Domestic and Cross-border Mergers, R&D and Profitability with Asymmetric R&D Spillovers

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Abstract

Mergers can make merged companies more profitable and competitive. Meanwhile, R&D has become a crucial factor for firm survival and growth. In various industries, such as the high-tech industries, mergers are considered as paths toward more R&D and higher profitability. The connections between mergers, R&D and profitability have become more important in recent years. In this paper, we develop a theoretical model with R&D and domestic/cross-border merger. In this two-stage model with homogenous product, firms choose R&D in the first stage and outputs in the second stage, and then the equilibriums are computed, analyzed and compared. Also, asymmetric R&D spillovers help to explain the process of cost-reducing innovation. Finally, this paper finds that mergers help increase R&D and profitability, but that cross-border mergers are not always a better option than domestic mergers.

Keywords: Domestic and cross-border mergers, R&D, R&D spillovers, Profitability
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1. Introduction

In mainstream economic history, mergers have a relatively short history. Horizontal mergers started in the early 1900s, after a period of economic expansion (Gaughan, 2007). The important feature of the first merger wave was the consolidation of manufacturers within the same industry. Although the magnitudes of the second and third merger waves were relatively smaller than the magnitude of the first, the concept of merger was enriched, and vertical integrations started to account for a larger portion after the second and third merger waves (Sudarsannam and Mahate, 2003). During the 1980s and 1990s, the world experienced the fourth and fifth merger waves of cross-border mergers and mega-mergers (Sudarsannam and Mahate, 2003; Belleflamme and Peitz, 2010).

The booming financial markets of the 1980s and 90s indeed provided favourable conditions for cross-border mergers. The Daimler–Chrysler merger, the Citibank-Travelers merger and the Exxon-Mobil alliance are just a few famous examples in this global merger trend (Qiu and Zhou, 2006; Qiu, 2010). In recent years, many well-known Canadian firms have been purchased. In 2014, Burger King purchased Tim Hortons for 11.4 billion US dollars (Baker et al., 2014). On February 3th, 2016, U.S. retailer Lowe's announced its intent to acquire Rona for 3.2 billion US dollars — a merger pending government approval. Is this global merger trend welfare increasing? What should governments do to prevent inefficiency in this trend?

Research and development (R&D) is essential for economic development. Firms and governments expect timely and specific returns from their infrastructure investments, human resources and market strategies. However, R&D does not generate profit immediately and this process usually involves high risks and unpredictable returns.
Competing in a more innovative environment, companies enjoy continuously new technical improvements more often. For example, new cars with high-tech features are continually being developed, and new models made by different manufacturers tend to benefit from the same high-tech improvements every year. The same trend is also observed in software engineering and the mobile app designing industries. Most software, for example, uses identical or nearly-identical user interfaces, and the same is true of mobile apps. Meanwhile, the importance of spillover effects cannot be ignored, as the R&D process has been accelerated so much in the past thirty years. Also, the industry R&D level has become an important measure of overall economic progress of a country. Not only governments and business leaders, but also consumers consider R&D essential, especially during recessions.

However, a fuller understanding of the relationship between mergers and R&D remains incomplete. The main purpose of this thesis is to shed some light on this relationship. Additionally, we will analyze merger profitability, since the first priority of companies is to maximize profits.

The model in this paper is based on a two-stage Cournot framework with asymmetric spillovers. Four firms, two domestic and two foreign firms, compete in a one-product market. Each firm makes R&D decisions preceding production decisions. Technological leakages (R&D spillovers) are allowed. In order to consider the effects of spillovers separately, both domestic and foreign R&D spillovers are applied in the model.

There are three main findings. First, cross-border mergers are not always a good choice. When the domestic R&D spillover is high and the foreign R&D spillover is low, governments and companies should pay more attention to domestic mergers. In a situation of low domestic R&D spillover and high foreign R&D spillover, domestic and cross-border mergers should both
be encouraged.

Second, the profitability of mergers is significant. In most situations, both domestic and cross-border mergers can make merged entities more profitable than outsiders. For merged firms, cross-border mergers can help make more profit than domestic mergers.

Third, for R&D pursuing firms, maintaining two independent R&D labs is more efficient and productive than having only one R&D lab.

This paper is structured as follows: Section 2 is a brief review of the literature. The papers discussed were all published in the last three decades and include both theoretical and empirical literature on domestic and cross-border mergers. Section 3 presents the basic model and assumptions. Sections 4 and 5 calculate and compare equilibrium R&D and profit from the different perspectives that firms and governments care about. Section 6 considers the case of a merged entity operating a single research lab. The last section, section 7, concludes.

2. Literature Review

Mergers have been a highly discussed topic during the last thirty years. The driving forces underlying the trend to mergers are complex and vary by sector, industry and country. Kang and Johansson (2000) researched the trends and drivers of mergers from 1991 to 1998 in both Europe and North America. They found that cross-border mergers consist of more than 85% of foreign direct investment, that they increased more than six-fold during that period and that they accounted for almost one-third of all mergers in the 1990s. This booming trend was influenced by a number of factors, including excessive capital for investment, increased competition in traditional industries and newly emerged opportunities in high-technology markets. The most important motive underlying mergers is the need to acquire complementary innovative assets.
Also, the authors found that the extent to which firms can benefit from mergers is highly related to regional and national regulations and policies. Zademach and Rodriguez-Pose (2009) analyzed domestic and cross-border mergers in Europe. They used the Mergermarket database from 1998-2003 and discovered that international mergers rose more rapidly than domestic mergers.

The Asia-Pacific sphere also followed this international merger trend. Chen and Findlay (2003) found that cross-border mergers increased rapidly among countries in the Asia-Pacific Economic Cooperation region (APEC), especially since the mid-1990s. This dramatic increase in cross-border mergers in the Asian and Pacific areas, they claim, can be explained by an integrated effect of free trade policy, the deregulation of planned economic systems and privatisation of the public sector. In the area of cross-border mergers, the APEC developing economies, such as China and India, have become more prominent since the late 1990s.

There is a large theoretical literature on the relationship between mergers and R&D. The traditional relations among firms are usually of a simple type, wholly cooperative or wholly noncooperative. Contrary to the traditional assumptions made in most oligopoly models, D’Aspremont and Jacquemin (1988) introduced the partly cooperative and noncooperative type of oligopoly to construct a two-stage model with R&D spillover. They came to the conclusion that, in some situations, firms compete in research, while cooperating in production. This paper mainly focused on two types of cooperative oligopoly. In the first type, firms cooperate in the first stage (R&D), but compete in the second stage (output). In the second type, firms cooperate in both research and outputs stages. Their model shows that cooperative behavior among firms can have a positive effect on innovation in industries with a few competitors and high R&D spillovers.

Atallah (2005a) studied mergers between arbitrary numbers of firms in oligopoly with R&D
spillovers. He mainly focused on merger profitability and found that in most situations the effects of R&D on merger profitability are negligible. His paper demonstrates that noncooperative R&D increases the profitability of mergers when spillovers are low or high, but decreases profits when spillovers are intermediate. With information barriers, cooperative R&D makes mergers profitable for low R&D spillovers, but unprofitable for high leakages.

In most previous theoretical papers, R&D spillovers are assumed symmetric. However, a few papers discussed mergers and R&D with asymmetric spillovers. Atallah (2005b) analysed R&D cooperation with asymmetric spillovers based on the framework of D’Aspremont and Jacquemin (1988). The critical idea here is that firms have different levels of R&D spillovers because the degree of technological leakage is not the same for all firms. There are mainly three findings in this paper. First, cooperation will have a positive effect on R&D when the average spillover rate is higher than 0.5. Second, for a wide range of spillover rates, R&D cooperation is not profitable to at least one of the firms. Thus, cooperation will not occur in most situations. Third, R&D cooperation with asymmetric spillovers reduces consumer surplus but increases welfare.

Atallah (2005c) studied the profitability of R&D cooperation under asymmetric spillover rates. The paper contributes in two aspects. For an individual firm, R&D competition is preferable to research joint venture (RJV) cartelization when its R&D leakage rate is low and the spillover rate of its competitor is high. And RJV cartelization is preferable to R&D cartelization when its R&D leakage rate is high and the spillover of its competitor is low. For the equilibrium configuration structure, the equilibriums are RJV cartelization, R&D competition and R&D cartelization when the degree of asymmetry in spillovers is low, intermediate and high.

Nikzad and Atallah (2011) focused on the effect of tariffs on R&D expenditure with
asymmetric R&D spillovers. They consider a three-stage game based on Cournot competition. In the first stage, the government sets the level of the tariff and the R&D subsidy. Firms choose their R&D expenditures in the second stage and outputs in the third stage. The first conclusion is that domestic firms can make more profits under R&D cartelization than under research joint ventures (RJV) cartelization. When foreign spillover is high, welfare is higher under R&D cartelization than it is under no cooperation. Also, they found that the home country can enjoy profit and welfare gains by using a tariff policy and an R&D subsidy policy simultaneously.

However, the majority of papers only studied domestic mergers, and the published theoretical works related to cross-border mergers or international mergers are still scarce. Most of these studies mainly paid attention to merger profitability and drivers of cross-border mergers, but neglected the R&D changes and R&D spillover effects.

Long and Vousden (1995) evaluated the effects of tariff reduction on mergers in a two-country Cournot oligopoly model. They observed that relaxed trading conditions help to increase merger activities, especially cost-reducing mergers. This result applies to both domestic and cross-border mergers.

Horn and Persson (2001) applied a model of endogenous merger formation to an international oligopoly model with trade barriers. In their model, high trade costs tend to induce cross-border mergers. Additionally, the market structure with only domestic mergers is never preferable when the government aims for the maximization of social welfare.

In a two-country model of oligopoly, Neary (2007) specified and solved a general equilibrium model by introducing trade conditions. In international mergers, the cases where low-cost firms buy out high-cost foreign opponents are profitable. Neary showed that the primary driver of cross-border mergers is to gain more market power and comparative
technological advantage. Also, he confirmed that trade liberalization can induce cross-border merger waves.

Heywood and McGinty (2011) identified the unique strategic issues of cross-border mergers in a mixed oligopoly model, and they also illustrated that the existence of a welfare maximizing public firm stimulates such mergers. With a public firm in the model, they examined two different forms of cost functions (linear and convex). The prominent merger paradox that two-firm mergers are rarely profitable is substantially relaxed in the cases of both linear and convex cost functions. In addition, the paper suggests that two-firm mergers are profitable when public firms participate in them.

Among empirical works, there are few studies that try to find a direct link between mergers and R&D. Views on how technological levels of firms are influenced by mergers are often conflicting. Moreover, most of such studies to date show high variance in regression results and find no robustness.

In the US, Hall (1988) assumed the merger and acquisition market is a matching process and estimated this matching model with a large cross-industry sample (2,519 manufacturing firms from 1976 to 1985) constructed from the Compustat data files. The main finding is that, with this matching model, firms of similar sizes and R&D levels are more likely to merge, and there is no evidence to show that the merger process has a negative effect on R&D. Also, she mentioned that relatively lower R&D intensities and weaker technological bases are found in foreign firms, which are involved in mergers in the domestic market. Later, Hall (1990) explored the general link between R&D intensity and merger activities. Finally, she suggested a possible positive relation between R&D intensity and merger activity.

However, by using data from Standard and Poor's Compustat research data files and
Moody’s Industrial Manual, Hitt et al. (1991) identified 278 mergers and obtained data on both R&D expenditures and patent data. The qualified data set covered 29 industries. Mainly, the authors concluded that mergers had negative effects on R&D intensity and patent intensity, which represent R&D inputs and outputs respectively.

Blonigen and Taylor (2000) focused their investigation on US companies in the electronics and electrical equipment industries. Usually, these high technology industries are not only R&D-intensive, but also tend to be involved in a lot of mergers. They sampled 217 electronics and electrical equipment firms from the Compustat database with continuous financial data from 1985 to 1993. This paper provided evidence of a significant negative relationship between R&D intensity and mergers in high-tech industries.

Hagedoorn and Duysters (2002) intended to further the research on the relationship of mergers and R&D in high-tech industries. They used data from the computer industry, involving companies that produce mainframes and other parts of computers, peripherals, software solutions, data communications and data processing services. Their sample consists of 201 global mergers during the period from 1986 to 1992. Their data showed that the mergers can truly improve the technological achievement of high-tech companies.

Cassiman et al. (2005) used European data to provide a more detailed picture. When merged entities are technologically complementary, the merger process results in a more active R&D performance for involved firms. In contrast, when merged entities are technologically substitutive, the merger process significantly decreases R&D. Furthermore, the efficiency of R&D increases more significantly when merged entities are technologically complementary than when they are substitutive.

In addition, the empirical works of Patel and Pavitt (1991) and Serapio and Dalton (1999)
found that international merged companies tend to increase their investment in R&D activities.

3. The Basic Model

Two Cournot duopolies, a domestic duopoly and a foreign duopoly, take decisions in the framework of a two-stage model. Considering R&D as a form of investment, the R&D decisions precede the production decisions; that is, firms first make decisions on R&D investments, and then choose outputs in the second stage to maximize profits.

In this paper, an important factor, asymmetric R&D spillovers, is considered in the analysis, since the difficulties of protecting intellectual property are different between domestic and international competitions. Detailed assumptions of the model are as follows.

There are two countries, A and B, and there are two identical companies in each country (firms 1 and 2 in country A; firms 3 and 4 in country B). All four firms are assumed to be the same size and to produce the same product. Also, the cross-border trading cost of this product is assumed to be zero.

The market inverse demand is assumed to be linear as:

\[ p = A - b \sum q_i \]  

(1)

where \( A \) is a constant, which is assumed to be sufficiently large so that all firms produce positive outputs in equilibrium. Here, all firms produce a homogeneous product and we set \( b = 1 \). \( p \) is the price of this product in both countries, and \( q_i \) represents firm \( i \)'s output.

The marginal cost function of firm \( i \) is given by:

\[ c_i = a - x_i - \beta \left( \sum x_d \right) - \alpha \cdot \beta \left( \sum x_f \right) \]  

(2)

where \( a \) is the initial marginal cost, which is the same for all firms since they are symmetric. \( x_i \) is
the R&D choice of firm $i$. $x_d$ is the R&D of domestic firms other than firm $i$ itself, and $x_f$ refers to the R&D of foreign firms. Thus, this cost function involves all the possible R&D cost reductions. Meanwhile, different R&D spillovers are taken into consideration. The spillover rate of the domestic industry is $\beta \in (0,1)$ and the parameter $\alpha \in (0,1)$ reflects the relatively lower spillover degree of the foreign industry. The product $\alpha \cdot \beta$ measures the R&D cross-border spillover effect.$^1$

The profit function of firm $i$ is given by:

$$
\pi_i = (p - c_i) y_i - \gamma x_i^2
$$

where $y_i$ is the output of firm $i$ and the last term, $\gamma x_i^2$ is the R&D cost, which is assumed to be quadratic. The quadratic assumption reflects diminishing returns to R&D expenditure. $\gamma$ is an R&D cost parameter, which is negatively related to the efficiency of research.

Based on these assumptions, three equilibrium cases — no merger, domestic merger and cross-border merger — are analyzed.

3.1 No Merger

If no merger occurs, the market consists of four firms in total. Since all firms are symmetric, the equilibrium is symmetric. By solving the maximization problem of the two-stage game, equilibrium R&D and profit are given by: $^2$

$$
x_i^* = \frac{(A-a)(4-\beta+2\alpha\beta)}{-4+(1+2\alpha)\beta(-3+\beta+2\alpha\beta)+25\gamma} \quad (4)$$

$$
\pi_i^* = \frac{(A-a)^2 \gamma \left(25\gamma - (-4+\beta+2\alpha\beta)^2\right)}{(-4+(1+2\alpha)\beta(-3+\beta+2\alpha\beta)+25\gamma)^2} - 1 - \frac{(-4+\beta+2\alpha\beta)^2}{25\gamma} \left(y_i^*\right)^2 \quad i = 1, 2, 3, 4 \quad (5)
$$

$^1$ By assumption of $\beta$ and $\alpha$, we have $\alpha \cdot \beta \in (0,1)$ and $\alpha \cdot \beta < \beta$.

$^2$ Computation details are given in Appendix and where $y_i^*$ is given by:

$$
y_i^* = \frac{5(A-a)\gamma}{-4+(1+2\alpha)\beta(-3+\beta+2\alpha\beta)+25\gamma} \quad i = 1, 2, 3, 4
$$
In most situations, R&D ($x_i^n$) decreases with both $\beta$ and $\alpha$, suggesting that both domestic spillover ($\beta$) and cross-border spillover ($\alpha$) primarily induce negative effects on R&D. R&D is illustrated in Figure 1. As the graph illustrates, the absolute value of $\frac{\partial x_i^n}{\partial \beta}$ is larger when $\alpha$ is closer to 1 and the absolute value of $\frac{\partial x_i^n}{\partial \alpha}$ is larger when $\beta$ is closer to 1. This is mainly due to the larger cost reduction effect of higher R&D spillovers. With high R&D spillovers, firms tend to count on sharing R&D progress with others, instead of doing research on their own. However, lower rates of both domestic spillover ($\beta$) and foreign spillover ($\alpha$) help firms keep a higher R&D.

**Figure 1** R&D of firm $i$ without merger

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3 Differentiating w.r.t $\alpha$ and $\beta$:

$$\frac{\partial x_i^n}{\partial \beta} = \frac{(A-a)(1+2\alpha)((-4+\beta+2\alpha\beta)^2-25\gamma)}{(-4+(1+2\alpha)\beta(-3+\beta+2\alpha\beta)+25\gamma)^2} < 0$$

$$\frac{\partial x_i^n}{\partial \alpha} = \frac{2(A-a)\beta((-4+\beta+2\alpha\beta)^2-25\gamma)}{(-4+(1+2\alpha)\beta(-3+\beta+2\alpha\beta)+25\gamma)^2} < 0$$

when $\gamma < \frac{(\beta+2\alpha\beta-4)^2}{25}$, then $\frac{\partial x_i^n}{\partial \beta} > 0$ and $\frac{\partial x_i^n}{\partial \alpha} > 0$.

4 To keep generality, all figures in this paper are drawn using the same parameterization standard, i.e. $A=1000$, $a=50$ and $\gamma=60$. 

---
Figure 2 illustrates profit ($\pi_i^n$) as a function of $\beta$ and $\alpha$. Profit first increases with $\beta$ and $\alpha$, but then starts to decrease with $\beta$ and $\alpha$ when $\beta$ and $\alpha$ are quite high and close to 1. In this region, R&D spillovers have a negative effect on profit (the cost reduction effect that spillovers bring is less than the loss from reduced own R&D).

![Profit of firm i without merger](image)

**Figure 2** Profit of firm $i$ without merger
3.2 Domestic Merger

Now, assume that firms 1 and 2 merge and become a domestic monopoly in country A. We assume that the merged entity maintains two research labs. Solutions to the R&D and profit of the domestic merged entity are given by:

\[ x_1^d = x_2^d = \frac{3 - 2\alpha\beta}{4\gamma} y_m^d \]  
\[ \pi_m^d = \left[ 1 - \frac{(3 - 2\alpha\beta)^2}{8\gamma} \right] (y_m^d)^2 \]

Solutions to the R&D and profit of foreign outsiders are given by:

\[ x_3^d = x_4^d = \frac{3 - \beta - \alpha\beta}{4\gamma} y_3^d = \frac{3 - \beta - \alpha\beta}{4\gamma} y_4^d \]
\[ \pi_3^d = \pi_4^d = \left[ 1 - \frac{(3 - \beta - \alpha\beta)^2}{16\gamma} \right] (y_3^d)^2 \]

In this case of a domestic merger, the R&D of the merged entity \((x_1^d = x_2^d)\) and outsiders \((x_3^d = x_4^d)\) are shown together in Figure 3, where both of them decrease with \(\beta\) and \(\alpha\). The R&D of the merged entity decreases more rapidly with \(\alpha\), while the R&D of outsiders decreases more rapidly with \(\beta\). The R&D of the merged entity is higher than the R&D of foreign outsiders.

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5 In section 6, we consider the case where the merged entity maintains only one research lab.

6 Computation details are given in Appendix and where \( y_m^d \) is given by:

\[ y_m^d = \frac{2(A - \alpha)(3 + \beta(2 - \beta + \alpha(-7 + \beta + 2\alpha\beta)) - 4\gamma)}{-9 + \beta(3(-2 + \beta) + \alpha(9 + \beta(7 - 2\beta + 2\alpha(-4 - 9\alpha + 2\alpha(1 + \alpha)\beta))) + 48\gamma + 4\beta(2 - \beta + \alpha(-16 + 5\alpha\beta))\gamma - 32\gamma^2} \]

7 Computation details are given in Appendix and where \( y_3^d = y_4^d \) are given by:

\[ y_3^d = y_4^d = \frac{4(A - \alpha)(3 + \alpha\beta(-5 + 2\alpha\beta) - 2\gamma)}{-9 + \beta(3(-2 + \beta) + \alpha(9 + \beta(7 - 2\beta + 2\alpha(-4 - 9\alpha + 2\alpha(1 + \alpha)\beta))) + 48\gamma + 4\beta(2 - \beta + \alpha(-16 + 5\alpha\beta))\gamma - 32\gamma^2} \]

8 For example, we check two points in Figure 3:

- where \( \beta = 0.03 \) and \( \alpha = 0.97 \), \( x_1^d = x_2^d = 2.94 > x_3^d = x_4^d = 2.91 \);
- where \( \beta = 0.05 \) and \( \alpha = 0.95 \), \( x_1^d = x_2^d = 2.91 > x_3^d = x_4^d = 2.87 \).
This gap between the two R&D increases when $\beta \to 1$ and $\alpha \to 0$. This is because the R&D spillovers gap (the gap between $\beta$ and $\alpha \cdot \beta$) increases when $\beta \to 1$ and $\alpha \to 0$ and more technological externalities are internalized by the merged entity under domestic mergers.

**Figure 3** R&D of the merged entity and outsiders with domestic merger

Figure 4 illustrates profits of both the merged entity ($\pi^d_m$) and foreign outsiders ($\pi^d_3 = \pi^d_4$) as a function of $\beta$ and $\alpha$. The profit of outsiders increases with both $\beta$ and $\alpha$. The profit of the merged entity stays at a relatively higher level than the profit of outsiders in most situations, but it becomes lower when $\beta$ and $\alpha$ are high. For outsiders, it is possible to gain more profit than the merged entity when the domestic spillover ($\beta$) is quite high (very close to 1).
3.3 Cross-border Merger

In this part, suppose that one domestic firm merges with a foreign firm, that is, firm 1 in country A merges with firm 3 in country B. The cross-border merged entity keeps R&D labs in both countries, and thus this merged entity can benefit from the R&D of other firms in both countries. Solutions to the R&D and profit of the merged entity are given by:

\[ x_1^c = x_3^c = \frac{(3 - \beta - \alpha \beta)}{4 \gamma} y_m^c \quad (10) \]

\[ \pi_m^c = \left( 1 - \frac{(3 - \beta - \alpha \beta)^2}{8 \gamma} \right) \left( y_m^c \right)^2 \quad (11) \]
Solutions to the R&D and profit of the two outsiders are given by:\textsuperscript{10}

\[
x_2^\varepsilon = x_3^\varepsilon = \frac{(3 - \beta - \alpha \beta)}{4\gamma} \\
y_2^\varepsilon = y_3^\varepsilon = \frac{(3 - \beta - \alpha \beta)}{4\gamma}
\]

(12)

\[
\pi_2^c = \pi_4^c = \left(1 - \frac{(3 - \beta - \alpha \beta)^2}{16\gamma}\right) y_2^c = \left(1 - \frac{(3 - \beta - \alpha \beta)^2}{16\gamma}\right) y_4^c
\]

(13)

By superposing R&D of insiders \((x_i^\gamma = x_i^\varepsilon)\) and outsiders \((x_2^\varepsilon = x_3^\varepsilon)\) in Figure 5, we can see that R&D investments of the merged entity and outsiders decrease with spillovers \((\beta\) and \(\alpha)\). The R&D of the merged entity is slightly higher than the R&D of outsiders in Figure 5.\textsuperscript{11}

![Figure 5 R&D of the merged entity and outsiders with cross-border merger](image)

Figure 5 shows the profits of the merged entity \((\pi_m^c)\) and outsiders \((\pi_2^c = \pi_4^c)\). On average,

\textsuperscript{10} Computation details are given in Appendix and where \(y_2^c = y_4^c\) are given by:

\[
y_2^c = y_4^c = \frac{2(A - a)(-6 - (1 + \alpha)\beta(-5 + \beta + \alpha \beta) + 4\gamma)}{(-1 + \beta)(-3 + \beta + \alpha \beta)^2(1 + \beta + \alpha \beta) - 8(-2 + \beta)(-3 + \beta + \alpha \beta)\gamma + 32\gamma^2}
\]

\textsuperscript{11} For example, we check two points in Figure 5:

where \(\beta = 0.50\) and \(\alpha = 0.40\), \(x_i^\varepsilon = x_i^\gamma = 2.30 > x_2^\varepsilon = x_2^\gamma = 2.28\);

where \(\beta = 0.70\) and \(\alpha = 0.50\), \(x_i^\varepsilon = x_i^\gamma = 1.95 > x_2^\varepsilon = x_2^\gamma = 1.93\).
the profit of the merged entity is higher than the profit of outsiders. But when domestic spillover ($\beta$) and cross-border spillover ($\alpha$) are both quite high (very close to 1), the outsiders’ profit is higher.

![Figure 6](image)

**Figure 6** Profits of the merged entity and outsiders with cross-border merger

4. Effects on R&D

Based on the discussion in the previous section, I compare and explain R&D of the merged entity, R&D of outsiders and domestic industry R&D.

4.1 R&D of the Merged Entity

As previously discussed in section 3, the merged entity has two research labs with the same R&D in equilibrium ($x_1^d = x_2^d$ and $x_1^c = x_2^c$), no matter whether the merger is domestic or cross-border.

Now, all the equilibrium R&D levels are drawn in Figure 7 to make a generalized comparison. Overall, the R&D of the merged entity with domestic merger ($x_1^d = x_2^d$) is highest,
especially when $\beta \to 1$ and $\alpha \to 1$. However, in the area where $\beta \to 0$ and $\alpha \to 1$, R&D of the merged entity with domestic merger ($x_1^d = x_2^d$) and cross-border merger ($x_1^c = x_3^c$) are the same.\(^{12}\) Thus, firms would prefer a domestic merger to gain a higher level of R&D when the domestic spillover rate ($\beta$) is high and foreign spillover rate ($\alpha$) is low. This is because more technological externalities are internalized under the domestic merger when domestic spillover is high. No merger is obviously the worst choice.

![Figure 7](image.png)

**Figure 7** R&D of the merged entity with no merger, domestic merger and cross-border merger

### 4.2 R&D of Outsiders

For R&D of outsiders, Figure 8 shows the generalized results by superposing all R&D of outsiders and also R&D without merger. From the graph on the right in Figure 8, we can see that the R&D of outsiders with domestic merger and the R&D of outsiders with cross-border merger

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\(^{12}\) For example, we check two points of the graph on the right in Figure 7:

where $\beta = 0.03$ and $\alpha = 0.97$, $x_1^d = x_2^d = 2.95 \approx x_1^c = x_3^c = 2.95$;

where $\beta = 0.05$ and $\alpha = 0.95$, $x_1^d = x_2^d = 2.91 \approx x_1^c = x_3^c = 2.91$. 

18
are at the same level.\textsuperscript{13} When a domestic or cross-border merger occurs, outsiders enjoy better achievement in R&D compared with no merger. Thus, no merger is once again the worst option. When mergers occur outsiders tend to invest more in R&D than when no merger occurs in order to compete with the merged entity.

![Figure 8 R&D of outsiders with no merger, domestic merger and cross-border merger](image)

\textbf{4.3 Domestic Industry R&D}

The domestic industry R&D is measured by aggregating R&D choices of all domestic firms, i.e. firms 1 and 2. In the case of no merger, the domestic industry R&D is given by:

$$D_n = x_1^n + x_2^n = \frac{2(A-a)(4-\beta+2a\beta)}{-4 + (1+2\alpha)\beta(-3 + \beta + 2a\beta) + 25\gamma}$$  \quad (14)$$

In the case of domestic merger, the domestic industry R&D is given by:

$$D_d = x_1^d + x_2^d = \frac{(3-2a\beta)}{2\gamma} y_m^d$$  \quad (15)$$

\textsuperscript{13} For example, we check two points of the graph on the right in Figure 8:

where $\beta = 0.50$ and $\alpha = 0.40$, $x_1^d = x_2^d = 2.28 \approx x_1^e = x_2^e = 2.28$;

where $\beta = 0.70$ and $\alpha = 0.50$, $x_1^d = x_2^d = 1.93 \approx x_1^e = x_2^e = 1.93$. 
In the case of cross-border merger, the domestic industry R&D is given by:

\[ D_c = x_1^c + x_2^c = \frac{(3 - \beta - \alpha \beta)}{4\gamma} y_m^c + \frac{(3 - \beta - \alpha \beta)}{4\gamma} y_2^c \]  
(16)

Figure 9 superposes the graphs of equations (14) and (15) in the left graph and equations (15) and (16) in the right graph. Overall, the domestic industry R&D is highest under domestic merger, followed by cross-border merger, followed by no merger \((D_d > D_c > D_n)\), especially when the domestic spillover rate \((\beta)\) is high and foreign spillover rate \((\alpha)\) is low.

Meanwhile, from the graph on the right in Figure 9, we can see that a low \(\beta\) and a high \(\alpha\) reduce the gap between \(D_c\) and \(D_n\). Therefore, a policy encouraging domestic merger would raise the domestic industry R&D level when the domestic spillover rate \((\beta)\) is high and the foreign spillover rate \((\alpha)\) is low. The reason why domestic merger leads to a higher level of domestic industry R&D than cross-border merger is that more technological externalities are internalized under domestic merger when the domestic spillover \((\beta)\) is high.

**Figure 9** Domestic industry R&D with no merger, domestic merger and cross-border merger
5. Effects on Profitability

The model analyzed in section 3 examined the profitability of domestic and cross-border mergers and found that the merged entity achieves a higher profit level than outsiders in most situations. From Figures 4 and 6, we can see that the profit levels with both domestic and cross-border merger are higher than the profit with no merger. Overall, the merged entity and outsiders benefit from mergers. But whether domestic or cross-border merger leads to more profit is ambiguous. Considering the merged entity and outsiders respectively, the profitability of domestic merger and cross-border merger are examined and compared in this section.

5.1 Profit of the Merged Entity

By superposing profits of the merged entity with domestic and cross-border merger in Figure 10, we can see that the profit of the merged entity with domestic merger ($\pi_m^d$) is higher than the profit of the merged entity with cross-border merger ($\pi_m^c$) for all possible $\beta$ and $\alpha$. For the merged entity, cross-border merger does lead to a more profitable equilibrium than domestic merger. This is because the merged entity with cross-border merger has more market power than with domestic merger. This market power gap increases with domestic spillover ($\beta$) and decreases with cross-border spillover ($\alpha$).

Under domestic merger, even though the merged entity benefits from the R&D investment of the foreign outsiders, the cross-border spillover has a relatively weak power to influence the profitability of the merged entity. However, in the case of cross-border merger, the R&D of the merged entity deceases with the cross-border spillover ($\alpha$) (see Figure 5), and this decrease of
R&D leads increased production costs and reduces the market share of the merged entity.

Figure 10 Profits of the merged entity with domestic merger and cross-border merger

5.2 Profit of Outsiders

In Figure 11, the profits of outsiders with domestic merger and cross-border merger are superposed. Unlike the profit of the merged entity, outsiders with domestic merger enjoy higher profits than outsiders with cross-border merger. This result can be explained by the market power of the merged entity. Since the merged entity with domestic merger has less market power than with cross-border merger, outsiders with domestic merger thus enjoy more market power than outsiders with cross-border merger. Moreover, the gap of outsiders’ profits between domestic and cross-border merger cases increases with domestic spillover ($\beta$) and decreases with cross-border spillover ($\alpha$).

Comparing Figures 10 and 11, we can see that the average profit gap of the merged entities ($\pi_m^d$ and $\pi_m^c$) is relatively larger than the average profit gap of outsiders ($\pi_3^d = \pi_4^d$ and
This is because the merged entities are monopolists with relatively higher market power than the outsiders.

Figure 11 Profits of outsiders with domestic and cross-border merger

6. Single Research Lab

In section 3.2, the model was analyzed with two R&D labs. But what would happen if a merged monopolist keeps only one R&D lab? In this section, we examine this scenario. Assume the domestic merged entity keeps only one R&D lab because it faces a homogenous product market and it does not need to keep a second R&D lab. Then the solution to the R&D and profit of the domestic merged entity are given by:\(^{14}\)

\[
\chi_m^d = \frac{3 - 2\alpha\beta}{4\gamma} \sigma_m^{d'}
\]

\(^{14}\) Computation details are given in Appendix and where \(\sigma_m^{d'}\) is given by:

\[
\sigma_m^{d'} = \frac{4(A-a)(3 + \beta(2 - \beta + \alpha(-7 + \beta + 2\alpha\beta)) - 4\gamma)\gamma}{-9 + \beta\left[3(-2 + \beta) + \alpha\left(9 + \beta\left(7 - 2\beta + 2\alpha\left(8 + \beta(-4 - 9\alpha + 2\alpha(1 + \alpha\beta))\right)\right)\right] + 60\gamma + 8\beta(2 - \beta + \alpha(-10 + 3\alpha\beta))\gamma - 64\gamma^2}
\]
\[
\pi_m^{dtr} = \left(1 - \frac{(3 - 2\alpha\beta)^2}{16\gamma}\right)\left(y_m^{dtr}\right)^2
\]  
\hspace{1cm} (18)

Solutions to the R&D investments and profits of foreign outsiders are given by:  
\[
x_3^{dtr} = x_4^{dtr} = \frac{(3 - \beta - \alpha\beta)}{4\gamma} y_3^{dtr} = \frac{(3 - \beta - \alpha\beta)}{4\gamma} y_4^{dtr}
\]  
\hspace{1cm} (19)
\[
\pi_3^{dtr} = \pi_4^{dtr} = \left(1 - \frac{(3 - \beta - \alpha\beta)^2}{16\gamma}\right)\left(y_3^{dtr}\right)^2 = \left(1 - \frac{(3 - \beta - \alpha\beta)^2}{16\gamma}\right)\left(y_4^{dtr}\right)^2
\]  
\hspace{1cm} (20)

Figure 12 shows the R&D of the domestic merged entity \((x_m^{dtr})\) and also the R&D of foreign outsiders \((x_3^{dtr} = x_4^{dtr})\). The shapes of these two surfaces are similar to those shown in Figure 3. R&D of both the domestic merged entity and foreign outsiders decrease with \(\beta\) and \(\alpha\). Also, the R&D of the merged entity is higher than the R&D of foreign outsiders in most cases. But when \(\beta \to 0\) and \(\alpha \to 1\) the two R&D investments converge to the same level.  

On average, the R&D of the merged entity is higher than the R&D of outsiders.

---

**Computation details** are given in Appendix and where \(y_i^{dtr} = y_i^{dtr}\) are given by:
\[
y_3^{dtr} = y_4^{dtr} = \frac{4(A - a)(3 + \alpha\beta(5 + \alpha\beta))}{-9 + \beta(3(-2 + \beta) + \alpha(9 + \beta(7 - 2\beta + 2\alpha(8 + \beta(-4 - 9\alpha + 2\alpha(1 + \alpha)\beta)))) + 60\gamma + 8\beta(2 - \beta + \alpha(-10 + 3\alpha\beta))}\gamma - 64\gamma^2}
\]

**For example,** we check two points in Figure 12:

where \(\beta = 0.03\) and \(\alpha = 0.97\), \(x_3^{dtr} = 2.92 \approx x_4^{dtr} = x_4^{dtr} = 2.92\);  

where \(\beta = 0.05\) and \(\alpha = 0.95\), \(x_3^{dtr} = 2.88 \approx x_4^{dtr} = x_4^{dtr} = 2.88\).
As for profit, Figure 13 illustrates profits of both the merged entity \( (\pi_m') \) and outsiders \( (\pi_i'' = \pi_i'^{d'}) \) as a function of \( \beta \) and \( \alpha \). The profit of the merged entity increases with \( \alpha \), while the profit of outsiders increases with \( \beta \). The profit of the merged entity stays at a lower level than the profit of foreign outsiders over the entire region. When \( \beta \to 1 \) and \( \alpha \to 0 \), the gap between these two profits increases. Thus, domestic mergers with only one research lab normally will not occur, since companies are averse to more profitable competitors and they pursue greater market power.
7. Conclusion

In this paper, I used a two-duopolies and two-stage model to analyze equilibrium R&D investments and profits in cases involving no merger, domestic merger and cross-border merger. Also, I used the equilibrium R&D choices to make comparisons with regard to R&D of the merged entity, outsiders and the domestic industry. Furthermore, I compared and analyzed the profitability of merged entities and outsiders. In addition, I considered the case where the domestic merged entity operated only one research lab. After detailed comparisons and analysis, the main findings of this study can be summarized as follows.

First, cross-border mergers sometimes reduce innovation. For governments and companies, domestic mergers should be encouraged in order to achieve high levels of R&D when the domestic R&D spillover is high and the foreign R&D spillover is low. Meanwhile, in cases of low domestic spillover and high foreign spillover, domestic mergers and cross-border mergers should both be encouraged. Also, mergers do help increase domestic industry’s R&D. This result
is consistent with numerous previous empirical and theoretical works (for example, Hagedoorn and Duysters, 2002; Cassiman et al., 2005; Cefis, 2010; Atallah, 2016).

Second, the profitability of mergers is significant. In domestic and cross-border mergers merged entities can make more profits than outsiders. Meanwhile, outsiders are more profitable when mergers occur than when no merger occurs. This conclusion is consistent with previous theoretical studies (for example, Atallah, 2005a; José, 2008). However, for profit-pursuing firms, cross-border mergers are more beneficial than domestic mergers. Also, this paper finds that the pursuit of profit can explain the trend of cross-border mergers and globalization in the past thirty years (for example, Kang and Johansson, 2000; Zademach and Rodrigueuz-Pose, 2009; Chen and Findlay, 2003).

Third, for companies who wish to achieve higher R&D levels, it is wiser to maintain two independent R&D labs than to retain only one. This is because keeping two R&D labs is more efficient and productive than having only one lab.
References


important case of nonglobalization.’ *Journal of International Business Studies* 22, 53–74


Appendix

Computations for section 3.1

The market inverse demand function is given by:

\[ p = A - \sum_{i=1}^{4} y_i \quad i = 1, 2, 3, 4 \]  

(21)

The marginal cost functions are given by:

\[ c_1 = \alpha - x_1 - \beta x_2 - \alpha \cdot \beta (x_3 + x_4) \]  

(22)

\[ c_2 = \alpha - x_2 - \beta x_1 - \alpha \cdot \beta (x_3 + x_4) \]  

(23)

\[ c_3 = \alpha - x_3 - \beta x_4 - \alpha \cdot \beta (x_1 + x_2) \]  

(24)

\[ c_4 = \alpha - x_4 - \beta x_3 - \alpha \cdot \beta (x_1 + x_2) \]  

(25)

The profit function of firm \( i \) is given by:

\[ \pi_i = (p - c_i) y_i - \gamma (x_i)^2 \quad i = 1, 2, 3, 4 \]  

(26)

By solving the maximization problem of the two-stage game, the equilibrium R&D investment and outputs without merger are given by (superscript \( n \) denotes no merger):

\[ y_i^n = \frac{5(A-a)\gamma}{-4 + (1+2\alpha)\beta(-3 + \beta + 2\alpha\beta) + 25\gamma} \quad i = 1, 2, 3, 4 \]  

(27)

\[ x_i^n = \frac{(A-a)(4 - \beta + 2\alpha\beta)}{-4 + (1+2\alpha)\beta(-3 + \beta + 2\alpha\beta) + 25\gamma} \cdot \frac{(4 - \beta + 2\alpha\beta)}{5\gamma} y_i^n \quad i = 1, 2, 3, 4 \]  

(28)

Computations for section 3.2

The market inverse demand now becomes (superscript \( d \) denotes domestic mergers):

\[ p = A - (y_m^d + y_3^d + y_4^d) \]  

(29)

where \( y_m^d \) denotes the domestic merged entity’s production, and marginal cost functions are given by:
\[ c_m = \alpha - x_1 - x_2 - \alpha \cdot \beta \cdot (x_3 + x_4) \]  
(30)

\[ c_3 = \alpha - x_3 - \beta x_4 - \alpha \cdot \beta (x_1 + x_2) \]  
(31)

\[ c_4 = \alpha - x_4 - \beta x_3 - \alpha \cdot \beta (x_1 + x_2) \]  
(32)

Then profit functions are given by:

\[ \pi_m = (p - c_m) y_m^d - \gamma x_1^2 - \gamma x_2^2 \]  
(33)

\[ \pi_3 = (p - c_3) y_3^d - \gamma x_3^2 \]  
(34)

\[ \pi_4 = (p - c_4) y_4^d - \gamma x_4^2 \]  
(35)

By solving the maximization problem of the two-stage game, the two R&D investments of the domestic merged entity are given by:

\[
X_1^d = X_2^d = \frac{(-3 + 2a \beta)(3 + \beta (-2 - \beta + a (-7 + \beta + 2a \beta))) - 4\gamma}{2(-9 + \beta(3(-2 + \beta) + a(9 + \beta(7 - 2\beta + 2a(8 + \beta(-4 - 9a + 2a(1 + a)\beta)))) + 48\gamma + 4\beta(2 - \beta + a(-16 + 5a \beta))\gamma - 32\gamma})}
\]
(36)

Solutions to the R&D investments for foreign outsiders are given by:

\[
X_3^d = X_4^d = \frac{(-3 + 2a \beta)(3 + \beta (-2 - \beta + a (-7 + \beta + 2a \beta))) - 2\gamma}{(-9 + \beta(3(-2 + \beta) + a(9 + \beta(7 - 2\beta + 2a(8 + \beta(-4 - 9a + 2a(1 + a)\beta)))) + 48\gamma + 4\beta(2 - \beta + a(-16 + 5a \beta))\gamma - 32\gamma})}
\]
(37)

The output of the merged entity is given by:

\[
y_m^d = \frac{((2(A - a)(3 + \beta(2 - \beta + a(-7 + \beta + 2a \beta))) - 4\gamma)\gamma}{(-9 + \beta(3(-2 + \beta) + a(9 + \beta(7 - 2\beta + 2a(8 + \beta(-4 - 9a + 2a(1 + a)\beta)))) + 48\gamma + 4\beta(2 - \beta + a(-16 + 5a \beta))\gamma - 32\gamma})}
\]
(38)

For the two symmetric outsiders, the outputs are given by:

\[
y_3^d = y_4^d = \frac{4(A - a)(3 + \beta(-5 + 2a \beta) - 2\gamma)\gamma}{(-9 + \beta(3(-2 + \beta) + a(9 + \beta(7 - 2\beta + 2a(8 + \beta(-4 - 9a + 2a(1 + a)\beta)))) + 48\gamma + 4\beta(2 - \beta + a(-16 + 5a \beta))\gamma - 32\gamma})}
\]
(39)

**Computations for section 3.3**

The market inverse demand is given by (superscript c denotes cross-border mergers):

\[ p = A - (y_m^c + y_3^c + y_4^c) \]  
(40)
The marginal cost and profit functions with cross-border merger are given by:

\[ c_m = \alpha - x_1 - x_3 - \beta(x_2 + x_4) \]  
\[ c_2 = \alpha - x_2 - \beta x_1 - \alpha \cdot \beta (x_3 + x_4) \]  
\[ c_4 = \alpha - x_4 - \beta x_3 - \alpha \cdot \beta (x_1 + x_2) \]  
\[ \pi_m = (p - c_m^c) y_m^c - \gamma x_1^2 - \gamma x_3^2 \]  
\[ \pi_2 = (p - c_2^c) y_2^c - \gamma x_2^2 \]  
\[ \pi_4 = (p - c_4^c) y_4^c - \gamma x_4^2 \]  

By solving the maximization problem of the two-stage game, the R&D investments of the cross-border entity are given by:

\[ x_1^c = x_3^c = \frac{(A-a)(3-\beta - \alpha \beta)(-1 + (2+\alpha) \beta (-3+\beta + \alpha \beta) - 4\gamma)}{2(-1+\beta)(-3+\beta + \alpha \beta)^2 (1+\beta + \alpha \beta) + 16(-2+\beta)(-3+\beta + \alpha \beta)\gamma - 64\gamma^2} \]  
\[ x_2^c = x_4^c = \frac{(A-a)(3-\beta - \alpha \beta)(6 + (1+\alpha) \beta (-5+\beta + \alpha \beta) - 4\gamma)}{2(-1+\beta)(-3+\beta + \alpha \beta)^2 (1+\beta + \alpha \beta) + 16(-2+\beta)(-3+\beta + \alpha \beta)\gamma - 64\gamma^2} \]  

The output of the merger is given by:

\[ y_m^c = \frac{2(A-a)\gamma (3-\beta (7-2\alpha + (-2+\alpha)(1+\alpha) \beta) - 4\gamma)}{(-1+\beta)(-3+\beta + \alpha \beta)^2 (1+\beta + \alpha \beta) + 8(-2+\beta)(-3+\beta + \alpha \beta)\gamma - 32\gamma^2} \]  

The outputs of outsiders are given by:

\[ y_2^c = y_4^c = \frac{2(A-a)\gamma (6 + (1+\alpha) \beta (-5+\beta + \alpha \beta) - 4\gamma)}{(-1+\beta)(-3+\beta + \alpha \beta)^2 (1+\beta + \alpha \beta) + 8(-2+\beta)(-3+\beta + \alpha \beta)\gamma - 32\gamma^2} \]  

**Computations for section 6**

The market inverse demand now becomes (superscript \(d'\) denotes domestic mergers with one R&D lab):
\[ p = A - (y''_m + y''_3 + y''_4) \]  \hspace{1cm} (51)

where \( y''_m \) denotes production of the merged entity, and marginal cost functions are given by:

\[ c_m = \alpha - x_m - \alpha \cdot \beta \cdot (x_3 + x_4) \]  \hspace{1cm} (52)

\[ c_3 = \alpha - x_3 - \beta x_4 - \alpha \cdot \beta (x_m) \]  \hspace{1cm} (53)

\[ c_4 = \alpha - x_4 - \beta x_3 - \alpha \cdot \beta (x_m) \]  \hspace{1cm} (54)

Then profit functions are given by:

\[ \pi_m = (p - c_m) y''_m - \gamma x^2_m \]  \hspace{1cm} (55)

\[ \pi_3 = (p - c_3) y''_3 - \gamma x^2_3 \]  \hspace{1cm} (56)

\[ \pi_4 = (p - c_4) y''_4 - \gamma x^2_4 \]  \hspace{1cm} (57)

R&D of the domestic merged entity is given by:

\[ x''_m = \frac{(A - a)(3 - 2\alpha \beta)(3 + \beta(2 - \beta + \alpha(-7 + \beta + 2\alpha\beta)) - 4\gamma)}{-9 + \beta(3(-2 + \beta) + a(9 + \beta(7 - 2\beta + 2\alpha(8 + \beta(-4 - 9\alpha + 2\alpha(1 + a)\beta))) + 60\gamma + 8\beta(2 - \beta + a(-10 + 3\alpha\beta))\gamma - 64\gamma} \]  \hspace{1cm} (58)

Solutions to the R&D investments for foreign outsiders are given by:

\[ x''_3 = x''_4 = \frac{(A - a)(3 - \beta - a\beta)(3 + a\beta(-5 + 2\alpha\beta) - 4\gamma)}{-9 + \beta(3(-2 + \beta) + a(9 + \beta(7 - 2\beta + 2\alpha(8 + \beta(-4 - 9\alpha + 2\alpha(1 + a)\beta))) + 60\gamma + 8\beta(2 - \beta + a(-10 + 3\alpha\beta))\gamma - 64\gamma} \]  \hspace{1cm} (59)

The output of the merged entity is given by:

\[ y''_m = \frac{4(A - a)(3 + \beta(2 - \beta + a(-7 + \beta + 2\alpha\beta)) - 4\gamma)}{-9 + \beta(3(-2 + \beta) + a(9 + \beta(7 - 2\beta + 2\alpha(8 + \beta(-4 - 9\alpha + 2\alpha(1 + a)\beta))) + 60\gamma + 8\beta(2 - \beta + a(-10 + 3\alpha\beta))\gamma - 64\gamma} \]  \hspace{1cm} (60)

For the two symmetric outsiders, outputs are given by:

\[ y''_m = y''_3 = \frac{4(A - a)(3 + a\beta(-5 + 2\alpha\beta) - 4\gamma)}{-9 + \beta(3(-2 + \beta) + a(9 + \beta(7 - 2\beta + 2\alpha(8 + \beta(-4 - 9\alpha + 2\alpha(1 + a)\beta))) + 60\gamma + 8\beta(2 - \beta + a(-10 + 3\alpha\beta))\gamma - 64\gamma} \]  \hspace{1cm} (61)