Earnings Inequality in the 1980s and the 1990s: A comparison between Canada and the US

by

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1. Introduction

Earnings inequality has been increasing in both Canada and the US in the 1980s and the 1990s. Various studies and researches have been done to examine the driving forces underlying this phenomenon. This paper presents several popular arguments about those forces, including skill premium, macroeconomic conditions and labor market institutions, etc. In either of these arguments, Canada and the US have performed differently, and therefore, the main causes of increased earnings inequality are different in the two countries. We use a computable general overlapping-generation model to explore the skill premium explanation in particular. We find that, domestically, either a skill-biased technological progress or a development in total factor productivity in the relatively skill-intensive industries would make both high-skilled and low-skilled workers better off, but at the same time raise the skill premium and hence earnings inequality. Besides, given the high economic integration between Canada and the US, one country benefits somewhat from the other's technological progress or productivity upgrading. Finally, we show that the observation of larger skill premium in US than in Canada in recent years can be explained by an asymmetric total factor productivity shock concentrating in the high-tech sector of the US economy.

Sector 2 provides overview of trends in earnings inequality and changes in the labor market conditions in Canada and the US in the last two decades as well as three explanations from the existing literature. Sector 3 details the general overlapping-generation model used in this paper, followed by data description in sector 4. Simulation results and interpretation are given in sector 5. Sector 6 concludes.

2. Increasing Earnings Inequality and Possible Explanations

2.1 Overview

As documented by many studies, over the 1980s and the 1990s, the US experienced a significant increase in inequality of annual employment earnings across individuals, especially before tax and among male workers. A similar phenomenon was observed in Canada during the same time periods, though the increase is not as dramatic as in the US. Table 2.1 shows the inequality trend in these two countries.

Table 2.1 Gini coefficient for all effective labor force participants, 1974, 1985, 1995, Canada and the US

	Canada	US
1974	0.407	0.436
1985	0.428	0.447
1995	0.423	0.467

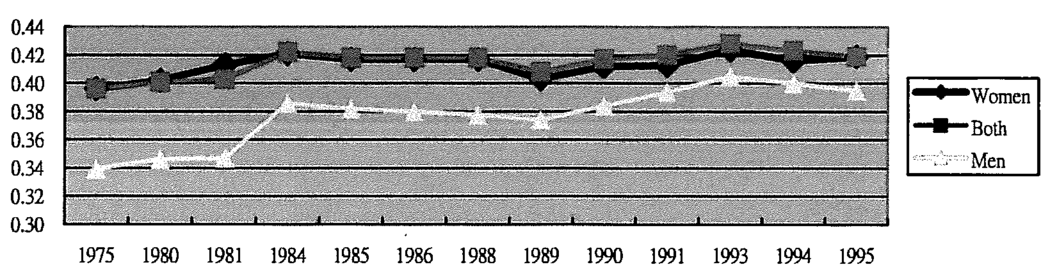
Source: Wolfson and Murphy (1998)

Effective labor force participant is defined as the proportion of individuals aged from 18 to 64 who have minimum $500 (in own country currency) earnings from both employment and self-employment.

The Gini coefficient, one of the most common measures of income inequality, increased in the US from 0.436 in the mid-1970s to 0.447 in 1985, and continued to increase sharply to 0.467 in 1995; Compared with that in the US, Canada's Gini coefficient rose more drastically from 0.407 to 0.428 over the 1974-85 period, and then remained relatively stable during the mid-1980s and the mid-1990s.

Being examined further, however, the relative stability in Canada's inequality trend is the result of several offsetting trends in earnings inequality of different subgroups of the total population. Figure 2.1 illustrates these trends by Gini coefficient. Namely, with inequality among all male workers increasing constantly since 1975, as demonstrated by Picott (1998), real earnings of lower paid (and presumably lower skilled) dropped, while those of high paid (high skilled) increased. On the contrary, inequality among all female workers trended upward over the mid 1970s and the early 1980s, and then remained virtually unchanged (or declined slightly if anything) in the remaining sample period. Moreover, the level of real earnings rose for females in all positions in the wage distribution, which contributed to the trend of stability in earnings inequality among all workers (male and female combined).

Figure 2.1 Gini Coefficient, among Canadian paid workers, 1975-1995



Source: Picot (1998)

How can one explain the trends of increased earnings inequality in Canada and the US? Are the driving forces behind inequality in these two countries the same? Why has Canada experienced a less dramatic increase in inequality? What are the possible causes for the difference in trends of inequality between male and female workers in Canada? Under the overall widening earnings distribution, were people on average better off, worse off, or the rich became richer while poor poorer? Before we start to explore answers to these questions, it might be helpful to keep in mind some major changes in the labor markets over the 1980s and the 1990s since many of them might influence the earnings distribution across individuals:

-The US was hit by a recession in 1980, while Canada experienced a recession two years later in 1982 with a sharp rising unemployment rate. During the following expansion, the unemployment rate fell gradually but rose again in the 1990-92 recession in Canada. And this time, the recession was deeper and the Canadian economy had an almost 5-year slump, accompanied by a much higher average level of unemployment rate.

-In terms of technological progress, both Canada and the US experienced fast changes.

-The proportion of female labor force participation rose, the percentage of female college/university attendance also increased.

-In Canada, the relative supply of highly-educated workers rose more quickly in the 1980s and the 1990s than in the previous decades, while it stagnated in the US throughout the same time periods.

-The degree of unionization fell in both countries, but much more quickly in the US, and it actually increased somewhat for the Canadian female workers in the 1980s.

Now we turn to look into various possible answers for the questions of interest mentioned above.

2.2 Skill Premium and Skill-Biased Technological Change

2.2.1Education Premium

Education is one of the major components of human capital and is tightly linked to individual earnings. Education premium, or in other words, wage differentials between more and less educated, will therefore influence earnings distribution across individuals.

There appeared to be a considerable increase in education-related earnings differentials in the US over the 1980s and the 1990s. On the contrary, education premium remained relatively stable in Canada during that time. Table 2.2.1 and Figure 2.2.1 presents different trends in education premium in Canada and the US according to Burbidge, Magee and Robb (2002). While it trended downward for less educated, the median earnings of university educated American male workers increased sharply by around 21 per cent over 1981 to 1999. For American female workers, on the other hand, the median earnings increased about 13 per cent for less educated, and it rose by 33 per cent for university educated. Overall, the earnings difference between more and less educated labors for both genders rose remarkably in the US during the last two decades.

Table 2.2.1a Median Weekly Earnings (1997 dollars in own country currencies) of Male workers aged 25-64, Canada and the US

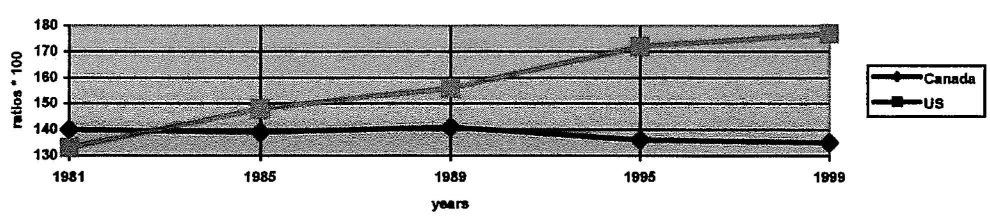
	UN		NONUN		Earnings ratio *100 UN/NONUN	
	Canada	US	Canada	US	Canada	US
1981	1063	814	761	611	140	133
1985	1051	890	756	603	139	148
1989	1046	934	744	598	141	156
1995	984	941	721	547	136	172
1999	936	982	693	556	135	177

Source: Burbidge, Magee and Robb (2002)

UN: university educated/NONUN: other (lower) than university educated

The **US data** is from CPS (US Current Population Surveys); The **Canadian data** from 1981 to 1995 is from SCF (Surveys of Consumer Finance); 1999 is from LFS (Labor Force Surveys)

Figure 2.2.1a Evolution of education premium (earnings ration of university to non-university) for males in Canada and the US



Source: Table 2.2.1a

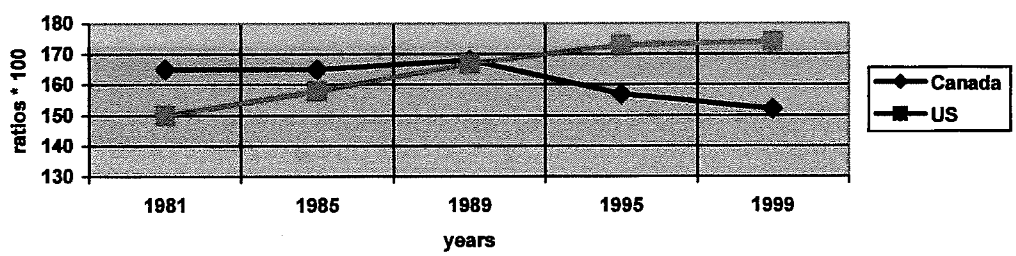
Compared with their US counterparts, however, the situation of Canadian workers was quite different. Real earnings of both more and less educated Canadian male workers fell over the 1980s and the 1990s. Real earnings of less educated Canadian females increased a little while it remained virtually stable for the highly educated females. Overall, the education premium among male workers does not show any upward trend and it actually declined for females in Canada in the sample periods.

Table 2.2.1b Median Weekly Earnings (1997 dollars in own country currencies) of Female workers aged 25-64, Canada and the US

	UN		NONUN		Earnings ratio *100 UN/NONUN	
	Canada	US	Canada	US	Canada	US
1981	776	509	471	339	165	150
1985	773	585	469	371	165	158
1989	796	623	475	374	168	167
1995	794	648	506	375	157	173
1999	768	678	506	389	152	174

Source: see Table 2.2.1a

Figure 2.2.1b Evolution of education premium (earnings ratio of university to non-university) for females in Canada and the US



Source: Table 2.2.1b

It is often argued that skill-biased technological change (SBTC) is the underlying force behind the growth in relative demand for highly educated workers and hence the education premium. Technological progress was obvious in both Canada and the US over the last two decades. If the technological change in these two countries is skill-biased, then the labor demand would undoubtedly shift in favor of more skilled and against less skilled workers with a resultant increase in skill premium for any given relative supply of skills. A number of studies agree that SBTC was a major source for changes in the relative demand for skilled labor in the US over the 1980s and the 1990s. Though there is no theoretical presumption that technological progress will lead to an increase in demand for more educated labors, as Murphy, Riddell and Romer (1998) argue, the historical data imply that it has done so. More and less educated workers are substitutes, but they are not perfect substitutes. Another example is as demonstrated by Machin and Reenen (1998). They examine among seven OECD countries the relationship between industry R&D (as a measure of technology) and non-production workers (as a representative of more-skilled) and find that SBTC was statistically significant in raising the relative demand for skills and the corresponding skill premium in the US. Unlike the quick increase in skill-upgrading requirements observed in the US, Gera, Gu and Lin (2001) conclude that skill upgrading on the aggregate level was less evident in Canada, but SBTC was an important driving force behind several within-industry skill upgrading.

In addition to relative demand, the relative supply of highly educated workers should be inspected as well to infer something about the education premium. As a measure of supply of education, school enrollment rate at every education level in Canada initially fell behind that in the US in the 1970s. During the 1980s and the 1990s, however, the enrollment rate increased substantially in Canada while it stagnated or grew more slowly in the US, mainly due to differences in government policies such as education subsidies. That is, the relative supply of university-educated labor has grown more quickly in Canada than in the US. To this point, one may naturally conclude that Canada would quite possibly have experienced a sharp increase in education premium similar to the US without its education expansion in the last two decades. This is also known as the race between education and technology, as discussed by Fortin (1999) and Murphy, Riddell and Romer (1998). Moreover, female working hours and the proportion of highly-educated female workers in the Canadian labor force rose considerably over the education expansion compared with previous decades, which somewhat caused stagnation and a drop in their education premium respectively in the 1980s and the 1990s, and therefore improved their average earnings while dampening the earnings inequality among them.

A widened earnings gap between highly educated and less educated workers will exacerbate earnings dispersion across individuals. The dynamic interaction between relative demand for and supply of more educated workers resulted in a large increase in education premium in the US, which is very likely to account for part of the growing earnings inequality in the 1980s and 1990s. In contrast, the increasing inequality in Canada over the same time periods, particularly among male workers, does not seem to be attributed to the relatively stable Canadian education premium.

2.2.2Experience Premium

Similar to education, experience is another major element of human capital. Return to experience also affects earnings distribution across individuals. Recent facts show that earnings differential and wage gap between younger and older workers have broadened, especially in Canada. This widening age-related dispersion is often viewed as an indicator of a rising experience premium.

However, Beaudry and Green (2000) demonstrate that during the 1980s and the early 1990s, regardless of their education level, the cohort-specific age-earnings profiles of Canadian male workers moved down over time and there was no evidence that increased returns to experience tended to offset this downward shift: entry-level wages for successive labor entrant cohorts were falling continuously; the age-earnings profiles were getting flatter in more recent years during the sample periods. They draw the conclusion that the widening age-related earnings differential is rather a reflection of deterioration in labor market conditions for successive cohorts of young

Canadians. In fact, earnings of younger Canadian workers first dropped during recession in the early 1980s and before recovering from that drop, it again declined greatly in the next big 1990s recession. The drop was about 36 per cent for males aged 18 to 24 and 29 per cent for young females over the 1980s and 1990s.

Picott (1998) provides various arguments about the possible reason for this decline in earnings of younger workers. Except for the SBTC-driven demand argument mentioned above, some suggest that the decrease in aggregate demand in recession time reduced labor demand and firms are likely to press down the entry- or near entry-level wages of the young. Others argue that with the aging of the baby-boom generation, who experienced a remarkable increase in educational attainment, the younger workers lost their education advantages they once had and, hence, the relative wages of the young tended to go down.

On the whole, in spite of the rising differentials in age-related earnings, one can hardly prove the occurrence of an increasing experience premium in Canada over the last two decades.

2.2.3 Summary of Skill Premium Explanation

Many studies believed that increased earnings inequality was attributable to increased skill premium. Specifically, skill-biased technological change resulted in an increase in the relative demand for skills of all types, including education and experience, and consequently the skill premium jumped up while the labor market position of less skilled workers worsened. Hence, the individual earnings dispersion widened.

However, regardless of its popularity, this argument is sometimes challenged. First, technological change may not be the only thing that shifted the skill premium and earnings distribution in Canada and the US. For example, based on regression results, Machin and Reenen (1998) concluded that other factors in addition to technology seemed likely to contribute to the declining labor market position of the less-skilled workers in the US during the 1980s and the early 1990s. Moreover, the SBTC argument and the resultant skill-biased demand shift are less convincing in Canada than in the US. Neither education nor experience premium can fully explain the inequality story in Canada during the 1980s and the 1990s. We now turn to the contributions of other factors to the inequality story, in particular, the macroeconomic conditions and labor market institutions.

2.3 Macroeconomic Conditions

In this section, we take a glance at the explanation power of macroeconomic conditions to the increased earnings inequality in Canada and the US, though this paper will not offer any quantitive examination on it.

2.3.1 Unemployment Rate--Cyclical Effects

In addition to the skill premium, changes in unemployment rates are widely accepted as another explanation for the rising earnings inequality. Namely, an increase in unemployment, which reflects a shrink in aggregate demand in recessions, is hypothesized to broaden the dispersion of earnings distribution across individuals.

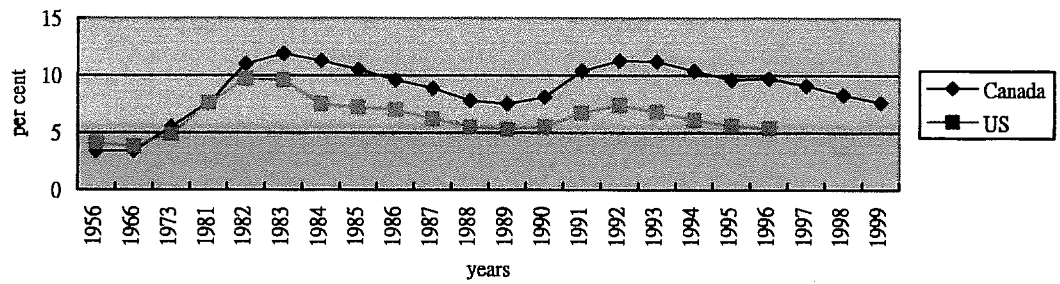
Apparently, the trend in earnings inequality in Canada revealed in Figure 2.1 did vary with cyclical fluctuations. During the 1982-83 recession, unemployment rate increased sharply from 7.5 per cent in 1981 to a peak of 11.8 per cent in 1983, and at the same time inequality went up quite a lot relative to previous years. After that, unemployment and inequality recovered gradually during expansion in the late 1980s before they experienced a much more significant increase in the 1990-92 recession, then a slower recovery in the following 5-year economic slump.

Theoretically, this positive relationship between the unemployment rate and earnings inequality is consistent with both competitive and segmented labor market models, as verified by Macphail (2000). From a competitive labor market perspective, assuming the firing and hiring cost are correlated with labor skills, a decline in the labor demand derived from a decreased aggregate demand will disproportionately depress wages in the lower tail of the wage distribution. As a result, the wage inequality increases and so does the annual earnings inequality. In a segmented labor market, alternatively, wage rates are relatively sticky, and firms respond to lower aggregate demand primarily by using more part-time or part-year workers (Osberg 1995). Consequently, annual earnings dispersion widens due to the increase in inequality in annual hours worked. In short, changes in the unemployment rate contribute to earnings inequality either through wages or the annual hours worked across individuals in the labor market.

Actually, many studies have proved that the cyclical effects raised to some extent the earnings inequality in both Canada and the US. According to Burtless (1990), changes in the unemployment rate explained about 20 per cent of the widening dispersion in annual earnings among the US males. Macphail (1998) proves that changes in the unemployment rate in Canada significantly and positively contributed to the increased annual earnings inequality (but not the hourly wage inequality) for both genders.

Unemployment rates in Canada and the US were about at the same level before the 1980s. After Canada experienced two much deeper recessions relative to the US in 1982-83 and 1990-92, the Canadian unemployment rate began to climb more quickly and it almost doubled that of the US in the late 1990s. Furthermore, the recession in 1990-92 in Canada was also much deeper than in 1982-83, and the subsequent recovery was much slower. Figure 2.3.1 illustrates the diverging trends in these two countries since the early 1980s.

Figure 2.3.1 Unemployment rates, Canada and the US, 1981-1996



Source: Data from 1956 to 1996 are from Riddell and Sharpe, the original sources of which are: Canada: 1956 data from *Historical Statistics of Canada*, second edition, Statistics Canada, 1984, cat. 11-516; 1966 and 1973 data from *Historical Labor Force Statistics*, Statistics Canada, February 1993, cat 71-201; post-1975 data from *Labor Force Historical Review*, Statistics Canada, CD ROM version, February 1997. United States: *Economic Report of the President*, February 1997; **Data from 1997 to 1999 for Canada** are the arithmetic average of monthly unemployment rate from Canadian Economic Online, the original source of which is Statistic Canada.

Analogously, changes in the employment to population ratio, either because of changes in labor demand or workers preferences, will also influence earnings distribution. Similar to unemployment, the employment rate in Canada dropped substantially as well relative to the US through the 1980s, and fell both in relative and absolute terms during the 1990s.

Then why has Canada had such a disappointing economic performance relative to the US (and actually in absolute term as well since the US economy also deteriorated relative to its previous decades) in the last two decades, especially the 1990s? Fortin (1999) explores the answer: the deadly interaction between overly tight monetary policy and overly slack fiscal policy is the main reason for the deep recessions, high unemployment rate and delayed recovery. The tight Canadian monetary policy, targeting on low inflation rate, led to increases in interest rates. On the other hand, the loose Canadian fiscal policy resulted in a highly accumulated public debt, which exaggerated the size of interest payments on the debt at the growing interest rates. The fiscal deficit thus grew bigger, which in turn raised public and external debt. In order to maintain order in both domestic and financial markets, Canadian monetary authorities were forced to raise the interest rates even higher. This circular process brought about recessions with a great cut in domestic aggregate demand and the derived labor demand. After the early 1990s' recession, the Canadian government tried to jump out from this circular process by raising taxation and reducing public program expenditures, which depressed further the domestic demand and output, discouraged employment and hence delayed the recovery.

To this point, it is clear that changes in unemployment rates during business cycles have an impact on earnings distribution and inequality in both countries. It is then natural to have such a question: What is the relationship between unemployment rate and inequality if the cyclical effects are removed? The answer is provided in the next section.

2.3.2 Unemployment Rate—Secular Trend

The "cyclical neutral" unemployment rates in both Canada and the US showed an upward trend over the sample periods, much more so in Canada. Table 2.3.2a and 2.3.2b offer average unemployment rates over decades and cyclical neutral periods.

Table 2.3.2a Average Unemployment Rate (%) over Decades, Canada and the US

Year	Canada	US
1970-79	6.7	6.2
1980-89	9.4	7.3
1990-96	10.1	6.2
1996-99	8.7	4.8

Table 2.3.2b Average Unemployment Rate (%) over Cyclical Neutral Periods, Canada

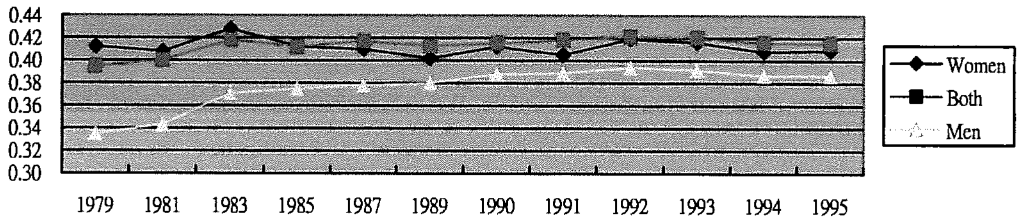
Year	Canada
1974-81	7.3
1982-89	9.8
1993-99	9.4

Source: 1970-1996 Data for Canada and the US are from Riddell and Sharpe, see Figure 2.3.1; **1996-99 Data**: Canada: see Figure 2.3.1; US: civilian labor force statistics, *US Department of Labor, Bureau of Labor Statistics (2001)* The *average* here is the arithmetic average of yearly unemployment rate.

Did the long-term upward tend in unemployment rate contribute to increased earnings inequality in the 1980s and the 1990s? Probably yes. Increased unemployment rate will make workers who worked previously become unemployed and have zero earnings, which possibly raises the earnings dispersion.

Except for table 2.3.2b which reveals an increasing unemployment rate in the long term in Canada, there are other investigations suggesting that, after removing the cyclical effects (not the long-term trend in unemployment rate), inequality in both Canada and the US in the last two decades still trended upward. For instance, Macphail (1998) regresses a time trend on the Canadian inequality through the 1980s and got an upward secular trend for males and a downward one for females. Picot (1998) also separates the effect on inequality of a long-term rise in unemployment rate from the cyclical fluctuation, and uncovers in Canada an increasing inequality with cyclical variation removed, as indicated by the following Figure 2.3.2.

Figure 2.3.2 Gini Coefficient without cyclical variation, Canadian Paid Workers, 1977-95



Source: Picot (1998)

2.3.3 Summary of the Macroeconomic Conditions Explanation

The unemployment rate, as a proxy of macroeconomic conditions, is believed to have raised the earnings inequality in Canada and the US over the last two decades. Either cyclical variations or a long-term time trend in unemployment rates are likely to change the earnings dispersion by depressing further the lower tail of the wage or working hour distribution. Canada suffered a much severe increase in unemployment rates than the US during the 1980s and the 1990s, mainly because of the two deep recessions and the delayed recovery. Canada's problematic fiscal and monetary policies were largely responsible for those recessions and slump.

Correcting these policies is expected to improve the economic performance, reduce unemployment rates, raise the employment to population ratio, and hence dampen earnings inequality in Canada.

Some studies argue that the explanation power of unemployment weakened since the latter 1980s. Nevertheless, this argument does not seem to be supported by Canadian evidence according to Macphail (1998). After regressing several interactive dummy variables on the annual earnings inequality, she concludes that, in contrast to the US, the unemployment-inequality relationship still held in Canada, the reason of which might be the differences in the labor market institutions between these two countries, such as unionization density and the minimum wage, which is going to be discussed in the next section.

2.4 Labor market institutions

Different labor market institutions obviously result in different labor market positions for skilled and unskilled workers. The stronger the institutions are, the better the unskilled workers are protected from being laid off and/or from low wages. As a result, the lower tail of the earnings distribution is less likely to be depressed. Overall, labor market institutions in the US weakened significantly in the last two decades compared with those in Canada. This institutional difference may be an important reason for the difference in earnings structures in these two countries, as suspected by Burbidge, Magee and Robb (2002).

2.4.1Unionization

Over the 1980s and the 1990s, unionization rates for Canadian males fell a little while they rose for Canadian females. Contrarily, it declined much more remarkably in the US. Unionization rates were once at the same level in both countries in the early 1970s, but the union density gap between Canada and the US began to widen dramatically since the 1980s and the gap was as large as 22 percentage points in 1994 (HRDC Canada).

Card (1992) does the estimate to find that the declined unionization explained about 20 per cent of the wages inequality among the US male workers. Machin and Reenen (1998) also suggest that, in addition to SBTC, the declining ability of institutions to set wages, affect training and protect unskilled workers was probably another major factor that contributed to the deterioration in labor market position of unskilled in the US.

For the Canadian case, MacPhail (2000) illustrates a significantly negative relationship between unionization and earnings inequality using a multiple regression model. It is explained that, the falling unionization degree might result in an increase in the use of part-time workers and thus the hourly wage rate variation, and/or an inequality in annual hours worked, either of which widens the earnings dispersion.

Given the status of unionization in Canada and the US in the sample time period, declined unionization for Canadian males and the US labor forces contributed to the increased earnings inequality; whereas increased unionization for Canadian females actually played a role to reduce the earnings dispersion, which is consistent with the fact that earnings inequality among this group remained stable or declined slightly if anything over the last two decades.

2.4.2 Minimum wages

As another aspect of the labor market institutions, changes in the minimum wages may have an impact on the wage distribution across individuals. As presented by MacPhail (2000), if a decline in minimum wages occurs, it is expected to accelerate the increase in earnings inequality by depressing further the wages in the lower tail of the wage distribution.

2.5 Skill premium examined in this paper

We have known that skill premium is an important contributor to changes in earnings inequality. Some of the existing literature point out, however, that skill premium has developed divergently in Canada and the US in the last two decades and, hence contributed differently to earnings inequality in these two countries. A computable general overlapping-generation model is used in this paper to explore the relationships between skill premium and skill-biased technological change and total factor productivity, and to examine the contribution of skill premium to increased inequality. Moreover, because of the high integration between Canadian and the US economy, it is very likely that shocks to one may influence the other. Then why have skill premiums shown up different trends in these two countries? How does skill-biased technological change affect relative demand for skills and thus the skill premium? Have we missed anything else that might have impact on skill premium? Other than differences in the increase in the relative supply of highly educated, is there any other possible cause for the divergent trends? From the next section, we will start to look into this skill premium explanation more deeply.

3. Model Description

We offer in this section the theoretical description of the computable general overlapping-generation model that is used for simulation experiments. This model is a slight modification of the model developed by Mérette and Mercenier (2003). The modification was operated on the demand of the firms for labor to permit the simulations of skill-biased technological changes. This model has a multi-country feature to investigate the effect of technological changes occurring in one country on the demand for high skilled and less skilled labor in another country.

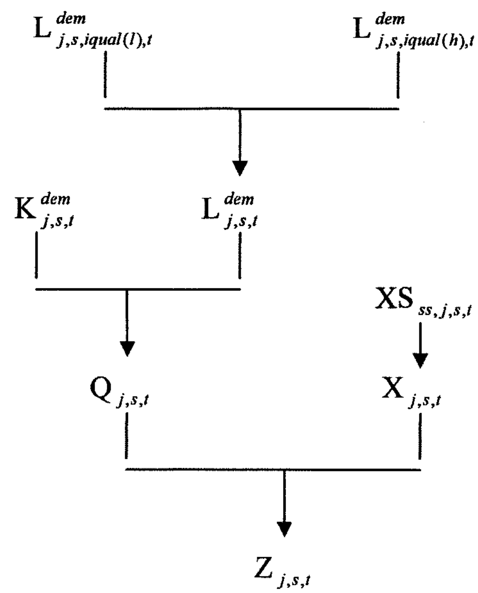
Assume that the world consists of 2 symmetric regions (the US and Canada), indexed by *i* or *j*, and the residual rest of the world, indexed by *row*. Denote I={ Canada, the US } and II={ Canada, the US, rest of the world }which is indexed by *ii* or *jj*. Price and income of the rest of the world are assumed to be constant. Moreover, the rest of the world does not borrow or lend internationally. In each region, there are 2 production sectors, indexed by *s*, *ss* (s=1, 2). For any time period *t*, there coexist 6 overlapping generations (*g*=0, 1,..., 5). That is, each individual lives 6 periods of life, 10 years per period (from age 15 to 74). Individuals work for the first five periods which are indexed by *gj* (*gj*=0, 1,..., 4) and retire at the last, indexed by *gm* (*gm*=5). Only two types of labor services are supplied: low-skilled and high-skilled.

3.1 regional (national) producers at time t

As shown in figure 3.1, a producer of region *j* at time *t* uses a combination of intermediate good inputs $X_{j,s,t}$ (with $XS_{ss,j,s,t}$ amount from each intermediate good sector *ss*) and $Q_{j,s,t}$ amount of value added to produce $Z_{j,s,t}$ amount of final good *s*. The two components of the value added are physical capital $K^{dem}_{j,s,t}$ and human capital

$L^{dem}_{j,s,t}$, the latter of which in turn includes two different types of labor services indexed by *iqual*. Namely, $L^{dem}_{j,s,iqual(h),t}$ denotes high skilled labor services and $L^{dem}_{j,s,iqual(l),t}$ for the low skilled.

Figure 3.1 Producer Technology



Inputs can be bought at market prices at $P^{c}_{j,ss,t}$, $Rent_{j,t}$, and $W_{j,iqual,t}$ for intermediate goods, physical capital and human capital respectively. The subscript *s* is omitted because though factors are region-specific, they are assumed mobile across sectors in each region.

Assume further the following for sector s of region *j* at time *t*:

(i) A Cobb-Douglas production function

$$Z_{j,s,t} = CD\,(X_{j,s,t}, Q_{j,s,t}; Sc^{Z}_{j,s,t}, \alpha^{Q}_{j,s})$$

(ii) Aggregate intermediate good $X_{j,s,t}$ is a CES function of individual intermediate good $XS_{ss,j,s,t}$ from each sector *ss*.

$$X_{j,s,t} = \text{CES}\,(XS_{ss,j,s,t}\,; \alpha^X_{ss,j,s}, \sigma^X_{j,s})$$

(iii) Value added $Q_{j,s,t}$ is a Cobb-Douglas mixture of its two components.

$$Q_{j,s,t} = \text{CD}\,(K^{dem}_{j,s,t}, L^{dem}_{j,s,t}\,; Sc^{Q}_{j,s,t}, \alpha^K_{j,s})$$

(iv) Aggregate human capital inputs $L^{dem}_{j,s,t}$ is a CES mix of different skill level of labor services $L^{dem}_{j,s,iqual,t}$.

$$L^{dem}_{j,s,t} = \text{CES}\,(L^{dem}_{j,s,iqual,t}\,; \alpha^L_{j,s,iqual}, \sigma^L_{j,s})$$

Sc is the scaling parameter representing the total factor productivity (TFP). α is the parameter measuring the expenditure share and $\alpha^L_{j,s,iqual}$ therefore reflects the exogenous skill-biased technological progress in region *j*. σ denotes the substitution elasticity. Both α and σ are assumed fixed over time.

Thus, the problem of producer *s* in region *j* at time *t* is to maximize the total profits subject to its technology that is characterized by above four constraints:

$$\underset{X_{ss,j,s,t},K^{dem}_{j,s,t},L^{dem}_{j,s,iqual,t}}{Maximize}\ P_{j,s,t}\ Z_{j,s,t} - \sum_{ss} P^C_{ss,j,s,t}\ XS_{ss,j,s,t} - \text{Rent}_{j,t}\ K^{dem}_{j,s,t} - \sum_{iqual} W_{j,iqual,t}\ L^{dem}_{j,s,iqual,t}$$

S.T. (i) ~ (iv)

The producer's optimal conditions are therefore the following[1]:

(1) $\mathbf{P}^C_{ss(1),j,s,t} =$

$$P_{j,s,t}\,(\frac{1-\alpha^Q_{j,s}}{\sigma^X_{j,s}})\ [\ \alpha^X_{ss(1),j,s} XS_{ss(1),j,s,t}{}^{\sigma^X_{j,s}} + (1-\alpha^X_{ss(1),j,s})XS_{ss(2),j,s,t}{}^{\sigma^X_{j,s}})\ ]^{(\frac{1-\alpha^Q_{j,s}}{\sigma^X_{j,s}})-1}$$

$$\alpha^{X}_{ss(1),j,s}\ \sigma^{X}_{j,s}\ \mathrm{XS}_{ss(1),j,s,t}{}^{\sigma^{X}_{j,s}-1}\ \mathrm{Sc}^{Z}_{j,s,t}\ \{\ \mathrm{Sc}^{Q}_{j,s,t}\ \mathrm{K}^{dem}_{j,s,t}{}^{\alpha^{K}_{j,s}}\ [\ \alpha^{L}_{j,s,iqual(h)}\mathrm{L}^{dem}_{j,s,iqual(h),t}{}^{\sigma^{L}_{j,s}} +^{1}$$

$$(1-\alpha^{L}_{j,s,iqual(h)})\mathrm{L}^{dem}_{j,s,iqual(l),t}{}^{\sigma^{L}_{j,s}}\ ]\ \}^{\frac{\alpha^{Q}_{j,s}(1-\alpha^{K}_{j,s})}{\sigma^{L}_{j,s}}}$$

$$\mathbf{P}^{C}_{ss(2),j,s,t} =$$

$$\mathrm{P}_{j,s,t}\left(\frac{1-\alpha^{Q}_{j,s}}{\sigma^{X}_{j,s}}\right)[\ \alpha^{X}_{ss(1),j,s}\ \mathrm{XS}_{ss(1),j,s,t}{}^{\sigma^{X}_{j,s}} + (1-\alpha^{X}_{ss(1),j,s})\mathrm{XS}_{ss(2),j,s,t}{}^{\sigma^{X}_{j,s}}\ )\ ]^{(\frac{1-\alpha^{Q}_{j,s}}{\sigma^{X}_{j,s}})-1}$$

$$(1-\alpha^{X}_{ss(1),j,s})\,\sigma^{X}_{j,s}\ \mathrm{XS}_{ss(2),j,s,t}{}^{\sigma^{X}_{j,s}-1}\ \mathrm{Sc}^{Z}_{j,s,t}\ \{\ \mathrm{Sc}^{Q}_{j,s,t}\ \mathrm{K}^{dem}_{j,s,t}{}^{\alpha^{K}_{j,s}}\ [\ \alpha^{L}_{j,s,iqual(h)}\mathrm{L}^{dem}_{j,s,iqual(h),t}{}^{\sigma^{L}_{j,s}} +$$

$$(1-\alpha^{L}_{j,s,iqual(h)})\mathrm{L}^{dem}_{j,s,iqual(l),t}{}^{\sigma^{L}_{j,s}}\ ]\ \}^{\frac{\alpha^{Q}_{j,s}(1-\alpha^{K}_{j,s})}{\sigma^{L}_{j,s}}}$$

$$\Rightarrow \frac{P^{C}_{ss(1),j,s,t}}{P^{C}_{ss(2),j,s,t}} = \frac{\alpha^{X}_{ss(1),j,s}\,XS_{ss(1),j,s,t}{}^{\sigma^{X}_{j,s}-1}}{(1-\alpha^{X}_{ss(1),j,s})\,XS_{ss(2),j,s,t}{}^{\sigma^{X}_{j,s}-1}}$$

$$\textbf{(2) Rent}_{j,t} = \mathrm{P}_{j,s,t}\,[\ \alpha^{X}_{ss(1),j,s}\ \mathrm{XS}_{ss(1),j,s,t}{}^{\sigma^{X}_{j,s}} + (1-\alpha^{X}_{ss(1),j,s})\ \mathrm{XS}_{ss(2),j,s,t}{}^{\sigma^{X}_{j,s}}\ ]^{\frac{(1-\alpha^{Q}_{j,s})}{\sigma^{X}_{j,s}}}$$

$$\mathrm{Sc}^{Z}_{j,s,t}\ \frac{\alpha^{Q}_{j,s}(1-\alpha^{K}_{j,s})}{\sigma^{L}_{j,s}}\ \{\ \mathrm{Sc}^{Q}_{j,s,t}\ \mathrm{K}^{dem}_{j,s,t}{}^{\alpha^{K}_{j,s}}\ [\ \alpha^{L}_{j,s,iqual(h)}\mathrm{L}^{dem}_{j,s,iqual(h),t}{}^{\sigma^{L}_{j,s}} + (1-\alpha^{L}_{j,s,iqual(h)})$$

$$\mathrm{L}^{dem}_{j,s,iqual(l),t}{}^{\sigma^{L}_{j,s}}\ ]\ \}^{[\frac{\alpha^{Q}_{j,s}(1-\alpha^{K}_{j,s})}{\sigma^{L}_{j,s}}]-1}\ \mathrm{Sc}^{Q}_{j,s,t}\ \alpha^{K}_{j,s}\ \mathrm{K}^{deam}_{j,s,t}{}^{\alpha^{K}_{j,s}-1}\ [\ \alpha^{L}_{j,s,iqual(h)}\mathrm{L}^{dem}_{j,s,iqual(h),t}{}^{\sigma^{L}_{j,s}} +$$

$$(1-\alpha^{L}_{j,s,iqual(h)})\ \mathrm{L}^{dem}_{j,s,iqual(l),t}{}^{\sigma^{L}_{j,s}}\ ]$$

$$\textbf{(3) W}_{j,iqual(h),t} = \mathbf{P}_{j,s,t}\ [\ \alpha^{X}_{ss(1),j,s}\mathrm{XS}_{ss(1),j,s,t}{}^{\sigma^{X}_{j,s}} + (1-\alpha^{X}_{ss(1),j,s})\ \mathrm{XS}_{ss(2),j,s,t}{}^{\sigma^{X}_{j,s}}\ ]^{\frac{(1-\alpha^{Q}_{j,s})}{\sigma^{X}_{j,s}}}$$

$$\mathrm{Sc}^{Z}_{j,s,t}\ \frac{\alpha^{Q}_{j,s}(1-\alpha^{K}_{j,s})}{\sigma^{L}_{j,s}}\,\mathrm{Sc}^{Q}_{j,s,t}\,\mathrm{K}^{dem}_{j,s,t}{}^{\alpha^{K}_{j,s}}\ \alpha^{L}_{j,s,iqual(h),t}\ \sigma^{L}_{j,s}\,\mathrm{L}^{dem}_{j,s,iqual(h),t}{}^{\sigma^{L}_{j,s}-1}\ \{\ \mathrm{Sc}^{Q}_{j,s,t}$$

[1] The results are derived in Appendix (1)

$$K^{dem\ \alpha^K_{j,s}}_{j,s,t} [\ \alpha^L_{j,s,iqual(h)} L^{dem\ \sigma^L_{j,s}}_{j,s,iqual(h),t} + (1-\alpha^L_{j,s,iqual(h)})\, L^{dem\ \sigma^L_{j,s}}_{j,s,iqual(l),t}\ ]\ \}^{[\frac{\alpha^Q_{j,s}(1-\alpha^K_{j,s})}{\sigma^L_{j,s}}]-1}$$

$$\mathbf{W}_{j,iqual(l),t} = \mathbf{P}_{j,s,t} [\ \alpha^X_{ss(1),j,s} \mathrm{XS}^{\ \sigma^X_{j,s}}_{ss(1),j,s,t} + (1-\alpha^X_{ss(1),j,s})\, \mathrm{XS}^{\ \sigma^X_{j,s}}_{ss(2),j,s,t}\ ]^{\frac{(1-\alpha^Q_{j,s})}{\sigma^X_{j,s}}}$$

$$\mathrm{Sc}^Z_{j,s,t} \frac{\alpha^Q_{j,s}(1-\alpha^K_{j,s})}{\sigma^L_{j,s}} \mathrm{Sc}^Q_{j,s,t} K^{dem\ \alpha^K_{j,s}}_{j,s,t} (1-\alpha^L_{j,s,iqual(h),t})\, \sigma^L_{j,s} L^{dem\ \sigma^L_{j,s}-1}_{j,s,iqual(l),t} \{\ \mathrm{Sc}^Q_{j,s,t}$$

$$K^{dem\ \alpha^K_{j,s}}_{j,s,t} [\ \alpha^L_{j,s,iqual(h)} L^{dem\ \sigma^L_{j,s}}_{j,s,iqual(h),t} + (1-\alpha^L_{j,s,iqual(h)})\, L^{dem\ \sigma^L_{j,s}}_{j,s,iqual(l),t}\ ]\ \}^{[\frac{\alpha^Q_{j,s}(1-\alpha^K_{j,s})}{\sigma^L_{j,s}}]-1}$$

$$\Rightarrow \frac{W_{j,iqual(h),t}}{W_{j,iqual(l),t}} = \frac{\alpha^L_{j,s,iqual(h),t} L^{dem\ \ \sigma^L_{j,s}-1}_{j,s,iqual(h),t}}{(1-\alpha^L_{j,s,iqual(h),t}) L^{dem\ \ \sigma^L_{j,s}-1}_{j,s,iqual(l),t}}$$

3.2 Regional (national) households at time t

In each region, an individual is born at age 15 (*g*=0) and dies at 74 (*g*=5), living 6 periods of 10 years with working in the first five periods (*g*=*gj*=0, 1,…,4) while retiring in the last (*g*=*gm*=5). For individual born in region *j* with *g*=0 at time *t*, his/her lifetime utility is assumed to be a CES function of the six-period consumptions and bequests left at *g*=5 at time *t*+5 (excluding leisure since labor supply is assumed exogenous):

$$\mathrm{U}\,(\mathrm{Con}_{j,g,t+g}, \mathrm{RBeq}_{j,g,t+g}) =$$

$$\frac{1}{1-\theta} \sum_{g=0}^{5} (\frac{1}{1+\rho_j})^g\ [\ (\mathrm{Con}_{j,g,t+g})^{1-\theta} + \beta^\theta_g (\mathrm{RBeq}_{j,g,t+g})^{1-\theta}\ ]$$

Where, $\theta > 0, \beta_{g\neq 5} = 0, \beta_{g=5} > 0$. $\mathrm{Con}_{j,g,t}$ is the consumption amount (aggregation of consumption expenditures on different final goods *s*) spent by individual living in region j with age g at time t. $\text{RBeq}_{j,g,t}$ is the bequest amount in real term.

The present value of lifetime income for each individual is a discounted sum of net-of-tax lifetime labor income, public old-age pensions and inheritances. Under the assumption of perfect capital markets, we have:

$$\text{Wealth}_{j,t} = \sum_{g=0}^{g=5} \left[\frac{1}{1+R\text{int}_{t+g}(1-\tau^K_{j,t+g})} \right]^g \left[\text{LInc}_{j,g,t+g} \; (1-\tau^W_{j,t+g} - \text{CtR}_{j,t+g}) + \right.$$

$$\left. (1-\tau^W_{j,t+g}) \; \text{Inh}_{j,g,t+g} + \text{Pens}_{j,g,t+g} \right]$$

Where, Rint_t stands for the real interest rate between period t-1 and t. It is the same across regions at any time period since financial assets are assumed mobile inter-regionally and perfectly substitutable. $\tau^K_{j,t}$ and $\tau^W_{j,t}$ are the tax rates charged on capital and labor income, respectively. $\text{LInc}_{j,g,t}$ is the individual's labor income. $\text{CtR}_{j,t}$ is the contribution rate to the pay-as-you-go public pension program. $\text{Inh}_{j,g,t}$ and $\text{Pens}_{j,g,t}$ denote inheritances and old-age pensions respectively.

The household's problem is thus to maximize lifetime utility under the inter-temporal budget constraint:

$$\underset{Con_{j,g,t},RBeq_{j,g,t}}{Maximize} \; \text{U}(\text{Con}_{j,g,t+g}, \text{RBeq}_{j,g,t+g}) =$$

$$\frac{1}{1-\theta} \sum_{g=0}^{5} \left(\frac{1}{1+\rho_j}\right)^g \left[\; (\text{Con}_{j,g,t+g})^{1-\theta} + \beta_g^\theta \, (\text{RBeq}_{j,g,t+g})^{1-\theta} \; \right] \quad \theta > 0, \beta_{g\neq 5} = 0, \beta_{g=5} > 0$$

S.T.

$$\sum_{g=0}^{5} \left[\frac{1}{1+R\text{int}_{t+g}(1-\tau^K_{j,t+g})} \right]^g P^{Con}_{j,g,t+g} \; \text{Con}_{j,g,t+g} \, (1+\tau^C_{j,t+g}) =$$

$$\sum_{g=0}^{5}\left[\frac{1}{1+R\mathrm{int}_{t+g}(1-\tau^{K}_{j,t+g})}\right]^{g}\ [\ \mathrm{LInc}_{j,g,t+g}(1-\tau^{W}_{j,t+g}-\mathrm{CtR}_{j,t+g})+(1-\tau^{W}_{j,t+g})\mathrm{Inh}_{j,g,t+g}+$$

$$\mathrm{Pens}_{j,g,t+g}\ ]$$

Where, $\mathrm{P}^{Con}_{j,g,t}$ is the aggregate consumer index. $\tau^{C}_{j,t+g}$ denotes consumption tax rate.

The first order conditions for individual consumption and bequest is therefore as following:

$$\begin{cases} Con_{j,g+1,t+g+1}=\left[\dfrac{\left[1+R\mathrm{int}_{t+g}(1-\tau^{K}_{j,t+g})\right](1+\tau^{C}_{j,t+g})P^{Con}_{j,g,t+g}}{(1+\rho_j)(1+\tau^{C}_{j,t+g+1})P^{Con}_{j,g+1,t+g+1}}\right]^{\frac{1}{\theta}} Con_{j,g,t+g} \\ RBeq_{j,g,t}=\beta_g Con_{j,g,t} \end{cases}$$

At any time *t*, there coexists 6 overlapping generations. The youngest five are in their working age groups (*g*=*gj*=0, 1,..., 4) and the oldest one (*g*=*gm*=5) is retired.

(i) Each working individual's labor income depends on not only how much labor service he/she supplies, but also his/her labor productivity. Assume that productivity is a quadratic function of age g, the maximum of which is achieved between one's midlife and retirement:

$$\begin{cases} LInc_{j,gj,t}=\sum_{iqual} W_{j,iqual,t} L^{\sup}_{j,iqual,gj,t} EP_{gj} \\ EP_g=\gamma+\lambda g-\psi g^2, \gamma,\lambda,\psi\geq 0 \end{cases}$$

Where, $\mathrm{L}^{\sup}_{j,iqual,gj,t}$ is the amount of exogenous labor supply provided by individual living in region j of working age *gj* at time *t*. EP_g is the labor productivity profile.

(ii) At the same time, individuals in their retiring life period gm get the public old-age pension benefits, which are proportional to the retiree's lifetime labor income:

$$\text{Pens}_{j,gm,t} = \text{PensR}_j \frac{1}{5}\sum_{gj} LInc_{j,gj,t-5+gj}$$

Where, PensR_j is the pension replacement rate of region j.

(iii) Inheritances $\text{Inh}_{j,gj,t}$ of working generations gj are obtained by equally distributing bequests of the oldest generation gn (here $gn=gm=5$):

$$\text{Inh}_{j,gj,t}\,\text{Pop}_{j,gj,t} = \frac{1}{5}\text{P}^{Con}_{j,gn,t}\,\text{RBeq}_{j,gn,t}\,\text{Pop}_{j,gn,t}$$

Where, $\text{Pop}_{j,g,t}$ is the population of age g at time t.

Furthermore, households living at time t allocate their consumptions across different goods s. Assume that the aggregate consumption of any individual is a CES combination of consumption expenditure on different final goods s:

$$\text{Con}_{j,g,t} = \text{CES}\,(\text{ConS}_{j,s,g,t};\alpha^{ConS}_{j,s,g},\ \sigma^{ConS}_{j,g.})$$

Thus, we have the following optimality conditions[2]:

$$\begin{cases} {P^{Con}_{j,g,t}}^{1-\sigma^{ConS}_{j,g}} = \sum_s \alpha^{ConS}_{j,s,g} {P^{C}_{j,s,t}}^{1-\sigma^{ConS}_{j,g}} \\ ConS_{j,s,g,t} = \alpha^{ConS}_{j,s,g}\left(\dfrac{P^{Con}_{j,g,t}}{P^{C}_{j,s,t}}\right)^{\sigma^{ConS}_{j,g}} Con_{j,g,t} \end{cases}$$

Where, $\text{ConS}_{j,s,g,t}$ is the consumption expenditure on good s made by individual living in region j of age g at time t, which is bought at its market price $\text{P}^{C}_{j,s,t}$.

Households in region j at time t invest in $\text{Bij}_{i,j,g,t}$ and $\text{Kij}_{i,j,g,t}$ amounts of bonds and physical capital goods, respectively, which are issued by producers and

[2] The results are derived in Appendix (2)

governments across regions. In other words, although purchased only by individuals living in region *j*, $Bij_{i,j,g,t}$ and $Kij_{i,j,g,t}$ include bonds and physical capital issued by both region *j* and *i*.

3.3 Regional (national) investors at time t

Denoting $Kstock_{j,t}$ as the capital stock of region *j* at the beginning of time *t*, we have for region *j*:

$$Kstock_{j,t+1} = Inv_{j,t} + (1-DepR_{j})\, Kstock_{j,t}$$

Where, $DepR_{j}$ represents the capital depreciation rate (constant). $Inv_{j,t}$ is the amount of aggregate physical capital investment in period *t*, which is constructed by aggregating through a CES aggregator the different market goods s that are assumed to be substitutable between each other:

$$Inv_{j,t} = CES\,(InvS_{j,s,t};\ \alpha_{j,s}^{InvS},\ \sigma_{j,}^{InvS})$$

Hence, investors' reach their optimality when:

$$\begin{cases} {P_{j,t}^{Inv}}^{1-\sigma_j^{InvS}} = \sum_s \alpha_{j,s}^{InvS} {P_{j,s,t}^{C}}^{1-\sigma_j^{InvS}} \\ InvS_{j,s,t} = \alpha_{j,s}^{InvS} \left(\dfrac{P_{j,t}^{Inv}}{P_{j,s,t}^{C}} \right)^{\sigma_j^{InvS}} Inv_{j,t} \end{cases}$$

Where, $P_{j,t}^{Inv}$ is aggregate investment market price. $InvS_{j,.s.t}$ is investment expenditures on good *s*.

At time *t*, return on physical capital bought at *t*-1 is realized. So the one period expected rate of return is:

$$\text{RRet}_{j,t} = \frac{\text{Re}\,nt_{j,t} + (1 + DepR_j)P^{Inv}_{j,t}}{P^{Inv}_{j,t-1}}$$

3.4 Regional (national) government at time t

Regional government revenue includes tax incomes on capital income, labor income and private consumption expenditures, and income from selling bonds. On the other hand, government expenditures consist of government consumption $\text{Gov}_{j,t}$ and interest payments on debt (which is assumed to be equal to the interest payments on government bonds issued at the previous time period). Therefore, Budget constrains of government of region j at time t is:

$$\text{P}^{Gov}_{j,t}\,\text{Gov}_{j,t} + \left[\frac{R\text{int}\,J_{j,t-1}P^{Gov}_{j,t}}{P^{Gov}_{j,t-1}}\right]P^{Gov}_{j,t-1}Bond_{j,t} = \text{P}^{Gov}_{j,t}\,\text{Bond}_{j,t+1} +$$

$$\sum_g Pop_{j,g,t}\ \{\ \tau^W_{j,t}(LInc_{j,g,t} + Pens_{j,g,t}) + \tau^C_{j,t}P^{Con}_{j,t}Con_{j,g,t} +$$

$$\tau^K_{j,t}\ [\ \sum_i(\frac{R\text{int}\,J_{i,t-1}P^{Gov}_{i,t}}{P^{Gov}_{i,t-1}} - 1)\,\text{P}^{Gov}_{i,t-1}\text{Bij}_{i,j,g,t} + \sum_i(R\text{Re}t_{i,t} - 1)P^{Inv}_{i,t-1}Kij_{i,j,g,t}\ \ ]\ \}$$

Where, $\text{P}^{Gov}_{j,t}$ is aggregate price index of government spending of region j at time t, which is the same as the price of government bond at time t. $\text{Bond}_{j,t}$ is the amount of bonds issued at time t-1 with fixed interest rate $\text{RintJ}_{j,t-1}$. $\text{Pens}_{j,g,t}$ is the amount of old-age pension transfers paid by regional government, which are financed by contribution rates on households' wage incomes:

$$\sum_{j} Pop_{j,gn,t} Pens_{j,gn,t} = CtR_{j,t} \sum_{j} \sum_{gj} Pop_{j,gj,t} LInc_{j,gj,t}$$

Again, we assume government of each region allocates its consumption Gov $_{j,t}$ across final good sectors s through a CES aggregator:

Gov $_{j,t}$ = CES (GovS $_{j,t}$; $\alpha_{j,s}^{GovS}$, σ_{j}^{GovS})

Where, GovS $_{j,t}$ denotes government consumption expenditures on good *s*. The first order condition gives the following optimal conditions for government of region *j* at time *t*:

$$\begin{cases} {P_{j,t}^{Gov}}^{1-\sigma_j^{GovS}} = \sum_{s} \alpha_{j,s}^{GovS} {P_{j,s,t}^{C}}^{1-\sigma_j^{GovS}} \\ GovS_{j,s,t} = \alpha_{j,s}^{GovS} \left(\dfrac{P_{j,t}^{Gov}}{P_{j,s,t}^{C}} \right)^{\sigma_j^{GovS}} Gov_{j,t} \end{cases}$$

3.5 International trade of region j at time t

Within region j, domestic aggregate demand for goods in sector *s* at time *t* is the sum of all individual demands from the producers, households, investors and government:

AgD $_{j,s,t} = \sum_{ss} XS_{ss,j,s,t} + \sum_{g} Pop_{j,g,t} ConS_{j,s,g,t}$ + InvS $_{j,s,t}$ + GovS $_{j,s,t}$

Among this basket of composite good s demanded, some are produced domestically while some are produced by other regions. Assume that for each region, an optimal basket is chosen across all rejoins using a CES aggregator CES (E $_{ii,j,s,t}$; $\alpha_{ii,j,s}^{E}, \sigma_{j,s}^{E}$). For product *s*, domestic price index and the aggregate demand are therefore reached under the following conditions:

$$\begin{cases} P^{C\ 1-\sigma^E_{j,s}}_{j,s,t} = \sum_{ii} \alpha^E_{ii,j,s} P_{ii,s,t}^{\ 1-\sigma^E_{j,s}} \\ E_{ii,j,s,t} = \alpha^E_{ii,j,s} \left(\frac{P^C_{j,s,t}}{P_{ii,s,t}} \right)^{\sigma^E_{j,s}} \left\{ \sum_{ss} XS_{ss,j,s,t} + \sum_{g} Pop_{j,g,t} ConS_{j,s,g,t} + InvS_{j,s,t} + GovS_{j,s,t} \right\} \end{cases}$$

Where, $P^C_{j,s,t}$ is the price index of goods in sector *s*. $E_{ii,j,s,t}$ refers to good *s* demanded by region *j* that is originated from region *ii.*

3.6 Rest of the world at time t

Holding its aggregate market price and income constant, rest of the world's demand for goods in sector *s* which are produced in region *j* depends on region *j*'s sectoral competitiveness. Hence, we have:

$$E_{j,row,s,t} = Sc^E_{j,row,s} \left(\frac{P_{row,s,t}}{P_{j,s,t}} \right)^{\eta_s} \qquad \eta^s > 0$$

For rest of the world as a whole, its aggregate demand for goods in sector *s* is $E_{ii,row,s,t}$ that is produced across the world (including rest of the world itself and the other two symmetric regions). On the contrary, $E_{row,ii,s,t}$ stands for optimal aggregate amount of goods in sector *s* that rest of the world chooses to produce. η^s represents the export price elasticity to the rest of the world.

Furthermore, assuming that rest of the world does not either borrow or lend worldwide, its trade with the other two regions are therefore balanced at any time period as below:

$$\sum_{s}\sum_{ii} P_{ii,s,t} E_{ii,row,s,t} = \sum_{s} P_{row,s,t} \sum_{ii} E_{row,ii,s,t}$$

3.7 Equilibrium Conditions

At any time *t*, the general equilibrium of this model holds when:

1) Market is clearing for all goods:

$$Z_{j,s,t} = \sum_{ii} E_{j,ii,s,t}$$

Output (supply) of composite good *s* in region *j* equals to the sum of domestic and foreign demand, the amount of which is optimally chosen.

2) No unemployment:

$$\sum_{gj} Pop_{j,gj,t} L^{sup}_{j,iqual,gj} EP_{gj} = \sum_{s} L^{dem}_{j,s,iqual,t}$$

3) Full occupation of physical capital:

$$\text{Kstock}_{j,t} = \sum_{s} K^{dem}_{j,s,t}$$

4) Perfectly integrated asset markets:

$$\frac{R\text{int}J_{j,t} P^{Gov}_{j,t+1}}{P^{Gov}_{j,t}} = \text{RetR}_{j,t+1}$$

$$\text{Rint}_t = \frac{R\text{int}J_{j,t} P^{Gov}_{j,t+1}}{P^{Gov}_{j,t}}$$

Whether investing in government bonds or physical capitals is no difference in terms of the rate of return.

4. Data

The data used in this paper are fictitious and taken from Mérette and Mercenier (2003). Consequently, there is no effort here to calibrate the model on real data. One region will be called Canada, another one the United States just for convenience. The main difference between the two regions is the size of GDP and population. We repeat here that our objective is to see if in a general equilibrium framework asymmetric stylized facts regarding skill premium can or cannot be explained by a combination of country-specific shocks and economic interdependence. We understand that the results of this paper will need to be tested further before making any firm conclusion. Below are the most important features of the computable model.

Two symmetric regions and the rest of the world as a residual region are included in the model. We denote region *1* as Canada, region *2* as the US, and *row* as the rest of the world. All parameters and exogenous variables are assumed same across these two regions. Labor services represent 65 percent of GDP which means that capital services amount to 35 percent of GDP. We have two types of labor services in each region: low-skilled labor service is denoted as *iqual (1)* and high-skilled *as iqual (2)*. For each region, private consumption, government consumption and investment represent respectively 60, 20 and 20 percents of GDP. Trade balances equal zero. The base-year parameters and exogenous variables displayed in the following table are assumed identical (except when notified) in the two symmetric regions:

Table 4.1 Parameters and Exogenous Variables in the base-year in region 1 & 2

$\tau^{W}_{j,t}$	0.318
$\tau^{K}_{j,t}$	0.382
$\tau^{C}_{j,t}$	0.234
$Q_{j,s,t}$	1 (Canada) 10 (US)
$1/\theta$	0.175
$\sigma^{ConS}_{j,s}$	2.500
$\sigma^{Inv}_{j,s}$	2.500
$\sigma^{Gov}_{j,s}$	2.500
$\sigma^{C}_{j,s}$	3.000
$\sigma^{L}_{j,s}$	3.000
$\sigma^{X}_{j,s}$	2.000
$RBeq_{j,gn,t}$	0.400
$Inh_{j,gj,t}$	0.250
$PensR_{j}$	0.300
$Rint_{t}$	0.038
$DepR_{j}$	0.051
$\sum_{g} Pop_{j,g,t}$	1 (Canada) 10 (US)
η_{s}	5.000

For each region, sector *1* uses low-skilled labor more intensively while sector

2 is biased to high-skilled labor. Share of the two types of labor services used in the each sectors are:

Table 4.2 Share of type of labor used $\alpha^{L}_{j,s,iqual}$

	Share of low-skilled	Share of high-skilled
Sector 1	0.75	0.25
Sector 2	0.25	0.75

5. Results

The simulation results are presented in this section. We report in the next sub-sections the impacts on wage levels with either symmetrical or asymmetrical changes in the skill-biased technological progress and total factor productivity. Note that all shocks are temporary and so do the impacts (which means that all changes will converge to their initial level eventually). All percentage changes of the results are with respect to the base-year values.

5.1 Shocks on Total Factor Productivity ($Sc^{Z}_{j,s,t}$)

An improvement in TFP results in higher level of GDP and better labor market position for both high-skilled and low-skilled workers.

5.1.1 Symmetric shock on TFP —both sectors and regions

We assume here that total factor productivity increases by 3 per cent everywhere for three decades. Because the shock is symmetric, changes are identical in both sectors and regions as expected. Results on changes in wage rates and on GDP are displayed in the tables below:

Table 5.1.1a Change (%) in $W_{j,iqual,t}$ when shocks on TFP for both sectors and regions: Region 1 and Region 2

Time Period T	Iqual (1)	Iqual (2)
T0	0.0000	0.0000
T1	4.1893	4.1893
T2	6.0299	6.0299
T3	7.5013	7.5013
T4	4.3014	4.3014
T5	3.5475	3.5475
T6	2.8915	2.8915
T7	2.3616	2.3616
T8	1.9181	1.9181
T9	1.5393	1.5393

Table 5.1.1b Change (%) in $Q_{j,s,t}$ when shocks on TFP for both sectors and regions: Region 1 and Region 2

Time Period T	Sector 1	Sector 2
T0	0.0000	0.0000
T1	0.0000	0.0000
T2	2.5544	2.5544
T3	4.6102	4.6102
T4	6.2545	6.2545
T5	5.1500	5.1500
T6	4.1918	4.1918
T7	3.4198	3.4198
T8	2.7748	2.7748
T9	2.2250	2.2250

Since higher TFP implies higher labor productivity, improving total factor productivity results in increases in the wage levels of both high- and low-skilled labor, as well as the GDP value ($Q_{j,s,t}$) of both sectors. The results show in table 5.1.1a and 5.1.1b are perfectly symmetric and are just like predicted by theory, which proves that the numerical model we constructed for this paper works well. Now we proceed to the next simulation scenario.

5.1.2 Asymmetric shock on TFP— both sectors but only in region 1

Total factor productivity is now assumed to increase by 5 per cent for both sectors in region 1 for four decades. The following results indicate that wage levels and GDP in region 1 show greater improvements compared with those in 5.1.1, consistently with a greater TFP improvement. In region 2, on the other hand, although no shock occurs, aggregate wage rates and GDP rise as well, to a lesser extent than those in region 1. The resultant outcomes are shown in the following tables:

Table 5.1.2a Change (%) in $W_{j,iqual,t}$ when shocks on TFP for both sectors in region 1:

Region 1

Time Period T	Iqual (1)	Iqual (2)
T0	0.0000	0.0000
T1	6.0005	6.0005
T2	8.0281	8.0281
T3	9.6458	9.6458
T4	10.9682	10.9682
T5	5.8395	5.8395
T6	5.2165	5.2165
T7	4.5870	4.5870
T8	4.0438	4.0438
T9	3.5069	3.5069

Table 5.1.2b Change (%) in $W_{j,iqual,t}$ when shocks on TFP for both sectors in region 1:

Region 2

Time Period T	Iqual (1)	Iqual (2)
T0	0.0000	0.0000
T1	0.9980	0.9980
T2	2.0826	2.0826
T3	2.9184	2.9184
T4	3.5067	3.5067
T5	2.7914	2.7914
T6	1.8808	1.8808
T7	1.1720	1.1720
T8	0.6806	0.6806
T9	0.3223	0.3223

Table 5.1.2c Change (%) in $Q_{j,s,t}$ when shocks on TFP for both sectors in region 1:

Region 1

Time Period T	Sector 1	Sector 2
T0	0.0000	0.0000
T1	0.0000	0.0000
T2	3.6157	3.6157
T3	5.9954	5.9954
T4	7.6749	7.6749
T5	8.9145	8.9145
T6	6.5422	6.5422
T7	4.9746	4.9746
T8	3.9319	3.9319
T9	3.1188	3.1188

Table 5.1.2d Change (%) in $Q_{j,s,t}$ when shocks on TFP for both sectors in region 1:

Region 2

Time Period T	Sector 1	Sector 2
T0	0.0000	0.0000
T1	0.0000	0.0000
T2	0.6399	0.6399
T3	1.6725	1.6725
T4	2.6761	2.6761
T5	3.6595	3.6595
T6	3.7531	3.7531
T7	3.3477	3.3477
T8	2.8756	2.8756
T9	2.3864	2.3864

Because of high economic integration between these two regions, higher TFP in region 1 not only raises returns to both types of skills and GDP level domestically, but also those in region 2. However, region 2 gets less benefit from region 1's TFP improvement. Still, the simulation experiment shows that a country can benefit significantly from a TFP improvement occurring in another country when the two economies are integrated through free trade of goods and capital assets.

5.1.3 Asymmetric shock on TFP— only sector 2 in region 2

This time, total factor productivity rises by 4 per cent for four decades for sector 2 in region 2 only. Wages and GDP value no longer change by the same amount across sectors as the positive shock only occurs in sector 2. Results are reported in tables below:

Table 5.1.3a Change (%) in $W_{j,iqual,t}$ when shocks on TFP for sector 2 in region 2:

Region 1

Time Period T	Iqual (1)	Iqual (2)	Iqual(2)/Iqual(1)
T0	0.0000	0.0000	0.00
T1	0.3377	0.4704	0. 13
T2	0.7748	0.9083	0. 13
T3	1.1091	1.2432	0. 13
T4	1.3303	1.4649	0. 13
T5	1.4950	1.6299	0. 13
T6	1.2206	1.2206	0.00
T7	0.8123	0.8123	0.00
T8	0.4927	0.4927	0.00
T9	0.2653	0.2653	0.00

Table 5.1.3b Change (%) in $W_{j,iqual,t}$ when shocks on TFP for sector 2 in region 2:

Region 2

Time Period T	Iqual (1)	Iqual (2)	%Iqual(2)/%Iqual(1)
T0	0.0000	0.0000	0.000
T1	1.6697	3.1380	1.44
T2	2.4443	3.9233	1.44
T3	3.0821	4.5698	1.44
T4	3.6262	5.1214	1.44
T5	4.0905	5.5920	1.44
T6	2.8298	2.8298	0.0000
T7	2.5252	2.5252	0.0000
T8	2.2519	2.2519	0.0000
T9	1.9985	1.9985	0.0000

Since sector 2 is high-skilled intensive (e.g. information technology industry), a positive TFP shock in sector 2 makes wages of high-skilled workers increase more than that of low skilled. In other words, skill premium increases and so does the earnings inequality. Without TFP improvement in either sector, region 1 is still positively influenced by the increased productivity that occurs in sector 2, region 2, though the effects are not as significant as those in region 2. Namely, in both regions (more so in region 2), $\mathbf{Q}_{j,s,t}$, $W_{j,iqual(h),t}$, $\mathbf{W}_{j,iqual(l),t}$ and the skill premium $\left(\frac{W_{j,iqual(h),t}}{W_{j,iqual(l),t}}\right)$ all levels up.

It is interesting to note that such a scenario has not been discussed in the literature on skill premium. The results of the simulation experiment clearly show that for the case of Canada and the US, this scenario may explain the divergent trends in skill premium quite well. Although Canada does share some benefits from positive shocks to TFP in the high-skilled intensive industries in the US, the impacts on the skill premium are not as significant. Moreover, comparing with the argument of SBTC-driven demand for more skilled, higher TFP in the high-skilled intensive sector provides another possibility that might drive demands in favor of more-skilled labor while against the less-skilled.

Table 5.1.3c Change (%) in Q $_{j,s,t}$ when shocks on TFP for sector 2 in region 2:

Region 1

Time Period T	Sector 1	Sector 2
T0	0.0000	0.0000
T1	-0.2611	0.2611
T2	0.0003	0.5246
T3	0.4232	0.9503
T4	0.8295	1.3595
T5	1.2058	1.7384
T6	1.8370	1.8370
T7	1.8203	1.8203
T8	1.6168	1.6168
T9	1.3839	1.3839

Table 5.1.3d Change (%) in Q $_{j,s,t}$ when shocks on TFP for sector 2 in region 2:

Region 2

Time Period T	Sector 1	Sector 2
T0	0.0000	0.0000
T1	-2.8354	2.8293
T2	-1.4211	4.3238
T3	-0.4844	5.3132
T4	0.1860	6.0211
T5	0.6980	6.5614
T6	4.0275	4.0275
T7	3.0015	3.0015
T8	2.3411	2.3411
T9	1.8761	1.8761

In the results reported above, it is interesting to note that the asymmetric sectoral shock has a temporary negative impact on sector 1 in both regions 1 and 2. Indeed, as the shock occurs in period *T1*, substitution effects are immediately at work and so consumption favors the good produced by sector 2 as its price is now lower, which is resulted from the higher TFP and more outputs in sector 2. In the medium run, sector 1 production increases again because of positive income effects.

5.2 Shocks on Skill-Biased Technological Change ($Sc^{Q}_{j,s,t} + \alpha^{L}_{j,s,iqual}$)

5.2.1 Symmetric shock on SBTC—both sectors and regions

Suppose there is a temporary skill-biased shock across sectors in both regions, which increases by 4 per cent for five periods. The shock results in higher productivities of physical capital and human capital combined in both sectors, but since it is biased towards higher skills, the relative demand for high-skilled labor increases in both sectors. That is, $Sc^{Q}_{j,s,t}$ becomes higher due to the positive shock, and $\alpha^{L}_{j,s,iqual(h)}$ rises because the shock is skill-biased.

We will find from the following simulation results that skill premium rises in both regions. Specifically, the wage level for higher skilled workers increases. Returns to low-skilled workers, on the other hand, decreases at first, which can be explained as a result from the shift in labor demand that is against them. However, since higher aggregate demand generated from the positive shock on productivity ultimately improves labor market positions of both types of skills, the wage rate level of the low-skilled workers gradually increases from the next time period, though to a much lesser extent than their high-skilled counterparts.

Table 5.2.1a Change (%) in $W_{j,iqual,t}$ when shocks on SBTC for both sectors and regions:

Region 1 and Region 2

Time Period T	Iqual (1)	Iqual (2)	Iqual(2)/Iqual(1)
T0	0.0000	0.0000	0.00
T1	-0.4479	4.5366	5. 01
T2	0.4150	5.4426	5. 01
T3	1.1089	6.1713	5. 01
T4	1.6541	6.7438	5. 01
T5	2.0999	7.2119	5. 01
T6	2.9319	2.9319	0.00
T7	2.4064	2.4064	0.00
T8	1.9676	1.9676	0.00
T9	1.6143	1.6143	0.00

Table 5.2.1b Change (%) in $Q_{j,s,t}$ when shocks on SBTC for both sectors and regions:

Region 1 and Region 2

Time Period T	Sector 1	Sector 2
T0	0.0000	0.0000
T1	2.5797	3.3376
T2	3.8629	4.6302
T3	4.8983	5.6733
T4	5.7140	6.4950
T5	6.3824	7.1684
T6	4.2507	4.2507
T7	3.4849	3.4849
T8	2.8467	2.8467
T9	2.3338	2.3338

5.2.2 Asymmetric shock on SBTC—both sectors in region 2 only

What will happen to the skill premium if the shock on SBTC only takes place in one of them? Assuming the shock happens across sectors in region 2, the skill premiums show divergent trends.

Table 5.2.2a Change (%) in $W_{j,iqual,t}$ when shocks on SBTC for both sectors in region 2:

Region 1

Time Period T	Iqual (1)	Iqual (2)	Iqual(2)/Iqual(1)
T0	0.0000	0.0000	0.00
T1	-0.6034	4.2053	4. 84
T2	0.2181	5.0666	4. 84
T3	0.8753	5.7556	4. 84
T4	1.3818	6.2866	4. 84
T5	1.7920	6.7166	4. 84
T6	2.7387	2.7387	0.00
T7	2.2021	2.2021	0.00
T8	1.7592	1.7592	0.00
T9	1.4102	1.4102	0.00

Table 5.2.2b Change (%) in $W_{j,iqual,t}$ when shocks on SBTC for both sectors in region 2:

Region 2

Time Period T	Iqual (1)	Iqual (2)	Iqual(2)/Iqual(1)
T0	0.0000	0.0000	0.00
T1	-0.2928	4.8665	5. 17
T2	0.6114	5.8175	5. 17
T3	1.3417	6.5856	5. 17
T4	1.9252	7.1993	5. 17
T5	2.4062	7.7052	5. 17
T6	3.1228	3.1228	0.00
T7	2.6089	2.6089	0.00
T8	2.1747	2.1747	0.00
T9	1.8175	1.8175	0.00

High-skilled labor in both regions benefits more than the lower skilled from the shock on SBTC that happens in region 2. Again, the wage level for the low-skilled workers decreases at first because of the decrease in relative demand for them, and then gradually rises before converging to its initial level. Changes in skill premium are more dramatic in region 2 than in region 1. This is very likely to contribute to the divergent trends in skill in Canada and the US. If SBTC progresses at a faster rate in the US than Canada, on one hand, there will be a spillover effect which makes both countries better off due to the high economic integration; on the other, Canada will benefit much less without its own improvement in SBTC. Consequently, skill premium widens in the US, but not so in Canada.

Table 5.2.2c Change (%) in $Q_{j,s,t}$ when shocks on SBTC for both sectors in region 2:

Region 1

Time Period T	Sector 1	Sector 2
T0	0.0000	0.0000
T1	2.8810	2.0443
T2	4.0185	3.1726
T3	4.9889	4.1352
T4	5.7712	4.9112
T5	6.4225	5.5572
T6	3.9819	3.9819
T7	3.3401	3.3401
T8	2.7587	2.7587
T9	2.2746	2.2746

Table 5.2.2d Change (%) in $Q_{j,s,t}$ when shocks on SBTC for both sectors in region 2:

Region 2

Time Period T	Sector 1	Sector 2
T0	0.0000	0.0000
T1	2.2815	4.6334
T2	3.7086	6.0935
T3	4.8079	7.2181
T4	5.6561	8.0859
T5	6.3409	8.7865
T6	4.5164	4.5164
T7	3.6269	3.6269
T8	2.9326	2.9326
T9	2.3913	2.3913

In region 2, higher $Sc^{Q}_{j,s,t}$ across sectors raises the GDP level in the short term in both sectors for any given inputs, more so in sector 2 since this higher productivity is biased towards high skilled. Output in the two sectors in region 1 rises somewhat as well temporarily from the spillover benefits originated in region 2. Sector 1 in region 1 enjoys greater output than sector 2 does because the high-skilled labor that is intensively used in sector 2 become a more expensive input than the low skilled and therefore, prices of goods in sector 2 jumps up with demand and production shifting toward goods in sector 1.

6. Conclusion

Increased skill premiums, poor macroeconomic conditions and deteriorated labor market institutions are argued to have either jointly or independently widened the earnings inequality in Canada and the US in the last two decades. As Canada and the US experienced different changes in skill premiums, macroeconomic conditions and labor market institutions, the reason for increased earnings dispersions is probably not the same in the two countries. This paper mainly focuses on the skill premium explanation by applying a computable general overlapping-generation model. We find out that, except for the skill-biased technological progress that is mentioned in the existing literature, changes in total factor productivity in the skill-intensive industries will also result in higher skill premiums for any given supply of skills. Furthermore, regardless of their high economic integration, a faster SBTC or productivity progress in the US will enlarge differences in skill premiums between

Canada and the US. To prevent the earnings inequality from increasing, and to make people better off in every position of the earnings distribution, we probably should figure out policies or solutions that could increase technology and productivity, improve economic performance, lift educational attainment and so on.

Appendix

(1) Producer's Problem:

$$\underset{X_{ss,j,s,t},K^{dem}_{j,s,t},L^{dem}_{j,s,iqual,t}}{Maximize}\ \pi = P_{j,s,t}\ Z_{j,s,t} - \sum_{ss} P^{C}_{ss,j,s,t}\ X_{ss,j,s,t} - R_{j,t}\ K^{dem}_{j,s,t} - \sum_{iqual} W_{j,iqual,t}\ L^{dem}_{j,s,iqual,t}$$

Subject to:

(i) $Z_{j,s,t} = Sc^{Z}_{j,s,t}\ (Q_{j,s,t})^{\alpha^{Q}_{j,s}}\ (X_{j,s,t})^{1-\alpha^{Q}_{j,s}}$

(ii) $X_{j,s,t} = (\alpha^{X}_{ss(1),j,s} X_{ss(1),j,s,t}{}^{\sigma^{X}_{j,s}} + \alpha^{X}_{ss(2),j,s}\ X_{ss(2),j,s,t}{}^{\sigma^{X}_{j,s}})^{\frac{1}{\sigma^{X}_{j,s}}}$

(iii) $Q_{j,s,t} = Sc^{Q}_{j,s,t}\ (K^{dem}_{j,s,t})^{\alpha^{K}_{j,s}}\ (L^{dem}_{j,s,t})^{1-\alpha^{K}_{j,s}}$

(iv) $L^{dem}_{j,s,t} = (\alpha^{L}_{j,s,iqual(h)}\ L^{dem}_{j,s,iqual(h),t}{}^{\sigma^{L}_{j,s}} + \alpha^{L}_{j,s,iqual(l)} L^{dem}_{j,s,iqual(l),t}{}^{\sigma^{L}_{j,s}})^{\frac{1}{\sigma^{L}_{j,s}}}$

Substitute (iv) into (iii):

$$Q_{j,s,t} = Sc^{Q}_{j,s,t}\ (K^{dem}_{j,s,t})^{\alpha^{K}_{j,s}} (\alpha^{L}_{j,s,iqual(h)}\ L^{dem}_{j,s,iqual(h),t}{}^{\sigma^{L}_{j,s}} + \alpha^{L}_{j,s,iqual(l)} L^{dem}_{j,s,iqual(l),t}{}^{\sigma^{L}_{j,s}})^{\frac{(1-\alpha^{K}_{j,s})}{\sigma^{L}_{j,s}}} \qquad (1)$$

Substitute (1) & (ii) into (i):

$Z_{j,s,t} =$

$$Sc^{Z}_{j,s,t}\ [\ Sc^{Q}_{j,s,t} (K^{dem}_{j,s,t})^{\alpha^{K}_{j,s}} (\alpha^{L}_{j,s,iqual(h)} L^{dem}_{j,s,iqual(h),t}{}^{\sigma^{L}_{j,s}} + \alpha^{L}_{j,s,iqual(l)} L^{dem}_{j,s,iqual(l),t}{}^{\sigma^{L}_{j,s}})\ ]^{\frac{\alpha^{Q}_{j,s}(1-\alpha^{K}_{j,s})}{\sigma^{L}_{j,s}}}$$

$$(\alpha^{X}_{ss(1),j,s} X_{ss(1),j,s,t}{}^{\sigma^{X}_{j,s}} + \alpha^{X}_{ss(2),j,s}\ X_{ss(2),j,s,t}{}^{\sigma^{X}_{j,s}})^{\frac{(1-\alpha^{Q}_{j,s})}{\sigma^{X}_{j,s}}} \qquad (2)$$

Substitute (2) into π and derive the first order condition with respect to $X_{ss,j,s,t}$, $K^{dem}_{j,s,t}$ and $L^{dem}_{j,s,iqual,t}$, respectively:

- $\dfrac{\partial \pi}{\partial X_{ss,j,s,t}} = 0 \Rightarrow$

$$\mathrm{P}^{C}_{ss,j,s,t} = \mathrm{P}_{j,s,t}\left(\frac{1-\alpha^{Q}_{j,s}}{\sigma^{X}_{j,s}}\right)\left(\alpha^{X}_{ss(1),j,s}\mathrm{X}_{ss(1),j,s,t}{}^{\sigma^{X}_{j,s}} + \alpha^{X}_{ss(2),j,s}\mathrm{X}_{ss(2),j,s,t}{}^{\sigma^{X}_{j,s}}\right)^{\left(\frac{1-\alpha^{Q}_{j,s}}{\sigma^{X}_{j,s}}\right)-1}$$

$$\alpha^{X}_{ss,j,s}\,\sigma^{X}_{j,s}(\mathrm{X}_{ss,j,s,t})^{\sigma^{X}_{j,s}-1}\,\mathrm{Sc}^{Z}_{j,s,t}\left[\mathrm{Sc}^{Q}_{j,s,t}(\mathrm{K}^{dem}_{j,s,t})^{\alpha^{K}_{j,s}}(\alpha^{L}_{j,s,iqual(h)}\mathrm{L}^{dem}_{j,s,iqual(h),t}{}^{\sigma^{L}_{j,s}}\right.$$

$$\left. + \alpha^{L}_{j,s,iqual(l)}\mathrm{L}^{dem}_{j,s,iqual(l),t}{}^{\sigma^{L}_{j,s}})\right]^{\frac{\alpha^{Q}_{j,s}(1-\alpha^{K}_{j,s})}{\sigma^{L}_{j,s}}}$$

- $\dfrac{\partial \pi}{\partial K^{dem}_{j,s,t}} = 0 \Rightarrow$

$$\mathrm{R}_{j,t} = \mathrm{P}_{j,s,t}(\alpha^{X}_{ss(1),j,s}\mathrm{X}_{ss(1),j,s,t}{}^{\sigma^{X}_{j,s}} + \alpha^{X}_{ss(2),j,s}\mathrm{X}_{ss(2),j,s,t}{}^{\sigma^{X}_{j,s}})^{\frac{(1-\alpha^{Q}_{j,s})}{\sigma^{X}_{j,s}}}$$

$$\mathrm{Sc}^{Z}_{j,s,t}\frac{\alpha^{Q}_{j,s}(1-\alpha^{K}_{j,s})}{\sigma^{L}_{j,s}}\left[\mathrm{Sc}^{Q}_{j,s,t}(\mathrm{K}^{dem}_{j,s,t})^{\alpha^{K}_{j,s}}(\alpha^{L}_{j,s,iqual(h)}\mathrm{L}^{dem}_{j,s,iqual(h),t}{}^{\sigma^{L}_{j,s}}\right.$$

$$\left. + \alpha^{L}_{j,s,iqual(l)}\mathrm{L}^{dem}_{j,s,iqual(l),t}{}^{\sigma^{L}_{j,s}})\right]^{\left[\frac{\alpha^{Q}_{j,s}(1-\alpha^{K}_{j,s})}{\sigma^{L}_{j,s}}\right]-1}\mathrm{Sc}^{Q}_{j,s,t}\,\alpha^{K}_{j,s}(\mathrm{K}^{deam}_{j,s,t})^{\alpha^{K}_{j,s}-1}$$

$$(\alpha^{L}_{j,s,iqual(h)}\mathrm{L}^{dem}_{j,s,iqual(h),t}{}^{\sigma^{L}_{j,s}} + \alpha^{L}_{j,s,iqual(l)}\mathrm{L}^{dem}_{j,s,iqual(l),t}{}^{\sigma^{L}_{j,s}})$$

- $\dfrac{\partial \pi}{\partial L^{dem}_{j,s,iqual,t}} = 0 \Rightarrow$

$$\mathrm{W}_{j,iqual,t} = \mathrm{P}_{j,s,t}(\alpha^{X}_{ss(1),j,s}\mathrm{X}_{ss(1),j,s,t}{}^{\sigma^{X}_{j,s}} + \alpha^{X}_{ss(2),j,s}\mathrm{X}_{ss(2),j,s,t}{}^{\sigma^{X}_{j,s}})^{\frac{(1-\alpha^{Q}_{j,s})}{\sigma^{X}_{j,s}}}\mathrm{Sc}^{Z}_{j,s,t}\frac{\alpha^{Q}_{j,s}(1-\alpha^{K}_{j,s})}{\sigma^{L}_{j,s}}$$

$$\mathrm{Sc}^{Q}_{j,s,t}(\mathrm{K}^{dem}_{j,s,t})^{\alpha^{K}_{j,s}}\alpha^{L}_{j,s,iqual,t}\,\sigma^{L}_{j,s}(\mathrm{L}^{dem}_{j,s,iqual,t})^{\sigma^{L}_{j,s}-1}\left[\mathrm{Sc}^{Q}_{j,s,t}(\mathrm{K}^{dem}_{j,s,t})^{\alpha^{K}_{j,s}}(\alpha^{L}_{j,s,iqual(h)}\mathrm{L}^{dem}_{j,s,iqual(h),t}{}^{\sigma^{L}_{j,s}}\right.$$

$$\left. + \alpha^{L}_{j,s,iqual(l)}\mathrm{L}^{dem}_{j,s,iqual(l),t}{}^{\sigma^{L}_{j,s}})\right]^{\left[\frac{\alpha^{Q}_{j,s}(1-\alpha^{K}_{j,s})}{\sigma^{L}_{j,s}}\right]-1}$$

(2) Households' optimal allocation between final goods s

$$\underset{ConS_{j,s,g,t}}{Maximize}\ \mathrm{Con}_{j,g,t} = \left(\sum_{s}\alpha^{ConS}_{j,s,g}ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}}\right)^{\frac{\sigma^{ConS}_{j,g}}{\sigma^{ConS}_{j,g}-1}} \quad (1)$$

$$\text{Subject to: } \mathrm{P}^{Con}_{j,g,t}\,\mathrm{Con}_{j,g,t} = \sum_{s}P^{C}_{j,s,t}ConS_{j,s,g,t} \quad (2)$$

$$\Leftrightarrow \text{Max } L = \left(\sum_s \alpha^{ConS}_{j,s,g} ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}} \right)^{\frac{\sigma^{ConS}_{j,g}}{\sigma^{ConS}_{j,g}-1}} - \lambda \left(\sum_s P^{C}_{j,s,t} ConS_{j,s,g,t} - P^{Con}_{j,g,t} \text{Con}_{j,g,t} \right)$$

F.O.C for any given s is:

$$\frac{\partial L}{\partial ConS} =$$

$$\frac{\sigma^{ConS}_{j,g}}{\sigma^{ConS}_{j,g}-1} \left(\sum_s \alpha^{ConS}_{j,s,g} ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}} \right)^{\frac{\sigma^{ConS}_{j,g}}{\sigma^{ConS}_{j,g}-1}-1} \frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}} \alpha^{ConS}_{j,s,g} ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}-1} - \lambda P^{C}_{j,s,t} = 0$$

$$\rightarrow \left(\sum_s \alpha^{ConS}_{j,s,g} ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}} \right)^{\frac{1}{\sigma^{ConS}_{j,g}-1}} \alpha^{ConS}_{j,s,g} ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}-1} = \lambda P^{C}_{j,s,t} \qquad (3)$$

Multiplying both sides in (3) by ConS $_{j,s,g,t}$, we get the following:

$$\left(\sum_s \alpha^{ConS}_{j,s,g} ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}} \right)^{\frac{1}{\sigma^{ConS}_{j,g}-1}} \alpha^{ConS}_{j,s,g} ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}} = \lambda P^{C}_{j,s,t} \qquad (4)$$

Taking sum of all s:

$$\left(\sum_s \alpha^{ConS}_{j,s,g} ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}} \right)^{\frac{1}{\sigma^{ConS}_{j,g}-1}} \sum_s \alpha^{ConS}_{j,s,g} ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}} = \lambda \sum_s P^{C}_{j,s,t} ConS_{j,s,g,t}$$

$$\rightarrow \left(\sum_s \alpha^{ConS}_{j,s,g} ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}} \right)^{\frac{\sigma^{ConS}_{j,g}}{\sigma^{ConS}_{j,g}-1}} = \lambda \sum_s P^{C}_{j,s,t} ConS_{j,s,g,t} \qquad (5)$$

Substituting (1) and (2) into (5), we have:

$$\text{Con}_{j,g,t} = \lambda \text{P}^{Con}_{j,g,t} Con_{j,g,t}$$

$$\rightarrow \lambda = \frac{1}{P^{Con}_{j,g,t}} \qquad (6)$$

Replacing λ in (4) by equation (6):

$$\left(\sum_s \alpha^{ConS}_{j,s,g} ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}} \right)^{\frac{1}{\sigma^{ConS}_{j,g}-1}} \alpha^{ConS}_{j,s,g} ConS_{j,s,g,t}^{\frac{\sigma^{ConS}_{j,g}-1}{\sigma^{ConS}_{j,g}}} = \frac{P^{C}_{j,s,t}}{P^{Con}_{j,g,t}} \text{ ConS}_{j,s,g,t} \qquad (7)$$

Multiplying both sides in (7) by exponential $\sigma_{j,g}^{ConS}$:

$$\left(\sum_s \alpha_{j,s,g}^{ConS} ConS_{j,s,g,t}^{\frac{\sigma_{j,g}^{ConS}-1}{\sigma_{j,g}^{ConS}}}\right)^{\frac{\sigma_{j,g}^{ConS}}{\sigma_{j,g}^{ConS}-1}} \alpha_{j,s,g}^{ConS} ConS_{j,s,g,t}^{\sigma_{j,g}^{ConS}-1} = \left(\frac{P_{j,s,t}^{C}}{P_{j,g,t}^{Con}}\right)^{\sigma_{j,g}^{ConS}} \text{ConS}_{j,s,g,t}^{\sigma_{j,g}^{ConS}}$$

$$\rightarrow \text{Con}_{j,g,t}\ \alpha_{j,s,g}^{ConS} ConS_{j,s,g,t}^{\sigma_{j,g}^{ConS}-1} = \left(\frac{P_{j,s,t}^{C}}{P_{j,g,t}^{Con}}\right)^{\sigma_{j,g}^{ConS}} \text{ConS}_{j,s,g,t}^{\sigma_{j,g}^{ConS}}$$

$$\rightarrow \text{ConS}_{j,s,g,t} = \alpha_{j,s,g}^{ConS} \left(\frac{P_{j,g,t}^{Con}}{P_{j,s,t}^{C}}\right)^{\sigma_{j,g}^{ConS}} \text{Con}_{j,g,t} \qquad (8)$$

From equation (2), we can get:

$$\text{P}_{j,g,t}^{Con} = \frac{\sum_s P_{j,s,t}^{C} ConS_{j,s,g,t}}{Con_{j,g,t}}$$

Substituting $\text{ConS}_{j,s,g,t}$ in the above expression by (8) gives:

$$(\text{P}_{j,g,t}^{Con})^{1-\sigma_{j,g}^{ConS}} = \sum_s \alpha_{j,s,g}^{ConS} P_{j,s,t}^{C\ \ 1-\sigma_{j,g}^{Con}}$$

Therefore, for households in region j, using a CES aggregator, the optimal allocation of expenditures between final good s is achieved when:

$$\begin{cases} P_{j,g,t}^{Con\,1-\sigma_{j,g}^{ConS}} = \sum_s \alpha_{j,s,g}^{ConS} P_{j,s,t}^{C\ \ 1-\sigma_{j,g}^{ConS}} \\ ConS_{j,s,g,t} = \alpha_{j,s,g}^{ConS} \left(\frac{P_{j,g,t}^{Con}}{P_{j,s,t}^{C}}\right)^{\sigma_{j,g}^{ConS}} Con_{j,g,t} \end{cases}$$

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