communication systems and programmable elastic optical networks (EONs).
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<td>3D</td>
<td>Three Dimensional</td>
<td>FEM</td>
<td>Finite Element Method</td>
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<td>AGD</td>
<td>Average Gain Difference</td>
<td>FIR</td>
<td>Finite Impulse Response</td>
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<td>AM</td>
<td>Amplitude Modulation</td>
<td>FSR</td>
<td>Free Spectral Range</td>
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<td>ASK</td>
<td>Amplitude Shift Keying</td>
<td>FV</td>
<td>Full Vectorial</td>
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<td>AWG</td>
<td>Arrayed Waveguide Grating</td>
<td>GMZI</td>
<td>Generalized Mach-Zehnder Interferometer</td>
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<td>AWGR</td>
<td>Arrayed Waveguide Grating Router</td>
<td>ICT</td>
<td>Information and Communications Technology</td>
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<tr>
<td>BOX</td>
<td>Buried OXide (SiO₂) layer</td>
<td>IIR</td>
<td>Infinite Impulse Response</td>
</tr>
<tr>
<td>BPSK</td>
<td>Binary Phase Shift Keying</td>
<td>IMDD</td>
<td>Intensity Modulation Direct Detection</td>
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<td>CMC</td>
<td>Canadian Microelectronics Corporation</td>
<td>IMEC</td>
<td>Inter-university Micro-Electronics Centre</td>
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<td>CO</td>
<td>Central Office</td>
<td>IMWP</td>
<td>Integrated MicroWave Photonic</td>
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<tr>
<td>CWDM</td>
<td>Coarse Wavelength Division Multiplexing</td>
<td>I/O</td>
<td>Input/Output</td>
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<td>DBR</td>
<td>Distributed Bragg Reflector</td>
<td>IPPAS</td>
<td>Integrated Photonic Phased Array Switch</td>
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<td>DFT</td>
<td>Discrete Fourier Transform</td>
<td>ISP</td>
<td>Internet Service Provider</td>
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<td>DL-DIDO</td>
<td>Double-Loop Double-Input-Doubl-</td>
<td>LTI</td>
<td>Linear Time Invariant</td>
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<td>DP-QPSK</td>
<td>Dual Polarization Quadrature Phase Shift Keying</td>
<td>ITU</td>
<td>International Telecommunication Union</td>
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<td>DWDM</td>
<td>Dense Wavelength Division Multiplexing</td>
<td>LCs</td>
<td>Liquid Crystals</td>
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<td>EG</td>
<td>Echelle Grating</td>
<td>MEMS</td>
<td>MicroElectroMechanical Systems</td>
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<td>EIM</td>
<td>Effective Index Method</td>
<td>MIMO</td>
<td>Multiple-Input-Multiple-Output</td>
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<tr>
<td>emi</td>
<td>electromechanics (in COMSOL Multiphysics)</td>
<td>MMI</td>
<td>Multimode Interference</td>
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<td>EON</td>
<td>Elastic Optical Network</td>
<td>MRR</td>
<td>MicroRing Resonator</td>
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<td>FDM</td>
<td>Finite Difference Mode</td>
<td>MZI</td>
<td>Mach-Zehnder Interferometer</td>
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<td>NEMS</td>
<td>Nano-ElectroMechanical-System</td>
<td>RN</td>
<td>Remote Node</td>
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<td>NAI</td>
<td>Network Artificial Intelligence</td>
<td>SDN</td>
<td>Software-Defined Networking</td>
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<td>NLCs</td>
<td>Nematic Liquid Crystals</td>
<td>Si</td>
<td>Silicon</td>
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<td>NRZ</td>
<td>Non-Return to Zero</td>
<td>Si₃N₄</td>
<td>Silicon Nitride</td>
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<td>OOK</td>
<td>On-Off Keying</td>
<td>SiO₂</td>
<td>Silicon Oxide</td>
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<td>OPA</td>
<td>Optical Phased-Array</td>
<td>SiON</td>
<td>Silicon OxyNitride</td>
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<td>PIC</td>
<td>Photonic Integrated Circuit</td>
<td>SOI</td>
<td>Silicon-On-Insulator</td>
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<tr>
<td>PLC</td>
<td>Planer Lightwave Circuit</td>
<td>SMF</td>
<td>Single Mode Fiber</td>
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<tr>
<td>PMF</td>
<td>Polarization Maintaining Fiber</td>
<td>SSC</td>
<td>Spot-Size Converter</td>
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<td>PMLs</td>
<td>Perfectly Matched Layers</td>
<td>TDM</td>
<td>Time Division Multiplexing</td>
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<td>PON</td>
<td>Passive Optical Network</td>
<td>TPA</td>
<td>Two-Photon Absorption</td>
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<td>PSK</td>
<td>Phase Shift Keying</td>
<td>WDM</td>
<td>Wavelength Division Multiplexing</td>
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<td>PTLaB</td>
<td>Photonic Technology Laboratory</td>
<td>WSS</td>
<td>Wavelength-Selective Switch</td>
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<td>Q</td>
<td>Quality factor</td>
<td>WXC</td>
<td>Wavelength Cross Connection</td>
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<td>QAM</td>
<td>Quadrature Amplitude Modulation</td>
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**Figure 2.18** COMSOL Multiphysics simulation of the displacement deformation of the designed suspended silicon beams for both fixed (left) and hinged (right) boundary conditions. $L_s = 5.4\ \mu\text{m}$, $W_{gs} = 230\ \text{nm}$, $W_{s0} = 100\ \text{nm}$, $h = 300\ \text{nm}$, and $V = 15\ \text{V}$.

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Chapter (5) Figures

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### Figure 5.13
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**Figure app.8** Cascade of two single-loop resonator feedback elements utilizing the -3dB 2×2 MMI coupler.

**Figure app.9** Dynamic zero-pole evolution diagram for the cascaded system of two single-loop feedback resonators of Figure app.8. $L_A = 200 \, \mu m$ and every feedback loop attenuation is 3 dB. $L_B$ equals to 200 $\mu m$ (up-left), 200.36 $\mu m$ (up-right), 200.18 $\mu m$ (down-left), and 230 $\mu m$ (down-right).

**Figure app.10** Symmetrical passive feedback DL-DIDO (de)multiplexer.

**Figure app.11** Dynamic zero-pole evolution diagram of $\tau_{22}$ of the DL-DIDO feedback element of Figure app.10. $L_1 = L_4 = 210 \, \mu m$. No attenuators included.

**Figure app.12** Dynamic zero-pole evolution diagram of $\tau_{22}$ of the DL-DIDO feedback element of Figure app.10. $L_1 = 210 \, \mu m$ and $L_4 = 210.36 \, \mu m$. No attenuators included.

**Figure app.13** Dynamic zero-pole evolution diagrams of $\tau_{22}$ of the DL-DIDO feedback element of Figure app.10. $L_1 = 210 \, \mu m$ and $L_4 = 260 \, \mu m$. No attenuators included.

**Figure app.14** Feedback 2nd order binary bandwidth variable 2×2 (de)multiplexer.

**Figure app.15** Dynamic zero-pole evolution diagrams of $\tau_{22}$ of the (de)multiplexer of Figure app.14. $L_A = 210 \, \mu m$ and $L_C = 315 \, \mu m$. $\Delta \phi_B$ is 0° (up-left), 180° (up-right), and 90° (down-left and down-right). No attenuators included.

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**Figure app.17** Envelope/Wavelength modulation feedback (de)multiplexer.

**Figure app.18** Dynamic zero-pole evolution diagrams of $\tau_{22}$ of the (de)multiplexer of Figure app.17. $L_A = L_C = 210 \, \mu m$, $\Delta L_B = 50 \, \mu m$, and $\Delta \phi_B = 0^\circ$. No attenuators included.

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**Figure app.21** Tunable single-stage FIR 2×2 WDM (de)multiplexer.

**Figure app.22** Universal 2nd order 2×2 FIR phase array processor.
CHAPTER 1
INTRODUCTION

1.1 Background and Motivation
Emerging applications like streaming video (e.g. Netflix), social networking (e.g. Facebook) and cloud computing (e.g. Amazon Web Services and Dropbox) are driving the exponential growth of the traffic carried over the world’s information and communications technology (ICT) networks. This growth has been sustained by the proliferation of data centers and the ingenuity of engineers to extract sufficient capacity from the installed base of optical fiber. Optical fiber exhaust has been staved off by incremental improvements combined with breakthroughs in the enabling technology associated with wavelength division multiplexing and digital coherent optical transmission using spectrally efficient modulation formats.

Wavelength division multiplexing (WDM) is a means of carrying multiple data streams using different colors of light which are analogous to the multiple lanes of a highway. Initial optical communication systems were analogous to the spark transmitters of the early 1900s where a noise source is turned on and off at a transmitter to encode data. Digital coherent optical transmission is the optical equivalent of modern wireless communication in which one can tune with high selectivity to the desired channel. Innovations in cellular radio also inspire spectrally efficient modulation formats which enable higher data rate signals to be packed into a smaller channel by making use of more than the two levels of digital on-off modulation.

Optical communications once thought to offer essentially infinite bandwidth, now face the challenge long faced by wireless of squeezing more capacity out of a crowded spectrum. Wireless communication and optical communication are converging in terms of methods and technology. Optical communication increasingly makes use of coherent transmission, advanced modulation formats, and sophisticated signal processing algorithms long used in wireless. Conversely, 5G wireless envisages optical fiber transmission for front-haul and back-haul, radio-over-fiber supported distributed antenna systems for massive multiple-input-multiple-output (MIMO) signal processing, and RF photonic technology for signal processing and the generation of spectrally pure carriers. The bottom line on increasing the capacity of a channel is Shannon’s channel capacity limit in which signal-to-noise ratio plays a primary role and channel bandwidth a secondary role.
Software-defined networking (SDN), elastic optical networks (EON) and network artificial intelligence (NAI) have gained wide acceptance as these technologies support network operators like Bell, Rogers, and Verizon in their quest to address the challenge of rapidly changing and demanding service requirements while making efficient use of network resources [1,2]. SDN allows programmability of network functions and protocols by decoupling the control plane and the data plane, i.e. by separating the equipment that controls the network from the equipment responsible for the transmission and routing of the data. This concept is rather like replacing a highway carrying vehicles with autonomous drivers with a railway network with a central control headquarters that transport vehicles and their drivers in its wagons (c.f. the UK channel tunnel). The SDN technology allows network operators to manipulate a logical map of the network and create multiple co-existing network slices (virtual networks) in technology and protocol agnostic manner. EON allows the allocation of an arbitrary and appropriate spectral range and modulation format to an optical path according to application bandwidth and quality of service requirements taking into account optical physical layer attributes such as impairments. NAI allows the network to automatically provision resources in response to current service requests while learning from the past to improve network efficiency and effectiveness.

To fully implement SDN and EON, all network elements must be software programmable, but traditionally this has not been the case in the optical transport layer which has remained opaque to higher layers. Recent advances in photonic network technology offer the promise of software-programmable photonic network elements [3-20]. These programmable features can be exploited with SDN to enhance capacity exploitation, efficient resource utilization, and energy efficiency. In particular, the growing technical capabilities of photonic integration with novel algorithms, architectures, and components are making coherent photonic integrated circuit (PIC) implementation practical. The capability to introduce a new and very interesting category of highly functional optical signal processing systems that may be SDN-NAI enabled and manufactured in volume at low-cost covers wide application including data center interconnection, wired and wireless communications, and quantum optics.

1.2 Flexible Bandwidth Provision
Dense wavelength division multiplexing (DWDM) operates by multiplexing different data rate channels of different bandwidths utilizing different modulation methods over the operating C-band
(1530-1565 nm) of optical fibers. The ITU G.694 industrial standard uses a frequency granularity of 12.5 GHz. The optical spectrum of the fixed bandwidth grid illustrated in part (a) of Figure 1.2 is divided into fixed-width divisions each of 50 GHz (4×12.5 GHz); typically supporting the multiplexing of up to 88 channels across the C-band for long haul transmission. Each channel carries either 10 Gb/s, 40 Gb/s or 100 Gb/s data rate services. The spectral efficiency of the modulation format used for a channel transmission is defined as the ratio of the transmission bit rate and the channel bandwidth. The 10 Gb/s service using non-return to zero (NRZ) on-off keying (OOK) intensity modulation direct detection (IMDD) fits easily into the 50 GHz channel using about half the available bandwidth. The remaining bandwidth of the channel is wasted. The corresponding spectral efficiency of 0.2 bps/Hz is low. The 40 Gb/s and 100 Gb/s data services that make use of coherent detection and multi-level modulation formats, as in dual-polarization quadrature phase-shift keying (DP-QPSK), use most of the 50 GHz fixed slot bandwidth providing improved spectral efficiencies of 0.8 bps/Hz and 2 bps/Hz, respectively.

The industry is moving towards even higher data rates; 400 Gb/s channel transmission systems are already in deployment, and 1 Tb/s channel rates are expected shortly. To maximize the spectral efficiency, each of these channels must be routed across the network as a single entity. Accommodating these high bit rate services within the 50 GHz bandwidth of the fixed grid is not practical as the advanced modulation format used must provide an excessive number of bits per symbol reducing the length of fiber over which the optical signal can be transmitted with sufficient signal to noise ratio. Therefore the adoption of more flexible grids that combine a variety of bandwidth divisions that can accommodate the multiplexing of contiguous super-channels while maintaining compatibility with the multiplexing of the existing lower data rates channels has become essential. A flexible grid, also referred to as gridless channel spacing, has already been adopted by the ITU G.694.1 standard which provides wider channel bandwidth divisions (e.g. 75 GHz and 150 GHz). For example, 400 Gb/s DP-QPSK is suited for transmission over long haul distances and may be accommodated by a 150 GHz (12×12.5 GHz) bandwidth channel.

Flexible grid multiplexing enables the transmission of different data rates and improves the spectral efficiency by removing the inter-channel guard bands. In applications, different levels of flexibility might be required in different parts of the network. In part (b) of Figure 1.1 a semi-flexible grid provides the multiplexing of fixed wide bandwidth divisions and standard 50 GHz fixed
bandwidth divisions. This scheme enables some super-channels of 400 Gb/s or even 1 Tb/s to be carried along with the regular multiplexing of smaller 10 Gb/s, 40 Gb/s and 100 Gb/s demands. In the parts of the network where the transmission of super bandwidth demands is not required, this multiplexing scheme can be adapted to increase the spectral efficiency for multiplexing the 40 Gb/s and 100 Gb/s demands in one bandwidth division size and the multiplexing of the 10 Gb/s demands in another smaller bandwidth division size.

Figure 1.1 Fixed grid and flexible grid bandwidth utilization in optical fibers [1].

In part (c) of Figure 1.1 a flexible hierarchical multiplexing scheme subdivides the optical spectrum into flexible primary and flexible secondary bandwidth divisions. The secondary bandwidth divisions can be divided into further bandwidth divisions as needed. In an example, the primary division can be set to accommodate a super bandwidth demand such as the 400 Gb/s or 1 Tb/s. Then
the equal bandwidth of the secondary divisions within each primary division can be set flexibly to accommodate one low data rate category such as the 100 Gb/s or 40 Gb/s as seen in the figure. The SDN programmable control plane can decide on the desired bandwidth of the secondary channels based on a customer request.

Part (d) of Figure 1.1 shows flexible multiplexing of different bandwidth demands of all sizes combined packing of the channels within the band in an arbitrary order. This flexible utilization of the spectrum should provide the capability to change the location of the channels within the band. Dropped channels leave empty slots which the SDN controller can try to fill with new demands. Inevitably gaps will only be partially filled, and it may not be possible to find an empty slot large enough to accommodate a new demand even though there is sufficient aggregate unused bandwidth. Similar to computer disk maintenance, the defragmentation of the multiplex is required. As a simple example, if one of the interior channels is dropped then the SDN controller might decide to translate all channels located to the right (left) side of the dropped channel to the left (right) side to remove the gap and therefore allow the addition of new demands without leaving unused bandwidth. As a consequence, the (de)multiplexing elements used in EONs must offer: (i) the steering of the (de)multiplexed channels over the whole band, (ii) changing the (de)multiplexing channel bandwidth, and (iii) (de)multiplexing channels with a mix of bandwidths.

1.3 Integrated Circuit Coherent Optical Signal Processors

The coherent optical processor PIC architectures presented in the thesis are programmable extensions of a class of optical filters that includes Arrayed Waveguide Gratings (AWGs), Echelle Gratings (EGs), (Generalised) Mach-Zehnder Interferometers ((G)MZIs) [21-26], and Microring Resonators (MRRs); see Figure 1.2. Optical circuits inherently introduce a discretization in space that is analogous to the discretization in time inherent in digital signal processing. Madsen and Zhao [27] draw on this analogy to formulate an approach to optical filter design and analysis based on signal processing concepts thereby offering a methodology to tailor the filtering characteristic of photonic processors in accordance with the frequency response of known finite impulse response (FIR) and infinite impulse response (IIR) digital filters architectures.

AWG, EG, and GMZI are distinct embodiments of a linear time-invariant (LTI) system that forms a linear combination of a set of discrete regularly spaced samples of a windowed input signal. The samples are provided by a set of delay lines and combined by a discretized Fourier transform
[27-29], which gives rise to an equivalent interpretation of the system as an optical phased array (OPA). The LTI system is completely characterized by its finite impulse response or equivalently by its frequency response that is periodic on account of the sampling. The period defines the free-spectral range (FSR), and the time duration of the window determines the highest resolution (minimum bandwidth) that can be achieved.

An AWG is a faithful embodiment of the system described. Light from an input waveguide is distributed via a first star coupler to an array of waveguides. The waveguide array is curved so that each waveguide differs from its adjacent guide by the same increment in its path length. The arrayed waveguides are connected to an array of output waveguides by a second star coupler. The star couplers perform a discretized Fourier transform via Fresnel diffraction in a free-space waveguide slab region that is regularly sampled by the input and output waveguides, which are tapered to optimize light collection to minimize overall insertion loss. The number $N$ of arrayed waveguides is typically chosen to be greater than the number $M$ of output waveguides. Typically the AWG is equipped with a single input, but it is not restricted to only one input. An arrayed waveguide grating router (AWGR) features $M$ inputs with each input providing a cyclic permutation of the center wavelengths of the $M$ output channels. Test structures may feature several input waveguides to align the channelized response to a frequency grid. Two or more inputs may be excited simultaneously for passband flattening [30-32]. The EG is similar in structure to the AWG folded back on itself. The
curved grating operates in reflection and provides the delays. The Fourier transform is again provided by a free-space waveguide slab region.

If \( N = M \) the discretized Fourier transform may be identified with an \( N \times N \) discrete Fourier transform (DFT) and, if \( N \) may be factored, may be further decomposed into a network of reduced dimension DFT blocks which can be identified with a network of optical delay, phase shift, and coupler components. If \( N=2^L \) the reduction may proceed down to \( 2 \times 2 \) couplers and the circuit rearranged into a functionally equivalent but simpler form more suited to implementation as a PIC. Every path from a specified input to a specified output of the decomposed system has the structure of a cascade of \( 2 \times 2 \) MZI sections each incorporates a delay line and phase shift element within one of its two arms. This process is well described by Hillerkuss et al. [29].

Doerr and Okamoto [32] and Bogaerts et al. [33,34] have compared AWG, EG & MZI filters. Bogaerts concludes that AWGs are suited to DWDM (i.e. 25/50 GHz channel spacing), EGs are suited to coarse wavelength division multiplexing (CWDM) (i.e. 20 nm wavelength channel spacing), and MZIs can offer lower insertion loss. Doerr and Okamoto report very impressive results for AWGs in terms of insertion loss, number of channels, channel spacing, and uniformity. This is even more impressive given that there is no means to adjust splitting ratios or phase-shifts to trim its operating state; it comes as is fabricated.

A cascade of MZI, in contrast, can be tuned by the phase shift elements. Moreover, the individual couplers of an MZI may be implemented as individual inner MZIs to compensate for errors in splitting ratios due to fabrication imperfections. Adjustable differential phase shifts between the arms of the inner MZIs providing variable splitting ratios combined with appropriate algorithms can then trim the outer MZI to ultra-high extinction [35-37], and thereby in the case of WDM can reduce the level of cross-talk between the frequency bins.

The observation that a DFT is equivalent to a bank of FIR filters leads to the consideration of using other filter types. An MRR (see [38] for a review) is equivalent to an IIR filter that has an all-pole frequency response\(^1\). These have the advantage that their finesse, i.e. the ratio of their bandwidth to their free-spectral range exceeds that of an FIR filter. Integrated MRRs with an intrinsic quality factor (Q) of 81 million have been reported [39], which is sufficient for a resolution

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\(^1\) The frequency response is given by the reciprocal of a trigonometric polynomial. The poles are located at the zeros of the trigonometrical polynomial.
of 2.5 MHz. The bandwidth of an MRR may be adjusted by using an MZI with phase shifters in its arms as a variable coupler into the ring. The resonant frequency of an MRR may be tuned using an intra-ring phase shifter. However, the introduction of any loss within the ring risks compromising the Q of the MRR impairing its selectivity.

One can place MRRs within the arms of an MZI to create one unit of a lattice filter with a frequency response defined by a rational function of trigonometrical polynomials with an equal number of poles and zeros\(^2\) \[40,41\]. This is useful when filters with flat-topped passband bands with steep sides and deep rejection bands are desired, e.g. to apply a divide-and-conquer strategy to the spectral analysis. The poles and zeros of each unit may in principle be freely located using phase shift elements permitting the frequency response to be programmable and optimized automatically by a suitable control algorithm \[41\] albeit required monitoring ports load the Q of the MRR.

The spectral frequency content of a signal is given by the Fourier transform of the signal in the time domain. The latter ideally involves integration over an infinite time duration. In practice, only a finite duration of the signal can be analyzed. The duration of the segment \(\Delta t\) determines the frequency resolution \(\Delta f \approx 1/\Delta t\). It follows that a spectral processor must have a means of storing a signal for a duration sufficient to meet the desired bandwidth resolution. In a PIC, the complex envelope of a sufficiently narrowband optical signal propagates at the group velocity \(v_g\) and hence storing a segment of this envelope corresponds to propagation over a path length \(\Delta l\) given by:

\[
\Delta l = v_g \Delta t = \left(c / n_g\right) \Delta t \approx \left(c / n_g\right) \left(1 / \Delta f\right)
\]

... Eq. (1.1)

where \(c = 3\times10^8\) ms\(^{-1}\) is the vacuum velocity of light and \(n_g\) is the group index of the waveguide. One arrives at the estimate \(\Delta l \approx 6\) mm, where the values of the parameters \(n_g = 5.06\) and \(\Delta f = 10\) GHz have been taken for estimation. The value of the waveguide group index for the integration platform used in the thesis is estimated at around the 1.55 \(\mu\)m center wavelength. The loss factor \(\alpha\) of the propagation waveguide delay line must satisfy:

\[
\alpha \Delta l < < 1 \Rightarrow \alpha < < 7.3\ \text{dB.cm}^{-1}
\]

... Eq. (1.2)

This loss includes bending loss in addition to waveguide loss. A moderately low-loss integration material platform is consequently an essential requirement.

\(^2\) Such filters are loosely called ‘all-pass filters’ but strictly the frequency response of an all-pass filter has a constant modulus that restricts the poles and zeros to occur in reciprocal conjugate pairs about the unit circle in the \(z\)-plane.
The wavelength dependence of the phase contributed by a delay line around a specified wavelength $\lambda_0$ is accurately described by the first two terms of its Taylor series. The first term involves the effective index of the mode and sets the phase bias at the design wavelength. The second term involves the group index of the mode and sets the FSR of the delay line length. The phase bias and the FSR are insensitive to the wavelength providing that the offset from the design wavelength is relatively small. The FSR is also insensitive to small changes to the physical lengths or group index of the waveguide delay lines. In contrast, the phase bias ranges over $[0, 2\pi]$ for a one wave change of optical path length. This will tune the transfer function of an MZI block with delay lines over a full FSR which, if not controlled, would lead to disastrous results for cascaded embodiments. This implies that the path length of the waveguides and couplers must be repeatable to deep sub-wavelength precision. GMZI-based processors may be tuned and phase errors due to process variations may be compensated by placing variable phase shifters in series with the delay lines. Circuits featuring such tuneable building blocks are referred in general to as integrated photonic phased array switches (IPPASs).

The geometry of AWG and EG components lends itself to a reproducible phase shift increment between adjacent paths. Consequently, the channelized frequency response profile is robust to process variations. It is more difficult to avoid an offset of the channel frequency grid of the fabricated AWG/EG from the desired frequency grid. This offset, however, can be corrected by shifting the position of the input waveguide which may be determined using test structures with more than one input. That position can then be retained in further fabrication runs if the process is sufficiently repeatable. The performance of AWG components and MZDI-based cascaded (de)multiplexers, e.g. [42], routinely demonstrated in low to medium confinement planar lightwave circuit (PLC) technologies such as the silicon nitride integration platform [43] is impressive given an absence of trimming, whereas trimming by thermo-optical or other means is mandatory for high confinement SOI integration platforms.

The high degree of flexibility to reconfigure and optimize the MZI might prove to be its Achilles’ heel in terms of the number of control lines needed, the complexity of control, and if thermo-optic phase shifters are used thermal cross-talk and excessive power consumption. The latter can be particularly serious. Photonic phase shifters, compared to electronic phase shifters, offer an ultra-compact form factor and higher bandwidth for applications in integrated microwave photonic
(IMWP) systems [44]. However, the mW range power [45-48] required for the operation of thermo-optic phase shifters can scale up the total power used in an integrated microwave system that deploys thousands of phase shifters into impractical levels (even in the kW range). This issue becomes more serious in cases when power resources available are limited as for a satellite in space. While it is essential that phase-shift elements used for data modulation are high speed, phase-shift elements used for setting operating points may be slow, but it is critical that they consume negligible power. The nano-electromechanical-system (NEMS) phase shifter considered in this thesis, which uses power in the µW range, offers a potential solution.

1.4 Aim and Objectives

This thesis aims to advance research on coherent optical PIC architectures towards the goal of a universal processor. The universal processors are intended to be programmable modular units that can perform different tasks in the network. It is envisaged that network devices may be constructed from these units to provide a comprehensive range of functions such as non-blocking broadband switches and adaptive (de)multiplexers/routers with the flexibility needed for elastic optical network applications. Ultimately, the network devices should be able to (de)multiplex different bandwidth demands, change the bandwidth of the links either smoothly in an analog manner or several preset discrete values, provide hitless wavelength steering of the optical spectrum and yield non-blocking wavelength cross-connect routing. Specific objectives are:

1- To conceive novel coherent optical processor circuit architectures capable of an advanced level of spectrum processing of WDM channels including the tunable (de)multiplexing of channels of different and similar bandwidths, routing, and bandwidth control.

2- To assess the viability of NEMS-operated phase shift elements to reconfigure or set the operating point of relatively large-scale coherent optical processor circuit architectures given available fabrication processes. This includes a performance evaluation in terms of the achievable phase shift range per unit length, required drive voltage, power consumption, optical loss, and footprint.

3- To design and optimize the simulated performance of the all components required to construct the programmable PICs proposed in this thesis including the single-mode waveguide, inverse-taper edge couplers, tapers, multimode interference couplers (MMI), and NEMS-operated phase shifters.
4- To design, verify by simulation, layout, and prototype a PIC test chip of coherent optical processors featuring feedforward and feedback GMZI (phased array) structures reconfigured by NEMS-operated phase shift elements.

1.5 Research Methodology

1.5.1 Choice of Integration Platform

The silicon on insulator (SOI) photonic integration platform offers very tight light confinement due to the high contrast between the refractive index of the silicon core layer and the refractive indices of the surrounding silica or air cladding. The SOI platform has, therefore, become the first choice for designers seeking photonic components and circuits with small footprints. SOI also offers active high-speed modulators and waveguide photodetectors. The possibility of etching the buried oxide layer under the silicon waveguides provides an opportunity to extend the device library to include active devices based on the NEMS-operated slotted waveguides. So the extended SOI platform can provide at low price in volumes compact ultra-low power consumption PICs, among other things, for switching, modulation, detection, filtering, and flexible WDM (de)multiplexing applications. The SOI integration platform with etching in the SiO$_2$ buried oxide (BOX) layer is chosen in this thesis to examine the possible construction of large PICs proposed for programmable processing of the light spectrum in EONs.

![Figure 1.3 Silicon-on-insulator technology.](image)

Figure 1.3 illustrates the features of an SOI integration platform. The light is confined in the silicon core layer which typically has a refractive index of about 3.45. The silicon core layer is sandwiched between the oxide layer beneath and usually air above. The oxide layer has a typical refractive index of about 1.44, and the air layer has a refractive index of 1. The thickness of the oxide layer is usually in the range of 1 µm to 2 µm. The thickness of the silicon core layer, on the other hand, has been
standardized to a small number of fixed values suited to the fabrication technology. The Inter-university Micro-Electronics Centre (IMEC) SOI nanophotonic platform conveniently uses a typical silicon core layer thickness of 220 nm. The Canadian Microelectronics Corporation (CMC) Microsystem’s NanoSOI nanomachining manufacturing process has two convenient SOI platforms with two different silicon core thicknesses of 145 nm and 300 nm. Changing the thickness of the silicon core implies different component dimensions and requires redesign and the running of new simulations. In this research, the silicon base layer thickness of 300 nm of the CMC Microsystem’s Soitech platform is adopted for the designs. The silicon core layer is supposed to be fully etched to form the boundaries of the designed photonic structures. Another important parameter in constructing SOI structures is the minimum feature size, which refers to the smallest design detail that can be manufactured. The minimum feature size for the mentioned manufacturing technologies is typically 100 nm or less. The slot width of the NEMS-operated slot waveguides is 100 nm matching this limitation. The dimensions of all components considered in this thesis, including the delay lines used in the (de)multiplexers, are chosen to be within the manufacturer’s tolerances.

The SOI template utilized in this research is suitable for the telecom band applications at low optical power levels. At high optical power levels of few tens mW the two-photon absorption (TPA) and its interaction with the free charge carrier absorption can cause heating in narrow width waveguides and change their refractive index. This effect can impair the performance of MZI based switches and WDM (de)multiplexers. The silicon nitride ($\text{Si}_3\text{N}_4$) platform provides an alternative solution for high power applications because it virtually has zero TPA due to its large bandgap [49]. However, the moderate contrast in the refractive index of the $\text{Si}_3\text{N}_4$ platform of around 2 in the core versus 1.5 in the cladding results in larger system dimensions. Although the study provided in the thesis is examined for the SOI platform, the modeling and analysis of the flexible processors could be applied to other platforms in the future. Other issues associated with the SOI integration platform are the sidewall surface roughness and polarization dependence of the components. The sidewall surface roughness of the waveguides causes scattering loss. The rms roughness of the sidewall surface can be reduced in the fabrication process as by treating the silicon structure with hydrogen annealing [50]. The operation of the coherent PIC processors presented in the thesis requires stability of the input polarization state. In a controlled lab environment, external polarization controllers and polarization-maintaining fibers (PMFs) can be used to maximize the power detected at the outputs of
the processor under test. In practice, the received polarization state is unknown and fluctuates with time. This requires the equipment that processes the light spectrum to be polarization transparent. The utilization of polarization diversity schemes, where both received orthogonal polarizations are converted into one output polarization, is proposed in the literature, e.g. see [51-53]. The inclusion of an integrated polarization controller, e.g. see [54], that is programmable and can be set dynamically to the operating point following a proper algorithm is advantageous to the equipment used in EONs, but the utilization of the selected integration platform to achieve this goal is left for the future.

The robustness of the fabrication process variations of the waveguide delay lines used in the MZI-WDM (de)multiplexers is the same as in AWGs requiring subwavelength precision. The wavelength dependence due to the delay lines is determined by the first two terms of a Taylor expansion of the phase delay about a nominal center frequency. The first-order term determines the FSR and is robust to process variations. The second-order term is a phase bias at the center frequency. A variation of the delay line length for the selected integration platform in the order of ±360 nm turns the zeroth-order phase into a full wrap of ±180º, and therefore the length differences of the waveguide imbalances require accurate subwavelength precision. Tunable MZI phased array systems provide trimming capabilities to compensate for the phase errors due to fabrication process variations. High order phased array systems composed of many cascaded stages require a large number of tuning elements creating three major issues: (i) excessive power consumption and consequent cross-talk, (ii) excessive number of off-chip lines with electrode alignment difficulties, and (iii) excessive phase control complexity of the system. The use of the NEMS-operated phase shifter demonstrated in the thesis which requires one electrode and uses low power of less than 20 µW estimated at 10 MHz switching speed gives relief to both points (i) and (ii).

1.5.2 Verification by Simulation

Optical designs are verified by industry standard commercial software tools augmented by custom code and scripting. The principal tools used are the Photon Design Suite (FIMMPROP), Comsol Multiphysics (FEM), MATLAB and Visual Basic for custom code, and DW-2000 for layout. The complex finite-difference mode three-dimensional full-vectorial (FDM-3D-FV) eigenmode solver of FIMMPROP was used to determine material loss and, with the aid absorbing boundaries of perfectly matched layers (PMLs), radiation loss. The scattering parameters of the simulated MMI couplers and other components evaluated at the center wavelength of 1550 nm are found in agreement with those
published by other researchers. Therefore, it is expected that the results presented in this thesis are reproducible using alternative available simulation tools. The basic components are optimized to reach favorable figure-of-merit parameters aiming to reduce system losses and footprint. The dimensions of the different components are selected properly to make them compatible with each other for the construction of the variety of photonic processors presented in the thesis. Novel arrangements for the construction of the designed NEMS-operated phase shifters are carefully tailored to simplify electrical and optical access to the structures. The overall system response is determined by performing the transfer matrix analysis of the designed PICs utilizing the appropriate scattering. The wavelength scanning of the complex scattering parameters of all of the components used in the construction of the PICs is utilized to present realistic wavelength-dependent responses of the proposed PICs. The system losses, imbalances, and phase errors are self-contained in the simulated wavelength-dependent scattering parameters of the basic construction components and are reflected in the depicted transmission responses. No approximations are used in determining the demonstrated transmission responses for accurate performance evaluation of the studied processors. The phase response characteristic for all of the studied processors is also determined using a custom numerical script, but results are not shown throughout the thesis for brevity.

The transmission and phase modulation characteristics of the PICs are illustrated using both constellation diagrams and wavelength response graphs. Compact photonic processing elements are proposed in strategic steps to achieve specific functions including (de)multiplexing of similar and different bandwidth demands, conversion between broadband and WDM responses, comparing blocking with non-blocking switching, bandwidth variable control of the (de)multiplexed channels, and steering of the spectrum. The designed PICs presented in this thesis are chosen native to standard SOI fabrication platforms of commercial foundries and are measurable utilizing the photonic facilities available in standard photonic laboratories. The quantitative parameters of loss, crosstalk, quality factor, and imbalances in the (de)multiplexed channels are either indicated in the text wherever applicable or can be observed in the displayed transmission characteristic. In semi-elastic utilization of the spectrum, it is expected to need to multiplex permutations of different bandwidth demands. For example in one part of the network 400 Gb/s super demands might be multiplexed with 100 Gb/s demands, whereas in another part of the network 1 Tb/s super demands might be multiplexed with 100 Gb/s demands. Elasticity also means accepting any data rates,
removing the boundaries between standard bandwidth demands. Therefore, the investigation presented in this thesis is kept in a general sense to explore the principle of operation of the proposed flexible photonic processors.

1.6 Structure of the Thesis

The thesis is organized to consider first the design, verification, optimization and performance evaluation of the basic components that constitute an IPPAS element then to consider processor architectures composed of IPPAS elements within cascaded feedforward (FIR) and feedback (IIR) circuits that perform programmable functions applicable to current WDM networks and next-generation EONs. The thesis comprises four principal chapters (2-5) in addition to the introduction chapter (1) and the general conclusions and future work chapter (6).

Chapter (2) presents the study and design of the components that constitute an IPPAS element. The chapter splits into two main parts. The first part of the chapter briefly covers the passive components used in the construction of the presented PICs, including the single-mode waveguide, inverse-taper edge coupler, dual inverse-taper edge coupler, and MMI couplers. A -3dB 2×2, crossover 2×2, -6dB 4×4 and -3dB 4×4 MMI couplers that are used as splitters and combiners are conveniently designed to typical performance parameters. The designed MMI couplers are optimized for a minimum cost that combines and can trade-off excess loss and imbalance. The second part of the chapter presents the analysis, design, and simulation of the NEMS-operated phase shifter. An integral combination of simulations and numerical analysis is used in the design of 180° and 360° capable phase shifters. Simulations of the phase shift element of cascaded slot waveguides and the transition slot couplers are presented. The NEMS-operated phase shifters are used as tunable elements to control the demand bandwidth and spectrum steering.

Chapter (3) introduces the single-stage IPPAS element. This chapter provides an understanding of the structure and features of IPPAS elements functioning as either basic broadband switch elements or as tunable WDM (de)multiplexers. The chapter splits into two main parts. The first part covers the single-stage broadband 2×2 and 4×4 IPPAS switch elements. To explain the novel architecture of the broadband IPPAS elements their schematic diagrams are outlined with typical dimensions indicated on them. To signify the limited blocking switching capability available for a 4×4 single-stage broadband IPPAS element the available single-throw, double-throw, and broadcast switching operations are outlined. The second part of the chapter introduces the design of
tunable WDM (de)multiplexers. The wavelength steering of the transmission interferograms by applying a progressive phase shift to the NEMS-operated phase shift array in a single-stage 4×4 WDM (de)multiplexer is demonstrated.

Chapter (4) investigates the architectures of phased array systems made of cascaded FIR sections utilized to achieve tunable wavelength-selective routing for potential application in EONs. Five different cascaded architectures of non-blocking 4×4 broadband switches that are key elements in constructing universal broadband/WDM processors are demonstrated. The cascaded binary bandwidth variable (de)multiplexers are introduced to explain the objective of controlling the bandwidth of cyclic channels. Binary bandwidth variable (de)multiplexing is a key element in constructing logically operated flexible wavelength selective switches (WSSs) and wavelength cross-connections (WXC s). The envelope/wavelength modulation (de)multiplexers are introduced to meet the objective of (de)multiplexing different bandwidth channels. The chapter concludes to the construction of universal 2×2 and 4×4 broadband/WDM processors that constitute key elements for providing wavelength-selective bandwidth-controlled flexible routing of the channels.

Chapter (5) provides an expanded study into feedback phased array systems utilizing MMI couplers. The analysis and response of single feedback components with multiple loopback paths and MIMO ports are demonstrated. A passive (de)multiplexer utilized to (de)multiplex either similar or different bandwidth demands is constructed of a single -6dB 4×4 MMI coupler and two feedback loops. In analogous with Chapter (4), binary bandwidth variable (de)multiplexing, envelope and wavelength modulations (de)multiplexing, and universal processing are demonstrated utilizing cascaded IIR flexible processors.

The thesis also contains appendices supporting the studies presented in the different chapters. The details of some mathematical derivations are drawn to the appendices in the interest of brief description in the chapters while providing the needed details for future investigations. Some of the studied processors related to chapters (3-5) are also relegated to the appendices for the same reason. The last appendix documents the analysis of photonic system roots in the complex plane using dynamic zero-pole evolution diagrams.

1.7 Achievements and Original Contributions

The research achievements can be categorized into two major groups, one at the component level and the other one at the system level. Several components known in the prior art used in the
construction of IPPAS architectures are simulated to meet performance targets following the state of the art when implemented using the CMC Microsystem’s NanoSOI integration platform with the silicon core thickness option of 300 nm. These components include the single-mode waveguide, inverse-taper edge coupler, linear adapter taper and MMI couplers of different numbers of I/O ports. Two other innovative components, including the NEMS-operated phase shifter and dual inverse-taper edge coupler, are also presented in this research. The novel NEMS-operated phase shifter is compact, power-efficient, can achieve up to 180° or 360° of phase shift differences and requires one biasing electrode that is electrically isolated from the rest of the system structure and located at one side of the phase shifter. The proposed dual inverse-taper edge coupler, which consists of a dual inverse taper followed by a high efficiency balanced transition double slot coupler, can mitigate the low power coupling efficiency issue found in conventional inverse-taper edge couplers without the need to use a nonstandard fabrication process.

On the system side, the broadband 2×2 and 4×4 IPPAS single-stage elements are first demonstrated. Their transmission-phase modulation characteristic is explained utilizing constellation diagrams. The wavelength response of these switches is also used to quantify their performance. The available different broadcast routing operations of the 4×4 broadband switches are identified. The inclusion of both delay lines and NEMS-operated phase shifters in the interconnection network between the splitter and combiner units is verified for the construction of tunable 2×2 and 4×4 WDM (de)multiplexers. The application of a progressive phase shift through the NEMS-operated phase shift elements of a 4×4 WDM (de)multiplexer is utilized to steer the transmission responses over the wavelength range smoothly.

In this research, a broadband section refers to a network with or without NEMS-operated phase shifters and having interconnection paths of similar length between the two splitter and combiner units that are usually implemented as MMI couplers. On the other hand, a WDM section refers to a network of different length path imbalances and with or without NEMS-operated phase shifters interconnecting the splitter and combiner units. The thesis reports research findings on cascading broadband and WDM sections configured to provide solutions to a variety of advanced engineering applications. For instance, findings in the prior art [21-25] include: (i) the cascading of two broadband 4×4 sections can be utilized to increase the number of single-throw routing states, (ii) the cascading of a single 4×4 GMZI broadband switch with four banks of 2×2 MZI broadband switches
located at its input and output sides constructs a non-blocking switch, and (iii) the interconnection of two 4×4 GMZI broadband switches through a block of dual 2×2 MZI broadband switch also constructs a non-blocking switch. Lagali [26] also outlined the cascading of a 4×4 GMZI, 3×3 GMZI, and 2×2 MZI broadband switches to construct another non-blocking 4×4 GMZI switch but this choice is not compatible with the IPPAS architecture, and therefore it is excluded from this research investigation. In this thesis, other architectures are examined for constructing non-blocking 4×4 broadband switches in addition to the other priorly mentioned topologies that are compatible with the IPPAS architecture. These architectures include the cascading of three broadband 4×4 sections and the cascading of a unit of two cascaded 4×4 sections with a bank of dual 2×2 IPPAS broadband switch. All of the 24 single-throw, 18 double-throw and broadcast routing operations available for the studied 4×4 non-blocking IPPAS broadband switch architectures are determined. The non-blocking broadband switches are key elements in the construction of ROADMs, WXC, and universal router network nodes.

Although the architecture of cascading FIR and IIR photonic filter sections is known, e.g. [27], the advantages of cascading broadband and WDM photonic sections of MIMO ports for providing flexible (de)multiplexing and routing in EONs are not fully covered in the literature. In this research, the principle for constructing processors that can convert between the broadband routing and tunable WDM (de)multiplexing of the signals is examined. For example, in current optical networks, an Optical Time-Domain Reflectometry (OTDR) test conducted from the central office (CO) site is used to diagnose faulty conditions in the drop fibers at the remote nodes (RNs) by usually deploying bypass circuit components in parallel with the WDM (de)multiplexers. The broadband/WDM conversion principle can provide a direct solution to reach the drop fibers at the RNs without the traditional need to use the bypass circuit components.

In next-generation flexible optical communication networks, it is desirable to (de)multiplex narrowband and wideband channels to accommodate different signal transmission rates and to provide the capability to relocate the different demands over the wavelength range as needed at the routers and gateways. In this regard, the two techniques described in this research are the envelope modulation and wavelength modulation of the transmission interferograms. In envelope modulation, a fast ripple interferogram response is amplitude modulated by another slow varying transmission component yielding the (de)multiplexing of narrowband channels and contiguous wideband
channels. In wavelength modulation, the width of the (de)multiplexed channels is made variable over the wavelength range. The cascading of three FIR sections is tailored to achieve either envelope or wavelength modulation of the transmission responses. The steering capability of both of the fast ripple channels and the slow envelope/wavelength modulating functions along the wavelength range each in separate is shown possible. Another approach to alternatively (de)multiplex wideband and narrowband channels of respective Chebyshev-like and inverse Chebyshev-like equi-ripple characteristic with providing the spectrum steering capability is also presented. Another three cascaded architecture is utilized to achieve binary bandwidth variable (de)multiplexing. The construction of universal SOI 2×2 and 4×4 broadband/WDM processors is proposed in this research. A universal processor can be tuned to perform as a broadband switch providing different kinds of single and multiple throw routing operations and can be tuned to perform as a tunable WDM (de)multiplexer. A high-order universal processor also supports controlling the channel bandwidth for several set values and yields additional (de)multiplexing schemes.

The utilization of MMI couplers in the construction of useful feedback systems for the flexible (de)multiplexing and routing is yet to receive attention in the literature. Using the -3dB 2×2 and -6dB 4×4 MMI couplers in the construction of feedback elements with MIMO ports and multi-loopback paths is investigated in the thesis. The equivalent lengths of the MMI couplers based on their scattering phase characteristics are used in determining the total lengths of the feedback paths for accurate estimation of the resulted FSR values. A single -6dB 4×4 MMI coupler feedback component equipped with two balanced equal-length loopback paths is demonstrated to act as a 2×2 AWG (de)multiplexer for two photonic signals. The introduction of an imbalance in the lengths of the loopback paths of the same single coupler (de)multiplexer is demonstrated to produce wavelength modulation of the transmission interferograms resulting in cyclic changing in the bandwidth of the (de)multiplexed channels. Novel feedback (de)multiplexers that can switch between several FSR values following the logic rule of $2^N$ (i.e. 2, 4, 8, 16, etc.), where $N$ is the number of interconnection networks of dual NEMS-operated phase shifter used in the constructed cascaded phased structures, are demonstrated. The capability to construct feedback (de)multiplexers that can introduce either envelope or wavelength modulation of the transmission interferograms and the construction of a feedback universal 2×2 processor are also demonstrated.
A list of the original literature contributions resulted from this research is included separately on page xii of the thesis. It has to be also emphasized that although FIR and IIR architectures of cascaded photonic phased array sections are known in the art, oriented mostly to mimic the characteristics of known digital signal processing filters, the scope of the binary bandwidth variable (de)multiplexers, envelope/wavelength modulation (de)multiplexers, and universal processors is widely covered in this thesis.

1.8 Summary
In this chapter, an introduction to the utilization of photonic phased array FIR and IIR processors in EONs has been given. The requirements of flexible bandwidth utilization in EONs are explained. The novel features of the IPPAS architecture presented in this thesis have been outlined. The parameters of the SOI integration platform have been explained. The rationale of the research investigation and the organization of the thesis structure have been described. The thesis research focuses on 2×2 and 4×4 phased array architectures. The expansion of the study to larger dimensions such as the 8×8 is straightforward in theory but is more challenging in terms of fabrication. The advantages of utilizing IPPAS cascaded FIR and IIR architectures in EONs are outlined. Different degrees of flexibility in processing the photonic spectrum within the C-band that are essential requirements in EONs are summarized and related to the forthcoming chapters including wavelength steering of the spectrum, bandwidth variable control of the demands and (de)multiplexing of different bandwidth channels. To enable the realization of IPPAS cascaded FIR and IIR architectures used in providing answers for the desired objectives, the next chapter of the thesis starts with introducing the design of the construction components. The simulated wavelength-dependent scattering parameters of the designed components are used in providing realistic qualification of the behavior of the studied photonic processors.
CHAPTER 2
CONSTRUCTION COMPONENTS

2.1 Introduction

It is indicated in Chapter (1) that one of the important objectives of the thesis is to reflect the effect of the wavelength-dependent errors in phase and transmission responses of the construction components on the performance of the coherent processors used for the advanced elastic spectral processing of the optical signal. It is also indicated in Chapter (1) that the performance of the phase shift element is critical to the operation of the studied coherent processors. Since the phase shifters are used for setting the operating points and not for data modulation they do not need to be fast but because so many are used in advanced coherent processors they need to be power efficient and of compact size. The NEMS-operated phase shifter meets this requirement.

This chapter addresses the objective of studying, designing, and simulating the SOI components that are essential for the construction of the photonic processors presented in the next chapters. The chapter is organized into two parts. The first part (2.2) deals with interconnection components including the waveguide, waveguide taper, inverse-taper edge coupler, dual inverse-taper edge coupler, and MMI couplers of 2×2 and 4×4 sizes that are used as splitters and combiners. The second part (2.3) deals with the NEMS-operated phase shifter used as the tuning component. The choice of the integration platform with the silicon core thickness of 300 nm implies simulating all of the construction components including those that are conventionally designed to achieve performance comparable to their counterparts found in the literature based on different integration platforms. The dual inverse-taper edge coupler proposed in this chapter based on the SOI template provides mitigation to the low coupling efficiency of conventional edge couplers. Although MMI couplers are well known in the literature [34,55-79] the research supplements on optimizing the design as a tradeoff between the requirements for minimizing losses and imbalances. The chapter also presents an extension to the work of Acoleyen et al. [80,81] for constructing NEMS-operated phase shifters that can achieve up to 180º and 360º of voltage-activated phase shifts. The nanoelectromechanical phase shifters presented in this chapter possess novel features promoting them for the construction of large photonic integrated processors. The simulations presented in this chapter are based on the complex FDM-3D-FV solver of FIMMPROP.
2.2 Part 1: Interconnection Components

2.2.1 Introduction of Part 1

This part of the chapter presents the study and simulation of interconnection components used in photonic phased array switches and feedback integrated circuits. The list of components dealt with in this part includes the waveguide, taper adapter, inverse-taper edge coupler, and MMI couplers. The only interconnection component excluded from this part is the transition slot coupler, which is included in Part (2.3) as it is part of the NEMS-operated phase shifter. Nevertheless, a special balanced version of the transition slot coupler is used in the construction of the dual inverse-taper edge coupler presented in this part of the chapter.

Waveguides are used to interconnect the PIC components. The waveguide designed here supports the propagation of the fundamental \( TE \)-like mode. The width of the access ports to the MMI couplers is usually wider than the width of the waveguides. This is necessary to avoid exciting many higher-order modes in the multimode region for reduced modal phase errors and better self-imaging of the input mode profiles at the output ports. Therefore, a taper is required to adapt coupling the mode between the two distinct widths. The study of adiabatic and near-adiabatic tapers is reported in the literature [82-84]. The taper used in this study has simple linear geometry with its length chosen large enough to justify the near adiabatic performance.

To couple the light from optical fibers to the photonic integrated chip traditionally either grating surface couplers or inverse-taper edge couplers are used. Grating couplers provide the capability for near-vertical coupling to the chip surface, making it easy to reach the embedded structures from anywhere on the chip as desired. To utilize grating couplers, shallow etching of the silicon core layer has to be available in the foundry. An example is the 70 nm depth of shallow etching in the silicon core layer used in the IMEC fabrication process. However, in addition to the high excess coupling loss figure (typically around 6 dB) and the polarization dependence of a conventional grating coupler, the main drawback is in the limitation of its bandwidth response. Whether utilized as a broadband switch or as a WDM-like structure, an IPPAS element usually desires a broadband response to utilize the available spectrum.

Inverse-taper edge couplers provide a wider spectral response compared to grating couplers. They also have less polarization dependence. Physically, an edge coupler can only provide access to the sides of the chip. One can access an embedded structure from any two sides of the chip to
provide both input and output ports. Usually, two opposite sides of the chip are utilized to ease testing in the lab environment. Alternatively, access from one side of the chip is also possible utilizing a loopback arrangement in the layout of the structure. In such a case, using an external fiber array coupling arrangement is necessary. The main drawback of a traditional inverse-taper edge coupler is its high excess coupling loss figure (typically around 6 dB). In this thesis, a dual inverse-taper edge coupler achieving excess coupling loss of less than 3 dB is reported based on simulation results. The designed dual inverse-taper edge coupler uses the standard SOI fabrication process and does not need any special requirements. For the two I/O ports of a structure, the dual inverse-taper edge coupler is expected to save more than 6 dB of the excess coupling loss that is wasted when using traditional single inverse-taper edge couplers.

The excellent properties of SOI-MMI couplers, including compact size, low loss, low reflection, low crosstalk, and wide bandwidth response, have encouraged the use of SOI-MMI couplers in several PICs. A glance at the literature [34,55-79] provides examples of utilizing SOI-MMI couplers in applications that include electro-optic switches, MZI-based switches, polarization splitters/converts, MMI coupler based microring resonators, optical 90° hybrids, phased array wavelength division (de)multiplexers, and spectral filters. This work is based on high accuracy simulations using the complex FDM-3D-FV solver of FIMMPROP and follows credible procedures to produce MMI couplers of typical standards. The design of a variety of MMI couplers of different types and sizes is briefly presented in this part of the chapter including the -3dB 2×2, crossover 2×2, -6dB 4×4, and -3dB 4×4. All the MMI couplers presented in this chapter are simulated and designed at an operating optical center wavelength of $\lambda = 1550$ nm. The wavelength dependence of characteristic of the designed MMI couplers is demonstrated in the next chapter due to its relevance to determining the response of the constructed systems. The studied MMI couplers are used in the next chapters in constructing broadband phased array switches and tunable WDM (de)multiplexers of single-stage, cascaded, and feedback architectures.

2.2.2 Photonic Waveguides

SOI waveguides are used to interconnect the different components used in building PICs. Waveguide bends composed of circular arc segments are used in this work to create the path length differences required by phased array interferometers, to interconnect the ports of components displaced laterally from each other, and to form the feedback paths in some considered architectures. 
The topology of the waveguide depends on the integration platform. The structures of two variant waveguides are illustrated in Figure 2.1. Full etching of the silicon layer to form the waveguides is taken as the norm in this thesis. In this respect, the height $h$ of the waveguide represents the thickness of the silicon core layer, which is laid on top of the SiO$_2$ buried oxide (BOX) layer of the integration platform. Examples of SOI templates are the IMEC fabrication process with the 220 nm of silicon core thickness on top of a 2 μm thick BOX layer and the CMC Microsystem’s NanoSOI wafers with the option of 300 nm of silicon core thickness on top of a 1 μm thick BOX layer. The width of the photonic waveguide $W_g$ is usually adjusted to support efficient propagation of the TE-like fundamental mode (also called the dominant mode) and suppress the propagation of higher-order modes. The filtering of the higher-order modes within a short length (about few microns) of the waveguide by their radiation into the surrounding cladding is desirable to ensure coupling of only the fundamental mode between the distinct components of the PIC. The selection of $W_g$ depends on the wafer structure for different templates. The structure depicted by the left-hand panel of Figure 2.1 has an air cladding on the top and sides of the silicon waveguide. Whereas, for the structure depicted by the right-hand panel of the same figure, a passivation layer is deposited to form an over-cladding for the silicon waveguide. The over-cladding could be formed, for example, of silicon dioxide (SiO$_2$) as in [85] or silicon oxynitride (SiON) as in [86].

![Figure 2.1 Structure of waveguides for SOI templates with air (left) and SiO$_2$ (right) upper and side cladding.](image-url)

The simulation of the confinement factor for the first five encountered modes for the NanoSOI nanomachining template offered by CMC Microsystems having a silicon core thickness of 300 nm with air over-cladding has revealed that a single-mode waveguide requires a width within the range from 300 to 400 nm. In this thesis, the width of 350 nm is adopted for waveguides and bends that are used in the designed photonic circuits.
Figure 2.2 Simulated optical power distributions inside a straight single-mode waveguide of 500 µm length situated laterally shifted by 2 µm from another similar waveguide. $h = 300$ nm and $W_g = 350$ nm.

The high index contrast of the SOI integration platform results in small width waveguides and allows using tight bends enabling the construction of dense PICs. However, the high index contrast results in more scattering loss of the light due to surface roughness of the sidewalls. The propagation loss of Si waveguides reported in the literature ranges within about 0.5 to 3 dB/cm [87-92], with low loss figures requiring the best in the class fabrication process. Tran et al. [92] reported ultra-low propagation loss utilizing shallow rib Si waveguides. The shallow etching of the Si waveguide instead of the deep etching allows lower interaction of the optical mode with the sidewalls reducing the scattering loss. In the thesis, the delay lines and feedback loops are within the sub-millimeter range length short enough that ultra-low propagation loss is not necessary. The propagation loss due to surface roughness can be estimated using [90-92]:

$$\alpha_r = A(\sigma, L_c, n_e) \cdot \frac{\partial n_e}{\partial w},$$

where $\sigma$ is the rms sidewall surface roughness, $L_c$ is the correlation length, $n_e$ is the effective index, $w$ is the waveguide width, and $A$ is a proportionality factor that depends on $\sigma$, $L_c$, and $n_e$. Typical $\sigma$ and $L_c$ values used in an example by Tran et al. for theoretical estimations were 5 nm and 60 nm, respectively, but the rms sidewall surface roughness can be reduced in the sub-nanometer range as in [93] reducing the sidewall surface roughness loss in the sub-dB/cm range. In this thesis, taking default simulation parameters for the designed straight waveguide of 350 nm width yields around 2.5 dB/cm of propagation loss obtained by inspecting the resulted scattering parameters. The simulated optical
power distribution inside a straight waveguide of length 500 µm and width 350 nm based on the silicon core thickness of 300 nm is depicted in Figure 2.2. In this simulation, another similar waveguide is placed laterally shifted by 2 µm from the excited waveguide. Inspection of the scattering parameters resulted from this simulation reflects the fact that the crosstalk between the adjacent waveguides of the phased array structures presented in this thesis is never effective up to such a long length of interaction that is not even used.

2.2.3 Coupling off the SOI Chip

The compact size, low loss, and compatibility with CMOS technology of SOI-PICs have attracted many contemporary researchers. In this thesis, the deployment of nano-electromechanical phase shifters of compact size enables the construction of large SOI-based photonic processors utilizing current foundry specifications targeting applications in current WDM networks and next-generation EONs. However, the main issue known is the coupling of SOI-PICs to fibers used in optical communication networks and equipment. Both grating and edge couplers have been proposed in the literature to enable coupling to SOI chips with each method having its merits and demerits. Grating couplers are reached from the top surface of the photonic chip for near-vertical coupling. They avoid the need for cleaving the edge facet of the chip required in the case of edge couplers and provide a convenient way for coupling from anywhere on the chip area. The performance of the surface grating coupler depends on the specifications of the integration platform, design of the grating coupler, adding of a passivation layer for the antireflection or modification of the diffraction properties of the grating waveguide, and adding of a distributed Bragg reflector (DBR) bottom mirror; e.g. see [94-96]. Standard grating couplers are polarization-sensitive and respond over a small wavelength range compared to edge couplers. The design, simulation, fabrication, and testing of both conventional and improved grating couplers with high coupling efficiencies, reduced reflections, and wider wavelength responses achieved at the cost of more complicated structures and nonstandard fabrication processes have been reported in contemporary literature; e.g. [97-108]. Edge couplers, on the other hand, are polarization-insensitive and have a wider wavelength response. However, edge couplers are only reachable from the edges of the chip. The excess coupling loss for standard grating couplers and standard edge couplers is high in the order of 5 to 8 dB per coupler. Survey of contemporary literature revealed that reducing the coupling loss might require novel procedures that can add more challenges in the simulation, design, and manufacturing steps as in
spot-size converters (SSCs), overlaid three-dimensional taper adapters, V-groove edge couplers, and subwavelength grating mode adapters as in [85,86,109-119].

Dewanjee et al. [120] presented a compact broadband silicon bilayer inverse-taper edge coupler achieving 1.7 dB coupling loss to optical fibers with 5 µm mode diameter. However, the bilayer inverse-taper edge coupler of Dewanjee et al. requires a shallow etching step of 70 nm for the coupler inverse-taper section located close to the edge of the chip which is a requirement that might not be available to all foundries. Yoo et al. [121] demonstrated a high-efficiency silicon inverse-taper edge coupler with two sets of silicon or metal Bragg reflectors located near the chip edge to focus the mode incident from the fiber to the tip of the inverse-taper coupler. Although the structure of Yao et al. meets standard foundries, the optimization of the design parameters of the reflectors is expected to be challenging in the simulation. The other major drawback of the edge coupler demonstrated by Yao et al. is that its bandwidth response is too narrow compared to standard inverse-taper edge couplers due to the use of the Bragg reflectors that are wavelength-dependent in design which is the same issue experienced with conventional surface grating couplers. In next-generation EONs maximization of the utilized spectral bandwidth is demanded. A structure that combines simplicity of design, standard fabrication, wide bandwidth response, and mitigation to the coupling efficiency of conventional single-tip inverse-taper edge couplers was proposed first time by Tao et al. [122]. The double-tip edge coupler of Tao et al. comprises two inverse-taper sections that are longitudinally parallel to each other and are coupled at the back using s-bend waveguides to superpose the guided mode. The edge coupler demonstrated by Tao et al. was based on a silicon nitride (SiN) template and could achieve up to 2 dB of improvement in coupling efficiency than a single-tip coupler. The advantage for utilizing the double-tip inverse-taper edge coupler for the SOI template has not yet received full consideration in the literature.

The objective of this section is to investigate improving the off-coupling efficiency with the SOI template of 300 nm of silicon core thickness. A focused optical mode using a lensed fiber is used as a source of excitation for the studied edge couplers to obtain reliable numerical results for the coupling losses. This concept is examined to determine the coupling loss for a conventional inverse-taper edge coupler yielding standard results well known in the literature. Subsequently, the design of an innovative dual inverse-taper edge coupler comprising two longitudinally parallel inverse tapers coupled at the back using a balanced transition double slot coupler is proposed based
on the selected SOI template. The optimization in dimensions of the proposed dual inverse-taper edge coupler indicates possible improvement in coupling loss over 3 dB per facet than a single inverse-taper edge coupler.

2.2.3.1 Coupling to Fiber Probes and Array Waveguide Connectors

The coupling efficiency to an SOI integration platform is affected by a number of factors including the design of the on-chip coupling component, type and dimensions of the probing fiber, mismatch in the field profiles of the overlapped modes, geometrical misalignment between the fiber and the on-chip coupler, polarization mismatch between the overlapped modes, and impedance mismatch which causes reflection. The misalignment loss is caused by off-axis lateral shifts between the probing fiber and the on-chip coupler, focal (depth) mismatch, and wavefront angular-tilt mismatch. The polarization mismatch can be controlled with the use of polarization-maintaining single-mode fibers (SMFs), external polarization controllers, and polarization-insensitive chip couplers [110]. On the other hand, array waveguide connectors provide a convenient way to access the MIMO photonic processors presented in this thesis.

In end-fire coupling, the optical mode of the fiber is focused on the end face of the edge coupler. In this process, usually, a lensed fiber is used to reduce the diameter of the coupled mode size. The working distance of a lensed optical fiber represents the distance separating the focused light beam from the tip of the fiber. Lensed optical fiber probes are commercially available with a wide range of working distances up to 50 µm; see, for example, Nanonics Imaging Ltd. [123]. The diameter of the focused beam which is usually referred to as the spot size is also commercially available within a range from 2 to 7 µm. A focused fiber beam can be matched easier to the edge coupler mode since the later has a maximum diameter limited by the smallest tip width of the edge coupler depending on the manufacturer smallest feature size.

When single-fiber probes are used, usually one input and one output ports of an on-chip PIC are reached. However, multiple input/output ports of a PIC can be reached simultaneously using fiber array connectors. An example is the PLC transposer [124,125] available in our Photonic Technology Laboratory (PTLab). The PLC transposer has eight square output waveguides for simultaneous coupling with eight SOI edge couplers. The glass waveguides of the PLC-transposer fan from the 127 µm pitch separating the array fibers down to 20±0.05 µm pitch between the coupling square waveguides. The spot size of each square outlet waveguide of the PLC transposer
was measured in our PTLab and was estimated to be within the range from about 3 to 5 µm in diameter. In conclusion, in the forthcoming simulations of the studied edge couplers, a tapered fiber producing a spot size diameter within the range from 2.5 µm to 5 µm is required to match specifications of traditionally available off-chip probes.

### 2.2.3.2 Standard Inverse-Taper Edge Coupler

Figure 2.3 shows the structure of a conventional inverse-taper edge coupler of either one section or two cascaded sections. The inverse-taper section has a tip width $W_e$ and length $L_e$. The small tip width provides better matching for the incident fiber mode. The rest of the linear inverse-taper section of slowly variable-width $W_t$ allows for the gradual confinement of the matched mode into the waveguide. The second taper section, when included, is used to expand the confined mode from the width $W_{t1}$ to the regular mono mode waveguide width $W_g$ in a shorter length. The width $W_{t1}$ should be large enough such that it is due to a high waveguide confinement factor and low power coupling efficiency with the incident fiber mode. Simulation results of this research have confirmed that the addition of the expansion taper section does not improve the coupling efficiency, but rather it allows for the design of a shorter edge coupler.

![Figure 2.3](image)

**Figure 2.3** Structure of a standard inverse-taper edge coupler of either one section (a) or two cascaded sections (b).

The incident fiber mode is usually assumed Gaussian within the focus range of the fiber beam. Mostly only the field profile of the edge coupler part located within the focus range of the incident
fiber beam is effective in determining the coupling efficiency. The total coupling efficiency is affected by the tapering shape of the edge coupler; e.g. [114]. Only linear inverse-taper couplers are considered in this research for simplicity of design and simulations. A simple recommendation to maximize the coupling efficiency for the design of a single section linear inverse-taper edge coupler is to adjust the tip width to the feature size of the fabrication process and relax the length of the coupler. For the CMC Microsystem’s Soitech platform with the silicon base layer thickness of 300 nm and upper air cladding typically used for the design of the PICs presented in this thesis, an inverse-taper edge coupler with a tip width of 100 nm and a length of 500 µm is used for the coupling with lensed fiber probes.

The simulation of the optical power distribution for the designed edge coupler is depicted in Figure 2.4. A tapered strip fiber is used as an excitation source and is located at the left side of the inverse-taper edge coupler in the simulation of Figure 2.4. The structure of the excitation source is composed of a stripped fiber with standard silica refractive index followed by a fiber cone and an air gap. The cone of the fiber has a length of 3 µm, and the fiber diameter is taken as 8 µm. The resulting beam of the excitation source has a spot size diameter close to 2.5 µm, a working distance within about the

Figure 2.4 Simulation of the optical power distribution for the coupling between a tapered fiber and an inverse-taper edge coupler. $h = 300$ nm, $W_e = 100$ nm, $L_e = 500$ µm, and $W_g = 350$ nm.
range from 0.5 to 1.3 µm measured from the tip of the fiber cone, and a range of focus of about 0.8 µm. In general, the working distance and the spot size can be tuned by adjusting the length of the tapered cone and diameter of the fiber. The air gap length is tuned to focus the waist of the Gaussian beam of the excitation source at the tip region of the inverse-taper edge coupler within the working distance of the focused beam to maximize the coupling efficiency. The inspection of the scattering parameters of the simulation results indicates coupling loss in the order of around 6 dB per facet. This is equivalent to around 25 % of power coupling efficiency per facet. In conclusion, a standard single inverse-taper edge coupler has a simple structure and wide bandwidth response, and it is simple to design and fabricate, but its low coupling efficiency hinders its utilization in constructing processors used in advanced optical networks. A mitigation to this drawback is proposed in the next section utilizing a similar simulation procedure.

2.2.3.3 Dual Inverse-Taper Edge Coupler

![Figure 2.5](image.png)

Figure 2.5 Geometry of a dual inverse-taper edge coupler.

To mitigate the low power coupling efficiency issue found in conventional single inverse-taper edge couplers, without the need to use a nonstandard fabrication process, the concept of utilizing a dual inverse-taper edge coupler is examined in this section [122]. The proposed SOI-based edge coupler, having the geometry illustrated in Figure 2.5, consists of a dual inverse taper followed by a high-efficiency balanced transition double slot coupler. Each inverse taper is misaligned from the centerline of the incident fiber beam by an amount of \((W_g+300)/2 = 325\) nm, where \(W_g = 350\) nm is the width of the single-mode waveguide designed for the template with a silicone core thickness of \(h = 300\) nm. Each inverse taper has a tip width of \(W_e = 100\) nm and a length of \(L_e = 500\) µm and ends up with a base width equals to the waveguide width \(W_g\) at the side of the balanced transition coupler. Each slot of the balanced transition coupler has a fixed width of 100 nm. The length of the balanced
transition coupler is relaxed to $L_c = 100 \, \mu m$ to achieve high coupling efficiency. The single-mode outputs appearing at the wide bases of the dual inverse tapers are combined constructively by the balanced transition coupler into the output single-mode waveguide. When the centerline of the incident fiber beam matches the centerline of the construct, the single-mode outputs of the dual inverse tapers become similar.

The simulation of the optical power distribution for the designed dual inverse-taper edge coupler is depicted in Figure 2.6. The same tapered fiber used in the last section is taken as the excitation source and is located at the left side of the dual inverse-taper edge coupler as appeared in Figure 2.6. The waist of the beam spot of the coned fiber is tuned at the location of the two tips of the dual inverse-taper edge coupler to maximize the coupling efficiency. Inspection of the scattering parameters of simulation results has indicated a power coupling efficiency of 55% for the construct. This is equivalent to an excess coupling loss of around 2.6 dB. Therefore, the dual inverse-taper edge coupler provides an improvement in the power coupling efficiency by a factor of at least 2.2 compared to the single inverse-taper edge coupler. This improvement may be related to a better field distribution of the construct mode expanded at the tips of the inverse tapers yielding a better match with the mode incident from the fiber. The coupled-mode power splits gradually into equal portions in the bodies of the two dual inverse tapers. The resulted outputs modes of the dual inverse tapers are similar in intensity and phase at the exact alignment of the fiber. The output power of the high efficiency balanced transition coupler equals to the coupled fiber mode power minus the small loss it introduces. These simulation results depicting improvement in the coupling efficiency of the dual inverse-taper edge coupler presented in this section are consistent with the simulation and testing results demonstrated by Tao et al. in [122]. The double-tip edge coupler of Tao et al. was based on a SiN template with SiO$_2$ top and bottom cladding layers. The application of a top cladding layer over the entire chip is not compatible with the use of NEMS-operated phase shifters unless if the passivation layer is applied locally to the area of the edge couplers. The two inverse tapers of the construct of Tao et al. were coupled at the backside using s-bend waveguides. Tao et al. reported about 2.2 dB of improvement in the coupling efficiency for the TE-mode per facet for their constructed double inverse-taper edge coupler compared to a standard single inverse-taper edge coupler based on the same foundry. In conclusion, the difference made by the dual inverse-taper edge coupler is to reform the coupling mode of the construct for a better match with the incident
fiber mode. Another advantage expected for the dual inverse-taper edge coupler is the provision of extra relief to the misalignment loss along the lateral $x$-direction.

![Simulated optical power distribution for the coupling of the dual inverse-taper edge coupler with a lensed fiber.](image)

**Figure 2.6** Simulated optical power distribution for the coupling of the dual inverse-taper edge coupler with a lensed fiber. $h = 300$ nm, $W_e = 100$ nm, $L_e = 500$ $\mu$m, $W_g = 350$ nm, and $L_c = 100$ $\mu$m.

### 2.2.4 Multimode Interference Couplers

The development of photonic processors of cascaded FIR and IIR architectures characterized by having MIMO ports require the use of combiners and splitters with MIMO ports in general rather than 2×2. In this regard, the use of the 4×4 MMI coupler provides an advantage in comparison with alternative components such as the use of a directional coupler, which is a 2×2 component. Additionally, the use of MMI couplers with the properly selected separation distance between the adjacent I/O ports eliminates the need to involve waveguide bends in the core of a broadband IPPAS element as demonstrated in the next chapter for which case the phase-shifting elements are aligned with the ports of the MMI couplers. In this thesis, the pitch size between the adjacent ports of all designed MMI couplers is set fixed to 2 $\mu$m which represents a reasonable choice to make all of the MMI couplers of different sizes compatible with each other to form any complicated architecture while minimizing the need to involve waveguide bends between the splitter/combiner junctions. Another motivation is the need to use crossover 2×2 MMI couplers to exchange the inner and outer
interconnection lines located between the splitter and combiner units in 4×4 IPPAS elements achieved at minimum loss and crosstalk penalties in comparison to the alternative use of crossover waveguides. The use of crossover waveguides introduces unavoidable crosstalk and requires using waveguide bends that inhabit additional loss. Providing the above advantages of MMI couplers, it is expected that the wavelength-dependent performance deterioration of MMI couplers including imbalances in the split ratios, losses for the different inputs, and phase synthesis errors for the different I/O ports would affect the response of the photonic processors that are intended to operate over the entire C-band. Therefore, it is planned in this research to study the effect of the simulated characteristic of the designed conventional 2×2 and 4×4 MMI couplers on the resulted transmission response of the photonic processors proposed in this thesis. This provides a realistic study helping to address the challenges and requirements for future work to achieve the advanced processing required in next-generation flexible optical communication networks.

An MMI coupler is designed such that the field profile of the input ports are reproduced at the desired output ports. The reproduced field profiles at the output ports are referred to as Talbot or self-images of the input field profiles [126-128]. Tapers are usually used to adapt the width of the interconnecting single-mode waveguides to the wider width of the I/O access ports of the MMI coupler. Controlling the width of the access ports to an MMI coupler is of great importance to minimize imbalances in the split ratios and excess losses for the different inputs to the MMI coupler [79]. The geometries of the different 2×2 and 4×4 MMI couplers considered in this thesis are shown in Figure 2.7. The MMI coupler section is rectangular with width \(W_c\) and length \(L_c\). The entrance face of the \(N\times N\) MMI coupler is divided into \(N\) access sections each of width \(W_{am}\) corresponding to the absolute maximum possible access port width. Each I/O access port has width \(W_a\), and it is centered exactly at the midpoint of the access section.

Linear tapers, each of length \(L_t\), are used to adapt the width \(W_g\) of the single-mode access waveguides to the width \(W_a\) of the access ports of the MMI coupler. The inputs (outputs) at the MMI coupler ports are designated as \(I_1\) to \(I_4\) \((O_1\) to \(O_4\)). The used separation between the adjacent waveguides and the relative dimensions of the adjacent tapers are confirmed through simulation not to cause any effective direct coupling between them. Adjusting the length and width of the rectangular MMI coupler and the width and lateral position of the access ports affect the resulted imbalance, loss, and phase error characteristics.
Figure 2.7 Geometries of a -3dB 2×2 MMI coupler (up), crossover 2×2 MMI coupler (middle), and 4×4 MMI coupler (down).

Manufacturing tolerances and discrepancies from the assumptions of simulation cause deviations from reproducing perfect self-images at the output ports yielding non-zero imbalance and excess loss in actual MMI couplers. The imbalance and excess loss depend on the way the input power from an
input excess port is partitioned into the modes excited in the MMI section [79] and the length of the MMI coupler \(L_c\). This partitioning is driven by the relative width \(W_a/W_{am}\) and positioning of the access ports of the MMI coupler.

The well-known simplified theory of MMI couplers based on the effective index method (EIM), as was summarized by the thesis author in [129], provides great insight into the physics of MMI couplers. The beat length of an MMI section, \(L_\pi = 4n_eW_c^2/3\lambda\), is defined such that the phase shift between the two lowest order modes excited in the MMI section is \(\pi\); where \(n_e\) is the effective index in the multimode region and \(\lambda\) is the free-space wavelength. Single and multiple self-images of the optical input field are formed at the MMI coupler lengths, \(L_c = 3L_\pi(p/q)\), where \(p \geq 0\) and \(q \geq 1\) are integers with no common divisor [56]. The integer \(q\) represents the number of images formed in the extended field. For a -3dB 2×2 MMI coupler, two self-images are formed at the shortest coupler length of value: \(L_c = 3L_\pi/2\) (i.e. \(p = 1\) and \(q = 2\)). For a crossover 2×2 MMI coupler, a mirrored single image is formed at the coupler length of value: \(L_c = 3L_\pi\) (i.e. \(p = 1\) and \(q = 1\)). For a -6 dB 4×4 MMI coupler, four self-images are formed at the shortest coupler length of value: \(L_c = 3L_\pi/4\) (i.e. \(p = 1\) and \(q = 4\)).

The wavelength-dependent performance parameters of an MMI coupler that should be accounted for in the design procedure include the imbalances, excess losses, and phase errors. These parameters are also equivalent to the determination of the transmissions and phase relations of the scattering parameters for the different I/O ports of an MMI coupler. Optimum self-images result at minimum excess loss points. The set of parameters that result in minimum imbalances and minimum excess losses for the different inputs of 4×4 MMI couplers do not exactly match each other. Optimization functions for the minimization of the non-uniformity in the output powers and the minimization of power losses for the different inputs of \(N\times N\) MMI couplers are formulated by the thesis author in [129] and used for the design of 4×4 MMI couplers. For a simplified design, the length of the multimode section and the width of the I/O access ports of an MMI coupler are determined at the center operating wavelength of \(\lambda = 1550\) nm for a selected multimode section width. The design procedure starts with estimating the beat length of the multimode section using the effective index obtained utilizing simulation for a given width of the rectangular multimode section. In the next design procedure step, simulations of imbalances in the split ratios and excess losses for the different inputs within a small range around the expected value of \(L_c\) based on the beat length.
that was determined in the first design procedure step are used to predict the optimized value of the multimode section length. Once the dimensions of an MMI coupler are optimized for reducing imbalances and excess losses at $\lambda = 1550$ nm, its transmission and phase characteristics are scanned over the entire C-band, and the effect of their wavelength-dependent deviations on the performance of the photonic processors presented in the consequent chapters is studied. The complex FDM-3D-FV solver of FIMMPROP is used in performing the simulations presented in this section. The resolution of the simulation parameters is saturated such that further refining is found not affecting the convergence of the simulation results. Up to 64 modes were taken in the simulation of every designed multimode section and found not to make a significant difference in the simulation results compared to taking about half that number of modes. The increase in the number of considered modes for the multimode section exhausts more computer resources and memory and increases the simulation time needed to perform the numerical computations making it harder to reach the optimization design objectives.

The thickness of the BOX layer is taken as 1 $\mu$m, and the thickness of the Si core layer is taken as 300 nm for all simulations. The effect of changing the multimode section width on the imbalances and excess losses was first investigated taking the -6dB 4×4 MMI coupler and compared with the parameters reported in literature, e.g. [64,65,67,70,74], and then the maximum access port width used for all of the MMI couplers designed in this thesis of $W_{am} = 2$ $\mu$m is taken providing small footprint at comparable performance. This sets the width of the -3dB 2×2 MMI coupler to 4 $\mu$m and the width of the 4×4 MMI couplers of two different types to 8 $\mu$m. The width of the crossover 2×2 MMI coupler is set to $W_c = 3.49$ $\mu$m keeping the same separation between the centers of its access ports of 2 $\mu$m to have it native to the IPPAS fabrics.

The simulation of imbalances versus the access port width revealed a value of $W_a = 1.49$ $\mu$m for minimizing imbalances in the power split ratios of the MMI couplers presented in this thesis. -3dB 2×2 and -6dB 4×4 MMI ideal couplers split the power at any of the inputs into equal portions at the output ports. An ideal crossover 2×2 MMI coupler produces 0dB of relative power at the output access port located at the side opposite to the input access port. The length of a crossover 2×2 MMI coupler is ideally twice the length of a -3dB 2×2 MMI coupler having the same width. A -3dB 4×4 MMI coupler is designed around the coupler length $L_c = 3L_{w}/2$. An ideal -3dB 4×4 MMI coupler produces -3dB of relative power at each of outputs $O_1$ and $O_4$ when applying the input to either $I_1$ or
and also produces -3dB of relative power at each of outputs $O_2$ and $O_3$ when applying the input to either $I_2$ or $I_3$. The lengths of the -3dB 2×2 MMI coupler, crossover 2×2 MMI coupler, -6dB 4×4 MMI coupler, and -3dB 4×4 MMI coupler optimized in this research for the minimization of power losses and imbalances are found to be 60.66 µm, 94.55 µm, 122.2 µm, and 243.46 µm, respectively. The simulated optical power distribution inside the designed MMI coupler of different sizes are shown in Figure 2.8 taken for input $I_1$ for the -3dB and crossover 2×2 MMI couplers and taken for inputs $I_1$ (left) and $I_3$ (right) for the 4×4 MMI couplers. The simulation of the loss characteristic of the taper used to adapt the width $W_g = 350 nm$ of the single-mode waveguide to the width $W_a = 1.49 \mu m$ of the MMI coupler access port indicates that a length of $L_t = 10 \mu m$ of the adapter taper is proficient for near adiabatic expansion of the mode.

In this thesis the imbalance in the fundamental mode power of outputs $k$ and $l$ due to input $j$ is denoted as $I_{k/l,j}$ and the excess loss due to input $j$ is denoted as $L_j$. The -3dB 2×2 MMI coupler simulated in the upper-left part of Figure 2.8 has imbalance of $I_{1/2,1} = 0.053$ dB and excess loss of $L_1 = 0.11$ dB. A -3dB 2×2 MMI coupler can be designed for lower values of imbalance and excess loss compared to a larger size -6dB 4×4 MMI coupler of similar $W_{am}$ width since the ratio $W_a/W_c$ is maximized yielding the excitation of a less number of modes of effective weights in the multimode section. The reconstruction of the input field self-images at the output ports is more accurate for a less number of interfered multimode fields subjected to less phase error deviations in the reconstruction modes.

The crossover MMI coupler simulated in the upper-right part of Figure 2.8 has an excess loss of only 0.122 dB and crosstalk of -31.9 dB. Therefore, the crossover 2×2 MMI coupler merits quality performance of low excess loss and low crosstalk. Moreover, it has a wide bandwidth response over the entire C-band as will be depicted in the next chapter addressing it as a perfect choice compared to the alternative use of crossover waveguides.

The -6dB 4×4 MMI coupler simulated in the middle part of Figure 2.8 has a maximum imbalance of 0.074 dB and maximum excess loss of 0.377 dB. In comparison with the parameters of -6dB 4×4 MMI couplers reported in the literature, it is desirable in general to achieve imbalance level in the output power split ratios below about 0.1 dB and excess loss level below about 0.5 dB. Therefore, the presented -6dB 4×4 MMI coupler merits quality performance of perfectly optimized uniformity and loss metrics at the center wavelength $\lambda = 1550$ nm. However, the performance of the
conventional -6dB 4×4 MMI coupler is not uniform over the entire wavelength range of the C-band as will be demonstrated in the next chapter.

**Figure 2.8** Simulated optical power distributions inside the designed -3dB 2×2 MMI coupler (up-left), crossover 2×2 MMI coupler (up-right), -6dB 4×4 MMI coupler (middle left and right), and -3dB 4×4 MMI coupler (down left and right) of Figure 2.7.
The -3dB 4×4 MMI coupler simulated in the lower part of Figure 2.8 has maximum values of imbalance, loss, and crosstalk of 0.285 dB, 1.47 dB, and -28.9 dB, respectively. The level of crosstalk is acceptable, but the imbalance and loss figures are high in this case. Although the characteristics of the -3dB 4×4 MMI coupler is worse compared to the -6dB 4×4 MMI coupler, it is still useful to demonstrate the construction of photonic processors with reasonable overall performance.

2.2.5 Conclusions of Part 1

A number of passive components used in the construction of the PICs presented in the next chapters based on the silicon base layer thickness of 300 nm of the SOI template are studied including the mono mode waveguide, inverse-taper edge coupler, dual inverse-taper edge coupler, linear taper adapter, and MMI couplers. The width of the waveguide is taken as 350 nm to support the propagation of the $TE$-like fundamental mode. A tapered fiber producing Gaussian-like field profile with a spot size of around 2.5 $\mu$m is used in the simulation to estimate the coupling efficiency with inverse-taper edge couplers. This simulation arrangement proved to provide reliable estimation for the coupling efficiency with standard single inverse-taper edge couplers and therefore used to determine the coupling efficiency with an innovative dual inverse-taper edge coupler. The proposed dual inverse-taper edge coupler is demonstrated eligible to mitigate the low coupling efficiency issue to the figure of 55% compared to the typical figure of 25% known for conventional single inverse-taper edge couplers and conventional surface grating couplers. The targeted testing of the proposed dual inverse-taper edge coupler would provide an experimental evaluation of its characteristics. More investigation in future is advised looking into further increasing the coupling efficiency with SOI chips that can host nano-electromechanical structures fabricated using standard procedures for potential applications in flexible optical communication networks and adaptive high-resolution photonic motion sensing [130-132].

Accurate simulations using the complex FDM-3D-FV solver of FIMMPROP are used in designing several MMI couplers that are used in building the phased array systems presented in the next chapters. Although, the designed MMI couplers exhibit imbalance, loss, and crosstalk levels comparable to benchmarks found in the literature, it is well known that conventional MMI couplers exhibit phase errors and they have limitations on their bandwidth responses. For simplicity of the design procedure, the optimization of the MMI couplers is based in this chapter on the minimization.
of non-uniformity and loss metrics evaluated at the center wavelength of the C-band. In general, the optimization objective can be based to account for errors in the phase synthesis relations of the designed MMI couplers estimated over the entire operating bandwidth. Studying the characteristic of the MMI couplers presented in this part reveals the fact that MMI couplers are the major components expected to affect the performance of the PICs proposed in this thesis due to imbalances, losses, and phase errors that are wavelength-dependent and are different for the different inputs. In the future, further investigation to design MMI couplers with improved figures of merit and broader bandwidth responses is advised for applications in broadband phased array systems. The dimensions and merit parameters of the MMI couplers exhibited in this chapter lead the organization and performance of the IPPAS elements presented in the next chapter. In the next part of this chapter, the study, simulation, and design of the NEMS-operated phase shifter based on the same SOI integration platform are presented. NEMS-operated phase shifters that can achieve up to 180º and 360º of voltage-controlled phase shifts are demonstrated.

2.3 Part 2: Nano-Electromechanical Phase Shifters

2.3.1 Introduction of Part 2

As introduced in Chapter (1), the central object of study of the thesis is a new class of programmable optical coherent processors to be described in detail in Chapters (3-5). The processors consist of active phase shift elements interconnected by waveguide splitters and combiners and implemented as a photonic integrated circuit. The previous part of this chapter described the rationale for the choice of multimode interference couplers as the splitters and combiners and elucidated their design. A significant feature is their ultra-compact footprint compared to the footprint of conventional active phase shift elements. In order not to compromise this compactness, a phase shift element with commensurate footprint size was sought and found in the work of Acoleyen et al. [80,81]. The phase shift element constructed by Acoleyen et al. was composed of only three cascaded suspended slot waveguide sections. The application of a voltage difference between the two beams of the phase shift line modulates the width of the slot between the suspended silicon beams. This, in consequence, modulates the effective index of the slot waveguide and results in a phase-shift difference. The most important feature of the phase shift element described by Acoleyen et al. is that it has a small footprint and requires low power consumption, but it could achieve up to around 40º of maximum phase shift differences, whereas in practice phased array systems require up to a maximum phase
shift differences of 180° or 360°. A second issue with the phase shift element demonstrated by Acoleyen et al. is that it requires two electrodes located at two opposite sides of the construction with each electrode connected to an opposite end of one of the two suspended silicon beams of the phase shift element. That biasing arrangement enables testing of the phase shift element as a standalone component, but it is not possible to be integrated into a phased array system which usually requires the engagement of several electrically isolated phase shift elements.

In this research, the cascading of 15 suspended slot waveguide sections to form a phase shift element is considered to extend the achieved phase shift range up to 180° or 360°. An innovative transition slot coupler achieving high coupling efficiency of ~97% is designed to bias the phase shift element using only one side located electrode that is electrically isolated from the rest of the silicon structure, which is taken as the ground. The structure and principle of operation of the NEMS-operated suspended slot waveguide acting as a phase shift element are illustrated in this part of the chapter. The state-of-the-art in active phase shift technology is first described in Section (2.3.2) to set the context and justify the rationale for the choice of the NEMS phase shift element. The behavior of the suspended slot waveguide is described in Section (2.3.3) in terms of numerical analysis utilizing the simulation of the effective index of the suspended slot waveguide as a function of its voltage-controlled slot width. The results of simulation and numerical analysis are then used in the design procedure described in Section (2.3.4). The structure and simulation of the transition slot couplers are also presented in this section. Simulation results of COMSOL Multiphysics are also presented in this section to confirm the accuracy of the results of the custom script, which is based on the presented numerical analysis.

2.3.2 Overview of Alternative Refractive Index Modulation Methods

The amplitude-phase modulation of an optical carrier using SOI-based devices is usually achieved using MZI structures equipped with electro-optical phase shift elements that may operate at radio frequencies extending into the microwave region. An SOI phase shifter is usually constructed of an optical waveguide of length \( L \) sufficient to achieve the desired phase shift difference of \( \Delta \phi = \frac{2\pi}{\lambda} \Delta n_e L \) resulted from modulating the effective index difference \( \Delta n_e \) of the waveguide. A variety of conventional SOI phase shifters has been reported in contemporary literature, e.g. [45,133-141], based on modulating the refractive index of silicon using the free carrier plasma dispersion effect or the thermo-optic effect for modulating the refractive index of silicon.
An applied electric field induces a change in the refractive index via the electro-optic effect, and this change of refractive index induces a change in the phase of an optical waveguide. The electrically induced phase modulation may then be converted to an amplitude modulation using an interferometer such as the ubiquitous MZI Interferometer. In general, the electric field effects for modulating the refractive index include the Pockels effect, Kerr effect, and Franz-Keldysh effect. The crystalline silicon used in SOI templates does not exhibit the linear Pockels effect. The nonlinear Kerr effect in crystalline silicon has a refractive index coefficient of $\Delta n \approx 10^{-8}$ for an applied electric field intensity of $10^5$ V/cm close to the silicon breakdown potential [133]. The Kerr effect is relatively small compared to the plasma dispersion effect. The Franz-Keldysh effect gives rise to the real part of the refractive index (electro-refractive), and it gives rise to the imaginary part of the refractive index (electro-absorption). The electro-refractive coefficient of the Franz-Keldysh effect in crystalline silicon is in the order of $\Delta n \approx 10^{-5}$ for an applied voltage of $10^5$ V/cm [133]. In conclusion, the electro-optic effect is weak in crystalline silicon and requires the application of electric field intensity close to the breakdown potential of the silicon to reasonably modulate the refractive index of the silicon.

The carrier plasma dispersion effect is based on the depletion and injection of carriers in the waveguide region. This effect is polarization-independent and can operate in the GHz range. The control of the carrier concentrations results in changing both of the absorption coefficient $\Delta \alpha$ and refractive index $\Delta n$. At the telecom operating wavelength of 1.55 µm, an electro-refractive coefficient of $\Delta n \approx -2 \times 10^{-3}$ is produced for a variation in the carrier concentrations of $\Delta N \approx 10^{18}$ carriers/cm$^3$ [133]. The carrier plasma dispersion effect is many multiple times stronger than the electro-optic effect in crystalline silicon, and hence, it dominates the process of the phase shift control. The serious drawbacks of the phase shifters constructed based on the carrier plasma dispersion effect are the large footprint and free-carrier absorption loss. The refractive index change and absorption loss are also nonlinear functions of the applied voltage. The free-carrier absorption loss results in intensity modulation due to modulating the carrier concentrations that competes with the effect of the electro-refractive phase shift control in MZI modulators. The tendency to build compact phase shifters motivated several researchers to integrate the phase shift line within the feedback loop of a microring resonator, e.g. see [142]. A microring resonator is a single-loop feedback element that results in a wavelength selective transmission response of usually narrow
bands. Wideband MZI-based switch elements cannot utilize the advantage of the small footprint of ring resonator-based phase shifters.

The thermo-optic effect provides another effective method for modulating the refractive index of silicon. The thermo-optic modulation coefficient for the crystalline silicon is in the order of $\frac{dn}{dT} \approx 1.86 \times 10^{-4} \text{ K}^{-1}$ near the 1.55 $\mu$m operating wavelength [143]. Conventional SOI thermo-optic straight phase shifters have a large footprint, require electrical power in the mW range (more likely a good fraction of Watt) to operate them, and have a slow response time in the $\mu$s range. Other known issues of the thermo-optic phase shifters are the requirements for the localized application of the heating powers to the phase shift elements, the provision of thermal isolation between the adjacent phase shift elements, and the chip substrate has to be thermostated to below $\pm0.1 \text{ C}$.

Another alternative method to modulate the effective index in photonic phase shifters and wavelength-tunable optical filters include the hybrid use of SOI-based slot waveguides and thin-film organic nematic liquid crystals (NLCs) [144-146]. The liquid crystals (LCs) consist of rod-like molecules having their long axes aligned parallel to each other in the nematic phase. The LCs thin film forms the upper and side claddings of the waveguide structure which could be a slot waveguide as in [144] or a directional coupler structure of a straight waveguide and a ring resonator as in [145,146]. The fundamental mode field propagating in the waveguide structure has evanescent tails that penetrate in the LCs cladding. In consequence, the orientation of the LCs-director with respect to the direction of the electric field intensity lines of the fundamental mode affects the index of the waveguide. In the ordinary (extraordinary) state the director is perpendicular (parallel) to the electric field lines of the mode resulting in the lowest (highest) index value. The director of the LCs is oriented under the influence of an electric field intensity applied by localized electrodes to modulate the index of the waveguide resulting in a phase shift difference. Although a hybrid phase shifter of SOI slot waveguide and organic NLCs thin film can save on the device length [144] the localization of the electric field intensity to orient the LCs for each phase shift element in a dense photonic processor deploying many adjacent phase shifters is not examined yet. Another issue this type of phase shifters might suffer from is the need to apply threshold voltage to overcome the possible elastic forces that can hold the director of the LCs [146]. The alignment of the LCs in a structured PIC rather than a single phase shift element in a planar silicon chip is expected to be difficult and tricky and can give rise to serious crosstalk issues.
2.3.3 Structure and Analysis of a Suspended Slot Waveguide

In this research, the development of the NEMS-operated phase shifters is based on the NanoSOI nanomachining fabrication process offered by CMC Microsystems; by utilizing their Soitech wafer with the option of 300 nm thickness of silicon base layer on the buried oxide of 1 µm thickness. The resist is patterned using e-beam lithography achieving small feature size in the order of 100 nm. Both metal patterning and etching under the silicon base layer in the buried oxide have made it possible to support developing the active voltage-controlled phase shifters using the Soitech platform. The wavelength of operation is set to $\lambda = 1.55$ µm in the design procedure.

The NEMS-operated phase shift element uses suspended slotted photonic waveguides with an electrostatically controlled variable slot width. The slotted photonic waveguide, in essence, consists of two nanoscale silicon beams with sub-micron separation (100 nm) that are rendered free to move laterally relative to each other by using an under-etch step in the fabrication process to remove the oxide layer underneath the silicon beams.

The use of a single suspended slot waveguide sections and, later, a cascade of sections was introduced by Acoleyen et al. in references [80,81,147]. The structure and dimensions of the suspended slot waveguide used in this research are illustrated in Figure 2.9. It consists of two suspended silicon beams each of width $W_{gs}$ (230 nm) and height $h$ (300 nm) equals the thickness of the silicon base layer of the SOI wafer. The length of the suspended slot waveguide section is $L_s$. The slot width between the undeformed suspended silicon beams when no potential difference is applied between the silicon beams is $W_{s0}$ (100 nm). An applied voltage difference causes the slot width between the suspended silicon beams $W_s$ to vary as a function of the distance as a consequence of electrostatic attraction. The excursion width $W_{ex}$ may be defined by:

$$W_{ex} = \frac{W_{s0} - W_s}{2}$$  

... Eq. (2.1)

The distributed electrostatic force due to the voltage difference applied between the suspended silicon beams of the under-etched slot waveguide modulates the slot width. The excursion results in a large modulation of the effective index based on which principle the phase shifter works. At the pull-in voltage, the system becomes unstable, and the two suspended beams of the slot waveguide are drawn together until they reach direct contact. This is expected to cause the device to fail.
The positive $z$-axis is taken as the direction of propagation. The phase difference $\Delta \phi$ introduced due to the modulation of the slot width of the NEMS-operated slot waveguide is given by:

$$\Delta \phi = \frac{2\pi}{\lambda} \int_0^L n_e dz - n_{e0} L_z$$

... Eq. (2.2)

In Eq. (2.2), the effective index $n_e$ is a function of the slot width $W_s$ which by itself is a function of the propagation distance $z$; i.e. $n_e(z) = n_e[W_s(z)]$. The term $(2\pi/\lambda)n_{e0} L_s$ corresponds to the phase without applying voltage to the under-etched slot waveguide. Therefore, $n_{e0}$ represents the effective index when the slot width equals to $W_{s0}$; i.e. $n_{e0} = n_e(W_{s0})$. The dependency on $W_s$ and $z$ is removed from Eq. (2.2) for simplicity. The wavelength $\lambda$ is taken here as the free-space wavelength.

When applying voltage to the silicon beams, the lateral forces due to the electrostatic attraction that cause the excursion of the beams can be described by the Euler-Bernoulli beam equation, which provides a means of calculating small beam deflections due to an applied distributed load. The axial stress is neglected in the formulation. Further relating the distributed load to the distributed electrostatic force between the silicon beams [80,81,147] reveals the following nonlinear fourth-order differential equation for beam excursion:

$$\frac{d^4 W_{ex}}{dz^4} = \frac{\varepsilon_e h V^2}{2EI(W_{s0} - 2W_{ex})^2}$$

... Eq. (2.3)

$V$ is the voltage applied across the slot between the two suspended silicon beams. $E$ is Young’s modulus and is equal to 169 GPa in the core silicon in the direction of interest. The beam area moment of inertia ($I$) in the direction of interest is given in terms of the beam width ($W_{gs}$) and thickness ($h$) by [80,81,147]:

Figure 2.9 Geometry of the under-etched slot waveguide.
The solution of Eq. (2.3) is achieved numerically subject to fixed and hinged boundary conditions. Fixed boundary conditions allow no displacement and no rotation at the clamping points:

\[ W_{ex} = 0, \quad \frac{dW_{ex}}{dz} = 0 \quad \text{at} \quad z = 0, L_s \]

... Eq. (2.5)

Hinged boundary conditions allow no displacement, but do allow rotation around the clamping points:

\[ W_{ex} = 0, \quad \frac{d^2W_{ex}}{dz^2} = 0 \quad \text{at} \quad z = 0, L_s \]

... Eq. (2.6)

The simulation of the effective index as a function of the slot width and the numerical solution of Eq. (2.3) allows the determination of the phase difference achieved by the voltage-activated suspended silicon beams of the under-etched slot waveguide based on Eq. (2.2). The numerical solution of Eq. (2.3) followed in this research is detailed in Appendix (1) explained for both of the fixed and hinged boundary conditions.

The speed of the nanomechanical suspended silicon beams of the under-etched slot waveguide depends on their mechanical relaxation time, which is determined by the fundamental resonance frequency of the structure[80,81] given by:

\[ f_o = \frac{K^2}{2\pi L_s^2} \sqrt{\frac{EI}{\rho h W_{gs}}} \]

... Eq. (2.7)

where \( \rho = 2.329 \text{ g.cm}^{-3} \) is the silicon mass density. The factor \( K \) equals to 4.73 for fixed boundary conditions and equals to \( \pi \) for hinged boundary conditions. For the used integration template with a silicon core thickness of \( h = 300 \text{ nm} \), each suspended silicon beam has a fixed width of \( W_{gs} = 230 \text{ nm} \). Therefore, the fundamental resonance frequency \( f_o \) in Eq. (2.7) depends only on the length of the suspended silicon beams \( L_s \). Eq. (2.7) is used here to determine \( f_o \) as a function of \( L_s \) and is always verified in the design procedure to make the proper selection for the value of \( L_s \). In general, for typical dimensions, the resonant frequency is expected to be in the MHz range.

A second significant factor affecting the speed of the NEMS-operated phase shifter is the \( RC \) time constant circuit of the electrical feed from the electrodes to the suspended slot. The distributed
capacitance of the structure is composed of three contributions: (i) the voltage-dependent capacitances of the suspended silicon beams of the cascaded under-etched slot waveguide sections, (ii) the fixed-value capacitances of the slot waveguide support segments between the cascaded under-etched slot waveguide sections, and (iii) the fixed-value capacitances of the voltage feeding arrangements and transition slot couplers. The overall capacitance of the designed NEMS-operated phase shifter is estimated to be in the fF range (around 10 fF).

The $RC$ lowpass filtering limit affecting the speed of the system might be relieved in the manufacturing process if the doping level of the silicon core of the used wafer is made high enough to reduce the overall resistance between the electrodes and the NEMS-operated phase shift line. In such a case, the only factor affecting the speed would be due to the frequency of resonance of the nanomechanical suspended silicon beams. Examining the $RC$ speed limiting factor versus the nanomechanical frequency of resonance is possible when testing the fabrication variants of some designed NEMS-operated phase shifters.

### 2.3.4 Design of the NEMS-operated Phase Shifter

The design of the NEMS-operated slot waveguide phase shifter is based on the numerical approach presented by Acoleyen et al. [81,86,147], which was explained in the preceding section (2.3.3), and the simulation of the effective index of the slot waveguide using FIMMPROP. Acoleyen designed, fabricated, and tested a NEMS-operated phase-shifting element, composed of three cascaded suspended slot waveguide sections, which could achieve up to about 40º of phase shift difference. The test results of Acoleyen, indicate that the electromechanical behavior of the NEMS-operated slot waveguide is located (about midway) between fixed and hinged boundary conditions of the freestanding silicon beams. Here, a similar procedure is followed to design a single suspended slot waveguide based on the Soitech wafer with the 300 nm of silicon thickness option. Then, 15 suspended slot waveguide sections interconnected via high-efficiency transition slot waveguides are cascaded to form a 180º phase shift element. The simulations of the designed full NEMS-operated phase shifter, as well as the transition slot couplers, are presented in this section. It is also proposed that a phase shift element composed of 15 cascaded suspended slot waveguide sections capable of a 360º phase shift range is feasible through a slight modification of the design parameters of the consistent suspended slot waveguide sections. A full phase (360º) capable phase shifter provides wider control of the amplitude-phase relations in IPPAS elements.
Figure 2.10  Simulation of the effective index for the fundamental mode of the under-etched slot waveguide versus the slot width. $W_{gs} = 230$ nm, $W_{s0} = 100$ nm, and $h = 300$ nm.

Figure 2.11  Predicted phase difference versus the length of the NEMS-operated slot waveguide for an operating voltage of 15 V. $W_{gs} = 230$ nm, $W_{s0} = 100$ nm, and $h = 300$ nm.

The width of each one of the suspended silicon beams ($W_{gs} = 230$ nm) and the slot width ($W_{s0} = 100$ nm) are selected to support the propagation of the fundamental mode in the suspended slot waveguide. The effective index $n_e$ of the suspended slot waveguide for the fundamental mode as a function of the modulated slot width predicted by simulation is shown in Figure 2.10. Here, a minimum slot width $W_{s,min}$ of 20 nm is allowed as a guard band to avoid the applied voltage required reaching the pull-in voltage. Utilizing the data of the effective index versus slot width presented in Figure 2.10 and the numerical solution of Eq. (2.3), the phase change predicted by Eq. (2.2) as a
function of the length of the NEMS-operated slot waveguide, for an operating voltage of 15 V applied to the suspended silicon beams, is shown in Figure 2.11. From this figure, the length of the suspended slot waveguide section is selected as 5.4 µm. Each suspended slot waveguide section of this length might achieve a phase shift that falls midway between fixed and hinged boundary conditions of about 16.6º. Therefore, at least about 11 cascaded under-etched slot waveguides are needed to achieve up to 180º of phase shift at an operating voltage of 15 V. However, a fairly large margin is considered here by cascading 15 suspended slot waveguide sections.

Figure 2.12 Distribution of the excursion width along the length of the under-etched slot waveguide of length \( L_s = 5.4 \) µm when applying a voltage of 15 V. \( W_{gs} = 230 \) nm, \( W_{s0} = 100 \) nm, and \( h = 300 \) nm.

The distribution of the excursion width along the length of the suspended slot waveguide section for both of the fixed and hinged boundary conditions for the design length of \( L_s = 5.4 \) µm, when applying a maximum operating voltage of 15 V, is depicted in Figure 2.12. This figure shows that the maximum excursion width is \( W_{ex,max} = 8.7 \) nm for hinged boundary conditions. This is equivalent to a minimum slot width of \( W_{s,min} = 82.6 \) nm obtained when applying a maximum voltage of 15 V with hinged boundary conditions. Therefore, there is a clearance of 82.6% of the spacing between the suspended silicon beams of the suspended slot waveguide when confining the applied voltage to a maximum of 15 V. The stress due to the bending of the suspended Si beams can be doubted to induce birefringence and other nonlinear optical effects that can cause self-phase modulation in the slot waveguide. However, due to the large ratio of the chosen length of the suspended Si beams to the maximum possible excursion width, the bending curvature is very small as can be inferred from
the results of Figure 2.12. Therefore, the effect of the stress-induced birefringence is expected to be minimal in contrast to changing the effective index due to modulating the gap width between the suspended Si beams of the slot waveguide.

Figure 2.13 displays the maximum width excursion occurring at the midpoint of the suspended silicon beams of the under-etched slot waveguide versus the section length $L_s$ obtained when applying a maximum voltage of 15 V. In this figure the length $L_s = 5.86 \mu m$ is about the maximum possible length of the suspended slot waveguide before the onset of pull-in failure for an applied voltage of 15 V. Therefore, there is a guard of about 7.85% of the selected design length of $L_s = 5.4 \mu m$, for the maximum operating voltage of 15 V, compared to the pull-in length of the suspended slot waveguide.

![Figure 2.13 Predicted maximum excursion width versus the design length of the under-etched slot waveguide when applying a voltage of 15 V. $W_{gs} = 230$ nm, $W_{so} = 100$ nm, and $h = 300$ nm.](image)

The phase shift characteristic of a single suspended slot waveguide section, at the design length $L_s = 5.4 \mu m$, as a function of the voltage applied to the suspended silicon beams for both fixed and hinged boundary conditions is depicted in Figure 2.14. From this figure, it is found that the possible pull-in voltage at hinged boundary conditions (worst-case premise) is about 17.7 V.

The fundamental resonance frequency of the nanomechanical suspended silicon beams as a function of the length of the under-etched slot waveguide is displayed in Figure 2.15 based on Eq. (2.7). At the design length of the suspended slot waveguide section of 5.4 $\mu m$, the fundamental resonance frequency is about 50 MHz. Increasing the fundamental resonance frequency of the
suspended silicon beams is possible by decreasing the length of the suspended slot waveguide section. However, a smaller length yields less phase shift capability of each suspended slot waveguide section which then mandates an increased number of sections, increasing the excess loss of the NEMS-operated phase shifter. There is, therefore, a tradeoff between speed and loss.

Figure 2.14 Predicted phase difference versus the voltage applied to a NEMS-operated slot waveguide of length $L_s = 5.4 \, \mu m$. $W_{gs} = 230 \, nm$, $W_{s0} = 100 \, nm$, and $h = 300 \, nm$.

Figure 2.15 Fundamental resonance frequencies versus the under-etched slot waveguide length for fixed and hinged boundary conditions. $W_{gs} = 230 \, nm$, $W_{s0} = 100 \, nm$, and $h = 300 \, nm$.

The schematic diagram of the designed 15 section 180º capable phase shift element with the I/O straight transition slot couplers is shown in Figure 2.16. The designed 180º capable NEMS-operated phase shifter is just 349 \, \mu m, more compact than thermo-optic effect, carrier dispersion effect, and
electro-optic effect counterparts. Simulations of the optical power distribution inside the designed 180° capable NEMS-operated phase shifter and transition slot couplers, at an operating wavelength of $\lambda = 1550$ nm, are depicted in Figure 2.17. The resolution of the simulation parameters is increased to assure reaching the convergence of the simulation results. Based on simulation statistics, the designed 180° capable phase shifter introduces a total excess loss of about 1.1 dB, and each additional suspended slot section adds an excess loss of about 0.055 dB. For the designed transition slot coupler length of 100 µm, the optical mode coupling efficiency of each transition slot coupler predicted by simulation is about 97%.

As explained in the following chapters, a 360° capable phase shifter is required for the full coverage of the constellation space of the amplitude-phase relations of the outputs of the studied photonic phased array switches. The designed phase shifter of the dimensions indicated in Figure 2.37 should already achieve a maximum phase shift larger than 180° due to the relatively large margins that have been accounted for in its design. Increasing the number of sections to increase the phase capacity of the cascade would also excessively increase the excess loss. Alternatively, a true 360° capable phase shifter is argued feasible, without increasing the number of cascaded under-etched slot waveguides, by slightly increasing the length of each under-etched slot waveguide to $L_s = 5.8$ µm. This increase in $L_s$ increases the phase shift that can be achieved by each section to about 31°. Only 12 sections are then needed for a total 360° phase shift capability. However, up to 15 cascaded sections in the design are used to provide a margin. This results in a slight increase in the excess loss predicted to about 1.2 dB evaluated at the central operating wavelength of 1.55 µm. The fundamental resonance frequency of the nanomechanical structure is also reduced to about 43 MHz as in Figure 2.15. The decrease in
the fundamental resonance frequency might not matter because the speed of the phase shifter is expected to be limited mostly by the RC time constant of the electrical feed. The overall length of the designed 360° capable phase shifter is also slightly increased to 355 µm.

To confirm the accuracy of the numerical analysis presented in this section, the simulation suite COMSOL Multiphysics is used to study the electromechanical behavior of the designed under-etched slot waveguide segment of 5.4 µm length. The electromechanics (emi) physics is adopted in
the simulations which combines both solid mechanics and electrostatics physics. The geometry of the studied structure is constructed of two suspended polysilicon beams modeled as both electrical and linear elastic materials and surrounded in an air block that extends to 1 µm from the outermost surfaces of the beams in each orthogonal direction.

![Figure 2.18 COMSOL Multiphysics simulation of the displacement deformation of the designed suspended silicon beams for both fixed (left) and hinged (right) boundary conditions.](image)

$L_s = 5.4$ µm, $W_{gs} = 230$ nm, $W_{s0} = 100$ nm, $h = 300$ nm, and $V = 15$ V.

The response for fixed boundary conditions is simulated by applying fixed mechanical constraints to the four side boundary surfaces of the two silicon beams. The response for hinged boundary conditions is simulated by allowing each beam to rotate around a z-oriented center axle constraint at each side boundary area of the two beams. The simulations of the deformation in the silicon beams due to the electrostatic distributed force when applying a voltage of 15 V between the silicon beams taken for fixed and hinged boundary conditions are depicted in the respective left and right parts of Figure 2.18. The simulated maximum displacements of this figure agree with those predicted in Figure 2.12 based on numerical analysis of 1.35 nm for fixed boundary conditions and 8.7 nm for hinged boundary conditions.

### 2.3.5 Conclusions of Part 2

The structure and analysis of a suspended slot waveguide section acting as a phase shift segment have been reviewed. Simulation of the effective index of the suspended silicon beams of the slot waveguide and numerical analysis are used in the design procedure. COMSOL Multiphysics simulations are also used to confirm the accuracy of the achieved numerical analysis. The cascading
of 15 suspended slot waveguide sections is used in constructing phase shift elements that can achieve up to 180° and 360°. Innovative transition slot couplers that are 97% efficient are used to couple the phase shift element and provide biasing using a single isolated electrode. The designed phase shifters have a maximum operating voltage of 15 V, pull-in voltage of 17.7 V, and overall length of around 350 µm and they require a small amount of power to operate them. Although the fundamental resonance frequency of the designed nanomechanical structures is more than 40 MHz, the \(RC\) low pass filtering effect of the biasing circuit of the phase shift element is expected to play a significant role in lowering the speed of the designed NEMS-operated phase shifters. The compact footprint of the demonstrated full capable 360° NEMS-operated phase shift element of length less than 355 µm and its need for a single electrode located at one side encouraged investigating the construction of relatively large photonic processors that are presented in the following chapters. Making the length of the phase shift element even shorter for dense integration is also possible by reducing the length of the transition slot couplers for slight increase in their losses and by slightly increasing the length of the sections of suspended silicon beams such that the reduction in their nanomechanical resonance frequency matches the speed limit imposed by the \(RC\) lowpass filtering effect. This increase in the length of the under-etched section increases the maximum phase shift that can be achieved by each section and hence requires less number of cascaded sections and in result reduces the overall length of the phase shift element and possibly lowers the overall optical loss. The bandwidth response of the NEMS-operated phase shifter, depicted in the next chapter, is found to be reasonably flat over the entire C-band. This in addition to the compact size and other novel features entitles the NEMS-operated phase shifter as the optimum choice for applications in next-generation elastic optical networks as far as a switching speed in the fractional \(\mu\)s range suffices. The redesign of the voltage-controlled NEMS-operated phase shifter utilizing the same SOI integration template can also be configured for adaptive high-resolution photonic acceleration sensing targeting wide range applications that could cover ultrasound imaging, sonar, seismic wave detection, automobile industry, aerospace guidance and orientation detection in electronic mobile devices. The detection sensitivity of an adaptive photonic acceleration sensor can be voltage-controlled within a wide dynamic range. Acceleration sensing levels ranging from tiny vibrations of less than \(1 \text{ ms}^{-2}\) to harsh accelerations up to \(10^4 \text{ ms}^{-2}\) can be conveniently detected covering wide range applications. Photonic motion sensors have the advantage of achieving sensitivity and resolution levels higher than those of
their counterparts of microelectromechanical systems (MEMS) motion sensors. The former method converts the mechanical motion into a modulation of an aspect of the light such as intensity or wavelength shift of the spectrum whereas the latter method is conventionally based on capacitive motion sensing which can be obscured by the stray capacitance of the feeding arrangements.

2.4 Summary

The chapter investigated the components used in the construction of the photonic integrated processors presented in the following chapters. The covered construction components include the waveguide, edge coupler, taper adapter, MMI couplers, and nano-electromechanical phase shifter. The simulation results obtained based on the parameters of the designed construction components are used in determining the response of the PICs presented in the following chapters based on the transfer function method.
CHAPTER 3
SINGLE-STAGE FIR PHASED ARRAY ELEMENTS

3.1 Introduction

In Chapter (1) the argument was made to utilize PIC phased array architectures for the flexible spectral processing of the light signal in EONs. Both broadband switching operations and tunable wavelength-selective switching operations are required in integrated elastic optical systems. It was indicated in Chapter (1) to drive the study in steps, and consequently, this chapter introduces the single-stage phased array architectures which are used in the next two chapters as building blocks for the advanced cascaded and feedback elastic spectrum processors.

An integrated single-stage photonic FIR phased array circuit is constructed of two $N \times N$ splitter/combiner MMI couplers interconnected by a network of phase shifters and path length imbalances. Photonic FIR digital signal processing systems have the merit of possessing linear phase responses within the passband channels. When several phased array networks interconnect more than two MMI couplers, the structure is referred to as multistage or cascaded. This chapter considers single-stage FIR phased array structures, whereas the next two chapters delve through the advantages of multistage FIR and IIR phased array photonic circuits. When the path length imbalances are eliminated from the interconnection circuit of a single-stage FIR phased-array circuit, the transmission responses for the different inputs become flat in the ideal case as functions of wavelength. In this case, the photonic phased array circuit is referred to as a broadband switch element in the thesis. The inclusion of unequal path length imbalances in the interconnection circuit modulates the transmission responses for the different inputs as functions of wavelength. The transmission responses for the different inputs become wavelength-selective in this case, and the resulted FIR phased array circuit can be used for the wavelength (de)multiplexing/filtering of several optical signals. The transmission responses, in this case, can also be steered along the wavelength scale by applying proper phase relations through the used NEMS-operated phase shifters of the interconnection circuit. The single-stage FIR photonic integrated circuit, in such a case, is referred to as a tunable WDM phased-array element in the thesis.

In the previous chapter, the construction components used in the building of photonic phased array elements were introduced and simulated using FIMMPROP. In this chapter, these same
construction components are used to form single-stage phased array elements. The transfer function method is used to numerically determine the transmission responses for the constructed photonic switch elements utilizing the simulated parameters of the construction components. The chapter splits into two parts. The first part (3.2) deals with the study of single-stage broadband IPPAS elements of 2×2 and 4×4 sizes. The second part (3.3) presents tunable single-stage WDM phased array elements of 2×2 and 4×4 sizes. The novel features of IPPAS elements are outlined in this chapter and explained using schematic diagrams showing the typical dimensions of the components and switch structures presented in this study. The single-stage 4×4 phased-array architecture provides only four switching states out of the total 24 possible switching permutations, and therefore it is referred to as a blocking broadband switch. The blocking switching operations of the broadband 4×4 IPPAS element are emphasized in Part (3.2) explaining the necessity for the cascaded architectures presented in Chapter (4) to create non-blocking broadband switches. Quantifying the performance of the blocking switching operations of the 4×4 IPPAS element is important to the development of the advanced flexible-spectrum processors which are constructed of units of the blocking switch elements. The capability to steer the transmission responses of the transfer functions of 2×2 and 4×4 single-stage WDM phased array elements by applying progressive phase shifts through the NEMS-operated phase shifters is demonstrated in Part (3.3). In Chapter (4) single-stage FIR elements of 2×2 and 4×4 sizes are cascaded to form more complicated photonic integrated circuits that can achieve more advanced functions. Therefore, this chapter establishes the background needed to understand how the simpler IPPAS single-stage FIR elements of GMZI architectures behave.

3.2 Part 1: Broadband FIR Phased Array Switch Elements

3.2.1 Introduction of Part 1

The single-stage broadband FIR 4×4 switch architecture, usually referred to as the generalized MZI modulator, was introduced as a generalization to the 2×2 MZI modulator; e.g. [26,27,148,149]. The architecture of the generalized MZI modulator is shown in Figure (1.2) of Chapter (1). The studied characteristic of the generalized single-stage 4×4 switch element in [149] demonstrated its blocking behavior of being able to achieve only four single-throw routing operations out of the 24 comprehensive sets of single-throw routing operations that should be available to a non-blocking 4×4 switch fabric. The deployment of the innovative NEMS-operated phase shifters in phased array
switches benefits from the novel feature for placing the phase shift elements at the outermost branches of the interconnection network between the splitter and combiner 4×4 MMI couplers. Each NEMS-operated phase shifter is biased by a single electrode located at one side of the switch structure away from the switch core. This novel feeding arrangement imposes rearranging the order of the photonic signal paths that are applied to the inputs of the combiner 4×4 MMI unit. In consequence, the resulted single-stage broadband 4×4 IPPAS element presented in this research exhibits four single-throw routing permutations that are different from those exhibited by the conventional switch architecture presented by Cahill [149]. The responses of the broadband IPPAS elements designed in this research are re-evaluated, taking into account the actual effect of every component used in the architectures to provide accurate estimations of their characteristics. Additionally, the double-throw and broadcast switching operations for the broadband 4×4 IPPAS element are also determined in this chapter for their importance in predicting the possible wavelength (de)multiplexing schemes for several signals in the cascaded architectures presented in the next two chapters.

In this chapter, the wavelength-dependent transmission and phase responses of the transfer functions of single-stage 2×2 and 4×4 IPPAS switch elements are studied as functions of the control phase-shift differences of the interconnection network. The transmission and phase responses of the IPPAS elements are explained using the following three ways:

1. Using phase-dependent transmission graphs evaluated at a single wavelength of 1.55 µm. In this case, the transmission graph of the transfer function is drawn as a function of the control phase differences applied through the NEMS-operated phase shifters of the interconnection network. The wavelength, in this case, is usually fixed to the center wavelength of 1.55 µm. This method is suitable to explore the transmission response of broadband switches to the control phase shifts of the interconnection network, but it pays no attention to the degradation in the transmission characteristic over the wavelength range, and it provides no information for how the phase of the optical signal changes as a function of the control phases.

2. Using transmission-phase constellation diagrams. As used in communications theory, each point in the constellation diagram provides information for both of the transmission and phase of the transfer function. The locus of these points is drawn as a function of the control phase-shift differences of the interconnection network. The wavelength of operation, in this case, is
also usually fixed at 1.55 µm. This method is suitable to explore the transmission and phase modulation of the optical signal as a function of the control phases for applications of the broadband switches in modulation techniques such as in amplitude shift keying (ASK), phase shift keying (PSK), and quadrature amplitude modulation (QAM). It also helps to understand how single-stage switch architectures function when they are used in cascaded and feedback architectures. However, this method also does not pay attention to the wavelength dependence of the transmission and phase responses of the optical carrier. The other drawback in this method is to visually hardly relate each point of the transmission and phase of the modulated optical signal to the particular values of the applied control phases unless if some properties of the plotted characteristic such as the size or color are driven in accordance to an applied control variable of the phase shift array.

3. Using broadband wavelength-dependent transmission graphs. In this case, the transmission of the transfer function is drawn versus wavelength over the entire C-band. This method is suitable to observe the degradation in the characteristic of switches on the broad wavelength range basis. It is also the only way to display the transmission characteristic of tunable WDM elements and wavelength-selective filters. Drawing the wavelength-dependent transmission characteristic for several discrete control phase shift differences of the interconnection network is also feasible.

The simulations of the wavelength-dependent magnitudes and wavelength-dependent phases of the forward scattering parameters of the construction components of Chapter (2) are displayed in this chapter and are used in determining the wavelength responses of the phased array systems throughout the rest of the thesis. The phase control of the transmission, while keeping the phase of an output photonic signal of a broadband IPPAS element unchanged, is illustrated utilizing a transmission-phase constellation diagram.

### 3.2.2 Forward Scattering Parameters of the Construction Components

It was emphasized in Chapter (1) that the wavelength-dependent impairments in the phase and transmission characteristics of the scattering parameters of the construction components affect the operation of the processors studied in the thesis. In principle, the architectures are developed to achieve certain objectives based on the assumption of using ideal components. In the thesis it is planned to reflect the effect of the impairments on the proposed processors that are presented in this
chapter and the next two chapters. Therefore, the determination of the magnitudes and phases of the forward scattering parameters of the different components used in the construction of the phased array systems is presented in this section and used in the forthcoming results. The symbols of the 2×2 and 4×4 MMI couplers of different kinds are explained in Figure 3.1.

Figure 3.1 Symbols of the -3dB 2×2, crossover 2×2, -6dB 4×4, and -3dB 4×4 MMI couplers as arranged from left to right, respectively.

The forward scattering matrix of the 2×2 MMI couplers presented in the thesis is symmetrical due to the lateral structure symmetry around the centerline and is expressed here as:

\[
\begin{bmatrix}
t_{11} & t_{12} \\
t_{21} & t_{22}
\end{bmatrix}
= 
\begin{bmatrix}
t_{11} & t_{12} \\
t_{21} & t_{22}
\end{bmatrix}
\]

... Eq. (3.1)

Each forward scattering parameter \( t_{kl} = T_{kl} \exp(j \theta_{kl}) \) has a magnitude \( T_{kl} \) and phase \( \theta_{kl} \) relating output \( k \) to input \( l \). In Eq. (3.1) due to the symmetry: \( t_{12} = t_{21} \) and \( t_{22} = t_{11} \). The symmetry in these parameters is due to the symmetrical design of the MMI couplers presented in the thesis and such that all corresponding input and output ports are at the same distance from the longitudinal axial line. The forward scattering parameters for the designed -3dB 2×2 MMI coupler, obtained from the simulation at the wavelength 1.55 µm, are given by \( T_{11} \approx 0.7, \theta_{11} \approx -142.21^\circ, T_{21} \approx 0.696, \) and \( \theta_{21} \approx 126.79^\circ \). The absolute values of the \( \theta_{11} \) and \( \theta_{21} \) phases have no significance on the response of balanced forward IPPAS elements, but rather only the phase difference \( \theta_{11} - \theta_{21} = 91^\circ \), which has an error of 1° compared to the ideal value of 90°, is important. The forward scattering matrix of the 4×4 MMI couplers presented in the thesis is also symmetrical for the same reason and is given by:

\[
\begin{bmatrix}
t_{11} & t_{12} & t_{13} & t_{14} \\
t_{21} & t_{22} & t_{23} & t_{24} \\
t_{31} & t_{32} & t_{33} & t_{34} \\
t_{41} & t_{42} & t_{43} & t_{44}
\end{bmatrix}
= 
\begin{bmatrix}
t_{11} & t_{13} & t_{14} \\
t_{21} & t_{23} & t_{31} \\
t_{31} & t_{33} & t_{21} \\
t_{41} & t_{43} & t_{11}
\end{bmatrix}
\]

... Eq. (3.2)
In Eq. (3.2), the forward scattering parameters are stated in terms of inputs $I_1$ and $I_3$ utilizing symmetry. The forward scattering parameters for the designed -6dB 4×4 MMI coupler, obtained from the simulation at the wavelength 1.55 µm, are listed in Table 3.1.

$$\begin{array}{ccc}
kl & T_{kl} & \theta_{kl} \\
11 & 0.476 & 110.09^\circ \\
21 & 0.480 & -24.01^\circ \\
31 & 0.478 & 155.31^\circ \\
41 & 0.481 & 110.92^\circ \\
13 & 0.478 & 155.32^\circ \\
23 & 0.483 & 110.29^\circ \\
33 & 0.479 & 109.39^\circ \\
43 & 0.480 & -24.02^\circ \\
\end{array}$$

**Table 3.1** Forward scattering parameters for the designed -6dB 4×4 MMI coupler of the middle parts (left and right) of Figure 2.8.

The following phase differences result from the simulated phases of Table 3.1 for the designed -6dB 4×4 MMI coupler of the middle parts (left and right) of Figure 2.8. The errors in the phase differences are compared with the ideal phase difference values:

$$\theta_{11}-\theta_{13} = -45.23^\circ$$ (error: -0.23°, ideal: -45°),
$$\theta_{21}-\theta_{23} = -134.30^\circ$$ (error: 0.7°, ideal: -135°)

$$\theta_{31}-\theta_{33} = 45.92^\circ$$ (error: 0.92°, ideal: 45°),
$$\theta_{41}-\theta_{43} = 134.94^\circ$$ (error: 0.06°, ideal: 135°)

The forward scattering parameters for the designed -3dB 4×4 MMI coupler, obtained from the simulation at the wavelength 1.55 µm, are listed in Table 3.2.

$$\begin{array}{ccc}
kl & T_{kl} & \theta_{kl} \\
11 & 0.643 & 67.49^\circ \\
21 & 0.033 & -110.47^\circ \\
31 & 0.036 & -160.49^\circ \\
41 & 0.629 & -22.03^\circ \\
13 & 0.036 & -160.43^\circ \\
23 & 0.606 & -27.32^\circ \\
33 & 0.586 & 70.02^\circ \\
43 & 0.033 & -110.28^\circ \\
\end{array}$$

**Table 3.2** Forward scattering parameters for the designed -3dB 4×4 MMI coupler of the lower parts (left and right) of Figure 2.8.

The following phase differences result from the simulated phases of Table 3.2 for the designed -3dB 4×4 MMI coupler of the lower parts (left and right) of Figure 2.8. The errors in the phase differences are compared with the ideal phase difference values:

$$\theta_{11}-\theta_{14} = \theta_{11}-\theta_{41} = 89.52^\circ$$ (error: -0.48°, ideal: 90°)
$$\theta_{22}-\theta_{23} = \theta_{33}-\theta_{23} = 97.34^\circ$$ (error: 7.34°, ideal: 90°)

The simulated phase difference errors of the designed MMI couplers are less than or equal to 1° except for the relatively large value of $\theta_{22}-\theta_{23}-90^\circ = 7.34^\circ$ for the -3dB 4×4 MMI coupler.

The simulated magnitude and phase wavelength responses of the forward scattering parameters for input $I_1$ of the designed -3dB and crossover 2×2 MMI couplers, for the wavelength range 1.53-1.57 µm, are displayed in Figures 3.2 and 3.3, respectively. The simulated magnitude and phase
wavelength responses of the forward scattering parameters for inputs $I_1$ and $I_3$ of the designed -6dB and -3dB 4×4 MMI couplers, for the wavelength range 1.53-1.57 µm, are displayed in Figures 3.4 through 3.7.

Figure 3.2 Simulated magnitude (left) and phase (right) wavelength responses of the forward s-parameters for input $I_1$ of the designed -3dB 2×2 MMI coupler. The $kl$ legend represents output $k$ and input $l$.

Figure 3.3 Simulated magnitude (left) and phase (right) wavelength responses of the forward s-parameters for input $I_1$ of the designed crossover 2×2 MMI coupler. The $kl$ legend represents output $k$ and input $l$.

The wavelength response of the designed crossover 2×2 MMI coupler, as appeared in Figure 3.3, is broad. For an ideal crossover 2×2 MMI coupler, the light of input $I_1$ should be all yielded to output $O_2$. By other words, $I_1$ should ideally produce zero transmission value at output $O_1$. As seen in the results of Figure 3.3, input $I_1$ produces wavelength-dependent crosstalk at output $O_1$. The crosstalk for the designed crossover 2×2 MMI coupler of Figure 2.22 is about -26 dB estimated at the wavelength 1.57 µm. The responses of the other designed MMI couplers deteriorate in the extremities of the wavelength range. Each of inputs $I_1$ and $I_4$ ($I_2$ and $I_3$) of an ideal -3dB 4×4 MMI coupler produces -3dB of equal transmissions at outputs $O_1$ and $O_4$ ($O_2$ and $O_3$). In the real case, the
wavelength-dependent crosstalk is identified when any of inputs \( I_1 \) or \( I_4 \) \((I_2 \text{ or } I_3)\) produces nonzero optical signals at any of outputs \( O_2 \) or \( O_3 \) \((O_1 \text{ or } O_4)\). The maximum crosstalk for the designed -3dB 4×4 MMI coupler of Figure 2.29 is about -12 dB estimated at the wavelength 1.57 \( \mu \text{m} \).

Figure 3.4 Simulated magnitude (left) and phase (right) wavelength responses of the forward s-parameters for input \( I_1 \) of the designed -6dB 4×4 MMI coupler. The \( kl \) legend represents output \( k \) and input \( l \).

Figure 3.5 Simulated magnitude (left) and phase (right) wavelength responses of the forward s-parameters for input \( I_3 \) of the designed -6dB 4×4 MMI coupler. The \( kl \) legend represents output \( k \) and input \( l \).

Improvement of the responses of the MMI couplers can result in PICs operating at wider wavelength ranges. Examples of improved MMI couplers of possible wider wavelength responses are those reported in [150] that use subwavelength patterning of the multimode section to engineer the strict quadratic dependence of the effective index on mode order required for perfect Talbot imaging of the field profiles over a wider wavelength range. Nevertheless, such a goal is kept for future investigation, and only the conventional solid MMI couplers designed in Chapter (2) are used to examine the properties of the different PICs considered in this study.
Figure 3.6 Simulated magnitude (left) and phase (right) wavelength responses of the forward s-parameters for input $I_1$ of the designed -3dB 4×4 MMI coupler. The $kl$ legend represents output $k$ and input $l$.

Figure 3.7 Simulated magnitude (left) and phase (right) wavelength responses of the forward s-parameters for input $I_3$ of the designed -3dB 4×4 MMI coupler. The $kl$ legend represents output $k$ and input $l$.

The simulated wavelength-dependent magnitude responses of the forward scattering parameters of the NEMS-operated phase shifter and of the taper that is used at the I/O ports of the MMI couplers are depicted in Figures 3.8 and 3.9, respectively, and are taken into account in this thesis to accurately determine the transfer function responses of the designed IPPAS elements. As seen in the figures, these components are naturally broadband. The magnitude wavelength response of the taper is almost flat over the entire C-band, and therefore its wavelength dependency might be ignored for simplicity of the numerical analysis. The limited ripple observed in the magnitude response of the NEMS-operated phase shifter is due to its complex construction of concatenated under-etched and supported slot waveguide sections presenting periodic discontinuity in the under cladding combined with the wavelength response of the transition slot couplers.
Usually, in this thesis, there is only one phase shifter and an equal number of tapers engaged in each signal path of the designed well balanced IPPAS elements. Therefore, the determination of the wavelength-dependent phase responses of the forward scattering parameters of the NEMS-operated phase shifter and of the taper that is used at the I/O ports of the MMI couplers is not needed for determining the transfer function responses of the IPPAS elements presented in this thesis.

Considering the impairments of the construction components presented in this section is of great importance on evaluating the validity of the processors proposed in the thesis. These impairments can accumulate large computational errors in processing the light spectrum in advanced elastic processors that are made of many cascaded stages. Within the current context of the thesis, the studied behavior of the components is considered good enough for many of the introduced
coherent processors but still not close to the ideal case. Future studies should look into remedying the impairments in the responses of the construction components.

3.2.3 Broadband FIR 2×2 Phased Array Switch Element

Although a broadband FIR 2×2 IPPAS element shares the characteristic of a traditional MZI modulator, it inherits the distinct novel features of the IPPAS class of switches that use a single electrode per phase shift element located at one side of the switch structure and eliminate the need for engaging waveguide bends to interconnect the phase shift array. Figure 3.10 shows the schematic diagram of the broadband 2×2 IPPAS element introduced in this work. The switch element consists of two -3dB 2×2 MMI couplers with two NEMS-operated phase shifters interconnecting them. A -3dB 2×2 MMI coupler splits the optical power at an entrance port equally into the two arms. The optical beams in the two arms of phase shift elements recombine at the exit -3dB 2×2 MMI coupler arms. Since no path length imbalances are included in the interconnection network between the splitter and combiner 2×2 MMI couplers, the element performs as a broadband switch. The phase shift elements in the two arms adjust the interference of the two beams on recombination such that light may be switched between either one of the two output ports of the switch element. This basic switch element is also characterized by the capability to change the phase modulation due to the voltage applied at the electrodes into both amplitude and phase modulation of the light transferred through the silicon switch.

![Figure 3.10](image)

**Figure 3.10** Schematic diagram of a single-stage broadband 2×2 FIR-IPPAS element.

As detailed in Chapter (2), The NEMS-operated phase shifter used in the switch element of Figure 3.10 is composed of 15 cascaded slot waveguides estimated to provide a phase shift of more than either 180° or 360° at an applied voltage of about 15 V. The switch element uses no waveguide bends in the interconnecting branches between the ports of the MMI couplers. Near vertical grating couplers or edge couplers can be used to couple the light in and out the switch fabric. One of the
silicon beams of each NEMS-operated phase shifter is connected to the common ground electrode through the rest of the silicon waveguide circuitry as outlined in the schematic diagram of Figure 3.10. The other silicon beam is connected to an isolated electrode located on one side of the phase shift element. This switch element is compact, has a simple layout, and provides an effective test harness to measure the phase difference introduced by the NEMS-operated phase shifters using its MZI-like behavior. In Figure 3.10, \( \Delta \phi_1 \) and \( \Delta \phi_2 \) are the phase shift differences introduced by the two NEMS-operated phase shifters due to the external voltage applied to electrodes 1 and 2, respectively.

The length of the 180° capable NEMS-operated phase shifter with the I/O straight transition slot-waveguide couplers is 349 µm. The entire length of the core of the switch element of this figure is around 520 µm excluding the I/O coupling arrangements to the optical fibers of near-vertical grating couplers or inverse-taper couplers and excluding the crossover 2×2 MMI couplers at the output ports used to facilitate connecting the silicon structure to the ground electrodes. This length of the switch element core is much smaller than that of a traditional MZI modulator; the length of a traditional phase shift element alone is in the millimeter range.

Figure 3.11 Symbolic diagram of the single-stage broadband 2×2 FIR-IPPAS element.

The symbolic diagram for the core of the broadband 2×2 IPPAS element of Figure 3.10 is shown in Figure 3.11 for convenience. This switch configuration represents the basic broadband MZI modulator with the capability to modulate the magnitudes and phases of the outputs as functions of the phase shift differences \( \Delta \phi_1 \) and \( \Delta \phi_2 \) of the phase shifters inserted into the two arms between the splitter and combiner 2×2 MMI couplers. The forward scattering matrix \([ t_{kl} ]\) of the 2×2 FIR-IPPAS element of Figure 3.12 is given by:

\[
[O_k] = [t_{kl}] [I_l]
\]

\[
[t_{kl}] = T_0 e^{-j\phi} [\kappa_{kl}] [t_{kl}]
\]

... Eq. (3.3)

\([O_k] = [O_1, O_2]^T\) is the vector of outputs, \([I_l] = [I_1, I_2]^T\) is the vector of inputs, \([t_{kl}]\) is the forward scattering matrix of the 2×2 MMI coupler as introduced in Eq. (3.1), and \([\kappa_{kl}]\) is a diagonal matrix introducing the synthesized phase shift differences:
\[
\mathbf{K} = \begin{bmatrix}
\kappa_{11} & \kappa_{12} \\
\kappa_{21} & \kappa_{22}
\end{bmatrix} = \begin{bmatrix}
e^{i\phi} & 0 \\
0 & e^{i\phi_0}
\end{bmatrix}
\]

... Eq. (3.4)

The factor \( T_o \) models all wavelength-dependent losses caused by the tapers and one phase shifter that are engaged into each signal path. The angle \( \theta_o \) models the accumulated phase shift due to any balanced signal path lengths. Expansion of Eq. (3.3) and splitting the complex terms into real and imaginary parts yields:

\[
\begin{bmatrix}
O_1 \\
O_2
\end{bmatrix} = 2T_oT_{11}T_{22}e^{j(\psi_o+\Delta\phi_{av})} \left[ \begin{array}{c}
\left( \frac{T_{11} + T_{21}}{T_{21}} \right) \cos \left( \frac{\Delta\phi}{2} + \Delta\theta \right) + j \left( \frac{T_{11} - T_{21}}{T_{21}} \right) \sin \left( \frac{\Delta\phi}{2} + \Delta\theta \right) \right] \frac{I_1}{2} + \cos \left( \frac{\Delta\phi}{2} \right) I_2 \\
\cos \left( \frac{\Delta\phi}{2} \right) I_1 + \left( \frac{T_{11} + T_{21}}{T_{21}} \right) \cos \left( \frac{\Delta\phi}{2} - \Delta\theta \right) - j \left( \frac{T_{11} - T_{21}}{T_{21}} \right) \sin \left( \frac{\Delta\phi}{2} - \Delta\theta \right) \right] \frac{I_2}{2}
\]

... Eq. (3.5)

\( \psi_o = \theta_o + \theta_{11} + \theta_{12}, \Delta\phi_{av} = (\Delta\phi_1 + \Delta\phi_2)/2, \Delta\phi = \Delta\phi_1 - \Delta\phi_2, \) and \( \Delta\theta = \theta_{11} - \theta_{21}. \) Both Equations (3.3) and (3.5) represent the exact evaluation for the response of the broadband 2×2 IPPAS element. However, for the ease of interpretation of the response of the basic MZI modulator, some simplifications are considered here. For well balanced 2×2 MMI couplers the imaginary terms in Eq. (3.5) might be ignored. Furthermore, assuming ideal 2×2 MMI couplers with null imbalances, null excess losses, and null phase errors imposes \( T_{11} = T_{12} = 1/\sqrt{2} \) and \( \Delta\theta = 90^\circ. \) The assumption of lossless broadband tapers and phase shifters sets \( T_o = 1. \) The accumulated constant phase \( \psi_o \) can be dropped from the expression of Eq. (3.5), which reduces after using the simplifications above into:

\[
\begin{bmatrix}
O_1 \\
O_2
\end{bmatrix} = e^{i\Delta\phi_{av}} \left[ \begin{array}{c}
- \sin \left( \frac{\Delta\phi}{2} \right) I_1 + \cos \left( \frac{\Delta\phi}{2} \right) I_2 \\
\cos \left( \frac{\Delta\phi}{2} \right) I_1 + \sin \left( \frac{\Delta\phi}{2} \right) I_2
\end{array} \right]
\]

... Eq. (3.6)

The transfer function responses of the broadband 2×2 IPPAS element to the voltage-controlled phases of the NEMS-operated phase shifters is sinusoidal as predicted by the simplified relation of Eq. (3.6). The phase difference \( \Delta\phi \) modulates the magnitudes of the 2×2 IPPAS element outputs. Whereas, the average phase \( \Delta\phi_{av} \) modulates the phases of the 2×2 IPPAS element outputs. If the average phase \( \Delta\phi_{av} \) is kept constant, the phase term \( \exp(j\Delta\phi_{av}) \) can be dropped from the expressions for simplicity.

Using 180° capable NEMS-operated phase shifters, the transmission responses for both outputs \( O_1 \) and \( O_2 \) and inputs \( I_1 \) and \( I_2 \) of the 2×2 IPPAS element, versus the phase shift difference \( \Delta\phi, \) are
depicted in Figure 3.12 at the wavelength 1.55 µm. Full switching of the magnitudes of the outputs is achievable with the use of the 180° NEMS-operated phase shifters, as seen in the figure. The transmission responses of this figure sound to be typical to any traditional MZI switch element. Nevertheless, the determined numerical results of this figure reflect the effect of the power losses and imbalances in the power split ratios of the -3dB 2×2 MMI couplers and the effect of the power losses in the NEMS-operated phase shifters and tapers used in the construction of the broadband 2×2 IPPAS element on the resulted transmission responses for the different inputs and outputs. The simulated switch element introduces a loss of around 1.7 dB and crosstalk of less than -35 dB at single-throw routing operations evaluated at $\lambda = 1.55$ µm.

**Figure 3.12** Transmission responses of the 2×2 IPPAS element versus the phase difference $\Delta \phi$.

**Figure 3.13** Simulated wavelength-dependent transmission responses for input $I_1$ and both outputs $O_1$ (left) and $O_2$ (right). The legend represents the value of the phase shift difference $\Delta \phi$. 
The wavelength-dependent transmission responses of the 2×2 IPPAS element for input \( I_1 \) and both outputs \( O_1 \) and \( O_2 \) are numerically determined based on the simulated wavelength-dependent scattering parameters of the construction components and are depicted in Figure 3.13, taking the phase shift difference \( \Delta \phi \) as a parameter. The wavelength responses of the 2×2 IPPAS element is broad in the entire C-band as seen in Figure 3.13 even though the effects of bandwidth limitations and ripples due to the splitter and combiner 2×2 MMI couplers, tapers, and NEMS-operated phase shifters are recognizable. The average loss of the simulated switch element over the entire wavelength range displayed in this figure is around 2.1 dB.

The transmission-phase relations for switching, phase-reconfiguration, and detection processing can be viewed using constellation graphs. Each point in the constellation graph represents the transmission-phase combination obtained for a certain phase synthesis state of the NEMS-operated phase shifters evaluated at a given wavelength. The distance from each point in the graph to the origin of the graph represents the transmission value, and therefore, both of the horizontal and vertical axes have the same unit of transmission. The line drawn from the point to the origin of the graph makes an angle measured from the right axis representing the phase of the considered output state. The horizontal and vertical axes of a constellation diagram represent the real and imaginary parts of the transfer function, expressed as transmission and phase, for a given input and output. The pattern drawn in a constellation diagram can be rotated by any angle due to the accumulated phase of common waveguide lengths. This rotation is not important but rather the relative location of the output states with respect to each other matters. The importance of these traces is to inform the space within which the transmission and phase relative relations of the outputs can be controlled for a given switch element. They also visually allow quantifying the relative errors in the output states with respect to each other when used to demonstrate a switching operation for a single-phase control state as demonstrated in the next section of this chapter.

Different phase synthesis relations yield different transmission-phase traces. To determine the full space in the constellation diagram within which the transmission-phase points of the optical signal can be modulated, the phases of the NEMS-operated phase shifters of the switch element have to be scanned to all possible values within the range of the maximum phase shift that can be achieved by the phase shifter. The transmission-phase traces resulted from the full-range-scan of the phases of the NEMS-operated phase shifters for the 2×2 IPPAS switch element for both cases of
using the 180° and 360° NEMS-operated phase shifters are compared in the respective left and right sides of Figure 3.14 taken for input $I_1$ at an operating wavelength of 1.55 µm.

![Figure 3.14](image)

Figure 3.14 Simulated full-range-scan transmission-phase constellation graphs for input $I_1$ of the 2×2 IPPAS element using 180° (left) and 360° (right) capable NEMS-operated phase shifters.

Although, using 180° capable phase shifters enables the full switching of the inputs to the outputs as demonstrated in Figure 3.12, such that $I_1$ is yielded to $O_2$ when $\Delta \phi = 0$ and $I_1$ is yielded to $O_1$ when $\Delta \phi = \pm 180°$, the transmission-phase relations do not cover the entire constellation space as seen in the left drawing of Figure 3.14. For input $I_1$ and using the 180° capable phase shifters, output $O_1$ has the fan-shaped trace and output $O_2$ has the crescent-shaped trace with center peak. The use of the 360° phase shifters enables covering the entire transmission-phase space, as seen in the right drawing of Figure 3.14. Therefore, in applications where the full-range configuration of the phase of the optical signal is desired, the use of 360° capable NEMS-operated phase shifters is required.

The differential-balanced control of the phases of the NEMS-operated phase shifters is defined here such that the average phase $\Delta \phi_{av}$ is kept constant and equal to the mid-range value of the maximum phase that can be achieved by the NEMS-operated phase shifter while allowing the differential phase $\Delta \phi$ to vary. In this case, the amplitude of each output optical signal of the 2×2 IPPAS element is modulated due to $\Delta \phi$, and the phase of each output optical signal is held constant due to the fixed value of $\Delta \phi_{av}$. Figure 3.15 depicts the transmission-phase traces of the 2×2 IPPAS
element outputs taken for input \( I_i \) resulted when using the differential-balanced phase synthesis of the 180° and 360° NEMS-operated phase shifters.

![Figure 3.15 Simulated transmission-phase constellation graphs for input \( I_i \) of the 2×2 IPPAS element using the differential-balanced phase synthesis of the phases of the 180° (left) and 360° (right) NEMS-operated phase shifters.](image)

From the study of the aforementioned transmission-phase responses, it is evident that utilizing the 180° phase shifters can achieve full binary switching capability of the outputs, amplitude (intensity) modulation (AM), amplitude shift keying (ASK) modulation, and binary phase shift keying (BPSK) modulation. The NEMS-operated phase shifters, however, are low-speed elements compared to those based on the carrier dispersion effect and electro-optic effect and therefore their utilization in modulation can only be useful for low data rate transmission such as the sending of signaling information in the optical network. Nevertheless, the sub-microsecond speed of the NEMS-operated phase shifters used in broadband IPPAS elements entitles them to be eligible candidates in the construction of optical routers. The broadband 2×2 IPPAS element is a non-blocking switch element that can perform the two single-throw and one double-throw (broadcast) routing operations that are illustrated graphically in Figure 3.16 achieved utilizing 180° capable NEMS-operated phase shifters.

Based on the simulation results, the quality of the routing operations represented in Figure 3.16 for the 2×2 IPPAS element is perfect, as can be inferred from the characteristics of Figures 3.12 and 3.13. The extinction ratio is very high and almost ideal when switching one input to one output. The
bandwidth of the transmission responses is also wide within the C-band. However, the construction of a switch fabric with I/O port sizes more than 2×2 utilizing several 2×2 broadband IPPAS elements introduces structural difficulties for locating the electrodes of the inner switching elements of the switch fabric and might require the use of crossover waveguide connections. This motivates the introduction of the broadband 4×4 IPPAS element in the next section to expand the size of the I/O ports to 4×4 with the use of facilitated electrodes located on two sides of the switch structure and avoids the engagement of crossover waveguides.

Figure 3.16 Graphical representations of the single-throw (a, b) and double-throw (c) routing operations for the broadband 2×2 IPPAS element of Figure 3.11.

### 3.2.4 Broadband FIR 4×4 Phased Array Switch Element

A broadband FIR 4×4 IPPAS switch element is shown in Figure 3.17. This switch element has four NEMS-operated phase shifters. Each phase shifter is attached to one of the four branches interconnecting the I/O MMI couplers. Two intermediate crossover 2×2 MMI couplers are used to toggle the interconnecting paths such that all of the four phase shifters are located on the sides of the fabric as seen in the figure. The interconnecting paths are interlaced with each other as they are connected to the combiner MMI coupler. That is $O_{a1}$, $O_{a2}$, $O_{a3}$, and $O_{a4}$ are connected to $I_{c2}$, $I_{c1}$, $I_{c4}$, and $I_{c3}$ through $\Delta\phi_1$, $\Delta\phi_2$, $\Delta\phi_3$, and $\Delta\phi_4$, respectively. The crossover 2×2 MMI couplers located at the output ports are used to facilitate connecting the ground electrodes to the core structure of the switch. This broadband switch element is capable of providing four single-throw routing operations, two double-throw routing operations and a broadcast routing operation with the use of -6dB 4×4 splitter and combiner MMI couplers.

Figure 3.17 Schematic diagram of a single-stage broadband 4×4 FIR-IPPAS element.
As indicated in Figure 3.17, the length of the core of this broadband switch element with the -6dB 4×4 MMI couplers is around 1120 µm, which is possibly still shorter than just the length of a traditional electro-optic or thermo-optic phase shift element. It should be mentioned here that the inner edges of the crossover 2×2 MMI couplers are separated from each other by a distance of 0.51 µm; quite enough to avoid direct coupling of their internal fields. The pitch size between the centers of the adjacent I/O ports of the intermediate crossover 2×2 MMI couplers is the same as those for the I/O MMI splitter and combiner. Thanks to this novel feature in the design, the need to use any waveguide bends to interconnect the I/O MMI couplers is eliminated. This indeed is a strong merit given the difficulty of achieving control of the path length imbalances when resorting to waveguide bends. The electrodes are spaced from each other sufficiently that the electric field intensities developed between them when applying external voltages remain within safe limits. This broadband 4×4 switch design has the merits of compact footprint, good organization, low power consumption, and low excess loss.

![Figure 3.18 Symbolic diagram of the single-stage broadband 4×4 FIR-IPPAS element using -6dB 4×4 MMI couplers for the splitter and combiner units.](image)

The symbolic diagram of the 4×4 IPPAS element, using -6dB 4×4 MMI couplers for both of the splitter and combiner units, is shown in Figure 3.18. The forward scattering matrix \([\tau_{kl}]\) of the 4×4 IPPAS element of this figure is described by:

\[
[O_k] = [\tau_{kl}] [I_l] = T_o e^{i\theta_o} [t_{a,kl}] [\kappa_{b,kl}] [\kappa_{a,kl}] [t_{a,kl}] [I_l]
\]

... Eq. (3.7)

\([O_k] = [O_1, O_2, O_3, O_4]^T\) is the vector of outputs. \([I_l] = [I_1, I_2, I_3, I_4]^T\) is the vector of inputs. \(T_o\) and \(\theta_o\) are as defined before. \([t_{a,kl}]\) is the forward scattering matrix of the -6dB 4×4 MMI coupler used for the splitter and combiner units as described by Eq. (3.2). \([\kappa_{a,kl}]\) and \([\kappa_{b,kl}]\) are diagonal matrixes describing the phase synthesis introduced by the four NEMS-operated phase shifters given by:
The matrix $[\chi_{b,kl}]$ introduces the crossovers of the signal paths at the mid of the branches between the splitter and combiner units as described by:

$$
[\chi_{b,kl}] = \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & 0 \\
0 & 0 & e^{i\Delta\phi_k} \\
\end{bmatrix},
$$

... Eq. (3.8)

Symmetry is utilized in the description of the forward scattering matrix $[t_{b,kl}]$ of the crossover $2\times2$ MMI couplers used in the formation of Eq. (3.9). When assuming ideal crossover $2\times2$ MMI couplers that are wavelength-independent and lossless, Eq. (3.9) is simplified into:

$$
[t_{b,kl}] = \begin{bmatrix}
t_{b,11} & 0 & 0 \\
0 & t_{b,21} & 0 \\
0 & 0 & t_{b,11} \\
\end{bmatrix}
$$

... Eq. (3.9)

Furthermore, to yield simplified relations, the forward scattering parameters of the -6dB $4\times4$ MMI couplers are assumed to be ideally broadband with null excess losses, null imbalances, and null phase errors. The phase shifters and tapers are also assumed to be broadband and lossless. Consequently, the expansion and simplification of Eq. (3.7) yield the magnitudes of the forward scattering parameters documented in Appendix (2) for the $4\times4$ IPPAS element of Figure 3.18. The magnitude of each forward scattering parameter $\tau_{kl}$ in Appendix (2) is expressed as a function of the phase differences $\Delta\phi_{k,l} = \Delta\phi_k - \Delta\phi_l$, where $\Delta\phi_k$ and $\Delta\phi_l$ are the phases of the $k$'th and $l$'th NEMS-operated phase shifters. The combination of phase differences $(\Delta\phi_1, \Delta\phi_2, \Delta\phi_3, \Delta\phi_4)$ represents a phase synthesis state. A phase synthesis state can be any set of phases of the NEMS-operated phase shifters due to the voltages applied through the electrodes of the switch element aimed to control the relations between the optical signals at the I/O ports of the switch element. The useful single-throw, double-throw, and broadcast broadband switching operations are produced by applying certain phase synthesis states such that each phase difference $\Delta\phi_k$ ($k = 1, 2, 3$ or 4) is set to either $0^\circ$, $90^\circ$, $180^\circ$, or
270° as can be inferred from the relations in Appendix (2). Due to the symmetry relations of the transfer function elements in Appendix (2), the single-stage 4×4 GMZI of Figure 3.18 can achieve only four distinct single-throw routing operations. The phase synthesis states required to achieve an available switching operation can be predicted from the expressions of Appendix (2).

![Figure 3.19](image)

**Figure 3.19** Simulated wavelength-dependent transmission responses for input \(I_1\) of the 4×4 IPPAS element of Figure 3.18 taken for the phase synthesis state of \((0°, 0°, 0°, 0°)\).

The wavelength-dependent transmission responses case of the 4×4 IPPAS element for input \(I_1\) when the NEMS-operated phase shifters are at reset, i.e. for the phase synthesis state of \((0°, 0°, 0°, 0°)\), is depicted in Figure 3.19. In this case, each of inputs \(I_1\) and \(I_4\) splits about equally between outputs \(O_2\) and \(O_3\), whereas each of inputs \(I_2\) and \(I_3\) splits about equally between outputs \(O_1\) and \(O_4\). This phase synthesis state, therefore, presents an example of a double-throw switching operation. The loss in the outputs at the center wavelength of \(\lambda = 1.55\) μm is around 2.3 dB, and the crosstalk over the entire C-band is less than -22 dB. The wavelength-dependent maximum imbalance between the yielded outputs is less than 1.1 dB.

Figure 3.20 depicts the constellation diagram responses for inputs \(I_1\) and \(I_3\) of the broadband 4×4 IPPAS element of Figure 3.18 obtained for the phase synthesis state of \((180°, 180°, 0°, 0°)\). In this case, inputs \(I_1\) through \(I_4\) are routed to outputs \(O_1\) through \(O_4\), respectively. In ideal single-throw operations of a 4×4 broadband switch, each input optical signal should be routed to the desired output at no power loss cost, with no crosstalk to the other ports, and the transmission of the routed signals should be all the same. As seen in the results of Figure 3.20 numerically evaluated at the wavelength \(\lambda = 1.55\) μm, input \(I_1\) (\(I_3\)) produces non-zero optical signals at outputs \(O_2\), \(O_3\), and \(O_4\) (\(O_1\), \(O_2\), and \(O_4\)). However, the maximum crosstalk for this switching case is about -36 dB. The
excess loss of the switch core, excluding the off-chip coupling losses, is about 2.58 dB. The imbalance between outputs $O_1$ and $O_3$ is about 0.07 dB.

**Figure 3.20** Transmission-phase constellation diagram for inputs $I_1$ (left) and $I_3$ (right) of the 4×4 IPPAS element of Figure 3.18 determined for the phase synthesis state of (180º, 180º, 0º, 0º).

Setting the phase synthesis state of the 4×4 IPPAS element of Figure 3.18 to (180º, 0º, 180º, 0º) routes inputs $I_1$ through $I_4$ to outputs $O_4$, $O_3$, $O_2$, and $O_1$, respectively. The routing of inputs $I_1$ through $I_4$ to outputs $O_2$, $O_4$, $O_1$, and $O_3$, respectively, can be achieved by applying the phase synthesis state of (90º, 0º, 0º, 90º). The phase synthesis state of (0º, 90º, 90º, 0º) routes inputs $I_1$ through $I_4$ to outputs $O_3$, $O_1$, $O_4$, and $O_2$, respectively. The constellation diagrams for these three single-throw routing operations are not shown for brevity. All other possible phase synthesis states of single-throw routings using combinations of 0º, 90º, 180º, and 270º yield the same four broadband switching operations mentioned above.

**Figure 3.21** Graphical representations of the single-throw (a-d), double-throw (e-f) and broadcast (g) routing operations for the broadband 4×4 IPPAS element of Figure 3.20.
In summary, the $4 \times 4$ IPPAS element of Figure 3.18 provides four different single-throw routing operations including $(S_{11}, S_{22}, S_{33}, S_{44})$, $(S_{14}, S_{23}, S_{32}, S_{41})$, $(S_{13}, S_{21}, S_{34}, S_{42})$, and $(S_{12}, S_{24}, S_{31}, S_{43})$. Each $S_{kl}$ here is a complex number with magnitude and phase corresponding to output $k$ due to input $l$. The output switching state of a $4 \times 4$ broadband IPPAS element is described by $(O_1, O_2, O_3, O_4)$, where $O_k$ is given by:

$$O_k = \sum_{l=1}^{4} S_{kl} = \sum_{l=1}^{4} \tau_{kl} I_l$$

... Eq. (3.11)

$\tau_{kl}$ is the forward scattering parameter of the broadband IPPAS element for output $k$ and input $l$ as defined before. In general, a non-blocking $N \times N$ broadband switch requires $N!$ of distinct single-throw routing operations. For a $4 \times 4$ broadband switch, the total number of non-blocking single-throw distinct routings is 24. The single-stage broadband $4 \times 4$ IPPAS element of Figure 3.18 is a blocking switch unit that exhibits the four single-throw, two double-throw, and broadcast routings depicted graphically in Figure 3.21. The construction of non-blocking broadband switches requires the cascading of broadband sections as investigated comprehensively in Chapter (4).

### 3.2.5 Conclusions of Part 1

In this part of the chapter, the novel features of broadband IPPAS elements are clarified. The IPPAS elements save on the footprint due to the use of compact nano-electromechanical phase shifters allowing for the construction of the large PICs presented in the next two chapters. No waveguide bends are needed in the core structure of a broadband FIR-IPPAS element. The electrodes of an IPPAS element are located on two sides of the switch structure, making the wiring arrangements to the photonic chip more facilitated.

Utilizing the simulated wavelength-dependent scattering parameters of the construction components presented in Chapter (2), the transfer function responses of the single-stage broadband $2 \times 2$ and $4 \times 4$ IPPAS elements are numerically determined. The broadband $2 \times 2$ IPPAS element has the response of a basic MZI modulator. It represents a non-blocking switch providing two single-throw routings and one broadcast routing in addition to its capability to modulate both of the transmission and phase characteristics of the output optical signals as demonstrated in this chapter utilizing transmission-phase constellation diagrams.

The blocking characteristic of the single-stage broadband $4 \times 4$ IPPAS element is identified in this chapter. The phase synthesis states required to achieve the four single-throw, two double-throw,
and one broadcast routing operations for the single-stage broadband $4 \times 4$ IPPAS element are determined. In general, any single-stage broadband $N \times N$ IPPAS element with $N>2$ is a blocking switch element capable of providing only $N$ single-throw routing states out of the total number of $N!$ non-blocking routing states.

The replacement of one or both of the splitter and combiner -6dB $4 \times 4$ MMI couplers of Figure 3.18 with the -3dB $4 \times 4$ MMI coupler produces a new broadband IPPAS element with a different response. In an example the replacement of both of the splitter and combiner units with the -3dB $4 \times 4$ MMI coupler produces a dual $2 \times 2$ broadband IPPAS element where inputs $I_1$ and $I_4$ ($I_2$ and $I_3$) with outputs $O_2$ and $O_3$ ($O_1$ and $O_4$) form a $2 \times 2$ MZI switch separately controlled by the phase shifts $\Delta \phi_1$ and $\Delta \phi_4$ ($\Delta \phi_2$ and $\Delta \phi_3$). These options were inspected in this research, but the simulation results are not shown for brevity. The variety of broadband switch elements can provide flexibility in future studies to construct larger signal processing PICs.

It is known that the broadband $4 \times 4$ IPPAS element is a generalization to the MZI modulator. However, the basic MZI modulator is a non-blocking switch element, whereas the $4 \times 4$ GMZI is a blocking switch element. This justifies one of the objectives of the next chapter to explore new architectures for the construction of non-blocking broadband $4 \times 4$ switches made of cascaded stages of $2 \times 2$ and $4 \times 4$ broadband switch sections. The broadband $2 \times 2$ and $4 \times 4$ IPPAS elements presented in this chapter are used as building blocks for the larger photonic processors presented in the next two chapters. In the next part of this chapter, path length imbalances are added in the interconnection circuit between the splitter and combiner units to enable the construction of simple single-stage tunable wavelength division (de)multiplexers.

### 3.3 Part 2: Tunable FIR-WDM Phased Array Elements

#### 3.3.1 Introduction of Part 2

The demand for high capacity transmission of multiuser data has motivated the utilization of wavelength division multiplexing (WDM) in passive optical networks (PONs). A WDM-PON provides better security for having each subscriber using an allocated wavelength-band channel, requires a simplified point-to-point or point-to-multipoint operation, and provides an enhanced reach as it does not suffer from the power-splitting losses compared to a time division multiplexing (TDM) PON [151,152]. Arrayed waveguide gratings (AWGs) constructed using splitter/combiner multimode interference (MMI) couplers interconnected by a network of path length imbalances are
reported in the literature for the realization of wavelength routing (de)multiplexers [26,76,153-159]. The introduction of the generalized broadband MZI interference structures promised the capability to implement MIMO light routing devices [21-23,26,148,149].

In this part of the chapter, the concept of tunable WDM light routing is illustrated. Tunable single-stage $N \times N$ WDM (de)multiplexers are generalized MZI structures having both phase shifters and path length imbalances in the interconnection network between the splitter and combiner MMI couplers. They provide the capability to steer the (de)multiplexed channels over the wavelength range for the utilization of flexible passband channel routing. The interconnection network used in the tunable $4 \times 4$ WDM (de)multiplexer introduced in this work has an interlaced path topology due to the crossover $2 \times 2$ MMI couplers used to facilitate the placement of the phase shifters at the exterior paths of the network. This yields changing the order of the (de)multiplexed channels compared to those of a traditional topology (de)multiplexer.

The responses of $2 \times 2$ and $4 \times 4$ tunable WDM (de)multiplexers are studied. Utilization of the wavelength shift in the fringes of the transmission response of the single-stage $2 \times 2$ WDM (de)multiplexer for measuring the phase shift introduced by the NEMS-operated phase shifters is associated. Steering of the transmission interferograms of the single-stage $4 \times 4$ (de)multiplexer by applying a progressive phase shift to the NEMS-operated phase shifters is shown. The sensitivity of the tunable $4 \times 4$ (de)multiplexer to a variation in the length of one of the delay lines and the trimming of this effect is demonstrated. The construction of a tunable $4 \times 4$ (de)multiplexer that can provide two different free-spectral-range (FSR) values for the transmission interferograms of the (de)multiplexed optical signals is also demonstrated utilizing a -6dB $4 \times 4$ MMI coupler splitter and -3dB $4 \times 4$ MMI coupler combiner.

As in Part (3.2) of this chapter, all of the depicted wavelength-dependent transmission responses of the studied single-stage tunable PICs are determined using the transfer function method using the simulated wavelength-dependent scattering parameters of the construction components presented in Chapter (2). The losses introduced by the phase delay lines are accounted for in the numerical evaluation of the transmission responses, but the loss imbalance due to the difference in length of the delay lines is found to be minimal. Therefore, this effect is not shown in the analysis presented here for simplicity of the expressions. This part of the chapter mainly aims to explain how the inclusion of path length imbalances in the interconnection network of a single-stage IPPAS
element results in wavelength-selective transmission responses that can be utilized in the wavelength (de)multiplexing and filtering of optical signals. It also aims to explain the principle of utilizing the application of a progressive phase shift through the NEMS-operated phase shifters to steer the (de)multiplexed transmission interferograms for the different MIMO ports of the (de)multiplexer element. The same principle for steering the transmission interferograms learned in this part of the chapter is utilized in fact in the next two chapters.

### 3.3.2 Phase Shift Contributed by a Path Length Imbalance

The periodic response of passive and tunable WDM (de)multiplexers is due to the wavelength-dependent phase shifts contributed by the path length imbalances of the network interconnecting the splitter and combiner units. The interference at the combiner unit creates the cyclic interferograms at the outputs. The path length imbalances are single-mode waveguides of different lengths. The phase shift $\Delta \phi$ introduced by a path length imbalance $\Delta L$ is given by:

$$\Delta \phi = 2 \pi n_e(\lambda) \frac{\Delta L}{\lambda}$$

---

**Figure 3.22** Simulation of the wavelength-dependent effective index of the waveguide imbalance of width 350 nm and silicon base layer thickness of 300 nm.

In Eq. (3.12), $n_e(\lambda)$ is the wavelength-dependent effective index for the fundamental TE-like mode of the waveguide path imbalance. The simulation of the effective index $n_e(\lambda)$ versus wavelength for the waveguide of width 350 nm taken for the SOI template with silicone base layer thickness of 300 nm is depicted in Figure 3.22. This simulation with Eq. (3.12) are used to determine the phase shifts.
contributed by the path length imbalances used in the WDM (de)multiplexers of this chapter and of Chapter (4), and are used in determining the phase shifts contributed by the loopback paths used in the feedback systems of Chapter (5). The wavelength-dependent characteristic of the effective index shown in Figure 3.22 is well fitted by a straight line relation which corresponds to a constant group index over the C-band and hence a constant FSR.

### 3.3.3 Tunable 2×2 FIR-WDM (de)Multiplexer

A single-stage tunable FIR-WDM (de)multiplexer is obtained by adding voltage-controlled phase shift elements in the interconnection circuit of path length imbalances between the splitter and combiner units of a passive AWG photonic circuit of similar topology. The added phase shift elements enable the steering of the transmission interferograms that are formed due to the AWG structure. A tunable WDM (de)multiplexer can be utilized for example as a building block in Wavelength-Selective Switches (WSSs) used in Multi-degree Reconfigurable Optical Add/Drop Multiplexers (M-ROADMs) in a broadcast-and-select architecture [2]. The WSS is also deployed in tunable (de)multiplexers and tunable wavelength filters used in colorless ROADM. Tunable WDM (de)multiplexers also provide the capability to compensate for the characteristic deterioration due to fabrication tolerances and computational simulation deficiencies. In this section, the single-stage 2×2 WDM (de)multiplexer is presented to provide a simple understanding of the principle of operation. In particular, the steering capability of the transmission interferograms for the single-stage 2×2 WDM (de)multiplexer is studied in this section. The 2×2 WDM (de)multiplexer is an optimum construct for the testing of the designed NEMS-operated phase shifters as the amount of wavelength shift in the transmission interferograms is proportional to the phase shift difference introduced by the phase shift array element of the (de)multiplexer.

The schematic diagram of the tunable single-stage 2×2 WDM (de)multiplexer constructed in this work is depicted in Figure 3.23 with the proper dimensions of the components indicated in the figure. The symbolic diagram equivalent to the core of the tunable (de)multiplexer is also shown in the figure. Edge couplers, as shown in Figure 3.23, or grating couplers, can be used to couple the light in and out of the photonic chip. The crossover 2×2 MMI couplers at the output ports facilitate coupling the ground electrodes to the PIC structure. The overall structure is symmetrical, and losses are well-balanced for all paths of the interconnection network between the splitter and combiner units. The tunable WDM (de)multiplexer of Figure 3.23 has similar features to broadband IPPAS
elements of compact overall size and good organization. It uses compact NEMS-operated phase shifters derived by single electrodes located at two sides of the structure core.

Figure 3.23 Schematic realization (up) of the tunable 2×2 WDM (de)multiplexer and the equivalent symbolic diagram (down) of its core structure.

The different path length imbalances \( \Delta L_1 \) and \( \Delta L_2 \) correspond to the wavelength-dependent phase shifts \( \Delta \phi_1 \) and \( \Delta \phi_2 \), respectively. Every path length imbalance has the same number of waveguide bends of similar radii to have well-balanced losses of the interfering signals. The forward scattering matrix \([\tau_{kl}]\) of the tunable 2×2 WDM of Figure 3.23 is given by:

\[
[\tau_{kl}] = T_o e^{i \theta_o} [t_{kl}] [\kappa_{kl}] [t_{kl}]
\]  
... Eq. (3.13)

\([t_{kl}]\) is the forward scattering matrix of the 2×2 MMI coupler as given in Eq. (3.1) and \([\kappa_{kl}]\) is a diagonal matrix introducing the phase shifts \( \Delta \psi_1 \) and \( \Delta \psi_2 \) of the two arms of the MZI structure:

\[
[\kappa_{kl}] = \begin{bmatrix}
0 & 0 \\
0 & e^{i \Delta \psi_2}
\end{bmatrix}
\]

\( \Delta \psi_k = \Delta \phi_k + \Delta \phi_k \) 
... Eq. (3.14)

\( \Delta \phi_k \) is the wavelength-dependent phase shift introduced by the path length imbalance number \( k \), and \( \Delta \phi_k \) is the phase shift difference introduced by the NEMS-operated phase shifter number \( k \). The factor \( T_o \) in Eq. (3.13) models all wavelength-dependent losses caused by the tapers and waveguide bends that are engaged into each signal path. The angle \( \theta_o \) models the accumulated phase shift due to any balanced signal path lengths. The assumption of lossless components and both null imbalances
and null phase errors in the outputs of the 2×2 MMI couplers yields the following simplified expression for the forward scattering matrix of the tunable 2×2 WDM (de)multiplexer:

\[
\begin{bmatrix}
\cos\left(\frac{\Delta \psi}{2}\right) & \sin\left(\frac{\Delta \psi}{2}\right) \\
-\sin\left(\frac{\Delta \psi}{2}\right) & \cos\left(\frac{\Delta \psi}{2}\right)
\end{bmatrix} e^{j\Delta \phi_{av}}
\]

… Eq. (3.15)

All of the wavelength-dependent accumulated phase shift terms are dropped from the expression of Eq. (3.15). \(\Delta \psi = \Delta \phi + \Delta \phi \), \(\Delta \phi = \Delta \phi_1 - \Delta \phi_2 = 2 m \lambda \Delta L / \lambda \) is the wavelength-dependent phase shift difference of the path length imbalance \(\Delta L = L_1 - L_2\), \(\Delta \phi = \Delta \phi_1 - \Delta \phi_2\) is the phase shift difference of the NEMS-operated phase shift element, and \(\Delta \phi_{av} = (\Delta \phi_1 + \Delta \phi_2) / 2\) is the modulated average phase introduced by the NEMS-operated phase shift element.

According to the simplified expression of Equation (3.15) each transmission response belonging to one of the inputs and one of the outputs of the tunable 2×2 WDM (de)multiplexer is sinusoidal. The free spectral range (FSR) in wavelength units denoted as \(\Lambda\) represents the distance between two successive peaks of the transmission response taken for one of the inputs and one of the outputs. According to the simplified expression of Eq. (3.15), if a transmission peak occurs at some wavelength \(\lambda\), then the next peak occurs at the wavelength \(\lambda + \Lambda\) such that the argument \((\Delta \phi/2)\) of the sinusoidal transmission function is decreased by \(\pi\). Therefore, the following expression can be derived to describe the FSR:

\[
\frac{n_e(\lambda + \Lambda)}{\lambda + \Lambda} = \frac{n_e(\lambda)}{\lambda} \frac{1}{\Delta L}
\]

… Eq. (3.16)

\(n_e(\lambda + \Lambda)\) is the effective index of the waveguide imbalance determined at the wavelength \(\lambda + \Lambda\), and \(n_e(\lambda)\) is the effective index of the waveguide imbalance at \(\lambda\). Eq. (3.16) can be evaluated numerically utilizing the characteristic of Figure 3.22. Eq. (3.16) explains the dependency of \(\Lambda\) on \(\Delta L\); that is a larger value of the path length imbalance yields a smaller FSR and vice-versa. Eq. (3.16) also indicates that the FSR is wavelength-dependent. The frequency dependence of the FSR rises from the frequency dependence of the group index of the delay line. However, the change of \(\Lambda\) with \(\lambda\) is a slow function within the investigated optical telecommunication wavelength range of 1.52-1.58 \(\mu\)m as numerical results show. A simplified relation between \(\Lambda\) and \(\Delta L\) can be derived by recognizing that the effective index of the waveguide in the simulation of Figure 3.22 is almost a linear function.
of wavelength within the C-band range. Therefore, substituting $n_e(\lambda + A) = n_e(\lambda) + A(dn_e/d\lambda)$ into Eq. (3.16) yields:

$$\Lambda = \frac{\lambda^2}{n_g \Delta L}$$

... Eq. (3.17)

where $n_g = n_e - \lambda(dn_e/d\lambda)$ is the group index, and the path length difference is such that $\Delta L \gg \lambda/n_g$.

The FSR is the spacing in frequency or wavelength between two successive intensity maxima or minima of an optical transmission response. The bandwidth of the multiplexed channels can be related to the FSR in frequency units. In the thesis, the bandwidth $B_{FSR}$ equivalent to the FSR wavelength span $\Lambda$ referenced to the light propagation speed in vacuum $c$ is given by:

$$B_{FSR} = \frac{c\Lambda}{\lambda^2} = \frac{c}{n_g \Delta L}$$

... Eq. (3.18)

About half the bandwidth indicated by Eq. (3.18) is available for each (de)multiplexed channel for the tunable 2×2 WDM (de)multiplexer. Equation (3.18) can be used to determine $\Delta L$ required to achieve a desired (de)multiplexing channel bandwidth. The wavelength dependency of the group index of the waveguide used in this work is depicted in Figure 3.24 determined based on the simulated characteristic of Figure 3.22. According to Eq. (3.18) the smallest available channel bandwidth can be estimated at the highest value of $n_{g,max} \approx 5.06$, which occurs around the center wavelength of the C-band of $\lambda = 1.55 \mu$m.

![Figure 3.24 Simulation of the wavelength-dependent group index of the waveguide imbalance of width 350 nm and silicon base layer thickness of 300 nm.](image)
According to the simplified expression of Eq. (3.15) the transmission responses for all inputs and outputs are shifted simultaneously over the wavelength range due to the phase shift difference $\Delta \phi/2$. The wavelength shift $\Delta \lambda_s$ of the sinusoidal transmission characteristic due to the applied phase shift difference can be shown to be given by $\Delta \lambda_s/\Lambda = \Delta \phi/(2\pi)$. Therefore, the wavelength steering of the multiplexed channels is a linear function of the applied phase shift difference $\Delta \phi$. Strictly the phase shift $\Delta \phi$ is also wavelength dependent but the dependence is small given the small length of the phase-shift element. At the full-steering range, the passband channels of the transmission interferograms are shifted by $\pm 1/2$ times the FSR. The full steering capability of the transmission interferograms requires a driven change in the phase of each sinusoidal transmission characteristic curve by $\pm 90^\circ$. Therefore, it is adequate to use $180^\circ$ capable phase shifters to achieve a full wavelength steering capability of the transmission interferograms for a tunable $2 \times 2$ WDM (de)multiplexer.

![Simulated wavelength-dependent transmission responses for input $I_1$ of the tunable $2 \times 2$ WDM (de)multiplexer of Figure 3.23 when $\Delta \phi = 0^\circ$ (up) and $\Delta \phi = 180^\circ$ (down). $\Delta L = 198 \mu m$.](image)

**Figure 3.25** Simulated wavelength-dependent transmission responses for input $I_1$ of the tunable $2 \times 2$ WDM (de)multiplexer of Figure 3.23 when $\Delta \phi = 0^\circ$ (up) and $\Delta \phi = 180^\circ$ (down). $\Delta L = 198 \mu m$. Taking the example of setting the bandwidth of each (de)multiplexed channel to 150 GHz, Eq. (3.18) requires using $\Delta L \approx 198 \mu m$. The upper part in Figure 3.25 depicts the simulation of the resulted
transmission responses taken for input $I_1$ when the NEMS-operated phase shifters are at reset conditions ($\Delta \phi = 0^\circ$). The resulted FSR, in this case, is about $\Lambda \approx 2.4$ nm, and the loss introduced by the (de)multiplexer core circuit is about 1.7 dB at close to the center wavelength of the C-band. The lower part in the same figure demonstrates the steering of the transmission responses also taken for input $I_1$ when applying a phase shift difference of $\Delta \phi = 180^\circ$. The transmission responses are shifted in this case to the right side by half the FSR value ($\Delta \lambda_s \approx 1.2$ nm) as seen in the figure such that the locations of the cyclic channels of outputs $O_1$ and $O_2$ for the same input are replacing each other compared with the case before the steering. According to these demonstrated simulation results, the routing of the cyclic channels for both inputs to outputs $O_1$ and $O_2$ can be adjusted to any desired location along the wavelength range using the voltage-controlled NEMS-operated phase shifters of the tunable WDM (de)multiplexer.

### 3.3.4 Tunable 4×4 FIR-WDM (de)Multiplexer

Among the requirements in developing elastic optical networks is the flexibility to relocate the WDM (de)multiplexed channels over the wavelength range. A tunable single-stage $N \times N$ FIR-WDM (de)multiplexer provides the wavelength steering capability for $N$ (de)multiplexed cyclic channels. The schematic and symbolic diagrams of a tunable single-stage 4×4 WDM (de)multiplexer are shown in Figure 3.26. The crossover $2 \times 2$ MMI couplers at the mid of the interconnection network between the splitter and combiner units are used to switch the inner and outer waveguides such that the NEMS-operated phase shifters and the path length imbalances for each interconnection line are located at the outer sides of the phased array. The other coupler crossovers at the output ports are used to facilitate connecting the ground contact to the structure. The losses for all photonic signal paths are well balanced in this design architecture.

The transfer function response of the tunable 4×4 WDM (de)multiplexer of Figure 3.26 is described by the same mathematical model of Equations (3.7) through (3.10) and the approximate equations of Appendix (2) that express the magnitudes of the transfer function scattering parameters assuming ideal components and after replacing each $\Delta \phi_k$ in those expressions with $\Delta \psi_k = \Delta \phi_k + \Delta \phi_k$. To figure out the operation of a WDM (de)multiplexer not only the magnitudes of the transfer function scattering parameters are needed, but also the relative wavelength shift between the transmission responses for the different inputs and outputs are needed to be known. Appendix (3) lists the transfer function scattering parameters for the (de)multiplexer of Figure 3.26 each
represented as a sum of four phasors obtained by the inspection method or by the expansion of Equations (3.7) through (3.10). This appendix does also summarize the symmetry relations between the transfer function scattering parameters when ideal system components are assumed. Due to these symmetry relations of the transfer function elements, up to four separate sets of wavelength-shifted transmission interferograms are available to the inputs and outputs when the four path length imbalances are taken different from each other.

Figure 3.26 Schematic realization (up) of the tunable 4×4 WDM (de)multiplexer and the equivalent symbolic diagram (down) of its core structure.

In theory for the assumption of ideal components the (de)multiplexing of four alternative cyclic channels that are equally spaced in the wavelength range is possible based on the mathematical model of Appendix (3) by setting a progressive path length difference: \(L_1-L_2 = L_2-L_3 = L_3-L_4 = \Delta L\). In this case, each resulted interferogram has the same \(\Lambda\) and \(B_{FSR}\) values derived by the argument of the slowly rotating exponential term in the expressions of Appendix (3) that correspond to the shortest path length difference \(\Delta L\). The resulted \(\Lambda\) and \(B_{FSR}\) values are as described by Equations (3.17) and (3.18), respectively, and the available channel bandwidth is about one-quarter times the FSR equivalent bandwidth \(B_{FSR}\). In reality, the phase errors of the scattering parameters of the used MMI couplers create imbalance, change the bandwidth, and wavelength-shift the (de)multiplexed channels. The imbalance in the (de)multiplexed channels can be corrected by slightly adjusting the
lengths of the path length differences around their values based on the constant pitch size required for a given (de)multiplexing channel bandwidth. Also ideally, based on the same mathematical model of Appendix (3) it is possible to wavelength steer the four cyclic alternative channels by applying a progressive phase shift through the NEMS-operated phase shifters of $\Delta \phi = \Delta \phi_1 - \Delta \phi_2 = \Delta \phi_2 - \Delta \phi_3 = \Delta \phi_3 - \Delta \phi_4$. The progressive phase shift modifies the argument of each exponential term in the expressions of Appendix (3) by a proportional amount causing uniform steering of the transmission responses over the wavelength range for the assumption of ideal system components. The wavelength shift in the steered transmission interferograms is as given before for the tunable 2×2 WDM (de)multiplexer: $\Delta \lambda_s / \Lambda = \Delta \phi / (2 \pi)$.

**Figure 3.27** Simulated wavelength-dependent transmission responses for input $I_1$ of the tunable 4×4 WDM (de)multiplexer of Figure 3.26 when the phase shifters are at reset conditions (up) and when applying a progressive phase shift of $\Delta \phi = 120^\circ$ (down). $L_1-L_4 = 299.95$ μm, $L_2-L_4 = 200$ μm, and $L_3-L_4 = 100.14$ μm.

Taking the example of (de)multiplexing 150 GHz passband channels the step increase in the path length imbalance according to Eq. (3.18) is close to $\Delta L \approx 100$ μm. The resulted FSR, in this case, is around $\Lambda \approx 4.75$ nm. The upper part of Figure 3.27 depicts the simulation of the transmission
responses for input $I_1$ resulted when numerically adjusting the path length differences for reduced channel imbalance to the values: $L_1-L_4 = 299.95 \, \mu m$, $L_2-L_4 = 200 \, \mu m$, and $L_3-L_4 = 100.14 \, \mu m$. The NEMS-operated phase shifters are taken for this part of the figure at reset conditions ($\Delta \phi = 0^\circ$). The signal attenuation is about 3.7 dB close to the center wavelength of the C-band ($\lambda = 1.55 \, \mu m$). The lower part of the same figure depicts the steering of the transmission responses also taken for input $I_1$ towards the right side of the wavelength range resulted when applying a progressive phase shift of $\Delta \phi = 120^\circ$. The steering of the transmission responses achieved in this case is about one-third the FSR value ($\Delta \lambda_s \approx 1.58 \, nm$).

**Figure 3.28** Simulated wavelength-dependent transmission responses for input $I_1$ of the tunable $4 \times 4$ WDM (de)multiplexer of Figure 3.26 when $\Delta \phi_1$ equals $0^\circ$ (up) and $50^\circ$ (down). $L_1-L_4 = 299.85 \, \mu m$, $L_2-L_4 = 200 \, \mu m$, $L_3-L_4 = 100.14 \, \mu m$, and $\Delta \phi_2 = \Delta \phi_3 = \Delta \phi_4 = 0^\circ$.

A full-steering capability along the wavelength range of $\pm 1/2$ times the FSR is reached when applying a progressive phase shift of $\Delta \phi = \pm 180^\circ$. This requires the use of 360$^\circ$ full capable NEMS-operated phase shifters for the case of (de)multiplexing four alternative cyclic channels. The use of 180$^\circ$ capable phase shifters, in this case, can only provide partial steering of the transmission interferograms over the wavelength range of about $\pm 1/6$ times the FSR. Therefore, the use of 180$^\circ$
capable phase shifters covers only one-third the full steering range when used in a tunable 4×4 WDM (de)multiplexer that (de)multiplexes four alternative cyclic channels.

The lengths of the delay lines in a tunable (de)multiplexer require subwavelength accuracy. Using the same design parameters of Figure 3.27 the upper part in Figure 3.28 demonstrates the sensitivity of the 4×4 (de)multiplexer taken for the example of detuning the path difference \( L_1-L_4 \) by a large increment of -100 nm (i.e. \( L_1-L_4 = 299.85 \) µm). In this case, in order to rectify the non-uniformity in the channel levels and trim the effect of spectral corruption, the application of the phase shift difference \( \Delta \phi_1 = 50^\circ \), estimated at the center wavelength of 1.55 µm, is required to compensate the reduction in \( L_1 \) as demonstrated in the lower part of the same figure.

Despite the fact that the available Half-Power-Bandwidth (HPB) of the passbands for a tunable 2×2 WDM (de)multiplexer is about half the FSR and that the HPB of the passbands for a tunable 4×4 WDM (de)multiplexer that (de)multiplexes four alternative cyclic channels is about quarter the FSR, the transmission channels are not flat within their available full passband ranges as they are sinusoidal in nature. The flattening of the cyclic transmission passbands is possible utilizing FIR and IIR cascaded filter structures [27], but the wavelength steering of the passband channels for high-performance filtering characteristic might be restricted. Another important point that should be addressed for the WDM multiplexing characteristic is the uniformity of the channel levels. A figure of merit that can be inferred from [160] is the average gain difference (AGD) of the channels. The AGD for \( N \) number of channels can be defined here for sinusoidal (de)multiplexing as the average of the absolute differences of the channel peaks from their average value:

\[
AGD = \frac{1}{N} \sum_{n=1}^{N} |G_{n,\max} - G_{av}|,
\]

where \( G_{av} = \frac{1}{N} \sum_{n=1}^{N} G_{n,\max} \) ...

Eq. (3.19)

Where \( G \) here corresponds to transmission and \( G_{\max} \) is the peak value of a transmission channel. The AGD has to be as small as possible for better performance.

3.3.5 Tunable Dual-Bandwidth 4×4 FIR-WDM (de)Multiplexer

The demand for having flexible wavelength grids in Elastic Optical Networks (EONs) [1] where signals at different rates requiring different bandwidths can be (de)multiplexed and re-locate in the wavelength range have motivated the need for constructing versatile (de)multiplexing elements. The replacement of either one or both of the splitter and combiner -6dB 4×4 MMI couplers with overlapping -3dB 4×4 MMI couplers modifies the processor of Figure 3.26 to include two cyclic
(de)multiplexing characteristic groups of different FSR values. Each group of different FSR value has a different channel bandwidth. The (de)multiplexed channels of each FSR group can be wavelength steered using two distinct NEMS-operated phase shifters.

The replacement of both of the splitter and combiner units of Figure 3.26 with the overlapping -3dB 4×4 MMI couplers designed in Chapter (2) results in the construction of a dual 2×2 WDM (de)multiplexer. In this case inputs $I_1$ and $I_4$ ($I_2$ and $I_3$) and outputs $O_2$ and $O_3$ ($O_1$ and $O_4$) are (de)multiplexed together controlled separately by the path length difference $\Delta L_{1-4} = L_{1-4}$ ($\Delta L_{2-3} = L_{2-3}$) and phase shift difference $\Delta \phi_{1-4} = \Delta \phi_{1-4}$ ($\Delta \phi_{2-3} = \Delta \phi_{2-3}$). The achievement of different channel bandwidths and separate wavelength steering functions of the interferograms for the separately (de)multiplexed groups of inputs and outputs is handy in this case. This option is studied, but the results of simulations are not shown for brevity.

Figure 3.29 Symbolic diagram of a tunable 4×4 WDM (de)multiplexer with -6dB 4×4 MMI splitter and -3dB 4×4 MMI combiner.

A second option demonstrated here for the sinusoidal (de)multiplexing of the four input and four output optical signals at two different channel bandwidth values providing that one channel bandwidth value is associated with each set of two outputs and any of the four inputs. This design variant, shown in Figure 3.29, is obtained by using a -6dB 4×4 MMI coupler as the splitter unit and a -3dB 4×4 MMI coupler as the combiner unit. The operation of this (de)multiplexer can be illustrated in terms of the scattering parameters of its transfer function derived here for the assumption of ideal system components:

$$
\begin{align*}
\tau_{11} &= \tau_{42} = \tau_{43} = -\tau_{14} = 2 j \nu_1 \nu_2 e^{j \frac{\Delta \psi_{2-3}}{4}} \sin \left( \frac{\Delta \psi_{2-3} + \pi}{2} \right) \\
\tau_{21} &= -\tau_{32} = \tau_{33} = \tau_{24} = -2 j \nu_1 \nu_2 e^{j \frac{\Delta \psi_{1-4}}{4}} \sin \left( \frac{\Delta \psi_{1-4} - \pi}{2} \right)
\end{align*}
$$
\[ \tau_{31} = \tau_{22} = -\tau_{23} = \tau_{34} = 2 j \nu_1 \nu_2 e^{j \frac{\Delta \psi_{14}}{2} + \frac{\pi}{4}} \sin \left( \frac{\Delta \psi_{14}}{2} + \frac{\pi}{4} \right) \]

\[ \tau_{41} = -\tau_{12} = -\tau_{13} = -\tau_{44} = 2 j \nu_1 \nu_2 e^{j \frac{\Delta \psi_{13}}{2} + \frac{\pi}{4}} \sin \left( \frac{\Delta \psi_{13}}{2} + \frac{\pi}{4} \right) \] ...

Eq. (3.20)

\[ \Delta \psi_{k,1} = \Delta \psi_{k} - \Delta \psi_{1}, \Delta \psi_{k+1} = \Delta \psi_{k} + \Delta \psi_{1}, |\nu_1| = 0.5, \text{ and } |\nu_2| = 1/\sqrt{2} \approx 0.707. \] The phases of \( \nu_1 \) and \( \nu_2 \) that mainly model the phases of the -6dB and -3dB 4×4 MMI couplers, respectively, are linear functions of wavelength with negative slopes for the assumption of ideal components. Eq. (3.20) describes that all inputs are sinusoidally (de)multiplexed at outputs \( O_1 \) and \( O_4 \) (\( O_2 \) and \( O_3 \)) at FSR value derived by the path length difference \( \Delta L \) of \( \Delta L_{2,3} \) (\( \Delta L_{1,4} \)) as given by Equations (3.17) and (3.18) with the channel bandwidth equals to half the \( B_{FSR} \) value as in a tunable 2×2 WDM (de)multiplexer. All of the channels (de)multiplexed for the four inputs and outputs \( O_1 \) and \( O_4 \) (\( O_2 \) and \( O_3 \)) can be wavelength steered separately by the phase shift difference \( \Delta \phi_{2,3} \) (\( \Delta \phi_{1,4} \)). The use of 180° phase shifters, in this case, provides full-wavelength steering capability to \( \pm 1/2 \) times the FSR value for each group of similar bandwidth channels.

An illustrative example to enable the (de)multiplexing of the inputs and outputs for two channel bandwidths of 150 GHz and 100 GHz is given here. The path length difference \( \Delta L_{1,4} \) (\( \Delta L_{2,3} \)) is set according to Eq. (3.18) to 198 \( \mu \)m (297 \( \mu \)m) to enable the (de)multiplexing of the four inputs at outputs \( O_2 \) and \( O_3 \) (\( O_1 \) and \( O_4 \)) at the 150 GHz (100 GHz) bandwidth resulting in an FSR value according to Eq. (3.17) of about 2.4 nm (1.6 nm). The upper part of Figure 3.30 depicts the simulation of the transmission characteristic for this case taken for input \( I_1 \) when all phase shifters are at the reset condition. The signal fading is about 6 dB around the wavelength \( \lambda = 1.55 \) \( \mu \)m. The lower part of the same figure demonstrates the steering of the channels at outputs \( O_2 \) and \( O_3 \) (\( O_1 \) and \( O_4 \)) to the right (left) side of the wavelength range by quarter the associated FSR value of about \( \Delta \lambda_s \approx 0.6 \) nm (\( \Delta \lambda_s \approx 0.4 \) nm) by applying \( \Delta \phi_{1,4} = 90^\circ \) (\( \Delta \phi_{2,3} = -90^\circ \)).

In conclusion, the transmission interferograms associated with every two outputs of the studied single-stage 4×4 (de)multiplexer of Figure 3.29 can be set in the design to a different channel bandwidth and can be dynamically steered separately over the wavelength range providing a different type of structural flexibility for building components used for applications in EONs. The addition of cascaded broadband switch(s) at the outputs of this (de)multiplexer can be used to choose between the two different available channel bandwidths.
Figure 3.30 Simulated wavelength-dependent transmission responses for input $I_1$ of the tunable 4×4 WDM (de)multiplexer of Figure 3.29 when applying $\Delta \phi_{1,4} = \Delta \phi_{2,3} = 0^\circ$ (up) and when applying $\Delta \phi_{1,4} = 90^\circ$ and $\Delta \phi_{2,3} = -90^\circ$ (down). $\Delta L_{1,4} = 198$ µm and $\Delta L_{2,3} = 297$ µm.

3.3.6 Conclusions of Part 2

Part 2 of this chapter studied the utilization of single-stage 2×2 and 4×4 tunable WDM elements as building blocks for devices used in EONs to provide the flexibility to wavelength steer the transmission interferograms and to enable the (de)multiplexing of optical signals at different passband values of their cyclic channels. The application of a progressive phase shift through the NEMS-operated phase shifters added to a GMZI with progressive path length imbalances of similar pitch size is shown to enable the wavelength steering of the transmission characteristics. It has been shown that the use of the 180° capable phase shifter suffices to provide full-range wavelength steering capability of the transmission interferograms for a tunable 2×2 WDM (de)multiplexer. Whereas, it has been shown that the use of the 360° capable phase shifter is required to provide full-range wavelength steering capability of the transmission interferograms for a tunable 4×4 WDM (de)multiplexer that (de)multiplexes four alternative cyclic channels. The shift along the wavelength range of the transmission interferograms of a 2×2 WDM (de)multiplexer is planned to be utilized in
this research for the measurement of the characteristic of the designed NEMS-operated phase shifters. The possibility to (de)multiplex the four inputs and each set of two outputs for two different channel bandwidths is demonstrated by changing the combiner unit into a -3dB 4×4 MMI coupler while keeping the splitter unit as a -6dB 4×4 MMI coupler in a tunable single-stage 4×4 (de)multiplexer. Tunable WDM (de)multiplexers are candidate elements in the construction of Wavelength-Selective Switches (WSSs) used in EONs. However, single-stage tunable WDM elements cannot provide the additional flexibility needed in advanced EONs such as changing the bandwidth of the (de)multiplexed channels on demand, (de)multiplexing different bandwidth channels, and enabling the (de)multiplexer unit to turn into a broadband switch when needed. A study into providing these capabilities is provided in the next two chapters requiring the construction of more complicated PICs of cascaded FIR and IIR architectures that can provide flexible operation in advanced EONs.

3.4 Summary

Single-stage photonic processors acting as either broadband switch elements or acting as tunable WDM (de)multiplexers of 2×2 and 4×4 MIMO port sizes are studied in this chapter. The novel aspects of the proposed IPPAS architectures are illustrated using schematic diagrams of the designed switch elements. The blocking switching states of the proposed broadband 4×4 IPPAS element are identified. Transmission-phase constellation diagrams and wavelength-dependent transmission responses are used in studying the characteristics of the switch elements. The steering of the transmission responses of a single-stage 4×4 WDM (de)multiplexer by applying a progressive phase shift to its NEMS-operated phase shifters is demonstrated. The spectral sensitivity to changes in the lengths of the delay lines is also verified for the tunable 4×4 WDM (de)multiplexer. A single-stage WDM element that can achieve the (de)multiplexing at two different FSR values providing the flexibility to independently wavelength steer each group of similar bandwidth channels is proposed. The chapter outlines the limitations in processing the optical spectrum when using single-stage photonic processors, such as the incapability to control the bandwidth of the (de)multiplexed channels for applications in EONs. These limitations in processing the photonic signal are resolved in the next two chapters.
CHAPTER 4
CASCADED FIR PHASED ARRAY ARCHITECTURES

4.1 Introduction
In the previous chapter, the characteristic of a novel class of integrated photonic phased array processors of single-stage architecture was numerically demonstrated. The introduced single-stage processors can only achieve basic functions for responding as either broadband switches or as tunable WDM (de)multiplexers. The blocking single-stage 4×4 broadband switch is found capable of achieving only four switching states out of the complete set of 24 distinct broadband single-throw switching operations.

It is expected that applications in SDN of the evolving EONs would require different levels of flexibility in processing the optical spectrum. Based on the SOI template, the construction of all-glass flexible photonic processors achieving different levels of advanced functionalities is possible throughout the deployment of cascaded architectures. This chapter of the thesis presents a comprehensive investigation into flexible categories of cascaded FIR phased array processors. Specific objectives of the chapter are:

1- To construct non-blocking 4×4 broadband switches that are used in the thesis as building elements for the universal processors.
2- To construct WDM (de)multiplexers that can change the bandwidth of the cyclic channels into several binary values with providing wavelength steering capability of the channels.
3- To construct WDM (de)multiplexers that combine narrowband and wideband cyclic signals by utilizing envelope/wavelength modulation of the transmission responses with providing the capability to wavelength steer the channels.
4- To achieve optimum (de)multiplexing of wideband and narrowband channels of respective Chebyshev-like and inverse Chebyshev-like equi-ripple transmission characteristics with providing wavelength steering capability of the channels.
5- To reach the ultimate goal of constructing universal networking elements for applications in SDN-EONs. These universal optical layer networking processors are programmable modular units providing comprehensive flexibility to act as non-blocking 4×4 broadband switches and
to act as variable-mode tunable WDM (de)multiplexers that are capable of dealing with different bandwidth channels providing wavelength steering capability of the spectrum.

The cascaded photonic phased array architecture is composed of more than two MMI couplers each acting as a splitting/combining unit interconnected in cascaded by networks of waveguides, tapers, phase shifters, and path length imbalances. In general, the cascaded sections can be of similar or different $N \times N$ dimensions, and they can be of similar or different broadband and WDM section types. Depending on the architectures of the interconnection networks, the response of a cascaded phased array element can be broadband in nature or can achieve cyclic wavelength division (de)multiplexing of several photonic signals. The cascading of broadband and WDM phased array sections can provide solutions to several addressed issues and expand the usability of the phased array PICs. Several papers of Cahill [21,22,24,149] demonstrated the cascading of $4 \times 4$ and $2 \times 2$ broadband sections to increase the number of possible single-throw routing operations of a $4 \times 4$ switch architecture and to construct non-blocking $4 \times 4$ broadband switches. Lu et al. [25] experimentally demonstrated a non-blocking $4 \times 4$ broadband switch constructed of cascaded $2 \times 2$ and $4 \times 4$ broadband sections and utilizing thermo-optic phase shifters. Lagali [26] also demonstrated another general scheme for the construction of non-blocking $N \times N$ broadband switches composed of cascaded descending size broadband switches. For example, a $4 \times 4$ non-blocking switch can be constructed by cascading three GMZI switches of $4 \times 4$, $3 \times 3$ and $2 \times 2$ sizes. However, this architecture would yield imbalance in the outputs since the optical signals undergo different path losses. It is also not easily compatible with the novel features of the IPPAS architecture presented in this thesis which is mainly based on pushing the location of the electrodes of the used NEMS-operated phase shifters to the exterior sides of the phased array switch. The study of Madsen and Zhao [27] provides detailed insight into the construction of FIR and IIR optical filters of cascaded architectures from a signal processing analogy prospective with digital filters. However, this thesis is not looking at this stage into the perfection of achieving high filtering characteristics but rather into studying the construction of more flexible processing units that can (de)multiplex optical signals of different bandwidths and provide the capability to wavelength steer the spectrum. In this regard tailoring a cascaded architecture targeting flexible (de)multiplexing and switching can be completely different from engineering the same structure to achieve specific filtering characteristics that might not regard flexible operation of the unit.
In WDM-PONs it is desirable to have the capability to diagnose fault conditions of the telecommunication fibers between the optical routers and the fault condition of the drop fibers at the Remote Nodes (RNs) [151]. Optical Time-Domain Reflectometry (OTDR) is the technique used to detect both losses and breaks in optical fiber systems. In the OTDR technique, a short optical pulse is launched into the fiber, and the analysis of the back-reflection can localize the fault. The wavelength-selective nature of the (de)multiplexers used at the RNs makes it not possible, utilizing the OTDR testing, to directly reach the drop fibers from the site of the remote Central Office (CO). The use of a broadband $1\times N$ coupler bypassing the WDM (de)multiplexer is suggested by Hilbk et al. [161], among the other available solutions, to couple the OTDR pulses to the drop fibers at the RNs. The OTDR pulses, in this case, have to operate at wavelengths different from those of the (de)multiplexed optical signals. Instead, the capability to reconfigure the (de)multiplexers used at the RNs to operate in both of the broadband and WDM modes as needed provides a direct solution to this issue without the need for any additional bypass hardware and the OTDR pulses in result can operate within the same wavelength range of the (de)multiplexed optical signals. The building of optical switch elements that can be tuned into the broadband and WDM responses utilizing the cascading of broadband/WDM sections is possible as demonstrated inclusively in this research. A broadband/WDM convertible cascaded switch element can also provide the option to steer the passband transmission channels over the wavelength range.

The cascaded architecture can also be used to envelope-modulate or wavelength-modulate the transmission responses of the (de)multiplexers and provides the capability to steer the fast ripple channels and the slow modulating functions over the wavelength range. The envelope/wavelength modulation of the transmission interferograms can be utilized for the tunable (de)multiplexing of narrowband and wideband channels. It can also be utilized to form a desired spectral filtering characteristic for the photonic signal. The cascading of WDM sections for the (de)multiplexing of Chebyshev-like wideband channels and inverse Chebyshev-like narrowband channels is also demonstrated. The construction of five different IPPAS architectures of cascaded broadband sections, three of them newly sighted, are demonstrated in this chapter to perform as non-blocking $4\times4$ broadband switches. All of the single-throw, double-throw, and broadcast routing operations and the phase synthesis states available for the proposed five different non-blocking $4\times4$ broadband switches are predicted in this research. This chapter also presents for the first time the construction
of universal 2×2 and 4×4 phased array fabrics. A universal phased array fabric is capable of functioning as a non-blocking broadband router and can be reconfigured to function as a WDM (de)multiplexer with steering capability of the transmission interferograms. Universal phased array processors of high orders can also provide the (de)multiplexing of photonic signals set for different channel bandwidths. The features available utilizing cascaded photonic phased array architectures are explained in this chapter through several numerical design examples.

The chapter is organized to at first in Section (4.2) explain the layout of cascaded integrated photonic phased array processors such that to accommodate the specifications of current standard SOI foundries. Then, achieving the preceding list of photonic signal processing objectives is demonstrated in the next sections. Section (4.3) demonstrates improving the switching blocking characteristic by cascading two broadband 4×4 sections. Then in Section (4.4) the non-blocking broadband 4×4 phased array switch architectures are presented. Section (4.5) presents a new category of cascaded processors that can (de)multiplex photonic signals at a binary number of different channel bandwidths. Section (4.6) demonstrates the envelope/wavelength modulation FIR (de)multiplexers proposed in this research for semi-elastic (de)multiplexing of narrowband and wideband channels. The chapter concludes in Section (4.7) to the presentation of the universal modular processors of 2×2 and 4×4 sizes. Additionally, the case of achieving the (de)multiplexing of narrowband and wideband channels with equi-ripple characteristics in the stopbands and passbands, respectively, providing spectrum steering is relegated to Appendix (4).

4.2 Layouts and Responses of Cascaded Phased Array Elements
The schematic diagrams of cascaded 2×2 and 4×4 WDM-IPPAS elements each comprising two sections are shown in Figure 4.1. Each cascaded structure is composed of three -3dB 2×2 MMI couplers or three -6dB 4×4 MMI couplers interconnected by two networks. In the full configuration, each interconnection network incorporates the NEMS-operated phase shifters and path length imbalances. In the various design examples, however, either the NEMS-operated phase shifters or path length imbalances might be eliminated from any interconnection network. The layout of the cascaded 4×4 WDM element and other long cascaded elements can be arranged into a structure of parallel layers of components connected by waveguides of similar lengths as shown in the lower part of Figure 4.1 for the utilization of a limited length fabrication patterning area.
Figure 4.1 Schematic diagrams of 2×2 (up) and 4×4 (down) FIR integrated photonic phased array elements of two cascaded sections.

Imbalances in the losses of the delay lines of different lengths are accounted for all of the studied cascaded FIR (de)multiplexers, but this effect is found to be minimal. Therefore, the path loss of the delay lines is not shown in the analysis. The response of the cascaded 2×2 WDM-IPPAS element of Figure 4.1 can be described by the forward scattering matrix \([\tau_{kl}]\) given by:

\[
\begin{bmatrix}
\tau_{kl}
\end{bmatrix} = T_e e^{i\phi_a} \begin{bmatrix} f_{kl} \end{bmatrix} \begin{bmatrix} \kappa_{A,kl} \end{bmatrix} \begin{bmatrix} \kappa_{B,kl} \end{bmatrix} \begin{bmatrix} f_{kl} \end{bmatrix}
\]

… Eq. (4.1)

\([f_{kl}]\) is the forward scattering matrix of the 2×2 MMI coupler as given in Eq. (3.1). \([\kappa_{A,kl}]\) and \([\kappa_{B,kl}]\) are diagonal matrices introducing the corresponding wavelength-dependent phase shift differences.
due to the path length imbalances and NEMS-operated phase shifters of sections $A$ and $B$ of the interconnecting networks:

$$[\kappa_{s,kl}]=\begin{bmatrix} e^{i\Delta \psi_{s1}} & 0 \\ 0 & e^{i\Delta \psi_{s2}} \end{bmatrix}$$

... Eq. (4.2)

$\Delta \psi_{kx} = \Delta \phi_{kx} + \Delta \phi_{kx}$, $k$ is either 1 or 2, and $x$ corresponds to either section $A$ or section $B$ of the interconnection networks. The factor $T_o$ models all wavelength-dependent losses caused by the tapers, waveguide bends, and NEMS-operated phase shifters that are engaged into each signal path. The angle $\theta_o$ models the accumulated phase shift due to any balanced signal path lengths.

The response of the cascaded 4×4 WDM (de)multiplexer of Figure 4.1 is described by the forward scattering matrix $[\tau_o]$ given by:

$$[\tau_o]=T_o e^{i\theta_o} [t_{a,kl}] [\kappa_{B1,kl}] [\kappa_{B2,kl}] [\kappa_{A1,kl}] [\kappa_{A2,kl}]$$

... Eq. (4.3)

The factors $T_o$ and $\theta_o$ are as before. $[t_{a,kl}]$ represents the forward scattering matrix of the -6dB 4×4 MMI coupler as given in Eq. (3.2). The diagonal matrices introducing the wavelength-dependent phase shift differences of the path length imbalances and NEMS-operated phase shifters of the different sections of the interconnection networks are given by:

$$[\kappa_{s1,kl}]=\begin{bmatrix} e^{i\Delta \psi_{s1}} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\Delta \psi_{s1}} \end{bmatrix}, \quad [\kappa_{s2,kl}]=\begin{bmatrix} e^{i\Delta \psi_{s2}} & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & e^{i\Delta \psi_{s2}} \end{bmatrix}$$

... Eq. (4.4)

$\Delta \psi_{kx} = \Delta \phi_{kx} + \Delta \phi_{kx}$, $k$ is an integer from 1 to 4 and $x$ signifies either section $A$ or section $B$ of the interconnection networks as before. The matrix of crossover exchanges of the interconnection waveguides $[\chi_{b,kl}]$ is as given as in Equations (3.9) and (3.10).

### 4.3 Broadband FIR 4×4 Switch of Two Cascaded Sections

This part explores the ability of a photonic broadband switch made of two 4×4 cascaded sections to route the inputs to the outputs. A routing state of the inputs to the outputs might be achieved for several distinguishable phase synthesis states of the NEMS-operated phase shifters. The phase synthesis states resulting in similar routings of the inputs to the outputs might yield different relative phase relations between the routed photonic signals. However, the focus here is only on the transmission routings of the inputs to the outputs. An input photonic signal might be routed to a
single output or a group of outputs. The work in this thesis describes single-throw routing operations where each input is routed to one of the outputs, double-throw routing operations where each input is routed in equal portions to two outputs, and all-throw (broadcast) routing operations where each input is routed in equal portions to all of the outputs of the broadband switch.

In Chapter (3) it was found that the single-stage broadband 2×2 IPPAS element, which represents the basic MZI switch configuration, is a non-blocking switch providing all possible routing states of its inputs to the output ports. It was also found that the single-stage broadband 4×4 IPPAS element is a blocking switch due to the symmetry relations of the elements of its transfer function providing only four single-throw and two double-throw routing operations in addition to the broadcast routing operation. Cahill [21] proposed the increase in the number of cascaded 4×4 stages to improve the blocking characteristic of generalized broadband switches. This section demonstrates the extended routing capabilities of the broadband 4×4 IPPAS element of two cascaded sections of Figure 4.2. The expansion of Eq. (4.3) or the use of the inspection method reveals the following general expression for the transfer function elements \( \tau_{ij} \) (\( i = 1 \text{ to } 4, j = 1 \text{ to } 4 \)) of the switch:

\[
\tau_{ij} = \sum_{n=1}^{4} \sum_{m=1}^{4} \rho_{kl, mn} e^{i(\Delta\phi_{mn} + \delta\phi_{kl})} \quad \ldots \text{Eq. (4.5)}
\]

where each \( \rho_{kl, mn} \) is a constant phasor. The full expansion of Eq. (4.5) is determined and used to predict the phase synthesis states required to achieve the available switching operations. The use of the second cascaded section breaks the symmetry in the overall transfer function elements compared to those of a single-stage 4×4 GMZI switch such that the overall available single-throw routing permutations are increased to 16 due to the four phase shifters of section-A and the additional control plane of four phase shifters of section-B (i.e. \( 4 \times 4 = 16 \)).
In general for the cascaded broadband 4×4 photonic phased array switch of Figure 4.2, a switching state of \((\sum S_{1k}, \sum S_{2k}, \sum S_{3k}, \sum S_{4k})\), where \(k = 1 \text{ to } 4\), results from the application of a phase synthesis state of \((\Delta \phi_{1A}, \Delta \phi_{2A}, \Delta \phi_{3A}, \Delta \phi_{4A}) (\Delta \phi_{1B}, \Delta \phi_{2B}, \Delta \phi_{3B}, \Delta \phi_{4B})\). Figure 4.3 depicts the single-throw routing of input \(I_1\) to output \(O_4\) resulted when setting the phase synthesis state of \((90^\circ, 0^\circ, 90^\circ, 0^\circ)\) \((90^\circ, 0^\circ, 90^\circ, 0^\circ)\). In this phase synthesis case, inputs \(I_1\) through \(I_4\) are routed to outputs \(O_4\), \(O_1\), \(O_2\), and \(O_3\), respectively. The crosstalk, in this case, is below -13.5 dB. The signal attenuation is about 6.1 dB estimated at \(\lambda = 1.55\ \mu\text{m}\).

**Figure 4.3** Simulated wavelength-dependent transmission responses for input \(I_1\) of the broadband 4×4 FIR-IPPAS element of two cascaded sections of Figure 4.2 when setting the phase synthesis state of \((90^\circ, 0^\circ, 90^\circ, 0^\circ)\) \((90^\circ, 0^\circ, 90^\circ, 0^\circ)\).

A double-throw routing operation of all inputs of the broadband switch of Figure 4.2 results when applying the phase synthesis state of \((90^\circ, 0^\circ, 90^\circ, 0^\circ)\) \((180^\circ, 0^\circ, 0^\circ, 180^\circ)\). Figure 4.4 depicts the simulation of the transmission responses for this case taken for input \(I_1\). In this phase synthesis state,
each of inputs $I_1$ and $I_4$ is routed in equal portions to outputs $O_1$ and $O_2$, whereas each of inputs $I_2$ and $I_3$ is routed in equal portions to outputs $O_3$ and $O_4$. The crosstalk for this routing case is lower than -24 dB, and the loss estimated at $\lambda = 1.55 \mu m$ is about 7.1 dB.

Figure 4.5 Graphical representations of the single-throw (a-p) and double-throw (q-x) routing operations for the broadband 4×4 FIR-IPPAS element of two cascaded sections of Figure 4.2.

The symmetry relations obtained from the expansion of Eq. (4.5) confirmed with the numerical inspection of the quality of the resulted switching permutations when applying control phases of 0°, 90°, 180°, and 270° has shown that the broadband 4×4 switch element of two cascaded sections of Figure 4.2 exhibits the 16 single-throw routing operations and eight double-throw routing operations that are shown graphically in Figure 4.5 in addition to the broadcast routing operation. The single-section broadband 4×4 switch element has about 83.3% of blocking single-throw routing operations, whereas the broadband 4×4 switch element of two cascaded sections has only about 33.3% of blocking single-throw routing operations. In conclusion, the cascading of broadband sections of a similar kind can be deployed to reduce the number of blocking single-throw and double-throw routing states.
4.4 Non-blocking Broadband FIR 4×4 Cascaded Switches

This section presents the design of five non-blocking broadband 4×4 phased array switches of different architectures utilizing NEMS-operated phase shifters. The combination of 4×4 switch elements with banks of 2×2 switch elements to construct non-blocking 4×4 routers was studied by Cahill [21,22,149] and Lagali [26]. The proposed generalized broadband switches of Cahill and Lagali, however, used the traditional architecture of phased array switches. The experimental characterization of a non-blocking broadband 4×4 router utilizing the cascading of a 4×4 switch element between four banks of 2×2 MZI switches has been recently demonstrated by Lu et al. [25]. The experimental structure of Lu et al., however, uses thermo-optic phase shifters in the construction of the traditional broadband switch architecture. The broadband 4×4 switches presented here have the novel features of the IPPAS architecture for having all of the NEMS-operated phase shifters that use single biasing electrodes located at two sides of the switch structure.

Figure 4.6 Symbolic diagram of the non-blocking broadband FIR 4×4 phased array switch made of three similar cascaded sections.

In general, the cascading of \( k \) broadband \( N \times N \) phased array sections increases the single-throw routing permutations to \( N^k \) providing that the maximum number of non-repeated single-throw transmission routings does not exceed \( N! \). Therefore, a non-blocking 4×4 phased array switch can be constructed by cascading three broadband 4×4 phased array switch sections, as shown in Figure 4.6. The transfer function elements for all inputs and outputs for this case is derived using the inspection method as in Eq. (4.5), and used to determine the phase synthesis states required to achieve the different permutations of single-throw, double-throw, and broadcast routings. The resulting wavelength-dependent transmission responses taken for input \( I_i \) of the non-blocking 4×4 switch fabric of Figure 4.6 are demonstrated in Figure 4.7 in the case of the phase synthesis state of \((90^\circ, 90^\circ, 0^\circ, 0^\circ)\), \((0^\circ, 0^\circ, 90^\circ, 270^\circ)\), \((180^\circ, 0^\circ, 0^\circ, 0^\circ)\) for which case inputs \( I_1 \) through \( I_4 \) are routed to outputs \( O_4, O_2, O_1, \) and \( O_3 \), respectively. The crosstalk level for this routing case is below -13.8 dB and the signal attenuation is about 8.4 dB at around \( \lambda = 1.55 \, \mu\text{m} \).
Figure 4.7 Simulated wavelength-dependent transmission responses for input $I_1$ of the broadband 4×4 FIR-IPPAS fabric of three cascaded sections of Figure 4.6 when setting the phase synthesis state of $(90^\circ, 90^\circ, 0^\circ, 0^\circ) (0^\circ, 0^\circ, 90^\circ, 270^\circ) (180^\circ, 0^\circ, 0^\circ, 0^\circ)$.

The structure of the non-blocking broadband 4×4 switch of Figure 4.6 is built of similar-kind cascading sections. Alternatively, a non-blocking broadband 4×4 switch can also be built by properly cascading sections of different kinds. The non-blocking broadband 4×4 switch of Figure 4.8 is constructed by cascading four 2×2 IPPAS elements with a 4×4 IPPAS element. Two broadband 2×2 IPPAS elements precede the broadband 4×4 IPPAS element, and two other broadband 2×2 IPPAS elements follow the broadband 4×4 IPPAS element. The broadband 4×4 IPPAS element is a blocking switch providing only the four single-throw routing states documented in Figure 3.21. Each of the four broadband 2×2 IPPAS elements at the input and output ports of the broadband 4×4 IPPAS element is used either in the straight-throw or crossover-throw to generate all of the 24 single-throw switching operations for the compound broadband switch fabric.

The phase synthesis state for the non-blocking broadband FIR 4×4 switch of Figure 4.8 is designated as $(\Delta \phi_{1A}, \Delta \phi_{2A}, \Delta \phi_{3A}, \Delta \phi_{4A})(\Delta \phi_{1B}, \Delta \phi_{2B}, \Delta \phi_{3B}, \Delta \phi_{4B})(\Delta \phi_{1C}, \Delta \phi_{2C}, \Delta \phi_{3C}, \Delta \phi_{4C})$. The resulted transmission responses for input $I_1$ of the non-blocking 4×4 switch fabric of Figure 4.8 are depicted in Figure 4.9 taken for the phase synthesis state of $(0^\circ, 0^\circ, 0^\circ, 0^\circ) (180^\circ, 0^\circ, 180^\circ, 0^\circ) (0^\circ, 0^\circ, 180^\circ, 0^\circ)$ where inputs $I_1$ through $I_4$, in this case, are routed to outputs $O_2$, $O_4$, $O_2$, and $O_1$, respectively. The crosstalk level
for this routing case is below -31 dB and the signal attenuation is about 6.4 dB at around $\lambda = 1.55$ µm. This non-blocking 4×4 switch performs better routing operations for negligible crosstalk and lower signal attenuation level compared with the non-blocking 4×4 switch of three cascaded sections of similar type of Figure 4.6.

Figure 4.9 Simulated wavelength-dependent transmission responses for input $I_1$ of the cascaded broadband 4×4 FIR-IPPAS fabric of Figure 4.8 when setting the phase synthesis state of (0º, 0º, 0º, 0º) (180º, 0º, 180º, 0º) (0º, 0º, 180º, 0º) (0º, 180º, 0º).

Figure 4.10 Symbolic diagrams of non-blocking broadband FIR 4×4 phased array switches of cascaded 2×2 and 4×4 switch elements.

Another alternative solution to construct a non-blocking broadband 4×4 switch is to precede or follow the broadband 4×4 switch element of two cascaded sections of Figure 4.2 with two broadband 2×2 switch elements as shown in Figure 4.10. In these cases, the additional two broadband 2×2 switch elements would expand the 16 single-throw switching states of the two-section cascaded broadband 4×4 switch element into the full set of 24 single-throw routing operations. Figure 4.11 depicts the transmission responses for input $I_1$ for the broadband switch in the lower part of Figure
4.10 when applying the phase synthesis state of \((90^\circ, 0^\circ, 90^\circ, 0^\circ)\) \((0^\circ, 90^\circ, 0^\circ, 270^\circ)\) \((0^\circ, 0^\circ, 180^\circ, 0^\circ)\) for which case inputs \(I_1\) through \(I_4\) are routed to outputs \(O_2\), \(O_1\), \(O_4\), and \(O_3\), respectively. The crosstalk level in this routing case is lower than -15 dB, and the signal attenuation is about 8.5 dB around the wavelength \(\lambda = 1.55\, \mu\text{m}\). This performance level is comparable to that of the 4×4 switch of three cascaded sections of similar type of Figure 4.6.

\[\text{Figure 4.11} \quad \text{Simulated wavelength-dependent transmission responses for input } I_1 \text{ of the cascaded broadband 4×4 FIR-IPPAS fabric of the lower part in Figure 4.10 when setting the phase synthesis state of } (90^\circ, 0^\circ, 90^\circ, 0^\circ) \quad (0^\circ, 90^\circ, 0^\circ, 270^\circ) \quad (0^\circ, 0^\circ, 180^\circ, 0^\circ).\]

\[\text{Figure 4.12} \quad \text{Symbolic diagram of a compound non-blocking broadband FIR 4×4 phased array switch made of two single-stage 4×4 switch elements interconnected by a bank of two 2×2 switch elements.}\]

\[\text{Figure 4.13} \quad \text{Simulated wavelength-dependent transmission responses for input } I_1 \text{ of the cascaded broadband 4×4 FIR-IPPAS fabric of Figure 4.12 when setting the phase synthesis state of } (180^\circ, 180^\circ, 0^\circ, 0^\circ) \quad (0^\circ, 0^\circ, 0^\circ, 0^\circ) \quad (180^\circ, 180^\circ, 0^\circ, 0^\circ).\]
Figure 4.14 Graphical representations of the comprehensive 24 single-throw (a-x) routing operations and 18 double-throw (aa-rr) routing operations that are available for the non-blocking broadband 4×4 switch fabrics of Figures 4.6, 4.8, 4.10 and 4.12.
When two or more single-stage 4×4 full-structured broadband switches of Figure 3.18 are straight connected in cascade, the overall fabric exhibits just the same four known symmetrical single-throw routings of Figure 3.21. However, the interconnection of two single-stage 4×4 IPPAS elements through a bank of two 2×2 IPPAS elements as in Figure 4.12 builds another non-blocking broadband switch. Figure 4.13 depicts the transmission responses for input $I_1$ of the compound non-blocking switch fabric of Figure 4.12 when applying the phase synthesis state of $(180^\circ, 180^\circ, 0^\circ, 0^\circ) (0^\circ, 0^\circ, 0^\circ, 0^\circ) (180^\circ, 180^\circ, 0^\circ, 0^\circ)$ for which case inputs $I_1$ through $I_4$ are single-throw routed to outputs $O_1$ through $O_4$, respectively. The crosstalk for the simulation of Figure 4.13 is below -24 dB, and the signal fading is about -7 dB at around $\lambda = 1.55$ µm. Likewise the non-blocking switch fabric of Figure 4.8 the switch fabric of Figure 4.12 is better than the switch fabric of three similar cascaded sections of Figure 4.6 and better than the non-blocking switches of Figure 4.10 in terms of exhibiting lower crosstalk and signal fading levels.

In summary, the non-blocking broadband 4×4 phased array switches of Figures 4.8 and 4.12 exhibit negligible crosstalk and lower signal attenuation for their single-throw routing operations, and in result, they are considered better in performance compared to those of Figures 4.6 and 4.10. In general, the increase in the number of concatenated cascaded broadband $N\times N$ IPPAS sections to increase the available switching permutations yields more deterioration in performance due to the accumulation in phase errors of the cascaded MMI couplers.

Figure 4.14 depicts the graphical representations of the comprehensive set of 24 single-throw routing operations and 18 double-throw routing operations that are available for the non-blocking broadband 4×4 switch fabrics of Figures 4.6, 4.8, 4.10, and 4.12. The comprehensive lists of phase synthesis states required for performing the single-throw, double-throw, and broadcast routing operations for the non-blocking broadband 4×4 switches of Figures 4.6, 4.8, 4.10, and 4.12 based on applying the discrete phases of $0^\circ, 90^\circ, 180^\circ,$ and $270^\circ$ are tabulated in this work but they are not documented in the thesis for brevity.

**4.5 Binary Bandwidth Variable FIR 2×2 (de)Multiplexers**

Binary bandwidth variable (de)multiplexers, also called binary switchable-FSR (de)multiplexers, are processors capable of changing the FSR of two (de)multiplexed cyclic photonic signals into a binary number of values controlled by the phase shift elements of the phased array circuit. The binary number of available different channel bandwidths depends on the number of cascaded sections of the
processor and the setting of the used path length differences. These processors are expandable in theory to any desired larger size at the expense of increased signal loss, and they provide the capability to wavelength steer the (de)multiplexed channels. They can be used as construction elements in larger fabrics for the flexible (de)multiplexing of groups of variable bandwidth channels, as explained in part (c) of Figure 1.2 in Chapter (1). They can also be set to behave as universal binary controlled processors, as explained in this section.

An $N^{th}$ order $2 \times 2$ binary bandwidth variable (de)multiplexer is constructed of $N$ number of tunable $2 \times 2$ WDM (de)multiplexers of different path length imbalances interconnected by networks that contain only phase shifters. The networks of interconnecting phase shifters control the binary switching between the available different FSR values, whereas the phase shifters of the tunable WDM (de)multiplexers control the wavelength steering of the (de)multiplexed channels. The maximum number of different bandwidths follows the binary rule of $2^{N-1}$. Figure 4.15 shows the circuit of a $2^{nd}$ order FIR $2 \times 2$ binary bandwidth variable (de)multiplexer made of two tunable $2 \times 2$ WDM (de)multiplexers interconnected by a bank of two phase shifters.

![Figure 4.15 Symbolic diagram of the 2nd order 2×2 FIR binary bandwidth variable (de)multiplexer.](image)

The useful operation of the (de)multiplexer of Figure 4.15 results when the binary control phase shift difference $\Delta \phi_B = \Delta \phi_{B1} - \Delta \phi_{B2}$ is set to either 0º or 180º. In the following analysis, the operation of this (de)multiplexer is clarified by assuming all system components to be ideal. When $\Delta \phi_B = 0^\circ$ the magnitudes of the elements of the transfer function of the (de)multiplexer are given by:

$$|r_{11}| = |r_{22}| = \cos\left(\frac{\Delta \psi_A - \Delta \psi_C}{2}\right), \quad |r_{21}| = |r_{12}| = \sin\left(\frac{\Delta \psi_A - \Delta \psi_C}{2}\right)$$

... Eq. (4.6)

where $\Delta \psi_x = \Delta \phi_x + \Delta \phi_t$, $\Delta \phi_t = 2\pi n_e AL_x/\lambda$, $\Delta L_x = L_{1x} - L_{2x}$, $\Delta \phi_t = \Delta \phi_{1x} - \Delta \phi_{2x}$, $n_e$ is the effective index of the waveguide and $x$ is either $A$ or $C$. In this case, the resulted FSR is driven by the magnitude of the difference between the path length imbalances of sections $A$ and $C$ of the (de)multiplexer. The FSR length and its equivalent bandwidth are described as in Equations (3.17) and (3.18), respectively, after substituting $\Delta L = |\Delta L_A - \Delta L_C|$ and the resulted channel bandwidth is half the $B_{FSR}$ value. Eq. (4.6)
also indicates that the (de)multiplexed transmission interferograms for this binary phase control state of $\Delta \phi_B = 0^\circ$ can be wavelength steered by the subtraction in phase differences of sections $A$ and $C$; i.e. by $\Delta \phi_A - \Delta \phi_C$.

In the second useful binary phase control state of $\Delta \phi_B = 180^\circ$, the elements of the transfer function for the assumption of ideal system components are given by:

$$|r_{11}| = |r_{22}| = \cos \left( \frac{\Delta \psi_A + \Delta \psi_C}{2} \right), \quad |r_{12}| = |r_{21}| = \sin \left( \frac{\Delta \psi_A + \Delta \psi_C}{2} \right)$$

... Eq. (4.7)

In this case, the FSR is derived by the sum of path length differences of sections $A$ and $C$. $A$ and $B_{FSR}$ are determined as in Equations (3.17) and (3.18), respectively, after substituting $\Delta L = \Delta L_A + \Delta L_C$ and the resulted channel bandwidth is also half the $B_{FSR}$ value. The wavelength steering of the channels, in this case, is controlled by the sum of phase shift differences of sections $A$ and $C$; i.e. by $\Delta \phi_A + \Delta \phi_C$.

Letting the two desired FSR equivalent bandwidths to be $B_{FSR1}$ and $B_{FSR2}$ such that $B_{FSR1} > B_{FSR2}$ and taking $\Delta L_A > \Delta L_C$ the path length differences of sections $A$ and $C$ are given by:

$$\Delta L_A = \frac{c}{2n g} \left( \frac{1}{B_{FSR1}} + \frac{1}{B_{FSR2}} \right), \quad \Delta L_C = \frac{c}{2n g} \left( \frac{1}{B_{FSR2}} - \frac{1}{B_{FSR1}} \right)$$

... Eq. (4.8)

Taking the example of setting the switchable channel bandwidths to 150 GHz and 100 GHz Eq. (4.8) requires $\Delta L_A = 247.1$ µm and $\Delta L_C = 49.42$ µm. In this case, the resulted long and short FSR values are about 2.4 nm and 1.6 nm, respectively. The upper (lower-middle) part of Figure 4.16 depicts the simulation of the transmission responses for input $I_1$ for this study example when the bandwidth is set to 150 GHz (100 GHz) by applying $\Delta \phi_B = 0^\circ$ ($\Delta \phi_B = 180^\circ$). The upper-middle (lower) part of the same figure depicts the steering of the same (de)multiplexed channels to the right side of the wavelength range by half the FSR value for this binary control state of about $\Delta \lambda_s \approx 1.2$ nm (0.6 nm) when applying $\Delta \phi_A - \Delta \phi_C = 180^\circ$ ($\Delta \phi_A + \Delta \phi_C = 180^\circ$).

The setting of $B_{FSR1} = \infty$ for the 2$^{nd}$ order (de)multiplexer of Figure 4.15 yields according to Eq. (4.8) equal path length imbalances for sections $A$ and $C$ of: $\Delta L_A = \Delta L_C = c/(2 n g B_{FSR2})$. In this case, the binary phase control state of $\Delta \phi_B = 0^\circ$ yields the following elements of the transfer function according to Eq. (4.6) for the assumption of ideal system components:
Figure 4.16 Simulated wavelength-dependent transmission responses for input \( I_1 \) of the binary bandwidth variable (demultiplexer of Figure 4.15 for the phase difference combinations: \( \Delta \phi_A = \Delta \phi_B = \Delta \phi_C = 0^\circ \) (up), \( \Delta \phi_A = -\Delta \phi_C = 90^\circ \) & \( \Delta \phi_B = 0^\circ \) (up-middle), \( \Delta \phi_A = \Delta \phi_C = 0^\circ \) & \( \Delta \phi_B = 180^\circ \) (down-middle), and \( \Delta \phi_A = \Delta \phi_C = 90^\circ \) & \( \Delta \phi_B = 180^\circ \) (down). \( \Delta L_A = 247.1 \) \( \mu \)m and \( \Delta L_C = 49.42 \) \( \mu \)m.
\[
|r_{11}| = |r_{22}| = \cos\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right), \quad |r_{21}| = |r_{12}| = \sin\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right)
\]

... Eq. (4.9)

Eq. (4.9) describes the behavior of a broadband MZI modulator providing intensity control of the transmission responses controlled by the phase difference \(\Delta \phi_A - \Delta \phi_C\). When setting \(\Delta \phi_B = 180^\circ\) tunable WDM (de)multiplexing is resulted according to Eq. (4.7) providing wavelength steering capability controlled by \(\Delta \phi_A + \Delta \phi_C\). This special case of taking \(\Delta L_A = \Delta L_C\) of the (de)multiplexer of Figure 4.15 in fact represents one of the possible architectures for constructing a universal 2×2 processor that is capable of behaving as both a broadband MZI modulator and tunable WDM (de)multiplexer. This study case is considered, but simulation results are not shown here for brevity. However, a different architecture of a universal 2×2 processor of similar behavior is studied and demonstrated in Section (4.7.1).

After explaining the operation of the 2\(^{nd}\) order binary bandwidth variable 2×2 (de)multiplexer of Figure 4.15 a generalization of the theory to any \(N^{th}\) order binary (de)multiplexer is derived here and the general condition to construct a universal binary processor is concluded. Figure 4.17 presents the block diagram of an \(N^{th}\) order cascaded FIR binary bandwidth variable 2×2 (de)multiplexer. In this figure, each tunable 2×2 WDM (de)multiplexer of the \(k^{th}\) stage has a wavelength-dependent phase: \(\Delta \psi_k = \Delta \phi_k + \Delta \phi_k\); where \(\Delta \phi_k\) is due to the path length difference \(\Delta L_k\). Each \(k^{th}\) and \(k+1^{th}\) WDM stages are connected by two phase shifters described by the binary control phase difference \(\Delta \theta_k\) and binary switching operator \(\kappa_k = T \exp(j \Delta \theta_k)\); providing that \(\kappa_k\) has only the two binary states of either \(\kappa_k = -1\) when \(\Delta \theta_k = 0^\circ\) or \(\kappa_k = 1\) when \(\Delta \theta_k = 180^\circ\).

**Figure 4.17** Block diagram of an \(N^{th}\) order 2×2 FIR binary bandwidth variable (de)multiplexer.

The derivation of the system transfer function with the assumption of ideal system components and taking only the binary control states yields the following magnitudes of the transfer function elements that represent the generalization of Equations (4.6) and (4.7):
This system supports setting the permutations of the channel bandwidths to \( N_{ch} = 2^{N-1} \) of all different or a mix up of different and similar values depending on the selection of path length differences of the tunable WDM stages. Any binary control state that sets \( \Delta \phi = 0 \) requiring \( \Delta L = 0 \) turns the device into a broadband MZI modulator with the output intensities controlled by \( \Delta \phi \) as in Eq. (4.10).

Therefore, a universal 2×2 binary processor can be constructed by setting \( \Delta L = 0 \) for one of the binary control permutations for which case the processor functions as a broadband switch with intensity control. Whereas, the other binary control permutations can be tailored to support tunable WDM (de)multiplexing for \( N_{ch} = 2^{N-1} - 1 \) of different channel bandwidths providing wavelength steering of the transmission responses controlled by \( \Delta \phi \). A 3rd order universal binary processor is inspected, but results of the simulation are not shown for brevity.

**Figure 4.18** Re-outlining of the \( N^{th} \) order 2×2 FIR binary bandwidth variable (de)multiplexer of Figure 4.17. In order to clarify the difference in operation between the binary bandwidth variable 2×2 (de)multiplexer and the operation of the universal 2×2 processors presented in Section 4.7.1 one should notice that in Figure 4.17 each two \( k^{th} \) binary control phase shifters of \( \Delta \theta_k \) (or \( \kappa_k \)) form with the two 2×2 MMI couplers attached to them a broadband MZI switch. Therefore, under the condition of binary operation of the interconnecting switches, it is possible to outline the same general \( N^{th} \) order architecture of the binary (de)multiplexer of Figure 4.17 as shown differently in Figure 4.18. The architecture of this figure clarifies why the broadband switches, in this case, can only operate either in the forward switching mode (i.e. when \( \Delta \theta_k = 180^\circ \) and \( \kappa_k = 1 \)) or in the crossover switching mode (i.e. when \( \Delta \theta_k = 0^\circ \) and \( \kappa_k = -1 \)). The role of the binary operation of the broadband switches in this architecture is to control the permutations of interconnecting the paths that each contains a path length imbalance and wavelength steering phase shifter between the first and last 2×2 MMI couplers of the (de)multiplexer. The overall architecture forms one tunable 2×2 WDM (de)multiplexer of switchable controlled paths of length imbalances and steering phase shifters. In the special case
when one of the binary permutations yields a zero path length imbalance between the two compound arms that are interconnected between the first and last 2×2 MMI couplers a universal 2×2 binary processor results.

4.6 Envelope/Wavelength Modulation FIR (de)Multiplexers

As mentioned in Chapter (1) envelope and wavelength modulation of the transmission responses is proposed in this research to enable the (de)multiplexing of different bandwidth channels. In envelope modulation, a fast ripple transmission response is envelope modulated by a slow ripple transmission component yielding one major transmission lobe in each long FSR cycle of the transmission response. These major cyclic lobes accommodate the (de)multiplexing of wider bandwidth channels with the narrower bandwidth channels of the fast ripple transmission component. Envelope modulation (de)multiplexing has the disadvantage of having the notch bandstop filtering not deep enough at the boundaries between the wideband transmission lobes and the surrounding narrowband channels due to the sinusoidal envelope modulation effect. In wavelength modulation, on the other hand, the bandwidth of the (de)multiplexed channels is periodically modulated resulting in the (de)multiplexing of several wider band channels and narrower band channels per cycle of the long FSR transmission modulating component. The notch bandstop filtering is maintained proficiently deep between the wavelength modulated channels. Similar PIC architectures can be utilized to achieve both kinds of the envelope and wavelength modulation, but tailoring of the photonic circuit parameters is different for each kind of transmission modulation.

![Symbolic diagram of the 2×2 envelope and wavelength modulation (de)multiplexer of three cascaded sections.](image)

In this section, the envelope and wavelength modulation (de)multiplexing of different bandwidth channels are illustrated utilizing the tunable 2×2 FIR (de)multiplexer of three cascaded sections of Figure 4.19. This (de)multiplexer is composed of four -3dB 2×2 MMI couplers interconnected in cascade by three networks each has both NEMS-operated phase shifters and path length imbalances. To understand the operation of this (de)multiplexer one may notice that it is constructed by adding path length imbalances to section-B of the binary (de)multiplexer of Figure 4.15. Therefore, the
middle stage acts as a wavelength-selective switch with FSR value that depends on its path length difference. The middle stage sinusoidally controls the switching between the (de)multiplexing for the sum and absolute of subtraction of the path length differences of sections $A$ and $C$ as in a binary (de)multiplexer. When the path length differences of sections $A$ and $C$ are equal the zero difference between them yields one wide lobe in each wavelength cycle of the middle stage switching function, and envelope modulation of the transmission responses results. When the path length differences of sections $A$ and $C$ are not equal wavelength modulation of the transmission responses due to switching between the fast and slow ripple components is resulted.

In the following the operation of the (de)multiplexer of Figure 4.19 is illustrated analytically for the assumption of ideal system components. The design rules and quantitative parameters of the (de)multiplexer are concluded from the simplified analysis. Study cases are taken, and numerical simulation results are used to qualify the tunable (de)multiplexers. The derivation of the magnitudes of the transfer function elements yields:

\[
|t_{11}|^2 = |t_{22}|^2 = \cos^2\left(\frac{\Delta \psi_A - \Delta \psi_C}{2}\right) \cos^2\left(\frac{\Delta \psi_B}{2}\right) + \cos^2\left(\frac{\Delta \psi_A + \Delta \psi_C}{2}\right) \sin^2\left(\frac{\Delta \psi_B}{2}\right)
\]

\[
|t_{21}|^2 = |t_{12}|^2 = \sin^2\left(\frac{\Delta \psi_A - \Delta \psi_C}{2}\right) \cos^2\left(\frac{\Delta \psi_B}{2}\right) + \sin^2\left(\frac{\Delta \psi_A + \Delta \psi_C}{2}\right) \sin^2\left(\frac{\Delta \psi_B}{2}\right)
\]

… Eq. (4.11)

where $\Delta \psi_x = \Delta \phi_x + \Delta \phi_x$, $\Delta \phi_x = 2 \pi n e \Delta L_x / \lambda$, $\Delta L_x = L_{1x} - L_{2x}$, $\Delta \phi_x = \Delta \phi_{1x} - \Delta \phi_{2x}$, $n_e$ is the effective index of the waveguide, and $x$ is either $A$, $B$ or $C$. The system of Eq. (4.11) might be utilized in different ways. However, in the following explanation, it is assumed that $\Delta L_B << \Delta L_A + \Delta L_C$. Every sinusoidal term with argument $\Delta \psi_B$ represents a wavelength selective switching function with an FSR value of $\Lambda_B$ determined according to Eq. (3.17) based on the path length imbalance $\Delta L_B$. The sinusoidal terms with the argument difference $\Delta \psi_A - \Delta \psi_C$ represent slow ripple transmission components with an FSR value of $\Lambda_{A-C}$ determined according to Eq. (3.17) based on the absolute of the path length difference $|\Delta L_A - \Delta L_C|$. The sinusoidal terms with the argument sum $\Delta \psi_A + \Delta \psi_C$ represent fast ripple transmission components with an FSR value of $\Lambda_{A+C}$ determined according to Eq. (3.17) based on the path length sum $\Delta L_A + \Delta L_C$. Every transmission response sinusoidally switches between the slow and fast ripple components over the wavelength range. The wavelength selective switching terms of section $B$ are steered by $\Delta \phi_B$. The slow ripple transmission components are steered by $\Delta \phi_{A-C}$ and the fast ripple transmission components are steered by $\Delta \phi_A + \Delta \phi_C$. Additional to specifying the FSR values of the
narrowband and wideband channels it is possible to estimate the number of narrowband and wideband channels that are (de)multiplexed within one FSR cycle of the wavelength selective switching function from the bandwidth ratios.

As mentioned earlier, envelope modulation results when setting $\Delta L_A = \Delta L_C$. In this case each slow ripple term in Eq. (4.11) produces one transmission lobe (i.e. one wideband channel that is either on or off) per cycle of $\Lambda_B$. Eq. (4.11) in this case can be re-expressed as follows to reflect the envelope modulation of the transmission responses:

$$|r_{11}|^2 = |r_{22}|^2 = \cos^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right) + \sin^2\left(\frac{2 \Delta \phi_A + \Delta \phi_A + \Delta \phi_C}{2}\right) - \cos^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right) \sin^2\left(\frac{\Delta \psi_B}{2}\right)$$

$$|r_{21}|^2 = |r_{12}|^2 = \sin^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right) + \cos^2\left(\frac{2 \Delta \phi_A + \Delta \phi_A + \Delta \phi_C}{2}\right) - \sin^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right) \sin^2\left(\frac{\Delta \psi_B}{2}\right)$$

To examine the selection of system parameters and demonstrate the flexible control features available for the envelope modulation (de)multiplexing processor of Figure 4.19 an example is taken here for the cyclic (de)multiplexing of an elephant demand of 500 GHz and standard demands each of 50 GHz. This sets the equivalent bandwidth of section $B$ to $B_{FSR,B} = 1000$ GHz and the FSR equivalent bandwidth for the combination of sections $A$ and $C$ to $B_{FSR,A+C} = 100$ GHz. Using Eq. (3.18) the required path length imbalances are: $\Delta L_B = c/(n_g B_{FSR,B}) \approx 59.31$ $\mu$m and $\Delta L_A = \Delta L_C = c/(2n_g B_{FSR,A+B}) \approx 296.53$ $\mu$m. This selection according to Eq. (3.17) sets the modulation switching FSR length to $\Lambda_B \approx 8$ nm and the fast ripple FSR length to $\Lambda_{A+C} \approx 0.8$ nm. The upper part of Figure 4.20 depicts the simulation of the transmission responses for this study case for input $I_1$ when the NEMS-operated phase shifters are at reset conditions. The signal attenuation is about 4.6 dB at around $\lambda = 1.55$ $\mu$m. Increasing (decreasing) the phase difference $\Delta \phi_B$ steers the slow envelope functions that modulate the transmission responses to the right (left) side of the wavelength range as indicated in Eq. (4.12). The upper-middle part of Figure 4.20 depicts the steering of the slow envelope modulating functions taken for input $I_1$ to the right side of the wavelength range by half the $\Lambda_B$ value (i.e. by about 4 nm) due to setting $\Delta \phi_B = 180^\circ$. Maintaining the phase difference $\Delta \phi_A - \Delta \phi_C$ constant and equally increasing (decreasing) both of $\Delta \phi_A$ and $\Delta \phi_C$ steers the fast ripples of the transmission interferograms for each input to the right (left) side of the wavelength range derived by the sum $\Delta \phi_A + \Delta \phi_C$ as indicated in Eq. (4.12). The slow envelope modulation functions of the transmission interferograms stay not shifted when $\Delta \phi_B$ is maintained constant.
Figure 4.20 Simulated wavelength-dependent transmission responses for input \( I_1 \) of the envelope modulation 2×2 (de)multiplexer of Figure 4.19 when: \( \Delta \phi_A = \Delta \phi_B = \Delta \phi_C = 0^\circ \) (up), \( \Delta \phi_A = \Delta \phi_C = 0^\circ \) \& \( \Delta \phi_B = 180^\circ \) (up-middle), \( \Delta \phi_A = \Delta \phi_C = 90^\circ \) \& \( \Delta \phi_B = 180^\circ \) (down-middle), and \( \Delta \phi_A = -\Delta \phi_C = 90^\circ \) \& \( \Delta \phi_B = 180^\circ \) (down). \( \Delta L_A = \Delta L_C = 296.53 \) µm and \( \Delta L_B = 59.31 \) µm.
The lower-middle part of Figure 4.20 depicts the shifting of the fast ripple channels of the upper-middle part of the same figure by half the $A_{t+c}$ value (i.e. by about 0.4 nm) when applying $\Delta \phi_{A} = \Delta \phi_{C} = 90^\circ$ for the same case of taking $\Delta \phi_{B} = 180^\circ$. The lower part of Figure 4.20 depicts the simulated transmission interferograms for input $I_{t}$ when setting $\Delta \phi_{A} = 90^\circ$ and $\Delta \phi_{C} = -90^\circ$ keeping $\Delta \phi_{B} = 180^\circ$. This part of figure 4.20 can be compared with the upper-middle part of the same figure to notice that for input $I_{t}$ the wideband channels of outputs $O_{1}$ and $O_{2}$ are exchanged with each other. The elephant demand at input $I_{t}$ is directed to output $O_{1}$ in the upper-middle part, whereas it is directed to output $O_{2}$ in the lower part of the figure. The location of the fast ripple channels in the lower part of Figure 4.20 is the same as those in the upper-middle part of the same figure since the sum $\Delta \phi_{A} + \Delta \phi_{C} = 0^\circ$ is not affected. The number of narrowband channels in each cycle frame is around $\Delta L_{A}/\Delta L_{B} \approx 5$. The utilization of steering both of the envelope modulating functions and the channels of fast ripples of the transmission interferograms makes it possible to steer the overall transmission patterns along the wavelength range maintaining the exact locations of the fast ripples relative to the envelope modulating functions. In conclusion, this study example has demonstrated the utilization of envelope modulation of the transmission responses to (de)multiplex narrowband and wideband channels providing the freedom to steer the envelope modulating functions and narrowband channels in separate. Switching the wideband channels between the two outputs is also facilitated. Based on Eq. (4.12) it is also possible to use $\Delta L_{B}$ and $\Delta L_{A} = \Delta L_{C}$ to present the respective fast and slow ripple components of the transmission responses. This option is investigated, but simulation results are not shown for brevity.

The utilization of the FIR-WDM processor of three cascaded sections of Figure 4.19 for wavelength modulation (de)multiplexing achieved by taking $\Delta L_{A} \neq \Delta L_{C}$ as described by Eq. (4.11) is demonstrated here by taking the example of setting the bandwidths of the narrowband and wideband channels to 50 GHz and 150 GHz, respectively. This sets the FSR equivalent bandwidth of the small demands to $B_{FSR,A+C} = 100$ GHz and that of the large demands to $B_{FSR,A-C} = 300$ GHz. The same format of Eq. (4.8) can then be used to determine the required path length imbalances of sections $A$ and $C$ of $\Delta L_{A} = 395.37$ µm and $\Delta L_{C} = 179.68$ µm. The path length imbalance of section $B$ determines the number of (de)multiplexed narrowband and wideband channels per cyclic frame. There are around $|\Delta L_{A} + \Delta L_{C}|/(2\Delta L_{B})$ of small bandwidth demands and around $(\Delta L_{A} - \Delta L_{C})/(2\Delta L_{B})$ of large bandwidth demands per cyclic frame.
Figure 4.21 Simulated wavelength-dependent transmission responses for input $I_1$ of the wavelength modulation 2×2 (de)multiplexer of Figure 4.19 when: $\Delta \phi_A = \Delta \phi_B = \Delta \phi_C = 0^\circ$ (up), $\Delta \phi_A = \Delta \phi_C = 0^\circ$ & $\Delta \phi_B = 180^\circ$ (up-middle), $\Delta \phi_A = \Delta \phi_C = 90^\circ$ & $\Delta \phi_B = 180^\circ$ (down-middle), and $\Delta \phi_A = -\Delta \phi_C = 90^\circ$ & $\Delta \phi_B = 180^\circ$ (down). $\Delta L_A = 395.37 \mu m$, $\Delta L_B = 24.71 \mu m$ and $\Delta L_C = 197.68 \mu m$. 
In this study example, the selection of $\Delta L_B = 24.71 \, \mu m$ sets about 12 narrowband channels and about four wideband channels in each full cyclic frame. The selected path length imbalances of sections $A$, $B$, and $C$ of this study example yields the following respective FSR values of transmission response components of the narrowband channels, wideband channels, and wavelength modulating functions: $\Lambda_{A+C} \approx 0.8 \, \text{nm}$, $\Lambda_{A-C} \approx 2.4 \, \text{nm}$, and $\Lambda_B \approx 19.22 \, \text{nm}$. The upper part of Figure 4.21 depicts the simulation of the transmissions taken for input $I_1$ when all phase shifters are at the reset condition. The upper-middle part demonstrates the steering of the envelope modulating functions to the right side of the wavelength range by $\Lambda_B/2 \approx 9.61 \, \text{nm}$ when applying $\Delta \phi_B = 180^\circ$ keeping $\Delta \phi_A = \Delta \phi_C = 0^\circ$. The lower-middle part demonstrates the individual steering of the small bandwidth channels by $\Lambda_{A+C}/2 \approx 0.4 \, \text{nm}$ when applying $\Delta \phi_A = \Delta \phi_C = 90^\circ$ keeping $\Delta \phi_B = 180^\circ$. This part of the figure should be compared with the upper-middle part of the same figure to notice the relative wavelength shift in the location of only the small demands. The lower part of the figure demonstrates the steering of the wide bandwidth channels to the right side of the wavelength range by $\Lambda_{A-C}/2 \approx 1.2 \, \text{nm}$ when applying $\Delta \phi_A = -\Delta \phi_C = 90^\circ$ keeping $\Delta \phi_B = 180^\circ$. This part of the figure should also be compared with the upper-middle part of the same figure to notice the relative wavelength shift in the location of only the large demands. In summary, this study case has demonstrated the utilization of wavelength modulation of the transmission responses using the FIR cascaded processor of Figure 4.19 for the (de)multiplexing of two cyclic groups of different channel bandwidths providing the capability to individually steer each of the narrowband channels, wideband channels and wavelength modulating functions.

4.7 Universal FIR Phased Array Processors

A universal phased array processor is described in this thesis such that it is capable of functioning as both a broadband switch with intensity control and as a tunable WDM (de)multiplexer. In the broadband switching operation mode, the universal unit should be able to route the input signals to the desired outputs satisfying either partially blocking or fully non-blocking characteristics. In the WDM (de)multiplexing operation mode, the universal unit should be able to (de)multiplex the input and output signals with providing the steering capability of the transmission responses over the wavelength range. Universal processors can be engineered to support changing the bandwidth of the (de)multiplexed channels as in binary bandwidth variable (de)multiplexers. Universal $4 \times 4$ phased processors can support the simultaneous (de)multiplexing of different bandwidth signals. They also
provide the facility to change the sequence of the (de)multiplexed signals. In the next four sections, both universal 2×2 and 4×4 processors are demonstrated.

4.7.1 Universal 2×2 FIR Phased Array Processors

In this section, at first, a universal 2×2 phased array processor of the 1st order that functions both as a tunable WDM (de)multiplexer supporting only one channel bandwidth setting and as a controllable broadband switch is demonstrated. The structure of the universal processor supports the steering of the formed WDM interferograms over the wavelength range as well as controlling the intensities of the outputs when it is turned into the broadband regime. The architecture of the universal 2×2 processor of the 1st order is illustrated in Figure 4.22. The universal processor consists of three cascaded sections of four 2×2 MMI couplers and three interconnection networks; the first and last of them are broadband equipped with NEMS-operated phase shifters, and the middle one has both path length imbalances and NEMS-operated phase shifters.

![Figure 4.22 Symbolic diagram of a 1st order universal 2×2 phased array processor.](image)

To understand the operation of the 1st order universal 2×2 FIR phased array processor of Figure 4.22 one should notice that the first and second MMI couplers with the two phase shifters between them form a broadband MZI switch. Similarly, the third and fourth MMI couplers with the two phase shifters between them form another broadband MZI switch. Each one of the described broadband switch sections can be turned into the single-throw switching mode of either crossover or forward routing when applying a phase shift difference of either 0° or ±180°, respectively, between the NEMS-operated phase shifters of that section. When one or both of the described broadband MZI switches are at the crossover or forward routing mode, each output of the universal 2×2 phased array switch due to either one of the two inputs is derived by one of the two intermediate signals $U_1$ or $U_2$ resulting in an overall broadband response of the switch fabric. In such a case, the phase shifters of the second interconnection network $\Delta\phi_{1B}$ and $\Delta\phi_{2B}$ become paralyzed, and each input is routed to the outputs in the broadband regime. Furthermore, in the broadband mode operation while one of the broadband MZI switches is set into the crossover or forward routing mode the phase shift difference
of the other broadband MZI switch can be used to control the power split ratio of the inputs to the outputs. On the other hand, when both of the broadband MZI stages are not in the crossover or direct routing modes, the universal 1st order 2×2 phased array processor is at least engaged partially into the WDM regime. A full engagement into the WDM regime happens when turning each broadband MZI stage into the broadcasting mode by setting $|\Delta \phi_A| = |\Delta \phi_C| = 90^\circ$. In the WDM mode, $\Delta \psi_B$ determines the FSR value and enables the wavelength steering of the transmission responses as in a tunable single-stage WDM (de)multiplexer. To mathematically state the operation conditions of the universal 2×2 processor of Figure 4.22 the transfer function elements are expressed here for the assumption of ideal system components:

$$
|r_{11}|^2 = |r_{22}|^2 = \cos^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right) + \cos^2\left(\frac{\Delta \phi_A + \Delta \phi_C}{2}\right) - \cos^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right)^2 \sin^2\left(\frac{\Delta \psi_B}{2}\right)
$$

$$
|r_{21}|^2 = |r_{12}|^2 = \sin^2\left(\frac{\Delta \phi_A + \Delta \phi_C}{2}\right) + \sin^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right) - \sin^2\left(\frac{\Delta \phi_A + \Delta \phi_C}{2}\right)^2 \cos^2\left(\frac{\Delta \psi_B}{2}\right)
$$

… Eq. (4.13)

where $\Delta \psi_B = \Delta \psi_C + \Delta \psi_B$ is as always defined before. In broadband operation the ripple terms in Eq. (4.13) have to be nulled. This requires both:

$$
\cos^2\left(\frac{\Delta \phi_A + \Delta \phi_C}{2}\right) = \cos^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right), \quad \sin^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right) = \sin^2\left(\frac{\Delta \phi_A + \Delta \phi_C}{2}\right)
$$

… Eq. (4.14)

Within the phase range from -180° to 180° the condition of Eq. (4.14) requires having either one or both of $\Delta \phi_A$ and $\Delta \phi_C$ set to either 0° or ±180°. This means as indicated before that one or both of the broadband switch sections have to be in the crossover or forward switching status. Eq. (4.13) for the broadband condition of Eq. (4.14) is reduced to:

$$
|r_{11}| = |r_{22}| = \cos\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right), \quad |r_{21}| = |r_{12}| = \sin\left(\frac{\Delta \phi_A + \Delta \phi_C}{2}\right)
$$

… Eq. (4.15)

Eq. (4.15) describes the response of a broadband MZI modulator with intensity control. When the broadband condition of Eq. (4.14) is not met (i.e. when both $\Delta \phi_A$ and $\Delta \phi_C$ are not equal to 0° or ±180°) the universal processor is engaged at least partially into the WDM mode. The conditions for full engagement into the WDM mode based on Eq. (4.13) require either:

$$
\cos^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right) = \sin^2\left(\frac{\Delta \phi_A + \Delta \phi_C}{2}\right) = 1, \quad \cos^2\left(\frac{\Delta \phi_A + \Delta \phi_C}{2}\right) = \sin^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right) = 0
$$

… Eq. (4.16)
or:

\[
\cos^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right) = \sin^2\left(\frac{\Delta \phi_A + \Delta \phi_C}{2}\right) = 0, \quad \cos^2\left(\frac{\Delta \phi_A + \Delta \phi_C}{2}\right) = \sin^2\left(\frac{\Delta \phi_A - \Delta \phi_C}{2}\right) = 1
\]

… Eq. (4.17)

The condition of Eq. (4.16) requires \(\Delta \phi_A = \Delta \phi_C = \pm 90^\circ\). In this case, Eq. (4.13) is reduced to:

\[
|r_{11}| = |r_{22}| = \left|\cos\left(\frac{\Delta \psi_B}{2}\right)\right|, \quad |r_{21}| = |r_{12}| = \left|\sin\left(\frac{\Delta \psi_B}{2}\right)\right|
\]

… Eq. (4.18)

The condition of Eq. (4.17) requires \(\Delta \phi_A = -\Delta \phi_C = \mp 90^\circ\). In this case, Eq. (4.13) is reduced to:

\[
|r_{11}| = |r_{22}| = \left|\sin\left(\frac{\Delta \psi_B}{2}\right)\right|, \quad |r_{21}| = |r_{12}| = \left|\cos\left(\frac{\Delta \psi_B}{2}\right)\right|
\]

… Eq. (4.19)

Both of Equations (4.18) and (4.19) describe the response of a tunable 2×2 WDM (de)multiplexer with FSR value set by the path length difference \(\Delta L_B\) as in Eq. (3.17) providing wavelength steering capability of the transmission responses driven by the phase difference \(\Delta \phi_B\). The difference between Equations (4.18) and (4.19) is to switch the cyclic transmission channels for each input at outputs \(O_1\) and \(O_2\) with each other.

To demonstrate the operation of the universal processor of Figure 4.22 an example is taken here for setting the channel bandwidth to 150 GHz. The required path length difference of section B, according to Eq. (3.18), is set to \(\Delta L_B = 198\ \mu\text{m}\). This results, according to Eq. (3.17), in \(A \approx 2.4\ \text{nm}\).

The upper part of Figure 4.23 shows the crossover broadband routing operation resulted when setting \(\Delta \phi_A = 0^\circ\) and \(\Delta \phi_C = 180^\circ\). The phase difference \(\Delta \phi_B\), in this case, does not matter since it is paralyzed in the broadband regime. The signal attenuation around \(\lambda = 1.55\ \mu\text{m}\), in this case, is about 4.2 dB. The crosstalk for this crossover broadband routing operation is less than about -30 dB. The upper-middle part demonstrates turning the processor in the WDM mode when setting \(\Delta \phi_A = \Delta \phi_C = 90^\circ\) taken for \(\Delta \phi_B = 0^\circ\) to keep the transmission responses at the reset position. The lower-middle part of Figure 4.23 demonstrates steering the transmission responses of the upper-middle part to the right side of the wavelength range by quarter the FSR length \((\Delta \lambda_s \approx 0.6\ \text{nm})\) when applying \(\Delta \phi_B = 90^\circ\) keeping \(\Delta \phi_A = \Delta \phi_C = 90^\circ\) as before. The lower part of the figure demonstrates switching the passband channels of outputs \(O_1\) and \(O_2\) of the simulation case of the lower-middle part of the same figure with each other when setting \(\Delta \phi_A = -\Delta \phi_C = 90^\circ\) keeping \(\Delta \phi_B = 90^\circ\) as in the previous step to cause shifting the spectrum as before by quarter the FSR value.
Figure 4.23 Simulated wavelength-dependent transmission responses for input $I_1$ of the universal 2×2 FIR phased array processor of Figure 4.22 when: $\Delta \phi_A = \Delta \phi_B = 0^\circ$ & $\Delta \phi_C = 180^\circ$ (up), $\Delta \phi_A = \Delta \phi_C = 90^\circ$ & $\Delta \phi_B = 0^\circ$ (up-middle), $\Delta \phi_A = \Delta \phi_B = \Delta \phi_C = 90^\circ$ (down-middle), and $\Delta \phi_A = \Delta \phi_B = -\Delta \phi_C = 90^\circ$ (down). $\Delta L_B = 198 \, \mu m$. 

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The universal 2×2 phased array processor of Figure 4.22 can also be designed to exhibit WDM (de)multiplexing characteristics when all of the NEMS-operated phase shifters are at the reset conditions. This can be achieved by adding path length imbalances to the interconnection networks of the first and third cascaded stages as in Figure 4.19. In such a case the path length imbalances of the first and third cascaded sections should be set to $|ΔL_A| = |ΔL_C| = \lambda/(4n_e) \approx ±0.18$ µm to result into the ±90º of phase shift differences at $\lambda = 1.55$ µm required to fully engage the universal 2×2 phased array processor of Figure 4.22 into the WDM mode. This option is investigated, but simulation results are not shown for brevity.

The generalization of the 1st order universal 2×2 FIR processor of Figure 4.22 is shown in Figure 4.24. This figure shows the block diagram of a general $N^\text{th}$ order universal 2×2 FIR processor composed of $N$ number of WDM interconnection sections of path length imbalances and steering phase shifters sandwiched between $N+1$ number of broadband MZI switches. The driving phase shift difference of each $k^{th}$ broadband switch is denoted as $Δθ_k$. The wavelength dependent argument of each $k^{th}$ interconnecting WDM section is denoted as $Δψ_k = Δϕ_k + Δφ_k$ as used before. One can notice that the architecture of the universal 2×2 FIR processor of Figure 4.24 is an extension to that of the binary bandwidth variable processor of Figure 4.18 such that the first and last MMI couplers in Figure 4.18 have been replaced with full MZI broadband switches. The universal processor of Figure 4.24 can provide more permutations to control the bandwidth of the (de)multiplexed channels compared to the architecture of the binary processor of Figure 4.18 for the same number of used cascaded sections as explained herein.

![Figure 4.24 Block diagram of an $N^{th}$ order universal 2×2 FIR processor.](image)

To operate the universal processor of Figure 4.24 in the broadband regime every broadband switch except one has to be turned into the crossover or forward single-throw switching modes. In this case, the optical signal at every WDM section passes through one of its two path length imbalances and in result no ripple is produced in the transmission responses. The only one broadband switch that is not fixed turned into the crossover or forward modes is used to control the intensities of the output signals as in any broadband MZI modulator.
Table 4.1 Comparison of the number of channel bandwidth permutations available for the universal 2×2 binary processor of Figure 4.18 and the universal 2×2 processor of Figure 4.24.

The $N^{th}$ order universal processor of Figure 4.24 is engaged into the full WDM regime for the (de)multiplexing of two alternative cyclic channels utilizing sinusoidal filtering characteristic when any two of the broadband switches are turned into the broadcast double-throw switching mode keeping all other broadband switches operating into the binary mode of crossover or forward switching status. The WDM sections of different path length imbalances located between the two broadband switches that are turned into the broadcast switching mode determine the available permutations of channel bandwidths. Therefore, the total number of available channel bandwidth permutations $N_{ch}$ is given by:

$$N_{ch} = \sum_{m=0}^{N-1} 2^m (N - m)$$

… Eq. (4.20)

Table 2.1 provides a comparison of the available number of channel bandwidth permutations for the universal processor of Figure 4.24 and the universal binary processor of Figure 4.18 assuming that one binary switching permutation of the universal binary processor is reserved for the broadband operation mode. The comparison is based on using the same number of sections in the two FIR cascaded processors. When the smallest size of three cascaded sections is used, both processors can produce only one channel bandwidth. For any odd number of cascaded sections over three, the universal processor of Figure 4.24 produces a larger number of channel bandwidth permutations compared with the binary processor of Figure 4.18. The difference in the number of channel bandwidth permutations between the two processors increases with the increase in the number of used cascaded sections as in Table 4.1. The wavelength steering of the (de)multiplexed transmission
responses for all bandwidth permutations of the $N^{th}$ order universal 2×2 FIR processor of Figure 4.24 is driven by the phase shifters of the WDM sections. The transmission responses of universal 2×2 FIR processors of orders higher than three are examined in this research, but simulation results are not shown here for brevity.

4.7.2 Universal 1st Order 4×4 FIR Phased Array Processors

The construction of universal 4×4 FIR phased array processors that can operate in the WDM regime besides being able to function in the broadband switching mode is possible like the construction of the universal 2×2 FIR phased array processors. Figure 4.25 illustrates the requirements of a universal 1st order 4×4 FIR phased array processor composed of two 4×4 broadband switches interconnected by a network of four path length imbalances, four phase shifters and two crossover 2×2 MMI couplers. The broadband 4×4 phased array switches can be made blocking of simple single stage architecture, partially blocking of two cascaded sections or non-blocking of advanced architecture. The universal 1st order 4×4 phased array processor functions as a broadband router for the inputs/outputs when one of the two broadband switches is turned into the single-throw mode. When the two broadband switches are both not operating in the single-throw mode, the universal 1st order processor (de)multiplexes the inputs/outputs depending on the design of the intermediate network of path length imbalances and on the exact switching states of the two broadband stages located on both sides of the WDM intermediate section. The phase shifters of the intermediate interconnection network are used for the wavelength steering of the transmission interferograms. Simulation results for universal 4×4 FIR phased array processors are investigated in this research for different designs of simple and complex architectures. However, for the brevity reason, the comprehensive simulation results are not disclosed here. Instead, in the next section, the performance of a compact universal 1st order 4×4 FIR phased array processor that provides the full set of 24 non-blocking broadband routing states is explored.

**Figure 4.25** Block diagram of a universal 1st order 4×4 FIR phased array processor.
Figure 4.26 Some compact non-blocking universal 1st order 4×4 FIR phased array processors.

The construction of a compact universal 1st order 4×4 FIR phased array processor that provides the full set of 24 non-blocking switching operations can be achieved by using one non-blocking 4×4 broadband switch on one side of the WDM section and a simple single-stage 4×4 broadband switch on the other side of the WDM section with the capability to wisely swap the cascaded sections. Figure 4.26 depicts some of the different available structures that function as compact non-blocking universal 1st order 4×4 FIR phased array processors. Each one of these architectures has been faithfully studied and simulated. Some other different design variants can be derived from these
structures. Each one of these compact universal 1\textsuperscript{st} order 4×4 phased array processors can be properly designed to (de)multiplex four cyclic channels with providing the capability to steer the transmission responses over the wavelength range in addition to exhibiting the full set of 24 single-throw broadband switching operations of Figure 4.14. Each one of these relatively large designs can be built on a limited patterning chip area utilizing a snake-multipath layout pattern, as illustrated in the lower part of Figure 4.1.

**4.7.3 Compact Universal 1\textsuperscript{st} Order 4×4 FIR Phased Array Processor**

Every universal 1\textsuperscript{st} order FIR 4×4 phased array processor of Figure 4.26 requires the deployment of 20 NEMS-operated phase shifters. This section studies the performance of the compact universal 1\textsuperscript{st} order FIR 4×4 phased array processor illustrated in Figure 4.27, which requires the deployment of 16 total NEMS-operated phase shifters. This universal 1\textsuperscript{st} order 4×4 FIR phased array processor is constructed based on adding a WDM section in the mid of the non-blocking 4×4 switch of Figure 4.12 between the 4×4 broadband switch elements.

![Figure 4.27 Compact non-blocking universal 1\textsuperscript{st} order 4×4 FIR phased array processor.](image)

All of the 24 single-throw broadband routing states are available to this universal 4×4 processor. It provides the advantages of facilitated operation and negligible crosstalk. It is also capable of either (de)multiplexing four alternative similar bandwidth channels at one FSR value or (de)multiplexing different bandwidth channels for selected combinations of the inputs and outputs. It also provides the wavelength steering capability of the transmission responses for all (de)multiplexing modes. Some simulation examples for this most compact universal processor which provides many switching, filtering, and (de)multiplexing operations are demonstrated here.

To demonstrate the operation of the universal 4×4 processor of Figure 4.27 an example is taken here for the (de)multiplexing of four alternative cyclic channels each has the same 150 GHz bandwidth. As explained before in Section (3.3.4) of Chapter (3) the pitch in the progressive path length imbalances of the WDM section required to achieve the 150 GHz channel bandwidth is about \(\Delta L \approx 100\ \mu\text{m}\). In order to reduce the issue due to the existing phase errors of the used MMI couplers
that cause imbalances in the (de)multiplexed four alternative cyclic channels of similar bandwidth
the path length imbalances of the WDM section are numerically adjusted to the values: $L_{1C}-L_{4C} = 300 \, \mu m$, $L_{2C}-L_{4C} = 200.35 \, \mu m$ and $L_{3C}-L_{4C} = 100 \, \mu m$.

The voltage-controlled biasing of each one of the three cascaded broadband switches to turn
each of them into one of the single-throw broadband routing states results in one of the available 24
single-throw broadband routing operations. As an example the application of the phase synthesis
state of $(180^\circ, 180^\circ, 0^\circ, 0^\circ)$ $(180^\circ, 0^\circ, 180^\circ, 0^\circ)$ $(0^\circ, 0^\circ, 0^\circ, 0^\circ)$ $(180^\circ, 180^\circ, 0^\circ, 0^\circ)$ yields the respective
straight broadband routing of inputs $I_1$ through $I_4$ to outputs $O_1$ through $O_4$. Figure 4.28 depicts the
transmission response for the broadband routing of input $I_1$ to output $O_1$ for this case. As seen in the
figure, this clean routing operation has minimal crosstalk over a wide wavelengths range around the
center telecom operating wavelength of 1.55 $\mu m$. The signal attenuation for this case routing is about
8 dB at $\lambda = 1.55 \, \mu m$, and the crosstalk is less than -26.8 dB.

The proper turning of both of the 4×4 broadband switches into corresponding broadcasting routing
states of equal output split ratios keeping the phases of the two 2×2 broadband intermediate switches
at the reset conditions yields the alternative (de)multiplexing of four cyclic channels for this
processor. There is a number of distinguishable phase synthesis states that can turn each one of the
two 4×4 broadband switches of the universal structure of Figure 4.27 into the broadcast state
including $(0^\circ, 90^\circ, 270^\circ, 0^\circ)$, $(0^\circ, 270^\circ, 90^\circ, 0^\circ)$, $(90^\circ, 0^\circ, 0^\circ, 270^\circ)$ and $(270^\circ, 0^\circ, 0^\circ, 90^\circ)$. The different
combinations of these phase synthesis settings though are examined to yield one of two possible

Figure 4.28 Simulated wavelength-dependent transmission response for input $I_1$ of the universal 4×4 phased
array processor of Figure 4.27 when setting the phase synthesis state of $(180^\circ, 180^\circ, 0^\circ, 0^\circ)$ $(180^\circ,$
$0^\circ, 180^\circ, 0^\circ)$ $(0^\circ, 0^\circ, 0^\circ, 0^\circ)$ $(180^\circ, 180^\circ, 0^\circ, 0^\circ)$. $L_{1C}-L_{4C} = 300 \, \mu m$, $L_{2C}-L_{4C} = 200.35 \, \mu m$, and $L_{3C}-$
$L_{4C} = 100 \, \mu m$. 
sequencing operations for the four cyclic output channels of either 1-4-2-3 or 1-3-2-4 for input $I_I$. The application of progressive phase shift differences to the NEMS-operated phase shifters of the WDM section (section-C) for the case of alternative (de)multiplexing of four channels enables the steering of the transmission channels along the wavelength range.

Figure 4.29 Simulated wavelength-dependent transmission responses for input $I_I$ of the universal 4×4 phased array processor of Figure 4.27 when setting the phase synthesis states of (0º, 270º, 90º, 0º) (0º, 0º, 0º, 0º) (0º, 0º, 0º, 0º) (0º, 270º, 90º, 0º) for up, (0º, 90º, 270º, 0º) (0º, 0º, 0º, 0º) (0º, 0º, 0º, 0º) (0º, 270º, 90º, 0º) for middle, and (0º, 90º, 270º, 0º) (0º, 0º, 0º, 0º) (270º, 180º, 90º, 0º) (0º, 270º, 90º, 0º) for down. $L_{1C-L_{4C}} = 300$ μm, $L_{2C-L_{4C}} = 200.35$ μm, and $L_{3C-L_{4C}} = 100$ μm.
The upper part in Figure 4.29 depicts the (de)multiplexing characteristic, taking input \( I_1 \) into consideration, resulted when applying the phase synthesis state of \((0^\circ, 270^\circ, 90^\circ, 0^\circ) (0^\circ, 0^\circ, 0^\circ, 0^\circ) (0^\circ, 0^\circ, 0^\circ, 0^\circ) (270^\circ, 90^\circ, 0^\circ)\). The FSR value, in this case, is about \( \Lambda = 4.75 \text{ nm} \) corresponding to the targeted channel bandwidth of 150 GHz. Attenuation and channel level imbalances result as seen in the figure due to the non-ideal components. The middle part of the same figure depicts the change in the sequencing of the passband channels resulted when applying the phase synthesis state of \((0^\circ, 90^\circ, 270^\circ, 0^\circ) (0^\circ, 0^\circ, 0^\circ, 0^\circ) (0^\circ, 0^\circ, 0^\circ, 0^\circ) (90^\circ, 270^\circ, 90^\circ, 0^\circ)\). The lower part of the same figure depicts the steering of the (de)multiplexed transmission passbands of the middle part towards the right side of the wavelength range by quarter the FSR value \( (\Delta \lambda_s \approx 1.2 \text{ nm}) \) resulted when applying a progressive phase shift of \( 90^\circ \); i.e. when the phase synthesis state is set to \((0^\circ, 90^\circ, 270^\circ, 0^\circ) (270^\circ, 180^\circ, 90^\circ, 0^\circ) (0^\circ, 270^\circ, 90^\circ, 0^\circ)\).

The upper and upper-middle parts of figure 4.30 depict the (de)multiplexing characteristics for inputs \( I_1 \) and \( I_3 \), respectively, resulted when resetting all of the NEMS-operated phase shifters. In this case, the entrance 4×4 broadband switch splits inputs \( I_1 \) and \( I_4 \) into equal portions at its 2\textsuperscript{nd} and 3\textsuperscript{rd} outputs resulting into the (de)multiplexed interferograms at outputs \( O_2 \) and \( O_3 \) influenced by the path length imbalance \( \Delta L_{2,3} = L_2 - L_3 = 100.35 \text{ \mu m} \). This \( \Delta L_{2,3} \) results in FSR value of about \( \Lambda \approx 4.73 \text{ \mu m} \) corresponding to a channel bandwidth of about \( B_{ch} \approx 296 \text{ GHz} \) as in any 2×2 (de)multiplexer according to Equations (3.17) and (3.18), respectively. The entrance 4×4 broadband switch in this case also splits inputs \( I_2 \) and \( I_3 \) into equal portions at its 1\textsuperscript{st} and 4\textsuperscript{th} outputs resulting in the (de)multiplexed interferograms at outputs \( O_1 \) and \( O_4 \) influenced by the path length imbalance \( \Delta L_{1,4} = L_1 - L_4 = 300 \text{ \mu m} \) which results into FSR value of about \( \Lambda \approx 1.58 \text{ \mu m} \). For this reason, the bandwidth of the (de)multiplexed channels for inputs \( I_2 \) and \( I_3 \) and outputs \( O_1 \) and \( O_4 \) is about one third that for inputs \( I_1 \) and \( I_4 \) and outputs \( O_2 \) and \( O_3 \) for this phase synthesis control case.

The setting of the intermediate two 2×2 broadband switches to both produce crossover routings by applying the phase synthesis state of \((180^\circ, 0^\circ, 180^\circ, 0^\circ)\) to section-B turns over the rolls such that inputs \( I_1 \) (\( I_2 \)) and \( I_4 \) (\( I_3 \)) are (de)multiplexed at outputs \( O_1 \) (\( O_2 \)) and \( O_4 \) (\( O_3 \)) at the small (large) channel bandwidth of 99 GHz (296 GHz). The lower-middle part of Figure 4.30 depicts the (de)multiplexing of input \( I_1 \) into outputs \( O_1 \) and \( O_4 \) for this case.
Figure 4.30 Simulated wavelength-dependent transmission responses for inputs $I_1$ and $I_3$ of the universal 4×4 processor of Figure 4.27 when setting the phase synthesis states of reset condition for up and up-middle, (0°, 0°, 0°, 0°) (180°, 0°, 180°, 0°) (0°, 0°, 0°, 0°) (0°, 0°, 0°, 0°) for middle-down, and (0°, 90°, 270°, 180°) (0°, 0°, 0°, 0°) (0°, 0°, 0°, 0°) (0°, 90°, 270°, 180°) for down. $L_{1C}$-$L_{4C} = 300$ μm, $L_{2C}$-$L_{4C} = 200.35$ μm, and $L_{3C}$-$L_{4C} = 100$ μm.
Reconfiguring the universal processor to (de)multiplex inputs $I_1$ and $I_4$ at outputs $O_2$ and $O_3$ for the small channel bandwidth of 99 GHz influenced by the long path length imbalance $\Delta L_{1-4}$ and the (de)multiplexing of inputs $I_2$ and $I_3$ at outputs $O_1$ and $O_4$ for the large channel bandwidth of 296 GHz influenced by the small path length imbalance $\Delta L_{2-3}$ can be achieved in an example by applying the phase synthesis state of $(0^\circ, 90^\circ, 270^\circ, 180^\circ) \ (0^\circ, 0^\circ, 0^\circ, 0^\circ) \ (0^\circ, 0^\circ, 0^\circ, 0^\circ) \ (0^\circ, 90^\circ, 270^\circ, 180^\circ)$. The lower part of Figure 4.30 depicts the simulation for the (de)multiplexing of input $I_1$ and outputs $O_2$ and $O_3$ for this case. Separate steering of the transmission interferograms for the different simulation cases of Figure 4.30 can be achieved utilizing the corresponding two NEMS-operated phase shifters that are involved in the signals pathways of the WDM section as in any tunable 2×2 (de)multiplexer. The simulation results for the steering are not shown for brevity.

Figure 4.31 Simulated wavelength-dependent transmission responses for input $I_1$ of the universal 4×4 phased array processor of Figure 4.27 resulted when setting the phase synthesis state of $(0^\circ, 90^\circ, 270^\circ, 0^\circ) \ (0^\circ, 0^\circ, 0^\circ, 0^\circ) \ (0^\circ, 0^\circ, 0^\circ, 0^\circ) \ (0^\circ, 90^\circ, 270^\circ, 180^\circ)$. $L_{1C-L_{4C}} = 300 \ \mu\text{m}$, $L_{2C-L_{4C}} = 200.35 \ \mu\text{m}$, and $L_{3C-L_{4C}} = 100 \ \mu\text{m}$.

Figure 4.31 depicts another (de)multiplexing scheme resulted when applying the phase synthesis state of $(0^\circ, 90^\circ, 270^\circ, 0^\circ) \ (0^\circ, 0^\circ, 0^\circ, 0^\circ) \ (0^\circ, 0^\circ, 0^\circ, 0^\circ)$ for which case input $I_1$ is (de)multiplexed both at outputs $O_2$ and $O_3$ for the small channel bandwidth of 99 GHz and outputs $O_1$ and $O_4$ for the large channel bandwidth of 296 GHz. In this case, the entrance 4×4 broadband switch is turned into the broadcast routing of its inputs. The output 4×4 broadband switch of the structure is turned into one of the two available double-throw routing operations of the inputs. The resulted channel bandwidth of the transmission interferograms at each output due to each input is dependent on comparing the signals at the 1st and 4th or 2nd and 3rd intermediate interconnection branches of the WDM section (section-C). Steering of the transmission interferograms is also
facilitated in this case as before. The larger universal 4×4 phased array processors of Figure 4.26 yield more permutations for comparing the path length differences of the WDM section, and in result, they provide more (de)multiplexing schemes at the expenses of increasing the system footprint and loss due to the additional cascaded sections.

4.7.4 Compact Universal 2\textsuperscript{nd} Order 4×4 FIR Phased Array Processor

The universal 1\textsuperscript{st} order processor of the last section has one 4×4 WDM section of progressive path length imbalance. Therefore, as demonstrated in the last section, a 1\textsuperscript{st} order 4×4 processor is capable of (de)multiplexing four alternative cyclic channels for only one available channel bandwidth. In theory, as in universal 2×2 processors, the cascaded architecture of universal 4×4 FIR processors can be expanded to any desired $N^{th}$ order constructed of $N$ number of WDM sections each has four path length imbalances and four phase shifters sandwiched between $N+1$ number of broadband switches. An $N^{th}$ order 4×4 universal processor can support at least $N$ different bandwidths for the standard (de)multiplexing of four alternative cyclic channels. Many other (de)multiplexing schemes are also supported in addition to providing the wavelength steering capability of the transmission responses to all of the available (de)multiplexing modes. An $N^{th}$ order 4×4 universal processor can also be operated in the broadband routing mode. However, the major limitation that can be addressed here is the footprint of high order universal 4×4 processors. In theory, the schemes proposed in the thesis can be expanded to any $N^{th}$ order universal $M\times M$ processor. This provides an ultimate solution to (de)multiplex $M$ number of alternative cyclic channels for at least $N$ different bandwidths in addition to supporting a multitude of other operation modes.

![Block diagram of a 2\textsuperscript{nd} order universal 4×4 FIR processor (up) and compact implementation of the universal processor using single-stage 4×4 broadband switches (down).](image_url)

**Figure 4.32** Block diagram of a 2\textsuperscript{nd} order universal 4×4 FIR processor (up) and compact implementation of the universal processor using single-stage 4×4 broadband switches (down).
Due to the limitation in the available patterning area of current foundries only a compact implementation of a 2nd order universal 4×4 processor is briefly demonstrated in this section. The upper part of Figure 4.32 shows the general block diagram of a 2nd order universal 4×4 processor. Each broadband switch in the lower part of the same figure is replaced with a single-stage 4×4 broadband switch to result in the smallest possible processor size.

To at least demonstrate the capability of the processor fabric of the lower part of Figure 4.32 to support two bandwidths for the (de)multiplexing of four alternative channels an example is taken here for the channel bandwidths of 100 GHz and 150 GHz. The required progressive path length imbalance for each of sections $B$ and $D$ can be determined from: $\Delta L_x = c/(4n g B_{ch,x})$. Here $x$ refers to either section $B$ or $D$. The channel bandwidth $B_{ch,x}$ is set by section $x$ when the two switches around it are turned into the broadcast broadband modes, and the other third switch is turned into one of the available four single-throw broadband routing modes. The progressive path length imbalances of sections $B$ and $D$ are set to $\Delta L_B = 150 \, \mu m$ and $\Delta L_D = 100 \, \mu m$ for the bandwidths of $B_{ch,B} = 100$ GHz and $B_{ch,D} = 150$ GHz, respectively. The path length imbalances used in the simulations are slightly detuned to compensate for the effect of phase errors of the MMI couplers to better balance the (de)multiplexed channels: $L_{1B}-L_{4B} = 450 \, \mu m$, $L_{2B}-L_{4B} = 300.35 \, \mu m$, $L_{3B}-L_{4B} = 150 \, \mu m$, $L_{1D}-L_{4D} = 300 \, \mu m$, $L_{2D}-L_{4D} = 200.35 \, \mu m$, and $L_{3D}-L_{4D} = 100 \, \mu m$. The upper part of Figure 4.33 depicts the simulation of the (de)multiplexed four alternative channels each of 100 GHz bandwidth taken for input $I_1$ when setting the phase synthesis state of: $(0º, 270º, 90º, 0º) (0º, 0º, 0º, 0º) (0º, 270º, 90º, 0º) (0º, 0º, 0º, 0º) (0º, 270º, 90º, 0º)$. The (0º, 270º, 90º, 0º) phase synthesis state turns the broadband switch into the broadcast broadband switching mode whereas the (180º, 180º, 0º, 0º) phase synthesis state turns the broadband switch into the straight-through single-throw broadband switching mode. The FSR value, in this case, is about $A_B \approx 3.17$ nm. The signal attenuation is about 12 dB around the wavelength $\lambda = 1.55 \, \mu m$, and there is channel imbalance. The lower part of Figure 4.33 depicts the simulation of the transmission responses for input $I_1$ when the bandwidth of the (de)multiplexed four alternative channels is switched to 150 GHz for which case the FSR value is set to $A_B \approx 4.75$ nm. This lower part of the figure is simulated when applying the phase synthesis state of: $(180º, 180º, 0º, 0º) (0º, 0º, 0º, 0º) (0º, 270º, 90º, 0º) (0º, 0º, 0º, 0º) (0º, 270º, 90º, 0º)$. The steering of the spectrum can be achieved by applying a progressive phase shift through the WDM section associated with each bandwidth setting. The other three available single-throw routing modes can be used to re-sequence...
the order of the (de)multiplexed channels. Many other (de)multiplexing and filtering schemes that have been investigated in this research are available to this powerful processor, but simulation results are not shown for brevity.

Figure 4.33 Simulated wavelength-dependent transmission responses for input $I_1$ of the 2nd order universal 4×4 phased array processor of the lower part of Figure 4.32 when setting the phase synthesis states of (0°, 270°, 90°, 0°) (0°, 0°, 0°, 0°) (180°, 180°, 0°, 0°) for up and (180°, 180°, 0°, 0°) (0°, 0°, 0°, 0°) (0°, 270°, 90°, 0°) (0°, 0°, 0°, 0°) (0°, 270°, 90°, 0°) for down. $L_{1B}-L_{4B} = 450 \, \mu m$, $L_{2B}-L_{4B} = 300.35 \, \mu m$, $L_{3B}-L_{4B} = 150 \, \mu m$, $L_{1D}-L_{4D} = 300 \, \mu m$, $L_{2D}-L_{4D} = 200.35 \, \mu m$, and $L_{3D}-L_{4D} = 100 \, \mu m$.

4.8 Conclusions

The utilization of cascaded architectures of integrated photonic phased array circuits to achieve the objectives indicated in the introduction section of this chapter introduces challenges for constructing large PICs were performance errors in the construction components are expected to affect the overall response of the advanced photonic processors. In this regard, the S-parameters of the components designed in Chapter (2) reliably simulated using FIMMPROP are used to numerically determine the transmission responses of the cascaded structures studied in this chapter utilizing the well-known
transfer function method. This study, therefore, provides realistic consideration for possible
deterioration in the performance of the investigated cascaded FIR structures due to bandwidth
limitations in both of the magnitude responses and most importantly phase error responses of the
different construction components. The usually noticeable ripple and deviations close to the end
regions of the C-band in the transmission characteristic are due to this fact; unlike some other studies
that might, for example, take the responses of the components to be ideal. In large cascaded FIR
(de)multiplexers the increased ripple worse the channel uniformity and puts a cap on the maximum
number of cascaded sections that can be utilized. The principle of operation supported with detailed
analysis of each photonic processor is presented in each associated section. The path length
imbalances are theoretically determined to achieve the desired channel bandwidths, and the
quantitative parameters are obtained from the simulation results of each processor. It is always
advantageous to have a trimming phase shifter in cascade with any used delay line to manage to
rectify the transmission response sensitivity to fabrication tolerances.

The single-throw, double-throw, and broadcast broadband routing operations available to the
cascading of two broadband 4×4 phased array sections are figured out. The single-stage broadband
4×4 IPPAS element, two cascaded stages broadband 4×4 IPPAS element and single-stage broadband
2×2 IPPAS element are used to construct five different architectures of non-blocking broadband 4×4
switches. The phase synthesis states for each non-blocking broadband 4×4 switch to provide the
available different single-throw, double-throw and broadcast routing operations are comprehensively
predicted. The crosstalk, loss, and ease of operation merits are used to differentiate the studied five
broadband 4×4 switch architectures. The cascading of broadband and WDM phased array sections
provides several utilities in the design of broadband switches and tunable WDM (de)multiplexers.
The interconnection of WDM sections by broadband switches sandwiched between a splitter and
combiner MMI couplers is demonstrated to construct binary bandwidth variable (de)multiplexers
that are capable to change the channel bandwidth to several values following a binary rule. The
(de)multiplexed bandwidth-controlled channels are also wavelength steerable. The binary bandwidth
variable (de)multiplexers can also be designed to behave as universal switches supporting operation
in the broadband routing mode. The envelope modulation of the fast ripple transmission responses
by slowly varying envelope functions utilizing three cascaded WDM sections is demonstrated to
yield the cyclic (de)multiplexing and filtering of one narrowband channel and several wideband
channels within each long FSR period. The same architecture of three cascaded WDM sections is also tailored to wavelength-modulate the transmission responses for the cyclic (de)multiplexing of several narrowband and wideband channels within each long FSR cycle. Both of the fast ripples of the (de)multiplexed transmission interferograms and the envelope/wavelength modulating functions can be steered independently over the wavelength range. The optical transmission spectrum can, therefore, be steered keeping the same relative wavelength locations of the narrowband and wideband channels. In Appendix (4) the same structure of three cascaded 2×2 WDM sections is tailored to (de)multiplex Chebyshev-like wideband channels of equal ripples in the passbands and inverse Chebyshev-like narrowband channels with equal ripples in the stopbands. The steering of the wideband and narrowband channels is also shown facilitated in this case with proper phase shift control of the three cascaded sections. Switching of the (de)multiplexed wideband and narrowband channels to either one of the two outputs of the (de)multiplexer is also available in this case. These processors can also be utilized in semi-elastic networking. The concept for the design of 2×2 and 4×4 universal processors that can operate as non-blocking broadband routers as well as being reconfigurable to operate as tunable (de)multiplexers is demonstrated in this chapter. Universal processors can provide comprehensive flexibility to deal with the spectra of photonic signals. The universal 4×4 processor is also shown to enable the (de)multiplexing of different bandwidth signals as well as enabling the resequencing of the order of the (de)multiplexed signals for passband routing flexibility. High order universal processors support the (de)multiplexing of several alternative passband channels switchable to different bandwidths. Although the sizes of universal 2×2 processors are reasonable for fabrication and experimental testing, the proposed compact universal 4×4 processors demonstrated in the thesis have large sizes that would be more challenging in the fabrication process and experimental lab testing.

Among the possible applications of cascaded PICs is the utilization of envelope/wavelength modulation of the transmission responses for semi-elastic (de)multiplexing of slow data rate and fast data rate channels carried on one optical link. The relocation of the (de)multiplexed channels along the wavelength range is a facilitated function in EONs as well. The equi-ripple (de)multiplexing scheme of wideband and narrowband channels presented in Appendix (4) can also serve this objective. It has been outlined that a universal processor can achieve the functions of a non-blocking broadband switch and tunable bandwidth variable WDM (de)multiplexer. Eventually, it will become
vital to have comprehensive programmable PIC processors to accommodate the emerging needs of future optical networks. The capability to switch between the broadband routing mode and tunable bandwidth variable WDM (de)multiplexing mode enables reaching the drop fibers at stub-ends of the network remotely from the central office during times of using OTDR testing. It also enables either providing broadband routing at the network nodes or providing WDM (de)multiplexing and channel routing at the network nodes supporting different data rates and wavelength-relocation of the channels during other times of providing networking services. The modularity to switch between different modes entitles universal processors to be reused in different kind equipment of future flexible networks. A universal processor can also be constructed from two separate units of a non-blocking broadband switch and tunable WDM (de)multiplexer having their inputs and outputs coupled using two-way broadband switching units. However, in such a case the resulted structure would be more massive compared to the compact universal architectures presented in this chapter and crossover waveguide arrangements at the exit ports would also be needed increasing system complexity, losses, and crosstalk.

Although the cascading of FIR sections could provide answers to many objectives of photonic signal processing some control styles of the shape of the transmission response within the passband channels such as achieving delta narrowband filtering functions are facilitated utilizing feedback IIR photonic integrated circuits. Furthermore, the cascading of IIR sections can be computationally more efficient than the cascading of FIR sections to achieve the same signal processing objective such that less number of construction components is used in the design. The study of feedback in flexible photonic phased array circuits is presented in the next chapter.
CHAPTER 5
FEEDBACK ARCHITECTURES UTILIZING MMI COUPLERS

5.1 Introduction
The single-stage integrated photonic phased array processors presented in Chapter (3) can mainly offer either performing as broadband switches of non-blocking 2×2 or blocking 4×4 switching states or performing as tunable WDM (de)multiplexers providing wavelength steering of the spectrum. The cascading of integrated photonic phased array sections presented in Chapter (4) provides more advanced flexibility for the routing, (de)multiplexing and filtering of optical signals targeting possible applications in SDN-EONs. The cascading of 2×2 and 4×4 three broadband switch stages is used in Chapter (4) for the construction of non-blocking switches. Several cascaded structures are also proposed in that chapter to enable the conversion between the broadband response providing switching operations and intensity control of the outputs and the tunable WDM (de)multiplexing of the I/O signals. The envelope and wavelength modulations of the transmission interferograms using cascaded structures are also utilized in the previous chapter for the (de)multiplexing of narrowband and wideband channels. The utilization of universal cascaded processors of 2×2 and 4×4 sizes supporting setting the bandwidth of the (de)multiplexed alternative channels for different data rates is also demonstrated in the previous chapter.

Although it is known in system theory that both systems with and without feedback can achieve similar functions, optical feedback systems have the advantage of facilitating certain signal processing functions using less number of components compared with cascaded FIR architectures. This means that a photonic feedback system can have a less complicated structure, uses a smaller footprint, and be more suitable for the construction of dense multifunction PICs. As an application example, the simple feedback element of a straight waveguide coupled to a ring waveguide resonator or equivalently a single -3dB 2×2 MMI coupler equipped with a loop feedback path can achieve narrowband cyclic delta filtering of the spectrum of an optical signal. It can be shown that the cyclic narrowband filtering of an optical signal can also be achieved using a cascaded structure without feedback but at the expense of using several cascaded stages that would be more massive in comparison with the simple ring resonator structure if only passive spectral filtering is required. Another example is the possibility to shape the cyclic transmission response into a periodic sawtooth
pattern utilizing a -6dB 4×4 MMI coupler equipped with a single feedback path. This particular study case is considered in this research but not included in the thesis for brevity. The cyclic sawtooth transmission response can be used in converting a wavelength shift in the optical carrier into intensity modulation as in remote optical sensing of a moving object utilizing Doppler effect. The sawtooth shaping of a transmission response using a cascaded structure without feedback is not handy to design.

The utilization of envelope and wavelength modulations of the transmission responses in the forward-path cascaded structures of Chapter (4) demonstrates the capability to (de)multiplex several narrowband and wideband channels within each complete frame of the periodic transmission interferogram. In this chapter, it is shown that using simpler feedback structures constructed of less number of cascaded IIR components it is possible to achieve both functions of envelope and wavelength modulations of the transmission responses for the cyclic (de)multiplexing of narrowband and wideband channels. The compact universal processors presented in Chapter (4) can provide extreme modular flexibility to perform as both broadband routers and bandwidth variable tunable (de)multiplexers but still at the expense of large structures. A universal 1st order forward-path 2×2 processor, for example, requires the cascading of three sections. In this chapter, a simpler 2×2 feedback structure of a single interconnection section is demonstrated to behave similarly as a universal processor using less number of components. Also, different kinds of binary bandwidth variable 2×2 (de)multiplexers utilizing different architectures of cascaded IIR sections are covered in the chapter. In these (de)multiplexers the bandwidth of the (de)multiplexed channels can be changed into a binary number of different values depending on the type and number of cascaded IIR sections.

The specific objectives of the chapter are:

1- To study the effect of adding attenuation in the feedback path of a single-loop feedback component that utilizes the -3dB 2×2 MMI coupler on tuning the filtering characteristic of its transmission response.

2- To exploit the deployment of multi-feedback paths utilizing the -6dB 4×4 MMI coupler for the (de)multiplexing and filtering of transmission responses, including wavelength modulated functions.

3- To demonstrate the construction of compact binary bandwidth variable integrated phased array (de)multiplexers.
4- To utilize feedback for the construction of tunable envelope and wavelength modulation (de)multiplexers that are used for the (de)multiplexing of narrowband and wideband channels using less number of cascaded stages and less number of components compared to their cascaded FIR counterparts.

5- To construct compact universal feedback processors of smaller footprints compared to their FIR counterparts.

As it is known in systems theory, a system with all forward signal paths exhibits a transfer function characterized by a finite impulse response (FIR). On the other hand, a system provided with feedback signal paths in addition to the forward signal paths exhibits a transfer function characterized by an infinite impulse response (IIR). The number of transfer function zeros and poles characterizes the degree of the system which depends on the hardware complexity of the system derived by the number of paths, number of (de)multiplexing points, number of cascaded stages, and circuit topology of the system. The number of zeros and poles and their relative locations in the complex plane of analysis determines the transfer function response of the system. All of the PICs studied in the previous chapters contain only forward optical signal paths and therefore have transfer functions featuring zeros only. The inclusion of feedback paths introduces poles in the transfer functions that add an additional degree of diversity and freedom to the design of optical systems including the (de)multiplexers, filters, signal processors and spectral (de)modulators as demonstrated utilizing analysis and simulations in this chapter.

As mentioned earlier, feedback in optical systems is advantageous in constructing more compact signal processors. The feedback in optical systems means the feeding of one or more of the optical output signals of a component or system to one or more of the inputs. In the general context, the deployment of single or multiple forward and feedback paths in a MIMO system can shape the desired response and behavior depending on the targeted application. The microring resonators well known in contemporary literature based on either directional couplers or MMI couplers are examples of the deployment of feedback in optical systems. A directional coupler based microring resonator is built of a directional coupler component and a feedback path which is usually but not necessarily formed into a circular pathway that connects one of the outputs to one of the inputs of the ring resonator. The directional coupler is constructed of two waveguides placed in proximity to each other to form efficient direct coupling between the two waveguides forming a coupling device with
two inputs and two outputs. The coupling factor between the two waveguides of the directional coupler can be adjusted by controlling the gap width between the two waveguides and by controlling the length of interaction between the two waveguides. The length of the feedback path determines the FSR of the resultant transmission response. For a specified length of the feedback path, the circular pathway allows for the utilization of the maximum possible curvature radius of the waveguide bend used and hence reduces the losses introduced by the feedback path. This is the reason why a circular pathway has been the preferred option. Transmission responses, usually with either narrow passbands or narrow stopbands, might be achieved utilizing simple systems of directional coupler based microring resonators.

A directional coupler is a 2×2 coupling device, and therefore it enables only one feedback loop to be attached to it. Due to this fact and the fact that directional coupler based microring resonators have been already covered widely in contemporary literature by many researchers, this chapter does not consider ring-resonator implementations of the feedback circuits demonstrated here although the same methodology may be applied. Several types of research encountered in contemporary literature have brought the deployment of MMI couplers into the construction of ring resonators and feedback circuits. In an example, the work of Le et al. [162] utilized the -3dB 2×2 MMI coupler which enables the construction of a feedback component with a single feedback loop, one input, and one output but no attenuator was deployed in the feedback path to tune the filtering characteristic of the transmission interferogram. In replacement to the use of an attenuator in the feedback loop of a single MMI coupler, one can use an MZI as a tuning coupling element instead of the MMI coupler with fixed coupling ratio. However, this increases the system complexity, introduces more loss, which affects the performance of high-quality resonance circuits, and increases the length of the feedback loop, which decreases the FSR. Although photonic IIR signal processing systems are known, e.g. [27,163], the deployment of feedback for the construction of flexible spectral processors for applications in EONs providing comprehensive capabilities to steer the spectrum, change the channel bandwidth, (de)multiplex different bandwidth channels and operate at different modes still needs attention in the literature. Many applications require narrow bandwidth filtering at the level of 100 MHz as in microwave photonics and wireless communications. To achieve narrow bandwidth processing at high-quality factor resonance very low waveguide loss at the level of 0.1 dB/cm is required providing that the length of the used delay lines is in the centimeter range. Li at al. [164]
demonstrated IIR processors that are compact and can achieve high resolution and large FSR based on using microring architectures. However, the high-quality resonance circuits of Li at al. are found sensitive to the fabrication process. The demand for agility to control bandwidth and FSR for applications in microwave photonics including radio communications, radar, and terahertz imaging has motivated researches to examine the development of advanced PIC architectures constructed of meshes of interconnected FIR and IIR MZI-based units constructed of directional couplers and ring resonators as in [165,166]. In conclusion, the race to explore the proper circuit topologies to tune the light spectrum for specific applications and to remedy the degradation effects of losses, phase errors, and non-uniformities in split ratios of the components has already started, and it is expected to continue inspiring more researchers.

This chapter delves deep into the study of photonic MIMO feedback components and flexible processors that utilize MMI couplers, and it establishes the methodology of analysis for the utilization of more advanced feedback processors in EONs. Feedback components using 2×2 and 4×4 MMI couplers with systems of single and multiple feedback loops equipped with attenuators if needed to control the amount of the applied feedback and the availability of single and multiple inputs and outputs are analyzed, and their corresponding transmission responses are simulated. The utilization of the studied feedback components into the construction of flexible photonic phased array processors is demonstrated. The photonic feedback phase-controlled systems are also utilized to produce envelope modulation and direct wavelength modulation of the transmission responses to enable the (de)multiplexing of narrowband and wideband cyclic channels with the provision of wavelength steering capability. The construction of binary bandwidth variable feedback (de)multiplexers of 2×2 sizes is also presented. These (de)multiplexers can deal with photonic signals of different bandwidths. The design of a universal feedback photonic processor that can operate as a broadband router and as a tunable (de)multiplexer is also presented in the chapter. The versatility to (de)multiplex signals of different bandwidths, convert between the broadband and WDM regimes, switch the bandwidth of the (de)multiplexed channels, filter the optical spectrum, and steer the channels entitle the flexible feedback structures presented in this research as candidate building elements for the equipment used in EONs.

As in the previous chapters, the transfer function for each studied feedback photonic processor presented in this chapter is derived and used to determine the transmission responses numerically.
The analysis based on the assumption of ideal system components is also used to explain the principle of operation and describe the quantitative parameters of the proposed processors. The wavelength-dependent S-parameters of the construction components of Chapter (2) simulated using FIMMPROP are used in this chapter as in Chapters (3) and (4) to account for any possible phase errors and bandwidth limitations in the responses of the components. The path loss of the feedback loops without adding attenuators, and the path loss of the delay imbalances used in the forward path coupling networks, are accounted in the simulated transmission responses. The accounted path loss includes the simulated loss figure of the straight waveguides of 2.5 dB/cm and the additional loss due to the waveguide bends. However, this loss effect is found minimal because it is much smaller than the loss introduced by the other components used in the system, such as the tapers, phase shifters, and MMI couplers. The chapter is organized to introduce the -3dB 2×2 MMI coupler with a single feedback loop at first in Section (5.2.1) to study the effect of including attenuation in the feedback path. Multiple feedback loops utilizing a single -6dB 4×4 MMI coupler are introduced next in Section (5.2.2). A feedback binary bandwidth variable (de)multiplexer supporting four different channel bandwidths is demonstrated in Section (5.3). A cascaded feedback processor that can be engineered to achieve either envelope or wavelength modulation (de)multiplexing is presented in Section (5.4). Finally, in Section (5.5) a feedback cascaded processor of compact size equipped with phase shifters in its feedback loops is utilized for the implementations of a universal 2×2 processor and binary bandwidth variable (de)multiplexing with wavelength steering of the channels.

5.2 Single Feedback Components Utilizing MMI Couplers

This section of the chapter presents the utilization of -3dB 2×2 and -6dB 4×4 MMI couplers for the construction of feedback elements constructed of single MMI couplers and single or multiple feedback paths. These single component feedback elements can be used as standalone processing units for the filtering and (de)multiplexing of similar and different bandwidth channels. They can also be deployed in integrated cascaded phased array architectures to form more advanced signal processors as demonstrated in next sections of the chapter.

5.2.1 Feedback Component Utilizing the -3dB 2×2 MMI Coupler

Figure 5.1 shows the schematic details of the basic feedback component considered in this section with single-loop single-input-single-output (SL-SISO) utilizing the -3dB 2×2 MMI coupler. Tapers are used at the input and output ports of the MMI coupler. Two of the tapers are in fact part of the
loopback path that connects the second output to the second input of the MMI coupler. An attenuator is added in the loopback path to adjust the extent of the optical feedback signal to bring the transfer function zero on the unit circle for deep notch filtering of the spectrum. The equivalent simple symbolic diagram of the feedback component is shown in the same figure. The parameters of the feedback component used in the analysis of its response are indicated in the same figure. In a ring resonator circuit that utilizes a directional coupler the transmission response can be brought in the over, critical, and under coupling regimes by controlling the coupling coefficients of the directional coupler and providing whether gain is deployed or not in the feedback loop. The SL-SISO element of Figure 5.1 uses an MMI coupler with a fixed split ratio, and therefore the filtering is controlled by the attenuator added in the feedback loop.

Figure 5.1 Schematic (left) and symbolic (right) diagrams of a feedback component with single-loop, single-input, and single-output utilizing the -3dB 2×2 MMI coupler.

The total length of the feedback loop $L$ indicated in Figure 5.1 includes the lengths of the loop straight waveguides, lengths of the bend waveguides of radius $r$, equivalent lengths of the two tapers that are part of the feedback loop and equivalent length of the attenuator. The FSR is specified by $L$ and the equivalent length of the MMI coupler $L_{mmi}$. The equivalent lengths of the tapers can be taken as their physical lengths for simplicity. The wavelength-dependent phase contributed by the feedback path is given as in Eq. (3.12) by $\phi = (2\pi/\lambda)n_eL$, where $n_e$ is the wavelength-dependent effective index of the waveguide. The losses of the bends and straight waveguides, as well as the loss introduced by the attenuator, are all accounted for by the magnitude forward scattering parameter $T$ of the feedback loop. The radius of the waveguide bends is usually taken large enough such that $T$ corresponds mainly to the attenuator loss. The wavelength-dependent small loss due to each taper of the feedback loop is accounted for separately by the magnitude forward scattering parameter $T_{tp}$. Each taper can be substituted in analysis with a zero-length device that contributes
with the wavelength-dependent loss for the forward transfer relations corresponding to $T_{tp}$ and a lossless waveguide of equivalent length which might be approximated to the taper physical length. The $T_{tp}$ contributions of the hypothetical zero-length tapers can be attributed within the MMI coupler forward scattering relations as managed in the symbolic diagram of Figure 5.1.

The two tapers at the input and output ports of the feedback component as well as any additional straight waveguides that might be attached to them contribute with an accumulated phase shift that can be dropped from the forward scattering relations. This is true if the absolute phase shift contributed by the feedback component is not significant in the design of the photonic system, which is usually the case in well balanced generalized MZI structures. The accumulated phase shift is only significant if the MZI-PIC has unbalanced branches containing different types of components, a case which is never mistaken in the thesis. Therefore, the accumulated phase shift is dropped from the derived relations herein. Based on the schematic diagram of the left part in Figure 5.1, one can write the following forward scattering relations:

\[
\hat{I}_1 = T_{tp} I_1 \quad \ldots \text{Eq. (5.1)}
\]

\[
O_1 = T_{tp} \hat{O}_1 \quad \ldots \text{Eq. (5.2)}
\]

\[
[O] = \begin{bmatrix} \hat{O}_1 \\ \hat{O}_2 \end{bmatrix} = [r][I] = \begin{bmatrix} t_{11} & t_{12} \\ t_{12} & t_{11} \end{bmatrix} \begin{bmatrix} \hat{I}_1 \\ \hat{I}_2 \end{bmatrix} \quad \ldots \text{Eq. (5.3)}
\]

$t_{kl} = T_{kl} \exp(j \theta_{kl})$ are the MMI coupler forward scattering parameters. In Equations (5.1) and (5.2) the accumulated phase shift factor $\exp(j \theta_{tp})$ due to each one of the input and output tapers is omitted from the expressions. The loopback relation of the feedback component is expressed as:

\[
\hat{I}_2 = T_{tp}^2 T e^{j \varphi} \hat{O}_2 \quad \ldots \text{Eq. (5.4)}
\]

As mentioned before the wavelength-dependent phase shift $\varphi$ accounts for the total equivalent length of the loopback path, including the waveguides, bends, tapers, and attenuator. Alternatively, based on the symbolic equivalent diagram of the right part in Figure 5.1, one can write the following forward scattering relations for the device:

\[
[O] = \begin{bmatrix} O_1 \\ O_2 \end{bmatrix} = T_{wp}^2 [r][I] = T_{wp}^2 \begin{bmatrix} t_{11} & t_{12} \\ t_{12} & t_{11} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} \quad \ldots \text{Eq. (5.5)}
\]

The feedback relation, in this case, is expressed as:
The solution of Equations (5.1) through (5.4) and the solution of Equations (5.5) and (5.6) both yield the same result for the forward scattering parameter $\tau = O/I = O_1/I_1$ of the feedback device given by:

$$\tau = T_{11}^2 + \frac{TT_{12}^2 e^{j\phi}}{1 - TT_{12}^2 e^{j\phi}}$$

... Eq. (5.7)

The quantitative parameters for the transmission responses of the feedback element of Figure 5.1 can be determined based on the assumption of ideal system components for which case Eq. (5.7) may be given by:

$$\tau = j\kappa \cdot \frac{1 - j2\kappa T e^{j\phi}}{1 - j\kappa T e^{j\phi}} = \frac{je^{j\phi_{\text{mmi}}}}{\sqrt{2}} \cdot \frac{1 - j\sqrt{2} T e^{j\phi_r}}{1 - j\frac{1}{\sqrt{2}} T e^{j\phi_r}}$$

$$\kappa = \frac{1}{\sqrt{2}} \exp\left(j \frac{2\pi}{\lambda} n_e L_{\text{mmi}} \right), \quad \varphi_T = \varphi + \varphi_{\text{mmi}} = \frac{2\pi}{\lambda} n_e (L + L_{\text{mmi}})$$

... Eq. (5.8)

where $\kappa$ is a scattering parameter due to the MMI coupler and $\varphi_T$ is the phase contributed due to the sum of the length of the loop waveguide $L$ and the equivalent length of the MMI coupler $L_{\text{mmi}}$. The magnitude of the transfer function element $\tau$ of Eq. (5.8) may be given by:

$$|\tau|^2 = 2 \cdot \frac{T^2 + \sqrt{2} T \sin \varphi_T + 1/2}{T^2 + 2\sqrt{2} T \sin \varphi_T + 2}$$

... Eq. (5.9)

Based on Eq. (5.9) the transmission pattern minima occur when $\sin \varphi_T = -1$:

$$|\tau|^2_{\text{min}} = 2 \cdot \frac{T^2 - \sqrt{2} T + 1/2}{T^2 - 2\sqrt{2} T + 2}$$

... Eq. (5.10)

Nulling the transmission pattern minima in Eq. (5.10) requires setting the loop attenuation to the value of $T = 1/\sqrt{2} \approx 0.7071$ (i.e. $T_{\text{dB}} \approx -3$ dB). Also based on Eq. (5.9) the transmission pattern maxima occur when $\sin \varphi_T = 1$:

$$|\tau|^2_{\text{max}} = 2 \cdot \frac{T^2 + \sqrt{2} T + 1/2}{T^2 + 2\sqrt{2} T + 2}$$

... Eq. (5.11)

Therefore, for the optimum setting of $T = 1/\sqrt{2}$ (-3 dB) the transmission pattern maxima for the ideal case of ignoring the losses introduced due to the scattering parameters of the MMI coupler is about 0.888 (-0.512 dB). The FSR length $\Lambda$ and its equivalent bandwidth $B_{\text{FSR}}$ of the periodic transmission...
pattern due to Equations (5.7) through (5.9) is given as in Equations (3.17) and (3.18), respectively, after replacing $\Delta L$ with $L + L_{\text{mmi}}$. The magnitude of the transfer function element $\tau$ of Eq. (5.9) for the optimum setting of $T = 1/\sqrt{2}$ is reduced to:

$$|\tau|^2 = 2 \cdot \left( \frac{1 + \sin \varphi_r}{2.5 + 2 \sin \varphi_r} \right) \quad \ldots \text{Eq. (5.12)}$$

Eq. (5.12) can be used to determine the half-power bandwidth $B_{ch}$ of the filtered passband channels providing that $|\tau|^2 = |\tau|^2_{\text{max}}/2$ at the half-power wavelength points: $B_{ch} = \zeta B_{\text{FSR}}$; where $\zeta = \Delta \theta / \pi \approx 0.7951672$ and $\Delta \theta \approx 143.13^\circ$. Here also $Q_S = B_{\text{FSR}}/(B_{\text{FSR}} - B_{ch}) = 1/(1 - \zeta) \approx 4.882$ represents the quality factor for the stopband filtering characteristic. The length $L_{\text{mmi}}$ is equivalent to the length of a waveguide of index $n_e$ and group index $n_g$. The length $L_{\text{mmi}}$ can be estimated from the simulated phase characteristic of the MMI coupler that is almost a linear function of wavelength using either one of the following linear relations taking the phases of the scattering parameters of the MMI coupler as $\Delta \phi_1$ and $\Delta \phi_2$ at the respective wavelengths of $\lambda_1$ and $\lambda_2$:

$$L_{\text{mmi}} = \frac{1}{2\pi} \cdot \frac{\Delta \phi_1 - \Delta \phi_2}{n_{\lambda_1} - n_{\lambda_2}} \quad \text{and} \quad L_{\text{mmi}} = \frac{\Delta \phi_1 - \Delta \phi_2}{\lambda_2 - \lambda_1} \cdot \frac{\lambda^2}{2\pi n_g} \quad \ldots \text{Eq. (5.13)}$$

where $\lambda_1 < \lambda_2$ and $n_g \approx 5.06$ at $\lambda = 1.55$ µm based on the simulation characteristic of Figure 3.24. The equivalent length of the -3dB $2 \times 2$ MMI coupler based on the simulated phase characteristic of Figure 3.2 is about $L_{\text{mmi}} = 46$ µm.

If the loss of the feedback path mainly contributed by the attenuator in dB units is given as $l$, then $T$ is given by $T = 10^{l/20}$. Practically, the smallest feedback path length $L_{\text{min}}$ is given by $L_{\text{min}} = L_c + 2L_t + 2\pi r$, where $L_c$ is the length of the -3dB $2 \times 2$ MMI coupler, $L_t$ is the taper length, and $r$ is the bend waveguide radius as mentioned before. As an example, taking a bend radius of $r = 5\sim10$ µm, using the -3dB $2 \times 2$ MMI coupler designed in Chapter (2) of length $L_c = 60.66$ µm, and taking the taper length of $L_t = 10$ µm, the minimum loopback length is about $L_{\text{min}} \approx 112.1\sim143.5$ µm. This explains the challenge in utilizing feedback in optical networks when large FSR values are required since longer values of the loopback length results in smaller FSR values.
Figure 5.2 Simulated wavelength-dependent transmission response of the feedback component of Figure 5.1 when the attenuation is set to 0 dB (up), 3 dB (middle), and 100 dB (down). $L = 268.38 \mu m$.

Based on the presented analysis, an example is taken here for setting the passband channel bandwidth to $B_{ch} = 150$ GHz. The required length of the feedback loop is $L = 268.38 \mu m$. This results in an FSR value of $A = 1.51$ nm. Figure 5.2 depicts the resulted transmission responses for three loopback path losses of 0 dB, 3 dB, and 100 dB. The attenuation of 0 dB corresponds to the maximum level of feedback resulting in the filtering characteristic with poor stopband responses that are seen in the top part of Figure 5.3. The inclusion of the 3 dB attenuation improves the shape of the filtering transmission characteristic with stopband minima closer to the null values as seen in the
middle part of Figure 5.3. The setting of the loopback attenuation to a high level such as the 100 dB disables the feedback and hence the component behaves just like a -3dB 2×2 MMI coupler without feedback as seen in the result of the lower part of Figure 5.3. The filtering characteristic when the loopback attenuation is set to 3 dB has periodic delta stopbands, and the signal attenuation, in this case, is about 0.8 dB around the center wavelength of 1.55 µm.

The attenuation included in the feedback path expressed in terms of the factor $T$, which has a range of values from 0 to 1 with the value of $T = 1$ corresponds to no attenuation and with the value of $T = 0$ corresponds to infinite attenuation, controls the location of the system roots. One should notice that the scattering parameter of Eq. (5.7), which represents the transfer function of the SL-SISO element of Figure 5.1, has one pole located inside the unit circle and one zero located outside the unit circle of the complex $z$-plane when no attenuation is included, and therefore the feedback element behaves close to an all-pass filter in this case. Both of the zero and pole of the system are approximately located on one radial line (i.e. both have the same angle) if the MMI coupler is assumed to have ideal split ratios and null phase errors. Increasing the loopback attenuation moves the pole and zero of the system on the same radial line towards the center of the $z$-plane. When the system zero is relocated close to the circumference of the unit circle, the periodic transmission characteristic is tuned perfectly to bandstop-filter the optical spectrum at the periodic wavelength points of intersection of the system zero and the rotating $z$ point, as seen in the middle part of Figure 5.2. When the attenuation is increased to infinity both of the zero and pole of the system are relocated at the center point of the $z$-plane neutralizing their effect such that the system shows no wavelength selectivity any more.

### 5.2.2 Double-Loop Feedback Component Utilizing the -6dB 4×4 MMI Coupler

This section considers the response for two inputs and two outputs of the -6dB 4×4 MMI coupler utilized as a feedback component comprising a maximum of two feedback loops. In particular, the laterally symmetrical configuration of the double-loop double-input-double-output (DL-DIDO) feedback component shown in Figure 5.3 utilizing the -6dB 4×4 MMI coupler is considered here. One feedback loop connects output $O_1$ to input $I_1$, and the second feedback loop connects output $O_4$ to input $I_4$, forming a unique vertically and horizontally symmetrical structure. This structure enables using the minimum possible lengths of the feedback loops with no restrictions compared with other possible configurations. The minimum feedback path length is given by $L_{min} = L_c + 2L_t + 2\pi r$, where
\( L_c \) is the length of the -6dB 4×4 MMI coupler, \( L_t \) is the taper length, and \( r \) is the bend waveguide radius as before. In an example, for a bend radius of \( r = 5-10 \ \mu m \), a taper length of \( L_t = 10 \ \mu m \), and using the -6dB 4×4 MMI coupler designed in Chapter (2) of length \( L_c = 122.2 \ \mu m \), the minimum length of the feedback paths is about \( L_{min} \approx 174-205 \ \mu m \).

![Figure 5.3 Symmetrical feedback component with double-loops, two inputs, and two outputs utilizing the -6dB 4×4 MMI coupler.](image)

The forward scattering relations for the feedback component of Figure 5.3 are expressed by:

\[
\begin{bmatrix}
O_1 \\
O_2 \\
O_3 \\
O_4
\end{bmatrix} = T_\phi^2 \begin{bmatrix} t_{11} & t_{43} & t_{13} & t_{41} \\ t_{21} & t_{33} & t_{23} & t_{31} \\ t_{31} & t_{23} & t_{33} & t_{21} \\ t_{41} & t_{13} & t_{43} & t_{11} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}
\]

... Eq. (5.14)

The feedback relations for the two loops of the feedback component of Figure 5.3 are given by:

\[
I_1 = T_1 e^{j\phi} O_1, \quad I_4 = T_4 e^{j\phi} O_4
\]

... Eq. (5.15)

Here \( T_1 \) and \( T_4 \) model the total losses of the two feedback loops providing the fact that the effect of the loopback path loss without adding attenuators as used to be the case in the forthcoming examples is minimal. The substitution of Eq. (5.15) into Eq. (5.14) and the solution for the final scattering transfer relations of the feedback component yields:

\[
\begin{bmatrix} O_2 \\ O_3 \end{bmatrix} = \begin{bmatrix} \tau_{22} & \tau_{23} \\ \tau_{32} & \tau_{33} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix}
\]

\[
\begin{align*}
\tau_{22} &= T_\phi^2 (t_{33} + T_2 t_{21} \chi_{12} e^{j\phi} + T_4 t_{31} \chi_{42} e^{j\phi}) , \\
\tau_{23} &= T_\phi^2 (t_{23} + T_3 t_{21} \chi_{13} e^{j\phi} + T_4 t_{13} \chi_{43} e^{j\phi}) , \\
\tau_{32} &= T_\phi^2 (t_{23} + T_3 t_{31} \chi_{13} e^{j\phi} + T_4 t_{21} \chi_{43} e^{j\phi}) , \\
\tau_{33} &= T_\phi^2 (t_{33} + T_3 t_{31} \chi_{13} e^{j\phi} + T_4 t_{13} \chi_{43} e^{j\phi})
\end{align*}
\]

... Eq. (5.16)
The transfer scattering parameters $\chi_{12} (= O_2/I_2)$, $\chi_{13} (= O_1/I_3)$, $\chi_{42} (= O_4/I_2)$, and $\chi_{43} (= O_4/I_3)$ for this case are determined from the solution of:

$$\begin{bmatrix}
 a_1 & b_1 & \chi_{12} & \chi_{13} \\
 a_2 & b_2 & \chi_{42} & \chi_{43}
\end{bmatrix} = \begin{bmatrix}
 c_1 & d_1 \\
 c_2 & d_2
\end{bmatrix}$$

... Eq. (5.17)

The parameters $a_k, b_k, c_k$ and $d_k$ ($k = 1, 2$) for this case are given by:

$$a_1 = 1 - T_1 T_2^2 t_1 e^{i/\phi}, \quad b_1 = -T_4 T_2^2 t_4 e^{i/\phi}, \quad c_1 = T_2^2 t_4, \quad d_1 = T_2^2 t_1$$

$$a_2 = -T_2 T_4^2 t_4 e^{i/\phi}, \quad b_2 = 1 - T_4 T_2^2 t_1 e^{i/\phi}, \quad c_2 = T_2^2 t_4, \quad d_2 = T_2^2 t_1$$

... Eq. (5.18)

The solution of Eq. (5.17) can be derived using Cramer’s rule [167] into the form:

$$\chi_{12} = \frac{O_1}{I_2} = \frac{c_1 b_1 - c_2 b_2}{a_1 b_2 - a_2 b_1}, \quad \chi_{13} = \frac{O_1}{I_3} = \frac{d_1 b_1 - d_2 b_2}{a_1 b_2 - a_2 b_1}, \quad \chi_{42} = \frac{O_4}{I_2} = \frac{a_2 c_1 - a_1 c_2}{a_2 b_2 - a_1 b_1}, \quad \chi_{43} = \frac{O_4}{I_3} = \frac{a_2 d_1 - a_1 d_2}{a_2 b_2 - a_1 b_1}$$

... Eq. (5.19)

The performance of the DL-DIDO (de)multiplexer of Figure 5.3 depends on the selection of the lengths $L_1$ and $L_4$ of the feedback loops. To simplify explaining the operation of the feedback (de)multiplexer and describe its quantitative parameters the assumption of ideal components is used in the following analysis. The losses of the feedback loops are assumed null: $T_1 = T_4 = 1$. When the lengths of the feedback loops are taken equal (i.e. $L_1 = L_4 = L$), the elements of the transfer function can be given by the following simplified expressions:

$$\tau_{22} = \tau_{33} = \sin \left[ \frac{m \lambda}{\lambda} (L + L_{\text{mmi}}) + \frac{\pi}{\lambda} \right], \quad \tau_{32} = \tau_{23} = j \cos \left[ \frac{m \lambda}{\lambda} (L + L_{\text{mmi}}) + \frac{\pi}{\lambda} \right]$$

... Eq. (5.20)

where $L_{\text{mmi}} \approx 92.4 \mu m$ represents the equivalent length of the -6dB 4×4 MMI coupler determined using Eq. (5.13) and the simulated phase characteristics of Figures 3.4 and 3.5. The accumulated wavelength-dependent common phase shift is dropped from the expressions of the transfer function elements of Eq. (5.20). The expressions of Eq. (5.20) describe the performance of a traditional 2×2 WDM (de)multiplexer. The channel bandwidth is fixed over the wavelength range in this case. The FSR and $B_{FSR}$ are given as in Equations (3.17) and (3.18) after replacing $\Delta L$ in those equations with the total length $L + L_{\text{mmi}}$ of the two equal-length feedback loops. The channel bandwidth, in this case, is half the equivalent FSR bandwidth: $B_{ch} = B_{FSR}/2$. 

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In the second case of taking different feedback loop lengths (i.e. \(L_1 \neq L_4\)) the transfer function elements for the assumption of ideal components and considering lossless loops can be given as follows after dropping the accumulated common phase shift:

\[
\tau_{22} = \tau_{33} = 1 - \frac{e^{-\frac{j 2\pi}{\Lambda} (L_1 + L_4)} - \sqrt{2} e^{-\frac{j 2\pi}{\Lambda} L_{\text{mmi}}}}{e^{-\frac{j 2\pi}{\Lambda} (t_1 + t_4)} - j e^{\frac{j \pi}{\Lambda}} e^{\frac{2 j \pi}{\Lambda} L_{\text{mmi}}}} e^{-\frac{j 2\pi}{\Lambda} L_{\text{mmi}}} \cos\left(\frac{m e}{\Lambda} \Delta L_{4-1}\right) - e^{-\frac{j 2\pi}{\Lambda} L_{\text{mmi}}} \cos\left(\frac{m e}{\Lambda} \Delta L_{4-1}\right)
\]

\[
\tau_{32} = \tau_{23} = 1 - \frac{e^{-\frac{j 2\pi}{\Lambda} (t_1 + t_4)} + \sqrt{2} e^{-\frac{j 2\pi}{\Lambda} (t_1 + t_4)} - \sqrt{2} e^{-\frac{j 2\pi}{\Lambda} L_{\text{mmi}}}}{e^{-\frac{j 2\pi}{\Lambda} (t_1 + t_4)} - j e^{\frac{j \pi}{\Lambda}} e^{\frac{2 j \pi}{\Lambda} L_{\text{mmi}}}} e^{-\frac{j 2\pi}{\Lambda} L_{\text{mmi}}} \cos\left(\frac{m e}{\Lambda} \Delta L_{4-1}\right) + e^{-\frac{j 2\pi}{\Lambda} L_{\text{mmi}}} \cos\left(\frac{m e}{\Lambda} \Delta L_{4-1}\right)
\]

... Eq. (5.21)

where \(\Delta L_{4-1} = L_4 - L_1\) represents here the difference between the lengths of the two feedback loops. If the path length difference \(\Delta L_{4-1}\) is large enough the expressions of Eq. (5.21) exhibit the wavelength modulation of the channel bandwidth of two (de)multiplexed sinusoidal transmission responses. The term \(\cos(\pi n e \Delta L_{4-1}/\lambda)\) represents the function that wavelength modulates the transmission responses. The FSR length of the wavelength modulating function is given as in Eq. (3.17): \(\Lambda_m = \lambda^2/(n_g \Delta L_{4-1})\).

The slowest ripple in the transmission responses (i.e. the widest channel bandwidth) results when \(\cos(\pi n e \Delta L_{4-1}/\lambda) = 1\). In this particular case, Eq. (5.21) is reduced to:

\[
\tau_{22} = \tau_{33} = \sin\left[\frac{m e}{\Lambda} \left(L_1 + L_4 - L_{\text{mmi}}\right) + \frac{\pi}{8}\right], \quad \tau_{32} = \tau_{23} = j \cos\left[\frac{m e}{\Lambda} \left(L_1 + L_4 + L_{\text{mmi}}\right) + \frac{\pi}{8}\right]
\]

... Eq. (5.22)

The accumulated common phase shift is removed from Eq. (5.22). In this case the wide FSR length \(\Lambda_w\) and its equivalent bandwidth \(B_{\text{FSR},w}\) are given respectively as in Equations (3.17) and (3.18) after replacing \(\Delta L\) in those equations with the length \((L_1 + L_4)/2 + L_{\text{mmi}}\). The wide channel bandwidth, in this case, is given by \(B_{\text{ch},w} = B_{\text{FSR},w}/2\). On the other inverse limit, the fastest ripple in the transmission responses (i.e. the narrowest channel bandwidth) results when \(\cos(\pi n e \Delta L_{4-1}/\lambda) = 0\). In this particular case, Eq. (5.21) is reduced to:

\[
\tau_{22} = \tau_{33} = \sin\left[\frac{m e}{\Lambda} \left(L_1 + L_4 + 2L_{\text{mmi}}\right)\right], \quad \tau_{32} = \tau_{23} = j \cos\left[\frac{m e}{\Lambda} \left(L_1 + L_4 + 2L_{\text{mmi}}\right)\right]
\]

... Eq. (5.23)

The accumulated common phase shift is also removed from Eq. (5.23). In this case the narrow FSR length \(\Lambda_n\) and its equivalent bandwidth \(B_{\text{FSR},n}\) are given respectively as in Equations (3.17) and
(3.18) after replacing $\Delta L$ in those equations with the length $L_1 + L_4 + 2L_{\text{mmi}}$. The narrow channel bandwidth, in this case, is given by $B_{\text{ch,N}} = B_{\text{FSR,N}}/2$. Therefore, the narrow channel bandwidth $B_{\text{ch,N}}$ is always half the wide channel bandwidth $B_{\text{ch,W}}$. If the path length difference $\Delta L_{4,1}$ is taken small enough, the function $\cos(\pi n_e \Delta L_{4,1}/\lambda)$ becomes almost constant over the entire C-band. For the particular case of setting $\Delta L_{4,1} = \pm 0.36$ $\mu$m the modulating function $\cos(\pi n_e \Delta L_{4,1}/\lambda) \approx 0$ over the entire C-band and therefore the transfer function elements are described as in Eq. (5.23). In this case, the channel bandwidth is set constant to $B_{\text{ch,N}}$ over the entire C-band.

Three simulation examples for the (de)multiplexer of Figure 5.3 without including attenuators in the feedback loops are considered here. To result in a wavelength-independent channel bandwidth of $B_{\text{ch}} = 100$ GHz utilizing equal-length feedback loops the previous analysis requires: $L_1 = L_4 = c/(2n_g B_{\text{ch}})T_{\text{mmi}} \approx 204.13$ $\mu$m. The upper part in Figure 5.4 depicts this case simulation taken for input $I_2$. This case results in FSR value of $A \approx 1.6$ nm, and the signal attenuation based on simulation is about 0.8 dB at around $\lambda = 1.55$ $\mu$m. In the simulation of the middle part of the same figure also taken for input $I_2$ the lengths of the feedback loops are detuned to $L_1 = (L_1 + L_4)/2 - 0.18 = 203.95$ $\mu$m and $L_4 = (L_1 + L_4)/2 + 0.18 = 204.31$ $\mu$m such that the average length $(L_1 + L_4)/2 = 204.13$ $\mu$m as before. In this case, the short length difference of the feedback loops $\Delta L_{4,1} = 0.36$ $\mu$m turns the (de)multiplexer to the narrow bandwidth of 50 GHz which is half that in the upper part of the same figure. In this case, the FSR value is $A \approx 0.8$ nm, and the signal attenuation based on simulation is about 1.1 dB at around $\lambda = 1.55$ $\mu$m.

The (de)multiplexer in the lower part of Figure 5.4 is set to wavelength modulate the channel bandwidth between the two extreme demands of 100 GHz and 50 GHz. In this case, the average length of the two feedback loops is such that: $(L_1 + L_4)/2 = c/(2n_g B_{\text{ch,W}}) - L_{\text{mmi}} = 204.13$ $\mu$m. The difference in the lengths of the loops might be set to control the number of (de)multiplexed wideband and narrowband channels per complete cycle of the wavelength modulating function. In this simulation example $\Delta L_{4,1}$ is set such that $A_W/A_W = 10$; a condition which requires $|\Delta L_{4,1}| = [(L_1 + L_4)/2 + L_{\text{mmi}}]/(A_W/A_W) \approx 29.65$ $\mu$m. This setting is equivalent to the (de)multiplexing of five wideband equal-bandwidth channels and ten narrowband equal-bandwidth channels per modulating wavelength cycle. The two mentioned design conditions require $L_1 = 189.3$ $\mu$m and $L_4 = 218.95$ $\mu$m.
The signal attenuation estimated based on simulation for this case is about 1 dB at around the center wavelength $\lambda = 1.55 \, \mu\text{m}$.

Figure 5.4 Simulated wavelength-dependent transmission responses of the symmetrical feedback component of Figure 5.3 for input $I_2$ when setting $L_1 = L_4 = 204.13 \, \mu\text{m}$ (up), $L_1 = 203.95 \, \mu\text{m} \& L_4 = 204.31 \, \mu\text{m}$ (middle), and $L_1 = 189.3 \, \mu\text{m} \& L_4 = 218.95 \, \mu\text{m}$ (down). No attenuators included.

It was found in the previous chapter that achieving the wavelength modulation of the transmission responses to enable the (de)multiplexing of narrowband and wideband channels without the use of feedback structures requires the cascading of three sections. Therefore, the passive feedback (de)multiplexer of Figure 5.3 greatly saves on the footprint using only one -6 dB 4×4 MMI coupler.
equipped with two feedback loops providing versatile (de)multiplexing capabilities. This studied (de)multiplexer is also used as a building block to construct advanced tunable processors that are presented in the next sections of this chapter.

5.3 Binary Bandwidth Variable IIR (de)Multiplexers

The cascading of several DL-DIDO symmetrical feedback components each have two equal-length loops interconnected by networks of phase shifters forms a binary IIR (de)multiplexer that can switch the channel bandwidth of two (de)multiplexed transmission responses into several binary values. However, the wavelength steering of the transmission responses is not supported in this case.

To illustrate the operation of the binary bandwidth variable IIR (de)multiplexers proposed in this section the simplified analysis of the 3\textsuperscript{rd} order (de)multiplexer shown in Figure 5.5 is considered at first based on the assumption of ideal system components. This (de)multiplexer is composed of three cascaded stages of balanced symmetrical DL-DIDO 2×2 elements designated as $A$, $C$ and $E$ interconnected by two networks of phase shifters designated as $B$ and $D$. In the following analysis $\tau_{kl(N)}$ represents the transfer function element for output $k$ and input $l$ for the first $N$ cascaded stages. The values of $k$ and $l$ here can be either 2 or 3 as can be seen in the figure. The transfer function elements taken for the first two cascaded stages, i.e. for a 2\textsuperscript{nd} order (de)multiplexer, and taken for all of the three cascaded stages, i.e. for a 3\textsuperscript{rd} order (de)multiplexer, can be given by the following expressions after removing common phases:

\[
\begin{align*}
\tau_{22(2)} &= \tau_{33(2)} = \tau_{22A} \tau_{32C} e^{j \frac{\Delta \phi_2}{2}} + \tau_{32A} \tau_{23C} e^{-j \frac{\Delta \phi_2}{2}}, \quad \tau_{32(2)} = \tau_{23(2)} = \tau_{22A} \tau_{32C} e^{j \frac{\Delta \phi_2}{2}} + \tau_{32A} \tau_{33C} e^{-j \frac{\Delta \phi_2}{2}}
\end{align*}
\]

\[
\begin{align*}
\tau_{22(3)} &= \tau_{33(3)} = \tau_{22A} \tau_{22E} e^{j \frac{\Delta \phi_0}{2}} + \tau_{32A} \tau_{23E} e^{-j \frac{\Delta \phi_0}{2}}, \quad \tau_{32(3)} = \tau_{23(3)} = \tau_{22A} \tau_{32E} e^{j \frac{\Delta \phi_0}{2}} + \tau_{32A} \tau_{33E} e^{-j \frac{\Delta \phi_0}{2}}
\end{align*}
\]

\[
\text{… Eq. (5.24)}
\]

![Figure 5.5 Third order binary bandwidth variable IIR (de)multiplexer.](image)

where $\tau_{klX}$ here represents the transfer function element for output $k$ and input $l$ of the symmetrical 2×2 (de)multiplexer of stage $X$; taking $X$ as either $A$, $C$ or $E$. The (de)multiplexing of full sinusoidal
transmission responses is achieved when each of the control phases $\Delta \phi_B = \Delta \phi_{1B} - \Delta \phi_{2B}$ and $\Delta \phi_D = \Delta \phi_{1D} - \Delta \phi_{2D}$ is set to either 0º or ±180º. Using Eq. (5.20) into Eq. (5.24) yields the following two binary bandwidth control states for a 2nd order (de)multiplexer:

When $\Delta \phi_B = 0º$:

$$\tau_{22(2)} = \tau_{33(2)} = j \cos \left[ \frac{m}{\lambda} (L_A + L_C + 2L_{\text{mmi}}) + \frac{\pi}{4} \right], \quad \tau_{32(2)} = \tau_{23(2)} = \sin \left[ \frac{m}{\lambda} (L_A + L_C + 2L_{\text{mmi}}) + \frac{\pi}{4} \right]$$

When $\Delta \phi_B = \pm180º$:

$$\tau_{22(2)} = \tau_{33(2)} = \cos \left[ \frac{m}{\lambda} (L_A - L_C) \right], \quad \tau_{32(2)} = \tau_{23(2)} = j \sin \left[ \frac{m}{\lambda} (L_A - L_C) \right]$$

where losses of the loops are assumed null and $L_{\text{mmi}} = 92.4$ µm. A 2nd order (de)multiplexer supports two different bandwidths with $\Lambda$ and $B_{FSR}$ determined as in Equations (3.17) and (3.18), respectively, having $\Delta L$ replaced with either $L_A + L_C + 2L_{\text{mmi}}$ when $\Delta \phi_B = 0º$ or $|L_A - L_C|$ when $\Delta \phi_B = \pm180º$. The channel bandwidth $B_{ch}$ is taken as half $B_{FSR}$ as usual. Also, using Eq. (5.20) into Eq. (5.24) yields the following four binary bandwidth control states for a 3rd order (de)multiplexer:

When $\Delta \phi_B = 0º$ and $\Delta \phi_D = 0º$:

$$\tau_{22(3)} = \tau_{33(3)} = \sin \left[ \frac{m}{\lambda} (L_A + L_C + L_E + 3L_{\text{mmi}}) + \frac{3\pi}{8} \right], \quad \tau_{32(3)} = \tau_{23(3)} = j \cos \left[ \frac{m}{\lambda} (L_A + L_C + L_E + 3L_{\text{mmi}}) + \frac{3\pi}{8} \right]$$

When $\Delta \phi_B = 0º$ and $\Delta \phi_D = \pm180º$:

$$\tau_{22(3)} = \tau_{33(3)} = j \sin \left[ \frac{m}{\lambda} (L_A + L_C - L_E + L_{\text{mmi}}) + \frac{\pi}{8} \right], \quad \tau_{32(3)} = \tau_{23(3)} = \cos \left[ \frac{m}{\lambda} (L_A + L_C - L_E + L_{\text{mmi}}) + \frac{\pi}{8} \right]$$

When $\Delta \phi_B = \pm180º$ and $\Delta \phi_D = 0º$:

$$\tau_{22(3)} = \tau_{33(3)} = j \sin \left[ \frac{m}{\lambda} (L_A - L_C - L_E - L_{\text{mmi}}) - \frac{\pi}{8} \right], \quad \tau_{32(3)} = \tau_{23(3)} = \cos \left[ \frac{m}{\lambda} (L_A - L_C - L_E - L_{\text{mmi}}) - \frac{\pi}{8} \right]$$

When $\Delta \phi_B = \pm180º$ and $\Delta \phi_D = \pm180º$:

$$\tau_{22(3)} = \tau_{33(3)} = \sin \left[ \frac{m}{\lambda} (L_A - L_C + L_E + L_{\text{mmi}}) + \frac{3\pi}{8} \right], \quad \tau_{32(3)} = \tau_{23(3)} = j \cos \left[ \frac{m}{\lambda} (L_A - L_C + L_E + L_{\text{mmi}}) + \frac{3\pi}{8} \right]$$

Here again for each binary phase control state $\Lambda$ and $B_{FSR}$ are determined as in Equations (3.17) and (3.18), respectively, having $\Delta L$ replaced with the absolute of the length indicated in the arguments of the sinusoidal terms in Eq. (5.26). The proposed binary feedback (de)multiplexer of Figure 5.5 is expandable to any desired $N^{th}$ order providing up to $2^{N-1}$ of possible different bandwidth control states. Figure 5.6 shows the photonic circuit of an $N^{th}$ order 2×2 binary bandwidth variable feedback (de)multiplexer. The symmetrical feedback (de)multiplexers and the phase shifter elements next to
them are numbered from 1 to $N$, as seen in the figure. The transfer function elements can be given by the backward recursive relations:

$$
\tau_{22(N)} = \tau_{33(N)} = \tau_{22(N-1)} \tau_{22N} e^{j \Delta \phi_{N-1}/2} + \tau_{32(N-1)} \tau_{33N} e^{-j \Delta \phi_{N-1}/2}
$$

$$
\tau_{32(N)} = \tau_{22(N-1)} \tau_{32N} e^{j \Delta \phi_{N-1}/2} + \tau_{32(N-1)} \tau_{33N} e^{-j \Delta \phi_{N-1}/2}
$$

... Eq. (5.27)

**Figure 5.6** General $N^{th}$ order binary bandwidth variable IIR (de)multiplexer.

Here $\Delta \phi_m$ represents the phase shift difference introduced by the $m^{th}$ phase shift element. The magnitudes of the transfer function elements of Eq. (5.27) can be described by simplified relations for the assumption of ideal system components as follows:

$$
|\tau_{22(N)}|^2 = |\tau_{33(N)}|^2 = \cos^2 \left[ \frac{m \pi}{\lambda} L_T + \theta_T + N \frac{\pi}{2} \right],
$$

$$
|\tau_{32(N)}|^2 = |\tau_{33(N)}|^2 = \sin^2 \left[ \frac{m \pi}{\lambda} L_T + \theta_T + N \frac{\pi}{2} \right] 
$$

$$
L_T = \sum_{k=1}^{N} \left( L_k + L_{mm} \right) \prod_{l=k}^{k-1} e^{j \Delta \phi_l},
\theta_T = \sum_{k=1}^{N} \frac{\pi}{8} \prod_{l=k}^{k-1} e^{j \Delta \phi_l}
$$

... Eq. (5.28)

In Eq. (5.28) each phase difference $\Delta \phi$ is assumed to be set to either 0º or ±180º. The total length $L_T$ for each binary phase control state determines the channel bandwidth: $B_{\text{ch}} = B_{\text{FSR}}/2 = c/(2n g |L_T|)$. When one of the binary phase control states yields a zero total length ($L_T = 0$) then the transfer function elements become wavelength-independent, and the inputs of the (de)multiplexer are broadband routed to the outputs as indicated by Eq. (5.28).

The 3rd order binary bandwidth variable (de)multiplexer of Figure 5.5 is considered here for a demonstration of simulation results. The mathematical model of Eq. (5.26) implies three length variables ($L_A, L_C, L_E$) and four possible different channel bandwidths. Therefore, one can design the three different lengths of the feedback loops to satisfy three required channel bandwidths, and the fourth channel bandwidth is implied according to the selected lengths of the loops.
Figure 5.7 Simulated wavelength-dependent transmission responses of the binary bandwidth variable (de)multiplexer of Figure 5.5 for input $I_2$ when applying the phase synthesis states $(\Delta \phi_B, \Delta \phi_D)$ of $(0^\circ, 0^\circ)$ (up), $(\pm 180^\circ, 0^\circ)$ (up-middle), $(0^\circ, \pm 180^\circ)$ (down-middle), and $(\pm 180^\circ, \pm 180^\circ)$ (down). $L_A = 204.13 \, \mu m$, $L_C = 401.81 \, \mu m$, and $L_E = 302.97 \, \mu m$. No attenuators included.
The design procedure might be carried out in different ways utilizing the four control states of Eq. (5.26). The lengths of $L_A = 204.13 \mu m$, $L_C = 401.81 \mu m$, and $L_E = 302.97 \mu m$ are determined here to satisfy the channel bandwidths of 25 GHz ($\Lambda = 0.4$ nm), 50 GHz ($\Lambda = 0.8$ nm), 75 GHz ($\Lambda = 1.2$ nm), and 150 GHz ($\Lambda = 2.4$ nm) when applying the binary phase control states ($\Delta \phi_B$, $\Delta \phi_D$) of (0º, 0º), (±180º, 0º), (0º, ±180º), and (±180º, ±180º), respectively. Figure 5.7 depicts the simulation of the transmission responses for input $I_2$ for the four indicated phase control states resulting into the four channel bandwidths of 25 GHz, 50 GHz, 75 GHz, and 150 GHz shown in the upper, upper-middle, lower-middle, and lower parts of the figure. In summary, the binary bandwidth variable feedback (de)multiplexers demonstrated in this section have a simple structure, use less number of cascaded stages and result into lower signal fading when compared with their counterparts demonstrated in the previous chapter, but steering of the spectrum is not supported.

5.4 Envelope/Wavelength Modulation IIR (de)Multiplexers

This section demonstrates the envelope/wavelength modulation of the transmission responses providing wavelength steering capability of the modulating functions using the phased array IIR (de)multiplexer of Figure 5.8 which consists of two balanced symmetrical DL-DIDO components interconnected by a network of two path length imbalances and two NEMS-operated phase shifters. This compact IIR (de)multiplexer achieves envelope/wavelength modulation of the transmission responses using less number of cascaded stages and less number of components compared with that of Figure 4.19 without feedback. However, the feedback (de)multiplexer of Figure 5.8 provides wavelength steering capability for only the modulating functions, whereas the (de)multiplexer of Figure 4.19 provides wavelength steering capabilities for both of the modulating functions and the fast ripple channels as explained in Section (4.6).

![Figure 5.8 Envelope/Wavelength modulation IIR (de)multiplexer.](image)
Each symmetrical DL-DIDO component is equipped with two equal-length feedback loops. The losses of all loops are taken null in the following simplified analysis that is carried out based on the assumption of ideal system components. The three sections of the (de)multiplexer are designated as $A$, $B$, and $C$, as seen in the figure. The elements of the transfer function after removing the common phase component can be expressed as:

\[
\tau_{22} = \tau_{33} = \tau_{22A} \tau_{22C} e^{-j \frac{\Delta \psi_2}{2}} + \tau_{32A} \tau_{32C} e^{-j \frac{\Delta \psi_2}{2}}, \quad \tau_{32} = \tau_{23} = \tau_{22A} \tau_{32C} e^{-j \frac{\Delta \psi_2}{2}} + \tau_{32A} \tau_{33C} e^{-j \frac{\Delta \psi_2}{2}}
\]

\[
\Delta \psi_B = \Delta \phi_B + \Delta \phi_W, \quad \Delta \phi_B = \frac{2 m_e}{\lambda} \Delta L_B, \quad \Delta L_B = L_{1B} - L_{2B}
\]

... Eq. (5.29)

where the transfer function element $\tau_{kX}$ is defined as before for each $X$ ($A$ or $C$) symmetrical DL-DIDO element of equal-length loops. The substitution of Eq. (5.20) into Eq. (5.29) yields the following simplified expressions:

\[
\tau_{22} = \tau_{33} = j \cos \left( \frac{m_e}{\lambda} L_N + \frac{\pi}{4} \right) \cos \left( \frac{m_e}{\lambda} L_m + \frac{\Delta \phi_B}{2} \right) + \cos \left( \frac{m_e}{\lambda} L_m + \frac{\Delta \phi_B}{2} \right) 
\]

\[
\tau_{32} = \tau_{23} = \sin \left( \frac{m_e}{\lambda} L_N + \frac{\pi}{4} \right) \cos \left( \frac{m_e}{\lambda} L_m + \frac{\Delta \phi_B}{2} \right) + j \sin \left( \frac{m_e}{\lambda} L_m + \frac{\Delta \phi_B}{2} \right)
\]

\[
L_N = L_A + L_C + 2 L_{nni}, \quad L_W = L_A - L_C, \quad L_m = \Delta L_B
\]

... Eq. (5.30)

In Eq. (5.30) the sinusoidal terms with the $\Delta \psi_B/2$ argument represent the modulating functions of the transmission response components. Each modulating function has FSR length of $A_m = \lambda^2/(n_g L_m)$, and it can be wavelength steered by the phase difference $\Delta \phi_B$ as indicated in Eq. (5.30). The modulating functions sinusoidally alter the transmission responses between the (de)multiplexing of narrowband channels of bandwidth $B_{ch,N} = c/(2 n_g L_N)$ and the (de)multiplexing of wideband channels of bandwidth $B_{ch,W} = c/(2 n_g L_W)$ over the wavelength range. Therefore, Eq. (5.30) in the general sense attributes wavelength modulation of the transmission responses yielding the (de)multiplexing of several wideband channels and several narrowband channels per cycle of $A_m$. In the special case of taking equal loop lengths of $L_A = L_C$, the length $L_W = 0$ and the bandwidth of the wideband channels is set to infinity. In this case of equal loop lengths, Eq. (5.30) indicates that only one wideband channel is produced in the forward routing path. The bandwidth of the wideband channel, in this case, might be taken as equal to half the equivalent bandwidth of the modulating function: $B_{ch,W} = B_{FSR,m}/2 = c/(2 n_g L_m)$. Envelope modulation of the transmission responses results in this case since
the low ripple components of the transmission responses turn into just slowly varying modulating functions.

Figure 5.9 Simulated wavelength-dependent transmission responses of the envelope-modulation (de)multiplexer of Figure 5.8 taken for input $I_2$ when the phase difference $\Delta \phi_B$ is set to 0º (up) and 90º (down). $L_A = L_C = 204.13$ µm and $\Delta L_B = 59.31$ µm. No attenuators included.

To demonstrate the envelope modulation of the transmission responses with slow envelope functions the length parameters of the (de)multiplexer of Figure 5.8 are taken as $L_A = L_C = 204.13$ µm and $\Delta L_B = 59.31$ µm. These lengths are determined based on the presented analysis to achieve the (de)multiplexing of 50 GHz ($\Lambda_N \approx 0.8$ nm) narrowband demands and 500 GHz wideband demands. Each wideband demand spans over about 4 nm in wavelength. The taken ratio of $B_{ch, W}/B_{ch, N} = L_N/L_m = 10$ implies the (de)multiplexing of about five narrowband channels with one wideband channel per cycle of the modulating function which has FSR length of $\Lambda_m \approx 8$ nm. The upper part of Figure 5.9 depicts the simulation of the transmission responses for this case taken for input $I_2$ when the two phase shifters are at reset conditions. The lower part of the same figure depicts the shift in the envelope modulating functions by quarter the FSR length of $\Lambda_m = \Lambda_m/4 \approx 2$ nm to the right side of the wavelength range due to applying $\Delta \phi_B = 90º$. 
Figure 5.10 Simulated wavelength-dependent transmission responses of the wavelength-modulation (de)multiplexer of Figure 5.8 taken for input $I_2$ when the phase difference $\Delta\phi_B$ is set to $0^\circ$ (up) and $180^\circ$ (down). $L_A = 352.39$ µm, $L_C = 204.13$ µm, and $\Delta L_B = 37.07$ µm. No attenuators included.

To demonstrate the use of the same (de)multiplexer to achieve wavelength modulation of the transmission responses an example is taken here for setting the bandwidth of the narrowband channels to 40 GHz ($\Lambda_N \approx 0.64$ nm) and the bandwidth of the wideband channels to 200 GHz ($\Lambda_W \approx 3.2$ nm)). The ratio $\Lambda_m/\Lambda_W$ is set to 4 yielding the (de)multiplexing of about two wideband channels and about ten narrowband channels per cycle of the wavelength modulating function which has FSR length of $\Lambda_m \approx 12.8$ nm. These design parameters result in the lengths of: $L_A = 352.39$ µm, $L_C = 204.13$ µm and $\Delta L_B = 37.07$ µm. The upper part of Figure 5.10 depicts the simulation of the transmission responses for input $I_2$ for this case example when both phase shifters of section $B$ are at reset. The lower part of the same figure demonstrates the steering of the wavelength modulating functions to the right side of the wavelength range by half the FSR length of $\Lambda_s = \Lambda_m/2 \approx 6.4$ nm when applying the phase difference $\Delta\phi_B = 180^\circ$.

A comparison between the results of Figure 5.9 and Figure 5.10 confirms the advantage of utilizing direct wavelength modulation of the transmission responses over the envelope modulation.
method to enable the (de)multiplexing of wideband and narrowband channels. In envelope modulation, only one wideband channel is (de)multiplexed with several narrowband channels for every interval of the periodic transmission interferogram. Whereas, in wavelength modulation, several wideband channels can be (de)multiplexed with several narrowband channels within each wavelength period. On the performance side, the minima of the narrowband channels in the envelope modulation of Figure 5.9 are not all deep proficiently. Whereas the filtering characteristic for both of the wideband and narrowband channels in the wavelength modulation of Figure 5.10 is more proficient, and the wavelength boundaries of all channels are better defined. Therefore, the wavelength modulation of the transmission responses with the wavelength steering capability is advised here as a better method for the cyclic (de)multiplexing of wideband and narrowband demands for applications in flexible SDN-EONs.

5.5 Feedback Phased Array Processor with Loop Phase Shifters

This section demonstrates the possibility to construct a 2×2 MZI like broadband switch built of a cascaded feedback phased circuit that is still convertible to the (de)multiplexing of the two inputs at a fixed FSR value with the provision of steering capability of the interferograms. Such a structure represents a universal feedback processor. The same proposed feedback circuit topology is also redesigned to behave as a bandwidth variable (de)multiplexer supporting four different channel bandwidths with the provision of the wavelength steering capability of the transmission responses. These objectives are achieved by adopting the architecture of the feedback processor of Figure 5.11 which has NEMS-operated phase shifters in the loops of the two DL-DIDO elements that are interconnected in cascade by a network of only path length imbalances.

![Figure 5.11 Feedback phased array processor with loop phase shifters.](image-url)
The inclusion of the NEMS-operated phase shifters in the feedback paths increases the minimum length of the loops to \( L_{\text{min}} = 2L_{\text{ph}} + 2\pi r L_c \), where \( L_{\text{ph}} \) is the length of the NEMS-operated phase shifter, \( r \) is the radius of the waveguide bends used in forming the feedback loops, and \( L_c \) is the length of the -6dB 4×4 MMI coupler. Using the length of the 360º NEMS-operated phase shifter designed in Chapter (2) of 355 µm, assuming \( r = 5 \sim 10 \) µm, and letting \( L_c = 122.2 \) µm as in Chapter (2), the minimum length of the loops is about 619~651 µm. The path length imbalances of the interconnection network between the cascaded feedback elements is intended to produce 180º of phase shift evaluated at the center wavelength of 1.55 µm in order to correspond the FSR value of the (de)multiplexed transmission responses to the absolute of the subtraction of the loop lengths of sections A and C of the processor. Therefore, the path length difference of section B is always set to \( \Delta L_B = L_{1B} - L_{2B} = 0.36 \) µm. In the following simplified analysis, which is based on the assumption of ideal system components, the two loops of each DL-DIDO element are set equal in length and their losses are assumed null. The elements of the transfer function of the processor of Figure 5.11 by always taking \( \Delta \phi_B = 180^\circ \) are given by:

\[
\tau_{22} = \tau_{33} = \tau_{22,4} \tau_{22,3} - \tau_{32,4} \tau_{32,3} \\
\tau_{32} = \tau_{23} = \tau_{22,4} \tau_{32,3} - \tau_{32,4} \tau_{33,3} 
\]

... Eq. (5.31)

The elements of the transfer functions of sections A and C can be given by the following expressions after removing the common phase component:

\[
\begin{align*}
\tau_{22, X} &= \tau_{33, X} = \frac{1}{2} \left( e^{j \left( \frac{4\pi L_{\text{X}}}{X} + \Delta \phi_{4, X} \right)} - j\sqrt{2} e^{j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)} e^{j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)} e^{-j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)} \right) \\
&\quad - e^{j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)} e^{-j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)} e^{-j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)} e^{j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)}
\end{align*}
\]

\[
\begin{align*}
\tau_{32, X} &= \tau_{23, X} = \frac{1}{2} \left( e^{-j \left( \frac{4\pi L_{\text{X}}}{X} + \Delta \phi_{4, X} \right)} + j\sqrt{2} e^{-j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)} e^{-j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)} e^{j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)} \\
&\quad - e^{-j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)} e^{j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)} e^{j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)} e^{-j \left( \frac{2\pi L_{\text{X}}}{X} + \frac{\Delta \phi_{4, X}}{2} \right)}
\end{align*}
\]

\[
\Delta \phi_{4, X} = \Delta \phi_{4, X} + \Delta \phi_{4, X}, \quad \Delta \phi_{4, -X} = \Delta \phi_{4, X} - \Delta \phi_{4, X} 
\]

... Eq. (5.32)

where \( X \) refers to either section A or C of the processor. The useful operation of the processor to always (de)multiplex sinusoidal transmission responses results when the phase difference \( \Delta \phi_{4, X} \) of each of sections A and C is set to either 0º or ±180º. Eq. (5.32) yields the following elements for the transfer function of section X after removing the common phase:
When $\Delta \phi_{1T4X} = 0^\circ$:

$$\tau_{22x} = \tau_{33x} = \sin \left[ \frac{m_x}{\lambda} \left( L_x + L_{num} \right) + \frac{\pi}{8} + \frac{\Delta \phi_{14X}}{4} \right]$$

$$\tau_{32} = \tau_{23} = j \cos \left[ \frac{m_x}{\lambda} \left( L_x + L_{num} \right) + \frac{\pi}{8} + \frac{\Delta \phi_{14X}}{4} \right]$$

When $\Delta \phi_{1T4X} = \pm 180^\circ$:

$$\tau_{22x} = \tau_{33x} = \sin \left[ \frac{2m_x}{\lambda} \left( L_x + L_{num} \right) + \frac{\Delta \phi_{14X}}{2} \right]$$

$$\tau_{32} = \tau_{23} = j \cos \left[ \frac{2m_x}{\lambda} \left( L_x + L_{num} \right) + \frac{\Delta \phi_{14X}}{2} \right]$$

The expressions of Equations (5.31) and (5.33) are used to determine the simplified transmission responses in the next two subsections. The solution of Equations (5.31) and (5.33) evaluated for the operational useful phase control states when setting $\Delta \phi_{1T4X}$ of sections $A$ and $C$ to either $0^\circ$ or $\pm 180^\circ$ can be given in general as follows after removing the common phase:

$$\tau_{22} = \tau_{33} = \cos \left[ \frac{m_x}{\lambda} L_T + \theta_T + \Delta \phi_S \right]$$

$$\tau_{32} = \tau_{23} = j \sin \left[ \frac{m_x}{\lambda} L_T + \theta_T + \Delta \phi_S \right]$$

where $L_T$ represents the total length responsible to set the FSR length and channel bandwidth of the (de)multiplexed sinusoidal transmission responses: $\Lambda = \lambda^2/(n_g|L_T|)$ and $B_{ch} = B_{FSR}/2 = c/(2n_g|L_T|)$. The phase shift $\theta_T$ is constant for each phase control state, and $\Delta \phi_S$ represents a voltage-controlled phase shift responsible for the wavelength steering of the transmission responses. The expressions of the three parameters $L_T$, $\theta_T$, and $\Delta \phi_S$ depend on the loop lengths and phase control states of sections $A$ and $C$, as stated in the following two sections.

**5.5.1 Universal Feedback Processor with Loop Phase Shifters**

In this thesis a universal $2 \times 2$ processor is described such that at least it is capable of functioning as both a non-blocking broadband switch to route each input signal to any desired output and of functioning as a tunable WDM (de)multiplexer providing wavelength steering of the (de)multiplexed channels. In this section, to achieve the universal routing and (de)multiplexing behavior for the processor of Figure 5.11 equal loop lengths of $L_A = L_C = L$ have to be used. In such a case, the parameters of the general solution of Eq. (5.34) are described by the expressions in Table 5.1 taken for the four useful phase control states $\Delta \phi_{1T4X}$ of sections $A$ and $C$.

As stated in Table 5.1 the universal processor of Figure 5.11 behaves as a broadband MZI switch when setting the phase control states $(\Delta \phi_{14A}, \Delta \phi_{14C})$ to either $(0^\circ, 0^\circ)$ or $(\pm 180^\circ, \pm 180^\circ)$. In both cases, the intensity of the broadband transmission responses is controlled by $\Delta \phi_S$. On the other
hand the universal processor is turned to WDM (de)multiplex two cyclic channels when setting the phase control states ($\Delta \phi_{1-4A}$, $\Delta \phi_{1-4C}$) to either ($\pm 180^\circ$, $0^\circ$) or ($0^\circ$, $\pm 180^\circ$). In both cases, the channel bandwidth is the same set by the length $L_T = L + L_{\text{mi}}$. The transmission responses are steered differently for the two cases by $\Delta \phi_S$, as stated in table 5.1.

<table>
<thead>
<tr>
<th>$\Delta \phi_{1-4A}$</th>
<th>$\Delta \phi_{1-4C}$</th>
<th>$L_T$</th>
<th>$\theta_T$</th>
<th>$\Delta \phi_S$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$0^\circ$</td>
<td>$0^\circ$</td>
<td>0</td>
<td>0</td>
<td>$\frac{\Delta \phi_{1+4A} - \Delta \phi_{1+4C}}{4}$</td>
</tr>
<tr>
<td>$\pm 180^\circ$</td>
<td>$0^\circ$</td>
<td>$L + L_{\text{mi}}$</td>
<td>$-\frac{\pi}{8}$</td>
<td>$\frac{2\Delta \phi_{1+4A} - \Delta \phi_{1+4C}}{4}$</td>
</tr>
<tr>
<td>$0^\circ$</td>
<td>$\pm 180^\circ$</td>
<td>$-(L + L_{\text{mi}})$</td>
<td>$\frac{\pi}{8}$</td>
<td>$\frac{\Delta \phi_{1+4A} - 2\Delta \phi_{1+4C}}{4}$</td>
</tr>
<tr>
<td>$\pm 180^\circ$</td>
<td>$\pm 180^\circ$</td>
<td>0</td>
<td>0</td>
<td>$\frac{\Delta \phi_{1+4A} - \Delta \phi_{1+4C}}{2}$</td>
</tr>
</tbody>
</table>

Table 5.1 Parameters of Eq. (5.34) for the universal processor of Figure 5.11 when $L_A = L_C = L$.

An example is taken here to set the loop length of $L = 620$ $\mu$m close to the minimum possible length when the bend radius is taken close to $5$ $\mu$m. This selected value of $L$ sets the channel bandwidth to about 42 GHz and the FSR length to about 0.67 nm. In the simulated broadband transmission responses in the upper part of Figure 5.12 taken for input $I_2$ the phase synthesis state ($\Delta \phi_{1A}$, $\Delta \phi_{4A}$, $\Delta \phi_{1C}$, $\Delta \phi_{4C}$) of ($180^\circ$, $180^\circ$, $0^\circ$, $0^\circ$) is applied to crossover switch input $I_2$ to output $O_3$. In the middle part of the same figure, the same simulation is taken for the phase synthesis state of ($180^\circ$, $0^\circ$, $0^\circ$, $0^\circ$) to WDM (de)multiplex the input and output signals. In the lower part of the same figure, the (de)multiplexed transmission responses of the middle part are shifted to the left side of the wavelength range by half the FSR value of $\Lambda_s = \Lambda/2 \approx 0.33$ nm due to applying the phase synthesis state of ($180^\circ$, $0^\circ$, $180^\circ$, $180^\circ$).

The only clear drawback of this universal feedback processor is that the channel bandwidth cannot be made larger than about 42 GHz due to the minimum possible length of the loops; that is the Achilles Heel of the loop implies small FSR values in this case. The only mitigation to this problem is to utilize the design of a shorter NEMS-operated phase shifter. However, for the same possible channel bandwidth, the universal feedback processor of Figure 5.11 uses a smaller structure compared to a replacement cascaded universal processor that does not use feedback.
Figure 5.12 Simulated wavelength-dependent transmission responses of the universal feedback processor of Figure 5.11 for input $I_2$ when setting the phase synthesis state ($\Delta\phi_{1A}$, $\Delta\phi_{2A}$, $\Delta\phi_{1C}$, $\Delta\phi_{2C}$) of $(180^\circ, 180^\circ, 0^\circ, 0^\circ)$ for up, $(180^\circ, 0^\circ, 0^\circ, 0^\circ)$ for middle, and $(180^\circ, 0^\circ, 180^\circ, 180^\circ)$ for down. $L_A = L_C = 620 \, \mu m$ and $\Delta L_B = 0.36 \, \mu m$. No attenuators included.

5.5.2 Binary Bandwidth Variable (de)Multiplexer with Loop Phase Shifters

A recognized drawback of the binary bandwidth variable feedback (de)multiplexers demonstrated in Section (5.3) is that they do not provide wavelength steering capability for the (de)multiplexed channels. In this section, the architecture of Figure 5.11 is used to construct a 3rd order binary bandwidth variable (de)multiplexer that supports wavelength steering of the spectrum using even a
smaller structure that comprises two cascaded DL-DIDO elements instead of three. To achieve this objective different loop lengths of \( L_A \neq L_C \) are used. In such a case, the parameters of the general solution of Eq. (5.34) are described by the expressions in Table 5.2 taken for the four useful phase control states \( \Delta \phi_{1-4X} \) of sections \( A \) and \( C \) of the processor. As stated in Table 5.2 the channel bandwidth of the two (de)multiplexed cyclic signals can be controlled by the four different values of the length \( L_T \) resulted due to the four different binary settings of the phases (\( \Delta \phi_{1-4A}, \Delta \phi_{1-4C} \)). Like the previous section, the transmission responses can be steered by \( \Delta \phi_S \) that is stated in table 5.2. The setting of \( L_T \) to zero for any phase synthesis state (\( \Delta \phi_{1-4A}, \Delta \phi_{1-4C} \)) yields broadband routing of the inputs to the outputs for that binary state.

<table>
<thead>
<tr>
<th>( \Delta \phi_{1-4A} )</th>
<th>( \Delta \phi_{1-4C} )</th>
<th>( L_T )</th>
<th>( \theta_T )</th>
<th>( \Delta \phi_S )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0º</td>
<td>0º</td>
<td>( L_A - L_C )</td>
<td>0</td>
<td>( \Delta \phi_{1-4A} - \Delta \phi_{1-4C} ) / 4</td>
</tr>
<tr>
<td>( \pm 180º )</td>
<td>0º</td>
<td>( 2L_A - L_C + L_{\text{nni}} )</td>
<td>( -\pi / 8 )</td>
<td>( 2\Delta \phi_{1-4A} - \Delta \phi_{1-4C} ) / 4</td>
</tr>
<tr>
<td>0º</td>
<td>( \pm 180º )</td>
<td>( L_A - 2L_C - L_{\text{nni}} )</td>
<td>( \pi / 8 )</td>
<td>( \Delta \phi_{1-4A} - 2\Delta \phi_{1-4C} ) / 4</td>
</tr>
<tr>
<td>( \pm 180º )</td>
<td>( \pm 180º )</td>
<td>( 2(L_A - L_C) )</td>
<td>0</td>
<td>( \Delta \phi_{1-4A} - \Delta \phi_{1-4C} ) / 2</td>
</tr>
</tbody>
</table>

**Table 5.2** Parameters of Eq. (5.34) for the bandwidth variable (de)multiplexer of Figure 5.15 when \( L_A \neq L_C \).

An example is taken here for setting the lengths of the loops of the processor of Figure 5.11 to the values of \( L_A = 718.84 \) \( \mu \text{m} \) and \( L_C = 620 \) \( \mu \text{m} \) determined based on the \( L_T \) values of Table (5.2) in order to set the channel bandwidth to the values of 300 GHz \( (\Lambda \approx 4.8 \text{ nm}) \), 33 GHz \( (\Lambda \approx 0.52 \text{ nm}) \), 48 GHz \( (\Lambda \approx 0.77 \text{ nm}) \), and 150 GHz \( (\Lambda \approx 2.4 \text{ nm}) \) when the phase synthesis state (\( \Delta \phi_{1-4A}, \Delta \phi_{1-4C} \)) is set to (0º, 0º), (180º, 0º), (0º, 180º), and (180º, 180º), respectively, as demonstrated respectively in the simulations of Figure 5.13. In conclusion, the bandwidth variable (de)multiplexer of Figure 5.11 is more compact, can be easily designed to support typical channel bandwidth settings and provides wavelength steering of the transmission responses.
Figure 5.13 Simulated transmission responses of the binary bandwidth variable (de)multiplexer of Figure 5.11 for input $I_2$ when setting the phase synthesis states ($\Delta \phi_{I_A}$, $\Delta \phi_{I_B}$, $\Delta \phi_{I_C}$, $\Delta \phi_{I_D}$) of (0°, 0°, 0°, 0°) for up, (180°, 0°, 0°, 0°) for up-middle, (0°, 0°, 180°, 0°) for down-middle, and (180°, 0°, 180°, 0°) for down. $L_A = 718.84 \, \mu m$, $L_C = 620 \, \mu m$, and $\Delta L_B = 0.36 \, \mu m$. No attenuators included.
5.6 Conclusions

The study of photonic feedback components and photonic feedback phased systems utilizing MMI couplers are presented in this chapter. Several MIMO feedback elements with single and multiple feedback loops are studied. A -3dB attenuator deployed in the loopback path of an SL-SISO feedback component utilizing the -3dB 2×2 MMI coupler is shown necessary to enhance the cyclic stopband notch filtering characteristic of the feedback component. The symmetrical DL-DIDO feedback element utilizing the -6dB 4×4 MMI coupler is demonstrated to perform as a balanced 2×2 passive WDM (de)multiplexer. The imbalance in the lengths of the loopback paths of the feedback DL-DIDO component is demonstrated to result in the wavelength modulation of the transmission responses yielding the (de)multiplexing of narrowband and wideband channels. The cascading of the symmetrical DL-DIDO (de)multiplexers interconnected by networks of NEMS-operated phase shifters is utilized for the construction of bandwidth variable WDM (de)multiplexers. Those processors can change the channel bandwidth to a binary number of values set as needed at different times in flexible optical networks. The inclusion of both NEMS-operated phase shifters and path length imbalances in the interconnection network between two DL-DIDO components is tailored to achieve envelope and wavelength modulations of the transmission responses utilized for the cyclic (de)multiplexing of narrowband and wideband channels with providing the wavelength steering capability of the slowly varying modulating functions.

The architecture of a phased processor constructed of two cascaded DL-DIDO elements each equipped with two NEMS-operated phase shifters interconnected by two path length imbalances that introduce ±180° of phase shift between the transported signals is tailored to either act as a universal 2×2 feedback processor or to act as a binary bandwidth variable (de)multiplexer. The constructed universal feedback processor can both operate in the broadband switching mode providing intensity control of the transmission responses and operate in the WDM (de)multiplexing mode providing one channel bandwidth with wavelength steering of the spectrum. The proposed universal IIR processor has a smaller structure compared with a universal FIR processor of the same order and the same number of I/O ports. However, the universal feedback processor cannot be designed to (de)multiplex wide bandwidth demands. Although a universal 2×2 feedback processor could be demonstrated here, the construction of a larger universal 4×4 feedback processor could be more challenging, and it is left for future work. The proposed binary bandwidth variable feedback processor with loop phase
shifters provides wavelength steering of the transmission responses using compact architecture. It is found that the design of this (de)multiplexer with loop phase shifters is facilitated to control the channel bandwidth for four different standard demands.

In summary, the utilization of feedback in optical systems integrated on the SOI template is found advantageous over forwarding path systems for achieving similar signal processing functions utilizing smaller structures that reduce signal attenuations. However, the main drawback for some of the feedback processors is the possible restrictions on setting the bandwidths of the (de)multiplexed channels due to the restrictions on the minimum lengths of the feedback loops used in the feedback architectures. Feedback architectures are computationally more efficient, but they can also yield more nonlinearities in phase responses.

As a last important comment related to this chapter the understanding of the principle of operation of the photonic feedback processors presented in the thesis can also be related to controlling the behavior of locating the dynamic zeros and poles of the system transfer function in the complex plane of analysis. Appendix (6) in this thesis provides insight into the dynamic complex plane zero-pole analysis of the transfer functions for a number of the proposed feedback processors presented in this chapter as well as touching on some of the cascaded forward-path processors of the previous chapter. The responses of the photonic feedback systems are viewed using wavelength-dependent zero-pole evolution diagrams in that appendix for the first time. This study is expected to pave the way for future related investigations of photonic systems and help to build a better understanding of their operation.
CHAPTER 6
GENERAL REMARKS, CONCLUSIONS, AND FUTURE WORK

The growing need to transport large and small bandwidth demands of industrial enterprises and small businesses by the Internet service providers (ISPs) at different parts of the backbone and distribution networks as well as controlling the quality of service by controlling the transmission bandwidth has motivated this research to utilize the concept of flexible bandwidth (de)multiplexing in EONs. The thesis has mainly studied the utilization of several newly proposed FIR and IIR architectures of phased array processors of MIMO ports that can provide flexible spectral processing and wavelength-selective routing of the photonic signal. This chapter of the thesis derives general remarks and conclusions of the previous chapters providing some directions for possible future improvements in Section (6.1), and it provides some additional guidelines for possible extended future work branching from this research in Section (6.2).

6.1 General Remarks and Conclusions

The underlying industrial technology enabling the fabrication of SOI-based nano-dimension components that can be integrated into advanced photonic processors has made it possible in this thesis to address achieving elastic WDM (de)multiplexing and wavelength-selective routing for better utilization of the telecommunication spectrum available within the C-band. The thesis assumes the utilization of a standard fabrication process enabling the construction of known conventional components of waveguides, tapers, edge couplers, MMI couplers, and NEMS-operated phase shifters that require under-etching in the fabrication step in the box layer underneath the silicon core layer. In particular, the investigation presented in this thesis is devoted to the proposal of novel integrated photonic phased array processors of 2×2 and 4×4 MIMO ports and is devoted to the utilization of cascaded FIR and IIR architectures for constructing advanced photonic processors that can find applications EONs.

The thesis has aimed to provide a realistic study for the performance of the proposed elastic (de)multiplexing and routing processors based on achieving accurate simulations for the wavelength-dependent scattering parameters of the construction components that are designed to meet typical merit figures. All dimensions of the components and lengths of the delay line imbalances and feedback paths that are used in simulations are taken within the manufacturer accuracy, and a chip
has been fabricated based on this fact. The simulated MMI couplers of 2×2 and 4×4 I/O ports, in general, achieve imbalances in the order of 0.1 dB and excess losses of less than 0.5 dB. The straight waveguide has an excess loss of about 2.5 dB/cm. Each linear taper used to couple a waveguide to an input or output port of an MMI coupler introduces a wavelength-dependent average excess loss of about 0.1 dB. Each NEMS-operated phase shifter introduces an average wavelength-dependent excess loss of around 1.27 dB. The NEMS-operated phase shifter has a small footprint of about 350 µm in length encouraging the construction of large PICs. The thesis has also investigated the proposal for utilizing the structure of a dual inverse-taper edge coupler to improve the coupling efficiency of traditional inverse-taper edge couplers built on the same fabrication template by a factor of at least twice. Losses due to the straight waveguides and waveguide bends of the delay line imbalances and feedback paths that are used in all studied processors are included in the numerical computation of the transmission responses. However, this effect is found to be minimal for the studied (de)multiplexers, and therefore, it can be ignored in future studies providing that the waveguide bend radius is relaxed. Losses of the used micrometer length feedback loops are small compared to losses of the MMI couplers and the tapers used at their ports over which feedback is applied. This makes the path loss effect minimal as can be inferred in an example from Eq. (5.7) or Eq. (app.12). The path loss, however, can be of significant effect in high-quality-factor ring resonators and should not be neglected. The delay line imbalances are kept at proximity from each other in the layout of the different studied processors to reduce the effect of variation in fabrication tolerances at the different areas of the chip.

The spectral responses of the studied processors depend on the architectures of the processors and the simulated scattering parameters of the individual construction components. In this regard, conventional plain rectangular MMI couplers are found to be the neck of the bottle components that mostly affect the transmission response of the proposed processors in the wide wavelength range and can worsen it in specific at the wavelength regions close to the edges of the C-band. Therefore, future studies may focus more on improving the frequency response of MMI couplers or even consider replacing them with alternative MIMO port splitting/combining components of small footprints and wide bandwidth responses.

Accommodating the transmission of higher data rates may be achieved by increasing the transmission bandwidth, using higher inter-symbol modulation methods, and utilizing diversity
(de)multiplexing of the signals carrying the data. Some typical optical channel bandwidths might be utilized for the different data rates such as 10 GHz, 50 GHz, 75 GHz, 100 GHz, 150 GHz, and 300 GHz. However, in an elastic optical communication, the channel bandwidth may be adjusted to any in-between values as required in a software-defined network. The transition from the ITU fixed grid networking of 50 GHz slot widths to bandwidth variable and wavelength-steerable (de)multiplexing of the channels is categorized in this thesis into partially-elastic and fully-elastic phases. It is expected that not all segments of an elastic optical network would need to handle non-determinant bandwidth demands but rather several possible channel bandwidths depending on the location of the network segment in the metropolitan optical network.

The flexible utilization of the optical spectrum has been related in the thesis in general to the capabilities to adjust the bandwidth of the channels to several set values, (de)multiplexing of different bandwidth channels, and wavelength steering of the channels. In this regard different (de)multiplexing modes have been studied including combining one wideband channel and several narrowband channels utilizing envelope modulation of the transmission responses, combining several wideband and several narrowband channels utilizing wavelength modulation of the transmission responses, and the categorized (de)multiplexing of groups of binary bandwidth-controlled channels. The direct wavelength modulation of the bandwidth of the (de)multiplexed channels is found advantageous over envelope modulation for maintaining better channel uniformity and providing proficient isolation between the adjacent channels.

The single-stage FIR-MZI element can achieve basic functions for acting as either a broadband switch or tunable WDM (de)multiplexer supporting one channel bandwidth and providing steering capability of the spectrum. The cascaded FIR and IIR phased array architectures presented in the thesis can achieve more advanced elastic spectral processing features. The envelope/wavelength modulation FIR (de)multiplexer presented in the thesis is constructed of three cascaded stages. It has simple architecture and yields sinusoidal modulation of the transmission responses by slowly varying envelope or wavelength modulation functions. The transition between the different bandwidths of the channels is gradual, and noticeably includes distortion in the transition regions. The architecture of a general $N^{th}$ order binary bandwidth variable (de)multiplexer formed of cascaded broadband switches and WDM sections of path length imbalances and phase shifters sandwiched between a splitter and combiner MMI couplers is proposed. The architecture of a
general $N^{th}$ order universal processor constructed of cascaded broadband phased array switches interconnected by WDM sections of path length imbalances and phase shifters is described. It has been found that a universal processor built of more than three sections provides more switching permutations to control the (de)multiplexing bandwidth compared to a binary bandwidth variable (de)multiplexer of a similar number of sections.

The proposed universal processors support multimode operation enabling intensity control and broadband routing of the I/O signals and enabling tunable WDM (de)multiplexing of wavelength-steerable and bandwidth variable channels. However, the architecture of the $4\times4$ FIR cascaded universal processors is large and is expected to present challenges when subjected to fabrication and testing in the lab environment. The utilization of IIR sections that utilize feedback in the cascaded architectures of phased array processors is found advantageous to reduce the design footprint but it might presents difficulties to set the (de)multiplexing channel bandwidth beyond certain limits due to the restrictions on the minimum possible lengths of the feedback loops. Investigating the construction of more advanced flexible photonic processors that for example support shaping the wavelength-switching functions and passband channels into more flattened responses rather than the sinusoidal functions currently described in the thesis is possible but it is expected to result in more complicated architectures.

The thesis has been based on performing detailed and credible simulations of the construction components using FIMMPROP. Analysis of the proposed processors based on the transfer function method without making significant approximations and the use of the simulated wavelength-dependent scattering parameters of the construction components has allowed achieving credible investigation for the performance of the newly proposed flexible processors. The imbalances, excess losses, crosstalk, and other outlined parameters are based on this realistic procedure. Simplified modeling and analysis based on the assumption of ideal system components are utilized to both explain the operation of the proposed phased array processors and to enable the design for certain (de)multiplexing schemes. Real examples for taking certain channel bandwidths are taken in the study cases presented in the previous chapters. The thesis has studied several newly proposed elastic spectral processing units, but yet the deployment of these elements into integrated devices achieving, for example, hierarchical (de)multiplexing of groups of different bandwidth demands is still not verified and is left for future studies.
It seemed that current industrial standards support the construction of flexible spectral processing units of large sizes for elastic (de)multiplexing of varying bandwidth demands. It is recommended to keep the number of cascaded FIR and IIR sections of a flexible processor low to maintain low levels of system loss and signal distortion. Increased losses of high order processors made of many cascaded sections can exacerbate the advantage of flexible bandwidth processing. Overall isolation level of adjacent filtered channels of more than about 15 dB is experienced in the demonstrated simulation results throughout the thesis. Achieving the desired bandwidths of the (de)multiplexed demands by setting the lengths of the MZI path imbalances and the lengths of the feedback loops is found agreeing accurately with the simulation results. The wavelength-dependent imbalances in the split ratios and phase errors of the MMI couplers bring unwanted ripple and distortion in the transmission characteristics. This issue is more apparent in 4×4 MIMO systems. The distortion in the transition regions of wavelength-modulated transmission characteristics contributes to wasting part of the utilized spectrum. The binary bandwidth variable and universal processors seem to provide better harness of the spectrum compared to envelope/wavelength modulation processors since the former methods do not include transition regions. Cascaded FIR architectures provide more freedom in design to set the desired channel bandwidths, and therefore they are expected to dominate the design of high order processors used in EONs. Future improvements in the design of the construction components to greatly reduce the lengths of the phase shift elements and splitting/combining units may eventually favor utilizing cascaded IIR architectures for reduced losses and footprints of the processors.

6.2 Some Suggestions for Future Studies and Related Work
Efficient utilization of the available optical communication bandwidth for applications in SDN-EONs requires variable bandwidth control and wavelength relocation of the (de)multiplexed channels. The relocation of a service channel along the wavelength range should not affect the quality of service provided to a customer at any time. Therefore, with the advent of the generation of SDN-EONs with NAI, it has become evident that further investigations are needed to produce novel components approaching ideal performance for responding over the entire C-band with flat transmission and linear phase characteristic. Improvements on the design of 2×2 and 4×4 MMI couplers or the use of alternative splitting and combining elements with broadband responses and minimal phase errors are required in future studies. Efforts to improve the coupling to SOI chips of
photonic integrated circuits with NEMS-operated elements should also be continued. Future investigations for the construction of programmable polarization controllers using the integration platform chosen in the thesis is also necessary for practical applications of studied systems. Further investigations into the architectures of photonic feedback integrated phased array processors can yield to further structural simplifications and therefore reducing the size of the processors used to achieve certain objectives required in SDN-EONs.

![Figure 6.1 Image from the mask design of the fabricated chip.](image)

This thesis work is complemented with the mask design of a photonic chip based on the CMC’s fabrication process offered for the silicon base layer thickness of 300 nm. Figure 6.1 shows an image from the mask design of the chip which utilizes a patterning area of around $2\times2$ mm$^2$. The design variants include several components, an AWG passive (de)multiplexer, and three single-stage phased array elements. The components include the single-mode waveguide, inverse-taper edge coupler, dual inverse-taper edge coupler, waveguide bend, -3dB $2\times2$ MMI coupler, crossover $2\times2$ MMI coupler, -6dB $4\times4$ MMI coupler, NEMS-operated phase shifter and two transition slot couplers connected back-to-back. A $4\times4$ AWG made of two -6dB $4\times4$ MMI couplers acting as the splitter and combiner units interconnected by four path length imbalances is also included to investigate the resulted characteristic of a basic passive (de)multiplexer. One broadband $2\times2$ IPPAS element and one $2\times2$ WDM (de)multiplexer are included on the chip to facilitate measuring the phase shift.
characteristics of the designed 180° capable NEMS-operated phase shifters. Direct measurement of the phase shift is possible by observing the proportional steering in the peaks of the (de)multiplexed transmission interferograms. Finally, one 4×4 single-stage WDM (de)multiplexer is also included to examine the steering of the transmission responses achieved by applying progressive phase shifts through the four NEMS-operated phase shifters of the (de)multiplexer. The targeted data of lab testing should be used to qualify the performance of the designed components and phased array systems as well as addressing future challenges for the building of the larger PICs explored in this thesis. In the future, the study of the cascaded and feedback architectures presented in this thesis should be verified experimentally. Successful fabrication of the designed nanophotonic chip of Figure 6.1 has been achieved. Some microscopic magnified images of the structures and components of the fabricated chip are depicted in Appendix (5).

The utilization of under-etching in the box layer of the SOI integration template to construct nano-electromechanical components can also be utilized for the development of photonic motion sensors with controlled sensitivity and speed of detection. The design of these sensors can follow similar guidelines used in developing the phased array photonic systems presented in the thesis. The NEMS-operated photonic elements can provide better motion-sensing accuracy compared to the use of capacitive-based MEMS motion sensors. The formerly mentioned method can also provide better voltage-controlled motion-sensing adaptation. At the small nanometer dimension of the separation between the silicon beams of a suspended slot waveguide, the electrostatic forces are effective to control the motion characteristic of the nano-electromechanical sensor. The SOI template allows the construction of small footprint NEMS-operated phased array photonic motion sensors at economical prices. An SOI-based integrated phased array photonic sensor is also compatible with the CMOS technology for the integration of other electronic devices.
REFERENCES


[61] Xu Xue-Jun, Chen Shao-Wu, Xu Hai-Hua, Sun Yang, Yu Yu-De, Yu Jin-Zhong, and Wang Qi-Ming, “High-Speed 2×2 Silicon-Based Electro-optic Switch with Nanosecond Switch


[70] Sawsan Abdul-Majid, Ramón Maldonado-Basilio, Chengmin Lei, Hazem Awad, Imad Hasan, Winnie N. Ye, and Trevor J. Hall, “Performance Analysis of a Photonic Integrated


[163] Wim Bogaerts, Peter De Heyn, Thomas Van Vaerenbergh, Katrien DeVos, Shankar Kumar Selvaraja, Tom Claes, Pieter Dumon, Peter Bienstman, Dries Van Thourhout, and Roel Baets,


APPENDIX 1

Numerical Solution of Eq. (2.3)

The numerical solution of Eq. (2.3) followed in this research is carried out by sampling the excursion width $W_{ex}$ into $N+1$ number of points along the length $L_s$ of the suspended silicon beams. The central difference approximation of Eq. (2.3) is given by:

$$
\frac{W_{ex,k-2} - 4W_{ex,k-1} + 6W_{ex,k} - 4W_{ex,k+1} + W_{ex,k+2}}{z_o^4} = \frac{\varepsilon_o h V^2}{2EIw_{so}^2 \left(1 - 2W_{ex,k} / w_{so}\right)^2} \quad \ldots \text{Eq. (app.1)}
$$

$k$ in Eq. (app.1) is the sampling integer. $z_o = L_s/N$ is the sampling pitch along the length of the suspended silicon beams. The two edge points of the suspended silicon beams are assumed to be fixed for all kinds of boundary conditions; i.e. $W_{ex,0} = W_{ex,N} = 0$. Examining Eq. (app.1) for the values of the integer $k$ from 2 to $N-2$ yields the following set of $N-3$ respective expressions:

$$
-W_{ex,1} + 6W_{ex,2} - 4W_{ex,3} + W_{ex,4} = \frac{\varepsilon_o h V^2 z_o^4}{2EIw_{so}^2 \left(1 - 2W_{ex,2} / w_{so}\right)^2} \quad \ldots \text{Eq. (app.2)}
$$

$$
-W_{ex,3} + 6W_{ex,4} - 4W_{ex,5} + W_{ex,6} = \frac{\varepsilon_o h V^2 z_o^4}{2EIw_{so}^2 \left(1 - 2W_{ex,3} / w_{so}\right)^2}
$$

$$
-W_{ex,5} + 6W_{ex,6} - 4W_{ex,7} + W_{ex,8} = \frac{\varepsilon_o h V^2 z_o^4}{2EIw_{so}^2 \left(1 - 2W_{ex,5} / w_{so}\right)^2}
$$

$$
\vdots
$$

$$
-W_{ex,N-3} + 6W_{ex,N-2} - 4W_{ex,N-1} = \frac{\varepsilon_o h V^2 z_o^4}{2EIw_{so}^2 \left(1 - 2W_{ex,N-2} / w_{so}\right)^2} \quad \ldots \text{Eq. (app.2)}
$$

The fixed boundary conditions require $W_{ex,1} = W_{ex,N+1} = 0$. Examining Eq. (app.1) for the fixed boundary conditions for the two values of the integer $k$ of $l$ and $N-l$ yields the following two respective expressions:

$$
6W_{ex,1} - 4W_{ex,2} + W_{ex,3} = \frac{\varepsilon_o h V^2 z_o^4}{2EIw_{so}^2 \left(1 - 2W_{ex,1} / w_{so}\right)^2} \quad \ldots \text{Eq. (app.3)}
$$

$$
-W_{ex,N-3} - 4W_{ex,N-2} + 6W_{ex,N-1} = \frac{\varepsilon_o h V^2 z_o^4}{2EIw_{so}^2 \left(1 - 2W_{ex,N-1} / w_{so}\right)^2}
$$

The hinged boundary conditions require $W_{ex,1} = -W_{ex,1}$ and $W_{ex,N+1} = -W_{ex,N-1}$. Examining Eq. (app.1) for the hinged boundary conditions for the two values of the integer $k$ of $l$ and $N-l$ yields the following two respective expressions:
Equations (app.2) and (app.3) constitute the solution for the excursion width of the suspended silicon beams for fixed boundary conditions. Similarly, Equations (app.2) and (app.4) provides the solution for the hinged boundary conditions. The samples of the suspended silicon beams excursion width are determined from:

\[ [W_{ex,k}] = [U_{\chi}]^{-1} [f(W_{ex,k})] \]

\[ [f(W_{ex,k})] = \frac{\varepsilon_0 h V^2 z_{o}^4}{2E I W_{so}^2 (1-2W_{ex,1}/W_{so})^2} \cdot \frac{1}{(1-2W_{ex,2}/W_{so})^2} \cdot \cdots \cdot \frac{1}{(1-2W_{ex,N-1}/W_{so})^2} \]

\[ U_{\chi} = \begin{bmatrix} \chi & -4 & 1 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -4 & 6 & -4 & 1 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 & \ldots & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \ldots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 1 & -4 & 6 & -4 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 1 & -4 & 6 & -4 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 1 & -4 & 6 & -4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \ldots & 0 & 0 & 0 & 0 & 1 & -4 & 6 & -4 \end{bmatrix} \]

In Eq. (app.5), \( \chi = 6 \) for fixed boundary conditions, and \( \chi = 5 \) for hinged boundary conditions. For any other approximation in-between the fixed and hinged boundary conditions, the solution coefficient \( \chi \) is in the range between 5 and 6. When the boundary conditions coefficient \( \chi \) is closer to either 6 or 5, the behavior of the suspended silicon beams is biased towards either the fixed or hinged boundary conditions, respectively.

**APPENDIX 2**

**Scattering Matrix Magnitudes of a Single-Stage 4×4 FIR-IPPAS Element**
The magnitudes of the scattering parameters of the single-stage broadband 4×4 FIR-IPPAS element of Figure 3.18 assuming ideal components are given by:

\[ |r_{11}| = |r_{22}| = |r_{33}| = |r_{44}| = \frac{1}{2} \left| \cos^2 \frac{\Delta \phi_{1-2}}{2} + \sin^2 \frac{\Delta \phi_{1-3}}{2} + \sin^2 \frac{\Delta \phi_{1-4}}{2} + \sin^2 \frac{\Delta \phi_{2-3}}{2} + \sin^2 \frac{\Delta \phi_{2-4}}{2} + \cos^2 \frac{\Delta \phi_{3-4}}{2} \right|^{1/2} \]

\[ |r_{21}| = |r_{42}| = |r_{13}| = |r_{34}| = \frac{1}{2} \left| \sin \Delta \phi_{1-2} + \frac{1}{2} \sin \Delta \phi_{1-3} + \cos^2 \frac{\Delta \phi_{1-4}}{2} + \cos^2 \frac{\Delta \phi_{2-3}}{2} - \frac{1}{2} \sin \Delta \phi_{2-4} - \frac{1}{2} \sin \Delta \phi_{3-4} \right|^{1/2} \]

\[ |r_{31}| = |r_{12}| = |r_{43}| = |r_{24}| = \frac{1}{2} \left| -\frac{1}{2} \sin \Delta \phi_{1-2} - \frac{1}{2} \sin \Delta \phi_{1-3} + \cos^2 \frac{\Delta \phi_{1-4}}{2} + \cos^2 \frac{\Delta \phi_{2-3}}{2} + \frac{1}{2} \sin \Delta \phi_{2-4} + \frac{1}{2} \sin \Delta \phi_{3-4} \right|^{1/2} \]

\[ |r_{41}| = |r_{32}| = |r_{25}| = |r_{14}| = \frac{1}{2} \left| \sin^2 \frac{\Delta \phi_{1-2}}{2} + \cos^2 \frac{\Delta \phi_{1-3}}{2} + \sin^2 \frac{\Delta \phi_{1-4}}{2} + \sin^2 \frac{\Delta \phi_{2-3}}{2} + \cos^2 \frac{\Delta \phi_{2-4}}{2} + \sin^2 \frac{\Delta \phi_{3-4}}{2} \right|^{1/2} \]

where \( \Delta \phi_{k-l} = \Delta \phi_k - \Delta \phi_l \) is the difference between the phases of the \( k \)th and \( l \)th NEMS-operated phase shifters.

**APPENDIX 3**

**Scattering Matrix of a Single-Stage 4×4 FIR-WDM (de)Multiplexer**

The scattering matrix parameters of the single-stage 4×4 FIR-WDM (de)multiplexer of Figure 3.26 assuming ideal components are given by:

\[ \tau_{11} = \tau_{22} = -\tau_{33} = -\tau_{44} = je^{j\pi/4} \psi^{2} \left( e^{j\Delta \psi_1} + e^{j\Delta \psi_2} - e^{j\Delta \psi_3} - e^{j\Delta \psi_4} \right) \]

\[ \tau_{21} = j \tau_{42} = j \tau_{13} = \tau_{34} = \psi^{2} \left( -j e^{j\Delta \psi_1} + e^{j\Delta \psi_2} + e^{j\Delta \psi_3} - je^{j\Delta \psi_4} \right) \]

\[ \tau_{31} = -j \tau_{12} = -j \tau_{43} = \tau_{24} = -\psi^{2} \left( je^{j\Delta \psi_1} + e^{j\Delta \psi_2} + e^{j\Delta \psi_3} + je^{j\Delta \psi_4} \right) \]

\[ \tau_{41} = -\tau_{32} = \tau_{23} = -\tau_{14} = je^{j\pi/4} \psi^{2} \left( -e^{j\Delta \psi_1} + e^{j\Delta \psi_2} - e^{j\Delta \psi_3} + e^{j\Delta \psi_4} \right) \]

\[ \text{... Eq. (app.7)} \]

where \( \Delta \psi_k = \Delta \phi_k + \Delta \phi_k \), \( \Delta \phi_k = 2\pi n_k \Delta L_k / \lambda \), \( |\psi| = 0.5 \) and the phase of \( \psi \) is a linear function of wavelength with a negative slope.

**APPENDIX 4**

**Tunable Equi-Ripple (de)Multiplexing of Narrowband and Wideband Channels**

This appendix presents the (de)multiplexing of two cyclic channels of Chebyshev-like and inverse Chebyshev-like transmission responses. This scheme is shown to enable the (de)multiplexing of alternative wideband and narrowband channels. The wideband channels have equi-ripples in the passbands whereas the narrowband channels have equi-ripples in the stopbands. The condition to maintain the equi-ripple characteristic and enable the wavelength steering of the (de)multiplexed channels is derived mathematically and demonstrated with simulations. The bandwidths of the
narrowband and wideband channels are also formulated in terms of the FSR length of the periodic transmission responses and are used in a real (de)multiplexing scheme taken as an example to perform the simulations. The same 2×2 FIR (de)multiplexer of three cascaded sections of Figure 4.19 is used here and is shown again in Figure app.1.

Figure app.1 Symbolic diagram of a tunable 2×2 FIR (de)multiple xer of three cascaded sections used to achieve equi-ripple (de)multiplexing of narrowband and wideband channels.

To achieve equi-ripple (de)multiplexing of narrowband and wideband channels, the path length imbalances of sections A, B and C have to be set such that: \( \Delta L_A = \Delta L_C \) and \( \Delta L_B = \Delta L_A + 0.18 \, \mu \text{m} \). The extra length of \( \Delta L_B \) of \( \lambda/4n_e \approx 0.18 \, \mu \text{m} \) is needed to introduce a phase shift difference for section B of \( \pi/2 \) estimated at \( \lambda = 1.55 \, \mu \text{m} \) compared with sections A and C. If the path length imbalances of the three sections are equal, then the 90° phase shift difference for section B can be introduced by \( \Delta \phi_B \). For this selection of path length imbalances the (de)multiplexer has the following transfer function elements derived for the assumption of ideal components:

\[
|r_{11}| = |r_{22}| = \cos^2 \left( \frac{\Delta \phi_A - \Delta \phi_C}{2} \right) + \frac{1}{2} \cos^2 \left( \Delta \phi_A + \frac{\Delta \phi_A + \Delta \phi_C}{2} \right) - \cos^2 \left( \frac{\Delta \phi_A - \Delta \phi_C}{2} \right) \left[ 1 + \sin(\Delta \phi_A + \Delta \phi_B) \right]
\]

\[
|r_{21}| = |r_{12}| = \sin^2 \left( \frac{\Delta \phi_A - \Delta \phi_C}{2} \right) + \frac{1}{2} \sin^2 \left( \Delta \phi_A + \frac{\Delta \phi_A + \Delta \phi_C}{2} \right) - \sin^2 \left( \frac{\Delta \phi_A - \Delta \phi_C}{2} \right) \left[ 1 + \sin(\Delta \phi_A + \Delta \phi_B) \right]
\]

Eq. (app.8) exhibits equi-ripple (de)multiplexing of narrowband and wideband channels when all phase shifters are at the reset condition of \( \Delta \phi_A = \Delta \phi_B = \Delta \phi_C = 0^\circ \). One of the choices available to maintain the equi-ripple characteristic and steer the transmission responses is to set equal phase differences: \( \Delta \phi_A = \Delta \phi_B = \Delta \phi_C \). Under this condition, Eq. (app.8) is reduced to:

\[
|r_{11}|^2 = |r_{22}|^2 = 1 - |r_{21}|^2 = 1 - \frac{1}{2} \sin^2(\Delta \phi_A + \Delta \phi_B) - \frac{1}{2} \sin^2(\Delta \phi_A + \Delta \phi_C)
\]

\[
|r_{21}|^2 = |r_{12}|^2 = 1 - |r_{11}|^2 = \frac{1}{2} \sin^2(\Delta \phi_A + \Delta \phi_B) + \frac{1}{2} \sin^2(\Delta \phi_A + \Delta \phi_C)
\]

Eq. (app.9) describes an inverse Chebyshev-like transmission response with narrow passband channels and equi-ripple characteristic in the stopbands. \( |t_{11}|^2 \) in Eq. (app.9) describes a Chebyshev-
like transmission response with wide passband channels and equi-ripple characteristic in the passbands. The transmission responses can be wavelength steered by the equal phase as indicated by Eq. (app.9). The FSR length $\Lambda$ of the periodic patterns of Eq. (app.9) as in a single-stage WDM (de)multiplexer with path length difference of $\Delta L_A$ is given by $\Lambda = \frac{\lambda^2}{n_g \Delta L_A}$. The equivalent bandwidth of $\Lambda$ is given as before by $B_{FSR} = \frac{c \Lambda}{\lambda^2} = \frac{c}{n_g \Delta L_A}$. The bandwidth of the narrowband channels $B_{chN}$ can be derived from Eq. (app.9) such that $|\tau_2|^2$ is dropped to 0.5 at the two cutoff wavelengths around the transmission peak: $B_{chN} = \zeta B_{FSR}$; where $\zeta = \Delta \theta / \pi \approx 0.2276961$ and $\Delta \theta \approx 40.9853^\circ$. Here also $Q_N = B_{FSR}/B_{chN} = 1/\zeta \approx 4.39$ represents the quality factor for the filtering characteristic of the narrowband channels. The bandwidth of the wideband channels $B_{chW}$ is given by: $B_{chW} = 1 - B_{chN} = (1 - \zeta) B_{FSR}$. The ratio of the bandwidth of the wideband channels to the bandwidth of the narrowband channels is always: $B_{chW}/B_{chN} = (1 - \zeta)/\zeta \approx 3.392$.

![Figure app.2](image.png)

**Figure app.2** Simulated wavelength-dependent transmission responses for input $I_1$ of the cascaded 2×2 (de)multiplexer of Figure app.1 when the NEMS-operated phase shifters are at reset (up) and when setting $\Delta \phi_A = \Delta \phi_B = \Delta \phi_C = 90^\circ$ (down). $\Delta L_A = \Delta L_C = 135.04 \mu m$ and $\Delta L_B = 135.22 \mu m$.

Taking the example of setting the bandwidth of the narrowband channels to 100 GHz the bandwidth of the wideband channels is set to about 339.2 GHz. For this (de)multiplexing scheme the path
length differences based on the presented analysis are required to be: $\Delta L_A = \Delta L_C = 135.04 \, \mu m$ and $\Delta L_B = 135.22 \, \mu m$. For this selection of the path length differences, the FSR length is given by about $A \approx 3.52 \, \text{nm}$. The upper part of Figure app.2 depicts the simulated transmission responses taken for input $I_1$ when all phase shifters are at reset. The lower part of the same figure depicts the steering of the transmission responses to the right side of the wavelength range by quarter the FSR value of about $\Delta \lambda_s = 0.88 \, \text{nm}$ resulted when applying an equal phase shift difference to the three cascaded sections of $\Delta \phi_A = \Delta \phi_B = \Delta \phi_C = 90^\circ$. Imbalances in the equi-ripples due to the phase errors of the 2×2 MMI couplers can be noticed in the simulation results.

**Figure app.3** Simulated wavelength-dependent transmission responses for input $I_1$ of the cascaded 2×2 (de)multiplexer of Figure app.1 when the NEMS-operated phase shifters are set to: $\Delta \phi_A = \Delta \phi_C = 0^\circ$ & $\Delta \phi_B = 180^\circ$ (up) and when $\Delta \phi_A = \Delta \phi_B = 0^\circ$ & $\Delta \phi_C = 180^\circ$ (down). $\Delta L_A = \Delta L_C = 135.04 \, \mu m$ and $\Delta L_B = 135.22 \, \mu m$.

The equi-ripple characteristic providing wavelength steering capability can also be maintained with the choice of $\Delta \phi_A = \Delta \phi_B$ and $\Delta \phi_C = \Delta \phi_A \pm \pi$. Under this condition, Eq. (app.8) is reduced to:

$$|r_{11}|^2 = |r_{22}|^2 = 1 - |r_{21}|^2 = 1 - \frac{1}{2} \sin^2 (\Delta \phi_A + \Delta \phi_A) + \frac{1}{2} \sin^2 (\Delta \phi_A + \Delta \phi_A)$$
\[ |\tau_{21}|^2 = |\tau_{12}|^2 = 1 - |\tau_{11}|^2 = \frac{1}{2} \sin^2(\Delta\phi_A + \Delta\phi_A) - \frac{1}{2} \sin^2(\Delta\phi_A + \Delta\phi_A) \]

Eq. (app.10) has the effect of reassembling the transmission responses described by Eq. (app.9) shifted in the wavelength range by ±Λ/2. This effect is demonstrated in the upper part of Figure app.3, which depicts the simulated transmission responses for input \( I_1 \) when applying the phase differences: \( \Delta\phi_A = \Delta\phi_B = 0^\circ \) and \( \Delta\phi_C = 180^\circ \). This part of the figure should be compared with the upper part of Figure app.2 to notice the shift in the transmission responses by half the FSR length of 1.76 nm. Another choice available to keep the equi-ripple characteristic and steer the spectrum is to set: \( \Delta\phi_A = \Delta\phi_B = \Delta\phi_C \pm \pi \). In this case the expressions of \( |\tau_{11}|^2 = |\tau_{22}|^2 \) and \( |\tau_{21}|^2 = |\tau_{12}|^2 \) in Eq. (app.9) are exchanged with each other. This results in switching the transmission responses of outputs \( O_1 \) and \( O_2 \) with each other taken for one input. The lower part of Figure app.3 depicts this case when setting: \( \Delta\phi_A = \Delta\phi_B = 0^\circ \) and \( \Delta\phi_C = 180^\circ \). This part of the figure should also be compared with the upper part of Figure app.2 to notice that the passbands and stopbands of outputs \( O_1 \) and \( O_2 \) for input \( I_1 \) replace each other. The two effects demonstrated in Figure app.3 can be combined in addition to the capability to steer the spectrum, as demonstrated in Figure app.2.

In conclusion, the general condition to maintain equi-ripple characteristic and provide steering capability of the spectrum requires \( \Delta\phi_B = \Delta\phi_A \pm k\pi \) and \( \Delta\phi_C = \Delta\phi_A \pm l\pi \), where each of \( k \) and \( l \) is either 0 or 1. Similar performance of the equi-ripple (de)multiplexer of Figure app.1 can be achieved by setting the path length imbalances of the three cascaded sections such that: \( \Delta L_A = \Delta L_C = \Delta L_B + 0.18 \) µm. One can also set \( \Delta L_A = \Delta L_B = \Delta L_C \) and turns \( \Delta\phi_A = \Delta\phi_C = 90^\circ \) to obtain similar sense results. These options have been inspected, but results are not documented for brevity.

**APPENDIX 5**

**Images from the Fabricated Chip**

Some of the magnified microscopic images taken for the components of the chip, which is fabricated using the mask of Figure 6.1, are displayed here. The taken images show that the waveguides, edge tapers, and other components appear to be intact and that the resist is removed after the release process. The release regions at the under-etched areas in the box layer around the suspended slot waveguides were imaged at 100X and show that the BOE process was run long enough to release the
suspended slot waveguides. The images could not be performed using SEM as the devices are expected to become electrostatically sensitive and could collapse under e-beam imaging.

**Figure app.4** The above images show parts from the NEMS-operated phase shifters. The under etching areas in the BOX layer around the suspended silicon beams of the slot waveguides can be seen.

**Figure app.5** The above images show the fabricated processors and components. In the top-left image, the 2×2 and 4×4 WDM (de)multiplexers can be seen. In the top-right image, the broadband and WDM 2×2 (de)multiplexers can be seen. In the lower images, the tip part of the inverse-taper edge couplers can be seen.
APPENDIX 6

Insight into the Dynamic Zero-Pole Evolution Analysis of Photonic Systems

In the theory of system analysis, it is known that the magnitude and phase responses of a simple linear time-invariant system can be determined from the relative location of the zeros and poles in the complex plane of analysis. As an example, a discrete-time system with either finite or infinite impulse response and regular sampling time-interval is characterized by periodic magnitude and phase responses of its frequency-dependent transfer function. In the complex z-plane, the increase in the normalized frequency rotates the z-point in the counterclockwise direction around the unit circle. Conventionally, the zeros and poles of a discrete-time system are located in fixed positions in the z-plane. The change in the position of the z-point as a function of the normalized frequency with respect to the fixed-place zeros and poles determines the periodic magnitude and phase responses of the system. The magnitude response of the system can be obtained by determining the multiplication of the lengths of the distances from the z-point to the system zeros divided by the multiplication of the lengths of the distances from the z-point to the system poles.

In this study the representation of zeros and poles of a photonic system is achieved by defining the z-variable as \( z = \exp(-j\phi) = \exp(-j2\pi n_e L/\lambda) \), where \( n_e \) is the effective index of the used single-mode waveguide, \( \lambda \) is the operating wavelength and \( L \) is a common waveguide length. With this definition, the z-point of a photonic system has a magnitude of unity and phase of \(-2\pi n_e L/\lambda\). Therefore, the z-point rotates with the increase of \( \lambda \) in the counterclockwise direction along the unit circle in the complex z-plane. The relative location of the zeros and poles of a photonic system in the z-plane is taken wavelength dependent. As wavelength increases, the dynamic zeros and poles run into trajectories that could be circular or take other shapes. In the general sense, the zeros and poles rotate into orbits in the clockwise direction around the center point of the z-plane with the increase in \( \lambda \), but they could also reverse their direction of rotation with the increase in \( \lambda \) depending on the design parameters of the photonic system. For each wavelength value, when the z-point becomes closer to one of the zeros (poles) of the photonic system, a transmission pattern minima (maxima) results. A zero or pole is referred in general here as a system root. The roots of the numerator (denominator) polynomial of the transfer function represent the zeros (poles) of the photonic system. When a system root is located into the center point of the z-plane or when a system zero is located at
a very far distance from the unit circle it becomes paralyzed from affecting the wavelength-
dependency of the transmission response of the photonic system based on the rotation of the z-point. 
A system root located at the origin point is always at a unit distance from the z-point for any value of 
\( \lambda \), and hence it does not affect the transmission value. When a system zero is located at a far
distance from the unit circle it becomes almost at the same distance from the z-point for any value of 
\( \lambda \) and in result the rotation of the z-point around the unit circle does not produce ripple in the
transmission pattern due to the distant zero. However, the radial motion of a zero outside or inside
the unit circle can modulate the transmission value of a system transfer function as explored in some
of the forthcoming study examples. When both a zero and pole of a photonic system join together
the same trajectory points into the z-plane for some values of \( \lambda \) they cancel out each other in the
mathematical expression of the system transfer function for those values of \( \lambda \). An auxiliary complex
u-plane might also be necessary to complement the description of the wavelength-dependent
transmission response of some photonic systems. In such a case the rotations of the z-point and u-
point around the unit circles of their complex planes as well as the motion of the system roots in both
planes are synchronized together to produce the periodic transmission response.

This motion behavior of the system roots in a complex plane similar to a constellation of
celestial bodies running into orbits around a central point determines the wavelength-dependent
periodic transmission response of the photonic system. The styles of motion patterns of zeros and
poles could be simple or complicated depending on the photonic system structure complexity and
phase control parameters. The visual control of the relative location of the zeros and poles can teach
the rules to tailor the desired system response. A numerical based software method is used in this
research to observe the behavior of the studied photonic systems. To enable documenting the results
of the study into the thesis using static figures wavelength-dependent zero-pole evolution diagrams
are used to illustrate the motion of the system roots and the motion of the z-point for part of the
wavelength range as needed. This motion feeling is achieved by increasing the width of the root
trajectory with the increase of \( \lambda \). The instances of similar-width trajectories of the different system
roots and z-point correspond to the same \( \lambda \) value. Therefore, the observer can relate the wavelength-
dependent evolution history of the system roots and z-point to the resulted transmission response.
The end simulation points of the zeros, poles, and z-point are also made dark in color when needed
in the forthcoming figures to bring awareness to their relative locations. Additionally, the
transmission pattern resulted from the relative motion of the system zeros and poles with respect to
the \( z \)-point for the considered simulation wavelength range is computed directly from the zero-pole
evolution diagram and depicted underneath it to confirm the validity of the demonstrated results.

The location of the roots of photonic processors, especially those with complicated transfer
functions of high-order polynomials, can be simplified if the MMI couplers are assumed to be ideal
just for the theoretical study purpose. Based on the wavelength-dependent characteristics depicted in
Chapter (3) for the MMI couplers that are designed in Chapter (2), a -3dB 2×2 and -6dB 4×4 MMI
couplers have ideal transfer functions that can be given by:

\[
\begin{bmatrix}
\tau_{11} & \tau_{21} \\
\tau_{21} & \tau_{11}
\end{bmatrix}_{3dB,2×2} = \kappa \begin{bmatrix}
e^{j\pi/2} & 1 \\
1 & e^{j\pi/2}
\end{bmatrix}
\]

\[
\begin{bmatrix}
\tau_{11} & \tau_{21} & \tau_{31} & \tau_{41} \\
\tau_{21} & \tau_{31} & \tau_{43} & \tau_{44} \\
\tau_{31} & \tau_{43} & \tau_{33} & \tau_{31} \\
\tau_{41} & \tau_{43} & \tau_{43} & \tau_{11}
\end{bmatrix}_{6dB,4×4} = \nu \begin{bmatrix}
e^{j3\pi/4} & 1 & -1 & e^{j3\pi/4} \\
1 & e^{j3\pi/4} & e^{j3\pi/4} & -1 \\
-1 & e^{j3\pi/4} & e^{j3\pi/4} & 1 \\
e^{j3\pi/4} & -1 & 1 & e^{j3\pi/4}
\end{bmatrix}
\]

… Eq. (app.11)

The common phasor \( \kappa \) of the -3dB 2×2 MMI coupler has a magnitude of \( 1/\sqrt{2} \approx 0.7071 \) and phase
that is almost a linear function of \( \lambda \) with negative slope as can be predicted from the simulation
results of Figure 3.2. The common phasor \( \nu \) of the -6dB 4×4 MMI coupler has a magnitude of 0.5
and phase that is also almost a linear function of \( \lambda \) with negative slope as can be predicted from the
simulation results of Figures 3.4 and 3.5. The wavelength dependency of the phases of the \( \kappa \) and \( \nu \)
phasors is what rotates the zeros and poles of the system transfer function in the clockwise direction
in the complex \( z \)-plane. The rotation rate and the shape of the trajectory along which a root follows
depend on the structural complexity of the processor.

To explain the motives of the \( z \)-plane evolution diagrams of the dynamic zeros and poles easily
the single-loop resonator feedback element of Figure 5.1, which is shown in Figure app.6 for
convenience, is considered here at first for its simplicity. All system components are assumed to be
ideal, and the attenuation \( T \) of the feedback path is accounted for in the following analysis.
Substituting \( z = \exp(-j\phi) \) in Eq. (5.7) and utilizing Eq. (app.11) yields the simplified \( z \)-plane transfer
function of the feedback element:
This system transfer function has a single zero located at \( q = 2Tt_{11} \) and a single pole located at \( p = q/2 \). As mentioned in Chapter (5) the zero and pole of the single-loop resonator are always located on the same radial line drawn from the center of the \( z \)-plane. The pole is always located at half the distance to the zero measured from the center of the \( z \)-plane. The zero and pole roots are both rotating in the clockwise direction by the same phase angle of \( t_{11} \). When no attenuation is included in the loopback path and neglecting the waveguide and bend losses (i.e. \( T = 1 \)) the zero runs in the clockwise direction in a circle with a radius of \( \sqrt{2} \approx 1.4142 \) outside the unit circle of the \( z \)-plane. It is therefore obvious why the transmission pattern minima are not nulled in this case.

The increase in the attenuation of the feedback loop reduces the radius of the circle trajectory of the system zero. At the exact value of 3 dB of feedback loop attenuation, the system zero runs exactly on the unit circle, since \( |q| = 1 \), as shown in the zero-pole evolution diagram of Figure app.7. In this figure, the circle radius of the system zero is increased by an amount of 0.1 from the actual value to visibly separate the trajectory of the zero from the \( z \)-point path which also runs on the unit circle in the counterclockwise direction. The loopback path length is taken as 200 \( \mu \)m, and only part of the full wavelength range is used in this simulation for clarity of the zero-pole evolution diagram. Otherwise, the zero, pole, and \( z \)-point keep rotating to the end of the full simulation wavelength range of 1.52-1.58 \( \mu \)m. The transmission determined directly from the motion of the system roots and the motion of the \( z \)-point of the zero-pole evolution diagram is also shown in this figure for the same part of the used simulation wavelength range.

Another feature learned from this figure is that the \( z \)-point rotates with the increase in \( \lambda \) at a faster angular rate in a reverse direction compared to that of the zero and pole of the single loop resonator. This is the reason why the thickness of the \( z \)-point trajectory looks almost the same around
since the $z$-point completes more than one round in the meantime the zero and pole of the system complete about only one round of the simulation displayed over part of the wavelength range. This characteristic is true for the other photonic processors presented here. For every value of $\lambda$ at which the $z$-point intersects with the rotating system zero, a transmission pattern null point is produced. However, the non-ideal characteristic of the MMI coupler and other components can weaken this perfection of the transmission pattern nulls. When the attenuation of the feedback loop is increased to infinity both of the zero and pole sink into the origin point ($q = p = 0$) of the $z$-plane making them not effective in shaping the transmission response and the transfer function is simplified to $\tau = t_{11}$ according to Eq. (app.12) as expected.

**Figure app.7** Dynamic zero-pole evolution diagram in the $z$-plane for the single-loop resonator of Figure app.6. $L = 200$ µm and the loop attenuation is 3 dB.

In the next step, the cascade of two single-loop resonator elements of different parameters, as shown in Figure app.8, is taken as a case study to gain skills with the $z$-plane analysis management of photonic systems. Each of the transfer functions of the two cascaded sections can be given as in Eq. (app.12) after replacing $z$ with $z_A = \exp(-j\varphi_A) = \exp(-j2\pi n e L_A/\lambda)$ for section $A$ and replacing $z$ with $z_B = \exp(-j\varphi_B) = \exp(-j2\pi n e L_B/\lambda)$ for section $B$. 

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Figure app.8 Cascade of two single-loop resonator feedback elements utilizing the -3dB 2×2 MMI coupler.

The difference between the $z_A$ and $z_B$ complex planes is due to the possible difference in the lengths of the feedback paths of the two cascaded elements. Each of the $z_A$ and $z_B$ points rotates at a different rate in its complex plane when $L_A \neq L_B$. Allowing the feedback loop losses to be also different (i.e. $T_A \neq T_B$) also imposes different root values for the two cascaded sections of $q_A = 2T_A t_{11}$, $p_A = q_A/2$, $q_B = 2T_B t_{11}$, and $p_B = q_B/2$. Therefore, the total transfer function of the two cascaded single-loop resonators is given by:

$$
\tau = t_{11}^2 \frac{z_A z_B - q_B z_A - q_A z_B + q_A q_B}{z_A^2 - p_A z_A - p_A z_B + p_A p_B}
$$

… Eq. (app.13)

The system zeros and poles can be represented in one complex plane by combining the $z_A$ and $z_B$ planes into one $z$ plane. The transformation $z^2 = z_A z_B$ is substituted in Eq. (app.13) to achieve this objective. The following system transfer function results:

$$
\tau = t_{11}^2 \frac{z^2 + b_q z + c_q}{z^2 + b_p z + c_p} = t_{11}^2 \frac{(z-q_1)(z-q_2)}{(z-p_1)(z-p_2)}
$$

$$
b_q = -q_B \exp(j \pi \Delta L / \lambda), \quad b_p = -p_B \exp(j \pi \Delta L / \lambda), \quad c_q = q_A q_B, \quad c_p = p_A p_B, \quad \Delta L = L_B - L_A
$$

… Eq. (app.14)

The $z$-plane system transfer function of Eq. (app.14) has two zeros $q_{1,2}$ and two poles $p_{1,2}$. The trajectories of the two zeros and two poles are dependent on the values of $T_A$, $T_B$, $L_A$, and $L_B$. When the feedback loop attenuation of the two cascaded sections are set the same (i.e. $T_A = T_B = T$), then $q_A = q_B = q = 2T t_{11}$, $p_A = p_B = p = q/2$, $b_q = -2q \cos(\pi \Delta L / \lambda)$, $b_p = -2p \cos(\pi \Delta L / \lambda)$, $c_q = q^2$, and $c_p = p^2$. Furthermore, when the lengths of the same-attenuation feedback loops of the two cascaded sections are equal (i.e. $\Delta L = 0$), then Eq. (app.14) yields the roots of $q_{1,2} = q$ and $p_{1,2} = p$. In this case, the system has both of its zeros located at the same $z$-point and both of its poles located at the same $z$-point as well.
Figure app.9 Dynamic zero-pole evolution diagram for the cascaded system of two single-loop feedback resonators of Figure app.8. $L_A = 200$ µm and every feedback loop attenuation is 3 dB. $L_B$ equals to 200 µm (up-left), 200.36 µm (up-right), 200.18 µm (down-left), and 230 µm (down-right).
For this reason, the resulted transmission response, in this case, has the same FSR value of a single-section single-loop resonator. On the other hand, when the difference in length of the same-attenuation feedback loops of the two cascaded sections is such that \( m_n \Delta L / \lambda = \pi / 2 \) requiring \( \Delta L = \lambda / (2n_e) \) which is around 0.36 \( \mu \)m, then the system zeros and poles are located at \( q_{1,2} = \pm j q \) and \( p_{1,2} = \pm j p \), respectively. In this case, the two zeros are running on the same circle in the clockwise direction but are separated by 180° of rotation angle from each other for all values of \( \lambda \). The two poles are also rotated by 180° from each other, and they run on a circle with a radius that is half that of the zeros. Each pole runs in-line with one of the two zeros forming a zero-pole dual. The rotation of the system second zero-pole dual by 180° with respect to the first zero-pole dual reduces the FSR length of the transmission response by a half.

The value of \( \Delta L \) can also be adjusted to other values rather than 0.36 \( \mu \)m to control the rotation angle between the system two zero-pole duals to any desired value within the range from 0° to 360° but such that the two zero-pole duals are still rotating by about the same speed with the increase in \( \lambda \). This creates a filtered transmission response of unequal-bandwidth lobes and unequal peak values of the lobes. When \( \Delta L \) is increased much greater than \( \lambda / n_e \) causing the rotation angle to be much greater than 360°, then one of the two zero-pole duals rotates faster than the other one causing modulating the net rotation angle between the two zero-pole duals versus wavelength. This causes wavelength-modulating the filtering characteristic of the transmission pattern.

To explain these principles, some numerical simulations of zero-pole evolution diagrams are depicted here for the different cases taking the example of \( L_A = 200 \) \( \mu \)m and same attenuation of 3 dB of the feedback loops of the two cascaded sections. Each of the trajectories of the two zeros (two poles) are separated from each other in radius differences of \( \pm 0.1 \) (\( \pm 0.05 \)) from their actual paths for clarity of the figures. The upper-left part of Figure app.9 depicts the zero-pole evolution diagram when \( L_B = L_A \) for which case the two zero-pole duals of the system occupy the same location in their circular trajectories. The upper-right part of the same figure depicts the zero-pole evolution diagram when \( L_B = L_A + 0.36 \) \( \mu \)m for which case the two zero-pole duals of the system are separated by 180° from each other. The lower-left part of the same figure depicts the case when \( L_B = L_A + 0.18 \) \( \mu \)m for which case the second zero-pole dual is separated by 90° from the first zero-pole dual of the system transfer function. The lower-right part of the same figure depicts the case when taking \( L_B = 230 \) \( \mu \)m for which case one zero-pole dual rotates at a higher speed with the increase in \( \lambda \) than that of the
other zero-pole dual. Although the simulation of the zero-pole evolution diagram for this case is displayed for part of the wavelength range, the transmission response is forced to be shown for the full wavelength range of 1.52-1.58 µm to make the overall filtering characteristic clear.

As a third case study, the symmetrical DL-DIDO (de)multiplexer of Figure 5.3, which is shown again for convenience in Figure app.10, is considered here. To represent the transfer functions of the system in the complex $z$-plane for the two input ports and two output ports the phasors $z_1 = \exp(-j\varphi_1) = \exp(-j2\pi n_e L_1/\lambda)$ and $z_4 = \exp(-j\varphi_4) = \exp(-j2\pi n_e L_4/\lambda)$ corresponding to the two feedback loops of the (de)multiplexer and the transformation $z^2 = z_1 z_4$ are used in Equations (5.16) through (5.19). The attenuation of both feedback loops is considered null for simplicity and Eq. (app.11) is also used to simplify the resulted expressions further. Consequently, the $z$-plane transfer functions of the DL-DIDO feedback element of Figure app.10 are given by:

\[
\tau_{22} = \tau_{33} = \frac{z^2 + b_{q(22)} z + c_{q(22)}}{z(z - p_2)} = t_{33} \frac{(z - q_{i(22)})(z - q_{2(22)})}{(z - p_1)(z - p_2)}
\]

\[
\tau_{23} = \tau_{32} = \frac{z^2 + b_{q(23)} z + c_{q(23)}}{z(z - p_2)} = t_{23} \frac{(z - q_{i(23)})(z - q_{2(23)})}{(z - p_1)(z - p_2)}
\]

\[
b_{q(22)} = 2(t_1 t_{33}/t_{33} - t_{11}) \cos(\pi n_e \Delta L / \lambda), \quad c_{q(22)} = -4t_1 t_{33} t_{11} / t_{33}
\]

\[
b_{q(23)} = 2(t_1 t_{23}/t_{23} - t_{11}) \cos(\pi n_e \Delta L / \lambda), \quad c_{q(23)} = -4t_1 t_{23} t_{11} / t_{23}
\]

\[
p_1 = 0, \quad p_2 = 2t_{11} \cos(\pi n_e \Delta L / \lambda), \quad \Delta L = L_4 - L_1
\]

… Eq. (app.15)

The four $z$-plane transfer functions of Eq. (app.15) have the same two poles; one pole located at the origin point of the $z$-plane ($p_1 = 0$) and another pole located at $p_2$. However, the locations of the
zeros of $\tau_{22} = \tau_{33}$ are different than those of $\tau_{23} = \tau_{32}$. When the lengths of the two feedback loops are equal (i.e. $\Delta L = 0$), the transfer functions of Eq. (app.15) are simplified to:

$$\tau_{22} = \tau_{33} = t_{33} \frac{z - q_{(22)}}{z}, \quad \tau_{23} = \tau_{32} = t_{23} \frac{z - q_{(23)}}{z}$$

$$q_{(22)} = -2t_{13}t_{31} / t_{33}, \quad q_{(23)} = -2t_{13}t_{21} / t_{23}$$

... Eq. (app.16)

**Figure app.11** Dynamic zero-pole evolution diagram of $\tau_{22}$ of the DL-DIDO feedback element of Figure app.10. $L_1 = L_4 = 210 \mu$m. No attenuators included.

In this case of $\Delta L = 0$, each transfer function has a single pole located at the origin point of the $z$-plane. The zeros of $\tau_{22} = \tau_{33}$ are shifted by 180° from those of $\tau_{23} = \tau_{32}$. Therefore, in this case, the I/O optical signals are conventionally WDM (de)multiplexed such that the bandwidth of the channels is a constant function of $\lambda$ as seen for example in the simulation results of the upper part of Figure 5.4. Whereas, when $\Delta L \neq 0$ and $\Delta L \gg \lambda / n_e$, then according to Eq. (app.15) the bandwidth of the (de)multiplexed signals is wavelength modulated as can be seen, for example, in the simulation results of the lower part of Figure 5.4. However, when $\Delta L \neq 0$ and $\Delta L < \lambda / n_e$, the difference in length
of the feedback loops rotates the two zero-pole duals from each other by a constant angle for the entire C-band.

Figure app.12 Dynamic zero-pole evolution diagram of $\tau_{22}$ of the DL-DIDO feedback element of Figure app.10. $L_1 = 210 \, \mu m$ and $L_4 = 210.36 \, \mu m$. No attenuators included.

Figure app.11 depicts the simulation of the zero-pole evolution diagram of $\tau_{22}$ when $L_1 = L_4 = 210 \, \mu m$ taken as an example of equal-length loops. As in Eq. (app.16) one single pole is located at the origin point, which has no effect on the transmission response, and one zero rotates along the unit circle, which forms equal bandwidth channels. The zero location is shifted from the unit circle by an increase of 0.1 in its trajectory radius for clarity of the figure.

The $\tau_{22}$ zero-pole evolution diagram of Figure app.12 depicts the case when $\Delta L = 0.36 \, \mu m$. The lengths of $L_1 = 210 \, \mu m$ and $L_4 = 210.36 \, \mu m$ are taken here as an example. One pole is located at the origin point having no effect on the transmission response. The other pole moves in a trajectory very close to the origin point due to the small value of the difference $\Delta L$, and hence, it has minimal effect on shaping the transmission response. The two zeros rotate at the same speed versus $\lambda$ around the unit circle, maintaining 180º of rotation angle between them. This has the effect of decreasing the
bandwidth of the channels to half compared to Figure app.11. The two zeros are shifted by ±0.1 of radius differences from their actual trajectories for clarity.

To explain the wavelength modulation of the transmission interferograms of the lower part of Figure 5.4, according to Eq. app.15 the loop lengths of \( L_1 = 210 \) µm and \( L_4 = 260 \) µm are taken here as an example. The zero-pole evolution simulations of \( \tau_{22} \) for this case example are shown in Figure app.13 for wavelength ranges of different increasing lengths. The reason for displaying these images is to explain the nature of movements of the zeros and poles that create the wavelength modulated transmission responses as in the lower part of Figure 5.4. The two system zeros always rotate in the clockwise direction around the unit circle. The trajectories of the system zeros in the upper-left, upper-right and lower-left parts of the figure are shifted by ±0.1 of radius differences from their actual tracks on the unit circle for clarity. The trajectories of the system zeros in the lower-right part of the figure are shifted by ±0.05 of radius differences from their actual tracks on the unit circle and are made thinner for clarity.

The two zeros change their relative angular speeds as functions of wavelength such that one of the two zeros rotates faster than the other one for part of the wavelength range and gets closer to the other zero from one side. Then the other zero speeds up while the first zero slows down until the other zero gets closer to the first zero from the other side, and this continues with the increase in \( \lambda \) throughout the C-band. Either one of the two zeros does never cross the other one. This behavior can be clearly seen by observing the last simulation points that are made darker in color in the upper-left, upper-right, and lower-left parts of Figure app.13. In the upper-left part of Figure app.13, the second zero appears ahead of the first zero at the end simulation point. In the upper-right part, both zeros appear at the same distance from each other at the end simulation point. In the lower-left part, the first zero appears ahead of the second zero at the end simulation point. The first system pole is always located at the origin point as in Eq. (app.15) and it does not affect shaping the transmission response. As seen in the different parts of Figure app.13, the second pole runs as a function of wavelength into rotating circular trajectories drawn between the origin point and the perimeter of the unit circle. This is due to sinusoidally modulating the radial distance of the second pole from the origin point while it continually rotates in the clockwise direction. This pole meets with one of the system zeros when it gets close to the unit circle.
Figure app.13 Dynamic zero-pole evolution diagrams of $\tau_{22}$ of the DL-DIDO feedback element of Figure app.10. $L_1 = 210 \mu$m and $L_4 = 260 \mu$m. No attenuators included.
The visiting pole and zero meet exactly at the point where the pole touches the unit circle at a unit radial distance from the origin as can be seen in the upper-left part of Figure app.13. This temporary zero-pole merging for part of the wavelength range cancels out both of the merged zero and pole roots from the mathematical expression of the transfer function and therefore increases the FSR value to double (i.e. doubles the channel bandwidth) compared to the other wavelength ranges when the pole trajectory becomes closer to the origin point. This is because only one zero remains effective in the transfer function for the wavelength ranges when the other zero merges with the visiting pole. The visiting pole acts as the wavelength-dependent modulating function for the bandwidth of the transmission channels. The visiting pole alternatively meets with the two zeros at the unit circle with the increase in $\lambda$. The lower-right part of Figure app.13 displays the overall trajectory movement of the switching pole for the wavelength range 1.52-1.58 µm. In the four parts of Figure app.13, the $z$-point is disabled (not shown) for clarity of the figures.

![Figure app.14 Feedback 2nd order binary bandwidth variable 2×2 (de)multiplexer.](image)

In the next step of the zero-pole analysis, the two stages 2nd order binary bandwidth variable 2×2 (de)multiplexer shown in Figure app.14 is considered. The system transfer function $\tau_{22}$ for this (de)multiplexer can be expressed as:

$$\tau_{22} = \tau_{22A} e^{i\Delta \phi_{22A}} + \tau_{22C} e^{i\Delta \phi_{22C}}$$

... Eq. (app.17)

The lengths of the feedback loops of each DL-DIDO stage are taken similar (i.e. $\Delta L_A = \Delta L_C = 0$) as seen in the figure. Therefore, each of $\tau_{22A}$, $\tau_{22C}$, $\tau_{32A}$, and $\tau_{32C}$ are given as in Eq. (app.16). The utilization of the transformation $z^2 = z_A z_C$ and the simplifications of Eq. (app.11) yield the following $z$-plane $\tau_{22}$ transfer function:

$$\tau_{22} = 2i_3 e^{i\Delta \phi_{22}} \frac{a_4 z^2 + b_2 z + c_4}{z^2} = 2i_3 a_4 e^{i\Delta \phi_{22}} \frac{(z - q_1)(z - q_2)}{(z - p_1)(z - p_2)}$$
In Eq. (app.18) both poles are always located at the origin point of the z-plane (i.e. \( p_1 = p_2 = 0 \)), and they both do not affect shaping the transmission response. One should also notice that \( a_q \) is a wavelength-independent factor. Therefore, only the motions of the two zeros in their trajectories as functions of wavelength play the role of determining the transmission response of the binary bandwidth-switchable (de)multiplexer. When the voltage-controlled phase shift difference \( \Delta \phi_B \) is set to \( 0^\circ \) the expression of Eq. (app.18) is reduced to:

\[
\tau_{22} = 2t_{33}^2 e^{j\phi_{\text{avg}}} \frac{z^2 + q_{22}^2}{z^2}
\]

… Eq. (app.19)

In this case, the system two zeros are located at \( q_{1,2} = \pm jq_{22} \). These two zeros are rotated by \( 180^\circ \) from each other for all \( \lambda \) values, and they both rotate on the unit circle in the clockwise direction with the increase in \( \lambda \) as shown in the zero-pole evolution diagram of the upper-left part of Figure app.15. The two zeros are drawn shifted by \( \pm 0.1 \) of radius differences from their actual trajectories for clarity of the figure. The equally-spaced two zeros halve the FSR value of the transmission response compared to that of a single zero system. The equal spacing of the two zeros on the unit circle implies the same bandwidth of the (de)multiplexed channels.

In Eq. (app.18), since \( z^2 = z_A z_C = \exp[-j2m_e(L_A + L_C)/\lambda] \), then the FSR of the transmission responses depends on the sum \( L_A + L_C \) as indicated in Chapter (5). The FSR, in this case, depends on the wavelength-dependent angular speeds of the z-point and system zeros. The angular speed of the zeros versus \( \lambda \) is fixed since it depends on the linear phase shift relations of the scattering parameters of the MMI coupler. Therefore, only the sum \( L_A + L_C \), in this case, plays the role of controlling the FSR value. The other useful phase control state of the (de)multiplexer of Figure app.14 results when applying \( \Delta \phi_B = 180^\circ \). In this case, Eq. (app.18) is reduced to:

\[
\tau_{22} = -j4d_{22}^2 t_{33}^2 e^{j\phi_{\text{avg}}} \frac{\cos(m_e \Delta L / \lambda)}{z}
\]

… Eq. (app.20)
Figure app.15 Dynamic zero-pole evolution diagrams of $\tau_{z2}$ of the (de)multiplexer of Figure app.14. $L_A = 210 \ \mu m$ and $L_C = 315 \ \mu m$. $\Delta \phi_0$ is $0^\circ$ (up-left), $180^\circ$ (up-right), and $90^\circ$ (down-left and down-right). No attenuators included.
The $z$-plane function of Eq. (app.20) has one non-effective pole at the origin point. The two zeros of the system transfer function in the $z$-plane disappear from this equation. The zero-pole evolution diagram for this case is shown in the upper-right part of Figure app.15. The transmission response, in this case, is formed due to the cosine term in Eq. (app.20). The term to the left side of the cosine function in this equation has a magnitude of unity for the assumption of ideal MMI couplers. Therefore, the FSR value of the (de)multiplexed transmission interferograms, in this case, depends, as per Eq. (app.20), on the absolute of the difference in lengths of the feedback loops of sections $A$ and $C$, i.e. $|\Delta L| = |L_A - L_C|$, of the (de)multiplexer. Eq. (app.20) also explains the fact that when $\Delta L = 0$, then the setting of $\Delta \phi_B = 180^\circ$ yields a constant $\tau_{22}$ value of unity if the MMI couplers are considered to be ideal and therefore the signal at input $I_2$ is passed to output $O_2$ in a straight-throw broadband routing mode as indicated before in Chapter (5).

When $\Delta \phi_B$ is not equal to either $0^\circ$ or $180^\circ$, the two zeros of the system run as functions of wavelength into different circular trajectories of variable radial distances from the origin point to reflect the gradual transition between the two control states of $\Delta \phi_B = 0^\circ$ and $\Delta \phi_B = 180^\circ$. The trajectories of the two zeros get inside and outside the unit circle while rotating in the clockwise direction with the increase in wavelength. The two lower parts of Figure app.15 depict this case taken for $\Delta \phi_B = 90^\circ$. The lower-right part shows the simulation for the full wavelength range of 1.52-1.58 $\mu$m. The lower-left part shows the simulation for a shorter part of the wavelength range to ease observing the relative location of the two zeros in their wavelength-dependent motion.

As indicated before, the transfer function $\tau_{22}$ of Eq. (app.20) has no wavelength-dependent effective roots in the complex $z$-plane. However, the wavelength-dependent cosine term in that equation, which describes the transmission response relating output $O_2$ to input $I_2$, can still be represented by zeros and poles in an auxiliary complex $u$-plane. The $u$-point is defined for this study case by $u = \exp(-j \Delta \varphi / 2)$, where $\Delta \varphi = 2 \pi n_e \Delta L / \lambda$. The $u$-point rotates versus $\lambda$ in the $u$-plane around the unit circle in the counterclockwise (clockwise) direction when $L_C > L_A$ ($L_C < L_A$) and it becomes at standstill at the point $u = 1$ for all values of $\lambda$ when $L_C = L_A$ ($\Delta L = 0$). Accordingly, Eq. (app.20) can be represented in the complex $u$-plane as:

$$
\tau_{22} = \frac{\tau_2}{z} e^{j \Delta \varphi} = \frac{u^2 + 1}{u}
$$

... Eq. (app.21)
Figure app.16 Dynamic zero-pole evolution diagrams of $\tau_{22}$ of the (de)multiplexer of Figure app.14 in the auxiliary $u$-plane. $\Delta \phi_B = 180^\circ$, $L_A = 210$ $\mu$m & $L_C = 315$ $\mu$m for left and $L_A = L_C = 210$ $\mu$m for right. No attenuators included.

Eq. (app.21) still reflects the fact that $\tau_{22}$ has no zeros and one non-effective pole located at the origin point in the $z$-plane. In the auxiliary $u$-plane, $\tau_{22}$ also has a non-effective pole located at the origin point. However, $\tau_{22}$ has two effective zeros located at $q_{1,2} = \pm j$ in the $u$-plane. These two zeros are $180^\circ$ apart from each other, and both of them are at a standstill for all $\lambda$ values in the C-band. For the same simulation parameters of Figure app.15 of $L_A = 210$ $\mu$m, $L_C = 315$ $\mu$m, and $\Delta \phi_B = 180^\circ$, the left part of Figure app.16 depicts the zero-pole evolution diagram in the $u$-plane for the binary bandwidth-controlled (de)multiplexer of Figure app.14 for this case. The $u$-point rotates in the counterclockwise direction with the increase in $\lambda$ since $L_C > L_A$ in this case. The rotating $u$-point produces a transmission pattern null of $\tau_{22}$ each time it passes one of the two zeros that are located on the unit circle assuming the MMI couplers to be ideal. The speed of rotation of the $u$-point over the wavelength range and therefore the resulted FSR of the transmission response depends on $\Delta L$ in this case. The right part of Figure app.16 depicts the zero-pole evolution diagram for the example of taking $L_A = L_C = 210$ $\mu$m ($\Delta L = 0$). As indicated before, in this case of taking equal length of the
feedback loops of sections \( A \) and \( C \) of the (de)multiplexer the \( u \)-point freezes at the point \( u = 1 \) for all \( \lambda \) values. The \( u \)-point, in this case, is at the same distance of \( \sqrt{2} = 1.4142 \) from both zeros of the system. Therefore, assuming the MMI couplers to be ideal, the magnitude of \( \tau_{22} \) based on Eq. (app.21) is unity for this case. This again indicates the broadband coupling of input \( I_2 \) and output \( O_2 \) of the (de)multiplexer for the phase control state of \( \Delta \phi_B = 180^\circ \) and having \( L_A = L_C \).

\[ T_2'[t], L_A, \phi_A, T_A \quad L_C, \phi_C, T_C \]

**Figure app.17** Envelope/Wavelength modulation feedback (de)multiplexer.

As a last zero-pole case study case of a photonic feedback processor, the envelope/wavelength modulation (de)multiplexer of Figure 5.8, which is shown in Figure app.17 for convenience, is considered here. This (de)multiplexer is similar to the previous one of Figure app.14 with the addition of path length imbalance at section \( B \). Therefore, in a similar manner to Eq. (app.18) the simplified system transfer function \( \tau_{22} \) in the complex \( z \)-plane is given by:

\[
\tau_{22} = 2t_{13}^2 e^{j\Delta \phi_{av}} e^{j\Delta \phi_{1,B}} \frac{a_q z^2 + b_q z + c_q}{z^2} = 2t_{13}^2 a_q e^{j\Delta \phi_{av}} e^{j\Delta \phi_{1,B}} \frac{(z-q_1)(z-q_2)}{(z-p_1)(z-p_2)}
\]

\[
a_q = \cos\left(\frac{\Delta \phi_B + \Delta \phi_B}{2}\right), \quad b_q = -j2q_{22} \sin\left(\frac{\Delta \phi_B + \Delta \phi_B}{2}\right) \cos(\pi nL_2 / \lambda), \quad c_q = q_{22}^2 a_q
\]

\[
\Delta \phi_{B,av} = \pi n(L_1 + L_2) / \lambda, \quad \Delta \phi_{1,B} = (\Delta \phi_{1,B} + \Delta \phi_{2,B}) / 2, \quad \Delta L = L_C - L_A
\]

\[
\Delta \phi_B = 2 \pi n \Delta L / \lambda, \quad \Delta \phi_B = \Delta \phi_{1,B} - \Delta \phi_{2,B}, \quad \Delta L = L_1 - L_2, \quad q_{22} = -2t_{13} t_{31} / t_{33}
\]

… Eq. (app.22)

As in the previous (de)multiplexer, the two system poles in the \( z \)-plane are not effective since they are located at the origin point \((p_{1,2} = 0)\) for all \( \lambda \) values. Unlike the case in Eq. (app.18), the separate \( a_q \) term in Eq. (app.22) is wavelength-dependent. The separate \( a_q \) term can be represented in an auxiliary \( u \)-plane using the definition \( u = \exp(-j\Delta \phi_B/2) \). Therefore, Eq. (app.22) can be re-expressed in both of the complex \( z \)-plane and complex \( u \)-plane as:
\[\tau_{22} = l_3^2 e^{i\Delta \phi_B} e^{i\Delta \phi_z} \cdot \frac{u^2 + e^{i\Delta \phi_z}}{u} \cdot \frac{z^2 + (b_q / a_q)z + (c_q / a_q)}{z^2} \]

\[= l_3^2 e^{i\Delta \phi_B} e^{i\Delta \phi_z} \cdot \frac{(u - q_{1u})(u - q_{2u})}{u} \cdot \frac{(z - q_1)(z - q_2)}{z^2}\]

… Eq. (app.23)

The \textit{u}-term of Eq. (app.23) has one non-effective pole at the origin point and two zeros located at wavelength-independent positions of \(q_{1u,2u} = \pm j \exp(j \Delta \phi_B/2)\). The rate at which the \(z\)-point rotates versus \(\lambda\) in the \(z\)-plane depends on the sum \(L_A + L_C\). The rate at which the \(u\)-point rotates versus \(\lambda\) in the \(u\)-plane depends on \(\Delta L_B\). In Eq. (app.22) the factors \(a_q\), \(b_q\), and \(c_q\) appear to be affected by \(\Delta L_B\). Therefore, the wavelength-dependent motions of the system zeros \(q_{1,2}\) in the complex \(z\)-plane are affected by both \(\Delta L_B\) and \(\Delta L\). Consequently, it is concluded that there is a state of coordination between the rotation of the \(u\)-point and the location of the system zeros in the \(z\)-plane. The filtering characteristic of the system transmission response depends on the selection of the length parameters \(L_A\), \(L_C\), and \(\Delta L_B\). With the selection of \((L_A, L_C, L_A+L_C) \gg \Delta L_B\) the \textit{z}-term of Eq. (app.23) results in fast ripple in the transmission response when the system zeros \(q_{1,2}\) are located close to the unit circle in the \(z\)-plane. The resulted FSR, in this case, depends on the length sum \(L_A+L_C\). On the other hand in the \(z\)-plane when one system zero is at a close distance from the origin point while the other system zero is at a far distance from the unit circle the fast ripple component of the transmission response generated due to the rotation of the \(z\)-point is attenuated. In this case modulating the radial distance of the two zeros in the \(z\)-plane as well as the coherent passage of the \(u\)-point on the unit circle around one of the system zeros in the \(u\)-plane can be controlled to produce a low ripple transmission pattern component with FSR value depending on the absolute length difference \(|\Delta L| = |L_C-L_A|\). In the case of equal-length feedback loops of sections \(A\) and \(C\) (i.e. \(L_A = L_C\)) one broadband transmission pattern lobe is resulted when in the \(z\)-plane one zero heads to the origin point while the other zero heads to a distant location from the unit circle while the \(u\)-point passes through one of the \(u\)-plane zeros. Proper selection of the lengths \(L_A\), \(L_C\), and \(\Delta L_B\) can be used to adjust the characteristic of the transmission response by coherently controlling the motion of the system zeros and the \(z\)-point in the \(z\)-plane and the motion of the \(u\)-point in the \(u\)-plane.
Figure app.18 Dynamic zero-pole evolution diagrams of $\tau_{22}$ of the (de)multiplexer of Figure app.17. $L_A = L_C = 210 \mu m$, $\Delta L_B = 50 \mu m$, and $\Delta \phi_B = 0^\circ$. No attenuators included.
As demonstrated in Chapter (5), the (de)multiplexer of Figure app.17 envelope-modulates the transmission responses when selecting equal lengths of the feedback loops of sections A and C; i.e. when $\Delta L = 0$. The lengths of $L_A = L_C = 210 \, \mu m$ and $\Delta L_B = 50 \, \mu m$ are taken here as an example. The zero-pole evolution diagrams in the $z$-plane and $u$-plane for this case are depicted in Figure app.18 taken for setting $\Delta \phi_B = 0^\circ$. The upper two parts of this figure are taken for the same partial length of the wavelength range, whereas the lower two parts of the figure are taken for the full simulation wavelength range of 1.52-1.58 $\mu m$. The trajectories of the $z$-plane two zeros and the coherent rotation of the $u$-point in this figure explain how envelope modulation of the transmission response is formed. The first $z$-plane zero moves into rotational cyclonic trajectories each one originates from the origin point and heads into a distant radial location from the unit circle. The second zero moves into another set of rotational cyclonic trajectories originating from a distant radial location from the unit circle and sinking into the origin point as clearly seen in the upper part of the figure. The set of cyclonic trajectories of each $z$-plane zero is formed while the zero rotates in the clockwise direction with the increase in wavelength. The trajectories of the first (second) $z$-plane zero form the shape of a right-handed (left-handed) hurricane-like pattern. As $\lambda$ increases the evolved hurricane-like patterns of both $z$-plane zeros appear to spin simultaneously in the counterclockwise direction. The motions of the two zeros in the $z$-plane are synchronized such that when the first zero is at the origin point, then the other zero is located at a distant location from the unit circle and vice versa. The two $z$-plane zeros also cross the unit circle simultaneously, and they are always at 180$^\circ$ apart from each other for all $\lambda$ values. The rotation of the $u$-point along the unit circle is also synchronized with the motion of the $z$-plane zeros in their cyclonic trajectories. When one $z$-plane zero is at the origin point, and the other $z$-plane zero is at a very distant location from the unit circle (theoretically at infinity), the $u$-point is located at one of the $u$-plane zeros. In this case the $z$-plane zero at infinity and the $u$-plane zero crossed by the $u$-point cancel out each other’s effect in the transmission response formula such that the multiplication of the lengths from their locations to the corresponding $z$-point and $u$-point is equal to unity for the $\tau_{22}$ transfer function. On the other hand, when both of the $z$-plane zeros are located close to the unit circle, the $u$-point is located close to one of the two midway points between the two zeros on the unit circle of the $u$-plane. In this case, the $u$-term of Eq. (app.23) is multiplied by a factor close to unity and the rotation of the $z$-point produces the fast ripple pattern of the transmission response. The $z$-point produces a pattern null each time it passes one of the $z$-plane
zeros. The cyclonic motion modulates the radial distance of the system zeros in the $z$-plane in a manner synchronized with the rotation of the $u$-point to produce the envelope-modulated transmission response. The modulation of the radial distance of the $z$-plane zeros in their cyclonic paths sinusoidally damps the fast ripple component of the transmission response as clearly seen in the two mentioned figures. The FSR value of the fast ripple component of the transmission response, in this case, depends on the length $L_A + L_C = 2L_A$. The path length difference $\Delta L_B$ controls the rate at which the system transition between the wideband-lobe and the fast ripple transmission responses. This means that $\Delta L_B$ controls the periodic length of the envelope modulating function of the transmission response for this case of taking $L_A = L_C$.

The wavelength modulation of the transmission interferograms of the (de)multiplexer of Figure app.17 can be achieved when taking $\Delta L \neq 0$. The system parameters of $L_A = 210 \ \mu$m, $L_C = 315 \ \mu$m, and $\Delta L_B = 23 \ \mu$m are taken here as an example. Different simulation instances of zero-pole evolution diagrams in the $z$-plane and $u$-plane for this case are shown in Figure app.19 and Figure app.20. The different parts of these two figures are taken for increasing simulation wavelength ranges. The lower part of Figure app.20 is taken for the full wavelength range of 1.52-1.58 $\mu$m. The simulated motion of the system zeros in their trajectories in the $z$-plane for the increase in wavelength for this case example animates the formation of a tornado-like pattern with turbulent trajectories. The two zeros are always at 180° of rotation angle apart from each other for all $\lambda$ values as before. To understand the operation of the (de)multiplexer for this case, one should recall that when the two zeros are running for a while on the unit circle or close to it the transmission pattern ripples at a small FSR value depending on the sum $L_A + L_C$ as indicated before. On the other hand, when one zero dives inside the unit circle towards the origin point while the other zero moves far away outside the unit circle the fast ripple of the transmission pattern is attenuated. For the parts of the wavelength range when the fast ripples of the transmission response are encountered the $z$-plane zeros of the system rotate around and close to the unit circle as can be seen for example in the upper parts of both mentioned figures. On the other hand, for the other parts of the wavelength range, the two $z$-plane zeros move in and out the unit circle in turbulent trajectories as can be seen in both of the mentioned figures. The turbulent trajectories could be round as can be seen in all parts of the two figures or overshotting between the origin point and distant locations from the unit circle as can be seen at the end simulation points of the lower parts of both figures.
Figure app.19 Dynamic zero-pole evolution diagrams of $\tau_{22}$ of the (de)multiplexer of Figure app.17. $L_A = 210 \ \mu\text{m}$, $L_C = 315 \ \mu\text{m}$, $\Delta L_B = 23 \ \mu\text{m}$, and $\Delta \phi_B = 0^\circ$. No attenuators included.
Figure app.20 Dynamic zero-pole evolution diagrams of $\tau_{22}$ of the (de)multiplexer of Figure app.17. $L_A = 210$ $\mu$m, $L_C = 315$ $\mu$m, $\Delta L_B = 23$ $\mu$m, and $\Delta \phi_B = 0^\circ$. No attenuators included.
The modulation of the radial distances of the two \( z \)-plane zeros as they move in their turbulent trajectories and the synchronized rotation of the \( u \)-point result in the low ripple component of the transmission response. The FSR of the low ripple component of the transmission pattern depends on the absolute length difference \(|\Delta L|\) as mentioned before. The path length difference \( \Delta L_B \) controls the periodic length of the wavelength modulation function of the transmission response.

The zero-pole analysis can also be drawn for non-feedback photonic systems. A simple example is taken here to at first further explain the principle of operation of the single-stage tunable 2×2 WDM (de)multiplexer of Figure 3.23, which is also shown in Figure app.21 for convenience. Ignoring the loss of all components and dropping any accumulated constant phase, the transfer function \( \tau_{11} \) can be given as in Eq. (3.13) by:

\[
\tau_{11} = t_1^2 e^{j\Delta \psi_1} + t_2^2 e^{j\Delta \psi_2}
\]

\[
\Delta \psi_1 = \Delta \phi + \Delta \varphi_1, \quad \Delta \psi_2 = \Delta \phi + \Delta \varphi_2, \quad \Delta \varphi_1 = 2\pi n_1 L_1 / \lambda, \quad \Delta \varphi_2 = 2\pi n_2 L_2 / \lambda
\]

… Eq. (app.24)

Substituting \( z_1 = \exp(-j\varphi_1) \) and \( z_2 = \exp(-j\varphi_2) \) in Eq. (app.24), utilizing the ideal 2×2 MMI-coupler properties of Eq. (app.11), and using the transformation \( z^2 = z_1 z_2 \) yields:

\[
\tau_{11} = \frac{2}{z} e^{j\Delta \phi} t_{11}^2 \sin\left(\frac{\Delta \phi + \Delta \varphi}{2}\right)
\]

\[
\Delta \varphi = (\Delta \phi + \Delta \varphi) / 2, \quad \Delta \phi = \Delta \phi_1 - \Delta \phi_2, \quad \Delta \varphi = \Delta \varphi_1 - \Delta \varphi_2 = 2\pi n_1 \Delta L / \lambda \quad \Delta L = L_1 - L_2
\]

… Eq. (app.25)

In the \( z \)-plane the transfer function \( \tau_{11} \) of Eq. (app.25) has a single non-effective pole located at the origin point. The sine term in this equation forms the sinusoidal transmission response of \( \tau_{11} \) of the tunable (de)multiplexer. The \( \Delta \varphi \) is the wavelength-dependent term, and therefore the FSR value of the formed transmission response depends on the length difference \( \Delta L \). Since \( \Delta \phi \) represents a constant phase in this equation it therefore only shifts the transmission response of \( \tau_{11} \) over the wavelength range when \( \Delta L \neq 0 \). Eq. (app.25) also illustrates the fact that when \( \Delta L = 0 \) then \(|\tau_{11}|\)
becomes constant of \( \lambda \) (i.e. broadband response). In this last case \( \Delta \phi \) modulates the intensity of the output optical signal for the resulted 2\( \times \)2 broadband-MZI switch. The sine term in Eq. (app.25) can still be represented by zeros and poles in an auxiliary \( u \)-plane defined as before by \( u = \exp(-j\Delta \phi/2) \).

The expression of \( \tau_{11} \) in the \( u \)-plane is given by:

\[
\tau_{11} = \frac{-e^{j\Delta \phi}e^{-j\Delta \phi/2}t_{11}^2}{z} \frac{u^{2} - e^{j\Delta \phi}t_{11}^2}{u} = \frac{-e^{j\Delta \phi}t_{11}^2}{z} \frac{u^{2}}{u} \]

... Eq. (app.26)

The transfer function \( \tau_{11} \) of Eq. (app.26) has one non-effective pole in the \( u \)-plane located at the origin point. It also has two zeros \( q_{1,2} = \pm \exp(j\Delta \phi/2) \) in the \( u \)-plane located at fixed opposite positions on the unit circle for all \( \lambda \) values. The \( u \)-point rotates around the unit circle in the counterclockwise (clockwise) direction when \( \Delta L > 0 \) (\( \Delta L < 0 \)) producing the sinusoidal transmission response. The wavelength-independent \( \Delta \phi \) can be used to rotate the two zeros to new positions on the unit circle to wavelength steer the transmission response. When \( \Delta L = 0 \) then the \( u \)-point freezes on the unit circle at \( z = 1 \) for all \( \lambda \) values. In this last case, the transmission of \( \tau_{11} \) is constant versus \( \lambda \) controlled by the rotation angle of the two zeros.

A second zero-pole analysis example of a forward-path system is taken here to explain the principle of operation of the universal 2\( \text{nd} \) order 2\( \times \)2 FIR phased array processor of Figure 4.22 which is shown again in Figure app.22 for convenience.

![Figure app.22 Universal 2\( \text{nd} \) order 2\( \times \)2 FIR phase array processor.](image)

Utilizing the simplified scattering parameters of the ideal 2\( \times \)2 MMI coupler of Eq. (app.11) the transfer function \( \tau_{11} \) for the photonic processor of Figure app.22 is given here in the \( z \)-plane by:

\[
\tau_{11} = k_{11} \cos \left( \frac{\Delta \phi_{1}}{2} \right) \cos \left( \frac{\Delta \phi_{2}}{2} \right) z^{2} + \tan \left( \frac{\Delta \phi_{1}}{2} \right) \tan \left( \frac{\Delta \phi_{2}}{2} \right) z
\]

\[
k_{11} = -4t_{11}^{4} e^{j(\Delta \phi_{1} + \Delta \phi_{2} - 2\gamma)}, \quad z = e^{-j\gamma/2}
\]
\[
\begin{align*}
\Delta \phi_{A,av} &= \frac{\Delta \phi_{A_1} + \Delta \phi_{A_2}}{2}, \quad \Delta \phi_{C,av} = \frac{\Delta \phi_{C_1} + \Delta \phi_{C_2}}{2}, \quad \Delta \psi_{1B, av} = \frac{\Delta \psi_{1B} + \Delta \psi_{2B}}{2} \\
\Delta \psi_{1B} &= \Delta \phi_{1B} + \Delta \phi_{A_1}, \quad \Delta \psi_{2B} = \Delta \phi_{2B} + \Delta \phi_{A_2}, \quad \Delta \phi_A = \Delta \phi_{A_1} - \Delta \phi_{A_2}, \quad \Delta \phi_C = \Delta \phi_{C_1} - \Delta \phi_{C_2} \\
\Delta \psi_A &= \Delta \psi_{1B} - \Delta \psi_{2B} = \Delta \phi_B + \Delta \phi_A, \quad \Delta \phi_B = \Delta \phi_{B_1} - \Delta \phi_{B_2} = 2 \pi n \Delta L_B / \lambda, \quad \Delta \phi_b = \Delta \phi_{B_1} - \Delta \phi_{B_2}
\end{align*}
\]

... Eq. (app.27)

The wavelength-dependent phase factor \(k_{11}\) has a magnitude of unity (\(|k_{11}| = 1\)). According to the definition of the \(z\)-point in Eq. (app.27), \(z = \exp[-j(\Delta \phi_B + \Delta \phi_B)]\), the \(z\)-point rotates with the increase of \(\lambda\) in the counterclockwise direction at a rate depending on the length difference \(\Delta L_B = L_{1B} - L_{2B}\). This rotation of the \(z\)-point creates a ripple in the transmission characteristic of \(\tau_{11}\) whenever the two system zeros are not located at neither the origin point nor at \(\pm j\infty\). The FSR of the transmission pattern ripple depends on \(\Delta L_B\). The phase shift difference \(\Delta \phi_B\) rotates the loci of the \(z\)-point for all \(\lambda\) values by a constant angle causing wavelength-steering the transmission response, as indicated before in Chapter (4). The expression of \(\tau_{11}\) in Eq. (app.27) has one non-effective pole located at the origin point and two zeros located at \(q_{1,2} = \pm j[\tan(\Delta \phi_A/2)\tan(\Delta \phi_C/2)]^{1/2}\). When either one or both of the phase shift differences \(\Delta \phi_A\) and \(\Delta \phi_C\) equals to 0°, the two system zeros are located at the origin point \((q_{1,2} = 0)\), and therefore the rotation of the \(z\)-point versus \(\lambda\) does not ripple the transmission response. In this case, the universal photonic processor of Figure app.22 functions as a broadband-MZI modulator based on the reduction of Eq. (app.27) to:

\[
\tau_{11} = \begin{cases} 
  k_{11}z \cos\left(\frac{\Delta \phi_C}{2}\right), & \text{when} (\Delta \phi_A = 0^\circ) \\
  k_{11}z \cos\left(\frac{\Delta \phi_A}{2}\right), & \text{when} (\Delta \phi_C = 0^\circ)
\end{cases}
\]

... Eq. (app.28)

On the other hand, when either one of the phase shift differences \(\Delta \phi_A\) and \(\Delta \phi_C\) equals to 180°, the two system zeros are located at \(q_{1,2} = \pm j\infty\). Again, the universal photonic processor behaves in this case as a broadband MZI modulator based on the reduction of Eq. (app.27) to:

\[
\tau_{11} = \begin{cases} 
  k_{11} \sin\left(\frac{\Delta \phi_C}{2}\right), & \text{when} (\Delta \phi_A = 180^\circ) \\
  k_{11} \sin\left(\frac{\Delta \phi_A}{2}\right), & \text{when} (\Delta \phi_C = 180^\circ)
\end{cases}
\]

... Eq. (app.29)
In-between these two formerly described cases when both of the phase differences $\Delta \phi_A$ and $\Delta \phi_C$ are not equal to neither $0^\circ$ nor $180^\circ$ and such that $(\Delta \phi_A, \Delta \phi_C) \in [0^\circ, 180^\circ]$ the two system zeros are located at opposite directions on the imaginary axis at an equal distance from the origin point. In this case, the rotation of the $z$-point versus $\lambda$ ripples the transmission response of $\tau_{11}$. The transmission pattern ripple is maximized for full WDM (de)multiplexing characteristic when the two system zeros are located on the unit circle at $q_{1,2} = \pm j$. According to Eq. (app.27), full WDM (de)multiplexing is achieved by adjusting the phase shift differences of sections $A$ and $C$ to $\tan(\Delta \phi_A/2) = \cot(\Delta \phi_C/2)$. A convenient choice that is taken in Chapter (4) is to set equal phase differences of $\Delta \phi_A = \Delta \phi_C = 90^\circ$ for which case the transfer function $\tau_{11}$ of Eq. (app.27) is reduced to:

$$\tau_{11} = \frac{k_{11}}{2} \cdot \frac{z^2 + 1}{z}, \quad \text{when}(\Delta \phi_A = \Delta \phi_C = 90^\circ)$$

... Eq. (app.30)

The transmission pattern of the transfer function $\tau_{11}$ of Eq. (app.30) ripples at full strength within the range from 0 to 1 with FSR value set by $\Delta L_B$. The phase shift difference $\Delta \phi_B$ embedded in $z$ in the expression of Eq. (app.30) steers the ripples of the transmission response along $\lambda$.

In conclusion, the zero-pole evolution analysis of photonic processors can be achieved by representing the system transfer function into a principal $z$-plane in addition to possible one or more auxiliary complex planes. The wavelength-dependent motion of the system roots and the rotation of the complex points along the unit circles of the principal and auxiliary planes are found to be synchronized to produce the expected transmission responses of the photonic processors studied in this thesis. The learning experience gained to control the relative locations of system roots in their wavelength-dependent trajectories can be further utilized in future studies. Although the zero-pole evolution analysis is useful to explain the operation of all other FIR and IIR photonic processors that are presented in this thesis, the investigation achieved currently is considered sufficient to explain the concept and directions of the study.
Ali Abdulsattar Hussein Mohammed al-Wasfi was born in Basrah, Iraq, in 1965. He received his B.Sc. and M.Sc. degrees in Electrical Engineering from the University of Basrah, Iraq, in 1987 and 1991, respectively. In his first master's degree he researched the microstrip patch antennas and used them in constructing phased array antennas arranged in linear and planar configurations and fed with power using the method of inter-injection locking of canonical microwave oscillators providing steering capability of their radiation beams by means of using some injection phase-controlled current sources placed at strategic points in the phased power distribution system [168]. He received his M.A.Sc. degree in Electrical and Computer Engineering from the University of Ottawa, Canada, in 2014. Since then he has been working towards obtaining his Ph.D. degree in Electrical and Computer Engineering from the University of Ottawa, Canada. His research in both of his second master's degree and the Ph.D. phase has been with the study of photonic phased array integrated circuits utilizing nano-electromechanical optical phase shifters integrated on a silicon-on-insulator platform. His research covers applications for constructing non-blocking broadband photonic routers and tunable WDM (de)multplexers. He demonstrated the concept of constructing universal phased array processors that can operate as both broadband routers and tunable WDM (de)multplexers. He utilized cascaded FIR and IIR architectures of flexible photonic phased array integrated circuits targeting applications in Elastic Optical Networks. He is a recipient of two awards for his scientific achievements. On the academic experience side, from 1993 to 1994 he was an assistant lecturer in the Electrical Engineering Department at the University of Basrah, Iraq. From 1996 to 2002 he was a lecturer in the Electrical and Computer Engineering Department at Almargep University, Khoms, Libya. From 2004 to 2010 he was instructing at Humber College, Toronto, Canada. From 2006 till recently he is instructing at George Brown College, Toronto, Canada. Also, since 2017 he has been instructing at Seneca College, Toronto, Canada. He is a licensed professional engineer in Ontario, Canada, since 2010 and he has industrial experience. He asks Allah to accept this work, and he thanks Allah for the successful completion of the thesis.