The cost of children in non-cooperative marriage models

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1. Introduction

Marriage is one of the most important institutions in human society. Strikingly, however, there has been a significant decline in the marriage rate in many advanced economies over the past number of years (Stevenson and Wolfers 2007). In Canada, married couples constituted approximately 92% of the 1961 census families compared to 67% in 2011\(^1\). Moreover, the composition of the family has evolved. Although the traditional family comprised of heterosexual parents with 2.2 children is still the most prevalent family form, blended families and other family structures are increasingly common in advanced economies.

Given the importance of marriage and family structure in determining outcomes such as birth rates, individual well-being, and economic inequality, it is not surprising that economists have a longstanding interest in explaining the process of household formation. Whereas the prevalent view in society at large, at least as expressed in the popular press or reflected in marriage vows (Goode 1959, Keri and Lobo 2008) is that the family should function as a single unit on the basis of mutual responsibility (love and respect), economists generally take a more analytic view of marriage. In particular, they believe that marriage is an important economic institution, and that decisions about forming households, and about the allocation of effort within the household, are guided at least in part by the desire to improve economic outcomes and welfare for the participating parties.

While married couples still represent the most common family structure, it is also a fact that nearly half of all marriages end in divorce. The increase in the rate of divorce in the U.S. and other countries over the past fifty years has inspired researchers to focus specifically on the economics of divorce. One possible explanation for this increase in the divorce rate is that the

true quality of the partners is only revealed at a later stage in marriage (Roth 1996, Smith 2000); yet, other economists argue that the increase is due to exogenous factors and the changes in traditional household roles and income distributions (Chiappori and Weiss 2000).

Another major consideration brought by the literature is the impact of children on individuals’ decisions to marry, stay married or even divorce. The presence of children can considerably affect intra-household resource allocation and the division of outputs between the partners. In most societies, spousal support payments are transferred to the lower-wage partner if divorce occurs. However, the presence of children may increase the cost of divorce beyond spousal support. It has been shown that an increase in the cost of divorce can significantly reduce the probability of divorce (Lundberg and Pollack 1993, Chiappori and Weiss 2000). As a result, when modeling the family, it is crucial to consider the role of children in the household decision-making process. However, there has not been any focus on the impact of the cost of children on the decisions to marry and divorce in non-cooperative marriage models.

Barham, Devlin and Yang (2009) construct a non-cooperative model of household formation and dissolution and demonstrate that equal sharing of household private goods and identical preferences are economic behaviours that prompt spouses to either get married and/or stay married. However, one limitation of this paper is that it abstracts from the consideration of children as members of the household.

Consequently, the following paper extends the basic model presented in Barham, Devlin and Yang (2009) where the authors consider only two goods: a composite private good and a local public good. In my model, children are added as a second public good within the household and are produced at a positive probability $p > 0$, which is determined exogenously. Accordingly, the presence of children affects the division of the private good between partners. If divorce
occurs, children are shared between partners in proportion to the relative investment of each spouse in the production of the local public good. Divorce also implies a payment of child support in addition to the spousal support to the lower-income person, which further affects the division of outputs upon marital dissolution.

The structure of this paper is organized as follows. In Section 2, we conduct a literature review outlining a number of papers and approaches that have been influential in shaping the economic analysis of the family. Section 3 presents the model. Section 4 is devoted to the equilibrium analysis. Finally, Section 5 concludes the paper with the major results of our analysis.
2. Literature Review

There is a very extensive literature on the economics of the family, and there are a number of excellent survey articles. Weiss (1993) focuses specifically on models of the formation and dissolution of households, and the various rationales that have been provided for the decision to marry, alternative explanations of who marries whom, and the way these decisions are affected by the possibility of divorce. A more recent survey by Lerman (2002) pays specific attention to the economic well-being of families with children.

Given the availability of these (amongst other) survey articles, the objective of the literature review presented below is merely to highlight a number of papers and approaches that have been influential in shaping the economic analysis of the family. We first focus on the economic reasons for marriage, using Becker’s (1973) seminal work as the point of departure. We subsequently review alternative approaches to modeling the benefits that arise from sharing public goods within the household, exploring the different ways in which the problem of intra-household resource allocations is treated in both cooperative and non-cooperative models of the family. We conclude the first part of the survey by discussing the implications of Becker’s theory on love and sharing for reductions in the cost of resource transfer within the household and examine the importance of risk sharing and imperfect credit markets.

The second part of this review focuses more on the dynamics of the decisions to marry, divorce and to have children. Search markets and uncertainty about the quality of the match, the integration of women into the labour force as well as the differences in productivity and unequal share of private goods are revealed as major factors leading to marital instability. We are interested in looking at the impact of children on marital decisions by firstly considering children as public goods within the household, then showing the impact of their presence on the cost of
divorce and finally evaluating the cost of children in collective models of the family. At the end of this review, we highlight the gap in the existing literature where there has not been any focus on the impact of the cost of children on the decisions to marry and divorce in non-cooperative marriage models.

**Economic reasons for marriage**

In his seminal article “Theory of marriage Part I” Becker (1973) seeks to explain who matches with whom. His model is based on two main assumptions: (i) that marriage is always voluntary, and consequently individuals marry only when their utility after marriage is higher than their utility if they remain single, and (ii) that a marriage market exists and will always achieve equilibrium. Becker also introduces the concept of assortative mating and shows the importance of the quality of matching for the decision to marry. He shows that the gain from marriage depends on the type of inputs used in household production, and notes that non-economic factors – such as intelligence, race, and education – may also influence the decision. Partners choose counterparts with similar traits when the traits are complements (e.g. education, height, beauty) whereas the contrary occurs when traits are substitutes (e.g. wage rate and other determinants of earning power).

Becker’s theory sparked many researchers to also study the economic role of marriage. One particularly telling criticism is that it neglects strategic considerations (Tomassi 1995 and Westley 2012). Smith (2006) develops Becker’s model of assortative mating by considering search frictions on the marital market. The objective of his paper is to show how the introduction of search costs affects the determination of acceptable partners. Whereas Becker’s model is based on transferable utility, Smith assumes that utility is non-transferable, and shows that the
complementarities of traits are not enough to explain positive assortative mating. Atakan (2006) also addresses the nature of matching choices in the context of the marriage market, and shows that assortative matching exists in any competitive equilibrium with complimentary, joint production and core allocations in transferable utilities.

A very different approach is taken in Barham, Devlin and Yang (2009) who build a non-cooperative model of household formation and dissolution. They show that a Pareto efficient outcome may characterize the equilibrium, if there is similarity of spousal preferences, and an equal division of private goods. The model indicates that equal sharing of household private goods and identical preferences are economic behaviours that prompt spouses to either get married and/or stay married. In contrast, if potential partners have different preferences, and if there is a trend of unequally sharing household income to acquire private composite goods, then they are likely to divorce or to remain single. Similar results are found by Browning et al. (2011), who study this issue in the context of a cooperative model of household formation and dissolution.

The literature covering the importance of the quality of matching also considers the role of the optimal sorting of mates on the decision to marry. In his paper, Becker (1973) presents a payoff matrix showing the maximum output that can be produced within a household by any combination of mates. The optimal sorting is defined as being the matching that maximizes the sum of outputs produced over all marriages and not the maximum output produced within one marriage. A marriage is thus optimal if mates cannot find another division of their output (outside their current union) that would make them better off. In 1974, Becker extends his theory on optimal sorting and assortative matching by considering the impact of income and the age at marriage. He suggests that, at an optimum, men with high wages marry
earlier and expect higher gains from marriage because they will enjoy greater returns from specialization by marrying low wage females who specialize in the production of household commodities.

This prediction, however, seems at odds with what is observed in the marriage market, more specifically with regards to the age at marriage and its correlation to income, which has encouraged other researchers to develop alternative approaches. Keely (1977) expands Becker’s model through the inclusion of education, and finds that this will lead to a negative correlation between age at first marriage and income. Subsequently, Bergstrom and Bagnoli (1993) explain the age gap between males and females on the marriage market and provide a reason for its diminishment over time while giving an insight into why it is more common in some societies compared to others. They show that men who look after their prospects will wait until their economic success is revealed before choosing their partners and hence will marry at an older age.

The more traditional idea of marriage based on love and sharing was nonetheless acknowledged and explained by Becker (1974) as a factor that plays a significant role in spousal selection and marriages. Marriage and caring reduce the cost of resource transfer within households, therefore allowing couples to acquire more of the produced collective good that is now consumed jointly. As a result, love and care motivate the decision to marry by allowing partners to maximize joint incomes and their produced outputs. Other more prosaic factors that may also significantly reduce the cost of resource transfer within the household include risk sharing and imperfect credit markets. Weiss (1997) suggests that when credit markets are imperfect, the coordination of investment activities within the household creates an incentive to marry. The main finding relies on the fact that the family will invest in the schooling of the
person with the highest return on human capital, thus generating another gain from marriage (i.e.
education) via cost reduction.

**Marriage, Divorce and Children**

The increase in the rate of divorce in the U.S. and other countries over the past fifty years has inspired researchers to focus specifically on the economics of divorce. One approach is to assume that, although individuals do not marry unless the utility expected from marriage is higher than what these individuals expect to obtain were they to remain single, the *ex post* quality of the match may not be as high as initially anticipated; dissolution then occurs when the utility from marriage falls below the expected utility from divorce. Becker and Landes (1977) suggest that the majority of divorces result from uncertainty and unfavorable outcomes. If the division of output is flexible, divorce will occur only if the combined wealth from marriage is less than the combined wealth from dissolution; if one partner expects higher wealth from separation, the other partner can propose an alternative division of household resources that would compensate him/her for foregone outside opportunities, and encourage them to remain married. Becker and Landes (1977) show that an increase in search costs can lead to a point where outcomes from a first marriage become less than the minimum acceptable offer. They also argue that an increase in search costs could be an indicator of the existence of a preferable match which can explain the prevalence of extramarital affairs. Hence, an increase in search costs increases the probability of dissolution.

Additionally, when individuals marry, they have limited information regarding the quality of their mates. Over time, however, both spouses acquire better information, and this may ultimately result in a decision to divorce. Roth (1996) tries to identify the benefits and
disadvantages of marital dissolution when the true quality of the partners is revealed. The objective of his paper is to prove that the revelation of the true quality of partners, at a certain point in marriage, affects the level of effort in the household production of each of the spouses, hence affecting the stability of marriage.

Besides search markets and uncertainty about the quality of the match, economists argue that the integration of women into the labour market has been a major causal factor driving the increase in divorce rates. Chiappori and Weiss (2000) contend that the increase in divorce rates is due to exogenous factors and the changes in traditional household roles and income distributions. Nowadays, women are more economically independent. The fact that women now typically earn income outside the home has increased their decision-making power and affected the way in which household resources are allocated and controlled.

Another key consideration is the impact of children on individuals’ decisions to marry, stay married or even divorce. In the literature covering households with children, the latter are often modeled as public goods and parents care about their welfare. The utility of children depends on both their consumption and the time spent with each of their parents whereas the utility of parents depends on their consumption and their children's utility. Folbre (1994) discusses the issue of resource-allocation in the presence of children and shows that the high wage parent specializes in market labor and the low wage partner in household production (due to the existence of absolute advantages) in order to invest in their children. Weiss and Willis (1985) also view children as public goods within the family and discuss the impacts of divorce and marital dissolution on household members, more specifically children. Following divorce, the inefficient allocation of labor time and in some cases, the incapacity of spending money on children, concludes that the latter are always made worse off when the family splits. They find
that having custody can make one of the partners worse off comparatively to the non-custodial parent, and free-riding opportunities may arise.

Moreover, the presence of children in the household may increase the cost of divorce. In fact, Becker was the first to introduce children as marital-specific investments. In his paper, he explains the importance of children on the frequency of divorce and shows that the greater the marriage-specific investments, the lower is the incentive to separate. Furthermore, children are considered as a source of friction on search markets that may impact the likelihood of finding an acceptable partner post-divorce which may be a factor which explains why the rate of divorce increases with the number of remarriages (Roth 1996).

One might expect that the existence of child support payments will impact marital decisions. Lundberg and Pollack (1993) demonstrate the distributional effect of child support, allowance, and subsidy payment schemes on divorce rates and show that an increase in the costs of divorce can significantly reduce the probability of divorce. Similarly, Chiappori and Weiss (2000) demonstrate how the determination of child custody, alimony, and fund transfers reduces the divorce rate in equilibrium. Browning, Chiappori and Weiss (2011) also discuss how the presence of a child can impact spousal decisions towards marriage or divorce. They demonstrate that a possible decrease in utility for a single parent with children (usually the mother) could cause her to rethink a decision to divorce, and compromise to stay with her current partner knowing that she might get lesser resource transfer from the father of her child, if they divorce or separate.

Finally, a number of researchers have also considered the cost of children. Deaton et al. (1989) discuss the theoretical basis for measuring child costs in a unitary framework through the
use two methods, the Engel model and the Rothbarth model, which lead to different results of the sharing rule in the framework of resource allocation. However, by imposing an assumption of “demographic separability”, all models under the unitary model using equivalence scales yield the same output and reveal the effect of children on adult welfare. Bourguignon (1999) introduces an alternative approach to the estimation of the “cost” of children. However, instead of using the common unitary approach, he uses the collective model of the family and identifies the sharing rule regarding resource allocation between household members without imposing the assumption of “demographic separability”.

In contrast, there has not been similar attention given to the impact of the cost of children on the decisions to marry and divorce in non-cooperative marriage models. Consequently, this paper extends the model presented in Barham, Devlin and Yang (2009). Children are added as a second public good within the household. If divorce occurs, this implies a payment of child support in addition to the spousal support to the lower-income person, which further affects the division of outputs upon marital dissolution.
3. The model

In this section, we extend the model presented in Barham and Devlin (2009) by introducing children as a second public good produced within the household.

We consider three goods: A composite private good (X), a local public good (G) and children (C). The utility function $U_i(X^i, G, C)$ is concave with continuous first and second order derivatives with respect to each good.

The local public good G is quasi-durable and can represent both the physical assets (e.g. home, furniture, cooked meals) and non-physical assets (e.g. companionship). The amount of the local public good available to each partner at period $t$ is given by $G_t$, and is a function of the depreciated stock from the previous period and an investment at period $t$ of each of the husband and the wife:

$$G_t = \delta G_{t-1} + f(k_{mt}, k_{wt})$$

Where $0 < \delta < 1$ is the depreciation rate and $k_{mt}$ and $k_{wt}$ are the investments of the man and the woman, respectively, in the production of the local public good. The production of the public good within the household does not necessarily require both the husband's and wife's production efforts: $f(k_{mt}, 0), f(0, k_{wt}) > 0$.

There is a fixed amount of labor available to either spouse for investment in the production of the local public good and the acquisition of the composite private commodity. The decision of each household member on how much labour to allocate to the acquisition of the public good is made in a non-cooperative manner. The outside productivity of each of the partners is given by $v_i(i = m, w)$. The labour-market incomes are pooled within the household. Therefore, the total amount of private good available at period $t$ is given by:
\[ X_t = v_m (L - k_{mt}) + v_w (L - k_{wt}) \]

We assume that, if the couple is married, children can be obtained with a positive probability \( p \), which is determined exogenously. This assumption is made for simplicity and is important for the results of the paper, more specifically to the model in period one. To eliminate uncertainty in period one, we assume that children arrive before partners choose their levels of investment in the production of the local public good. This approach is also intended to capture the notion that there are no direct costs associated with having children, but that there is some uncertainty regarding fertility.

The division of \( X_t \) between the partners is affected by the presence of children: Children receive a share of the private good. Consequently, we denote by \( \hat{\theta}_m \) and \( \hat{\theta}_w \) the shares of the husband and the wife, respectively such that \( \hat{\theta}_m + \hat{\theta}_w \leq 1 \). The private good consumption of both partners in period \( t \) can be expressed as:

\[
X_{mt} = \hat{\theta}_m [v_m (L - k_{mt}) + v_w (L - k_{wt})] \\
X_{wt} = \hat{\theta}_w [v_m (L - k_{mt}) + v_w (L - k_{wt})]
\]

As a result, the presence of children reduces the shares of the private good \( \theta_m \) and \( \theta_w \), compared to the basic model without children.

We consider a two-period game to model household formation and dissolution. At the beginning of period one, each individual is offered a mate and must decide whether to marry or remain single. If both partners agree to marry, each one of them receives his respective amount of the local public good and the private good as outlined above. If not, then they remain single and each individual must provide independently both the goods.
At the end of the first period, each partner decides whether to remain married or divorce. Unlike marriage, divorce is a unilateral decision and once the decision is taken, each partner receives a share $\lambda_i$ of the local public good that is predetermined by law such that $\lambda_m + \lambda_w \leq 1$. In the case of unequal earning power, and more specifically when the divorced male earns more than the female, he must pay a spousal support for his wife denoted by $\theta_D$. Moreover, in the presence of children, he must also pay a child support amount given by $\propto \geq 0$. In general, spousal support and child support payments are determined as a function of the differences in the incomes of the two partners. Given our assumption that the male earns more than the female, we assume for simplicity, the latter payments are proportional to the productivity of the person paying it (i.e. the male partner).

The game ends after two periods and we assume that if the decision to divorce is taken at the end of period one, individuals cannot remarry and remain single for the rest of their lives. If individuals remain married, then they must decide non-cooperatively on how much time to allocate to the production of the local public good and to acquire the private good, in a similar fashion than that outlined above.

4. **Equilibrium analysis**

4.1. **The model in period 2**

4.1.1. *Period two: Collective good production in a one-person household*

To solve this game, we use the concept of the subgame perfect Nash equilibrium.

In this section, we distinguish between three cases: never-married individuals, divorced individuals without children and divorced individuals with children.
We first start by showing the decision problem facing a never-married individual at the beginning of period two. As noted, in a single person household, each individual allocates his available time to both the production of the local public good and the acquisition of the private good while maximizing his utility:

$$\max_{k_{i2}} U^i(x_i, G_2) = U^i(v_i (L - k_{i2}), \delta G_1 + f(k_{i2}, 0)).$$

At an optimum, the level of investment allocated to the production of the local public good ($\bar{k}_{i2}$) is given by the Kuhn-Tucker conditions:

$$-v_i U^i_x + U^i_G f_{k2} \leq 0 \quad (1)$$

$$k_{i2} \geq 0 \quad (2)$$

$$k_{i2}[-v_i U^i_x + U^i_G f_{k2}] = 0 \quad (3)$$

In the case of divorced individuals without children, there are no child support payments to make. Thus, assuming that the male earns more than the female, he is only responsible for paying the spousal support amount ($\theta_D$) to the female partner.

The decision problem facing the divorced male without children is given by:

$$\max_{k_{m2}} U^m(X_m; G_2) = U^m(v_m(L - k_{m2})(1 - \theta_D); \lambda G_1 + f(k_{m2}; 0)$$

At an optimum, it must be true that:

$$-v_m U^m_x (1 - \theta_D) + U^m_G f_{km} \leq 0 \quad (4)$$

$$k_{m2} \geq 0 \quad (5)$$

$$k_{m2}[-v_m U^m_x (1 - \theta_D) + U^m_G f_{km}] = 0 \quad (6)$$

Equations (4) -(6) represent the Kuhn-Tucker conditions at the subgame Nash equilibrium, in the case of a divorced male without children. Hence, at an optimum, the level of investment
allocated to the production of the local public good for a divorced male without children will be determined by these conditions.

The decision problem facing the divorced female without children is expressed as:

$$\max_{k_{w2}} U^w(X_w; G_2) = U^w(v_w(L - k_{m2}) + \theta_D v_m(L - k_{m2}); \delta(1 - \lambda)G_1 + f(0; k_{w2})$$

At an optimum, it must be true that:

$$-v_w U^w_X + U^w_G f_{kw} \leq 0 \quad (7)$$

$$k_{w2} \geq 0 \quad (8)$$

$$k_{w2}[-v_m U^w_X + U^w_G f_{kw}] = 0 \quad (9)$$

Thus, by resolving the following decision problems, the solutions to (4)-(6) and (7)-(9) are given by $k_{m2}^D$ and $k_{w2}^D$.

In a similar manner, we set the maximization problem that belongs to divorced individuals with children at period two, taking into account the respective amounts of child support ($\propto$) and spousal support ($\theta_D$) determined by Family Law. Assuming that the male’s income is higher than that of his ex-partner, he will be responsible of paying both amounts to the female, upon divorce. Children are added to the model and are shared between partners in proportion to the relative investment of each spouse in the production of the local public good: $\frac{k_{m1}}{k_{m1} + k_{w1}}$.

The decision problem facing the divorced male with children is given by:

$$\max_{k_{m2}} U^m(X_m; G_2; C) = U^m(v_m(L - k_{m2})(1 - \theta_D - \alpha); \delta\lambda G_1 + f(k_{m2}; 0); \left(\frac{k_{m1}}{k_{m1} + k_{w1}}\right)C)$$

At an optimum, it must be true that:

$$-v_m U^m_X (1 - \theta_D - \propto) + U^m_G f_{km} \leq 0 \quad (10)$$

$$k_{m2} \geq 0 \quad (11)$$

$$k_{m2}[-v_m U^m_X (1 - \theta_D - \propto) + U^m_G f_{km}] = 0 \quad (12)$$
We assume that the woman has custody. This implies that her consumption of the private good is reduced by a proportion \((1 - \theta_c)\) to cover the child’s expenses. The decision problem facing the divorced female with children is expressed as:

\[
\max_{k_w} U_w^w (X_w; G_2; C) =
\]

\[
U_w \left( 1 - \theta_c \right) v_w (L - k_w) + v_m (L - k_m) (\theta_D + \alpha); \quad \delta (1 - \lambda) G_1 + f(0; k_w), \left( \frac{k_w}{k_m + k_w} \right) C
\]

At an optimum, it must be true that:

\[
-(1 - \theta_c) v_w U_X^w + U_G^w f_{k_w} \leq 0 \quad \text{(13)}
\]

\[
k_{w2} \geq 0 \quad \text{(14)}
\]

\[
k_{w2} \left[-(1 - \theta_c) v_w U_X^w + U_G^w f_{k_w}\right] = 0 \quad \text{(15)}
\]

Thus, by resolving the following decision problems, the solutions to (10)-(12) and (13)-(15) are given by \(k_{m2}^{DC}\) and \(k_{w2}^{DC}\).

**Comparative statics**

It is interesting to see how the presence of children affects the levels of investments in the local public good for each of the partners. To do so, we look at the impact of the cost of children \((\theta_c)\) and child support payments \((\infty)\) on the amounts of \(k_{m2}^{DC}\) and \(k_{w2}^{DC}\).

From (13) we have:

\[-(1 - \theta_c) v_w U_X^w + U_G^w f_{k_w} = 0\]

By differentiating the first-order condition above we obtain that:

\[
\frac{d k_{w2}^{DC}}{d \theta_c} = \frac{-(1 - \theta_c) v_w \{U_X^w + U_{XX}^w [v_w (L - k_w) + v_m (L - k_m) (\theta_D + \alpha)]\}}{\left( (1 - \theta_c) v_w \right)^2 U_X^w + U_{GG}^w f_{k_w}}
\]

and
\[
\frac{dK_{w2}^{DC}}{d\theta_C} = \frac{(1 - \theta_C) v_w U_{XX}^w [(1 - \theta_C) v_m (L - k_{m2})]}{((1 - \theta_C) v_w)^2 U_{XX}^w + U_{GG}^w f_{kw}}
\]

In general, the impact of child costs on the level of investment in the production of the local public good is ambiguous for the divorced female with children and depends on the concavity of her utility function. Note, however, that if the utility function \( U_X^w \) is strongly concave, then \( \frac{dK_{w2}^{DC}}{d\theta_C} < 0 \) which implies that an increase in the child costs reduces the level of contribution of the separated female to the local public good. On another hand, it is clear that the amount of child support received by the female has a positive impact on her level of investment in the local public good, therefore allowing her to increase her contribution to the production of the local public good.

In a similar manner, we look at the impact of child support payments on the level of investment in the local public good for the divorced male.

From (10), we have that:

\[
-v_m U_{X}^m (1 - \theta_D - \alpha) + U_{G}^m f_{km} = 0
\]

By differentiating the FOC above we obtain that:

\[
\frac{dK_{m2}^{DC}}{d\theta_C} = \frac{-v_m U_{X}^m + v_m (1 - \theta_D - \alpha) U_{XX}^m [v_m (L - k_{m2})]}{(v_m (1 - \theta_D - \alpha))^2 U_{XX}^m + U_{GG}^m f_{km}}
\]

The impact of child support payments on the level of investment in the production of the local public good is ambiguous for the divorced male with children and also depends on the concavity of his utility function. That is, if the utility function \( U_X^m \) is strongly concave \( \frac{dK_{m2}^{DC}}{d\theta_C} < 0 \). As a result, an increase in the child support payment reduces the level of contribution of the male partner to the local public good.
4.1.2. Period two: collective good production in a two-person household

This scenario presents the decision problem of individuals whose marriage survived until period two. We distinguish between two cases: married partners without children and married partners with children. In both cases, the decision regarding how much effort to allocate between the production of the local public good and the acquisition of the private commodity is taken, by each of the partners, in a non-cooperative framework. Individuals take into account the expected contribution of their opposite mate in the production of the local public good.

Barham, Devlin and Yang (2009), present the basic model without children as follows:

$$\max_{k,m} U^i(X, G_2) = U^i(\hat{\theta}_i(v_m(L - k_m) + v_w(L - k_w), \delta G_1 + f(k_m, k_w))$$

At an optimum, it must be true that:

$$-\hat{\theta}_m v_m U^m_X + U^m_G f_{km} \leq 0$$  \hspace{1cm} (16)

$$k_m \geq 0$$  \hspace{1cm} (17)

$$k_m[-\hat{\theta}_m v_m U^m_X + U^m_G f_{km}] = 0$$  \hspace{1cm} (18)

$$-\hat{\theta}_w v_w U^w_X + U^w_G f_{kw} \leq 0$$  \hspace{1cm} (19)

$$k_w \geq 0$$  \hspace{1cm} (20)

$$k_w[-\hat{\theta}_w v_w U^w_X + U^w_G f_{kw}] = 0$$  \hspace{1cm} (21)

The solution to this system of equations is denoted by $(\hat{k}_m, \hat{k}_w)$.

In a similar fashion, the decision problem for married partners with children is given by:

$$\max_{k,l} U^i(X, G_2, C) = U^i(\hat{\theta}_i(v_m(L - k_m) + v_w(L - k_w), \delta G_1 + f(k_m, k_w), C)$$

We assume that there is no cost associated with the production of children in a two-person household. However, the division of the private good between the partners is affected by the presence of children such that $\hat{\theta}_m + \hat{\theta}_w \leq 1$. 

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At Nash equilibrium, it is given that:

\[-\hat{\theta}_m v_m U_X^m + U_G^m f_{km} \leq 0 \quad (22)\]

\[k_{m2} \geq 0 \quad (23)\]

\[k_{m2} [-\hat{\theta}_m v_m U_X^m + U_G^m f_{km}] = 0 \quad (24)\]

\[-\hat{\theta}_w v_w U_X^w + U_G^w f_{kw} \leq 0 \quad (25)\]

\[k_{w2} \geq 0 \quad (26)\]

\[k_{w2} [-\hat{\theta}_w v_w U_X^w + U_G^w f_{kw}] = 0 \quad (27)\]

Therefore, the optimal investment levels required for the husband and the wife to produce the local public good at period two are given by \(\hat{k}_{m2}^c\) and \(\hat{k}_{w2}^c\), respectively.

We are now interested in analyzing the impact of children on the levels of investments in the production of the local public good, in the case of a married couple with children.

From the first-order conditions above we have that:

\[-\hat{\theta}_m v_m U_X^m + U_G^m f_{km} = 0\]

\[-\hat{\theta}_w v_w U_X^w + U_G^w f_{kw} = 0\]

To investigate the impact of children on the levels of investments in the production of the local public good, we use Cramer’s method to solve this system of equations and find \(\frac{dk_{m2}^c}{dC}\) and \(\frac{dk_{w2}^c}{dC}\).

\[
\begin{bmatrix}
(\hat{\theta}_m v_m)^2 U_X^m + U_G^m f_{km} & \hat{\theta}_m \hat{\theta}_w v_m v_w U_X^m \\
\hat{\theta}_m \hat{\theta}_w v_m v_w U_X^w & (\hat{\theta}_w v_w)^2 U_X^w + U_G^w f_{kw}
\end{bmatrix}
= \begin{bmatrix}
\hat{\theta}_m v_m U_X^m \\
\hat{\theta}_w v_w U_X^w
\end{bmatrix}
\]

\[
\frac{dk_{m2}^c}{dC} = \frac{\begin{vmatrix}
\hat{\theta}_m v_m U_X^m & \hat{\theta}_m \hat{\theta}_w v_m v_w U_X^m \\
\hat{\theta}_w v_w U_X^w & (\hat{\theta}_w v_w)^2 U_X^w + U_G^w f_{kw}
\end{vmatrix}}{\begin{vmatrix}
(\hat{\theta}_m v_m)^2 U_X^m + U_G^m f_{km} & \hat{\theta}_m \hat{\theta}_w v_m v_w U_X^m \\
\hat{\theta}_m \hat{\theta}_w v_m v_w U_X^w & (\hat{\theta}_w v_w)^2 U_X^w + U_G^w f_{kw}
\end{vmatrix}}
\]

\[
\frac{dk_{w2}^c}{dC} = \frac{\begin{vmatrix}
\hat{\theta}_m v_m U_X^m & \hat{\theta}_m \hat{\theta}_w v_m v_w U_X^m \\
\hat{\theta}_w v_w U_X^w & (\hat{\theta}_w v_w)^2 U_X^w + U_G^w f_{kw}
\end{vmatrix}}{\begin{vmatrix}
(\hat{\theta}_m v_m)^2 U_X^m + U_G^m f_{km} & \hat{\theta}_m \hat{\theta}_w v_m v_w U_X^m \\
\hat{\theta}_m \hat{\theta}_w v_m v_w U_X^w & (\hat{\theta}_w v_w)^2 U_X^w + U_G^w f_{kw}
\end{vmatrix}}
\]

Applying the Cramer’s rule for the female yields:
\[
\begin{bmatrix}
(1 - \theta_C) v_w^2 U_{XX}^w + U_G^m f_{km} - \theta_m \hat{v}_w v_m v_w U_{XX}^m \\
\hat{\theta}_m \hat{v}_w v_m v_w U_{XX}^m - (\theta_m v_m)^2 U_{XX}^m + U_G^m f_{kw}
\end{bmatrix} =
\begin{bmatrix}
\hat{\theta}_w v_w U_{XC}^w \\
\hat{\theta}_m \hat{v}_w v_m v_w U_{XC}^m - (\theta_m v_m)^2 U_{XX}^m + U_G^m f_{kw}
\end{bmatrix}
\]

\[
\frac{d \hat{k}_{w2}}{d c} = \frac{\begin{bmatrix}
\hat{\theta}_w v_w U_{XC}^w \\
\hat{\theta}_m \hat{v}_w v_m v_w U_{XC}^m - (\theta_m v_m)^2 U_{XX}^m + U_G^m f_{kw}
\end{bmatrix}
}{\begin{bmatrix}
(1 - \theta_C) v_w^2 U_{XX}^w + U_G^m f_{km} - \theta_m \hat{v}_w v_m v_w U_{XX}^m \\
\hat{\theta}_m \hat{v}_w v_m v_w U_{XX}^m - (\theta_m v_m)^2 U_{XX}^m + U_G^m f_{kw}
\end{bmatrix}}
\]

In general, the signs of \( \frac{d k_{m2}^C}{d c} \) and \( \frac{d k_{w2}^C}{d c} \) are ambiguous and depend on the sign of the determinant in the numerator. However, starting with the male partner, and if we assume that \( \theta_m v_m U_{XC}^m \left[ \frac{\partial^2 U_w^m}{\partial k_{w2}^2} \right] > \hat{\theta}_w v_w U_{XC}^w \left[ \frac{\partial^2 U_m^w}{\partial k_{m2} \partial k_{w2}} \right] \), then \( \frac{d k_{m2}^C}{d c} > 0 \) and therefore his investment in the production of the local public good increases with the number of children. In the same way, if \( \hat{\theta}_w v_w U_{XC}^w \left[ \frac{\partial^2 U_m^w}{\partial k_{m2}^2} \right] > \theta_m v_m U_{XC}^m \left[ \frac{\partial^2 U_w^m}{\partial k_{m2} \partial k_{w2}} \right] \), this implies that \( \frac{d k_{w2}^C}{d c} > 0 \) and therefore the female’s contribution to the production of the local good increases with the number of children.

**Proposition** Suppose that the stock of the local public good at period \( t - 1 \), \( G_{t-1} \), is the same in the two-person household with children and in two-person household without children. Then, in period \( t \), the stock of the local public good available to the partners will be (weakly) higher in the case of a married couple without children than in the case of a married couple with children.

**Proof** The decision problem facing married couples without children is given by:

\[
\max_{k_{l2}} U^i (X, G_2) = U^i (\theta_i (v_m (L - k_{m2}) + v_w (L - k_{w2}), \delta G_1 + f (k_{m2}, k_{w2}))
\]

The Kuhn-Tucker conditions at the subgame Nash equilibrium imply that:

\[
-\theta_m v_m U_{X}^m + U_G^m f_{km} = 0
\]

\[
-\theta_w v_w U_{X}^w + U_G^w f_{kw} = 0
\]

The amount of contributions to the production of the local public good are given by \( (\hat{k}_{m2}^C, \hat{k}_{w2}^C) \).

Thus, the amount of public good available is
\( \bar{G}_t = \delta G_{t-1} + \hat{f}(\bar{k}_{mt}, \bar{k}_{wt}) \)

By contrast, the decision problem facing married couples with children is given by:

\[
\max_{k_{12}} U^i(X, G_2, C) = U^i(\hat{\theta}_m (v_m (L - k_{m2}) + v_w (L - k_{w2}), \delta G_1 + \hat{f}(k_{m2}, k_{w2}), C)
\]

If we derive the first-order conditions, we find that

\[
\begin{align*}
-\hat{\theta}_m v_m U_X^m + U_G^m f_{km} &= 0 \\
-\hat{\theta}_w v_w U_X^w + U_G^w f_{kw} &= 0
\end{align*}
\]

The amount of contributions to the production of the local public good in this case are given by \((\bar{k}_{m2}^c; \bar{k}_{w2}^c)\). Thus, the amount of public good available is

\( G_t = \delta G_{t-1} + \hat{f}(\bar{k}_{mt}^c, \bar{k}_{wt}^c) \)

Suppose that in equilibrium we have \( G_t > \bar{G}_t \), then it must be true that

\[ U_G^m < U_G^m \]

Now, if we take the case where \( \frac{d\bar{k}_{m2}^c}{dC} > 0 \), we know that

\[ \bar{k}_{mt}^c > \bar{k}_{mt} \]

Considering that the production function has decreasing returns to scale, we have that

\[ f_{\bar{k}_{mt}}^c < f_{\bar{k}_{mt}} \]

Therefore, we obtain that

\[ U_G^m f_{\bar{k}_{mt}} > U_G^m f_{\bar{k}_{mt}} \]

However, we have that

\[
\begin{align*}
-\theta_m v_m U_X^m &\geq -\hat{\theta}_m v_m U_X^m \\
\Rightarrow -\theta_m v_m U_X^m + U_G^m f_{\bar{k}_{mt}} &\geq -\hat{\theta}_m v_m U_X^m + U_G^m f_{\bar{k}_{mt}}^c = 0
\end{align*}
\]
which is impossible. Thus, it must be true that $\tilde{G}_t > G_t$. This means that, if the existing stock of public good in period $t-1$ is the same in both marriages with and without children, then the level of the local public good available to the partners in period $t$ will be higher (weakly) in the two-person household without children than in the two-person household with children.

The result above suggests that in non-cooperative models of the family, the presence of children not only affects the division of the private good between the partners, but also the total amount of the local public good available to them. The intuition behind this result stems from the fact that public goods that are now produced in the household are no longer exclusive to the partners, but rather shared with their children. The differences in the amount of the local public good for couples without children and those with children could be interpreted as marital-specific investments that partners choose to allocate to the children’s well-being (e.g. education, extracurricular activities etc.).

4.2. Period one decision: to marry or remain single

After showing the possible scenarios that couples may face in period two, either by choosing to remain single or to divorce, we now move on to period one where we consider the decision problems of individuals who expect to remain single for the rest of their lives, and those who anticipate that their marriage will last or end in period two. It is important here to understand how the addition of children to the model can impact the partners’ decisions and note the changes to the first order conditions compared to the latter in the basic model without children.

First, let us consider the case of single individuals in period one, prior to exposing the two scenarios that married couples face at the beginning of this period.
4.2.1. *Forever single*

As outlined in the model, individuals who do not marry in period one remain single for the rest of their lives. When deciding on the amount of labor to allocate between the production of the private good and the local public good, individuals anticipate the impact of their decision on their options in period two. In the decision problem below, $\emptyset$ denotes the individual’s time preference and $V_i$ represents their first period utility.

Barham, Devlin and Yang (2009) characterize the decision problem facing a single individual in period one as

$$\max_{k_{i1}} V_i(x_{i1}, G_1) + \emptyset U^i(x_{i2}, G_2) = V_i(L - k_{i1}), f (k_{i1}, 0) + \emptyset U^i(v_i(L - \tilde{k}_{i2}), \delta G_1 + f(\tilde{k}_{i2}, 0))$$

At an optimum, it must be true that

$$-v_i V_k^i + U_{G}^i f_k + \emptyset U_{G}^i f_k = 0$$

(28)

Thus, the required level of investment required to produce the local problem good by either of the partners is given by $\tilde{k}_{i1}$.

4.2.2. *Just married*

In this part, we look at individuals who decide to marry at the beginning of the first period and get children. In this case, two scenarios should be taken into account separately. The first, assuming that individuals expect that their marriage will survive until the second period and the second in which individuals expect to divorce at the beginning of the second period. We know that children are produced at a certain probability $p > 0$ which is determined exogenously; however, to eliminate uncertainty in period 1, we assume that children arrive before partners choose their levels of investment in the production of the local public good $k_{m1}$ and $k_{w1}$.
If individuals anticipate they will remain married, the decision problem facing the male partner is given by:

$$\max_{k_{m1}} V^m(X_m, G_1, C) + \emptyset U^m(X_{m2}, G_2, C)$$

$$= V^m(\tilde{\theta}_m[v_m(L - k_{m1}) + v_w(L - k_{w1})], f(k_{m1}, k_{w1}), C)$$

$$+ \emptyset U^m(\tilde{\theta}_m[(v_m(L - \hat{k}_{c_{m2}}) + v_w(L - \hat{k}_{c_{w2}})], \delta G_1 + f(\hat{k}_{c_{m2}}, \hat{k}_{c_{w2}}), C)$$

The first order conditions are given by:

$$-v_m \tilde{\theta}_m V^m_X + V^m_G f_{km} + \emptyset \frac{\partial \hat{k}_{c_{w2}}}{\partial k_{m1}} (-v_m \tilde{\theta}_m U^m_X + U^m_G f_{kw}) + \emptyset \delta U^m_G f_{km} \leq 0$$ (29)

$$k_{m1} \geq 0$$ (30)

$$k_{m1} \left[-v_m \tilde{\theta}_m V^m_X + V^m_G f_{km} + \emptyset \frac{\partial \hat{k}_{c_{w2}}}{\partial k_{m1}} (-v_m \tilde{\theta}_m U^m_X + U^m_G f_{kw}) + \emptyset \delta U^m_G f_{km}\right] = 0$$ (31)

In a similar manner, the decision problem facing the female partner is given by:

$$\max_{k_{w1}} V^w(X_w, G_1, C) + \emptyset U^w(X_{w2}, G_2, C)$$

$$= V^w(\tilde{\theta}_w[v_m(L - k_{m1}) + v_w(L - k_{w1})], f(k_{m1}, k_{w1}), C)$$

$$+ \emptyset U^w(\tilde{\theta}_w[(v_m(L - \hat{k}_{c_{m2}}) + v_w(L - \hat{k}_{c_{w2}})], \delta G_1 + f(\hat{k}_{c_{m2}}, \hat{k}_{c_{w2}}), C)$$

The first order conditions are given by:

$$-v_w \tilde{\theta}_w V^w_X + V^w_G f_{kw} + \emptyset \frac{\partial \hat{k}_{c_{m2}}}{\partial k_{w1}} (-v_m \tilde{\theta}_w U^w_X + U^w_G f_{km}) + \emptyset \delta U^w_G f_{kw} \leq 0$$ (32)

$$k_{w1} \geq 0$$ (33)

$$k_{w1} \left[-v_w \tilde{\theta}_w V^w_X + V^w_G f_{kw} + \emptyset \frac{\partial \hat{k}_{c_{m2}}}{\partial k_{w1}} (-v_m \tilde{\theta}_w U^w_X + U^w_G f_{km}) + \emptyset \delta U^w_G f_{kw}\right] = 0$$ (34)

Therefore, the optimal investment levels required for the husband and the wife to produce the local public good at period one are given by \(\hat{k}_{c_{m2}}\) and \(\hat{k}_{c_{w2}}\), respectively.
We now look at the decision problems facing the partners if they anticipate that they will divorce at the beginning of period two. In this case, children are shared at period two following the outlined sharing rule and assuming that the male partner earns more than the female, he will have to share a part of his labour-income with his ex-partner by paying her both child and spousal support.

Thus, the decision problem facing the male partner if he anticipates to divorce is given by:

\[
\max_{k_{m1}} V^m(X_m, G_1, C) + \emptyset U^m(X_{m2}, G_2, C)
\]

\[
= V^m(\tilde{\theta}_m[v_m(L - k_{m1}) + v_w(L - k_{w1})], f(k_{m1}, k_{w1}), C)
\]

\[
+ \phi U^m(v_m(L - k_{m1}^{DC})(1 - \theta_D - \alpha), \delta \lambda G_1 + f(k_{m1}^{DC}, 0), \left(\frac{k_{m1}}{k_{m1} + k_{w1}}\right) C)
\]

The Nash equilibrium solution to this problem is obtained through the Kuhn-Tucker conditions below:

\[
-v_m \tilde{\theta}_m V_m^m + V_g^m f_{k_m} + \phi \delta \lambda m U_g^m f_{k_m} + \phi U_c^m \left[\frac{k_{w1}}{(k_{m1} + k_{w1})^2} C\right] \leq 0
\] (35)

\[
k_{m1} \geq 0
\] (36)

\[
k_{m1} \left\{ -v_m \tilde{\theta}_m V_m^m + V_g^m f_{k_m} + \phi \delta \lambda m U_g^m f_{k_m} + \phi U_c^m \left[\frac{k_{w1}}{(k_{m1} + k_{w1})^2} C\right]\right\} = 0
\] (37)

In a similar fashion, the decision problem facing the female partner if she anticipates to divorce is given by:

\[
\max_{k_{w1}} V^w(X_w, G_1, C) + \emptyset U^w(X_{w2}, G_2, C)
\]

\[
= V^w(\tilde{\theta}_w[v_m(L - k_{m1}) + v_w(L - k_{w1})], f(k_{m1}, k_{w1}), C)
\]

\[
+ \phi U^w \left( (1 - \theta_c)[v_w(L - k_{w2}^{DC}) + v_m(L - k_{m2}^{DC})(\theta_D + \alpha)], \delta \lambda w G_w + f(0, k_{w2}^{DC}), \left(\frac{k_{w1}}{k_{m1} + k_{w1}}\right) C\right)
\]

At optimum, it must be true that:
The subgame perfect Nash equilibrium investment levels required to produce the local public good are given by \((\tilde{k}^{DC}_{m1}, \tilde{k}^{DC}_{w1})\).

### 4.3. The impact of children on the decision to marry or remain single

After all, the question of interest is to see whether children can create an incentive for individuals to marry, even though the latter anticipate divorcing at some point. Children clearly are a natural attraction to marriage, as stated by Becker, and partners may be able to benefit from goods and services that cannot be purchased on the market; however, the benefits from having children can be attenuated by their costs, and this may create an incentive for individuals to choose to remain single. For couples to marry regardless of the anticipated divorce option, it must be true that

\[ U^i_{m\text{(marriage,divorce)}} > U^i_{m\text{(single,single)}} \text{ for both } m, f \]

To analyze how the presence of children can impact the decision to marry or remain single, we examine how the gap between the payoff to marriage-divorce and to single-single evolves in response to changes in the value of \(C\).

The gaps for each partner can be expressed as

\[
\Delta^m = V^m \left( \theta_m \left[ v_m \left( L - \tilde{k}^{DC}_{m1} \right) + v_w \left( L - \tilde{k}^{DC}_{w1} \right) \right] , f \left( \tilde{k}^{DC}_{m1}, \tilde{k}^{DC}_{w1} , C \right) \right.
\]

\[ + \phi U^m \left( v_m \left( L - k^{DC}_{m2} \right) \left( 1 - \theta - \alpha \right) , \delta \lambda G_1 + f \left( k^{DC}_{m2}, 0 \right) , \left( \frac{\tilde{k}^{DC}_{m1}}{\tilde{k}^{DC}_{m1} + \tilde{k}^{DC}_{w1}} \right) \right) \]
We can then study how the presence of children can affect the tradeoff between choosing marriage over remaining single by differentiating these gaps with respect to $C$.

\[
\frac{d\Delta^m}{dC} = V_c^m + \phi U_c^m \left( \frac{\hat{k}_{DC}^m}{\hat{k}_{m1}^m + \hat{k}_{w1}^m} \right) + \frac{d\hat{k}_{m1}^m}{dC} \left( V_G^m \hat{k}_{m1}^m + V_X^m \hat{k}_{m1}^m \right)
\]

\[
\frac{d\Delta^w}{dC} = V_c^w + \phi U_c^w \left( \frac{\hat{k}_{DC}^w}{\hat{k}_{m1}^w + \hat{k}_{w1}^w} \right) + \frac{d\hat{k}_{w1}^w}{dC} \left( V_G^w \hat{k}_{w1}^w + V_X^w \hat{k}_{w1}^w \right)
\]

We see that the presence of children affects the gap between the payoffs to marriage/divorce and single/single and this effect depends on the signs of $\frac{d\hat{k}_{DC}^m}{dC}$ and $\frac{d\hat{k}_{DC}^w}{dC}$. In fact, if $\frac{d\hat{k}_{DC}^m}{dC} > 0$ and $\frac{d\hat{k}_{DC}^w}{dC} > 0$, that is the level of contribution to the production of the local public good in period 1 of the male (female) partner increases with the number of children, then it must be true that children have a positive impact on the gap between the two outlined payoffs, and thus, encourage the male (female) partner to marry even though he (she) anticipates to divorce at a later stage of the union.
5. Conclusion

In this paper, we build a model that includes children as public goods within the household and find the impact of their costs on the decision to marry or to divorce in non-cooperative models of the family. We find that the presence of children reduces the sharing of the composite private good between the partners and furthermore affects the levels of investments in the local public good for each one of the partners.

If individuals choose to divorce, the impact of the cost of children on the levels of investments in the production of the local public good is ambiguous. However, by imposing some restrictions on the concavity of the utility function of the partners, an increase in children costs reduces the level of contribution of both partners to the production of the local public good. Regarding spousal supports, and assuming that the male earns more than the female, an increase in child support payments increases the level of investment of the female to the production of the local public good, whereas it reduces the contribution of the male partner to the production of the local public good.

If partners remain married, children increase the levels of investments of both partners to the production of the local public good. Interestingly, we also prove that the stock of the local public good available to the partners will be weakly higher in the case of a married couple without children than in the case of a married couple with children.

The analysis also considers the impact of children on the decision to marry or remain single. Our results confirm Becker’s theory stating that children are a natural attraction to marriage; even though divorce remains an option at a later stage in the union, children encourage the partners to marry rather than remaining single.
A limitation of this model is that it does not consider the possibility of remarriage following a divorce. A more promising avenue for extending the model in this paper would be through the consideration of the cost of children in models of the family where the decision to remarry is an option, as this might generate greater insights on the impact of children on divorce rates in subsequent unions.
6. References


