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Algorithms for Survivable Routing and Wavelength Assignment in
WDM Wavelength-Routed Mesh Networks

By
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To the memory of my father Eswei Eshoul and my mother Aesha Hablous
Abstract

Due to the huge amount of data that can be lost and the large number of users that can be disrupted during the times of failure in wavelength division multiplexing (WDM) optical networks, reliable optical layer that can quickly and efficiently respond to failures such as fiber cuts is a critical issue to users and service providers. The major challenge in survivable mesh networks is the design of resource allocation algorithms that allocate network resources efficiently while at the same time are able to recover from a failure quickly. This issue is particularly more challenging in optical networks operating under wavelength continuity constraint, where the same wavelength must be assigned on all links in the selected path. This thesis presents a number of schemes to solve the problem of survivable routing and wavelength assignment (RWA) in wavelength-routed WDM networks. The focus is on developing algorithms for survivable RWA in mesh networks that address both resource efficiency and restoration speed under static and dynamic traffic environments. Two different approaches (diverse routing and p-cycles) are pursued to solve the problem of survivable RWA. Their performances and complexities are also compared. Under static traffic, new algorithms have been developed to formulate the RWA as an Integer Linear programming problem using both diverse routing and p-cycles. Under dynamic traffic, new heuristic algorithms are developed to solve the survivable RWA using both approaches and their performances are evaluated and compared using simulations.
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Acronyms

BLSR  Bidirectional Line Switching Rings
ER   Efficiency Ratio
FF   First Fit
FWC  Full Wavelength Conversion
ILP  Integer Linear Programming
ITSA Iterative Two Step Approach
LBR  Load balance Routing
LP   Linear Programming
MBCF Minimum Backup Capacity First
NP   Nondeterministic Polynomial
NSF  National Science Foundation
NWC  No Wavelength Conversion
OLPBC Off Line Planning of Backup Capacity
QoS  Quality of Service
RRBC Realtime Reconfiguration of Backup Capacity
RSE  Route Sensitive Efficiency
RSS  Route Sensitive Score
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<tr>
<td>RWA</td>
<td>routing and wavelength assignment</td>
</tr>
<tr>
<td>s-d</td>
<td>source-destination</td>
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<tr>
<td>SAD</td>
<td>Survivable Algorithm for Dynamic RWA</td>
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<tr>
<td>SLPR</td>
<td>Straddling Links Priority Routing</td>
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<tr>
<td>SP</td>
<td>Shortest Path</td>
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<tr>
<td>SRLG</td>
<td>Shared Risk Link Group</td>
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<tr>
<td>TE</td>
<td>Topology Efficiency</td>
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<tr>
<td>TRE</td>
<td>Traffic Routing Efficiency</td>
</tr>
<tr>
<td>TRS</td>
<td>Traffic Routing Score</td>
</tr>
<tr>
<td>TS</td>
<td>Topology Score</td>
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<tr>
<td>VWP</td>
<td>Virtual Wavelength path</td>
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<td>WDM</td>
<td>Wavelength Division Multiplexing</td>
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<td>WP</td>
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<td>WRON</td>
<td>Wavelength Routed Optical Networks</td>
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\( A_e \) summation of all the primary lightpaths on link \( e \),

\( B_{tk}^{dw} \) objective function variable to represent the \( d^{th} \)
backup route to protect the \( k^{th} \) primary route of
the \( i^{th} \) s-d pair on wavelength \( \omega \),

\( C_e \) the cost of using a wavelength on link \( e \),

\( D \) maximum number of backup path candidates for
each primary path candidate,

\( E \) number of directed links in the network,

\( K \) maximum number of primary candidate routes,

\( L_{ij} \) directed link from node \( i \) to node \( j \),

\( M \) number of selected unidirectional cycles to be used
in the formulation,

\( N \) number of nodes in the network,

\( P_{tk}^{w} \) objective function variable to represent the \( k^{th} \) pri-
mary route of the \( i^{th} \) s-d pair on wavelength \( \omega \),
\( P_{ik} \) objective function variable used to represent the number of channels assigned to the \( k^{th} \) primary route of the \( i^{th} \) s-d pair (used for FWC in the p-cycle approach),

\( Q \) number of s-d pairs,

\( S_{e} \) summation of the backup lightpaths on link \( e \),

\( W \) link capacity,

\( W_{e} \) number of channels on link \( e \),

\( W_{ik} \) the higher index of the wavelengths range which the lightpaths between the \( i^{th} \) s-d pair are allowed to use on the \( k^{th} \) primary route,

\( X_{j} \) objective function variable that represents the number of channels required on p-cycle \( j \) (used for FWC in the p-cycle approach),

\( X_{j,\omega} \) binary objective function variable (used for NWC) for the p-cycle approach,

\( Z \) number possible unidirectional cycles,

\( \Lambda_{i}^{e} \) number of channels assigned to the \( i^{th} \) s-d pair on link \( e \),

\( \Lambda_{i} \) number of channels requested by the \( i^{th} \) s-d pair,

\( \Phi_{i}^{e} \) indicator function which takes the value of 1 if the \( i^{th} \) s-d pair uses the link \( e \) on its working path; otherwise it takes the value of 0; only used for the p-cycle approach,
\( \Psi_{je} \) indicator function which takes the value of 1 if the 
\( j^{th} \) cycle pass through link \( e \); otherwise it takes the 
value of 0; used only for the p-cycle approach,

\( \chi_{ik}^{e} \) indicator function that takes the value of 1 if the 
\( k^{th} \) primary route of the \( i^{th} s-d \) pair passes through 
link \( e \); otherwise it takes the value of 0,

\( \varphi_{ik}^{de} \) indicator function that takes the value of 1 if the 
\( d^{th} \) backup route to protect the \( k^{th} \) primary route 
of the \( i^{th} s-d \) pair passes through link \( e \); otherwise it 
takes the value of 0,

\( \varphi_{je} \) indicator function which takes the value of 1 if the 
\( j^{th} \) cycle protects the \( e^{th} \) link; otherwise it takes 
the value of 0,

\( \xi_{e}^{\omega} \) objective function variable in the shared protection 
formulation,

\( b_{ik} \) number of routes to backup the \( k^{th} \) primary path 
of the \( i^{th} s-d \) pair,

\( d \) backup path index,

\( e \) Link index,

\( i \) s-d pair index,

\( j \) cycle index,

\( k \) primary path index,

\( p_{i} \) number of primary paths between the \( i^{th} s-d \) pair,
$w_{ik}$ the lower index of the wavelengths range which the lightpaths between the $i^{th}$ $s-d$ pair are allowed to use on the $k^{th}$ primary route,
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2 National Science Foundation (NSF) network

3 The ARPANET network topology

4 Test network

5 Flow chart for cycles generation
Chapter 1

Introduction

1.1 Background

For a source and destination \((s,d)\) pair to communicate in wavelength-routed optical networks (WRON), a lightpath in the optical layer between the two nodes must be set up. Lightpath establishment, also known as Routing and Wavelength Assignment (RWA), is accomplished by selecting a route between the two end nodes and assigning a suitable wavelength. The aim of the RWA process is to find routes and assign wavelengths for connection requests in a way that minimizes the consumption of network resources, while at the same time ensuring that no two lightpaths are assigned the same wavelength on a shared link. Furthermore, in networks with no wavelength conversion capability (NWC), a lightpath must be assigned the same wavelength on all the links along its path, a constraint known as wavelength continuity constraint.

The traffic applied to wavelength-routed WDM networks is mainly confined to two types: static traffic and dynamic traffic. Under the static traffic environment, all connection requests are known in advance, so it is required to set up all the required
lightpaths while at the same time optimizing a certain objective such as minimizing the number of wavelengths needed. On the other hand, under the dynamic traffic environment, connection requests arrive at and depart from the networks at random times. So the objective of the dynamic RWA algorithm is to minimize the blocking rate of connection requests.

A node failure or a fiber cut in WRON can cause the breakdown of all lightpaths that traverse the failed node or broken link. Due to the huge amount of data that can be lost and the large number of users that can be disrupted as a result of a fiber cut or node failure, network survivability has become a key issue in the RWA process. Network survivability requires the protection of lightpaths against failures by reserving spare capacity during the connection setup and the restoration during which the spare capacity is utilized upon the occurrence of a failure. The prime objective of most survivable routing algorithms is to minimize the consumption of network resources and reduce the restoration time during a failure.

There are two different approaches to the survivability in mesh networks [GDC+02] [GR00][Qia02]: the diverse routing approach and the p-cycle approach. Using the diverse routing approach, in addition to setting up a working lightpath (primary lightpath) to carry traffic during the normal operation, a backup lightpath is also required to carry traffic in case the primary lightpath fails. The routes and the wavelengths of the primary and backup lightpaths are selected during the RWA process. The routes of the working and the backup lightpaths must be link-disjoint in order to protect against fiber cut or node-disjoint to protect against node failure. Based on the rerouting choice, diverse routing protection schemes can either be link-based, where the traffic is rerouted around the end nodes of the failed link, or path-based, where a backup lightpath is pre-determined between the source and the
destination nodes. Furthermore, diverse routing protection schemes are classified based on the possibility of resource sharing as dedicated protection and shared protection. Dedicated protection schemes have fast restoration times at the expense of higher resource redundancy. In contrast, shared protection schemes reduce resource redundancy significantly at the expense of increased restoration time. Resource redundancy is the ratio of total spare capacity to the total working capacity in a network.

On the other hand, the p-cycle approach is link-based and can only be applied to shared protection schemes. P-cycles are pre-configured protection cycles formed in the spare capacity of the network. In order for a mesh network to be efficiently protected against any single span failure, one or more p-cycles, which may overlap with each other, are required. The major advantages of P-cycles protection schemes over the diverse routing protection schemes are their ability to achieve both good resource efficiency and fast restoration times simultaneously.

1.2 Motivation and Objectives

The current skyrocketing demands for bandwidth have resulted in the transition from point-to-point WDM transmission lines to all-optical backbone networks. Motivated by the recent advancement and evolution of optical technology, the objective of the thesis is to contribute to the ongoing research and developments in the area of RWA for wavelength routed WDM mesh networks. This thesis focuses on developing algorithms for RWA in mesh networks that address survivability, capacity efficiency, and restoration speed under static and dynamic traffic environments. The way that the RWA is solved can play a significant role in improving the efficiency and the reliability of the network. Increasing network efficiency enables service providers
to accommodate more lightpaths and reduce blocking in times of congestion. Furthermore, providing a reliable lightpath in the optical layer that can respond very quickly to failures, such as fiber cuts, is a crucial issue to users and service providers, due to the huge loss of data and the number of users that can be disrupted during times of failure.

Optimal solutions to the survivable RWA problems under static traffic require the use of integer linear programming (ILP) [MHZW06][WVV02] [SGA02][KS01b]. However, the complexity of formulating survivable RWA as an ILP problem increases with the size of the network. The number of variables that are used to formulate the RWA problem is one of the contributing factors to its complexity. Other parameters such as the constraints equations combined with the number of variables give a more accurate assessment of the problem complexity. The problem can be simplified by carefully dividing it into smaller sub-problems without affecting the optimality of the final solution. However, this optimality depends solely on how these sub-problems are solved.

Using diverse routing, survivable RWA can be divided into the routing and the wavelength assignment sub-problems. On the other hand, when using p-cycles, the survivable RWA problem can be divided into routing the working lightpaths and finding the minimum spare capacity of backup p-cycles. The overall problem can then be solved either jointly, where the working and backup capacities are formulated together or non-jointly, where each problem is solved separately. In all cases, the objective is to simplify the complexity of the survivable RWA problem while at the same time increasing the chances of generating the optimum solution.

The problems of solving the survivable RWA under dynamic traffic lie on the random arrival and departure of connection demands, as well as the difficulties to
1.2. MOTIVATION AND OBJECTIVES

rearrange the connections of existing lightpaths during the setup of a newly arriving request, due to the QoS requirements. Therefore, several important issues must be addressed during the design of survivable RWA algorithms. The dynamic nature of the traffic is one of the constraints which needs to be considered, because it severely limits the complexity of the algorithm. Another issue which needs to be addressed is scalability which includes the size of the network and the characteristics of the traffic.

Using diverse routing techniques to solve the survivable RWA problem requires finding the working and backup paths for each individual connection demand as it arrives at the network. Known algorithms, such as the iterative two step approach [HM04b], have only considered survivable RWA in networks with full wavelength conversion capability. However, to find the optimum working and backup paths under the wavelength continuity constraint, the entire wavelength space may have to be searched during the calculations of both working and backup paths. Therefore, the objective is a tradeoff between increasing the efficiency of the network by reducing the blocking probability and simplifying the complexity of the algorithm.

The research in the area of dynamic RWA using p-cycle has not matured and has a long way to go yet. This research investigates the different options of using p-cycle in survivable RWA. The difficulties lie on the inability to solve the RWA jointly (routing the working demands and configuring the backup cycles). The dependency between routing the working paths and configuring the necessary p-cycles further complicates the problem. The possibility of re-optimizing the locations and the number of p-cycles to adapt to changes in demand and its impact on the blocking performance is thoroughly investigated.
1.3 Thesis Contributions

To address the above mentioned objectives, the thesis is divided into three parts. In the first part, the problem of survivable RWA is solved using diverse routing techniques and the contributions and accomplishments are the following:

- A new algorithm has been developed to solve the RWA under static traffic using ILP. To increase the chances of obtaining the optimum solution to the RWA problem while keeping the complexity of the problem under control, the algorithm generates the most likely candidate routes for the primary and their corresponding backup lightpaths. The number of candidate routes used in the formulation of the problem can be selected to compromise between the complexity of the problem and the optimality of the solution. The inputs to the algorithm are the set of demands and the network topology on which the demands are to be realized. The output of the algorithm is an ILP problem formulated and ready to be solved by any CPLEX solver. The algorithm can be used to formulate dedicated or shared protection schemes in mesh WDM networks.

- A new heuristic scheme has been introduced to solve the survivable RWA for shared protection using diverse routing under dynamic traffic. The scheme is called survivable algorithm for dynamic RWA (SAD-RWA). The scheme finds the two least costly link-disjoint lightpaths by searching the entire wavelength space. The wavelength continuity constraint is satisfied for both the working and backup lightpaths. To reduce the computational complexity, the first fit FF scheme has been proposed for wavelength assignment.

- A simulation tool has been developed to evaluate the performance of the above
mentioned algorithm and compare its performance with other algorithms using p-cycle techniques.

The second part of the thesis addresses the problem of survivable RWA using p-Cycle techniques, and the contributions include the following:

- A new ILP formulation algorithm for the survivable RWA in mesh networks under static traffic has been proposed. To reduce the number of candidate cycles in the formulation, a new metric -Route Sensitive Efficiency (RSE)- has been introduced to pre-select a reduced number of high merit cycle candidates. The RSE ranks each cycle based on the relative cost of each primary candidate route that it can protect. The inputs to the algorithm are the set of demands and the network topology on which the demands are to be realized. The algorithm first generates all possible cycles in the network and calculates the primary route candidates for each s-d pair. It then sorts the cycles based on the RSE ranking and selects the top ranked subset of cycles to be used in the formulation. The cardinality of the subset is a tradeoff between the complexity of the problem and the optimality of the solution. The output of the algorithm is an ILP problem formulated in a way that is acceptable to any CPLEX solver. The new algorithm is evaluated and compared with other previously developed algorithms.

- To further reduce the complexity of the problem, a new formulation procedure is developed. The idea is to first find the minimum and sufficient backup capacity against any single span failure. Then the working lightpaths problem is formulated as an ILP problem. However, this necessitates the introduction of new rules for the routing algorithm to avoid generating useless routes because of the conflicts between the working routes and the backup p-cycles, due to
the wavelength continuity constraints. A new constrained K-shortest route
algorithm is introduced to generate the candidate primary routes to be used
in the \textit{ILP} model.

- Two schemes to solve the \textit{RWA} using p-cycles have been proposed. The first
  scheme is based on \textit{Real-time Reconfiguration of Backup Capacity (RRBC)}
presented in [ZZ05]. The idea is to reconfigure the existing backup capacity
whenever necessary without the disruption of working lightpaths in progress.
The second is based on the work presented in [HFS05]. The idea is to plan
the backup capacity off-line (\textit{Off-line Planning of Backup Capacity OLPBC})
by reserving the minimum backup capacity to ensure full protection against
any single span failure. As a result, the only necessary real-time action is to
find a working route for the newly arrived demand from the available working
capacity.

- To improve the blocking performance of the \textit{OLPBC}, a new routing strategy
called \textit{Straddling Links Priority Routing (SLPR)} is introduced. \textit{SLPR} takes
advantage of the full link capacity that is available in the straddling links for
the working lightpaths. The idea to encourage the use of straddling links in
the working lightpaths by assigning them a lower cost than the on-cycle links.

- A simulation tool has been developed to evaluate and compare the perform-
ances of the two schemes (\textit{RRBC and OLPBC}) together with a number of
routing schemes for the working paths.

Finally, a comparative study has been conducted to compare the performances of
the diverse routing algorithms and the p-cycles techniques.
1.4 Thesis Outline

The rest of the dissertation is organized as follows. In the next chapter, a brief description of the various approaches to achieve survivable RWA in WDM networks is presented. Then an overview of the significant previous research related to the RWA in optical networks is given. An algorithm to formulate survivable RWA as ILP problem under static traffic using diverse routing for both dedicated and shared protection schemes is presented in Chapter 3. Chapter 4 presents the proposed SAD-RWA algorithm for dynamic RWA using diverse routing. Also presented in this chapter is the performance evaluation of the RWA algorithm on several networks topologies. Chapter 5 investigates the use of p-cycle techniques in survivable RWA under static traffic. The chapter starts by presenting the problem and outlining the different solution options. It then presents the formulation of the static survivable RWA using p-cycle as an ILP problem for the joint and non-joint approaches. The chapter also proposes a new ranking procedure to select a subset of high merit cycles to simplify the complexity of the joint approach problem and increase the chances of obtaining the optimum solution. Chapter 6 introduces two ways to configure p-cycles under dynamic traffic and compares their performances. A new scheme for routing working lightpaths is also presented and its performance is compared with the performances of known schemes. In Chapter 7, a comparative study to compare the performances of diverse routing and p-cycles in survivable RWA under static and dynamic traffic is presented. Chapter 8 concludes the thesis and proposes some future research work.
Publications


Chapter 2

\textit{RWA in Mesh Networks}

2.1 Introduction

The aim of the \textit{RWA} process is to find routes and assign wavelengths for connection demands in an efficient manner. Furthermore, the \textit{RWA} algorithm must ensure that no two lightpaths are assigned the same wavelength on a shared fiber link. Additionally, in the lack of wavelength converters, a lightpath must assign the same wavelength to all the links in its path, a condition known as the wavelength continuity constraint. Therefore, effective algorithms for the \textit{RWA} process in wavelength routed WDM networks are critically important for utilizing network resources efficiently. Numerous research studies have investigated the \textit{RWA} problem under two different traffic environments: static and dynamic.

2.2 \textit{RWA} under Static Traffic

Under the static traffic environment, all connection requests are known in advance, so the typical objective is to set up all the required lightpaths while at the same time
minimizing the number of wavelengths needed. The optimal solution to the RWA problem for small to moderate size networks under static traffic is best obtained using ILP tool. One way to formulate the RWA problem is to use the multi-commodity flow technique, where the demand of each s-d pair is considered a different commodity. Moreover, a unique variable must be used to represent the flow of each commodity along each directional link on each wavelength. As a result, the number of variables required to formulate the RWA, which represents the size of the problem, increases with the size of the network. The size of the problem depends on the number of links in the network, the number of lightpaths and the number of wavelengths per link. The inputs to the RWA problem are a set of connection demands between the various s-d pairs and the network topology which is represented by a number of nodes interconnected by a set of unidirectional links. The ILP finds routes and assigns wavelengths to the given set of lightpaths in the most optimal way subject to various constraints such as flow, capacity and wavelength continuity constraints.

Due to the NP completeness of the RWA problem [ZJM00], this problem may be simplified by partitioning it into two sub-problems: the routing sub-problem and the wavelength assignment sub-problem. Although partitioning the problem decreases the number of variables required, it does not always guarantee the optimum solution. For example, the solution obtained by partitioning the problem may require more resources (larger number of wavelengths) than does the optimum solution to satisfy a certain demand matrix. The main reason lies on the reduced search space which restricts lightpaths to pass through a limited subset of links as dictated by the routing tables. As a result, during the optimization process, some of the possible routes are not considered as part of the search space. Consequently, if the excluded routes are part of the optimum solution, the generated solution will not be
optimum. Several heuristic algorithms have been proposed to solve more complex RWA problems under static traffic [MPRC00][ZCM03].

2.3 RWA under Dynamic Traffic

Driven by the rapid growth of the Internet, the increasing demand and the nature of traffic, WDM is now beginning to expand from a network core technology towards the metropolitan and access networks. It is envisioned that fiber will be extended to homes and small businesses, thereby making efficient use of the increasing number of wavelengths available, so dynamic lightpath establishment is essential to respond quickly and economically to customer demands.

Dynamic RWA involves the setup and tear down of lightpaths in real-time with the objective of reducing blocking rates. As a result, one of the challenges confronting wavelength routed networks with dynamic traffic is the complexity of developing efficient techniques for establishing lightpaths. In order to reduce connection blocking and utilize network resources efficiently, the routing and wavelength assignment decisions must be made based on the latest network state information such as traffic congestion and wavelength usages. Moreover, due to the QoS requirements, the RWA is not only required to utilize network resources efficiently during the setup of lightpaths for newly arriving demands, but it is also required not to interrupt lightpaths that are in progress. These constraints dictate the optimization of the RWA for each individual connection request separately as it arrives with the objective of allocating minimum bandwidth based on the network status at the time of the arrival.
2.4 Survivable RWA

A fiber cut or a node failure in WDM network can cause the loss of a huge amount of data and the disruption of a large number of users. As a result, network survivability has become a key issue during the RWA process. In addition to the setup of a working lightpath (primary lightpath) to carry traffic during the normal operation, network survivability requires the setup of a backup lightpath to carry traffic in case the primary lightpath fails. The working lightpath and the backup lightpath must be link-disjoint in order to protect against fiber cut or node-disjoint to further protect against node failure. However, due to the internal redundancy, most researchers assume WDM nodes to be very reliable. Consequently, researchers have put more emphasis on the protection against link failures. Furthermore, due to the remote probability of dual failures, most of the research in the area of protection and restoration, has only considered single failure scenarios. However, for larger networks, where dual failures are much less probable than single failures, dual failures have become a real concern in the last few years [Sch03b][WS03][DG05].

Based on the rerouting choice, protection schemes can be either link-based, where the traffic is rerouted around the end nodes of the failed link or path-based where a backup lightpath is pre-determined between the source and the destination nodes [RM99a; DS00]. Figure 2.1 shows how the two schemes work in a scenario where a failure occurs in the intermediate link 2-4 in a lightpath between nodes 1-6. The figure clearly shows the advantage of path based protection over link based protection in resource efficiency. However, the restoration time is significantly longer in path-based protection depending on the location of the failure relative to the end nodes. Furthermore, protection schemes are classified based on the possibility of resource sharing as dedicated protection or shared protection.
2.4.1 Dedicated Protection

Dedicated protection requires the configuration of both the working and backup paths for each request. In this manner, the resources along the backup path are dedicated for this request and it cannot be shared with other backup paths of other requests. The fastest restoration time is achieved in the case of 1+1 dedicated protection, where data is transmitted simultaneously along both paths and the destination node can choose the better of the two. Although dedicated protection provides fast restoration time, the ratio of redundancy of the backup paths to the working paths is at least 100%.

2.4.2 Shared Protection

Shared protection schemes allow resource sharing among several backup lightpaths as long as their corresponding working paths are not in the same risk link group (SRLG). Shared protection schemes significantly reduce resource redundancy at the expense of increased restoration time. The restoration period depends on the
2.4. SURVIVABLE RWA

signaling protocols used to execute the restoration process as well as on the location of the failure relative to the end nodes. The objectives of shared protection schemes are the following:

1. to minimize the capacity redundancy, by minimizing the backup resources required for given demand matrix.

2. to minimize the restoration time, by reducing the time required to notify failure and activate restoration path.

Achieving both objectives simultaneously is the ultimate goal.

As Chapters 3 and 4 will demonstrate, solving the RWA for shared protection scheme is more complicated (even in the case of static traffic) than it is for dedicated protection scheme. The complication is mainly due to the resource sharing constraint. Resource sharing implies that the cost of choosing a link for the protection path of a request depends on the physical location of its working path. Moreover, a wavelength in a link may protect many different lightpaths if they do not share a common link in their corresponding working paths. Under dynamic traffic, the RWA problem for the shared protection is decomposed into two sub-problems: The routing sub-problem and the wavelength assignment sub-problem. However, the routing sub-problem is not straightforward due to the fact that the link state during the calculation of the backup path differs from that used to calculate its corresponding working path. Consequently, known algorithms to find the two disjoint paths such as Dijkstra and Suurballe [ST84] can not be applied.
2.4.3 Survivability using p-Cycles

A new paradigm for mesh restorable networks is the cycle oriented pre-configuration of spare capacity. The idea, first proposed in 1998 by D. Grover and D. Stamatelakis [GS98], is now widely known as P-cycle. P-cycle is a pre-configured protection cycle formed in the spare capacity of the network. In order for a mesh network to be efficiently protected against any single span failure, one or more p-cycles, which may overlap with each other, are required. The interesting and exciting feature of the P-cycles is their ability to achieve both resource efficiency and fast restoration time simultaneously. This feature constitutes the main objectives of shared protection scheme in mesh networks. Until the development of p-cycles, it was widely recognized that one objective (restoration speed or capacity redundancy) can be optimized on the expense of the other. P-cycle can retain the capacity efficiency of mesh restorable networks while at the same time providing the speed and switching simplicity of bidirectional line switching rings (BLSR).

The main advantage of p-cycles over conventional ring networks in terms of the

![Diagram of p-cycles](image)

Figure 2.2: Basic features of p-cycles
capacity efficiency lies on their ability to protect against straddling spans failures as well as the usual ring protection against the on-cycle spans failure as shown in Figure 2.2b. Straddling spans, shown in Figure 2.2a, are those spans which have their end-nodes on the cycle, but are not part of the p-cycle. Furthermore, in the case of straddling span failure, the restoration path can be routed in both directions around the p-cycle as shown in Figure 2.2b. Also, unlike conventional rings, p-cycles allow the routing of working paths using the shortest path or any other routing scheme.

The improvement of restoration speed over other mesh restoration schemes is derived from the reduced signaling and activation required along the restoration path which is essentially the same as in the BLSR. As Figure 2.2b shows, on the occurrence of a span failure, only the end nodes of the failed span are required to do any real time switching and no switching actions are required by the intermediate nodes along the restoration path. The fundamental design problems in p-cycle based networks lie on the optimum selection of working and protection paths, the complexity of the algorithms used to efficiently implement the design, and the flexibility of these algorithms to adapt to changing demands. Solving the problem under static traffic requires finding the most efficient p-cycles in a topology to satisfy a given set of demands. However, dynamic traffic may also requires that the locations and the number of p-cycles should be easily re-optimized to adapt to changes in demand.

2.5 Related Research on the RWA

2.5.1 RWA without Protection

Research on the RWA for WDM wavelength-routed mesh networks has come a long way over the past few years evidenced by the recent publications in the area. Under
static traffic environment, where all connection requests are known in advance, the
typical objective of the RWA process is to set up all the required lightpaths while
at the same time minimizing the number of wavelengths needed. Researchers in
[KS01b] solved the problem of logical topologies over wavelength-routed optical net-
works by formulating it as an optimization problem. In addition to the wavelength
continuity constraint, the formulation also took into account the maximum number
of hops a lightpath is allowed to take with the objective of minimizing congestion.

Researchers in [ZJM00] solved the RWA problem by formulating it as Integer
Linear Programming problem (ILP). The objective is to minimize the flow in each
link by minimizing the number of lightpaths passing through each link. This ap-
proach is used to find the minimum number of wavelengths required to establish a
certain set of connection requests, by performing an iterative search starting with a
small number of wavelengths. If a solution is not feasible, the number of wavelengths
is incremented and the procedure is repeated until a feasible solution is found. The
approach uses more flow conservation constraint equations than actually needed by
considering each individual lightpath separately. The number of flow conservation
equations required can be reduced by considering the number of lightpaths between
each s-d together as one commodity which may also decrease the number of vari-
ables required to formulate the problem. The only variables that are necessary to
formulate the problem are the flow variables $F_{ek}^\lambda$ which represent the flow in link $e$
from the request $k$ on wavelength $\lambda$.

Due to the NP completeness of the RWA problem, the paper proposes partition-
ing the RWA problem into two sub-problems: The routing sub-problem and the
wavelength assignment sub-problem. The routing sub-problem is solved by formu-
lating it as ILP, and it is further simplified by reducing the search space in which
only a limited subset of links are considered. Although the idea reduces the complexity of the problem, the generated solution is not always optimum.

Similar to [ZJM00], the work in [KS01a] presents joint ILP formulations for the RWA problem to maximize the number of lightpaths that can be setup given a set of wavelengths per fibre. The paper also gives a brief description of an algorithm used to approximate an integer solution to the problem based on the LP relaxation to the ILP. Researchers in [HMM97][MA98] presented an analytical method to compute approximate blocking probabilities for fixed and fixed alternate routing in addition to investigating the benefits of using multi fibres per link to reduce blocking probability. To improve the performance of fixed-alternate routing, researcher in [HTM02] proposed a capacity balanced alternate routing in which the routing of lightpaths takes advantage of the load-balancing characteristic of the alternate paths. The work in [Esh02][ZJS+01] [LJO+03] provides a thorough review of the control and signalling mechanisms for the RWA under dynamic traffic. They also provide a comparative study between several RWA schemes using distributed control algorithms. The work in [HM02c] presented a routing and signalling protocol called asynchronous criticality avoidance (ACA) for networks operated under fully distributed control environment. The idea of the protocol is to improve the blocking performance by reducing the mutual interference between different lightpaths from different s-d pairs. When the available number of wavelengths between an s-d pair is reduced to a certain threshold, the protocol labels these wavelengths as critical and passes the information to other nodes in the network to avoid assigning these wavelengths as much as possible.
2.5.2 Survivable RWA Using Diverse Routing

2.5.2.1 Under Static Traffic

The authors of [DS00] give a general overview of survivability in optical networks and take a closer look at the techniques employed to achieve survivability. To protect from single failure, numerous protection and restoration schemes have been developed. In [SRM02], researchers consider solving the RWA problem for shared protection by formulating it as ILP. The idea is to partition the problem into two sub-problems: The routing sub-problem and the wavelength assignment sub-problem. The path flow technique is used, where only one variable for each chosen route is used instead of using link flow where a different variable must be used to represent the flow of each s-d pair on each bidirectional link in the network. Moreover, the constraint equations for the flow conservation and wavelength continuity constraint in the intermediate nodes are not required. Although partitioning the problem reduces the complexity of the problem by reducing the number of variables and the number of constraint equations required to formulate the problem, it does not always guarantee the optimum solution. Furthermore, the work in [SRM02] did not show how the alternate routes are determined as they greatly affect the complexity of the problem as well as the optimality of the solution. They have also assumed that the alternate routing table for each s-d pair consists of the k shortest link-disjoint routes, where k=3 may be sufficient for most networks. However, for better results, the choice of k must also take into account the traffic distribution.

In order to make a network survivable against duct failure, researchers in [ZCM01] solve the RWA problem in a WDM mesh network under duct-layer constraints for different path-protection schemes. All links that are buried in the same duct under the ground belong to the same Shared Risk Link Group (SRLG). The work pre-
presented a combined \textit{ILP} formulation for the \textit{RWA} with the shared-path protection scheme. The objective is to minimize the total number of wavelength-links in the network required to establish a given set of connection demands. The formulation simultaneously finds the optimum solution for the routing and wavelength assignment problems. However, the large number of variables required to formulate the problem which exponentially increases the search space to solve it, limits the practical use of this approach to very small-sized networks. In order to make the problem more tractable, the authors in this paper have solved the problem in two steps. In the first, the routing problem is solved by formulating the problem as \textit{ILP} to find the two duct-disjoint paths for each \textit{s-d} pair. In the second step, the wavelength assignment problem is solved by formulating it as \textit{ILP} following exactly the same procedure as that presented in [SRM02]. The solution to the second problem assigns a wavelength to each primary path and a number of backup paths may share a single wavelength on a link subject to the \textit{SRLG} constraint. Limiting the solution of the routing problem to only two duct-disjoint paths significantly reduces the possibility to find the optimum solution to the \textit{RWA} especially when the traffic is not uniformly distributed. In addition to formulating the \textit{RWA} problem as \textit{ILP}, the authors propose a three stage heuristic to solve the \textit{RWA} problem. In the first stage, the heuristic computes two duct-disjoint routes for each source-destination pair and in the second stage, it assigns a wavelength to each path. In the final stage, the heuristic performs an iterative optimization by rerouting some of the paths.

Similar to [SRM02][ZCM01], researchers in [RSM03][RM99a] solve the problem of \textit{RWA} for both dedicated protection and shared protection schemes by partitioning it into two sub-problems. For the routing sub-problem, they assume that a set of alternate routes between each \textit{s-d} pair is known. However, the study provided
no details about the number of alternate routes, the way they are generated and whether they are link-disjoint. Researchers in [OB03] have also formulated the RWA problem as ILP where an integer optimal solution can be obtained in most cases of interest by solving the corresponding relaxed linear programming model using efficient commercial or special purpose simplex methods with fast running times. This result motivates us to consider solving the RWA jointly without having to partition it into smaller sub-problems. Formulating the RWA problem jointly as one problem guarantees an optimal solution (assuming there is a feasible solution).

Researchers in [RM99a] have proposed distributed control protocols for link and path restoration schemes. Numerical results obtained by simulations indicate that while path restoration schemes outperform link restoration schemes in terms of spare capacity utilization, link restoration schemes have shorter restoration time. In order to achieve fast recovery times, the authors of [CHS+04b] have proposed a new protection scheme called sub-path protection. The idea of sub-path protection is to partition the network into several domains and apply path protection under two constraints: Lightpaths within intra-domain that do not use resources from other intra-domains and inter-domain lightpaths that exit and enter other domains through a common egress (or ingress) domain border nodes. The work has also presented a two step ILP formulation to solve the RWA of the sub-path protection in small sized networks, and proposed a heuristic algorithm for larger networks. Results show that while sub-path protection schemes achieved better recovery times than path protection schemes, they are less resource efficient. Furthermore, the inter-domain border nodes must have the capability of wavelength conversion.

To protect network against double link failure scenarios, the research in [WS03] has formulated the problem as an ILP for both dedicated and shared protection
schemes. The idea is to reserve two link-disjoint lightpaths for each working lightpath. In this manner there must be at least 3 link-disjoint paths for each s-d pair and as a result the nodal degree for any node in the network must not be less than three. In this paper, the author proposes to solve the routing problem by generating 3 link-disjoint routes for each s-d pair. However, nothing has been said about how the routing tables are generated. Furthermore, applying dedicated protection against double link failure consumes far too many resources and significantly deteriorates the efficiency of the network. On the other hand shared protection scheme can be applied to protect a network against double link failure with a moderate increase in backup capacity.

2.5.2.2 Under Dynamic Traffic

Under dynamic traffic, survivable RWA implies allocating minimum resources for both the working and backup lightpaths of the arriving request. A number of optimization schemes have proposed the use of ILP to optimize the RWA for each request separately. These schemes guarantee allocating minimum bandwidth for both working and backup paths by jointly optimizing the selection of both paths. However, since the optimization schemes consider one request at a time without interrupting the connections in progress, they can not guarantee the optimum solutions to the RWA for subsequent demands. Furthermore, the solutions of ILP problems, especially for large networks, usually take a long time making them unsuitable for the dynamic traffic. Consequently, numerous heuristic algorithms to solve the RWA under dynamic traffic environment have been proposed.

To reduce the blocking probability, the selection of the working and backup paths must be based on some link state information. Therefore, it is desirable to have
full knowledge about the routing and wavelength assignment of existing lightpaths. However, due to the significant control overhead involved, complete information may not be feasible in all network topologies. Researchers in [KL00][QD02][YWHM04] have addressed three different scenarios: Complete link state information, partial link state information and no link state information. Researchers in [XQX02] have proposed a novel heuristic algorithm to find a pair of disjoint paths for every arriving request. The purpose is to select the working path based on a link metric that takes into account the impact of choosing the working path on the amount of resource sharing which will be available for the backup path.

The studies in [BLRC02][HTM01][HM02a][XYDQ02] [XQX02] inspect the k-shortest paths between each s-d pair. First the k shortest paths are generated (one of them will eventually be used as working path) and then for each one of them, the backup path is derived. Out of the k choices, the most optimum pair (in terms of the total cost of the working and backup paths) is selected. The authors in [HTM04] have proposed the Iterative Two-Step Approach (ITSA) to find the best (in terms of the total bandwidth allocated) working and backup paths for the on line connections based on the link state at the time of the connection request arrival. The algorithm iteratively inspects up to k candidate routes for the working path in an ascending order of their cost. The algorithm invokes the two-step approach, at the beginning of each iteration, to find the optimum backup path for the corresponding candidate working path. The algorithm then ends when the optimality criterion is met or when all k routes are inspected. The algorithm does not always have to inspect all k-shortest routes. For example, the algorithm quits if the cost of the invoked working path is greater than the best total cost of the working and backup paths found from previous iterations.
To decrease the computational complexity, the same researchers proposed the maximum likelihood relaxation scheme (MLR) to improve on computational time of the ITSA. The main idea of the MLR is to use a new link cost to derive the working path. The cost of the working link is considered as a function of the working link cost and the number of links without enough sharable capacity if that link is selected. This link cost encourages having as many links with enough sharable capacity to be considered for the backup path as possible, while at the same time discouraging having working paths with longer routes. However, as the MLR scheme improves the computational efficiency over other schemes proposed to derive the pair of disjoint paths, it cannot guarantee the generation of the optimum pair.

To avoid enumerating the k-routes and overcome trap-topology problem, the work in [CHS+04a] has proposed a new approach (CAFES) to calculate the two link-disjoint paths. The idea of the CAFES is to use backtracking to compute the backup paths in trap topologies. In situations where the backup path can not be computed, CAFES identifies the back haul links, increases their cost to larger values (the sum of the costs of all links in the network) and restarts the two step approach. Moreover, CAFES can be applied to avoid backup-sharing-caused trap, where the backup path can not be found due to the unavailability of spare or sharing capacity. The work also proposes a heuristic algorithm (OPT) to find the optimum pair of working and backup paths.

The research in [BLE+02] has used a stochastic approach to determine channel sharing during the computation of backup paths. The merit of this method is that it has faster computation times than other deterministic methods. The stochastic technique derives its faster computation times from the reduced level of information required to determine the backup path compared to the deterministic method.
where the level of information is proportional to the active lightpaths. The article in [HM04b] gives a general background on the design principals of shared protection in survivable WDM optical networks. The article also elaborates on some of the solutions to the problem of diverse routing for both path-based and segment-based shared protection schemes. Since improving resource sharing may be achieved on the expense of longer backup paths and consequently longer restoration times, researchers in [RBS+01] have examined the relation between the hop distance of the backup paths and resource sharing. Similar research in [BLRC02][XXQ03] have concluded that the hop distance of the backup paths is a tradeoff between restoration times and resource sharing.

The work in [AQ00] has proposed a new heuristic algorithm for dynamic establishment of protection paths under incremental traffic. The concept is to allow the rearrangement of protection paths (both route and wavelength) to accommodate new lightpaths. Upon arrival of a new connection request, the algorithm assigns an available wavelength for the pre-determined working path. The list of available wavelengths includes wavelengths already assigned to existing protection paths. The algorithm assigns the first fit wavelength so that all existing protection paths can still be established even if it is required to rearrange some of them. The work also carried out a comparative study between the static and dynamic establishment of protection paths. The study considers protection against a single link failure using 1:1 path protection. Three different routing strategies have been tested in the study: First is the fixed routing, where a single working and a single protection path are selected for each s-d pair. The second routing strategy is the single working path and multiple protection paths. Last, the multiple working paths and multiple protection paths are examined. Rearranging protection paths do not cause any service
disruption; however, the numerical results obtained by simulations show that the static establishment of protection paths, in which protection paths are allowed to change once they are setup, performs better than the dynamic establishment.

To decrease the restoration time, researchers in [HM02b][SR02] [HTM02] divide the working path into a number of segments, and find the backup path for each segment separately. Since the signals to notify failure and activate restoration only travel within the affected segment, the restoration time can be adjusted to meet certain QoS requirements. The restoration time requirements are guaranteed by adjusting the length of each working path segment and as a result the time to notify failure and activate restoration. Another advantage of this approach is that segmenting the working path increases resource sharing during the generation of the corresponding backup segment, because the SRLG constraint is only imposed on the links belonging to the corresponding segment. However, the computation complexity of the backup paths and the signaling overhead are increased considerably. Furthermore, it imposes the wavelength continuity constraint on the working path and all the backup paths for the different working segments. Moreover, all working paths that are sharing some resources on their backup paths as well as all their backup path segments must be assigned the same wavelength.

To alleviate the wavelength continuity constraint, researchers in [HM04a] have proposed a novel survivable routing scheme called optimal self-healing loop allocation (OSHLA) to use segment protection with partial wavelength conversion. The research in [NM04] investigates the impact of multi-link failure on mesh restoration schemes using analytical and simulation models.
2.5. RELATED RESEARCH ON THE RWA

2.5.3 Survivable RWA Using P-cycle

2.5.3.1 Under Static Traffic

Researchers in [GS98] have presented an ILP formulation for the design of p-cycle based networks. First the set of demands are routed using the shortest path or any other routing scheme and then the set of all simple distinct cycles up to some limiting size is generated from the spare capacity in the network topology. Finally the ILP minimizes the number of copies of each cycle required to achieve full restorability. The formulation does not however take into consideration the wavelength continuity constraint imposed by wavelength-routed optical networks. Furthermore, since the routing of the working paths and the selection of the required p-cycle are optimized separately, the final solution may not be optimal.

Schupke and Gruber [SGA02] have developed optimization models using ILP for the configuration of p-cycles in WDM mesh networks with and without wavelength converters. The models showed high efficiency, especially in the case of networks with wavelength converters, when applied to the pan-European network. First the demands are routed using the shortest route algorithm and then the spare capacity of the network is obtained. Two cost metrics, equal link cost and the reciprocal of free capacity in a link are used for the routing of working paths. The spare capacity in each unidirectional link is calculated by subtracting the total working capacity assigned to all demands on the link from the link capacity. A set of p-cycles is generated from the spare capacity using a breadth first search algorithm. The set of p-cycles is chosen such that every working connection is protected by a p-cycle of corresponding capacity. The routing tables and the p-cycles are then used as inputs to the ILP to find the number of copies of each cycle required for full restorability. In case a set of protecting p-cycle is not possible, the routing of the demands has
to be adapted. Although the model takes into account the wavelength continuity constraint, the optimization problem is divided into two sub-problems which may reduce the efficiency of the final solution. Furthermore, the number of cycles in the list exponentially increases with the number of nodes, the average nodal degree and the traffic distribution which in turn increases the problem complexity.

To reduce the number of candidate cycles in the formulation which effectively reduces the problem complexity, researchers have proposed different ways for pre-selecting a reduced number of high merit p-cycles [GD02][SG03]. Researchers in [GD02] have used two pre-selection metrics for cycle ranking: the topological score ($TS$) and the apriori efficiency ($AE$). The topological score of a cycle is the number of links that the cycle protects in case of failure. The topological score of a cycle grows with the cycle size. In some applications the cycle size may be restricted to a pre-defined number of hops or to pre-defined physical distance. However, the $TS$ and $AE$ metrics select cycles based purely on the topology of the network and do not take into consideration the traffic distribution. As a result, cycles that offer little or no protection to the applied traffic can still be selected while cycles that offer considerable protection to the applied traffic may not. To alleviate this problem, researchers in [SG03] have proposed a new selection strategy where the p-cycles are selected based on the amount of protection they offer to the applied traffic.

Researchers in [ZDM04] have proposed a heuristic method for the design of survivable wavelength-routed networks with p-cycle protection. The method is based on the efficiency ratio ER of each possible unity p-cycle. It first routes all lightpath demands using any efficient routing algorithm and then finds the list of all possible unity cycles in the network topology from the residual capacity. The method then calculates the efficiency score (ER) for each candidate cycle based on the working
links that the cycle can protect. Unlike other selection metrics such as $TS$ and $AE$
described above, the ER score takes into account both the network topology and
the actual working units that are protected. The cycle with the highest ER is se-
lected and the working links that are protected by it are removed. The procedure is
repeated until all working paths are protected and the list of cycles selected is the
required spare capacity p-cycles for the set of demands. However, this algorithm
can be used with full wavelength conversion. Furthermore, even with full wave-
length conversion the algorithm may sometimes use more backup capacity than it
is actually needed and as a result increases the redundancy ratio.

The work in [SG04] investigates the use of Hamiltonian p-cycles in homogeneous
networks. A network is Hamiltonian if it is possible to construct a cycle that en-
ters and leaves every node in the network exactly once. Homogeneous networks,
also known as flat capacity networks, employ equal capacity in each span regard-
less of any future demand anticipation. Although the assumption of homogeneous
networks may not be economically practical, the paper shows that using a single
Hamiltonian p-cycle is the optimal (minimum cost) solution to protect the network
against any single span failure. However, homogeneity restricts the working capacity
of straddling spans to that of the capacity of the p-cycle. On the other hand, the
paper finds that the optimal solution to a capacitated network involves a number of
p-cycles, few of which may be Hamiltonian cycles. The paper derives a lower bound
for the capacity redundancy in the case of homogeneous networks to be $\frac{2}{d}$, where $d$
is the average nodal degree in the network. A better lower bound of $\frac{1}{d-1}$ is derived for
special semi-homogeneous networks, where the straddling spans of the Hamiltonian
p-cycle are allowed to take working loads twice the capacity of the p-cycle.

The research in [Sch03a; Sch03b] focus on multi-failures survivability in WDM
networks with p-cycles. Networks, protected by multiple p-cycles, survive multiple failures provided that at most one failure occurs in any individual p-cycle. The work in [Sch03b] analyzes the dual failures cases within a single p-cycle which can not be survived. In order to reduce the risk of multiple failures in individual p-cycles, the physical length or the hop count of individual p-cycles must be minimized. However, minimizing the physical length or the hop count of individual p-cycle tends to increase the number of p-cycles required to make the network survivable against single failure which in turn increases the required spare capacity. An analysis of the tradeoff between the number of deployed p-cycles and survivability to dual failures is presented in [Sch03b].

Researchers in [GD02; ST04] have presented an ILP formulation to jointly solve the problem of routing the working paths and selecting the required backup p-cycles. Results from both studies show substantial savings on capacity under the joint formulation compared to the non-joint formulation. The work in [GD02] further investigates the impact of the joint optimization on the efficiency of a p-cycle network and to what extent the routing solution deviates from the shortest route solution used in the non-joint formulations. The work in [ST04] also proposed the column generation algorithm which enables the solution of the problem by generating only a fraction of the possible p-cycles.

Shen and Grover [SG03] have extended the p-cycle technique to protect any flow segment along the working path as well as the original span protection. Flow, in this case, is defined as any single contiguous segment of the working path. So, flow could be a single span, a sequence of spans or the entire working path. A cycle can offer protection to flow segments that intersect it. "A path intersects a cycle if the two have at least two common nodes" [SG03]. The work also presented
a different scoring credit which can be used for the cycle pre-selection strategy. Unlike the selection strategy proposed for span protecting p-cycle which take into account only the topological issues of the network, the selection strategy proposed in [SG03] for path protecting p-cycle also takes into account the traffic demand pattern. The paper has also presented an ILP formulation to optimize the spare capacity placement of a flow p-cycle network. It is a non-joint formulation, where the working lightpaths are first routed using any routing algorithm such as the shortest route and then the most optimal path p-cycles are formed from the spare capacity using the formulation. Since it is a non-joint formulation, the solution found may not be the optimal solution. Furthermore, the formulation did not take into consideration the wavelength continuity constraint imposed by wavelength routed networks. To alleviate the wavelength continuity constraint imposed between the working span and the p-cycle used for its protection, researchers in [SSG03] have proposed a strategy for wavelength conversion in WDM p-cycle networks. The work focuses on the tradeoffs between costs associated with wavelength converters and the spare capacity needed for protection. The work in [GDC+02] provides more insights to mesh restorability and how the p-cycle concept performs compared to conventional methods.

2.5.3.2 Under Dynamic Traffic

While most of the research conducted in the method of p-cycle protection dealt with fairly static traffic, with lightpaths being applied for long periods of time, especially in long haul-networks, it is expected that, as the traffic continues to scale up and become more "bursty" in nature, connection requests will become more dynamic, with higher arrival rates and shorter holding time periods. Under such
traffic environments, a higher degree of multiplexing, flexibility and reconfiguration is required in the optical layer. Each intermediate node is required to switch a wavelength from an input port to an output port according to a given routing decision. Researchers in [HFS05] have developed a new p-cycle based method to address mesh networks survivability under dynamic traffic. The method first finds a set of cycles with minimum total length such that the two end nodes of each link in the network appear together at least on one cycle. The paper used ILP to find the optimal sub-set of p-cycles out of the set of all simple distinct cycles in the graph. Half of the capacity on the selected cycles is reserved for protection purposes and the arriving connection requests are set up using the remaining capacity. Three routing schemes are proposed to route the working paths, namely the shortest route routing, the least loaded routing and the most free routing. Their performances have been evaluated by simulation and the numerical results have shown that while most free routing required less capacity than the other two schemes, it has similar blocking performance to the least loaded routing scheme. Although this paper gives a general framework on the application of p-cycle protection in mesh network under dynamic traffic, further investigations are required. For example, the idea has been only tested under incremental traffic where connection requests arrive at the network nodes and stay connected for a very long time. Furthermore, in addition to evaluating the performance using fixed alternate routing method, the idea must also be evaluated using dynamic routing as it improves the blocking performance significantly.

The work in [ZZ05] has considered three different strategies for dynamically configuring p-cycles. The first strategy routes a new request on the residual and backup capacities. The request is accepted if a working path is found and a set of p-cycles to
protect the new working capacity is generated and configured; otherwise the request is blocked. This strategy requires the generation of all possible cycles on the residual capacity after a working path for the new request is found. Furthermore, it requires the selection of an optimum subset of cycles for the backup. All these computations must be performed during the inter-arrival times which restricts the dynamicity of the traffic. To reduce the computational complexity, the second strategy routes the new request on the residual capacity only. If a working path for the new request is found, the algorithm checks whether the existing p-cycles can provide the necessary backup for the new request. If not, new p-cycles are configured and added to the already configured p-cycles. Depending on the dynamic nature of the traffic, the two strategies offer a tradeoff between the computational complexity and the blocking performance. To benefit from the advantages of both strategies, the third strategy employs the first strategy first and if it is not successful, the second strategy is tried.
Chapter 3

Static Survivable \textit{RWA} using Diverse Routing

3.1 Introduction

Optimization is the art of allocating scarce resources for the best possible effect. In network optimization problems, it is required to optimize a certain objective function, such as minimizing cost or maximizing output, subject to a number of constraints. Most network problems usually require finding the optimum solution out of all feasible solutions to the problem. For example, a typical optimization problem is to find the least-cost route out of all feasible routes that satisfies the constraint equations in the network. In Linear programming (\textit{LP}) optimization problems, both the objective function and the constraint equations are linear. Linear programming problems are easily solved using fast techniques such as the Simplex method. Linear programming solvers calculate the value of the objective function at a finite number of points called the feasible corner points and terminate when it reaches optimality.
All network optimization problems considered in this thesis are linear problems. However, they have an added constraint that requires all variables to take integer values. This type of optimization problem is known as Integer Linear Programming problems (ILP). The solution of an ILP problem requires the search for the most optimum integer solution out of all possible integer combinations that the variables can take. As a result of the large search space, solving an ILP problem can take a considerable amount of time depending on the number of variables used in the formulation of the problem. In some cases, the search space becomes prohibitive and other faster heuristic algorithms which can generate sub-optimal solutions are used to solve the problem instead.

Network optimization usually aims to minimize the consumption of network resources, such as bandwidth subject to capacity and demand constraints. Minimizing the number of wavelengths per fiber required to establish a given set of lightpaths is a typical example of the objective of an optical network optimization problem. Optimization techniques are also used to find the shortest path and the k-shortest link-disjoint paths in a network.

### 3.2 Related Theory

The multi-commodity flow approach is employed to formulate most network optimization problems. A multi-commodity network flow problems imply the flow of several different commodities (connection requests in our case) in various parts (links) of the network simultaneously. These flows are subject to some constraints such as link capacity and flow conservation, to name few examples. A multi-commodity network flow problem is defined on a directed graph \( G(N, E) \) where \( N \) and \( E \) are the number of nodes and the number of directed edges in the network respectively.
(links and edges are used interchangeably throughout).

It is important to clarify that link $L_{ij}$ represents the directed link connecting node $i$ to node $j$ whereas, $L_{ji}$ represents the directed link connecting node $j$ to node $i$; so that link $L_{ij}$ is used to transfer flow from node $i$ to node $j$ and link $L_{ji}$ is used to transfer flow from node $j$ to node $i$. In the context of connection-oriented communications networks, different commodities correspond to different connection requests (demands). For the simple circuit-switched networks or optical networks with full wavelength conversion, one only needs to distinguish between the flows of different connection requests inside the network. For example, $F_{ij}^k$ represents the amount of flow from request $k$ in link $L_{ij}$, whereas $F_{ij}^n$ represents the amount of flow from request $n$ in link $L_{ij}$. However, in the more complicated case of optical networks with wavelength continuity constraints, it is also necessary to distinguish between the flows of the same connection request in the same link on different wavelengths. For example, $F_{ij}^{wk}$ represents the amount of flow from lightpath $k$ on wavelength $w$, in link $L_{ij}$. Conversely, $F_{ij}^{ck}$ represents the amount of flow from lightpath $k$ on wavelength $c$, on link $L_{ij}$. Although the flow belongs to the same connection request and it is going through the same link, different variables are used to distinguish between the same lightpath on different wavelengths to ensure the wavelength continuity constraint. Therefore, every lightpath is considered a different flow, and every s-d pair may request an integer number of lightpaths.

Each link in the network has an associated cost $C_{ij}$ of transferring one unit of flow through it. The cost of transferring one unit of flow through a link can vary according to the length of the link, the amount of traffic on the link or it can be constant throughout the network. The maximum capacity on each link in the network is also given. Similar to link cost, link capacity can vary throughout the
network from one link to another, but for simplicity it is assumed to be constant. Finally, the set of demands between each \( s-d \) pair is given in a two dimensional array \((N \times N)\). The entry \( \Lambda_{ij} \) denotes the number of lightpaths that must be set up from source \( i \) to destination \( j \) in the network. Assigning the flow variables is an essential step towards formulating an optimization problem for a network. Moreover, the complexity of the problem is a measure of the number of variables used in the formulation. As the number of variables increases, the complexity of the problem increases and it becomes more difficult and, consequently, it takes longer to solve.

The most general form of a network optimization problem is as follows:

\[
\text{minimize} \sum_{ij \in E} C_{ij} F_{ij}
\]

where \( F_{ij} \) denotes the total flow in link \( L_{ij} \) and \( C_{ij} \) denotes the cost of transferring one unit of flow along link \( L_{ij} \).

The Minimization is subject to the following constraints:

1. Link capacity constraint which is mathematically denoted as: \( F_{ij} \leq W \)

2. Flow conservation constraints which implies the total flow into a network node equal to the total flow out of it.

### 3.2.1 Reducing the number of variables

The most practical network problems involve the flow of several connection demands on various links of the network simultaneously. The optimization algorithm is employed to find routes and allocate network resources to the various demands in an optimum way that satisfies the given constraints. As explained above, to ensure the optimum solution to the problem, one needs to denote the flow of each connection
3.2. RELATED THEORY

demand on each link of the network by a different variable [EM04c]. Although this way ensures the optimum solution (if feasible), it involves the employment of too many variables even for small sized problems. Furthermore, for each demand, there are certain links that would not be utilized to get from its source to its destination. Therefore, any link that is not part of any possible route between the source and the destination of a demand can be excluded from the formulation without affecting the optimality of the overall solution. As a result, a variable is used to denote the flow of an s-d pair on each link only if the link is part of any possible route between its source and its destination.

Another more efficient way to reduce the number of variables is to use larger granularity representation of the flows in the network. The idea is to employ the flow per route instead of employing the flow per directional link. However, this implies solving the RWA problem in two steps. In the first step, the routing problem is solved by generating the most likely candidate routes that an s-d pair may use for its lightpaths. Then these candidate routes are used in the formulation to find the optimum wavelength assignment for the overall problem. In this manner, only one variable is required to denote the flow of each s-d pair on each candidate route instead of requiring one variable to denote the flow of each s-d pair on each directional link. Therefore, if all candidate routes for an s-d pair can be generated prior to the formulation, the optimum solution to the problem can still be found. At the same time, the number of required variables can be reduced to the number of candidate routes per demand, instead of the number of links in the network per demand. In this manner, one variable is required to represent the flow of a connection demand for each candidate route in comparison to the previous method which requires one variable to represent the demand for each bidirectional link in
the network. Furthermore, path flow has the following advantages over link flow:

1. Only one flow conservation constraint equation is required per s-d pair when the path flow is employed in comparison to $N$ equations per s-d pair when the link flow is employed.

2. No wavelength continuity constraint equations are required in the case of path flow in comparison to $N \times W \times S$ constraint equations when the link flow is employed.

Where:

- $N$ = number of nodes in the network
- $W$ = number of channels on a link
- $S$ = number of s-d pairs

On the other hand, in the case of path flow, the optimality of the solution especially under unbalanced load, depends largely on the number of candidate routes per s-d pair and on the way they are generated. Moreover, the number of variables to formulate the problem increases with the number of candidate routes per s-d pair. Therefore, the number of candidate routes for each s-d pair is sometimes a tradeoff between the optimality of the solution and the complexity of the problem.

### 3.3 Rationale

This section demonstrates how the optimality of the solution and the complexity of the problem depends significantly on the way the routing problem is solved. This dependency is particularly more evident in networks with high average nodal degree, where multiple routes of equal cost between several s-d pairs are possible. The following example on the European COST239 network shown in Figure 1 illustrates
the idea. In this example, there are two $s$-$d$ pairs (9-7 and 11-5) requesting two lightpaths each. Shared protection schemes are used and all links in the network are assumed to have a fixed cost of one unit and an equal capacity of one wavelength. It is assumed that the shortest routes are generated in a similar way to the idea of flooding. The idea is to flood a search signal from the source node into the network and the $k$ shortest routes are those of the first $k$ signals to arrive at the destination node. The order in which the search signal is flooded from a node to its neighbors' dictates the order in which the shortest routes are listed in the case of multiple routes with equal costs. Searching signals can be flooded in ascending or descending order of the neighbor index. Alternatively, random or any other order can be selected. Depending on the network topology, each rule may produce different routing tables. Furthermore, it is not known in advance which rule produces the best solution, because the solution of the RWA depends on the traffic distribution as well as the choice of routing tables.

In this example, it is assumed that the ascending order flooding rule is used to generate the routing tables. Based on this rule, and in order to find a feasible solution to the RWA problem, the first three shortest primary candidate routes for each $s$-$d$ pair need to be generated. Furthermore, for each primary candidate route, the first four corresponding shortest backup candidate routes need to be generated as shown in Table 3.1. The optimum solution to the problem uses 10 wavelength-links and selects the following routes: For $s$-$d$ pair 9-7, the primary routes are 9-7 and 9-10-7 and the backup route is 9-6-7. For $s$-$d$ pair 11-5, the primary routes are 11-5 and 11-8-5 and the backup route is 11-9-6-5. As a result of the way the routes were generated, at least 3 primary routes must be generated for each $s$-$d$ pair. Additionally, at least 4 backup routes for each primary candidate must be generated.
3.4. SOLVING THE ROUTING PROBLEM

to guarantee the optimum solution, when in fact, only two primary routes with two
backup routes for each primary candidate are sufficient to guarantee the optimum
solution had a different way been used to generate the routing table.

<table>
<thead>
<tr>
<th>s-d pair</th>
<th>primary candidate</th>
<th>backup candidate</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>9-7</td>
<td>[9-7]</td>
<td>[9-6-7]</td>
</tr>
<tr>
<td>9-6-7</td>
<td>[9-7]</td>
<td>[9-10-7]</td>
</tr>
<tr>
<td>9-10-7</td>
<td>9-7</td>
<td>[9-6-7]</td>
</tr>
<tr>
<td>11-8-5</td>
<td>[11-5]</td>
<td>[11-9-6-5]</td>
</tr>
<tr>
<td>11-8-2-5</td>
<td>[11-5]</td>
<td>[11-9-6-5]</td>
</tr>
</tbody>
</table>

3.4 Solving the Routing Problem

To ensure an optimum solution to the RWA problem for a given set of demands in a
given network topology, the routing problem must be solved so that the most likely
candidate routes are included in the formulation. To include the most likely can-
didate routes for survivable RWA, the routing problem is solved by first generating a
set of possible routes between every s-d pair to be used as candidates routes for the
primary lightpath. Then, for each primary candidate route, a set of possible can-
didate routes for its backup lightpath is generated. Each primary candidate route
must be link-disjoint or node-disjoint as required with each backup candidate route
in its corresponding set. The following two-step algorithm is proposed to generate
the most likely candidate routes between each s-d pair:

1. Working path candidates

   For each s-d pair, generate the k-shortest routes to be considered as candidate
   paths for the primary lightpath(s) of that s-d pair in the formulation of the
3.4. SOLVING THE ROUTING PROBLEM

problem. $K$ is an integer variable ($K = 1, 2, 3, \ldots$) that may be adjusted to reduce the complexity of the problem, achieve the optimum solution and suit load distribution. The candidate routes for the primary paths of all s-d pairs are tabulated as shown in Table 3.2. The candidate routes for the primary lightpath do not have to be link or node disjointed. Any K-shortest routing algorithm such as Yen’s algorithm [ST84] may be used. The primary routing matrix has a maximum size of $E$ by $N(N - 1)K$, where $E$ is the number of directional links in the network and $N(N - 1)$ is the maximum number of s-d pairs and $K$ is the maximum number of primary candidate routes for an s-d pair. The entry $I_{ij}$ takes a value of 1 if the $j^{th}$ link is part of the primary route corresponding to row $i$; otherwise, $I_{ij}$ is set to 0.

2. Backup path candidates

For each candidate of the primary routes generated in step one, all possible corresponding link-disjoint routes up to a maximum of $D$ is derived. First, the cost of the bidirectional links making the candidate primary route is set to infinity so that these links would not be part of any of the backup routes’ can-

<table>
<thead>
<tr>
<th>s-d pair index</th>
<th>route index</th>
<th>link index</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1 2 3 4 5 6</td>
</tr>
<tr>
<td>1</td>
<td>1</td>
<td>1 0 0 1 0 0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>0 1 0 1 0 0</td>
</tr>
<tr>
<td></td>
<td>.</td>
<td>. . . . .</td>
</tr>
<tr>
<td></td>
<td>K</td>
<td>0 0 0 0 1 0 0</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1 0 0 1 0 0</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>2 0 1 0 1 0</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>0 0 0 0 1 0 0</td>
</tr>
<tr>
<td>.</td>
<td>.</td>
<td>. . . . .</td>
</tr>
<tr>
<td>N(N-1)</td>
<td></td>
<td>. . . . .</td>
</tr>
</tbody>
</table>
3.4. SOLVING THE ROUTING PROBLEM

didates. Then, the k-shortest algorithm is invoked to derive up to \( D \) shortest paths. Each one of the \( D \) routes is link-disjoint with its corresponding primary route candidate. In situations where it would not be possible to generate all \( D \) possible link-disjoint routes, it suffices to use all possible link disjoint routes. For example, in the network shown in Figure 3.1, the second candidate for the primary route between \( s-d \) pair 1-2 is \([1-3-2]\). The only backup route candidate which is link-disjoint with this primary candidate is \([1-2]\). The candidate backup routes for each primary candidate are tabulated in a similar fashion to the primary routing table shown in Table 3.2.

There are two reasons for considering more than one primary path; the first is that for some trap network topologies such as that shown in Figure 4.1a, there exists a primary route that does not have a corresponding link-disjoint route to be considered as a backup path. Secondly and more importantly, it is not known prior to the optimization process which of these routes would be selected as part of the optimum solution when all lightpaths of all \( s-d \) pairs are considered jointly. For this research, a software tool is developed to formulates the RWA problem for a given set of demands on a given mesh network topology for both dedicated and shared protection schemes.

\[ \text{Figure 3.1: six node network} \]
3.5 Problem formulation

This section illustrates how the routing tables, generated in Section 3.4, can be used to formulate the overall problem to find the optimum solution.

3.5.1 Notations

- Network $G(N,E)$, where:
  - $N$ is the number of nodes in the network.
  - $E$ is the number of directed links in the network.

- $Q$ is the number of $s$-$d$ pairs, $1 \leq Q \leq N(N-1)$

- $W$ is the number of wavelengths per link (link capacity).

- $K$ is the maximum number of primary paths that an $s$-$d$ pair can have.

- $D$ is the maximum number of backup paths that a primary path can have.

- $C_e$ is the the cost of using a wavelength on link $e$.

- $L_{ij}$ is usually used to denote the directed link from node $i$ to node $j$, but for convenience a single identifier is used instead. All links in the network are sequentially numbered from 1 to $E$. The lower indexes are assigned to the links outgoing from the lower indexed nodes. For example, the network shown in Figure 3.1 consists of six nodes and 20 directional links. So $L_{12}$ is assigned index 1, $L_{13}$ is assigned index 2, $L_{21}$ is assigned index 3, $L_{65}$ is assigned index 20 and so on.

Similarly, each $s$-$d$ pair is given a single identifier and all $s$-$d$ pairs are indexed from 1-$Q$. For example, index 1 would be assigned to $s$-$d$ pair 1-2 if there
are any connection demands between them. Only node pairs that requested a connection between them are indexed.

- \( A_i \) is the number of lightpaths requested by the \( i^{th} \) s-d pair; \( i \leq Q \).

- \( p_i \) is the number of primary paths between the \( i^{th} \) s-d pair; \( p_i \leq K \).

- \( b_{ik} \) is the number of routes to backup the \( k^{th} \) primary path of the \( i^{th} \) s-d pair;
  \( 1 \leq k \leq p_i \) and \( b_{ik} \leq D \).

- \( A_e \) is the summation of all the primary paths that are assigned a wavelength (active lightpaths) on link \( e \); \( A_e \) is the same for dedicated and shared protection formulation.

- \( S_e \) is the summation of the variables representing the backup capacity that assigned a wavelength on link \( e \). \( S_e \) represents different variables for the dedicated and shared protection formulation.

- \( P_{ik}^\omega \) is used as an objective function variable to represent the \( k^{th} \) primary route of the \( i^{th} \) s-d pair on wavelength \( \omega \). In the final solution \( P_{ik}^\omega \) takes the value of 1 if the \( k^{th} \) primary route of the \( i^{th} \) s-d pair is assigned wavelength \( \omega \); otherwise it takes the value of 0.

- \( B_{ik}^{d\omega} \) is used as an objective function variable in the dedicated protection formulation. It represents the \( d^{th} \) backup route to protect the \( k^{th} \) primary route of the \( i^{th} \) s-d pair on wavelength \( \omega \). In the final solution, \( B_{ik}^{d\omega} \) takes the value of 1 if the \( d^{th} \) backup route employs wavelength \( \omega \) to protect the \( k^{th} \) primary route of the \( i^{th} \) s-d pair; otherwise it takes the value of 0.

- \( \xi_e^\omega \) is used as an objective function variable in the shared protection formulation. \( \xi_e^\omega \) takes the value of 1 if the wavelength \( \omega \) is assigned to any backup
path on link \( e \); otherwise it takes the value of 0.

- \( e \) is used to index a link; \( 1 \leq e \leq E \).

- \( i \) is used to index an \( s-d \) pair; \( 1 \leq i \leq Q \).

- \( k \) is used to index a primary path; \( 1 \leq k \leq K \).

- \( d \) is used to index a backup path; \( 1 \leq d \leq D \).

- \( \chi_{ik}^e \) is an indicator function that takes the value of 1 if the \( k^{th} \) primary route of the \( i^{th} s-d \) pair passes through link \( e \); otherwise it takes the value of 0.

- \( \varphi_{ik}^{de} \) is an indicator function that takes the value of 1 if the \( d^{th} \) backup route to protect the \( k^{th} \) primary route of the \( i^{th} s-d \) pair passes through link \( e \); otherwise it takes the value of 0.

### 3.5.2 Dedicated Protection Formulation

1. **Objective function:**

   
   
   Minimize \( \sum_{e=1}^{E} C_e (A_e + S_e) \) \hspace{1cm} (3.1)

   On any link \( e \), \( A_e \) is the summation of the objective function variables which are represented by the working path variables that pass through link \( e \). From the primary routing matrix, \( A_e \) can be easily calculated by summing all working path variables with entries of 1 in the column \( e \) for all wavelength planes.

   \[
   A_e = \sum_{i=1}^{Q} \sum_{k=1}^{K} \sum_{\omega=1}^{W} P_{ik}^{\omega} \chi_{ik}^e \quad \text{for} \ 1 \leq e \leq E \tag{3.2}
   \]

   Similarly, \( S_e \) is the summation of the objective function variables which are represented by the backup path variables that pass through link \( e \). From the
backup routing matrix, $S_e$ can be easily calculated by summing all the backup paths variables with entries of 1 in the column $e$ for all wavelength planes.

$$S_e = \sum_{i=1}^{Q} \sum_{k=1}^{p_i} \sum_{d=1}^{b_{ik}} \sum_{\omega=1}^{W} B_{ik}^{d\omega} \varphi_{ik}^{d\omega} \quad for \ 1 \leq e \leq E$$ (3.3)

2. Constraints:

(a) Demand Constraint

The demand constraint is used to ensure that the number of lightpaths requested by each $s-d$ pair is satisfied.

$$\Lambda_i = \sum_{k=1}^{p_i} \sum_{\omega=1}^{W} P_{ik}^{\omega} \quad \forall \ i \in Q$$ (3.4)

(b) Capacity Constraint

The capacity constraint is employed to ensure that a wavelength $\omega$ on any link can only be assigned to a primary lightpath or to a backup lightpath.

$$\sum_{i=1}^{Q} \sum_{k=1}^{p_i} \left( P_{ik}^{\omega \times \lambda_{ik}} + b_{ik} \right) \leq 1 \quad \forall \ e \in E \ and \ \forall \ \omega \in W$$ (3.5)

Although all research papers that have been encountered apply another capacity constraint to ensure that the number of lightpaths on a link is bounded by its capacity which can be written as $A_e + S_e \leq W$, it has been found that this constraint is redundant, because it is satisfied under the above constraint. Therefore, it is not included in the formulation.

(c) Protection Constraint

The protection constraint ensures that if the primary candidate route of an $s-d$ pair is assigned a wavelength, then one of its corresponding back
3.5. PROBLEM FORMULATION

up routes must also be assigned a wavelength to protect it.

\[
\sum_{\omega=1}^{W} p_{ik}^\omega = \sum_{d=1}^{b_{ik}} \sum_{\omega=1}^{W} b_{ik}^d \quad \forall k \in p_i \text{ and } \forall i \in Q \tag{3.6}
\]

The protection constraint can also be written as follows:

\[
\sum_{d=1}^{b_{ik}} \sum_{\omega=1}^{W} b_{ik}^d \geq \sum_{\omega=1}^{W} P_{ik}^\omega \tag{3.7}
\]

(d) Integer Constraint (all variables are binary numbers)

3.5.3 Shared Protection Formulation

1. Objective function:

\[
\text{Minimize } \sum_{e=1}^{E} C_e (A_e + S_e)
\]

\(A_e\) is calculated in a similar way to equation (3.2). On the other hand, \(S_e\) for shared protection is different because a number of backup paths can share the same resource on link \(e\). \(S_e\) represents the summation of the objective function variable \(\xi_e^\omega\) if any of the backup routes passing through link \(e\) is assigned wavelength \(\omega\).

\[
S_e = \sum_{\omega=1}^{W} \xi_e^\omega \quad \forall e \in E
\]

\(\xi_e^\omega\) represents a fixed cost (1 unit) incurred for assigning wavelength \(\omega\) on link \(e\) to one or more backup lightpaths. This cost does not increase with the number of backup lightpaths that are assigned wavelength \(\omega\) on link \(e\).
2. Constraints:

(a) Demand Constraint

It is the same as the demand constraint for dedicated protection and is represented by equation (3.4).

(b) Capacity Constraint

It is similar to the capacity constraint in dedicated protection. However, in the shared protection case, a wavelength may be assigned to more than one backup lightpath on a link as long as their corresponding working lightpaths are not in the same SRLG. Therefore, a wavelength on a link can only be assigned to a primary lightpath or to one or more backup lightpaths. These constraint equations can be easily generated from the primary routing matrix entry of each link column.

\[
\left\{ \sum_{i=1}^{Q} \sum_{k=1}^{p_i} P_{ik}^{e} x_{ik}^{e} \right\} + \xi_e^{\omega} \leq 1 \quad \forall e \in E \text{ and } \forall \omega \in W
\]  

(3.8)

(c) Fixed Cost Constraint

It is used to indicate if wavelength \(\omega\) is assigned to any backup path on link \(e\).

\[
\left\{ \sum_{i=1}^{Q} \sum_{k=1}^{p_i} \sum_{d=1}^{b_{ik}} \varphi_{ik}^{de} \right\} \xi_e^{\omega} \geq \sum_{i=1}^{Q} \sum_{k=1}^{p_i} \sum_{d=1}^{b_{ik}} B_{ik}^{dw} \varphi_{ik}^{de}
\]  

(3.9)

\forall e \in E \text{ and } \forall \omega \in W

In case all candidate backup routes passing through link \(e\) are selected and allowed to share the same wavelength \(\omega\), the indicator function \(\xi_e^{\omega}\) must be multiplied by an integer number greater than or equal to the total number of candidate backup routes passing through link \(e\). This explains the need for the summation in both sides of the equation. It is
worth mentioning that all previous formulations employ equation (3.10) in addition to equation (3.9) to satisfy the fixed cost constraint.

\[ \xi_{\omega}^{\omega} \leq \sum_{i=1}^{Q} \sum_{k=1}^{n_{i}} \sum_{d=1}^{B_{ik}} b_{ik}^{\omega} \varphi_{i k}^{d} \]  \hspace{1cm} (3.10)

However, it was found out that equation (3.10) is redundant and that equation (3.9) is sufficient to satisfy the fixed cost constraint.

(d) Protection Constraint

It is the same as the protection constraint employed for dedicated protection. Equation (3.6) can be used to generate it.

(e) Sharing Constraint

It is employed to ensure that a number of backup lightpaths can only share a wavelength on a common fiber if their corresponding working paths are fiber-disjoint. In this case, fiber implies a bidirectional link, where both directional links are considered to pass through the same fiber. To generate all the equations for this constraint, the following procedure has been developed:

i. From the routing tables, construct the SRLG set \( \Omega_{s} \) for each span \( s \). Each \( \Omega_{s} \) has a cardinality equal to the total number of backup candidate routes in the routing table. The following rule is used to construct each \( \Omega_{s} \): for both directional links of the span \( s \), if \( \chi_{i k}^{s} = 1 \), then assign 1 to all the entries corresponding to the \( b_{ik} \) backup routes in the entries of the set \( \Omega_{s} \); otherwise assign 0. Any combination of the backup paths with entries of 1 in \( \Omega_{s} \) can not share the same resource on any link \( e \) because of the possibility of a cut in span \( s \).
ii. Each directional link \( e \) in the network has a corresponding column in the backup routing table. Each column has one entry for each backup path. The entry indicates whether the corresponding backup path passes through the link. The objective is to find out all the different combinations of the backup paths that can not share the same resource while passing through the link. In order to generate all the different combinations of the sharing constraint equations in a link \( e \), a logical AND operation is carried out between the entries of column \( e, 1 \leq e \leq E \) in the backup routing table and each \( \Omega_s \). If the results of any AND operation has more than one entry with 1, then the backup paths corresponding to those entries must not share a common resource on link \( e \).

To illustrate the idea, four connection demands are requested between the s-d pairs 1-6, 2-6, 3-4 and 5-1 in the network shown in Figure 3.1. Suppose the number of primary candidate routes for each s-d pair is set to three and for each primary candidate, up to three candidate backup paths are generated. The candidate routing tables for this example are shown in Table 3.3. For each span \( s \), the SRLG set \( \Omega_s \) is generated and are shown in Table 3.4. For illustration, only the AND operations with the entries of link 2 – 4 are shown. Link 2 – 4 has the entries [0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 0, 1, 0, 1, 0, 0, 0, 0, 1, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 1, 0] in the backup routing table. Therefore, \( B^3_{12}, B^2_{13}, B^1_{22}, B^2_{23}, B^1_{31}, B^3_{31}, B^2_{33}, B^3_{33} \) and \( B^2_{43} \) pass through link 2-4. The AND operations between the entries of link 2 – 4 and the \( \Omega_s \), generates the following inequalities:
Table 3.3: primary and backup candidate routing tables

<table>
<thead>
<tr>
<th>s-d pair</th>
<th>primary candidate</th>
<th>backup candidate</th>
</tr>
</thead>
<tbody>
<tr>
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<td>Route</td>
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</tr>
<tr>
<td></td>
<td>V Route</td>
<td>V Route</td>
</tr>
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<td>$B_{11}^1$ [1-3-5-6]</td>
</tr>
<tr>
<td></td>
<td>$P_{12}$ [1-3-4-6]</td>
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<tr>
<td></td>
<td>$P_{22}$ [2-5-6]</td>
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<tr>
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<td>$P_{42}$ [5-3-1]</td>
<td>$B_{42}^1$ [5-2-1]</td>
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<td></td>
<td>$P_{43}$ [5-3-2-1]</td>
<td>$B_{43}^1$ [5-4-3-1]</td>
</tr>
</tbody>
</table>

\[
\begin{align*}
B_{13}^2 + B_{43}^2 & \leq 1 \text{ with } \Omega_1 \\
B_{23}^2 + B_{43}^2 & \leq 1 \text{ with } \Omega_3 \\
B_{13}^1 + B_{22}^1 & \leq 1 \text{ with } \Omega_5 \\
B_{12}^3 + B_{33}^2 + B_{31}^1 + B_{31}^3 & \leq 1 \text{ with } \Omega_6 \\
B_{33}^2 + B_{33}^3 + B_{43}^2 & \leq 1 \text{ with } \Omega_7 \\
B_{33}^2 + B_{33}^3 & \leq 1 \text{ with } \Omega_8 \\
B_{12}^3 + B_{22}^3 & \leq 1 \text{ with } \Omega_9 \\
B_{13}^2 + B_{22}^1 & \leq 1 \text{ with } \Omega_{10}
\end{align*}
\]

The same inequalities may be generated more than once because the corresponding working paths of their members share more than one link and as a result appear in more than one SRLG. The third and the last inequalities highlights this repetition. To setup the lightpaths given in the example, at least two channels per link were required. The optimum solution is given below and the primary and the backup lightpaths are shown in Figure 3.2.
### Table 3.4: SRLG table

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<th>$\Omega_1$</th>
<th>$\Omega_2$</th>
<th>$\Omega_3$</th>
<th>$\Omega_4$</th>
<th>$\Omega_5$</th>
<th>$\Omega_6$</th>
<th>$\Omega_7$</th>
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primary backup

$P_{13}^1$ $B_{13}^{31}$
$P_{21}^2$ $B_{21}^{31}$
$P_{31}^2$ $B_{31}^{21}$
$P_{42}^1$ $B_{42}^{21}$

56
3.6 Numerical Results

3.6.1 Dedicated protection

The equations presented in Section 3.5.2 were applied to formulate a number of dedicated protection problems. In each problem it was required to find the optimum setup for a given demand matrix on a given network topologies with a given link capacity. The network topologies and the set of demands are shown in Appendix 8.2 and 8.2 respectively. For each $s$-$d$ pair, the shortest three routes were used as primary candidate routes. For each primary candidate route, the shortest three link-disjoint routes were used as the backup candidates. The CPLEX 7.0 ILP solver was used to solve the formulated problems on a 2.4 GHz Pentium IV machine. Table 3.5 shows the final solution of each formulated problem represented as the cost in Channel-Link to set up each demand matrix on its respective network topology.
3.6. NUMERICAL RESULTS

<table>
<thead>
<tr>
<th>Network</th>
<th>Figure</th>
<th>Demand</th>
<th>Capacity (Channel)</th>
<th>No. of Variables</th>
<th>Total cost (channel-link)</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
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<td>40</td>
<td>95080</td>
<td>1369</td>
<td>43609.63</td>
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</table>

The table also includes the link capacity, the number of variables and the amount of time required to find the optimum solution.

Table 3.5 clearly shows the relationship between the number of variables used to formulate the problem and the time required to solve it. As the number of variables required to formulate the problem increases, the time required to solve the problem also increases. However, the solution time increases at a much higher rate than the number of variables. The number of variables required to formulate the dedicated protection problem is a function of the number of s-d pairs, the number of channels per link and the number of primary and backup candidate routes.

3.6.2 Shared protection

Due to the increased complexity of the shared protection scheme, a number of small-sized problems were selected for illustration. Each problem was first formulated using the equations presented in Section 3.5.3, and the resulted formulation was then fed into the CPLEX 7.0 ILP solver to generate the optimum solution. Table 3.6 shows the generated solution for each case. The last column in the tables indicates how far the generated solution is from the optimum solution. By comparing the times required to generate a solution in the dedicated and shared protection schemes, it is observed that the solution time in the case of shared protection is much longer than that of the dedicated protection. For example, the solver required
559800 seconds to generate a feasible solution with a gap of 7.12% for a shared protection problem with 8000 variables, compared to only 61.43 seconds to generate the optimum solution for a dedicated protection problem with 15168 variables. The exponential increase in the solution time in the shared protection scheme is mainly due to the fixed cost constraint rather than the increase in the number of variables. The fixed cost constraint was used in the shared protection formulation to ensure that a backup channel incurs a fixed cost of one unit, regardless of the number of backup paths sharing it.

<table>
<thead>
<tr>
<th>Network</th>
<th>Figure</th>
<th>Demand</th>
<th>Capacity (Channel)</th>
<th>No. of Variables</th>
<th>Total cost (channel-link)</th>
<th>Time (sec)</th>
<th>Gap (%)</th>
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Chapter 4

Dynamic Survivable RWA using Diverse Routing

4.1 Introduction

Due to the dynamic nature of the traffic and the quality of service (QoS) requirements, optimization schemes such as ILP can only consider each demand separately as it arrives. Although these schemes can find the minimum resources required for the demand under consideration, they cannot guarantee the optimum selections of resources for subsequent requests. Moreover, the solutions of ILP problems, especially for large networks, usually take a long time which makes them unsuitable for the dynamic traffic. Consequently, numerous researches proposed heuristic algorithms to solve the dynamic RWA problem. To reduce the blocking probability, the selection of the working and backup lightpaths must be based on some link state information. Therefore, it is desirable to have full knowledge about the routing and wavelength assignment of existing lightpaths. However, due to the significant control overhead involved, complete information may not be feasible in all network
topologies. Another concern which must be taken into consideration during the development of the dynamic RWA algorithm is the complexity and the scalability of the algorithm. In fact, the practical applications of any dynamic RWA algorithm depends a great deal on its complexity and scalability.

Most researchers solved the RWA problem for shared protection by decomposing it into two smaller sub-problems, the routing sub-problem and the wavelength assignment sub-problem. However, the routing sub-problem is not straightforward, because the cost of using a link for a backup path differs from the cost of using the same link for a working path due to the possibility of resource sharing during the calculation of backup paths. Consequently, known algorithms to find the disjoint paths such as Dijkstra and Suurballe can not be applied. So in order to solve the routing problem for shared protection, a straightforward way is to use a two-step approach to generate the two disjoint paths. In the first step, the working path is calculated using any shortest path algorithm based on network status at the time of the arrival. The network status includes the reserved capacity for both working and backup lightpaths at the time of the request arrival. The spare link state of the network is then calculated based on the location of the working lightpath. From the spare link-state, the protection path is derived by removing the spans traversed by the working lightpath. Following this, the determination of resource sharing status will update the costs of all other links.

However, in some trap network topologies, such as that shown in Figure 4.1a, where the two disjoint routes between node pair 1-11 are feasible, the two-step approach can not find them. Furthermore, even where the two disjoint routes are feasible and can be selected using the two-step approach, they may not be the most optimum pair as is the case where the two-step approach is used to find the two
shortest link-disjoint routes between node pair 1-5 in Figure 4.1b. As a result, most researchers proposed inspecting the k-shortest link-disjoint routes.

4.2 Rationale

All heuristic schemes such as ITS$A$ that are based on inspecting the k-shortest link-disjoint routes do not address the wavelength continuity constraint imposed by wavelength-routed optical networks. During the derivation of the working path and in addition to finding the shortest route in terms of some link metrics such as number of hops, the same wavelength must be available on all links along the selected route. Furthermore, after finding the working route and assigning a suitable wavelength for the working lightpath, the new link state of the network does not
only depend on the location of the working path (such as the case on networks with full wavelength conversion capability), but it also depends on the wavelengths status throughout the network. As a result, calculating the new link state for the network under wavelength continuity constraint is significantly more complicated than it is for networks with full wavelength conversion FWC. As shown in Figure 4.2, under the wavelength continuity constraint where there may be sharable capacities on a number of links along the backup path, they can not be all utilized when they are of different wavelengths. Consequently, the sharable capacities that are

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure4_2.png}
\caption{Three-dimensional view of the link state in a network}
\end{figure}

\begin{align*}
F &= \text{Free (available)} \\
NS &= \text{Not Sharable} \\
RW &= \text{Reserved for working lightpath} \\
S &= \text{Sharable}
\end{align*}
on different wavelengths can not be all chosen as part of the backup path. One way to address this problem is to calculate the sharable capacity of each wavelength plane separately. It requires the derivation of a link state for each wavelength plane and then the route with a minimum cost on any wavelength is selected as the backup route. Another problem which has to be addressed is which wavelength should be assigned to the working and the backup lightpaths when multiple paths of equal cost are available in several wavelengths. The wavelength assignment scheme plays an important role in improving network efficiency. Wavelengths should be assigned in a way that keeps as many free wavelengths as possible for subsequent demands and at the same time increase channel sharing among backup channels. For example, during the setup of the primary lightpath, which wavelength out of the available ones should be assigned? Also, during the setup of the backup lightpath, which wavelength out of the equally sharable wavelengths should be assigned? In situations where there are no sharable channels available, the important question again is which wavelength out of the available one should be designated.

4.3 Survivable Algorithm for Dynamic RWA

\textit{(SAD-RWA)}

The algorithm is an extension to the iterative two step approach (\textit{ITSA}) which is based on the two-step algorithm. To ensure the optimum choice of the working and backup light paths for the online connection as well as wavelength continuity constraint, the algorithm inspects the k-shortest routes for the working lightpath on all wavelength planes. The algorithm can be implemented by the following steps:
1. Link state

The link status in the network is updated whenever an establishment or a tear down of a lightpath takes place. The state of each wavelength $\lambda$, in the network is represented by a double dimensional array as shown in the array (Wavelength state). The array is an $E$ column by $E + 2$ rows, where $E$ is the number of links in the network. The top square array ($E$ by $E$) represents the spare capacity status of $\lambda$ in the network. For example, the entry $S_{ij}$ is in Reserved state if $\lambda$ is assigned on link $j$ to protect a working lightpath passing through link $i$ and it is in Free-state otherwise. The bottom two rows are used to indicate if $\lambda$ is assigned to a working lightpath, a backup lightpath or Free. For example, on link $i$, $\lambda$ is assigned to a primary lightpath if the entry $P_i$ is set to the Reserved state. It is assigned to backup lightpath(s) if the entry $B_i$ is set to Reserved state (note that $B_i$ is set to the Reserved state if one or more entries in column $i$ are set to Reserved state). $\lambda$ is in the Free state if both entries $P_i$ and $B_i$ are set to Free state. At any time only $P_i$ or $B_i$ can be in the Reserved state.

$$
\begin{bmatrix}
S_{11} & S_{12} & S_{13} & S_{14} & \cdots & S_{1E} \\
S_{21} & S_{22} & S_{23} & S_{24} & \cdots & S_{2E} \\
S_{31} & S_{32} & S_{33} & S_{34} & \cdots & S_{3E} \\
\ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
\ddots & \ddots & \ddots & \ddots & \ddots & \ddots \\
B_1 & B_2 & B_3 & B_4 & \cdots & B_E \\
P_1 & P_2 & P_3 & P_4 & \cdots & P_E \\
\end{bmatrix}
$$

(Wavelength state)
2. Primary route calculation

For the sake of a primary route calculation, the state of a wavelength $\lambda$ on a link can either be *Reserved* or *Free*. It is *Reserved* if it is assigned to a working lightpath or to some backup lightpaths; it is *Free* otherwise.

$$\lambda \text{ state on a link } = \begin{cases} 
\text{Reserved} & \text{if } \lambda \text{ is assigned} \\
\text{Free} & \text{otherwise}
\end{cases}$$

Based on the wavelength state, the primary link cost for $\lambda$ is calculated according to the following rule:

$$\text{Link cost } = \begin{cases} 
\text{actual link cost} & \text{if } \lambda \text{ is Free} \\
\infty & \text{if } \lambda \text{ is Reserved}
\end{cases}$$

The working cost of wavelength $\lambda$ on any link $i$ in the network can simply be determined by inspecting the $B_i$ and $P_i$ entries in its (Wavelength state) array. If any of the two entries is set to *Reserved*, the working cost of $\lambda$ on link $i$ is infinity. If on the other hand both entries are set to *Free*, the working cost is equal to the actual link cost. Upon arrival of a connection demand, the k-shortest path algorithm is invoked to find the k-shortest routes for each $\lambda_i$; $i = \{1, 2, ..., W\}$ based on its link cost calculated above.

3. Link spare capacity state

For each working route of the $K$-possible candidates along which the working lightpath may be routed on wavelength $\lambda_p$, the new wavelength state for each $\lambda$ in the network is derived based on the location of the working candidate route. In order to find the most optimum backup route for that primary candidate route, the Wavelength state array is used to calculate the backup link cost for
4.3. **SURVIVABLE ALGORITHM FOR DYNAMIC RWA**

(SAD-RWA)

each λ in the network according to the following rule:

(a) Find the sub set \( P \in E \) of the link indices of the bidirectional links making up the candidate primary route. Where \( E = \) set of links \( \{1, 2, \ldots, E\} \) in the network and \( P \in E = \{a, c, \ldots\} \). The idea is to calculate the cost of assigning wavelength \( \lambda \) to a backup lightpath in link \( i \) based on the entries \( ji \) as well as the entries \( Bi \) and \( Pi \) in its Wavelength state array, where \( j \in P \).

(b) Disjointness

To disjoin the working and backup routes, the costs of all bidirectional links making up the working route candidate are set to \( \infty \).

(c) Cost of working capacity

The cost of a wavelength \( \lambda \) on link \( i \) is \( \infty \) if \( \lambda \) is assigned to a primary lightpath in progress on link \( i \). The entry \( Pi \) in the Wavelength state array for \( \lambda \) indicates if \( \lambda \) is assigned to a working lightpath on link \( i \).

(d) Cost of non-sharable capacity

The cost of \( \lambda \) on link \( i \) is set to \( \infty \) if any of the entries \( ji \; \forall (j \in P) \) of its Wavelength state array is in the *Reserved* state. \( \lambda \) is said to be in the non-sharable state in \( i \) because of the *SRLG* constraint.

(e) Cost of residual capacity

The cost of \( \lambda \) on link \( i \) is set to the working link cost if the entries \( Bi \) and \( Pi \) of its Wavelength state array are in the *Free* state. \( \lambda_b \) is said to be a residual capacity.

(f) Cost of sharable wavelength

The cost of \( \lambda \) on link \( i \) is set to a very small value (\( << 1 \)) if all entries
$j_i \forall (j \in p)$ of its Wavelength state array are in the $Free$ state and the entry $B_i$ is in the $Reserved$ state. $\lambda$ is said to be in the sharable state.

Cost of $\lambda$ on link $i = \begin{cases} \infty & \text{if } W_i \text{ Reserved} \\ \text{link cost} & \text{if } B_i \text{ and } P_i \text{ are in Free state} \\ <<1 & \text{if entries } j_i \forall j \in p \text{ are in Free state} \end{cases}$

For each working path candidate, provided that its cost is less than the total cost of the working and backup routes of the candidates before it, all wavelength planes are searched to find the shortest backup route if possible. The least costly combination of working and backup paths are selected for the current working wavelength.

4. Step 4

Repeat steps 1-3 for all wavelength planes and select the least costly combination of the working and the backup routes.

A flowchart showing the various steps of the algorithm is shown in Figure 4.3.
4.3. SURVIVABLE ALGORITHM FOR DYNAMIC RWA
(SAD-RWA)

Figure 4.3: Flowchart for the SAD-RWA
4.4 Performance Evaluation

A simulation tool has been developed to evaluate the performance of the SAD-RWA algorithm. The objective is to study the blocking performance of the SAD-RWA algorithm by varying the load and link capacity in a number of different network topologies. Furthermore, the tool also evaluates the performance of the FF wavelength assignment scheme in conjunction with the algorithm. Several network topologies, shown in Appendix 8.2 are chosen to carry out the evaluation experiments due to their well known characteristics and popularity among researchers. The tool developed in this section is used to carry out all tests that form the remainder of this Chapter. It is also used for the performance comparison in Chapter 7.

4.4.1 Simulation Assumptions

To focus on the performance of the SAD-RWA algorithm, all the simulation experiments consider the following simplifying assumptions:

1. Network topology

   The network consists of $N$ switching nodes interconnected by WDM bidirectional fiber-optic links to construct an irregular mesh topology. The physical topology has an $N \times N$ distance matrix $D$, where each element in the matrix represents the cost of using the link between the corresponding pair of nodes in a lightpath. For example, the element $D_{ij}$ is the matrix entry in row $i$ and column $j$, represents the cost of having link $i - j$ as a part of a lightpath. $D_{ij}$ is set to an actual cost, such as the physical distance between the two nodes if they are neighbors. Otherwise, it is set to infinity. Each fiber link can
carry traffic up to \( W \) lightpaths and there is no wavelength conversion. Consequently, the same wavelength must be assigned on all links of a lightpath. At each node, a signal can either be received locally (if it is intended for the node) or switched to one of the outgoing links on the same wavelength. The network may be considered as consisting of a control network and a data network. The control network may be an in-band or out-of-band IP network, consisting of electronic switch controllers at all nodes and is used to exchange control packets and to broadcast network state information. The data network, consisting of optical switches and data channels is used to transfer data and it operates in circuit switching mode. The physical topology of the network does not change throughout the simulation period.

2. Traffic Models

The applied traffic is dynamic in nature, where connection requests arrive at each network node randomly and independently according to some stochastic process such as Poisson’s process with an average arrival rate \( \lambda \). The average arrival rate can vary from one node to another. The destination of a connection demand is uniformly distributed to all other nodes with probability of \( \frac{1}{N-1} \) and the connection holding time is exponentially distributed with mean \( \frac{1}{\mu} \). Blocked connections are dropped and do not return.

3. Control

There is a central control entity which calculates the routes and keeps the network state information. The algorithm can as easily be used in the case of distributed control networks.
4.4.2 Numerical Results and Analysis

4.4.2.1 Blocking Performance with Different Links capacities

The objective of this experiment is to evaluate the blocking performance of the SAD-RWA algorithm on the National Science Foundation network (NSF) with different link capacities. The NSF network has 14 nodes and 42 unidirectional links. All links have an equal cost of one unit and all nodes have no wavelength conversion capability. The algorithms examines the shortest three primary routes \((K = 3)\) to select the least costly primary and backup paths for each arriving request. Additionally, the algorithm employs the first fit \(FF\) wavelength assignment scheme to select an available wavelength for both the working and the backup lightpaths.

Figure 4.4 shows the variations of the blocking probability with the load for different link capacities. The load range shown in the graph is the total load applied

![Blocking probability vs. load](image)

Figure 4.4: Blocking performance of the SAD-RWA algorithm
to the network. As expected, the graph shows an improvement of the blocking probability with the increase in link capacity.

4.4.2.2 Performance Comparison with Full Wavelength Conversion

The aim of this experiment is to compare the blocking performances of the SAD-RWA algorithm with known algorithms for the survivable RWA with FWC capability. The experiment is carried out on the NSF network with a link capacity of twelve channels. Figure 4.5 shows that at low load (< 82 Erlang), the network has similar blocking performances with and without wavelength conversion. The main reason for the similarity in the blocking performances at low load is that the network has more resources than is required to satisfy the demands. Therefore, the network can still achieve similar blocking performance with NWC even though, on

![Blocking probability vs. load](image)

Figure 4.5: Blocking performance in the NSF with link capacity of 12 channels
4.4. PERFORMANCE EVALUATION

average, it consumes more resources per demand than when the FWC is employed. However, as the load increases, the advantages of the FWC over the NWC becomes apparent. The deterioration of the blocking performance with NWC, as the load increases, is due to the wavelength continuity constraint. However, one might argue that with NWC, demands require longer routes to satisfy the wavelength continuity constraint. As a result, they consume more resources. Consequently, networks with NWC have higher blocking probability. On the hand, another valid argument is that under NWC, requests with longer routes tend to be blocked more often than requests with shorter routes. Therefore, this argument questions the fairness and the accuracy of the comparison. Consequently, for more accurate comparison, other experiments are carried out to compare the blocking performances more accurately.

Figure 4.5 also shows that at higher load, the FWC has similar blocking performance to that of the NWC. As the load increases further, it becomes the dominant factor of the blocking, which explains the reason for the similarity in the blocking performance at high load.

Figure 4.6 shows the variations of the average cost per demand with the load for both the FWC and NWC. The total cost to establish a lightpath is the sum of the costs of its working and backup paths. The cost of a lightpath is measured in the number of channel links used. For example, if the working lightpath consists of three links and uses a full channel, then its cost is three channel links. From the graph, it can be seen that at very low load, the NWC requires more resources than does the FWC, especially for the working paths because of the wavelength continuity constraint. This finding is intuitive because in order to satisfy the wavelength continuity constraint, the routing scheme often has to select longer routes. At low load, it is possible for the routing scheme to select longer routes with a high
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![Average path cost vs. load](image)

WFWC = average cost of working path for FWC
TFWC = average total cost of working and backup paths for FWC
WNWC = average cost of working path for NWC
TNWC = average total length of working and backup paths NWC

Figure 4.6: Average path cost in the NSF network with 12 channel link capacity

probability of success due to the availability of resources.

Under FWC, the average length of the working paths increases gradually with the load before starting to decline at higher load as shown in Figure 4.6. The gradual increase in the length of the working paths is partly due to the increase in load and partly due to the selection metric of the routing scheme. During the setup of lightpaths, the routing scheme selects the least costly working and backup routes for the lightpath. As a result, the routing algorithm often selects longer routes for the working paths, even if shorter routes are available, because of the very low cost of their corresponding backup paths due to availability of shared resources. For this
reason, the gradual increase in the cost of the working lightpaths is followed by a gradual decrease in the average total cost of the demand due to the increase in the use of resource sharing during the calculations of backup paths.

In contrast, using $NWC$ causes a sharper decrease in the average length of the working paths with the load, because of the wavelength continuity constraint. Accordingly, as the load increases only those demands with shorter routes have any real chance of being admitted into the network because of the remote probability to find the same channel available on longer routes. Additionally, those demands that are admitted into the network, for the most part, use shared resources for their backups as evidenced by the sharp reduction in the average total cost shown in the graph. In fact, the graph shows that the average working path cost is well below the average shortest route of the network.

Finally, the graph clearly shows a large difference between the $FWC$ and $NWC$ in the average total consumed resources under the same load. The $FWC$, for the most part, uses more resources than the $NWC$ which results in better resources utilization. This finding suggest that the difference between the blocking performances is even bigger and that Figure 4.5 does not reflect the true difference in the blocking performances. Therefore, it can be claimed with certainty that the increase of the blocking probability in the $NWC$ is not due to the consumption of more resources per demand as evidenced by the reduction of the average total cost per demand. Rather, it is mainly due to the wavelength continuity constraint. In fact, the network resources are under utilized with $NWC$. The graph also raises the question of fairness, since only those demands with shorter routes have any real chance of being admitted into the network, especially at higher load. The problem of unfairness can be alleviated by employing other reservation algorithms [PSM05] which reserve part
of the wavelengths on each link for longer routed lightpaths. Although these reservation schemes can improve the blocking probability of longer routed lightpaths, they increase the overall blocking probability of the network.

### 4.4.2.3 Blocking Performance in Different Topologies

In this subsection, the blocking performances of the the SAD-RWA is evaluated in a number of network topologies (COST239, NSF and ARPANET). The objective is to compare and analyze the blocking performance of different network topologies so that ways to improve network performance can be suggested. The same load is applied to all three topologies. Each topology is assumed to have a link capacity of 12 channels.

The simulation results, shown in Figure 4.7 suggest that the COST239 network

![Blocking probability vs. load](image)

*Figure 4.7: Blocking performance comparison with link capacity of 12 channels*
has better blocking performance than both the NSF and the ARPANET networks. The figure also suggests that the ARPANET has better blocking performance than the NSF network. The variations in blocking performance could be due to the variations in capacity, average nodal degree or the number of nodes in each network. Although the effects of the capacity on the blocking performance are obvious, the effects of the nodal degree and the number of nodes are not straightforward.

The number of nodes and the average nodal degree of a network can jointly have a direct impact on the average resources each s-d pair needs to establish a connection, which in turn has a direct impact on the blocking performance of the network. The capacity of a network is a measure of the number of Link-Channel in the network. Since all links carry equal capacities, the COST239, NSF and ARPANET networks have the following capacity proportion 52:42:64. Although the ARPANET has more capacity than the COST239 network, its blocking performance is not as good. This confirms that the variation in capacity is not the only cause of the variation in the blocking performances shown in Figure 4.7. Accordingly, the other parameters need to be investigated thoroughly to find out other contributors to the variations in the blocking performances of the different network topologies.

Figure 4.8 shows the variations of the average working route cost with the load for each network topology. Also shown is the variations of the average total cost (costs of working and backup paths) of a demand with the load. From the figure, it can be seen that while the COST239 network consumes the least amount of resources per demand, the ARPANET consumes more resources per demand which is partly responsible for its poor blocking performance. The reason for this poor performance is its large size (20 nodes) and its low average nodal degree which result in longer routes and thus more resources per s-d pair. This is no surprise, since the average
shortest route of the ARPANET network is the highest among the three networks. Another interesting observation from the graph is the amount of shared resources between the backup paths in each network. While it is possible for the COST239 network to effectively share its backup resources between its backup lightpaths as evidenced by its low average backup cost, the NSF and ARPANET networks are not as effective in sharing their backup resources between their corresponding backup paths especially at low load. The main reason for this is that the COST239 network has shorter average working paths than do the other two networks. Shorter working routes have smaller number of SRLG to satisfy and as a result, are less constrained to share backup resources for their backup lightpaths. Thus the number of sharable
resources per demand increases and eventually the required overall backup resources decrease. Finally, at high load, the performances of all three networks deteriorate because only those demands with short routes are admitted due to the remote possibility of finding the same wavelength available in longer routes.

4.4.2.4 The Choice of Wavelength Assignment Scheme

In order to improve the complexity of the SAD-RWA algorithm, the first fit FF wavelength assignment scheme is chosen because of its low computational cost and complexity. In FF, all wavelengths are listed and numbered from zero to $W-1$, where $W$ is the number of wavelengths. When the control entity attempts to assign a wavelength, it searches the ordered list sequentially from a certain starting point and assigns the first available wavelength. The idea is to pack all wavelengths that are in use toward one end of the wavelength space, while at the same time reusing wavelengths as many times as possible. The list of wavelengths is searched twice. The first time is to find an available wavelength for the working lightpath and the second time is to find an available wavelength for the backup lightpath. For no added complexity, two different assignment policies are tested and their performance are compared. The first assignment policy searches the list for both the working and backup lightpaths, in ascending order. The second policy uses an ascending order search to find an available wavelength for the working lightpath and a descending order search to find an available wavelength for the backup lightpath. In all searches, the scheme assigns the first available wavelength.

Figure 4.9 shows a comparison between the blocking performances of the two different search policies of the FF wavelength assignment scheme. The figure shows an improvement in the blocking performance when the opposite starting search points
4.4. PERFORMANCE EVALUATION

![Graph showing Blocking probability vs. load]

```
AFF = The same starting search point
DFF = Opposite starting search points
```

Figure 4.9: Blocking performance comparison between AFF and DFF wavelength assignment schemes

are used. The reason for the improved performance is the increasing possibilities of resource sharing since the wavelength assigned for backup lightpaths are packed toward one end of wavelength space. The improvement is expected to increase with the increase of link capacity.

4.4.3 Computational Complexity of SAD-RWA

The practical implementation of any algorithm for the dynamic RWA depends a great deal on its computational complexity and the dynamic nature of the traffic. The computational complexity of Yen’s algorithm to calculate the k-shortest route is
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\(O(N^2 \log N)\) and the computational complexity of Dijkstra's algorithm to calculate the shortest route is \(N \log N\). Therefore, assuming \(K\) is small, the computational complexity of the SAD-RWA algorithm is calculated to be:

\[
\text{Computational complexity} = O(WN^2 \log N + W^2 N \log N).
\]

Where:

\[
\begin{align*}
N &= \text{Number of nodes in the network} \\
W &= \text{Link capacity (number of wavelengths)}
\end{align*}
\]

Therefore,

\[
\text{Computational complexity} = \begin{cases} 
O(WN^2 \log N) & \text{if } N \gg W \\
O(W^2 N \log N) & \text{if } N \ll W \\
O(N^3 \log N) & \text{if } N \simeq W 
\end{cases}
\]

In order for the algorithm to be of any practical use, most of the calculations have to be carried out during the inter-arrival times between connection requests. Therefore, the practical application of the algorithm largely depends on the dynamic nature of the traffic.
Chapter 5

Static Survivable $RWA$ using P-Cycles

5.1 Background

P-Cycles have the advantage of fast restoration time since the only real time switching required upon link failure is between the end nodes of the failed link. Moreover, P-Cycles can reach good resource redundancy compatible to that of conventional survivable schemes used in mesh networks. In $WDM$ wavelength-routed optical mesh networks, P-Cycle techniques can be applied to ensure survivability against span failures (fiber cuts) under static and dynamic traffic environments.

For a mesh network to be protected against any single span failure, a subset of p-cycles are pre-configured in the spare capacity. However, the number of possible cycles in a mesh network increases rapidly with the number of nodes and the average nodal-degree of the network. As a result, selecting a subset of the most efficient cycles that provides full backup is crucial to make the solution of the $RWA$ problem possible and improve the efficiency of the network. Cycle efficiency can be thought
of as a proportion of the number of links the cycle protects to the number of links making the cycle. Efficiency measurement can be based on the topology score which is the number of links a cycle can protect, or traffic score which is the number of working links the cycle is actually protecting.

Most researchers consider nonsymmetric traffic in both directions between any pair of nodes [SGA02][ZDM04]. As a result, only nonsymmetric traffic which necessitates the use of unidirectional p-cycles is considered throughout the thesis. Similar to [ZDM04], unity p-cycle is used to denote a unidirectional p-cycle that has a one unit of wavelength on every span it traverses. As a result, unity p-cycle protects one working unit in the opposite direction to the cycle for every on-cycle span and two working units for every straddling span.

### 5.2 Solution Options

Under static traffic environment, all connection requests are known in advance, and the objective is to set up all the required lightpaths together with the required P-cycles to provide full backup against any single span failure while at the same time minimizing the number of wavelengths needed. The static RWA problem using p-cycles can be solved non-jointly, where either the working lightpaths or the backup p-cycles are established first and then either the corresponding minimum spare capacity or the working lightpaths problem is solved using an optimization tool such as ILP. Alternatively, for a more optimum solution, the problem can be solved jointly, where the working routes and the backup p-cycles are jointly formulated as an ILP problem to minimize the total capacity required.
5.2.1 Non-Joint solution

The non-joint approaches reduce the complexity of the problem significantly and as a result, decrease the time to find the solution. All previously proposed non-joint approaches [GS98][SG03] consider solving the routing problem first and then finding the minimum backup p-cycles capacity required for a single span failure. However, solving the routing problem without taking the required backup capacity into consideration may result in not finding the adequate backup cycles. The problem is even worse and it gets more complicated to solve under wavelength continuity constraint because of the conflicts between routing the working paths and finding a backup p-cycle with the same wavelength to protect them. For example, if the same wavelength on the two directional links in a single span is assigned to some working paths, these two directional links can not be protected as on-cycle links. On the other hand, heuristic algorithms tend to decrease network efficiency and are not suitable under wavelength continuity constraint. Consider for example the network shown in Figure 5.1 in which there are two demands between s-d pairs 1–6 and 3–1. Using the heuristic method proposed in [ZDM04], the routing problem is solved by selecting either the routes [1–2–4–6] or [1–3–5–6] for the s-d pair 1–6 and the route [3–1] for the s-d pair 3–1. If the routes [1–2–4–6] and [3–1] are selected, the heuristic method generates the backup cycles [1–3–4–2–1] and [4–5–6–4] of a total cost of seven links; whereas the optimum single p-cycle is [1–3–5–6–4–2–1] of a cost of six links. The reason for this is that the cycle [1–3–4–2–1] has an efficiency ratio of 0.75, which is higher than the efficiency ratio of the cycle [1–3–5–6–4–2–1] of 0.666. As a result, cycle [1–3–4–2–1] is selected first which prevents the method of considering the cycle [1–3–5–6–4–2–1].
5.2. SOLUTION OPTIONS

![Diagram](image)

Figure 5.1: Failure to select the optimum solution

### 5.2.1.1 Minimum Backup capacity First \textit{MBCF}

Since routing the working lightpaths before finding their adequate backups may block the formation of the required backup p-cycles, the problem can be solved by first reserving the minimum backup capacity required against any single span failure. Then the working lightpaths can be established on the residual capacity. Routing the working lightpaths can be performed either using heuristic methods or it can be formulated as an \textit{ILP} problem. Additionally, the minimum backup capacity problem, for any network topology, can be solved by formulating it as \textit{ILP} problem [HFS05]. The input to the problem is a set of all possible candidate cycles in the network and the solution is a subset of the candidate p-cycles, with minimum length, that provide full protection against any single span failure. The subset of p-cycles is selected so that each link in the network is either a link that is a part of at least one cycle or a link that straddles at least one cycle [HFS05].

Unless the length constraint is imposed on the selected cycles, the result is always hamiltonian cycles in networks where hamiltonian cycles are feasible. For example, for the network shown in Figure 5.1, one feasible solution would be to select the cycles $PC_1 [1-2-4-6-5-3-1]$ and $PC_2 [1-3-5-6-4-2-1]$. To ensure full protection against any single span failure and assuming network links can carry
up to $W$ channels (assuming $W$ is even), the minimum backup capacity is reserved according to the following rule:

1. Channels $1$ to $\frac{W}{2}$ are reserved along cycle $PC_1$

2. Channels $\frac{W}{2} + 1$ to $W$ are reserved along cycle $PC_2$

5.2.1.2 Constrained K-shortest Routes Algorithm

Since the backup capacity of any possible working lightpath has been taken care of in advance (subsection 5.2.1.1), the rest of the problem comes down to solving the RWA of the working lightpaths from the residual capacity in the most efficient way. The RWA of the working lightpaths can be solved by formulating it as an ILP problem. The RWA problem can be further simplified by finding the most likely candidate routes to be used in the formulation for the overall problem. However, the candidate routes must be chosen so that a single route does not have links that are part of two complementing backup p-cycles. The following example explains the problem.

Consider the network shown in Figure 5.1 with several $s$-$d$ pairs, among them the $s$-$d$ pair 2-6. Suppose that the selected backup p-cycles are $PC_1 [1-2-4-6-5-3-1]$ and $PC_2 [1-3-5-6-4-2-1]$ and that the backup capacities are reserved according to the rule given in subsection 5.2.1.1. Suppose also that it is required to generate the shortest three routes between each $s$-$d$ pair to be used in the formulation. Although the route $2-4-5-6$ is a possible candidate, it is in fact useless because there is no single available wavelength on all its links to be used. Furthermore, no backup p-cycles of the same wavelength are available to back it up. Consequently, the routing algorithm, used for calculating the K-shortest routes, must be modified accordingly to avoid selecting useless routes.

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5.2. SOLUTION OPTIONS

The idea is to add the necessary constraints to the K-shortest routes algorithm so that the choices of the shortest routes are limited to those routes that do not span over two complementing cycles. The routing algorithm should be prohibited from selecting two links, that belong to two complementary backup p-cycles, in the same route. This added constraint can be easily satisfied when at the first inclusion of a link that is part of a backup p-cycle in a route, all the links that belong to the complementary p-cycles are opened before including any subsequent links in the same route. As a result, any primary candidate route can only be one of the following:

1. The primary route has one or more of its links on one of the backup p-cycles.
   
   If this primary route is selected to carry traffic, it can only be protected by the complementary p-cycle. As a result, if a working lightpath is to use this route, it must be assigned one of the wavelengths that are assigned to the complementary p-cycle.

2. The primary route has all of its links straddling the backup p-cycles. Therefore, if this primary route is selected to carry traffic, it can be protected by any p-cycle where the failed link is straddling. Consequently, this primary route can be assigned any wavelength.

5.2.2 Joint solution

For a more optimum solution, the problem can be solved jointly, where the candidate working routes and the candidate backup p-cycles are jointly formulated as an ILP problem to minimize the total capacity required. However, the number of possible p-cycle candidates grow exponentially with the average nodal degree and the number of nodes in the network. Furthermore, the number of variables used to represent flow
due to each lightpath demand increases with the number of links and the number of wavelengths. As a result, the complexity of the problem grows very rapidly, which makes the solution time of the problem unacceptable. To reduce the complexity of the problem and to make it more tractable, the routing problem is first solved using path flow technique. The idea is to generate the $K$ shortest routes for each $s$-$d$ pair to be used as candidate routes in the optimization model. The candidate primary routes together with the candidate cycles are then used to formulate the overall problem as an ILP. To reduce the number of candidate p-cycles in the formulation, a novel pre-selection algorithm has been developed to select a reduced number of high merit cycles. Consequently, the final solution to the problem is a tradeoff between the optimality of the solution and the complexity of the problem.

5.3 Notations

The following notations are used throughout the rest of this chapter.

- $E$ is the number of unidirectional links in the network.

- $Q$ is the number of $s$-$d$ pairs; $1 \leq Q \leq N(N - 1)$.

- $Z$ is the number possible unidirectional cycles.

- $M$ is the number selected unidirectional cycles to be used in the model.

- $K$ is the number of primary candidate routes between each $s$-$d$ pair.

- $C_e$ is the cost of using a wavelength on link $e$.

- $W_e$ is the number of channels on link $e$.

- $\Lambda_i$ is the number of lightpaths requested between the $i^{th}$ $s$-$d$ pair.
• $\Lambda^e_i$ is the number of channels assigned to the $i^{th}$ s-d pair on link $e$.

• $e$ is used to index a link; $1 \leq e \leq E$.

• $i$ is used to index an s-d pair; $1 \leq i \leq N(N - 1)$.

• $k$ is used to index a primary path; $1 \leq k \leq K$.

• $j$ is used to index a cycle; $1 \leq j \leq Z$.

• $P_{ik}$ is an objective function variable used to represent the number of channels assigned to the $k^{th}$ primary route of the $i^{th}$ s-d pair (used for FWC).

• $P_{ik}^\omega$ is a binary objective function variable. It takes a value of 1 if the $k^{th}$ primary route of the $i^{th}$ s-d pair is assigned wavelength $\omega$ in the final solution; otherwise it takes the value of 0 (used for the NWC).

• $X_j$ is an objective function variable that represents the number of channels required on p-cycle $j$ (used for FWC).

• $X_{j\omega}$ is a binary objective function variable. It takes a value of 1 if the $j^{th}$ cycle is assigned wavelength $\omega$; otherwise it takes the value of 0 (used for NWC).

• $w_{ik}$ is the lower index of the wavelengths range which the lightpaths between the $i^{th}$ s-d pair are allowed to use on the $k^{th}$ primary route.

• $W_{ik}$ is the higher index of the wavelengths range which the lightpaths between the $i^{th}$ s-d pair are allowed to use on the $k^{th}$ primary route.

• $\varphi_{je}$ is an indicator function which takes the value of 1 if the $j^{th}$ cycle protects the $e^{th}$ link; otherwise it takes the value of 0.
5.4. CYCLE RANKING

- \( \Phi_i^e \) is an indicator function which takes the value of 1 if the \(i^{th}\) s-d pair uses the link \(e\) on its working path; otherwise it takes the value of 0.

- \( \Phi_{ik}^e \) is an indicator function which takes the value of 1 if the \(k^{th}\) primary route of the \(i^{th}\) s-d pair passes through link \(e\); otherwise it takes the value of 0.

- \( \Psi_{je} \) is an indicator function which takes the value of 1 if the \(j^{th}\) cycle pass through link \(e\); otherwise it takes the value of 0.

- \( A_e \) is the summation of all the primary paths that are assigned a wavelength (active lightpaths) on link \(e\).

- \( S_e \) is the summation of the variables representing the backup p-cycles that are assigned a wavelength on link \(e\).

5.4 Cycle Ranking

The idea of cycles ranking is to sort the set \(Z\) of all possible cycles in the network in a descending order according to some efficiency metric. Then, a subset \(M\) of the best cycles is selected to be used in the optimization model \((M \subseteq Z)\). The cardinality of \(M\) is adjusted to compromise between the optimality of the solution and the complexity of the problem. Cycle ranking can be based on one of two metrics: Topology Efficiency \(TE\) or Traffic Routing Efficiency \(TRE\). In the \(TE\) metric, cycles are merited according to their topology score \(TS\). The Topology score of a cycle \(TS\) is the number of links that the cycle can protect, regardless of whether the links are being used by any working path. The following rule is applied to compute \(TE\) of a cycle \(j\):

\[
TE = \frac{TS}{\text{cost of cycle}}
\]  

(5.1)
Where:

\[ TS = \sum_{e=1}^{E} \tilde{\varphi}_{je} \tag{5.2} \]

However, TE metric is based purely on the topology of the network and does not take traffic distribution into consideration. Hence it is not good selecting a cycle which can protect many links but none of them are employed in any of the working lightpaths. Consider for example, the working lightpath routed along link 9 – 10 in the network shown in Figure 1. Based on TE metric, cycle [0 – 1 – 3 – 4 – 5 – 7 – 10 – 8 – 6 – 3 – 0] is ranked well ahead of the most efficient backup cycle [8 – 10 – 9 – 8], even though it does not provide any backups for the working path 9 – 10.

One way to alleviate the problem is to use traffic routing efficiency TRE metric, where cycles are ranked based on their traffic routing score TRS. The traffic routing score TRS of a cycle is the total amount of traffic on the links which the cycle is protecting. The following rule is used to compute the traffic routing efficiency TRE of any cycle \( j \):

\[ TRE = \frac{TRS}{\text{cost of cycle}} \tag{5.3} \]

Where:

\[ TRS = \sum_{i=1}^{Q} \sum_{e=1}^{E} \Phi_{ie} \tilde{\varphi}_{je} \Lambda_{ii}^e \tag{5.4} \]

TRE selection metric is particularly effective in the non-joint formulation case where cycle selection is performed after the actual routing of working lightpaths is carried out. As a result, cycles can be selected based on the amount of backup capacity each cycle can provide to actual working traffic. However, in the joint formulation case, multiple candidate primary routes are nominated for each s-d pair. As a result, not all candidate routes carry traffic in the final solution of the problem. Therefore,
using $TRE$ selection metric may favor cycles that provide very little or no backups to candidate routes that are part of the optimum solution which results in increasing the number of cycles required to obtain the optimum solution.

## 5.5 Rationale

This subsection shows how ranking cycles based on the routing efficiency metric $TRE$, for the joint formulation case, increases the number of candidate cycles required in the formulation of the overall problem and decreases the likelihood of obtaining the optimum solution. As explained above, in the joint formulation case, a number of candidate primary routes for each $s$-$d$ pair are used in the optimization model. Among the set of primary candidate routes, an optimum subset which satisfies the given constraint are selected as the working paths. However, this optimum subset of working routes are not known prior to the optimization process. Consequently, this optimum subset of working routes cannot be used as basis to select a subset of the possible cycles for the joint optimization model. On the other hand, using all primary candidate routes to compute $TRE$ does not produce the most effective subset of cycles. The main problem in assigning equal weights to all primary candidate routes is that it will result in selecting cycles that provide backup for longer routes due to their increased number of links. Since the optimization model aims to minimize the overall capacity requirements, shorter routes have higher priority to be part of the optimum solution than longer routes. As a result, more candidate cycles are required in order to include those cycles that provide backups for the shorter candidate routes.

One way to overcome the shortcomings of the $TRE$ scheme is to assign higher priority to cycles that provide backups to the shorter routes. The following example
illustrates the idea.

Consider the network shown in Figure 5.2. Two lightpaths are requested between $s-d$ pairs 1–4 and 4–3. As shown in the figure, the candidate primary routes for $s-d$ pair 1–4 are [1-2-4], [1-3-4] and [1-3-2-4] and the candidate primary routes for $s-d$ pair 4–3 are [4-3], [4-2-3] and [4-5-3]. There are 38 possible candidate cycles. By inspection however, the optimum solution to this problem is to select [1-2-4] and [4-3] as the two working paths and $[1 - 3 - 4 - 2 - 1]$ as the backup p-cycle. Accordingly, in an effective selection metric, cycle $[1 - 3 - 4 - 2 - 1]$ should be ranked in the top of the list of the subset. However, employing the $TE$ selection metric ranks the optimum cycle any where between the $13^{th}$ and the $26^{th}$ position in the ordered list. With $TRE$ metric the ranking will improve to the $8^{th}$ position. Even using $TRE$ metric in this simple example requires the use of at least eight cycles in the optimization model to obtain the optimum solution whereas one cycle is sufficient to generate the same optimum solution. The following subsection presents a new selection metric that can greatly reduce the number of cycles required to produce the optimum solution.

![Figure 5.2: Cycles ranking based on multiple primary route candidates](image-url)
5.6 Route Sensitive Efficiency \((RSE)\)

In this subsection, a new metric that is sensitive to the cost of each candidate route is proposed. \(RSE\) can be effectively used to pre-select a reduced number of high merit cycle candidates. \(RSE\) is computed based on a sensitive traffic routing score \(RSS\). The route sensitive score \(RSS\) of a cycle due to a link which the cycle is protecting reflects the relative cost of the candidate primary routes traversing the link.

In order to calculate an effective sensitive score, two scoring functions are employed in the calculation of the \(RSS\). The first scoring function reflects how good the candidate route is, relative to the shortest route in its s-d pair routing table \((Low\ Ratio)\); while the second scoring function reflects how good the candidate route is relative to the longest route in its s-d pair routing table \((High\ Ratio)\).

\[
Low\ Ratio = \frac{\text{Cost of shortest route}}{\text{Cost of the candidate route}}
\]

\[
High\ Ratio = \frac{\text{Cost of Longest route}}{\text{Cost of the candidate route}}
\]

The \(Low\ Ratio\) and the \(High\ Ratio\) of the \(k^{th}\) primary candidate route of the \(i^{th}\ s-d\) pair are denoted by \(R_{ik}\) and \(R^{ik}\) respectively. The \(R_{ik}\) and \(R^{ik}\) are calculated using Equation (5.5) and Equation (5.6) respectively.

\[
R_{ik} = \frac{\sum_{e=1}^{E} \Phi_{ik}^{e} C_{e}}{\sum_{e=1}^{E} \Phi_{ik}^{e} C_{e}} \quad (5.5)
\]

\[
R^{ik} = \frac{\sum_{e=1}^{E} \Phi_{ik}^{e} C_{e}}{\sum_{e=1}^{E} \Phi_{ik}^{e} C_{e}} \quad (5.6)
\]

The idea is to assign a score to a cycle for each link it protects that is dependent on the relative cost of each primary route candidate traversing the link as well as
the amount of traffic requested by the corresponding \( s-d \) pair. As a result, \( RSS \) is not only sensitive to the traffic demand distribution in the network, but it is also sensitive to the priority and cost of each primary candidate route in the routing table. The \( RSS \) of a cycle due to a link \( l \) depends on the following:

1. Whether link \( l \) is protected by the cycle.

2. The number of the candidate routes traversing link \( l \).

3. The Low Ratio and High Ratio of each candidate route traversing link \( l \).

4. The number of cycles protecting the link.

5. The number of channels requested by the \( s-d \) pairs of the candidate routes traversing link \( l \).

The following rule is used to compute the sensitive routing efficiency \( SRE \) of cycle \( j \):

\[
SRE = \frac{RSS}{\text{cost of cycle}} \quad (5.7)
\]

Where:

\[
RSS = \sum_{e=1}^{E} \sum_{i=1}^{Q} \sum_{k=1}^{K} \frac{\phi_{ik}^{e} \varphi_{je} A_{i} R_{ik} R_{ik}}{\sum_{z=1}^{z} \varphi_{ze}} \quad (5.8)
\]

Equation (5.8) ensures assigning bigger scores to routes of low cost relative to the other candidate routes of the same \( s-d \) pair. Furthermore, routes of equal cost of the same \( s-d \) pair would be assigned equal scores regardless of their position in the routing table. Also, the score of a cycle \( j \) for protecting link \( e \) is inversely proportional to the number of possible cycle protecting that link.
5.7 Problem Formulation

5.7.1 Non-Joint Formulation

It is assumed that the minimum backup capacity for any single span failure has been reserved and configured as described in Subsection 5.2.1.1. Additionally, the K-shortest candidate routes for each s-d pair have been generated as described in Subsection 5.2.1.2.

1. Objective function:

\[
\text{Minimize} \quad \sum_{e=1}^{E} C_e A_e \tag{5.9}
\]

Where:

\[
A_e = \sum_{i=1}^{Q} \sum_{k=1}^{K} \sum_{\omega=\omega_{i,k}} W_{i,k} P_{i,k}^\omega \Phi_{i,k}^e \quad \forall \ e \in E \tag{5.10}
\]

2. Constraints:

(a) Demand constraints

\[
\Lambda_i = \sum_{k=1}^{K} \sum_{\omega=\omega_{i,k}} W_{i,k} P_{i,k}^\omega \quad \forall \ i \in Q \tag{5.11}
\]

(b) Capacity constraints

A wavelength on a link can only be assigned to a working path.

\[
\sum_{i=1}^{Q} \sum_{k=1}^{K} P_{i,k}^\omega \Phi_{i,k}^e \leq 1 \tag{5.12}
\]

\[ \forall (e \in E) \text{ and } \forall (w_{i,k} \leq \omega \leq W_{i,k}) \]
5.7. PROBLEM FORMULATION

(c) Integer constraints

All variables take binary values

5.7.2 Joint Formulation

All $Z$ possible cycles are generated and sorted based on some scoring criterion such as $RSE$. A subset $M$ of the best merited cycles are then selected to be used in the formulation. Additionally, the K-shortest candidate primary routes are generated for each $s-d$ pair. For more details on cycles generation refer to Appendix 8.2.

5.7.2.1 Under Full Wavelength Conversion FWC

1. Objective function:

$$\text{Minimize} \quad \sum_{e=1}^{E} C_e (A_e + S_e) \quad (5.13)$$

Where:

$$A_e = \sum_{i=1}^{Q} \sum_{k=1}^{K} P_{ik} \Phi_{ik}^e \quad (5.14)$$

$$S_e = \sum_{j=1}^{M} X_j \Psi_{je} \quad (5.15)$$

2. Constraints:

(a) Demand constraints

$$\Lambda_i = \sum_{k=1}^{K} P_{ik} \quad \forall \ i \in S \quad (5.16)$$

(b) Capacity constraints

The capacity used by the working traffic and the backup p-cycle passing
through link $e$ must not exceed the capacity of the link

$$\left\{ \sum_{j=1}^{M} X_{je}^{j} \geq \sum_{i=1}^{Q} \sum_{k=1}^{K} P_{ik}^{e} \Phi_{ik}^{e} \right\} \leq W_{e} \quad \forall \ e \in E \quad (5.17)$$

(c) Protection constraint

$$\sum_{j=1}^{M} X_{je} \geq \sum_{i=1}^{Q} \sum_{k=1}^{K} P_{ik} \Phi_{ik}^{e} \quad \forall \ e \in E \quad (5.18)$$

(d) Integer constraints

All variables must take integer values

5.7.2.2 Under No Wavelength Conversion NWC

In the absence of wavelength converters, all the links of a working route must be assigned the same wavelength. Furthermore, since p-cycle protection is based on the link protection method, both a backup p-cycle and all the working paths it protects must be assigned the same wavelength. As a result, two bidirectional cycles on a shared link must not be assigned the same wavelength if any of them is to protect traffic traversing the shared link. Moreover, in order to provide on-cycle protection for a bidirectional working traffic on a link, the two bidirectional lightpaths on that link must not be assigned the same wavelengths. Consider the example shown in Figure 5.3, where two bidirectional p-cycles $C_1$, $C_2$ and two working lightpaths $P_1$, $P_2$ are shown. P-cycle $C_2$ can not protect $P_2$ if both $C_1$ and $C_2$ are assigned the same wavelength. Similarly, $P_1$ and $P_2$ can not be protected if both are assigned the same wavelength.

1. Objective function:

$$\text{Minimize} \quad \sum_{e=1}^{E} C_{e}(A_{e} + S_{e}) \quad (5.19)$$
Figure 5.3: Wavelength continuity constraint in P-cycles

Where:

\[ A_e = \sum_{i=1}^{Q} \sum_{k=1}^{K} \sum_{\omega=1}^{W} P_{ik}^\omega \Phi_{ik}^\omega \quad (5.20) \]

\[ S_e = \sum_{j=1}^{M} \sum_{\omega=1}^{W} X_{j\omega} \Psi_{je} \quad (5.21) \]

2. Constraints:

(a) Demand constraints

\[ \Lambda_i = \sum_{k=1}^{K} \sum_{\omega=1}^{W} P_{ik}^\omega \quad \forall \; i \in Q \quad (5.22) \]

(b) Capacity constraints

A wavelength on a link can only be assigned to a working path or to a backup p-cycle

\[ \left\{ \sum_{j=1}^{M} X_{j\omega} \Psi_{je} + \sum_{i=1}^{Q} \sum_{k=1}^{K} P_{ik}^\omega \Phi_{ik}^\omega \right\} \leq 1 \quad (5.23) \]

\[ \forall \; e \in E \; \text{and} \; \forall \; \omega \in W \]
(c) Protection constraint

\[
\sum_{j=1}^{M} X_{j\omega} \varphi_{j\omega} \geq \sum_{i=1}^{Q} \sum_{k=1}^{K} P_{ik}^{\omega} \Phi_{ik}^{\omega}
\]  \hspace{1cm} (5.24)

\forall \ e \in E \text{ and } \forall \omega \in W

(d) Integer constraints

All variables take binary values

5.8 Numerical results

The performances of the joint and non-joint approaches are evaluated and compared. For the joint approach, the proposed RSE scoring scheme is evaluated for both FWC and NWC. In order to prove the credibility of the RSE, its performance is compared with the performances of previously proposed schemes. The evaluations are carried out under different traffic loads on several network topologies. Table 5.1 summarizes the specifications of each network used in the evaluation.

<table>
<thead>
<tr>
<th>Network</th>
<th>Figure</th>
<th>Number of nodes</th>
<th>Number of unidirectional links</th>
<th>Number of possible cycles</th>
</tr>
</thead>
<tbody>
<tr>
<td>ARPANET</td>
<td>3</td>
<td>20</td>
<td>64</td>
<td>2722</td>
</tr>
<tr>
<td>COST239</td>
<td>1</td>
<td>11</td>
<td>52</td>
<td>7062</td>
</tr>
<tr>
<td>NSFNET</td>
<td>2</td>
<td>14</td>
<td>42</td>
<td>278</td>
</tr>
<tr>
<td>Testnet</td>
<td>4</td>
<td>15</td>
<td>52</td>
<td>1040</td>
</tr>
</tbody>
</table>

5.8.1 traffic loads

In order to evaluate the performance of the RSE scheme under different traffic loads, two randomly generated sets of demands were applied to each network topology. The first set is generated with a maximum load of 3 lightpaths to each s-d pair while the
second set is generated with a maximum load of 7 lightpaths to each s-d pair. The generated static traffic is shown in Tables 2-9

### 5.8.2 Non-joint Formulation Results

In order to solve the survivable RWA using the non-joint scheme MBCF described in Subsection 5.2.1.1, the backup p-cycles are setup for each network topology as shown in Table 5.2. The shortest three routes for each s-d pair are generated using the routing scheme described in Subsection 5.2.1.2. The problem is then formulated using the non-joint formulation described in Subsection 5.7.1. The results are obtained by solving the problem using CPLEX version 7.0 and are shown in Table 5.3. The major advantages of this method are its simplified complexity and fast solution time as shown in Table 5.3. However, these advantages come at a small compromise to the optimality of the solution.

Table 5.3 shows the minimum working capacity required to establish each set of demand. The total cost is the sum of the minimum working capacity and the backup capacity. If the backup cycles used were hamiltonian cycles, then Equation (5.25) can be used to calculate the total capacity required to setup each demand.

<table>
<thead>
<tr>
<th>Network</th>
<th>Backup p-cycles</th>
<th>Channel</th>
</tr>
</thead>
<tbody>
<tr>
<td>COST239</td>
<td>1,3,2,5,8,11,9,6,7,10,4,1</td>
<td>1 - $\frac{W}{2} + 1$ - W</td>
</tr>
<tr>
<td></td>
<td>1,4,10,7,6,9,11,8,5,2,3,1</td>
<td>($\frac{W}{2} + 1$) - W</td>
</tr>
<tr>
<td>NSF</td>
<td>1,3,6,11,9,14,13,12,10,4,5,7,8,2,1</td>
<td>1 - $\frac{W}{2}$</td>
</tr>
<tr>
<td></td>
<td>1,2,8,7,5,4,10,12,13,14,9,11,6,3,1</td>
<td>($\frac{W}{2} + 1$) - W</td>
</tr>
<tr>
<td>TestNet</td>
<td>1,2,4,6,10,13,12,14,8,7,5,9,15,3,11,1</td>
<td>1 - $\frac{W}{2}$</td>
</tr>
<tr>
<td></td>
<td>1,1,3,15,9,5,7,8,14,12,13,10,6,4,2,1</td>
<td>($\frac{W}{2} + 1$) - W</td>
</tr>
<tr>
<td>ARPANET</td>
<td>4,2,1,3,11,15,20,14,13,19,18,17,16,12,9,8,10,7,6,5,4</td>
<td>1 - $\frac{W}{2}$</td>
</tr>
<tr>
<td></td>
<td>4,5,6,7,10,8,9,12,16,17,18,19,13,14,20,15,11,3,1,2,4</td>
<td>($\frac{W}{2} + 1$) - W</td>
</tr>
</tbody>
</table>
5.8. NUMERICAL RESULTS

Table 5.3: Calculated minimum working cost for each demand matrix

<table>
<thead>
<tr>
<th>Network</th>
<th>Demand</th>
<th>Capacity (Channel)</th>
<th>Working cost (channel-link)</th>
<th>Time (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>COST239</td>
<td>Table 2</td>
<td>10</td>
<td>250</td>
<td>0.03</td>
</tr>
<tr>
<td>COST239</td>
<td>Table 3</td>
<td>20</td>
<td>603</td>
<td>0.2</td>
</tr>
<tr>
<td>ARPANET</td>
<td>Table 4</td>
<td>30</td>
<td>561</td>
<td>3.6</td>
</tr>
<tr>
<td>ARPANET</td>
<td>Table 5</td>
<td>84</td>
<td>1711</td>
<td>64.79</td>
</tr>
<tr>
<td>NSF</td>
<td>Table 6</td>
<td>32</td>
<td>646</td>
<td>3.26</td>
</tr>
<tr>
<td>NSF</td>
<td>Table 7</td>
<td>74</td>
<td>1381</td>
<td>8.8</td>
</tr>
<tr>
<td>TestNet</td>
<td>Table 8</td>
<td>38</td>
<td>739</td>
<td>1.25</td>
</tr>
<tr>
<td>TestNet</td>
<td>Table 9</td>
<td>98</td>
<td>1857</td>
<td>7.08</td>
</tr>
</tbody>
</table>

matrix.

Total capacity = Minimum working cost + (N x W) \hspace{1cm} (5.25)

Table 5.4 shows the capacity redundancy calculated for each case. The capacity redundancy is the proportion of the backup capacity to the working capacity. The COST239 and the TESTNET showed good capacity redundancy, especially at high load while the ARPANET had very poor capacity redundancy, especially at low load.

Table 5.4: Capacity redundancy

<table>
<thead>
<tr>
<th>Network</th>
<th>Demand</th>
<th>Capacity redundancy (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>COST239</td>
<td>Table 2</td>
<td>44</td>
</tr>
<tr>
<td>COST239</td>
<td>Table 3</td>
<td>36.48</td>
</tr>
<tr>
<td>ARPANET</td>
<td>Table 4</td>
<td>107</td>
</tr>
<tr>
<td>ARPANET</td>
<td>Table 5</td>
<td>98.19</td>
</tr>
<tr>
<td>NSF</td>
<td>Table 6</td>
<td>69.34</td>
</tr>
<tr>
<td>NSF</td>
<td>Table 7</td>
<td>75</td>
</tr>
<tr>
<td>TestNet</td>
<td>Table 8</td>
<td>77.13</td>
</tr>
<tr>
<td>TestNet</td>
<td>Table 9</td>
<td>39.58</td>
</tr>
</tbody>
</table>
5.8.3 Joint Formulation Results

Table 5.5 shows the minimum number of candidate cycles that are required by each ranking scheme to generate a certain feasible solution. Several solutions are generated to set up different demand matrices on the Cost239 European network with different link capacities. The demand matrices used are shown in Tables 2 and 3. Solutions are also generated for the network with and without full wavelength conversion capability. For example, to realize the demand set given in Table 2 for the COST239 network with eight channels and NWC, the TE, TRE, RSE required 268, 23, 7 ordered cycles respectively to generate a feasible solution at a cost of 338 channel-link.

<table>
<thead>
<tr>
<th>Demand</th>
<th>Constraint</th>
<th>W</th>
<th>Number of cycles</th>
<th>Total cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>TE</td>
<td>TRE</td>
</tr>
<tr>
<td>Table 2</td>
<td>FWC</td>
<td>8</td>
<td>268</td>
<td>15</td>
</tr>
<tr>
<td>Table 3</td>
<td>FWC</td>
<td>18</td>
<td>477</td>
<td>90</td>
</tr>
<tr>
<td>Table 2</td>
<td>NWC</td>
<td>8</td>
<td>268</td>
<td>23</td>
</tr>
<tr>
<td>Table 3</td>
<td>NWC</td>
<td>18</td>
<td>475</td>
<td>98</td>
</tr>
</tbody>
</table>

To further encourage the selection of cycles that protect the shorter routes, the effects of high and low ratios $R_{ik}$ and $R_{ik}$ on the score of a cycle are increased by taking them up to some power, say $x$ and $y$ respectively. Experimenting with several values of $x$ and $y$ improved the final solution and reduced the execution time. Table 5.6 shows the results for $x=2$ and $y=6$ with the set of demands shown in Table 3 on the COST239 network. The table shows that while it is possible to generate a feasible solution using as little as two cycles when the RSE scoring scheme is used to rank them, it has taken at least 23 cycles to generate a feasible solution when the TRE ranking scheme is used. Furthermore, for a fixed number of cycles, the RSE scoring scheme generates a solution that is closer to the optimum than TE and
5.8. NUMERICAL RESULTS

Table 5.6: Number of cycles versus minimum cost

<table>
<thead>
<tr>
<th>Number of cycles</th>
<th>Solution</th>
<th>TE cost</th>
<th>TE Gap</th>
<th>TRE cost</th>
<th>TRE Gap</th>
<th>RSE cost</th>
<th>RSE Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>2</td>
<td>NF</td>
<td>-</td>
<td>NF</td>
<td>-</td>
<td>820</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>NF</td>
<td>-</td>
<td>NF</td>
<td>-</td>
<td>810</td>
<td>0.86</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>NF</td>
<td>-</td>
<td>NF</td>
<td>-</td>
<td>798</td>
<td>2.69</td>
<td></td>
</tr>
<tr>
<td>23</td>
<td>NF</td>
<td>-</td>
<td>800</td>
<td>0.83</td>
<td>778</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>30</td>
<td>NF</td>
<td>-</td>
<td>799</td>
<td>0.75</td>
<td>778</td>
<td>0.4</td>
<td></td>
</tr>
<tr>
<td>60</td>
<td>NF</td>
<td>-</td>
<td>789</td>
<td>0.98</td>
<td>778</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>186</td>
<td>NF</td>
<td>-</td>
<td>778</td>
<td>0.16</td>
<td>778</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>500</td>
<td>778</td>
<td>0.16</td>
<td>778</td>
<td>0</td>
<td>778</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>

NF = Not Feasible

TRE scoring schemes.

Table 5.7 shows a comparison between the final solutions of setting up the demand matrix shown in Table 6 in the NSFNET using the three scoring schemes. The table shows that the first 3 and 4 cycles are enough to generate a feasible solution when the cycles are ranked using the RSE and the TRE scoring schemes respectively. The RSE scoring scheme has not only obtained a feasible solution with less number of cycles, but its solution is also closer to the optimum solution. In fact for the

<table>
<thead>
<tr>
<th>Number of cycles</th>
<th>Minimum cost</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TE</td>
</tr>
<tr>
<td>3</td>
<td>NF</td>
</tr>
<tr>
<td>4</td>
<td>NF</td>
</tr>
<tr>
<td>5</td>
<td>1043</td>
</tr>
<tr>
<td>10</td>
<td>1039</td>
</tr>
<tr>
<td>15</td>
<td>1038</td>
</tr>
<tr>
<td>20</td>
<td>1038</td>
</tr>
<tr>
<td>25</td>
<td>1037</td>
</tr>
<tr>
<td>35</td>
<td>1036</td>
</tr>
<tr>
<td>38</td>
<td>1028*</td>
</tr>
<tr>
<td>39</td>
<td>1028</td>
</tr>
</tbody>
</table>

NF = Not Feasible, * = Optimum solution

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same number of cycles (4 cycles), the RSE scoring scheme obtains a solution that is only 1.95% away of the optimum solution whereas the TRE obtained a solution that is 9.3% away from the optimum solution. All in all, the RSE, TRE, and the TE require 20, 39 and 38 cycles respectively to obtain the optimum solution. Another important observation is that the TE ranking metric, in some cases, produced better solution than the TRE metric in the NSF network. This is because of the big differences in the number of hops between the generated shortest K candidate routes for each s-d pair in the NSF network. Since the TRE metric encourages the selection of cycles that protect longer routes, its solution is pushed further away from the optimum solution.

5.8.4 Comparison between Joint and Non-Joint Approaches

Table 5.8 shows a comparison between the final solutions obtained by the joint and non-joint approaches. The first row of the table shows the variations of the solution when the set of demands given in Table 2 are set up on the COST239

<table>
<thead>
<tr>
<th>Network</th>
<th>Demand</th>
<th>Approach</th>
<th>M</th>
<th>W</th>
<th>TC</th>
<th>NV</th>
<th>T</th>
</tr>
</thead>
<tbody>
<tr>
<td>COST239</td>
<td>Table 2</td>
<td>Non-joint</td>
<td>2</td>
<td>10</td>
<td>360</td>
<td>1520</td>
<td>.03</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Joint</td>
<td>7</td>
<td>8</td>
<td>338</td>
<td>1952</td>
<td>56.53</td>
</tr>
<tr>
<td>NSFNET</td>
<td>Table 6</td>
<td>Non-joint</td>
<td>2</td>
<td>32</td>
<td>1094</td>
<td>7008</td>
<td>3.3</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Joint</td>
<td>39</td>
<td>32</td>
<td>1028</td>
<td>14784</td>
<td>26950</td>
</tr>
</tbody>
</table>

M    = Number of p-cycles used in the formulation
NV   = Number of variables
T    = Solution time (sec)
TC   = Total cost (link-channel)
W    = Number of channels
network. The table shows that while it is possible for the joint approach to generate a feasible solution with only eight channels, the non-joint approach requires at least ten channel to generate a feasible solution. Furthermore, the joint approach consumes only 338 units to set up the demand set while the non-joint approach consumes a total of 360 units. On the other hand, the non-joint approach requires 1520 variables and takes 0.03 sec while the joint approach requires 1952 variables and takes 56.53 sec to solve the problem.

Although at first glance, the joint approach seems to have a reasonable number of variables and solution time for this small problem, as the complexity of the problem increases rapidly, it consequently takes considerably longer to solve as shown in the last row of Table 5.5. Therefore, for more complex problems, it is recommended to use the non-joint approach in order to generate a feasible solution in an acceptable time despite the small penalty in network efficiency.
Chapter 6

Dynamic Survivable RWA using P-Cycles

6.1 Problem Definition

Survivable RWA under dynamic traffic involves the setup and tear down of lightpaths dynamically at random times with the objective of reducing blocking rates. Due to the unawareness of future demands, and the inability to reroute existing lightpaths, the RWA of each demand is considered separately at the time of the arrival. Consequently, first instinct suggests that using p-cycles under dynamic traffic may require the re-optimization of the locations and the number of p-cycles to adapt to changes in demand to improve the blocking performance of the network. However, reconfiguring backup p-cycles comes with high computation complexity which can not be tolerated with highly dynamic traffic. Furthermore, routing each individual lightpath demand separately without taking into consideration future demands, blocks the forming of efficient p-cycles especially under wavelength continuity constraint. As a result, the benefit of reconfiguring existing p-cycles is severely limited. The
network shown in Figure 6.1 illustrates a simple example of backup cycles blocking. The figure shows four lightpath demands \((L_1, L_2, L_3, L_4)\) arriving to the network at random times \((t_1, t_2, t_3, t_4)\) respectively. Here \(t_1 < t_2 < t_3 < t_4\). Assume the shortest route algorithm and the first fit (FF) are used. At time \(t_1\), lightpath \(L_1\) uses the route \([1 - 2 - 4]\) and channel \(\lambda_1\). The p-cycle \([1 - 3 - 4 - 2 - 1]\) is configured on channel \(\lambda_1\) to protect \(L_1\). At time \(t_2\), \(L_2\) arrives at node 3 and selects route \([3 - 1]\) and channel \(\lambda_1\). There is no need to configure any new p-cycles to protect \(L_2\) since the already configured p-cycle \([1 - 3 - 4 - 2 - 1]\) can be used. At time \(t_3\), \(L_3\) arrives at node 3 and selects route \([3 - 5]\) and channel \(\lambda_1\). Since the already configured p-cycles can not protect \(L_3\), the p-cycle \([2 - 5 - 3 - 2]\) is configured on channel \(\lambda_1\) to protect \(L_3\). At time \(t_4\), \(L_4\) arrives at node 6 and selects route \([6 - 5]\) and channel \(\lambda_1\). At this point if the system reconfigures the existing backup p-cycles, the new p-cycle \([4 - 5 - 6 - 4]\) is added to protect \(L_4\). However, the p-cycle \([1 - 3 - 5 - 6 - 4 - 2 - 1]\) would have been configured instead to protect \(L_1, L_2\) and \(L_4\) had \(L_3\) been assigned a different channel or better still a different route such as \([3 - 4 - 5]\). For the latter case, cycle \([1 - 3 - 5 - 6 - 4 - 2 - 1]\) is enough to protect all four lightpaths.
6.2 Solution Options

6.2.1 Real-Time Reconfiguration of Backup Capacity (RRBC)

Reconfiguration of existing p-cycles may be feasible under low dynamic traffic, where the inter-arrival times between connection requests are sufficiently long to re-optimize the number and locations of existing p-cycles if required. Depending on the dynamic nature of traffic, reconfiguring existing backup capacity can be performed on the arrival of each new connection demand, every time a working path can not be found or every time a backup capacity for newly arrived demand can not be found. Reconfiguration of the backup capacity is accomplished in three steps:

1. Release of existing p-cycles
   
   All backup capacity occupied by p-cycles already configured are released and made available in addition to the free capacity for the new demand to route its working path.

2. Routing the new demand
   
   A working path is calculated for the new demand from the free capacity. The free capacity includes the free capacity at the time of the demand arrival as well as the capacity freed by the release of the existing backup p-cycles. Any routing algorithm such as shortest path may be used.

3. Calculating an Optimum backup capacity
   
   Any optimization algorithm can be used to find the optimum backup cycle to provide full protection against any single span failure to the lightpaths already in progress as well as the new demand. The optimization algorithm can be either an ILP such as that used for the non joint formulation in the static
traffic case or a heuristic algorithm such as that proposed in [ZZ05].

4. The new request is blocked when it is not possible to find a working route or enough backup capacity for the new working lightpaths.

### 6.2.2 Off-Line Planning of Backup Capacity (OLPBC)

Since no information is known about the connection demands in advance, the off-line planning of backup capacity aims at reserving minimum backup capacity to ensure full protection against any single span failure. Fortunately, one of the merits of p-cycle protection is the ability to calculate the minimum backup capacity required to provide full protection against any single span failure. The minimum backup capacity problem, for any network topology, can be solved by formulating it as ILP as illustrated in Subsection 5.2.1.1.

The major advantage of planning the backup capacity off-line is the fact that no computations are required during the setup of connection demands. All that is required is to find a working route from the available working capacity for a demand upon its arrival. The required backup capacity for any possible working path regardless of its location has been taken care of beforehand. Furthermore, the blockage to form optimum p-cycles by the established lightpaths in the case of real-time reconfiguration no longer exist. However, the optimality of the off-line planning of backup capacity depends significantly on the distribution of the traffic.

### 6.3 Routing Primary Lightpaths

Under dynamic traffic, the routing schemes for working demands play a significant role on the blocking performance of a network. However, most of the time, the
choice of a routing scheme is a tradeoff between the improvement in performance and the complexity of the scheme. Furthermore, the solution option imposes some constraints on the routing choice. For example, in the real-time reconfiguration of existing backup capacity, the instantaneous capacity of a link changes whenever the backup capacity of the network is reconfigured or whenever a lightpath is setup or torn down. As a result, it is not possible to base routing decisions on the instantaneous capacity of the network. In contrast, under the off-line planning of backup capacity, the backup capacity is fixed throughout. As a result, it is easy to make more intelligent routing decisions based on the traffic load at the time of arrival.

Figure 6.2 shows the instantaneous capacity of a link for the off-line planning of backup capacity. At any instant, if a link is part of any backup cycles, half of its capacity would always be reserved for backup purposes. The second half of its capacity is available for working traffic as shown in Figure 6.2a. On the other hand,

Figure 6.2: Instantaneous link capacity distribution
all the capacity of a straddling link is available for working traffic as shown in Figure 6.2b. In both types of links, the working capacity may at any time be partially or totally used by existing working lightpaths. Since the straddling links have more working capacity than do links on the cycles, it would be more beneficial to employ a routing scheme that favors using the straddling links over the on-cycle links. In the next sections a novel routing scheme is proposed, based on the off-line planning of backup capacity.

6.3.1 Load Balanced Routing (LBR)

The idea of load balancing is to reduce blocking probability by balancing the load over the network and thus reducing congestion in bottleneck links. Load balanced routing is useful when the off-line planning of backup capacity is used due to the big difference between the working capacity offered by straddling links and on cycle links. It is also very useful when network nodes receive different amount of traffic making certain links heavily congested. The scheme is basically similar to the work proposed in [TVJ03]. It uses the shortest path algorithm to calculate a working route for connection demand upon its arrival. However, the dynamic cost of any channel \( \lambda \) on a link is calculated dynamically based on the state of \( \lambda \) (Free or Reserved) and on the level of congestion on the link \( \rho \). The level of congestion \( \rho_i \) on a link \( L_i \) is a proportion between the reserved and the total capacity on \( L_i \). When \( \rho_i \) on \( L_i \) passes a certain threshold \( \rho_i^{MAX} \), the link cost starts to increase proportionally to divert traffic away. The state of a channel \( \lambda \) on a link is either Reserved or Free. It is Reserved if it is assigned to a working lightpath or to some backup p-cycle;
otherwise, it is $\textit{Free}$.

\[
\lambda \text{ state on a link } = \begin{cases} 
\text{Reserved} & \text{if } \lambda \text{ is assigned to a backup p-cycle or a lightpath} \\
\text{Free} & \text{otherwise}
\end{cases}
\]

The cost of $\lambda$ on a link is calculated according to the following rule:

\[
\text{Cost of } \lambda \text{ on a link } = \begin{cases} 
1 + \rho_i & \text{if } \rho_i \geq \rho^{\text{MAX}} \\
1 & \text{if } \rho_i < \rho^{\text{MAX}} \\
\infty & \text{if } \lambda \text{ is Reserved}
\end{cases}
\]

Where:

\[
\rho_i = \frac{\text{Number of Reserved channels on } L_i}{\text{Number of Channels on } L_i}
\]

The value of $\rho^{\text{MAX}}$ is selected based on the applied load. For example, low $\rho^{\text{MAX}}$ is chosen for low load ranges where traffic is allowed to take longer routes using less congested links. On the other hand, low values of $\rho^{\text{MAX}}$ for high traffic ranges tend to increase blocking probability because demands tend to consume more resources.

6.3.2 Straddling Links Priority Routing (SLPR)

Straddling links priority routing gives straddling links higher priority of being selected as part of working routes than the on-cycle links. The idea is to reduce blocking probability by taking advantage of straddling links capacity (full link capacity) that is available for routing working lightpaths as shown in Figure 6.2. This is accomplished by assigning lower cost to straddling links than to the on-cycle links during the calculation of working routes.
6.4 Numerical Results and Analysis

A simulation tool has been developed to evaluate and compare the performance of the RRBC and OLPBC as well as the proposed routing schemes. The tool is designed such that the system and load parameters can be easily varied to study the behavior of wavelength-routed WDM optical networks. The system parameters include network topology, link bandwidth and physical distance. The load parameters include the rate of arrivals, average holding-time period, and the distribution of the inter-arrival time.

In each simulation run, the effect of the initial transient is eliminated by discarding enough observations at the start of the simulation. The length of the initial transient is decided by experimenting with different values until a satisfactory steady state is reached. The batch approach is used to make a succession of seven runs for each experiment. Each run is carried out for long enough time to make sure that the calculated average blocking probabilities for each simulation period are not correlated. Then the mean of the seven observations is calculated with a confidence interval of 95%. More details about the confidence interval are found in Appendix 8.2. Several network topologies, shown in Appendix 8.2, are chosen due to their well-known characteristics and popularity among researchers.

6.4.1 Comparison of Blocking Performance between RRBC and OLPBC

Figure 6.3 shows a comparison of the blocking performance between RRBC and OLPBC as a function of the applied load per node. The simulation is carried out on the COST239 European network with the link capacity of eight channels. The
Figure 6.3: Dynamic p-cycle algorithms blocking performance comparison

The graph shows that at low loads, the OLPBC has better blocking performance than the RRBC. The OLPBC derives its improved performance from using the minimum backup capacity required for any single span failure. Unlike RRBC, OLPBC does not cause any p-cycle blockage during the setup of primary paths. The main cause of blocking, in the case of RRBC, is that primary lightpaths and the backup cycles are set up independently. As a result, setting up a lightpath using a certain channel may block the forming of backup p-cycles for many future connection demands. On the other hand, as the load increases, the difference in blocking probability between RRBC and OLPBC decreases because the main cause of blocking is the heavy load rather than the conflict caused by the setup of primary lightpaths.

The RRBC scheme does not only perform worse than the OLPBC scheme, but in addition to its computational complexity, it is required to maintain a complete
knowledge of the network state information. As a result, the network requires the broadcast of update messages whenever there is a change in the network state. Consequently, significant control overhead may occur which may result in incorrect decisions based on outdated information, especially during high arrival rates of connection requests.

### 6.4.2 Routing Schemes Blocking Performance Comparison

The performance of the three routing schemes (\textit{SP, LBR} and \textit{SLPR}) are evaluated and compared under different traffic loads. Using \textit{SP}, all links are assigned a fixed cost of one unit. Using load balance routing (\textit{LBR}), the rule given in Subsection 6.3.1 is used to calculate each link cost with different threshold values $\rho_{\text{MAX}}$ for different traffic loads. Finally, for straddling links priority routing (\textit{SLPR}), several cost values of $<1$ are tested for straddling links, and all of the on-cycles links are assigned a fixed cost of one unit each. Similar to Subsection 6.4.1, the simulation is carried out on the \textit{COST 239} European network with a link capacity of eight channels.

To better understand the behavior of the network and observe its blocking performance under different load distributions, the network performance is evaluated under both balanced and unbalanced loads. Under balanced load, each node in the network receives an equal number of connection demands with exponentially distributed holding times. The destination of any connection request is randomly selected with a probability function $\left(\frac{1}{N-1}\right)$ that is uniformly distributed over all destinations. In contrast, for unbalanced load, only a subset of nodes receives connection demands with different loads. Additionally, each node in the subset is only allowed to connect to another subset of nodes (for convenience, even indexed nodes
are used as destinations).

Figures 6.4 and 6.5 show a comparison of the blocking performance between the three dynamic routing schemes at low loads. To show and compare the blocking probability, two load ranges are plotted for the equal and unbalanced loads as shown in Figure 6.4 (a) and (b) and 6.5 (a) and (b). For LBR, a maximum threshold ($\rho^{MAX}$) of 0.3 is used. For the straddling priority routing SLPR, the straddling links are given a cost of 0.95 and the on-cycle links have a cost of one unit each.

Figure 6.4 (a) and (b) show the blocking performance of the routing schemes when equal loads are applied to each node in the network. The figure shows that the LBR has a better blocking performance than SLPR and SP. The advantage of the LBR over the other two routing schemes is credited to its ability to divert congestion away from the more congested links. Having more congested links than others, even though all nodes received equal load, is a direct consequence of the difference in nodal degree between nodes. Since nodes send and receive an equal amount of traffic and have a different number of physical links to accomplish this, links that are connected to nodes with smaller nodal degree tend to be more congested than others. From the figure, it can be also seen that SLPR has a better blocking performance than the SP due to its ability to balance traffic at low loads.

Figure 6.5 (a) and (b) show the blocking performance of the routing schemes under unbalanced loads. The total applied load is the same as in the balanced load case; however, connection demands only arrive at the following source nodes [1,3,6,7,9,11] of the network shown in Figure 1. To unbalance the load even further, connection demands arrive at the source nodes according to the following proportion [3,4,5,3,2,5].

Intuitively, the first observation from Figure 6.5 is that under the same total
Figure 6.4: Blocking performance of the routing schemes under equal low loads
Figure 6.5: Blocking performance of the routing schemes under unbalanced low loads
load, the unbalanced load causes more blocking than in the case of the equal loading. The main reason for this blocking is the congestion in a number of links caused by the unbalanced load. It is the same reason why the \textit{LBR} tends to perform better than the \textit{SLPR} and \textit{SP} schemes due to its ability to divert traffic away from the congested links. Another interesting observation is that the \textit{SLPR} does not seem to perform as well as it does under the equal loading case. The reason for this is that \textit{SLPR} assigns a slightly lower cost to straddling links, which encourages their selection in the working lightpaths. As a result, \textit{SLPR} increases the congestion on the straddling links, which subsequently has a negative impact on the blocking probability of future demands. However, \textit{SLPR} still performs better than the \textit{SP} under low traffic due to the larger working capacity that straddling links possess over the on-cycle links.

Figure 6.6 shows the blocking performance of the routing schemes under moderate and higher load ranges with equal loads being applied to each node in the network. The load shown in the graph represents the total load applied to the \textit{COST 239} European network with a link capacity of eight channels. For the \textit{LBR}, the maximum threshold ($\rho^{MAX}$) is set to 0.6 and for the \textit{SLPR}, the straddling link cost is set to 0.95 as opposed to a cost of one unit for the on-cycle links. The graph shows that the blocking probability increases with the load and that when equal loads are applied to each network node, the \textit{SLPR} routing scheme has similar blocking performance to the \textit{LBR} routing scheme. The reasons for this similarity in blocking performance are that the heavy load is the dominant factor of blocking and that the load is well balanced throughout the network. As a result, the blocking caused by the load outweighs the blocking caused by any unbalance in the load that may have taken place. The unbalance of load may have been the result of the
Figure 6.6: Blocking performance of the routing schemes under moderate equal loads
6.4. NUMERICAL RESULTS AND ANALYSIS

differences in the nodal degrees of the network nodes or the result of favoring the straddling links over the on-cycle links in the case of SLPR. Consequently, the LBR has no big advantage over the SLPR at heavy load.

Figure 6.7 shows the blocking performance of the routing schemes on the COST239 European network under heavy load. The total applied load is the same as that applied in Figure 6.6b. However, the load is distributed between the source nodes [1,3,6,7,9,11] according to the proportion [3,4,5,3,2,5]. Additionally, only even-indexed nodes are selected as destination nodes with equal probability. The graph shows that blocking performance of the three routing schemes deteriorates under the heavy and unbalanced loads and that the degree of unbalance of the load is overwhelming even for the LBR.

![Blocking probability vs. load](image)

Figure 6.7: Blocking performance of the routing schemes under heavy and unbalanced load

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6.4.3 Blocking Performances of Different Topologies

In this subsection, the blocking performances of the OLPBC scheme are analyzed to understand its behaviors under dynamic traffic when applied to a number of network topologies (COST239, NSF and ARPANET). Simulation results, shown in Figures 6.8-6.10, suggest that the COST239 network has better blocking performance than the NSF and the ARPANET networks. Figure 6.8 shows the blocking performances of the three networks under equal loads and with equal link capacity on each network. The COST239 network derives its improved blocking performance partly from its large offered capacity. However, one may argue that the ARPANET has more capacity than the COST239 and that its blocking performance is not as good. A simple answer to this problem is that the total capacity of a network is only one of

![Blocking probability vs. load](image)

Figure 6.8: Blocking performances under equal loads
the factors that affect the blocking performance of a network.

To better investigate the other factors affecting the blocking performance, the impact caused by the difference in capacity is eliminated by applying a load that is proportional to the capacity of each network. As a result, each network topology receives a load that is proportional to its capacity. So, if the load $L$ is normalized on the capacity of the COST239, the ARPANET network receives $\frac{32}{26}L$, the NSF network receives $\frac{21}{26}L$ and the COST239 network receives $L$. The results shown in Figure 6.9 show the blocking performance when equal loads per unit capacity are applied. As expected, the figure shows that while there is no change in the performance of the COST239 network and considerable improvement in the performance of the NSF network, the performance of the ARPANET has noticeably deteriorated.

![Blocking probability vs. load](image)

Figure 6.9: Comparison between the blocking performances under equal load per unit capacity
due to the increase in load. Knowing that each network has received a load proportional to its capacity, the other factor that may be partly responsible for the big variations in the blocking performance is the difference in the reserved backup capacity in each network. For the ARPA\textit{NET}, the NSF and the COST239 networks to be fully protected against any single span failure, a 31.25\%, 33.33\% and 21.15\% of their respective capacity must be reserved as backup. Although the reserved percentage of capacity for backup in the ARPA\textit{NET} network is less than that in the NSF network, its blocking performance has worsened, compared to that of the NSF network which has considerably improved. Therefore, one can correctly claim that the difference in capacity is not the only factor responsible for the variations in the blocking performance and that there are other factors which need to be analyzed.

Therefore, to eliminate the effect of the variations in the working capacity, an equal load per unit of working capacity is applied to find out the other important factors which still affect the blocking performances in the different networks. For this experiment, the load is normalized to the working capacity of the COST239 network. So, the ARPA\textit{NET} network received $\frac{34}{41}L$, the NSF network received $\frac{28}{41}L$ and the COST239 network received $L$. The results shown in Figure 6.10 show the blocking performances of the three networks when equal loads per unit of working capacity are applied. The figure shows that although the applied load is proportional to the working capacity of each network and that some improvement of the blocking performances in both the NSF and the ARPA\textit{NET} networks has been gained, there are still considerable variations in the blocking performances. Therefore, it is correct to dismiss the load and the difference in working capacity as the main causes for the variations between the blocking performances of the three networks.

In order to get to the bottom of the problem and find out the main causes
Figure 6.10: Blocking performance under equal load per unit of working capacity behind the variations in the blocking performances of the three networks, further analysis to the specifications of each network topology has been carried out. Topology specifications include parameters such as the number of nodes and the average nodal degree. However, as Table 5.1 shows, the nodal degrees of the NSF, the ARPA NET and the COST239 networks are 3, 3.2 and 4.727 respectively, which suggests that the ARPA NET network should have better blocking performance than the COST239 network, contrary to the results shown in Figure 6.10. Consequently, the average nodal degree alone cannot be the cause of the variations in the blocking performance. The roles of the average nodal degree and the number of the nodes in the network combined are further examined to investigate how they can affect the average length of the working paths in each topology. The average length of the working lightpaths directly reflects the average amount of resources required
per demand which, in turn, has a direct impact on the blocking performance of the network.

The graph in Figure 6.11 shows the variations of the average length of working lightpaths with the load for each network topology. The load applied to each network is proportional to its working capacity. The average length of working lightpaths is measured in the number of links. The graph clearly shows that the ARPANET network has the highest average length of working lightpaths, which accounts for its poor blocking performance, since on average, it consumes more resources per demand than do the other two networks. The graph also shows that, at low loads, the average length of working paths in each network approaches the average length of the shortest route in the network. This is because resources are readily available and the routing algorithm always selects the shortest path for each demand. The

![Average working path cost vs. load](image)

Figure 6.11: Average length of working paths in the different topologies

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average shortest paths for the the COST239, the NSF, and the ARPANET networks are calculated to be 1.56, 2.14 and 2.74 links respectively. As the load starts to gradually increase, the routing algorithm is forced to choose longer routes because of the increase in load and the wavelength continuity constraint which causes a slight increase in the average length of working paths in each case. However, as the load increases, which further causes the blocking probability to increase, demands with longer routes tend to be blocked more often than demands with shorter routes. As a result, requests with shorter routes contribute more to the average length of working paths than do requests with longer routes, eventually causing the average length of working paths to decrease.
Chapter 7

Performance Comparison between Diverse Routing and P-Cycle in Survivable RWA

In this chapter a comparison between the two approaches (diverse routing and p-cycles) to solve the survivable RWA problem is presented. The comparison includes the complexities of the algorithms that have been developed to solve the problems as well as their performances.

7.1 Comparison between Static Traffic Algorithms

Intuitively, the computational time of any computing problem increases with the amount of input data that describes the problem. Consequently, the time complexity of a computing problem is expressed as a function of the amount of data required to describe the problem. Accordingly, the complexity of the static traffic algorithms is expressed as a function of the number of variables required to formulate the problems.
Table 7.1: Number of variables required in the static traffic algorithms

<table>
<thead>
<tr>
<th>Number of variables</th>
<th>P-cycle</th>
<th>Diverse Routing</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Joint</td>
<td>non-Joint</td>
</tr>
</tbody>
</table>

Where:
- $D$ = number of candidate backup routes per primary route
- $E$ = number of directional links in the network
- $K$ = number of candidate primary routes per s-d pair
- $M$ = number of high merit candidate cycles used
- $Q$ = number of s-d pairs
- $W$ = number of channels on a link

as ILP. Table 7.1 shows a comparison between the number of variables required to formulate the survivable RWA using diverse routing and p-cycle approaches under static traffic. Table 7.1 states that the p-cycle approach uses more variables than the diverse routing approach only if $M > (K \times Q \times D + E)$.

Other important variables in the comparison include the solution time and the optimality of the solution in each case. Table 7.2 shows a performance comparison between the diverse routing and the p-cycle approaches when used to set up two

Table 7.2: Comparison between the final solutions of each approach

<table>
<thead>
<tr>
<th>Demand</th>
<th>Approach</th>
<th>W</th>
<th>NV</th>
<th>TC</th>
<th>T</th>
<th>Gap</th>
</tr>
</thead>
<tbody>
<tr>
<td>Table 2</td>
<td>P-cycles</td>
<td>Joint</td>
<td>8</td>
<td>1952</td>
<td>338</td>
<td>56.53</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-joint</td>
<td>10</td>
<td>1520</td>
<td>360</td>
<td>0.03</td>
</tr>
<tr>
<td></td>
<td>Diverse routing</td>
<td>8</td>
<td>11088</td>
<td>366</td>
<td>559800</td>
<td>7.12</td>
</tr>
<tr>
<td>Table 3</td>
<td>P-cycles</td>
<td>Joint</td>
<td>18</td>
<td>5274</td>
<td>779</td>
<td>165</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Non-joint</td>
<td>20</td>
<td>3660</td>
<td>823</td>
<td>0.2</td>
</tr>
<tr>
<td></td>
<td>Diverse routing</td>
<td>18</td>
<td>21240</td>
<td>869</td>
<td>468830</td>
<td>11</td>
</tr>
</tbody>
</table>

$NV$ = Number of variables
$T$ = Solution time (sec)
$TC$ = Total cost (link-channel)
$W$ = Number of channels
different load matrixes on the \textit{COST239} network. The comparison includes the required number of variables, the solution time and the optimality of the solution in each case. From the comparison, it can be seen that the diverse routing approach takes significantly longer to solve than the p-cycle approach. The considerable solution time in the case of the diverse routing approach is mainly due to the fixed cost constraint, which is only required in the diverse routing formulation. The choice between the joint and the non-joint formulations in the case of the p-cycle approach is a tradeoff between the optimality and the time of the solution. For larger problems, the non-joint approach is recommended due to its fast solution time despite the small degradation in the network efficiency.

### 7.2 Comparison between Dynamic Traffic Algorithms

Table 7.3 shows a comparison between the computational complexity of the \textit{SAD-RWA} and the \textit{OLPBC} algorithms. The table clearly shows the huge advantage

<table>
<thead>
<tr>
<th>Computational complexity</th>
<th>SAD-RWA</th>
<th>OLPBC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$O(WN^2 \log N + W^2N \log N)$</td>
<td>$O(N \log N)$</td>
<td></td>
</tr>
</tbody>
</table>

Where:

$N$ = number of nodes in the network

which the \textit{OLPBC} algorithm has over the \textit{SAD-RWA} algorithm in terms of the computational complexity. In order to compare the blocking performances of the two approaches, a number of simulation experiments have been carried out on several network topologies. All network topologies are assumed to have equal link capacities.

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of twelve channels. In order not to give the OLPBC algorithm an advantage over the SAD-RWA algorithm, the shortest path $SP$ is employed to calculate the working path for the OLPBC algorithm. Figure 7.1 shows a comparison of the blocking performances between the SAD-RWA and the OLPBC algorithms on the COST239 network (shown in Figure 1). The figure shows that the two algorithms have similar blocking performances with a slight advantage to the SAD-RWA at low loads. This slight advantage becomes clearer when the blocking probability is plotted against a low load range, as shown in Figure 7.2.

In order to compare the blocking performances of the two approaches more accurately and understand the reasons behind the difference in performances, other blocking performance comparisons for different network topologies have been carried

![Blocking probability vs. load](image)

Figure 7.1: Blocking performance comparison between the SLPR and SAD-RWA algorithms on the COST239 network with 12 channel link capacity at wide load range
Figure 7.2: Blocking performance comparison between the SLPR and SAD-RWA algorithms on the COST239 network with 12 channel link capacity at low load.

out. Figure 7.3a shows a comparison between the blocking performance of the two approaches on the NSF network (shown in Figure 2). Similar to the blocking performance of the COST239 network, the SAD-RWA algorithm has better blocking performance than the OLPBC algorithm. In fact, the difference in the blocking performance between the two approaches for the NSF network is even bigger.

In contrast, the p-cycle approach has less blocking than the diverse routing approach on the ARPANET network (shown in Figure 3), especially at low load as shown in Figure 7.3b. A similar comparison, shown in Figure 7.4, carried out on a test network (shown in Figure 4) suggests that the SAD-RWA algorithm has better blocking performance than the OLPBC algorithm at low load while the OLPBC algorithm has better blocking performance at higher loads.
(a) NSF network with 12 channel link capacity

(b) ARPANET network with 12 channel link capacity

Figure 7.3: Blocking performance comparison between the OLPBC and SAD-RWA algorithms
Figure 7.4: Blocking performance comparison between the *OLPBC* and *SAD-RWA* algorithms on the *TEST* network with 12 channel link capacity.

Although most of the blocking performance comparisons give the impression that the *SAD-RWA* algorithm has better blocking performance than the *OLPBC* algorithm, this conclusion cannot be claimed before other important factors have been thoroughly investigated. Therefore, in order to draw a more accurate conclusion, a deeper analysis have been carried out to examine the effects of the average number of resources consumed per demand on the blocking performance in each case.

Intuitively, the *SAD-RWA* algorithm consumes more resources per demand for its working paths than the *OLPBC* algorithm. This intuition is based on the mechanism by which the *SAD-RWA* algorithm calculates its working paths. The *SAD-RWA* selects the least costly working and backup paths out of a given number of choices per demand. Therefore, due to the possibility of resource sharing, the *SAD-RWA*
algorithm may select a pair of paths (working and backup) even if the working path is longer than other possible working paths in other pairs.

Figure 7.5 shows a comparison between the average working paths for the SAD-RWA and the OLPBC algorithms. The figure shows that at low load (< 148 Erlang), the SAD-RWA algorithm consumes more resources for its working path than the OLPBC algorithm. This finding comes as no surprise since the SAD-RWA may select longer working routes if they result in a lower overall cost. As expected, the figure further shows that at very low loads, the average working path for the OLPBC is approximately equal to the calculated average shortest route of the network (1.56 link). Additionally, the figure shows that the average working path of the SAD-RWA algorithm decreases slightly as the load increases from 66 to 132 Erlang. The

![Average cost of working path vs. load](image)

Figure 7.5: Average working path cost on the COST 239 network with 12 channel link capacity
reason for this slight decrease is the improvement in resource sharing, which makes it possible to share backup resources without having to consider longer routes for the working paths. The increase in resource sharing also tends to decrease the effect of the applied load. At this low load range, both approaches have similar blocking performance as shown in Figure 7.1.

On the other hand, as the load increases beyond the 148 Erlang, Figure 7.5 shows an increase in the average working paths for both algorithms. Furthermore the figure shows that the OLPBC algorithm consumes more resources for its working paths than the SAD-RWA algorithm. An increase is expected in the required resources for the working paths of both algorithms with an increase in the load. It is also expected that as the load increased, resources become scarcer, and it becomes more difficult to satisfy the wavelength continuity constraint. Consequently, the routing algorithm may have to settle for longer routes to set up the incoming demands. However, what is not expected, because of the mechanism of the selection criteria of the SAD-RWA algorithm, is to observe that the SAD-RWA algorithm consumes fewer resources for its working paths than the OLPBC algorithm. The main reason for this unexpected observation is that the SAD-RWA algorithm has to satisfy the wavelength continuity constraint twice during the RWA process. Since the wavelength continuity constraint must be satisfied in the working and backup paths, demands with longer working paths stand less of a chance to succeed with the SAD-RWA algorithm than they do with the OLPBC algorithm. As a result, the average working path length is shorter when the SAD-RWA algorithm is employed than it is when the OLPBC algorithm is employed. Similar experiments have been carried out on the NSF, ARPANET and the test networks; their simulation results, shown in Figures 7.6-7.7, suggest identical findings.
Figure 7.6: Average working path cost with 12 channel link capacity
Figure 7.7: Average working path cost on the Test network with 12 channel link capacity

Based on the above discussion, the complexity comparison shown in Table 7.3 and on the simulation results shown in Figures 7.1-7.7, the following points can be claimed with high degree of certainty:

1. The OLPBC algorithm, using the p-cycles approach, has much lower computational complexity than the SAD-RWA algorithm, using the diverse routing approach. The lower computational complexity of the OLPBC algorithm makes it more suitable for the dynamic traffic than the SAD-RWA algorithm, especially at higher arrival rates. Furthermore, the OLPBC algorithm scales well with the network size and link capacity.

2. Although the SAD-RWA algorithm gives a false impression of having better blocking performance than the OLPBC algorithm, further investigation reveals
that the overall blocking performance of the OLPBC algorithm is in fact better. The SAD-RWA algorithm gives a false impression for the blocking performance because of its inability to accommodate demands with longer paths due to the wavelength continuity constraint in both the working and backup paths.

3. The blocking performance of the OLPBC algorithm can be enhanced further by employing the SLPR routing scheme instead of the SP routing scheme at no added cost.

4. The OLPBC algorithm is fairer to the demands with longer routes than the SAD-RWA algorithm as shown in Figures 7.5-7.6.

5. Taking the above points into consideration, the p-cycle approach through the OLPBC algorithm presents itself as a much better alternative than the diverse routing approach to solve the problem of survivable RWA under dynamic traffic in WDM wavelength-routed networks.
Chapter 8

Conclusion and Future Research

8.1 Concluding Remarks

The prime objective of this thesis is to study the problem of survivable RWA in WDM wavelength-routed mesh networks and to develop new algorithms that can allocate network resources efficiently while at the same time are able to recover from a failure quickly by rerouting the failed connection using the reserved spare capacity. The efficiency and the reliability of the network depend on the solution approach of the RWA problem. Improving network efficiency enables service providers to accommodate more lightpaths and reduce blocking at times of congestion. Furthermore, providing a reliable lightpath in the optical layer that can respond very quickly to failures, such as fiber cuts, is critically important to users and service providers, because it prevents the huge loss of data and the disruption of a large number of users.

A number of algorithms have been proposed to solve the problem of survivable RWA under static and dynamic traffic environments. Under both traffic environments, two different approaches have been pursued to solve the survivable RWA
and their performances are evaluated and compared. The first is based on diverse routing approach, where a working path and a backup path are setup for each lightpath. The second is based on the p-cycles approach, where a number of p-cycles are pre-configured in the spare capacity of the network to carry traffic during times of failure. Under static traffic, the survivable RWA is formulated as an ILP problem using both approaches.

For the diverse routing, an algorithm has been developed to solve the RWA problem. For the p-cycle approach, two different schemes have been developed to solve the static survivable RWA problem. The first scheme is called MBCF, and it employs a non-joint approach where the working capacity and the backup capacity problems are solved separately. Alternatively, for a more optimum solution, the RWA problem is formulated jointly. To reduce the number of candidate p-cycles in the formulation, a new ranking metric called RSE has been developed to select a subset of high merit p-cycles. Numerical results have confirmed that the RSE drastically reduces the number of cycles required for the formulation without compromising the optimality of the solution. The advantages of the MBCF scheme are its simplified complexity and fast computational times. However, its advantages are sometimes a tradeoff for the optimality of the solution.

A computer tool has been developed to perform the formulation for the static RWA problem for the diverse routing and the p-cycle approaches. The tool has been tested and the numerical results suggest that the solution computational time depends on the number of variables and the type of constraints used to formulate the problem. Shared protection problems using the diverse routing approach take a lot longer to solve than the dedicated protection and the p-cycles approach problems. The main reason for the longer time is the fixed cost constraint, rather than the
increase in the number of variables.

A new algorithm called SAD-RWA have been developed for survivable RWA under dynamic traffic with wavelength continuity constraint. The algorithm can find the optimum pair of working and backup paths for each individual demand. Simulation results have shown good blocking performance when the FF wavelength assignment scheme is used with the algorithm. However, the issue of computational complexity and the unfairness to connection demands with long routes are a big concern. Other assignment schemes have been proposed in the literature to alleviate the unfairness problem, but at the expense of increasing the computational complexity.

Two solution options have been adopted to solve the dynamic RWA using the p-cycle approach. The first option is based on the real-time reconfiguration of backup capacity (RRBC). Its objective is to re-optimize the locations and the number of p-cycles to adapt to changes in demand and reduce blocking probability. However, due to its high computational complexity, its practical application depends on the dynamic nature of the traffic. In contrast, the second option is based on the off-line configuration of backup capacity (OLPBC). Its objective is to reduce the computational complexity by configuring sufficient backup capacity to ensure full protection against any single span failure. Consequently, the OLPBC has much lower computational complexity than the RRBC which makes it more suitable for dynamic traffic, especially at higher arrival rates. Furthermore, simulation results have suggested that the OLPBC has better blocking performance than the RRBC.

To further improve the blocking performance of the OLPBC, a new routing scheme called SLPR have been proposed and its performance has been evaluated and compared with the shortest path (SP) and the load balanced routing (LBR) schemes.
The straddling link priority routing (SLPR) assigns a lower cost to straddling links during the calculation of working routes. Although simulation results have shown that the LBR has a slightly better blocking performance than the SLPR, its large control overhead and the computational complexity makes it unsuitable for highly dynamic traffic. On the other hand, the SLPR has no penalty for its improved blocking performance over the SP.

Finally, a comparative study has been carried out to evaluate and compare the performances of the two approaches (diverse routing and p-cycles) to solve the survivable RWA. For static traffic RWA, the complexity of formulating the shared protection scheme as an ILP problem using both approaches has been calculated and compared. For dynamic traffic RWA, the p-cycles approach does not only have a much lower computational complexity than the diverse routing approach, but it also has a better blocking performance. The low computational complexity of the p-cycles approach makes it more suitable for dynamic traffic, especially at higher arrival rates. Furthermore, the p-cycles approach can scale better with the network size than the diverse routing approach. The p-cycle approach has presented itself as a much better option than the diverse routing to solve the problem of survivable RWA under static and dynamic traffic environments in WDM wavelength-routed networks.

8.2 Future Research

The fact that all the variables used to formulate the static survivable RWA problem under the wavelength continuity constraint are binary variables makes the solution of the problem easier than if they were general integers. Furthermore, it is found that in most cases of interest, the optimum solution obtained when the integrity constraint
is relaxed (using LP solver) is the same as the optimum integer solution. This is possible in problems with multiple optimum solutions. Although the solutions obtained when the integrity constraint is relaxed have some non-integer variables, the LP solver may still be able to generate the other optimal solutions which contain at least one integer solution. Generating the integer solution using an LP solver is a lot faster than using the ILP solvers. However, more research is required to fully investigate the problem and identify the problems that may be solved using an ILP solver and still generate an integer solution.

All the algorithms for the survivable RWA developed in this thesis provide solutions for network survivability in the optical layer. These algorithms can not offer any backup against failures in the upper layers. An integrated multi-layer survivable scheme can improve network efficiency even further by reducing the overlap of backup resources in the different layers. However, the scheme has to address issues like the location of the failure and how to propagate it to the corresponding layer without creating chaos in the network.

It has been observed during the testing of the proposed survivable RWS algorithms that some degree of multiple failure protection is possible. Therefore, a possible future research can address the problem of multiple failure and how much extra resources are required to fulfill it. It is believed that the proposed algorithm can be extended to achieve survivability against possible multiple failure scenarios.
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Network Topologies

Figure 1: European COST239 network
Figure 2: National Science Foundation (NSF) network

Figure 3: The ARPANET network topology
Figure 4: Test network
Cycles Generation

All possible simple cycles in a mesh network can be generated using a breadth first search algorithm. The nodes of the network are ordered sequentially in ascending order according to their indexes from 0 to $N - 1$. To construct all possible cycles, a cycles search must be initiated at each node. However, since it takes at least three nodes to construct a simple cycle, only the first $N - 2$ nodes are considered as cycle initiators. At each cycle initiator, all possible cycles starting from that node are generated by recursively inspecting its neighbor nodes. In order to avoid generating cycles that have been already generated by previous cycle initiators, only neighbors with indexes higher than the index of the current cycle initiator are considered. Furthermore, to generate simple cycles, each node with the exception of the initiator node is visited only once in any cycle. Figure 5 shows a flowchart for cycle generation in a mesh network.
Figure 5: Flow chart for cycles generation

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Random variable Generation

The computer simulation of any random process requires the generation of random variables. For example, the simulation of the different events in a communication network involves generating the time between the arrivals of connection requests (inter-arrival time) as well as the holding time of each connection request. These random quantities are generated according to a certain random process. For example, throughout this thesis, it is assumed that connection requests arrive at network nodes according to Poisson process. Poisson random variable arises in situations where the events occur completely at random in time. Poisson arrivals imply that inter-arrival and holding times are exponentially distributed. The cumulative distribution function (cdf) for the exponential distribution is given by the equation

\[ D(T) = 1 - e^{\lambda T} \]  \hspace{1cm} (1)

Where;
\[ D(T) = P[\text{inter-arrival time }] \leq t \]
\[ \lambda = \text{average arrival rate} \]
\[ D(T) \text{ is uniformly distributed in}[0, 1] \]

To generate a random value for the inter-arrival time \( t \), we need to generate a random value for its corresponding probability \( D(T) \) and then use the inversion method on
equation 1 to calculate the random value for the inter-arrival time $t$.

From equation 1

$$D(T)^{-1} = t = \frac{-1}{\lambda} \ln (1 - D(T))$$ (2)

\[\therefore (1 - D(T)) \text{ is also uniformly distributed in}[0, 1]\]

$$t = D(T)^{-1} = \frac{-1}{\lambda} \ln U$$

Where $U$ is a random number uniformly distributed between $0 - 1$ and can either be generated using a built in function in the system or by a special function. A similar approach is used to generate the holding time using the average holding time
Confidence Intervals

Simulated quantities such as blocking probability are measured by taking the mean of a succession of \( n \) runs, each of long enough time to ensure uncorrelated results. All runs are identical and independent from each other. The \( n \) independent results will be represented by \( B_1, B_2, B_3, \ldots, B_{n-1}, B_n \).

Where \( B_i \) is the average blocking probability obtained from the simulation run \( i \).

\[
\text{The mean } \bar{B} = \frac{1}{n} \sum_{i=1}^{n} B_i
\]  

However, the mean of the independent simulation runs \( \bar{B} \) provide us with a single numerical value for the estimate of the expected value \( E[B] = \mu \). In order to know how good is the estimate provided by \( \bar{B} \) for the simulation results, it is necessary to compute the variance \( \text{V}_B^2 \).

\[
\text{The Variance } \text{V}_B^2 = \frac{1}{n-1} \sum_{i=1}^{n} (B_i - \bar{B})^2
\]  

Small \( \text{V}_B^2 \) indicates that the results are tightly clustered around \( \bar{B} \) and we can be confident that \( \bar{B} \) is close to the \( E[B] \). On the other hand, if \( \text{V}_B^2 \) is large, the results are widely dispersed about \( \bar{B} \) and we cannot be confident that \( \bar{B} \) is close to the \( E[B] \). Instead of seeking a single value to estimate the \( E[B] \), we can specify an
interval of values that is highly likely to contain the true value of the parameter. We begin by specifying some high probability, say $1 - \alpha$, we then find an interval $[L(B), U(B)]$ such that:

$$\text{The Probability } P[L(B) \leq \mu \leq U(B)] = 1 - \alpha \quad (5)$$

This interval contains the true value of the parameter with probability $1 - \alpha$. Such an interval is a $(1 - \alpha) \times 100\%$ confidence interval.

Using the standard deviation and the $t$ distribution table, the lower and upper limits of the 95% confidence interval can be calculated as follows:

$$\text{Lower Limit } L(B) = \bar{B} - \frac{\sigma t_{1-\frac{\alpha}{2}, n-1}}{\sqrt{2}} \quad (6)$$

$$\text{Upper Limit } U(B) = \bar{B} + \frac{\sigma t_{1-\frac{\alpha}{2}, n-1}}{\sqrt{2}} \quad (7)$$

Where:

$\alpha = 0.05$

$n = \text{number of observations}$

$\bar{B} = \text{sample average}$

$\sigma = \text{sample standard deviation} = \sqrt{V_b^2} = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (B_i - \bar{B})^2}$

The confidence interval means that 95% of the simulation results falls within the interval. Throughout this thesis the confidence interval is computed based on 7 independent runs. From the table of the $t$ distribution, the $t_{1-\frac{\alpha}{2}, n}$ is found to be 2.447. It was observed that more than 95% of the results were within the calculated confidence interval for each experiment. Table 1 shows an example of how the confidence interval is calculated. The table show the blocking performance of the off-line planning of backup capacity of p-cycles algorithm when simulated on the
COST239 European network with a capacity of 12 channels per unidirectional link. The probabilities of seven independent runs are shown together with the calculated average $\bar{B}$, the lower and upper values of the interval $L(B) - U(B)$. Therefore, it can be concluded that all simulation results carried out in this thesis have 95% confidence.

Table 1: Example of Confidence interval calculations

<table>
<thead>
<tr>
<th>Load (Erlang)</th>
<th>$B_1$</th>
<th>$B_2$</th>
<th>$B_3$</th>
<th>$B_4$</th>
<th>$B_5$</th>
<th>$B_6$</th>
<th>$B_7$</th>
<th>$\bar{B}$</th>
<th>$\sigma$</th>
<th>L(B)</th>
<th>U(B)</th>
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# Static demand matrices

**Randomly generated set of demands on COST239 network**

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Table 7: NSFNET demand matrix with a maximum of 7 lightpaths

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Table 8: Test network demand matrix with a maximum of 3 lightpaths

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Table 9: Test network demand matrix with a maximum of 7 lightpaths

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