Mathématiques en français, Math in English:

Discourse in an Elementary School French Immersion Classroom

Carrie Engelbrecht Learned

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Abstract

French immersion is a program that some non-native French parents in Canada choose for their children. Some time allocation models for French immersion programs mean that students in French immersion classes will study mathematics in French and in English at different times during their educational careers. This study follows an elementary class of French immersion students from grade three, when the language of mathematics instruction is French, to grade four, when the language of mathematics instruction changes to English. Using Sfard's four categories of discourse: routine, endorsement, visual mediators, and word use, transcripts of audio recordings of teacher and student language in the classroom were analysed. The characteristics of the teacher and student discourse, as well as the similarities and differences between mathematical discourse in the French language and English language mathematics classrooms were described. The data was characterised by two routines: a question-response-endorsement routine, and an exploratory routine. Although both routines were found in both the French language and the English language classrooms, there were differences as well as similarities in the routines, as well as in word use, visual mediators, and endorsement, between the teacher and the student language, as well as between the two language settings. Limitations to this study, as well as the role of talk in the mathematics classroom, are discussed.

Chapter One: Introduction

Statement of research problem and objectives of research

In many elementary schools in Canada today, Anglophone students' first day in grade one, including literacy instruction, mathematics lessons, snack time, physical education, and art, is conducted entirely in French. These students are in French immersion, an elective program that is designed to help students acquire fluency in French, one of Canada's official languages.

As an experienced French immersion teacher, I have taught a wide variety of subjects in French to students in upper elementary grades. As a lifelong lover of languages it was a natural fit for me. Although I was never passionate about learning mathematics as a student, through teaching mathematics I gained a deep appreciation for its logic and interconnectedness and I am always eager to share its beauty with my students. Disappointingly for me, I have seen my mathematics teaching assignment dwindle to almost nothing since 2007, when the Ottawa-Carleton District School Board decreed that beginning in grade four all mathematics instruction would be in English. At a time when the school board was putting pressure on teachers to ensure that their practices were research-based, I questioned the reason for this decision, and I am still inquisitive about the decision to discontinue mathematics instruction in French after grade three.

Specific time allocation models for French immersion programs vary from school board to school board in Canada, but in the Ottawa-Carleton District School Board (OCDSB), where this study took place, the early French immersion program currently begins in senior kindergarten. In French immersion senior kindergarten, 50% of a student's daily instruction is in French. In grade one, the percentage of instruction in French changes to 100%. Instruction in English language arts is introduced for sixty minutes per day in grades two and three, which represents 20% of instructional time. This percentage is increased to 40% in grades four to six, which includes sixty minutes of English language arts instruction per day, as well as sixty minutes of mathematics instruction in English per day (Ottawa-Carleton District School Board, 2010). Further changes in the same vein for the 2016-2017 school year have recently been approved. Beginning in September 2016 instruction in all junior and senior kindergarten classes will be 50% English and 50% French, and beginning in grade one, all mathematics instruction for French immersion students will be in English. English language arts instruction for students in grades one and two has been proposed to begin at a later date, possibly September 2018.

In the spring of 2012, I was involved as a research assistant in a larger study about mathematics learning in a variety of second language settings. I participated in the data collection in a grade three French immersion mathematics classroom, where the language of instruction was French. It was very interesting to be in another teacher's classroom as a researcher, listening to language with a view to analysis rather than instruction and assessment. This led me to wonder what the language of teaching and of learning mathematics would be like one year later, when the students would be in grade four and the language of mathematics instruction would change to English. For this study, I built on the data that was collected in 2012 in order to gain a more complete picture of the language employed during mathematics lessons of a French immersion class, both before and after the change to English as a language of instruction.

As will be seen in Chapter Two's literature review, French immersion is an educational option that many families choose. The OCDSB, however, is making changes to this program that have the potential to affect many students, their families, and teachers. There have been quantitative studies of French immersion students' achievement in mathematics, some of which compare the results of French immersion students with the results of their non-immersion peers, others which investigate what happens when the language of testing and/or instruction differs, and still others which examine the effect of the number of hours of instruction in French. For all these studies, however, and the debate about which decisions result in the best outcomes, there is little information available about what the language used by students and teachers in French immersion mathematics classrooms actually looks like. Data such as hours and language of instruction and testing results are easily shared and often publicly available. Despite the fact that the vast majority of people spent many years in classrooms, once they leave school what goes on inside of classrooms is largely inaccessible to most people. As a teacher-researcher, I was fortunate enough to have permission to peek inside the classroom and to gather data. As a researcher, I was listening differently than I would as a teacher, but as a teacher, a familiar face to the students, I tried to disrupt their daily routine and interactions as little as possible. This point of view is uncommon, yet important, and I strove to record and analyse the data accurately. Talk in classrooms is increasingly being recognised as instrumental for student learning, and this study of the characteristics of and the similarities and the differences in the language in mathematics classrooms gives a window into the learning and the teaching of mathematics in both French and English.

Chapter Two of this study is a literature review. In this review I address the areas of bilingualism and French immersion in a Canadian setting, the importance accorded to mathematics and mathematics education in today's society, diverse linguistic realities in mathematics around the world, and finally mathematics in a French immersion context. Chapter Three outlines the theoretical framework of the study, and presents the research questions, and

Chapter Four explains the methodology used. Findings are presented in Chapter Five, and are followed by a discussion and conclusions in Chapter Six.

Chapter Two: Literature review

Review of relevant literature

In order to set the stage for the present study I have reviewed literature in two areas. The first area examines notions related to the importance of mathematical education, and the issues surrounding the teaching and learning of mathematics in general. The increased complexity when learning mathematics in a second-language setting, and the associated questions of studying learning in these settings, are also explored. The second area looks at French-English bilingualism in Canada, and examines issues related to French immersion education, particularly the study of mathematics.

Mathematics: Both essential and challenging

Mathematics education is recognized as an essential component of children's education. Shapka (2006) examined the notion of mathematics as a "critical filter" to jobs requiring competence in quantitative skills, and found that, in a Canadian context, young adults' achievement in grade nine mathematics courses has important implications for the level of prestige of the job to which they later aspire. Despite, or perhaps because of the proven importance of the study of mathematics, many teachers, students, and parents, cite mathematics education, homework, and results as a significant source of stress. According to Drummond (2004), parents of grade three students report helping their children less often with mathematics homework than with reading homework, citing feelings of inadequacy in the domain of mathematics. A considerable number of pre-service primary teachers have mathematics anxiety (Uusimaki, 2004, Ertekin, 2010), and mathematics anxiety in students has been documented as early as grade three (Gierl, 1995). These studies have examined mathematics in a first-language setting.

Mathematics is widely recognized as vital to students' future success, but mathematics teaching and learning can be a significant source of stress for many parties involved. To add to the complexity of these issues, students all over the world are increasingly studying mathematics in a second language setting.

Diverse multilingual mathematics realities

Mathematics classrooms in the French immersion program, such as the one in this study, are part of a bigger, more diverse, and increasingly common picture of multilingual mathematics education worldwide. The diversity of this community means that the mathematics class in this study shares both similarities and differences with other multilingual mathematics classes around the world. In each of these classrooms students' and teachers' level of proficiency in the classroom language varies, which leads to particular discursive practices which many of these communities have in common. It is essential to be able to describe both the level of proficiency attained in the classroom language as well as the level of proficiency required, as there are important relationships between students' level of proficiency in the classroom language and students' level of achievement in mathematics. Research has indicated a number of strategies which can help students in these unique circumstances to master and demonstrate their understanding of essential concepts in mathematics.

Diversity in the multilingual mathematics classroom

There is a multitude of circumstances which may be described as multilingual mathematics classrooms: immigrant and refugee children from a variety of linguistic, socioeconomic, and educational backgrounds learning mathematics in English from a native English speaker in England while speaking their mother tongues at home, Pakistani children with a variety of home languages learning mathematics in English from a teacher with a similar linguistic background in English-medium schools in Pakistan, and Welsh students learning mathematics in Welsh from Welsh teachers, all of whom speak English, at a bilingual Welsh-English school in the United Kingdom (Barwell, 2009), dual language education programs, usually Spanish-English, in the United States (Gomez, Freeman, & Freeman, (2005), and multilingual primary mathematics classrooms in South Africa, where teachers and students speak English and Setswana during mathematics lessons (Setati, 2005). The French immersion classroom in this study, in which students from Anglophone backgrounds learn mathematics in French from a fluent speaker of the language, is part of this diversity of multilingual mathematics classrooms. Each of these specific linguistic settings shares some points in common. In each classroom, students are expected to learn mathematics in a language which is not their home language. Curriculum objectives in these situations, however, are not expected to be any different than students who are learning mathematics in their home language in the same region. In the case of the grade three French immersion class in this study, students are learning mathematics in French, while the curriculum expectations are identical to those of Anglophone students who are learning mathematics in English. Although these multilingual mathematics classrooms are similar at first glance, they differ from each other in important ways. In the French immersion classroom in this study, the students and the teacher are all fluent in a common language, English. In most cases this is their home language. This would not be the case in the class of immigrant and refugee children, nor in the Pakistani classroom described above. This difference has important implications for options for word choice of students and teachers in the French immersion classroom in this study, as will be seen in Chapter Five. Another important difference concerns language policy. Although I have not found an explicit written policy, I know from personal experience that teachers in French immersion classes are expected to make every effort to ensure that all interactions in the classroom take place in French. Although teachers and students all speak English, teachers are expected to use gesture, reformulation, and physical objects to make their meaning clear in French rather than speaking English in the classroom. This differs significantly from policies, written or unwritten, in other settings. In the bilingual Welsh school, teachers can schedule instructional time in English to serve the needs of those who are less fluent in Welsh (Jones, 2009). In Pakistan, teachers and students regularly move between English and other languages they have in common (Halai, 2009), as do the teachers and students in South Africa. Immigrant and refugee students may use languages they have in common with other students during the lesson. Moschkovich (2010) cautions that researchers must be clear in their descriptions of bilingual or multilingual classrooms, learners, and, I would add, teachers. Care must be taken not to over-generalize findings from one multilingual setting to another, and not to assume that findings, conclusions, and recommendations that are made in particular multilingual mathematics settings are applicable to the classroom, students, and teachers in my study. There is not a great deal of research available on learning mathematics in a French immersion setting, and the research that has been done is often based on acheivement results (i.e. Turnbull, Lapkin, and Hart, 2001), rather than classroom-based research. Due to this fact, when reviewing the available literature, I often read studies in multilingual mathematical learning settings which differed in important ways from the study that I did, or which used quantitative rather than qualitative data to answer very different research questions than the ones that I posed. I therefore need to use the literature that I reviewed in order to learn about issues and research into second language learning and mathematics, but not to assume that the conclusions that have been reached in other settings apply to the present one. This lack of classroom-based research in French immersion mathematics classrooms underlines the potential contributions of the present study. As will be seen later on in the second part of the literature review, French immersion continues to grow in popularity in Canada, and as was seen in the introduction, important policy decisions are being made about that learning, and therefore more needs to be known about mathematics learning in this specific multilingual mathematics setting.

Linguistic proficiency in the mathematics classroom

Given that there is a great deal of diversity in multilingual mathematics settings, it stands to reason that both teachers and students may have differing levels of proficiency in the written and oral comprehension and expression of the classroom language or in other languages. Recognizing and describing these differences is essential as linguistic proficiency is linked to achievement in mathematics, as will be discussed below. These differences play out in different ways in different multilingual mathematics settings. Anecdotally, the students in the present study had noticeably different levels of proficiency in oral French, as well as varying levels of confidence and motivation to volunteer to participate in mathematics lessons in French. Differences in the level of competence in Welsh in the Welsh-English bilingual school were expected, and these differences influenced teachers' decisions about the language of teaching, as will be further discussed below (Jones, 2009). Students and teachers may have a language or languages in common at school or at home, as in the French immersion class in this study where students and teacher were all fluent in English and the students had all followed a similar academic path in the early French immersion program. Students and teachers would also have common languages in the Welsh-English bilingual school, as well as in the South African classroom. Alternatively, a student may be the only one in the classroom with proficiency in a given language, as may be the case of an immigrant or refugee student.

When two or more members of the mathematics classroom have a language in common, code-switching often takes place in the classroom. Given the diversity of multilingual mathematics classrooms, code-switching, the use of more than one language in a single communicative episode (Moschkovitch, 2007) can take many different forms and can take place for many different reasons. Researchers have tackled the question of why participants engage in code-switching. They have found that code-switching may be a deliberate, planned choice, as when bilingual English-Welsh teachers in a bilingual Welsh school plan separate times for instruction in each language in order to accommodate differing levels of proficiency in Welsh (Jones, 2009). On the other hand, code-switching may be more spontaneous, as during group work among Pakistani students during a mathematics lesson in an English-medium school in Pakistan (Halai, 2009). Code-switching may also indicate that different languages are being used for different purposes in the classroom. In a South African context, Setati (2005) found that although both students and teachers used English and Setswana in the mathematics classroom, they were not used in the same contexts. Both English and Setswana were used for regulatory purposes, such as classroom management, although Setswana was used as a language of solidarity and English was presented as the language of authority. Setswana was used for contextual purposes (understanding the problem), as well as conceptual purposes (understanding and justifying mathematical ideas). English was used in procedural contexts, such as algorithms to find the correct answer. As English procedural discourse was the only one used for assessment, English was seen as the language of higher status in the classroom. Similar negotiations between power, policy, and language use were seen in the different ways in which students and teachers used English and Urdu in the mathematics classroom in Pakistan. Although, according to policy, English was the official language of instruction in the mathematics classroom, Urdu was commonly used by teachers and students. Urdu was often used during group work, but English was the expected language when presenting results, and permission was asked if students felt the need to use Urdu at that time. The question of who code-switches has also been addressed. It may involve only students, as when Spanish-English bilingual students enrolled in an English-language mathematics course move between languages while solving a problem (Moschkovich, 2007) or the teacher may be involved as well, as in the examples in Wales and in Pakistan. In the mathematics classrooms in this study, code-switching was evident, yet its incidence varied greatly depending on the language setting and the type of discourse, as will be discussed in the findings. In whatever context it occurs, care must be taken not to view it as part of a deficit model of bilingualism, using incidences of code-switching to make negative judgements or predictions about a learner's level of language proficiency and therefore about his or her level of mathematic achievement (Moschkovich, 2007).

It is important to address issues of proficiency in language, as proficiency of the language of teaching and learning mathematics has meaningful relationships with achievement in mathematics. Abedi and Lord (2001) found that English Language Learners (ELL's) scored lower on mathematics word problems than students who were classified as proficient in English. As linguistic proficiency, especially that which is required for academic success, is difficult to quantify or describe, some researchers have made efforts to develop models to describe the full range of linguistic proficiency.

Describing mathematical language

Cummins (1982) is one researcher who has developed a model to describe linguistic proficiency in the classroom. He describes linguistic competence using two parameters: context (embedded or reduced), and cognitive demand (demanding or undemanding). These two parameters are used to create four quadrants which can be used to describe linguistic tasks in second language settings.

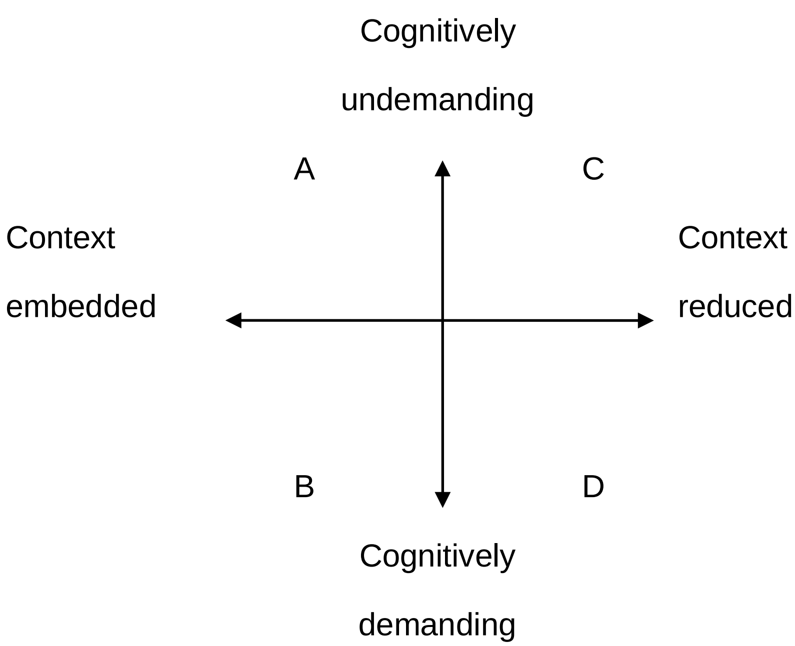

Figure 1. Cummins' Four Quadrant model

Adapted from Language, Power and Pedagogy: Bilingual Children in the Crossfire (p. 68), by J. Cummins, 2000, Tonawanda, NY; Multilingual Matters Ltd. Copyright 2000 by Jim Cummins

In quadrants A and C there are tasks which fall into the category of basic interpersonal communicative skills (BICS). These are cognitively undemanding tasks which have more or less context, such as having a concrete conversation about a familiar topic with someone, either in person (more context – quadrant A) or on the telephone (less context – quadrant C). Quadrants B and D contain tasks which are deemed to require cognitive academic language proficiency (CALP) in order to successfully complete. These tasks are cognitively more demanding, and may have more context, such as basic math computations (quadrant B), or less context, such as a standardised math test (quadrant D). In a classroom setting of a traditional school, such as the one in this study, curriculum demands dictate that teachers do their best to prepare students for quadrant D activities, which are highly cognitively demanding and often low in context. In addition to describing linguistic tasks and the language needed to accomplish them, Cummins' four quadrants have also been used to plan mathematics instruction for second language learners, as is described later in this section.

Cognitive academic language proficiency is essential to traditional academic success, as many academic tasks, such as standardised tests and textbook work, are found in Cummins' quadrants B and D. Describing this cognitive academic language proficiency specifically in the mathematics classroom is of interest to researchers at the intersection of language and mathematics. Pimm (1987) sees mathematics itself as a register, with a unique set of word meanings, symbols, and rules for social use of language in this specific setting. Sfard (2001b) makes a case for the supported development of a "literary" (formal) mathematics discourse from a colloquial (informal) one. This distinction can be seen as analogous to Cummins' division between BICS and CALP. Barwell (2009) highlights the tension between formal and informal mathematics language, and ties this tension to Adler's (1997, 1998, 1999) dilemma of mediation. It is acknowledged (i.e. Sfard, 2001b, and Adler, 1997) that part of a mathematics teacher's task is to assist students in practising and mastering the mathematics register, yet how best to do so is difficult to describe. Adler (1997) describes this difficulty as a series of dilemmas. The dilemma of mediation, which applies specifically to this context, illustrates the conflict between the teacher's needs to both validate students' points of view and to encourage independent participatory problem solving, while simultaneously stepping in to model and to encourage students to practice a more standard mathematical register. This task is made more complex when it takes place in multilingual mathematics classrooms, as students and teachers may or may not know the formal language of mathematics in the students' home language or languages. This was indeed the case in the mathematics classrooms in this study. In the chapter on Findings there are examples of efforts on the part of both teachers to assist students in mastering the mathematics register while the teachers navigate the dilemma of mediation.

Achievement in the multilingual mathematics classroom

Efforts made to describe students' linguistic proficiency naturally lead to questions about how linguistic proficiency affects mathematical competencies. As stated above, there is a documented connection between linguistic proficiency in the classroom language and some aspects of achievement in mathematics (Abedi & Lord, 2001). Given the diversity of multilingual mathematics classrooms, however, relationships between language and assessment, evaluation and achievement in mathematics are difficult to quantify. Research reports mixed results on questions of student achievement in second-language mathematics contexts. Clarkson (1992, 2007) and Clarkson and Gailbraith (1992) found that multilingual upper elementary students who had a lower score on proficiency tests in two of their languages performed less well in some aspects of mathematical testing. Discouragingly for our present context of elementary students learning mathematics in a second language that is not often spoken at home, Howie (2003) finds that learners' proficiency in the classroom language and frequency that the language of learning and teaching (LoLT) is spoken at home is a strong predictor of success in mathematics in South Africa. Brown and Rushowy (2001, in Setati, 2005) report that it was second language learners in applied level mathematics courses in Ontario who struggled the most to demonstrate an understanding of the material.

However Setati (2005) also affirms that multilingualism per se does not impede mathematics learning. Research by Clarkson (1992) has shown that bilingual students who score more highly on language tests in both of their languages also score more highly on mathematics assessments. Kempert, Saalbach, and Hardy (2011) reported similar findings. In their study of mathematical word problem solving by Turkish immigrant students in German schools, they found that students' command of the language of instruction (German) was highly predictive of their performance on the mathematical testing in that language. Although students with weaker German language skills were at a disadvantage when solving regular German mathematical word problems, these same students, however, showed an advantage when solving German mathematical word problems designed to test executive control, a set of skills such as organization, memory, and sustained attention, that is essential to the completion of cognitive tasks . When tested on questions that contained irrelevant numerical information, the students with weaker German language skills performed as well as their monolingual peers. This result is taken as evidence that bilingualism, even when the two languages do not approach native-like proficiency, can provide advantages in areas of executive control.

Despite evidence that multilingualism in the mathematics classroom is not necessarily a disadvantage, and can even be beneficial for mathematics learning, it has nevertheless been demonstrated that instructional and assessment accommodations can be helpful in improving multilingual students' achievement in mathematics. Moschkovich (1999, 2000, 2002) and Lager (2006) draw attention to the need for specific strategies when teaching mathematics to English Language Learners, claiming that generic ESL strategies are not sufficient, especially when teachers do not have sufficient mathematical content knowledge. Some researchers offer specific strategies to help students who are learning mathematics in another language to fully comprehend and demonstrate understanding of complex notions in mathematics. Abedi Lord (2001) found that changes in syntax and non-math vocabulary on standardised mathematics test questions in the United States led to moderate improvements in performance on these questions, especially for English language learners (ELL's) with previous low performance in mathematics, low socioeconomic status, and English Language Learners. Yeong and Chang (2014) focused instruction in Cummins' high cognitive demand/high context quadrant B through use of a second language framework (sheltered instruction observation protocol – SIOP) to develop and analyze a repertoire of strategies to teach cognitively challenging mathematical concepts to Korean language learners. They developed lesson plans using a variety of strategies such as visual or physical activities, scaffolding, and graphic organizers to increase context while maintaining cognitive demand by using questions requiring higher order thinking skills. The Ontario Elementary Mathematics Curriculum document (Ontario Ministry of Education, 2005) recommends similar strategies such as use of visual cues, manipulatives, and simplified text for English as a Second Language learners in order to maximize success in mathematics classrooms.

Multilingual mathematics classrooms are increasingly common around the world (Barwell, 2009, Gomez, Freeman, & Freeman, 2005, Setati, 2005). Although these settings have many common elements, it is also important to recognise and describe the ways in which they differ. Describing students' level of proficiency in the language of mathematics, as well as describing the complex mathematical language needed for success is also essential, as there are significant relationships between students' proficiency in the classroom language and their achievement in mathematics (Abedi & Lord, 2001). Researchers (Yeong & Chang, 2014) have found effective strategies which can be used in order to promote mastery of mathematical concepts and success in the mathematics classroom.

In this first part of the literature review, I have examined questions about the importance of mathematics and issues in multilingual mathematics classrooms on a global scale. In the second part of the review, I will be looking at similar issues in a specifically Canadian context. French-English bilingualism, especially in a French immersion context, raises important areas of inquiry, as does the growing field of mathematics teaching and learning in a French immersion context.

Bilingualism and French immersion in Canada

Bilingualism, especially in a French immersion context, is a unique and complex reality for a growing number of students, both in Canada (Statistics Canada 2010; Ottawa-Carleton District School Board 2011), and abroad (Jäppinen, 2005; de Courcy and Burston, 2000). Mathematics enjoys the privilege and the burden of being viewed as an essential yet challenging academic subject. There is a small but growing area of inquiry, to which my present study will contribute, which has examined various facets of mathematics learning and teaching in a French immersion context.

It has been over 40 years since the passing of Canada's Official Languages Act in 1969, which gave English and French equal status as official languages in Canada. The Canadian Council on Learning report on French immersion in Canada (2007) states that although rates of bilingualism have steadily increased since the 1970's, still today fewer than 20% of Canadians are bilingual. Nearly half of Canadian Francophones speak English. Fewer than 10% of Canadian Anglophones, however, speak French.

This is an unfortunate situation for Canada's Anglophones, as bilingualism has a number of important benefits. For example, bilingualism has been found to convey positive cognitive benefits for both children and adults. Bilingual children have been found to have advantages in cognitive processing, specifically on tasks which require attention to relevant information in the presence of misleading distractions, which has useful repercussions, even in areas other than language-based tasks (Bialystok, 2001). Although much of Bialystok's work in this area has focused on studying executive functioning in balanced bilinguals, i.e. those who are equally proficient in both of their languages, Kempert, Saalbach, and Hardy (2011) have found similar advantages in executive functioning in dominant bilinguals, who have stronger language skills in one language than the other. Bilingual children have also demonstrated advantages in the area of working memory, especially in the presence of other executive function demands (Morales, Calvo, and Bialystok, 2013). Both fully bilingual and partially bilingual children show greater metalinguistic awareness than unilingual children (Bialystok, 1991). The benefits of bilingualism have been found to extend to adulthood. Studies have indicated that bilingualism may delay the onset of symptoms of Alzheimer's disease. In a study by Schweizer (2012) which used brain scans to examine the brains of people with and without Alzheimer's disease, it was found that bilingual people could sustain up to twice as much damage to their brains as unilingual people before they started showing symptoms of dementia.

Cognitive benefits are not the only advantages for Canadians who speak two languages. There are important economic benefits to bilingualism as well. In the Official Languages and the Economy Report of 1997, Ghislain Savoie quotes Vaillancourt (1988) as finding that Canadians who speak both French and English have a higher rate of participation in the labour force. Christofides and Swidinsky (2010) found that bilingual men outside of Quebec have an average income 3.8% higher than English monolinguals, while bilingual women enjoyed salaries that were 6.6% higher than their monolingual English counterparts. French-English bilingualism also forms an important part of our national identity. As CBC News reported in 2007, a 2006 poll by the Centre de recherche sur l'opinion publique (CFORP) states that 81% of Canadians support the idea that Canada is a bilingual country.

Perhaps seeking these cognitive and economic benefits of bilingualism, some Anglophone parents choose to enrol their children in French immersion programs. In Canada as a whole, French immersion enrolment has been rising steadily, despite national declines in overall school enrolments (Statistics Canada, 2010). Canada-wide, enrolment in French immersion programs was up almost 10% from the 2000-2001 academic year to the 2006-2007 academic year (Statistics Canada, 2010). This increase is perhaps due in part to the "second wave" of immersion students, children of parents who were among the first to attend French immersion programs in Canada, and now hope that their children will experience the advantages of bilingualism as well (Friesen, 2013). According to the Ottawa-Carleton District School Board, 50% of all eligible elementary students in the Ottawa-Carleton District School Board (OCDSB) are currently enrolled in French immersion, and this percentage is also on the rise (Ottawa-Carleton District School Board, 2011). This enrollment, however, is not evenly distributed across all eligible students. Allen (2004) noted several differences between students who are enrolled in French immersion programs and those who are not. For example, there is a higher percentage of girls enrolled in French immersion programs, and students in these programs tend to come from higher socio-economic backgrounds and to have parents who are more likely to have a post-secondary education. Conclusions about achievement in French immersion programs must be interpreted with an eye to these differences in enrollment patterns.

Curriculum expectations in all subjects except French language arts, however, are identical for students in French immersion and students in the English stream. Students and teachers in French immersion programs therefore have the dual task of ensuring that curriculum content in other subjects is mastered in addition to acquiring a high level of linguistic competence in the French language.

There are numerous studies which examine the tension between teaching the linguistic conventions of the French language and teaching curriculum content in a French immersion setting (e.g. Lapkin & Swain, 2004; Laplante, 2000; Lyster, 1998; Swain & Lapkin, 1998). For example, Allen (2004) noted that reading scores in English are significantly higher for French immersion students than non-immersion students. She stated that the higher socio-economic background of French immersion students alone could not account for this difference. In addition to the fact that there is a gender imbalance in French immersion programs, and that they are more readily accessible in urban, affluent neighbourhoods, Allen noted that more self-selection or a positive peer effect may also be responsible for the increased scores. Lazaruk (2007) neatly summarized the benefits of French immersion in a Canadian context. He reviewed the available literature and concluded that French immersion students demonstrate significant linguistic, academic, and cognitive benefits, compared to their peers who are not in immersion programs. These benefits exist even despite that fact that immersion students' receptive skills of listening and reading may approach native-like fluency, productive skills, especially speaking, almost invariably lags behind that of the students' Francophone peers. Despite this lag in productive language skills, Lazaruk confirms that enrolment in a French immersion program does not impede progress in L1, nor is learning of content in other subjects (including mathematics) adversely affected. It is important, however, to keep in mind the differences, especially in gender and socio-economic status, between students that enrol in French immersion programs and those that do not. In this study, Lazaruk's comparisons of French immersion students with their "non-immersion peers" do not take into account differences in enrolment profiles, which, anecdotally, may be considerable, even among students in immersion and non-immersion programs at the same school.

It is evident that French-English bilingualism is an increasingly important and beneficial part of life in Canada. While the goal and outcomes of French immersion programs are not a native-like fluency in French, a substantial body of research exists which indicates that for Canadian children whose first language is not French, the French immersion program is a viable route to acquiring facility in French as well as the attendant benefits of bilingualism, while simultaneously mastering curriculum expectations in other subjects.

Mathematics in a French immersion context

Although evidence of the benefits of bilingualism and of the attractiveness and efficacy of the French immersion program may be convincing, many stakeholders are still wary of the idea of teaching mathematics in a second language. French immersion is a unique context of instruction and learning of and in a second language. Although the emerging body of research in the area of mathematics instruction in immersion or immersion-like settings is not extensive there are some recent studies that have examined this area. Studies like the present one, which examined student and teacher language in the mathematics classroom both before and after the change in language of instruction, will contribute to the understanding of learning mathematics in two languages in a French immersion context.

In a context which is specific to Ontario, Turnbull, Lapkin, and Hart (2003 & 2001) have found that French immersion students perform as well or better than non-immersion students on mathematics EQAO testing in grades three and six. Although the data used in the study is cross-sectional, rather than longitudinal, the authors do wonder about reasons for differing achievement levels other than second language instruction programs. In addition to the possibility of cognitive and academic benefits derived from the immersion program, the authors also suggest that immersion students may have greater academic potential than non-immersion students, or that attrition of weaker students out of immersion programs may affect testing results. Jäppinen (2005) studied the mathematics learning of Finnish students who were enrolled in a "content and language integrated learning" (CLIL) program which was modeled on Canadian French immersion programs. Students for whom Finnish was the first language enrolled in this program received mathematics instruction in English, French, or Swedish. Their mastery of mathematics concepts was tested in Finnish at three different stages (ages 7-9, 10-12, and 13-15) and their results on the mathematics assessment were compared with students who had received mathematics instruction in Finnish. There was no significant difference in the results for the youngest and oldest cohorts, but the students in the 10-12 immersion age group scored significantly better on the mathematics assessment than those students who had learned mathematics in their first language. Jäppinen saw this as evidence that CLIL and other immersion programs are indeed effective environments for the learning of mathematics. She cautions that despite the lack of significant differences in results, such an environment may be more challenging at the beginning, and that very young learners benefit from more concrete topics. Overall, the findings support the belief that the "demanding and language-enriched environment" of such programs has a "positive effect on cognitional development" (p. 163). In addition to this positive effect noted by Jäppinen, the positive results of the testing in the first language shows that the Finnish students enrolled in the study were clearly able to transfer their mathematical knowledge from their second language into their first language.

De Courcy and Burston (2000) examined mathematics learning in Australia in an early partial French immersion program (55% English and 45% French, the latter including instruction in mathematics). Although this study also demonstrates evidence of the transfer of knowledge from first language to second language, there are significant differences in the design of the two studies. In contrast to Jäppinen's study, de Courcy and Burston did not compare results on mathematics tests of immersion students with those of non-immersion students, but rather tested some immersion students in French and compared these results with immersion students who had been tested in English. De Courcy and Burston found that although the difference was not significant, the performance of immersion students tested in English was slightly better than those tested in French. They hypothesize that at the time of testing (years three and four) the students had perhaps not had enough exposure to French, or more specifically, academic French. They recommend that students being tested in the L2 be allowed more time to complete tests, and that immersion programs place more evidence on reading in order to improve students' performance on more text-rich questions. De Courcy and Burston felt strongly that students were capable of transferring knowledge from one language to another, and that there was no need to re-teach topics when transitioning between languages.

Bournot-Trites and Reeder (2001) conducted a study of mathematics in a Canadian French immersion context where once again there is the possibility of examining the transfer of mathematical knowledge from a second to the first language. Two groups of French immersion students were given an English language mathematics test. One group had a time allocation model very similar to that of the OCDSB, switching to mathematics instruction in English in grade four. The second group, the "second-language intensification" group, kept the percentage of French language instruction (including mathematics) at 80% through to the end of grade seven. The group which had a higher percentage of instruction in French achieved comparable or even better results on the mathematics test in English. Bournot-Trites and Reeder also counted increased motivation, being more accustomed to more challenging work, or a different pedagogical approach in the French textbook as possible reasons for the difference. Once again, there is evidence that students are able to effectively transfer mathematical knowledge from one language to another.

Culligan's work (2010) focuses specifically on the transition from mathematics instruction in a French immersion setting to English language mathematics instruction and perhaps corresponds most closely with my project. Although the transition from French mathematics instruction was optional and at a much later date than the present study (grade eleven or twelve, as opposed to grade four) the study is similar in that students had received mathematics instruction and evaluation in French, and then transitioned to English instruction and evaluation. In speaking with students who had transitioned to English language instruction and evaluation, Culligan found that students found subtle yet discernible differences in their experiences in the mathematics classroom. Although there was little to no change in their marks, students reported an increased sense of ease in mathematics, with increased focus, comfort, and ease of expression.

These studies show that there is evidence that bilingualism has many benefits, both cognitive and economic. Many parents in a non-native French Canadian context choose French immersion programs for their children. Proficiency in the classroom language, however, has been shown to be a predictor of success in mathematics (Howie, 2003). A great deal of importance is placed on the acquisition of numeracy skills (Shapka, 2006), and many people feel pressure and stress when faced with teaching or learning mathematics (Uusimaki, 2004, Ertekin, 2010, Gierl, 1995). There is, however, evidence that mathematics can be effectively learned in a second language (i.e. Setati, 2005).

The studies on mathematics learning in a French immersion setting are consistent in some of their findings. In none of the studies mentioned above was there evidence to indicate that there was any negative impact on students' learning of mathematics due to the fact that they were enrolled in French immersion programs. These studies also often involved students transferring their mathematical knowledge into their L1 for the purposes of mathematics testing. Jäppinen found that one of the three age groups in her study performed better when taught mathematics in L2 but were tested in L1. Similarly, De Courcy and Burston found that students' results in a comparable situation (teaching in L2, testing in L1) were slightly better than their counterparts who were both taught and tested in L2. In Bounot-Trites and Reeder's work, students who had more hours of instruction in L2 performed better on a mathematics test in L1 than students who had less hours of instruction in L2. Culligan's students who switched from L2 instruction and testing to L1 instruction and testing did not experience any quantitative change in their mathematics achievement, but reported some positive qualitative changes. Turnbull, Lapkin, and Hart, reporting on EQAO data for French immersion learners, stated that there is "virtually no support for the argument that testing in English disadvantages immersion students" (2001, p. 22), and that in both grades three and six, French immersion students' results on EQAO mathematics tests are "almost identical" (2002, p. 23) to their peers in the English program. Contrary to any fears of students not mastering mathematical concepts because they are in French immersion, these studies give evidence of students' abilities to both master mathematical concepts in French and also to transfer that knowledge back to L1 for testing purposes.

These studies, however, focus on the quantitative results of testing after the teaching and learning activities in the classroom. Although these methods yield important findings about the final outcomes of the language choices during the teaching and testing of mathematics, they give little insight into the teaching itself. The above studies can allay fears of students not mastering essential concepts in mathematics due to instruction in an immersion setting, but do not account for the perhaps counter-intuitive notion that achievement may even be improved through increased instruction in a second language.

The present study fills a gap in the literature, in that it is a qualitative study of student and teacher language in a French immersion mathematics classroom setting both before and after the transition from French to English language instruction. This perspective is absent in the studies that were already available. Jäppinen (2005), de Courcy Burston (2000), and Bournot-Trites and Reeder (2001) all studied students' mathematical achievement using quantitative methods, rather than an examination of students' mathematical language using qualitative methods as I did. Jäppinen and Bournot-Trites and Reeder tested students' abilities to transfer their mathematical knowledge from L2 to L1, while my project focused on the transition period from one language of instruction to another. Culligan (2010) studied the period after transition, rather than both before and after transition, through a series of subjective questions she posed to students in an interview setting, while I used observations of classroom language of teachers as well as students. Although useful information can be gleaned from the above studies, important questions about classroom language in this important period of transition still needed to be addressed.

In this chapter I have highlighted several important areas of research. It is evident from the literature that rates of bilingualism in Canada are increasing, as are rates of enrollment in French immersion programs, and that there are many benefits to bilingualism, both economic and cognitive. Worldwide there are many examples of students learning mathematics in a second-language setting. In some cases, lower achievement in mathematics may be linked to a lack of mastery of the language of teaching and learning of mathematics, but in the case of Canadian French immersion students, mathematics instruction in French has not been shown in any way to hinder mathematics achievement, and may even enhance it. I identified a gap in the literature which I propose to fill, namely a qualitative study of classroom language of teachers and students both before and after the transition from French-language to English-language mathematics instruction.

In Chapter Three, I introduce the theoretical framework I have chosen to examine this question. Quite apart from quantitative studies of achievement in mathematics, there is a wide-ranging area of study in the area of language in mathematics classrooms. Anna Sfard (2005, 2007) has designed a framework specifically to examine mathematical classroom language, and it is through the lens of the four interrelated categories of her framework: word use, visual mediators, routines, and endorsed narratives, that I propose to examine the classroom language of elementary French immersion students and teachers.

Chapter Three: Theoretical framework and research questions

The Study of Mathematical Discourse in the Classroom

In the studies reviewed in Chapter Two there was quantitative evidence that mathematical curricular objectives can be mastered in a French immersion setting. This quantitative data, however, does not describe the language used by teachers and students, both during instruction in French and after a shift to English-language mathematics instruction. For this reason, questions about classroom language, which are the focus of this study, require a different theoretical framework than the studies outlined in the literature review in Chapter Two. In order to answer these questions, a framework and methodology which is focused on classroom discourse rather than quantitative results on tests of mathematics achievement will be necessary. In this study, discourse will be defined in Sfard's words: "The different types of communication that bring some people together while excluding some others" (Sfard, 2007, p. 573). It can be "...any specific instance of communicating, whether diachronic or synchronic, whether with others or with oneself, whether predominantly verbal or with the help of any other symbolic system." (Sfard & Kieran, 2001b, p. 47). Mathematical discourse, in particular, is one that "features mathematical words, such as those related to quantities and shapes" (Sfard, 2005, p. 245). These two lines of inquiry, quantitative results and qualitative discourse, are not as strongly divided as it might seem. Kempert, Saalbach, and Hardy (2011) remark that the learning of mathematics is even more strongly related to language processes than was previously assumed. They cite Bialystok (2001), saying that "mathematics is a domain where cognitive effects in bilinguals are likely to occur as language and mathematics share common critical features such as abstract mental representation, conventional notations, and interpretive function" (p 548). Far from being a new topic of investigation, language in mathematics classrooms has long been an area of research.

As Morgan, Craig, Shuette, and Wagner (2014) note, as far back as 1979, Austin and Howson published a review of the previous two decades' research into language in mathematics classrooms. They note the increased interest into this area of investigation after the publication of Pimm's "Speaking Mathematically" in 1987. In the early 1990's, pedagogy and curriculum increasingly focused on student-centred, sociocultural, and constructivist theories, and research in this area shifted towards naturalistic language, with a greater emphasis on student discourse and student-teacher interaction. Methods of analysis changed as well, as new frameworks were needed to analyze the new kinds of data.

Although research into language in mathematics classrooms has evolved to place a greater emphasis on student-centred approaches, the trajectory of the implementation of such mathematical discourse in the classroom has not been so smooth. A traditional mathematics classroom brings to mind images of "chalk and talk", a direct, transmissive, teacher-led style of instruction. According to Elbers and Streefland (2000), however, these traditional approaches are not suitable for the curricular reforms in mathematics widely proposed in the 1990s and early 2000s. Brown and Hirst (2007) list a number of curriculum initiatives in the United States, Canada, and Australia which challenge teachers and students to move toward a more sociocultural practice of mathematics, in which mathematical talk among students, as well as mediated talk between students and teachers, serves to help students to construct mathematical knowledge and to master practices of the mathematical community. In these mathematics classrooms student discourse plays a more prominent role than in a traditional mathematics classroom. These initiatives have often proved to be divisive, as teachers are asked to teach in ways that differ dramatically from both their current and former practices, as well as from their own experiences as students in the classroom (Mitchell and Knuth, 2003). Viewing the traditional and reform approaches to mathematical instruction through a binary lens leads to tensions and divisions which need to be carefully managed in order to lead to more effective implementation of reforms in the classroom, hopefully followed by an anticipated increase in student success. De Haan, Elbers, and Leseman (2014) address the division between more traditional, teacher-centred transmissive classrooms and more constructivist approaches. They cite Winsler and Carleton (2003) as they wonder, “where along the continuum of ‘child-centered’ to ‘teacher-directed’ it is best to define the role of the teacher for optimizing children’s [development]”(p. 44). Adam and Chigeza (2015) address the debate by proposing a more balanced approach, which acknowledges the contributions from both sides of the spectrum. They suggest that a binary approach, in which “recognition of opposition and engagement with division” (p. 108) in the area of the nature of mathematical knowledge, can lead to “cooperation and deeper understanding” (p. 108), and as a result, more effective pedagogy in the classroom. This tension between traditional teacher-centred pedagogy and newer, more student-centred methods is worthwhile keeping in mind when analysing and interpreting the data in the current study. There is little indication of “where along the continuum of child-centred to teacher-directed” the teachers in this study would situate themselves. Although the data in this study consists solely of the discourse of both students and teachers, there are few clues to indicate the teachers’ beliefs about the value and effectiveness, nor their level of comfort with social constructivism in the mathematics classroom. Teachers’ beliefs and comfort level could play a role in their decisions surrounding the use of social constructivist activities in the classroom, and thus affect the quantity and quality of data available in this study. Such factors need to be taken into consideration when analysing, interpreting, and discussing the data.

Theoretical framework and epistemology

An understanding of social constructivism is important in order to comprehend the shift in focus to more student-centred approaches in both research and practice in mathematics classrooms. From a social constructivist view, the importance of social interaction through language in the classroom is necessarily emphasized and valued. In a study such as this one, where second language plays an important role, Vygotsky's (1986) views on language and social constructivism are especially relevant. According to his theory, the teacher facilitates the active construction of new understandings by the students themselves, by providing students opportunities to talk about, to think about, to engage with, to represent, and finally to integrate newly acquired concepts into their world view. Thinking is considered inseparable from language. It is perhaps more intuitive to realize that one cannot think without language, but it is also true that it is not possible to develop language without simultaneously developing the concepts that accompany it. In the context of the mathematics classroom, children cannot fully grasp new concepts if they do not have the language with which to express these new ideas, and they will not be capable of effectively using new vocabulary if they do not deeply understand the ideas about which they are speaking. They are using language to help construct their thinking, and using thinking to help construct their language.

Mercer (2000) subscribes wholly to a social constructivist view of language. In both theoretical and practical settings, he elaborates on the essential nature of communication in learning in the home as well as in the classroom. He believes that the "guided construction of knowledge" (1995) by teachers, parents, and peers, can aid in the development of critical thinking skills. Mercer has done extensive work on what he terms exploratory talk, when students engage critically but constructively with each others' ideas (2000). Exploratory talk has been applied and studied in classrooms, such as through Mercer's project "Thinking Together". For example, Mercer (2006) provided teachers of Year 5 students in the UK with in-service and lesson plans detailing how to teach students to use exploratory talk to solve problems in the mathematics classroom. Compared to a control group, students demonstrated more improvement on a post-test of their mathematics skills, and teachers reported more collaborative, enthusiastic, and productive group work. Similar results have been found in science classrooms (Mercer, N., Dawes, L., Wegerif, R., & Sams, C., 2004). Mercer's studies, especially in the mathematics classroom, show the effectiveness of social constructivist methods in this setting. This evidence of efficacy serves to corroborate the valuable role of discourse in the mathematics classroom, which is the focus of this study. His analysis, however, is different from that of this project. As is the case in previously mentioned studies of mathematics (i.e. Turnbull, Lapkin, and Hart, 2001), an element of Mercer's studies has been the analysis of quantitative data, based on student achievement in mathematics, which is not the focus of this study. Although he has also analysed student and teacher talk in the mathematics classroom, his analysis of this talk focuses on the degree to which it resembles the target, which is exploratory talk, the features of which have been carefully defined. While Mercer's work with discourse in the mathematics classroom is similar in many ways to the present study, I still needed a framework which was more descriptive, rather than evaluative, in nature, one which could be used to describe naturalistic language, rather than language after a planned intervention.

Sfard's framework for use with mathematic discourse (2005, 2007) is well-suited to the present study. She characterized this framework as "commognitive", emphasizing the unity of communication and cognition. Sfard (2007) takes the importance of a communicative community in learning one step further, contrasting an acquisitionist viewpoint with one that is participationist. Rather than simply viewing individuals as acquiring mathematical concepts or ideas, a participationist viewpoint emphasizes the collective nature of learning which integrates individuals into a community of mathematical practice. Sfard emphasizes the role of communication in these interactions, privileging it to the point where it is the very essence of learning. Learning, she posits, is a question of "modifying and extending one's discourse" (Sfard, 2007, p. 567). Interaction between teacher and student is viewed as important, as is interaction between students. Sfard also draws attention to thinking, which she defines as communicating with oneself. Sfard declines a binary approach to any aspect, and designs her frameworks to reflect the duality of mathematical objects and discourse, as well as communication and thinking.

Sfard has developed a framework specific to mathematical learning in the classroom which is designed to accommodate the wealth of increasingly detailed data available to researchers through greater use of audio and video recorders. This framework is useful in conceptualizing the wide range of communicative practices that Sfard includes in her definition of mathematical discourse. Her framework can also be applied in order to interpret the increasingly detailed data to which she refers. One of the basic commognitive tenets is that mathematics is a form of discourse. In this framework, the discourse of mathematics is made distinct by four interrelated features: routines, endorsed narratives, visual mediators, and word use.

The first feature of Sfard's description of mathematical discourse is *routines*, an overarching category which partially overlaps the subsequent three features and also expands on them. Sfard defines routines as "well-defined repetitive patterns in interlocutors' actions" (p. 574). These discursive patterns, which form a set of rules to be used in mathematical settings, help to categorize thinking and view similarities and differences in situations. Sfard states that these repetitive behaviours are part of human nature, and that this repetition makes communication possible, progressive, and intelligible. Although participants in many discursive routines follow these patterns instinctively, without conscious thought, Sfard proposes that a defining feature of mathematical discourse is that some of these routines do evolve into explicit rules. These routines are said to be meta-level discursive routines, as opposed to object-level discursive routines, which focus on the mathematical content of the routine, rather than the set of rules governing the routine. The aim of the present study can now be more precisely stated: to describe and classify the routines present in the teachers' mathematical discourses of functions in the grade three and four French immersion classroom.

After routine, the second of Sfard's categories used in the description of mathematical discourse is *endorsed narratives*. These are viewed as mathematical facts: descriptions of objects or of their relationships or activities. These narratives, which can be endorsed as true or false by the participants in the discourse, include mathematical theories, definitions, proofs, and theorems. In the case of young learners, endorsed narratives may be restricted to number facts, or may include "locally endorsable narratives" (2005, p. 246), which apply to a specific situation and respond to a given prompt.

In the context of mathematical discourse Sfard uses the term *visual mediators* to signify mathematical symbols, formulae, and graphs, drawings, and diagrams which are used to "identify the object of (their) talk and coordinate (their) communication" (p. 573). Sfard emphasizes that these "symbolic artefacts, created specially for this form of communication" (p. 573) are not merely representations of previously conceived thoughts, but are independent evidence of thought. Such symbolic representation may be absent, however, from the mathematical discourse of young children, who may still use concrete objects, which are usually a characteristic of colloquial, rather than mathematical, discourse, to identify and coordinate their mathematical discourse. With young learners, these physical objects may even be manipulated as part of the mathematical discourse. In Sfard' and Lavie's 2005 paper the young girls use marbles as visual mediators for their mathematical discourse, and the students in the present study use many different visual mediators. Some, such as fraction tiles, are specifically designed for teaching and learning mathematics, while others, such as a classroom clock, are everyday items found in the classroom.

Finally, Sfard uses the term *word use* where others may use the term "word meaning". By *word use* she means the specific vocabulary of the topic at hand and the ways in which it is used, which give a window into the participants' world view. Children studying mathematics may learn new mathematical vocabulary. Sfard (2007) gives "negative two" as an example of such lexical innovation. In the present study, students had likely never used the word "obtus" before. Children may also learn a new, more specialised use for a term they already used in a more colloquial way. Sfard (2007) suggests that shape and number related words, such as triangle or square, may develop a new, more disciplined use through the study of mathematics. A word such as "half", which students had almost certainly used before the lesson, takes on a much more precise meaning during lessons on fractions in the present study.

The presence of these four interconnected categories renders mathematical discourse unique. In Sfard's theoretical framework, mathematical learning corresponds to a change in the four categories of mathematical discourse. This framework encourages a broad conceptualization of mathematical discourse, including not only student and teacher utterances but also patterns of communication, visual supports, and commonly held definitions and assumptions.

There have been some studies that have used various versions of Sfard's categories to describe mathematical discourse in the classroom. Viirman (2014) used Sfard's categories to describe and name as many discursive routines as possible as used by a group of university mathematics instructors. Gücler (2013) analysed video-taped classroom observations, a diagnostic student survey, audio-taped task-based interviews, and students' written work in a university-level calculus class. She used Sfard's categories to describe differences between teacher and student discourse about limits, highlighting communicational breakages and putting forth the hypothesis that it is possible for teachers to enhance classroom communication by explicitly attending to metarules and other elements of discourse. These studies were helpful to me as they gave practical examples of other researchers interpreting and using Sfard's categories, as opposed to Sfard herself. Their usefulness was limited, however, as they both took place in unilingual university settings, rather than the multilingual elementary school setting of this study.

Other researchers have created categories similar to Sfard's, but have used different vocabulary. While investigating the negotiation progess between students and teacher, for example, Gellert (2014) noted previous researchers' attempts to describe the interactions between students and teachers. Bauersfeld (e.g. 1980, cited in Gellert, 2014) described "funnelling" routines, which were more teacher-centred, while Wood (e.g. 1994, cited in Gellert, 2014) used the term "focusing" to describe a more constructivist approach to these interactions.

Rojas-Drummond, Mercer, and Dabrowski (2001) draw similar distinctions. They use Mercer's earlier terminology of "loop IRF's" to describe closed, quiz-style questions in a direct transmissional style of teaching and "spiral IRF's" to describe a more scaffolded, social constructivist approach. They note that the style of teaching associated with greater use of spiral IRFs is associated with the development of more competent and independent problem solving abilities.

The benefits of applying Sfard's theoretical framework in an elementary French immersion classroom by using it to analyze rich data collected in such a setting are twofold. First of all, such an application will shed light on the feasibility of Sfard's framework in an elementary classroom setting, which was not used in a multilingual context in her 2007 article. Such a concrete application may inform changes or additions which would make the framework more practical in analyzing actual classroom data. Secondly, using this framework to analyze data collected will provide researchers and readers with a common language and parameters which would be useful in comparing and contrasting data from different settings. By analyzing the discourse of the students and the teacher in the mathematics class, I will have a window into the mathematical learning taking place in the elementary school classroom.

Research questions

French immersion is an area of growth in Canada and, happily for all stakeholders in this educational context, it has been well established in the literature that curriculum content in other subjects can be mastered in a French immersion setting. This includes, despite many concerns, the mathematics curriculum. It is evident from the literature that students can transfer mathematical learning from one language to another, yet the period of transition from one language of instruction to another had been largely unexplored. Culligan (2010) took some preliminary steps in this area of inquiry by interviewing secondary students about this transition period. I have built on her work by studying the classroom language of students and teachers while they are engaged in teaching and learning of mathematics both before and after this transition. Sfard's four features of mathematical discourse: routines, endorsed narratives, visual mediators, and word use provided a structure with which to frame this work. This framework was an excellent match for the project that I did, as Sfard designed it specifically for analysing available language in mathematics classrooms. The four categories encompass a wide range of relevant discourse in the classroom, including visual as well as oral components of discourse. This framework allowed me to capture, describe, analyse, and compare the complete extent of language in the mathematics classroom. A brief initial trial of this framework on the discourse of a grade three French immersion class learning mathematics in French indicated that Sfard's four categories were a feasible lens through which to examine classroom discourse. I then further developed this work by following the same class to grade four, where mathematics instruction was in English.

The research questions that guided this research were therefore:

- What are the characteristics of the discourse of teachers and students in an elementary French immersion mathematics classroom?
- What are the similarities and differences between the discourses when the instruction is in French as opposed to when the same students are taught mathematics in English?

Chapter Three introduced the area of qualitative research into language in mathematics classrooms. There is a potential for division in the linguistic and philosophical approach used in mathematics classrooms, and questions about how to best balance the role of teacher and student language in order to optimize learning. Mercer favours a socio-constructive approach, and Sfard furthers his view. Her commognitive framework, combining cognition and communication, provides four categories: routines, endorsed narratives, visual mediators, and word use, which will be used to analyze classroom discourse in the present study.

Chapter Four outlines the specific methodology involved in carrying out this study.

Chapter Four: Methodology

Methodological approach

For this project I intentionally focused on the oral language of the students and mathematics teacher, as I felt that the spontaneous nature of oral classroom language would best capture the subtle and fleeting changes in discourse which Sfard believes is indicative of mathematical learning. Thus, the unit of analysis was the *mathematical discourse*, including both discourse by students and by the teacher. This choice of unit of analysis is based on a suggestion by Sfard and Lavie (2005), who state that although *number concept* would seem to be a good candidate for the unit of analysis, they ultimately deem it to be too "confining", and propose that "*the entire discourse on numbers*...should be considered as the proper unit to be studied by those interested in the development of numerical thinking" (p. 248). For this project I have extended this unit of analysis to include *the entire mathematical discourse*, as I could not predict whether the mathematics classes that I visit would include discourse on numbers, or whether the class would be working on a mathematical concept, such as identifying geometric shapes, that did not necessarily involve the explicit use of numbers. I planned to use Sfard's four categories of mathematical discourse to analyse the student and teacher utterances during whole class mathematics lessons and during collaborative work in small groups or partners, but I also felt that information from interviews with students and with the teacher would lead to interesting insights into the research questions. This framework was used to analyse data from two different years, both before and after a transition in the language of instruction in mathematics.

Data collection

As part of a larger study, in the spring of 2012, I participated in the data collection of language surrounding French language mathematics lessons in a grade three French immersion class in Ottawa. The students attended a suburban kindergarten to grade eight school in a middle class neighbourhood in the city of Ottawa. Most of the nineteen students who participated in the project had been together since kindergarten, and most, but not all, speak English at home. For my project I extended this data collection to follow the same class to their grade four mathematics class in the spring of 2013, where the instruction was in English, in order to capture insights into classroom language both before and after a change in the language of mathematics instruction from French to English. Although I do not teach in the primary division or grade four at this school, the students and I were somewhat familiar with each other before beginning the project from casual encounters in the hallway, doing yard duty, or from having taught older siblings of the students in the class. The two teachers in the project very readily agreed to take part. The teacher of the French-language mathematics class was a long-term occasional teacher who had begun teaching the class in November of the school year. He was a young native Greek speaker from Montreal, fluent in both French and English. The teacher of the English-language mathematics class was an experienced teacher. She had been at the school for a number of years, and was a unilingual English speaker.

In the initial phase of the project, when the mathematics instruction was in French, the data included digital audio recordings of whole class mathematics lessons and recordings of students working with partners or in small groups. Samples of student worksheets were also collected, as were still images of environmental print, and notes were taken during observation periods, to ensure that relevant contextual cues were recorded. Post-observation interviews about language and mathematics at school and at home with pairs of all of the nineteen participating students as well as with the classroom teacher were also recorded. For this phase of the project, I analysed data from all three lessons which were recorded, including whole class instruction and partner work. I chose to work with three lessons in order to have enough data to get an impression of the typical day-to-day functioning of the classroom, while maintaining a workable amount of data to transcribe and analyse. When recording whole class lessons I sat at a table at the back of the room, taking notes and recording with a small portable digital recorder. This did not seem overly distracting to the students, who carried on with work of varying levels of focus and appropriateness. When it was time to record partner work, I looked for partners that were speaking as they worked, in order to obtain usable data. Both partners had to have permission to participate in the project, and I asked for their consent each time before I placed the digital recorder on the table or desk where they were working. The two student-led recordings that I did involved the same pair of students, as they both had permission and gave consent to participate, and were consistently engaged in oral discourse as they worked. Aside from some initial excitement, they carried on with their work as the recording progressed.

In the second phase of the project, when mathematics instruction was in English, the class was also observed, recorded, and documented during three fifty-minute periods of mathematics. The recording of three lessons in the English-language mathematics classroom, as had been done in the French-language mathematics classroom, ensured an equitable amount of French and English data in the project, while seeking to balance sufficient data with feasibility of transcription and analysis. As in the first phase, these periods included whole-class teacher directed lessons as well as work in partners on teacher-directed tasks. Both whole-class teacher-directed lessons and work in partners was recorded in order to get a more complete picture of the range of language in different classroom settings. Teacher language included formal lessons, answers to individual questions, and prompts. Student language included responses to questions by teachers, questions about the work posed to the teacher by students, and student-student interactions during work on the teacher-directed tasks. During the lesson the language of the teacher and of all the students was recorded. Following a suggestion from the committee, the work of several partnerships on the same task at the same time was recorded during the partner work in order to get a more representative sample. Students were more talkative this year than in the previous year. I tried to get recordings from as many different partnerships as possible, and succeeded in recording five different partnerships, involving seven different students, including the partnership from the French-language student-led recording. As before, each group had to have permission and give consent to participate in the project before I placed the recorder on the desk or floor where they were working. Again, after a very few moments, the partners worked without distraction of the recorder. Due to the fact that there was more than one recorder, I necessarily circulated in the classroom while recording. I occasionally interacted with partners during the recording. This was sometimes initiated by me, if I had questions about a students' mathematical thinking, and sometimes by them, sometimes about mathematics and sometimes not. During the recording of whole class lessons, I was seated off to the side. The students did not seem distracted by my presence.

Ideally, the three observation sessions would have taken place during lessons on the same topic as the first phase of the project, but this is not vital, as my work focused on classroom language rather than the content of the lessons. As it happened, one of the grade three lessons was on fractions, the same topic as all of the grade four lessons. This was sufficient to confirm that it was not necessary to analyze lessons on the same topic in order to reach conclusions about mathematical discourse in these settings.

I was present during the audio recording sessions in order to note any relevant additional relevant observations, such as gestures and writing, which would not be captured by the audio recording, as Sfard's category of visual mediators is an important part of mathematical discourse.

The audio recordings in the classrooms are summarized in the following two tables.

Table 1

French recordings and transcripts: May 2012

Code	Length (minutes & seconds)	Topic	Type of discourse
CG3MF12May02a	25:10	Capacity	Teacher-led
CG3MF12May 02b	15:50	Capacity	Student-led
CG3MF12May 07a	26:34	Fractions	Teacher-led
CG3MF12May14a	21:45	Angles	Teacher-led
CG3MF12May 14b	14:27	Angles	Student-led

Table 2

English recordings and transcripts: May 2013

Code	Length (minutes & seconds)	Topic	Type of discourse

CG4ME13May22a	24:22	Mixed and proper fractions	Teacher-led
CG4ME13May 22b	16:18	Mixed and proper fractions	Student-led
CG4ME13May 23a	3:58	Converting improper fractions to mixed fractions	Teacher-led
CG4ME13May 23b	10:27	Equivalent fractions	Teacher-led
CG4ME13May 23c	20:07	Equivalent fractions I	Student-led
CG4ME13May 23d	20:46	Equivalent fractions II	Student-led
CG4ME13May 27a	13:37	Equivalency I	Student-led
CG4ME13May 27b	12:30	Equivalency II	Student-led
CG4ME13May 27c	9:41	Equivalency	Teacher-led

The two teachers, one in the French-language classroom and one in the English-language classroom, and pairs of students were then interviewed at the end of the data collection period. I conducted the interviews hoping to gain additional information about the points of view of students and teachers language in a French immersion mathematics classroom, both before and after the transition to English-language mathematics instruction. Teachers were interviewed individually in their classrooms after class. Students were interviewed in partners in a small room down the hall from their classroom.

After recording, audio data in the form of classroom language from teachers and students was transcribed. The following transcription conventions emerged during the lengthy transcription process. Some conventions were adapted from existing conventions I had seen during my reading for the literature review and the theoretical framework, while others were conceived to fulfill a particular need relating to these specific transcripts.

Table 3

Transcription conventions

Convention	**Meaning**
Professeur / Teacher /	The mathematics teacher in the French/English mathematics class
Élève / Student	An individual student who is not identifiable on the recording
Élève 1 / Student 1	An individual, unidentifiable student who reliably has repeated utterances in the same routine
Classe / Class	Many students in the class speaking at once, individual students unidentifiable
...	A break in the transcription – irrelevant utterances have been cut from the excerpt
Indistinct / unintelligible	Utterance heard, unable to understand words
Pause	A pause in the discourse
(repeated)	The same utterance is repeated many times
Tout! / Tout! (repeated)	Many students are calling out the same answer.
Oui. / Oui. / Non. / Pas dans a). / Oui	Many students are talking over each other
/	A student begins talking before another is finished.

Student names were all changed in the transcripts. The French-language mathematics teacher was referred to as "Monsieur P", as in the classroom he referred to himself as Monsieur followed by an initial. The English-language mathematics teacher was referred to Mrs. Telfer, a pseudonym, as her students called her Mrs., followed by her last name.

Data analysis

After the transcription, the data was analysed based on Sfard's theory of commognition: routines, endorsed narratives, visual mediators, and word use. I further differentiated the transcriptions into teacher-led routines and student-led routines, in both the French and English-language classrooms, in order to highlight similarities and differences in the discourse. Sfard's framework was purposefully chosen to describe discourse in a mathematics classroom. Although I was careful to apply the four categories in an accurate manner, this practical application of her framework to my own data led to some adaptations and interpretations of the terminology.

Once the transcriptions were complete I began to analyze them using Sfard's categories. Throughout the analysis process I consistently kept a copy of Sfard's own definitions of her four categories (Sfard, 2007) close at hand, either in the document itself or as a document in synchronous view. I first examined, category by category, two of the transcripts, one in French, which was teacher-led, and one in English, which was student-led. In keeping with my research questions, I was seeking to use Sfard's categories to describe teacher and student discourse in the two classrooms, and then to compare and contrast the mathematical discourse used in the two language settings. In these two initial transcripts I found it was possible to use the four categories as a framework to describe the teacher and student discourse, and initial similarities and differences in the discourse between the two classrooms were noted using Sfard's terminology.

As this preliminary analysis was successful, this process was repeated with two new transcripts, this time a French student-led transcript and a teacher-led English transcript. Once again, carefully observing the definitions, it was possible to use Sfard's categories to describe, compare, and contrast different types of mathematical discourse collected in different settings, confirming that Sfard's framework was practicable for the data that I had collected and for the research questions I was asking.

After this initial successful analysis of four different texts (teacher-led and student-led, in French and in English), I took a more global look at the transcripts. It is at this stage of the analysis, as I looked to answer the research questions about the nature of student and teacher discourse and the inherent similarities and differences that the importance of the true definition of Sfard's category of "routine" began to emerge. It became clear that the characteristics, similarities, and differences I found in the other three categories of word use, visual mediators, and endorsement fulfilled Sfard's definition of routine: "well-defined, repetitive patterns in interlocutors' actions...partially overlap(ing) the previous three features and also expand(ing) on them, help(ing) to categorize thinking and view similarities and differences in situations" (Sfard, 2007, p. 574). I divided the discourse in the transcripts into discrete events which satisfy Sfard's definition of routines. It was now time to mine all of the possible routines out of the transcripts. I exhausted the transcripts looking for routines that satisfied Sfard's definition, setting the threshold for a distinct routine at a minimum of four examples. Taking a cue from Sfard, I decided to give a descriptive name to each routine I found. I initially identified and named nine discursive routines which occurred at least four times. These nine routines were eventually grouped together into two distinct routines which shared important characteristics. The first four original routines were combined into the "Question-response-endorsement" (QRE) routine. This routine consisted of simple, isolated questions and answers, each question and answer set complete unto itself. The remaining five original routines were combined to form the “exploratory” routine. The questions, answers, and discussion in these routines were often interconnected, directed towards an understanding of a more complex concept.

Table 4

Evolution of routine names

Original routine	**Combined routine**
Question-response-endorsement Question-response-non-endorsement Question-teacher answer Question-reformulation	Question-response-endorsement (QRE) routine
Question-response-endorsement-probe-exploratory dialogue-endorsement Question -incomplete response-push Question-incomplete response-teacher explanation Question-exploratory dialogue-teacher summation Question-exploratory dialogue-no teacher summation	Exploratory routine

The description and naming of these routines led naturally to a reading of aspects of others' terms to describe questioning routines in the mathematics classroom. Other researchers have categorized what Sfard would describe routines using different terminology, some of which was similar to the present study. Gellert (2014) investigated the negotiation process between students and teachers. He notes previous researchers' attempts to describe the interactions between students and teachers. Bauersfeld (e.g. 1980) described "funnelling" (teacher-centred), while Wood (e.g. 1994) used the term "focusing" to describe a more constructivist approach to these interactions. Rojas-Drummond, Mercer, and Dabrowski (2001) draw the same distinctions. They use Mercer's earlier terminology of "loop IRF's" to describe closed, quiz-style questions in a direct transmissional style of teaching and "spiral IRF's" to describe a more scaffolded, social constructivist approach. They note that the style of teaching associated with greater use of spiral IRFs is associated with the development of more competent and independent problem solving abilities.

In the current project, a "funnelling" or "loop IRF" corresponds most closely to what I have termed a QRE routine, and a "focusing" or "spiral IRF" corresponds to an exploratory routine. The fact that Rojas-Drummond, Mercer, and Dabrowski found that the equivalent of exploratory routines were associated with the development of more effective problem-solving skills is important in the present context. Once the routines were established and described in both English and French classrooms, in both student and teacher-led discourse, I sorted all of the routines I had noted in the transcripts into QRE or exploratory, teacher-led or student-led, in English or in French. I then analysed the endorsement, visual mediators and word use in each routine, seeking to fully describe and compare every category of Sfard's in each language context. Once the findings were articulated, examples were found to illustrate the findings.

There are many excerpts in the Findings section: one to typify each of the two distinct types of routine (QRE and exploratory), and one in French and one in English to illustrate each of Sfard's four categories. Both teacher-led discourse and student-led discourse were both illustrated with excerpts in this way. Excerpts are essential to this project, but it was essential to choose them prudently. The excerpts chosen spoke to me; I often remember thinking that they were significant as they were being recorded. Ideally they were as concise as possible, and clearly illustrated the desired aspect of discourse. This was possible when students spoke clearly on the recording, and it was also helpful if it was in the student's nature to "think aloud". When I had a choice between two excerpts, I looked at the table of times that I had chosen excerpts from each transcript, ensuring that each transcript had been represented in the findings. Due to the differences in nature and in length of each transcript, the number of excerpts chosen from each transcript is not equitable. The fact that each transcript has been represented, however, gives credence to the fact that the findings and the conclusions that have been drawn are representative of the mathematical discourse in these two classrooms, and are not isolated, rare occurrences. In the chapter on Findings, each excerpt has been described, and its significance in that category and in the larger context of the project has been explained.

Table 5

Excerpts used to illustrate Sfard's categories

Transcript Code	**Teacher-led excerpts**		**Student-led excerpts**	
	In QRE routines	**In exploratory routines**	**In QRE routines**	**In exploratory routines**
French Transcripts				
CG3MF12May02a	I,V,V,V,W, E	R,V,V,W,W		

CG3MF12May 02b	E,E		I	R,E,E,V,V, V,W,W
CG3MF12May 07a	E,E,E,E V,W,W	I,R,E,V,V, V,W,W,W,W		
CG3MF12May14a	R,V,V,W,W	V,W		
CG3MF12May 14b				W
English Transcripts				
CG4ME13May22a	E,V	R,E,V,W,W		
CG4ME13May 22b			I	E,V,W
CG4ME13May 23a	R,V			
CG4ME13May 23b	V,W	V		
CG4ME13May 23c				R,W
CG4ME13May 23d				E
CG4ME13May 27a				V,V
CG4ME13May 27b				W
CG4ME13May 27c	V	I.R,V,V,W		I

Legend:

Letter	**Meaning**
I	Introduction to section
R	Routine
E	Endorsement
V	Visual mediator
W	Word use

Despite the fact that students and teachers were interviewed as part of the data collection, these interviews are not part of the data that was analysed. Upon listening to the interviews, I was unable to find insights that were relevant to my research questions in the student and teacher responses to the interview questions. This was unexpected, but upon reflection, perhaps not surprising. My research questions are concerned with describing, comparing, and contrasting student and teacher discourse in two different language settings in mathematics classrooms. The students and teachers I interviewed, however, do not have this description and comparison as a goal. The teachers involved in the study do not spend time in each others' classrooms, and the students seemed to accept the reality of their mathematics instruction without reflecting upon the language of learning and teaching. Due to these realities, perhaps it should not have been surprising to me that the interviews had little to contribute to my search for answers to my research questions.

Sfard's four categories of routine, endorsement, visual mediators, and word use were part of the original theoretical framework, and were explicitly chosen in order to describe the mathematical discourse in the two classrooms where I collected data. Even though these four categories seemed to be clearly described at the outset, there were still decisions to be made during the data analysis regarding Sfard's terminology.

One such decision involved the use of the term "visual mediators". A wide variety of visual representations of the mathematics being done was used in both classrooms. The range of such visual representations included written representations, such as worksheets and student work, manipulatives, such as fraction strips and dice, and concrete objects, such as clocks and pies, some of which were actually present in the classroom and some of which were not. Sfard defines visual mediators as "mathematical symbols, formulae, and graphs, drawings, and diagrams which are used to “identify the object of (their) talk and coordinate (their) communication” (Sfard, 2007, p. 573). Although she categorizes the use of concrete objects as a characteristic of colloquial, as opposed to mathematical discourses, the use of concrete objects as visual mediators in the mathematics classroom is a defining characteristic of mathematical discourse in these two particular settings, and they are therefore “part and parcel of the act of communication and thus, in particular, of thinking processes” (Sfard, 2007, p. 572). Although these artifacts are neither symbolic, nor “created specially for the sake of this particular form of communication” (Sfard, 2007), this particular use of these quotidian objects is specific to the mathematical discourse in the classroom, and essential to communication and cognition, especially with these young learners. Jäppinen (2005) supports this idea, suggesting that more concrete topics may be helpful for younger learners with less-developed language skills in the medium of instruction. It is for these reasons that I decided to include concrete objects in my discussion of visual mediators.

Occasionally, rather than justifying my use of a term of modifying its definition to fit the circumstances, as I have done with visual mediators, I felt it was more appropriate and accurate to modify the term itself, or invent a new term. A new term that was not based on one of Sfard’s categories was “chained scaffolding”. I used this term to describe a type of exploratory routine that occurred in the French language mathematics classroom. French exploratory teacher-led routines were often begun by more closed questions than the ones which began exploratory teacher-led routines in the English-language classroom. Contrary to a QRE routine, however, once the initial question was answered, in the exploratory routines he continued to ask a series of closed questions on the same topic, which eventually led to a final answer. I termed this series of questions “chained scaffolding”, as the teacher seemed to be scaffolding the dialogue in order to support the students as they worked to demonstrate their understanding of a more complex concept.

One example of a term that I expanded upon is “negative endorsement”. Endorsed narratives is one of the original categories from Sfard. They are mathematical facts which can be verified as true or false by the participants in the discourse. As I felt it was important to describe the ways that teachers and students reacted when they did not agree with a proposed answer, I coined the term “negative endorsement” to describe this situation, and left the term “endorsement” to refer to an answer with which the interlocutor agreed.

It was also necessary to expand upon the Sfard’s term “routine”. Sfard’s definition, “well-defined repetitive patterns in interlocutors’ actions” (Sfard, 2007, p. 574) fit in well with what I was finding in the transcripts, and logically served as the foundation for my analysis, as routines were said to overlap and expand upon the other three categories. As I combed through the transcripts, looking for routines, I found that I needed to come up with descriptive terms to capture the nature of the various routines that I was identifying. I originally identified nine routines, which were eventually distilled into what I termed “question-response-endorsement (QRE) routines” and “exploratory routines”.

Chapter Four outlined the methodology used to plan the study, gather the data, and use the framework to analyse the data. A prudent choice of framework which was designed for the analysis of mathematical discourse, careful and logical application of the chosen methodology, as well as considered application of Sfard’s own definitions of her four categories of routine, endorsement, visual mediators, and word use all contributed to findings which clearly represent and consistently describe the nature of the mathematical discourse of teachers and students in two different language contexts.

This search for a suitable framework, which was appropriate for the analysis of classroom mathematical discourse, as well as the careful use of the framework, were both essential to the rigour of this project. Sfard's framework, designed to be used with mathematical discourse, helped to ensure that the appropriate data gathered would help to answer the specific research questions that have been asked. This choice of framework helped to assure the validity of the project, ensuring that the findings would clearly communicate answers to the research questions. Reliability, a measure of the consistency of the results, was achieved through a very careful expression of the definitions of different categories in the framework, and a precise application of the categories to the data. The use of strict rigour criteria ensures that findings which can be depended upon to be thorough and accurate.

Chapter Five outlines these findings, and supports them with excerpts from the transcripts. The findings are organized and presented by the two different routines, first question-response-endorsement, and then exploratory routines. Each routine is first described, and then the other categories of endorsement, visual mediators, and word use are examined, in that order. Teacher-led examples are presented first, followed by student-led examples in each language setting. A discussion of the findings, centred on the themes of terminology and the place of discourse in the classroom, follows.

Chapter Five: Findings

Findings

The findings yield many observations about teacher and student discourse during instruction in French and instruction in English. There is evidence of similarities as well as differences between teacher and student discourse in different language settings.

Routines are described by Sfard as the overarching category of analysis. As she states, routines help to categorize thinking and view similarities and differences in situations (Sfard, 2007, 2005). The findings in the current study are therefore organized by routine. Much of the data collected and transcribed can be divided into two routines, a question-response-endorsement routine and an exploratory dialogue routine. In addition to the use of the categories of these two routines, the similarities and differences of mathematical discourse, whether led by teachers or students, in French or in English, can be described using Sfard's other three categories: endorsement, visual mediators, and word use.

In this chapter I begin with question-response-endorsement routines in the data. I first define the routine, and then describe its essential features in teacher-led and student-led discourse in first the French-language and the English-language mathematics classrooms. I then discuss and give examples of Sfard's other categories of endorsement, visual mediators, and word use in the routine for teacher-led and student-led discourse in each linguistic setting. This process is then repeated for the exploratory routine: a definition, followed by central characteristics in teacher-led and student-led discourse in the French-language and English-language classrooms. Examples and discussion of routine, endorsement, visual mediators, and word use for exploratory dialogue in teacher-led then student-led discourse in each language setting follow.

QRE Routine

The first routine which will be described and discussed is familiar to anyone with experience in a traditional classroom. It begins with a brief question, which is followed by a similarly short response, which is then endorsed (positively or negatively) by the person who posed the initial question. Other researchers have noted the presence of this routine. Sfard mentions the "initiation-response-evaluation pattern" (IRE) as described by Sinclair and Coulthard (1975) as "pervasive in traditional classrooms (or)...any interaction where there is intentional teaching" (as cited in Sfard, 2008). Mercer (1995) also mentions the IRE pattern, as well as initiation-response-feedback (IRF), and notes that these patterns have been recorded by language researchers all over the world.

The following excerpts show typical examples of this routine from the data from the present study:

Professeur	…combien de millilitres avant d'avoir un litre? Kyle.
Classe	Oh! Oh! Je sais! Mille.
Kyle	Mille.
Professeur	Mille. Exactement.

CG3MF12May02a (Capacity, teacher-led), lines 12-15

Teacher	So I could say what, is equal to one whole? Kathleen?
Kathleen	Two halves.
Teacher	Two halves.

CG4ME13May27c (Equivalency, teacher-led), lines 3-5

In these examples, one each from the French-language and English-language mathematics classrooms, the elements of the QRE routine are evident. The initial questions are brief: «...combien de millilitres avant d'avoir un litre? » and "…what is equal to one whole?" The responses are also short: «Mille. » and "Two halves." The answers are then endorsed. The

French teacher repeats the answer and adds a positive comment: «Mille. Exactement. », while the English teacher simply repeats the answer: "Two halves" in a tone of approval.

In the current study the term QRE will be used, as the use of the term "endorsement" reflects the vocabulary of Sfard's categories, which are being used as a framework for analysis. In all of these patterns the turns by each interlocutor are brief, and it is understood by both parties that the one who poses the question has more knowledge and understanding of the topic at hand, including the answer to the question that has been asked. There are numerous possible purposes to such a routine, many of them outlined by Mercer (1995). At the beginning of a lesson, it can establish continuity between the current lesson and previous ones. A QRE routine can elicit what the questioner has deemed to be the crucial features of the learning, test recall, and provide feedback to those who are responding. QRE routines can also work to establish shared ownership of the knowledge at hand.

QRE in teacher-led discourse. In the classroom, the role of the teacher is central in the QRE routine. Both Mercer and Sfard describe this type of exchange as being initiated by the teacher. The response is then given by the pupils, and some form of endorsement is then provided by the teacher. It is not surprising, then, that the QRE routine is commonly found in both the French teacher-led discourse and the English teacher-led discourse data collected for this study. These routines were examined for similarities and differences between the two language settings, as were patterns of endorsement, visual mediators, and word choice.

Routine in QRE teacher-led discourse. The QRE routine in teacher-led scenarios has similar characteristics in both English and French. In both languages, this routine is often seen at the beginning of a lesson, in order to test recall and to establish continuity between present and previous lessons. In both language settings this routine begins with less complex, closed questions, which require only a short word or phrase, a number, or yes or no, as the correct answer, which is consistently known to the teacher. Students in these classrooms are expected to demonstrate their understanding of the material to the teacher by answering the questions correctly. They recognize and respect these discursive patterns and participate actively in the routine. The routine may or may not end with an endorsement of the answer to the initial question.

In the French-language mathematics classroom a series of simple, closed questions on the same theme from the teacher and answers from the class culminates with an endorsement from the teacher. Each pair of questions and answers within the series is complete unto itself, although related to the initial question.

Professeur	…Maintenant – le b). C'est quelle forme, le b)?
Classe	….Un triangle!
Professeur	Est-ce qu'il y a un angle obtus là-dedans?
Classe	Non! / Non!
Professeur	Un angle droit?
Classe	Non! / Non!
Professeur	Un angle aigu?
Classe	Oui! / Oui! / Oui!
Professeur	C'est lequel qui est un angle aigu? /
Classe	Tout! / Tout!/ Tout!
Professeur	Tout? Exactement!

CG3MF12May14a (Angles, teacher-led), lines 207-217

Similarly, in the English-language mathematics classroom, there are many examples of teacher-led QRE routines. The teacher asks a closed question ("...larger or smaller...?"), gets the correct answer from a student, and then endorses the answer.

Teacher	...Then you count the shaded pieces and that goes on the top. And the number of shaded pieces will be larger or smaller than the denominator? (Pause) Who's listening? How will they be – it's an *improper* fraction?

Student	Larger.
Teacher	Larger. 'Kay?

CG4ME13May23a (Improper to mixed fractions, teacher-led), lines 7-9

Although the QRE routine itself has similar characteristics in the French language and in the English language classrooms, differences within the QRE teacher-led routine are observed in Sfard's other three categories of mathematics classroom discourse.

Endorsement in QRE teacher-led discourse. Endorsement, the second of Sfard's categories, is different in the QRE routine in the two language settings. These differences are not unexpected, as Sfard does indeed state that "...terms and criteria of endorsement may vary considerably from discourse to discourse" (Sfard, 2007, p. 574). In the French language mathematics lessons, the correct answer is sometimes endorsed by the teacher. The endorsement "Exactement" was used on a regular basis, although other terms such as "Très bien", or a repetition of the correct answer, were also observed, as in the following examples:

Professeur	Ça va être ou, un quart d'un litre? Julius, peux-tu me montrer?
Julius	(Pause) Ici?
Professeur	Exactement. Et combien est-ce un quart d'un millilitre, combien c'est en millilitres?

CG3MF12May02a (Capacity, teacher-led), lines 47-49

Professeur	Donc, qui peut venir nous montrer un demi-litre ici? Adam.
Adam	Un demi? (Se lève, hésite, et montre un demi-litre sur l'affiche)
Professeur	Oh – un et demi. Un demi-litre.
Adam	(Pause, indistinct)
Professeur	On le laisse.
Adam	(Indistinct)
Professeur	Très bien.

CG3MF12May02a (Capacity, teacher-led), lines 73-79

Professeur	Deuxième, c'est combien de livres? (…)
Élève	Cinquante.
Professeur	Cinquante.

CG3MF12May02a (Capacity, teacher-led), lines 63-67

Although these brief endorsements were common in the French-language mathematics classroom, a correct answer was not necessarily always explicitly endorsed. After some QRE exchanges, the teacher simply moves on to the next question, as in the next example. This phenomenon was mentioned by Sfard.

Professeur	Si…si je voulais mesurer un pupitre, est-ce que c'est litres ou kilogrammes que j'utiliserais?
Classe	Kilogrammes! / Kilogrammes!
Professeur	Si je voulais mesurer…ummmm… la pluie qui a tombé.

CG3MF12May07a (Fractions, teacher-led), lines 290-291

After several students shouted out the correct response of "kilogrammes", the teacher simply initiated a new QRE routine by asking a new question in the same line of inquiry.

If the response was not correct, the French teacher was seen to negatively endorse a response, that is, to indicate that the answer was not satisfactory. In order to indicate that the answer was endorsed negatively, the teacher asked a probing question in order to prompt the desired response. The original answer received a conditional acknowledgement ("Litres, ou…? » or « Okay, mais...") that clearly indicated to students that a different answer was expected, as in the following examples:

Professeur	Très bien. S'il pleut dehors, avec quoi est-ce que je peux mesurer la pluie? On lève notre main, Easton. Adam.
Adam	Litres?
Professeur	Litres, ou…? Combien de millilitres est-ce que j'ai besoin pour avoir un litre?

CG3MF12May07a, (Fractions, teacher-led), lines 5-7

Professeur	Oui? Après, après demi, y a quoi?
Élève	Trois quarts?
Professeur	Okay, mais c'est quoi le mot que j'ai utilisé?

CG3MF12May07a (Fractions, teacher-led), lines 94-96

It is evident that students understood this conditional endorsement, and provided additional detail to demonstrate their understanding.

One student was even seen to mimic this very common technique of conditional endorsement in the French language mathematics classroom:

Professeur	Grammes. Si je veux mesurer de la terre…
Classe	Grammes / grammes / grammes!
Professeur	…de la terre.
Kyle	Kilogrammes.
Élève	Grammes et kilogrammes.
Professeur	Et? Quoi?
Kyle	Kilogrammes.
Professeur	Et?
Kyle	Grammes.
Professeur	Exactement. Okay…
Kyle	Eeeeet…kilogrammes. Eeeet…kilogrammes. (Rire).

CG3MF12May07a (Fractions, teacher-led), lines 311-321

He exaggerates the word “et”, which the teacher often used to elicit additional detail, laughing at his own impersonation of the teacher’s technique of endorsement.

In the English-language mathematics classroom, as in the French-language mathematics classroom, endorsements using words or short phrases, such as “That’s right” or “Okay” were often used to endorse a correct response. In contrast to the French classroom, a correct answer is consistently explicitly endorsed in the English-language lessons. The following example demonstrates the consistent endorsement which is a feature of teacher-led QRE routines in the English-language mathematics classroom.

Teacher	…thirty divided by three.
Class	Ten. (repeated)
Teacher	All right... Eighteen divided by three.
Student	Six?
Teacher	That’s right. Okay. Twelve divided by three?
Class	Two. / Four (repeated).
Teacher	Four. That’s right. Okay. Nine divided by three?
Class	Three (repeated).
Teacher	Fifteen divided by three.
Class	Four. / Five (repeated).
Teacher	Five, yeah. If it’s got a five in it then it’s gotta have a five somewhere in

	that answer, right?

CG4ME13May 22a (Mixed and proper fractions, teacher-led), lines 9-19

As seen the excerpt above, some additional explanation is sometimes given in the English-language mathematics classroom ("If it's got a five in it then it's gotta have a five somewhere in that answer, right?"). Such explanation is not seen in these routines in the French-language classroom. When more than one answer is proposed, the English teacher specifically endorsed the correct answer. ("Two. / Four. Four. That's right.")

Visual mediators in QRE teacher-led discourse. The use of visual mediators, the third of Sfard's four categories, is another area where there were both similarities and differences in the teacher-led QRE routines in the French and English classes.

As can be seen in the first examples of QRE routines for each language used in this section, often no visual mediators are used in QRE routines:

Professeur	…combien de millilitres avant d'avoir un litre? Kyle.
Classe	Oh! Oh! Je sais! Mille.
Kyle	Mille.
Professeur	Mille. Exactement.

CG3MF12May02a, (Capacity, teacher-led), lines 12-15

Teacher	So I could say what, is equal to one whole? Kathleen?
Kathleen	Two halves.
Teacher	Two halves.

CG4ME13May27c (Equivalent fractions, teacher-led), lines 3-4

As QRE routines are often a quick series of simple, closed questions, teachers often carried out the routine without the use of any visual mediators. When visual mediators were used, however, teachers in both classrooms used a variety of tools and techniques in order to coordinate mathematical discourse in their classrooms.

Photocopied published worksheets were a common visual mediator in the French-language mathematics classroom. The mathematical drawings and symbols on them were used to “identify the object of (their) talk and coordinate (their) communication” (Sfard, 2007, p. 571). The work and discussion surrounding the problems posed on the worksheets served to give the teacher a window into the thinking processes of the students in the classroom. The teacher in the French-language mathematics classroom often asked students to work on them in pairs at the end of a lesson in order to practice and consolidate learning. The teacher often went over the questions with the students before they began the worksheets. In the following example the teacher is orally reviewing the work to be done on a worksheet involving types of angles before the students work on the assignment in pairs.

Professeur	Très bien! Maintenant – le b). C’est quelle forme, le b)?
Classe	….Un triangle!
Professeur	Est-ce qu’il y a un angle obtus là-dedans?
Classe	Non! / Non!
Professeur	Un angle droit?
Classe	Non! / Non!
Professeur	Un angle aigu?
Classe	Oui! / Oui! / Oui!
Professeur	C’est lequel qui est un angle aigu/
Classe	Tout! / Tout!/ Tout!
Professeur	Tout? Exactement!

CG3MF12May14a (Angles, teacher-led), lines 207-217

In the above series of QRE exchanges about a shape on a worksheet, the teacher uses the triangle on the worksheet to coordinate discussion about the angles in a specific triangle while gaining insight into the thought processes of the students in the class. The class excitedly shouts out their correct responses.

During work on the same worksheet, the class was attempting to describe the angles in parallelograms. Despite the fact that there was a parallelogram on the worksheet, the students struggled to answer questions about the angle types. The teacher then decided to present the mediator in a different way, by drawing it on the blackboard.

Professeur	Donc : si on regarde, est-ce qu'il y a des angles droits dans un parallélogramme?
(Does not get expected answer - unintelligible)	
Professeur	Est-ce qu'il y a des angles droits? Est-ce qu'il y a des angles aigus?
Classe	Oui. / Oui. / Non. / Pas dans a). / Oui.
Professeur	Y a, est-ce que, est-ce que, quelles sortes d'angles est-ce qu'il y a, Lee?
Lee	Ahhh, oh oui, il y a des angles (unintelligible).
Professeur	Okay. Sont où, sont où, les…'gardez, les amis? Si je dessine un beau parallélogramme ici… (teacher reproduces parallelogram from worksheet on blackboard)
Classe	(Class talks among themselves)
Professeur	Ça c'est où, les angles, les angles aigus? Qui peut venir me le, me le montrer?
Élève	Ohhhhh.
Professeur	Lee. Montre-moi les angles aigus.
(Lee does so correctly)	
Professeur	Okay – qui peut venir me montrer les angles droits ou obtus?
Élève	Oh!
Professeur	Cole.
(Cole does so correctly)	
Professeur	Très bien! Maintenant – le b).

CG3MF12May14a (Angles, teacher-led), lines 181-207

Although the teacher in the French-language mathematics classroom rarely uses the blackboard, here we see that his decision to present the visual mediator in a different way, one where everyone's attention is focused on the same image (not simply a copy of the same image, as when the parallelogram is on the worksheet) garners more accurate results from the students.

The teacher in the French-language mathematics classroom often used concrete objects as visual mediators. Sometimes these objects were in the classroom, and sometimes the objects were familiar to the students, but not actually present in the classroom.

Although Sfard categorizes the use of concrete objects as a characteristic of colloquial, as opposed to mathematical discourses, the use of concrete objects as visual mediators in the mathematics classroom is a defining characteristic of mathematical discourse in these two particular settings, and are “part and parcel of the act of communication and thus, in particular, of thinking processes” (Sfard, 2007, p. 572). Although these artifacts are neither symbolic, nor “created specially for the sake of this particular form of communication” (Sfard, 2007, p. 571), this particular use of these quotidian objects is meaningful to the mathematical discourse in the classroom, and essential to communication and cognition. When working with young learners. Sfard mentions the importance of concrete visual mediators, as well as the value of physically manipulating these concrete objects (Sfard, 2005). In the next example, the teacher uses two different everyday objects as visual mediators in order to coordinate the mathematical discourse in his classroom.

Professeur	Si…si je voulais mesurer un pupitre, est-ce que c’est litres ou kilogrammes que j’utiliserais?
Classe	Kilogrammes! / Kilogrammes!
Professeur	Si je voulais mesurer…umm…
Élève	Moi.
Professeur	Si je voulais mesurer…
Classe	(Unintelligible suggestions)
Professeur	la pluie qui a tombé.
Kyle	Mi-li-litres!

CG3MF12May07a (Fractions, teacher-led), lines 290-297

In the exchange above, the teacher begins his questioning using a concrete object in the classroom (a student desk), and then moves on to something less concrete, but still visible and familiar to the students (rain). The students are able to correctly choose the appropriate units of measurement for both types of visual mediators. This movement from the concrete towards the symbolic is necessary for the deep understanding of mathematical concepts.

Unique to the French language classroom, the teacher often made posters as concrete visual mediators to which the students were expected to refer. The teacher often used these posters in the QRE routines, as in the example below:

Professeur	Mille. Exactement. Si vous vous regardez ici aujourd'hui j'ai (…) un ici …les unités de mesure. Donc, on a on a déjà couvert la distance, qui peut me dire les, les unités de mesure que j'ai indiqué ici? Emily. (…)
Emily	Millimètres, centimètres, mètres, et kilomètres.

CG312May02a (Capacity, teacher-led), lines 15-18

The teacher in the French-language mathematics classroom consistently drew students' attention to the posters as a way of focusing and coordinating discourse. He used phrases such as "Si vous regardez ici aujourd'hui..." and "les unités de mesure que j'ai indiqué ici?" as well as gesture in order to guide students' attention to the posters. Although the posters are "giving expression to pre-existing thought" (Sfard, 2007, p. 572) of the teacher, the way in which he invites the students to use them is essential to their mathematical thinking processes.

For example, in addition to asking questions about what was on the posters, the teacher would also ask students to demonstrate their understanding by pointing at the answer on the poster. This technique was also seen above, when the teacher used a parallelogram which he had drawn on the blackboard as a visual mediator.

Professeur	Exactement. Donc, les amis, assieds-toi, donc, les amis, si on regarde ici, j'ai indiqué où se trouve le 500ml et le 1000ml. Qui peut, qui peut venir me montrer où est-ce qu'on mettrait un quart d'un litre?
Élève	Oh! Je sais.
Professeur	Je sais que vous l'avez couvert l'année passée avec Mme Lemoy et avec Mme Thibodeau. Donc un quart d'un litre.
Élève	Oh, M. P!
Professeur	Ça va être où, un quart d'un litre? Julius, peux-tu me montrer?
Julius	(Pause) Ici?
Professeur	Exactement.

CG312May02a (Capacity, teacher-led), lines 43-49

In the French-language mathematics classroom there is evidence of initial questions in the teacher-led QRE routine involving visual mediators that require simple, non-verbal answers in order to demonstrate understanding. The teacher uses "...qui peut venir me montrer...?", and "...peux-tu me montrer?" to ask questions and the students demonstrate their understanding by pointing at the appropriate place on the poster. Gesture, therefore, is another possible answer to specific teacher-led QRE questions in the French-language mathematics classroom, along with short word or phrase, a number, or yes or no, as were mentioned when describing QRE routines.

Compared to the French-language mathematics classroom, there is less use of visual mediators in QRE routines in the English-language mathematics classroom. The teacher in the English-language mathematics classroom did use photocopied published worksheets, albeit less often than the teacher in the French-language mathematics classroom. In the excerpt below the teacher is focusing on the worksheet in a series of QRE exchanges while attempting to clarify the instructions printed on the sheet. This is different from the teacher in the French-language mathematics classroom, who used the practice worksheets to review concepts and test student knowledge.

Teacher	*Read*, read, read, read...Yes, sir. This is very confusing – I'm gonna take yours, okay. Okay, um, everybody, if you have not done, um, the second one, which is number eleven twelve, thirteen, where it says "Write the improper fraction represented by each group of diagrams", this is very confusing. Because all they really want you to do here is count one of these shapes how many pieces it is and this one's eight, even though I told Kier, incorrectly, that it was seven. So that's you what number? Where does it belong in your fraction? Yes?
Student 1	Top?
Teacher	No. Just this here.
Student 1	Oh, uh, bottom?
Teacher	The bottom. Okay – this is the *denominator* because it's how many pieces each thing is divided into. So then you count out of these two shapes all of the shaded ones. And I only figured that out because it says "improper fraction". I would've counted *all* of the pieces and I woulda

	said sixteen just like somebody else did, Julius, because it doesn't make sense to me, but, um , and I could be just the one missing out here, but, um so you count one shape, how many we have, that's your de-?
Class	-nominator? (repeated)
Teacher	The *bottom* number. Then you count the shaded pieces and that goes on the top. And the number of shaded pieces will be larger or smaller than the denominator? (Pause) Who's listening? How will they be – it's an *improper* fraction?

CG4ME13May23a (Improper to mixed fractions, teacher-led), lines 1-7

In this exchange the teacher can be seen to be attempting to establish shared ownership of the knowledge, one of the functions of QRE discourse as described by Mercer (1995). In using phrases such as "...all they really want you to do here..." (line 1) and ""...I woulda said sixteen just like somebody else did..." can serve to give students the impression that they are working together with the teacher to master challenging objectives related to fractions, thus possibly increasing student motivation.

As in the French-language mathematics classroom, the teacher in the English-language mathematics classroom made occasional use of the blackboard in order to communicate visual mediators. In this example, she is drawing circular representations of improper fractions (in this case 6/4) as a way to help students understand how to convert them to mixed numbers.

Teacher	(Counting and drawing on board.) So you took the six: one, two, three, four, five, six, and you took four away, and that gave you four over four, and that equals...?
Class	Two! / One. (repeated)
Teacher	*One.* How could it equal two? Four over four equals...?
Class	One. / Equals one.

CG4ME13May22a (Mixed and proper fractions, teacher-led), lines 220-223

Unlike the French-language mathematics classroom, where the teacher used the blackboard when students were struggling to demonstrate their understanding using the worksheet, the teacher in the English-language mathematics classroom used the blackboard to help students to understand how to complete an activity using manipulatives and document it in their notebooks. The

English-language mathematics teacher uses the blackboard proactively, using the blackboard as a first choice to share a visual mediator of her own creation with the class. This is in contrast to the reactive use of the blackboard in the French-language mathematics class, when the teacher used it to reproduce a shape from a worksheet after the students did not give the expected answer to his question.

Although worksheets and the blackboard were used as visual mediators in both language settings, there were two types of visual mediators that were used regularly in the French-language mathematics classroom that were not used in the English-language mathematics classroom. Neither concrete objects in the classroom, such as desks and the clock, nor teacher-generated posters were used as visual mediators by the English-language mathematics teacher in QRE routines. As in the French-language mathematics classroom, concrete objects from outside the classroom (such as the rain in the above example) were also used in the English-language mathematics classroom, often in conjunction with a visual mediator which was not seen in the French-language mathematics classroom: traditional mathematics manipulatives.

Teacher	Just hold on. So I'm gonna give you a whole pie, right here. See it?
Student	Thank you Mrs. Telfer!
Teacher	It's, um, actually cherry pie, my favourite.
Class	(Gasps) No! (repeated)
Teacher	Shhh! Excuse me, if you don't want a piece you don't have to have one.
Class	Ooooh! (repeated)
Teacher	If I cut four, if I cut it into four, ah, let's do this one, this is fun – I like the pink one 'cause it's more like cherries. Actually it's a strawberry-rhubarb pie. Um, I've got two pieces of strawberry-rhubarb pie that are halves, and they are exactly the same, they are exactly the same, as, my cherry pie. The same size, I can't eat all of that at one meal – it makes me sad, but I just can't; it would make me sick.
	...
Student	Mrs. Telfer – make a blueberry pie!
	...
Teacher	Um, here we go. This is the classic...

Class	Lemon meringue! / Lemon meringue pie!
Teacher	...lemon meringue pie.
Class	Yeah! (Cheers)
Teacher	And, I'm only going to have a piece.
Student	What about pecan pie?
Teacher	But that's the same thing – shhh! That's the same thing as one half of my cherry pie. Okay? So two, I'm getting two pieces of lemon meringue and only one of cherry which piece is the biggest?
Class	(Pause) They're equal. / Equal!

CG4ME13May23b (Equivalent fractions, teacher-led), lines 34-62

In this excerpt the teacher is using fraction circles on the overhead to help explain the concepts surrounding equivalent fractions. She is also using a common concrete visual mediator from outside of the classroom (pies) to help the students master the concepts. She is making every attempt to match the flavour of the pie to the colour of the fraction circles. The students are responding enthusiastically to her visual mediators, and answer the question in the QRE routine ("...which piece is the biggest?") correctly. It is possible that at this age, a year later than in the French-language classroom and into the junior, as opposed to primary grades, students are ready for more abstract, as opposed to concrete visual mediators.

Word use in QRE teacher-led discourse. Mathematical word use, the final category from Sfard, is similar in the French and the English language classrooms. During QRE routines, word use in both the English and French lessons, by both teachers and students, is standard English or French mathematical vocabulary, such as can be found in the corresponding mathematics curriculum documents. In fact, teachers in both languages were seen to make intentional, explicit efforts to ensure that students mastered mathematical vocabulary, and questions in the QRE routine sometimes centred upon vocabulary mastery. In Sfard's view, these words may be lexical innovations for the students, as they may never have previously encountered the specific mathematical terms, or they may be words with which the students are already familiar, for which they are now learning more specialized, specific meanings. In addition to new words or new meanings, in the French-language mathematics classroom, there is the additional possibility of mathematical vocabulary which is known to the student in English, but for which they are learning the French term.

In the excerpt below, the students likely already knew the terms “secondes, minutes, et heures” in French and in English. It is likely that they are learning an academic use of the words, as opposed to a more colloquial use, such as “Attends une minute/seconde!” or “On marche depuis des heures!” This movement from informal to formal use of mathematical words is evident in the excerpt below:

Professeur	Donc, on a on a déjà couvert la distance, qui peut me dire les, les unités de mesure que j’ai indiqué ici? (...) Pour la durée? Steven.
Steven	Secondes, minutes, et heures.
Professeur	Exactement.

CG3MF12May02a (Capacity, teacher-led), lines 15, 21-22

The teacher in the French-language mathematics classroom has made a poster about units of measurement, and works toward student mastery of this vocabulary by a series of QRE review questions. The student in this excerpt responds correctly, with correct pronunciation, despite the fact that “secondes” and “minutes” are very similar to the English words.

In addition to making posters and reviewing them, the teacher in the French-language mathematics classroom helps students to make connections in order to facilitate vocabulary acquisition:

Professeur	Exactement. Exactement. C’est pas liquide. Liquide – lorsque vous pensez « litre » j’aimerais que vous faites le lien avec « liquide ».

CG3MF12May07a (Fractions, teacher-led), line 276

In making the link between “liquide” and “litre”, he is helping students to acquire a mnemonic device to remember the appropriate unit of measure. He is not relying on the similar English vocabulary, perhaps recognizing that the concepts, as well as the vocabulary, may be new to the students in both English and French.

As will be further examined in the section below on exploratory dialogue, students in the French-language mathematics classroom tend to use code-switching and speak English when addressing each other. When engaging in teacher-led QRE routines, however they will respect classroom norms and expectations by seeking to answer questions in French, even if it is grammatically non-standard, as in the following example:

Kyle	Choisir moi – je sais, je sais!

CG3MF12May07a (Fractions, teacher-led), line 251

The student's message is perfectly clear, despite the grammatical error. His enthusiasm and eagerness to demonstrate his understanding are not affected by the fact that he is expected to communicate in French, his second language.

When students are not directly addressing the teacher, they did code-switch and use English during a class less on types of angles, as is seen in the next example.

Professeur	Qui peut me nommer la forme, la, le prochaine polygone? Comment s'appelle la prochaine polygone?
Élève 1	Oooh! Un quadrilatère…
Professeur	Un trapèze.
Élève 2	I said un trapèze.

CG3MF12May14a, lines 229-232

Professeur	Peux-tu aller me les écrire? Avec un a), où se trouvent les angles aigus?
Élève 1	Aigu!
Sara	Ummm…
Élève 2	Oh, I know where those angles (French) are.

CG3MF12May14a (Angles, teacher-led), lines 243-246

Interestingly, the mathematical vocabulary (trapèze, angle) is preserved in French, despite the student addressing his peer group at large in English. Other students have already attempted to answer the questions, but they do not answer correctly. Student 2 seems to address his peer group at large, appearing to still seek to demonstrate his understanding of the concepts.

In the English-language mathematics classroom there was no evidence of code-switching, despite the fact that this is their first year learning mathematics in English since kindergarten. There is, however, as in the French-language mathematics classroom, evidence of explicit teaching of mathematical vocabulary, such as the word "equivalent" in the excerpt below:

Teacher	But that's the same thing – shhh! That's the same thing as one half of my cherry pie. Okay? So two, I'm getting two pieces of lemon meringue and only one of cherry: which piece is the biggest?
Class	(Pause) They're equal. / Equal!
Teacher	They're equal. So they're called – what is the word that we use? Yes?
Class	Equal. (repeated) / Um, same? / Equality. / Equivalent.
Teacher	Equivalent. "Same" means the same thing but we have big words in math. Fun words. I'm going to write it down.

CG4ME13May23b (Equivalent fractions, teacher-led), lines 61-65

The teacher acknowledges that "we have big words in math", and writes the word on the blackboard. She does not, however, as was done in the French-language classroom, explicitly discuss mnemonic strategies to remember the mathematical vocabulary.

QRE in student-led Discourse. This routine of question-response-evaluation, so ubiquitous in teacher-led interactions in both languages of instruction, was not found in the student-led interactions in the data for the present study. This is not surprising, as both Mercer and Sfard describe this form of exchange as being initiated by the teacher. There are instances of student-led discourse that satisfy some aspects of a QRE routine, but none in the present data that satisfy enough criteria to qualify as a true QRE routine.

The following excerpt from the French-language classroom satisfies some of the criteria for a QRE routine. Although Kyle, who is working with Sara, seems very confident in the mathematics that he is doing ("It's easy.", "Just make him bring four..."), he does not, as is characteristic of a QRE routine, know the correct answer at the outset. Rather than posing the question to Sara, as would occur in a QRE routine, he is sharing his answer with her. Sara is not trying to demonstrate her knowledge, as she might have had the teacher posed the question in a QRE routine. Instead, she seems to be trying to catch up, and make sense of the question on her own ("I, I didn't even hear.", "(Reading) « …de kilogrammes… »"):

Kyle	Quatre boîtes. It's easy. (Reading) « Tim veut plus d'un kilogramme de biscuits pour partager avec ses amis. Combien de boîtes a-t-il besoin d'acheter? » He wants more than a kilogram.
Sara	I, I didn't even hear.
Kyle	Just make him bring four – quatre boîtes. Quatre boîtes ou trois boîtes.
Sara	(Reading) « …de kilogrammes… »
Kyle	Kilograms – he wants more than one kilogram. Four boxes, that's two kilograms...

CG3MF12May02b (Capacity, teacher-led), lines 107-111

Although this is a short exchange, with a closed initial question and a student who takes a leadership role, it cannot be defined as a QRE routine. Kyle's intention is not to verify Sara's understanding, nor does Sara seek to demonstrate her mastery of the material.

Student-led QRE routines are not a part of the English-language classroom either. In the excerpt below, featuring James and Kim, the initial question is not part of the discourse, as it has already been asked by the teacher. The students have used dice to roll a four and a six, and are now trying to convert the improper fraction that can be formed with the two numbers (6/4) into a mixed number. The question seems simple and closed, reminiscent of the example from the same classroom used in the section above on endorsement in teacher-led QRE routines ("...thirty divided by three"). In this exchange, however, James, who initiates the routine by "asking" the question ("Four plus six."), does not yet know the answer to the question.

James	Four plus six.
Kim	That would be, that would be, um...

James	Ones, and then there...
Kim	Then there'd be two, that would be two on four.
James	Yeah.
Kim	Because there's two...
James	Yeah.
Kim	Remaining in six, with the four. (Pause)

CG4ME13May22b (Mixed and proper fractions, student-led), lines 84-91

The resulting efforts by both students, combined with their endorsement (of an incorrect answer – 2/4) at the end of the routine ("Yeah." "Remaining in six, with the four.") are more characteristic of an exploratory dialogue, which will be defined and discussed in detail in the next section.

Despite some elements of the above excerpts that are similar to teacher-led QRE routines, there are no clear examples of QRE discourse in student-led routines. Even when the routine begins with a short, closed question, and even when a student is able to take on a leadership role because he/she feels that he/she has found the answer quickly and confidently, the dynamic between the student partners has not been one where one is testing the knowledge, and the other is demonstrating it. The collaborative, or at the very least parallel nature of the work in student pairs in these classrooms in order to complete the task precludes any student-led QRE routines in the present data.

QRE routine summary. Routines that are characterized as question-response-endorsement (QRE) are common in learning situations in a wide variety of settings. They are used in settings where one interlocutor, the one posing the questions and making decisions about endorsement, is more "powerful", through greater knowledge and/or decision-making authority. This routine is often used to establish continuity, emphasize a set of salient points, to test recall, to provide feedback, and to create a sense of shared ownership of knowledge among the community of learners (Mercer, 1995). The details of the routine itself can be described, as can the particularities of endorsement, visual mediators, and word use within the routine.

As in classroom settings around the world, QRE routines are common features in the mathematics discourse in both the French and English language mathematics classrooms. The features of the routines themselves in teacher-led settings: an initial short, closed rhetorical question for which there is one answer, a brief response by a student, often followed by an endorsement by the teacher are common to both the English and the French language classrooms. A concern for correct mathematical word use is also seen in both classrooms. Differences are seen, however, in the other two categories of discourse analysis. One of these areas is endorsement, where there has been evidence of consistent endorsement in the English classroom, but less regular endorsement in the French classroom. Visual mediators are different in the two classrooms as well, as the two teachers often use different visual mediators in their teaching practice. The ubiquitous nature of the QRE routine in teacher-led discourse stands in contrast to its absence in student-led discourse. When student peers worked together, in either French or English language mathematics classes, this routine was not in evidence. This is perhaps due to a lack of a power imbalance in peer relationships, or the reluctance of one peer to play the role of questioner to another peer. Social norms in these classrooms preclude one peer testing recall, providing feedback, or endorsing another in a naturalistic classroom setting.

The QRE routine, therefore, is very teacher-centred in the settings of this study. Despite its utility, as outlined in its many uses above, QRE routines do have their drawbacks. Mercer (1995) cautions that in the scope of these routines there is little opportunity for students to make an active contribution to learning. During QRE routines only a few students typically contribute, and they are often stronger students, chosen by the teacher to reinforce the goals of the lesson and of the routine. However, Mercer asserts that there is no need to dismiss QRE routines all together and adopt exclusively student-centred teaching styles or techniques, as students benefit from a wide, balanced repertoire of communicative activities. In addition to the teacher-centered, ritualized routine of QRE, students require opportunities to "...recall and apply relevant knowledge without teacher prompting" (Mercer, 1995, p. 38). As is shown in the next section, there is ample evidence in this study of a different routine in both classrooms which has the potential to allow students to do just that.

Exploratory Discourse Routine

In contrast to the QRE routine is the routine of exploratory dialogue. Unlike the short, well-defined routine of Question-Response-Endorsement, an exploratory dialogue routine is longer and less prescriptive. An exploratory dialogue begins, like the QRE exchange, with a question, which is then followed by a response. Rather than the endorsement (positive or negative) which follows the response in a QRE routine, the response in an exploratory dialogue is followed by a series of probes by the person who asked the initial question and further responses, which may be made by the person who answered the original question, or by other members of the class. At the end of the exchange, as in the QRE routine, a response may or may not be endorsed.

The excerpt below is an example of an exploratory routine from the French-language mathematics class. The initial question, posed by the teacher following another discussion, was how many times the weight of Timothy, a student in the class, does a car weigh? The class begins to propose answers ("trois cent...cinq"), and the teacher responds ("Cinq cent? Ou cinq? Cinq kilogrammes?"). The dialogue continues, with students and teachers offering answers ("Cinq cent."), further questions ("…penses-tu qu'un Smart Car peut peser le même que moi?"), and clarifications (“a limo?” “Oui, une limousine, exactement.”). The exchange ends with an (improbable) endorsement («…peut être deux cent kilogrammes »). Although the French teacher-led exploratory discourse routine begins similarly to the teacher-led QRE routine, with a closed question requiring a simple, one-word answer (a number, in the example below), there are important differences between the teacher-led exploratory routine and the QRE routine in the French-language mathematics classroom. In the QRE routine, the questions, as well as the answers, were so simple that no follow up explanations were necessary. In an example of teacher-led QRE routine in the section on QRE routines above, the initial question “Maintenant – le b). C’est quelle forme, le b)?” was quickly followed by the correct response “Un triangle!”. In the example of teacher-led exploratory routine below, the teacher quickly provides details about his initial question (“Si ça c’est trente ou trente-cinq kilos...”) before the students even begin answering, perhaps recognizing the increased complexity of the initial exploratory question “Donc, on pense que Timothy pèse à peu près trente, ou trente cinq, si on pense à une voiture, combien de fois pensez-vous?” as opposed to the initial QRE question of “Maintenant – le b). C’est quelle forme, le b)?”. In immediately adding detail, even in the absence of obvious cues from students, he is perhaps anticipating questions and proactively providing clarification. Other important differences between the two routines emerge as the exploratory routine progresses. After the initial question, teacher-led exploratory routines continue with a series of follow-up questions, or probes. QRE routines may also feature a series of questions after the initial question, but these follow-up questions are very different from those in an exploratory routine. Although the questions that followed the initial question in the QRE routine cited above are on the same topic as the initial question, each question and answer pair can stand alone, as its own QRE routine, such as: «Est-ce qu’il y a un angle obtus là-dedans? Non! / Non! », « Un angle droit? Non! / Non! », and « Un angle aigu? Oui! / Oui! / Oui! ». In contrast, the follow-up questions after the initial question in the exploratory routine in the French-language mathematics classroom do not stand alone. They serve to guide the discussion about the initial question, in a way that I have termed "chained scaffolding". The follow-up questions in this exploratory routine, such as "Cinq cent? Ou cinq? Cinq kilogrammes?" and "Donc, penses-tu qu'un Smart Car peut peser le même que moi?" cannot stand alone in this context, but can be seen as a way of leading the discussion towards a final answer.

Professeur	Donc, on pense que Timothy pèse à peu près trente, ou trente cinq, si on pense à une voiture, combien de fois pensez-vous? Si ça c'est trente ou trente-cinq kilos...
Classe	(Indistinct buzz)
Professeur	…pour Timothy. Combien pensez-vous?
Classe	…trois cent / cinq…
Professeur	Cinq cent? Ou cinq? Cinq kilogrammes?
Kyle	Cinq cent.
Professeur	Cinq cent?
Classe	Pourquoi?
Professeur	Ça dépend aussi…vous êtes supposés de dire à Monsieur P. « Quelle sorte de voiture? » Si c'est une voiture Smart, pensez-vous qu'elle va être le même poids qu'un...je sais pas…
Emily	A limo?
Professeur	Oui, une limousine, exactement.
Classe	Non! Smart car's only like…a truck…
Professeur	Ben non, ça se peut pas...
James	A Ford F-1 50.
Professeur	M. P. pèse à peu près ca, malheureusement, donc penses-tu que je suis la même grandeur que…
James	You weigh…
Professeur	Non, pas vraiment. J'ai dit à peu près. Donc, penses-tu qu'un Smart Car peut peser le même que moi?
Classe	Non!
Professeur	Non.
James	But a Ford F-1 50…
Professeur	Peut-être deux cent kilos…peut être deux cent kilogrammes; kilos kilogrammes c'est la même chose.

CG3MF12May07a (Fractions, teacher-led), lines 57-77

The exploratory routine also occurred in the English-language mathematics classroom. In the excerpt below, the initial question about the definition of the word “denominator” is once again posed by the teacher. A number of different students propose definitions (“...the fraction you’re using...”, “...the whole number”), followed each time by a comment from the teacher (“You’re on the right track. How do I find out what the whole number is?”). At the end of the routine, she endorses an answer (“Count *all* of the parts.”).

Teacher	The denominator, what is the denominator, can someone give me the definition? Yes?
Claire	Ah, the, the...fraction that you’re using, like fifths or sixths.
Teacher	You’re on the right track. Yeah?
Student 1	It’s the whole number.
Teacher	The whole number. How do I find what the whole number is?
Student 2	Oh!
Teacher	Yes?
Student 2	Count every single one?
Teacher	Count *all* of the parts. Okay? ...

CG4ME13May27c (Equivalent fractions I, student-led), lines 39-47

The exploratory routine is longer, more variable, and more complex than the QRE routine, and is therefore worthy of close examination in teacher-led and student-led discourse, in both French-language and English-language classrooms.

Teacher-led exploratory discourse. Exploratory dialogue is common in teacher-led discourse in both the French-language and the English-language mathematics classrooms in this study. When compared to QRE routines, exploratory routines are longer and more variable. In contrast to some of the QRE routine’s more common goals of eliciting known information, confirming knowledge, or recapping shared knowledge (Mercer, 1995), exploratory routines use different techniques to help students learn in a more constructivist manner, what Mercer calls the guided construction of knowledge. An analysis using Sfard’s four categories reveals the similarities and differences of this routine in the two language contexts.

Routine in teacher-led exploratory discourse. Although exploratory dialogue forms may be more variable than QRE patterns, the routine of teacher-led exploratory dialogues in French and English language mathematics classrooms share some important characteristics. In both French and English teacher-led exploratory dialogues the teacher initiates the routine with a question, a student is chosen to respond, and then the teacher chooses and initiates a probe. At this point, in both the French and English classrooms, other students may contribute to the routine. A variable number of additional probes and answers follows, and the dialogue may or may not end with an endorsement from the teacher.

The excerpt below, from the French-language mathematics classroom, is a clear example of the exploratory routine of initial question, initial response, a series of probes and responses, and an endorsement:

Professeur	Combien d'adultes est-ce que j'aurai besoin pour mesurer une voiture?
Classe	Oh! / Je sais! / Huit!
Professeur	Shhhh! Shhhh! On lève notre main.
Student	Monsieur!
Professeur	Regardez, les amis : pour, parce que pas tout le monde a leur main levée. Si une voiture est égale, donc ca balance avec quatre adultes, et deux enfants…
Student	Oh!
Professeur	…mesurent le même (assieds-toi, Benjamin) que un adulte, combien d'enfants est-ce que j'ai besoin pour …
Classe	Oh! / Je sais! Je sais! / Monsieur!
Professeur	…pour mesurer le même qu'une voiture?
Kyle	Oh! Je sais! Je sais! Huit!
Professeur	Mais regardez : il y a un piège, les amis. Moi je veux savoir quelle *stratégie* vous avez utilisée.
Classe	Oooh! / Je sais! / Oh!
Professeur	Donc – pas juste une réponse, je l'ai dit d'avance, j'ai dit pas juste votre réponse…
Classe	Je sais! / Je sais!/ Je sais!
Professeur	Mais je veux la stratégie.
Classe	Je sais! / Oooh! / La stratégie! /Monsieur! Monsieur! Monsieur!
Professeur	Melissa.
Classe	Ahhhh. /Awwww.

Professeur	Combien de, combien d'enfants est-ce que j'ai besoin?
Melissa	(Inaudible)
Professeur	Huit. Et quelle stratégie est-ce que t'as utilisée?
Melissa	J'ai fait, ah, deux, quatre,… huit.
Professeur	Okay, donc, elle…
Classe	Je sais, je sais!
Professeur	C'est comme une suite, okay : deux, mais le deux représente quoi?
Classe	Deux enfants. / Un, a, un adulte.
Professeur	Un adulte? Qui est égale a deux enfants. Donc, vous avez deux, qu'est-ce qu'il fait après deux?
Classe	Quatre. / Quatre. / Quatre, six, huit.
Professeur	Donc, ce qu'elle a fait, elle a compté par deux, donc tu as fait des groupes de deux enfants pour chaque adulte. Est-ce qu'on a une autre stratégie qu'on peut utiliser, les amis?
Classe	Oh! / Oh!/ Oh! / Monsieur!
Professeur	Adam.
Adam	Two times two…(?)…um, quatre fois deux.
Professeur	Quatre fois deux. Donc, ou on peut dire deux fois…?
Classe	Quatre. / Quatre.
Professeur	Donc, au lieu de dire « fois », on peut, on peut remplacer le mot « fois » avec quoi?
Student	Oooooohhh!
Professeur	Deux, deux groupes de…?
Student	Quatre.
Professeur	Deux groupes de quatre, exactement.
Lee	C'est la même chose, mais c'est deux fois parce que c'est deux égale un.
Professeur	Exactement. Maintenant : je veux le faire un peu plus, je veux voir…ça.

French: CG3MF12May07a (Fractions, teacher-led), lines 342-376

In the excerpt above we see clearly see the initial question from the teacher, a series of answers from a variety of students and a series of probes, such as “Donc – pas juste une réponse”, and “ ... mais le deux représente quoi?” from the teacher, as well as a final endorsement (“Exactement”) from the teacher. The endorsement in this case indicates the end of this routine, which is significantly longer than the QRE routines, and the “Maintenant...” signals the beginning of a new exchange.

The excerpt below, from the English-language mathematics classroom, is a similarly clear and typical example of teacher-led exploratory dialogue, following the routine of initial question, initial response, a series of probes and responses, and endorsement:

Teacher	The numerator is the top number and what did, what did, what's the definition of the numerator?
James	(Whisper) What did Easton do?
Student 1	Oh!
Teacher	Put your chair down and sit it up there, please! Turn it in there, now. (Pause) What does the top number tell us? Kyle.
Kyle	Um, it's the...
Teacher	Shhh.
Kyle	...it's the part of the fraction that's missing? That's missing one? Or two?
Teacher	Okay, you're so close to getting it.
Student 1	Oh!
Teacher	I know you understand what it is. Yes?
Student 1	Whatever number after you take...whatever number you take away it's the number that's left?
Teacher	Right, that's what Kyle said, but we could even describe it differently. Claire?
Claire	Um, the number on, the numerator is the number of, the number of frac, um denominators that are under, like, six, um, five sixths, for example, there's five of five sixths because...
Teacher	Yes, there is.
Claire	...you can take away one.
Teacher	I know, but I don't want to know the numbers I want words to say what that represents 'cause that could be any number. Yes?
Claire	It repre...the num, the number that you take away...
Teacher	We already said that, yes.
Claire	...you, um, you subtract it from the whole number, then the answer of your subtraction is the numerator.
Teacher	Right! Okay, I could say it in a simpler way, but it's very good, you *do* understand it. Yes.
Lee	It's the portion.
Teacher	Thank you!!
Lee	I just (unintelligible)
Teacher	Okay? This is the number of portions, right?
Student	(Laughing) It's the portion!
Teacher	The number of portions that's left. And nobody just said that but you all understand it, those people that spoke definitely understand it. So, (laughs), all right, the denominator now – what's that?

English: G4ME13May22a (Mixed & proper fractions, teacher-led), lines 144-169

As far as routine is concerned, the previous example from the English-language mathematics classroom is very similar to the above excerpt from the French-language mathematics classroom: an initial question from the teacher, a series of answers from a variety of students and a series of probes from the teacher, such as as “Okay, you’re so close to getting it” and “...but we could even describe it differently”, which serve to signal to students to continue to hone their answers. The routine finishes with a final endorsement (“The number of portions that’s left.”) from the teacher, which indicates that she is satisfied with the answer to her original question, and that she will be moving on to a new routine. Despite these general similarities the exploratory dialogue in the French and English language mathematics classroom is different in each of Sfard’s four categories of analysis.

Upon closer examination of the routine itself in the French language classroom, the initial question is closed, requiring a specific, short, one-word, numerical or action-based answer. The expected answer is prescribed – the range of acceptable responses is very narrow. The teacher often repeats or reformulates the question, without an explicit request from the students, as is seen in the following example:

Professeur	Qui peut, qui peut venir me montrer ou est-ce qu’on mettrait un quart d’un litre?
Classe	Oh! Je sais.
Professeur	Je sais que vous l’avez couvert l’année passe avec Mme Lemoy et avec Mme Thibodeau. Donc un quart d’un litre.
Classe	Oh, M. P!
Professeur	Ça va être ou, un quart d’un litre? Julius, peux-tu me montrer?
Julius	(Pause) Ici?
Professeur	Exactement. Et combien est-ce un quart d’un millilitre, combien c’est en millilitres?
	(Indistinct)
Professeur	Peux-tu, peux-tu me l’écrire, s’il te plaît? Très bien. (Pause) Pourquoi est-ce que c’est 250, et non…200?
	Oh ! Oh!
Professeur	Qui peut me dire pourquoi c’est 250 et non 200? Joël?
Eleve 1	Um, tu prends 25 plus 25…
Professeur	Mmm-hmm.
Eleve 1	C’est 50, so then 200 et 50 c’est deux cent et cinquante est égale à 500.
Professeur	Exactement. Tu as une bonne stratégie. Donc, Joël a enlevé l’unité, donc il a, il a vraiment compté le 25 et le 25 qui donne 50 et ensuite il a rajouté l’unité qui va à la fin. Est-ce que quelqu’un a une autre stratégie

pour utiliser?

CG3MF12May02a (Capacity, teacher-led), lines 43-57

In the above example the teacher in the French-language classroom has asked a closed question to which he wants the students to respond by showing the answer on a poster about capacity which he has made. He reformulates the question many times, after the initial question : “Qui peut, qui peut venir me montrer où est-ce qu’on mettrait un quart d’un litre? », saying « Donc, un quart de litre. », and « Ça va être où, un quart d’un litre? Julius, peux-tu me montrer? ». After beginning his exploratory dialogue with a simple, closed question, the teacher follows this question up with other similar simple, closed questions, asking: “Et combien est-ce un quart d’un millilitre, combien c’est en millilitres?” and “Peux-tu, peux-tu me l’écrire, s’il te plaît?” This style of exploratory questioning routine, a series of simple, closed questions on the same topic, is typical of the French-language mathematics classroom. It can be described as “chained scaffolding”. Even in the absence of a student request or an evident indicator that students are having difficulty, the French-language mathematics teachers guides the exploratory routine in a way that the English-language mathematics teacher did not. Through the use of closed questions, reformulation, and use of short, closed questions on the same topic, the French-language mathematics teacher leads the learning routine in a much more direct way than the English-language mathematics teacher.

In contrast, in the English language mathematics classroom an exploratory dialogue begins with a question that is very different from the ones that begin exploratory dialogues in the French classroom. These questions are often more open, requiring a more complex and detailed answer, and therefore render a wider range of responses acceptable. These questions also stand out in direct contrast to the teacher’s own QRE questions, in form, answer required, and timing within the lesson. In the following example, the initial question, as well as the following probes, are very general:

Teacher	How do I find the *denominator,* let's go there.... Yes?
Student	It's the, uh, the ones that *aren't* shaded.
Teacher	The denominator, what is the denominator, can someone give me the definition? Yes?
Student	Ah, the, the...fraction that you're using, like fifths or sixths.
Teacher	You're on the right track. Yeah?
Student	It's the whole number.
Teacher	The whole number. How do I find what the whole number is?
Student	Oh!
Teacher	Yes?
Student	Count every single one?
Teacher	Count *all* of the parts. Okay? The ones that are shaded.

English transcript: CG4ME13May27c (Equivalent fractions, teacher-led), lines 37-47

In the above example the question "How do I find the denominator?" is open, inviting a large range of responses to the question. Open questions such as these, and the exploratory routines that accompany them, appear later in the lesson than many of the questions in QRE routines in both the English and the French-language mathematics classrooms.

Endorsement in teacher-led exploratory discourse. Endorsement is another area where exploratory teacher-led discourse differs between the two language settings. As was noted in the QRE routine, the responses to the exploratory dialogue in the French language classroom are not consistently endorsed. From a researcher's point of view, it was evident that when an answer in a QRE routine went unendorsed, it was given as understood that the answer given was correct, and the teacher was therefore moving on to the next query. Unendorsed exploratory dialogues, however, seemed "unfinished", and lacking in either an explicit or implicit conclusion.

In the example below, the teacher in the French-language mathematics classroom the teacher asks a question which immediately engages the students. There is a long discussion about which weighs more, an elephant or a car, but a final answer is never endorsed.

Professeur	On lève notre main. T'as la bonne réponse. Hmmm….Lequel pèse plus : un éléphant ou une voiture : Lequel pensez-vous pèse le plus?
Classe	Un éléphant!
Professeur	Un éléphant? Est-ce qu'un éléphant peut peser 200 grammes?
Classe	(indistinct)
Professeur	Deux cent grammes.
Classe	Non…mmm…
Professeur	'Gardez.
James	It's impossible!
Professeur	Ca c'est notre deux cent cinquante grammes.
James	They can't, they can't, they…weigh grams! Its way, way...!
Professeur	Deux cent cinquante grams, c'est ça. Est-ce qu'un éléphant pèse ça?
Classe	Non, non…
Élève	Like a billion of those!
Élève	Not even!
Élève	Two billion!
Professeur	Si ça c'est un kilogramme, combien de kilogrammes vous pensez que Timothy pèse? Si ça c'est un kilogramme?
James	How much do you weigh, Timothy?
Professeur	Combien de poids d'un kilogramme (donne ça à...), combien de fois pensez-vous pour avoir…
Kyle	Ahh…deux mille.
Professeur	Combien de fois? Pas deux mille!
Élève	A hundred and twenty-eight pounds!
Élève	Cinq!
Professeur	Combien?
Élève	Vingt cinq.
Professeur	On a vu qu'un bébé pèse combien de kilogrammes?
Classe	Sept! Six! Douze! Trois. Cinq!
Profeseur	Trois ou quatre!
Classe	No!...
Profeseur	Quatre c'est un grand bébé! Mais! Donc, si un bébé est à peu près trois kilogrammes, alors trois fois ça, combien pensez-vous que Timothy pèse?
Élève	Vingt-cinq grammes. (?)
Professeur	Combien? Oui. Vingt huit?
Élève	Vingt cinq.
Professeur	Très bien. Combien pensez-vous de celui-là on a besoin de kilogrammes pour…
Classe	Moi! Monsieur!!

Professeur	Donc, on pense que Timothy pèse à peu près trente, ou trente cinq, si on pense à une voiture, combien de fois pensez-vous? Si ça c'est trente ou trente-cinq kilos...
Classe	(indistinct)
Professeur	…pour Timothy. Combien pensez-vous?
Classe	…trois cent…cinq…
Professeur	Cinq cent? Ou cinq? Cinq kilogrammes?
Kyle	Cinq cent.
Professeur	Cinq cent?
Classe	Pourquoi?
Professeur	Ça dépend aussi…vous êtes supposés de dire à Monsieur P. « Quelle sorte de voiture? » Si c'est une voiture Smart, pensez-vous qu'elle va être le même poids qu'un...je sais pas…
Emily	a limo?
Professeur	Oui, une limousine, exactement.
Classe	Non! Smart car's only like…a truck…
Professeur	Ben non, ça se peut pas...
James	A Ford F-1 50.
Professeur	Monsieur P. pèse à peu près ça, malheureusement, donc penses-tu que je suis la même grandeur que…
James	You weigh…
Professeur	Non, pas vraiment. J'ai dit à peu près. Donc, penses-tu qu'un Smart Car peut peser le même que moi?
Classe	Non!
Professeur	Non.
James	But a Ford F-1 50…
Professeur	Peut-être deux cent kilos…peut être deux cent kilogrammes; kilos kilogrammes c'est la même chose.
Classe	Kilos/kilogrammes…
James	Kilo (F) is a kilogramme
Professeur	Exactement.
James	Exact same thing.
Professeur	Exactement.
James	If you guys were listening you would know that.
Professeur	D'accord, ce qu'on va faire aujourd'hui, je vais vous passer quelques feuilles, et vous allez les faire. Mais avant de faire ça…

French: CG3MF12May07a (Fractions, teacher-led), lines 23-84

As in the first example of exploratory routine in this section, about how many more times Timothy weighs than a car, we see the typical exploratory routine in the French-language mathematics classroom. The above routine began with a closed question requiring a short answer: "Lequel pèse plus: un éléphant ou une voiture », which is immediately followed by a reformulation: "Lequel pensez-vous pèse le plus?" . This initial question is once again followed by chained scaffolding, a series of related questions such as "Est-ce qu'un éléphant peut peser 200 grammes? ", and "On a vu qu'un bébé pèse combien de kilogrammes?", attempting to lead the discussion towards a final answer. As in the first example, about Timothy and the car, this question does not end in an endorsement of the original question about the elephant and the car. In the above example, as well as in the first one, it is possible that the teacher is unsure what the answer is, and is therefore unable to endorse a correct answer. Below is a shorter example, where the correct answer is surely known by the teacher; however the definitive answer to the original question remains unendorsed.

Professeur	...un demi…plus…deux quarts.
Classe	(indistinct)
Professeur	'Garde sur le deux quarts – c'est où le demi?
Classe	(indistinct)
Professeur	Non, sur le quart. Je l'ai séparé en quatre parties.
Classe	(indistinct)
Professeur	Exactement. Deux quarts est équivalent, est égale à quoi, deux quarts?
Classe	Un demi! / Demi. / Deux quarts à un demi.
Professeur	Exactement.
Classe	(indistinct)
Professeur	Donc, qui veut venir me montrer, si je prends un litre...moins un tiers de litre, j'aurais combien?

French: CG3MF12May07a (Fractions, teacher-led), lines 189-227

With the lack of an endorsed answer to the original question ("un demi plus deux quarts") the exploratory routine in the above transcript has an unfinished, unsatisfying feel to it, as it is difficult to understand the place of these routines in the classroom if a final answer is not endorsed, nor any sort of a summary given.

In contrast, answers to questions in exploratory dialogues were consistently endorsed in the English classroom, as they were in QRE routines. In the excerpt below, as in many of the teacher-led exploratory dialogues in the English-language classroom, the endorsement is clear and is directly related to the original question:

Teacher	Anything that does not have a portion to it is a whole number. Okay, so what is, um, a fraction?
Student	Two hundred and fifty-seven.
Teacher	Kathleen.
Kathleen	Something over something.
Teacher	Something over something, help me out, do you have something else?
Student	(Unintelligible)
Teacher	Pardon?
Student	(Unintelligible)
Teacher	Kyle?
Kyle	Four over five?
Teacher	No, I didn't ask you to give me a fraction, I said to tell me what a fraction is. Yes? You know what, that's the same thing that she said, go ahead.
Student 1	An example would be...
Teacher	No, I don't want an example I want a definition.
Student 1	Four pieces of a pie.
Teacher	Pardon?
Student 1	Four pieces of a pie.
Teacher	Four pieces of a pie? How, how do I know what a whole pie is? ...'kay, I'm still looking for a clear definition, shout it out there. ...Put that in your desk now. Did you put yours...Everybody put their CASI tests in their desk. That's what I said five minutes ago.
Student	I was reading a book...
Claire	Something, something that doesn't have a portion to it?
Teacher	I think you're close to what I'm looking for as an answer. Yes?
Claire	Something that isn't divided.
Teacher	A fraction is something that isn't divided?
James	No, a fraction...
Teacher	Shhh! You didn't put up your hand. Yes? Are you, do you want...?
Claire	A whole is something that isn't, in half, and there's nothing missing from it. It...
Teacher	There's nothing missing from something that's a whole. Okay, before I eat my orange it's a whole. Keep going. Shhh. Do you have more to say? James? A fraction, you're talking about a fraction. Yes?
James	Ummm, if you ate a piece it would go down, to, ah, to maybe half, and then you would have a portion.
Teacher	Whoever's talking, stop. Oh, there's that word – portion. So a fraction is...?
Student 2	A portion.
Teacher	A portion of something. So it's something that's not a whole. Yes, anything else?
Student 2	Well, it's like, a whole...
Teacher	Put this down and don't play with it.
Student 2	Well, it's like, say you have a (pie?), and you eat half of it, that'd be half of a...

Teacher	And that's a fraction of it? Because it's just a...?
Class	Half. / Portion.
Teacher	Portion. Yes?
Kathleen	It's like, if I have eight of something, and then I take away two, it's six over eight.
Teacher	And that is a...?
Class	Portion! (whisper). Fraction.
Teacher	Fraction or a portion. So now we know that a fraction of something, a portion of something is not the *whole* thing. It's only part of it. So a fraction is a part of something. All right: we have two numbers in a fraction...you all know these two numbers...so; if we have something that's complete it's a whole. If we have something that's a fraction...

English: G4ME13May22a (Mixed & proper fractions, teacher-led), lines 72-112

In the English-language classroom the initial question ("Okay, so what is, um, a fraction?) and the endorsement ("So a fraction is a part of something....") are closely linked. Using language from the students' responses ("something", "portion", whole"), the teacher reformulates the various responses to endorse a definitive answer to her original question. This "finishes" this portion of the discourse and providing a foundation for further learning.

Visual mediators in teacher-led exploratory discourse. The category of visual mediators is another area in which exploratory dialogue routines differ in French and English language classrooms. The teacher in the French language mathematics classroom made extensive use of visual mediators during exploratory dialogue routines. As in the QRE routine, he consistently made reference to concrete physical objects (standardised weights), students themselves, himself, and often made anchor charts which he encouraged students to use as reference as well as to demonstrate their understanding. He also referred to objects which were not in the classroom. Also as in the QRE routines, these concrete objects are essential to the communication and cognition of young learners.

In the excerpt below from the French-language mathematics classroom, the teacher is making use of a visual mediator he has made, an anchor chart of capacity measurement using litres and millilitres:

Professeur	Qui peut, qui peut venir me montrer ou est-ce qu'on mettrait un quart d'un litre?
Classe	Oh! Je sais.
Professeur	Je sais que vous l'avez couvert l'année passe avec Mme Lemoy et avec Mme Thibodeau. Donc un quart d'un litre.
Classe	Oh, M. P!
Professeur	Ça va être ou, un quart d'un litre? Julius, peux-tu me montrer?
Julius	(Pause) Ici?
Professeur	Exactement. Et combien est-ce un quart d'un millilitre, combien c'est en millilitres?
	(Indistinct)
Professeur	Peux-tu, peux-tu me l'écrire, s'il te plaît? Très bien. (Pause) Pourquoi est-ce que c'est 250, et non…200?
	Oh ! Oh!
Professeur	Qui peut me dire pourquoi c'est 250 et non 200? Joël?
Eleve 1	Um, tu prends 25 plus 25…
Professeur	Mmm-hmm.
Eleve 1	C'est 50, so then 200 et 50 c'est deux cent et cinquante est égale à 500.
Professeur	Exactement. Tu as une bonne stratégie. Donc, Joël a enlevé l'unité, donc il a, il a vraiment compté le 25 et le 25 qui donne 50 et ensuite il a rajouté l'unité qui va à la fin. Est-ce que quelqu'un a une autre stratégie pour utiliser?

CG3MF12May02a (Capacity, teacher-led), lines 43-57

In the above example, the teacher in the French-language mathematics classroom is inviting students to demonstrate their understanding by indicating the answer on a poster he has made ("...peux-tu me montrer?") or to write it on the blackboard ("...peux-tu me l'écrire, s'il te plaît?").

The teacher also encourages his students to refer to anchor charts in order to solve problems about capacity, as in the transcript below:

Professeur	Donc, je prends un litre, j'enlève un demi-litre. 'Gardez, les amis,

	'gardez l'affiche.

CG3MF12May07a (Fractions, teacher-led), line 250

In saying "'Gardez, les amis, gardez l'affiche." the teacher promotes the use of the anchor charts to solve problems, perhaps sensing that his students need prompting in order to make use of these visual mediators in this way when an answer is not immediately forthcoming.

In this next excerpt, the teacher is once again using visual mediators to assess student understanding. This time he uses concrete, everyday objects rather than an anchor chart:

Professeur	Donc, si vous regardez on, on (unintelligible) les choses dans cette classe et j'aimerais que vous me dites, je vais vous demander de me donner les choses qui ont un angle *aigu* dans cette classe. Regardez autour de vous. Qu'est-ce qui a un angle aigu?
Benjamin	Monsieur! Monsieur P!
Professeur	Benjamin.
Benjamin	Le, uh, (en haut?) comme ça?
	…
Professeur	Oh, tu veux dire les aiguilles? Les aiguilles de l'horloge? Okay – lesquelles sont aigus présentement? …Maintenant, lesquelles sont, sont aigus?
Élève	Le rouge, le rouge et le long.
Professeur	Celui de l'heure, ou de minute?
Classe	Minute. / Minute. / Minute.
	…
Professeur	Exactement. Un et deux. Donc…

CG3MF12May14a (Angles, teacher-led), lines 105-153

In the above excerpt, a student has correctly pointed to the hands of the clock on the wall in order to demonstrate his understanding of acute angles.

In the French-language mathematics classroom, visual mediators were not only used to assess student understanding. The teacher also used them to present, explain, and clarify problems that he is presenting to the class. In the following example from the French-language mathematics classroom, the teacher draws a picture on the blackboard which he uses as a visual mediator, to help coordinate mathematical discourse in the classroom:

Professeur	…les amis. Si j'ai une balance ici…
Kyle	Oh man – I want to get a drink!
Classe	(indistinct)
Kyle	Hey! One of them's me! That's exactly how I draw myself!
Classe	(indistinct)
Kyle	One of them's me!
Professeur	Okay: si une voiture mesure quatre adultes – ça c'est une très petite – Benjamin – si une voiture mesure quatre adultes comme Monsieur P., on va dire une petite auto, pas une grande auto…
Classe	(indistinct)
Kyle	Pas un Ford F-1 50! (Rire).
Professeur	Sssshhhh!
Classe	No Rams / My Dad's Ram's…/ a huge truck / a tow truck

CG3MF12May07a (Fractions, teacher-led), lines 322-333

In the excerpt above, the teacher in the French-language mathematics class is presenting a problem about equivalency of mass by drawing on the blackboard. He draws a see-saw type balance with various vehicles and/or people on each side. The students are engaged and curious about his drawings. This line of questioning continues for quite some time, as is demonstrated by the excerpt below, which comes from later during the same lesson:

Professeur	Exactement. Maintenant : je veux le faire un peu plus, je veux voir…ça.
Classe	Oh monsieur! / Race car! / Rocket! / An airplane! / Un avion! / An airplane! / Un avion!
Kyle	Pèse combien de voitures?
Élève	A rocketship.
Kyle	A couple of Ford F-1 50's would do the trick.
Classe	...Monsieur...
Élève	Ford F-1 50's suck!
Professeur	C'est supposé être un avion, les amis!
Kyle	A Ford F-1 50! A Ford F-1 50!
Classe	It's not a car! / Shut up! / It's a truck!
Professeur	Shhhhh!
Classe	...truck...
Professeur	Okay, les amis, ça c'est une bicyclette, d'accord?
Classe	D'accord.
Professeur	Il faut dire que Monsieur P n'est pas bon en dessin.
Kyle	Oui! Si tu veux je peux faire la bicyclette!
Classe	(indistinct)
Kyle	I'm okay at bicyclettes
Classe	I am…/ I'm really good at drawing…/ drawing a bicyclette…/ motorcycles are simple… / I know how to draw a bike fine. / Avions are

	better. /
Kyle	Too bad!
Professeur	Et un camion est égale a quatre bicyclettes…ça, non - motocyclettes. ..
Kyle	Motorcycles?! Motorcycles?!
Professeur	…combien de motocyclettes j'ai besoin pour un avion?

CG3MF12May07a (Fractions, teacher-led), lines 376-399

Despite the fact that the teacher is not skilled at drawing (« M. P n'est pas bon en dessin."), the students remain engaged while he works to present the problem using his chosen visual mediator.

The teacher in the French-language mathematics class also uses photocopied worksheets as visual mediators to present problems, as in the excerpt below:

Professeur	Ah – tu veux dire deux cent cinquante? Okay les amis – est-ce que tout le monde a sorti leurs duotangs? D'accord : qui veut commencer à lire pour moi, s'il vous plaît?
Classe	(indistinct)
Professeur	Shhhh…? à ta place….Okay : qui veut lire la question? Shhh…est-ce que – les amis, les amis – je ne peux pas entendre, je ne peux pas entendre Emily. Vas-y, Emily.
Emily	(Reads inaudibly)
Professeur	Donc - Suzanne : Pourquoi est-ce que celui-là est tellement lourde? Et c'est quoi la réponse de Suzanne? Qui peut me lire la réponse de Suzanne? Sara.
Sara	Parce que ? a donné un trois fois plus lourde sac que régulier.
Professeur	Donc – parce que je t'ai donné un trois fois plus lourde qu'un sac régulier. Qui peut me dire c'est quoi en kilogrammes les sacs qu'il est en train de…/
Eleve	Dix.
Professeur	…Qui peut me dire c'est quoi, combien de kilos sont ces sacs-la? Oui. Est-ce que c'est juste dix?
Classe	Non. / Non. / Trente!
Professeur	Trente kilogrammes? Donc, si elle dit un trois fois plus lourde sac qu'un sac régulier, combien pèse un sac régulier?
Classe	Dix. / Dix. / Dix.
Professeur	Dix kilogrammes. Donc, s'il vous plaît, pouvez-vous tourner à la première page, s'il vous plaît? …Donc, la question ici c'est : la capacité d'une bouteille est d'un litre…?...qu'il peut contenir plus qu'un litre. Donc, vous allez/

CG3MF12May02a (Capacity, teacher-led), lines 155-167

The teacher asks a student to read a question from the worksheet, which begins an exploratory dialogue surrounding the question. After the student reads, the teacher asks, "Pourquoi est-ce que celui-là est tellement lourde?", and the routine continues from there.

Although the teacher in the English-language mathematics class used many of the same visual mediators as the teacher in the French-language mathematics classroom, visual mediators in general were less common in the English language mathematics classroom. During the observation period she did not make and use anchor charts, in the way that the teacher in the French-language mathematics classroom did. Depending on the lesson, the teacher occasionally used the blackboard, or demonstrated with mathematical manipulatives on the projector. One of the most common visual mediators she did use was concrete everyday objects. Even though they were not present in the classroom, this teacher frequently used a variety of objects to coordinate mathematical talk in her classroom. In the following excerpt , the teacher is using coloured fraction strips on the overhead projector in order to talk about equivalent fractions. In addition to the fraction circles, she uses pie flavours which correspond to the colours of the fraction circles as additional visual mediators:

Teacher	So two, I'm getting two pieces of lemon meringue and only one of cherry which piece is the biggest?
Class	(Pause) They're equal. / Equal!
Teacher	They're equal. So they're called – what is the word that we use? Yes?
Class	Equal. (repeated) / Um, same? / Equality. / Equivalent.
Teacher	Equivalent. "Same" means the same thing but we have big words in math. Fun words. I'm going to write it down.

CG4ME13May23b (Equivalent fractions, teacher-led), lines 61-65

As the lesson progresses, the teacher continues to use pies as a popular visual mediator when using coloured fraction circles. The next visual mediator that the teacher chooses, her grandchildren, is similar to the pies in the above example in that the visual mediator is not in the classroom. This time, however, there is no concrete link in the classroom to the visual mediator, such as the fraction circles in the previous example. The students nevertheless respond positively to her choice:

Teacher	...it says, "Estimate". What does "estimate" mean. Steven?
Steven	Guess?
Teacher	Guess. An estimate isn't *just* a guess, though. Like, um, I *guess* that my new grandchild will be born on November twenty-seventh, but I can't be sure, but it's pretty close. (Student claps)
Teacher	Thank you. (More clapping) Yes, I have a new grandchild coming. Anyway...
Student	Really?
Teacher	Yeah.
Student	But now you're gonna have...
Teacher	Six!
Student	Again?
Teacher	Six! You know, another family. (More clapping) Anyway, um, I guess that baby will be born in November, 'cause that's what I was told about it. It's an educated guess, 'cause I have information.
Student	What about, what about, baby...
Teacher	If I - *excuse* me. If I, if I said, like, um, I'm getting a new bike and I guess it will be purple, I don't have any information to go on about my new bike, like, okay, nothing, so it's totally a guess. I don't even know if they have purple bikes...So, an estimate is a guess that you make that's what we call an educated guess. You have other information to help you make the guess, and sometimes your guess might be right on. You might be exactly right. A guess could be. Lots of times an estimate is something close.

CG4ME12May27c (Equivalency, teacher-led), lines 9-21

As in the above example, the teacher in the English-language mathematics classroom often used real-life visual mediators, such as pie, her grandchildren, or a new bicycle that were not in the classroom. Students were often motivated, if also temporarily sidetracked, by these visual mediators.

In the excerpt below, the same teacher uses the blackboard to draw and to write while a student is explaining her thinking. Two dice have been rolled, yielding a six and a four. The class has worked together to form a proper fraction (4/6) and an improper fraction (6/4), and are now working to convert the improper fraction into a mixed fraction:

Teacher	We're going to make this (6/4 from dice roll) into a mixed fraction, who knows what to do there? Kathleen?
Kathleen	You put a one and then you put two over four
Teacher	But, how did you come up with that *right* answer?
Kathleen	Um, because there was two, because there was one (?) that was whole and there was two left over.
Teacher	So you took the six: one, two, three, four, five, six, and you took four away, and that gave you four over four, and that equals...? (Counting and drawing on board)
Class	2! / One. (repeated)
Teacher	*One*. How could it equal two? Four over four equals...?
Class	One. / Equals one.
Teacher	Because it's a complete pie. Then I have two left, and that's left over four. There we go. (Writes on board)

CG4ME13May22a (Mixed & proper fractions, teacher-led), lines 216-224

In this example, the teacher in the English-language mathematics classroom is using the blackboard as a visual mediator in order to coordinate discourse and model an activity which the students will later do independently, emphasizing and rendering visible the thought process that the student used in order to arrive at the correct answer.

Although not as often as in the French-language mathematics classroom, there was use of photocopied worksheets in the English-language mathematics classroom. Unlike in the French-language mathematics classroom, where the exploratory dialogue centred around the answers to the questions on the photocopied worksheets, in the English-language mathematics classroom it was the problems themselves on the worksheet that were the springboard to exploratory discourse using writing and drawing on the blackboard as a visual mediator, as in the excerpt below:

Teacher	If I - *excuse* me. If I, if I said, like, um, I'm getting a new bike and I guess it will be purple, I don't have any information to go on about my

	new bike, like, okay, nothing, so it's totally a guess. I don't even know if they have purple bikes. Excuse me, hand *down* (Student speaking) 'cause we're not having a discussion. *You* are to be quiet, and not interrupt again. So, an estimate is a guess that you make that's what we call an educated guess. You have other information to help you make the guess, and sometimes your guess might be right on. You might be exactly right. A guess could be. Lots of times an estimate is something close. Um, so, is the shaded part greater or less than one third? You have to imagine, I don't have enough of these manipulatives to give you all circles, but you could imagine what a third would look like, and then you would guess, is it close or not? So you're gonna have to maybe draw it, and guess that way. Um, and, so each of those questions refers to that. "Write a fraction for each of the shaded parts." You can be *accurate* with this, exact, because you know how many parts there are. What part is going to go on the top? What is going to be the numen, numerator? Lee?
Lee	The smaller one.
Teacher	The smaller what?
Lee	The smaller number.
Teacher	The smaller number of what?
Lee	Of the fraction...
Teacher	Yes, but how am I gonna find out what it is?
Lee	Ah...
Teacher	Benjamin?
Benjamin	Of the portions?
Teacher	Of the portions. But how am I gonna know which portion is the numerator? It's said in the sentence. Yes, Lee?
Lee	You'll know because it's not shaded?
Teacher	Okay, the, the *shaded* part is n, is the numerator?
Lee	(Inaudible)
Teacher	*Really?* It says, "Write a fraction for each shaded part. Circle the greater fraction in each pair."
Benjamin	Uh, the one that's *not* shaded?
Teacher	'Kay. There are some that are dark and some that are light. How do I find the *denominator*, let's go there. (Pause) You have those pages; get them out, James, please. (Pause) Yes?
Benjamin	It's the, uh, the ones that *aren't* shaded.
Teacher	The denominator, what is the denominator, can someone give me the definition? Yes?
Claire	Ah, the, the...fraction that you're using, like fifths or sixths.
Teacher	You're on the right track. Yeah?
Student	It's the whole number.
Teacher	The whole number. How do I find what the whole number is?
Student	Oh!
Teacher	Yes?
Student	Count every single one?

Teacher	Count *all* of the parts. Okay? The ones that are shaded: So, it says, “Write a fraction for each shaded part.” I think that they’re all shaded; some are dark and some are light, but they’re asking you to do *two* shaded parts. So, I’ll try to emulate this so that we can do one of our own, without stealing information there.
James	Ruler, Mrs. Telfer.
Teacher	How many are there: How many *parts,* for the denominator. Kim?
Kim	Seven.
Teacher	Seven. So: where does the denominator go: on top or on the bottom?
Student	Bottom!
Teacher	Bottom. All right, so, and if I was writing a fraction for the shaded parts what would it be? Kyle.
Kyle	Three.
Teacher	Is that a fraction?
Kyle	No! Three over seven!
Teacher	Thank you! So you have to count *all* of the parts to get the bottom number. So it, I *can’t* write a fraction for this that’s like this: it’s wrong! There are three shaded and three that are black, or four that are black, but that is not a fraction! How many, um, what is the fraction of the pieces that are, um, black? Kim?
Kim	Four over seven?
Teacher	Four over seven. Is there anybody who doesn’t know where seven comes from? Who *knows* where seven comes from? Easton, do you know where seven comes from? No? Okay, thank you. Could, put it down, put it down. You always have to count *all* the parts to get the denominator. That’s the number on the bottom. ‘Cause if I counted all of those I’ve got seven. If they were *all* coloured white, then I’d have seven over seven and that would equal one.
Easton	Okay.
Teacher	Okay? So then once I’ve made some of them, once I’ve made some of them, now, grey, I have: the ones that are coloured white, that’s three out of the seven, the ones that are coloured black-ish are *four* out of seven. But you *always* have to count all of the...?
Student	Portions/ number/ all of the pieces.
Teacher	All of the pieces, all of the portions, it’s okay, to say pieces or portions, yes? All right. So when I give this to you this is what you’re going to be doing for that number. I’m gonna give it and let you do a little bit of this today, and see how you get. ‘Kay? (Pause) James, start working on yours. You have it already. Put your name on it, please, and today’s date, May 27th. (Pause) This goes into what, which section of your binder? (Pause) Which section – yes?

CG4ME13May27c (Equivalency, teacher-led), lines 21-63

In the above excerpt the teacher in the English-language mathematics classroom leads an exploratory dialogue about how to solve the problem presented on the worksheet, rather than a discussion about the solution to these problems. This occurs before the students have completed the worksheet, rather than after, as in the French-language classroom. She uses the blackboard to coordinate this discussion, although a projector is available to her. Although this discourse contains many elements of QRE discourse within it, it is still considered to be an exploratory routine. Once the initial question ("Write a fraction for each of the shaded parts.") is asked, the elements that resemble a QRE discourse, such as "So: where does the denominator go: on top or on the bottom? Bottom! Bottom. All right, so..." are part of the larger question about how to complete the worksheet, rather than simply reviewing, checking understanding, or consolidating understanding, as would be the purpose for a purely QRE routine. These QRE-type questions serve to lead the discourse towards an understanding of how to complete the worksheet, much like the chained scaffolding questions which were more commonly seen in the French-language exploratory discourse. In this English-language exploratory discourse, however, we see a clear endorsement at the end ("All right. So when I give this to you this is what you're going to be doing for that number. I'm gonna give it and let you do a little bit of this today, and see how you get. 'Kay?"), an element which is not always present in the French-language exploratory discourse.

In addition to references to concrete, everyday objects from outside the classroom, the blackboard, and worksheets, the teacher in the English-language classroom also used dice and fraction strips in order to coordinate discourse during the mathematics activities in her classroom.

Word use in teacher-led exploratory discourse. Sfard's final category, word use, also varied between the classrooms in the teacher-led exploratory dialogue routines.

In the French language mathematics classroom, the teacher consistently used standardised French mathematics vocabulary, as would be used in the provincial mathematics curriculum document. The French-language mathematics teacher made regular efforts to explicitly teach French mathematical vocabulary, such as "litres" and "tiers", as in the examples below:

Professeur	Exactement. Exactement. C'est pas liquide. Liquide – lorsque vous pensez « litre » j'aimerais que vous faites le lien avec « liquide ».

CG3MF12May07a (Fractions, teacher-led), line 276

Professeur	…Après, après demi, y a quoi?
Classe	Trois quarts?
Professeur	Okay, mais c'est quoi le mot que j'ai utilisé?
James	It's not trois quarts…
Professeur	Qui peut me le lire?
Classe	…
Professeur	Jacob.
Jacob	Tiers.
Professeur	Tiers. Et regardez, la lettre « t », on peut faire le lien avec quel chiffre?
Classe	Trois?
P	Trois – exactement. Et le dernier?
Classe	Quart?
Professeur	Quart. Et on peut faire le lien avec quel chiffre?
Classe	Quatre. /Quatre. /Quatre.
Profeseur	Quatre – exactement.

CG3MF12May07a (Fractions, teacher-led), lines 94-108

As seen in the excerpts above, the teacher in the French-language mathematics classroom made consistent efforts to teach his students standard mathematical vocabulary.

The students in this classroom, used standard French vocabulary for answers which were one word or numerical, or answers to yes/no questions. Code-switching, however, was regularly observed when students wished to express a more complex thought.

Professeur	Lequel pèse plus : un éléphant ou une voiture : lequel pensez-vous pèse le plus?

Classe	Un éléphant!
Professeur	Un éléphant? Est-ce qu'un éléphant peut peser 200 grammes?
Classe	(indistinct)
Professeur	Deux cent grammes.
Classe	Non…mmm…
Professeur	'Gardez.
Élève 1	It's impossible!
Professeur	Ça c'est notre deux cent cinquante grammes.
Élève 1	They can't, they can't, they…weigh grams! Its way, way...!
Professeur	Deux cent cinquante grammes, c'est ça. Est-ce qu'un éléphant pèse ça?
Classe	Non, non…
Élève	Like a billion of those!
Élève	Not even!
Élève	Two billion!
Professeur	Si ça c'est un kilogramme, combien de kilogrammes vous pensez que Timothy pèse? Si ça c'est un kilogramme?

CG3MF12May07a (Fractions – teacher-led), lines 23-38

In this typical example, the French language mathematics teacher consistently uses standard mathematics vocabulary, such as peser and grammes. The students use standard French vocabulary when answering yes or no questions ("Non...") and when giving one-word answers ("Un éléphant!"). There is then evidence of code-switching when they want to express more complex thoughts ("They can't...It's way, way...!" "Like a billion of those!").

When speaking directly to the teacher, students will make an effort to avoid code-switching and express themselves in French. As is seen in the following excerpt, the teacher supports these efforts:

Professeur	Regardez autour de vous. Qu'est-ce qu'il a un angle aigu?
Classe	Monsieur! Monsieur P!
Professeur	Benjamin.
Benjamin	Le, uh, en haut? comme ça?…
Professeur	: Oh, tu veux dire les aiguilles? Les aiguilles de l'horloge? Okay – lesquelles sont aigues présentement? …Maintenant, lesquelles sont, sont aigues?
Benjamin	Le rouge, le rouge et le long.
Professeur	Celui de l'heure, ou de minute?
Classe	Minute. / Minute. / Minute.…

CG3MF12May14a (Angles, teacher-led), lines 105-153

Whereas students regularly and readily engaged in code-switching when they were calling out answers in a group, they did so less often when they were chosen by the teacher to answer a particular question. When a student said, pointing at the clock, "Le, uh, en haut? Comme ça?" the teacher easily understood him and provided the French words "aiguilles" and "horloge". The communication of the student's mathematical message was not impaired.

In making an effort to speak French to the teacher, students often made ungrammatical utterances when what they had to say exceeded a one-word answer, as in Kyle's sentences below:

Professeur	Donc, je prends un litre, j'enlève un demi-litre. 'Gardez, les amis, 'gardez l'affiche.
Kyle	Choisir moi – je sais, je sais!
Professeur	Je prends un litre, j'enlève un demi-litre…
Kyle	Monsieur P!
Professeur	…et ensuite …
Kyle	Je sais, je sais – choisir moi!
Professeur	…je (?) un quart d'un litre : combien est-ce que j'aurai?
Kyle	Choisir moi, Monsieur – je sais!

CG3MF12May07a (Fractions – teacher-led), lines 250-257

Kyle is extremely motivated to be called on to answer the teacher's question. He tries very hard to attract the teacher's attention, saying "Choisir moi!" several times. His message, even though grammatically incorrect, is clear, and is not corrected by the teacher or his peers.

As in seen in the excerpt below, it is not only the students who used code-switching. Very occasionally, the teacher in the French-language mathematics classroom used code-switching himself:

Professeur	Ça vous demande : « Résous les problèmes. » Donc le premier problème c'est : (Reading) « Un seau peut contenir (un seau en anglais c'est a bucket) donc, un seau peut contenir trois litres de sable. Combien de litres de sable est-ce qu'il faut pour remplir cinq seaux? » Deuxième

	question c'est : (reading) « Joseph boit un demi litre de lait par jour. Combien de jours est-ce qu'il faut pour finir quatre litres de lait? »

CG3MF12May02a (Capacity, teacher-led), line 192

Interestingly, the teacher is unprompted when he provides the word "bucket" for "seau" in the excerpt above. As he is reading aloud a problem from a worksheet, perhaps he did so in the interests of expediency, quickly translating a word that is useful in the context of the question, but is not, strictly speaking, a "mathematical" word.

When confronted with vocabulary which is unfamiliar to the students, it was much more common for the teacher in the French-language mathematics classroom to explain the meaning of the word in French, as in the excerpt below:

Professeur	Deux mille. Donc, la dernière page ça vous demande : (reading) « Résous les problèmes ». Vous (?) chaque problème, et à la fin il y a une question qui est en prime.
Classe	…?...
Professeur	C'est comme un boni.
Classe	Oh.
Professeur	C'est comme un boni, à la fin.
Classe	Oh.
Professeur	Donc vous allez répondre aux questions et…
Élève 1	Where's the boni?
Professeur	…à la fin il y a un boni.
Élève 1	Where's the boni?
Élève 2	At the fin.
Professeur	Juste ici. Une prime. Donc je vais vous dire avec qui vous allez travailler.

CG3MF12May02a (Capacity, teacher-led), lines 272-283

Contrary to the previous example of seau/bucket, the questioning this time about "une question qui est en prime" is initiated by several students in the class. The teacher explains the expression with a synonym "un boni". Even after this explanation, a student is still unsure, and twice uses code-switching within a single sentence to pursue his line of questioning: "Where's the boni?" He receives more than one answer; two in French, from the teacher: "à la fin il y a un boni" and

Juste ici. Une prime", and one from a peer, who himself uses code-switching in a single sentence: "At the fin". Despite the questioning, synonym, and answers in French and in English, it is unclear whether all students have understood the concept of "en prime" by the end of this exchange.

In the English-language mathematics classroom there was evidence of explicit vocabulary teaching in the English-language classroom, such as was seen in the French-language mathematics classroom. In the following example, the teacher in the English-language mathematics classroom attempts to ensure that the students understand clearly what is expected of them when they are required to estimate on a worksheet:

Teacher	...When you go to the next page, we're now not just finding equivalents, we're *comparing*. And they have pictures for you, um, it says, "Estimate". What does "estimate" mean. Steven?
Steven	Guess?
Teacher	Guess. An estimate isn't *just* a guess, though. Like, um, I *guess* that my new grandchild will be born on November twenty-seventh, but I can't be sure, but it's pretty close. (Student claps)
Teacher	Thank you. (More clapping) Yes, I have a new grandchild coming. Anyway...
Steven	Really?
Teacher	Yeah.
Steven	But now you're gonna have...
Teacher	Six!
Steven	Again?
Teacher	Six! You know, another family. (More clapping) Anyway, um, I guess that baby will be born in November, 'cause that's what I was told about it. It's an educated guess, 'cause I have information.
Steven	What about, what about, baby...
Teacher	If I - *excuse* me. If I, if I said, like, um, I'm getting a new bike and I guess it will be purple, I don't have any information to go on about my new bike, like, okay, nothing, so it's totally a guess. I don't even know if they have purple bikes. Excuse me, hand *down* (Student speaking) 'cause we're not having a discussion. *You* are to be quiet, and not interrupt again. So, an estimate is a guess that you make that's what we call an educated guess. You have other information to help you make the guess, and sometimes your guess might be right on. You might be

	exactly right. A guess could be. Lots of times an estimate is something close. ...

CG4ME13May27c (Equivalency, teacher-led), lines 9-21

Contrary to the French-language mathematics classroom, there was regular evidence of non-standard mathematical language in the English-language mathematics classroom. In the transcripts from this study there was no evidence of either students or teachers using non-standard mathematical vocabulary during mathematics lessons in French. When confronted with a mathematical concept which they had trouble expressing in a way that they could be easily understood, students, and more rarely the teacher, would use techniques such as code-switching, circumlocution, or gesture. These techniques were not seen in the English-language mathematics class. Rather, the use of non-standard mathematical language was initiated by both students and teachers, and both students and teachers would perpetuate the use of the word once it had been initiated.

In the excerpts below there is evidence of both students and teachers initiating and perpetuating the use of non-standard mathematical words:

Teacher	That is a whole number, but I said what a whole number *is*. So actually what I'm wanting is a definition. So I didn't say tell me a whole number, I, four over four is four parts of, um a whole that's divided into four parts, right? Okay. Yes? ...
Teacher	One is a whole number. Are there any other whole numbers?
Student	Two? No...
Teacher	No?
Student	Two. No, a hundred...
Teacher	Two is a whole number.
Student	...tens.
Teacher	It's two completes, right.

CGME13May22a (Mixed and proper fractions, teacher-led), lines 61-69

Teacher	Okay, so what is, um, a fraction?
Student	Two hundred and fifty-seven.

	...
Student	Something over something.
Teacher	Something over something, help me out, do you have something else?
Student	Four over five?
Teacher	No, I didn't ask you to give me a fraction; I said to tell me what a fraction is…
Student	Four pieces of a pie.
	...
Student	Ummm, if you ate a piece it would go down, to, ah, to maybe half, and then you would have a portion.
Teacher	…Oh, there's that word – portion. So a fraction is...?
	...
Teacher	So now we know that a fraction of something, a portion of something is not the whole thing. It's only part of it. So a fraction is a part of something.

CG4ME13May22a (Fractions, teacher-led), lines 59-112

In the above example from the transcripts, there are numerous examples of non-standard language which is initiated and perpetuated by both the teacher and the students during teacher-led exploratory dialogue in the English-language mathematics classroom. The teacher introduces the word "completes" to describe whole numbers, and "portions" to describe fractions, the latter being used later in the transcript by various students. A student-proposed example of non-standard language is "something", when the student attempts to give a definition of a fraction as "something over something". The teacher repeats the student's attempt, and continues her quest for a clear definition of a fraction.

Although exploratory discourse forms may be more variable than QRE patterns, the routine of teacher-led exploratory discourse in French and English language mathematics classrooms share some significant characteristics. In both French-language and English-language teacher-led exploratory mathematical discourse the teacher initiates the routine with a question, a student is chosen to respond, and then the teacher chooses and initiates a probe. At this point, in both the French-language and English-language mathematics classrooms, other students may contribute to the routine. A variable number of additional probes from the teacher and answers from students follows, and the discourse may or may not end with an endorsement from the teacher.

Student-led exploratory discourse. In contrast to the differences between the French and English-language mathematics classrooms in QRE discourse and in the teacher-led exploratory discourse, student-led exploratory discourse in the French-language and English-language mathematics classrooms were very similar, even unexpectedly so, when analysed using Sfard's four categories. The exploratory routine in student-led discourse shares some essential elements with the teacher-led exploratory routines, yet also differs in important ways.

Routine in student-led exploratory discourse. Unlike QRE routines, there was evidence of exploratory routines in both teacher-led and student-led discourse. Student-led exploratory discourse, like teacher-led exploratory discourse, was more variable than QRE routines, yet there were still commonalities between the teacher-led routines and the student-led routines. There were also similarities between student-led exploratory routines in the French-language and English-language mathematics classrooms.

In the student-led discourse, there were significant departures from the basic exploratory routine which was found in the teacher-led discourse, i.e. an initial question, a response, a number of additional probes and answers, and the possibility of a final endorsement. In the teacher-led exploratory routines the initial question is, by definition, posed by the teacher at the beginning of the routine, either orally, or in writing. In student-led exploratory routines, the origin of the question is still with the teacher. As the students already know the question, because it has been asked by the teacher, the students do not necessarily consistently repeat it at the beginning of the student-led routine. Students may be working on a specific question, as presented by the teacher, or they may independently chunk the given question into a more manageable inquiry. Given the important difference from the teacher-led exploratory dialogue that neither student participant has completed the task or reliably knows the answer to the question at the outset, the rest of the routine looks different from the teacher-led exploratory routine. Rather than a series of answers and probes, the routine becomes a series of steps towards both students writing down an answer. One student or the other may take the lead, there may be parallel work as they each seek to complete the task independently, or there may be collaboration as they work to find the answer. As in the teacher-led exploratory dialogue, endorsement is variable. This endorsement may be positive or negative, correct or not, and may relate to the answer, the completion of the task, or to the learning skills demonstrated during the work. Despite the significant differences between the teacher-led and the student-led exploratory dialogue, the two routines share enough commonalities to satisfy the definition of an exploratory routine.

In the following excerpt from the French-language classroom, the two students are working on a question about capacity and fractions from a worksheet. They are to colour the correct volume in the containers drawn on the worksheet. They read the instructions from the worksheet as the initial question:

Kyle	Okay, here's one thing. It says "colour".
Sara	(Unintelligible)
Kyle	Oh, yeah. The ketchup bottle?
Sara	Let's just, let's just go back to that.
Kyle	The bucket.
Sara	Umm...is it (?)?
Kyle	'Cause real buckets are, like, the size of this. Two thousand...
Sara	This is so heavy.
Kyle	I know.
Sara	Okay, read this thing. It's like a bathtub.
Kyle	You know how like when you go to the spas? It's like those things that you put your feet in. I think it's one of those.
Sara	The bath thing..?
Kyle	Yeah!
Sara	But definitely not. I guess...It looks so weird
Kyle	I already know this one. a) is the first,um trois quarts, b) is the ...c) is the ...

Sara	So just colour...
Kyle	I'll go get colours. (Pause) The tape was there, I think. Hand it over.
Sara	(Indistinct)
Kyle	Encore, so, right there.
Sara	Right here?
Kyle	One quarter, so...shhh! The first row, 'kay?
Sara	Ooo-kay...
Kyle	Right there! ...Un demi-litre..? And then un demi-litre so it's right there.
Sara	Here?
Kyle	Yep. So you know what we're doing, right?
Sara	Yep.

CG3MF12May02b (Capacity, student-led), lines 1-26

After the initial question, the first part of the routine deals with making real-world connections to the questions, searching for context ("...real buckets are..." and "It's like those things you put your feet in."). As the routine progresses, Kyle believes he already knows the answer, so he takes the lead, and shows Sara where to colour ("The first row, 'kay?"). She follows his instructions. The initial closed question, typical of the French-language classroom, perhaps allows Kyle to believe he knows the answer right away, and thus permits him to take on such a leadership role. In this excerpt, the two students endorse the fact that they now both understand the work they have done ("So you know what we're doing, right?" "Yep.")

In the next excerpt taken from the English-language mathematics classroom, the two students are working together, using fraction strips to find all the possible equivalents to one half. In the initial question, Kyle breaks down the larger, open-ended question, turning it into a less complex, closed question. Taking two different fraction strips, he asks: "Is this a half of this?":

Kyle	...So let's do, so a half, is this a half of this?
Kim	Wait, yeah.
Kyle	Wait! This thing, I think this thing, I think this might be a half. Give me another brown? Yep.
Kim	Do we have to draw that? Yeah, I think so.
Kyle	I'm pretty sure we do. So the one would just be, like, zoom! Wait – how

	long is the one blocks? How many blocks are the one?
Kim	I'm gonna trace it.
Kyle	Well I was tracing the line. And that will be big enough. And I'll put in four blocks down. And now...you can take one and trace it. There you go. That'll be good enough. So one whole and one half. There you go.
Kim	You have to do *both* halves.

CG4ME13May23c (Equivalent fractions, student-led I), lines 7-14

In this excerpt, the second part of the routine centres on how to complete the task. Neither student takes on a leadership role. With the more open-ended questions that are typical of the English-language mathematics classroom, it is possible that Kyle is not able to complete the question with a rapid mental calculation, and thus is forced to work on the question at the same time as his partner, if not collaboratively, then in a parallel fashion. They work simultaneously, each in his or her own way. Kyle rushes to be finished, while Kim ensures that the work is completed properly. Kyle measures the two strips, and concludes that "Yep.", one is one half of the other. Kim wonders if they have to draw their answers, and they both agree that they do. Kim decides to trace the entire strip, while Kyle just traces the length. He concludes that "one whole and one half" is sufficient to answer that part of the question. Kim negatively endorses his answer, saying "You have to do *both* halves."

Although the routine is necessarily and understandably different from the teacher-led exploratory dialogue, exploratory dialogue is in fact evident in the student-led dialogue in both languages. Unlike the QRE student-led discourse, which did not exist when neither participant knew the answer at the outset, the fact that neither student has completed the work at the beginning of the dialogue is not an impediment to the existence of exploratory student-led discourse.

Endorsement in student-led exploratory discourse. There are many examples of endorsement in exploratory student-led discourse in both the French-language and English-language mathematics classrooms. When the discourse is student-led it is not up to one partner or the other to do the endorsing, as it is up to the teacher in the teacher-led dialogues. Both partners endorse many different aspects of the student-led mathematical discourse. In both classrooms, the students use endorsement – negative or positive - to ensure that they both agree on the procedure being used to solve the problems, to finalize the answer to the problem (even if it may turn out to be incorrect), and to ensure that they are completing the work as set out by the teacher.

Student-led endorsement is not limited to endorsing "object level" narratives, or "stories about mathematical objects", in Sfard's terminology. In the excerpt below from the French-language classroom, two students pause as they work together to complete a worksheet, "endorsing" the fact that they are working well together. This can be viewed as an example of what Sfard terms "meta level" endorsement, a narrative about how mathematics is done:

Sara	I don't know.
Kyle	Is there any way we could…?
Sara	(Reading) "...deux boites de biscuits. Est-ce qu'un sac de biscuits pèse, le, pèse moins qu'une demi-kilogramme?
Kyle	Just write down « No. »…Actually, it does, so we…
Sara	Um, it only weighs a bit less than two of them. So if you take away one of them....
Kyle	Oui. Wow – we actually work good together.

CG3MF12May02b (Capacity, student-led), lines 114-119

In saying "Wow" and "actually", Student 1 recognizes, with some surprise, that he and his partner are working well together, and are completing the assigned work.

Despite the fact that Kyle feels that they are working well together, Sara and Kyle do not automatically endorse every object level narrative that his or her partner makes. In the excerpt below, continued from the same French-language mathematics classroom transcription, we see each student ensuring that she or he is ready to endorse each step as they work to figure out how many groups of four are needed to make twenty:

Sara	Should be like...four.
Kyle	Except he just needs to have six. Okay, so um ah, four times six equals twenty.
Sara	Twenty. It does?
Kyle	Twenty! That's easy.
Sara	Sorry – I wrote two.
Kyle	You wrote two?
Sara	Yeah.
Kyle	It's twenty. That was easy!
Sara	Uhhhh...it's actually six, though, 'cause, how many times. We already know it's twenty.
Kyle	Four times six...Whatever. Six
Sara	Do you get it?
Kyle	Oh yeah.

CG3MF12May02b (Capacity, student-led), lines 120-131

At each step in the problem the two students question in turn the proposed answer of the other, using "Except...", "It does?", and "Uhhh, it's actually..." These can be viewed as examples of negative ("non") endorsement. They eventually endorse an answer, even though it is incorrect: "Do you get it?" "Oh yeah".

In the excerpt below from the English-language mathematics classroom, students are each rolling a die and using the numbers to make first proper and then improper fractions, the latter of which is then expressed as a mixed fraction. As in the excerpt above from the French-language mathematics classroom, they work to endorse many small steps along the way before they both endorse a final answer:

Kim	So it's three...
James	So it's three on four.
Kim	Three on four...that's four plus three.
James	And then what do we /
Kim	/so do you...? That'd be one, that'd be, like, one three remaining one.

James	One third...
Kim	‘Cause in four there’s only one three, so...
James	Yeah.
Kim	...so there’s one left.
James	So that’d be one third remaining one.
Kim	One third.
James	One third....one third, one.
Kim	One third of, of, four?
James	What?
Kim	Of four, one third of four?
James	Yeah – one third of four. (Pause) ‘Kay?

CG4ME13May22b (Mixed and proper fractions, student-led), lines 12-28

In this excerpt, students use endorsement at several points during their work towards a final answer, saying “Yeah”, and repeating each other’s words (“One third.” / One third...”) Once again students end by endorsing an incorrect answer, using unusual visual mediators and non-standard mathematical language. The partners continue to check that they are in agreement: “’Kay?”

As in the French-language mathematics classroom, it is not only the object level narratives, such as the answers to mathematical problems or the steps involved in finding these answers which are endorsed by students during student-led exploratory dialogue. In the excerpt below, students in the English-language mathematics classroom are using fraction strips to find fractions that are equivalent to one half. When they believe that they have found all of the equivalent fractions, they endorse the fact that their work is complete, an endorsement of a meta-level narrative:

Benjamin	How ‘bout three. Let’s see how many we can fit into the three.
Kathleen	...four, five...it’s four times two – we haven’t done that one yet.
Benjamin	(Writing) Four times two...no, that’s all you can do for, um...
Kathleen	I think that’s all you can do.
Benjamin	Yeah.
Kathleen	Yep. That’s all we can do. And that...
Benjamin	Yeah.

Kathleen	That's all we can do for a half.

CG4ME13May23d (Equivalent fractions, student-led II), lines 150-157

Despite not having a strategy for ensuring or checking that they have indeed found all of the fractions that are equivalent to one half, the two students both more than once endorsed the fact that they had indeed completed the assigned task.

Endorsement is consistently evident in both the French-language and English-language student-led discourse. These endorsements vary between object-level endorsements, about the mathematical content, or meta-level endorsements, about the way that an answer is reached. In contrast to the teacher-led exploratory dialogues, where endorsement was not regular, especially in the French-language classroom, student-led discourse in both language settings was regularly endorsed by the students themselves.

Visual mediators in student-led exploratory discourse. Visual mediators were regularly used during student-led exploratory discourse in both the French and English-language classrooms. In the French-language mathematics classroom in particular, student-led exploratory dialogue was elicited when students worked together to complete photocopied worksheets. Both students consistently referred to these worksheets, which consistently kept them working on the same question at the same time, fulfilling the visual mediator role of coordinating mathematical talk.

While doing this type of work in the French-language mathematics classroom, one student referred to a concrete object that was not in the classroom, a mini milk carton, as a visual mediator. This use of concrete objects which were not in the classroom was seen in both French and English-language QRE and exploratory routines. As in teacher-led QRE and exploratory routines, these concrete objects can be viewed as "independent evidence of thought" (Sfard, 2007), and thus part of the communication of their mathematical thinking. In the following example Kyle uses a small milk carton as a visual mediator to coordinate their talk about the problem, even though his belief that it is a "demi-litre" is incorrect. When Kyle uses the mini milk carton as a visual mediator, it is clear that he is simultaneously thinking about the solution to a novel problem as well as communicating his thinking to his partner, as well as to himself.

Kyle	No! No. If it's a ...if he does...how, if he does, a de-, a half of a milk carton a day, if he does one...like, you know how you have those, you guys have those, you know how we have those mini...
Sara	Yeah.
Kyle	...milk cartons? That's a half, a demi-litre. If you drink four of those, how much is it?

CG3MF12May02b (Capacity, student-led), lines 57-59

Kyle also refers to a concrete visual mediator which is present on the worksheet, questioning the validity of a "bag" of cookies:

Kyle	(Reading) "Un sac de biscuits pèse..." Um....un sac de biscuits? »

CG3MF12May02b (Capacity, student- led), line 113

In questioning, by saying "Um", pausing, and using a questioning tone of voice in the excerpt above, Kyle underlines the importance of choosing a visual mediator. In order for the visual mediator to play a role in coordinating the thought of a person or persons engaged in mathematical discourse and be "part and parcel of the act of communication and thus, in particular, of thinking processes, (Sfard, 2007, p. 573), the visual mediator must be a plausible representation of that person's thought process. The importance of concrete visual mediators, perhaps especially in the mathematical thinking of and with young children, is seen when Kyle objects to a visual mediator that has been in effect chosen for him, by virtue of the fact that it comes from a question on a photocopied worksheet. Presumably, if visual mediators, even concrete objects such as this one, were not essential to his mathematical thinking and communication, he would not have objected to the choice of visual mediator that was provided to him.

This is not to say that there was no use of standard mathematical symbolic artefacts or visual mediators in student-led mathematical discourse in the French-language mathematics classroom. For example, in the excerpt below, Kyle coached his partner how to write an answer using proper mathematical notation:

Kyle	C'est deux, um, ah, litres. (Writing?) "Deux litres". Just do two and a capital L.

CG3MF12May02b (Capacity, student-led), line 55

His use of the phrase "Just do..." implies an interest in the expediency of using mathematical symbols to express mathematical thinking.

Visual mediators were also common in student-led exploratory routines in the English-language mathematics classroom. The tasks that the dyads were working on when the sessions were observed and recorded required the use of specific assigned mathematical manipulatives, such as dice and also fraction strips. In the excerpt below, students are using fraction strips to find fractions which are equivalent to one half.

Kim	Orange, orange, orange! I think that orange will work. Yep, it works.
Kyle	Three of them! Okay, I gotta get a new page.
Kim	Just get one of these, and then measure this, and then (?).
Kyle	(Singing) Eight times eleven equals eighty-eight.
Kim	And three of these, (pause) that's four, it's four on each side, it's a square.

CGME13May27a (Equivalency, student-led I), lines 46-50

Although fraction strips are once again concrete objects, which are actually present in the classroom this time, they are indeed "symbolic artifacts, created specially for the sake of this particular form of communication" (Sfard 2007, p. 571), and thus fulfill the role of visual mediator.

Students worked hard to record their results on lined paper in notebooks, but as is evidenced in the excerpt below, some students struggled to write down their answers in an efficient and standard fashion:

Kim	(Pause) I'm making a circle around it, like, to like, separate it from others. It could get confusing.

CG4ME13May27a (Equivalency, student-led I), line 19

In making an effort to preserve a clear record of her thoughts, Kim demonstrates that she recognizes the power of the symbols and diagrams that she has chosen as visual mediators to be "independent evidence of (her) thought" (Sfard, 2007, p. 573) when solving the assigned problem.

The use of lined notebook paper in the English-language mathematics classroom, rather than the photocopied worksheets or teacher-generated anchor charts in the French-language mathematics classroom, provides increased insight into the mathematical thinking of students. Like the more open questions in the exploratory routines in the English-language mathematics classroom, the lined paper allows for more free expression of students' mathematical thinking, opening the door for different representations. These expressions may be visual, as in the excerpt below, or in the specific word use, as seen in the next example:

Kim	Three on four...that's four plus three.
James	And then what do we /
Kim	/so do you...? That'd be one, that'd be, like, one three remaining one.
James	One third...
Kim	'Cause in four there's only one three, so...

James	Yeah.
Kim	...so there's one left.
James	So that'd be one third remaining one.

CG4ME13May22b (Mixed and proper fractions, student-led), lines 14-22

In the excerpt above, the two students had rolled a three and a four on their dice. They made the proper fraction ¾ and the improper fraction 4/3 and were working to convert the improper fraction to a mixed number. When James said "That'd be one, that'd be, like, one three remaining one." he wrote "1 3 R 1" in his notebook, meaning that there was one group of three and one left over. His non-standard visual representation initially confused his partner. She soon endorsed his representation ("So that'd be one third remaining one.") as well as his non-standard mathematical language, which is discussed below, in the section on word use.

Visual mediators are common elements in the student-led exploratory discourse in both the French-language and the English-language classrooms. Although many of the visual mediators are concrete objects, it does not preclude these items as visual mediators, as the students involved in the routines use them as tools for understanding and expressing mathematical thinking. As students progress from grade three, the French-language classroom, to grade four, the English-language classroom, there is a change in the visual mediators that they use. In grade three, the concrete objects used as visual mediators are often present in the classroom, such as posters or desks. In grade four, the concrete objects used are not usually present in the classroom (pies, grandchildren), and there is a move to more symbolic traditional mathematics manipulatives. When working with two four-year-olds, Sfard (2005) remarks that these young children "quite understandably" (p. 245) use concrete, rather than symbolic, visual mediators, and that "(s)uch symbolic mediation, however, is still absent from the incipient numerical talk of our young interviewees" (p. 245). If symbolic visual mediators are "still" absent, this implies that the use of such symbolic mediators will come at a later age. It is possible that the change in visual mediators between grade three and grade four is evidence of the shift from concrete towards more symbolic visual mediators as the primary grade three students become junior division students in grade four.

Word use in student-led exploratory discourse. Despite the fact that students have made the change from learning mathematics in a second language in grade three to a first language in grade four, there were many similarities in the category of word use between the French-language and the English-language mathematics classrooms. When the students in the French-language mathematics classroom worked together to complete worksheets, there was ample evidence of code-switching. Code-switching in the student-led exploratory dialogue was ubiquitous, and much more common that in either the teacher-led QRE or exploratory dialogues. As in the excerpt below, from a student-led exploratory dialogue in the French-language mathematics classroom, the students spoke French when referring to the worksheet to be completed, either reading aloud verbatim or finalising an answer to be written down in French, as is required of students in the French-language mathematics classroom. There were very few instances of students spontaneously using French with their peers in the French-language mathematics classroom:

Kyle	Okay – what the heck is this? (Reading) « Joseph boit un demi-litre de lait par jour. Combien de jours...
	...
Kyle	Oh, I know what he...I know what this is! It's um, ah, ah, ah, um...grrr! I know it's something around...To drink ...'kay, Joseph...Oh! demi-litre...Two! C'est deux, um, ah, litres. (Writing) "Deux litres". Just do two and a capital L.

CG3MF12May02b (Capacity, student-led), lines 49-55

When working to complete the worksheets the students regularly spoke French when reading the questions aloud ("Joseph boit un demi-litre de lait par jour..."). Although both students had a copy of the worksheet and were able to read it, this was a common strategy when working in pairs in the French language mathematics classroom. Students also often used French mathematical vocabulary when preparing to write the answers in French, as in "Oh! demi-litre...Two! C'est deux, um, ah, litres. (Writing) "Deux litres".

A rare example of spontaneous use of French mathematical vocabulary outside of these two situations occurred when students used the word "obtus" to label an angle. Although the word is very close to the English term "obtuse", it is reasonable to suppose that the students did not know the English vocabulary, and thus used the French term:

Sara	Oh my God. Okay, the roof is...
Sara	...obtus...
Sara	O, we have to put the O.

CG3MF12May14b (Angles, student-led), lines 32-39

Students in the French-language mathematics classroom usually spoke English with each other when they were working to complete tasks, such as in this classroom photocopied worksheets. When speaking English, there was one instance of a non-standard mathematical term in English, "calcumalations":

Kyle	Aaahhh! So four litres is, like, up to here, four litres is two sizes of this... If he drinks that...
Sara	(Inaudible)
Kyle	On my calcumalations...
Sara	But we're already at two.
Kyle	Oh, I know what he...I know what this is!

CG3MF12May02b (Capacity, student-led), lines 51-55

"Calcumalations", although unusual, is not a word on which the lesson is focused. This variant, at the lexical level, does not elicit a reaction from the student working with Kyle. Although she does not begin to use the word herself, Sara's tacit acceptance of the word can be viewed as a signal to what Mercer (2005) calls “cumulative talk... (which) is based on ground rules which encourage joint, additive contributions to the talk and relatively uncritical acceptance of what partners say” (p. 33) He describes two girls’ use of another nonsense word - “fantabuloso” - as part of the “contextual foundation” of their cumulative talk, which can be valuable in group work situations, such as the one in which Kyle and Sara are working.

This use of English non-standard mathematical word use is more common in the English-language mathematics classroom. Although students did not seem to have any difficulty expressing themselves or understanding one another, many of the student-led exploratory dialogues in the English-language mathematics classroom had striking examples of non-standard mathematical terms at the syntactical level, as in the excerpt below:

James	Three.
Kim	Four.
Kim	So it’s three...
James	So it’s three on four.
Kim	Three on four...that’s four plus three.
James	And then what do we /
Kim	/so do you...? That’d be one, that’d be, like, one three remaining one.
Kim	One third...
Kim	‘Cause in four there’s only one three, so...
James	Yeah.
Kim	...so there’s one left.
James	So that’d be one third remaining one.
Kim	One third.
James	One third....one third, one.
Kim	One third of, of, four?
James	What?
Kim	Of four, one third of four?
James	Yeah – one third of four. (Pause) ‘Kay?

CG4ME13May22b (Mixed and proper fractions, student-led), lines 6-28

The utterance “one third remaining one” does not contain any non-standard mathematical terms at the lexical level, as each word is a standard mathematical term. The way in which the students have used and endorsed the phrase, to mean “one group of three and one left over”, or, in standard mathematical terms “one and one third” (which is the correct answer for 4/3 expressed as a mixed number) is unusual at the level of syntax, rather than at the lexical level, as was seen in English in the French-language mathematics classroom.

Another example of non-standard mathematical discourse at the syntactical level is found in an excerpt from another pair of students during another activity. Using fraction strips, the partners are trying to find all of the equivalents to one half. At this point, they have been working on the activity for a while, and are trying to decide if they have found all of the equivalents and are now finished:

Lee	Okay, now I’ll do a half. Wait – how many thirds can you do? Where’s the other half? Where’d the other half go? (Rummaging) It didn’t work. The really long one. We used that... (rummaging) oh! Got it! A half equals...did we do a half in twelves?
Julius	No, not yet.
Lee	Oh, we already did eights. Is it, did we, a quarter?
Julius	Quarter?
Lee	Did we do quarters?
Julius	Yeah.
Lee	Did we?
Julius	Just look at your paper.
Lee	Oh, I didn’t write them. Is that a half?
Julius	Yeah.
Lee	One half. (Writes) (Pause) What one’s this? Is that a fifth?
Julius	It’s a...quarter.
Lee	No, that one/
Julius	No, a third. A third.
Lee	No, a third’s unequal with a half.
Julius	Oh yeah.
Lee	So, would it be this?
Julius	Oh – sixth. (Pause) Sixth.
Lee	Wait – it might be a fifth! ‘Cause I think a fifth works. A fifth works. (Pause) Oh, no, five doesn’t work; it’s (?).
Julius	(To self) Five tens.

CG4ME13May27b (Equivalency, student-led II), lines 15-34

The students show an interesting mix of standard and non-standard terminology for fractions. The students both use a variety of standard mathematical terms for fractions, such as "half", "third", "quarter", "fifth", and "sixth". They each also use some non-standard mathematical terms for fractions, such as "eights" (for "eighths"), tens (for "tenths"), and "twelves" (for "twelfths"). Even though the latter group of words is used in a non-standard way, the pair does use them in the same manner as they have used the standard terms for fractions, implying that the non-standard use does not indicate a lack of understanding about the concepts underlying fractions. The non-standard usage does not give pause to either student, and the problem-solving discussion continues. Lee also utters another non-standard syntactical form when he says "a third's unequal with a half", meaning "it is impossible to form an equivalent fraction to one half using thirds", which is in fact true. His partner endorses his statement ("Oh yeah."), suggesting that this non-standard language use does not interfere with the discussion of mathematical facts either.

Other non-standard word use emerges when a student is attempting to narrate his use of a visual mediator to write down his answers:

Kyle	I found a half of this. (Pause) That guy found a half. (Pause) Push! Got a half! (Pause) I got a half, so, okay, so square, I'll try this. (Pause) One slash two and one slash (?) one slash six. There we go! How's yours doing?

CG4ME13May23c (Equivalency, student-led I), line 15

He uses the word "slash", to indicate the line used in fractions as he searches for equivalents to one half. Once again, this non-standard usage does not seem to interfere with either communication of completion of the task.

Exploratory Discourse Summary. Unlike QRE routines, there was ample evidence of exploratory dialogue in all situations that were examined: teacher-led and student-led, and French-language and in English-language mathematics classrooms. When analyzed using Sfard's four categories there were differences between the teacher-led exploratory dialogues in the English-language and French-language mathematics classrooms, whereas the student-led exploratory dialogues in the two language settings were quite similar.

Despite superficial similarities in the routines of teacher-led exploratory dialogue in the French and English language mathematics classrooms it is the differences that stand out upon analysis. Although both teachers begin this routine with a question, the nature of the questions is very different. In the French classroom, the teacher asked closed questions, requiring short, simple, specific answers. These questions are often repeated or reformulated by the teacher. The initial question is often the first in a series of chained scaffolding, often offering clarification and guidance regarding the initial question in advance of any student request. In the English classroom, the teacher asked more open and complex questions, leading to more discussion and a wider variety of acceptable answers. Exploratory discourse including chained scaffolding was less common in the English-language mathematics classroom. Final answers to initial questions in exploratory discourse are consistently endorsed by the teacher in the English language classroom; however this endorsement is not the case in the French language classroom. A wide variety of concrete visual mediators are used by the teacher in the French language mathematics classroom. Fewer visual mediators are used by the teacher in the English language mathematics classroom; their use is less formalized, and the visual mediators used are often abstract references. Word use in the two classrooms differs as well. The French-speaking teacher consistently uses standard French mathematical vocabulary, as do the students when they are giving short, concrete answers using one or two words, numbers, or yes or no response. Code-switching to English by students is often seen in the French-language mathematics classroom when students wish to express a more complex thought. The English-speaking teacher uses both standard and non-standard mathematical language, as do her students. Despite the students' early mathematical education in French, there is no evidence of code-switching in the English-language mathematics classroom. These striking and consistent differences may have implications for the mathematics learning in the two classrooms.

In contrast to the differences found between the teacher-led exploratory dialogues in the two languages, the student-led exploratory dialogue in the French-language mathematics classroom and the English-language mathematics classroom shared many similar characteristics. The basic exploratory dialogue routine was comparable in the two classrooms, although the initial question was more concrete in the French classroom. Students in both classrooms regularly endorsed these dialogues, ensuring that they were both on track to complete the task set by the teacher. Visual mediators were common in both classrooms, although which mediators were used varied, and was often dependent upon the task set by the teacher. Despite the different language of instruction, word use in the two classrooms was very similar. Students in both classes used English to express the complex thinking required when completing the more involved tasks characteristic of exploratory dialogue. Despite the differences, exploratory student-led routines were quite similar in the different language settings.

Summary of Findings

In Chapter Five I have described the findings that became evident after examining the transcriptions of the recordings I did in the French-language and English-language mathematics classrooms. Taking a cue from Sfard, who described the category of routine as being "over-arching", I organised my findings into two main groups: question-response-endorsement (QRE) routines, and exploratory routines. I further sub-divided each group into teacher-led and student-led routines, in both French-language and English-language classrooms, and then used Sfard's other three categories of endorsement, visual mediators, and word use to describe the two routines in each different setting. Descriptions of the findings and the commentary that accompanied the descriptions were supported by many illustrative examples from the transcriptions of classroom recordings.

I found that there was evidence for both routines, QRE and exploratory, in both language settings. Both classrooms also had examples of teacher-led routines and student-led routines. Exploratory dialogue was found in both teacher-led and student-led discourse in both language settings, but QRE routines were found exclusively in teacher-led discourse in both classrooms. There was no evidence of QRE routines in student-led discourse in either language.

QRE routines, which were therefore all teacher-led, shared many common characteristics as described using Sfard's categories of endorsement, visual mediators, and word use in the two classrooms. Teachers in both language settings usually used standard mathematical vocabulary, and they both used visual mediators, albeit the use of specific objects varied somewhat. The most obvious difference was in the area of endorsement. The English-language mathematics teacher consistently endorsed answers, while the French-language mathematics teacher did not.

Exploratory routines were found in both teacher-led and student-led discourse. In the teacher-led exploratory discourse, comparisons between the two language settings shared some similarities with findings from the teacher-led QRE routines. The English-language mathematics teacher continued to use endorsement on a regular basis, while the French-language mathematics teacher did not. Both teachers continued to use visual mediators; however, the French-language mathematics teacher used a wider variety of visual mediators more often, and for more varied purposes than the English-language mathematics teacher. Both teachers sought to explicitly teach mathematical vocabulary, yet the English-language mathematics teacher used more non-standard language during teacher-led exploratory dialogue. The differences between teacher-led exploratory discourse, however, began at the initial question. This initial question in the French-language mathematics classroom was closed, and was often repeated or reformulated even in the absence of a student request. This initial question was often followed by other questions on the same topic, guiding students through the use of chained scaffolding. The initial question in the English-language mathematics classroom, by contrast, was consistently more open.

Student-led exploratory discourse was very similar between the two language settings. Students in both language settings used visual mediators to coordinate their talk, and they used meta-level and object level endorsements in both classrooms. Some differences were observed in the category of word use. In the French-language classroom there was extensive use of code-switching as students worked together to complete tasks. In the English-language mathematics classroom, there was more evidence of non-standard mathematical language use, as well as non-standard mathematical notation.

Discussion of the Findings

The main topic of discussion relating to these findings concerns questions surrounding the place of discourse in the classroom. This discussion will balance previous researchers’ thoughts on this subject as presented in the literature review and the theoretical framework, and my own use of and reflection on this topic.

In general, the receptive and expressive language requirements in French for students in the French-language mathematics classroom were less demanding than that in the English-language mathematics classroom. In the French-language mathematics classroom, questions from the teacher were often closed, as opposed to open-ended, the questions were often repeated or reformulated, and required answers, even in an exploratory routine, were often short, consisting of one-word answers, numbers, yes/no questions, or gestures. This is understandable, perhaps, given that the students are working in a second language, but it is also potentially troubling, given the research of a number of scholars in the field of classroom discourse.

It is evident that Sfard, with her theory of commognition, places a high value on communication in the classroom. If learning, as she writes, is a question of "modifying and extending one's discourse" (Sfard, 2007, p. 567), then it follows that discourse is essential to learning. If the opportunities for rich discourse are not available to students, due to a lack of open-ended questions in the French-language mathematics classroom, then presumably opportunities for deep understanding will not present themselves in that environment either.

Mercer also subscribes to the notion that classroom discourse plays an essential role in learning. Similar to Sfard's combination of communication and cognition, Mercer believes that the cultural function (communicating) and psychological tool (thinking) are not really separate. Although he acknowledges that it is challenging for teachers to balance opportunities for open-ended exploration and discussion while achieving curriculum goals (without even addressing the challenges that a second language environment presents), he is firm in his belief that exploratory-type routines are essential to deeper understanding. Mercer (1995, p. 38) allows that there is a place for QRE-type routines; he states that students need exploratory routines in order to make coherent, independent sense of what they are being taught. If they do not have to recall and apply relevant knowledge without teacher prompting, then they are unlikely to be able to consolidate their understanding. Students need to develop and practise their own ways of using language as a social mode of thinking, by using language to reason, argue, and explain. These assertions about the role that complex language plays in developing a deep understanding of concepts raise concerns about the lack of open-ended questions in the French-language mathematics classroom. Are closed questions, or even a series of them, requiring one word, yes/no, or gestural answers going to lead to mastery of complex mathematical ideas?

Cummins (1984) also makes connections between communication and cognition, specifically in the area of second language learning. Cummins' four-quadrant framework differentiates context, which is the second language aspect of the French-language mathematics classroom, with cognitive demand, which is the mathematical content-related factor. The challenge for all teachers, especially in second-language content classes, such as the French-language mathematics classroom, is to ensure that students master cognitively demanding topics while ensuring that enough linguistic context has been provided so that this understanding can occur. The search for this balance could explain the fact that more visual mediators, in both frequency and type, are used in the French-language mathematics classroom, as well as the fact that questions are more closed, repeated, and reformulated. In general, the language in the French-language mathematics classroom could be characterised as high context, low cognitive demand, while the language in the English-language mathematics classroom would be low context, high cognitive demand. It is possible that the high cognitive demand in the English-language mathematics classroom could explain the increased use of non-standard language. Perhaps the fact that the students are struggling with mathematics questions of high cognitive demand means that they are therefore less able to express their thoughts in standard English. This could be because they have less cognitive reserves left to monitor their language usage, or simply that they are dealing with new concepts, they also need new vocabulary, the latter possibility again lending credence to Sfard's and Mercer's assertions that communication and cognition are intimately linked.

The need to avoid a deficit model (Moschkovich, 2007) is again important. Far from being a sign of a lack of understanding or ability, the use of non-standard language and notation in the English-language mathematics classroom can be viewed as evidence of learning. As Sfard states, a change in mathematical discourse is a change in mathematical thinking. Studying mathematics in a second language, however, does not mean that students should be sacrificing opportunities to be stretched cognitively.

Variations in classroom language have been noted in other contexts. Culligan (2010) was one researcher that noted changes in language when Anglophone high school students switched from learning mathematics in French to learning mathematics in English. Although there were no quantitative changes to the grades in mathematics, the students reported feeling more at ease in an English-language mathematics classroom, and that they felt that they could express themselves more freely. Despite the lack of changes in quantitative results on mathematics assessments, it is possible that this ease of expression led to more and deeper communication about, and therefore understanding of, the mathematical concepts.

Another variation in classroom language is seen in the area of code-switching. In the present study, code-switching was only seen in the French-language mathematics classroom, very rarely by the teacher, occasionally by the students in teacher-led dialogue, and almost constantly in the student-led dialogue. Anecdotally, from personal experience, code-switching is largely frowned upon in the French immersion classroom. As was seen, however, in the literature review chapter, code-switching must not be seen as part of a deficit model. In Planas and Setati (2009) it was shown that Latin American learners switched to Spanish, their dominant language, as soon as the conceptual level of the mathematical explanation they were giving increased. The switch to the dominant language was thus associated with the experience and performance of more difficult mathematics. Thus, code-switching in the French immersion classroom may be a way to access the essential and complex social construction of understanding in a second language setting.

Language tensions, similar to those found in the French immersion mathematics class, were also found in an English-medium mathematics class in Malta. Farrugia (2009) found that despite a policy expectation of consistent English usage by teachers and students, tensions existed between the mathematics content and English use in the classroom, as well as tensions between the promotion of student talk and the consistent use of English in the classroom. Farrugia found that these tensions played out by the younger students in grade three, who had fewer English skills used more QRE-type routines, thus perhaps avoiding code-switching, while older (grade six) students who had better English skills engaged in more exploratory-type discussions, but code-switched from English to Maltese more frequently. These tensions - fewer complex routines in classes with fewer second language skills, and code-switching during more complex discourse - are all similar to findings in the present study.

Given the importance of complex communication in the construction of mathematical understanding as well as the attendant challenges and tensions in providing students with opportunities to engage in such discourse in second language mathematics classrooms, what is the best way to balance the exigencies of delivering mathematical curriculum, encouraging deep discourse, and developing second language skills? After reviewing research literature on mathematics education, Hiebert and Grouws (2007, in Moschkovich, 2010) suggested two important points in effectively delivering mathematics curriculum. The first point was that teachers and students must attend explicitly to mathematical concepts, and the other is that students must wrestle with important mathematical concepts. In Cummins' terms, cognitive demand must be kept high, regardless of the language context. The decision, conscious or not, of a teacher to pre-emptively simplify the mathematical discourse in a classroom, due perhaps to realisations or concerns about second-language competence, does not seem to be a reasonable solution. Teachers' efforts in both the French and English mathematics classrooms in this study to explicitly teach mathematical vocabulary in order to help students master the register of mathematics are not a sufficient solution either. Planas and Setati-Phakeng (2014) suggested that a shift in perspective may lead to more productive practices that address linguistic tensions in second language mathematics classrooms. They suggested that rather than viewing non-native language skills among second language learners a problem in the mathematics classroom, the use of other languages in the classroom should be seen as a right, and that the knowledge of these languages can even be seen as a resource in the mathematics classroom, and should be treated as such. Solano-Flores (2010) sums it up nicely, saying that "effective learning is more likely to occur when students are allowed to use their linguistic resources in full" (p. 123).

If this “full use of their linguistic resources” were to become reality in French immersion classrooms in Canada, the linguistic landscape of these classrooms would look very different from what it does today. Although I did not find any official policy documents on the use of French and English in French immersion classes in Canada, such as the ones suggesting “consistency in the use of language” in Maltese classrooms, there is an expectation among teachers, students, parents, and administration that efforts will be made to speak French exclusively in French during French instructional time in French immersion classrooms. If students (and teachers) were allowed or encouraged to use English during mathematics lessons, would there be a corresponding jump in mathematics understanding and achievement? What would happen to French language skills? Which would stakeholders value more: a deeper understanding of concepts in mathematics, or increased fluency in French? It remains to be seen if this is a choice that Canadians will have to make, or whether this really is a zero sum question.

The discussion portion of this chapter was centered on the broader topic of the study and role of discourse in the classroom, more specifically in the mathematics and second language classrooms.

Chapter Six will build on the findings as well as the discussion, and will present conclusions by answering the research questions presented in Chapter Three. Anticipated contributions to knowledge, limitations to this study, as well as personal and professional reflections on this project, will also be put forward.

Chapter Six: Conclusions

Analysis of the findings has led to answers to the two research questions.

1. What are the characteristics of the discourse of teachers and students in an elementary French immersion mathematics classroom?

When examined using the four categories of Sfard's framework, teacher and student discourse in the mathematics classrooms shared some characteristics, yet differences were also noted. Teachers and students both used exploratory questioning routines when working to find answers to mathematical questions, whereas QRE routines were only used in teacher-led discourse in the mathematics classrooms. Although both teachers and students initiated and participated in exploratory questioning routines, the nature of these exploratory routines differed whether they were led by teachers or by students. When teachers led the exploratory routines, they often already knew the answers, and regularly sought to guide the student participants to an answer to the question, and frequently ended the routine with an object-level endorsement about the mathematical concept. In student-led exploratory routines, the initial question was provided by the teacher, and particular students did not consistently take on a leadership role. Unlike the predictable nature of a teacher-led exploratory routine, the nature of student-led routines varied. Sometimes a student would take the lead on a particular question, and give the partner the answer. At other times, the students worked in a more parallel manner, each working towards a solution in an individual manner. Sometimes the exploratory routine was more authentically collaborative, and the students worked together to solve the questions.

As in the category of routines, there were also similarities and differences in Sfard's other three categories. When endorsing a narrative, students' types of endorsements were more varied.

Students sometimes used object-level endorsements, as did their teachers, but they were seen to endorse incorrect narratives, while the teachers did not. Students also endorsed meta-level narratives, in which they came to conclusions about the nature of the way that they were doing mathematics. This phenomenon was not used by teachers. Both teachers and students used a variety of visual mediators in the mathematics classroom. Use of visual mediators was often directed by the teacher, even during student-led routines, as they distributed worksheets and math manipulatives. The only visual mediators that students were seen to initiate were concrete objects that were not in the classroom, such as mini milk cartons. Perhaps this teacher-directed use of concrete visual mediators is due to the nature of the classroom, where students' use of visual mediators was seen by students and teachers to be the domain of the teachers. Students were not encouraged to use the blackboard, and were not offered a choice in manipulatives. When examining word use, teachers and students were both heard to use non-standard as well as standard mathematical language in the classrooms. Code-switching, a very particular kind of word use, was much more common among the students than the teachers.

Although there were distinct characteristics between the discourse of teachers and of students in the mathematics classroom, the two groups also shared some similarities in their discourse. This pattern of similarities and differences persists in the conclusions about the second research question.

2. What are the similarities and differences between the discourses in an elementary French immersion mathematics classroom when the instruction is in French as opposed to when the same students are taught mathematics in English?

Despite the fact that there are some similarities between the discourse in the French-language and the English-language mathematics classes in this study, the differences are much more prominent. Both the QRE and the exploratory routines exist in the two language settings, however the characteristics of the routines are different. The most striking difference was in the initial question, specifically in exploratory dialogue. Questions in the English-language classroom tended to be much more open-ended, and so there was a wider range of acceptable answers, and more opportunity for authentic discussion and debate. French questions, even in exploratory dialogue, tended to be more closed, and correct answers were often very simple, in the form of a single word, yes or no, a number, or a gesture. Even questions that began exploratory routines in the French-language classroom were often simple, leading to a "chained scaffolding" exploratory routine, consisting of many simple, guided questions on the same topic, as opposed to one open-ended question followed by discussion as seen in the English-language mathematics classroom. QRE routines were much more similar in the two classrooms. Endorsement in both routines was seen on a more regular basis in the English-language mathematics classroom, especially in the exploratory routine, where it could be argued that it is more important. In the French-language classroom there was more use of visual mediators in the classroom, and there was also a wider variety of visual mediators used. When examining word use, it was evident that code-switching from French to English was used in the French-language mathematics classroom, but not in the English-language classroom. Although non-standard mathematics language was seen in both classrooms, it was more common in the English-language classroom. In both classrooms, there were explicit efforts to teach mathematical vocabulary. Although there were similarities in Sfard's categories between the two language settings, the differences stood out much more clearly.

Anticipated Contributions to Knowledge

Communication in the mathematics classroom is of the utmost importance in learning mathematics. Pimm (1987) devoted his entire landmark book to the topic, and Barwell (2009) has described a wide variety contexts of learning mathematics in second language settings. Communication also appears frequently in the current Ontario Mathematics curriculum document (Ontario Ministry of Education, 2005), in both the Process Expectations and in the Achievement Chart for Mathematics. According to the Ontario Ministry of Education, communication is deemed essential for students to master as they work to achieve the curriculum expectations, and for teachers as they plan instruction, provide meaningful feedback to students, and assess and evaluate student learning.

The current study systematically describes student and teacher language in a French immersion mathematics class and compares and contrasts these findings with a description of student and teacher language when the language of mathematics instruction is English. Patterns, similarities and differences between student and teacher communication in the two languages were analyzed. In contrast to much of the available research on mathematics in French immersion contexts (i.e. Turnbull, Lapkin, and Hart, 2001), this project is based on a qualitative analysis of classroom discourse, rather than a quantitative examination of testing results, which lends a different perspective to the field of study. This analysis provides a framework and common vocabulary for further research and discussion in this area. It is a much-needed look at communication in a mathematics classroom in a French immersion context, and could be used to raise awareness about and begin dialogue surrounding issues of characteristics and importance of classroom discourse for pre-service and in-service teachers, as well as to help inform decisions surrounding French immersion programming, such as the ones currently facing the OCDSB.

Finally, it also tested the practicality of Sfard’s theoretical commognitive framework in an analytical second-language context.

Limitations to the Study

Limitations to the study can be categorized into two groups: concerns about the design of the study and limitations related to the interpretation and reporting of the data.

In the area of research design there are limitations that exist in both phases of the study and limitations that exist because the study was conducted over the span of two years. In both years of the study the very fact that the researcher is in the room recording the lesson may have led to the Hawthorne effect, where participants may have changed their behaviour simply because they were aware of being studied. Also, in the portion of the observation when the researcher recorded several pairs of students during the teacher-directed task, the language of the other student pairs was overlooked, simply because it was not possible to record all of the pairs, due to consent and/or equipment issues. An alternative to this scenario would have been to observe and record each consenting pair one at a time, so that the language of each pair of students was recorded. This option, however, was judged as too much of a departure from regular classroom interaction and would also have introduced an unwanted time differential between the pairs. When analyzing and comparing French and English language in the classroom, it must also be remembered that neither English nor French is the home language of some of the students in the classroom.

There are inherent limitations to conducting a study with a significant time lapse, especially when children are among the subjects. When examining language patterns, the fact that children’s language is expected to continue to evolve at this age must be taken into account.

It is also important to keep in mind that the students had a different mathematics teacher in grade three and in grade four. Discourse varies naturally between speakers, and is not necessarily due to differences in language of instruction. Comparisons between the languages in the two scenarios may also have been hampered by the fact that the two teachers are unlikely to be covering the same topic during the data collection period. One topic may be relatively more difficult, or vocabulary in one area may be more complex. The limitations due to this time lapse are preferable to those which would have been present if two different grade three classes, one in the French immersion program, and thus receiving mathematical instruction in French, and one in the regular English program, receiving mathematical instruction in English were studied. In addition to differences between individual students and the challenges between individual teachers remaining, there is a very real possibility of a significant socio-economic difference between the two classes (See Statistics Canada, 2004).

Despite valid concerns about limitations I feel fortunate to have had access to naturalistic student and teacher classroom language two years in a row in two different languages. Such a situation can be logistically difficult to set up, so I am grateful to have been able to take advantage of this opportunity. Due to the relatively small scale of this project (one class), the reliability and external validity of the results would not be as high as they would be if the sample size were larger and more varied. These concerns, however, are somewhat mitigated as there are gains in the ecological validity of the project, as much of the data collection takes place during regular lessons, which gives credence to conclusions drawn about mathematical discourse in the elementary French immersion classroom.

Reflections on Teaching

Interwoven with the literature review, methodology, analysis, conclusions, and many more aspects of this thesis are the implications / reflections on my own teaching practice. It was indeed my own frustrations with the implications of new policy on my own teaching practice – the fact that as a French immersion teacher I was no longer able to teach my own mathematics – that originally provided the impetus to begin my M.A. studies in the area of mathematics and second language education. Sometimes these reflections seemed very far from my studies, while at other times my own reflections closely paralleled what I was learning. Now, as this study draws to a close, I find that these reflections have led to some action in my own classroom.

My own personal question during my studies, quite apart from my research questions, has been: In a French immersion classroom, should mathematics be taught in French or in English? Despite my love for teaching mathematics and my contract as a French immersion teacher, as well as the ensuing dissatisfaction with my new teaching assignment, I wanted to know if teaching mathematics in English was somehow “better” for the students.

Of course I am aware, as are many French immersion teachers and other teachers in similar linguistic situations, of the tensions between teaching the linguistic conventions of the French language and teaching curriculum content in a French immersion setting (e.g. Lyster, 1998; Laplante, 2000; Swain & Lapkin, 1998; and Lapkin & Swain, 2004), as mentioned in Chapter 2. There is often worry from stakeholders that if one subject is seen as a priority, then the other one will necessarily suffer; an assumption in this setting that if the emphasis is put on mastery of French language skills than achievement in mathematics will suffer, and vice versa. If there is a trade-off, then I suppose someone – teachers? parents? the school board? the Ministry? has to decide what the priority is.

My own literature review says that this is not an either-or decision. I cited numerous studies that said that French immersion students do as well or better than their peers who study mathematics in English, that even students who are not fully bilingual can benefit from bilingualism, that bilingualism itself is good for our brains. Despite the fact that the students who have fewer skills in the language of teaching and learning of mathematics have the lower levels of achievement in mathematics, the evidence is that it is not bilingualism per se that is the cause of these shortcomings.

When I read these studies I was glad. I was happy to cite them. I felt vindicated in my frustration over not being able to continue to teach mathematics in French as I had been doing with enthusiasm and success and an ever-growing sense of mastery, for almost a decade. I headed off to the classroom to collect data, to listen to children learn mathematics in French much as they did in English.

Over the next four years I recorded mathematics lessons in French and in English, I transcribed them, I analysed them, sometimes word by word, and then stepped back to look for the stories that they told. I'm not sure that I found what I thought I would. I found students who worked together with their classmates to answer a challenging question in mathematics, who had to try several times in order to come up with a satisfactory answer and who sometimes used words and symbols in new ways in order to express their new understanding. In Mercer's words, the students had opportunities to "...recall and apply relevant knowledge without teacher prompting".

What most surprised me was the fact that the vast majority of these opportunities were given in the English-language mathematics classroom. In the French-language mathematics classroom, even exploratory routines were very closed and scaffolded, affording little opportunity for error or experimentation.

These observations are broad generalisations, and are of course specific to the two individual teachers that I observed. But as a French immersion teacher, I can relate to the degree of linguistic scaffolding that we do on a daily basis, especially in the primary grades. It's a common way to ensure that students are able to understand and express themselves even with limited linguistic means. But now I'm wondering whether this level of linguistic scaffolding is interfering with learning opportunities in mathematics, whether this pre-emptive simplification precludes a development of deeper understanding. To use Cummins' terminology, are our immersion students marooned in cognitively undemanding learning situations in an effort to retain the high context necessary in order to facilitate communication?

I don't feel that these generalisations about discourse in French and English mathematics classrooms contradict what I learned in the literature review, that doing mathematics in a bilingual setting is possible, even beneficial. I do, however, feel that there is a gap between what is known about effective talk in classrooms and what actually happens there. What impressed me the most as a teacher in doing this project has been the importance of student talk in the classroom. In reading about communication, about cognition, about commognition, in listening to students and teachers talk in the classroom, and in analysing their discourse, I have been convinced that talking is thinking, not just a participation comment in the learning skills section of the report card. Knowledge does not flow directly from teacher to student, no matter how much the teacher talks. It needs to go through something on the way: not a filter, not a mixer. That something is talking or thinking, or the same thing, if you're Sfard.

I think that it is possible to close the gap between knowledge about and practice of student talk in the classroom. I've been working on it in my classroom this year. We can explicitly teach students to talk together, and we as second language teachers can get more comfortable, and help students to feel more comfortable, with "messier" communication: pauses, false starts, grammatical errors, neologisms, debate, code-switching, gesture, justification, and noise. The sound of students making full use of their linguistic resources can be louder than a traditional classroom. In so doing, however, student can attack big questions in mathematics together, while at the same time practising their French more often. It's a win-win, no choice required.

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