Omemah Rajab Gliyah

Auteur de la thèse / Author of thesis

M.A.Sc. (Chemical Engineering)

Grade / Degree

Department of Chemical Engineering

Faculté, École, Département / Faculty, School, Department

Water Extraction from the Atmosphere

Titre de la thèse / Title of thesis

Dr. J. Thibault

Directeur (Directrice) de la thèse / Thesis supervisor

D. B. Kruczek

Co-directeur (Co-directrice) de la thèse / Thesis co-supervisor

Examinateurs (Examinatrices) de la thèse / Thesis examiners

Dr. K. Kennedy

Dr. D. McLean

Gary W. Slater

Le doyen de la Faculté des études supérieures et postdoctorales / Dean of the Faculty of Graduate and Postdoctoral Studies
Water Extraction from the Atmosphere

by

Omemah Rajab Gliah

A thesis submitted to the
Faculty of Graduate and Postdoctoral Studies
in partial fulfillment of the requirements for the
Master of Applied Science degree
in Chemical Engineering

Department of Chemical Engineering
Faculty of Engineering
University of Ottawa
Ottawa, Canada

© Omemah Rajab M Gliah, Ottawa, Canada, 2008
NOTICE:
The author has granted a non-exclusive license allowing Library and Archives Canada to reproduce, publish, archive, preserve, conserve, communicate to the public by telecommunication or on the Internet, loan, distribute and sell theses worldwide, for commercial or non-commercial purposes, in microform, paper, electronic and/or any other formats.

The author retains copyright ownership and moral rights in this thesis. Neither the thesis nor substantial extracts from it may be printed or otherwise reproduced without the author's permission.

In compliance with the Canadian Privacy Act some supporting forms may have been removed from this thesis.

While these forms may be included in the document page count, their removal does not represent any loss of content from the thesis.

AVIS:
L'auteur a accordé une licence non exclusive permettant à la Bibliothèque et Archives Canada de reproduire, publier, archiver, sauvegarder, conserver, transmettre au public par télécommunication ou par l'Internet, prêter, distribuer et vendre des thèses partout dans le monde, à des fins commerciales ou autres, sur support microforme, papier, électronique et/ou autres formats.

L'auteur conserve la propriété du droit d'auteur et des droits moraux qui protège cette thèse. Ni la thèse ni des extraits substantiels de celle-ci ne doivent être imprimés ou autrement reproduits sans son autorisation.

Conformément à la loi canadienne sur la protection de la vie privée, quelques formulaires secondaires ont été enlevés de cette thèse.

Bien que ces formulaires aient inclus dans la pagination, il n'y aura aucun contenu manquant.
Abstract

This investigation is driven by the desire to find an alternative method of obtaining fresh water. More specifically, fresh water is to be obtained by condensation of atmospheric water vapor on a surface maintained below the dew point by a radiative heat loss from its surface to the night sky. To properly simulate the condensation process, an accurate estimation of the effective sky temperature is required.

To estimate the effective night sky temperature an experimental system, consisting of a series of metal plates embedded on a heat transfer panel, a weather station and a control system, was built. The temperature of each plate was controlled using an electric flat-plate heater. A control loop was implemented to ensure that the heat transfer panel was adjusted automatically to always be oriented perpendicular to the wind direction. In addition to the effective night sky temperature, the experimental system was designed to estimate the convective heat transfer coefficient between the plates and ambient air as well as the surface emissivity of the plates. The results obtained from theoretical analyses and preliminary experiments are presented and discussed. Special emphasis was given to mathematical solution of the system of equations that need to be solved to obtain the effective sky temperature. It was shown that, although the solution of the system of equations was straightforward for the direct heat transfer problem, there was a serious difficulty to solve the inverse heat transfer problem to retrieve the desired parameters.
Résumé

Ce projet de recherche est motivé par le désir de développer des méthodes alternatives pour obtenir une source supplémentaire d’eau potable. Plus spécifiquement, cette méthode prévoit récupérer l’eau contenue dans l’atmosphère par condensation sur une surface maintenue sous le point de rosée par les pertes de chaleur occasionnées par le rayonnement thermique dans des conditions de ciel clair. Pour être en mesure de simuler le phénomène de condensation, une estimation relativement précise de la température effective du ciel doit être disponible.

Pour estimer la température effective du ciel, un système expérimental a été construit. Il comprend une série de plaques métalliques disposées sur un panneau de transfert de chaleur, une station météorologique et un système de contrôle. Chaque plaque est encastrée dans un panneau d’isolation et placée sur une plaque chauffante électrique afin de contrôler sa température. Une boucle de contrôle a aussi été mise en œuvre pour s’assurer que toutes les plaques métalliques soient orientées dans une direction perpendiculaire au vent. En plus de la température effective du ciel, le système expérimental permet l’estimation du coefficient de convection de transfert de chaleur ainsi que de l’émissivité des plaques métalliques. Les résultats des études expérimentales et théoriques sont présentés et discutés. Une emphase particulière est mise sur la solution du système d’équations nécessaire pour estimer la température effective du ciel. Il est montré que le système d’équations se solutionnent aisément pour le problème direct de transfert de chaleur mais il est presqu’impossible de solutionner le problème inverse qui permettrait de déterminer les paramètres désirés.
Acknowledgement

First, I would like to thank my supervisors, Dr. Jules Thibault and Dr. Boguslaw Kruczek for giving me the opportunity to work on this project. In addition, I would like to thank them for their valuable guidance and suggestions throughout this thesis.

I would also like to thank Gerard Nina, Louis Tremblay, Frank Ziroldo and all the staff of the Chemical Engineering Department for making this journey a pleasant one.

I would like to offer my special thanks to my husband and parents for their endless support and encouragement which truly contributed to the completion of this project.

I would like to thank my kids for their patience, siblings and friends.

Finally, I would like to offer my sincere gratitude to the Libyan High Education Board for giving me the opportunity to continue my education and their financial support.
# Nomenclature

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>Surface area of the plate</td>
<td>m²</td>
</tr>
<tr>
<td>c(_p)</td>
<td>Heat capacity</td>
<td>J/kg.K</td>
</tr>
<tr>
<td>D(_{AB})</td>
<td>Diffusivity</td>
<td>m²/s</td>
</tr>
<tr>
<td>Gr</td>
<td>Grashof number</td>
<td>-</td>
</tr>
<tr>
<td>g</td>
<td>Gravitational constant</td>
<td>m/s²</td>
</tr>
<tr>
<td>H</td>
<td>Humidity</td>
<td>kg H₂O/kg dry air</td>
</tr>
<tr>
<td>h</td>
<td>Convective heat transfer coefficient</td>
<td>W/m².K</td>
</tr>
<tr>
<td>I</td>
<td>Electric current</td>
<td>A</td>
</tr>
<tr>
<td>k</td>
<td>Heat conductivity coefficient</td>
<td>W/m.K</td>
</tr>
<tr>
<td>K(_c)</td>
<td>Mass transfer coefficient</td>
<td>m/s</td>
</tr>
<tr>
<td>K</td>
<td>Mass transfer coefficient</td>
<td>kg/m².s</td>
</tr>
<tr>
<td>L</td>
<td>Length of plate</td>
<td>m</td>
</tr>
<tr>
<td>M</td>
<td>Molar mass</td>
<td>kg/mol</td>
</tr>
<tr>
<td>Nu</td>
<td>Nusselt number</td>
<td>-</td>
</tr>
<tr>
<td>Pr</td>
<td>Prandtl number</td>
<td>-</td>
</tr>
<tr>
<td>P</td>
<td>Pressure</td>
<td>kPa</td>
</tr>
<tr>
<td>Q</td>
<td>Energy flow</td>
<td>J/s</td>
</tr>
<tr>
<td>Ra</td>
<td>Rayleigh number</td>
<td>-</td>
</tr>
<tr>
<td>Re</td>
<td>Reynolds number</td>
<td>-</td>
</tr>
<tr>
<td>RH</td>
<td>Relative humidity</td>
<td>%</td>
</tr>
<tr>
<td>S</td>
<td>Radiation</td>
<td>W/m²</td>
</tr>
<tr>
<td>Sc</td>
<td>Schmidt number</td>
<td>-</td>
</tr>
<tr>
<td>St</td>
<td>Stanton number</td>
<td>-</td>
</tr>
<tr>
<td>t</td>
<td>Time</td>
<td>s</td>
</tr>
<tr>
<td>T</td>
<td>Temperature</td>
<td>°C or K</td>
</tr>
<tr>
<td>V</td>
<td>Voltage</td>
<td>V</td>
</tr>
<tr>
<td>v</td>
<td>Velocity</td>
<td>m/s</td>
</tr>
<tr>
<td>Symbol</td>
<td>Definition</td>
<td>Unit</td>
</tr>
<tr>
<td>--------</td>
<td>------------</td>
<td>------</td>
</tr>
<tr>
<td>(\beta)</td>
<td>Volumetric coefficient of expansion</td>
<td>(K^{-1})</td>
</tr>
<tr>
<td>(\varepsilon)</td>
<td>Emissivity</td>
<td>-</td>
</tr>
<tr>
<td>(\lambda)</td>
<td>Heat of condensation or evaporation</td>
<td>(\text{kJ/kg})</td>
</tr>
<tr>
<td>(\rho)</td>
<td>Density</td>
<td>(\text{kg/m}^3)</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>Stefan-Boltzmann constant</td>
<td>(5.67 \times 10^{-8} \text{W/m}^2.\text{K}^4)</td>
</tr>
<tr>
<td>(\nu)</td>
<td>Kinematic viscosity</td>
<td>(\text{m}^2/\text{s})</td>
</tr>
<tr>
<td>(\mu)</td>
<td>Dynamic viscosity</td>
<td>(\text{kg/m.s})</td>
</tr>
<tr>
<td><strong>Subscripts</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>a</td>
<td>Air</td>
<td></td>
</tr>
<tr>
<td>Avg.</td>
<td>Average</td>
<td></td>
</tr>
<tr>
<td>cond.</td>
<td>Condensation</td>
<td></td>
</tr>
<tr>
<td>conv.</td>
<td>Convection</td>
<td></td>
</tr>
<tr>
<td>d</td>
<td>Dew point</td>
<td></td>
</tr>
<tr>
<td>f</td>
<td>Forced convection</td>
<td></td>
</tr>
<tr>
<td>H(_2)O</td>
<td>Water</td>
<td></td>
</tr>
<tr>
<td>n</td>
<td>Natural convection</td>
<td></td>
</tr>
<tr>
<td>p</td>
<td>Plate</td>
<td></td>
</tr>
<tr>
<td>rad.</td>
<td>Radiation</td>
<td></td>
</tr>
<tr>
<td>s</td>
<td>Saturation</td>
<td></td>
</tr>
<tr>
<td>S, sky</td>
<td>Sky (use interchangeably)</td>
<td></td>
</tr>
<tr>
<td>V</td>
<td>Vapor</td>
<td></td>
</tr>
<tr>
<td>w</td>
<td>Condition at 100% humidity</td>
<td></td>
</tr>
</tbody>
</table>
# Table of Contents

Abstract ................................................................................................................................. i  
Résumé .................................................................................................................................. ii  
Acknowledgement .................................................................................................................. iii  
Nomenclature .......................................................................................................................... iv  
Table of Contents ................................................................................................................... vi  
List of Tables ........................................................................................................................... viii  
List of Figures ........................................................................................................................ vix  

## Chapter 1 Introduction

1.1 Water extraction from the atmosphere ............................................................................. 1  
1.2 Effective sky temperature ................................................................................................. 6  
1.3 Objectives ......................................................................................................................... 7  
1.4 Organization of the thesis ................................................................................................. 8  

## Chapter 2 Theoretical background

2.1 Estimation of the sky emissivity ......................................................................................... 9  
2.2 Energy balance on a horizontal plate insulated at the bottom ........................................ 17  

## Chapter 3 Simulation of atmospheric water collection

## Chapter 4 Experimental system for the evaluation of the effective sky temperature

4.1 The concept of a six-plates system for the measurement of the effective sky temperature ........................................................................................................ 34  
4.2 The design and description of six-plates system ............................................................... 38  
4.2.1 Selection of the plates ................................................................................................. 38  
4.2.2 Arrangement of the plates ......................................................................................... 39  
4.2.3 Configuration of the system ...................................................................................... 40  
4.2.4 Specifications of system components ....................................................................... 42  

## Chapter 5 Results and Discussion

5.1 Theoretical Analysis .......................................................................................................... 43
<table>
<thead>
<tr>
<th>Section</th>
<th>Title</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1.1</td>
<td>Power required for a fixed plate temperature</td>
<td>44</td>
</tr>
<tr>
<td>5.2</td>
<td>Emissivity determination</td>
<td>47</td>
</tr>
<tr>
<td>5.3</td>
<td>Estimation of power losses</td>
<td>48</td>
</tr>
<tr>
<td>5.4</td>
<td>Experimental data analysis</td>
<td>50</td>
</tr>
<tr>
<td>5.4.1</td>
<td>Dynamic energy balance under natural convection</td>
<td>51</td>
</tr>
<tr>
<td>5.4.1.1</td>
<td>Experiment at low temperature of surroundings</td>
<td>52</td>
</tr>
<tr>
<td>5.4.1.2</td>
<td>Experiment at room temperature</td>
<td>53</td>
</tr>
<tr>
<td>5.4.2</td>
<td>Steady-state energy balance under natural convection</td>
<td>54</td>
</tr>
<tr>
<td>5.4.2.1</td>
<td>Six non-linear equations with six unknowns</td>
<td>54</td>
</tr>
<tr>
<td>5.4.2.2</td>
<td>Eight linear equations with eight unknowns</td>
<td>59</td>
</tr>
<tr>
<td>5.4.2.3</td>
<td>Three linear equations with three unknowns</td>
<td>61</td>
</tr>
<tr>
<td>5.4.2.4</td>
<td>Two linear equations with two unknowns</td>
<td>62</td>
</tr>
<tr>
<td>5.4.2.5</td>
<td>One equation with one unknown</td>
<td>64</td>
</tr>
<tr>
<td>Chapter 6</td>
<td>Conclusions</td>
<td>65</td>
</tr>
<tr>
<td>References</td>
<td></td>
<td>67</td>
</tr>
<tr>
<td>Appendix A</td>
<td>Calculation of air properties</td>
<td>70</td>
</tr>
<tr>
<td>Appendix B</td>
<td>Prediction of convective heat transfer coefficient</td>
<td>73</td>
</tr>
</tbody>
</table>
List of Tables

Table 4.1 Specification of the properties of the metal plates ------------------------ 38
Table 4.2 Summary of instrumentation in the six-plate system --------------------- 42
Table 5.1 Estimates of the emissivity of the three plates used in this investigation--- 48
Table 5.2 Conditions for the simulation of the six-plate system ------------------ 56
# List of Figures

- Figure 2.1 Clear sky emissivity as a function of dew point temperature .......................... 12
- Figure 2.2 Deviation of effective sky emissivity obtained with different correlations at 85% relative humidity and in the range of 283-293 K air temperature .......................... 13
- Figure 2.3 Effective sky temperature at 85% RH versus the air temperature .......................... 14
- Figure 2.4 Comparison of different correlations for the prediction of the sky emissivity versus the relative humidity at $T_a = 293$ K .......................................................... 15
- Figure 2.5 Variation of the effective sky temperature associated with the sky emissivity of Figure 2.4 for different correlations at 293 K air temperature and as a function of the relative humidity .......................................................... 15
- Figure 2.6 Heat transfer phenomena taking place on a horizontal plate, insulated at the bottom and exposed to clear night sky .......................................................... 17
- Figure 3.1 Effect of the effective sky temperature on the equilibrium plate temperature for three different air temperatures .......................................................... 26
- Figure 3.2 Effect of the effective sky temperature on the water condensation rate for three different air temperatures .......................................................... 27
- Figure 3.3 Effect of air temperature on the water condensation rate for two correlations to predict the effective sky emissivity .......................................................... 29
- Figure 3.4 Effect of the relative humidity on the water condensation rate for two correlations to predict the effective sky emissivity .......................................................... 30
- Figure 3.5 Effect of wind speed on the water condensation rate for two correlations to predict the effective sky emissivity .......................................................... 31
- Figure 3.6 Effect of plate emissivity on the water condensation rate for two correlations to predict the effective sky emissivity .......................................................... 32
- Figure 3.7 Effect of the plate's characteristic length on the water condensation rate ............ 33
- Figure 4.1 Dimensions of heat transfer panel .......................................................... 39
- Figure 4.2 Overall configuration of the six-plate system .......................................................... 40
- Figure 4.3 Electrical connections of one plate of the six-plate system .......................................................... 41
Figure 5.1 The effect of the plate temperature on the power required to keep a steady-state plate temperature. Individual contributions of convective and radiative heat transfer are shown. 45
Figure 5.2 The effect of the plate emissivity on the power required to maintain the plate temperature at 303 K when the air temperature is 295 K. 45
Figure 5.3 The effect of the wind speed on the required power when the plate temperature is 303 K, the air temperature at 295 K and the effective sky temperature at 275 K. 46
Figure 5.4 The effect of the effective sky temperature on the required power. 47
Figure 5.5 The temperature profile versus time when the upper plate surface is covered with an insulating board. 49
Figure 5.6 Rate of heat loss versus plate temperature. 50
Figure 5.7 Comparison of the predicted and experimental plate temperatures as a function of time under conditions of a low temperature of surroundings. 52
Figure 5.8 Predicted and measured plate temperature versus time for an experiment performed at room temperature. 53
Figure 5.9 Number of occurrences of the sky temperature for 1000 simulations. 57
Figure 5.10 The two heat transfer coefficients as a function of the sky temperature for the 1000 simulations. 58
Figure 5.11 Estimations of the three plate emissivities as a function of the sky temperature for the 1000 simulations. 58
Figure 5.12 Plot of the objective function as a function the estimated sky temperature for the 1000 simulations trials. 59
Figure 5.13 Effect of a small error in the power input on the value of the effective sky temperature at many values of the plate emissivity. 63
Figure 5.14 Effect of a small error in the power input on the value of the convective heat transfer coefficient for many values of the plate emissivity. 63
Figure 5.15 The sensitivity of effective sky temperature versus the error in input energy. 64
Figure A.1 Air thermal conductivity versus temperature. 70
Figure A.2 Air density versus temperature. 71
Figure A.3 Air kinematic viscosity versus temperature. 71
Figure A.4 Air dynamic viscosity versus temperature 72
Figure A.5 Air heat capacity versus temperature 72
Figure B.1 A schematic of a horizontal insulated plate exposed to a clear sky 73
Figure B.2 Effect of the difference between the air temperature and the plate temperature on the convective heat transfer coefficient at different wind velocity 75
Figure B.3 Effect of the wind velocity on the heat transfer coefficient at different values of the difference between air temperature and plate temperature 76
Chapter 1

Introduction

1.1 Water Extraction from the Atmosphere

Water, more precisely fresh water, is a basic human necessity to support life, as important as air and food. Fresh water is becoming scarce and this scarcity is becoming one of the world's biggest problems. The lack of fresh water is currently one of the main obstacles to the economic development of many countries, a problem that does not only belong to developing countries but is widespread across the globe. Recent studies show that an inhabitant of a modern town consumes 100-400 liters of water daily. This is of course an inhabitant of "rich" countries as, even now, in some developing countries the average consumption does not exceed 20-30 liters per day. Furthermore, annual water consumption continues to grow, increasing four fold over the last 50 years to stand at over $3 \times 10^{15}$ liters ($3,000$ km$^3$). This level of consumption is unlikely to decrease (Beysens and Milimouk, 2000).

Water comes in many forms: spring water, streams, seas, rivers, lakes, underground aquifers, rain, fog, and water vapour. Surface water and in some countries underground waters need to be treated before they can be used for humans. In many countries conventional water purification systems, such as reverse osmosis, may be adequate if water source and utilization locations are geographically near to one another. In most geographic areas, however, impaired water sources are likely to be far from the actual utilization point. In addition there are several issues facing the purification technologies such as corrosion problems and high energy consumption. In such case, being able to
extract water from air offers substantial advantages because the water does not have to be transported from a distant source to a local storage facility. It is possible to establish a sustainable potable water supply at virtually any location if one can develop a technology that efficiently harvests water from air (Sciperio, 2005).

Extraction of water from the atmospheric air is considered one of the potential methods of fresh water supply, because air, as a source of water, is renewable, clean and exists anywhere (Sultan, 2004). Air, composed primarily of nitrogen (78%) and oxygen (21%), contains a varying amount of water in vapour form, depending on its temperature and degree of saturation. The amount of water in the atmosphere is calculated from its partial pressure \((p_v)\) within the air mass. At a given temperature, the partial pressure cannot exceed a certain level without condensation occurring; this is the saturation pressure \((p_s)\). The relative humidity \((RH)\) is then defined as the ratio of partial pressures \((RH = p_v/p_s)\). The \(p_v\) rises in conjunction with an increase in air temperature and the water mass capacity of 1 m\(^3\) of air also rises (Beysens and Milimouk, 2000). The atmosphere contains a large quantity of water in the form of vapour in varying amounts, ranging from between 10.7 and 27 mg per kg of dry air at saturation temperatures between 15°C and 30°C. This endless source of water can be recovered for general use (Gandhidasan and Abualhamayel, 1996).

The extraction of water from the atmospheric air can be accomplished using several methods. The investigations performed for the extraction of water can be classified into two main groups. The first group deals with cooling of air, while the second group deals with the absorption-regeneration of moisture directly from the surrounding air (Sultan, 2004).
The extraction of water from the atmospheric air can be achieved by passing humid ambient atmospheric air over a cooling coil and the moisture is condensed if the coil temperature is less than the dew point temperature of air. This condensation unit comprises a compressor, a fan, a cooling coil, a refrigerant and a collection reservoir. This principle was experimentally studied (Nebbia, 1961) to obtain fresh water from the atmosphere. It is reported that the energy consumption was high due to low heat exchange process and the amount of water obtained was low with small units. This method was proposed for emergency use.

Another approach for extraction of water from the atmosphere is by absorption of water from the moist air into a solid or liquid desiccant with subsequent separation of the water from the desiccant. A non-conventional system was proposed (Sultan, 2004) for obtaining water from the outdoor air based on condensation of humidity by using a liquid desiccant. For a solid desiccant, it was observed that as the time progresses, the efficiency of the desiccant bed will be reduced due to dust and foreign matter deposited in the pores. To overcome this problem, an air filter can be added but only at the cost of an additional air pressure drop through the system. These drawbacks make the desiccant system less attractive (Gandhidasan and Abualhamayel, 1996). These systems are more appropriate to reduce the humidity content of the air but are not really efficient for the recovery of water from air.

Another method can be used to collect atmospheric water which uses nocturnal radiative cooling. This approach uses a condenser made from special foil that can benefit from radiative cooling. Water vapour is condensed on the upper surface of the foil when its temperature becomes lower than the dew point temperature. The advantage of this
method is that the labour and energy input is practically negligible, furthermore atmospheric water is high quality and can be used for drinking. On the other hand, quantity of water that can be collected is very low.

For the purposes of condenser optimization, meteorological parameters such as air temperature, air humidity (that define the dew temperature), and the sky radiation (cloud cover) are imposed and, therefore, cannot be adjusted. In order to increase the yield of dew harvesting, it is however, possible to (i) maximize the long wave-length emitting properties of the condenser surface (near infra-red), (ii) minimize the short wave-length absorption (sun visible light), (iii) lower the wind velocity on the condenser surface, (iv) increase the condensation time, and (v) recover most of the water drops (Beysens et al. 2003).

Khedari et al. (2000) studied variables that affect the night cooling radiation. Since the amount of radiant energy emitted by a plate depends on the temperature difference between the plate surface and sky, an examination of four types of radiators under different sky conditions was performed. The depression of the surface temperature was in the range of 1-6°C depending on:

- The thermal emissivity of the exposed surface material; material with a high emissivity emits more radiation and, therefore, achieves better cooling.
- The heat capacity decreases as the thickness of the plate decreases. In daytime, the radiator stores less energy; this means that shorter time is needed for radiation heat exchange to cool the radiator when the sun has set.
- The percentage of cloud coverage as the surface temperature increases when the sky is covered by cloud, thus limiting the night radiation cooling.
• The water condensation will give away the heat of condensation to the plate such that the temperature of the plate will be higher compared to a plate where condensation occurs. It is desired to have as much condensation as possible but condensation inhibits further condensation.

To properly design a condensation system, accurate measurements of the above variables is required. Since the radiation phenomenon strongly contributes to the condensation process, an accurate relationship for radiation parameters is needed in order to predict the rate at which heat is transferred by radiation. This accurate prediction can only be achieved if a good estimation of sky temperature is available. Unfortunately, as it will be shown later, available correlations to predict the effective sky temperature show significant variability.

In this study, an experimental system was designed to determine the effective sky temperature and then to derive a correlation that would allow its reliable prediction under various atmospheric conditions. As an additional benefit, the experimental system was designed to also allow:

• Derivation of correlations to estimate the convective heat transfer coefficient.
  
  Results are compared with those obtained by other researchers.

• Estimation of the emissivity of the plate surface.
1.2 Effective sky temperature

The determination of the effective sky temperature has recently become an important research area. Global warming/cooling may adversely affect human settlements by modifying sea levels as well as regional climatic patterns (Dhirendra et al., 1995). The prediction of the effective sky temperature is required to perform an overall heat balance around the earth’s atmosphere. In this thesis, the effective sky temperature is required because it is desired to simulate and design the most suitable atmospheric water recovery system. As it will be discussed shortly. The value of the effective sky temperature is required for the prediction of the radiative heat loss from the water collection plate. The effective sky temperature is almost invariably lower than the atmospheric temperature because the atmospheric temperature decreases with elevation and the atmosphere is partly transparent to radiation of certain wave-bands within the infrared region of the spectrum. On the planetary scale, the infrared exchange between the earth and sky and between the atmosphere and space allows the earth to maintain an equilibrium temperature by emitting large quantities of heat gained each day from the sun. The use of cold sky as a heat sink for radiating bodies on the earth's surface may provide a promising cooling technique. A quantitative understanding of sky radiation is necessary for the design of radiant cooling systems. For example, the ability to predict sky radiation accurately, along with knowledge of the radiative and other heat transfer characteristics of surfaces, can be used to help designing atmospheric vapour condensation systems. It can also be used to help designing buildings, which would remain cool without mechanical conditioners (Berdahl and Fromberg, 1982).
The net radiation loss \( Q_{\text{rad}} \) from the outer surface of bodies exposed to the sky can be expressed by:

\[
Q_{\text{rad}} = \varepsilon_p \sigma A (T_p^4 - T_s^4)
\]

(1.1)

where, \( \varepsilon_p \) is the emissivity of the outer emitting surface, \( \sigma \) is Stefan Boltzmann constant, \( A \) is the surface area, \( T_p \) is the outer surface temperature, and \( T_s \) is the effective sky temperature. The effective sky temperature is commonly estimated using the following equation:

\[
T_s = (\varepsilon S T_a^4)^{1/4}
\]

(1.2)

where, \( \varepsilon S \) is sky emissivity, and \( T_a \) is air temperature. All temperatures in Eqs. 1.1 and 1.2 are in degrees Kelvin (Mills, 1999). If the emitted radiation of a surface exceeds the absorbed radiation, the surface will cool (Berdahl and Fromberg, 1982) and, if the temperature of the surface decreases below the dew point, condensation will occur on the upper surface of the emitting body.

1.3 Objectives

The main objective of the thesis is to design and build an experimental system that will allow determination of the effective night sky temperature under various conditions and deriving a correlation to accurately predict its value. As a secondary objective, the experimental system was also designed to estimate, at the same time, the emissivities of the exposed plates and the heat transfer coefficient.
1.4 Organization of the thesis

The thesis is divided into six chapters. Chapter 2 provides the theoretical background to perform complete steady and dynamic-state heat balances. In this chapter, an emphasis is placed on the existing correlations of the effective sky temperature to clearly show the variability of its estimation. Chapter 3 presents the simulation of a potential atmospheric water collection system under various operating conditions. Chapter 4 describes the experimental system that was designed in this research project. Chapter 5 presents and discusses the results obtained from theoretical analyses and with the experimental system. An emphasis is placed in this chapter on the mathematical solution of the system of equations that need to be solved to obtain the effective sky temperature. It is shown that although the system of equations works well for the direct heat transfer problem, there is serious difficulty to solve the inverse heat transfer problem to retrieve the desired parameters. Finally Chapter 6 presents the main conclusions of this investigation.
Chapter 2

Theoretical Background

This chapter presents a brief literature review of the main correlations that were proposed in the literature to estimate the effective sky temperature. These correlations are then compared under conditions that would realistically prevail for an atmospheric water collection system to show their significant variability and unequivocally justify designing and constructing an experimental system and performing experiments to determine the effective sky temperature. For this investigation, it will be assumed that the plate that would be used for collecting water is a horizontal flat plate and the basic heat balance on this particular geometry will be presented and briefly discussed.

2.1 Estimation of the sky emissivity

The emissivity ($\varepsilon$) is a radiative property of the surface, and dimensionless quantity, with values in the range $0 \leq \varepsilon \leq 1$. It provides a measure of how efficiently a surface emits energy relative to a black body (Incropera and DeWitt, 1996). The black surface is defined as a surface that absorbs all incident radiation, reflecting none. So all of the radiation leaving a black surface is emitted by the surface (Mills, 1999).

Sky emissivity is a measure of the atmosphere’s ability to transfer heat by radiation. It depends on atmospheric temperature and water vapour content (cloud cover and humidity) (Chen et al., 2008).

A large number of semi-empirical equations have been proposed in an attempt to best represent experimental measurements of clear sky radiation. The most commonly
used variables are the dew point temperature $T_d$ and water vapour partial pressure $P_v$. Angstrom’s equation was introduced in 1916 (Berdahl and Fromberg, 1982) and relates the effective sky emissivity with an exponential function of the water vapour partial pressure $P_v$.

$$\varepsilon_s = A - B \exp\left(-CP_v\right) \quad (2.1)$$

where $A$, $B$, and $C$ are empirical constants which have been assigned values in the ranges (0.75—0.82), (0.15—0.33) and (0.09—0.22), respectively. The most frequently used values are 0.806, 0.236 and 0.092, respectively for $A$, $B$ and $C$ when the partial pressure $P_v$ is evaluated in mbar. Knowing the effective sky emissivity, the effective sky temperature is calculated using Eq. 1.2.

A similar relationship derived from experimental data was given by Brunt (1932) as a function of the square root of water vapour partial pressure.

$$\varepsilon_s = A + BP_v^{1/2} \quad (2.2)$$

It has the advantage to only contain two adjustable parameters instead of three. The value for $A$ falls in the range $(0.34 - 0.71)$ and the value for $B$ lies in the range $(0.023 - 0.110)$. Sellers (Berdahl and Fromberg, 1982) suggested using values of $(A = 0.605)$ and $(B = 0.048)$ for equation (2.2).

$$\varepsilon_s = 0.605 + 0.048P_v^{1/2} \quad (2.3)$$

Brunt (1932) obtained $(A = 0.55)$ and $(B = 0.065)$, based on data collected by Dines 1927.

$$\varepsilon_s = 0.55 + 0.056P_v^{1/2} \quad (2.4)$$

Three additional studies performed after 1955 provide values of $A$ and $B$ close to those found by Brunt.
Brunt’s argument for using the square root of the water vapour pressure is based on an analogy between heat transfer by conduction and heat transfer by radiation. For transient conduction, the average heat flux can be proportional to the square root of the heat conductivity. In light of the modern theory of the transport of atmospheric radiation, there is no reason to expect a dependence on the square root of the water vapour pressure (Berdahl and Fromberg, 1982). As a result, Brunt developed a new relationship (Mills, 1999):

\[ \varepsilon_s = 0.55 + 1.8 \left( \frac{P_v}{P} \right)^{0.5} \]  

(2.5)

In 1942, Elsasser developed an expression for the sky emissivity as a function of the logarithm of the water vapour pressure.

\[ \varepsilon_s = 0.21 + 0.22 \ln P_v \]  

(2.6)

Bliss (1961) proposed a different correlation where the sky emissivity was a function of the dew point temperature.

\[ \varepsilon_s = 0.8004 + 0.00396T_d \]  

(2.7)

Clark and Allen (1978) also employed the dew point temperature \( T_d (^\circ C) \), to encapsulate the results of 800 clear-sky night-time experiments.

\[ \varepsilon_s = 0.787 + 0.764 \ln \left[ \frac{T_d + 273}{273} \right] \]  

(2.8)

They simplified the above equation to:

\[ \varepsilon_s = 0.787 + 0.0028T_d \]  

(2.9)

Berdahel and Fromberg (1982) proposed yet another correlation:

\[ \varepsilon_s = 0.734 + 0.0061T_d \]  

(2.10)
Burger et al. (1984) introduced an equation, which is very similar to Eqs. 2.7, 2.9, 2.10.

\[ \varepsilon_s = 0.77 + 0.00387T_d \]  

Chen et al., 2008 at the University of Nebraska developed another equation. The sky emissivity value was obtained from an Eppley pyrgeometer, which measures night sky radiation in W/m², and then the sky emissivity can be calculated from:

\[ S = \varepsilon_s \sigma T_a^4 \]  

where \( S \) is the sky radiation in W/m², \( \varepsilon_s \) is sky emissivity, \( \sigma \) is Stefan-Boltzmann constant \( 5.67 \times 10^{-8} \) w/m².K⁴, \( T_a \) is the ambient temperature in degree Kelvin.

Figure (2.1) shows the values of the sky emissivity versus dew point temperature in degree Celsius. The linear equation is given by:

\[ \varepsilon_s = 0.736 + 0.00577T_d \]  

Figure 2.1 Clear sky emissivity as a function of dew point temperature (Chen et al., 2008).
A comparison of results obtained by applying the above correlations was performed to illustrate the difference in the predicted values of the sky emissivity and thus the resulting effective sky temperature.

Figure 2.2 shows the difference in the values of the effective sky emissivity obtained by applying the above different correlations as a function of air temperature. As the air temperature increases the sky emissivity increases and, as a result, the effective sky temperature also increases.

![Figure 2.2 Deviation of effective sky emissivity obtained with different correlations at 85% relative humidity and in the range of 283-293 K air temperature.](image)

Figure 2.3 illustrates the variation in the effective sky temperature using values of the sky emissivity, which were calculated from the different correlations and as a function of the air temperature. This figure clearly shows the significant variability in the prediction of the effective sky temperature using the various correlations that were proposed in the literature.
Figure 2.3 Effective sky temperature at 85% RH versus the air temperature.

Figure 2.4 shows the differences observed in the values of the sky emissivity that were obtained with the different proposed correlations for different values of the relative humidity. When the relative humidity increases, the sky emissivity and the effective sky temperature increase. This increase is caused by the increased opacity of the atmosphere to infrared radiation as the quantity of water in the air increases. Figure 2.5 illustrates the variation of the effective sky temperature, using the estimated sky emissivity of Figure 2.4, calculated with the different correlations as a function of the relative humidity.
Figure 2.4 Comparison of different correlations for the prediction of the sky emissivity versus the relative humidity at \( T_a = 293 \) K.

Figure 2.5 Variation of the effective sky temperature associated with the sky emissivity of Figure 2.4 for different correlation at 293 K air temperature and as a function of the relative humidity.
It is clear that the different correlations for the prediction of the sky emissivity provide values encompassing a very wide range. As a result, the prediction of the effective sky temperature shows a very significant variability. To properly design an efficient atmospheric water recovery system, a reliable correlation to predict the effective sky temperature is required. In this study, an experimental system was specifically designed to develop a new correlation to estimate the effective sky temperature. As it will be shown in Chapter 5, this task proved to be more difficult than anticipated because the set of equations that needed to be solved was poorly conditioned.
2.2 Energy balance on a horizontal plate insulated at the bottom

Consider a thin horizontal plate with an insulated bottom surface presented schematically in Figure 2.6. The top surface of this plate is exposed to the atmosphere and sees only a clear night sky.

A simple energy balance was performed on the plate assuming steady-state conditions.

\[ Q_{\text{rad.}} + Q_{\text{conv.}} = Q_{\text{ext.}} \]  \hspace{1cm} (2.12)

Three modes of energy transfer prevail: heat transfer by convection, radiation, and the addition of heat from external energy sources. The latter energy additions are produced by condensation if the plate temperature is below the dew point temperature or via an imposed heat source such as an electric heater that could be used in an experimental study to prevent dew formation if desired.
The rate of heat transfer by convection away from the plate is expressed as:

\[ Q_{\text{conv}} = hA(T_p - T_a) \]  

(2.13)

The plate area, plate temperature, and air temperature are easy to measure and are readily available. The heat transfer coefficient \((\bar{h})\) is the only unknown in this equation, which can be predicted from the average Nusselt number (Mills, 1999).

\[ \frac{\bar{Nu}}{k} = \frac{\bar{h}L}{k} \]  

(2.14)

In this heat transfer problem from a horizontal plate exposed to atmosphere, natural and forced convections can coexist. The average Nusselt number is calculated as follows to consider both modes of heat transfer:

\[ \overline{\frac{\nu_{1/2}}{\nu^{1/2}}} = \overline{\nu_{f}^{1/2}} \pm \overline{\nu_{n}^{1/2}} \]  

(2.15)

The positive sign of Eq. 2.15 is applicable to a heated plate facing upwards, whereas the negative sign prevails for a cooled plate facing upwards. The Nusselt number for forced convection can be predicted using correlations that involve the Reynolds number and the Prandtl number:

\[ \overline{\nu_{f}} = 0.664 \nu_{e}^{1/2} \nu_{r}^{1/3}, \quad \nu_{e} > 0.5 \text{ & } \nu < 5 \times 10^5 \]  

(2.16)

\[ \overline{\nu_{f}} = 0.664 \nu_{e}^{1/2} \nu_{r}^{1/3} + 0.036 \nu_{e}^{0.8} \nu_{r}^{0.43} \left[ 1 - \left( \frac{\nu_{e}}{\nu_{r}} \right)^{0.8} \right], \quad 5 \times 10^5 < \nu < 3 \times 10^7 \]  

(2.17)

The Reynolds number is defined as follows:

\[ \nu = \frac{\nu L^{*}}{\nu} \]  

(2.18)

where, \(\nu\) is the wind velocity parallel to the plate surface, \(\nu\) is the kinematic viscosity of air, and \(L^{*}\) equals to the summation of the actual length of the plate and the length of the
unheated leading edge ($\xi$) preceding the plate. The Prandtl number of the surrounding air is found in tables in common heat transfer textbooks or can be calculated as follows:

$$\text{Pr} = \frac{\frac{\nu p c_p}{k}}{\frac{c_p \mu}{k}} = \frac{c_p \mu}{k} \quad \text{Pr} \approx 0.69 \quad \text{for air} \quad (2.19)$$

The Nusselt number for forced convection should be corrected, because of the unheated leading edge of the system.

$$\overline{\text{Nu}}_f = \overline{\text{Nu}}_{f|\xi=0} \left[ \frac{L^*}{L^* - \frac{\xi}{L^*}} \left( 1 - \left( \frac{\xi}{L^*} \right)^{\frac{p+1}{p+2}} \right)^p \right] (2.20)$$

where, $\xi$ is the unheated length prior to the beginning of the plate, $P = 2$ for laminar flow and $P = 8$ for turbulent flow. From the corrected Nusselt number, the heat transfer coefficient for forced convection is calculated as follows:

$$\overline{h}_f = \frac{\overline{\text{Nu}}_f k}{L^*}, \quad L^* = L + \xi \quad (2.21)$$

The Nusselt number for natural convection can be estimated with a correlation that involves the Rayleigh number:

$$\overline{\text{Nu}}_n = 0.54 Ra^{1/4}, \quad 10^5 < Ra < 2 \times 10^7 \quad (2.22)$$

$$\overline{\text{Nu}}_n = 0.14 Ra^{1/3}, \quad 2 \times 10^7 < Ra < 3 \times 10^{10} \quad (2.23)$$

The Rayleigh number is equal to the product of the Grashof and Prandtl numbers:

$$Ra = Gr \text{ Pr} \quad (2.24)$$

The Grashof number is calculated from the following equation:

$$Gr = \frac{\beta(T_p - T_a)gL_c^3}{\nu^2} \quad (2.25)$$
where $\beta$ is volumetric expansion coefficient, and $L_c$ is defined as the characteristic length of the plate, which equals $(L^2/4L)$ (Incropera, 1996). With the Nusselt number for natural convection, the heat transfer coefficient can be calculated:

$$\overline{h_n} = \frac{\overline{Nu_n} k}{L_c}, \quad (2.26)$$

The thermo-physical air properties, which are involved in the calculation of Re, Nu, Pr, Ra, and Gr numbers, can be determined from Appendix A.

From Appendix B, and considering the difference in the characteristic length used for forced and natural convections, the average heat transfer coefficient can be obtained:

$$\overline{h} = \overline{h_f} \pm \overline{h_n} \quad (2.27)$$

The heat transfer by radiation between the plate and the sky is given as follows:

$$Q_{rad} = e_p \sigma A (T_p^4 - T_s^4) \quad (2.28)$$

where,

$$T_s = (e_s T_s^4)^{0.25} \quad (2.29)$$

From the comparison of the available correlation for predicting sky emissivity, which was preformed in section 2.1, two correlations were retained to estimate the sky emissivity. These are Eqs. 2.5 and 2.10 as they were judged to be more reliable because they lie in the middle regions of the values of the predicted sky emissivity, and are based on different variables. A lower value of $e_s$ leads to a lower value of $T_s$ and a higher heat transfer by radiation.

If the plate temperature is lower than the dew point temperature, condensation will occur. If condensation occurs, the energy of condensation will be given to the plate.
The heat transfer due to the water vapour condensation on the upper surface of the plate is expressed as follows:

\[ Q_{\text{cond.}} = \dot{\lambda}_w AK (H - H_w) \]  \hspace{1cm} (2.30)

\( \dot{\lambda}_w \) is the latent heat of vaporization or condensation and can be approximated by the following equation that was fitted from steam tables:

\[ \dot{\lambda}_w = 2500.3 - 2.257 T_d - 0.00188 T_d^2 \] \hspace{1cm} (2.31)

The difference between the humidity of air and the humidity at the plate surface is the driving force for mass transfer. The humidity of the air at the surface of the plate corresponds to the saturation humidity evaluated at the plate temperature. The humidity is calculated using the partial pressure of water vapour, which is a function of the temperature of the air.

\[ H = \frac{M_{H_2O} P_{H_2O}}{M_a (P - P_{H_2O})} \] \hspace{1cm} (2.32)

The relative humidity can be calculated from the ratio of the partial pressure of the water vapour to the partial pressure of the water vapour at saturation.

\[ RH = \left( \frac{P_{H_2O}}{P_{H_2O_s}} \right) \times 100 \] \hspace{1cm} (2.33)

From Eqs. (2.32) and (2.33), the driving force for mass transfer could be calculated as:

\[ H - H_w = \frac{M_{H_2O}}{M_a} \left[ \frac{RH \times P_{H_2O_s} \{T_a\}}{100 \left( P - \frac{RH \times P_{H_2O_s} \{T_a\}}{100} \right)} - \frac{P_{H_2O_s} \{T_p\}}{P - P_{H_2O_s} \{T_p\}} \right] \] \hspace{1cm} (2.34)

where \( P_{H_2O_s} \{T_a\} \) and \( P_{H_2O_s} \{T_p\} \) are saturated water vapour pressure at air and plate temperature respectively.
The saturated water vapour pressure can be obtained at any temperature from the Antoine equation (Felder and Rousseau, 1986):

$$\log_{10}(P_{H_2O}) = A - \frac{B}{T + C}$$

(2.35)

where $P$ in kPa, and $T$ in °C.

For $0 < T < 60^\circ$C, $A = 7.2324, B = 1750.286, C = 235.0$

So,

$$P_{H_2O} = 10^{A - \frac{B}{T + C}}$$

(2.36)

Substituting in equation 2.34

$$H - H_w = \frac{M_{H_2O}}{M_a} \left( \frac{RH \times 10^{\left(\frac{A - B}{T + C}\right)}}{100} - \frac{10^{\left(\frac{A - B}{T + C}\right)}}{P - 10^{\left(\frac{A - B}{T + C}\right)}} \right)$$

(2.37)

The rate of condensation is proportional to the rate of water vapour mass transfer at the surface of the plate. The mass transfer coefficient ($K$ in kg/m$^2$.s) can be predicted from the Chilton-Colburn analogy (Edwards et al., 1979):

$$St Pr^{3/2} = St_m Sc^{3/2}$$

(2.38)

where Stanton numbers and Schmidt number are given as follows:

$$St = \frac{h}{\rho v c_p}$$

(2.39)

$$St_m = \frac{K}{\rho v}$$

(2.40)
\[ Sc = \frac{\nu}{D_{AB}} \]  

(2.41)

The diffusion coefficients for air and water vapour are calculated from the following equation (Edwards et al., 1979):

\[ D_{AB} = 1.97 \times 10^{-5} \left( \frac{P}{P_0} \right)^{1.685} \left( \frac{T}{T_0} \right)^{1.285} \]  

(2.42)

where \( P_0 = 1 \) atm and \( T_0 = 256 \) K.

The only unknown for the mass transfer term is the mass transfer coefficient.

\[ K = \frac{\bar{h} \Pr^{2/3}}{c_p Sc^{2/3}} \]  

(2.43)

Substituting in equation 2.12

\[ e_p c_p (T_p - T_s) \frac{\bar{h}}{c_p Sc} A \left( \frac{P}{T_s + C} \right)^{2/3} \times \frac{M_{H,C}}{M_o} \left( \frac{RH \times 10^{10} \left( \frac{A - B}{T_s + C} \right)}{P \left( \frac{RH \times 10^{10}}{100} \right)} \right) \]  

(2.44)

In the case in which it is desired to prevent condensation, energy should be provided to the plate by an external heater to control the temperature above the dew point temperature. In the designed experimental system, an electrical element was used to add energy to the bottom of the plate. The energy added by this external source can be calculated by measuring the voltage and the current to the heating element as a function of time.

\[ Q_{ext} = \frac{\int_{t=0}^{t_f} IV \, dt}{\int_{t=0}^{t_f} dt} \]  

(2.45)
The complete list of energy terms necessary to perform a heat balance on the horizontal plate has now been defined along with the known and unknown parameters. Under steady state, the overall heat balance on the horizontal plate becomes:

$$
\varepsilon \sigma A (T_p^4 - T_s^4) - \bar{h}A(T_a - T_p) = Q_{\text{ext}}
$$

(2.46)
Chapter 3

Simulation of Atmospheric Water Collection

In this chapter, a theoretical study on the water collection system is presented to evaluate the effects of the effective sky temperature, atmospheric conditions and plate properties on the condensation rate on a horizontal plate. This study allows predicting the amount of water that can be condensed during a similar situation.

The determination of the condensation rate \( \dot{m} \) on a horizontal plate, whose top surface is exposed to night sky while the bottom surface is perfectly insulated, is a two-step process, which involves Eq. 2.44 derived in Chapter 2.

First, the equilibrium plate temperature \( T_p \) under given operating conditions is calculated from Eq. 2.44. In this calculation, it is assumed that the condensate forming on the surface of the plate is immediately removed from the surface such that the surface emissivity remains constant.

Moreover, this calculation requires specification of the following parameters: surface emissivity of the plate \( e_p \), effective sky temperature \( T_{sky} \), ambient air temperature \( T_a \), wind velocity \( v \), the size of the plate, which determines the characteristic length \( L_c \) that is required for the evaluation of the heat transfer coefficient \( \bar{h} \) and the relative humidity \( RH \). Once \( T_p \) is evaluated, the individual modes of heat transfer in the energy balance on the plate can be evaluated. This includes heat transfer by condensation \( Q_{cond} \), which is represented by the entire term on the right hand side of Eq. 2.44. Since \( Q_{cond} = \lambda_w \dot{m} \), the condensation rate can be calculated from Eq. 3.1. Unless otherwise stated, the following set of parameters will be used in the foregoing simulations:
\[ \dot{m} = \frac{Q_{\text{cond}}}{\lambda_w} = A \left( \frac{\Pr}{c_p Sc} \right)^{2/3} \times \frac{M_{H_2O}}{M_a} \left\{ \begin{array}{l} RH \times 10^A \left( \frac{B}{T_a+C} \right) \left( \frac{T_s}{T_a+C} \right)^{100} P \left( \frac{RH \times 10^A}{100} \right) \left( \frac{B}{T_a+C} \right) \left( \frac{T_s}{T_a+C} \right) \left( \frac{B}{T_a+C} \right) \left( \frac{T_s}{T_a+C} \right) \frac{P - 10^A}{100} \end{array} \right\} \]

(3.1)

The effective sky temperature is not specified, because as shown in Chapter 2, \( T_{sky} \) is correlated with \( T_a \) and \( RH \). However, since for a given \( T_a \) and \( RH \) one can predict, depending on a given correlation, a wide range of \( T_{sky} \), \( T_{sky} \) will be dissociated from \( T_a \) and \( RH \) to show its effect on \( T_p \) and \( \dot{m} \).

Figure 3.1 presents the effect of \( T_{sky} \) on \( T_p \) for three different ambient temperatures. It is evident that \( T_p \) increases with \( T_{sky} \); also the greater is \( T_a \) for a given \( T_{sky} \), the greater is \( T_p \). As long as \( T_{sky} < T_a \), \( T_p \) is lower than \( T_a \). However, in the limiting case of \( T_{sky} = T_a \), the plate would reach a thermal equilibrium, i.e., \( T_p = T_{sky} = T_a \).

![Figure 3.1](image-url)  
Figure 3.1 Effect of the effective sky temperature on the equilibrium plate temperature for three different air temperatures.
Condensation can only occur if the plate temperature is less than or equal to the dew point temperature \((T_p \leq T_d)\), which in turn depends on the relative humidity. Figure 3.2 illustrates the impact of the effective sky temperature on the water condensation rate for three ambient temperatures when \(\varepsilon_p, \nu, L_c\) and \(RH\) are fixed. It can be noticed that \(\dot{m}\) rapidly decreases as \(T_{sky}\) increases. This is a direct effect of an increase in \(T_p\) with \(T_{sky}\) which leads to an increase in \(H_w\) for a given \(T_a\) and hence a decrease in the driving force for condensation \((H - H_w)\). On the other hand, for a given \(T_{sky}\), the condensation rate increases with \(T_a\). At first this might seem surprising because as shown in Figure 3.1, for a given \(T_{sky}\), \(T_p\) increases with \(T_a\) and thus \(H_w\) increases. On the other hand, the simulation in Figure 3.2 was performed for a constant \(RH\) and therefore, an increase in \(T_a\) increases \(H\), and the increase in \(H\) must be greater than \(H_w\), and consequently, \((H - H_w)\) must increase with \(T_a\) leading to a greater \(\dot{m}\) at higher ambient temperatures as seen in Figure 3.2.

![Figure 3.2 Effect of the effective sky temperature on the water condensation rate for three different air temperatures.](image-url)
In reality $T_{sk}$ depends on $T_a$ and $RH$ and it cannot be freely varied as shown in Figures 3.1 and 3.2. However, because of numerous empirical correlations for the prediction of $T_{sk}$, one can obtain quite different values of this very important parameter. In the following simulations, the effects of all variables appearing in Eq. 2.44 on the condensation rate will be examined individually, while relating $T_{sk}$ to $T_a$ and $RH$ through two correlations presented in Chapter 2 that yield extreme values for the sky emissivity and hence for $T_{sk}$ under given conditions. These correlations are given by Eqs. 2.1 and 2.7, respectively.

Figure 3.3 presents the effect of ambient air temperature on the condensation rate. It is evident that different behaviour is observed for the two different correlations for the sky emissivity. With Eq. 2.1 the condensation rate increases with $T_a$ for the entire temperature range shown in this figure. This is because while both $H$ and $H_w$ increase with $T_a$ the former increases more, leading to an increase in the driving force for condensation. It is important to emphasize that, as for Figure 3.2, the relative humidity in Figure 3.3 is also kept constant. On the other hand, with Eq. 2.7 the condensation rate is not only much lower than that for Eq. 2.1, but also it reaches a maximum value after which it decreases with an increase in $T_a$. These remarkably different effects of $T_a$ on $\dot{m}$ with Eqs. 2.1 and 2.7 arise from the fact that for the latter equation there is a value of $T_a$ above which $T_a$ has a stronger influence on $H_w$ than on $H$, leading to a decrease in the driving force for condensation.
The condensation rate is proportional to the relative humidity. When the relative humidity is higher, the driving force for condensation process is higher and a higher water condensation rate is observed. Figure 3.4 shows the effect of the relative humidity on the water condensation rate. Unlike Figure 3.3, where using Eqs. 2.1 and 2.7 led to a different relationship between $m$ and $T_a$, the condensation rate for both correlations in Figure 3.4 increases with relative humidity. However, for Eq. 2.7, the condensation rate begins only for $RH > 65\%$, which means that up to this $RH$ the equilibrium plate temperature must be greater than the dew point temperature. According to Fig. 3.4 there is a range of $RH$ of roughly 20% for the minimum $RH$ at given conditions required for the condensation to commence.
Wind speed is an important parameter which affects the water collection process. It controls both the convective heat and mass transfer coefficients. The wind velocity affects the plate surface temperature and, as a result, the condensation process. Figure 3.5 shows the effect of the wind velocity on the amount of condensed water. At low wind speed, an increase in wind velocity leads to an increase in the condensation rate because of the increasing driving force associated with the heat transferred by radiation and condensation due to an increase in the convective heat and mass transfer coefficients. However, at one point, the increase of the plate temperature due to an increase in the convective heat transfer limits the driving force associated with the condensation process and the opposite effect is observed. This behaviour is observed for both Eqs. 2.1 and 2.7. However, there is a remarkably large difference between the condensation rates when
these two equations are used. For the range of wind speeds used in Figure 3.5, this difference increases with an increase in wind speed.

![Diagram](image)

**Figure 3.5** Effect of wind speed on the water condensation rate for two correlations to predict the effective sky emissivity.

The properties of the plate play an important role in the condensation process. The most important factor affecting the radiation phenomenon is the surface emissivity. Figure 3.6 shows an increase in the condensation rate with an increase in the plate emissivity ($\varepsilon_p$). This is because as the plate emissivity increases, the radiative heat emitted by the surface of the plate increases, thus, leading to a lower plate temperature and an increase in the driving force for condensation. For the purpose of water collection, it is highly favourable to use a plate with a high emissivity in order to condense as much water as possible. Again, the choice of the correlation for the prediction of the sky emissivity and hence $T_{sky}$ has a major effect on the condensation rate. According to
Figure 3.6 the minimum $\varepsilon_p$ required to commence condensation differs by roughly 15%.

Moreover, the difference in $\dot{m}$ according to Eqs. 2.1 and 2.7 increase with an increase in $\varepsilon_p$.

The characteristic length of the plate also affects the condensation process. Figure 3.7 clearly shows that for the lower range of the characteristic length, an increase in the characteristic length of the plate has a major effect on the water condensation rate. This is because an increase in $L_c$ leads to a decrease in $\bar{h}$. Conversely, as $\bar{h}$ decreases $T_p$ decreases, thus increasing the driving force for condensation. For $L_c$ greater than 0.5 m its influence on $\bar{h}$ becomes insignificant leading to practically constant $\dot{m}$.
As shown in Figures 3.3-3.7, there is a large difference in the predicted water collection rate using the two correlations. These results point to the necessity of obtaining a reliable correlation to predict the effective sky emissivity in order to more accurately design an efficient atmospheric water recovery system.

In Chapter 4, an experimental system, specifically designed to estimate the effective sky temperature, is described.
Chapter 4

Experimental System for the Evaluation of the Effective Night Sky Temperature

It is apparent from the analysis presented in Chapter 3 that $T_{sky}$ is a key parameter in the prediction of the condensation rate due to the night radiation cooling. On the other hand, as shown in Chapter 2, there are numerous empirical correlations for the estimation of the sky emissivity ($e_{sky}$) from which $T_{sky}$ can be evaluated, and which lead to significant differences in the evaluated $T_{sky}$ at given conditions. In this chapter, a conceptual design of an experimental system for the measurement of $T_{sky}$ is presented, followed by a description of the system that was designed and built for the purpose of this project.

4.1 The concept of a six-plates system for the measurement of effective sky temperature

Suppose that $Q_{ext}$ in Figure 2.5, which presents heat transfer phenomena taking place on a horizontal plate exposed to a night sky, is a controllable parameter. In practice, this can be achieved by attaching a heating element to the bottom surface of the plate. By controlling the power input to the heating element, it is possible to adjust the plate temperature. If the power input is such that $T_p > T_{dp}$, there will be no condensation occurring on the plate. For a given $T_a$, $T_{dp}$ depends on the relative humidity of air and in the extreme case of $RH = 100\%$, $T_{dp} = T_a$. Therefore, to prevent condensation on the plate at any atmospheric conditions the power input to the plate should be such that $T_p > T_a$, and in this case the energy balance given by Eq. 2.44 becomes:
\[ \varepsilon_p \sigma A (T_p^4 - T_{sky}^4) + \bar{h} A (T_p - T_a) = Q_{ext} \]  \hspace{1cm} (4.1)

Note that, unlike the case in which \( Q_{ext} \) represents the power input due to condensation, the plate is losing rather than gaining the heat by convection, that is, convection and radiation are acting in the same direction.

In principle, knowing the plate emissivity, external power input and the resulting plate temperature one could evaluate \( T_{sky} \) by solving Eq. 4.1 if \( \bar{h} \) were known. As discussed in Chapter 2, knowing \( T_p, T_a \) and wind velocity, \( \bar{h} \) can be evaluated from appropriate empirical correlations. However, these correlations, as any empirical correlations are associated with a certain degree of uncertainty, which in turn would be transferred into the calculated \( T_{sky} \).

To overcome potential problems associated with an error in \( T_{sky} \) associated with an uncertainty in \( \bar{h} \), the latter should also be treated as an unknown. In this case, however, there are two unknowns, \( \bar{h} \) and \( T_{sky} \), in Eq. 4.1. To be able to solve for these two unknowns, one needs another independent equation without introducing any new unknown. The simplest way to accomplish this is to use a second plate having a different surface emissivity, which would however be maintained at the same temperature as the first plate. This would lead to the following set of equations:

\[ \varepsilon_{p1} \sigma A (T_p^4 - T_{sky1}^4) + \bar{h} A (T_p - T_a) = Q_{ext1} \]  \hspace{1cm} (4.2a)

\[ \varepsilon_{p2} \sigma A (T_p^4 - T_{sky2}^4) + \bar{h} A (T_p - T_a) = Q_{ext2} \]  \hspace{1cm} (4.2b)

The required power to the two plates would be different because of their different emissivities. On the other hand, maintaining the plates at the same temperatures would ensure the same convective heat transfer coefficient for both plates, provided that the plates have the same characteristic length \( (L_c) \) for convection. There are two potential
problems with this approach. To have the two plates at the same temperature would require a manual adjustment of the external power input to the plates. These power inputs would be subject to change with changing external conditions, that is, a successful determination of $\bar{h}$ and $T_{sky}$ would require a prolonged period of constant external conditions. Another and perhaps a more serious problem associated with this approach is the error associated with an uncertainty in the exact values of surface emissivity of the plates. While the surface emissivity is a material property, it strongly depends on the finishing of the surface. The latter is subject to changes due to surface oxidation, fouling, etc.

If the surface emissivity was considered to be an unknown in Eqs. (4.2a) and (4.2b), there would be four unknowns and only two equations, and consequently $T_{sky}$ could not be determined. Consequently, it is necessary to introduce more independent equations to match up the number of equations with the number of unknowns.

If in addition to the two plates, two identical plates with the respective surface emissivity of $\varepsilon_1$ and $\varepsilon_2$ were used at a different temperature than the original plates, one could write four independent equations:

$$\varepsilon_{p1} \sigma A(T_{p1}^4 - T_{sky}^4) + \bar{h}_1 A(T_{p1} - T_a) = Q_{ext1} \quad (4.3a)$$

$$\varepsilon_{p2} \sigma A(T_{p2}^4 - T_{sky}^4) + \bar{h}_2 A(T_{p2} - T_a) = Q_{ext2} \quad (4.3b)$$

$$\varepsilon_{p1} \sigma A(T_{p2}^4 - T_{sky}^4) + \bar{h}_1 A(T_{p1} - T_a) = Q_{ext3} \quad (4.3c)$$

$$\varepsilon_{p2} \sigma A(T_{p2}^4 - T_{sky}^4) + \bar{h}_2 A(T_{p2} - T_a) = Q_{ext4} \quad (4.3d)$$

Heat transfer coefficient depends on the surface temperature; therefore, since the plates are maintained at two different temperatures, the corresponding heat transfer
coefficients will be different. On the other hand, the dependence of $\bar{h}$ on the surface temperature is only significant in the case of natural convection; in the case of forced convection, $\bar{h}$ is practically independent of the surface temperature. Therefore, for the special case of forced convection $\bar{h}_1 \approx \bar{h}_2$ and there would be only four unknowns in Eqs. 4.3a-d. Solving this set of equations would allow determining all unknowns. However, when the convective heat transfer to the plate is dominated by natural convection, there are five unknowns in the above set of equations. To match the number of unknowns with the number of equations, it is necessary to add one additional pair of plates having a different surface emissivity ($\varepsilon_3$), which are operated at $T_1$ and $T_2$. This way one introduces one new unknown and two independent equations. Thus the resulting set of equations becomes:

\begin{align}
\varepsilon_{p1}\sigma A(T_{p1}^4 - T_{sky}^4) + \bar{h}_1 A(T_{p1} - T_a) &= Q_{ext1} \quad (4.4a) \\
\varepsilon_{p1}\sigma A(T_{p2}^4 - T_{sky}^4) + \bar{h}_2 A(T_{p2} - T_a) &= Q_{ext2} \quad (4.4b) \\
\varepsilon_{p2}\sigma A(T_{p1}^4 - T_{sky}^4) + \bar{h}_1 A(T_{p1} - T_a) &= Q_{ext3} \quad (4.4c) \\
\varepsilon_{p2}\sigma A(T_{p2}^4 - T_{sky}^4) + \bar{h}_2 A(T_{p2} - T_a) &= Q_{ext4} \quad (4.4d) \\
\varepsilon_{p3}\sigma A(T_{p3}^4 - T_{sky}^4) + \bar{h}_1 A(T_{p3} - T_a) &= Q_{ext5} \quad (4.4e) \\
\varepsilon_{p3}\sigma A(T_{p2}^4 - T_{sky}^4) + \bar{h}_2 A(T_{p2} - T_a) &= Q_{ext6} \quad (4.4f)
\end{align}

Accordingly, the above equations represent a set of six independent equations with six unknowns, $\varepsilon_1$, $\varepsilon_2$, $\varepsilon_3$, $\bar{h}_1$, $\bar{h}_2$, $T_{sky}$. Solving this system of equations simultaneously allows the determination of all unknowns, including $T_{sky}$. The only measurable variables in the above system of equations are the power inputs to the plates required to maintain them at
specific temperatures, the actual plate temperatures and the air temperature. In addition to having the same surface area, each plate should have the same characteristic length for heat transfer.

4.2 The design and description of the six-plate system

4.2.1 Selection of the plates

As discussed above, the six-plate system requires three pairs of plates having different surface emissivity. Considering that the measurement of $T_{sky}$ requires steady-state conditions, which are beyond the experimenter's control, the dynamics of the system should be minimized. Consequently, it was decided to use thin metal plates with high thermal diffusivity. The size of the plates was set on the basis of the size of available foil heaters. The latter, Mod-Tronic Thermofoil Heaters (model # HR5560 R12.2L 12A), had the dimensions of 152.4x152.4 mm. Consequently, it was decided to use slightly larger plates (160x160 mm), with each plate having the same thickness of 2 mm. Table 4.1 summarizes the properties of the plates selected for the six-plate system. The surface emissivity provided in this table represents textbook values for the plate materials (Karkelar and Desmond, 1982).

<table>
<thead>
<tr>
<th>Metal</th>
<th>Dimension (cm)</th>
<th>Mass (kg)</th>
<th>Specific heat (J/kg K)</th>
<th>Thermal conductivity (W/m K)</th>
<th>Emissivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chromium</td>
<td>16x16</td>
<td>0.370</td>
<td>465</td>
<td>54</td>
<td>0.08-0.27</td>
</tr>
<tr>
<td>Nickel</td>
<td>16x16</td>
<td>0.552</td>
<td>383.1</td>
<td>386</td>
<td>0.05-0.49</td>
</tr>
<tr>
<td>Copper</td>
<td>16x16</td>
<td>0.552</td>
<td>383.1</td>
<td>386</td>
<td>0.02-0.76</td>
</tr>
</tbody>
</table>
4.2.2 Arrangement of the plates

One of the critical requirements of the system is to ensure that the heat provided to the plates by the respective heaters is dissipated only through their top surface. Consequently, the bottom surface and the sides of each plate must be well insulated. For this purpose, a 5 cm-thick Styrofoam sheet having the thermal resistance of 1.32 K.m²/W was selected. Six square grooves were made on the top surface of the Styrofoam sheet, with dimensions equal to the size of the metallic plates. Each of the six plates was embedded with a thin heater sheet and deposited in a groove, so the plate’s surfaces and the Styrofoam sheet surface are at the same level. Figure 4.1 presents the dimensions of a heat transfer panel with three pairs of the metal plates installed on the Styrofoam sheet.

![Figure 4.1 Dimensions of heat transfer panel.](image)

To ensure the same convective heat transfer coefficients, the plates were maintained at the same temperature, the characteristic length for each plate must be identical. In the case of natural convection, since the plates have the same dimensions, the characteristic length for each plate will be the same. On the other hand, in the case of
forced convection, the plates will have the same characteristic length only when the longer edge of the heat transfer panel is oriented perpendicular to the wind direction.

4.2.3 Configuration of the system

To ensure perpendicular orientation of the panel with respect to the wind direction, the panel is installed on a steel frame mounted to a beam, which can rotate 360° by means of a small step motor. The motor is attached at the pivot point, which is located in a tri-pod stand beneath the heat transfer panel. The direction of wind with an accuracy of ± 3° is determined by a weather station (Vaisala Weather Transmitter, Model: WXT510), which also records wind speed to ± 0.3 m/s, ambient temperature to ± 0.3°C, and the relative humidity to ± 3%. The signals from the weather station, including wind direction, are sent to the data acquisition system. Wind direction is then translated to the motor movement, which adjusts the angle of the panel accordingly. The weather transmitter constantly monitors all elements in real time. The configuration of the six-plate system is presented in Figure 4.2. The weather transmitter is situated approximately a meter away from the main heat transfer apparatus.

![Figure 4.2 Overall configuration of the six-plate system.](image-url)
The layout of electrical connections for one plate of the six-plates system is presented in Figure 4.3. Each metallic plate has two thermocouples secured on the top facing surface on opposite sides of the square metallic plate. These thermocouples record the temperature of the plate to ± 0.5°C. Thermal information from the thermocouples is sent via analog input (AI) to a data acquisition box where it is then recorded by the laptop via USB cable. Beneath each one of the metallic plates, there is a heater element which is connected to the data acquisition and Toshiba laptop, from which the temperature set-point can be controlled. Therefore, each of the 6 metallic plates can be set to individual temperatures above air temperature to a maximum of approximately 70°C. Each piece of equipment is connected in a particular format for proper communication. In order for all electrical equipment to function properly, an assortment of different wires and types of electrical contacts connect unique electrical instruments as illustrated in Figure 4.3.

Figure 4.3 Electrical connections of one plate of the six-plate system.
4.2.4 Specifications of system components

The Summary of instrumentation of the six-plate system is given in Table 4.2.

<table>
<thead>
<tr>
<th>Name</th>
<th>Quantity</th>
<th>Specifications</th>
<th>Manufacturer</th>
</tr>
</thead>
<tbody>
<tr>
<td>Copper plates</td>
<td>2</td>
<td>Dimensions (m): 0.16×0.16×0.002</td>
<td></td>
</tr>
<tr>
<td>Chromium on steel plates</td>
<td>2</td>
<td>Dimensions (m): 0.16×0.16×0.002</td>
<td></td>
</tr>
<tr>
<td>Nickel on copper plates</td>
<td>2</td>
<td>Dimensions (m): 0.16×0.16×0.002</td>
<td></td>
</tr>
<tr>
<td>Styrofoam sheet</td>
<td>1</td>
<td>Dimensions (m): 1.31×0.26×0.5 Thermal resistance: 1.32 K.m²/W</td>
<td></td>
</tr>
<tr>
<td>Thermofoil Heaters, HR5560 R12.2L 12A</td>
<td>6</td>
<td>Dimensions (m): 0.1524×0.1524, effective area: 0.02206m². The wattage output at particular resistance depends on the voltage applied as per Ohm's law.</td>
<td>Mod-Tronic Instruments Limited</td>
</tr>
<tr>
<td>Thermocouples STC-TT-T-30-72</td>
<td>12</td>
<td>Diameter of 0.25 mm, Teflon insulated thermocouple, type T</td>
<td>Omega</td>
</tr>
<tr>
<td>Relays DR-OAC</td>
<td>6</td>
<td>120/240 VAC Isolated Output Module, Maximum current 5A. Turn-on/off time is 8.33 ms; Operating temperature is from 30 to 80°C.</td>
<td>Techmatron Instruments Inc</td>
</tr>
<tr>
<td>Data acquisition NI USB-6259</td>
<td>1</td>
<td>32 inputs, 16 bits</td>
<td>National Instruments</td>
</tr>
<tr>
<td>Computer</td>
<td>1</td>
<td>Portable laptop computer</td>
<td>Toshiba</td>
</tr>
<tr>
<td>Motor step P/N 3863h</td>
<td>1</td>
<td></td>
<td>FAULHABER</td>
</tr>
<tr>
<td>Power supply 2208249</td>
<td>1</td>
<td>Converts 120 VAC 60 Hz to 12.5 volts DC nominal. Output current is 6 A maximum.</td>
<td>NexxTech</td>
</tr>
<tr>
<td>Weather Transmitter WXT510</td>
<td>1</td>
<td>Dimensions: height: 0.24 m, diameter: 0.12 m, mass: 650 g. Input voltage: 5-30 VDC. Equipped with: WINDCAP sensor for horizontal wind speed (0 - 60 m/s) and direction measurements; THERMOCAP sensor for ambient temperature (-52°C to 60°C) measurement; BAROCAP pressure sensor 60 to 110 kPa measurement; HUMICAP sensor for the relative humidity (0 – 100%) measurement.</td>
<td>Vaisala</td>
</tr>
</tbody>
</table>
Chapter 5

Results and Discussion

This chapter presents the main results, both theoretical and experimental, that were obtained throughout this investigation. A large number of experiments were performed with the six-plate experimental system in the laboratory. It was the intention, following this experimental validation to install the system on the roof of the building to perform the final series of experiments that would have allowed measuring the effective sky temperature. Unfortunately, as it will be shown in this chapter, the inherent difficulty in solving the set of heat balance equations prevented completion of the initially proposed experimental plan.

It is interesting to note that most heat transfer problems, such as the results of Chapter 3, are called direct problems and they consist of determining temperature profiles and/or heat fluxes in a system subject to prescribed initial and boundary conditions. Inverse problems are usually concerned with finding boundary conditions and heat transfer parameters which give rise to a measured temperature distribution. While direct problems are characterized by stable unique solutions (i.e., they are well-posed), inverse problems are ill-posed and typically very sensitive to small variations in the input data. This was the case in this investigation and therefore, only a small portion of the experimental results that were obtained will be presented. In this chapter, a theoretical analysis on the mathematical solution of the system of equations will be presented with the various steps that were taken to find a reliable solution procedure. Some experimental results will then be presented to illustrate that despite the good prediction of the temperature profiles, estimation of the effective sky temperature remains very difficult.
5.1 Theoretical Analysis

5.1.1 Power required for a fixed plate temperature

The basic concept, for measuring the effective sky temperature, is to perform a steady-state energy balance on a horizontal plate at a constant temperature. In order to study the sensitivity of the numerous parameters that are involved in this experimental system, a set of realistic data was theoretically created to calculate the power required to maintain the plate at a steady-state temperature while varying the various system parameters. Effects of the desired plate temperature, the plate emissivity, the surrounding air temperature, the wind speed, and the effective sky temperature on the required power are presented in the following figures.

Figure 5.1 shows the influence of the plate temperature on the required power using plate emissivity of 0.3, and an effective sky temperature of 275 K when only natural convection prevails. This figure also shows the individual contribution of the convective and radiative heat transfer. As the plate temperature increases, the difference between the plate and air temperatures increases, so the driving forces for convective heat transfer, as well as, the radiative heat transfer increase. Thus, the power required to maintain the plate temperature at steady state also increases.

Figure 5.2 shows the effect of the plate emissivity on the power required to maintain the plate temperature at 303 K when the air temperature is 295 K and the effective sky temperature equals 275 K. It is clear that plate emissivity has a major effect on the power required because the contribution of the radiative transfer increases with the emissivity.
Figure 5.1 Effect of the plate temperature on the power required to keep a steady plate temperature when $\varepsilon = 0.3$ and $T_S = 275K$. Individual contributions of convective and radiative heat transfer are shown.

Figure 5.2 Effect of the plate emissivity on the power required to maintain the plate temperature at 303 K when the air temperature is 295 K.

Figure 5.3 illustrates the effect of the wind speed on the power required to maintain the plate at a constant temperature (303 K). As the wind speed increases, the
convective heat transfer increases and, thus heat losses by convective heat transfer increases and more power is required to keep the plate at a constant temperature.

Figure 5.3 Effect of the wind speed on the required power when the plate temperature is 303 K, $\varepsilon = 0.3$, the air temperature at 295 K and the effective sky temperature at 275 K.

Figure 5.4 illustrates the impact of the effective sky temperature on the power required to maintain the plate temperature at 303 K when the air temperature equals 295 K and only natural convection prevails. As the effective sky temperature increases, the driving force for radiative heat transfer decreases such that less energy is lost by radiation which in turn decreases the power required for the plate to remain at a steady state temperature. This figure also shows that as the plate emissivity increases the effect of the effective sky temperature on the required power becomes more important.
The main purpose of the experimental project was to calculate the effective sky temperature. Using the available equipment, it is possible to record with a good precision the values of many variables that are necessary for the calculation of the effective sky temperature. These are the plate temperature, air temperature, wind speed, and the power required by the heating element to maintain a constant plate temperature. If the emissivity of the plate and the convective heat transfer coefficient are known, a simple energy balance should allow the estimation of the effective sky temperature.

5.2 Emissivity determination

The emissivity of the three plates used in this investigation was estimated using an infrared camera (P60, FLIR). To perform these tests, each plate was heated to a set temperature and the emissivity of the plate was determined by selecting the emissivity on
the camera dial such that the recorded temperature coincided with the actual plate temperature. The estimated emissivities as well as common values listed in the literature for each type of material are presented in Table 5.1 (Karkelar and Desmond, 1982). The estimated values are within the range of values that are reported in the literature. It is however difficult to quantify the state of the actual surface. Ideally, it would have been interesting to completely characterize more precisely the surface to determine the average emissivity but it has not been possible to find a laboratory in the Ottawa region to perform these tests.

Table 5.1 Estimates of the emissivity of the three plates used in this investigation.

<table>
<thead>
<tr>
<th>Plate material</th>
<th>Estimated emissivity</th>
<th>Listed emissivity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Chromium</td>
<td>0.18</td>
<td>0.08-0.27</td>
</tr>
<tr>
<td>Copper:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Highly polished</td>
<td></td>
<td>0.02</td>
</tr>
<tr>
<td>Polished</td>
<td>0.16</td>
<td>0.04</td>
</tr>
<tr>
<td>Dull</td>
<td>0.15</td>
<td></td>
</tr>
<tr>
<td>Black oxidized</td>
<td>0.76</td>
<td></td>
</tr>
<tr>
<td>Nickel:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Polished</td>
<td>0.24</td>
<td>0.05-0.07</td>
</tr>
<tr>
<td>Oxidized</td>
<td></td>
<td>0.35-0.49</td>
</tr>
</tbody>
</table>

5.3 Estimation of power losses

The foil heating sheet was added to the experimental system to control the temperature of the metal plate. Even though insulation is used to minimize heat losses to the environment, a fraction of the heat is nevertheless lost. It is therefore necessary to estimate the amount of heat that is lost in order to determine as exactly as possible the amount of heat that was actually provided to the plate. To estimate the power loss, a sheet of Styrofoam was used to cover and insulate the top of the plates from the surrounding air. The insulation cover was made of the same insulating material that was underneath
the plates. The plate temperature was then controlled at a given temperature. When the plate reached steady state, the average power required to maintain a constant temperature was recorded. The temperature was then increased by steps of 10°C to calculate the power loss at different temperature levels. Figure 5.5 shows the change of the plate temperature as a function of time.

![Plate temperature versus time](image)

Figure 5.5 Plate temperature versus time when the upper plate surface is covered with an insulating board.

The average power input that is required to maintain a constant temperature is therefore equal to the amount of heat conducted through the top and bottom sheet of insulation and then lost to the environment. It is assumed that heat is lost equally from the two surfaces of the boards such that half of the value for a given plate temperature will account for the heat lost during all experimental runs. At each temperature plateau, the power input can be calculated as the fraction of the time that the heater was switched on multiplied by the power supplied by the heater. Figure 5.6 shows the amount of heat lost
as a function of the plate temperature based only on one side of the plate. It appears that the rate of heat loss increases linearly with the plate temperature which seems to indicate that heat is lost predominantly by conduction through the insulating material. This power loss was used to correct for the power input recorded during each experiment.

![Graph showing rate of heat loss versus plate temperature](image)

Figure 5.6 Rate of heat loss versus the plate temperature.

The power loss can be approximated using the following equation:

\[ Q_{\text{loss}} = 0.0493 \times T - 1.1414 \]  

(5.1)

where, \( Q_{\text{loss}} \) is in Watt and \( T \) in °C.

### 5.4 Experimental data analysis

For each experiment, the input power, the plate temperatures, and the surrounding air conditions were recorded and stored every 10 s. Steady-state and dynamic-state energy balances were performed for each plate. Each energy balance equation has three unmeasured parameters: the plate emissivity \( (\varepsilon_p) \), the convective heat transfer coefficient
(\bar{h})$, and the effective sky temperature ($T_S$). A simple heat balance accounts for the rate of change in the internal energy as being equal to energy input minus the energy output. The input energy is the heat provided by the heater that was corrected for heat losses whereas the output energy corresponds to convection and radiation heat transfer. This simple energy balance is expressed by the following equation:

$$mc_p \frac{dT_p}{dt} = Q_{ext} - \left[\varepsilon_p \sigma A(T_p^4 - T_S^4) + \bar{h}A(T_p - T_a)\right]$$ \hspace{1cm} (5.2)

### 5.4.1 Dynamic energy balance under natural convection

A dynamic energy balance was performed for each plate when only natural convection prevailed. The heat transfer coefficient was calculated from the correlation presented in Chapter 2. The emissivity values obtained from the infrared camera estimations were used. At any instant ($t+\Delta t$), using Euler approximation, the plate temperature is:

$$T_{p(t+\Delta t)} = T_{p(t)} + \frac{\Delta t}{mc_p} \left[Q_{ext} - \varepsilon_p \sigma A(T_{p(t)}^4 - T_S^4) - \bar{h}A(T_{p(t)} - T_a)\right]$$ \hspace{1cm} (5.3)

This equation was solved with an optimization algorithm in such a way to find the value of the effective sky temperature that minimizes the sum of squares of residuals. When only the sky temperature had to be determined, the predicted value was reasonable and close to the expected temperature.
5.4.1.1 Experiment at low temperature of surroundings

Amongst the many experiments that were conducted to examine the potential of this experimental system for the estimation of the effective sky temperature, one experiment was performed in the laboratory where the experimental system was covered with an aluminium plate maintained at a temperature near 0°C. To simulate a low sky temperature, an ice-water mixture was used to cover the top surface of the Aluminium plate to achieve a temperature close to 0°C. A measurement of the cover plate temperature revealed that it could be maintained at approximately 2°C. A dynamic energy balance using Eq. 5.3 was preformed on the cooling period of this experiment to solve for the sky temperature (i.e. in the absence of heating). By minimizing the sum of squares of the errors, a temperature of the bottom surface of the Aluminium plate of 2°C was determined. Figure 4.7 illustrates the agreement of the actual cooling curve and the calculated one.

Figure 5.7 Comparison of the predicted and experimental plate temperatures as a function of time under conditions of a low temperature of surroundings.
5.4.1.2 Experiments at room temperature

A series of experiments were done inside the laboratory at room temperature, the laboratory ceiling acting as the sky and its temperature obviously being close to the room temperature. The heater, underneath a plate that has been at steady state at room temperature for some time, was switched on to increase the temperature of the plate. When steady state at the higher temperature was achieved, the heater was switched off and the plate cooled off. Throughout this experiment, the plate temperature was measured. Figure 5.8 illustrates the comparison between the predicted and experimental plate temperatures as a function of time (using Eq. 5.3).

Figure 5.8 Predicted and measured plate temperature versus time for an experiment performed at room temperature.

Figure 5.8 shows that the predicted temperature is higher than the experimental one during the heating phase. This deviation may be due to the assumed power loss from
the plate, the measurement of the power input to the electric heater underneath the plate, and to the value of the heat capacity obtained from the literature. On the other hand, the predicted temperature during the cooling phase was very close to the actual temperature. Since there is no power to the heater, this source of error has been eliminated and a much better prediction is obtained. This result points more particularly to a relative error in the estimation of the power input to the heater that is determined from a measurement of the voltage between the two extremities of the heater of known resistance.

5.4.2 Steady-state energy balance under natural convection

Steady-state energy balances were done for each plate. Under steady-state, the power required to maintain a given plate at a constant temperature must equal the total of the energy lost by radiation and convection.

\[ Q_{\text{ext}} = \varepsilon_p \sigma A (T_p^4 - T_s^4) + \bar{h} A (T_p - T_a) \]  

(5.4)

This equation was used as the basis for designing the experimental system. It was hypothesized that, given the uncertainty associated with the plate emissivity and the heat transfer coefficient, an experimental system could be designed to measure these variables at the same time as the effective sky temperature. For this purpose, a system of three sets of two identical plates (2 chromium, 2 copper and 2 nickel), as described in Chapter 4, was built. The different attempts to resolve this heat transfer problem are discussed in this section.

5.4.2.1- Six non-linear equations with six unknowns

All experiments, performed inside the laboratory using the six plates simultaneously and the ceiling acting as the sky, led to a system of six equations and six
unknowns. In these experiments, three plates of a different surface finish were set at a constant temperature and the other three plates were set at a different temperature. The power required to maintain each plate at the desired temperature was measured and corrected for heat losses. It was therefore possible to write a heat balance (Eq. 5.4) for each plate. Solving the nonlinear system of six equations with six unknowns (three plate emissivities, heat transfer coefficients at two different temperatures, and sky temperature) was unfortunately not successful. The system of equations was found to be very sensitive to small errors in measured variables.

To better comprehend this sensitivity problem, a theoretical study was undertaken whereby pure simulation was used to generate data that was then solved in the same manner. The advantage of simulations is the possibility of generating data that perfectly satisfy heat balances in the absence of noise and measurement error. Many simulations were performed and the system of six equations and six unknowns was solved. Results in all cases showed that it is possible to estimate very accurately the effective sky temperature but all the other five unknowns were far from their original values that were used in the simulation. The reason why the sky temperature was very well estimated is probably because it appears in all six equations. In theory, this would not be too problematic because the information that is truly desired is the effective sky temperature and it is measured very accurately. However, under a realistic situation where inherent noise and measurement errors prevail, the estimation of the sky temperature becomes impossible as it will be shown in the following paragraphs.

One simulation of the six plates was performed using Eq. 5.4 and the conditions given in Table 5.2. For this simulation, the wind speed was 5 m/s, the air temperature was
282 K, and the sky temperature was set at 260 K. The nonlinear regression was performed 1000 times to estimate the six unknown variables and, each time, the values of the power required ($Q_{ext}$) to maintain a plate at constant temperature for all six plates were corrupted by a Gaussian random noise with a standard deviation of 5% of the calculated values (Table 5.2).

Table 5.2 Conditions for the simulation of the six-plate system.

<table>
<thead>
<tr>
<th>Plate</th>
<th>$T$ (K)</th>
<th>$\varepsilon$</th>
<th>$h$ (W/m$^2$K)</th>
<th>$Q_{con.}$ (W)</th>
<th>$Q_{rad.}$ (W)</th>
<th>$Q_{ext}$ (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>285</td>
<td>0.85</td>
<td>19.1</td>
<td>1.468</td>
<td>2.501</td>
<td>3.969</td>
</tr>
<tr>
<td>2</td>
<td>295</td>
<td>0.85</td>
<td>19.2</td>
<td>6.374</td>
<td>3.705</td>
<td>10.079</td>
</tr>
<tr>
<td>3</td>
<td>285</td>
<td>0.20</td>
<td>19.1</td>
<td>1.468</td>
<td>0.589</td>
<td>2.056</td>
</tr>
<tr>
<td>4</td>
<td>295</td>
<td>0.20</td>
<td>19.2</td>
<td>6.374</td>
<td>0.872</td>
<td>7.246</td>
</tr>
<tr>
<td>5</td>
<td>285</td>
<td>0.03</td>
<td>19.1</td>
<td>1.468</td>
<td>0.088</td>
<td>1.556</td>
</tr>
<tr>
<td>6</td>
<td>295</td>
<td>0.03</td>
<td>19.2</td>
<td>6.374</td>
<td>0.131</td>
<td>6.505</td>
</tr>
</tbody>
</table>

Figure 5.9 presents the number of occurrences of the estimated sky temperature along with a Gaussian fit of the distribution. Values of the sky temperature as low as 215 K and as high as 305 K were obtained. The Gaussian fit gave an average value of approximately 258 K and a standard deviation of 13 K. It is interesting to note that the standard deviation corresponds to 5% of the 260 K simulated value, i.e. the identical percentage of the standard deviation that was used to artificially corrupt the power required for each heater. These results truly show the sensitivity of the estimate of the sky temperature on the accuracy of the power required by the heaters and the difficulty to achieve an accurate estimation.
Figures 5.9 Number of occurrences of the sky temperature for 1000 simulations.

Figures 5.10 and 5.11 present the estimated values of the additional five unknowns as a function of the sky temperature for the 1000 regressions that were performed with the noisy data of Table 5.2. These results clearly show the high correlations that exist amongst the estimated variables. It is interesting to note that the emissivity of the two plates assumed values that were as high as 2. It would have been possible to place a constraint on the value to ensure that it remains below unity but it was decided to let it float to better analyze the sensitivity of its value. The highly correlated estimates add to the difficulty to obtain a proper value of the sky temperature and all the other unknowns.
Figure 5.10 The two heat transfer coefficients as a function of the sky temperature for the 1000 simulations.

Figure 5.11 Estimations of the three plate emissivities as a function of the sky temperature for the 1000 simulations.
The objective function that was minimized in order to obtain an estimate of the six unknown variables for the 1000 simulation runs was calculated as the sum the differences between the predicted and simulated power required to maintain the plate at a constant temperature. Figure 5.12 presents the plot of this objective function as a function of the sky temperature for each simulated run. This plot shows that, over a wide range of the sky temperature, the objective function is nearly the same. In that range, a change in the sky temperature is compensated by a change of the other variables involved due to high correlations that exist amongst all the parameters.

Figure 5.12 Plot of the objective function as a function the estimated sky temperature for the 1000 simulations trials.

5.4.2.2- Eight linear equations with eight unknowns

In an attempt to overcome the difficulty discussed in the previous section, the heat balance equation was linearized.

\[ Q_{\text{ext}} = h_r A (T_p - T_s) + h A (T_p - T_a) \]  

(5.5)

59
with

\[ h_r = \varepsilon_p \sigma (T_p + T_s) \left( T_p^2 + T_s^2 \right) \]  

(5.6)

To use this new scheme, two sets of three identical plates would be required. Each set consists of three identical plates with the same emissivity that would be kept at three different temperature levels in the presence of forced convection. Under forced convection, the heat transfer coefficient is nearly independent of the plate temperature thereby reducing the number of unknowns. For each of the six plates, a heat balance can be written. Plates 1, 2 and 3 would have the same emissivity \( \varepsilon_1 \) and would be maintained at temperature \( T_1 \), \( T_2 \) and \( T_3 \). Plates 4, 5 and 6 would have the same emissivity \( \varepsilon_2 \) and would also be maintained at temperature \( T_1 \), \( T_2 \) and \( T_3 \). This system of six equations has eight unknowns: the convective heat transfer coefficient, the effective sky temperature, and the six radiative heat transfer coefficients.

To obtain two additional equations, Eq. 5.6 can be used to establish a relationship between the radiative heat transfer coefficients since two plates would be maintained at the same temperature.

\[ \frac{h_{r_1}}{h_{r_4}} = \frac{h_{r_2}}{h_{r_5}} = \frac{h_{r_3}}{h_{r_6}} = \frac{\varepsilon_1}{\varepsilon_2} = \text{constant} \]

Therefore, two additional equations can be used to have a system of eight equations and eight unknowns.

\[ \frac{h_{r_1}}{h_{r_4}} = \frac{h_{r_2}}{h_{r_5}} = \frac{h_{r_3}}{h_{r_6}} \]
Using this new procedure, it was still not possible to solve these eight equations to estimate all the unknowns. This transformation did not modify the highly sensitive nature of this system.

5.4.2.3- Three linear equations with three unknowns

As shown previously, solving the set of six and eight equations proved to be impossible. A new experiment was therefore conducted in the presence of forced convection with only one plate. Under the same experimental conditions, the plate temperature was set in turn to three different steady-state temperatures. For this experiment, it was therefore possible to generate three heat balance equations. Since the emissivity and the heat transfer coefficient are the same for the three equations, it is possible to solve for three unknowns, the sky temperature being the third unknown.

Linearizing the heat balance equation, it follows:

$$ \bar{h}A(T_p - T_a) + \varepsilon_p \sigma AT_p^4 - \sigma AT_a = Q_{ext} \quad (5.7) $$

with

$$ Y = \varepsilon_p T_S^4 $$

However, due to measurement errors, this transformation did not lead to a better solution. Since the system of equations is linear, it can be theoretically analyzed and, more particularly, it is possible to examine the matrix conditioning to assess the difficulty in solving this apparently simple problem. For a specific example of this three temperature level system, the matrix is as follows:

$$ \begin{bmatrix}
A(T_{p1}-T_a) & \sigma AT_{p1}^4 & -\sigma A \\
A(T_{p2}-T_a) & \sigma AT_{p2}^4 & -\sigma A \\
A(T_{p3}-T_a) & \sigma AT_{p3}^4 & -\sigma A
\end{bmatrix} \begin{bmatrix}
0.2048 \\
0.4608 \\
0.7168
\end{bmatrix} = \begin{bmatrix}
12.258 \\
13.958 \\
15.828
\end{bmatrix} - \begin{bmatrix}
1.45152 \times 10^{-9} \\
1.45152 \times 10^{-9} \\
1.45152 \times 10^{-9}
\end{bmatrix}.$$
The Eigenvalues for this matrix are: -0.1943, 14.357 and 2.275×10^{11}. The huge differences in these values are a clear indication of the ill-conditioning of the matrix, and point to the same sensitivity problem noted in the previous discussion.

5.4.2.4 - Two linear equations with two unknowns

To continue the analysis, the problem was reduced by one dimension whereby the plate emissivity was assumed to be known. The energy balance equation required to represent a plate, under the same experimental conditions but at two temperature levels, is given in Eq. (5.8).

\( \overline{h} A(T_p - T_a) - \sigma A Y = Q_{ext} - \varepsilon_p \alpha A T_p^4 \)  \hspace{1cm} (5.8)

The matrix for this system becomes:

\[
\begin{bmatrix}
A(T_{p1} - T_a) & -\sigma A \\
A(T_{p2} - T_a) & -\sigma A
\end{bmatrix}
\begin{bmatrix}
0.2048 \\
0.4608
\end{bmatrix} = 
\begin{bmatrix}
0.2048 \\
0.4608
\end{bmatrix} -1.45152\times10^{-9}
\]

The Eigenvalues for this matrix are: 0.204 and 1.8144×10^{-9}. The Eigenvalues for this matrix are also very different, such that the problem cannot be solved because a small error in the measured power leads to a major difference in the obtained solution.

Figure 5.13 shows the effect of a small error in the power input on the value of effective sky temperature at many values of the plate emissivity. In this calculation, the effective sky temperature is 270 K. Solving the above matrix with ±5\% error in \( Q_{ext} \) results in a large error in the effective sky temperature. The best solution for the effective sky temperature is obtained at a plate emissivity equal to 0.25 but this is only circumstantial for the particular set of values that were used in this example.

Figure 5.14 shows the effect of a small error in the power input on the value of convective heat transfer coefficient at many values of the plate emissivity. The original
value of the convective heat transfer coefficient, used for generating the simulated results, is 19.68 W/m²K. These results clearly show again the sensitivity of the system being considered and the main effect coming from the radiation term.

Figure 5.13 Effect of a small error in the power input on the value of the effective sky temperature at many values of the plate emissivity.

Figure 5.14 Effect of a small error in the power input on the value of the convective heat transfer coefficient for many values of the plate emissivity.
5.4.2.5- One equation in one unknown

Reducing the number of unknowns to one by considering the sky temperature only as unknown and assuming that the plate emissivity is known and the convective heat transfer coefficient can be calculated from the correlation presented in Chapter 2. From the steady state energy balance, it is possible to write

\[
Q_{\text{ext}} = \varepsilon_p \sigma A (T_p^4 - T_s^4) + \bar{h} A (T_p - T_a)
\]

\[
T_s = \left( T_p^4 - \left( \frac{Q_{\text{ext}} - \bar{h} A (T_p - T_a)}{\varepsilon_p \sigma A} \right) \right)^{0.25}
\]

(5.9)

This equation is also greatly affected by small errors in the power input. An error of ±20% causes a large error in the value of the effective sky temperature. Figure 5.15 presents the variation the estimated sky temperature as a function of an error in the power input.

Figure 5.15 The sensitivity of effective sky temperature versus the error in input energy.
Chapter 6

Conclusions

Water extraction from atmosphere is a potential method to produce fresh water in regions where water sources are scarce or non-existent. In the literature, there are several attempts to obtain water from atmosphere either by passing humid air over cooling coils or by absorption regeneration of moisture directly from air using a desiccant. An alternative method was considered in this study which benefits from the night cooling phenomenon. At night time, where there is no solar energy, a surface emits energy to a relatively cool sky and its temperature cools down. If its temperature drops below the dew point temperature of the air, moisture will condense on that surface. A simulation on a water collection system was preformed to study the influence of the atmospheric conditions and the surface properties on the condensation process.

The simulation showed the importance of the value of the effective sky temperature and its impact on the amount of collected water. In the literature, there exist many correlations to estimate the sky emissivity and then calculate the effective sky temperature. Applying some of those correlations led to a large difference in the estimation of the amount of condensed water.

To design a system for water collection purpose, an accurate sky temperature is required. To estimate the effective night sky temperature, an experimental system consisting of a series of six insulated plates has been built. Each plate was embedded in an insulating and placed on a heater to maintain its temperature at a desired value. Two thermocouples were added on the upper surface of each plate to measure its temperature. The weather conditions, plate temperatures and power inputs were monitored and
recorded every 10 seconds. The heat transfer panel was adjusted automatically to be always oriented perpendicular to the wind direction to ensure all plates were exposed to the same conditions. In addition to the effective sky temperature, the experimental system was designed to estimate the convective heat transfer coefficient between a plate and ambient air as well as estimate the surface emissivity of the plates.

Inside the laboratory, many experiments were performed using the six plates simultaneously and the roof acting as the sky, which led to a system of six equations and six unknowns. Solving the nonlinear system of six equations with six unknowns (three plate’s emissivities, heat transfer coefficient at two different temperatures, and sky temperature) was unfortunately not successful. The system of equations was found to be very sensitive to small errors in measured variables.

A theoretical analysis of the various steps in the mathematical solution of the system of equations was performed to find a suitable solution the problem observed. The results obtained from theoretical analyses and preliminary experiments showed that, although the system of equations works well for the direct heat transfer problem, there is a serious difficulty to solve the inverse heat transfer problem to retrieve the desired parameters. This is due to the inherent error associated with the measurements.
References


Appendix A

A.1 Calculation of air properties

The thermo-physical properties of air were taken from Mills (1999) in the range of 260 to 300 K. Plots of the data points along with the best-fit curve of each physical property as a function of temperature are presented in Figs. A-1 to A-5.

Figure A.1 Air thermal conductivity versus temperature.
Figure A.2 Air density versus temperature.

Figure A-3 Air kinematic viscosity versus temperature.
Figure A-4 Air dynamic viscosity versus temperature.

Figure A-5 Air heat capacity versus temperature.
Appendix B

B.1 Prediction of convective heat transfer coefficient

The heat transfer coefficient is used to calculate the heat transfer between a moving fluid and a solid surface. The heat transfer coefficient is often estimated from correlations that involve the Nusselt number. There exist many heat transfer correlations for different liquids, flow regimes, and thermodynamic conditions. The schematic diagram of a typical thin horizontal plate with an insulated bottom, as used in this investigation, is presented in Figure B.1. This plate is exposed to the atmosphere and only sees a clear night sky.

Figure B.1 A schematic of a horizontal insulated plate exposed to a clear sky.

The convective heat transfer coefficient can be calculated using the empirical correlations that were presented in Chapter 2.
The convective heat transfer coefficient can be directly calculated by applying equation 2.14 considering only the actual length of the heated plate.

\[ \overline{\text{Nu}} = \frac{\tilde{h}L}{k} \]  

(B.1)

A second way to calculate the convective heat transfer coefficient is using equation 2.15.

\[ \overline{\text{Nu}}^{7/2} = \overline{\text{Nu}}_{f}^{7/2} \pm \overline{\text{Nu}}_{n}^{7/2} \]  

(B.2)

\[ \left( \frac{\tilde{h}L}{k} \right)^{7/2} = \left( \frac{\tilde{h}_{f}(L + \xi)}{k} \right)^{7/2} \pm \left( \frac{\tilde{h}_{n}(L/4)}{k} \right)^{7/2} \]  

(B.3)

Where, \( \xi \) is the unheated edge can be defined as a function of \( L \).

\[ \xi = \chi L, \quad 0 \leq \chi \leq 1 \]

(B.4)

Multiplying both sides by \((k/L)^{7/2}\)

\[ \tilde{h}^{7/2} = \left( (1 + \chi) \overline{h}_{f} \right)^{7/2} \pm \left( 0.25 \overline{h}_{n} \right)^{7/2} \]  

(B.5)

A third way to calculate the heat transfer coefficient is using equation B.2 but neglecting the difference in the characteristic length of forced and natural convection.

\[ \overline{h}^{7/2} = \overline{h}_{f}^{7/2} \pm \overline{h}_{n}^{7/2} \]  

(B.6)

Comparing the values of the convective heat transfer coefficient obtained from equations B.1, B.5 and B.6, it is found the results, obtained from Equation B.6, is more convenient because both terms can be calculated independently. Results are shown in Figures B.2 and B.3.

In the presence of a sufficient wind, it clearly appears that the convective heat transfer coefficient is strongly affected by the forced convection. Figure B.2 shows that, at wind velocity equals to zero (only natural convection), the heat transfer coefficient
increases with an increase in the difference between the air temperature and the plate temperature, but, as the wind velocity increases the forced convection takes an increasing role and eventually dominates. Indeed, as shown in Figure B.3, the convective heat transfer coefficient becomes independent of the temperature difference between the plate and the air.

![Figure B.2 Effect of the difference between the air temperature and the plate temperature on the convective heat transfer coefficient at different wind velocity.](image)
Figure B.3 Effect of the wind velocity on the heat transfer coefficient at different values of the difference between air temperature and plate temperature.