

Introducing Population Growth and Demography in Demand-led Models of Growth and
Accumulation

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In 1960, demographer Ansley J. Coale asserted that “Ever since Keynes set forth his theory of income determination, demographic variables have been discussed as possible determinants of effective demand, and consequently the level of unemployment” (Coale, 1960b, page 352). Unfortunately, this discussion seems more likely to have taken place in the departments of demography than in the well-established departments of economics, despite the inherent economic nature of the material. Economists of the then dominant neoclassical synthesis were more apt to borrow from Malthus or Walras than from Keynes when discussing population growth. Illustratively, two economists who would later go on to win the Nobel Memorial Prize in Economic Sciences contributed to the same conference from which Coale's statement originates. The purpose of the conference, according to Coale (1960a, p. 3) was “to discuss the mutual influences in industrially advanced countries between changes in national population and changes in national economies”. Neither future Nobel laureate even considered effective demand. Gary Becker's (1960) piece discussed the influence of economic factors on demography and presented a hedonic model of fertility choice where household fertility decisions are akin to consumer choice. In both cases, the end goal is the acquisition of utility. Simon Kuznets (1960) did examine the effects of population growth on aggregate output. However, effective demand plays no part in his analysis. Indeed, Kuznets assumes full employment from the beginning! Population affects output, but only as an input in a production function both as the labour input, and as a determinant of productivity. Nowhere, however, does it affect demand. Importantly then, neither of the contributions of these prominent economists deals with traditionally Keynesian concerns. Indeed, only Coale's submission to the conference deals with the possible macroeconomic effects that demographic change may bring about through effective demand.¹

The economic mainstream has continued to follow in the footsteps of Kuznets. Population growth may have growth effects by entering into the aggregate production function, but it does *not*, as

¹ Crockett (1960) and Ferber (1960) are possible exceptions as both examine the effects of population change on consumption patterns. The focus in both, however, is microeconomic.

Coale believed, have possible aggregate demand effects. Instead, population growth is said to occur in the fabled “long run”, where unemployment does not exist.

Meanwhile, for economists for whom effective demand is a main concern – that is, post-Keynesian and other heterodox strands – population growth and demography are almost completely overlooked!² Demography plays no role in heterodox short run models of effective demand, perhaps considered to be too inconsequential to include. Meanwhile, the structure of heterodox models of accumulation left no room for population growth to be included as a determining factor. Robinson (1956, p. 68) states clearly that “it seems best to treat accumulation and growth in the labour force as two independent factors”. As such, population growth and demography were left aside.³

The present work tries to rectify this deficiency on the part of the economics profession, and in particular heterodox strands. It tries to demonstrate that demography, which up until now has been largely overlooked in the heterodox literature, conforms to a heterodox preoccupation with effective demand, and thus belongs in heterodox models of growth and accumulation. In particular we suggest that population growth may act as a main determinant of the rate of economic growth through a “supermultiplier” process. We also examine several possible methods by which demography is able to affect the level of employment. We then introduce a method by which these processes could be included into standard Keynesian models of accumulation with the use of a stock flow consistent model. The rest of the paper is laid out as follows. Section 2 examines the “growth problem” – i.e., what determines the rate of growth of the economy – and introduces the “supermultiplier” approach. Section 3 details the possibility of population growth acting as the engine of growth. Section 4 details the possible effects of demography on the rate of employment. Section 5 introduces a simple stock-

2 Two notable exceptions are of course the work of Alvin Hansen (Hansen, 1939) which will be discussed later, and the work done by Codrina Rada (cf. Rada, 2012)

3 Note here that often population growth is contained as a factor influencing the “natural rate” of growth, and thus may have indirect effects on the actual growth rate. See Lavoie (2014, pp. 411-15) for a summary of post-Keynesian works which attempt to reconcile the growth of aggregate demand with the “natural rate” of growth.

flow consistent model to simulate the ideas of the earlier sections and to draw out their conclusions. Finally, section 6 concludes.

2 The Growth Problem

Questioning what leads an economy to grow seems rather commonplace in our discipline. One generally begins by making reference to the full title of Adam Smith's celebrated 1776 classic thus demonstrating the persistence with which this question preoccupies the economics profession. For economists of the neoclassical tradition, the basics of the growth problem were easily solved. Given that markets are able to fully employ resources in the long run, then growth is determined by the growth of those things that create output: land, labour, capital, and technology.

However, as soon as one throws out the market-clearing axiom, this whole edifice collapses. Indeed, unless one subscribes to a general equilibrium market clearing process, it is evident that the level of output is demand determined. Likewise, unless one believes that similar market clearing processes take place in the long-run, it must be the case that the growth of output is determined by the growth of demand. The growth problem then becomes more specific; no longer is it “what causes an economy to grow?” but rather “what causes demand to grow?” The problem of the cause of the wealth of nations is solved; what is needed is an inquiry into the nature and causes of demand growth.

Demand in a simple Keynesian framework, is easily expressed as a relation between the level of investment and the rate of saving. Thus:

$$Y = I/s \tag{1}$$

One can simply turn this equation into rates of growth which would yield:

$$\hat{Y} = \hat{I} - \hat{s} \tag{2}$$

where hats represent growth rates. Assuming that there is no secular tendency for the rate of saving to

change, $\hat{s} = 0$, we are left with the simple conclusion that output grows at the rate of investment. Of course, this explanation is insufficient on its own. Any clever person would be inclined to ask what then determines the rate of growth of investment. The acceleration hypothesis argues that accumulation keeps pace with the rate of growth of the economy. In this case, we have sort of a circular model in which $\hat{Y} \rightarrow \hat{I} \rightarrow \hat{Y} \dots$. Indeed, left so simply any combination of \hat{Y} and \hat{I} are acceptable so long as $\hat{Y} = \hat{I}$. In the words of Joan Robinson “Carrying itself by its own bootstraps is just what a capitalist economy *can* do” (Robinson 1962, p. 13).

Given that there is an infinite number of combinations of \hat{Y} and \hat{I} , the problem is in determining what specific value the pair will take. One might posit that investment depends on the expected rate of profit as in the canonical Robinsonian growth model or on the level of capacity utilization as in the canonical Kaleckian model. In either case, we are left in a situation where investment determines its own rate of growth. That is, investment determines the profit rate through the Cambridge equation, or the level of utilization through a modified Cambridge equation. In both cases, assuming the appropriate stability conditions, we end up with a unique rate of investment, and thus a unique rate of output growth.⁴ In both cases however, the system is self propelling; capitalism pulls itself by its own bootstraps.

This raises two important points. First, note that certain parameter changes are able to change the rate of economic growth. For example, an increase in “animal spirits” is able to raise both the rate of investment, and the profit rate. Similarly, the Kaleckian model famously demonstrates that an increase in the wage share increases the rate of growth. Second, given that the rate of output growth can take a great number of values, there is nothing leading it to grow at the same rate as population. This seems to be the typical heterodox position. From Robinson (1956, p 68):

4 See Lavoie (2014, Ch. 6) for an excellent pedagogical presentation of these two models.

But the recruitment of labour may run ahead or fall short of the pace of accumulation... Thus it seems best to treat accumulation and growth in the labour force as two independent factors which may or may not be in harmony with each other.

Or in her *Essays in the Theory of Economic Growth* (1962, p. 14):

No doubt there is a connection between the rate of growth of the population and the standard of life but it is unreliable and apt to go in the contrary direction. We must allow the growth of the labour force to follow its own path. Should we then postulate that the stock of capital tends to grow in step with it, in such a way that a constant ratio of employment to population is always preserved, with a constant ratio of capital to labour?... This is easily said but where is the world that it is supposed to describe? When did the right stock of capital come into existence, and what mechanism, supposing that it did, keeps accumulation going at the right rate?

Thus, in these models of accumulation, it is possible for output growth to run ahead of or behind the rate of population growth. Unless the rate of output growth is equal to the sum of population growth and labour productivity, and assuming a relatively constant labour force participation rate, then the rate of employment will either be constantly rising or falling.

Note that this contradicts an oft quoted stylized fact of the neoclassical school: that economies tend to grow at rates which roughly maintains employment to population ratios. In the words of Arrow, as quoted in Serrano (1995, p. 68, footnote 2):

The US[. . .]created many more jobs in the last ten or fifteen years than Europe has. [. . .]the US labor force had during these years grown a lot more than the European has. And that is not a coincidence! If the Europeans had a lot more people looking for jobs, there would be more jobs.

Mainstream economists such as Arrow are quick to cite such observations as they conform to their theoretical propositions: this coincidence of growth rates is brought about by the ability of markets to absorb excess labour supplies in the long-run through market clearing. However, at least on the surface, this coincidence does seem to exist. We rarely see *secular* increases or decreases in the unemployment rate. To be sure, there are changes, but rarely do these tend to move in one direction for long periods of time.

Those who subscribed to earlier models of demand-led growth *à-la* Robinson or Kalecki countered that this is an illusion brought about by labour adapting to demand conditions, rather than through market clearing processes. To quote Garegnani (1992, p. 116):

That rough coincidence may in fact result from employment-seeking labour adjusting to employment opportunities rather than the reverse, with the labour 'endowment' being a determined rather than a determining magnitude in the system. [We] might indeed easily indicate the massive migrations of workers from country to country that have steadily accompanied the economic development of market economies in the last centuries... [or] point to the adaptation implicit in the so-called 'dualism' of many economies, in which a sector using advanced techniques coexists with sectors using the traditional methods, which provide much lower incomes to the producers and release labour in step with the needs of the advanced sector.

Or from Robinson (1962 p. 15):

Capitalist industry does not employ the whole work force in any country. Domestic service, paid or unpaid, jobbing work and small-scale trade, and, in most countries, agriculture, hold a reservoir of labour which fills up when regular employment is not expanding as fast as the population.

The implication then is that it is either the labour force participation rate or influxes of labour due to immigration that tend to adjust to secular increases in the rate of output growth above the rate of population growth. There seems to be undeniable truth to this; one need only look at the behaviour of the participation rate in the U.S. economy following the great financial crisis. The unemployment rate has fallen, but mainly due to decreases in the rate of labour force participation. However, such labour force adjustment has upper bounds determined by, in the first instance, the amount of “hidden” unemployment in the system, and, in the last instance, by the availability of working age population both domestically and abroad. As such, the arguments of these proponents of early demand-led models are not wholly satisfying.

There exists a second strand of heterodox literature which conceives of the growth process in a different way. We will call this the “supermultiplier” theory. The name comes from Hicks's (1950) conception of long-run effective demand which incorporates both the multiplier and accelerator. This

approach has become popular with certain Sraffians (cf. Serrano 1995)⁵ but also with adherents to the Kaldor/Thirlwall model of export-led growth, although the latter's interpretation is slightly different (cf. McCombie 1985).⁶

The supermultiplier is easily derived. We start with the determinants of effective demand in a closed economy⁷:

$$Y = C + I + G \quad (3)$$

We then assume specific functional forms for consumption and investment:

$$C = \alpha_c + (1-s)Y \quad (4)$$

$$I = \alpha_I + v \Delta Y^e = \alpha_I + v g_y^e Y \quad (5)$$

Where v is the desired ratio of capital to output and g_y^e is the expected growth rate of output.

Importantly, α_I and α_c are the exogenous components of investment and consumption respectively – that is, these are consumption and investment expenditures which do not vary directly with output.

Solving the model yields:

$$Y = \frac{\alpha_c + \alpha_I + G}{s - v g_y^e} \quad (6)$$

Defining the exogenous components of demand as

$$\alpha_Y = \alpha_c + \alpha_I + G \quad (7)$$

we get:

5 See also (Cesaratto 2012) for a discussion of Kaleckian vs. Sraffian growth models, where the latter includes the supermultiplier framework.

6 We should also mention here the growth model of Godley and Lavoie (2012, Ch. 11) in which long run growth is determined by increases in government expenditures, as well as the work on the irreversibility of consumption done by Trezzini and Garegnani (Garegnani and Trezzini 2010, Trezzini 2011).

7 Introducing an open economy changes the analysis to some extent, as now exports enter as a fourth exogenous component of demand. See footnote 10.

$$Y = \frac{\alpha_Y}{s - v g_y^e} \quad (8)$$

If we assume a process wherein the expected rate of growth is brought into equality with the actual rate, and the parameter values to be relatively constant, then the growth rate of output is

$$\hat{Y} = \hat{\alpha}_Y \quad (9)$$

i.e. the growth rate of output is equal to the growth rate of the exogenous components of effective demand.⁸

Note that there are two important differences here. First, investment is now made up of two parts: induced investment, and exogenous investment. The latter constitutes any investment expenditure not directly linked the level of output, whereas the former is generally assumed to be investment aimed at keeping the rate of utilization constant. The latter might include such things as R&D expenditure or advertising. Second, exogenous consumption and government expenditures are included.

Importantly, note that the inclusion of exogenous demand leads to a scenario in which the rate of investment is *determined* by the rate of output growth, and not vice versa. Here, the bootstraps of capitalism are being pulled by exogenous expenditures, while capital is the torso trying to keep up. Capitalists are reactive and not proactive. Indeed, note that if investment in one period is to run ahead of the growth rate of exogenous expenditures, this will mean that investment has also run ahead of the growth rate of output (the latter being a weighted average of the various growth rates which make up its components). Thus, to maintain a specific K/Y ratio, investment will have to be brought down in the following period to compensate.⁹ Not only does capitalism *not* pull itself by its own bootstraps, but it is

8 Lavoie (2014, pp. 405-409) offers a similar analysis in which exogenous expenditures are entered into a Kaleckian investment function.

9 Note that in the Robinsonian case, investment is not necessarily reacting to keep up with output changes, but rather is proactively predicting changes in output, which, due to the simple conception of output, are necessarily verified.

incapable of doing so so long as there are expenditures whose rate of growth is not tied directly to the rate of growth of output.

3 Population Growth as determinant of the growth of exogenous expenditures

What is it then that determines $\hat{\alpha}_y$, the rate of growth of exogenous expenditures? We contend that the major determinant is the rate of population growth.¹⁰ Alvin Hansen, arguably the first to work on the effects of demography on output from a Keynesian perspective, was well aware of the effect population growth could have on effective demand. In his famous speech to the American Economic Association, Hansen (1939, p. 8) wrote:

Obviously the growth of population affects capital formation most directly in the field of construction, especially residential buildings. From decade to decade, the increase in the number of dwellings had maintained a close relation to the increase in population... But the effect of population growth on capital formation is, of course, felt in other spheres as well. This is notably true for all the various municipal and public utilities, and also in the manufacture of essential consumers' goods.

Here, Hansen outlines three channels through which population growth affects effective demand:

residential capital formation, government expenditures, and production of essential consumer goods.

New families or adult individuals require living spaces and some basic level of subsistence

consumption, while government programs which provide universal benefits necessarily increase with

the population (examples of the latter might include health care spending, roads to new communities,

municipal and public utilities, new schools, etc.) Note that these three categories also correspond to our

three categories of exogenous expenditure: α_C , α_I , and G .

10 Note that this is not fully the case if we include exports as a fourth component of exogenous demand. In this case, effective demand need not be growing at the same rate as population growth, as one term in the exogenous component of effective demand (the growth rate of exports) is determined independently of domestic population growth. However, we give a precursory rebuttal: (i) much trade is between similarly developed countries with similar population growth rates, (ii) in the case where export demand outstrips domestic supply, the export sector can often address the supply constraints by shifting some production abroad, and (iii) in the case where export demand grows more slowly than domestic demand, eventually the export sector is reduced to such a small percentage of the overall economy that its effects on economic growth are minute.

Our definition of α_C , the exogenous component of demand, differs slightly from the classical Keynesian definition. It is not necessarily the level of spending if income were zero (indeed, this is a rather strange counterfactual). Rather, we interpret α_C as that element of consumption demand which will increase without first requiring an increase in income. Without its exogenous component, consumption may increase either if employment increases or if there is a rise in wages and consumers look for new outlets to spend their larger incomes. However, we might also consider cases where consumption increases *before* there is a corresponding increase in income. For example, social trends might dictate increased spending on electronics as new expensive smartphones are released, a trend which consumers will try to follow even without a requisite increase in income. Similarly, and more to our point, population growth entails new human life born into a social surrounding in which it must seek to emulate currently existing consumption patterns. From birth onwards, there is a necessity to keep up with the Joneses as best as one can. This, no doubt, calls forth a certain level of consumption spending, regardless of income (although this is no doubt is modified depending on one's level of income, represented here by the propensity to consume). Note then that we take what Hansen termed “essential consumers' goods” to be largely socially determined.

We mark the start of this process “at birth” as expenditures on children no doubt represent an increase in consumption not brought about by increases in income. Coale (1960, p. 354) agrees when discussing the effects of dependents on consumption:

Suppose a larger population is compared with the given population, the extra persons all being under 17 or over 70. Suppose further that the additional dependents are pictured as belonging to existing households. The most likely effect upon household budgets at any given disposable income would be an increase in expenditures. True, there might be an offsetting tendency to save specifically for the future welfare of children... On balance, however, most households would find their consumption enlarged and their saving reduced by an extra member.

Based on survey results (albeit, outdated now), Coale (1960) quotes that consumption increased by 1%

for every 6% increase in population when this increase is entirely made up of dependents.

As young adults, these children will continue to maintain a certain standard of living whether this is accomplished through student loans, work, or intergenerational transfers from family to children. The important concept to grasp however is that in these cases consumption *precedes* income. No doubt, it's level is modified by income (a recent graduate with a full time job is likely to spend more than an unemployed graduate, but both will maintain similar lifestyles none-the-less) but there must exist some basic socially determined consumption level which will go on (although perhaps not indefinitely) even without income.¹¹

Here it is important to make a small note about the importance of consumer credit. Barba and Pivetti (2009) and Setterfield and Kim (2013) attach a particular importance to the financing of consumption expenditures through credit by some households. Although consumer credit is no doubt important and indeed, as Barba and Pivetti (2009) show, its importance has been growing, it is not a prerequisite for consumption-led growth. Indeed, households may just as easily draw on their savings to increase their consumption. This in turn calls forth new investment which will increase the funds available for new savings. The model we simulate in section 5 does not include credit for consumption but still demonstrates consumption-led growth.

The matter of residential investment is very similar to that of autonomous consumption. As Hansen makes clear, the increase in dwellings generally follows the rate of increase in population. Larger families require homes with more rooms, students require apartments which are often funded by student loans, intergenerational transfers, or at least cosigned by family members, and young families take out mortgages to purchase houses. Such simple correlation is superficially confirmed by

11 No doubt there is a bi-causal relationship here. What is socially determined is easily modified by experience. Thus, if recent graduates as a whole begin to find it harder and harder to find employment, there may be a revision of expectations leading to a more frugal consumption pattern. Graduates might be more likely to live with their parents than on their own, for example. What is socially acceptable is generally determined, in part, by what is affordable. (Affordable at least to some group if we take the case of pecuniary emulation). However, it seems likely that in most cases social norms are “sticky” and are not apt to change quickly.

examining the housing boom in the 1970s which coincided with the Baby-Boom generation coming of age (Russell, 1982).

We should also note here that residential investment is obviously not the totality of exogenous investment expenditures. Indeed, it is evident that there must be some exogenous investment expenditure *not* tied to population increases – for example, R&D spending or advertising. Similarly, not all aspects of government expenditure will keep up with population growth. We would assume that defence spending, for example, is largely determined by a country's involvement in military activity rather than by the population level. These aspects of exogenous expenditure might be better treated as “shocks” to the economy rather than as engines of growth.

One final caveat is perhaps required regarding government expenditures. It is not necessarily the case that even what we term the “endogenous” components of government expenditures will grow at the same pace as the population. For example, Russell (1982) notes that as the baby boom generation became school-aged, US government officials sought cost-cutting measures in the education system out of fear that the growing student population would over-burden government coffers. Consequently, government expenditure on education was intended to grow slower than the growth rate of population. We might counter, however, that government officials were frightened into this rash action by what they saw as an unfamiliar and overwhelming situation. In normal times, when population growth rates are progressing at a steady pace, it is uncommon, unless politically motivated, for governments to engage in such austerity.

Note the important consequences of a supermultiplier model in which the rate of growth of exogenous demand is determined by the rate of population growth. First, no longer can we affirm, as did Robinson, that population growth and output growth are two separate phenomena which need not be in harmony. Indeed, in this formulation, the two are inextricably intertwined. In the absence of productivity changes, labour demand will grow at roughly the same rate as labour supply in the long

run. It is important to note however that this is *not the same as affirming that labour supply will be equal to labour demand*, a point which we discuss further after presenting our stock-flow consistent model.

Second, as has been shown by Allain (forthcoming), a supermultiplier approach such as this is able to address the problem of non-normal rates of capacity utilization, explaining the popularity of the supermultiplier approach among the Sraffian strand. Whereas in the Kaleckian model, firms as a whole are incapable of achieving “normal” or “target” capacity rates except by chance, this is not the case in models operating within a supermultiplier framework. The difference comes from the fact that, with a growing autonomous demand, firm investment can increase without leading to a proportional increase in output. As such, the firm sector can select any K/Y ratio that it wants. Although this was always a theoretical appeal of the supermultiplier approach, we suspect that it was not sufficient to convince many heterodox scholars of the superiority of this approach. First, there was some question as to the stability of the supermultiplier and even its ability to achieve normal rates of utilization (cf. Trezzini 1995). Allain (forthcoming) shows both concerns to be unwarranted. Second, we suggest that until now, the possible determinants of autonomous demand growth have been unconvincing by themselves. If, for example, like Allain, we assume that the endogenous component of demand is government expenditure, what is leading government expenditure to grow? We hope that the introduction of population growth helps to solve this second shortcoming, and lends further appeal to this approach.

4 Demography and Unemployment

From our above discussion, it may begin to become apparent that demography is able to affect the composition and level of demand, and thus affect the level of employment. Indeed, demonstrating this was one of the main purposes of Hansen's (1939, p. 7) paper. He writes:

Now the rate of population growth must necessarily play an important rôle in determining

the character of output; in other words, the composition of the flow of final goods. Thus a rapidly growing population will demand a much larger per capita volume of new residential building construction than will a stationary population. A stationary population with its larger proportion of old people may perhaps demand more personal services; and the composition of consumer demand will have an important influence on the quantity of capital required. The demand for housing calls for large capital outlays, while the demand for personal services can be met without making large investment expenditures.

Hansen wished to explain the “secular stagnation” which afflicted the United States in the mid 1930s.

Taking it for granted that

the maximum currently attainable income level cannot be reached in the modern free enterprise economy without a volume of investment expenditures adequate to fill the gap between consumption expenditures and that level of income which could be achieved were all the factors employed (Hansen 1939, p. 5)

he set out to discover what accounted for the dearth of investment in the 1930s. Hansen concluded that the dearth in investment was not simply a mirror of a dearth of business confidence. Kaldor (1939, p. 92) summarizes this idea in his review of Hansen's longer presentation, *Full Recovery or Stagnation*.

The inference is obvious: the failure of the recovery in the 'thirties to carry production to pre-depression levels was not due to any “want of confidence” on the part of business men, since business capital expenditures expanded with consumption in much the same way as in the 'twenties. It was due to the absence of those extraordinary and non-recurrent investment opportunities which sustained the prosperity of the previous decade.

In terms of “extraordinary and non-recurrent” investments, Hansen (1939) lists three: population growth; the opening up of new territory and the discovery of new resources; and technical innovations. Chief among these three for our purposes is, of course, the first.

If it is true that a growing population calls forth a growing housing stock, then it is also true that *the faster is the growth of population, the greater will be the residential investment demanded per capita*. If a city adds two thousand new residents requiring housing, then the per capita spending on housing will be greater than if the same city adds only one thousand new residents. Incidentally, through a simple multiplier process, faster population growth is able to increase total output, and

decrease unemployment.

We might also recall Coale's (1960) proposition that greater levels of dependents tend to call forth greater levels of demand. In this case, however, we may say that it implies an increase in the average propensity to consume, rather than a direct increase in the level of investment. However, the distinction is not necessarily clear when one considers the case of an ageing population. Retirees generally engage in dissaving, receive pensions, and often receive transfers from the government (Rada 2012). All of these are likely to increase the amount of spending *relative to the current labour force*. *Ceteris paribus* then we would expect the unemployment rate to decrease. An ageing population need not be something to be feared as the current discourse would have you believe, but in fact may be beneficial in an economy with excess capacity.

5 The Model

Matrices

We present the above ideas using a simple closed economy stock-flow consistent (SFC) model. As will be shown, in this model population increases act as the engine of economic growth. The main innovation of this model is to include population growth through an overlapping generations framework. As far as we can tell, this is the first attempt to create an overlapping generations SFC model although other post-Keynesian and heterodox scholars have created overlapping generations models (cf. Skott and Ryoo 2011, Rada 2012). Indeed SFC theorists seem unsure how to address labour dynamics. In the canonical SFC growth models of Lavoie and Godley (2001-2002) and Godley and Lavoie (2012), labour supply was either infinitely elastic, or labour supply is constant, while productivity grows at an exogenous rate.

In true SFC form, we begin by presenting stock-flow matrices. Table 1 shows the balance sheet matrix of the model, while table 2 gives the transactions flow matrix.

Table 1: Balance Sheet of Model PG

	Households	Production Firms	Banks	Σ
Money Deposits	+M _H	+M _F	-M _S	0
Loans	-L _H	-L _F	+L _S	0
Fixed Capital		+K		+K
Housing	+H			+H
Balance	-V _H	-V _F	0	-(V _H +V _F)
Σ	0	0	0	0

Table 1 shows the balance sheets of the three sectors in the model. Assets are entered as positives, while liabilities are negatives. Firms take out loans to pay for capital, and hold some undistributed profits. Households symmetrically take out loans to pay for housing (mortgages) and hold money balances. Banks make no profits and simply provide loans and accept money balances on demand.

Table 2: Transactions Flow Matrix of Model PG

	Households		Production Firms		Banks		Σ
	Current	Capital	Current	Capital	Current	Capital	
Consumption	-C		+C				0
Investment (firms)			+I _F	-I _F			0
Investment (households)		-I _H	+I _H				0
[Production]			[Y]				
Wages	+WB		-WB				0
Distributed Profits	+FD		-FD				0
Depreciation Allowances			-AF	+AF			0
Mortgage Payments	-MP	+MP					0
Interest on Loans	-r _{t-1} · L _{H-1}		-r _{t-1} · L _{F-1}		+r _{t-1} · L _{S-1}		0
Interest on deposits	+r _{m-1} · M _{H-1}		+r _{m-1} · M _{F-1}		-r _{m-1} · M _{S-1}		0
Change in Loans		+ΔL _H		+ΔL _F		-ΔL _S	0
Change in Deposits	-ΔM _H		-ΔM _F			+ΔM _S	0
Σ	0	0	0	0	0	0	0

Table 2, the transactions flow matrix, demonstrates the flows of funds which occur from period to period. Note the symmetry in the firm and household sectors. Both take on loans to pay for physical capital, and both make payments related to this capital – in the case of the firm sector, these are depreciation allowances, while in the case of the household sector these are mortgage payments. As will be shown, to simplify the analysis we assume that both sectors make these payments at the rate of depreciation. Thus, loans are always equal to the value of physical capital. Note also that the banking sector makes no profits, meaning that the rate of interest on loans (assumed the same for both household and firm sector) must be equal to the rate of interest on deposits. The transactions-flow matrix will appear more simple once we examine the model equations.

Equations

We start with equations related to the firm.

$$Y = C + I_H + I_F \quad (M1)$$

$$WB = W \cdot N_d \quad (M2)$$

$$N_d = Y/pr \quad (M3)$$

$$W = pr/(1+\psi) \quad (M4)$$

$$F_T = Y - WB = (\psi/1 + \psi) \quad (M5)$$

$$FD = (1-s_f) \cdot F_{T-1} \quad (M6)$$

$$FU = F_T - FD - r_{l-1} \cdot L_{F-1} + r_{m-1} \cdot M_{F-1} - AF \quad (M7)$$

$$\Delta M_F = FU \quad (M8)$$

$$AF = \delta \cdot K_{-1} \quad (M9)$$

These first eight equations details the firm sector's current account. In all equations the price level is set fixed at 1. The firm receives payments from consumption and both firm and household investment. The wage bill, WB , is simply the (real) wage, W , multiplied by labour demand. Labour demand, N_d , meanwhile is determined by the amount of output produced divided by worker productivity, pr . The firm receives gross profits, F_T , which are equal to total receipts less the wage bill. The real wage, W , is

determined by a simple markup equation (M4) (remember the price level is fixed at 1(!)) and the markup is equal to ψ . Distributed profits, FD , are determined as a percentage of last period's gross profits. Retained earnings, FU , which are held as money balances (M8), are equal to the remainder of income after the payment of wages, dividends, interest payments, and amortization funds necessary to cover the cost of depreciation. AF represents the depreciation allowance, or amortization fund, which is simply equal to the depreciation rate, δ , times the stock of capital from the previous period. The variables r_l and r_m are the rates of interest on loans and money respectively.

$$K^T = \kappa \cdot Y_{-1} \quad (\text{M10})$$

$$I_F = \gamma \cdot (K^T - K_{-1}) + AF \quad (\text{M11})$$

$$\Delta K = I_f - \delta \cdot K_{-1} \quad (\text{M12})$$

$$L_F = L_{F-1} + I_F - AF = L_{F-1} + \Delta K \quad (\text{M13})$$

Equations M10-12 deal with investment behaviour of the firm sector. The firm sector is trying to hit a targeted capital to output ratio, κ . To do this, firms use a partial adjustment function M11. Equation M12 says that the change in capital is thus equal to total investment less depreciation. Equation M13 says that all new capital formation is financed by a creation of loans. As was discussed above, this implies that $L_F = K$.

We next turn to equations dealing with consumer behaviour.

$$YD = WB + FD + r_{m-1} \cdot M_{H-1} - r_{l-1} \cdot L_{H-1} - MP \quad (\text{M14})$$

$$C = \alpha_0 + \alpha_1 \cdot YD + \alpha_2 \cdot M_{H-1} \quad (\text{M15})$$

$$\Delta M_H = YD - C \quad (\text{M16})$$

Equation M14 represents the budget constraint of the household sector. The household sector receives wages, distributed profits, and interest payments on its money holdings, and pays mortgage payments as well as interest payments on its mortgages. The remainder of income, YD is then either consumed or saved as per M15-16.

Before dealing with household investment, it is necessary to examine the population dynamics of the model. In our model, households experience a Sisyphean existence. They are born, at which point they take out a mortgage to buy a house. Households live for five years over which time they pay down their mortgage, and then die. This is represented by the following equations:

$$N_s = N_{s-1} + BR - BR_{-5} \quad (M17)$$

$$BR = BR_{-1} \cdot (1+g) \quad (M18)$$

$$I_H = BR \cdot \beta \quad (M19)$$

$$MP = \sum_{t=1}^5 \frac{I_{H-t}}{5} \quad (M20)$$

$$L_H = L_{H-1} + I_H - MP \quad (M21)$$

$$\alpha_0 = \mu \cdot N_s \quad (M22)$$

Equation M17 is the labour supply equation. Each period the supply of labour increases by the number of births that period, BR , and decreases by the cohort born five periods prior. Furthermore, we assume that the number of births is growing at a fixed rate, g . Finally, since all members of a cohort invest in housing upon birth, then housing investment is equal to the number of births multiplied by the value of residential investment per household, β . Mortgage payments are made evenly over households' lifespan. Each household pays off 1/5 of its mortgage each period. This is given by equation M20. The value of outstanding household loans then is last periods' loans, plus new household investment, less mortgage payments. Finally, equation M22 says that the exogenous component of consumption is proportional to the population.

As stated above, interest rates on money deposits are equal to interest rates on loans meaning the banking sector makes no profits. This is represented in equation M23.

$$r_l = r_m = \bar{r} \quad (M23)$$

Finally, it should be noted that the supply and demand for all variables save for labour are

equalized via quantity adjustments. That is to say, the firm sector provides consumption and investment goods on demand. Similarly, banks provide loans and money deposits on demand.¹² In the case of labour, however, there is no equilibrating process. We thus define an employment rate variable, ER , equal to the ratio between labour demand (employed labour) and labour supply:

$$ER = N_d / N_s \quad (M24)$$

Results

We simulate the above equations and demonstrate some results. First and foremost note that output converges to a steady growth rate equal to the growth rate of population. This is shown in figure 1 which shows the logarithm of output and labour supply. This simply follows the supermultiplier approach outlined in sections 2 and 3. Net household investment (i.e., household investment less mortgage payments) quickly converges to a growth rate of g . Similarly, by construction α_0 is growing at a rate of g . Save for their exogenous components, consumption and investment meanwhile are endogenous and thus converge to growth rates of g as well.

This same result can be seen in figure 2, which shows the employment rate over time. Given that

$$\frac{\dot{ER}}{ER} = \frac{\dot{N}_D}{N_D} - \frac{\dot{N}_S}{N_S} \quad (M24.5)$$

we can see that the employment rate will be stable only if labour demand grows at the same rate as labour supply. Since labour demand is proportional to output for a given productivity level, the growth rate of labour demand must be equal to the growth rate of output. Thus, equation M24.5 could equally be rewritten to say that the employment rate is stable when output grows at the same rate as population.

¹² Godley and Lavoie (2012) make explicit reference to these relationships in their equations. Thus, for example, they have separate variables for loans demanded and loans supplied. We forgo these for simplicity.

Figure 2 shows that this does indeed happen after some initial adjustment.

What then is the effect of a change in the long run growth rate of population, g ? This is detailed in figures 3 and 4. Figure 3 shows the logarithm of output following an increase in g . Not surprisingly the growth rate of output also increases, converging to the same rate. Note the difference here relative to the views of earlier Keynesians, for whom population growth and output growth were unrelated. In the latter's models, if population growth increased, there was no reason why output growth should increase by a similar rate, and thus unemployment would either increase, labour force participation would decrease (effectively leading to disguised unemployment), or there would be out-migration of the population. Our own model suggests no such outcome. Instead, because there is some part of household spending which *precedes* the acquisition of income, increases in population are, to some extent, able to create their own demand! Note that this is not Say's Law proper, nor are we positing that there will be a long-run coincidence of labour supply and labour demand (instead realize that in all simulations the unemployment rate is always positive). This pseudo-Say's Law instead posits that, neglecting technological change, there is some long run tendency for output to grow at the same rate as population. As such, increases in labour supply may lead to proportional increases in labour demand.¹³

There is another interesting facet of an increase in population growth which plays out through a Hansenian mechanism. Figure 4 shows the employment rate following an increase in population growth. Note that it *increases*; the increase in population growth has not lead to lower employment, as earlier Keynesians may have posited, but instead leads to an increase in the rate of employment. This, of course, occurs for reasons Hansen (1939) laid out in his speech to the American Economic

¹³ Labour supply is not ex ante spending all the income necessary for it's full employment. Instead, the increase in labour supply leads to an increase in a portion of demand – the exogenous share related to households – which must rely on the various multiplier and accelerator relationships to turn this initial increase into a larger increase. It is the value of the parameters determining the multiplier and accelerator relationships, as well as the other exogenous components of demand, which may or may not lead to the full employment of labour.

Association, and discussed in section 4. Faster growth of population means that household investment per capita will be greater. This is shown in figure 5 which shows net household investment per capita.

The result demonstrated in figures 4 and 5 is particularly important as it contrasts our results with those of the neoclassical growth model. In both models, abstracting from technological change the long run rate of growth of output will be equal to the rate of growth of population. However, whereas in the neoclassical growth model, increases in population growth lead to *less* output per head, here we have the contrary; here, increases in population growth lead to *greater* output per head, which in our case is expressed through greater employment.¹⁴

Here also the long run rate of growth is impervious to various parameter changes that in earlier Keynesian models would lead to long run changes in the growth rate. As an example, the effects of a decrease in the propensity to save (an increase in α_1) on output growth are demonstrated in figure 6. Note that this achieves a *temporary* growth effect. Output growth initially exceeds the rate of population growth, g , but then over time converges again to this rate. Incidentally, as figure 7 shows, this produces an increase in the employment rate. Note that in both Robinsonian and Kaleckian growth models an decrease in the propensity to save (or equivalently an increase in labour's share of income) leads to an increase in the growth rate of the economy. Here there is no such long run effect.

Introducing an Ageing Population

To introduce an ageing population we now include a set of households who have retired and thus no longer are in the labour force, but are still consuming. The new story runs as follows: households are born, work for five periods, are retired for one period over which time they consume their accumulated savings, and then die. Whereas our labour supply function is still valid, we must now introduce a population function, which is simply the labour supply function plus that cohort born 5 periods prior.

¹⁴ However, note that this is only true if there exists labour market slack.

Thus:

$$Pop = N_s + BR_{-5} \quad (M25)$$

Next, we must alter the household consumption function to take account of the consumption of retirees. Thus:

$$C = \alpha_0 + \alpha_1 \cdot YD + \alpha_2 \cdot (1 - \theta) \cdot M_{H-1} + \theta \cdot M_{H-1} \quad (M15A)$$

where

$$\theta = \frac{Pop - N_s}{Pop} \quad (M26)$$

The variable θ represents the proportion of retirees in the population. Evidently θ is determined by the dynamics of population growth. Thus, the slower the population grows, the greater is θ .

With this new model formulation, we simulate the effects of a decrease in the population growth rate. Figure 8 demonstrates the effects on θ , which increases, implying an ageing population. Figure 9 meanwhile shows the effects of the population decrease on three main determinants of effective demand per worker in the labour force: household investment, firm investment, and consumption out of savings. We do not include consumption out of current income here as it is simply a multiple of the above components. Note that although, as we would expect, household investment per worker decreases, this is partly compensated for by the rise in consumption out of savings brought about by the increase in retired workers. Consumption out of savings only starts to level out five periods after the population increase, at which point the last cohort to be born under the higher growth rate has reached retirement. Here, as one might infer, θ reaches its peak.

Finally, note that the wealth to disposable income ratio necessarily falls for households as a whole.¹⁵ The greater proportion of retirees in the population changes the effective wealth to income

¹⁵ In our model, although the wealth target decreases, the actual ratio of wealth to income *increases* as, due to the partial adjustment function which defines households' accumulation of wealth (see Godley and Lavoie 2012, ch. 3), the resulting slower growth in income allows households to closer approximate their wealth to income target. If we were to

target of the population. The wealth to income target of workers is still the same as our model without an ageing population. Modifying the presentation of Godley and Lavoie (2012) by including the α_0 term, this is given by:

$$M_H^* = \frac{1-\alpha_1}{\alpha_2} \cdot YD - \frac{\alpha_0}{\alpha_2} \quad (\text{M27})$$

$$M_H^* = \alpha_3 \cdot YD - \frac{\alpha_0}{\alpha_2} \quad (\text{M27A})$$

where the * superscript represents a long run target value. However, once we introduce demographic effects on the rate of consumption out of savings, the wealth target is modified:

$$M_H^* = \frac{1-\alpha_1}{\alpha_2 + \theta \cdot (1-\alpha_2)} \cdot YD - \frac{\alpha_0}{\alpha_2 + \theta \cdot (1-\alpha_2)} \quad (\text{M28})$$

Note that the derivative $\delta M_H^* / \delta \theta$ will be negative if:

$$(1-\alpha_1) \cdot YD > \alpha_0 \quad (\text{M29})$$

that is, if saving out of disposable income is greater than the exogenous component of demand. The fact that an ageing population is able to affect the overall long run wealth target of households may go some way to explaining the decrease in the realized wealth to disposable income ratio of the private sector in both the US and the UK noted by Martin (2012).

These features illustrate the possibility of a second important demographic factor in the determination of effective demand. Whereas a rapidly growing population may, as Hansen suggested, contribute to greater effective demand through residential construction, it may also imply effects acting in the opposite direction. A faster growing population means also a younger population. This in turn

change the formulation such that households target growth rates of money rather than levels (as suggested by Flemming 1976), we would see a decrease in actual wealth to disposable income ratios as well.

limits effective demand by decreasing consumption out of accumulated savings.^{16 17}

6 Conclusion

We have presented here an argument for the inclusion of population growth in long period models of effective demand. Importantly, building on the existing “supermultiplier” literature, we suggest that population growth may make up an important engine of effective demand growth. That population growth might in the first instance drive economic growth has several important implications. For one, it suggests a new explanation for the rough coincidence of labour supply and labour demand often noted by neoclassical economists. Indeed, as such it equally suggests a new solution to the problem of reconciling long run aggregate demand and aggregate supply with which several heterodox authors have grappled. In addition, the supermultiplier approach on which we base our analysis is able to overcome another problem of earlier demand-led growth models: the problem of normal rates of capacity. The supermultiplier approach further alters the conclusions of traditional demand-led growth models in that, since these newer models are no longer investment-led, parameter changes which in the short-run lead to greater investment do *not* in the long run lead to higher growth rates. Instead, in all cases, the long run growth rate is given by the rate of population growth.

We have also shown how the addition of demography is able to enrich long run models. For example, the addition of Hansenian dynamics suggests that, contrary to the results of the neoclassical growth model, higher rates of population growth may lead to *higher* levels of output per head.

Although the introduction of an ageing population weakens the Hansenian argument, it does suggest that there are benefits in terms of effective demand of an ageing population, with its greater proportion

16 In the first instance, however, this may not be the case. A faster growing population will in the first instance *increase* the amount of dependents relative to the working age population.

17 Keen readers may be aware of the fact that Hansen did make reference to the effects of an older population in one of the above quotes. However, Hansen makes no reference to the average propensity to consume in this respect, but rather notes that an older population will likely demand less capital intensive products. The possibility of multiple sectors with different capital to output ratios was not considered here.

of dissavers and its greater use of governments' spending power.

We have also shown here how population dynamics might be added into stock-flow consistent models. These were not present in the earlier growth models of Lavoie and Godley (2001-2) and Godley and Lavoie (2012) likely because (A) it was not the focus of the models and (B) like other Keynesian-Kaleckian writers, they saw no reason why population growth and economic growth should be related.¹⁸ Our main purpose here is to show the real and financial effects that arise when population growth is able to fuel economic growth when consumers make some spending decisions *prior* to the receipt of income.

The model presented above is, however, still incomplete. There are three main ingredients which are missing for a fully satisfying model: a government sector, changes in labour productivity, and international trade. We do not think that the inclusion of any of these three need modify the basic premise of the model, however, they may lead to different and more interesting results. They were left out here due first to concerns about complexity of the ensuing model, and second due to constraints on space. Future research should seek to address these added complexities.

In the whole, we hope that this paper has made the case for the inclusion of population and demographics in long run models of growth and accumulation. Heterodox authors in particular should find interest in the effects these may have on effective demand, as well as the potential solutions offered for theoretical problems in the traditional growth-led models.

18 Godley and Lavoie (2012) however is interesting given that, as noted above, it is essentially a supermultiplier model where government expenditures act as the exogenous component of effective demand. This seems to be the logical long-run extension of Godley's "short-run" economics in which government expenditures are the main determinant of steady-state output levels. However, the rate of growth of the effective labour supply shows no direct relation to the rate of output growth and therefore the model "describes a growing economy which does not spontaneously find a steady state even in the long run, but which requires active management of fiscal and monetary policy if full employment without inflation is to be achieved." As such, the rate of government expenditures must be growing at the rate of effective labour growth.

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Appendix – Figures

Figure 1

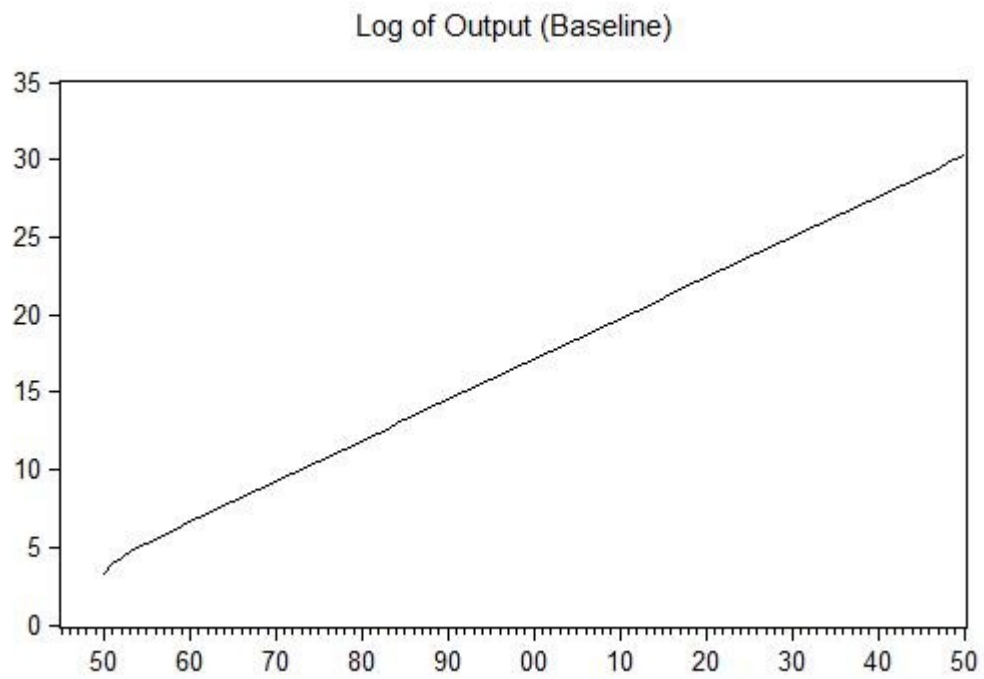


Figure 2

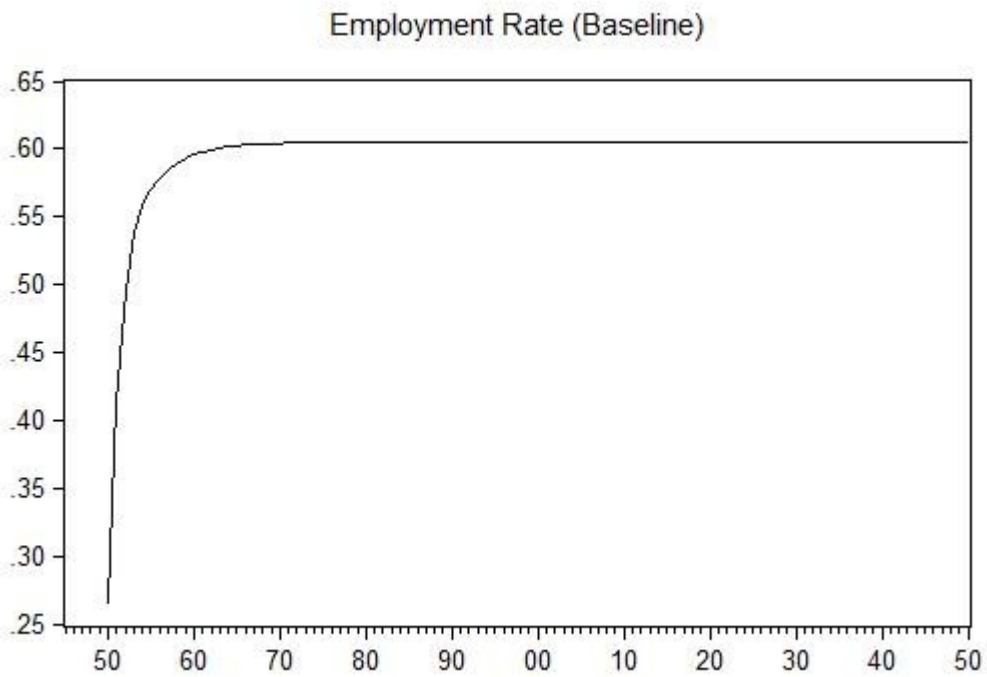


Figure 3

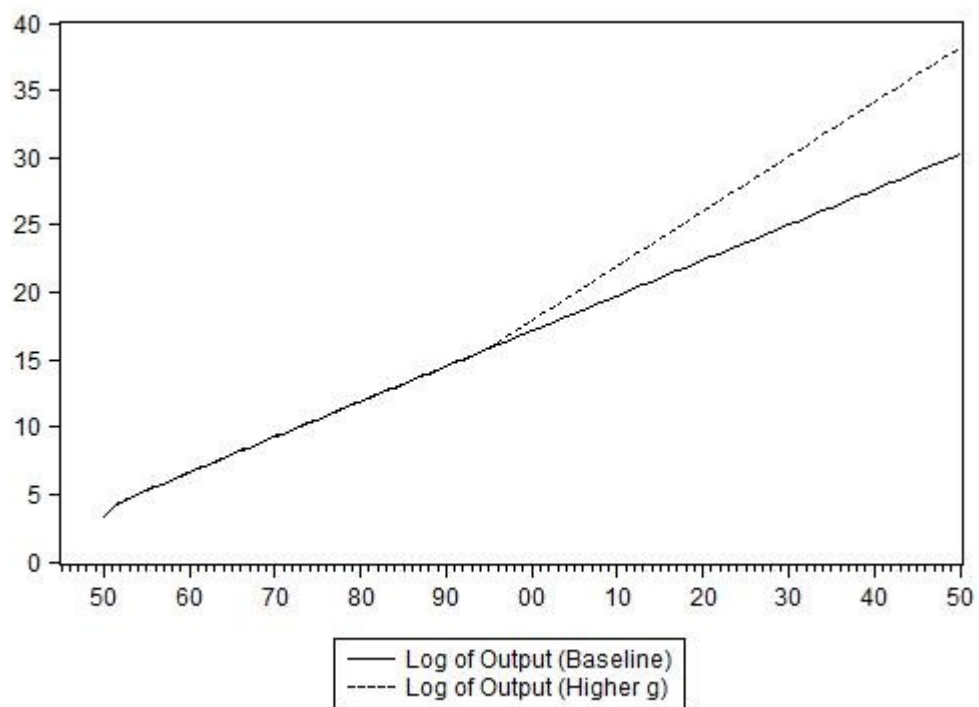


Figure 4

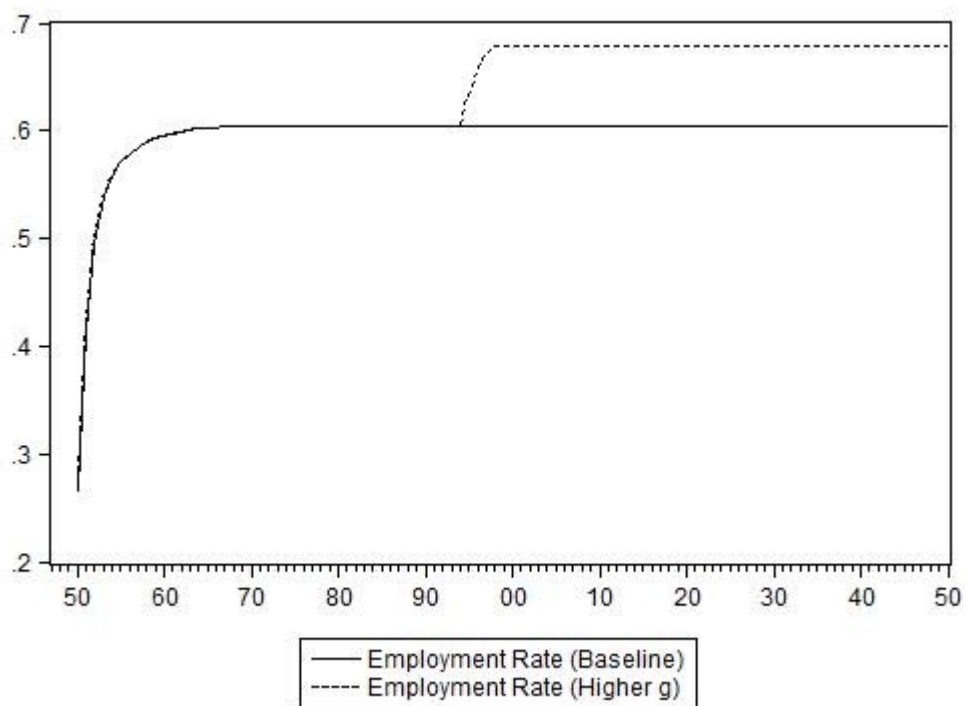


Figure 5

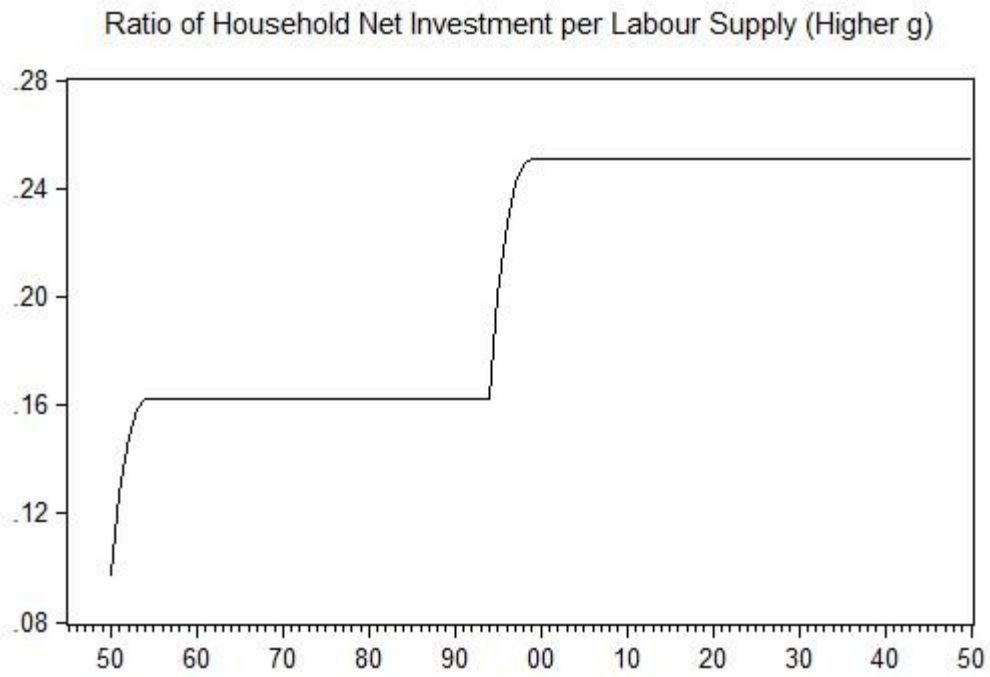


Figure 6

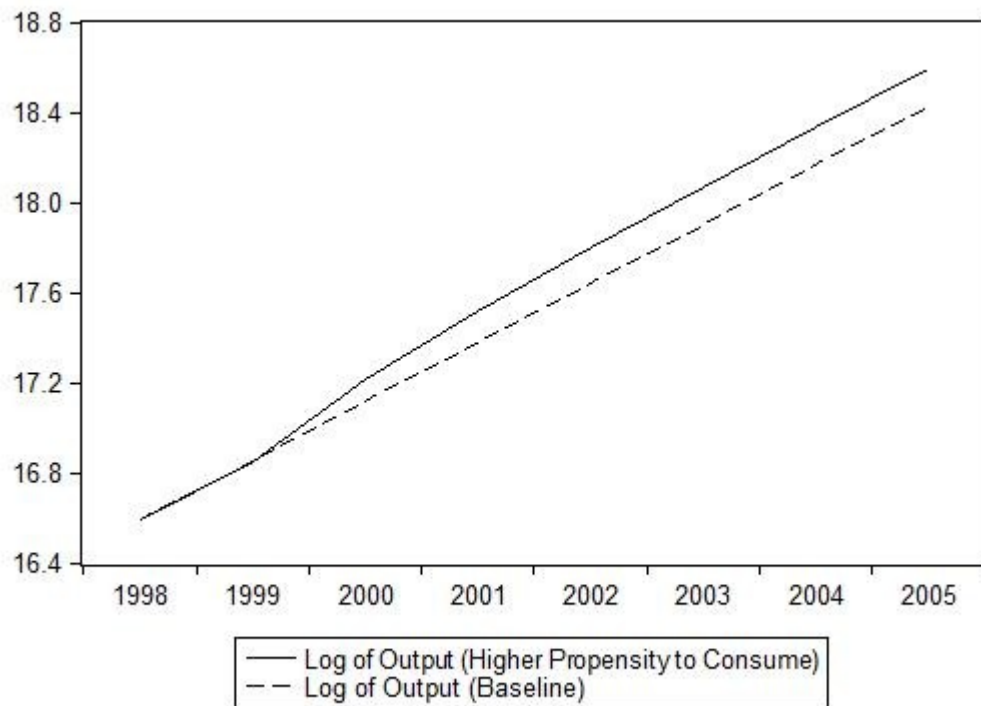


Figure 7

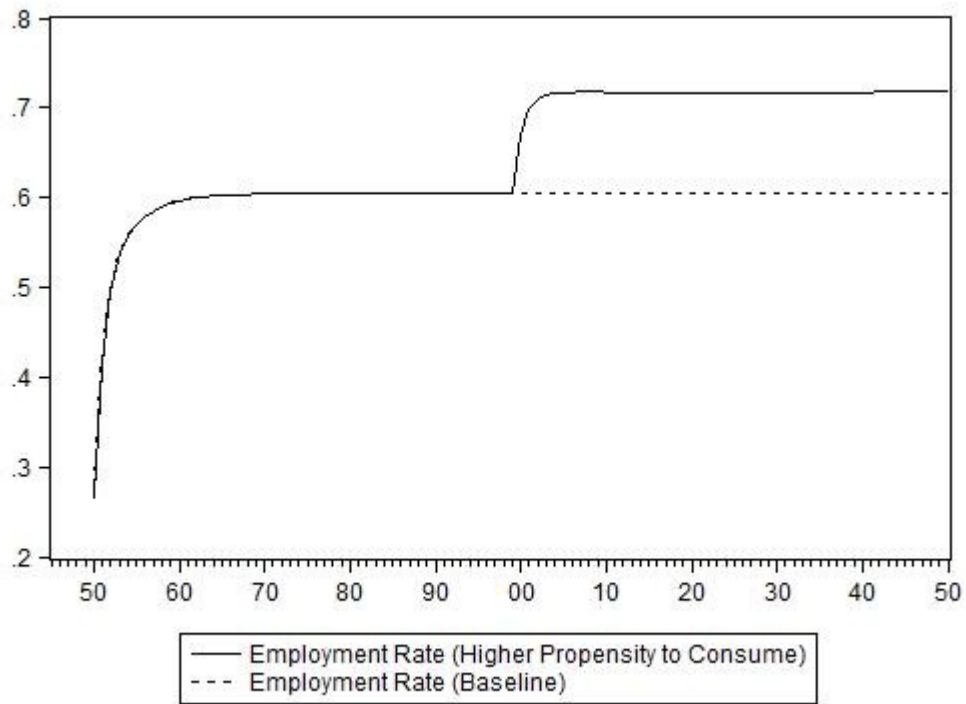


Figure 8

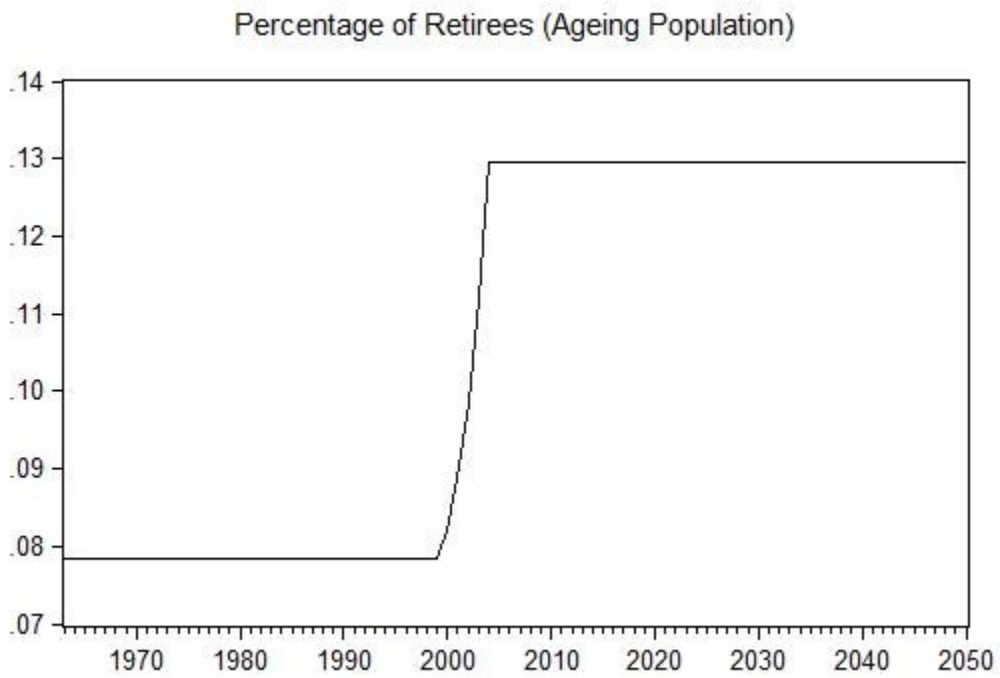


Figure 9

