The Taylor Rule: An exploration in Theory and Practice

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Major Paper presented to the
Department of Economics of the University of Ottawa
in partial fulfillment of the requirements of the M.A. Degree

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Ottawa, Ontario
November, 2002
Acknowledgements

The author is grateful to Professor Marc Lavoie for his supervision and for his insightful ideas and comments.
Abstract

Among various monetary policy rules, the Taylor rule has been most popular and has attracted a good deal of attention in recent years. This paper explores the theoretical background of the Taylor rule by introducing the IS-MP-IA model. The development of modern macroeconomics suggests that the IS-MP-IA model is superior to the IS-LM-AS model and as a result, the Taylor rule is well founded on the theoretical level. The empirical study in this paper also shows that a revised version of the Taylor rule performs very well in describing the behavior of the Bank of Canada from 1993 to 2002. As a policy rule of thumb, the Taylor rule provides a useful guideline for central banks to implement their monetary policy and maintain price stability and output stability.
I. Introduction

Whether central banks should use rules and, if they do, what kind of rules should be used in the implementation of monetary policy are old and important issues in macroeconomics. After the collapse of the gold standard regime, the monetary authorities have been facing the question of how to conduct monetary policy to stabilize prices, output and employment.

There are various approaches to conduct monetary policy. It is well accepted that the two extremes of monetary policy-making are rules and discretion. In McCallum’s words, “The basic distinction between rule-based and discretionarv behavior is that the former ignores current conditions in designing the relationship of variables to current and past conditions, whereas the latter does not ignore current conditions and treats past conditions as bygones” (2000, p.6). However, some economists reject the rules vs. discretion dichotomy. Bernanke, Laubach, Mishkin, and Posen (1999) defined inflation targeting as a policy framework and suggested that inflation targeting “combines some of the advantages traditionally ascribed to rules with those ascribed to discretion” (1999, p.6).

Rudebusch and Svensson (1999) and Svensson (1997) introduced two classes of rules: instrument rules, which “express monetary policy instrument as an explicit function of available information” (Rudebusch and Svensson, 1999, p.204), and targeting rules, which is “represented by the assignment of a loss

According to Gramlich (1998), an unconditional rule is the simplest one, such as the constant rate of growth rule proposed by Milton Friedman. A target rule is a totally different approach, such as inflation targeting, which is used in a number of industrialized countries (New Zealand, Canada, the United Kingdom, Sweden, Australia, Finland, Spain, and Israel). Rather than follow a simple rule such as one that has some monetary quantities grow at a constant rate, with inflation targeting the monetary authorities have great discretion to pursue the ultimate goal of monetary policy -- a stable overall price level or stable inflation rates.

Even though inflation targeting has advantages such as transparency and accountability, it has drawbacks as well. Some academic papers criticize inflation targeting for ignoring output and making rigid central bank policy. A similar target rule is nominal income targeting suggested by Hall and Mankiw (1994). Nominal income targeting is a better choice under price shocks, while inflation targeting is better under output productivity shocks (Gramlich, 1998). Another target rule is money supply targeting, under which the monetary authorities are concerned only with money but ignore all other information contained in non-money variables. Some authors believe that money supply targeting can be also regarded as the limiting case of inflation targeting. When
there is a one-to-one mapping between the growth of money and inflation, the inflation-target regime collapses to that of money supply targeting (King, 1997). Money supply targeting is important since the IS-LM-AD model, which has been an essential tool of macroeconomics for many years, is built under the assumption that central banks follow money supply targeting.

A feedback rule is regarded as an intermediate approach between the two types of monetary policy rule we have talked above. “Under this approach policy objectives, or targets, might be specified in the rule and the authorities would respond in a regular way to deviations between actual values and the target levels of these variables” (Gramlich, 1998, p.2).

As the representative of the feedback rules, the Taylor rule has attracted a lot of attention and has played a very important role in recent years. Many academic works focus on comparing the performance of simple rules, like the Taylor-type rules, with that of complex optimal rules under uncertain circumstances. Levin, Wieland, and Williams (1999), Taylor (1999) and Rudebusch and Svensson (1999) showed that simple rules are nearly as robust or more robust than complex optimal rules across models. Some authors, however, extended the Taylor rule to incorporate other related factors. Clarida, Gali, and Gertler (1998) and Sroul (2001) advocated including the lagged interest rate term to get a better monetary policy rule. Ball (1999) and Svensson (2000) argued that the open-economy rule, which includes an
exchange rate term in the Taylor rule, would be more appropriate for small open economies.

The Taylor rule has monetary authorities conduct monetary policy in response to the gaps between actual and target values of key objective variables, and in this way, these key objective variables can feed back onto monetary policy. More precisely, the Taylor rule can be expressed in an algebraic formula where the real short-term interest rate is set following a reaction function to inflation and the output gap. The inclusion of inflation and output gaps in the Taylor rule reflects the concerns of the monetary authorities for both inflation and output, and their desire to maintain low deviations of inflation rates and output levels from their target or potential values.

As Yellen (1996) pointed out, the Taylor rule being a useful tool of conducting monetary policy, it has several desirable features. The first one is that it provides a built-in anchor with an explicit inflation target. To follow this rule would lead the economy to the target value. Furthermore, the credibility that central banks have earned in the successful conducting of monetary policy can help them to resist the fluctuations of real output and employment caused by business cycles. The second feature is that the Taylor rule is somewhat flexible in dealing with the deviation of inflation from its target value. Sometimes a central bank lowers the short-term real interest rate to combat a rise in the unemployment rate, even if the inflation rate is above the target value. The acceptance of a specific short-run tradeoff between the
two objectives is one advantage of having multiple objectives. The third desirable feature of the Taylor rule is that it has been shown to be robust across different econometric models and to have a pretty good performance under a wide variety of shocks in uncertain circumstances.

In this paper, we focus on the Taylor rule and demonstrate its feasibility and validity from both theoretical and practical aspects. The structure of the rest of this paper is as follows. Section II introduces the IS-MP-IA model and discusses why it is superior to the IS-LM-AS model. Section III presents empirical studies, where we use the historical data of Canada to test how well the Taylor rule is fitted. Section IV delivers the conclusion.

II. The IS-MP-IA model

As we have mentioned in section I, the money targeting rule is a kind of monetary policy rule that requires central banks to adjust the short-term interest rate to keep the money supply as close as possible to the target value. That means that central banks conduct monetary policy by focusing on the money supply. Under this assumption, the IS-LM-AS model is built and has been an essential tool in macroeconomics for many years. Some monetary policy rules, such as the Taylor rule, belong to real interest rules. Real interest rate rules require central banks to conduct monetary policy by focusing on the real interest rate and manipulating the short-term nominal interest rate to achieve their goals for inflation and output. Here the real interest rate is
always expressed as a function of macroeconomic variables, such as inflation rate, output, unemployment, and exchange rate. The belief that most monetary authorities follow real interest rules rather than a money supply rule brings about an alternative macroeconomic model --- the IS-MP-IA model. Based on the analysis in Romer (1999) and Romer (2000), we will discuss how the IS-MP-IA model works and why it is superior to the traditional IS-LM-AS model in this section.

As a widely used macroeconomic tool, the IS-LM-AS model occupies most macroeconomics textbooks, and actually it is rather powerful for analyzing macroeconomic fluctuations. The IS-LM-AS model consists of two components --- the IS-LM model and the AS-AD model. The basic version of the IS-LM model employs two relationships, which are denoted as the IS curve and LM curve, to describe the macroeconomics in a output–interest rate space.

The IS curve is an equilibrium line that concerns the goods market. A higher interest rate reduces the demand for investment goods. In an open economy with floating exchange rates, a higher interest rate also bids up the value of domestic currency and thereby reduces net exports. The relatively lower demand corresponds to a lower supply when the goods market is in equilibrium. Thus we get a negative relationship between interest rates and output in the goods market and this relationship is the IS curve in the model.
The LM curve, similarly, is an equilibrium line related to the money market. The quantity of money demanded is a positive function of income and a negative function of the interest rate. With a fixed money supply, a rise in output or aggregate income rises the demand for money, and therefore increases the interest rate at which the money demand equals the money supply. The positive relationship between output and interest rates in the equilibrated money market at a given money supply is denoted as the LM curve.

![Figure 1. The IS-LM diagram](image)

Figure 1 shows the basic version of the IS-LM model. In the intersection point of the IS and LM curves, both the goods market and money market are in equilibrium. This equilibrium point E in Figure 1 indicates the interest rate and output in the economy. An increase in government expenditures or a decrease in taxes shifts the IS curve to the right, and thus raises the equilibrium interest rate and output along the LM curve. Similarly, an increase...
in the money supply also shifts the LM curve to the right, and this change leads to a lower interest rate and a higher output level in the new equilibrium.

One obvious drawback of the IS-LM model is that it assumes a fixed price level. The IS-LM model is thus extended to the AD-AS model, where price changes are considered. The IS and LM curves are combined to form a new relationship in the output-price level space when the price level is taken into account in the analysis. Given a fixed money supply, an increase in the price level raises the money demand and thus raises the interest rate at which money demand equals money supply. Consequently, the LM curve shifts up and the IS and LM curves cross at a new intersection that corresponds to a lower output level than before. This inverse relationship between output and the price level is known as the aggregate demand curve, i.e. the AD curve. In the AD-AS model, AS means aggregate supply.

The long-run AS curve is a vertical line, where output equals its potential level. In the short-run without complete nominal flexibility, however, the AS curve is upwards sloping in the output-price level space. That is, a higher output level implies a higher price level. The new model formed by the AS and AD curves in an output-price level space, which is shown in Figure 2, is useful in some cases for analyzing short term macroeconomic fluctuations.

Increases in consumption, investment or net exports raise the equilibrium price level at a given output level and thus shift the AD curve upwards. As a
consequence, both the price level and output rise in the short-run. On the contrary, decreases in consumption, investment, or net exports lower the price level and output in the short-run. Some other factors, a shock in cost for example, shift the AS curve in a similar way. A reduction in cost reduces the equilibrium price level at a given output and thus shifts the AS curve downwards. The new equilibrium predicts a higher output but lower price level than before, while if the cost goes up, the AS curve shifts upward, the price level goes up and the output falls in the short-run.

![AD-AS Diagram](image)

Figure 2. The AD-AS diagram

Compared to its merits, the problems of the IS-LM-AS model are now considered more dominant. Rather than controversial topics, such as the assumption of sticky prices and the lack of microeconomic foundations, the IS-LM-AS model is now facing some more fatal difficulties. According to Romer (2000), there are three problems with the model.
Firstly, the definition of interest rates is inconsistent within the model. The demand for goods is related to the real interest rate and therefore, the IS curve is tied to the real interest rate; contrarily, the demand for money is related to the nominal interest rate and therefore, the LM curve is tied to the nominal interest rate. Secondly, the result we can get from the model is not what we want. The AS curve and the AD curve are relationships linking real output and the price level, whereas what we are interested in is the relationship between output and inflation. Thirdly, the IS-LM-AS model is based on an unrealistic assumption. The model assumes that central banks follow a money targeting rule and set a fixed money supply. In practice, however, most central banks pay little attention to money supply when making monetary policy.

Given these drawbacks with the IS-LM-AS model, it is natural to present an alternative model in the following part. This new model is based on the assumption that central banks follow a real interest rate rule rather than the unrealistic assumption that they target money supply. The new approach is known as the IS-MP-IA model, where the IS curve still represents the goods market; the MP curve represents the monetary policy, and the IA curve represents the inflation adjustment.

Like the IS-LM-AS model, the IS-MP-IA model is also composed of two parts. The counterpart of the IS-LM diagram in the IS-MP-IA model is the IS-MP diagram. As we have mentioned before, the LM curve is based on the assumption that central banks target the money supply. Considering that most
central banks, especially those of the industrialized countries, are following a real interest rate rule, we adopt the MP curve to replace the LM curve.

The MP curve describes how central banks conduct monetary policy. In this particular case, the MP curve describes how central banks adjust the real interest rate in response to changes in output and inflation. It is well accepted that central banks set the real interest rate as an increasing function of inflation. Since in the output-real interest rate space, one MP curve corresponds to a certain level of inflation rate, central banks shift the MP curve upwards when inflation rises. Regarding the change in output, however, there are different assumptions on how central banks conduct their monetary policy in response.

In Romer (2000), the author mainly presents an IS-MP model where the MP curve is horizontal. This means that central banks will not adjust the real interest rate in response to changes in output, and the real interest rate is a function only of the rate of inflation: \( r = r(\pi) \). But here, we prefer the MP curve with an upward slope since it is more realistic and a better description of what central banks are doing in the real world. For a given rate of inflation, the story is that central banks raise the real interest rate when inflation and output rise, and reduce the real interest rate when inflation and output fall. This relationship can be expressed algebraically as the following equation: \( r = r(Y, \pi) \), where \( r \) is the real interest rate, \( Y \) is output, and \( r(Y, \pi) \) is an increasing function.
What the monetary authorities want is a higher level of output and a lower inflation rate. It is natural that when output falls, they reduce the real interest rate to spur investment and consumption and thereby to raise the output. But why do they raise the real interest rate when output rises?

Firstly, there is limited room for central banks to reduce the real interest rate, and thus they can not reduce the real interest rate continually. Secondly, the goal of a low and stable rate of inflation is predominant among all the objectives of central banks. An output that is higher than its potential level usually leads to an increase in the inflation rate. In order to prevent the inflation rate from increasing further, central banks also need to raise the real interest rate when output rises beyond its potential. It will be too late if central banks adjust the real interest rate when the inflation rate begins to rise or fall. The monetary policy of adjusting the real rate of interest in response to the change of output can smooth down the volatility of inflation, output and real interest rate.

Figure 3. The IS-MP diagram
In Figure 3, the upward-sloping MP curve and the IS curve together form the IS-MP diagram, and the intersection of the IS and MP curves gives us the real rate of interest and output in the economy. This simple model can be used to analyze short-run economic fluctuations in practice.

However, the analysis within the IS-MP diagram does not take into account the role of inflation. To incorporate inflation, it is necessary to extend the IS-MP model to include the IA curve. The new model is called the IS-MP-IA model.

The first step is to combine the IS curve and the MP curve into one relationship in the output-inflation space. As we have seen, one MP curve in the IS-MP diagram corresponds to a certain level of inflation. When inflation rises, the monetary authorities raise the real rate of interest in response, i.e. the MP curve shifts up. The economy moves up along the IS curve, and output falls. Thus we obtain a negative relationship between the output level and inflation --- an aggregate demand curve or AD curve. This process is shown in Figure 4.

This new relationship is named the aggregate demand curve because in the IS-MP diagram the output is determined by aggregate demand in the economy. However, it is different from the traditional AD curve that we got by combining the IS curve and the LM curve.
Figure 4. The aggregate demand curve

The traditional AD curve is in an output-price level space, and the mechanism is that a higher price level reduces the real money stock and thereby reduces the equilibrium output. The new AD curve, however, is in an output-inflation space and the mechanism is that higher inflation causes central banks to
increase the real rate of interest and thereby reduces output. Having obtained the AD curve in an output-inflation space, we then construct the IA curve. In this model, IA means inflation adjustment.

Following Taylor (1998), we have the following assumptions regarding inflation. First, the rate of inflation is given at any point in time, or in other words, inflation does not respond immediately to economic fluctuations. Second, without inflation shocks, inflation rises when output is above its potential level and it falls otherwise. Therefore, the IA curve is a horizontal line in the output-inflation space --- that is, inflation does not depend on output at a point in time. The output can affect inflation only by causing it to rise or fall gradually over time.

Figure 5. The process of the AD-IA diagram adjusting to long-run equilibrium
In Figure 5, the IA curve indicates the initial inflation level and the AD curve shows output for all possible inflation rates. The behavior of the IA curve depends on whether the original output is above or below its potential level $Y^*$, which is indicated by a dashed vertical line in Figure 5. If the output is above its potential level for example, as is the case in the figure above, inflation goes up, i.e. the IA curve shifts upwards. This process continues until output reaches its long-run equilibrium, a point labeled $E_{LR}$ in Figure 5. At this point output equals its potential level.

There are some differences between the IS-MP model in a closed economy and that in an open economy with floating exchange rates. An obvious one is that in the version of open economy with floating exchange rates, both the IS and the MP curves are flatter than those in the closed economy version. The reason is that in the open economy, a fall in the real rate of interest not only raises investment, just as it does in the closed economy, but it also causes the exchange rate to depreciate and thus raises net exports. Consequently, the change in the real rate of interest can have a larger effect on short-run output in an open economy with floating exchange rates than in a closed economy.

This difference between the two IS-MP diagrams under different circumstances results in a similar difference between the two AD-AS diagrams under these circumstances: the AD curve is flatter in an open economy than in a closed economy.
So far we have constructed the AD-IA diagram and understood its mechanism of adjusting to long-run equilibrium. The IS-MP diagram and the AD-IA diagram together form the IS-MP-IA model, which can be used to analyze how various shocks affect the economy.

First, we analyze the effects of changes in fiscal policy on the economy. The strategies of fiscal policy are the adjustments of government expenditures and taxes. An increase in government expenditure or a reduction in taxes is often called an expansionary fiscal policy, while the opposite is called a deflationary fiscal policy.

![Diagram of IS-MP model showing the effects of an expansionary fiscal policy](image)

Figure 6. The effects of an expansionary fiscal policy in the IS-MP diagram

We assume that the economy was in its long-run equilibrium initially at point E. A move to an expansionary fiscal policy, such as an increase in government expenditures or a decrease in taxes, raises the expected expenditure for a given
level of income, and thus raises equilibrium output level for a given real interest rate in the goods market. In term of the IS-MP diagram, this change shifts the IS curve to the right, and the equilibrium point moves upwards along the MP curve. Then the economy reaches point A with a higher output and a higher real interest rate in the IS-MP diagram. That is a move to an expansionary fiscal policy raises both the output and the real interest rate in the short-run. This process is shown in Figure 6.

The MP curve is unchanged as long as the inflation rate remains at its original value. The new intersection of the IS and MP curves shows a higher output level which, at a given inflation rate, corresponds to a rightward shift of the AD curve in the AD-AS diagram. That is, an expansionary fiscal policy shifts the AD curve to the right immediately in the AD-IA diagram.

![Diagram](image)

Figure 7. The effects of an expansionary fiscal policy in the AD-IA diagram
Since inflation does not respond to economic fluctuations immediately, an expansionary fiscal policy shifts the AD curve to the right and leaves the IA curve unchanged in the short-run, or equivalently, it raises the output and leaves inflation unchanged. Thus the economy jumps to a new point A with an output above its potential level, and hence inflation rises gradually, i.e. the IA curve moves up gradually until the economy reaches its new long-run equilibrium $E_{LR}$ where output equals its potential level. This process is depicted in Figure 7.

One advantage of the AD-IA diagram is that one can trace out the immediate impacts as well as gradual effects of an economic fluctuation, rather than the traditional AS-AD diagram, in which one can only analyze the short-run effects of an economic fluctuation. We can also depict this process in the IS-MP diagram, in which the behavior of central banks is interpreted by the MP curve.

An expansionary fiscal policy raises output at a given real interest rate and central banks raise the real interest rate immediately in response. Central banks raise the real interest rate further as long as output is above its potential level, or equivalently, as long as inflation rises. In the IS-MP diagram, central banks shift the MP curve upwards continually towards its new long-run equilibrium, with real interest rates rising and output falling. In the new long-run equilibrium in Figure 6, output equals its potential level but the real rate of interest is higher than before. This result is consistent with the common
understanding that an expansionary fiscal policy raises the real rate of interest in the long run. By a similar argument we can show that a deflationary fiscal policy reduces inflation and the real interest rate in the long run.

In the case of an open economy, the effects of an expansionary fiscal policy or a deflationary fiscal policy are similar to those in a closed economy, apart from the quantitative difference. Since the IS, MP and AD curves are all flatter in an open economy than in a closed economy, the same expansionary fiscal policy causes the same rightward shift of the IS curve and of the AD curve and thus leads to a larger rise of the output in the short-run in both the IS-MP and the AD-IA diagrams. In the new long-run equilibrium, inflation and the real interest rate are higher in an open economy than in a closed economy.

Figure 8. The effects of a tighter monetary policy in the IS-MP diagram
Another important governmental policy tool for influencing the economy is monetary policy. Suppose the government moves to a tighter monetary policy, which means that the central bank sets a higher real interest rate at a given level of inflation and output than before; as a result, the MP curve shifts upwards in the IS-MP diagram. With the IS curve unchanged, the economy jumps to point A in Figure 8, which indicates a higher real interest rate and a lower output level than before. That is a tighter monetary policy raises the real rate of interest and reduces the output level in the short run.

![Diagram](image)

Figure 9. The effects of a tighter monetary policy in a AD-IA diagram

A tighter monetary policy shifts the MP curve upwards and this shift leads to a lower equilibrium output at a given inflation rate. In the AD-IA diagram, the
AD curve shifts to the left as a consequence. Since inflation does not respond immediately, the economy jumps to point A in Figure 9, where the output is below its potential level. Thus inflation begins to fall and the output begins to rise. This process continues until the IA curve and the AD curve intersect at the long-run equilibrium $E_{LR}$, where output equals its potential level. Figure 9 shows the effects of a tighter monetary policy on the economy in the AD-IA diagram.

The AD-IA diagram shows that a tighter monetary policy reduces inflation in the long run. By means of the IS-MP diagram, we can see how a tighter monetary policy affects the real interest rate in the long run. An upward shift of the MP curve reduces output and raises the real interest rate immediately. Since the output is below its potential level at that time, inflation falls. The central bank reduces the real interest in response and thus, the MP curve shifts downwards gradually until the economy returns to its potential level of output. That is, a tighter monetary policy has no effect on output and the real interest rate in the long run. By a similar argument, one can see that a loose monetary policy only raises inflation and has no effect on the real interest rate in the long run. The effect of a tighter monetary policy on the economy in the IS-MP diagram is shown in Figure 8.

The result that monetary policy determines inflation but does not affect any real variables in the long run is useful in practice. Because inflation is harmful to the economy, keeping a low and stable inflation has been a key objective of
most central banks in the world, who design their monetary policy with a focus on inflation. This result suggests that central banks can achieve a lower inflation rate without having any impact on real variables in the long run. A tighter monetary policy has a cost only in the short run, during which output is lower than its potential level.

The above story occurs in a closed economy as well as in an open economy. The only thing different is that the same tighter monetary policy costs more in an open economy than in a closed economy. In an open economy, a rise in the real interest rate at a given output not only reduces investment, just as it does in a closed economy, but also bids up the value of domestic currency and thereby reduces net exports. That is, in an open economy, a tighter monetary policy crowds out not just investment, as it does in a closed economy, but also net exports. The monetary policy works through two channels in an open economy, rather than through one channel in a closed economy. As a result, a tighter monetary policy has a larger effect on output and thus results in a lower output level in the short run in an open economy than in a closed economy.

Another explanation may be more straightforward. Since the IS, MP and IA curves are flatter in an open economy than in a closed economy, the same upward shift of the MP curve causes a larger fall in output in an open economy in the short run.
Besides the government, whose policy changes can cause short-run fluctuations, other economic components, such as investors and consumers, can also cause economic fluctuations. Behavior changes of both investors and consumers cause changes in the IS curve. An increase in consumer and investor confidence leads to a higher output level than before for a given real rate of interest, and thus shifts the IS curve upwards. On the other hand, a decline in consumer and investor confidence shifts the IS curve downwards. The remaining analysis is similar to what we did before. As a result, an increase in consumer and investor confidence has the same effects as an increase in government expenditures or a decrease in taxes.

The changes we discussed above are all shocks on the aggregate demand side since they shift the AD curve in the AD-IA diagram. In the following part we will discuss two major shocks on the aggregate supply side: inflation shocks and supply shocks.

According to Romer (1999, p.48), “an inflation shock is a disturbance to the usual behavior of inflation that shifts the inflation adjustment line”. Normally, inflation rises, or the IA curve moves up, when the output is above its potential level, and falls, or the IA curve moves down, when the output is below its potential level. An inflation shock is an event that shifts the IA curve upwards or downwards even if output equals its potential. Indeed, inflation shocks can arise from many events, such as changes in prices of inputs, in productivity, or in the competitive environment.
One example of inflation shocks that cause the IA curve to shift upwards (this sort of inflation shocks is also called unfavorable shocks) is the Oil Crisis of the 1970’s. As an essential industrial material, oil is wildly used in many fields and therefore, a sharply increased oil price leads to a dramatic rise of the overall price level as in the 1970’s. This rise in inflation is due to the increase in oil price and has nothing to do with the output level.

![Diagram of AD-IA model]

Figure 10. The effects of an unfavorable inflation shock in the AD-IA diagram

In terms of the AD-IA diagram, this inflation shock shifts the IA curve upwards. The economy moves up along the AD curve to point A in Figure 10, with the output lower than its potential level. Thus inflation begins to fall. In Figure 10, the IA curve moves down gradually along with the rise of the
output. This process continues until the output returns to its potential level. In the meantime, inflation also goes back to its initial level.

Monetary authorities raise the real rate of interest immediately in response to the inflation jump caused by the unfavorable inflation shock, and then reduce the real rate of interest gradually as inflation falls. Finally, the real interest rate returns to its initial level. Figure 11 describes this process in the IS-MP diagram. In summary, an unfavorable inflation shock only causes a period of high inflation and low output, and has no effect on the real interest rate, output and inflation in the long run.

![IS-MP Diagram](image)

**Figure 11.** The effects of an unfavorable inflation shock in the IS-MP diagram
A supply shock is a change in the potential level of output or unemployment. Suppose there is a decline in the potential level of output. Since this change does not affect the IS, MP and IA curves directly, neither the real interest rate nor output and inflation change immediately. But this supply shock makes the potential level of output lower than before. That is, the output is above its potential level after this shock. Consequently, inflation rises and the IA curve shifts upwards gradually. The economy moves up along the AD curve with inflation rising and the output falling. In the new equilibrium, the output equals its new potential level and inflation is higher than before. Figure 12 summarizes this process.

Figure 12. The effects of a decline in the potential level of output in the AD-IA diagram
In terms of the IS-MP diagram, the central bank raises the real interest rate by shifting the MP curve upwards when inflation rises. The output falls towards its new potential level. The new equilibrium in Figure 13 reports a higher real interest rate than before. Thus, a decline in the potential level of output raises the real interest rate and inflation in the long run.

Figure 13. The effects of a decline in the potential level of output in the IS-MP diagram

As an alternative to the traditional IS-LM-AS model, the IS-MP-IA model is developed and proven to be powerful in analyzing the effects of various macroeconomic shocks. This new approach has many advantages over the
traditional one. Specifically, this new approach avoids the three problems mentioned before that the IS-LM-AS model faces.

Firstly, in the IS-MP-IA model, the concept of interest rate is consistent. Unlike the IS-LM-AS model, where the IS curve is related to the real interest rate and the LM curve is related to the nominal interest rate, in the new model, both the IS and the MP curves are related to the real interest rate. Secondly, in the new model, the AD curve directly describes the relationship between output and inflation, and this relationship is what we are interested in. In the traditional approach, however, the AD curve depicts the relationship between output and price level. Thirdly, the assumption on which the new approach is developed is realistic and reasonable. Rather than assuming that central banks target the money supply, the new approach assumes that central banks follow a real interest rate rule and this assumption is consistent with the behavior of most central banks in the real world.

The main objective of this paper is to test whether the Taylor rule is a suitable monetary policy rule. The analysis in the IS-MP-IA model is meaningful and helpful in later studies. In the IS-MP-IA model, the upward-sloping MP curve depicts the behavior of central banks and it is exactly a graphic expression of the Taylor rule.
III. Empirical Study

The new IS-MP-IA model consists of three curves, which correspond to three relationships in the economy. First, the downward sloping IS curve involves the negative relationship between output and the real interest rate in the goods market. The simplest algebraic form would be:

\[ y = -a \times r + u, \quad (1) \]

where \( y \) is the deviation of real output from its potential level, \( r \) is the real interest rate, \( u \) is a shift term. Second, the MP curve describes the monetary policy by which central banks set the real interest rate as an increasing function of output and inflation. This relationship could be expressed as:

\[ r = b \times \pi + c \times y + v, \quad (2) \]

where \( \pi \) is inflation rate, \( v \) is a shift term. Combining the IS curve and the MP curve together, one can obtain the AD curve, which interprets the negative relationship between inflation and output. This relationship can also be shown algebraically by combining the two equations above:

\[ y = -\frac{a \times b}{1 + a \times c} \times \pi - \frac{a \times v}{1 + a \times c}. \quad (3) \]
Third, the IA curve involves the relationship between inflation and output. The inflation rises when the output is above its potential level, and it falls otherwise. The change of inflation depends on the output gap of the period before. In terms of algebraic form, this relationship could be:

\[ \pi - \pi_{-1} = d \times y_{-1} + w, \]  

(4)

where \( w \) is a shift term.

Given these three equations, the three variables \( y, r \) and \( \pi \) are connected with each other. Central banks can affect inflation and output, and ultimately, achieve their goal of low and stable inflation by adjusting the real rate of interest. Indeed, most researchers and central banks use this type of model that contain these three relationships in their monetary policy evaluations. It would be helpful to derive the optimal monetary policy rule in such an economic model. Here a simple closed economy model that contains two equations is used. Suppose the real rate of interest affects the output with one period lag: the output level is determined by the real interest rate of the period previous. This simple model is as follows:

\[ y_{t+1} = -a \times r_t + u, \]  

(5)

\[ \pi_{t+1} = \pi_t + d \times y_t + w. \]  

(6)
where equation (5) represents the IS curve and equation (6) represents the IA curve. Substituting (5) into (6) results in:

\[ \pi_{t+2} = \pi_{t+1} - d \times a \times r_t + d \times u + w. \]  \hspace{1cm} (7)

Substituting (6) into (7) and taking expectations, equation (7) becomes:

\[ E_t(\pi_{t+2}) = \pi_t + d \times y_t + w - d \times a \times r_t + d \times u + w \]  \hspace{1cm} (8)

One can notice that the real rate of interest affects inflation with a two-period lag through its effect on output. Thus the optimal rule is the one that sets the real interest rate such that the expected inflation rate two periods ahead equals its target value. That is \( E_t(\pi_{t+2}) = \pi^* \). Then we have:

\[ \pi^* = \pi_t + d \times y_t - d \times a \times r_t + d \times u + 2 \times w. \]  \hspace{1cm} (9)

Rearranging and combining terms, we get an equation for the real rate of interest \( r_t \):

\[ r_t = \alpha + \beta \times (\pi_t - \pi^*) + \gamma \times y_t, \]  \hspace{1cm} (10)

The optimal monetary policy rule we get in this simple case has the same form as the popular monetary feedback rule that John Taylor proposed in 1993.
Taylor (1993) presented a simple monetary policy rule to describe the behavior of the Federal Reserve from 1987 to 1992:

\[ r_t = r_n + \alpha \times (\pi_t - \pi_t^*) + \beta \times y_t, \]  

(11)

where \( r_t \) is real interest rate at period \( t \), \( r_n \) is the long-run equilibrium or neutral real rate of interest and it is defined as the real interest rate when the economy is in equilibrium, that is when output equals its potential, or \( y_t \) equals zero and inflation equals its target value, \( \pi_t \) and \( \pi_t^* \) are the actual inflation rate and the inflation target at period \( t \) respectively, \( y_t \) is the output gap at period \( t \).

In the Taylor rule, the real interest rate responds contemporaneously to the inflation gap and the output gap. If inflation equals its target value and output equals its potential level, central banks set the real interest rate \( r_t \) equal to the neutral real interest rate \( r_n \), which is assumed to be 2 in this case. Furthermore, both coefficients, \( \alpha \) and \( \beta \), are assumed to be 0.5.

Central banks raise the real interest rate in response to a rise in inflation or output, while they reduce it otherwise. This behavior of central banks interpreted in the Taylor rule is consistent with what is discussed in detail within the IS-MP-IA model in the last section. As a consequence, the real rate of interest should be higher than the neutral rate if inflation is above its target value or output is above its potential level, and it should be lower than
the neutral rate otherwise. In this way, central banks can keep the values of inflation and of output as close as possible to their target or potential value.

Here we will explain why central banks prefer a low and stable inflation rate. On the one hand, a higher inflation rate reduces investment in both physical and human capital, and thereby reduces productivity growth. Historical evidence also suggests that hyperinflation is harmful to the economy. On the other hand, a negative inflation rate damages the economy as well by discouraging consumption. Taking into account the upward bias in measures of inflation, which is widely accepted to be less than two percent, a low inflation rate is a reasonable target for central banks. The Bank of Canada, for example, sets 2 percent as its inflation target.

The Taylor rule works pretty well in the case of the United States. Some empirical studies, such as Howard and Owens (1996), also indicate that the Taylor rule works well in some other industrialized countries, except that the estimated coefficients are different. Here, "the Taylor rule" refers to the type of monetary policy rules that have the form of equation (10). In other words, in this paper the Taylor rule refers to the monetary policy rule in which the short term real interest rate is expressed as a linear function of inflation and the output gap. In this section, we will test the Taylor rule for Canada.

In order to test equation (10), one needs to collect data on interest rates, inflation rates and output. Since the earliest target value of inflation we can
obtain is that of the first quarter in 1993, the data set of the test is quarterly data from 1993:1 to 2002:1. The real rate of interest is the difference between the nominal one and the inflation rate. The nominal rate used in this paper is the interest rate of the 3-month Treasury bill, which is obtained from the Bank of Canada (Series No. B14060). Because the interest rate provided by the Bank of Canada is that of the last day of each month rather than of each quarter, we first compute the average values of every 3-months as the quarterly data.

One can not obtain directly the data of inflation either. Instead, we can get the data of the Consumer Price Index (CPI) from Statistics Canada. From the web site of the Bank of Canada we can see that it adopted an inflation target in February 1991. Operationally, the Bank of Canada uses a measure of "core" inflation as the guide for conducting the monetary policy, where "core" inflation is defined as the year-over-year increase rate of CPI excluding food, energy and the effects of changes in indirect taxes (CPIXFET).

The reasons why the Bank of Canada focuses on core inflation are as follows. Firstly, since some goods, such as food and energy, have very volatile prices, while monetary policy affects the economy with lags, responding to these short-run fluctuations of prices is not necessary and can cause volatility in both inflation and real economic activity. Secondly, since an indirect tax leads to a proportional increase in the price level and thereby
results in a temporary increase in the inflation level in its first-round effects, core inflation is a better measure in that it also excludes this first-round effects of indirect taxes.

As Macklem (2001) mentions, starting May 2001, the Bank of Canada adopted a new measure of core inflation. This new measure is known as CPIX and it excludes the eight most volatile components of the CPI and the effects of indirect taxes. Despite some minor differences on the components they exclude, CPIXFET and CPIX move in a similar fashion over time. In order to be consistent, the CPIX data are used throughout in this paper. We can obtain the monthly CPIX data of 1996 basket content from Statistics Canada (Table 176-0003). We first compute the average values of every 3-months to get the quarterly data and then compute the 12-month growth rates of CPIX to get the inflation rate of each quarter. Inflation target data can be obtained directly from the Bank of Canada. The difference between the actual inflation rate and the target inflation rate shall be called the “inflation gap”.

For the output gap, we use seasonally adjusted real GDP data by North American Industry Classification System (NAICS) from Statistics Canada (Table 379-00181). Since there is an obvious trend in real GDP value, we first do a linear regression of these data and thus obtain the estimated data of each period as the potential level of GDP for that particular period. Then we compute the deviations of the actual values of real GDP for each quarter
from their estimated potential value to get the output gap. We do the stationarity tests for all the time series data used as independent variables in this paper and the results are shown in the Appendix. We can see that all variables, but one used in Table 5, are stationary. The regression result of equation (11) is reported in Table 1.

<table>
<thead>
<tr>
<th>variable</th>
<th>coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>2.725584</td>
<td>4.818034</td>
<td>0.0000</td>
</tr>
<tr>
<td>inflation gap (1)</td>
<td>-0.424947</td>
<td>-1.062697</td>
<td>0.2964</td>
</tr>
<tr>
<td>output gap (1)</td>
<td>0.499784</td>
<td>1.620231</td>
<td>0.1156</td>
</tr>
<tr>
<td>ar(1)</td>
<td>1.151853</td>
<td>6.505559</td>
<td>0.0000</td>
</tr>
<tr>
<td>ar(2)</td>
<td>-0.360757</td>
<td>-1.871735</td>
<td>0.0710</td>
</tr>
</tbody>
</table>

adjusted R-squared 0.783509
S.E. of regression 0.661767
Durbin-Watson stat 2.069361

*Note: The above regression result is obtained by using Least Squares in Eviews 3.1.*

Here ar(1) and ar(2) denote the process of the first-order and second-order autoregression, respectively. They are added because of the problem of autocorrelation. The existence of these autoregression terms in the regression implies that the Bank of Canada smoothes the real interest rate in conducting monetary policy.

Unfortunately, the estimated coefficient of the inflation gap has the wrong sign! The regression contradicts the theoretical analysis. One reasonable
explanation is that central banks tend to smooth the nominal rate of interest and thus when the inflation rate goes up in the short run, real rates of interest would decline. If we believe that the theoretical analysis is correct, a possible way out is that the Bank of Canada targets the forward inflation gap instead of the contemporaneous value.

Since monetary policy affects the economy with lags, it would be too late for the monetary authorities to adjust the short-term real interest rate when the inflation rate has started to rise or fall. It is reasonable for the monetary authorities to look ahead and implement monetary policy in response to the inflation gap in the future. Clarida and Gertler (1996) pointed out that Germany's Bundesbank adjusted the real interest rate responding to expected, not current, inflation.

Indeed, the contemporaneous output gap term in the Taylor rule has already reflected the forward-looking character. In the beginning of this section we showed algebraically that output fluctuations affect inflation in the next period rather than within the period. Thus the inclusion of the contemporaneous output gap in the Taylor rule implies that central banks are concerned with future inflation and are willing to act in advance.

Therefore, we attempt to use the inflation gap of two quarters ahead instead of the contemporaneous one to do the test again. To simplify the analysis, we assume that the Bank of Canada has "perfect expectations". That is, the
forecast inflation rate of two quarters ahead equals the realized value in the future, and hence the regression equation has the following form:

\[ r_t = r_n + \alpha \times (\pi_{t+2} - \pi_{t+2}^*) + \beta \times y_t. \]  
(12)

The regression result is shown in Table 2. We can see that the estimated coefficients of the explanatory variables have correct signs once we change the inflation gap from the current values to two periods ahead. This result suggests that the Bank of Canada focuses more on the future inflation gap than on the current one for the purpose of controlling inflation. This result also demonstrates that the assumption of central banks targeting future inflation gaps appears to be reasonable both theoretically and practically.

<table>
<thead>
<tr>
<th>Table 2. Regression results of equation (12)</th>
<th>dependent variable: ( r_t )</th>
</tr>
</thead>
<tbody>
<tr>
<td>variable</td>
<td>coefficient</td>
</tr>
<tr>
<td>constant</td>
<td>3.203183</td>
</tr>
<tr>
<td>inflation gap (2)</td>
<td>0.228743</td>
</tr>
<tr>
<td>output gap (1)</td>
<td>0.563584</td>
</tr>
<tr>
<td>( \text{ar}(1) )</td>
<td>1.096492</td>
</tr>
<tr>
<td>( \text{ar}(2) )</td>
<td>-0.400931</td>
</tr>
<tr>
<td>adjusted R-squared</td>
<td>0.744418</td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>0.645069</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>2.096395</td>
</tr>
</tbody>
</table>

*Note: This regression result is obtained by using Least Squares in Eviews 3.1.*
However, this result is not satisfactory since the estimated coefficient of the inflation gap is statistically insignificant at a 90 percent confidence level. More convincing evidence on the validity of the Taylor rule in Canada requires new measurements of the variables involved.

Using the same CPIX data, we try an alternative way to compute the inflation rate. Rather than computing the year over year inflation rate, here we compute the quarterly growth rates of CPIX and then annualize them to obtain the inflation rate of each quarter. The explanatory variables are still the inflation gap of two quarters ahead and the current output gap, with the latter taking the same values as before. The regression result is shown in Table 3.

Table 3. Regression result of equation (12) by using an alternative measure of the inflation rate dependent variable: n

<table>
<thead>
<tr>
<th>variable</th>
<th>coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>3.278489</td>
<td>9.007034</td>
<td>0.0000</td>
</tr>
<tr>
<td>inflation gap (3)</td>
<td>0.462548</td>
<td>2.412578</td>
<td>0.0222</td>
</tr>
<tr>
<td>output gap (1)</td>
<td>0.535874</td>
<td>2.076864</td>
<td>0.0465</td>
</tr>
<tr>
<td>ar(1)</td>
<td>0.445371</td>
<td>2.581468</td>
<td>0.0150</td>
</tr>
</tbody>
</table>

| adjusted R-squared | 0.411382    |
| S.E. of regression  | 1.150510    |
| Durbin-Watson stat  | 2.075583    |

*Note: This regression result is obtained by using Least Squares in Eviews 3.1.*
This time we get a very good result: the estimated coefficients of the inflation gap and the output gap have the correct signs and both of them are statistically significant at a 95 percent confidence interval. This result implies that the Bank of Canada is looking at the latest trends of the inflation changes. Additionally, the estimated coefficients of both the inflation gap and the output gap are close to 0.50, a figure originally suggested by Taylor (1993). The constant term in the regression result refers to the estimated value of natural or neutral real interest rate. Here the estimated neutral real interest rate is 3.28, which is greater than 2 -- the value that Taylor (1993) suggested in the case of the US in the period of 1987 to 1992.

Besides the seasonally adjusted real GDP data, unadjusted real GDP data also can be used in the following tests to enrich the evidence. The desired data can be obtained from the same source as the adjusted ones. Unlike the seasonally adjusted data, the unadjusted data exhibit obvious seasonal fluctuations: the third quarter reports the highest value, and then follows the fourth, the second and the first quarter in almost all the years in our data set. The reason might be twofold.

First, the real GDP values tend to increase as time passes and thus, they are always higher in the second half of the year than that in the first half. Second, since some industries are closely related to the weather -- the real GDP declines when it becomes cold in winter, the real GDP for the third quarter is significantly higher than that of the fourth quarter, and the real
GDP for the fourth quarter is higher than that of the first quarter in the next year. It is these mixed effects that make the real GDP data show these seasonal fluctuations in practice.

Consequently, the method used previously to compute the output gap is not proper any more in this particular case. A useful method to deseasonalize the real GDP data is suggested by *Managerial Economics* (Keat and Young, 2002). The detailed process is described in Chapter 6 of this book. We can use the deseasonalized data to do linear regression and obtain their potential values, just as we did before. Thus a new data set for the output gap can be used to do the regression again. Table 4 shows the regression results.

<table>
<thead>
<tr>
<th>variable</th>
<th>coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>3.278516</td>
<td>9.135193</td>
<td>0.0000</td>
</tr>
<tr>
<td>inflation gap (3)</td>
<td>0.457382</td>
<td>2.287478</td>
<td>0.0294</td>
</tr>
<tr>
<td>output gap (2)</td>
<td>0.375215</td>
<td>1.717836</td>
<td>0.0961</td>
</tr>
<tr>
<td>ar(1)</td>
<td>0.421526</td>
<td>2.414600</td>
<td>0.0221</td>
</tr>
</tbody>
</table>

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>adjusted R-squared</td>
<td>0.381692</td>
<td></td>
<td></td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>1.179169</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>2.100474</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

*Note: This regression result is obtained by using Least Squares in Eviews 3.1.*
The estimated coefficients of the inflation gap and the output gap are 0.46 and 0.38 respectively, and they have the correct signs and are statistically significant at a 90 percent confidence interval. Furthermore, the estimated natural or neutral real rate of interest is 3.28, the same value as that of the last test.

An alternative way to offset seasonal fluctuations is simpler. We can do the regression for the data of different quarters individually. By doing a linear regression four times, we obtain the estimated real GDP values for each quarter as its potential output. This new measure of the output gap appears to be non-stationary (see the Appendix), but we still provide the regression results for comparison purposes. The regression equation is still equation (12) and the result is reported in Table 5.

<table>
<thead>
<tr>
<th>variable</th>
<th>coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>3.285010</td>
<td>8.697152</td>
<td>0.0000</td>
</tr>
<tr>
<td>inflation gap (3)</td>
<td>0.491254</td>
<td>2.645822</td>
<td>0.0128</td>
</tr>
<tr>
<td>output gap (3)</td>
<td>0.571635</td>
<td>2.238601</td>
<td>0.0328</td>
</tr>
<tr>
<td>ar(1)</td>
<td>0.474335</td>
<td>2.808562</td>
<td>0.0087</td>
</tr>
</tbody>
</table>

adjusted R-squared  0.427386
S.E. of regression   1.134762
Durbin-Watson stat   2.077441

Note: This regression result is obtained by using Least Squares in Eviews 3.1.
The results are nearly identical to those of Table 3. The estimated coefficients of the inflation gap and the output gap have the correct signs and they are significant at a 95 percent confidence interval. Once again, the estimated neutral real interest rate is 3.29 -- a value close to that of the last two tests. The estimated coefficients of the two explanatory variables are 0.49 and 0.57 respectively, which are also similar to the results of the last two tests. The regression results of the last three tests show that different measures of the output gap have no significant effect on the regression results. Additionally, the estimated coefficients of both the inflation gap and the output gap in those tests are all close to the value of 0.50 suggested by Taylor (1993) for the US case.

These empirical results are inconsistent, however, with what is suggested by Côté, Kusczczak, Lam, Liu and St-Amant (2002b) and Armour, Fung, and Maclean (2002). In Côté, Kusczczak, Lam, Liu and St-Amant (2002b), the authors evaluate seven simple monetary policy rules in 12 private and public sector models of the Canadian economy by conducting dynamic simulations. They find that although none of these simple policy rules is robust to model uncertainty, a simple Taylor type rule with 2 as the coefficient of the inflation gap and 0.5 as that of the output gap performs relatively well in certain models. In Armour, Fung, and Maclean (2002), the authors conclude that a simple Taylor type rule, in which the coefficients of the inflation gap and the output gap are 3 and 0.5 respectively, performs well in the Quarterly Projection Model.
Another point we want to mention here is the estimated neutral or natural real interest rate. The neutral short term real interest rate could vary across countries according to some other empirical studies. Goldman Sachs estimated 2 percent for the US and 3.5 percent for some European countries, such as Britain, France and Germany. Our tests suggest that the estimated neutral real rate is about 3.28 in Canada. According to the last section’s analysis, a tighter monetary policy leads to a lower inflation rate but has no effect on any real variables in the long run. Therefore, this result implies that the Bank of Canada followed a tighter monetary policy to maintain a lower inflation rate than the Federal Reserve did in the US. In practice, the average inflation rates, which are obtained by annualizing the quarterly growth rates of the Consumer Price Index (Table 387-00071 from Statistics Canada), of Canada and the US in our sample period are 1.60 and 2.44, respectively.

Chart 1. Actual and predicted values of real interest rate

1993:1 -- 2002:1
Based on the result of Table 4, Chart 1 shows that the predicted values of the real interest rate trace out the main fluctuations of the actual values. The actual real interest rates are significantly higher than their predicted values for three periods of time, which are 1994:1, 1995:1 and 1995:4, while the opposite occurred in 1993:2, 1996:4 and 2001:2.

Ball (1999) and Svensson (2000) suggest that the Taylor rule should be modified for a small open economy like Canada by incorporating the exchange rate. Ball (1999) presents a monetary policy rule that includes the exchange rate gap, which is expressed as the deviation of the real exchange rate from its equilibrium value, as the third explanatory variable. Since it is difficult to obtain the long-run equilibrium value of exchange rates, the quarterly growth rate data of the actual exchange rate are employed as the additional explanatory variable in the following test.

The original data of exchange rates are obtained from Statistics Canada (Table 387-00061), where the exchange rate is expressed as Canadian dollars per US dollar. Here we are interested in the exchange rate of the Canadian dollar with respect to the US dollar since the US is the largest trade partner of Canada and its economy has huge effects on the Canadian economy. A rise in the exchange rate represents a depreciation of the Canadian dollar. Thus the estimated coefficient of the exchange term is expected to be positive. The regression equation takes the following form:
$$r_i = r_n + \alpha \times (\pi_{t+2} - \pi_{t+2}^*) + \beta \times y_i + \gamma \times e_i,$$

(13)

where \( e_i \) is the growth rate of exchange rate in period \( t \), while other variables are defined as in Table 4. Table 6 reports the regression result.

<table>
<thead>
<tr>
<th>Table 6. Regression results of equation (13)</th>
<th>dependent variable: ( r_i )</th>
</tr>
</thead>
<tbody>
<tr>
<td>variable</td>
<td>coefficient</td>
</tr>
<tr>
<td>constant</td>
<td>3.320379</td>
</tr>
<tr>
<td>inflation gap (3)</td>
<td>0.454021</td>
</tr>
<tr>
<td>output gap (2)</td>
<td>0.373851</td>
</tr>
<tr>
<td>( c_t )</td>
<td>-0.067749</td>
</tr>
<tr>
<td>ar(1)</td>
<td>0.422932</td>
</tr>
<tr>
<td>adjusted R-squared</td>
<td>0.368332</td>
</tr>
<tr>
<td>S.E. of regression</td>
<td>1.191841</td>
</tr>
<tr>
<td>Durbin-Watson stat</td>
<td>2.062891</td>
</tr>
</tbody>
</table>

*Note: This regression result is obtained by using Least Squares in Eviews 3.1.*

This result indicates that the exchange rate term has no explanatory power: the estimated coefficient is statistically insignificant and has the wrong sign. Moreover, the value of the adjusted R-square statistic falls after the introduction of this new term. That is, the Bank of Canada seems to pay little attention to the exchange rate and Ball’s rule works poorly for Canada. Indeed, the Bank of Canada lets exchange rates be freely floating since the 1990’s.
Considering the huge impacts of the US economy on the Canadian economy, it is reasonable for some people to believe that the Bank of Canada conducts monetary policy following that of the Federal Reserve of the US. It is necessary to do additional tests to verify this point. Among the three tests, in which the contemporaneous US real interest rate and the values of one quarter and two quarters before are used as the new explanatory variable individually, the one that employs the US real interest rate of one quarter before yields the best result. The regression equation, which extends equation (12) to include a new term of the US real interest rate of one quarter before, is shown as equation (14):

\[ r_t = r_n + \alpha \times (\pi_t - \pi_{t-2}) + \beta \times y_t + \gamma \times rus_{t-1}, \]  

(14)

where \( rus_{t-1} \) is the US real interest rate in period t-1 and it is the difference of the nominal rate of 3-month Treasury bill, which is obtained from the website of the Federal Reserve, and the US inflation rate, which is obtained from Statistics Canada (Table 387-00071), in period t-1, while other variables are defined as in the last test. The regression result is reported in Table 7.

In this case, the coefficient of the US real interest rate term has the correct sign even though it is not statistically significant at a 90 percent confidence interval. The adjusted R-square statistic goes up from 0.38 to 0.40 after the new explanatory variable is added. This result means that the US real interest rate term does have explanatory power. In other words, the belief
that the Bank of Canada watches the real interest rate of the US is supported by our empirical evidence.

<table>
<thead>
<tr>
<th>variable</th>
<th>coefficient</th>
<th>t-statistic</th>
<th>p-value</th>
</tr>
</thead>
<tbody>
<tr>
<td>constant</td>
<td>2.760017</td>
<td>5.535586</td>
<td>0.0000</td>
</tr>
<tr>
<td>inflation gap (3)</td>
<td>0.402407</td>
<td>1.934435</td>
<td>0.0629</td>
</tr>
<tr>
<td>output gap (2)</td>
<td>0.419690</td>
<td>2.011055</td>
<td>0.0537</td>
</tr>
<tr>
<td>rus</td>
<td>0.247905</td>
<td>1.383108</td>
<td>0.1772</td>
</tr>
<tr>
<td>ar(1)</td>
<td>0.364761</td>
<td>1.974323</td>
<td>0.0578</td>
</tr>
</tbody>
</table>

adjusted R-squared          0.398014
S.E. of regression           1.163502
Durbin-Watson stat           2.044717

Note: This regression result is obtained by using Least Squares in Eviews 3.1.

So far we have tested the original and the modified versions of the Taylor rule by using the historical data of Canada. The results suggest that a revised version of the Taylor rule, in which the inflation gap of two quarters ahead replaces the current one, describes the behavior of the Bank of Canada from 1993:1 to 2002:1 pretty well.

However, our objective is to explore a useful monetary policy rule to guide the behavior of central banks. The usefulness of the Taylor rule in conducting the monetary policy depends on whether central banks can predict future inflation precisely. In practice, perfect forecasting is
impossible and the Taylor rule is not easy to apply because of this uncertainty. Additionally, the estimation of the potential level of output is another difficulty. In this paper we apply linear regression to estimate the potential level of output, but others may not accept this approach. Clearly, the Taylor rule could not be applied mechanically with these limitations.

IV. Conclusion

In this paper, we studied the feasibility of the Taylor rule at both the theoretical level and the practical level. Based on the realistic assumption of central banks following a real interest rate rule, the IS-MP-IA model is developed. This new model is powerful in explaining the effects of various macroeconomic shocks and shows some essential advantages over the traditional IS-LM-AS model. In the IS-MP-IA model, the monetary authorities adjust the short-term real interest rate in response to changes in inflation and real output. This behavior of central banks is consistent with what is depicted in the Taylor rule.

The Taylor rule performs well in describing the behavior of the Federal Reserve and several other central banks according to some former literature. However, the situation is a little different in the case of the Canadian experience. The empirical study in this paper shows that a revised version of the Taylor rule, in which the current inflation gap is replaced by the inflation
gap of two quarters ahead, can approximate the behavior of the Bank of Canada from 1993:1 to 2002:1. The estimated coefficients of both the inflation gap and the output gap are statistically significant and close to 0.5, a value suggested by Taylor (1993). The estimated neutral real interest rate is higher than what Taylor originally suggested, implying that the Bank of Canada follows a tighter monetary policy than does the Federal Reserve.

The usefulness of the Taylor rule in describing the behavior of the Bank of Canada does not mean that it is easy to apply in practice since both forecasts of future inflation and estimates of potential output are challenging tasks. To follow a simple monetary policy rule mechanically, like the Taylor rule, is almost always unwise and dangerous. Indeed, the monetary authorities are constantly striving to improve their understanding of the economy's structure, to uncover the source of shocks and to devise policies to accomplish more precisely their objectives (Yellen, 1996). Nonetheless, as a rule of thumb, the Taylor rule provides a simple but useful guideline to conduct monetary policy in a complex and uncertain economic environment.
Appendix: Stationarity tests

<table>
<thead>
<tr>
<th>variable</th>
<th>ADF statistic</th>
<th>10% critical value</th>
<th>result</th>
</tr>
</thead>
<tbody>
<tr>
<td>inflation gap (1)</td>
<td>-3.030465</td>
<td>-2.6118</td>
<td>stationary*</td>
</tr>
<tr>
<td>inflation gap (2)</td>
<td>-2.736498</td>
<td>-2.6148</td>
<td>stationary</td>
</tr>
<tr>
<td>inflation gap (3)</td>
<td>-5.691575</td>
<td>-2.6148</td>
<td>stationary**</td>
</tr>
<tr>
<td>output gap (1)</td>
<td>-2.612338</td>
<td>-2.6118</td>
<td>stationary</td>
</tr>
<tr>
<td>output gap (2)</td>
<td>-2.907651</td>
<td>-2.6118</td>
<td>stationary</td>
</tr>
<tr>
<td>output gap (3)</td>
<td>-2.131645</td>
<td>-2.6118</td>
<td>non-stationary</td>
</tr>
<tr>
<td>et</td>
<td>-3.414773</td>
<td>-1.6208</td>
<td>stationary**</td>
</tr>
<tr>
<td>rus</td>
<td>-3.577087</td>
<td>-2.6118</td>
<td>stationary*</td>
</tr>
</tbody>
</table>

*Note: *refers to the rejection of hypothesis of a unit root at 5% critical level;

**refers to the rejection of hypothesis of a unit root at 1% critical level.
References:


