Assessing the Economic Impact of Foreign Aid in Benin

A Dynamic General Equilibrium Analysis

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Major Paper presented to the
Department of Economics of the University of Ottawa
in partial fulfillment of the requirements of the M.A. Degree

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August 2008
Abstract:

This paper examines the impacts of foreign aid in a low-income developing economy, which is Benin. A multisector dynamic general equilibrium model is developed and is used to evaluate the aggregate, sectoral and growth impacts of a permanent increase in foreign aid in that country. A distinctive feature of the model is its combination of several desirable characteristics related to the tradable nature of all sectors, the composite nature of investment good, the presence of capital installation costs and the endogeneity of labour supply. In addition to the traditional Dutch disease insights, the model results suggest that the increase in foreign aid increases consumption, decreases investment and reduces labour supply. While households enjoy a higher level of utility, the significant decrease in investment has a negative impact on growth as GDP drops in relation to the reference case.
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1. Introduction

Foreign assistance started after the Second World War, and since then has been steadily, but not predictably, increasing. This continually increasing monetary flow from the developed to the developing world has brought about a large body of literature surrounding the question of the efficiency of foreign aid. Researchers have sought to ascertain whether these funds have assisted in increasing growth in the recipient country.

Foreign assistance is considered efficient if it has a positive impact on economic development in the recipient country. In the literature that emerged over the past half century, there is no systematic answer to this question. The sources of the main potential inefficiencies of aid are its fungibility as well as to a phenomenon known as Dutch Disease. The former phenomenon refers to the potential of recipient governments to use aid for purposes other than intended, for example, when aid is consumed rather than invested (Pack & Pack, 1993; Durbarr, 2004). The latter refers to the possibility of aid to decrease the size of the traditional export sector, through its effect on relative prices (Hjertholm, Laursen, & White, 2000; Adam & Bevan, 2003; Torvik, 2001). If a portion of foreign aid is spent on non-tradable goods, it increases relative price of non-tradables, induces real exchange rate appreciation and a reallocation of resources towards the non-tradable sector (Agénor, Bayraktar, & Aynaoui, 2008). Dutch Disease is likely to result in decreased competitiveness in the recipient country, due to higher remuneration in the non-tradable sector. The decrease in competitiveness is problematic since it goes against the initial intent of foreign aid (Hjertholm, Laursen, & White, 2000).

Several papers have studied the relationship between foreign aid and growth in the recipient economy. Empirical research has nonetheless led to no conclusive evidence. There is however, a growing credence that aid does promote growth under certain conditions. This view is advocated by Burnside and Dollar (2000) and Lensink and White (2001) among others.

Most recently, a new segment of the literature pertaining to the effectiveness of foreign aid has emerged. This new research branch suggests that aid volatility is an additional source of aid inefficiency. Since developing countries have limited access to world financial markets, and lack substantial policy tools to counter business cycles, the cost of business cycles in these countries tends to be greater (Bulir & Hamann, 2003). This implies that additional sources of uncertainty are harmful to the economy, as aid volatility is a source of uncertainty.
Aid is highly volatile, in many cases more volatile than revenue, unpredictable, and generally speaking procyclical (Bulir & Hamann, 2003). The unpredictability of aid refers to the inability of recipient countries to rely on aid commitments by donors. The unpredictability of aid coupled with its volatility creates the potential for aid shocks, which, as with any shock, can negatively affect the recipient economy. The procyclicality of aid suggests that aid disbursements are below their trend when GDP is below its’ trend. This indicates that foreign assistance is not, empirically, an insurance mechanism against other economic shocks (Dhasmana, 2008). The volatility of foreign aid has detrimental impacts on the recipient economy, as aid shocks tend to be unpredictable and quite large. Both positive and negative aid shocks trigger important consequences for the economy (Hudson and Mosley, 2008). The negative shocks are detrimental since they require governments to rely on often costly, inefficient policy instruments to counter the shock. The decreased aid receipt results in increased taxes, or decreased government spending, both of which can further amplify the effect of the negative aid shock. Positive shocks are harmful because of the absorptive capacity of the economy, only some of the unexpected increased aid is productively used.

Empirical studies quantifying the costs of aid volatility are scarce, but exist in both regression analysis and general equilibrium framework. Using a two-sector general equilibrium framework, Arellano et al. analyze the impacts of aid volatility on growth. In this context, aid volatility negatively affects economic welfare; but does not offset the positive impact of the aid inflow itself. Because of the sectoral nature of the model in Arellano et al., the authors are able to find that Dutch Disease is not simply a static response to increased foreign inflows, but also dynamic.

Extending the model by Arellano et al., Dhasmana includes capital adjustment costs. This implies that there is a cost associated with making capital productive, in addition to the price of the investment good. Ortiguera and Santos (1997) show that capital adjustment costs are important in endogenous growth models, as they play a significant role in the dynamics of the model. "Changing the level of capital services at a business generates disruption costs during installation of any new or replacement capital and costly learning must be incurred as the structure of production may have been changed. The inclusion of capital adjustment costs is important in quantifying the negative impact of aid volatility. The costs of aid volatility proved much greater in this context, as demonstrated by Dhasmana, 2008."
In a paper not related to aid volatility, Chatterjee and Turnovsky (2006) show that in order to capture the dynamic adjustment of a recipient country consequent to an aid inflow, endogenous labour supply must be included. Exogenous labour supply models allow instantaneous adjustment, and do not capture the transmission mechanisms pertaining to the tradeoff between leisure and consumption. The labour leisure decision is important in capturing these transmission mechanisms, as aid inflows cause the price of consumption good relative to leisure to change. A change in the demand for leisure has an impact on labour supply as total time endowment is fixed. The result is that with flexible labour supply, increased aid inflow reduces labour supply and decreases the growth rate. However, Chatterjee and Turnovsky’s model, includes only one single sector economy, and ignores the potential Dutch Disease effect, as seen in models that include two or more sectors.

The present paper fills an existing gap in the literature by combining some of the important features of the papers on foreign aid mentioned above in a more complex setting. The framework is extended to a four-sector model. To my knowledge, this is the first paper that examines aid increases in this context. A similar setup to that of Arellano et al. is utilized, as the government sector is consolidated with the household sector. However, aid inflow is not provided in the form of goods, as in Arellano et al., but rather it appears in the private agent’s budget constraint. This implies that aid is viewed as an additional source of income, which can be spent the same way as any other form of income. Capital adjustment costs and endogenous labour supply are included. Moreover, the current analysis assumes that the investment good is a composite of goods from various sectors in the economy. This is in contrast to previous studies that treat investment as a single good, most commonly the tradable good. This is a realistic assumption, as many traditional investment goods, for example infrastructure, are non-tradable goods. Furthermore, this study includes intermediate inputs as production factors.

The expectation is that the loss created by the inclusion of capital adjustment costs will decrease the demand for investment goods and increase the demand for consumption goods, relative to the case of abstaining from their inclusion. Including endogenous labour supply is expected to magnify the negative impact of aid volatility on growth through its effect on decreased labour, due to increased income. Investment as a composite good is projected to increase the adverse effects of Dutch Disease, allowing an additional component of domestic
demand to vary with relative prices. The inclusion of intermediate inputs is expected to have a similar effect as the diversification of investment.

The remainder of the paper is organized as follows. Section 2 presents a literature review pertaining to aid volatility. Section 3 outlines the model used in the paper. The fourth section discusses the data and calibration of the model. Section 5 outlines the various simulations and results, in section 6, sensitivity analysis is undertaken and section 7 concludes.

2. Literature Review

The effectiveness of foreign aid and the sources of its inefficiencies have been analyzed using various methodologies. Econometric analyses, using both exogenous and endogenous growth models have yet to yield any conclusive evidence. In recent years, several general equilibrium models have been developed to analyze the same topic. This section is organized as follows. First, the evidence pertaining to Dutch Disease is reviewed. Second, the question of whether or not aid is efficient in promoting growth is assessed, initially in the context of econometric studies, and second in the context of general equilibrium models. Third, some of the evidence pertaining to aid volatility and its quantification is outlined. Fourth and finally, the impact of aid volatility is surveyed, also through econometric analyses and in the context of general equilibrium models.

One of the causes of the ineffectiveness of foreign aid is Dutch Disease. As explained earlier, this occurs when increased aid flow results in increased domestic demand for non-tradable goods, whereby increasing their relative prices. This causes higher remuneration in this sector, and a reallocation of resources towards it, along with real exchange rate appreciation. The overall negative impact is the loss of competitiveness in the world market. This demand side effect is most commonly discussed in the literature. According to this argument, the long-run impact will be a contraction of the tradable sector, and a decline in growth, relative to the situation with no aid (Adam and Bevan, 2004).

Agénor et al. (2006) point out two supply side effects that can counter the traditional impact of Dutch Disease, and increase the benefits of foreign aid. First, increased aid flow can increase productivity through a learning-by-doing effect. Second, when foreign aid contributes to increasing public investment on infrastructure, it can counterbalance the traditional contraction of the tradable sector.
The learning-by-doing effect implies that there is a productivity spillover from the donor country to the recipient country. When this occurs, productivity increases in the recipient economy, typically in the tradable sector. Torvik (2001) points out that when there are productivity spillovers between sectors, the effect of the increased aid on the exchange rate is ambiguous. The demonstration is that increased productivity can increase supply in the non-tradable sector, whereby lowering the price of non-tradables, which counteracts the conventional demand side effect of Dutch Disease.

The second supply side effect is the potential increase in public investment. If foreign aid increases public investment in infrastructure, and infrastructure is a productive complement to other forms of privately supplied capital, the resulting effect will be an increase in private investment. Since the two forms of capital are complements, there will be an increase in the productivity of private capital, whereby increasing the return to private capital. Adam and Bevan (2004) demonstrate that the demand side impact of Dutch Disease can be minimized by this supply side effect. In their study, they generate a benchmark case, where public infrastructure has no effect on private capital productivity. This benchmark scenario allows them to isolate the demand side effects of increased aid. As expected, in this case increased aid decreases investment and tradable output over time, generating a decrease in exports and a real exchange rate appreciation. In a second simulation, the model assumes that increases in public infrastructure affects private capital productivity in all sectors uniformly. This stimulates private sector investment. In the medium term, the results paint a very different picture, there is an increase in the volume of exports (Adam and Bevan, 2004). Moreover, most of the real exchange rate appreciation is reversed. Thus, the demand side effect of Dutch Disease is at least partially counterbalanced.

Historically the Dutch Disease effect takes account of the demand side effect. Recently, it has been demonstrated however, that if foreign aid can induce productivity gains through a learning-by-doing effect, or increases in public investment, then the supply side effects may be enough to offset the traditional demand side effects, and the long run effect of aid on GDP may be positive.

Many studies attempt to verify empirically whether the effect of aid on growth is positive. Chatterjee and Turnovsky (2007) is an example among many others. They try to determine whether the additional monetary inflow is used for investment purposes, and whether
this investment increases productivity, to determine whether aid flows are helpful to the
developing world.

The earliest studies that looked at the efficiency of foreign aid used the Harrod-Domar
model of economic development. This model stresses the importance of physical capital
accumulation, and investment productivity as central to economic growth (Hjertholm et. al,
The two gaps are the savings gap, and the trade gap. This framework suggests that any rate of
economic growth is attainable by appropriate choice of investment level. Savings, which is
comprised of both domestic and foreign savings, is chosen to achieve that level of investment. A
savings gap arises when domestic savings is insufficient to reach the desired level of investment.
A trade gap arises because it is not possible to produce all investment goods domestically,
therefore, some must be imported. According to this model, foreign aid should lead to a direct
increase in investment, as fungibility is not considered.

One of the most cited studies employing the Harrod-Domar two-gap theoretical
framework is Papanek (1973). The authors used the following regression equation.

\[ G_i = \alpha_0 + \alpha_2 S_i + \alpha_3 A_i + \alpha_4 P_i + \alpha_4 O_i + \varepsilon_i \]

where \( G \) is the growth rate, \( S \) is domestic savings, \( A \) is foreign aid inflows, \( P \) is private capital
flows, and \( O \) is other foreign capital flows.

Papanek (1973) is among the first authors who disaggregate aid from other forms of
capital inflow. His cross-country regression is based on 34 observations from the 1950s, and 51
observations from the 1960s. The main results stemming from this influential paper are that
savings, aid, and other capital inflows account for one third of growth, while two-thirds is
explained by other factors, such as resource endowments. The study suggests that the coefficient
of aid is nearly twice that of other inflow variables, indicating a positive link between aid and
growth.

Many of the studies employing this methodology use regression equations very similar to
that used by Papanek (1973), but obtain very different results. Rahman (1968) and Griffin
(1970) among others find a negative relationship between foreign aid and domestic savings,
while Gupta (1970) finds that the coefficient is not significantly different from zero. Dowling
and Hiemenz (1982), along with Gupta and Ismal (1983) find that aid has a positive impact on
growth, while Mosley (1980) and Voivodas (1973) find that there is no correlation between aid
and growth. These studies demonstrate that there is no irrefutable evidence that aid has a positive impact on savings, or that aid increases growth.

Hansen and Tarp (2000) comment on these earlier studies, suggesting that the concern is not whether the coefficient between aid and savings is negative, but whether it is less than negative one. This is because a coefficient between zero and one indicates a positive aid investment relationship. Though aid displaces domestic savings when the coefficient is between zero and negative one, an increase in overall savings is achieved (McGillivray et al., 2006). In a survey of 39 first generation models, many applying the framework discussed above, only one displays an aid coefficient significantly less than negative one (Hansen and Tarp, 2000). The overall conclusion brought forward by Hansen and Tarp (2000) is that while the correlation may not be one for one, generally these earlier studies indicate a positive relationship between foreign aid and economic growth.

More recently, some studies used endogenous growth models as their theoretical underpinning to assess the effectiveness of foreign aid. Using very different explanatory variables from the studies discussed previously, these studies often come to new and insightful conclusions. The explanatory variables used in the regressions that were previously ignored often allow for difference in policy environment or climatic and geographic characteristics. Also, individual country characteristics are often included.

Many of these third generation studies used equations similar to the one stated below (McGillivray et al., 2006):

\[ G_i = \alpha_0 + \alpha_1 A_i + \alpha_2 A_i^2 + \alpha_3 P_i + \alpha_4 A_g P_g + \epsilon_i \]

The aid-squared variable allows for the existence of a non-linear relationship between aid and economic growth. The other terms are interaction terms between aid and other characteristics of the recipient country. Some examples are the interaction between aid and policy, or aid and climactic conditions. In addition, these more recent studies often consider the potential endogeneity of aid (Hansen and Tarp, 2000). The endogeneity of aid refers to the situation in which higher aid provides a higher growth rate, but that countries with higher growth rates receive higher aid disbursements.

Because of the nature of the econometric equations, these more recent studies generally yield conclusions that indicate conditions under which aid is effective. For example, Guillaumont and Chauvet (2001) include an aid-policy interaction term and an aid-climate
interaction term, but exclude the aid-squared explanatory variable. This specification of model, and the data set, suggests that the effectiveness of aid is dependent upon the climatic conditions of the recipient country. The evidence is inconclusive as different studies yield different conditions under which aid is effective using various regression equations and different data sets. A number of studies, including Burnside and Dollar (2000) find that the aid policy interaction term is important in cross country growth regressions, while another group of studies (for example Hudson and Mosley, 2001; Islam, 2002) find that this term is insignificant.

Generally, econometric analyses, using both exogenous and endogenous growth models, indicate that aid can be efficient at increasing growth in the recipient country. These studies yield various conditions under which this is the case. There is no consensus on these conditions.

A more recent branch of economic modeling, general equilibrium modeling, has also been used to analyze the efficiency of foreign aid. Chatterjee and Turnovsky (2007) use a general equilibrium framework to analyze the effectiveness of foreign aid. The authors disaggregate aid into two categories, tied and untied aid. A single representative consumer has preferences over consumption and leisure. The consumption good is a single tradable good. The production function is a constant elasticity of substitution (CES) function that combines labour, public capital and private capital. Labour is combined with public capital to produce labour effective units in the production function. The model includes capital adjustment costs. Aid is modeled as additional units of government revenue. Aid is considered tied, if it goes directly to augmenting the stock of public capital. The model is calibrated to a benchmark representation of a developing economy.

The simulation is to increase aid from 0% of GDP to 5% of GDP. The authors find that the degree to which aid is tied will have a largely influence the effectiveness of aid. For tied aid, increased aid flows will directly increase the stock of public capital. The larger stock of public capital will increase the marginal productivity of private capital and labour. This will cause an increase in labour supply, and an increase in private capital accumulation. The economy becomes more productive, and the long run growth rate increases. In the case of untied aid, the additional inflow is used for debt reduction, causing an increase in consumption. The increased consumption increases the marginal benefit of leisure, which causes a decrease in labour supply. This causes a decrease in the marginal productivity of both types of capital, which reduces the growth rate.
The general equilibrium model developed by Moreira and Bayrakater (2008) is a single sector, two-good economy. The productive process in this model combines educated labour, public capital and private capital in health and infrastructure. The productive process has two levels, initially, skilled labour is combined with public health, according to a CES function. This produces effective labour units, which are then combined with private capital, according to a CES function. The composite from this second level CES function, is then combined with public infrastructure. Aid is modeled as a grant to the government. The simulation is to increase the aid to GDP ratio by 5%. The result is an increase in public investment, which causes an increase the public stock of capital. This induces a productivity increase for private capital, thereby increasing the level of private investment. In their paper, the Dutch Disease effect materializes only in the short run. In the long run, the supply side effect of increased investment offsets the initial rise in prices. This study demonstrates the potential for increased aid to increase the GDP in the recipient economy, caused by an increase in public capital.

These general equilibrium analyses demonstrate the potential for increased aid to have benefits for the recipient economy. If the supplementary aid increases the stock of capital, then the long run effect on economic performance will be positive. If given to the private economy in the form of grants, it will serve to increase consumption, and have a negative long run impact on the economic performance of the recipient economy.

Recently, the literature pertaining to the effectiveness of foreign aid has focused on a new issue: the volatility of aid disbursements. Bulir and Hamann (2003) conduct an empirical study, which yields conclusions regarding the overall volatility of foreign aid. The regression is based on 72 countries from 1975 to 1997. The main findings of this paper are important for subsequent research analyzing the impact of aid volatility. The study demonstrates that aid is highly volatile, in most cases more volatile than revenue. Additionally, aid volatility increases with aid dependence. This indicates that countries, which rely most heavily on aid disbursements, have to deal with the most unpredictability. The measure of aid volatility almost doubles when the sample is narrowed to countries with an aid to revenue ratio larger than 50%. Additionally, the study finds that aid and revenue deviate from their respective trends coincidentally. This is worrisome, because it indicates that aid is procyclical to the business cycle, and therefore often aid disbursements are not increased when the country is in recession and needs it most.
Selected results from Bulir and Hamman (2003) are presented in the table 1. The countries for which results are presented are countries used in studies overviewed here, and Benin, used in the current paper. The conclusions of Bulir and Hamann (2003) indicate that aid volatility may have important impacts. A limited number of studies have attempted to quantify the impact of foreign aid volatility. A number of these studies are the subject of what follows. Two methodologies are employed in the quantification of aid volatility’s impact on GDP growth. The first two studies employ regression analysis while the last three use a general equilibrium framework.

Table 1: Volatility of Aid in a Sample of Countries

<table>
<thead>
<tr>
<th>Country</th>
<th>In percent of GDP</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Aid Volatility</td>
<td>Revenue Volatility</td>
</tr>
<tr>
<td>Benin (1985-1997)</td>
<td>2.26</td>
<td>0.72</td>
</tr>
<tr>
<td>Burkina Faso (1992-1997)</td>
<td>2.23</td>
<td>0.45</td>
</tr>
<tr>
<td>Cote d’Ivoire (1988-1997)</td>
<td>12.91</td>
<td>3.82</td>
</tr>
</tbody>
</table>

Source: Bulir and Hamann (2003)

One of the earliest studies that attempt to quantify the effects of aid variability is the study by Lensink and Morrissey (2000). They conduct a cross-country regression analysis to determine the effect of foreign aid volatility on economic growth. Aid volatility is measured in three different ways for each of the 75 countries in the study.

\[
AID_t = \alpha_1 + \alpha_2 T + \alpha_3 AID_{t-1} + \alpha_4 AID_{t-2} + \varepsilon_t
\]

\[
AID_t = \alpha_5 + \alpha_6 AID_{t-1} + \alpha_7 AID_{t-2} + \varepsilon_t
\]

\[
AID_t = \alpha_8 + \alpha_9 T + \alpha_{10} T^2 + \varepsilon_t
\]

Where T is a time trend variable. The first two equations capture unexpected instability while the third is a measure of instability around a time trend. GDP growth is then regressed on
initial level of GDP, secondary school enrollment, and aid. In a second set of equations, investment is added to the regression equation, with the potential specification problem of investment being correlated with aid. The results of these base equations show that aid has no statistically significant impact on GDP growth. The same regression equations (with and without investment) are then augmented to include each of the measures of aid volatility in turn. Once the first or second measure of aid volatility is included, the coefficient for aid becomes positive and significant; however its’ significance decreases in equations where investment is included. Both the first and second measures of aid volatility are negative and significant when included in the regression. In a separate regression equation, a positive link between aid and investment is found. The conclusion is that aid, increases growth through its positive effect on investment. However, aid uncertainty, as with most sources of economic uncertainty, deters investment and is harmful for growth.

Hudson and Mosley (2008), use a dataset of 131 countries to conduct an econometric analysis of the impact of foreign aid volatility on macroeconomic variables. The dependent variables are the main components of GDP, when calculated according to expenditures, notably consumption, investment, government spending, imports and exports. Each of the GDP components, as well as GDP itself is regressed on aid, and aid volatility, among other variables. The coefficient of positive volatility is negatively significant (at the 5% significance level) for all components of GDP except for exports, where the coefficient is negative, however statistically insignificant. Negative volatility is harmful for investment and government expenditure. The coefficient of imports and exports is negative, though not statistically significant. Consumption share increases following negative volatility. “The impact of negative volatility in increasing consumption simply implies that the impact of negative volatility on consumption is less than on other components of GDP, rather than consumption responds positively to negative volatility” (Hudson and Mosley, 2008). In both cases, volatility has a negative and significant impact on government spending and investment. This indicates that governments are unable to rearrange their spending patterns subsequent to a change in aid receipts. Through its’ effect on government spending and investment, both positive and negative foreign aid volatility have an adverse effect on GDP.
The following studies all attempt to quantify the impacts of foreign aid volatility on growth in a general equilibrium framework. In these models, countries do not have access to international capital markets, which implies that they are not able to borrow money when aid disbursements are lower than expected. This assumption implies that these countries will need to use other fiscal, and monetary policies to adjust to the lower than expected aid receipt. This assumption also implies that the only way that individuals can save is by investing in the domestic capital stock.

Neanidis and Varvarigos (2007) attempt to quantify the impact of aid volatility by disaggregating aid into productive and unproductive aid. In their study, aid is modeled as an inflow of funds to the government. The government then allocates a fraction, \( \delta \) to the accumulation of public capital, which is labeled productive aid, while the remainder (1-\( \delta \)) is disbursed to the private economy in the form of grants. They note that empirically, productive aid is nearly three times more volatile than unproductive aid and accounts for 90% of all aid flows.

The study uses the following regression equation. This equation separates productive from unproductive aid, and has a variable for each of their variability. This allows the author to determine whether productive aid volatility has the same impact as unproductive aid volatility.

\[
g_{it} = a + \beta_p a_{it}^p + \beta_u a_{it}^u + \beta_r \ln r_{it} + \sum_{j=p}^U V_{jt}^j + \sum_{k=1}^M Y_k X_{k, it} + \sum_{l=1}^n \mu_l D_{l, it} + \epsilon_{it}
\]

The dependent variable is the country’s average GDP growth rate. The first two explanatory variables represent productive aid and unproductive aid, followed by aid repayment. \( V_{ij} \) is a vector of volatilities for the two types of aid. \( X \) is a matrix of other explanatory variables, and \( D \) is a matrix of regional dummies. Among the most important empirical findings of this paper is that while productive aid helps growth (\( \beta_p > 0 \)), unproductive aid disbursements are found to have a negative impact on growth (\( \beta_u < 0 \)). Their volatilities are found to have opposite effects. The volatility of productive aid has been found to be harmful to growth, whereas the volatility of unproductive aid was determined to affect growth positively. The authors also note that when the volatility of foreign aid is considered, they are unable to reject
the null hypothesis that the sum of the effects of aid and its’ volatility is negative on the recipient economy’s growth.

Based on these main findings, a theoretical model is built; it allows the authors to explain the empirical results. The model is comprised of a single productive sector, combining public and private capital using a Cobb-Douglas production function to produce output. Both private and public capital are assumed to depreciate fully between periods, and the dynamics of the model occur through investment. Aid flows are a function of the recipient economy’s revenue, and a random variable, which embeds volatility in the model. The model generates a number of propositions, and one corollary.

The authors are able to reproduce the empirical results noted above. Among the propositions of the model are that an increase in productive foreign aid along with a decrease in its volatility increase the growth rate of the economy, while an increase in unproductive aid along with a decrease in its volatility impede the growth rate of the economy. The corollary is that “the effect of foreign aid on the recipient’s growth rate may be negative if and only if aid is volatile” (Neanidis & Varvarigos, 2007). This reproduces the empirical finding that aid may have a negative relationship on growth when the coefficient of aid and its volatility are jointly considered.

The papers by Arellano et al. (2008) and Dhasmana (2008) share many common properties. Aid inflows are modeled as additional units of tradable goods. Consumption is a composite of tradable and non-tradable goods according to a CES consumption function. While production functions are Cobb-Douglas forms with labour and capital as inputs. Labour supply is assumed exogenous, implying that leisure does not enter the intertemporal utility function. Capital is assumed sector specific, implying that it is not perfectly mobile. Imperfect capital mobility is modeled using a constant elasticity of transformation function for capital supply. All investment is tradable goods.

Arellano et al. (2008) is a two-sector model, which includes technological shocks. The model is calibrated to Côte d’Ivoire, one of the countries of the West African Economic and Monetary Union. As mentioned by the authors, Côte d’Ivoire is an aid dependent country that has had to face volatile aid inflows.
The dynamics of the model exhibit Dutch Disease so that increased aid is translated into lower prices for tradable goods, and increased production of the higher priced non-tradable goods. The results of the model demonstrate that increased aid results in increased volatility of all macroeconomic variables. Increased aid also decreased the private propensity to save as reliance on aid inflows for investment purposes is increased. "In summary, we find that the large productivity fluctuations typical of aid-dependent countries such as Côte d'Ivoire introduce high volatility in all macroeconomic aggregates, even in the absence of aid volatility. However, in this framework, aid volatility further exacerbates macroeconomic fluctuations" (Arellano et al.).

The authors find that if aid could be disbursed in a stable manner, the same level of welfare could be achieved by reducing aid by 8%. This result indicates that aid volatility does in fact reduce the efficiency of aid disbursements in terms of welfare. The authors also look at aid as an insurance mechanism against productivity shocks and find that if foreign aid could be provided to insure against productivity shocks, it could be reduced by 64% while still maintaining the same level of welfare. Regression results from a sample of 73 aid dependent countries demonstrate that the results of the model are robust to empirical findings.

Dhasmana (2008) further extends the model by including capital adjustment costs and terms of trade shocks. The goal is to determine the potential benefits associated with aid acting as an insurance mechanism against terms of trade shocks in this theoretical context. The empirical study leading to the motivation for the model is based on 72 aid-dependent countries. For the 72 countries, aid is on average four times more volatile than revenue. In addition, the volatility of aid is an increasing function of the level of aid dependency of the country. This indicates that countries with the highest reliance on foreign aid, are the most likely to encounter a sudden unexpected decline in aid receipts.

The extent to which countries rely on a small number of exports is also explored. It is demonstrated that developing economies are highly reliant on one or a few exports as a primary source of income. Due to this fact, including terms of trade shocks, to quantify welfare gains of aid as an insurance mechanism against these sudden price changes. For example, 14 of the 17 countries have more than 20% of the export revenue from one product, while 15 of them rely on only three goods for 30% of their export revenues. Additionally the price of commodities which
are generally exported from developed countries (cotton, sugar, coffee etc.) are found to be highly volatile, inducing terms of trade shocks that are likely to have important impacts.

The choice of the variable used to measure of terms-of-trade shocks can be problematic. Though indexing against terms of trade shocks at international prices presents no moral hazard issues, it may not provide full insurance as this would not take into account transport costs, or differentiable product (Dhasmana, 2008). The next possible way to index aid discussed by the authors is that of indexing aid against the value of exports. When the value of exports fall below a certain level of their steady state value, an inflow of aid is provided to insure against this shock. The third possible way discussed to index aid is against GDP. The two last measures are likely to be subject to problems of moral hazard, as the recipient economy could alter its behavior to ensure that it receives higher aid receipts. There is a tradeoff between full insurance against the shock and full information.

The model differs slightly from that of Arellano et al. It is a three-sector model, in which the tradable sector is further decomposed into exports and imports. The consumption function is a CES composite of tradable and non-tradable goods, where-by the former are a CES composite of domestically produced goods tradable goods (export sector) and imports. There are two productive sectors in this economy, which are the non-tradable sector and the export sector. The model includes terms of trade shocks and productivity shocks.

The country chosen in the study (Dhasmana, 2008) is Burkina Faso, also belonging to the West African Economic and Monetary Union. Burkina Faso‘s main export good is cotton, which makes up 65% of total export revenue, and 9% of GDP. The price of cotton is also highly volatile. Additionally, government revenue seems to be positively correlated with cotton prices. Aid flows were also found to be positively correlated to the world price of cotton, indicating that aid is not provided as an insurance against negative terms of trade shocks. The potential adverse effects of aid volatility are compounded by its’ procyclical nature with terms of trade shocks. Together these two effects may be among very important sources of business cycles in the developing world.

As with Arellano et al., there is evidence of the Dutch Disease phenomenon, as increased aid flows caused the real exchange rate to appreciate, decreased the price of export goods relative
to non-tradable goods, causing a reallocation of resources in the economy towards the non-tradable sector. These authors show that in the presence of capital adjustment costs and terms of trade shocks, the negative effect of foreign aid volatility is approximately 20 times higher than in the study by Arellano et al.

The study also finds that there is a potential benefit associated with aid indexation, and the possibility of using foreign aid as an insurance against other shocks. If aid is indexed to terms of trade shocks, consumption volatility in the recipient country can be reduced by about 10%. This translates into a decrease of 4% on the compensating variation of the cost of business cycle fluctuations. The same level of welfare in the recipient country could be obtained by reducing aid flows by 3% if these were indexed to terms of trade shocks.

The literature overview suggests that aid volatility is a reality, and is problematic. It seems that aid volatility reduces the effectiveness of aid. Aid volatility is a source of uncertainty in developing economies and may additionally be a source of business cycles. The volatility of aid has been demonstrated to be procyclical, implying that it does not seem to act as an insurance mechanism against other forms of economic uncertainty. The present paper will discuss these effects in a general equilibrium framework that differs quite substantially from those currently being used to answer these questions.

3. Theoretical Framework

The model is a forward-looking, dynamic general equilibrium model. There is one utility maximizing agent, and four profit maximizing productive sectors. The four sectors are the primary sector including agriculture, livestock and mining, the secondary sector, which encompasses all manufacturing, and the tertiary sector that embodies most services. The fourth sector, named trade, is the re-exportation sector. The latter activity is excluded from the services sector at large because its responses to policy changes are likely to be very different from the remainder of the services that are not largely traded. The trade service is naturally traded with the rest of the world; it is used by firms as intermediate good and is not used a final demand good.
The infinitely lived representative agent maximizes an intertemporal utility function subject to a budget constraint. The intertemporal utility function is a discounted sum of momentary utility functions that have aggregate consumption and leisure as arguments. The agent’s intertemporal decision determines the quantity of aggregate consumption, investment and labour at each time period, \( t \). Once the aggregate quantities of investment and consumption have been chosen, the intra-temporal decision, at every period, is to determine the composition of each bundle in terms of the quantity of each of the three goods that make up aggregate demand.

In the model, capital is sector specific, leading to a different rate of return on capital in each of the four sectors. Capital supply to each of the four sectors comes from revenue maximization on the part of the capital owner. Additionally, there are capital adjustment costs. This means that the purchase of one unit of investment good translates into less than one unit of productive capital in the next period; there is therefore a loss associated with capital installment. Each of the four sectors produces goods from value added, which is a composite of labour and capital, and intermediate inputs. Additionally the economy is assumed a small open economy. The assumption of small economy in the world market implies that the country is unable to affect the world price of goods through demand or supply shocks. Each good demanded domestically, whether for consumption, investment or intermediate input in the productive process, is a composite of domestically produced and imported goods. A cost minimization principle according to the Armington assumption (Armington, 1969) is used to determine the quantity of each good that will be supplied by the domestic market and the quantity, which will be purchased from abroad. Firms maximize revenue in their decision to supply their goods on the domestic or international markets.

The equations used in the final model are shaded, and included in the Appendix to this paper.

3.1. The intertemporal decision of the utility maximizing agent

The arguments of the intertemporal utility are consumption and leisure. Total time is assumed to be one unit and \((1 - L_t^S)\) is assumed to be the proportion of time spent on leisure activities, where \(L_t^S\) is the labour supply at time \( t \). The representative agent maximizes utility subject to a budget constraint, a capital accumulation constraint subject to adjustment costs.
The budget constraint requires in each period that the sum of expenditures on consumption and investment be equal to total income, which is the sum of labour income, capital income and foreign capital inflows. The capital accumulation constraint describes the motion law of physical capital from one period to the other. The current level of capital stock is equal to the sum of last period’s net investment and capital stock net of depreciation. Investment is subject to convex capital installation costs. Hence, the volume of investment (paid for) in the budget constraint \(JInv\) is larger than the volume \(Inv\) that effectively increases capital stock. We consider a quadratic adjustment cost that has the following expression: 
\[
JInv = \left(\frac{\beta}{2}\right)[Inv_T]^2
\]
where \(\beta\) is a parameter.

The consumer’s intertemporal maximization problem is as follows:

\[
\max_{\{C_t, L_t, Inv_t, K_{t+1}\}} U \{C_t, L_t^S\} = \sum_{t=0}^{\infty} \left(\frac{1}{1+\rho}\right)^t [\ln C_t + \varphi \ln(1 - L_t^S)]
\]
(1a)

s.t.

\[
PC_t \times C_t + PInv_t \times JInv_t = w_t \times L_t^S + R_t \times K_t^S + X_t
\]
(1b)

\[
KTOT_{t+1} = (1 - \delta)KTOT_t + Inv_t
\]
(1c)

\[
JInv_t = Inv_t + \left(\frac{\beta}{2}\right)[(Inv_t)^2]
\]
(1d)

Where \(\left(\frac{1}{1+\rho}\right)^t\) is the discount factor. \(C_t\) is the aggregate consumption good and \(PC_t\) is the price of the aggregate consumption good. \(Inv_T\) is the aggregate investment good and \(PInv_t\) is the price of the aggregate investment good. Both consumption and investment are composites of goods from the three sectors (trade sector excluded). The re-exportation sector is not used for consumption or investment purposes; its only domestic component is the demand for intermediate inputs. \(R_t\) is the aggregate rental rate of capital, which is an index of the rental rates in each of the four sectors. \(KTOT_t\) is the total stock available at time \(t\), augmented by investment, and declining according to depreciation. \(w_t\) is the wage rate, and is the same across sectors. \(X_t\) represents capital inflow that encompasses foreign transfers \(TROW_t\) and foreign aid \(FAID_t\), both of which are exogenous in the current framework. By combining equations (1b) and (1d), the Lagrangian for this problem is:
\[ \mathcal{L} \{ C_t, L_t, Inv_t, K_{t+1} \} \]
\[ = \sum_{t=0}^{\infty} \left( \frac{1}{1 + \rho} \right)^t \left[ \ln C_t + \varphi \ln(1 - L_t^\varphi) \right] \]
\[ + \sum_{t=0}^{\infty} \lambda_t \left[ w_t \times L_t^\varphi + R_t \times K_t^\varphi + X_t - PC_t \times C_t - PLinv_t \times Inv_t + \left( \frac{C}{2} \right)(Inv_t)^2 \right] \]
\[ + \sum_{t=0}^{\infty} \mu_t [ KTOT_{t+1} - (1 - \delta) KTOT_t - Inv_t ] \]

The first order conditions of the maximization are the following:

\[ \frac{\partial \mathcal{L}}{\partial C_t} = 0 \Rightarrow \left( \frac{1}{1 + \rho} \right)^t \frac{1}{C_t} = \lambda_t PC_t \quad (1.1) \]

\[ \frac{\partial \mathcal{L}}{\partial L_t} = 0 \Rightarrow \left( \frac{1}{1 + \rho} \right)^t \frac{\varphi}{1 - L_t^\varphi} = \lambda_t w_t \quad (1.2) \]

\[ \frac{\partial \mathcal{L}}{\partial Inv_t} = 0 \Rightarrow -\lambda_t PLinv_t(1 + \beta Inv_t) = \mu_t \quad (1.3) \]

\[ \frac{\partial \mathcal{L}}{\partial K_{t+1}} = 0 \Rightarrow -\lambda_{t+1} R_{t+1} + \mu_t = (1 - \delta) \mu_{t+1} \quad (1.4) \]

Equation 1.1 and 1.2 state the marginal utility of consumption and leisure respectively. By combining these two equations, we can obtain an equation for the arbitrage condition between consumption and leisure.

\[ \frac{1}{PC_t C_t} = \frac{\varphi}{w_t(1 - L_t^\varphi)} \]

By appropriate substitution, using equations 1.1, 1.2, and 1.4 we can obtain the intertemporal Euler equation. This equation gives the tradeoff of consumption between two consecutive periods and is indicative of the consumer’s willingness to smooth consumption over time.

\[ \frac{C_{t+1}}{C_t} = \left( \frac{1}{1 + \rho} \right)^t \times \frac{PC_t}{PC_{t+1}} \times \left[ \frac{R_{t+1} + PLinv_{t+1}(1 - \delta)(1 + \beta \times Inv_{t+1})}{PLinv_t(1 + \beta \times Inv_t)} \right] \]

A permanent increase in the return to investment will increase the demand for consumption in all periods, because the increased rental rate of capital augments the agent’s
budget constraint. A temporary increase in the return to investment will have a similar, but smaller effect than a permanent increase, since the one-time increase is dispersed over time, reflecting consumption smoothing behaviour. The discount factor, \( \frac{1}{1+\rho} \), in an indication of time preference, the higher is this value, the more impatient the representative agent is, and the higher will be consumption in the current period relative to consumption in future periods.

The budget constraint, and the capital accumulation constraint, as listed in the problem definition also appear in the system of equations, which characterize the optimal solution.

3.1 The intra-temporal decision of the utility maximizing agent

From the intertemporal problem, aggregate quantities of consumption and investment, and capital have been determined. At each time period, \( t \), the agent must decide how to allocate aggregate consumption and investment among the three goods, and how to supply aggregate capital to the three industries. Each aggregator is a constant elasticity of substitution (CES) aggregator, and the agent’s goal is to minimize cost subject to this CES function. The intratemporal problem of the representative agent with respect to consumption is the following:

\[
\begin{align*}
\min_{\{c_t\}} & \quad P_t C_t = \sum_i P_t^i C_t^i \\
\text{s.t.} & \quad C_t = A_c \sum_i (\alpha_t^i C_t^{i-\mu_c})^{\frac{1}{\mu_c}}
\end{align*}
\tag{2a}
\tag{2b}
\]

Where \( C_t^i \) is the consumption of good \( i \), and \( P_t^i \) is the price of good \( i \). \( \alpha_t^c \) is an indication of the weight associated to each good (\( \sum_i \alpha_t^i = 1 \)), and \( \sigma_c = \frac{1}{1+\mu_c} \) is the elasticity of substitution. The Lagrangian for this minimization problem is:

\[
L \{C_t\} = \sum_i P_t^i C_t^i + \lambda_t \left[ A_c \sum_i (\alpha_t^i C_t^{i-\mu_c})^{\frac{1}{\mu_c}} \right]
\]

The marginal rates of substitution between each pair of consumption goods is obtained from the first order conditions and expressed by the following equation:

\[
\frac{c_t^i}{c_t^j} = \left( \frac{\alpha_t^i p_t^j}{\alpha_t^j p_t^i} \right)^{\frac{1}{1+\mu_c}}
\tag{2.1}
\]
From the marginal rates of substitution, and appropriate algebraic manipulation, the following demand for consumption goods and the dual price of consumption are obtained:

\[
C_t = A_c^{\sigma_c-1}C_t \left\{ \frac{\alpha_c \times PC_t}{P_t^l} \right\}^{\sigma_c}
\]

\[
PC_t = \frac{1}{\lambda_c} \left\{ \sum_i \alpha_c^{i \sigma_c} \times P_t^{1-\sigma_c} \right\}^{\frac{1}{1+\sigma_c}}
\]

The cost minimization problem for determining the composition of the aggregate investment good is analogous to that of the aggregate consumption good.

\[
\min_{(Inv_t)} PInv_t lnv_t = \sum_i P_i^l lnv_t^i \tag{2c}
\]

s.t.

\[
lnv_t = A_{inv} \sum_i (\alpha_i^lnv lnv_t^{-\mu_{inv}})^{-1} \tag{2d}
\]

Where \(lnv_t^i\) is the volume of good \(i\) demanded for investment purposes. \(P_i^l\) is the price of good \(i\), and is the same whether the good is used for consumption or investment purposes.

\[
Inv_t = A_{inv}^{\sigma_{lnv}-1} \left\{ \frac{\alpha_{lnv} \times PInv_t}{P_l^i} \right\}^{\sigma_{lnv}} \times flnv_t
\]

\[
PInv_t = \frac{1}{A_{inv}} \left\{ \sum_i \alpha_i^lnv \sigma_{lnv} \times P_t^{1-\sigma_{lnv}} \right\}^{\frac{1}{1+\sigma_{lnv}}}
\]

Capital is supplied to each of the four sectors to maximize its rental rate. We assume that the aggregate capital stock is a constant elasticity of transformation (CET) function of sectoral capital stocks. The optimization problem of the capital owner is the following:

\[
\max_{(K_t^S_j)} R_t KTOT_t = \sum_i R_t^j K_t^S_j \tag{3a}
\]

s.t.

\[
KTOT_t = A_k \left\{ \sum_i \eta_k^j \times K_t^{S_j} \frac{1+\sigma_k}{\frac{\sigma_k}{\frac{1+\sigma_k}{\Sigma}}} \right\}^{\frac{\sigma_k}{1+\sigma_k}}\tag{3b}
\]
Where $K_t^{S_j}$ is the quantity of capital supplied to industry $j$, and $R_t^j$ is the rental rate of capital in industry $j$. $S$ in superscript indicates supply of capital stock that is different from capital demand, which stems from profit maximization that is discussed later.

The Lagrangian of this problem is the following:

$$L\{K_t^j\} = \sum_t R_t^j K_t^{S_j} + \lambda \left[ KTOT_t - A_k \left\{ \sum_l \eta_k^l K_t^{S_j \frac{1+\sigma_k}{\sigma_k}} \right\} \right]$$

The following are the first order conditions for the revenue maximizing capital owner, there are $j$ first order conditions of the same form.

$$\frac{\partial L}{\partial K_t^j} = 0 \Rightarrow R_t^j = \lambda \left[ A_k \left\{ \sum_l \eta_k^l K_t^{S_j \frac{1+\sigma_k}{\sigma_k}} \right\} \right] \frac{1}{K_t^{S_j \frac{1+\sigma_k}{\sigma_k}}} (3.1a)$$

By algebraic manipulation of the first order conditions with respect to $K_t^j$ and $K_t^l$, the following equation can be obtained. This equation states the marginal rate of substitution between supplying capital to different industries at time $t$.

$$\frac{k_t^j}{k_t^l} = \left( \frac{\eta_k^{R_t^j}}{\eta_k^{R_t^l}} \right)^{\sigma_k} \quad (3.1b)$$

Along with the transformation constraint, the marginal rate of substitution can be used to find optimal supply to each industry at time $t$, as well as a dual price for capital. The solution to this optimization problem is stated below; the second expression is the dual price of capital, which is a composite of rental rates in the four sectors.

$$R_t^j = A_k^{-1-\sigma_k} \times KTOT_t \left\{ \frac{R_t^j}{\eta_k^{R_t}} \right\}^{\sigma_k}$$

$$R_T = \frac{1}{A_k} \left[ \sum_l \eta_k^{l-\sigma_k} \frac{1}{R_t^{l+\sigma_k}} \right]^{1+\sigma_k}$$
3.2 Profit maximizing firms

Each of the three productive sectors maximizes profit. The technology is characterized by nested production functions with two levels. At the highest level, final production is a composite of value added and intermediate inputs. At the second level, value added is a composite of labour and capital, while intermediate inputs is a composite of goods from all four sectors. The production process is demonstrated by the hierarchy figure below.

At the highest level, firms maximize profit, subject to a technological constraint. This yields a demand for value added and intermediate inputs. Subsequently, given the amount of value added, and intermediate inputs, firms minimize costs to choose the level of labour, capital, and intermediate inputs from each industry. The following is the set of equations that characterizes the optimization problem of firms.

\[
\max_{\{VA^i_t, Int^i_t\}} \pi_t = P Y_t^j Y_t^j - R_t^j K_t^j - w_t L_t^j
\]  \hspace{1cm} (4a)

s.t.
\[
Y_t^j = A_P^j \left\{ VA_t^{\sigma_P} \times Int_t^{(1-\sigma_P)} \right\}
\]  \hspace{1cm} (4b)

\[
\min_{\{e_t^j, k_t^j\}} PVA_t^j VA_t^j = R_t^j K_t^j + w_t L_t^j
\]  \hspace{1cm} (5a)

s.t.
\[
VA_y^j = A_V^j \left\{ K_t^{\sigma_V} \times L_t^{(1-\sigma_V)} \right\}
\]  \hspace{1cm} (5b)
\[
\begin{align*}
\min_{\{nt_t^{i,j}\}} PInt_t^{i,j} & \quad \text{Int}_t^j = \sum_i P_t^i \text{Int}_t^{i,j} \\
\text{s.t.} & \quad \text{Int}_t^j = \min\left[\frac{\text{Int}_t^{i,j}}{\alpha_{nt}}\right] 
\end{align*}
\] (6a)

Where \( Y_t^j \) is output in industry \( j \), and \( PY_t^j \) is its price. \( VA_t^j \) is value added demanded by industry \( j \), and \( PVA_t^j \) is the price of value added in industry \( j \). \( Int_t^j \) is the quantity of intermediate inputs demanded by industry \( j \) and \( PINT_t^j \) is the price of the aggregate intermediate input of industry \( j \). \( A_p^j \) is an indication of the technology used in final production in industry \( j \) and \( \alpha_p^j \) is the share of value added in total production in industry \( j \). \( A_{VA}^j \), and \( \alpha_{VA}^j \) are analogous for the value added function, \( \alpha_{VA}^j \) is associated with capital. The value of \( a_{nt}^{i,j} \) from the intermediate input function is the amount of input i used to produce one unit of good j.

The derivation of Cobb-Douglas and Leontief functions is common in economics, and thus only the demand equations and the dual prices are stated here without derivation. A superscript of D indicates that it is the demand for the input.

\[
\begin{align*}
PY_t^j &= \frac{1}{A_p^j} \left[ PVA_t^j \right]^{\alpha_p^j} \left[ \frac{PINT_t^j}{(1-\alpha_p^j)} \right]^{1-\alpha_p^j} \\
\text{Int}_t^j &= \frac{(1-\alpha_p^j)PY_t^j}{PINT_t^j} \\
VA_t^j &= \frac{a_{VA}^j \alpha_{VA}^j}{PVA_t^j} \\
PVA_t^j &= \frac{1}{A_{VA}^j} \left[ \frac{RU_t^j}{\alpha_{VA}^j} \right]^{\alpha_{VA}^j} \left[ \frac{W_t}{(1-\alpha_{VA}^j)} \right]^{1-\alpha_{VA}^j} \\
R_t^{D,j} &= \frac{a_{VA}^j PVA_t^j VA_t^j}{RU_t^j} \\
I_t^{D,j} &= \frac{(1-\alpha_p^j)PVA_t^j VA_t^j}{W_t} \\
PInt_t^j &= \sum_{i=1}^3 a_t^{i,j} P_t^i \\
V_t^{i,j} &= a_t^{i,j} \text{Int}_t^j
\end{align*}
\]
3.3. **Relationship with the rest of the world**

The economy is a small open economy. This implies that a proportion of domestically produced goods are sold in the international market, while a fraction of domestically consumed goods is purchased from the international market. There are two types of capital inflow in the mode: foreign savings, and foreign aid. Using the balance of payment equilibrium, the current account balance must be equal to the value of foreign savings, plus the value of other capital inflows and aid. The variable $X$ is used as the aggregate of all capital inflows, as indicated in the definition of the consumer’s intertemporal problem.

$$\sum_t PM_t^i M_t^i - \sum_t PEX_t^i EX_t^i - TROW_t = X_t$$

Where $M_t^i$ is the quantity of imports of good $i$, $PM_t^i$ is the import price of good $i$, $EX_t^i$ is the quantity of exports of good $i$, and $PEX_t^i$ is the export price of good $i$.

Domestic demand for each commodity includes consumption, investment and intermediate inputs, such that the following relationship holds:

$$XTD_t^i = C_t^i + Inv_t + \sum_j V_t^{i,j}$$

Domestically consumed goods are composites of domestic production and international production. We use Armington (1969) to distinguish demand for imports from demand for domestically produced goods. This assumption is useful because it allows cross-hauling, which refers to importing and exporting gross substitutes. Agents minimize cost associated with demand for each good subject to a constant elasticity of substitution constraint, between domestically produced and imported goods. The cost minimization problem is the following:

$$\min_{(XDD_t^i, M_t^i)} P_t^i XTD_t^i = Pd_t^i XDD_t^i + PM_t^i M_t^i$$  \hspace{1cm} (7a)

s.t.

$$XTD_t^i = A_t^i \{\alpha_t^i M_t^{i-\mu M} + (1 - \alpha_t^i) XDD_t^{i-\mu M}\}^{-\frac{1}{\mu}}$$  \hspace{1cm} (7b)

Where $XDD_t^i$ represents domestic demand of good $i$, and $XTD_t^i$ represents total demand of good $i$. $Pd$ indicates the price of the domestic good, $PM$ indicated the price of the imported good, $M$ represents import good demand, and $XDD$ represents demand for domestically produced
goods. The first order conditions, which characterize the cost minimization problem, can be stated by the following equation.

\[
\frac{M_t^M}{XDD_t^M} = \left[ \frac{\alpha_M^t Pd_t^M}{(1-\alpha_M^t) PM_t^M} \right]^{\sigma_M^t} \tag{7.1}
\]

Each good that is consumed by domestic agents is a composite of domestically produced and imported goods implies that the price at time \( t \), the price of that composite good is a composite of the prices of domestically produced and imported goods.

From the first order conditions, the following demand for domestically produced and imported goods, as well as an equation for the dual price of domestic demand can be obtained.

\[
M_t^M = A_t^M \sigma_M^M - 1 XTD_t \left[ \frac{\alpha_M^t P_t^l}{P_{M,t}^l} \right]^{\sigma_M^M}
\]

\[
XDD_t^M = A_t^M \sigma_M^M - 1 XTD_t \left[ \frac{(1-\alpha_M^t) P_t^l}{P_{d,t}^l} \right]^{\sigma_M^M}
\]

\[
P_t^l = \frac{1}{A_t^M} \left[ \alpha_M^t \sigma_M^M PM_t^l - 1 - \sigma_M^M + (1-\alpha_M^t) \sigma_M^M Pd_t^l - \sigma_M^M \right]^{-\frac{1}{\sigma_M^M}}
\]

Each sector supplies goods on the domestic and international markets according to a constant elasticity of transformation function to achieve maximum revenue. The following is an outline of the revenue maximization problem solved by the firm in their decision to supply on the domestic and international markets. Total supply of a good produced by industry \( j \), is indicated as \( Y_t \), as it was in the production equations.

\[
\max_{(EX_t^j, XDS_t^j)} \frac{P_t^j Y_t^j}{Y_t^j} = PEX_t^j EX_t^j + PPD_t^j XDS_t^j \tag{8a}
\]

\[
\text{s.t.} \quad Y_t^j = A_{EX}^j \left\{ \omega_{EX}^j EX_t^j^{\frac{1+\sigma_{EX}}{\sigma_{EX}}} + (1-\omega_{EX}^j) XDS_t^j \right\}^{\frac{\sigma_{EX}}{1+\sigma_{EX}}} \tag{8b}
\]

Where \( PEX \) is the price received on the international market, and \( PD \) is the same as in the previous set of equations, and indicates the price on the domestic market. \( P \) is a composite of the
prices received on the two markets. The marginal rate of substitution, obtained from the first order conditions pertaining to this problem is the following:

$$\frac{EX^j_t}{XDS^j_t} = \left[ \frac{(1-\omega^j_{EX})P_{EX}^j}{\omega^j_{EX}P_{d}^j} \right]^{\sigma^j_{EX}}$$  \hspace{1cm} (8.1)$$

The solution to the revenue optimization results in an equation for export supply, domestic supply, and the dual price of each good $j$.

$$EX^j_t = A^j_{EX}^{-\sigma^j_{EX} - 1} XTS^j_t \left[ \frac{P_{EX}^j}{\omega^j_{EX} P^j_t} \right]^{\sigma^j_{EX}}$$

$$XDS^j_t = A^j_{EX}^{-\sigma^j_{EX} - 1} XTS^j_t \left[ \frac{P^j_t}{(1-\omega^j_{EX}) P^j_t} \right]^{\sigma^j_{EX}}$$

$$P^j_t = \frac{1}{A^j_{EX}} \left[ \omega^j_{EX}^{-\sigma^j_{EX}} P_{EX}^j (1 + \sigma^j_{EX}) + (1-\omega^j_{EX})^{-\sigma^j_{EX}} P_{d}^j (1 + \sigma^j_{EX}) \right]^{1 + \sigma^j_{EX}}$$

3.4. Equilibrium Conditions

A solution to this problem requires that at each time period $t$, all agents are behaving optimally, implying that all first-order conditions (FOCs) are simultaneously respected, and that prices adjust so that all markets are in equilibrium.

The equilibrium conditions pertaining to the model are outlined below. The first indicates that capital supply must equal capital demand for each of the four sectors. Meeting this condition will determine the rental rate of capital for each of the four sectors. The second condition ensures that the sum of labour demanded by the four sectors is equal to the supply of labour. Since labour is perfectly mobile across sectors, there will only be one ensuing wage rate. Finally, total demand for domestically produced goods must equal total supply for domestically produced goods, in each sector.
The final two equilibrium conditions listed ensure that the price of international goods on the domestic market and of domestic goods on the international market properly reflect the exchange rate.

\[ K^I_{s,T} = K^I_{D,T} \]
\[ \sum_j L^I_{D,T} = L_{s,T} \]
\[ XTD^I_T = XTS^I_T \]
\[ PM^T_t = PWM^T_t \times ER_t \]
\[ PEX^T_t = PWEX^T_t \times ER_t \]

3.5. Steady State

The steady state of this economy is one in which dynamic variables remain stable over time. This indicates that the steady state implies that capital stock, and therefore investment is constant over time.

4. Data and Calibration

4.1. Benin at a glance

Benin was colonized by the French, and remained a French colony until 1960 when it gained independence as the Republic of Dahomey. The country was renamed the People’s Republic of Benin in 1975. Benin is currently one of the countries belonging to the West African Monetary and Economy Union (WAMEU), and is one of the original countries belonging to this group.

The World Bank classifies Benin as a very poor country, with a per capita income of US$560 in 2006. As can be seen by Figure 1, GDP growth in Benin over the past two decades has been unpredictable, and at many times, very slow, or even negative.
Benin is a small open economy. Its' main export sector is re-exportation (trade sector), followed by the cotton industry. Re-exportation was the source of 54% of total exports in 2005, and cotton was the source of 19% of exports in 2005.

Benin has received large aid inflows since the post World War II period. Aid inflows have been increasing along a trend, though deviations from this trend have been large and unpredictable. At times, the ratio of aid to GDP was almost 20%. In recent years, this ratio has been closer to 10%. Figure 2 shows the aid to GDP ratio for the period of 1960 – 2005.

4.2. Data

The data used in this study come from various sources. Data regarding the composition of GDP between the four sectors of production, and the current account balance is obtained from the Institut national de la statistique et de l’analyse économique (INSAE). The World Bank is the source of data pertaining to the amount of aid provided, as a percentage of GDP. From the current account balance, the terms of trade data (obtained from exports and imports), and the amount of foreign aid, we were able to calculate other forms of capital inflow. A fifteen sector
SAM, from the INSAE is the source of data pertaining to value added, which encompasses salaries and profit from capital, and intermediate inputs. This is also the source of imports, exports, consumption, and investment by sector. The aggregation of production sectors was required, as the SAM mentioned above was disaggregated into 15 productive sectors. Table 2 presents the social accounting matrix (SAM) used for the calibration of the benchmark scenario.

Table 2: Social Accounting Matrix for Benin, 2003 (millions of FCFA)

<table>
<thead>
<tr>
<th></th>
<th>LABOUR</th>
<th>CAPITAL</th>
<th>AGRIC</th>
<th>MANUF</th>
<th>SERV</th>
<th>TRADE</th>
<th>CONS</th>
<th>AID</th>
<th>ROW</th>
<th>ACCUM</th>
</tr>
</thead>
<tbody>
<tr>
<td>LABOUR</td>
<td></td>
<td></td>
<td>231.7</td>
<td>72.0</td>
<td>273.3</td>
<td>69.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CAPITAL</td>
<td></td>
<td></td>
<td>144.5</td>
<td>75.6</td>
<td>31.3</td>
<td>93.0</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>AGRIC</td>
<td></td>
<td></td>
<td>73.4</td>
<td>146.4</td>
<td>0.0</td>
<td>1.9</td>
<td>290.5</td>
<td></td>
<td></td>
<td>2.4</td>
</tr>
<tr>
<td>MANUF</td>
<td></td>
<td></td>
<td>33.4</td>
<td>235.3</td>
<td>85.8</td>
<td>11.8</td>
<td>243.5</td>
<td></td>
<td></td>
<td>72.9</td>
</tr>
<tr>
<td>SERV</td>
<td></td>
<td></td>
<td>1.3</td>
<td>15.8</td>
<td>64.4</td>
<td>51.8</td>
<td>355.4</td>
<td></td>
<td></td>
<td>21.2</td>
</tr>
<tr>
<td>TRADE</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1.7</td>
<td></td>
<td></td>
<td>167.1</td>
</tr>
<tr>
<td>CONS</td>
<td>646.4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>79.9</td>
<td>96.1</td>
</tr>
<tr>
<td>AID</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>79.9</td>
</tr>
<tr>
<td>ROW</td>
<td>72.3</td>
<td>255.0</td>
<td></td>
<td>10.6</td>
<td></td>
<td>12.5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>ACCUM</td>
<td>277.5</td>
<td>0.0</td>
<td></td>
<td>-131.3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Source: INSAE, World Bank, IMF

4.3. Model Calibration

As the economy is assumed to be in a steady state and the population stock has been normalized to unity, all variables are constant and equal to the value of the benchmark year chosen for calibration (2003). The model calibration consists of determining the parameter values to reproduce the benchmark situation. Some of the parameters must be set exogenously and are often taken from empirical studies or from other studies. The remaining parameters are computed to reproduce the reference case using the FOCs.

The rate of time preference is set to 0.05, and the depreciation of capital is 0.15 to fulfill the steady state condition related to the Euler equation. The amount of available work time is normalized to one, and the proportion of available time devoted to labour is set to 0.7.

Consistent with Dhasmana(2008), estimates from Ostry and Rienhart (1992) pertaining to the intra-temporal elasticity of substitution in the consumption function are used. The value of $\sigma_c$ is assumed to be 1.279, as this is the estimate provided for Africa (Ostry & Reinhart, 1992). We use the same value for the elasticity of substitution in the investment function.
The value for the elasticity of transformation in the capital supply function is taken from Mendoza and Uribe (2001) and is equal to 0.10. The value of the elasticity of substitution in the CES function for import demand and the CET function for export supply vary by sector are taken from Decaluwe et al. (2004). We chose this source because it pertains to the WAMEU region. For the agricultural sector, the elasticities of substitution in both the CES function and the CET function are 1.5. For the manufacturing and services sector, both values are set to 2. For the trade sector, both values are 4. The trade sector is primarily exported, and this is why its’ elasticity of substitution is relatively higher than the other three sectors. Table 3 presents the all of the values imposed on the model.

The model is solved numerically using the GAMS (general algebraic modeling system) software package, and the CONOPT solver and using a 50-year time horizon. Non-reported simulations have shown that this period length is sufficient for the economy to reach a new steady state after an initial shock.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Definition</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Time preference parameter</td>
<td>0.05</td>
</tr>
<tr>
<td>$(1 - L_0)$</td>
<td>Proportion of available time on leisure activities, in benchmark</td>
<td>0.3</td>
</tr>
<tr>
<td>$\beta_y$</td>
<td>Adjustment costs parameter</td>
<td>1.3</td>
</tr>
<tr>
<td>$\sigma_c = \frac{1}{1 + \mu_c}$</td>
<td>Elasticity of substitution in the CES consumption aggregator</td>
<td>1.279</td>
</tr>
<tr>
<td>$\sigma_{inv} = \frac{1}{1 + \mu_{inv}}$</td>
<td>Elasticity of substitution in the CES investment aggregator</td>
<td>1.279</td>
</tr>
<tr>
<td>$\sigma_K$</td>
<td>Elasticity of substitution in CET capital supply function</td>
<td>0.1</td>
</tr>
<tr>
<td>$\sigma_M^A$ and $\sigma_X^A$</td>
<td>Elasticity of substitution in CES Armington function and CET supply function – Agriculture</td>
<td>1.5</td>
</tr>
<tr>
<td>$\sigma_M^M$ and $\sigma_X^M$</td>
<td>Elasticity of substitution in CES Armington function and CET supply function – Manufacturing</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma_M^S$ and $\sigma_X^S$</td>
<td>Elasticity of substitution in CES Armington function, and CET supply function – Services</td>
<td>2</td>
</tr>
<tr>
<td>$\sigma_M^T$ and $\sigma_X^T$</td>
<td>Elasticity of substitution in CES Armington function, and CET supply function – Trade</td>
<td>4</td>
</tr>
</tbody>
</table>

5. Simulation results

In this study, we run two simulations in which we increase permanently the ratio of foreign aid to current GDP, respectively by, 10%, and 15%. In the reference case, we assume the ratio of
foreign aid to GDP to be equal to its 2005 value of 8%. This implies that in the first simulation aid increases to 8.8% of GDP at current value in each of the subsequent periods, in the second simulation, aid increases to 9.2% of GDP at current value. This implies that at each period, aid is a proportion of GDP at current prices. We present results as percentage deviations from the benchmark situation, unless otherwise noted.

Results for the two simulations are presented for each component of GDP, prices of consumption and investment goods, as well as the results for the labour and capital markets. The analysis will also discuss the overall impact on GDP, both at market prices and factor costs, and a brief welfare analysis is presented.

Higher foreign aid inflows directly increase the disposable income of the representative agent. It is thus not surprising that an increase in aid increases the consumption expenditures in all sectors. The largest increase is in the secondary sector, which has the largest proportion of imports. As will be discussed, domestic prices increase, causing the relative price of the secondary sector to decrease. This is because the proportion of imports is high in this sector, and the relative price of imports has decreased, compared with the benchmark situation.

Changes in investment levels (saving) make it possible for the consumer to smooth consumption over time. Increased levels of investment will increase capital stock, which will increase income in subsequent periods. If there is a temporary change in income, this is reflected in the investment level, to smooth the effects of the temporary shock over time. In the current analysis, there is a permanent increase in income. This increase allows a higher level of consumption in every period, and less investment is required to obtain this level of consumption. For this reason, investment decreases in both investment sectors. The decrease is smallest in the secondary sector, because of the relative price effect discussed previously. Results pertaining to investment and consumption by sector are presented table 4. Results pertaining to aggregate levels of consumption and investment are presented in table 8.

It is interesting to note that the ratio of the absolute value of the percentage change in demand for investment goods to percentage change in aid to GDP is a decreasing function of the percentage change in aid to GDP. For example, this ratio is -0.0692 when there is a 10%
increase in the aid to GDP ratio. This value is -0.0689 when there is a 15% increase in the aid to GDP ratio.

Table 4: Percentage Deviations from Benchmark Situation – Consumption and Investment

<table>
<thead>
<tr>
<th>Aid (% of current value GDP)</th>
<th>Period</th>
<th>Primary</th>
<th>Secondary</th>
<th>Tertiary (excluding re-exportation)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation 1: 8.8</td>
<td>Initial</td>
<td>0.68</td>
<td>1.12</td>
<td>0.69</td>
</tr>
<tr>
<td></td>
<td>Long run convergence</td>
<td>0.61</td>
<td>1.03</td>
<td>0.66</td>
</tr>
<tr>
<td>Simulation 2: 9.2</td>
<td>Initial</td>
<td>1.26</td>
<td>1.47</td>
<td>0.99</td>
</tr>
<tr>
<td></td>
<td>Long run convergence</td>
<td>1.08</td>
<td>1.25</td>
<td>0.91</td>
</tr>
<tr>
<td>Investment</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Simulation 1: 8.8</td>
<td>Initial</td>
<td>-0.69</td>
<td>-0.26</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Long run convergence</td>
<td>-0.67</td>
<td>-0.25</td>
<td></td>
</tr>
<tr>
<td>Simulation 2: 9.2</td>
<td>Initial</td>
<td>-1.03</td>
<td>-0.38</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Long run convergence</td>
<td>-1.00</td>
<td>-0.37</td>
<td></td>
</tr>
</tbody>
</table>

The increased aid also causes an increase in the demand for leisure. As income increases and consumption increases, the arbitrage condition between consumption and leisure leads to an increase in the demand for leisure. Consequently, there is a decrease in the supply of labour. The values of the percentage deviations for the benchmark situation are presented in Table 8.

Physical capital is augmented by investment, net of capital adjustment costs. The decreased demand for investment goods causes a decrease in the capital stock in the next period. The following figure demonstrates the dynamics of the percentage change in the capital stock from the benchmark situation. The magnitude of the effect is larger in the case of a larger increase in the aid to GDP ratio, as is the case with demand for investment goods.

Figure 3: Percentage change in capital stock from benchmark situation
The decreased total stock of capital causes decreases in capital supply to each of the three sectors. On the production side of the economy, there is evidence of Dutch Disease. The decrease in production is largest in the trade sector, which is by far the largest export sector. This is because the domestic price of goods increases (discussed below) and therefore factors of production relocate towards this sector to take advantage of higher remuneration in the less traded sectors. An interesting dynamic feature of this model is that while the primary sector's output is initially higher than that in the benchmark situation, the long run convergence is that this sector's output is actually a decrease in production compared to the benchmark situation. In the short term, production in both the primary and tertiary sector increases. The long run convergence however yields an increase in production only in the services sector, the least traded sector. Moreover, as expected, this phenomenon intensifies when the aid to GDP ratio increases by 15% rather than 10%. The decreased capital supply and labour supply explain the long-term decrease in production for most sectors. The remaining labour and capital supply move toward the secondary sector, as discussed, amplifying the effect of decreased production in the other sectors.

<table>
<thead>
<tr>
<th>Aid (% of current GDP)</th>
<th>Period</th>
<th>Primary</th>
<th>Secondary</th>
<th>Tertiary (excluding re-exportation)</th>
<th>Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.8</td>
<td>Initial</td>
<td>0.0304</td>
<td>-0.275</td>
<td>0.2912</td>
<td>-1.149</td>
</tr>
<tr>
<td></td>
<td>Long run convergence</td>
<td>-0.025</td>
<td>-0.323</td>
<td>0.2582</td>
<td>-1.215</td>
</tr>
<tr>
<td>9.2</td>
<td>Initial</td>
<td>0.0458</td>
<td>-0.410</td>
<td>0.437</td>
<td>-1.723</td>
</tr>
<tr>
<td></td>
<td>Long run convergence</td>
<td>-0.0370</td>
<td>-0.482</td>
<td>0.388</td>
<td>-1.821</td>
</tr>
</tbody>
</table>

Overall, the changes in production affect the demand for value added, and intermediate inputs. The demand for capital goods decreases in the long run relative to the benchmark situation in all sectors. The dynamics of the decrease in certain sectors are noteworthy. Initially, the demand for capital increases in the primary, secondary and tertiary sectors, in the long run, however, the demand for capital decreases in all sectors. This is explained by the decrease in capital causing an increase in the relative price of capital, as a factor of production. Labour demand decreases in the primary, secondary and trade sectors. In the tertiary sector, there is a substitution away from capital towards labour. There is, consequently, increased
labour demand in order to increase production, as previously discussed. The qualitative effect is the same whether the aid to GPD ratio increases by 10% or 15%. As aid grows, the quantitative effect is amplified.

The demand for intermediate inputs decreases in the secondary and trade sectors, but increases in the agricultural and services sectors. There is evidence that the latter sectors have substituted value added for intermediate inputs in their production. This is demonstrated by a decline in production, coupled with an increase in demand for intermediate inputs.

The changes on the supply side create changes on both the capital and labour markets, affecting the wage rate and the rental rate of capital. The overall effect of decreased capital supply to all markets is an increase in the rental rate of capital specific to the agricultural, manufacturing and services sector. The only sector where the decreased demand more than offsets the decreased supply, causing a decline in the specific rental rate of capital, is the trade sector. The overall effect is an increase in the dual price of capital. Perfectly mobile labour in the model implies that there is only one wage rate in the economy; otherwise, agents would be forever changing employment to take advantage of higher wage rates. The decreased supply of labour, coupled with decreased demand from all sectors, except the services sector, cause the wage rate to increase, implying that the overall decrease in supply is greater than the overall decrease in demand.

Table 6: Percentage Deviations from Benchmark Situation – Rental Rates and Wage Rate

<table>
<thead>
<tr>
<th>Variable</th>
<th>Aid to GDP ratio (%)</th>
<th>Primary</th>
<th>Secondary</th>
<th>Tertiary</th>
<th>Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_t^I$</td>
<td>8.8%</td>
<td>Initial</td>
<td>0.75</td>
<td>0.26</td>
<td>0.93</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Long run convergence</td>
<td>0.83</td>
<td>0.33</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>9.2%</td>
<td>Initial</td>
<td>1.13</td>
<td>0.38</td>
<td>1.40</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Long run convergence</td>
<td>1.25</td>
<td>0.50</td>
<td>1.51</td>
</tr>
<tr>
<td>$R_t$</td>
<td>8.8%</td>
<td>Initial</td>
<td></td>
<td></td>
<td>0.19</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Long run convergence</td>
<td></td>
<td></td>
<td>0.27</td>
</tr>
<tr>
<td>$w_t$</td>
<td>8.8%</td>
<td>Initial</td>
<td></td>
<td></td>
<td>0.87</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Long run convergence</td>
<td></td>
<td></td>
<td>0.80</td>
</tr>
<tr>
<td></td>
<td>9.2%</td>
<td>Initial</td>
<td></td>
<td></td>
<td>1.31</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Long run convergence</td>
<td></td>
<td></td>
<td>1.20</td>
</tr>
</tbody>
</table>
The increased demand for consumption goods, decreased demand for investment goods, and change in the demand for intermediate inputs by all sectors, causes changes in the price of domestic goods for all sectors. In all sectors, domestic prices increase subsequent to an increase in foreign aid. Sector specific results are presented in table 7.

Exports in all sectors decrease, which is explained by the revenue-maximization process. To maximize revenues, firms prefer to supply more to the domestic market, where there has been an increase in the price level. The opposite effect occurs for imports. Since the domestic price has gone up, agents will prefer to purchase imported goods, since their price is relatively less expensive after the increased foreign aid, while leaving other forms of foreign inflow constant.

Table 7: Percentage Deviations from Benchmark Situation – Domestic Prices

<table>
<thead>
<tr>
<th>Aid to GDP ratio (%)</th>
<th>Primary</th>
<th>Secondary</th>
<th>Tertiary</th>
<th>Trade</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>8.8%</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial</td>
<td>0.855</td>
<td>0.592</td>
<td>0.7394</td>
<td>0.190</td>
</tr>
<tr>
<td>Long run convergence</td>
<td>0.839</td>
<td>0.589</td>
<td>0.697</td>
<td>0.191</td>
</tr>
<tr>
<td><strong>9.2%</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Initial</td>
<td>1.288</td>
<td>0.891</td>
<td>1.114</td>
<td>0.286</td>
</tr>
<tr>
<td>Long run convergence</td>
<td>1.264</td>
<td>0.888</td>
<td>1.050</td>
<td>0.289</td>
</tr>
</tbody>
</table>

The price of final demand goods, an aggregate of the domestic and world price of imports, increases. This increase is smaller than the increase in domestic prices, because the import price component remains constant. The discrepancy between the change in domestic price and aggregate consumer price is thus the smallest for the services sector, where the proportion of imports is very small.

The overall effect is a decrease in GDP at market price, as well as at factor costs. Table 8 presents the results for changes to the aggregates of each component of both calculations of GDP. The results presented are the long run convergence, in percentage deviation, from the benchmark situation.

The result demonstrates the demand side effects of Dutch Disease. The increased aid flow is largely spent on consumption. The increased consumption is at least partially spent on less traded goods, inducing their price increase to be larger than in the tradable sectors. This induces a shift in resources from other sectors to the non tradable sector, whereby instigating a real exchange rate appreciation, a decrease in exports, a loss in competitiveness and finally a decrease in GDP. The effect of Dutch Disease increases with the amount of aid provided.
Table 8: Percentage Deviations from Benchmark Situation – Aggregate Components of GDP

<table>
<thead>
<tr>
<th>Component of GDP</th>
<th>Simulation 1 – Aid to GDP ratio: 8.8%</th>
<th>Simulation 2 – Aid to GDP ratio: 9.2%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Aggregate Consumption</td>
<td>0.75</td>
<td>1.12</td>
</tr>
<tr>
<td>Aggregate Investment</td>
<td>-0.26</td>
<td>-0.38</td>
</tr>
<tr>
<td>Aggregate Exports</td>
<td>-1.45</td>
<td>-2.18</td>
</tr>
<tr>
<td>Aggregate Imports</td>
<td>1.11</td>
<td>1.68</td>
</tr>
<tr>
<td>Labour</td>
<td>-0.24</td>
<td>-0.36</td>
</tr>
<tr>
<td>Capital</td>
<td>-0.15</td>
<td>-0.22</td>
</tr>
</tbody>
</table>

For the calculation of GDP at factor cost, the decrease in both aggregate capital and labour cause a decrease in GDP at factor cost. GDP at factor cost is a feature that is unique to this analysis regarding changes in aid disbursements, as the endogeneity of labour supply is required for this calculation.

Figure 4: Annual GDP at market cost – 2003 prices

Though the analysis of GDP, both at market prices and factor costs suggests that increases in aid disbursements have negative effects on the recipient economy, a welfare analysis provides a different interpretation. We use compensating variation in the welfare analysis. Since the compensating variation is positive, we know that an increase in aid increases the level of welfare of the representative consumer. This is caused by the increase in both arguments of the intertemporal utility function: consumption and leisure.
6. Sensitivity Analysis

The parameters of the model are altered, to verify the robustness of the results. Table 9 displays the results of the model to various changes in parameters. All results presented pertain to long run convergence, in the case of a 10% increase in the aid to GDP ratio.

<table>
<thead>
<tr>
<th>Change of parameter</th>
<th>Consumption</th>
<th>Investment</th>
<th>Exports</th>
<th>Imports</th>
</tr>
</thead>
<tbody>
<tr>
<td>No change</td>
<td>0.75</td>
<td>-0.26</td>
<td>-1.45</td>
<td>1.11</td>
</tr>
<tr>
<td>$\sigma_K$ increased to 2</td>
<td>0.72</td>
<td>-0.37</td>
<td>-1.72</td>
<td>0.82</td>
</tr>
<tr>
<td>$\sigma_C$ and $\sigma_{INV}$ decreased to 0.76</td>
<td>0.75</td>
<td>-0.26</td>
<td>-1.48</td>
<td>1.09</td>
</tr>
<tr>
<td>$\sigma_{M}$ and $\sigma_{I}$ decreased to 0.5 and 0.6</td>
<td>0.75</td>
<td>-0.26</td>
<td>-1.45</td>
<td>1.11</td>
</tr>
</tbody>
</table>

First, the elasticity of substitution of capital is increased. The overall effect of the various components of GDP is a smaller decrease in GDP at market prices. The qualitative effects of the model remain the same.

The second sensitivity analysis pertains to the elasticity of substitution in the CES consumption and investment functions. The value is decreased to 0.76, which is the same as in the study by Arellano et al. The results for the aggregate components of GDP are very similar to those produced by the initial parameter values.

The next change is to reduce the elasticities of substitution for the CES import demand, and CET export supply functions. The elasticities are reduced to 0.5 for the CES function, and 0.6 for the CET function, in accordance with Devarajan and Go (1998). The results demonstrate that the qualitative results of the model are not overly sensitive to these changes.

The sensitivity analysis demonstrates that the overall results of the model are robust to various choices of parameters.

7. Conclusion and Suggestions for Further Research

As discussed in the literature review, aid is unlikely to stimulate growth unless there is a positive impact on investment. Additionally, unless there are supply side effects, Dutch Disease is also likely to be an impediment to growth. In the model, productivity spillovers are not included, and therefore unable to induce supply side effects. In addition, there was no government in the
model and therefore no increase in public capital. Additionally, the unpredicted aid flow actually decreases investment. Thus, there are no supply side effects coming from increased investment.

The overall conclusion is that in our theoretical context, additional unpredicted aid decreases investment and GDP in the economy, while increasing welfare as it allows higher levels of consumption and leisure. In this context, aid is modeled as an entirely untied grant to the recipient economy. There are no restrictions regarding the way that aid is to be utilized. The economy becomes more dependent upon imports relative to domestic production with increased aid. This is because the additional aid causes a real exchange rate appreciation, and a reallocation of resources towards the non-tradable sectors. The demand for tradable goods therefore has to be satisfies by increased imports. This result is consistent with that found in Arellano et al. (2008). The increased overall level of consumption, and decrease in investment following an aid increase is also consistent with previous findings for studies of this type.

The endogeneity of the labour supply demonstrates an additional channel through which increased aid can have negative effects on growth, and positive effects on welfare. This study demonstrates that the dynamic implications associated with endogenous labour supply are important.

Changes could be made to the model, or simulations that could further advance empirical research. Future research should attempt to include endogeneity of technological progress, allowing for the learning by doing effect to take place, and for one of the supply side effects to be incorporated. In addition, it would be interesting to include a government, who receives the aid inflow and invests, at least a portion of it in the accumulation of public capital. If public capital is complementary to private capital in the productive process, then another supply side effect would be captured. These two additions to the model would minimize the effects of Dutch Disease, and perhaps change the results obtained.

Another interesting addition to the model would be to include not only a permanent change in aid, but also various changes in aid over time. To do this, it would be necessary to determine the historic trend of foreign aid, and the deviations from this trend. This would make it possible to include the process followed by foreign aid into the model. During some periods, aid relative to current GDP would increase, while in others there would be negative volatility.
Overall, the research pertaining to aid volatility, including the current paper, demonstrates that aid is highly volatile, and that sudden unexpected changes in aid disbursements have negative economic impacts on recipient economies.
Appendix A: Variables and Parameters

1 - Endogenous Variables

- $P^i_t$: Price of good i
- $PC_t$: Dual price of aggregate consumption at time t
- $C_t$: Aggregate consumption at time t
- $PINV_t$: Dual price of aggregate investment
- $JINV_t$: Total expenditures on investment good, including adjustment costs
- $INV_t$: Investment
- $w_t$: Wage rate
- $L^S_t$: Labour supply
- $R^j_t$: Rental rate of capital in industry j
- $R_t$: Dual price of capital
- $KTOT^S_t$: Capital supply
- $PY^j_t$: Price of output j
- $PVA^j_t$: Dual price of value added in industry j
- $PIN^j_t$: Dual price of intermediate inputs in industry j
- $X_t$: Foreign capital inflows
- $K^{D,j}_t$: Demand for capital goods by industry j
- $L^{D,j}_t$: Labour demand by industry j
- $V^{i,j}_t$: Intermediate input purchased from industry i, by industry j
- $M^i_t$: Import demand of commodity i
- $XDD^i_t$: Domestic demand of
- $P^i_t$: Dual price of good i
\[ Pd_t^i \quad \text{Price of domestic good } i \]
\[ PM_t^i \quad \text{Price of import good } i \]
\[ EX_t^j \quad \text{Supply of export good } j \]
\[ XDS_t^j \quad \text{Domestic supply of good } j \]
\[ PEX_t^j \quad \text{Price of export good } j \]
\[ Pd_t^j \quad \text{Price of domestic good } j \]

2 - Exogenous variables

\[ FAID_t \quad \text{Foreign aid} \]
\[ FCAP_t \quad \text{Other forms of capital inflow} \]
\[ FSAV_t \quad \text{Foreign Savings} \]
\[ PWM_t \quad \text{World Price of Imports} \]
\[ PWEX_t \quad \text{World Price of Exports} \]
\[ ER_t \quad \text{Exchange Rate} \]

3 - Parameters

\[ \rho \quad \text{Time preference parameter} \]
\[ (1 - L_0) \quad \text{Proportion of time allocated to leisure in benchmark} \]
\[ \varphi \quad \text{Importance of leisure in the intertemporal utility function} \]
\[ \beta_\nu \quad \text{Adjustment cost parameter} \]
\[ \sigma_c \quad \text{Elasticity of substitution in CES consumption aggregator} \]
\[ \alpha_c^\nu \quad \text{Share parameter in CES consumption aggregator} \]
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
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</thead>
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<tr>
<td>$A_c$</td>
<td>Shift parameter is CES consumption aggregator</td>
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<td>$\sigma_{inv}$</td>
<td>Elasticity of substitution in CES investment aggregator</td>
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<td>$\alpha_{inv}$</td>
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<td>$A_{inv}$</td>
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<td>$A_k$</td>
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<td>Share of value added in Cobb-Douglas final production of good $j$</td>
</tr>
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<td>$A_p^j$</td>
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<td>$\alpha_{VA}^i$</td>
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<tr>
<td>$\alpha_{int}^{i,j}$</td>
<td>Share parameter in Leontief intermediate inputs of good $i$ to industry $j$</td>
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<tr>
<td>$\sigma_{i}^m$</td>
<td>Elasticity of substitution CES import demand of good $i$</td>
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<tr>
<td>$\alpha_{i}^m$</td>
<td>Share of imports in CES import demand function of good $i$</td>
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<tr>
<td>$\sigma_{X}^i$</td>
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<tr>
<td>$\omega_{X}^i$</td>
<td>Share of exports in CET export supply function</td>
</tr>
<tr>
<td>$A_{X}^i$</td>
<td>Shift parameter in CET export supply function</td>
</tr>
</tbody>
</table>
Appendix B: Equations

\[ PC_t \times C_t + PInvt \times JInvt = w_t \times L_t^S + R_t \times K_t^S + X_t \]  
(1)

\[ KTOT_{t+1} = (1 - \delta)KTOT_t + Invt \]  
(2)

\[ JInvt = Invt + \left( \frac{R_t}{2} \right) (Invt)^2 \]  
(3)

\[ \frac{1}{PC_tC_t} = \frac{\phi}{w_t(1-L_t^S)} \]  
(4)

\[ \frac{C_{t+1}}{C_t} = \left( \frac{1}{1+\rho} \right)^t \times \frac{PC_t}{PC_{t+1}} \times \left[ \frac{R_{t+1} + PInvt_{t+1}(1-\delta)(1+\beta \times Invt_{t+1})}{PInvt(1+\beta \times Invt)} \right] \]  
(5)

\[ C_t^I = A_c^{-1} C_t \left( \frac{\alpha_t^I \times PC_t}{P_t^I} \right)^\sigma_c \]  
(6)

\[ PC_t = \frac{1}{\lambda_C} \left( \sum_i \alpha_t^I \sigma_c \times P_t^{1-\sigma_c} \right)^{\frac{1}{1+\sigma_c}} \]  
(7)

\[ Invt^I = A_{Invt}^{-1} \left( \frac{\alpha_t^I \times PInvt}{P_t^I} \right)^\sigma_{Invt} \times JInvt \]  
(8)

\[ PInvt = \frac{1}{\lambda_{Invt}} \left( \sum_i \alpha_t^I \sigma_{Invt} \times P_t^{1-\sigma_{Invt}} \right)^{\frac{1}{1+\sigma_{Invt}}} \]  
(9)

\[ R_t^I = A_K^{-1-\sigma_K} \times KTOT_t \left( \frac{R_t^I}{\eta_t R_t} \right)^\sigma_K \]  
(10)

\[ R_t = \frac{1}{\lambda_K} \left[ \sum_j \eta_t^j \sigma_K \right]^R_t \]  
(11)

\[ P^j = \frac{1}{\lambda_p} \left[ \frac{\alpha_p^I \times PInvt}{(1-a_p)} \right]^\sigma_p \]  
(12)

\[ Int^I = \frac{(1-a_p^I)P^IY_t^I}{PInvt^I} \]  
(13)

\[ VA_t^I = \frac{\alpha_p^I \times V_t^I}{PV^I} \]  
(14)

\[ PV^I = \frac{1}{\lambda_{VA}^I} \left[ \frac{R_t^I}{\alpha_{VA}^I} \right]^{-1} \]  
(15)
\[ k^D_{t,j} = \frac{a^j_{t,A} PAV_t^i Y_t^i}{r^A_t} \]  
(16)

\[ l^D_{t,j} = \frac{(1-a^j_{t,A}) PAV_t^i Y_t^i}{w_t} \]  
(17)

\[ PInt_t^i = \sum_{i=1}^{3} a^{i,j}_t P_t^i \]  
(18)

\[ V_t^{i,j} = a^{i,j}_t Int_t^i \]  
(19)

\[ \sum_i PM_t^i M_t^i - \sum_i PEX_t^i EX_t^i = FSAV_t + FCAP_t + FAID_t \]  
(20)

\[ M_t^i = A^i M^{i-1}_t XTD_t^i \left[ \frac{a^i_{t,p} p_t^i}{p_t^i} \right]^{a^i_M} \]  
(21)

\[ XDD_t^i = A^i M^{i-1}_t XTD_t^i \left[ \frac{(1-a^i_{t,p}) p_t^i}{p_t^i} \right]^{a^i_M} \]  
(22)

\[ p_t^i = \frac{1}{A^i} \left[ \alpha^i_{t,M} PM_t^{1-\sigma^i_M} + (1 - \alpha^i_{t,M})^\sigma^i_M \right]^{\frac{1}{1-\sigma^i_M}} \]  
(23)

\[ EX_t^i = A^i_{EX} -\sigma^i_{EX} XTS_t^i \left[ \frac{PEX_t^i}{\omega^i_{EX} p_t^i} \right]^{a^i_{EX}} \]  
(24)

\[ XDS_t^j = A^j_{EX} -\sigma^j_{EX} XTS_t^j \left[ \frac{p_t^i}{(1-\omega^j_{EX}) p_t^i} \right]^{a^j_{EX}} \]  
(25)

\[ p_t^j = \frac{1}{A^j_{EX}} \left[ \omega^j_{EX} PEX_t^{1+\sigma^j_{EX}} + (1 - \omega^j_{EX})^{-\sigma^j_{EX}} Pd_t^{1+\sigma^j_{EX}} \right]^{\frac{1}{1+\sigma^j_{EX}}} \]  
(26)

\[ K_{S,T} = K_{D,T} \]  
(27)

\[ \sum_j L_{D,T}^j = L_{S,T} \]  
(28)

\[ XTD_t^j = XTS_t^j \]  
(29)

\[ PM_t^i = PWM_t^i \times ER_t \]  
(30)

\[ PEX_t^i = PWEX_t^i \times ER_t \]  
(31)
Bibliography


*For example.*


