A Non-Uniform User Distribution and its Performance Analysis on K-tier Heterogeneous Cellular Networks Using Stochastic Geometry

by

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Abstract

In the cellular networks, to support the increasing data rate requirements, many base stations (BSs) with low transmit power and small coverage area are deployed in addition to classical macro cell BSs. Low power nodes, such as micro, pico, and femto nodes (indoor and outdoor), which complement the conventional macro networks, are placed primarily to increase capacity in hotspots (such as shopping malls and conference centers) and to enhance coverage of macro cells near the cell boundary. Combining macro and small cells results in heterogeneous networks (HetNets).

An accurate node (BS or user equipment (UE)) model is important in the research, design, evaluation, and deployment of 5G HetNets. The distance between transmitter (TX), receiver (RX), and interferer determines the received signal power and interference signal power. Therefore, the spatial placement of BSs and UEs greatly impacts the performance of cellular networks. However, the investigation on the spatial distribution of UE is limited, though there is ample research on the topic of the spatial distribution of BS. In HetNets, UEs tend to cluster around BSs or social attractors (SAs). The spatial distribution of these UEs is non-uniform. Therefore, the analysis of the impact of non-uniformity of UE distribution on HetNets is essential for designing efficient HetNets. This thesis presents a non-uniform user distribution model based on the existing K-tier BS distribution. Our proposed non-uniform user distribution model is such that a Poisson cluster process with the cluster centers located at SAs in which SAs have a base station offset with their BSs. There are two parameters (cluster radius and base station offset) the combination of which can cover many possible non-uniformity. The heterogeneity analysis of the proposed non-uniform user distribution model is also given.

The downlink performance analysis of the designed non-uniform user model is investigated. The numerical results show that our theoretical results closely match the simulation results. Moreover, the effect of BS parameters of small cells such as BS density, BS cell extension bias factor, and BS transmit power is included. At the same time, the uplink coverage probability by the theoretical derivation is also analyzed based on some simplifying assumptions as a result of the added complexity of the uplink analysis due to the UEs’ mobile position and the uplink power control. However, the numerical results show a small gap between the theoretical results and the simulation results, suggesting that our simplifying assumptions are acceptable if the system requirement is not very strict. In addition to the effect of BS density, BS cell extension bias factor, and BS transmit power, the effect of fractional power control factor in the uplink is also introduced. The comparison between the downlink and the uplink is discussed and summarized at the end.

The main goal of this thesis is to develop a comprehensive framework of the non-uniform user distribution in order to produce a tractable analysis of HetNets in the downlink and the uplink using the tools of stochastic geometry.
Acknowledgements

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Table of Contents

List of Tables xi

List of Figures xii

List of Abbreviations xix

List of Symbols xxii

1 Introduction 1
   1.1 Problem Definition 4
   1.2 Motivation 5
   1.3 Contributions 5
   1.4 Thesis Organization 8
   1.5 Publications 9

2 Background and Literature Review 10
   2.1 Classical Poisson Distribution 11
   2.2 Temporal Point Process 12
   2.3 Spatial Point Process 12
      2.3.1 Poisson Point Process (PPP) 13
         2.3.1.1 Homogeneous PPP 13
         2.3.1.2 Heterogeneous PPP 16
         2.3.1.3 Practical Scenarios and Examples 17
2.3.2 Transforming a Spatial Point Process ........................................ 17
   2.3.2.1 Displacement ......................................................... 17
   2.3.2.2 Thinning ............................................................ 19
   2.3.2.3 Superposition ....................................................... 19
   2.3.2.4 Practical Scenarios and Examples ............................... 20
2.3.3 Hard Core Point Process (HCPP) ........................................... 22
   2.3.3.1 Hard Core Point Process Introduction ............................ 22
   2.3.3.2 Practical Scenarios and Examples ............................... 25
2.3.4 Poisson Cluster Process (PCP) ............................................ 25
   2.3.4.1 Poisson Cluster Process Introduction ............................ 25
   2.3.4.2 Practical Scenarios and Examples ............................... 27
2.4 Literature Review of Non-Uniform UE Distribution ...................... 28

3 System Model ................................................................. 32
   3.1 Introduction .............................................................. 32
   3.2 BS Distribution .......................................................... 32
      3.2.1 Related Work on BS Distribution in the Literature ........ 32
      3.2.2 BS Distribution Model Used In the Thesis .................... 33
   3.3 User Distribution ........................................................ 34
      3.3.1 Non-Uniform User Distribution in a Single Tier BS System 35
         3.3.1.1 Set Initial BS Density ........................................ 35
            3.3.1.1.1 UE Distribution Pattern ............................ 35
            3.3.1.1.2 Coverage Map and Cell Boundaries .................. 36
            3.3.1.1.3 Performance ........................................ 37
         3.3.1.2 Increasing Macro BS Density ............................... 38
            3.3.1.2.1 UE Distribution Pattern ............................ 38
            3.3.1.2.2 Coverage Map and Cell Boundaries .................. 39
            3.3.1.2.3 Performance ........................................ 39
3.3.2 Non-Uniform User Distribution in a 2-tier BS System
  3.3.2.1 Replacing Macro BSs with Pico BSs
    3.3.2.1.1 UE Distribution Pattern
    3.3.2.1.2 Coverage Map and Cell Boundaries
    3.3.2.1.3 Performance
  3.3.2.2 Increasing Pico BS Density
    3.3.2.2.1 UE Distribution Pattern
    3.3.2.2.2 Coverage Map and Cell Boundaries
    3.3.2.2.3 Performance
3.3.3 Non-uniform User Distribution in a 3-tier BS System
  3.3.3.1 Replacing Pico BSs with Femto BSs
    3.3.3.1.1 UE Distribution Pattern
    3.3.3.1.2 Coverage Map and Cell Boundaries
    3.3.3.1.3 Performance
3.3.4 Summary of User Distribution
3.4 User Connectivity
  3.4.1 Connectivity Rule
  3.4.2 Connectivity Model
3.5 Conclusions

4 User Model and Heterogeneity Analysis
  4.1 Introduction
  4.2 Non-Uniform User Distribution
    4.2.1 Definition
    4.2.2 Parameters
    4.2.3 Attraction Level and Dependency Level
      4.2.3.1 Attraction Level
      4.2.3.2 Dependency Level
5 Downlink Performance Analysis

5.1 Introduction ................................. 69

5.2 Model Performance Analysis ..................... 72
  5.2.1 Distance Characterization ..................... 73
    5.2.1.1 PDF and CCDF of Random Variable \( V \) .... 73
    5.2.1.2 PDF and CCDF of Random Variable \( L \) .... 74
  5.2.2 Interference Characterization .................. 77
  5.2.3 Downlink Coverage Probability for Case1 (associated with the BS to which the UE belongs) .... 78
  5.2.4 Downlink Coverage Probability for Case2 (associated with a BS in all tiers except the BS to which UE belongs) .... 79
  5.2.5 Overall Downlink Coverage Probability ............ 80

5.3 Simulation Results ................................ 81
  5.3.1 Matching between Simulation and Theoretical Analysis 83
  5.3.2 Effect of Base Station Offset .................... 84
  5.3.3 Effect of System Parameters .................... 84
    5.3.3.1 Effect of BS Density Ratio between Small Cells and Macro Cells ................. 85
    5.3.3.2 Effect of Cell Extension of Small Cells .... 88
    5.3.3.3 Effect of BS Transmit Power of Small Cells .. 91

5.4 Conclusions .................................... 93
6 Uplink Performance Analysis

6.1 Introduction ................................................. 94
6.2 Simplifying Assumptions ................................. 99
6.3 Uplink Power Control ..................................... 101
6.4 Model Performance Analysis .............................. 102
  6.4.1 Distance Characterization ......................... 104
  6.4.2 Interference Characterization .................. 104
  6.4.3 Uplink Coverage Probability ................... 105
  6.4.4 Overall Uplink Coverage Probability ........... 107
6.5 Simulation Results .......................................... 107
  6.5.1 Matching between Simulation and Theoretical Analysis 108
  6.5.2 Effect of Base Station Offset .................. 110
  6.5.3 Effect of System Parameters .................... 111
    6.5.3.1 Effect of BS Density Ratio between Small Cells and Macro Cells 111
    6.5.3.2 Effect of Cell Extension of Small Cells ...... 114
    6.5.3.3 Effect of BS Transmit Power of Small Cells .. 116
    6.5.3.4 Effect of Fractional Power Control Factor .. 117
6.6 Conclusions .................................................. 118

7 Comparison between Downlink and Uplink ................ 120

7.1 Introduction ................................................ 120
7.2 Non-uniform UE Model ..................................... 120
7.3 Coverage Probability Performance Analysis ............. 121
  7.3.1 Coverage Probability Definition ................ 122
    7.3.1.1 Downlink .................................... 122
    7.3.1.2 Uplink ..................................... 123
    7.3.1.3 Difference .................................. 123
B Uplink Coverage Performance Analysis 147
  B.1 Proof of Lemma 6-4 147

C Simulation Setting 148
  C.1 Downlink Coverage Probability 148
  C.2 Uplink Coverage Probability 150
  C.3 Heterogeneity Analysis based on CoV 151

References 153
List of Tables

1.1 Typical parameters of different tiers of cells ........................................ 3
5.1 PDF and CCDF of the random variable $L$ .............................................. 76
5.2 Parameters about BSs of a 2-tier system ............................................. 82
5.3 Parameters of effect of BS density ratio between small cells and macro cells .......................................................... 85
5.4 Parameters of effect of BS cell extension bias factor of small cells 88
5.5 Parameters of effect of BS transmit power of small cells ................ 91
List of Figures

1.1 Network architecture for 5G HetNets .................................. 2

2.1 Probability of Poisson distribution ..................................... 11
2.2 Realization of homogeneous PPP and heterogeneous PPP .... 16
2.3 Example of the Voronoi region of homogeneous PPP and heterogeneous PPP ......................................................... 17
2.4 Realization 1 of displacement (points are displaced by shifting) 18
2.5 Realization 2 of displacement (points are displaced by contraction) ................................................................. 18
2.6 Realization of thinning (black points are thinned with the probability of 70%) .......................................................... 19
2.7 Realization of superposition (superposition of two point patterns with the same density) ........................................... 20
2.8 Example of displacement points and their Voronoi region .... 20
2.9 Example of thinning points and their Voronoi region ........... 21
2.10 Example of generating non-uniform users using thinning .... 21
2.11 Example of superposition points and their Voronoi region .... 22
2.12 Rule to select HCPP points .............................................. 23
2.13 Realization of hard core point process ............................... 24
2.14 Example of hard core point process and their Voronoi region. 25
2.15 PDF distribution example in PCP ...................................... 26
2.16 Realization of PCP ...................................................... 27
2.17 Example of PCP and their Voronoi region ........................... 28
<table>
<thead>
<tr>
<th>Section</th>
<th>Description</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.1</td>
<td>A realization of a 3-tier BS distribution</td>
<td>34</td>
</tr>
<tr>
<td>3.2</td>
<td>UE distribution pattern with uniform UEs and non-uniform UEs. Red triangles are macro BSs. Blue curves are cell boundaries. Black points are uniform UEs in (a) while magenta points are non-uniform UEs in (b).</td>
<td>36</td>
</tr>
<tr>
<td>3.3</td>
<td>Coverage map (a) and cell boundaries (b) in a single tier BS system (3 BSs) where the transmit power of BS1, BS2 and BS3 is the same.</td>
<td>37</td>
</tr>
<tr>
<td>3.4</td>
<td>Coverage map (a) and cell boundaries (b) in a single tier BS system (3 BSs) where the transmit power of BS2 and BS3 is the same and the transmit power of BS1 is 4 times of the transmit power of BS2 and BS3.</td>
<td>37</td>
</tr>
<tr>
<td>3.5</td>
<td>Performance of DL and UL in a signal tier (3 macro BSs, the non-uniform UE cluster centers are randomly distributed.)</td>
<td>38</td>
</tr>
<tr>
<td>3.6</td>
<td>UE distribution pattern with non-uniform UEs where three additional macro BSs are deployed in the UE cluster centers.</td>
<td>38</td>
</tr>
<tr>
<td>3.7</td>
<td>Coverage map (a) and cell boundaries (b) in a single tier BS system (6 BSs where 3 macro BSs are put randomly and another 3 macro BSs are put in the user cluster centers)</td>
<td>39</td>
</tr>
<tr>
<td>3.8</td>
<td>Performance of DL and UL in a signal tier (red curve is for 3 macro BSs in a single tier where 3 macro BSs are independent with UE cluster distribution, black curve is for 6 macro BSs in a single tier where 3 macro BSs are independent with UE cluster distribution and another 3 macro BSs are deployed in UE cluster centers. )</td>
<td>39</td>
</tr>
<tr>
<td>3.9</td>
<td>UE distribution pattern with non-uniform UEs in which pico BSs are deployed in the UE cluster centers.</td>
<td>40</td>
</tr>
<tr>
<td>3.10</td>
<td>Coverage map (a) and cell boundaries (b) in a 2-tier BS system (6 BSs where 3 macro BSs (red triangles) are put randomly and 3 pico BSs (black triangles) are put in the user cluster centers)</td>
<td>41</td>
</tr>
</tbody>
</table>
3.11 Performance of DL and UL (blue curve is for 3 macro BSs in a single tier where 3 macro BSs are independent with UE cluster distribution, red curve is for 6 macro BSs in a single tier where 3 macro BSs are independent with UE cluster distribution and another 3 macro BSs are deployed in UE cluster centers, black curve is for 3 macro BSs and 3 pico BSs in a 2-tier system.)

3.12 Coverage probability of cluster users (magenta points) when $P_1 = 2P_2$, $P_1 = 10P_2$, and $P_1 = 20P_2$ in a 2-tier BS system.

3.13 Coverage probability of cluster users (magenta points) with different path loss exponents in a 2-tier BS system when $P_1 = 10P_2$.

3.14 UE distribution pattern with non-uniform UEs in which new 6 pico BSs are deployed in these additional 6 UE cluster centers in green.

3.15 Coverage map (a) and cell boundaries (b) in a 2-tier BS system (9 BSs where 3 macro BSs (red triangles) are put randomly, 3 pico BSs (black triangles) are put in the user cluster centers in magenta and 6 pico BS (black triangles) are put in the user clusters in green.)

3.16 Performance of DL and UL in a 2-tier BS system (red curve is for 3 macro BSs and 3 pico BSs in a 2-tier system and black curve is for 3 macro BSs and 9 pico BSs in a 2-tier system.)

3.17 UE distribution pattern with non-uniform UEs in which new 6 femto BSs are deployed in these additional 6 UE cluster centers in green. Red triangles are macro BSs. Black triangle are pico BSs. Blue triangles are femto BSs.

3.18 Coverage map (a) and cell boundaries (b) in a 3-tier BS system (12 BSs where 3 macro BSs (red triangles) are put randomly, 3 pico BSs (black triangles) are put in the user cluster centers in magenta and 6 femto BSs (blue triangles) are put in the user clusters in green.)

3.19 Performance of DL and UL in a 2-tier and a 3-tier (red curve is for 3 macro BSs and 9 pico BSs in a 2-tier system and black curve is for 3 macro BSs, 3 pico BS and 6 femto BSs in a 3-tier system.)
3.20 Cell coverage boundaries diagram for a 2-tier cellular network based on user connectivity rule of maximum received power. 
(a) $P_1 = 1000P_2$, $\lambda_1 = 4\lambda_2$. (b) $P_1 = 100P_2$, $\lambda_1 = 4\lambda_2$. (c) $P_1 = 1000P_2$, $\lambda_1 = 8\lambda_2$. (d) $P_1 = 100P_2$, $\lambda_1 = 8\lambda_2$. 

3.21 Cell extension example. (a) and (b) have the exactly same BS locations. (a) is $\{C_1, C_2\} = \{1, 1\}$ without cell extension and (b) is $\{C_1, C_2\} = \{1, 6.3\}$ with cell extension. The area is 1 km by 1 km.

4.1 Non-uniform user distribution example ($D_2$ is 55 m with orange line and it is Matern cluster process with radius $R = 40$ m).

4.2 Range of attraction level and dependency level.

4.3 A user distribution realization of the non-uniform model (attraction level = 0.75 for (a), attraction level = 0.6 for (b) and attraction level = 0 for (c), dependency level = 1, blue curves are assumed to be cell boundaries).

4.4 A user distribution realization of the non-uniform model with a dependency level is 0.6 (attraction level is 0.8 for (a), attraction level is 0.7 for (b), attraction level is 0.5 for (c), blue curves are assumed to be cell boundaries).

4.5 Non-uniform user distribution Scenario 1 (a-f). The area is 1 km by 1 km. Green triangles are macro BSs and black triangles are small BSs.

4.6 Non-uniform user distribution Scenario 1 (g-l). The area is 1 km by 1 km. Green triangles are macro BSs and black triangles are small BSs.

4.7 Non-uniform user distribution Scenario 2. There are non-uniform cluster users (magenta points) distributed for small cells. Magenta circles are UE cluster boundaries distributed for small cells. Black diamonds are UE cluster centers. There are also almost uniform users (black points) distributed for macro cells in (b). Black circles are UE cluster boundaries for macro cells. The blue curves are the cell boundaries. The area is 1 km by 1 km. Green triangles are macro BSs and black triangles are small BSs.
4.8 Two ways to capture the traffic property in 1D ............... 62
4.9 Pattern of 1D from left to right with sub-Poisson ($0 < C_E < 1$), Poisson ($C_E = 1$) and super-Poisson ($C_E > 1$) ............... 63
4.10 Pattern of 2D from left to right with sub-Poisson ($0 < C_E < 1$), Poisson ($C_E = 1$) and super-Poisson ($C_E > 1$) ............... 64
4.11 Example of Voronoi and Delaunay tessellations (Delaunay cell edges with blue lines and Voronoi cell edges with black lines, and the pattern points with red diamonds). ............... 65
4.12 Measured normalized CoV for Delaunay cell edge length, Delaunay cell area, and Voronoi cell area (without base station offset between SAs and BSs.) ............... 66
4.13 Measured CoV based on the attraction level and the dependency level ............... 67
4.14 Measured coverage probability based on the attraction level and the dependency level ............... 67
5.1 Example of spatial distances between UE, BS, and SA and the related notation in a 2-tier downlink system ............... 70
5.2 Calculation example of PDF and CCDF of Random Variable $L$ in the $m$th tier ............... 74
5.3 PDF and CCDF example of $L$ ($R = 100, D = 50$) ............... 75
5.4 Downlink coverage probability of our designed non-uniform user distribution model, in which non-uniform users are distributed in small cells ............... 83
5.5 Effect of base station offset $D_2$ on the downlink coverage probability ............... 84
5.6 Downlink coverage probability as a function of BS density ratio $\frac{\lambda_2}{\lambda_1}$ between small cells and macro cells ............... 86
5.7 Downlink coverage probability as a function of BS cell extension bias factor $C_2$ in small cells ............... 89
5.8 Downlink coverage probability as a function of BS transmit power $P_2$ in small cells ............... 91
A.4 PDF and CCDF when $D_m \geq R_m$ and $D_m + R_m \leq l$  
A.5 PDF and CCDF when $D_m < R_m$ and $l \leq R_m - D_m$  
A.6 PDF and CCDF when $D_m < R_m$ and $R_m - D_m < l \leq \sqrt{R^2_m - D_m^2}$  
A.7 PDF and CCDF when $D_m < R_m$ and $\sqrt{R^2_m - D_m^2} < l \leq \sqrt{R^2_m + D_m^2}$  
A.8 PDF and CCDF when $D_m < R_m$ and $\sqrt{R^2_m + D_m^2} < l < R_m + D_m$  
A.9 PDF and CCDF when $D_m < R_m$ and $R_m + D_m \leq l$  
A.10 PDF and CCDF when $D_m = 0$ and $l \geq R_m$  
A.11 PDF and CCDF when $D_m = 0$ and $l < R_m$  

C.1 An Example of BS Pattern (Red triangles are BSs in the first tier with $\lambda_1 = 3 \times 10^{-6}$. Black triangles are BSs in the second tier with $\lambda_2 = 3 \times 10^{-6}$. Cyan triangles are BSs in the third tier with $\lambda_3 = 6 \times 10^{-6}$. The area is a square of 1 km by 1 km. Blue curves are cell boundaries.)
### List of Abbreviation

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>2D</td>
<td>Two Dimension</td>
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<tr>
<td>3GPP</td>
<td>Third Generation Partnership Project</td>
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<td>5G</td>
<td>Fifth Generation</td>
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<tr>
<td>AWGN</td>
<td>Additive White Gaussian Noise</td>
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<tr>
<td>BLER</td>
<td>Block Error Rate</td>
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<tr>
<td>BRP</td>
<td>Bias Received Power</td>
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<tr>
<td>BS</td>
<td>Base Station</td>
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<tr>
<td>BSO</td>
<td>Base Station Offset</td>
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<tr>
<td>CDF</td>
<td>Cumulative Distribution Function</td>
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<tr>
<td>CCDF</td>
<td>Complementary Cumulative Distribution Function</td>
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<tr>
<td>CDMA</td>
<td>Code Division Multiple Access</td>
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<tr>
<td>CoV</td>
<td>Coefficient of Variation</td>
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<td>CRE</td>
<td>Cell Range Extension</td>
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<td>DL</td>
<td>Downlink</td>
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<tr>
<td>DSPP</td>
<td>Doubly Stochastic Poisson Process</td>
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<tr>
<td>EE</td>
<td>Energy Efficiency</td>
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<tr>
<td>GSM</td>
<td>Global System for Mobile Communication</td>
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<td>HCN</td>
<td>Heterogeneous Cellular Network</td>
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<td>HCPP</td>
<td>Hard Core Point Process</td>
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<td>HetNets</td>
<td>Heterogeneous Networks</td>
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<tr>
<td>IAT</td>
<td>Inter Arrival Time</td>
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<td>LOS</td>
<td>Line of Sight</td>
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<tr>
<td>Acronym</td>
<td>Definition</td>
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<tr>
<td>LT</td>
<td>Laplace Transform</td>
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<td>LTE</td>
<td>Long Term Evolution</td>
</tr>
<tr>
<td>NLOS</td>
<td>Non Line of Sight</td>
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<tr>
<td>MIMO</td>
<td>Multi Input Multi Output</td>
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<td>MMWave</td>
<td>Millimeter Wave</td>
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<tr>
<td>OFDM</td>
<td>Orthogonal Frequency Division Multiplexing</td>
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<tr>
<td>PCP</td>
<td>Poisson Cluster Process</td>
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<tr>
<td>PDF</td>
<td>Probability Density Function</td>
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<td>PGFL</td>
<td>Probability Generating Functional</td>
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<td>PHP</td>
<td>Poisson Hole Process</td>
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<tr>
<td>PP</td>
<td>Point Process</td>
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<tr>
<td>PPP</td>
<td>Poisson Point Process</td>
</tr>
<tr>
<td>RX</td>
<td>Receiver</td>
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<tr>
<td>SA</td>
<td>Social Attractor</td>
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<td>SE</td>
<td>Spectral Efficiency</td>
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<tr>
<td>SG</td>
<td>Stochastic Geometry</td>
</tr>
<tr>
<td>SINR</td>
<td>Signal to Interference Plus Noise Ratio</td>
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<tr>
<td>SNR</td>
<td>Signal to Noise Ratio</td>
</tr>
<tr>
<td>SPPP</td>
<td>Spatial Poisson Point Process</td>
</tr>
<tr>
<td>TX</td>
<td>Transmitter</td>
</tr>
<tr>
<td>UE</td>
<td>User Equipment</td>
</tr>
<tr>
<td>UL</td>
<td>Uplink</td>
</tr>
<tr>
<td>UAV</td>
<td>Unmanned Aerial Vehicle</td>
</tr>
<tr>
<td>UMTS</td>
<td>Universal Mobile Telecommunications System</td>
</tr>
<tr>
<td>WCDMA</td>
<td>Wideband Code Division Multiple Access</td>
</tr>
</tbody>
</table>
List of Symbols

\( \alpha_i \) \hspace{1cm} \text{path loss exponent in the } i\text{th tier}
\( B_{ji} \) \hspace{1cm} \text{coordinates of the } i\text{th BS in the } j\text{th tier}
\( B_0 \) \hspace{1cm} \text{coordinates of the associated BS}
\( BS_i^j \) \hspace{1cm} j\text{th BS in the } i\text{th tier}
\( C_i \) \hspace{1cm} \text{cell extension bias factor in the } i\text{th tier}
\( \text{CoV} \) \hspace{1cm} \text{ratio of the standard deviation of the measurement to the mean of the measurement}
\( C_I \) \hspace{1cm} \text{Normalized Cov in 1D time domain}
\( C_E \) \hspace{1cm} \text{Normalized Cov in 2D space domain}
\( D_i \) \hspace{1cm} \text{base station offset in the } i\text{th tier}
\( E \) \hspace{1cm} \text{rate parameter of exponential cluster process}
\( _2F_1 \) \hspace{1cm} \text{Gauss hypergeometric function}
\( \bar{F}(g) \) \hspace{1cm} \text{CCDF of } g
\( f_u(r) \) \hspace{1cm} \text{probability density function (PDF) of UEs}
\( f(g) \) \hspace{1cm} \text{PDF of } g
\( G_i \) \hspace{1cm} \text{channel random power gain in the } i\text{th tier}
\( I_k \) \hspace{1cm} \text{Inter arrival time}
\( I_{rj} \) \hspace{1cm} \text{total received interference from the } j\text{th tier}
\( K \) \hspace{1cm} \text{total number of tiers}
\( k \) \hspace{1cm} \text{index of tier}
\( \kappa \) \hspace{1cm} \text{subset of cell tiers}
\( L_0 \) \hspace{1cm} \text{path loss at a reference distance 1 m}
\( L \) distance between the UE and the BS that the UE belongs to
\( N_m \) average number of users in each cluster
\( P_i \) BS transmit power in the \( i \)th tier
\( P_{dl,c} \) the overall downlink coverage probability of the designed non-uniform distribution model
\( P_{ul,c} \) the overall uplink coverage probability of the designed non-uniform distribution model
\( R \) cluster radius of linear or Matern cluster process
\( r_{ji} \) distance of the user at the origin to the \( i \)th BS in the \( j \)th tier (i.e. \( B_{ji} \))
\( S \) subset of small cell tiers
\( S \) total number of small cell tiers distributed with non-uniform user
\( SA^j_i \) \( j \)th SA in the \( i \)th tier
\( T_i \) SINR coverage target of the \( i \)th tier
\( V \) the distance between a UE and the closest BS
\( x_i \) coordinates of a random point in a subset
\( \beta(x,y) \) density function at the coordinates of \((x,y)\)
\( \lambda \) point density
\( \sigma^2 \) power of the additive white Gaussian noise (AWGN)
\( \phi \) spatial Poisson point process
\( \sigma \) variance of Thomas cluster process
\( \epsilon \) fractional power control factor
Chapter 1

Introduction

The remarkable increase of mobile users as well as applications for video streaming, social media, and data cloud services have created the need for higher data rates and increased system capacity. For example, data traffic increased 11-fold between 2013 and 2018 [1]. A 1000-fold increase of traffic demand as well as an increase of connected devices to 100 billion is predicted between 2020 and 2030 [2]. The exponentially increased traffic demand raises severe challenges to future cellular networks.

The 5G and future networks will be heterogeneous and include both macro and small cells, such as micro, pico, and femto cells. Heterogeneous Networks (HetNets) are defined as networks that are made up of different types of cells [3–9]. In 3GPP standard body, HetNets are proposed as a key candidate for solving the traffic demand challenge and for accommodating the increased number of mobile users. Some technically challenging issues of HetNets have been widely discussed (e.g., see [10–13]). Even higher densification of base stations (BSs) would be built into HetNets of 5G. Therefore, HetNets in 5G would provide significant improvement in both spectral efficiency (SE) and energy efficiency (EE). Network densification is adding more cell sites in order to increase network capacity. Densification is not a new paradigm in the cellular networks of 5G; it has existed since 1G [2]. In 1G voice only networks, a BS can cover a cell radius of about 10 miles and the densification is splitting the cell coverage into two or more smaller cells so as to combat path loss and accommodate more users. In 2G networks with low 64kbps speed data service, a macro BS can cover a cell radius a hundred meters to a few kilometers and the densification is such that small cells are deployed in order to offload the traffic data from macro cells. In 3G and 4G networks with high speed data services (up to 2Mbps for 3G and up to 1Gbps for 4G), small cells [14,15] are covered by low-power radio access nodes that help provide cellular services to both indoor and outdoor areas from an open outdoor environment to
indoor buildings and homes. Small cells are set up to enhance range and capacity in areas that contain densely distributed users in urban areas and bring the dual benefit of high capacity and improved mobile traffic. In 5G networks with data services of extremely high speeds (above 1Gbps), such as in the fiber-like experience, the density of small cells will accelerate even faster in contrast to 4G in order to provide extremely high data rates and capacity.

HetNets are multi-tier cellular networks where macro cells and small cells (micro cells, pico cells, and femto cells) coexist. Figure 1.1 presents the network architecture of 5G HetNets. For example, the user equipment named UE A is in the coverage of the macro cell. UE A will communicate with other users through the macro cell’s BS and core networks. Similarly, users named UE B, UE C, and UE D are in the coverage of the micro cell, pico cell, and femto cell respectively. Macro cells supported by high transmit power BSs cover a wide area (particularly in rural areas or along highways) while small cells are deployed in hot spots in order to improve the capacity, enhance coverage of macro cells near cell boundaries, and increase the frequency reuse factor. A micro cell is used in a densely populated urban area. Pico cells are used in areas smaller than micro cells, such as a large office, a mall, or train station. Femto cells are used at homes or small offices. Femto cells can be installed by any consumer while macro cells, micro cells, and pico cells can be deployed only by operators. User-deployed femto cell can have open access, which allows any user to access the femto cell, or closed access where only certain users are permitted to access the femto cell. In Table 1.1, the parameters of different cell tiers are provided. For example, the transmit power of macro BS is around 46 dBm, the transmit powers of
micro BS and pico BS are around 16 dB lower than that of macro BS. Transmit power of a femto BS is around 26 dB lower than that of macro BS. A conventional macro cell network or a small cell network makes up a network tier. The conventional macro cell network is called a single-tier network. We call a network consisting of macro cells and any type of small cells as a 2-tier network. And then the networks consisting of multiple type of cells as multi-tier networks. Multi-tier networks support expansion in conventional cellular network with a flexible and economic architecture design [16].

Differences between macro cell, micro cell, pico cell, and femto cell are shown in the following table (referred from [17] and link: //www.rfwireless-world.com/Tutorials/femtocell-vs-picocell-vs-microcell.html).

Table 1.1: Typical parameters of different tiers of cells

<table>
<thead>
<tr>
<th></th>
<th>macro cell</th>
<th>micro cell</th>
<th>pico cell</th>
<th>femto cell</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Transmit power</strong></td>
<td>46 dBm</td>
<td>30 dBm</td>
<td>30 dBm</td>
<td>20 dBm</td>
</tr>
<tr>
<td><strong>Coverage radius</strong></td>
<td>1 km to 20 km</td>
<td>500 m to 2 km</td>
<td>50 m to 500 m</td>
<td>4 m to 10 m</td>
</tr>
<tr>
<td><strong>Deployment location</strong></td>
<td>outdoor</td>
<td>outdoor</td>
<td>outdoor as well as indoor</td>
<td>indoor</td>
</tr>
<tr>
<td><strong>Installation</strong></td>
<td>by operator</td>
<td>by operator</td>
<td>by operator</td>
<td>by user</td>
</tr>
</tbody>
</table>

All of the network tiers reuse the available spectrum to improve the spatial spectrum efficiency and network capacity. However, in doing so, cross-tier and co-tier interference is created. The spatial distribution of BS and user equipment (UE) has a big impact on the performance of cellular networks because the received signal power and interference signal power depend on the distance between transmitter (TX), receiver (RX), and interferer. Emerging network paradigms, such as femto cells, which are overlaid upon conventional cellular networks, improve the system capacity and throughput. In these new paradigms, BSs and UEs appear to be randomly located. Therefore, it is important to create accurate spatial models to evaluate the impact of the cross-tier and co-tier interference on the overall performance of the network that occur in HetNets. An accurate node (BS or UE) model is a key factor in the research and design of 5G heterogeneous cellular networks.
1.1 Problem Definition

In single-tier networks, the distribution of BSs is assumed to be a regular grid model, in which BSs are deployed on a two-dimensional grid (e.g., a square lattice or a hexagonal lattice). Researchers analyze single-tier networks by either Monte Carlo simulations [18] or simplifying assumptions that inter-cell interference only comes from two neighboring cells [19]. Nowadays, the distribution of BS is becoming more heterogeneous due to the introduction of small cell tiers such that Poisson point process (PPP) distribution of BSs is assumed where the distance between BSs is totally random. However, operators do not randomly place their macro cell BSs over a given area. To tackle the distribution model of macro BSs, hard core point process (HCPP) rather than PPP distribution is considered as a candidate, in which there is a defined minimum distance between any two macro BSs. Therefore, for macro BSs distribution model, there are two main approaches according to the above discussion. The first approach is a PPP distribution while the second approach is a HCPP. For the other tiers of the networks, such as small cell tiers, there are also two main approaches. The first approach is also a PPP, in which different tiers of BSs have different parameters (e.g., transmit power, density, and signal to interference plus noise ratio (SINR) coverage threshold) while the second approach is a Poisson cluster process (PCP) to specify the cluster property of small BSs. Although the second approaches of both macro BSs and small BSs are more practical, we still choose the first PPP approaches of both macro BSs and small BSs as the basic assumptions in this thesis. The reasons are as follows. First, it is possible to achieve a tractable closed-form performance analysis for BSs using the PPP distribution. Second, the first PPP approaches of both macro BSs and small BSs are very popular assumptions in the literature. We may take the BS distribution assumption as PPP and focus our research on non-uniform UE distribution. Finally, we can take our research as the baseline or lower bound of practical BS cases (for instance, HCPP, or PCP distribution) and extend our research to more complicated and practical BS distribution in future work. Hence, in this thesis we make the assumptions of BS distribution as the first PPP approaches for both macro BSs and small BSs. This is called the K-tier model, which is a popular multi-tier BS distribution model of HetNets in the literature. In K-tier networks, each tier of BSs, such as macro BSs and small BSs, are distributed according to a homogeneous spatial Poisson point process (SPPP) [20–22]. SPPPs are independent between tiers. BS locations in the $k$th tier are generated by SPPP with density $\lambda_k$, transmit power $P_k$, Signal to interference plus noise ratio (SINR) coverage threshold $T_k$, and path loss exponent $\alpha_k$.

In the literature, the locations of BSs are modeled as K-tier PPP in the spatial domain.
and the locations of UEs are usually modeled by another independent PPP or are called uniform distribution\(^1\). But in practice, UEs often concentrate or cluster near some social attractors (SAs), such as bus stops, shopping malls, or convention centers rather than distributed randomly. Moreover, there is a spatial dependency between BSs and SAs locations and a simple PPP does not represent the UE distribution. Hence, the model or the framework with non-uniform UE distribution\(^2\) in heterogeneous networks must be a realistic scenario, which can be represented by spatial dependency between UEs, SAs, and BSs.

### 1.2 Motivation

5G supports the proliferation of devices that need a mobile connection. The devices can be phones, computers, home appliances, cars, wearables, and other devices that are connected to the multi-tier networks. As a result, the complex networks and the volume of devices make 5G difficult to analyze. Accurate performance analysis is important for enabling system developers to design networks that meet quality of service requirements. The performance analysis of the networks is normally obtained through time-consuming system simulation based on simple assumptions of the location of BSs and UEs. Analysis provides the design baseline, system insight, and parameter dependencies for network operators and carriers. However, the mathematical performance analysis of networks are challenging. Presently, stochastic geometry is adopted as a new approach to model the multi-tier cellular network. It provides the tools and techniques for computing the distribution of SINR, especially with the node distribution of PPP, which causes the possibility of the mathematical analysis of coverage probability in the downlink and the uplink.

### 1.3 Contributions

On the design and research of HetNets, there are two aspects that we should tackle. The first aspect is to build system models that cover the heterogeneity and irregularity in BS distribution and UE distribution. The system models should be realistic and simple enough to be used. The second aspect is to carry out the performance analysis like coverage probability as a function of SINR such that we better understand the system design foundation.

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\(^1\)In this thesis, the uniform UE distribution means that the average UE density is the same.  
\(^2\)In this thesis, the non-uniform UE distribution means that the average UE density is different.
and system infrastructure of HetNets. The main goal of this thesis is to develop a comprehensive framework of the non-uniform UE distribution such that it has tractable analysis in HetNets. The main contributions are summarized as follows.

- **New non-uniform UE model based on K-tier BS HetNets**
  
  - **New non-uniform UE model.** We develop a flexible and practical non-uniform UE model, where UEs are distributed around cluster centers called social attractors (e.g., SAs, such as bus stops, shopping malls, conference centers). Here UEs are symmetrically, independently, and identically distributed around SAs, which is Possion cluster process (PCP) to capture the correlation between UEs and SAs. SAs have base station offset (BSO) with their BSs to capture the correlation between SAs and BSs.

  * **Parameters in new non-uniform UE model.** There are two parameters of our non-uniform UE model. One parameter is the cluster radius, which determines the correlation (represented as attraction level) between UEs and SAs and the other parameter is the base station offset, which determines the correlation (represented as dependency level) between SAs and BSs. The designers can adjust two parameters to obtain their preferred and specific heterogeneity. This non-uniform UE model is tunable such that it can cover the wide possibilities from homogeneous to heterogeneous.

  * **Definition of attraction level and dependency level.** We define the attraction level and the dependency level, which are determined from the parameters of the cluster radius and the base offset. The attraction level and the dependency level are one possible way to express these two parameters. Because we confine the value range from 0 to 1, the attraction level and dependency level better represent the degree of correlation. In the simulations and visualized figures of this thesis, we use the attraction level and dependency level to stand for the characteristic of non-uniform user model. The concept of the attraction level is not new and it has been utilized in the research [23, 24]. High attraction level stands for the case of high heterogeneity around SAs (low cluster radius), while low attraction level stands for the case of approaching uniform distribution with low heterogeneity. However, the dependency level is novel in this thesis in that it indicates the placement relationship between SAs and BSs. Because of physical constraints, BSs can not be deployed exactly in the cluster centers
(e.g., SAs). This thesis considers the joint relationship between BSs, UEs, and SAs.

- **Heterogeneity analysis of non-uniform UE distribution.** The relationship between UE heterogeneity and designed user model’s characteristics, such as the attraction level and dependency level are presented based on the normalized coefficient of variation (CoV), which is the standard deviation of measured Voronoi cell area are divided by the mean of measured Voronoi cell area.

- **Performance analysis**

  This thesis carries on the research of both downlink and uplink performance analysis for non-uniform UE distribution.

- **Downlink.**
  
  * **Downlink coverage probability.** In this thesis, the performance analysis of the downlink coverage probability based on our non-uniform UE model is derived step by step.
  
  * **Derivation challenges.** Because the coverage probability is the complementary cumulative distribution function of SINR, the challenging part of the performance analysis is the characterization of the total interference. And the derivation of the probability density function (PDF) and complementary cumulative distribution function (CCDF) of the distance between a UE and its associated BS is also crucial.

  * **Simulations.** The simulation results demonstrate the accuracy of the performance analysis. In practice, we can do away with the time consuming system simulations and use the equations given in this thesis to provide the downlink coverage performance.

  * **System Insight.** Based on the derived performance analysis, the effect of system parameters is shown. The performance is quite different in contrast to the well researched case on the uniform UE distribution. This justifies the reason to seek for a system insight, such as the effect of system parameters on the non-uniform UE distribution. For example, for uniform UE distribution on K-tier HetNets, the increase of the density of small BS will not decrease the downlink coverage probability (e.g., more BSs can be deployed without decreasing the downlink coverage probability while the capacity increases with the increase of the density of small BSs). But for
non-uniform UE distribution, the above conclusion does not hold. Therefore, the system insight from the derived performance equations is helpful to understand the networks, especially since it will show that the system insight does not align with our expectation and is different from the system insight on the widely researched uniform UE distribution in the literature.

- **Uplink**

  - **Uplink coverage probability.** The performance analysis of the uplink coverage probability for the non-uniform UE model is also derived.

  - **Derivation challenges.** The uplink performance analysis is more challenging than the downlink case due to the characterization of the uplink interference and the uplink power control. We make some assumptions so that it is possible to obtain the performance analysis of the uplink coverage probability since the characterization of the uplink interference is not available and the uplink power control causes UE transmit power to fluctuate.

  - **Simulations.** The simulation results have limited mismatch (less than 1 dB) with the analytical equations due to the assumptions made. It still can be regarded as a reference if the system requirement is not very strict.

  - **System insight.** The effect of system parameters in the uplink is also obtained.

### 1.4 Thesis Organization

The organization is as follows.

Chapter 2 provides the background on stochastic geometry, which focuses on point processes. Then, a literature review about non-uniform UE distribution is provided.

Chapter 3 presents the system model, which includes BS distribution, user distribution, and UE connectivity. We also introduce the coverage map, cell boundaries and performance of non-uniform users in single tier or multi-tier BS systems to demonstrate the benefits of deploying small cells.

Chapter 4 defines a spatial distribution of non-uniform UE model. In the model, the UEs are distributed in a cluster with the center being SA, which has base station offset to its BS. The corresponding UE patterns or distribution examples are demonstrated in this chapter. Finally, the heterogeneity analysis of non-uniform UE distribution is illustrated.
Chapter 5 explains the downlink coverage probability analysis. The analytical performance expression of the coverage probability is achieved through mathematical derivation. The simulation results included demonstrate the accuracy of the analytical expression. The effect of system parameters based on the above theoretical analysis is given.

Chapter 6 shows the uplink coverage probability analysis. The analytical performance equations of the uplink coverage probability are given. The effect of system parameters are also investigated.

Chapter 7 summarizes the comparison between the downlink and the uplink analysis. Chapter 8 concludes the thesis and explores possibilities for future work.

1.5 Publications

- Chao Li, Abbas Yongacoglu, and Claude D’Amours, Downlink Coverage Probability with Spatial Non-uniform User Distribution Around Social Attractors, IEEE 24th International Conference on Telecommunication (ICT), Cyprus, May, 2017
- Chao Li, Abbas Yongacoglu, and Claude D’Amours, Heterogeneous Cellular Network User Distribution Model, IEEE 8th Latin-American Conference on Communications (LATINCOM), Colombia, Nov, 2016
- Chao Li, Abbas Yongacoglu, and Claude D’Amours, Mixed Spatial Traffic Modeling of Heterogeneous Cellular Networks, IEEE International Conference on ubiquitous Wireless Broadband (ICUWB), Canada, Sep, 2015
- Chao Li, Abbas Yongacoglu, and Claude D’Amours, Spatial traffic modeling with adjustable heterogeneity based on signal to interference ratio, 2nd International Conference on Telecommunication Systems and Networks in Istanbul (MIC-Telecom), Turkey, Dec, 2014
Chapter 2

Background on Stochastic Geometry and Literature Review of Non-Uniform User Models

Stochastic geometry is the study of random spatial realizations [25, 26]. It is a powerful mathematical tool to model single-tier and multi-tier cellular networks for wireless networks with random topologies. Before the stochastic geometry became popular, the mathematical performance analysis depended on overly-simplified assumptions. For example, the Wyner model in [27] assumes only one or two interferers, while in [18] the authors consider the total interference as a random variable and the interferers in [28] are assumed to be the same distance from the transmitter (TX). Using stochastic geometry, the spatial randomness is reflected and tractable analysis\(^1\) is also achieved. By averaging over many random spatial realizations, stochastic geometry gives a method of defining the characteristics of networks, whose nodes are placed according to some probability distribution.

In wireless networks, such as cellular networks, the geometry (the locations of BSs and UEs) plays a key role because of the random interference. The assumption of simple traditional single-tier BSs is the hexagonal grid [29–31], which leads to either time-consuming analysis or inaccurate results. In practice, the position of BSs follows a random pattern as opposed to the grid pattern. Multi-tier cellular networks where each tier is deployed independently have even more spatial randomness. The received interference signal power can limit the system performance. The received signal power and interference signal power are random variables due to the random placement of BSs, the random placement of UEs, and

\(^1\)Tractable analysis means that the analysis could be handled or managed through mathematical derivations. The analysis could be expressed with closed-form mathematical equations.
the random fading power gain that is present in the channel. Dealing with the above random items, the techniques based on stochastic geometry provide the performance analysis, such as connectivity, capacity, coverage probability, and other performance metrics. [32–40] also give more details about stochastic geometry. Normally, the location of the nodes is modeled as a point process. A point process is one of the important aspects in stochastic geometry. A point process is a kind of random process for which one pattern makes up of a set of isolated points and is a countable random collection of points that reside in some measure space. A point process (PP) can capture the network properties. A temporal point process is a collection of mathematical points with random intervals defined in one dimension, such as the time of arrival points. A spatial point process is a collection of mathematical points randomly located in a space, which is a multiple dimensional domain. A spatial point process is frequently used in geographical models, such as positions of towns, cities, BSs, or UEs.

2.1 Classical Poisson Distribution

The Poisson distribution is the basis of most point processes.

![Poisson distribution (PDF)](image)

Figure 2.1: Probability of Poisson distribution

**Definition 2-1.** A Poisson distribution is defined as follows.
Let $K$ be the number of events in a given time period. If the mean number of events in a given time period is $\lambda$, the probability of observing $k$ events in a given time period is:

$$P(K = k) = e^{-\lambda} \frac{\lambda^k}{k!}.$$  

(2.1)

The random variable $K$ is said to follow a Poisson distribution with a mean parameter $\lambda$, denoted $K \sim \text{Poi}(\lambda)$. Figure 2.1 gives examples of Poisson distributions with different values of $\lambda$.

### 2.2 Temporal Point Process

A temporal point process is a stochastic, or random process composed of isolated events that occur in continuous time [41].

**Definition 2-2.** A temporal point process is a random process whose realizations consist of the times $\tau_j$, $\tau_j \in \mathbb{R}$, $j = 0, \pm 1, \pm 2$ of isolated events scattered in time [42].

Normally, there are three types of representations of temporal point process:

- **Step function:** the total number of events since the time $\tau_j$.
- **Dirac delta function:** dirac delta function is presented at $\tau_j$ where the event occurs.
- **Event intervals:** the time interval from the current event $\tau_j$ to the previous event $\tau_{j-1}$.

### 2.3 Spatial Point Process

According to the network topology assumed, a matching spatial point process is chosen to represent the network nodes, such as BSs and UEs. In the following subsections, the most popular spatial point processes in cellular networks are listed. The practical scenarios and examples based on BS distribution are also given.
2.3.1 Poisson Point Process (PPP)

Poisson point process plays a fundamental role in the theory of PPs. Since they carry the property of complete spatial randomness, the mathematical model is highly tractable. But it is not realistic to assume a complete random model. However, it can be regarded as a reference or a baseline.

**Definition 2-3.** A point process is a PPP if and only if [32]

- The number of points $N(B_1),\ldots,N(B_k),\ldots,N(B_K)$ in disjoint subsets $B_1,\ldots,B_k,\ldots,B_K$ is a Poisson random variable.
- The number of points $N(B_1),\ldots,N(B_k),\ldots,N(B_K)$ in disjoint subsets $B_1,\ldots,B_k,\ldots,B_K$ are independent.

2.3.1.1 Homogeneous PPP

**Definition 2-4.** A PPP is a homogeneous PPP (i.e. uniform) if the density $\lambda$ is the same in all subsets. The mean number of points in disjoint subsets $B_1,\ldots,B_k,\ldots,B_K$ is $A(B_1)\lambda,\ldots,A(B_k)\lambda,\ldots,A(B_K)\lambda$. $A(B_k)$ is the area in subset $B_k$.

The probability when the number of points in the subset $B_k$ is $n$ is

$$P(n, B_k) = \frac{(A(B_k)\lambda)^n \exp(-A(B_k)\lambda)}{n!}.$$  \hspace{1cm} (2.2)

The complete independent property of homogeneous PPP leads to some useful properties, such as the superposition and thinning properties. The superposition of $n$ homogeneous PPs with the densities $\lambda_1$ to $\lambda_n$ is a homogeneous PPP with the density $\lambda_T = \sum_{j=1}^{n} \lambda_i$. Furthermore, randomly thinning a homogeneous PPP with the density $\lambda$, where each point is independently removed with some probability $p$, forms another homogeneous PPP with the density $\lambda_T = (1-p)\lambda$.

The homogeneous PPP of a cellular network proves to be more tractable than a hexagonal model [43,44] because the Campbell’s theorem and Probability Generating Functional (PGFL) based on PPP simplify the calculation and result in the closed-form equations. Traditionally, the positions of BSs are well represented in a hexagonal model. However, the
problem with the hexagonal model is that the performance analysis is intractable\textsuperscript{1}. The resulting SINR with hexagonal model is still a random variable in the case of shadowing and fading and it is difficult to match the distribution of the related SINR random variable. Homogeneous PPPs were introduced due to their well-defined tractability.

The tractability of homogeneous PPP comes from the two properties explained in [45]. These two properties can transform the unlimited sum or the unlimited product over PPP into tractable mathematical integral expression. Let \( \phi \) be a homogeneous PPP set with mean measure \( \lambda \) and let \( f(x) \) be a function in \( \mathbb{R}^2 \).

**Property 2-1.** Sums over PPP (Campbell’s Theorem):

\[
\mathbb{E}\left[ \sum_{x_i \in \phi} f(x_i) \right] = \lambda \int_{\mathbb{R}^2} f(x) dx, \quad (2.3)
\]

- \( f(x_i) \) can be any function.
- The expectation of the function summed over a PPP \( \phi \) can be obtained through Campbell’s theorem.
- The expectation is equal to an integral involving the mean measure \( \lambda \) of the point process.

An example in wireless network communication is when a BS sends a signal to a UE, all other BSs can be regarded as interferers. If the locations of the interfering BSs form a PPP, the expectation of the sum of the interference can be calculated by an integral using Campbell’s theorem, which is the stochastic geometry model of wireless networks.

**Example 2-1.** Calculate the expectation of total interference power at the origin from the BSs with homogeneous PPP if path loss exponent is \( \alpha \) and constant transmit power of BSs is \( P \). Assume the distance \( \|x_i\| \) between the interference and origin is \( d_{\text{min}} \leq \|x_i\| < d_{\text{max}} \).

\textsuperscript{1}Intractable analysis means that the analysis could be not handled or managed through mathematical derivations. The analysis could not be expressed with the closed-form mathematical equations.
The total interference is

\[ I_r = \sum_{x_i \in \phi} f(x_i) = \sum_{x_i \in \phi} P\|x_i\|^{-\alpha}. \]  

(2.4)

According to Campbell’s theorem, the expectation of the total interference is

\[ \mathbb{E}[I_r] = \mathbb{E}[\sum_{x_i \in \phi} P\|x_i\|^{-\alpha}], \]

\[ = \lambda \int_{\mathbb{R}^2} P\|x\|^{-\alpha} dx, \]

\[ = \lambda \int_{d_{\min}}^{d_{\max}} \int_0^{2\pi} P r^{-\alpha} d\theta dr, \]

\[ = 2\pi \lambda \int_{d_{\min}}^{d_{\max}} P r^{-\alpha+1} dr, \]

\[ = \frac{2\pi \lambda P}{2-\alpha} (d_{\max}^{2-\alpha} - d_{\min}^{2-\alpha}). \]

(2.5)

Property 2-2. If \( \phi \) is a Poisson point process with mean measure \( \lambda \), products over \( \text{PPP} \) for any function \( f(x_i) \) (Probability Generating Functional (PGFL)) is:

\[ \mathbb{E}[\prod_{x_i \in \phi} f(x_i)] = \exp(-\lambda \int_{\mathbb{R}^2} (1 - f(x)) dx). \]

(2.6)

Example 2-2. Calculate the Laplace transform of the total interference at the origin.

\[ \mathbb{E}[e^{-sI_r}] = \mathbb{E}[e^{-s\sum_{x_i \in \phi} P\|x_i\|^{-\alpha}}], \]

\[ = \mathbb{E}[\prod_{x_i \in \phi} e^{-sP\|x_i\|^{-\alpha}}], \]

\[ = \exp(-\lambda \int_{\mathbb{R}^2} (1 - e^{-sP\|x\|^{-\alpha}}) dx), \]

\[ = \exp(-2\pi \lambda \int_{d_{\min}}^{d_{\max}} (1 - e^{-sP r^{-\alpha}}) r dr). \]

(2.7)

The Laplace transform of the total interference is the key step in the closed-form tractable analysis of a homogeneous PPP. For example, the coverage probability analysis depends on SINR, which is the ratio between the received signal to the total interference.
plus noise. If the Laplace transform of the total interference is characterized by an integral as in Equation 2.6, it is easy to determine the coverage probability.

### 2.3.1.2 Heterogeneous PPP

**Definition 2-5.** A PPP is a heterogeneous PPP or inhomogeneous PPP if the density is not equal among each subset. Assume that the density function is $\beta(x, y)$, $x, y \in \mathbb{R}^2$. The density measure $\lambda(B_k)$ over $B_k$ is $\int_{B_k} \beta(x, y) dxdy$. The mean number of points in subsets $B_1, ..., B_k, ..., B_K$ is $A(B_1)\lambda(B_1), ..., A(B_k)\lambda(B_k), ..., A(B_K)\lambda(B_K)$.

The probability when the number of points in the subset $B_k$ is $n$ is

$$P(n, B_k) = \frac{(A(B_k)\lambda(B_k))^n \exp(-A(B_k)\lambda(B_k))}{n!}. \quad (2.8)$$

![Figure 2.2: Realization of homogeneous PPP and heterogeneous PPP](image)

Figure 2.2: Realization of homogeneous PPP and heterogeneous PPP

Figure 2.2 (a) shows a realization of a homogeneous PPP while Figure 2.2 (b) shows a realization of a heterogeneous PPP. A realization of a spatial point process is considered as spatial point pattern with countable collection of points. The square is 10 m by 10 m and split into 100 grid squares of 1 m by 1 m. In this realization, the density $\lambda$ is 1 user per square meter for homogeneous PPP and the density function $\beta$ is $\frac{\exp(1+0.3x)}{10}$ users per square meter ($x$ is the horizontal coordinate) for heterogeneous PPP.
2.3.1.3 Practical Scenarios and Examples

Figure 2.3: Example of the Voronoi region of homogeneous PPP and heterogeneous PPP

PPP, especially homogeneous PPP, is a very popular distribution for modeling the nodes, such as BSs and UEs. Figure 2.3 gives an example of points with PPP distribution and their Voronoi cells. If we assume the points as BSs, curves are cell boundaries with the closest BS association rule. In Figure 2.3 (a), the mean density is the same across the whole area such that it is possible to get the tractable performance analysis using the Campbell’s theorem and PGFL. In Figure 2.3 (b), the mean density is not the same across the whole area. The density is higher when approaching the right side. It models the scenario that more UEs need to be served on the right side if points are assumed as BSs. However, it is challenging for heterogeneous PPP to get a tractable analysis.

2.3.2 Transforming a Spatial Point Process

For different conditions we may want to modify a uniform PPP to obtain a new PP. This new point process is obtained by transforming an existing point process. It includes displacement, thinning, and superposition [46].

2.3.2.1 Displacement

Displacement is the mapping of points in a point process to the same or another space by a defined mapping rule.
Figure 2.4 and Figure 2.5 show the realizations of displacing a point process to another point process in two dimensions. The mapping rule of Figure 2.4 is from \((x, y)\) to \((x + 10, y + 10)\). The mapping rule of Figure 2.5 is from \((x, y)\) to \((0.6x + 2, 0.6y + 2)\). \((x, y)\) are the original point coordinates.

Independent displacement consists of independently mapping each point of the original PPP to a random placement. The result of independent displacement of PPP is still a PPP.

The displacement of PPP has two advantage. The first advantage is that the displacement of PPP would result in the expected user distribution, such as more users around the
cell center. The second advantage is that the performance analysis is potentially tractable if it is displaced from a tractable homogeneous PPP.

2.3.2.2 Thinning

Thinning of a point process is to remove some points according to defined thinning rule. There is independent thinning and dependent thinning. If the original point process is homogeneous PPP, the remaining independent points also belong to a homogeneous PPP.

Figure 2.6 sketches a realization of thinning. Thinning rule in this example is that each point is retained with the probability of 30%. In Figure 2.6, (a) shows original points in black. (b) includes the same points as in (a) except that retained points are labeled in red during thinning. (c) includes retained points in red.

![Figure 2.6: Realization of thinning (black points are thinned with the probability of 70%)](image)

2.3.2.3 Superposition

Superposition is the operation of the sum of two or more point processes. The superposition of two independent PPP with a density $\lambda_1$ and $\lambda_2$ is still a PPP with a density $\lambda_1 + \lambda_2$.

Figure 2.7 shows a realization of superposition of two PPP. (a) is PPP with a density $\lambda = 1$ in black. (b) is PPP with a density $\lambda = 1$ in red. (c) is the resulting superposition of (a) and (b). (c) is still a PPP with a density $\lambda = 2$. 

![Figure 2.7: Realization of superposition (black points are superposed with the probability of 70%)](image)
2.3.2.4 Practical Scenarios and Examples

Figure 2.8 gives an example of point distribution of displacement and the Voronoi region of points. The original point process is a homogeneous PPP. Any mapping or displacement rule can be applied to obtain the resulting distribution. In this example, the resulting distribution of displacement is shifting the points position toward to the area center according to a parameter. The cell boundaries of original PPP BS distribution and displaced BS distribution are all shown in this figure if we assume the points are BSs with closest BSs association rule. The cell coverage area are the Voronoi region. This example represents the scenario where more users are distributed at the center area and fewer users are distributed at the edge if points are assumed as BSs. Hence, more BSs should be distributed.
at the center area so that the cell radius of BSs at the center becomes smaller such that it serves and supports more users. Resulting displacement is obtained from homogeneous PPP so that it is possible to get the closed-form performance analysis.

![Figure 2.9: Example of thinning points and their Voronoi region](image1)

Figure 2.9 indicates an example of point distribution of thinning and the Voronoi region of points. The original point process is also a homogeneous PPP. Thinning process retains each point independently with a given probability. The retained or kept BSs will have larger cell coverage areas if we assume the retained points are BSs. The retained points distribution is still homogeneous PPP. Figure 2.10 gives an example how to generate non-uniform user distribution using thinning process. Blue triangles are BSs, black points are users and red curves are cell boundaries. The users are distributed uniformly and independently in the original point process (a). The third BS is thinned and the users associated with the third BS is also thinned shown in (b). There are only two BSs left.

![Figure 2.10: Example of generating non-uniform users using thinning](image2)
after thinning process. New cell boundaries are shown in (c) where the user distribution is non-uniform. This has been widely discussed in [47].

Figure 2.11: Example of superposition points and their Voronoi region

Figure 2.11 presents an example of point distribution of superposition and their Voronoi region. Both original points 1 and original points 2 are a homogeneous PPP. The superposition of original points 1 and original points 2 is still a homogeneous PPP. The Voronoi region is shown with closest BSs association if we assume points as BSs. In the literature, this superposition of multi-homogeneous PPPs with different BS density in each tier is called K-tier networks [20]. If the cell association rule is such that a UE connects to the BS with maximum SINR, the cell coverage area in each tier will be different due to different BS transmit powers in each tier (e.g., the cell coverage area of BSs with low transmit power will be relatively small.)

2.3.3 Hard Core Point Process (HCPP)

2.3.3.1 Hard Core Point Process Introduction

The hard core point process (HCPP) is a repulsive point process, in which the distance between any two points is more than the prescribed hard core threshold [26]. HCPP belongs to the thinning point process introduced in subsection 2.3.2.2. Because HCPP is a practical and important node model, for instance any two BS nodes or two UE nodes cannot be closer than a certain distance in real deployment, here we introduce HCPP as a separate subsection.

Definition 2-6. A PP $\phi = \{x_i, i = 1, 2, 3, \ldots\} \subset R$ is hard core point process if and only if $\|x_i - x_j\| \geq d, \forall x_i, x_j \in \phi, i \neq j.$
The rule used to select which points are retained and which points are removed is as shown in Figure 2.12.

```plaintext
UE_x; (the horizontal coordinate of UE is given)
UE_y; (the vertical coordinate of UE is given)
Number_{UE}; (Number of UE is given)
UE_{x_{retained}} = []; (the horizontal coordinate of retained UE)
UE_{y_{retained}} = []; (the vertical coordinate of retained UE)
Number_{UE_{retained}} = 0; (Number of retained UE)
Distance = 0; (distance between points)
Radius = 1; (the prescribed minimum distance between hard core points)
Indication = 0; (Indication if the point is removed. 0 — retained and 1 — removed.)
for i = 1 : (Number_{UE} — 1)
    for j = (i + 1) : Number_{UE}
        Distance = \sqrt{{(UE_x(i) — UE_x(j))^2 + (UE_y(i) — UE_y(j))^2}}
        if Distance < Radius
            Indication = 1;
        else
            end
        end
    if indication == 1
    else
        Number_{UE_{retained}} = Number_{UE_{retained}} + 1;
        UE_{x_{retained}}(Number_{UE_{retained}}) = UE_x(i);
        UE_{y_{retained}}(Number_{UE_{retained}}) = UE_y(i);
    end
end
```

Figure 2.12: Rule to select HCPP points

To put it simply, HCPP has a prescribed minimum distance between the points. A realization is shown in Figure 2.13. (a) shows the original points in red. (b) labels points black if there are other points inside the circle of radius $R_h$. Black points are removed to form the resulting hard core points in (c). If two points have a distance less than the prescribed threshold, any point of two can be chosen to remove in this example. The distance between any two kept points is larger then $R_h$ in (c). The radius of circle is $\frac{R_h}{2}$. In this example, $R_h = 1$. 
Figure 2.13: Realization of hard core point process
2.3.3.2 Practical Scenarios and Examples

Figure 2.14: Example of hard core point process and their Voronoi region

Figure 2.14 introduces an example of point distribution of hard core point process. If we assume the points as BSs, the labeled red BSs are removed in the right figure since these BSs have a distance of less than a prescribed minimum distance with other BSs in the left figure. It is practical BS distribution because the distance between any two BSs should be more than a prescribed minimum distance while it is possible for any two BSs to have zero distance when BS distribution is PPP.

2.3.4 Poisson Cluster Process (PCP)

2.3.4.1 Poisson Cluster Process Introduction

Poisson cluster process is a random pattern produced by random clusters [48]. PCP generates homogeneous and independent clusters. The PCP has two types of points, parent points and daughter points. Parent points are PPPs and form the center of each cluster. Daughter points are generated spatially around each parent point according to some distribution function. PCP assumes that daughter points are symmetrically, independently, and identically distributed (i.i.d.) around parent points.

There are many types of PDF distribution for daughter points. For example, in a Matern cluster process, daughter points are uniformly distributed around parent points in a circle of a predefined radius $R$ where the PDF is $\frac{1}{\pi R^2}$. In a Thomas cluster process, daughter points are distributed around parent points by an uncorrelated two dimensional normal
distribution with a predefined variance $\sigma$ where the PDF is $\frac{1}{2\pi\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right)$. In a linear cluster process, daughter points are distributed around parent points by linearly decreasing probability within a circular disc of predefined radius $R$ where the PDF is $\frac{3}{\pi R^2} \left(1 - \frac{r}{R}\right)$. In an exponential cluster process, daughter points are distributed around parent points by exponentially decreasing probability of predefined rate parameter $E$ where the PDF is $\frac{E^2}{2\pi} \exp\left(-Er\right)$. $r$ is the distance between the daughter point and the parent point. In Figure 2.15, an example of the above four PDFs is shown. For Matern cluster process, the radius is $R = 50$ m. For Thomas cluster process, the variance is $\sigma = 40$. For linear cluster process, the radius is $R = 50$ m. For exponential cluster process, the rate is $E = 0.05$.

![Figure 2.15: PDF distribution example in PCP](image)

Figure 2.16 shows a realization of PCP. (a) shows parent points with PPP. (b) is PCP based on (a). The black points are daughter points and the red points are parent points. The daughter points follow Matern cluster process. The area is 10 m by 10 m.
2.3.4.2 Practical Scenarios and Examples

Figure 2.17 illustrates an example of point distribution of PCP. If we assume the points as BS, this example represents the scenario where users concentrate together and more BSs are deployed around the area where cluster users are distributed so as to serve and support cluster users. The Voronoi region represent the coverage area with closest distance association. In this example, a single tier of BSs is assumed. For situations where multi-tier BSs exist, the coverage area will be different, which will be discussed in the following chapter.
The use of spatial random point processes to model UE in HetNet has been considered extensively. In the literature, the user spatial distribution is mostly assumed to be distributed as an PPP (uniform) independent of the distribution of BS. However, in HetNets, users tend to be spatially non-uniform and have an attraction to their associated BSs, especially in small cells [49,50]. Non-uniform spatial distribution of users reflects the user-centric capacity-driven deployment of small cells. The literature focused on non-uniformly distributed users is very limited. In the literature, the performance analysis of the non-uniform user distributions is summarized in the following two categories.

1. The first category of research work are studies where coverage probability is determined through time-consuming system level simulations.

In [51], the authors uses the doubly stochastic Poisson process (DSPP) to model the user location, which is also known as the Cox process. Cox process considers a second layer of randomness, for example by generalizing the constant user density into a random variable. Coefficient of variation (CoV) is taken as an indication of heterogeneity. The authors of [51] give system level simulations that show that the network performance deteriorates greatly with increased heterogeneity (CoV) of UEs and the non-correlation of the UE location and the BS location. Nevertheless, the performance enhances if small cell BSs are deployed at the centers of the user hot-spots determined by cluster analysis algorithm. The model used in [52] also indicates a similar conclusion that the performance improves when users are clustered.
around the BSs in WCDMA networks. While in [53], the model simulates a conventional CDMA system on uniform distributed users and clustered distributed users. It shows that the two distributed users have similar mean values of measured carrier to interference as well as similar outage. The clustered distributed users have higher standard deviation of measured carrier to interference ratio. The work in [54] demonstrates that 3GPP standard body requires users with the high density in the vicinity of small cell BSs. In [55,56], the performance is obtained through system simulation while the traffic load is a random variable with log-normal or Weibull distribution.

The authors of [57] describe a non-uniformly distributed spatial user model in cellular CDMA systems. The density of users per unit area as a function of the distance from the base station have been modeled in different ways, such as linear exponential, and Gaussian. However, this model is for conventional macro cells, i.e. single tier systems; and the performance analysis and results are obtained by Monte Carlo simulations rather than analysis using stochastic geometry.

A user model for heterogeneous user distribution in heterogeneous wireless cellular networks (thus called HetHetNets) is introduced and discussed in [58]. Users are attracted to social attractors (SAs), such as buildings, bus stations, shopping centers, restaurants, and other social places. Attraction of SAs to BSs is also considered. Performance metrics are determined by simulation methods. This paper considers the correlation between BSs, UEs, and SAs. However, an analytical evaluation is not presented in [58].

2. The second category of research work is performed using the tools of stochastic geometry.

In [59], we presented a user model, in which the user’s position relative to the BS is determined by a parameter $p$, which varies between 0 and 1. The parameter $p$ represents a correlation between the location of the UE and the BS. For $p = 0$, all UEs are randomly placed in the cell according to a PPP, while when $p = 1$, all UEs are positioned at the BS. Therefore, the model created situations where there are very few UEs near the cell boundaries when $p$ is high. In [60], we deployed a density function as opposed to a correlation parameter $p$ as in [59] to describe the non-uniformity to avoid the unlikely scenarios where there are no users near a cell boundary. The work of [60] provided a general non-uniform user model with heterogeneous PPP, in which the density is dependent on the distance to the associated BS. The density function can be any function, such as monotonically increasing, decreasing, or other trend.

The authors of [61] presented a random heterogeneous node distribution based on
a neighborhood-dependent thinning of a homogeneous PPP. This user model is heterogeneous and clustered to some degree. However, it is impossible to predict the position of the cluster center. The positions of cluster center in this model are completely random or unrelated to the position of BS. However, rather than being totally random, users in practical HetNets are attracted to BSs. Furthermore, this model is hard to analytically obtain the derived performance metrics, such as the coverage probability.

In [47] another kind of thinning model is described. Each base station is independently retained with probability $P_r$ and thinned (i.e. removed) with probability $1 - P_r$. The users inside the coverage of the thinned base station are also thinned. The retained base stations determine the new coverage boundaries. In [47], a tractable performance analysis is given; but again, this model is only for single-tier macro cells. In [47], both user and BS locations are modeled by homogeneous PPPs with different densities in the first step; and heterogeneous user distribution is obtained by conditional thinning of BSs and the corresponding users in the Voronoi cells of thinned BSs. This method facilitates the deduction of analytical expressions. However, the generated user distributions may not be entirely realistic because of the absence of users in the thinned areas and the identical density of users in the remaining areas. Figure 2.10 gives an example of this thinning process.

The research in [23, 24] introduces a case of user-centric deployment, in which the UE locations and BS locations are correlated. The user distribution is a Poisson cluster process (PCP) where cluster centers are BSs. PCP assumes that the users in the cluster are symmetrically, independently, and identically distributed (i.i.d.) around the center. The corresponding performance analysis of the downlink coverage probability is obtained.

In [62], we introduced a mixed traffic model including the uniformly and non-uniformly distributed users at the same time. The non-uniform user distribution follows the same idea as in [59], in addition to which we extend the non-uniform distribution to include mixed distributions.

The authors of [63] provided a simple non-uniform distribution model, which only focuses on indoor communications. More practical non-uniform distribution model over HetNets should also be considered. The networks with the non-uniform user distribution in device-to-device communication is also discussed in [64–68].

The above research focuses on the downlink. In the literature, the uplink is also investigated. In the downlink the interference comes only from BSs of all tiers while
in the uplink the interference comes from all active UEs. The location of BSs is fixed, but the location of UEs is moving. In addition, the uplink power control of UE side should be considered. Covering BS point process and UE point process together into an uplink analysis brings many challenges. In the uplink reference, the uplink coverage probability is obtained over single tier, multi-tier, other fading type, decoupled uplink-downlink biased cell association, and so on [69–74]. However, the user distribution is still uniform. We extend our non-uniform user distribution to the uplink research in this thesis.

The literature on non-uniform user distribution is very limited. It is especially lacking in research on non-uniform user distribution on heterogeneous networks in both downlink and uplink. Therefore, developing the non-uniform user model framework, delivering the performance analysis in both downlink and uplink, and showing system insight on HetNets is the main goal of this thesis.
Chapter 3

System Model

3.1 Introduction

The performance of HetNets critically depends on the spatial distribution of BSs and UEs, which plays a key role in the research, design, evaluation, and deployment of 5G HetNets. In HetNets, the spatial distribution of BS and UE is generally non-uniform. Analysis of the impact of non-uniform user distribution based on a heterogeneous cellular network is essential for designing efficient HetNets. In this chapter the system model – which deals with BS distribution (Section 3.2), user distribution (Section 3.3), and user connectivity (Section 3.4) – will be explained.

3.2 BS Distribution

3.2.1 Related Work on BS Distribution in the Literature

There are many papers that focus on the research of the spatial distribution of BS with the assumption of uniform user distribution. Some of these papers are:

- The work presented in [20,21] focuses on the downlink of a heterogeneous cellular network (HCN) consisting of K tiers of randomly located BSs with different cell sizes and capabilities.

- The work in [22,75–79] extends the idea of K-tier networks to include flexible cell association and load-aware policies.
• The work in [80] considers a K-tier network where the BS distribution includes a minimum distance policy while the work in [81] considers a K-Tier network where the BSs are distributed according to a Matern process.

• The work in [82–84] presents a BS distribution model where small cells are deployed outside of exclusion zones around macro cells. The exclusion zones are referred to as holes and the BS distribution is performed using a Poisson hole process (PHP).

• In [85–89], small cells are modeled by clusters using Poisson cluster process (PCP).

• In [90,91], the outage probability of BS models with the general point process can be obtained with a fixed positive SINR shift from the PPP model.

3.2.2 BS Distribution Model Used In the Thesis

Throughout this thesis, the basic assumption is that the BS distribution follows the K-tier network described in [20]. In K-tier networks, cells in the same tier have approximately the same BS transmit powers, densities, SINR coverage thresholds, and path loss exponents. However, BSs from different tiers have different transmit powers, densities, SINR coverage thresholds and path loss exponents. The BS distribution in each tier is an independent spatial Poisson point process (SPPP). We assume that BSs in the $k$th tier are generated by SPPP $\phi_k$ with the density $\lambda_k$, transmit power $P_k$, SINR coverage threshold $T_k$, and path loss exponent $\alpha_k$.

Figure 3.1 gives a realization of a 3-tier BS distribution. Figure 3.1 (a-c) represents a BS realization of 1st tier, 2nd tier and 3rd tier separately, in which each tier is an independent PPP with different BS density. The final BS realization (d) of 3-tier is the superposition of (a), (b), and (c). We name the first tier of BS the macro cell tier, the second tier of BS the pico cell tier, and the third tier of BS the femto cell tier. In Figure 3.1 (e), the BS realization of 3-tier with cell boundaries (blue curves) is given. In (e), the BS transmit power of the macro cell is 10 times greater than that of the cell pico and 100 times greater than that of the femto cell. The area is a square of 1 km by 1 km, in which there are 3 macro BSs, 6 pico BSs, and 18 femto BSs.

Based on the assumption of BS distribution of the above K-tier networks, this thesis proposes the non-uniform user distribution model and the corresponding performance analysis, such as the downlink and the uplink coverage probability using stochastic geometry. The downlink and the uplink coverage probability is obtained by considering the comple-
Figure 3.1: A realization of a 3-tier BS distribution

mentary cumulative distribution function (CCDF) of the signal to interference plus noise ratio (SINR).

3.3 User Distribution

Realistic user behavior is hard to model because the motions and positions of people with mobile devices are difficult to predict and control. However, most of the research in the literature is based on uniform user distribution models. We can take the uniform user distribution model as a benchmark. In this section, we introduce the concept of user distribution, including uniform and non-uniform cases, in single tier and multi-tier BS systems. In order to evaluate the benefit of deploying small cells, the simulation results of the coverage probability in the downlink and the uplink are shown in the following subsection.

The parameters of the 1-tier BS system are as follows: There are three macro BSs in the area of a square of 1 km by 1 km, which is equivalent of the macro BS density $\lambda_1 = \frac{3}{10^6}$. The path loss exponent is normally in the range of 2.5 to 4. Here we choose the path loss exponent $\alpha_1 = 3$. According to 3GPP TR 36.942, the maximum transmit power for a 10
MHz channel bandwidth is 46 dBm so we take the macro BS transmit power $P_1 = 45$ dBm.

The parameters of the 2-tier BS systems are as follows: There are three macro BSs and three pico BSs in the area of a square of 1 km by 1 km, which is equivalent to the macro BS density $\lambda_1 = \frac{3}{10^6}$ and the pico BS density $\lambda_2 = \frac{3}{10^6}$. We take the same path loss exponent for both tiers, which is $\{\alpha_1, \alpha_2\} = \{3, 3\}$. Lastly the transmit power of macro BSs is 10 times greater than that of pico BSs, which is $\{P_1, P_2\} = \{45, 35\}$ dBm and $P_1 = 10P_2$.

The parameters of the 3-tier BS system are as follows: There are three macro BSs, three pico BSs and six femto BSs in the area of a square of 1 km by 1 km, which is equivalent to the macro BS density $\lambda_1 = \frac{3}{10^6}$, the pico BS density $\lambda_2 = \frac{3}{10^6}$ and the femto BS density $\lambda_3 = \frac{6}{10^6}$. We take the same path loss exponent for three tiers, which is $\{\alpha_1, \alpha_2, \alpha_3\} = \{3, 3, 3\}$. The transmit power of macro BSs is 10 times greater than that of pico BSs and 100 times greater than that of femto BSs, which is $\{P_1, P_2, P_3\} = \{45, 35, 25\}$ dBm and $P_1 = 10P_2 = 100P_3$.

A procedure of designing the simulation setting of the following figures is given in Appendix C.1 and C.2.

3.3.1 Non-Uniform User Distribution in a Single Tier BS System

3.3.1.1 Set Initial BS Density

The UE distribution pattern of uniform and non-uniform, coverage map and coverage boundaries, and performance based on three macro BSs are shown.

3.3.1.1.1 UE Distribution Pattern

Figure 3.2 shows a uniform UE distribution in (a) and a non-uniform UE distribution in (b), in which the user cluster centers of the non-uniform UEs are distributed randomly and they have not any correlation between BSs and the user cluster centers.
3.3.1.1.2 Coverage Map and Cell Boundaries

In Figure 3.3, (a) gives the actual cell map with the maximum SINR association rule and (b) indicates the cell boundaries with blue curve. (a) shows the coverage map in which the relative SINR coverage threshold is labeled. The coverage map is the contour with the measured SINR of 0, 10, 20 and 30 dB. If a UE is randomly distributed, it will fall into a BS’s associated area. However, the coverage area depends on the SINR coverage threshold. For example, in (a), most area is in coverage if the SINR coverage threshold is 0 dB. In a single tier system, the cell boundaries are the exact same as the Voronoi cell edges if the transmit power of all BSs is the same as shown in Figure 3.3. In Figure 3.4, the transmit power of BS1 is 4 times of the transmit power of BS2 and BS3. The cell boundaries is different with Figure 3.3. In Figure 3.4, we just give one example that the BSs in the same tier transmit with different power. In other figures of this thesis, we assume the BSs in the same tier transmit with the same power.
Figure 3.3: Coverage map (a) and cell boundaries (b) in a single tier BS system (3 BSs) where the transmit power of BS1, BS2 and BS3 is the same.

Figure 3.4: Coverage map (a) and cell boundaries (b) in a single tier BS system (3 BSs) where the transmit power of BS2 and BS3 is the same and the transmit power of BS1 is 4 times the transmit power of BS2 and BS3.

3.3.1.1.3 Performance

Figure 3.5 shows the coverage probability of the downlink and the uplink with uniformly distributed UEs and non-uniformly distributed UEs. The figures present the performance of the uniform UEs is the same as the performance of the non-uniform UEs if the non-uniform UE cluster centers are randomly distributed and do not have any correlation with BSs.
3.3.1.2 Increasing Macro BS Density

In the following part, we evaluate the benefit by deploying additional macro BSs in the UE cluster centers. In this example, three additional macro BSs are deployed in the cluster centers in order to support the dense UEs.

3.3.1.2.1 UE Distribution Pattern

Figure 3.6 gives UE distribution pattern in which three additional macro BSs are deployed in the UE cluster centers.

Figure 3.6: UE distribution pattern with non-uniform UEs where three additional macro BSs are deployed in the UE cluster centers.
3.3.1.2.2 Coverage Map and Cell Boundaries

Figure 3.7: Coverage map (a) and cell boundaries (b) in a single tier BS system (6 BSs where 3 macro BSs are put randomly and another 3 macro BSs are put in the user cluster centers)

3.3.1.2.3 Performance

Since three additional macro BSs are deployed in three UE cluster centers, the coverage probability has tremendous improvement in the downlink and the uplink.

Figure 3.8: Performance of DL and UL in a signal tier (red curve is for 3 macro BSs in a single tier where 3 macro BSs are independent with UE cluster distribution, black curve is for 6 macro BSs in a single tier where 3 macro BSs are independent with UE cluster distribution and another 3 macro BSs are deployed in UE cluster centers.)
3.3.2 Non-Uniform User Distribution in a 2-tier BS System

3.3.2.1 Replacing Macro BSs with Pico BSs

In previous example, macro BSs are deployed in the UE cluster centers. However, due to the small UE cluster radius and power efficiency, it is better to deploy pico BSs rather than macro BSs. The system including macro BSs and pico BSs are named as a 2-tier BS system here.

3.3.2.1.1 UE Distribution Pattern

![UE distribution pattern with non-uniform UEs in which pico BSs are deployed in the UE cluster centers.](image)

Figure 3.9: UE distribution pattern with non-uniform UEs in which pico BSs are deployed in the UE cluster centers.

3.3.2.1.2 Coverage Map and Cell Boundaries
3.3.2.1.3 Performance

After replacing macro BSs with pico BSs, the performance decreases as shown in Figure 3.11. But it saves the power consumption and the coverage area of pico BSs is enough to cover the cluster UEs.
Figure 3.12: Coverage probability of cluster users (magenta points) when $P_1 = 2P_2$, $P_1 = 10P_2$, and $P_1 = 20P_2$ in a 2-tier BS system.

Figure 3.12 shows the coverage probability of a cluster of users with a different power ratio, in which the transmit power of macro BSs is 2 times, 10 times and 20 times greater than the transmit power of pico BSs, respectively. When the power ratio is low, such as $P_1 = 2P_2$, the clustered users are served by pico BS with high SINR so that the coverage probability is higher compared with other two power ratios.

Figure 3.13: Coverage probability of cluster users (magenta points) with different path loss exponents in a 2-tier BS system when $P_1 = 10P_2$.

Figure 3.13 gives the coverage probability of cluster users with different path loss exponents between macro cells and pico cells. When $\alpha_1 = 3.5$, $\alpha_2 = 2.5$, the cluster users served
by pico cells have higher SINR since $\alpha_1 = 3.5$ results in higher path loss from macro BSs and $\alpha_2 = 2.5$ results in lower path loss from pico BSs so that the cluster users in pico cells have high coverage probability. The users in the macro cells propagate through space with large power attenuation due to the large coverage area and more blocking while the users in the pico cells propagate with low power attenuation due to the small coverage area and less blocking. The users from macro BSs experience the high path loss and the users from pico BSs experience the low path loss. Therefore, the case of $\alpha_1 = 3.5, \alpha_2 = 2.5$ is the most realistic among the three path loss exponent ratios.

3.3.2.2 Increasing Pico BS Density

The additional 6 UE clusters are distributed with smaller radius. In Figure 3.14, these additional 6 UE clusters are labeled with green points.

3.3.2.2.1 UE Distribution Pattern

Figure 3.14: UE distribution pattern with non-uniform UEs in which new 6 pico BSs are deployed in these additional 6 UE cluster centers in green.

3.3.2.2.2 Coverage Map and Cell Boundaries
3.3.2.2.3 Performance

It is obvious that the performance is improved after deploying new 6 pico BSs in these additional 6 small UE clusters in green in Figure 3.14. But due to the number of green cluster limited, the performance improvement is not substantial.

Figure 3.16: Performance of DL and UL in a 2-tier BS system (red curve is for 3 macro BSs and 3 pico BSs in a 2-tier system and black curve is for 3 macro BSs and 9 pico BSs in a 2-tier system.)
3.3.3 Non-uniform User Distribution in a 3-tier BS System

3.3.3.1 Replacing Pico BSs with Femto BSs

Due to the small radius of new green cluster, femto BSs are enough to cover the green UE cluster.

3.3.3.1.1 UE Distribution Pattern

![UE distribution pattern with non-uniform UEs in which new 6 femto BSs are deployed in these additional 6 UE cluster centers in green. Red triangles are macro BSs. Black triangle are pico BSs. Blue triangles are femto BSs.](image)

Figure 3.17: UE distribution pattern with non-uniform UEs in which new 6 femto BSs are deployed in these additional 6 UE cluster centers in green. Red triangles are macro BSs. Black triangle are pico BSs. Blue triangles are femto BSs.

3.3.3.1.2 Coverage Map and Cell Boundaries
3.3.3.1.3 Performance
After replacing the 6 pico BSs with 6 femto BSs in order to cover small UE clusters in green, the coverage probability decrease. However, femto BSs are enough to cover the small green UE clusters in Figure 3.17 and low power femto BSs would save power consumption.

Figure 3.19: Performance of DL and UL in a 2-tier and a 3-tier (red curve is for 3 macro BSs and 9 pico BSs in a 2-tier system and black curve is for 3 macro BSs, 3 pico BS and 6 femto BSs in a 3-tier system.)
3.3.4 Summary of User Distribution

From the above subsection on non-uniform user distribution in a single tier, 2-tier and 3-tier BS system, we summarize that deploying small BSs at the user cluster centers can enhance the capacity (small BSs provide coverage to a cluster of users in a hot spot within macro cells which in turn increases the overall system capacity) and improve the coverage probability. Deploying small BSs with high transmit power will bring more performance improvement for users in small cells, but it may cause high deployment cost and high interference to other users. When the cluster radius is less than 100 m and the transmit power is about 30 dBm, deploying pico cells is a good choice, while when the cluster radius is less than 30 m and the transmit power is about 20 dBm, deploying femto cells is a good choice (referred from the link: //www.rfwireless-world.com/Tutorials/femtocell-vs-picocell-vs-microcell.html). In the examples of this chapter, the UE cluster (magenta points) radius in pico cells is 100 m and the UE cluster (green points) radius in femto cells is 50 m.

Small BSs are used to improve the cell range, capacity and coverage performance in hot spots or other densely distributed user areas. Therefore, it is reasonable to assume that the user distribution model is that in small cells the users are non-uniformly distributed and clustered around a cluster center. Furthermore, we may extend the discussion to the mixed user distribution, in which uniformly distributed users and non-uniformly distributed users coexist. In mixed user distribution, uniform users are distributed around the whole area, while non-uniform users are distributed with high attraction level in small cells.

3.4 User Connectivity

3.4.1 Connectivity Rule

User connectivity is the process that specifies the BS to which a user is associated with. Usually, the rule of the user connectivity is that a user connects only to one BS at a time. For conventional networks, the user connectivity is normally based on maximum received power or minimum distance. However, such a connectivity rule is not suitable for heterogeneous K-tier networks. In HetNets, one reason for deploying small cells is to expand indoor as well as macro cell edge coverage. Since the transmit power of BSs in small cells is low and the coverage area is small, these cells have lighter loads. Other connectivity rules, such as the bias connectivity rule should be considered to tackle the lighter load issue.
In Figure 3.20, we illustrate four examples of the cell coverage boundaries for a 2-tier (macro cells and small cells) HetNets. These examples show the related cells, the distributed users should associate with, according to the user connectivity rule of maximum received power. The macro cell BSs are represented by red triangles, while small cell BSs are represented by black triangles. The BS transmit power in the macro cell, $P_1$, is assumed to be 1000 or 100 times greater than the BS transmit power in the small cell, $P_2$. We have considered 2 BS density ratios. In Figure 3.20, the density of the small cells is 4 times greater than the macro cells: $P_1 = 1000P_2$ in (a) and $P_1 = 100P_2$ in (b). In Figure 3.20, the density ratio is 8 for $P_1 = 1000P_2$ in (c) and $P_1 = 100P_2$ in (d). When the BS density is the same, the cell coverage region varies as a function of the transmit power ratio between macro cell BS and small cell BS. As the transmit power ratio drops, the small cells cover
larger areas. The small cell coverage regions are larger when the small cells are located far from the macro cell center.

### 3.4.2 Connectivity Model

In HetNets, deploying small cells brings the benefits of expanding cell edge coverage. However, the transmit power of BSs in small cells is relatively low and the corresponding coverage area is small, such that these cells have lighter loads. Hence, we need a cell extension bias that enables UE to access the small cells even though UE receives a stronger signal from a macro cell than a small cell. The above rule of associating a user with a bias is controlled by a cell extension bias factor $C_j$. $C_j > 1$ is used to extend the cell coverage of the $j$th tier. This bias connectivity rule is called cell range extension (CRE) [22, 78, 92, 93].

![Cell extension example](image)

Figure 3.21: Cell extension example. (a) and (b) have the exactly same BS locations. (a) is $\{C_1, C_2\} = \{1, 1\}$ without cell extension and (b) is $\{C_1, C_2\} = \{1, 6.3\}$ with cell extension. The area is 1 km by 1 km.

In Figure 3.21, the two figures have the exact same BS densities, BS transmit powers and BS locations. In the left figure, there is no cell extension ($C_2=1$) for small cells. In the right figure, there is cell extension ($C_2=6.3$) for small cells. In Figure 3.21, the blue curves are the cell boundaries. After changing cell extension bias factor $C_2$ from 1 to 6.3, the cell coverage areas of the small cells increase. Using the cell extension $C_j$ parameter to enforce a larger coverage area for small cells brings the drawback that the users near the small cell boundary have a lower SINR as opposed to no cell extension. However, the benefit of cell extension is higher capacity in small cells. It is a balance between the users’ SINR and small cell capacity. How to find the optimal cell extension parameter $C_j$ is not
Definition 3-1. In this thesis, the maximum average bias received power (BRP) connectivity model [75] is used, which is defined as follows.

The user is associated with the BS, which can provide maximum average bias received power. Let $P_{BRP_j} = P_j C_j r_j^{-\alpha_j}$ (the path loss $L_0$ at a reference distance 1 m is ignored since it is a constant). The distance between a user and its associated BS is $r$.

$$r = \arg \max_{B_j \in \phi_{BS}} \{P_j C_j r_j^{-\alpha_j}\}. \quad (3.1)$$

3.5 Conclusions

In this chapter, we introduced the system model used in this thesis, which includes BS distribution, user distribution, and user connectivity. We mainly discussed the non-uniformly distributed users. BS distribution is assumed as a K-tier network, in which each tier of BS is an independent PPP. We also introduced user distributions from a single tier BS system to a 3-tier BS system and from uniform to non-uniform distributions. We showed that deploying small BSs at the user cluster will improve the coverage probability and system capacity such that it is reasonable to assume that the non-uniform users are distributed as a cluster around small BSs. The designed non-uniform user model in this thesis will be introduced in the next chapter. Finally, user connectivity model of the maximum average bias received power is presented. This user connectivity model supports the cell association and it is equivalent to maximum average received power rule when cell extension bias factor $C = 1$. In this thesis, we refer to this maximum average bias received power as the user connectivity rule.
Chapter 4

Designed User Distribution Model and Heterogeneity Analysis

4.1 Introduction

The criteria to design a user distribution model in this thesis is as follows:

- In each tier, user distributions are not uniform as they tend to converge towards social attractors (SAs). Different attraction levels between UEs and SAs should be presented and included.

- In each tier, there should be a parameter that controls spatial dependency or correlation between the location of SAs and BSs. All kinds of dependency should be presented and included.

- In the macro cell tier, the users tend to be distributed with low attraction\(^1\) between UEs and SAs and low dependency\(^2\) between SAs and their BSs.

- In the small cell tier, the users tend to be distributed with high attraction\(^3\) between UEs and SAs and high dependency\(^4\) between SAs and their BSs.

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\(^1\)Low attraction means that users have low spatial correlation with their SAs. The user distribution density over the given area is almost the same.

\(^2\)Low dependency means that SAs have low spatial dependency with their BSs. The placement of SAs and BSs is almost independent.

\(^3\)High attraction means that users have high spatial correlation with their SAs. There is a high user density around SAs.

\(^4\)High dependency means that SAs have high spatial dependency with their BSs. The placement of SAs and BSs is dependent.
• Users in different tiers may have different attraction and dependency levels.

• Through the tools of stochastic geometry, the theoretical performance analysis or closed-form equations about performance, such as coverage probability, can be obtained for the designed user model.

Based on the available K-tier BS model, we design a spatially non-uniform user distribution model. Here the non-uniform model (Section 4.2) is a Poisson cluster process (PCP) with the cluster centers located at SAs and SAs have an offset with their BSs. We apply the same user distribution model for each tier. However, in small cells we can choose the user model with high attraction and high dependency while we can choose the user model with low attraction and low dependency for macro cells. Finally, the heterogeneity analysis of non-uniform user distribution is discussed (Section 4.3). All downlink and uplink analysis assume that the channel experiences Rayleigh fading.

4.2 Non-Uniform User Distribution

4.2.1 Definition

Definition 4-1. A non-uniform user distribution in each tier is defined as follows.

• A Poisson cluster process with the cluster centers located at the SAs.

• SAs have a base station offset (BSO) which is the distance between SAs and their BSs.

Figure 4.1 gives an example of this non-uniform user distribution model. Black triangles are BSs of small cells. The blue curves are cell boundaries. Magenta points are users in clusters. Green diamonds are SAs, which are cluster centers. Green circles are the boundaries of clusters with Matern cluster process. $D_2$ is the base station offset between the SA and its BS in small cells.
4.2.2 Parameters

- Model of UEs Clustering around SAs

UEs in small cells tend to gather in clusters while UEs in macro cells tend to be distributed more uniformly. Our non-uniform user distribution is a PCP with SAs at the cluster centers. We define a subset $S(\subset \kappa)$ where non-uniform UEs are distributed. The total number of tiers of $S$ is $S$. In the $k$th tier ($k \in S$), UEs follow our defined non-uniform distribution. It is possible for small cell tiers and macro cell tiers to coexist in the subset $S$, in which they would have their preferred attraction level depending on different parameters.

We assume that the distribution of a UE’s location in a cluster is described by a probability density function (PDF) $f_u(r)$. Here $r$ is the distance of a UE from the cluster center (in our case the SA). $f_u(r)$ in each tier may be different.

We considered four PDFs:

(i) Matern cluster: The density of UEs is uniformly within a circular disc with radius $R$. Here, $R$ is the parameter to control the level of attraction between UEs and SAs. A small value of $R$ means the attraction level is high, and vice versa.

$$f_u(r) = \frac{1}{\pi R^2}, \quad 0 \leq r \leq R. \quad (4.1)$$
(ii) **Thomas cluster**: The density of UEs is with Gaussian spatial distribution, in which $\sigma$ is the standard deviation. Here, $\sigma$ is the parameter to control the level of attraction between UEs and SAs. A low value of $\sigma$ means the attraction level is high, and vice versa.

$$f_u(r) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{r^2}{2\sigma^2}\right) \quad 0 \leq r \leq \infty. \quad (4.2)$$

(iii) **Linear cluster**: The density of UEs is linearly decreasing within a circular disc with radius $R$. Here, $R$ is the parameter to control the level of attraction between UEs and SAs. A small value of $R$ means the attraction level is high, and vice versa.

$$f_u(r) = \frac{3}{\pi R^2} \left(1 - \frac{r}{R}\right) \quad 0 \leq r \leq R. \quad (4.3)$$

(iv) **Exponential cluster**: The density of UEs is exponentially decreasing, in which $E$ is the rate parameter. Here, $E$ is the parameter to control the level of attraction between UEs and SAs. A high value of $E$ means the attraction level is high, and vice versa.

$$f_u(r) = \frac{E^2}{2\pi} \exp(-E \cdot r) \quad 0 \leq r \leq \infty. \quad (4.4)$$

In this thesis, we design our non-uniform user distribution model with Matern cluster process. It is easy to extend to Thomas cluster, linear cluster and exponential cluster. The Matern cluster radius in the $k$th tier ($k \in S$) between UE and its SA is defined as $R_k$. If $R_k$ is high, the attraction level is low, vice versa. There is the same cluster radius $R$ for each BS in the same tier. $R_k$ is the parameter that determines the attraction level between UE and SA in the $k$th tier.

- **Model of SAs dependent on BSs**

Our non-uniform user distribution is modeled by a PCP with the cluster center located at SAs rather than being at BSs. The base station offset between SAs and their BSs represents the dependency level between SAs and BSs.

The base station offset, which is the distance in the $k$th tier ($k \in S$) between SAs and their BSs, is defined as $D_k$. If $D_k$ is high, the dependency level is low, and vice versa. There is the same $D$ for each BS in the same tier. Hence, the base station offset $D_k$ is the parameter that determines the dependency level between
SAs and BSs in the $k$th tier.

- Parameters summary of the user model

There are two parameters in our designed user distribution model:

- Cluster radius $R$
  It is used to control the attraction level between UEs and their SAs.

- Base station offset $D$
  It is used to control the dependency level between SAs and their BSs.

We can choose the combinations of the cluster radius $R$ and base station offset $D$ to create the specific user distribution we need.

### 4.2.3 Attraction Level and Dependency Level

![Figure 4.2: Range of attraction level and dependency level.](image)

#### 4.2.3.1 Attraction Level

The attraction level represents the attraction degree between UEs and SAs. The range of the attraction level is from 0 to 1. The value 0 means that there is no attraction and the distribution of UEs approaches to be uniform. The value 1 means that UEs are extremely attracted to a SA and UEs are located at the same position with the SA.

Let $R$ be the Matern cluster radius of the non-uniform user distribution model. When the cluster radius is large enough, the user distribution converges to a uniform distribution. $R_{uni}$ is the minimum Matern cluster radius where the performance of the non-uniform distribution based on $R_{uni}$ is equivalent to the performance of the uniform distribution. The value of $R_{uni}$ can be measured.

**Definition 4-2.** The attraction level is defined as follows.
\[ \text{Attraction Level} = \begin{cases} 
1 - \frac{R}{R_{\text{uni}}} & \text{if } R < R_{\text{uni}}, \\
0 & \text{if } R \geq R_{\text{uni}}. 
\end{cases} \tag{4.5} \]

### 4.2.3.2 Dependency Level

The dependency level we define is used to represent the dependency degree between SAs and BSs. The range of the dependency level is also from 0 to 1. The value 0 means there is no dependency and the position of SAs is randomly and independently distributed with the position of BSs. The value 1 means that SAs are extremely dependent to BSs (SAs and BSs are in the same position).

Let \( D \) be the base station offset between SAs and BSs in the non-uniform user distribution model. \( D_{\text{uni}} \) is the minimum base station offset where the performance of the non-uniform user distribution based on \( D_{\text{uni}} \) is equivalent to the performance of the uniform distribution. The value of \( D_{\text{uni}} \) can be measured.

**Definition 4-3.** The dependency level is defined as follows.

\[ \text{Dependency Level} = \begin{cases} 
1 - \frac{D}{D_{\text{uni}}} & \text{if } D < D_{\text{uni}}, \\
0 & \text{if } D \geq D_{\text{uni}}. 
\end{cases} \tag{4.6} \]

### 4.2.4 UE Realization

![Figure 4.3: A user distribution realization of the non-uniform model](image)

Figure 4.3: A user distribution realization of the non-uniform model (attraction level = 0.75 for (a), attraction level = 0.6 for (b) and attraction level = 0 for (c), dependency level = 1, blue curves are assumed to be cell boundaries).
Figure 4.3 shows examples of user distribution realization. Figure 4.3 is a Matern cluster PDF with the cluster radius parameter $R = 50$ for (a), $R = 80$ for (b) and $R = 200$ for (c). The cluster center is BS and the base station offset is zero between the cluster center (SA) and BS such that the dependency level is 1. We measure $R_{uni} = 200$. Then the attraction level for $R = 50$ is 0.75, the attraction level for $R = 80$ is 0.6, and the attraction level for $R = 200$ is 0. The blue curves are the cell boundaries. The green square is a small BS. The area is 400 m by 400 m. If $R$ is small, such as 50 in (a), there is a high attraction level of 0.75 between the cluster center and UEs, which means that UEs converge to the cluster center, vice versa. UEs are assumed with high attraction level for small cells, such as (a) while UEs are assumed with low attraction level for macro cells, such as (c). The large number of UEs in Figure 4.3, which is just for better visualizing the UE spatial distribution and attraction levels, can be ignored.

![Figure 4.3: Examples of user distribution realization.](image)

Figure 4.4: A user distribution realization of the non-uniform model with a dependency level is 0.6 (attraction level is 0.8 for (a), attraction level is 0.7 for (b), attraction level is 0.5 for (c), blue curves are assumed to be cell boundaries).

In Figure 4.4, (a) is $R = 40$ m (attraction level is 0.8), (b) is $R = 60$ m (attraction level is 0.7), (c) is $R = 100$ m (attraction level is 0.5). The base station offset between small BS (green square) and cluster center (blue triangle) is 40 m (dependency level is 0.6, measure $D_{uni} = 100$). The area is 400 m by 400 m.
Figure 4.5: Non-uniform user distribution Scenario 1 (a-f). The area is 1 km by 1 km. Green triangles are macro BSs and black triangles are small BSs.
CHAPTER 4. USER MODEL AND HETEROGENEITY ANALYSIS

Figure 4.6: Non-uniform user distribution Scenario 1 (g-l). The area is 1 km by 1 km. Green triangles are macro BSs and black triangles are small BSs.
Figure 4.7: Non-uniform user distribution Scenario 2. There are non-uniform cluster users (magenta points) distributed for small cells. Magenta circles are UE cluster boundaries distributed for small cells. Black diamonds are UE cluster centers. There are also almost uniform users (black points) distributed for macro cells in (b). Black circles are UE cluster boundaries for macro cells. The blue curves are the cell boundaries. The area is 1 km by 1 km. Green triangles are macro BSs and black triangles are small BSs.

In Figure 4.5 and Figure 4.6, there are 3 macro BSs and 8 small BSs. Non-uniform UEs (magenta points) are distributed in small cells. The transmit power of macro BSs is 100 times greater than the transmit power of small BSs. We measure $R_{uni} = 200$ and $D_{uni} = 100$. In Figure 4.5 and Figure 4.6, the sub figures in each column have the same base station offset $D$, such as $D = 0$ (dependency level is 1) for sub figures (a, c, e), $D = 40$ (dependency level is 0.6) for sub figures (b, d, f), $D = 60$ (dependency level is 0.4) for sub figures (g, i, k) and $D = 80$ (dependency level is 0.2) for sub figures (h, j, l). In each column, the radius $R$ from top to bottom is $R = 40$ (attraction level is 0.7), $R = 60$ (attraction level is 0.8), $R = 80$ (attraction level is 0.6) separately. Each sub figure is a combination of different $R$ and $D$, which would present plenty of non-uniform user distribution cases.

Figure 4.7 shows the mixed user model. In Figure 4.7 (a) there are non-uniformly distributed users ($R = 60$, $D = 60$, the attraction level is 0.7, and the dependency level is 0.4) with magenta points in small cell and in Figure 4.7 (b) there are almost uniformly distributed users ($R = 500$, $D = 0$, the attraction level is 0, and the dependency level is 1) with black points in macro cells besides the non-uniform users in small cells. It indicates the flexibility that this user model can represent different kinds of attraction levels and...
4.3 Heterogeneity Analysis of Non-Uniform User Distribution

The attraction and the dependency levels defined above are good indicators of heterogeneity. However, these two levels represent either the relationship between SAs and UEs or the relationship between SAs and BSs. We need another indicator to express the overall heterogeneity. In this section, the heterogeneity analysis is based on the coefficient of variation (CoV). CoV is a good overall indicator of users’ spatial heterogeneity. It can be regarded a parameter of user’s heterogeneity as path loss exponent in large scale fading.

4.3.1 Definition of CoV (Coefficient of Variation)

For the user distribution model, a suitable statistical measure is required to capture the traffic property, i.e. heterogeneity. Lots of traffic measures exist. In time domain, inter-arrival time is a good measurement. In space domain, there are the density based measures, such as Ripley’s K-function and pair correlation function, which measures the user density within a given disc or ring, but it needs to be parameterized by the window size, such as the radius of the given disc. Another popular measurement is distance-based. For example, the nearest-neighbor measurement is the simplest candidate. However, only the nearest neighbor is considered in this measurement. Then there are some other proposed distance-based measures, such as Voronoi cell areas, Delaunay cell edge lengths, and Delaunay cell areas.

4.3.1.1 One Dimension (1D) or Time Domain

In 1D point pattern or the time domain, there are two ways to capture the traffic property.

- The first way is to count the number of arrival points during a specific interval. This is a density-based metric, in which deciding the interval length is challenging. Figure 4.8 (a) shows an example of the first way.

- Second way is Inter-Arrival Time (IAT) \( I_k = \text{Time}_{k+1} - \text{Time}_k \), which is a popular measure. The IAT is the consumed time for any point to arrive compared with the
previous point, which is distance-based. Figure 4.8 (b) shows an example of the second way.

![Diagram](image)

**Figure 4.8: Two ways to capture the traffic property in 1D**

Define $C_I$ as normalized coefficient of variation (CoV), which is the ratio of standard deviation of $I_k$ and the mean of $I_k$. For $C_I = 0$, it is 1D lattice distribution. For $C_I = 1$, it is 1D Poisson distribution. For $0 < C_I < 1$, it is sub-Poisson distribution. For $C_I > 1$, it is super-Poisson distribution. Figure 4.9 gives an example of pattern of 1D.

**4.3.1.2 Two Dimensions (2D) or Space Domain**

Statistics, such as the probability density function (PDF) and auto-correlation function of spatial user are required for good modeling. But it is hard to find a perfect match between the model and the practical scenario. Hence, simplified models only consider the first few moments of the user spatial distribution. The non-uniform user spatial placement in HetNets normally is modeled by two dimension (2D), in which Voronoi cell areas, Delaunay cell edge lengths and Delaunay cell areas [58,94] associated with the points can be measured as equivalents of IAT in the time domain.
Figure 4.9: Pattern of 1D from left to right with sub-Poisson \((0 < C_E < 1)\), Poisson \((C_E = 1)\) and super-Poisson \((C_E > 1)\)

According to the reference [95], we take \(C_E\), which is a normalized CoV of the second-order as the desired statistics of user model.

\[
\text{CoV} = \frac{\sigma_E}{\mu_E}, \tag{4.7}
\]

where \(E\) is statistic measurement, such as Voronoi cell areas, Delaunay cell edge lengths and Delaunay cell areas. \(\sigma_E\) is the standard deviation of \(E\) and \(\mu_E\) is the mean of \(E\). \(C_E\) is normalized second order statistics CoV of user model, which demonstrates the deviation from homogeneity using inter-point distance metrics.

The \(C_E\) in 2D is the analog of \(C_I\) in 1D to capture heterogeneity of user distribution. It is shown in [94] that for a Poisson distribution \(C_E = 1\), for a sub-Poisson distribution (more homogeneous than Poisson) \(0 < C_E < 1\), and for a super-Poisson distribution (more heterogeneous than Poisson) \(C_E > 1\).
Figure 4.10 gives examples of user distribution pattern. If the pattern is the perfect 2D lattice, the statistic measurement is the same for all points so that $C_E = 0$. Hence a constant distance between points creates the perfect homogeneity. But the Poisson distribution of $C_E = 1$ creates the complete randomness, which is shown in the middle of Figure 4.10. Sub-Poisson pattern is created from a perfect lattice and then apply a random displacement, which is shown in the left of Figure 4.10 with $0 < C_E < 1$. Sub-Poisson pattern is with more homogeneity than Poisson pattern. Super-Poisson pattern is created, such as hierarchical randomness and Markov models. Super-Poisson pattern is with more heterogeneity than Poisson, which is shown in the right of Figure 4.10.

![Figure 4.10: Pattern of 2D from left to right with sub-Poisson ($0 < C_E < 1$), Poisson ($C_E = 1$) and super-Poisson ($C_E > 1$)](image)

### 4.3.2 CoV Measurement

The proposed distance-based measurement is Voronoi cell areas, Delaunay cell edge lengths, and Delaunay cell areas. The definition of Voronoi tessellation and Delaunay tessellation is given in the following.

**Definition 4-4.** Voronoi tessellation is defined as follows.

For a point pattern $P = \{p_1, p_2, ..., p_k\}$ in a D-dimensional Euclidean space $\mathbb{R}^D$, the Voronoi tessellation $V = \{V_{p_1}, V_{p_2}, ..., V_{p_k}\}$ is the set of cells so that any location $y \in V_{p_i}$ is closer to $p_i$ than any other point in $P$ which has the equation as

$$V_{p_i} = \{y \in \mathbb{R}^D : |y - p_i| \leq |y - p_j| \text{ for } i, j \in 1, ..., k\}. \quad (4.8)$$
**Definition 4-5.** Delaunay tessellation is defined as follows.

$D$ is space dimension. If $D + 1$ Voronoi cells have an intersection, the corresponding Delaunay cell is made up of these $D + 1$ points from the pattern $P = \{p_1, p_2, ..., p_k\}$.

Voronoi tessellation and Delaunay tessellation are said to be dual.

![Figure 4.11: Example of Voronoi and Delaunay tessellations (Delaunay cell edges with blue lines and Voronoi cell edges with black lines, and the pattern points with red diamonds).](image)

In the following, we evaluate measured normalized CoV based on Voronoi cell areas, Delaunay cell edge lengths, and Delaunay cell areas. In the simulations, we generate 1000 BS patterns and we then generate the non-uniformly distributed users on each BS pattern where a CoV values based on Voronoi cell areas, Delaunay cell edge lengths, and Delaunay cell areas are calculated separately. Normalized CoV is the measured CoV divided by CoV when PPP. For the detailed simulation setting, please refer to Appendix C.3. The area is a square of 1 km by 1 km. There are 3 macro BSs and 6 small BSs, which is equivalent to $\lambda_1 = \frac{3}{10^6}$ and $\lambda_2 = 2\lambda_1$. The path loss exponent is $\{\alpha_1, \alpha_2\} = \{3.5, 3.5\}$ and the transmit power $\{P_1, P_2\} = \{45, 35\}$ dBm.
Figure 4.12 indicates the measured normalized CoV based on the attraction level in our designed user model. This figure shows that normalized CoV is high when the attraction level is high. CoV is a good indicator of users’ spatial heterogeneity. High CoV represents high users’ spatial heterogeneity, and vice versa. Hence, high attraction level results in high CoV such that high users’ spatial heterogeneity. It also shows that Voronoi cell area is preferable to Delaunay cell edge length and Delaunay cell area since there is large CoV range and the slope is moderate. Therefore, Voronoi cell area is used as the traffic CoV statistic measurement in the following simulation results.

### 4.3.3 CoV Simulation Results

We take the normalized CoV, $C_E$, as an indication of users’ heterogeneity. The relationship between normalized CoV and designed user model’s properties, such as the attraction level and the dependency level, is given from Figure 4.13. It can be observed that the dependency level has little impact for normalized CoV and the attraction level has big impact for normalized CoV. Normalized CoV is very high when the attraction level is high. In Figure 4.14, the coverage probability based on the attraction level and the dependency level is
shown. When both the attraction and the dependency are high, the coverage probability is high. Otherwise the performance is almost the same as the uniform case.

![Normalized CoV](image1.png)

(a) Normalized CoV

![View from top](image2.png)

(b) View from top

**Figure 4.13:** Measured CoV based on the attraction level and the dependency level

![Coverage probability](image3.png)

(a) Coverage probability

![Coverage Probability](image4.png)

(b) View from top

**Figure 4.14:** Measured coverage probability based on the attraction level and the dependency level

### 4.4 Conclusions

In this chapter, we introduced our non-uniform user distribution model. In our model, there is a parameter called the cluster radius $R$, which controls the attraction level between UEs and SAs. There is also another parameter called the base station offset $D$, which controls the dependency level between BSs and SAs. This user model is a Poisson cluster process (PCP) with the cluster centers located at the SAs and SAs may have a base station offset with their BSs. UEs in each tier may have their own attraction level and dependency level.
For example, in small cells UEs are non-uniformly distributed with high attraction level and high dependency level while in macro cells UEs are almost uniformly distributed with low attraction level and low dependency level.

The heterogeneity analysis is also given based on the normalized CoV of Voronoi cell area. It shows that the normalized CoV is high when the attraction level is high regardless of the dependency level. However, the coverage probability depends on both the attraction level and dependency level. Hence, high CoV (high heterogeneity) does not result in high coverage probability.
5.1 Introduction

In the first part of this chapter, we give the performance analysis of the downlink coverage probability for the designed non-uniform user distribution model. In the second part of this chapter, simulation results are provided to verify the accuracy of the performance analysis derived in the first part of this chapter.

Before we introduce the derivation of the downlink coverage probability, some notations are given:

- In our non-uniform user model, each SA has a base station offset with a BS. A UE cluster is distributed around each SA. The base station offset between each SA and its BS in the same tier is the same. In this thesis, the distance between the SA and its BS is called the base station offset.

- $BS^j_i$ refers to the $j$th BS in the $i$th tier.

- Similarly, $SA^j_i$ refers to the SA that has a base station offset $D_i$ with $BS^j_i$.

- A UE of interest in the cluster is distributed around $SA^j_i$ and $SA^j_i$ has a base station offset $D_i$ with $BS^j_i$. Since it is not certain that the UE of interest in the cluster has the nearest distance with $BS^j_i$ or the UE of interest is associated with $BS^j_i$, we can not name $BS^j_i$ as the nearest BS or associated BS and we just name $BS^j_i$ as the BS that the UE of interest belongs to spatially.
Figure 5.1 shows an example of spatial distances between UE, BS, and SA and the related notation in a 2-tier downlink system. The 1st tier such as macro cell tier is in red and the 2nd tier such as small cell tier is in blue. In the 1st tier, there are three BSs ($BS_1^1$, $BS_2^1$ and $BS_3^1$) and three corresponding SAs ($SA_1^1$, $SA_2^1$ and $SA_3^1$). The three corresponding SAs have a base station offset $D_1$ with the three BSs. In the 2nd tier, there are four BSs ($BS_1^2$, $BS_2^2$, $BS_3^2$ and $BS_4^2$) and the four corresponding SAs ($SA_1^2$, $SA_2^2$, $SA_3^2$ and $SA_4^2$). The four corresponding SAs have a base station offset $D_2$ with the four BSs. In our non-uniform user distribution, a SA has a base station offset $D_i$ with a $i$th tier BS. $D_i$ is the parameter that determines the dependency level of a SA to its BS in the $i$th tier. There is a cluster of UE that surrounds each SA. The UEs are distributed uniformly within the disc of the radius $R_i$ in the $i$th tier, which is called Matern cluster process. $R_i$ is the parameter that determines the attraction level of a UE and its SA in the $i$th tier. The random variable $L_i$ is the distance between the UE of interest and the BS that the UE belongs to spatially in the $i$th tier. The random variable $V_i$ is the distance between the UE of interest and its closest BS in the $i$th tier.

Let us analyze the performance for a UE of interest, which is the black point in Figure 5.1. This black point UE is uniformly distributed inside the red disc with the radius $R_1$. The BS that the black point UE belongs to spatially is $BS_1^1$. The UE is distributed around the cluster center $SA_1^1$. $SA_1^1$ has a base station offset $D_1$ to $BS_1^1$. The possible BS that the
UE of interest will be associated with can be described into two cases. Case1 is that the UE of interest is associated with the $BS_1^1$ that it belongs to spatially. Case2 is that the UE of interest is associated with other BS, such as BSs in the 1st tier ($BS_1^2$ and $BS_1^3$), or BSs in the 2th tier ($BS_2^1$, $BS_2^2$, $BS_2^3$ and $BS_2^4$). Therefore, let us summarize the two cases of the possible BS that the UE of interest is associated with. The UE of interest in the cluster of $SA_i^j$ may be associated with the $BS_i^j$ (case1) or associated with another BS $k^m$ (case2). This $BS_k^m$ may be of the same tier as $i$ ($m = i, k \neq j$), or it may be in another tier ($m \neq i$).

Let $m = 1, 2, ...k$ be the tier of interest. The procedure to get the downlink coverage probability $P_{dl,c}$ is as follows:

- Find the probability density function (PDF) $f_L^{(m)}(l)$ and complementary cumulative distribution function (CCDF) $F_L^{(m)}(l) = \mathbb{P}^{(m)}(L > l)$ of random variable $L$. Let $L$ be the distance between a UE and the BS that the UE belongs to in the $m$th tier.
- Find the PDF $f_V^{(m)}(v)$ and CCDF $F_V^{(m)}(v) = \mathbb{P}^{(m)}(V > v)$ of random variable $V$. Let $V$ be the distance between a UE and the closest BS in the $m$th tier.
- The possible BS that the UE of interest (e.g. black point UE in Figure 5.1) in a given $m$th tier is associated with can be summarized into two cases.
  - Derive $\mathbb{P}^{(m)}(A_{case1})$, which is the probability that a UE of interest is associated with the BS to which the UE belongs. In Figure 5.1, it means that the black point UE is associated with $BS_1^1$.
  - Derive the downlink coverage probability $\mathbb{P}^{(m)}[SINR_m(l) > T_m|A_{case1}]$ conditioned on the black point UE associated with $BS_1^1$.
  - Derive the downlink coverage probability $\mathbb{P}_{dl,c,case1}^{(m)} = \mathbb{P}^{(m)}(A_{case1}) \ast \mathbb{P}^{(m)}[SINR_m(l) > T_m|A_{case1}]$ that the black point UE is associated with $BS_1^1$ to which this UE belongs.

Case2: associated with a BS in all tiers except the BS to which UE belongs (e.g. except $BS_1^1$)

- Derive $\mathbb{P}^{(m)}(A_{k,case2})$, which is the probability that a UE is associated with a $k$th tier BS.
- Derive the downlink coverage probability $\mathbb{P}^{(m)}[SINR_k(v) > T_k|A_{k,case2}]$ conditioned on a UE associated with a $k$th tier BS.
Derive the downlink coverage probability $\mathbb{P}_{dl,ck,case2}^{(m)} = \mathbb{P}^{(m)}(A_{k,case2}) \cdot \mathbb{P}^{(m)}[SINR_k(v) > T_k | A_{k,case2}]$ of associating with a $k$th tier BS.

Summary

- $\mathbb{P}^{(m)}(A_{case1})$ and $\mathbb{P}^{(m)}(A_{k,case2})$ is the probability that a UE of interest in the $m$th tier is associated with the BS to which UE belongs and a $k$th tier BS. $\mathbb{P}^{(m)}(A_{case1}) + \sum_{k=1}^{K} \mathbb{P}^{(m)}(A_{k,case2}) = 1$. $K$ is the total number of BS tiers.

- The downlink coverage probability for a UE distributed in the $m$th tier is $\mathbb{P}_{dl,c}^{(m)} = \mathbb{P}_{dl,c,case1}^{(m)} + \sum_{k=1}^{K} \mathbb{P}_{dl,ck,case2}^{(m)}$.

- The overall coverage probability of the designed non-uniform distribution model is

$$\mathbb{P}_{dl,c} = \sum_{m \in S} \frac{N_m \lambda_m}{\sum_{m \in S} N_m \lambda_m} \mathbb{P}_{dl,c}^{(m)}, \quad (5.1)$$

where $N_m$ is the average number of users in each cluster of the $m$th tier. $\lambda_m$ is the BS density of the $m$th tier. There are total of $S$ tiers with a subset $S$ distributed with our designed non-uniform user model.

### 5.2 Model Performance Analysis

In this section, we derive the downlink coverage probability. The definition of the downlink coverage is as follows.

**Definition 5-1.** The downlink SINR at a typical UE located at the origin in the $k$th tier is

$$SINR_k(r) = \frac{P_k G_k r^{-\alpha_k}}{I_r + \frac{\sigma^2}{L_0}}, \quad (5.2)$$

$$I_r = \sum_{j=1}^{K} I_{rj} = \sum_{j=1}^{K} \sum_{B_{ji} \in \phi_j \setminus B_0} P_j G_{ji} r_{ji}^{-\alpha_j}, \quad (5.3)$$

- $P_k$ is BS transmit power in the $k$th tier.
• $G_k$ denotes the channel random power gain in the $k$th tier. Let the complex channel coefficient between the $k$th tier BS and the user be denoted as $h_k$ (Rayleigh fading is assumed in this thesis). Then we define $G_k = |h_k|^2$, which is an exponential distribution with unity mean.

• $\alpha_k$ is the path loss exponent in the $k$th tier.

• $r_{ji}$ is the distance of the typical user to the $i$th BS in the $j$th tier (i.e. $B_{ji}$).

• $I_r$ is the total received interference from all BSs (except the associated BS at $B_0$) and $I_{rj}$ is the total received interference from the $j$th tier (see (5.3)).

• $\sigma^2$ is the power of the additive white Gaussian noise (AWGN).

• $K$ is the total number of tiers.

• $\phi_i$ is SPPP with a density $\lambda_i$ in the $i$th tier.

• $L_0$ is the path loss at a reference distance of 1 m.

**Definition 5-2.** A UE in the $k$th tier is in the coverage if the downlink SINR from the associated BS with the maximum downlink average bias received power is more than the coverage threshold $T_k$ ($\phi_j$ is an independent homogeneous PPP).

\[
SINR_k = \frac{P_k G_k r_{\alpha_k}}{\sum_{j=1}^{K} \sum_{B_{ji} \in \phi_j \setminus B_0} P_j G_j r_{\alpha_j} + \frac{\sigma^2}{L_0}} > T_k. \tag{5.4}
\]

### 5.2.1 Distance Characterization

#### 5.2.1.1 PDF and CCDF of Random Variable $V$

$V$ is a random variable that represents the distance between a UE of interest and the closest BS. The distance between the UE of interest and other BSs must be farther than $V$. Because the BS distribution in each tier is an independent PPP, the probability density function of $V$ is obtained by the conclusion of the null probability of a PPP. The distribution of $V$ is given from [24] and [96].
Remark 5-1. The distribution of $V$ in the $m$th tier is

PDF: \[ f^{(m)}_V(v) = 2\pi\lambda_m v \exp(-\lambda_m \pi v^2) \], \hspace{1cm} (5.5a)

CCDF: \[ \bar{F}^{(m)}_V(v) = \exp(-\lambda_m \pi v^2) \], \hspace{1cm} (5.5b)

where $\lambda_m$ is the BS distribution density of the $m$th tier.

5.2.1.2 PDF and CCDF of Random Variable $L$

$L$ is a random variable that represents the distance between a UE of interest and the BS that the UE of interest belongs to spatially. Matern cluster process has a PDF of $f_u(r) = \frac{1}{\pi R^2_m} (0 \leq r \leq R_m)$, which means that the clustered UEs are uniformly distributed within a circular disc with the radius $R_m$ around the SA in the $m$th tier. $D_m$ is the base station offset between the surrounded SA and the BS that the UE belongs to in the $m$th tier.

Figure 5.2: Calculation example of PDF and CCDF of Random Variable $L$ in the $m$th tier

Figure 5.2 gives an example of PDF and CCDF calculation in the $m$th tier. We assume that $SA_m$ is located at the origin and $(x, y)$ are the coordinates of a UE of interest in the Matern cluster (the black disc with the radius $R_m$). $L = \sqrt{(x-D_m)^2 + y^2}$ is the random distance variable between the UE of interest and $BS_m$ to which the UE of interest belongs. The red circle is the position where the distance to $BS_m$ is $l$ so that CCDF $\bar{F}_L^{(m)}(l) = \mathbb{P}^{(m)}(L > l)$ is the proportion of the shadowed area to the whole black cluster disc area since the UE of interest is uniformly distributed within the black cluster disc. PDF $f_L^{(m)}(l)$ is the proportion of the red arc length of the shadow part to the whole black cluster disc area.
CHAPTER 5. DOWNLINK PERFORMANCE ANALYSIS

According to Figure 5.2, we get the following equations of the coordinates \((x_0, y_0)\) of the joint point between the black user cluster disc and the red circle which will be used in the following derivation.

\[
x_0^2 + y_0^2 = R_m^2,
\]
\[
(D_m - x_0^2) + y_0^2 = l^2,
\]
\[
\Rightarrow x_0 = \frac{D_m^2 + R_m^2 - l^2}{2D_m},
\]
\[
\Rightarrow y_0 = \frac{1}{2D_m} \sqrt{4D_m^2 R_m^2 - (D_m^2 + R_m^2 - l^2)^2}.
\]

(5.6)

**Corollary 5-1.** PDF \(f^{(m)}_L(l)\) and CCDF \(\bar{F}^{(m)}_L(l)\) of the random variable \(L\) in the \(m\)th tier is in Table 5.1.

For proof of Table 5.1 see Appendix A.1.

![PDF and CCDF example of L (R = 100, D = 50)](image)

Figure 5.3: PDF and CCDF example of \(L\) \((R = 100, D = 50)\)

Figure 5.3 shows an example for PDF and CCDF of \(L\). The figure on the left is about PDF where the blue bins are from simulations and the red curve is from the above derived equations in Table 5.1. It shows that the simulations and derivations match closely. The curve on the right is about CCDF where the black curve is from simulations and the red curve is from the above derived equations in Table 5.1. The simulation curves and derivations also match closely.
### Table 5.1: PDF and CCDF of the random variable $L$

<table>
<thead>
<tr>
<th>Scope</th>
<th>$f_{L_m}(l)$</th>
<th>$F_{L_m}(l)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l \leq D_m - R_m$</td>
<td>$\theta_1$</td>
<td>NA</td>
</tr>
<tr>
<td>$D_m - R_m &lt; l \leq D_m + R_m$</td>
<td>$D_m - R_m &lt; l \leq l \leq D_m + R_m$</td>
<td>$\frac{2l}{R_m}$</td>
</tr>
<tr>
<td>$l \leq R_m - D_m$</td>
<td>$R_m - D_m &lt; l \leq R_m + D_m$</td>
<td>$\frac{2R_m}{R_m}$</td>
</tr>
<tr>
<td>$l \geq R_m$</td>
<td>$l \leq l &lt; l &lt; l \leq l$</td>
<td>$\frac{2R_m}{R_m}$</td>
</tr>
</tbody>
</table>

When $D_m \geq R_m$

When $D_m < R_m$

When $D_m = 0$

$x_0 = \frac{D_m^2 + R_m^2}{2D_m}$ and $y_0 = \frac{1}{2D_m} \sqrt{14D_m^2 R_m^2 - (D_m^2 + R_m^2 - l^2)^2}$. $a \% b$ is the remainder when dividing $a$ by $b$. 

\[
\theta_1 = \arctan \left( \frac{y_0}{x_0} \right) + \pi
\]
5.2.2 Interference Characterization

The total downlink interference of the UE of interest from BSs in all tiers except the associated BS $B_0$ is

$$I_r = \sum_{j=1}^{K} I_{rj} = \sum_{j=1}^{K} \sum_{B_{ji} \in \phi_j \setminus B_0} P_j G_j r_{ji}^{-\alpha_j}.$$ (5.7)

The basic reason that we can analyze the performance theoretically using the tools of stochastic geometry is that the distribution of BS in each tier is an independent PPP. And we can transform the unlimited sum of interference from BSs in one tier into an integral expression utilizing the property of products over PPP (probability generating functional (PGFL)). If $\phi$ is a PPP with the density $\lambda$, this expression is

$$E[\prod_{x \in \phi} f(x)] = \exp(-\lambda \int_{\mathbb{R}^2} (1-f(x))dx).$$

Specifically, the Laplace transform $\mathcal{L}_x(s) = E_x[e^{-sx}]$ ($E_x[\ast]$ is the expectation of $\ast$ over $x$) of the interference is very useful during the theoretical derivation.

**Lemma 5-1.** Calculate the Laplace transform of the total interference in the $j$th tier.

$$\mathcal{L}_{I_{rj}}(s) = E_{I_{rj}}[e^{-sI_{rj}}],$$

$$= E_{\phi_j}[e^{-s\sum_{B_{ji} \in \phi_j \setminus B_0} P_j G_j r_{ji}^{-\alpha_j}}],$$

$$= E_{\phi_j}\left[\prod_{x \in \phi_j} e^{-sP_j G_j r_{ji}^{-\alpha_j}}\right],$$

$$(a) = \exp(-\lambda_j \int_{\mathbb{R}^2} (1 - \mathcal{L}_{G_j}(e^{-sP_j G_j r_{ji}^{-\alpha_j}}))dx),$$

$$(b) = \exp(-2\pi \lambda_j \int_{d_{\text{min}}}^{\infty} (1 - \mathcal{L}_{G_j}(s P_j r^{-\alpha_j})rdr),$$

where (a) comes from the property of product over PPP (PGFL). (b) follows the Laplace transform of the Rayleigh fading in which the fading power $G_j$ is the exponential distribution with unity gain and the Rayleigh fading envelope is $\sqrt{G_j}$. $d_{\text{min}}$ is the distance to the
closest interferer in the \( j \)th tier. The above procedure follows the same derivation as [75].

### 5.2.3 Downlink Coverage Probability for Case1 (associated with the BS to which the UE belongs)

In this subsection, we derive the downlink coverage probability that a UE of interest in a cluster of the \( m \)th tier associates with the BS to which the UE belongs. The SA in the cluster center has the distance \( D_m \) with the BS to which the UE belongs. In Figure 5.1, it means that a UE of interest (black point) clustered around \( SA_1 \) is associates with the red triangle \( BS_1 \).

**Lemma 5-2.** The probability that a UE of interest in the \( m \)th tier is associated with the BS that the UE belongs to is (conditioned on the distance \( l \) between the UE of interest and the BS that the UE belongs to)

\[
P^{(m)}(A_{case1}) = \prod_{i=1}^{K} \exp(-\lambda_i \pi \left( \frac{P_i C_i}{P_m C_m} \right)^{\frac{2}{\alpha_i}} \cdot l^{\frac{2\alpha_m}{\alpha_i}}).
\]

(5.9)

For proof see Appendix A.2.

**Lemma 5-3.** The coverage probability associated with the BS that a UE of interest belongs to is (conditioned on the distance \( l \) between the UE of interest and the BS that the UE belongs to)

\[
P^{(m)}(SINR_m(l) > T_m | A_{case1}) = \exp\left(-\frac{\sigma^2}{L_0} T_m P_m^{-1} l^{\alpha_m}\right) \cdot \prod_{i=1}^{K} \mathcal{L}_{I_i}(T_m P_m^{-1} l^{\alpha_m}),
\]

(5.10)

where \( T_m \) is SINR coverage threshold in the \( m \)th tier. For proof see Appendix A.3.

For the details of \( \mathcal{L}_{I_i}(T_m P_m^{-1} l^{\alpha_m}) \), the reader is referred to Equation 5.8 and [75]. \( \mathcal{L}_{I_i}(s) \) is the Laplace transform of the total interference in the \( i \)th tier.
\[ \mathcal{L}_{r_i}(T_m P_m^{-1} l^\alpha_m) \]
\[ = \exp(-2\pi \lambda_i \int_{d_{\text{min}}}^{\infty} \left( \frac{T_m P_m^{-1} l^\alpha_m P_i^{-\alpha_i}}{1 + T_m P_m^{-1} l^\alpha_m P_i^{-\alpha_i}} \right) r dr), \]
\[ = \exp(-2\pi \lambda_i \int_{d_{\text{min}}}^{\infty} \left( \frac{1}{1 + (T_m l^\alpha_m P_i P_m^{-1} l^\alpha_i)^{-1} r^\alpha_i} \right) r dr), \]
\[ = \exp\left\{ -\pi \lambda_i \left( \frac{P_i}{P_m} \right)^{\frac{2}{\alpha_i}} Z(T_m, \alpha_i, \frac{C_i}{C_m}) \cdot l^{\frac{2\alpha_m}{\alpha_i}} \right\}, \]

where \( d_{\text{min}} = \left( \frac{P_i C_i}{P_m C_m} \right)^{\frac{1}{\alpha_i}} \cdot l^{\frac{\alpha_m}{\alpha_i}} \) from Appendix A.2.

\[ Z(T_m, \alpha_i, \frac{C_i}{C_m}) = \frac{2T_m}{\alpha_i - 2} \left( \frac{C_i}{C_m} \right)^{\frac{2}{\alpha_i} - 1} \cdot 2F_1[1, 1 - \frac{2}{\alpha_i}; 2 - \frac{2}{\alpha_i}, -\frac{T_m C_m}{C_i}] \text{ from [75]}. \]

where \( 2F_1 \) indicates the Gauss hypergeometric function.

**Theorem 5-1.** The coverage probability that a UE of interest in the \( m \)th tier is associated with the BS that the UE of interest belongs to is

\[ \mathbb{P}^{(m)}_{\text{df}, \text{case1}} = \int_{0}^{\infty} \mathbb{P}^{(m)}_{\text{t}}[\text{SINR}_m(l) > T_m | A_{\text{case1}}] \cdot \mathbb{P}^{(m)}_{\text{t}}(A_{\text{case1}}) \cdot f_L^{(m)}(l) \cdot dl, \]  

where \( f_L^{(m)}(l) \) comes from subsection 5.2.1.2, which is the PDF of \( L \) in the \( m \)th tier. \( L \) is the random distance variable between a UE of interest and the BS that the UE belongs to. Each tier may have their own cluster radius \( R \) and base station offset \( D \) such that the PDF of \( L \) \( (f_L^{(m)}(l)) \) in each tier may be different.

**5.2.4 Downlink Coverage Probability for Case2 (associated with a BS in all tiers except the BS to which UE belongs)**

In this subsection, we derive the coverage probability of a UE of interest in a cluster of the \( m \)th tier associated with a BS in all tiers except the BS that UE belongs to. In Figure 5.1, it means that a UE of interest (black point) clustered around \( SA_1^1 \) is associated with a BS except the red triangle \( BS_1^1 \).

**Lemma 5-4.** The probability that a UE of interest is associated with a BS in the \( k \)th tier rather than the BS to which the UE belongs is (conditioned on the distance \( v \) between the
UE of interest and the closest BS in the \( k \)th tier

\[
\mathbb{P}^{(m)}(A_{k,\text{case}2}) = \tilde{F}_L^m\left(\left(\frac{P_mC_m}{P_kC_k}\right)^{\frac{1}{\alpha_m}} \cdot \frac{v^{\alpha_k}}{v^{\alpha_m}}\right) \cdot \prod_{i=1, i \neq k}^{K} \exp\left(-\lambda_i \pi \left(\frac{P_iC_i}{P_kC_k}\right)^{\frac{2}{\alpha_k}} v^{\frac{2 \alpha_k}{\alpha_i}}\right). \tag{5.13}
\]

For proof see Appendix A.4.

**Lemma 5-5.** The coverage probability associated with a \( k \)th tier BS rather than the BS that the UE of interest belongs to is (conditioned on the distance \( v \) between the UE of interest and the closest BS in the \( k \)th tier)

\[
\mathbb{P}^{(m)}|v[SINR_k(v) > T_k|A_{k,\text{case}2}] = \exp\left(-\frac{\sigma^2}{L_0} T_k P_k^{-1} v^{\alpha_k}\right) \prod_{i=1}^{K} \mathcal{L}_{\nu_i}(T_k P_k^{-1} v^{\alpha_k}). \tag{5.14}
\]

For the proof the reader can refer to Equation 5.10.

**Theorem 5-2.** The coverage probability that a UE of interest in the \( m \)th tier is associated with a \( k \)th tier BS rather than the BS that the UE of interest belongs is

\[
\mathbb{P}^{(m)}_{dl,ck,\text{case}2} = \int_{0}^{\infty} \mathbb{P}^{(m)}|v[SINR_k(v) > T_k|A_{k,\text{case}2}] \cdot \mathbb{P}^{(m)}(A_{k,\text{case}2}) \cdot f_v^{(k)}(v) \, dv, \tag{5.15}
\]

where \( f_v^{(k)}(v) = 2\pi \lambda_k v \exp(-\lambda_k \pi v^2) \) is the PDF of the distance \( V \) between the UE of interest and the closest BS in the \( k \)th tier with a PPP distribution. \( dv \) is the derivative operation over \( v \).

### 5.2.5 Overall Downlink Coverage Probability

If the UE of interest is distributed around a SA, which has the base station offset \( D_m \) with the BS that the UE of interest belongs to, Equation (5.12) and Equation (5.15) give
the coverage probability associated with the BS that the UE of interest belongs to (case1) or other BS (case2). When $D_m = 0 \ m$, this special case is the non-uniform user model discussed in [23] and [24].

**Lemma 5-6.** The downlink coverage probability for a non-uniform UE distributed in a $m$th tier is

$$ P_{dl,c}^{(m)} = P_{dl,c,\text{case1}}^{(m)} + \sum_{k=1}^{K} P_{dl,ck,\text{case2}}^{(m)}. $$ (5.16)

**Theorem 5-3.** The overall coverage probability is

$$ P_{dl,c} = \sum_{m \in S} \frac{N_m \lambda_m}{\sum_{m \in S} N_m \lambda_m} P_{dl,c}^{(m)}, $$ (5.17)

where $N_m$ is the average number of users in each cluster of the $m$th tier. $\lambda_m$ is the BS density of the $m$th tier. There are total of $S$ tiers with a subset $S$ distributed with our designed non-uniform user model.

### 5.3 Simulation Results

BSs are deployed in a square area of 2 km by 2 km. A 2-tier system is simulated consisting of macro and small BSs. Different tiers are independent PPPs with their own BS densities, BS transmit powers, and path loss exponents. Define $\kappa = \{1, 2\}$ as the indices of the 2 tiers (macro cell and small cell tiers). Our non-uniform user distribution model is applied over the small cells so that the subset $S = \{2\}$. The base station offset between SA and its BS in the small cells is defined as $D_2$. The fading channel between BSs and UEs is assumed to be Rayleigh fading with unity power gain. Firstly, we generate a BS pattern, and then a SA is selected uniformly around the circle with the radius $D_2$ to each BS. A cluster of UEs is generated with Matern cluster process around each SA. The average number of UEs, $N_m$, in a cluster is taken as 20. SINR of each UE in the cluster is measured. We repeat to generate BS pattern, SA, and UE cluster and calculate their SINRs until the number of BS pattern reaches 1,000. We then compare the above simulated coverage probability and the theoretical analysis from Equation (5.17). The detailed simulation setting is discussed
in Appendix C.1.

For the parameters, we follow the same setting as [75]. The thermal noise is assumed to be -104 dBm for all cases. The path loss $L_0$ at a reference distance 1 m is assumed to be -38.5 dB. In the 2-tier BS system, there are 12 macro BSs and 120 small BSs, which is equivalent that the BS density $\lambda_2$ of the small cell tier is 10 times more than the BS density $\lambda_1$ of the macro cell tier (the BS density of the macro cell tier is $\lambda_1 = \frac{3}{10^6}$). We choose the path loss exponent as 3.5 for both tiers. The transmit power of macro BSs is 100 times greater than the transmit power of small BSs, which is $\{P_1, P_2\} = \{53, 33\}$ dBm. As we know the maximum transmit power of LTE on a 10MHz bandwidth is about 46 dBm according to 3GPP TR 36.942, but here we set it to 53 dBm. One reason is that we want to follow the same parameter setting as [75] as our baseline. The other reason is rather than the actual transmit power that determines the coverage probability it is the the power ratio between macro BSs and small BSs. There is no cell extension bias factor for both tiers, which is $\{C_1, C_2\} = \{1, 1\}$. The summary table of used parameters is shown in Table 5.2.

There is no cell extension bias factor in the subsection 5.3.1 and 5.3.2. The effect of cell extension bias factor will be discussed in the subsection 5.3.3. We also discuss the effect of system parameters, such as BS density ratio between small cells and macro cells, and BS transmit power of small cells. For the path loss exponents, it is hard to determine that the path loss exponent of small cells is higher or lower than the path loss exponent of macro cells. Hence, the different relationships of the path loss exponents between the macro cells and the small cells are discussed in the subsection 5.3.3 of effect of system parameters.

Table 5.2: Parameters about BSs of a 2-tier system

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS density</td>
<td>${\lambda_1, \lambda_2} = {\frac{3}{10^6}, \frac{3}{10^5}}$</td>
</tr>
<tr>
<td>Path loss exponent</td>
<td>${\alpha_1, \alpha_2} = {3.5, 3.5}$</td>
</tr>
<tr>
<td>Cell extension bias factor</td>
<td>${C_1, C_2} = {1, 1}$</td>
</tr>
<tr>
<td>BS transmit power</td>
<td>${P_1, P_2} = {53, 33}$ dBm</td>
</tr>
<tr>
<td>$R_{uni}$</td>
<td>200 m</td>
</tr>
<tr>
<td>$D_{uni}$</td>
<td>100 m</td>
</tr>
</tbody>
</table>
5.3.1 Matching between Simulation and Theoretical Analysis

Figure 5.4 demonstrates the simulation results and the theoretical results. The theoretical results are obtained from the above Section 5.2. The cluster process is Matern cluster. The attraction level is 0.85 (the UE cluster radius $R$ is 30 m, high attraction, green curve), 0.75 ($R$ is 50 m, high attraction, red curve), 0.6 ($R$ is 80 m, medium attraction, black curve), and 0 ($R$ is 200 m, low attraction, blue curve). The uniform result (cyan curve) has also been included in order to compare with the above non-uniform user distribution. When the attraction level is high, such as 0.85 or 0.75, the coverage probability is high since the user density is high near the cell center. When the attraction level is very low, the coverage probability of the non-uniform user distribution converges to the uniform distribution (cyan curve). The base station offset $D_2$ is 0 m, which means the dependency level is 1. The effect of the base station offset is discussed in the following subsection 5.3.2. The non-uniform UEs are distributed only in small cells.
5.3.2 Effect of Base Station Offset

In this subsection, we keep the user distribution as Matern cluster (attraction level = 0.75, UE cluster radius $R = 50$ m) and vary the base station offset $D_2$ as \{0, 30, 50, 100\} m, which corresponds to the dependency level as \{1, 0.7, 0.5, 0\}. When the dependency level is high, such as 1 or 0.7, SA has a high dependency with its small BS so that the coverage probability is high. When the dependency level is low, such as 0, the dependency decreases and the coverage probability converges to the uniform case (cyan curve). Figure 5.5 demonstrates these results.

5.3.3 Effect of System Parameters

The non-uniform user distribution model has been introduced and the theoretical analysis of the downlink coverage probability has also been derived in previous Section 5.2. In this subsection, the effect of system parameters, such as BS density ratio between small cells and macro cells, the cell extension bias factor of small BSs and the transmit power of small BSs is given. For the reference and comparison, the performance based on the uniform user model is also included in the following figures. In the literature of the uniform user model, there are some conclusions that give more system insight. Here are two examples. The first example comes from [20], which has the conclusion that "more BSs can be added
in any tier without affecting the coverage and hence the network capacity can be increased linearly with the number of BSs" when SINR threshold is the same in all tiers. The second example is from [75], which concludes "When the low-tier BSs experience higher path loss, the outage and average rate improve as the BSs are added". The above conclusions are true for uniform user cases. However, these conclusions do not hold for the non-uniform user cases. It is essential to give the performance analysis for the non-uniform user model over HetNets, which will provide more system insight and understanding for HetNets design.

We follow the same simulation parameters as the above subsection. When calculating the downlink coverage probability, the SINR coverage threshold is 0 dB. Since we cannot determine the relationship between the path loss exponent of macro cells and the path loss exponent of small cells, we give three curves for each kind of attraction. For each attraction level (uniform, low attraction, medium attraction and high attraction), we show three curves. The first red curve is when the path loss exponent of macro cell is higher than that of small cells ($\{\alpha_1, \alpha_2\} = \{3.5, 3\}$). The second blue curve is when the path loss exponent of macro cell is equal to that of small cells ($\{\alpha_1, \alpha_2\} = \{3.5, 3.5\}$). The third black curve is when the path loss exponent of macro cell is lower than that of small cells ($\{\alpha_1, \alpha_2\} = \{3.5, 4\}$). The base station offset between SAs and BSs is assumed to be 0 m. In the following figures, we set the low attraction level as 0.1, the medium attraction level as 0.6, and the high attraction level as 0.75.

5.3.3.1 Effect of BS Density Ratio between Small Cells and Macro Cells

Table 5.3: Parameters of effect of BS density ratio between small cells and macro cells

<table>
<thead>
<tr>
<th></th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>BS density</strong></td>
<td>$\lambda_1 = \frac{3}{10^5}$, $\lambda_2 = 1$(0 dB) to 100 (20 dB)</td>
</tr>
<tr>
<td><strong>Path loss exponent</strong></td>
<td>${\alpha_1, \alpha_2} = {3.5, 3.5}$</td>
</tr>
<tr>
<td><strong>Cell extension bias factor</strong></td>
<td>${C_1, C_2} = {1, 1}$</td>
</tr>
<tr>
<td><strong>BS transmit power</strong></td>
<td>${P_1, P_2} = {53, 33}$ dBm</td>
</tr>
</tbody>
</table>
The effect of BS density ratio between small cells and macro cells is shown in Figure 5.6.

From Figure 5.6 we can observe:

- When the BS density ratio $\frac{\lambda_2}{\lambda_1}$ between small cells and macro cells is low, such as $0 - 5$ dB, the case of high attraction has the high coverage probability. Since the coverage area in small cells is large (as shown in Figure 3.20) at the low BS density ratio $\frac{\lambda_2}{\lambda_1}$, the UE cluster may be in the coverage area. Therefore, the high attraction case has high performance.

- When the BS density ratio $\frac{\lambda_2}{\lambda_1}$ between small cells and macro cells is low, such as $0 - 5$ dB, the case experiencing low path loss exponent in each attraction may have the high coverage probability since the received signal power from the associated BS may be high and the received signal power from interfering BS may be low due to longer distance.
• When the BS density ratio $\frac{\lambda_2}{\lambda_1}$ between small cells and macro cells is high, such as 15 – 20 dB, all kinds of attraction cases have almost the same performance. When the density ratio is high, the coverage area in the small cells might be small such that all kinds of attraction cases will have the user distribution over the whole coverage area and approach to be uniform.

• When the BS density ratio $\frac{\lambda_2}{\lambda_1}$ between small cells and macro cells is high, such as 15 – 20 dB, the case experiencing high path loss exponent in each attraction has the high coverage probability.

• For medium attraction and high attraction, the coverage probability increases and then decreases as the BS density ratio increases. The turning point of the BS density ratio from increasing to decreasing is around 12dB in this simulation setting. If deploying small BSs with the same simulation setting for medium attraction and high attraction, the density ratio between small cells and macro cells is preferred to be 12 dB since the coverage probability is optimum at the turning point 12 dB.

• BS deployment suggestion:
  
  – For low attraction and uniform case experiencing high path loss exponent, the BS density ratio between small cells and macro cells can be increased as much as possible since the coverage probability does not decrease as the increase of the BS density ratio.

  – For medium attraction and high attraction case, the BS density ratio is preferred to be deployed at the density of the turning point (e.g. 12 dB in this simulation setting) since the coverage probability has the optimum or peak value at the turning point.

The summary of the effect of the BS ratio between small cells and macro cells is in the following table.
CHAPTER 5. DOWNLINK PERFORMANCE ANALYSIS

Summary

Effect of $\frac{\lambda_2}{\lambda_1}$

<table>
<thead>
<tr>
<th>Summary</th>
<th>Effect of $\frac{\lambda_2}{\lambda_1}$</th>
<th>Effect of Path Loss Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low attraction</td>
<td>High path loss: ↑</td>
<td>High path loss exponent has high performance.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: ↑</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ↑</td>
<td></td>
</tr>
</tbody>
</table>

Medium attraction

<table>
<thead>
<tr>
<th>Summary</th>
<th>Effect of $\frac{\lambda_2}{\lambda_1}$</th>
<th>Effect of Path Loss Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High path loss: ↑</td>
<td>Low path loss exponent has high performance at low BS density ratio and high path loss exponent has high performance at high BS density ratio.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: ↑→↓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ↑→↓</td>
<td></td>
</tr>
</tbody>
</table>

High attraction

<table>
<thead>
<tr>
<th>Summary</th>
<th>Effect of $\frac{\lambda_2}{\lambda_1}$</th>
<th>Effect of Path Loss Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>High path loss: ↑→↓</td>
<td>Same as medium attraction case.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: ↑→↓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ↑→↓</td>
<td></td>
</tr>
</tbody>
</table>

Note:

↑ means that the coverage probability increases with the increase of BS density ratio $\frac{\lambda_2}{\lambda_1}$.

↑→↓ means that the coverage probability increases, and then decreases with the increase of BS density ratio $\frac{\lambda_2}{\lambda_1}$ after a turning point.

5.3.3.2 Effect of Cell Extension of Small Cells

Table 5.4: Parameters of effect of BS cell extension bias factor of small cells

<table>
<thead>
<tr>
<th>Value</th>
<th>BS density</th>
<th>Cell extension bias factor</th>
<th>BS transmit power</th>
</tr>
</thead>
<tbody>
<tr>
<td>BS density</td>
<td>${\lambda_1, \lambda_2} = {\frac{3}{10^5}, \frac{3}{10^6}}$</td>
<td>$C_1 = 1, C_2 = 1$ (0dB) to 100 (20dB)</td>
<td>${P_1, P_2} = {53, 33}$ dBm</td>
</tr>
</tbody>
</table>

The effect of BS cell extension bias factor $C_2$ of small cells is shown in Figure 5.7.

From Figure 5.7 we can observe:
The effect of BS cell extension bias factor $C_2$ of small cells is shown in Figure 5.7.

From Figure 5.7 we can observe:

- For low attraction, the performance is almost the same as the uniformly distributed cases.

- For low attraction and medium attraction, the coverage probability decreases with the increase of BS cell extension bias factor $C_2$ since more users will associate with small cells and this will result in low SINR level.

- For the high attraction, the coverage probability remains almost the same as the increases of $C_2$ since the users are extremely clustered around the cell center and few users will be impacted by increasing $C_2$. 

Figure 5.7: Downlink coverage probability as a function of BS cell extension bias factor $C_2$ in small cells.
• For uniform and low attraction, the case experiencing high path loss exponent has high coverage probability.

• For medium and high attraction, the case experiencing low path loss exponent has high coverage probability.

• BS deployment suggestion:
  
  – For low attraction, the case experiencing high path loss exponent is preferred since it will have high coverage probability.
  
  – For medium attraction and high attraction, the case experiencing low path loss exponent is preferred because it will have high coverage probability.
  
  – For high attraction experiencing low path loss exponent, the cell extension bias factor can be increased as much as possible since the coverage probability still remains the same.

The summary of the effect of BS cell extension bias factor $C_2$ in small cells is in the following table.

<table>
<thead>
<tr>
<th>Summary</th>
<th>Effect of $C_2$</th>
<th>Effect of Path Loss Exponent</th>
</tr>
</thead>
</table>
| Low attraction   | High path loss: ↓  
|                  | Same path loss: ↓  
|                  | Low path loss: ↓  | High path loss exponent has high performance.                    |
| Medium attraction| High path loss: ↓  
|                  | Same path loss: ↓  
|                  | Low path loss: ↓  | Low path loss exponent has high performance.                     |
| High attraction  | High path loss: ~  
|                  | Same path loss: ~  
|                  | Low path loss: ~  | Low path loss exponent has high performance.                     |

Note:

↓ means that the coverage probability decreases with the increase of BS cell extension bias factor $C_2$.

~ means that the coverage probability does not vary with the increase of BS cell extension bias factor $C_2$. 
5.3.3.3 Effect of BS Transmit Power of Small Cells

Table 5.5: Parameters of effect of BS transmit power of small cells

<table>
<thead>
<tr>
<th>BS density</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( {\lambda_1, \lambda_2} = { \frac{3}{10^7}, \frac{3}{10^7} } )</td>
<td></td>
</tr>
<tr>
<td>Cell extension bias factor</td>
<td>( {C_1, C_2} = {1, 1} )</td>
</tr>
<tr>
<td>BS transmit power</td>
<td>( P_1 = 53 \text{ dBm}, \ P_2 = 20 \text{ dBm to 45 dBm} )</td>
</tr>
</tbody>
</table>

Figure 5.8: Downlink coverage probability as a function of BS transmit power \( P_2 \) in small cells

The effect of BS transmit power \( P_2 \) of small cells is shown in Figure 5.8.

From Figure 5.8 we can observe:
• The performance of low attraction is almost the same as the uniform case.

• The coverage probability remains the same when experiencing low path loss exponent with low attraction.

• The coverage probability increases with the increase of $P_2$ when experiencing high or the same path loss exponent with low attraction.

• The coverage probability increases with the increase of BS transmit power $P_2$ for medium attraction and high attraction.

• The case experiencing low path loss exponent has high coverage probability for medium attraction and high attraction.

• BS deployment suggestion:
  
  – The transmit power $P_2$ of small cells can be increased as much as possible since the coverage probability will increase or remain the same with the increase of $P_2$.

The summary of the effect of BS transmit power $P_2$ in small cells is in the following table.

<table>
<thead>
<tr>
<th>Summary</th>
<th>Effect of $P_2$</th>
<th>Effect of Path Loss Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low attraction</td>
<td>High path loss: ↑</td>
<td>High path loss exponent has high performance.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: ↑</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ~</td>
<td></td>
</tr>
<tr>
<td>Medium attraction</td>
<td>High path loss: ↑</td>
<td>Low path loss exponent has high performance.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: ↑</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ↑</td>
<td></td>
</tr>
<tr>
<td>High attraction</td>
<td>High path loss: ↑</td>
<td>Low path loss exponent has high performance.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: ↑</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ↑</td>
<td></td>
</tr>
</tbody>
</table>

Note:

↑ means that the coverage probability increases with the increase of $P_2$.

∼ means that the coverage probability does not vary with the increase of $P_2$. 
5.4 Conclusions

In this chapter, we derived the downlink coverage probability of our designed non-uniform user distribution model. It can be described into two cases. The first case (case1) is to derive the downlink coverage probability when the UE of interest is associated with the BS that the UE of interest belongs to, and the second case (case2) is to derive the downlink coverage probability when the UE of interest is associated with other BS except the BS that the UE of interest belongs to. Then the overall downlink coverage probability is obtained.

The simulation results follow the derived performance analysis and confirm that our derived theoretical analysis is very accurate. Hence, the performance can be obtained using our derived equations rather than carrying out time-consuming simulations for any researchers, operators who rely on the non-uniform user model to verify the downlink HetNet system.

The effect of the base station offset between SA and BS is evaluated. It indicates that the performance converges to the uniform case when the base station offset is large enough. The effect of system parameters, such as the BS density ratio between small cells and macro cells, the cell extension bias factor of small cells and the BS transmit power of small cells is also discussed. For example, for the effect of BS density ratio between small cells and macro cells in medium attraction and high attraction case, there is a turning point (here 12 dB) where the coverage probability is optimum so that the operators should consider this turning point as design reference when deploying small BSs. For the effect of cell extension bias factor $C_2$, the coverage probability could decrease as the increase of $C_2$ in most cases. However, the performance is not decreased when the path loss exponent of small cells is low and the attraction level is high such that we can consider to increase $C_2$ when deploying small BSs. For the effect of BS transmit power $P_2$, the coverage probability increases when the transmit power $P_2$ increases. Hence, increasing $P_2$ can bring benefit. But it will bring more interference to other UEs and high power consumption. Then the balance should be taken between increasing the coverage probability and increasing the interference and it will not be covered in this thesis.
Chapter 6

Uplink Coverage Probability Performance Analysis

6.1 Introduction

The downlink analysis is based on the received SINR of a UE from the associated BS with the interference from all other BSs in the downlink direction. The uplink analysis is based on the received SINR of the associated BS from its active UE with the interference from all other active UEs in the uplink direction. There are more challenges for the uplink analysis than the downlink analysis. The first reason is that the interference in the uplink is coming from the randomly located UEs while the interference in the downlink is coming from the BSs with the fixed placement. The second reason is that there is the uplink power control for the UEs in order to mitigate the near-far effect. This makes the UE transmit power to fluctuate and then an exact characterization of the total uplink interference from all interfering UEs is not available. Hence, we have to make some simplifying assumptions in order to obtain the uplink coverage probability performance using the tools of stochastic geometry. In this chapter, the uplink analysis depends on the same non-uniform UE distribution and the same K-tier BS distribution as the downlink analysis in Chapter 5. The non-uniformly distributed UE model has been introduced in Chapter 4.

The maximum average bias received power connectivity model is assumed for both downlink and uplink in Section 3.4.2. In this connectivity model, UEs are associated or served by the BSs from which the received average bias power is maximum. Figure 6.1 shows a system model example with the above connectivity rule. There are one macro BS and three small BSs, and non-uniform UEs are distributed around small BSs with cluster
center being SAs and SAs have a base station offset with their BSs. Using the maximum average bias received power connectivity model some UEs in the cluster are associated with the BSs that they belong to and other UEs in the same cluster are associated with other BSs. Figure 6.1 has three small BSs, separately represented with a black square, a red square and a blue square, and one macro BS represented with a green square. The UEs are represented as points. The cell boundaries are represented as curves with the same color of the corresponding BS square. For instance, for small BS of black square, there is a UE cluster. The UEs in the cluster associated with the small BS of black square are labeled in black and the remaining UEs in the cluster associated with other BS, such as macro BS of green square are labeled in green. The small BS of black square is the BS that this UE cluster belongs to. In this thesis we assume the UEs are always distributed in cluster. In further research, we propose the non-uniformly distributed clusters as well as the uniformly distributed UEs, i.e. mixed UE distribution should be studied.

![Figure 6.1: System model example with the same cell extension association for both downlink and uplink](image)

Figure 6.2 gives an example with SINR in the downlink. The typical UE receives the signal from its associated small BS of red square with distance $L$ and also receives the interference signals from all interfering BSs with distance $V_1$, $V_2$ and $V_3$. BSs in the same tier have the same downlink transmit power.
Figure 6.3 shows an example with SINR in the uplink. The associated small BS of black square receives the signal from the typical active UE with distance $L$ and also receives the interference signal from all interfering active UEs with distance $V_1$ and $V_2$. In a given resource block of LTE (Long Term Evolution), there is only one active UE in each cell, which is the typical UE or the interfering UEs labeled in maroon in Figure 6.3. The uplink transmit power of each UE is variable. It depends on the power control policy and the distance $L_1$, $L_2$, $V_1$ and $V_2$. 
Figure 6.4 shows an example of spatial distances between UE, BS, and SA and the related notation in a 2-tier uplink system. The 1st tier such as the macro cell tier is in red and the 2nd tier such as the small cell tier is in blue. The BS and UE notations are the same as Figure 5.1. In the 1st tier, there are three BSs ($BS_1$, $BS_2$, and $BS_3$) and three corresponding SAs ($SA_1$, $SA_2$, and $SA_3$). The three corresponding SAs have a base station offset $D_1$ with the three BSs. In the 2nd tier, there are four BSs ($BS_1$, $BS_2$, $BS_3$, and $BS_4$) and the four corresponding SAs ($SA_1$, $SA_2$, $SA_3$, and $SA_4$). The four corresponding SAs have a base station offset $D_2$ with the four BSs.

Let us analyze the uplink performance for a BS of interest, such as $BS_1$ in Figure 6.4. The active UE that is associated with $BS_1$ can be described into two cases. Case1 is that the active UE such as UE1 is located in the cluster around $SA_1$, or we can say the active UE (UE1) belongs to the BS of interest (e.g. $BS_1$). Case2 is that the active UE such as UE2 or UE3 is located in the cluster around other SAs rather than $SA_1$. Therefore, let us summarize the two cases that the possible active UE associated with the BS of interest, $BS_i$, is as follows. The active UE is located in the cluster of $SA_i$ (case1) or the active UE is located in the cluster of other $SA_k$ (case2). This $SA_k$ may be of the same tier as $i$ ($m = i, k \neq j$), or it may be in another tier ($m \neq i$).

Let $m = 1, 2, \ldots k$ be the tier of interest. The procedure to get the uplink coverage probability $P_{ul,c}$ is as follows:
• Find the probability density function (PDF) \( f_L^{(m)}(l) \) and complementary cumulative distribution function (CCDF) \( \bar{F}_L^{(m)}(l) = \mathbb{P}^{(m)}(L > l) \) of random variable \( L \). Let \( L \) be the distance between an active UE and its associated BS that the active UE belongs to in the \( m \)th tier.

• The BS of interest (e.g. \( BS_1 \) in Figure 6.4) in a given \( m \)th tier can be associated with an active UE in two cases.
  
  Case1: an active UE that belongs to the BS of interest.
  
  - Derive \( \mathbb{P}^{(m)}(A_{\text{case1}}) \), which is the probability that the active UE is associated with the BS of interest that the active UE belongs to in the \( m \)th tier.
  
  - Derive the uplink coverage probability \( \mathbb{P}^{(m)}(\text{SINR}_{m}(l) > T_m | A_{\text{case1}}) \) conditioned on the active UE associated with \( BS_1 \).
  
  - Derive the uplink coverage probability \( \mathbb{P}^{(m)}_{ul,c,\text{case1}} = \mathbb{P}^{(m)}(A_{\text{case1}}) \cdot \mathbb{P}^{(m)}(\text{SINR}_{m}(l) > T_m | A_{\text{case1}}) \) that the active UE is associated with \( BS_1 \).

  Case2: an active UE that belongs to other BS rather than the BS of interest.
  
  - Ignored since a simplifying assumption made in the following section, which is the active UE belongs to the BS of interest.

Summary

- \( \mathbb{P}^{(m)}(A_{\text{case1}}) \) is the probability that the active UE that belongs to the BS of interest is associated with the BS of interest.

- \( \mathbb{P}^{(m)}_{ul,c,\text{case1}} \) is the coverage probability associated with the BS of interest.

- The uplink coverage probability for a BS of interest distributed in a \( m \)th tier based on the second simplifying assumption that the active UE belongs to the BS of interest is \( \mathbb{P}^{(m)}_{ul,c} = \frac{\mathbb{P}^{(m)}_{ul,c,\text{case1}}}{\mathbb{P}^{(m)}(A_{\text{case1}})} \).

- The detail of the second simplifying assumption will be explained in the following Section 6.2.

• The overall uplink coverage probability is

\[
\mathbb{P}_{ul,c} = \sum_{m \in \mathcal{S}} \frac{N_m \lambda_m}{\sum_{m \in \mathcal{S}} N_m \lambda_m} \mathbb{P}^{(m)}_{ul,c},
\]

(6.1)

where \( \mathcal{S} \) is the subset of tiers, in which the non-uniform user is distributed. \( N_m \)
is the average number of users in each cluster of a \( m \)th tier. \( \lambda_m \) is the BS density of a \( m \)th tier.

Section 6.2 introduces the simplifying assumptions of the uplink analysis in this thesis. The uplink power control is shown in Section 6.3. The uplink coverage probability analysis is derived in Section 6.4. Simulation results and the effect of system parameters are presented in Section 6.5.

### 6.2 Simplifying Assumptions

As mentioned above, the uplink analysis brings more challenges since the uplink power control makes the UE transmit power variable and an exact characterization of the total uplink interference is not available. We have to make some assumptions to simplify the analysis. In Section 6.5, there are the simulation results to demonstrate that the mismatch due to simplified assumptions is less than 1 dB.

The assumptions are as follows:

- In each cell, there is only one active UE. LTE of orthogonal multiple access technique is assumed. In a given resource block, there is one active UE in each cell so that the density of the active UE is the same as the density of the BS. The uplink analysis is based on active UEs. In each tier, the density of active UEs in the \( i \)th tier is assumed to be \( \lambda_i \), which is the same as the density of BS in the \( i \)th tier.

- An active UE associated with \( BS_i^j \) comes from the UEs in the cluster around \( SA_i^j \) and other active UEs associated with \( BS_i^j \) in the cluster around other SA rather than \( SA_i^j \) are not considered in the uplink analysis.

  - In Figure 6.4, \( SA_1^1 \) has a base station offset \( D_1 \) with \( BS_1^1 \). Some UEs in the cluster around \( SA_1^1 \) are associated with \( BS_1^1 \), and other UEs in the cluster around \( SA_1^1 \) are associated with other BSs. Or we can also say that in the coverage area of \( BS_1^1 \), some UEs are from the cluster around \( SA_1^1 \) and other UEs are from the cluster around other SAs.

  - We make an assumption that an active UE such as UE1 comes from the cluster around \( SA_1^1 \) (and ignore the UE such as UE2 or UE3 from the cluster around other SAs such as \( SA_2^2 \) or \( SA_3^3 \)) when we take \( BS_1^1 \) as a BS of interest. The
Reason of the above assumption is that the PDF and CCDF of the distance between the UE1 and the BS of interest (e.g. $BS_1^1$) is available from Section 5.2.1.2. The characterization of the distance between the UE2 (or UE3) and the BS of interest (e.g. $BS_1^2$) is not available.

Figure 6.5 gives an example how we assume an active UE in the uplink analysis. In (a), it lists the non-uniformly distributed clusters of our designed user model in small cells. Black points are UEs associated with the small BS1 and green points are UEs associated with other BSs. There are three UE clusters. The center SA1 of UE cluster1 has a base station offset with small BS1. The center SA2 of UE cluster2 (or the center SA3 of UE cluster3) has the base station offset with other BS, which is not included in Figure 6.5. According to our assumption, the active UE is assumed from UE cluster1. The UE cluster2 and UE cluster3 are not considered due to the unavailability of the distance characterization between UEs in UE cluster 2 (or UE cluster 3) and small BS1. An active UE will be got randomly among the black points in (b). These black points from the UE cluster1 are in the coverage area of small BS1. In (c), the active UE is got among black points of (b) randomly.

![Figure 6.5: Assumption of an active UE](image)

- The base station offset is less than a threshold.

  When the base station offset between SA and its BS is high, most of UEs are located far away from BSs or around the cell edge, which results in the performance will be even worse than the uniform scenario. We assume in the uplink research that the base station offset is less than a threshold. It is reasonable since the scenario
of the non-uniform UE distribution above this threshold will never be deployed by operators.

- Active UEs in each tier form a homogeneous and independent PPP.

When deriving the characterization of the interference from all interfering active UEs, we make another assumption that the active UEs in each tier form a homogeneous and independent PPP. Since we assume that the distribution of active UEs is PPP, the uplink interference of active UEs can be simplified by the property of probability generating functional (PFGL) using the tools of stochastic geometry.

### 6.3 Uplink Power Control

Power control is important in the system design of cellular networks. The relevant uplink power control policy is one of the key features of CDMA, UMST or OFDM. The purpose of power control is noted in the following [97–100]:

- Mitigate the near-far problem. If UEs at different locations transmit at the same power, BS potentially fails to decode the signals from the UE far away because the UE close to the BS causes a much higher interference power. This means that the propagation path loss between UEs and BSs should be considered.

- Power control can help save energy cost, which is an important part of green communication. For UEs, the power control is intended to extend battery life through minimum possible power while stable communications are still maintained. For BSs, reasonable power control can maintain interference at a low level while the overall service to UEs are hold.

- Power control provides protection against shadowing and fast fading.

Wireless channels experience both short term and long term fading. We may consider two main power control mechanisms [101]:

- Open loop power control:

  The initial power is set at which the UE should transmit considering path loss. The power control has no feedback.
• Closed loop power control:
  
  – Inner (fast) power control:
    Inner power control is a kind of fast closed loop power control to combat fast fading and short term distortion. Inner fast power control maintain the constant SINR target. This power control happens in high rate.
  
  – Outer (slow) power control:
    Outer power control is a kind of slow closed loop power control to combat slow fading and long term distortion. Outer slow power control maintain the constant transport block error rate (BLER) target. This power control happens in slow rate.

However, it is difficult to do a tractable analysis of the above power control mechanisms. [102] gives the fractional power control maintaining the constant interference power at BS. The author in [103] proposes an analytical approach depending on the fractional power control. Hence, in this thesis, the fractional power control is considered. The fractional power control proposes a compensation which is proportional to the path loss. The transmit power of the UE using the fractional power control is \( r^{\alpha\epsilon} \), where \( \epsilon \in [0, 1] \) is the fractional power control factor, \( r \) is the distance between the UE and BS, and \( \alpha \) is path loss exponent. When a UE is close to the associated BS, the required uplink transmit power decreases while remaining the same received signal power at the associated BS. The uplink power control helps to combat the near-far effect and extends UE device battery life. When \( \epsilon = 1 \), the fractional power control compensates the complete path loss while when \( \epsilon = 0 \) there is no power control, UEs transmit with the same power.

6.4 Model Performance Analysis

In this section, we derive the uplink coverage probability. The definition of the uplink SINR and the uplink coverage is as follows. There are total of \( K \) tiers of BSs with a subset \( \kappa \) and there are total of \( S \) tiers with a subset \( S \) distributed with our designed non-uniform user model.

**Definition 6-1.** The uplink SINR at an associated BS \( B_0 \) located at the origin in the \( k \)th tier is
\[ \text{SINR}_k(L) = \frac{r^{\alpha_k} G_k r^{-\alpha_k}}{I_r + \frac{\sigma^2}{L_0}}, \]  
(6.2)

\[ I_r = \sum_{j=1}^{S} I_{rj} = \sum_{j=1}^{S} \sum_{i \in \phi_j} l_{ji}^{\alpha_k} G_j r_{ji}^{-\alpha_j} \]  
(6.3)

- \( r \) is the distance between the associated BS \( B_0 \) and its active UE. \( l_{ji} \) is the \( i \)th distance between the interfering active UEs and their associated BSs in the \( j \)th tier. \( r_{ji} \) is the \( i \)th distance between the interfering active UEs in the \( j \)th tier and the associated BS \( B_0 \).

- \( G_k \) denotes the random channel power gain in the \( k \)th tier. Let the complex channel coefficient between the \( k \)th tier BSs and the UEs be denoted as \( h_k \) (Rayleigh fading is assumed in this thesis). Then we define \( G_k = |h_k|^2 \), which is an exponential distribution with unity mean.

- \( \alpha_k \) is \( k \)th path loss exponent in the \( k \)th tier.

- \( I_r \) is the total uplink received interference from all interfering active UEs. \( I_{rj} \) is the total uplink received interference in the \( j \)th tier.

- \( \sigma^2 \) is the power of the additive white Gaussian noise (AWGN).

- \( S \) is the total number of tiers distributed with our non-uniform UEs.

- \( \phi_j \) is SPPP with the density \( \lambda_j \) in the \( j \)th tier.

- \( L_0 \) is the path loss at a reference distance of 1 m.

**Definition 6-2.** An active UE in the \( k \)th tier is in the coverage if the uplink SINR at the BS of interest with the maximum average bias received power is more than the coverage threshold \( T_k \).

\[ \text{SINR}_K = \frac{r^{\alpha_k} G_k r^{-\alpha_k}}{\sum_{j=1}^{S} \sum_{i \in \phi_j} l_{ji}^{\alpha_k} G_j r_{ji}^{-\alpha_j} + \frac{\sigma^2}{L_0}} > T_k. \]  
(6.4)
6.4.1 Distance Characterization

Based on the assumption that an active UE associated with $BS_i^j$ comes from the UEs in the cluster around $SA_i^j$, the received signal power of the associated BS from its active UE depends on the random distance variable $L$ between the associated BS and its active UE. $L$ has the same PDF and CCDF as given in Section 5.2.1.2.

6.4.2 Interference Characterization

The uplink total interference from all interfering active UEs in tiers distributed with our designed non-uniform user model is

$$I_r = \sum_{j=1}^{S} I_{rj} = \sum_{j=1}^{S} \sum_{\phi_j} l_{ji}^{\alpha_j} G_j r_{ji}^{-\alpha_j}.$$  \hspace{1cm} (6.5)

We have made the assumptions in Section 6.2 that there is an active UE in each cell and active UEs in each tier form a PPP. Then we can use stochastic geometry to simplify the uplink interference characterization. So we can change the unlimited sum of interference from all interfering active UEs into an integral expression utilizing the property of products over PPP (probability generating functional (PGFL)), which is $\mathbb{E}[\prod_{x \in \phi} f(x)] = \exp(-\lambda \int_{\mathbb{R}^2} (1 - f(x)) dx)$ if $\phi$ is PPP. Specifically, the Laplace transform $L_x(s) = \mathbb{E}_x[e^{-sx}]$ ($\mathbb{E}_x[*]$ is the expectation of $*$ over $x$) of the interference is very useful.

**Lemma 6-1.** Calculate the Laplace transform of the total uplink interference in the $j$th tier distributed with our non-uniform UE model.
\[ L_{rj}(s) = \mathbb{E}_{rj}[e^{-sI_{rj}}], \]
\[ = \mathbb{E}_{\phi_j}[e^{-s\sum_{l\in\phi_j} G_{jl}r_{jl}^{-\alpha_j}}], \]
\[ = \mathbb{E}_{\phi_j,l_j}\prod_{l\in\phi_j} e^{-sG_{jl}r_{jl}^{-\alpha_j}}, \]
\[ \equiv (a) \mathbb{E}_{l_j}[\exp(-\lambda_j \int_{\mathbb{R}^2} (1 - G_j(e^{-sG_jx^{-\alpha_j}}))dx)], \]
\[ = \mathbb{E}_{l_j}[\exp(-2\pi\lambda_j \int_{d_{\min}}^{\infty} (1 - \mathcal{L}_G_j(sG_j^{-\alpha_j}))rdr)], \]
\[ \equiv (b) \mathbb{E}_{l_j}[\exp(-2\pi\lambda_j \int_{d_{\min}}^{\infty} (1 - \frac{1}{1 + sG_j^{-\alpha_j}}))rdr)], \]
\[ = \mathbb{E}_{l_j}[\exp(-2\pi\lambda_j \int_{d_{\min}}^{\infty} \frac{sG_j^{-\alpha_j}}{1 + sG_j^{-\alpha_j}})rdr)]. \]

where (a) comes from the property of product over PPP (PGFL). (b) follows the fading power \( G_j \) of the exponential distribution with the unity gain. \( d_{\min} \) is the distance to the closest interferer in the \( j \)th tier, which is the same as the downlink case. This Laplace transform is the expectation over \( l_j \), which is the random distance variable between UEs and their associated BSs in the \( j \)th tier.

### 6.4.3 Uplink Coverage Probability

Based on the simplifying assumptions we made in Section 6.2, the active UE associated with the BS of interest is in the cluster around a SA, which has a base station offset with the BS of interest. We call that this active UE belongs to the BS of interest spatially. It means that we only consider the case1 and ignore the case2 (case1 and case2 are discussed at the beginning of this chapter). In Figure 6.4, if we take \( BS_1 \) as the BS of interest, UE1 is assumed as the active UE associated with the BS of interest rather than UE2 and UE3.

**Lemma 6-2.** The probability that the active UE is associated with the BS of interest that the active UE belongs in the \( m \)th tier (conditioned that the distance between the active UE and the BS of interest is \( l \)) is

\[ P_{l}^{(m)}(A_{\text{case1}}) = \prod_{i=1}^{K} \exp(-\lambda_i \pi \frac{P_i G_i}{\tau_i C_i} l^2 \frac{2\alpha_m}{\alpha_i}), \] (6.7)
where \( l \) is the random distance variable between the active UE and the BS of interest that the active UE belongs to in the \( m \)th tier. For proof it is the same as Equation 5.9 since the downlink and the uplink analysis use the same user connectivity rule, which is the downlink maximum average bias received power.

**Lemma 6-3.** The probability that the active UE is associated with the BS of interest that the active UE belongs to in the \( m \)th tier

\[
\mathbb{P}^{(m)}(A_{\text{case}1}) = \int_0^\infty \mathbb{P}^{(m)}(A_{\text{case}1}) \cdot f_L^{(m)}(l) \cdot dl,
\]

\[
= \int_0^\infty \prod_{i=1}^K \exp\left(-\lambda_i \pi \left(\frac{P_{Ce_{m,m}}}{T_m C_{m,m}}\right)^{\frac{2}{\alpha_i}} \cdot l^{2\alpha_m} \right) \cdot f_L^{(m)}(l) \cdot dl,
\]

where \( f_L^{(m)}(l) \) comes from subsection 5.2.1.2.

**Lemma 6-4.** The coverage probability of the BS of interest conditioned on the distance between the active UE and the BS of interest as \( l \) is

\[
\mathbb{P}^{(m)}[\text{SINR}_m(l) > T_m | A_{\text{case}1}],
\]

\[
= \exp\left(-\sigma^2 \frac{L_0}{L} T_m^{\alpha_m(1-\epsilon)} \cdot \prod_{i=1}^S \mathcal{L}_{l_i}(T_m^{\alpha_i(1-\epsilon)}),
\]

where \( T_m \) is SINR coverage threshold in the \( m \)th tier. For proof see Appendix B.1.

For the details of \( \mathcal{L}_{l_i}(T_m^{\alpha_i(1-\epsilon)}) \) the reader is referred to Equation 6.6. \( \mathcal{L}_{l_i}(s) \) is the Laplace transform of the uplink interference in the \( i \)th tier.

\[
\mathcal{L}_{l_i}(T_m^{\alpha_i(1-\epsilon)})
\]

\[
= \mathbb{E}_{l_i}[\exp(-2\pi \lambda_i \int_{d_{\text{min}}}^\infty \frac{T_m^{\alpha_i(1-\epsilon)}}{1 + T_m^{\alpha_i(1-\epsilon)}} l^{\alpha_i(1-\epsilon)} r^{\alpha_m(1-\epsilon)} dr)],
\]

\[
= \exp(-2\pi \lambda_i \int_{d_{\text{min}}}^\infty \int_{d_{\text{min}}}^\infty \frac{T_m^{\alpha_i(1-\epsilon)}}{1 + T_m^{\alpha_i(1-\epsilon)}} l^{\alpha_i(1-\epsilon)} r^{\alpha_m(1-\epsilon)} f_L^{(i)}(l_i) dr dl_i).
\]

**Theorem 6-1.** The coverage probability associated with the BS of interest in the \( m \)th
tier, in which the active UE belongs to the BS of interest is

\[ P_{ul,c,case1}^{(m)} = \int_0^\infty p_{ul,c}^{(m)}(l) \cdot \mathbb{P}_{|\Gamma}^{(m)}[\text{SINR}_m(l) > T_m | A_{case1}] \cdot \mathbb{P}_{|\Gamma}^{(m)}(A_{case1}) \cdot f_L^{(m)}(l) \cdot dl, \quad (6.11) \]

where \( f_L^{(m)}(l) \) comes from subsection 5.2.1.2 and \( L \) is the random distance variable between the UE and the BS of interest in the \( m \)th tier. Each tier may have different parameters of the radius \( R \) and the base station offset \( D \) such that PDF of \( L \), \( f_L^{(m)}(l) \), in each tier may be different.

### 6.4.4 Overall Uplink Coverage Probability

**Lemma 6-5.** The uplink coverage probability for a BS of interest distributed in a \( m \)th tier with the simplifying assumption that the active UE belongs to the BS of interest is

\[ P_{ul,c}^{(m)} = \frac{p_{ul,c,case1}^{(m)}}{p_{ul,c,case1}^{(m)}}, \quad (6.12) \]

where \( p_{ul,c,case1}^{(m)} \) comes from Equation 6.8 and \( p_{ul,c,case1}^{(m)} \) comes from Equation 6.11.

**Theorem 6-2.** The overall uplink coverage probability is

\[ P_{ul,c} = \sum_{m \in S} \frac{N_m \lambda_m}{\sum_{m \in S} N_m \lambda_m} p_{ul,c}^{(m)} \quad (6.13) \]

where \( S \) is the subset of tiers, in which the non-uniform UE is distributed. \( N_m \) is the average number of UEs in each cluster of the \( m \)th tier. \( \lambda_m \) is the BS density of the \( m \)th tier.

### 6.5 Simulation Results

A 2-tier system consists of macro and small BSs. Each tier BS is deployed as an independent PPP with the specific BS density, BS transmit power and path loss exponent. The channels between all BSs and the UEs are independent Rayleigh fading. The average fading power
gain is unity. We generate a BS and UE pattern. According to the maximum downlink average bias received power, we choose an active UE in each cell. We repeat the above steps until the number of BS patterns reaches 1,000 and measure the SINR of the BS of interest in each pattern. Then we calculate the simulated uplink coverage probability using the above recorded SINR and compare with the analytical results of the designed non-uniform UE distribution model in Equation 6.13. For detail simulation setting and method, please refer to Appendix C.2.

For the parameters, we follow the same setting as the downlink case in the previous chapter (Table 5.2) and [75] except that $D_{uni} = 80$ m.

6.5.1 Matching between Simulation and Theoretical Analysis

Figure 6.6 gives the simulation results and theoretical results obtained from the above Section 6.4. The solid lines are for analysis using Equation 6.13. The dash lines are for the simulations. It shows that the analysis and simulation results have some discrepancy because of our simplifying assumptions made in Section 6.2. One assumption is that an active UE associated with $BS_i^j$ comes from the UEs in the cluster around $SA_i^j$. Another assumption is that the active UEs form a PPP. These assumptions can help us achieve the theoretical performance analysis. But they also cause some mismatch. From Figure 6.6, we can observe that the mismatch between analysis and simulation is less than 1 dB. The analytical method still can be regarded as a reference if the system requirement is not very strict.
The cluster process is Matern cluster. The attraction level is 0.85 (the radius $R$ is 30 m, green curve), 0.75 (the radius $R$ is 50 m, red curve), 0.6 (the radius $R$ is 80 m, black curve) and 0 (the radius $R$ is 200 m, blue curve). The uniform result (cyan curve) has been also included in order to compare with the above non-uniform UE distribution. When the attraction level is high, such as 0.85 or 0.75, the uplink coverage probability is high since the active UE is chosen near the cell center. When the attraction level is low, the coverage probability of non-uniform UE distribution converges to uniform distribution (cyan curve). The base station offset $D_2$ is 0 m which means the dependency level is 1. The effect of the base station offset is discussed in the following subsection. In order to evaluate the performance of our designed non-uniformly distributed UE model, we put non-uniform UEs only in small cells.
6.5.2 Effect of Base Station Offset

In this subsection, we show that the effect of the base station offset \( D_2 \) in the uplink coverage probability. We remain the UE distribution as Matern cluster (the attraction level \( = 0.75, \ R = 50 \) m) and vary the distance \( D_2 \) from \{0, 30, 50, 80\} m, which corresponds to the dependency level from \{1, 0.625, 0.375, 0\}. We can observe that there is no obvious difference between \( D_2 = 0 \) m and \( D_2 = 30 \) m. However, in the downlink analysis from Figure 5.5, the downlink coverage probability of \( D_2 = 30 \) m is obviously worse than that of \( D_2 = 0 \) m. It is due to the fact that the active UE position is almost the same when the base station offset \( D_2 \) is less than the Mater cluster radius \( R \) (here \( D_2 = 0 \) m, 30 m and \( R = 50 \) m) in the uplink analysis. When \( D_2 = 50 \) m in the uplink, the dependency decreases and then the uplink coverage probability also decreases. And the coverage probability converges to the uniform case (cyan curve) when \( D_2 = 80 \) m in the uplink. In the downlink, the overall performance for the whole cluster UEs are considered while only an active UE in one cell are considered in the uplink. The clustered UEs are located around the cell edge when the base station offset is high, such as \( D_2 = 80 \) m. If we still continue to increase the base station offset, such as \( D_2 = 100 \) m, the uplink coverage probability would be worse than the uniform case. The scenario of the high base station offset doesn’t bring any

Figure 6.7: Effect of the base station offset \( D_2 \) on the uplink coverage probability
benefits and it will also never be deployed by operators. Then we assume that an active UE associated with $BS_i^j$ comes from the UEs in the cluster around $SA_i^j$ with reasonable base station offset between $BS_i^j$ and $SA_i^j$ (for example, the base station offset is less than $D_2 = 80$ m in the above simulation setting).

6.5.3 Effect of System Parameters

The theoretical analysis of the uplink coverage probability has been derived. This subsection describes the effect of system parameters, such as the BS density ratio between small cells and macro cells, the cell extension bias factor of small cells, the transmit power of small BSs, and the fractional power control factor.

We follow the same simulation parameters as the downlink case. When calculating the uplink coverage probability, the SINR coverage threshold is 0 dB and the fractional power control factor $\epsilon$ is 0.5. Since we cannot determine the relationship between the path loss exponent of macro cells and the path loss exponent of small cells, we give three curves for each kind of attraction. For each attraction (uniform, low attraction, medium attraction and high attraction), we show three curves. The first red curve is that the path loss exponent of macro cell is higher than that of small cells ($\{\alpha_1, \alpha_2\} = \{3.5, 3\}$). The second blue curve is that the path loss exponent of macro cell is equal to that of small cells ($\{\alpha_1, \alpha_2\} = \{3.5, 3.5\}$). The third black curve is that the path loss exponent of macro cell is lower than that of small cells ($\{\alpha_1, \alpha_2\} = \{3.5, 4\}$). The base station offset between SAs and BSs is assumed to be 0 m. In the following figures, the low attraction level is assumed as 0.1, the medium attraction level is assumed as 0.6, and the high attraction level is assumed as 0.75.

6.5.3.1 Effect of BS Density Ratio between Small Cells and Macro Cells

The parameter of effect of BS density ratio between small cells and macro cells are the same as Table 5.3 in the downlink case.
Figure 6.8: Uplink coverage probability as a function of BS density ratio $\frac{\lambda_2}{\lambda_1}$ between small cells and macro cells

The effect of BS density ratio between small cells and macro cells is shown in Figure 6.8.

From Figure 6.8 we can observe:

- When the BS density ratio $\frac{\lambda_2}{\lambda_1}$ between small cells and macro cells is low, such as $0 - 5$ dB, the number of small cells or the number of active UEs is also low. This results in less interference to the BS of interest such that the uplink coverage probability might be very high.

- When the BS density ratio $\frac{\lambda_2}{\lambda_1}$ between small cells and macro cells is high, such as $15 - 20$ dB, the density of small cells may be very high and then the coverage area of small cell is small. So the cases of low attraction, medium attraction and high attraction converge to the uniform case when the BS density ratio is high.
• The case experiencing high path loss exponent always has the high uplink coverage probability.

• The general trend is that the uplink coverage probability decreases as $\frac{\lambda_2}{\lambda_1}$ increases.

• The uplink coverage probability of low attraction does not converge to that of the uniform case when the BS density ratio is low. When the BS density ratio is low, the coverage area of small cells is very large and the equivalent radius of the coverage area may be more than the UE cluster radius of low attraction in this simulation setting. Hence, the performance of the uniform case will be worse than the performance of low attraction when the BS density ratio is low.

• BS deployment suggestion:
  
  – Deploying small cells with the low BS density ratio will have high uplink coverage probability.
  
  – Deploying the case experiencing high path loss exponent will have high uplink coverage probability.

The summary of the effect of BS density ratio between small cells and macro cells is in the following table.

<table>
<thead>
<tr>
<th>Summary</th>
<th>Effect of $\frac{\lambda_2}{\lambda_1}$</th>
<th>Effect of Path Loss Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low attraction</td>
<td>High path loss: ↓</td>
<td>High path loss exponent has high performance.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: ↓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ↓</td>
<td></td>
</tr>
<tr>
<td>Medium attraction</td>
<td>High path loss: ↓</td>
<td>High path loss exponent has high performance.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: ↓</td>
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<td></td>
<td>Low path loss: ↓</td>
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<td></td>
<td>Same path loss: ↓</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ↓</td>
<td></td>
</tr>
</tbody>
</table>

Note:
$\downarrow$ means that the coverage probability decreases with the increase of the BS density ratio $\frac{\lambda_2}{\lambda_1}$. 
6.5.3.2 Effect of Cell Extension of Small Cells

The parameter of effect of BS cell extension of small cells are the same as Table 5.4 in the downlink case.

The effect of BS cell extension bias factor $C_2$ of small cells is shown in Figure 6.9.

From Figure 6.9 we can observe:

- The case experiencing high path loss exponent always has the high uplink coverage probability. In the case experiencing high path loss exponent, the active interfering UEs might generate less interference so that the uplink coverage probability might be high in this simulation setting.

- For low attraction, the uplink coverage probability decreases as the increase of $C_2$. When $C_2$ increases, the coverage area in small cells increases. So the active UEs may have long distance with their associated BSs. Long distance may cause high transmit power from interfering active UEs in uplink depending on the uplink power control policy. Hence, they might raise high interference such that the uplink coverage probability may decrease.

- For medium attraction and high attraction, the uplink coverage probability remains the same as the increase of $C_2$. When $C_2$ increases, the coverage area in small cells increases. However, the UE cluster is still in the coverage, the position of the active UE may not be changed so that the uplink coverage probability remains.

- BS deployment suggestion:
  - Deploying small cells with high attraction will have high uplink coverage probability.
  - Deploying the case experiencing high path loss exponent will have high uplink coverage probability.
  - Deploying low attraction with low $C_2$ will have high coverage probability.
The summary of the effect of BS cell extension bias factor $C_2$ in small cells is in the following table.

<table>
<thead>
<tr>
<th>Summary</th>
<th>Effect of $C_2$</th>
<th>Effect of Path Loss Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low attraction</td>
<td>High path loss: ↓</td>
<td>high path loss exponent has high performance.</td>
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<td></td>
<td>Same path loss: ↓</td>
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<td></td>
<td>Low path loss: ∼</td>
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<tr>
<td>Medium attraction</td>
<td>High path loss: ∼</td>
<td>high path loss exponent has high performance.</td>
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<tr>
<td></td>
<td>Same path loss: ∼</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ∼</td>
<td></td>
</tr>
<tr>
<td>High attraction</td>
<td>High path loss: ∼</td>
<td>high path loss exponent has high performance.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: ∼</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ∼</td>
<td></td>
</tr>
</tbody>
</table>
Note:
↓ means that the coverage probability decreases with the increase of BS cell extension bias factor $C_2$.

~ means that the coverage probability does not vary with the increase of BS cell extension bias factor $C_2$.

6.5.3.3 Effect of BS Transmit Power of Small Cells

The parameter of effect of BS transmit power of small cells are the same as Table 5.5 in the downlink case.

The effect of BS transmit power $P_2$ of small cells is shown in Figure 6.10.

![Figure 6.10: Uplink coverage probability as a function of BS transmit power $P_2$ in small cells](image)

From Figure 6.10 we can observe:
• Increasing $P_2$ is equivalent to increase $C_2$. For instance, increasing $P_2$ will cause the larger coverage area which has the same result as the increase of $C_2$.

• It can be observed that Figure 6.9 and Figure 6.10 have the almost same curve.

• Hence, the observation and BS deployment suggestion is the same for the effect of BS cell extension bias factor $C_2$ and the effect of BS transmit power $P_2$.

The summary of the effect of BS transmit power $P_2$ in small cells is in the following table.

<table>
<thead>
<tr>
<th>Summary</th>
<th>Effect of $P_2$</th>
<th>Effect of Path Loss Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low attraction</td>
<td>High path loss: $\downarrow$</td>
<td>High path loss exponent has high performance.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: $\downarrow$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: $\sim$</td>
<td></td>
</tr>
<tr>
<td>Medium attraction</td>
<td>High path loss: $\sim$</td>
<td>High path loss exponent has high performance.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: $\sim$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: $\sim$</td>
<td></td>
</tr>
<tr>
<td>High attraction</td>
<td>High path loss: $\sim$</td>
<td>High path loss exponent has high performance.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: $\sim$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: $\sim$</td>
<td></td>
</tr>
</tbody>
</table>

Note:

$\downarrow$ means that the coverage probability decreases with the increase of BS transmit power $P_2$.

$\sim$ means that the coverage probability does not vary with the increase of BS transmit power $P_2$.

6.5.3.4 Effect of Fractional Power Control Factor

For the parameters, we follow the same setting as the downlink case in the previous chapter (Table 5.2) and [75]. Matern cluster process is with the radius $R = 50$ m and the base station offset is $D = 0$ m.
Figure 6.11 gives the effect of fractional power control $\epsilon$. When $\epsilon = 0$, it represents the fixed transmit power. The case of $\epsilon = 0$ has the highest uplink coverage probability when SINR is high, such as SINR > 14 dB in this simulation setting. The cases of $\epsilon = 0.25$ and $\epsilon = 0.5$ have the highest uplink coverage probability when SINR is low, such as SINR < 14 dB. The difference between $\epsilon = 0.25$ and $\epsilon = 0.5$ is negligible when SINR is low, such as SINR < 14 dB. The uplink coverage probability of $\epsilon = 0.25$ is a little higher than that of $\epsilon = 0.5$ when SINR is high, such as SINR > 14 dB. The case of $\epsilon = 0.75$ is almost the same as the case of fixed transmit power $\epsilon = 0$ when SINR is low, such as SINR < 10 dB. The case of $\epsilon = 0.75$ has lower uplink coverage probability than the case of $\epsilon = 0$, $\epsilon = 0.25$ and $\epsilon = 0.5$ when SINR is high, such as SINR > 10 dB. The case of full path loss compensation $\epsilon = 1$ provides the lowest uplink coverage probability across all SINR coverage thresholds. Hence, the coverage probability of $\epsilon = 0$ is the best when SINR > 14 dB while the coverage probability of $\epsilon = 0.25$ or $\epsilon = 0.5$ is the best when SINR < 14 dB.

6.6 Conclusions

In this chapter, we derived the uplink coverage probability of our designed non-uniform UE distribution. The uplink analysis is more complex than the downlink analysis since the uplink power control makes the UE transmit power variable and the characterization of the
uplink interference from all interfering active UEs is not available. Therefore, we propose some assumptions in order to make our analysis using the tools of stochastic geometry. One assumption is that an active UE associated with $BS_i^j$ comes from the UEs in the cluster around $SA_i^j$. Then the above assumption makes it possible to get the PDF and CCDF of the distance between the BS of interest and the associated active UE. The uplink coverage probability is derived based on the proposed assumptions. The simulation results do not match closely with the derived analysis because of the simplifying assumptions we made. However, the mismatch between the analysis and the simulation is less than 1 dB. This shows that our assumptions are acceptable if the system requirement is not very strict. Hence, we can obtain the performance using our derived equations rather than carrying out time-consuming simulations for the designed non-uniform user model to verify uplink HetNet performance.

The effect of the base station offset between SA and BS is evaluated. It shows that there is no obvious difference when the base station offset $D$ is less than the Matern cluster radius $R$. If we still continue to increase the base station offset, the uplink coverage probability would be worse than the uniform case. The operators will never deploy small cells with high base station offset since it will not bring any benefits. So we make an assumption that the base station offset is less than a threshold. The effect of system parameters, such as the BS density ratio between small cells and macro cells, the BS cell extension bias factor of small cells, the BS transmit power of small cells, and the fractional power control factor are also discussed. For example, for the effect of BS density ratio, when the BS density ratio is low, such as $0 - 5$ dB between small cells and macro cells, the number of small cells or the number of active UEs is also low. This results in less interference so that the uplink coverage probability might be very high. For the effect of BS cell extension bias factor $C_2$, the case experiencing high path loss exponent always has the high uplink coverage probability. The case experiencing high path loss exponent will generate less interference so that the uplink coverage probability might be high in this simulation setting. For the effect of BS transmit power $P_2$, increasing $P_2$ will increase the coverage area, which is equivalent to increase $C_2$. Finally, the effect of different fractional power control factor is introduced.
Chapter 7

Analysis Comparison between Downlink and Uplink

7.1 Introduction

In this chapter, the analysis comparison between the downlink and the uplink is summarized from the work covered in the previous two chapters. Firstly, Section 7.2 introduces the proposed non-uniform UE model used in the downlink and the uplink analysis. Then Section 7.3 represents the coverage probability performance analysis including the coverage probability definition, the assumptions, the coverage probability derivations and the simulation results. Finally, Section 7.4 indicates the effect of system parameters.

7.2 Non-uniform UE Model

In this thesis, a non-uniform UE model is introduced. This non-uniform UE model is a Poisson cluster process with the cluster centers located at SAs. SAs have the base station offset with their BSs. There are two parameters in this UE distribution model. They are the cluster radius $R$ and the base station offset $D$. $R$ is used to determine the attraction level between UEs and their SAs. $D$ is used to determine the dependency level between SAs and their BSs. The combination of the attraction level and the dependency level can cover most of the non-uniform scenarios. When operators carry out the evaluation of HetNet design based on the non-uniform UE model, they can choose any combination of two parameters to create the specific non-uniform UE distribution. In our designed non-uniform UE model, different cell tier may have different parameters. In this thesis, we
focus our designed non-uniform UE model with a user’s location in a cluster with a PDF of Matern cluster process. It is easy to extend to other PDFs. Our non-uniform UE model is introduced in Chapter 4.

The heterogeneity of this non-uniform UE model is also shown in Chapter 4. The heterogeneity analysis is based on coefficient of variation (CoV). Voronio cell area is used as the CoV statistic measurement of inter-point distance metrics. In this thesis, Cov is defined as the standard deviation of Voronio cell area measurement divided by the mean of Voronio cell area measurement. CoV is a good indication of user’s spatial heterogeneity. From Chapter 4, the dependency level has little impact for the heterogeneity and the attraction level has big impact for heterogeneity.

The downlink coverage probability performance analysis in Chapter 5 and the uplink coverage probability performance analysis in Chapter 6 are all based on the non-uniform user model introduced in Chapter 4.

7.3 Coverage Probability Performance Analysis

Based on our designed non-uniform UE distribution model, the downlink and the uplink coverage probability performance analysis are derived step by step. During the evaluation that researchers or operators carry out on HetNets based on the non-uniform UEs, the derived equations of the downlink coverage probability (Equation 5.17) and the uplink coverage probability (Equation 6.13) can be used directly rather than time consuming simulations. The same connectivity rule is applied for the downlink and the uplink analysis, which is the downlink maximum average bias received power. We also set the same simulation parameters (e.g., BS density ratio, BS transmit power, BS cell extension bias factor) in the downlink of Chapter 5 and the uplink of Chapter 6 in order to demonstrate clear difference between them.
7.3.1 Coverage Probability Definition

The downlink and the uplink research is carried out based on K-tier BS system. Figure 7.1 shows an example of coverage probability definition.

7.3.1.1 Downlink

The downlink coverage probability is defined as the average coverage probability over all UEs in the downlink. Firstly, find the associated BS for each UE. Secondly, calculate
the received signal from the associated BS and the total interference power from all other interfering BSs. Thirdly, calculate the downlink SINR for each UE. Finally, the downlink coverage probability is obtained by the complementary cumulative distribution (CCDF) of all calculated SINR. In Figure 7.1 (a), it is an example of the downlink. There are one macro BS (in green) and three small BSs (in black, blue and red). UEs are represented as points and are labeled by the color that is the same as the associated BS. A downlink SINR is calculated in Figure 7.1 (a), in which a black point user is called as the typical UE. The received signal power of the typical UE comes from the associated BS in red and the interference power of the typical UE experiencing comes from the other two small BSs and the macro BS. All UEs will do the same calculation of SINR. The downlink coverage probability is the average coverage probability over all UEs from the BSs in the downlink.

### 7.3.1.2 Uplink

The uplink coverage probability is defined as the average coverage probability over all BSs in the uplink. Firstly, find the active UE for each BS and there is only one active UE in each cell in a given resource block since LTE of orthogonal multiple access technique is assumed. Secondly, calculate the received signal from the active UE and total interference power from all other active UEs. Thirdly, calculate the uplink SINR for each BS. Finally, the uplink coverage probability is obtained by the complementary cumulative distribution (CCDF) of all calculated SINR. In Figure 7.1 (b), it is an example of the uplink. In each cell, an active UE in maroon is chosen randomly. An uplink SINR is calculated in Figure 7.1 (b), in which there is a typical UE. The received signal power of the associated BS comes from the typical UE and the interference power comes from the other two interfering active UEs. All BSs will do the same calculation of SINR. Hence, the uplink coverage probability is the average coverage probability over all BSs from the active UEs in uplink.

### 7.3.1.3 Difference

The non-uniform UEs are from our designed non-uniform UE model with specific parameters. All UEs are used to calculate the downlink coverage probability while only active UEs are used to calculate the uplink coverage probability.

### 7.3.2 Assumptions

The assumptions of the performance analysis are summarized as follows.
7.3.2.1 Downlink

- The channel is Rayleigh fading.

7.3.2.2 Uplink

- The channel is Rayleigh fading.
- In each cell, there is one active UE.
- An active UE associated with $BS_i^j$ comes from the UEs in the cluster around $SA_i^j$ and other active UEs associated with $BS_i^j$ in the cluster around other SA rather than $SA_i^j$ are not considered.
- The base station offset is less than a threshold.
- Active UEs in each tier form a homogeneous and independent PPP.

7.3.2.3 Difference

There are more assumptions in uplink since the positions of UEs are totally random and the uplink power control makes the uplink transmit power of UEs to fluctuate.

7.3.3 Coverage Probability Derivation

7.3.3.1 Downlink

Let $m = 1, 2, \ldots k$ be the tier of interest. The procedure to get the downlink coverage probability $P_{dl,c}$ is as follows:

- Find the probability density function (PDF) $f_L^{(m)}(l)$ and complementary cumulative distribution function (CCDF) $\bar{F}_L^{(m)}(l) = P^{(m)}(L > l)$ of random variable $L$. Let $L$ be the distance between a UE and the BS that the UE belongs to in the $m$th tier.
- Find the PDF $f_V^{(m)}(v)$ and CCDF $\bar{F}_V^{(m)}(v) = P^{(m)}(V > v)$ of random variable $V$. Let $V$ be the distance between a UE and the closest BS in the $m$th tier.
- The UE of interest (e.g. block point UE in Figure 5.1) in a given $m$th tier can be associated with a possible BS into two cases.
  Case1: associated with the BS to which the UE of interest belongs.
- Derive $P^{(m)}(A_{case1})$, which is the probability that a UE of interest is associated with the BS to which the UE belongs. In Figure 5.1, it means that the black point UE is associated with $BS_1$.
- Derive the downlink coverage probability $P^{(m)}[\text{SINR}_m(l) > T_m|A_{case1}]$ conditioned on the black point UE associated with $BS_1$.
- Derive the downlink coverage probability $P^{(m)}_{dl,c,case1} = P^{(m)}(A_{case1}) * P^{(m)}[\text{SINR}_m(l) > T_m|A_{case1}]$ that the black point UE is associated with $BS_1$ to which this UE belongs.

Case 2: associated with a BS in all tiers except the BS to which UE belongs (e.g. except $BS_1$)
- Derive $P^{(m)}(A_{k,case2})$, which is the probability that a UE is associated with a $k$th tier BS.
- Derive the downlink coverage probability $P^{(m)}[\text{SINR}_k(v) > T_k|A_{k,case2}]$ conditioned on a UE associated with a $k$th tier BS.
- Derive the downlink coverage probability $P^{(m)}_{dl,ck,case2} = P^{(m)}(A_{k,case2}) * P^{(m)}[\text{SINR}_k(v) > T_k|A_{k,case2}]$ of associating with a $k$th tier BS.

Summary
- $P^{(m)}(A_{case1})$ and $P^{(m)}(A_{k,case2})$ is the probability that a UE of interest in the $m$th tier is associated with the BS to which UE belongs and a $k$th tier BS. $P^{(m)}(A_{case1}) + \sum_{k=1}^{K} P^{(m)}(A_{k,case2}) = 1$. $K$ is the total number of BS tiers.
- The downlink coverage probability for a UE distributed in the $m$th tier is $P^{(m)}_{dl,c} = P^{(m)}_{dl,c,case1} + \sum_{k=1}^{K} P^{(m)}_{dl,ck,case2}$.

7.3.3.2 Uplink

Let $m = 1, 2, ..., k$ be the tier of interest. The procedure to get the uplink coverage probability $P_{ul,c}$ is as follows:

- Find the probability density function (PDF) $f^{(m)}_L(l)$ and complementary cumulative distribution function (CCDF) $\bar{F}^{(m)}_L(l) = P^{(m)}(L > l)$ of random variable $L$. Let $L$ be the distance between an active UE and its associated BS that the active UE belongs to in the $m$th tier.
• The BS of interest (e.g. $BS_1^1$ in Figure 6.4) in a given $m$th tier can be associated with an active UE in two cases.

Case 1: an active UE that belongs to the BS of interest.

- Derive $P^{(m)}(A_{\text{case 1}})$, which is the probability that the active UE is associated with the BS of interest that the active UE belongs to in the $m$th tier.
- Derive the uplink coverage probability $P^{(m)}[\text{SINR}_m(l) > T_m|A_{\text{case 1}}]$ conditioned on the active UE associated with $BS_1^1$.
- Derive the uplink coverage probability $P_{ul,c,\text{case 1}}^{(m)} = P^{(m)}(A_{\text{case 1}}) \cdot P^{(m)}[\text{SINR}_m(l) > T_m|A_{\text{case 1}}]$ that the active UE is associated with $BS_1^1$.

Case 2: an active UE that belongs to the other BS rather than the BS of interest.

- Ignored since the simplifying assumption, which is the active UE belongs to the BS of interest.

Summary

- $P^{(m)}(A_{\text{case 1}})$ is the probability that the active UE that belongs the BS of interest is associated with the BS of interest.
- $P_{ul,c,\text{case 1}}^{(m)}$ is the coverage probability associated with the BS of interest.
- The uplink coverage probability for a BS of interest distributed in a $m$th tier with the simplifying assumption that the active UE belongs to the BS of interest is $P_{ul,c}^{(m)} = \frac{P_{ul,c,\text{case 1}}^{(m)}}{P^{(m)}(A_{\text{case 1}})}$.

7.3.3.3 Difference

A UE has associated with a BS into two cases. The first case is that the UE is associated with the BS that the UE belongs to (case 1) and the second case is that the UE is associated with other BSs (case 2). For downlink analysis, both case 1 and case 2 are considered. For uplink analysis, only case 1 is considered due to the simplifying assumptions.

7.3.4 Simulation Results

For the downlink and the uplink analysis, the simulation results and the theoretical results are based on the same simulation parameters.
7.3.4.1 Downlink

In the downlink, the simulation results and the theoretical results match closely, which proves that our analysis is accurate. For effect of the base station offset, the low base station offset has the high downlink coverage probability while the high base station offset has the low downlink coverage probability and converges to the uniform case.

7.3.4.2 Uplink

The simulation results and the theoretical results do not match closely because of the assumptions made in the uplink performance analysis. However, the gap is less than 1 dB, which proves that our assumptions are acceptable if the system requirement is not very strict. For effect of the base station offset, we can observe that there is no obvious difference between the low base station offset. And the coverage probability converges to the uniform case when the base station offset arrives a threshold in uplink. If we still continue to increase the base station offset, such as $D = 100$ m in our simulation setting, the uplink coverage probability would be worse than the uniform case. Therefore, we have an assumption that the base station offset is less than a threshold.

7.3.4.3 Difference

The theoretical analysis in the downlink is the accurate performance and the theoretical analysis in uplink is the estimated performance. The downlink and the uplink analysis have different effect from the base station offset since all UEs are calculated in the downlink and only an active UE in one cell is calculated in the uplink.

In the following, we indicate the comparison of the downlink and the uplink performance.

- Comparison with the reference literature [69]

Based on our theoretical analysis, Figure 7.2 gives the performance of the downlink and the uplink, which has the same simulation setting as [69] in a single tier system. In Figure 7.2, the UE distribution converges to be uniform and the coverage probability based on the analysis in this thesis matches with the results in [69]. It confirms that our simulation method and theoretical analysis are solid and we can extend our research to multi-tier system.
The interference in the downlink comes from all interfering BSs and the interference in uplink comes from all interfering active UEs. From Figure 7.2, we can observe that the coverage probability of the downlink is higher than that of the uplink.

Another observation from Figure 7.2 is that the effect of the fractional power control factor $\epsilon$. Low $\epsilon$ has the high coverage probability, which aligns with the results of Subsection 6.5.3.4.

![Figure 7.2: Comparison of the coverage probability of the downlink and the uplink in a single tier system (compared with the reference literature)](image)

- **Comparison between downlink and uplink**

Figure 7.3 gives the comparison between downlink and uplink. The simulation results are collected from Figure 5.4 and Figure 6.6. The coverage probability of the uplink is better than that of the downlink. This conclusion is inconsistent with the comparison with the reference literature of single tier in the above subsection. It shows that the relationship between the downlink and the uplink performance is not fixed (e.g., the downlink is always better or the uplink is always better). It is because the downlink coverage is calculated by the receiving signal from the associated BS and the receiving interference from all other BSs on the view of a typical user in the downlink and the uplink coverage is calculated by the receiving signal from the associated active user.
and the receiving interference from all other active users on the view of a BS in the uplink.

![Image](image.png)

Figure 7.3: Comparison of the coverage probability of the downlink and the uplink in a 2-tier system

### 7.4 Effect of System Parameters

For the downlink and the uplink analysis, the effect of system parameters is based on the same simulation parameters.

#### 7.4.1 Effect of BS Density Ratio between Small Cells and Macro Cells

The summary of the effect of the BS ratio between small cells and macro cells in small cells is in the following table.
### Summary

<table>
<thead>
<tr>
<th>Summary</th>
<th>Effect of $\frac{\lambda_2}{\lambda_1}$</th>
<th>Effect of Path Loss Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Downlink</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
| Low attraction | High path loss: ↑  
Same path loss: ↑  
Low path loss: ↑ | High path loss exponent has high performance. |
| Medium attraction | High path loss: ↑  
Same path loss: ↑→↓  
Low path loss: ↑→↓ | Low path loss exponent has high performance at low BS density ratio and high path loss exponent has high performance at high BS density ratio. |
| High attraction | High path loss: ↑→↓  
Same path loss: ↑→↓  
Low path loss: ↑→↓ | Same as medium attraction case. |
| **Uplink** | | |
| Low attraction | High path loss: ↓  
Same path loss: ↓  
Low path loss: ↓ | High path loss exponent has high performance. |
| Medium attraction | High path loss: ↓  
Same path loss: ↓  
Low path loss: ↓ | High path loss exponent has high performance. |
| High attraction | High path loss: ↓  
Same path loss: ↓  
Low path loss: ↓ | High path loss exponent has high performance. |

Note:

↑ means that the coverage probability increases with the increase of BS density ratio $\frac{\lambda_2}{\lambda_1}$.

↓ means that the coverage probability decreases with the increase of the BS density ratio $\frac{\lambda_2}{\lambda_1}$.

↑→↓ means that the coverage probability increases, and then decreases with the increase of BS density ratio $\frac{\lambda_2}{\lambda_1}$ after a turning point.

### 7.4.2 Effect of Cell Extension of Small Cells

The summary of the effect of BS cell extension bias factor $C_2$ in small cells is in the following table.
CHAPTER 7. COMPARISON BETWEEN DOWNLINK AND UPLINK 131

Summary

<table>
<thead>
<tr>
<th>Effect of $C_2$</th>
<th>Effect of Path Loss Exponent</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Downlink</strong></td>
<td></td>
</tr>
<tr>
<td>Low attraction</td>
<td>High path loss: ↓</td>
</tr>
<tr>
<td></td>
<td>Same path loss: ↓</td>
</tr>
<tr>
<td></td>
<td>Low path loss: ↓</td>
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</tr>
<tr>
<td></td>
<td>Same path loss: ↓</td>
</tr>
<tr>
<td></td>
<td>Low path loss: ↓</td>
</tr>
<tr>
<td>High attraction</td>
<td>High path loss: ~</td>
</tr>
<tr>
<td></td>
<td>Same path loss: ~</td>
</tr>
<tr>
<td></td>
<td>Low path loss: ~</td>
</tr>
</tbody>
</table>

| **Uplink**      |                             |
| Low attraction  | High path loss: ↓            |
|                 | Same path loss: ↓            |
|                 | Low path loss: ~             |
| Medium attraction | High path loss: ~        |
|                 | Same path loss: ~            |
|                 | Low path loss: ~             |
| High attraction  | High path loss: ~           |
|                 | Same path loss: ~            |
|                 | Low path loss: ~             |

Note:
↓ means that the coverage probability decreases with the increase of BS cell extension bias factor $C_2$.
∼ means that the coverage probability does not vary with the increase of BS cell extension bias factor $C_2$.

7.4.3 Effect of Transmit Power of Small Cells

The summary of the effect of BS transmit power $P_2$ in small cells is in the following table.
CHAPTER 7. COMPARISON BETWEEN DOWNLINK AND UPLINK

<table>
<thead>
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<th>Summary</th>
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</tr>
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<td>High path loss exponent has high performance.</td>
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<tr>
<td></td>
<td>Same path loss: ↑</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ~</td>
<td></td>
</tr>
<tr>
<td>Medium attraction</td>
<td>High path loss: ↑</td>
<td>Low path loss exponent has high performance.</td>
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<td>Same path loss: ↑</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ↑</td>
<td></td>
</tr>
<tr>
<td>High attraction</td>
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<td>Low path loss exponent has high performance.</td>
</tr>
<tr>
<td></td>
<td>Same path loss: ↑</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ↑</td>
<td></td>
</tr>
<tr>
<td><strong>Uplink</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Low attraction</td>
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<td>High path loss exponent has high performance.</td>
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</tr>
<tr>
<td></td>
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<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ~</td>
<td></td>
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<tr>
<td></td>
<td>Same path loss: ~</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Low path loss: ~</td>
<td></td>
</tr>
</tbody>
</table>

Note:

↑ means that the coverage probability increases with the increase of $P_2$.

↓ means that the coverage probability decreases with the increase of $P_2$.

~ means that the coverage probability does not vary with the increase of $P_2$.

7.5 Conclusions

This chapter summarized the difference between the downlink and the uplink analysis. The same non-uniform UE model is used in the research of the downlink and the uplink analysis. For the downlink, SINR of each UE in the cluster from its associated BS and all other interfering BSs in the downlink is calculated and the average downlink coverage probability is obtained by the CCDF of all calculated downlink SINRs. For the uplink, SINR of each BS from the associated active UE and all other interfering active UEs in the
uplink is calculated and the average uplink coverage probability is obtained by the CCDF of all calculated uplink SINRs. So the downlink analysis is dependent on all UEs in the cluster while the uplink analysis is only dependent on the active UEs.

The comparison between the downlink and the uplink is also given, such as the coverage definition, assumptions and coverage probability derivation. The simulation results are shown to confirm the relevant analysis. The downlink analysis is the accurate analysis while the uplink analysis is the approximated analysis due to the simplifying assumptions. Finally, the effect of BS system parameters is listed. The effect in the downlink and the uplink is quite different.
Chapter 8

Conclusions and Future Work

8.1 Conclusions

5G networks demand high data capacity. Small cell deployment is a great solution to this demand. Conventional networks combined with small cells, such as pico cells, femto cells consist of HetNets. Distribution models of BS and UE are significant for system design and evaluation. In the literature, the non-uniform UE distribution model is very limited. This thesis designed a comprehensive framework of the non-uniform UE distribution to have a tractable downlink and uplink analysis in HetNets. It shown that deploying small BSs at the UE cluster centers can improve the coverage probability. Physical constraints let small BSs deployment challenging such that small BSs are placed at the position with a base station offset to the UE cluster center. To address the existing problems, in this thesis: (1) A new non-uniform UE model based on K-tier BS HetNets was proposed. (2) The downlink and uplink coverage probability was analyzed.

We proposed a new non-uniform UE distribution model based on the existing K-tier BS distribution. The UE model is PCP, in which the UEs are symmetrically distributed around SAs that have a base station offset with their BSs. The UE distribution model with the combination of the attraction level and the dependency level covers most of the practical scenarios. The heterogeneity of the non-uniform UE model based on coefficient of variant (CoV) was illustrated. Voronoi cell area was chosen as the CoV statistic measurement confirmed by the simulation results. CoV in this thesis is the standard deviation of Voronoi cell area divided by the mean of Voronoi cell area. Normalized CoV can represent the UE pattern is sub-Poisson, Poisson or super-Poisson. As the path loss exponent is a good indication of the large scale fading, normalized CoV can be regarded as a good indication
of user’s spatial heterogeneity. The relationship between the normalized CoV and the UE model’s parameters was discussed. The relationship between the coverage probability and the UE model’s parameter was also discussed.

This thesis derived the theoretical analysis of the coverage probability for the non-uniform UE distribution model in the downlink and the uplink. The researchers can rely on equations we derived instead of time-consuming simulations to know the performance based on the non-uniform UE distribution.

In the downlink, SINR of all UEs in a cluster is used to calculate or derive the coverage probability. The UE of interest is located inside a UE cluster, which has the base station offset with a BS. This UE is called to belong to this BS. The BS that the UE of interest is associated with can be described into two cases. Case1 is that the UE is associated with the BS that the UE of interest belongs to and case2 is that the UE is associated with other BS. The key part of the performance derivation is the characterization of the distance between the UE of interest and the BS that the UE of interest belongs to and the distance between the UE of interest and the closest BS in different tier. The simulation results were implemented to show the closely matching with our theoretical analysis. The effect of base station offset was also given. Low base station offset will have high coverage probability, vice versa. When the base station offset is approaching a threshold, the performance converges to the uniform case, which means that deploying a large base station offset will not bring any benefit. Hence, all research in this thesis focus on a reasonable base station offset less than the threshold, which still brings benefits compared with the uniform case. Furthermore, the effect of BS system parameters was demonstrated (e.g., the BS density ratio between small cells and macro cells, the BS cell extension bias factor of small cells and the BS transmit power of small BSs). The conclusions from our research of the non-uniform UE distribution were quite different compared with the uniform cases that had introduced in the literature. For example, [20] had the conclusion about the uniform case that "more BSs can be added in any tier without affecting the coverage and hence the net network capacity can be increased linearly with the number of BSs" when SINR threshold is the same in all tiers. But the conclusion was different when the non-uniform user distribution was introduced, which had the conclusion in our thesis that the coverage probability increased, then decreased after a turning point with the increase of the BS density ratio between small cells and macro cells. Hence, this turning point is an optimum deployed density ratio.

In the uplink, SINR of an active UE randomly chosen in one cell was used to calculate or derive the coverage probability since we assume orthogonal multiple access technique.
Unlike the downlink analysis, all interference power comes from BSs with fixed position, such as PPP, all interference power in the uplink comes from UEs with mobile position so that it is challenging to get the characterization of all uplink interference power. Another reason is the uplink power control, which causes the UE transmit power to fluctuate. Therefore, this thesis makes some assumptions in order to get a tractable analysis or closed-form equations. The assumptions made in this thesis were that (1) there is one active UE in each cell, (2) an active UE is assumed in the cluster that has a base station offset with the BS of interest, (3) the base station offset is less than a threshold and (4) the active UEs in each tier form a homogeneous and independent PPP. Because of the above assumptions, the simulation results do not closely match with the analysis results. However, the gap was less than 1 dB, which was acceptable if the system requirement was not tight. The effect of base station offset was also presented. It can be observed that there was no obvious difference when the base station offset $D$ was less than Matern cluster radius $R$. And the coverage probability converged to the uniform case at the high base station offset. The effect of BS system parameters was simulated, too. For example, the coverage probability always decreased with the increase of the density ratio so that we cannot deploy small cells as much as possible if considering the uplink performance.

Finally, the comparison of the coverage probability definition, the assumptions, the coverage probability derivation and simulation results between the uplink and the downlink was summarized. In addition, the comparison of the effect of BS system parameters was also discussed.

This thesis filled the gap of the non-uniform UE distribution. The combination of the attraction level and the dependency level can cover most of non-uniform UE scenarios. The target customers of this non-uniform UE model are researchers or operators who design and evaluate HetNets. Once the specific non-uniform UE distribution was defined, the downlink or the uplink coverage probability would be obtained by our theoretical analysis equations derived in this thesis. The distribution of BSs is assumed as PPP in this thesis, which has the possibility that the distance between two BSs is zero. We should consider more practical scenarios of BS distribution in future work. However, our research of the non-uniform UE model and its downlink/uplink performance analysis in this thesis still can be regarded as the baseline of the research of the non-uniform UE distribution.

8.2 Future Work

- **Other metrics and mobility**: In this thesis, we derived the downlink and the
uplink coverage probability for the spatial UE model using the tools of stochastic geometry. In future work, other performance metrics rather than the coverage probability should be researched, such as the ergodic rate, the effective rate, the symbol error rate (SER). At the same time, the UEs may be mobile so we should research on spatiotemporal model using stochastic geometry and queuing theory.

- **General channel model:** All theoretical results and simulations in this thesis assumed that the fading type was Rayleigh. In the future, more general fading should be considered, such as Nakagami-m fading as in [104]. Nakagami-m is accurate and flexible to match a real scenario compared with Rayleigh, Rician, etc. Rayleigh fading, one-sided Gaussian and Rician distribution are included in Nakagami-m. When $m = 1$, it is Rayleigh fading. When $m = \frac{1}{2}$, it is one-sided Gaussian fading. When $m > 1$, it is equivalent to Rician fading. Beyond Nakagami-m fading, we should continue to research the UE behavior based on more general fading [44,105].

- **General BS spatial distribution:** In this thesis, we carried out the research about the UE spatial distribution model and remain the BS assumption as a K-tier distribution. In further research, we should study non-uniform UE over more general BS spatial distribution, such as determinantal process, Matern process, PCP, and PHP. At the same time, three dimensional (3D) BSs should also been considered. Most of research about BS spatial distribution is based on 2D. In practice, BSs can not be placed directly on the ground and they should have height. Moreover, the height of BSs may be variable, which results in more complicated analytical derivation. In the literature, there is some research based on drone or unmanned aerial vehicle (UVA) [106–109], in which BSs have variable height.

- **Multi-input multi-output (MIMO) and millimeter Wave (mmWave):** In 5G, another two key features are MIMO and mmWave. Combining these two features into the UE spatial model is very practical. For MIMO, the channel power gain from associated BSs and interfering BSs is quite different, which has been proven in [110–113] so that we need to pay attention to this new phenomenon when designing and analyzing the UE models. For mmWave, the operating frequency is normally above 30 GHz, which leads the blocking of transmit wave. The popular modeling of link is to separate modeling into Line-of-Sight (LOS) and non-LOS (NLOS). The wireless link is a combination of LOS and NLOS with their own probability. How to shape LOS and NLOS link [114,115] is what we should focus on research of the UE spatial behavior under mmWave.
Appendix
Appendix A

Downlink Coverage Performance Analysis

A.1 Proof of Corollary 5-1

When $D_m \geq R_m$,

- $l \leq D_m - R_m$

![Figure A.1: PDF and CCDF when $D_m \geq R_m$ and $l \leq D_m - R_m$](image)

From Figure A.1, the random distance $L$ of all users in black disc to $BS_m$ is more than $l$. So CCDF $F_L^{(m)}(l) = P^{(m)}(L > l) = 1$. PDF $f_L^{(m)}(l) = 0$.

- $D_m - R_m < l \leq \sqrt{D_m^2 + R_m^2}$
Then, $\theta_1 = \arctan \frac{y_0}{x_0}$, $\theta_2 = \arctan \frac{y_0}{D_m-x_0}$.

So CCDF $\bar{F}_L^{(m)}(l) = \mathbb{P}(m)(L > l) = 1 - \frac{\theta_1 R_m^2 + \theta_2 l^2 - D_m y_0}{\pi R_m^2}$ in which $\theta_1 R_m^2 + \theta_2 l^2 - D_m y_0$ is the joint area between the black circle and the red circle. PDF $f_L^{(m)}(l) = \frac{2\theta_2 l}{\pi R_m^2}$.

- $\sqrt{D_m^2 + R_m^2} < l < D_m + R_m$

Then, $\theta_1 = \arctan \frac{y_0}{x_0} + \pi$, $\theta_2 = \arctan \frac{y_0}{D_m-x_0}$.

So CCDF $\bar{F}_L^{(m)}(l) = \mathbb{P}(m)(L > l) = 1 - \frac{\theta_1 R_m^2 + \theta_2 l^2 - D_m y_0}{\pi R_m^2}$. PDF $f_L^{(m)}(l) = \frac{2\theta_2 l}{\pi R_m^2}$.

- $D_m + R_m \leq l$
APPENDIX A. DOWNLINK COVERAGE PERFORMANCE ANALYSIS

Figure A.4: PDF and CCDF when $D_m \geq R_m$ and $D_m + R_m \leq l$

So CCDF $\bar{F}_L^{(m)}(l) = \mathbb{P}^{(m)}(L > l) = 0$. PDF $f_L^{(m)}(l) = 0$.

When $D_m < R_m$,

- $l \leq R_m - D_m$

Figure A.5: PDF and CCDF when $D_m < R_m$ and $l \leq R_m - D_m$

So CCDF $\bar{F}_L^{(m)}(l) = \mathbb{P}^{(m)}(L > l) = 1 - \frac{l^2}{R_m^2}$. PDF $f_L^{(m)}(l) = \frac{2l}{R_m^2}$. 
• \( R_m - D_m < l \leq \sqrt{R^2 - D_m^2} \)

\[\begin{align*}
\text{Figure A.6: PDF and CCDF when } D_m &< R_m \text{ and } R_m - D_m < l \leq \sqrt{R_m^2 - D_m^2} \\
\text{Then, } \theta_1 = \arctan \frac{y_0}{x_0}, \quad \theta_2 = \arctan \frac{y_0}{D_m - x_0} + \pi.
\end{align*}\]

So CCDF \( \bar{F}_L^{(m)}(l) = \mathbb{P}(L > l) = 1 - \frac{\theta_1 R_m^2 + \theta_2 l^2 - D_m y_0}{\pi R_m^2} \). PDF \( f_L^{(m)}(l) = \frac{2\theta_2 l}{\pi R_m^2} \).

• \( \sqrt{R_m^2 - D_m^2} < l \leq \sqrt{R_m^2 + D_m^2} \)

\[\begin{align*}
\text{Figure A.7: PDF and CCDF when } D_m &< R_m \text{ and } \sqrt{R_m^2 - D_m^2} < l \leq \sqrt{R_m^2 + D_m^2} \\
\text{Then, } \theta_1 = \arctan \frac{y_0}{x_0}, \quad \theta_2 = \arctan \frac{y_0}{D_m - x_0}.
\end{align*}\]

So CCDF \( \bar{F}_L^{(m)}(l) = \mathbb{P}(m)(L > l) = 1 - \frac{\theta_1 R_m^2 + \theta_2 l^2 - D_m y_0}{\pi R_m^2} \). PDF \( f_L^{(m)}(l) = \frac{2\theta_2 l}{\pi R_m^2} \).

• \( \sqrt{R^2 + D_m^2} < l < R_m + D_m \)
Then, $\theta_1 = \arctan \frac{y_0}{x_0} + \pi$, $\theta_2 = \arctan \frac{y_0}{D_m - x_0}$.

So $\text{CCDF} \ F_L^{(m)}(l) = P(L > l) = 1 - \frac{\theta_1 R_m^2 + \theta_2 l^2 - D_m y_0}{\pi R_m^2}$. PDF $f_L^{(m)}(l) = \frac{2\theta_2 l}{\pi R_m^2}$.

- $R_m + D_m \leq l$

So $\text{CCDF} \ F_L^{(m)}(l) = P(m)(L > l) = 0$. PDF $f_L^{(m)}(l) = 0$.

When $D_m = 0$,

- $l \geq R_m$
APPENDIX A. DOWNLINK COVERAGE PERFORMANCE ANALYSIS

Figure A.10: PDF and CCDF when $D_m = 0$ and $l \geq R_m$

So CCDF $\bar{F}_L^{(m)}(l) = \mathbb{P}^{(m)}(L > l) = 0$. PDF $f_L^{(m)}(l) = 0$.

• $l < R_m$

Figure A.11: PDF and CCDF when $D_m = 0$ and $l < R_m$

So CCDF $\bar{F}_L^{(m)}(l) = \mathbb{P}^{(m)}(L > l) = 1 - \frac{l^2}{R_m^2}$. PDF $f_L^{(m)}(l) = \frac{2l}{R_m^2}$.

A.2 Proof of Lemma 5-2

The probability that a UE of interest in the $m$th tier is associated with the BS that the UE belongs to is (conditioned on the distance $l$ between the UE of interest and the BS that the UE belongs to)
\[ \mathbb{P}^{(m)}(A_{case1}) = \mathbb{P}[P_{RP_m} > \text{any } P_{RP_i}] = \prod_{i=1}^{K} \mathbb{P}[P_{RP_m} > P_{RP_i}], \]
\[ = \prod_{i=1}^{K} \mathbb{P}[P_{mC_ml^{-\alpha_m}} > P_iC_iV^{-\alpha_i}], \]
\[ = \prod_{i=1}^{K} \mathbb{P}[V_i > (\frac{P_iC_i}{P_mC_m})^\frac{1}{\alpha_i} \cdot l^{\frac{\alpha_m}{\alpha_i}}], \]
\[ \overset{(a)}{=} \prod_{i=1}^{K} \exp(-\lambda_i \pi (\frac{P_iC_i}{P_mC_m})^\frac{2}{\alpha_m} \cdot l^2), \]

where for step (a), the BS spatial distribution is PPP in the \( i \)th tier and it is from Equation 5.5 (CCDF of \( V \) in the \( i \)th tier).

### A.3 Proof of Lemma 5-3

The coverage probability associated with the BS that a UE of interest belongs to is (conditioned on the distance \( l \) between the UE of interest and the BS that the UE belongs to)

\[ \mathbb{P}^{(m)}[SINR_m(l) > T_m | A_{case1}] = \mathbb{P}^{(m)}[\frac{P_mG_ml^{-\alpha_m}}{I_r + \frac{\sigma^2}{L_0}} > T_m], \]
\[ = \mathbb{P}^{(m)}[G_m > (I_r + \frac{\sigma^2}{L_0})T_mP_m^{-1}l^{\alpha_m}], \]
\[ \overset{(a)}{=} \mathbb{E}_{I_r}[\exp(-\frac{\sigma^2}{L_0}T_mP_m^{-1}l^{\alpha_m})], \]
\[ = \exp(-\frac{\sigma^2}{L_0}T_mP_m^{-1}l^{\alpha_m}) \cdot \mathbb{E}_{I_r}[\exp(-I_rT_mP_m^{-1}l^{\alpha_m})], \]
\[ \overset{(b)}{=} \exp(-\frac{\sigma^2}{L_0}T_mP_m^{-1}l^{\alpha_m}) \cdot \prod_{i=1}^{K} \mathcal{L}_{I_i}(T_mP_m^{-1}l^{\alpha_m}), \]
where (a) is from the assumption that the channel fading envelope $\sqrt{G_m}$ is Rayleigh distribution and the channel fading power $G_m$ is exponential distribution. (b) gives the Laplace transform $L_x(s)$ of the total interference.

A.4 Proof of Lemma 5-4

The probability that a UE of interest is associated with a BS in the $k$th tier rather than the BS to which the UE belongs is (conditioned on the distance $v$ between the UE of interest and the closest BS in $k$th tier)

$$P_{|v|}^{(m)}(A_{k,\text{case2}})$$

$$= P_{|v|}(P_{RP_k} > \text{any } P_{RP}),$$

$$= P_{|v|}(P_{RP_k} > P_{RP_m}) \cdot \prod_{i=1,i \neq k}^{K} P_{|v|}(P_{RP_k} > P_{RP_i}),$$

$$= P_{|v|}(P_k C_k v^{-\alpha_k} > P_m C_m L^{-\alpha_m}) \cdot \prod_{i=1,i \neq k}^{K} P_{|v|}(P_k C_k v^{-\alpha_k} > P_i C_i V_i^{-\alpha_i}),$$

$$= P_{|v|}(L > \left( \frac{P_m C_m}{P_k C_k} \right)^{\frac{1}{\alpha_m}} \cdot \frac{\alpha_k}{v^{\alpha_m}}) \cdot \prod_{i=1,i \neq k}^{K} \bar{F}_V^{(i)}(\left( \frac{P_i C_i}{P_k C_k} \right)^{\frac{1}{\alpha_i}} \cdot \frac{\alpha_k}{v^{\alpha_i}}),$$

$$= \bar{F}_L^{(m)}(\left( \frac{P_m C_m}{P_k C_k} \right)^{\frac{1}{\alpha_m}} \cdot \frac{\alpha_k}{v^{\alpha_m}}) \cdot \prod_{i=1,i \neq k}^{K} \exp(-\lambda_i \pi \left( \frac{P_i C_i}{P_k C_k} \right)^{\frac{2}{\alpha_i}} \cdot \frac{2\alpha_k}{v^{\alpha_i}}),$$

where step (a) is from the distance between the UE of interest and the closest BS in the $i$th tier being $V_i$ and the distance between the UE of interest and the BS to which the UE of interest belongs being $L$ in the $m$th tier. For step (b), $\bar{F}_L^{(m)}(l) = P[L > l]$ is complementary cumulative distribution function (CCDF) of random variable $L$ in the $m$th tier from 5.2.1.2. $F_V^{(i)}(v) = P[V_i > v]$ is CCDF of of random variable $V_i$ in the $i$th tier from 5.2.1.1.
Appendix B

Uplink Coverage Performance Analysis

B.1 Proof of Lemma 6-4

The coverage probability of the BS of interest conditioned on the distance between the active UE and the BS of interest as \( l \) is

\[
\mathbb{P}_{|l}^{(m)}[\text{SINR}_m(l) > T_m | \text{Case} 1]
= \mathbb{P}_{|l}^{(m)}\left[\frac{I_m^{\alpha_m(\epsilon-1)} G_m}{I_r + \frac{\sigma^2}{L_0}} > T_m\right],
= \mathbb{P}_{|l}^{(m)}[G_m > (I_r + \frac{\sigma^2}{L_0}) T_m l^{\alpha_m(1-\epsilon)}],
\overset{(a)}{=} \mathbb{E}_{I_r}\left[\exp\left(-\left(I_r + \frac{\sigma^2}{L_0}\right) T_m l^{\alpha_m(1-\epsilon)}\right)\right],
= \exp\left(-\frac{\sigma^2}{L_0} T_m l^{\alpha_m(1-\epsilon)}\right) \cdot \mathbb{E}_{I_r}\left[\exp\left(-I_r T_m l^{\alpha_m(1-\epsilon)}\right)\right],
\overset{(b)}{=} \exp\left(-\frac{\sigma^2}{L_0} T_m l^{\alpha_m(1-\epsilon)}\right) \cdot \prod_{i=1}^{S} \mathcal{L}_{I_{r_i}}(T_m l^{\alpha_m(1-\epsilon)}),
\]

where (a) is from the assumption that the channel fading envelope \( \sqrt{G_m} \) is Rayleigh fading. (b) gives the Laplace transform \( \mathcal{L}_x(s) \). \( L_0 \) is the path loss at a reference distance of 1 m. \( S \) is the total number of tiers distributed with our designed non-uniform UEs.
Appendix C

Simulation Setting

This chapter gives the simulation setting when evaluating the non-uniform distribution.

C.1 Downlink Coverage Probability

In this section, the simulation setting of the downlink coverage probability is given.

- **Step1: Generate a BS pattern**
  In a given square (such as a square of 1 km by 1 km or a square of 2 km by 2 km), generate a BS pattern or a BS realization of K-tier. BSs in each tier have their own density (in the $i$th tier, the BS density is $\lambda_i$). The distribution of BSs in each tier is an independent homogeneous PPP. An example of BS pattern is shown in Figure C.1. The procedure to generate a BS pattern in the $i$th tier is:
  
  - The area of the given square is $A_{si}$. The mean number of distributed BSs in the given square of the $i$th tier is $A_{si} \cdot \lambda_i$.
  
  - Generate a random number of Poisson distribution with the mean of $A_{si} \cdot \lambda_i$. Matlab code is `poissrnd(A_{si} \cdot \lambda_i)`. The generated random number is the real number of BSs of the given square of the $i$th tier in this BS pattern.

  - Generate the distributed BSs in the given square based on the above real number of BSs. The distribution of BSs in the given square is uniform. Matlab code to generate a horizontal or vertical coordinates is `random('unif',0,Side length of the square,1,real number of BS)`.
Step 2: Generate a UE pattern

The UE model is that the UE concentrates around the social attractor (SA) and SA has a base station offset to its BS. The procedure to generate UE pattern of the $i$th tier is:

- For each BS in the $i$th tier, generate a SA which is uniformly distributed around a BS of the radius $D_i$.
- Generate clustered UEs with the center being the above generated SA.
- The cluster UEs are uniformly distributed within a circular disc with radius $R_i$ which is called as Matern cluster.

Step 3: Calculation of SINR

Based on BS pattern and UE pattern generated above, calculate instantaneous SINR for each UE. We assume the distance between the UE and its associated BS as $d$ and the distance between the UE and the $j$th interfering BS $BS_{ij}$ in the $i$th tier as $d_{ij}$. The path loss exponent in the $i$th tier is $\alpha_i$. The transmit power of the $i$th tier BS is $P_i$. The random amplitude fading type is Rayleigh fading with unity mean power gain which means the power is exponential distribution with unity.

- Generate random exponential power gain $G$ for each UE with Matlab code:
  \[ \text{random('exp',1)}. \]
APPENDIX C. SIMULATION SETTING

- Received signal power from associated BS $B_0$ is $P_{d}^{-\alpha}G$. The received signal power considers the large scale fading which is dependent on distance $d^{-\alpha}$ and small scale fading which is the random variable $G$. Shadowing is not considered in this thesis.

- Interference power from other interfering BS $B_{ij}$ is $P_i d_{ij}^{-\alpha_i}G_{ij}$.

- The instantaneous SINR is
  \[
  SINR = \frac{P_{d}^{-\alpha}G}{\sum_{i=1}^{K} \sum_{B_{ij} \in \phi_i \setminus B_0} P_i d_{ij}^{-\alpha_i}G_{ij} + Noise}.
  \]

- Step4: Calculation of downlink coverage probability
  Repeat Step1 to Step3 for 1,000 times and record the calculated SINR for every UE. The downlink coverage probability is the CCDF of the recorded SINRs.

C.2 Uplink Coverage Probability

In this section, the simulation setting of the uplink coverage probability is given.

- Step1: Generate a BS pattern
  It is the same to Step1 in the simulation setting of the downlink coverage probability.

- Step2: Generate a UE pattern
  The UE model is that the UEs concentrate around the social attractor (SA) and SA has a base station offset to its BS. The procedure to generate the active UE pattern of the $i$th tier is:

  - For each BS in $i$th tier, generate a SA which is uniformly distributed around the BS with radius $D_i$.
  - Generate the clustered UEs with the cluster center being the above generated SA.
  - The clustered UEs are uniformly distributed within a circular disc of the radius $R_i$ which is called Matern cluster process.
  - In the coverage area of each cell, choose an active UE randomly.

- Step3: Calculation of SINR
  Based on BS pattern and active UE pattern generated above, calculate the instantaneous SINR for the BS of interest. We assume $L$ is the distance between the BS of interest and the associated active UE. $L_{ji}$ is the distance between the $i$th active UE
and their associated BSs in the $j$th tier. $V_{ji}$ is the distance between the $i$th active UE in the $j$th tier and the BS of interest. $G_i$ denotes the channel random power gain in the $i$th tier. The path loss exponent in the $i$th tier is $\alpha_i$. The transmit power of the active UE associated with the BS of interest is $L_\alpha^k \epsilon^\alpha$ and the transmit power of the $i$th active UE in the $j$th tier is $L_\alpha^j \epsilon$ which considers the uplink fractional power control policy with factional factor $\epsilon$. The random amplitude fading type is Rayleigh fading with unity mean power gain which means the power is exponential distribution with unity.

- Generate random exponential power gain $G_k$ with Matlab code: `random('exp',1).

- Received signal power from the active UE associated with the BS of interest is $L_\alpha^k (\epsilon - 1) G_k$.

- Interference power from all other interfering active UEs is $L_\alpha^j \epsilon G_j V_{ji}^{-\alpha_j}$

- The instantaneous uplink SINR is $\frac{L_\alpha^k (\epsilon - 1) G_k}{\sum_{j=1}^{S} \sum_{\phi_j} L_\alpha^j \epsilon G_j V_{ji}^{-\alpha_j} + \text{Noise}}$

- Step4: Calculation of uplink coverage probability

Repeat Step1 to Step3 for 1,000 times and record the calculated SINR. The uplink coverage probability is the CCDF of the accumulated SINR.

C.3 Heterogeneity Analysis based on CoV

In this section, the simulation setting of the heterogeneity analysis based on CoV is given.

- Step1: Generate a BS pattern

It is the same to Step1 in the simulation setting of the downlink coverage probability.

- Step2: Generate a UE pattern

It is the same to Step2 in the simulation setting of the downlink coverage probability.

- Step3: Calculate the CoV of Voronoi cell area

For the UE pattern, we generate the Voronoi and Delaunay tessellations and find the relative Voronoi cell area. For each UE pattern, we obtain one measured CoV of Voronoi cell area. CoV is the standard deviation divided by the mean of measured Voronoi cell area.
• Step 4: Generate the final mean CoV
  Repeat Step 1 to Step 3 for 1,000 times and calculate the mean CoV among 1,000 measured CoV.

The procedure generating CoV of Delaunay cell edge length and Delaunay cell area is the same as that of Voronoi cell area.
References


