

PUBLIC POLICY AND POVERTY IN QUEBEC FROM 1996 TO 2006

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Abstract

Using cross-section Survey of Labour Income Dynamics (SLID) data from 1996 to 2006 period, I examine how poverty in Quebec has changed over time, and relative to the other provinces. To better understand the changes, I decompose Foster-Greer-Thorbecke (FGT) index by provinces and family types. Poverty and inequality are ranked with restricted stochastic dominance. I find that Quebec did a better job at reducing poverty than most of other Canadian provinces over the ten years' time span. I also find that lone mothers have the lowest income, and single senior women are the second worst group. Finally, I find that single women have the highest risk of falling in poverty.

Key words: poverty and inequality, restricted stochastic dominance, FGT index, decomposition, Quebec.

1 Introduction

Poverty has received a great deal of attention in the field of economic growth in the last two decades and income inequality has become even more pertinent after the recent “Occupy Wall Street” movements. Poverty and income inequality have been the barriers of economic development and the main reason of social unrest. As a leading province in the design of poverty reduction programs in Canada, the government of Quebec has spent billions of dollars fighting to reduce poverty. More precisely, poverty reduction became a cornerstone of government policy with the introduction of Bill 112, i.e. *An Act to Combat Poverty and Social Exclusion*. The goal of Quebec’s legislation is to make Quebec the province with the least poverty among industrialized nations.¹

There are two different views concerning Quebec’s public policy towards poverty. The first opinion is that Quebec has successfully won the campaign against poverty and that Quebec is now a much more equal society. Pierre Fortin (2009) concluded:

“Quebecers enjoy more free time, much of it by free choice: they work fewer hours per week, fewer weeks per year and fewer years in their careers. Incorporating the value of the increased leisure time together with cash income into an overall estimate of ‘true’ standard of living leads to one basic conclusion: the average standard of living is now about the same in Quebec as in Ontario. This result comes with a second surprise: among regions of Canada, Quebec is a leader in the fight against poverty and inequality.”

Couturier and Schepper (2010) challenged the above statements by using the Gini index to show that Quebecers worked longer hours per year and enjoyed less free time, but were worse off in terms of income and had the biggest income gap between the rich and the poor from 1996 to 2006. Kozhaya (2006) also agreed that Quebec had big gaps were compared to the Canadian average and to Ontario in terms of GDP per capita and individual income per capita in 2004.

¹ *An Act to Combat Poverty and Social Exclusion*, R.S.Q. L-7, c. II.s.4.

Researchers have argued about whether poverty in Quebec is decrease or not. Some of them found that poverty was not decrease in Quebec; on the other hand, others believed that Quebec was getting worse. There are several factors that determine the results of poverty research, such as choice of poverty line, units of analysis, and poverty indices.

Individual income rose with economic growth (Adams, 2003), which means that both the poor and the rich social members get benefits from economic development. Poverty decreases when individual income increases, but income inequality may not decrease, since income distribution may not change equally as Zyblock and Tyrrell stated (1997). This paper used income survey data from Statistics Canada, which is reported as family income but was calculated by family weight into individual income.

According to Sen (1976), poverty research must answer two questions: one is who the poor are and the other is how poor they are. These are poverty identification question and the aggregation poverty measurement question.

The first decision is the choice of poverty line that is the threshold to identify the poor. The second decision is the choice of poverty index that is the measure of degrees of poverty (Atkinson, 1987).

There is no official poverty line published by the provincial or the federal governments in Canada. In this paper, poverty lines are calculated from total income instead of well-known Low-Income Cut-off (LICO). One is the extreme poverty line, defined as 30 percent of the median of total income after taxes and government transfers of 2006; and the other one is the moderate poverty line, defined as 50 percent of the median of the same total income.

There are different poverty indices that can be applied to poverty measurement, and different poverty indices may lead to different conclusions with nature of poverty. Furthermore, a poverty index only summarizes poverty measures as a whole numerically, but fails to indicate any details within different income subgroups and income levels, or across distributions (Maasoumi & Heshmati, 2000).

These shortcomings show that poverty indices are not accurate enough for acknowledging aggregate poverty reduction to the public and in supporting policy decisions. For example, it is hard for the public to understand poverty improvement only with summarized numbers and it is either difficulty for the policy makers to improve the social welfare without aggregate poverty information.

In this paper, stochastic dominance is employed to find poverty and inequality dominances. A stochastic dominance test compares two income distributions more robustly, since this method is free from the specific value of poverty line.

Although a high correlation exists between poverty and income inequality, they are different economic concepts and have distinctive extensions (Sen, 1981).

Townsend (1979) defined poverty as:

“Individuals, families and groups in the population can be said to be in poverty when they lack the resources to obtain the type of diet, participate in the activities and have the living conditions and amenities which are customary, or are at least widely encouraged or approved, in the societies to which they belong. Their resources are so seriously below those commanded by the average individual or family that they are, in effect, excluded from ordinary living patterns, customs and activities”.

As a result, poverty is a situation or conditions when an individual lacks resources to achieve the “minimum necessities” (Sen, 1981). Inequality is a concept related to income distribution.

Poverty can actually perpetuate and even increase inequality in a society while income inequality has a direct and immediate impact on the degree of poverty within the poor population. Some studies separate poverty from inequality. In this paper, poverty is measured with inequality simultaneously.

Incomes in Quebec are compared with those of other provinces across Canada to study the efficiency of poverty reduction in Quebec. Furthermore, population subgroups such as seniors,

couples, and lone parents are taken in the account in poverty index decomposition to reference the aggregate tendency of intensity of poverty in Quebec.

The purpose of this paper is to examine whether poverty has changed in Quebec compared to other provinces. For this analysis, I rely on cross-sectional data from the Survey of Labour Income Dynamic (SLID); more precisely, I use the economic family files from 1996 to 2006.

There is a time lag to access SLID data from Statistics Canada which explains why this paper only covers the 1996 to 2006 period. Poverty status in Quebec during the recent 2008-2009 global recession will therefore not be discussed in this paper.

Although, some researchers pointed out that taxes and government transfers are not the only way to reduce poverty; economic growth plays a large role of poverty elimination (Admas, 2003). Furthermore, some economists suggested that the strategy to reduce poverty should include “increase the labour market participation of those who are able to work” (Collin, 2007).

I do not discuss in this paper other factors that affect poverty and income inequality, such as employment and economic growth. Taxes and government transfers are examined in this paper to evaluate poverty reduction in terms of social welfare and public policy.

I find that Quebec did a good job in reducing poverty from 1996 to 2006. It had less intensity of poverty and more income equality than most other Canadian provinces in 2006. Furthermore, inequality of some subgroups decreased from 1996 to 2006 and the welfare of some population subgroup improved significant in Quebec.

The structure of the present paper is as follows: Section 2 reviews the literature; section 3 describes the data sources; section 4 lays out the methodology, and section 5 indicates the results. Finally section 6 concludes.

2 Literature Review

2.1 Introduction

According to Sen (1979), the foundation of poverty research is the answers of two questions: one is “Who are the poor?” and another is “How poor are they?” The first question is the identification problem and the second question is the aggregation problem (Brochu, Makdissi, & Taohan, 2011). In other words, these questions link how one defines poverty and how one measures it.

Sen (1981) pointed out that a strong correlation existed between poverty and inequality. Although the two issues are interconnected, they are still different concepts, and we cannot confuse one with the other.

Most researchers study poverty and income inequality separately. In this paper, following Duclos and Makdissi (2004), poverty and income inequality are studied simultaneously. This section focuses on three questions: the first one is what the relationship is between poverty, inequality, and public policy; the second one is the selection of poverty lines; the last one is the choice of poverty indices

Arguments can be made on the selection of poverty lines and poverty indices. Stochastic dominance is one way to quench the arguments. Stochastic dominance is a technique leading to the robust poverty rankings and inequality orderings. For any given poverty ranking, stochastic dominance is not based on the specific value of poverty line and poverty index functionally, but rather on the social judgements axioms and principles which poverty index family should follow (Araar, 2006).

This section is organized as follows: section 1 expresses the interconnection between poverty, inequality, and public policy; section 2 introduces poverty lines; section 3 lays out some poverty indices used in this paper; section 4 introduces stochastic dominance and the techniques of ordering poverty and inequality dominance.

2.2 Poverty, Inequality and Public Policy

Some researchers declared that the essential of poverty is inequality, but Sen (1981) argued that although poverty and inequality have common characteristics, they are essentially different concepts. Poverty is identified as individuals whose income is below poverty line while inequality is the lack of equality of income redistribution. For instance, absolute poverty will vanish when the minimum income is equal to the absolute poverty line. Relative poverty exists as long as an income gap exists among individuals (Duclos & Araar, 2006).

Total income is divided by poverty line into two subsets, an upper set and a lower set. Define income distribution below poverty line as the lower set, and income distribution over poverty line as the upper set, such that

$$S_L = \{y_p \mid y_p \leq Z, p \in \mathbb{N}\},$$

$$S_U = \{y_i \mid y_i > Z, i = p + 1, p + 2, \dots, n\},$$

where Z is poverty line and y_p is the individual's income. Ordering y_p , so that it is in increasing in p , i.e. $y_1 \leq y_2 \leq \dots \leq y_p \leq Z < y_{p+1} \leq \dots \leq y_n$, \mathbb{N} is the set of nature numbers.

Income inequality will vanish when the income gap is equal to zero. Inequality changes with movements of any income transfer. If set I is an income transfer function, then income inequality decreases whenever I is transferred to a poorer person from a richer person, but it increases whenever I is added to a richer person from a poorer person in S_L . Inequality can happen both in S_L and S_U (Duclos & Araar, 2006).

Poverty only occurs in S_L . Poverty does not change when income transfers happen within the upper income set, since the income transfer is over poverty line and it does not influence the income of the poor person.

Poverty decreases whenever any income is transferred from the upper set to the lower set, since the income coming from the rich is added to the poor. This holds true as long as this transfers does not make the income of the rich go below poverty line. In the lower set, poverty decreases when income transfers is added to a poorer person from the less poor, and poverty is intensive when any income transfers is added to a poor from a poorer person (Duclos & Araar, 2006).

Public policy focuses on two purposes: poverty reduction and welfare enhancement (Lubrano, 2011). Government income transfers are a direct way to decrease poverty, but the effect on inequality is not clear. For example, the children benefits program helps families with children; however, the degree of poverty reduction is unknown, and the outcome needs to be estimated statistically. That is the purpose of this paper, to estimate poverty and inequality in Quebec from 1996 to 2006.

2.3 Poverty Lines

Poverty line is a criterion to identify who is poor and who is not. However, “poor” is a word developed within time and space. It is changing with the standard of living, personal preference, and individual satisfaction. Sen (1981) believed that poverty line can only be a hypothesis in any given society at any given time, and that any given poverty line is a distortion against reality.

Individual inference is too subjective to be a measurement of poverty. The alternative is to measure the standard of living. The standard of living changes with both time and space. It is easy to understand that the minimum living conditions on which is an individual can survive are different from town to town, country to country, and time to time. For example, the standard of living cannot be the same in Toronto downtown as in a small village of rural Manitoba.

Following Atkinson (1987), monetary income is chosen to define poverty line in this paper. Poverty line is the threshold for poverty identification, where the individual whose income is lower than the line is identified as poor. According to Sen (1979), the advantage of using an income poverty line is that it is easy to calculate the income shortfalls, because this method applies the numerical metric of income gap from poverty line.

Researchers and policy makers face a dilemma: setting a low poverty line may underestimate poverty in a society. The cost of poverty reduction decreases because there are fewer populations who are identified as poor. However, welfare may also decrease since part of the poor population is excluded from the social assistance class. On the other hand, setting a high poverty line may overestimate poverty in a society and expand the cost of poverty reduction.

According to Sen (1981), poverty has an absolute “core”, because those who are in absolute poverty cannot survive due to a lack of primary needs. The absolute poverty line is the minimum amount of income required to satisfy a given set of basic needs, the minimal standard of food, clothing, healthcare and shelter.

The income that is below an absolute poverty line is identified as absolute poverty. A relative poverty line is the threshold set as a ratio of the income distribution, such as the median. Income that is below relative poverty line is identified as relative poverty (Chiappero-Martinetti & Civardi, 2006).

There is no official poverty line in Canada. However, several unofficial low-income lines are regularly used in Canada poverty studies such as the Low Income Cut-offs (LICO) and the Low Income Measures (LIM), which are published by Statistics Canada.

“A low income cutoff is an income threshold used below which a family is likely to spend significantly more of its income on food, shelter, and clothing than the average family” (Cotton, 2002). LICOs focus on the family spending but rather the income share of food, shelter, and clothing. A family is identified as low income if its income is lower than the cutoff. Low-income family spend larger income share in food, shelter, and clothing than average family.

“Low Income Measures (LIMs) are defined as 50% of median adjusted family income” (Cotton, 2002). LIMs are used in international comparisons commonly while LICOs are used in Canada usually.

Human Resources and Skill Development Canada (HRSBC) publish the Market Basket Measure (MBM). The MBM is a measurement of cost of goods and services that are necessary to maintain physical health and social connection (Zhang, 2010).

Families may fall into poverty if their income is lower than LICOs, but sometimes they are not poverty at all, since low income and poverty are different situations and they have different identification criterion. LIM represents half of the contemporary median adjusted family income. Both measurements use the family as a unit of observation. This paper focuses on individual income since welfare cares individual benefits; therefore, individual income is the unit of observation in this paper.

Instead of using low-income lines, I use the method which sets poverty line as a “constant proportion” (Ravallion, 1996) of the median of income. There are two relative poverty lines used in this paper. One is the extreme poverty line, which is 30 percent of the median income; another is the moderate poverty line, which is 50 percent of the median income. The deflator is used since the data covers a ten-year time span. Here, individual income is the observation instead of family total income, such as LIMs and LICOs.

2.4 Poverty Indices

There are a great numbers of poverty indices used to estimate poverty reduction, but is there any one better than others are? The answer is yes and no. Whether a poverty index is good enough to be a poverty indicator depends on which results the researcher expected. Sometimes the simplest index, the Headcount index, is strong enough to indicate poverty incidence. However, poverty studies should discover more facts than incidence, such as the intensity and the severity.

2.4.1 The Headcount Index

The Headcount index, denoted as H , is the simplest index.

$$H = \frac{1}{n} \sum_{i=1}^n \lambda(y_i < Z), \lambda = \begin{cases} 1, & \text{if } y_i < Z \\ 0, & \text{otherwise} \end{cases}$$

where λ is the parameter whose value is 0 or 1. n is the total population, and Z is poverty line (Makoka & Kaplan, 2005). Poverty population is the population of people whose income is below line, and H is an indicator of poverty incidence. The main advantage of the Headcount ratio is its easy structure and the fact that it is easy to understand. On the other hand, its simplicity is the disadvantage of the Headcount index. This poverty measure indicates poverty incidence but does not demonstrate any other poverty characteristics such as the intensity of poverty, the severity of poverty, and income inequality.

The Headcount index cannot uncover the intensity of poverty. For example, assume two income metrics $A = (10, 10, 20, 20)^T$, and $B = (5, 5, 20, 20)^T$. If one sets $Z = 15$, then $H_A = H_B = 50\%$, but it is clear that poverty is more intensive in B than in A . In addition, if the poor individual becomes worse off, then poverty is more intensive; however, the poor population does not change, since the poor is still poor. Therefore, the Headcount index does not change either.

The unit used for the Headcount index should be the individual instead of the household, because the individual population is always greater than the total number of households. Poverty will be underestimated if the household is the unit of observation.

In this paper, family size is the coefficient as equivalence scale. Since this paper focuses on individual income but rather than family total income.

2.4.2 The Income Short-fall Index

The Income Short-fall index can capture the depth of poverty, because it is an index that indicates the gap between the individual's income and poverty line. In other words, this index is a measure about the "average distance" (Makdissi & Wodon, 2004) between individual income and poverty line. It shows poverty contribution from the poor individual

The income vector of n individual is $y_i = (y_1, y_2, \dots, y_n)^T$, $i = 1, 2, \dots, n$, where n is the population size and y_i , is the income of the i^{th} individual; the income of the poor individual is

$y_p = (y_1, y_2, \dots, y_q)^T$, where q is the size of the poor population. Poverty line is Z and both y_n and y_p are in increasing order, i. e. $y_1 \leq y_2 \leq \dots \leq y_q < Z \leq y_{q+1} \leq \dots \leq y_n$. The total income gap is

$$G = \sum_{i=1}^q (Z - y_i).$$

If the average income of the poor is μ_p and the average poverty gap is μ_G , then we get

$$\mu_p = (\sum_{i=1}^q y_i) / q,$$

$$G = q * (Z - \mu_p),$$

$$\mu_G = G / q = (Z - \mu_p),$$

both G and μ_G indicate the gap, with the former showing the total gap and the later showing the average gap. Let I be the Income Short-fall ratio,

$$I = G / (q * Z) = 1/q \sum_{i=1}^q [(Z - y_i) / Z] = \mu_G / Z,$$

where $(Z - y_i) / Z$ is the Income Short-fall ratio of the i^{th} poor individual. Holding $0 \leq I \leq 1$, a greater I indicates the deeper poverty. As shown in the equation above, there is a positive relationship between I and μ_G and a negative one with Z (Makoka & Kaplan, 2005).

The Income Short-fall index is not sensitive for the severity of poverty. For example, assume two income vectors, A is $(3, 3, 3, 5, 5, 5, 8)$ and B is $(4, 4, 4, 4, 4, 4, 8)$, and poverty line is $Z = 5.5$. Both metric A and metric B have the same Headcount ratio, $H_A = H_B = 6/7$ and the Income Short-fall ratio is $I_A = I_B = 3/11$. But the distribution of these two metrics is different. The maximum gap in metric A is 5, while the maximum gap of B is 4. We cannot say that metric A has the same income distribution as metric B , although they have the same H and I index. Furthermore, if the crucial standard of living income is 4, there is 3/7 population of A which do not have enough income for basic food, clothing, healthcare and shelter, but there is no population of B which falls into the vital situation.

2.4.3 Foster-Greer-Thorbecke (FGT) Index

Social welfare can link and integrate inequality under some restricted conditions. Following some social ethic principle, such as Pigou-Dalton transfer principle, we can find the positive relationship between welfare and income and negative relationship between welfare and poverty gaps (Duclos & Araar, 2006)

Following these principles and axioms, we also can construct poverty indices without poverty line. One way to construct poverty indices is using the distribution of poverty gaps. We can find aggregation of poverty as long as the distribution is known (Duclos & Araar, 2006).

Foster-Greer-Thorbecke (FGT) index is a class of poverty gap indices. FGT index has advantages that other poverty indices do not possess. It indicates poverty and inequality with different parameters. It is a “distributional sensitive measure” as Foster and Shorrocks (1988 a, 1988 b) pointed out. The FGT index is also convenient to do the decomposition test. Furthermore, the FGT index can be the equivalent of the stochastic dominance under certain conditions.

Under the assumption of continuity and differentiable (Atkinson, 1970), the FGT index is defined as:

$$P_{\alpha} = \int_0^z \left(\frac{z-y}{z}\right)^{\alpha} dF(y),$$

where Z is poverty line, y is any individual income in the income set, $F(y)$ is income distribution function and α is a poverty parameter and $\alpha \geq 0$ (Lubrano, 2011).

With $\alpha = 0$, $P_{\alpha} = \int_0^z \left(\frac{z-y}{z}\right)^0 dF(y) = \int_0^z dF(y)$ is the Headcount index. This index is the indicator of poverty incidence. It indicates poverty population ratio that shows how many people are poor. Practically, this poverty measure can get poverty sample size if it multiplies total sample size (Lubrano, 2011).

The first limitation of this index is it cannot show the welfare amelioration that happens under poverty line. On other words, the Headcount ratio does not decrease some time when the welfare is greater. For example, a richer person transfers some income to a poorer person, according to the Pigou-Dalton principle, the welfare is greater (Duclos & Araar, 2006). On the other hand, poverty population does not change if the sum of this transfer does not make the income of the poorer person over poverty line.

The second limitation is it cannot show the severity of poverty. This poverty measure cannot apply any information about inequality among the poverty. The higher order test is following, since the limitation of this index cannot give an aggregate poverty measure beside poverty incidence ratio.

With $\alpha=1$, $P_\alpha = \int_0^z \left(\frac{z-y}{z}\right)^1 dF(y) = \int_0^z \left(1 - \frac{y}{z}\right)^1 dF(y)$ is the Income Short-fall index. This index demonstrates the depth of poverty additively. It shows poverty degree, since it displays the gap between the average income of the poor and the alternative poverty line. This index displays how far the poor are away from poverty line, and consequently, it demonstrates the minimum budget of poverty elimination (Osberg & Xu, 1998).

It is notable that the FGT index at the first-order aggregates both Headcount ratio and Income Short-fall, but not the severity. This poverty measure fulfills the transfer ethical principle. Wellbeing is increased and poverty is decrease with any income transfers to the poor individual.

When $\alpha =2$, $P_\alpha = \int_0^z \left(\frac{z-y}{z}\right)^2 dF(y)$ is the income gap squared index. This poverty measure fulfills the Pigou-Dalton welfare transfers among income distribution segment that is below poverty line. Poverty declines when income is transferred from a poor person to a poorer person, and thus welfare is greater. The reverse transfer ethical principle is satisfied among the upper income distribution. An income transfer from a rich person to a richer person will improve the welfare (Duclos & Araar, 2006).

The FGT index at the second-order demonstrates inequality elimination when the monetary flow from an individual whose income just below poverty line to a poorest individual while poverty is better off.

Foster and Schorrocks (1988 a) pointed out that the FGT indices are good enough to imply poverty rank between income distributions under given poverty line, since this index class is an approach of poverty gap. Poverty orders may be not consistency under different poverty lines (Lubrano, 2011). To approach the consistency of poverty ordering, the stochastic dominance is introduced to test poverty rank robustly with a restricted range of poverty line.

2.5 Stochastic Dominance

Stochastic dominance was used as a methodology in finance research, particularly in risk management and in portfolio investment. Atkinson (1970) introduced this method to compare income distributions in the spirit of risk measurement under uncertainty (Lubrano, 2011). Foster and Schorrocks (1988 a; 1988 b) restricted the conditions to practice poverty dominance. Davidson and Duclos (2000) discussed the bounds of stochastic dominance and statistical inference. Finally, Duclos and Makdissi (2004) reviewed the robustness of restricted and unrestricted stochastic dominance for welfare and income inequality. The main contribution of stochastic dominance is the robustness of poverty orderings and social welfare rankings following ethical judgments (Araar, 2006).

2.5.1 Defining Stochastic Dominance

Assume that there are two Cumulative Distribution Functions (CDFs), F_A and F_B , which are from income distributions A and B respectively. F_A and F_B are smooth all over the domain, and let x is any individual income,

$$D_A^1(x) = F_A(x), \frac{dF(x)}{dx} = f(x)$$

$$D_A^s(x) = \int_0^x D_A^{(s-1)}(y) dy$$

for any integer $s \geq 2$, and we can denote $D_B^s(x)$ following the same analogy. For any order s , we also can rewrite $D^s(x)$ as:

$$D^s(x) = \frac{1}{(s-1)!} \int_0^x (x-y)^{(s-1)} dF(y).$$

If $D_A^s(x) \geq D_B^s(x)$ holds for all $x \in \mathcal{R}$, then we infer that distribution B stochastically dominates distribution A at order s (Davidson & Duclos, 2000).

First-order stochastic dominance is when $s=1$. if $D_A^1(x) \geq D_B^1(x)$ for all $x \leq Z$, then distribution B stochastically dominates A, since $F_A(x) \geq F_B(x)$ at $s=1$ which means that the proportion of poverty in population A is greater than that in B, then distribution B has less poverty than A has. We can say that distribution B is stochastic dominance distribution A. Stochastic dominance with $s=1$ is the equivalent of Headcount ratio of the FGT index with $\alpha = 0$, since $\alpha=s-1$ (Davidson & Duclos, 2000).

When $s=2$, it is said the second-order stochastic dominance, and if $D_A^2(x) \geq D_B^2(x)$ holds, then we can get

$$\int_0^x (x-y) dF_A(y) \geq \int_0^x (x-y) dF_B(y).$$

For all $x \leq Z$, we can define the income gap, g , as:

$$g = Z - y.$$

If $x \geq 0$ and $g \geq 0$ hold for poverty line Z and an individual's income (assume that poverty gap exists in the distribution, and poverty line is greater than 0), then we could infer that B dominates A stochastically at second-order, which indicates the average income gap of A is greater than that of B.

When we move to the third-order, $s=3$, if $D_A^3(x) \geq D_B^3(x)$, $\int_0^x (x-y)^2 dF_A(y) \geq \int_0^x (x-y)^2 dF_B(y)$ holds, then we can declare that distribution B dominates distribution A stochastically at third-order. And the higher order than 3 can be arrived at analogically (Davidson, 2006).

2.5.2 Poverty and Stochastic Dominance

Following Atkinson (1970), the social welfare function is assumed to be concave, continuous, and homogeneous. We can define a poverty function with respect to poverty gap, $p(\cdot)$, as follows,

$$\begin{cases} p(y, z) \geq 0, & \text{if } y \leq z, \\ p(y, z) = 0, & \text{if } y > z. \end{cases}$$

If $p(\cdot)$ is differentiable, then we can define poverty indices family with respect to the income gap, g , as

$$P(z) = \int_0^z p(g(z, y)) dF(y).$$

The welfare dominance can be tested as the equivalents of the FGT poverty indices with parameter α . The first-order welfare dominance is the equivalent of the FGT poverty index with $\alpha = 0$, which is the Headcount index.

Social welfare is greater in B than in A if the Headcount ratio is greater in A than in B . Numerically, $P_A(Z, \alpha = 0) \geq P_B(Z, \alpha = 0)$, if and only if $D_A^1(Z) \geq D_B^1(Z)$ holds and Z is between 0 and infinity (Duclos & Araar, 2006). However, there are only two poverty lines that are chosen in this paper, and then we employ stochastic dominance to test the robustness of the social welfare rankings and poverty orders under the two poverty lines.

The FGT poverty index at first-order, P^1 , is the equivalent of second-order stochastic dominance. The second-order welfare dominance is following Pigou-Dalton transfer principle (Atkinson, 1987). We can find P^s following the same mathematical analogy for all $P(\cdot) \in P^s$ if and only if the stochastic dominance holds for all $x \leq Z$ and s (Davidson & Duclos, 2000).

Assume that a maximum income y^* exists in the income distribution; we can define poverty line as

$$Z_s = \inf\{x > 0 \mid D_A^s \geq D_B^s, s=1, 2, \dots\}.$$

Technically, we can test poverty dominance running all poverty lines infinitely, but practically, we can restrict poverty line Z in a range $[Z^-, Z^+]$ and denote it as Z^* . Using stochastic dominance test, we can find poverty dominance and welfare dominance robustly.

The strategies are: if poverty dominance between the distribution A and B is not clear at order s with poverty line Z_1^* , then we can move to order $s+1$ dominance test or move to poverty line Z_2^* , until we find distinct poverty dominance between distribution A and B (Davidson & Duclos, 2000). The higher the s -order is the more attention paid to the poorest population located at the left tail of income distribution (Araar, 2006).

It is a convenient technique using the FGT curves to infer the stochastic dominance of two income distributions, since the FGT index with parameter $\alpha = s-1$ is the equivalent of the stochastic dominance with order $s = \alpha+1$. Graphically, for any two income distributions A and B , if the FGT curve of A is above the FGT curve of B everywhere, then the distribution of B dominates the distribution of A stochastically (Araar & Duclos, 2009).

Two FGT curves may intersect, and the intersection is called “critical points” (Araar, 2006). The “critical points” can be chosen as the moderate poverty line that identifies the pertinent poverty population without underestimating or overestimating the poor population (Lubrano, 2011).

2.5.3 Inequality and Stochastic Dominance

One of the most commonly used methods of testing inequality with respect to stochastic dominance in two income distributions is the comparison of normalized Lorenz Curve $L(p)$ at the second-order. Atkinson (1970) pointed out that the Pigou-Dalton principle is the ethical doctrine for indices to demonstrate inequality. If the normalized Lorenz curves of A , $L_A(p)$ is above

normalized Lorenz curve of B everywhere, $L_B(p)$, and then the inequality in A is more oppressive than that of B (Araar, 2006).

Only the normalized Lorenz curve can be used as the equivalent of a stochastic dominance test, but it is not the technique used in this paper. In the present paper, the FGT index with $\alpha = 0, 1,$ and 2 is compared with two income distributions for income inequality in poverty population. The FGT index with $\alpha = 0, 1,$ and 2 is the equivalence of stochastic dominance with $s=1, 2,$ and 3 .

Following Araar and Duclos (2009), if FGT curve has any intersect, the intersection of the FGT curves leads to the fuzzy rank of poverty dominance. We can move to higher order test or choose another alternative poverty line to find poverty dominance.

3 Data Sources

3.1 SLID Data

The data used in this paper is from the Survey of Labour Income Dynamics (SLID) economic family files. The SLID data provides the income information for individuals and their families from all ten Canadian provinces. The SLID is a longitudinal survey that was started in 1993, and it follows individuals over a six-year window. A new panel is introduced every three years, and as such, the panel overlap.

For this paper, I rely on the public-use files. Unlike the master files,² the public-use files are cross-sectional in nature. The variable that allows the researcher to follow an individual across time is masked in the public-use files. A second limitation of the public-use files is that they are only available as of 1996. My analysis therefore covers the 1996 to 2006 period. Data on the recent recession were not available at the time the empirical work was carried out which explains why 2006 is my end year. To simplify the presentation of equations that follow, I have decided to use the same variable names as presented in the SLID dataset.

² The master files are only available through the Research Data Centers (RDCs).

3.2 Description of the Variables

3.2.1 Family Size

In this paper, family size is calculated as:

$$FS = \sqrt[2]{fmsz}$$

fmsz is the number of family members from the SLID economic family files. The employment of *FS* counteracts the size gap between big and small family.

3.2.2 Family Weight

This paper is interested in the individual's income and the welfare of individuals. However, the unit of SLID dataset is family, and under the assumption that the income should be shared by each members of the family (Osberg & Xu, 1998), the simplest way is using an equivalence scale to recalculate the income shares of each family member.

Family weight is equivalence scale in this paper and it is calculated following this formula:

$$FW = icswt * 100000 * fmsz$$

where *icswt* is the code of Regular cross-sectional weight and *fmsz* is the number of family members.

3.2.3 Real Income Coefficient

Real income coefficient is used to normalize income in provincial level and ten years' time. The coefficient can offset the impact of inflation and provincial economy diversity during time.

The paper relies on data that spans a ten-year period, and comparisons are made every year in Quebec and other Canadian provinces. In this paper, 2006 is the base year, if I want to compare

income in 2006 with income in 1996, then income must be normalized. Since one thousand dollars in 2006 maybe was equal to one thousand and eighty dollars in 1996 or only nine hundreds and ninety dollars in 1996 depending on an inflation rate.

Inflation is one of the factors that influence the real income. The poor are worse off by inflation than the rich are because the poor hold larger cash share than that of the rich in portfolios. Furthermore, the poor depend on income more than the rich do. On the other hand, inflation reduces taxes and government transfers in terms of real income. For example, inflation offsets the real value of the children tax benefits and the seniors pensions (Easterly & Fischer, 2001).

Additionally, each province has economy variety, consumption diversity, and income distributions. For example, the minimum income rate is different among provinces; furthermore, inflation rates are different among provinces also, and then I choose provincial price deflator instead of Canadian price deflator.

The coefficient is following as the formula:

$$D = \frac{CPI_{97}^{ON}}{CPI_t^{ON}} * \frac{CPI_t^r}{CPI_{97}^r} * Deflator_{97}^r$$

where CPI_{97}^{ON} is the customer price index for Ontario in 1997 as the base year. The subscript r also represents provinces. Finally $Deflator_{97}^r$ is the consumption price deflator for Canadian provinces in 1997.

3.2.4 Total Income

Although Sen (1981) suggested that consumption was the best indicator for poverty research and income just was the "second best", but in this paper, income has advantages to define poverty line rather than consumption. Since one of hypothesis of this paper is welfare policy and taxes and government transfers play a large role in poverty reduction; therefore, income is a better observation to imply poverty gaps between different income distribution levels.

Following Atkinson (1987), income is easy to compare numerically. Another advantage is it is easy to calculate the income gap. Furthermore, it is a direct way to estimate the cost of poverty reduction. This paper focuses on the welfare policy and taxes and government transfers, then income comparisons of before and after taxes and government transfers are an efficient way to evaluate public policy.

I code total incomes before taxes and government transfers as BTIC and the formula is following as:

$$BTIC = (ttinc - gtr) / (FS * D)$$

where *ttinc* is used as the same code as SLID used for total income before tax and *gtr* is total government transfers from SLID also. The total government transfers includes: federal provincial program general child tax benefits, Canada pension plan and Quebec pension plan benefits, federal goods and services tax/harmonized sales tax credits, Old Age Security benefits, Provincial tax credits, social assistance, employment insurance benefits, and Workers' compensation benefits.

I code total incomes after taxes and government transfers as ATIC and the formula is following as:

$$ATIC = atinc / (FS * D)$$

where *atinc* is used as the same code as SLID used for total income after taxes, *FS* is the calculated family size, and *D* is the price deflator.

3.2.5 Poverty Line

There are difficulties in choosing a poverty line. The first one is the choice between an absolute and a relative poverty line. An absolute poverty line is usually chosen for developing country studies while a relative poverty line is ordinarily chosen for developed country research (Beyene, 2009). Relative poverty line is chosen in this paper.

This paper uses two relative poverty lines: one is the extreme poverty line, Z_E , and another is moderate poverty line, Z_M . They are defined as following:

$$Z_E = (CPI_t/CPI_{2006}) * 30\% * \text{median of } ATIC_{2006}$$

$$Z_M = (CPI_t/CPI_{2006}) * 50\% * \text{median of } ATIC_{2006}$$

where CPI_t is the customer price index of Canada from 1996 to 2006, $ATIC_{2006}$ is total income after taxes and government transfers of 2006.

3.3 Confidence Interval

To test the hypothesis that one income distribution dominates another, I use a 95 percent confidence interval.

4 Methodology

4.1 The FGT Index and Stochastic Dominance

FGT indices indicate poverty ranks summararily, which is limited by specific value of poverty line. Stochastic dominance indicates the probability of being poverty with one income distribution is not larger than that with another income distribution. Following Foster, Greer, and Thorbecke (1984), it is easy to link income distribution orders and FGT indices when poverty parameter $\alpha = s-1$ (Lubrano, 2011).

Income distributions are tested at the extreme poverty level and the moderate poverty level. The comparisons are made between Quebec and the other nine Canadian provinces from 1996 to 2006. In Quebec, the income is compared between 1996 and 2006.

Population subgroups incomes are also compared to identify which group is worse off and which group is better off over ten years in Quebec.

Pigou-Dalton social ethic principle is the main axiom for which stochastic dominance is following. Any income transfers from a richer individual to a poorer individual will increase the welfare and decreases poverty (Atkinson, 1987).

Graphically, we can find poverty and welfare dominance by comparing the FGT curves. However, the intersection of two curves raises the argument that the results of poverty and inequality comparisons are not always reliable. Under this situation, the test must move to higher order until the results are robust.

Another method to get consistency poverty ranking is to modify the value of poverty line. These tests should go through the entire income domain and estimate all possible poverty lines. It takes time to test all poverty lines, and these tests need a large sample size (Lubrano, 2011). Therefore, the stochastic dominance is introduced to compare two income distributions robustly and effectively.

4.2 Group Decomposition of the FGT Index

The FGT poverty index can be decomposed by population subgroup, income component, growth, and redistribution components. In this paper, the FGT poverty index is decomposed by population subgroups, since this paper focuses on the welfare of individuals instead of the income structure and impact of economic growth.

The total FGT index can be formed as the weighted sum of the FGT index of each subgroup. The decomposition of FGT index by the population subgroup defined as:

$$P(Z, \alpha) = \sum_{g=1}^G \varphi(g)P(Z, \alpha, g),$$

where g is the number of subgroups, Z is poverty line, $\varphi(g)$ is the population share of the subgroup, and $P(Z, \alpha, g)$ is the FGT index of the subgroup. Furthermore, $\varphi(g)P(Z, \alpha, g)$ is the absolute poverty contribution of the subgroup while $\frac{\varphi(g)P(Z, \alpha, g)}{P(Z, \alpha)}$ is relative poverty contribution

of the subgroup, where $P(Z, \alpha)$ is the FGT index of the total income distribution (Araar & Duclos, 2009).

The decomposition of the FGT index demonstrates poverty contribution of the subgroup, and it is easy to find out which subgroup is poverty-stricken and needs more welfare improvement. According the subgroup-decomposable principle, each subgroup is independent from other groups in welfare improvement. Income transfers to any subgroup will improve the aggregate poverty (Duclos & Araar, 2006).

In this paper, the population of Quebec is separated into segments by gender, age, marital situation and the number of children. Some subgroups are expected to have intense poverty such as lone-parents and senior single women. Poverty indices are compared every year in a ten-year period.

4.3 Restricted Stochastic Dominance Procedure

The FGT index is strictly equivalent of stochastic dominance at $\alpha = s-1$. Restricted by two poverty lines, stochastic dominance is used to find poverty dominance without running the FGT index by for all alternative poverty lines.

The test is to compare total income after taxes and government transfers with total income before taxes and government transfers. The efficiency of taxes and government transfers has to be taken into account and it is compared between homogeneous subgroups (Lubrano, 2011).

From the above test, we can find the difference of the FGT index that indicates poverty reduction tendency. Therefore, we can evaluate the efficiency of public policy and net poverty reduction.

The stochastic dominance procedure includes comparisons of the province of Quebec with Canada in 1996 and in 2006; comparisons of different population subgroups within the province of Quebec; comparisons of income of 1996 with 2006 in the province of Quebec. Both poverty lines are used for above tests where the parameter $\alpha = 0, 1, \text{ and } 2$.

5 Results and Discussions

The results are present from the most general tendency to the most specific situations, and from cross-provinces to within the province of Quebec.

5.1 Poverty and Inequality at the Extreme Poverty Level

The total income after taxes and government transfers is compared between Quebec and Canada from 1996 to 2006. The FGT index with $\alpha = 0, 1, 2$ is the equivalent with the stochastic dominance strictly with $s=1, 2, 3$. The FGT index curves are compared to find the robust poverty dominance.

Figure 1 shows the FGT index with $\alpha = 0$, where the Quebec poverty population ratio is compared with that of Canada. The Quebec curve is located everywhere below the Canadian curve while the two have the same decreasing tendency. Both curves decline steeply before 1999 and become flatter after 2000.

However, there was a 7.2 percent increase in poverty population ratio from 1999 to 2000 in Canada. The ratio grew sharply to 12.1 percent in 2000 and dropped to 4 percent in 2001. Without any calculation error, the exact reason is unknown for this abrupt jump, but one of them is the original data published by SLID is not decent and this part is far away to be controlled by users. In Quebec, the ratio had a 0.4 percent increase in 2000 when compared to that of 1999.

Quebec had a smaller poverty population ratio than Canada from 1996 to 2006. Poverty population ratio kept decreasing from 1996 until 2006. Poverty population ratio rose in 2000 in both Quebec and Canada. However, the rise in Quebec was more modest.

Compared to Canada, Quebec holds stochastic dominance. Because the Quebec curve is below the Canadian curve everywhere and there is no intersection at the first-order, Quebec definitely has a smaller poverty population ratio.

For Canada, poverty population ratio was 5.8 percent in 1996, dropping down to just less than 4 percent in 10 years. The biggest ratio was at 12.1 percent in 2000, and declining all the way to 4 percent in only a year, in 2001.

Poverty population ratio curve was flat after 2001, and there was a small rise to 4.3 percent in 2004, while the ratio was declining continuously until 2006 when it stopped to 3.8 percent.

For Quebec, poverty population ratio had the same tendency as Canada's, but the degree of fluctuation is much smaller than that of Canada. The ratio was 3.8 percent in 1996 with a slight ascent to 4 percent in 1997, and then a decline to 3.3 percent by 1999.

In 2000, there was a 0.4 percent ascent from 1999, and then a bigger descent of 1.1 percent in 2001. Poverty population ratio went down to 2.1 percent in 2004 and remained steady until 2006.

Figure 2 shows the FGT index curves with $\alpha = 1$. This poverty index indicates the average income gap. The FGT index with α equals to one is the equivalent to stochastic dominance with $s = 2$. It demonstrates the average distance between the income of the poor and poverty line. It is also an indicator of the minimum cost of poverty reduction.

Quebec holds a stochastic dominance at order two, since the FGT curve for Quebec is everywhere below the Canadian curve. Both Quebec and Canada had a decreasing tendency of the income gap from 1996 to 2006. Quebec had less intensive poverty and smaller income gap than that of Canada. The welfare of Quebec was better than that of Canada.

For Canada, there was a 10.8 percent decrease in the average income gap from 1996 to 2006. A sharp jump of 6.2 percent still existed in 2000 as compared to 1999. The average income gap decreased from 14 percent in 2001 to 11.9 percent in 2006.

Quebec had a similar decreasing tendency as Canada had, from 17.5 percent dropping to 9.1 percent. The decline is much smoother. The decreasing tendency stopped at 9.1 percent in 2004. After decreasing constantly in nine years, the income gap had a 0.9 percent increase in 2005. It approached 9.2 percent in 2006 with a 0.8 percent decline from 2005.

Figure 3 shows the FGT index curve where α equals to 2. The FGT index test is the equivalent of restricted stochastic dominance test at the third-order. This Figure shows the severity of poverty. The squared income gap demonstrates inequality of poverty income distribution.

Quebec did a better job in income inequality elimination than Canada did, and Quebec holds stochastic dominance at third order. The Quebec curve is everywhere below the Canadian curve and the FGT index difference is significant.

For Canada, the notable inequality increasing was in 2000. Income inequality was increasing from 2.1 percent in 1999 to 9.5 percent in 2000. Inequality dropped back to 1.6 percent in 2001. After that, the fluctuation of severity is fairly flat. The FGT index was 1.5 percent in 2006. Again, without any calculation errors, the reason for this abrupt is hard to trace, but one of tentative reason is the SLID published data is not decent for 2000. The further research in the future will explain the theoretic assumption.

Quebec has less severe poverty than Canada but the movement of poverty changes is undulating with slight up and down wave. The main tendency is decreasing; from 1.4 percent in 1996 to 0.7 percent in 2006. There were three slight ascents, 0.5 percent in 1997, 0.07 percent in 2000, and 0.4 percent in 2005 compared with previous year respectively. Inequality in poverty distribution did not change much from one year to another.

There was a recession during 2000-2001, with some economists declaring that this recession was very mild in Canada. However, Figure 1 shows that the recession impacted in poverty incidence ratio is much more serious in Canada than in Quebec. The recession did not impact the province of Quebec in terms of poverty population ratio.

Figure 2 shows that there are no reasons to infer that the 2000-2001 recession made any impact on the average income gap of Quebec. The depth of poverty did not change the path from decreasing, and poverty was sustained with a smooth decline from 13.5 percent in 1999 down to 9.2 percent in 2006.

For Canada, the abrupt changed inequality of 2000 might be associated with the recession of 2000-20001. For instance, the layoff may be a reason of short-term shortage of income or the lower income, which can be the reason for poverty severity. The explanation may form another analysis of employment and poverty, which is beyond the scope of this paper.

Another explain for this sharp jump happened in 2000 in Canada is the original data is not decent published by SLID.

Although poverty is not intensive in Quebec, but there are some space to reduce it, relieve its severity, and eliminate inequality, which would increase the welfare in Quebec.

5.2 Poverty and Inequality at the Moderate Poverty Level

Figures 4 to 6 are the FGT index curves with $\alpha = 0, 1, \text{ and } 2$ at the moderate poverty line level from 1996 to 2006. Total income after taxes and government transfers of Quebec compares with Canada as a whole.

Figure 4 shows the FGT index where α equals 0. The province of Quebec dominates Canada stochastically, since poverty population ratio curve is everywhere below Canadian curve.

Both Quebec and Canada had bigger poverty population ratio with the moderate poverty line than with the extreme poverty line. On the other hand, both Quebec and Canada had a poverty decreasing tendency from 1996 to 2006. A 20 percent increasing income gap leads to more population falling into poverty.

For Canada, poverty population ratio was 6.2 percent in 1996 and it decreased to 3.8 percent in 2006. Poverty population ratio increased to 12.4 percent in 2000. It dropped to 4.3 percent in 2001 and it was 3.8 percent in 2006.

In Quebec, there was 2.6 percent of descent from 1996 to 2006. There were three increases happened in 1997, 2000, and 2005 respectively, but all these raises are slight.

Poverty population ratio at the moderate poverty line is greater than that at the extreme poverty line in both Quebec and Canada, the ratio decreased in faster speed. For example, in Quebec, a 1.8 percent population ratio decreased from 1996 to 2006 with the extreme poverty line. The descent of population ratio with the moderate poverty line was 2.5 percent from 1996 to 2006.

Figure 5 shows the FGT index where α equals 1. Quebec had smaller income gap than Canada from 1996 to 2006. The decreasing tendency of Quebec is fairly flat. Canada's average income gap increased both in 1997 and 2000 sharply. After 2001, the FGT curve showed that the intensity of poverty was fairly flat in Canada.

Canada had a big income gap in 1997. The average income gap is 58.8 percent and the difference is significant. The gap fell to 3.5 percent in 1999, rising to 10.9 percent in 2000 again. After 2001, it was fairly flat until 2006.

The Quebec curve approached the Canada curve but they did not intersect. The gap between them is not significant after 2001. The average income gap of the poor is converging to poverty line in both Canada and Quebec.

Figure 6 presents the FGT curve where α equals 2. It is the equivalent to stochastic dominance at third order. The province of Quebec dominates Canada stochastically, since the Quebec curve is everywhere below the Canadian curve across the income distribution.

In Canada, the worst inequality was 24 percent in 1997 and the second worst was 11 percent in 2000. Inequality had improved to 2.4 percent in 2001. After 2001, it was fairly flat. For example, it was 1.2 percent in 2006.

Inequality of Canada is worse than that of Quebec. The worst inequality was only 4.4 percent in 1997 in Quebec. Canadian had more than 19.6 percent intensity than Quebec in 1997 and 9.9 percent in 2000.

5.3 Overall Income in Quebec and Canada

Figure 7 and 8 present comparisons between the total income both after taxes and government transfers and before taxes and government transfers. They are comparisons of Quebec's income and Canadian's from 1996 to 2006.

Figure 7 shows the income tendency in Quebec. The total income after taxes and government transfers are compared to the total income before taxes and government transfers. The two income curves have the same increasing tendency, since the total income increased during 1996 to 2006.

The total income before taxes and government transfers curve was above the total income after taxes and government transfers curve until 2004. They have 28.76 dollars of difference in 2004 and the total income after taxes and government transfers curve was over its contradistinction in 2005, with the gap expanding in 2006.

Although there was a rise of total income before taxes and government transfers in 2000, the intensity of poverty is increase in Quebec in 2000 when the total income after taxes and government transfers decreased.

2005 is a notable year in Quebec. Total income after taxes and government transfers is greater than total income before taxes and government transfers. In 2005, the Quebec government introduced some new programs to improve the working poor. One of the programs is Work Premium. It replaced the parental wage assistance program that "was an earning supplementation measures with restrictive rules and an associated low take-up rate" (Torjman, 2010). The work premium pays refundable tax credit even to workers whose income is lower than the taxpaying threshold (2010).

The Headcount ratio had risen slightly and poverty is severe disregarded in terms of both the extreme poverty lines and the moderate poverty line.

With the extreme poverty line, the Headcount ratio rose 1.9 percent in 2005 in comparison to that of 2004. With the average income distance between the income of the poor and poverty line increasing 0.9 percent, the poor were worse off in 2005. Inequality among the poor had risen 0.3 percent in Quebec.

With the moderate poverty line, the Headcount ratio increased 5 percent compared to that of in 2004. The average income gap of the poor had risen 1.5 percent while inequality among poverty population increased 0.8 percent in 2005 compared to that of in 2004. The higher poverty line intensified the aggregate poverty.

Figure 8 shows the total income of Canada. Total income after taxes and government transfers curve is everywhere below the curve of total income before taxes and government transfers. Both curves were almost parallel.

Another notable tendency of the two curves was that they had simultaneous turning points. Two rises were seen in both 1999 and 2001 connected by one drop in 2000 and all three changes were small.

The comparison of Canada's total income curves showed a negative relationship between the intensity of poverty and total incomes. Aggregate poverty decreased while the welfare increased as the total income had risen. On the other hand, poverty increased when the welfare was worse off while the total income declined. This circumstance was true for total incomes both after and before taxes and government transfers.

5.4 Poverty and Inequality in Quebec

Figure 9 is present the FGT index with parameter 0, 1, and 2 and the extreme poverty line and the moderate poverty line are chosen. The comparison is between the total income after taxes and government transfers in 1996 and 2006.

The situation is very clear from Figure 9. The province of Quebec did a good job to reduce poverty from 1996 to 2006. Intensity of poverty was lower in 2006 than that in 1996 and the

difference is statistically significant with 95 percent confidence interval. All the Headcount ratio, the Income Short-fall index, and the squared income gap are much lower in 2006 than they were in 1996.

With the extreme poverty line, the Headcount ratio dropped 1.51 percent, the average income gap decreased 0.7 percent since 1996, and the squared of average income gap had decreased by 1.78 percent in ten years. With the moderate poverty line, the Headcount ratio dropped 8.4 percent, the income gap decreased 2.6 percent, and the squared income gap declined 1.5 percent from 1996 to 2006.

Higher value of poverty line pushed more people into poverty population. The province of Quebec dragged higher population ratio out of poverty. There was a greater increasing in welfare and a decreasing in income inequality than that of the lower value of poverty line.

In comparing the extreme poverty line, the province of Quebec had lower intensity of poverty than most of the other provinces in both 1996 and 2006. The province of Quebec did a good job in contributing to lower population ratio, less intensity of poverty, and lower inequality.

Figure 10 shows FGT index comparisons of ten provinces' total income after taxes and government transfers at the extreme poverty level in 1996. New Brunswick had smaller population ratio and less intensity of poverty than that of Quebec, but the difference is not significant statistically. There are five provinces that had less inequality than Quebec in 1996. The gaps were from 0.1 percent to 0.5 percent respectively, thus the gaps were not significant statistically.

Figure 11 shows the FGT index comparison of ten provinces' total income after tax and government transfer at the moderate poverty level in 1996. The province of Quebec still had some advantages in reducing the intensity of poverty. Prince Edward Island, New Brunswick and Alberta were better off than Quebec with Headcount ratio and the maximum difference was 4.5 percent.

These three provinces were also better off than Quebec in terms of intensity of poverty; the maximum gap was 0.9 percent. Prince Edward Island and New Brunswick had less inequality than Quebec did; the difference was a maximum of 0.6 percent. However, all these difference are not significant statistically.

Figures 12 and 13 present the FGT index of total income after taxes and government transfers at the extreme poverty level and the moderate level in 2006 respectively. The figures imply that the province of Quebec did better in poverty reduction in 2006 than it did in 1996. Furthermore, it did better than other provinces in 2006 as well.

With the extreme poverty line, Quebec had the least intensity of poverty among all of other nine provinces. Prince Edward Island had a smaller average income gap than Quebec did, but the gap was only 0.13 percent, which was not statistically significant.

According to Osberg and Xu (1998), Prince Edward Island did a very good job at the beginning of the 90's in reducing intensity of poverty. Quebec was already better than PEI in 2006.

With the moderate poverty line, only Alberta had a smaller income gap than Quebec and the difference was 0.24 percent. Therefore, it was not statistically significant either.

5.5 Group Decomposition of the Poverty

5.5.1 Decomposition of Total Income after Taxes and government transfers

The decomposition of the FGT index decentralizes the total poverty index into the sum of the index of population subgroups. The entire poverty index is decentralized in terms of subgroups' population shares and poverty contribution to total poverty.

Figure 14 and 15 present the decomposition of Quebec's total income after taxes and government transfers from 1996 to 2006.

Table 1 and 2 lay out poverty indices reduction comparing total income before and after taxes and government transfers in 2006 in Quebec. The two poverty lines are used, and the parameter equals 0, 1, and 2.

Figure 14 shows the decomposition of the Quebec total income after tax and government transfer in 10 years. Focusing on the working group, this paper decentralizes the family type into two groups. One group is of people who are younger than 65, then it is the working age group. Another group is of those who are equal or older the 65 and it is the non-working age group.

In the working age group, single mothers have the highest risk to fall below poverty line, since they have the lowest total income after the taxes and government transfers. Following single mothers, single females are the second weakest group to poverty, since their incomes are as low as the lone mothers'. As shown in Figure 14, single males are the high risk group is easy to fall into poverty, because their income is the third lowest.

Married people are wealthy. Couples who do not have children at home have the highest income since 1996, and they still have the significant increasing tendency. Couples who have children at home have the second highest income over ten years.

Single fathers had the third highest income in 1997. After 1999, their income caught up the level of single male, and the difference was not very significant.

The income gap was 15,314 dollars between the single mother and the couples who have children in 2006, and the gap was 12,410 dollars in 1996. It is easy to see that the gap is extended to almost 3,000 dollars in 10 years.

In working age group, females are "the most vulnerable to poverty" (Bönke & Schröder, 2010). Both single mothers and single women are in the highest risk of poverty. They have the lowest income compared to other families.

Focusing on the non-working age group, Figure 15 implies that single women had the lowest income since 1996, and they remained in the low income category for 10 years. They are still the poorest group in 2006.

Their income gap is 7,088 dollars compared to couples who do not have children at home and there are 1,597 dollars compared to males in 2006. The gap was 6,220 dollars and 150 dollars in 1996 respectively. The income gap had steeply been extending.

5.5.2 Decomposition of the FGT Index Reduction

Table 1 and 2 lay out the FGT index reduction by the Quebec taxes and government transfers in 2006. Table 1 is at the extreme poverty level and Table 2 is at the moderate poverty level. The decomposition of the FGT index is with parameter equals 0, 1, and 2. The FGT index is decentralized by the family types. The reduction of the Headcount ratio, the average income gap, and the squared income gap are depicted. As shown from Table 1, there are noticeable divergences across family types.

Focusing on the Headcount ratio, senior single females are the most favourable group for the taxes and government transfers, since their extreme poverty population ratio is 68 percent decline. Senior single males eliminated 64 percent extreme poverty population ratio by the taxes and government transfers.

For working age group, lone mothers are the most favourable group who had a decrease of 31 percent of the extreme poverty population ratio. Single fathers are the second most favourable group, whose poverty population ratio was eliminated by almost 17 percent in 2006.

On the other hand, couples who do not have children at home experienced the least reduction of poverty population ratio in 2006. However, they have the highest income out of all the subgroups.

As shown in Table 1, lone mothers had experienced almost a 19 percent reduction of the average income gap. The intensity of poverty had improved in income distribution of lone mothers.

Taxes and government transfers benefitted lone mother a 19 percent reduction of the average income gap in 2006.

Non-senior single women had the second most poverty reduction based on the tax and government transfer. They have almost 18 percent of intensity reduction.

For seniors, both single males and females have greater intensity of poverty; they reduce 56 percent and 48 percent of average income gap respectively. The couples have 31 percent of intensity of poverty reduction among their income distribution.

Focusing on income inequality, working age group, single females have the most reduction of the squared income gap, with a 17 percent decrease in the severity of poverty among their income redistribution.

The income redistribution also helps lone mothers eliminate over 14 percent of the severity of poverty. Lone fathers experienced a 7.6 percent improvement in the severity of poverty.

For seniors, single females saw a 50 percent improvement in the squared income gap. The income redistribution benefits the poorer part of income distribution of senior women. Senior men's income distribution was also improved by almost a 44 percent reduction in the severity of poverty.

Seniors, all single males, single females, and couples, had the most to gain from taxes and government transfers and this was evident in the significant reduction of their aggregate extreme poverty.

After seniors, single mothers subgroup were the second highest benefit holders of poverty reduction in 2006 according to the all subgroups, and they had the biggest poverty reduction according to the working age group.

Table 2 presents the decomposition of the FGT index reduction by family types at the moderate poverty level. Again, the reduction is based on taxes and government transfers in 2006.

Focusing on the Headcount ratio, among the working age group, lone mothers experienced a reduction of 30 percent of poverty population ratio, and lone fathers saw over 19 percent reduction in poverty population ratio.

All seniors shared over 60 percent of poverty population ratio reduction due to taxes and government transfers. Single women benefitted 63 percent, couples had 62 percent reduction, and single males saw an improvement of 61 percent.

Focusing on lone mothers, their reduction in intensity of poverty was at 24 percent; the welfare was greater in their income distribution. Income inequality was improved by 19 percent, and they were the most favourable group in eliminating the severity of poverty.

Non-senior single women experienced a 14 percent reduction in intensity of poverty and a 17 percent improvement in the severity of poverty. Their average income gap was 14 percent smaller than that of before tax and government transfer while 17 percent of inequality decreased in their income distribution. Their welfare is much greater due to the income redistribution.

All seniors are better off from the income redistribution, since there is elimination of both intensity of poverty and severity of poverty. Single women are the most favourable group having experienced over a 62 percent reduction of the average income gap and 57 percent improvement of equality among their income distribution.

6 Conclusions

Quebec did a good job in poverty reduction. In the ten years from 1996 to 2006, the province of Quebec had a smaller Headcount ratio, less intensity of poverty, and was able to reduce income inequality.

In 1996, there were some provinces that did better than Quebec, such as PEI and Alberta, but Quebec caught up and won the campaign to reduce poverty. It not only reduced poverty among its population but it was also able to reduce the intensity as well. Welfare is now greater than it was in 1996 and as compared to the other provinces.

The 2000-2001 recession did not impact on Quebec as much. On the other hand, while Quebec was affected by that recession trivially, poverty reduction had not changed its declining path by 2000. The intensity of poverty was reduced during that recession. Although, the Headcount had slightly risen, it recovered in the next year.

Income policy played a large role in poverty reduction, particularly the income after taxes and government transfers. It may be that it makes some subgroup worse off, but this situation cannot be found from the Quebec taxes and government transfers. The government transfers, such as the Children Tax benefits, reduced poverty intensity, making certain groups better off.

Quebec poverty intensity fluctuated over time, but the movements were marked by flat trends, when compared to poverty intensity of Canada. Although poverty reduction initiatives were successful, the speed at which poverty was reduced was slow, which means that it takes a long time to combat poverty. Since poverty population, which is just below poverty line has survived and left, the rest remain far from poverty line, and it takes more cost and longer time to eliminate poverty and inequality.

The FGT decomposition showed some important results for population subgroups. Women are the group that deserves the most attention. Lone mothers have the lowest income, and the income gap is getting extended when compared to with the previous year and other groups.

Older single women are also a group that suffers poverty. They hold less income and have higher intensity of poverty. The severity of poverty is worse in their income distribution.

“The most vulnerable group to poverty” (Bönke & Schröder, 2010) experienced some improvement from poverty reduction due to the Quebec taxes and government transfers, but the income gaps are still larger in 2006 when compared with 1996.

Seniors are another notable subgroup. They are better off due to the Quebec taxes and government transfers. They improve the aggregate poverty. Senior single women are in the highest risk of falling into poor population, but public policy reduced their risks in the intensity of poverty and severity of poverty.

Couples are a fortunate group in Quebec. Based on both poverty lines, they contribute small intensity of poverty at total income before and after taxes and government transfers. Couples have the best income among all population subgroups. They experienced fewer improvements from poverty reduction, since they have the lowest Headcount ratio, the smallest average income gap, and the smallest squared income gap.

The improvement from poverty reduction among the non-working group shows that the Quebec taxes and government transfers are the main reason for poverty reduction among seniors.

For the working age group, it is more complicated to find direct methods to eliminate aggregate poverty. Since the income structure of the working group is more complex than that of seniors. However, taxes and government transfers are still a conspicuous method to reduce poverty and inequality. For example, lone mothers have greater welfare due to the taxes and government transfers. Working age single women also benefitted from the Quebec taxes and government transfers.

The Headcount ratio, the intensity of poverty and the severity of poverty have a strong relationship. High poverty population ratio always accompanies high intensity of poverty, while the severity of poverty is also high. The larger population has more different income subsets, and inequality is more divergent.

If the Headcount ratio decreases, then the intensity and the severity decline as well. Smaller poverty population has less income classes, and the income gap is smaller. Therefore, the intensity of poverty is reduced. Inequality also can be relieved, but the degree depends on the income redistribution

These connections can be seen across all the Canadian provinces. However, the degree of the rise or fall of poverty is not as the same as in these three orders poverty measures. For the Headcount ratio it is higher than that of the average income gap and the squared average income gap.

Income fluctuation is one of the reasons for the changing tendency of the Headcount ratio. It is harder to improve the average income gap, since the improvement involves in the total income of

the entire poverty population. Furthermore, the fluctuation of the squared average income gap is most steady among the three poverty measures.

7 Bibliography

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8 Appendix

8.1 Tables

Table 1: Decomposition of poverty by family types, the extreme poverty line, reduction of FGT index, 2006, Quebec

Family Types	$\alpha=0$	$\alpha=1$	$\alpha=2$
single male ≥ 65	0.6434	0.4873	0.4369
single female ≥ 65	0.6868	0.5610	0.5052
single male < 65	0.0952	0.1094	0.1083
single female < 65	0.1429	0.1774	0.1756
couple with no children	0.0500	0.0352	0.0308
couple with children	0.0670	0.0326	0.0242
couple ≥ 65	0.4754	0.3174	0.2630
lone mother	0.3110	0.1881	0.1424
lone father	0.1691	0.1055	0.0764

Table 2: Decomposition of poverty by family types, the moderate poverty line, reduction of FGT index 2006, Quebec

Family Type	$\alpha=0$	$\alpha=1$	$\alpha=2$
single male ≥ 65	0.6124	0.5507	0.4953
single female ≥ 65	0.6350	0.6246	0.5666
single male < 65	0.0634	0.0930	0.1031
single female < 65	0.0588	0.1439	0.1663
couple with no children	0.0533	0.0405	0.0356
couple with children	0.0971	0.0540	0.0372
couple ≥ 65	0.6189	0.4132	0.3324
lone mother	0.3096	0.2364	0.1924
lone father	0.1942	0.1424	0.1083

8.2 Figures

8.2.1 The FGT Index, the Extreme Poverty Line, Quebec and Canada, 1996 to 2006

Figure 1: Total income after taxes and government transfers, the FGT index, $\alpha = 0$.

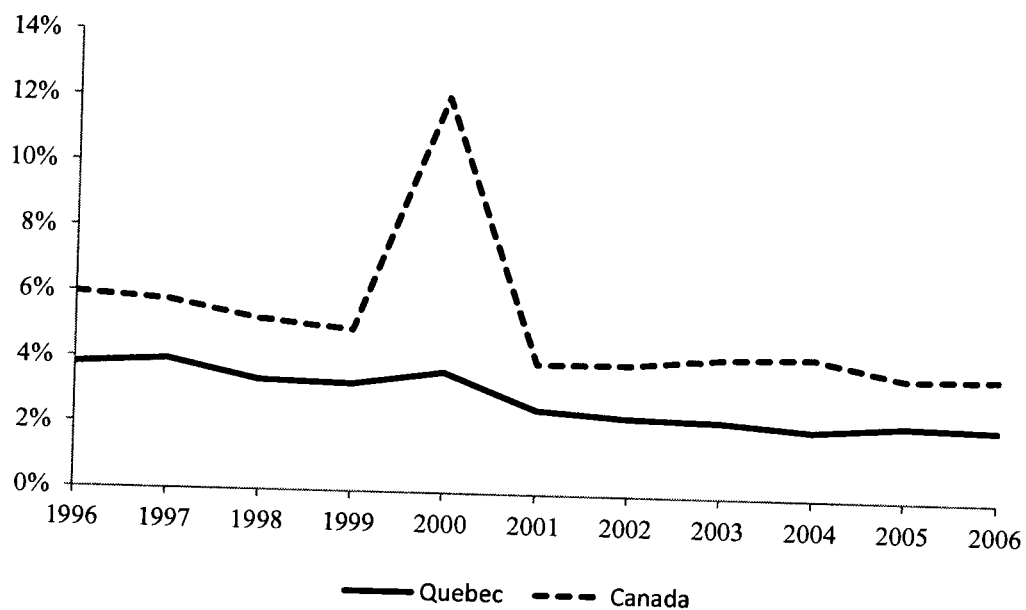


Figure 2: Total income after taxes and government transfers, the FGT index, $\alpha = 1$

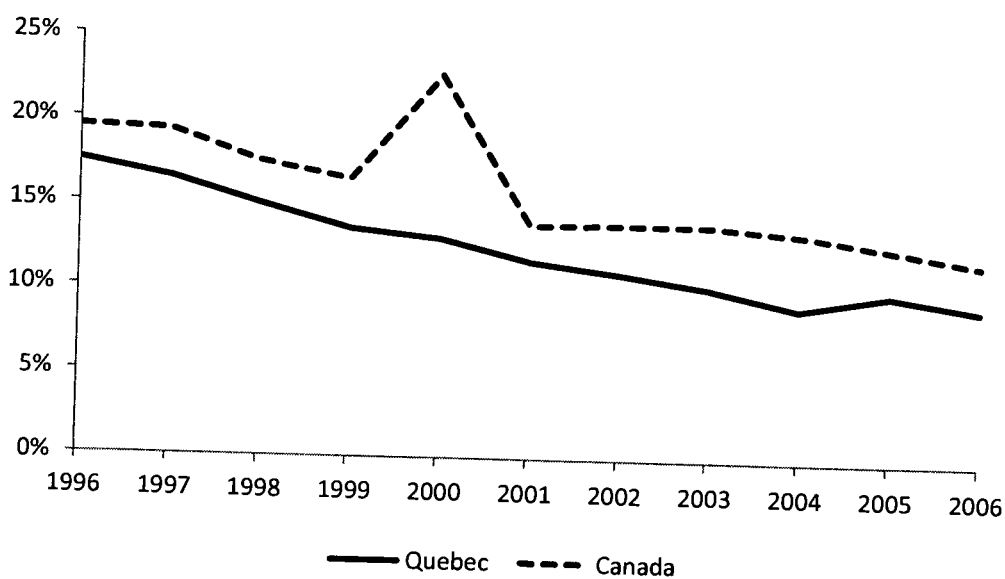
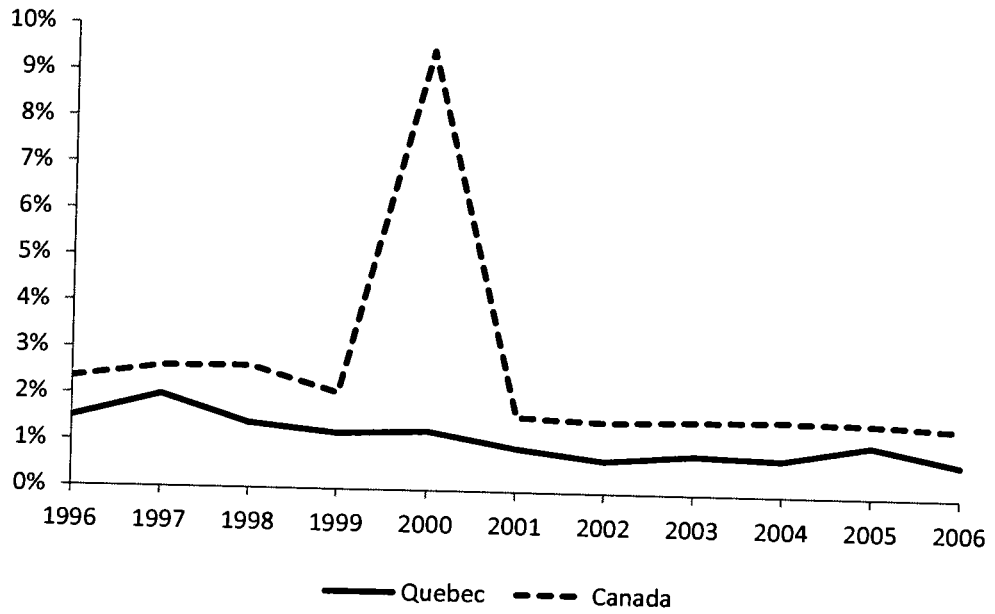


Figure 3: Total income after taxes and government transfers, the FGT index, $\alpha = 2$



8.2.2 The FGT Index, the Moderate Poverty Line, Quebec and Canada, 1996 to 2006

Figure 4: Total income after taxes and government transfers, the FGT index, $\alpha = 0$

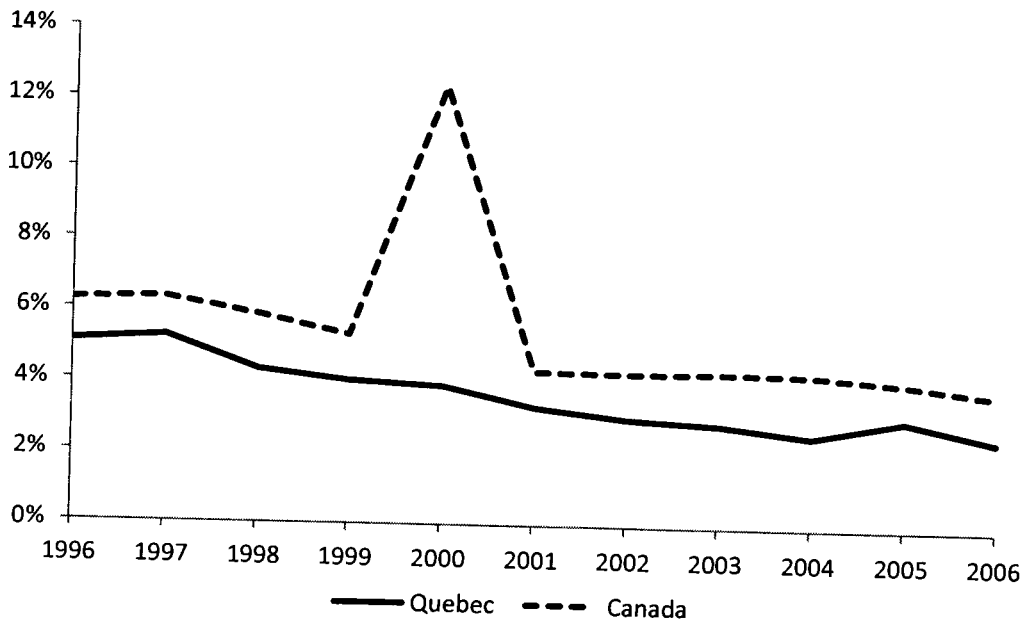


Figure 5: Total income after taxes and government transfers, the FGT index, $\alpha = 1$

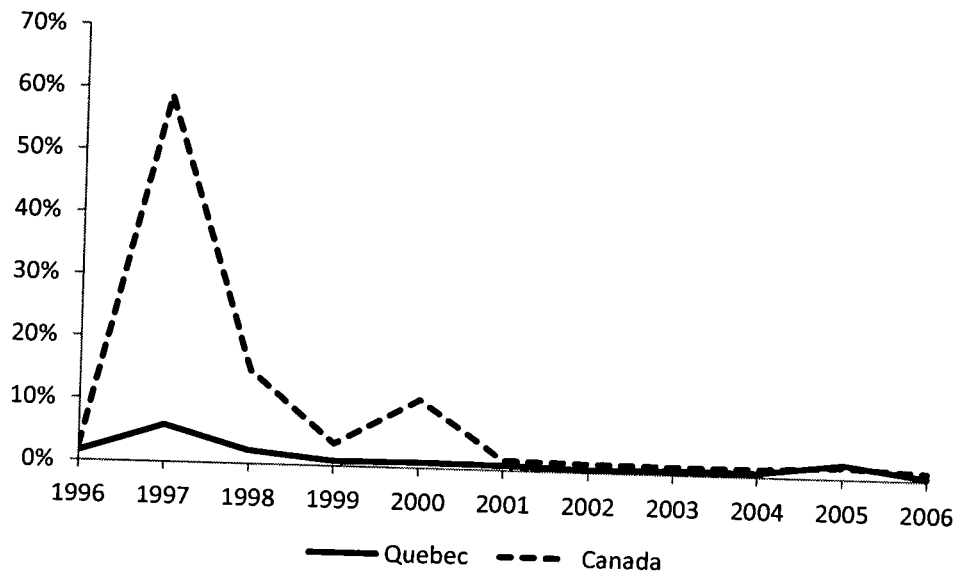
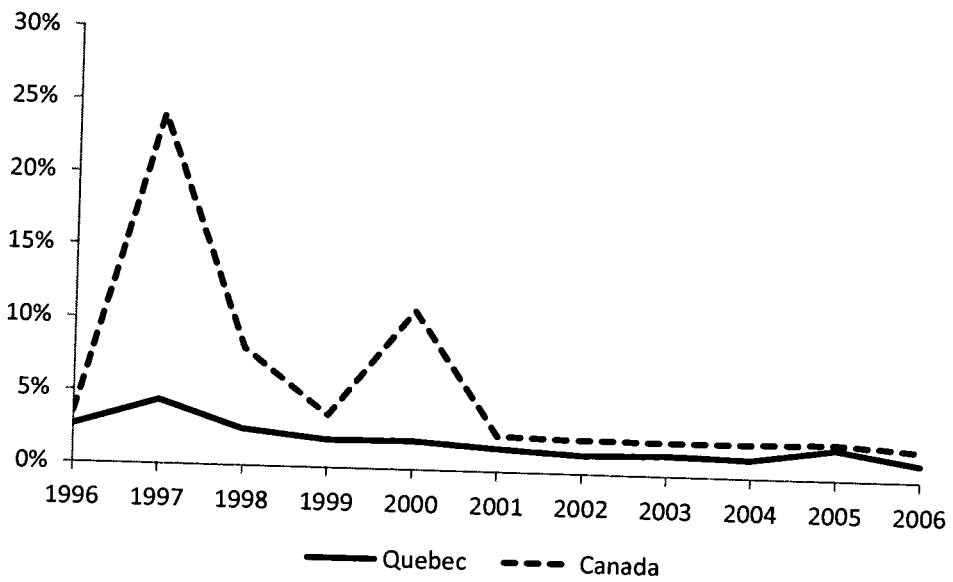


Figure 6: Total income after taxes and government transfers, the FGT index, $\alpha = 2$



8.2.3 Total Income, Quebec and Canada, 1996 to 2006

Figure 7: Total income before vs. after taxes and government transfers, Quebec, 1996 to 2006

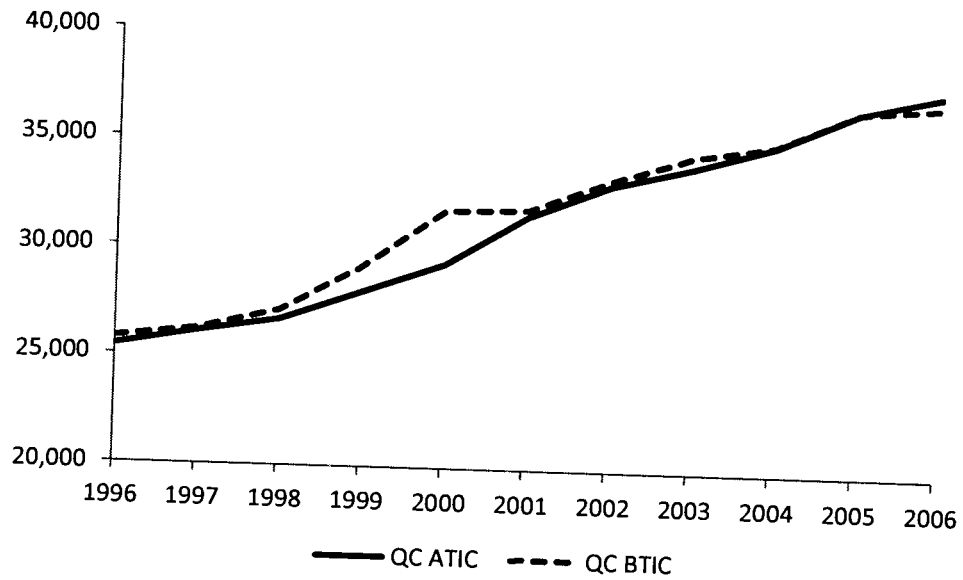
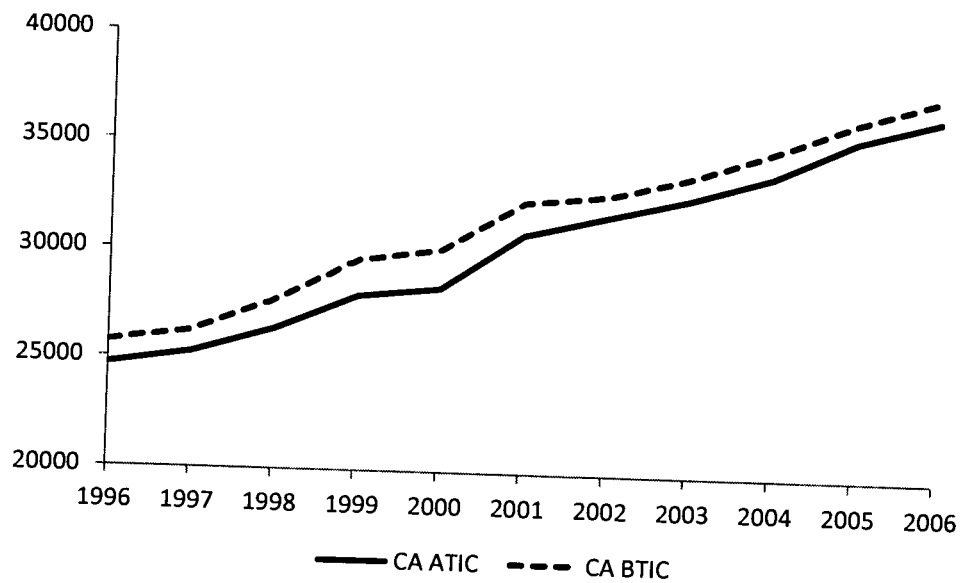


Figure 8: Total income before vs. after taxes and government transfers, Canada, 1996 to 2006



8.2.4 The FGT Indices, the Extreme Poverty Line and the Moderate Poverty Line, Quebec, 1996 and 2006,

Figure 9: The FGT index, $\alpha = 0, 1, 2$, two poverty lines, total income after taxes and government transfers, Quebec, 1996 and 2006

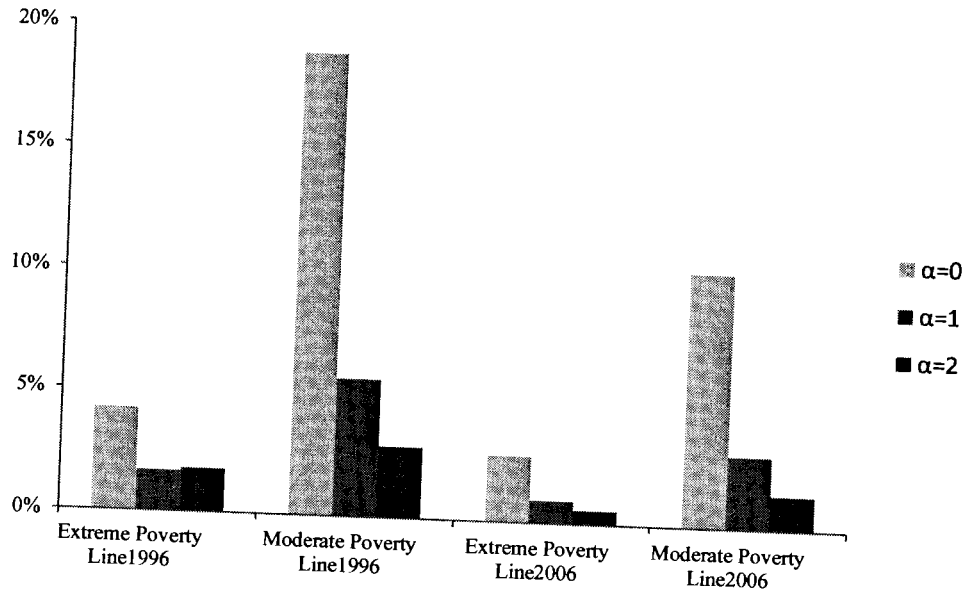


Figure 10: The FGT index, $\alpha = 0, 1, 2$, the extreme poverty line, total income after taxes and government transfers, 10 provinces, 1996

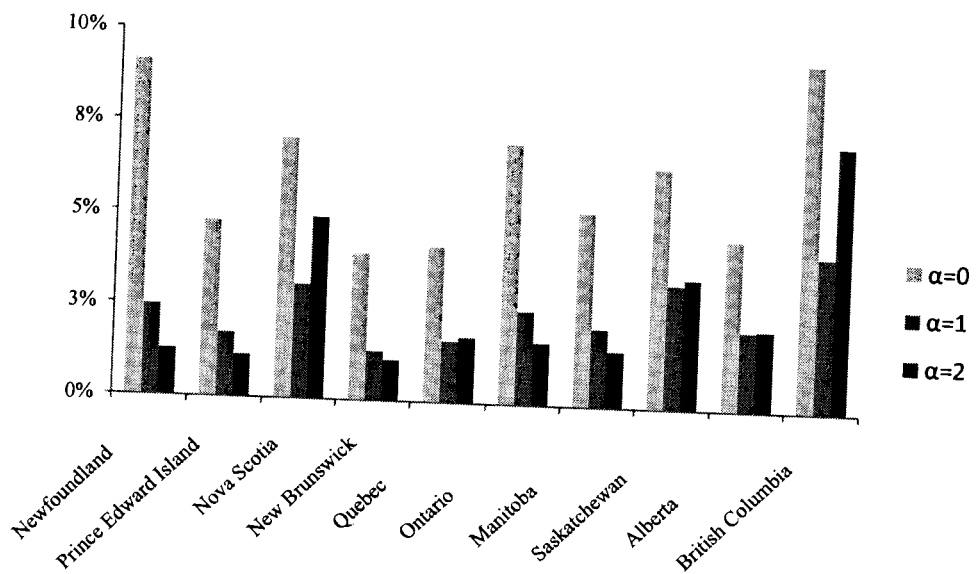


Figure 14: Decomposition by family types, equal or older than 65, total income after taxes and government transfers, Quebec, 2006

