The Inter-Temporal Trade-Off of Human Capital Investment in a Two-Region Dynamic Model

M.A. MAJOR PAPER*

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* I am thankful to God that He made it possible for me to write this paper till the end, giving as a supervisor such helpful, supportive and understanding Professor Marcel Mérette. All errors are mine.

Abstract

I use a two-region overlapping generations dynamic model to investigate the inter-temporal trade-off of public investments in education. Using simulation exercises, I apply two sequential shocks to government spending on education and to the efficiency of the labor force. I suppose that government increases invests in education by 10 per cent (first shock) during 3 consecutive periods. This shock is followed by a 10 per cent increase in the efficiency of labor. We examine the effects of this exercise on a number of economic variables.

Section 1

Introduction

Nowadays we have "an ocean" of research carried out by economists on the issues of economic development. New fields were discovered and new theories were developed as a result of qualitative changes that have occurred in the world during the last decade. One of the most significant events that changed the world was the formation of new independent states all over the world, in particular after the breakdown of USSR into 15 new independent countries. Democracy, cooperation and market-oriented economics were chosen as the fundamental instruments of the development of the nations in the hope of achieving prosperity, peace and efficiency.

In my paper, I would like to investigate the role of education or, more broadly, the issue of availability and quality of human capital in a country. This topic still remains controversial in terms of the appropriate degree of the government's policy response. I use a two-region model, that was developed by Mercenier J., and M. Merette (2002), in which producers maximize profits, and consumers desire to be on the optimal level of utility. By means of GAMS software, we conduct simulations with the variables related to the education and investment into human capital.

In this two-region model, we will investigate the short and the long-run effect of the investments in the human capital. We will assume that one country invests in human capital while the other does not. As a result we will see what happens to the economic performance of both countries in the short and long run.

The experiment may be interesting to examine in terms of current events occurring in developing countries regarding human capital policies. Some countries, like

Uzbekistan for example, invest heavily in human capital by undertaking major reforms in education system. Since attaining independence, more than 4000 colleges were opened, and several Universities were established and equipped with up-to-date facilities. It is the only country in Central Asian region that developed a long-run national program on human capital development. At the same time, its neighbor Tajikistan has not made any substantial changes and innovations in the education system. The level of investment in the human capital has dropped to a level that is even lower than it was under the Communist rule. Given the differences in the national policies regarding human capital in these two countries, it seems interesting to know what would be the results of each strategy.

In my attempt to investigate the above-mentioned issues and to respond to a number of other questions related to the subject, the paper is organized in the following manner. Section 2 contains a brief review of literature and research related to the question of human capital investments. Section 3 consists of a detailed description of the model that we use for simulation analysis. It will also describe the data and calibration procedures of the model. Section 4 contains the presentation and interpretation of simulation results. And finally, section 5 provides some conclusive remarks.

Section 2

Literature review.

It is known that efficient allocation of available resources is a major problem of developing countries. It means that the governments are not able to attain the potential aggregate production. As we know, the fact that a country is rich with natural resources does not automatically imply that the per capita income will increase and the standard of living will rise. For example, countries of Central Asian region are considered to be leaders in terms of their endowments of natural gas, oil and gold. Despite this factor, the GDP level for all 5 countries (Kazakhstan, Turkmenistan, Kyrgyz Republic, Tajikistan and Uzbekistan) is decreasing, and the standard of living is worsening each year. So the availability of resources may be a necessary condition, but it does not constitute a sufficient condition for the growth and development. One should consider the fact that there are a lot of different types of resources that could be exploited. So a challenge that must be faced by the government is to assign certain priorities to the development of the different type of the resources. In other words, governments have to decide how much of their time, effort and resources should be invested into the development of each type of resources. It is a challenge for the countries to elaborate a program that will give a list of specific actions and procedures that increases the efficiency of the economy. As stated by Veena (1987) "… Most of the empirical studies commonly indicate that education is one of the basic inputs in economic development. It is shown that the stock of educated human capital in less developed countries is very low in comparison to the developed countries and, therefore, an optimum utilization of physical capital and maximum exploitation of natural resources could not be achieved. Hence, there is considerable need to undertake studies on manpower and education planning".

In this paper I examine the role of the human capital in the process of development of a country. I will try to explain the way human capital affects the growth rate of GDP and what steps should the governments take in order to efficiently allocate its human capital.

In most cases, human capital has positive effect on the growth rate of GDP. What still remains unexplored in this subject is the economic variables that are directly affected by human capital and how this influence should be incorporated in a dynamic growth model, like the one used in this paper. According to some economists, the effects that are stemming from human capital could be summarized by the so called "residual factor'. If we look at the Cobb-Douglas production function, the constant term "A" is the above-mentioned residual factor. It is assumed that this residual factor reflects the technological changes. If there is a shock in technology, such as computers being introduced into production process, there will be an increase in productivity, even though the level of capital and labor inputs are unchanged.

In addition to technological changes, residual factor reflects the effects of the changes in the quality of human capital (Veena, 1987). Moreover the estimate that is assigned to that residual varies between 46% (Kendrick, 1961) and 80% (Bowen, 1964). This means that, for example, 80% rise in output growth is attributed to the residual factor.

As mentioned earlier, increasingly it has been found that an important element of the residual factor is education. The magnitude of the impact of the human capital policy of the country varies with demographic, geographical, socio-cultural and developmental factors, the relationship between education and economic development, the rates of return from education, openness of economy and the level of liberalization of economy.

Another group of economists gives different explanation how the human capital changes can affect the GDP level. In their opinion the estimation should explicitly include a control variable that reflect the changes in quality of labor from time to time. One of the representatives of this school of thought, Alwyn Young, criticizes the earlier theory and indicates that the economists assume that there is no systematic or predictable change in human capital when there is no change in the government spending on education. He claims that: "…it is worth addressing a common misconception concerning growth accounting adjustments for the "quality" of labor and capital input; i.e., that these adjustments implicitly incorporate any embodiment of technological change in those inputs. Fundamentally, the growth accounting procedure assumes that input i today is the same as it was yesterday; i.e., that a 25-years-old female worker with a secondary school education today is identical to a 25-year-old female worker yesterday with the same education level. In so doing, the procedure places any increase in the productivity of the input into the residual" (Young, 1995).

In other words, he assumes that those factors that can change the quality of the labor in the long run should be explicitly included into the model. If the government invests in human capital development, there is going to be a substantial change in the quality of labor. The explanatory power of production function with an explicit labor efficiency variable will be higher. So the government will have opportunity to estimate the exact change in productivity that is consequent to their human capital investment decisions. Keeping this in mind, next I present the simulation model to investigate the inter-temporal effect of public investment into human capital in an international context.

Section 3

The model

I am using overlapping generation model in order to examine the effects of imposed shocks on different economic variables in dynamic. The model was developed by Mercenier J. and M. Merette (2002).

I have grouped the population of each country into 5 generations or time periods, assuming that each period lasts for 12 years. For the regions, we have Uzbekistan, which is one of the 5 Central Asian countries with an economy in transition, and the rest of the world that will include several countries.

producers

We employ the Cobb-Douglas type of production function that is specified as follows:

(1) $$Q(j,t) = A * K(j,t)^{\alpha} * L(j,t)^{(1-\alpha)}$$

where:

$Q(j,t)$ is the amount of produced goods by each region in time t, A is a technology or input allocation coefficient, which is exogenous for the model, K is the capital stock that is available for the whole economy in each region "j" and each period of time "t", and L is the labor force available for the producers.

It is estimated as the level of population at the working age multiplied by the coefficient of labor efficiency. In mathematical notation we have:

1a) $L(j,t) = \sum_{g=1}^{5} pop(j,t,g) * EP(j,g)$

The efficiency profile is an exogenous variable that assigns different levels of efficiency or productivity to the different age groups. For example, we assume that younger are less efficient compared to older people due to the lack of experience. We may say that the given production function is characterized by the constant returns to scale because the sum of the coefficients of K and L is a unity.

To find optimal levels of input use, we use the first-order condition of the firm's profit maximization problem from the production function. We have :

(2) $\frac{wage(j,t)}{p(j,t)} = (1-\alpha) * A * K(j,t)^{\alpha} * L(j,t)^{-\alpha}$

where $wage(j,t)$ is the general wage rate for the economy of each region in time t, and $p(j,t)$ is the general price level in the economy. This is the derivative from profit maximization problem with respect to labor. This gives the producer's labor demand which is optimized when $Marginal\ cost = Marginal\ revenue\ product$ or as in left hand side of our equation real wage rate is equal to marginal revenue obtained from additional labor unit, holding capital fixed.

The capital demand equation is determined as follows. This time I take the derivative from profit maximization problem with respect to K(capital). The first order conditions require that real rate of return of using capital must be equal to the productivity obtained from one additional unit of capital, holding labor fixed:

(3) $\frac{rent(j,t)}{p(j,t)} = \alpha * A * K(j,t)^{\alpha-1} * L(j,t)^{1-\alpha}$

The left hand side of equation is the real rate of return on capital for the economy of each region. The above equations give us the solution for the profit maximizing problem of producer.

household

Household budget :

Equation (4) below consists of the expenditure side of the household's budget constraint (left hand side from the equation sign), showing that the households consume goods-$Con(j,t,g)$, at the general price level-Pcon(j,t) for each region, plus tax on consumption goods-$ConTxR(j,t)$. As part of their expenditure, we have the total amount of savings that is used to purchase government issued bonds (financial capital) and capital stock (physical capital)-$Lend(j,t+1,g+1)$, in certain proportion.

(4) $(1 + ConTxR(j,t)) * Pcon(j,t) * Con(j,t,g) + Lend(j,t+1,g+1) =$

$= (1 - WtxR(j,t) - CTR(t)) * Wage(j,t) * EP(j,g)+$

$+ \sum_{g=1}^{5}(Rintj(i,t-1) * \frac{p(i,t)}{p(i,t-1)}) * p(i,t-1) * Bij(i,j,t,g))-$

$$-KTxR(j,t)*\sum_{g=1}^{5}(Rintj(i,t-1)*\frac{p(i,t)}{p(i,t-1)}-1)*p(i,t-1)*Bij(i,j,t,g))+$$

$$+Rret(j,t)*pInv(j,t-1)*K(j,t,g)-KTxR(j,t)*$$

$$*Rrent(j,t)-1)*pInv(j,t-1)*K(j,t,g)+Inh(j,t,g)-Beq(j,t,g)$$

For the income side we consider several different sources. As the main source of income available for every citizen we have net wage income that equals the wage rate minus the wage tax rate and the rate of contribution to the pension fund. Economic agents in our model hold government bonds, so the net income coming from interest payments on those bonds equals the gross income earned from holding bonds minus tax rate on the net profit from that investments. We should also consider the rate of return on capital stock (physical capital) purchased by households that is left after deduction of tax rate on net income from that capital. In addition, we have the benefits coming from the pension plan and income from inheritance. And finally we deduct the contribution of the households to the bequest fund which will be left for the next generation at the end of the life cycle of each household.

Appealing to the cycle theory of savings, the oldest generation does not have any interest to save besides leaving a bequest. Therefore the last generation's budget constraint is somewhat different. Expenditure and the income sides of the equation are basically the same, except that we take the data that relates to the last generation of households, and their expenditures do not include savings. In fact, the last generation will try to derive as much utility as they can from earned wealth, and they dissave by not spending on capital stock or bonds. Here is mathematical presentation of last generation's budget constraint:

(4a) $$(1 + ConTxR(j,t)) * Pcon(j,t) * Con(j,t,gn) =$$

$$= (1 - WtxR(j,t) - CTR(t)) * Wage(j,t) * EP(j,gn) *$$

$$* \sum(Rintj(i,t-1) * \frac{p(i,t)}{p(i,t-1)}) * p(i,t-1) * Bij(i,j,t,gn)) - KTxR(j,t) *$$

$$* \sum_{g=1}^{5}(Rintj(i,t-1) * \frac{p(i,t)}{p(i,t-1)} - 1) * p(i,t-1) * Bij(i,j,t,gn)) +$$

$$+ Rret(j,t) * pInv(j,t-1) * K(j,t,gn) - KTxR(j,t) * Rrent(j,t) - 1) *$$

$$* pInv(j,t-1) * K(j,t,gn) + Inh(j,t,gn) - Beq(j,t,gn)$$

Bequests:

From utility maximization, bequest is modeled as a certain proportion of our consumption and the rate of contribution into the bequest fund expressed by an exogenously determined variable: $Beqr(j,g)$ is the bequest rate.

(5) $$Beq(j,t,g) = Beqr(j,g) * Pcon(j,t) * Con(j,t,g)$$

Inheritance:

Inheritance received by the population of each region depends on how much of our population had been retired and the amount of money that is left as a bequest for younger generations multiplied by exogenously determined coefficient $InhR(j, g)$.

(6) $$Pop(j,t,g)*Inh(j,t,g) = InhR(j,g)*\sum_{g=1}^{5}(Pop(j,t,gm)*Beq(j,t,gm))$$

Consumption:

The next equation is the inter-temporal first order condition derived from the consumer's problem:

(7) $$\frac{Con(j,t+1,g+1)}{Con(j,t,g)} = \left(\frac{((1+(1-KTxR(j,t))*(Rret(j,t)-1))*Pcon(j,t)}{((1+DiscR(j))*Pcon(j,t+1))}\right)^{\delta(j)}$$

To interpret the equation (7) we will combine the numerator and denominator of the RHS into r and ρ respectively. So we have :

$r = (1+(1-KTxR(j,t))*(Rret(j,t)-1))$ is the interest rate in the economy,

$\rho = 1 + DiscR(j)$ is the rate of time preference (impatience coefficient),

and $\delta(j)$ is the inter-temporal elasticity of substitution.

This equation states that for the generation born at time t, future consumption is greater then current consumption when the interest rate is higher than the rate of time preference. How much higher depends on the inter-temporal elasticity of substitution of future and current consumption.

As there is no technological progress in the model, the steady state condition for the consumption (7a) states that consumption of generation g will be the same in $t+1$ and t time periods. The consumption of each generation in steady state is constant.

(7a) $$Con(j,t+1,g) = Con(j,t,g)$$

In a second step of the optimization problem, households have to allocate their consumption expenditure across the different goods in the economy. I assume that a CES function represents the inter-regional preferences of households. Accordingly, the inter-regional first order conditions stipulate that a good produced in region i consumed by an individual living in region j of age g ($ConI(i,j,g)$) is determined by the following expression:

(8) $$ConI(i,j,t,g) = \gamma_{(i,j)} * (\frac{Pcon(j,t)}{P(i,t)})^{\delta con(j)} * con(j,t,g)$$

where $\gamma(i,j)$ is a preference coefficient.

The price of consumption ($PCon$) is a non-linear weighted average of local prices:

(9) $$Pcon(j,t)^{(1-\delta con(j))} = \sum_{i=1}^{j}(\gamma_{(i,j)} * P(i.t)^{(1-\delta con(j))})$$

Price of consumption (9) in the region, for example 1, positively depends on the exogenous coefficient $\gamma(i,j)$ of consumption preference or the allocation between region 1 and 2.

Total demand in the region j for the products from region i equals consumption level of households in j of the goods from i multiplied by the total population of

j:

10) $$ECon(i,j,t) = \sum_{g=1}^{5}(Pop(j,t,g) * ConI(i,j,t,g))$$

Demand of region j for bonds issued by the government i depends on the number of bonds purchased in j from i over total holdings of bonds issued domestically and externally. We multiply this ratio by the amount of savings remaining after spending on physical capital stock:

11) $$\frac{Bij0(i,j)}{\sum_{i=1}^{j}(Bij0(i,j))} * (Lend(j,t+1,g+1) - Pinv(j,t) * K(j,t+1,g+1)) = \\ = P(i,t) * Bij(i,j,t+1,g+1)$$

We assume that physical capital can be purchased or sold only within the economy of one region (no mobility of physical capital). So the demand for physical capital in time $t+1$ of generation $g+1$ positively depends on the product of the total capital stock available in the economy and total savings of households of the same economy and depends negatively on the sum of the product between the total population and part of savings each household decides to spend to these investments:

12) $$K(j,t+1,g+1) = \frac{(Kstock(j,t+1) * Lend(j,t+1,g+1))}{\sum_{g=1}^{5}(Pop(j,t+1,g+1) * Lend(j,t+1,g+1))}$$

The rate of return on capital positively depends on the present price of the capital multiplied by the value of depreciation rate and negatively relates to the initial or actual price of that capital:

13) $Rret(j,t) = \frac{(rent(j,t)+(1-DepR(j))*Pinv(j,t))}{Pinv(j,t-1)}$

Equation (14) states that bonds and capital are perfect substitutes, so expected returns on bonds equal expected return on capital shares:

14) $RintJ(j,t) * \frac{P(j,t+1)}{P(j,t)} = Rret(j,t+1)$

As the regional stock of physical capital is a composite of the two regional final goods, a CES function describes the composition of regional investment ($EInv(i,j,t)$), taking a form similar to the composition of regional consumption, that is:

15) $EInv(i,j,t) = \lambda(i,j) * (\frac{Pinv(j,t)}{P(i,t)})^{\delta inv(j)} Inv(j,t)$

where $\lambda(i,j)$ is a parameter of the CES investment technology and δInv is the corresponding elasticity of substitution. Accordingly, the price of aggregate investment, $Pinv$, is determined by an equation similar to equation (9) for the price of consumption:

(16) $Pinv(j,t)^{(1-\delta inv(j))} = \sum_{i=1}^{j}(\lambda(i,j) * P(i,t)^{(1-\delta inv(j))})$

The future capital stock in the economy (physical capital) equals the amount of investment into physical capital plus the value of capital stock left after deducting for the depreciation rate of capital.

(17) $Kstock(j,t+1) = Inv(j,t) + (1 - DepR(j)) * Kstock(j,t)$

In the steady state, investment in capital will depend on the real population growth rate plus the value of current capital left after reducing it by the rate of capital depreciation:

$$Inv(j,t) = ((NN(j,t) - 1) + DepR(j)) * Kstock(j,t) \qquad (18)$$

The budget of the government is formed by different sources of income. The biggest part of that income is due to the tax collections imposed on the economic agents. The budget constraint of the government consists of the following elements. The RHS is the income part: income from the amount of bonds sold by price $P(j,t)$ plus income from taxes imposed on wages, consumption, net income from interest on bonds, net income from investment into physical capital, and income from pension plans. The LHS consists of the government expenditures on health, education and other expenses plus payments of dividends to the households holding government issued bonds:

$$\begin{aligned} &P(j,t)*Bond(j,t+1)+\textstyle\sum_{g=1}^{5}(Pop(j,t,g)*(WTxR(j,t)*wage(j,t))* \\ &*EP(j,g)+ConTxR(j,t)*Pcon(j,t)*Con(j,t,g)+KTxR(j,t))* \\ &*\textstyle\sum_{i=1}^{j}(RintJ(i,t-1)*\frac{P(i,t)}{P(i,t-1)}-1)*P(i,t-1)*Bij(i,j,t,g))+ \\ &+KTxR(j,t)*(Rret(j,t)-1)*Pinv(j,t-1)*K(j,t,g) = P(j,t)* \\ &(Gov(j,t)+ \\ &+GovH(j,t)+GovE(j,t))+(RintJ(j,t-1)*\frac{P(j,t)}{P(j,t-1)})*P(j,t-1)* \\ &Bond(j,t) \end{aligned} \qquad (19)$$

In a steady state the value of government bonds grows at the constant population growth rate. Hence, in steady state, the government budget constraint becomes:

(20) $NN(j,t) * P(j,t) * Bond(j,t) + \sum_{g=1}^{5}(Pop(j,t,g) * (WTxR(j,t) * wage(j,t))*$

$$*EP(j,g) + ConTxR(j,t) * Pcon(j,t) * Con(j,t,g) + KTxR(j,t))*$$

$$* \sum_{i=1}^{j}(RintJ(i,t-1) * \frac{P(i,t)}{P(i,t-1)} - 1) * P(i,t-1) * Bij(i,j,t,g))+$$

$$+KTxR(j,t) * Rret(j,t) - 1) * Pinv(j,t-1) * K(j,t,g))) =$$

$$= P(j,t) * (Gov(j,t) + GovH(j,t) + GovE(j,t)) + (RintJ(j,t-1) * \frac{P(j,t)}{P(j,t-1)})*$$

$$*P(j,t-1) * Bond(j,t)$$

We assume in the simulation experiments that government will balance their budget every period by adjusting the wage tax rate. Consequently, the capital and the consumption tax rates will be exogenous, and future supply of bonds will grow in the same proportion as the rate of population growth:

(21) $Bond(j,t+1) = NNP(j,t) * Bond(j,t)$

Markets:

Total output of the economy is equal to sum of total consumption, total investment and total government spending :

(22) $Q(j,t) = \sum_{i=1}^{j}(ECon(j,i,t)+EInv(j,i,t))+(Gov(j,t)+GovH(j,t)+GovE(j,t))$

According to the interest rate parity theory, interest rates across regions must be the same as financial capital is assumed to be perfectly mobile across regions:

(23) $Rint(t) = RintJ(j,t) * \frac{P(j,t+1)}{P(j,t)}$

Another way of interpreting that equation is that financial markets are fully integrated. So total supply of bonds over the 2 regions must equal total demand.

(24) $\sum_j \sum_{g=1}^{5}(Pop(j,t+1,g+1) * Lend(j,t+1,g+1)) =$

$= \sum_j(P(j,t) * Bond(j,t+1) + Pinv(j,t) * Kstock(j,t+1))$

Data and calibration procedures

The values for the exogenous parameters differ from region to region or/and by generations. We summarize this information in the following table:

<table>
<tr><td>Parameters</td><td colspan="5">Uzbekistan</td><td colspan="5">The world</td></tr>
<tr><td>ALFA(J) production function parameter</td><td colspan="5">0.28</td><td colspan="5">0.28</td></tr>
<tr><td>B(J) net government debt</td><td colspan="5">0.0055</td><td colspan="5">0.0445</td></tr>
<tr><td>WTAX(J) wage-income tax rate</td><td colspan="5">0.300</td><td colspan="5">0.313</td></tr>
<tr><td>CTAX(J) consumption tax</td><td colspan="5">0.170</td><td colspan="5">0.200</td></tr>
<tr><td>NGR(J) population growth rate</td><td colspan="5">1.156</td><td colspan="5">1.156</td></tr>
<tr><td>FA(J) foreign assets</td><td colspan="5">0.0347</td><td colspan="5">-0.0347</td></tr>
<tr><td>GHEA(J) government spending on health</td><td colspan="5">0.00246</td><td colspan="5">0.0224</td></tr>
<tr><td>GEDU(J) government spending on education</td><td colspan="5">0.00147</td><td colspan="5">0.0134</td></tr>
<tr><td>Generations</td><td>G1</td><td>G2</td><td>G3</td><td>G4</td><td>G5</td><td>G1</td><td>G2</td><td>G3</td><td>G4</td><td>G5</td></tr>
<tr><td>NC1(J,G) population</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td><td>1</td></tr>
<tr><td>BEC(J,G) bequest parameters</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0.3</td><td>0</td><td>0</td><td>0</td><td>0</td><td>0.3</td></tr>
<tr><td>Generations</td><td colspan="2">G1</td><td colspan="2">G2</td><td colspan="2">G3</td><td colspan="2">G4</td><td colspan="2">G5</td></tr>
<tr><td>DH(G) distribution parameter of health care by age</td><td colspan="2">0.1096</td><td colspan="2">0.1213</td><td colspan="2">0.1213</td><td colspan="2">0.2043</td><td colspan="2">0.4249</td></tr>
<tr><td>DE(G) distribution parameter of education by age</td><td colspan="2">0.330</td><td colspan="2">0.360</td><td colspan="2">0.190</td><td colspan="2">0.10</td><td colspan="2">0.10</td></tr>
<tr><td>Parameters</td><td colspan="2">Uzbekistan</td><td colspan="2">The world</td><td colspan="2"></td><td colspan="2"></td><td colspan="2"></td></tr>
<tr><td>Generations</td><td colspan="2">G1</td><td colspan="2">G2</td><td colspan="2">G3</td><td colspan="2">G4</td><td colspan="2">G5</td></tr>
<tr><td>IDC(G) inheritance distribution</td><td colspan="2">1/4</td><td colspan="2">1/4</td><td colspan="2">1/4</td><td colspan="2">0</td><td colspan="2">0</td></tr>
<tr><td>EP9(G) human capital profile</td><td colspan="2">1.199</td><td colspan="2">1.322</td><td colspan="2">1.369</td><td colspan="2">0</td><td colspan="2">0</td></tr>
</table>

The data is an approximation, obtained from related online databases and other sources.

In order to find the general equilibrium of the economy, we solve the problems of the firms and consumers simultaneously, taking into account the changes in population and of governmental spending (on education, health and other administrative expenditures).

Mercenier J. and M., Merette (2002) developed the calibration procedure that I am using in this paper. It is carried out as follows. First, we determine the factor prices using the values obtained from the supply side of the economy. Second, using factor prices determined in the first step, we calculate consumers' preference parameters, and governments' income and expenditure, such that we have saving equal to investment spending. Third, we calibrate the distribution of education and health care by age group. We assume that education spending is sensitive to the size of young age groups, where health care is sensitive to the size of the old age group.

Section 4

Simulation analysis.

The simulation exercises assume that a public investment in education generates future benefits that will take the form of a more productive labor force. More precisely, the simulation experiment consists of a 10 percent increase in governmental expenditure on education in Uzbekistan, followed by the 10 percent increase in the efficiency of the labor force during the next three periods. This scenario is consistent with the methodology of Young (1995). I also would like to analyze the inter-temporal tradeoffs involved. Indeed, the investment in education during the first three periods imposes an immediate fiscal burden on the government, whereas the benefits are expected to occur after. On the horizontal axis I plot the time scale where each period equals 12 years. The public investments in education occur unexpectedly at period 5.

The following are the results of the simulation for the GDP in Uzbekistan:

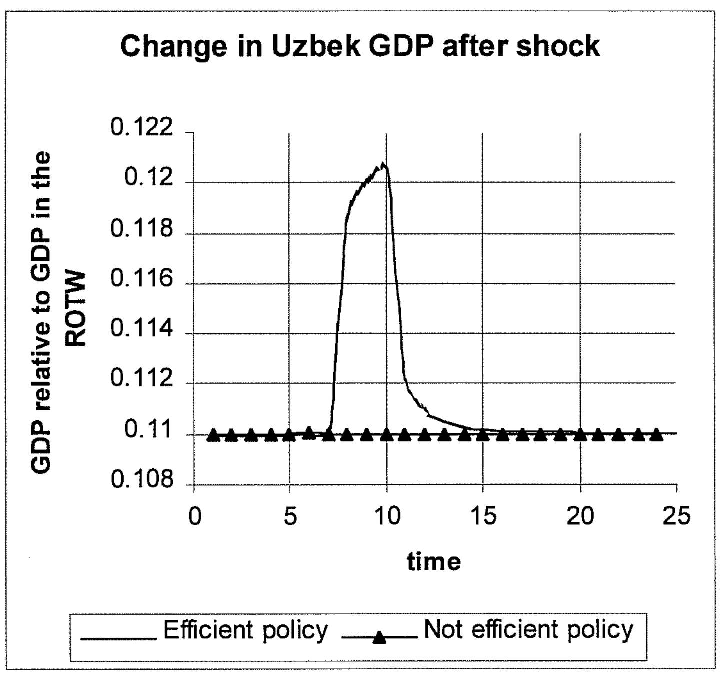

Chart 1

Chart 1 shows how the shock affects the GDP of the Uzbek economy. Our model is two-region model (Uzbekistan and The Rest-of-the World). Obviously, we assume that the GDP level for Uzbekistan would be smaller than that of the Rest-of-the-World (ROTW). On the graph above the level of Uzbekistan's GDP starts from 11, which could be interpreted as the 11 percent portion of model's total GDP.

As we can see from the Chart 1, there is a positive relationship between the government spending on education, efficiency of labor and GDP. The level of after-shock GDP is above the initial level. Right after the shock that occurred from the 5th period and lasted till the 11th, we can see that there is a tendency for GDP to increase, reaching the maximum value of 0.122. The rise in GDP during the 1st part is due to the increase in aggregate demand that results from the rise in government spending. It is followed by the positive effect that results from a more productive labor force. Note that the Uzbek economy goes back close to its initial steady state as we assume that government's investment in education is temporary.

Increase in government spending in Uzbek economy has a positive effect to the economy of the ROTW. Growth in the level of GDP could alternatively be interpreted as the increase in income per capita (as we do not have any demographic change). From theory we know that increase in income will increase overall demand for goods. This increase in demand relates not only to the domestically produced goods, but also to the imported goods. So increase in demand within one economy has positive effect to the economy of the second country In our case it is ROTW economy. The Chart 2 below shows the increase in GDP of the ROTW economy. The increase is proportionally smaller that for Uzbekistan, but it is still significant.

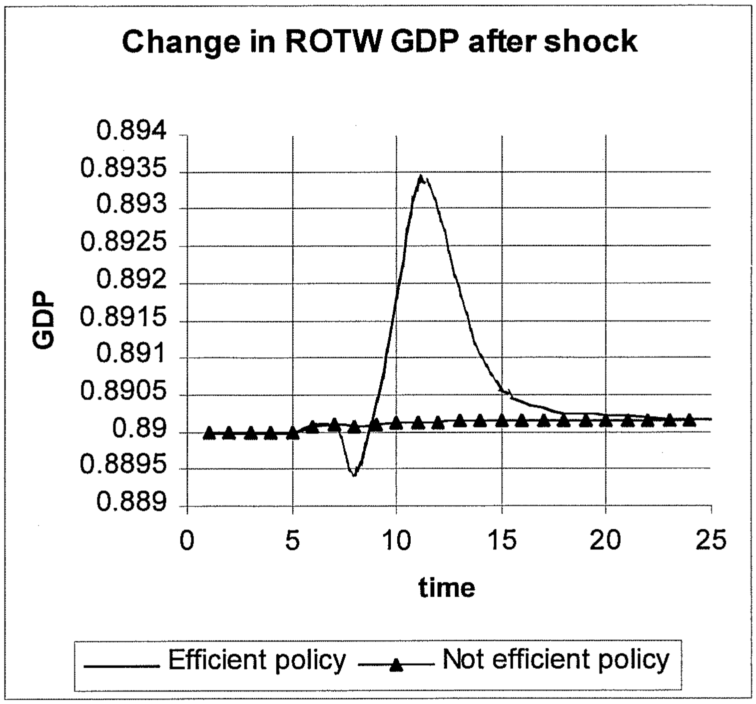

Chart 2

On the charts above one can see that depending on how efficiently government uses its investments, the effect of that investment can be totally different. The efficiency of investments means that the human capital policy of the government is directed towards improvements that are indeed effective. For example, instead of spending money on building 10 new schools in a small village with the population of 500 people, the government spends on innovating the existed schools by equipping them with new computers. The quality of students, will, most likely, increase as a result of computerization, rather than from building schools that, eventually, will be half empty. As an alternative scenario representing an inefficient policy, we simulate the government education investment but this time assuming that the investment is not followed by an improvement in the efficiency of labor. In the case of efficient investment policy, there are substantial changes in GDP level. Inefficient investment policy does not bring major change in GDP, as shown in Charts 1 and 2. This could imply that investment by themselves do not guarantee the success.

To show which region benefits more from those human capital investments, on the Chart 3, we compare (in percentage terms) the impacts of the shock on the GDP of Uzbek and on the ROTW economies for the efficient policy scenario. We can see that the economy in which the shock occurred (Uzbekistan) benefits from the investment. Note also that ROTW's GDP starts to increase when Uzbek economy is actually on its way back to initial values.

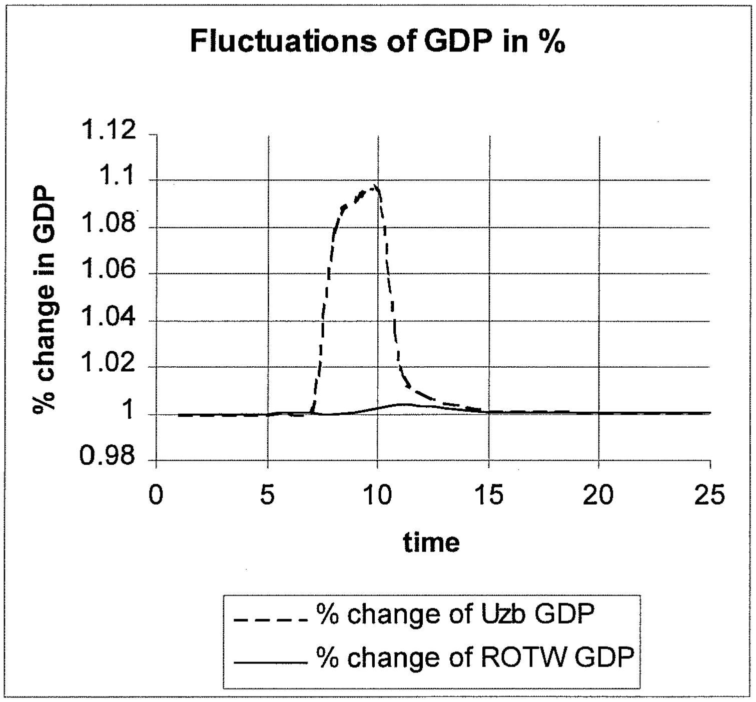

Chart 3.

This phenomenon could be explained by the lag in the price adjustment mechanism. Looking at the results of price adjustment (Chart 4), we can observe that output price in Uzbekistan starts to increase because of the increase of the aggregate demand that follows the increase in government spending. The productivity of labor increases. At a subsequent stage increase in aggregate supply pushes aggregate output up and output price down. After a while, the positive productivity shock starts to disappear. The shock I am imposing is temporal, and as a result output price increases back to its initial level. The citizens of Uzbekistan have then an incentive to consume more of the imported goods, as the domestic goods become relatively more expensive.

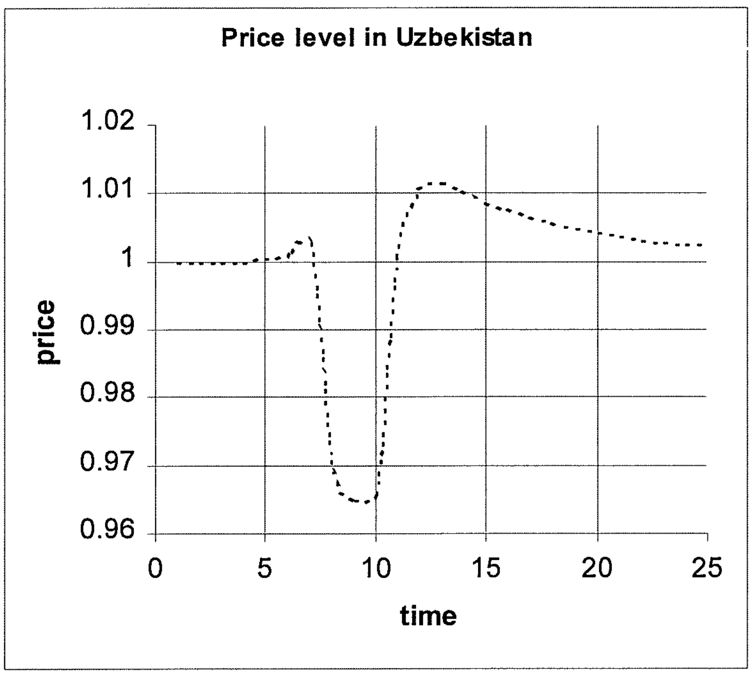

Chart 4.

The labor market is always in equilibrium in this model, so it is the nominal wage per unit of labor that absorbs the shock on the economy. As reported in Chart 5, we see that wage rate in Uzbekistan first increases then declines, following the sequential shocks scenario (positive on aggregate demand followed by a positive shock on aggregate supply). In contrast, wage rate rises temporarily in ROTW.

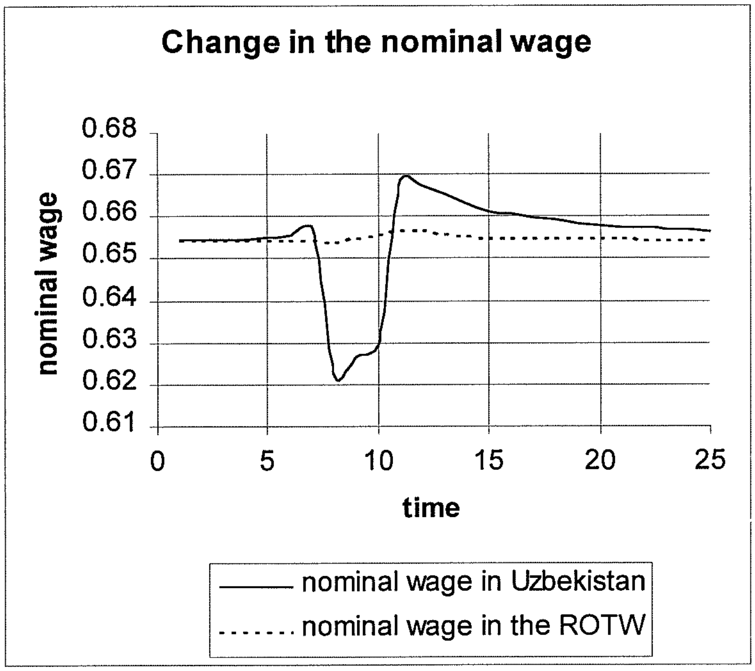

Chart 5.

The wage fluctuations that are shown on the next chart give us a better idea of the evolution of the labor earnings of different age groups. As one can see, the shock that had been imposed affects each age-group generation in equal proportion. However, this does not imply that each household will benefit from this policy equally. For instance, those members of the age group G4 in period 7 will benefit from the wage increase only one period. In contrast, members of age group G1 in period 7 will benefit from the wage increase for their entire working lives. Their successors, like their ancestors, would not be able to benefit as much.

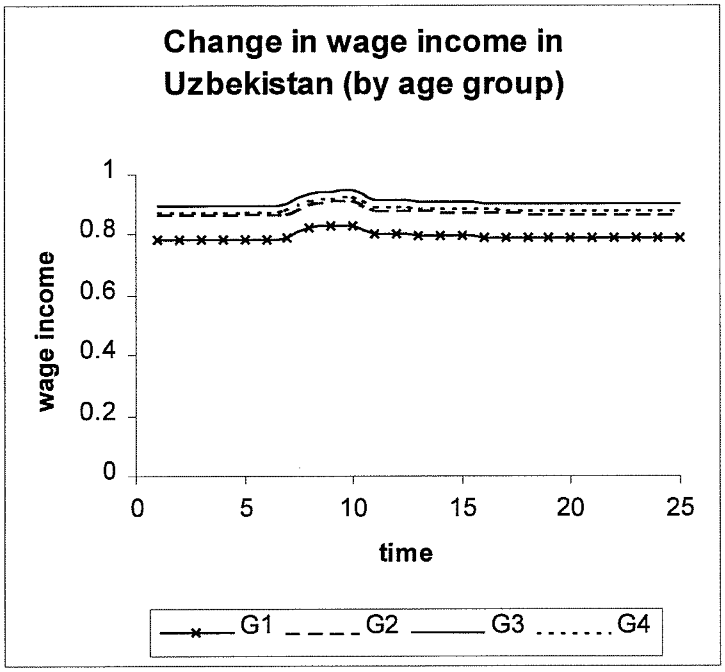

Chart 6.

The evolution of the wage income in the ROTW follows the same pattern as shown by Chart 7.As it is illustrated, the variations in the wage income are smaller in magnitude.

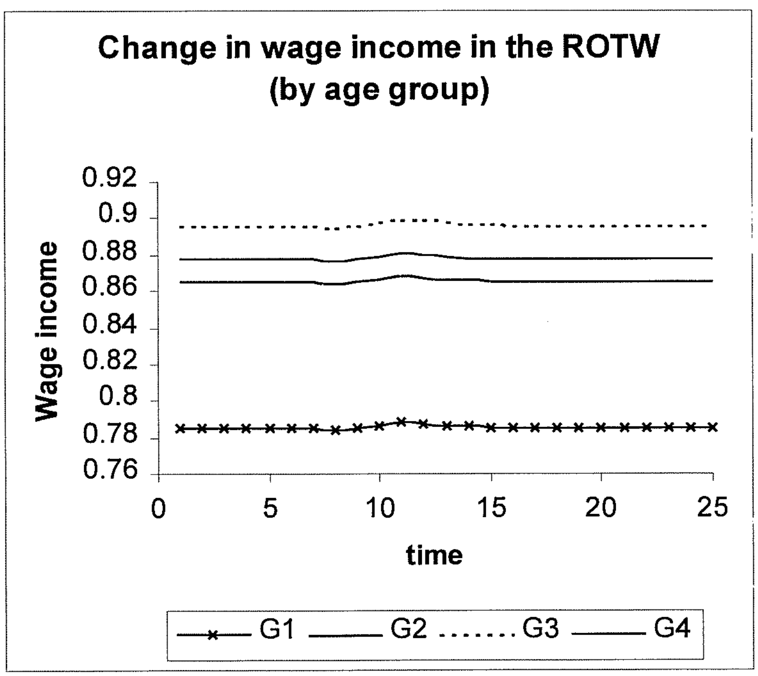

Chart 7.

Another important feature is the examination of the evolution of the wage tax rate. The Uzbek government finances its increase in education expenditure through adjustment of the taxation of the wage income. As illustrated in Chart 8, after a slight increase, the wage tax rate declines as a result of the enlargement of the labor tax base that follows the improvement of the productivity of the workers. The positive supply shock drives the output price level in Uzbekistan down. This makes Uzbek goods more attractive not only to domestic but also to foreign consumers. Exports increase and the expansion of the economy helps the government to balance its budget.

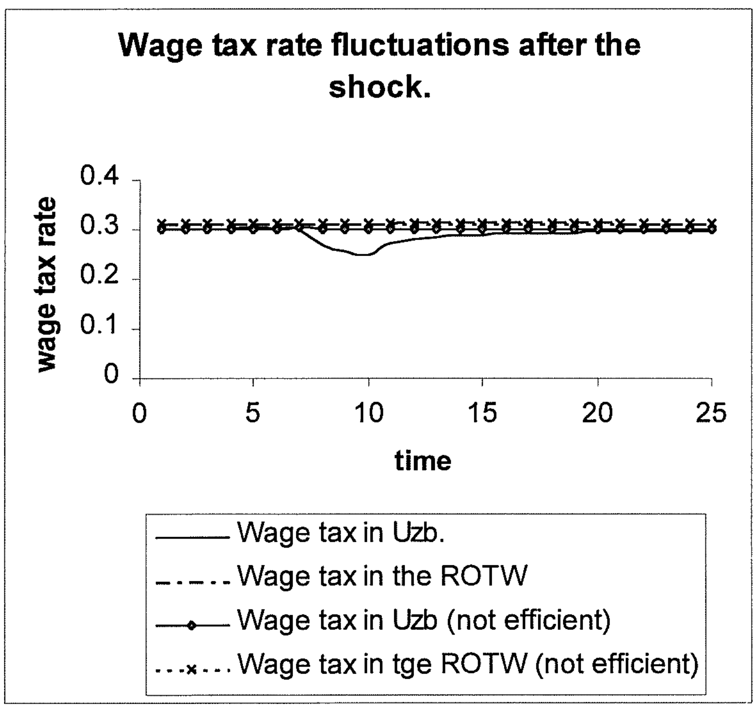

Chart 8.

In the case of inefficient use of investments, is no decline in the wage tax rate. Charts 9 and 10 confirm the above argument. In Chart 9 we see that the domestic and foreign demand for Uzbek goods increases. During the same period, demand for foreign goods by Uzbek consumers remains unchanged. Notice from Chart 8 that the wage tax rate does not go above the initial level in the long run. This implies that the burden of increase in government spending in education is covered by profits, earned during the boom.

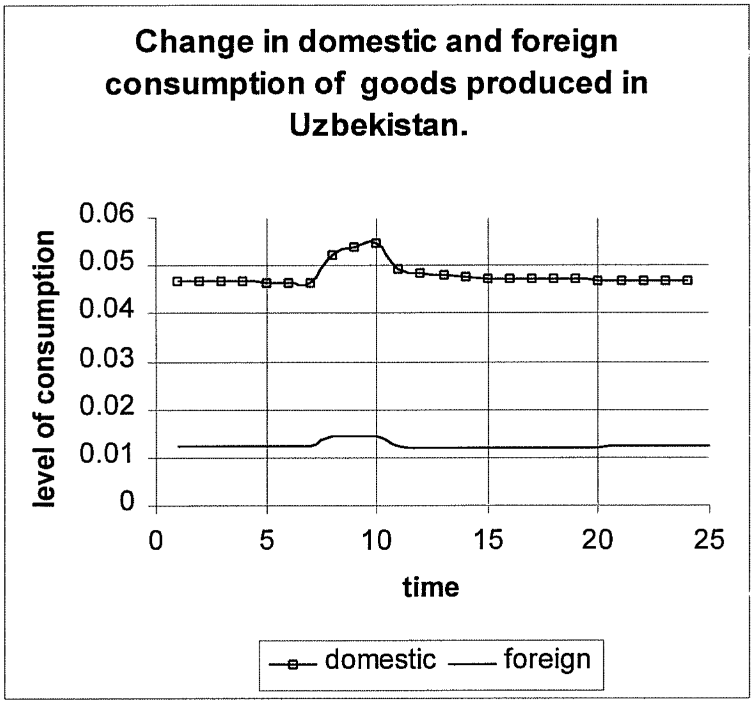

Chart 9.

We consider this as an important finding because it might encourage the governments of developing countries like Uzbekistan to invest in human capital with higher intensity. The results seem to be feasible or achievable, even though we induced some limitation into our model by not including some variables that could affect the outcome of the model. For example, we assume that there are no trade barriers between our two countries; there are no tariffs, no quotas and other restrictions that could negatively affect the demand of foreign consumers for Uzbek goods. We have free trade assumption, which reflects a goal of any intergovernmental negotiations more than the reality.

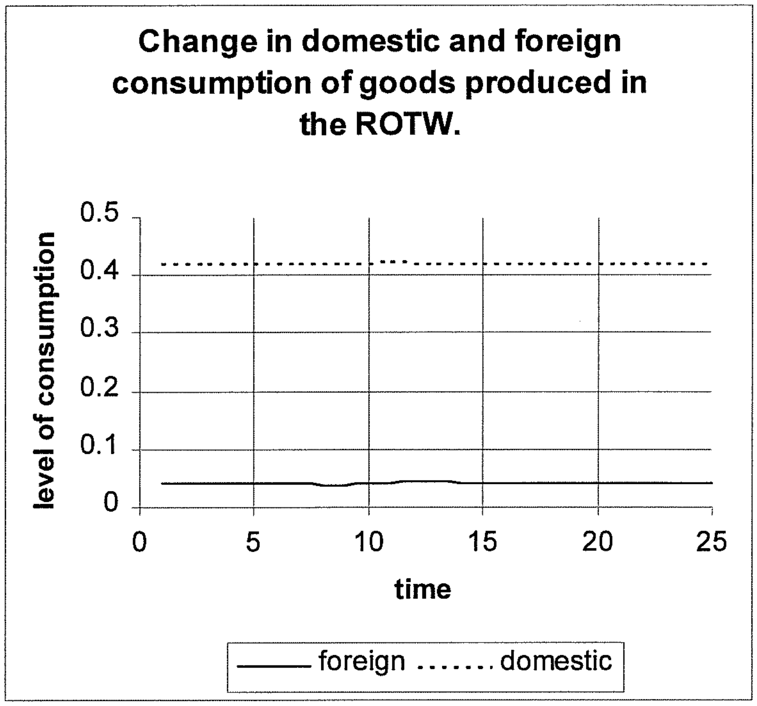

Chart 10.

Next we look at the effects on the investment market. Chart 11 gives us a picture of how the interest rate is affected by the sequential shocks.

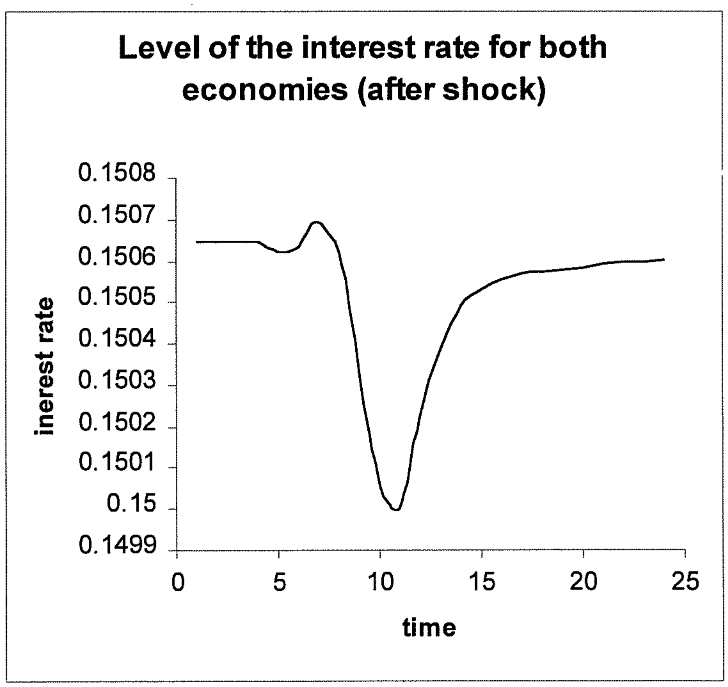

Chart 11.

The interest rate first increases as a result of the rise in the aggregate demand, and then declines as output expands. The evolution of the interest rate is common to both countries, as the capital market is assumed perfectly integrated. The overall effect on the interest rate is rather small as the sequential shocks occur in the smaller country.

There is a number of interest rates that differently affect the lending and borrowing decisions of economic agents. If there is a shock in one of the economies that pushes the interest rate up, foreign and domestic investors will try to invest their liquidities in the country with higher interest rate, or in other words, they will shift their activities to the economy with higher rate of return to their investments. On the other hand for the borrowers prefer a declining interest rate. For them interest rate is part of the cost of their investments. The interest rate that is used to analyze the effect on investments decisions is the average interest rate of the whole economy. As we can see from Chart 12 below, the rate of investments increases/decreases with decrease/increase of interest rate.

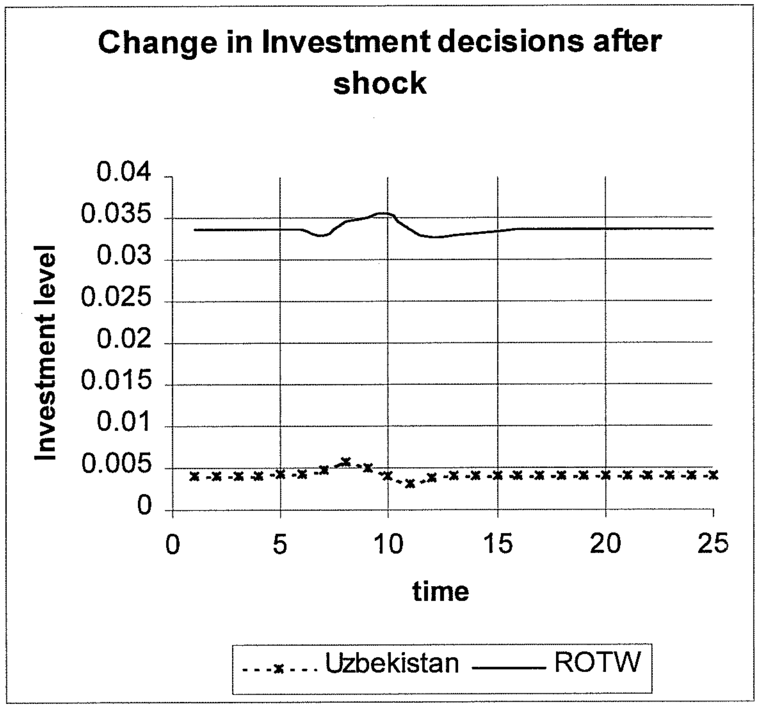

Chart 12.

The last variable to be interpreted in this paper is the Trade balance. As we mentioned before, we have two-country world model. Exports of one country are imports for the other. On the Chart 13, we can see that curves are the mirror reflection of each other, with persistently negative for the Uzbek balance. During the boom period, the deficit of Uzbek trade balance decreases, but in the long run it returns to its initial value as the rest of the real variables like GDP, Consumption, Investment, interest rate and others return to the steady state values.

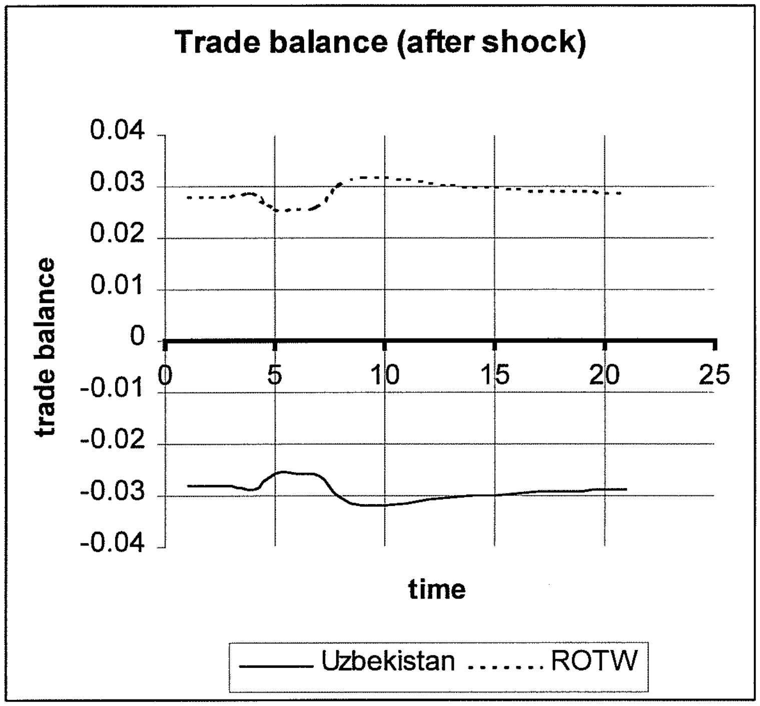
Trade balance (after shock)
0.04
0.03
0.02
0.01
0
-0.01
-0.02
-0.03
-0.04
trade balance
0
5
10
15
20
25
time
Uzbekistan
ROTW

Chart 13

Section 5.

Conclusion.

Nowadays the interest to the questions related to the topic of our paper is increasing rapidly for several reasons. First of all, these countries are experiencing intensive reduction of natural resources. In order to prolong its availability, they need specific means that could help to utilize materials with high efficiency, such us improvements in the production process. Hence, each particular government should invest in both technological innovations and human capital development. I suggest that development of human capital be deeper and more fundamental. Secondly, taking into account problems like ageing of the labor force in OECD countries, investment in human capital may increase the mobility of the labor force. For developing countries these investments could create additional sources of foreign currency from remittances by establishing contract employment of extra labor force.

In my paper, I use a two-region dynamic model. As a simulation exercise, I apply a positive government expenditure shock, which eventually causes an increase of efficiency of labor force. As a result, I found that there is positive relationship between government spending, consumption, wage income and investments. The effect on the country with a shock has greater magnitude, whereas in the second country the relationship between variables is the same, but with less impact.

In general, both regions benefit and I conclude that it pays for the governments to spend on education. Our results are consistent with the theory. However, I admit that our model has some limitations. To better reflect economic reality, we should introduce unemployment, inflation and border effects into the model that somehow may change my initial results.

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Annex

```


$OFFSYMXREF OFFSYMLIST OFFUELLIST OFFUELXREF
OPTION DECIMALS=6;


 SETS      G     GENERATIONS            / G1 * G5 /
         GI(G)    FIRST GENERATION
        GJ(G)    GENERATIONS WORKING  / G1 * G4 /
        GM(G)    RETIRED GENERATIONS  /  G5 /
        GN(G)    LAST GENERATION
;
 GN(G) = YES$(ORD(G) EQ CARD(G));
 GI(G) = YES$(ORD(G) EQ(1));


 SET J / Uzbekistan , Rest-of-the-world  /;

 ALIAS (I,J), (I,II), (J,JJ), (G,GG);

Scalars

 SIGC    inter-temporal rate of substitution
 RR      final interest rate
;

 SIGC = 1.1*2;
 RR = 0.3231*2.5 ;
*RR represents 7.25% rate per year

 PARAMETERS
 ALFA(J)     production function parameter
 B (J)       net government debt

 WW(J)       final wage rate
 WTAX(J)     wage-income tax rate
 KTAX(J)     capital tax
 CTAX(J)     consumption tax
 NC1(J,G)    population
 HAC(J,G)    financial assets accumulated
 BEC(J,G)    bequest parameter
 IDC(G)      inheritance distribution
 EP9(G)      human capital profile
 BR          benefit ratio
 NGR(J)      population growth rate (is determined in the demog.gms file when the
*             latter is used; otherwise is fixed at 1 or 1.156)
 FA(J)       foreign assets (must sum to zero)
 GHEA(J)
 GEDU(J)

;

  ALFA("Uzbekistan") = 0.280; ALFA("Rest-of-the-world") = 0.280;

  WTAX("Uzbekistan") = 0.300; WTAX("Rest-of-the-world") =0.313;
  KTAX("Uzbekistan") = 0.200; KTAX("Rest-of-the-world") = 0.562;
  CTAX("Uzbekistan") = 0.170; CTAX("Rest-of-the-world") = 0.200;
```

```
  BEC(J,G) = 0;  BEC(J,"G5")=.3;
  IDC(G) =1/4; IDC(GM)=0;
  EP9(G)  =1+.237*ORD(G)-.038*(ORD(G)**2);
  BR      =.3216*EP9("G3");
*  NC(J,G) = Pop(J,"T6",G) ;
  NGR(J) =1.156;
  FA("Uzbekistan") = .0347; FA("Rest-of-the-world") = -.0347;
*  KTAX(J) = 0;
*  BEC(J,G) = 0;
*  IDC(G)=0;
*  BR =0;
  NGR(J) =1.0;
*  FA(J) =0;
   GHEA("Uzbekistan")=.00246; GHEA("Rest-of-the-world")=.0224;
   GEDU("Uzbekistan")=.00147; GEDU("Rest-of-the-world")=.0134;

PARAMETER DH(g) distribution parameter of health care by age;
    DH(GI)=.1096; DH("G2")=.1213;DH("G3")=.1399;DH("G4")=.2043;DH("G5")=.4249;

PARAMETER DE(G) distribution parameter of health care by age;
    DE(GI)=.330; DE("G2")=.360;DE("G3")=.190;DE("G4")=.10;DE("G5")=.065;


 POSITIVE VARIABLES

 RC(J)      rental cost of capital
 LL(J)      labour supply
 KK(J)      capital stock
 YY(J)      output
 CC(J,G)    consumption level
 WC(J)      initial wage rate
 BEQC(J,G)  bequest
 INHC(J,G)  inheritance
 TT(J)      tax income
 AA(J)      scalar in the production function
 GEXP(J)    government expenditure
 NNJ(J)     population level adjustment (useful to normalize popul.
*           when demog.gms is used; otherwise fix to 1)
 NC(J,G)    population
 HEAC(J,G)
 EDUC(J,G)
 CTRC

Free variables

 DEP(J)        physical depreciation rare of capital
 DELTAC(J)     consumers rate of time preference
 SC(J,G)       private savings
 HA(J,G)       households assets
 DIC(J,G)      after tax income
 WALO(J)       walras variable
 OBJ

EQUATIONS

 EL1(J)
 EL2(J,G)
```

```

 E1(J)
 E2(J)
 E3(J)

 EC1(J,G)
 EC2(J,G)
 EC3(J,G)
 EC4(J,G)
 EC5(J,G)
 EC6(J,G)
 EC7(J,G)
 EC8(J,G)

 EE1(J)
 EE2(J)

 EDH1(J,G)
 EDH2(J)
 EDE1(J,G)
 EDE2(J)
 EG1(J)
 EG2(J)
 EG3

 OBJeq
;

*      POPULATION CALIBRATION

 EL1(J)..           LL(J)       =E= SUM(GJ, NNJ(J)*NC(J,GJ)*EP9(GJ) )
;
 EL2(J,G+1)..     NC(J,G+1) =E= NC(J,G) / NGR(J)
;

*      FIRM PROBLEM

 E1(J)..            WC(J) =E= (1-ALFA(J))*AA(J)*(KK(J)/LL(J))**ALFA(J)
;
 E2(J)..            RC(J) =E=    ALFA(J)  *AA(J)*(LL(J)/KK(J))**(1-ALFA(J))
;
 E3(J)..            YY(J) =E=   AA(J)* KK(J)**ALFA(J)* LL(J)**(1-ALFA(J))
;

*      CONSUMER PROBLEM

 EC1(J,G+1)..     CC(J,G+1) =E= ( (1+RR*(1-KTAX(J))) / (1+DELTAC(J)) )**SIGC *CC(J,G)
;
 EC2(J,GJ)..      DIC(J,GJ) =E= WW(J)*EP9(GJ)*(1-WTAX(J)-CTRC) + RR*HA(J,GJ)*(1-KTAX(J)) - CTAX(J)*CC(J,GJ)
;
 EC3(J,GM)..      DIC(J,GM) =E= WW(J)*BR*(1-WTAX(J))      + RR*HA(J,GM)*(1-KTAX(J)) - CTAX(J)*CC(J,GM)
;
 EC4(J,G)..       SC(J,G)    =E= DIC(J,G) + INHC(J,G) - CC(J,G) - BEQC(J,G)
;
```

```
 EC5(J,GN)..      SC(J,GN)   =E= -HA(J,GN)
;
 EC6(J,G+1)..     HA(J,G+1) =E= HA(J,G) + SC(J,G)
;
 EC7(J,G)..       BEQC(J,G) =E= BEC(J,G)*CC(J,G)
;
 EC8(J,G)..       INHC(J,G) =E= IDC(G)*SUM(GM, NC1(J,GM)*BEQC(J,GM) ) / NC1(J,G)
;

* AGGREGATION AND EQUILIBRIUM CONDITIONS

 EE1(J)..        KK(J)+ FA(J)    =E= SUM(G, NC1(J,G)*HA(J,G) ) - B(J)
;
 EE2(J)..        WALO(J) =E= YY(J) -(NGR(J)-1+DEP(J))*KK(J)-(GEXP(J)+GHEA(J)+GEDU(J))
          - SUM(G, NC1(J,G)*CC(J,G) )- RR* (B(J)+KK(J)-SUM(G,NC1(J,G)*HA(J,G)))

;

* GOVERNMENT

 EDH1(J,G).. HEAC(J,G+1) =e= (DH(G+1)/DH(G))*HEAC(J,G)
;
 EDH2(J)..     SUM(G,NC1(J,G)*HEAC(J,G)) =e= GHEA(J)
;
 EDE1(J,G).. EDUC(J,G+1) =e= (DE(G+1)/DE(G))*EDUC(J,G)
;
 EDE2(J)..     SUM(G,NC1(J,G)*EDUC(J,G)) =e= GEDU(J)
;

 EG1(J)..        TT(J) =E= SUM(GJ, NC1(J,GJ)*EP9(GJ)*WW(J)*WTAX(J) )+
                           SUM(GM, NC1(J,GM)*BR     *WW(J)*WTAX(J) )+
                           SUM(G,  NC1(J,G) *RR     *HA(J,G)*KTAX(J) )+
                           SUM(G,  NC1(J,G) *CC(J,G) *CTAX(J) )
;
 EG2(J)..       GEXP(J)+GHEA(J)+GEDU(J) =E= TT(J) - (RR-(NGR(J)-1))*B(J)

;
 EG3..         SUM((J,GM),NC1(J,GM)*BR*WW(J)) =E= CTRC*SUM(J,LL(J)*WW(J))
;
 OBJeq..         OBJ =E= 0
;

*  Restrictions

 HA.FX(J,GI)=  0.0;
 YY.FX("Uzbekistan")=0.110; YY.FX("Rest-of-the-world")=0.890;
 YY.FX(J) =YY.L(J);
 KK.FX(J) = YY.L(J)*0.3;
* KK.FX("Uzbekistan")=YY.L("Uzbekistan")*0.75; KK.FX("Rest of the world")=YY.L("Rest of the world")*0.9;
 LL.FX(J)=   YY.L(J)*1.1;
 B("Uzbekistan")=YY.L("Uzbekistan")*0.05; B("Rest-of-the-world")=YY.L("Rest-of-the-world")*0.05;
*Federal debt is assumed at 74% GDP and has been distributed in proportion to regional GDP
*This will permit to simulate the recent debt reduction policy
```

```
*  Initial guesses
 NNJ.FX(J)=1;
* NGR.l(J)   =  NNGR(J,"T6") ;
 NC.L(J,G)  = 1;
 RC.L(J) = .12;
 LOOP(GJ, HA.L(J,GJ+1) = HA.L(J,GJ)+.4);  HA.L(J,"G3") = .5*HA.L(J,"G2");
 LOOP(GM, HA.L(J,GM+1) =.5*HA.L(J,GM));
 DELTAC.L(J) = .11;
 WC.L(J)      = .7;
 CC.L(J,GI) = .1; LOOP(G,CC.L(J,G+1)= ((1+RR)/(1+DELTAC.L(J)))**SIGC*CC.L(J,G));
 BEQC.L(J,G) = 0; BEQC.L(J,GN)= .5*CC.L(J,GN);
 INHC.L(J,G) = IDC(G)*NC.L(J,"G5")*BEQC.L(J,"G5")/NC.L(J,G);
 DIC.L(J,GJ) =     WC.L(J)*(1-WTAX(J))+RR*HA.L(J,GJ)*(1-KTAX(J)) -
                   CC.L(J,GJ)*CTAX(J) + INHC.L(J,GJ);
 DIC.L(J,GM) = BR*WC.L(J)*(1-WTAX(J))+RR*HA.L(J,GM)*KTAX(J) - CC.L(J,GM)*CTAX(J)
             - BEQC.L(J,GM);
 SC.L(J,G)    = DIC.L(J,G)-CC.L(J,G);
 TT.L(J)       = 0.17;
 GEXP.L(J)     = 0.17;
 AA.L(J)       = 1.25;
 WAL0.FX(J)     = 0;
 CTRC.L        = .10;


*The model CA1 serves to determines demographic and production parameters and prices

 MODEL CA1 /
             E1,E2,E3,EL1,EL2, OBJeq /
;
 OPTIONS SOLPRINT=OFF, LIMCOL=0, LIMROW=0;
 SOLVE CA1 USING NLP MINIMIZING OBJ;

* THROW FACTOR PRICES DETERMINED BY CA1 INTO CONSUMER PROBLEM
* FIX POPULATION STRUCTURE AS DETERMINED BY CA1

 NC1(J,G) = NNJ.L(J)*NC.L(J,G);
 WW(J)      = WC.L(J);
 DEP.FX(J) = RC.L(J)-RR ;

 DISPLAY WW,NC1,DEP.L,AA.L,RC.L;

 MODEL CA2 /
             EC1,EC2,EC3,EC4,EC5,EC6,EC7,EC8,EE1,EG1,EG2,EG3,EE2
     OBJeq /
;

 MODEL CA3 / EDH1,EDH2,EDE1,EDE2,OBJeq/
;

  CA2.HOLDFIXED = 1;
 OPTIONS SOLPRINT=OFF, LIMCOL=0, LIMROW=0, DECIMALS=6;
 SOLVE CA2 USING NLP MINIMIZING OBJ;

 CA3.HOLDFIXED = 1;
 OPTIONS SOLPRINT=OFF, LIMCOL=0, LIMROW=0, DECIMALS=6;
```

```
SOLVE CA3 USING NLP MINIMIZING OBJ;

PARAMETER
DEF(J)         DEFICIT-GDP RATIO
TR(J)          TRANSFER TO THE OLD
BX(J)          BEQUEST CHECK
PSR(J)         PRIVATE SAVING RATE
NS(J)          NATIONAL SAVING RATE
SA(J,G)        SAVING BY AGE
CCI(I,J,G)     INTER-REGIONAL CONSUMPTION
INVI(I,J)      INTER-REGIONAL INVESTMENT
NX(J)          NET EXPORTS
WALOX(J)       GOODS MARKET BALANCE
;

DEF(J)    = (NGR(J)-1)*B(J);
TR(J)     = SUM(GM,NC1(J,GM))*WW(J)*BR;
BX(J)     = SUM(G,BEQC.L(J,G)*NC1(J,G))-SUM(G,INHC.L(J,G)*NC1(J,G));
PSR(J)    = (SUM(G,NC1(J,G)*SC.L(J,G))+DEP.L(J)*KK.L(J))/YY.L(J);
NS(J)     = (YY.L(J)-SUM(G,CC.L(J,G)*NC1(J,G))-(GEXP.L(J)+GHEA(J)+GEDU(J))
                          +RR*FA(J))/YY.L(J);
SA(J,G)= (SC.L(J,G)+BEQC.L(J,G))/(DIC.L(J,G)+INHC.L(J,G));

*SMALL MODEL TO CALIBRATE TRADE FLOWS MATRIX BETWEEN REGIONS

VARIABLE
E9(I,J)  TRADE FLOW MATRIX BETWEEN REGION I AND J
;

EQUATIONS
EXeq1(J)
EXeq2(J)

;

EXeq1(I)..
          SUM(J,E9(I,J)) =E= YY.L(I)
;
EXeq2(I)..
          SUM(J,E9(I,J)) - SUM (J,E9(J,I)) =E= -RR*FA(I)
;

MODEL EXMOD /EXeq1, EXeq2, OBJeq/;

*INITIALISATION

E9.L(I,J) = .1*( SUM(GG,NC1(J,GG)*CC.L(J,GG))+(NGR(J)-1+DEP.L(J))*KK.L(J) ) ;
*E9.LO(I,J) = .02*( SUM(GG,NC1(J,GG)*CC.L(J,GG))+(NGR(J)-1+DEP.L(J))*KK.L(J) ) ;
E9.L(J,J) = ( SUM(GG,NC1(J,GG)*CC.L(J,GG))+(NGR(J)-1+DEP.L(J))*KK.L(J) +
                 (GEXP.L(J)+GHEA(J)+GEDU(J)) - RR*FA(J) ) ;
E9.FX("Uzbekistan","Uzbekistan")=.878*E9.L("Uzbekistan","Uzbekistan");
E9.L("Rest-of-the-world","Rest-of-the-world")=.915*E9.L("Rest-of-the-world","Rest-of-the-world");
E9.L("Uzbekistan","Rest-of-the-world")=.0737*E9.L("Uzbekistan","Uzbekistan");

OPTION ITERLIM=1000;
SOLVE EXMOD MAXIMIZING OBJ USING NLP ;
```

```
DISPLAY E9.L;
E9.L(I,I)=E9.L(I,I)-(GEXP.L(I)+GHEA(I)+GEDU(I)); DISPLAY E9.L;
CCI(I,J,G) = E9.L(I,J)*CC.L(J,G)/
           ( SUM(GG,NC1(J,GG)*CC.L(J,GG))+(NGR(J)-1+DEP.L(J))*KK.L(J) );
INVI(I,J) = E9.L(I,J)*(NGR(J)-1+DEP.L(J))*KK.L(J)/
           ( SUM(GG,NC1(J,GG)*CC.L(J,GG))+(NGR(J)-1+DEP.L(J))*KK.L(J) );
E9.L(I,J) = SUM(G,NC1(J,G)*CCI(I,J,G))+INVI(I,J); DISPLAY E9.L;
E9.L(I,I)=E9.L(I,I)+(GEXP.L(I)+GHEA(I)+GEDU(I)); DISPLAY E9.L;

 NX(J)    = SUM((G,I), NC1(I,G)*CCI(J,I,G))+SUM(I,INVI(J,I)) -
            SUM((G,I), NC1(J,G)*CCI(I,J,G))-SUM(I,INVI(I,J))
;

 WALOX(J)= YY.L(J)-SUM((I,G), NC1(I,G)*CCI(J,I,G) ) -(GEXP.L(J)+GHEA(J)+GEDU(J))
              -  SUM(I, INVI(J,I) ) ;

DISPLAY DEF,TR,BX,PSR,NS,SA,NX,WALOX,CCI,DELTAC.L,EP9,CTRC.L ;
```

```
SET TTP       TOTAL TIME HORIZON                          /T1  * T30 /
    TP(TTP)   PERIODS OF PREVIOUSLY BORN                  /T1  * T4  /
    TP1(TTP)  PERIODS OF POST STEADY-STATE BIRTH          /T26 * T30 /

    T(TTP)     PERIODS OF ENDOGENOUS BIRTH                /T5 * T25 /
    TI(T)      FIRST PERIOD OF ENDOGENOUS BIRTH
    TN(T)      LAST TWO PERIODS OF ENDOGENOUS BIRTH       /T24 , T25 /

;
  TI(T) = YES$(ORD(T) EQ (1));

*+++++++++++
 PARAMETERS
*+++++++++++

 RkG(G)
*--->Producers J
 AlQ(J,TTP)
 AlK(J)
*--->Households J
 Sig(J)
 DiscR(J)
 AlConI(I,J)
 SigCon(J)
 BeqR(J,G)
 InhR(J,G)
 NN(J,TTP)
 TPOP(J,TTP)
 NNP(J,TTP)
 Pop(J,TTP,G)
 EP(J,TTP,G)
 BSh
 KSh(I)
 Bij0(I,J)
 DepR(J)
 AlInv(I,J)
 SigInv(J)
*--->Government J
 KTxR(J,TTP)
 ConTxR(J,TTP)
 PensR(J)
 ExoBonds(J,TTP)       Choose tax (1) vs Bond (0) financed deficits
 GovH(J,TTP)
 GovE(J,TTP)
;


*++++++++++
 VARIABLES
*++++++++++
*--->Producers J
 Ldem(J,TTP)
 Kdem(J,TTP)
 Q(J,TTP)
*--->Households J
```

```
 Lend(J,TTP,G)
 K(J,TTP,G)
 BondD(J,TTP,G)
 Bij(I,J,TTP,G)
 Con(J,TTP,G)
 Pcon(J,TTP)
 ConI(I,J,TTP,G)
 Beq(J,TTP,G)
 Inh(J,TTP,G)
 Pens(J,TTP,G)
 ECon(I,J,TTP)
 Investor J
 Inv(J,TTP)
 PInv(J,TTP)
 EInv(I,J,TTP)
 RRET(J,TTP)
*--->Government J
 Bond(J,TTP)
 Gov(J,TTP)
 WTxR(J,TTP)
 CTR(TTP)
*--->Markets
 Wage(J,TTP)
 Kstock(J,TTP)
 Rent(J,TTP)
 P(J,TTP)
 Rint(TTP)
 RintJ(J,TTP)

 EXC(J,TTP)  EXCESS WALRAS VARIABLE FOR THE CASE IN WHICH POPUL. GROWTH IS
*            DIFFERENT ACROSS REGION AT THE INITIAL EQUILIBRIUM
;
 RkG(G) = ORD(G);

*++++++++++
 EQUATIONS
*++++++++++
*--->Producers J
 WageEq(J,TTP)
 RentEq(J,TTP)
 QEq(J,TTP)
*--->Household J
 HBudgEq1(J,TTP,G)
 HBudgEq2(J,TTP,G)
 BeqEq(J,TTP,G)
 InhEq(J,TTP,G)
 PensEq(J,TTP,G)
 ConEq(J,TTP,G)
 ConSSEq(J,TTP,G)
 PConEq(J,TTP)
 ConIEq(I,J,TTP,G)
 EConEq(I,J,TTP)
 BijEQ(I,J,TTP,G)
 KEq(J,TTP,G)
 RRetKeq(J,TTP)
 InvEq(J,TTP)
```

```
 PInvEq(J,TTP)
 EInvEq(I,J,TTP)
 KstockEq(J,TTP)
 KstockSSEq(J,TTP)
*--->Government J
 GBudgEq1(J,TTP)
 GBudgSSEq(J,TTP)
 GBudgEq2(J,TTP)
 GBudgEq3(J,TTP)
 GPENS(TTP)
*--->Markets
 PEq(J,TTP)
 RintEq(J,TTP)
 RintJIntEq(TTP)
;

*+++++++++
* MODEL
*+++++++++

*=============
* Producers J
*=============

* Labor
 WageEq(J,TTP)      $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
 Wage(J,TTP)/P(J,TTP) =E= (1-AlK(J))*AlQ(J,TTP)*(Kstock(J,TTP)/SUM(G,Pop(J,TTP,G)*EP(J,TTP,G)))**AlK(J)
;
* Capital
 RentEq(J,TTP)      $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
 Rent(J,TTP)/P(J,TTP) =E= AlK(J)*AlQ(J,TTP)*(Kstock(J,TTP)/SUM(G,Pop(J,TTP,G)*EP(J,TTP,G)))**(AlK(J)-1)
;
* Output
 QEq(J,TTP)         $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
 Q(J,TTP) =E= AlQ(J,TTP)*Kstock(J,TTP)**AlK(J)*SUM(G,Pop(J,TTP,G)*EP(J,TTP,G))**(1-AlK(J))
;
*=============
* Household J
*=============

* Budget constraint
 HBudgEq1(J,TTP+1,G+1)  $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
 (1+ConTxR(J,TTP))*Pcon(J,TTP)*Con(J,TTP,G)+Lend(J,TTP+1,G+1)
 =E=
   (1-WTxR(J,TTP)-CTR(TTP))*Wage(J,TTP)*EP(J,TTP,G) + (
                  SUM(I,  (RintJ(I,TTP-1)*P(I,TTP)/P(I,TTP-1)   )*P(I,TTP-1)*Bij(I,J,TTP,G) )
  - KTxR(J,TTP)*SUM(I,  (RintJ(I,TTP-1)*P(I,TTP)/P(I,TTP-1)-1)*P(I,TTP-1)*Bij(I,J,TTP,G) )
  +                RRET(J,TTP)    *PInv(J,TTP-1)*K(J,TTP,G)
  - KTxR(J,TTP)*(RRET(J,TTP)-1)*PInv(J,TTP-1)*K(J,TTP,G)
                                               )$(ORD(G) GT 1)
 +
```

```
  (1-WTxR(J,TTP))*Pens(J,TTP,G)+Inh(J,TTP,G)-Beq(J,TTP,G)
;
 HBudgEq2(J,TTP,GN) $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..

 (1+ConTxR(J,TTP))*Pcon(J,TTP)*Con(J,TTP,GN)
 =E=
   (1-WTxR(J,TTP)-CTR(TTP))*Wage(J,TTP)*EP(J,TTP,GN) +
                   SUM(I,  (RintJ(I,TTP-1)*P(I,TTP)/P(I,TTP-1)   )* P(I,TTP-1)*Bij(I,J,TTP,GN) )
 - KTxR(J,TTP)*SUM(I,  (RintJ(I,TTP-1)*P(I,TTP)/P(I,TTP-1)-1)* P(I,TTP-1)*Bij(I,J,TTP,GN) )
 +                   RRET(J,TTP)   *PInv(J,TTP-1)*K(J,TTP,GN)
 - KTxR(J,TTP)*(RRET(J,TTP)-1)*PInv(J,TTP-1)*K(J,TTP,GN)
 +
  (1-WTxR(J,TTP))*Pens(J,TTP,GN)+Inh(J,TTP,GN)-Beq(J,TTP,GN)
;
* Bequests
 BeqEq(J,TTP,G)          $(( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+
                             CARD(T) ) AND BeqR(J,G) NE 0)..
 Beq(J,TTP,G) =E= BeqR(J,G)*Pcon(J,TTP)*Con(J,TTP,G)
;
* Inheritance
 InhEq(J,TTP,G)          $(( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+
                             CARD(T) ) AND InhR(J,G) NE 0)..
 Pop(J,TTP,G)*Inh(J,TTP,G) =E= InhR(J,G)*SUM(GM,Pop(J,TTP,GM)*Beq(J,TTP,GM))
;
* Pensions
 PensEq(J,TTP,GM)        $(( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+
                             CARD(T) ) AND PensR(J) NE 0)..
 Pens(J,TTP,GM) =E= PensR(J)*SUM(GJ, Wage(J,TTP-(ORD(GM)-1+ORD(GJ)))  )/CARD(GJ)
;
* Consumption
 ConEq(J,TTP+1,G+1) $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
 Con(J,TTP+1,G+1)/Con(J,TTP,G) =E=
 (
 ( ( 1 + (1 - KTxR(J,TTP)) * (RRET(J,TTP)-1) ) * PCon(J,TTP)    ) /
 ( ( 1 + DiscR(J)                              ) * PCon(J,TTP+1) )
 )**Sig(J)
;
 ConSSEq(J,TTP+1,G)    $( ORD(TTP) EQ CARD(TP)+CARD(T)-1 AND ORD(G) LT CARD(G))..
 Con(J,TTP+1,G) =E= Con(J,TTP,G)
;
* Price of consumption
 PConEq(J,TTP)       $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
 PCon(J,TTP)**(1-SigCon(J)) =E= SUM(I$(AlConI(I,J) GT 1.E-13),
                                AlConI(I,J)*P(I,TTP)**(1-SigCon(J)))
;
* Composition of consumption (countries I of origin)
 ConIEq(I,J,TTP,G)     $( (ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T))
                         AND AlConI(I,J) GT 1.E-13)..
 ConI(I,J,TTP,G) =E= AlConI(I,J)*(PCon(J,TTP)/P(I,TTP))**SigCon(J)*Con(J,TTP,G)
;
 EConEq(I,J,TTP)     $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
 ECon(I,J,TTP) =E= SUM(G,Pop(J,TTP,G)*ConI(I,J,TTP,G))
;
* Holding of bonds
 BijEq(I,J,TTP+1,G+1) $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
```

```
 P(I,TTP)*Bij(I,J,TTP+1,G+1) =E= Bij0(I,J)/SUM(II,Bij0(II,J))*
                                (Lend(J,TTP+1,G+1)-PInv(J,TTP)*K(J,TTP+1,G+1))
;
* Holding of physical capital
 KEq(J,TTP+1,G+1)         $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT
                             CARD(TP)+CARD(T) )..
 K(J,TTP+1,G+1) =E= ( Kstock(J,TTP+1)*        Lend(J,TTP+1,G+1))/
                      SUM(GG,Pop(J,TTP+1,GG+1)*Lend(J,TTP+1,GG+1))
;
* Rate of return on capital
 RRETKeq(J,TTP)      $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
 RRET(J,TTP) =E= (Rent(J,TTP)+(1-DepR(J))*PInv(J,TTP))/PInv(J,TTP-1)
;
* Investment
 InvEq(J,TTP+1)      $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
 RintJ(J,TTP)*P(J,TTP+1)/P(J,TTP) =E= RRET(J,TTP+1)
;
* Price of investment
 PInvEq(J,TTP)      $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
 PInv(J,TTP)**(1-SigInv(J)) =E= SUM(I$(AlInv(I,J) GT 1.E-13),AlInv(I,J)*P(I,TTP)*
*(1-SigInv(J)))
;
* Composition of investment (countries I of origin)
 EInvEq(I,J,TTP)  $( (ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T)) AND
                      AlInv(I,J) GT 1.E-13 )..
 EInv(I,J,TTP) =E= AlInv(I,J)*(PInv(J,TTP)/P(I,TTP))**SigInv(J)*Inv(J,TTP)
;
 KstockEq(J,TTP+1)  $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
 Kstock(J,TTP+1) =E= Inv(J,TTP)+(1-DepR(J))*Kstock(J,TTP)
;
 KstockSSEq(J,TTP)      $( ORD(TTP) EQ CARD(TP)+CARD(T) )..
 Inv(J,TTP) =E=  ((NN(J,TTP)-1)+DepR(J))*Kstock(J,TTP)
;
*==============
* Government J
*==============

* Budget constraint
 GBudgEq1(J,TTP+1) $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) ) ..
 P(J,TTP)*Bond(J,TTP+1) + SUM(G,POP(J,TTP,G)*(
     WTxR(J,TTP)*Wage(J,TTP)*EP(J,TTP,G)
  +  WTxR(J,TTP)*Pens(J,TTP,G) + ConTxR(J,TTP)*Pcon(J,TTP)*Con(J,TTP,G)
  + KTxR(J,TTP)*SUM(I, (RintJ(I,TTP-1)*P(I,TTP)/P(I,TTP-1)-1)*P(I,TTP-1)*Bij(I,J,
TTP,G) )
  + KTxR(J,TTP)*(RRET(J,TTP)-1)*PInv(J,TTP-1)*K(J,TTP,G)
                                                  ))
 =E=
 P(J,TTP)*( Gov(J,TTP) + GovH(J,TTP) + GovE(J,TTP) )
  +(RintJ(J,TTP-1)*P(J,TTP)/P(J,TTP-1))*P(J,TTP-1)*Bond(J,TTP)
;
 GBudgSSEq(J,TTP)        $( ORD(TTP) EQ CARD(TP)+CARD(T) ) ..
  NN(J,TTP)*P(J,TTP)*Bond(J,TTP)   + SUM(G,POP(J,TTP,G)*(
    WTxR(J,TTP)*Wage(J,TTP)*EP(J,TTP,G)
  + WTxR(J,TTP)*Pens(J,TTP,G) + ConTxR(J,TTP)*Pcon(J,TTP)*Con(J,TTP,G)
  + KTxR(J,TTP)*SUM(I, (RintJ(I,TTP-1)*P(I,TTP)/P(I,TTP-1)-1)*P(I,TTP-1)*Bij(I,J,
TTP,G) )
  + KTxR(J,TTP)*(RRET(J,TTP)-1)*PInv(J,TTP-1)*K(J,TTP,G)
```

```
))
 =E=
 P(J,TTP)*( Gov(J,TTP) + GovH(J,TTP) + GovE(J,TTP) )
 +(RintJ(J,TTP-1)*
                          P(J,TTP)/P(J,TTP-1))*P(J,TTP-1)*Bond(J,TTP)
;
 GBudgEq2(J,TTP+1)       $( (ORD(TTP) GT CARD(TP) AND ORD(TTP) LT
                           CARD(TP)+CARD(T)) AND ExoBonds(J,TTP+1) EQ 1 )..
 Bond(J,TTP+1) =E= NNP(J,TTP)*Bond(J,TTP)
;
 GBudgEq3(J,TTP)         $( (ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+
                            CARD(T)) AND ExoBonds(J,TTP+1) EQ 0 )..
 WTxR(J,TTP) =E= WTxR(J,TTP-1)
;
 GPENS(TTP)    $ ( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+
                            CARD(T))..
 SUM((J,GM),Pop(J,TTP,GM)*Pens(J,TTP,GM)) =E=
                          CTR(TTP)*SUM((J,G),Pop(J,TTP,G)*EP(J,TTP,G)*Wage(J,TTP))
;
*=========
* Markets
*=========
* Goods
 PEq(J,TTP)         $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+CARD(T) )..
 Q(J,TTP) =E= SUM(I, ECon(J,I,TTP) + EInv(J,I,TTP) ) +
        ( Gov(J,TTP) + GovH(J,TTP) + GovE(J,TTP) ) + EXC(J,TTP)
;

* Integrated asset markets
 RintEq(J,TTP+1)     $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
 Rint(TTP) =E= RintJ(J,TTP)*P(J,TTP+1)/P(J,TTP)
;
 RintJIntEq(TTP+1)   $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T) )..
 SUM((J,G),Pop(J,TTP+1,G+1)*LEND(J,TTP+1,G+1)) =E=
 SUM(J,P(J,TTP)*Bond(J,TTP+1)+PInv(J,TTP)*Kstock(J,TTP+1))
;

 MODEL OLGMultiR /

*---> Producers J
 WageEq, RentEq, QEq
*--->Household J
 HBudgEq1, HBudgEq2,
 BeqEq, InhEq, PensEq
 ConEq,
 ConSSEq,
 PConEq,
 ConIEq,
 EConEq
 BijEQ
  KEq,
  RRetKeq, InvEq,PInvEq,EInvEq
  KstockEq,
  KstockSSEq
*--->Government J
```

```
  GBudgEq1, GBudgSSEq,GBudgEq2,GBudgEq3,GPENS
*--->Markets
  PEq,
  RintEq
* RintJIntEq
 OBJEQ          /
;
 OLGMultiR.HOLDFIXED = 1;
 OLGMultiR.OPTFILE   = 1;

 FILE MAR /Margins.chk/;

*++++++++++++++++++++++++++++++++++++++++++++++
* Initialisation (from Marcel's single country)
*++++++++++++++++++++++++++++++++++++++++++++++

*===Parameters
*--->Producers
 AlQ(J,TTP)    = AA.L(J);
 AlK(J)        = Alfa(J);
*--->Households J
 Sig(J)        = SIGC;
 DiscR(J)      = DELTAC.L(J);
* DiscR(J)      = DELTAC.L;
 SigCon(J)     = 4.5;
 BeqR(J,G)     = BEC(J,G);
 InhR(J,G)     = IDC(G);
*Population is assumed to be constant from period T1 to T6
 NN(J,TTP)     = NGR(J) ;
 Pop(J,"T5",G)= NC1(J,G);
 LOOP(TTP$(ORD(TTP) GT CARD(TP)) , Pop(J,TTP+1,G) = Pop(J,TTP,G)*NN(J,TTP) ) ;
 POP(J,TTP,G)$(ORD(TTP) LE CARD(TP))=POP(J,"T5",G);
 TPop(J,TTP)= SUM(G,Pop(J,TTP,G));
 LOOP(TTP, NNP(J,TTP) = TPop(J,TTP+1)/TPop(J,TTP)) ;
 DISPLAY TPop,NNP;
 EP9(GM) = 0;
 EP(J,TTP,G)        = EP9(G);
 DepR(J)       = DEP.L(J);
 SigInv(J)     = 4.5;
*--->Government J
 KTxR(J,TTP)      = KTAX(J);
 ConTxR(J,TTP)    = CTAX(J);
 PensR(J)         = BR;
 ExoBonds(J,TTP)  = 1;
 GovH(J,TTP)      = SUM(G,POP(J,TTP,G)*HEAC.L(J,G));
 GovE(J,TTP)      = SUM(G,POP(J,TTP,G)*EDUC.L(J,G));
 DISPLAY GovH, GovE;


*===Variables
*--->Producers
 Q.L(J,TTP)          = YY.L(J);
 LOOP(TTP $(ORD(TTP) GT CARD(TP) ),Q.L(J,TTP+1)=NNP(J,TTP)*Q.L(J,TTP));
*--->Households J
 Con.L(J,TTP,G)    = CC.L(J,G);
 Lend.L(J,TTP,G)   = HA.L(J,G);
```

```
 Pcon.L(J,TTP)     = 1;
 ConI.L(I,J,TTP,G)= 0;  ConI.L(I,J,TTP,G) = CCI(I,J,G);
 Beq.L(J,TTP,G)    = BEQC.L(J,G);
 Inh.L(J,TTP,G)    = INHC.L(J,G);
 Pens.L(J,TTP,GM) = WW(J)*BR;  Pens.FX(J,TTP,GJ) = 0;

 ECon.L(I,J,TTP)   = SUM(G,Pop(J,TTP,G)*ConI.L(I,J,TTP,G));
 Inv.L(J,TTP)      = ((NGR(J)-1)+DEP.L(J))*KK.L(J);
 LOOP(TTP $(ORD(TTP) GT CARD(TP) ),Inv.L(J,TTP+1)=NNP(J,TTP)*Inv.L(J,TTP));
 PInv.L(J,TTP)     = 1;

*--->Government J
 Bond.L(J,TTP)     = B(J);
 LOOP(TTP $(ORD(TTP) GT CARD(TP) ),Bond.L(J,TTP+1)=NNP(J,TTP)*Bond.L(J,TTP));
 Gov.FX(J,TTP)      = GEXP.L(J);
 LOOP(TTP $(ORD(TTP) GT CARD(TP) ),Gov.FX(J,TTP+1)=NNP(J,TTP)*Gov.L(J,TTP));
 WTxR.L(J,TTP)     = WTAX(J);
 EInv.L(I,J,TTP)= 0;        EInv.L(I,J,TTP) = INVI(I,J);
 LOOP(TTP $(ORD(TTP) GT CARD(TP) ),EInv.L(I,J,TTP+1)=NNP(J,TTP)*EInv.L(I,J,TTP));
 CTR.L(TTP) = CTRC.L;

*--->Markets
 Wage.L(J,TTP)     = WW(J);
 Kstock.L(J,TTP)  = KK.L(J);
 LOOP(TTP $(ORD(TTP) GT CARD(TP) ),Kstock.L(J,TTP+1)=NNP(J,TTP)*Kstock.L(J,TTP));
 Rent.L(J,TTP)     = RR+DEP.L(J);
 P.L(J,TTP)        = 1;
 Rint.L(TTP)       = 1+RR;
 K.L(J,TTP+1,G+1)$SUM(GG,Pop(J,TTP+1,GG+1)) = (Kstock.L(J,TTP+1)*
     Lend.L(J,TTP+1,G+1))/SUM(GG,Pop(J,TTP+1,GG+1)*Lend.L(J,TTP+1,GG+1)) ;
 BondD.fx(J,TTP,G) = 0;

*-------------> Choosing an (arbitrary) matrix of bilateral bond holding consistent with data
   PARAMETER Bij0(I,J);
   VARIABLES Bij0v(I,J);   EQUATIONS PortEQ1(I,T),PortEQ2(I,T),OBJPortEQ;
   Bij0v.L(I,I)= .8*B(I); Bij0v.L(I,J)$(ORD(I) NE ORD(J)) = (B(I)-Bij0v.L(I,I))/(CARD(J)-1);
   PortEQ1(I,TI)..            Bond.L(I,TI) =E= SUM(J, Bij0v(I,J) );
   PortEQ2(J,TI)..     SUM(I, Bij0v(I,J) ) =E= SUM(G,Pop(J,TI,G)*(Lend.L(J,TI,G)-PInv.L(J,TI)*K.L(J,TI,G)));
   OBJPortEQ..                          OBJ =E= SUM(I, (Bij0v(I,I)-.8*B(I))*(Bij0v(I,I)-.8*B(I)) );
   MODEL PORTF0 / PortEQ1, PortEQ2, OBJPortEQ / ;
   PORTF0.OPTFILE   = 1;
*   Bij0v.LO(I,J) = .02*B(J);Bij0v.UP(I,J) = .99*B(J);
   OPTION NLP=MINOS5;
   OPTION SOLPRINT=ON, LIMROW=0, LIMCOL=0, ITERLIM=5000;
   SOLVE PORTF0 MINIMIZING OBJ USING NLP;
   Bij0(I,J)              = Bij0v.L(I,J);
   Bij.L(I,J,TTP+1,G+1) = Bij0(I,J)/SUM(II,Bij0(II,J))* (Lend.L(J,TTP+1,G+1)-PInv.L(J,TTP)*K.L(J,TTP+1,G+1));
   Bij.L(I,J,TTP+1,G+1) = Bij.L(I,J,TTP+1,G+1)/P.L(I,TTP);
DISPLAY Bij.L, Bij0;
*-------------<
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 Lend.FX(J,TTP,GI)  = 0;
 K.FX(J,TTP,GI)     = 0;
 Bij.FX(I,J,TTP,GI) = 0;

*===OTHER PARAMETERS
  AlConI(I,J)   = SUM((TI,GI), ConI.L(I,J,TI,GI)/Con.L(J,TI,GI) );
  AlInv(I,J)    = SUM(TI, EInv.L(I,J,TI)/Inv.L(J,TI) );


 LOOP(TTP $(ORD(TTP) GE CARD(TP)),
  RRET.L(I,TTP) = (Rent.L(I,TTP)+(1-DepR(I))*PInv.L(I,TTP))/PInv.L(I,TTP-1) ;
     );

 Lend.FX(J,TTP,G) $(ORD(TTP) LE CARD(TP)+1) = Lend.L(J,TTP,G);
 K.FX(J,TTP,G)     $(ORD(TTP) LE CARD(TP)+1) = K.L(J,TTP,G);
 Bij.FX(I,J,TTP,G)$(ORD(TTP) LE CARD(TP)+1) = Bij.L(I,J,TTP,G);
 Beq.FX(J,TTP,G)  $(BeqR(J,G) EQ 0) = 0;
 Inh.FX(J,TTP,G)  $(InhR(J,G) EQ 0) = 0;
 Pens.FX(J,TTP,G) $(PensR(J)  EQ 0) = 0;

* Gov.FX(J,TTP) = Gov.L(J,TTP);
 BOND.FX(J,TTP)$(ORD(TTP) LE CARD(TP)+1) = BOND.L(J,TTP);
 WTxR.FX(J,TTP)$(ORD(TTP) LE CARD(TP))   = WTAX(J);

 Wage.FX(J,TTP)$(ORD(TTP) LE CARD(TP))   = Wage.L(J,TTP);
 Kstock.FX(J,TI)     = Kstock.L(J,TI);

 Rint.FX(TTP)$(ORD(TTP) EQ CARD(TP))     = Rint.L(TTP);

 ConI.FX(I,J,TTP,G) $(AlConI(I,J) LT 1.E-13) = 0;
 P.FX("rest-of-the-world",TTP)                          = 1;
 P.FX(J,TTP)$(ORD(TTP) LE CARD(TP))          = 1;

 RintJ.L(J,TTP)     = Rint.L(TTP);
 RintJ.FX(J,TTP)$(ORD(TTP) EQ CARD(TP))  = Rint.L(TTP);

 Con.LO(J,TTP,G) = .50*Con.L(J,TTP,G);  Con.UP(J,TTP,G) = 1.50*Con.L(J,TTP,G);
 EInv.FX(I,J,TTP) $(AlInv(I,J) LT 1.E-13)    = 0;
 EInv.LO(I,J,TTP)    = .10*EInv.L(I,J,TTP)   ; EInv.UP(I,J,TTP)    = 2.00*EInv.L(I,J,
TTP)   ;

* Inv.LO(J,TTP)    = .50*Inv.L(J,TTP);    Inv.UP(J,TTP)    = 1.50*Inv.L(J,TTP);
 PInv.FX(J,TTP)$(ORD(TTP) LE CARD(TP))        = PInv.L(J,TTP);

 EXC.FX(J,TTP)$(ORD(TTP) LE CARD(TP)) = 0;
 EXC.FX(J,TTP)$(ORD(TTP) GE 12) = 0;
 EXC.FX(J,TTP) = 0;

 OPTIONS SOLPRINT=On, LIMROW=9999, LIMCOL=0, ITERLIM=10000, RESLIM=20000;
 OPTION NLP=CONOPT2 ;
 SOLVE OLGMultiR USING NLP MINIMIZING OBJ;
 DISPLAY Q.L, P.L;
*+++++++++++++++++++
*TEST of NUMERAIRE
*+++++++++++++++++++
$ONTEXT
 P.FX("REST-OF-THE-WORLD",TTP)                          = 1.2;
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 P.FX(J,TTP)$(ORD(TTP) LE CARD(TP))          = 1.2;
 Lend.FX(J,TTP,G)   $(ORD(TTP) LE CARD(TP)+1) = 1.2*Lend.L(J,TTP,G);
 Wage.FX(J,TTP)$(ORD(TTP) LE CARD(TP))       = 1.2*Wage.L(J,TTP);
 PInv.FX(J,TTP)$(ORD(TTP) LE CARD(TP))       = 1.2*PInv.L(J,TTP);
 Rent.FX(J,TTP)$(ORD(TTP) LE CARD(TP))       = 1.2*Rent.L(J,TTP);


 OPTIONS SOLPRINT=OFF, LIMROW=0, LIMCOL=0, ITERLIM=10000, RESLIM=20000;
 OPTION NLP=CONOPT2;
 SOLVE OLGMultiR USING NLP MINIMIZING OBJ;

 OPTIONS SOLPRINT=Off, LIMROW=0, LIMCOL=0, ITERLIM=10000, RESLIM=20000;
 OPTION NLP=MINOS5;
 SOLVE OLGMultiR USING NLP MINIMIZING OBJ;
$OFFTEXT

*+++++++
* SHOCKS
*+++++++

*$ONTEXT

* (temporary unexpected) Productivity shock on all countries:
* AlQ(J,TTP)   $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+3 ) = 1.01*AlQ(J,
TTP);
* AlQ("EA",TTP)$( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+5 ) = 1.03*AlQ("
EA",TTP);
 GovE("Uzbekistan",TTP) $( ORD(TTP) GT CARD(TP) AND ORD(TTP) LE CARD(TP)+3 ) = 1.
1*GovE("Uzbekistan",TTP);
 EP("Uzbekistan",TTP,G)   $( ORD(TTP) GT CARD(TP)+3 AND ORD(TTP) LE CARD(TP)+6 ) =
1.1*EP("Uzbekistan",TTP,G);
*++++++++++++++++
* Policy options
*++++++++++++++++

* Choose periods of BOND-financed government deficits (default is tax financing)
* ExoBonds(J,TTP)$( (ORD(TTP) GT CARD(TP)+1) AND (ORD(TTP) LE CARD(TP)+5)) = 0;

*++++++++++
* Solving
*++++++++++

 OPTIONS SOLPRINT=OFF, LIMROW=0, LIMCOL=0, ITERLIM=10000, RESLIM=20000;
 OPTION NLP=CONOPT2;
 SOLVE OLGMultiR USING NLP MINIMIZING OBJ;

 OPTIONS SOLPRINT=OFF, LIMROW=0, LIMCOL=0, ITERLIM=10000, RESLIM=20000;
 OPTION NLP=MINOS5;
 SOLVE OLGMultiR USING NLP MINIMIZING OBJ;

PARAMETER WAGINC(J,TTP,G)
          NETWAGINC(J,TTP,G);
 WAGINC(J,TTP,G) = Wage.L(J,TTP)*EP(J,TTP,G) ;
 NETWAGINC(J,TTP,G)= (1-WTxR.L(J,TTP)-CTR.L(TTP))*WAGINC(J,TTP,G);

 DISPLAY Q.L, P.L,Wage.L,WAGINC,NETWAGINC, WTxR.L, RINT.L, INV.L, ECON.L, EINV.L ;
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*$OFFTEXT


*+++++++++++++++++++++++++++
* VARIOUS EQUILIBRIUM TESTS
*+++++++++++++++++++++++++++
*$ONTEXT
 PARAMETER TRADEBAL(I,TTP);
 TRADEBAL(I,TTP)$(ORD(TTP) GE CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T)) =
                     SUM(J$(ORD(J) NE ORD(I)), P.L(I,TTP)*(ECon.L(I,J,TTP)+EInv.L(I,J,TTP)))-
                     SUM(J$(ORD(J) NE ORD(I)), P.L(J,TTP)*(ECon.L(J,I,TTP)+EInv.L(J,I,TTP)));
 TRADEBAL(J,T)$(ABS(TRADEBAL(J,T)) LT 1.E-7) = 0; DISPLAY TRADEBAL;

 PARAMETER WTRADEBAL(TTP);
 WTRADEBAL(TTP)   = SUM(I,TRADEBAL(I,TTP));
 WTRADEBAL(TTP)$(ABS(WTRADEBAL(TTP)) LT 1.E-7) = 0; DISPLAY WTRADEBAL;

 PARAMETER WALRASJ(J,TTP);
 WALRASJ(J,T)     = Q.L(J,T)-SUM(I,ECon.L(J,I,T)+EInv.L(J,I,T))-Gov.L(J,T);
 WALRASJ(J,T)$(ABS(WALRASJ(J,T)) LT 1.E-7) = 0; DISPLAY WALRASJ;

 PARAMETER WASSETBAL(TTP);
 WASSETBAL(TTP+1)$(ORD(TTP) GE CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T)) =
     SUM((J,G),Pop(J,TTP+1,G+1)*LEND.L(J,TTP+1,G+1))-
     SUM( J   ,P.L(J,TTP)*Bond.L(J,TTP+1)+PInv.L(J,TTP)*Kstock.L(J,TTP+1));
 WASSETBAL(TTP)$(ABS(WASSETBAL(TTP)) LT 1.E-7)  = 0; DISPLAY WASSETBAL;

 PARAMETER WASSETBAL(TTP);
 WASSETBAL(TTP+1)$(ORD(TTP) GE CARD(TP) AND ORD(TTP) LT CARD(TP)+CARD(T)) =
     SUM((I,J,G),Pop(J,TTP+1,G+1)*P.L(I,TTP)*Bij.L(I,J,TTP+1,G+1))-
     SUM( J   ,P.L(J,TTP)*Bond.L(J,TTP+1));
 DISPLAY WASSETBAL;

*$OFFTEXT

*$INCLUDE OLG6-RCan\0801\Pprint-OLG2.INC

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