

Indirect Tax Reform and Redistribution in Tlemcen

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### **Abstract**

The aim of this paper is to identify poverty reducing tax reforms in Tlemcen using normalized consumption dominance curves (CD-curves). The normalized CD-curves were first derived by Makdissi and Wodon (2002) and then improved by Duclos, Makdissi and Wodon (2008) as a method for testing the impact of marginal tax reform on poverty under budget neutrality. Normalized CD-curves are unlimited to the ethical order of the restricted stochastic dominance.

**KEYWORDS:** Normalized consumption dominance curves, critical poverty line, poverty-reducing, welfare-improving, indirect tax reforms.

## I. Introduction

The fact that personal income is unequally distributed has become one of the most crucial problems for economic and social relationships. In 1993, Atkinson commented that “Everyone is aware of the existence of inequality, but its precise extent is difficult to determine”. To find an appropriate economic instrument to reduce inequality, Makdissi and Wodon (2002) presented the consumption dominance curve (CD-curve) to test the “impact on poverty of marginal tax reforms for pairs of commodities” (Makdissi and Wodon 2002). Distinguished from the former theories, which could only test up to the second order of dominance, the CD-curve tests the impact of an indirect tax reform on poverty in unlimited order of restricted stochastic dominance.

Later on, Duclos, Makdissi and Wodon (2008) extended this previous paper in order to illustrate whether commodity tax changes are “socially improving”. In addition, the paper also demonstrates how to use estimators for critical poverty lines and economic efficiency ratios to characterize socially improving tax reforms.

The data analyzed in this paper is from Tlemcen. The observed variables in the dataset include household size, household consumption and the contribution to household spending for each individual. Specifically, spending is divided into food expenditures such as bread, fruit, beverage, meat, and nonfood expenditures like housing, health care, education and leisure. Based on the dataset and the methodology of the normalized CD-curves, this paper will try to conclude whether an indirect tax reform would reduce

poverty and improve social welfare in Tlemcen.

The paper is divided into several sections. Section II outlines previous literature on the analysis of inequality and poverty in the recent decades, as well as theories that relate to the study of marginal tax reform. Section III presents the normalized CD-curves methodology proposed by Makdissi and Wodon (2002) and further developed by Duclos, Makdissi and Wodon (2008). Section IV is devoted to the analysis based on the data from Tlemcen and contains two parts. The first part provides a brief introduction of the province of Tlemcen and the data collected by Professor Samir Maliki. The second part analyses the data and discusses the results by comparing the normalized CD-curves on food expenditures with those on nonfood expenditures at different orders of stochastic dominance. Another comparison is also made between the normalized CD-Curves of health care spending and other nonfood commodities spending. Section V presents a conclusion, along with the possible influences of an indirect tax reform on poverty and social welfare in Tlemcen.

## **II. Previous literature.**

### **i. Inequality**

Economic inequality is an extremely complex phenomenon. Tawney (1964) points out the difficulty of interpreting the concept, and “the controversies surrounding it arise partly, at least, because the same term is employed with different connotations”. As a result, it is hard to conclude a completely

attained equality in a unique way. Therefore, activities which will lead to greater equality are sincerely sought with the use of different reasonable economics instruments.

Lorenz Curves and the S-Gini indices are conventional measures that outline a desirable distribution of income. Atkinson (1970) also focused on the relationship between the measurement of inequality and the socially desired income distribution. He argues, however, Lorenz Curves, the S-Gini indices are not appropriate measures about a desirable distribution of income. Moreover, he introduces the concept of the conventional summary measures and the use of the equally distributed equivalent measure.

## **ii. Poverty**

The common component in any measure of poverty is defined as whether households or individuals have enough resources or abilities today to meet their needs.<sup>1</sup> Poverty is measured by computing poverty indices. A poverty index is computed as the difference between the equally distributed equivalent income and the poverty line. The latter term refers to the minimum level of income deemed necessary to achieve an adequate standard of living in a given country.

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<sup>1</sup> Poverty analysis of The World Bank <http://go.worldbank.org/8GKDUJWK20>

Public interest in regards to poverty has risen in the past decades. Abel-Smith and Townsend (1965) noticed that, in Britain, two million persons' incomes were lower than the basic national assistance scale in 1960, and that a further five and a half million were living at less than 40 per cent above the basic scale. In addition, the extent of poverty among families with children was first brought to light in their study. Poverty is found widely among all major groups in the population and in all parts of the world, even in the wealthiest developed countries. "The poor may not be in absolute poverty (the most basic of provisions may be obtainable for many) or their level of poverty may be a lot higher than those in developing countries, but in terms of their standing in society, their relative poverty can also have serious consequences such as deteriorating social cohesion, increasing crime and violence, and poorer health".<sup>2</sup>

In the context of poverty alleviation, Besley and Kanbur (1988) addressed the question of the "optimal pattern of food subsidies" under budget constraints. In this paper, the authors assume that it is not cost-effective to identify households below the poverty line in the medium to long run. Thus, the subsidies on commodities consumed by both rich and poor are appealing. Furthermore, the paper discusses subsidies on both the marginal unit of consumption and infra-marginal units in order to analysis the impact of food

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<sup>2</sup> Anup Shah *Poverty Around The World 2004*  
<http://www.globalissues.org/article/4/poverty-around-the-world>

subsidies. Based on the framework of modern public finance literature, Besley and Kanbur derived rules for reallocating food subsidies in terms of poverty alleviation.

**iii. Theories that relate to the study of marginal tax reform**

To alleviate poverty and tackle inequality, economists never stop searching for economic instruments and strategies. As mentioned earlier, measurements such as Lorenz Curves and S-Gini indices have been proposed to address poverty and inequality. However, to compute these indices, one needs to set up a specified poverty line to identify poverty and social welfare. Therefore, a new method which avoids the process of setting up a poverty line and targets on choosing a particular index is required. In other words, this new methodology does not require a specific social welfare function and is applicable to a wider range of functions.

Yitzhaki and Slemrod (1991) proposed a method to test the impact of commodity-tax changes. With this specific method, the “commodities that a large class of social-welfare functions would reveal as being worth subsidizing or taxing” could be identified, and therefore, the cost for an optimal tax reform would be close to zero. In other words, “...nearly all social-welfare functions would indicate that the taxation of one commodity should be reduced in favor of heavier taxation on another” (Yitzhaki and

Slemrod 1991). As mentioned before, Atkinson (1970) was the first to use stochastic dominance in the context of income distribution analysis; Yitzhaki and Slemrod were the first to use stochastic dominance in the context of indirect tax reforms. In their work, the “second-degree stochastic dominance” criterion is such that the investigator makes the assumption of nonnegative and nonincreasing marginal utility of income in order to rank portfolios. Yitzhaki and Slemrod tended to assess the tax reforms of commodities based on this criterion. It is worth noting that three main changes had been made to the methodology: a) rather than ranking portfolios according to the expected utility, they rank income distributions; b) they focus on dominance at the margin, that is, checking the improvement on social welfare instead of on expected utility; c) the efficiency implications of the reform is taken into account. Next, they develop the analysis by comparing the shifted concentration curves and conclude that “if and only if concentration curves do not intersect will all additive social-welfare functions show that the tax change increases welfare” (Yitzhaki and Slemrod 1991).

The following sections of this paper provides the intuition and proof for these “marginal conditional stochastic dominance” rules (MCSD rules), relates MCSD rules to “a methodology based on the decomposition of the Gini coefficient”, and illustrates the methodology based on the data from



Israel.

Instead of testing the Dalton-improving marginal tax reform with only two goods, Mayshar and Yitzhaki (1995) extend this previous methodology to identify a Dalton-improving marginal tax reform with multiple commodities. The authors state that if and only if the post-reform effective income distribution dominates the initial distribution, a Dalton improvement will be achieved. In other words, a tax reform is welfare improving if it distributes from the higher ranking (the rich) to the lower ranking (the poor) without altering the rankings of the two individuals. Later on, using the same methodology, Yitzhaki and Lewis (1996) demonstrate the search for the impact of Dalton-improving tax reforms. They use the data from Indonesia to illustrate the methodology and conclude that, “given the structure of demand for different energy products, equity could be improved further by reducing the gasoline tax, increasing the subsidy to kerosene, and imposing a tax on electricity” (Yitzhaki and Lewis 1996). However, the article is portrayed by the concentration curve. If one tries to adapt such approach to test a tax reform on poverty, must limit the tests to second order dominance.

Makdissi and Wodon (2002) presented a new tool “which can be used to test for the impact of an indirect tax on poverty for any order of restricted stochastic dominance”. They also extended the result presented by Besley

and Kanbur (1988) “by considering a larger class of additive poverty measures”. In this article, the author assumes an additive index of poverty, determined by income and a defined poverty line, which is non-negative for all individuals and zero for those with income above the poverty line. Moreover, the poverty measure is assumed to be “a continuous function s-time differentiable”. The magnitude of these two assumptions is obvious: in the process of the dominance conditions proof, continuity and zero poverty measure for income above the poverty line ensures that testing the indirect tax reform on poverty for an order higher than 2 is possible. With respect to these important assumptions, Makdissi and Wodon (2002) compute the CD-Curve of order  $s$  and indicate that “a tax reform will decrease poverty for the class of poverty measures if the two CD-Curves associated with this class of measures are not intersecting before the maximum poverty line”. An empirical illustration follows, which demonstrates the use of CD-Curves based on a 1999 Bolivian survey.

Using the methodology derived by Makdissi and Wodon (2002), Duclos, Makdissi and Wodon (2008) broaden the analysis of social improvement. The article focuses on the effect of marginal tax reforms to evaluate the distributive impact of marginal tax reforms. This assumption simplifies the process by eliminating the estimation of individual demand and utility function. Instead of choosing a social evaluation function, the authors

“define classes of social evaluation indices that incorporate increasingly stronger judgments on the importance of distributive issues in designing tax policy” (Duclos, Makdissi and Wodon 2008). By increasing the ethical order, one can show whether the reform is Pen-improving, Dalton-improvement, Kolm-improving and higher-order welfare-improvement tax reforms. Referring to the impact on tax revenues of changes in indirect taxation, Duclos, Makdissi and Wodon (2008) try to estimate the critical efficiency ratio as well as the critical poverty lines, with which policy makers could conclude the tax reform leads to socially improving and poverty improving if the critical value is above the actual efficiency ratio and the critical poverty lines are also high enough to exceed all plausible poverty lines. One of the most crucial parts of this article is that it checks social efficiency by extending the use of consumption dominance curve. Specially, if the policy maker is going to increase tax on good  $j$  and decrease tax on good  $l$ , poverty improvement is obtained if the normalized consumption dominance curve of good  $l$  is higher than the normalized consumption dominance curve of good  $j$  “at every threshold under a maximum poverty line, when that maximum poverty line extends to infinity, the tax reform can be called welfare improving” (Duclos, Makdissi and Wodon 2008). It is worth notice that the total tax revenues do not change under the tax reform--conditions of revenue neutrality hold. In the last section of their paper, Duclos, Makdissi and Wodon (2008) illustrate the methodology based on a nationally

representative survey of Mexico (1996). They apply the tools described in the paper, including consumption dominance curves, critical poverty lines and critical economic efficiency ratios, to analyze the impact of an indirect tax reform.

### III. The methodological framework

#### i. Notation and definitions

This section will present the methodological framework developed in Duclos, Makdissi and Wodon (2008). To determine how consumer welfare is affected by a marginal change in tax rates, one needs to assume firstly that there are  $K$  goods with consumer prices denoted by vector  $q$  and tax rates denoted by vector  $t$ . Here, set the producer prices to be constant and equal to 1, and, it is independent to the changes in  $t$ . Define the indirect utility function as  $v(y, \theta, q)$  where  $y$  and  $\theta$  respectively denote the nominal income and consumers' preferences. With the reference prices vector  $q^R$  and the post-reform real income vector  $y^R$ , Duclos, Makdissi and Wodon (2008) set  $y^R=y$  and  $q^R=q$ , then define  $y^R$  by both the implicit function  $v(y^R, \theta, q^R) = v(y, \theta, q)$  and the real income function  $y^R = \rho(y, \theta, q, q^R)$ , thus, the indirect utility function could be written as  $v(y^R, \theta, q^R) = v(\rho(y^R, \theta, q, q^R), \theta, q)$ .

To determine how consumer welfare is affected by a marginal change in tax

rates, let  $x_k(y, \theta, q)$  be the consumption of good k, where q is the price of good k. As mentioned above, assume reference prices equal to pre-reform prices, Besley and Kanbur (1988) adopt Roy's identity and get

$$\left. \frac{\partial p(y, \theta, q, q^R)}{\partial q_k} \right|_{q=q^R} = -x_k(y, \theta, q^R).$$

This equation says that "observed pre-reform consumption of good k is a sufficient statistic to show the impact on consumer welfare of a marginal change in the price of good k" (Duclos, Makdissi and Wodon 2008).

Suppose preferences  $\theta$  and nominal income  $y$  are jointly distributed and denote the distribution function with  $F(y, \theta)$ , the conditional distribution of  $\theta$  given  $y$  is therefore  $F(\theta|y)$ , the marginal distribution of nominal income, on the other hand, is  $F(y)$ . Let preferences  $\theta$  belong to a set  $\Theta$  and  $y$  is distributed over  $[0, a]$ , the expected consumption of good k will take the following form:

$$x_k = E[x_k(y, \theta, q)] = \int_{\Theta} x_k(y, \theta, q) dF(\theta | y).$$

Define  $X_k(q) = \int_0^a x_k(y, q) dF(y)$ , where  $X_k(q)$  is the per capita consumption of good k. Notice that  $X_k(q)$  is the average cost of increasing the price of good k<sup>3</sup>. Accordingly, the consumption of good k as a

<sup>3</sup> According to  $\left. \frac{\partial p(y, \theta, q, q^R)}{\partial q_k} \right|_{q=q^R} = -x_k(y, \theta, q^R)$ .

proportion of per capita consumption can be written as  $\bar{x}_k(q) = x_k(y, q)/X_k(q)$ .

Government tries to maintain the commodity tax revenues level during a marginal tax reform. Specifically, consider per capita commodity tax revenues  $R(q)$  is expressed as  $R(q) = \sum_{k=1}^K t_k X_k(q)$ , government tends to subsidize good 1 by increasing the tax on good  $j$ , with commodity tax revenues unchanged. A tax reform under this condition is called the revenue neutrality of tax reform and revenue neutrality can be expressed as:

$$dR(q) = \left[ X_j(q) + \sum_{k=1}^K t_k \frac{\partial X_k(q)}{\partial q_j} \right] dq_j + \left[ X_1(q) + \sum_{k=1}^K t_k \frac{\partial X_k(q)}{\partial q_1} \right] dq_1 = 0.$$

Lastly, define the economic efficiency ratio  $\gamma_{1j}$  as:

$$\gamma_{1j} = \frac{\frac{X_1 + \sum_{k=2}^K t_k \frac{\partial X_k}{\partial q_1}}{X_1}}{\frac{X_j + \sum_{k=1}^K t_k \frac{\partial X_k}{\partial q_j}}{X_j}}$$

This can be interpreted as ‘the differential efficiency cost of raising one dollar of public funds, by taxing the  $j^{\text{th}}$  commodity and subsidizing the  $i^{\text{th}}$  commodity’ (Wildasin, 1984). The numerator represents the inverse of the marginal economic efficiency cost of funds (MECF<sub>1</sub>) from taxing good 1. The denominator represents the inverse of the marginal economic efficiency cost of funds (MECF<sub>j</sub>) from taxing good  $j$ . “The higher the value of  $\gamma_{1j}$ , the

less economically efficient is taxing good j” (Duclos Makdissi and Wodon 2008). Combine the revenue neutrality condition with the economic efficiency ratio, revenue neutrality can be rewritten as:

$$dq_j = -\gamma_{ij} \left( \frac{x_i}{x_j} \right) dq_i.$$

**ii. Poverty index, social welfare function and the normalized CD-curve**

Makdissi and Wodon (2002) made two assumptions in their work on illustrating the Consumption Dominance Curve methodology. The first is the assumption of an additive index of poverty denoted by

$$P(F, z) = \int_0^a p(y^E(q, y), z) dF(y),$$

while the second is the s-time (piecewise) differentiable on that poverty measure. These two assumptions ensure that testing the indirect tax reform on poverty for an order higher than 2 is possible. Similarly, Duclos Makdissi and Wodon (2008) assume

$$\text{that poverty indices can be expressed as } P(z) = \int_0^a p(y, z) dF(y),$$

furthermore, they indicate the class of poverty measures  $\Pi^s$  should take the form:

$$\Pi^s = \left\{ P(z) \left| \begin{array}{l} p(y, z) = 0 \quad \text{if } y > z, p(y, z) \in \bar{C}^s \\ (-1)^i p^{(i)}(y, z) \geq 0 \text{ for } i = 0, 1, \dots, s, \\ p^{(t)}(z, z) = \text{ for } t = 0, 1, \dots, s - 2 \text{ when } s \geq 2 \end{array} \right. \right\},$$

where  $\bar{C}^s$  is a set of functions that are s-time piecewise differentiable over  $[0, z]$  and s represents the ethical order.

Next, consider the social welfare functions such that  $U = \int_0^a u(y) dF(y)$ .

Similarly, let social welfare functions belong to the set  $\Omega^s$  which is expressed as

$$\Omega^s = \{U | u(y) \in C^s(\infty), (-1)^{i+1} u^{(i)}(y) \geq 0 \text{ for } i = 1, 2, \dots, s\}.$$

In order to derive the  $\overline{CD}$  curves, the discussion must initially focus on the concept of stochastic dominance first. As mentioned previously, Duclos, Makdissi and Wodon (2008) set  $q^R=q$  while defining the post-reform real income vector  $y^R$ . Under this assumption, the stochastic dominance can be expressed as:

$$D^s(z) = \frac{1}{(s-1)!} \int_0^z [z-y]^{(s-1)} df(y).$$

Or, by sequential integration:

$$D^s(z) = \begin{cases} F(y) & \text{for } s = 1 \\ \int_0^z D^{s-1}(y) dy & \text{for } s = 2, 3, \dots, s \end{cases}$$

In real income space, since we have  $q = q^k$ , then:

$$D^s(z) = \frac{1}{(s-1)!} \int \int_0^{\eta(y^R, \theta, q, q^R)} [z - p(y, \theta, q, q^R)]^{(s-1)} dF(y, \theta).$$

Using Roy's identity, we know that the negative of the demand for good k equals to the differential in equivalent income in respect to a marginal



change in tax. Combine this condition with the stochastic dominance, we get:

$$\frac{\partial D^s(z)}{\partial t_k} \Big|_{q=q^R} = \begin{cases} x_k(z, q^R) f(z) & \text{if } s = 1 \\ \frac{1}{(s-2)!} \int_0^z x_k(y, q^R) (z-y)^{s-2} dF(y) & \text{if } s = 2, 3, \dots, \end{cases}$$

where  $f(z)$  is the density function of income at poverty line  $z$ . The Consumption Dominance Curve is therefore:

$$CD_k^s(z) = \frac{\partial D^s(z)}{\partial t_k}, \quad s=1, 2, \dots,$$

Ultimately, we define the normalized Consumption Dominance Curve as

$$\overline{CD}_k^s(z) = \frac{CD_k^s(z)}{x_k(q)}.$$

For ethical order  $s=1$ , the interpretation of  $\overline{CD}_k^1$  is the ratio, given consumer income  $y$ , of consumption of good  $k$  over the aggregate consumption of that good. For ethical order  $s=2$ , the interpretation of  $\overline{CD}_k^2$  is the consumption of good  $k$  consumed by the consumers with income lower than  $y$  in terms of the total consumption. For any higher ethical order, the normalized consumption dominance curve is derived by integrating the  $\overline{CD}$  curves at order  $s-1$ .

### iii. Intuition

In Duclos Makdissi and Wodon(2008)'s paper, poverty index, social welfare function and the normalized CD-curve are very important in analyzing the impact of a marginal tax reform. Here, the discussion will be first aimed on different classes of indices for poverty and social welfare.

For ethical order  $s \geq 1$ , as an individual's income increases, "poverty indices weakly decrease whereas welfare indices weakly increase" (Duclos Makdissi and Wodon 2008). It is worth noting that poverty indices belong to  $\Pi^1$  and social welfare functions belong to  $\Omega^1$ . Not only do they obey the Pareto principle, but also conform to the symmetric or anonymity axiom<sup>4</sup>. To compare poverty indices, one must check the distribution of living standards within the poverty range  $[0, z]$ . The first order welfare improving tax reform is also called the Pen-improving tax reform.

By second-order derivative, "poverty indices are convex and welfare indices are concave" (Duclos Makdissi and Wodo 2008). Poverty indices and social welfare functions respectively belong to  $\Pi^2$  and  $\Omega^2$ . Moreover, they obey the Pigou-Dalton principle of transfers<sup>5</sup>. The second-order welfare improving tax reforms are denoted as "Dalton-improving tax reforms" by Mayshar and Yitzhaki (1995).

<sup>4</sup> Symmetry or anonymity axiom: interchanging any two individuals' incomes leaves unchanged the poverty and social welfare indices.

<sup>5</sup> It implies that a mean-preserving transfer of income from a higher order individual to a lower order individual ranked by income construct a social improvement that will increase welfare or reduce poverty.

Referring to  $s=3$ , once again, poverty indices and social welfare functions respectively belong to  $\Pi^3$  and  $\Omega^3$ . Kolm (1976) states that the two instruments mentioned above are sensitive to favorable composite transfers. For simplicity, the third-order welfare improving tax reform is also called Kolm-improving tax reforms.

Turning to the use of normalized CD-curve to identify the socially improving tax reforms, Duclos Makdissi and Wodon(2008) provide the following two theorems to show CD curves are an instrument capable of testing whether tax reforms are poverty or welfare improving.

Theorem I (Duclos Makdissi and Wodon 2008): A necessary and sufficient condition for a marginal tax reform,  $dq_j = -\gamma_{ij} \left( \frac{x_i}{x_j} \right) dq_i > 0$ , to be  $s$ -order poverty reducing, that is to say, to decrease poverty weakly for all  $P(z) \in \Pi^s$ , for all  $z \in [0, z^+]$  and for a given  $s \in \{1, 2, 3, \dots\}$ , is that

$\overline{CD}_1^s(y) - \gamma_{ij} \overline{CD}_1^s(y) \geq 0, \forall y \in [0, z^+]$ , where  $\gamma$  is the economic efficiency ratio mentioned before.

Theorem II (Duclos Makdissi and Wodon 2008): A sufficient condition for a marginal tax reform,  $dq_j = -\gamma_{ij} \left( \frac{x_i}{x_j} \right) dq_i > 0$ , to be  $s$ -order welfare

improving, that is, to increase social welfare weakly for all  $W \in \Omega^s$  and for a given  $s \in \{1,2,3, \dots, \}$  is that

$\overline{CD}_1^s(y) - \gamma_j \overline{CD}_j^s(y) \geq 0$  ,  $\forall y \in [0, \infty)$  , where  $\gamma$  is the economic efficiency ratio mentioned before.

“The only difference between the social improvement conditions of Theorems I and II is that the social-welfare test extends over the entire space  $[0, \infty)$ , whereas the poverty test is limited to the range of potential poverty lines  $[0, z^+]$ ”(Duclos Makdissi and Wodon 2008) (Duclos Makdissi and Wodon 2008). Theorem I gives the poverty-reducing condition while Theorem II provides a criterion to identify welfare-increase. More precisely, when  $\gamma = 1$ , if the normalized CD-curve of good 1 and the normalized CD-curve of good j do not intersect at order  $s=1, s=2, s=3$ , the tax reform is considered to be poverty-reducing at order  $s=1, s=2, s=3$  respectively; if the two normalized CD-curves do have intersections but the first intersection is above the critical poverty line, the tax reform will still be poverty-reducing at order  $s=1, s=2, s=3$  respectively. In short, the tax reform will be poverty-reducing if the normalized CD-curve of good 1 is everywhere above the normalized CD-curve of good j for all poverty indices belong to  $\Pi^s$  and for any poverty line under the critical poverty line  $z^+$ . If the normalized CD-curve of good 1 and the normalized CD-curve of good j do not intersect at order  $s=1, s=2, s=3$  under any poverty line, the tax reform is considered to

be welfare increasing. In other words, the tax reform will be Pen-improving, Dalton-improving, Kolm-improving if the normalized CD-curve of good  $l$  is everywhere above the normalized CD-curve of good  $j$ , for all social welfare functions belonging to  $\Omega^S$ .

For  $\gamma \neq 1$ , to identify a poverty-reducing tax reform or a welfare-improving tax reform, one needs to multiply the normalized CD-curve of good  $j$  by the economic efficiency ratio, then compare it with the normalized CD-curve of good  $l$ . Recall that  $\gamma_{lj}$  is the economic efficiency cost of taxing  $j$  relative to that of taxing  $l$ :  $MECF_j / MECF_l$ . The remaining steps for measurement will then be similar as the described for  $\gamma = 1$ .

**iv. Distributive benefit, critical efficiency ratio and critical poverty.**

According to Theorem I and Theorem II, we have

$$\overline{CD}_l^s(y) - \gamma_{lj} \overline{CD}_j^s(y) \geq 0, \quad \forall y \in [0, z^+] \text{ and } \overline{CD}_l^s(y) - \gamma_{lj} \overline{CD}_j^s(y) \geq 0$$

,  $\forall y \in [0, \infty)$ . Duclos, Makdissi and Wodon (2008) state in their paper that

one can rewrite the conditions of Theorem I and Theorem II by checking

whether  $\delta^s(z) \geq \gamma$  for all  $z \in [0, z^+]$  and for all  $z \in [0, \infty)$  respectively. To

see why, we need to first introduce the concept of a distributive benefit ratio

“ $\delta$ ”. Denote this distributive benefit ratio as  $\delta^s(z) = \overline{CD}_l^s(z) / \overline{CD}_j^s(z)$ . The

intuition behind this equation is the benefit obtained from taxing good  $j$  rather than taxing good  $l$ . Notice when  $\overline{CD}_j^s(z) = 0$ ,  $\delta^s(z)$  will tend to be infinite. Here define  $\delta^s(z)$  as  $\gamma^{++}$  when it tends to be infinite, and the distributive benefit ratio is therefore:

$$\delta^s(z) = \begin{cases} \frac{\overline{CD}_l^s(z)}{\overline{CD}_j^s(z)} & \text{if } \overline{CD}_j^s(z) \neq 0 \\ \gamma^{++} & \text{if } \overline{CD}_j^s(z) = 0 \end{cases} .$$

Thus, the progress of identifying the socially improving tax reforms is converted into comparing distributive benefit  $\delta^s(z)$  with the economic efficiency ratio  $\gamma$ . Specifically, a tax reform is  $s$ -ordered socially improving if its distributive benefit is larger than the economic efficiency ratio for all poverty indices belong to  $\Pi^s$  and for any poverty line under the critical poverty line  $z^+$ .

Again, using Theorem I, suppose a given value of  $\gamma = \gamma_0$  such that  $\overline{CD}_l^s(z) - \gamma_0 \overline{CD}_j^s \geq 0, \forall z \in [0, z^+]$ , notice this equation will hold for all  $\gamma \leq \gamma_0$ . Hence, there must exist a critical value of  $\gamma$  for all  $z \in [0, z^+]$ , beyond which the equation above will not hold. This critical efficient ratio is denoted by  $\gamma_s(z^+)$  and defined by  $\gamma_s(z^+) = \inf \{\delta^s | z \in [0, z^+]\}$ . Particularly,  $\gamma_s(z^+)$  is the upper bound of the economic efficiency ratio

where  $\overline{CD}_1^s(z)$  and  $\gamma \overline{CD}_j^s$  intersects at  $z=z+$ . When  $\gamma > \gamma^0$ , on the other hand, if and only if  $\delta^s(z) = \gamma++$  for all  $z \in [0, z+]$  the marginal tax reform will be poverty-reducing.

A critical upper poverty line of  $z_s(\gamma^+)$  is defined, similarly, by  $z_s(\gamma^+) = \sup \{z | \delta^s(y) \geq \gamma^+, y \in [0, z], z \leq z^{++}\}$ .

Equation  $\overline{CD}_1^s(y) - \gamma_j \overline{CD}_j^s(y) \geq 0$ ,  $\forall y \in [0, z^+]$  holds for any  $z$  above  $z_s(\gamma^+)$ . In other words, if the two normalized CD-curves intersects at a level larger than the critical poverty line, the tax reform will be poverty-reducing.

“For a given  $\gamma+$  and  $z+$ ,  $z_s(\gamma+)$  and  $\gamma_s(z+)$  respectively give the critical upper poverty line and the critical economic efficiency ratio up to which the tax reform is necessarily s-order poverty improving” (Duclos, Makdissi and Wodon 2008).

#### IV. Empirical Illustration.

##### i. Tlemcen economy and the data.

Tlemcen is a province in north-western Algeria and the word Tlemcen in Tifinagh meaning “the dry spring”. Its population is 945,525 as of the 2008 census. It is divided into 20 districts and 53 municipalities. The capital of the province is of the same name and developed leather, carpet, and textile

industries, where goods were shipped to the port of Rashgun for export. Samir B.E. Maliki, Abderrezak Benhabib and Jacques Charmes (2009) stated that “the legal monthly minimum wage of 8000 AD applied since January 2001 represents approximately 4 times the poverty line”.

The dataset analyzed in this paper contains 471 households<sup>6</sup>. It was collected by Professor Samir Maliki according to a field survey and shows expenditures on different commodities for each household. More specifically, the variables include “Spending on bread and cereals” (BS); “Spending on fruits and vegetables” (FV); “Spending on meat, poultry and fish” (MPF); “Spending on other food items and beverages” (OB); “Spending on housing, heating and electricity” (HHE); “Spending on durables” (D); “Spending on health care and hygiene” (HH); “Spending on transportation” (T); “Spending on education, culture and leisure” (ECL). The dataset also contains the size of each household and “Contribution to household spending”. To obtain total household expenditure, “contribution to household spending” is added up. Referring to the 2000 AD poverty line outlined by Maliki, Benhabib and Charmes (2009), 41.83% of the sample are poor.

Next, consider the process of deriving expenditure on food and nonfood

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<sup>6</sup> There are 500 households in the original dataset with which 29 of them have missed information. The author excluded those 29 while doing the Consumption-Dominance analysis.



products. To do this, one can simply add up the spending on all food commodities to get “aggregate food expenditure”, and, remove it from household total household expenditure, to get “aggregate nonfood expenditure”. It should be noted that there are negative values when implementing this process, meaning some households are spending more than what they have in terms of “aggregate nonfood expenditure”. To adjust for this, those with negative values of “aggregate nonfood expenditure” are discarded from the dataset. Therefore, the dataset size was reduced from 471 to 408.

Lastly, dividing “aggregate food expenditure” and “aggregate nonfood expenditure” by the household size<sup>7</sup>, yields per capita spending of food and nonfood, denoted by “PF” and “PNF”. The analysis will also evaluate per capita health care spending and per capita spending on other nonfood commodities. Once again, dividing the total health care expenditures by household size, per capita health care spending will be denoted as PH. Excluded PH from PNF will give per capita spending of other nonfood commodities, denoted as PN. For example, there are 3 individuals in household 22, their total food expenditure is 10,500 and total non food expenditure is 22,500. Thus,  $PF_{22}$  is yielded by  $10,500/3$  equals to 3,500 and  $PNF_{22}$  is 7,500. Per capita expenditure of household 22 is 13,333.33 and

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<sup>7</sup> All individuals in the household are included.

their spending on health care is 5,000. Therefore,  $PH_{22}$  equals to 1,666.67 and  $PN_{22}$  equals to  $7,500 - 1,666.67 = 5,833.33$ . In other words, 12.5% of the spending in this household is for health care, and 43.75% of the spending goes to other non food products.

**ii. Empirical analysis.**

With the adjusted dataset and the methodology provided in section III, the following section will test and evaluate the impact of a marginal tax reform for two different cases in Tlemcen.

Firstly, consider a marginal tax reform that increases the tax rate on nonfood commodities and uses the proceeds to decrease the tax rate on food. Table 1 presents estimated critical values of poverty lines  $z_s(\gamma)$  for different economic efficiency ratios  $\gamma$  and different ethical orders  $s$  on food and nonfood products.

When  $\gamma=1$ , the first intersection of  $\overline{CD}^1$  curves for PF and PNF is at  $\hat{z}_1(1)=3,122.36$ , with standard error of the sampling distribution of  $\hat{z}_1(1)$  estimated to be 153.99. Thus, the confidence interval for the estimated critical poverty line  $\hat{z}_1(1)$  with level of significance  $\alpha=0.05$  and  $t\text{-ratio}=1.96$  is [2,820.54, 3,424.18]. In other words, increasing the tax on nonfood and using the proceeds to reduce the tax on food will reduce

poverty for any poverty index that is monotonic and symmetric. Figure 1 shows the Normalized CD-Curves for PF and PNF when  $s=1$ . Since the two  $\overline{CD}^1$  curves cross—Normalized CD-Curves for food is not everywhere above Normalized CD-Curves for nonfood—theorem II is not satisfied. In short, the marginal tax reform is not first-order welfare-improving or Pen-improving tax reform.

For ethical order  $s = 2$ , there is no critical poverty line due to the fact that  $\overline{CD}^2$  curves for PF and PNF do not intersect. As a result, for any poverty line, the marginal tax reform is poverty-reducing as well as welfare-improving at the second ethical order. Specifically, if the government imposes an indirect tax reform which increases the tax on non-food expenditures and decreases the tax on food expenditures, it will reduce poverty for any poverty line and for any poverty index that is monotonic, symmetric and averse to inequality and will also increase social welfare for any social welfare index that is monotonic, symmetric and averse to inequality. Likewise, no critical poverty line is obtained at the third order with economic efficiency ratio equal to 1. Therefore, a similar marginal tax reform mentioned above will decrease poverty and improve social welfare without any limitation on a critical upper poverty threshold value.

Thus, with a unit economic efficiency ratio, increasing the tax on nonfood commodities and providing subsidies for food expenditures is poverty-reducing for  $s = 1, 2$  and  $3$ . Moreover, this indirect marginal tax reform is Dalton-improving and Kolm-improving.

Now consider economic an efficiency ratio  $\gamma$  equal to  $0.5$ . From Table 1, the two  $\overline{CD}^1$  curves for PF and PNF across at  $5,687.05$ , with a standard error estimated to be  $106.90$ . Thus, the confidence interval for the estimated critical poverty line  $\hat{z}_1(0.5)$  with level of significance  $\alpha=0.05$  and  $t$ -ratio= $1.96$  is  $[5,477.53, 5,896.57]$ . Therefore, increasing the tax on nonfood commodities, while providing subsidies for food commodities, will be first-order poverty-reducing if the poverty line is assigned below  $5,477.53$ . Figure 4 shows the Normalized CD-Curves for PF and PNF when  $s=1$  and  $\gamma=0.5$ . According to the graph, the Normalized CD-Curves for food is not everywhere above Normalized CD-Curves for nonfood. As a result, theorem II is not satisfied—the marginal tax reform is not first-order welfare-improving or, Pen-improving. Keep the same economic efficiency ratio  $\gamma=0.5$  and increase the ethical orders, no critical poverty line is found,  $\overline{CD}^s$  ( $s=2, 3$ ) curve for PF is everywhere above  $\overline{CD}^s$  ( $s=2, 3$ ) curve for PNF.

Therefore, for any poverty line, the indirect marginal tax reform of

subsidizing food by increasing tax on nonfood is Dalton-improving and Kolm-improving.

With an economic efficiency ratio  $\gamma$  equals to 1.5, the two  $\overline{CD}^1$  curves cross at 1,894.15, with a standard error estimated to be 294.94, the confidence interval for the estimated critical poverty line  $\hat{z}_1(1.5)$  with level of significance  $\alpha=0.05$  and t-ratio=1.96 is [1,316.07, 2,472.23] (shown graphically in Figure 5). For any poverty line between 0 and 1,316.07, increasing the tax on nonfood and using the proceeds to reduce the tax on food will reduce poverty. For  $s=2$ , the two  $\overline{CD}^2$  curves for PF and PNF have an intersection at 2,717.03, with a standard error estimated to be 682.95, the confidence interval for the estimated critical poverty line  $\hat{z}_2(1.5)$  with level of significance  $\alpha=0.05$  and t-ratio=1.96 is [1,378.45, 4,055.61]. When  $s=3$ , the two  $\overline{CD}^3$  curves for PF and PNF cross at 3,739.94, with a standard error estimated to be 445.08, the confidence interval for the estimated critical poverty line  $\hat{z}_3(1.5)$  with level of significance  $\alpha=0.05$  and t-ratio=1.96 is [2,867.58, 4,612.30]. These results lead to the conclusion that for  $\gamma=1.5$ , poverty will be reduced by the tax reform for all poverty indices below the lower bound of the confidence interval for  $s = 1, 2, \text{ or } 3$ , respectively. However, the tax reform is not Pen-improving, Dalton-improving, and Kolm-improving due to the fact that the two

$\overline{CD}^s$  curve for PH and PNF intersect for  $s = 1, 2,$  or  $3.$

The second case targets on “Per capita spending on health care and hygiene” (PH) and “Spending on other nonfood commodities” (PN). Consider the growing support for expanding health insurance coverage by a tax subsidy, the assessment of comparing PH and PN is crucial in evaluating the impact of a marginal tax reform.

Similarly, consider the tax reform that increases the tax rate on all nonfood commodities other than health care, and uses the proceeds to decrease the tax rate on health care. Table 2 presents estimated critical values of poverty lines  $z_s(\gamma)$  for different economic efficiency ratios  $\gamma$  and different ethical orders  $s$  on health care and other nonfood products.

For simplicity, the following discussion will first focus on  $s=1.$  When  $\gamma=1,$  the first intersection of  $\overline{CD}^1$  curves for PH and PN is at  $\hat{z}_1(1)=2,977.47,$  with standard error of the sampling distribution of  $\hat{z}_1(1)$  is estimated to be 179.225. Thus, the confidence interval for the estimated critical poverty line  $\hat{z}_1(1)$  with level of significance  $\alpha=0.05$  and  $t$ -ratio=1.96 is [2,626.20, 3,328.74]. In other words, with economic efficiency ratio equals to 1, if the poverty line is assigned below 2,626.20, increasing the tax on all nonfood

commodities other than health care while providing subsidies for health care will be first-order poverty-reducing. Figure 6 shows the Normalized CD-Curves for PH and PN when  $s=1$ . Since the two  $\overline{CD}^1$  curves do have intersection, theorem II is not satisfied, which means the marginal tax reform is not first-order welfare-improving or, Pen-improving.

For economic efficiency ratio  $\gamma=0.5$ , an intersection of  $\overline{CD}^1$  curves for PF and PNF occurs at  $\hat{z}_1(0.5)=2,977.47$ , with the standard error of the sampling distribution of  $\hat{z}_1(0.5)$  estimated to be 179.22. Thus, with economic an efficiency ratio equal to 0.5, if the poverty line is assigned below 2,626.20 (assuming a 95% interval), increasing the tax on nonfood items other than health care, while providing subsidies for health care expenditures will be first-order poverty-reducing. Figure 9 shows the Normalized CD-Curves for PH and PN when  $s=1, \gamma=0.5$ .

Increasing the economic efficiency ratio to 1.5, Figure 10 gives the shape of the two Normalized CD-Curves under the condition of unit ethical order. The first intersection is at  $\hat{z}_1(1.5)=2,094.67$ , with the standard error of the sampling distribution of  $\hat{z}_1(1.5)$  estimated to be 178.36. Likewise, if the poverty line is assigned below 1,745.08 (assuming a 95% interval), increasing the tax on nonfood items excluding health care, while providing

subsidies for health care expenditures will be first-order poverty-reducing.

Notice that no Pen-improving result occurs for different economic efficiency ratios when  $s=1$  since Theorem II is not satisfied. However, the third column in table 2, where  $\gamma=1$  displays an absence of the critical poverty value when  $s=2$  and 3. Therefore, welfare-improving is observed while increasing the tax on nonfood items and using the proceeds to reduce the tax on health care for second-order dominance and third-order dominance (See Figure 7 and Figure 8 in appendix for more details).

## **V. Conclusion**

This paper tested indirect tax reforms in Tlemcen using normalized consumption dominance curves derived by Makdissi and Wodon (2002) and improved by Duclos, Makdissi and Wodon (2008). With different ethical orders and economic efficiency ratios, conditions for poverty-reduction and welfare-improvement were illustrated in the empirical work. The appropriate taxation strategy in Tlemcen is to subsidize food items by increasing tax on non-food items. Additionally, a health care subsidy is also recommended in Tlemcen as being socially improving.



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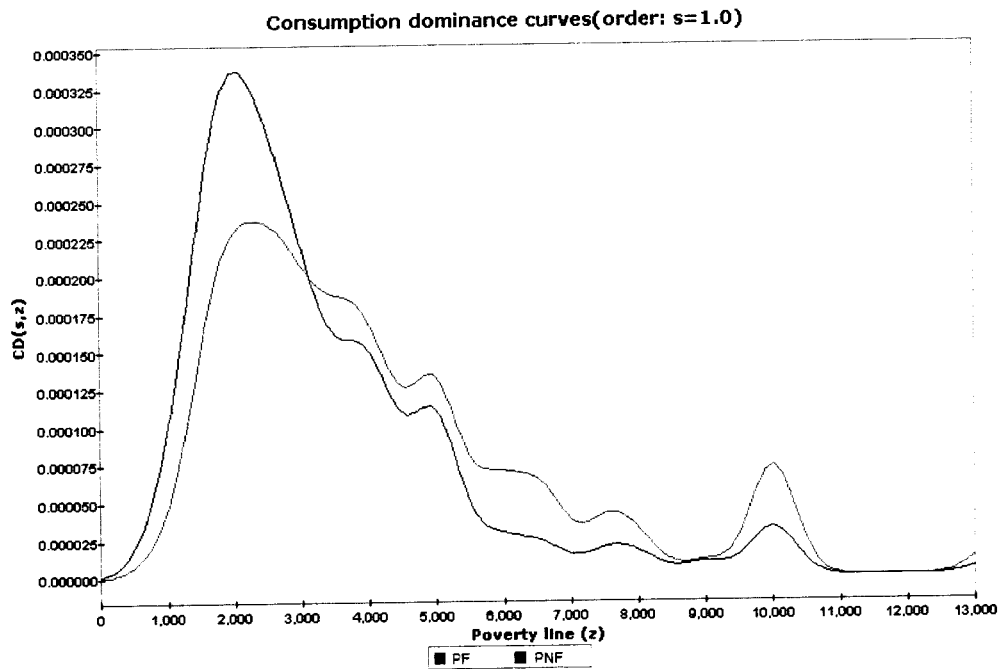
**APPENDIX**

**TABLE 1**

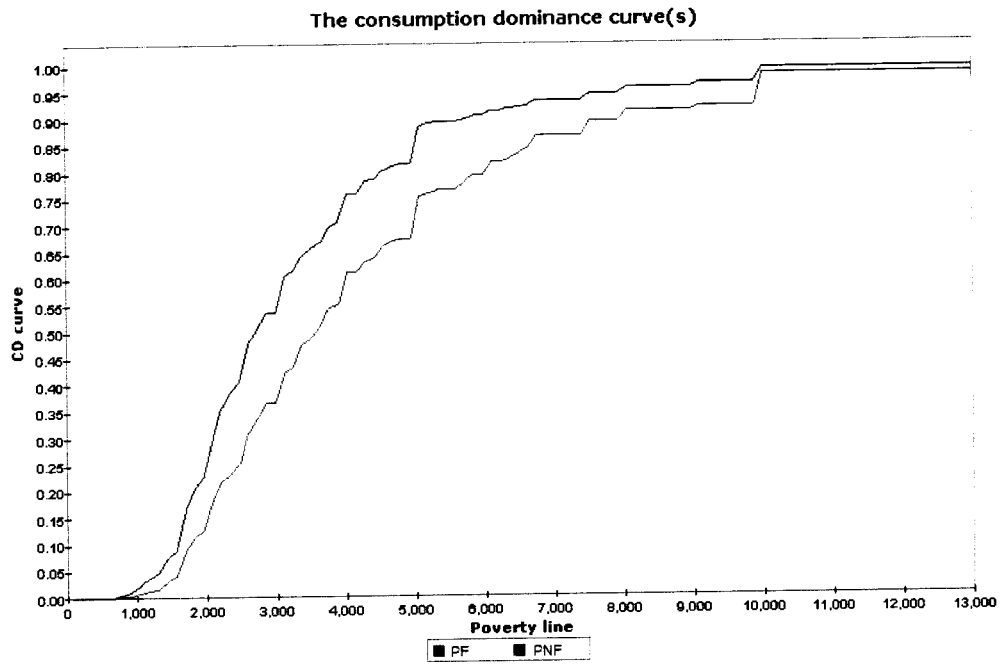
**Indirect Marginal Tax Reform  
For Food Expenditure Versus Nonfood Expenditure in Tlemcen**

Critical poverty line $z_s(\gamma)$ for different ratios of economic efficiency ratio $\gamma$ and for different orders of dominance $s$			
	$\gamma=0.5$	$\gamma=1$	$\gamma=1.5$
$z_1(\gamma)$	5687.05 (106.90)	3122.36 (153.99)	1894.15 (294.94)
$z_2(\gamma)$	–	–	2717.03 (682.95)
$z_3(\gamma)$	–	–	3739.94 (445.08)

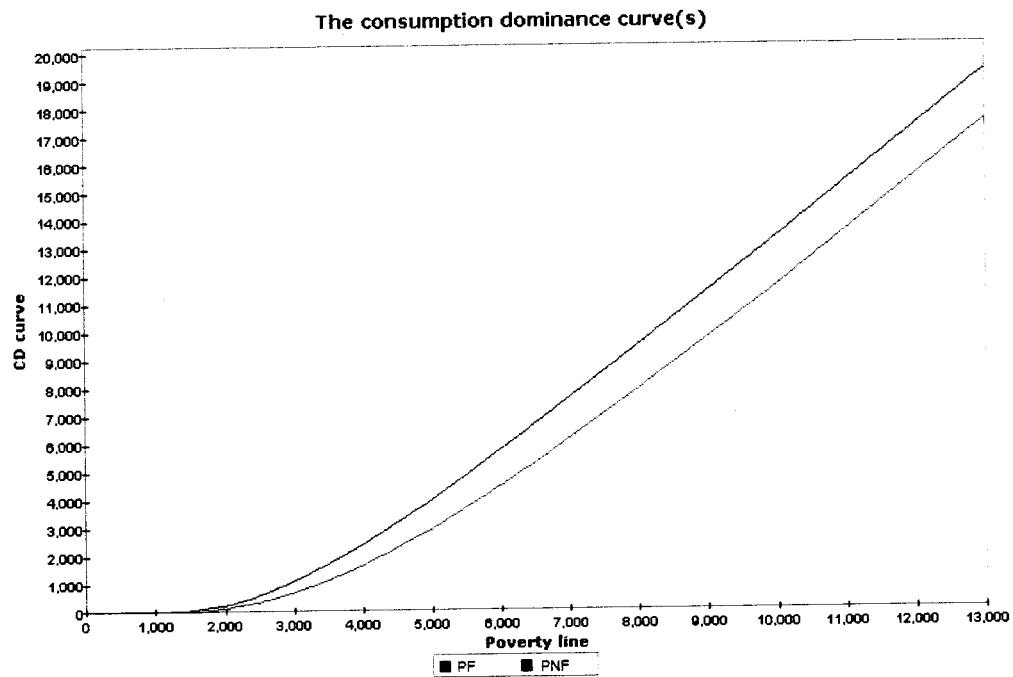
**FIGURE 1 Normalized CD-Curves for PF and PNF  $s=1, \gamma=1$**



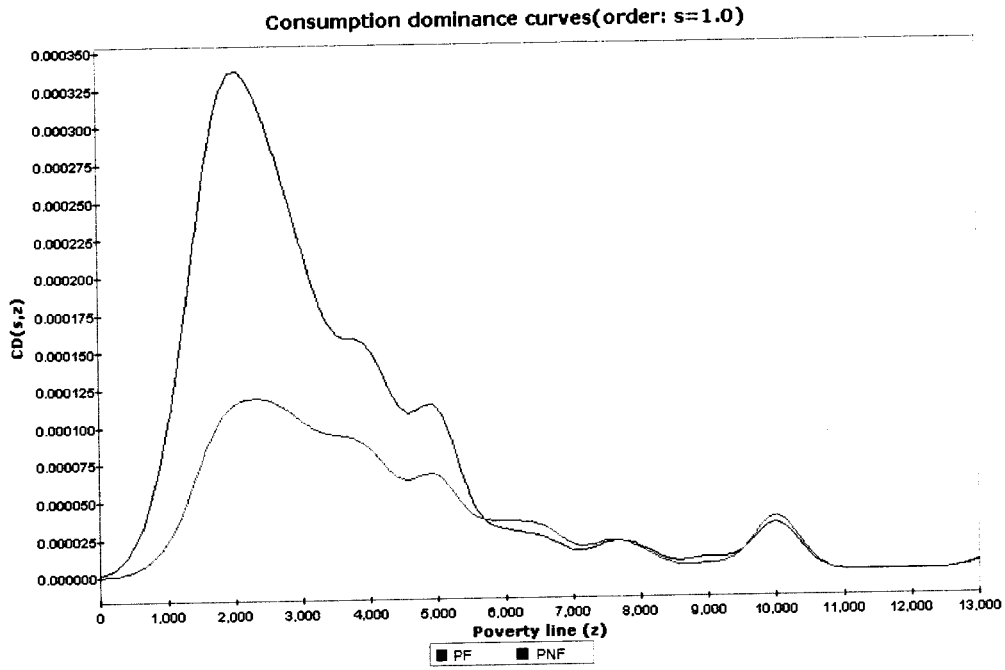
**FIGURE 2 Normalized CD-Curves for PF and PNF  $s=2, \gamma=1$**



**FIGURE 3 Normalized CD-Curves for PF and PNF  $s=3, \gamma=1$**



**FIGURE 4 Normalized CD-Curves for PF and PNF  $s=1, \gamma=0.5$**



**FIGURE 5 Normalized CD-Curves for PF and PNF  $s=1, \gamma=1.5$**

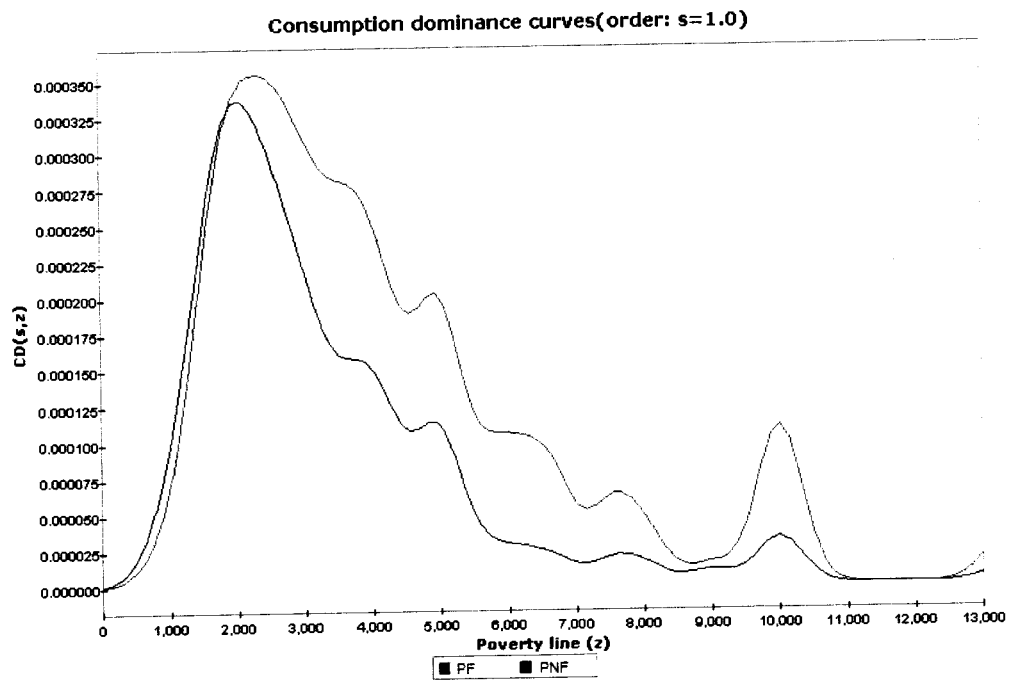
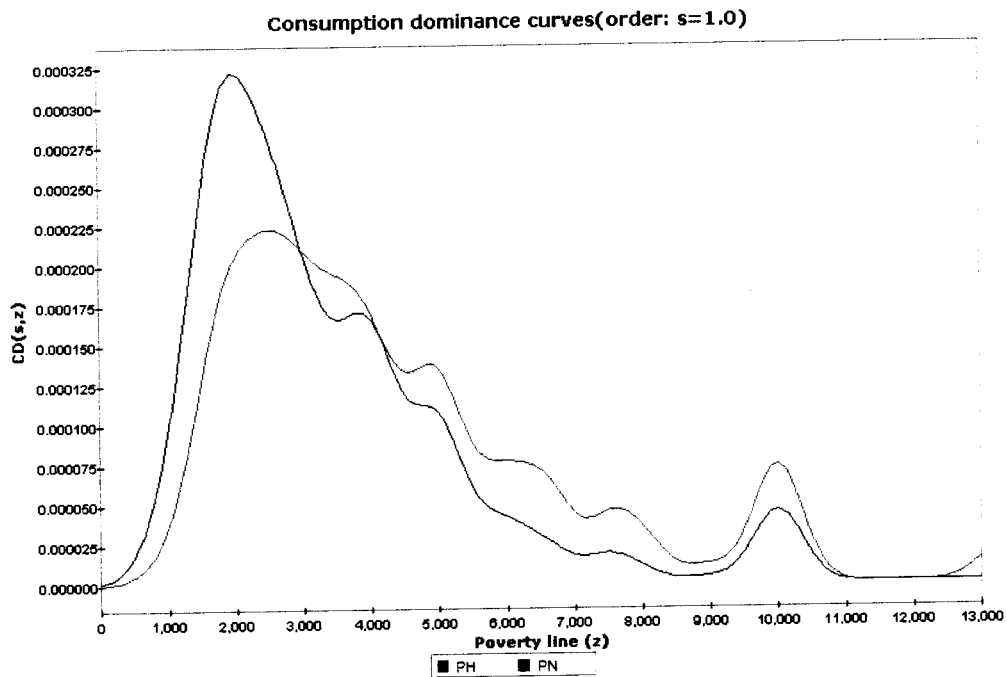


TABLE 2

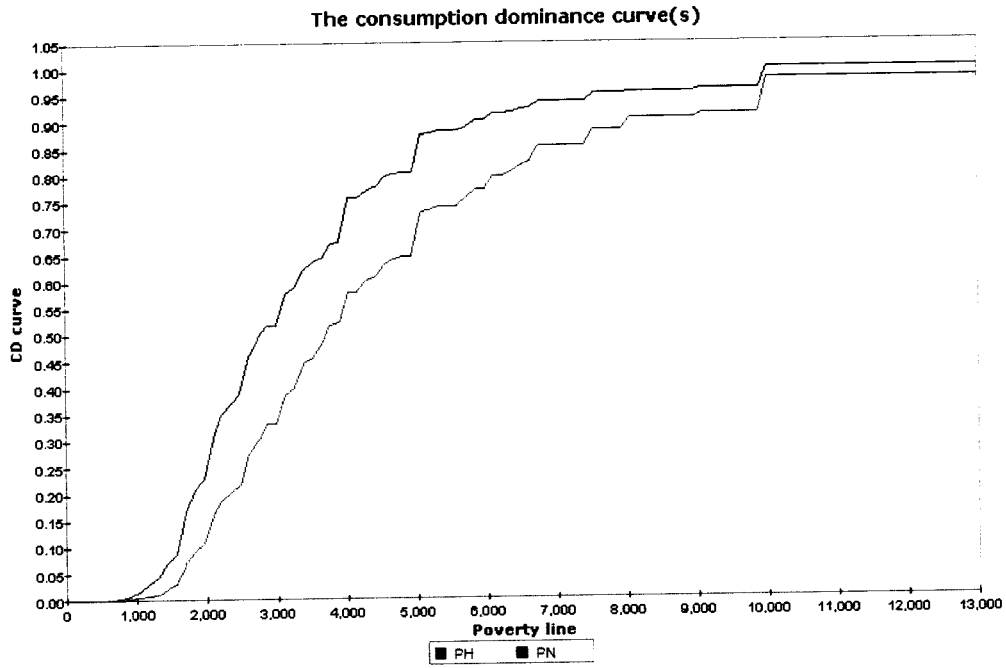
Indirect Marginal Tax Reform  
For Health Care and Hygiene Expenditure Versus Other Nonfood Expenditure  
in Tlemcen

	Critical poverty line $z_s(\gamma)$ for different ratios of economic efficiency ratio $\gamma$ and for different orders of dominance $s$		
	$\gamma=0.5$	$\gamma=1$	$\gamma=1.5$
$z_1(\gamma)$	6193.13 (286.00)	2977.47 (179.22)	2094.67 (178.36)
$z_2(\gamma)$	-	-	2998.35 (-)
$z_3(\gamma)$	-	-	4349.99 (632.73)

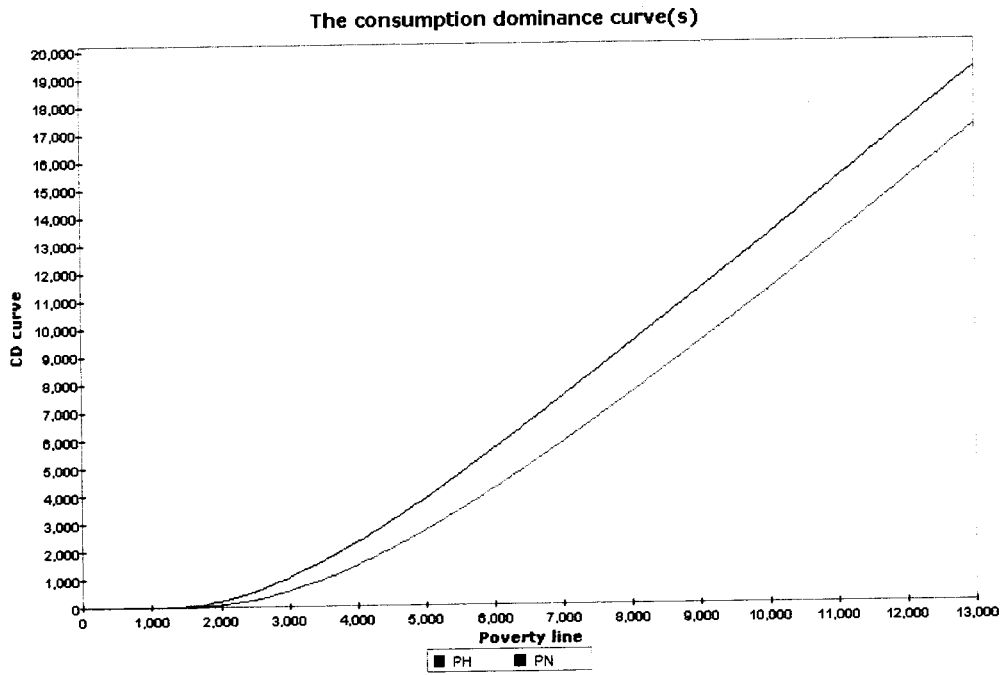
FIGURE 6 Normalized CD-Curves for PH and PN  $s=1, \gamma=1$



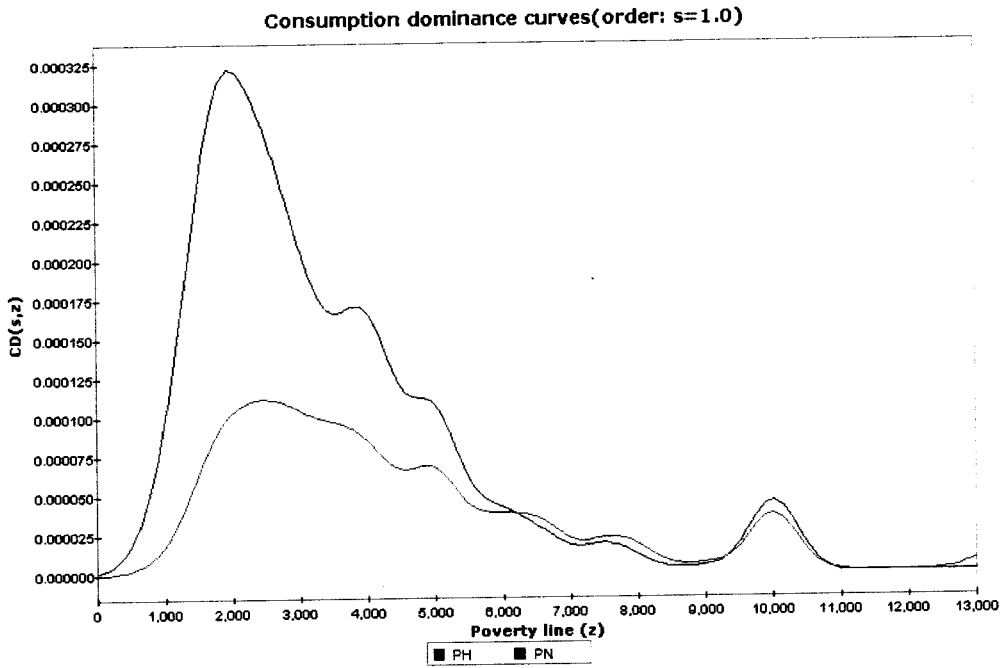
**FIGURE 7 Normalized CD-Curves for PH and PN  $s=2, \gamma=1$**



**FIGURE 8 Normalized CD-Curves for PH and PN  $s=3, \gamma=1$**



**FIGURE 9 Normalized CD-Curves for PH and PN  $s=1, \gamma=0.5$**



**FIGURE 10 Normalized CD-Curves for PH and PN  $s=1, \gamma=1.5$**

