In his masterpiece, *The Elements*, Euclid laid down the foundations of the geometry of space by starting from a handful of self-evident axioms and postulates. One of these postulates, now known as the parallel postulate, states that for a given line $l$ and a point $p$ not lying on $l$, there is precisely one line through $p$ that is parallel to $l$. While the parallel postulate appears to be intuitively true, mathematicians after Euclid were uncomfortable with the fact that the parallel postulate was not as self-evident as the other postulates. Indeed, there were countless failed attempts to derive the parallel postulate as a consequence of the other axioms. It was only in the 19th century that European mathematicians began to entertain the idea of a geometry that did not satisfy the parallel postulate. Such investigations eventually led to the invention of hyperbolic geometry - a geometry that fails to satisfy the parallel postulate and yet is internally consistent. Recent developments have produced many remarkable and deep connections between hyperbolic geometry and the theory of abstract mathematical objects known as groups. In particular, there are fascinating ways in which certain algebraic gadgets known as modular groups can be used to study hyperbolic geometry.

**ABSTRACT**

We begin the fundamental domain for the action of the modular group on the upper half-plane. This fundamental domain can be defined as

$$ \mathcal{U} = \{ \tau \in \mathbb{H} | \text{Im}(\tau) > 1, \text{Im}(\tau) > 1 \} $$

The set $\mathcal{U}$ is geometrically represented as the shaded region in the following diagram.

Using the familiar technique of integration by parts, we can compute this area. This might seem a bit surprising at first considering the fact that the shaded area appears to be unbounded and intuition tells us that the area should thus be infinite. However, just as we have a different notion of line and length in hyperbolic geometry, so too do we have a definition of area that varies from the ordinary Euclidean conception. Using the same method, we can actually compute areas of fundamental domains of different congruence subgroups of the modular group. The goal of these computations was to detect patterns that may underline the areas of the congruence subgroups. To carry out the computations, it was first necessary to visualise the fundamental domains in the upper half-plane. The Java applet Fundamental Domain Drawer, developed by H.A. Verrill at Louisiana State University, proved to be a very effective tool for this purpose and a few of the results produced by the applet are shown below.

Although the computations of the fundamental domains were not difficult, it was hard to detect patterns in their areas. The initial hypothesis that there are connections between the areas of fundamental domains of congruence subgroups and the Riemann zeta function is supported in the paper by Cazacu and Ghisa, where the authors use much broader techniques and theorems to establish some remarkable properties concerning fundamental domains. However, for the present, there are not many results apart from the calculations concerning the domains.

**REFERENCES**


**ACKNOWLEDGEMENTS**

I would like to thank Professor Hadi Salmassian for the invaluable guidance he has provided throughout the project and I would especially like to acknowledge his role as a supportive and understanding mentor. I am also grateful to the University of Ottawa and the Undergraduate Research Opportunity Program for providing me with the opportunity to pursue the research I have undertaken.