Interest rate, debt, distribution and growth: A post-Keynesian model in a coherent stock-flow monetary framework

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Abstract

In this paper, the author attempts to build a more complete Kaleckian distribution and growth model based on Lavoie's (1995) 'Minsky-Steindl-model', which was further elaborated in Hein (1999, 2005). It extends the analysis of the impact of variations in the interest rate on the dynamics of indebtedness, wealth distribution and capital accumulation towards equilibrium in both the short run and the long run. The author sets solid foundations by introducing an explicitly coherent stock-flow monetary framework into the new model. By modifying the investment and savings functions in the model, the effects of the variation of interest rate on both the short run and the long run are fully analyzed, with respect to the necessary bounded conditions, especially an implicit maximum condition that is ignored by Lavoie and Hein. Finally, the author shows that, the effects of interest variations on the endogenously determined equilibrium values of the model depend not only the parameter values of the saving and investment functions, as analyzed in Lavoie and Hein, but also on the interest elasticity of distribution as proposed by Hein, as well as the initial interest rate and the initial debt-capital ratio relative to the maximum.

Keywords: debt-capital ratio, Interest rate, Capital accumulation

1. Introduction

For decades, the impact of monetary variables has barely caught the attention that it deserves from post-Keynesian authors. Monetary variables were ignored and hence not taken into consideration in their models. Consequently, for years, numerous criticisms were addressed against these models because of this failure. As pointed out by Kregel (1985, p.133), it seems that, "Money plays no more than a perfunctory role in the Cambridge theories of growth, capital and distribution." However, since the 1980s, this situation has changed gradually. Post-Keynesians have started to introduce monetary variables into their growth and distribution models, mainly based on the so-called 'horizontalist' monetary approach[1] assumption: the rate of interest is considered to be an exogenous variable for production and accumulation, under the control of the central bank, whereas the supply of credit and money is endogenous and accommodates to demand.

In particular, in 1995, Lavoie first incorporated the effects of debt ratios on capital accumulation into what he called the Minsky-Steindl model based on the above 'horizontalist' assumption. He showed that under the assumption of stable equilibrium conditions, in the short run, with a constant debt-capital ratio, the direct impact of higher interest rates causes a "puzzling" increase in the rates of capital accumulation, capacity utilization and profit; moreover, in the long run, the higher interest rates result in a further increase of these variables through the indirect effects of the gradually increasing debt-capital ratio. This was followed by Hein's studies (1999, 2005), who developed the original model by modifying the investment function according to his argument that the

[1] See, among others, Kaldor(1970, 1982), Moore (1988), Wray L.R. (1990), and Lavoie (1992).

profit rate should be decomposed into the profit share reflecting the development of unit costs and the rate of capacity utilization indicating the development of demand. Hein also simplified the saving function by assuming away firms' equities, as well as introducing more complete considerations on a markup pricing. Consequently, he reached the conclusion that, beside what had been concluded by Lavoie, the effects of interest variations on the endogenous model variables also depend on the interest elasticity of distribution, as well as the initial interest rate and debt-capital ratio, which were not discussed in Lavoie (1995).

However, I found that the analysis could be further extended, based on the following considerations. First of all, while the modification of the investment function by Hein is a worthwhile consideration, it is not quite practical to assume away the firms' equities from the model. Thus I tried to derive a more proper saving function by introducing the explicitly coherent stock-flow monetary framework into the model, which turned out to be the same as the counterpart in Lavoie's (1995). Secondly, incorporating the considerations of a varying markup proposed by Hein, I reached a slightly different conclusion: beside what was concluded by Lavoie and Hein, in fact, the effects of interest variations on the endogenous model variables also depend on the initial debt-capital ratio relative to the maximum ratio constrained by a given economic environment, instead of the initial debt-capital ratio itself. Furthermore, more importantly, both Lavoie and Hein did not analyze the hidden maximum bounded condition, which indicates that the value of the initial equilibrium debt-capital ratio relative to the maximum value is crucial in determining the stability of the long run equilibrium in the model. Consequently, in my paper, I found that the condition for stable long run equilibrium in Lavoie's (1995) and

Hein's models is actually only a necessary but not a sufficient condition, which needs to be modified accordingly.

With respect to the above considerations, setting the model on a more solid foundation of a coherent stock-flow monetary framework, I explore a more complete analysis of the effects of variations in the interest rate on the rates of capital accumulation, capacity utilization and profit in short run equilibrium, as well as in the long run equilibrium, on the basis of the previous model. The structure of the paper is as follows. In Section 2 we firstly introduce a coherent stock-flow monetary framework, and then describe the basic assumptions and the main background information of our model. In section 3, we will derive the short run equilibrium solution. In section 4, we will analyze the long run effects of variations of interest rate on the equilibrium debt-capital ratio. In Section 5 we will deal with the complete dynamics of the model. Section 6 concludes.

2. The basic model

2.1 A coherent stock-flow monetary framework

The coherent stock-flow monetary framework, based on the social accounting matrices (SAM), is developed by Godley and Cripps (1983) and Godley (1993, 1996, 1999) while the Kaleckian models of growth originate from the works of Rowthorn (1981) Dutt (1990) and Lavoie (1995). With double entry bookkeeping used to organize national income and flow of funds concepts, the framework applies a consistent set of sectoral and national balance sheets where every financial asset has a counterpart liability, and budget constraints for each sector describe how the balance sheets between flows of expenditure, factor income, and transfers generate counterpart changes in stocks of assets and liabilities. Under the comprehensive approach, all stocks and flows can be fitted into matrices in which columns and rows all sum to zero,[2] avoiding accounting errors and their unacceptable implications.

The methodology applied to analyze the growth of the model would be similar to that introduced by Lavoie/Godley (2001) in a post-Kaleckian model extending Kaldor's 1966 model. To take money into consideration, Lavoie/Godley presented explicit budget constraints with money stocks for both firms and households, and successfully developed a Kaldorian view including characteristics of markup pricing, endogenous growth, flexible rates of utilization, as well as endogenous credit money and exogenous interest rates.

To make the model the most simple while still holding to these unique Kaldorian features, we need to make several assumptions and simplifications. Firstly, the economy has neither a foreign sector nor a government. Secondly, banks have zero net worth and no loans are made to households. Thirdly, firms issue no bonds, only equities, and hold no money balance. Finally, we assume that there is no inflation.

The balance sheet matrix of the economy is shown in the Table 1 below, whereas the transaction matrix is presented in Table 2, describing transaction flows between the three sectors of the economy, as well as recognizing the distinction between a current account and a capital account within a sector in the cases of banks and firms.

Symbols with plus signs represent source of funds, whereas the negative signs indicate uses of funds. The financial balance of each sector, that is, the gap between its income and expenditure, as shown vertically in every columns of the table, is always equal to the total of its transactions in financial assets, so every column in the table represents a

[2] This method was firstly put to use by Backus et al. (1980).

Table 1:	Balance sheets			
	Households	Firms	Banks	$\sum$
Money	$+M_d$		$-M_s$	0
Equities	$+e_d p_e$	$-e_s p_e$		0
Capital		$+K$		$+K$
Loans		$+B_d$	$+B_s$	0
$\sum$	$+V$	$K-(B_d+e_s p_e)$	0	$+K$

Table 2:	Transactions matrix					
	Households	Firms		Banks		$\sum$
		Current	Capital	Current	Capital	
Consumption	$-C_d$	$+C_s$				0
Investment		$+I_s$	$-I_d$			0
Wages	$+W_s$	$-W_d$				0
Net profit	$+F_D$	$-(F_U+F_D)$	$+F_U$			0
Interest on loans		$-i_l B_{d(-1)}$		$+i_l B_{s(-1)}$		0
Interest on deposits	$+i_m M_{d(-1)}$			$-i_m M_{s(-1)}$		0
Δ In Loans			$+\Delta B_d$		$-\Delta B_s$	0
Δ In Money	$-\Delta M_d$				$+\Delta M_s$	0
Issue of equities	$-\Delta e_d p_e$		$+\Delta e_s p_e$			0
$\sum$	0	0	0	0	0	0

corresponding budget constraint. The watertight accounting of the model implies that the value of any one variable is logically implied by all the other variables taken together. It also implies that any one of the columns in Table 2 is logically implied by the sum of the other four columns. Under the sealed accounting system, it is clearly indicated that everything comes from somewhere and everything goes somewhere and will guarantee that there will be no black hole in the economy.

2.2 The Basic background

The basic background of the model is similar to those in Lavoie (1995) and Hein (2005), except for the modifications we mentioned previously, which will be explained in more detail in due course. In our model, technical change is assumed to be exogenous, and thus it will not be explicitly considered. For simplification, we also assume that there is just one type of commodity produced that can be used for both consumption and investment purposes.

Assuming that there is a constant relationship between the employed number of labor (L) and real output (Y), the productivity of labor (P_L) is constant up to full capacity output. We get a constant labor-output ratio (l) as following:

$$l = \frac{1}{P_L} = \frac{L}{Y} = cons \text{, where } P_L = \frac{Y}{L} \text{, which is the productivity of labor.} \tag{1}$$

The capital to full capacity output ratio (v), also called the capital-potential output ratio, which describes the relationship between the real capital stock (K) and potential real output (Y^V), is also assumed to be some constant:

$$v = \frac{K}{Y^V} = cons \tag{2}$$

The capital stock is assumed not to depreciate. The rate of capacity utilization (u) is given by the relation between actual real output (Y) and potential real output (Y^V):

$$u = \frac{Y}{Y^V} \tag{3}$$

The debt-to-capital ratio or leverage ratio is denoted by λ. It is defined as the amount of loans (B) contracted by firms over the replacement value of capital (K). While this ratio is considered as given in the short run, it is changing over time in the long run, unless a steady state is reached:

$$\lambda = \frac{B}{PK} \tag{4}$$

In our model, through decisions on profit markup, production, investment and saving, firms play a crucial role in determining the growth dynamics of the economy. The markup set by firms[3] determines the share of income between profits flowing to firms and wages going to households. Under our assumptions of constant unit costs up to full capacity output and constant returns to scale of production, firms set their prices (p) according to a mark-up (m) on unit direct costs that consist entirely of wages. As in Hein (2005), we assume that the mark-up in the price equation is mainly determined by the degree of price competition in the goods markets and by the relative powers of capital and labor in the labor market. Then we have a simple markup equation:

$$p = (1+m)wl, \qquad m > 0 \tag{5}$$

Where, p the price level, w the nominal wage rate, m the markup, and l is the labor-output ratio. From this we get the profit share (h), which is the proportion of profits (F_T) and the nominal output (PY):

$$h = \frac{F_T}{PY} = \frac{(mwl)Y}{PY} = \frac{m}{(1+m)} \tag{6}$$

The markup has been considered as exogenous and thus assumed to be some unchanged constant in Lavoie (1995). However, this assumption may be considered an extreme one to make. As analyzed in Hein (2005), the mark-up and the profit share consist of two

[3] See Lavoie (1992, ch 3).

parts: a part containing the profits of enterprises and the other part covering the cost of debt represented by interest payments. The mark-up may, but need not increase, when overhead costs, including interest costs, increase. Therefore, for simplification, we define a linear simple markup function, in which the profit share may but need not respond to a variation in the interest rate:

$$h = h(i), \quad \text{where } \frac{\partial h}{\partial i} \geq 0, \quad \frac{\partial^2 h}{\partial i^2} = 0 \tag{7}$$

Obviously, the influence on income distribution of variations in interest rates will be rather different depending on what kind of markup we are considering. Firstly we consider a regime where an interest-inelastic mark-up dominantes, which is written as: $h = h(i)$, where $\frac{\partial h}{\partial i} = 0$, $\frac{\partial^2 h}{\partial i^2} = 0$. Under this environment, the real wage will not be affected by variations of the rate of interest. As a result, the variations of the rate of interest do not change the distribution of income between wages and profits; however, it does affect the distribution between profits of enterprise and incomes of rentier households. Then we consider the other case, the interest-elastic markup, which can be presented as: $h = h(i)$, where $\frac{\partial h}{\partial i} > 0$, $\frac{\partial^2 h}{\partial i^2} = 0$. In this case, variations of the rates of interest will directly affect the real wage through the corresponding change in the markup. According to the assumption we have made, an increase in the rates of interest causes an increase in the mark-up, then the rising markup pushes up the price level, finally leading to falling real wages even though the nominal wages are unchanged. Based on this analysis, we can easily find that the variations of the rates of interest affect the distribution of income between profits and wages, whereas the profits of enterprise remain constant due to a flexible markup. Through the above discussion, we complete the argument with respect to markup pricing.

Depending on the endogenously determined profit share (h) we have defined, the rate of capacity utilization (u) and the capital-potential output ratio (v), the profit rate (r) describes the relationship between the annual flow of profits (F_T) to the nominal capital stock (PK). It can be written as:

$$r = \frac{F_T}{PK} = \frac{(mwl)Y}{PY} \frac{PY}{PY^V} \frac{PY^V}{PK} = hu\frac{1}{v} \qquad (8)$$

Setting a coherent stock-flow monetary framework as we have described in the first part, we introduce monetary variables into the model, following the post-Keynesian 'horizontalist' monetary view developed by Kaldor (1970, 1982), Lavoie (1984, 1992, pp. 149-216, 1996) and Moore (1988, 1989) which states that the monetary interest rate is an exogenous variable in the dynamic process whereas the supply of credit and money are determined endogenously by economic activity. According to the horizontalist view, the central bank controls the base rate of interest through open market operations; commercial banks take the base rate as given, however they set market rates by marking up on the base rate and then supply the credit demand that creditworthy consumers and investors are asking for at the market rate. Additionally, the central bank accommodates the necessary amount of cash to maintain the market balance.

We further assume that the monetary circuit will be closed in each period. Furthermore, we assume only a single interest rate determined by the policy of the central bank.

Now we are ready to derive the investment function and the saving function in our model.

Following previous presented coherent stock-flow monetary framework in part one, we can solve the model through the budget constraints for each sector by summing up all items in the respective column of either matrix. First of all, taking the first columns of Table 2, we will get the budget constraint for households:

$$[- C_d + W_s + F_D + i_m M_{d(-1)} - \Delta M_d - \Delta e_d p_e] = 0 \tag{9}$$

Rearrange the equation: $$[W_s + F_D + i_m M_{d(-1)}] - C_d = \Delta M_d + \Delta e_d p_e \tag{10}$$

The equation shows that, households must finance their investment (equities and money deposits) though their saving (household income minus their consumption) under the assumption in our model that loans are not granted to households. Actually, if we let S_h represent households' saving investment, the relationship can be shown as:

$$\Delta M_d + \Delta e_d p_e = Y_d - C_d = [+W_s + F_D + i_m M_{d(-1)}] - C_d = S_h \tag{11}$$

Similarly, the budget constraints for firms is obtained by combining the 2nd & 3rd columns in Table 2:

$$[+C_s + I_s - W_d - (F_U + F_D) - i_l B_{d(-1)}] + [- I_d + F_U + \Delta B_d + \Delta e_s p_e] = 0 \tag{12}$$

Rearrange the equation: $$[\Delta B_d + \Delta e_s p_e] = [(W_d + F_D + i_l B_{d(-1)} - C_s)] \tag{13}$$

From Table 2, we know: $W_d = W_s$, $C_s = C_d$, $i_l = i_m$, $B_{d(-1)} = M_{d(-1)}$.

The above equation then becomes:

$$[\Delta B_d + \Delta e_s p_e] = [(W_d + F_D + i_l B_{d(-1)} - C_s)] = [W_s + F_D + i_m M_{d(-1)}] - C_d = S_d \tag{14}$$

It indicates that, firms are able to finance their investment by issuing new equities and obtaining new loans from banks only if households are willing to invest in either equities or money deposits, or both.

Considering the capital section of firms (the 3rd column in Table 2), we obtain the budget constraint as: $[- I_d + F_U + \Delta B_d + \Delta e_s p_e] = 0$ (15)

Rearrange: $I_d = F_U + [\Delta B_d + \Delta e_s p_e] = F_U + S_h = S$ (16)

This equation tells us that, firms' investment is financed by the total saving of the economy, the firms' saving and the households' saving.

Considering the current section of firms (the 2nd column in Table 2), we obtain the budget constraint as: $[C_s + I_s - W_d - (F_U + F_D) - i_l B_{d(-1)}] = 0$ (17)

Rearrange: $I_s = W_d + (F_U + F_D) + i_l B_{d(-1)} - C_s$ (18)

Combining all sectoral constraint equations together (from (9) to (18)), then we can get the saving function as following:[4]

$$I_s = W + F_U + F_D + iB - C \tag{19}$$

Since firms have borrowed an amount (B) from households , either directly in the form of equities issued by firms or indirectly in the form of loans through banks where households deposit their money, they must pay out an amount of interest payment (iB) and thus this amount will be distributed to households holding equities or money deposits. The profits net of this amount may be either distributed to households holding firms' equities in the form of dividends (F_D) or be saved by firms as retained profits (F_U). Therefore, firms' total profits can be shown as follows:

$$F_T = F_U + F_D + iB \tag{20}$$

In order to keep the argument simple, we suppose a classical saving hypothesis: labor

[4] The values for previous period and current period will be the same in a continuously developing model when we assume that the number of small periods goes to infinite, thus the subscripts indicating periods can be ignored.

does not save, that is, all wages will be consumed. Furthermore, let, s_f, which connects to firms' saving behavior, the retention rate out of profits net of interest payments; s_S, the propensity to save of households holding shares out of firms' dividends; s_b, the propensity to save of households holding banks deposits. We assume that the propensity to save of households holding shares out of firms' dividends is equal to the propensity to save of households holding bonds or banks deposits, that is, $s_b = s_S$. To avoid unnecessary confusion, let $s_Z = s_b = s_S$. Under these assumptions, there is a single propensity to save by households, s_Z. Moreover, under this assumption, we do not need to distinguish between creditor households receiving interest income, on the one hand, and shareholder households receiving dividend income, on the other hand, and their different savings propensities, as in Lavoie (1995).[5] And we can find out the consumption of households under these assumptions: $C = W + (1 - s_Z)(F_D + iB)$ (21)

Substituting into our saving function, then: $I_s = F_U + s_Z (F_D + iB)$ (22)

The equation is rather intuitive: the total savings are made up of three parts: retained profits that are completely saved as firms' savings by definition; savings out of interest income that is saved by households holding bonds or deposits; and savings out of dividends income that is saved by households holding shares. Taking equations (3), (4) and (5) into account, for the savings rate (g^s) which is the ratio of total savings to the nominal capital stock, by definition, we get:

$$g^s = \frac{S}{PK} = \frac{I_S}{PK} = \frac{F_U + S_Z}{PK} = \frac{F_U + s_z(F_D + iB)}{PK}$$

[5] Capital gains are omitted, or it is assumed that they are entirely saved.

$$= s_T \frac{hu}{v} - s_f(1 - s_z)\lambda i \tag{23}$$

where $s_T = [1-(1-s_f)(1-s_z)]$, $0 < s_f < 1$, $0 < s_Z < 1$.

The saving function shows that, at a given rate of profit (r), a given debt-capital ratio (λ) and a given propensity to save of households (s_Z), as well as a given retention rate (s_f), the higher the interest (i) rate the lower will be the savings rate (g^s), and vice versa. The rationality underlining the conclusion is quite simple: the part of profits that would have been saved completely by firms as retained profits otherwise, now is distributed from firms to households who consume at least part of this income and thus save only a proportion of it. As indicated by the form of the equation, variations of debt-capital ratio will have similar effects on the savings rate for the same reason.

Now we turn to the investment behavior of firms. For the accumulation function, many non-monetary models based on the principle of effective demand, which rely on an extension of the function proposed by Bhaduri/Marglin (1990), claim that decisions to invest are assumed to depend on the rate of profit. In Lavoie (1995), the investment function has been assumed as following:

$$g^i = \gamma + g_r \frac{hu}{v} - g_l \lambda i \tag{24}$$

The underlining logic is based on the principle of effective demand as well as the elegance of the symmetry with the form of the savings function. As pointed out by Lavoie (1995), a Minskian investment function can be justified by showing that investment would have a positive function of tranquility and a negative function of financial fragility, the latter being connected to interest payments.

Following Kalecki's (1937) principle of increasing risk, provided that firms have to at least partially finance their net investment spending by credit, we know that the firms' capability of access to credit is positively correlated with the firms' internal means of finance and negatively with their debt-capital ratios. The higher the amount of the firms' own capital, the higher the amount of debt capital that can be obtained for investment. From these arguments it follows, that the rate of interest and the debt-capital ratio have a negative impact on investment.

Without disagreement regarding other arguments shown in Lavoie (1995), however, we would still insist that firms may not react with equal intensity to an increase in the profit rate which is due to a decrease in real wages or to an increase in capacity utilization. In fact, under the assumption of the constant technical conditions of production, the profit rate can be decomposed into the profit share reflecting the development of unit costs and the rate of capacity utilization indicating the development of demand. Therefore, a simple linear function for the accumulation rate relating net investment to the capital stock can be formulated as below:

$$g^i = \frac{\Delta K}{K} = \alpha + \beta u + \tau h - \theta \lambda i \qquad \text{where } \alpha, \beta, \tau, \theta > 0. \tag{25}$$

The parameter α stands for the motivation to accumulate which derives from the competition of firms independently of the influence of distribution, effective demand, monetary or financial variables. The intensity of the influence of effective demand is indicated by β, whereas τ shows the weight of the income distribution variable and θ the impact of debt and the interest rate. Obviously, to induce investors to demand real capital goods instead of financial assets, the expected rate of profit has to exceed the rate of interest in financial markets. This investment equation can be found in Hein (2005), and this is the investment equation that will be adopted here.

3. Short run equilibrium

Following Lavoie (1995), we take the debt-capital ratio as a given constant in the short run, however, it becomes a variable to be endogenously determined in the long run. The short run equilibrium requires the adjustment of production and capacity utilization to effective demand in the goods market. Therefore, the equilibrium condition is given by:

$$g^i = g^s \tag{26}$$

The short run equilibrium will be stable, if the savings function responds less elastically than the investment function to a variation in the profits rate (or in the rate of capacity utilization). It requires that:

$$\frac{\partial g^s}{\partial u} - \frac{\partial g^i}{\partial u} > 0, \tag{27}$$

Since $\frac{\partial g^s}{\partial u} = s_T \frac{h}{v}$, and $\frac{\partial g^i}{\partial u} = \beta$, Therefore:

$$\frac{\partial g^s}{\partial u} - \frac{\partial g^i}{\partial u} > 0, \text{ if and only if } s_T \frac{h}{v} - \beta > 0 \tag{28}$$

Following the equilibrium condition and combining equations (23) and (25), then we get the equilibrium values (*) for capacity utilization, capital accumulation and the rate of profit in the short run which are as follows:

$$u^* = \frac{\lambda i[s_f(1-s_Z)-\theta]+(\alpha+\tau h)}{s_T\dfrac{h}{v}-\beta} \tag{29}$$

$$g^* = \frac{\lambda i\{\beta[s_f(1-s_Z)]-s_T\dfrac{h}{v}\theta\}+s_T\dfrac{h}{v}(\alpha+\tau h)}{s_T\dfrac{h}{v}-\beta} \tag{30}$$

$$r^* = \frac{\dfrac{h}{v}\{\lambda i[s_f(1-s_Z)-\theta]\}+\dfrac{h}{v}(\alpha+\tau h)}{s_T\dfrac{h}{v}-\beta} \tag{31}$$

Under the given assumption that the debt-capital ratio (λ) is constant in the short run, taking the derivatives of equations (29), (30) and (31) with respect to the rate of interest, we obtain the following reactions of the equilibrium variables from variations in the rate of interest:

$$\frac{\partial u^*}{\partial i} = \frac{\lambda[s_f(1-s_Z)-\theta]+\dfrac{\partial h}{\partial i}(\tau-s_T\dfrac{h}{v})}{s_T\dfrac{h}{v}-\beta} \tag{32}$$

$$\frac{\partial g^*}{\partial i} = \frac{\lambda\{\beta[s_f(1-s_Z)]-s_T\dfrac{h}{v}\theta\}+\dfrac{\partial h}{\partial i}\dfrac{1}{v}s_T(\tau h-\beta u)}{s_T\dfrac{h}{v}-\beta} \tag{33}$$

$$\frac{\partial r^*}{\partial i} = \frac{\dfrac{h}{v}\lambda[s_f(1-s_Z)-\theta]+\dfrac{\partial h}{\partial i}\dfrac{1}{v}s_T(\tau h-\beta u)}{s_T\dfrac{h}{v}-\beta} \tag{34}$$

Considering the cases that provide for short-run stability, from equations (32) to (34), we can find that, in the short term, effects of variations in the rate of interest on these variables are uncertain, since they depend on the interest rate elasticity of the mark-up, on the value taken by the debt-capital ratio (λ), as well as on the values taken by the parameters in the savings function, i.e. the propensity to save of households (s_Z), and the retention rate (s_f), on one hand, and in the investment function, i.e., the elasticity of investment with respect to the debt and the interest rate (θ), to the profit share (τ) and to capacity utilization (β), on the other hand.

First of all, suppose a regime where an interest-inelastic mark-up dominates. The dynamics of the equilibrium values of the model variables to variations in the interest rate is mainly determined by households' propensity to save, the retention rate of firms, the debt and the debt service elasticity of investment. In the so called 'normal' regime in post-Keynesian models where, usually, households' savings propensity is rather high, the retention rate of firms is rather low, and the debt service elasticity of investment is rather high as well, then an increase in the rate of interest will be more likely to decrease the model variables: rates of capacity utilization, profit and capital accumulation.

On the other hand, in a regime that Lavoie (1995) calls the 'puzzling' regime, we observe a completely different development: there is a positive relationship between the real rate of interest and the rate of accumulation, at least in the short run. In this regime, investment usually is hardly affected by debt and debt servicing, which implies the debt service elasticity of investment is rather low, the propensity to save out of interest and dividends income is also relatively low and the retention rate of firms is rather high. In such a case, an increase in the rate of interest will generally cause an increase in the values of the rates of capacity utilization, profit and capital accumulation. Theses results are similar to Lavoie's (1995). With an interest-inelastic mark-up, the debt-capital ratio does not play a crucial role in determining the direction of development of the equilibrium values, however, it does play an important way in deciding the "speed" of the dynamics of the equilibrium values, or more precisely, the extent of the change. From equations (32) to (34), it is not difficult for one to reach the conclusion that: the lower the debt-capital ratio, the smaller will be the effects of variations in the rate of interest, and vice versa. Especially, when the debt-capital ratio equals zero, under the assumptions of our model, variations in the interest rate will not affect the short run equilibrium position at all, as long as the interest rate is still lower than the profit rate (This is a condition that keeps firms in business). (Table 3)

Table 3: Dynamics of the model to variations in the rate of interest to a stable short run equilibrium, with an interest-inelastic mark-up			
$\frac{\partial h}{\partial i}=0$	$s_f(1-s_Z)<\theta$	$\theta<s_f(1-s_Z)<\frac{s_T\frac{h}{v}}{\beta}\theta$	$\frac{s_T\frac{h}{v}}{\beta}\theta<s_f(1-s_Z)$
Assuming short run equilibrium stable $(s_T\frac{h}{v}-\beta>0)$	$\frac{\partial u}{\partial i}<0$; $\frac{\partial r}{\partial i}<0$; $\frac{\partial g}{\partial i}<0$;	$\frac{\partial u}{\partial i}>0$; $\frac{\partial r}{\partial i}>0$; $\frac{\partial g}{\partial i}<0$;	$\frac{\partial u}{\partial i}>0$; $\frac{\partial r}{\partial i}>0$; $\frac{\partial g}{\partial i}>0$;

In the following paragraph, we will consider the case of an interest-elastic mark-up, which was not discussed in Lavoie (1995). In this case, the debt-capital ratio does play an utmost role in determining the direction of development of the equilibrium values, as well as the "speed" of the dynamics of the equilibrium values. As shown in model dynamics equations from (32) to (34), we can divide the effects of variations of the rate of interest into two parts: the general direct effects which is the same as the one in the interest-inelastic case, and the indirect effects which take place through a change in the markup. If these effects are in the same direction, the debt-capital ratio will play a similar role as it does in the interest-inelastic markup case: it only has an impact on the extent of the change but not on its direction. However, when these two effects are in the opposite directions, the debt-capital ratio becomes extremely important since it determines not only the extent but also the direction of the change. Suppose a regime, when the

Table 4: Dynamics of the model to variations in the rate of interest to a stable short run equilibrium, with an interest-elastic mark-up				
$\frac{\partial h}{\partial i}>0$		$s_f(1-s_Z)<\theta$	$\theta<s_f(1-s_Z)<\frac{s_T\frac{h}{v}}{\beta}\theta$	$\frac{s_T\frac{h}{v}}{\beta}\theta<s_f(1-s_Z)$
$\frac{\partial u}{\partial i}$	$\tau<s_T\frac{h}{v}$	$\frac{\partial u}{\partial i}$<0	Uncertain^{u*}	Uncertain^{u*}
	$\tau>s_T\frac{h}{v}$	Uncertain^{u*}	$\frac{\partial u}{\partial i}$>0	$\frac{\partial u}{\partial i}$>0
$\frac{\partial r}{\partial i}$	$\tau h<\beta u$	$\frac{\partial r}{\partial i}$<0	$\frac{\partial r}{\partial i}$<0	Uncertain^{r*}
	$\tau h>\beta u$	Uncertain^{r*}	Uncertain^{r*}	$\frac{\partial r}{\partial i}$>0
$\frac{\partial g}{\partial i}$	$\tau h<\beta u$	$\frac{\partial g}{\partial i}$<0	Uncertain^{g*}	Uncertain^{g*}
	$\tau h>\beta u$	Uncertain^{g*}	$\frac{\partial g}{\partial i}$>0	$\frac{\partial g}{\partial i}$>0

Where: Uncertain^{u*}: $\frac{\partial u}{\partial i}$>0, if and only if $\lambda[s_f(1-s_Z)-\theta]+\frac{\partial h}{\partial i}(\tau-s_T\frac{h}{v})$

Uncertain^{r*}: $\frac{\partial r}{\partial i}$>0, if and only if $\frac{h}{v}\lambda[s_f(1-s_Z)-\theta]+\frac{\partial h}{\partial i}\frac{1}{v}s_T(\tau h-\beta u)>0$

Uncertain^{g*}: $\frac{\partial g}{\partial i}$>0, if and only if $\lambda\{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta\}+\frac{\partial h}{\partial i}\frac{1}{v}s_T(\tau h-\beta u)>0)$

debt-capital ratio is rather low, the positive direct effects reacting to the variations of interest rate on investment would be less than the negative indirect effects exerted by the redistribution of income between profits and wages (due to the changes in markup caused by variations of interest rate) on investment, thus the indirect effects would dominate and we observe a negative relationship between interest rates and accumulation. However, if the debt-capital ratio is rather high, the positive direct effects reacting to the variations of interest rate on investment may be much stronger than the negative indirect effects caused by the redistribution of income between profits and wages on investment, such that the direct effects are overwhelming and thus we observe a positive relationship between interest rates and capital growth. A similar analysis will also apply to the influence of the debt-capital ratio on the rates of capacity utilization and the profit rate. Therefore, in the case of an interest-elastic mark-up, the debt-capital-ratio is of utmost importance, as it determines the extent and the direction of the change imposed by an increase in interest rates. (Table 4)

4. Long run effects of variations in the interest rate on the leverage ratio

In the long run, the value of the leverage ratio is an endogenous variable, and thus will be affected gradually by variations in the interest rate. The dynamics of the leverage ratio can be found by taking the logarithmic derivative of equation (4). Assuming away inflation, the price level will not change, even though the mark-up may do so. The dynamics of the leverage ratio is given by:

$$\hat{\lambda} = \hat{B} - \hat{K} = \hat{B} - g \tag{35}$$

With the above assumptions, the additional long-term credit granted to firms, or put it another way, the new loans contracted by firms, in each period corresponds to the savings of the bonds holders (households that hold deposits) in the period. In growth terms, we get:

$$\hat{B} = s_Z i \tag{36}$$

In long run equilibrium, the endogenously determined debt-capital-ratio equals zero:

$$\hat{\lambda} = \hat{B} - \hat{K} = 0 \tag{37}$$

Integrating this condition into equation (35) and making use of equations (30) and (36) we get the long run equilibrium debt-capital-ratio, which should be between zero and one, by definition:

$$\lambda^* = \frac{s_Z i(s_T \frac{h}{v} - \beta) - s_T \frac{h}{v}(\alpha + \tau h)}{i\{\beta[s_f(1 - s_Z)] - s_T \frac{h}{v}\theta\}} \tag{38}$$

This equilibrium will be stable, if $\frac{\partial \hat{\lambda}}{\partial \lambda} < 0$ as asserted by Lavoie (1995, p. 168)). This condition is related to the relative sensitivity of the savings and investment functions. Making use of equation (35) and applying equations (30) and (36) yields:

$$\frac{\partial \hat{\lambda}}{\partial \lambda} = -\frac{i\{\beta[s_f(1 - s_Z)] - s_T \frac{h}{v}\theta\}}{s_T \frac{h}{v} - \beta} < 0 \tag{39}$$

When the short run stable equilibrium holds, we have: $s_T \frac{h}{v} - \beta > 0$, thus the long run stability condition can be expressed as:

$$\frac{\partial \hat{\lambda}}{\partial \lambda} < 0, \text{ if } \beta[s_f(1 - s_Z)] - s_T \frac{h}{v}\theta > 0. \tag{40}$$

The long run equilibrium stable condition would hold if the bonds holders' savings propensity is relatively low and investment decisions are very elastic with respect to changes in capacity utilization but very inelastic with respect to changes in debt services. As in Lavoie (1995), this is the case that favors a 'puzzling' positive effect of the increasing of the rate of interest on capacity utilization, capital accumulation and the profit rate in the short run. However, if bond holder households' savings propensity is rather high and investment decisions are very inelastic with respect to demand but very elastic with respect to debt and debt services, then the long run equilibrium stable condition would be violated and the economy tends to be unstable. If the stable long run equilibrium condition does not hold, rising interest rates may trigger either a falling equilibrium debt-capital ratio until it reaches zero or a rise in the ratio until it becomes unity, as in Lavoie (1995). Thus deviations from equilibrium will generate a long run debt-capital ratio of either unity or zero. The conditions for long run instability are associated with short run 'normal' negative effects which refer to cases in which an increase of the rate of interest cause a fall in the rates of capacity utilization, capital accumulation and profits.

The effects of variations in the rate of interest on the equilibrium debt-capital-ratio can be derived from equation (38):

$$\frac{\partial \lambda}{\partial i} = \frac{s_Z(s_T \frac{h}{v} - \beta) - \lambda\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\} + \frac{\partial h}{\partial i}\frac{s_T}{v}[i(\theta\lambda + s_Z) - \alpha - 2\tau h]}{i\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\}} \qquad (41)$$

Let's start with the case of an interest-inelastic mark-up ($\frac{\partial h}{\partial i} = 0$). When $\frac{\partial \lambda}{\partial i}$=0, the economy reaches its maximum value of long run equilibrium debt-capital ratio, that is equation (41) equals zero. With an interest-inelastic mark-up ($\frac{\partial h}{\partial i} = 0$), we have the following:

$$\frac{\partial \lambda}{\partial i} = \frac{s_Z(s_T \frac{h}{v} - \beta) - \lambda\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\}}{i\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\}} = 0 \qquad (42)$$

Solving for the maximum value of the long run equilibrium debt-capital ratio, we get[6]:

[6] Hein (2005) pointed out that initial debt-capital ratio should play a role, but he did not analyze the maximum ratio.

$$\lambda_{\max}* = \frac{s_Z(s_T\frac{h}{v} - \beta)}{\beta[s_f(1-s_Z)] - s_T\frac{h}{v}\theta} \tag{43}$$

Normally, if without disrupting economic shocks, we should expect that: $\lambda \le \lambda_{\max}*$.

Naturally, we may take it for granted that the value of actual debt-capital ratio should lay between zero and one, that is, $0 < \lambda < 1$. If $0 < \lambda < 1$ is regarded as a natural bounded condition for the economy, then $\lambda \le \lambda_{\max}*$ should be taken as an endogenously (hidden) bounded condition. One should easily find that, as long as the maximum value of long run equilibrium debt-capital ratio lies out of the range from zero to one, that is, $\lambda_{\max}* < 0$ or $\lambda_{\max}* > 1$, the hidden condition will be excluded by the natural condition and thus need not be considered additionally. However, if the maximum value lies right between zero and one, that is, $0 < \lambda_{\max}* < 1$, then the hidden condition desires additional attention. More details will be discussed in the next chapter.

Given the conditions for long run stability, increasing interest rates will have a negative or positive impact on the equilibrium debt-capital ratio, depending on the initial level of the debt-capital-ratio the economy begins with. Firstly, suppose that the initial debt-capital ratio is so high that it exceeds the maximum value ($\lambda_0 > \lambda_{\max}*$), the rising interest rate will decrease the equilibrium debt-capital ratio. As the debt-capital ratio decreases gradually as a result of the increased interest rate, the economy will be pushed toward the direction of a lower long run equilibrium debt-capital ratio. And finally the economy will stay at the maximum level of equilibrium debt-capital ratio in the long run.

On the other hand, when the initial debt-capital ratio is such that it is below the maximum value ($\lambda_0 < \lambda_{max}*$) (probably most cases), the increase in the rate of interest will have a positive effect on the long run equilibrium debt-capital ratio. Similar to the above discussion, but in an opposite direction, as the debt-capital-ratio adjusts slowly to the increased interest rate, it will approach the long run equilibrium debt-capital ratio continuously. And finally the higher interest rate will lead to an increase in the debt-capital ratio until it reaches its equilibrium level in the long term.

The above consideration regarding the endogenous (hidden) condition is absent from Lavoie (1995) and Hein (2005), and hence this result is different from the unique positive relationship between the interest rate and the debt-capital ratio that was derived by Lavoie (1995) in the case of a stable long run equilibrium. However, we find that the unique case studied in Lavoie (1995) is a only special case in our model, when the maximum value of the debt-capital ratio is greater than one ($\lambda_{max}* > 1$). As we will discuss in the next chapter, the unstable case in Lavoie's (1995) model turns out to be another special case when the maximum value of the debt-capital ratio is less that zero ($0 > \lambda_{max}*$). Without additional considerations for the hidden bounded condition, as a result, only these two special cases, in which the hidden bounded condition is excluded by the natural bounded condition ($0 < \lambda < 1$) and thus need not to be considered, are actually analyzed in Lavoie (1995), and hence the other case ($0 < \lambda_{max}* < 1$) is ignored.

Nevertheless, we can also show that, only the stable case discussed by Lavoie is likely to be feasible in practice, because of the following:

$$\lambda^* = \frac{s_Z i(s_T \frac{h}{v} - \beta) - s_T \frac{h}{v}(\alpha + \tau h)}{i\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\}} = \lambda_{\max}{}^* - \frac{1}{i}\frac{s_T \frac{h}{v}(\alpha + \tau h)}{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta} \le \lambda_{\max}{}^* \quad (44)$$

Mathematically, under the given assumptions, which imply that: $\frac{s_T \frac{h}{v}(\alpha + \tau h)}{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta} > 0$. Only when the rate of interest tends to infinite (that is, $\lim_{i \to \pm\infty} \frac{1}{i} = 0$), can the stable equilibrium debt-capital ratio equal the maximum value:

$$\lim_{i \to \pm\infty} \lambda^* = \lambda_{\max}{}^* - \left(\lim_{i \to \pm\infty} \frac{1}{i}\right)\left(\frac{s_T \frac{h}{v}(\alpha + \tau h)}{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta}\right) = \lambda_{\max}{}^* \quad (45)$$

Otherwise, the long run stable equilibrium debt-capital ratio is always less than the maximum value. Certainly, an infinite interest rate is a rather ludicrous assumption. Therefore, though all cases may hold in theory, however, only the stable case discussed in Lavoie (1995) is feasible in reality.

Now we consider the case of an interest-elastic mark-up ($\frac{\partial h}{\partial i} > 0$), which is not discussed in Lavoie (1995), but discussed in Hein (2005). We find that, in this case, the value of the debt-capital ratio plays a much more crucial role in our model: it determines not only the extent of the change caused by effects of variations in the interest rate, but also the direction of the change. If the conditions for stable long run equilibrium hold, while investment decisions are rather inelastic with respect to unit labor costs and the initial interest rate is rather high, so that $[i(\theta\lambda + s_Z) - \alpha - 2\tau h] > 0$, the positive direct effect from the variation of the interest rate on the debt-capital ratio would be re-enforced by the indirect effect arising from the higher markup resulting from the increase in the interest rate. However, if investment decisions are rather elastic with respect to unit labor costs and the initial interest rate is rather low, so that $[i(\theta\lambda + s_Z) - \alpha - 2\tau h] < 0$, then the negative indirect effect of the higher markup induced by higher interest rates may dominate, dampening and even reversing the positive direct effect exercised by the higher interest rate on the debt-capital ratio. If the long run equilibrium is unstable, low unit labor cost elasticity of investment and a high interest rate in initial equilibrium, so that $[i(\theta\lambda + s_Z) - \alpha - 2\tau h] > 0$, the negative direct effect of the variations of the rate of interest on the long run equilibrium debt-capital ratio will be reinforced by the negative indirect effect of the higher markup. If the unit labor cost elasticity of investment is very high and increasing interest rates start from a low level, so that $[i(\theta\lambda + s_Z) - \alpha - 2\tau h] < 0$, the final impact is determined by the dominant effect between the positive indirect effect of the rising markup and the negative direct effect caused by the variation in the interest rate on the debt-capital ratio.

As discussed above, in the model, under the natural bounded condition and the hidden condition, the relation between the interest rate and the equilibrium debt-capital ratio does not only depend on the parameters of the savings and the investment function, but also on the initial value of the debt-capital ratio, in the case of a stable long run equilibrium, as well as the initial value of the interest rate if we consider the case with the interest-elastic mark-up. The conclusion in our model is quite similar to what has been studied in Lavoie (1995) and Hein (2005), but with more complete considerations.

5. The complete dynamics of the model

In this chapter, we are ready to explore the complete dynamics of our model in detail. In the previous two chapters, we have shown respectively that the long run and short run effects of variations in interest rate depend on parameters in the investment and savings function, such as households' propensity to save, the retention rate of firms, the elasticity of investment with respect to debt and the interest rate, the reaction of investment to capacity utilization and to unit labor costs, interest rate elasticity of the mark-up, and the initial values of the interest rate and the equilibrium debt-capital ratio. The following equations show the long run effects of variations in the rate of interest on the model variables:

$$\frac{\partial\lambda}{\partial i}=\frac{s_Z(s_T\frac{h}{v}-\beta)-\lambda\{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta\}+\frac{\partial h}{\partial i}\frac{s_T}{v}[i(\theta\lambda+s_Z)-\alpha-2\tau h]}{i\{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta\}} \tag{46}$$

$$\frac{\partial u^*}{\partial i}=\frac{(\lambda+i\frac{\partial\lambda}{\partial i})[s_f(1-s_Z)-\theta]+\frac{\partial h}{\partial i}(\tau-s_T\frac{h}{v})}{s_T\frac{h}{v}-\beta} \tag{47}$$

$$\frac{\partial g^*}{\partial i}=\frac{(\lambda+i\frac{\partial\lambda}{\partial i})\{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta\}+\frac{\partial h}{\partial i}\frac{1}{v}s_T(\tau h-\beta u)}{s_T\frac{h}{v}-\beta} \tag{48}$$

$$\frac{\partial r^*}{\partial i}=\frac{\frac{h}{v}(\lambda+i\frac{\partial\lambda}{\partial i})[s_f(1-s_Z)-\theta]+\frac{\partial h}{\partial i}\frac{1}{v}s_T(\tau h-\beta u)}{s_T\frac{h}{v}-\beta} \tag{49}$$

To start with the dynamics of the short and long run effects of changing interest rates in the model, we will only consider cases where short run stability holds, since it will be meaningless to discuss the long run equilibrium if the model is in disequilibrium even in the short run. For simplification, only the case of an interest-inelastic mark-up is explicitly discussed. However, one could expect that the result, for the case of an interest-elastic mark-up, will be similar only with more complicated dynamic equations.

With short run stability being assumed, long run stability requires the following condition:

$$\frac{\partial \hat{\lambda}}{\partial \lambda} < 0 \text{, if and only if } \beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta > 0. \tag{40}$$

Denoting the maximum equilibrium debt-capital ratio as: $\lambda_{\max}{}^* = \dfrac{s_Z(s_T \frac{h}{v} - \beta)}{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta}$,

then, with an interest-inelastic mark-up, we get:

$$\frac{\partial g}{\partial i} = \frac{\lambda\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\}}{s_T \frac{h}{v} - \beta} = \frac{s_Z \lambda}{\lambda_{\max}{}^*} \quad \text{(Short run effect)} \tag{50}$$

$$\frac{\partial g}{\partial \lambda} = \frac{i\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\}}{s_T \frac{h}{v} - \beta} = \frac{s_Z i}{\lambda_{\max}{}^*} \quad \text{(Long run effect)} \tag{51}$$

$$\frac{\partial \lambda}{\partial i} = \frac{1}{i}\left[\frac{s_Z(s_T \frac{h}{v} - \beta)}{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta} - \lambda\right] = \frac{\lambda_{\max}{}^* - \lambda}{i} \quad \text{(Long run effect)} \tag{52}$$

$$\frac{\partial g}{\partial i} = \frac{(\lambda + i\frac{\partial \lambda}{\partial i})\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\}}{s_T \frac{h}{v} - \beta} = s_Z \quad \text{(Long run effect)} \tag{53}$$

$$\frac{\partial \hat{\lambda}}{\partial \lambda} = -\frac{i\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\}}{s_T \frac{h}{v} - \beta} = -\frac{s_Z i}{\lambda_{max}*} \qquad (54)$$

From this simplified dynamics equation, we can easily deal with the model dynamics of these dependent variables resulting from variations in the rate of interest both in the short run and in the long run. (Table 5)

Table 5 Dynamics of the model with an interest-inelastic markup when the interest rate changes permanently (Assuming short run stability)

$\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta$		Negative	Positive		
Long run Stability		Unstable	Unstable		Stable
Initial value of λ_0		$\lambda_{max}*<0<\lambda_0*$	$0<\lambda_{max}*<\lambda_0*$	$\lambda_1*<\lambda_{max}*<1$	$\lambda_0*<1<\lambda_{max}*$
A (50)	$\frac{\partial g}{\partial i}$, $\lambda=\bar{\lambda}$	Negative	Positive	Positive	Positive
B (52)	$\frac{\partial \lambda}{\partial i}$	Negative	Negative	Negative	Positive
C (51)	$\frac{\partial g}{\partial \lambda}$, $i=\bar{i}$	Negative	Positive	Positive	Positive
D (53)	$\frac{\partial g}{\partial i}$	Positive	Positive	Positive	Positive
E (54)	$\frac{\partial \hat{\lambda}}{\partial \lambda}$, $i=\bar{i}$	Positive	Negative	Negative	Negative

On one hand, in the case where, $\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta>0$, which is the stable condition for the long run equilibrium debt-capital ratio, as in Lavoie (1995), the economy would behave in such a way that households' propensity to save is rather low while firms are not sensitive to changes in leverage ratios or interest payment. Such behaviors imply that, relative to saving, the investment elasticity is rather high with respect to capacity utilization but rather low with respect to interest payments and debts. Therefore, the condition implies that, in the regime when the debt-capital ratio changes due to variations in the rate of interest, creditors do not change their willingness to engage in long term finance and debtors do not change their willingness to invest very much. Then investment decisions are determined by sales expectations of firms, rather than by the risks associated with increasing debt finance. This particular phenomenon may be more consistent with what we observe in bank-based finance systems that are characterized by more stable long-term relations between firms and creditors, instead of the short-term relations dominating in capital-market based systems (Grabel (1997)).

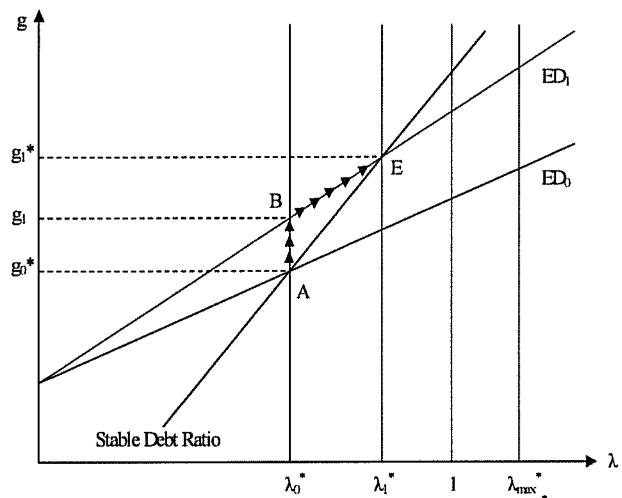

Figure 1 the stable long-run equilibrium dynamics with a "regular" initial λ_0^*

However, with respect to the hidden bounded condition that is not considered in Lavoie (1995) and Hein (2005), the above condition: $\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta>0$ is a necessary but not sufficient condition for the long run stable equilibrium for our model since the hidden bounded condition implies that the starting value of the initial equilibrium dept-capital ratio has to be considered. As long as the model starts with a "high" initial equilibrium dept-capital ratio relatively to a given maximum ratio, the stable long run equilibrium will not exit. Thus the stable long run equilibrium will exist only when, as most cases are likely to be, the model starts with a "regular" initial equilibrium debt-capital ratio relatively to the maximum ratio.

Firstly we consider the stable case when the model starts with a "regular" initial equilibrium debt-capital ratio relatively to the given maximum ratio ($\lambda_0 < 1 < \lambda_{max}*$), which is also the stable case analyzed in Lavoie (1995) because the hidden bounded condition is excluded by the natural condition and thus need not to be considered. Under the stable long run equilibrium, the effective demand curve ED has a positive slope, thus it rotates upwards as the interest rate increases permanently, as shown in Figure 1. In the short run, with a constant debt-capital-ratio, the rate of capital accumulation will increase. This is the so-called short-term "puzzling" case, which shows a counterintuitive positive relation between accumulation and interest rates. In the long run, the higher interest rate induces a gradual increase in the debt-capital ratio of firms. Consequently, the increasing debt-capital ratio of firms will push up the accumulation rate until it reaches the long run stable equilibrium debt-capital ratio. Therefore, as pointed out by Lavoie (1995), in this case, variations in the rate of interest have positive impacts on the long run equilibrium debt-capital ratio, as well as on the accumulation rate. (Figure 1, Table 5)

Secondly, we consider one of other two possible (long-run) unstable cases when the model starts with a too "high" initial equilibrium debt-capital ratio relatively to the given maximum ratio. With long run stability, as in the previous case, the effective demand curve ED has a positive slope, thus it rotates upwards as the interest rate increases permanently. In the short run, with a constant debt-capital ratio, the rate of capital accumulation will increase.

However, in the long run, while the equilibrium debt-capital ratio is shifted to the right, suppose that it moves from λ_0^* to λ_1^* in Figure 2, the actual debt-capital-ratio is induced to a gradual decrease by the higher interest rate under the hidden bounded condition ($\lambda \leq \lambda_{max}$*), which drives the economy in the opposite direction and thus pulls it away from its long run equilibrium. That is, the decreasing debt-capital ratio will induce the

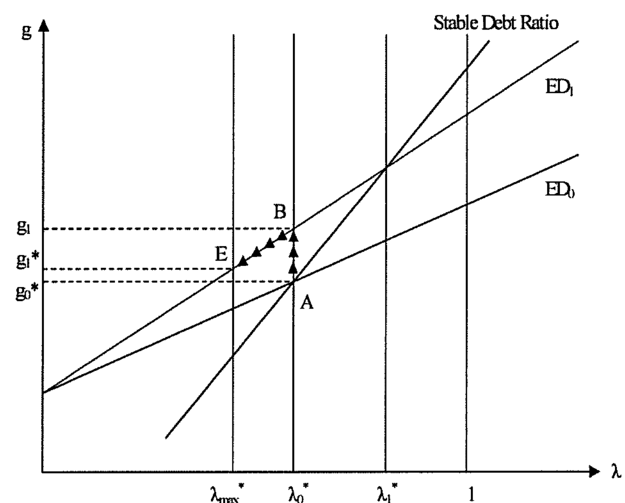

Figure 2 the unstable long-run equilibrium dynamics with a too "high" initial λ_0^*

economy to move to the given maximum debt-capital ratio (λ_{max}*) along the effective demand curve. Consequently, due to a positive slope of the effective demand curve in this case, the lower debt-capital ratio of firms will push down the accumulation rate along the effective demand curve until it reaches the upper bound of the equilibrium debt-capital ratio. As long as the long run negative effect on accumulation resulting from the decrease of the debt-capital ratio is greater than the positive one caused by the increasing interest rate in the short run, there the higher interest rate will have a negative impact. Otherwise the long run impact of the increase in the interest rate on the capital accumulation rate will be positive. This case is similar to the unstable case when interest rate decrease, but it ends up with a lower accumulation rate due to the positive slope of the ED curve, and a slightly different result to some extent: the economy finally stops at the position of its bounded maximum debt-capital ratio instead of zero. However, the rationality underlining the phenomenon is the same, which will be fully discussed later. (Figure 2)

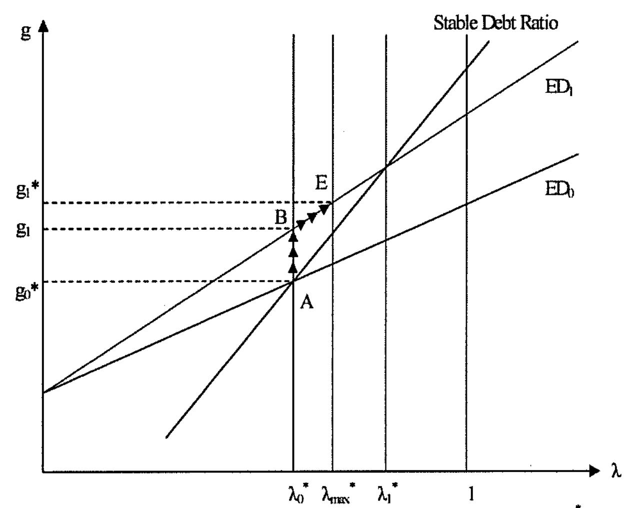

Figure 3 the unstable long-run equilibrium dynamics with a rather "high" initial λ_0^*

Thirdly, we consider the last possible long-run unstable case, when the model starts with a rather "high" initial equilibrium debt-capital ratio relatively to the given maximum ratio, which is such that the new equilibrium debt-capital ratio resulting from the higher interest rate will be greater than the maximum value ($\lambda_1^* < \lambda_{max}^* < 1$). With long-run stability, the effective demand curve ED still has a positive slope, thus it rotates upwards as the interest rate increases permanently. In the short run, with a constant debt-capital ratio, the rate of capital accumulation will increase. This is also consistent with the short run "puzzling" case. However, in the long run, while the equilibrium debt-capital ratio is shifted to the right, suppose from λ_0^* to λ_1^* in Figure 3, the actual debt-capital-ratio is also induced to rise gradually by the higher interest rate, which will drive the economy to move along the positive effective demand curve. However, as long as the economy reaches the position of the given maximum debt-capital ratio, it will be bounded there and thus the new long run equilibrium debt-capital ratio resulting from the increasing of the interest rate can never be fulfilled, under the hidden bounded condition ($\lambda \leq \lambda_{max}$*). As a result, an increase of the interest rate will finally cause an increase in both the debt-capital ratio and the accumulation rate. (Figure 3)

Though the above three cases all assume that, $\beta[s_f(1-s_Z)] - s_T\frac{h}{v}\theta > 0$, only the case where the model starts with a "regular" initial equilibrium debt-capital ratio relative to the given maximum ratio ($\lambda_0 < 1 < \lambda_{max}$*) is stable, which implies that the stable condition in Lavoie (1995) turns out to be only a necessary but not a sufficient condition.

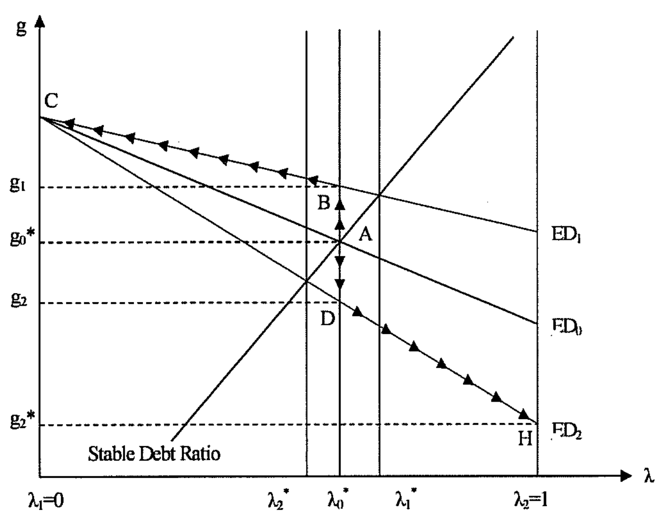

Figure 4 the long-run equilibrium dynamics of unstable leverage rate

On the other hand, if $\beta[s_f(1-s_Z)] - s_T\frac{h}{v}\theta < 0$, the economy would possibly behave in a completely opposite way such that: households' propensity to save is rather high and firms are rather sensitive to changes in leverage ratios or interest payment. Under these conditions, investment elasticity is rather low with respect to capacity utilization but rather high with respect to interest payments and debt. As a result, when the debt-capital ratio varies resulting from variations in the rate of interest, creditors' willingness to engage in long-term finance would be influenced significantly. Then investment decisions are determined by the risks associated with increasing debt finance, instead of sales expectations of firms, as was the case with long-run stability. These features would be found more likely in capital-market based finance systems in which the short-term unstable relations dominate. (Grabel (1997))

If interest rates increase under a long run unstable debt-capital-ratio condition, in the short run, with a constant debt-capital-ratio, capital accumulation will decrease because the effective demand curve ED has a negative slope now and thus it rotates downwards as the interest rate increases permanently. The long run variable debt-capital-ratio will be shifted to the left, suppose from λ_0 to λ_2^* (as in Figure 4). However, the actual debt-capital-ratio is induced to a gradual increase by the higher interest rate, which drives the economy to the opposite direction and thus pulls it away from its long run equilibrium. Due to the negative slope of the effective demand curve in this case, the gradually increasing debt-capital ratio will drive the debt-capital ratio towards unity, while the growth rate falls continuously. If interest rates decrease permanently, by contrast, short run capital accumulation will improve at a given debt-capital-ratio. Meanwhile, the long run equilibrium debt-capital ratio will be shifted to the right, say from λ_0 to λ_1^*. As the actual debt-capital ratio gets modified, the economy moves towards the left-hand side of

Figure 5, getting away from its equilibrium long run value. Moving gradually along the effective demand curve, in the long run, higher and higher levels of accumulation will be accompanied with lower debt-capital ratios, which will eventually reach their lower bound: zero. Therefore, with long-run instability, economic forces will be such that the economy flows away from its long-run equilibrium position. The above dynamics finally drive the economy to either end of the bounds for debt-capital ratios: zero or unity. (Table 5) The description of this paradoxical situation can also be found in Steindl (1952, pp. 114-118).

As discussed in the above dynamics, starting from a long run equilibrium position, a permanent increase of interest rates has a negative effect on the equilibrium debt-capital ratio in the long run. As a result, the actual debt-capital-ratio diverges from its new long run equilibrium ratio. As firms try to provide remedy to what is going on, the actual debt-capital-ratio moves up even further, which once again, results in a new round of reactions, and thus make the situation worse and worse until the debt-capital ratio finally approaches unity. The disequilibrium process is characterized by falling accumulation rates and rising debt-capital ratios, both triggered by an increasing interest rate.

This is the so called macroeconomic "paradox of debt": as the rate of interest rises, naturally, firms attempt to reduce leverage ratios and thus interest payments by cutting down investment. However, the reactions are actually pushing firms to an unexpected situation: their actual debt-capital ratio and hence interest payments are going up. The higher undesired debt-capital ratio induces firms to further cut down investment. Thus the divergence continues automatically and never stops unless it reaches the bound of the economy. When interest rates decrease, the disequilibrium process happens in the opposite direction: the impact of decreasing interest rates pulls the debt-capital ratio down to its bottom and pushes up the rate of capital accumulation.

6. Conclusions

In this paper, we build a more complete Kaleckian distribution and growth model based on the model first built by Lavoie (1995) and further developed by Hein (1999, 2005). We extend the analysis of the impacts of variations of interest rate on indebtedness, wealth distribution and capital accumulation in both short run and long run equilibria by taking worthwhile factors, which had been ignored completely or partially in previous models of both Lavoie and Hein and thus not fully discussed, such as initial values of the debt-capital ratio relative to the maximum debt-capital ratio. This (theoretical) maximum ratio plays an important role in determining the model dynamics. In my paper, I have shown that the condition for stable long run equilibrium in Lavoie's (1995) and Hein's (2005) models is only a necessary but not sufficient condition and thus needs to be modified accordingly with the stricter condition.

On the one hand, in the short run, with a fixed debt-capital ratio assumption, the effects of interest variations on the stable equilibrium rates of capacity utilization, capital accumulation and profit in our model will depend on the given economic behavior through investment and savings functions, such as households' savings propensity, firms' retention rate and the elasticity of investment with respect to debt and capacity utilization, as well as the interest rate elasticity of the mark-up [as introduced by Hein(2005)]. Generally speaking, the effects of variations of the interest rate on the real equilibrium may be either negative ('normal case') or positive ('puzzling case'), depending on the exact economic behavior, which is represented by the values of the above parameters. In particular, when, under a given situation, the agents in the economy react in such a way that both the households' savings propensity and the responsiveness of investment with respect to debt are rather low, while both firms' retention rate and the responsiveness of investment with respect to capacity utilization are rather high, then more likely, the effect between the interest rate and the rates of capacity utilization, capital accumulation and the profit rate is positive. And vice versa. Under the assumption of an interest-inelastic mark-up, the initial debt-capital ratio will not affect the direction of the impacts of variations in the interest rate on the above dependent variables, which is consistent with the conclusion of Lavoie and Hein. In the case of an interest-elastic mark-up, as proposed by Hein (2005) but excluded from Lavoie (1995), then the initial debt-capital ratio relative to the maximum debt ratio that can be achieved under given economic behavior will have an influence on the direction of the impacts of variations in interest rates on the model equilibrium, whereas Hein (2005) stated that only the initial debt-capital ratio would play a role.

On the other hand, in the long run, when debt-capital ratio varies endogenously, in the stable case, we draw the same conclusion as Lavoie and Hein did: long-run stability is associated with a long run positive relation between the interest rate and the equilibrium rates of capacity utilization, capital accumulation and profit, as well as the short run "puzzling" positive relationship. In this case, the agents in the economy react in such a way that both the households' savings propensity and the responsiveness of investment with respect to debt are rather low, while both the firms' retention rate and the responsiveness of investment with respect to capacity utilization are rather high. However, the long run stability condition suggested in Lavoie (1995) and Hein (2006) turns out to be only a necessary but not sufficient condition since the explicitly hidden maximum bounded condition is not considered at all. The more strict stable equilibrium condition would be connected to a "regular" initial degree of indebtedness relative to the endogenous maximum level of indebtedness. Otherwise, the model equilibrium will be unstable too, as I have shown in the analysis of the two possible additional unstable cases that are said to be stable by Lavoie and Hein.

Nevertheless, the analysis of the unstable case is consistent: as shown by Lavoie and Hein, this case is associated with a short run negative ('normal') relation between the interest rate and the equilibrium rates of capacity utilization, capital accumulation and profit, but with long run instability. In this case, the agents in the regime respond in such a way that, both the households' savings propensity and the responsiveness of investment with respect to debt are rather high, while both the firms' retention rate and the responsiveness of investment with respect to capacity utilization are rather low. This case clearly corresponds to the so-called 'paradox of debt': Rising interest rates cause rising debt-capital ratios and falling rates of capital accumulation. If taking the case of an interest-elastic mark-up into consideration as proposed by Hein (2005), which is not explicitly discussed in the present paper, the conclusion most likely would be similar. The main difference would be that the initial debt-capital ratio relative to the maximum debt ratio will have an influence on the direction of the impacts of variations in the interest rate on the model equilibrium, instead of the initial debt-capital ratio itself as suggested in Hein (2005).

Finally, in the paper, I demonstrate that, the effects of interest variations on the endogenously determined equilibrium values of the model depend not only on the parameter values in the savings and investment functions, as analyzed in Lavoie and Hein, but also on the interest elasticity of distribution as proposed by Hein (2005), as well as the initial interest rate and the initial debt-capital ratio relative to the maximum. This is similar, but not quite the same, as the argument made by Hein (2005), while the argument was ignored altogether in Lavoie (1995).

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Appendix I: Model dynamics of the stable long run equilibrium resulting from changing interest rate, with an interest-inelastic mark-up

The model dynamics equations can be written as:

$$\frac{\partial \lambda}{\partial i} = \frac{s_Z(s_T \frac{h}{v} - \beta) - \lambda\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\} + \frac{\partial h}{\partial i}\frac{s_T}{v}[i(\theta\lambda + s_Z) - \alpha - 2\tau h]}{i\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\}}$$

$$\frac{\partial g^*}{\partial i} = \frac{(\lambda + i\frac{\partial \lambda}{\partial i})\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\} + \frac{\partial h}{\partial i}\frac{1}{v}s_T(\tau h - \beta u)}{s_T \frac{h}{v} - \beta}$$

$$\frac{\partial u^*}{\partial i} = \frac{(\lambda + i\frac{\partial \lambda}{\partial i})[s_f(1-s_Z) - \theta] + \frac{\partial h}{\partial i}(\tau - s_T \frac{h}{v})}{s_T \frac{h}{v} - \beta}$$

$$\frac{\partial r^*}{\partial i} = \frac{\frac{h}{v}(\lambda + i\frac{\partial \lambda}{\partial i})[s_f(1-s_Z) - \theta] + \frac{\partial h}{\partial i}\frac{1}{v}s_T(\tau h - \beta u)}{s_T \frac{h}{v} - \beta}$$

We assume a stable short run goods market equilibrium as well as a stable long run debt-capital ratio and an interest-inelastic mark-up, that is, $s_T\frac{h}{v} - \beta > 0$, $\beta[s_f(1-s_Z)] - s_T\frac{h}{v}\theta > 0$ and $\frac{\partial h}{\partial i} = 0$. Firstly, substituting $\frac{\partial h}{\partial i} = 0$ into the above model dynamic equations, we get:

$$\frac{\partial \lambda}{\partial i} = \frac{s_Z(s_T \frac{h}{v} - \beta) - \lambda\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\}}{i\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\}} = \frac{1}{i}\left[\frac{s_Z(s_T \frac{h}{v} - \beta)}{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta} - \lambda\right]$$

Integrating the above result into other dynamics equations yields:

$$\frac{\partial g^*}{\partial i} = \frac{(\lambda + i\frac{\partial \lambda}{\partial i})\{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta\}}{s_T \frac{h}{v} - \beta} = s_Z$$

$$\frac{\partial u^*}{\partial i} = \frac{(\lambda + i\frac{\partial \lambda}{\partial i})[s_f(1-s_Z) - \theta]}{s_T \frac{h}{v} - \beta} = \frac{s_Z[s_f(1-s_Z) - \theta]}{\beta[s_f(1-s_Z)] - s_T \frac{h}{v}\theta}$$

$$\frac{\partial r^*}{\partial i}=\frac{\frac{h}{v}(\lambda+i\frac{\partial\lambda}{\partial i})[s_f(1-s_Z)-\theta]}{s_T\frac{h}{v}-\beta}=\frac{\frac{h}{v}s_Z[s_f(1-s_Z)-\theta]}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}$$

Since $s_T\frac{h}{v}-\beta>0$ and $\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta>0$, as soon as the real interest rate is positive, we know that the sign of $\frac{\partial\lambda}{\partial i}$ will only depend on the value of the initial debt-capital ratio (λ_0), which can be written as:

$$\frac{\partial\lambda}{\partial i}<0\text{, if and only } \lambda_0>\frac{s_Z(s_T\frac{h}{v}-\beta)}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta},$$

$$\frac{\partial\lambda}{\partial i}=0\text{, if and only } \lambda_0=\frac{s_Z(s_T\frac{h}{v}-\beta)}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta},$$

$$\frac{\partial\lambda}{\partial i}>0\text{, if and only } \lambda_0<\frac{s_Z(s_T\frac{h}{v}-\beta)}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta},$$

Since $\frac{\partial g^*}{\partial i}=s_Z>0$, in the stable long run, rising rates of interest always have a positive effect on the equilibrium rate of capital accumulation.

However, the signs of $\frac{\partial u^*}{\partial i}$ and $\frac{\partial r^*}{\partial i}$ can not be determined unless further information of the economies is provided. The effect of λi on u is ambiguous since it is involved in not only the investment side but also the saving. A bigger debt burden reduces investment demand through the coefficient ($-\theta$) in the accumulation function on one hand, but it also cuts into the available savings for firms' finance through the counterpart $-s_f(1-s_Z)$ in saving function on the other hand. In the so call "debt-led" economies, the first effect will dominate since $s_f(1-s_Z)>\theta$. Otherwise, the economies with coefficients $s_f(1-s_Z)<\theta$ will be called "debt-burden".

So, in a "debt-led" economy with $s_f(1-s_Z)>\theta$, combining given assumptions yields:

$$\frac{\partial u^*}{\partial i}=\frac{s_Z[s_f(1-s_Z)-\theta]}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}>0, \quad \text{and} \quad \frac{\partial r^*}{\partial i}=\frac{\frac{h}{v}s_Z[s_f(1-s_Z)-\theta]}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}>0$$

However, in a "debt-burden" economy with $s_f(1-s_Z)<\theta$, combining given assumptions yields:

$$\frac{\partial u^*}{\partial i}=\frac{s_Z[s_f(1-s_Z)-\theta]}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}<0, \quad \text{and} \quad \frac{\partial r^*}{\partial i}=\frac{\frac{h}{v}s_Z[s_f(1-s_Z)-\theta]}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}<0$$

Therefore, the variations in the rate of interest have ambiguous effect on the dynamics of the rates of capacity utilization and profit, depending on the type of economies we study. In so called "debt-led" economies, the increasing the rate of interest has a positive effect on the stable long run equilibrium of both rates, while the rising interest rate has a negative effect on the stable long run equilibrium of the rates of both capacity utilization and profit.

Appendix II: Model dynamics of the unstable long run equilibrium resulting from changing interest rate, with an interest-inelastic mark-up

The model dynamics equations can be written as:

$$\frac{\partial \lambda}{\partial i}=\frac{s_Z(s_T\frac{h}{v}-\beta)-\lambda\{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta\}+\frac{\partial h}{\partial i}\frac{s_T}{v}[i(\theta\lambda+s_Z)-\alpha-2\tau h]}{i\{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta\}}$$

$$\frac{\partial g^*}{\partial i}=\frac{(\lambda+i\frac{\partial \lambda}{\partial i})\{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta\}+\frac{\partial h}{\partial i}\frac{1}{v}s_T(\tau h-\beta u)}{s_T\frac{h}{v}-\beta}$$

$$\frac{\partial u^*}{\partial i}=\frac{(\lambda+i\frac{\partial \lambda}{\partial i})[s_f(1-s_Z)-\theta]+\frac{\partial h}{\partial i}(\tau-s_T\frac{h}{v})}{s_T\frac{h}{v}-\beta}$$

$$\frac{\partial r^*}{\partial i}=\frac{\frac{h}{v}(\lambda+i\frac{\partial \lambda}{\partial i})[s_f(1-s_Z)-\theta]+\frac{\partial h}{\partial i}\frac{1}{v}s_T(\tau h-\beta u)}{s_T\frac{h}{v}-\beta}$$

The assumptions of a stable short run goods market equilibrium, a unstable long run debt-capital ratio equilibrium and an interest-inelastic mark-up imply that, $s_T\frac{h}{v}-\beta>0$, $\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta<0$ and $\frac{\partial h}{\partial i}=0$. Substituting $\frac{\partial h}{\partial i}=0$ into the above model dynamic equations, we get:

$$\frac{\partial\lambda}{\partial i}=\frac{s_Z(s_T\frac{h}{v}-\beta)-\lambda\{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta\}}{i\{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta\}}=\frac{1}{i}\left[\frac{s_Z(s_T\frac{h}{v}-\beta)}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}-\lambda\right]$$

Integrating the above result into other dynamics equations yields:

$$\frac{\partial g^*}{\partial i}=\frac{(\lambda+i\frac{\partial\lambda}{\partial i})\{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta\}}{s_T\frac{h}{v}-\beta}=s_Z$$

$$\frac{\partial u^*}{\partial i}=\frac{(\lambda+i\frac{\partial\lambda}{\partial i})[s_f(1-s_Z)-\theta]}{s_T\frac{h}{v}-\beta}=\frac{s_Z[s_f(1-s_Z)-\theta]}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}$$

$$\frac{\partial r^*}{\partial i}=\frac{\frac{h}{v}(\lambda+i\frac{\partial\lambda}{\partial i})[s_f(1-s_Z)-\theta]}{s_T\frac{h}{v}-\beta}=\frac{\frac{h}{v}s_Z[s_f(1-s_Z)-\theta]}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}$$

Since $s_T\frac{h}{v}-\beta>0$ and $\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta<0$ under given consumptions, taking all assumptions on involved coefficients into consideration, we know that the sign of $\frac{\partial\lambda}{\partial i}$ will always be negative:

$$\frac{\partial\lambda}{\partial i}=\frac{1}{i}\left[\frac{s_Z(s_T\frac{h}{v}-\beta)}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}-\lambda\right]<0$$

Therefore, increasing the rate of interest will always have a negative effect on the unstable long run equilibrium, regardless the level of the initial debt-capital ratio that the economy starts with.

When $\frac{\partial g^*}{\partial i}$ will always be positive ($\frac{\partial g^*}{\partial i}=s_Z>0$), implying that rising rates of interest always have a positive effect on the unstable long run equilibrium rate of capital accumulation. However, the signs of $\frac{\partial u^*}{\partial i}$ and $\frac{\partial r^*}{\partial i}$ can not be determined unless further information of the economies is provided. The effect of λi on u is ambiguous since it is involved in not only the investment side but also the saving. Instinctively, a bigger debt burden reduces investment demand through the coefficient ($-\theta$) in the accumulation function on one hand, but it also cuts into the available savings for firms through the counterpart $-s_f(1-s_Z)$ in saving function on the other hand. In the so call "debt-led" economies, the first effect will dominate since $s_f(1-s_Z)>\theta$. Otherwise, the economies with coefficients $s_f(1-s_Z)<\theta$ will be called "debt-burden".

So, in a "debt-led" economy with $s_f(1-s_Z)>\theta$, combining given assumptions yields:

$$\frac{\partial u^*}{\partial i}=\frac{s_Z[s_f(1-s_Z)-\theta]}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}<0,$$

$$\frac{\partial r^*}{\partial i}=\frac{\frac{h}{v}s_Z[s_f(1-s_Z)-\theta]}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}<0$$

However, in a "debt-burden" economy with $s_f(1-s_Z)<\theta$, combining given assumptions yields:

$$\frac{\partial u^*}{\partial i}=\frac{s_Z[s_f(1-s_Z)-\theta]}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}>0,$$

$$\frac{\partial r^*}{\partial i}=\frac{\frac{h}{v}s_Z[s_f(1-s_Z)-\theta]}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta}>0$$

Therefore, the variations in the rate of interest have ambiguous effect on the dynamics of the rates of capacity utilization and profit in the unstable long run equilibrium, depending on the type of economies we study. In so called "debt-led" economies, the increasing the rate of interest has a negative effect on the unstable long run equilibrium of both rates, while the rising interest rate has a positive effect on the unstable long run equilibrium of the rates of both capacity utilization and profit.

Actually, summarizing the Model dynamics of variations of interest rate, with an interest-inelastic mark-up all dynamics effects, this is displayed in the following table:

<table>
<tr><td colspan="6">Model dynamics of the long run equilibrium resulting from changing interest rate, with an interest-inelastic mark-up</td></tr>
<tr><td colspan="2">Type of Economies
Long-run Stability</td><td colspan="2">"Debt-led" Economies
$(s_f(1-s_Z)>\theta)$</td><td colspan="2">"Debt-burden" Economies
$(s_f(1-s_Z)<\theta)$</td></tr>
<tr><td rowspan="3">On the stable long run equilibrium
$(\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta>0)$</td><td>$\lambda_0>\lambda_{\max}*$</td><td>$\frac{\partial\lambda}{\partial i}<0$</td><td rowspan="3">$\frac{\partial g}{\partial i}>0$,
$\frac{\partial u}{\partial i}>0$,
$\frac{\partial r}{\partial i}>0$;</td><td>$\frac{\partial\lambda}{\partial i}<0$</td><td rowspan="3">$\frac{\partial g}{\partial i}>0$,
$\frac{\partial u}{\partial i}<0$,
$\frac{\partial r}{\partial i}<0$;</td></tr>
<tr><td>$\lambda_0=\lambda_{\max}*$</td><td>$\frac{\partial\lambda}{\partial i}=0$</td><td>$\frac{\partial\lambda}{\partial i}=0$</td></tr>
<tr><td>$\lambda_0<\lambda_{\max}*$</td><td>$\frac{\partial\lambda}{\partial i}>0$</td><td>$\frac{\partial\lambda}{\partial i}>0$</td></tr>
<tr><td colspan="2">On the unstable long run equilibrium
$(\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta<0)$</td><td colspan="2">$\frac{\partial\lambda}{\partial i}<0,\frac{\partial g}{\partial i}>0$,
$\frac{\partial u}{\partial i}<0\ \frac{\partial r}{\partial i}<0$;</td><td colspan="2">$\frac{\partial\lambda}{\partial i}<0,\frac{\partial g}{\partial i}>0$
$\frac{\partial u}{\partial i}>0,\frac{\partial r}{\partial i}>0$;</td></tr>
</table>

Note: $$\lambda_{\max}* = \frac{s_Z(s_T\frac{h}{v}-\beta)}{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta};$$

λ_0 is the value of the initial debt-capital ratio.

Appendix III: Feedback effects of changing debt-capital-ratios on the equilibrium rates of capacity utilization, growth rate and profit rate

Starting from equilibrium equations

$$u^* = \frac{\lambda i[s_f(1-s_Z)-\theta]+(\alpha+\tau h)}{s_T\frac{h}{v}-\beta}$$

$$g^* = \frac{\lambda i\{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta\}+s_T\frac{h}{v}(\alpha+\tau h)}{s_T\frac{h}{v}-\beta}$$

$$r^* = \frac{\frac{h}{v}\{\lambda i[s_f(1-s_Z)-\theta]\}+\frac{h}{v}(\alpha+\tau h)}{s_T\frac{h}{v}-\beta}$$

Feedback effects of changing long run debt-capital-ratios on the equilibrium rates of capacity utilization, capital accumulation and profit have to be determined. We assume that changing debt-capital-ratios, for the reasons given in the text, will have no direct feedback effects on the interest rate or on the mark-up. Variations in indebtedness will hence only affect the distribution of profits between retained earnings and households' income. This will in turn affect households' consumption demand and firms' investment. From equations (11) - (13) we get:

$$\frac{\partial g}{\partial \lambda} = \frac{i\{\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta\}}{s_T\frac{h}{v}-\beta}$$

$$\frac{\partial u}{\partial \lambda} = \frac{i[s_f(1-s_Z)-\theta]}{s_T\frac{h}{v}-\beta}$$

$$\frac{\partial r}{\partial \lambda} = \frac{\frac{h}{v}i[s_f(1-s_Z)-\theta]}{s_T\frac{h}{v}-\beta}$$

If only stable equilibrium (at least satisfying short run stability condition, i.e. $s_T\frac{h}{v}-\beta>0$) and a positive rate of interest is considered, we can present feedback effects as follows:

Feedback effects of changing debt-capital-ratios on the equilibrium rates of capacity utilization, growth rate and profit rate		
Type of Economies / Long-run Stability	"Debt-led" economies ($s_f(1-s_Z)>\theta$)	"Debt-burden" economies ($s_f(1-s_Z)<\theta$)
On the unstable long run equilibrium ($\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta<0$)	$\frac{\partial u}{\partial\lambda}>0, \frac{\partial g}{\partial\lambda}<0, \frac{\partial r}{\partial\lambda}>0$;	$\frac{\partial u}{\partial\lambda}<0, \frac{\partial g}{\partial\lambda}<0, \frac{\partial r}{\partial\lambda}<0$;
On the stable long run equilibrium ($\beta[s_f(1-s_Z)]-s_T\frac{h}{v}\theta>0$)	$\frac{\partial u}{\partial\lambda}>0, \frac{\partial g}{\partial\lambda}>0, \frac{\partial r}{\partial\lambda}>0$;	$\frac{\partial u}{\partial\lambda}<0, \frac{\partial g}{\partial\lambda}>0, \frac{\partial r}{\partial\lambda}<0$;

Under the given assumptions, changing debt-capital-ratios have the same effects as changing interest rates when the mark-up is interest-inelastic. On one hand, similar to the effect of increasing interest rate, rising debt-capital-ratios is associated with increasing values for the real equilibrium variables of rates of capacity utilization and profit in the "debt-led" economies in which a low households' savings propensity and a low elasticity of investment with respect to debt services prevail. However, if the households' savings propensity is rather high and associated with a high elasticity of investment with respect to debt services, rising indebtedness will rather have a negative effect on the real variables of rates of capacity utilization and profit. Whereas, if the responsiveness of investment to capacity utilization is relatively higher than the counterpart of saving, a positive relation between indebtedness and capital accumulation will be maintained, even in the "debt-burden" economies in which a rather high households' savings propensity and a high elasticity of investment with respect to debt services prevail. On the other hand, under the stable long run equilibrium condition, a low responsiveness of investment to demand induces a negative impact between indebtedness and capital accumulation regardless the overwhelming effect of a low households' savings propensity and a low elasticity of investment with respect to debt services in the so called "debt-led" economies.