Analyzing Canadian Regional Business Cycles Using Logistic Smooth Transition Autoregressive (LSTAR) Models

by

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Abstract

This paper estimates Logistic Smooth Transition Autoregressive (LSTAR) models to analyze nonlinearities and asymmetries in Canadian regional business cycles. We use quarterly data of real personal income for the ten Canadian provinces for the period starting 1960:1 until 2000:2. The modelling cycle for nonlinear models proposed by Granger and Teräsvirta (1993) and Teräsvirta (1994) is used to analyze the growth rates of real personal income. Null hypothesis of linearity $AR(p)$ model is strongly rejected for all ten provinces. Then we estimate a LSTAR model for each province. Diagnostic tests for residual autocorrelation, parameter constancy and remaining nonlinearity are applied and all models passed these tests succesfully. Estimated transition functions and estimated parameters show particular dynamics in the business cycles for each province. Furthermore, evolution of the estimated transition functions over time allow us to identify principal peaks and troughs in provincial economies.

Contents

1 Introduction

Using the definition of Burns and Mitchell (1946), "business cycles are a type of fluctuations found in the aggregate economic activity of nations that organize their work mainly in business enterprises; a cycle consists of expansions occurring at about the same time in many economic activities followed by similarly general recessions, contractions, and revivals which merge into the expansion phase of the next cycle. (p.3)". Hence, this definition possesses two basic features. The first is the comovement among individual economic variables, taking into account possible leads and lags in timing, including the historical concordance of hundred of series such as commodity output, income, prices, interest rates, banking transactions, and transportation services. The second feature is that how business cycles are divided into separate phases or regimes. The authors treat expansions separately from contractions. For example, certain series are classified as leading or lagging indicators of the cycle, depending on the general state of business conditions.

From a related perspective, it is agreed that business cycles have two important features such as nonlinearities and asymmetries. Asymmetry implies that contractions in business cycles are on average shorter and steeper than expansions (see, for example, Zarnowitz, 1992). One of the first econometric models in measuring business cycles was Tinbergen's model (1939) who used linear difference equations as an instrument of analysis. This empirical work was generally focused on the time-series properties of just one or a few macroeconomic indicators and the structure was linear without possibility to separate between recessions and expansions. In other hand, Lucas (1976) proposed the use of dynamic models to analyze movements in the economy using autocorrelation and spectral densities functions as the two more important econometric tools.

However, there are some agreement to accept importance of nonlinearities in the economy. For example, according to Zarnowitz (1992) nonlinear models can explain endogenously the existence and amplitude of cycles. In last period, a lot of research consider theoretical and empirical aspects of modeling nonlinearities and asymmetries. For example, these type of new models allow to calculate duration of recessions and expansions. Interesting contributions are made by Neftci (1984) and Stock and Watson (1991) who used a probabilistic and a dynamic factor models, respectively. Other type of models assume that the transition between regimes is caused by exogenous but not observable (or unknown events) variables. This is the case of Markov Switching (MS) model, originally proposed by Hamilton (1989). He used an AR(4) model for the growth rate of US output allowing for a changing mean and a constant variance. The results allow to identify phases of recession very close to the dates identified by the NBER using a large set of leading indicators. This model has been extended in many forms allowing for changing variance, varying transition probabilities, multivariate analysis, among others. For other applications using MS models to analyze output growth, see Godwin (1993), Bodman and Crosby (2000) and the references mentioned there.

A drawback in the MS approach and other nonlinear approaches is the step where testing for linearity is done. Because there is a problem of nuisance parameters not identified under the null hypothesis, standard limiting distributions and analytical expressions are not available in most cases. However, it is possible to test the null hypothesis of linearity against an alternative hypothesis of Smooth Transition Autoregressive (STAR) models. In the more simple version, a STAR model allows to change from one regime to another regime according to a transition variable. Hence, the transition mechanism is endogenous and in many cases, the transition variable is the dependent variable lagged d times, where this delay is selected from the linearity tests for different values of d.

There are two types of STAR models according to the transition function models and double stochastic models (see Teräsvirta *et al,* 1994).

In this section, we present a selected survey of nonlinear models with an emphasis in methodological issues. For a more extensive survey, see Granger and Teräsvirta (1993), Teräsvirta, Tjostheim and Granger (1994) and van Dijk, Teräsvirta and Franses (2001). In this section we also focus on presentation of Markov Switching (MS) models and Smooth Transition Autoregressive (STAR) models. At the end, we briefly mention some other class of nonlinear models. Most of nonlinear models assume that regime switches are caused by certain past values of the same dependent variable itself (y_{t-d}, say). Hence, regime changes are assumed to be endogenous as in STAR models. But we may assume that such shifts are exogenous and perhaps caused by external effects such as wars or oil crisis. It is the case of MS models where researcher assumes that such shifts are exogenous but at the same time the model does not assume a priori knowledge of the location of the shifts.

2.1 Markov Switching (MS) Models

Let y_t be our dependent variable for all following discussion. Using notation from Kim and Nelson (2000), in general, an autoregressive model of order p with first-order, M-state MS mean and variance may be written as[1]

$$\phi(L)(y_t - \mu_{s_t}) = e_t, \tag{1}$$

where $e_t \sim N(0, \sigma^2_{s_t})$. The model is complete specifying that $Pr[s_t = j | s_{t-1} = i] = p_{ij}$ for $i, j = 1, 2, ..., M$; $\sum_{j=1}^{M} p_{ij} = 1$ and

$$\mu_{s_t} = \mu_1 s_{1t} + \mu_2 s_{2t} + ... + \mu_M s_{Mt}, \tag{2}$$

$$\sigma^2_{s_t} = \sigma^2_1 s_{1t} + \sigma^2_2 s_{2t} + ... + \sigma^2_M s_{Mt}, \tag{3}$$

[1] In the seminal paper of Hamilton (1989) a constant variance and a fourth-order autoregressive process was considered in analyzing the growth rate of US output. See section 4 for more examples of empirical applications using MS models.

where $s_{mt} = 1$, if $s_t = m$, and $s_{mt} = 0$, otherwise. In the above model, if s_t, for $t = 1, 2, ..., T$, were known a priori, this variable could be considered as a dummy variable and the construction of the likelihood function would be straightforward. Unfortunately, s_t is an unobserved variable. Let ψ_{t-1} be the set of all past information up to time $t-1$. What we need is to write the density of y_t given past information and to do that we need s_t and s_{t-1}, which are unobserved. To solve this problem, instead of considering a joint density of y_t and s_t, we consider the joint density of y_t, s_t and s_{t-1}. Basically, it can be done in two steps as shown in Kim and Nelson (2000). In the first step, we derive the joint density of y_t, s_t and s_{t-1}, conditional on ψ_{t-1}:

$$f(y_t, s_t, s_{t-1}|\psi_{t-1}) = f(y_t|s_t, s_{t-1}, \psi_{t-1}) \Pr[s_t, s_{t-1}|\psi_{t-1}],$$

where

$$f(y_t|\psi_{t-1}, s_t, s_{t-1}) = (2\pi\sigma_{s_t}^2)^{-1/2} \exp\{-\frac{[\phi(L)(y_t - \mu_{s_t})]^2}{2\sigma_{s_t}^2}\}.$$

In the second step, to get $f(y_t|\psi_{t-1})$, we have to integrate s_t and s_{t-1} out of the joint density by summing the joint density over all possible values of s_t and s_{t-1}:

$$\begin{aligned} f(y_t|\psi_{t-1}) &= \sum_{s_t=1}^{M} \sum_{s_{t-1}=1}^{M} f(y_t, s_t, s_{t-1}|\psi_{t-1}) \\ &= \sum_{s_t=1}^{M} \sum_{s_{t-1}=1}^{M} f(y_t|s_t, s_{t-1}, \psi_{t-1}) \Pr[s_t, s_{t-1}|\psi_{t-1}]. \end{aligned}$$

Then, the marginal density $f(y_t|\psi_{t-1})$ is a weighted average of M^2 conditional densities. Finally, the log likelihood is given by:

$$\ln L = \sum_{t=1}^{T} \ln\{\sum_{s_t=1}^{M} \sum_{s_{t-1}=1}^{M} f(y_t|s_t, s_{t-1}, \psi_{t-1}) \Pr[s_t, s_{t-1}|\psi_{t-1}].$$

To complete the above procedure, we still need to deal with the problem to calculate $\Pr[s_t = j, s_{t-1} = i|\psi_{t-1}]$, that is, the weights. This is known as the filtering procedure which consists also of two steps. In the first step, given $\Pr[s_{t-1} = i|\psi_{t-1})$, $i = 1, 2, ..., M$, at the beginning of time t, we can calculate the weights by

$$\Pr[s_t = j, s_{t-1} = i|\psi_{t-1}] = \Pr[s_t = j|s_{t-1} = i] \Pr[s_{t-1} = i|\psi_{t-1}]$$

where the first term in the last expression are the transition probabilities. In the second step, once y_t is observed at the end of time t, we can update the probability terms by

$$\begin{aligned}
\Pr[s_t &= j, s_{t-1} = i|\psi_{t-1}] = \Pr[s_t = j, s_{t-1} = i|\psi_{t-1}, y_t] \\
&= \frac{f(s_t = j, s_{t-1} = i, y_t|\psi_{t-1})}{f(y_t|\psi_{t-1})} \\
&= \frac{f(y_t|s_t = j, s_{t-1} = i, \psi_{t-1}) \Pr[s_t = j, s_{t-1} = i|]}{\sum_{s_t=1}^{M} \sum_{s_{t-1}=1}^{M} f(y_t|s_t, s_{t-1}, \psi_{t-1}) \Pr[s_t, s_{t-1}|\psi_{t-1}]},
\end{aligned}$$

with $\Pr[s_t = j|\psi_t] = \sum_{s_{t-1}=1}^{M} \Pr[s_t = j, s_{t-1} = i|\psi_t]$. Iterating these two steps for $t = 1, 2, ..., T$ allows us the appropriate weighting terms to use in the log likelihood.

Construction of the log likelihood and filtering steps are known as the Hamilton filter which is a modification of the Kalman filter. In fact, one of the principal outputs from Hamilton filter is the so called filtered probabilities, which are inferences about s_t using information until time t. But it is also possible to obtain smoothed probabilities which are inferences about s_t using information until time T, that is, total information. This is found using the Kim's smooth algorithm, see Kim (1994).

Other extensions to the MS models are to allow to change the autoregressive coefficients according to the unobserved variable s_t. Another possibility is to allow for time varying transition probabilities as suggested by Filardo (1994).

Because nonlinearity and structural instability may be regarded as two competing alternatives to linearity, the MSTAR model have been extended to allow for time-varying characteristics giving the so called time-varying MSTAR model (TVSTAR). This model is obtained when one of the transition variables in (7) is taken to be time implying that the time series y_t follows a STAR model at all times, with smooth change in the autoregressive parameters in all regimes.

Extension of univariate STAR models to the multivariate framework creates the so called vector STAR models. It is equivalent to use expression (4) but replacing scalars by vectors and the coefficients for matrices of coefficients. In this case the regimes are common to the k variables because only one and the same transition variable determines the prevailing regime and the switches between regimes in all k equations of the model. However, it is possible to allow for equation-specific regime-switching by introducing equation-specific transition functions.

Finally, another interesting extension is the smooth transition error-correction model (STECM), where the model takes into account nonlinear or asymmetric adjustment (see Granger and Swanson, 1996). A STECM can be represented by

$$\begin{aligned} \Delta Y_t &= (\Phi_{1,0} + \alpha_1 z_{t-1} + \sum_{j=1}^{p-1} \Phi_{1,j} \Delta Y_{t-j})(1 - F(Y_{t-d}; \gamma, c)) \qquad (8) \\ &\quad + (\Phi_{2,0} + \alpha_2 z_{t-1} + \sum_{j=1}^{p-1} \Phi_{2,j} \Delta Y_{t-j}) F(Y_{t-d}; \gamma, c)) + \varepsilon_t \end{aligned}$$

where $\alpha_i, i = 1, 2$ are $kx1$ vector and $z_t = \beta' Y_t$ for some $kx1$ vector β denote the error correction term, that is, z_t is the deviation from the equilibrium relationship which is given by $\beta' Y_t = 0$. An additional extension of this model incorporates multiple equilibrium relationships.

For a more detailed discussion of this model and other models mentioned above, see Lundbergh, Teräsvirta and van Dijk (2000) and van Dijk, approximated by high choice of p. According to Teräsvirta *et al* (1994), the small sample sizes frequently available in economic data implies a limited p, say one or two. The goal is to keep the number of parameters to be estimated at a reasonable level. However, in theory, p should be selected using some stopping criterion or goodness of fit measure.

Other popular class of nonlinear models are the time varying parameters (TVP). In this class of models the parameters of interest depend on time and the model has to be estimated using the Kalman filter. Sometimes these models are called random coefficient models, because parameters of these models change smoothly. TVP models, as any models that include non observed components, can be represented in a space state form which consists of an observed equation and a transition equation. The first equation relates dependent variable with explanatory variables which are associated to a random coefficient. The behavior of this random coefficient is specified by the transition equation. Hence, a TVP model can be expressed by

$$\begin{aligned} y_t &= \beta_t x_{t-1} + \varepsilon_t \\ \beta_t &= m + \alpha \beta_{t-1} + \eta_t \end{aligned} \tag{11}$$

where η_t is a vector of white noise not necessarily correlated with ε_t.

3 Building, Estimating and Evaluating LSTAR Models

As it is mentioned by Granger (1993), construction of nonlinear models consists of two steps. In the first step, a test for linearity is performed. If the null hypothesis of linearity is rejected, a second step is necessary. In this step a nonlinear model is chosen, estimated, analyzed and evaluated. Generally, at this step, a decision is done, for example between an ESTAR or a LSTAR model. However, in this paper we select directly the LSTAR model given our interest to identify asymmetries.

The linearity tests can be classified in two broad categories. In the first category, tests are derived without assuming a specific nonlinear alternative. According to Granger and Teräsvirta (1993) a few tests of this category may also be interpreted as a score or Lagrange multiplier (LM) tests. Unlike this class of tests, in the second category tests are derived against a specific alternative. These tests are known as LM-type tests where the specific alternative may consists of a regression where the dependent variable is regressed on its own lagged values and other explanatory variables adding nonlinear terms for which the test is applied[6,7]. A more extensive discussion of the different tests with comparison of size and power using Monte Carlo simulations can be found in Lee *et al.* (1993).

In most of cases, it is strongly recommended to have a specific strategy for building nonlinear time-series models. Teräsvirta (1994) proposed the following data-based modeling cycle for STAR models. In the first step, we estimate a linear AR model of order p. In general the linear model is represented in the following way:

$$A(L)y_t = u_t \tag{12}$$

where $A(L) = 1 - a_1L - a_2L^2 - .... - a_pL^p$ is a finite polynomial in the lag operator with all roots outside of the unit circle. This is customary to select p based on the Akaike Information Criteria (AIC). Sometimes, it is also possible to use the Schwarz Information Criterion (BIC) or the Ljung-Box statistic. The second step of the modelling cycle consists of testing the null hypothesis of linearity against the specific alternative of STAR nonlinearity.

[6] A classical example of a test for linearity is the statistic of Ramsey (1969), known as the *RESET* test, where the nonlinear terms of the alternative may be the powers of the lagged dependent and explanatory variables.

[7] There are also non-standard tests, for example, proposed by Brock, Deckert and Scheinkman (1987), known as the BDS test.

and in most cases analytical expressions are not available and critical values have to be calculated by simulation and for each particular case.

In the LSTAR framework, Luukkonen, Saikkonen and Teräsvirta (1988) analyzed the issue of testing for linearity[8]. They suggested to replace the transition function $F(s_t; \gamma, c)$ by a convenient Taylor series approximation. Because in the new parameterization there is no identification problems, linearity can be tested using a LM statistic with a standard asymptotic χ^2 distribution under the null hypothesis. As it is known, in a LM statistic the model under the alternative hypothesis is not needed to be calculated which gives an additional advantage to this case. To calculate the LM-type statistic, we start to estimate the following regression as suggested by Granger and Teräsvirta (1993):

$$y_t = \beta_0 + \beta_1' x_t + \sum_{j=1}^{p} \beta_{2j} y_{t-j} y_{t-d} + \sum_{j=1}^{p} \beta_{3j} y_{t-j} y_{t-d}^2 + \sum_{j=1}^{p} \beta_{4j} y_{t-j} y_{t-d}^3 + v_t \quad (15)$$

where the null hypothesis is $H_0 : \beta_{2j} = \beta_{3j} = \beta_{4j} = 0, j = 1, ..., p$. In (15), the transition variable s_t has been replaced by y_{t-d}, with d as the delay parameter. In order to specify the value of d, it is appropriate to carry out the test for a range of values $1 \leqslant d \leqslant p$. If we reject H_0 for more than one value of d , then d is determined as $\widehat{d} = \arg\min\, p(d)$ for $d \in [1, p]$ where $p(d)$ is the $p-$value of the selected test[9].

Although we assume that the correct nonlinear model under the alternative hypothesis is the LSTAR model, it is interesting to explain the selection process between ESTAR and LSTAR models. This can be done by a sequence of tests within (15) as is discussed in many papers; see for example Teräsvirta (1990). It is possible to specify the following set of null hypotheses:

[8]Note that the LM type tests of testing nonlinearity differs for both LSTAR and ESTAR models. The practical ways of carrying out tests for both types of models are given in Terasvirta (1994).

[9]The test has maximum power if d is chosen correctly, otherwise an incorrect choice of d weakens the power of the test.

$$
\begin{aligned}
H_{04} &: \beta_{4j} = 0 \\
H_{03} &: \beta_{3j} = 0 | \beta_{4j} = 0 \\
H_{02} &: \beta_{2j} = 0 | \beta_{3j} = \beta_{4j} = 0
\end{aligned}
$$

where for all cases, $j = 0, 1, 2, ..., p$. The logic behind this sequence is based on interpreting the coefficients β_{ij} as functions of the parameters of the original model, that is, the expression (4); see Teräsvirta and Anderson (1992). If the model follows an ESTAR specification, then $\beta_{4j} = 0$, for $j = 1, ..., p$, but $\beta_{3j} \neq 0$ for at least one j if $\phi_2 \neq 0$. Furthermore, if the model follows a LSTAR specification, $\beta_{2j} \neq 0$ for at least one j if $\phi_2 \neq 0$. Thus, if H_{04} is rejected we choose the LSTAR model. If H_{04} is not rejected and H_{03} is rejected, the ESTAR model is selected. If H_{04} and H_{03} are not rejected and H_{02} is rejected, it leads to a LSTAR model. The inconclusive case is the one in which both H_{03} and H_{02} are rejected. Details how to proceed in this case are given in Teräsvirta and Anderson (1992).

After that transition variable s_t and the transition function $F(s_t; \gamma, c)$ have been selected, estimation of the model is the next step. Estimation of a model given by (13) may be considered as an application of nonlinear least squares (NLS). Because we are assuming a two regime STAR model, the set of parameters $\theta = (\phi_1, \phi_2, \gamma, c)'$ may be estimated as

$$
\widehat{\theta} = \arg\min_{\theta} Q_T(\theta) = \arg\min_{\theta} \sum_{t=1}^{T} (y_t - G(x_t; \theta))^2, \tag{16}
$$

where $G(x_t; \theta) = \phi_1' x_t (1 - F(s_t; \gamma, c)) + \phi_2' x_t F(s_t; \gamma, c)$[10]. Assuming that errors are normally distributed, then NLS estimates are equivalent to those obtained from maximum likelihood. If this is not the case, NLS estimates are interpreted as quasi maximum likelihood estimates. Some details about

[10] This term is known as the skeleton of the model. See van Dijk, Terasvirta and Franses (2001).

the limiting distribution of θ and estimation of the covariance-matrix are available in Wooldridge (1994). Notice that the estimation procedure can be simplified by concentrating the sum of squares function. More details are given in van Dijk, Teräsvirta and Franses (2001).

In the evaluation step, the residuals from the estimated model are submitted to some diagnostic tests. The first is a LM test to verify for q-th order serial dependence in ε_t which is a generalization of the LM test for serial correlation in an $AR(p)$ model by Godfrey (1979). It is distributed asymptotically as a χ^2 with q degrees of freedom. If we consider the model given by (13), this test consists of a regression of $\widehat{\varepsilon}_t$ on $\partial G(x_t;\theta)/\partial\theta$ and q lagged estimated residuals with $\theta = (\phi_1, \phi_2, \gamma, c)'$, all values are calculated under the null hypothesis.

A second diagnostic test is a LM test is designed to verify the hypothesis of no remaining nonlinearity. It was proposed by Eitheim and Teräsvirta (1996) to test the two regime LSTAR model (expression (14), for example) against the alternative of an additive STAR model which can be consisted of a three regime LSTAR with an additional function $F_2(s_t;\gamma_2, c)$; hence, $H_0 : \gamma_2 = 0$. Because in this case we have again identification problems under the null hypothesis, the new transition function $F_2(s_t;\gamma_2, c)$ is replaced by a Taylor series approximation around $\gamma_2 = 0$. Residuals from this extended model are regressed against the parameters in the two regime model (evaluated under H_0) and auxiliary regressors (power of products between x_t and s_t). The statistic constructed as nR^2 has an χ^2 distribution with $3(p+1)$ degrees of freedom.

Another diagnostic test is designed to verify the hypothesis of parameter constancy. The null hypothesis is constancy of parameter in the two regime STAR model (expression (13) and the alternative hypothesis is smoothly changing parameters, that is $s_t = t$. The limiting distribution is the same as for the tests of remaining nonlinearity.

4 Business Cycles and Nonlinear Models

As mentioned in Introduction, one of the first econometric models in measuring business cycles was proposed by Tinbergen (1939) who used linear difference equations as instrument of analysis. He applied to time series of some macroeconomic indicators. Unfortunately, the linear structure of the model did not consider any nonlinearities of the business cycles. Recently there has been a growing interest in applying nonlinear models to many macroeconomic and financial time series. The idea that the growth rates of output is affected in a different way and magnitude from negative and positive shocks (asymmetries) has supported the interest to apply nonlinear models to analyze the business cycle issue. Additionally, researchers used these tools to calculate the average time of a recession and expansion, dates and the adjustment process. Many models have been used to pursue these goals, but here, we select some of them, specially those related to the use of MS models and STAR models.

During 1980's the empirical analysis of cyclical asymmetries with new contributions in nonlinear modeling. Most empirical business cycle modelling focuses on the growth rate of output of the United States. By estimating an univariate nonlinear model for US GNP, Potter (1995) found evidence in favor of a SETAR model. His results prove that linear methods can hide much interesting economic structure. Estimating nonlinear models for post-1945 US GNP, Potter (1995) suggests that even if the economy was under the influence of negative shocks such as the Great Depression, the output level returns to its trend quickly.

One of the most applied approaches to analyze business cycles in empirical work is MS models. It was proposed by Hamilton (1989) when he analyzed the growth rates of the US output allowing for a changing mean which was dependent of a state variable (s_t). However, one of the shortcomings of his work was that he did not tested the linearity of the data. This was done later by Hansen (1992) who developed appropriate tests which have non standard limiting distribution. Other applications have followed in the literature as for example Godwin (1993) who analyzed business cycles for growth rates of output in G7 countries using a MS model with two regimes. Bodman and Crosby (2000) analyzed business cycle for Canada also using a MS model imposing two different regimes. The results of this model show that the Canadian economy was in recession in 1951:3, even though negative growth was not recorded in the period. A quarter of well below-average growth after a period of very large quarterly decline (greater than minus 4 percent) was followed during the recession of 1953:3-1954:4. It is important to note that the MS model with two regimes could not find the periods of small negative growth in 1956:4 and 1957:1 as a recession and instead identifies the quarter 1957:2 as a peak followed by two quarters of negative growth as a recession.

Bodman and Crosby (2000) extended the model to allow for three regimes in the growth rate of the output of Canada. All three states of this model can be explained as a low-growth (recession) state, a high-growth state, and an intermediate state. The estimates show that the annualized average growth rate in the second regime ("normal times"), $\mu_2 = 4.62$ percent while the recession state is characterized by a negative annual growth rate of -2.77 percent. The growth rate in the recovery phase, μ_1, is estimated to be around 7.82 percent. The expected duration for each period are around 2.5 quarters, 2.7 quarters and 19.2 quarters, respectively.

Some other empirical applications are mentioned by Kim and Nelson (2000). The more important shortcoming in MS models is the test for linearity. Traditional tests, such as Wald statistics, are not available because nuisance parameters are not present under the null hypothesis as we explained in last section. Overall, decision of how many regimes or states exist in the growth rates of output (or any other variable) is imposed arbitrarily. In some other cases, this choice is done using a statistic proposed by Davies (1987)[11].

Examples of applications of STAR models can be found in Granger and Teräsvirta (1993), Teräsvirta and Anderson (1992) and Anderson and Teräsvirta (1992). In most of these cases, the dependent variable is the quarterly differences of the logarithm of the industrial production for 13 OECD countries. They reject the linearity tests for most of these series and ESTAR and LSTAR models are chosen for the different variables.

Another application of STAR model but with two additive smooth transition function was estimated by Öcal and Osborn (2000). In this model, with two thresholds, they found that industrial production, contrary to consumption, is explained by three regimes of recession, normal growth and high growth. The transitions from recession to expansion for both variables are similar showing possibility that asymmetries are not very important.

According to our knowledge, there is no empirical application of STAR models to regional data in Canada. We cover this space estimating LSTAR models for the growth rate of the personal income for each Canadian province. Details of the estimation, testing and all results are presented in next section.

5 Empirical Results

Quarterly data of real income for the ten Canadian provinces is used and data spans from 1960:1 until 2000:2. The application of standard statistics proposed by Dickey and Fuller (1979) and Said and Dickey (1984) shows that all variables are non stationary in logarithm levels but stationary in first differences. In consequence, subsequent analysis is performed using the first differences of the logarithm of real personal income. Evolution of the series over time are presented in Figures 1.1 and 1.2.

We follow the nonlinear modelling cycle proposed by Granger and Teräsvirta

[11] See Garcia and Perron (1996) for an example of the use of this statistic to choose between two and three regimes for the real interest rate and inflation rates of US. Another example can be found in Demers and Rodríguez (2001) who applied this test to inflation and growth output rates of Canada.

(1993) but if we reject the null hypothesis of linearity we proceed to estimate a LSTAR model. In applying the LM test for linearity, we used a maximum lag $p = 8$. Table 1 presents the minimum p-value obtained from the application of linearity tests using $d \in [1, 8]$. Overall, the evidence suggests strong rejection of the null hypothesis of linearity. Table 1 also presents the corresponding delay selected for each of the ten Canadian provinces. In most of cases, $d = 1$ is chosen, while $d = 4$ is selected for British Columbia and Saskatchewan and $d = 5$ is chosen for New Foundland and Nova Scotia.

Estimates of LSTAR models for ten provinces are shown in Tables 2.1 until 2.10. For all cases the ratio of variances of a nonlinear model with respect to that of a linear model is less than unity which means that estimating nonlinear models allows us to achieve efficiency gains. Most of the coefficients are significant using customary levels of significance. As we saw before, parameter γ indicates the smoothness degree and the parameter c is the threshold parameter. These parameters are significant and show different patterns in the behavior of the growth rates for each province. Provinces where the transition from one to other regime is very smooth are Alberta, British Columbia, Manitoba, New Foundland, Nova Scotia and Prince Edward Island. The provinces where the adjustment is abrupt are New Brunswick and Quebec. Intermediate cases with a tendency to fast transition are Ontario and Saskatchewan.

Tables 3.1 until 3.10 show probability values from diagnostic tests applied to each LSTAR model. The first panel in each table are the results of testing the null hypothesis of non autocorrelation using $q = 1, 2, ..., 8$. What these results inform is that in all cases, we can not reject the null hypothesis and then there is not autocorrelation in the residuals obtained from each LSTAR model. The second test verifies the null hypothesis of parameter constancy. Using 5% of significance, in any case, we can not reject the null hypothesis that the parameters in each LSTAR model are constant. If we use 10% of significance, we find evidence of non constancy for British Columbia. The third test verifies the null hypothesis of remaining non linearity in the residuals of each LSTAR models. The results show that in no case, except Saskatchewan, we can reject this null hypothesis. Using a 10% of significance, there are few cases where we should reject this null hypothesis, as for example Alberta and British Columbia. In other cases, except Saskatchewan, there is no evidence of remaining nonlinearity at 10% of significance.

The estimated parameter c is also different for all of the ten provinces. For most of provinces this parameter is negative around -1.5%. Cases where this parameter is positive are Quebec, Saskatchewan, Prince Edward Island, Nova Scotia and New Foundland. Given that the threshold parameter indicates the point where the economy passes from one regime (recession) to another regime (expansion), it is possible to see that from the first set of the mentioned provinces (where $y_{t-d} < c$) this passage is done before the other provinces. This is shown in Figures 2.1 until 2.10, where the transition function $\widehat{F}(.)$ is graphed against the transition delay variable. In these figures, each point corresponds to each observation and then it is possible to see their distribution. For example, look at the Figure 2.7, the transition function for Ontario, most of observations are concentrated after the threshold parameter indicating that there are not many observations in the regimen zero, that is, in a recession. British Columbia (Figure 2.2) is similar but with a smooth adjustment in comparison to Ontario. Figures for provinces such New Foundland, Nova Scotia and Prince Edward Island show a more concentrated distribution of observations between each regime.

A final output from the estimation of LSTAR models is identification of the periods of recession and recovery for the ten provinces. This is done from the estimates of the transition function $F(.)$ over time (quarters) which is shown in Figures 3.1 until 3.2. Notice that when the transition variable is larger than the threshold the transition function is closer to the value of one. In the converse case ($y_{t-d} < c$), the value of the transition function $\widehat{F}(.)$ is close to zero. This case corresponds to the periods where the economy is in a recession. Following other studies (see for example Franses 2000), we proceed to identify peaks and troughs for each of ten Canadian provinces. There is no 'practical' definition to use to identify duration of a recession. For example in Franses and Paap (1998), they consider that a through consists of two consecutive quarters where $F(.)$ is below the value of the threshold parameter. In other practical guide a trough is defined as two consecutive quarters when economy experienced negative growth rates (see Bodman and Crosby, 2000). From our results (Figures 3.1 until 3.2) there are many provinces where it is difficult to have two consecutive quarters where $\widehat{F}(.) \leq 0.5$ because the changes are faster. However, we will account the observations closer to $\widehat{F}(.) = 0$ as a trough. The results are shown in Tables 4.1-4.4. It is possible to see that troughs are distributed in different ways in each province. Overall, recessions of around 1973, 1980, 1990 are shown for most of provinces indicating that total country was in a recession. The dates are closely related to those from Bodman and Crosby (2000) for total Canada. There are also other dates of recession that correspond to circumstances which can be qualified as more provincial showing that each province has particular cycles.

6 Conclusions

The goal of this paper is the estimation and the analysis of business cycles in the ten Canadian provinces. Because we are interested to identify asymmetries in business cycles, we apply a LSTAR model. Quarterly data for growth rates of the personal income is used. By testing for linearity we are able to reject the null hypothesis of linearity. Given this result, we proceed to estimate a LSTAR model for each province allowing for one threshold in the specification. Different degrees of adjustment and threshold parameters are found showing the different dynamics in each provincial economy. Transition function are calculated to show the speed of adjustment and the evolution of these estimations over time allows us to estimate the principal periods of recession and recovery for each economy.

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Table 1. Linearity Tests

Provinces	Minimum $p-$value	Corresponding delay	Corresponding k
Alberta	0.008	1	1
British Columbia	0.003	4	2
Manitoba	0.027	1	3
New Brunswick	0.035	2	1
New Foundland	0.016	5	3
Nova Scotia	0.003	5	2
Ontario	0.004	1	3
Prince Edward Island	0.037	1	3
Quebec	0.012	1	2
Saskatchewan	0.000	4	2

Table 2.1. LSTAR Model for Alberta

$$
\begin{aligned}
\Delta y_t &= \underset{(0.001)}{0.012} + \underset{(0.23)}{0.66}\ \Delta y_{t-2} - \underset{(0.33)}{1.21}\ \Delta y_{t-3} + \underset{(0.3)}{1.03}\ \Delta y_{t-4} - \underset{(0.33)}{0.84}\ \Delta y_{t-5} \\
&\quad - \underset{(0.25)}{0.69}\ \Delta y_{t-8} + (-\ \underset{(0.26)}{0.67}\ \Delta y_{t-2} + \underset{(0.34)}{1.43}\ \Delta y_{t-3} - \underset{(0.33)}{1.06}\ \Delta y_{t-4} + \\
&\quad \underset{(0.35)}{0.71}\ \Delta y_{t-5} + \underset{(0.06)}{0.1}\ \Delta y_{t-7} + \underset{(0.26)}{0.61}\ \Delta y_{t-8}) \\
&\quad \times(1 + \exp(-\underset{(0.79)}{2.75} \times(\Delta y_{t-1} + \underset{(0.002)}{0.0068})/\hat{\sigma}(\Delta y_{t-1})))^{-1} + u_t \\
s^2/s_L^2 &= 0.88,\ AIC = -8.57,\ SIC = -8.29, \\
HQ &= -8.46,\ \ R^2 = 0.23,\ \ \bar{R}^2 = 0.19
\end{aligned}
$$

Table 2.2. LSTAR Model for British Columbia

$$
\begin{aligned}
\Delta y_t &= \underset{(0.056)}{0.1} + \underset{(1.11)}{2.54}\ \Delta y_{t-4} + (\underset{(0.069)}{-0.11} - \underset{(0.96)}{1.97}\ \Delta y_{t-4}) \\
&\quad \times(1 + \exp(-\underset{(0.6)}{1.38} \times(\Delta y_{t-4} + \underset{(0.008)}{0.017})/\hat{\sigma}(\Delta y_{t-4})))^{-1} + u_t \\
s^2/s_L^2 &= 0.93,\ \ AIC = -8.28,\ \ SIC = -8.16, \\
HQ &= -8.23,\ R^2 = 0.09,\ \bar{R}^2 = 0.07
\end{aligned}
$$

Table 2.3. LSTAR Model for Manitoba

$$\begin{aligned}
\Delta y_t &= \underset{(0.000)}{0.004} + \underset{(0.7)}{1.4}\ \Delta y_{t-3} + \underset{(0.38)}{0.71}\ \Delta y_{t-8} + \\
&\quad (\underset{(0.07)}{0.28}\ \Delta y_{t-1} - \underset{(0.71)}{1.4}\ \Delta y_{t-3} - \underset{(0.39)}{0.77}\ \Delta y_{t-8}) \\
&\quad \times(1+\exp(-\underset{(1.53)}{3.48} \times(\Delta y_{t-1} + \underset{(0.002)}{0.016})/\hat{\sigma}(\Delta y_{t-1})))^{-1} + u_t \\
s^2/s_L^2 &= 0.92,\ AIC = -8.91,\ SIC = -8.74, \\
HQ &= -8.84,\ R^2 = 0.1,\ \bar{R}^2 = 0.05
\end{aligned}$$

Table 2.4. LSTAR Model for New Brunswick

$$\begin{aligned}
\Delta y_t &= \underset{(0.002)}{0.02} - \underset{(0.11)}{0.56}\ \Delta y_{t-1} - \underset{(0.13)}{0.32}\ \Delta y_{t-3} + \\
&\quad (-\underset{(0.003)}{0.016} + \underset{(0.13)}{0.57}\ \Delta y_{t-1} + \underset{(0.08)}{0.31}\ \Delta y_{t-2} + \underset{(0.14)}{0.32}\ \Delta y_{t-3}) \\
&\quad \times(1+\exp(-\underset{(252.62)}{125.61} \times(\Delta y_{t-2} + \underset{(0.000)}{0.002})/\hat{\sigma}(\Delta y_{t-2})))^{-1} + u_t \\
s^2/s_L^2 &= 0.93,\ AIC = -8.33,\ SIC = -8.15, \\
HQ &= -8.26,\ R^2 = 0.11,\ \bar{R}^2 = 0.10
\end{aligned}$$

Table 2.5. LSTAR Model for New Foundland

$$\begin{aligned}
\Delta y_t &= \underset{(0.09)}{0.22}\ \Delta y_{t-1} + \underset{(0.05)}{0.09}\ \Delta y_{t-2} - \underset{(0.10)}{0.23}\ \Delta y_{t-5} - \underset{(0.05)}{0.07}\ \Delta y_{t-8} + \\
&\quad (\underset{(0.005)}{0.022} - \underset{(0.17)}{0.59}\ \Delta y_{t-1} - \underset{(0.13)}{0.38}\ \Delta y_{t-4}) \\
&\quad \times(1+\exp(-\underset{(1.9)}{3.19}\times(\Delta y_{t-5} - \underset{(0.004)}{0.0084})/\hat{\sigma}(\Delta y_{t-5})))^{-1} + u_t \\
s^2/s_L^2 &= 0.95,\ AIC = -7.45,\ SIC = -7.27, \\
HQ &= -7.37,\ R^2 = 0.08,\ \bar{R}^2 = 0.03
\end{aligned}$$

Table 2.6. LSTAR Model for Nova Scotia

$$\begin{aligned}
\Delta y_t &= -\underset{(0.09)}{0.19}\ \Delta y_{t-1} - \underset{(0.11)}{0.75}\ \Delta y_{t-5} + \\
&\quad (\underset{(0.004)}{0.01} + \underset{(0.14)}{0.34}\ \Delta y_{t-1} - \underset{(0.08)}{0.41}\ \Delta y_{t-2} + \underset{(0.09)}{0.47}\ \Delta y_{t-4} - \underset{(0.22)}{0.56}\ \Delta y_{t-5}) \\
&\quad \times(1+\exp(-\underset{(0.9)}{3.36}\times(\Delta y_{t-5} - \underset{(0.001)}{0.0052})/\hat{\sigma}(\Delta y_{t-5})))^{-1} + u_t \\
s^2/s_L^2 &= 0.67,\ AIC = -8.89,\ SIC = -8.71, \\
HQ &= -8.82,\ R^2 = 0.24,\ \bar{R}^2 = 0.22
\end{aligned}$$

Table 2.7. LSTAR Model for Ontario

$$
\begin{aligned}
\Delta y_t &= \underset{(0.05)}{-0.36} - \underset{(3.22)}{21.72}\, \Delta y_{t-1} + \underset{(3.4)}{19.89}\, \Delta y_{t-2} - \underset{(3.51)}{21.93}\, \Delta y_{t-3} + \\
&\quad (\underset{(0.05)}{0.37} + \underset{(3.22)}{22.08}\, \Delta y_{t-1} - \underset{(3.4)}{19.85}\, \Delta y_{t-2} + \underset{(3.51)}{22.12}\, \Delta y_{t-3}) \\
&\quad \times (1 + \exp(-\underset{(7.5)}{15.02} \times (\Delta y_{t-1} + \underset{(0.001)}{0.0121}) / \hat{\sigma}(\Delta y_{t-1})))^{-1} + u_t \\
s^2/s_L^2 &= 0.79, AIC = -9.40, SIC = -9.21, \\
HQ &= -9.32,\ R^2 = 0.35,\ \bar{R}^2 = 0.34
\end{aligned}
$$

Table 2.8. LSTAR Model for Prince Edward Island

$$
\begin{aligned}
\Delta y_t &= \underset{(0.003)}{0.017} - \underset{(0.1)}{0.44}\, \Delta y_{t-2} + \\
&\quad (-\underset{(0.01)}{0.028} + \underset{(0.35)}{0.34}\, \Delta y_{t-1} + \underset{(0.28)}{0.67}\, \Delta y_{t-2} - \underset{(0.15)}{0.38}\, \Delta y_{t-6} \\
&\quad \times (1 + \exp(-\underset{(1.48)}{2.55} \times (\Delta y_{t-1} - \underset{(0.007)}{0.017}) / \hat{\sigma}(\Delta y_{t-1})))^{-1} + u_t \\
s^2/s_L^2 &= 0.86,\ AIC = -7.80, SIC = -7.64, \\
HQ &= -7.73,\ \ R^2 = 0.14,\ \bar{R}^2 = 0.11
\end{aligned}
$$

Table 2.9. LSTAR Model for Quebec

$$
\begin{aligned}
\Delta y_t &= \underset{(0.00)}{0.003} + \underset{(0.07)}{0.56}\,\Delta y_{t-1} + \underset{(0.05)}{0.23}\,\Delta y_{t-2} + \\
&\quad (\underset{(0.005)}{0.0005} - \underset{(0.24)}{0.57}\,\Delta y_{t-1}) \\
&\quad \times(1+\exp(-\underset{(33.13)}{44.58}\times(\Delta y_{t-1} - \underset{(0.0003)}{0.015})/\hat{\sigma}(\Delta y_{t-1})))^{-1} + u_t \\
s^2/s_L^2 &= 0.89,\ AIC = -9.49, SIC = -9.35, \\
HQ &= -9.43,\ R^2 = 0.24,\ \bar{R}^2 = 0.23
\end{aligned}
$$

Table 2.10. LSTAR Model for Saskatchewan

$$
\begin{aligned}
\Delta y_t &= \underset{(0.001)}{0.008} - \underset{(0.05)}{0.2}\,\Delta y_{t-4} + \underset{(0.04)}{0.13}\,\Delta y_{t-5} + \\
&\quad (-\underset{(0.01)}{0.044} + \underset{(1.75)}{3.48}\,\Delta y_{t-2} - \underset{(0.87)}{1.33}\,\Delta y_{t-5} - \underset{(1.56)}{2.6}\,\Delta y_{t-6} - \underset{(0.11)}{0.24}\,\Delta y_{t-8}) \\
&\quad \times(1+\exp(-\underset{(0.42)}{2.13}\times(\Delta y_{t-4} - \underset{(0.01)}{0.056})/\hat{\sigma}(\Delta y_{t-4})))^{-1} + u_t \\
s^2/s_L^2 &= 0.74,\ AIC = -8.03,\ SIC = -7.82, \\
HQ &= -7.94,\ R^2 = 0.50,\ \bar{R}^2 = 0.48 \qquad (1)
\end{aligned}
$$

Table 3.1. Diagnostic Tests of LSTAR for Alberta.

Tests for q−th order serial correlation								
q	1	2	3	4	5	6	7	8
p−value	0.772	0.694	0.644	0.790	0.890	0.871	0.786	0.872

Tests for Parameter Constancy			
Functional form	All model	Linear	Nonlinear
p−value	0.302	0.183	0.283

Tests for remaining nonlinearity	
Lag	p−value
Δy_{t-1}	0.361
Δy_{t-2}	0.249
Δy_{t-3}	0.074
Δy_{t-4}	0.288
Δy_{t-5}	0.323
Δy_{t-6}	0.192
Δy_{t-7}	0.218
Δy_{t-8}	0.428

Table 3.2. Diagnostic Tests for British Columbia

Tests for q−th order serial correlation								
q	1	2	3	4	5	6	7	8
p−value	0.863	0.967	0.938	0.948	0.927	0.968	0.976	0.991

Tests for parameter constancy			
Functional form	All model	Linear	Nonlinear
p−value	0.353	0.053	0.054

Tests for remaining nonlinearity	
Lag	p−value
Δy_{t-1}	0.478
Δy_{t-2}	0.212
Δy_{t-3}	0.052
Δy_{t-4}	0.055
Δy_{t-5}	0.967
Δy_{t-6}	0.821
Δy_{t-7}	0.601
Δy_{t-8}	0.829

Table 3.3. Diagnostic Tests for Manitoba

Tests for q-th order serial correlation								
q	1	2	3	4	5	6	7	8
p-value	0.340	0.639	0.540	0.807	0.898	0.947	0.963	0.963

Tests for parameter constancy			
Functional form	All model	Linear	Nonlinear
p-value	0.120	0.265	0.363

Tests for remaining nonlinearity	
Lag	p-value
Δy_{t-1}	0.105
Δy_{t-2}	0.615
Δy_{t-3}	0.679
Δy_{t-4}	0.866
Δy_{t-5}	0.860
Δy_{t-6}	0.839
Δy_{t-7}	0.227
Δy_{t-8}	0.146

Table 3.4. Diagnostic Tests for New Brunswick

Tests for q−th order serial correlation								
q	1	2	3	4	5	6	7	8
p−value	0.293	0.454	0.397	0.563	0.380	0.235	0.274	0.357

Tests for parameter constancy			
Functional form	All model	Linear	Nonlinear
p−value	0.356	0.158	0.122

Tests for remaining nonlinearity	
Lag	p−value
Δy_{t-1}	0.137
Δy_{t-2}	0.733
Δy_{t-3}	0.723
Δy_{t-4}	0.547
Δy_{t-5}	0.746
Δy_{t-6}	0.661
Δy_{t-7}	0.857
Δy_{t-8}	0.120

Table 3.5. Diagnostic Tests for New Foundland

Tests for q−th order serial correlation								
q	1	2	3	4	5	6	7	8
p−value	0.318	0.523	0.585	0.689	0.473	0.602	0.717	0.761

Tests for parameter constancy			
Functional form	All model	Linear	Nonlinear
p−value	0.130	0.098	0.159

Tests for remaining nonlinearity	
Lag	p−value
Δy_{t-1}	0.874
Δy_{t-2}	0.683
Δy_{t-3}	0.372
Δy_{t-4}	0.115
Δy_{t-5}	0.841
Δy_{t-6}	0.848
Δy_{t-7}	0.401
Δy_{t-8}	0.147

Table 3.6. Diagnostic Tests for Nova Scotia

Tests for $q-$th order serial correlation								
q	1	2	3	4	5	6	7	8
$p-$value	0.103	0.182	0.257	0.422	0.397	0.553	0.601	0.471
Tests for parameter constancy								
Functional form	All model		Linear			Nonlinear		
$p-$value	0.611		0.739			0.841		
Tests for remaining nonlinearity								
Lag	$p-$value							
Δy_{t-1}	0.984							
Δy_{t-2}	0.348							
Δy_{t-3}	0.916							
Δy_{t-4}	0.721							
Δy_{t-5}	0.897							
Δy_{t-6}	0.902							
Δy_{t-7}	0.962							
Δy_{t-8}	0.518							

Table 3.7. Diagnostic Tests for Ontario

Tests for $q-$th order serial correlation								
q	1	2	3	4	5	6	7	8
$p-$value	0.497	0.791	0.710	0.401	0.418	0.504	0.634	0.680

Tests for parameter constancy			
Functional form	All model	Linear	Nonlinear
$p-$value	0.558	0.490	0.489

Tests for remaining nonlinearity	
Lag	$p-$value
Δy_{t-1}	0.917
Δy_{t-2}	0.752
Δy_{t-3}	0.885
Δy_{t-4}	0.827
Δy_{t-5}	0.607
Δy_{t-6}	0.688
Δy_{t-7}	0.735
Δy_{t-8}	0.282

Table 3.8. Diagnostic Tests for Prince Edward Island

Tests for $q-$th order serial correlation								
q	1	2	3	4	5	6	7	8
$p-$value	0.591	0.468	0.571	0.462	0.692	0.730	0.278	0.457

Tests for parameter constancy			
Functional form	All model	Linear	Nonlinear
$p-$value	0.393	0.631	0.195

Tests for remaining nonlinearity	
Lag	$p-$value
Δy_{t-1}	0.841
Δy_{t-2}	0.384
Δy_{t-3}	0.972
Δy_{t-4}	0.991
Δy_{t-5}	0.897
Δy_{t-6}	0.425
Δy_{t-7}	0.503
Δy_{t-8}	0.371

Table 3.9. Diagnostic Tests for Quebec

Tests for $q-$th order serial correlation								
q	1	2	3	4	5	6	7	8
$p-$value	0.971	0.669	0.376	0.437	0.482	0.614	0.360	0.464

Tests for parameter constancy			
Functional form	All model	Linear	Nonlinear
$p-$value	0.338	0.144	0.423

Tests for remaining nonlinearity	
Lag	$p-$value
Δy_{t-1}	0.166
Δy_{t-2}	0.102
Δy_{t-3}	0.357
Δy_{t-4}	0.677
Δy_{t-5}	0.181
Δy_{t-6}	0.914
Δy_{t-7}	0.244
Δy_{t-8}	0.563

Table 3.10. Diagnostic Tests for Saskatchewan

Tests for $q-$th order serial correlation								
q	1	2	3	4	5	6	7	8
$p-$value	0.301	0.319	0.334	0.133	0.157	0.251	0.173	0.108
Tests for parameter constancy								
Functional form	All model		Linear		Nonlinear			
$p-$value	0.126		0.316		0.275			
Tests for remaining nonlinearity								
Lag	$p-$value							
Δy_{t-1}	0.175							
Δy_{t-2}	0.644							
Δy_{t-3}	0.676							
Δy_{t-4}	0.022							
Δy_{t-5}	0.395							
Δy_{t-6}	0.357							
Δy_{t-7}	0.257							
Δy_{t-8}	0.198							
Δy_{t-9}	0.769							

Table 4.1. Business cycle peaks and troughs*

Alberta		British Columbia		Manitoba	
Peak	Trough	Peak	Trough	Peak	Trough
1974.2	1975.1	1967.2	1967.4	1978.4	1979.2
1982.2	1984.1	1982.2	1983.3	1986.3	1987.1
1986.2	1987.1			1990.4	1991.2
1988.4	1989.2				
1991.2	1991.4				
1998.2	1999.1				

*A peak is defined as the last observation before a recession. A trough is the last quarter of a recession. A recession is defined when $\widehat{F}(\Delta y_{t-d}) \leq 0.5$

Table 4.2. Business cycle peaks and troughs*

New Brunswick		New Foundland	
Peak	Trough	Peak	Trough
1968.4	1969.2	1963.3	1964.2
1975.1	1975.4	1968.1	1968.3
1978.1	1978.3	1969.4	1970.4
1982.4	1983.2	1971.4	1972.2
1988.1	1988.3	1972.4	1973.2
1991.3	1992.1	1975.2	1975.4
1995.4	1996.2	1978.3	1979.3
		1980.4	1981.4
		1985.4	1986.3
		1987.2	1988.1
		1992.4	1993.3
		1997.1	1997.3

*See notes of Table 4.1

Table 4.3. Business cycle peaks and troughs

Nova Scotia		Ontario		Prince Edward Island	
Peak	Trough	Peak	Trough	Peak	Trough
1963.4	1964.2	1974.4	1975.2	1962.4	1963.4
1967.4	1968.2	1982.3	1983.1	1964.3	1965.3
1971.3	1972.2	1990.4	1991.1	1966.1	1966.3
1977.3	1978.2			1970.2	1971.1
1979.4	1980.2			1974.1	1975.2
1982.1	1983.3			1979.3	1980.2
1987.2	1988.1			1983.2	1984.2
1990.1	1990.4			1989.1	1989.4
1992.1	1992.4			1994.3	1995.1
1994.3	1995.3			1995.2	1996.1
1996.2	1997.4			1998.2	1999.4

*See notes of Table 4.1.

Table 4.4. Business cycle peaks and troughs*

Quebec		Saskatchewan	
Peak	Trough	Peak	Trough
1962.3	1963.4	1964.1	1965.4
1965.2	1965.4	1966.1	1971.4
1967.1	1968.2	1972.1	1975.4
1969.1	1971.2	1977.1	1986.4
1972.2	1973.1	1987.1	1990.4
1974.3	1975.4	1991.4	1994.1
1976.3	1978.3	1995.3	1996.4
1979.3	1980.4		
1981.2	1983.2		
1986.1	1987.2		
1988.2	1994.1		
1994.4	1998.2		

*See notes of Table 4.1.

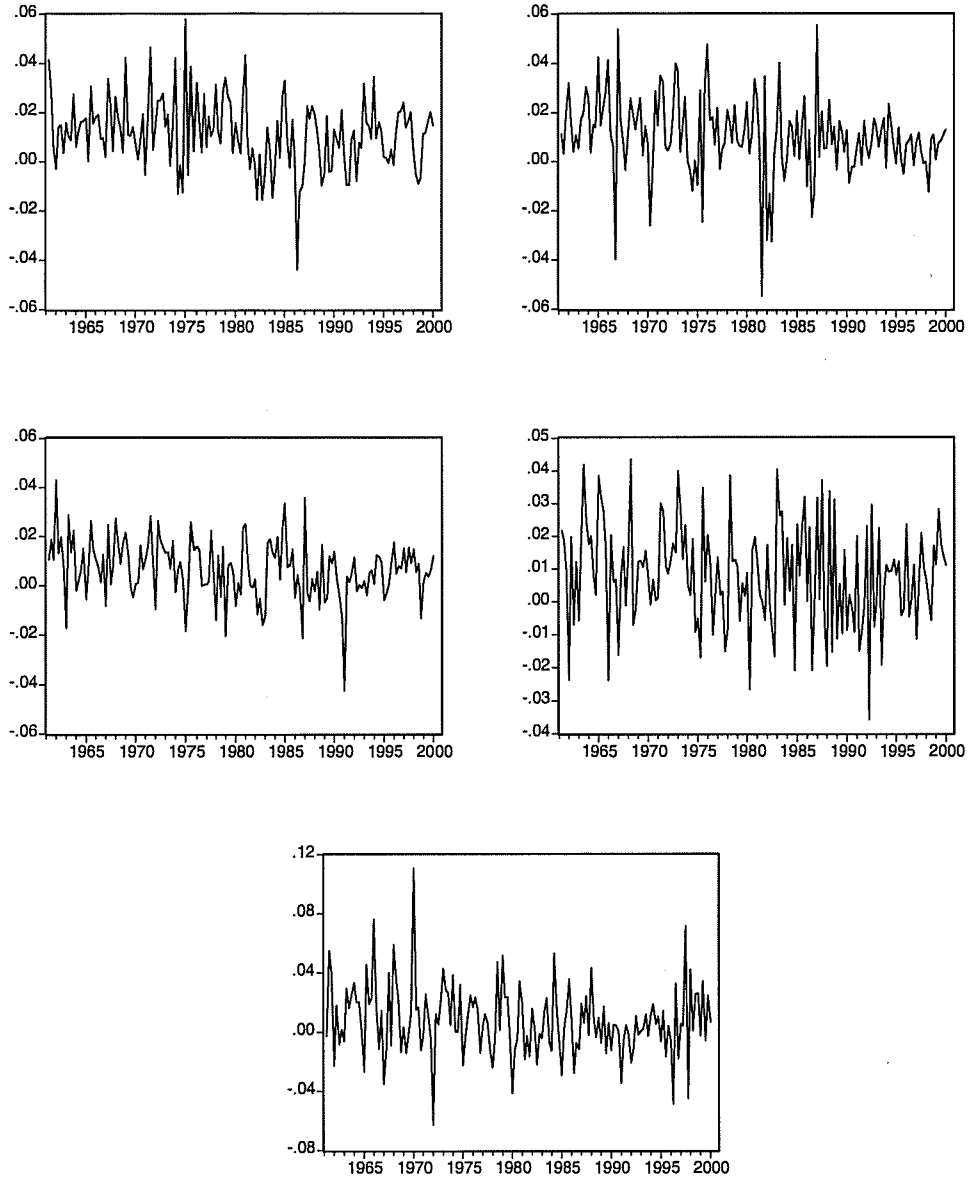

Figure 1.1. Growth Rates of Personal Income of Alberta, British Columbia, Manitoba, New Brunswick and New Foundland

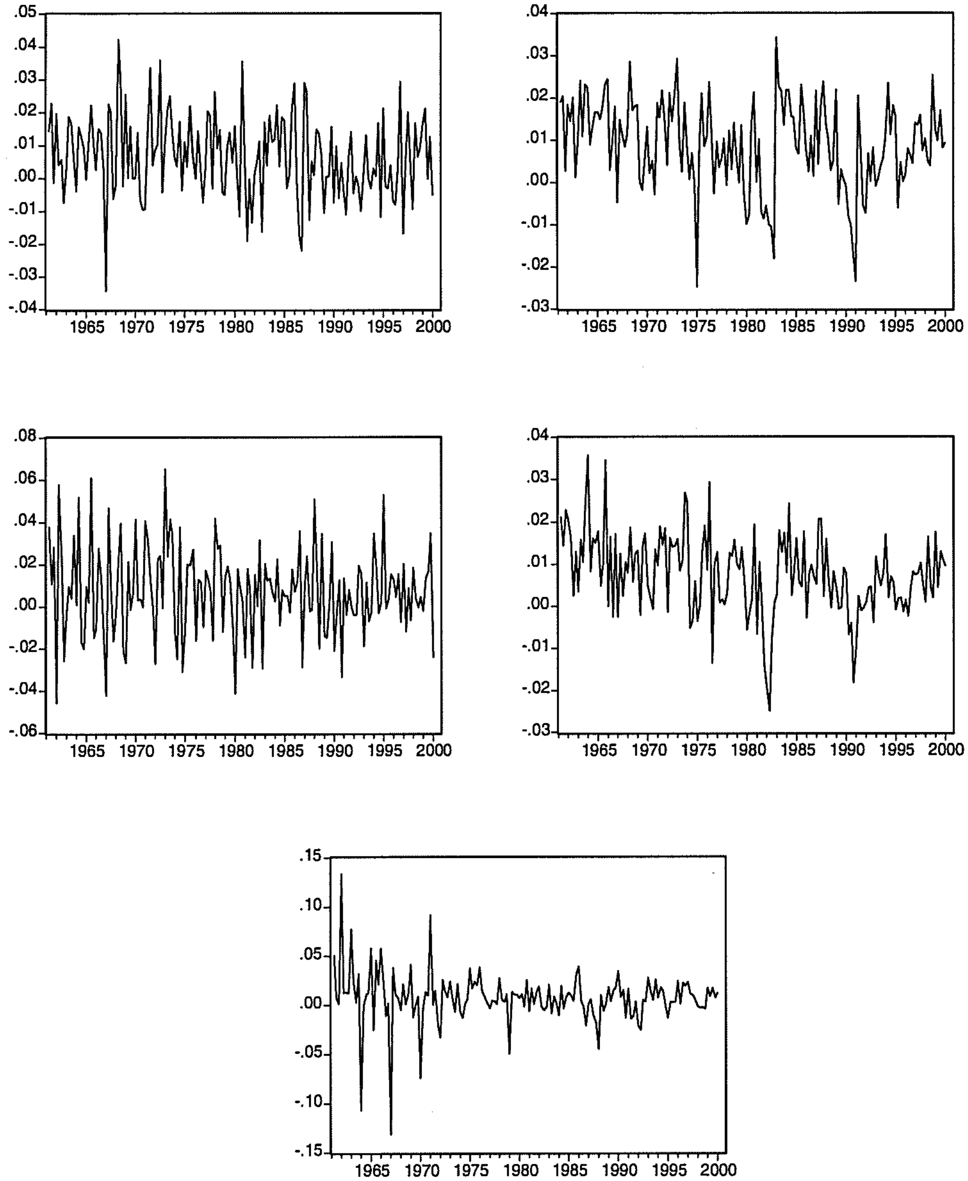

Figure 1.2. Growth Rates of Persona Income for Nova Scotia, Ontario, Prince Edward Island, Quebec and Saskatchewan

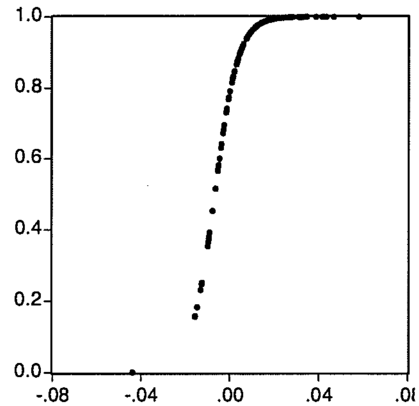

Figure 2.1. Transition Function against Δy_{t-1} for Alberta

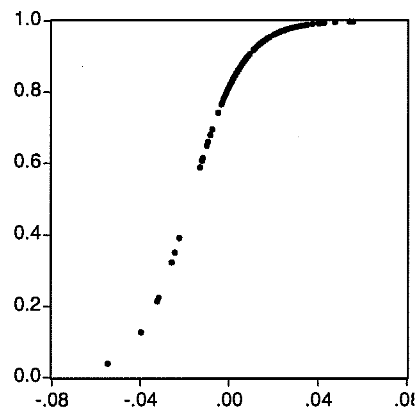

Figure 2.2. Transition Function against Δy_{t-4} for British Columbia

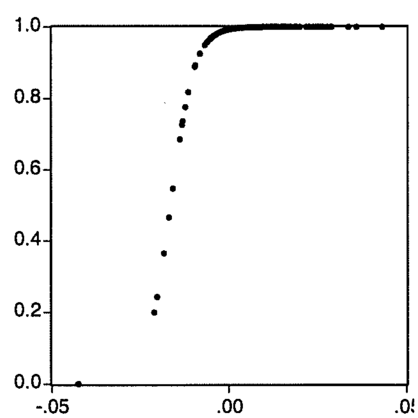

Figure 2.3. Transition Function against Δy_{t-1} for Manitoba

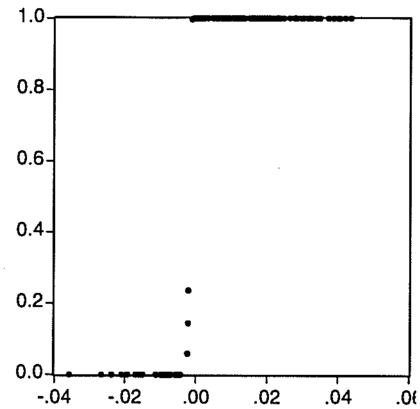

Figure 2.4. Transition Function against $\Delta y_{t-2}$for New Brunswick

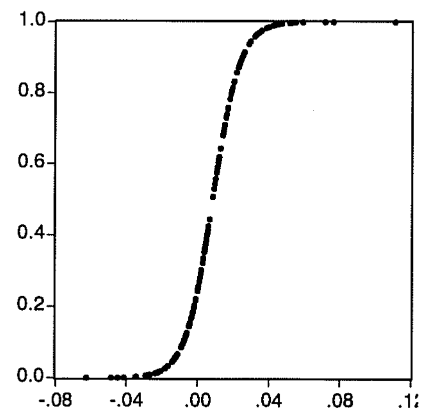

Figure 2.5. Transition Function against Δy_{t-5} for New Foundland

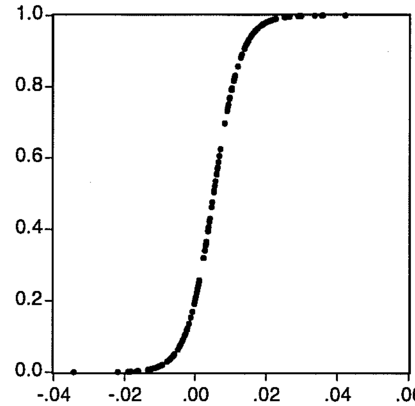

Figure 2.6. Transition Function against Δy_{t-5} for Nova Scotia

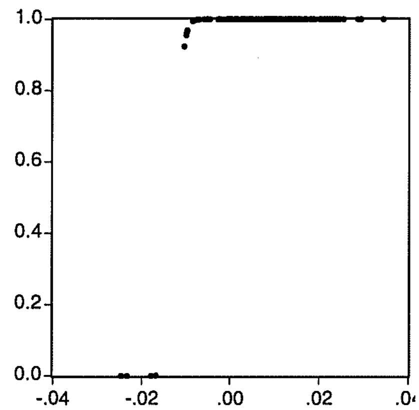

Figure 2.7. Transition Function against Δy_{t-1} for Ontario

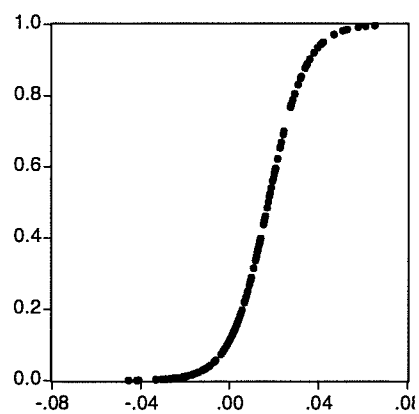

Figure 2.8. Transition Function against Δy_{t-1} for Prince Edward Island

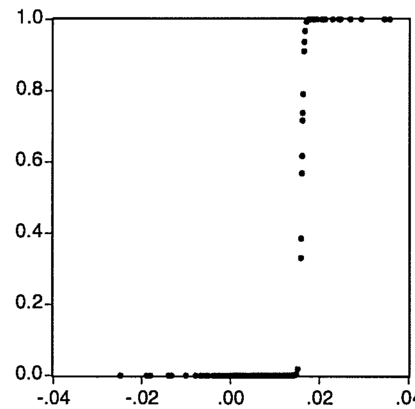

Figure 2.9. Transition Function against Δy_{t-1} for Quebec

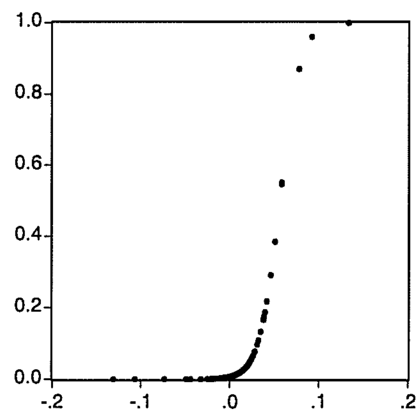

Figure 2.10. Transition Function against Δy_{t-4} for Saskatchewan

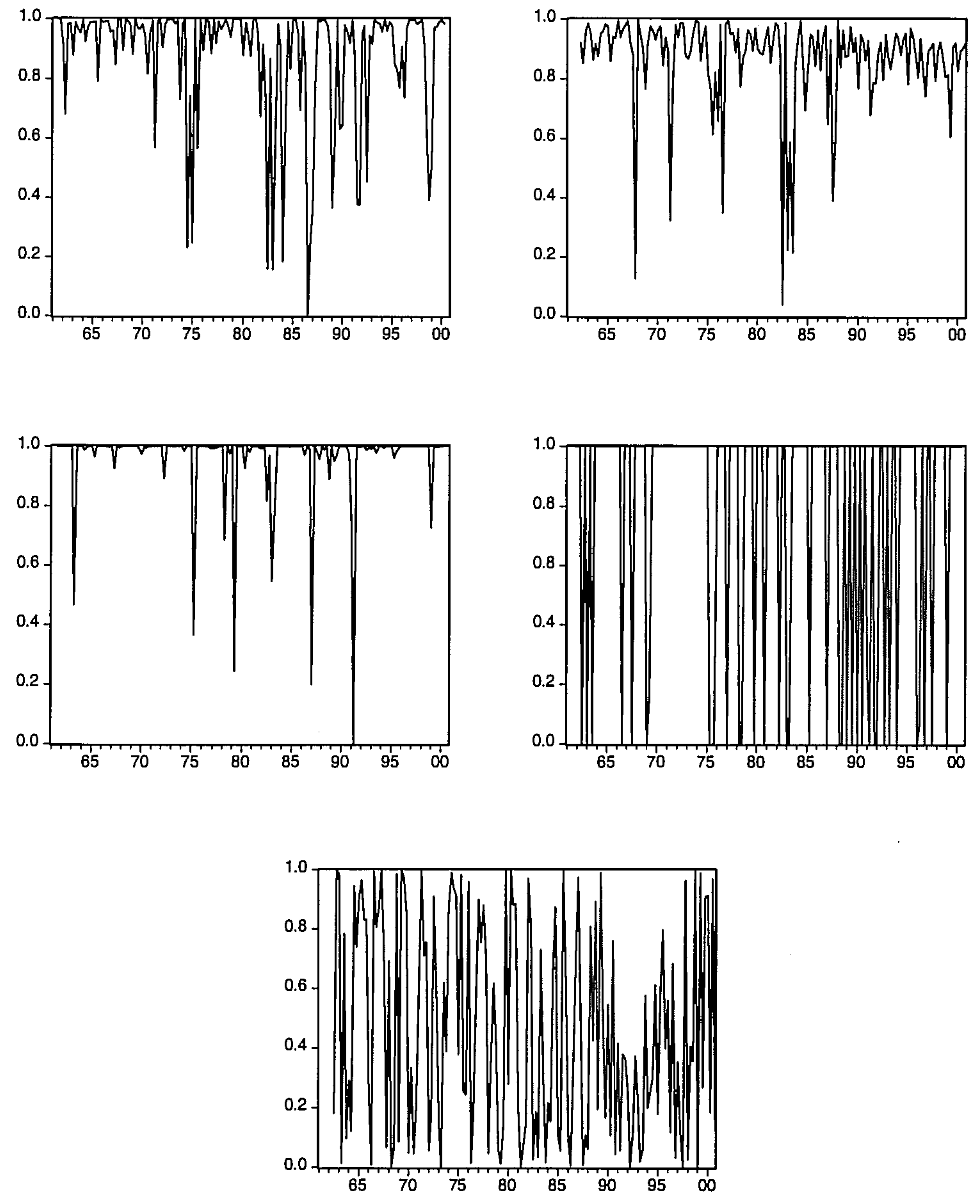

Figure 3.1. Transition Function against Time for Alberta, British Columbia, Manitoba, New Brunswick and New Foundland

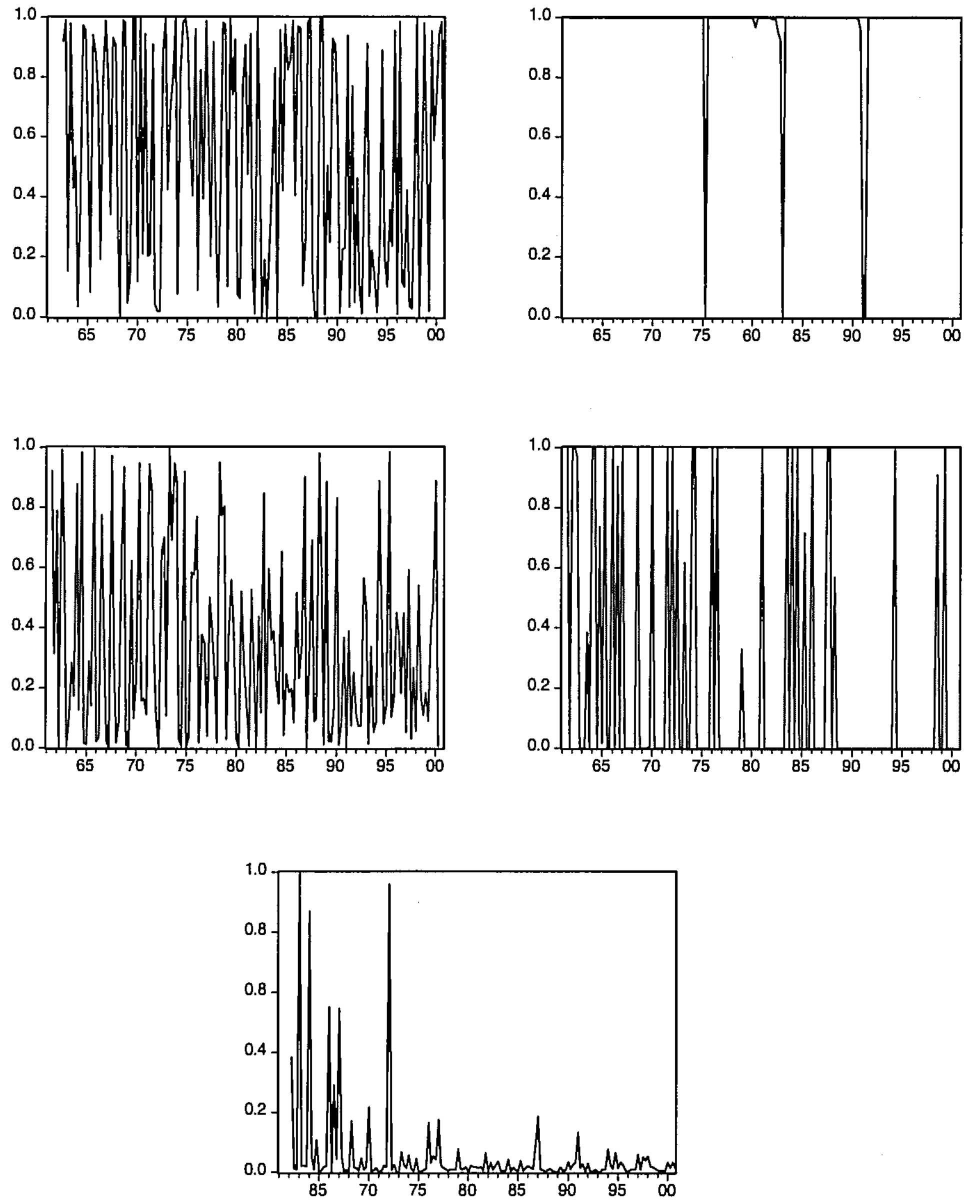

Figure 3.2. Transition Function against Time for Nova Scotia, Ontario, Prince Edward Island, Quebec and Saskatchewan