KEYWORDS: Virtual currencies, cryptocurrencies, blockchain, Bitcoin, GARCH model

ABSTRACT

The rapid advancement in encryption and network computing gave birth to new tools and products that have influenced the local and global economy alike. One recent and notable example is the emergence of virtual currencies, also known as cryptocurrencies or digital currencies. Virtual currencies, such as Bitcoin, introduced a fundamental transformation that affected the way goods, services, and assets are exchanged. As a result of its distributed ledgers based on blockchain, cryptocurrencies not only offer some unique advantages to the economy, investors, and consumers, but also pose considerable risks to users and challenges for regulators when fitting the new technology into the old legal framework.

This paper attempts to model the volatility of bitcoin using 5 variants of the GARCH model namely: GARCH(1,1), EGARCH(1,1) IGARCH(1,1) TGARCH(1,1) and GJR-GARCH(1,1). Once the best model is selected, an OLS regression was ran on the volatility series to measure the day of the week the effect. The results indicate that the TGARCH (1,1) model best fits the volatility price for the data. Moreover, Sunday appears as the most significant day in the week. A nontechnical discussion of several aspects and features of virtual currencies and a glimpse at what the future may hold for these decentralized currencies is also presented.
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INTRODUCTION

*Virtual currency,* also known as cryptocurrency and digital currency, is a medium of exchange not authorized or adopted by a government. Virtual currencies share several properties common to physical currencies or fiat money without being in physical format. Many different digital currencies are being used over the Internet but Bitcoin is the most common cryptocurrency. In fact, according to Coinmarketcap (2017), hundred types of cryptocurrencies exist that are crowding the virtual currency scene and are a rapidly evolving topic of interest for many stakeholders. Virtual currencies such as World of Warcraft Gold and online payments such as Paypal are not new and have existed for decades. But what make cryptocurrencies, such as Bitcoin, special is that, unlike online currencies or online payment systems, they are decentralized, peer-to-peer networks that allow for the proof and transfer of ownership of virtual currencies without the need for an intermediary such as a bank. By removing the need for intermediaries, digital currencies can provide a more efficient infrastructure for the transfer of money, allowing for cheaper and faster payments.

Cryptocurrencies are widely viewed as a disruptive technology that raise both hopes and fears in the minds of consumers, businesses, investors, charities, and regulators. In fact, virtual currencies offer several potential advantages to different stakeholders and to the wider economy as a method of payment. Moreover, the ‘distributed ledger’ technology that supports virtual currencies has substantial potential for innovative applications of the technology across financial services and beyond. Yet, virtual currencies present some existing and potential risks that could harm consumers, financial systems, and even the national security, which is now an important matter specially to developed countries facing an enhanced threat of terrorism.

Not surprisingly, several countries, including the United States, are trying to introduce regulations aimed at creating a hostile environment for illicit users of digital currencies, while providing a supportive environment for their legitimate use to flourish. As U.S. Senator Tom Carper, the Homeland Security Committee’s chairman, notes: “rather than play ‘whack-a-mole’ with the latest website, currency, or other method criminals are using…we need to develop thoughtful, nimble and sensible policies that protect the public without stifling innovation and economic growth” (*The Financial Times* 2013).

Despite the potential of virtual currencies, only a relatively small number of consumers and businesses worldwide have currently adopted them. This situation is due to several factors
including the lack of a regulatory framework needed to enhance the credibility and legitimacy of
digital currencies. This lack of a regulatory framework continues to cause some uncertainty for
businesses. Another factor is the excessive volatility of their value. For instance, investors who
believed in Bitcoin when it first emerged made millions of dollars due to the rapid increase of its
value. However, those who invested later lost millions of dollars because of its quick decline in
value. Bitcoin’s volatility proved to be a double-edged sword for investors. Negative media
coverage of arrests related to illicit financial activities and the security problems that led to
different breaches affected volatility in the dollar-to-Bitcoin exchange rate. For example, a
breach involving Mt. Gox occurred in February 2016 resulted in losses of $500 million.
Businesses investing in Bitcoin are also finding difficulties opening bank accounts as some
banks are still treating virtual currencies transactions as fraudulent.

Beyond negative media coverage, other factors can explain the high volatility of Bitcoin.
For instance, because both the technology and the industry that has grown up around Bitcoin are
still in an early stage, large trades of Bitcoins can substantially affect exchange rates.
Additionally, many people still do not know or fully understand what virtual currencies are,
which gives rise to noise trading as opposed to informed trading that are based on fundamentals.

On the positive side, virtual currencies have witnessed the emergence of private funds
and alternative virtual currencies’ investment. Amongst many things, the creation of hedge funds
that encompass strategic trading on virtual currencies’ volatility has become a trend. One of
those entities called Global Advisors Bitcoin Investment Funds (GABI) received regulatory
approval from the Jersey Financial Services Commission (JFSC) in July 2014, making it the first
regulated virtual currencies investment fund. Also on February 25, 2015, the New York Stock
Exchange (NYSE) announced that it is investing in the U.S.-based Bitcoin exchange and wallet
service called Coinbase. Coinbase Exchange is the first regulated Bitcoin exchange, providing
what seems to be a reliable and secure platform for Bitcoin trading that is backed by investors
such as the New York Stock Exchange. On March 24, 2015, Noble Markets, a platform for
trading Bitcoin, announced that it is adopting the same software used by major securities
exchanges around the world, provided by Nasdaq OMX Group Inc. Embracing Bitcoin by the
NYSE and NASDAQ, which are the biggest U.S. stock exchange operators, is a sign that the
digital currency is emerging from underground and is here to stay.
This thesis has the following organization. The next section briefly discusses the most common cryptocurrencies (i.e., Bitcoin, Darkcoin, Peercoin, Primecoin, and Dogecoin), followed by a discussion about their nature and classification. The thesis then explains how Bitcoin works and discusses the transaction costs associated with it. Next, the paper provides a discussion of advantages and disadvantages of digital currencies and how they are regulated. The following section analyses and discusses the results of the estimated models based on the sample data and select the best model using criteria such as AIC and log likelihood. Then a volatility series is constructed based on the selected model to run a OLS regression to test the day of the week effect. Lastly, the thesis concludes with closing remarks about the future of virtual currencies and in particular Bitcoin, along with a discussion of the political impact on the Bitcoin volatility.

MOST PROMOMENT DIGITAL CURRENCIES

Although there are more than 800 types of cryptocurrencies are available for trade in online markets, only 26 have market capitalizations greater than $10 million (Coinmarketcap 2017). The following sections discuss some of the most common cryptocurrencies.

**Bitcoin**

Created by anonymous group of developers (Nakamoto 2008), Bitcoin is the most prominent virtual currency. It has been on the market since 2009 and is created through a “mining” process with every transaction saved in the transaction log or blockchain. Like any other currency, Bitcoin is divided in sub-units called milli-Bitcoin, micro-Bitcoin, and Satoshi. Since its inception, the popularity of Bitcoin has increased among both businesses and consumers. In fact, Bitcoin’s market capitalization has increased from roughly $144 million on January 4, 2013 to beyond an all-time high of $16 billion on January 4, 2017 (Blockchain.info 2017).

The main differences between Bitcoin and known fiat money are:

- Bitcoins are generated by a unique code and transactions are stored in a public ledger. In other words, transactions are made public and are traceable.
- The whole Bitcoin transaction process is decentralized (i.e., with no central repository or administration) and is not under the control of any bank, state or country.
A total of 25 Bitcoins are mined every 10 minutes and an estimated maximum of 21 million Bitcoins will be reached between 2110 and 2140.

Anyone can participate in the mining process by dedicating processor power to the open-source code. Users who do so are called miners and are rewarded for their efforts with Bitcoins.

Bitcoin does not have any physical form, and thus its environmental impact is limited to electricity (albeit a high 1.46 terawatt-hours per year, which corresponds approximately to the consumption of 135,000 average American homes) (The Economist 2015).

Litecoin
In 2011, a former Google engineer invented Litecoin, the second most popular virtual currency after Bitcoin (Rao 2014). Similar to Bitcoin, Litecoin is a decentralized system that can be generated electronically by anyone around the world who is running a computer. It contains an open source global payment system, which means that it can be owned by anyone, and it does not require any form of monitoring. The key difference between Bitcoin and Litecoin is that Litecoin allows for faster transaction confirmation as it processes a blockchain roughly four times faster than Bitcoin (i.e., 2.5 minutes instead of 10 minutes in the Bitcoin network). Moreover, the Litecoin network produces about four times more currency units than produced by the Bitcoin network. Despite this competitive edge over the Bitcoin, the Litecoin network still has fewer users than the Bitcoin network (Gibbs and Yordchim 2014).

Darkcoin
Because transactions are traceable in the blockchain, the use of the Bitcoin network can have implications for users’ personal privacy. Contrary to Bitcoin, Darkcoin allows its users to maintain a high level of privacy. In fact, Darkcoin is the first privacy centric cryptographic currency. To anonymize transactions, Darkcoin uses a decentralized implementation protocol, which Duffield and Hagan (2014) name "DarkSend.” However, because of the decentralized implementation of DarkSend, the Darkcoin network faces the risk of being hacked by rogue users who can modify the software and prevent transactions from being completed.
Peercoin
Peercoin, also known as P2P or PPCoins, started in 2012 and was also inspired by the framework of Bitcoin. It consumes less energy than the Bitcoin network. Moreover, it does not have a fixed upper limit to the coins produced, which makes it an inflationary currency (King and Nadal 2012).

Dogecoin
Dogecoin is a degenderized peer-to-peer network with an uncapped supply. An advantage of Dogecoin is that it deals with large volume of coins with relatively low individual value. Such a feature makes Dogecoin more accessible to new users.

Primecoin
Introduced in July 2013, Primecoin offers great security and strong monitoring over the network, which makes the mining process quite easy and fast. According to King (2013), the speed of the payment through a Primecoin network is 10 times faster than in the Bitcoin network. Similar to Peercoin, no predefined ultimate number of coins exists for Primecoin.
ARE VIRTUAL CURRENCIES COMMODITIES, CURRENCIES OR BOTH?

Since their inception, cryptocurrencies have been the subject of debate with respect to their nature and classification: Are they currencies, commodities, both or another class of asset altogether. Yermack (2014) contends that Bitcoin does not qualify as currency but it is rather a speculative investment. Moreover, unlike commodities, cryptocurrencies are not tangible and have no inherent value by design given that they are not backed by a government or commodity. Yet they share a number of properties common to currencies and commodities. As Grinberg (2012, p. 201) notes:

   Bitcoin is similar to the dozens of gold-backed digital currencies that already exist, such as Pecunix or GoldMoney, because it is liquid, digital, easy for end users to exchange with one another, generally anonymous, and popular among government-distrusting “gold bugs.” However, Bitcoin is different in several key ways: (1) there is no central authority that can issue new currency or defraud holders of the currency (e.g., by holding fractional reserves while promising to hold full reserves), (2) it is fiat money rather than commodity money, and (3) it may be difficult to regulate because there is no centrally controlling authority.

Similar to the vast majority of currencies and commodities, cryptocurrencies provide their holders with the right to use them in any way they want. Holders can also write contract involving them (Grinberg 2012). As for their perceived lack of inherent value, some contend that cryptocurrencies, such as Bitcoin, do have value given their fixed level of supply and over time rarity. In September 2015, the U.S. Commodity Futures Trading Commission (CFTC) ruled that virtual currencies should be considered as commodities. The slow adoption of virtual currencies as commodities by the CFTC is due to the lack of clarity involving which jurisdiction virtual currencies should fall. The International Monetary Fund (IMF) (2016, p. 24) summarizes the status of virtual currencies (VCs) as follows:

   VCs combine properties of currencies, commodities, and payments systems, and their classification as one or the other will often have implications for their legal and regulatory treatment—in particular, in determining which national agencies should regulate them. Finding a consistent classification for VCs even within the same jurisdiction has proven difficult, as different competent authorities may
classify them according to their own policy priorities. For example, the U.S. tax authority, the IRS, has classified VCs as “property” for the purpose of federal taxation, whereas the Treasury Department’s FinCEN has classified VCs as “value” for the purpose of AML/CFT obligations. Other jurisdictions have taken a different approach, avoiding a formal classification and focusing instead on the nature or type of transaction being conducted. This disparity of treatment within and among jurisdictions may hamper coordination and may lead to inconsistencies.

To date, the United States is the only country that considers virtual currencies as commodities. In Canada, the Canadian Revenue Agency treats virtual currencies as barter transactions for tax purposes. However, other developed countries may follow the lead of the United States in classifying virtual currencies as commodities just like crude oil or wheat. Because Bitcoin dominates the universe of virtual currencies as reflected in the research output and media coverage on virtual currencies, the rest of the thesis focuses on Bitcoin.

**HOW DOES BITCOIN WORK?**

Unlike traditional currencies, Bitcoin does not rely on central banks but is supported by its users through a peer-to-peer computer network that allows the transfer of Bitcoins between users and the generation of new Bitcoins. Bitcoins are created through a procedure known as *Bitcoin mining*, which is essentially a mathematical execution of difficult number-crunching problems in the Bitcoin network (*The Economist* 2013). Bitcoins generators, called *miners*, are compensated with Bitcoin transaction fees. The mathematical tasks used to generate Bitcoins are designed so that they become increasing difficult over time, while setting a cap on the total number of Bitcoins that can be mined of around 21 million.

Anyone can become a Bitcoin miner by running software with specialized hardware. The Bitcoin software is easily available via Bitcoin’s official website (https://bitcoin.org/en/choose-your-wallet). Bitcoin software has many features including digital wallet, which stores information including user’s Bitcoin balance. The wallet creates an address randomly using a combination of letters and numbers that requires using the network (Böhme, Christin, Edelman,
To enhance privacy, users can delve into the Bitcoin system using nicknames or Bitcoin addresses, so that users do not have to reveal their identity or disclose their place of residence to create their own wallet (Blockchain.info 2017). Users can obtain several unique addresses in which they do not need to provide personal information. Users can transfer Bitcoins to each other for a certain service or goods by scanning the barcode of the address that appears in the digital wallet in any device (Androulaki, Karame, Roeschlin, Scherer, and Capkun 2013). Bitcoin uses a key system that allows users to store and spend Bitcoin money. Each Bitcoin address contains a private key and a public key. The private key unlocks the wallet to allow receiving and sending Bitcoins, while the public key confirms all payment process sent from that address. Anyone can find the inside address, but only the private key can unlock the wallet to allow receiving and sending Bitcoins.

In the Bitcoin network, blockchain records all historical transactions (Fink and Johann 2014). Bitcoin continuously groups all transactions and every new transaction becomes part of this group and is called a block. These blocks validate the authority and ensure that no unauthorized transactions occur. The process of confirming transaction is a continuous process that occurs almost every 10 minutes (Böhme et al. 2015).

In the Bitcoin database, each transaction includes the input (sender) address, output (recipient) address, number of Bitcoins sent to the receiver, and time at which the transaction was added to the network. Generally, any transaction may contain one sender (input) or multiple senders, the same situation applies to the receiver (output), in which the receiver can be one or more individuals (Fink and Johann 2014). The amount of Bitcoins sent is usually related to the amount of Bitcoins received previously by the same client. Assume person A received five Bitcoins from person B who receives seven Bitcoins from person C. The five and seven Bitcoins are the output in the previous transactions and serves as the input in the following transaction such as person A sending two Bitcoins to person D. This transaction must have one input (either the five or seven Bitcoins) and three outputs. One of the outputs should be assigned to person D by Bitcoins; the second output should be allocated as a transaction fee by, for example, 0.01 Bitcoins to the miner who verifies the transaction. 2.99 Bitcoins should assign the last output to person A as a charge. Unlike other popular online payment systems, Bitcoin network cannot reverse the payment once it is made even in case of a wrong transaction (Robleh, Bahrdear, Clews, and Southgate 2014a; Böhme et al. 2015).
TRANSACTION COSTS OF BITCOIN

Bitcoin has three types of transaction costs: (1) search and information costs, (2) bargaining and decision costs, and (3) policing and enforcement costs. Currently, Bitcoin has no formal transaction fees. When a transaction takes place, any portion of the transaction not received by the recipient is considered a fee and is given to the miner who solved that transaction block as a reward. Thus, cryptocurrencies offer a way to exchange money while foregoing the laws involved in assuring the customer’s privacy. From a retailers’ perspective, the main driver of accepting electronic currencies is the promise of lower transaction fees. However, the marginal cost of verifying transactions are actually higher for miners than centralized payment systems. Additionally, the costs of verifying transactions are expected to rise in electronic currencies while remaining constant in traditional payment systems. As Robleh, Barrdear, Clews, and Southgate (2014b, p. 280) state, “The low transaction fees for digital currency payments are largely driven by a subsidy that is paid to transaction verifiers (miners) in the form of new currency. The size of this subsidy depends not only on the current price of the digital currency, but also on miners’ beliefs about the future price of the digital currency.”

As more people begin to support electronic currencies, demand will increase allowing miners to continue to accept low transaction fees due to the prospect of future increases in system usage. However, because the supply of electronic currencies is usually fixed, sustaining a subsidy to miners in the long run will be impossible, which will likely result in higher transaction fees.

BENEFITS AND RISKS OF DIGITAL CURRENCIES

Cryptocurrencies in general and Bitcoin in particular have some unique advantages and disadvantages for the economy, investors, and consumers. Being a peer-to-peer network system, all transactions in the Bitcoin network occur without passing through a third-party such as a bank, state or government. Thus, Bitcoin is an extremely disruptive technology in the financial sector, which some compare to the advent of personal computers in 1975 or the Internet in 1993. Eliminating the traditional middleman will reduce the total cost of payments and increase the efficiency of the financial world, but will cause enormous losses to the banks and governments, who are the middlemen. The existence of cryptocurrencies would ultimately lead to more
competition among banks and traditional financial players to lower their costs for a share of the market. The increasing competition in the financial sector should also lead to better service for consumers. Moreover, because Bitcoin is not under the control of any central bank or government, no “printing press” entity exists that can create additional Bitcoins and inflate the consumers’ saving value, which is another huge benefit to the economy. As discussed later, governments are trying to catch-up and regulate the cryptocurrency industry, whereas banks are readying their strategies to deal with cryptocurrencies as soon as the regulation is approved.

The peer-to-peer network system eliminates transaction costs that consumers pay today with credit cards (usually around 3 percent) and will improve the economy overall, with more money in the consumers’ pockets. According to a report by the British government’s Her Majesty's Treasury (2015), the lower transaction costs associated with digital currencies can translate into billions of dollars of savings to businesses. In fact, a peer-to-peer network system enables businesses to monetize very low cost goods and services and accept micro-payments. These savings can be reinvested to create more jobs or can help reduce the price of goods. By removing the need for traditional intermediaries, cryptocurrencies provide a more efficient infrastructure for the transfer of money, allowing for faster payments. The traditional interbank payment systems can take several hours, if not days, to move money between accounts. Faster settlement has cash flow benefits for businesses by allowing corporate capital to be put to more productive use than at present. The lower cost and faster payment associated with cryptocurrencies are even more pronounced in the context of cross-country transactions in which settlement often involves high transaction fees of around 8 percent and payment can easily take several days to be processed. This feature is particularly important for small businesses facing cash flow problems.

Another advantage of cryptocurrencies is their availability. In fact, while conventional payment services are available only during banking hours, cryptocurrencies are available 24 hours a day. Such property is even more desirable in the context of unbanked or under-banked countries where the absence of a traditional banking system deprives people in remote regions from payments and other financial services. Moreover, transactions made using digital currencies prevent the possibility of fraud through using fake payments or double spending of digital currency units. Another major advantage with cryptocurrencies is that their associated technologies have several possible applications beyond retail payment services. In fact, the
distributed ledgers based on block chain can be used, for instance, to record and transfer the ownership of financial instruments such as bonds and shares.

Bitcoin consumers use the cryptocurrency as a peer-to-peer cash system that is more transparent than most other types of payments. The parties involved in the transaction are clearly identified and the transaction details are stored in the Bitcoin ledger. However, those transactions are irreversible. Although consumers might consider this irreversibility a disadvantage, merchants consider it an advantage because it removes the risk of any reversal of payment. Moreover, because cryptocurrency networks allow for lower costs and faster settlement times, they attract both legitimate and illegitimate users. In fact, digital currencies network can be used to trade illicit goods and services, engage in money laundering or even to finance terrorists. However, in October 2014, the National Crime Agency (NCA) concluded that the risk of criminal use of digital currencies is essentially limited to the sale and purchase of illicit goods and services (Her Majesty's Treasury 2015). The risks that digital currencies pose to a country’s monetary and financial stability should be low because of the very low volume of digital currencies circulating at present compared to fiat currencies.

Another risk associated with virtual currency stems the unsupervised e-currency trading platforms. The issue of arbitrage and trading collusion surfaces when using e-currency trading platforms. Bitcoin, for instance, are traded through an online exchange. From an investor’s perspective, these exchanges provide online platforms for transferring cash for the equivalent in virtual currencies and vice-versa. Because many of these platforms operate simultaneously, they can arguably create arbitrage opportunities. Given that the same Bitcoin can be bought on one platform and sold on another, sometimes with fees and delays involved, arbitrage opportunities might exist. If two of those exchanges collude, therefore removing a potential barrier of fiat transfer between the two for an exclusive segment of privileged customers or owners, such unethical trading could take place for the exclusive benefit of certain persons.
VOLATILITY OF BITCOIN PRICE

Bitcoin has some unique features that differentiate it from fiat currencies. For instance, Bitcoin is not considered as a payment instrument in the sense of the monetary and financial code because it is issued without a money deposit. Moreover, contrary to fiat currencies, Bitcoin does not endow the trust of central banks, with irreversible transactions that do not explicitly identify the payer or the payee. Another important feature of Bitcoin is its highly volatile value. During the first two years since its creation, the parity of Bitcoin with the U.S. dollars was about 0.10. By February 7, 2011, the price of Bitcoin reached $1. Bitcoin experienced a second surge on June 6, 2011 reaching $22. Following multiple surges, Bitcoin reached a peak of $979 on November 25, 2013. However, the price of Bitcoin decreased sharply in 2014 due to the bankruptcy of a Bitcoin exchange based in Tokyo, named Mt. Gox. After reaching a low of $232 on August 17, 2015, Bitcoin has continued to gain momentum and was priced at $1,069 on January 4, 2017. Figure 1 illustrates the high volatility of Bitcoin between 2012 and 2015 compared to the price of gold, two broad stock markets indices (i.e., MSCI Emerging Markets Index and MSCI Global Market Index), and two currencies (the Euro and Yen).

Table 1 provides summary statistics for all the series, along with the Jarque-Bera test for normality. In dollar terms, Bitcoin’s realized monthly volatility between May 2012 and May 2015 was 265 percent, which is much higher than gold’s 118 percent. This high volatility hinders Bitcoin ability to be used as a medium of exchange. Table 1 shows that kurtosis is positive and high for all variables, suggesting presence of fat tails. The Euro and Chinese exchange rate for gold and Bitcoin is positively skewed but negatively skewed for stock markets. This leptokurtic excess and asymmetry are consistent with the Jarque–Bera test, justifying the rejection of normality.

As previously discussed, a major difference between Bitcoin and fiat currencies is that no centralized authority regulates Bitcoin. This lack of control both attracts people to and scares them away from Bitcoin. Although the possibility exists for large gains in exchange rates, such rates could also fall dramatically. One way that banks maintain a stable currency is to control the currency supply by increasing or decreasing inflation. Bitcoin, however, cannot employ the same strategy. Every four years, the number of Bitcoins is increased by a half. This increase in the currency supply coupled with a decreasing or stable trade activity results in substantial volatility. Like any currency or commodity, a highly unstable supply of and demand for Bitcoins led to
higher volatility of its market value. Because Bitcoins are infrequently traded, large trades can dramatically influence exchange rates.

The fact that this digital currency is still in the infancy of its market evolution further exacerbates the exchange rate volatility of Bitcoin’s price. Similarly, relevant news related to Bitcoin can affect its exchange rate. Positive news might dramatically increase the exchange rate while negative news could send it plummeting. Also, many people still do not know or understand the nature of Bitcoin. Therefore, news introduces them to the currency, inevitably increasing Bitcoin demand and likely helping to stabilize its volatility (The Economist 2015).

What most merchants do not realize about Bitcoin is that it can be used simply as a payment system that enables them to convert customers’ payment in Bitcoins to a stable currency. However, the 1 percent transaction fee that companies charge is insufficient to cover the exchange rate risk of Bitcoin. A possible remedy is to engage in a swap or futures contract. The use of derivatives instruments can lessen Bitcoin’s volatility and enhance the development of its network’s infrastructure (Brito, Shadab, and Castillo 2014).

Due to its finite supply, Bitcoin is more like a commodity than a currency. Given its similarities to commodities, the continuing decline in the rate of supply and a tendency for demand to increase as stability takes hold, can lead to an increase in the exchange rate between Bitcoins and dollars in the long run. However, such an increase assumes that education and awareness of Bitcoin continue to increase and no massive speculative selling by traders occurs.

Academic research dealing with cryptocurrencies has mainly focused on their ability to become an alternative monetary system (Segendorf 2014; Dwyer 2015). A few research studies examine Bitcoin’s volatility. Some researchers postulate that demand for Bitcoins heavily influences the volatility (Buchholz, Delaney, Warren, and Parker 2012) while others contend that the Bitcoin’s value has nothing to do with economic theories but rather speculation drives its value (Kristoufek 2015). Ciaian, Rajcaniova, and Kancs (2016) suggest that market fundamentals and Bitcoin’s attractiveness to investors affect the price formulation of Bitcoin. Users of Bitcoin anticipate that the exchange rate will increase and do not want to sell at a low value. However, they may not realize that by holding Bitcoins they are causing a decrease in the demand for the currency eventually decreasing its exchange rate.

Few studies examine cryptocurrencies as hedging and diversification tools. Investors look for alternative investment instruments as part of diversified investment portfolios. For example,
during the early 2000s, investors considered commodity instruments effective for portfolio diversification because of their lower correlation with stock returns. Today, cryptocurrencies represent such an alternative because of their high average return and low correlation with financial assets (Bouoiyour and Selmi 2014).

Guesmi, Ftiti, Saadi, and Abid (2016) examine the conditional cross effects and volatility spillover between Bitcoin and financial indicators using different multivariate Generalized Autoregressive Conditional Heteroskedasticity (GARCH) specifications. They find that a Dynamic Conditional Correlation (DCC)-GARCH is the best-fit model for modeling the joint dynamics of different financial variables and Bitcoin. In particular, they find that unpredicted changes in the returns on different financial indicators significantly affect the conditional volatility of Bitcoin returns. Thus, a shock to financial variables, regardless of their signs, implies an increase in the volatility of Bitcoin returns. Their evidence also shows that a short position in the Bitcoin market allows hedging the risk investment for all different financial assets. Additionally, hedging strategies involving gold, oil, stocks of developing markets, and Bitcoin considerably reduce the portfolio’s risk, as measured by variance, compared to the portfolio risk consisting of only gold, oil, and stocks of developing markets.

**REGULATION OF CRYPTOCURRENCIES**

The gradual acceptance of the electronic currencies is likely to eventually result in their financial product securitization. Considering the perceived risks associated with virtual currencies, notably the hacking risks and the tax/security/money laundering concerns, the next logical step is to enact clear and stringent legislation to enable the full potential of virtual currencies and to reduce undesirable use. Although digital currencies are unregulated in most countries, some major economies have regulation in place and others are studying the potential need to do so (Fuller 2014). As outlined in a comprehensive study of Bitcoin conducted by the Law Library of the U.S. Congress (2014, p. 1), “there is widespread concern about the Bitcoin system’s possible impact on national currencies, its potential for criminal misuse, and the implications of its use for taxation.”
The United States
One country that regulates digital currencies is the United States. In November 2013, the U.S. Senate Committee on Homeland Security and Governmental Affairs held two days of hearings on the different risks and potentials of digital currencies (U.S. Senate Committee on Homeland Security and Governmental Affairs, 2013). In January 2013, the Financial Crimes Enforcement Network of the U.S. Department of the Treasury published how it will approach individuals and businesses that invest in Bitcoin and other virtual currencies. The goal of creating guidelines for the currency was to enable innovators to explore opportunities with the technology while also following federal law. The agency provided additional guidance in October 2014 when it ruled that custodial Bitcoin exchanges and payment processors may be considered money services businesses under law (Financial Crimes Enforcement Network 2014). In fact, the U.S. government has transacted in Bitcoins when it auctioned off 10 blocks of Bitcoin that it had confiscated when it shut down the online black market Silk Road in 2013 (Coindesk 2014a).

Individual states are also beginning to push forward with regulations related to digital currencies, as evidenced by a recent move toward tougher regulation in New York State that is considered to be more stringent than the regime that pertains to banks (Forbes 2014a). Other states, however, such as Utah and New Hampshire, are considering legislation more favorable to digital currencies that would allow taxpayers in those states to pay taxes in Bitcoin (Bitcoin Magazine 2015).

On July 17, 2014, the New York Department of Financial Services (NYDFS) proposed a regulatory framework for virtual currency firms, called BitLicence, which includes several requirements that are usually found in securities law (BitLicence, 2014).

Brazil
In October 2013, Brazil enacted a law facilitating the creation of electronic currencies, including Bitcoin. In April 2014, the government declared that gains in Bitcoins would be considered taxable income and announced that it will require holders of Bitcoins to recognize capital gains similar to requirements for capital gains on securities (Forbes 2014b).

Canada
The Canadian government followed the U.S. government’s footsteps and recently established regulatory guidelines classifying Bitcoin as a decentralized virtual currency, and Bitcoin miners
as Money Service Businesses (MSBs) that are subject to legal obligations and registration to The Financial Transactions and Reports Analysis Centre of Canada. In 2015, the Québec Autorité des marchés financiers (AMF) amended the Money-Services Businesses Act to force virtual currency automated teller machine owners to obtain a license in conformity to the Act (Autorité des Marchés Financiers 2015).

**China**
In December 2013, the central bank of China issued the “Notice on Precautions against the Risks of Bitcoins,” in which it defined Bitcoin as a “virtual commodity” and, as a result, should not be circulated in the market as a currency. The Notice also prohibited banks from dealing in Bitcoins and strengthened oversight of Internet websites related to it (Regulation of Bitcoin in Selected Jurisdictions 2017).

**Russia**
In October 2014, Russia’s Finance Ministry proposed fines of up to 1 million rubles for those producing and/or carrying out transactions with digital currencies. This action followed on the Bank of Russia’s statement in January 2014 that dealing in Bitcoins would be considered as involvement in suspicious activity according to Russia law related to crime and terrorist financing. Whether this law has been enacted by early 2017 is unclear but at least a de facto ban is in place on the currency (Regulation of Bitcoin in Selected Jurisdictions 2017).

**United Kingdom**
A comprehensive study of digital currencies in the United Kingdom, during the summer of 2013 raised concerns but ultimately left the currency unregulated at that time (Robleh et al. 2014b). More recently, in November 2014, the Bank of England issued a call for information on the benefits and risks of digital currencies as well as to get the public’s views on whether regulation of digital currencies is required. In a discussion paper released in February 2015, the Bank of England raised the question of whether central banks should use such technology to issue digital currencies (Bank of England 2015). In the United Kingdom, Bitcoins have been classified as “single purpose vouchers,” meaning that they are subject to a 10 to 20 percent tax.
European Union
In the European Union (EU), the European Central Bank released reports on virtual currencies in October 2012 and in December 2013, and its regulatory agency, the European Banking Authority, issued a warning on the dangers associated with dealing in virtual currencies (European Central Bank 2012; European Banking Authority 2013). This warning outlined that since Bitcoin is unregulated; consumers are unprotected and may be liable for taxes. However, the EU has not passed legislation related to Bitcoins.

Taxation and Bitcoins
Regarding taxation, many countries, including the United States, Japan, Finland, and Germany, currently tax Bitcoins, Ecuador was the first country to issue its own digital currency with a launch date of February 26, 2015. The Ecuador’s Central Bank noted at the time that the currency is intended to support the country’s dollar-based monetary system and not to replace it.

ALTERNATIVE VIRTUAL CURRENCIES’ INVESTMENT

During 2014 and 2015, various entities were created to benefit from the expansion of virtual currencies. For example, some hedge funds now engage in strategic trading on virtual currencies’ volatility and private equity firms invest in small portfolios of virtual currencies’ companies. One of these entities called Global Advisors Bitcoin Investment Funds (GABI) received regulatory approval from the Jersey Financial Services Commission (JFSC) in July 2014, making it the first investment fund in virtual currencies to be regulated, albeit regulated in a commonly known tax-heaven jurisdiction (Coindesk 2014b). GABI describes virtual currencies as Internet money a store of value, a medium of exchange and a unit of account, all digitized.

On February 25, 2015, the NYSE announced its investment in the U.S.-based Bitcoin exchange and wallet service Coinbase (The Financial Times 2015). In 2015, major banks such as Goldman Sachs, Morgan Stanley, BNP Paribas, and Société General became involved in a new derivative-trading platform specializing in futures and options tied to the price of Bitcoin. Such derivative-trading platforms provide businesses with additional solutions in managing daily transactions, especially those that encompass an international or a speculative component. The
main concerns of Bitcoin merchants are the strong volatility of the currency coupled with tax concerns. Given the functioning of Bitcoin, a merchant can accept a Bitcoin within a 10-minute timeframe and reconvert this Bitcoin within the timeframe of 10 minutes. Thus, the daily fluctuation risk is reduced to an overall 20 minutes timeframe risk of volatility. Combined with an appropriate system, the use of any electronic currencies as a medium of exchange has the potential to radically cut costs for businesses.

On March 10, 2017, however, The Securities and Exchange Commission (SEC) rejected an application by the stock exchange Bats BZX Exchange to create the first exchange-traded fund (ETF) for Bitcoin (The Washington Post 2017). The news caused an abrupt decline in Bitcoin price by more than 16 percent. The SEC denied the application because Bitcoin exchanges are not regulated and are susceptible to fraudulent or manipulative acts and practices.

ANALYSES AND DISCUSSION OF THE RESULTS

Data and methodology

The data used in this work consist of the daily bitcoin prices from July 18, 2010 to December 30, 2016 (from http://www.coindesk.com/). Assuming the price of bitcoin changes continuously, the daily rate of return, \( r_t \), of the closing prices is computed using the log return formula that is then used for the analysis, that is,

\[
 r_t = \ln \left( \frac{P_t}{P_{t-1}} \right),
\]

Where \( P_t \) is the current price and \( P_{t-1} \) is the previous price.

I apply five GARCH-type models (GARCH, EGARCH, TGARCH, IGARCH, and GJR-Garch) to model bitcoin volatility and assume \( p = 1 \) and \( q = 1 \). Hansen and Lunde (2005) compare more than 300 types of GARCH (1, 1) model including my candidate models and find that GARCH (1,1) to be the best model to forecast volatility (for their dataset). The variables of some variant of the GARCH model like GARCH(1,2) and GARCH(2,2) are more significant than that of the GARCH(1,1) model. However, in terms of forecasting, the GARCH(1,1) model is preferable based on the data. Using SPLUS 8.2 to find the distribution error that best fits the data, the models are analyzed under three error distributions: normal distribution, Student’s
distribution, and generalized error distribution (GED). Using Akaike information criterion (AIC) and log likelihood to measure the relative quality of the statistical model, I select the best model and use it to measure the day of the week effect on the price volatility of bitcoin, that is, the impact of each given day on the change in bitcoin price.

**Analyses**

**GARCH(1,1)**

The GARCH(1,1) model is a statistical model that predicts the conditional variance of an independent variable based on the past volatility, $\sigma^2_{t-1}$, and the past error term, $e^2_{t-1}$. The GARCH(1,1) model is written as follows: $(\sigma^2_t) = \alpha_0 + \alpha_1 e^2_{t-1} + \beta_1 \sigma^2_{t-1}$.

where $\alpha_0, \alpha_1,$ and $\beta_1$ are the estimated parameters. When past volatility, $\sigma^2_{t-1}$, increases by one unit, future volatility will increase or decrease by $\beta_1$ units, on average. When the past error term, $e^2_{t-1}$, increases by one unit, future volatility will increase or decrease by $\alpha_1$ units, on average. The constant term, $\alpha_0$, is the average volatility of bitcoin over the analyzed period. I note that the standard GARCH model treats the news symmetrically, in that the negative and positive errors have the same impact on future volatility. The equations of all the models discussed in this work are derived from Zivot and Wang (2006).

a. **GARCH(1,1), normal error distribution**

I first run a GARCH(1,1) model with the normal error distribution ($e_t$ has normal distribution), and I obtain the following equation: $(\sigma^2_t) = 0.000017 + 0.226387 e^2_{t-1} + 0.772613 \sigma^2_{t-1}$. The $p$-values for the coefficients ($p = .000$) are less than the 1% significance level, so the coefficients are all statistically significant. The estimated model is written as $(\sigma^2_t) = \alpha_0 + \alpha_1 e^2_{t-1} + \beta_1 \sigma^2_{t-1}$ ($\alpha_0 = 0.000017, \alpha_1 = 0.226387, \text{and } \beta_1 = 0.772613$). These estimates allow me to predict the rate of change of the future volatility. For instance, if the volatility of today’s bitcoin price increases by 1 unit, tomorrow’s volatility will increase by 0.772613 only. Moreover, the 1-unit rise in today’s error term will increase the next day’s volatility by 0.226387. The average volatility for the sample data is estimated to be 0.000017. The AIC is -12,029.42, and the log likelihood is 6,014.712 (see Table 2).
b. GARCH(1,1), Student’s t error distribution

I then run a GARCH(1,1) model with the Student’s t error distribution. The model’s equation is still given by as \( (\sigma_t^2) = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \), where \( e_t \) has a Student’s t distribution. The estimated model is \( (\sigma_t^2) = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \) (\( \alpha_0 = 0.000006, \alpha_1 = 0.223320, \) and \( \beta_1 = 0.775680 \)). The \( p \)-values (\( p = .000 \)) indicate that all coefficients are statistically significant (at least) at the 1% level. These results provide evidence that past volatility and error term of bitcoin affect future volatility. The AIC and the log likelihood are -13,026.97 and 6,513.485, respectively (see Table 2).

c. GARCH(1,1), GED

I perform the GARCH(1,1) model with GED, \( e_t \). The results show that the \( p \)-values for all the coefficients are small (\( p = .000 \)), and all are less than 1%, indicating that all the coefficients are all statistically significant (at least) at the 1% level. The estimated model is written as \( (\sigma_t^2) = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2 \) (\( \alpha_0 = 0.00001, \alpha_1 = 0.23721, \) and \( \beta_1 = 0.76179 \)). These results show that past volatility and the error term of bitcoin affect future volatility. The AIC and the log likelihood are -13,127.24 and 6,563.62, respectively (see Table 2).

IGARCH(1,1)

The integrated generalized autoregressive conditional heteroscedasticity (IGARCH) model is a restricted version of the GARCH model, where the persistent parameters are positive and sum to one. Like the standard GARCH model, the IGARCH(1,1) model attempts to predict the future volatility based on the past error term and the past volatility. According to the model, when the past error increases by 1 unit, the future volatility will also increase or decrease by \( \alpha_1 \), on average.

\[
(\sigma_t^2) = \alpha_0 + \alpha_1 e_{t-1}^2 + \beta_1 \sigma_{t-1}^2,
\]

where \( \alpha_0 + \alpha_1 + \beta_1 = 1 \).

a. IGARCH(1,1), normal error distribution

The estimated model is written as

\[
(\sigma_t^2) = 0.000017 + 0.227573 e_{t-1}^2 + 0.772427 \sigma_{t-1}^2.
\]
The analysis shows that all the coefficients in the model are statistically significant ($p < .05$) at the 5% level, at least. These results are not surprising since this model is relatively close to the GARCH(1,1) model. The AIC is -12,029.76, and the log likelihood is 6,014.879 (Table 3).

b. IGARCH(1,1), Student’s error distribution

The estimated model is written as

$$(\sigma_t^2) = 0.000006 + 0.223929 e_{t-1}^2 + 0.776071 \sigma_{t-1}^2.$$  

The sample data provide strong evidence that the model is significant ($p < .05$) with respect to the constant and the variables. The AIC is -13,027.66, and the log likelihood is 6,513.831. (Table 3).

c. IGARCH(1,1), GED

The estimated model is written as

$$(\sigma_t^2) = -0.00001 + 0.23759 e_{t-1}^2 + 0.76241 \sigma_{t-1}^2.$$  

The $p$-value of the constant term appears to be not significant at the 5% level. The constant term summarizes the data, in that it is an average volatility of the past bitcoin prices. All the other coefficients are significant. The AIC is -13,127.77, and the log likelihood is 6,563.885 (Table 3).

EGARCH(1,1)

The exponential generalized autoregressive conditional heteroscedastic (EGARCH) model proposed by Nelson (1991) is used to model the logarithm of the volatility of a random process (including stock price and bitcoin price). This model is useful because it takes into account the impact of news on volatility (which is absent in the standard GARCH model) by introducing a leverage parameter in the model. The EGARCH(1,1) model has the general equation

$$\log (\sigma_t^2) = \alpha_0 + \alpha_1 e_{t-1} + \alpha_2 (|e_{t-1}| - \frac{\sqrt{2}}{\pi}) + \beta_1 \log (\sigma_{t-1}^2).$$  

The model predicts the log conditional variance based on the lag 1 volatility, $\sigma_{t-1}^2$, and current error term, $e_t$. $|e_{t-1}|$ represents the absolute value of the error term of the volatility for the previous period of time $t-1$. The parameters of the model are $\alpha_0, \alpha_1, \alpha_2$, and $\beta_1$. The constant term $\alpha_0$ is the average volatility of bitcoin over the period of study. The coefficients $\alpha_1$ and $\alpha_2$
are, respectively, the previous error term (which can be positive or negative) and the leverage term, which measures the asymmetric impact of the news. When the past volatility rises by 1%, the future volatility will also increase by $\beta_1 \%$.

a. EGARCH(1,1), normal error distribution

I analyze the EGARCH(1,1) model with the normal error distribution ($e_t$ has normal distribution) and obtain the following equation:

$$
\log (\sigma_t^2) = -0.491746 + 0.021372e_{t-1} + 0.427155(\left|e_{t-1}\right| - \sqrt{\frac{2}{\pi}}) + 0.929106\log (\sigma_{t-1}^2).
$$

The $p$-values for the coefficients are $p = .000, p = .16476, p = .000$, and $p = .000$, respectively. Thus the coefficients are all significant at least at the 1% level, except for the past error term (meaning the lag 1 volatility of bitcoin has an impact on future volatility), which is insignificantly different from zero. When volatility rises by 1%, the future volatility will (on average) rise by about 0.93%. The significance of the leverage term indicates that good news (positive error) has more impact on volatility than does bad news. The AIC and the log likelihood are -12,024.436 and 6,012.218, respectively (Table 4).

b. EGARCH(1,1), Student’s error distribution

I run the EGARCH(1,1) model with the error distribution ($e_t$ has normal distribution) and obtain the following estimated equation:

$$
\log (\sigma_t^2) = -0.303416 + 0.034341e_{t-1} + 0.742511(\left|e_{t-1}\right| - \sqrt{\frac{2}{\pi}}) + 0.955063\log (\sigma_{t-1}^2).
$$

The $p$-values for the coefficients are $p = .000, p = .332552, p = .012794$, and $p = .000$, respectively. Hence all the coefficients are statistically significant, except for the past error term. My findings indicate that the previous volatility of bitcoin prices has an impact on the future volatility. Moreover, the AIC is -13,099.23, and the log likelihood is 6,549.615 (Table 4).

c. EGARCH(1,1), GED

Lastly, I estimate the EGARCH(1,1) model using a GED ($e_t$ has GED), and the estimated model is written as
\[
\log (\sigma_t^2) = 0.003861 + 0.030652e_{t-1} + 0.316978(|e_{t-1}| - \sqrt{\pi}) + 0.999221\log (\sigma_{t-1}^2).
\]

The \(p\)-values for the coefficients are \(p = .48302\), \(p = .27682\), \(p = .000\), and \(p = .000\). This implies that the constant and the past error term have no significant effect in predicting the volatility of future price of bitcoin. More specifically, when the past volatility increases by 1%, the log of future volatility will increase by about 0.999221%. Moreover, the AIC is -12,988.264, and the log likelihood is 6,494.132 (Table 4).

**TGARCH(1,1)**

The threshold GARCH(1,1) model proposed by Zakoian (1994) is used to model the standard deviation of the log return price according to the following equation:

\[
(\sigma_t) = \alpha_0 + \alpha_1^+ (e_{t-1}^+) + \alpha_1^- (e_{t-1}^-) + \beta_1(\sigma_{t-1}).
\]

\[
e_{t-1}^+ = e_{t-1} \text{ and 0 otherwise}
\]

\[
e_{t-1}^- = e_{t-1} \text{ and 0 otherwise}
\]

This model also uses a leverage effect to measure the asymmetric impact of news. The error terms \(e_{t-1}^+\) and \(e_{t-1}^-\) are variables that refer to the value of the past error, \(e_{t-1}\), depending on the sign of the error. If the error is positive (good news), then \(e_{t-1}^+ = e_{t-1} \text{ and 0 otherwise}\), and if the error is negative (bad news), then \(e_{t-1}^- = e_{t-1} \text{ and 0 otherwise}\). The model attempts to predict the square root of the variance based on the sign of the past error term (leverage effect). Four parameters are involved in the estimation of the model, namely, \(\alpha_0, \alpha_1^+, \alpha_1^-, \text{ and } \beta_1\). The constant term \(\alpha_0\) refers to the average volatility of bitcoin over the period analyzed.

a. **TGARCH(1,1), normal error distribution**

I analyze the TGARCH(1,1) model with the normal error distribution (\(e_t\) has normal distribution), and I obtain the following equation:

\[
(\sigma_t) = 0.0011713 + 0.2536273(e_{t-1}^+) - 0.0128515(e_{t-1}^-) + 0.7832436(\sigma_{t-1}).
\]

The \(p\)-values for the coefficients are \(p = .000\), \(p = .000\), \(p = .739\), and \(p = .000\), respectively, which are less than 1% significance level, except for the \(p\)-value of the negative error term (\(p =\) ...
.739, \( p > .05 \), which is statically insignificant. So the future volatility of the bitcoin price is significantly affected by the positive error term and the past volatility. The results indicate that that future volatility of bitcoin price is independent of the negative error term (or is not affected). There is a positive leverage effect: good news affects volatility more than does bad news. Moreover, the AIC is -11,216.42, and the log likelihood is 5,608.521. Increasing the past standard deviation of the bitcoin price by 1 unit will, on average, increase the future standard deviation (which is a measure of volatility) by about 0.7832436 (Table 5).

b. TGARCH(1,1), Student’s error distribution

I analyze the TGARCH(1,1) model with Student’s error distribution (\( e_t \) has Student’s distribution), and I obtain the following equation:

\[
(\sigma_t) = 0.00000002613 + 1 (e_{t-1}^*) - 0.05379 (e_{t-1}^-) + 0.6285(\sigma_{t-1}).
\]

The \( p \)-values for the coefficients are, respectively, \( p = .000 \), \( p = .000 \), \( p = .174 \), and \( p = .000 \) and are less than 1% significance level, except for the \( p \)-value of the negative error term, which is statically insignificant. This means that the past negative error term has no effect on volatility. The findings indicate that the lag 1 volatility and the positive error term of bitcoin affect future volatility. The quality criteria of the model are AIC -13,473.64, and log likelihood 6,736.823 (Table 5).

c. TGARCH(1,1), GED

Lastly, I estimate the TGARCH(1,1) model using a GED (\( e_t \) has GED), and the estimated model is written as

\[
(\sigma_t) = 0.0006279 + 0.2457 (e_{t-1}^*) - 0.01661 (e_{t-1}^-) + 0.7971(\sigma_{t-1}).
\]

Like before, the \( p \)-values for the coefficients (\( p = .000 \), \( p = .000 \), \( p = .736 \), and \( p = .000 \)) are less than 1% significance level, except for the \( p \)-value of the negative error term, which is statically insignificant. The data show that the lag 1 volatility and positive error term of bitcoin price affect future volatility. Moreover the AIC is -12,928.93, and the log likelihood is 6,464.466 (Table 5).

GJR-GARCH(1,1)
Similar to the TGARCH model, the GJR-GARCH(1,1) model was proposed by Glosten, Jagannathan, and Runkle (1993). The model examines the asymmetry of news in the ARCH process. It describes the square of volatility as a function of the square of both past volatility and error term:

\[
\sigma_t^2 = \alpha_0 + \alpha_1 (e_{t-1}^2) + \alpha_2 (e_{t-1}^2) I_{t-1} + \beta_1 (\sigma_{t-1}^2),
\]

where

\[
I_{t-1} = \begin{cases} 
1, & \text{if } e_{t-1} < 0 \\
0, & \text{if } e_{t-1} > 0.
\end{cases}
\]

The binary variable \(I\) takes the value 1 if the past error is negative and 0 otherwise (error is positive). It should be noted that this model always returns a positive volatility because it analyzes a squared variable.

a. GJR-GARCH(1,1), normal error distribution

The estimated model based on the available data is written as

\[
\sigma_t^2 = 0.00001656 + 0.2592 (e_{t-1}^2) - 0.05453 (e_{t-1}^2) I_{t-1} + 0.7636 (\sigma_{t-1}^2).
\]

The \(p\)-values for the coefficients are \(p = .000, p = .000, p = .072145, \) and \(p = .000\), respectively. All coefficients are statistically significant at 1% level, except the leverage coefficient which is significant only at 10% level. The values of AIC and log likelihood are -12,036.91 and 6,018.457, respectively (Table 6).

b. GJR-GARCH(1,1), Student’s error distribution

The estimated model (using a Student’s error distribution) based on the available data is written as

\[
\sigma_t^2 = 0.0000000006830 + 1 (e_{t-1}^2) - 0.1012 (e_{t-1}^2) I_{t-1} + 0.6580 (\sigma_{t-1}^2).
\]

All coefficient terms of the model are significant at the 1% level at least (i.e., \(p = .000\)). The negative value of the leverage factor (-0.1012) indicates that bad news affect volatility more than does good news. The AIC is -13,173.2, and the log likelihood is 6,586.605 (Table 6).

c. GJR-GARCH(1,1), GED

The estimated model based on the available data is written as
\((\sigma_t^2) = 0.000006354 + 0.2573 (e_{t-1}^2) - 0.06781 (e_{t-1}^2) I_{t-1} + 0.7584 (\sigma_{t-1}^2).\)

All coefficients, except for \(\alpha_2 (p = .0786, p > .05)\), are significant at the 5% level, with emphasis on the fact that the insignificant \(p\)-value is relatively low. This could be due to the sample. More data could improve the model and make the effect of \(\alpha_2\) on volatility significant at the 5% level. The AIC is -13,020.86, and the log likelihood is 6,510.429 (Table 6).

FIEGARCH(1,1)

The FIEGARCH(1,1) model is used to analyze the long-term memory properties of financial data. FIEGARCH model captures the relation between the future volatility and the past volatility in the long-term as opposed to short-term, where it is assumed that in the long-run, volatilities are independent. The model equation (with normal error distribution) is written as

\[(1 - \phi_1 L)(1 - L)\delta \log [(\sigma_t^2)] = a_0 + a_1 \frac{e_{t-1}}{\sigma_{t-1}} + b_1 \frac{e_{t-1}}{\sigma_{t-1}}.\]

The parameters of the model are \(\phi_1, d, a_0, a_1,\) and \(b_1\). \(b_1\) is the leverage term; it measures the effect of negative news or error terms on the future volatility of bitcoin. If \(b_1\) is negative (positive) and statistically significant, negative news will affect volatility more (less) than will positive news. The term \(a_0\) is the constant of the model; \(a_1\) is the volatility term; \(\phi_1\) denotes the error term; and \(d\) is a fraction that measures the long-term effect.

a. FIEGARCH(1,1), normal error distribution

The estimated model is written as

\[(1 - 0.47290L)(1 - L)^{0.45288} \log [(\sigma_t^2)] = -0.44080 + 0.40739 \frac{e_{t-1}}{\sigma_{t-1}} + 0.04915 \frac{e_{t-1}}{\sigma_{t-1}}.\]

All coefficient terms of the model are significant at the 1% level at least (i.e., \(p = .000\)). My findings suggest that there is a long-term effect on the volatility of bitcoin price levels. The asymmetry of news is also confirmed. The positive value of the leverage term indicates that good news influences the volatility more than does bad news. The AIC is -12,129.33, and the log likelihood is 6,064.665 (Table 7).

FIGARCH(1,1)
The FIGARCH(1,1) model is used to analyze the long-term memory properties of financial data. It also captures the relation between the future volatility and the past volatility in the long-term as opposed to short-term, where it is assumed that in the long-run, volatilities are independent. It differs from the FIEGARCH model in that there is no leverage effect, so that news is treated symmetrically. The model equation using the normal distribution for the error term is written as

\[(1 - b_1 L)(\sigma_t^2) = a_0 + (1 - b_1 L)e_t^2 - (1 - \phi_1 L)(1 - L)^d(e_t^2).\]

The parameters of the model are \(d, a_0, a_1,\) and \(b_1,\) where \(a_0\) is the constant term, \(d\) is the fractional term, \(\phi_1\) represents the error term, and \(b_1\) is the volatility coefficient. The square of the error term in the equation model indicates the absence of leverage.

a. FIGARCH(1,1), normal error distribution

Using the sample data, I find the following estimated model:

\[(1 - 0.700000000L)(\sigma_t^2) = 0.00001818 + (1 - 0.700000000L)e_t^2 - (1 - 0.400000000L)(1 - L)^{0.50000000}(e_t^2).\]

All coefficient terms of the model are significant at the 1% level at least (i.e., \(p = .000\)). There is a long-term effect that is slightly greater than that in the FIEGARCH model. The AIC is -11,977.12, and the log likelihood is 5,988.5 (Table 8).

Discussion

The maximum likelihood method is used as a quality criterion to select the best-fit model. The method is based on computing the likelihood that a given data set is sampled from a specified statistical model. The model with the highest likelihood number fits the data more than do the others. Throughout this work, I compared five models under three different error distributions. The candidate models are shown in Table 9.

My results indicate that the TGARCH(1,1) model (with Student’s error distribution) is a better fit for the bitcoin volatility than the other models, with a log likelihood index of 6,736.823. The best model equation is written as

\[(\sigma_t) = \alpha_0 + \alpha_1 (e_{t-1}^+ + \alpha_2 (e_{t-1}^-) + \beta_1 (\sigma_{t-1}),\]
where $\alpha_0$ is the constant term, $\beta_1$ is the past volatility coefficient, $\alpha_2$ denotes the leverage term, and $\alpha_1$ is the coefficient of the positive error term. The estimated model is written as follows:

$$(\sigma_t) = 0.000879 + 1(e^+_{t-1}) - 16.621637(e^-_{t-1}) + 2.312122(\sigma_{t-1}).$$

The leverage effect parameter $\alpha_2$ is equal to -16.62, but is statistically insignificant ($p = .174$). This indicates that there is a symmetric effect of the news on the bitcoin volatility. This result is agrees with the findings of Kosapattarapim, Lin, and McCrae (2011), who find that the best model does not necessarily provide statistical significance for all independent variables. The coefficient $\beta_1$ of the past volatility is statically significant, and indicates that if the previous volatility changes by one unit, the future volatility will increase by 2.312 units.

The models that I have tested so far are short-term volatility models. They test the short-term effect of the independent variables (previous error term or previous news, past volatility) on the future volatility. However, some models, such as FIGARCH and FIEGARCH, allow me to measure the long-term effect of the previous news and volatility. The FIEGARCH model differs from the FIGARCH model in that it also measures the leverage effect. Does the available data for bitcoin support the evidence of a long-term effect? In the next section, I will test the two models.

The TGARCH(1,1) model is followed by the next best-fitting models: the GJR-GARCH(1,1) model with the Student’s error (LL = 6,586), the GARCH(1,1) model with the GED error (LL = 6,563), the IGARCH(1,1) model with the GED error (LL = 6,563), and the EGARCH(1,1) model with the Student’s error (LL = 6,549). I first observe that the log likelihood of these models is relatively close to one another. The independent variables in the GJR-GARCH(1,1), GARCH(1,1), and IGARCH(1,1) models are all significant at the 1% level. This shows that when fitting the model with the standard GARCH(1,1) model, the news affects the future volatility, regardless of whether it is good news (positive error term) or bad news (negative error). This result also explains why the IGARCH model is significant for all the variables in the model, since the two models are closely related: the IGARCH model is a constrained standard GARCH model, where the coefficients sum to 1. The GJR-GARCH model is the next-best significant model, following the TGARCH model. This outcome is not surprising since the two models are also closely related. Finally, the EGARCH(1,1) model is significant for all the coefficients, except for the past error term. The leverage is positive and significant,
indicating asymmetry in the effect of news on volatility. More precisely, when using the EGARCH(1,1) model, good news has a greater effect on the volatility of the price level of bitcoins than does bad news.

The FIGARCH and FIEGARCH models with the normal distribution are not among the best five models. So I also tested these models with respect to the Student’s t distribution and the GED and found the same results as under the normal distribution.

**Day effect on bitcoin volatility**

In this section, I examine the weekday effects in the daily volatility. In particular, I use the best model obtained from the previous analysis, the TGARCH(1,1) model with Student’s distribution, to compute estimate values of the volatility of bitcoin on the corresponding days of the sample data. The volatility time series is used as the dependent variable, and the 7 days of the week are the independent variables. To examine whether volatility changes across day of the week, I introduce the week-day dummies into the conditional variance equation of the selected TGARCH(1,1) model. The statistical model depends on the choice of a reference or base day. I set Monday as the base, the model is:

\[
\text{Volatility} = a_1 \text{Tues} + a_2 \text{Wed} + a_3 \text{Thurs} + a_4 \text{Fri} + a_5 \text{Sat} + a_6 \text{Sun}
\]

The regression coefficients measure the change in the volatility as I go from Monday (base) to any other day. For instance, if today is Monday, I expect the volatility to increase or decrease by \(a_1\) units on Tuesday (depending on the sign). If today is Monday, I expect the volatility to increase or decrease by \(a_4\) units on Friday (depending on the sign).

Using Monday as a reference or base, I run the regression model on the data and obtain the following estimated model (Table 10):

\[
\text{Volatility} = -0.0251397 \text{Tues} - 0.0375799 \text{Wed} - 0.0257352 \text{Thurs} - 0.0409717 \text{Fri} - 0.049336 \text{Sat} - 0.0560666 \text{Sun}.
\]

The model coefficients are all negative and significant at the 1% level, indicating that Monday has the highest level of the volatility compared to other 6 days of the week. The highest change in volatility occurs from Monday to Sunday, with a significant decrease of 0.0560666. This means Sunday has the lowest volatility.
After running the TGARCH model and finding evidence of day of the week effects in the short-term, I run the FIEGARCH model (using Monday as the base day) to determine which day of week would be the most significant in the long-run. The results from Table 10 indicate that Monday still has the most impact on the bitcoin volatility compared to other days. In fact, all the coefficients are negative and significant at the 1% level. The estimated model equation is written as

\[
\text{Volatility} = -0.0113857 \text{ Tues} - 0.0194565 \text{ Wed} - 0.0133107 \text{ Thurs} - 0.0204791 \text{ Fri} - 0.0243206 \text{ Sat} - 0.0272436 \text{ Sun}.
\]

The weekday effects in bitcoin price volatility can be explained by the fact that the volatility on Monday is affected by the weekend political and economic news (good or bad). For psychological reasons, news will impact the decision-making of investors on Monday, regardless of the real available information. Moreover, investors’ expectations can increase the volatility on Monday. Since there is less information during the weekend, investors tend to overreact to the new information available on Monday. That is, the high volatility on Monday could be due to the overflow of information on Monday.

The low volatility on Sunday can also be explained by the fact that the volume of bitcoin transactions in China is relatively high compared to other parts of the world. Chinese treat weekend days as regular days, something that is not the case in most countries in North America and in Europe (Solomon 2014). Also Bitcoin is regulated in China.

**Political effects on bitcoin price volatility**

This section examines the effect of political events as a driver of bitcoin price volatility. Since its introduction, bitcoin has witnessed major developments both in terms of trade volume and price. This has resulted in increased attention from the media, regulators, and investors. The increased levels of attention have resulted in increased price volatility for bitcoin (Vigna, Paul, and Michael 2015).
Political events often cause uncertainty as a result of doubts about the direction of future policy decisions. Political events make the financial markets significantly volatile, more so in proximity to close elections or those that might result in radical policy alterations. These events also affect the price volatility of bitcoin either directly through creating uncertainty on the policy direction regarding the cryptocurrency or indirectly through the turmoil created in the financial markets that make investors consider bitcoin to be a safer investment (Taleb, Nassim, and Antifragile 2014).

Bitcoin experienced a year of significant maturity in 2016, and this was, in part, due to the advances made in bitcoin’s underlying technology but also its lower price volatility. The cryptocurrency also started reacting to macroeconomic factors, showing that traders of bitcoin consider the role of bitcoin in the world beyond just the cryptocurrency system as a form of currency hedge. Bitcoin experienced the effects of economic factors on its price, via events such as a hike in the Federal Reserve rates, for the first time in 2016. Bitcoin also experienced an increase in demand as a result of the indication of Greece’s intention to exit from the European Union (Bouoiyour and Selmi, 2016). A similar situation occurred when Britain voted to exit the European Union, and when Donald Trump defied the odds, expectations, and polls to win the U.S. Presidential election. Recent economic surprises in major economies, such as India, Venezuela, and China, have threatened the stability of their fiat currencies and resulted in an increased interest in the digital alternative available. Bitcoin started to act more like gold, that is, as a hedge against the geopolitical, systemic, and monetary risks.

On November 9, 2016, bitcoin prices increased by more than 3% as Donald Trump, a controversial businessman and the Republican presidential nominee, clinched the election against the former U.S. Secretary of State Hillary Clinton. As Trump’s victory became apparent, the global stock markets sharply fell. That Trump was to be the next president of the United States created turmoil in the financial markets world; however, the Bitcoin trading thrived in such an environment, at least until the financial markets recovered from the shock. It is the reason that Donald Trump’s inauguration, in contrast, had little effect on the prices of bitcoin. The prices of bitcoin reached a low of $882.30 hours before Trump’s inauguration (Aitken 2017).
Bitcoin prices often rise as a result of some form of crisis in the financial markets. Incidents that threaten the stability of fiat currencies result in increased demand for the cryptocurrency. It is also clear that political events, either directly or indirectly, exert a significant level of influence on the stock markets and, consequently, bitcoin prices (Bouoiyour, Selmi, Tiwari, and Olayeni 2016). The price volatility over Donald Trump’s victory in the U.S. presidential elections offers a perfect example of when political events affect financial markets and, consequently, bitcoin prices (Moshinsky 2017).

When People bank of China ok’s the use of bitcoin in November 2013, the price level of bitcoin went from 641.23$ to 1075.16$. This significant increase was lowered by the regulations imposed by the Chinese government a month later, to reach 839.93$. In March 2014, the news according to which the IRS would apply a tax on bitcoin led to a decrease of the bitcoin value (see Figure 2).

**SUMMARY AND CONCLUSIONS**

Virtual currency, including bitcoin, is a medium of exchange not adopted by a government. Virtual currencies have several properties common to fiat currencies and commodities. Although virtual currencies and their associated technology offer much potential to investors, consumers, businesses, and government entities, they are also the source of risks and challenges to both users and regulators. One of the risks associated with virtual currencies is the high volatility of their corresponding market value.

This thesis has attempted to find the best model to forecast bitcoin volatility using five concurrent GARCH(1,1) models with three different types of error distributions and apply them to the bitcoin closing price. My results show that the TGARC(1,1) model has the best performance with the Student’s t error distribution. To examine the day of the week effect in the long-term and short-term volatility model, I estimated the FIEGARCH(1,1) and TGARCH(1,1) models. The results designate Monday as the day of week with the highest volatility in both long- and short-term setting.

Following the election of president Trump, bitcoin prices have varied greatly. For instance, according to 99bitcoins.com, as of August 14 2017, the bitcoin price is about 4100$ as opposed to 800$, when I started this work in December 2016. Future work on modeling
Bitcoin volatility should use more recent data to investigate whether the TGARCH model is still the best model for analysis.

An important question remains: what does the future hold for virtual currencies? Cryptocurrencies have a dark history of mishaps, especially with the Mt. Gox scandal in which hackers stole hundreds of millions of dollars through bitcoin transactions. However, the future of bitcoin recently brightened with the U.S. and Canadian governments leading the way in the early stages of regulation and taxation. For instance, the Bank of Montreal announced that it will accept cryptocurrencies as soon as they are regulated and reliable. According to Ben Bernanke, the former Chairman of the Federal Reserve, “[Virtual currencies] may hold long-term promise, particularly if the innovations promote a faster, more secure and more efficient payment system” (Forbes 2014c). In fact, the recent implementation of tokenization in Apple Pay and the endorsement of tokenization by Visa show a clear trend for this new practice, which has been at the heart of every cryptocurrency transaction since its inception. This fact reinforces the future of cryptocurrencies and highlights the advantages of this cryptographic encryption process.

Another potential future of cryptocurrencies is the advent of smart property. Although still at the conceptual level, smart property is likely to change current ownership concepts. Through smart property, an entity can electronically pay to access a cryptographic key, thereby giving users access to a property, either temporarily or permanently. Examples encompass any kind of property, whether nonphysical (e.g., rights, patents, and trademarks) or physical (e.g., automobile rental and resort time-sharing). As an example, in the future world of smart cars, a cryptographic key can ensure that the smart property is secure and that no theft can occur. Hence, although cryptocurrencies are very volatile and risk extinction, the technology behind them is here to stay and may become a major player in the future smart world. Some contend that the chances of an evolved form of cryptocurrency will replace paper money by 2050 if that form is backed by a solid commodity or a set of commodities. The adoption of the technology behind the cryptocurrencies has the potential to validate any single transaction every person makes on the planet.

In the future, money may be likely treated as bits of information. If and when that happens, the exchange of money will be similar to information exchange. The next generation of developers is creating cryptographic breakthroughs that revolutionize financial transactions.
Although the current concept of cryptocurrency will continue to evolve over time, the technology and innovation that come with it will open the way to something much bigger, a revolution not only in financial transactions but also in every transaction.
REFERENCES


Appendix

Figure 1 Time Series Depiction of Bitcoin Daily Price and Other Financial Series
This figure depicts the daily price level of the following series between January 9, 2011 and January 11, 2015: MSCI Emerging Markets Index (EMG), MSCI Global Market Index (WRD), the Euro (EURO) and Chinese exchange rate (YEN), gold bullion (GOLD), Bitcoin (BTC). Data are drawn from DataStream, Eurostat, and blockchain.info.
Figure 2 Time Series Depiction of Bitcoin average Daily Price

This figure depicts the average daily price level of the following series between July 2010 and October 2017. Data are drawn from 99bitcoins.com.
**Table 1 Descriptive Statistics of Bitcoin Daily Price and Other Financial Series**

This table presents the descriptive statistics of the daily price level of the following series between January 9, 2011 and January 11, 2015: Bitcoin (BTC), gold bullion (GOLD), MSCI Emerging Markets Index (Stock-EMG), MSCI Global Market Index (Stock-DVP), the Euro (EURO) and Chinese exchange rate (YEN). Data are drawn from DataStream, Eurostat, and blockchain.info.

<table>
<thead>
<tr>
<th></th>
<th>BTC</th>
<th>GOLD</th>
<th>Stock-EMG</th>
<th>Stock-DVP</th>
<th>EURO</th>
<th>YEN</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean</td>
<td>229.59</td>
<td>1455.91</td>
<td>1011.15</td>
<td>1553.96</td>
<td>0.80</td>
<td>6.42</td>
</tr>
<tr>
<td>Median</td>
<td>143.99</td>
<td>1359.81</td>
<td>1020.22</td>
<td>1602.17</td>
<td>0.78</td>
<td>6.40</td>
</tr>
<tr>
<td>Maximum</td>
<td>1165.96</td>
<td>1944.13</td>
<td>1133.93</td>
<td>1865.12</td>
<td>0.98</td>
<td>6.60</td>
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<tr>
<td>Minimum</td>
<td>2.31</td>
<td>1100.97</td>
<td>794.85</td>
<td>1106.63</td>
<td>0.70</td>
<td>6.21</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>242.77</td>
<td>226.50</td>
<td>65.61</td>
<td>217.02</td>
<td>0.06</td>
<td>0.09</td>
</tr>
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<td>Skewness</td>
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<td>–0.32</td>
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<tr>
<td>Kurtosis</td>
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<td>1.66</td>
<td>3.65</td>
<td>1.69</td>
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<td>1.99</td>
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<td>Jarque-Bera</td>
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<td>128.29</td>
<td>102.84</td>
<td>199.81</td>
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</tr>
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<td>(P-value)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
<td>(0.000)</td>
</tr>
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</table>
Table 2 Volatility model of bitcoin using GARCH(1,1) model
This table models the volatility of bitcoin based on the standard GARCH(1,1) using three error distributions (Normal, student t, GED) using data from July 2010 to December 2016.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>GARCH (1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Normal</td>
</tr>
<tr>
<td>$\alpha_0$</td>
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</tr>
<tr>
<td>$\alpha_1$</td>
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<td>$\beta_1$</td>
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</tr>
<tr>
<td>P value</td>
<td></td>
</tr>
<tr>
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</tr>
<tr>
<td>$\alpha_1$</td>
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</tr>
<tr>
<td>$\beta_1$</td>
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<tr>
<td>Log Likelihood</td>
<td>6014.712</td>
</tr>
<tr>
<td>AIC</td>
<td>-12029.42</td>
</tr>
</tbody>
</table>
Table 3 Volatility model of bitcoin using IGARCH(1,1) model

This table models the volatility of bitcoin based on the IGARCH(1,1) using three error distributions (Normal, student t, GED) using data from July 2010 to December 2016.

*means not applicable;

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Normal</th>
<th>Student t</th>
<th>GED</th>
</tr>
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<tbody>
<tr>
<td>( \alpha_0 )</td>
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<td>0.000006</td>
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<td>( \alpha_1 )</td>
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<td>0.223929</td>
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<tr>
<td>( \beta_1 )</td>
<td>0.772427</td>
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<td>0.76241</td>
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</table>

<table>
<thead>
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<th>P value</th>
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<tr>
<td>( \alpha_0 )</td>
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<td>( \alpha_1 )</td>
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<td>0.00000</td>
</tr>
<tr>
<td>( \beta_1 )</td>
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<td>NA*</td>
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Log Likelihood

<table>
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</tr>
</thead>
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<tr>
<td>Log Likelihood</td>
<td>6014.879</td>
<td>6513.831</td>
<td>6563.885</td>
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</tbody>
</table>

AIC

<table>
<thead>
<tr>
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<th>Student t</th>
<th>GED</th>
</tr>
</thead>
<tbody>
<tr>
<td>AIC</td>
<td>-12029.76</td>
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<td>-13127.77</td>
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</table>
Table 4 Volatility model of bitcoin using EGARCH(1,1) model

This table models the volatility of bitcoin based on the EGARCH(1,1) using three error distributions (Normal, student t, GED) using data from July 2010 to December 2016.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>Normal</th>
<th>Student t</th>
<th>GED</th>
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</thead>
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</tr>
<tr>
<td>$\alpha_2$</td>
<td>0.427155</td>
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<tr>
<td>$\beta_1$</td>
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<table>
<thead>
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<table>
<thead>
<tr>
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<td></td>
<td>6012.218</td>
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<td>6494.132</td>
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<table>
<thead>
<tr>
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<th>GED</th>
</tr>
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<tbody>
<tr>
<td></td>
<td>-12024.436</td>
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<td>-12988.264</td>
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</table>
Table 5 Volatility model of bitcoin using TGARCH(1,1) model

This table models the volatility of bitcoin based on the TGARCH(1,1) using three error distributions (Normal, student t, GED) using data from July 2010 to December 2016.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>TGARCH (1,1)</th>
<th></th>
<th></th>
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<tbody>
<tr>
<td></td>
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<td>GED</td>
</tr>
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<td>$\alpha_0$</td>
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<td>0.0006279</td>
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<tr>
<td>$\alpha_1$</td>
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<td>1.000</td>
<td>0.2457</td>
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<tr>
<td>$\alpha_2$</td>
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<td>-0.01661</td>
</tr>
<tr>
<td>$\beta_1$</td>
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<td>0.6285</td>
<td>0.7971</td>
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<tr>
<td>P value</td>
<td></td>
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</tr>
<tr>
<td>$\alpha_0$</td>
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</tr>
<tr>
<td>$\alpha_1$</td>
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<td>$\alpha_2$</td>
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<tr>
<td>Log Likelihood</td>
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<td>6736.823</td>
<td>6464.466</td>
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<tr>
<td>AIC</td>
<td>-11216.42</td>
<td>-13473.64</td>
<td>-12928.93</td>
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</table>
Table 6 Volatility model of bitcoin using GJR-GARCH(1,1) model

This table models the volatility of bitcoin based on the GJR-GARCH(1,1) using three error distributions (Normal, student t, GED) using data from July 2010 to December 2016.

*means not applicable;

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>GJR-GARCH (1,1)</th>
<th>Normal</th>
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</tr>
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<td>$\alpha_2$</td>
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<td>-0.06781</td>
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</tr>
<tr>
<td>$\beta_1$</td>
<td>0.7636</td>
<td>0.6580</td>
<td>0.7584</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P value</th>
<th>GJR-GARCH (1,1)</th>
<th>Normal</th>
<th>Student t</th>
<th>GED</th>
</tr>
</thead>
<tbody>
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<td>0.000</td>
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</tr>
<tr>
<td>$\alpha_1$</td>
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<td>0.000</td>
<td></td>
</tr>
<tr>
<td>$\alpha_2$</td>
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<td>0.00477</td>
<td>0.0786</td>
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</tr>
<tr>
<td>$\beta_1$</td>
<td>0.000000</td>
<td>NA*</td>
<td>0.000</td>
<td></td>
</tr>
</tbody>
</table>

Log Likelihood | 6018.457 | 6586.605 | 6510.429 |

AIC            | -12036.91 | -13173.2 | -13020.86 |
**Table 7 Volatility model of bitcoin using FIEGARCH(1,1) model**

This table models the volatility of bitcoin based on the long term model FIEGARCH(1,1) using the normal error distributions and data from July 2010 to December 2016.

<table>
<thead>
<tr>
<th>Coefficients</th>
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</tr>
</thead>
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<td></td>
<td></td>
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<td>( a_1 )</td>
<td>0.40739</td>
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<tr>
<td>( \phi_1 )</td>
<td>0.47290</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>0.04915</td>
</tr>
<tr>
<td>( d )</td>
<td>0.45288</td>
</tr>
<tr>
<td>( a_0 )</td>
<td>-0.44080</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P value</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>( a_1 )</td>
<td>0.000000</td>
</tr>
<tr>
<td>( \phi_1 )</td>
<td>0.000000</td>
</tr>
<tr>
<td>( b_1 )</td>
<td>0.000003414</td>
</tr>
<tr>
<td>( d )</td>
<td>0.000000</td>
</tr>
<tr>
<td>( a_0 )</td>
<td>0.000000</td>
</tr>
</tbody>
</table>

Log Likelihood | 6064.665
AIC            | -12129.33
Table 8 Volatility model of bitcoin using FIGARCH(1,1) model

This table models the volatility of bitcoin based on the long term model FIGARCH(1,1) using the normal error distributions and data from July 2010 to December 2016.

<table>
<thead>
<tr>
<th>Coefficients</th>
<th>FIGARCH (1,1)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Normal</td>
<td></td>
</tr>
<tr>
<td>$b_1$</td>
<td>0.70000000</td>
</tr>
<tr>
<td>$\phi_1$</td>
<td>0.40000000</td>
</tr>
<tr>
<td>$d$</td>
<td>0.50000000</td>
</tr>
<tr>
<td>$a_0$</td>
<td>0.00001818</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>P value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$b_1$</td>
</tr>
<tr>
<td>$\phi_1$</td>
</tr>
<tr>
<td>$d$</td>
</tr>
<tr>
<td>$a_0$</td>
</tr>
</tbody>
</table>

| Log Likelihood | 5988.5 |
| AIC           | -11977.12 |
Table 9 Five best GARCH models based on bitcoin volatility
This table summarizes the volatility of bitcoin based on the five best models.

<table>
<thead>
<tr>
<th>Model</th>
<th>Error distribution</th>
<th>Log-likelihood</th>
<th>AIC</th>
</tr>
</thead>
<tbody>
<tr>
<td>TGARCH (1,1)</td>
<td>Student t</td>
<td>6736.823</td>
<td>-13173.2</td>
</tr>
<tr>
<td>GJR-GARCH (1,1)</td>
<td>Student t</td>
<td>6586.605</td>
<td>-13173.2</td>
</tr>
<tr>
<td>IGARCH (1,1)</td>
<td>GED</td>
<td>6563.885</td>
<td>-13127.77</td>
</tr>
<tr>
<td>GARCH (1,1)</td>
<td>GED</td>
<td>6563.62</td>
<td>-13127.24</td>
</tr>
<tr>
<td>EGARCH (1,1)</td>
<td>Student t</td>
<td>6549.615</td>
<td>-13099.23</td>
</tr>
</tbody>
</table>

Table 10 the day-of-the-week effect on bitcoin volatility
This table summarizes the results of the day of the week effect on the volatility using the best of both the short and long term model.

<table>
<thead>
<tr>
<th>Volatility</th>
<th>TGARCH</th>
<th>FIEGARCH</th>
</tr>
</thead>
<tbody>
<tr>
<td>Days</td>
<td>Coefficients</td>
<td>P-value</td>
</tr>
<tr>
<td>Tues</td>
<td>-0.0251397</td>
<td>0.000</td>
</tr>
<tr>
<td>Wed</td>
<td>-0.0375799</td>
<td>0.000</td>
</tr>
<tr>
<td>Thurs</td>
<td>-0.0257352</td>
<td>0.000</td>
</tr>
<tr>
<td>Fri</td>
<td>-0.0409717</td>
<td>0.000</td>
</tr>
<tr>
<td>Sat</td>
<td>-0.049336</td>
<td>0.000</td>
</tr>
<tr>
<td>Sun</td>
<td>-0.0560666</td>
<td>0.000</td>
</tr>
<tr>
<td>Cons</td>
<td>0.0726925</td>
<td>0.000</td>
</tr>
</tbody>
</table>