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LA THÈSE A ÉTÉ MICROFILMÉE TELLE QUE NOUS L'AVONS RECUE
HUMAN CAPITAL AND DISTRIBUTION OF PERSONAL INCOME: A THEORETICAL ANALYSIS AND AN EXAMINATION OF SOME ASPECTS OF INCOME DISTRIBUTION IN SRI LANKA

by

Leslie L. Gunaratne

Thesis presented to the School of Graduate Studies of the University of Ottawa as partial fulfillment of the requirements for the degree of Doctor of Philosophy (Economics)

Ottawa, Canada, 1982

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CURRICULUM STUDIORUM

Lokupitiyage Leslie Gunaratne was born in Colombo, Sri Lanka. He received the Bachelor of Science degree with honours in Mathematics and Physics from the University of Ceylon, Peradeniya, Sri Lanka, in 1964. In 1972 he received a Diploma in Mathematical Statistics from the Vidyodaya University of Ceylon, Nugegoda, Sri Lanka. He won a Commonwealth Scholarship tenable in Canada, in 1973. In 1974, he received a Master of Mathematics degree from the University of Waterloo, Waterloo, Ontario, Canada.
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CHAPTER I

INTRODUCTION

The pursuit of equality has a high value for the great majority of mankind. At first, there were many who advocated universal equality, by which they meant that all men should be equal in all respects. Today, there are many who advocate equalizing, not in all, but in certain respects, as the ideal state towards which society should move. In a prelude to his book, Equality, R. H. Tawney, the British egalitarian, remarked:

So to criticize inequality and to desire equality is not, as is sometimes suggested, to cherish the romantic illusion that men are equal in character and intelligence. It is to hold that, while their natural endowments differ profoundly, it is the mark of a civilized society to aim at eliminating such inequalities as have their source, not in individual differences, but in its own organization, and that individual differences, which are a source of social energy, are more likely to ripen and find expression if social inequalities are, as far as practicable, diminished.¹

Equality has long been a matter of social concern and a source of political controversy. Some 2500 years ago, Confucius observed that "inequality is to be lamented more than scarcity" ² Aristotle remarked that,"inequality is generally at the bottom of internal warfare in states, for it is in their striving for what is fair and equal that men become divided" (Aristotle, Politics, V.I). It aroused the interest of the classical economists during the time of Ricardo, who wrote to Malthus: "Political economy, you think, is an enquiry into the nature and causes of wealth—I think it should be called an enquiry into the laws which determine the


division of the produce of industries amongst the classes who concur in its formation.\textsuperscript{3} The Great Depression, World War II and the Keynesian Revolution brought about a marked decline in professional concern about distribution theory, for some time, until its revival during the recent years.

1. The Dichotomy between Equality and Efficiency

The political and social institutions of modern market economies provide universally distributed rights and privileges that proclaim equality of all citizens. Nevertheless, the egalitarian sentiments of these societies are juxtaposed with their inequalitarian realizations, making the pursuit of equality as elusive as the pursuit of well-being. Contemporary views on the optimal distribution of economic rewards in society are diverse and often contradictory. There is disagreement as to the fairness or justice of greater equality. Those who are opposed to egalitarian policies often claim that there is a sharp distinction between considerations of fairness and considerations of equality. By contrast, egalitarians believe that justice involves equality, and that greater fairness involves a diminution of social inequalities.\textsuperscript{4} Some reject reward for contribution as an ethical principle, but accept it, within limits, as a pragmatic necessity. In so doing, they compare the degree of efficiency, equality, and freedom in current market economies with that obtainable from full-fledged socialism.

"Modern society", writes Raymond Aron in a related work, "seems to conform to two imperatives: to produce as much as possible through mastery of


\textsuperscript{4} For the purpose of this thesis, it is assumed that less inequality means greater equity.
the forces of nature and to treat its members as equals. It is Aron's view that there is some tension between the two imperatives and, in addition, certain contradictions between different egalitarian ideals. The principle of equality often conflicts in its policy implications with other values like economic efficiency, individual freedom and the maintenance of legitimate expectations. Commenting on the dichotomy between the egalitarian ideal and inequalitarian realities of modern societies, Arthur M. Okun aptly states: "Such is the double standard of a capitalist democracy, professing and pursuing an egalitarian political and social system and simultaneously generating gaping disparities in economic well-being! through a market system that allows the big winners to "feed their pets better than the losers can feed their children." Such society is, in a sense, a "split-level institutional structure". Radical economists view inequality in market economies as a structural necessity, providing incentives, rewards, and penalties that are instrumental in promoting efficiency in the use of resources and contribute toward generating a larger, and a growing national product. According to them, a society premised on notions of liberty and equal justice is juxtaposed with an economic system in which inequality is functional.

To the extent that the system of incentives, rewards and penalties operating in a market economy succeeds as an effective allocative mechanism, it helps to generate an efficient economy. But, that pursuit of efficiency creates inequalities which can be, and often are, extremely costly in social and political

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7 Ibid., p. 1.

8 Ibid., p. 4.
terms. Efforts to promote equality, by public policy, represent a deliberate interference with the results generated by the market place; and they too are rarely costless. Such policies can weaken incentives to produce and otherwise impair economic efficiency. Society, thus, confronts a tradeoff between equality and efficiency. The choice is between somewhat more equality at the expense of efficiency or somewhat more efficiency at the expense of equality. Ironically, the actual tradeoff preferred by society had, often, been subject to conjecture more than value judgements on the part of the society.

Modern economic theory has concentrated on the pursuit of efficiency almost as wholeheartedly as it has avoided the issue of equity. Mainstream economic thought has paid very little attention even to the definition of the concept of equity, or the just distribution of economic resources in society. Harry G. Johnson has discarded the notion of equity in economics as a type of neurosis, attributing such misguided preoccupations to a "naive and basically infantile anthropomorphism". Milton Friedman saw very little relevance in the concept, and instead substituted the goal of freedom:

"But freedom for the pike is death for the minnows", says R. H. Tawney. It is possible that equality is to be contrasted, not with liberty, but only with a particular interpretation of it.

---


Often quoted criticism of mainstream welfare economics is that it avoids
judgements on the distribution of economic rewards, altogether. The concept of
Pareto optimality, a cornerstone of contemporary welfare economics is devoid of
value judgements as to the desired or just distribution of economic resources in
society. There is an infinite set of Pareto optimal points open to society; some of
them representing extreme cases of inequality, nonetheless, efficient in Pareto
sense. A decision to cut into the affluence, to make the lot of the poor any better
can lead to a Pareto inferior situation, although such decision can, nevertheless,
bring forth a more equitable distribution and an increase in general economic well-
being. The pre-eminence of Paretian dogma in modern welfare economics has
made it unsuitable for investigating problems of inequality. It is indeed pertinent,
in this context, to quote D. M. Winch, who asserts:

The Paretian value judgement is only a value judgement. It may well be
rejected by some. But before it is rejected by economists, either
explicitly or by implication, it must be remembered that virtually the
entire edifice of economic theory as we know it today is built on Paretian
premises. If those premises are rejected, that theory becomes irrelevant
to the world in which we live.12

Another frequently employed measure of social welfare, the level and
rate of growth of GNP, totally disregards the interpersonal distribution of the
national product. Growth without any consideration to the concomitant
distribution, does not and cannot guarantee a higher standard of material living for
all. A larger product does not necessarily mean prosperity for all.

The relative neglect of the equity issue has not, of course, passed
completely unnoticed, and in recent times, it has been one of the main criticisms
of 'mainstream' economics made by radical economists. J. Weeks argues in a
recent work, that "inequality is what economics should be all about. But, in fact,

12 D. M. Winch, Analytical Welfare Economics (Harmondsworth: Penguin,
economics as it is taught and practised by economists deals very little with inequality.13 Ignorance is seldom arbitrary and the oversight exists for some highly rational reasons. As Gunnar Myrdal has put it:

People who are better off usually have done their best to keep their minds off the equality issue. . . . In every country there have been whole systems of psychological and ideological barriers protecting the well-to-do from knowledge of facts which would be embarrassing to them.14

The dominance of the efficiency criterion to the exclusion of equity considerations is justified by some, on the grounds that efficiency constitutes a value-free, non-controversial goal, universally desired by society-at-large. After all, the idea that more is better would seem to be incontrovertible, and, as long as economic analysis concerns itself with producing more aggregate income, the equity question of who should get what can safely be left in the hands of political scientists, theologians, and citizens, groups that are presumably meant to feel more at ease with value judgements.15 Whether the efficiency goal is, in fact, value free is another issue, altogether. It is often argued that the efficiency goal is far from being value free and is highly dependent upon the underlying prior distribution. The distributional factors, among other things, determine the production possibility frontier of an economy, the preference function, and ultimately the production mix of a society.

There are also some conceptual flaws in this vision of reality. The criterion of efficiency entrenched in contemporary welfare economics is hardly


unambiguous; and the dichotomy between dynamic efficiency and static efficiency is not merely a question of semantics. A distinction is often made between dynamic efficiency which is synonymous with maximizing the rate of growth of GNP and static efficiency which is closely related to the competitive world of neoclassical analysis. The criterion of efficiency in terms of the growth objective, "a sustained and significant rise in product per capita",\(^{16}\) which Kuznets suggested, offers a pragmatic definition of efficiency, in the context of countries that are at a low level of development.

In recent times, considerable attention has been directed to the hypothesis that, at the lowest levels of development, growth tends to increase inequality. A broad generalization would point to the fact that, in the poorest countries, growth works against the poorest segments of the population. It has also been observed that countries experiencing rapid rates of economic growth suffer from increasing inequality. Thus, the evidence available on recent growth experience in the developing world suggests that there is an inevitable, and rather severe, conflict between the goal of equity and growth performance of these economies. This gave an impetus for renewed interest in the study of the tradeoff between equity and growth in these countries. While the egalitarians claim the necessity to sacrifice growth in order to achieve better distribution, the proponents of economic growth contend that it is only through an increase in the product that there will be anything significant to distribute. The choice of equity becomes more attractive as an economy develops and, at a relatively low level of development, raising of minimum standards of living should take precedence over equity. In recent years, however, there is growing skepticism about the "grow now,

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redistribute later" package. Expressing concern over such policies, Irma Adelman and others claim:

After two decades of concern with the problem of raising per-capita GNP in low-income developing countries, the development community of the 1970s has shifted its focus to the challenge of increasing the equity of the distribution of income. The shift proved to be dramatically needed, as the empirical studies of the distribution of the benefits from economic growth showed that the expected trickle-down was not taking place. More serious, a number of studies indicated a systematic worsening, both relative and absolute, in the position of the poorest stratum of income recipients.  

Kuznets, the protagonist of the tradeoff hypothesis, claims that the relationship between levels of economic development and equity is inverse U-shaped, with more egalitarian income distributions being characteristic of both extreme economic underdevelopment and high levels of economic development. Between these extremes, however, the relationship is, for the most part, inverse up to a point; rapid growth characterized by a worsening of income distribution. After a "middle" development level is reached, further growth is associated with increasingly equal distribution of income. He concludes that, as income increases from very low levels of development toward higher levels, the distribution of income must first worsen before it can improve. The inverse U-shaped relation between growth and equity was the "normal science" (in Kuhn's sense) in growth-equity studies, until the emergence of counter-evidence in recent growth experience in some countries. The evidence coming from the deviant countries, mainly Korea and Taiwan, could well "blur" the tradeoff paradigm (in the sense used by Kuhn). This phenomenon marks an epoch in the study of the growth-equity...


relationship, of growth with equity optimism; the optimism that there is no inevitable conflict between the societal objectives of growth and equity. The hope is now stronger that a society can achieve one objective without constraining the path to the achievement of the other objective. Given this experience, an improved understanding of the causal relationships between growth and equity would point out the extent to which societies that are not yet on the frontier could, in fact, improve their position with respect to both objectives. It would provide greater hope to these countries, that obstacles can be identified and eventually removed by the administration of suitable corrective policies.

Even though a full understanding of the interrelationships between growth and equity is still elusive, identifying factors that are conducive to a softening or even an elimination of the observed conflict between growth and distribution objectives may be well worth the effort. It is often claimed that the increasing inequalities during the early stages of development are not a result of economic growth per se. They are rather a result of uneven development, a scenario characteristic of the developing world. Rapid growth usually proceeds in a dualistic fashion, so that growth in the leading sectors is more rapid than that in the lagging sectors. Rapid economic growth in some sectors, regions, or groups of the population often goes hand in hand with the stagnation of other sectors. Those endowed with resources (physical and human) grow and prosper, while the less fortunate must merely wait for the results of economic progress to "trickle down". The uneven distribution of human resources, it is often asserted, also causes wide disparities in productivity and thus in income.

Policies aimed at reducing dualism by widening the base for economic growth are, thus, conducive to greater income equality, but even when a transition from sharply dualistic growth to more broadly based economic growth is accomplished, the poorest segments of the population still typically benefits from
economic growth only if widespread efforts are made to improve the human resource base. A favourable impact on distribution is likely only when growth is achieved through a broadly based strategy predicated upon the wide spread and the application of educational skills. At all levels, increased access to the acquisition of human capital contributes to improving the income distribution through its favourable impact on the share of the middle-income groups. A major spread of education together with effective use of the increased stock of human capital in a labour and human capital intensive growth strategy, coupled with a significant reduction in dualism, are often cited as the factors that contributed to the success story of the deviant countries.

The foregoing brief review indicates how a widespread and full utilization of human capital have helped some countries to achieve unprecedented growth rates, while at the same time maintaining a relatively less unequal income distribution. It is, therefore, incumbent on those countries that desire both these goals without a possible tradeoff, to examine to what extent a similar environment is present in them, and if such a conducive environment is not present, possible ways and means of developing it. Needless to say, these lessons are of paramount importance to Sri Lanka, a country that aspires to achieve rapid economic growth within its egalitarian traditions. It is, therefore, prudent, from an analytical point of view, to review critically the growth path and dynamics of growth in Sri Lanka's economy; particularly its relationship to equity, with a view to identifying salient factors in its growth process.

19 However, increasing incomes of middle class does not seem to eliminate poverty of the lowest income groups, but to the extent that the low income groups move into the middle class, this is one way of dealing with it.

2. Growth and Equity in Sri Lanka:

A Historical Perspective

In contemporary Sri Lanka the issue of growth and equity must be viewed in a historical context, in order to discern the underlying long term causal relationship. Here, three questions are of particular interest. First, has the inequality in distribution of income increased in the course of the country's economic growth? Second, whether growth objectives and distribution objectives, viewed together, have constituted tradeoffs or complements. Third, what factors determine the secular level and trends of income inequalities?

The first step towards defining the dimensions of this problem is to consider the extent of inequality in the course of the country's economic growth. Table 1 presents data, for Sri Lanka, showing GNP growth rate and the Gini coefficient. The GNP growth rate is computed as the average of real GNP growth rates in the five years preceding that for which distribution is measured. The Gini coefficient is in respect of the distribution of spending unit\textsuperscript{21} income.

A tradeoff between growth and equity is discernible during the period 1953-1979. It is observed that the periods characterized by rapid rates of economic growth suffer from increasing inequality. In the five year period preceding 1953, Sri Lanka's economy experienced unprecedented growth rates averaging 7.1 percent. Fast growth during that period, however, was not costless in terms of its toll on income distribution. Rapid growth went hand in hand with increasing inequality. The country recorded the highest Gini in 1953. The growth rate of 4.2 percent in 1973 compared to that in 1953 shows a decline of 40.8 percent, while the Gini coefficient too declined by a remarkable 24.0 percent,

\textsuperscript{21} Spending units are smaller groups (subsets) within a household that act as more or less independent units for spending purposes.
showing a substantial improvement in income distribution during the 1953-1973 period. A trend toward greater inequality along with economic growth also characterized the 1973-1979 period. These data point to the increasing inequality accompanying rapid growth. The Gini of 0.40 in 1978/79 represents an increase of 14.3 percent over that in 1973. Indeed, there is also a hopeful side to this scenario.

Table 1.- GNP-Growth Rate and Gini Coefficient in Sri Lanka, By Selected Years

<table>
<thead>
<tr>
<th>Year</th>
<th>Growth Rate of GNP</th>
<th>Gini Coefficienta</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953</td>
<td>7.1</td>
<td>0.46</td>
</tr>
<tr>
<td>1963</td>
<td>5.2</td>
<td>0.45</td>
</tr>
<tr>
<td>1973</td>
<td>4.2</td>
<td>0.35</td>
</tr>
<tr>
<td>1978/79</td>
<td>4.9</td>
<td>0.40</td>
</tr>
</tbody>
</table>


a The Gini coefficient is defined in conjunction with the Lorenz curve which relates the proportion of income-recipient units to the proportion of total income, as the ratio of the area between the Lorenz curve and the diagonal representing the line of perfect equality—which measures the departure of the Lorenz curve from complete equality—to the area of maximum inequality, which is given by the triangle under the line of equal distribution (diagonal). It ranges from 0 for complete equality to 1 for maximum inequality. The larger the ratio, the greater is the inequality of distribution of income. For simplicity, the term "Gini" is used to mean Gini coefficient. The data on Gini coefficient are from the abovementioned survey reports, and are based on grouped observations.
During the five-year period preceding 1978/79, the economy experienced modest growth rates averaging 4.9 percent, while at the same time maintaining a relatively less unequal distribution compared to other countries in the developing world. The Gini of 0.40, undoubtedly, is lower than that for most contemporary developing countries. Suffice it to say, that Sri Lanka has managed to maintain a relatively low Gini, in the face of very high growth rates of 8.2 and 6.2 percent respectively in 1978 and 1979.

Turning to Taiwan and Korean experience, their levels of Gini are lower (in the 0.3 range rather than in the customary 0.5 range) and their Ginis have not risen during the first phase of rapid transition growth. Rapid growth was compatible with favourable levels and trends in the distribution of income. Of special interest here, is the issue, whether the Taiwan and Korean pattern could be imitated in Sri Lanka. Whether, subject to its mixed-economy constraint, Sri Lanka can maintain growth and development within a market-oriented structure of rewards as incentives and innovation and yet provide for distributive justice, by a suitable choice of policy directed toward changing the nature of the growth path rather than intervention to achieve redistribution by direct government action to replace the market.

It is pertinent in this respect to identify the egalitarian forces in the growth path of Sri Lanka's economy, that have tended to generate a relatively less unequal income distribution. It would also be useful to examine why a reversal of the egalitarian trend occurred during the subsequent period of rapid growth. Another matter of particular interest is whether the reversal of the egalitarian

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22 As a spending unit is a subset of household and, presumably, there are wider disparities of spending unit income within household, it is reasonable to assume, a priori, that inequality of distribution of household income is even less than that of spending unit income.
trend is a transient phenomenon consequent on imbalances accompanying modern fast growth or the beginning of an adverse long-term trend. A broad outline of Sri Lanka's social and economic development since the immediate post-war period might shed some light on the causality underlying its growth-equity relationship.

A significant feature of post-war growth experience in Sri Lanka was its dualistic nature, partly attributable to the legacy of the colonial era. The modern sector, centred around the plantations, but also including a variety of supporting commercial, financial, transportation, communications, and other service activities, experienced rapid sustained economic growth. The impressive development of the modern sector, however, had little immediate impact on the traditional sector. Thus, growth proceeded in a dualistic fashion causing greater inequality of income. This effect, however, was partly offset by an improvement in the human resource base, the result of widespread education. Enumerating the salient features characteristic of the Sri Lanka economy, in the post-war period, Gunnar Myrdal states:

With a flourishing and efficiently run plantation industry, Ceylon has for some time enjoyed much higher levels of living than India. Malnutrition is far less common and the level of literacy in colonial times was the highest in South Asia, except for the Phillipines. Inequalities are generally smaller than in India. In particular, the social stratification in Ceylon is more egalitarian and women are less subjugated. The existing inequalities are less rigidly rooted and unyielding. ... 23

He also saw a wide spread of education and commented: "...from the beginning levels of literacy and education generally were relatively high, and educational opportunities more democratically dispersed. ..." 24

Continuous efforts were made by successive governments to rid the economy of its duality by widening the growth base. At the same time, policies


24 Ibid., 3:1635-1636.
were directed towards increasing investment in education with a view to improving the general skill level of the labour-force and achieving a better spread of education. These policies were complemented by the use of labour and human capital-intensive technology in its growth path.

The Free Education Act of 1945 provided a land-mark in the history of educational development in Sri Lanka. The scheme allowed for universal free education from kindergarten to the university, and had an enormous impact on the general level and distribution of education. "Although it cannot be said that all children have equal access to a good education even today," writes Gavin Jones, "a great deal of progress has been made, and in terms of equality of educational opportunity it is doubtful whether Sri Lanka can be matched by any other developing Asian country." To support his argument, Jones quotes from the Report on the Preliminary Survey of Education, prepared by the World Bank:

To judge from its education pyramid alone it might be said that Ceylon, after Japan, had the best developed education system in Asia. A very high proportion of children attend school, the wastage rates are relatively low, the numbers of girls are little lower than those of boys, schools at the secondary level are well attended and very large numbers take the school certificate examination at the end of the tenth year.... Moreover, Ceylon has provided this education free at all levels, so that it is not surprising that the proportion of its GNP spent on education is almost 5 percent and the highest in Asia.

Under a major educational reform in 1960, the principle of providing equal opportunity for schooling was further enhanced. Consequent on this policy, all schools (except for a small number accounting for 3 percent of the total enrollment) were brought under comprehensive state financing, and overall state management.


Formal education constitutes a large component of public and private spending in Sri Lanka. Ever since the introduction of the free-education scheme, the amount of resources allocated to education each year has been increasing. In 1960, the total outlay on formal education was Rs 296 million compared with a total gross material capital formation for that year of Rs 978 million. The estimate, however, does not include private spending on formal education and earnings foregone, which can be substantial. Using the conventional concept of material capital, 15.6 percent of GNP is accounted for in gross fixed capital formation, while public spending on formal education constitutes 4.7 percent of GNP. If the direct outlays on formal education are considered as part of capital formation in a broad sense, that increase the real gross capital formation to 20.3 percent of GNP.

Data on capital formation (see Table 2) show the growth of human capital in relation to material capital, over the years. The investments in human capital have been of a magnitude to alter radically the conventional measures of the share of GNP allocated to gross capital formation. The data point to the fact that the investment in human capital relative to income had been increasing up to 1963, maintaining an average of 4.5 percent during the 1959-1973 period. This increase explains the modest growth rates experienced by the economy (see Table 1) in spite of a relatively low rate of material capital formation. It is also to be observed that investment in human capital had been rising relative to investment in material

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capital, up to 1963. This long term trend, ceteris paribus, explains the increase in earnings relative to property income. To the extent that the more equal distribution of investment in human capital equalizes earnings among human agents, these changes in the investment in human capital are a major factor reducing inequality in the distribution of personal income.

Table 2—Capital Formation in Sri Lanka, By Selected Years\textsuperscript{a}

<table>
<thead>
<tr>
<th>Year</th>
<th>Material Capital</th>
<th>Human Capital</th>
<th>Total Capital (Material &amp; Human)</th>
<th>Human Capital as a Percentage of Material Capital</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953</td>
<td>11.3</td>
<td>2.5</td>
<td>13.8</td>
<td>21.9</td>
</tr>
<tr>
<td>1963</td>
<td>17.0</td>
<td>4.6</td>
<td>21.6</td>
<td>27.2</td>
</tr>
<tr>
<td>1973</td>
<td>19.2</td>
<td>4.4</td>
<td>23.6</td>
<td>23.0</td>
</tr>
<tr>
<td>1978/79</td>
<td>20.3</td>
<td>3.3</td>
<td>23.6</td>
<td>16.5</td>
</tr>
</tbody>
</table>

Sources: Donald R. Snodgrass, Ceylon: An Export Economy, Tables A2, A5.


\textsuperscript{a} Data represent averages for the preceding five years.

Needless to say, investment in human capital has been an "engine of growth" and a strong egalitarian force in Sri Lanka. The wide spread of education, undoubtedly, was made possible by the advent of free education. H. N. S. Karunatilake voiced a similar point of view:

Although the economic implications of free education were not examined in the early years, it was, however, the government's view that a scheme of free education would enable the great majority of children to benefit
from the scheme. Thus, many children, especially from the rural areas, who could not afford an education at fee levying schools, had access to free education and opportunities of securing positions of standing in the country's administration and business. A large number of students who are now in the universities, possibly more than 85 percent, are those who have benefited from free education; the bulk of those who have secured jobs in the public service since 1965 are in this category.  

He further asserts "Implications of the free education scheme for development of human resources and manpower have been very significant. Despite any defects in the scheme, the principle that every individual in the country should have access to education is one of the fundamental principles..." But quite contrary to the principle, its realizations fell short of its objective of equalizing opportunities for all, to secure education to make them capable of fulfilling their personal differences. Despite huge chunks of money being spent on education in successive years, there are still glaring disparities between the rich and the poor; urban and rural; elite and the under-privileged. Although it was expected that much would flow from the full implementation of the 1945 Free Education Act, designed to reduce the inequalities in educational opportunity, many children still do not have access to a good education. Thus, the realization of the equality of opportunity concept cherished in the free-education scheme is far more elusive than that of equality of income. It is pertinent, in this respect, to investigate whether the free-education scheme, in fact, provided equal opportunity for all children. On this point, none other than Gavin Jones' observations are illuminating:

Illiteracy rates in Sri Lanka are among the lowest in the region, and secondary education is well developed. However, retention rates from grade to grade at the primary level in Sri Lanka are lower than those of a number of Asian countries: grade 8 enrollments are only about 50 percent of the enrollment of the same cohort in grade 1, even though education has been free in Sri Lanka since 1952.

29 Ibid., p. 155.
Educational statistics also show that a fair proportion of the children leaving elementary schools do not pass on to some form of secondary and post-secondary education. In reality, the free education system, in spite of its beneficial effects, had until recently done little to make such education accessible to the children of the poorest. It is claimed that lower retention rates are the result of the inability of some parents to send their children to school due to poverty and backwardness more than of any lack of innate abilities of the children. It has often been pointed out that some children are taken out of school by their parents in order that they may work and help support family. These children are of considerable economic value as workers, and some parents are keeping them from school for that reason. It is possible that very few bright and promising children thus have their education cut short, but farm families in rural areas, particularly, still place a considerable value on the work that their children could do for them.

Any programme aimed at equalizing access to education has to reckon the cost entailed in the earnings that older children will have to forego. With this accompanying cost of foregone earnings, children from an impoverished background, particularly those from rural areas, confront higher real costs than their counterparts from well-to-do families. Besides, the children from families with low incomes and with little wealth, in general cannot afford, even if they wanted, to finance their education out of their own funds. The general ability of these children or of their parents to borrow funds in the capital markets for investment in education, is also limited. Consequently, the low-income level of the parents in the lower income groups and the associated low educational levels would make for a much lower investment in their children than would be true of higher income groups. Thus, it is seen that, the poor and under-privileged are faced with a vicious circle. They face a severe budget constraint in acquiring human capital. Real costs including earnings foregone are higher. The result is the paradox of the
poor and under-privileged investing less in their human capital; the rich investing more. The distribution of human capital becomes progressively dispersed. The least investment is done by the very groups that must invest the most if the inequality of income distribution is to be narrowed. If left to take its own course, investment in human capital is a process of "circular and cumulative causation" which tends to award its favours to those who are already well endowed.

There are, indeed, challenges to the dominant views concerning equality of opportunity for education, provided by the free-education scheme. These challenges, however, are not to equality of opportunity as a guiding social principle, but to some factors inhibiting the realization of this idea. Certainly, it was thought, talent should be rewarded, but it cannot be rewarded properly unless it has a true opportunity to express itself. It is often argued that, the present inequalities are the result, not of opportunities denied, but of the inability of some to take advantage of opportunities offered due to disabilities imposed on them by the environment.

In reality, the opportunities for access to a good education are not uniformly available in all sectors of society. They depend on geographical areas, rate of urbanization, the composition of the population, and so forth. Poor neighbourhoods and rural areas have fewer opportunities than other better endowed areas. Regional imbalances in the distribution of opportunities for education do exist and are significant. The best educational facilities have tended to be concentrated in the south-west maritime districts and in the northern peninsula part of the country. Social disparities and differences between privileged and under-privileged are even more pervasive and significant. Thus, it is seen that the wealth, education, location of residence of parents and a host of other socio-economic factors determine a student's access to a secondary and post-secondary education. Family background and other environmental factors, thus, open a good
education to some and close it to others. It is not surprising to see, in this context, that most of the students who gain admission to the coveted science, medicine and engineering places in the university are drawn from well-to-do strata. Financial and other obstacles to education confronted by some children are important barriers to the achievement of the goal of equality of opportunity. Given these barriers, the principle of equality of opportunity, even when properly applied, is consistent with a large amount of actual inequality.

This tension in the traditional notion of equality seems to call for a reinterpretation of the concept, in view of the recent challenges to implications of the free-education scheme. The free-education scheme is, indeed, an unfortunate example for the advocates of equality of opportunity. The provision of universal free education does not and cannot guarantee equal opportunities for unequal people. Education should be universal, that is, it should be provided for all. But universal educational services is not the same thing as equal opportunity. Offering the same opportunity to every child brings out inequalities. For example, under the free-education scheme, children from poor families cannot reap the full benefits because of disabilities from which they suffer, while the well-to-do enjoy the lion's share. The publicly supported free-education scheme is more intensively used by the privileged sectors. This represents, as Karl Marx pointed out, subsidies to the well-to-do by the poor. Under these circumstances, unequal opportunities for unequal persons give a nearer approach to equality than equal opportunities for unequal persons.

Equality of opportunity is, basically, rooted in the notion of a fair race, where everyone is even at the starting line. It is impossible to have equality of opportunity if people start out highly unequal. R. H. Tawney, the British egalitarian, remarked "The existence of such opportunities in fact, and not merely
in form, depends, not only upon an open road, but upon an equal start". The traditional notion of equality of opportunity has proved to be inadequate in ensuring equal access to education in contemporary societies. This is the rationale for Tawney's contention that:

...equality of opportunity is not simply a matter of legal equality. Its existence depends, not merely on the absence of disabilities, but on the presence of abilities. It obtains in so far as, and only in so far as, each member of a community whatever his birth, or occupation, or social position, possesses in fact, and not merely in form, equal chances of using to the full his natural endowments of physique, of character, and of intelligence. In proportion as the capacities of some are sterilized or stunted by their social environment, while those of others are favoured or pampered by it, equality of opportunity becomes a graceful, but attenuated, figment.  

In the absence of equality of circumstances, the notion of equality of opportunity becomes illusory.

Thus, from the point of view of providing equal access to education, the classical liberal principle of "la carrière ouverte aux talents", entrenched in the free education ideal is shown to be devoid of merit. According to Hayek, this principle:

...was a demand that all man-made obstacles to the rise of some should be removed, that all privileges of individuals should be abolished, and that what the state contributed to the chance of improving one's conditions should be the same for all... It was understood that the duty of government was...merely to make available to all on equal terms those facilities which in their nature depended on government action.

Although Hayek was not explicit in what he meant by saying "the state contributed to the chance of improving one's conditions", it is clear that the state's contribution was not "the same for all", precisely because an individual's chance of

32 R. H. Tawney, Equality, pp. 103–104.
benefiting depended on other factors, which differed among people. It was shown that, given the provision of free education "to all on equal terms", without making other compensatory adjustments, the chance of benefiting from the state's contribution was strongly affected by family environment. Does this mean that society should adopt what Meade calls the "distributional principle" which "...would be to use the available resources in education in such a way as to equalize the future earning power of different students".\(^{34}\) A criticism of the "distributional principle" is that it conflicts with the goal of efficiency. Okun argues that both efficiency and economic equality can be increased by correcting existing inequalities of opportunity arising from differences in environmental factors. The present inequalities can, undoubtedly, be reduced by public policies. Proponents of this argument claim that familial disadvantages can in principle be offset at a cost if society accepts Rawls' principle of redress. Such policies would enhance both equality of income and efficiency. "In such cases, society can get more of two good things rather than sacrificing one for the other."\(^{35}\)

It is, therefore, incumbent on those who endeavour to promote the general well-being of society, in terms of increased equality and economic efficiency, to look at ways and means of reducing the environmental barriers to equal opportunity. It is also pertinent, in this context, to examine the policies required to neutralize all the constraints on the opportunity structure and to open access for all those children whose education now finishes when it has hardly begun; and the policies that will ensure the uniform distribution of opportunities, their multiplication according to needs, and the correction of the specific chains of


\(^{35}\) Arthur M. Okun, Equality and Efficiency, p. 76.
success or failure. This can be achieved, it is often claimed, by correcting the regional and sectoral imbalances in the distribution of educational facilities, and through a compensatory education scheme for the children of disadvantaged families, coupled with minimum schooling legislation. Welfare-economic considerations require that the government's subsidy to parents should vary inversely with income, with a view to providing adequate finance for families in lower income brackets. These compensatory programmes would, however, entail additional costs to society. Then, there is the question of "who should pay what"? Towards this end, it is also necessary to identify the flaws in the present scheme whereby the well-to-do get a "free ride" which enables them to enjoy full benefits, while the less well-endowed are caught up in a vicious circle which prevents them from obtaining any benefit.

3. **Focus of Thesis**

The focus of this thesis is two fold. Firstly, it examines the positive aspects or the descriptions of the way the world is and why, with a view to ascertaining the dimensions of the problem of inequality in income distribution, as it exists in Sri Lanka. For the purpose of this investigation it is necessary to measure the extent of inequality, in some objective sense, in relation to other countries, and over time. The study also attempts to identify the sources and causes of inequality, in the domain of the study covering the positive aspects. Secondly, the study seeks to shed some light on the normative aspects of the problem or how the world ought to be. The issue raised here is how to achieve a more egalitarian society, while at the same time maintaining a decent standard of living obtainable through an adequate growth of the economy. The theme of the present study is that this objective can be achieved by deliberate government action directed at correcting the existing imbalances in the distribution of human
capital. The study reflects the belief that increases in human capital investment per se would, however, be of little help, unless complemented by government action to ensure its fair distribution.

The schema of the study is as follows:

Chapter II presents a review of literature on human capital theory, theories of personal distribution of income, the human capital approach to income distribution theory, income distribution models, and measures of income inequality.

Chapter III discusses some theoretical issues in the analysis of personal income distribution. A model which purports to describe the observed income distribution is presented. Its properties and methods of estimation are discussed. The chapter concludes with a test of validity for the model.

Chapter IV embodies an empirical analysis of personal income distribution in Sri Lanka. The inter-temporal and inter-spatial inequalities are ascertained. The extent of inequalities in some selected countries is presented with a view to comparison. The analysis rests on the model developed in Chapter III.

Chapter V presents a model which relates income inequality to years of schooling completed and the rate of return to investments in schooling. A test criterion for the specification of earnings models is introduced. An inter-temporal and inter-spatial comparison of inequalities is made, using the model. In the schooling model, the distribution of income (earnings) is made to depend on the distribution of schooling (human capital in general), but the latter is unexplained. An alternative approach that takes into account the precise functional form of the distribution of schooling is introduced with a view to explaining inequalities. The chapter concludes with a discussion on limitations of the human capital approach.

Chapter VI discusses some policy issues. This chapter emphasizes the need for a unified approach to economic development, in which the goals of both equity and efficiency are considered. A generalized process of capital
accumulation is an essential pre-requisite for such an approach. An optimal control model is developed to determine the optimal mix of physical and human capital, so as to maximize a general welfare goal of growth and equity.

Chapter VII presents a synopsis of findings and concluding remarks.
CHAPTER II

REVIEW OF THE LITERATURE

1. Human Capital Theory

The application of conventional capital theory to decisions on individual development, and in particular improvement of earning capacity, has provided an analytical framework for explaining such activities as education, on-the-job training, medical care, and mobility, as rational investment of current resources for the procurement of future returns. Interest in studies related to investment in human capital embodied in human beings, can be traced both to the finding that a substantial growth in income in Western societies remains unexplained after the growth in material capital and in the size of the labour-force has been accounted for, and to a new emphasis on the importance of education in promoting economic development. It was this (accompanying) unexplained rise in the economic value of people that led to the concept of Human capital. The theory has since then encompassed many new vistas and evolved into a general theory of investment in human capital with ramifications in a variety of economic phenomena. Today, human capital theory makes up the core of the economics of human resources and has silently revolutionized such traditional subjects as growth economics, labour economics, international trade, and public finance.

Human capital theory encompasses all processes connected with the augmentation of productive skills, talents, and knowledge. The justification for the expression "human capital" is the fact that skills, talents and knowledge, like other assets, contribute to the production of future income. It is "human" because it is an integral part of a person. It is a form of capital because it is a source of future
earnings, or of future satisfactions, or both. According to this concept, the skill and education embodied in workers are envisaged as particular types of capital equipment employed in the production process, in the sense that their capacity to make a contribution to the productive process is improved by a process of investment, (representing a sacrifice of current resources for future returns) incurred in the formal education system and through on-the-job training. Such investments yield returns over the life-time of the individual concerned. These acquired abilities are obviously not free. Scarce resources are allocated to their acquisition. In a broad generalization of the concept, the domain of human capital theory extends beyond formal education and training to such activities as migration, medical care and economic information.

Human capital theory rejects the simplistic assumption of homogenous labour. Investment in human capital enhances the productivity of a person's acquired capabilities. This has led to the development of measurement techniques with a view to quantifying changes in the quality of labour. If the productive abilities of all workers were identical, simple enumeration would provide an adequate measure of the productive capacity of the labour-force. Either workers or workhours could be counted. This procedure, however, does not recognize labour's heterogeneity. All workers and workhours are not equivalent; one person is more productive than another, and one workhour is more productive than another. The concept of human capital provides an alternative measure of labour's contribution to the productive process. An adjustment is made for the quality of labour input. The adjustment takes into account the human capital embodied in labour. With steadily increasing levels of resources devoted to the improvement of quality of labour through the processes of formal and informal education, training and medical care, the shift towards treating the worker more precisely as a capital good seems virtually inevitable. Thus, in its conceptual aspect, the most
fundamental implication of human capital theory, is that every skilled worker in a modern economy has typically acquired a capital asset. He has become a "capitalist", not from a diffusion of the ownership of corporate stocks, but from the acquisition of knowledge, skill, and the capabilities that have economic value. In the classical tradition, the concept of capital encompassed and unified two distinct ideas: that of a claim on future income and that of a means of future production. Admittedly, educated and skilled workers do not control, much less own, the means of production. Yet it is precisely this latter, classical sense of the word capital which provides the ideological impact of the statement that every educated and skilled worker is now a "capitalist".

It is now realized that the growth in investment in human beings has improved markedly the quality of work entering into economic endeavour, and these improvements in the quality of the labour force with the resulting increase in the productivity of labour, have been a major source of economic growth. Consequently, the traditional distinction between labour as an original factor and capital as a produced factor has become increasingly unrealistic. Despite these developments, neoclassical tradition continues to dominate the theory and policy of economic growth. Indeed, this tradition has been powerfully reinforced by the impact of Keynes' General Theory, with its emphasis on fixed capital investment as the key variable in the macro-economic system, and its assumption of a homogenous labour force of a given quality, as well as by the subsequent transition of the Keynesian short-run equilibrium model into the Harrod growth model. One of the reasons why neoclassical growth models are lacking in explanatory power when applied empirically is the omission of human capital, for it is this capital that has augmented the quality of the labour-force. As already indicated, the limitations of accumulation of material capital as an explanation of economic growth in modern societies have prompted the contemporary interest in human
capital formation, and set the analytical stage for a generalized theory of capital accumulation to include investment in all types of capital formation. However, in using a generalized concept of capital formation, it is helpful to begin with a dichotomy consisting of human and non-human (material) capital in which a distinction is made between the two classes of capital.

Analyses between human capital and material capital led to the development of an all-inclusive concept of capital. The similarities between the two classes of capital, however, should not blind one to peculiarities of human capital. Human capital has some distinctive attributes. Human beings are first and foremost the end to be served by economic endeavour; they are not property or marketable assets, slavery aside. Their productive services are, however, sold in the market place. Consequently, human capital can be measured in terms of the value of its productive services. The value of a worker's productive services through his probable lifetime is discounted to provide a monetary estimate of the present value of his human capital. An important implication of this attribute is that human capital has no legal status as property. The most critical attribute of human capital arises from the fact that the person and his human capital are inseparable. The fusion of human capital with the personality of its owner raises, among other things, the problem of how far expenditure on the creation of human capital should be accounted as investment, and how far it should be classed as consumption. To a large degree the consumption and investment components of a person's human capital are inextricably bound up. Thus, there is the question of inseparability of the consumption and investment aspects of expenditures on education, training, medical care, etc. Much of what was called consumption, hitherto, in conventional theory, however, constitutes investment in human capital.

The generalization of the concept of capital is a culmination of the idea that early post-war emphasis on investment in material capital in the methodology
of economic planning was grossly inadequate and that economic development depends on the creation of both human and material capital. From a somewhat different perspective of planning economic development, nonetheless, within the ambit of neoclassical theory, efficient development involves the allocation of investment resources according to the priorities set by the relative rates of return on alternative investment opportunities.

The reorientation of economic thinking on the lines of a generalized theory of capital accumulation sets the stage for analyzing the effects of the formation and utilization of human capital upon the economic system, particularly, on capital formation, economic growth, structure of earnings and income distribution. Human capital has grown in modern societies at a much faster rate than conventional (material) capital, and its growth may well be the most distinctive feature of the economic system. Since, in modern industrialized economies, more than three-fourths of aggregate income is attributable to the contributions of human agents, and hence a fourth or less to natural and man-made capital assets, it should be obvious that the formation and utilization of human capital is of major economic importance.

Many paradoxes and puzzles about the dynamics of modern growth can be resolved in terms of an all-inclusive concept of capital in which human and material capital are treated as alternative forms of capital in general. Such a conceptualization provides a unifying principle for the statistical explanation of past growth and the formulation of policy for future growth or plans for economic development. Furthermore, a broadly conceived concept of capital accumulation is considered as a key to a rational explanation of the patterns of capital accumulation and income distribution, in the growth path of modern economies.

Turning to the questions closely connected with the riddle of economic growth and income distribution, it has been observed that the pace of capital
formation in advanced countries has been slowing down, which means that net capital formation has been declining relative to the increases in national income, contrary to the belief that these countries, endowed with more material capital relative to labour, would employ such capital intensively because of its growing abundance. Paradoxically, the evidence points to the fact that less of such capital tends to be employed relative to income as economic growth proceeds. Meanwhile, it has been observed that the stock of human capital has been rising relative to income, notwithstanding. In pursuance of this trend, the proponents of human capital theory advance the hypothesis that, with the inclusion of human capital under a generalized concept of capital accumulation, the ratio of "all capital" to income is not declining. The decline in the conventional capital-income ratio is simply a signal that human capital has been increasing relatively not only to conventional capital but also to income. In a related theoretical issue in international trade, the concept of human capital points to the solution of the Leontief paradox, showing why capital-rich countries nevertheless export labour-intensive goods. Human capital embodied in labour entering into these goods goes a long way towards resolving this paradox. In solving the long-standing puzzle of the "residual", where the rate of increase in output exceeds the rate of increase in inputs, an all-inclusive concept of capital accumulation in which human and material capital are treated as alternative forms of capital, provides a rational explanation of most of the observable economic growth.

Lastly, with regard to the large increases in earnings of workers and the reduction of inequalities of income distribution experienced in the advanced countries since the turn of the century, these trends have depended in large part on the growth of human capital vis-à-vis material capital. It has frequently been pointed out that the observed growth in productivity per unit of labour is simply a consequence of assuming that the unit of labour is constant over time although in
fact this unit of labour has been increasing as a result of the growth of human capital embodied in the labour-force. Investment in human capital, particularly that in education, has risen at a rapid rate and by itself may well account for a substantial part of the otherwise unexplained rise in earnings. On the question of income distribution, it has been observed that the distribution of personal income has become more equal in western societies since the beginning of this century, despite the fact that wealth distribution in these countries has remained very unequal. The general extension of education and the additional earnings from these forms of human capital have been a major factor in changing the distribution of personal income. Not only has the supply of educational opportunities increased markedly over time, but the inequality in its distribution has been reduced. To the extent that personal income from human capital (earnings) is more equally distributed than that from material capital, the growth of human capital in relation to material capital provides a rational explanation of the favourable trends of distribution of personal income.

Investment in human capital can conveniently be classified into investment in (1) schooling and higher education; (2) post-school training and learning; (3) pre-school learning activities; (4) migration; (5) medical care and nutrition; and (6) the search for information. With respect to each of these investment activities there are unsettled questions of economic efficiency and of equity. The theoretical issues of particular interest on this score are the effects of investment in human capital on productivity and growth, and the relationship between investment in human capital and income distribution. Since the human capital revolution in the late 1950's, there have been important advances in economic thinking with respect to investment in human capital. The theory has since then been extended to a variety of subfields falling within or overlapping the boundaries of the domain of human capital. They include, among the others, (1)
manpower studies; (2) economics of discrimination, particularly its effects upon the formation and utilization of human capital; (3) migration within a country (job mobility); (4) migration between countries; (5) the theory of allocation of time (earnings foregone); (6) international trade implications of the acquisition of human capital; (7) the sources of poverty associated with lack of schooling; (8) the sources of changes in the structure of wages and salaries; (9) the factors that account for the tendency towards more equal distribution of personal income in advanced countries; (10) the economic connections between functional and personal distribution of income; (11) the allocation of resources in the production of educational services and other forms of human capital; (12) the effects of investment in human capital on productivity and growth. The focus of the present study is on investment in education. Since schooling and higher education encompass the most important set of programmes contributing to the formation of human capital and, furthermore, since far more intensive work has been done on education as an investment than on any of the other components of human capital, the ensuing survey of it will be much more extensive than that of the others.

The recent revival of interest in the concept of human capital began with the work of Gary S. Becker\(^1\) and Theodore W. Schultz.\(^2\) Although the work of these two authors was a landmark in the historical development of the theory of human capital, the treatment of currently or potentially productive human beings as capital and/or wealth has a long history in economic literature.

Evidently, the mercantilists recognized the value of investment in man, for they laid great emphasis on the importance of "art and ingenuity", or skilled

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manpower, as a key to growth in national wealth. Sir William Petty\(^3\) placed a monetary value on labourers. Labour, to him, was the "father of wealth" and must therefore be included in any estimate of national wealth. Petty's calculations of the money value of a human being is considered as the first attempt to estimate the capital embodied in man. Mercantilists, however, were not human capital theorists for their analysis was not based on a cost-benefit framework. Adam Smith\(^4\) reversed the mercantilist view concerning the economic value of man, for he believed that economic progress depended on division of labour and that it would reduce rather than raise supply of human skills and demand for them. He thought only machines could increase market production. Both Smith\(^5\) and Malthus,\(^6\) however, favoured increasing public education for the betterment of man, not for the creation of human resources. Their main concern was the nonmarket benefits of education. Smith, however, at one point, drew an analogy between men and machines, where acquisition of skills was viewed as an investment, and his sage remarks vividly summarized this vision:

The wages of labour vary with the easiness and cheapness, or the difficulty and expense of learning the business... A man educated at the expense of much labour and time to any of those employments which require extraordinary dexterity and skill, may be compared to one of those expensive machines. The work which he learns to perform, it must be expected, over and above the usual wages of common labour, will replace to him the whole expense of his education, with at least the ordinary profits of an equally valuable capital.\(^7\)

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5 Ibid., pp. 733-738.


Jean Baptiste Say asserted likewise that since skills and abilities are acquired at a cost and tend to increase worker productivity, they should be regarded as capital. This was also the contention of John Stuart Mill. He, however, insisted that:

...the people of a country are not to be counted in its wealth. They are that for the sake of which its wealth exists. The term wealth is wanted to denote the desirable objects which they possess, not inclusive of, but in contradistinction to, their own persons.9

This view contrasted sharply with that of Léon Walras, who included all persons capable of yielding personal incomes or services of persons, as a class of capital, namely personal capital. He further asserted that "...although personal capital is not subject to purchase and sale, labour or personal services are offered and demanded every day on the market, so that personal capital can, and often should, at least, be evaluated".10 Ricardo, and, following him, McCulloch considered labour as the only valid standard for the value of exchange goods. According to their labour theory of value, men were most assuredly capital. This was the rationale for McCulloch's contention that "...man himself should...be considered as forming a part of the national capital".13 Despite their references to the concept, at many points in the work of classical economists, the idea of human


13 Ibid., p. 66.
capital did not receive any systematic treatment in either classical or Marxian tradition. This oversight existed for some highly rational reasons. As Samuel Bowles put it:

The absence of any systematic treatment of human capital in either the classical or the Marxist scheme results from the fact that both Marx and the classical writers defined their factors of production in terms of the way they perceived the class structure of the period. (This) is...a result of a conscious attempt to portray the class structure as they saw it, coupled with the fact that the role of education and skills in the economy was considerably less than today...14

Economic thought as it has been carried on in the English tradition has moved steadily away from treating labour as a produced means of production.

In contrast, H. von Thünen15 in his "Der isolierte Staat" introduces some reflections upon man viewed as capital:

The more highly developed population can with the same material goods produce a greater income than the undeveloped population, and this higher development can be achieved only through a training which requires a greater consumption of material goods. As a result there remains for such a people a greater capital, the use of which manifests itself in a greater work product.16

He recognized the reluctance of some to treat human beings as capital and commented:

An inner reluctance seems to hold the writers — and in fact all of them — from facing what human costs are and what capital is contained in a man...But from this reluctance spring an obscurity and a confusion of the concepts concerning one of the most vital points of economics, and conversely it is proved that freedom and dignity of man can be victorious even when man is subjected to the laws of capital.17


16 Ibid., p. 358.

17 Ibid., p. 358.
Von Thünen's idea seems to have stimulated later thought, especially in Alfred Marshall, and is not without practical significance today. Marshall presents very similar considerations:

We have already defined Personal wealth to consist firstly of those energies, faculties and habits which directly contribute to making people industrially efficient... if they are to be reckoned as wealth at all, they are also to be reckoned as capital.\(^{18}\)

Although Marshall admitted that "the most valuable of all capital is that invested in human beings",\(^{19}\) he later discarded the notion as "unrealistic". He held that, while human beings are incontestably capital from an abstract and mathematical point of view, it would be out of touch with the language of the market place to treat them as capital in practical analyses. Irving Fisher,\(^{20}\) in his all-inclusive concept of capital, brought the "human" component of capital fully into the fold. Capital, he asserted, is something (a stock) that yields a flow of services over time. Whether the physical entity in which the capital stock is embodied is marketable, is a matter of degree. Resources spent on education, among other things, are investments in the acquisition of potential future income streams, whether looked at from the individual or from the societal point of view. This is a kind of capital formation. It is the formation of capital embodied in man. Fisher's work on capital and income was carried forward by Frank H. Knight\(^{21}\) who perceived clearly and cogently, both the improvements in the "quality" of the labour-force and the economic contributions from the advances in the sciences and their effects on the rate of returns to investment.


\(^{19}\) Ibid., p. 564.


Despite these developments, during the first half of the twentieth century, most economists, following Alfred Marshall, have shown a tendency to use the concept of capital as applicable to that portion of the non-human, material, produced means of production. Investment in human beings has accordingly seldom been incorporated in the mainstream of economic thought, until the "rebirth" of the concept in the late 1950's.

Turning now to education, J.R. Walsh in his study of the applicability of the capital concept to man, took up the subject of the economic importance of education, now being treated by Schultz, Becker, and others. He attempted to determine whether money spent in acquiring training (schooling of a particular kind) is, in a strict sense, "a capital investment made in a profit-seeking, equalizing market, in response to the same motives which lead to the creation of factories, machinery, and the like". This view contrasted sharply with that of earlier writers, who treated all men as capital. Walsh asserted:

Further education means income foregone; and the longer formal schooling continues, the larger this loss to him will be. Professional training in particular will be at the expense of a substantial opportunity cost, which future income will be expected to repay. The student, in short, will tend to view this cost as the capital investment it undoubtedly is.

In a much more elaborate study of incomes of doctors and dentists in independent professional practice, Milton Friedman and Simon Kuznets attempted to explain income differences by differences in ability and monopolistic constraints on entry into training for the higher paying professions.


23 Ibid., p. 256.

24 Ibid., p. 257.

As mentioned earlier, the theory of capital applied to human beings, had largely disappeared from the main stream of economics by the 1930's. Antithesis came with Keynes' General Theory. Following Keynes' tradition, the emphasis of a whole generation of economists was shifted from viewing labour as an active agent of production to viewing labour as a passive agent that would find employment only if there were a high enough rate of "investment" and, most especially, of investment in the production of physical producer goods. Out of Keynes' writings, that dealt primarily with economic fluctuations and persistent unemployment, came some remarkable progeny — long-term "growth" theories in which growth was explained by the amount of material capital and its rate of increase. In attempting to explain the post-war growth experience of advanced economies, the econometricians discovered that their growth models premised on conventional capital-output ratios were not behaving properly. The explanatory power of these models was greatly hampered by the limiting assumptions of labour and capital homogeneity. Moses Abramovitz showed that the growth in labour and capital as conventionally measured failed dismally in explaining the growth in per capita incomes during the twentieth century in the United States. Abramovitz's work set the stage for his paper announcing "The Residual". Following Abramovitz, John W. Kendrick and Edward F. Denison carried forward the work on the "residual". Denison, using the residual approach, estimated the effect of advances


27 Ibid.


in knowledge by simply subtracting the rate of growth attributable to all other
inputs from the total rate of growth. Zvi Griliches and Dale W. Jorgenson, in their
search for the sources of productivity growth, fittingly close their paper with the
remark: "In explaining economic growth we suggest greater reliance than
heretofore on the twin pillars of human and non-human capital, each supporting an
important part of the capital structure".30

Theodore W. Schultz is unique among those who have analysed sources of
economic growth in that he explicitly linked his analysis with the theme of
investment in human beings. This led him to measure the value of foregone
earnings as a student input into the educational process. His conservative
estimates showed that earnings foregone accounted for over half of the total costs
of secondary and higher education in the United States. Commenting on the
growing disbelief in GNP and flaws in its measurement he asserted:

The GNP is indeed only a partial concept in the way it is specified and
estimated at present. A major overhauling is overdue, and a large part
of what is missing in GNP consists in the components of human
capital.31

Schultz pointed out that many paradoxes and puzzles about the United States
economy, such as the problem of the "residual" and the Leontief paradox might be
resolved once human capital is taken into account.

The empirical study of investment in human capital that stimulated
widespread interest was that of Gary S. Becker.32 Becker's analysis views
investment in human beings in a micro-decision theoretic framework. As such,

30 Zvi Griliches and Dale W. Jorgenson, "Sources of Measured
50-61.

31 Theodore W. Schultz, Human Resources (New York: National Bureau

32 Gary S. Becker, "Underinvestment in College Education", American
there is no "growth" in Becker's analysis. Decision theory in this context is micro
cost-benefit analysis applied to decisions on investment in human capital. Using an
"internal rate of return" approach, he attempted to determine whether there was
under-investment in college education, compared to alternative investments. Becker's
subsequent work gives an overview of the pervasiveness of human
capital analysis and reveals many of its applications. These studies set out the
basic theoretical framework within which investments in human capital embodied
in individuals and in society can be analysed. In essence, Becker applies, with some
modifications, conventional capital (or investment) theory to investment in human
beings.

The work of Schultz and Becker provided the impetus for a proliferation
of interest in the subject of human capital. There was also complementary
evidence of a different sort, coming from the work of Herman P. Miller, H.S.
Houthakker, and from the discovery of the relevance of the 1945 study by Milton
Friedman and Simon Kuznets, Jacob Mincer's study of human capital and the
distribution of personal income provided still another type of supporting evidence.
In a significant development of Becker's theory, Mincer treats "training on the
job" as an investment that employees make in themselves. He defined and

33 Gary S. Becker, Human Capital.

34 Herman P. Miller, "Annual and Lifetime Income in Relation to

35 H. S. Houthakker, "Education and Income", Review of Economics and

36 Milton Friedman and Simon Kuznets, Income from Profession.

37 Jacob Mincer, "Investment in Human Capital and Personal Income

38 Jacob Mincer, "On-the-Job Training: Costs, Returns, and Some
estimated on-the-job training investment in terms of its cost in foregone earnings. The treatment by George J. Stigler39 of the search for information about jobs in terms of the costs of acquiring such information and the return it fetches, and a comparable treatment of migration by Larry A. Sjaastad40 present new hypotheses based on investment in human capital, for analyzing interindustry shifts. Herbert G. Grubel and Anthony D. Scott41 apply human capital theory in analysing the international migration in selected professions or the problem of "brain drain". Burton A. Weisbrod42 examines external benefits of education where he makes a distinction between individual and societal benefits (and costs). Giora Hanoch43 estimates male expected earnings in the United States in 1959, by age and education, after standardizing for other relevant factors, such as expected secular growth, and expected improvements in the quality of education. Meanwhile, in a theoretical paper, Yoram Ben-Porath44 relates the shape of individual's earning profiles to properties of the production function of human capital. In an endeavour to explain the differences in per capita income among nations, Anne O. Krueger, in her paper on factor endowments and per capita income, concludes "that the difference in human resources between the United States and the less-developed


countries accounts for more of the differences in per capita income than all the other factors combined. Lastly, in an article on a related field, Peter B. Kenen brings up to date the findings pertaining to the role of human capital in determining the comparative advantage of nations.

2. Theories of Personal Distribution of Income

The position taken in the present study is that equity is the dominant criterion for appraisal of income distribution. If provision for ultimate consumers is accepted as the ultimate goal of economic activity, distribution should rank high in economists' interest. Yet most of the work in economics that goes by the name of "Incomes distribution theory" has focused on the distribution of income among factors of production, rather than on distribution among persons. This orientation dates back at least to the time of classical economists and may have been appropriate to the class structure of their day. The classical writers, including Ricardo and Marx, were primarily concerned with how national income was divided among land, labour and capital. While the behaviour of distributive shares gives some insight into the understanding of the distribution among persons, it is by no means sufficient to explain it fully. Today, the link between the shares of factors of production in national income and the personal distribution of income is more complex. Thus, the normative significance of the functional distribution of income, for inequality as a social problem is rather limited. The traditional factor-share approach continues to flourish in the literature despite the blurring of distinct social class identifications with specific factors of production, and despite


the recognition that in modern economies the variance in labour incomes is the dominant component of total income inequality.

The mainstream of economic theory has concerned itself only with functional distribution of income, though not always with the normative implications. As against this, no generally accepted theory of personal distribution of income exists and most textbooks on economics make only passing reference, if any, to this aspect of distribution. The lack of a well-developed theory, however, contrasts sharply with empirical research in the area. More importantly, it is the empirical data that have prompted many of the attempts at theoretical analysis of the personal distribution of income. The pursuit of empirical research in this area has often been ethically motivated. Notwithstanding, study of the consequences of the distribution pattern for different aspects of economic activity also reflects a dominant tendency on the theoretical side.

The heterogeneity of rewards to individual workers did not escape the attention of classical economists; notwithstanding their preoccupation with factor shares. Their comments are summed up in two famous principles: First, Smith's "compensatory principle", which is conditional on the strength of competitive forces in the labour market. Labour mobility produces earnings differentials which tend to equalize "net advantages and disadvantages" of work. Second, Mill's and Cairnes' doctrine of "noncompeting groups" proclaims in effect the absence of labour mobility resulting in real income differences, produced and perpetuated by socially, legally and culturally imposed and inherited stratifications. It explains why equalization is incomplete and in the process stresses "institutional" factors in the determination of income differences. The doctrine of noncompeting groups is rarely accepted in the extreme form enunciated by Mill. Nonetheless it has contributed, by its emphasis on a variety of environmental factors, to a pragmatic statistical approach. In this spirit, such institutional, demographic or socio-
economic factors as sex, age, occupation, education, location, and parental wealth provide meaningful leads to the determination of total income inequality.

The problem of formulating a mathematical expression to describe the frequency distribution of income has attracted the interest of many economists and statisticians. Following the terminology of Bjerke,47 one may classify these economists and statisticians into two main schools. The first comprises those who regard income formation as the product of certain stochastic processes. Bjerke refers to this group as the theoretic-stastical school. The other school has, to a greater degree, considered the problem from a sociological point of view, pointing out that income differentials are the result of social stratification. This, according to Bjerke's terminology, is the sociological school. A serious omission from Bjerke's classification, however, is the human capital school. The human capital school has attempted to explain income differentials in terms of the distribution of human capital, in particular, the distribution of education and training. In its simplest form, the human capital approach consists of a series of definitions and hypotheses of lifetime income maximization. In the ensuing survey, the order of exposition is both methodological and chronological.

The first scientific and empirical generalization regarding the functional relationship between size of income and number of recipients was that of Pareto. The regularity of pattern in the various distributions Pareto studied, led him to formulate what has subsequently been referred to as Pareto's Law. Pareto's Law asserts that if \( y \) is the level of income and \( N \) the number of income recipients with income \( y \) or above, the relationship between \( y \) and \( N \) can be expressed as:

\[
(2.2.1) \quad N = Ay^{-\alpha}, \text{ for all } y \geq y_o > 0,
\]

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where $A$ and $\alpha$ are statistical parameters to be estimated from empirical data. The latter provides a measure of income inequality — the higher the value of $\alpha$ the lower is the inequality. While Pareto's law gave a fairly adequate description of the upper tail of income distribution, it gave inconsistent results when applied to lower income ranges. The inadequacy of Pareto's equation as a general law of income distribution is therefore obvious.

The apparent stability and regularity of income distributions have suggested to several authors (theoretic-statistical school) that the distribution might be the steady state solution of some stochastic process. The basic stochastic model dates back to Gibrat,\textsuperscript{48} who noted that the product of a large number of normal and independent random variables tends toward the lognormal distribution, which has the positive skewness displayed by the data. This multiplicative central limit theorem leads naturally to a simple Markov model. The rationale applied to this model is a special formulation of the central limit theorem as expressed in the "law of proportionate effect". The law states that if individuals are subjected to the play of "chance" (random shock), starting with some initial distribution, and if they experience percentage changes of income unrelated to income levels, then the process converges to a lognormal distribution, regardless of the form of the initial distribution. Since income at any time is portrayed as a product of uncorrelated random shocks in all previous periods, the implication of Gibrat's model is that the variance of the logarithm of income must continually increase, contrary to the evidence. Michal Kalecki\textsuperscript{49} suggested a method of dealing with the problem of exploding inequality by introducing a negative correlation between the income of


the preceding period and the random shock. The operation of this negative correlation has a stabilizing influence on the variance. Developing his argument in an economic context, Kalecki postulated that variations in the inequality of incomes are to a great extent determined by economic forces. A critic of Kalecki, however, claims that economically negative correlation between prior income and random shock means that the probability that income will rise by a given percentage is lower for the rich than for the poor, which is far from true.

D.G. Champernowne uses postulates similar to those underlying Gibrat's Law of Proportionate Effect. The essence of these models is the assumption that the incomes of individuals follow a Markov process; the percentage change in income during each interval of time being a random variable. Champernowne generated a model of income distribution, through a stochastic process, which converges to a Pareto distribution after a long sequence of transitions. In his model, the income intervals defining each class are assumed to form a geometric progression rather than the customary arithmetic progression. No individual is assumed to move more than one interval upwards, whereas he can move down several intervals. Income is assumed not to increase indefinitely. Finally, he assumes that the law of proportionate effect applies to the transition probabilities and that for every income recipient who dies or disappears there is a successor to his income in the following year. Champernowne, thus, demonstrates that the key assumptions leading to the Pareto distribution are those of proportionate effect and of geometric progression. J. Aitchison and J.A.C. Brown have investigated the effect of dropping the assumption of geometric progression. They consider


first income intervals of equal size and later a model with infinitesimal income intervals. The usual assumptions for the law of proportionate effect are, however, retained. Aitchison and Brown point out that the assumption of the law of proportionate effect requires homogenous groups. They show that, even if the law of proportionate effect generated a lognormal distribution within homogenous subgroups ("trades"), the aggregate distribution would remain lognormal only if the variances in the component distributions are equal and the means of the components are lognormally distributed.

Gibrat's principle has been further worked out by R.S.G. Rutherford, who cites a number of instances in which empirical distributions of income do not seem to fulfill the assumptions of Champernowne or those of Gibrat. Rutherford specifies a new model, effecting a transformation of the lognormal, which turns out to be a Gram-Charlier distribution (an S-shaped distribution) of income-power. Rutherford attempts to explain the generation of an income distribution by adding birth and death considerations to the customary random shock type of model. Rutherford's model has a simplified stochastic foundation and is supported by empirical observations. Unlike Gibrat's assumption of a linear relation between income-power and the standardized normal abcissa, Rutherford finds two or three ranges over which different linear relations hold. The departure from strict linearity may reflect, according to Rutherford, either discontinuities or the possibility that the relation is a cubic or a polynomial of higher degree. From his set of assumptions, Rutherford arrives at the specification of a four-parameter model. The variance of income-power is a function of three components, namely, (1) the income-power variance of new entrants; (2) life expectancy at entry; and (3) the variance of the shock system.

Changes in any of these components will induce changes in the variance of income-power. Rutherford considers that, in the short term, these components are relatively stable.

Empirical evidence shows that the Pareto distribution is the model of high income groups. It is overwhelmingly supported by theoretic and quantitative research. Davis\textsuperscript{54} substantiates the necessity of specifying a more complete model, which should include that of Pareto, to fully understand the phenomena associated with the distribution of total income. The Pareto convergence property is called the weak form of the Pareto law by Benoit Mandelbrot,\textsuperscript{55} another proponent of the Pareto distribution. This property has far-reaching theoretico-empirical implications. Paul Lévy\textsuperscript{56} proved the existence of a class of stable probability density functions other than the class of normal (Laplace-Gauss) distributions, called the Pareto-Lévy class by Mandelbrot.

When a model of income distribution fulfills the weak form of the Pareto law, it is important to determine whether it also belongs to the class of stable Pareto-Lévy distributions. If that is the case, and different categories of income are analysed, each category being distributed according to a law of the Pareto-Lévy class, then the distribution of the sum of incomes is also a member of the same class.


\textsuperscript{54} H. T. Davis, \textit{The Analysis of Time Series}, p. 387.

\textsuperscript{55} Benoit Mandelbrot, "The Pareto-Lévy Law"; p. 81.

\textsuperscript{56} Paul Lévy, \textit{Calcul des probabilités} (Paris: Gauthier-Villars, 1925).
An important digression from the stochastic approach, nonetheless rich in economic content, is that of Milton Friedman. The model is premised on the notion that economic analysis of income distribution must rest on the implications of the theory of rational choice — choice among unsure prospects — just as the analysis of other economic processes does. The model is stochastic to the extent that it is generated from a random process. But, unlike the other stochastic models, differing propensities to undertake risk (or risk aversion) influence the distribution of income. Friedman shows that risk-taking behaviour can produce an overall pattern of income distribution which, like the observed distribution, is positively skewed and humped. The rationale for his model is that, in a situation of risk, each person pays both to avoid risk (insurance) and to incur risk (gambling); the latter set of actions is in the nature of participation in a lottery. The greater the number of people who like risk, the greater is likely to be the inequality of income, since risk takers have as much chance of being unfavourably as favourably surprised. If the lottery has only a few winners of very large prizes, the resulting overall distribution is positively skewed with an elongated upper tail.

The sociological school takes a general view regarding the shape of the income distribution, which differs widely from that of the theoretic-statistical school, in that they stress the importance of a number of institutional factors which influence the income level in different trades, in addition to the distribution of talents. The income differentials arising from institutional stratification, in conjunction with the distribution of talents, are a contributory factor in producing a positively skewed income distribution. If changes occur in the underlying factors, such as sex composition, age, occupation, education, trade, geographical differences and the distribution of wealth; and, if these changes affect income

levels, the shape of the income distribution will be changed. There is, thus, a divergence in the views of the two schools. The theoretic-statistical school assumes a process leading to convergence towards a definite type of distribution, whilst the sociological school views the shape of the income distribution, at any given point in time, as the result of a historical process. The shape of the income distribution is subject to change depending on the changes in the relative importance of the different institutional factors.

Theorizing of this sort appears to have been started by A.C. Pigou,58 who noted that incomes follow a skewed distribution while abilities are apparently normally distributed. He also attempted to reconcile this contradiction, which he characterized as a "paradox". Pigou considered the personal attributes additively, which is implicit in the assumption of normality in the distribution of abilities. But income distribution also depends on the distribution of wealth which has been observed to be highly skewed. Secondly, Pigou suggested that the overall distribution of income might be skewed because it is an amalgam of the distributions within "noncompeting" subgroups of the population. He speculated that the distribution of income within each subgroup might be normal and yet the overall distribution could be skewed. Pigou's theory stands out as a progeny of the classical doctrine of "noncompeting groups", elevated to a theory of income distribution. In an attempt to find an answer to Pigou's question of whether a skewed distribution of earnings can be associated with an allegedly normal distribution of abilities, C.H. Boissevain59 and, following him, A.D. Roy60 argue


that earnings are proportional to ability. But ability is a multi-dimensional concept. If component abilities combine multiplicatively, as "random shock" does in stochastic models, the resulting implications are: Symmetric distributions of component abilities produce positively skewed aggregates, hence earnings. If the component abilities are positively intercorrelated, skewness is augmented, and if the variances of the components are unequal, humpedness will result.

H. Lydall has constructed a similar model, though with a much stronger economic motivation. In analysing earnings distributions, he stresses four factors: (1) Differences in innate abilities; (2) the influence of social status (environmental factors); (3) differences in education; and (4) the hierarchical structure. While the first three of these factors would lead to lognormal leptokurtic distributions, the last would apply to the upper tail of the earnings distribution which approximates to a Pareto distribution. Persons with higher salaries are, in Lydall's view, not paid according to their abilities, but according to their position in the social hierarchy. He argues that in hierarchical organizations a person's earnings depend largely on the number of people he supervises. With these and a few other assumptions, Lydall was able to set up a model that generates a Pareto distribution, for the upper tail of the earnings distribution.

Lydall's reasoning runs parallel to that of the functionalist argument, first made famous by K. Davis and W.E. Moore. The thesis, quite simply, is that society needs for its survival certain skills, that occupations may be ranked on a hierarchy of structural significance, and that the unequal distribution of income and status insure that the high-level, low-supply skills are provided. Inequality of


income is a function of the unequal structural significance of occupations. According to Samuel Bowles and Herbert Gintis, the protagonists of the "radical school", income inequality emerges as a structural necessity of market economies. It is built into the property and production arrangements of capitalist societies as an integral part of the institutional framework. On a more specific level of reasoning, income inequality can be seen as directly and immediately functional in the growth process: It enforces labour discipline, is a crucial ingredient in the incentive apparatus, and produces those levels of private profit that are essential to capitalist growth.

The authors discussed so far have viewed the determinants of the distribution of income as the result of certain stochastic processes or as being institutionally determined. On the contrary, Jan Tinbergen presents a model which meets more fully the criteria of economic theory. The model emphasizes the rational choice of both those who provide and those who demand the various attributes required in economic activity. A person's income, according to Tinbergen, is dependent on the tension between requirements and the availability of a set of attributes. These tensions eventually determine the shape of the income distribution. Tinbergen asserts that income distribution as a whole is explained by the sum of a large number of terms, each representing the income distribution for a separate productive attribute. These attributes are assumed to have been normally distributed. Tinbergen further shows that a Gibrat distribution results if the spread of the attributes supplied does not differ from the spread of the attributes demanded.

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3. The Human Capital Approach to Income Distribution Theory

Interest among economists in the distributional aspects of education has as long a history as modern economics itself. The recognition of the fact that education increases earnings by improving productivity dates back to the time of the classical economists. Ever since, economists have argued that earnings differentials could be reduced by the spread of education. McCulloch, in his Principles of Political Economy, was the first to perceive that "a better system of education and a better law of inheritance are two of the most powerful means of reducing inequalities of income".65 Hugh Dalton recognized the importance of education as a means to reform. His sage remark summarizes this vision:

The most important direction for practical reform (in decreasing inequality of earnings) is to increase the ability of the great majority of workers at the beginning of their working lives, to make a freer choice than at present between occupations... The main instrument for increasing this freedom is better and more equally distributed facilities for education and training.66

A more systematic linking of education and income is, however, of more recent origin. Jacob Mincer's model67 focusing on choice with respect to occupation is the first of the "human capital models" which seek to explain heterogeneity of labour incomes in terms of the investment behaviour of individuals. The model, premised on the basic tenet of "rational behaviour", nevertheless shows that even when the exercise of rational choice by individuals is not in any way constrained, differences in income must arise as compensation for differences in the length of training and the consequent postponement of earnings. When the analysis is extended to include on-the-job training, which augments investment embodied in an


66 Quoted in Ibid., pp. 13-14.

67 Jacob Mincer, "Investment in Human Capital".
individual, an additional factor is introduced to explain the dispersion in earnings distribution.

In a subsequent study, Becker develops an analytical approach, bringing economic theory to bear on the effects of human capital on the distribution of personal income. Becker and, following him, Barry R. Chiswick incorporate the effect on income distribution of various "institutional" factors such as parental wealth, inheritance of property, distribution of abilities, unequal opportunities and interrelations among them. In the human capital model, "rational behaviour" occupies a central role, as the process of income determination is made to depend on a series of definitions and hypotheses of lifetime income maximization. In so far as each person's demand for human capital is contingent upon his ability, the human capital model can subsume most of the ability-related theories. Through the supply side, it is also capable of accounting for capital market imperfections and the interaction of inherited wealth (and other environmental factors) with access to education. While the model cannot generate a precise functional form for the income distribution, it does, however, explain positive skewness and certain relationships among the distribution of ability, education, training, and earnings. Furthermore, the model is superior to most of the models considered so far, in that it lends itself readily to empirical verification and theoretical generalization.

In the exposition of Becker's theory, a distinction is made between the "egalitarian" and "elite" approaches to income distribution. Becker identifies the egalitarian view with supply conditions; the objective is to reduce the inequality


resulting from differences in the supply of educational opportunities. The elite view, on the other hand, rests on the assumption that supply conditions are identical and that demand conditions alone determine the actual investment and differences in earnings, arising from differences in the capacity of individuals to benefit from investment in education and from other forms of human capital. In the more general case, however, both supply and demand conditions vary, and the combined effect determines the varying amounts of investment in human capital and hence income of different members of a society.

The principal characteristic that distinguishes human from other forms of capital is that, by definition, the former is embodied in the person in whom the investment is made. This embodiment of human capital explains why marginal benefits decline as additional capital is accumulated. One obvious implication of embodiment is that, since physical and cognitive capacities of each investee are limited, eventually diminishing returns set in from producing additional capital. The result is an increasing marginal cost of producing an extra dollar of returns.

Closely related to the embodiment of human capital, is the rising cost of foregone earnings with increase in investment and imperfect substitution of direct for time costs. Furthermore, the finiteness of working lives makes later investments less profitable (attractive) than the earlier ones. Thus the demand curve \((D_i)\) relating an individual's investment in himself to marginal rate of return on that investment, is negatively sloped. On the supply side, it is highly unlikely that the capital market will operate in a perfect manner, especially in the case of acquisition of human capital. The market for human capital financing is extremely segmented and is subject to capital rationing. The result is that although certain sources of funds are cheaper than others, the persons accumulating human capital must shift from the cheaper sources to more expensive sources, as additional units of capital are acquired. The supply curve \((S_i)\) relating funds obtainable for financing
investment to marginal interest costs is positively sloped, as the expansion of capital investment requires resort to more expensive sources of funds. The positions and shape of the demand and supply curves depend on "ability" and ease of access to funds respectively. A person of high ability might be hypothesized to benefit more from additional investments and accordingly the demand curve corresponding to an "abler" individual is higher than that for an average individual. Individuals facing a lower supply curve can obtain low-cost financing compared to those facing a higher supply curve. The distribution of investment and of earnings, therefore, depends on the distributions and shapes of these demand and supply curves.

The demand and supply curves embody the effects of differences in capacities ("elite" approach) and unequal opportunities ("egalitarian" approach). The "elite" approach implicitly assumes that actual investments and earnings differ primarily because of differences in the capacity to benefit from investment in human capital: some persons are abler and form an "elite" (see Figure 1). At the other end of the spectrum is the "egalitarian" approach which attributes differences in investment and earnings to unequal opportunities, implicitly assuming individuals to have equal capacities (see Figure 2). Egalitarians assume that investment and earnings differ because of differences in the environment, family wealth, family connections, subsidies, etc. Eliminating environmental differences inhibiting equal opportunities would eventually eliminate the important differences in investment and earnings. The combined effect of unequal opportunity and unequal abilities is shown in Figure 3. The nature of the observed relationship depends on the shape of the supply and demand functions, and the interconnection between them. It can be seen that the distributions of investment and earnings would be more unequal and more skewed, the greater the elasticities of supply and demand curves, and the more unequal and skewed their distributions.
Figure 1.- Human Capital and Income Distribution: "Elite" Approach

Figure 2.- Human Capital and Income Distribution: "Egalitarian" Approach

Figure 3.- Human Capital and Income Distribution: Generalized Approach
A positive correlation between supply and demand conditions would further augment inequality and skewness of the distribution of investment and earnings. A positive correlation, implying more favourable supply conditions associated with more favourable demand conditions, seems a plausible assumption, in view of the fact that "abler" persons normally can obtain finance on more favourable terms.

4. Income Distribution Models

The systematic quantitative analysis of personal distribution of income was initiated by V. Pareto,70 motivated by his ideological polemic with the Italian and French socialist schools, concerning ways and means of achieving a less unequal income distribution. Out of Pareto's polemic came some quite remarkable progeny, models of size distribution of income, of which his was one.

The Pareto Model

Pareto specified his model (Type I), on assumptions based on the observed regularity and permanence of the elasticity of the upper tail of the income distribution. The model was further generalized with the specification of Types II and III. The Pareto Type I model has the following mathematical form:

\[(2.4.1) \quad G(y) = 1 - F(y) = \left(\frac{y}{y_0}\right)^\alpha, \quad y \geq y_0 > 0, \quad \alpha > 1,\]

where \(F(y) = P(Y \leq y)\) is the cumulative distribution function (cdf). The Pareto Type II model is obtained after replacing \(y\) and \(y_0\) by \((y - \theta)\) and \((y_0 - \theta)\) respectively, i.e.,

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(2.4.2) \[ G(y) = 1 - F(y) = \frac{(y - \theta)/(y_\circ - \theta))^{-\alpha}}{\alpha}, \quad 0 < y_\circ \leq y \]

Multiplication of Type II by a factor \( \exp(-\beta(y - y_\circ)) \), \( \beta > 0 \), generates the Pareto Type III model. Therefore:

(2.4.3) \[ G(y) = 1 - F(y) = \frac{(y - \theta)/(y_\circ - \theta))^{-\alpha}}{\alpha} \exp(-\beta(y - y_\circ)) \]

All three models are zeromodal and strictly decreasing. The truncated character of Pareto's models results from the type of data used, which at that time consisted of the incomes of persons that exceeded a certain limit, \( y_\circ \), fixed by taxation rules. Further theoretical development and empirical research,\(^{71}\) led to the acceptance of the Pareto law as the model of high income groups. The convergence to the Pareto law for high levels of income, is considered as an important property to be fulfilled by alternative models of income distribution.

Gamma Function

Pareto's contribution stimulated further research in the specification of new models to fit the whole range of incomes. Several probability distribution functions (pdf) were specified as models of income distribution. O. Ammon\(^{72}\) appears to be the first to propose the gamma probability distribution function as a descriptive model of income distribution. The gamma function, a contribution of L. Euler (1707-1783), is defined by the integral:

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\(^{71}\) See H. T. Davis, *The Analysis of Time Series*; and B. Mandelbrot, "The Pareto-Lévy Law".

(2.4.4) \[ \Gamma(n) = \int_0^\infty y^{n-1}e^{-y}dy, \ n > 0 \]

It is a continuous function of the parameter \( n \) and has continuous derivatives of all orders. It can be shown, by partial integration, that the following finite difference equation holds:

(2.4.5) \[ \Gamma(n + 1) = n\Gamma(n), \ n > 0 \]

In 1898, L. March\textsuperscript{73} fitted the gamma model to wage distribution of several professional categories in France, Germany and the United States. A generalized gamma distribution was developed by L. Amoroso\textsuperscript{74} in 1925, and applied to describe the income distribution in Prussia. Its probability density function (pdf) is given by:

(2.4.6) \[ f(y; \alpha, \lambda, h, s) = [\alpha^\lambda/(\Gamma(\lambda))]y^{\lambda/(s-1)} \exp(-\alpha(y-h)^{1/s}), \]
\[ y > h, \ (\alpha, \lambda) > 0, \ s \neq 0, \ \alpha + s > 0 \]

The gamma probability density function (2.4.4) is a particular case of the generalized gamma, obtained when \( s=1 \) and \( h=0 \).

In 1974, A.B.Z. Salem and T.D. Mount\textsuperscript{75} fitted the gamma probability density function to income data in the United States. C.P.A. Bartels\textsuperscript{76} made a


\textsuperscript{76} C. P. A. Bartels, Economic Aspects of Regional Welfare (Leiden: Martinus Nijhoff Social Sciences Division, 1977).
detailed comparative study of the gamma, lognormal and other probability density functions as models of income distribution.

Lognormal Distribution

D. McAlister,77 inspired by Francis Galton, appears to have been the first person to study and specify the lognormal distribution. However, it was R. Gibrat's research on income distribution and inequality, published in 1931,78 that brought to the fore lognormal distribution as a model of income distribution, widely accepted for almost three decades. Its probability density function is given by:

\[
(2.4.7) \quad f(y) = \begin{cases} 
(1/\sqrt{2\pi\sigma^2}) \exp\left((-1/2\sigma^2)(\log y - \mu)^2\right), & y > 0 \\
0, & y \leq 0
\end{cases}
\]

The poor fit of the lognormal distribution necessitated research on alternative models of income distribution. The result was the development and application of a large number of competing income distribution models. H.T. Davis79 provided a theoretical derivation of a model based entirely on probability considerations. It converges to the Pareto Law. The model, however, has no empirical support. The stochastic foundations of the research on income distribution summarized in Mandelbrot,80 as well as those specified by Champernowne,81 which are very similar, also lack empirical validation.


78 R. Gibrat, "On Economic Inequalities".


80 Benoit Mandelbrot, "The Pareto-Lévy Law".

81 D. G. Champernowne, "A Model of Income Distribution".
Rutherford\textsuperscript{82} specified a new model, performing a transformation of the lognormal, which turned out to be a Gram-Charlier distribution of income power. His model has a simplified stochastic foundation and is supported by empirical observations. In 1970, Thurow\textsuperscript{83} proposed the beta distribution, a variant of which was adopted by Kakwani and Podder\textsuperscript{84}.

Singh and Maddala Model

S.K. Singh and G.S. Maddala\textsuperscript{85} developed a model based on the concept of hazard or failure rate, which has been widely applied to deduce distributions in reliability theory and distribution of life times. On the basis of available empirical evidence, these authors specified an increasing and bounded proportionate failure rate. The solution of the resulting differential equation generates a three-parameter model given by:

\begin{equation}
F(y) = 1 - (1 + ay^b)^{-c}
\end{equation}

The model can be related to Burr's\textsuperscript{86} distribution. The two-parameter loglogistic model, proposed and applied by P.R. Fisk\textsuperscript{87} and Weibull distribution\textsuperscript{88} are particular cases of the Singh-Maddala model.

\textsuperscript{82} R. S. G. Rutherford, "Income Distributions: A New Model".


\textsuperscript{88} See C. P. A. Bartels, \textit{Economic Aspects of Welfare}. 
Generalized Logistic-Burr's System

Of particular interest is a class of distributions generated by the Generalized Logistic-Burr's System. The specification of this system considers the characteristics of regularity and permanence of the income-elasticity of observed income distribution. The elasticity is postulated to be a bounded and decreasing function of the cumulative distribution function (cdf). Hence, it takes the form:

\[(2.4.9) \quad d \log(F(y) - \alpha) = \phi(y) \left[ 1 - ((F(y) - \alpha) / (1 - \alpha))^{1/\beta} \right] dy\]

\[\beta > 0, \quad \phi(y) > 0 \quad \text{and} \quad F(y) = P(Y \leq y) > \alpha\]

The solution of this differential equation is:

\[(2.4.10) \quad F(y) = \alpha + (1 - \alpha) \left( 1 + \lambda \exp(-1/\beta) \int \phi(y) dy \right)^{-\beta}\]

where \(\lambda\) is a positive constant of integration and \(F(.)\) is the cumulative distribution function (cdf).

Camilo Dagum,\(^{89}\) the chief proponent of the generalized logistic approach, in 1975 specified a three-parameter loglogistic model. This model was fitted to a sample of income earners from the City of Toronto. In 1977 Dagum\(^{90}\) deduced and applied the generalized logistic Type I model, followed by the Type II model. Both Type I and Type II models were used by Dagum to study intra- and inter-income inequalities in Canada and the United States. In a recent

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development, Dagum\textsuperscript{91} deduced a variant of the generalized logistic family (Type III), to deal with strictly positive income above a threshold \( y_o > 0 \). The specifications of the models are given below. A detailed exposition of Dagum's models appears in Chapter 3 of the present study.

\[(2.4.11) \quad F(y) = \begin{cases} 
(1 + \lambda y^{-\delta})^{-\beta}, & y > 0, \quad (\beta, \lambda, \delta) > 0 \\
0, & y \leq 0 
\end{cases} \quad \text{(Dagum Type I)}
\]

\[(2.4.12) \quad F(y) = \alpha + (1 - \alpha)(1 + \lambda y^{-\delta})^{-\beta}, \quad y \geq 0 \quad (\beta, \lambda, \delta) > 0, \quad 0 < \alpha < 1
\]

\[(2.4.13) \quad F(y) = \alpha + (1 - \alpha)(1 + \lambda y^{-\delta})^{-\beta}, \quad y \geq y_o > 0, \quad F(y_o) = 0 \quad \alpha < 0, \quad (\beta, \lambda, \delta) > 0,
\]

which can be written in the form:

\[(2.4.14) \quad F(y) = (1 + \lambda (y - y_o)^{-\delta})^{-\beta} \quad \text{(Dagum Type III)}
\]

**Model Foundations**

The mathematical form of the income distribution models proposed in the literature, can be grouped according to the characteristics of its generating structure, into three main classes,\textsuperscript{92} namely:

---


(1) stochastic
(2) logico-empirical, and
(3) ad-hoc (specification)

An income distribution model has a stochastic foundation, when its mathematical form is the outcome of an a priori set of probability assumptions. The income distribution models developed by Gibrat, Davis, Champernowne, Rutherford, Mandelbrot, and Singh-Maddala belong to this class. The functional form of the logico-empirical class is based on the characteristics of regularity and permanence of the observed income distribution. Models with a logico-empirical foundation are those specified by Pareto, Fisk and Dagum. Models proposed by Gibrat and Rutherford have both stochastic and logico-empirical foundations. Finally, when a model is proposed for the sole purpose of fitting to observed income distributions, providing neither a plausible probability theoretical basis nor a logico-empirical foundation, such a model belongs to the class of ad-hoc models. This is the largest class of income distribution models, and comprises the gamma model proposed by Ammon and its generalization by Amoroso, the Weibull distribution proposed and applied by Bartels, the hyperbolic distribution proposed by Champernowne, the beta proposed and applied by Thurow; and its variant due to Kakwani and Podder.

5. Measures of Income Inequality

A number of measures of inequality have been proposed in the literature, for inter-temporal and inter-spatial comparisons of income inequality. These measures fall broadly into two classes, namely, positive measures, which make no explicit reference to any ethical evaluation of the distribution, and normative measures which are based on some normative notion of social welfare such that a higher degree of inequality corresponds to a lower level of social welfare for a given total income. In the latter case, inequality ceases to be an objective notion,
and the problem of measurement is enmeshed with that of ethical evaluation. The review that follows discusses the two types of measures in turn.

**Lorenz Curve**

The Lorenz curve, widely used to analyse the inequality in distribution of income and wealth, is defined as the relationship between the cumulative proportion of income-recipient units and the cumulative share of income, when income-recipient units are ranked in terms of their income. Lorenz\(^93\) proposed this curve in 1905 for inter-temporal and inter-spatial comparisons of inequality in wealth. Ever since, it has been employed as a convenient graphical device to analyse size distribution of income and wealth. A striking (and useful) feature of the Lorenz curve, is that it can simultaneously take into account differences in income and population, by putting on a comparable basis, communities with diverse structural characteristics in terms of income and population.

**A Formal Definition of the Lorenz Curve**

The Lorenz Curve, \( q = L(y) \), is a real-valued function which maps the cumulative distribution of recipient units, \( p = F(y) \), ranked according to the size of their income, onto the cumulative distribution of their corresponding aggregate income, that is:

\[
(2.5.1) \quad L(y) = \frac{\int_0^y x \, dF(x)}{E(y)}
\]

where \( y \) is a non-negative income variable, for which the mathematical expectation exists. \( L(y) \) is the first moment distribution function of \( F(y) \), and is, therefore, a

cumulative distribution function. Since both L and F are cumulative distribution functions:

\[ (2.5.2) \quad L : (0,1) \rightarrow (0,1) \]

where L is a bijective (one-to-one and onto) function. Its parametric representation:

\[ (2.5.3) \quad L = \{(F(y), L(y)) \mid y \in (0, \infty)\} \]

follows from the definition of the Lorenz curve. Hence the Cartesian representation is:

\[ (2.5.4) \quad L(p) = \int_0^p F^{-1}(q) \, dq / E(y) \]

for all F, being a continuous and strictly increasing function of income, as are all the specified models of income distribution. The Cartesian representation can easily be obtained when \( p = F(y) \) has an explicit mathematical solution for \( y \) in function of \( p \).

Since the Lorenz curve displays the deviation of each individual income from perfect equality, it captures, in a sense, the essence of inequality. The Lorenz curve could, therefore, be used as a criterion for ranking income distributions. One criticism is, however, that the Lorenz curve provides only a partial ranking. When the Lorenz curve of one distribution lies strictly inside that of another, it can safely be inferred that the former is more equal than the latter. The real problem occurs when the Lorenz curves of two distributions intersect; then, neither distribution can be said to be more equal than the other. In such
cases, ranking in terms of social welfare involves a value judgement, which is subjective.

Whilst the Lorenz curve provides only a partial ordering of distributions, measures of income inequality have been devised to provide complete orderings; of all these measures the Gini ratio is the most widely used. Meanwhile, several other scalar measures of income inequality have been proposed in the literature. Of these, coefficient of variation, variance of the logarithm of income, Theil's index, Hirschman's index and Generalized information index, together with the Gini ratio, belong to the class of positive measures of income inequality. Normative measures include Dalton's ratio, Atkinson's ratio and Kolm's ratio. Although a distinction is made between the two classes, the line of demarcation is not very clear. Even when inequality is taken as an objective notion, interest in its measurement must relate to one's normative concern with it. By the same token, if one takes a normative view of the measures of income inequality, this is not necessarily meant to catch the totality of his ethical evaluation. It would presumably emphasize one particular aspect of the normative comparison which will depend on the objective features of the inequality problem, to the exclusion of other considerations.

The Gini Ratio

The ratio as proposed by Gini\(^\text{94}\) in 1912, measures the relative degree of departure of a population from the situation of perfect equality, i.e., when all income recipient units have the same income. Gini introduced his measure of income inequality as a function of his measure of dispersion, or the Gini mean difference.

Let $\Delta$ symbolize the Gini mean difference, defined as the mathematical expectation of the absolute difference of all the possible pairs of variate-values of income $y$. Hence:

\[(2.5.5) \quad \Delta = E(|Y - X|) = \int_0^\infty \int_0^\infty dF(y) dF(x) \int_0^\infty |y - x| dF(x)\]

and the Gini ratio $G$ is defined as:

\[(2.5.6) \quad G = \frac{\Delta}{2E(Y)} = \frac{\Delta}{2\mu}\]

where $\mu = E(Y)$; $X$ and $Y$ are identically distributed random variables. In the discrete case:

\[(2.5.7) \quad \Delta = \frac{1}{n(n-1)} \sum_{i=1}^{n} \sum_{j=1}^{n} |y_i - y_j|\]

$y_i$ being the income of the $i$th unit, and $n$ the total number of units.

In 1914, Gini proved the important theorem that relates his mean difference to the "area of concentration", i.e., the area between the equidistribution function $L=F$ and the Lorenz curve.

The geometrical interpretation of the Gini ratio is facilitated with the help of the Lorenz curve diagram. (See Figure 4). It is an increasing function of the area enclosed between the Lorenz curve $L=L(F)$ and the identity function $L=F$ (the equidistribution line). Hence, it is an increasing function of the "degree of departure" of the Lorenz curve from the equidistribution function. The geometrical interpretation is given by the ratio of the "area of concentration" (the area between $L=F$ and $L=L(F)$) and the area of triangle OAB. As the area of the triangle is equal to $1/2$, the Gini ratio is equal to twice the area between $L=F$ and
L = L(F), i.e., twice the shaded area in Figure 4. It follows that the Gini ratio takes values in the interval (0,1). Since F and L are integrable functions, the mathematical representation for the Gini ratio G is given by:

\[ G = 2 \int_0^1 (F - L) \, dF = 1 - 2 \int_0^1 L \, dF \]

Integrating by parts, gives:

\[ G = -1 + 2 \int_0^1 F \, dL \]

and adding (2.5.8) and (2.5.9):

\[ G = \int_0^1 F \, dL - \int_0^1 L \, dF \]
The three formulae (2.5.8) - (2.5.10) are equivalent representations of the same concept. Using the concept of the integral as an area under a curve, each one of them has a neat geometrical interpretation in Figure 4. It follows that the Gini ratio is a functional\footnote{A functional is a correspondence which assigns a definite (real) number to each function (or curve) belonging to some set.} that maps the set \{L(F)\} of Lorenz curves onto the set (0,1) of real numbers, i.e.,

\[(2.5.11) \quad G: \{L(F)\} \rightarrow (0,1)\]

Undoubtedly, one appeal of the Gini ratio lies in the fact that it is a direct measure of income difference, taking account of differences between every pair of incomes. In taking differences over all pairs of incomes, it avoids total concentration being calculated on deviations from the mean which the coefficient of variation or the variance of the logarithm of income characterizes. The Gini ratio has the property of being sensitive to transfers from the rich to the poor at every level. Its welfare implications have, however, been questioned on the ground that a transfer of a given amount of income from the richest equally to everyone else would have the same impact as a transfer of an equal amount from everyone to the poorest, even though the change in income will add proportionately much more to the poor than it subtracts from the rich. Furthermore, the Gini ratio gives more weight to transfers near the mode of the distribution than at the tails.

Coefficient of Variation

The coefficient of variation as a measure of dispersion, suggested by Karl Pearson,\footnote{Karl Pearson, "Regression, Heredity and Panmixia", \textit{Philosophical Transactions of the Royal Society, Series A} 187 (1896): 276-277.} is defined as:
(2.5.12) \( CV = \sigma / \mu \)

where \( \mu = E(Y) \), and \( \sigma \) is the standard deviation of the distribution given by:

\[
(2.5.13) \quad \sigma^2 = \int_0^\infty (y - \mu)^2 f(y) \, dy
\]

(discrete case)

\[
\sigma^2 = \int_0^\infty (y - \mu)^2 f(y) \, dy
\]

(continuous case)

As the coefficient of variation is not directly related to the Lorenz curve, it does not offer a visual picture relating to the Lorenz diagram. The coefficient of variation has the property of being sensitive to income transfers for all income levels, a serious criticism of it is, however, that it is equally sensitive to transfers at all levels of income. Proponents of this view attach greater importance to income transfers at the lower end of distribution. One such measure that has this property is the variance of the income power, or the logarithm of income.

**Variance of the Logarithm of Income**

A measure of income inequality that has the property that a transfer of income carried out in the low income brackets would be quantitatively more effective in reducing inequality than if the transfer were carried out in the high income brackets, is the variance of the logarithm of income. It is expressed by:

\[
(2.5.14) \quad V = \int_0^\infty (\log y - \log G)^2 dF(y)
\]

---

97 Using the Stieltjes-Riemann integral to represent both the continuous and discrete distributions.

where $G$ is the geometric mean. A desirable property of the variance of the logarithm of income is its invariance to the choice of units. In as much as its strength is that it highlights differences at the lower end of the scale, it has the shortcoming of not being sensitive to transfers among high income units. Furthermore, it shares with the coefficient of variation the limitation of taking differences only from the mean.

Theil's Index

H. Theil proposed an inequality measure that is based on the notion of entropy in information theory. It derives from the measure of disorder in thermodynamics. In information theory, one is concerned with the problem of "valuing" the information that a certain event has occurred out of a large number of possibilities. When $p$ is the probability that a certain event will occur, the information content, $h(p)$, of noticing that the event has in fact occurred must be a decreasing function of $p$. One relationship which has this property is the logarithm of the reciprocal of $p$.

\[(2.5.15) \quad h(p) = \log(1/p)\]

When there are $n$ possible states (events) with probability $p_i$, such that $p_i \geq 0$, $\forall i \in \{1, 2, ..., n\}$ and $\sum_{i=1}^{n} p_i = 1$, the weighted sum of all the information values for the various events, each event weighted by the respective probabilities gives the entropy or the expected information content of the system:

(2.5.16) \[ H(p) = \sum_{i=1}^{n} p_i h(p_i) \]
\[ = \sum_{i=1}^{n} p_i \log(1/p_i) \]

Theil argued that the entropy concept provides a useful device for measurement of inequality. He interpreted \( n \) possible events as \( n \) income-recipient units, and \( p_i \) the share of unit \( i \) in total income. Further, if \( y_i \) is the income of the \( i \)th unit and \( \bar{y} \) the mean income, then:

(2.5.17) \[ p_i = y_i / n\bar{y} \text{ so that } \sum p_i = 1 \]

Theil's index is obtained by subtracting the entropy, \( H(p) \), of the income distribution from its maximum value which occurs when everyone gets an equal share, i.e., \( p_i = 1/n \):

(2.5.18) \[ T = \sum_{i=1}^{n} \frac{1}{n} h(1/n) - \sum_{i=1}^{n} p_i h(p_i) \]
\[ = \left( \frac{1}{n} \right) \sum_{i=1}^{n} \frac{y_i}{\bar{y}} \log \left( \frac{y_i}{\bar{y}} \right) \]

or in general:

(2.5.19) \[ T = \int_{0}^{\infty} \frac{y}{\mu} \log \frac{y}{\mu} \, dF(y) \]

Turning to the sensitivity of Theil's index, a shift from a richer to a poorer person lowers \( T \), i.e., Theil's index satisfies the Pigou-Dalton condition.

---

Another attractive feature is that it can be aggregated in a simple manner over groups.

**Hirschman's Index**

Hirschman's index is a special case of the Generalized information index discussed in the following subsection. In the discrete case it is expressed as:

\[
H = (1/2) \left( \sum_{i=1}^{n} p_i^2 - (1/n) \right)
\]

or in general:

\[
H = (1/2) \int (y/\mu) (y/\mu - 1) dF(y)
\]

Another variation, the Herfindhal's index, is simply:

\[
HF = \sum_{i=1}^{n} p_i^2
\]

or in general:

\[
HF = \int (y^2/\mu^2) dF(y)
\]

**Generalized Information Index**

An interpretation of the "social analogue" of the information function \( h \), points to the fact that in Theil's index, \( h = \log(1/p) \), belongs to a much wider class of functions given by:

---


(2.5.24) \[ h(p) = -(1/\beta) p^\beta \]

Deriving an inequality measure in exactly the same way as before, gives for different values of \( \beta \), a generalized information index which may be written in the form:

(2.5.25) \[ I_\beta = 1/(\beta(\beta + 1))( \sum_{i=1}^{n} p_i^{\beta+1} - n^{-\beta} ) \]

The special case \( I_0 \) (that is, where \( \beta = 0 \)) is simply the measure \( T \) (Theil's index). Another special case, Hirschman's index, is obtained by setting \( \beta = 1 \). So by choosing an appropriate "distance function", a particular "information theoretic" measure of income inequality can be generated. Using the Stieltjes-Riemann integral to represent both continuous and discrete distributions, the Generalized information index is expressed as:

(2.5.26) \[ I_\beta = 1/(\beta(\beta + 1)) \int_0^\infty (y/\mu)((y/\mu)^\beta - 1) \, dF(y) \]

Dalton's Ratio

In the preceding review, only the positive measures of income inequality were discussed, and their welfare implications examined. Hugh Dalton\(^{103}\) was the first to argue that any measure of income inequality must be derived directly from a social welfare function. He proposed a measure of equality that is defined as the ratio of actual social welfare to the maximal social welfare:

(2.5.27) \[ E = \left( \frac{\sum_{i=1}^{n} U(y_i)}{nU(\mu)} \right) \]

---

\(^{103}\) H. Dalton, "The Measurement of Inequality".
The maximum welfare is obtained when all incomes are equal. Dalton's measure of income inequality (Dalton's ratio) is defined as the proportional welfare loss resulting from income inequality, i.e.,

\[(2.5.28) \quad D = 1 - \left( \frac{\sum_{i=1}^{n} U(y_i)}{nU(\mu)} \right) / nU(\mu)\]

In general,

\[(2.5.29) \quad D = 1 - \int_{0}^{\infty} U(y) dF(y) / U(\mu)\]

Dalton's ratio, however, is not independent of the unit of measurement. A.B. Atkinson\(^{104}\) criticized this measure on the grounds that it is not invariant with respect to any positive linear transformations of the utility function. Countering Atkinson, Sen\(^{105}\) argued that the ordering of Dalton's ratio is not affected by positive linear transformations, and that the ordering property is a significant factor.

Atkinson's Ratio

Atkinson\(^{106}\) specified a measure of income inequality which unlike Dalton's is invariant with respect to any positive linear transformation of the individual utility functions. It is derived from the concept of equally distributed equivalent level of income, \((y^*)\), the level if received by every individual, would


\(^{105}\) Amartya Sen, *On Economic Inequality*, p. 37.

\(^{106}\) A. B. Atkinson, "On the Measurement of Inequality".
result in the same level of social welfare as the total welfare generated by the actual income distribution, i.e.,

\[(2.5.30) \quad U(y^*) = (1/n) \sum_{i=1}^{n} U(y_i)\]

Atkinson's measure of income inequality is:

\[(2.5.31) \quad A = 1 - (y^*/\mu)\]

which is equal to one minus the ratio of the equally distributed equivalent level of income to the mean income of the actual distribution. Camilo Dagum\textsuperscript{107} shows that it is in fact equal to one minus the ratio of the mean generating function of order less than one (\(\varepsilon > 0\)) and the arithmetic mean. For a specific form of the utility function, Atkinson obtained the precise formula for A, given by:

\[(2.5.32) \quad A(\varepsilon) = 1 - \left( \sum_{i=1}^{n} (y_i/\mu)^{1-\varepsilon} \right)^{1/(1-\varepsilon)}\]

where \(\mu\) denotes the proportion of the population with incomes in the \(i\)th range. The parameter \(\varepsilon\) measures the degree of inequality aversion, and is subjectively determined. Clearly, if income is equally distributed, then \(y^*\) is equal to \(\mu\), and the value of Atkinson's ratio will be zero. For a given distribution the value of A must lie between zero and one. Using the Stieltjes-Riemann integral to represent both continuous and discrete distributions:

\[(2.5.33) \quad A(\varepsilon) = 1 - \left( \int_{0}^{\infty} y^{1-\varepsilon} dF(y) \right)^{1/(1-\varepsilon)} / \mu\]

Kolm's Ratio

S.C. Kolm\textsuperscript{108} suggested a measure of income inequality based on the assumption of constant absolute inequality aversion, so that as a person's income, $y$, is increased by one unit of income, his welfare weight, $U'(y)$, drops by a percent where $\alpha$ is the constant amount of absolute inequality aversion. In general, his approach leads to an inequality measure which does not satisfy the principle of scale independence. Kolm's ratio is given by:

\begin{equation}
K(\alpha) = 1 - (1/\alpha) \log \sum_{i=1}^{n} \exp(-\alpha(y_i - \mu)) f(y_i)
\end{equation}

In general,

\begin{equation}
k(\alpha) = 1 - \log \int_{0}^{\infty} \exp(-\alpha(y - \mu)) dF(y) / \alpha
\end{equation}

6. Measures of Inequality between Income Distributions

The measures of income inequality discussed hitherto; proposed by Gini, Theil and others, account for the degree of income inequality within a given population of income-recipient units. There also exists a class of income inequality indices designed to measure the degree of inequality between income distributions. Following Camilo Dagum's terminology,\textsuperscript{109} the former is called the "intra-income distribution inequality ratio", while the latter is referred to as the "inter-income


distribution inequality or economic distance ratio. Dagum\textsuperscript{110} has proposed a generalized mathematical form of the economic distance ratio. Two particular forms of economic distance ratios were identified, and were presented under both the discrete form, which is distribution-free (for a direct application to observed income distributions), and the parametric form associated with a given model of income distribution.\textsuperscript{111}

In order to measure the degree of inequality between income distributions, the population of income-recipient units must be partitioned according to given socio-economic characteristics such as ethnic groups, regions, social classes, and sex. The new concept of economic distance ratio measures the degree of relative affluence of one population with respect to another. In the theoretical development of the concept, Dagum derives two classes of economic distance $d_1$ and $d_1^r$, which he generalizes to a class of economic distance functions $d_r$, for all real $r$. The concept of economic distance, $d_r$, however, is not invariant with respect to the choice of units. The class of economic distance ratios, $D_r$, obtained as a transformation of the economic distance, $d_r$, is dimensionless and takes values in the unit interval. These are two desirable properties of any inter- and intra-income distribution inequality measure.

Economic Distance $d_1$

Let $X$ and $Y$ be two income variables with cumulative distribution functions (cdf), $F_1(x) = P(X \leq x)$ and $F_2(y) = P(Y \leq y)$ respectively. Their probability density functions (pdf) are $f_1(x)$ and $f_2(y)$. Let $M_1$ and $M_2$ be their corresponding means, $E(X)$ and $E(Y)$ (also denoted by $E_1(Y)$ and $E_2(Y)$ respectively).

\textsuperscript{110} Ibid.

\textsuperscript{111} See Camilo Dagum, "A New Model", pp. 413-437.
For any pair of observed income distributions, the subscript 2 is attached to the
distribution with the larger mean income. Hence without loss of generality,
$M_2 > M_1$. A subscript 1 or 2 is attached to the mathematical expectation operator,
$E$, as the weighting function are the probability density functions, $f_1(x)$ or $f_2(y)$,
respectively. It follows from the relation $M_2 > M_1$, that the population with the
cumulative distribution function (cdf) $F_2(y)$ is more affluent than that with cdf
$F_1(x)$. The meaning of affluence is used in an average sense. The economic
distance, $d_1$, is defined with a view to assessing the relative degree of "economic
affluence". For the purpose, an indicator function, $I(Y - X)$, which assumes the
value 1 for all $Y > X$, the value $1/2$ for all $Y = X$, and the value 0 for all $Y < X$, is
introduced. The economic distance $d_1$ between the income distributions $F_1(x)$ and
$F_2(y)$ is then defined to be the weighted sum of the income difference $Y - X$ for all
$Y > X$, given that $M_2 > M_1$. The weighting factor is the joint pdf $f_1(x)f_2(y)$. Using
the indicator function, it can be shown that:

\[
(2.6.1) \quad d_1 = E((Y - X)I(Y - X)|M_2 > M_1) = \int_{0}^{\infty} dF_2(y) \int_{0}^{y} (y-x) dF_1(x)
\]

Hence,

\[
(2.6.2) \quad d_1 = E_2(\gamma F_1(Y)) + E_1(\gamma F_2(Y)) - E_1(Y)
\]

Economic Distance Ratio $D_1$

The economic distance ratio $D_1$ is a transformation of economic distance
$d_1$ such that $D_1$ is dimensionless and takes values only in the unit interval. It is
defined as:

\[
(2.6.3) \quad D_1 = (d_1 - d^\dagger)/(A_1 - d^\dagger)
\]
The mathematical forms of \( d_1 \) are given in (2.6.1) and (2.6.2). The definitions of \( d^*_1 \) and \( \Delta_1 \) are given below.

**Definition of \( \Delta_1 \)**

\( \Delta_1 \) is defined as the unconditional expectation of the absolute value of the income difference. Thus:

\[
(2.6.4) \quad \Delta_1 = E( | Y - X | )
\]

By definition, \( \Delta_1 \) is the Gini mean difference between two income variables. It can be proven that:

\[
(2.6.5) \quad \Delta_1 = 2E_2(YF_1(Y)) + 2E_1(YF_2(Y)) - E_1(Y) - E_2(Y)
\]

It follows from (2.6.2) and (2.6.5) that:

\[
(2.6.6) \quad \Delta_1 = 2d_1 + E_1(Y) - E_2(Y) < 2d_1
\]

**Definition of \( d^*_1 \)**

The income differential \( d^*_1 \) is obtained from (2.6.1) under the assumption that \( X \) and \( Y \) are identically distributed income variables. By applying the definition of \( d^*_1 \) to the distribution \( F_1(x) \), it can be deduced that:

\[
(2.6.7) \quad d^*_1(x) = \int_0^{\infty} \int_0^{y} (y-x) \ dF_1(x)
\]

and for the distribution \( F_2(y) \):
\[(2.6.8) \quad d_1^*(2) = \int_0^\infty dF_2(y) \int_0^y (y-x) dF_2(x) = 2E_2(YF_2(Y)) - E_2(Y)\]

\[(2.6.9) \quad d_1^* = \min\{d_1^*(1), d_1^*(2)\}\]

In general, \(d_1^*(1) < d_1^*(2)\).

Therefore:

\[(2.6.10) \quad d_1^* = 2E_1(YF_1(Y)) - E_1(Y)\]

Substituting (2.6.2), (2.6.5), and (2.6.10) in (2.6.3):

\[(2.6.11) \quad D_1 = \frac{E_1(YF_2(Y)) + E_2(YF_1(Y)) - 2E_1(YF_1(Y))}{2E_1(YF_2(Y)) + 2E_2(YF_1(Y)) - 2E_1(YF_1(Y)) - E_2(Y)}\]

The Parametric Form of Economic Distance Ratio \(D_1\)

The parametric form of the economic distance ratio, \(D_1\), for Dagum's three-parameter and four-parameter models, is deduced (using 2.6.11) in Chapter III.

The Non-Parametric Form of Economic Distance Ratio \(D_1\)

Observed income distributions always belong to the discrete type of distribution functions. This means that the income range is partitioned into a finite number of intervals. Thus, the economic distance ratio \(D_1\) has to be calculated using a non-parametric, distribution-free method.
In general, it is assumed that the observed distributions, \( F_1(x) \) and \( F_2(y) \), are not similarly partitioned. Let \( F_1(x) \) be partitioned into \( h \) intervals and \( F_2(y) \) into \( k \) intervals, \( h \neq k \). It can then be deduced that:

\[
(2.6.12) \quad d_1 = \sum_{j=1}^{h} \sum_{x_i \leq y_j} (y_j - x_i) \cdot f_1(x_i) \cdot f_2(y_j)
\]

\[
(2.6.13) \quad \Delta_1 = \sum_{j=1}^{h} \sum_{i=1}^{k} (|y_j - x_i|) \cdot f_1(x_i) \cdot f_2(y_j)
\]

\[
(2.6.14) \quad d_1^* = \sum_{j=1}^{h} \sum_{i=1}^{k} (x_j - x_i) \cdot f_1(x_i) \cdot f_1(x_j)
\]

(2.6.12), (2.6.13) and (2.6.14) are the discrete forms of \( d_1 \), \( \Delta_1 \) and \( d_1^* \) respectively.

Substituting (2.6.12), (2.6.13) and (2.6.14) in (2.6.3), the value of \( D_1 \) for the distribution-free case can be obtained.

Testing the Statistical Significance of Economic Distance

In order to decide whether the economic distance ratio \( D_1 \) is significant, the null hypothesis that the two distributions are identical, i.e., \( H_0: F_1(y) \equiv F_2(y) \), must be tested against the alternative hypothesis that they are different. The differences in the distributions can arise in a number of ways, such as changes in means (shift), or variances and asymmetries (shape) of the income distributions \( F_1(y) \) and \( F_2(y) \). The one-sided two-sample Kolmogorov-Smirnov test is adequate, as a test of significance in this case.

Given two random samples of income receivers, one of size \( m \) from the population \( Q_1 \) and another of size \( n \) from the population \( Q_2 \), the one-sided Kolmogorov-Smirnov statistic is:
\( (2.6.15) \quad D^+ = \sup_{\{y\}} \left( F_{1n}(y) - F_{2n}(y) \right) \)

It can be shown that the asymptotic distribution of \( 4(D^+)^2mn/(m+n) \) is a chi-squared distribution with two degrees of freedom. Whence,

\( (2.6.16) \quad 4(D^+)^2mn/(m+n) \sim \chi^2(2) \)

for large samples, which is the case with observed income distributions.

If the statistic obtained by application of (2.6.16) is less than the critical value \( \chi^2_\alpha(2) \) for an \( \alpha \) level of significance, the null hypothesis is accepted. This means that the income inequality between the two distributions considered is not significant. The Kolmogorov-Smirnov statistic \( D^+ \) can be computed from either the observed or the parametric estimates of the income distribution functions.
CHAPTER III

THEORETICAL ANALYSIS OF PERSONAL INCOME DISTRIBUTION

The theoretical development of the tools of analysis and concepts employed in Chapter IV, in particular the theoretical model on which that chapter rests, is presented here. As a prelude to this exposition some issues relating to the measurement of inequalities in the distribution of income are discussed in section 1. They include a discussion of problems of measurement, problems of data, parametric approach to measurement of inequality and a set of properties for income distribution models. In section 2, the model used in the empirical analysis is presented, followed by a brief discussion of methods of parameter estimation, and test of goodness of fit criteria for income distribution models. The closing subsection is devoted to the validation process of the model.

1. Measurement of Inequalities in the Distribution of Income

Problems of Measurement

In attempting to measure inequality, a problem frequently confronted is the choice of an appropriate index of inequality (I(Y)) that can be computed when the pattern of income distribution \( Y = (Y_1, Y_2, \ldots, Y_n) \) is given for \( n \) recipient units. Many alternative indexes of inequality were discussed in Chapter II. For reasons spelt out in that chapter, the Gini ratio is chosen to measure income
inequalities in the present study. The variance of income power is computed in order to facilitate the analysis in terms of the human capital earnings function.

In the spatial comparison of income inequalities, an attempt is made to differentiate various types of income-recipient units. For the purpose, the global pattern of income \((Y)\) is segmented as follows:

\[(3.1.1) \quad Y = \{Y_1, Y_2, \ldots, Y_n\}; \quad S = \{S_1, S_2, \ldots, S_r\}\]

Where \(S_i, \forall i \in \{1, 2, \ldots, r\}\) are mutually exclusive and jointly exhaustive subsets of \(Y\), \(r\) represents the number of homogenous groups. Hence, if \(Y\) represents the set of incomes in the population, the spatial context implies the study of disjoint subsets:

\[(3.1.2) \quad S_i \subseteq Y, \quad \forall i \in \{1, 2, \ldots, r\}\]

and

\[(3.1.3) \quad S_i \cap S_j = \emptyset, \quad \forall i, j \in \{1, 2, \ldots, r\}, i \neq j\]

Each subset, \(S_i\), corresponds to a particular class of income-recipient units. For example, when \(r\) is 2, \(S_1\) and \(S_2\) can represent male and female incomes. The classification suggests that all income-recipient units within each \(S_i\) can be referred to as a homogenous group. Consequently, in this abstract formulation, which can be referred to as the segmentation model, an index of inequality can be

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1 In the context of the present study, the term "spatial" is defined to include not only physical (geographical) space, i.e., regions, but also other kinds of socio-economic, demographic and institutional space, such as sectors, sex, age and educational levels.
defined for all income recipients \( I(Y) \), as well as for each homogenous group \( I(S_i); \forall i \in \{1, 2, \ldots, r\} \) (intra-income distribution inequality ratio). Inequality between homogenous groups (inter-income distribution inequality ratio) can also be identified. Thus, in the segmentation model, overall inequality, \( I(Y) \), can be explained in terms of the intra-group effect, \( I(S) \), and the inter-group effect. The Gini ratio is used to measure the intra-group inequality. Dagum's concept of "economic distance ratio",\(^2\) given by:

\[
D(S_i, S_j), \forall i, j \in \{1, 2, \ldots, r\}, i \neq j
\]

measures the inter-group effect.

Problems of Data

In Chapter IV, the concepts introduced in the present chapter will be studied in an empirical context. Consequently, the availability and quality of data become critical issues. The foregoing discussion of the problem of measurement has already suggested the kind of data needed for the present study. The segmentation model requires, for example, data on the global income pattern \( Y \) and on the patterns of income \( Y_i \) for various "homogenous" groups. All such data must normally come from household surveys.

Data drawn from a series of consumer finance surveys,\(^3\) covering the period 1953-1979, undertaken by the Central Bank of Ceylon, form the basis for

\(^2\) See Camilo Dagum, "Inequality Measures", pp. 1791-1803. 

empirical analysis. The Consumer Finance Survey of 1953 was the first in a
planned regular series undertaken at ten-year intervals up to 1973, and at five-year
intervals thereafter. Accordingly, the last survey of the series was conducted in
1978/79. The survey of Consumer Finances is perhaps the single most important
source of information, on a historical basis, as regards the distribution of income
by size and by detailed socio-economic characteristics. The data have a good
coverage and are representative. The specific dimensions of the data had,
however, to be recognized and addressed. There are problems associated with the
randomness of sampling and the biased non-responses among sample returns. For
those who do respond, under-reporting of income; particularly certain income types
such as interest, dividends, property income and transfer income, is not uncommon.
As a result, there is a tendency for the data to produce observed distributions
which fail to capture the precise shape of the income distribution, particularly its
upper and lower tails.

The data used for the United Kingdom, German Democratic Republic,
Pakistan and Singapore are drawn from Household Income and Expenditure
Statistics published by the International Labour Office in Geneva. The source of
information for Canada is Income Distribution by Size in Canada published by
Statistics Canada. The data for the United States are based on the Current
Population Survey conducted by the U.S. Bureau of the Census.

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4 International Labour Office, Household Income and Expenditure

5 Statistics Canada, Income Distribution by Size in Canada, 1979
(Catalogue 13-207) (Ottawa, 1980).

in 1978 of Families and Persons in the United States (Series P-60, N. 123)
(Washington D.C., 1980).
Parametric Approach to Measurement of Inequality

Section 5 of Chapter II, concentrated on defining a set of inequality measures which used only the general notion of a discrete or continuous random variable, without explicitly specifying its associated probability density function. In section 4, however, a number of alternative probability density functions (models) that purport to describe the observed income distribution, was discussed. The search for analytic distributions resembling the observed income distribution can be justified from several points of view.

Empirical work in the field of personal income distribution, is frequently confronted with data in grouped form. Data on observed income distribution are normally available for predefined income classes. However, if a theoretical function can be specified which provides a reasonably close approximation to the true distribution, one obtains a device for interpolation within income classes and for extrapolation in the lowest and/or highest classes. The mechanism readily enables one to calculate the number of recipients in arbitrarily selected income brackets and their income. Furthermore, measures of inequality are normally defined in terms of individual incomes, as has been demonstrated earlier. The use of grouped data to compute such measures, therefore, requires some additional assumptions with regard to within-class incomes. Suffice it to say, that the value of a measure of inequality obtained by using grouped data neglects part of the variation existing (within income classes) in reality. A bias in estimation emerges as a result. However if a well-fitting analytic model is obtained, measures of income inequality can frequently be derived out of the estimated parameter values of the distribution. These parameters may be estimated without the additional restrictive assumption of complete within-class equality. Hence, the parametric approach provides a more realistic estimate of actual inequality. Another advantage of this approach is that, given a specific functional form, relationships
between different inequality measures may be derived. A function representing
the distribution of income may also be used to smooth out irregularities in the
observed income distribution caused by misreporting of income. In this role it is
similar to the graduation formulae used in demographic analysis to correct an age
distribution distorted by misstatements of age. The parametric approach has
normative implications, too. A well-fitting distribution function can provide the
basis for specification of a simulation model of income distribution.

The above, succinctly, illustrates that the search for an analytic
distribution function is valid for both positive and normative purposes. The
question arises, then, as to the kind of approaches that can be followed to select
the analytic distribution functions, and criteria that may be used in this selection
process.

A Set of Properties for Income Distribution Models

The models developed for the analysis of income distribution are not
based on a causal explanation. They are simply univariate models that purport to
describe accurately the phenomenon in question, namely, the observed pattern of
income distribution. The choice of a specific form, therefore, depends on a set of
desirable properties that it is expected to fulfill. These properties are well
accepted in the literature and have, by and large, motivated the specification of a
number of income distribution models. However, no single model fulfills all of
them.

P.1 Model foundations

This property requires that the mathematical form of an income
distribution model be derived from realistic elementary assumptions. The word

7 See Camilo Dagum, "Generating Systems and Properties of Income
"elementary" is used in the context of both its meanings: "simple" from a technical point of view and "fundamental" from the point of view of model building. The mathematical form of the income distribution models can be grouped according to the characteristics of its generating structure, into three main classes, namely, stochastic, logico-empirical, and ad-hoc (specification). A model derived by means of a given stochastic process (a set of probability assumptions) has a stochastic foundation. When the generating structure is based on the characteristics of regularity and permanence of the observed income distributions, the model is said to have a logico-empirical foundation. Mathematical forms proposed solely because they provide a satisfactory fit to the observed income distributions, have an ad-hoc model specification.

P.2 Convergence to the Pareto distribution

Empirical evidence suggests that the Pareto distribution is the model of high income groups. The Pareto convergence property or the weak Pareto law, as it is called by Mandelbrot, requires the upper tail of income distribution models to approximate to the Pareto law.

P.3 Economic significance of the parameters

This property requires an explicit economic meaning to be ascribed to the parameters of an income distribution model. The model and its parameters must, therefore, have meaningful economic interpretations in terms of characteristics of the observed income distribution.

8 Benoit Mandelbrot, "The Pareto-Lévy Law".
P.4 Good fit of the whole income range

A model must have the power to accurately describe the whole range of an observed income distribution. An accurate description of the observed distribution is a pre-requisite, if the model is to be employed for any analytical purpose or policy making, such as (1) an accurate measurement of inequality; (2) support for a given social and income policy; or (3) planning the taxation structure.

P.5 Good fit of both unimodal and zeromodal distributions

The ability of a model to describe both unimodal and zeromodal distributions has implications for inter-country comparisons, for zeromodal income distributions can be observed in the less affluent countries. Zeromodal distributions can also be present in subpopulations of economic units and are always found in the distribution of wealth.

P.6 Good fit of distributions with zero or negative incomes

Zero and negative incomes result from the definition of income adopted in household surveys in some countries, Canada and the United States, in particular. A negative income can arise from net income from self-employment and net investment income. All sources of income can be responsible for the existence of a finite frequency of recipient units with zero income.

P.7 Good fit of the whole income range of distribution starting from an unknown positive origin

The model should be able to describe an observed distribution with a positive minimum income that is not predetermined. This is the case when a social security system with universal coverage is implemented with a view to guaranteeing a minimum income.
P.8 Derivation of the mathematical form of the Lorenz curve from the model of income distribution and conversely

This property has both analytical and policy implications. Knowledge of the mathematical form of the income distribution, and its associated Lorenz curve, facilitate both analysis in terms of welfare ranking of income distributions and the concept of stochastic dominance.

P.9 Explicit mathematical solution of the Gini income inequality ratio

As in P.8 above, an explicit mathematical form for the Gini ratio provides an important tool of analysis with both positive and normative implications.

P.10 The capability of the Gini ratio to account for intersecting Lorenz curves

The Gini ratio is widely employed in the measurement of income inequality. This ratio, however, produces an ambiguous measure of change in inequality, if it is a function of a single parameter. This ambiguity is a consequence of the mathematical limitations of the income distribution model being identified rather than a limitation of the Gini measure. The Gini ratio as a function of more than one parameter is free from this defect, and is a powerful tool in the analysis of differences in income distribution. Here, the analysis is carried out in terms of the quantiles that improve their shares as well as those that suffer losses. Its implications for the analysis of poverty is of the utmost importance.

P.11 The Principle of parsimony

The rationale of this property is to reduce the number of parameters of a given model, to a minimum without negating any of the other properties.
Evaluation of the Income Distribution Models

The models discussed in chapter II were evaluated according to the foregoing criteria. Table 3 presents these models and their properties. An X represents fulfilment of a given property. A dash (-) stands for either "no fulfilment" or "not yet known", and n/a for not applicable.

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</tbody>
</table>

Of the models evaluated, Dagum's Type II model fulfils all the properties except the principle of parsimony. The model ranks first, followed by Dagum's Type I model. Empirical work undertaken hitherto, using Dagum's Type II model suggests that the model fits well the whole range of both standard type distributions (P. 4) and distributions with special characteristics (P. 6 and P. 7). Although no weight is attached to each of the properties discussed, property 4 is by far the most important. Models that do not fit well the whole range of observed
income distributions lose analytical power. Only three out of the 17 models included in Table 3 fulfil P. 4. Based on prior evidence on the comparison of goodness of fit for various models, Dagum's Type I and Type II are ranked first, followed by Singh-Maddala's model. However, the actual performance of Dagum's model vis-à-vis other models in a given situation, has to be evaluated (for that situation) in terms of a suitable test of goodness of fit.

2. The Model

Dagum's model, both Type I and Type II with its variant (Type III) to deal with strictly positive income above a threshold income, is employed in the ensuing analysis.

Model Specification

The model specification is the theoretic representation of the characteristics of regularity and permanence of income elasticity in observed income distributions. The systematic analysis of the observed income distributions shows that the income elasticity of the cumulative distribution function, \( F(y) \), is a monotonic decreasing function of \( F(y) \) itself. In general, this elasticity is a concave function of \( F \). The mathematical specification of this elasticity is:

\[
\varepsilon(F, y) = \frac{d \log F(y)}{d \log y} = \beta_1 (1 - (F(y))^{\beta_2}), \quad y > 0, \quad (\beta_1, \beta_2) > 0
\]

The solution of the differential equation (3.2.1) gives the three-parameter model:

\[
F(y) = \begin{cases} (1 + \lambda y^{-\delta})^{-\beta} & , \quad y > 0, \quad (\beta, \lambda, \delta) > 0 \\ 0 & , \quad y \leq 0 \end{cases} \quad \text{Dagum Type I}
\]

where $\lambda$ is a scale parameter. $\lambda$ is positive since it is the exponential of a constant of integration, and:

\begin{equation}
\beta = 1/\beta_2, \quad \delta = \beta_1\beta_2
\end{equation}

are inequality parameters.

A more general specification of the model is obtained when the deviations of $F(y)$ from an arbitrary origin, $\alpha < 1, \alpha \neq 0$, is considered. The parameter $\alpha$ plays an important role in the modification. If there exists a finite frequency of recipient units with zero and negative income, this implies that $\alpha$ is significantly greater than zero (i.e. $0 < \alpha < 1$). The economic interpretation of $\alpha$ is that it accounts for the unemployed economic units not covered by any social security system (null income), and the proprietors of unincorporated enterprises with a net loss (negative income) during the reference period. If a universal unemployment insurance coverage or a guaranteed minimum income programme exists, then the observed income distribution starts from a positive income $y_0$. This is also the case when the sample covers only the employed economic units with a positive minimum income $y_0$. Hence, $y \geq y_0 > 0$, where $y_0$ is the minimum income, which implies that $\alpha < 0$. Furthermore, in this case $\alpha = F(0)$, which is the extrapolation of $F(y)$, for $y = 0$; and $y_0$ is the solution of $F(y_0) = 0$, that is, when $F(y)$ intersects the abcissa $y$.

Hence, the mathematical representation of the elasticity is as follows:

\begin{equation}
\varepsilon(F, \alpha, y) = \frac{d \log (F(y) - \alpha)}{d \log y} = \beta_1(1 - \frac{F(y) - \alpha}{1 - \alpha})^{\beta_2},
\end{equation}

$y \geq y_0 > 0$, $\alpha < 1$, $(\beta_1, \beta_2) > 0$,

$F(y_0) = 0$ if $\alpha < 0$, and $y_0 = 0$ if $0 \leq \alpha < 1$. 
The solution of the differential equation (3.2.4) gives the four-parameter model:

(3.2.5) \[ F(y) = \alpha + (1 - \alpha) (1 + \lambda y^{-\delta})^{-\beta}, \quad y \geq y_0 \geq 0, \]

(3.2.6) \[ \beta = 1/\beta_2, \quad \delta = \beta_1 \beta_2, \quad \lambda > 0, \quad \alpha < 1 \]

Model (3.2.5) generates three important cases of functional forms for describing observed income distributions.

Case 1: \( \alpha = 0 \) (The three-parameter Dagum Type I model).

This is the same as the model generated by the differential equation (3.2.1). The range of income is the open infinite interval \((0, \infty)\).

Case 2: \( 0 < \alpha < 1 \) (The four-parameter model to deal with null and negative incomes — Dagum Type II model).

This case is relevant when a finite proportion (estimated by \( \alpha \)) of economic units with zero and negative incomes exists. The model specification corresponding to this case is given by the differential equation (3.2.4) with the added constraint \( 0 < \alpha < 1 \). Its solution is given by (3.2.5) and taking into account that \( \alpha \) is positive and less than one:

\[
(3.2.7) \quad F(y) = \begin{cases} 
\alpha + (1 - \alpha)(1 + \lambda y^{-\delta})^{-\beta}, & y > 0, \quad 0 < \alpha < 1, \quad \beta, \lambda, \delta > 0 \\
\alpha, & y = 0 \\
0, & y < 0
\end{cases} \]

Dagum Type II

The specification of the model is also possible using the Jordan decomposition theorem,\(^10\) which states that every probability distribution is a convex combination of a discrete and a continuous distribution, i.e.,

(3.2.8) \( F(y) = \alpha F_d(y) + (1 - \alpha)F_C(y) \), \( 0 \leq \alpha \leq 1 \)

It follows from (3.2.7) and (3.2.8)

(3.2.9) \( F_d(0) = 1, \quad F_d(y) = 0, \quad y < 0 \)

(3.2.10) \( F_C(0) = 0, \quad \text{and} \quad F_C(y) = (1 + \lambda y^{-\delta})^{-\beta}, \quad y > 0 \)

Case 3: \( \alpha < 0 \) (The four-parameter model to deal with strictly positive income above a threshold \( y_0 > 0 \) —Dagum Type III model).

The empirical foundations for this case were discussed earlier. Its mathematical representation is given by the differential equation (3.2.4), with the constraint \( \alpha < 0 \). Hence, its solution is:

(3.2.11) \( F(y) = \alpha + (1 - \alpha)(1 + \lambda y^{-\delta})^{-\beta}, \quad y \geq y_0 > 0, \quad F(y_0) = 0, \quad \alpha < 0, \quad (\beta, \lambda, \delta) > 0 \),

which can be written in the form:

(3.2.12) \( F(y) = (1 + \lambda(y - y_0)^{-\delta})^{-\beta}, \quad \alpha < 0, \quad (\beta, \lambda, \delta) > 0 \)  \( \text{Dagum Type III} \)

Probability Density Function (pdf)

Case I

(3.2.13) \( f(y) = \begin{cases} \frac{\beta \lambda \delta y^{-\delta-1}(1 + \lambda y^{-\delta})^{-\beta-1}}{y_0}, & y > 0 \\ 0, & y \leq 0 \end{cases} \)

Case II

(3.2.14) \( f(y) = \begin{cases} \alpha, & y = 0 \\ (1 - \alpha) \frac{\beta \lambda \delta y^{-\delta-1}(1 + \lambda y^{-\delta})^{-\beta-1}}{y_0}, & y > 0 \\ 0, & y < 0 \end{cases} \)
Case III

(3.2.15) \[ f(y) = \begin{cases} 
\beta \delta (y - y_0)^{-\delta - 1} (1 + \lambda (y - y_0)^{-\delta})^{\beta - 1}, & y > y_0 \\
0, & y \leq y_0 
\end{cases} \]

(3.2.16) \[ y_0 = \lambda^{1/\delta} \left( (1 - (1/\alpha))^{1/\beta} - 1 \right)^{-1/\delta} \]

Mode and Percentiles of the Distribution

The modes (interior maxima) for the distribution function in cases 1 and 2 are obtained as the solutions of the equation \( F''(y) = 0 \), since \( F(y) \) and \( f(y) = F'(y) \) are continuously differentiable functions for all \( y > 0 \). It can be shown that \( F(y) \) is unimodal (has a unique interior maximum) when \( \beta \delta > 1 \) and is zeromodal (has no interior maximum) when \( \beta \delta \leq 1 \). In the latter case the maximum occurs at \( y = 0 \), hence the probability density function is a strictly decreasing function. The solution of the equation \( F''(y) = 0 \) gives the unique maximum \( y_M \) of the distribution in both cases 1 and 2:

(3.2.17) \[ y_M = \begin{cases} 
\lambda^{1/\delta} \left( (\beta \delta - 1) / (\delta + 1) \right)^{1/\delta}, & \beta \delta > 1 \\
0, & \beta \delta \leq 1
\end{cases} \]

Similarly, it can be deduced that the distribution function in Case 3 has a unique maximum \( y_{M3} \) (unimodal if \( \beta \delta > 1 \) and zeromodal if \( \beta \delta \leq 1 \)):

(3.2.18) \[ y_{M3} = \begin{cases} 
y_0 + y_M, & \beta \delta > 1 \\
y_0, & \beta \delta \leq 1
\end{cases} \]

Hence, the maximum for case 3 is obtained by adding \( y_0 \) units to the corresponding maximum for cases 1 and 2 given in (3.2.17).

The \( p \)th percentile \( y_p \) is the solution of the equation \( F(y_p) = p \). It then follows from (3.2.2), (3.2.7) and (3.2.12).
Case I

(3.2.19) \( y_p = \lambda^{1/\delta} (\rho^{-1/\beta} - 1)^{-1/\delta} \), \( \alpha = 0 \)

Case II

(3.2.20) \( y_p = \begin{cases} \lambda^{1/\delta} ((1 - \alpha) / (p - \alpha))^{1/\beta} - 1)^{-1/\delta}, & 0 < \alpha < p \\ 0, & p \geq \alpha < 1 \end{cases} \)

Case III

(3.2.21) \( y_p = y_0 + \lambda^{1/\delta} (\rho^{-1/\beta} - 1)^{-1/\delta}, \quad y_0 > 0 \)

The median of the distribution occurs when \( p = 1/2 \).

Moments of the Distribution

Case I \( (\alpha = 0) \)

Using (3.2.13) and the definition of moments of order \( r \) about the origin, and performing the substitution:

(3.2.22) \( x = \lambda y^{-\delta} (1 + \lambda y^{-\delta})^{-1} \)

\[ dy = (\lambda^{1/\delta} / \delta) x^{-1-(1/\delta)} (1-x)^{-1+(1/\delta)} \, dx \]

It can be deduced that:

(3.2.23) \( \mu_r = E(Y^r) = \beta \lambda^{r/\delta} \int_0^1 x^{-r/\delta} (1-x)^{\beta+(r/\delta)-1} \, dx \)

\[ = \beta \lambda^{r/\delta} B(1-(r/\delta), \beta+(r/\delta)), \quad r < \delta \]

That is, the moment of order \( r \) exists for all \( r < \delta \). Making \( r = 1 \) the mathematical expectation of income or the first moment of income can be obtained:
\[(3.2.24) \mu = E(Y) = \beta \lambda^{1/\delta} B(1 - (1/\delta), \beta + (1/\delta)), \delta > 1\]

where \(B(1 - (1/\delta), \beta + (1/\delta))\) is the Euler beta function:

\[(3.2.25) \quad B(m, n) = \int_0^1 y^{m-1} (1 - y)^{n-1} \, dy, \quad (m, n) > 0\]

with parameters \(m = 1 - (1/\delta)\) and \(n = \beta + (1/\delta)\)

Case II \((0 < \alpha < 1)\)

Using \((3.2.14)\) and the definition of moments of order \(r\) about the origin, it can be deduced that:

\[(3.2.26) \quad \mu_r = E(Y^r) = (1 - \alpha) \beta \lambda^{r/\delta} \int_0^\infty y^{r-\delta-1} (1 + \lambda y^{-\delta})^{\beta-1} \, dy\]

Performing the substitution \((3.2.22)\):

\[(3.2.27) \quad \mu_r = (1 - \alpha) \beta \lambda^{r/\delta} B(1 - (r/\delta), \beta + (r/\delta)), \quad r < \delta\]

The mathematical expectation is obtained when \(r = 1:\)

\[(3.2.28) \quad \mu = E(Y) = (1 - \alpha) \beta \lambda^{1/\delta} B(1 - (1/\delta), \beta + (1/\delta))\]

Case III \((\alpha < 0)\)

When \(\alpha < 0\), the range of income is \(y_0 < y < \infty\), with \(y_0 > 0\). Hence, it starts from a positive value of income, \(y_0\). It follows from \((3.2.15)\) and the definition of moments of order \(r\) about the origin, that:
\[(3.2.29) \quad \mu_r = E(Y^r) = \beta \lambda \delta \int_{y_0}^{\infty} (y - y_0)^{\beta-1} (1 + \lambda(y - y_0)^{-\delta})^{-\beta-1} dy\]

Performing the substitution \( u = y - y_0 \) and

\[(3.2.30) \quad x = \lambda u^{-\delta}(1 + \lambda u^{-\delta})^{-1}\]

where \( du = (\lambda^{1/\delta} / \delta) x^{-1-(1/\delta)} (1-x)^{-1+(1/\delta)} \, dx \)

It can be deduced that:

\[(3.2.31) \quad \mu_r = \sum_{j=0}^{r} \binom{r}{j} y_0^{r-j} \lambda^{j/\delta} B(1-(j/\delta), \, \beta+(j/\delta)), \quad r < \delta\]

For \( r = 1 \)

\[(3.2.32) \quad \mu = E(Y) = y_0 + \beta \lambda^{1/\delta} B(1-(1/\delta), \, \beta+(1/\delta)), \quad \delta > 1\]

which is equal to the mathematical expectation corresponding to Case I (\( \alpha = 0 \)) deduced in (3.2.24) plus the displacement \( y_0 \) of \( y \).

The Lorenz Curve

Case I (\( \alpha = 0 \))

Solving for \( y \) in (3.2.2)

\[(3.2.33) \quad y = \lambda^{1/\delta} (\beta^{-1/\beta} - 1)^{-1/\delta}\]

Using (3.2.2), (3.2.24) and (3.2.33) in (2.5.4) and performing the substitution:
\( (3.2.34) \quad Z = F^{1/\beta}, \quad dF = \beta Z^{\beta-1} dZ \)

It can be deduced that:

\( (3.2.35) \quad L(F) = B(F^{1/\beta}; \beta + (1/\delta), 1 - (1/\delta)), \quad \delta > 1 \)

where \( B(.) \) is a beta cumulative distribution function of the form:

\( (3.2.36) \quad B(Z; m, n) = \frac{Z^m \int_0^Z x^{m-1} (1 - x)^{n-1} dx}{B(m, n)} \)

with respect to the variable \( Z = F^{1/\beta} \)

Case II \((0 < \alpha < 1)\)

Solving for \( y \) in \((3.2.7)\):

\( (3.2.37) \quad y = \lambda^{1/\delta} \left( \frac{(1 - \alpha) / (F - \alpha)}{(1 - \alpha) / (F - \alpha) + 1} \right)^{-1/\delta}, \quad F \geq \alpha \)

Using \((3.2.7), (3.2.28)\) and \((3.2.37)\) in \((2.5.4)\) and performing the substitution:

\( (3.2.38) \quad Z = \left( (F - \alpha) / (1 - \alpha) \right)^{1/\beta} \)

\( dF = (1 - \alpha) \beta Z^{\beta-1} dZ \)

It can be shown:

\( (3.2.39) \quad L(F) = B \left( \left( (F - \alpha) / (1 - \alpha) \right)^{1/\beta}; \beta + (1/\delta), 1 - (1/\delta) \right), \quad \delta > 1 \)
Hence, the Lorenz curve is a cumulative distribution function with respect to the variable $Z'$ as specified in (3.2.38).

Case III ($\alpha < 0$)

Using (3.2.12), (3.2.32) in (2.5.4), and applying similar arguments to those used before, it can be shown that, when $\delta > 1$:

$$L(F) = \frac{y_0^F}{\mu} + \frac{\beta \lambda^{1/\delta}}{\mu} B(\beta + (1/\delta), 1 - (1/\delta)) B(F^{1/\beta}; \beta + (1/\delta), 1 - (1/\delta)),$$

where $\mu = \mu(1; \alpha < 0)$

It can also be shown that:

$$\mu = \mu(1; \alpha < 0) = y_0 + \mu(1; \alpha = 0)$$

Hence, for $\delta > 1$,

$$L(F) = \frac{y_0^F}{y_0 + \mu(1; \alpha = 0)} + \frac{\mu(1; \alpha = 0)}{y_0 + \mu(1; \alpha = 0)} B(F^{1/\beta}; \beta + (1/\delta), 1 - (1/\delta)),$$

where $\mu(1; \alpha = 0)$ symbolizes (3.2.24).

The Gini Ratio

Case I ($\alpha = 0$)

Differentiating (3.2.35) with respect to $F$:

$$dL = \frac{F^{1/\beta \delta} (1-F^{1/\beta})^{-1/\delta}}{\beta B(\beta + (1/\delta), 1 - (1/\delta))} dF$$
and substituting (3.2.43) in (2.5.9):

\[
(3.2.44) \quad G = -1 + \frac{2}{\beta \psi'(\beta + (1/\delta), 1 - (1/\delta))} \int_0^1 F^{1/\beta \delta} (1 - F^{1/\delta})^{-1/\delta} \, dF
\]

Performing the substitution \( x = F^{1/\beta} \),

\[
(3.2.45) \quad G = -1 + B(\beta, 2\beta + (1/\delta)) / B(2\beta, \beta + (1/\delta))
\]

It follows from (3.2.45), and the interpretation of beta distribution in terms of the gamma distribution, namely:

\[
(3.2.46) \quad B(m, n) = \Gamma(m) \Gamma(n) / \Gamma(m+n),
\]

that:

\[
(3.2.47) \quad G = G(\alpha = 0) = -1 + B(\beta, \beta) / B(\beta, \beta + (1/\delta)),
\]

where \( G(\alpha = 0) \) is the Gini ratio for the three-parameter model (Case I).

**Case II \( (0 < \alpha < 1) \)**

Differentiating (3.2.39):

\[
(3.2.48) \quad dL = \frac{(F - \alpha)^{1/\beta \delta} ((1 - \alpha)^{1/\beta} - (F - \alpha)^{1/\beta})^{-1/\delta}}{(1 - \alpha) \beta B(\beta + (1/\delta), 1 - (1/\delta))} \, dF
\]

Substituting (3.2.48) in (2.5.9) and performing the substitution:

\[
(3.2.49) \quad x = (F - \alpha)^{1/\beta} / (1 - \alpha)^{1/\beta}
\]
It can be deduced that:

\[(3.2.50) \ G = (2\alpha - 1) + (1 - \alpha) \ B(\beta, \beta) / B(\beta, \beta + (1/\delta)) \]

\[(3.2.47)\) is a particular case of \((3.2.50)\) obtained when \(\alpha = 0\).

**Case III (\(\alpha < 0\))**

Differentiating \((3.2.42)\), and using \((3.2.24)\), \((3.2.32)\) and \((3.2.41)\), it can be deduced that:

\[(3.2.51) \ dL = \frac{1}{y_0 + \mu(1; \alpha = 0)} \ (y_0 + \lambda^{1/\delta} F^{1/\delta} (1 - F^{1/\delta})^{-1/\delta}) \ dF \]

It follows from \((2.5.9)\) and \((3.2.51)\) and performing the substitution \(x = F^{1/\delta}\), that:

\[(3.2.52) \ G = G(\alpha < 0) = \frac{1}{y_0 + \mu(1; \alpha = 0)} \ (-\mu(1; \alpha = 0) + 2B\lambda^{1/\delta} B(2\beta + (1/\delta), 1 - (1/\delta))) \]

Multiplying both sides of \((3.2.45)\) by \(\mu(1; \alpha = 0)\), whose mathematical expression is given in \((3.2.24)\):

\[(3.2.53) \ \mu(1; \alpha = 0) \ G(\alpha = 0) = -\mu(1; \alpha = 0) + 2B\lambda^{1/\delta} B(2\beta + (1/\delta), 1 - (1/\delta)) \]

It follows from \((3.2.52)\) and \((3.2.53)\) that:

\[(3.2.54) \ G(\alpha < 0) = \frac{\mu(1; \alpha = 0) \ G(\alpha = 0)}{y_0 + \mu(1; \alpha = 0)} \]
Economic Distance Ratio $D_1$

It was shown in Chapter II, that:

\[
(3.2.55) \quad D_1 = \frac{E_1(YF_2(Y)) + E_2(YF_1(Y)) - 2E_1(YF_1(Y))}{2E_1(YF_2(Y)) + 2E_2(YF_1(Y)) - 2E_1(YF_1(Y)) - E_2(Y)}
\]

Case I ($\alpha = 0$)

It can be deduced, using the model specification in (3.2.2), that:

\[
(3.2.56) \quad E_2(YF_1(Y)) = \int_0^\infty y F_1(y) dF_2(y)
\]

\[
= \beta_2^{1/\delta_2} \lambda_2^{1/\delta_2} \int_0^1 \frac{1}{t^{\lambda_2}} (1 - t)^b ((1 - t)^\delta + at^\delta)^{-\beta_1} dt,
\]

where

\[
(3.2.57) \quad b = \beta_1 \delta + \beta_2 + (1/\delta_2) - 1, \quad \delta = \delta_1 / \delta_2, \quad a = \lambda_1 \lambda_2^{-\delta}
\]

$E_1(YF_2(Y))$ is obtained by interchanging the subscripts 1 and 2 in (3.2.56) and (3.2.57). The subscript 2 in (3.2.56) is replaced by 1 to obtain $E_1(YF_1(Y))$. $E_2(Y)$ is obtained from (3.2.23), making $r = 1$ and noting that the weighting function is the probability density function (pdf) $f_2(y)$. The subscript 1 in the parameter vector $(\beta, \lambda, \delta)$ identifies the cdf, $F_1(x)$, and the subscript 2 the cdf, $F_2(y)$.

Case II ($0 < \alpha < 1$)

Using the model specification in (3.2.7), it can be shown that:

\[
(3.2.58) \quad E_2(YF_1(Y)) = \int_0^\infty y F_1(y) dF_2(y)
\]

\[
= \alpha_1 E_2(Y) + (1 - \alpha_1)(1 - \alpha_2) \beta_2^{1/\delta_2} \lambda_2^{1/\delta_2} \int_0^1 \frac{1}{t^{\lambda_2}} (1 - t)^b ((1 - t)^\delta + at^\delta)^{-\beta_1} dt,
\]

where $b$, $\delta$, and $a$ are as in (3.2.57).
As before, $E_1(YF_2(Y))$ is obtained by interchanging the subscripts 1 and 2 in (3.2.58) and (3.2.57).

Methods of Parameter Estimation

The methods of parameter estimation are given for Dagum's Type II (four-parameter) model. The same scheme can be followed for Type I and Type III models. Dagum's model belongs to the class of transcendent functions, besides being nonlinear. It cannot be linearized after performing a logarithmic transformation. Hence, the least square (LS) and the maximum likelihood (ML) methods of parameter estimation require the solution of a system of nonlinear (transcendent) equations.

In order to obtain parameter estimates some method of estimation must be selected. Possible candidates involve the method of maximum likelihood, minimization of some measure of discrepancy between actual and theoretical frequencies, and some iterative methods. Several iterative methods are discussed elsewhere.\footnote{Camilo Dagum, "A New Model", pp. 413-437.

The methods of maximum likelihood (ML)

The maximum likelihood (ML) method chooses the k-parameter vector $\hat{\theta}$ as an estimate of $\theta$ that maximizes the likelihood $L(\theta; y)$ with $y$, a vector of observations and $L(y; \theta)$ equal to the joint density of the observations. In practice the logarithm of likelihood is used in the estimation. The observed income distribution data are customarily published in grouped form. The random sample of size n consists of individual observations on income which are grouped into m
mutually exclusive and exhaustive income intervals. The joint frequency or likelihood function of the sample is the multinomial probability function:

\begin{equation}
L(\theta; y) = \frac{n!}{n_1! n_2! \ldots n_m!} \prod_{i=1}^{m} \left( p_i(\theta) \right)^{n_i}
\end{equation}

where \( n_i (i = 1, 2, \ldots, m) \) denotes the observed class frequency and \( p_i(\theta) \) the corresponding theoretical probability. The \( k \)-vector \( \theta \) indicates the set of unknown parameters of the theoretical distribution considered, in the present case \( k = 4 \) and \( \theta = (\alpha, \beta, \lambda, \delta) \). The theoretical probability is defined as:

\begin{equation}
p_i(\theta) = \frac{a_{i+1}}{a_i} \int f(y; \theta) \, dy
\end{equation}

where \( y \) is the income variable, \( (a_i, a_{i+1}) \) is the \( i \)-th interval and \( f(y; \theta) \) is the density of the theoretical distribution given in (3.2.14). In general, the integral in (3.2.60) is not easily available in analytical form. However, using a standard numerical integration procedure,\(^{12}\) approximations to a very high accuracy may be obtained. Alternatively, in the particular case of the Dagum Type II model, the analytical form of (3.2.60) for that model given in (3.2.61) may be used, i.e.,

\begin{equation}
p_i(\theta) = \frac{a_{i+1}}{a_i} \int f(y; \theta) \, dy = (1 - \alpha) \left( \left( 1 + \frac{\lambda a_i^{-\delta}}{a_{i+1}} \right)^{-\beta} - \left( 1 + \frac{\lambda a_{i+1}^{-\delta}}{a_i} \right)^{-\beta} \right)
\end{equation}

The maximization of the log likelihood of (3.2.59) with respect to \( \theta \) may be done with a routine that yields a local optimum of a function of several variables.

\(^{12}\) The procedure for numerical integration is an extension of the Gaussian quadrature formulae as proposed by T. N. L. Patterson, "On some Gauss and Lobatto Based Integration Formula", Mathematics of Computation 22 (1968): 877-881.
without use of the derivatives. Ideally, a direct search procedure such as the method of conjugate directions due to Powell,\(^\text{13}\) may be employed for the purpose.

With regard to the properties of the estimators, it can be proved that under certain general conditions, the ML estimators are consistent, and first order efficient in the sense of Rao,\(^\text{14}\) and their asymptotic distribution is \(k\)-variate normal with mean \(\theta.\)

Minimization of Pearson's chi-square distance

An alternative to the ML method is given by the minimization of some measure of discrepancy between the observed and theoretical absolute frequencies. The theoretical absolute frequencies are obtained as \(n_p(\theta).\) In this approach, one seeks the vector \(\theta\) that minimizes Pearson's chi-square distance given by:

\[
(3.2.62) \quad \chi^2(\theta) = \sum_{i=1}^{m} \frac{(n_i - n_p(\theta))^2}{n_p(\theta)}
\]

This can be interpreted as a generalized least squares or minimum distance criterion, compared to the classical least squares criterion which involves the minimization of \(\sum_{i=1}^{m} (n_i - n_p(\theta))^2.\) For large samples, the criterion follows a chi-square distribution\(^\text{15}\) with \(m-k-1\) degrees of freedom. The minimization of (3.2.62)


\(\text{15 Ibid., pp. 361-362.}\)

with respect to $\theta$ may be done using the direct search method of Powell. The $\chi^2$-estimators are consistent and first order efficient in the sense of Rao.

Method of unconstrained least squares

The method involves locating the vector $\theta^* = (\alpha^*, \beta^*, \lambda^*, \delta^*)$, that minimizes the sum of the squares of deviations of the observed from the fitted values of the cumulative distribution function given by:

$$
(3.2.63) \quad \Psi(\theta) = \Psi(\alpha, \beta, \lambda, \delta) = \sum [F(y) - \alpha - (1 - \alpha)(1 + \lambda y^{-\delta}) - \beta] \cdot 2
$$

The minimization of $\Psi(\alpha, \beta, \lambda, \delta)$ belongs to the domain of classical least squares estimation. The specific minimization problem can be stated as follows: Find the argument $\theta^*$ of the scalar function $\Psi(\theta)$ ( $\theta$ is an $k$-vector), which yields the minimum value for $\Psi$. The function $\Psi(\theta)$ is called the criterion or objective function. The problem in question is that of finding the solution of the unconstrained minimization of $\Psi(\theta)$.

The numerical solution of the unconstrained minimization problem has been extensively studied and a variety of algorithms have been developed. It is generally conceded, however, that no single one of these algorithms is the best under all possible circumstances, i.e., "shapes" for the criterion function $\Psi(\theta)$. This suggests that it is not prudent to rely on a single algorithm for locating the minimum of $\Psi(\theta)$. A degree of flexibility for the search procedure is essential, and such flexibility can be achieved through the use of a software package, which places at the user's disposal several different minimization algorithms. The "OPTPAK" iterative package, initially developed for the use on an IBM 360/65

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computer, has these capabilities and can best be adapted to find the solution of the unconstrained minimization of $\Psi(\theta)$.

In its present structure the package can accommodate problems with up to eight parameters. It provides the user with eight multidimensional minimization algorithms which can be divided into two classes, referred to as gradient dependent, and gradient independent. The class of gradient dependent algorithms requires gradient information about the function being minimized. This information can be generated via a finite difference approximation provided by the package. The original software package developed for the unconstrained minimization of $\Psi(\theta)$ was handled in a batch computing environment, in which a user was obliged to accept significant delays between minimization experiments. This difficulty can be overcome through the use of an interactive approach. The software package "EPID" (Econometric Package for Income Distribution) was developed with this problem in mind. EPID is designed to use on an AMDAHL 1700 computer (compatible with an IBM 370). A Datamedia CRT (or equivalent) graphics display unit is utilized as an on-line input-output device. Presently, the package provides on-line interactive editing capabilities and facilities for job submission and retrieval from remote terminals.

Besides Dagum's model (all three types), EPID also estimates the parameters of Singh-Maddala,18 Lognormal,19 and Gamma20 models. The package also computes several goodness of fit statistics; and inter- and intra-income inequality measures. Finally, EPID provides plotting capabilities on a CALCOMP

18 See S. K. Singh and G. S. Maddala, "A Function for Size Distribution".
19 See R. Gibrat, "On Economic Inequalities".
plotter, for the histogram and the density functions of the distributions estimated with the four models.

Test of Goodness of Fit

In order to determine the appropriateness of a given model to be used in the empirical analysis, some indicators that summarize its goodness of fit, as compared with the actual frequency distribution, are needed. A test of goodness of fit is concerned with the agreement between the distribution of a set of sample (observed) values and a theoretic distribution. Interest will be focused on determining whether or not sample data are compatible with the hypothesis that they are drawn from a population that follows some specified functional form. The test statistics, used for the purpose do not depend upon either the explicit form of or the value of certain parameters in the distribution of the population. Such tests are called non-parametric or distribution-free tests.

For a measure of closeness of the fit, some distance measure between the sampled and fitted distribution has to be chosen. The $\chi^2$ test is, perhaps, the most commonly used. The $\chi^2$ measure, however, implies a very specific weighting scheme, the large deviations are weighted more heavily than the small; and the smaller the $p_i(q)$, the more heavily a deviation will be weighted. Furthermore, the $\chi^2$ test assumes that the number of observations is large enough, so that $\chi^2$ distribution provides a good approximation as the distribution of the test statistic.

An alternative distribution-free test of goodness of fit has been suggested by A. Kolmogorov and N. Smirnov.\textsuperscript{21} The relative merits of the $\chi^2$ and the Kolmogorov-Smirnov tests of goodness of fit have been discussed by a number of authors.\textsuperscript{22}


Kolmogorov-Smirnov test may be preferred to the $\chi^2$ test for goodness of fit, because it is exact even for small samples. The test requires only the relatively modest assumptions that sampling is random and that the sampled population is continuous. The Kolmogorov-Smirnov (K-S) test is more powerful than the $\chi^2$ test in most situations. The rationale of the test is as follows.

Let $F(y)$ be the cumulative probability distribution function of a continuously distributed variate $Y$ upon which $n$ random observations have been made, and let $S_n(y)$ be the empirical cumulative probability distribution function of the $n$ observations on $Y$. Finally, let $F(y) - S_n(y)$ be the ordinatewise difference between the two cumulatives at a common abscissa value. Then the K-S test statistic, $T$, is the greatest vertical distance between $F(y)$ and $S_n(y)$, i.e.,

$$T = \sup_{\{y\}} \{|F(y) - S_n(y)|\}$$

The greatest vertical distance is obtained as the supremum over all $y$, of the absolute value of the difference between $F(y)$ and $S_n(y)$. To test the null hypothesis $H_0$ that $F(y) = F_0(y)$, where $F_0(y)$ is a completely specified cumulative distribution hypothesized to be that of the sampled variate, $Y$, $F_0(y)$ is substituted for $F(y)$ in (3.2.64) and the resulting value of the test statistic $T$ is compared with its acceptance limit or the critical value.

Validation Process of the Model

In Chapter II, a set of income distribution models was discussed. Their properties, particularly those of Dagum's model, were discussed elsewhere in the present chapter. According to the properties enumerated there, Dagum's model ranked first, followed by the Singh-Maddala model. The evaluation of property P.4, namely the goodness of fit, was based on prior information. It is, therefore,
<table>
<thead>
<tr>
<th>Country and Year</th>
<th>Mean Square Error (MSE)</th>
<th>Kolmogorov-Smirnov Test Statistic (K-S)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lognormal</td>
<td>Gamma</td>
</tr>
<tr>
<td>GDR 1976</td>
<td>19.652 (6)</td>
<td>9.283 (6)</td>
</tr>
<tr>
<td>UK 1975</td>
<td>31.624 (13)</td>
<td>6.653 (13)</td>
</tr>
<tr>
<td>Canada 1979</td>
<td>30.815 (15)</td>
<td>6.372 (13)</td>
</tr>
<tr>
<td>USA 1978</td>
<td>16.215 (18)</td>
<td>2.248 (18)</td>
</tr>
<tr>
<td>Singapore 1972/73</td>
<td>6.394 (8)</td>
<td>22.259 (8)</td>
</tr>
<tr>
<td>Pakistan 1971/72</td>
<td>4.784 (10)</td>
<td>16.160 (10)</td>
</tr>
<tr>
<td>Sri Lanka 1978/79</td>
<td>1.521 (22)</td>
<td>11.376 (22)</td>
</tr>
</tbody>
</table>

a The figures in parentheses represent degrees of freedom (df) (equal to the number of class intervals minus the number of parameters in the model minus one).

b The acceptance limit (or critical value) for the K-S test statistic at the 5% significance level is $136/\sqrt{n}$, in percentage terms, where $n$ is the sample size.
<table>
<thead>
<tr>
<th>Year</th>
<th>Mean Square Error (MSE)</th>
<th>Kolmogorov-Smirnov Test Statistic (K-S)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Lognormal</td>
<td>Gamma</td>
</tr>
<tr>
<td>1953</td>
<td>1.681 (17)</td>
<td>10.916 (17)</td>
</tr>
<tr>
<td>1963</td>
<td>1.006 (24)</td>
<td>11.070 (24)</td>
</tr>
<tr>
<td>1973</td>
<td>1.324 (24)</td>
<td>6.303 (24)</td>
</tr>
<tr>
<td>1978/79</td>
<td>1.521 (22)</td>
<td>11.376 (22)</td>
</tr>
</tbody>
</table>

a The figures in parentheses represent degrees of freedom (df) (equal to the number of class intervals minus the number of parameters in the model minus one).

b The acceptance limit (or critical value) for the K-S test statistic at the 5% significance level is $136/\sqrt{n}$, in percentage terms, where $n$ is the sample size.
prudent to determine the validity of the models in terms of their goodness of fit with respect to the present data base, before any empirical analysis is attempted using them. With this end in mind, the four models, lognormal, gamma, Singh-Maddala and Dagum, were analysed using the Kolmogorov-Smirnov (K-S) test criterion. In the K-S test, the theoretical distribution is the corresponding fitted model. In order to show the degree of agreement between the observed and theoretic models, the size of mean square error (MSE) is also given. (See Tables 4 and 5). Another check on the fit is to determine how accurately the models predict the summary statistics such as the mean and median. (See Tables 6 and 7). Finally, with a view to checking how accurately the fitted model tracks the observed

<table>
<thead>
<tr>
<th>Country and Year</th>
<th>Mean Income</th>
<th>Median Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Estimated</td>
</tr>
<tr>
<td></td>
<td>Percentage</td>
<td>Difference</td>
</tr>
<tr>
<td>GDR 1976</td>
<td>-</td>
<td>1332</td>
</tr>
<tr>
<td>UK 1975</td>
<td>73</td>
<td>72</td>
</tr>
<tr>
<td>Canada 1979</td>
<td>24245</td>
<td>23779</td>
</tr>
<tr>
<td>USA 1978</td>
<td>20090</td>
<td>20110</td>
</tr>
<tr>
<td>Singapore 1972/73</td>
<td>591</td>
<td>636</td>
</tr>
<tr>
<td>Pakistan 1971/72</td>
<td>-</td>
<td>309</td>
</tr>
</tbody>
</table>

a A dash (-) represents data not available.
frequency distribution, observed and fitted or predicted cumulative frequency
distributions are compared in Table 8.

The results show that Dagum's model performs better than the other
models, in terms of overall performance. It can be seen from Table 4 (inter-
country analysis) and Table 5 (inter-temporal analysis) that, in terms of the
Kolmogorov-Smirnov (K-S) test statistic, Dagum's model is accepted in all cases
except GDR and the U.K. The fit as judged by the mean square error (MSE) is also
very good. The mean square errors (MSE) reported in Tables 4 and 5 show an
exceptional goodness of fit for Dagum's model, in all cases studied. Clearly, the
model fits the data better than the competing models, except in the case of Singh-
Maddala which performs marginally better in some cases. It is in fact observed
that the mean square error is several times greater for the lognormal and gamma
models than for Dagum's model.

The model also predicts the mean and median of the distributions within
less than five percent departure from the observed values. Two exceptions are,
however, Singapore 1972/73 (see Table 6) and Sri Lanka 1953. (See Table 7). In
Table 8, the model displays its ability to track the observed data to which it was

TABLE 7.-Mean and Median - Observed and Estimated Values
Using Dagum's Model, Sri Lanka, By Selected Years

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean Income</th>
<th>Median Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Observed</td>
<td>Estimated</td>
</tr>
<tr>
<td>1953</td>
<td>169</td>
<td>151</td>
</tr>
<tr>
<td>1963</td>
<td>385</td>
<td>384</td>
</tr>
<tr>
<td>1973</td>
<td>622</td>
<td>613</td>
</tr>
<tr>
<td>1978/79</td>
<td>920</td>
<td>876</td>
</tr>
</tbody>
</table>
fitted, fairly accurately compared to the competing models. The ability of the
model to reproduce the sample data is demonstrated clearly by Figure 5, in which
the observed (histogram) and predicted frequencies of spending unit income
distribution of Sri Lanka for 1978/79 are illustrated. Table 9 presents the
estimated models. The Kolmogorov-Smirnov (K-S) test statistic and mean square
error (MSE) are also given for easy comparison. The asymptotic critical value of
the K-S statistic is $136/\sqrt{n} = 136/\sqrt{8384} = 1.485$, in percentage terms, at the five
percent significance level.

<table>
<thead>
<tr>
<th>Income (in Rs.1000)</th>
<th>Observed CDF</th>
<th>Fitted Cumulative Distribution Function (CDF)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Lognormal</td>
</tr>
<tr>
<td>&lt;0.05</td>
<td>0.26</td>
<td>0.04</td>
</tr>
<tr>
<td>&lt;0.10</td>
<td>0.83</td>
<td>0.19</td>
</tr>
<tr>
<td>&lt;0.15</td>
<td>2.42</td>
<td>1.16</td>
</tr>
<tr>
<td>&lt;0.20</td>
<td>4.97</td>
<td>3.39</td>
</tr>
<tr>
<td>&lt;0.25</td>
<td>8.19</td>
<td>6.94</td>
</tr>
<tr>
<td>&lt;0.30</td>
<td>12.19</td>
<td>11.55</td>
</tr>
<tr>
<td>&lt;0.35</td>
<td>16.61</td>
<td>16.88</td>
</tr>
<tr>
<td>&lt;0.40</td>
<td>21.64</td>
<td>22.60</td>
</tr>
<tr>
<td>&lt;0.45</td>
<td>27.81</td>
<td>28.44</td>
</tr>
<tr>
<td>&lt;0.50</td>
<td>33.45</td>
<td>34.22</td>
</tr>
<tr>
<td>&lt;0.60</td>
<td>44.23</td>
<td>45.06</td>
</tr>
<tr>
<td>&lt;0.70</td>
<td>54.09</td>
<td>54.56</td>
</tr>
<tr>
<td>&lt;0.80</td>
<td>62.34</td>
<td>62.60</td>
</tr>
<tr>
<td>&lt;0.90</td>
<td>69.02</td>
<td>69.28</td>
</tr>
<tr>
<td>&lt;1.00</td>
<td>74.41</td>
<td>74.76</td>
</tr>
<tr>
<td>&lt;1.25</td>
<td>83.34</td>
<td>84.44</td>
</tr>
<tr>
<td>&lt;1.50</td>
<td>88.65</td>
<td>90.23</td>
</tr>
<tr>
<td>&lt;1.75</td>
<td>91.89</td>
<td>93.75</td>
</tr>
<tr>
<td>&lt;2.00</td>
<td>93.86</td>
<td>95.91</td>
</tr>
<tr>
<td>&lt;2.25</td>
<td>95.85</td>
<td>98.15</td>
</tr>
<tr>
<td>&lt;3.00</td>
<td>97.15</td>
<td>99.11</td>
</tr>
<tr>
<td>&lt;4.00</td>
<td>98.29</td>
<td>99.76</td>
</tr>
<tr>
<td>&lt;5.00</td>
<td>98.89</td>
<td>99.92</td>
</tr>
<tr>
<td>&lt;10.00</td>
<td>99.67</td>
<td>99.99</td>
</tr>
<tr>
<td>≥</td>
<td>100.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>
Figure 5.- Observed (Histogram) and Predicted Frequencies of Spending Unit Income Distribution in Sri Lanka for 1978/79
Table 9.—Spending Unit Income Distribution — Sri Lanka, 1978/79
Estimated Models (Income in Rs 1000)

<table>
<thead>
<tr>
<th>Model</th>
<th>Density Function</th>
<th>Parameters</th>
<th>MSE</th>
<th>K-S</th>
</tr>
</thead>
<tbody>
<tr>
<td>Lognormal</td>
<td>$f(y) = \exp\left(-\left(\log y - \mu\right)^2 / 2\sigma^2\right) / (\sqrt{2\pi} \sigma y),$</td>
<td>$\mu = -0.4306$, $\sigma^2 = 0.4169$</td>
<td>1.521</td>
<td>2.301</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Gamma</td>
<td>$f(y) = \lambda^n y^{n-1} \exp(-\lambda y) / \Gamma(n)$</td>
<td>$\alpha = 3.8108$, $\lambda = 2.7242$</td>
<td>11.376</td>
<td>5.698</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Singh-Maddala</td>
<td>$F(y) = 1 - (1 + ay^b)^{-c}$</td>
<td>$a = 3.4769$, $b = 2.6042$, $c = 0.9023$</td>
<td>0.086</td>
<td>0.508</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dagum Type I</td>
<td>$F(y) = (1 + \lambda y^{-\delta})^{-\beta}$</td>
<td>$\delta = 2.4512$, $\beta = 1.0799$, $\lambda = 0.3202$</td>
<td>0.114</td>
<td>0.615</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Dagum Type II</td>
<td>$F(y) = \alpha + (1 - \alpha)(1 + \lambda y^{-\delta})^{-\beta}$</td>
<td>$\alpha = 0.0039$, $\delta = 2.4298$, $\beta = 1.1380$, $\lambda = 0.3029$</td>
<td>0.105</td>
<td>0.570</td>
</tr>
</tbody>
</table>

MSE = mean squared error, K-S = Kolmogorov-Smirnov test statistic.
CHAPTER IV

EMPIRICAL ANALYSIS OF PERSONAL INCOME DISTRIBUTION

This chapter covers two main aspects. Firstly it aims at measuring the extent of income inequalities in Sri Lanka vis-à-vis some selected countries. With a view to this comparison, it brings together data on the size distribution of income for some selected countries, within a common framework. An attempt is also made to determine the inter-temporal changes in income distribution in Sri Lanka. Secondly, a spatial comparison\(^1\) of income inequalities in Sri Lanka is undertaken, with a view to identifying the sources of inequalities in the distribution of income.

1. Extent of Income Inequalities in Sri Lanka

In Chapter III, Dagum's model was proposed as a mechanism to describe the personal distribution of income. The model is applied to the household income distributions of some selected countries, and to the spending unit\(^2\) income distribution in Sri Lanka, for some selected years. This section examines the extent of income inequalities in Sri Lanka vis-à-vis the selected countries, and the

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1 The term "spatial" is used in a broad context to include regional, sectoral, sex-wise, age-wise and education-wise comparisons. See also footnote 1 in Chapter III.

2 Spending units are smaller groups (or subsets) within a household that act as more or less independent units for spending purposes. For instance, an employed son living with his parents can be part of the household to which his parents belong, but his spending can be quite independent of the others'. In this example, the son will form a different spending unit. A spending unit is either the whole household or part of it.
inter-temporal changes in income inequality as reflected in quantile shares, and
other inequality measures computed from the parametric estimates of Dagum's
model. All inequality measures, summary measures, such as mean, median, quintile
and decile shares, are, therefore, calculated using the parametric approach
discussed in the preceding chapter. The parameters of the model were estimated
using the method of unconstrained least squares. The estimates are obtained by an
iterative algorithm\textsuperscript{3} that locates the solution to the unconstrained minimization of
the sum of squared deviations of the actual from the fitted values, using a
multidimensional search procedure.

**Inter-Country Comparison of Income Inequality**

In the cross-country comparison of income inequality, the structure of
income distribution in seven countries is examined. The choice of the countries
drawn into the study was based on their politico-social structure, and the stage of
development. There are several difficulties associated with a study of this nature,
the most notable being the problem of comparability of data. The data available
for these countries are judged to be reasonably comparable, subject to the
variability of the definitions of income-recipient units, which are spelt out in the
footnote reference to Table 10. The conclusions from the inter-country
comparison are, therefore, subject to this limitation.

The data on the size distribution of income are available in the form of a
frequency distribution, giving (1) the total number of units with income in different
ranges, and (2) the total income in each range. The distributions were not directly
comparable in their original form, because the income ranges varied from one
source to another. With a view to ensuring comparability, data were transformed

\[\text{3 See L. G. Birta, "OPTPAK; A Program for Minimization".}\]
into estimates of income shares accruing to different quintiles of the population. This was done using the parametric approach reported in the preceding chapter. The estimated nonlinear parameter values are reported in Table 10, together with

Table 10.-Estimated Parameters and Gini Ratios for Dagum's Model, By Selected Countries

<table>
<thead>
<tr>
<th>Country and Year</th>
<th>Estimated Parameters</th>
<th>Gini Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>α</td>
<td>β</td>
</tr>
<tr>
<td>GDR 1976</td>
<td>0.0007</td>
<td>0.3629</td>
</tr>
<tr>
<td>UK 1975</td>
<td>-0.0459</td>
<td>0.2253</td>
</tr>
<tr>
<td>Canada 1979</td>
<td>-0.0136</td>
<td>0.2814</td>
</tr>
<tr>
<td>USA 1978</td>
<td>-0.0184</td>
<td>0.3346</td>
</tr>
<tr>
<td>Singapore 1972/73</td>
<td>-0.4081</td>
<td>0.3585</td>
</tr>
<tr>
<td>Pakistan 1971/72</td>
<td>0.0024</td>
<td>2.1312</td>
</tr>
<tr>
<td>Sri Lanka 1978/79</td>
<td>0.0039</td>
<td>1.1380</td>
</tr>
</tbody>
</table>

Definition of income-recipient unit and income:

GDR Households of employees, co-operative farmers and pensioners. Household net income per month. (Income in Marks 103)

UK Households—all types. Household cash income per week. (Income in £10)

Canada Families—all types. Family income per year. (Income in Can $ 104)

USA Families—all types. Family income per year. (Income in US $ 104)

Singapore Households—all types of households of 2 persons or more. Household cash income per month. (Income in Singapore $ 102)

Pakistan Households—all types. Household income per year. (Income in Pakistan Rs 102)

Sri Lanka Spending-units (defined elsewhere)—all types. Spending unit income per month. (Income in Sri Lanka Rs 103)
the values of the Gini ratio which were also estimated by resorting to the parametric approach. Table 11 presents the quintile shares of income in the seven countries, together with the shares of the lowest and top five percent.

The results show significant differences among countries with respect to the degree of income inequality, which is higher in the three developing countries than in the developed countries reported in the study. Among them, the Gini ratio is highest in Singapore followed by Sri Lanka. The results, however, should be interpreted with caution, as the country data refer to different time periods. For instance, Sri Lanka reports a Gini ratio of 0.404 for 1978/79, compared with Pakistan, whose Gini ratio is 0.365 for 1971/72. It can be seen from Table 12, that the Gini ratio for Sri Lanka in 1973 was only 0.348. In contrast to the developing countries, the three developed countries report lower Gini ratios in the 0.3 range. Among the developed countries, Canada has the lowest Gini ratio (0.307) compared to 0.357 for the USA. GDR, the only socialist country reported in the study, has the lowest degree of overall inequality in the distribution of income. This is clearly seen, both in terms of the Gini ratio (Table 10) and quintile shares (Table 11). Although these results lend support to the Kuznets’ hypothesis of greater inequality in developing countries, the differences in inequality between the Asian countries included in the study and the developed countries are too small to warrant any strong conclusions. It is indeed pertinent, in this respect, to point out that income inequalities are significantly lower in Asian developing countries than in other developing countries.4

The income shares presented in Table 11 further highlight the structure of income distribution of the seven countries reported in the study. The choice of the

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4 See Nanak C. Kakwani, Income Inequality and Poverty: Methods of Estimation and Policy Applications (Oxford: Oxford University Press, 1980), pp. 388-389, wherein Kakwani reports the following average Gini ratios: Developing countries -- Africa 0.487, Asia 0.407, Latin America 0.515. Developed countries -- 0.386.
## Income Shares of the Quintiles and the Lowest and Top Five Percent, By Selected Countries

<table>
<thead>
<tr>
<th>Country and Year</th>
<th>Lowest 5%</th>
<th>Income Shares of Quintiles</th>
<th>Top 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>GDR 1976</td>
<td>1.50</td>
<td>10.06 16.09 19.83 23.46 30.56</td>
<td>9.38</td>
</tr>
<tr>
<td>UK 1975</td>
<td>0.72</td>
<td>5.42 11.90 18.23 25.20 39.25</td>
<td>13.36</td>
</tr>
<tr>
<td>Canada 1979</td>
<td>0.81</td>
<td>6.42 13.14 18.81 24.79 36.84</td>
<td>12.26</td>
</tr>
<tr>
<td>USA 1978</td>
<td>0.66</td>
<td>5.41 11.72 17.55 24.34 40.98</td>
<td>14.98</td>
</tr>
<tr>
<td>Pakistan 1971/72</td>
<td>1.26</td>
<td>7.52 11.59 15.35 20.96 44.58</td>
<td>19.92</td>
</tr>
<tr>
<td>Sri Lanka 1978/79</td>
<td>0.79</td>
<td>5.97 10.71 15.03 21.29 47.00</td>
<td>21.27</td>
</tr>
</tbody>
</table>

Income shares approach calls for some explanation. The Gini ratio provides a summary measure of inequality over the entire range of the population and as such may be insensitive to the degree of inequality in particular ranges. The income shares approach makes it possible to bring out these specific differences in the structure of income distribution. It is seen from Table 11 that major differences between countries emerge in terms of the share of the top quintile, the top five percent in particular. The share of the upper income groups (top quintile and five percent) is distinctly larger in the developing countries than in the developed countries. The top quintile has a share ranging from 30.56 percent in GDR to 50.26 percent in Singapore, whilst the share of the top five percent ranges from 9.38 percent in GDR to 24.37 percent in Singapore.
With respect to the share of the bottom income group (bottom forty percent), the differences between the developed countries and the Asian developing countries do not appear significant. The average income share of the bottom forty percent in the developed countries, amounts to about 18 percent, which is considerably lower than the average of 26 percent for GDR (socialist country), but marginally better than the average of 17 percent for the developing countries. The income share of the intermediate group (third and fourth quintiles combined) is appreciably larger in the developed than in the developing countries, and is relatively stable. In GDR, UK and Canada, this group receives slightly over 43 percent of the total income, compared with 42 percent for USA. The average income share of the intermediate group in developing countries reported in the study is 36 percent.

Although it does not stand out from Table 1, a closer look would reveal that for a number of country-by-country comparisons of the distributions, the corresponding Lorenz curves would intersect making judgement of inter-country differences in inequality difficult. It can be seen, for example, that a low share in the lower quintiles is not always associated with a high share in the higher quintiles. This is clearly demonstrated by the Lorenz curves for Singapore and Sri Lanka. One possible explanation of this phenomenon is that the definition of households in Singapore precludes single person households which can seriously overestimate the share of the lower tail, whilst in Sri Lanka the distribution of spending unit income can seriously underestimate the share of this group due to the existence of spending units (within households) receiving very low income and the differences in household size, particularly in the lower percentiles. A negative correlation between household size and household income can overestimate inequality in the lower tail of spending unit income distribution in Sri Lanka vis-à-vis the household income distribution in Singapore. In terms of the Gini ratio,
income inequalities are greater in Singapore than in Sri Lanka, more so in relation to the Gini ratio reported for the comparable year in Table 12.

It is worth noting that overall income inequality in the developing countries is associated with markedly larger income shares for upper income groups (top quintile and top five percent). These countries show high inequality in terms of low income shares for the middle income groups, lending support to Kuznets' assertion that higher income inequality in developing countries is due to greater inequality between the top and middle income groups.

In making these comparisons, it should be emphasized that disparities reported in the study might be accentuated or weakened, if strictly comparable data were available. As pointed out earlier, the periods for which the data are given, particularly those of Pakistan and Singapore, are different from the period for Sri Lanka. Furthermore, in the case of Singapore only households of two persons or more are included in the sample. This can introduce a significant downward bias into the inequality estimates for that country, particularly in the low income range. In Sri Lanka, the spending unit is the recipient-unit, not the household. As a spending unit is a subset of a household, and, presumably there are variations of spending unit income within households, it can be conjectured that inequality of household income is even less than that of spending unit income reported in the study.

Inter-Temporal Comparison of Income Inequality in Sri Lanka

Data on the distribution of spending unit income by size are available for survey years from 1953 on. As the data are drawn from the same series of surveys,

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the statistical series are directly comparable. An examination of the trends in inequality over the last twenty-five years shows marked changes in income inequality between survey years. (See Table 12 and 13). The Gini ratio for the successive survey years presented in Table 12 discloses that it increased by 14 percent during the period 1953-1963, followed by a dramatic decline of 27 percent during the subsequent intersurvey period. Income distribution during the ten year period beginning in 1963, moved markedly towards greater equality both by a significant reduction of the Gini ratio and the share received by the top quintile. The data for 1978/79, however, disclose a reversal of this egalitarian trend, indicative of a shift towards greater income inequality during the five years preceding that year.

Table 12.-Estimated Parameters and Gini Ratios for Dagum's Model, Sri Lanka, By Selected Years

<table>
<thead>
<tr>
<th>Year</th>
<th>α</th>
<th>γ</th>
<th>β</th>
<th>λ</th>
<th>δ</th>
<th>Gini Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953</td>
<td>0.0071</td>
<td>1.2065</td>
<td>0.0044</td>
<td>2.3581</td>
<td>0.415</td>
<td></td>
</tr>
<tr>
<td>1963</td>
<td>0.0041</td>
<td>0.9931</td>
<td>0.0574</td>
<td>2.1250</td>
<td>0.473</td>
<td></td>
</tr>
<tr>
<td>1973</td>
<td>0.0013</td>
<td>1.0160</td>
<td>0.1325</td>
<td>2.8808</td>
<td>0.348</td>
<td></td>
</tr>
<tr>
<td>1978/79</td>
<td>0.0039</td>
<td>1.1380</td>
<td>0.3029</td>
<td>2.4298</td>
<td>0.404</td>
<td></td>
</tr>
</tbody>
</table>

The data on income shares by quintiles, presented in Table 13, reveal that during 1963-1973 there was a significant redistribution of income in favour of the lower and middle income groups at the expense of the highest quintile. A reduction in inequalities was thus achieved by an across-the-board redistribution. A sharp reversal of this trend has occurred during the five year period following
Table 13.-Income Shares of the Quintiles and the Lowest and Top Five Percent, Sri Lanka, By Selected Years

<table>
<thead>
<tr>
<th>Year</th>
<th>Lowest 5%</th>
<th>Income Shares of Quintiles</th>
<th>Top 5%</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>1953</td>
<td>0.71</td>
<td>5.75</td>
<td>10.49</td>
</tr>
<tr>
<td>1963</td>
<td>0.50</td>
<td>4.43</td>
<td>9.02</td>
</tr>
<tr>
<td>1973</td>
<td>1.06</td>
<td>7.21</td>
<td>12.13</td>
</tr>
<tr>
<td>1978/79</td>
<td>0.79</td>
<td>5.97</td>
<td>10.71</td>
</tr>
</tbody>
</table>

1973. All lower quintiles have lost in favour of the top quintile, which saw its income share increase by five percentage points to 47 percent. The share of the poorest forty percent declined from 19.3 percent to 16.7 percent, whilst that of the intermediate group (fortieth to eightieth percentile) declined from 38.4 percent to 36.3 percent during the same period. An examination of income shares received by the top five percent reveals that its share increased by as much as 21.7 percent compared to the 32.5 percent it lost in favour of the lower percentiles during the preceding inter-survey period, 1963-1973. The poorest five percent, which saw its income share more than double during the ten years preceding 1973, experienced a 25.5 percent decline in its share during the five year period following 1973.

Income Inequalities amongst Spending Units and Income Receivers

There are numerous factors influencing the distribution of income amongst spending units. In making inter-temporal comparisons in the foregoing

6 A person who has received an income during the six months immediately prior to the survey period is referred to as an income receiver.
subsection, particularly for the lower quintiles, it should be emphasized that the
data refer to spending unit income, and that at least some of the differences shown
may be a result of variations in spending unit size. It is often conjectured that
there is a negative correlation between spending unit income and the size of a
spending unit, which can have an equalizing effect on spending unit income. This is
possible with a varying ratio of income receivers to dependents within spending
units, the lower income groups having a larger ratio. Furthermore, there can be
considerable differences in income among income receivers within spending units,
which can have a strong dis-equalizing effect on income receivers. A priori, one
would expect greater inequalities amongst income receivers than amongst spending
units. The pattern is discernible in Table 14, in which spending unit and income
receiver income distributions are compared. Notwithstanding, both spending unit
and income receiver income distributions show the same trend in inequalities, over
time.

The Gini ratio for income receivers has declined by 23.2 percent during
the period 1963-1973, as compared to a 26.4 percent reduction of the Gini ratio for
spending units. The Gini ratio for 1978/79 compared to that in 1973 shows a 17.8
percent increase for income receivers. The corresponding increase for spending
units was 16.1 percent. A closer look at the income shares of the different
quintiles reveals a strong equalizing effect for spending unit income, particularly in
the lower quintiles, at the expense of the top quintile. For instance, in 1978/79,
the income share of the lowest quintile of income receivers was 3.84 percent
compared to the top quintile whose share was 52.2 percent. The corresponding
shares for the spending units were 6.0 percent and 47.0 percent respectively. Thus,
the data support the hypothesis that income inequalities are even greater when
distributions are based on individual income receivers rather than spending units (or
households).
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowest 5%</td>
<td>0.71</td>
<td>0.89</td>
<td>0.50</td>
<td>0.33</td>
<td>1.00</td>
<td>0.71</td>
<td>0.79</td>
<td>0.40</td>
</tr>
<tr>
<td>First</td>
<td>5.75</td>
<td>4.96</td>
<td>4.43</td>
<td>3.54</td>
<td>7.15</td>
<td>4.90</td>
<td>5.97</td>
<td>3.84</td>
</tr>
<tr>
<td>Second</td>
<td>10.49</td>
<td>8.72</td>
<td>9.02</td>
<td>7.78</td>
<td>12.14</td>
<td>10.29</td>
<td>10.71</td>
<td>8.77</td>
</tr>
<tr>
<td>Third</td>
<td>14.79</td>
<td>13.03</td>
<td>13.54</td>
<td>12.18</td>
<td>16.31</td>
<td>16.00</td>
<td>15.03</td>
<td>13.88</td>
</tr>
<tr>
<td>Fourth</td>
<td>21.11</td>
<td>19.78</td>
<td>20.43</td>
<td>19.20</td>
<td>22.02</td>
<td>23.41</td>
<td>21.29</td>
<td>21.31</td>
</tr>
<tr>
<td>Fifth</td>
<td>47.86</td>
<td>53.51</td>
<td>52.58</td>
<td>57.30</td>
<td>42.38</td>
<td>45.40</td>
<td>47.00</td>
<td>52.20</td>
</tr>
<tr>
<td>Top 5%</td>
<td>22.04</td>
<td>27.30</td>
<td>25.91</td>
<td>30.56</td>
<td>17.58</td>
<td>18.35</td>
<td>21.27</td>
<td>24.70</td>
</tr>
<tr>
<td>Gini Ratio</td>
<td>0.415</td>
<td>0.477</td>
<td>0.473</td>
<td>0.527</td>
<td>0.348</td>
<td>0.405</td>
<td>0.404</td>
<td>0.477</td>
</tr>
</tbody>
</table>
As can be seen in Table 14, income distribution varies according to the definition of income-recipient unit. Furthermore, in conformity with the accepted hypothesis, the income distribution of spending units was found to be less unequal than that of individual income receivers. If a study is discerning purchasing power differentials (or for that matter welfare differentials), the logical unit to choose would be the spending unit since this unit appropriately measures income requirements and since consumption is carried out by and for the spending unit. On the contrary, if the purpose is to measure the capacity for income generation of persons with different levels of education, age or sex, the logical unit is the income receiver. As shown earlier, the distribution of individual incomes is usually less equitable than that of spending units, mainly because, when the spending unit is used, a number of income receivers are included in each unit, thus increasing the income of the average unit and reducing the weight of low-income earners, many of whom are secondary earners within the spending unit. In the ensuing analysis of sources of income inequalities, both individual and spending unit distributions are presented for economic sectors. The analysis of income distribution by other socio-economic characteristics is confined to the distribution of income receivers.

2. Sources of Income Inequalities in Sri Lanka

In this section, an attempt is made to identify the sources of income inequalities in Sri Lanka. With this purpose in mind, the pattern of total income \( Y \) is segmented into homogenous subgroups. Each subpopulation represents a particular class of income-recipient units. The classification is made according to given socio-economic characteristics. In the present study, the population is partitioned into economic sectors, regions, sex, age and educational level. In order to trace the factors contributing to the observed overall inequalities, both inter- and intra-income distribution inequality ratios for various subpopulations are
computed. A comparison is made between the years 1973 and 1978/79, with a view to identifying the factors that contributed to increased overall inequalities during the period 1973-1978/79, marking a reversal of the egalitarian trend observed in the decade preceding 1973. The purpose of the inter-spatial income inequality comparison (the term inter-spatial is used here in its broad sense to encompass all cross-section comparisons) that follows is to trace the disequalizing influence, be it the result of increased differentials (distance) among subpopulations, or increased differentials within subpopulations (intra-group effect), or both.

Inter- and Intra-Sectoral Income Inequalities

Tables 15 and 16 summarize the intra- and inter-sectoral spending unit income distribution in Sri Lanka. As will be readily noticed, the degree of income concentration in the urban sector substantially exceeds that in the rural and plantation sectors. Table 15 shows higher Gini ratios in urban than in rural or plantation sectors. The data for 1978/79 reveal that the top five percent of the spending units in the urban sector controls about a fourth of the total income in that sector, whilst the top quintile controls slightly over one half of the total income. In the same year, the poorest forty percent received only about 15 percent of the total income. The share of the intermediate group (fortieth to eightieth percentile) was 35 percent. The scenario emerging from the rural and plantation sectors is quite different. The Gini ratio is lowest in the plantation sector. A closer look at income shares reveals that income is more equitably distributed in that sector than in the urban sector and rural sector. The Gini ratio for the rural sector is 0.396 compared to the global estimate of 0.404. The income share of the highest income group in the rural sector is significantly larger than the share of the same group in the plantation sector, but is smaller than that of the urban sector.
Table 15.-Gini Ratio and Income Shares of Spending Units, 1973 and 1978/79, By Sectors

<table>
<thead>
<tr>
<th>Quintile/Percentile</th>
<th>1973</th>
<th></th>
<th>1978/79</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Urban</td>
<td>Rural</td>
<td>Plantation</td>
<td>Urban</td>
<td>Rural</td>
<td>Plantation</td>
</tr>
<tr>
<td>Lowest 5%</td>
<td>0.83</td>
<td>1.04</td>
<td>1.14</td>
<td>0.67</td>
<td>0.80</td>
<td>1.24</td>
</tr>
<tr>
<td>First</td>
<td>6.13</td>
<td>7.40</td>
<td>8.54</td>
<td>5.07</td>
<td>6.06</td>
<td>8.32</td>
</tr>
<tr>
<td>Third</td>
<td>15.62</td>
<td>16.72</td>
<td>17.18</td>
<td>13.94</td>
<td>15.28</td>
<td>17.31</td>
</tr>
<tr>
<td>Fourth</td>
<td>21.86</td>
<td>22.29</td>
<td>22.13</td>
<td>20.56</td>
<td>21.53</td>
<td>22.43</td>
</tr>
<tr>
<td>Fifth</td>
<td>45.27</td>
<td>41.07</td>
<td>38.67</td>
<td>50.87</td>
<td>46.21</td>
<td>38.57</td>
</tr>
<tr>
<td>Top 5%</td>
<td>19.60</td>
<td>16.52</td>
<td>15.10</td>
<td>24.61</td>
<td>20.53</td>
<td>14.81</td>
</tr>
<tr>
<td>Gini Ratio</td>
<td>0.387</td>
<td>0.333</td>
<td>0.298</td>
<td>0.450</td>
<td>0.396</td>
<td>0.300</td>
</tr>
</tbody>
</table>
As regards the inter-income distribution inequalities, (see Table 16), mean income in the urban sector substantially exceeds that in the rural and plantation sectors. In terms of the mean income differentials, the urban sector is, again, the most affluent sector, followed by the rural sector. The economic distance ratios corresponding to the binary combinations of sectors reported in Table 16 give a measure of degree of affluence of each sector vis-à-vis the other sectors. It is evident from the results that the inter-income distribution inequality between urban and plantation is significantly greater than that between urban and rural or rural and plantation.

Table 16.- Economic Distance Ratio (D1) and Mean Monthly Income of Spending Units, 1973 and 1978/79, By Sectors

<table>
<thead>
<tr>
<th></th>
<th>1973</th>
<th></th>
<th>1978/79</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rural</td>
<td>Plantation</td>
<td>Mean</td>
<td>Rural</td>
</tr>
<tr>
<td>Urban</td>
<td>0.414</td>
<td>0.489</td>
<td>394</td>
<td>0.298</td>
</tr>
<tr>
<td>Rural</td>
<td>0.073</td>
<td>288</td>
<td></td>
<td>0.227</td>
</tr>
<tr>
<td>Plantation</td>
<td></td>
<td>275</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Turning to between year comparisons of income inequalities, all the summary measures indicate that by 1978/79 the distributions in both urban and rural sectors had become more unequal, although the rural sector was still relatively less unequal than the urban sector. The Gini ratio in the rural sector rose by 18.3 percent compared to a 16.3 percent increase in the urban sector. The Gini ratio for the plantation sector was stable over the 1973-1978/79 period. The most striking features of the detailed income shares in Table 15 (see also Table 17 for comparative changes) are, first, the magnitude of the changes in both the
urban and rural sectors, and second the stability of the income shares in the plantation sector. The inequalities in the intermediate group (third and fourth quintiles combined) appear to have increased the most. The income share of this group in 1973 ranged between 37.5 percent for the urban sector to 39.3 percent in the plantation sector. The share of the rural sector was comparable to that in the plantation sector. By 1978/79, the share in the urban and rural sectors had fallen to 34.5 percent and 36.8 percent respectively, while that in the plantation sector was relatively stable at 39.7 percent. In percentage terms, however, the lowest income groups in the urban and rural sectors appear to have lost the most ground as revealed by Table 17. So, to summarize, a sharp reversal of the egalitarian trend achieved by an across-the-board redistribution in the 1960's, had occurred over the period 1973-1978/79.

Table 17.- Improvement in Income Shares, 1978/79 over 1973, for Spending Units, By Sectors

<table>
<thead>
<tr>
<th>Quintile/Percentile</th>
<th>1978/79 Income Share as a Percentage of 1973 Share</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Urban</td>
</tr>
<tr>
<td>Lowest 5%</td>
<td>80.72</td>
</tr>
<tr>
<td>First</td>
<td>82.71</td>
</tr>
<tr>
<td>Second</td>
<td>85.97</td>
</tr>
<tr>
<td>Third</td>
<td>89.24</td>
</tr>
<tr>
<td>Fourth</td>
<td>94.05</td>
</tr>
<tr>
<td>Fifth</td>
<td>112.37</td>
</tr>
<tr>
<td>Top 5%</td>
<td>125.56</td>
</tr>
</tbody>
</table>

As regards the between year changes in inter-income distribution inequalities, by 1978/79 the economic distance ratio between the urban and rural
sectors had diminished substantially, while that between the urban and plantation sectors had increased. The distance between the rural and plantation sectors had experienced a more than three-fold increase (from 0.07 to 0.23). The rural sector recorded almost a three-fold increase in its mean income, compared to the urban sector whose mean income rose by 174 percent. The corresponding increase in the plantation sector was only 160 percent.

As discussed earlier, the shape of income distribution is influenced by the definition of the recipient-unit. This is clearly seen in the comparison of the distribution of income receivers (Tables 18 and 19) with that of the spending units presented in Tables 15 and 16. Although the pattern of income distribution among income receivers is the same as with spending units, income inequalities among the former are accentuated. An exception to the rule is the urban sector which records a lower Gini ratio for income receivers than for spending units (in 1973). As regards between-year comparisons, intra-income inequalities have increased both in the urban and rural sectors in conformity with the trends observed in the spending unit income distribution. The plantation sector, however, shows a reduction in inequality in the distribution of income among income receivers as compared to the disequalizing trend observed for spending unit income distribution in that sector. The economic distance ratios presented in Table 19 are greater than those for spending units, with the exception of the distance between urban and rural sectors in 1973. The economic distance ratios reported for 1978/79 are more or less of the same order of magnitude as those reported for 1973. Hence, the between-year changes are not as significant as they were for the spending units. The magnitude of the economic distance ratios for spending units and income receivers points to the fact that inter-income distribution inequalities are greater for income receivers than for spending units.

The fitted sectoral spending unit income distributions for 1978/79 are shown in Figure 6.

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Lowest 5%</td>
<td>0.54</td>
<td>0.66</td>
<td>0.79</td>
<td>0.33</td>
<td>0.32</td>
<td>0.98</td>
</tr>
<tr>
<td>First</td>
<td>5.19</td>
<td>5.45</td>
<td>7.61</td>
<td>3.93</td>
<td>3.66</td>
<td>7.48</td>
</tr>
<tr>
<td>Second</td>
<td>11.20</td>
<td>11.62</td>
<td>12.45</td>
<td>8.86</td>
<td>9.23</td>
<td>13.31</td>
</tr>
<tr>
<td>Third</td>
<td>16.53</td>
<td>17.26</td>
<td>16.11</td>
<td>13.69</td>
<td>14.92</td>
<td>17.37</td>
</tr>
<tr>
<td>Fourth</td>
<td>23.27</td>
<td>24.00</td>
<td>21.43</td>
<td>20.86</td>
<td>22.65</td>
<td>22.61</td>
</tr>
<tr>
<td>Fifth</td>
<td>43.81</td>
<td>41.67</td>
<td>42.40</td>
<td>52.66</td>
<td>49.54</td>
<td>39.23</td>
</tr>
<tr>
<td>Top 5%</td>
<td>17.53</td>
<td>15.65</td>
<td>18.10</td>
<td>25.53</td>
<td>21.67</td>
<td>15.15</td>
</tr>
<tr>
<td>Gini Ratio</td>
<td>0.384</td>
<td>0.363</td>
<td>0.343</td>
<td>0.480</td>
<td>0.455</td>
<td>0.314</td>
</tr>
</tbody>
</table>
Table 19.- Economic Distance Ratio (D₁) and Mean Monthly Income of Income Receivers, 1973 and 1978/79, By Sectors

<table>
<thead>
<tr>
<th></th>
<th>1973 Economic Distance</th>
<th>Mean Income</th>
<th>1978/79 Economic Distance</th>
<th>Mean Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Rural</td>
<td>Plantation</td>
<td></td>
<td>Rural</td>
</tr>
<tr>
<td>Urban</td>
<td>0.375</td>
<td>0.908</td>
<td>307</td>
<td>0.317</td>
</tr>
<tr>
<td>Rural</td>
<td>0.795</td>
<td>228</td>
<td></td>
<td>0.706</td>
</tr>
<tr>
<td>Plantation</td>
<td></td>
<td>111</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Regional Income Inequalities

A frequently important aspect of a country's overall income distribution—at a point of time and over time—is the distribution among and within regions. A considerable variation in income levels within and between regions does exist, and a study on inter- and intra-regional inequalities at a given point of time provides a useful input into the analysis of overall inequalities. Furthermore, the evidence as to how these inequalities have been changing over the years helps to identify the factors that have contributed to changes in overall inequalities. The information on regional inequalities also provides a useful input into the analysis that should underlie regional policy.

Tables 20 and 21 present intra- and inter-regional income distribution, for 1973 and 1978/79. A broad generalization emerging from the data is that both

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7 The country is divided into five principal zones. Zone 1 comprises the maritime districts in the Western and Southern provinces (excluding the metropolis of Colombo). Zone 2 and Zone 3 comprise the dry zone districts in the Soth, North, North-Central and Eastern provinces. The division of the dry zone into Zone 2 and Zone 3 is based on ethnicity, Zone 2 comprising mainly the majority community (Sinhalese) and Zone 3 comprising minorities (Tamils and Moors). Zone 4 consists of plantation districts in the central part of the country. Zone 5 consists of the metropolis of Colombo.
Figure 6.- Fitted Sectoral Spending Unit Income Distributions in Sri Lanka for 1978/79
inter- and intra-regional inequalities were lower in 1973 than in 1978/79. Lending support to the hypothesis\(^8\) that inequalities are greater in urban areas than elsewhere, Zone 5 which includes the metropolis of Colombo, records the highest Gini ratio in 1978/79. It is often claimed that urban/rural/plantation differentials may have ramifications on regional inequalities. It was shown earlier that a substantial gap in mean income between sectors exists, and income inequality in the urban sector is highest. As differences in the proportion of rural, urban, and plantation populations in different regions do exist, this means that some regional differences merely reflect differences in the rural, urban and plantation weights of each region. This clearly is the case with Zone 4 which has a sizeable plantation population with relatively low mean income compared to the urban and rural population in that region. The high Gini ratio reported for Zone 4 may be the result of these differences. Zone 2, which had a Gini ratio of 0.34 in 1973, records a 51.2 percent increase in its Gini over the 1973-1978/79 period. Zone 1 records the lowest Gini ratio for 1978/79.

A disequalizing regional trend is discernible in Tables 20 and 21. Although the data show no simple pattern, there appears to be some tendency towards an increase of both inter- and intra-regional differences, over time. Paradoxically, both Zone 2 and Zone 3, two regions that are predominantly rural, record the highest percentage increase in the Gini ratio. Zone 1 records the lowest percentage increase in the Gini ratio, showing greater socio-economic stability.

As regards inter-regional disparities, the most significant feature is the substantial increase in the economic distance ratio between Zone 5 and the other

---

zones. The disparities between Zone 1, Zone 2 and Zone 3, as revealed by economic distance ratios, are relatively small. The economic distance ratios between Zone 4 and the other regions (with the exception of Zone 5) have diminished.

The overall picture suggests that the increase in global inequalities is mostly a result of the increase of intra-regional inequalities. A noteworthy aspect of inter-regional disparities, is the substantial increase of the economic distance ratio of Zone 5 vis-à-vis the other regions. The mean income of this region almost quadrupled during the 1973-1978/79 period, compared to Zone 1, whose mean income rose only by 137 percent. The magnitude of these changes in regional inequalities warrants further investigation, which will be undertaken in Chapter V.

Table 20.--Gini Ratio for Income Receivers, 1973 and 1978/79, By Regions

<table>
<thead>
<tr>
<th>Region</th>
<th>1973</th>
<th>1978/79&lt;sup&gt;a&lt;/sup&gt;</th>
</tr>
</thead>
<tbody>
<tr>
<td>Zone 1</td>
<td>0.413</td>
<td>0.446 (7.8)</td>
</tr>
<tr>
<td>Zone 2</td>
<td>0.342</td>
<td>0.517 (51.2)</td>
</tr>
<tr>
<td>Zone 3</td>
<td>0.320</td>
<td>0.463 (44.7)</td>
</tr>
<tr>
<td>Zone 4</td>
<td>0.413</td>
<td>0.475 (15.0)</td>
</tr>
<tr>
<td>Zone 5</td>
<td>0.406</td>
<td>0.571 (40.6)</td>
</tr>
</tbody>
</table>

<sup>a</sup> The figures in parentheses represent the percentage increase of the Gini ratio over 1973.
Table 21.- Economic Distance Ratio ($D_1$) and Mean Monthly Income of Income Receivers, 1973 and 1978/79, By Regions

<table>
<thead>
<tr>
<th></th>
<th>1973</th>
<th></th>
<th>1978/79</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Economic Distance</td>
<td>Mean Income</td>
<td>Economic Distance</td>
<td>Mean Income</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Zone 2</td>
<td>Zone 3</td>
<td>Zone 4</td>
<td>Zone 5</td>
<td>Zone 2</td>
<td>Zone 3</td>
<td>Zone 4</td>
</tr>
<tr>
<td>Zone 1</td>
<td>0.027</td>
<td>0.068</td>
<td>0.370</td>
<td>0.124</td>
<td>253</td>
<td>0.108</td>
<td>0.120</td>
</tr>
<tr>
<td>Zone 2</td>
<td>0.045</td>
<td>0.398</td>
<td>0.110</td>
<td>258</td>
<td>0.004</td>
<td>0.274</td>
<td>0.434</td>
</tr>
<tr>
<td>Zone 3</td>
<td>0.442</td>
<td>0.071</td>
<td>266</td>
<td>0.300</td>
<td>0.430</td>
<td>670</td>
<td></td>
</tr>
<tr>
<td>Zone 4</td>
<td>0.442</td>
<td>182</td>
<td></td>
<td>0.635</td>
<td>503</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zone 5</td>
<td>280</td>
<td></td>
<td></td>
<td>1069</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Income Inequalities by Sex

The Gini ratios for male and female income receivers are reported in Table 22. Income tends to be more unequally distributed among females than among males. This conjecture is supported by the data reported in the study. However, the Gini ratio for females has declined by 4.7 percent over the 1973-1978/79 period, indicating a reduction of inequalities within that group. During the same period, inequalities amongst males have increased by 25.1 percent. There are also persistent income differentials between males and females. In 1973 the mean monthly income of males was more than double that for females (see Table 23). The economic distance ratio was 0.602. By 1978/79, male-female income differentials have declined as reflected in the reduction of the economic distance ratio by 9.5 percent. In spite of this decline, the mean monthly income of a female was only 56 percent of the mean male income.

<table>
<thead>
<tr>
<th>Sex</th>
<th>1973</th>
<th>1978/79</th>
</tr>
</thead>
<tbody>
<tr>
<td>Male</td>
<td>0.362</td>
<td>0.453</td>
</tr>
<tr>
<td>Female</td>
<td>0.513</td>
<td>0.489</td>
</tr>
</tbody>
</table>

As males comprise 70 percent of all income receivers, the increase in inequality within this group has a great impact on global inequalities. Furthermore, considering the magnitudes of the percentage changes in intra-income inequalities, the effect of the disequalizing trend among males more than outweighed the equalizing trend observed among females.
Table 23.--Economic Distance Ratio (D₁) and Mean Monthly Income of Income Receivers, 1973 and 1978/79, By Sex

<table>
<thead>
<tr>
<th></th>
<th>1973</th>
<th>1978/79</th>
</tr>
</thead>
<tbody>
<tr>
<td>Economic Distance</td>
<td>0.602</td>
<td>0.545</td>
</tr>
<tr>
<td>Females</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mean Income</td>
<td>257</td>
<td>678</td>
</tr>
<tr>
<td>Females</td>
<td>142</td>
<td>379</td>
</tr>
</tbody>
</table>

Income Inequalities by Age

It is frequently claimed that part of overall income inequality is due to the effect of the age-income profile. This clearly is the case in a country undergoing rapid demographic transition. The conjecture is the subject of inquiry in this subsection. Tables 24-26 present data on inter- and intra-income distribution inequalities for different age groups. Table 24 indicates that intra-age

Table 24.--Gini Ratio for Income Receivers, 1973 and 1978/79, By Age

<table>
<thead>
<tr>
<th>Age (Years)</th>
<th>1973</th>
<th>1978/79²</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-18</td>
<td>0.351</td>
<td>0.355 (1.1)</td>
</tr>
<tr>
<td>19-25</td>
<td>0.436</td>
<td>0.416 (-4.6)</td>
</tr>
<tr>
<td>26-35</td>
<td>0.372</td>
<td>0.422 (13.4)</td>
</tr>
<tr>
<td>36-45</td>
<td>0.363</td>
<td>0.438 (20.7)</td>
</tr>
<tr>
<td>46-55</td>
<td>0.371</td>
<td>0.440 (18.6)</td>
</tr>
<tr>
<td>&gt; 55</td>
<td>0.424</td>
<td>0.503 (18.6)</td>
</tr>
</tbody>
</table>

² The figures in parentheses represent the percentage increase of the Gini ratio over 1973.
group income inequalities in 1973 were in the 0.3 range except in the case of 19-25 and over 55 years age groups. A sharp increase in intra-age group inequalities is shown for all age groups over 25 years, over the 1973-1978/79 period. The 19-25 years age group shows a decline in inequality, while inequality is almost invariant in the 14-18 years age group. Inequality within the over 55 years age group remains high, and in 1978/79 this age group reports the highest Gini ratio.

Table 25.-Economic Distance Ratio (D1) and Mean Monthly Income of Income Receivers, 1973, By Age

<table>
<thead>
<tr>
<th></th>
<th>Economic Distance</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Mean Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>19-25</td>
<td>26-35</td>
<td>36-45</td>
<td>46-55</td>
<td>&gt;55</td>
<td></td>
</tr>
<tr>
<td>14-18</td>
<td>0.687</td>
<td>0.744</td>
<td>0.787</td>
<td>0.810</td>
<td>0.806</td>
<td>83</td>
</tr>
<tr>
<td>19-25</td>
<td></td>
<td>0.481</td>
<td>0.621</td>
<td>0.661</td>
<td>0.558</td>
<td>144</td>
</tr>
<tr>
<td>26-35</td>
<td></td>
<td></td>
<td>0.282</td>
<td>0.350</td>
<td>0.216</td>
<td>209</td>
</tr>
<tr>
<td>36-45</td>
<td></td>
<td></td>
<td></td>
<td>0.093</td>
<td>0.055</td>
<td>263</td>
</tr>
<tr>
<td>46-55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>0.141</td>
<td>281</td>
</tr>
<tr>
<td>&gt; 55</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>251</td>
</tr>
</tbody>
</table>

The inter-income distribution inequalities reported in Tables 25 and 26 aptly demonstrate the effect of the age-income profile. An increasing pattern of mean income is discernible. The economic distance ratios between the dominant age group (26-35 comprising 26 percent of all income receivers), and the other age groups appear to have changed marginally, over the years. The economic distance ratio estimates for the 14-18 age group indicate a persistent gap between that age group and others. Notwithstanding, there has been a significant reduction in its economic distance ratios, over the years. The economic distance ratios of the
most affluent age group in 1978/79 (over 55 years) vis-à-vis the other age groups (with the exception of the 14-18 and 46-55 years age groups) have increased significantly. The 46-55 age group saw a closing of disparities with the over 55 age group, and by 1978/79 the degree of affluence in the two age groups was almost the same.

Table 26.-Economic Distance Ratio (D1) and Mean Monthly Income of Income Receivers, 1978/79, By Age

<table>
<thead>
<tr>
<th></th>
<th>Economic Distance</th>
<th>Mean Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>19-25</td>
<td>0.576</td>
<td></td>
</tr>
<tr>
<td>26-35</td>
<td>0.807</td>
<td></td>
</tr>
<tr>
<td>36-45</td>
<td>0.838</td>
<td></td>
</tr>
<tr>
<td>46-55</td>
<td>0.860</td>
<td></td>
</tr>
<tr>
<td>&gt;55</td>
<td>0.865</td>
<td>181</td>
</tr>
<tr>
<td>19-25</td>
<td>0.579</td>
<td></td>
</tr>
<tr>
<td>26-35</td>
<td>0.728</td>
<td></td>
</tr>
<tr>
<td>36-45</td>
<td>0.252</td>
<td></td>
</tr>
<tr>
<td>46-55</td>
<td>0.358</td>
<td></td>
</tr>
<tr>
<td>&gt;55</td>
<td>0.343</td>
<td></td>
</tr>
<tr>
<td>19-25</td>
<td>0.112</td>
<td></td>
</tr>
<tr>
<td>26-35</td>
<td>0.110</td>
<td></td>
</tr>
<tr>
<td>36-45</td>
<td>0.005</td>
<td>685</td>
</tr>
<tr>
<td>46-55</td>
<td>755</td>
<td></td>
</tr>
<tr>
<td>&gt;55</td>
<td>759</td>
<td></td>
</tr>
</tbody>
</table>

The effect of the age-income profile on the overall inequalities at a point of time comes into play via the age-income differentials (measured by the economic distance ratio) and the composition of the population. Besides the effect of age-income profiles, differences within age groups also contribute to overall inequality. The precise effect of inter- and intra- components on overall inequality, however, depends on the age structure of the population (population pyramid). Changes in overall inequalities are, therefore, the result of changes in the inter- and intra-income inequalities and changes in the structure of the population. As the age structure of the income receiver population reveals
relatively young composition (see Table 27), inequalities within younger age groups have a greater impact on overall inequality than do those within older groups. Table 27 reveals that as much as half the income receiver population is below 36 years of age, compared to 15 percent in the oldest age category (over 55 years). An overview of inter-income distribution inequalities among different age groups does not provide conclusive evidence as to the effect of changes in the economic distance ratio on overall inequality. Intra-age group inequalities, particularly, in all age groups over 25 years, have increased substantially, which may have a significant impact on overall inequality.

Table 27.-Composition of the Income Receiver Population 1973 and 1978/79, By Age

<table>
<thead>
<tr>
<th>Age (Years)</th>
<th>1973</th>
<th>1978/79</th>
</tr>
</thead>
<tbody>
<tr>
<td>14-18</td>
<td>3.50</td>
<td>5.32</td>
</tr>
<tr>
<td>19-25</td>
<td>15.26</td>
<td>16.92</td>
</tr>
<tr>
<td>26-35</td>
<td>25.38</td>
<td>26.04</td>
</tr>
<tr>
<td>36-45</td>
<td>24.13</td>
<td>21.28</td>
</tr>
<tr>
<td>46-55</td>
<td>15.45</td>
<td>15.71</td>
</tr>
<tr>
<td>&gt; 55</td>
<td>16.28</td>
<td>14.73</td>
</tr>
<tr>
<td>All Age Groups</td>
<td>100.00</td>
<td>100.00</td>
</tr>
</tbody>
</table>

Income Inequalities by Educational Level

A good predictor of a person's income level, particularly his or her earnings, is his or her educational attainment. This variable is known to explain a good part of income inequality. As suggested in Chapter II, unequal distribution of investment in human capital is, perhaps, one of the most important causes of the
dispersion of income. This conjecture is the domain of inquiry in Chapter V. The brief analysis that follows is a prelude to the detailed inquiry envisaged.

The differentials within and between educational levels are summarized in Tables 28-30. The data reveal clearly the familiar tendency for income to increase with education (see Tables 29 and 30). Notwithstanding, there are considerable inequalities within educational levels, as shown in Table 28. These variations in income within educational categories are a major part of total inequality, and may be attributable to the existence of pronounced age-income profiles and a host of other factors. These other factors include differences in ability, inter-regional, inter-sectoral differences and the male-female income differentials discussed earlier.

Table 28.-Gini Ratio for Income Receivers, 1973 and 1978/79, By Educational Level

<table>
<thead>
<tr>
<th>Educational Level</th>
<th>1973</th>
<th>1978/79</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Schooling</td>
<td>0.419</td>
<td>0.438 (4.5)</td>
</tr>
<tr>
<td>Primary</td>
<td>0.364</td>
<td>0.459 (26.1)</td>
</tr>
<tr>
<td>Secondary</td>
<td>0.367</td>
<td>0.460 (25.3)</td>
</tr>
<tr>
<td>GCE (Ord. Level)b</td>
<td>0.339</td>
<td>0.415 (22.4)</td>
</tr>
<tr>
<td>GCE (Adv. Level)c</td>
<td>0.426</td>
<td>0.432 (1.4)</td>
</tr>
<tr>
<td>University Degree</td>
<td>0.325</td>
<td>0.513 (57.8)</td>
</tr>
</tbody>
</table>

a The figures in parentheses represent the percentage increase of the Gini ratio over 1973.

b General Certificate of Education (Ordinary Level), taken at the end of the tenth year.

c General Certificate of Education (Advanced Level), taken at the end of the twelfth year.
In 1978/79, university graduates record the highest Gini ratio of 0.513. This group, incidentally, reports the highest percentage increase of Gini ratio, over 1973, as well. All other groups have lower Gini ratios than the global average and are in the 0.4 range. Besides university graduates, the persons whose formal education ended before the sixth grade (primary), tenth grade (secondary) and those who have completed grade 10 (GCE Ord. level) record significant increases in the Gini ratio over the 1973-1978/79 period.

Table 29.-Economic Distance Ratio (D$_{ij}$) and Mean Monthly Income of Income Receivers, 1973, By Educational Level

<table>
<thead>
<tr>
<th>Economic Distance</th>
<th>Primary</th>
<th>Secondary (Ord. level)</th>
<th>Secondary (Adv. level)</th>
<th>GCE</th>
<th>GCE (Ord. level)</th>
<th>GCE (Adv. level)</th>
<th>University Degree</th>
<th>Mean Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Schooling</td>
<td>0.358</td>
<td>0.568</td>
<td>0.704</td>
<td>0.815</td>
<td>0.930</td>
<td></td>
<td></td>
<td>149</td>
</tr>
<tr>
<td>Primary</td>
<td>0.353</td>
<td>0.632</td>
<td>0.733</td>
<td>0.888</td>
<td></td>
<td></td>
<td></td>
<td>201</td>
</tr>
<tr>
<td>Secondary</td>
<td></td>
<td>0.402</td>
<td>0.567</td>
<td>0.787</td>
<td>266</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>GCE (Ord. level)</td>
<td></td>
<td></td>
<td></td>
<td>0.157</td>
<td>0.653</td>
<td></td>
<td></td>
<td>361</td>
</tr>
<tr>
<td>GCE (Adv. level)</td>
<td></td>
<td></td>
<td></td>
<td>0.487</td>
<td>564</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>University Degree</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>835</td>
</tr>
</tbody>
</table>

A closer look at the inter-income distribution inequality ratios reveals that except for the university graduates who have become more affluent with respect to all other categories (as reflected in the increase of its economic distance ratio in relation to all other groups), the remaining groups show an equalizing trend. Two minor exceptions are, however, the economic distance ratios between "no schooling and primary", and between "no schooling and GCE (Ord. level)", which have increased marginally. The economic distance ratio
between those who have obtained the GCE (Ordinary level) diploma and those who have obtained the GCE (Advanced level) diploma shows a significant reduction from 0.557 to 0.059. In 1978/79, of the fifteen binary combinations for which the economic distance ratio is calculated, 10 show values greater than 0.5, pointing to the fact that significant income differentials do exist between educational levels.

Table 30.- Economic Distance Ratio (D1) and Mean Monthly Income of Income Receivers, 1978/79, By Educational Level

<table>
<thead>
<tr>
<th>Economic Distance</th>
<th>Primary</th>
<th>GCE Secondary (Ord. level)</th>
<th>GCE (Adv. level)</th>
<th>University Degree</th>
<th>Mean Income</th>
</tr>
</thead>
<tbody>
<tr>
<td>No Schooling</td>
<td>0.382</td>
<td>0.562</td>
<td>0.764</td>
<td>0.749</td>
<td>0.972</td>
</tr>
<tr>
<td>Primary</td>
<td>0.225</td>
<td>0.543</td>
<td>0.559</td>
<td>0.909</td>
<td>503</td>
</tr>
<tr>
<td>Secondary</td>
<td></td>
<td></td>
<td>0.366</td>
<td>0.402</td>
<td>0.806</td>
</tr>
<tr>
<td>GCE (Ord. level)</td>
<td></td>
<td></td>
<td>0.059</td>
<td>0.680</td>
<td>872</td>
</tr>
<tr>
<td>GCE (Adv. level)</td>
<td></td>
<td></td>
<td>0.570</td>
<td></td>
<td>916</td>
</tr>
<tr>
<td>University Degree</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1791</td>
</tr>
</tbody>
</table>

An overview of changes in inter- and intra-income inequalities suggests that economic distance ratios have not increased much. Although, the economic distance ratios between university graduates and other categories have increased, this category comprises only one percent of all income receivers. In a number of cases the economic distance ratio has decreased, which has an equalizing tendency on overall inequality. Income inequalities within different educational levels (measured by the Gini ratio) have increased significantly. These results have clear implications for the disequalizing trend in overall inequality observed over the 1973-1978/79 period.
CHAPTER V

CAUSES OF INCOME INEQUALITIES IN SRI LANKA

Conventional wisdom proclaims that more education has been an egalitarian force in many countries. Empirical work has been attempted elsewhere, notably in the United States, which lends support to this hypothesis. Yet surprisingly little empirical knowledge exists about this question in the developing countries. On this score Sri Lanka is no exception. Despite the disequalizing trend in income distribution observed in the recent past, Sri Lanka's income distribution has shown an egalitarian trend since the 1960's. The country's income inequalities are still low by developing countries' standards. In the background of this scenario is Sri Lanka's universal free-education scheme which appears to have brought about a relatively equal distribution of human capital, over the years. Given the premise that unequal distribution of human capital is one of the most important causes of disparities in earnings, and considering the fact that in Sri Lanka 78.7 percent of the total personal income consists of earnings, the distribution of human capital has far-reaching implications for the overall inequality in the distribution of income. The present chapter embodies an inquiry into this aspect of Sri Lanka's income distribution.

Education is frequently presented as an important determinant of distribution patterns. Over the past two decades, the "human capital" school has emerged with the proposition that differences in education are a prime explanation of differences in income. Viewed as a hypothesis amenable to empirical examination, this proposition states that a generalized earnings function can account for all of the human capital components, namely, schooling and higher
education, post-school and also pre-school attainments, and other forms of human capital. Such a function would explain most of the distribution of personal income in a given country at a specific point in time, and its inter-temporal and inter-spatial changes. Considered a milestone in the evolution of distribution theory, the human capital approach has its roots in the work of Jacob Mincer,1 Gary S. Becker2 and Barry R. Chiswick.3 Their studies rely on investment in human capital as the leading single cause from which many of the observed regularities in earnings data are derived. Mincer4 finds that in the United States, one-half of the dispersion of earnings among male workers (adjusted for length of work year) is explained by different levels of formal schooling, experience, and on-the-job training.

1. The Human Capital Approach to Income Distribution

The Schooling Model

Mincer, Becker and Chiswick considered a model in which individuals invest in themselves by undergoing training (schooling). During the time devoted to schooling their earnings are zero. All schooling costs are viewed by the individual as time costs. The model takes the length of schooling as the basic source of heterogeneity of earnings. Schooling raises productivity, but the time spent in schooling necessitates postponement of earnings to a later age. Individuals undertake various amounts of schooling in the expectation that their future

1 Jacob Mincer, "Investment in Human Capital", pp. 281-302.
2 Gary S. Becker, "Human Capital and Distribution of Income".
earnings will be sufficiently large to compensate for the cost of schooling. They are, therefore, assumed to maximize the present value of their expected future earnings streams, subject to the usual assumptions of competitive equilibrium, such as the absence of capital market imperfections. Under these conditions, equilibrium occurs when the present values of future earnings streams are equalized at the time schooling begins.

Let

\[ y_s = \text{income per unit of time of a person with } s \text{ years of schooling}, \]
\[ V_s = \text{present value at the start of schooling of earnings stream } y_s, \]
\[ r = \text{rate of discount}, \]

and

\[ T = \text{economic life span, assumed to be same for all individuals. } T \text{ is defined as the length of the schooling period plus the length of the earning period}. \]

Then, using continuous discounting, the present value \( V_s \) is obtained as:

\[
(5.1.1) \quad V_s = e^{-rs} \int_0^{T-s} e^{-rt} y_s \, dt = r^{-1} e^{-rs} y_s (1 - e^{-(T-s)})
\]

Now, in equilibrium, according to the strong assumptions of the neoclassical school, \( V_s \) is the same for all \( s \). Suppose there is an occupation which requires no education. Putting \( s = 0 \) in (5.1.1),

\[
(5.1.2) \quad V_0 = r^{-1} y_0 (1 - e^{-rT})
\]

Equilibrium occurs when \( V_s = V_0 \), i.e.,

\[
(5.1.3) \quad e^{-rs} y_s (1 - e^{-(T-s)}) = y_0 (1 - e^{-rT})
\]
Rearranging (5.1.3):

\[(5.1.4) \quad \frac{y_s}{y_0} = e^{rs} \frac{(1 - e^{-rT})}{(1 - e^{-r(T-s)})} \]

As a first order approximation, the right hand side of the expression (5.1.4) approaches $e^{rs}$ if $T - s$ is large.

\[(5.1.5) \quad \frac{y_s}{y_0} \approx e^{rs} \]

Taking logarithm of (5.1.5)

\[(5.1.6) \quad \log y_s = \log y_0 + rs \]

In (5.1.6), $r$ stands for the market discount rate or for the internal rate of return on the differential investment which it must equal. If the competitive assumptions are relaxed, internal rates of return cannot be equated with the market rate of interest and generally differ among individuals. The equation (5.1.6) can remain valid, however, with $r$ interpreted as a group average internal rate of return on schooling, while individual differences in $r$ are considered random and compounded in the statistical residual.

The expression in (5.1.6) describes a cause-and-effect relationship, which has a theoretical foundation. It follows from (5.1.6) that the distribution of earnings depends on the distribution of education. For example if education is normally distributed, then the logarithm of earnings will be normally distributed; or in other words a normal distribution of education results in a lognormal distribution of income. Of particular interest here is the implication of (5.1.6) that both the skewness of the distribution and the degree of dispersion of earnings will be
greater, the greater the dispersion in years of schooling. Indeed, a positive skewness of earnings cannot be avoided given that the observed distribution of schooling is skewed in the positive direction. Furthermore, both dispersion and skewness will be larger when the rate of return is higher. Thus, if education is unequally distributed and the rate of return in the economy is high, a high degree of concentration of earnings is to be expected.

Returning to (5.1.6) in which \( r \) represents the average rate of return on schooling for the group or population, and placing deviations from this population average, in the residual \( U \), the equation reads as:

\[
(5.1.7) \quad \log y_s = \log y_0 + rs + U
\]

Regression estimate of the rate of return is obtained via equation (5.1.7) by regressing the natural logarithm of earnings on years of schooling completed:

\[
(5.1.8) \quad \log y_s = (\log y_0) + \hat{r}s + \hat{U}
\]

where \( \hat{r} \) and \( (\log y_0) \) are the least-square linear regression estimates of the average rate of return on schooling and the logarithm of zero schooling level of earnings respectively. \( \hat{U} \) is the residual. The implications for inequality are shown by taking the variance of both sides of the regression equation (5.1.7):

\[
(5.1.9) \quad \sigma^2(\log y) = r^2\sigma^2(s) + \sigma^2(U)
\]

The product \( r^2\sigma^2(s) \) is called the education component of earnings inequality. The variance of the logarithm of earnings (earnings-power) is dimensionless and can be compared inter-temporally and inter-spatially. The variance of schooling, \( \sigma^2(s) \) is
not without dimension; hence the value of the index is sensitive to the definition of
the numeraire. Besides, it is also dependent on the level of schooling. It is
customary to undertake comparisons in terms of the education component which is
dimensionless. In the present study, the conclusions as to the relationship between
schooling inequality and earnings inequality are reached using the coefficient of
variation of schooling given by:

\[(5.1.10) \quad CV(s) = \sigma(s) / \mu,\]

corresponding to the following specification of \((5.1.7)\),

\[(5.1.11) \quad \log y_s = \log y_0 + r(s/\mu) + \epsilon\]

where \(\mu\) and \(\sigma(s)\), are the mean and the standard deviation of the distribution of
schooling. Thus, the study of the implications for inequality within (otherwise)
homogenous groups is based on the variance of earnings-power and the coefficient
of variation of schooling. An indirect approach is used to determine the effect of
inter-group inequalities in education, on inter-earnings distribution inequalities, by
recourse to the fundamental human capital identity:

\[(5.1.12) \quad y = y_0 + rH,\]

\(y\) representing the earning capacity of a typical individual, as the sum of the base
earnings which he would receive without investing in human capital plus the returns
on his previous investments in human capital. \(y_0\) and \(r\) for each subpopulation are
obtained as regression estimates in \((5.1.8)\). Mean years of schooling completed is a
good proxy for H in each cell. The inequalities in the distribution of education between groups, can thus be ascertained by a comparison of these three variables.

**Validation Process of the Model**

The schooling model in (5.1.6) has a theoretical foundation. The logical implications of the theory were discussed. The next step is to determine whether the model provides a good fit to the data. The conventional goodness of fit statistic, the coefficient of determination $R^2$ has been used hitherto, as a measure of explanatory power of the model. The present study proposes a test criterion which uses the information in the data set to determine whether the data confirm the specification of the model (5.1.6) or some other functional form. The purpose of the test is to ascertain whether the model has a sound empirical foundation in a given situation, notwithstanding prior considerations of its theoretical validity. The test is a logical consequence of an analysis of transformations due to G. E. P. Box and D. R. Cox.\(^5\)

Box and Cox contend that, in regression studies, it is sometimes necessary to take an entirely empirical approach to the choice of a relation. The situation is slightly different, however, when prior reasoning (theory) as to the nature of the functional form exists. Here, it is customary for the model user to treat his model as the null hypothesis and test its validity against alternative parametric representations. The Box-Cox analysis is concerned with a parametric family of transformations from $y$ to $y(\lambda)$, the parameter $\lambda$ defining a particular transformation. It can be shown that the model represented by (5.1.6) is a particular case of the parametric family proposed by Box and Cox. The test criterion proposed in the present study examines whether the value of $\lambda$ for the

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particular case in (5.1.6) is supported by the data set, at a specified significance level.

In their analysis, Box and Cox considered the parametric family of transformations from $y$ to $y^{(\lambda)}$, where $y^{(\lambda)}$ is a monotonic function of $y$ over the admissible range, given by:

$\tag{5.1.13} y^{(\lambda)} = (y^\lambda - 1)/\lambda$

Suppose the vector of observations on the dependent variable is $y = (y_1, y_2, \ldots, y_n)$, the transformed regression model with $y_i > 0$, $i \in \{1, 2, \ldots, n\}$ is given by:

$\tag{5.1.14} (y_1^{\lambda} - 1)/\lambda = \beta_1 + \beta_2 x_{2i} + \ldots + \beta_k x_{ki} + U_i$

where $\beta$'s and $\lambda$ are unknown parameters, the $x$'s are observations on explanatory variables, and $U_i$ is the $i$th error term. They assume that for some unknown value of $\lambda$ the transformed observations $y_i^{(\lambda)}$, $i \in \{1, 2, \ldots, n\}$ satisfy the standard assumptions of the normal multiple regression model; that is, they are normally and independently distributed with constant variance $\sigma^2$. Thus, by assumption, for some value of $\lambda$ a transformation on the dependent variable is assumed (a) to induce normality, (b) to stabilize the variance, and (c) to induce simplicity of structure in the sense that:

$\tag{5.1.15} E((y_1^{\lambda} - 1)/\lambda | x_{2i}, \ldots, x_{ki}) = \beta_1 + \beta_2 x_{2i} + \ldots + \beta_k x_{ki}$

is a simple function of the $\beta$'s and $x$'s. The particular transformation in (5.1.13) has the following properties: For $\lambda = 1$, $(y_1^{\lambda} - 1)/\lambda = y_i - 1$, and the model (5.1.14) is
linear in the $y_i$. For $\lambda > 0$, \(\lim (y_i^\lambda - 1) / \lambda = \log y_i\), and thus the model in (5.1.14) has $\log y_i$ as the dependent variable. For other values of $\lambda$, powers of $y_i$ appear as the dependent variable. Since $\lambda$ is an unknown parameter, it will have to be estimated along with the other unknown parameters, the $\beta$'s and $\sigma$. The most desirable property of this transformation is that information in the data is used to determine the appropriate transformation for the dependent variable. The estimation problem can be solved by using the maximum likelihood (ML) method. In matrix notation (5.1.14) can be written as:

\[(5.1.16)\quad y^{(\lambda)} = X\beta + U,\]

where $y^{(\lambda)}$ is a column vector of transformed observations; $X$ is an $n \times k$ matrix, with rank $k$, of given observations on $k - 1$ explanatory variables; and $U$ is a column vector of error terms. The joint probability density function for the transformed observations, and hence the likelihood in relation to the original observations, is obtained by multiplying the normal density by the Jacobian of the transformation.

The likelihood $l(\lambda, \beta, \sigma^2 | y)$ is thus:

\[(5.1.17)\quad l(\lambda, \beta, \sigma^2 | y) = \exp\left\{ -\frac{(y^{(\lambda)} - XB)'(y^{(\lambda)} - XB)}{2\sigma^2} - \frac{1}{2} (y^{(\lambda)} - XB)' J(\lambda; y)(y^{(\lambda)} - XB) \right\} \cdot \frac{1}{\sigma^2 \cdot \sqrt{2\pi}},\]

where

\[(5.1.18)\quad J(\lambda; y) = \prod_{i=1}^{n} \frac{\partial y_i^{(\lambda)}}{\partial \lambda} \frac{\partial y_i}{\partial y_i},\]

and since $\frac{\partial y_i^{(\lambda)}}{\partial \lambda} = y_i^{\lambda - 1}$, it can be shown that:

\[(5.1.19)\quad J(\lambda; y) = \left( \prod_{i=1}^{n} y_i \right)^{\lambda - 1} = y^{n(\lambda - 1)}\]
where \( y = (\prod_{i=1}^{n} y_i)^{1/n} \), the geometric mean of the \( y \)'s.

\( J(\lambda; y) > 0 \), since by assumption \( y_i > 0 \), \( i \in \{1, 2, \ldots, n\} \). The log likelihood function is obtained by letting \( L = \log J \)

\[(5.1.20) \quad L = \text{constant} + \log J - (n/2) \log \sigma^2 - (y^{(\lambda)} - X\hat{\beta})' (y^{(\lambda)} - X\hat{\beta}) / 2\sigma^2 \]

On differentiating with respect to \( \beta \) and \( \sigma^2 \) to obtain the maximum likelihood estimates \( \hat{\beta} \) and \( \hat{\sigma}^2 \):

\[(5.1.21) \quad \hat{\beta}(\lambda) = (X'X)^{-1}X'y^{(\lambda)} \]

and

\[(5.1.22) \quad \hat{\sigma}^2(\lambda) = (y^{(\lambda)} - X\hat{\beta})' (y^{(\lambda)} - X\hat{\beta}) / n \]

If \( \lambda \) were known, \( (5.1.21) \) and \( (5.1.22) \) could be computed and would be ML estimates. However, since \( \lambda \) is assumed to be unknown, \( (5.1.21) \) and \( (5.1.22) \) are substituted in \( (5.1.20) \) to obtain the maximized log likelihood function, \( L_{\max}(\lambda) \) which is given by:

\[(5.1.23) \quad L_{\max}(\lambda) = \text{constant} + \log J - (n/2) \log \sigma^2 \]

Substituting \( (5.1.19) \) in \( (5.1.23) \)

\[(5.1.24) \quad L_{\max}(\lambda) = \text{constant} + (\lambda - 1) \sum_{i=1}^{n} \log y_i - (n/2) \log \sigma^2(\lambda) \]
$L_{\text{max}}(\lambda)$ in (5.1.24) is evaluated for various values of $\lambda$ and plotted. The value, say $\hat{\lambda}$ for which (5.1.24) attains its maximal value, is the ML estimate for $\lambda$. Then (5.1.21) and (5.1.22), evaluated at $\lambda = \hat{\lambda}$ are ML estimates for $\beta$ and $\sigma^2$, respectively. Inferences about the parameters can be made using large-sample maximum likelihood theory. This approach leads directly to point estimates of the parameters (as discussed above) and to approximate tests and confidence intervals based on the chi-squared ($\chi^2$) distribution. The theory stipulates that $2(L_{\text{max}}(\hat{\lambda}) - L_{\text{max}}(\lambda))$ is approximately distributed as $\chi^2$ with one degree of freedom. The approximate $100(1 - \alpha)$ percent confidence interval can be constructed from:

$$L_{\text{max}}(\hat{\lambda}) - L_{\text{max}}(\lambda) < (1/2)\chi^2(\alpha).$$

where $\alpha$ is the significance level.

The test has the capability of analysing a general family of earnings functions with several explanatory variables, such as schooling, experience, and for each-given combination of explanatory variables, different functional forms. The schooling model in (5.1.6) is a particular case of the general family (5.1.14), when $x_{2i} = S$, $x_{r} = 0$, $\forall r > 2$, and $\lambda = 0$.

The proposed test criterion is applied in the analysis of earnings functions. The dependent variable is earnings and years of schooling completed is the explanatory variable. Table 31 gives point estimates and confidence intervals for lambda ($\lambda$), the parameter of transformation. The analysis points to an optimal value of $\lambda = 0.04$, for 1953. The graph of maximized likelihood gives an approximate 95 percent confidence interval for $\lambda$ extending from -0.01 to 0.09. The semi-log transformation which has a natural appeal for the analysis of earnings, since it is open to a simple interpretation in terms of internal rate of
Table 31.-Test Criterion for Model Specification, Point Estimates and Confidence Intervals for Lambda, By Selected Years

<table>
<thead>
<tr>
<th>Year</th>
<th>Value of Lambda</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Point Estimate</td>
<td>Confidence Interval</td>
<td></td>
</tr>
<tr>
<td>1953</td>
<td>0.04</td>
<td>(-0.01, 0.09)</td>
<td></td>
</tr>
<tr>
<td>1963</td>
<td>-0.38</td>
<td>(-0.78, 0.02)</td>
<td></td>
</tr>
<tr>
<td>1973</td>
<td>-0.56</td>
<td>(-1.01, -0.11)</td>
<td></td>
</tr>
<tr>
<td>1978/79</td>
<td>-0.36</td>
<td>(-1.04, 0.32)</td>
<td></td>
</tr>
</tbody>
</table>

return, is in fact supported by data. The coefficient of determination ($R^2$) for transformed observations in Table 32 further confirms this evidence. As regards the other years, the value of $\lambda$ corresponding to the schooling model, namely, $\lambda = 0$,

Table 32.-Coefficient of Determination ($R^2$) for Transformed Observations for Some Selected Values of Lambda, By Selected Years

<table>
<thead>
<tr>
<th>Lambda</th>
<th>Coefficient of Determination ($R^2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-1.50</td>
<td>0.864</td>
</tr>
<tr>
<td>-1.00</td>
<td>0.926</td>
</tr>
<tr>
<td>-0.50</td>
<td>0.977</td>
</tr>
<tr>
<td>0.00</td>
<td>0.999</td>
</tr>
<tr>
<td>0.50</td>
<td>0.980</td>
</tr>
<tr>
<td>1.00</td>
<td>0.919</td>
</tr>
<tr>
<td>1.50</td>
<td>0.837</td>
</tr>
</tbody>
</table>
clearly lies within the 95 percent confidence interval. A marginal exception is, however, the year 1973. The semi-log schooling model also has high values for the coefficient of determination for all years.

2. Empirical Analysis of Income and Schooling

The relationship between earnings inequality, and the rate of return on schooling and the variance of schooling was the focal point of the schooling model (5.1.7) presented in the preceding section. The model provides some insights into the theoretical structure of earnings inequality at a point of time. In the ensuing analysis, the schooling model is applied in an empirical context in an attempt to explain earnings inequalities in Sri Lanka. Before embarking on this analysis some comments on the errors of measurement are noted.

Errors of Measurement

Data on income by level of schooling are contained in the Consumer Finance Survey Reports. Total income, rather than earnings, is recorded in these reports. Since the schooling model in (5.1.7) is developed to represent observed earnings, use of total income data has a tendency to bias the estimates of returns on schooling. It is frequently claimed that there is a positive correlation between property income and the level of schooling attained. If a positive correlation does exist, the inclusion of non-earnings income will bias the estimated returns on schooling upward. Another source of bias is the use of cross-section data on people of various ages to approximate lifetime earnings of an individual. The study treats the cross-section income-schooling data for a given year as though the age patterns in that year represent the income path that would be traced out through real time (or had been so traced) by any cohort of persons with the indicated schooling.
There are several additional qualifications to the empirical estimates reported in the study, arising from the measurement of education achieved. The education achieved is measured by the number of school grades completed in the formal education system. Years of schooling completed is a convenient unit of measurement, nonetheless, it is far from ideal for the purpose. First, there are vast differences in the quality of schooling at all levels, among regions, sectors etc.; and therefore an across-the-board comparison of the unit of measurement should be pursued with caution. Years of schooling completed without reference to the kinds of schooling pursued can introduce a serious bias into the estimates. A good part of the returns reported may, in fact, be attributable to differences in the quality of schooling. Second, no explicit correction is made for the average days of school attendance. A school year is far from constant across-the-board and over time. Third, the study does not explicitly separate out "employment" effects. The conditions of the economy and of the labour markets, in particular, affect income, and hence the estimates.

Rate of Return on Schooling: Trends over Time

In the preceding section, the implications of rate of return and dispersion of schooling on earnings inequality were discussed in a theoretical context. It was shown that, the higher the rate of return, and/or greater the dispersion of schooling, the greater are the inequalities in earnings. The analysis that follows is an empirical verification of this hypothesis.

This subsection investigates changes over time in the economic effects of schooling. Table 33 provides estimated rates of return on schooling and mean years of schooling completed, for the income receivers. The data indicate that the mean years of schooling completed has nearly doubled over the twenty-five year period under study. The rapid secular growth in schooling reveals the progress in mass
education, consequent on the policy of free-education introduced in 1951. On the demand side, anticipated rates of return may have motivated the acquisition of increasing amounts of education, over the years.

Table 33—Means and Rates of Return on Schooling, By Selected Years

<table>
<thead>
<tr>
<th>Year</th>
<th>Mean Schooling in Years</th>
<th>Rate of Return on Schooling (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1953</td>
<td>2.865</td>
<td>16.03</td>
</tr>
<tr>
<td>1963</td>
<td>3.570</td>
<td>13.91</td>
</tr>
<tr>
<td>1973</td>
<td>4.210</td>
<td>11.77</td>
</tr>
<tr>
<td>1978/79</td>
<td>5.235</td>
<td>9.96</td>
</tr>
</tbody>
</table>

The data clearly support the hypothesis that the secular increase in the relative supply of more educated persons reduces the gains from education. The decline in the rates of return on schooling over time, is indicative of a general secular decline in the monetary gain from education. Table 33 shows that the rate of return declined about 6 percentage points from 1953 to 1978/79. The high rate of return of 16 percent in 1953 reveals that there was considerable under-investment in education in the 1950's. With the advent of free education, more persons motivated by high rates of return pursued some kind of formal schooling. An ever-increasing proportion of the population completed additional years of schooling with the result that the educational attainment of the population grew.

considerably during the past twenty-five years. However, the economic advantages associated with the completion of additional years of schooling have diminished, as shown by the decline in rates of return on schooling. (See Table 33).

Inequalities in Income and Schooling: Inter-Temporal Changes

Table 34 presents data on mean and variance of income power (logarithm of income) and schooling, together with the estimated rates of return and coefficient of variation of schooling, for the years 1953, 1963, 1973 and 1978/79. The data are given in respect of income receivers. The results clearly indicate a reduction in inequality during the 1963-1973 period, followed by increasing inequality over the 1973-1978/79 period. An analysis of changes in the variance of income power, and coefficient of variation of schooling sheds some light on the effects of schooling on inter-temporal changes in income inequality.

The data in Table 34 show that the coefficient of variation of schooling has declined over the years, indicating a secular decline in the inequality in education. Clearly, the decline in inequalities in education and the accompanying decline in the rate of return have an equalizing effect on the overall inequality in income. The increase in income inequality over the 1973-1978/79 period, supposedly, represents a cyclical phenomenon, brought about by the spurt of fast economic growth experienced during the last two years of that period. Normally in a cyclical upswing, low and decreasing rates of unemployment are expected to lower income inequality by reducing the pool of unemployed poor and by increasing the bargaining power of labour relative to capital. Quite contrary to this expectation, in Sri Lanka, a reduction in the unemployment rate may have the perverse effect of increasing income inequality, through employment creation in low income pursuits, particularly in self employment. The unemployed, mostly educated young people who remain at home with their parents, and are counted as
unemployed during a period of high unemployment, find employment in a cyclical upswing. The absorption into the workforce of these low-income secondary wage earners causes income inequality to increase. However, this phenomenon provides only a partial explanation, as in any case, spending unit (family) income inequalities have increased. (See Table 14).

It is frequently alleged that fast growth in developing countries is characterized by uneven development, with concomitant adverse effects on income inequality. Rapid growth usually proceeds in a dualistic fashion, so that growth at the centre is more rapid than growth at the periphery. Closely associated with this phenomenon of unbalanced growth are the major shifts in population to urban and development project areas which can also cause overall inequality to rise. These factors alone are enough to ensure that, ceteris paribus, income distribution becomes more unequal during periods of rapid growth. The increase in inequalities over the 1973-1978/79 period warrants further investigation, which is undertaken in the spatial analysis that follows.

Sectoral Inequalities in Income and Schooling

In 1978/79, income inequalities in the urban and rural sectors, measured in terms of the variance of income power are more than double that in the plantation sector. (See Table 35). A noteworthy feature in the ranking of sectoral income distributions for 1978/79, is that the inequality in the rural sector is greater than the inequality in the urban sector. This represents a reversal of the ranking

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7 In 1973, the unemployment rate was estimated to be 24.0 percent of the workforce. Unemployment among the 19-25 years age group was 47.5 percent. Of the educational categories, those who have passed grade 10, suffered the highest unemployment rate of 47.4 percent. In 1978/79, the overall unemployment rate was estimated at 14.8 percent. Unemployment among the 19-25 years age group declined to 31.1 percent and that among the "passed grade 10" category was 27.6 percent. See Central Bank of Ceylon, Consumer Finance 1978/79.
Table 34.-Analysis of Income and Schooling, Income Receivers, By Selected Years

<table>
<thead>
<tr>
<th>Year</th>
<th>Income Power</th>
<th></th>
<th>Schooling</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Variance</td>
<td>Mean</td>
<td>Variance</td>
<td>Coefficient of Variation</td>
<td>Rate of Return (%)</td>
<td>Mean Income$^a$</td>
</tr>
<tr>
<td>1953</td>
<td>-0.417</td>
<td>0.712</td>
<td>2.865</td>
<td>9.038</td>
<td>1.049</td>
<td>16.03</td>
<td>0.956</td>
</tr>
<tr>
<td>1963</td>
<td>-0.207</td>
<td>0.896</td>
<td>3.570</td>
<td>12.195</td>
<td>0.978</td>
<td>13.91</td>
<td>1.239</td>
</tr>
<tr>
<td>1973</td>
<td>0.492</td>
<td>0.652</td>
<td>4.210</td>
<td>12.105</td>
<td>0.826</td>
<td>11.77</td>
<td>2.185</td>
</tr>
<tr>
<td>1978/79</td>
<td>1.336</td>
<td>0.909</td>
<td>5.235</td>
<td>13.320</td>
<td>0.697</td>
<td>9.96</td>
<td>5.878</td>
</tr>
</tbody>
</table>

$^a$ Income in Rs 10^2. Mean incomes in Tables 34-38 are calculated from grouped data.
implied by the Gini ratio. The inequality of income in the plantation sector, which increased marginally over the 1973-1978/79 period, remains relatively low. As the rural sector, comprising 63.5 percent of the total number of income receivers, is the dominant sector, greater inequalities within that sector have clear implications for global inequality. Income inequality in the rural sector increased by 59.1 percent, compared with the 39.4 percent increase in global inequality, over the 1973-1978/79 period. During the same period, inequalities of income in the urban and plantation sectors rose by 43.0 and 3.8 percent respectively.

As regards inter-sectoral inequalities in income, mean income in all three sectors increased by 161 percent, keeping the relative affluence of each sector vis-à-vis other sectors unchanged. An overview of inter-temporal changes in sectoral inequality suggests that the increase in inequalities within sectors, particularly within urban and rural sectors, has a disequalizing effect on overall inequality. Inter-sectoral inequalities remain invariant and therefore the effect of this component on overall inequality is neutral.

Turning to intra-sector inequalities in schooling, it appears that the distribution of schooling within the urban sector is relatively less unequal than that in the rural sector. Paradoxically, the inequality in schooling is highest in the plantation sector which records the lowest inequality in income. Another striking feature of the plantation sector is its low level of schooling coupled with low mean income power. These features have much to do with the type of technology

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8 It is possible that in the urban sector income transfers have occurred at high income levels in favour of the richest. The Gini ratio, which is sensitive to income transfers at all levels, ranks the urban sector as the sector with the highest inequality. The variance of income power (logarithm of income) has the shortcoming of not being sensitive to transfers among high income units, that is, when \( y > eM_g \), where \( M_g \) is the geometric mean and \( e \) is the base of the natural logarithm.

9 See also Table 19. The binary combinations of the economic distance ratio in the sectoral analysis are invariant.
Table 35.- Analysis of Income and Schooling, Income Receivers, 1973 and 1978/79, By Sectors

<table>
<thead>
<tr>
<th>Sector</th>
<th>Income Power</th>
<th>Schooling</th>
<th>Mean Income</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Variance</td>
<td>Mean</td>
</tr>
<tr>
<td>Urban</td>
<td>0.819</td>
<td>0.621</td>
<td>5.603</td>
</tr>
<tr>
<td>Rural</td>
<td>0.565</td>
<td>0.602</td>
<td>4.401</td>
</tr>
<tr>
<td>Plantation</td>
<td>-0.097</td>
<td>0.397</td>
<td>2.151</td>
</tr>
<tr>
<td>Whole Country</td>
<td>0.492</td>
<td>0.652</td>
<td>4.210</td>
</tr>
</tbody>
</table>

1978/79

| Urban           | 1.617        | 0.888     | 6.564       | 13.203     | 0.554                    | 11.02              | 7.702       |
| Rural           | 1.341        | 0.958     | 5.417       | 12.715     | 0.658                    | 6.85               | 5.925       |
| Plantation      | 0.895        | 0.412     | 2.458       | 5.645      | 0.966                    | 8.15               | 2.943       |
| Whole Country   | 1.336        | 0.909     | 5.235       | 13.320     | 0.697                    | 9.96               | 5.878       |

a Income in Rs $10^2$
employed in that sector. Except for the manufacturing phases of production of tea, rubber and coconut, the bulk of the operations in the plantations are labour intensive. The type of jobs prevalent in that sector do not demand high levels of education. The workforce (excluding the management and factory personnel) is relatively homogenous in terms of level of income. Almost all plantations are state-owned and workers are guaranteed a minimum wage. It is, therefore, not surprising that the plantation sector with the lowest mean schooling and highest coefficient of variation of schooling is also the sector with the lowest mean and variance of income power. The results clearly support the hypothesis that inequalities in schooling are negatively correlated with the level of development (measured in terms of level of schooling and income power). The levels of income (power) and schooling are positively correlated.

The data on the rate of return on schooling and the stock of education per income receiver explain the between-sector income differences, as hypothesized earlier. According to Table 35, in 1978/79, the rate of return and mean years of schooling are highest in the urban sector which reports the highest mean income. The mean years of schooling in both urban and rural sectors is more than double that in the plantation sector. This explains, among other things, why incomes are higher in the urban and rural sectors than in the plantation sector, in spite of somewhat higher rates of return on schooling in the plantations. The estimated rate of return of 25.7 percent for the plantation sector in 1973 appears to be an "outlier". The data in their aggregate form do not provide a satisfactory explanation for this paradox. However, a closer examination of sample data10

10 Estimates are highly sensitive to sampling variations. It is seen that the estimated rates of return vary considerably according to whether or not management personnel are included in the sample. In 1978/79 this category is not represented in the sample, and so the rate of return estimate for the plantation sector is comparable with that of the other sectors.
reveals that, due to some plantation managers, who are traditionally paid above their marginal product, being included in the sample, the rate of return estimate is unusually high. One of the basic assumptions of the schooling model is that individuals are paid according to their marginal product. A possible divergence between earnings and marginal product results in unreliable estimates of rates of return on schooling. This aspect is discussed in detail under limitations of the schooling model. The bias is also accentuated due to the inherent weakness of the type of data employed in the regressions. The grouped data rather than individual observations on income and schooling are used. As such, they are not weighted according to the number of income receivers in each cell. The rate of return estimate, partially corrected for this source of bias, was found to be 17.3 percent.

In examining the effect of schooling on inter-temporal changes in income inequality, it appears that inequalities in schooling among income receivers in all three sectors have declined. Notwithstanding, income inequalities have increased across-the-board. The data on mean years of schooling depict the long term trend in increase of stock of education per income receiver consequent on the implementation of the policy on free education. The increase in income inequality in spite of these equalizing forces, points to the fact that these changes are mainly the result of rapid economic growth which was largely related to the expansion of certain sectors of the economy. This is clearly seen in considerable increases in income inequality in both the urban and rural sectors. The urban sector responded fast to the far-reaching policy changes introduced in 1977. The removal of artificial controls, opening up of the economy, and the favourable investment climate brought about by the liberal policies, among other things, contributed to a boom in economic activity. The people in the expanding part of the economy benefited most from the income growth. A large and persisting disparity in labour productivity between industries resulted. In the rural sector, where several multi-
purpose development projects got under way, inequalities increased due to growth of income in and around these growth centres. Migration to urban and development project areas also accentuated whatever tendencies there are for income inequality to increase.

Regional Inequalities in Income and Schooling

In the regional analysis of inequalities, it is observed that inequality of income in 1978/79 is highest in Zone 2 (see Table 36), the region in which most of the major development projects are located. The second in ranking of inequality is Zone 5, consisting of the whole of the metropolis of Colombo. The lowest inequality of income is recorded in Zone 3. However, this region records an almost 100 percent increase in inequality of income, from 1973 to 1978/79. The inequality of income in Zone 2 more than doubled, while that in Zone 5 increased by 34.7 percent over the 1973-1978/79 period. During the same period, the inequality in Zone 4 rose by 41.6 percent. The results clearly support the conjecture that increase in overall inequality is the result of growth of income in the expanding sectors of the economy, particularly in the urban sector and development project areas in the rural sector.

In 1978/79, the sharpest regional difference in income is observed between Zone 5 and Zone 4. The differences in mean income between Zone 1, Zone 2 and Zone 3 are small. However, the mean income of each of these regions is considerably lower than that in Zone 5. As regards the growth of income, Zone 3, the most affluent region in 1973, records the lowest increase in mean income (142.7 percent). The mean income in the least affluent region in 1973, namely Zone 4, rose by 184.7 percent. The mean income in Zone 5 grew by 250 percent, compared to the 176 percent increase in mean income in Zone 2. An overview of regional inequalities suggests that the increase in overall inequality is more a
<table>
<thead>
<tr>
<th>Region</th>
<th>Income Power</th>
<th>Schooling</th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean(^a)</td>
<td>Variance</td>
<td>Mean</td>
<td>Variance</td>
<td>Coefficient of Variation</td>
<td>Rate of Return (%)</td>
<td>Mean Income(^a)</td>
</tr>
<tr>
<td>1973</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zone 1</td>
<td>0.593</td>
<td>0.702</td>
<td>5.231</td>
<td>12.759</td>
<td>0.683</td>
<td>12.38</td>
<td>2.459</td>
</tr>
<tr>
<td>Zone 2</td>
<td>0.707</td>
<td>0.537</td>
<td>4.325</td>
<td>11.514</td>
<td>0.785</td>
<td>6.13</td>
<td>2.530</td>
</tr>
<tr>
<td>Zone 3</td>
<td>0.794</td>
<td>0.399</td>
<td>4.108</td>
<td>9.917</td>
<td>0.767</td>
<td>6.02</td>
<td>2.670</td>
</tr>
<tr>
<td>Zone 4</td>
<td>0.282</td>
<td>0.608</td>
<td>3.349</td>
<td>10.505</td>
<td>0.968</td>
<td>12.25</td>
<td>1.757</td>
</tr>
<tr>
<td>Zone 5</td>
<td>0.678</td>
<td>0.704</td>
<td>5.083</td>
<td>13.332</td>
<td>0.718</td>
<td>10.67</td>
<td>2.591</td>
</tr>
<tr>
<td>1978/79</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zone 1</td>
<td>1.411</td>
<td>0.868</td>
<td>5.973</td>
<td>12.560</td>
<td>0.593</td>
<td>9.32</td>
<td>6.089</td>
</tr>
<tr>
<td>Zone 2</td>
<td>1.388</td>
<td>1.146</td>
<td>5.093</td>
<td>12.576</td>
<td>0.696</td>
<td>2.74</td>
<td>6.980</td>
</tr>
<tr>
<td>Zone 3</td>
<td>1.502</td>
<td>0.762</td>
<td>5.577</td>
<td>13.925</td>
<td>0.669</td>
<td>10.23</td>
<td>6.482</td>
</tr>
<tr>
<td>Zone 4</td>
<td>1.191</td>
<td>0.861</td>
<td>4.463</td>
<td>12.679</td>
<td>0.798</td>
<td>9.65</td>
<td>5.002</td>
</tr>
<tr>
<td>Zone 5</td>
<td>1.738</td>
<td>0.948</td>
<td>6.631</td>
<td>13.896</td>
<td>0.562</td>
<td>14.91</td>
<td>9.064</td>
</tr>
</tbody>
</table>

\(^a\) Income in Rs \(10^2\)
result of an increase in inequalities within regions than of an increase in inequalities between regions. The growth of mean income does not provide any conclusive evidence as to the effect of changes in inter-regional inequalities on overall inequality.

Data for 1978/79 indicate that the level of income and the level of schooling are positively correlated, and each is negatively correlated with the coefficient of variation of schooling. Variation in schooling is lowest in Zone 5, the most developed region, while that in Zone 4 is highest. Zone 4 (comprising the central parts of the country) consists of two heterogeneous subpopulations, namely, the plantation population, with a relatively low level of education and the non-plantation (urban and rural) population with a relatively high level of education. Hence, the variations in schooling are unusually high in that region.

An examination of data on schooling reveals a secular increase in the stock of human capital per income receiver in all regions. The inequalities in schooling have declined considerably in all regions. The rate of return on schooling in Zone 1, Zone 2 and Zone 4 has declined three percentage points, while that in Zone 3 and Zone 5 has increased by 4 percentage points. These trends point to the fact that the egalitarian forces consequent on wide spread of education is continuing. Notwithstanding, the inequalities in income have increased. The results vindicate the hypothesis that the disequalizing trends are the results of uneven growth experienced during the 1973-1978/79 period.

Inequalities in Income and Schooling by Sex

Table 37 presents data on income and schooling classified by sex. These data reveal a narrowing of male-female income differentials. In 1973; the mean female income was 50 percent of male income. During the 1973-1978/79 period, the mean female income grew 196.9 percent compared with a growth of 173.4
percent in the mean male income. In 1978/79, the mean female income was 54 percent of the mean male income. The evidence emerging from the study (see Table 37) shows a tendency towards a greater increase of dispersion of income amongst males than among females. The inequality in income amongst males rose by 68.3 percent over the 1973-1978/79 period. The corresponding increase for females was only 28.2 percent. The overall picture suggests that increase in "within inequalities", particularly the substantial increase in inequality amongst males may have caused global inequality to increase. The narrowing of male-female income differentials has an equalizing effect on global inequality.

Table 37.—Analysis of Income and Schooling, Income Receivers, 1973 and 1978/79, By Sex

<table>
<thead>
<tr>
<th>Sex</th>
<th>Income Power</th>
<th>Schooling</th>
<th>Income¹</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean¹</td>
<td>Variance</td>
<td>Mean</td>
</tr>
<tr>
<td>1973</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Males</td>
<td>0.698</td>
<td>0.473</td>
<td>5.067</td>
</tr>
<tr>
<td>Females</td>
<td>-0.125</td>
<td>0.680</td>
<td>4.168</td>
</tr>
<tr>
<td>1978/79</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Males</td>
<td>1.534</td>
<td>0.796</td>
<td>5.851</td>
</tr>
<tr>
<td>Females</td>
<td>0.883</td>
<td>0.872</td>
<td>5.213</td>
</tr>
</tbody>
</table>

¹ Income in Rs 10²

Turning to data on schooling, the mean level of schooling for both males and females has increased, while the difference in level of schooling between the
two groups has declined. The data also show an appreciable reduction in inequalities of schooling within each group. These clearly show an equalizing tendency on overall income inequality. In spite of these egalitarian tendencies, the inequalities in income have increased over the 1973-1978/79 period, suggesting the fact that factors other than education are responsible for the increase in income inequalities during that period.

It was shown that, on an average, females receive little over one half of the male income. This differential, rather than being an accurate reflection of the difference of average effective abilities of males and females, are partly institutionally or sociologically determined. Females are segregated into certain jobs and are considered a source of relatively cheap labour. The male-female income differential cannot, as Table 37 shows, be explained away by saying that females have less education than males. The educational attainment of females is not far short of male educational attainment. However, among females the association between education and financial returns tends to be less clear-cut than among males. This is due to several reasons. Firstly, wage differentials favouring males do exist. There is discrimination in pay, which means females receive, in general, a lower wage than male workers. Secondly, there is discrimination in access to jobs in the form of de facto restrictions on the sorts of work for which females can be employed. Thirdly, as a rule, a female, regardless of her education, in seeking a job, is bound to the location where her husband works. Fourthly, most of the employed females fall into the category of secondary wage earners. These factors alone explain why female incomes are generally lower than male incomes, and why variations in income and schooling are higher among females.

Inequalities in Income and Schooling by Age.

The data on income and schooling for income receivers classified by age are presented in Table 38. The table, read down the columns, gives data for
differences cohorts at a point of time (cross-sectional changes), and not those of a
given cohort aging over time. The longitudinal or time series changes of fixed age
groups are discernible in a comparison of data for fixed age groups at two different
points of time, namely, 1973 and 1978/79. The table clearly shows that income
tends to be relatively low at the beginning of labour-force participation, rises

Table 38.-Analysis of Income and Schooling, Income Receivers,
1973 and 1978/79, By Age

<table>
<thead>
<tr>
<th>Age</th>
<th>Income Power</th>
<th></th>
<th>Schooling</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>Variance</td>
<td>Mean</td>
<td>Variance</td>
<td>Coefficient of Variation</td>
<td>Mean</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14-18</td>
<td>-0.403</td>
<td>0.428</td>
<td>5.699</td>
<td>9.427</td>
<td>0.539</td>
<td>0.813</td>
</tr>
<tr>
<td>19-25</td>
<td>0.017</td>
<td>0.558</td>
<td>5.872</td>
<td>11.773</td>
<td>0.584</td>
<td>1.316</td>
</tr>
<tr>
<td>26-35</td>
<td>0.477</td>
<td>0.573</td>
<td>4.929</td>
<td>12.790</td>
<td>0.726</td>
<td>2.081</td>
</tr>
<tr>
<td>36-45</td>
<td>0.712</td>
<td>0.531</td>
<td>3.783</td>
<td>11.376</td>
<td>0.892</td>
<td>2.579</td>
</tr>
<tr>
<td>46-55</td>
<td>0.775</td>
<td>0.497</td>
<td>3.418</td>
<td>10.614</td>
<td>0.953</td>
<td>2.720</td>
</tr>
<tr>
<td>&gt;55</td>
<td>0.569</td>
<td>0.684</td>
<td>2.591</td>
<td>9.403</td>
<td>1.184</td>
<td>2.377</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1978/79</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14-18</td>
<td>0.347</td>
<td>0.579</td>
<td>6.290</td>
<td>9.133</td>
<td>0.480</td>
<td>1.851</td>
</tr>
<tr>
<td>19-25</td>
<td>0.834</td>
<td>0.761</td>
<td>6.576</td>
<td>11.227</td>
<td>0.510</td>
<td>3.271</td>
</tr>
<tr>
<td>26-35</td>
<td>1.368</td>
<td>0.763</td>
<td>6.313</td>
<td>12.672</td>
<td>0.564</td>
<td>5.626</td>
</tr>
<tr>
<td>36-45</td>
<td>1.568</td>
<td>0.788</td>
<td>4.953</td>
<td>13.386</td>
<td>0.739</td>
<td>7.004</td>
</tr>
<tr>
<td>46-55</td>
<td>1.662</td>
<td>0.764</td>
<td>4.053</td>
<td>11.644</td>
<td>0.842</td>
<td>7.589</td>
</tr>
<tr>
<td>&gt;55</td>
<td>1.560</td>
<td>0.878</td>
<td>3.431</td>
<td>11.409</td>
<td>0.985</td>
<td>7.420</td>
</tr>
</tbody>
</table>

\[a\] Income in Rs 10^2
throughout later ages, until a peak is reached in the 46-55 age group, and declines in the last age group. The results indicate the existence of pronounced age-income profiles. As a result of the marked secular rise in the level of schooling, the younger cohorts appear to be endowed with more educational capital than the older ones. The rapid upward trends in years of schooling attainment are reflected in Table 38.

The inequality in the distribution of schooling is lower within younger cohorts than within older ones. The data for 1978/79 show that inequality in income increases systematically with age, save for a marginal deviation from this pattern in the 46-55 age group. As regards inter-temporal change, the mean years of schooling has increased, while inequality of schooling has decreased for all age groups, over the 1973-1978/79 period. These results show the persistent equalizing trend of schooling. Mean incomes have increased substantially in all age groups, the older age groups gaining the most. The data indicate systematically higher income growth rates with increasing age. For instance, the mean income of the over 55 age group increased by 212.2 percent over the 1973-1978/79 period, compared to a 127.6 percent increase in mean income in the 14-18 age group. The inequalities of income within age groups have increased in all age groups. The 46-55 age group records the highest increase in "within group inequalities" (53.7 percent), while the over 55 age group records the lowest increase (28.3 percent).

An overview of changes in inter- and intra-age group inequalities suggests that increases in both inter- and intra-age group inequalities have a clear disequalizing effect on overall inequality.

Analysis of Income and Schooling: A Parametric Approach

In the foregoing analysis of income and schooling, the distribution of income was made to depend on the distribution of schooling, but the latter was
unexplained. By implication, the distribution of schooling could take any form, which is far from ideal. In a more-complete model, the distributions of income and schooling would be jointly determined. The study proposes a simplistic application of this approach, in which Dagum's model is used as a mechanism to describe the distribution of income, as well as schooling. It is assumed that income is an increasing function of schooling and rate of return on schooling (this seems to be a plausible assumption in view of the received theory discussed hitherto). It can then be shown, intuitively, that the degree of inequality in income will be greater, the greater the inequality in years of schooling and/or the higher the rate of return on schooling.

Table 39.- Analysis of Income and Schooling: Gini Ratio and Rate of Return on Schooling, Income Receivers, By Selected Years

<table>
<thead>
<tr>
<th>Year</th>
<th>Gini Ratio</th>
<th></th>
<th>Rate of Return on Schooling (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Income</td>
<td>Schooling</td>
<td></td>
</tr>
<tr>
<td>1953</td>
<td>0.477</td>
<td>0.536</td>
<td>16.03</td>
</tr>
<tr>
<td>1963</td>
<td>0.527</td>
<td>0.531</td>
<td>13.91</td>
</tr>
<tr>
<td>1973</td>
<td>0.405</td>
<td>0.469</td>
<td>11.77</td>
</tr>
<tr>
<td>1978/79</td>
<td>0.477</td>
<td>0.402</td>
<td>9.96</td>
</tr>
</tbody>
</table>

The Gini ratio for income and schooling, based on the estimated parameters of Dagum's model, is presented in Table 39, along with the estimates of the rate of return on schooling. The data clearly show the secular declining trend of both inequality in schooling and rate of return on schooling, brought about by the advent of free education. The impact of the free-education scheme, which was implemented in 1951, is clearly shown in the remarkable improvement of income
distribution over the 1963-1973 period. If the first age-cohort who typically benefitted from the free-education scheme entered the workforce with a lag of 12-15 years, then the trend shown in the 1963-1973 period is not surprising. The inequality of income has increased over the 1973-1978/79 period, in spite of the continuation of the favourable trend in schooling. Given this favourable trend, the recent worsening of income distribution cannot be expected to contribute significantly to income disqualification in the coming years. As conjectured earlier, the recent adverse trends reflect a cyclical phenomenon, which is expected to dampen over a long-run growth path.

A Critique of the Schooling Model

At this point it may be useful to emphasize that, although considerable evidence supports the schooling model, the real relationship between earnings and schooling is, perhaps, not so simple as it appears in (5.1.6). Implicit in the model is the positive relation between an individual's schooling and his subsequent earnings which reflect the productivity-augmenting effects of education. Schooling and education are, however, not synonymous. The educational content of time spent at school or the cognitive capacities acquired in the learning process differ a great deal among persons and places. Indeed, one might hypothesize that it is only what a student has learned, both in school and elsewhere, and not the amount of time spent in school, that influences earnings. There are differences in the quality of education at all levels, among regions, among sectors and between schools. Real differences do exist between urban and rural, developed and backward regions. The variations reported are, therefore, attributable to variations in the quantity of education absorbed rather than variations in years of schooling. Moreover, school is not the only training ground for shaping market productivities. The post-school investment plays an equally important role. Besides schooling and learning, on-the-
job training is likely to be an important means of acquiring knowledge and skills, useful for augmenting productivity. In the absence of data on post-school investment, the study was confined to the analysis of earnings and schooling.

There are also a number of important qualifications to be reckoned with in interpreting the results. First, data on earnings at a point of time are only rough indicators of future lifetime incomes. Second, data on total income rather than earnings are used for the dependent variable. The inclusion of property and transfer incomes increases the extent to which it could be claimed that in this analysis education is taking credit for things not truly the result of education. The distribution of wealth explains some portion of income differentials, particularly in the upper-tail of income distribution. Third, the predictive power of the schooling model is greatly hampered, if individuals are not paid according to their marginal product. This is particularly true in the plantation sector. The social relations of production characteristic in the plantation sector, the modern sector in general, is characterized by bureaucratic organization and hierarchical lines of authority. Occupational roles are grouped according to the degree of control exercised by the person holding the job. An important consequence is a widespread in pay scales, since hierarchical structures are based on the assumption that a superior must always have a higher salary than a subordinate, whatever their relative marginal productivities dictate. The salary structure is rigid because of status overtones. The existence of collective power resulting from the presence of organized power groups (or lack of it), causes actual earnings to be different from marginal productivities. Therefore, earnings differentials may not always be an accurate reflection of differences in marginal productivities. Regardless of its source, any divergence between earnings and marginal productivities can produce estimates of rate of return which can be misleading as a guide to positive or normative analysis.

Some of the differences in earnings may result from variations in seasonal unemployment and degree of underemployment; both visible and invisible.
Underemployment, a phenomenon observed in all sectors of the economy, exists when a person's employment is inadequate, in relation to specified norms or alternative employment, account being taken of his occupational skill in terms of training and work experience. The presence of underemployment weakens the predictive power of the schooling model and hence the results generated by the model are obscure. The differentials in income arising from shorter work weeks or differences in weeks worked, reflecting an insufficiency in the volume of employment (visible underemployment) can be accounted for by introducing length of work week and number of weeks worked as explanatory variables in a generalized earnings function. Ideally, a generalized earnings function may also contain as an explanatory variable, years of experience (a proxy for post-school investment). If the quality of schooling could be controlled, the explanatory power of the generalized earnings function would be further enhanced. The significance of years of schooling per se diminishes with the introduction of a measure of learning.

A major part of the explanation of income differentials must be sought in differences in abilities, both innate and acquired, and many other forces that impinge on the observed relationship between income and education. In an analysis of income and schooling, groups with differing amounts of education tend to differ systematically in terms of other attributes, which are also likely to influence relative earnings. An often quoted criticism of many studies purporting to show a positive correlation between income and schooling, is that they attribute results to education which were caused, at least in part, by differences in ability, motivations, family background, and so forth. The family environment and family institutional factors play an important role in determining the distribution of human capital and the distribution of income. It is often argued that factors such as parental wealth and income, parents' education, family connections, the occupational and cultural level of parents, family values, peers and community to
which household members are subject, and other factors which result from individual differences in ability and opportunity, have some influence on years of schooling. Furthermore, all these factors have a direct bearing on income, quite apart from years of preparation. Similarly, economic opportunities vary among sectors and regions, reflecting differential growth rates, wages and unemployment rates, differences in the mixes of skills demanded and supplied, access to knowledge about job opportunities, and the like. The foregoing critique leads to the conclusion that earnings are a function of many other factors, hence the need for specification and quantification of those factors. Such an exercise, however, cannot now be undertaken because of data limitations.

On the theoretic front, although the schooling model represents a useful departure from the neoclassical assumptions of labour homogeneity, the model, nevertheless, rests on strong assumptions of neoclassical theory for its derivation. The present value of future earnings streams for all educational levels would be equal in equilibrium. This means that the present value of future earnings of a university graduate, for instance, should be the same as that of a person with no schooling, which is far from true. There had also been some recent criticism to the narrowness exhibited in the human capital approach to the distribution of income. In the human capital formulation, the distribution of income is determined solely by the conditions of supply of human capital; demand conditions being considered given. Macroeconomic considerations, the structure of economic production, market structure, technical change, economic dualism and other presumably crucial aspects of the distribution problem are ignored. In a more complete model demand for human capital by the "organizers of production" must be introduced alongside of supply. According to the demand-supply theory11, (sometimes referred

to as "tension theory"), income is determined by the tension between demand and supply, arising from differences between the values of the demand factors and those of the supply factors. High incomes are paid for qualifications for which there is a high tension and low income for qualifications for which there is a low, even a "negative" tension, where supply exceeds demand. The income distribution may then be derived from the distribution of qualifications demanded and qualifications available.
CHAPTER VI

NORMATIVE ASPECTS OF HUMAN CAPITAL
AND INCOME DISTRIBUTION

Equality is the main theme of the present study. In the pursuit of equality, what is ultimately pursued is well being. Equality is only one of the arguments, admittedly a very significant one, in the total welfare function of a society. Putting equality in its place, and ascertaining its costs, are important considerations in planning economic development. There is, in a sense, a connection between development ideologies and equity goals. They are viewed as isomorphic with policy making and the instrumentalities for achieving distributive justice. In the policy facet, what is in fact attempted is, to introduce conscious objectives into the overall dynamic process of the society, with a view to changing its parameters. Discoursing on the role of the state in pursuit of equality, Rousseau remarked: "It is just because the pressure of events tends always to the destruction of equality that the force of legislation should always be directed to maintaining it" (Social Contract). Of the instrumentalities available to the planner, one that was considered most desirable in the present study is investment in human capital. The domain of the study covering positive aspects of the problem was concerned with ascertaining the magnitude of and relationship between the parameters of the dynamic process. The purpose of this chapter is to look at them in a normative sense.

Although the concept of human capital has become increasingly useful in economic analysis, in Sri Lanka's context, policy choices did not seem to reflect the theoretical and empirical work on investment in human capital. Due to lack of
research on human capital, changes in manpower policies, the adoption of new programmes, and policy decisions that determine the allocation of resources to alternative forms of human capital have, hitherto, been put into effect on an ad-hoc basis. Human capital is an economic concept dealing with costs and benefits essentially in economic terms. Nonetheless, it has implications for the development of social policy. If the facts and conclusions emerging from a well formulated human capital analysis, are presented clearly and persuasively, the analysts can have some impact on changes in manpower programmes and other investments in human resources. It is, therefore, incumbent on those who are engaged in policy formulation, to broaden the social context of human capital analysis, and to further empirical research on human capital with a view to influencing decision-makers.

I. Education—the "Great Equalizer"

As has been seen, education is frequently considered as an important determinant of distribution patterns, and one that can be incorporated into government policy. Unlike land reform and nationalization, which redistribute existing assets, the extension of education to the poor results in the creation of new (human) capital. The thrust of this chapter is primarily to clarify the investment processes and opportunities that provide incentives to invest in human capital. The main issue raised here is how the visible hand of government assigns priorities in its investment programmes in order to achieve the goals of both growth and equity. This chapter also discusses some issues of policy to ensure a fair distribution of human capital. As a foundation for the discussion of policy measures to narrow differentials, the existing magnitude and character of inequality in the distribution of income were discussed in the preceding chapters. A discussion of causes of inequalities followed. That discussion raised issues about
the relationship between equality of income and equality of opportunity. The present chapter focuses on inequalities of opportunity and their consequences for both efficiency and equality of income.

The empirical analysis in Chapter V suggests (though not in a definitive manner) how a major spread of education consequent on a policy of free education has helped to bring about a relatively less unequal income distribution, while at the same time perpetuating income growth. However, a positive correlation between educational effort and distributional equity does not necessarily imply that when education is expanded, equity is promoted. It was pointed out that it is the quality of education that counts, not its expansion per se. Two main features of education are likely to determine its distributional impact. First, is inequality in schooling reduced by expanding education? Second, do less-educated groups receive a greater (rate of) return for the lower investment in their education than that received by the better educated groups? Evidence of past trends in income differentials associated with various levels of education is of some relevance in this connection. In the empirical analysis, both these conjectures were supported by time series results. The spatial analysis, however, provided only inconclusive evidence. Although the evidence can only be speculative, it is, indeed, needed to establish the facts as they are, as well as to guide policy making.

An important welfare goal of society is to reduce inequalities in the distribution of income. Lessening the degree of inequality, while achieving a desirable rate of economic growth, is synonymous with improving social welfare and reducing the incidence of poverty. As a means of redistribution, society relies heavily on inheritance and progressive income taxation. Given public revenue from these sources, public investment in human capital, particularly that entering into general education, is an effective and efficient instrument for attaining the goal of equity, without constraining the goal of efficiency. When there is underinvestment
in schooling in backward sectors or among depressed groups, a policy to invest in more schooling of high quality contributes both to economic efficiency and to reducing the inequality in income. In Sri Lanka almost the whole of the education industry is collectively provided and financed. This makes it easy to appraise the efficiency with which resources are allocated within the education industry, and in relation to competing investment opportunities. If education is to be the principal means of reducing inequality, while sustaining economic growth, two decisions have to be taken. First, how much, or what proportion of the national income should be spent on education? Second, how best should a given expenditure be distributed, to attain the objective?

2. An Optimal Control Approach to Investment, Growth and Equity

The conception of economic development as a generalized process of capital accumulation involving various types of capital has important implications for the efficient programming of investment in economic development. These implications centre on the relationships of complementarity and substitutability in the use of alternative forms of capital provided by different investment processes, and the consequent desirability of achieving balanced investment in the development process. Balanced investment, in the context of the present study, involves the optimum mix of alternative forms of capital, e.g., material and human capital, in accordance with the priorities set by the societal welfare goals of growth and equity. In a somewhat broad sense, economic growth is necessarily the result of accumulation of capital, or of investment, but investment in this context is defined to include the acquisition of material and human capital. Unlike material capital, human capital is less prone to concentration. If income distribution is functionally related to concentration of capital, capital defined in a broad sense, then reliance on human capital is likely to stimulate greater equity.
The ensuing formulation is an attempt to construct a theory of optimal investment in material and human capital in a neoclassical growth model. Long-run optimum welfare is measured by an inter-temporal welfare function with two arguments, namely, average per capita consumption and the distribution of income. As a first step, the distribution of income is ignored as a policy objective and consequences for optimal choice between material and human capital investments are identified. The analysis is then extended to include in the welfare function an index of inequality in the distribution of income.

An Optimal Control Model

The study proposes an optimal control model to determine the optimal allocation of resources between consumption, investment in material capital and investment in human capital. In the language of optimal control theory, the problem is how to design a set of control variables (instruments), in the form of saving devoted to material capital formation and human capital formation, to bring the economic system from a given initial state to a final state at some future date, so as to maximize a long-run general welfare goal of growth and equity. As a first stage of analysis, it is assumed that the maximization of per capita consumption is the sole objective of the planning authority. This simplistic formulation assumes the absence of income distribution effects and consumption externalities on the social welfare function. It is also assumed that education is the only source of human capital creation.

List of symbols:

\[ Y \quad \text{national income} \]
K  material capital stock

L  labour-force; \( L(t) = L_0 e^{nt} \), the size of the labour-force is considered to be growing at the constant rate \( n \) (in the absence of migration)

\( C_e \)  total cost of the education programme (\( c_e \) constant average cost per graduate in the stock of educated manpower)

C  total consumption, with \( c = C/L \) = per capita consumption

\( \delta_1 \)  attrition rate due to retirement, obsolescence and death of graduates

\( \delta_2 \)  depreciation rate of material capital

t  time period

\( \dot{K} = \frac{dK}{dt} \)  time derivative, e.g., \( \dot{K} = \frac{dK}{dt} \) = investment in material capital.

The total labour-force, \( L(t) \), at any given time \( t \) is partitioned into skilled workers, \( L_e(t) \), and unskilled workers, \( L_u(t) \). The skilled workers constitute a constant proportion, \( w \), of the total labour-force, i.e., \( L_e(t) = w(t)L(t) \). Therefore, the unskilled workers, \( L_u(t) = (1-w(t))L(t) \). The model is, thus, highly aggregate, in that there is only one type of training (education): workers are either skilled or
unskilled. The analysis can, however, be easily extended to encompass alternative types of training (education).

The total costs of education, $C_e$, and of material capital accumulation are defrayed by the portion of national income, $Y$, saved and allocated to each programme. (See $u_1$ and $u_2$ in equations (6.2.5) and (6.2.6) below). Of course, more of one means less of the others; more current consumption means less savings available to defray investment in material and human capital; more allocated to the acquisition of machines means less left over for the education programmes. The problem is one of optimal allocation of scarce resources among these competing needs.

The following strong assumptions of neoclassical theory are assumed to hold:

A.1 Production is subject to constant returns to scale.

A.2 All factors are necessary, i.e., in equation (6.2.3a) below,

$$f(0, l_u, k) = f(l_e, 0, k) = f(l_e, l_u, 0) = 0$$

A.3 The production function in equation (6.2.3a) is well behaved, i.e.,

$$f_e(\infty, l_u, k) = f_u(l_e, \infty, k) = f_k(l_e, l_u, \infty) = 0 < f_i < f_e(0, l_u, k), f_u(l_e, 0, k), f_k(l_e, l_u, 0) = \infty$$

with $f_{ij}$ negative definite, $\forall i, j \in \{1, 2, 3\}$. $f_i$ and $f_{ij}$ are partial derivatives, e.g. $f_e = \frac{\partial f(l_e, l_u, k)}{\partial l_e}$, etc.

A.4 Social welfare, $W$, is a function of per capita consumption, $c$, with diminishing but non-vanishing marginal utility, i.e., for all consumption levels $c \in (0, \infty)$ in equation (6.2.4) below, $W''(c) < 0$ and $0 = W'(\infty) < W'(c) < W'(0) = \infty$. 
Now

(6.2.1) \( L(t) = L_e(t) + L_u(t) \)

(6.2.2) \( C_e(t) = c_e (\dot{L}_e + \delta \dot{L}_e) = c_e (\omega \dot{L} + \omega L + \delta \dot{L}) \)

(6.2.3) \( Y(t) = F(L_e, L_u, K) \)

(6.2.3a) \( y = Y / L = f(l_e, l_u, k) \) by A.1

where \( l_e = L_e / L = \omega; l_u = L_u / L = 1 - \omega; k = K/L \)

(6.2.4) \( W = W(c) \), with assumed properties listed under A.4.

The control variables \( u_1 \) and \( u_2 \) are:

(6.2.5) \( u_1(t) = \frac{C_e}{Y} = \frac{c_e}{\dot{L}} [((\delta \dot{L} + n)l_e + \dot{l}_e] \)

(6.2.6) \( u_2(t) = \frac{\dot{K} + \delta_2 K}{Y} = \frac{1}{\dot{L}} [((\delta + n)k + \dot{k}) \]

with \( 0 \leq u_i \leq 1; \ 0 \leq \sum_{i=1}^{2} u_i \leq 1, \ i \in \{1, 2\} \)

Also

(6.2.7) \( \dot{c}(t) = \frac{C(t)}{L(t)} = (1 - u_1 - u_2) f(l_e, l_u, k) \)

The objective functional is
\begin{align*}
(6.2.8) \quad J &= \int_0^T \mathcal{W}(c) e^{-rt} \, dt \\
&= \int_0^T \mathcal{W}((1 - \sum_{i=1}^{2} u_i)(l_e, l_{u}, k)) e^{-rt} \, dt
\end{align*}

The dynamical system implied by the saving-investment conditions \((6.2.5)\) and \((6.2.6)\) can be written more explicitly as:

\begin{align*}
(6.2.9) \quad \dot{l}_e &= \frac{u_1}{c_e} f - (\delta_1 + n)l_e \\
(6.2.10) \quad \dot{k} &= u_2 f - (\delta_2 + n)k,
\end{align*}

with boundary conditions (initial and terminal time conditions):

\begin{align*}
(6.2.11) \quad l_e(0) &= l_{e0} \quad ; \quad l_e(T) = l_{eT} \\
&\quad \quad k(0) = k_0 \quad ; \quad k(T) = k_T
\end{align*}

In \((6.2.9)\) and \((6.2.10)\), the rate of change of the state variables, \(l_e\) and \(k\), depend upon the values of the state variables, \(l_e\) and \(k\), and upon the values of the controls, \(u_1\) and \(u_2\), being applied.

The optimal control programme then consists of finding the optimal control functions, \(u^* = (u_1^*, u_2^*) \in U\), to transfer the dynamical system \((6.2.9), (6.2.10)\) from its initial configuration \((l_e(0), k(0))\) to its final configuration \((l_e(T), k(T))\), in such a way as to impart a maximum value to the objective functional, \(J\), in equation \((6.2.8)\). This is a typical optimal control problem, which can be best solved by applying Pontryagin's Maximum Principle.\(^1\) In this case, the Hamiltonian

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H takes the form:

\[(6.2.12) \quad H \equiv W((1 - \frac{2}{i=1} u_i) e^{-rt} + \lambda_1 \frac{u}{e} f - (\delta_1 + n) l_e) e^{-rt} + \lambda_2 (u_2 f - (\delta_2 + n) k) e^{-rt}\]

The problem consists of choosing the control process, \(u^* \in U\), such that \(H\) is maximized. \(\lambda(t) = (\lambda_1(t), \lambda_2(t))\), is a non-zero continuous vector function of dynamic multipliers.

In an effort to extend this analysis to encompass the income distribution effects on social welfare, the inevitable dilemma of the choice of a criterion to measure long-run welfare has to be reckoned with. \(W\) can be reduced to a welfare function with a measure of inequality in the distribution of income, and the average per capita consumption as arguments. For simplicity in mathematical manipulation, the variance of income is chosen as the index of inequality in income. The criterion of welfare, \(J\), is taken to be the integral of an undiscounted welfare function, \(W'\), where,

\[(6.2.13) \quad J = \int_0^T W'(v, c) \, dt\]

It is also assumed that \(W'\) has the property of additive separability, according to which:

\[(6.2.14) \quad W' = \gamma c - (1 - \gamma)v\]

where \(\gamma\) denotes the preference for economic growth, and \((1 - \gamma)\) the preference for income equality.\(^2\) The weights are non-negative and for mathematical

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convenience, add up to one. The criterion for optimality is:

\( (6.2.15) \quad J = \int_{0}^{T} (\gamma c - (1 - \gamma)v) \, dt, \)

with \( c \) as given in (6.2.7), and

\( (6.2.16) \quad v = \int_{0}^{\infty} (y - \mu)^2 \, dF(y) \)

with the usual notation. (See equation (2.5.13)).

The structure obviously has consequences for the interpretation of the optimal trajectories, in the maximization of the social welfare goals of growth and equity.

With all the above assumptions, the model cannot be more than a simplistic representation of reality. Yet an effort has been made in the analysis to capture some of the pertinent characteristics of the real world, with a view to providing a better understanding of the relation between investment in education, economic growth and income distribution, than at present. It goes without saying, that "the model is to be used, not to be believed". In using the model of course attention will have to be given to the limitations arising out of complexities of the real world which are not reflected in the model as presented with its simplistic assumptions.

3. Some Issues of Policy in the Distribution of Human Capital

This section discusses briefly the implications of equality of opportunity for equality of income and efficiency, and some issues of policy. The reason for

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advocating equality of opportunity is that it would promote distributive justice, while at the same time promoting economic efficiency. Lack of equality of opportunity, means that certain individuals, groups, regions or sectors lack certain opportunities. Such a state not only is "unfair" but also leads to inefficiency. Under the free-education scheme the state provides free education "to all on equal terms", from the kindergarten to the university. Despite its good effects, the chance of benefiting from the state's contribution has been strongly affected by family income. Due to family circumstances, the obstacles to access to a good education for many would remain, unless the state goes beyond the provisions of the free-education scheme in an effort to correct such disparities.

The emphasis now is on equality of opportunity in education. This like the distributive justice, is advocated on grounds of "fairness", but unlike distributive justice is also advocated on ground of efficiency. Equality of opportunity is interpreted in this study as the principle which demands the exclusion of such irrelevant factors as family circumstances in determining the "outcome" of a contest. Differential education may lead to differential "rewards" in terms of income. However, these rewards ought to be independent of certain factors, e.g., parental expenditure on education, and depend only on other factors, e.g., ability. Independence, in the context of the present study, means invariance of educational expenditure to parental wealth and income, while the childrens' preference functions and abilities are given. This implies that any two children with identical innate abilities and preferences would receive the same education, irrespective of their parents' resources or expenditure preferences.

Inequality of opportunity in education leads to inefficiency. If the family is the only source of educational expenditure, investment in education in different children may not be undertaken to the point at which the rates of return are equalized at the margin as well as the rate of return on other assets. Furthermore,
in the way the capital markets operate, investment in human capital tends to be inefficient by comparison with investment in material capital. Again, in a societal standpoint inefficiency results if the family is the sole source of educational finance. This is possible, due to the existence of externalities, defined as discrepancies between private and social costs and benefits. Private expenditure on education may generate external economies which are not taken into account in the private calculus of costs and benefits. As has been seen, the increase in public expenditure on education, particularly expenditure on elementary and secondary education has been an important factor in reducing inequalities in the distribution of income. In addition the study suggested the fact that increased expenditure on education increases the long-term growth prospects of the country. On the grounds of equity, efficiency and the existence of externalities, there is therefore, justification for the government to continue financing of pre-university education.

The study regards equality of opportunity as the proper aim of the policy for university finance. It is evident that many families, who have members enrolled in universities, now have the sufficient resources to finance the full cost of education. It is equally evident that many families who are not rich, in contrast, lack sufficient income and wealth to finance from their own resources the cost of university education. The ability of these families to borrow is also not promising in view of the imperfect character of the capital market serving the formation of human capital. Thus, it can be argued that preferential treatment of qualified students who are in need has a priority over the equal treatment for all students. Accordingly, the study advocates that the subsidies enjoyed by the children from well-to-do families be withdrawn. However, the government and the private sector, as beneficiaries of investment in human capital, must finance the capital costs of education. A special fund designated the "educational development fund" should be set up for the purpose. Resources earmarked for the educational
development fund may be used to defray the capital costs of universities and of establishing good schools in all parts of the country. The participation of the private sector is through a special educational development tax. The quota of the total resources channeled to this development fund by the government and private sector is to be determined according to the size of educated workforce employed in each sector. The study also advocates the following policies:

1. An education voucher scheme to support children from poor families at pre-university levels. As the study advocates, the continuation of free-education scheme up to Grade 12 (G.C.E. (Advanced level)), these vouchers should cover earmarked maintenance grants graded inversely with family income or according to need.

2. An education voucher scheme to support children from poor families attending universities. As the study advocates levying of tuition fees to defray current expenditure of universities, the education vouchers in this case must cover the full cost of education including tuition fees and maintenance grants. The value of vouchers should be inversely related to family income.

3. An "education bank" to provide equity-type loans for university students. Such a bank would go a long way in correcting the imbalances arising from imperfect capital markets, in the creation of human capital.

A voucher system would increase the efficiency of resource use by facilitating competition among the various institutions of higher learning and also decentralization in the administration of financing higher education.

The study reflects the belief that the policies envisaged would increase equality of environment, circumstance, and opportunity. In conclusion it is quite appropriate to quote Tawney again:

It may well be the case that capricious inequalities are in some measure inevitable, in the sense that, like crime and disease, they are a malady which the most rigorous precautions cannot wholly overcome. But, when crime is known as crime, and disease as disease, the ravages of both are
circumscribed by the mere fact that they are recognized for what they are, and described by their proper names, not by flattering euphemisms. And a society which is convinced that inequality is an evil need not be alarmed because the evil is one which cannot wholly be subdued. In recognizing the poison it will have armed itself with an antidote. It will have deprived inequality of its sting by stripping it of its esteem.\(^4\)

CHAPTER VII

CONCLUSIONS

Thus far the study has treated a variety of topics related to inequality in the distribution of income and education. In this concluding chapter an attempt is made to pull the threads together and assess what has been said.

The study started off with an assessment of the present state of knowledge in the economics of income distribution and human capital theory. A model which purports to describe the observed income distribution in Sri Lanka was presented. An empirical analysis was attempted using the model, to ascertain the extent and sources of inequality in the distribution of income in Sri Lanka. The study reflected the belief that the inequality in the distribution of human capital is an important cause of the inequality in income, earnings in particular. An empirical verification of this idea was attempted using the schooling model relating income power to years of schooling, and rate of return on schooling.

Synopsis of Findings

Subject to the qualifications discussed throughout this study, the review of evidence supports the following propositions:

1. A four-parameter model belonging to a generalized version of the Logistic-Burr's System (Dagum's model) was found to fit the observed income distribution data better than the competing models. The model has certain desirable properties, and in this respect it out-performed the other models.
The model was employed in the empirical analysis of personal income distribution. The findings enumerated below are based on parameter estimates of this model. Inequality of income is measured in terms of the Gini ratio.

2. It appears from the empirical analysis that inequality in the distribution of income in Sri Lanka is much higher than the inequality in one socialist country (GDR), (although, they are not strictly comparable, as personal income in socialist countries does not include profits), and moderately higher than that in the capitalist developed countries. Inequality of income in Sri Lanka for the comparable years is less than that in other Asian developing countries.

3. The distribution of income of spending units in Sri Lanka showed a marked improvement during the 1963-1973 period, but deteriorated in the following inter-survey period, 1973-1978/79.

4. Income distribution among spending units was found to be less unequal than that among individual income receivers. Nonetheless, both showed the same trend in inequality over time:

5. The degree of income concentration in the urban sector substantially exceeded that in the rural and plantation sectors. Inequality was lowest in the plantation sector. In terms of mean income differentials and economic distance ratios, the urban sector emerged as the most affluent sector, followed by the rural sector. In 1978/79, the degree of affluence of the urban sector as compared with that of the plantation sector was approximately twice as large as the degree of affluence of the urban sector as compared with that of the rural sector or the
degree of affluence of the rural sector as compared with that of the plantation sector.

6. A between-year comparison of income inequalities reflecting changes that occurred during the 1973-1978/79 period, reveals that the distributions in both urban and rural sectors had become more unequal. The distribution of income in the plantation sector was relatively stable during the same period. The inequality of income between the urban and rural sectors had diminished substantially, while that between the urban and plantation sectors had increased.

7. Zone 5, the region that includes the metropolis of Colombo, recorded the highest degree of concentration of income, followed by Zone 2. Zone 2 also recorded the highest increase in the degree of income concentration. Zone 1, the region that reported the least increase in inequality over the 1973-1978/79 period, was also the region with the lowest inequality. Zone 5 was the most affluent region, while Zone 4 was the least affluent.

8. The increase of global inequalities is mostly a result of the increase of intra-regional inequalities. A substantial increase of the economic distance ratio of Zone 5 vis-à-vis the other regions (inter-regional disparity) also contributed to the increase of overall inequality.

9. Income is more unequally distributed among females than among males. There are substantial income differentials between males and females.

10. Male-female income differentials declined during the 1973-1978/79 period. During the same period, the inequality of income amongst females
declined, while the inequality amongst males increased. As males comprise 70 percent of all income receivers, the increase in inequality within this group has a substantial impact on global inequality.

11. Inequality of income is highest in the over 55 age group, while it is lowest in the 14-18 age group. An increasing pattern of intra-age group inequality with age is discernible. Income tends to be relatively low at the beginning of labour-force participation, rises throughout later ages, until a peak is reached in the 46-55 age group, and declines in the last age group. The results indicate the existence of pronounced age-income profiles.

12. Intra-age-group inequalities, particularly in all age groups over 25 years, have increased substantially over the 1973-1978/79 period. Changes in inter-income distribution inequalities among different age groups do not provide conclusive evidence as to the effect of these changes on the overall inequality.

13. The data reveal the familiar tendency for income to increase with education. Notwithstanding, there are considerable inequalities within educational levels. Inequality of income among university graduates is highest, while those who have completed grade 10 record the lowest income inequality. Significant income differentials do exist between educational levels as revealed by the magnitude of economic distance ratios.

14. Income inequalities within different educational levels increased substantially over the 1973-1978/79 period. Inequalities between educational levels have not increased much.
15. A model which relates income inequality to years of schooling completed and the rate of return to investment in schooling (the Schooling Model) was supported by the data, both in terms of a new test criterion introduced in the study and in terms of the coefficient of determination.

The findings enumerated below are based on the schooling model. The inequality of income is measured in terms of the variance of income power (the logarithm of income). The inequality of schooling is measured in terms of the coefficient of variation.

16. The mean years of schooling completed has nearly doubled over the twenty-five year period, 1953-1978/79. However, the economic advantages associated with the completion of additional years of schooling have diminished, as shown by the decline in rates of return on schooling.

17. Inequality of income declined substantially over the 1963-1973 period. During the five-year period following 1973, income inequality increased. The mean years of schooling had increased and the inequality of schooling had declined throughout the entire span of the twenty-five year period, beginning 1953.

18. The increase in mean years of schooling and the decline in inequality and rate of return on schooling may have been brought about by the wide spread of education consequent on free education.

19. The decline in inequality of income over the 1963-1973 period may have been the result of egalitarian tendencies brought about by the free-education scheme.
20. In spite of the decline in the inequality in schooling and the accompanying decline in the rate of return on schooling, which may have had an equalizing effect on the overall inequality in income, inequality in income has increased during the 1973-1978/79 period. This, supposedly, represents a cyclical phenomenon, brought about by the spurt of fast economic growth experienced during the last two years of that period.

21. Income inequality in the urban and rural sectors are more than double that in the plantation sector. Paradoxically, inequality in schooling is highest in the plantation sector, which records the lowest inequality in income.

22. Barring some exceptions, the results generally support the hypothesis that inequalities in schooling are negatively correlated with the level of schooling.

23. All regions have experienced a secular increase in the stock of human capital per income receiver. The inequalities in schooling have declined. Zone 5, the region with the lowest inequality in schooling, is ranked second according to the degree of inequality in income.

24. In 1973, the mean female income was 50 percent of the mean male income. A narrowing of male-female income differential occurred during the 1973-1978/79 period. The inequalities in schooling within and between the two groups have declined.

25. The inequality in the distribution of schooling is lower within younger cohorts than within older ones. In 1978/79, the inequality of income followed the same pattern, save for a marginal exception.
26. A parametric approach to the analysis of income and schooling confirmed the results obtained before, showing a secular decline in the inequality of schooling. If the first age-cohort who typically benefitted from the free-education scheme entered the workforce with a lag of 12-15 years, then the egalitarian trend shown in the 1963-1973 period is hardly surprising. The study reflects the belief that the recent adverse trends reflect a cyclical phenomenon, which is expected to dampen over a long-run growth path.

Of the instruments available to the planner for achieving distributive justice, investment in human capital was considered the most desirable. A need was emphasized for balanced investment involving the optimum mix of alternative forms of capital in accordance with the priorities set by the societal welfare goals of growth and equity. This is possible through a generalized process of capital accumulation involving a well specified structure of both material and human capital. An optimal control approach to development helps to incorporate a general welfare goal of growth and equity within a generalized process of capital accumulation. A complete specification and analysis of a decision model for economic growth and income redistribution, within the optimal control approach are not the objectives of this study. It is however appropriate to present a brief outline of such a model as an integral part of the subject matter. Given that the time path of human capital investment is determined in an optimal control model, this should be complemented by deliberate government action (policies) to ensure that the distribution of human capital too is optimal, in achieving the goals of both growth and equity.
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ABSTRACT

HUMAN CAPITAL AND DISTRIBUTION OF PERSONAL INCOME: A THEORETICAL ANALYSIS AND AN EXAMINATION OF SOME ASPECTS OF INCOME DISTRIBUTION IN SRI LANKA

A theoretical and empirical treatment of income distribution theory with special emphasis on the human capital approach to personal income distribution theory is the main concern of the thesis.

The thesis covers the positive aspects of the distribution of income and the normative implications for income distribution of investment in human capital. The domain of the study covering positive aspects is designed to ascertain the dimensions of the problem of inequality in income, as it exists in Sri Lanka. For this purpose it is necessary to measure the extent of inequality, in some objective sense, in relation to other countries, and over time. It is equally important to determine the sources and causes of inequality. For a variety of reasons, it is preferable to use a model which attempts to describe observed income distribution in an empirical analysis of the extent and sources of inequality. Such a model is proposed as a mechanism to describe the observed income distribution in Sri Lanka. Its properties and methods of estimation are discussed.

The study reflects the belief that unequal distribution of human capital is a very important cause of inequality in income. An empirical verification of this idea is attempted using a model relating income power (the logarithm of income) to years of schooling and rate of return on schooling (the Schooling Model). A test criterion is proposed which uses the information of the data set to determine whether the data confirm the specification of the schooling model or an alternative functional form. In the schooling model, the distribution of income is made to
depend on the distribution of schooling, but the latter is unexplained. An alternative approach to the analysis of income and schooling that takes into account the precise functional form of the distribution of schooling is introduced with a view to explaining inequalities.

The main issue raised in the domain of the study covering normative aspects of the problem is how to achieve a more egalitarian society, while at the same time attaining a decent standard of living obtainable through an adequate growth of the economy. Of the instruments available to the planner for achieving this end, investment in human capital is considered the most desirable. The study emphasizes the need for balanced investment involving an optimal mix of alternative forms of capital in accordance with the priorities set by the societal welfare goals of growth and equity. The programming of balanced investment is facilitated through a generalized process of capital accumulation involving various forms of capital, e.g., material and human capital. The optimal control approach to development, by its versatility, provides capabilities for incorporating a general welfare function of growth and equity within a generalized capital accumulation framework. The thrust of the argument in this thesis is that even when the time path of human capital investment is determined in an optimal control model, ideally this should be complemented by deliberate government action (policies) to ensure that its distribution too is optimal, in achieving the goals of both growth and equity.