The Impacts of Population Aging on the Real Wage Rates and the Retirement Age: a General Equilibrium Assessment

By

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Abstract

Population aging is expected to create a huge retirement wave and generate a significant impact on labour force growth rate. One factor that may offset the negative effect on labour market is the rising wage rate with the scarcity of labour supply in the context of population aging. In this paper, we investigate the possible impacts of population aging on real wage rates of workers in different age groups with technologies that assume imperfect substitution between young and old workers from the point of view of the firms. We borrow a computable overlapping generations open-economy general equilibrium model to get the simulation results of the possible changes in the wage rates, when the age composition of the labour is modified. We prove the point that with the potential labour becoming scarcer and the size of the older workers becoming relatively larger over time, although population aging increases the wage rate on average because of an eventual slowdown in the labour force, this may not be the case for old workers. Consequently, labour market reaction to population aging may not generate economic incentives for old workers to return to or stick to the workforce. In such a context, there may imply a new trend of early retirement.
1. Introduction

Canada, like other developed countries, is facing a dramatic change in its population. The rapid increase in the proportion of its population that is retiring will create a huge retirement wave after 2011 when the first baby-boom generation reach the age of 65. The drop in fertility and more Canadians in their later phase of the lifecycle are the explanations of population aging. With fewer young people entering the working-age population replacing the tail end of the baby boomers, the demographic shock will have a great impact on a number of socioeconomic factors and pose serious challenges for Canadian economy.

In particular, the demographic shifts are likely to lower the real GDP and discourage the growth of living standards (Börsch-Supan, 2001; Disney, 1996; Fougere, Harvey, Mercenier and Merette, 2005). Population aging with a big drop in the size of working age population may affect not only the average hour worked per person, but also the skill accumulation process, which are important components influencing output per capita. Moreover, population aging would change the composition of consumption demand with increasing proportion of old consumers and impose great amount of adjustment costs on every aspect of social developments. Finally, many previous studies suggest that the scarcity of labour supply due to population aging will drives real wage rate up. According to the Labour Force Survey (LFS), the average age of retirement has declines from about 65 in the 1970s and early 1980s to 61 in 2005, which exacerbates the shortage
of labour supply and pose potential economic lost. The outcome of rising wage rate would be welcome in the context of aging since it can influence workers' retirement decision and the early withdrawal trend. Those in return would somehow offset the threat of labour force scarcity and generate benefits to economic activity. This argument seems reasonable if the assumption of perfect substitution between the young and the old is valid. However, the perfect substitution may not correspond to the reality of the labour market. If the elasticity of substitution between the young and the old is less than infinite, then the impact of aging on the real wage rate may differ across the various age groups of workers. In fact, we may observe an increase in the average wage rate, but an absolute decline in the wage rate offered to older workers since this type of labour will be in abundance relative to other groups in the context of ageing. If so, the retirement age may decline further and the early retirement problem is likely to be aggravated.

In this paper, we investigate the possible impacts of population ageing on real wage rates of workers in different age groups with technologies that assume imperfect substitution between young and old workers from the point of view of the firms. We borrow a computable overlapping generations open-economy general equilibrium model to get the simulation results of the possible changes in the wage rates, when the age composition of the labour is modified. We also look at the net wage rates which are adjusted after taxes and contributions to pensions in the context of aging. We finally analyze the general
equilibrium effects on workers’ participation rates.

This paper is organized as follows. In Section 2, we discuss the possible impacts of population aging on economy and retirement age. We examine the differences between the young and the old and argue that there are imperfect substitutes between different age groups from a labour demand point of view. Section 3 describes the computable general equilibrium model we use for this study. Section 4 reports and discusses the simulation results from three experiments we conduct. Section 5 gives some conclusion remarks.

2. Population Aging and Retirement Age

2.1 Population Aging and its Impacts on Economy

Canada, like other developed countries, is experiencing rapid change in its population. With the huge retirement wave and the drop in fertility, the decreasing working age population may not meet the demand requirements of workforce and lead to relatively slow or stagnant growth in living standards or tremendous adjustment costs.

In Canadian population history, the period of 1946-1965 is marked by an important increase in fertility rates and in the absolute number of births. With 426,000 average number of births per year in that period, the baby-boom cohort represents a significant size of the population. They were about 9.4 million in 2001, represents nearly one-third of the total population of Canada. However, when the baby boom was almost over in
1966, the fertility rate began to drop drastically. Although Canada's total population increased 50% from about 20 million to 30 million in the period of 1966-2001, the population aged 19 and under declined by 8% to 7.7 million. And especially in recent years, Canada experienced a rapid drop in the number of children aged four and under. The 2001 census reports only 1.7 million children in this age group, compared with 1.9 million in 1991. For older people, the population aged 65 and over more than doubled from 1.5 million to 3.9 million from 1966 to 2001 and will increase rapidly after 2011 when the first baby boomers born in 1946 reach 65. With low fertility and a large proportion of old people, Canada is aging rapidly. Additionally, the change in the nation's median age rose from 25.4 years at the end of the baby boom in 1966 to an all-time high of 37.6 years in 2001. Another indicator may be the amount change in working age population aged 15 to 64. The rate of growth of this group is on a declining trend and this group has already overweighed by the subgroup aged 45 to 64. In 2001, people aged 45 to 64 alone reach 7.5 million to account for one-quarter of Canada's total population and are expected to jump an additional 30% to reach about 9.5 million by 2011. With fewer young people entering the working-age population replacing the tail end of the baby boomers and the fact of an aging workforce, it will not be a surprise to see a huge retirement wave and a shortage of labour supply in the Canadian labour market.¹

¹ Data in this paragraph are found in Statistic Canada (2002), 2001 census: Analysis Series, Profile of the Canadian population by age and sex: Canada ages, Cat. 96F0030XIE2001002, Ottawa
Other industrialized countries, such as the United States, countries in EU and Japan, are also undertaking similar demographic shifts. In the United States, the population aged 65 or older was estimated to be 35 million in 2000 and is projected to double over 70 million by 2030. Within the EU, the workforce may begin to shrink and age significantly and the population of workers aged between 50 and 64 will increase by 25% over the next two decades. At the same time, the number of workers aged 20 to 29 will decline by 20%. Japan, a country facing the most serious challenge of an ageing population, has a large proportion of older people aged 65 and older, exceeding 20% of the total population at the end of 2005. The report prepared by the International Policy Coordination Division of Citizenship and Immigration Canada shows fertility in developed countries has decreased below the ratio of 2.1 children per woman, the number sufficient enough to replace present population. These problems of low fertility and population aging may have profound economic and social consequences on those countries.

These demographic shifts are likely to have great impacts on the world economy. One of those effects may even be a slowdown on the growth of living standards, measured by output per capita.

2 Data are found in U.S. Census Bureau.
One of determinants on output per capita is the ratio of the size of working-age population to the total population. In recent decades, this ratio rose dramatically with the baby-boom cohort entering the labour market and made a tremendous contribution to the rising living standard. Figure 1 shows that the ratio and real GDP per capita have the same trends and go upwards together from 1960s to 2000. However, with the first baby-boomers born in 1946 turning 60 years of age this year, this contribution would possibly be reversed and a downward trend of real GDP per capita may be foreseeable. When the year of the first baby-boom generation reaching 65 approaches, the vast numbers of workers heading towards retirement will leave the labour force and the grow of the working-age population will slow dramatically. Population projections for
Canada (2005) predicts that in all of its six scenarios, the proportion of the working-age population, approximately 70% in 2005, would decline steadily in the 2010s and 2020s and drop to about 62% by the early 2030s, then stabilize close to 60%, which levels off to the ratio of 1968 back to the Figure 1. Hence with the slowdown in the supply of labour resulted from low fertility, the big drop of this ratio would potentially drag down the real GDP per capita together and decrease the living standard in the years to come.

The average hour worked per person in the working–age population is also considered as a factor affecting output per capita. Over the past decades, the entry of women in the labour market led to an increase in the average hours worked of working-age population and favored rapid growth in output per capita. From the Labour Force Survey, adult women participation rate rose from 41.3% in 1976 to 61.3% in May 2006.\(^4\) Figure 2 indicates the trend of adult women participation rate and the average hours worked by persons per week in the working–age population in the period of 1976 to 2004. From this figure, we can see the greatly rising trend of adult women participation rate and the average hours worked increases as well. With the inevitable demographic shift, women participation can no longer be a strong contribution of rising average working hours. Therefore, the positive contribution of the rise in average hours worked will have to be replaced to sustain a positive growth rate on output per capita and living standard.

Finally, labour productivity, which mainly depends on the skills held by the workforce, is also an important component influencing output per capita. Generally, there are two broad sources of job skills: skill acquired through school attendance and skill acquired from training while in the workforce. In recent decades, soaring living standard has been largely attributed to the rapid increase in the skill level of labour force through these two sources. The entry of large baby-boom cohorts in the labour market, the increase in the schooling level of successive entry cohorts and more skill acquisition while in the workforce, all combine to contribute to the rising effective labour force. However, the demographic shifts give us several reasons to believe that it will be difficult to sustain such a rate of growth in the effective labour force.
First of all, older workers will represent a larger share of the labour force. The 1998 Adult Education and Training Survey from Statistics Canada (2001) shows a decrease in the job-related training from 30.6% of the 25-34 years age group to 23.7% of the 45-54 years age group and to 8.4% of the 55-64 years age group. Moreover, the average hours of training falls from 272 hours per year for the 25-34 years age group to 106 hours per year for the 45-54 years old age group. In this sense, relatively small share of younger cohorts and greater proportion of older workers in the workforce may result in a negative impact on overall skill accumulation. Unless the education levels of young generation strongly increases in the future decades, the pace of skills accumulation of total workforce might slow as the result of population aging.

Secondly, an older workforce may slow down the introduction of new technologies through "vintage effects" in human capital. Vintage effects are losses of economic value of human capital because of technological changes that make human capital partially or completely obsolete. Older workers may have an older average vintage, high rate of loss to obsolescence, and may not be provided enough opportunities to fresh their knowledge when new technologies are introduced. Baldwin and Peters (2001) provide evidence that training decisions may be part of employers' decision related to introducing innovative technologies. With the difficulties to recoup training investment

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from older workers' shorter remaining work life, employers may be unwilling to offer training to them. Therefore, with an aging workforce and the inadequate training for old workers, the pace of introducing new technologies may slowdown.

Another possible consequence of rising from population aging would be changes in the composition of consumption demand with increasing proportion of old consumers. Younger people consume different baskets of goods and services compared to older individuals. For example, younger households consume more education services for themselves and for their children, whereas older households consume more health services, traveling services, etc. According to Table 1 taken from Fougere, Harvey, Mercenier and Merette (2005, p. 20), which reports the dollar share of public health care and education spending for different age group, the age group 15-24 and 25-34 obtains 26% and 40% respectively of each dollar spent on public education per capita, whereas health care spending is allocated mainly to the older age groups: 27% for age group 65-74 and even 40% for age group 75-84. Therefore, with the aging-induced rising share of old people in total population, the final demand change in the consumption mix may significantly affect the sectoral composition of production in the future decades and consequently influence labour market flows. Table 2 shows the predictions of various sectors’ share output in total GDP from 2000 to 2050. Noticeably, sectors of finance, insurance and real estate, and communication, the sectoral share of health are projected to increase very fast in this period, by nearly 50% from 4.8% to 7% of total GDP. On the
other hand, the share of GDP of manufacturing and utilities, construction, education and wholesaling and retailing is expected to decline.

**Table 1: Public Expenditure on Health and Education per Age Group**

(Dollar share per capita, %)

<table>
<thead>
<tr>
<th>Age-group</th>
<th>15-24</th>
<th>25-34</th>
<th>45-54</th>
<th>35-44</th>
<th>55-64</th>
<th>65-74</th>
<th>75-84</th>
</tr>
</thead>
<tbody>
<tr>
<td>Health</td>
<td>3.0</td>
<td>3.0</td>
<td>7.0</td>
<td>9.0</td>
<td>11.0</td>
<td>27.0</td>
<td>40.0</td>
</tr>
<tr>
<td>Education</td>
<td>26.0</td>
<td>40.0</td>
<td>20.0</td>
<td>7.0</td>
<td>5.0</td>
<td>1.0</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Source: Fougere, Harvey, Mercenier and Merette (2005, p. 20)

**Table 2: Sectoral Impact of Population Ageing**

(Share of Sectoral Output in Total GDP)

<table>
<thead>
<tr>
<th>Sectors\Years</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary</td>
<td>0.034</td>
<td>0.034</td>
<td>0.035</td>
<td>0.035</td>
<td>0.035</td>
<td>0.034</td>
</tr>
<tr>
<td>Manufacturing and Utilities</td>
<td>0.220</td>
<td>0.219</td>
<td>0.218</td>
<td>0.216</td>
<td>0.213</td>
<td>0.208</td>
</tr>
<tr>
<td>Construction</td>
<td>0.156</td>
<td>0.154</td>
<td>0.151</td>
<td>0.148</td>
<td>0.146</td>
<td>0.146</td>
</tr>
<tr>
<td>Transportation</td>
<td>0.021</td>
<td>0.021</td>
<td>0.021</td>
<td>0.021</td>
<td>0.022</td>
<td>0.022</td>
</tr>
<tr>
<td>Communication</td>
<td>0.017</td>
<td>0.018</td>
<td>0.018</td>
<td>0.019</td>
<td>0.019</td>
<td>0.019</td>
</tr>
<tr>
<td>Wholesaling and Retailing</td>
<td>0.096</td>
<td>0.095</td>
<td>0.094</td>
<td>0.092</td>
<td>0.091</td>
<td>0.090</td>
</tr>
<tr>
<td>Finance, Insurance and Real Estate</td>
<td>0.113</td>
<td>0.117</td>
<td>0.120</td>
<td>0.121</td>
<td>0.120</td>
<td>0.116</td>
</tr>
<tr>
<td>Professional Services to Firms</td>
<td>0.050</td>
<td>0.049</td>
<td>0.049</td>
<td>0.048</td>
<td>0.047</td>
<td>0.047</td>
</tr>
<tr>
<td>Computer and other Firm Services</td>
<td>0.033</td>
<td>0.033</td>
<td>0.032</td>
<td>0.032</td>
<td>0.031</td>
<td>0.031</td>
</tr>
<tr>
<td>Public Administration</td>
<td>0.098</td>
<td>0.097</td>
<td>0.097</td>
<td>0.099</td>
<td>0.102</td>
<td>0.106</td>
</tr>
<tr>
<td>Education</td>
<td>0.036</td>
<td>0.034</td>
<td>0.032</td>
<td>0.030</td>
<td>0.030</td>
<td>0.031</td>
</tr>
<tr>
<td>Health</td>
<td>0.048</td>
<td>0.051</td>
<td>0.055</td>
<td>0.060</td>
<td>0.065</td>
<td>0.070</td>
</tr>
<tr>
<td>Accommodation and Leisure Services</td>
<td>0.014</td>
<td>0.014</td>
<td>0.014</td>
<td>0.015</td>
<td>0.015</td>
<td>0.015</td>
</tr>
<tr>
<td>Other Services</td>
<td>0.064</td>
<td>0.064</td>
<td>0.064</td>
<td>0.064</td>
<td>0.065</td>
<td>0.066</td>
</tr>
</tbody>
</table>

Source: Fougere, Harvey, Mercenier and Merette (2005, p. 25)
The current literature assesses the above issue from different perspectives and methodologies. Börsch-Supan (2001) develops a shift-share analysis of the age-characterizing distribution of consumption across various baskets of goods for Germany. Kuhn (2003) takes input-output considerations into account and argues that the change in consumption patterns may also influence the demand for intermediate goods by industries, which is not considered in a shift-share analysis. Mercenier, Mérette and Harvey (2003) develop a computable general equilibrium model (CGE) that is calibrated on Canadian data characterized by regions, sectors and occupations. The model provides a simulation analysis to explore the potential sectoral and occupational implications of possible changes on the composition of final demand in Canadian economy. In short, these aging-induced final demand changes may result in significant sectoral compositional shifts. Although these inter-sectoral and occupational effects may not be as dramatic as the labour supply shock, they still can generate influential transformations to our economy.

In addition to the effect on final demand changes affecting the Canadian economy and the labour market, population and workforce aging will require significant adjustments on every aspect of social developments. Concomitant cost of job mobility is one of them. Kuhn (2003) mentions that these adjustments will happen through a population aging channel, which operates through the composition of final demand and a workforce aging channel. Moreover, the OECD (1998) reports estimates that show that firms are
unwilling to hire older workers relative to younger workers. (pp. 144-145). It implies that higher level of labour mobility among young workers is required to maintain the present level of labour mobility. Correspondingly, costs related to the job movements may raise significantly.

Another related adjustment cost would be the foreseeable change of greatly increased retirement level. The retirement wave has started as the first members of the baby-boom generation reach 60 this year and is expected to continue to grow for the next twenty. Betcherman (1995) and MacKenzie (2003) both show a certain number of industries and occupations, such as health professionals and university professors, have a high concentration of older workers, and these industries and occupations may be vulnerable to the heavy replacement cost brought by the incoming intensive retirement wave. With few younger people entering labour market to replace the great number of baby-boom group nearing retirement, firms may find harder and harder to look for enough workers who have good quality in skills to meet their demand.

Other adjustment cost, like social benefit costs, implies additional labour market cost adjustments. An aging workforce with a greater share of older workers might mean higher burden costs to employers for the reason that older workers cost firms more due to the health problems or others factors. Therefore, the presence of demographic shifts may impose on the Canadian labour market significant social benefit costs.
Moreover, wage changes from the pressures of labour market are likely to be important as a consequence of the population aging. In general, scarcity of labour supply drives wages up. Particularly in the health sector, it is estimated that real wages may have to rise twice as fast as those in other occupations of the economy to prevent possible shortages of labour supply (Fougere, Harvey, Mercenier and Merette, 2005).

2.2 The Possibility of Decrease in Retirement Age

a. Current Retirement Situation and Related Analysis

Population aging may have large impacts on the labour market and socioeconomic development, while the widespread situation of early withdrawal of older workers may exacerbate the problem and become a serious concern.

According to the Labour Force Survey (LFS), the average age of retirement has declines from about 65 in the 1970s and early 1980s to 61 in 2005. Figure 3 shows that the decline situation was more intensive over the recent period 1997-2000, in which 43% of retirees retired before age 60 compared to 29% over the period 1987-1990. The effective age of retirement is significantly different across workers by industry and occupation. Table 3 reports that workers in primary industries tend to retire at a much older age, around age 66. By contrast, the effective age of retirement is at the lowest point, 57, across workers in social sciences, education, government and religion over the period 1996-2001.
Figure 3: Average Retirement Age in Canada

Table 3: Effective Age of Retirement by Occupation, 1996-2001

<table>
<thead>
<tr>
<th>Occupational Group</th>
<th>Average Retirement Age</th>
</tr>
</thead>
<tbody>
<tr>
<td>Management Occupations</td>
<td>60.4</td>
</tr>
<tr>
<td>Business, Finance and Administration</td>
<td>60.3</td>
</tr>
<tr>
<td>Natural and Applied Sciences</td>
<td>60.6</td>
</tr>
<tr>
<td>Health Occupations</td>
<td>61.0</td>
</tr>
<tr>
<td>Social Sciences, Education, Government and Religion</td>
<td>57.4</td>
</tr>
<tr>
<td>Art, Culture, Recreation and Sports</td>
<td>61.8</td>
</tr>
<tr>
<td>Sales and Services</td>
<td>62.0</td>
</tr>
<tr>
<td>Trades, Transport and Equipment Operators</td>
<td>62.4</td>
</tr>
<tr>
<td>Occupations in Primary Industries</td>
<td>66.3</td>
</tr>
<tr>
<td>Processing, Manufacturing and Utilities</td>
<td>61.4</td>
</tr>
</tbody>
</table>

Source: Labour Force Survey, National Occupation Classification Matrix
If current trends of early withdrawals persist, labour supply shortage and economic lost may be exacerbated in the coming years. Recently, a few studies calculate the economic cost of early retirement in terms of “unused production capacity” as proposed by Gruber and Wise (1999). By taking into account wages and incorporating the effect of increased employment on wages, Herbertsson and Orszag (2001) develop a simple framework to measure the unused productive capacity associated with early retirement in OECD countries. Rowe and Nguyen (2003) use the dynamic microsimulation model called LifePaths, calibrated on the Canadian Census, to estimate forgone earnings and evaluate the impact of early retirement. Fougere et al. (2005) work with a computable general equilibrium model with overlapping generations to evaluate the potential forgone benefits of early retirement in Canada in the context of population aging and indicated large economic costs of early retirement. For instance, additional economic benefits of about 3.5% of real per-capita GDP would be generated with the marginal effect of workers working one more year. And economic gains would be achieved equivalent to 12% of real per-capita GDP if average retirement age could reach and maintain at 65. In sum, these results suggest that extending the effective retirement age can significantly offset the consequences of ageing, reduce pressures of labour market and increase the real GDP per capita.

Specifically, many aspects from retaining older workers and postpone the effective retirement age would generate benefits. First, continued employment of older workers
would counteract the effect of decreasing average working hours caused by population aging and improve the level of hours worked per capita. Second, with more old people working in the labour market, the incentives of employers to invest in skills for older workers are likely to rise. And also for the old workers' character of the short time work life horizons, the investment in "risky" skills such as the skills that are the least portable and most vulnerable to obsolescence are preferred by firms, which may provide an opportunity for Canada to specialize in the acquisition and use of these "risky" skills. In addition, older workers are likely to prefer "casual" employment relationship with the employers, like part-time work or part-year work that is more suitable for older generation. Thus, retaining the older workers may require more flexible work environment which will also make older workers less vulnerable to economic shocks. Finally, Kuhn and Sweetman (2003) suggest that with a lower historical level of actual or perceived job security and a greater level of competition in labour market, the current older generation may do a better job than the previous generation. This improvement may contribute to the growth of labour productivity and to the increase in GDP with the retirement delay of older workers.

b. Factors Affecting the Retirement Decision from Labour Supply and Demand

As we have discussed, earlier retirement behaviour may aggravate labour supply shortage and generate larger economic costs in the context of ageing. Thus what factors affect retirement decisions become major concerns in the context of population aging. A large
body of research has described possible explanations for early retirement from the perspective of labour supply and demand.

On the labour supply side, wealth plays an important role in the retirement decision of older workers. Tompa (1999) uses longitudinal Canadian income tax data and proves that higher lifetime earnings increase the probability of early CPP/QPP take-up. Similarly, Burtless and Quinn (2001) find that the long-term increase in wealth in the U.S.A. is the main factor contributing to the decline of the retirement age. Moreover the wealth received after retirement, which is mainly composed of income security programs, defined-benefits versus defined-contributions employer pension and private pension plans, are important sources of incentives for early retirement. Gustman, Mitchell and Steinmeier (1994) provide evidence that workers tend to retire earlier in presence of generous pension plans. In other cases, old workers were offered sufficient incentives by early retirement programs and tend to leave the labour force at an earlier age (Hogarth, 1988; Lumsdaine, Stock and Wise, 1990).

Among other factors, an unexpected acceleration of the rate of technological change may lead to earlier retirement because of vintage effect and the possible uneasy learning new technology. Moreover, early retirement decision can also be explained by job loss, disability, family-related event, and slim probabilities of re-employment after displacement. In particular, for example, Tompa (1999) discovers that having a retired
spouse increases the probability of early CPP/QPP take-up. Some studies in the United States (Gorey Rice and Brice, 1992, Moen et al., 1994) show that particularly for women, taking care of family members may reduce labour force participation. On displacement issue, Ruhm (1991), Stevens (1997) and Chan and Stevens (2001) provide evidences that earnings recovery may be slowed and incomplete, especially for older workers. Arulampalam (2001) and Gregory and Jukes (2001) suggest that skill atrophy during long unemployment spells can be larger and substantial for older workers. Finally, a strong increase in the preference for leisure as we get older may have an impact on the timing of retirement. Therefore, it may be reasonable to assume that these individuals affected by those labour supply factors would quit earlier from the workforce.

On the labour demand side, the willingness of employers to retain or hire old workers may be one of major factors affecting old people’s early withdrawal from the workforce. Lazear’s (1979) model suggests that workers are paid below their marginal productivity and compensated by above marginal-productivity earnings in their later years. Therefore, firms may have incentives to prefer early retirement to dismissals if the gap between the actual wage rate and the marginal product of older workers keeps rising because of seniority. On the other hand, the reluctance of employers to hire old people due to their heavier benefit cost or possibility of unrecoverable investment (in job match; in training, etc.) may induce old workers to leave labour market earlier when they are displaced from the former job. It is not surprised to see labour force exit after an
unsuccessful period of job search for old people. Consequently, to achieve the goal of retaining more old workers in the workforce and increasing average retirement age, policy makers face great challenge in setting up effective policies.

c. Self-Solving Mechanism under the Assumption of Perfect Substitution between the Young and the Old Workers

If the assumption of perfect substitution between the young and the old is valid, it may not be necessary to implement new policies to increase the average retirement age. The market may give the signals by increasing wage rates and hence solve the early retirement problem by its own mechanism.

The scarcity of labour supply due to population aging and early retirement may imply an increase in future wages. Using a simple model with homogeneous labour and capital, Auerbach and Kotlikoff (1987) and Hamermesh (1993) demonstrate that an increase in wages and a decline in the return to capital may occur due to the relative scarcity of labour caused by population ageing. Similarly, Baker, Gruber and Milligan (2003) using administrative data for Canada find that older workers tend to delay their retirement if there are large financial incentives attached to additional years of work. Under the scenario of perfect substitution between the young and the old, employers are indifferent between old or young workers. Also as wealth accumulated during the working life plays an important role in the retirement decision of older workers, we may get the
conclusion that the rising wage may encourage older worker to stay or return in the workforce. In sum, with the assumption of perfect substitution between the young and the old, retirement decision may respond automatically to the market signals and average retirement age may increase consequently.

2.3 Differences between the Young and the Old

Under the perfect substitution assumption between the young and the old, the market mechanism may solve the early retirement problem by itself. However, the perfect substitution may not correspond to the reality of the labour market.

Although the current literature points out that there is little direct evidence indicating much difference between the young and the old, such as in the aspects of workers’ productivity or cognitive skills, etc., a large number of studies suggest that the young and old workers are quite different.

First of all, physiology and psychology studies show that a lot differences in physical and mental performance with people aging. As the examination of physical aspects, human physical performance declines after a certain peak age with the reducing of muscular power or strength. Moreover, older workers are more likely to suffer poor health and disability (OECD, 1998, p. 136).
Considering mental performance, over the past half century, psychologists collected an astonishing mass of experimental evidence of age-related declines across a very wide range of mental tasks, including memory, problem solving, cognition, concentration and spatial orientation. For example, the "working memory" required for multitasking, which is asked to store and process information simultaneously, is not likely to be an advantage for old workers (Hoyer, 2001). In contrast, psychology points out that the performance of certain types of tasks, like those that require softer personal skills such as wisdom and leadership, may survive in the aging challenge better than those that require multitasking and the rapid assimilation of new information. That is to say old workers have a comparative advantage in wisdom-intensive jobs. In addition, verbal knowledge and certain numerical abilities may be relatively stable in the aging process. In this aspect, Figure 4, taken from "classic aging pattern" of Papalia et al’s (2002, p. 201), shows that scores on verbal psychological test hold up much better than scores on "performance" tests with age. Finally, as psychologists’ Lifespan Theories of Cognitive Development (LTCD’s; see Lindenberger 2001) indicated, the stock of acquired knowledge ("crystallized intelligence") would continue to expand as long as some learning obtained with age even though his or her capacity to acquire new knowledge ("fluid intelligence") declines. In sum, as the research suggests above, the young and the old are fairly different from physiology and psychology sides.
In the view of skill differences, we first consider the skill type. As for the earlier discussion, younger workers may acquire greater portion of skills from previous school education, whereas older workers may obtain most of their skills from training while in the workforce. With regard to the different weight on theoretical education and practical instruction between schooling and work training, skills acquired through schooling and skills gained through training while in the workforce may have qualitative differences. Hence from the skill type's point of view, younger workers with relatively more “schooling-type” skills differ from older workers with more “training-type” skills.

Other than the dissimilarity in skill types, young workers may have lower rate of skill
loss to obsolescence than old workers. These vintage effects are likely to affect employers’ willingness to retain or hire older workers. Hamermesh (2001) finds that skills may have significant effects on the substitutability of workers in various age groups.

Additionally, there is maybe different training cost between the young workers and the old workers. Bartel and Lichtenberg (1987) use a model to exam “trainability” and to see what the differences in costs when firms provide training to workers with certain characteristics to gain the same increment of skill. To seek whether for young workers it is easier to learn some skills than for old workers could be a direct way to exam “trainability”. In particular, Borghans and ter Weel (2002) explore the question whether old workers have more difficulty learning to use a computer. They use the data of the Skills Survey of the Employed British Workforce and find out that the age group of 20–29 has less trouble learning than any other age groups. Thus, age may be a relevant factor resulting different cost of training inputs.

Moreover, old workers have shorter periods contributing to the work at their point of life, whereas relatively more time left to work for young workers. That seems to be a natural factor for employers to make hiring decisions between the young and the old. Also future expectations from work, intended goals for further development, cognition of work meanings, and working attitudes may vary considerably among young and old workers.
2.4 Consequences of Imperfect Substitution

From the existing research and the above discussions about the differences between the young and the old, we have reasons to believe that the substitution between workers young and old is not perfect in the labour market.

According to Hamermesh (2001), the evidence indicates that there is substantial imperfect substitution between workers at different ages. Unlike Mérette (2002), whose studies are based on the assumption of perfect substitutes for age groups, Card and Lemieux (2001) build a general framework in which different age and education groups are modeled upon imperfect substitutions for each other. They find that, in Canada, the U.S. and the U.K., the elasticity of substitution is about 4-5 range for different age groups. Wasmer (2001) finds an elasticity of substitution around 0.5 among age groups with different experience and knowledge. This 0.5 number means that the substitution between young and old workers is very low.

As we discussed above, the scarcity of labour due to population aging may lead to rapid rise in wages. Consequently, old workers may be willing to stay or go back to workforce and the problem of early retirement will disappear. However, if the elasticity of substitution between the young and the old is less than infinite, then the net result would be less clear. Thus, it is crucial to understand that, under imperfect substitution, what will be the consequences of population aging on the wages may differ across
workers of different ages. Disney (1996, p.159) shows that in the case of demographic changes, a non-infinite elasticity of substitution between old and young workers may have significant impacts on wage differences across age groups. Similarly, the model of Card and Lemieux (2001) can be used to estimate the relative wage changes of all age and education groups in front of the shifts of age composition.

Under imperfect substitution between old and young workers, the labour demand becomes specific to the various age groups of workers. Hence the impact of population ageing on the wage rates may differ across workers of different age groups. It should be expect that with the upcoming demographic changes, the share of young (old) workers with respect to the total labour force will decline (increase). Hence the wage rate changes of a specific age group of workers will depend on the impact of population ageing on total labour force, but also on the relative share with respect to the total labour force of this specific age group. Hamermesh (2001) mentions the possibility of a wage rise of prime-age workers accompanied by a decreasing wage for older workers. If so, the retirement age may decline further and the early retirement problem is likely to be exacerbated.

3. The Computable General Equilibrium Model

In order to conduct some simulations experiments we borrow the computable general equilibrium model developed by Fougere, Harvey, Mercenier and Mérette (2005).
this model, workers of different age groups are considered perfect substitute by firms.

Another important assumption in this model is the endogenous character of labour supply, so that the analysis of the labour supply reaction in the demographic change becomes possible. We modify slightly the Fougere et al. (2005) model by making labour demand a constant elasticity of substitution (CES) function of workers of different age groups.

The computable overlapping generations model with endogenous labour supply was developed to analyze numerical property of the issue on population aging in the perspective of imperfect substitutability across workers of seven different age groups. Assuming an open economy background, the rest-of-the-world is specified by two reduced form equations, representing trade balance and importation demand. The small open economy produces a particular good which is an imperfectly substitute to the one manufactured by the rest-of-the-world. The model only considers the adult activity, so the adult life span of a typical age group is 70 years from the age 15 to 84 and divided by seven periods of 10 years. In other words, the youngest individuals refer to the members of the 15-24 age group, while the oldest cohort are those people among the 75-84 age group. The way to divide those generations into seven groups (15-24, 25-34, 35-44, 45-54, 55-64, 65-74, 65-84) realistically reflects the potentially imperfect substitution across workers of different age groups.

\( a. \) Preliminaries on sets and indices

27
In each period (time is indexed by \( t \)), the last generation denoted by \( gr \) (\( g = g7584 \)) fully retires and does not work, so the potential labour is available from the first six imperfectly substitute age groups of workers denoted by \( gw \) (\( gw = g1524, g2534, g3544, g4554, g5564, g6574 \)). We first discuss the household problem, followed by the manufacturer problem, the rest-of-the-world, the government constraint, country's foreign trade and the equilibrium conditions.

**b. Household \( g \) at time \( t \)**

Households are specified as in an Allais-Samuelson overlapping generations framework, so the model is developed on the basis of the life-cycle theory of savings. Each adult individual joins the labour force at age 15 with perfect foresight and works for sequential six periods and fully retires at age 75, then dies at age 84. In each period, a new generation enters the workforce and the eldest generation \( gr \) dies. That is to say there are always seven generations alive at any point of time. Assumed to be fully dependent on their parents, younger children below 15 have no extra burden or benefits to their parents. A household’s problem is maximizing an inter-temporal utility function of consumption and leisure subject to a lifetime budget constraint (see equation 6 below).

The time-separable utility function takes the following constant-elasticity-of-substitution (CES) function form:

\[
U_t = \frac{1}{1-\theta} \sum_{g=1}^{7} \left( \frac{1}{1+\rho} \right)^g \left( Cons_{t+g-1,g}^{1-\varepsilon} + \alpha_g Leis_{t+g-1,g}^{1-\varepsilon} \right)^{1-\varepsilon} \quad \theta > 0, \alpha_g > 0,
\]
where $\rho$ symbolizes the pure rate of time preference, $\theta$ refers to the inverse of the inter-temporal elasticity of substitution $\sigma$ ($\sigma=1/\theta$), $\varepsilon$ the inverse of the intra-temporal elasticity of substitution $\eta$ ($\eta=1/\varepsilon$), and $\alpha_g$ is a parameter representing the intensity of household preferences for leisure relative to consumption. The terms $Cons_{t,g}$ and $Leis_{t,g}$ respectively denote consumption and leisure of an member of age group $g$ at time $t$ and the value of leisure ranges from 0 to 1. Here it is easy to understand that $(1-Leis)$ represents the participation rate in the workforce since $Leis$ equals the portion of time distributed to leisure activity. Equation (1) indicates that the total welfare of one person is a weighted sum of seven periods of leisure and consumption from age group 15-24 at period $t$ to age group 75-84 at $t+6$. With the addition of leisure into the utility function and the previous assumption of endogenous household’s labour supply, we can investigate the reaction of labour supply, particularly the supply of older workers facing the upcoming demographic shift.

Assuming perfect capital markets and no borrowing constraints, the present value of the household wealth is the discounted sum of lifetime labour income $LivInc_{g,t}$ minus taxes but including public old-age pensions $Pens_{g,t}$:

\[ W_t = \sum_{g=1}^{7} \left[ \prod_{g=1}^{7} \left( \frac{1 + R_t}{1 + R_{t+g-1}(1 - \tau_K)} \right) \right] \left[ LivInc_{t+g-1,k} \left(1 - \tau_{t+g-1} - contr_{t+g-1}\right) + \left(Pens_{t+g-1,g}\right) \right] , \]

where $R_t$ represents the rate of interest, $\tau_K$ is effective tax rates on capital and $\tau^*$ is the tax rates on labour income, $contr$ denotes the contribution rate to the pay-as-you-go
pension system.

The defined labour income earned by working age group \( gw \) at period \( t \) and pension benefits are expressed in equation (3) and (4) respectively:

\[
\begin{align*}
(3) & \quad LifInc_{t,gw} = \text{wage}_{t,gw}(1 - \text{Leis}_{t,gw}) \\
(4) & \quad Pens_{t,gr} = \text{PensR} \sum_{gw} LifInc_{t,gr+gw,gw},
\end{align*}
\]

where \( \text{wage}_{t,gw} \) is labour income, and retirees' pension benefits are the product of pension replacement rate \( \text{PensR} \) and labour income of the whole working life.

Substituting equation (3) and (4) into (2), we can see that the choices on leisure and consequently on labour market participation have significant effects on the amount of the future pensions and the household wealth. Through differentiating the household's utility function with respect to its lifetime budget constraint, we get a reservation wage that takes the future pension revenue into account and also considers the possibility that the proportion of time allocated to leisure may equal to upper bound 1. The reservation wage \( \text{ReWage} \) for each generation is written as:

\[
\begin{align*}
(5) & \quad \text{ReWage}_{t,gw} = \text{wage}_{t,gr}(1 - \tau_t - \text{contr}_t) + \prod_{i-t+gw}^{t+gr} \left( \frac{1}{1 + R_s} \right) \text{PensR} \cdot \text{wage}_{t,gw} + \lambda_{t,gw},
\end{align*}
\]

where \( \lambda_{t,gw} \) is a kuhn-tucker multiplier and will not equal to zero if the individual chooses to retire in period \( t \). This multiplier represents the extra money the individual would like to acquire in order not to retire from the workforce and supply a positive
amount of labour. The contribution rate contr, presented in equation (5) implies the assumption that workers consider social security contributions as marginal taxes, which provide no additional benefits in return for additional taxes paid.

The lifetime budget constraint now can be written as the following:

\[ W_t = \sum_{s=1}^{T} \left( \prod_{g=1}^{g} \left( \frac{1 + R_t}{1 + R_{t+g} (1 - \tau^K)} \right) \right) CP_{t+g-1} \left(1 + \tau_{t+g-1}^{\text{Cons}} \right) \text{Cons}_{s} \]

where \( \tau^{\text{Cons}} \) represents effective tax rate on consumption.

Before we state the household first order conditions, here we define a term as \( Z_{t, gw} \) to avoid the messy look of the one of the following first order conditions:

\[ Z_{t, gw} = \left[ 1 + \alpha_g R^{\text{ReWage}_{t, gw}} \left( \frac{1}{1 + \tau^{\text{Cons}} CP_t} \right)^{1-\eta} \right] \]

where \( CP_t \) refers to the composite price of output produced in Canada and in the rest-of-the-world. Now the intra-temporal and inter-temporal first order conditions of the household problem are generated as the following forms:

\[ \text{Leis}_{t, g} = \left[ \frac{\alpha_g (1 + \tau^{\text{Cons}} CP_t )^{\eta}}{\text{ReWage}_{t, g}} \right] \text{Cons}_{t, g} \]

\[ \text{Cons}_{t+1, g+1} = \left[ \frac{1 + R_{t+1} (1 - \tau^K) (1 + \tau^{\text{Cons}} CP_{t+1} )^{\eta/\gamma}}{(1 + \rho)(1 + \tau^{\text{Cons}} CP_{t+1} )} \right] \left( \frac{Z_{t+1, g+1}}{Z_{t, g}} \right)^{\eta - \sigma} \text{Cons}_{t, g} \]

Equation (8) suggests that a decline in leisure relative to consumption can be caused by an increase in the reservation wage \( \text{ReWage} \). Actually, with a change in the reservation
wage, the percentage change in the ratio of leisure-consumption \((Leis/Cons)\) equals to the intra-temporal elasticity of substitution \(\eta\). For equation (9), we can see that the intra- and inter-temporal effects counteract each other. To be more precise, if the \(ReWage\) increases over time, the term \(Z\) will increase respectively and also the ratio \((Leis/Cons)\) has a tendency to decline caused by the intra-temporal effect as mentioned in equation (8). On the other hand, from time to time, households begin to shift more weight to leisure by decreasing labour supply, which may be relatively high at households’ earlier years since they are likely to supply more labour in their early years to take advantage of the higher accumulated wage. Thus leisure increases over time. Therefore, because of the offset factor, the net impact on the pattern of consumption over the lifecycle \((Cong+I / Cong)\) is unclear. There are three possibilities in this case:

(a) if the intra-temporal effect dominates \((\eta > \sigma)\) when wage grows, equation (9) implies that consumption grows faster;

(b) in contrast, consumption grows more slowly when intra-temporal elasticity of substitution is less than the inter-temporal elasticity of substitution \((\eta < \sigma)\);

(c) additionally, if the intra-temporal elasticity of substitution equals to the inter-temporal elasticity substitution \((\eta = \sigma)\) or labour supply is exogenous \((\alpha_g = 0)\), then equation (9) has another standard form:

\[
(9') \quad Con_{i+1, g+1} = \left[ \frac{1 + R_{i+1} (1 - \tau_i^k) (1 + \tau_i^{Cons} CP_i)}{(1 + \rho) (1 + \tau_i^{Cons} CP_i)} \right]^{\sigma} Cons_{i, g} ,
\]

where the special case equation \((9')\) states that in the condition that the price of
consumption and the consumption tax rate stay constants, the consumption will increase over the life cycle if the interest rate net of capital taxes \( R(1-\tau^K) \) exceeds the pure rate of time preference \( \rho \). And also the increase rate depends on the inter-temporal elasticity of substitution \( \sigma \). Unlike the general case (9) which includes wage income on the pattern of consumption through inter- and intra-temporal substitution effects, the special case equation (9') does not involve the impact of wage income on the pattern of consumption over time since wage income does not influence the growth rate of consumption over the life cycle but does have impact on the consumption level through the budget constraint.

The pattern of consumption over the life cycle determines what amount of saving the households generate through the budget constraint. Households invest in physical capital, in bonds issued by firms and by the government, as discussed below.

c. Producers in Canada at time \( t \)

For producers’ problem, inputs including capital services \( K_{it}^{dem} \) and labour services \( L_{it}^{dem} \) distinguished by the various age groups of workers \( L_{t,g} \). Factors capital and labour are needed to produce the unique national good in quantity \( Q_t \). Capital services and labour services are bought by paying the capital rental rate \( R_{nt} R_t \) and wage rate for the six age groups of workers \( wage_{t,g} \) at the market prices. The aggregate price of labour is denoted by \( WLDem_t \). Producers solve the following problem by minimizing the cost function:
subject to the following constraints (11) that represent the technology of the firm. Here we normalize the price of output produced in Canada $P_t^{Can}$ to one for simplicity, so $P_t^{Can}$ can be omitted in equation (11):

$$Q_t = ScP \cdot K_t^{density} \cdot L_t^{density},$$

where equation (11) is the national production function taking a Cobb-Douglas form, using the capital input and labour input and specified with the scaling parameter $ScP$ and expenditure shares $\alpha$.

There is not much evidence that the age-composition of workers differs across sectors and across skills, so the assumption of a single production sector and a single skill for labour may be appropriate. Beckstead and Vinodrai (2003) point out that in the period of 1971 to 1996, the emergence and the growth of occupations linked to knowledge-based economy were widespread across regions and sectors of countries like Canada. Therefore, the well-believed saying that young workers are concentrated in the high-technology sectors may be not correct even though they are mostly hired by these occupations for their more connections to new technologies.

The first-order conditions from minimizing the producers’ problem subject to the two constraints are as the forms below:
(12) \( RentR_t \cdot K_i^{dem} = \alpha \cdot ScP \cdot Q_t \),
(13) \( WLDem_t \cdot L_t^{dem} = (1 - \alpha) \cdot ScP \cdot Q_t \).

Another producers’ problem can be expressed as the following:
(14) Minimize \( \sum_{gw} wage_{i,gw} \cdot L_{i,gw} \),

and the corresponding constraint is:
(15) \( L_t^{dem} = \left( \sum_{gw} \alpha p_{gw} \cdot L_{i,gw}^{n} \right)^{\frac{1}{\phi}} \),

where the labour demand function based on the impact substitution among workers of different age group of workers \( L_{i,g} \), is a CES function parameterized by share parameters \( \alpha p \) and an elasticity of substitution \( \sigma^{Ldem} = \frac{1}{1 - \phi} \).

Through minimizing the labour costs subject to the labour constraint, we can derive the following relation as equation (16) and (17) (see derivations in Appendix):
(16) \( L_{i,gw} = \alpha p_{gw} \left( \frac{WLDem_t}{wage_{i,gw}} \right)^{\sigma^{Ldem}} \cdot L_t^{dem} \)
(17) \( WLDem_t^{(1 - \sigma^{Ldem})} = \sum_{gw} \alpha p_{gw} \cdot wage_{i,gw}^{(1 - \sigma^{Ldem})} \),

where Equation (16) states that if its price (\( wage_{i,gw} \)) declines relative to the aggregate price of labour (\( WLDem_t \)), labour demand for age group \( gw \) increases with respect to total labour demand. Also the percentage change in the ratio \( \frac{L_{i,gw}}{L_t^{dem}} \) with respect to a change
in \( \frac{W_{dem_t}}{wage_{i,g}} \) depends on \( \sigma^{dem} \). Finally, for the relation of the aggregate price of labour \( W_{dem_t} \) and different age groups of workers \( wage_{i,g} \), it is not difficult to understand that the aggregate price of labour \( W_{dem_t} \) equals to a non-linear weighted sum of wages across age groups \( wage_{i,gw} \).

\( d. \) The rest of the world at time \( t \)

The rest-of-the-world described in a reduced form is introduced here to close the model; its prices and income are treated as exogenous and hold constant. Its demand \( D \) for our small economy good is based on the region’s sectoral competitiveness:

\[
D_{can,row,t} = ScP^D_{can,row} \left( \frac{P^row_{t}}{P^can_{t}} \right)^{\eta_{row}},
\]

where \( P^row_{t} \) is the price of output produced in the rest-of-the-world. Considered the description of the reduced form and assumed the rest-of-the-world neither borrows nor lends internationally, we impose that its trade account is as always in balanced at all \( t \):

\[
P^{can}_{t} \cdot D_{can,row,t} = P^{row}_{t} \cdot D_{row,can,t}.
\]

\( e. \) The government at time \( t \)

The revenue of the central government comes from the taxes on labour and capital incomes, as well as on consumption expenditures. Government consumption \( Gov_t \) and interest payments on the debt constitute government spending. When central government runs a deficit, tax revenues come short of expenditures, the government
issues new bonds, denoted as $Bond$, to satisfy its budget constraint. The following is the budget constraint of the government:

$$ (20) \ CP_t \cdot Bond_{t+1} + \sum_{g} \ Pop_{g,t} \left[ \pi_t \left( LifInc_{g,t} + Pens_{g,t} \right) + \tau_t^{Con} \cdot CP_t \cdot Cons_{t,g} + \tau_t^{K} \cdot R_{t-1} \cdot CP_{t-1} \cdot Bij_{t,g} + \tau_t^{K} \cdot R_{t-1} \cdot CP_{t-1} \cdot Kij_{t,g} \right] = CP_t \cdot Gov_t + (1 + R_t) \ CP_{t-1} Bond_t,$$

where $Pop_{g,t}$ represents the number of people living by age group $g$ at period $t$. The population growth rate is assumed as exogenous. The terms $Kij_{t,g}$ and $Bij_{t,g}$ are the investment in physical capital and bonds respectively. Additionally, pay-as-you-go pension benefits are obtained by contribution rates on wage earnings:

$$ (21) \ \sum_{gr} Pop_{t,gr} \cdot Pens_{t,gr} = contr \sum_{gw} Pop_{t,gw} \cdot LifInc_{t,gw}.$$  

\textit{f. Country's foreign trade in goods at time $t$}

Adding-up all individual demands, we get the aggregate demand for goods:

$$ (22) \ \sum_{g} Pop_{t,g} \cdot Cons_{t,g} + Inv_t + Gov_t.$$  

This demand can be allocated between domestic and the rest-of-the-world using the traditional Armington assumption. Goods of different regions are assumed differentiated in demand by their geographic origin although individual producers are treated as price takers. Therefore, a national importer using a CES ($D_{ii,j;i} ; \alpha_{ii,j} ^{D} , \sigma_{j} ^{D}$) aggregator makes optimal choices on the basket of domestic and international goods in each sector. Then the composite price of output $CP_t$ can be stated as a function of domestic and rest-of-the-world's producer price $P_{ii,j}$ (where $ii$ refers here to a region of the world):
\( CP^1_{j} = \alpha^D_{can,can} \cdot P^1_{can} + \alpha^E_{row,can} \cdot P^1_{row} \),
and the associated demand system can be expressed as:

\( D_{ii,j,t} = \alpha^D_{ii,j} \left( \frac{P^1_{j,t}}{P^{ii,t}} \right)^{\sigma_j} \left( \sum_g Pop_{j,t,g} \cdot Cons_{j,t,g} + Inv_{j,t} + Gov_{j,t} \right) \).

g. Equilibrium conditions

The equilibrium of the market clearing for goods is:

\( Q_{can,t} = D_{can,can,t} + D_{can,row,t} \),

and full employment of labour satisfies:

\( Pop_{t,gw} \cdot (1 - Leis_{t,gw}) = L_{t,gw} \).

The total capital demand is equal to the capital stock:

\( StocK_{can,t} = K^{dem}_{can,t} \).

The physical capital stock is composed of investment \( Inv_t \) made by households in capital subtracts the depreciation at constant rate \( DepR \):

\( StocK_{can,t} = Inv_t + (1 - DepR) StocK_{can,t} \).

Returns to bonds equal to returns to capital since bonds and physical capital ownerships are perfectly substitutes. The interest rate \( R_t \) is thus equivalent to the one period expected rate of return on one unit of physical capital bought at time \( t-1 \):

\( 1 + R_t = \left[ RentR_t + (1 - DepR) \right] \left( \frac{CP_t}{CP_{t-1}} \right) \).

At last, the equilibrium condition requires that total supply of assets in the financial market must be equal to the total demand:
(30) \[ \sum_{g} \text{Pop}_{t,g} \text{Lend}_{t+1,g+1} = \text{CP}_{t} \text{Bond}_{t+1} + \text{CP}_{t} \text{StocK}_{t+1}. \]

4. Simulation Results

In this section we report the results of some simulation experiments using the computable general equilibrium model described above. We conduct three experiments differing only by the value of the elasticity of substitution for labour demand (\(\sigma^{Ldem}\)) used. The demographic shock on the three experiments is similar.

Table 4 reports the growth rates imposed on the new cohorts’ from 1930 to 2000 that we imposed on the model. The numbers correspond in fact to one plus the growth rates of the 15-24 age group who enter the labour force in each decade. For instance, the number 1.045 for 1960 implies that the size of new cohort in 1960 is 4.5 percent larger than the size of the previous cohort. Notice that the growth rate increases first and then declines, as for the fertility rate of the baby boom after World War II.

### Table 4: New Cohorts’ Growth Rates
(one plus changes relative to previous period)

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<td></td>
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<td>1.015</td>
<td>1.025</td>
<td>1.045</td>
<td>1.040</td>
<td>1.025</td>
<td>1.005</td>
<td>1</td>
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The impact of those changes on the population for decades 2000 to decade 2070 is
reported in Table 5. As it can be seen in Table 5, the relative size of the older age
groups increases over time between 2000 and 2050 before stabilizing from year 2060.

This means that the population is aging over this period.

Table 5: Population from 2000 to 2070
(absolute numbers)

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
<th>2070</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-24</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
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<tr>
<td>25-34</td>
<td>0.1656</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
</tr>
<tr>
<td>35-44</td>
<td>0.1615</td>
<td>0.1656</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
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</tr>
<tr>
<td>45-54</td>
<td>0.1553</td>
<td>0.1615</td>
<td>0.1656</td>
<td>0.1664</td>
<td>0.1664</td>
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<td>0.1664</td>
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</tr>
<tr>
<td>55-64</td>
<td>0.1486</td>
<td>0.1553</td>
<td>0.1615</td>
<td>0.1656</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
</tr>
<tr>
<td>65-74</td>
<td>0.1450</td>
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<td>0.1553</td>
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<td>0.1656</td>
<td>0.1664</td>
<td>0.1664</td>
<td>0.1664</td>
</tr>
<tr>
<td>75-84</td>
<td>0.1429</td>
<td>0.1450</td>
<td>0.1486</td>
<td>0.1553</td>
<td>0.1615</td>
<td>0.1656</td>
<td>0.1664</td>
<td>0.1664</td>
</tr>
<tr>
<td>15 to 64</td>
<td>0.7974</td>
<td>0.8152</td>
<td>0.8263</td>
<td>0.8312</td>
<td>0.832</td>
<td>0.832</td>
<td>0.832</td>
<td>0.832</td>
</tr>
<tr>
<td>Total</td>
<td>1.0853</td>
<td>1.1088</td>
<td>1.1302</td>
<td>1.148</td>
<td>1.1591</td>
<td>1.164</td>
<td>1.1648</td>
<td>1.1648</td>
</tr>
<tr>
<td>15-64/Total</td>
<td>73.47%</td>
<td>73.52%</td>
<td>73.11%</td>
<td>72.40%</td>
<td>71.78%</td>
<td>71.48%</td>
<td>71.43%</td>
<td>71.43%</td>
</tr>
</tbody>
</table>

Table 6: Parameter Values

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inter-temporal elasticity of substitution ($\sigma$)</td>
<td>1.5</td>
</tr>
<tr>
<td>Intra-temporal elasticity of substitution ($\eta$)</td>
<td>1.15</td>
</tr>
<tr>
<td>Elasticity of substitution for labour demand ($\sigma_{Ldem}$)</td>
<td></td>
</tr>
<tr>
<td>First scenario</td>
<td>3.0</td>
</tr>
<tr>
<td>Second scenario</td>
<td>6.0</td>
</tr>
<tr>
<td>Third scenario</td>
<td>1.5</td>
</tr>
<tr>
<td>Replacement rate of wage income for pensions ($PensR$)</td>
<td>0.25</td>
</tr>
</tbody>
</table>

40
In Table 6, we state the values of several parameters, such as the inter-temporal elasticity of substitution, the intra-temporal elasticity of substitution, the elasticity of substitution for labour demand and the pension replacement rate ratio, which we used in the simulation experiments. Notice that the values of the elasticity of labour demand in the three scenarios range from 1.5 to 6.0.

The inter-temporal elasticity of substitution seems relatively high for an overlapping generations model. However, since every period of the model is 10 year long, the number implies a yearly elasticity of 0.15. This is more conservative than the number 0.25 used in Auerach and Kotlikoff (1987). The value of intra-temporal elasticity of substitution is smaller than the inter-temporal one, which implies a domination of the inter-temporal effect. Consequently leisure declines over time if wage income grows. Moreover, the replacement rate for pensions is a good approximation of the value used in the Canada Pension Plans and Quebec Pension Plans.

The elasticity of substitution for labour demand is usually assumed to be equal to infinite in the computable general equilibrium literature. However, Wasmer (2001) finds an elasticity of substitution around 0.5 among age groups with different experience and knowledge. Card and Lemieux (2001) find for Canada, the U.S. and the U.K., different age groups are close (elasticity of substitution in the 4-5 range), but not perfect substitutes for each other. Given the range of values found in the literature, we consider
three scenarios and compare the results accordingly.

Population aging like assumed in Table 4 and Table 5 has two implications. First although the total number of workers increases over time, its relative size with respect to the total population increases slightly from year 2000 to 2010. After that the potential labour force becomes relatively scarcer up to 2060. Indeed, the working age group represents 73.52% in 2010 to decline to 71.43% in 2060. Second, among the working age groups, the size of older workers increases relative to the size of younger workers over time. In year 2000, the oldest workers (55 to 64 years of age) represent 18.64 percent of the working age group and this number increases to 20 percent from year 2040 on. With perfect substitution of labour demand across workers of different age groups, the unique wage rate in the economy will likely be determined by the relative size of the working age group (15-64) with respect to total population. In this case, for a given labour demand, we should expect a decline in the wage rate from 2000 to 2010 and thereafter an increase. If workers are imperfect substitutes from the labour demand perspective, each age group would have its own wage rate and the changes in the wage rate will differ across age groups. In particular, as the older workers are becoming relatively larger, we should expect their corresponding wage rate to decline relative to younger workers.

a. Baseline Scenario: $\sigma^{Ldem} = 3$
We first exam the simulation results of this general equilibrium model when the elasticity of substitution for labour demand equals to 3. We consider this scenario as our benchmark. We first report the evolution of the wage rate in percentage variations relative to year 2000 for different age groups of workers in Table 7.

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
<th>2070</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-24</td>
<td>0.00</td>
<td>-0.08</td>
<td>0.26</td>
<td>0.52</td>
<td>0.73</td>
<td>0.85</td>
<td>0.85</td>
<td>0.83</td>
</tr>
<tr>
<td>25-34</td>
<td>0.00</td>
<td>-0.70</td>
<td>-0.76</td>
<td>-0.53</td>
<td>-0.31</td>
<td>-0.16</td>
<td>-0.16</td>
<td>-0.18</td>
</tr>
<tr>
<td>35-44</td>
<td>0.00</td>
<td>-1.16</td>
<td>-1.84</td>
<td>-1.79</td>
<td>-1.59</td>
<td>-1.44</td>
<td>-1.42</td>
<td>-1.44</td>
</tr>
<tr>
<td>45-54</td>
<td>0.00</td>
<td>-1.34</td>
<td>-2.49</td>
<td>-3.05</td>
<td>-3.04</td>
<td>-2.89</td>
<td>-2.85</td>
<td>-2.85</td>
</tr>
<tr>
<td>55-64</td>
<td>0.00</td>
<td>-1.00</td>
<td>-2.37</td>
<td>-3.40</td>
<td>-3.95</td>
<td>-3.96</td>
<td>-3.92</td>
<td>-3.89</td>
</tr>
<tr>
<td>65-74</td>
<td>0.00</td>
<td>-1.00</td>
<td>-2.37</td>
<td>-3.40</td>
<td>-3.95</td>
<td>-3.96</td>
<td>-3.92</td>
<td>-3.89</td>
</tr>
</tbody>
</table>

From Table 7, we can see that for 15-24 and 25-34 age groups, almost all the figures imply a positive percentage change of their real wage rate and that percentage with respect to year 2000 increases over time. Unlike the increasing trend of the real wage rates for young workers, the real wage growth rates for the relatively older age groups not only have the negative values, but also decline more and more significantly over the decades. These declines suggest a reduction in the opportunity cost of leisure for older workers.
To better estimate the opportunity cost of leisure, it is worthy to investigate the evolution of real wage net of taxes and the contribution rate on pensions. The following Table 8 gives the percentage changes of the real wage net of wage taxes and contributions relative to year 2000.

**Table 8: Real Wage Rates Net of Taxes and Contributions**

(Changes in percentage terms relative to 2000)

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
<th>2070</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-24</td>
<td>0</td>
<td>1.30</td>
<td>2.42</td>
<td>3.15</td>
<td>3.30</td>
<td>3.16</td>
<td>2.98</td>
<td>2.92</td>
</tr>
<tr>
<td>25-34</td>
<td>0</td>
<td>1.11</td>
<td>2.20</td>
<td>2.92</td>
<td>3.09</td>
<td>2.96</td>
<td>2.78</td>
<td>2.72</td>
</tr>
<tr>
<td>35-44</td>
<td>0</td>
<td>0.48</td>
<td>1.37</td>
<td>2.07</td>
<td>2.24</td>
<td>2.12</td>
<td>1.95</td>
<td>1.88</td>
</tr>
<tr>
<td>45-54</td>
<td>0</td>
<td>0.01</td>
<td>0.27</td>
<td>0.78</td>
<td>0.92</td>
<td>0.81</td>
<td>0.65</td>
<td>0.60</td>
</tr>
<tr>
<td>55-64</td>
<td>0</td>
<td>-0.18</td>
<td>-0.39</td>
<td>-0.52</td>
<td>-0.56</td>
<td>-0.67</td>
<td>-0.81</td>
<td>-0.84</td>
</tr>
<tr>
<td>65-74</td>
<td>0</td>
<td>0.19</td>
<td>-0.27</td>
<td>-0.88</td>
<td>-1.49</td>
<td>-1.77</td>
<td>-1.89</td>
<td>-1.90</td>
</tr>
</tbody>
</table>

Although Table 8 numbers exhibit similar trends to the numbers in Table 7, it is noticeable that the percentage changes in the net real wage rates are all larger. The possible reason may be due to the increasing working age-group population in period of 2000 to 2040 as Table 2 shows, which permits new entry of taxation revenue for the government. Under the assumption of constant government expenditure over these decades, the payroll taxes imposed on each worker decline. This pushes up real wage rates net of taxes and contributions and makes the net real wage rates for younger workers growing faster and that for older workers declining at a slower pace.
Net real wage rates reflect the real opportunity cost of labour. In Table 9 we report labour participation rate percentage changes. These changes are the result of inter-temporal and intra-temporal substitution plus income effects in the optimization problems of the various cohorts.

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
<th>2070</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-24</td>
<td>0</td>
<td>0.07</td>
<td>0.21</td>
<td>0.29</td>
<td>0.27</td>
<td>0.18</td>
<td>0.12</td>
<td>0.11</td>
</tr>
<tr>
<td>25-34</td>
<td>0</td>
<td>0.15</td>
<td>0.36</td>
<td>0.45</td>
<td>0.38</td>
<td>0.26</td>
<td>0.20</td>
<td>0.20</td>
</tr>
<tr>
<td>35-44</td>
<td>0</td>
<td>0.05</td>
<td>0.33</td>
<td>0.47</td>
<td>0.40</td>
<td>0.24</td>
<td>0.17</td>
<td>0.17</td>
</tr>
<tr>
<td>45-54</td>
<td>0</td>
<td>0.01</td>
<td>0.18</td>
<td>0.38</td>
<td>0.35</td>
<td>0.19</td>
<td>0.07</td>
<td>0.07</td>
</tr>
<tr>
<td>55-64</td>
<td>0</td>
<td>0.07</td>
<td>0.25</td>
<td>0.37</td>
<td>0.40</td>
<td>0.23</td>
<td>0.05</td>
<td>-0.01</td>
</tr>
<tr>
<td>64-75</td>
<td>0</td>
<td>0.90</td>
<td>1.33</td>
<td>1.45</td>
<td>1.26</td>
<td>1.09</td>
<td>0.89</td>
<td>0.75</td>
</tr>
</tbody>
</table>

From Table 9, first we can see a common trend of increasing labour participation rates from year 2000 to year 2030. Then in most cases, the participation rates decline. Notice it declines more rapidly for older workers. Second, the increase in participation rates is larger for the 25-34 and 35-44 age groups at each period. The exception is for the 65-74 age group, but in this age category, the initial equilibrium participation rate is so small, any increase would represent a more significant one in percentage terms.

b. Second Scenario: \( \sigma_{Ldem} = 6 \)
In this scenario, workers of different age group become more substitutable from the labour demand perspective as the elasticity of substitution now equals to 6. We comment our results of this second scenario by comparing them to the benchmark case. The real wage rates and the net real wage rates are reported in Table 10 and Table 11 respectively.

Table 10: Real Wage Rates
(Changes in percentage terms)

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
<th>2070</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-24</td>
<td>0</td>
<td>-0.27</td>
<td>-0.47</td>
<td>-0.47</td>
<td>-0.40</td>
<td>-0.32</td>
<td>-0.33</td>
<td>-0.34</td>
</tr>
<tr>
<td>25-34</td>
<td>0</td>
<td>-0.37</td>
<td>-0.58</td>
<td>-0.58</td>
<td>-0.50</td>
<td>-0.42</td>
<td>-0.42</td>
<td>-0.43</td>
</tr>
<tr>
<td>35-44</td>
<td>0</td>
<td>-0.71</td>
<td>-1.02</td>
<td>-1.03</td>
<td>-0.95</td>
<td>-0.86</td>
<td>-0.86</td>
<td>-0.87</td>
</tr>
<tr>
<td>45-54</td>
<td>0</td>
<td>-0.96</td>
<td>-1.60</td>
<td>-1.70</td>
<td>-1.64</td>
<td>-1.55</td>
<td>-1.54</td>
<td>-1.55</td>
</tr>
<tr>
<td>55-64</td>
<td>0</td>
<td>-1.60</td>
<td>-1.97</td>
<td>-2.42</td>
<td>-2.46</td>
<td>-2.37</td>
<td>-2.35</td>
<td>-2.35</td>
</tr>
<tr>
<td>64-75</td>
<td>0</td>
<td>-0.91</td>
<td>-2.04</td>
<td>-2.84</td>
<td>-3.24</td>
<td>-3.26</td>
<td>-3.23</td>
<td>-3.21</td>
</tr>
</tbody>
</table>

Table 11: Real Wage Rates Net of Taxes and Contributions
(Changes in percentage terms relative to 2000)

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
<th>2070</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-24</td>
<td>0</td>
<td>0.90</td>
<td>1.65</td>
<td>2.10</td>
<td>2.09</td>
<td>1.86</td>
<td>1.64</td>
<td>1.56</td>
</tr>
<tr>
<td>25-34</td>
<td>0</td>
<td>0.80</td>
<td>1.54</td>
<td>1.99</td>
<td>1.98</td>
<td>1.77</td>
<td>1.54</td>
<td>1.46</td>
</tr>
<tr>
<td>35-44</td>
<td>0</td>
<td>0.46</td>
<td>1.09</td>
<td>1.53</td>
<td>1.53</td>
<td>1.31</td>
<td>1.10</td>
<td>1.02</td>
</tr>
<tr>
<td>45-54</td>
<td>0</td>
<td>0.21</td>
<td>0.50</td>
<td>0.84</td>
<td>0.82</td>
<td>0.61</td>
<td>0.40</td>
<td>0.33</td>
</tr>
<tr>
<td>55-64</td>
<td>0</td>
<td>0.10</td>
<td>0.12</td>
<td>0.10</td>
<td>-0.02</td>
<td>-0.23</td>
<td>-0.43</td>
<td>-0.49</td>
</tr>
<tr>
<td>64-75</td>
<td>0</td>
<td>0.25</td>
<td>0.05</td>
<td>-0.33</td>
<td>-0.82</td>
<td>-1.14</td>
<td>-1.32</td>
<td>-1.36</td>
</tr>
</tbody>
</table>
In comparison to the numbers reported in Table 7, we can see that the real wage rates in Table 10 have now all negative values. As workers of various age groups are now more substitutable than before, the size of the total labour force matters more than before in determining the wage rates. And the size of the potential labour force (15-64 age group) increases over time as reported in Table 5 before stabilizing in 2040. That increase also pushes down tax and contributions rates. This is why in Table 11, real wage rates net of taxes are positive for most age groups. Notice, however, that the percentage changes are more important for younger workers, whereas they remain negative for older workers. It is noticeable that the increase in the homogeneity across workers of different age groups implies that the differences of percentage wage rate changes between the young and the old workers at different points in time are much less significant than in the baseline scenario. For instance at year 2070 in Table 10, the difference in real wage rates between 15-24 age group and 55-64 age group is 2.31 percent, smaller than the difference of the baseline scenario of 3.68, as it can be calculated from Table 7. A similar conclusion can be drawn from the real wage rate net of taxes and contributions.

Comparing the labour participation rates with the benchmark scenario, we find two interesting facts. First, participation rates of older workers (45-54 and 55-64) increase by more when the elasticity of substation is higher. Indeed, as expected, the older workers benefit more from the increase on average of the net of taxes and contribution real wage rates, when workers are more homogenous across age groups. Second, for
successive cohorts (numbers in diagonals in Table 12), the optimization solution results in a decline of labour supply when young compensated by a more significant positive change at older age.

**Table 12: Labour Participation Rates**
(Changes in percentage terms relative to 2000)

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
<th>2070</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-24</td>
<td>0</td>
<td>-0.11</td>
<td>-0.13</td>
<td>-0.14</td>
<td>-0.20</td>
<td>-0.29</td>
<td>-0.35</td>
<td>-0.37</td>
</tr>
<tr>
<td>25-34</td>
<td>0</td>
<td>-0.02</td>
<td>0.03</td>
<td>0.02</td>
<td>-0.09</td>
<td>-0.22</td>
<td>-0.29</td>
<td>-0.30</td>
</tr>
<tr>
<td>35-44</td>
<td>0</td>
<td>0.03</td>
<td>0.19</td>
<td>0.25</td>
<td>0.14</td>
<td>-0.03</td>
<td>-0.12</td>
<td>-0.13</td>
</tr>
<tr>
<td>45-54</td>
<td>0</td>
<td>0.09</td>
<td>0.28</td>
<td>0.45</td>
<td>0.40</td>
<td>0.23</td>
<td>0.10</td>
<td>0.07</td>
</tr>
<tr>
<td>55-64</td>
<td>0</td>
<td>0.26</td>
<td>0.64</td>
<td>0.96</td>
<td>1.03</td>
<td>0.87</td>
<td>0.66</td>
<td>0.56</td>
</tr>
<tr>
<td>64-75</td>
<td>0</td>
<td>1.29</td>
<td>2.54</td>
<td>3.57</td>
<td>3.96</td>
<td>3.96</td>
<td>3.68</td>
<td>3.45</td>
</tr>
</tbody>
</table>

**c. Third Scenario:** $\sigma_{Ldem} = 1.5$

With an elasticity of substitution for labour demand equals to 1.5, workers of different age groups are more heterogeneous than the two previous scenarios. Hence, not surprisingly, when we look at the percentage changes in the real wage rates and in the net of taxes and contribution real wage rates (see Table 13 and Table 14), the differences across age groups are much more important. In particular, notice that the changes of the net of taxes and contributions real wage rate are negative only for the older workers.
### Table 13: Real Wage Rates
(Changes in percentage terms)

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
<th>2070</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-24</td>
<td>0</td>
<td>0.71</td>
<td>1.37</td>
<td>1.97</td>
<td>2.36</td>
<td>2.54</td>
<td>2.55</td>
<td>2.54</td>
</tr>
<tr>
<td>25-34</td>
<td>0</td>
<td>0.38</td>
<td>0.98</td>
<td>1.59</td>
<td>1.99</td>
<td>2.18</td>
<td>2.18</td>
<td>2.16</td>
</tr>
<tr>
<td>35-44</td>
<td>0</td>
<td>-0.70</td>
<td>-0.41</td>
<td>0.17</td>
<td>0.58</td>
<td>0.78</td>
<td>0.78</td>
<td>0.75</td>
</tr>
<tr>
<td>45-54</td>
<td>0</td>
<td>-1.51</td>
<td>-2.27</td>
<td>-2.00</td>
<td>-1.62</td>
<td>-1.40</td>
<td>-1.38</td>
<td>-1.41</td>
</tr>
<tr>
<td>55-64</td>
<td>0</td>
<td>-1.77</td>
<td>-3.29</td>
<td>-4.02</td>
<td>-3.92</td>
<td>-3.70</td>
<td>-3.65</td>
<td>-3.66</td>
</tr>
<tr>
<td>64-75</td>
<td>0</td>
<td>-1.05</td>
<td>-2.81</td>
<td>-4.15</td>
<td>-4.88</td>
<td>-4.88</td>
<td>-4.82</td>
<td>-4.80</td>
</tr>
</tbody>
</table>

### Table 14: Real Wage Rates Net of Taxes and Contributions
(Changes in percentage terms relative to 2000)

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
<th>2070</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-24</td>
<td>0</td>
<td>1.90</td>
<td>3.55</td>
<td>4.64</td>
<td>5.03</td>
<td>5.02</td>
<td>4.93</td>
<td>4.92</td>
</tr>
<tr>
<td>25-34</td>
<td>0</td>
<td>1.56</td>
<td>3.16</td>
<td>4.25</td>
<td>4.65</td>
<td>4.65</td>
<td>4.56</td>
<td>4.54</td>
</tr>
<tr>
<td>35-44</td>
<td>0</td>
<td>0.47</td>
<td>1.74</td>
<td>2.79</td>
<td>3.20</td>
<td>3.22</td>
<td>3.12</td>
<td>3.09</td>
</tr>
<tr>
<td>45-54</td>
<td>0</td>
<td>-0.35</td>
<td>-0.17</td>
<td>0.56</td>
<td>0.95</td>
<td>0.98</td>
<td>0.91</td>
<td>0.88</td>
</tr>
<tr>
<td>55-64</td>
<td>0</td>
<td>-0.62</td>
<td>-1.21</td>
<td>-1.51</td>
<td>-1.41</td>
<td>-1.37</td>
<td>-1.41</td>
<td>-1.42</td>
</tr>
<tr>
<td>64-75</td>
<td>0</td>
<td>0.12</td>
<td>-0.711</td>
<td>-1.64</td>
<td>-2.39</td>
<td>-2.58</td>
<td>-2.61</td>
<td>-2.59</td>
</tr>
</tbody>
</table>

The numbers in Table 13 and Table 14 reflects a higher demand for younger than for older workers. Hence, wage rates are up for younger and down for older workers. Moreover, the wage rates increase in a fast speed for younger workers and decline dramatically for older workers over the decades. In this sense, although there is an increase in the wage rate on average because of population aging, this may not be the
case for old workers. Consequently labour market reaction to population aging may not generate economic incentives for old workers to return to or stick to the workforce and postpone retirement. Notice as well that in contrast to the second scenario, the differences of percentage wage rate changes between the young and the old in a certain decade are much greater than the differences in the baseline scenario.

Table 15: Labour Participation Rates
(Changes in percentage terms relative to 2000)

<table>
<thead>
<tr>
<th>Age Groups</th>
<th>2000</th>
<th>2010</th>
<th>2020</th>
<th>2030</th>
<th>2040</th>
<th>2050</th>
<th>2060</th>
<th>2070</th>
</tr>
</thead>
<tbody>
<tr>
<td>15-24</td>
<td>0</td>
<td>0.38</td>
<td>0.76</td>
<td>0.95</td>
<td>0.95</td>
<td>0.86</td>
<td>0.83</td>
<td>0.83</td>
</tr>
<tr>
<td>25-34</td>
<td>0</td>
<td>0.39</td>
<td>0.83</td>
<td>1.02</td>
<td>1.00</td>
<td>0.89</td>
<td>0.87</td>
<td>0.88</td>
</tr>
<tr>
<td>35-44</td>
<td>0</td>
<td>0.04</td>
<td>0.44</td>
<td>0.66</td>
<td>0.62</td>
<td>0.49</td>
<td>0.46</td>
<td>0.49</td>
</tr>
<tr>
<td>45-54</td>
<td>0</td>
<td>-0.20</td>
<td>-0.15</td>
<td>0.02</td>
<td>0.01</td>
<td>-0.14</td>
<td>-0.20</td>
<td>-0.17</td>
</tr>
<tr>
<td>55-64</td>
<td>0</td>
<td>-0.27</td>
<td>-0.51</td>
<td>-0.76</td>
<td>-0.84</td>
<td>-1.01</td>
<td>-1.11</td>
<td>-1.11</td>
</tr>
<tr>
<td>64-75</td>
<td>0</td>
<td>0.56</td>
<td>0.19</td>
<td>-0.56</td>
<td>-1.31</td>
<td>-1.62</td>
<td>-1.73</td>
<td>-1.78</td>
</tr>
</tbody>
</table>

Not surprisingly, over the periods, the participation rates are all positively growing for workers of age 15 to 44. For older workers, however, almost the figures are negative. Additionally, the growth rates of participation rates for the younger workers rise over time, but decline further for older workers. Under this scenario, the decreasing cost of leisure provides incentives for the earlier retirement and drives old workers out of the workforce. In this case, the participation rates of older workers would not increase but decline, in contrast to what the literature suggests.
5. Conclusion

In this paper, we borrow a computable overlapping generations model under the assumption of imperfect substitution between young and old workers. This assumption determines that from the labour demand perspective, each age group would have its own wage rate and the changes in the wage rate will differ across age groups. We impose the same demographic shock and conduct three experiments differing by the value of the elasticity of substitution for the labour demand \( \sigma^{Ldem} \) used.

Through comparing the three scenarios, first of all, we show that the higher the value of the elasticity of substitution of the labour demand, the more important the size of the total labour force in determining the wage rates. In other words, the increase in the homogeneity across workers of different age groups implies smaller differences of percentage wage rate changes between the young and the old workers at different points in time. Secondly, we prove the point that with the potential labour becoming scarcer and the size of the older workers becoming relatively larger over time, although population aging increases the wage rate on average because of an eventual slowdown in the labour force, this may not be the case for old workers. Consequently, labour market reaction to population aging may not generate economic incentives for old workers to return to or stick to the workforce and postpone retirement. In such a context, there may imply a new trend of early retirement.
Appendix

\[ \text{Min} \sum_{gw} \text{wage}_{t, gw} \cdot L_{t, gw} \]

Subject to:

\[ L_{t}^{\text{dem}} = \left( \sum_{gw} \alpha \rho_{gw} L_{t, gw}^{\rho} \right)^{\frac{1}{\rho}} \]

We built the Lagrange function:

\[ L = \sum_{gw} \text{wage}_{t, gw} L_{t, gw} - \lambda \left( \sum_{gw} \alpha \rho_{gw} L_{t, gw}^{\rho} \right)^{\frac{1}{\rho}} - L_{t}^{\text{dem}} \]

Differentiating the Lagrange with respect to labour demand for age group \( gw \), then the first order condition takes the following form:

\[ \text{wage}_{t, gw} = \lambda \left( \sum_{gw} \alpha \rho_{gw} L_{t, gw}^{\rho} \right)^{\frac{1}{\rho-1}} \cdot \alpha \rho_{gw} L_{t, gw}^{\rho-1} \]

By multiplying both sides of equation by \( L_{t, gw} \), we get:

\[ \text{wage}_{t, gw} \cdot L_{t, gw} = \lambda \left( \sum_{gw} \alpha \rho_{gw} L_{t, gw}^{\rho} \right)^{\frac{1}{\rho-1}} \cdot \alpha \rho_{gw} L_{t, gw}^{\rho} \]

A similar first order condition exists for the labour demand of each generation. Taking the sum over all generations give:

\[ \sum_{gw} \text{wage}_{t, gw} \cdot L_{t, gw} = \lambda \left( \sum_{gw} \alpha \rho_{gw} L_{t, gw}^{\rho} \right)^{\frac{1}{\rho-1}} \cdot \sum_{gw} \alpha \rho_{gw} L_{t, gw}^{\rho} \]

According to the assumption of the equilibrium of labour market, we have:

\[ \sum_{gw} \text{wage}_{t, gw} \cdot L_{t, gw} = \text{WLDem} \cdot L_{t}^{\text{dem}} \]

Substituting constraint (1.1) and equation (1.3) into the above equation (1.4) yields an
expression for Lagrange multiplier:

$$\lambda = WLDem_i$$

Replacing this multiplier into (1.2) by the above relationship and also using constraint, we obtain:

$$wage_{t,gw} \cdot L_{t,gw} = WLDem_i \cdot (L_{t,gw}^{dem})^{1-\varphi} \cdot \alpha p_{gw} \cdot L_{t,gw}^\varphi$$

Arranging the above equation, we get the express of labour demand for age group $gw$:

$$L_{t,gw} = (\alpha p_{gw})^{(1-\varphi)} \cdot \left( \frac{WLDem_i}{wage_{t,gw}} \right)^{1-\varphi} \cdot L_{t,gw}^{dem}$$

Since $\sigma^{dem} = \frac{1}{1-\varphi}$, we have:

$$L_{t,gw} = (\alpha p_{gw})^{\sigma_{dem}} \cdot \left( \frac{WLDem_i}{wage_{t,gw}} \right)^{\sigma_{dem}} \cdot L_{t,gw}^{dem} \quad (1.5)$$

Combining (1.5) with the constraint (1.1) gives the explicit form of the aggregate price of labour $WLDem_i$:

$$WLDem_i^{(1-\sigma_{dem})} = \sum_{gw} \alpha p_{gw} wage_{t,gw}^{(1-\sigma_{dem})} \quad (1.6)$$

These two equations (1.5) and (1.6) are derived using the above method.
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